

Effect of fracture fill on seismic attenuation and dispersion in fractured porous rocks

Liyun Kong,¹ Boris Gurevich,^{2,3} Tobias M. Müller,³ Yibo Wang⁴ and Huizhu Yang¹

¹Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China

²Department of Exploration Geophysics, Curtin University, GPO Box U1987, Perth, Western Australia 6845, Australia, E-mail: b.gurevich@curtin.edu.au

³CSIRO Earth Science and Resource Engineering, 26 Dick Perry Avenue, Kensington, Perth, Western Australia 6151, Australia

⁴Institute of Geology and Geophysics, Chinese Academy of Science, Beijing 100029, China

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SUMMARY

When a porous medium is permeated by open fractures, wave-induced flow between pores and fractures can cause significant attenuation and dispersion. Most studies of this phenomenon assume that pores and fractures are saturated with the same fluid. In some situations, particularly when a fluid such as water or carbon dioxide is injected into a tight hydrocarbon reservoir, fractures may be filled with a different fluid (with capillary forces preventing fluid mixing). Here we develop a model for wave propagation in a porous medium with aligned fractures where pores and fractures are filled with different fluids. The fractured medium is modelled as a periodic system of alternating layers of two types: thick porous layers representing the background, and very thin and highly compliant porous layers representing fractures. A dispersion equation for the P -wave propagating through a layered porous medium is derived using Biot's theory of wave propagation in periodically stratified poroelastic media. By taking the limit of zero thickness and zero normal stiffness of the thin layers, we obtain expressions for dispersion and attenuation of the P waves. The results show that in the low-frequency limit the elastic properties of such a medium can be described by Gassmann's equation with a composite fluid, whose bulk modulus is a harmonic (Wood) average of the moduli of the two fluids. The dispersion is relatively large when the fluid in both pores and fractures is liquid, and also when the pores are filled with a liquid but fractures are filled with a highly compressible gas. An intermediate case exists where the fluid in the pores is liquid while the fluid in the fractures has a bulk modulus between those of liquid and gas. In this intermediate case no dispersion is observed. This can be explained by observing that for uniform saturation, wave-induced compression causes flow from fractures into pores due to the high compliance of the fractures. Conversely, when pores are filled with a liquid but fractures are filled with gas, the fluid will flow from pores into fractures due to the high compressibility of gas. Thus, there exists an intermediate fracture fluid compressibility such that no flow can be induced and hence no dispersion or attenuation is observed.

Key words: Fracture and flow; Seismic anisotropy; Seismic attenuation.

1 INTRODUCTION

Wave-induced oscillatory flow of pore fluid is widely believed to be the main cause of attenuation and dispersion of elastic waves in porous rocks. When an elastic wave propagates through a fluid-saturated medium, it creates local pressure gradients within the fluid phase, resulting in fluid flow and corresponding internal friction until the pore pressure is equilibrated. The fluid flow can take place on various length scales. In particular, flow that occurs due to spatial variations of rock or fluid properties at the mesoscopic scale (larger than the pore size but smaller than the wavelength) is believed to be significant at seismic frequencies (White *et al.* 1975; Gurevich & Lopatnikov 1995; Pride *et al.* 2003; Müller *et al.* 2010).

The magnitude of attenuation and dispersion caused by mesoscopic wave-induced flow is proportional to the squared contrast (variance) of spatial variations of rock or fluid properties. Thus attenuation and dispersion are only significant if the contrast of spatial variations is large. In recent years, two situations with large contrast in rock/fluid properties have been identified: partial saturation and fractured rock. Partial saturation refers to the situation where a rock is saturated with a mixture of two immiscible fluids with a large difference between their

properties (say, liquid and gas). When an elastic wave propagates through such a rock, the patches of rock saturated with gas and liquid will deform differently, resulting in pressure gradients and fluid flow (White 1975; Gist 1994; Johnson 2001; Pride *et al.* 2004; Toms *et al.* 2007). Fractured rock refers to a situation where a porous rock is permeated by open fractures forming a fractured reservoir, such as a fractured carbonate, tight sandstone or a gas shale. When a wave propagates through such a rock, fractures will deform to a greater extent than the porous background, resulting in fluid flow between pores and fractures (Hudson *et al.* 2001; Chapman 2003; Jakobsen *et al.* 2003; Brajanovski *et al.* 2005; Galvin & Gurevich 2009; Gurevich *et al.* 2009). These situations (partial saturation and fractures) are usually treated separately: analysis of wave propagation in a partially saturated rock usually ignores variations in elastic properties of the solid frame (skeleton), while the porous rock permeated by fractures is usually assumed to be saturated with a uniform fluid. A simple superposition model of the two relevant configurations of wave-induced attenuation (patchy saturation and fractures) was proposed by Brajanovski *et al.* (2010). This model is, however, limited by the assumption that the medium is uniformly saturated with some fluid patches away from the fractures.

