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Well-posedness of bimodal state-based switched systems[☆]

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Abstract

In this work, we consider the well-posedness of state-based switched systems in the sense of piecewise classical solutions which commonly arise in the control of hybrid systems. We give some necessary and sufficient conditions for the well-posedness of this class of systems. These results can be used as tools for excluding the bimodal system having a Zeno state.

Keywords: State-based switched system; Well-posedness; Bimodal

1. Introduction

Hybrid systems, which incorporate both discrete event-driven and time-driven dynamics that interact at the event times, have been a very active research area over the past few decades [1–4]. Switched systems form a particular class of hybrid systems which consists of several subsystems and switching laws orchestrating an active subsystem at each time instant. Depending on the switching transition mode, these systems can be classified into two subclasses. The first one includes those where the mode transition is triggered by the external forces, i.e. the mode of the state will switch to another one at some time instant independent of the state itself. Research on the control of this class of systems has been summarized in [14]. The well-posedness of this class of systems can be easily demonstrated by extending well known results from the theory of standard ordinary differential equations.

The second class is one in which the mode transitions are triggered by an internal force, i.e. the mode of the state will switch to another one based on the state itself. We call this switched dynamical system a state-based switched system. Research on the control of these systems includes [7–10] to name just a few. In [8], Xu and Antsaklis consider the optimal control of this class of systems. They develop a computational solution method based on parametrization of the switching time instants. However, if the Zeno phenomenon of [11–13] exists, we cannot obtain an exact optimal solution. Thus, research into the well-posedness of these systems is needed. Results on this are limited and very basic in nature. In [5], Imura and Schaft have considered the well-posedness of the linear dynamical case. On the basis of a lexicographic inequality relation and the smooth continuation property, they gave some necessary and

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sufficient conditions for the well-posedness question in the sense of Carathéodory. However, only linear dynamics are considered. In [6], Imura extended results to the multi-modal case. Here, only sufficient conditions are obtained. In this work, we will consider the case of nonlinear dynamics and also the solution is in the piecewise classical sense. In [5], the solutions may have the Zeno phenomenon. We will exclude this phenomenon in our result. We will give some necessary and sufficient conditions for the well-posedness of the bimodal case.

2. Bimodal state-based switched system

Consider the system given by

$$\Sigma : \dot{x}(t) = \begin{cases} f_1(x), & \text{if } y = h(x) \geq 0, \\ f_2(x), & \text{if } y = h(x) \leq 0, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}$, $f_1, f_2 : \mathbb{R}^n \to \mathbb{R}^n$, and $h : \mathbb{R}^n \to \mathbb{R}$ are real functions defined on $\mathbb{R}^n$. In the following discussion, we suppose that $f_1, f_2, h \in C^1$, where C^1 is the set of all continuously differentiable functions.

Remark 2.1. For the system Σ, the notation suggests that $h(x) = 0$ can be allowed in both modes. However, sometimes there is only one mode that is attainable for the state $x(t)$ in some interval $t \in [T, T + \varepsilon)$, for some $\varepsilon > 0$. For example, consider the following system:

$$\begin{cases} \text{mode 1}: \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_1^2 \end{bmatrix}, & \text{if } x_1 \geq 0, \\ \text{mode 2}: \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1^2 - x_2^2 \end{bmatrix}, & \text{if } x_1 \leq 0. \end{cases} \tag{2}$$

Suppose that the initial state satisfies $x_1(0) = 0$ and $x_2(0) > 0$. Then, for some small $\varepsilon > 0$, $x_1(t) > 0$ in mode 1 and $x_1(t) > 0$ in mode 2 for $t \in (0, \varepsilon)$. Thus, only mode 1 is active in $(0, \varepsilon)$. In a similar way, we can show that only mode 2 is active for the case $x_1(0) = 0$ and $x_2(0) < 0$.

