Banks’ Efficiency and Credit Risk Analysis using By-Production Approach: the case of Iranian Banks

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Abstract

This article uses a by-production approach that integrates credit risk to monitor bank efficiency. The method overcomes the possible misspecification issues of the commonly assumed weak disposability of undesirable outputs. In addition, our measure extends the classic by-production approach by including statistical aspects through sub-sampling techniques. We have also provided an algorithm to correct related infeasibilities. Using this approach, we investigate the performance of Iranian banks and credit risk management in the sector for the period 1998–2012. Non-performing loans have been used as an undesirable output and proxy for credit risk in our models. Based on our empirical results, although the banks generally exhibited efficiency improvements over time, their credit risk performance deteriorated considerably after the regulatory changes introduced in 2005. These findings confirm that credit quality can be monitored more actively across Iranian banks.

Keywords: Data envelopment analysis; Efficiency; Banking; Credit risk; Undesirable output

JEL Classification: C61, G21
1. Introduction

Over the last two decades, the Iranian government has implemented a series of regulatory reforms in the banking sector with the aim of increasing the efficiency of financial markets and strengthening economic activity and entrepreneurship. The most significant policies removed barriers to entry for private banks in 2001 and imposed different deposit interest rates and conditions on government-owned and private banks in 2005.¹ The latter obliged all banks to reduce their lending rates significantly and forced government-owned banks to set their deposit and lending rates at least three percentage points lower than those offered by private banks. In addition, the Central Bank of Iran (CBI) mandated that government-owned banks give priority in lending to less-developed sectors such as small and medium enterprises (SMEs) after 2004.²

For instance, under Article 3 of the executive by-law supporting the expansion of SMEs, banks were required to allocate approximately 20 percent of their credit to support the expansion of SMEs in 2005; this was to be raised to 35 percent in 2006 and 50 percent after that (CBI 2006). However, Iran’s SMEs are underdeveloped, and without appropriate infrastructure, this degree of investment can increase credit risk through the inefficient allocation of resources and mismanagement in the banking system. According to CBI (2008; 2012), the ratio of government-owned commercial bank non-performing loans (NPLs) to total loans almost immediately increased from approximately 5 percent in 2005 to 7.2 percent in 2007 and 8.1 percent in 2009. As a result, between 2010 and 2012, the allocated share of SMEs as a proportion of banking loans decreased to 30 percent, and the ratio of government-owned commercial bank NPLs also dropped and remained steady at approximately 6 percent. In this article, we argue that although the reforms could substantially increase the level of competition in the market and help the government provide more job opportunities by developing new enterprises and improving SME performance, they may have encouraged banks to orient their businesses toward higher risk clients, which leads to higher credit risk and lower financial

¹ Ten private banks joined the market from 2001. The sector is currently dominated by 10 government-owned institutions, including six commercial banks and four specialized banks. The specialized banks mainly focus on special services in their specific areas of interest such as mining, manufacturing, agriculture, and housing.
² The CBI is responsible for supporting economic growth in Iran by implementing appropriate monetary and credit policies and assisting the government to create stabilization and economic development programs (CBI, 2006). As the CBI is not independent of the government, banks’ ability to manage their credit policies, specifically government-owned banks, is extremely limited.
efficiencies. Therefore, a study of bank efficiency and credit risk management in both the pre- and post-reform periods is particularly pertinent. Although the relationship between efficiency and risk is well documented, previous studies of banking efficiency in Iran have not allowed for credit risk (e.g. Arjomandi et al., 2011; 2012). As stated by Mester (1996, p.1026), “unless quality and risk are controlled for, one might easily miscalculate a bank’s level of inefficiency, e.g., banks scrimping on credit evaluations or producing excessively risky loans might be labelled as efficient when compared to banks spending resources to ensure their loans are of higher quality”. Therefore, this study estimates and compares Iranian banks’ traditional and credit-risk-adjusted technical efficiencies in the period 1998–2012. The credit risk is assessed by the levels of NPLs, which are included in the technology as undesirable outputs. These estimates are grounded in bias-corrected efficiency scores using the subsampling techniques of Simar and Wilson (2011).

The contribution of this study is manifold. First, we propose to use an innovative methodological approach to include undesirable outputs (such as NPLs) in the production technology modeling to measure banks’ technical inefficiency. This new measure extends the by-production model of Murty et al. (2012) by including statistical aspects and is adapted to the banking system. The uniqueness and contribution of this approach (by-production) lies in the estimation of two sub-technologies, one related to the production of good outputs and the other to the production of bad outputs. To assess the bank performance values even more precisely than the traditional approaches, we also used a model based on the enhanced Russell-based directional distance measures discussed in Chen et al. (2014). Overall, the by-production approach offers the advantage of easy inclusion of undesirable outputs by considering two distinct sub-production frontiers and also provides a good sketch of all the processes involved in a decision-making unit (DMU). From a technical perspective, the by-production model overcomes the misspecification issues related to the commonly assumed weak disposability (WDA) concept (Färe et al, 1989, Kuosmanen, 2009). Second, we examine Iranian banking sector efficiency for the period from 1998 to 2012. There is a lack of recent empirical studies analyzing the changes within the Iranian banking industry following the reform process of the mid-2000s. Third, we investigate the nexus between NPLs and bank efficiency, which allows us to quantify the impact of NPLs on bank efficiency in Iran; NPLs are increasing, and the

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3 See the theoretical discussions in Frisch (1965) and Førsund (2009) on the good representation of production processes.
4 Issues associated with WDA modeling, such as the wrong trade-offs between inputs and undesirable outputs and the violation of the materials balance principles, are comprehensively discussed in Coelli et al. (2007) and Murty et al. (2012).
The direct influence of NPLs on bank performance has not been addressed in the literature. Such analysis of NPLs is important for policy-makers because it can inform the development of an appropriate regulatory framework and support the more efficient functioning of the Iranian banking sector.

