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## **Disclosure Quality, the Cost of Capital and Strategic Correlation**

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# **Disclosure Quality, the Cost of Capital and Strategic Correlation\***

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## **Abstract**

We investigate the strategic role of correlation between disclosure error and payoff shock in affecting a firm's cost of capital or share price. We show that the correlation affects the relationship between disclosure quality and the cost of capital or share price. The standard result where disclosure quality unambiguously lowers the firm's cost of capital or share price can be observed only in the case where the correlation is zero. In the extreme case where the correlation is perfect, disclosure quality does not affect the cost of capital or share price. When compared to other non-perfect correlation cases, the extreme case where the correlation is perfect results in, on average, a higher share price. This implies that the firm can achieve a higher share price by influencing the correlation (i.e., making it nonzero) and suggests a new way as to how the effectiveness of a disclosure should be evaluated.

## 1. Introduction

The literature on the relationship of disclosure quality and the cost of capital is large but still growing. Conventional wisdom suggests that more disclosure reduces information asymmetry and thus lowers the risk premium embedded in the cost of capital. But research of the last few decades cannot come to any definite answer on this issue. This issue still remains controversial at both theory and empirical levels. For a recent review of this literature, see Artiach and Clarkson (2011).

Our paper takes a new theoretical approach to examine this issue. We use a simple setting where there is only a single firm with many investors in an infinite period world. There is only a single disclosure/signal /management earnings forecast made by the firm.<sup>1</sup> But this disclosure is not perfect and noisy. We consider the strategic role of the correlation of the noise of this disclosure with that of the firm's payoff which we label as payoff shock. The strategic role of this correlation has been largely ignored in the literature. We show that this strategic role turns out to be very important in determining the relationship between disclosure quality and the cost of capital. In particular, we show that some standard results that the literature has established in this setting are only valid when the correlation is zero. Compared to the zero-correlation case, we show that as long as the correlation is non-zero the share price is, on average, higher. In the extreme cases where the correlation is perfect, disclosure quality does not matter as it has no effect on the cost of capital.

The contribution of this paper is threefold. First, it is the first paper which highlights the strategic role of manipulating the correlation between the disclosure (or signal) noise and that of the firm's payoff in the information disclosure or signalling literature in general and the management earnings forecast literature in particular. We show that it is always desirable for the firm to make the signal noise correlated with that of its payoff because by doing so the share price will be, on average, higher. The literature usually measures the performance of a disclosure (or signal or management earnings forecast) in terms of bias (i.e., the difference between the disclosed (or forecasted) value and the actual value) or quality (i.e., precision). Our result opens up a new way to capture the "effectiveness" of disclosure (or management earnings forecast) by measuring the correlation between its bias and payoff shock because the higher the correlation is, the more effective the disclosure (or signal or forecast) will be. This measure is easy to use and interpret. Second, we also establish that the impact of disclosure quality on the firm's cost of capital is ambiguous and depends on the correlation. Our results nest that of Clinch (2013) which assumes that the correlation is zero and are consistent with that of Lambert et al. (2007). Third, we also derive in terms of the correlation a new set of conditions under which disclosure quality does not affect the cost of capital. These conditions reveal that disclosure quality does not matter if the correlation is perfect, (i.e., either positive one or negative one).

The rest of the paper is organized as follows. First, we briefly review the literature and compare our approach with other existing approaches. Second, the basic setting of our model is outlined.

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<sup>1</sup> In our setting, we use the terms 'disclosure', 'signal' and 'management forecast' interchangeably.

Third, we show and discuss the main results. Finally, we conclude the paper with some discussions.

## 2. Literature review

Lambert et al. (2007) show that there are two effects associated with a change in disclosure quality. First, there is a direct effect because higher (lower) quality disclosure reduces (increases) a firm's covariance with those of the market as perceived by investors. Second, an indirect effect is expected because higher (lower) quality disclosure also affects the firm's real decisions that impact its expected cash flows and their covariance. Note that the direct effect is non-diversifiable while the indirect effect is diversifiable. In general, an increase in disclosure quality may affect the cost of capital in an ambiguous way as its overall effect depends on the term  $\left(\frac{\mu}{\sigma} - \frac{\Delta\mu}{\Delta\sigma}\right)$  where  $\mu$  and  $\sigma$  represent the expected payoff of the firm and the covariance of the firm's payoff with those of the market (see Proposition 4, Lambert et al. (2007)). They also derive the cases under which disclosure quality leads to a decrease in the cost of capital. A well-known and much discussed case in the literature is found whenever disclosure quality only leads to an decrease in  $\sigma$  but no corresponding decrease in  $\mu$ , then a decrease in the firm's cost of capital is expected.<sup>2</sup>

However, Christensen et al. (2010) shows in a finite period model that ex ante cost of capital is not affected by disclosure quality. In their model, there are two offsetting effects associated with a change in disclosure quality. The first effect reduces the perceived risk in relation to the expected terminal payoff of the firm, while the second effect increases the volatility of the security price of the firm. They offset each other, resulting in little change in the cost of capital.

Clinch (2013) argues that Christensen et al. (2010)'s finite-period model cannot capture price uncertainty because an investor who lives and holds shares for the entire period will not face the price uncertainty arising from the final payoff of the firm. He comes up with an infinite period model and shows that ex ante cost of capital is decreasing in the quality of disclosure.

