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Risk Analysis of Animal Vehicle Crashes: A Hierarchical Bayesian Approach to Spatial Modelling

Andrew Murphy and Jianhong (Cecilia) Xia¹
Curtin University, Department of Spatial Sciences, Australia

Abstract

Driving along any rural road within Western Australia involves some level of uncertainty about encountering an animal whether it is wildlife, farm stock or domestic. This level of uncertainty can vary with depending on factors such as the surrounding land use, water source, geometry of the road, speed limits and signage. This paper aims to model the risk of animal-vehicle crashes (AVCs) on a segmented highway. A hierarchical Bayesian model involving multivariate Poisson lognormal regression is used in establishing the relationship between AVCs and the contributing factors. Findings of this study show that Farming on both sides of a road, a mixture of farming and forest roadside vegetation and roadside vegetation have significant positive effect on AVCs, while speed limits and horizontal curves indicate a negative effect. AVCs consist of both spatial and segment specific contributions, even though the spatial random error does not dominate model variability. Segment 15 is identified as the highest risk segment and its nearby segments also exhibit high risk.

Keywords

Animal-vehicle crashes; Hierarchical Bayesian model; Multivariate Poisson lognormal regression model; Spatial modelling; Markov chain Monte Carlo (MCMC) techniques; Road safety

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^{*} Corresponding author : Jianhong (Cecilia) Xia. Email: c.xia@curtin.edu.au

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1. Introduction

1.1 Rationale and Objectives

Animal-vehicle crashes (AVCs) are an important health issue worldwide. Thousands of deaths and injuries and billions of damage occur annually because of these events (Langley & Mathison, 2008). In the United States, Significant attention has been paid into preventing animal-related crashes due to extensive vehicle damage occurring yearly (Hedlund, Curtis, Curtis, & Williams, 2004). Vehicle collisions with moose are considered as one of the major safety issues in Sweden with around 4500 crashes per year, including 10–15 human fatalities (Seiler, 2005). Each year in Australia, many thousands of AVCs have also resulted in considerable damage to the society. So far the specific nature of AVCs has not been thoroughly studied (Rowden, Steinhardt, & Sheehan, 2008). Ramp *et al.* (2005) suggests identifying and targeting fatality hotspots is required to focus mitigation efforts, given enormously long roads (810,022km) in Australia. It is too costly to manage the AVC damage for each segment of the road.

Statistical modelling of vehicle crashes has been of interest to researchers for many years (Mitra & Washington, 2007). More advanced statistical methods have become widely used in recent years to determine the effectiveness of safety measures and to identify high risk traffic areas. Bayesian statistical techniques, in particular, can generate more real-world-condition statistical results than traditional methods (Schultz, Thurgood, Olsen, & Shane Reese, 2011).

The objective of this paper is to develop a Bayesian model to identify higher risk segments of a roadway susceptible to AVCs and significant contributing factors to the risk of AVCs. The hierarchical Bayesian multivariate Poisson lognormal regression model allows for both over dispersion and a possible correlation structure and has been central to many traffic crash studies (El-Basyouny & Sayed, 2009; Ma, Kockelman, & Damien, 2008; M. Maes, Dann, Sarkar, & Midtgaard, 2007). In particular, the model developed by Maes *et* al. (2009), which was applied to a dynamically segmented coastal road in Norway by Graf (2009), is modified and adapted for AVC counts in this study. This model incorporates direct correlation using the distance between the centroids of homogeneous segments of varying lengths.

The paper is organised as follows: firstly a literature review of existing studies and models of AVCs and general traffic crashes as part of this introduction. The methodology employed in this paper is described in section 2. This includes outlining the study area selected, description of how the data were sourced and prepared, explanation of the Bayesian principals and Markov chain Monte Carlo simulation techniques applied and the hierarchical multivariate Poisson lognormal model adopted and evaluated. It is then followed by the results and discussion of the model and segment ranking in section 3. Finally conclusions are drawn.

1.2 Literature Review of AVC Analysis

The study by Rowden *et al.* (2008) essentially used descriptive and exploratory statistics to highlight the importance of the issue, finding AVCs comprised 5.5% of total serious crashes in the study. This indicates far more interest should be generated to this issue by road safety funding bodies and researchers. However, research papers analysing AVCs are still not as prevalent as would be expected given the significance of the issue. Seiler (2005) applied an empirical multiple

logistic regression model to determine factors affecting moose vehicle collisions. The traffic volume, the occurrence of fences and vehicle speed were discovered as dominant factors and 72.7% of the crash sites were predicted by the model. Gunson et al. (2011) reviewed 24 published papers, which used generalized linear models to quantitatively analyse the influence of environmental predictors on the wildlife vehicle crash locations. Increased traffic volumes, speed limits and road width, decreased visibility, flat terrain and the presence of water sources increased the number of wildlife vehicle collisions among all species. A diagonal inflated bivariate Poisson model was fitted to two datasets by Lao et al. (2011) and illustrated the influences of geographic characteristics, geometric design and traffic elements on the AVC occurrences. The annual average daily traffic, speed limit, and shoulder width were also found to increase the numbers of AVCs. On the other hand, some geometric factors, such as rolling and mountainous terrain, were found to decrease the number of reported AVCs (Lao, Wu, Corey, & Wang, 2011). A common finding amongst the research papers mentioned is that identification of spatial clusters of AVCs is the key of AVC mitigation and its associated factors are geographic characteristics, geometric design and traffic elements (Clevenger, Chruszcz, & Gunson, 2003; Gunson et al., 2011; Joyce & Mahoney, 2001; Plug, Xia, & Caulfield, 2011; Puglisi, Lindzey, & Bellis, 1974).

