School of Education

The Impact of Lesson Study on Primary School Teachers' Knowledge and Skills in Differentiating Primary School Mathematics Instruction: A Digital Mixed Methods Approach

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Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Signature : ………………………

Date : ………………………

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Abstract

This study investigates the impact of Lesson Study on teacher knowledge in differentiating mathematics instruction using a digital mixed methods research approach. The study was conducted in two primary schools in Nagekeo Regency on Flores Island in Indonesia. The activities involved an initial session in each school about Differentiated Mathematics Instruction and Lesson Study. After the initial session, Lesson Study Groups produced Lesson Plans, which were taught by one member of the group and observed by the other members. Following this, each Lesson Study Group met to discuss the lesson and plan the next lesson. The lessons were videoed and these became the main data source, which was analysed using the interactive digital software application Multimodal Analysis Video. The Lesson Plans and teachers' observation during the lessons were employed to support the findings from these videos. According to findings in the videos, the teachers discussed the topic and strategies that they would teach in the Lesson Study Group Meetings, focusing on the need for differentiated mathematics instruction to cater for the different abilities of the students. They also reviewed and refined their teaching in these meetings. Although Differentiated Mathematics Instruction was a new approach for the teachers, they attempted to implement new strategies in the classrooms while building on their Mathematical Pedagogical Content Knowledge. The findings were derived by analysing the video recordings of the lessons using the Multimodal Analysis Video software, which transformed the qualitative data from the video analysis into quantitative data to study the relations between Mathematical Pedagogical Content Knowledge and the teachers' actions in implementing Differentiated Mathematics Instruction. Remarkably, the Multimodal Analysis Video software has facilities for visualising relations between teachers' knowledge and the differentiation in the classroom. The findings suggest that Lesson Study can be a vehicle for enhancing teachers' knowledge of mathematical content, pedagogical, and context to differentiate the content, process, product, assessment, and learning environment in mathematics lessons in heterogeneous classrooms.

Table of Contents

Acknowledgements .... **iii**
Abstract .... **v**
List of Figures .... **ix**
List of Tables .... **xi**
List of Appendices .... **xii**
Chapter 1 – Introduction to the Study **1**
1.1 Introduction 1
1.2 Background 2
1.2.1 Lesson Study 2
1.2.2 Mathematical Pedagogical Content Knowledge 3
1.2.3 Differentiated Mathematics Instruction 4
1.2.4 Multimodal Analysis Video (MMAV) 5
1.3 Context of the Study 5
1.3.1 Mathematics Education in Indonesian Primary Schools 5
1.3.2 Mathematics Content Knowledge of Primary School Mathematics Teachers in Indonesia 7
1.3.3 Professional Development for Teachers in Indonesia 8
1.3.4 Heterogeneous Classrooms 9
1.4 Problem Statement 10
1.5 Research Question 10
1.6 Conceptual Framework 11
1.7 Nature of the Study 14
1.8 Significance of the Study 15
1.9 Organisation of the Thesis 16
1.10 Summary 17
Chapter 2 – Literature Review **18**
2.1 Introduction 18
2.2 Teachers' Mathematics Pedagogy 19
2.2.1 Pedagogical Content Knowledge 19
2.2.2 Mathematical Pedagogical Content Knowledge 23
2.3 Differentiated Mathematics Instruction in Heterogeneous Classrooms 30
2.3.1 Characteristics of Differentiated Instruction 31
2.3.2 Factors that Influence Differentiation 33
2.3.3 Modification of the Teaching and Learning Process 36
2.3.4 Challenges and Benefits of Differentiated Instruction 41
2.4 Lesson Study as Professional Development in Schools 42
2.4.1 Understanding Lesson Study 43
2.4.2 Lesson Study Challenges and Benefits 45
2.4.3 Lesson Study and Its Impact on Teachers' Knowledge 46
2.4.4 Lesson Study in Indonesia 47
2.5 The Link between Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction 48
2.5.1 The Connection between Lesson Study and Teachers' Mathematical Pedagogical Content Knowledge 48
2.5.2 The Connection between Lesson Study and Differentiated Mathematics Instruction 48
2.5.3 The Connection between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction 49
2.6 Conclusion 50

Chapter 3 – Research Method **52**
3.1 Introduction 52
3.2 Research Questions and Objectives 52
3.3 Research Design 53
3.3.1 Description of Research Design 53
3.3.2 Model of Research Design 55
3.4 Research Methodology 56
3.4.1 Research Site 56
3.4.2 Participants 57
3.4.3 Ethical Considerations 59
3.5 Data Collection 59
3.5.1 Video Recording 60
3.5.2 Written Documents 60
3.6 Data Analysis 62
3.7 Summary 65
Chapter 4 – Results **66**
4.1 Introduction 66
4.2 School A: Overview of Lessons A1 and A2 73
4.2.1 Lesson Study Group Meeting 1 in School A (LSGM-A1) 73
4.2.2 Lesson A1: Acute, right and obtuse angles 77
4.2.3 Lesson Study Group Meeting 2 in School A (LSGM-A2) 88
4.2.4 Lesson A2: Acute, right and obtuse angles 92
4.2.5 Lesson Study Group Meeting 3 in School A (LSGM-A3) 98
4.2.6 Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in School A 99
4.3 School B: Overview of Lessons B1 and B2 107
4.3.1 Lesson Study Group Meeting 1 in School B (LSGM-B1) 108
4.3.2 Lesson B1: Commutative law 112
4.3.3 Lesson Study Group Meeting 2 in School B (LSGM-B2) 119
4.3.4 Lesson B2: Associative law 124
4.3.5 Lesson Study Group Meeting 3 in School B (LSGM-B3) 130
4.3.6 Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in School B 131
4.4 The Digital Mixed-Methods Research Design in Lesson Study 137
4.5 Conclusion 140
Chapter 5 – Discussion **141**
5.1 Introduction 141
5.2 The Process of Implementing Lesson Study 142
5.2.1 Planning 142
5.2.2 Teaching 143
5.2.3 Observing 143
5.2.4 Reviewing 144
5.2.5 Refining 145
5.3 The Influence of Lesson Study on Primary School Teachers' Mathematics Pedagogy 145
5.3.1 The Influence of Lesson Study on Mathematics Content Knowledge 146
5.3.2 The Influence of Lesson Study on Mathematical Pedagogical Knowledge 146
5.3.3 The Influence of Lesson Study on Context Knowledge 147

5.4 The Influence of Lesson Study on Curriculum Differentiation in Mathematics 147
5.4.1 The Influence of Lesson Study on Differentiating Content 148
5.4.2 The Influence of Lesson Study on Differentiating Process 148
5.4.3 The Influence of Lesson Study on Differentiating Product 149
5.4.4 The Influence of Lesson Study on Differentiating Assessment 149
5.4.5 The Influence of Lesson Study on Differentiating the Learning Environment 150
5.5 The Influence of Teachers' Mathematical Pedagogical Content Knowledge in Differentiating Mathematics Instruction..150
5.6 Potential Key Characteristics of Lesson Study as a Form of Professional Development 155
5.6.1 Cooperation 156
5.6.2 Effectiveness 156
5.6.3 Independence...157
5.6.4 Tolerance...158
5.6.5 Reflection 159
5.6.6 Refinement 159
5.6.7 Dedication 160
5.6.8 Continuity..160
5.7 Digital Mixed-Methods Approach for Studying the Impact of Lesson Study on Teachers' Knowledge and Skills in Differentiating Mathematics Instruction 161
5.8 Conclusion 162
Chapter 6 – Conclusions 163
6.1 Introduction...163
6.2 Lesson Study as a Professional Development Model to Change Primary School Teachers' Knowledge in Differentiating Mathematics Instruction ..164
6.2.1 Lesson Study Steps in this Study..164
6.2.2 Lesson Study and Mathematical Pedagogical Content Knowledge 165
6.2.3 Lesson Study and Differentiated Mathematics Instruction...................166
6.2.4 Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction..166
6.2.5 Potential Characteristics of Lesson Study..167
6.2.6 Digital Mixed Methods Research Design 168
6.3 Contributions to the Field of Mathematics Education................................168
6.4 Implications...169
6.5 Limitations and Suggestions for Future Research.......................................170
6.6 Summary...172
References 173
Appendices 191

List of Figures

Figure 1-1 Framework of this study 12

Figure 1-2 Connection between Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in this study..............13

Figure 2-1Model of Pedagogical Content Knowledge (this diagram is based on a combination of ideas from Grossman (1990), Shulman (1986, 1987), Gess-Newsome (1999a), Rittle-Johnson, Siegler, & Alibali (2001) and Park and Oliver (2008)) 22

Figure 2-2 Differentiated Instruction Model (adapted from Tomlinson (2003)) 32

Figure 2-3 Lesson Study Cycle (adapted from Baba (2007)) 43

Figure 2-4 Lesson Study cycle for differentiated mathematics teaching...................44

Figure 3-1 Mixed methods research design (adapted from the Digital Mixed Methods Design by O'Halloran et al. (2016))..54

Figure 3-2 Analysis page (screenshot) from Multimodal Analysis Video 63

Figure 3-3 Library folder (screenshot) of multimodal systems................................63

Figure 3-4 Sample visualisation of Multimodal Analysis Video (screenshot) 64

Figure 4-1 The steps of the study 66

Figure 4-2 Indicators of Mathematical Pedagogical Content Knowledge 67

Figure 4-3 Indicators of Differentiated Mathematics Instruction.............................68

Figure 4-4 Video analysis of a lesson..71

Figure 4-5 Transition diagrams of three states: (1) Mathematical Pedagogical Content Knowledge, (2) Differentiated Mathematics Instruction, (3) Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction..72

Figure 4-6 Lesson Plan A1..75

Figure 4-7 Mathematical Pedagogical Content Knowledge in Lesson A1 78

Figure 4-8 Teacher assists students in group (left) and individually (right) 79

Figure 4-9 Teacher helps a student to measure an angle 80

Figure 4-10 Homework for Lesson A1 81

Figure 4-11 Differentiated Mathematics Instruction in Lesson A1...........................82

Figure 4-12 Student points to an angle on the blackboard.......................................85

Figure 4-13 Worksheet for Lesson A1...87

Figure 4-14 Lesson Plan A2..91

Figure 4-15 Teacher doing something else while students are working in groups 91

Figure 4-16 Mathematical Pedagogical Content Knowledge in Lesson A2 93

Figure 4-17 Use of visual aids in Lesson A2 94

Figure 4-18 Homework question for Lesson A2 95

Figure 4-19 Differentiated Mathematics Instruction in Lesson A2 96
Figure 4-20 Teacher points to the building being observed by one group 97
Figure 4-21 Angles of 120^0 97
Figure 4-22 Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in School A: Lesson A1 104
Figure 4-23 Mathematical Pedagogical Content Knowledge Differentiated Mathematics Instruction in School A: Lesson A2 104
Figure 4-24 Relationship between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in School A...............................105
Figure 4-25 Lesson Plan B1 111
Figure 4-26 Teacher B wrote the commutative law formula 113
Figure 4-27 Mathematical Pedagogical Content Knowledge in Lesson B1 114
Figure 4-28 Differentiated Mathematics Instruction in Lesson B1 118
Figure 4-29 Lesson Plan B2 123
Figure 4-30 Mathematical Pedagogical Content Knowledge in Lesson B2 125
Figure 4-31 Rubric of Lesson B2 127
Figure 4-32 Differentiated Mathematics Instruction in Lesson B2 128
Figure 4-33 Teacher B proved associative law in addition...................................129
Figure 4-34 Differentiated Mathematics Instruction and Mathematical Pedagogical Content Knowledge in School B: Lesson B1 135
Figure 4-35 Differentiated Mathematics Instruction and Mathematical Pedagogical Content Knowledge in School B: Lesson B2 135
Figure 4-36 Relationship between Mathematical Pedagogical Content Knowledge (MPCK) and Differentiated Mathematics Instruction (DMI) in School B 136
Figure 4-37 Mathematical Pedagogical Content Knowledge (a); Differentiated Mathematics Instruction (b); connection between the two (c) in Lesson A1 ...138
Figure 4-38 Connections between mathematics teaching strategies knowledge and learning environment differentiation in Lesson A1 139
Figure 6-1 Lesson Study Cycles in this study 165

List of Tables

Table 3-1 Activities during field work and collected data..56

Table 3-2 Study participants..58

Table 3-3 Objectives of the study linked to the data sources....................................60

Table 4-1 Terms used in this study..70

Table 4-2 Mathematical Pedagogical Content Knowledge shown in Lesson A1 81

Table 4-3 Use of Differentiated Mathematics Instruction in Lesson A1...................88

Table 4-4 Mathematical Pedagogical Content Knowledge shown in Lesson A2 94

Table 4-5 Use of Differentiated Mathematics Instruction in Lesson A2...................98

Table 4-6 Mathematical Pedagogical Content Knowledge Indicators in Lessons A1 and A2 101

Table 4-7 Differentiated Mathematics Instruction Indicators in Lessons A1 and A2 102

Table 4-8 Mathematical Pedagogical Content Knowledge (MPCK) and Differentiated Mathematics Instruction (DMI) in School A 106

Table 4-9 Mathematical Pedagogical Content Knowledge shown in Lesson B1 115

Table 4-10 The use of Differentiated Mathematics Instruction in Lesson B1 119

Table 4-11 Mathematical Pedagogical Content Knowledge shown in Lesson 2 in School B 126

Table 4-12 Use of Differentiated Mathematics Instruction in Lesson 2 in School B 129

Table 4-13 Mathematical Pedagogical Content Knowledge indicators in Lessons B1 and B2..132

Table 4-14 Differentiated Mathematics Instruction indicators in Lessons B1 and B2 133

Table 4-15 Mathematical Pedagogical Content Knowledge (MPCK) and Differentiated Mathematics Instruction (DMI) in School B 137

Table 5-1 Connection between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction...152

List of Appendices

Appendix 1: Lesson Plan example of integers number 191
Appendix 2: Example of Worksheet 1 – Compare and Order Integers 214
Appendix 3: Example of Worksheet 2 – Addition with integers number (positive plus positive and negative plus positive) 218
Appendix 4: Example of Worksheet 3 – Addition with integers (positive plus negative and negative plus negative) 226
Appendix 5: Example of Worksheet 4: Subtraction with integers (positive take away positive and negative take away positive) 236
Appendix 6: Example of Worksheet 5: Subtraction with integer numbers (positive take away negative and negative take away negative)..245
Appendix 7: Lesson Study: Class observation form..251
Appendix 8: Lesson Study: Class observation guidelines 253
Appendix 9: Lesson Plan A1 (School A-Indonesian version) 254
Appendix 10: Lesson Plan A2 (School A-Indonesian version) 257
Appendix 11: Lesson Plan B1 (School B-Indonesian version) 260
Appendix 12: Lesson Plan B2 (School B-Indonesian version) 263
Appendix 13a: Approval Request Letter for Policy Maker in Nagekeo District (English version)...263
Appendix 13b: Approval request Letter for Policy Maker in Nagekeo District (Indonesian version)...268
Appendix 14a: Approval Request Letter for School Principals (English version) ..270
Appendix 14b: Approval Request Letter for School Principals (Indonesian version)..272
Appendix 15a: Participant Information Sheet (English version) 274
Appendix 15b: Participant Information Sheet (Indonesian version) 277
Appendix 16a: Participant Consent Form (English version) 280
Appendix 16b: Participant Consent Form (Indonesian version) 281
Appendix 17a: Participant Permission for Audio-visual recording (English version)..282
Appendix 17b: Participant Permission for Audio-visual recording (Indonesian version)..283
Appendix 18a: Parent/Guardian Information Sheet (English version) 284
Appendix 18b: Parent / Guardian Information Sheet (Indonesian version).............286
Appendix 19a: Parent/Guardian Consent Form (English version) 288
Appendix 19b: Parent / Guardian Consent Form (Indonesian version)...................289
Appendix 20a: Consent Form (English version) 291
Appendix 20b: Student Consent Form (Indonesian version) 292

Chapter 1 – Introduction to the Study

1.1 Introduction

In today's society, education is critical for individual countries and for the global economy as a whole (Mundy, 2007). As teachers are the main focus for educational improvement, their professional development and their knowledge of the areas they teach are central issues in education. Various types of professional development are offered to improve teachers' proficiency in pedagogy and subject knowledge. One type is Lesson Study, which has been chosen in this study as the vehicle for improving teachers' knowledge and skills in Indonesian primary schools.

In schools, students need to learn core subjects such as science, art, language, and mathematics. This study focuses on mathematics because it is a foundational subject that contributes to other areas such as the sciences, technology and engineering.

The need for accurate analysis in this study to understand the impact of Lesson Study on mathematics teaching and learning, specifically in relation to Differentiated Mathematics Instruction to address different student capabilities, led to the development of a new mixed methods digital research design for analysing classroom video data. In this study, this involved the use of Multimodal Analysis Video software for transforming qualitative analytical data into quantitative data (O'Halloran, Tan, Pham, Bateman, & Moere, 2016). Moreover, this software is able to visualise the results of the analysis of video data in order to discern patterns and trends.

This study integrates four major areas of education research:

- Lesson Study — a type of professional development.
- Mathematical Pedagogical Content Knowledge — the knowledge of content and the way of teaching that teachers need to possess in teaching mathematics.
- Differentiated Mathematics Instruction — a strategy in teaching mathematics that can meet the needs of diverse students in regular classroom.
- Multimodal Analysis Video — a new software that was used for analysing video data.

The study examines the impact of Lesson Study, as one type of professional development, on teachers' knowledge and skills in differentiating primary school mathematics instruction through a case study process. The study commenced with professional learning sessions about Lesson Study and Differentiated Mathematics Instruction that were delivered to groups of teachers in two Indonesian primary schools. Following the initial sessions, each group of teachers met to plan a lesson, which was then taught by one teacher and observed by the rest of the teachers in that group. Following this, each group of teachers met to review the lesson and refine the lesson plan for the next lesson, which in turn was delivered by one group member and reviewed by the others. In each case, lessons were video recorded and later analysed in order to study the impact of Lesson Study on Differentiated Mathematics Instruction.

The rest of this chapter covers the background and context of the study, statement of the research problem, the research questions, the nature and significance of the study, thesis organisation, and finally, a summary of all chapters in the thesis.

1.2 Background

1.2.1 Lesson Study

Teaching quality is critical in assisting students to achieve their learning goals; this quality is evident in teachers' pedagogical content knowledge. Teacher education and development is a key factor in improving the quality of teaching in the education sector (Ball & Cohen, 1999), and professional development has come to be seen as an important way of improving teachers' knowledge and practices in teaching. However, the development of teachers' knowledge and teaching skills cannot be achieved effectively through intermittent training (Hunzicker, 2011). Grossman and McDonald (2008) maintain that teachers' pedagogy needs to be enhanced through intense prolonged training. There has been much effort to improve teaching quality through professional development; in the last few decades, researchers and educators have become interested in developing more prolonged forms of professional development, including Lesson Study.

Lesson Study has been defined in several ways. For example Lewis, Perry, and Murata (2006, p. 3) say that Lesson Study is a “form of professional development that centres on collaborative study of live classroom lessons”. According to Cerbin and Kopp (2006, p. 250) “Lesson Study is a teaching improvement and knowledge building process”, and Dudley (2011, p. 2) writes that “Lesson Study is a highly specified form of classroom action research focusing on the development of teacher practice knowledge”.

In this study, Lesson Study is defined as professional development that offers continuity, collaboration, and refinement to enhance teachers’ knowledge and skills in order to facilitate students’ learning. Lesson Study has been practiced in Japan for more than a century (Murata, 2011), and it has extended to other countries including the United States (Lewis et al., 2006), Australia (Hollingsworth & Oliver, 2005; White & Lim, 2008), Malaysia (White & Lim, 2008), and Indonesia (Tatang, 2012). As a form of professional development, Lesson Study provides an opportunity to improve teachers’ knowledge and skills (Cerbin & Kopp, 2006) at every level and in every subject area, including mathematics.

1.2.2 Mathematical Pedagogical Content Knowledge

Shulman (1987, p. 8) defined pedagogical content knowledge as “the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction”. More recently, in a research context, Hashweh (2005) defined pedagogical content knowledge as:

> the set or repertoire of private and personal content-specific general event-based as well as story-based pedagogical constructions that the experienced teacher has developed as a result of repeated planning and teaching of, and reflection on the teaching of, the most regularly taught topics (p. 277).

Thus, teachers need to possess knowledge of both content and the way in which to deliver that content to students with different backgrounds and learning needs. In this study, Mathematical Pedagogical Content Knowledge is defined as the knowledge of mathematics content and how to deliver that content to diverse students in the classroom.