The basic setting of Lambert et al. (2007) has been extended to various directions or settings that highlight some new conditions under which disclosure quality may or may not affect the cost of capital. For examples, Clinch (2013) discusses the issue in a multi-firm setting, with multiple signals/disclosures, or with the presence of heuristic traders. Given the fact that Clinch (2013)'s approach is quite flexible and versatile, our paper employs its basic setting which is described as below.

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<sup>2</sup> Johnstone (2013a, 2013b) observe that the current literature focuses too much on disclosure quality ( $\sigma$ ) without a proper regard on how information can affect expected payoff ( $\mu$ ). Bad (good) information can affect both  $\mu$  and  $\sigma$  (and their relative change as well) at the same time. He shows that even in the framework of Lambert et al. (2007), information disclosure can affect both  $\mu$  and  $\sigma$  simultaneously, it is possible that information can lead to a greater certainty (quality) and a lower cost of capital or greater certainty (quality) and a higher cost of capital.

### 3. Model setting

We follow the basic setting of Clinch (2013) which assumes a single firm in a infinite-period model. The firm generates a payoff,  $x_t$ , which follows a random walk, that is:

$$x_t = x_{t-1} + \varepsilon_t, \quad (1)$$

where  $\varepsilon_t$  is a random payoff shock normally distributed with mean zero, variance,  $\sigma_\varepsilon^2$ , and independent over time. There is also a riskfree security that pays  $1 + r$  each period. Thus,  $r$  represents the risk-free rate of return. In each period  $t$ , there are  $n$  identical and price taking investors, each with negative exponential utility with risk aversion parameter  $\alpha$ . Investors are assumed to exist for only a single period, at the end of which they liquidate their positions and consume the proceeds, which comprise the firm's payoff  $x_t$ , plus the price,  $P_t$ , at which they sell their shares (to the next generation of investors). Finally, the supply of the firm's security is normalized to one (and constant over time).

**Remark 1:** In this setting, the firm's payoff  $x_t$  represents the dividend rather than firm earnings because any plow-backed earnings cannot be consumed by the investors at time  $t + 1$ .

**Remark 2:** In some cases, the amount of rights issues could exceed the distributed dividends. Hence, this may result in negative dividend payments.

We assume there are two types of players: the firm and investors.

1. The firm can observe the income at that time based on which they decide the dividend of next period  $x_{t+1}$ . Hence to the firm,  $x_{t+1}$  is observable at time  $t$ .
2. The investors do not know  $x_{t+1}$ .
3. Given  $x_{t+1}$ , the firm will disclose information with  $y_t$  (e.g., management earnings forecast), and

$$y_t = x_{t+1} + e_t, \quad (2)$$

which is the sum of  $x_{t+1}$  and noise  $e_t$ , where  $e_t$  is a normally distributed variable, with mean zero, variance  $\sigma_e^2$ , and independent over time. To the firm,  $y_t$ ,  $x_{t+1}$  and  $e_t$  are all observable at time  $t$ .

By equating (1) and (2) together, we get

$$y_t = x_{t+1} + e_t = x_t + \varepsilon_{t+1} + e_t$$

We can see that both the firm and the investors know  $x_t$  at time  $t$ , so  $x_t$  is a realized fact;  $\varepsilon_{t+1}$  is the increment of payoff from  $t$  to  $t + 1$ , it is known by the firm at time  $t$  and will be known by the investors at time  $t + 1$ , and  $e_{t+1}$  is the information that the firm wants to transfer to the investors at time  $t$ . The firm can choose to tell the investors the

next payoff  $x_t + \varepsilon_{t+1}$  exactly, or they can choose to add noise, which is the case that  $e_t$  is independent of  $\varepsilon_{t+1}$ . But the problem is how to control the amount of the noise contained in the signal  $y_t$ ? Basically, there are two ways, either the firm makes  $e_t$  correlated with  $\varepsilon_{t+1}$ , or adjust the standard derivation of  $e_t$ . The former is the focus of this paper while the latter is called information quality  $\sigma_\varepsilon$  in the literature. As the correlation  $\rho$  goes from zero to one, or from zero to negative one, the information contained in the signal becomes stronger. As  $\sigma_\varepsilon$  shrinks to zero, the noise contained in the signal disappears; as  $\sigma_\varepsilon$  becomes large, the noise will dominate the signal. Note that even though  $\varepsilon_{t+1}$  and  $e_t$  are perfectly correlated, it does not necessarily imply that  $\varepsilon_{t+1}$  and  $e_t$  are the same because  $\sigma_\varepsilon$  and  $\sigma_e$  may be different.

4. The investors can only observe  $y_t$ ,  $x_{t+1}$  and  $e_t$  will not be known until time  $t + 1$ . In the absence of signal  $y_t$ , Clinch (2013) shows that

$$P_t = \frac{1}{1+r} \left[ \mathbb{E}_t(P_{t+1} + x_{t+1}) - \frac{\alpha}{n} \text{Var}_t(P_{t+1} + x_{t+1}) \right]. \quad (3)$$

The equation suggests that the current share price is nothing more than the present value of the future payoff denoted by the two terms in the bracket. The first term in the bracket refers to the investors' expected payoff and the second term stands for the variance of such payoff. One may interpret the second term loosely as the discount for the risk premium perceived by the investors and view it as a measure of the cost of capital. Note that this measure of the cost of capital is inversely related to the share price. There are two factors that affect the cost of capital. They are aggregate (or average) investors' risk aversion ( $\alpha/n$ ), and the risk of the future payoff ( $\text{Var}_t(P_{t+1} + x_{t+1})$ ). As will be shown later, this risk depends on disclosure quality.