1.3 Literature Review of Modelling Methods of General Vehicle Crash

Research of general traffic crashes has been far more extensive and methodologies for predictive modelling have evolved over many years. Crash count models have been established and universally adopted, but also critiqued and reinvented. The primary purpose of these models is to estimate the risk at a particular entity of the road network where a count of crash occurrences can be made. Possible entities include road segments, intersections and suburbs. Initially models are relatively simple and based on linear regression. Two early studies by Jovanis and Chang (1986)

and Miaou and Lum (1993) investigated conventional linear regression and Poisson regression. Both studies criticised their linear regression models in favour of Poisson regression model. Caution should be taken in using the normal distribution because of problems associated with non-negativity and unequal variances. If the underlying crash frequency is functionally related to the variance (e.g. Poisson Distribution), parameters in a linear regression model will be unbiased but will have incorrect confidence limits (Jovanis & Chang, 1986). The conventional linear regression models were found by Miaou and Lum (1993) to be lack of the distributional property to describe "adequately random, discrete, non-negative and typically sporadic" vehicle events on the road. Also, the prediction of a AVC rate is sensitive to the of the road segment length considered (Miaou & Lum, 1993). The Poisson distribution assumes that the mean and variance are equal, while crash count data are often over dispersed with the mean less than the variance.

To accommodate over dispersion, various alternatives such as the Negative Binomial (NB), Zero Inflated Poisson (ZIP), Zero Inflated Negative Binomial (ZINB) and Poisson lognormal models were developed. Miaou (1994) recommended the Poisson regression model as a pioneer model for developing the relationship between crash counts and factors. The NB and ZIP regression models can be explored if the over dispersion of crash data are found to be moderate or high. The ZIP models were introduced to account for frequently occurring zero counts in crash data but were found to be misleading. Lord *et al.* (2005) argued that although zero-inflated models, such as the NB and ZIP, can improve the statistical fit to crash data in many cases, the essential assumption of a "dual state process" of these models is inconsistent with crash data. Recently studies have identified the need to account for spatial variation in crash occurrences. Maher (1987) argued that the true crash rates are spatially correlated, and it is then shown, by means of simulation results, that the selection process of 'neighbouring' sites (which are untreated and adjacent to a treated site) leads to bias in the comparison of their before and after crash frequencies. It is important to consider both

uncorrelated heterogeneity and spatial dependence concurrently among neighbouring units (Quddus, 2008). Quddus (2008) also states that because Bayesian hierarchical models have this capability, they are more suitable for identifying a relationship between area-wide VCs and their contributing factors related to traffic conditions, socioeconomic and infrastructure of the area.

1.4 Literature Review of Bayesian methods

Bayesian methods present many advantages over classical methods in that all aspects of uncertainty present can be quantified. These aspects can be decomposed into a series of simpler conditional models (data, process and parameters) using a hierarchical Bayes model. In a hierarchical Bayesian analysis, prior (before) information and all available data are integrated into posterior (after) distributions from which inferences can be made; therefore, all uncertainties are accounted for in the analyses (Schultz et al., 2011).

The Hierarchical Bayes *spatial* generalized model is a general model, which can cover many circumstances in which spatial structure needs to be integrated (Ghosh, Natarajan, Waller, & Kim, 1999). MacNab (2004) illustrated this broad application of Bayesian methods by applying an ecological regression model to motor vehicle crash injury among males aged 0 - 24 in British Columbia, Canada. He summarised the usefulness of the Bayesian ecological studies in the prediction of spatial patterns of vehicle crash and identification of "areas in need" (MacNab, 2004). Aguero-Valverde (2011) divided models of spatial correlation into two main groups: direct spatial and conditional models. In the former, the spatial correlation between two observations is a function of the distance between the entities (J. Aguero-Valverde, 2011). Conditional spatial models, on the other hand, rely on the adjacency matrix. Ghosh *et al.* (1999) along with many vehicle crash analysis studies (Jonathan Aguero-Valverde & Jovanis, 2006; Huang & Abdel-Aty, 2010; Miaou,

Song, & Mallick, 2003) adopted the conditional autoregressive (CAR) model to incorporate spatial dependence. Direct Correlation models have been adopted more recently (Mitra, 2009; Augero-Valverdo, 2011). Direct spatial correlation models assess the spatial correlation as a function of the distance between sites through the covariance function; hence, the strength of the spatial correlation is derived directly from the data (Augero-Valverdo, 2011).

Bayesian analysis can be complicated and computationally intense, in particular for the hierarchical Bayesian multivariate models applied in vehicle crash analysis. Feasibility of Bayesian analysis is largely due to advances in technology and the application of Markov chain Monte Carlo simulation algorithms, such as Gibbs Sampler and Metropolis-Hastings algorithms (Ma et al., 2008).

2. Methodology

2.1 Study Area

The study area chosen for this study is the Bussel Highway in the Southwest of Western Australia. The highway links the city of Bunbury to the small coastal town of Augusta and meanders through the popular tourist region of Magaret River. The roadway was selected for its variety in geometry and adjacent environments that could contribute to AVC risk. The Bussel Highway is 142km in length, includes curved as well as straight sections and passes through urban, forest and farming landscapes, often differing on either side of the road.