Definition 2.1. Consider an ordinary differential equation

$$\dot{x}(t) = f(t, x). \tag{3}$$

A curve $\varphi : [t_0, \omega) \to \mathbb{R}^n$, $t_0 < \omega$, is a classical solution of (3) if (i) it is differentiable in $(0, \omega)$, (ii) it satisfies (3) at all $t \in (0, \omega)$ and (iii) there exists $\dot{\varphi}^+(0) = f(t_0, \varphi(t_0))$.

A curve $\varphi : [t_0, \omega) \to \mathbb{R}^n$, $t_0 < \omega$, is a piecewise classical solution of (3) if (i) it is continuous, (ii) there exists a subset N_φ of $[t_0, \omega)$ such that N_φ is finite if $\omega < +\infty$, and N_φ is locally finite if $\omega = +\infty$, and (iii) φ is a classical solution of (3) in $[t_0, \omega) \setminus N_\varphi$.

A curve $\varphi : [t_0, \omega) \to \mathbb{R}^n$, $t_0 < \omega$, is a Carathéodory solution of (3) if (i) it is absolutely continuous and (ii) it satisfies (3) almost everywhere.

Definition 2.2. The system (3) is said to be well-posed if there exists a unique piecewise classical solution of (3) on $[0, \infty)$.

Clearly, a classical solution is a piecewise classical solution and a piecewise classical solution is a Carathéodory solution. In this work, we will focus on discussing well-posedness of (1) in the sense of piecewise classical solutions.

We assume that the following conditions are satisfied throughout this work:

Assumption 1. There exist constants a_i, b_i, $i = 1, 2$, such that $\|f_i(x)\| \leq a_i\|x\| + b_i$, $i = 1, 2$ for all $x \in \mathbb{R}^n$.

Assumption 2. For any $x_0 \in \mathbb{R}^n$, if $h(x_0) = 0$, then there exists an $\varepsilon > 0$ such that $h(x) \neq 0$ for any $x \in B(x_0, \varepsilon) \setminus \{x_0\} = \{x \in \mathbb{R}^n : 0 < \|x - x_0\| < \varepsilon\}$.

3. Main results

First, we give the following theorem on the non-existence of piecewise classical solutions of (1).

Theorem 3.1. *For dynamical system (1) with initial condition $x(t_0) = x_0$, if the following conditions hold:*

1. $h(x_0) = 0$;
2. $\nabla h(x_0) \cdot f_1(x_0) < 0$ *and* $\nabla h(x_0) \cdot f_2(x_0) > 0$;

then there exists no piecewise classical solution starting from x_0.

Proof. We assume that this is not the case. Then, there exists $\omega > 0$ such that φ is a piecewise classical solution of (1) in $[t_0, t_0 + \omega)$. Since φ is a piecewise classical solution and $t_0 + \omega < +\infty$, we suppose that $N_\varphi = \{t_1, t_2, \ldots, t_N\}$. Since $h(x)$ and $f_i(x), i = 1, 2$, are C^1, there exists $\varepsilon > 0$ such that $\nabla h(x) \cdot f_1(x) < 0$ and $\nabla h(x) \cdot f_2(x) > 0$ for all $x \in B(x_0, \varepsilon)$. Since $x = \varphi(t)$ is differentiable in $[t_0, t_1)$ and $\varphi(t_0) = x_0$, there exists $\tau < t_1$ such that $\varphi(t) \in B(x_0, \varepsilon)$ for all $t \in [t_0, \tau)$. Now we consider $\varphi(t)$ in the interval $[t_0, \tau)$. Since $\nabla h(x(t)) \cdot f_1(x(t)) < 0$ and $\nabla h(x(t)) \cdot f_2(x(t)) > 0$ for all $t \in [t_0, \tau)$, it is easy to see that $\varphi(t)$ cannot evolve according to either of the two modes of (1) in $[t_0, \tau)$ without a mode transition. That contradicts the assumption that $\varphi(t)$ has no mode transition in $[t_0, \tau)$. Thus, the result of the theorem is true. ■

Before proceeding the discussion, we give the following definition.