5. Given signal  $y_t$ , the investors forecast  $x_{t+1}$  by forming conditional (rather than unconditional) expectation (i.e., by making a projection) of  $x_{t+1}$  on  $y_t$ ,

$$x_{t+1} = a + by_t + u_{t+1}.$$

where  $u_{t+1}$  is the projection error, which is independent of  $y_t$ .

6. The key difference between this paper and Clinch (2013) is that we allow for the correlation  $\rho$  between  $e_t$  and  $\varepsilon_t$  while Clinch (2013) assumes  $e_t$  and  $\varepsilon_t$  are independent. To account for the possible relationship between the noise ( $e_t$ ) and the payoff shock ( $\varepsilon_t$ ), we set

$$e_t = \frac{\sigma_\varepsilon}{\sigma_e} (\rho \varepsilon_{t+1} + \sqrt{1 - \rho^2} \xi_t), \quad (4)$$

where  $\xi_t$  is normally distributed with mean zero, variance,  $\sigma_\varepsilon^2$ .  $\xi_t$  is independent of  $\varepsilon_t$ , and independent over time. This construction will assure that the correlation between  $e_t$  and  $\varepsilon_t$  is  $\rho$ , and the standard deviation of  $e_t$  is  $\sigma_e$ .

7. The investors will use the signal to form their expectation of the future payoff, and the

cost of capital and  $P_t$  will be determined accordingly.

8. As the signal affects the cost of capital and  $P_t$ , the firm is able to minimize the cost of capital and maximize  $P_t$  by choosing  $\rho$  and  $\sigma_\varepsilon$  optimally.<sup>3</sup>
9. Table 1 illustrates the basic structure of our model over the first few time periods. For example, given a payoff shock at time 1, the payoff at time 1 will be  $x_0 + \varepsilon_1$ , while the disclosure (or signal) will become  $x_1 + \varepsilon_2 + \frac{\sigma_\varepsilon}{\sigma_\xi} (\rho\varepsilon_2 + \sqrt{1 - \rho^2}\xi_1)$  in general and  $x_0 + \left(1 \pm \frac{\sigma_\varepsilon}{\sigma_\xi}\right) \varepsilon_2$  if  $\rho = \pm 1$ .

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<sup>3</sup> In Clinch (2013)'s setting, the firm can only choose  $\sigma_\varepsilon$  (the precision of the signal) but not  $\rho$  (the correlation between the signal error and the payoff shock).

**Table 1**

$t$	0	1	2	3
Payoff shock		$\varepsilon_1$	$\varepsilon_2$	$\varepsilon_3$
Payoff	$x_0$	$x_0 + \varepsilon_1$	$x_1 + \varepsilon_2$	$x_2 + \varepsilon_3$
Signal	$x_0 + \varepsilon_1 + \frac{\sigma_\varepsilon}{\sigma_\xi} (\rho\varepsilon_1 + \sqrt{1 - \rho^2}\xi_0)$	$x_1 + \varepsilon_2 + \frac{\sigma_\varepsilon}{\sigma_\xi} (\rho\varepsilon_2 + \sqrt{1 - \rho^2}\xi_1)$	$x_2 + \varepsilon_3 + \frac{\sigma_\varepsilon}{\sigma_\xi} (\rho\varepsilon_3 + \sqrt{1 - \rho^2}\xi_2)$	$x_3 + \varepsilon_4 + \frac{\sigma_\varepsilon}{\sigma_\xi} (\rho\varepsilon_4 + \sqrt{1 - \rho^2}\xi_3)$
Signal $\rho=\pm 1$	$x_0 + \left(1 \pm \frac{\sigma_\varepsilon}{\sigma_\xi}\right) \varepsilon_1$	$x_1 + \left(1 \pm \frac{\sigma_\varepsilon}{\sigma_\xi}\right) \varepsilon_2$	$x_2 + \left(1 \pm \frac{\sigma_\varepsilon}{\sigma_\xi}\right) \varepsilon_3$	$x_3 + \left(1 \pm \frac{\sigma_\varepsilon}{\sigma_\xi}\right) \varepsilon_4$

## 4. Main results

We state and discuss the main results in this section. The details of the proofs are shown in the Appendix.

First, we derive a useful lemma which will be used subsequently.

**Lemma 1:** If the firm's payoff  $x_t$  follows a random walk,  $x_t = x_{t-1} + \varepsilon_t$ , then the share price follows a random walk. In particular,  $P_t = P_{t-1} + \frac{1}{r} \varepsilon_t$ .

Second, we state two main theorems of the paper here.