In some situations, particularly when a fluid such as water or carbon dioxide is injected into a tight hydrocarbon reservoir, fractures may be filled with a different fluid. In this paper, we consider the simplest situation of this kind: a porous rock saturated with one fluid and permeated by a single set of aligned planar fractures filled with another fluid. For such a medium, we derive a dispersion equation following a method originally proposed by Brajanovski *et al.* (2005) for a porous fractured medium saturated with a single fluid.

The paper is organized as follows. First, we review the theory for the case of a medium saturated with a single fluid. Then we extend the method to the situation where the porous background and fractures are saturated with different fluids and derive the corresponding dispersion equation, which yields expressions for dispersion and attenuation due to wave induced flow between pores and fractures. To explore the behaviour of attenuation and dispersion, we explore various limiting cases and present several numerical examples. Finally, we discuss the physical nature of the results obtained.

2 LIQUID SATURATED POROUS AND FRACTURED MEDIUM

Brajanovski *et al.* (2005) developed a model for a porous medium with aligned fractures. The medium comprises a periodic (with spatial period H) stratified system of alternating layers: relatively thick layers of a background material (with a finite porosity ϕ_b) and relatively thin layers of a high-porosity ($\phi_c \rightarrow 1$) material representing the fractures. This double porosity model is a limiting case of a periodically layered poroelastic medium studied by White *et al.* (1975) and Norris (1993). Norris (1993) showed that for frequencies much smaller than Biot's characteristic frequency $\omega_B = \eta\phi/k\rho_f$ (Biot 1962), and also much smaller than the resonant frequency of the layering $\omega_R = V_p/H$, the compressional wave modulus of a periodically layered fluid-saturated porous medium composed of two constituents, b and c, can be written in the form:

$$\frac{1}{C_{33}} = \frac{h_b}{C_b} + \frac{h_c}{C_c} + \frac{\left(\frac{\alpha_b M_b}{C_b} - \frac{\alpha_c M_c}{C_c}\right)^2}{\sqrt{\frac{i\omega\eta_b M_b L_b}{C_b k_b} \frac{H}{2}} \cot\left(\sqrt{\frac{i\omega\eta_b C_b}{k_b M_b L_b} \frac{h_b H}{2}}\right) + \sqrt{\frac{i\omega\eta_c M_c L_c}{k_c C_c} \frac{H}{2}} \cot\left(\sqrt{\frac{i\omega\eta_c C_c}{k_c M_c L_c} \frac{h_c H}{2}}\right)}. \quad (1)$$

In eq. (1), both constituents are assumed to be made of the same isotropic grain material with bulk modulus K_g , shear modulus μ_g and density ρ_g , but they have different solid frame parameters: porosity ϕ , permeability k , dry bulk modulus K , shear modulus μ and thickness fraction h . The layers b and c may be saturated with different fluids with bulk modulus K_f , density ρ_f and dynamic viscosity η , as indicated by adding 'b' and 'c' in the subscript. Parameter C_j denotes the fluid-saturated P -wave modulus of layer j given by Gassmann's equation (Gassmann 1951; White 1983):

$$C_j = L_j + \alpha_j^2 M_j, \quad (2)$$

where

$$\alpha_j = 1 - \frac{K_j}{K_g} \quad (3)$$

is Biot's effective stress coefficient, M_j is pore space modulus

$$\frac{1}{M_j} = \frac{\alpha_j - \phi_j}{K_g} + \frac{\phi_j}{K_{fj}}. \quad (4)$$

and

$$L_j = K_j + 4\mu_j/3 \quad (5)$$

is the dry P -wave modulus of layer j .

To construct a model for a porous medium permeated by parallel fractures, Brajanovski *et al.* (2005) considered parameters with subscript 'b' to represent the porous background, and parameters with subscript 'c' to represent fractures (cracks). They then assumed fractures to be very thin and very compliant layers so that the volume fraction of the fractures (fracture porosity) h_c and bulk and shear moduli of the dry frame of the fracture material are small, $h_c \ll 1$, $K_c \ll K_g$ and $\mu_c \ll K_g$ such that both K_c/K_g and μ_c/K_g (and hence L_c/K_g) are $O(h_c)$. By

assuming that both pores and fractures are saturated with the same fluid with the viscosity $\eta_b = \eta_c = \eta$, Brajanovski *et al.* (2005) obtained the equation

$$\frac{1}{C_{33}} = \frac{1}{C_b} + \frac{\left(\frac{\alpha_b M_b}{C_b} - 1\right)^2}{\sqrt{\frac{i\omega\eta M_b L_b}{C_b k_b} \frac{H}{2}} \cot\left(\sqrt{\frac{i\omega\eta C_b}{k_b M_b L_b} \frac{h_b H}{2}}\right) + \frac{1}{Z_N}}, \quad (6)$$

where Z_N is the normal excess fracture compliance of the dry frame defined by (Schoenberg 1980; Schoenberg & Douma 1988)

$$Z_N = \lim_{h_c \rightarrow 0} \frac{h_c}{L_c}. \quad (7)$$

In the derivation of eq. (6) Brajanovski *et al.* (2005) stated that if $h_c \ll 1$, $K_c \ll K_g$ and $\mu_c \ll K_g$, then $C_c/M_c \rightarrow 1$. More precisely, when $K_c \ll K_g$ and $\phi_c \rightarrow 1$, eqs (2)–(5) give

