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## TIME DOMAIN ANALYSIS OF A SYNCHRONOUS GENERATOR IN DISTORTED POWER SYSTEMS

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### Abstract

Calculating the synchronous generator steady state operating conditions and load angle is essential for power system studies. Time and space harmonics can cause variations on power system operation. This paper investigates the impact of time and space harmonics on the synchronous generator inductances, rotor currents, produced stator fluxes and voltages, electromagnetic torque and the steady state rotor angle. Harmonic Domain and abc-frame of reference is used for nonlinear modeling of synchronous generator. The instantaneous electromagnetic torque is calculated using rotational voltage component. Simulation results show that the presence of harmonics can affect the synchronous generator steady state operation and damper windings have considerable impact on the produces stator fluxes and voltages in the presence of harmonics. Matlab/Simulink software and Simpowersystem toolbox are used for the simulation in time domain.

### 1. Introduction

With the wide usage of energy-efficient appliances and power-electronic based devices, the presence of current and voltage harmonics in the power system is considerable. On the other hand, stator and rotor windings of synchronous generator contain space harmonics which can also produce and inject time harmonics into the power system particularly in distributed generation (DG) and isolated systems which typically have low inertia and short circuit levels. Therefore, SG operation and system stability studies need to be explored under the influence of harmonics in non-sinusoidal operating environments. In system stability studies, it is necessary to find initial steady-state values of SG variables as a function of non-sinusoidal terminal quantities. Therefore, an accurate nonlinear model of synchronous generator is needed to investigate the steady state operation and stability of the harmonically polluted power systems. However, effects of harmonics on stability of synchronous generators have not been investigated in the literature.

In literatures there are number of linear and nonlinear models of SG. The SG model in qd-frame of reference is the most widely used [1-3]. This model is

appropriate for balance sinusoidal operating conditions. A number of SG models have been developed for harmonic analysis [4-7]. These models are based on modified park transformation [5], transient inductances [6], frequency domain including imbalance and saturation effects [7] and abc-frame of reference which can present the nature of time-varying inductances [8]. Harmonic Domain (HD) is widely used for modeling the power system elements [9-16]. The main advantage of using HD is that the linear time periodic systems will be transferred to linear time invariant systems. In this paper synchronous generator is modeled as a voltage source in harmonic domain and abc-frame of reference considering the impact of damper windings and rotor pole saliency.

Simulation results for SG stator fluxes and voltages, rotor currents, the instantaneous electromagnetic torque and steady state rotor angle show the presence of harmonics can cause variations on the steady state operation. The impact of damper windings is also investigated and it is shown that the damper windings have considerable impact on SG steady state operation in the presence of harmonics.

## 2. Synchronous Generator Model for Non-Sinusoidal Operation

Synchronous generator is modeled in Harmonic Domain (HD) and abc frame of reference [16]. This model is appropriate for unbalance operating conditions and includes damper windings and rotor saliency. Machine main equations in time domain are as follows:

$$\begin{cases} \dot{\psi}_r = V_r - R_r i_r \\ \dot{\psi}_s = V_s + R_s i_s \\ \psi_s = -L_{ss} I_s + L_{sr} I_r \\ \psi_r = -L_{rs} I_s + L_{rr} I_r \end{cases} \quad (1)$$

Where  $\psi_r$  and  $\psi_s$  are rotor and stator fluxes in time domain respectively. These equations are transferred into harmonic domain:

$$\begin{cases} D_r \Psi_r = V_r - R_r I_r \\ D_s \Psi_s = V_s + R_s I_s \\ \Psi_s = -L_{ss} I_s + L_{sr} I_r \\ \Psi_r = -L_{rs} I_s + L_{rr} I_r \end{cases} \quad (2)$$

Where  $D_r$  and  $D_s$  are the operational matrices of differentiation applied to the rotor and stator equations respectively. Rotor-self inductance ( $L_{rr}$ ) is a constant matrix.  $L_{ss}$ ,  $L_{rr}$  and  $L_{rs}$  are self and mutual inductances of stator, rotor and rotor-stator, respectively and are constant Toeplitz-type with the general form of:

$$[M] = \begin{bmatrix} a_0 & a_{-1} & \cdots & a_{-h} & & \\ a_1 & & & & a_{-h} & \\ \vdots & \ddots & \ddots & \ddots & \vdots & \\ a_h & & & & a_{-1} & \\ & a_h & \cdots & a_1 & a_0 & \end{bmatrix}$$

Space harmonics are included in the inductance matrices using abc-frame of reference. The stator ( $I_s$ ) and rotor currents ( $I_r$ ) contain fundamental and time harmonics.

