MERGA33 – 2010
Shaping the Future of Mathematics Education

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Editors
Len Sparrow, Barry Kissane, Chris Hurst
Preface

These collected papers are a record of the proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia entitled, *Shaping the Future of Mathematics Education*. The conference is held in Fremantle, as was the previous conference a decade earlier.

Keynote speakers are discussing issues that are emerging more sharply at the present time as the mathematics education community focus on the education of Indigenous children, the scope of the mathematics curriculum and ways in which the content might be taught effectively, and the significant role of technologies in teaching and learning in the twenty-first century.

Research papers are featured from mathematics educators from all states and territories of Australia, from colleagues in New Zealand, and from overseas — United Kingdom, Singapore, United States of America, India, Thailand, South Africa, and Indonesia. Some participants are not only new researchers but also new to the conference. Many have been aided by experienced colleagues via the supervision process within their university and also by the Early Bird system coordinated by MERGA and the Organising Committee. I wish to acknowledge the advice, comments, and suggestions given as part of the Early Bird system by experienced MERGA researchers. Their generosity in many ways repays the help and advice they received from the MERGA community earlier in their own careers.

All papers submitted for publication have been ‘double blind’ reviewed by at least two experienced researchers and writers organised into review groups for the purpose. In some cases a third person also reviewed the paper. Only papers that have been accepted by two reviewers are published in these proceedings. In the ‘double blind’ review process, neither the reviewer nor the author was named. Reviewers worked with a set of review guidelines produced by MERGA to work for consistency of outcome. My thanks are offered to the group coordinators, the reviewers, Emeritus Professor John Malone for his coordination of the process, and to the local Organising Committee.

This set of proceedings includes abstracts and full papers for refereed research presentations, short communications of developing research, roundtable discussions, and symposia. Thanks are due to the co-editors, Barry Kissan and Chris Hurst, for their diligence and support in editing and compiling these proceedings.

The conference has been organised by a group of mathematics educators from each of the five universities in Perth, the main education systems, and the Curriculum Council of Western Australia. There has been a wonderful sense of collaboration typical of mathematics educators in Western Australia. All has been achieved under the experienced and watchful eye of conference organiser, Dr. Jack Bana. My thanks and appreciation is offered to all.

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Chair, Conference Organising Committee
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Teachers' Perceptions of Geometry Instruction and the Learning Environment in Years 9-10 ESL Classrooms

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This paper describes the development of an instrument to assess teachers' views on their geometry instruction and their classroom learning environments in six government high schools in southwest Sydney. The sample consisted of 18 Years 9/10 ESL teachers from participating schools. The study involved completion of a survey form using a modified and expanded *What Is Happening In this Class (WIHIC)* questionnaire for teachers along with participant interviews. The findings indicated that there were positive associations between the learning environment and teachers' views on geometry instruction and the achievement of their classroom goals.

In studying geometry, students are encouraged by teachers to communicate their understanding of geometrical concepts and expressions using their own words, diagrams and the relationships between symbols and diagrams that form basic geometrical knowledge. English is a second language to many students in southwest Sydney schools, a majority of whom migrated from Asian and middle-eastern countries with their families in order to seek new opportunities in Australia. Consequently it is often difficult to teach geometry in this region of Sydney, although multicultural education is being used increasingly to provide a curriculum for the majority of migrant children, and this has been generally successful in enhancing their English-speaking ability while enabling them to retain and maintain their mother language.

This study was designed to examine how ESL teachers in Years 9-10 at six secondary schools located in the region view their geometry instruction and how the classroom learning environment (CLE) is influenced by, and also influences this experience. The need for an encouraging, positive CLE is regarded as of prime importance by ESL teachers in their efforts to assist their students in improving their achievement in geometry (Sperling, 2008; Wetzel, 2009). The study sought to assess teacher views on their CLE and to identify links between these perceptions and teacher success with geometry instruction.

The research questions of the study were:

1. Can an instrument based on a modified *WIHIC* questionnaire be developed and validated in order to assess teachers' perceptions of their geometry CLE?
2. What are the links between these perceptions and teachers' achievements in teaching geometry?