$$\alpha_c \rightarrow 1 \quad (8)$$

$$M_c \rightarrow K_{fc}, \quad (9)$$

and

$$C_c \rightarrow L_c + K_{fc}. \quad (10)$$

Thus $C_c/M_c \rightarrow 1$ if and only if $K_{fc} \gg L_c$. Therefore, implicit in the derivation of eq. (6) was the assumption

$$K_f \gg L_c, \quad (11)$$

which means that the fluid compressibility should not be too high (e.g. it might not be applicable if the fluid is gas). This assumption of Brajanovski *et al.* (2005) was not unreasonable since they considered only the case of the same fluid in the pores and fractures. As will be shown later in this paper, when the fluid in pores and fractures is the same, high fluid compressibility yields no dispersion or attenuation and thus is not of much interest. Using definition (7), condition (11) can be written in the form

$$K_f \gg h_c/Z_N. \quad (12)$$

When both pores and fractures are dry, the model represented by eq. (6) is equivalent to a transversely isotropic dry elastic porous material with linear-slip interfaces. When the medium is saturated with a liquid, it exhibits significant attenuation and velocity dispersion. However the model is limited to the case where the fluid is the same in both matrix pores and fractures, and there is an upper limit on the fluid compressibility. Below we develop a model that overcomes these limitations.

3 ARBITRARY FLUID IN THE FRACTURES

We now analyse eq. (1) in the case $h_c \ll 1$, $\phi_c \rightarrow 1$, $K_c/K_g = O(h_c)$, $\mu_c/K_g = O(h_c)$ and $L_c/K_g = O(h_c)$ without imposing restrictions on the fluid modulus. Combining eqs (8)–(10) with the parametrization (6), we can obtain approximations for some terms in eq. (1), which are:

$$\frac{h_c}{C_c} \approx \frac{Z_N}{1 + K_{fc}/L_c}, \quad (13)$$

$$\frac{\alpha_c M_c}{C_c} \approx \frac{K_{fc}/L_c}{1 + K_{fc}/L_c}, \quad (14)$$

$$\frac{M_c L_c}{C_c} \approx \frac{K_{fc}}{1 + K_{fc}/L_c}. \quad (15)$$

And

$$\sqrt{\frac{\eta_c C_c}{M_c L_c}} \approx \sqrt{\eta_c \frac{1 + K_{fc}/L_c}{K_{fc}}}. \quad (16)$$

Furthermore, $\cot(x) \approx 1/x$ for any complex x such that $|x| \ll 1$ and $\cot(\sqrt{i x}) \rightarrow -i$ for any real $x \gg 1$. Thus, substituting eqs (13)–(16) into eq. (1) in the limit $h_c \rightarrow 0$ yields

$$\frac{1}{C_{33}} = \frac{1}{C_b} + \frac{Z_N}{1 + K_{fc}/L_c} + \frac{\left(\frac{\alpha_b M_b}{C_b} - \frac{K_{fc}/L_c}{1 + K_{fc}/L_c}\right)^2}{\sqrt{\frac{i\omega\eta_b M_b L_b}{C_b k_b} \frac{H}{2}} \cot\left(\sqrt{\frac{i\omega\eta_b C_b}{k_b M_b L_b} \frac{H}{2}}\right) + \frac{K_{fc}/h_c}{1 + K_{fc}/L_c}}. \quad (17)$$

Eq. (17) is the approximation of eq. (1) for a porous and fractured medium with an arbitrary fracture fill. The main feature of eq. (17) in comparison with eq. (6) is that fracture porosity h_c cannot be eliminated without further information on the bulk modulus K_{fc} of the fracture

fluid. Further simplifications can be made if such information is known. Additionally, we can define low and high frequencies with respect to fluid flow between fractures and background. In the following section we derive and analyse some limiting cases of fluid compressibility and frequency.

4 LIMITING CASES

4.1 Fluid limits

4.1.1 Liquid in fractures

If the fluid in the fractures is liquid, then its bulk modulus can be assumed to be about one order of magnitude smaller than the mineral modulus K_g . If we assume that fracture porosity h_c is on the order of 10^{-3} (or smaller), then it follows that $K_{fc} \gg h_c K_g$ and thus $K_{fc} \gg L_c$. Then eq. (17) gives

$$\frac{1}{C_{33}} = \frac{1}{C_b} + \frac{\left(\frac{\alpha_b M_b}{C_b} - 1\right)^2}{\sqrt{\frac{i\omega\eta_b M_b L_b}{C_b k_b} \frac{H}{2}} \cot\left(\sqrt{\frac{i\omega\eta_b C_b}{k_b M_b L_b} \frac{H}{2}}\right) + \frac{1}{Z_N}}. \quad (18)$$

Note that eq. (18) is exactly the same as eq. (6). This shows that the result of Brajanovski *et al.* (2005) is valid not only if both pores and fractures are saturated with the same liquid, but also when the two fluids are different as long as both are liquids, or, more precisely, satisfy condition (11) (or eq. 12).