1.2.3 Differentiated Mathematics Instruction

Mathematics is intimately connected to our everyday lives and can assist us to solve problems (Bishop, 2003). It underpins most other disciplines, including science, technology, health and computing. Therefore, mathematics is a core subject in schools in every country (Orton & Frobisher, 2005), including Indonesia. With rapid changes in information and technology, knowledge develops all the time, and teachers need to modify their own knowledge and skills in order to prepare students to operate effectively in a rapidly changing environment. Indeed, knowledge is one important indicator of a professional teacher (Grossman, 1990; Shulman, 1986).

The diversity of students in a regular classroom is a major challenge. In this situation, differentiated instruction may provide a way to meet the variety of students' backgrounds. Differentiated instruction has been defined by Smit and Humpert (2012, p. 1153) as "an approach that enables teachers to plan strategically to meet the needs of every student" while Birnie (2014, p. 20) state "[d]ifferentiated instruction is teaching that accommodates all of the learning needs of all of the children in a class, enabling each child to attain the desired academic results".

In this study, Differentiated Mathematics Instruction is defined as a teaching strategy in a mathematics lesson that is designed to serve the learning needs of a variety of students in a regular classroom. However, teachers' knowledge and skills are the key factors in differentiation, and for this reason their knowledge and skills needs to be improved while they continue to play their role as a teacher. Lesson Study is a professional development model that can be implemented continuously while teachers are at work (Subadi, Khotimah, & Sutarni, 2013). In this study, while teachers improved their knowledge and skills through Lesson Study, they learned, planned and worked cooperatively in their attempts to differentiate mathematics instruction.

1.2.4 Multimodal Analysis Video (MMAV)

Video recording was the main data source in this study as this method can capture the teaching practices used throughout the entire lesson. To analyse the data from video recording, Multimodal Analysis Video (Multimodal Analysis Company, Singapore, 2013) was chosen, to assist as this software offers different procedures and results. Multimodal Analysis Video can be used to annotate videos using different theoretical frameworks, and in the process, transforms qualitative analyses into quantitative data (O'Halloran et al., 2016). The quantitative data is visualised in diagrams produced from this software, which can then be interpreted qualitatively and the video clips used to support the analysis.

1.3 Context of the Study

This study was set within an Indonesian context. Two primary schools from a rural area in Flores were chosen to conduct the research. The classes in those two schools are held in heterogeneous classrooms with diverse student needs. Even though Indonesian schools use a national curriculum, they may face different challenges in obtaining references and resources. Moreover, teachers may get different opportunities to develop their professional practice. For example, Lesson Study is known to some teachers in Indonesia, especially those in larger cities.

1.3.1 Mathematics Education in Indonesian Primary Schools

Mathematics is a core subject in the Indonesian curriculum. Many Indonesian students, however, are not interested in learning mathematics; this influences their mathematics learning and achievement (Aritonang, 2008). The lack of interest in learning mathematics may be due to a number of reasons, including teachers' qualifications, their knowledge about mathematics, their teaching methods for mathematics (Surya, 2010), and the personalities of students and teachers (Aritonang, 2008). Voinea and Purcaru (2014) found that students' interest in learning mathematics changed during the transition from primary to high school; this might be due to teachers' strategies and the way in which students were assessed. Aritonang (2008) believes that teachers play an important role in improving students' interest and motivation in learning mathematics. One effective way of improving student engagement may be for teachers to differentiate mathematics instruction so that it can meet the needs of all learners in regular classrooms and improve the learning outcomes of the students (Tomlinson & Eidson, 2003).

Although Indonesian primary school students perform well in international mathematics competitions (for example, the International Mathematical Olympiad (IMO), the International Mathematics Contest (IMC), and the International Mathematics and Science Olympiad (IMSO)), the average achievement of Indonesian primary school students in mathematics is low compared with other countries (Hendayana et al., 2011). For example, in the Trends in International Mathematics and Science Study (TIMSS) in 2007, Year 8 Indonesian students were ranked 36th out of 48 countries (Martin, Mullis, & Foy, 2008). In 2011, the same grade of Indonesian students were ranked 38th out of 42 countries in the TIMSS (Mullis, Martin, Foy, & Arora, 2012). There is a significant gap in mathematics achievement between the highest and lowest achieving students in Indonesia (Hadi, 2012).

Surya (2010) argued that the problem was not the students' capacity to learn but the quality of their teaching and instruction. According to Jalal et al. (2009) the low achievement of Indonesian students in the TIMSS reflected a low standard of teaching in Indonesian schools. Teachers' content knowledge of mathematics may also contribute to students' low performance; in the national civil service teachers' examination in 2004, teachers received an average score of 14.34 out of 40 on a range of mathematics questions (Jalal et al., 2009).

Many efforts have been made to improve Indonesian education quality in general, as well as mathematics education specifically. The Indonesian government has changed the curriculum ten times since the country's independence in 1945. In 2005 the government introduced a requirement for teacher certification in order to improve teachers' qualifications, but a recent report from the World Bank found that this has not had significant impact on students' achievements (*"Education in Indonesia: School's In"*, 2014). Traditionally, education in Indonesia has been teacher-centred

(Hendayana, Supriatna, & Imansyah, 2011). Hendayana et al. (2011) commented that, in a teacher-centred model, students are less active. Moreover, in teacher-centred education, knowledge or skills are transferred from the teachers to the students (Schuh, 2004). Students mostly follow teacher instructions, such as listening to the teacher and doing what the teacher asks, and keep quiet during the lesson (Daniels, Kalkman, & McCombs, 2001). On the other hand, student-centred education encourages students to engage actively during lessons; for example, by raising questions (Daniels et al., 2001). In a student-centred model, teachers act as facilitators, and this improves student learning (Granger et al., 2012).

Recently, the Indonesian government has required teachers to move from a teacher-centred approach to a student-centred approach; however, the curriculum still needs to be rearranged to better accommodate this (Lotan, 2006). Ultimately, though, the change in method to a more student-centred approach is implemented by individual teachers (Weimer, 2002).

1.3.2 Mathematics Content Knowledge of Primary School Mathematics Teachers in Indonesia

In teacher education institutions in Indonesia, primary school teachers are trained to teach all subjects. This may be one reason why primary teachers are not equipped with specialised knowledge for teaching mathematics. Knowledge of mathematical concepts is still lower than expected in the majority of Indonesian primary school teachers (Jalal et al., 2009). Many teachers also lack understanding of effective ways in which to teach mathematics (Fernandez, 2005). This has become a significant concern for the Indonesian government, which now wishes to improve teachers' knowledge and skills in teaching mathematics (Admin, 2013). Mathematics teachers have been introduced to a variety of strategies for teaching mathematics, such as realistic mathematics education for teaching geometry. However, the strategies in teaching mathematics that are available do not necessarily match the teachers' own mathematics knowledge. It is crucial to improve teachers' mathematics content knowledge, as Hill, Rowan, and Ball (2005) found that teachers' mathematical knowledge influences their students' achievement in mathematics.

1.3.3 Professional Development for Teachers in Indonesia

During a study of the training of science and mathematics teachers conducted in Indonesia in 2008–2009, many teachers expressed a strong desire to improve their professional competence but said that they needed assistance, support and opportunities (Surya, 2010). Surya (2010) recommended ongoing professional development to improve teachers' content and pedagogical knowledge and skills.

It is common for professional development in Indonesia to be delivered in a top-down style (Hadi, 2012). Teachers attend a workshop and learn new strategies or knowledge from the instructors, then return to their schools. However, there are limits as to how far this method of professional development can be taken. There is no consideration of whether the teachers apply the new knowledge and skill, nor is there evaluation about the effectiveness of the professional development on their content and pedagogical knowledge and skills. Therefore, more effective professional development is needed, and this needs to be measured and evaluated.

Teachers need to improve their professionalism by engaging in professional development consistently, not just occasionally. Most teachers engage in formal professional development, such as short courses, training or workshops to meet the institutional requirements (Friedman & Phillips, 2004) and to develop student achievement (Newmann, Rutter, & Smith, 1989). Teachers may either apply new knowledge after a one-off professional development session a few times, then revert to their previous approaches, or they may not use it at all (Supriatna, 2011). Soebari (2012) found that this happened because the material that teachers received from professional development may not appropriate to their schools' context; for example, differences in the number of students, or lack of resources and facilities. Ongoing professional development for teachers is important and needs to be undertaken while they continue their everyday duties (Jalal et al., 2009). Lesson Study is one type of professional development that can offer this continuity.

It is felt that continuous professional development based on action research is more effective and more likely to lead to student improvement (Bell, Aldridge, & Fraser, 2010). Dudley (2011) expresses the view that teachers will learn to conduct classroom action research through their use of Lesson Study. Thus, Lesson Study as a professional development, will improve teachers' quality in teaching and also assist Indonesian teachers to conduct classroom action research, as the research report is an element of Indonesian teacher certification (Sujanto, 2009). Since 2006, in order to improve the quality of teachers, the Indonesian government has required teachers to be certified through the *Teachers and Lecturers Law, Number 14, 2005,* as well as undertaking professional development activities (Fahmi, Maulana, & Yusuf, 2011).

Lesson Study may also help teachers to develop their professionalism without leaving their routine duties or requiring extra expenses, although Murata (2011) expressed concern that teachers might employ Lesson Study the same way as other professional development; that is, in limited sessions only. It may also be difficult to introduce a different kind of professional development such as Lesson Study because teachers have established their own teaching styles (Hadi, 2012). This is not unique to mathematics teachers. On the other hand, through effort and willingness to change the quality of mathematics education in Indonesia, Lesson Study could be an effective way to improve teachers' knowledge and skills in Differentiated Mathematics Instruction. Through Lesson Study, teachers share their knowledge and understanding and provide constructive support to their colleagues. Though teachers may struggle initially when they engage in Lesson Study activities if they lack sufficient knowledge (Chokshi & Fernandez, 2004), this can be addressed by enlisting external professional support (Dubin, 2010).

1.3.4 Heterogeneous Classrooms

Even though some schools have recently begun to offer streamed classes, especially acceleration for the most able students, Indonesian classes are usually mixed, resulting in a wide variety of students in regular classrooms. They have different backgrounds, abilities, interests, and readiness to learn. In regular classrooms, differentiated instruction may better meet student needs, particularly now the Indonesian government has mandated a student-centred learning approach. In this case, differentiated instruction can be an alternative way to enable students to achieve their learning goals and progress at their own rate (George, 2005).

1.4 Problem Statement

The spread and impact of Lesson Study globally is the motivating factor behind this study, which aims to investigate whether this approach to teachers' professional development can improve mathematics teaching with regard to differentiating mathematics. In particular, the study aims to investigate whether Lesson Study can improve mathematics teaching in a small town like the Nagekeo District on Flores Island in Indonesia. Although Lesson Study has been used in Indonesia since the early 2000s, not all teachers in this country are familiar with it. Lesson Study has been proposed as a professional development in this study for three reasons. First, it offers in-school learning, enabling teachers to learn while continuing with their teaching. Secondly, it opens opportunities to improve teachers' mathematics knowledge. It is believed that with deeper knowledge, teachers will be better equipped to assist their students to achieve their learning goals. Finally, Lesson Study enables teachers to learn ways to differentiate mathematics instruction to meet diverse of student learning needs in a regular classroom.

1.5 Research Question

The existing literature reveals that Lesson Study appears to be an effective strategy Study for assisting teachers to develop their knowledge and practice of mathematics differentiation. The characteristics and processes of Lesson Study have been explained in some studies. However, within a different study context, characteristics and processes may differ or be added during this study. Therefore, the primary research question in this study is: What impact does Lesson Study have on teachers' knowledge and skills in differentiating primary school mathematics instruction?

Based on this question, the objectives of this study are:

1. To describe the processes involved in the implementation of Lesson Study.
2. To investigate the influence of Lesson Study on primary school teachers' mathematics pedagogy.

3. To investigate the influence of Lesson Study on primary school teachers' curriculum differentiation in mathematics.
4. To investigate the connections between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction during Lesson Study.
5. To identify the key characteristics of Lesson Study as a form of professional development.

A second major research question that arises in the context of this study is a methodology for investigating the relationship between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction. Given the complexity of such an undertaking, a digital mixed methods research design is proposed, resulting in the following research question: What is the nature of a digital mixed methods research design for investigating the relation between teachers' Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction during Lesson Study?

Based on this question, a major objective of this study is to develop a digital mixed methods research approach for studying the impact of Lesson Study on teachers' knowledge and skills in differentiating mathematics instruction.

1.6 Conceptual Framework

This study involves three principal ideas: Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction. The relationship of these ideas can be seen by describing Lesson Study as the vehicle to improve teachers' Mathematical Pedagogical Content Knowledge for differentiating mathematics instruction, as shown in Figure 1-1.

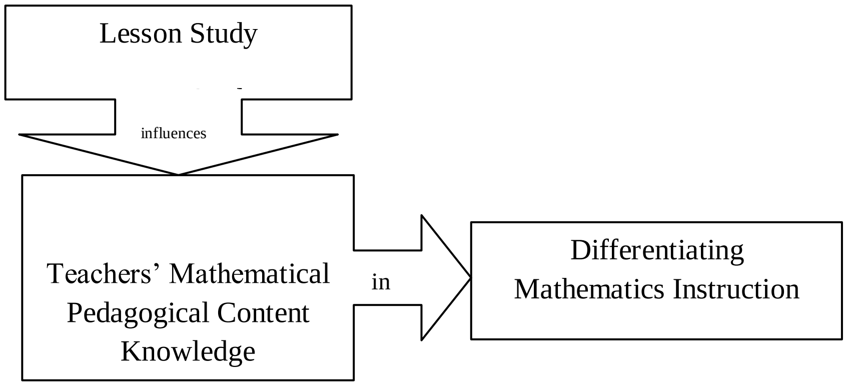


Figure 1-1 Framework of this study

As a professional development strategy, Lesson Study is used in this study to promote the development of teachers' Mathematical Pedagogical Content Knowledge and to assist them in differentiating mathematics instruction. Differentiated Mathematics Instruction was chosen as a strategy to facilitate meeting diverse student needs in a regular classroom. Thus, Lesson Study can be a vehicle to improve teacher knowledge and skills, which will impact the development of student learning, especially in mathematics (Yoshida, 2012), in this case through differentiated learning.

Mathematical Pedagogical Content Knowledge has been chosen as a key facet of this study because this knowledge is used to differentiate the mathematics instruction. In this study, Mathematical Pedagogical Content Knowledge includes mathematical content knowledge, mathematical pedagogical knowledge and mathematical context knowledge. Grossman, Wilson, and Shulman (1989) maintain that "content knowledge" consists of subject knowledge and its construction. Mathematical content knowledge, then, consists of mathematical conceptual knowledge, which is a teacher's' understanding of the concept of the topic being taught, and mathematics procedural knowledge, which shows a teacher's understanding of the procedure of the mathematics topic being taught (Ball, Hill, & Bass, 2005). Mathematical pedagogical knowledge is knowledge about teaching mathematics. It covers knowledge of students (Hill, Ball, & Schilling, 2008), mathematics teaching strategies and curriculum knowledge (Shulman, 1986). Knowledge of students is about knowing the way students think about the mathematics topic they are learning (Hill et al., 2008). Mathematical context knowledge is the knowledge that teachers need to have regarding the environment surrounding the classroom that may influence the process of teaching and learning (Barnett & Hodson, 2001). This knowledge includes central and local government regulations and knowledge of the community that may have an impact on student learning.

Differentiated Mathematics Instruction has been chosen as a strategy to facilitate meeting diverse student needs in a regular classroom. Differentiation includes content, process, product, assessment, and learning environment differentiation (Tomlinson, 2001, 2003), taking into account student profiles (Tomlinson et al., 2003), school environment and facilities, and the national curriculum.

Figure 1-2 shows the relationships between Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction. The diagram illustrates that, through professional development (Lesson Study), teachers can explore their mathematics knowledge to plan differentiated mathematics instruction and apply it in mathematics lessons.

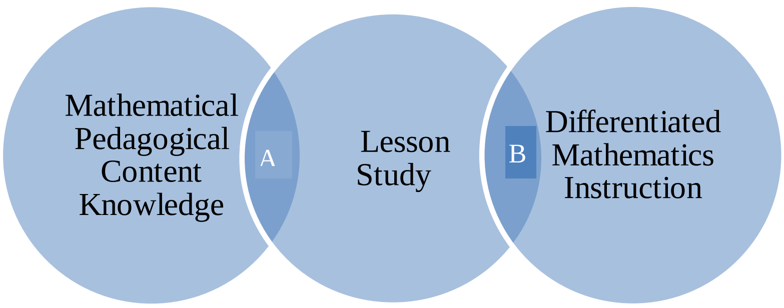


Figure 1-2 Connection between Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in this study

In Figure 1-2, the intersection of Mathematical Pedagogical Content Knowledge and Lesson Study (Area A) shows the way in which both concepts are used as the rationale in this study:

1. Lesson Study is believed to be able to improve teachers' content knowledge (Puchner & Taylor, 2006).
2. Based on the definition of pedagogical content knowledge that involves reflection on teaching which is the same as the step in Lesson Study that requires reflection for teachers to refine their practice (Hashweh, 2005).

Area B in Figure 1-2 shows the juncture of the concepts of Lesson Study and Differentiated Mathematics Instruction. The rationale for choosing Lesson Study for teachers to learn Differentiated Mathematics Instruction is given here:

1. "Planning for differentiated instruction is best done in collaboration with a trusted colleague or group of professionals" (O'Meara, 2010, p. 2). Lesson Study is a professional development program where teachers work as a team. Interaction between teachers formally or informally provides motivation for advancing development (Barnett & Hodson, 2001).
2. Differentiated instruction involves "lifelong learning" (O'Meara, 2010, p. 2). Lesson study is an ongoing learning process (Burghes & Robinson, 2009). It takes time to developing knowledge and skills of differentiation, and according to Lewis and Tsuchida (1999), Lesson Study is a continuous concept of study, as it is like a cycle in which teachers plan, apply and review to refine their teaching.
3. Differentiation is "a student-based process" (O'Meara, 2010). Lesson Study is also student centred, focusing on student development (Lewis, 2000).
4. Learning from previous experience. Assessment should be the basis for designing differentiated lessons (O'Meara, 2010). Lesson Study uses review of previous lessons (Baba, 2007).

1.7 Nature of the Study

This small-scale qualitative study employed a case study approach (Cohen, Manion, & Morrison, 2011; Creswell, 2013). A case study was used to investigate the impact of Lesson Study on the knowledge of teachers from two primary schools in Nagekeo District, Flores Island, Indonesia in differentiating mathematics instruction. Seven teachers from the two primary schools (School A and School B) participated in the study. Four teachers from School A and three teachers from School B formed two different Lesson Study Groups. The two groups worked at different times in their own schools. The use of an in-school professional development model enabled the teachers to save time and funds.

The data was collected mainly through video recording of the mathematics lessons and Lesson Study Group Meetings in two schools. Additional data was gathered from the teachers' observation forms from the lessons and the Lesson Plans created during the Lesson Study Group Meetings. The videos were analysed, using Multimodal Analysis Video software (O'Halloran, Tan, & E, 2015), to investigate the teachers' knowledge in differentiating mathematics instruction during the Lesson Study cycles. The videos from the lessons in both schools were transcribed and analysed based on the teachers' performance in differentiating the instruction and the Mathematical Pedagogical Content Knowledge demonstrated in the lessons.

1.8 Significance of the Study

There is no recent literature exploring the impact of Lesson Study on teachers' knowledge and skills in differentiating mathematics instruction in Indonesia; therefore, this study is significant to the Indonesian government's plan for educational reform. The findings of this study will benefit primary school mathematics teachers, who play an important role in assisting students in learning mathematics in regular classrooms.

Mathematics needs to be learnt by every student because of its role in everyday life. This study will contribute to the current knowledge on the influence of Lesson Study on teachers' knowledge and skills in differentiating mathematics instruction. Lesson Study has been implemented in parts of Indonesia since the early 2000s, but its impact on teacher knowledge and skills is not really understood. It is possible that the diverse needs of Indonesian students in regular classrooms may be met with Differentiated Mathematics Instruction, and that Lesson Study may be a vehicle for providing teachers with new knowledge and skills to plan and implement that differentiation.

There have been many studies investigating Lesson Study (for example, Lewis (2002), Takahashi and Yoshida (2004) and Murata, Bofferding, Pothen, Taylor, and Wischnia (2012)) and especially Lesson Study in Indonesia (for example, Marsigit (2007) and Suratno and Iskandar (2010)); Mathematical Pedagogical Content Knowledge (for example, Rittle-Johnson, Siegler, and Alibali (2001), Capraro,

Capraro, Parker, Kulm, and Raulerson (2005) and Kwong et al. (2007)); and Differentiated Mathematics Instruction (for example, Maes (2010) and Williams (2012)). However, studies on the link among all three concepts are hard to find. This small-scale case study thus provides a starting point for further research that examines the impact of Lesson Study on teachers' knowledge and the connection between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction.

