3C laboratory ultrasound: a new method for measuring elastic anisotropy of rocks

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Summary

We propose a new method for measuring velocities and polarizations of the compressional and shear waves. The method consists of measuring time dependent displacement of a particular point on the sample surface in three independent directions by a laser Doppler interferometer. The high accuracy of these measurements makes the determination of the polarization straightforward, and thus the identification the type of the waves unambiguous. These measurements can be used to estimate the stiffness tensor of anisotropic rock samples.

Introduction

Estimation of elastic anisotropy of a rock sample is an important problem in geophysics. Traditionally, in laboratories, the anisotropy of rock samples is determined by measuring the wave velocities using so called the "time-of-flight method". The compressional and shear velocities are measured at different directions; from these measurements the density scaled stiffness tensor is estimated. The velocities are determined from traveltimes of waves generated and recorded by ultrasonic piezoelectric transducers (e.g. Pros and Babuska, 1967; Jech, 1991; Rasolofosaon and Zinszner, 2002). Despite great advantage of implementations of piezoelectric transducers as sources and receivers in ultrasonics, there are at least two problems with this kind of experiments, namely, 1) uncertainty in determination of the time of shear wave arrivals, and 2) uncertainty of whether phase or group velocity is measured. In particular, if the size of the wave source is small compared to the sample dimensions, the measured velocity is the group velocity. If the size of the source is comparable to the sample size which is typical for ultrasonic experiments, the velocity of wave propagating in the direction perpendicular to the source surface is the wavefront velocity (Vestrum, 1994). Another drawback of traditional experimental set-up is that there are no "universal" ultrasonic transducers capable of measurement of both P and S waves at one small point on the sample's surface.

In seismic field measurements, the size of the receivers (geophones, hydrophones) is much smaller then the wavelength of the measured waves. This simplifies the estimation of the polarization, and thus determination of the wave type. There are a number of methods for estimations of anisotropy from seismic field measurements, which utilize wave polarizations (Dewangan and Grechka, 2003). In this paper, we propose a similar method for measuring anisotropy in laboratory experiments.

Nishizawa et al (1997) proposed a method shear waves detection in the laboratory using laser Doppler interferometer (LDI). The method is based on the measurement of the wave-induced movement of the small area of the sample's surface. To separate P and S waves, these authors performed measurements in two directions and then found projections of the displacement onto directions perpendicular and parallel to the surface. Fukushima et al. (2003) used LDI to investigate polarization of the shear waves in rock samples. The main advantage of using LDI in comparison with piezoelectric transducers is that the area of the measurement is much smaller than the wavelength, and that the full particle velocity vector can be recorded. In this paper we report our first results of P wave polarization measurements and describe an algorithm of stiffness tensor inversion from velocities and polarizations.

Concept

The propagation of the elastic waves within a sample is governed by the stiffness tensor and density of the material. Hence, the polarizations of these waves as well as their velocities and amplitudes depend on the stiffness tenor.
Using sufficient number of measurements of the polarizations and velocities of these waves, all (up to 21) coefficients of the stiffness tensor of sample can be obtained, as discussed in Bona et al. (2008). We note that there are two types of velocities that we can use: the group (ray) velocity and the wavefront (phase) velocity. These velocities are, in general, different.

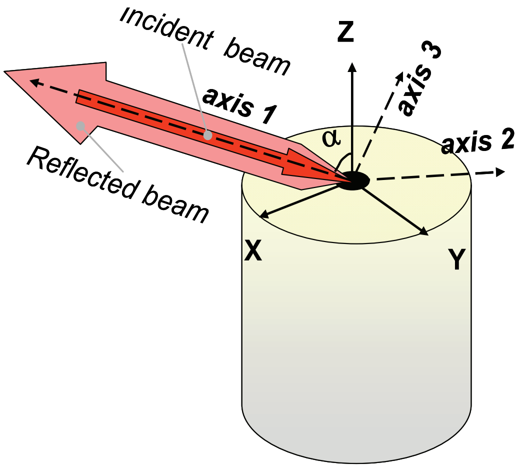


Figure 1. Schematic of the experimental set up.