4.1.2 Dry or nearly dry fractures

When fractures are dry or nearly dry ($K_{fc} \rightarrow 0$), eq. (17) gives

$$\frac{1}{C_{33}} = \frac{1}{C_b} + Z_N + \frac{\left(\frac{\alpha_b M_b}{C_b}\right)^2}{\sqrt{\frac{i\omega\eta_b M_b L_b}{C_b k_b} \frac{H}{2}} \cot\left(\sqrt{\frac{i\omega\eta_b C_b}{k_b M_b L_b} \frac{H}{2}}\right)}. \quad (19)$$

Eq. (19) gives the P -wave modulus for a porous medium with dry or gas-filled fractures, and it is quite different from eq. (6) for the liquid situation. To further analyse the reason for the difference, we will now derive the limiting cases of low and high frequencies.

4.2 Frequency limits

4.2.1 Low frequencies

In the low-frequency limit $\omega \rightarrow 0$, the cotangent function in eq. (17) can be replaced by the inverse of its argument. Thus, eq. (17) is reduced to

$$\frac{1}{C_{33}} = \frac{1}{C_b} + \frac{Z_N}{1 + K_{fc}/L_c} + \frac{\left(\frac{\alpha_b M_b}{C_b} - \frac{K_{fc}/L_c}{1 + K_{fc}/L_c}\right)^2}{\frac{M_b L_b}{C_b} + \frac{K_{fc}/h_c}{1 + K_{fc}/L_c}}. \quad (20)$$

In the low frequency limit, the fluid pressure should be fully equilibrated between pores and fractures. Thus in this limit the result must be consistent with the anisotropic Gassmann equations for a fractured medium saturated with a single composite fluid (see e.g. Gurevich 2003). The bulk modulus of this composite fluid should be the harmonic average of the moduli of the two fluids, weighted by their volume fractions S_{fc} and S_{fb} ,

$$\frac{1}{K_f^*} = \frac{S_{fb}}{K_{fb}} + \frac{S_{fc}}{K_{fc}}. \quad (21)$$

Eq. (21) is known as the Wood equation, and corresponds to so-called uniform saturation of the partial saturation theory (Mavko *et al.* 1998; Johnson 2001; Toms *et al.* 2007). In this theory, the low frequency (or uniform saturation) modulus is known as the Gassmann–Wood limit. However, in our case the solid frame of the medium is anisotropic, and thus the anisotropic Gassmann equation must be used (Gassmann 1951; Brown & Korrington 1975; Gurevich 2003). This equation can be obtained, for instance, by taking the low frequency limit of eq. (6). Thus, if we replace K_b with K_f^* in eq. (6), and then take the low frequency limit, we should get the same expression as given by eq. (20).

In a periodically stratified porous and fractured medium, saturations S_{fc} and S_{fb} are defined by

$$S_{fc} = \frac{h_c \phi_c}{(1 - h_c) \phi_b + h_c \phi_c} \quad (22)$$

and

$$S_{fb} = \frac{(1 - h_c) \phi_b}{(1 - h_c) \phi_b + h_c \phi_c}, \quad (23)$$

respectively. Thus, in the limit $h_c \rightarrow 0$, and $\phi_c \rightarrow 1$, we have $S_{fc} \rightarrow h_c/\phi_b$ and $S_{fb} \rightarrow 1$. Then, the Wood eq. (21) can be written as

$$\frac{1}{K_f^*} = \frac{1}{K_b} + \frac{h_c}{K_{fc} \phi_b}. \quad (24)$$

Substituting the effective fluid bulk modulus given by (24) into the definition of M_b (4), and then substituting the result into eq. (6) in the low frequency limit, we obtain

$$\frac{1}{C_{33}} = \frac{1}{C_b} + \frac{1}{\frac{1}{Z_N} + \frac{K_{fc}}{h_c}} + \frac{\left(\frac{\alpha_b M_b}{C_b} - \frac{\frac{K_{fc}}{h_c}}{\frac{1}{Z_N} + \frac{K_{fc}}{h_c}} \right)^2}{\frac{M_b L_b}{C_b} + \frac{\frac{K_{fc}}{h_c}}{1 + Z_N \frac{K_{fc}}{h_c}}}. \quad (25)$$

Note that eq. (25) is equivalent to (20).