**Theorem 1:** Under the above assumptions, if we allow for a non-zero correlation between  $\varepsilon_t$  and  $e_t$ , then the share price is

$$P_t = \frac{1}{r} \left[ x_t + b_*(y_t - x_t) - \frac{\alpha}{n} s_*^2 \right]$$

where  $b_* = \frac{\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_e}{\sigma_\varepsilon^2 + \sigma_e^2 + 2\rho \sigma_\varepsilon \sigma_e}$ , and  $s_*^2 = \frac{1}{r^2} [(1+r)^2 \sigma_\varepsilon^2 - [(1+r)^2 - 1] b_*(\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_e)]$ .

**Remark 3:** Note that now the variance of future payoffs ( $s_*^2$ ) is now a function of  $\rho$ .  $\rho$  can affect  $s_*^2$  (and hence share price) directly and indirectly via  $b_*$ .

**Remark 4:** If  $\rho = 0$ , then  $b_* = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_e^2}$ , and  $s_*^2 = \frac{1}{r^2} [(1+r)^2 \sigma_\varepsilon^2 - [(1+r)^2 - 1] b_* \sigma_\varepsilon^2]$ . The share price will degenerate into Clinch (2013)'s equation (2).

**Remark 5:** Note that  $b_*$  can affect both  $s_*^2$  and expected future payoff (i.e.,  $x_t + b_*(y_t - x_t)$ ) but in opposite ways. Expected future payoff is increasing in  $b_*$  but  $s_*^2$  is decreasing in  $b_*$ . This result is consistent with Johnstone (2013b)'s observation that information can affect both expected future payoff AND risk at the same time.

**Remark 6:** The firm will not choose a high  $y_t$  to boost up  $P_t$  because this is an infinitely repeated game where reputation does matter. If the firm cheats the investors by setting a high  $y_t$ , the investors will never believe the firm any more.

**Remark 7:** To understand the role of  $\rho$  in determining the cost of capital and the share price, it is illuminative to consider some special cases of  $\rho$ . They refer to the situations where  $\rho = -1, 0$ , and  $+1$ . It is easy to get the following results:

- If  $\rho = +1$ , then  $b_* = \frac{\sigma_\varepsilon}{\sigma_\varepsilon + \sigma_e}$ ,

$$s_*^2 = \frac{1}{r^2} [(1+r)^2 \sigma_\varepsilon^2 - [(1+r)^2 - 1] b_*(\sigma_\varepsilon^2 + \sigma_\varepsilon \sigma_e)], \text{ and}$$

$$\mathbb{E}(P_t) = \frac{1}{r} \left[ x_0 - \frac{\alpha}{n} \frac{1}{r^2} \sigma_\varepsilon^2 \right].$$

- If  $\rho = -1$ , then  $b_* = \frac{\sigma_\varepsilon}{\sigma_\varepsilon - \sigma_\xi}$ ,

$$s_*^2 = \frac{1}{r^2} [(1+r)^2 \sigma_\varepsilon^2 - [(1+r)^2 - 1] b_* (\sigma_\varepsilon^2 - \sigma_\varepsilon \sigma_\xi)], \text{ and}$$

$$\mathbb{E}(P_t) = \frac{1}{r} \left[ x_0 - \frac{\alpha}{n} \frac{1}{r^2} \sigma_\varepsilon^2 \right].$$

- If  $\rho = 0$ , then  $b_* = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\xi^2}$ ,

$$s_*^2 = \frac{1}{r^2} [(1+r)^2 \sigma_\varepsilon^2 - [(1+r)^2 - 1] b_* \sigma_\varepsilon^2], \text{ and}$$

$$P_t = \frac{1}{r} \left[ x_t + \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\xi^2} \left( \varepsilon_{t+1} + \frac{\sigma_\varepsilon}{\sigma_\xi} \xi_{t+1} \right) - \frac{\alpha}{n} \frac{1}{r^2} \left( (1+r)^2 \sigma_\varepsilon^2 - ((1+r)^2 - 1) \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\xi^2} \sigma_\varepsilon^2 \right) \right]$$

$$\mathbb{E}(P_t) = \frac{1}{r} \left[ x_t - \frac{\alpha}{n} \frac{1}{r^2} \left( (1+r)^2 \sigma_\varepsilon^2 - ((1+r)^2 - 1) \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\xi^2} \sigma_\varepsilon^2 \right) \right]$$

$$\leq \frac{1}{r} \left[ x_t - \frac{\alpha}{n} \frac{1}{r^2} \left( (1+r)^2 \sigma_\varepsilon^2 - ((1+r)^2 - 1) \sigma_\varepsilon^2 \right) \right] = \frac{1}{r} \left[ x_t - \frac{\alpha}{n} \frac{1}{r^2} \sigma_\varepsilon^2 \right]$$

The above inequality holds because  $b$  is always equal to or less than one by construction.

It is noteworthy that when  $\rho = \pm 1$ , the quality of the signal  $\sigma_\varepsilon$  does not enter into the pricing formula. That means, the investors place no concern over  $\sigma_\varepsilon$  as it does not matter. This result is in sharp contrast with the result of Clinch (2013) where the quality of the signal  $\sigma_\varepsilon$  has a role to play in determining the share price. In these two cases, only the payoff shock and its variance, together with other variables, determine the share price.

**Theorem 2:** If  $\varepsilon_{t+1}$  and  $e_t$  are perfectly correlated, namely  $\rho = \pm 1$ , the firm can experience, on average, a higher share price when compared to the other imperfect correlation cases. In particular,  $\mathbb{E}(P_t) \text{ if } \rho = \pm 1 > E(P_t) \text{ if } \rho \in (-1, 1)$ .