Definition 3.1. We say that a curve $\varphi : [t_0, \omega) \to \mathbb{R}^n$, $t_0 < \omega$, is a nearly classical solution of (3) if (i) it is continuous on $[t_0, \omega)$ and (ii) there exists $N_\varphi \subset [t_0, \omega)$ such that for all $\tau \in N_\varphi$, there exists a $\tau' \in N_\varphi$ such that φ is a classical solution of (3) in (τ, τ'). If $T_\varphi = \sup N_\varphi < \omega$, then φ is a classical solution of (3) in (T_φ, ω).

Clearly, any piecewise classical solution is also a nearly classical solution. In fact, the difference between nearly classical solutions and piecewise classical solutions is that the former includes a Zeno solution of (3) while the latter excludes it.

We give the following proposition without proof.

Proposition 3.1. *The following two statements are equivalent:*

- *for any initial state $x_0 \in \mathbb{R}^n$, there exists a nearly classical solution $\varphi(t)$ of system (1) on* $[t_0, +\infty)$;
- *for any initial state $x_0 \in \mathbb{R}^n$, there exists an $\varepsilon > 0$ such that system (1) has a classical solution on* $[t_0, t_0 + \varepsilon)$.

Let $\Delta = \{x \in \mathbb{R}^n : h(x) = 0\}$. That is, Δ is the set of all candidate mode transition points. We have the following result:

Proposition 3.2. *For any $x_0 \in \mathbb{R}^n$, if one of the following conditions holds, then system (1) with initial condition $x(0) = x_0$ has a nearly classical solution on* $[0, \infty)$*:*

1. $x_0 \notin \Delta$;
2. $x_0 \in \Delta$, $f_1(x_0) = f_2(x_0) = 0$;
3. *if $x_0 \in \Delta$, then $\nabla h(x_0) \cdot f_i(x_0) \neq 0$, $i = 1, 2$, and* $[\nabla h(x_0) \cdot f_1(x_0)][\nabla h(x_0) \cdot f_2(x_0)] > 0$;
4. *if $x_0 \in \Delta$, $\nabla h(x_0) \neq 0$, and $\nabla h(x_0) \cdot f_i(x_0) = 0$ for some $i \in \{1, 2\}$, then there exists $\varepsilon > 0$ and $\delta \in (0, 1)$ such that for all $x \in B(x_0, \varepsilon) \cap \Delta$, we have $\nabla h(x) \cdot f_i(x) = 0$, and if $\frac{x - x_0}{\|x - x_0\|} \cdot \frac{f_i(x_0)}{\|f_i(x_0)\|} > \delta$, we have $f_j(x) = f_i(x)$, $j = 1, 2$;*
5. *if $x_0 \in \Delta$ and $\nabla h(x_0) = 0$, there exists an $\varepsilon > 0$ such that $f_1(x) = f_2(x)$ for all $x \in B(x_0, \varepsilon)$.*

Proof. According to Proposition 3.1, we only need to show that for every $x_0 \in \mathbb{R}^n$, there exists an $\varepsilon > 0$ such that system (1) has a classical solution on $[t_0, t_0 + \varepsilon)$. For simplicity of notation, let $t_0 = 0$.

If $x_0 \notin \Delta$, then $h(x_0) \neq 0$. Suppose that $h(x_0) > 0$. Since $h \in C^1$, there exists $\varepsilon > 0$ such that $h(x) > 0$ for all $x \in B(x_0, \varepsilon)$. Since $f_1 \in C^1$, there exists $\tau > 0$ such that $\varphi(t) \in B(x_0, \varepsilon)$ and $\varphi(t)$ is a classical solution of $\dot{x} = f_1(x)$ with initial condition $x(0) = x_0$ on $[0, \tau)$. Clearly, $\varphi(t)$ is a classical solution of (1) on $[0, \tau)$. The alternative case (i.e. $h(x_0) < 0$) can be verified similarly.

If $x_0 \in \Delta$, $f_1(x_0) = f_2(x_0) = 0$, it is trivial to verify that $x = x_0$ is a classical solution of (1) on $[0, \infty)$.