The remainder of this paper is structured as follows: Section 2 provides a brief review of studies on the Iranian banking system. The methodology is presented in Section 3. Sections 4 and 5 discuss the data and the results, respectively, followed by concluding remarks in Section 6.

2 Empirical research on Iranian bank efficiency

To date, several studies have analyzed the performance of the Iranian banking industry. Hadian and Hosseini (2004) investigated the intermediation activities of six commercial and four specialized Iranian banks between 1997 and 1999, finding that the specialized banks were significantly more technically efficient than their commercial competitors. Hasanzadeh (2007) used the same approach as Hadian and Hosseini (2004) to estimate the technical efficiency of all Iranian banks between 1997 and 2003. Hasanzadeh found that private banks were more technically efficient than government-owned banks and observed that government control had a negative influence on the control of inputs and outputs within the government-owned banks.

The most recent studies of bank efficiency and productivity within the Iranian banking sector were undertaken by Arjomandi et al. (2011; 2012) for the pre- and post-regulation period (2003–2008) and used different efficiency and total factor productivity (TFP) indices. Arjomandi et al. (2011) group all of Iran’s authorized deposit-taking institutions into three categories—commercial banks, specialized banks and private banks—and estimate their intermediate efficiencies using a Malmquist TFP index under a variable returns to scale (VRS) assumption. They found that the technical efficiency of Iranian banks increased up to the point of the regulatory reform (in 2005) and then fell following the regulatory changes. Arjomandi et al. (2012) used a comprehensive decomposition of the Hicks-Moorsteer TFP index, proposed by O’Donnell (2012), to estimate the banks’ intermediate efficiencies and productivity changes and found similar results. Arjomandi et al. (2014) extended the findings of Arjomandi et al. (2012) by estimating both the intermediate and the operating performance of the same banks between 2003 and 2008. They found that irrespective of the considered approach, the industry experienced technical efficiency improvements after 2005 and some deterioration following the reforms. Other studies of the Iranian banking system have merely focused on the efficiency of a single bank’s branches (Hakimabady et al., 2006; Dadgar and
Nemat, 2007). Overall, in all of these studies, the banks’ most undesirable output—NPLs—is not taken into account. In addition, their databases only extend to the year 2008, which is only three years after the introduction of the most recent banking regulation and does not provide a sufficiently long period to accurately investigate the effect of the 2005 policies. To fill these important gaps in the literature, this study incorporates NPLs as a proxy of credit risk in the production process to provide a more accurate and realistic analysis of Iranian banks over the period from 1998 to 2012. Allowing for credit risk is particularly important in the case of banks because not only is it desirable for the banks to be efficient, but they must also be secure. Altunbas et al. (2000), Drake and Hall (2003), Pasiouras (2008) and Salim and Hoque (2010), inter alia, note that failure to adequately account for risk can have a major impact on relative efficiency scores.

3 Methodology and risk-adjusted efficiency appraisal

In this study, the nonparametric data envelopment analysis (DEA) programming technique is employed to estimate production frontiers and measure efficiency relative to these frontiers. This approach (DEA) has been extensively used to evaluate the technical performance of DMUs (Seiford, 1996; Cooper et al., 2007; Lampe and Hilgers, 2015). The literature on the efficiency of financial institutions using DEA has expanded rapidly in recent decades. Fethi and Pasiouras (2010), in their comprehensive survey of 196 bank performance studies, revealed that recent DEA studies have examined nearly all of the banking sectors in the world. After the widespread use of DEA in developed countries, it has also become a popular method used by banking researchers from developing countries to evaluate financial institutions. Sathye (2003), Ataullah and Le (2006), Drake et al. (2006), Isik (2008), and Pasiouras (2008) are among the studies from developing countries. The most relevant advantage of DEA, in the context of this study, is that it works well with small sample sizes. As there are currently only 20 banks in Iran’s banking system, the banking industry is less conducive to the use of parametric (econometric) analysis techniques. There are also other important advantages of this technique: there is no need to have a specific form for the production function; there is no restriction on the functional form of the production relationship; and it is able to use data on various inputs and outputs and indicate the magnitude of inefficiency.