For the purpose of the study, the Bussel Highway was divided into relatively homogeneous segments. In order to analyse contributing factors, the segments needed to be reasonably similar in terms of these factors. The beginning of a new segment was generally necessitated by a major

change in the characteristics of the roadway. The segments are illustrated in Figure 1.

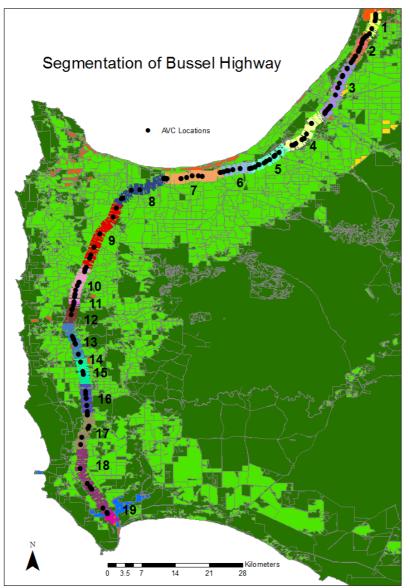


Figure 1: Segmentation of Bussel Highway and AVC locations

2.2 Data Preparation

Data was obtained from the Department of Main Roads Western Australia who maintain an Integrated Road Information System (IRIS). The IRIS uses a linear referencing system, Straight-line kilometer (SLK), which locates features along a roadway, such as crashes according to their relative position from the start of the roadway. Four datasets extracted from the IRIS by Main Roads were used in this paper including vehicle crash, road centreline and speed limit zone datasets for all Western Australian roads and a specific Geometry/Alignment dataset for the HO43 road (Bussel

Highway). The crash dataset contains thorough details of vehicle crashes occurring in the 10 year period of 2001 to 2010, sourced from police crash reports and the Insurance Commission of Western Australia. The details of vehicle crashes include crash location such as road name, SLK and geographic coordinates and crash nature such as hitting an animal. Crashes involving animals located on the Bussel Highway were filtered from the crash dataset and are presented on a map of the area in Figure 1.

Contributing factors to animal vehicle collisions can be categorised as either driver/vehicle related or road related. Driver/vehicle related factors such as the driver's level of fatigue or the condition of the vehicle's brakes apply homogenously to the entire road network. These factors are very difficult to address and require general strategies such as driver education and extra policing. Road related factors tend to be spatially variable and location specific.

Factors to be considered as possible covariates (independent variables) in the model include those that could contribute to the risk of animal presence near or on the road and those that could contribute to the risk of collision between vehicle and animal. Data used to describe these factors include the Geometry/Alignment and speed zone datasets obtained from main roads, Mean Annual Flow figures reported by Bowman (2007) and satellite imagery provided by Google Inc. (2012).

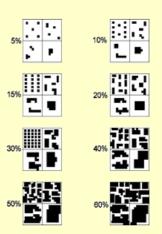
Only very small sections of roadway are perfectly homogeneous for each factor. These sections are also differing from factor to factor. An index to describe each factor was determined for each homogeneous section and then a weighted average of these indexes, using the proportional lengths of the sections as a weight, were assigned to the respective segment as a covariate value for the factor in the model.

Segment Factor Index =
$$\sum_{i=1}^{n} \frac{l_i}{l_i} \times I_i$$
 (1)

where l_i is the factor index determined for the i^{th} small homogeneous section of length l_i and L is the total length of n sections in the segment (also the length of the segment).

Adjacent land-use and roadside vegetation are considered to be possible contributing factors to the presence of an animal near or on the road, on the assumption that more animals are present in their natural environment. Indexes describing these factors were based on visual observation of satellite imagery. Six land-use covariates were investigated based on a combination of urban, farming or forest classification on either side of the road. For example, Roadside vegetation density within three-meter buffer alongside was measured using The Braun-Blanquet (1965) scale. Figure 2 illustrates the visual interpretation of the satellite imagery. The measure of the other land use variables was its proportion of total landuse. For example, the proportion of farming landuse was the percentage of Farming landuse size of the total area in a road segment.





Braun-Blanquet Scale Source: Braun-Blanquet J., 1965)

Figure 2: Satellite Imagery to determine Adjacent Landuse and Roadside Vegetation

Existing fencing is assumed to have a negative effect on the presence of the animal near or on the road as it may prevent animals from accessing to the road. The factor was not used as a covariate in the model due to its near perfect correlation with the farming land-use factor. Its effect can be inferred with that farming land-use covariate.

The availability of water is considered to be a possible contributing factor to the presence of the animal near or on the road as it is a requirement for survival for any animal. Appropriate data for this factor is the Mean Annual Flow of water within the area. The Cape to Cape area which surrounds the Bussel Highway has been divided into sub areas called the Whicher Catchments by the Department of Water of Western Australia. Mean annual flows are used to determine allocation limits for surface water use in the region (Bowman 2007). The Mean Annual Flow (MAF) figures provided an appropriate index to describe the availability of water for sections of roadway. A map of the Whicher Catchments and the path of segments through the sub areas are presented in Figure 3.

Horizontal and vertical curvature of the roadway are considered possible contributing factors to AVC due to the assumption that they affect the driver's vision and the animal awareness of oncoming traffic. The Geometry/Alignment dataset obtained from Main Roads include the radius of the curve of all horizontal sections of Bussel Highway. An index for horizontal curve was determined by the reciprocal of the radius of the curve due to an assumed inverse effect.