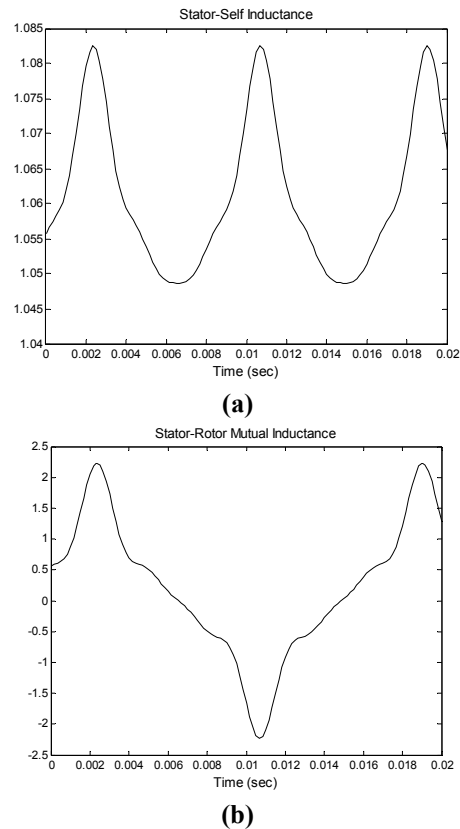
## 3. Synchronous Generator Operation in the Presence of Time and Space Harmonics

In this section the SG operation in the presence of time and space harmonics is investigated. The produced stator fluxes and voltages, rotor currents, electromagnetic torque and the rotor angle are

calculated in time domain. Space harmonics are considered in the stator-self and stator-rotor mutual inductances and stator currents include time harmonics. It is shown that there is considerable amount of induced currents on damper windings in the presence of harmonics and they can affect the steady state operating conditions. The simulated synchronous generator characteristics are given in Appendix A.

### 3.1. Stator and Rotor Inductances

Figure 1 illustrates synchronous generator inductances considering space harmonics up to 10<sup>th</sup> order with the magnitudes of 0.3, 0.2, 0.1 and 0.01 of the first and second harmonic order for Rotor-Stator mutual and stator self inductances, respectively.

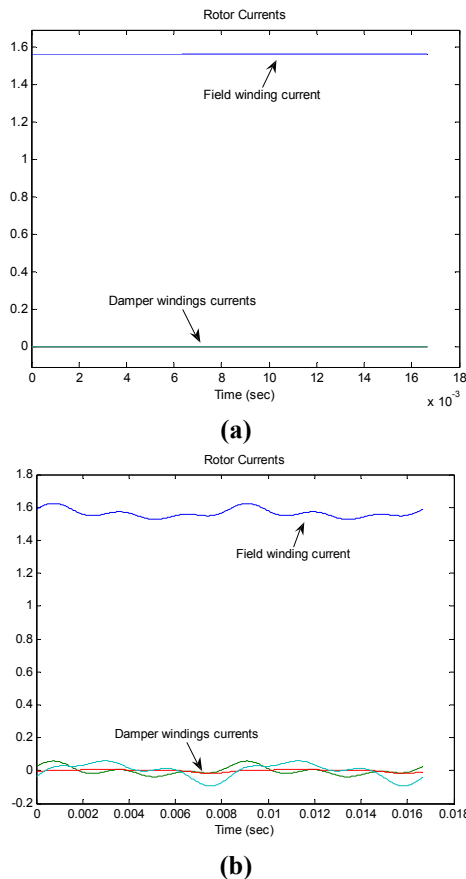


**Figure 1 Impact of space harmonics on (a) Stator-Self Inductance (b) Stator-Rotor Mutual Inductance**

### 3.2. Rotor Currents

Time and space harmonics can induce time harmonics in the rotor field and damper windings. These harmonics are of even orders and can be calculated by solving equation 2 in harmonic domain. Figure 2 shows currents in the field and damper windings in sinusoidal and harmonic conditions. These currents in return affect the produced stator fluxes and voltages.