**Remark 8:** The expected share price conditional on perfect correlation is always higher than that conditional on other imperfect correlation cases.

**Remark 9:** The implication of Theorem 2 is that it does not matter whether or not the disclosure or payoff forecast by managers is noisy (i.e.,  $e_t > 0$  and  $\sigma_\varepsilon^2 > 0$ ). As long as the managers can match this noise with that of the firm's payoff in such a way that their correlation is perfect, then disclosure quality  $\sigma_\varepsilon^2$  does not matter as it does not affect the cost of capital and therefore the share price. Two cases can be distinguished. In the first case where  $\varepsilon_{t+1} = e_t$  and  $\sigma_\varepsilon^2 = \sigma_\xi^2$ , the firm can be better off, in terms of reducing the cost of capital, by revealing the information to the investor in such a way that the information noise exactly matches the payoff shock. In the second case where  $\varepsilon_{t+1} \neq e_t$  and  $\sigma_\varepsilon^2 \neq \sigma_\xi^2$  but  $\rho = \pm 1$ , the firm may give either an optimistic forecast or pessimistic forecast as long as the firm does it consistently by matching the disclosure (or forecast) noise with payoff shock according to equation (4), then the disclosure quality  $\sigma_\varepsilon$  does not matter.

**Remark 10:** The prediction of Theorem 2 is consistent with the empirical evidence on the relationship between management earnings forecasts and the cost of capital, which generally finds that forecast precision does not affect the cost of capital (Kim and Shi, 2011; Baginski and Rakow, 2012; Larocque, Lawrence, and Veenstra, 2013).

**Remark 11:** This theorem suggests that one may use this correlation to evaluate the effectiveness of a particular measure of disclosure/management forecast/signal. The correlation measure has a number of advantages. First, it is easy to compute as long as data for actual payoff and disclosure/forecast/signal are available. Second, the interpretation of this correlation is straightforward; the higher the correlation is, the more effective the measure will be in revealing the information. Third, Theorem 2 provides an upper bound of the effectiveness of a particular disclosure/management forecast/signal; this bound is also ideal in the sense that it is deprived in a setting where we assume away other institutional or behavioural factors like agency costs that may move this correlation away from positive (or negative) one. Thus, in reality, the correlation is expected to be less than one. Empiricists may explore the possible determinants of this correlation.<sup>4</sup>

## 5. Conclusion

Based on a simple setting of Clinch (2013), we show that the correlation between disclosure error and payoff shock turns out to be an important factor that determines the cost of capital and share price. The higher the correlation is, the lower, on average, the cost of capital will be and therefore the higher the share price. We also show that in general the impact of disclosure quality on the cost of capital is ambiguous and depends on, *inter alia*, the correlation. In the extreme cases where the correlation is positively or negatively perfect, our model predicts that disclosure quality does not matter. This suggests a strategic role that this correlation may play in understanding the effectiveness of a disclosure/signal/earnings forecast.

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<sup>4</sup> We are preparing an empirical paper on this issue.



**Proof of Theorem 1.**

**Proof:**

$$P_t = \frac{1}{1+r} \left[ \mathbb{E}_t(P_{t+1}) + \mathbb{E}_t(x_{t+1}|y_t) - \frac{\alpha}{n} \text{Var}_t(P_{t+1} + x_{t+1}) \right] \quad (\text{A.1})$$

Since  $P_{t+1} = P_t + \frac{1}{r}\varepsilon_t$ , we have

$$\mathbb{E}_t(P_{t+1}) = P_t \quad (\text{A.2})$$

$$\mathbb{E}_t(x_{t+1}|y_t) = a + b_* y_t$$

where  $a$  is the intercept, and  $b_*$  is the slope. Hence, we have

$$b_* = \frac{\text{Cov}(x_{t+1}|y_t)}{\text{Var}(y_t)} = \frac{\text{Cov}(x_t + \varepsilon_{t+1} | x_t + \varepsilon_{t+1} + \theta_t)}{\text{Var}(x_t + \varepsilon_{t+1} + \theta_t)} = \frac{\sigma_\varepsilon^2 + \rho\sigma_\varepsilon\sigma_\theta}{\sigma_\varepsilon^2 + \sigma_\theta^2 + 2\rho\sigma_\varepsilon\sigma_\theta} \quad (\text{A.3})$$

Suppose that the best fitting line across  $(x_t, y_t)$  does exist, then

$$\mathbb{E}_t(x_{t+1}|y_t) = a + b_* y_t = x_t + b_*(y_t - x_t), \quad (\text{A.4})$$

where  $a = (1 - b_*)x_t$ ,

Denote  $\text{Var}_t(P_{t+1} + x_{t+1})$  as  $s_*^2$  and substitute (A.2) and (A.4) into (A.1),  $P_t$  can be simplified into

$$\begin{aligned} P_t &= \frac{1}{1+r} \left[ P_t + x_t + b_*(y_t - x_t) - \frac{\alpha}{n} s_*^2 \right], \\ P_t - \frac{1}{1+r} P_t &= \frac{1}{1+r} \left[ x_t + b_*(y_t - x_t) - \frac{\alpha}{n} s_*^2 \right], \\ P_t &= \frac{1}{r} \left[ x_t + b_*(y_t - x_t) - \frac{\alpha}{n} s_*^2 \right], \end{aligned}$$