If $x_0 \in \Delta$ and $\nabla h(x_0) \cdot f_i(x_0) \neq 0$, $i = 1, 2$, then $[\nabla h(x_0) \cdot f_1(x_0)][\nabla h(x_0) \cdot f_2(x_0)] > 0$. We suppose that $\nabla h(x_0) \cdot f_1(x_0) > 0$, $\nabla h(x_0) \cdot f_2(x_0) > 0$. Since $f_1, f_2, h \in C^1$, there exists $\varepsilon > 0$ such that $\nabla h(x) \cdot f_1(x) > 0$ and $\nabla h(x) \cdot f_2(x) > 0$ for all $x \in B(x_0, \varepsilon)$. In this case we only need to let mode 1 be active to easily obtain the result. We can treat the alternative case similarly.

Suppose that $x_0 \in \Delta$, $\nabla h(x_0) \neq 0$ and $\nabla h(x_0) \cdot f_1(x_0) = 0$. Consider $\dot{x} = f_1(x), x(0) = x_0$ in $B(x_0, \varepsilon) \cap \Delta$. Since $B(x_0, \varepsilon) \cap \Delta$ is a C^1 manifold and $\nabla h(x) \cdot f_1(x) = 0$ for all $x \in B(x_0, \varepsilon) \cap \Delta$, there exist a $t_\varepsilon > 0$ and a curve $\varphi : [0, t_\varepsilon) \to \mathbb{R}^n$ such that φ is a classical solution of the Cauchy problem and $\varphi(t) \in B(x_0, \varepsilon) \cap \Delta$. We note that there exists t'_ε such that for all $t \in [0, t'_\varepsilon)$, $\frac{\varphi(t)-x_0}{\|\varphi(t)-x_0\|} \cdot \frac{f_i(x_0)}{\|f_i(x_0)\|} > \delta$. If this was not the case, for any $1/n$, there would exist a $t_{\varepsilon n}$ such that $\frac{\varphi(t_{\varepsilon n})-x_0}{\|\varphi(t_{\varepsilon n})-x_0\|} \cdot \frac{f_i(x_0)}{\|f_i(x_0)\|} = \frac{t_{\varepsilon n}}{\|\varphi(t_{\varepsilon n})-\varphi(0)\|} \frac{\varphi(t_{\varepsilon n})-\varphi(0)}{t_{\varepsilon n}} \cdot \frac{f_i(x_0)}{\|f_i(x_0)\|} \leq \delta$. Letting $n \to \infty$, we have $1 \leq \delta$. This contradicts $\delta \in (0, 1)$. Thus, we have $f_1(\varphi(t)) = f_2(\varphi(t))$ for all $t \in [0, t'_\varepsilon)$ and $\varphi(t)$ is a classical solution of (1).

For Condition 5, consider the Cauchy problem $\dot{x} = f_1(x), x(0) = x_0$. There exist a $t_\varepsilon > 0$ and a curve $\varphi : [0, t_\varepsilon) \to \mathbb{R}^n$ such that φ is a classical solution of the Cauchy problem. Also, we can choose $t'_\varepsilon < t_\varepsilon$ such that for all $t \in [0, t'_\varepsilon)$, we have $\varphi(t) \in B(x_0, \varepsilon)$. According to the assumption, $f_1(\varphi(t)) = f_2(\varphi(t))$ for all $t \in [0, t'_\varepsilon)$. Thus, $\varphi(t)$ is a classical solution of (1). ■

We now assume that the following condition is satisfied.

Assumption 3. $\{x \in \Delta : \nabla h(x) f_1(x) \geq 0 \text{ and } \nabla h(x) f_2(x) \leq 0\} = \phi$, where ϕ is an empty set.

Theorem 3.2. *If Assumption 3 and the conditions of Proposition 3.2 hold, then the system (1) is well-posed.*

Proof. According to Proposition 3.2, there exists a nearly classical solution of (1). Next, on the basis of Assumption 3, we only need to show that this nearly classical solution is unique and that it is also a piecewise classical solution.