Charnes, Cooper and Rhodes (CCR) (1978) developed DEA and extended the economic aspect of linear programming, which was introduced by Farrell (1957) twenty years earlier. This technique constructs a nonparametric piece-wise surface or efficient frontier, and the efficiency measures of DMUs are then estimated relative to this frontier. The CCR model
assumed the existence of both constant returns to scale (CRS) and input orientation in calculating the resulting technical efficiency indices. The CRS assumption can be appropriate when all DMUs are functioning at an optimal scale. Nevertheless, due to imperfect competition, financial limitations, government control and regulation (such as the banking environment in Iran), a DMU may not actually perform at its optimal scale. Thus, the use of a CRS specification, when production is not at its optimal level, will yield distorted technical efficiency scores (Arjomandi, 2011; Salim et al., 2016). Therefore, subsequent studies, such as Banker, Charnes, and Cooper (1984) (known as the BCC model) and Färe et al. (1983), suggested an extension to account for VRS. The use of the VRS specification enables the calculation of technical efficiency free of these scale-efficiency distortion effects. In addition, as pointed out in Chambers and Pope (1996, p.1364), “while constant returns may make sense for some stylized ‘representative firm’ presumed to be in long-run equilibrium, it certainly does not make sense for most real-world observations”. Hence, we run our DEA models under the VRS assumption in this study.

Assaf et al. (2013) argue that both desirable and undesirable outputs should be present in a model and state that using only desirable outputs will fail to acknowledge a banks’ effort to reduce its undesirable outputs and may bias the results. For instance, if inefficiency exists in the production process, whereby final intermediation services are produced with an increase in NPLs, the production of NPLs is undesirable and must be reduced to improve performance. One may treat the undesirable outputs as inputs. However, Seiford and Zhu (2002) and Zhu (2009) state that this does not reflect the true production process.\(^5\) Some studies have examined the impact of credit risk on bank efficiency by including loan loss provisions as an additional input (see, \textit{inter alia}, Drake et al., 2006; Pasiouras, 2008). For the advocates of this process in the presence of undesirable outputs, in the case of bank efficiency estimation, NPLs can be viewed as a cost that is required to build up loan loss reserves. However, it may not reflect the quality of the credit risk management, particularly in the context of Iranian banks. NPLs have been considered as a control variable in specified efficiency functions in some cases, such as Mester (1996) and Berger and Mester (1997). However, we argue that NPLs are outputs of the banking production system and should not be considered as exogenous to the producing technology.

\(^5\) It should be noted that this idea has been rejected by Färe and Grosskopf (2003) as it violates the laws of thermodynamics and the materials balance principles.
In this study, following Pastor (1999), Chang and Chiu (2006), Chiu and Chen (2009), and Fukuyama and Weber (2010), NPLs are viewed as an undesirable by-product output arising from the production of loans and taken into account as a proxy of credit risk to obtain “risk-adjusted efficiency measures”. In this line, many studies are based on efficiency assessments using the WDA concept that consider NPLs to be a weakly disposable output, and thus, any effort to reduce the undesirable output will necessarily involve a proportional decrease in the good output (Park and Weber, 2006, Fukuyama and Weber, 2010, Barros et al., 2012). For instance, Epure and Lafuente (2014, p.1) recently extended the WDA based on the Kuosmanen and Podinovski (2009) approach, stating that their study “reflects the real banking system technology and accurately models the relationship between desirable and undesirable outputs”. Nevertheless, serious weaknesses associated with the use of the WDA have previously been discussed by Chen (2014) using an illustrative example. Further, the theoretical irrelevance of the WDA has been proved by Murty et al. (2012). Overall, it appears that the WDA does not provide the right trade-offs between the variables involved in the production technology.

As mentioned previously, this study relies on a by-production model that is based on the modeling of two sub-technologies (one for good output and the other for the unwanted outputs). Assume a set of \( n \) observations on the DMUs where each \( DMU_j (j = 1, \ldots, n) \) uses \( p \) inputs, \( x_{ij} (i = 1, 2, \ldots, p) \), to produce \( s \) good outputs, \( y_{ij} (r = 1, 2, \ldots, s) \), and \( u \) bad outputs, \( b_{qj} (q = 1, \ldots, u) \). Using the enhanced Russell-based directional distance measures (ERBDDM), discussed in Chen et al. (2014), and the by-production approach, the output-oriented technical inefficiency under the VRS model can be obtained by solving the following linear problem:

\[
\bar{D}(x, y, b; \tilde{g}_y, \tilde{g}_b) = \text{Max}^k = \frac{1}{2} \left[ \frac{1}{s} \sum_{r=1}^{s} \theta^k_r + \frac{1}{u} \sum_{q=1}^{u} \theta^k_q \right]
\]

Subject to

\[
\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} \quad i = 1, \ldots, p
\]

\[
\sum_{j=1}^{n} \lambda_j y_{rj} \geq \theta^k_r \tilde{g}_y + y_{rk} \quad r = 1, \ldots, s
\]

\[
\sum_{j=1}^{n} \lambda_j = 1
\]
\[
\sum_{j=1}^{n} \mu_j x_{ij} \geq x_{ik} \quad i = 1, \ldots, p
\]

\[
\sum_{j=1}^{n} \mu_j b_{qj} \leq b_{qk} - \theta^k_q \tilde{g}_b \quad q = 1, \ldots, u
\]

\[
\sum_{j=1}^{n} \mu_j = 1
\]

\[
\lambda_j, \mu_j \geq 0 \ (j = 1, \ldots, n); \quad \theta \geq 0
\]