Note that we can rewrite  $s_*^2$  as follows:

$$s_*^2 = \text{Var}(P_{t+1} + x_{t+1}) = \frac{1}{r^2} \left[ (1+r)^2 \sigma_\varepsilon^2 - [(1+r)^2 - 1] b_* (\sigma_\varepsilon^2 + \rho\sigma_\varepsilon\sigma_\theta) \right],$$

To prove this, recall (A.2) implies that

$$\mathbb{E}_t(P_{t+1}) = \mathbb{E}_t(P_{t+1}|y_t) = P_t,$$

We have

$$\mathbb{E}_t(P_{t+1} + x_{t+1}|y_t) = P_t + x_t + b_*(y_t - x_t),$$

Therefore,

$$P_{t+1} + x_{t+1} = P_t + x_t + b_*(y_t - x_t) + \tilde{u}_{t+1},$$

Using Lemma 1, we rewrite the LHS of the above equation as follows,

$$P_{t+1} + x_{t+1} = P_t + \frac{\varepsilon_{t+1}}{r} + x_t + \varepsilon_{t+1},$$

Hence,

$$P_t + x_t + b(y_t - x_t) + u_{t+1} = P_t + \frac{\varepsilon_{t+1}}{r} + x_t + \varepsilon_{t+1},$$

By cancelling out  $P_t$  and  $x_t$  from both sides of the equation, we get

$$x_t + b_*(y_t - x_t) + u_{t+1} = \frac{\varepsilon_{t+1}}{r} + x_t + \varepsilon_{t+1},$$

$$\begin{aligned} u_{t+1} &= \frac{1+r}{r} \varepsilon_{t+1} - b_*(y_t - x_t) \\ &= \frac{1+r}{r} \varepsilon_{t+1} - b_*(x_{t+1} + e_t - x_t) \\ &= \frac{1+r}{r} \varepsilon_{t+1} - b_*(\varepsilon_{t+1} + e_t) \end{aligned}$$

Therefore,

$$\begin{aligned} s_*^2 &= \text{Var}(P_{t+1} + x_{t+1}) \\ &= \text{Var}\left(\frac{1+r}{r} \varepsilon_{t+1} - b_*(\varepsilon_{t+1} + e_t)\right) \\ &= \left(\frac{1+r}{r}\right)^2 \sigma_\varepsilon^2 + b_*^2(\sigma_\varepsilon^2 + \sigma_e^2 + 2\rho\sigma_\varepsilon\sigma_e) - 2\left(\frac{1+r}{r}\right) b_* \text{Cov}(\varepsilon_{t+1}, \varepsilon_{t+1} + e_t) \\ &= \left(\frac{1+r}{r}\right)^2 \sigma_\varepsilon^2 + \left(\frac{\sigma_\varepsilon^2 + \rho\sigma_\varepsilon\sigma_e}{\sigma_\varepsilon^2 + \sigma_e^2 + 2\rho\sigma_\varepsilon\sigma_e}\right)^2 (\sigma_\varepsilon^2 + \sigma_e^2 + 2\rho\sigma_\varepsilon\sigma_e) \\ &\quad - 2\left(\frac{1+r}{r}\right) b_* \text{Cov}(\varepsilon_{t+1}, \varepsilon_{t+1} + e_t) \\ &= \left(\frac{1+r}{r}\right)^2 \sigma_\varepsilon^2 + b_*(\sigma_\varepsilon^2 + \rho\sigma_\varepsilon\sigma_e) - 2\left(\frac{1+r}{r}\right) b_*(\sigma_\varepsilon^2 + \rho\sigma_\varepsilon\sigma_e) \\ &= \frac{1}{r^2} [(1+r)^2 \sigma_\varepsilon^2 + [r^2 - 2(1+r)r] b_*(\sigma_\varepsilon^2 + \rho\sigma_\varepsilon\sigma_e)] \\ &= \frac{1}{r^2} [(1+r)^2 \sigma_\varepsilon^2 - [(1+r)^2 - 1] b_*(\sigma_\varepsilon^2 + \rho\sigma_\varepsilon\sigma_e)]. \end{aligned}$$

This completes the proof.

**Proof of Theorem 2.**

**Proof:**

$$P_t = \frac{1}{r} \left[ x_t + b_*(y_t - x_t) - \frac{\alpha}{n} s_*^2 \right].$$

Substitute  $b_*$  and  $s_*^2$  into  $P_t$ , we have,

$$\begin{aligned} P_t &= \frac{1}{r} \left( x_t + b_*(y_t - x_t) - \frac{\alpha}{n} \frac{1}{r^2} [(1+r)^2 \sigma_\varepsilon^2 - [(1+r)^2 - 1] b_*(\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta)] \right) \\ &= \frac{1}{r} \left( x_t + \frac{\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta}{\sigma_\varepsilon^2 + \sigma_\theta^2 + 2\rho \sigma_\varepsilon \sigma_\theta} \left( \varepsilon_{t+1} + \frac{\sigma_\theta}{\sigma_\varepsilon} (\rho \varepsilon_{t+1} + \sqrt{1-\rho^2} \xi_{t+1}) \right) - \frac{\alpha}{n} \frac{1}{r^2} [(1+r)^2 \sigma_\varepsilon^2 - [(1+r)^2 - 1] \right. \\ &\quad \left. \frac{\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta}{\sigma_\varepsilon^2 + \sigma_\theta^2 + 2\rho \sigma_\varepsilon \sigma_\theta} (\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta)] \right) \end{aligned}$$