Moreover, by using the Multimodal Analysis Video software, this study will develop a digital mixed methods research design for studying the impact of Lesson Study on teaching and learning mathematics. This design will contribute to the field of research that uses digital technology for the analysis of complex data. The Multimodal Analysis Video software produced quantitative data, which was analysed qualitatively in this study.

1.9 Organisation of the Thesis

This thesis is organised into six chapters.

Chapter 1 presents the background and context of the study, the problem statement, the research question, the nature and significance of the study, the organisation of the thesis and a summary of the chapter.

Chapter 2 reviews the literature relevant to this study, including both concepts and actual studies related to the theory. This chapter includes the topics of Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction, and the connection between these concepts.

Chapter 3 explains the research method used for this study and explores, in detail, the research question and objectives, research design, study methodology, data collection and data analysis.

Chapter 4 reports the results of this study from the two primary schools that participated in the Lesson Study cycles. This chapter describes the Lesson Study meetings and the mathematics lessons. In each Lesson Study meeting and mathematics lesson, the knowledge and skills of the teachers in differentiating mathematics instruction are identified. This is supported with the analysis outcomes from the Multimodal Analysis Video software with regards to the Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction. It also provides the means for making the connection between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction.

Chapter 5 discusses the results of this study. This chapter is organised according to the elements of the research question: the process involved in the implementation of Lesson Study in this study, the influence of Lesson Study on primary school teachers' mathematics pedagogy, the influence of teachers' Mathematical Pedagogical Content Knowledge in differentiating mathematics instruction, and the potential key characteristics of Lesson Study in this research.

Chapter 6 summarises this study and draws conclusions. This chapter also documents the contribution of this study to the field of mathematics education, its implications, limitations and suggestions for future research.

1.10 Summary

Providing effective professional development for teachers is a challenge for educators and governments. Teaching mathematics to a diversity of students in a regular classroom is a difficult task for mathematics teachers. Therefore, this study considers both teacher and student needs; that is, the need for teachers to improve their professionalism and the need of students for mathematics teaching that enables them to achieve their goals based on their own interests, readiness, and ability through Differentiated Mathematics Instruction. Lesson Study was investigated as a potential vehicle for meeting both of these needs.

Chapter 2 – Literature Review

2.1 Introduction

This study was inspired by the statement "one size doesn't fit all" (Gregory & Chapman, 2007, p. 1). In an interview much earlier, Howard Gardner claimed that "the biggest mistake of past centuries has been to treat all children as if they were variants of the same individual and thus to feel justified in teaching the same subjects in the same ways" (Gardner, Siegel, & Shaughnessy, 1994, p. 564). Each student is unique, therefore teachers need to differentiate their strategies when teaching different students (Forsten, Grant, & Hollas, 2002). Teachers need knowledge of both their students and pedagogy to differentiate their instruction (Ewens, 2014).

This chapter explores the ways in which teachers' mathematics pedagogical practice is built from theoretical perception, and how this knowledge may influence them in differentiating mathematics instruction. It begins with a brief discussion on Lesson Study as a professional development strategy to address the improvement of teachers' mathematics knowledge in differentiating mathematics instruction. The concepts of Lesson Study and its implementation are explored from the perspective of Shimahara (1998), Lewis (2002), Fernandez (2005), Saito, Harun, Kuboki, and Tachibana (2006) and Dudley (2014). This is followed by a concise discussion of pedagogical content knowledge based on Schwab (1978), Shulman (1986), Grossman (1990), Magnusson, Krajcik, and Borko (1999), Niess (2005) and Park and Oliver (2008). Next, the notion of Mathematical Pedagogical Content Knowledge is described based on the work of such professionals as Hiebert and Lefevre (1986), Carlsen (1999) and Schwartz (2008).

This chapter also investigates Differentiated Mathematics Instruction, mainly inspired by Tomlinson (1999, 2001, and 2003). The ideas of D'Amico and Gallaway (2008), Bender (2009), Small (2009), Williams (2012) and others are used to strengthen the concepts of differentiation. Finally, this chapter explores the links between the three major concepts used in this study: Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction.

2.2 Teachers' Mathematics Pedagogy

2.2.1 Pedagogical Content Knowledge

Numerous studies have explored pedagogical content knowledge; for example, Shulman (1986, 1987); Tamir (1988); Grossman (1990); Gudmundsdottir (1990); Magnusson, Krajcik and Borko (1999); Carlsen (1999); Niess (2005) and Park and Oliver (2008). Accordingly, a variety of definitions of the term have been suggested. Initially Shulman (1987, p. 8) defined pedagogical content knowledge as knowledge about how to manage and deliver a certain topic that is adjusted to students who have various "interests and abilities". Those two, interests and abilities are united to become "one body of understanding" (Shulman, 1986, p. 6).

Magnusson, Krajcik, and Borko (1999, p. 96) maintained that pedagogical content knowledge was a "teacher's understanding of how to help students understand specific subject matter". They explained that this knowledge includes an understanding of the way to manage, deliver and modify certain topics within a subject and present it for teaching students with various interests and abilities (Magnusson et al., 1999). Pedagogical content knowledge has also been described as "the intersection of knowledge of the subject matter with knowledge of teaching and learning" (Niess, 2005, p. 510). Park and Oliver (2008) further elaborated on this definition as a teacher's understanding and performance to assist a group of students to become knowledgeable about certain subject matter by employing various teaching strategies, depictions, and assessments while considering the students' learning circumstances of the students (Park & Oliver, 2008).

Shulman (1986, p. 9) states that pedagogical content knowledge consists of "the ways of representing and formulating the subject that makes it comprehensible to others" and is an "understanding of what makes the learning of specific topics easy or difficult". A teacher's pedagogical content knowledge is important and influences their students' improvement (Kleickmann et al., 2013). Moreover, academics and education policymakers believe that pedagogical content knowledge contributes to effective teaching and learning (Hill et al., 2008).

There is one important difference between Shulman (1986), and Gess-Newsome (1999a) and Grossman (1990) in positioning pedagogical content knowledge.

According to Shulman (1986, 1987) subject matter knowledge is positioned in a horizontal relationship to pedagogical content knowledge and curriculum knowledge to form the "categories of content knowledge" (Shulman, 1986, p. 9). In contrast to Shulman, Gess-Newsome (1999a) and Grossman (1990) refer to pedagogical content knowledge as a blend of subject matter knowledge, pedagogical knowledge and context knowledge.

Subject matter knowledge is an understanding of the content of a certain subject that will be taught and can be developed from teachers' own learning and experience through their education in institutional and professional development courses (Friedrichsen et al., 2009). The knowledge of the subject matter acquired during their own education contributes to their thinking. Shulman (1986) states that teachers need to know and understand the subject they teach. The knowledge of subject matter can be classified by its "substantive structure and syntactical structure" (Schwab, 1978, p. 25). Substantive knowledge includes "the key facts, concepts, principles and explanatory frameworks in a discipline". Syntactic knowledge is about "the nature of enquiry in the field, and how new knowledge is introduced and accepted in that community" (Rowland, Huckstep, & Thwaites, 2005, p. 255). Substantive refers to the content of the subject and syntactical refers to the processes used (Tamir, 1988).

Pedagogical knowledge is knowledge about how to deliver content (Shulman, 1986); the comprehensive theories and approaches of classroom management and organisation that go beyond subject matter (Shulman, 1987). Teachers need the ability to deliver a topic in a way that is both understandable for students and easy to accept (Shulman, 1986). Moreover, teachers are required to prepare their teaching thoroughly. How teachers deliver material to students originates from their "research" or "wisdom of practice" (Shulman, 1986, p. 9). They can gain these from experience and the motivation to improve their teaching quality. Thus, teaching experience contributes to developing teachers' knowledge (Shulman, 1986) and skills (Park & Oliver, 2008).

Context knowledge is information that teachers need to take into account when preparing their teaching. According to Grossman (1990), context knowledge influences pedagogical content knowledge and consists of knowledge about the state, community, district, school, and students. This knowledge includes the authority of the government in which the school is located, the character of the community and its culture (Shulman, 1987). Thus, context knowledge can be categorised as knowledge of central and local government regulations and community characteristics.

It can thus be said that pedagogical content knowledge is appropriate knowledge, understanding and skills that must be possessed by teachers to enable them to transmit content or topics in subject matter using a variety of teaching strategies to answer students' disparate needs in a heterogeneous class, taking into consideration the curricular requirements of the subject. Within pedagogical content knowledge, teachers need to acquire subject matter knowledge, which refers to the concepts and procedures of the material being taught; pedagogical knowledge, which refers to the way in which they deliver the material; and context knowledge, which refers to other factors influencing teaching and learning, such as regulation and environment. In the next section, these areas are explored more specifically in term of the mathematics lesson. They are defined as Mathematical Pedagogical Content Knowledge for the purposes of this study.

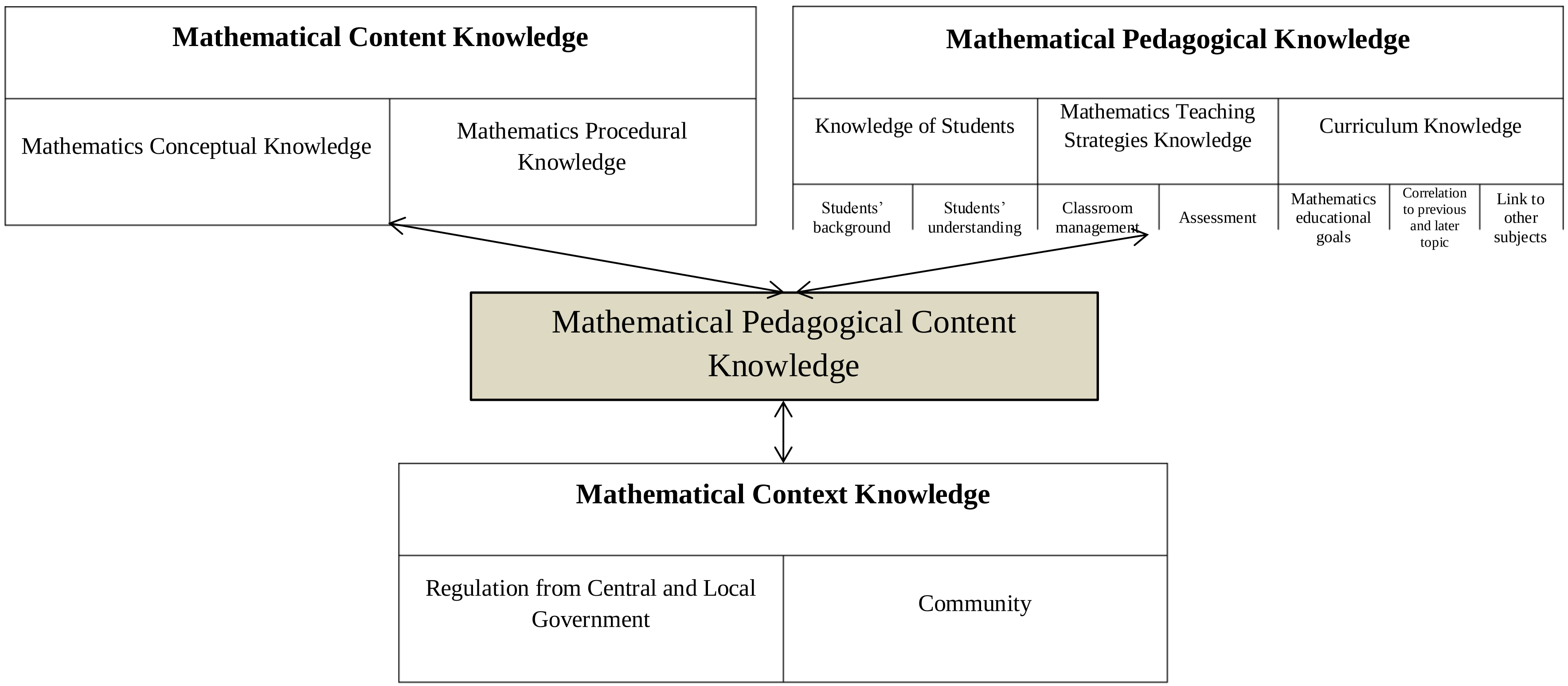


Figure 2-1Model of Pedagogical Content Knowledge (this diagram is based on a combination of ideas from Grossman (1990), Shulman (1986, 1987), Gess-Newsome (1999a), Rittle-Johnson, Siegler, & Alibali (2001) and Park and Oliver (2008))

2.2.2 Mathematical Pedagogical Content Knowledge

Many students and teachers see mathematics as abstract and difficult to understand. However, there is a relationship between mathematics as abstract knowledge and the "real world", the concrete experience of mathematics (Schwartz, 2008). Schwartz (2008) terms this connection "Mathematical Pedagogical Content Knowledge". This term can be understood as a combination of general pedagogical knowledge, approaches for the teaching of mathematics and content knowledge of mathematics (Kwong et al., 2007). Thus, Mathematical Pedagogical Content Knowledge is a blend of mathematical content knowledge, mathematical pedagogical knowledge, and mathematical context knowledge. Each component comprises different indicators, as shown in Figure 2-2.

Schwartz (2008) also suggests linking mathematics with other subjects and everyday life. This is important because many areas of knowledge and many occupations are based on the use of mathematics (Shi & Chen, 2006). For instance, students need to understand mathematics for learning economics, statistics, chemistry and biology. Moreover, by better understanding mathematics, students are more capable of solving daily problems related to mathematics, such as those involving time, money, and measurement: as an example, Aldo caught a train to Joondalup at 7.09 am yesterday. The trip took 1 hour and 12 minutes. At what time did Aldo arrive in Joondalup?

Mathematical Content Knowledge

Mathematical content knowledge is a comprehensive understanding of mathematics that has broadmindedness, depth, is linked, and is systematic (Ma, 2010). Understandings that can present connections between topics, that show concepts are similar and wider, and that demonstrate how topics or concepts are interrelated, become thorough information. Teachers can act as facilitators to assist their students to improve their knowledge and help prepare them for their future lives (Capraro et al., 2005).

There is a significant relationship between teachers' content knowledge and student achievement (Faulkner & R.Cain, 2013; Gess-Newsome, 1999b; Kleickmann et al., 2013; Mewborn, 2001). Teachers who lack subject matter knowledge tend to depend more on textbooks and less on student interaction (Lee, 1995). This means that students may not get the opportunity to explore the topic experientially. Mink (2010) advises teachers to avoid being highly dependent on the textbook.

Subject matter knowledge also influences teachers' strategies (Kleickmann et al., 2013) in delivering the topic being taught (Gudmundsdottir, 1990). Within the subject matter knowledge, teachers need to possess conceptual and procedural understanding in order to teach their students effectively (Hiebert & Lefevre, 1986; Hill & Lubienski, 2007; Rittle-Johnson et al., 2001). Conceptual and procedural knowledge are reciprocal; that is, they influence each other (Rittle-Johnson et al., 2001). Conceptual understanding and procedural skill are discussed further below.

Conceptual understanding is about knowing the theories in an area (Rittle-Johnson et al., 2001). Conceptual knowledge is characterised most clearly as knowledge that is rich in relationships (Hiebert & Lefevre, 1986). The pieces of information are interconnected with each other and when united become one whole knowledge. Mathematics teachers need to have a deep understanding of the mathematical concepts they teach. However, mathematics teachers tend to ask their students to memorise formulae and require the same procedure as they teach when solving a mathematics question or problem (Hadi, 2002). Moreover, Mewborn (2001) found that while teachers were generally competent to solve a problem within the topic area they taught, they were not capable of explaining the concepts they used to solve the problem. For instance, they could count the number of chicken wings if there were five chickens, but they could not differentiate whether the result come from 5 x 2 or 2 x 5.

In another example, most mathematics teachers, including those in Indonesia, ask their students to memorise the formulae for multiplication tables, but not to understand how they are derived or how to apply them. They are satisfied when their students can give the answer of 5 x 6 or 9 x 7 without necessarily understanding the meaning of those operations. Similarly, students are required to memorise that the area of triangle is half of the base multiplied by the vertical height, without understanding the logic of that formula. Teachers need to be able to explain the reasons for why something has happened logically (Shulman, 1986).

There is no alternative technique to restructure a procedure that cannot be remembered if only procedural knowledge is learnt (Schwartz, 2008). In fact, conceptual understanding is more vital, because a procedure can be forgotten. A procedure, however, can be restructured if the concepts that underlie it are properly understood (Schwartz, 2008).

Teachers also need to understand "students' common misconceptions" (Carlsen, 1999, p. 135). For instance, when calculating addition of decimal numbers, many students ignore the number of digits after the full stop and tend to add the numbers as they do in adding whole numbers; for example, $6.53 + 2.1 = 6.74$. Interestingly, a study found that students' misconceptions influence pedagogical content knowledge significantly (Park & Oliver, 2008). For example, when a teacher finds evidence of a student's misconception in adding two decimal numbers with different numbers of digits, the teachers may need to think more about what is behind the student's thinking and how to refine the misconception. Park and Oliver (2008) believe that when teachers identify their students' misconceptions, this impacts on their subsequent planning, teaching and assessment. Thus, this encourages teachers to seek deeper understanding of the misconceptions and, consequently, improves their knowledge.

Procedural skill is the capability to solve problems step by step and in order (Rittle-Johnson et al., 2001). This consists of knowledge about using symbols to represent mathematical concepts and about algorithms to solve mathematical problems (Hiebert & Lefevre, 1986). Thus, mathematical procedural knowledge is an understanding of operations to solve mathematical problems in order and to utilise mathematical symbols. Sometimes, mathematics teachers ask their students to copy the procedure to solve a problem; for example, multiplication with three-digit numbers, followed by a series of multiplication problems and ask their students to solve them by using the procedure that has been taught by the teachers in order to make students fluent in doing multiplication. However, Mink (2010) argues this way of teaching a mathematics procedure does not make students really understand what they are doing even if they can do all the questions correctly. "In order for students to make meaning in mathematics, they must understand both the how and why" (Mink, 2010, p. 125).

By mastering the content and skills of mathematics, teachers can recognise how deeply their students understand the concepts and can teach appropriately, building on the students' current levels of understanding. According to Schwartz (2008), these deficiencies can be addressed through practice. Mewborn (2001) also advises that trainee primary school mathematics teachers need to learn more mathematics. Improving teachers' knowledge and skills to recognise students' "mathematical work and thinking" now gets much attention in pre-service teacher education and professional development programs in California's Mathematics Professional Development Institutes (Hill et al., 2008, p. 373). Sometimes students solve or compute a problem by using methods different to the procedure that their teachers taught. In this case, teachers need to be able to evaluate whether the procedures that their students use are appropriate, can be generalised to other questions (Hill & Lubienski, 2007) and are valid. "Conceptual understanding" and "procedural skill" are needed in learning and doing mathematics (Rittle-Johnson et al., 2001, p. 346).

Mathematical Pedagogical Knowledge

Even if a teacher is knowledgeable about a subject, if they have difficulties in delivering the lesson this creates problems for the students in understanding the content. When teaching, teachers need to have a good grasp of the subject matter and effective teaching strategies, together with knowledge about their students' backgrounds. More frequently, when teaching a particular topic or material, a teacher will be more confident when they understand more, know the most appropriate strategies to be used, and are able to recognise what level of difficulty is suitable for their students' level of thinking. Many sources can be used to improve teachers' knowledge of teaching (Grossman, 1990), including research reports, other teachers' experience and reflection on the teachers' own experience.

Knowledge of students can be classified as knowledge about students' backgrounds and their understanding of a certain topic. Knowledge of students' backgrounds can be defined as an understanding of students' "learning difficulties, motivation, and diversity in ability, learning styles, interest, developmental level, and need" (Park &

Oliver, 2008, p. 266). Knowledge of students' understanding can be said to be knowledge about their previous knowledge, conceptions and misconceptions about the topic being taught (Grossman, 1990). These factors need to be considered when modifying teaching strategies. To develop a knowledge of students' understanding, Tamir (1988) advises teachers to respect students' answers. Attention that is given to students' answers, whether right or wrong, can help teachers to recognise and understand their students' conceptions and misconceptions. This is equally, or perhaps, more important than whether the final answer is "right" or "wrong".

Mathematics teaching strategies refers to the variety of approaches used to deliver mathematics concepts (Baker & Chick, 2006). Knowledge of teaching strategies is a foundation for improving students' knowledge of mathematical concepts (Bray, 2011). The use of various teaching strategies also helps students understand the content taught effectively (Baker & Chick, 2006). For instance, teachers may use "pictures or manipulatives" to assist in teaching the concepts and procedures of a certain topic in mathematics that is usually abstract for students (Hill & Lubienski, 2007, p. 753). Mathematics is more understandable and easier if every learner has the opportunity to move from the concrete to the abstract, no matter what level.