Section Index for Horizontal Curvature =
$$\frac{1}{Radius \ of \ Curve}$$
 (2)

The Geometry/Alignment dataset assigns vertically curved sections of roadway a K value which

represents the length of vertical curve in meters for 1% change of Grade (%). In order to determine a comparable index to that of the horizontal curve, an approximate circular radius can be applied to

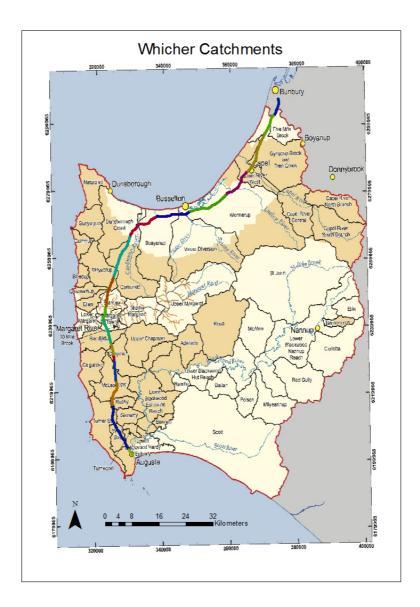


Figure 3: Segments passing through Whicher Catchments

the parabolic vertical curve using the K value multiplied by one hundred (Easa & Hassan, 2000). The following index formula for the vertical curve factor takes into consideration the conversion from meters to kilometers prior to multiplying by one hundred and the assumption of an inverse effect similar to the horizontal curve.

Section Index for Vertical Curvature =
$$\frac{10}{K}$$
 (3)

Based on the assumption that drivers ability to respond to an animal present on the road is affected by the speed of the vehicle, speed limits of a roadway is expected to contribute to AVC risk. The speed zone dataset obtained from Main Roads provided an overlay over the segments and value for the covariate.

Prior to using the covariate values in the model, scaling was required to ensure the values were within a similar range in order for resulting regression coefficients to be comparable. Without scaling, for example, the roadside vegetation values which range from 0 to 5, due to the Braun-Blanquet scale limits whereas the Mean Annual Flow values for water availability range from 6700 to 37770. Scaling factors and final covariate value ranges are presented in Table 1.

Table 1: Covariate information

Symbol	Name of Covariate	Definition and scaling	Min	Max
			Value	Value
x_1	Speed	Average Speed Limit / (50 km/hr)	1.34	2.2
x_2	Horizontal Curve	Average Inverse Radius (km) x 10	0.06	3.89
x_3	Vertical Curve	Average Inverse Radius (km) x 100	0	3.88
<i>x</i> ₄	Roadside Vegetation	Average Braun–Blanquet Scale	0	5
<i>x</i> ₅	Farming both sides	Proportion / 30%	0	3.33
<i>x</i> ₆	Forest both sides	Proportion / 20%	0	5
x ₇	Urban both sides	Proportion / 22%	0	3.28
<i>x</i> ₈	Urban/Farming	Proportion / 14%	0	3.07

<i>x</i> ₉	Urban/Forest	Proportion / 12%	0	3.04
x ₁₀	Farm/Forest	Proportion / 13%	0	3.3
x ₁₁	Water Availability	Mean Annual Flow / 10000	0.67	3.78

Traffic flow data was obtained from the South West Traffic Digest published by the Department of Main Roads Western Australia. The report provides the average number of vehicles at 38 locations along the Bussel Highway.

	AVC Count	Speed		Vertical Curve	Roadside Vegetation	Farming both sides	Forest both sides	Urban both sides	Urban /Farming	Urban /Forest	Farm /Forest	Water Availability
Segment 1	14	1.71	1.84	0.06	1.91	0.00	0.00	1.09	1.50	3.04	1.43	0.67
Segment 2	13	2.20	0.89	0.19	2.58	0.90	2.11	0.00	0.00	1.66	0.83	0.67
Segment 3	21	2.20	1.33	0.33	1.16	2.41	0.00	0.00	0.00	0.00	2.12	1.81
Segment 4	13	2.20	1.33	0.25	1.91	1.06	0.26	2.58	0.00	0.00	0.48	2.19
Segment 5	19	2.20	1.54	0.27	1.18	2.26	0.24	0.00	0.00	0.00	2.10	3.27
Segment 6	7	2.02	1.75	0.79	1.00	1.90	0.00	0.00	3.07	0.00	0.00	3.27
Segment 7	10	1.95	1.20	0.98	1.00	0.00	0.00	0.00	2.90	1.38	3.30	3.78
Segment 8	5	2.04	3.36	0.00	0.87	2.87	0.47	0.21	0.00	0.00	0.00	3.77
Segment 9	6	2.20	0.55	1.99	0.54	2.73	0.00	0.00	0.00	0.00	1.38	3.74
Segment 10	4	1.53	1.60	3.73	0.00	1.29	0.00	2.78	0.00	0.00	0.00	2.59
Segment 11	8	2.20	0.27	1.34	0.50	3.33	0.00	0.00	0.00	0.00	0.00	1.49
Segment 12	7	2.15	2.04	2.81	5.00	0.00	5.00	0.00	0.00	0.00	0.00	1.49
Segment 13	2	1.34	2.68	3.87	2.70	0.00	2.18	1.75	0.00	1.49	0.00	0.93
Segment 14	4	2.20	0.26	0.99	2.02	1.86	0.97	0.00	0.00	0.00	1.91	2.00
Segment 15	1	1.46	0.41	0.55	0.22	1.46	0.00	2.56	0.00	0.00	0.00	2.01
Segment 16	5	2.20	0.06	1.13	1.28	2.40	0.00	0.00	0.00	0.00	2.15	2.01
Segment 17	9	2.20	1.42	1.09	2.55	2.42	0.71	0.00	0.00	0.00	1.01	2.97
Segment 18	9	2.18	1.62	1.61	1.23	3.00	3.42	0.00	0.00	0.00	0.35	1.62
Segment 19	0	1.34	3.89	3.29	1.12	0.00	1.40	3.28	0.00	0.00	0.00	1.50