Uniqueness: We only need to show that, for any initial state $x_0 \in \mathbb{R}^n$, there exists an $\varepsilon > 0$ such that system (1) has a unique classical solution on $[t_0, t_0 + \varepsilon)$. That is, at any time, only one mode can be active. For any $x_0 \notin \Delta$, suppose that $h(x_0) > 0$. Then it is easy to show that only mode 1 can be active in $[t_0, \tau)$ for some $\tau > t_0$. Now suppose $x_0 \in \Delta$. If both modes can be active in $[t_0, \tau)$, we should have $\nabla h(x_0) f_1(x_0) \geq 0$ and $\nabla h(x_0) f_2(x_0) \leq 0$. Otherwise, suppose $\nabla h(x_0) f_1(x_0) \geq 0$ and $\nabla h(x_0) f_2(x_0) > 0$. Then, there exists a $\varepsilon > 0$ such that $h(x) \geq 0$ in mode 1 and $h(x) > 0$ in mode 2 over $[t_0, t_0 + \varepsilon)$. This contradicts the definition of the mode. The alternative case can be treated in a similar manner. Then, according to Assumption 3, the conclusion is satisfied.

Let φ be a nearly classical solution of (1) on $[0, \infty)$. Assume that φ is not piecewise classical. This implies that N_φ has at least one accumulation point. We suppose that $\bar{t}$ is an accumulation point of N_φ. Let $\bar{x} = \varphi(\bar{t})$. It is easy to show that $\bar{x} \in \Delta$ and $\nabla h(\bar{x}) f_i(\bar{x}) = 0, i = 1, 2$. In fact, let $t_{1,n}$ and $t_{2,n}$ be two sequences in N_φ and let $\dot{\varphi}(t) = f_i(\varphi(t)), t \in (t_{i,n}, t'_{i,n})$ with $h(\varphi(t_{i,n})) = h(\varphi(t'_{i,n})) = 0, i = 1, 2$. According to Rolle's theorem, there exists $t''_{i,n} \in (t_{i,n}, t'_{i,n})$ such that $\frac{d}{dt} h(\varphi(t''_{i,n})) = \nabla h(\varphi(t''_{i,n})) \cdot f_i(\varphi(t''_{i,n})) = 0$. Letting $n \to \infty$, we have $\nabla h(\bar{x}) f_i(\bar{x}) = 0, i = 1, 2$. This contradicts Assumption 3. Thus, every nearly classical solution of (1) is also a piecewise classical solution. ■

Assume f is an analytic function. We recursively define the Lie derivative of h along f, $L_f^m h : \mathbb{R}^n \to \mathbb{R}$ by

$$L_f^m h(x) = \begin{cases} h(x), & \text{if } m = 0, \\ \left(\dfrac{\partial}{\partial x} L_f^{m-1} h(x) \right) f(x), & \text{if } m > 0. \end{cases} \tag{4}$$

Let

$$S_1^+ = \left\{ x \in \mathbb{R}^n : L_{f_1}^i h = 0,\ L_{f_1}^{i+1} h > 0,\ \text{for some } i \in \mathbb{Z}_+ \cup \{0\},\ \text{or } L_{f_1}^i h = 0 \text{ for all } i \in \mathbb{Z}_+ \cup \{0\} \right\},$$
$$S_1^- = \left\{ x \in \mathbb{R}^n : L_{f_1}^i h = 0,\ L_{f_1}^{i+1} h < 0 \text{ for some } i \in \mathbb{Z}_+ \cup \{0\},\ \text{or } L_{f_1}^i h = 0 \text{ for all } i \in \mathbb{Z}_+ \cup \{0\} \right\},$$
$$S_2^+ = \left\{ x \in \mathbb{R}^n : L_{f_2}^i h = 0,\ L_{f_2}^{i+1} h > 0,\ \text{for some } i \in \mathbb{Z}_+ \cup \{0\},\ \text{or } L_{f_2}^i h = 0 \text{ for all } i \in \mathbb{Z}_+ \cup \{0\} \right\},$$
$$S_2^- = \left\{ x \in \mathbb{R}^n : L_{f_2}^i h = 0,\ L_{f_2}^{i+1} h < 0,\ \text{for some } i \in \mathbb{Z}_+ \cup \{0\},\ \text{or } L_{f_2}^i h = 0 \text{ for all } i \in \mathbb{Z}_+ \cup \{0\} \right\},$$