Knowledge of teaching strategies has two components, classroom management and assessment. Classroom management has a significant relationship to the way in which teachers deliver a lesson (Lee, 1995). When teachers are able to keep control in the classroom during a lesson, every single student may learn more effectively. Students with discipline problems are not only low achieving students, who may not be interested in the topic, but may also be high achieving students who do not get appropriate instruction that challenges them (Lee, 1995). When disruptive behaviours are not managed, for example, students chat rather than work on an assignment, this disturbs other students in the class. Birnie (2014) believes that when teachers are able to deliver instruction properly it is easier to control students, because they are engaged in the lesson. In addition, Brophy (2010) states that in managing a classroom is not just about how to force students to obey their rules, but more about how to create a conducive learning environment that requires students to actively engage in learning.

The second component of teaching strategies knowledge is knowledge of assessment. This includes knowledge about assessment methods, instruments, and approaches (Park & Oliver, 2008). Effective assessment also needs to consider the goal of the assessment and to reflect real student achievements. According to Wragg (2004), assessment methods can be differentiated into informal and formal methods. Informal methods can be used during the lesson by asking questions, observation, and monitoring (Wragg, 2004). Two other formats of assessment can be implemented during a lesson: peer-assessment, which can build student ability and tolerance for group work, and self-assessment, which can develop students' responsibility for their own learning progress (Black, 2010; Harrison, 2010; Tillema, 2010). Harrison (2010) classifies peer assessment and self-assessment as formal assessment. Moreover, he states that peer-assessment can be classified as formative or summative assessment depending on whether it is done during the lesson or at the end of the learning process. Formal methods are set in a certain timeframe and are done through written, practical, oral and standardised tests (Wragg, 2004).

There are many kinds of assessment instruments; for example, "rating or checklists or marking schemes" in peer and self-assessment (Harrison, 2010, p. 231), and portfolios for self-assessment (Klenowski, 2010). There are some assessment approaches that can be applied in the classroom; for example, a "dialogic approach" that requires students to be involved in discussion (Brown & Hodgen, 2010, p. 282). "Holistic or analytic approaches" consider the whole body of work of students, such as portfolios (Klenowski, 2010, p. 239), and these use a "cognitive approach [in] which assessment can be integrated with learning" (James, 2010, p. 167).

Curriculum knowledge is needed to determine the concepts to be taught and curriculum knowledge needs to be adjusted to the teaching strategies that will be used (Park & Oliver, 2008). Curriculum knowledge covers the subject's capacity and structure in conjunction with the information applied in the teaching process (Rowland et al., 2005). Curriculum resources, for example textbooks, are also considered in order to manage the program of study (Hill et al., 2005). In addition, the educational goals need to be considered when modifying teaching strategies (Park & Oliver, 2008). As well as understanding the specific topic being taught, teachers also need an understanding of the linkages between that topic and other topics in the same subject, and between the topic and other topics in other subjects (Shulman, 1987). For example, learning about multiplication in maths involves not only memorising the times table but also understanding the concept of multiplication. This is important for learning, not only in performing mathematical procedures such as calculating the area of a rectangle, but also in other subjects such as medicine, where multiplication is used to calculate the dosage of a medicine that should be consumed. Shulman (1987) argues that it is a teaching prerequisite for teachers to be able to associate old and recent knowledge that develops over time. Teachers need to think about the application of certain mathematical topics to other subjects that they teach, such as science or social studies (Schwartz, 2008). This can help students to learn that mathematics forms a basis for other knowledge.

Mathematical Context Knowledge

Context knowledge is about the learning environment that may influence the learning process. This can include the state, district, school, classroom and community (Barnett & Hodson, 2001; Park & Oliver, 2008). Barnett and Hodson (2001) classify those factors as external sources; an internal source of context knowledge may stem from a teacher's own experience in teaching.

Regulation by central and local government may influence teachers' strategies for improving their students' achievements. For example, in Indonesia, there is a national examination for Years 6, 9 and 12. The results from these examinations play a major role in determining whether or not students can move from one level to the next level, for example from primary school to junior high school, even though the national examination is supposed to constitute only a small proportion of a student's assessment compared with the overall school assessment (Wragg, 2004). Thus, teachers of these Years will focus on the topics that will be tested in the national examination exam. School policies are another source of context knowledge (Barnett & Hodson, 2001).

The community also influences the way in which teachers teach. Schwartz (2008) advises that mathematics teachers need to connect mathematics concepts to real life; when using an example in teaching mathematics, teachers need to consider whether it is relevant to students' daily life and their environment. For example, in low-lying land in a remote area, using an example of a well is more appropriate than using a lift to represent negative and positive numbers. When teaching about measurement, it is important to use the measurement system common to that country; for example, kilometres, metres, centimetres and millimetres in Indonesia and miles, yards, feet and inches in America. Knowing about the community is relevant to all educational matters (Barnett & Hodson, 2001).

In conclusion, Mathematical Pedagogical Content Knowledge is a teacher's knowledge about the content of mathematics, the method of teaching mathematics, and the circumstances in which they are teaching. Teaching experience is a key factor in improving teacher knowledge but this cannot be equated with teaching proficiency (Gess-Newsome & Lederman, 1995). However, Shulman (1986) states that teacher knowledge can improve through teaching. Teaching skills will also develop as teachers learn from their previous experience. Teaching is similar to learning, and can be said to be a learned career (Shulman, 1987).

2.3 Differentiated Mathematics Instruction in Heterogeneous Classrooms

As previously discussed, students in regular classrooms have diverse backgrounds and abilities. Differentiation in teaching can cater for that diversity in a heterogeneous classroom. Differentiated teaching is different from traditional teaching, and does not necessarily involve direct instruction. It uses similar content, but utilises diverse levels of cognitive thinking, for different groups (Bender, 2009). Teachers adjust their activities to cater for particular students' readiness, interests, and learning styles. Subsequently, they can deliver effective teaching, which can both improve students' interest and help them to realise their hidden interests (Tomlinson, 2003).

Teachers also need to consider standard curriculum expectations and school facilities. The curriculum, teaching processes, activities provided during the lesson and products of learning can all be modified, with the main objective being to maximise student learning (Tomlinson, 1999). It is also important that the methods for assessing student learning and the learning environment are modified to meet student needs (Tomlinson, 2003). Differentiation may differ from topic to topic because the interest and ability of students in those classrooms varies depending on the learning area (D'Amico & Gallaway, 2008). Even within the same topic, differentiation may vary when the school year changes and when it is being applied in different parallel classrooms (Birnie, 2014). This aspect of differentiation requires teachers to respond thoughtfully to their students' needs. The students' progress and response to lessons form the basis for modifying future lesson plans.

The diversity of students is also happen in Indonesian regular classroom. Indonesian schools try to assist the different level of academic ability students by providing remedial, enrichment, and acceleration program. However, those programs seem to group the students based on their academic achievement merely (Gunawan, 2007). In her study, Wilujeng (2012) stated that differentiated instruction that is used in a heterogeneous classroom was effective in a small class. The limitation of differentiated instruction study in Indonesia opens a lot of opportunity to develop this strategy in heterogeneous classrooms. Even though, Indonesia has applied inclusive education, frequently, the teachers still treat their students at the same way and less able to acknowledge students' needs and diversity (Rachmawati, Nu'man, Widiasmara, & Wibisono, 2016).

2.3.1 Characteristics of Differentiated Instruction

Differentiated instruction was initially designed to serve the needs of gifted students, but can also be applied in inclusive classrooms with diverse students who may be recognised as able, adequate students, less able students, and even students with special needs (Lawrence-Brown, 2004). Differentiated instruction's influence has been profound, and wherever it is used it has some common characteristics (O'Meara, 2010; Tomlinson, 1999). Both O'Meara (2010) and Tomlinson (1999) have stated that differentiated instruction meets the needs of all students and is differentiated by content, process, and product; is student centre; employs ongoing assessment to determine student achievement and provide records of students' readiness, interests, and learning preferences to design differentiated lessons; and uses flexible grouping.

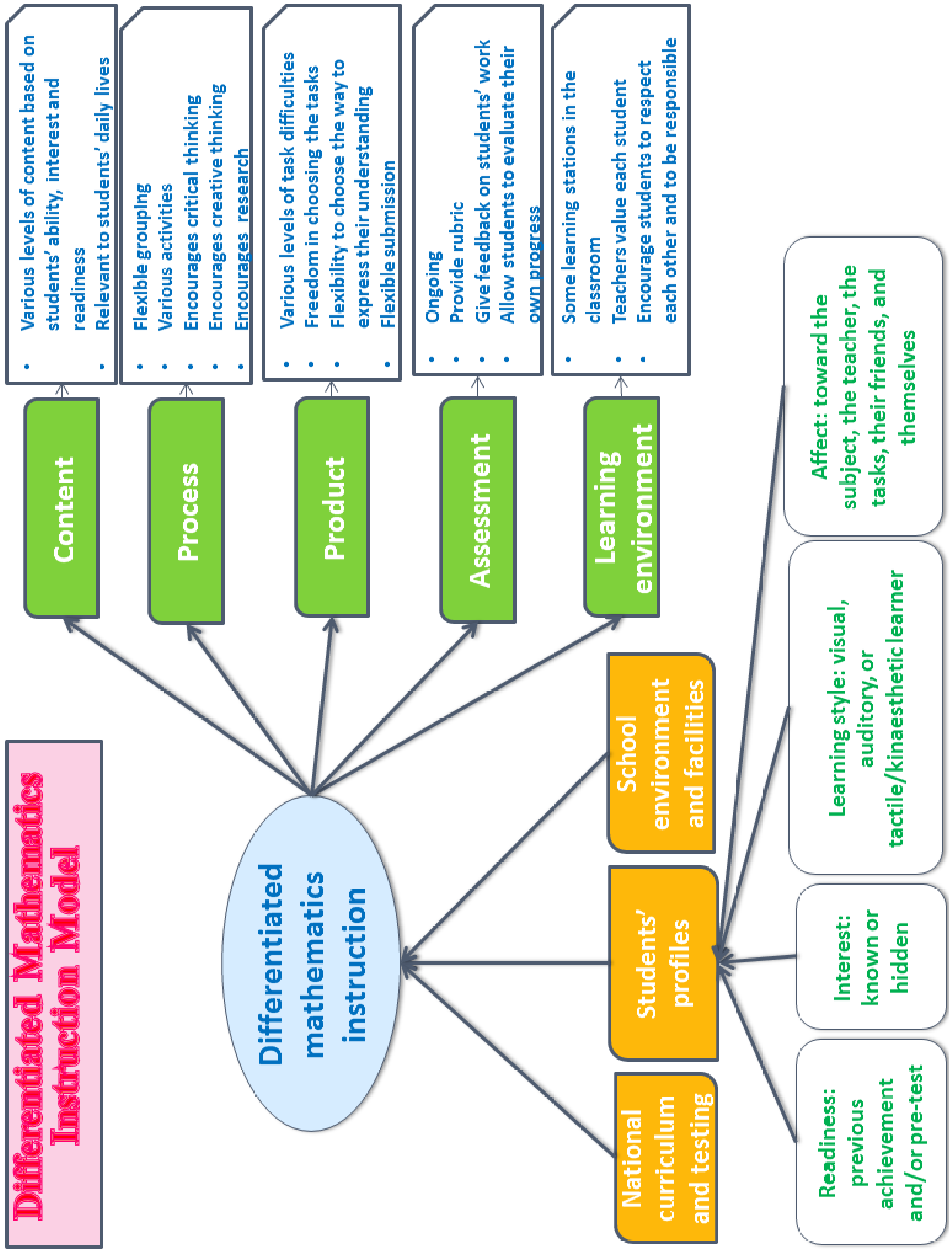


Figure 2-2 Differentiated Instruction Model (adapted from Tomlinson (2003))

2.3.2 Factors that Influence Differentiation

Three main factors must be considered in differentiated instruction: the national curriculum, including compulsory national tests and examinations, students' profiles, and the school environment and facilities.

National Curriculum

Most countries, including Australia, have a national curriculum that is intended to benefit students, teachers and economy ("National curriculum benefits kids, teachers, economy", 2010). "Standards-based curriculum" is another term that is frequently used and this describes basic knowledge that the national curriculum decrees is needed by all students (O'Meara, 2010, p. 17). It is argued that, with a national curriculum, students can adapt to lessons much more easily when they need to move to another school in a different place ("National curriculum benefits kids, teachers, economy", 2010). A national curriculum is also convenient for teachers who move interstate for promotion and other reasons.

A national curriculum may also imply national examinations. For example, in Indonesia, a compulsory national examination is used to gauge schools' and students' performance. During the final year of primary school, Year Six, students in Indonesia have to sit the National Examination as one of the requirements to decide whether they pass primary level and can continue to junior high school. Therefore, teachers need to provide a certain amount of national examination practice to accustom students to the national examination and possibly reduce their anxiety (Wragg, 2004). McTighe and Brown (2005, p. 237) state that students may achieve the standards of national examinations when teachers apply "diagnostic and ongoing assessment" which are the types of the differentiation used in assessment.

With a national curriculum, teachers have a guideline for preparing their lessons. They know what outcomes are expected of their students. Teachers can identify the teaching goals and can anticipate more appropriate strategies to use to achieve the objectives (O'Meara, 2010). O'Meara (2010) also states that recognised standards in the beginning are crucial because they influence differentiated instruction.

It seems that fully differentiated instruction is difficult to apply when education standards need to be met. However, McTighe and Brown (2005) strongly believe that education standards and differentiation can be used in conjunction to enable students to achieve their learning goals. McTighe and Brown (2005) also believe that teachers need knowledge of the curriculum and student readiness, together with the ability to design differentiation that will enable students to achieve the required standards as well as their own learning goals.

Student Profiles

The second point that needs to be considered in differentiated instruction is students' profiles, which include their readiness, interests, learning styles and affect (Tomlinson, 2003). Every student in a regular classroom is unique. They have different "learning styles, intelligence preferences, culture, and gender" (Tomlinson, 2003, p. 3). Students also have different interests (Bransford, 2000), talents, family backgrounds and capacity for learning. However, in Indonesian primary school classrooms, all students are usually required to learn the same topics at the same level of difficulty in the same period. Almost all Indonesian schools require teachers to base their teaching on the National Standard Curriculum, as students have to sit for the national examination at the end of Year Six. Birnie (2014) advises teachers to interview or observe their students and build conversations with parents or guardians, in order to gain more information regarding students' backgrounds that are related to the students' own study at school.

Student readiness refers to their earlier stage of knowledge or skill in a certain learning area (Tomlinson & Imbeau, 2013) and is not the same as student ability (Tomlinson, 2014). Student readiness can be determined by previous achievement. Diagnostic tests can also be used to recognise student readiness; teachers can then manage differentiation strategies for their lessons. For example, they may use a diagnostic test to group students and prepare tasks with different levels of difficulty. Tasks that are appropriate for students' readiness will help them to improve their learning (Tomlinson et al., 2003). "Tasks that are too easy become boring; tasks that are too difficult cause frustration" (Bransford, 2000, p. 61).

“Interest refers to a learner’s affinity, curiosity, or passion for a particular topic or skill” (Tomlinson, 2014, p. 19). In order for students to learn something, it is essential that it is based on their interest (Bransford, 2000), because only then will it increase their intellectual achievement (Tomlinson et al., 2003). It will also improve students’ motivation in studying (Tomlinson & Imbeau, 2013). Jones and Allebone (2000) state that students are more enthusiastic about completing work that attracts their interests. They say that teachers may group the students based on their interest. It is possible for students in one group to have the same level of interest but different levels of ability (Levy, 2008). There is a strong relationship between interest and motivation; this can help students improve their understanding and skills in learning (Turville, 2011). Turville (2011) advises the use of activities that can assist students to achieve learning goals based on their interests; activities such as “choice boards”, “cubing” and “learning contracts” allow students to choose the task that suits their interest.

Styles describes “a set of preferences” (Sternberg & Grigorenko, 2001). Learning styles assist in understanding students’ preferences in learning. The way in students learn is different. D’Amico and Gallaway (2008) described three kinds of learning styles. Visual learners prefer learning by seeing and tend to prefer to use, for example, videos, slides, pictures, charts, maps and diagrams. Auditory learners learn most effectively by listening; for example, listening to tapes, tutors, radio and other people. Auditory learners like discussion and hearing others’ opinions. Tactile or kinaesthetic learners improve their understanding by doing experiments, touching, and moving. Based on these types of learning, teachers can deliver material in various ways; for example, using videos or radio, or doing experiments. Levy (2008, p. 163) argues that students who can improve their understanding through a discussion can be classified as “verbal learners”. Students can be grouped in the same learning style, so they can learn the same way, or they can be put in a mixed group so they can assist each other (Levy, 2008).

Affect refers to students’ emotions towards the situation around them during a lesson (Tomlinson, 2003). Teachers also have a responsibility to be perceptive of students’ reactions that are brought about by conditions outside of themselves, such as a friend’s behaviour or the lesson workload. These issues should then be discussed with the students (Tomlinson, 2003).

School Environment and Facilities

“The environment has a great impact on student learning” (O'Meara, 2010, p. 92). For instance, a teacher may acknowledge that a traditional market near the school, the noise from vehicles passing the school or the sound of aeroplanes taking off or landing near the school will to affect the attention of the students. It is also important to consider students’ sense of safety, both physically and mentally (O'Meara, 2010). Disparity in regular classrooms needs to be appreciated, and student enquiries and errors respected, as these are a part of the learning process.

It is important to consider school facilities, especially in small or remote areas, as they influence student learning processes and achievements (Schneider, 2002). For example, if there is no computer in the school, teachers and students cannot participate in activities that require a computer in order to complete the task. Teachers may need to be creative; using available equipment or developing their own simple resources.

2.3.3 Modification of the Teaching and Learning Process

Based on the national curriculum, student profiles, and school environment and facilities, teachers can differentiate instruction. Differentiated instruction can be undertaken in relation to content, process, product, learning environment (Tomlinson, 2003) and assessment (Stefanakis, 2011; Tomlinson & Moon, 2013).

Content Differentiation

Content refers to material that students need to learn completely (Tomlinson, 1999) as part of the study process (Tomlinson & Eidson, 2003). Based on students’ levels of ability, interest and readiness, teachers can modify the content by providing various materials (Tomlinson, 1999) with different levels of difficulty and abstraction within the same topic (Conklin, 2010). For example, when teaching multiplication, teachers can provide one-, two- or three-digit examples based on students' readiness. Tomlinson (1999) also suggests that the content being taught needs to relate to daily life and present actual problems as well as exercises based on the set textbooks. For example, when teaching about whole numbers or introducing positive and negative numbers, teachers can use examples based on lifts in a tall building or other facilities near the school.

Process Differentiation

Process refers to activities within a lesson that assist students to achieve competencies required by the curriculum (Tomlinson & Eidson, 2003). The teaching and learning process is dynamic and can be differentiated through measures such as grouping students flexibly, providing various activities, encouraging critical and creative thinking, and promoting research. With flexible grouping, students are allowed to work alone or in a group (Strickland, 2007; Tomlinson, 1999, 2003; Tomlinson, Brimijoin, & Narvaez, 2008). They can work in pairs or with friends who have similar or different abilities and interests. Conklin (2010) advises changing the group regularly, maybe within a lesson, a week or a topic, to avoid labelling being attributed to specific groups; such as 'low ability' or 'high ability' in a homogeneous grouping. Flexible grouping enables students who may not be confident speaking in a big group to speak in a small group or with a single partner. Grouping students also allows them to practice a mathematics procedure that has just been taught (Mink, 2010). Teachers can still allocate time and manage discussion for a whole class at the beginning or end of the lesson; for example, at the end of the lesson, teachers can facilitate students' summaries of the topic they learnt that day.

Mcllvenny (2013, p. 18) defines "critical thinking (divergent thinking)" as occurring when "students recognise or develop an argument, use evidence in support of that argument, draw reasoned conclusions, and use information to solve problems". Students need to understand the content that they learn, so that they can argue in a positive way which criticises ideas, but does not attack personalities (Dinuta, 2015). Thinking critically can be improved in the learning process daily through discussions and problem solving which relies on mathematical content (Maricica & Spijunovicb, 2015). Teachers can also encourage students to create critical thinking questions for their classmates in paired or group discussion (King, 1995). In this growing era of technology, online discussion can be used to teach critical thinking (MacKnight, 2000). King (1995) suggests critical thinking questions such as asking the reason for something, comparing, requiring examples of an idea, and asking for the application of an idea.

Mcllvenny (2013, p. 18) defines “creative thinking (convergent thinking)” as occurring when “students learn to generate and apply new ideas in specific contexts, see existing situations in a new way, identify alternative explanations, and see or make new links that generate a positive outcome”. People think creatively when they have to solve a problem caused by a change in a certain situation (Monahan, 2002) and they think and problem solve in a different way to their customary technique (Low, 2006). In the classroom, when teachers demonstrate creative thinking, this encourages students to think creatively as well (Piggott, 2007). Piggott (2007, pp. 4-5) suggests three ways to boost students creative thinking: “posing problems”, which leads to investigation, “questioning” such as raising open questions, and “solutions”, where teachers allow students to answer in different ways as long as the answer is correct.