2.3 Hierarchical multivariate Poisson lognormal model

The fundamental theory of Bayesian Hierarchical models can be found in Haque, Chin, and Huang (2009), Huang and Abdel-Aty (2010) and (Andrew Gelman et al., 2013). This section will discuss the methods used for the AVC analysis based on Hierarchical multivariate Poisson lognormal model. The information on the AVC analysis is available on several levels and requires multi-level modelling to make accurate inferences. A hierarchical model involves multiple levels and considers all sources of uncertainty simultaneously. Three models at three different levels are applied to the AVCs along Bussel Hwy forming an overall hierarchical Poisson lognormal model.

At the lowest level is the data model, a likelihood function linking occurrences of AVC Y_i within a segment i with the length L_i and average daily traffic flow T_i during a period with a time t with the risk parameter θ_i .

$$Y_i \sim Poisson(\theta_i, L_i, T_i, t)$$
 (9)

The middle level is the process model formulating a logarithmic risk ω_i involving a general linear model that captures both spatially structured and spatially unstructured errors.

$$\omega_i = \log(\theta_i) = \mu_i + \varepsilon_i + \phi_i \tag{10}$$

 μ_i is the mean log risk, $\mu_i = \beta x_i$ where $x_i = \{1, x_{1i}, ..., x_{mi}\}$ representing m covariates characterizing segment i and $\beta = \{\beta_0, \beta_1, ..., \beta_m\}$ representing the respective regression coefficients. β_0 is the overall intercept and captures minimum level of risk for the entire highway.

 ε_i and ϕ_i are independent and identically distributed random variables representing the residual components. ε_i is the spatially unstructured error capturing uncorrelated error and ϕ_i is the spatially structured error capturing correlated error.

The highest level is the parameter models including hyper-parameters and assigned prior distributions.

The regression coefficients act as a hyper-parameter for the mean log risk. The vector of values for the regression coefficients are assumed to follow a multivariate normal distribution and hence assigned the non-informative prior distribution;

$$\{\beta_0, \beta_1, \dots, \beta_m\}^T \sim MV\mathcal{N}\left(\mu_{\beta_i}, \Sigma_{\beta}\right) \tag{11}$$

The values of the spatially unstructured error are assumed to follow a zero-mean normal Guassian distribution and hence assigned the prior distribution;

$$\varepsilon_i \sim \mathcal{N}(0, \sigma_{\varepsilon}^2)$$
 (12)

The variance of the spatially unstructured error, σ_{ε}^2 is analogous to the nugget effect and acts as a hyper-parameter to the spatially unstructured error ε_i providing precision. σ_{ε}^2 is assigned an inverse gamma distribution as a non-informative hyperprior.

The vector of values for the spatially structured error ϕ_i are assumed to follow a multivariate normal distribution and hence assigned the prior distribution;

$$\{\phi_1, \dots, \phi_m\}^T | \sigma_{\phi}^2, r_{\phi} \sim MV \mathcal{N}\left(\sigma_{\phi}^2, \Sigma\right)$$
(13)

The covariance matrix Σ incorporates isotropic spatial association that is conditional on the variance σ_{ϕ}^2 and correlation length r_{ϕ} and is represented by an exponential function of decay in correlation between pairs of segments according to the distance between their centroids:

$$\left(\Sigma\left(\sigma_{\phi}^{2}, r_{\phi}\right)\right)_{i,j} = \sigma_{\phi}^{2} e^{\left(-\frac{\left\|s_{i} - s_{j}\right\|}{r_{\phi}}\right)}$$

$$\tag{14}$$

This can be done on WinBUGS (Spiegelhalter et al. 2004) using the spatial exp function.

With the introduction of more parameters, such as hyper-parameters and respective prior distributions, the model presented in this paper exhibits a complex Bayesian statement compared to the equivalent simple Bayesian statement (8) presented in section 2.3.

$$\pi(\beta, \sigma_{\varepsilon}^{2}, \sigma_{\phi}^{2}, r_{\phi}|Y) \propto f(Y|\beta, \sigma_{\varepsilon}^{2}, \sigma_{\phi}^{2}, r_{\phi}) \times \pi(\beta) \times \pi(\sigma_{\varepsilon}^{2}) \times \pi(\sigma_{\phi}^{2}) \times \pi(r_{\phi})$$
(15)

When component distributions of a complex model, $\pi(\sigma_{\phi}^2)$ for example, are not available in closed form and hence neither is the normalising constant, the Gibbs sampling or Metropolis-Hastings alone are no longer suitable MCMC methods. In this case Gibbs samplers must be modified, which means accepting or rejecting at each step is based on the Metropolis-Hastings rule (A. Gelman, David, Huang, & John, 2008). The Metropolis-within-Gibbs MCMC algorithm is used to determine the posterior distribution of the hierarchical multivariate Poisson lognormal model adopted in this paper.

2.5 Model Evaluation

An important matter for using Markov Chain Monte Carlo (MCMC) methods is knowing when to stop sampling (Cowles & Carlin, 1996). The simplest way is to check convergence to the stationary posterior distribution in order to monitor the Monte Carlo (MC) error. Congdon (2010) suggests that the MC error should be less than 5% of the posterior standard deviation of a parameter.