Contrary to Murty et al. (2012), we do not use an input separation; we consider all inputs in both sub-frontier estimations\(^6\), and the sub-technologies are modeled using two different intensity variables (\(\mu_j\) and \(\lambda_j\)). Moreover, for the directional vectors, \(\tilde{g}_y\) and \(\tilde{g}_b\), following the recommendation of Chung et al. (1997), we use the observed vectors of the different outputs: \(\tilde{g}_y = \tilde{y}\) and \(\tilde{g}_b = \tilde{b}\). With these direction vectors, inefficiency scores can be easily derived. Given these transformations, the model can be specified as follows:

\[
\bar{D}(x, y, b; \tilde{g}_y, \tilde{g}_b) = \text{Max} \theta^k = \frac{1}{2} \left[ \frac{1}{s} \sum_{r=1}^{s} \theta^k_r + \frac{1}{u} \sum_{q=1}^{u} \theta^k_q \right]
\]

Subject to

\[
\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ik} \quad i = 1, \ldots, p
\]

\[
\sum_{j=1}^{n} \lambda_j y_{rj} \geq (1 + \theta^k_r) y_{rk} \quad r = 1, \ldots, s
\]

\[
\sum_{j=1}^{n} \lambda_j = 1
\]

\[
\sum_{j=1}^{n} \mu_j x_{ij} \geq x_{ik} \quad i = 1, \ldots, p
\]

\[
\sum_{j=1}^{n} \mu_j b_{qj} \leq (1 - \theta^k_q) b_{qk} \quad q = 1, \ldots, u
\]

\[
\sum_{j=1}^{n} \mu_j = 1
\]

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\(^6\) In Murty et al.’s model, the inputs associated with each sub-technology need to be identified beforehand, but we argue here that this way of proceeding is inappropriate for an analysis of the banking system.
\[ \lambda_j, \mu_j \geq 0 \ (j = 1, \ldots, n) \]

As presented, the directional distance function in the previous models assesses the inefficiency associated with each type of output and provides an average (arithmetic mean) inefficiency score. A non-radial efficiency score for each good output can be obtained using the following propositions:

\[ \Theta_r^k = \frac{1 + \theta_r^k}{y_{rk}} = \left(1 + \theta_r^k\right) \quad r = 1, \ldots, s \quad (3) \]

For the bad output, the efficiency can be computed as:

\[ \Theta_q^k = \frac{b_{qk}}{(1 - \theta_q^k)b_{qk}} = \frac{1}{\left(1 - \theta_q^k\right)} \quad q = 1, \ldots, u \quad (4) \]

A “global” efficiency score can then be obtained by using the arithmetic mean:

\[ \Theta^k = \frac{1}{2} \left[ \frac{1}{s} \sum_{r=1}^{s} \Theta_r^k + \frac{1}{u} \sum_{q=1}^{u} \Theta_q^k \right] \quad (5) \]

We measure the efficiency of banks with and without the undesirable output and use the ratio of the obtained technical efficiency and risk-adjusted technical efficiency to analyze the effect of credit risk on efficiency estimates. We designate this effect as the “risk effect”:

\[ \text{Risk Effect (RE)} = \frac{\text{output technical efficiency}}{\text{output risk-adjusted technical efficiency}} \]

We are using the non-radial output efficiency scores, which are all greater than or equal to one, and scores closer to unity indicate more efficient DMUs. Therefore, if RE=1, including NPLs in the model has no effect on the bank’s efficiency. If RE<1, the output risk-adjusted efficiency of a bank is higher than its technical efficiency, indicating that including risk deteriorated the efficiency of banks. If RE>1, the bank has performed well in terms of managing its credit risk.

For comparison purposes, we also estimate the model under the weak disposability assumption by using the non-uniform abatement factor model discussed in Kuosmanen et al. (2009) as an extension of the traditional WDA model (Färe et al., 2012). The estimated program is:

\[ \bar{D}(x, y, b; \bar{g}_y, \bar{g}_b) = \text{Max} \ \theta^k = \frac{1}{2} \left[ \frac{1}{s} \sum_{r=1}^{s} \theta_r^k + \frac{1}{u} \sum_{q=1}^{u} \theta_q^k \right] \quad (6) \]

Subject to
\[ \sum_{j=1}^{n} (\gamma_j + \tau_j) x_{ij} \leq x_{ik} \quad i = 1, \ldots, p \]
\[ \sum_{j=1}^{n} \gamma_j y_{rj} \geq (1 + \theta_r^k) y_{rk} \quad r = 1, \ldots, s \]
\[ \sum_{j=1}^{n} \gamma_j b_{aj} = (1 - \theta_q^k) b_{qk} \quad q = 1, \ldots, u \]
\[ \sum_{j=1}^{n} (\gamma_j + \tau_j) = 1 \]
\[ \gamma_j, \tau_j \geq 0 \quad (j = 1, \ldots, n) \]

Model (6) is quite similar to that used in Epure and Lafuente (2014), except that here we consider all good outputs to be associated with the NPLs.\(^7\)

As noted in Simar and Wilson (1998; 2000), the nonparametric frontier estimation based on the DEA methodology is a subset of the true frontier technology, and hence, the output efficiency scores can be biased downward. To address this issue, Simar and Wilson (1998) developed the bootstrap methodology to derive bias-corrected efficiency scores. However, this proposed procedure relies on the homogeneity assumption in the technology modeling, and this assumption is equivalent to the homoscedasticity in the econometric estimation. Therefore, Simar and Wilson (2011) have recently suggested using the repeated samples selection technique (i.e., subsampling), which consists in drawing without replacement \(m < n\) observations from the original sample to use as a benchmark to evaluate the efficiency levels of all observations. We extended this approach to the ERBDDM and obtained the corresponding bias-corrected measures.\(^8\) In fact, for the operationalization of the subsampling, we use the output efficiency scores (non-radial in this case) in Equations (3) to (5), which for practical purposes set all efficiency scores to be greater or equal to one.\(^9\) The bias-corrected efficiency scores are provided using the less conservative rule of Efron and Tibshirani (1993) that specifies whether we should use the corrected or the uncorrected results (as a rule of thumb,\(^7\) Epure and Lafuente (2014) have extended the Kuosmanen’s (2009) approach by separating the good outputs into two categories: those associated with the undesirable outputs and those that are not.