To clarify the physical meaning of the low-frequency eq. (20) we again consider liquid and dry (or gas) cases. For liquid-filled fractures (large K_{fc}), we have

$$\frac{1}{C_{33}} = \frac{1}{C_b} + \frac{\left(\frac{\alpha_b M_b}{C_b} - 1 \right)^2}{\frac{M_b L_b}{C_b} + \frac{1}{Z_N}} \quad (26)$$

while for $K_{fc} \rightarrow 0$

$$\frac{1}{C_{33}} = \frac{1}{L_b} + Z_N. \quad (27)$$

The liquid limit, eq. (26), corresponds exactly to the low frequency limit of the result of Brajanovski *et al.* (2005), with only the bulk modulus of the liquid in the pores affecting the overall modulus. This result may be understood from eq. (24), which shows that when the bulk moduli of the two fluids are comparable, the effect of the fracture fluid is negligible. In turn, eq. (27) is exactly the modulus of the dry medium (Schoenberg & Douma 1988; Brajanovski *et al.* 2005). This is because when K_{fc} is very small (much smaller than $h_c K_b$), Wood eq. (24) for the effective fluid modulus reduces to

$$\frac{1}{K_f^*} \approx \frac{h_c}{K_{fc} \phi_b} \quad (28)$$

and thus $K_f^* \rightarrow 0$, which means that the whole porous and fractured model can be treated as a dry or nearly dry medium. Physically, this is a consequence of the fact that at low frequencies, the pore pressure is equilibrated between pores and fractures, and when the fractures are dry (or nearly dry), the pressure in the fractures is zero, and thus is also zero in the pores. This is the drained—or dry—limit.

4.2.2 High frequencies

In the high-frequency limit $\omega \rightarrow \infty$, the cotangent function in eq. (17) can be replaced by $-i$. So, we can get the expression for the P -wave modulus at high frequencies

$$\frac{1}{C_{33}} = \frac{1}{C_b} + \frac{Z_N}{1 + K_{fc}/L_c}. \quad (29)$$

In liquid and gas cases we have

$$\frac{1}{C_{33}} = \frac{1}{C_b} \quad (30)$$

and

$$\frac{1}{C_{33}} = \frac{1}{C_b} + Z_N, \quad (31)$$

respectively.

Note that at high frequencies, the fluid pressure does not have time to equilibrate between pores and fractures, and thus they can be considered ‘hydraulically isolated’ (Gurevich 2003; Brajanovski *et al.* 2005). Thus the P -wave modulus in this limit corresponds to the modulus of a medium with isolated fractures. In the liquid case, the modulus given by eq. (30) is the same as if there were no fractures. This is because liquid can stiffen the otherwise very compliant fractures so that P -wave velocities for waves propagating parallel and perpendicular to layering are both approximately equal to the modulus of the background medium (Hudson 1980; Schoenberg & Douma 1988; Thomsen 1995; Brajanovski *et al.* 2005). Conversely, when fractures are dry, the P -wave modulus (31) is the same as for a medium with dry isolated fractures (*cf.* eq. 27).

5 NUMERICAL EXAMPLES

Our main result, eq. (17), shows that regardless of fluid saturation of fractures, the P -wave modulus is complex-valued and frequency dependent. This means that waves will have attenuation and dispersion.

To explore these effects, we compute the complex velocity in the direction normal to the fractures using the standard formula

$$V_p = \sqrt{\frac{C_{33}}{\rho_b}}, \quad (32)$$

where $\rho_b = (1 - \phi_b)\rho_g + \phi_b\rho_{fb}$ is the mass density of the fluid-saturated background (the effect of fractures on the density can be ignored as their volume fraction is negligibly small). Eq. (32) can be used to evaluate the frequency dependence of the P -wave phase velocity and attenuation for waves propagating perpendicular to fractures. The real part of the complex velocity is the P -wave phase speed, and twice the ratio of the imaginary to the real part of the complex phase velocity is the dimensionless attenuation (inverse quality factor) Q^{-1} .

Now, we rewrite eq. (17) as a function of normalized frequency Ω (Brajanovski *et al.* 2005)

$$\frac{1}{C_{33}} = \frac{h_b}{C_b} + \frac{1}{\frac{L_b(1-\delta_N)}{\delta_N} + K_{fc}/h_c} + \frac{\left(\frac{\alpha_b M_b}{C_b} - \frac{K_{fc}/h_c}{\frac{L_b(1-\delta_N)}{\delta_N} + K_{fc}/h_c}\right)^2}{L_b \sqrt{i\Omega} \cot\left(\frac{h_b C_b}{M_b} \sqrt{i\Omega}\right) + \frac{K_{fc}/h_c}{1 + \frac{\delta_N}{L_b(1-\delta_N)} K_{fc}/h_c}}, \quad (33)$$

where Ω is the normalized frequency and is defined by

$$\Omega = \frac{\omega H^2 \eta_b M_b}{4 C_b k_b L_b}. \quad (34)$$

and δ_N is a dimensionless (normalized) fracture weakness with values between 0 and 1 (Schoenberg & Douma 1988)

$$\delta_N = \frac{Z_N L_b}{1 + Z_N L_b}. \quad (35)$$

All our calculations are made for a water-saturated sandstone using quartz as the grain material ($K_g = 37$ GPa, $\rho_g = 2.65 \times 10^3$ kg m⁻³). The dependency of bulk and shear moduli of the dry background on porosity was assumed to follow the empirical model of Krief *et al.* (1990)

$$\frac{K_b}{K_g} = (1 - \phi_b)^{3/(1-\phi_b)}. \quad (36)$$

To explore the validity of our approximations, we compare the attenuation and dispersion results with the original Norris (1993) model, eq. (1). For the Norris model, we set fracture parameters to satisfy the assumptions of the approximation ($h_c = 0.001$, $\phi_c = 0.999$, $\delta_N = 0.2$, $k_c = 200$ mD). Viscosity η_{fc} of the fracture fluid is assumed to vary linearly with its compressibility. Then, the P -wave speeds and inverse quality factor Q^{-1} are calculated for different values of the compressibility of the fracture fluid. The results are shown in Fig. 1.