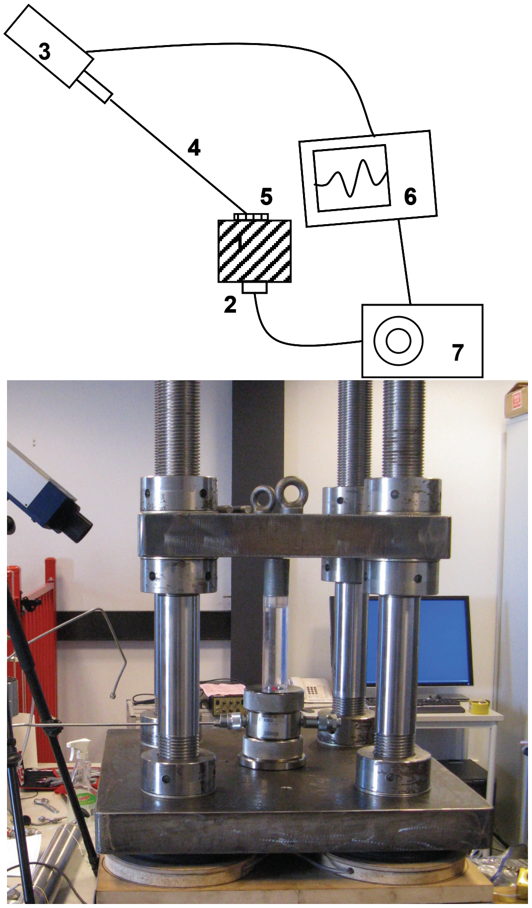


Figure 2. Experimental setup
1- sample; 2-source of elastic waves; 3- Laser Doppler Interferometer; 4- laser beams (emitted and back scattered) 5- reflective film; 6- acquisition system device; 7 - pulse generator.

If we measure the polarization of the displacement at an interface between two materials, we need to consider the amplitudes of the transmitted and reflected waves as well as the amplitude of the incoming wave. These amplitudes are related by the Zoeppritz equations.

Elastic body waves generated on the surface or inside the sample propagate thought the sample towards the sample surface. These waves induce displacement of individual points of the sample. The spatial components of the particle displacement and velocity in the wave are measured at different points on the surface in at least three directions (Fig.1). These directions have been chosen in such way as to span the three-dimensional space. By measuring at least 3 linearly independent components of velocity and/or displacement at a particular point at the surface, the polarization (direction of particle motion vs time) can be determined. Mathematically, this corresponds to finding a vector from its projections onto different planes. As results of these measurements, the times of arrivals of P and S waves as well as their polarisation (direction of particle motion) can be determined.

Experiment

To prove the idea described above, the experimental set up shown in Figure 2 was build. Our apparatus uses laser Doppler interferometer to find displacement of a point on a sample's surface in a given direction. Figure 3 shows the spatial arrangement of the experiment. The paper reinforced phenolic was chosen as the sample to test the presented method. This synthetic sample has a layered structure and P-wave velocity anisotropy of 18%: ultrasonic P velocities measured in the directions parallel and perpendicular to the layering plane are 3519 m/s and 2875 m/s, respectively. As the wave source, a 10 mm in diameter and 2 mm thick piezoceramic disk was glued by epoxy to the bottom side of the sample. Such a disc transducer has two resonance frequencies close to 1 MHz and 200 kHz, respectively. To generate ultrasonic wave in the sample, a 1 µs square 400 V electrical pulse was applied to the transducer. Reflective tape (3M) was glued on the sample's surface directly above the piezoceramic disk. This tape is made from 50 µm diameter micro glass beads; each glass bead reflects light backward. Reflective tape works ideally for the laser beam incident angles up to 80° (from the direction normal to the surface). We chose the incident angles of LDI to be 55° from the normal to sample's plane (Z axis). LDI and the sample were placed on the special vibro-isolating table. Experimental records of the particle displacement and velocity in one direction are shown in Fig. 4. LDI has a wide frequency range of measurements (from 1 Hz to 2.5 MHz). After measurements of the particle velocities in three linearly independent directions (axis 1, 2 and 3 in Fig. 1), we plotted in Fig. 5 the velocities of the surface particle motion versus time in Cartesian coordinate system X, Y and Z (with Z axis perpendicular to the surface, Fig. 3). Displacements as a function of time can be found by integration of the velocities; the trajectory of the particle (displacements) corresponding to the P wave is shown in Fig. 6. From Fig. 6b, c it is clear that the trajectory of particle motion in Y-Z plane is normal to sample's surface, while in X-Z plane it has a deviation of about 7° from the normal.

From these measurements, we are able to determine: 1) the angle between the particle movement and the direction of the wave propagation, i.e. the polarization, 2) the types of waves and 3) the arrival times of the wave at the point and thus the waves velocities.