Taking unconditional expectation of  $P_t$ ,

$$\mathbb{E}(P_t) = \frac{1}{r} \left( -\frac{\alpha}{n} \frac{1}{r^2} [(1+r)^2 \sigma_\varepsilon^2 - [(1+r)^2 - 1] \frac{\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta}{\sigma_\varepsilon^2 + \sigma_\theta^2 + 2\rho \sigma_\varepsilon \sigma_\theta} (\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta)] \right).$$

By taking the derivative of  $\mathbb{E}(P_t)$  with respect to  $\rho$ , we get the following:

$$\begin{aligned} \frac{\partial \mathbb{E}(P_t)}{\partial \rho} &= -\frac{1}{r} \frac{\alpha}{nr^2} \left( \frac{(2\sigma_\theta \sigma_\varepsilon ((1+r)^2 - 1) (\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta)^2)}{(\sigma_\varepsilon^2 + 2\rho \sigma_\varepsilon \sigma_\theta + \sigma_\theta^2)^2} - \frac{(2\sigma_\theta \sigma_\varepsilon ((1+r)^2 - 1) (\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta))}{\sigma_\varepsilon^2 + 2\rho \sigma_\varepsilon \sigma_\theta + \sigma_\theta^2} \right) \\ &= -\frac{1}{r} \times \frac{2\alpha \sigma_\theta \sigma_\varepsilon ((1+r)^2 - 1) (\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta) (\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta - (\sigma_\varepsilon^2 + 2\rho \sigma_\varepsilon \sigma_\theta + \sigma_\theta^2))}{nr^2 (\sigma_\varepsilon^2 + 2\rho \sigma_\varepsilon \sigma_\theta + \sigma_\theta^2)^2} \\ &= \frac{1}{r} \times \frac{2\alpha \sigma_\theta \sigma_\varepsilon ((1+r)^2 - 1) (\sigma_\varepsilon^2 + \rho \sigma_\varepsilon \sigma_\theta) (\sigma_\theta^2 + \rho \sigma_\varepsilon \sigma_\theta)}{nr^2 (\sigma_\varepsilon^2 + 2\rho \sigma_\varepsilon \sigma_\theta + \sigma_\theta^2)^2} \\ &= \frac{1}{r} \times \frac{2\alpha \sigma_\theta^2 \sigma_\varepsilon^2 ((1+r)^2 - 1) (\sigma_\varepsilon + \rho \sigma_\theta) (\sigma_\theta + \rho \sigma_\varepsilon)}{nr^2 (\sigma_\varepsilon^2 + 2\rho \sigma_\varepsilon \sigma_\theta + \sigma_\theta^2)^2}. \end{aligned}$$

$$\text{If } \rho = +1, \text{ then } \left. \frac{\partial \mathbb{E}(P_t)}{\partial \rho} \right|_{\rho=+1} = \frac{1}{r} \times \frac{2\alpha \sigma_\theta^2 \sigma_\varepsilon^2 ((1+r)^2 - 1) (\sigma_\varepsilon + \sigma_\theta) (\sigma_\theta + \sigma_\varepsilon)}{nr^2 (\sigma_\varepsilon^2 + 2\rho \sigma_\varepsilon \sigma_\theta + \sigma_\theta^2)^2} > 0.$$

$$\text{If } \rho = 0, \text{ then } \left. \frac{\partial \mathbb{E}(P_t)}{\partial \rho} \right|_{\rho=0} = \frac{1}{r} \times \frac{2\alpha \sigma_\theta^2 \sigma_\varepsilon^2 ((1+r)^2 - 1)}{nr^2 (\sigma_\varepsilon^2 + 2\rho \sigma_\varepsilon \sigma_\theta + \sigma_\theta^2)^2} > 0.$$

$$\text{If } \rho = -1, \text{ then } \left. \frac{\partial \mathbb{E}(P_t)}{\partial \rho} \right|_{\rho=-1} = \frac{1}{r} \times \frac{2\alpha \sigma_\theta^2 \sigma_\varepsilon^2 ((1+r)^2 - 1) (\sigma_\varepsilon - \sigma_\theta) (\sigma_\theta - \sigma_\varepsilon)}{nr^2 (\sigma_\varepsilon^2 + 2\rho \sigma_\varepsilon \sigma_\theta + \sigma_\theta^2)^2} < 0.$$

We define  $A \equiv \frac{1}{r} \times \frac{2\alpha \sigma_\theta^2 \sigma_\varepsilon^2 ((1+r)^2 - 1)}{nr^2 (\sigma_\varepsilon^2 + 2\rho \sigma_\varepsilon \sigma_\theta + \sigma_\theta^2)^2}$ ,  $B \equiv (\sigma_\varepsilon + \rho \sigma_\theta) (\sigma_\theta + \rho \sigma_\varepsilon)$ , then

$$\frac{\partial \mathbb{E}(P_t)}{\partial \rho} = A \times B,$$

Note that  $A$  is a (negative) function of  $\rho$ , with the value of  $A$  being always positive.  $B$  is a convex quadratic function of  $\rho$ , with two roots being  $-\frac{\sigma_\varepsilon}{\sigma_\varepsilon}$  and  $-\frac{\sigma_\varepsilon}{\sigma_\varepsilon}$ . Hence, no matter whether  $\sigma_\varepsilon < \sigma_\varepsilon$ , or  $\sigma_\varepsilon > \sigma_\varepsilon$ , one root  $\in (-1, 0)$ , the other  $\in (-\infty, -1)$ . As the domain of  $\rho$  in our problem is  $[-1, 1]$ , we are only interested in the root  $\in (-1, 0)$ .