Fig. 1 shows phase velocity and attenuation of P waves propagating along the symmetry axis (normal to fracture plane) for different values of a dimensionless parameter $F = K_{fc}/L_c$. Symbols show the values obtained by our approximation, eq. (17), while the curves correspond to the Norris (1993) general solution, eq. (1), computed using parameter values specified above. We see that for the whole range of parameter F , the approximation is accurate. Curves of dispersion and attenuation have a shape typical for a relaxation phenomenon. It is interesting that dispersion and attenuation is significant for both liquid-filled ($F = 100$, which corresponds to $K_{fc} = 16$ GPa) and nearly dry ($F = 0.001$) fractures, but is much lower for intermediate values of the fracture fluid compressibility. This somewhat surprising observation can be explained as follows. When both pores and fractures are filled with liquids, the compression caused by the wave will compress the fractures to a much greater extent than the background porous material (since fractures are much more compliant than pores), causing the fluid to flow from fractures into pores. Conversely, when the pores are saturated with a liquid and the fractures are dry (or filled with very compressible gas), the compression will cause the flow from pores into fractures (it will be easier to compress gas than deform the fractures). Thus, at some intermediate value of the fluid compressibility, there will be no flow at all, and hence no dispersion or attenuation. The physics of these processes is further analysed in the next section.

Fig. 2 shows the dispersion magnitude (difference between high- and low-frequency velocities) as a function of F . As F increases, the dispersion first decreases, drops to zero, and then increases again. We also see that the dispersion is almost insensitive to F both for very small and very large values of F (corresponding to highly compressible gases and liquids, respectively), but quite sensitive to F for values of F in a range around the critical value where dispersion reaches zero. This critical value can be obtained by equating low- and high-frequency asymptotes. This gives

$$F^* = \frac{\alpha_b M_b}{C_b - \alpha_b M_b}. \quad (37)$$

The value F^* given by eq. (37) corresponds to a critical fracture fluid modulus case, where there will be zero dispersion and attenuation in the general porous and fractured model. For the parameters used in the numerical example of Fig. 1, eq. (37) gives $F^* \approx 0.132$. This value is of the same order as 0.1, and thus we see very small dispersion and attenuation for $F = 0.1$.

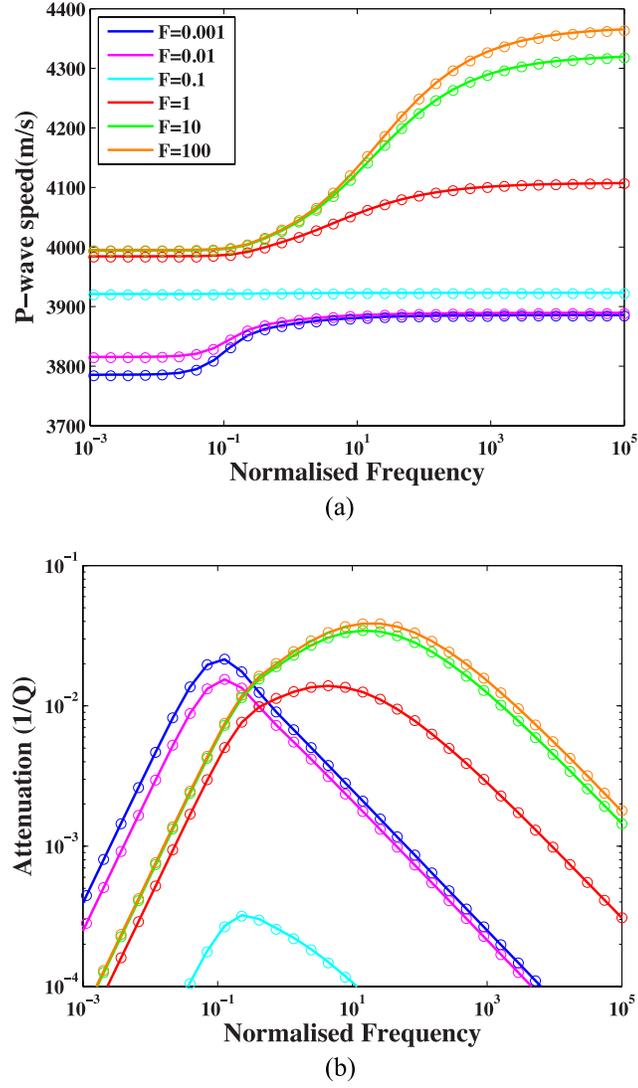


Figure 1. Frequency dependency of P -wave velocity (a) and inverse quality factor Q^{-1} (b) computed using our approximation (17) and Norris (1993) model for different values of the parameter F .