Space harmonics values are 0.003pu, 0.002pu, 0.001pu and 0.0001 and time harmonics are 0.05pu, 0.035pu, 0.025pu and 0.001 pu for the 3<sup>rd</sup> to 9<sup>th</sup> harmonics, respectively.



**Figure 2 Rotor currents (a) Sinusoidal conditions (b) in the presence of time and space harmonics**

### 3.3. Stator Three Phase Fluxes

Solving equation 2 for stator fluxes, results in additional harmonic components due to interaction between time and space harmonics. The generated stator flux ( $\psi_s$ ) includes fundamental and harmonic components which may not be balanced at three phases. The additional harmonic components produce rotating fields with different speeds with respect to the rotor and hence can not produce constant torques.

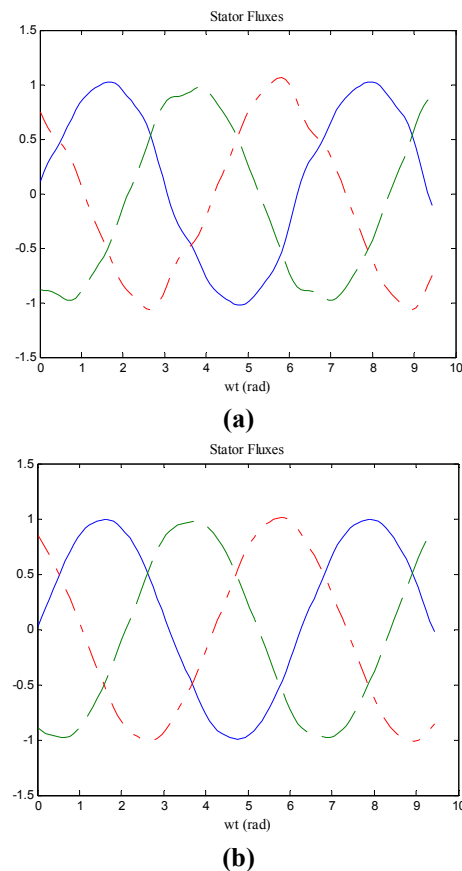
Figure 3 illustrates 3-phase stator fluxes with and without damper windings. Time and space harmonics are assumed at the same level as section 3.2.

It is observed that damper windings have considerable impact on the produced stator fluxes and can not be ignored in the harmonically polluted systems.

Harmonics can cause variations on the induced fundamental component of stator flux in SG which will result in addition or reduction of the generated power

and hence affect the rotor angle and dynamic behavior of the power system. They can also produce unbalance 3-phase fluxes in stator windings and inject zero sequence voltage into the system.

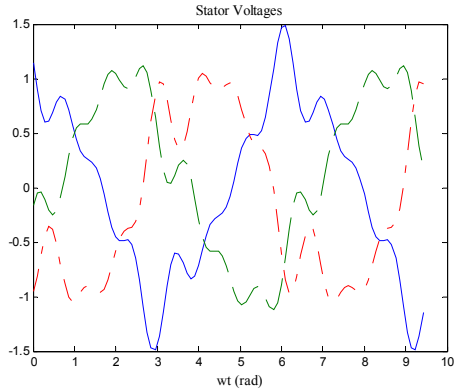
Solving equation 2 in harmonic domain and applying Park Transformation on the fundamental component of the stator flux results in the additional components in dq-axis ( $\psi_d^{due\ to\ harm}$ ,  $\psi_q^{due\ to\ harm}$ ) [17]. The values of the additional components depend on the time and space harmonics at special harmonic orders which contribute in producing forward rotating fundamental stator flux.