where $\mathbb{Z}_+$ is the set of positive integers.

If f_1, f_2, and h are analytic, we have the following theorem:

Theorem 3.3. *If f_1, f_2, and h are analytic and satisfy Assumption 3, then system (1) is well-posed if and only if $S_1^+ \cup S_2^- = \Delta$.*

Proof. First, we show that mode 1 is active if and only if $x_0 \in S_1^+$. If $x_0 \in S_1^+$, clearly mode 1 can be active. If mode 1 is active, we show that $x_0 \in S_1^+$ by contradiction. Suppose that mode 1 is active and $x_0 \notin S_1^+$. Then, there exists $i \in \mathbb{Z}_+$ such that $L_{f_1}^j h(x_0) = 0$, $L_{f_1}^i h(x_0) < 0$, $j = 1, \ldots, i-1$. Thus, there exists an $\varepsilon > 0$ such that $L_{f_1}^i h(x) < 0$ for all $x \in B(x_0, \varepsilon)$. Since system (1) is well-posed and mode 1 is active, there exists $\tau > 0$ such that the solution of (1) satisfies $\dot{\varphi}(t) = f_1(x(t))$ and $\varphi(t) \in B(x_0, \varepsilon)$, $t \in (0, \tau)$. Thus, $y = h(\varphi(x(t))) \geq 0$. According to $L_{f_1}^j h(x_0) = 0$ and $L_{f_1}^i h(x) < 0$ for all $x \in B(x_0, \varepsilon)$, $j = 1, \ldots, i-1$, we have $y = h(\varphi(x(t))) < 0$ for all $t \in (0, \tau)$. This contradicts $y = h(\varphi(x(t))) \geq 0$. In a similar way, we can show that mode 2 is active if and only if $x_0 \in S_2^-$.

(Only if). Suppose system (1) is well-posed, i.e. for every initial state x_0, only one of two modes can be active. Then, according to the above results, we have $S_1^+ \cup S_2^- = \Delta$.

(If). If $S_1^+ \cup S_2^- = \Delta$ and Assumption 3 is satisfied. For any $x_0 \in \mathbb{R}^n$, x_0 is only contained in S_1^+ or S_2^- by Assumption 3. We easily verify that there exists an $\varepsilon > 0$ such that the solution of (1) exists in $[t_0, t_0 + \varepsilon)$. According to Proposition 3.1, there exists a nearly classical solution with the initial state x_0. By a proof similar to that of Theorem 3.2, we can show that this nearly classical solution is also a piecewise classical solution and unique. This completes the proof. ■

We easily check that the system

$$\begin{cases} \text{mode } 1: \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_1 + x_2^2 + 0.1 \\ -x_1^2 + x_2 \end{bmatrix}, & \text{if } x_1 \geq 0, \\ \text{mode } 2: \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} e^{x_1 + x_2} \\ -x_1^2 + x_2^2 + e^{x_1} \end{bmatrix}, & \text{if } x_1 \leq 0 \end{cases} \tag{5}$$

satisfies Assumption 3. Thus, the system (5) is well-posed and has no Zeno state for any initial state $x_0 \in \mathbb{R}^n$.

4. Conclusion

In this work, we have considered the well-posedness of a class of bimodal state-based switched systems in the sense of piecewise classical solutions. We have given necessary and sufficient conditions for the well-posedness of this class of systems. However, our results only deal with the bimodal case. Extending these results to multi-modal cases is our intended future research.

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