A. Gelman, Carlin, Stern, and Rubin (2004) suggests simulating multiple chain sequences with differing starting values. Three Markov chains should converge to a stationary distribution irrespective of the initial starting values. WinBUGS calculates the Brooks-Gelman-Rubin diagnostic based on the ratio of between-within chain variances. This diagnostic should converge to 1.0 on convergence.

The model incorporated two types of error as per equation (10), spatially structured and unstructured error. The proportion of variability, α , in the model due to spatially structured error can be determined using the standard deviations of the posterior distributions of the error parameters ϕ and ε (Banerjee, Carlin, & Gelfand, 2003).

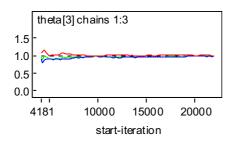
$$\alpha = \frac{sd(\phi|Y)}{sd(\phi|Y) + sd(\varepsilon|Y)}$$
(16)

The proportion of variability in the model due to spatially unstructured error can be determined by $(1 - \alpha)$. If the spatial random effects dominate, data errors or missing covariates that have spatial structure could occur (Law, Haining, Maheswaran, & Pearson, 2006).

3. Results and discussion

3.1 Model Diagnostics

For the purpose of attaining appropriate posterior distributions from the model, three separate chains with differing initial values were simulated to 100000 iterations. According to the Brooks-Gelman-Rubin statistic, convergence occurred after 15000 iterations. Figure 4 shows the plots of the Brooks-Gelman-Rubin statistic for parameters in the model representing the AVC risk for segments 3 and 4. The blue line represents the within-sample variance; the green line represents the estimated variance of the parameter involving both within and between variances and the red line represents the ratio diagnostic that converges to 1.0 on convergence. The first 15000 iterations are consequently referred to as "burn ins" and samples for posterior analysis are taken from the 15001st iteration.



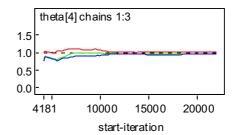


Figure 4: WinBUGS Report - Plots of the Brooks-Gelman-Rubin Statistic.

The proportion of variability due to spatially structured error was 36.58% indicating that AVCs consist of both spatial and segment specific contributions. The fact that the spatial random error does not dominate model variability indicates reasonable spatial data accountability.

3.2 Identification of Higher Risk Segments

In order to identify higher risk segments, the posterior distributions are best compared with the boxplot report as illustrated in Figure 5.

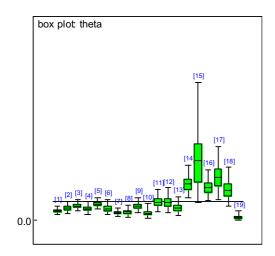


Figure 5: WinBUGS Report - Box Plots of Posterior Distributions for AVC Risk of 19 segments

Segment 15, even though it has the largest range, is clearly identifiable as the highest risk segment.

Segments close by also exhibit high risk. Using the medians of the posterior distributions, segments are categorised and mapped in Figure 6.

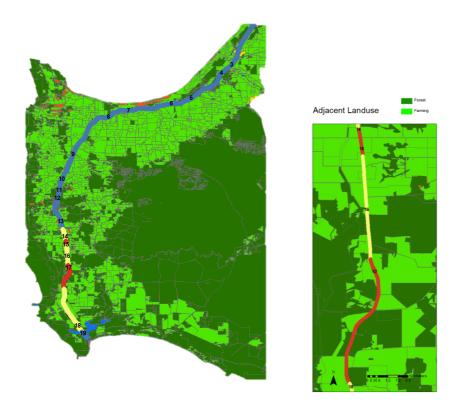


Figure 6 Map of AVC risks of the segments

The segment risk predicted by the model is the expected rate of AVCs per kilometre of roadway per 10000 daily vehicles. The actual count of AVCs may be higher in segments other than those indicated above but this is due to a larger traffic flow. Segments 1 to 3 involve dual lanes with segment 1, for example experiencing over ten times the traffic flow of segment 15. The model adopted in this paper assumes traffic flow as a certain contributing factor and is applied directly to the risk calculation. Traffic flow is often considered a covariate in prediction models and is repeatedly shown to be a significant positive factor. In agreement with numerous prior studies, traffic volumes on major and minor approaches to intersections are influential and positively correlated with crash occurrence (Mitra and Washington, 2012).

3.3 Identification of significant risk factors

Visual inspection of the zoomed insert in Figure 6 would suggest adjacent landuse may contribute to AVC risk with the higher risk segments passing through a mixture of farmland and forest.

WinBUGS can generate a simple statistical summary showing posterior mean, median and standard deviation with a 95% posterior credible interval. The summary of the regression coefficient parameters as presented in Table 2 is appropriate to recognise the significance of the contributing factors.

Table 2: WinBUGS output - Summary statistics for posterior distributions of regression coefficients.