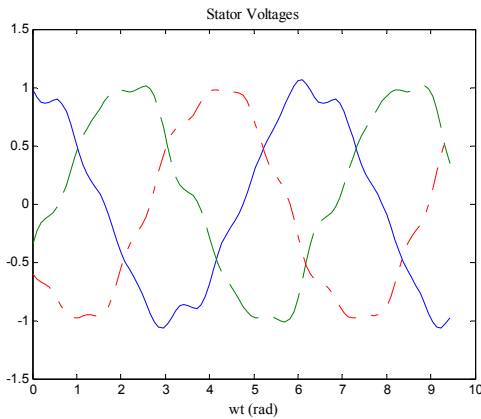
**Figure 3 Three phase stator fluxes in the presence of time and space harmonics, (a) without and (b) with damper windings**

### 3.4. Produced Stator Three-Phase Voltages

Figure 4 shows the impact of harmonics on the produced stator voltages. The produced voltage harmonics in stator can propagate in small systems and increase the level of total harmonic distortion. Comparing figures 4a and 4b it is observed that damper windings can affect the quality of the produced voltage.



(a)



(b)

**Figure 4 Three phase stator voltages in the presence of space harmonics, (a) without and (b) with damper windings**

### 3.5. Electromagnetic Torque

Electromagnetic torque is related to the instantaneous power flows across the airgap from stator to rotor via the actual rotor speed ( $\omega_r$ ) by the following equation [18]:

$$T_{em}(t) = P_{em}(t) / \omega_r \quad (3)$$

For all polyphase ac machines, the electromagnetic airgap power is defined as the product of the instantaneous rotational voltage component in each armature phase times the instantaneous phase current. Hence, the produced electromagnetic power is calculated using the following equation:

$$P_{em} = \hat{e}_a \cdot \hat{i}_a + \hat{e}_b \cdot \hat{i}_b + \hat{e}_c \cdot \hat{i}_c = \hat{E}_{abc} \cdot \hat{I}_{abc} \quad (4)$$

Where,  $\hat{e}_a$ ,  $\hat{e}_b$  and  $\hat{e}_c$  are the rotational phase voltage terms and  $\hat{i}_a$ ,  $\hat{i}_b$  and  $\hat{i}_c$  are the stator three-phase currents.

The rotational voltage components are defined as:

$$\hat{E}_{abc} = \dot{\hat{L}}_{abc} \cdot I \quad (5)$$

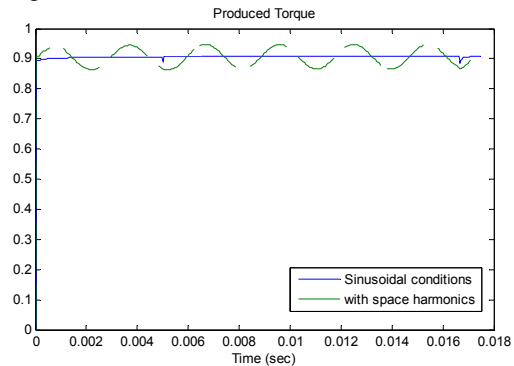
Where,

$$\dot{\hat{L}}_{abc} = dL_{abc} / dt$$

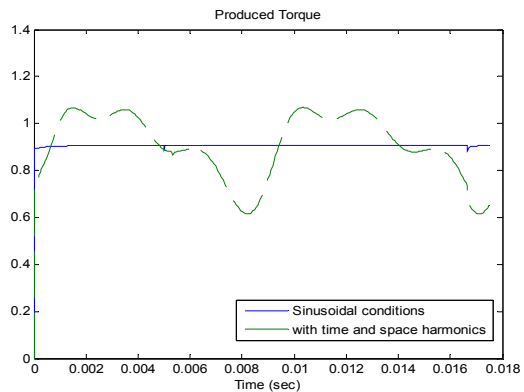
Writing equation 5, for phase (a) as an example, is as follows:

$$\hat{e}_a = (dL_{aa} / dt)i_a + (dL_{ab} / dt)i_b + (dL_{ac} / dt)i_c + (dL_{aF} / dt)i_F + (dL_{aD} / dt)i_D + (dL_{aG} / dt)i_G + (dL_{aQ} / dt)i_Q \quad (6)$$

Similar expressions can be written for  $\hat{e}_b$  and  $\hat{e}_c$ . Figure 5 demonstrates the instantaneous electromagnetic torque in sinusoidal and harmonic operating conditions considering the impact of damper windings. In figure 5a, the produced torque in sinusoidal system is compared with the one in the presence of space harmonics. It is observed that space harmonics can apply pulsation on the electromagnetic torque. Figure 5b shows that the interaction between time and space harmonics can cause higher electromagnetic torque perturbations.