\(^8\) To the best of our knowledge, this is the first time in the literature that the sub-sampling techniques have been extended to the models that include undesirable outputs and especially to the case of by-production.

\(^9\) This is beneficial for the computations because only one boundary for the efficiency measures is provided (rather than two when the score is lower than or equal to one, for which the two boundaries are zero and one). An additional advantage of this Farrell measure is that it prevents negative values in confidence intervals.
they specify not to correct the obtained values unless \( |\text{bias} \left( \hat{\delta}(x, y) \right) | > \frac{\text{std}(\hat{\delta}(x, y))}{4} \) (Daraio and Simar, 2007).\(^{10}\) The algorithm is run for several values of \( m \), and the appropriate value is selected given the data-driven rules discussed in Politis et al. (2001) and Bickel and Sakov (2008). Regarding these rules, the choice of \( m \) can be based on the minimization of a volatility criterion such as the standard deviation.\(^{11}\) Overall, the bias-corrected efficiency scores estimated for the measures in formulas (3) to (5) are associated to the DEA models (2) and (6).

An additional issue worth mentioning is that we consider an unbalanced sample in this study. As noted in Simar and Wilson (2011), when considering unbalanced subsampling, where \( m \) observations are randomly drawn from the original sample without accounting for the position of the DMU under evaluation, the model can result in some infeasibilities (an infinite value for the efficiency score). In this case, they recommend that these DMUs set the efficiency to 1. However, following the discussion by Lee et al. (2011), we decide to correct these infeasibilities due to the actual presence of super-efficiency on the input side (given that in this work, we are working on the output side). The correction we adopt is based on the “one model approach” of Chen and Liang (2011). Moreover, the use of a non-radial approach for the subsampling can produce other infeasibilities due to the lack of convergence of the simplex algorithm. This can also be corrected using the “one model approach”. We briefly present the algorithm that is used to solve this problem in Appendix A for all of the models.\(^{12}\) Given the special nature of the by-production approach based on the estimation of two distinct sub-technologies, two convergence rates are considered for the efficiency assessment.

### 4 Data

In terms of the specification of the inputs and outputs, in this study, we employ the intermediation approach, which focuses on bank services. This approach was firstly introduced by Sealey and Lindley (1977) and has been used by many subsequent studies. In this approach, banks are viewed as intermediaries of financial services that purchase inputs to generate earning assets. Hence, in our models, we include three inputs: labor (number of full-time employees); capital (book value of fixed assets); and different funding sources (time deposits, saving deposits, and other borrowed funds). We then include two desirable outputs, loans and

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\(^{10}\) \( \hat{\delta} \) represents the Farrell output efficiency score.

\(^{11}\) See Bickel and Sakov (2008) for more detail on the algorithm.

\(^{12}\) Such infeasibility correction and its extension to the cases of the by-production approach and the WDA approach is one of the contributions of this paper.
other earning assets, and one undesirable output, NPLs. Data for the period 1998 to 2012 are available in the banks’ annual reports and are collected from the Central Bank of Iran archives (CBI 2005; 2006; 2008; 2012). We consider all banks operating in the Iranian banking industry except those that were not homogenous in input and output mixes or were too new/small to warrant inclusion (such as Post Bank, Taat, Mehr, Hekmat and Dey). The banks analyzed here represent at least 94 percent of the total loans provided by the banking industry in Iran in each year. Table 1 provides a summary of the descriptive statistics for the inputs and outputs used in the analysis. One can observe that the variables have high standard deviations and the median values in almost all years are smaller than their mean values, indicating that the data are skewed to the right. This is mainly due to the existence of some large government-owned banks such as National Bank (Bank Melli) in the sector. Most of the maximum values of different variables belong to this bank. However, there are also some very small figures reported in Table 1 that show the entry of one or more new private banks to the market. Table 1 also shows the increase of deposits, loans and NPLs over time. We note, however, that labor show high volatility and decreased during the period 1998 to 2012.

5 Empirical results and discussion
We have estimated one frontier for the entire period of time to prevent small sample issues. It is then assumed that all deficiencies are due to technical inefficiencies.13 The sector’s original and bias-corrected efficiency scores, which are estimated based on three different models (efficiency without bad output, by-production efficiency, and weak disposability efficiency), are presented in Table 2. The interpretation is straightforward: a bank is efficient when its score equals one; it is inefficient when this score is greater than one. Depending on the approach used to account for the presence of undesirable outputs, the results presented in Table 2 indicate different implications.