Product Differentiation

Products determine students’ understanding and ability after they have been learning a certain topic or skills in a certain timeframe (Tomlinson & Eidson, 2003). To address different levels of student ability, tasks can be arranged in various levels of difficulty (Tomlinson, 2003). Appropriate levels of task complexity can accommodate different degrees of knowledge (Pnevmatikos & Trikkaliotis, 2012; Tomlinson, 1999). This can also ensure that students are being challenged (Strickland, 2007). Information about student readiness can be gleaned from previous school reports, observation, or, if needed, pre-tests.

Student can also be given freedom to show how they express their learning (Tomlinson, 1999). For example, they may explain their understanding orally, in written form or using charts. Flexibility in submitting a task is worth considering because students have different abilities in accomplishing tasks (Heacox, 2002; Pnevmatikos & Trikkaliotis, 2012; Tomlinson, 1999). Students who can complete their tasks correctly and quickly can then attempt more challenging tasks independently.

Assessment Differentiation

As opposed to conventional teaching, which usually assesses students the end of a unit, topic or period of time (summative assessment), in a differentiated classroom student progress is also assessed periodically during the learning process (formative assessment) (Strickland, 2007; Tillema, 2010; Tomlinson, 1999). Conklin (2010) suggests the use of both formative and summative assessment when differentiating instruction. Tomlinson (2014, p. 17) states that "in a differentiated classroom, assessment is diagnostic and ongoing". Diagnostic assessment affords everyday information about students' readiness, interests, and learning profiles (McTighe & Brown, 2005; Tomlinson, 1999). Portfolios can be used to inform teachers about their students' backgrounds (Stefanakis & Meier, 2010). Conklin (2010, p. 31) calls diagnostic assessment "pre-assessment" and explains that this can be done through "classroom discussions, quizzes, tests, or journals". Ongoing assessment means that teachers assess students in every lesson through measures such as portfolios, homework or teacher observation (Tomlinson, 2014). Portfolios in particular require a lot of teacher time to assess, but can really show students' work over a period, and Gardner argues that "[i]f you do not have time to look at your students' work, you should stop teaching" (Gardner et al., 1994, p. 564).

Other kinds of assessment that can be used to recognise students' knowledge and skills mathematically are "observation, interview, performance assessment, and work sample analysis" (Literacy and Numeracy Secretariat, 2008, p. 7). McTighe and Brown (2005, p. 236) suggest that in differentiation, "assessment should require students' demonstrations of understanding, not just recall of information or formulaic modelling".

In assessment differentiation, teachers can create "open and parallel tasks" to encourage students to think about and solve problems creatively (Literacy and Numeracy Secretariat, 2008, p. 7). Open-ended questions allow students to express their understanding using their own words and thinking (Wasik & Hindman, 2013). However, sometimes students encounter difficulties in answering open-ended questions, so teachers need to plan another strategy, such as, putting these students in a small group with the teacher to encourage them to speak (Wasik & Hindman, 2013).

Students’ products can be assessed by both the students themselves and the teachers (D'Amico & Gallaway, 2008). Self-assessment can build students’ self-confidence. Teachers can also do observations during the lesson time to assess their students’ progress. Formative assessment assists teachers to consider student understanding before the end of a unit (Conklin, 2010). The results of formative assessment are then used to modify the following lessons (Marshall, 2010).

Differentiated assessment assists teachers in planning the next differentiated instruction (Stefanakis, 2011) because they can recognise student learning styles and needs through the use of rubrics (Jackson & Larkin, 2002). Stefanakis (2011) suggests that teachers use rubrics to provide information for students in areas of strength and weakness. Rubrics are a guideline to assess students’ work (Jackson & Larkin, 2002).

Learning Environment Differentiation

Classroom environment also plays an important role in differentiation. This includes physical aspects, such as the arrangement of resources, and mental aspects, such as class roles (Tomlinson, 2003). A comfortable environment will support students in their learning.

Cox (2008, p. 54) suggests three kinds of physical classroom arrangement to differentiate the learning environment: “interest centers”, with an emphasis on the topic being taught, “learning centers”, which provide materials and activities to facilitate learning, and “learning stations” where students can go to work on assignments. In interest centres, the tasks are based on variation in student interest, but all cover the same topic (Cox, 2008; Tomlinson, 2000b). Learning centres facilitate students’ needs and develop their learning by allowing them to choose specific centres based on their interest (Sower & Warner, 2011). Different learning centres can accommodate different subjects such as mathematics, science and language (Pirozzo, 2014; Tomlinson, 1999). Learning stations provide assignments of varying levels of difficulty that allow students to visit each station or to skip one or more stations that they have mastered (Tomlinson, 1999). The assignments cover the same topic. In each station, students may meet and work with different friends and finish at different times (Tomlinson, 1999).

Learning environment also involves creating the right mental conditions for students to learn effectively. When teachers provide instruction that is appropriate for every student, they are meeting students' needs and student behaviour will improve (Birnie, 2014). An effective learning environment can also be created by teachers through modelling and expecting positive behaviours, such as respect for and valuing of each other (Brophy, 2010). When each student is valued, respected, and heard by their teachers and their peers, this strengthens their self-confidence and they learn more (Tomlinson & Imbeau, 2013).

2.3.4 Challenges and Benefits of Differentiated Instruction

Every effort to apply different teaching strategies has its own challenges and benefits, and this includes differentiated instruction. Studies such as those by Tomlinson (2000a), Huss-Keeler and Brown (2007) and Bondley (2011) have found various challenges and benefits in differentiating instruction in heterogeneous classrooms.

The use of differentiated instruction can increase teacher stress and workload initially (Bondley, 2011). Creating and applying differentiation in a regular classroom is a "time-consuming task" that requires strong teacher motivation (Smit & Humpert, 2012). Tomlinson (2000a, p. 26) states that differentiation is a complicated task for teachers and is "a long term change process". Harman (2014) finds that it is difficult to make time to discussion effectiveness of the differentiation approach with other teachers, because teachers already have a heavy schedule during school hours.

Bondley (2011) found that differentiated instruction contributed strongly to student learning. A study with Year 5 students at Turkish lesson in Turkey showed that differentiated instruction had a positive impact on student attitudes (Karadag & Yasar, 2010). In another study, differentiated instruction had a significant impact on student achievement in mathematics (Huss-Keeler & Brown, 2007). Differentiated instruction contributes to the development of gifted children intellectually and socially, rather than putting them in a homogenous classroom (Neihart, 2007). Maes (2010) found in her study that Differentiated Mathematics Instruction assisted all students in learning mathematics in a multi-age classroom. This classroom contained different ages and levels of study. It seems that differentiated instruction can fulfil the needs of various students in a heterogeneous classroom (Tomlinson, 2000a).

A study of differentiated instruction in 22 schools in Switzerland found that the teachers had not implemented a very detailed version of differentiated instruction. However, a survey found that they had a cohesive vision that may have empowered them to implement more than the elements observed in the study (Smit & Humpert, 2012). Moreover, they found that, on a standardised test, students in the classes that used differentiated instruction did not achieve any less than the control classes that did not use differentiated instruction. Another study in Cypriot primary schools showed that it is difficult to solve educational ineffectiveness by using differentiated instruction, but that differentiated instruction “can enhance the learning process and improve students’ achievement” (Stavroula, Leonidas, & Mary, 2011, p. 15). In a study of pre-service teachers in mathematics education in Mount St. Mary’s University, Maryland, United States, conducted in 2008, the results suggested that even though not all participants were pleased with differentiation, the group taught using differentiation performed better than another group that was not taught with differentiation (Butler & Lowe, 2010).

2.4 Lesson Study as Professional Development in Schools

Lesson Study has been applied broadly in several countries across the world. However, different countries use different terms, including “Learning Study” in Hong Kong (Ling et al., 2005); “Action Education” in China, which was initiated by Professor Gu Lingyuang and his team in Qingpu in 2002 (Paine & Fang, 2006); and, since 1998, “Learning Communities” in Singapore, where schools are clustered based on geographical location to motivate teachers to work collaboratively (Chua, 2009).

A study by Hurst, Armstrong and Young (2011) showed that professional learning could develop self-confidence, content and pedagogical knowledge, and engagement with students. Lesson Study is a school-based professional development and can be used for every subject at any level of education (Cheng & Yee, 2011/2012).

2.4.1 Understanding Lesson Study

Developing teachers' mathematical pedagogy is important in order to generate better learning from students (Murata et al., 2012). Lesson Study, as a professional development strategy, may be a way to improve both teachers' mathematical content knowledge (Fernandez, 2005; Murata et al., 2012; Taylor, Anderson, Meyer, Wagner, & West, 2005) and student outcomes (Stewart & Brendefur, 2005; Waterman, 2011). In addition, Lesson Study could be used to enhance teachers' pedagogical knowledge (Cerbin & Kopp, 2006; Taylor et al., 2005). Lesson Study has become popular in some countries in recent years; for example in the United States, where it has been used since the late 1990s (Murata, 2011).

Lesson Study began in the Meiji period in Japan in the late nineteenth century (Baba, 2007; Isoda, 2007). Some teachers from the same or different schools in a district worked together for the purpose of improving their teaching (Taylor et al., 2005). They discussed and planned a lesson, which was then implemented in the classroom of one teacher while the other teacher/s acted as observer/s. After the class session finished they met to review the lesson together. Baba described this as "preparation, actual class, and class review" (Baba, 2007, p. 2).

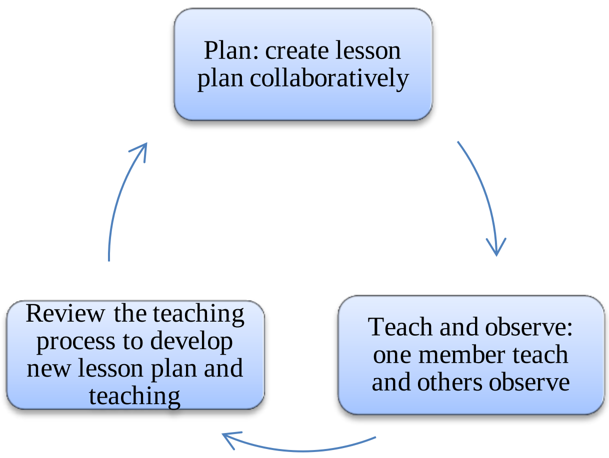


Figure 2-3 Lesson Study Cycle (adapted from Baba (2007))

In this study, the process of using Lesson Study as a vehicle for differentiating mathematics instruction in Indonesian primary schools is shown in Figure 2-4.

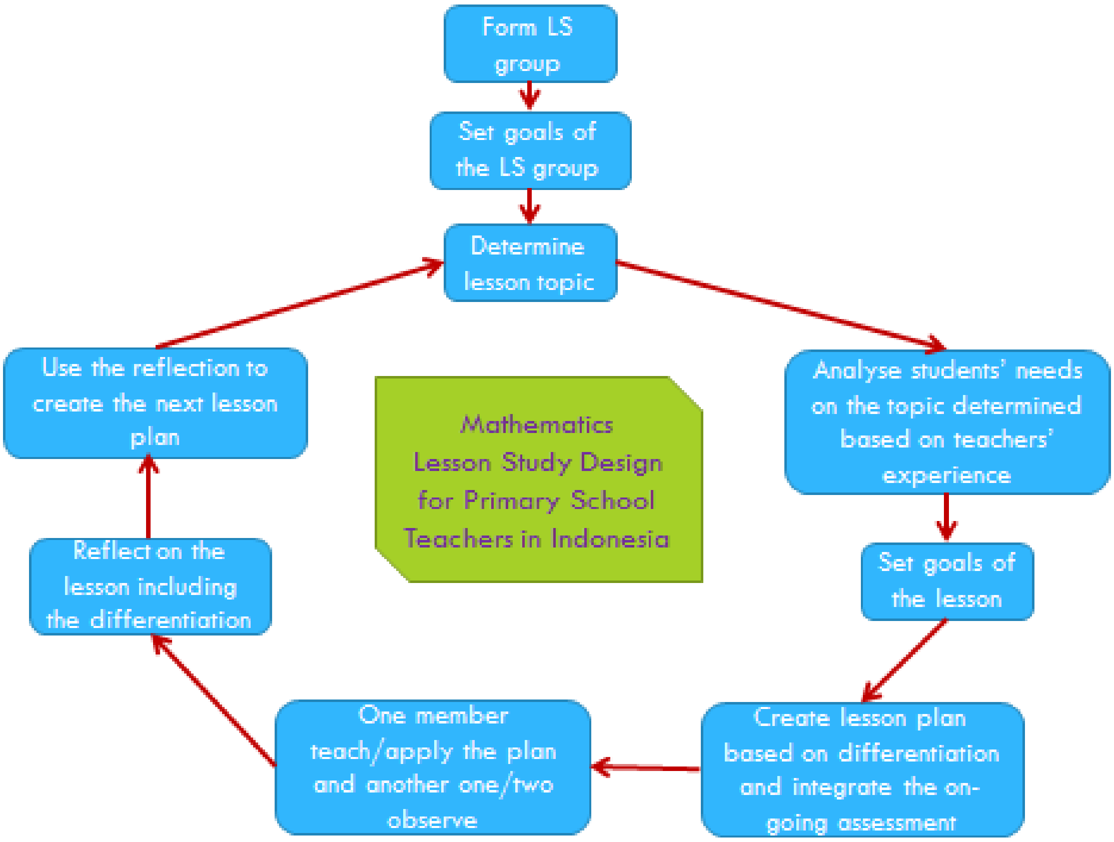


Figure 2-4 Lesson Study cycle for differentiated mathematics teaching

Professional learning starts when teachers establish a Lesson Study Group and set their goals. For example, in the study by Taylor et al. (2005), the teachers identified that their Lesson Study group's goal was "to give students the opportunity to share their math thinking with their classmates" (p. 19).

The group then starts the Lesson Study cycle by determining the lesson topic to be taught. After deciding which topic will be taught, the goals of the lesson can be generated from a general goal to more specific goals (Murata, 2011, p. 2). Examples of general goals of a lesson are "students know prime numbers", or "students understand multiplication". Specific goal may be "students can identify and name prime numbers between 20 and 30', or 'students can differentiate between 3 x 2 and 2 x 3'.

Based on the goals, teachers can start doing their lesson plans. Murata (2001) emphasised that the key purpose is not to design a perfect lesson but to investigate students’ ways of learning when teachers use a certain teaching strategy. Murata (2001) noted that this step provides an opportunity for teachers to develop their own knowledge of the content. In Lesson Study, teachers apply ongoing assessment that is consistent with the learning process in Differentiated Mathematics Instruction. This assessment is then be used to inform the next lesson plan (Bruce & Ladky, 2011; Tomlinson, 2001).

After lesson planning is finished, one teacher from the group teaches a class while one or more teachers observe the process (Murata, 2011). The observers fill in an observation checklist or form and/or make anecdotal notes. The data is then be discussed with the rest of the Lesson Study group members (Murata, 2011). From this discussion, a decision is made as to whether or not there is a need to revise and re-teach (Murata, 2011).

2.4.2 Lesson Study Challenges and Benefits

Lesson Study is time consuming, but according to Taylor et al. (2005) this ongoing professional learning will empower teachers and improve their professional development progressively. When Lesson Study group members are from different schools, they may need to travel, and may need additional time for the meeting and discussion. This can be overcome by populating the Lesson Study group from teachers within a school (Burghes & Robinson, 2009) or create “supportive schedule” (Huang & Shimizu, 2016, p. 402). Murata (2011) suggested that an in-school Lesson Study group is effective because teachers can share information about the students and the particular community.

Lesson Study also requires a lot of teacher energy (Lee, 2008). However, Lee (2008) suggests that the school administrator and the government can support the teachers as according to Huang and Shimizu (2016), Lesson Study is not a “voluntary activities of group of teachers” (p. 402). For example, the school may offer more time for Lesson Study professional development and the government may provide financial compensation (Lee, 2008).

Lesson Study contributes to student achievement (Yanping, Lee, & Haron, 2009), and can play an important part in improving mathematics and science teaching generally (Lewis, 2002). For example, in 2008, Cheng and Yee worked with a team that consisted of four mathematics teachers from Spring Hill Elementary (a neighbourhood public school in Singapore), the department head of mathematics and the level head of mathematics. They showed that, even though Lesson Study required much dedication of teachers' time and energy, there were some constructive results. First, the teachers realised that the use of mathematics terms needed to be consistent and accurate to help the students understand topics more easily. Second, Lesson Study encouraged "the teachers to reconstruct students' thinking and to plan lessons that address students' misconceptions based on their models of student thinking" (p. 53). Generally, Lesson Study enhanced "teachers' reflective thinking about teaching" (Cheng & Yee, 2011/2012, p. 54).

A group of secondary teachers in Hong Kong worked on Lesson Study (Lee, 2008). Even though the teachers had to work overtime after school hours, there were several positive outcomes. They became more independent because they could manage their own schedule and objectives. They learnt from their peers through the discussion in the Lesson Study meeting, and recognised their students' views.

2.4.3 Lesson Study and Its Impact on Teachers' Knowledge

Although initially teachers tend to pay attention to matters other than the subject matter knowledge that is considered essential by supervisors, administrators, and other educators, it is believed that Lesson Study has the power to develop teachers' content knowledge (Puchner & Taylor, 2006). In addition, Lesson Study can help teachers to improve their teaching knowledge (Huang & Shimizu, 2016).

A study with mathematics teachers from nine independent school districts in West Texas showed that, with Lesson Study, the educators were supported to think about their analysis and impact on teaching selection, and had the opportunity to alter their methods (Yarema, 2010). Yarema (2010) also argues that Lesson Study has the possibility of discovering teachers' view, which is usually influenced by the educational philosophy of the learning environment. Other research that used case studies in the United States showed that Lesson Study could change teachers' ways of teaching mathematics, and this influenced students' learning significantly (Puchner & Taylor, 2006).

In 2009, at least 60 of 328 primary and secondary schools in Singapore tried to apply Lesson Study for numerous subjects (Yanping & Lee, 2010). They found that Lesson Study developed teacher knowledge and skills. Moreover, with support from mathematics educators, Lesson Study was able to enhance teacher pedagogical content knowledge, impacting on the progress of students' understanding of the topics taught (Yanping & Lee, 2010).

2.4.4 Lesson Study in Indonesia

Lesson Study started around 2005 in Indonesia (Suratno & Iskandar, 2010) and this is a "top-down" professional development (Kusanagi, 2014). Although teachers found difficulty in allocating time, and Indonesian teachers are experience being interfered by the government (Kusanagi, 2014), this professional learning method developed participants' "teaching performance, variation of teaching methods/approaches, and collaboration" (Marsigit, 2007, p. 143). Moreover, a case study of cooperative Lesson Study between Indonesian mathematics and science teachers and university faculty academic staff concluded that there were improvements in teachers' knowledge of content and organisation of the lessons and the students' response throughout the lessons (Saito et al., 2006). This was supported by Fernandez (2005), who pointed out that Lesson Study could develop not only teachers' mathematical content knowledge but also their research experience.

In addition, Kusanagi (2014) found that through Lesson Study, Indonesian teachers may be able to discuss their lessons in the group to support the improvement of their pedagogies knowledge and teaching experience.

2.5 The Link between Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction

This chapter so far has explored each of the concepts Lesson Study, Mathematical Pedagogical Content Knowledge, and Differentiated Mathematics Instruction. The links between these concepts are explored in the following sections.

2.5.1 The Connection between Lesson Study and Teachers' Mathematical Pedagogical Content Knowledge

Discussing problems or questions raised by students in the Lesson Study meeting will develop teachers' knowledge of their students (Lewis, Friedkin, Baker, & Perry, 2011). Teachers will be forced to reflect upon and recognise students' abilities and their ways of thinking. Moreover, in the observation step, teachers who act as observers can learn from their colleagues' practices (Saito, Imansyah, Kubok, & Hendayana, 2007). Lewis et al. (2011) also note that when teachers review lessons, this will develop their mathematical knowledge. Moreover, they believe that discussion on reshaping the lessons will improve teacher performance. Tepylo and Moss (2011) state that when teachers have a good understanding of mathematics content it will help them to understand students' thinking about mathematics.

2.5.2 The Connection between Lesson Study and Differentiated Mathematics Instruction

In Lesson Study, teachers can discuss their teaching and learning, and this will improve their instruction (Stewart & Brendefur, 2005). Moreover, in Lesson Study meetings, teachers can learn together about how to differentiate instruction, which may be a new strategy for them (Hocket, 2010). As Lesson Study is a continuous process of teacher learning (Burghes & Robinson, 2009), this allows teachers to learn and develop differentiated instruction that may be new for teachers. Joseph, Thomas, Simonette, and Ramsook (2013) note that it takes time to be able to differentiate instruction effectively.