Parameter - Factor	mean	sd	MC error	2.5%	median	97.5%	start	sample
β_1 Speed Limit	-3.201	0.7683	0.03179	-4.829	-3.233	-1.764	15001	100000
β ₂ Horizontal Curve	-0.6824	0.1762	0.005091	-1.031	-0.6817	-0.3426	15001	100000
β ₃ Vertical Curve	-0.2992	0.1543	0.004435	-0.6028	-0.2994	0.004469	15001	100000
β_4 Roadside Vegetation	0.9012	0.2637	0.009883	0.3539	0.9035	1.392	15001	100000
β_5 Farming both sides	1.251	0.3686	0.01477	0.5412	1.256	1.961	15001	100000
β_6 Forest both sides	0.3229	0.1485	0.004208	0.02711	0.3252	0.6101	15001	100000
β_7 Urban both sides	0.4589	0.2558	0.009394	-0.03552	0.4586	0.9559	15001	100000
β ₈ Urban / Farming	0.2876	0.1823	0.005843	-0.06721	0.2883	0.644	15001	100000
β ₉ Urban / Forest	0.04497	0.2269	0.007847	-0.4068	0.04566	0.486	15001	100000
eta_{10} Farm / Forest	0.3779	0.1766	0.005839	0.03839	0.3741	0.7318	15001	100000
$eta_{\scriptscriptstyle 11}$ Water availability	0.1881	0.1586	0.005126	-0.1211	0.1866	0.5042	15001	100000

The low MC error for each parameter is evidence of convergence. To be a significant contributing factor, the 95% credible intervals for the parameter of the regression coefficient cannot include 0. The 95% credible intervals for parameters β_1 , β_2 , β_4 , β_5 , β_6 and β_{10} indicate that the respective factors speed limit, horizontal curve, roadside vegetation, forest on both sides and farm/forest have a relevant effect on the AVC risk. β_1 and β_2 indicate a negative effect for speed limit and horizontal curve whereas roadside vegetation ,farming on both sides, forest on both sides and the farm/forest (β_4 , β_5 , β_6 and β_{10}) appear to have a positive effect.

Speed limit and horizontal curvature of the road segment was considered as a possible contributing factor because it was expected to have a *positive* effect on the AVC risk. At higher speeds the driver has less time to react and curved sections of road restrict the vision of the driver hence increase the

liklihood of a collision with an animal on the road. Contrary to this, the model indicated both to have a significant *negative* effect. The covariate value for speed limit used the average speed limit for that segment of roadway and not the actual speed of drivers. Higher speed limits are assigned to the safer, more open sections of road and this could account for a lower AVC risk. On a curved section of road the animal could be alerted earlier because sound travels in a straight line and the vehicle is travelling around the longer curved distance.

The significant positive effect of farming on both sides of a road and a mixture of farming and forest is not surprising. Farming has created the perfect environment for kangaroos with vast pastures supplying the kangaroo's primary food source and the extra availability of water. Commercial grazing is possible over much of arid Australia due to a high density of artificial watering points. The broadscale supplementation of drinking water has not only enhanced densities of sheep (*Ovies aries*), cattle (*Bos taurus, Bos indicus*) and goats (*Capra hircus*), but has also contributed to increased populations of native kangaroos (*Macropus* spp.) since pre-European times (Fensham & Fairfax, 2008). Existing fencing was excluded as a covariate due to its high correlation with farming but should share the relevant effect. The model has again reported contrary effect to initial expectation. Fencing was initially considered to have a possible negative effect. Farm fences however restrict farm animals from the road and have little effect on wildlife, in particular kangaroos.

Although kangaroos graze on the farmland they still live in small pockets of natural habitat. Western grey kangaroos living in remnants of woodland often feed in adjacent farmland (Arnold *et al.* 1989). Roadside vegetation, however small, could provide the familiar safe habitat for kangaroos on the move and they may be distressed by the unfamiliar noise of traffic. The model indicating a significant positive effect of roadside vegetation was as expected.

4. Conclusion

This paper presents a hierarchical Bayesian multivariate Poisson lognormal regression model Bayesian model to identify the AVC risk for the segments of highways and its significant contributing factors. A case study of the Bussel Highway in the Southwest of Western Australia was developed to implement the model.

The literature suggests although its development of Bayesian methods in spatial epidemiology has been advanced, its application to spatial problems, including spatially structured and unstructured random effects, is still in its initial stages (Law et al. 2006). This study confirmed that AVCs consist of both spatial and segment specific contributions, even though the spatial random error does not dominate model variability. Models that can predict the most likely collision points can be of a practical value. They can be used to predict the location of road sections with the highest collision probability (Malo, SuÁRez, & DÍEz, 2004). If high risk locations and influential factors can be identified, simple strategies such as extra signage can be applied to prevent crashes and provide a safe driving environment. In precise locations of extreme risk more expensive but still feasible strategies are possible such as wildlife overpasses or underpasses, exclusion fencing and interactive intelligent warning signs. The segments identified in this paper will require such intervention methods particularly if a dramatic increase in traffic flow is perceived in the near future.

Analysis of segments is essentially an areal based analysis with the limitation of aggregation within each segment. The construction of an aggregated explanatory model for accidents involves any "classical" statistical problems associated with violations of regression assumptions such as misspecification of the model, autocorrelation of errors and instability or error variance, and

ecological fallacy (Thomas, 1996). The analysis of covariates would be more suited to smaller perfectly homogeneous segments. This would mean, however, extremely small impractical segments. The road geometry dataset for example describes sections of road as small as 20m.