(a)



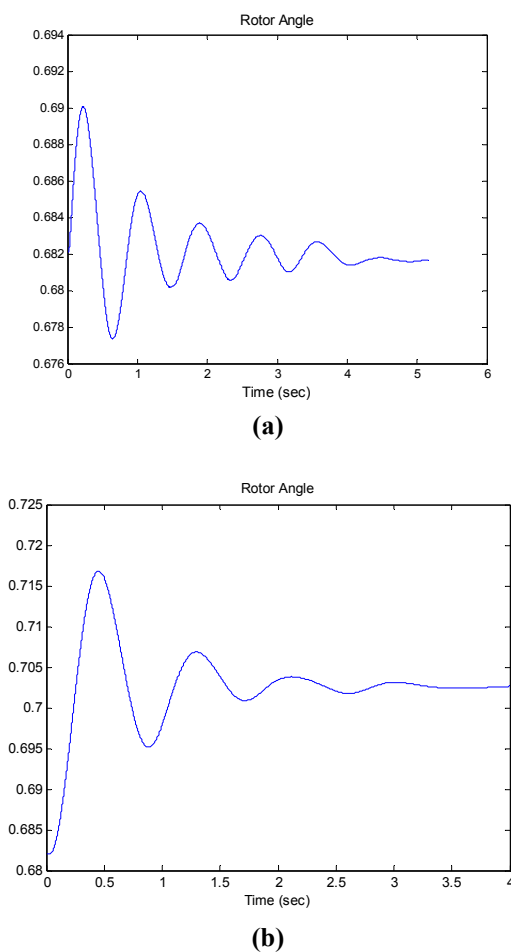
(b)

**Figure 5 Electromagnetic torques in the presence of (a) space (b) time and space harmonics**

#### 4. Load Angle

Synchronous generator rotor (load) angle, which is one of the most important parameters for power system stability studies, is determined by the active power demand by the load.

For power system studies, it is necessary to compute the initial steady-state load angle of synchronous generator as a function of non-sinusoidal terminal quantities. As it is shown in figure 6, additional terms of stator flux due to harmonics can change the steady-state load angle. The variations on rotor angle are related to the electromagnetic torque via the equation of motion.



**Figure 6 Rotor angle (a) Sinusoidal conditions (b) in the presence of time and space harmonics**

#### 5. CONCLUSION

A nonlinear model of synchronous generator in harmonic domain and abc-frame of reference considering the impact rotor pole saliency and damper

windings is used to demonstrate the impact of time and space harmonics on the steady state operation of the SG. Simulation results show the following main contributions:

- Time and space harmonics can cause variations on the produced stator fluxes and voltages.
- SG in the presence of time and space harmonics can produce unbalance stator voltages.
- Induced harmonic currents on damper windings are considerable.
- Harmonics can cause electromagnetic torque perturbation.
- Damper windings have considerable impact on the produced stator fluxes and voltages.
- Time and space harmonics will change the steady-state value of the synchronous generator load angle.

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## Appendix A

Synchronous machine parameters in per unit of machine rating (555MVA, 24kV, 0.9p.f, 60Hz, 3600RPM turbine generator) are:

- Stator Parameters:

$$L_s=1.060 \quad L_m=0.0140 \quad M_s=0.4550 \quad R_a=0.003$$

- Rotor Parameters:

$$R_F=0.0006 \quad L_F=1.551 \quad L_{FD}=1.386$$

$$R_D=0.0284 \quad L_D=1.5573 \quad L_{GQ}=1.344$$

$$R_G=0.00619 \quad L_G=2.0692 \quad L_{FG}=L_{FQ}=L_{DG}=L_{DQ}=0$$

$$R_Q=0.02368 \quad L_Q=1.469.$$

- Stator Rotor Mutual Parameters:

$$M_F=M_D=1.386 \quad M_G=M_Q=1.344$$