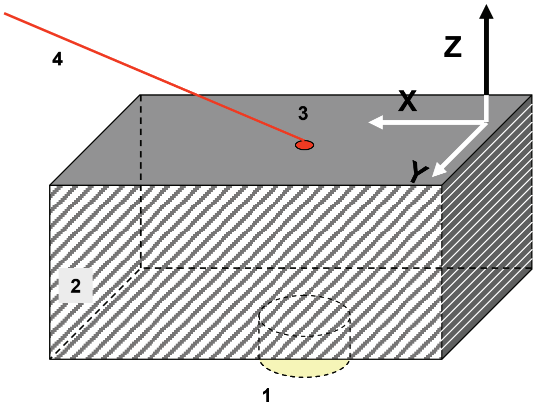


Figure 3. Validation experiment setup using paper reinforced phenolic sample:1 – ultrasonic transducer (pizoceramic disk 10 mm in diameter); 2 - 50 mm thick sample, layered structure is shown in the picture (cut at 45 degree to the "bedding plate"); 3 – measured area (0.3 mm in diameter); 4 –on eof the tree directions of measurements from and to the Laser Interferometer.

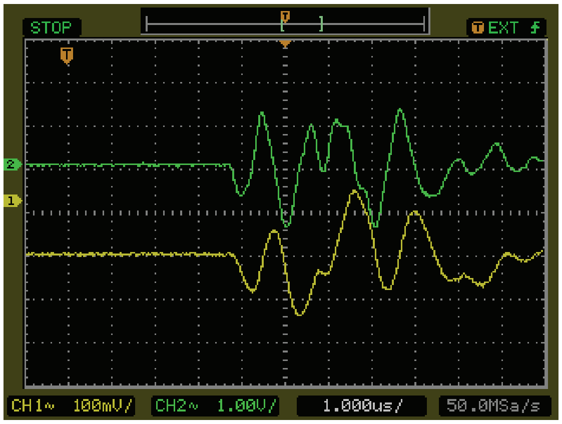


Figure 4. Typical oscilloscope traces directly recorded from LDI: green – velocity, yellow – displacement.

Methodology of Anisotropy Analysis

From the measured polarizations and velocities of the (quasi) compressional and (quasi) shear waves, we can find the density-scaled stiffness tensor of the sample. Our apparatus allows us to measure both the phase (wavefront) and the group (ray) velocities. The methods of finding the density-scaled stiffness tensor will be different for phase and group velocities. If we measure the phase (wavefront) velocities, we use the following expression:

$$a_{ijkl} p_i p_k A_j = A_l ,$$

where a is the density-scaled stiffness tensor, p is the phase slowness vector that has the direction of the wavefront-propagation and magnitude inversely proportional to the phase (wavefront) velocity, and vector A is the corresponding polarization vector. However, the measured amplitude is the sum of the amplitudes of the incoming, reflected and transmitted waves. These amplitudes are related to each other by the Zoeppritz equations that need to be included in the inversion of the density-scaled stiffness tensor a.