An assumption of the hierarchical Bayesian model is the existence of an underlying crash occurrence process based on a function of known and unknown factors that omits human intervention. A common shortfall of many of the previous studies is that they do not consider the effect of the drivers' characteristics (Abdel-Aty & Radwan, 2000). General traffic crashes involve more human intervention than AVCs. Two or more drivers are often involved and driver response is usually the major factor in a general traffic crash. Human factors were cited as probable causes in 92.6% of crashes investigated. Environmental factors were cited as probable cause in 33.8% of these crashes, while vehicular factors were identified as probably causes in 12.6% of crashes. The major human direct causes were excessive speed, improper lookout, improper evasive action, inattention, and internal distraction (Treat et al., 1979). An AVC usually only involves a single driver and involves external contributing factors that can affect the presence of the animal and the driver's ability to respond. In the future, we will investigate how human and animal characteristics could affect the risk of AVCs, especially in an Australian context. In addition, AVCs change not only over space but also over time. A spatio-temporal model that could identify higher risk segments and significant factors during specific time periods will also be developed.

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7. Appendix

WinBUGS Code

```
model;
{
         # for N road segments
         for( i in 1 : n ) {
                           #for all indicators
                           for (j in 1:ind){
                           mu[i,j] \leftarrow x[i,j]*Beta[j]
          }
                  #nugget
                   epsilon[i] ~ dnorm(0,sigma_e_i)
          Psi[i] <- sum(mu[i,1:ind])+epsilon[i]+phi[i]
                  log(theta[i]) <- Psi[i]
                  #occurence rate
                  V[i] <- theta[i]*L[i]*T[i]*time
                  #Poisson distribution
                  Y[i] ~ dpois(V[i])
         }
```

```
# Prior distribution
  r_i \sim dgamma(10,0.1)
  sigma_phi_i ~ dgamma(10,0.1)
  sigma_e_i \sim dgamma(10,0.1)
        #inverse gamma
        r <- 1/r i
        sigma_phi <-sqrt(1/sigma_phi_i)
        sigma_e<-sqrt(1/sigma_e_i)
        #Multivariate normal distribution of the regression coefficients
  Beta[1:ind] ~ dmnorm(MuB[1:ind],EB[1:ind,1:ind])
 #random field with zero mean, variation sigma_phi and a correlation length r
  phi[1:n] ~ spatial.exp(Mu0[1:n], XCentroids[], YCentroids[], sigma_phi_i ,r,1)
 #proportion random effects
 sd.h <- sd(epsilon[]) # marginal SD of heterogeniety effects
sd.c <- sd(phi[]) #marginal SD of clustering effects
alpha <- sd.c / (sd.h + sd.c)
}
# Data
list(
x=structure(.Data=c(1,1.711,1.8435,0.05575,1.911,0,0,1.088684043,1.498501499,3.044871795,1.42549757
9,0.67,
1,2.2,0.8915,0.1921,2.58,0.895170789,2.11130742,0,0,1.663722026,0.829029628,0.67,
```

- 1,2.2,1.3275,0.32695,1.162,2.413071895,0,0,0,0,2.123680241,1.81,
- 1,2.2,1.3315,0.25175,1.911,1.057098765,0.260416667,2.583122896,0,0,0.480769231,2.19,
- 1, 2.2, 1.5425, 0.2733, 1.184, 2.262773723, 0.243309002, 0, 0, 0, 2.096200636, 3.27,
- 1,2.015,1.748,0.7895,1,1.902961563,0,0,3.065082366,0,0,3.27,
- 1,1.951,1.1985,0.97735,1,0,0,0,2.89810103,1.375615407,3.301476977,3.777,
- 1,2.038,3.363,0,0.869,2.867084203,0.469728602,0.208768267,0,0,0,3.765,
- 1,2.2,0.5535,1.9851,0.541,2.733949301,0,0,0,0,1.383076923,3.742,
- 1,1.527,1.596,3.7337,0,1.293859649,0,2.781100478,0,0,0,2.59,
- 1,2.2,0.272,1.3397,0.5,3.3333333333,0,0,0,0,0,1.49,
- 1,2.153,2.0405,2.80895,5,0,5,0,0,0,0,1.49,
- 1,1.339,2.6795,3.8741,2.7,0,2.178571429,1.753246753,0,1.488095238,0,0.93,
- 1,2.2,0.2555,0.98525,2.022,1.862745098,0.966386555,0,0,0,1.906916613,2,
- 1,1.463,0.4065,0.54645,0.219,1.4571949,0,2.558370591,0,0,0,2.01,
- 1,2.2,0.062,1.1257,1.28,2.399797057,0,0,0,0,2.154314483,2.01,
- 1,2.2,1.4205,1.0929,2.551,2.420681551,0.710928319,0,0,0,1.012383621,2.97,
- 1,2.183,1.6215,1.60525,1.229,3.002421308,3.417634997,0,0,0,0.353883405,1.615,
- 1,1.338,3.885,3.28695,1.116,0,1.395348837,3.276955603,0,0,0,1.5), Dim = c(19, 12)),
- Y=c(14,13,21,13,19,7,10,5,6,4,8,7,2,4,1,5,9,9,0),
- XCentroids=c(5.505,11.16,20.36,31.155,39.59,46.255,53.965,64.285,76.585,84.695,87.73,92.43,97.27,101.
- 75,105.045,109.245,116.785,129.31,138.645),
- L = c(5.65, 5.66, 12.74, 8.85, 8.02, 5.31, 10.11, 10.53, 14.07, 2.15, 3.92, 5.48, 4.2, 4.76, 1.83, 6.57, 8.51, 16.54, 2.13),
- T=c(21697,13680,10240,10430,9670,10665,9246,4605,4300,4510,5641,
- 5270,6584,2530,1909,1760,1630,1536,1500),
- $\mathsf{Mu0} = \mathsf{c}(0.00, 0.$