6 DISCUSSION

According to Brajanovski *et al.* (2006), attenuation due to wave-induced pressure diffusion in fractured porous media entails three possible asymptotic frequency regimes. In order to analyse the asymptotic behaviour of the inverse quality factor we follow the approach of Brajanovski *et al.* (2006). The asymptotes at low, intermediate and high frequencies can be easily computed from eq. (17) to yield $Q_{\text{Low}}^{-1} \propto \omega$, $Q_{\text{Inter}}^{-1} \propto \omega^{1/2}$ and $Q_{\text{High}}^{-1} \propto \omega^{-1/2}$. By computing the intersections of these asymptotes, we obtain estimates of the corresponding cross-over characteristic frequencies. The cross-over frequency for the transition from the low to intermediate frequency regimes is

$$\omega_p = \frac{18D_b}{H^2} \left(1 + \frac{N_b \delta_N}{FL_b} \right)^4. \quad (38)$$

For the frequency of the maximum we obtain

$$\omega_M = 4 \left(\frac{C_b}{M_b} \right)^2 \delta_N^{-2} \frac{D_b}{H^2} \left(\frac{F}{1+F} \right)^2, \quad (39)$$

where the background diffusivity is $D_b = \frac{k_b N_b}{\eta_b}$ and $N_b = \frac{M_b L_b}{C_b}$.

By taking $F \rightarrow \infty$ in eqs (38) and (39) we recover two characteristic frequencies in the liquid limiting case as obtained by Brajanovski *et al.* (2006, their eqs (11) and (14)). All the three asymptotes and two characteristic frequencies (points P and M) are displayed in Fig. 3. Müller and Rothert (2006) relate these frequency regimes to the associated fluid fluxes $\frac{k_i}{\eta_i} \nabla p$ (where p denotes the fluid pressure). When fractures and pores are saturated with the same liquid as well as for large values of F , there is a flux inside the fracture (outgoing flux $\frac{k_e}{\eta} \nabla p$)

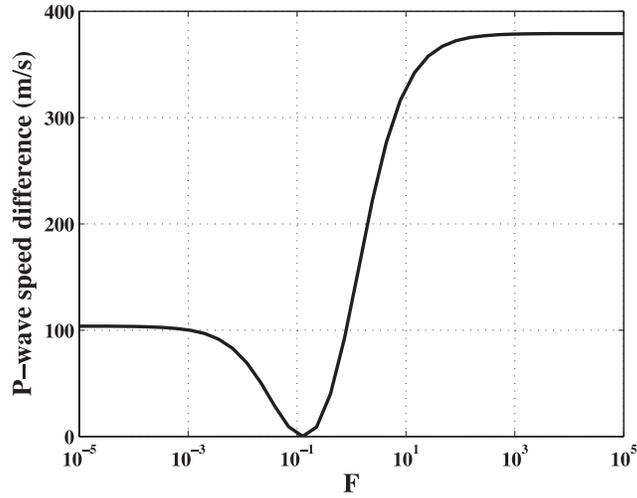


Figure 2. Dispersion magnitude (difference between high- and low-frequency velocities) as a function for parameter F . As F increases, the dispersion first decreases, reaches zero, and then increases again.

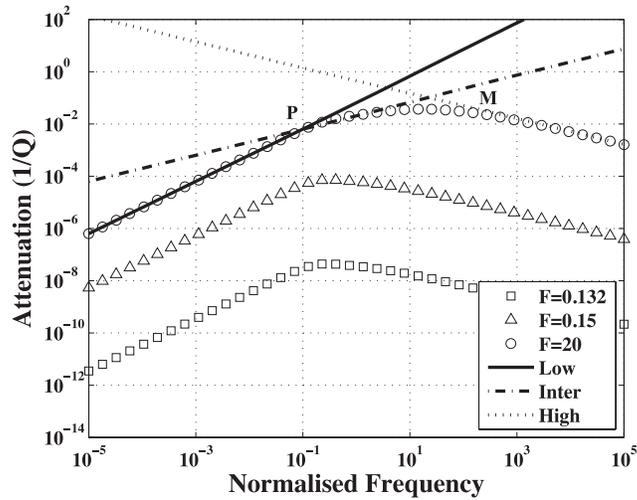


Figure 3. Asymptotic behaviour of Q^{-1} when the fluid in fractures is liquid. Three regimes can be clearly identified; at low frequencies Q^{-1} scales with ω , whereas at high frequencies it scales with $\omega^{-1/2}$. There are also intermediate frequencies where Q^{-1} scales with $\omega^{1/2}$.

and an incoming fluid flux in the background, $\frac{k_b}{\eta} \nabla p$ during the compression cycle. The pressure gradient points from the fracture to the background as fractures are highly compressible and thus the induced pressure is larger in the fracture. The pressure diffusion process starts with a sharp increase of the outgoing flux in the fracture while the incoming flux in the background is exponentially small. This characterizes the high frequency regime $Q_{\text{High}}^{-1} \propto \omega^{-1/2}$. The intermediate frequency regime is characterized by a decrease of the outgoing flux while the incoming flux in the background is still increasing. Finally, both fluxes decrease in the low frequency regime.