In comparison to the model in which the undesirable output is not taken into account, the weak disposability model indicates higher output efficiency scores, while the by-production model indicates higher levels of inefficiency for the banks over the sample period (Table 2). This result simply indicates the contrast between these models and is worth further discussion. The

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13 It is a strong assumption to maintain as lower efficiencies can be obtained in earlier periods in the case of technological progress. However, our choice of pooling the sample is the result of an arbitrage between discriminative efficiency scores and lack of this discriminatory power due to curse of dimensionality in small samples.
WDA approach states that reducing undesirable outputs is not costless to a firm and will also require decreasing the levels of good outputs by the same radial factor (Färe et al., 1996). Hence, the WDA results simply sketch the situation in which efficiency increases due to the introduction of additional constraints limiting the reference set attainable by inefficient banks (Table 2). However, as mentioned earlier, despite its popularity, this statement and the WDA approach’s ability to provide a good representation of undesirable output-generating technologies has been widely debated (Hailu and Veeman, 2001; Färe and Grosskopf, 2003). For instance, Murty et al. (2012) and Chen (2014) provide serious critiques and illustrative examples demonstrating that this approach does not yield the appropriate trade-offs between some inputs and the levels of bad outputs. Therefore, the by-production estimates, which capture the different trade-offs present in the production system more accurately, are considered to be appropriate alternatives to those from WDA in this study. Overall, our by-production results, presented in Table 2, show that the bank output-oriented efficiencies are generally lower compared to the case in which undesirable outputs are not considered, indicating that Iranian banks do not fully take advantage of the possible trade-offs between undesirable and good outputs. This issue will be discussed further later in the article.

Table 2 also reveals that the sector’s overall efficiency improves over time. For the sake of interpretation convenience, Figure 1 is provided depicting Table 2 mean efficiencies in graphical format. Figure 1 clearly shows that all of the models (with or without undesirable outputs) exhibit a decreasing tendency in the DEA scores over time. The scores are obtained based on non-radial output efficiencies; hence, the figure generally indicates an improvement in the technical efficiency of Iranian banks over the period of study. For instance, the estimated by-production efficiency line in Figure 1 shows that although the sector’s overall inefficiency showed peaks in 2000–2001 (coinciding with the entry of private banks), 2005 (the introduction of the banking reform) 15, and 2009 (when the ratio of government-owned commercial banks’ NPLs to total loans reached its maximum), the sector’s efficiency improved relatively over time, in particular after 2010. One may argue that the better performance of banks in 2011 and 2012 was mostly due to the growth in their deposits (coinciding with instability in the asset market and a sharp decline of GDP) and better control of NPLs as shown in Table 1.

14 The null-jointness property of both types of outputs is also assumed.
15 This finding is consistent with those of Arjomandi et al. (2011; 2012; 2014), who investigated the 2003–2008 period and found that banks’ technical efficiency fell considerably following the regulatory changes.
Table 3 displays the Risk Effect values (REs) associated with the WDA and by-production models. Table 3 exhibits a constant decreasing trend in by-production REs between 2001 and 2005 and below-unity by-production REs from 2004 to the end of the sample period. This finding indicates that the sector’s efficiency scores deteriorated when the undesirable output (the credit risk) was incorporated in the model. One could infer this as the banks’ inadequate control of credit risk in general and during the post-reform period in particular. As noted earlier, the WDA model indicated higher efficiencies over the sample period in comparison to the model that neglects the undesirable output. Hence, the WDA risk effects are greater than unity in all years. Apart from this unsurprising result, we can observe that the WDA REs also started to decline towards unity from 2002 and slightly deteriorated after 2006 (although they are still above unity). Therefore, based on the results obtained from both the by-production and the WDA REs, one may conclude that the banking sector’s credit risk increased after private banks gained access to the market and worsened relatively after the introduction of regulatory changes. Table 3 also shows that the Risk Effect values for government-owned and private banks are very close in most of the reported years under the by-production approach, particularly after 2006. This indicates that both groups have been affected nearly equally by the inclusion of NPLs in the model. The only exception is in 2006 (right after the regulatory changes), when the RE is 0.63 for government-owned banks but unity for private banks. This finding can be seen as a possible effect of the 2005 reforms on the government-owned banks, which obliged the banks to follow employment creation policies. But it does not provide a sufficient basis for us to conclude that the financial reforms affected the government-owned banks more negatively than the private banks as, for instance, in years 2008, 2011 and 2012, the government-owned banks show higher REs than those of private banks.

On the whole, the by-production results reported in Table 3 indicate that before the private banks were permitted entry into the market (before 2001), the management of credit risk was not a substantial issue in the sector. However, after 2001, most likely due to increased competition and the new banking reforms, banks did not perform very well in this regard. But again, the extent of the REs varies among the bank groups, and we cannot conclude which group has managed its credit risk better during the study period. It should be noted that, using the WDA approach provided in Table 3, one could reach a totally different conclusion with regard to the latter point: both bank groups have managed their credit risk very well (as REs
are greater than unity in all of the years) and private banks are consistently more efficient than their rivals in that sense.

[Table 3 about here]

6 Conclusion
This study has analyzed the efficiency of Iranian banks and their credit risk management using an innovative DEA by-production model covering the period 1998 to 2012. To demonstrate the effectiveness of our model, the by-production results are also compared with those of the WDA model. The two models showed different results for some aspects of bank efficiencies and risk effects. For instance, WDA findings indicated that both private and government-owned banks have managed their credit risk appropriately and that private banks were consistently more efficient than their competitors in terms of credit risk control, but we were not able to make the same inference using the results obtained under the by-production approach. Given the discussed theoretical drawbacks associated with the WDA model, we have argued that the WDA results can be misleading and have therefore mainly focused on the by-production approach-generated results to draw conclusions on the performance of Iranian banks.