Figure 1

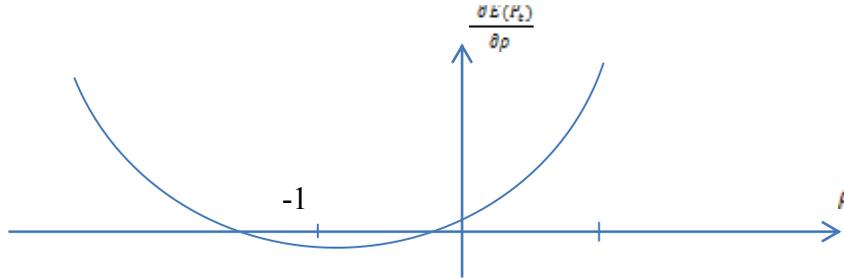


Figure 2

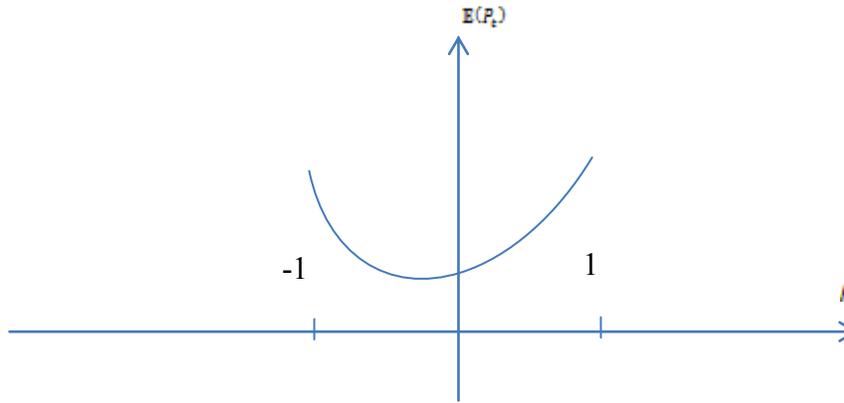


Figure 1 shows the relationship between  $\frac{\partial \mathbb{E}(P_t)}{\partial \rho}$  and  $\rho$ , from which we derive and show the implied relationship between  $\mathbb{E}(P_t)$  and  $\rho$  in Figure 2. As  $\rho$  goes from  $-1$  to  $0$ , Figure 1 shows that at first  $\frac{\partial \mathbb{E}(P_t)}{\partial \rho} < 0$ , and then it becomes  $\frac{\partial \mathbb{E}(P_t)}{\partial \rho} > 0$ ; based on the results in **Remark 7**, this implies that  $\mathbb{E}(P_t)$  goes from the maximum,  $\frac{1}{r} \left[ x_0 - \frac{\alpha}{n} \frac{1}{r^2} \sigma_\varepsilon^2 \right]$ , to a minimum and then increases to  $\frac{1}{r} \left[ x_0 - \frac{\alpha}{n} \frac{1}{r^2} \left( (1+r)^2 \sigma_\varepsilon^2 - ((1+r)^2 - 1) \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\varepsilon^2} \sigma_\varepsilon^2 \right) \right]$ . (See Figure 2)

As  $\rho$  goes from  $0$  to  $1$ , we always have  $\frac{\partial \mathbb{E}(P_t)}{\partial \rho} > 0$  (see Figure 1), then Figure 2 shows that  $\mathbb{E}(P_t)$  goes from  $\frac{1}{r} \left[ x_0 - \frac{\alpha}{n} \frac{1}{r^2} \left( (1+r)^2 \sigma_\varepsilon^2 - ((1+r)^2 - 1) \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_\varepsilon^2} \sigma_\varepsilon^2 \right) \right]$  to the maximum,  $\frac{1}{r} \left[ x_0 - \frac{\alpha}{n} \frac{1}{r^2} \sigma_\varepsilon^2 \right]$ .<sup>5</sup>

This completes the proof.

<sup>5</sup> Note that  $\frac{\partial \mathbb{E}(P_t)}{\partial \rho} = A \times B$ . Since both  $A$  and  $B$  are functions of  $\rho$ , and  $B$  is quadratic function of  $\rho$  while  $A$  is decreasing in  $\rho$ ,  $\frac{\partial \mathbb{E}(P_t)}{\partial \rho}$  is no longer a quadratic function. However, since  $A$  is always positive, the sign of  $\frac{\partial \mathbb{E}(P_t)}{\partial \rho}$  is consistent with the sign of  $B$ , hence the above analysis holds.

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