Literature Review

Previous research findings have shown that teachers affect students' learning and that the CLE and teacher differences also affect students' achievement (Fraser, 1994; Hill, Rowe, & Holmes-smith, 1995; Rawnsley, 1998). It has also been established that effectiveness in learning geometry is the result of the CLE, teacher influence on students' learning, and the quality of the teachers — factors that have special significance for those practitioners working with ESL students (Rawnsley, 1998; Sperling, 2008; Wetzel, 2009). Effectiveness in geometry teaching has been the subject of considerable theorising. Much research has investigated the validity of van Hiele's 1986 theory and has focused on teachers' emphasis on geometrical reasoning. Alternatively, Pusey (2003) considers that

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each of Piaget's five stages of development – the sensorimotor; iconic; concrete symbolic, formal, and post-formal stages – has an important role in learning geometry. Pusey claims that "the nature of students' development in their geometrical thinking happens over time as they grow older", the implication being that such development will be more prolonged for the ESL student and that the CLE plays an important role here (Pusey, 2003, p.4). Battista and Clement's (1995, p.425) recommendation that investigators use "a developmental sequence of reproducing geometrical figures focusing on memory; transformations involving rotation and visual perspective-taking" to examine children's actions and thoughts in the process of drawing shapes if they want them to organise spatial information in a meaningful way. This also has implications for the ESL student.

At the secondary school level in southwest Sydney, ESL teachers use instructional strategies for teaching geometry that involves drawing diagrams as well as guessing and matching words and geometrical figures; doing sample work on the blackboard and quizzes on paper to show step-by-step the explanations of mathematical problems; solving geometrical problems and brainstorming the meanings of key words and mathematical terminologies. In doing so they are utilizing the strategies proposed by Ding & Jones (2006) who make the point that teachers need to develop a sound pedagogy with considerable resources and activities if they are to improve their geometry teaching. The roles teachers play in adjusting to the interactions of the community through the process of following a curriculum and its associated cultures are most important (Tobin and Fraser, 1998), a sentiment that applies particularly to mathematics teachers dealing with ESL students. Often a lack of communication causes misunderstandings regarding students' behaviour, teachers' and students' interactions and geometrical instruction. Personal experience of the authors has shown that a positive, relaxed, supportive and focused CLE has a significant impact on these students.

One useful strategy used by Southwest Sydney teachers involves problem solving on related similar problems, and students are encouraged to group geometrical word problems into clusters for solving in accordance with a suggestion of Hinsley, Hayes, and Simons (1977). Such a successful pedagogical practice with a diverse classroom population helps to reinforce the ideas behind culturally relevant pedagogy being translated from theory into practice (Baker & Digiovanni, 2005). Successful teachers reflect upon classroom events to reconsider their own personal understandings of mathematics, and teaching mathematics (especially geometry) needs fluid and connected knowledge of mathematics (Bills, 1999).

Methodology

Instrumentation

In this study, a modified learning environment questionnaire that combined the WIHIC with items from another instrument – the My Classroom Inventory (MCI) – was administered to teachers. The instrument consisted of 54 items in nine scales. It measured teacher perceptions on nine scales of Student Cohesiveness and Satisfaction containing five items in each scale; Teacher support, Equity and Investigation containing six items in each scale; Task Orientation and Cooperation containing seven items in each scale; Involvement containing eight items; and Difficulty containing four items. The items in each scale were scored 1, 2, 3, 4 and 5 respectively for responses "almost never", "seldom", "sometimes", "often", and "almost always".

The 18 teachers (13 male and 5 females) were also interviewed. Classroom learning environment research has shifted from systematic observation to the use of a mixed
methodology involving quantitative and qualitative approaches to provide complementary perspectives on research problems (Rawnsley, 1998; Punch, 2000). The purpose of selecting this mixed-method approach was to enhance the quantitative component through the support of the qualitative data.

Data collection and analysis

All teachers completed the modified WIHIC questionnaire. Eight agreed to face-to-face interviews, and 10 responded to an interview form by correspondence. Interview methods have been used to investigate teachers' understanding of concepts in science and mathematics because they can reveal issues in students' thinking about these subjects as well as gauge their sensitivity to different teachers' ideas (Treagust, Duit, & Fraser, 1996). Each interview took approximately 10 to 15 minutes. The face-to-face teacher interviews were audio recorded and transferred to the computer for analyzing and backed up the findings of the quantitative data.