Our findings tend to support the idea that although the banks’ efficiency has improved over time, credit risk has had a negative influence on their performance. In fact, our findings reveal that both private and government-owned banks’ credit risk performance became relatively poorer in the post-regulatory era. We could argue that political interference can be the most important problem in the Iranian banking system; the lack of independence of the central bank and other banks has generally resulted in the implementation of financial policies that pay inadequate attention to their impact on the sensitivity of financial markets and the current market structure. As a result, the ability of banks to manage their credit policies is limited.

Overall, based on the findings of this study, one can suggest that Iranian banks may need to improve their monitoring mechanisms to assess their loan risk more closely. Additionally, central-bank independence as well as limited government-regulatory power in the industry may be seen as an important means of boosting the efficiency and stability of the banking sector.
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*Note:* The number of observations is 230 and figures are in million Rials except for Labor.
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<td>2010</td>
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<td>9.94</td>
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<td>4.85</td>
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<td>13.92</td>
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<td>2.05</td>
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<td>8.63</td>
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<tr>
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<td>1.66</td>
<td>1.74</td>
<td>3.53</td>
<td>4.22</td>
</tr>
<tr>
<td>Global (all years)</td>
<td></td>
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<td>13.53</td>
<td>4.25</td>
<td>4.52</td>
<td>23.96</td>
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</table>

Table 3. Risk effect results

<table>
<thead>
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<th>Years</th>
<th>Risk effect associated with weak disposability</th>
<th>Risk effect associated with the by-production approach</th>
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<tr>
<td></td>
<td>Private banks</td>
<td>Government-owned banks</td>
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<tr>
<td>1998</td>
<td>-</td>
<td>4.85</td>
</tr>
<tr>
<td>1999</td>
<td>-</td>
<td>3.13</td>
</tr>
<tr>
<td>2000</td>
<td>-</td>
<td>3.08</td>
</tr>
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<td>5.46</td>
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<td>2002</td>
<td>6.59</td>
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<td>2003</td>
<td>4.66</td>
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<td>2004</td>
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<td>2005</td>
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<td>2009</td>
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<td>2011</td>
<td>1.63</td>
<td>1.25</td>
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<tr>
<td>2012</td>
<td>1.51</td>
<td>1.21</td>
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<tr>
<td>Average</td>
<td>3.20</td>
<td>2.03</td>
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Figure 1. Different model efficiency changes over time
Appendix A: Infeasibilities correction

Infeasibility appears for some evaluated DMUs not in the reference set, i.e., the \( m \) observations that are drawn from the original sample and serve as a benchmark to compute the efficiency scores. A correction of these infeasibilities is provided using the following algorithm:

1- Solve the following model for all DMUs that have infeasibility in their efficiency or for which the simplex does not converge:

\[
\text{Min } \beta^k = \frac{1}{s} \sum_{r=1}^{s} \beta^k_r + M \sum_{i=1}^{p} \delta^k_i
\]

Subject to

\[
\sum_{j=1}^{m} \lambda_j x_{ij} \leq x_{ik}(1 + \delta^k_i) \quad i = 1, ..., p
\]

\[
\sum_{j=1}^{m} \lambda_j y_{rj} \geq (1 - \beta^k_r)y_{rk} \quad r = 1, ..., s
\]

\[
\sum_{j=1}^{m} \lambda_j = 1
\]

\[
\lambda_j \geq 0 \quad (j = 1, ..., m)
\]

In this model, \( \beta^k_r \) and \( \delta^k_i \) evaluate the output excess and input shortfalls, respectively, of the DMU that is evaluated. The model assesses the super-efficiency of \( DMU_k \). \( M \) is a sufficiently large number that needs to be prescribed by the user (Cook et al. (2008) used \( M = 10^5 \)).

2- Denote \( I = [i \mid \delta^k_i > 0] \) as the number of inputs for which shortfalls exist. The “super” efficiency of \( DMU_k \) can be computed as:

\[
\frac{1}{\beta^k} = \frac{1}{s} \sum_{i=1}^{s} (1 + \delta^k_i) + \frac{1}{B^k}, \quad \text{where } B^k = \frac{1}{s} \sum_{r=1}^{s} (1 - \beta^k_r).
\]

3- If \( I \) is empty (this situation corresponds to the non-convergence of the simplex algorithm), the efficiency can be evaluated as:

\[
\frac{1}{\beta^k} = \frac{1}{B^k}
\]
For the by-production model, given its particular nature (two sub-technologies), we need to introduce two distinct input inefficiencies. The model to solve can be written as follows:

\[ \text{Min } \Gamma^k = \frac{1}{2} \left[ \frac{1}{s} \sum_{r=1}^{s} \beta_r^k + \frac{1}{u} \sum_{q=1}^{u} \beta_q^k \right] + M \sum_{i=1}^{p} \delta_{i}^{k^+} + M \sum_{i=1}^{p} \delta_{i}^{k^-} \]