To prepare differentiated instruction, practice teachers need other teachers to complement their work (O'Meara, 2010; Roberts & Inman, 2013). Lesson Study is the place for discussing this as it is a collaborative professional learning model, where teachers can observe and critique one another in order to improve their teaching (Baba, 2007; Lewis & Tsuchida, 1999).

2.5.3 The Connection between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction

The main foundation for Differentiating Mathematics Instruction is for teachers to have a good understanding of the concepts and procedure of the topic they will teach (Literacy and Numeracy Secretariat, 2008). For example, when teaching multiplication, teachers need to know which problems can be categorised as easy or hard so they can match problems to student readiness. In addition, teachers may be able to create tasks that match student interests. Knowledge of student readiness, interest, and learning styles will lead teachers to differentiate as it will help teachers to tier levels of task difficulty and vary the materials (Tomlinson et al., 2003).

Knowing the curriculum will help teacher knowledge of the material to be taught and assist in achieving learning objectives, but this does not mean that teachers can plan and implement differentiation easily (Literacy and Numeracy Secretariat, 2008).

Tepylo and Moss (2011) contend that teachers' experience in observing students working in different ways to solve problems can force them to understand students' thinking and this will improve their mathematical knowledge. Giving students an opportunity to solve problems using their own understanding means that teachers will encourage creative thinking among their students. Creative thinking contributes to problem solving (Lumsdaine & Lumsdaine, Dec 1994-Jan 1995). This means that the teachers are employing process differentiation in the mathematics lesson, as creative thinking is one of the indicators of process differentiation.

2.6 Conclusion

Professional development is required by teachers from time to time to improve their professional practice. However, much of the professional development available to teachers has been single session, requires them to leave students for periods of time, and requires financial support to enable them to travel to another location. Lesson Study appears to be unique in professional development because it promises continuity: teachers do not need to leave their regular jobs, and require less financial support because they can participate in the school or among teachers in the local area.

Teachers' mathematics knowledge and skills are important to assist students in achieving their mathematics learning goals. Teachers need to not only know the material being taught but also need to be proficient in delivering lessons. Shulman (1986) named this 'pedagogical content knowledge'. Mathematical Pedagogical Content Knowledge, the focus of this study, requires mathematics teachers to understand mathematics content, ways of teaching mathematics, and the context influencing the teaching of mathematics.

Regular classrooms that contain a variety of students require teachers to adopt effective strategies to meet the need of all students and assist them in achieving their mathematics learning goals. Each student has a different background, learning ability, readiness and interest. Therefore, Differentiated Mathematics Instruction that is orientated towards meeting the needs of each student will accommodate the mathematics learning process in a heterogeneous classroom. Teachers can differentiate content, process, product, assessment, and learning environment. In differentiating mathematics instruction, teachers need to consider the curriculum that is being used, student profiles, and the school environment and facilities. It is not appropriate to imitate differentiated instruction methods used in other schools or countries. A differentiation strategy used one year may not apply the next academic year for the same teacher teaching the same level and topic. Every class has different students, who may need different strategies.

Lesson Study is a 'place' for teachers to share knowledge and ideas for designing teaching strategies that take the diverse learning needs of students into account.

Thus, Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction complement each other to enhance teachers' professional development that may affect student mathematics learning and achievement.

Chapter 3 – Research Method

3.1 Introduction

This chapter describes the methodology used to explore two Lesson Study groups from two primary schools in Nagekeo, Flores, Indonesia. First, the chapter addresses the research questions and objectives of the study. The research design and methodology used, and the ways in which the data was collected and analysed, are then described.

3.2 Research Questions and Objectives

The study was framed with the primary research question: What impact does Lesson Study have on teachers' knowledge and skills in differentiating primary school mathematics instruction?

Based on this question and the fact that the context of this study may differ from other Lesson Study research, the objectives of this study are:

1. To describe the processes involved in the implementation of Lesson Study
2. To investigate the influence of Lesson Study on primary school teachers' mathematics pedagogy.
3. To investigate the influence of Lesson Study on primary school teachers' curriculum differentiation in mathematics.
4. To investigate the connections between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction during Lesson Study.
5. To identify the key characteristics of Lesson Study as a form of professional development.

As this study used Multimodal Analysis Video software to transform the qualitative data analysis of teachers' Mathematical Pedagogical Content Knowledge and their practice in differentiating mathematics instruction into quantitative data, a second major research question arose in the context of this study, namely: What is the nature of a digital mixed methods research design for investigating the relations between teachers' Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction during Lesson Study?

Based on this question, a major objective of this study is to develop a digital mixed methods research approach for studying the impact of Lesson Study on teachers' knowledge and skills in differentiating mathematics instruction.

3.3 Research Design

3.3.1 Description of Research Design

This study was based on case studies of primary school teachers who participated in Lesson Study groups, supported by a digital mixed methods strategy. A case study was chosen because this type of research is "strong in reality", meaning that the case comes from a real problem that is close to daily life. Thus, the results of the study will be understood easily by readers from both academic and non-academic backgrounds, because it uses colloquial language, and its emphasis is on actual action that will support practitioners (Cohen et al., 2011, p. 292). This study focused on teaching that the participants usually perform in their routine duties in school. Although it cannot be generalised (O'Toole & Beckett, 2013) because it encapsulates a particular circumstance with detailed explanation (Merriam, 2009), a case study is appropriate when studying a program or an activity (Creswell, 2013). Moreover, the case study approach is consistent with Shulman's (1987) investigation about pedagogical content knowledge.

A case study is a suitable research design if there is limited influence from external factors (Merriam, 1998). For example, when investigating the impact of Lesson Study on teachers' knowledge in differentiating mathematics instruction, the researcher had little influence over the teachers apart from asking them to pay more attention to differentiating their teaching. The researcher sat in on the Lesson Study Group Meetings and conducted the video recording during mathematics lessons. Case study research was also chosen as a way to expand the existing theories (Merriam, 2009) as they relate to Mathematical Pedagogical Content Knowledge, Lesson Study, and Differentiated Mathematics Instruction. Case study was also beneficial as a way to describe a process (Stake, 2005) like Lesson Study in this study, as it involves a certain number of participants and time for observations (Merriam, 2009).

The qualitative data collected during the case study was transformed into quantitative data using Multimodal Analysis Video software; the results of the quantitative data can be seen from the visualisations in the software (O'Halloran, E, & Tan, 2014; O'Halloran et al., 2016). This research design was named 'digital mixed methods design' by its creators O'Halloran et al. (O'Halloran et al., 2016). The quantitative data from the software visualisations was then interpreted (O'Halloran et al., 2016) based on the framework of this study, which covers Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction during the Lesson Study cycles.

The software for this approach (Multimodal Analysis Video) was developed by O'Halloran and her colleagues at the National University of Singapore for analysing video data. The design of this part of the study is shown in Figure 3-1.

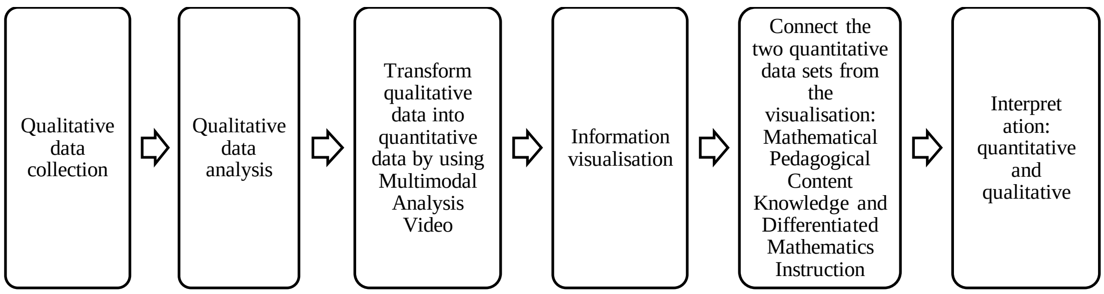


Figure 3-1 Mixed methods research design (adapted from the Digital Mixed Methods Design by O'Halloran et al. (2016))

This project was situated within the interpretive research paradigm (for example, Creswell (2013)). An interpretive research approach was chosen for its advantages; namely, it was conducted in the participants' natural setting, phenomena were not manipulated by the researcher, and the design allowed for "flexibility and tolerance" so that it could be modified while the research was being conducted (Wiersma & Jurs, 2005, p. 203). Moreover, researchers in an interpretive case study are the executors of data collection and its analysis (Creswell, 2013). Using this approach allowed for the real-world situation of participants to be studied as it unfolded and for immersion in the details of the generated data to highlight relationships, rather than testing a pre-determined hypothesis. Comprehensive descriptive data were collected in order to better understand the context of the phenomena being studied (Fraenkel & Wallen, 2009).

According to Crotty (1998), case study methods may use participant observation, data reduction, analysis of documents and interpretation of data. This supports the researcher's focus on investigating the impact of Lesson Study, an alternative model of professional development, on teachers' knowledge and skills in differentiating primary school mathematics instruction. From Lesson Study in this study, it was possible to make recommendations for the schools and system in the district for models of professional development.

3.3.2 Model of Research Design

This research started with two sessions of professional development. The first session was about Differentiated Mathematics Instruction and the second session was about Lesson Study. To support the professional development, documents about the professional development plan, handouts, PowerPoint presentations, and a lesson plan sample of integer numbers (see Appendix 1), including worksheets (see Appendices 2–6), were prepared as models for the participants to create their own lesson plans based on differentiated instruction.

Professional development, using a Lesson Study model, followed the conceptual framework outlined in Table 3-1. Two groups of primary school teachers participated in the professional development. Each group met and discussed planning for differentiation and devised an initial lesson plan. One group member taught the planned lesson while other group members observed and the researcher recorded the teaching process using a video recorder. Observations were discussed within each Lesson Study group. The lesson plans were modified based on the previous experience to create the next lesson plan for teaching the next lesson. In addition, teachers in each group were encouraged to extend the differentiation of their mathematics lessons with their own regular classes. Lesson Study Group Meetings were held before and after each teaching session.

Table 3-1 Activities during field work and collected data

Time	Activities	Data
Day 1	Brief meeting with the participants: • Schedule • Strategy • Data needed	
Day 2	Professional Development: • Differentiated Mathematics Instruction • Lesson Study • Lesson Study Group Meeting 1 • Discussion of Lesson Plan 1	Lesson Plan 1 and Video recording
Day 3	Mathematics Lesson 1 • Teaching • Observation	Video recording Observation report
	Lesson Study Group Meeting 2 • Review of lesson 1 • Discussion of lesson plan 2	Video recording and Lesson Plan 2
Day 4	Mathematics Lesson 2 • Teaching • Observation	Video recording Observation report
	Lesson Study Group Meeting 3 • Review of lesson 2	Video recording

3.4 Research Methodology

3.4.1 Research Site

The study was conducted in two primary schools in Nangaroro sub-district, Nagekeo District on Flores Island, East Nusa Tenggara Province, Indonesia. The schools are referred to as School A and School B.

Nagekeo is a new district, established in December 2007, and is in quite a small area on Flores. It is difficult for primary school teachers to meet in one place for discussions, as they usually teach full time during school hours. Transport is also sometimes a barrier. Thus, in this area, it was more effective for the Lesson Study groups to be established with teachers in the same school so that they could meet and discuss what they were doing easily. The teachers in this study had handwritten their lesson plans for effectiveness and efficiency because the only computer at the schools was used by administrative staff and the teachers could not type fluently. Additionally, there is no internet access at the schools, and no bookstore at Nangaroro. Consequently, the only resources the teachers use are the textbooks from the central government. School B had not yet received copies of the latest textbook of the 2013 curriculum at the time of the study, so they used the textbook from the previous curriculum. While the latest textbook could be accessed on the internet, from the Department of Education website, there was no internet access at the schools so this was not possible.

School A, however, had already received the latest textbook. Therefore, the mathematics topics taught during this study were different in the two schools. In School A, the participants taught angles, while in School B they taught commutative and associative laws of addition and multiplication.

3.4.2 Participants

The study involved seven teachers, four from School A and three from School B, as described in Table 3-2.

Table 3-2 Study participants

#	Teacher	Age (Years)	Teaching Experience (Years)	School	Role at School	Role in Lesson Study Group
1	Sis	47	6	School A	Year Four teacher	Teacher A
2	Ai	41	5	School A	Year Three teacher	Observer A1
3	Nur	39	8	School A	Year Six teacher but not teaching mathematics	Observer A2
4	Sef	58	39	School A	Principal and mathematics teacher for Year Six	Observer A3
5	Ros	57	36	School B	Year Four teacher	Teacher B
6	Lin	45	13	School B	Year Three teacher	Observer B1
7	Wal	52	28	School B	Principal and Year Six mathematics teacher	Observer B2

Students who attended the mathematics lessons in this study were at Year Four level and in the first semester of the 2014–2015 school year. There were 18 Year Four students from School A and 16 Year Four students from School B. However, the main focus of this study was on the teacher's activities during the mathematics lessons, not on the students.

3.4.3 Ethical Considerations

To gather the data needed, written permission was obtained from the local Department of Education, and from principals, teachers, students and parents of the students of the study participants. Participants were initially identified using a code (e.g. Teacher A, Observer A1); this allowed cross checking of data during the analysis stages to minimise misinterpretation. At the completion of the study all coding linking participants will be destroyed. Confidentiality is of paramount importance at all times and participants are de-identified or given pseudonyms in this thesis.

3.5 Data Collection

Data was collected through different sources to enable triangulation (Creswell, 2005; Yin, 2009). In this study, data was generated through video recordings, documents (lesson plans created by the teachers during the study), and observation. Initially, the researcher was involved with the Lesson Study group as a facilitator to deliver differentiated mathematics instruction and Lesson Study material. Then, the researcher attended the Lesson Study cycles and mathematics teaching process to record the activities using a video recorder.

Table 3-3 shows the objectives of this study linked to the data sources.

Table 3-3 Objectives of the study linked to the data sources

Objectives	Data Sources
1. To identify the key characteristics of Lesson Study as a form of professional development.	• Video recording of Lesson Study Group Meetings and mathematics lessons
2. To describe the processes involved in the implementation of Lesson Study.	• Video recording of Lesson Study Group Meetings and mathematics lessons
3. To investigate the influence of Lesson Study on Indonesian primary school teachers' mathematics pedagogy.	• Video recording of Lesson Study Group Meetings and mathematics lessons • Written documents of Lesson Plans and teaching observation sheets
4. To investigate the influence of Lesson Study on teachers' curriculum differentiation in mathematics.	• Video recording of Lesson Study Group Meetings and mathematics lessons • Written documents of Lesson Plans and teaching observation sheets
5. To develop a digital mixed-methods approach for studying the impact of Lesson Study on teachers' knowledge and skills in differentiating mathematics instruction.	• Video recording of Lesson Study Group Meetings and mathematics lessons

3.5.1 Video Recording

A video recorder was used to record the mathematics lessons and the Lesson Study meetings. There were two mathematics lessons and three Lesson Study Group Meetings video recorded for each school. Data from the video recordings was used to investigate the influence of the use of Lesson Study on the participants' mathematics pedagogy and their differentiation of the curriculum in mathematics. The researcher acted as the video recorder in the mathematics classroom but did not interfere in the teaching and learning processes.

3.5.2 Written Documents

The written documents collected were the lesson plans themselves, observation reports from the members of the Lesson Study groups, observation reports from the

Lesson Study Group Meetings, and summaries of Lesson Study Group Meetings. Creswell (2005) explains that documents are a good data source in qualitative research because they are presented in the participants' own "language and words" (p. 223) and do not need to be transcribed.

Lesson Plans

There were two mathematics lesson plans (see Appendices 9 and 10, in Indonesian) from each primary school, with two different topics taught. The topic taught at School A was about acute, obtuse, and right angles, and using a protractor to measure and draw an angle. The topic taught at School B was about commutative and associative laws of addition and multiplication. The lesson plans were developed during the Lesson Study Group Meetings. Group members used the format of the lesson plan that was usually used at their schools but they modified the content, with differentiated instruction being considered and included in the lesson plans.

Classroom Observations

As part of the Lesson Study process, one of the group members taught and other members acted as observers. The observation results were used as documentary data. The Participant-as-observer was chosen by consensus of the members of the Lesson Study groups. The observers sat at the back of the classroom and observed the classroom activities; the teacher knew that he or she was being observed for the benefit of the Lesson Study group (Fraenkel & Wallen, 2009).

Semi-structured observation schedules (see Appendix 7) were created as a guideline for the observers, to keep the observation process on track while they wrote down what they observed and considered the relevance of the lesson and differentiated mathematics instruction. For example, an observer might document what the teacher did in the lesson time, classroom management, conformity between the action and the lesson plan, strategies used, questions raised, and responses to students' queries. The observers also wrote about the students' behaviour during the lesson; for example, their responses, curiosity, questions and responses to the teachers' questions. Observation guidelines (see Appendix 8) were also created to assist the teachers to fill out the observation form in order to highlight Differentiated Mathematics Instruction.

3.6 Data Analysis

The triangulation achieved by using the various types of data collected was used to validate the data (Creswell, 2005; Creswell & Clark, 2011; Yin, 2009). As is common in the analysis of qualitative data, this occurred simultaneously with the data collection phase of the study (Wiersma & Jurs, 2005). This process allowed the researcher to provide "an accurate description and interpretation of the phenomenon" (Wiersma & Jurs, 2005, p. 206). The various types of data collection were analysed through three steps: "categorization", "description" and "synthesis" (Wiersma and Jurs, 2005, p. 207). This analysis enabled themes to emerge, which then enabled fine-tuning of the data collection process.

The video recordings enabled detailed analysis of the teachers' Mathematical Pedagogical Content Knowledge, as demonstrated during the mathematics lesson, in the Lesson Study Group Meeting. The recordings allowed analysis of both the teachers' teaching and the students' responses to the instructions, and provided data on the teachers' actions and the Lesson Study Group Meeting activities as a continuous flow of actions from which observations were created.

All lessons were videotaped and then analysed using Multimodal Analysis Video software. Figure 3-2 shows a screenshot from the Multimodal Analysis Video analysis page.

In the screenshot of the Multimodal Analysis Video in Figure 3-2, the video being analysed is visible in the player window [1]. While the video is playing, the transcription of the mathematics lessons can be entered using the facilities in the transcription window [2], by inserting nodes [5]. Timing of transcription entries is determined by the length of the nodes that are aligned with the film strip [3], sound strip [4], and the bar in dialog strip [5]. The strips [6] display the theoretical framework i.e. the Differentiated Mathematics Instruction and Mathematical Pedagogical Content Knowledge, which is used to analyse the video data. The analysis is undertaken by entering a node in the strips (see [5]) and entering the system choice [7] selected from the range of available options [8]. A description of each system choice is available [9]. The available system choices and the accompanying descriptions are entered in the Library page, which provides facilities for developing the theoretical framework (see Figure 3-3).

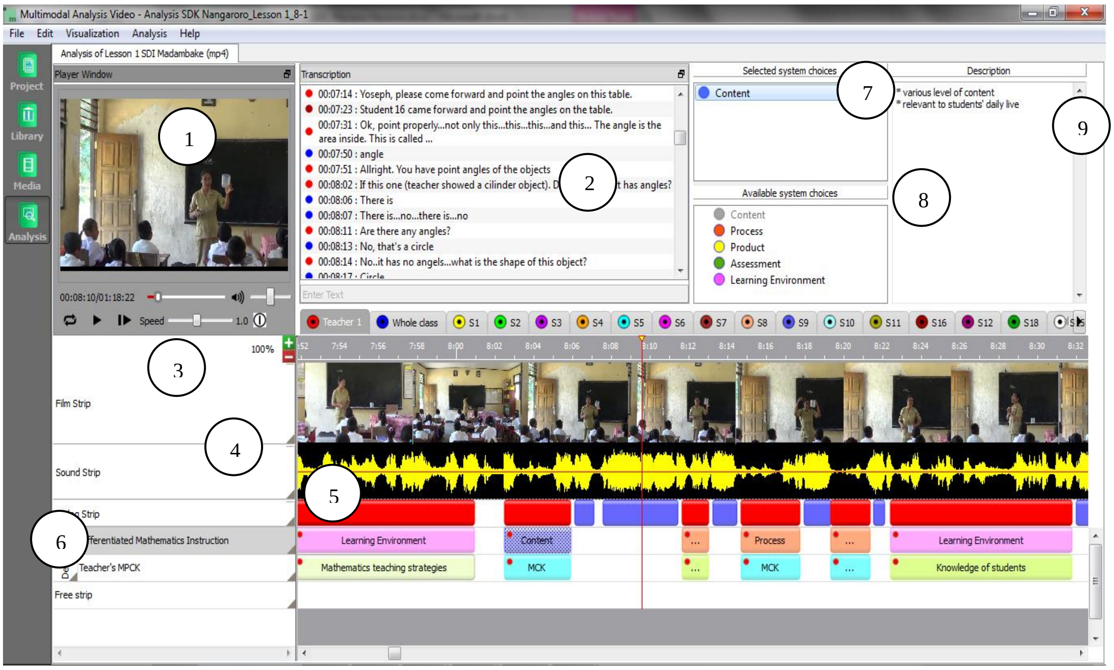


Figure 3-2 Analysis page (screenshot) from Multimodal Analysis Video

The categories in the available system choices were created by the researcher, based on the concepts of Differentiated Mathematics Instruction and Mathematical Pedagogical Content Knowledge. In Figure 3-3, the catalogue [1] shows the framework used in this study, configured from a range of systems [2]. The description of each system choice is entered in [3], in Figure 3-3.