When $F \rightarrow 0$, eqs (38) and (39) give $\omega_p \rightarrow \infty$ and $\omega_M \rightarrow 0$, respectively. This means that there is no intermediate frequency regime, as can be observed in Fig. 1. The case $F \rightarrow 0$ corresponds to dry fractures. In this case, no fluid pressure can be induced in the fracture and hence the fracture gradient points from the background into the fracture during the compression cycle. This is exactly opposite to the liquid case (i.e. for large F) discussed above. Additionally, the fluxes become synchronous. This means that in the initial stage of the pressure diffusion process there is a sharp increase in fluid flow for both fluxes corresponding to the high frequency regime. Then, they decrease after reaching the maximum value, which corresponds to the low frequency regime. The asymptotes and the cross-over point R are shown in Fig. 4.

From the above considerations it becomes clear that there exists an intermediate value of F such that the fluid pressures induced in the fractures and background equal each other, eq. (37). Then the pressure gradient and all fluxes vanish and no attenuation is observed. This means that in addition to the geometrical no-flow condition of the periodic medium there exists a fluid-contrast related no-flow condition. Interestingly, the wave-induced attenuation and dispersion characteristics are not symmetrical with respect to the critical F value (see Fig. 2). Fluxes and resulting attenuation occur in a broader frequency range if $F > F^*$ as compared to the $F < F^*$ regime.

This study is focused on the complex modulus, dispersion and attenuation for P -waves propagating perpendicular to fractures. Extension to other elastic constants and propagation angles (that give rise to frequency dependent anisotropy) can be obtained by using the knowledge of the solution in the low and high frequency limits (Lambert *et al.* 2005; Krzikalla & Müller, 2011; Carcione *et al.* 2013).

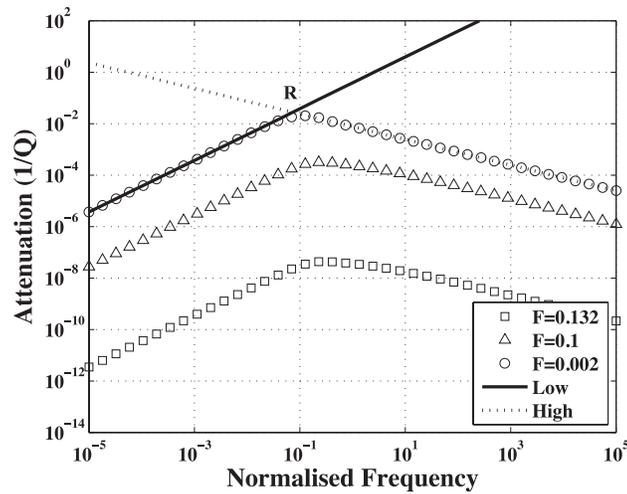


Figure 4. Asymptotic behaviour of Q^{-1} when the fluid in fractures is gas. Two regimes can be clearly identified; at low frequencies Q^{-1} scales with ω , whereas at high frequencies it scales with $\omega^{-1/2}$.

This study is focused on the case where pores and fractures are hydraulically connected but saturated with different fluids. For this situation to occur, the fluids must be prevented from mixing by capillary forces. Arguably, these capillary forces may have an effect not only on the spatial distribution of fluids, but also on the elastic properties and wave propagation (Tserkovnyak & Johnson 2003). This effect is beyond the scope of this paper but will be studied in the future.

7 CONCLUSIONS

We have developed a model for wave propagation in a porous medium with aligned fractures that allows pores and fractures to be filled with different fluids. The model treats a fractured medium as a periodic system of alternating layers of two types: thick porous layers representing the background, and very thin and highly compliant porous layers representing fractures. The results show that in the low-frequency limit the elastic properties of such a medium can be described by Gassmann's equation with a composite fluid, whose bulk modulus is a harmonic (Wood) average of the moduli of the two fluids. At higher frequencies, the model predicts significant dispersion and attenuation. The dispersion and attenuation are the highest when both pores and fractures are saturated with liquids; the results for this case are equivalent to those obtained by Brajanovski *et al.* (2005) for a porous and fractured medium saturated with a single liquid. The dispersion and attenuation are also significant (but somewhat weaker) when the pores are filled with a liquid but fractures are dry or filled with a highly compressible gas. However, there is an intermediate fracture fluid compressibility, where no dispersion is observed. This can be explained by observing that when the medium is uniformly saturated with a liquid, wave-induced compression causes flow from fractures into pores due to high compliance of the fractures. Conversely, when pores are filled with a liquid but fractures are filled with gas, flow will occur from pores into fractures due to the high compressibility of the gas. Thus an intermediate case exists where there is no flow and hence no dispersion or attenuation.

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