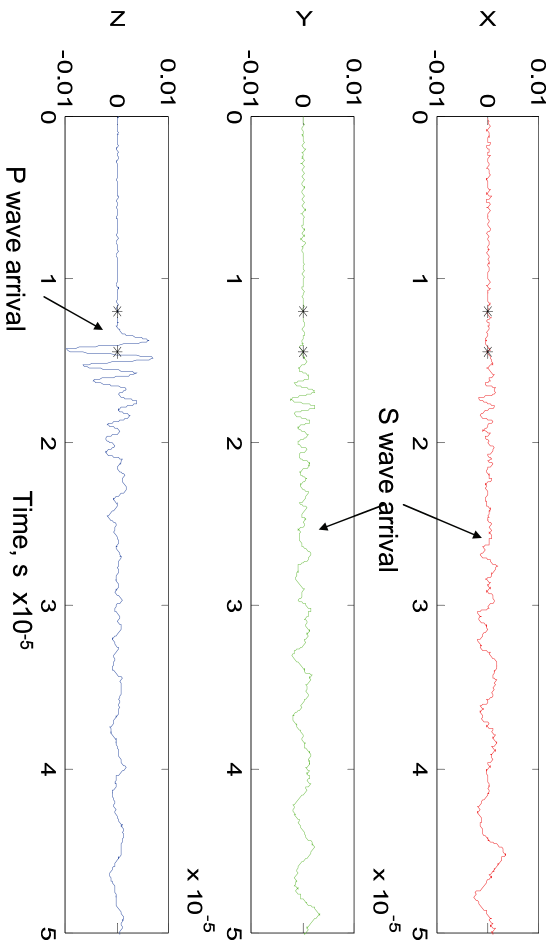
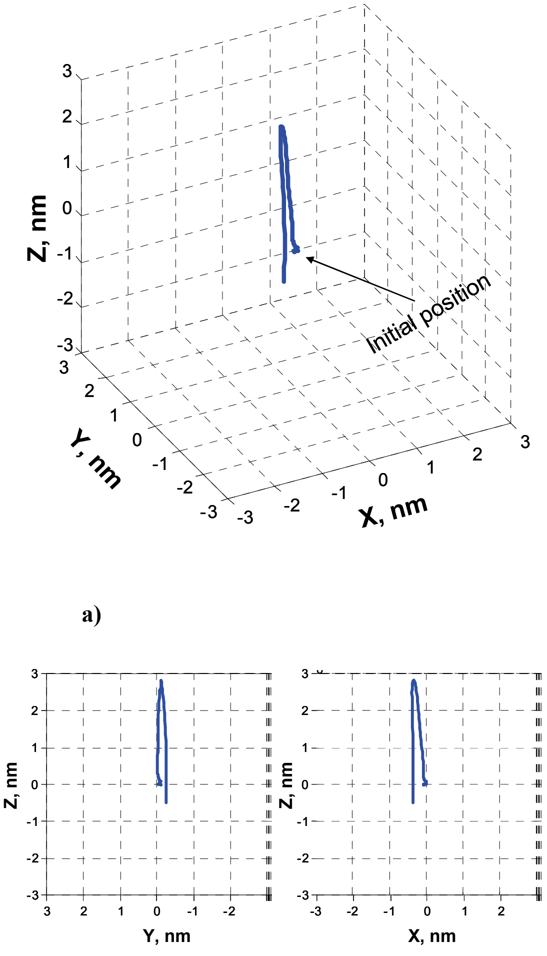


Figure 5. Velocities of the surfaces' point (arbitrary units) at directions X, Y and Z (shown at Fig. 3) vs time.



a)

b) c)_

Figure 6. a) 3D Trajectory of particles at the time interval 11~14.5 μs (between stars shown in the Figure 5), Hodogram of the same point in Z-Y plane (b) and X –Z plane.

We note that the Zoeppritz equations vary depending on the boundary condition at the interface: they are different for a free surface and for a welded contact interface.

If measured are group (ray) velocities, we use the following expression:

$$\Gamma(\Gamma^{-1}(A)v) = A\,,$$

where

$$\Gamma(x)_{ik} = a_{ijkl}x_j x_l\,,$$

with v being the group (ray) velocity, and vector A the corresponding polarization. Again, using different measurements, the above expressions together with the correct form of the Zoeppritz equations form a system of nonlinear equations for the unknown tensor a. This system can be solved by any method for solving systems of nonlinear equations.

Discussion

The measured angle between the direction of the wave propagation and the polarization of 7° (Fig. 6) is in a good agreement with the estimation of the polarization angle of the P wave propagating at 45° to the bedding plane in the media with similar anisotropic parameters (see White, 1987, page 42). However, up to now we are able to measure polarization of the waves on the surface only. The particle displacement at an interface differs form the displacement inside of the material. The displacement at an interface between two materials is influenced by the properties of both materials. The displacement is a combination of the displacements of the incoming wave, as well as reflected and transmitted waves. To obtain the displacements at the surface, it necessary to solve the Zoeppritz equations.

We note that the elliptical motion of the particles at an interface (shown at the Fig 6 b and c) is a demonstration of the phase differences caused by the differences of the slownesses of the incoming and the reflected/transmitted waves. It may be possible to utilize this phenomenon for the estimation of the elastic properties by using it as an additional constraint for the inversion.

More attention should be paid to the type of the wave source as well. Depending on the source type, the velocity is either the group or the wavefront velocity. In particular, if the size of the source is small compared to the sample dimensions, the velocity is the group velocity. If the size of the source is comparable to the sample size, the velocity of wave propagating in the direction perpendicular to the source surface is the wavefront velocity. If the size of the source is comparable to the wavelength, the diffraction effects should be taken into account as well.

Conclusion

We proposed and developed a new laboratory method for measurement of the displacement of waves propagating through a sample. These measurements can be used for estimation of anisotropy of rock samples based on the measurements of the velocities and polarizations of elastic waves. More theoretical analysis of the wave behavior at interfaces is necessary to fully unlock the potential of the presented method for stiffness tensor estimation. Also, measurements of anisotropy under triaxial stress will be the next step of this study (Fig. 2).

EDITED REFERENCES

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