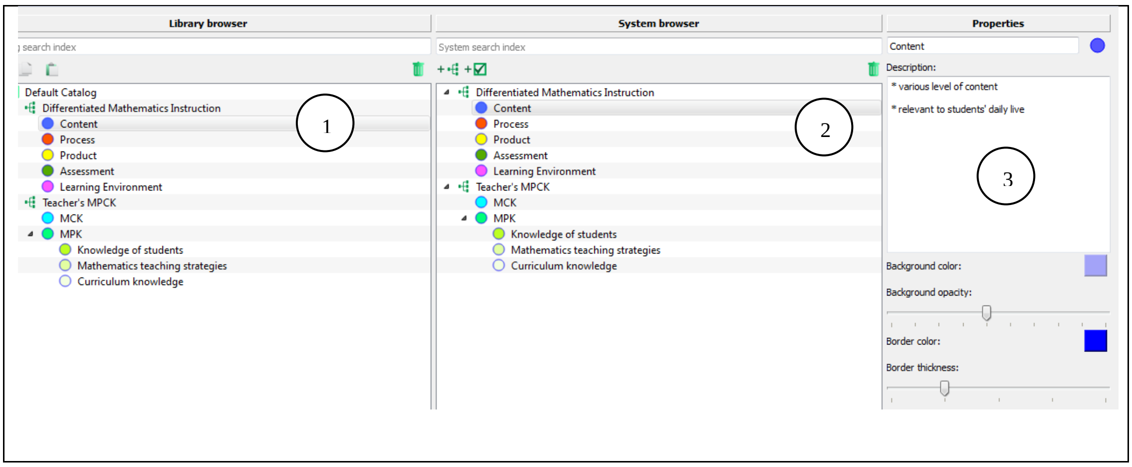


Figure 3-3 Library folder (screenshot) of multimodal systems

After the video is analysed, the results can be visualised using facilities in the software. A sample diagram is shown in Figure 3-4. The left panel shows the diagram produced from the categories selected in the video analysis. The right panel contains a video player where the video can be played and the results of the analysis displayed below. The red circle on the left panel [1] displays the analysis results (the system choices and the combination of system choices) for the current time in the video player as a relative percentage of the total video time. The black cirlces [2] show combinations of selected systems [4] (the "state") in the analysis, while the percentage of each state shows the time duration of the selected system [4] from the total video duration. The thick line [3] (the "transition") shows the current shift between states in the video clip. For example, in Figure 3-4, the thick line [3] shows that the teacher's action moved from an absent category (red circle [1]) to a state [2] of "process differentiation" [5] then moved to an absent category (red circle [1]) again. The total number of transitions between the different states is displayed in a "state transition" diagram. The state transition diagram is synchronised with the video player [5] so that the analyst can play the video and watch the analysis unfolding dynamically in the left panel.

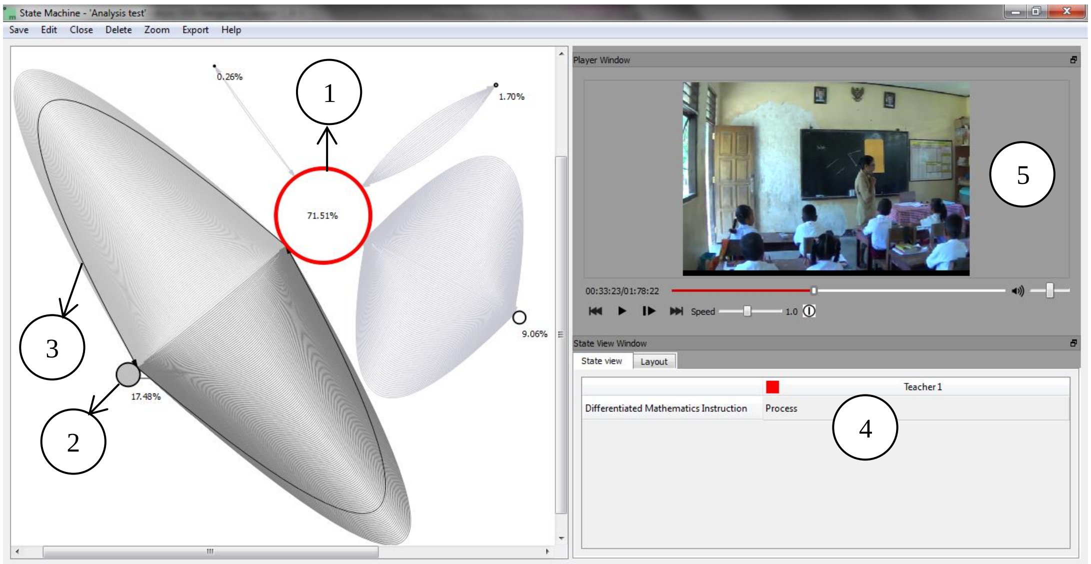


Figure 3-4 Sample visualisation of Multimodal Analysis Video (screenshot)

In this study, the state transition diagrams created using the facilities in Multimodal Analysis Video concerned Differentiated Mathematics Instruction, Mathematical Pedagogical Content Knowledge, and the combinations of these concepts that occurred in the lessons in the two primary schools.

3.7 Summary

This study adopted a case study approach supported by a digital mixed methods research approach that transformed the qualitative data analysis into quantitative data using Multimodal Analysis Video software. The design focussed on professional teacher development to investigate the influence of Lesson Study on teachers' mathematics pedagogy and curriculum differentiation in mathematics in two Indonesian primary schools in Nagekeo District on Flores Island. Data was collected through observation, documentation (lesson plans) and video recordings. The video recordings were analysed using the Multimodal Analysis Video software.

Chapter 4 – Results

4.1 Introduction

This chapter presents the analysis of two mathematics lesson cycles, Lesson Cycles 1 and 2, from primary schools A and B, to investigate the impact of Lesson Study on teachers' Mathematical Pedagogical Content Knowledge for differentiating mathematics instruction. Each Lesson Cycle began with a professional development session on Differentiated Mathematics Instruction and Lesson Study, provided by the researcher. After the session, a Lesson Study group was set up in Schools A and B, and members of each group met to plan for Lesson 1 (Lesson Study Meeting 1). A teacher in the Lesson Study Group taught Lesson 1, and following this, the group met a second time to discuss Lesson 1 and to plan for Lesson 2 (Lesson Study Meeting 2). After the same teacher from Schools A and B, respectively, taught Lesson 2, the group met again to discuss Lesson 2 (Lesson Study Meeting 3). The different stages of Lesson Cycles 1 and 2 are shown in Figure 4-1.

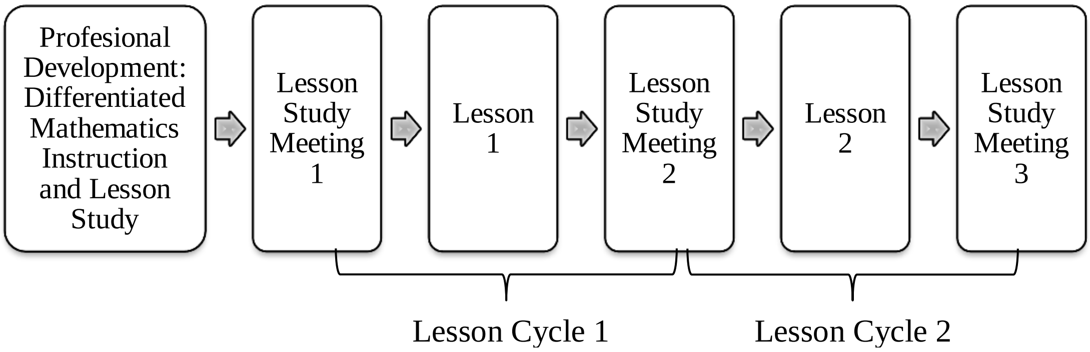


Figure 4-1 The steps of the study

Each stage of the Lesson Cycles in Schools A and B — that is, Lesson Study Meetings 1, 2 and 3 and Lessons 1 and 2 — were video recorded, and Lesson Plans 1 and 2 were collected and analysed by the researcher. Lessons 1 and 2 were analysed using Multimodal Analysis Video software to assess the impact of the Lesson Study meetings on the lessons. The results, in terms of the differentiation in Lessons 1 and 2 and the display of teachers' Mathematical Pedagogical Content Knowledge, are presented for Schools A and B below, together with a discussion of the Lesson Study

Group Meetings and the Lesson Plans. The indicators of Mathematics Pedagogical Content Knowledge and Differentiated Mathematics Instruction used in analysing the videos of the classroom lessons are given in Figure 4-2 and Figure 4-3. As shown in Figure 4-2, Mathematical Pedagogical Content Knowledge consists of mathematical content knowledge, mathematical pedagogical knowledge, and Context Knowledge. Each type of knowledge has indicators; for example, the indicators of mathematical content knowledge are mathematical conceptual and procedural knowledge (see Figure 4-2 for a detailed description).

The indicators of Mathematical Pedagogical Content Knowledge:

1. Mathematical content knowledge
    - Mathematical conceptual knowledge
    - Mathematical procedural knowledge
2. Mathematical pedagogical knowledge
    - Knowledge of students
        - Students' backgrounds
        - Students' understanding
    - Mathematics teaching strategies
        - Classroom management
        - Assessment
    - Curriculum knowledge
        - Mathematics educational goals
        - Correlation to previous and later topics
        - Link to other subjects
3. Context knowledge
    - Regulation from central and local government
    - Community

Figure 4-2 Indicators of Mathematical Pedagogical Content Knowledge

The indicators of Differentiated Mathematics Instruction:

1. Content differentiation
    - Varied levels of content
    - Relevance to students' daily lives
2. Process differentiation
    - Flexible grouping
    - Encourages creative thinking
    - Encourages critical thinking
    - Varied activities
    - Encourages research
3. Product differentiation
    - Various levels of task difficulty
    - Freedom in choosing the task
    - Flexible submission
4. Assessment differentiation
    - Ongoing
    - Provide rubric
    - Give feedback on students' work
    - Allow students to evaluate their own progress
5. Learning Environment differentiation
    - Learning centres in the classroom
    - Teacher values each student
    - Encourage students to respect each other
    - Encourage students to be responsible

Figure 4-3 Indicators of Differentiated Mathematics Instruction

Figure 4-3 shows the indicators of Differentiated Mathematics Instruction: content, process, product, assessment, and learning environment differentiation. Each element of differentiation in turn has indicators; for instance, the indicators of content differentiation are various level of content and relevance to students' daily lives (see Figure 4-3 for a detailed description), as discussed in Chapter 2.

For Schools A and B, the terms used to identify the various events in the Lesson Study Cycles and the participants are listed in Table 4-1:

Table 4-1 Terms used in this study

School	Lesson Study Group (LSG)	Teacher	Observers	Lessons	Lesson Study Group Meetings (LSGM)	Lesson Plan	Students
School A	LSG-School A	Teacher A	Observer A1 Observer A2 Observer A3	Lesson A1 Lesson A2	LSGM-A1 LSGM-A2 LSGM-A3	Lesson Plan A1 Lesson Plan A2	Student A1–Student A18
School B	LSG-School B	Teacher B	Observer B1 Observer B2	Lesson B1 Lesson B2	LSGM-B1 LSGM-B2 LSGM-B3	Lesson Plan B1 Lesson Plan B2	Student B1–Student B16

Multimodal Analysis Video software was used to analyse the video recordings. Figure 4-4 is a screenshot of the analytical process, showing the transcriptions and classification of Differentiated Mathematics Instruction and Mathematical Pedagogical Content Knowledge (see Chapter 3, page 52, for detailed description of software function). State transition results generated from the analysis are shown in Figure 4-5. In the results shown in Figure 4-4, there are three state transition diagrams for each lesson: (1) Mathematical Pedagogical Content Knowledge, (2) Differentiated Mathematics Instruction, and (3) Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction combined (see Figure 4-5). The analysis for each lesson, Lesson A1, A2, B1, and B2 generated three state transition diagrams. The percentage of each indicator is displayed in tables, together with examples of how those indicators were identified.

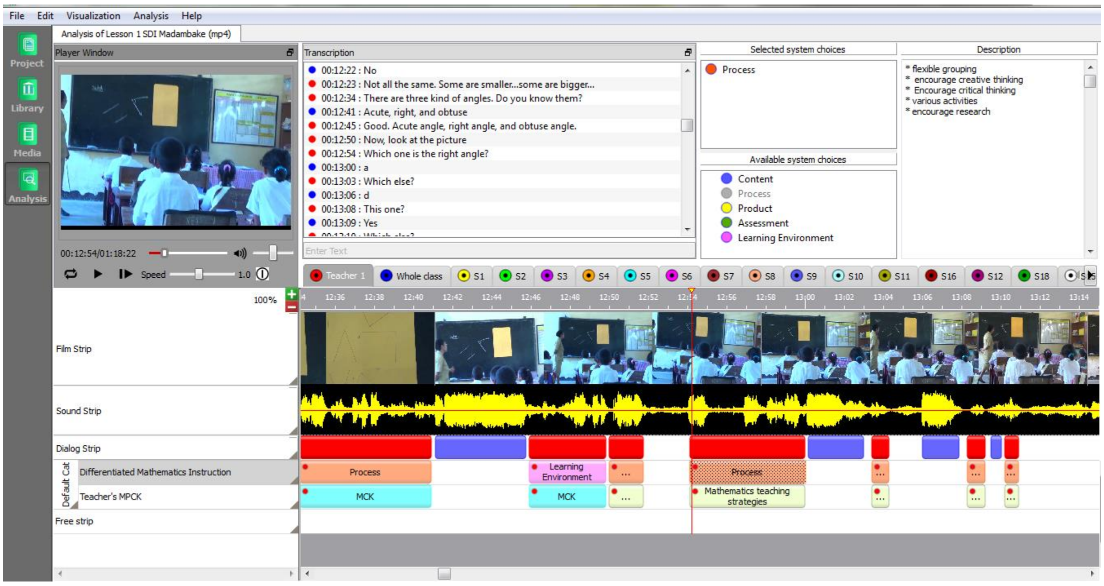


Figure 4-4 Video analysis of a lesson

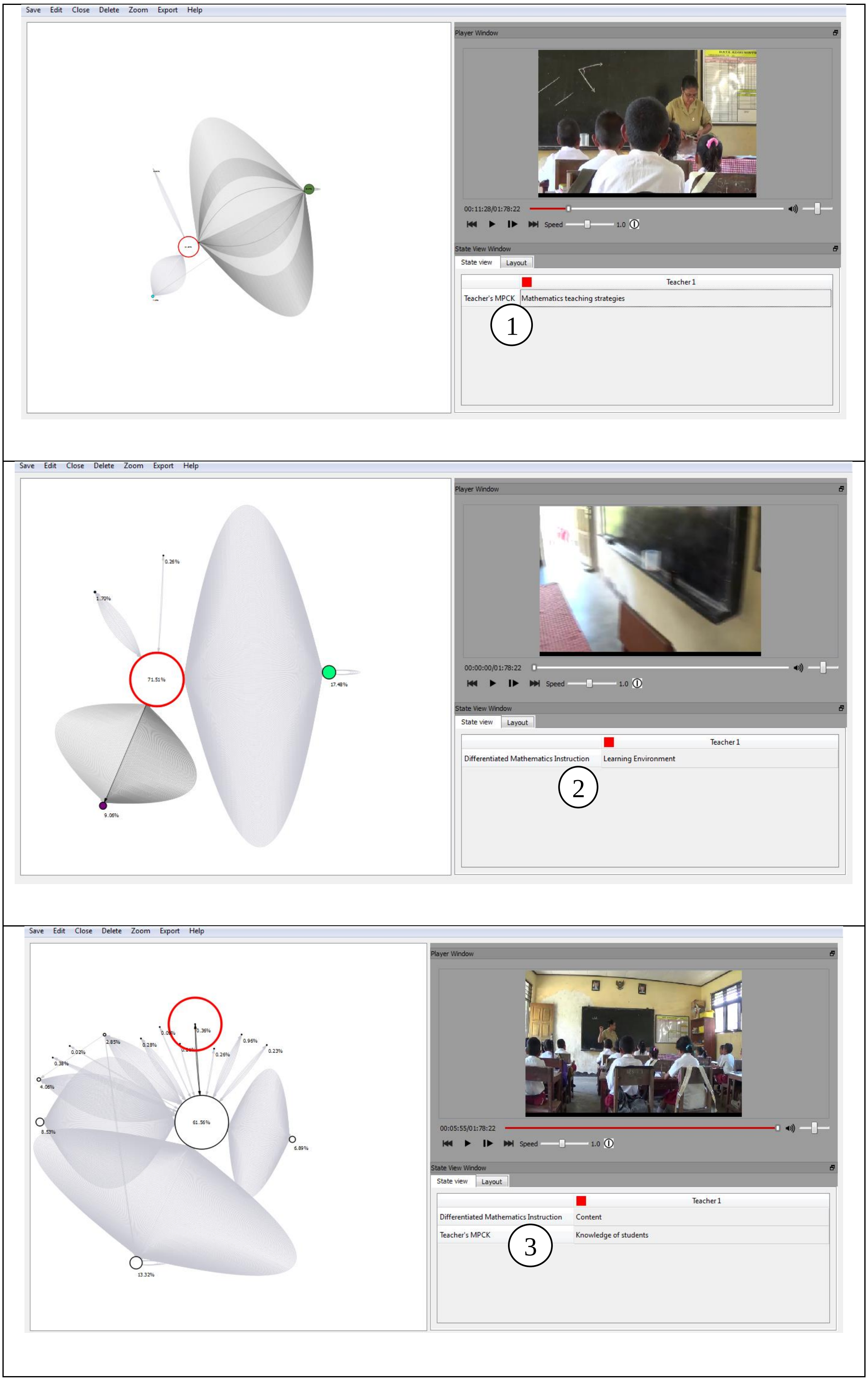


Figure 4-5 Transition diagrams of three states: (1) Mathematical Pedagogical Content Knowledge, (2) Differentiated Mathematics Instruction, (3) Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction

4.2 School A: Overview of Lessons A1 and A2

The topic taught in School A was about identifying and classifying different types of angles (acute, right and obtuse). In Lesson A1, Teacher A scaffolded the students' understanding of angles using objects in the classroom; for example, the blackboard, teacher's table, and wall. Teacher A then drew the different types of angles on cardboard, which was attached to the blackboard. Following this, she showed the students how to measure an angle by using a protractor. She then grouped the students into three groups of six to solve problems on the worksheets she provided. At the end of Lesson A1, the teacher assigned homework.

In Lesson A2, Teacher A started with a discussion of the homework from the first lesson. Students were then grouped into threes and asked to go outside to identify angles in buildings around the school. Afterwards, students drew the buildings that they observed and counted the number of angles in their pictures. They were then asked to classify the angles and submit their work. Teacher A attached some geometric shapes to the blackboard and asked the students to count the number of angles in each shape and classify the angles. She then named each vertex of the shape, using capital letters, and taught the students how to write the name of the angle using those letters. At the end of Lesson A2, Teacher A assigned homework.

Teacher A mostly taught what had been planned in both Lesson Study Group Meeting-A1 and A2, following Lesson Plans A1 and A2. Teacher A made changes to her teaching in Lesson A2 after the observers gave feedback about her first lesson in LSGM-A2. However, Teacher A also undertook some activities that were not included in the Lesson Plan; for example, the questions of homework were not included in the Lesson Plan A2. Moreover, the individual assessment that had been planned and written for Lesson A2 was deferred to the next lesson, due to time constraints in Lesson A2.

4.2.1 Lesson Study Group Meeting 1 in School A (LSGM-A1)

In LSGM-A1 the teachers worked together to produce the first Lesson Plan of School A (Lesson Plan A1). They used the same Lesson Plan structure that they usually use in the school. Figure 4-6 is an English-language version of Lesson Plan A1, which has been translated from the original version in Indonesian (see Appendix 9).