**Subject to**

\[ \sum_{j=1}^{m} \lambda_j x_{ij} \leq x_{ik}(1 + \delta_{i}^{k^+}) \quad i = 1,...,p \]

\[ \sum_{j=1}^{m} \lambda_j y_{rj} \geq (1 - \beta_{r}^{k}) y_{rk} \quad r = 1,...,s \]

\[ \sum_{j=1}^{m} \lambda_j = 1 \]

\[ \sum_{j=1}^{m} \mu_j x_{ij} \geq x_{ik}(1 - \delta_{i}^{k^-}) \quad i = 1,...,p \]

\[ \sum_{j=1}^{m} \mu_j b_{qj} \leq (1 + \beta_{q}^{k}) b_{qk} \quad q = 1,...,u \]

\[ \sum_{j=1}^{m} \mu_j = 1 \]

\[ \lambda_j, \mu_j \geq 0 \quad (j = 1,...,m) \]

Following the previous developments, let \( I^+ = [i \mid \delta_{i}^{k^+} > 0] \) and \( I^- = [i \mid \delta_{i}^{k^-} > 0] \).

The respective “super” efficiency score associated with each type of output of \( DMU_k \) can then be derived. For good outputs, we have:

\[ \frac{1}{\hat{\varepsilon}^k} = \frac{1}{I^+} \sum_{i=1}^{p} (1 + \delta_{i}^{k^+}) + \frac{1}{E^k} \]

where \( E^k = \frac{1}{s} \sum_{r=1}^{s} (1 - \beta_{r}^{k}) \). For bad outputs, we have:

\[ \frac{1}{\hat{\omega}^k} = \frac{1}{I^-} \sum_{i=1}^{p} (1 - \delta_{i}^{k^-}) + (1 + \Omega^k) \]
where $\Omega^k = \frac{1}{u} \sum_{q=1}^u \frac{1}{(1+\beta_q^k)}$ and $\hat{\Gamma}^k = \frac{1}{2} [e^k + \hat{\omega}^k]$. 

For the WDA model, the following model needs to be solved:

$$\text{Min } \Gamma^k = \frac{1}{2} \left[ \frac{1}{s} \sum_{r=1}^s \beta_r^k + \frac{1}{u} \sum_{q=1}^u \beta_q^k \right] + M \sum_{i=1}^p \delta_i^k$$

Subject to

$$\sum_{j=1}^m (y_j + \tau_j)x_{ij} \leq x_{ik}(1 + \delta_i^k) \quad i = 1, \ldots, p$$

$$\sum_{j=1}^m y_{ij}y_{rj} \geq (1 - \beta_r^k)y_{rk} \quad r = 1, \ldots, s$$

$$\sum_{j=1}^m y_{ij}b_{qj} = (1 + \beta_q^k)b_{qk} \quad q = 1, \ldots, u$$

$$\sum_{j=1}^m (y_j + \tau_j) = 1$$

$y_j, \tau_j \geq 0 \ (j = 1, \ldots, m)$

Contrary to the by-production model, we have here only one technology (even if there are two intensity variables). Under the WDA assumption, the bad outputs are modeled as joint products under the same technology. A global super-efficiency score can be computed as follows:

$$\frac{1}{\hat{\Gamma}^k} = \frac{1}{T} \sum_{i=1}^p (1 + \delta_i^k) + \frac{1}{\Pi^k}$$

where $\Pi^k = \frac{1}{2} \left[ \frac{1}{s} \sum_{r=1}^s (1 - \beta_r^k) + \frac{1}{u} \sum_{q=1}^u (1 + \beta_q^k) \right]$. 

Taking the same notations used for the by-production, we can write $\Pi^k = \frac{1}{2} [E^k + \Omega^k]$. Then,

$$\frac{1}{\hat{\Gamma}^k} = \frac{1}{T} \sum_{i=1}^p (1 + \delta_i^k) + \frac{2}{E^k + \Omega^k}$$

$$\frac{1}{\hat{\Gamma}^k} = \frac{2 + (E^k + \Omega^k) \cdot \frac{1}{T} \sum_{i=1}^p (1 + \delta_i^k)}{E^k + \Omega^k}$$
Hence,
\[
\hat{\pi}^k = \frac{E^k + \Omega^k}{2 + (E^k + \Omega^k) \ast \frac{1}{T} \Sigma_{i=1}^{p} (1 + \delta_i^k)}
\]

The good output “super”-efficiency is obtained by:
\[
\hat{\varepsilon}^k = \frac{2 \ast E^k}{2 + (E^k + \Omega^k) \ast \frac{1}{T} \Sigma_{i=1}^{p} (1 + \delta_i^k)}
\]

The bad output “super”-efficiency is computed using the following:
\[
\hat{\omega}^k = \frac{2 \ast \Omega^k}{2 + (E^k + \Omega^k) \ast \frac{1}{T} \Sigma_{i=1}^{p} (1 + \delta_i^k)}
\]

The global “super”-efficiency can be retrieved by computing the arithmetic mean of the two previous performance scores:
\[
\hat{\pi}^k = \frac{1}{2} [\hat{\varepsilon}^k + \hat{\omega}^k].
\]
Acknowledgement: The authors are grateful to the anonymous referee for helpful comments and suggestions, which tremendously improved the quality and presentation of this article. Financial support from Curtin Business School under the Journal Publication Scheme (JPS) are also gratefully acknowledged. However, usual disclaimer applies.

References