Results and Discussion

The index of internal consistency of the WIHIC instrument was measured by the Cronbach alpha coefficient (Cronbach, 1951) and the mean correlation with other scales was used as an index of discriminant validity by calculating the mean correlation of each scale with other scales toward teachers' perceptions of instruction in Years 9-10 geometry classrooms and their views of the CLE.

Internal Consistency Reliability of the Modified WIHIC Scales for Teachers

Data analysis of the inter-items correlation matrix in Table 1 and Figures 1 and 2 shows that the Cronbach alpha coefficient (α) of Satisfaction is 0.63 and the Cronbach Alpha value based on the standardised item (β) of Satisfaction is 0.62, which indicate that in the geometry class, teachers did not believe that students found their work too difficult, and were satisfied with the learning environment. Overall, Table 1 shows that the Difficulty scale values are negative both for the Cronbach alpha coefficient (α = -0.69) and Cronbach alpha value based on standardized items (β = -0.93). Teachers' perceptions are very strong in the Equity and the Investigation scales of the modified WIHIC based on standardized items (β = 0.94 and 0.91), and the Cronbach alpha (α = 0.94 and 0.91). The Cronbach alpha values of five scales (Student Cohesiveness, Teacher Support, Involvement, Task Orientation and Cooperation) are strongly positive (i.e. Student Cohesiveness = 0.74, Teacher Support = 0.89, Involvement = 0.83, Task Orientation = 0.77 and Cooperation = 0.86). Hence the results indicate that teachers create a strong positive influence in students' perceptions and attitudes towards geometry learning, thus suggesting a positive classroom environment.

Table 2 demonstrates that teachers' perceptions of teaching geometry are positive. The mean scores for all scales increased from 3.01 to 4.45 for the Teacher Support. The perceptions of teaching geometry in the classroom learning environments show a narrow standard deviation range of less than 1 (from 0.42 to 0.83). The Satisfaction scale indicates a mean of 3.40 and Standard Deviation of 0.57, showing that teachers' perceptions are very positive in the use of the modified WIHIC survey to differentiate between teachers' attitudes to teach geometry and the nature of the CLE. As an example, Teacher Support rates highly with a mean of 4.45.
Table 1.
Reliability Statistics of the modified WIHIC Scales for teachers

<table>
<thead>
<tr>
<th>WIHIC Scales</th>
<th>Cronbach’s Alpha (α)</th>
<th>Cronbach’s Alpha Based on Standardized Items (β)</th>
<th>No. of Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Cohesiveness</td>
<td>0.74</td>
<td>0.71</td>
<td>5</td>
</tr>
<tr>
<td>Teacher Support</td>
<td>0.89</td>
<td>0.89</td>
<td>6</td>
</tr>
<tr>
<td>Involvement</td>
<td>0.83</td>
<td>0.84</td>
<td>8</td>
</tr>
<tr>
<td>Investigation</td>
<td>0.91</td>
<td>0.91</td>
<td>6</td>
</tr>
<tr>
<td>Task Orientation</td>
<td>0.77</td>
<td>0.77</td>
<td>7</td>
</tr>
<tr>
<td>Cooperation</td>
<td>0.86</td>
<td>0.86</td>
<td>7</td>
</tr>
<tr>
<td>Equity</td>
<td>0.94</td>
<td>0.94</td>
<td>6</td>
</tr>
<tr>
<td>Satisfaction</td>
<td>0.63</td>
<td>0.62</td>
<td>5</td>
</tr>
<tr>
<td>Difficulty</td>
<td>-0.69</td>
<td>-0.93</td>
<td>4</td>
</tr>
</tbody>
</table>

** p < 0.01, the sample consisted of 18 Years 9 and 10 teachers in 16 classes

Figure 1. Alpha Reliability of the modified WIHIC scales for Teachers

Figure 2. Discriminant validity for teacher details recorded in Table1
Table 2.
Mean scores of modified WIHIC, Standard Deviation and Standard Error Mean for teachers

<table>
<thead>
<tr>
<th>WIHIC Scales</th>
<th>N</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Std. Error Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Cohesiveness</td>
<td>18</td>
<td>4.21</td>
<td>.47</td>
<td>.11</td>
</tr>
<tr>
<td>Teacher Support</td>
<td>18</td>
<td>4.45</td>
<td>.51</td>
<td>.12</td>
</tr>
<tr>
<td>Involvement</td>
<td>18</td>
<td>3.69</td>
<td>.56</td>
<td>.13</td>
</tr>
<tr>
<td>Investigation</td>
<td>18</td>
<td>3.21</td>
<td>.83</td>
<td>.20</td>
</tr>
<tr>
<td>Task Orientation</td>
<td>18</td>
<td>4.10</td>
<td>.52</td>
<td>.12</td>
</tr>
<tr>
<td>Cooperation</td>
<td>18</td>
<td>3.79</td>
<td>.67</td>
<td>.16</td>
</tr>
<tr>
<td>Equity</td>
<td>18</td>
<td>4.28</td>
<td>.81</td>
<td>.20</td>
</tr>
<tr>
<td>Satisfaction</td>
<td>18</td>
<td>3.40</td>
<td>.57</td>
<td>.13</td>
</tr>
<tr>
<td>Difficulty</td>
<td>18</td>
<td>3.01</td>
<td>.42</td>
<td>.10</td>
</tr>
</tbody>
</table>

Confirmation of Teachers’ Perceptions Toward Teaching Geometry Obtained through Interviews

The 18 teachers agreed to face-to-face interviews, and the responses of these teachers to the following four main questions provided information about their instructional strategies, roles and efforts to establish a positive CLE:

Q. 1: What strategies do you use to create a comfortable classroom environment for your students?
Q. 2: What teaching strategies do you use to control students who are not engaging in learning geometry in the mathematics classroom?
Q. 3: In what ways do you help students understand mathematics, especially geometry?
Q. 4: How do you know whether this class has achieved the goals you have set in learning geometry?

The majority of teachers’ responses to Question 1 indicated that their central intentions were to focus on the special needs of their students, and to have a sense of humour. They endeavoured to lead by example by preparing and organising their work and lesson preparation. Teachers used group discussions to promote understanding. Homework was recorded and records retained. The teachers generally considered that they developed a sense of responsibility among students by having them know the class rules and demanding respect for each other. They attempted to encourage students to work as a team and demonstrate proper behaviours, and they endeavoured to be tolerant. The latter quality was felt necessary to ensure understanding and to ensure good classroom management and positive attitudes among students.

Regarding Question 2, teachers used various strategies to control students who were not engaged in learning geometry. One teacher responded:

Well, I inform and remind students of their expectations and the importance of learning outcomes. I discipline students who misbehave when the class is learning. My work has to be prepared to an appropriate level for them to get on with (Teacher: T2; School: HS1).
Teachers used several strategies to control students who were not engaging in lessons by introducing group activities; using wait-time strategies questions (involving a definite pause between asking a question and requiring students to respond); seeking out reasons why students were not engaging; avoiding problems of lack of interest by making the lesson as interesting as possible, and providing feedback to students. Other strategies to counter misbehaviour involved using verbal warnings; having students work in isolation, lunchtime detention, and imposing school discipline policies:

I need to get students to work on task. To understand is in a student's interest and helps promoting interest in learning (Teacher: T3; School: HS5); and

To reprimand, make students responsible for their consequences. Isolate them to find out the reasons they misbehave; detention and letters home (Teacher: T9; School: HS3).

Responding to Question 3, teachers indicated that they helped students to understand the geometry, by using guided learning individually and by questioning and doing drill:

Individual help as requested; explanation with the class as a whole and individually as required, and examples are graded. Group work and paired work are used particularly for difficult examples by relating to practical problem in conjunction to systematic (structure) explanations in plain English. Practice lots of exercises, heavy use of examples and reinforcement using concrete materials, hand-on activity, various techniques. (Teacher: T1; School: HS1).

Answering Question 4, teachers said that they evaluated students by monitoring homework and class work; by observations and by asking questions to see if their classes achieved the goals they had for geometry. For example:

The first of students' work can tell me whether they understand or not. My role is to evaluate students by monitoring homework, class work, constant observations, tests, questioning, and listening to responses (Teacher: T1; School: HS1).

Teachers described their instructional strategies to assist students in geometry by explaining clearly and demonstrating a variety of techniques and new computer technology suitable for geometry teaching.

Table 1 and 2 demonstrate that the items of each scale in the survey support the instrument's internal consistency reliability and discriminant validity to distinguish between teachers' views towards their geometry teaching and the CLE. The findings of the qualitative component of the study (interviews) consistently supported the responses to the WIHIC questionnaire. The results confirm the validity and reliability of the WIHIC questionnaire that was used with the high school geometry teachers in Sydney. The Cronbach alpha coefficient (the internal consistency reliability) ranged from -0.69 to 0.94 and the discriminant validity ranged from -0.93 to 0.94. The results of the mean scores for all scales increased over the means from 3.01 to 4.45 and the simple correlation analyses from the nine scales of the modified WIHIC show that the associations between teachers' geometry instruction and their views towards their geometry CLE were statistically significant (p<0.01) regarding Satisfaction with learning geometry.

Accordingly, Research Question 1 can be answered in the affirmative. An instrument now exists to assist teachers' in assessing their geometry CLE.

In answering Research Question 2, the results of the study suggest that:

The links between teachers' perceptions of their CLE and their achievements in teaching geometry concern the need for teachers to address five areas (Teacher preparation, Teacher Support, Investigation, Cooperation and Equity) in order to enhance the achievement of students in geometry. These were the major components of a positive CLE. Teachers who emphasise these areas acknowledge the findings of other researchers
who suggest that a positive CLE is needed in addition to teaching skill (Bennett, 1988; Treaugust, Duit, & Fraser, 1996). Lessons began with revision to enforce understanding of the geometrical concepts taught earlier. Teachers usually controlled one third of the lesson time in any one period, spent approximately five to ten minutes for house-keeping, and then students spent the remaining time working independently. The teaching approach invariably utilised a traditional teaching method such as blackboard and chalk to display and explain concepts to students. In most schools, the period time for teaching was generally 45 minutes though some lessons were of 30 minutes duration.

Teachers mostly worked on explaining mathematical concepts, with much of their instruction relating to symbols. They considered that their lessons were generally well-structured and planned and took into account tasks and interactions aligned with the goals of geometry learning. The resources/materials that were employed in these lessons were often sparse, though the structure was adequate in providing sufficient time in the lesson for activities and for rounding off lessons with revision. During occasional observations of the teachers by the authors, they were professional in their approach, giving students' confidence in their ability to teach geometry, and their questioning strategies appeared to enhance the students' understanding and ability to solve problems. According to the teachers, all students interacted with them cooperatively and participated in the lessons, asking questions during lessons.

Students' Performances in Geometry

Many students learn geometry in the secondary school lacking the prior knowledge to do so successfully — a situation evident among the students involved in the present study. These students often solved geometrical problems well with visual but not with verbal cues. Geometrical terminology often caused confusion due to students' poor use of spoken English. The goals of geometry learning are to "develop thinking abilities as a foundation for the real world, and to convey the knowledge needed in geometry and to teach how to read and interpret mathematical arguments" (Board of Studies, New South Wales, 2002, p. 12). In the secondary school, geometry in the mathematical curriculum involves recognising and naming geometrical shapes, using the symbolism for geometrical concepts, developing skills with measurement and construction tools (i.e. compass, ruler and protractor), and using formulae in the measurement (Board of Studies, NSW, 2002). The goal in geometry education is for students to develop an understanding of the relevant concepts and communicate about quantities and unknown values through the use of signs, symbols, models, graphs, and mathematical terms. Students require a strong foundation in basic geometrical skills and they need to understand the meaning of mathematical contexts to assist their ability to discuss the subject purposefully.

Conclusion

Other suggestions from the teacher interviews which impact on learning and are realistic for ESL teachers to use in their classrooms are: (1) The use of posters in classrooms; (2) Allowing students to learn in a personally meaningful way, and maintain a positive and receptive climate for questions and answers; and (3) Ensuring that students are aware that they are accepted, respected and welcomed into each class. As a result, schools will have the chance of producing students who can achieve, who possess a sense of satisfaction with the school, and who have a positive perception of their classrooms (Young, 1998).
Outcomes of this study support the finding of Treagust, Duit, & Fraser (1996) who pointed out that using successful teaching methodologies to enhance the understanding of students’ learning mathematics will help students of all abilities to build onto their own knowledge of mathematics. Teaching methodologies are successful when students understand how to solve problems by applying the different aspects of concepts taught. Teachers need to accept students’ and colleagues’ ideas as central knowledge in mathematics in order to develop their own personal teaching approaches.

References


Tobin, K., & Fraser, B. J. (1998). Qualitative and quantitative landscapes of classroom learning environments. In B. J. Fraser & K. G. Tobin (Eds.), International Handbook of Science Education.