Lesson Plan A1

Subject: Mathematics
Year/Semester: 4/1
Time allocation: 70 minutes
Topic: Angles

1.1 Core Competence

Understand factual knowledge by observing (hear, see, read) and ask questions based on curiosity about him/herself, God's creatures and activities, and the objects he/she encountered at home, school, and playground.

1.2 Basic Competencies

- Recognise right angles through observation and compare to other kind of angles.
- Present acute and obtuse angles in 2-dimensional geometric shapes.

1.3 Indicators

1. Recognise acute, obtuse, and right angles.
2. Measure an angle by using a protractor.
3. Describe the types of angle.

1.4 Teaching Objectives

1. Students can differentiate acute, obtuse, and right angles.
2. Students can measure an angle using a protractor.
3. Students can describe the types of angles.

1.5 Subject Matter

Angles

1.6 Strategies

Differentiated Mathematics Instruction:

- Varied according to students’ ability, interest, and readiness.
- Relevant to students’ daily lives.
- Varied activities, encourages creative and critical thinking.
- Flexible grouping.
- Flexible submission of the task.

- Provide rubric for assessment.

Teaching Model

2.1 Teaching Steps

- Initial activities (10 minutes)
  - A. Apperception

    Students are asked to observe objects in the classroom.
  - B. Topic information

    Teacher teaches the material.
- Main activities (50 minutes)
  - Students are asked to explain their understanding of an angle.
  - Teacher explains the types of angles and draws some different angles.
  - Teacher raises guiding questions.
  - Students are grouped.
  - Students are asked to draw a certain angle.
  - Teacher refines the method of measuring an angle by using protractor appropriately.
  - Students differentiate the types of angles.
  - Students group the angles.
- Closing activities (10 minutes)
  - Teacher concludes the lesson.
  - Homework.

    Questions:

    1. Draw right, acute and obtuse angles.
    2. Draw an angle with 120^0.

2.2 References

- Curriculum 2013
- Teacher and student textbook

2.3 Assessment

Worksheet A1 (see Figure 4-13)

2.4 Ongoing assessment

Observation and portfolios

Figure 4-6 Lesson Plan A1

Mathematical Pedagogical Content Knowledge in LSGM-A1

With a focus on Mathematical Pedagogical Content Knowledge, the teachers in LSG-School A discussed mathematics teaching strategies and how to organise delivery of the topic about angles in LSGM-A1. Teacher A picked the topic, the core competence and the basic competencies as cited in the textbook referred to the national curriculum. Teacher A also determined the indicators and teaching objectives as agreed by other members of LSG-School A. In order to reach the first indicator, "Recognise acute, obtuse, and right angles", Observer A3 advised Teacher A to ask students to draw the different types of angles. From students' drawings, Teacher A could identify the students who recognised the angles.

At one point, the teachers also demonstrated their Mathematical Conceptual Knowledge in their discussion. For example, when Observer A2 said that two lines that form a straight line do not make an angle, Teacher A replied that the angle is 180°. From this discussion, it is evident that Lesson Study can help teachers to clarify concepts that will be used in a lesson. Teacher A also said that she would like to introduce right angles before acute and obtuse angles. She planned to use a protractor to introduce the right angle. She said, "a protractor is half of a circle and the angle is 180°. If the protractor divided by two, each angle will be 90°, which is a right angle." Observer A3 then advised Teacher A to draw some angles with different types to help the students learn to classify angles. Teacher A agreed with this idea, saying that after classifying the angles, she would continue using the drawings and ask the students to measure each angle.

Differentiated Mathematics Instruction in LSGM-A1

The four teachers from School A met to discuss and prepare the first lesson plan, with angles as the topic. They used the lesson plan (see Figure 4-6) format provided by the school, but modified it and paid more attention to differentiation in the lesson, attempting differentiate content, process, assessment and learning environment while they familiarised themselves with the new curriculum and textbook for 2013.

Evidence for differentiation in Lesson A1 can be seen Section IV of Lesson Plan A1 (see Figure 4-6). The teachers sought to focus on student ability, interests and readiness to learn. They planned to relate the learning to daily life, to vary the activities and to encourage the students to think creatively and critically by using flexible groupings and providing flexibility in their submissions. However, the teachers did not differentiate the product in the lesson; they prepared the task with the same level of difficulty for all students.

The teachers chose the topic as per the curriculum and then planned the lesson based on their experience with some modifications to the content, process, assessment and learning environment. For content differentiation, Teacher A planned to use objects in the classroom to examine the angles created by each object. She thought of this idea after reading the second basic competency (see Figure 4-6):“Present acute and obtuse angles in 2-dimensional geometric shapes”. This is relevant to students’ daily lives as the objects in the classroom are familiar to them. Other members of the LSG-School A supported the idea of using real objects in the classroom.

To differentiate the process, LSG-School A decided to group the students in the classroom into three groups of six students. The teachers used heterogeneous grouping, given the varying abilities of the students.

In assessing the students, LSG-School A decided to use a checklist for ongoing assessment. However, the indicators in the checklist — listening, non-verbal communication, participation and speaking fluently — focused on student activity during the lesson (that is, a participation checklist), and did not relate to the objectives of the lessons. For instance, indicators in listening are: a student always listens when another student is speaking; a student listens when another student is speaking but sometimes needs to be reminded to listen; and a student needs to be reminded to listen to other students who are speaking.

4.2.2 Lesson A1: Acute, right and obtuse angles

Mathematical Pedagogical Content Knowledge in Lesson A1

Figure 4-7 shows the analysis of Teacher A’s knowledge of content, students and teaching strategies, which was coded using Multimodal Analysis Video software and stored in the database. The most common occurrence of Mathematical Pedagogical Content Knowledge was in teaching strategies (29.94%), followed by knowledge of mathematics content about acute, obtuse and right angles (7.59%). Mathematics Content Knowledge consists of conceptual and procedural knowledge. Teacher A demonstrated her conceptual knowledge in Lesson A1. For example, when she explained the meaning of angle, she said, “angle is the meeting of two rays at the base or the meeting of two lines at a point. That is the definition of angle”.

In Lesson A1, Teacher A’s knowledge of the students was shown 0.85% of the time. In this case, for example, she knew that her students understood about the objects in the classroom that have angles. For example, she said “You have noticed and mentioned some objects; for example, board, door, desk, book, chair, window, cabinet.” However, the teacher’s knowledge of the curriculum was not obvious in her teaching in the lesson: she did not connect mathematical knowledge about angles to previous or later topics, nor did she link it to other subjects.

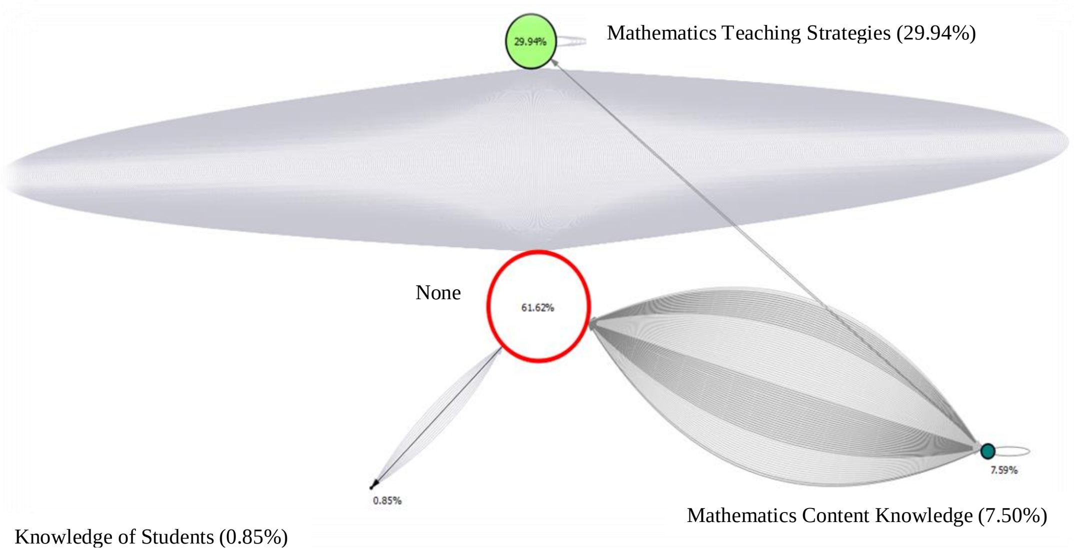


Figure 4-7 Mathematical Pedagogical Content Knowledge in Lesson A1

Teacher A demonstrated her ability in teaching strategies in the way she managed the students in the classroom. She grouped the students and gave them a task. Then, she moved from one group to another to assist the students, whether as a group or individually (see Figure 4-8).

Figure 4-8 Teacher assists students in group (left) and individually (right)

Teacher A was also able to control the class, in that the students appeared to pay attention to the lesson. They listened to Teacher A because she spoke loudly and with confidence. For example, when she saw that some of the students had started to lose concentration, she said loudly: "Now, look at here...". This direct strategy of control was used to attract and maintain interest and attention.

In terms of mathematics content knowledge, Teacher A showed her understanding of angles by providing the definition of an angle and explaining the difference between acute, obtuse and right angles. She also discussed with the students whether a cylinder has an angle or not. Her mathematics content knowledge was shown for 7.59% of the lesson. When identifying an angle as an acute, obtuse or right angle, Teacher A taught the students to measure the angle by using a protractor. However, the angles that she drew on a cardboard were difficult to see because they were unclear and not big enough (see Figure 4-9). For this reason, it was difficult for all students to see the procedure for measuring an angle.

Figure 4-9 Teacher helps a student to measure an angle

Table 4-2 summarises Teacher A's Mathematical Pedagogical Content Knowledge, as evidenced in Lesson A1. In each case, examples in the lesson are provided, as discussed above. During Lesson A1, Teacher A demonstrated her knowledge of mathematics pedagogy for 38.38% of the lesson. This means that the teacher showed her ability to understand the concepts to build students' understanding of the knowledge.

Table 4-2 Mathematical Pedagogical Content Knowledge shown in Lesson A1

#	Mathematical Pedagogical Content Knowledge	Examples of evidence	Proportion of lesson time (%)
1	Mathematics content knowledge	The teacher explained whether a cylinder has angles or not.	7.59
2a	Mathematical pedagogical knowledge: Knowledge of students	The teacher affirmed students' understanding about objects that have angles inside the classroom.	0.85
2b	Mathematical pedagogical knowledge: Mathematics teaching strategies	The teacher asked a student to come forward and point to the angles of the blackboard.	29.94
2c	Mathematical pedagogical knowledge: Curriculum knowledge	-	0
		Total	38.38

At the end of Lesson A1, Teacher A wrote homework (Figure 4-10) on the blackboard. She asked students to do the work at home and told them that the results would be discussed in Lesson A2.

1. Draw an angle of 120^0.
2. Draw an angle of 30^0.

Figure 4-10 Homework for Lesson A1

Differentiated Mathematics Instruction in Lesson A1

Figure 4-11 is the state transition diagram for the differentiation that occurred during Lesson A1. The results shown in Figure 4-11 (see also Table 4-3), indicate that Teacher A differentiated the lesson in content, process, product, and learning environment. She applied differentiation in almost all areas, with the exception of assessment differentiation.

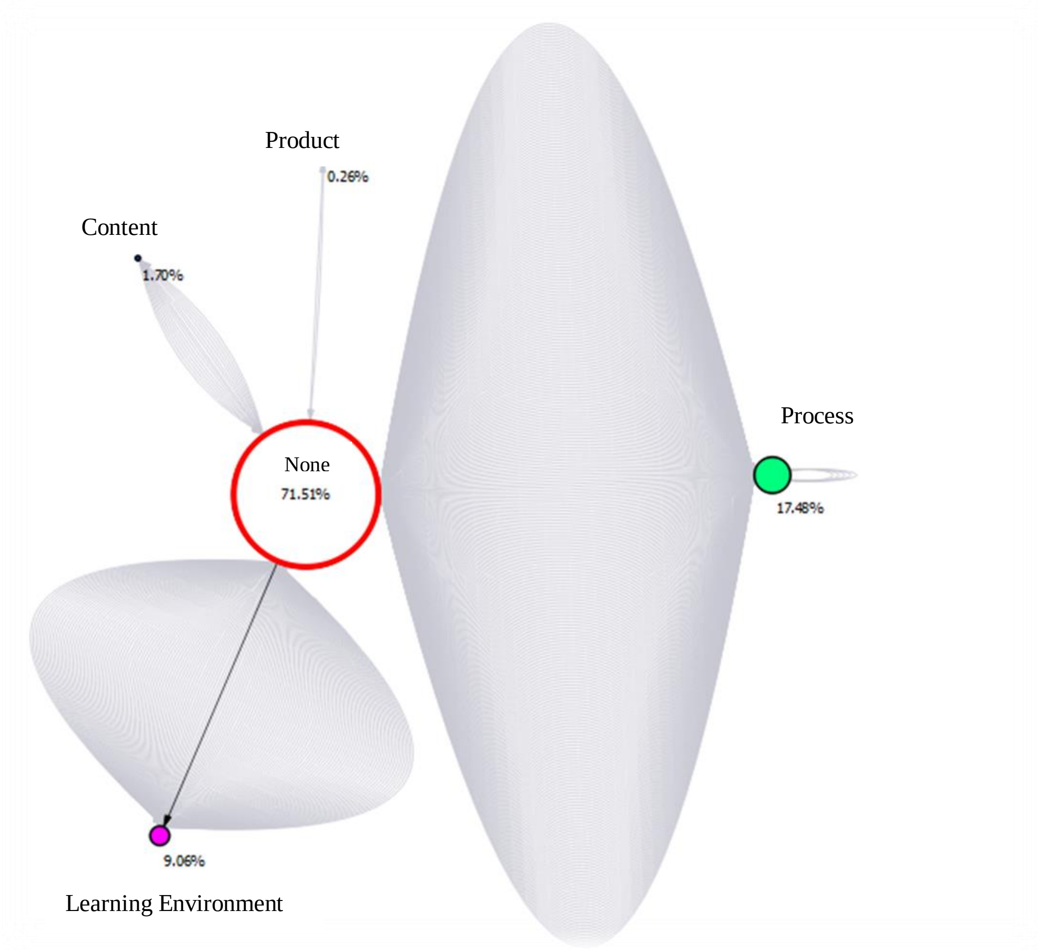


Figure 4-11 Differentiated Mathematics Instruction in Lesson A1

However, Teacher A did not apply any differentiation for 71.51% of the total class time. This means that the different levels of ability, readiness and interests of the students were not taken into account for nearly three-quarters of the lesson. For example, Teacher A asked students to repeat their answers as a group:

Teacher A:	Today, we will learn mathematics about ...(teacher wrote the word "angles" on the board)
Whole class:	Angles
Teacher A:	What is that?
Whole class:	Angles
Teacher A:	What did I write on the blackboard?
Whole class:	Angles
Whole class:	Angles
Teacher A:	Say it again properly. Again.

Whole class: Angles

Teacher A: Angles

Whole class: Angles

Teacher A: All right. Now, we will learn about angles.

It appears that Teacher A aimed to stress the concept through repetition; however, it is difficult to ascertain which students understood the concepts based on the group replies, because they all spoke together. It was noisy when the students did not say "angles" at the same time. For example, in the excerpt cited above, some students answered "angles" then other students repeated the word.

Moreover, when Teacher A asked questions which required more than one word as the answer (for example, the meaning of an angle), it was impossible to decipher the different responses and identify those students who answered correctly:

Teacher A: An angle is...

Whole class: The meeting of two rays at the base or at a point.

And again, when the teacher asked why some angles were classified as right angles:

Teacher A: Now, I would like to ask you. You said that a, d, e, and g are right angles. Why?

Whole class: Right angle is an angle that is ninety degrees.

Teacher A applied differentiation principles for 28.5% of the lesson time in Lesson A1. Although, the teacher taught the same level of content difficulty to all students, in her differentiation of content (1.70% of the lesson), she did seek to establish the relationship between the topic and the students' daily lives.

To do this, she asked students to indicate objects in the classroom that had angles, as seen in the following extract from Lesson A1:

Teacher A: Anything else? Objects that exist in this classroom?

Whole class: Board, cabinet.

Teacher A: Student 2, say one thing. Another thing?

Student A2: Door.

Student A3: Board.

Teacher A: Board. Student 4?

Student A4: Desk.

Teacher A: Desk. Student 5?

Student A5: Cabinet.

Teacher A: Cabinet.

Student A5: Door.

Teacher A: Door. Anything else...? Still some more...

Student A6: Chair.

Student A7: Window.

Student A8: Book.

Student A3: Drawing book.

Teacher A: Book. Say, that hasn't been mentioned yet.

Student A9: Eagle (symbol of Indonesian country)

Teacher A: Student 10. That is right...eagle. Student 10.

Student A10: Frame.

Teacher A: Frame...photo frame.

Whole class: Name board, chalk box, book.

Teacher A: Some more...some more...please look around. Common...teacher table, student desks, what is this? (Teacher A pointed at a chair.)

Whole class: Chair.

After this interaction, Teacher A asked some students to point to the angles of the objects. This required students to think deeply because the teacher had not yet defined angle. Figure 4-12 shows a student who was required to come and point out the angles on the blackboard.

Figure 4-12 Student points to an angle on the blackboard

Teacher A differentiated the process for 17.48% of the lesson time and this comprised most of the time spent in differentiation during this lesson. For example, she tried to encourage students to think critically. The following example is an interaction where the teacher challenged the students to think critically:

Teacher A:	If this one (teacher showed a circle object). Does this object have angles?
Some students:	Yes.
Some other students:	No.
Teacher A:	Are there any angles?
One student:	No, that's a circle.
Teacher A:	No, it has no angles... What is the shape of this object?
Whole class:	Circle.
Teacher A:	This is a circle and has no ...?
Whole class:	Angle.
Teacher A:	Good. Now you know the angles of an ...?
Whole class:	Object.

Another example of process differentiation occurred when Teacher A required one student to point to the angles of the blackboard in the classroom before she explained the meaning of angle. She then asked: “Who can define…what the meaning of angle is?" This question was posed after the teacher asked students to point to some objects in the classroom that had angles and point the angles in the objects. She encouraged students to define "angle" from their perspective after they had identified objects in the classroom that had angles.

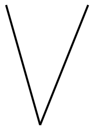
After Teacher A led class discussion to define an angle, she grouped students in groups of six and she distributed Worksheet A1 (Figure 4-13). Teacher A created a task with the same level of difficulty for all students. However, for product differentiation (0.26% of lesson time), Teacher A did, for example, provide for flexible submission (an indicator of product differentiation), by giving one group extra time to complete the task.

Worksheet A1

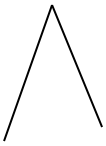
Consider the pictures below.

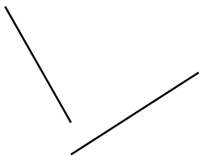
1. Which angles are right angles?

a. 

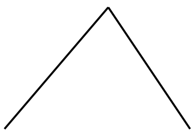
b. 

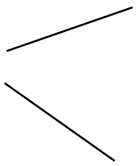
c. 

d. 

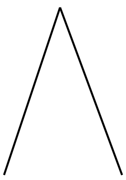
e. 

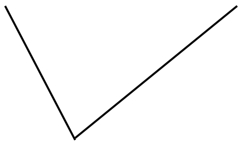
2. Which angles are acute angles?

a. 

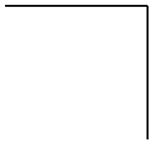
b. 

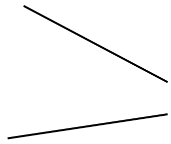
c.

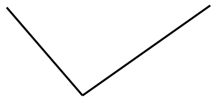
d. 

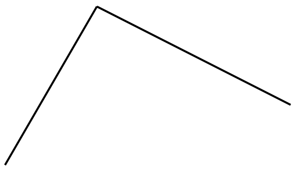
e. 

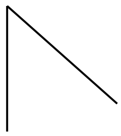
3. Which angles are obtuse angles?

a. 

b. 

c. 

d. 

e. 

4. Measure and determine each angle.

Figure 4-13 Worksheet for Lesson A1

Differentiation of assessment did not occur in Lesson A1. In this lesson, assessment of student understanding of the topic was undertaken through group discussion. The teacher allocated marks based on the number of questions that the students answered correctly.

Learning environment differentiation (9.06% of the time) was evident in Teacher A’s appreciation of appropriate responses to questions. Not only did she say “good” to the student, but also sometimes asked the whole class to clap their hands in appreciation. For example, after student A12 answered a question correctly the teacher said: “Good...clap hands for student A12”. In addition, the teacher assisted students on an individual basis when they had difficulty solving a problem. She moved from one group to another to help them students one by one. In this way, the teacher encouraged students to respect the rights of other students with regard to receiving help and assistance to learn in the class.

Table 4-3 lists evidence of the use of Differentiated Mathematics Instruction obtained from the video analysis of the first mathematics lesson about angles.

Table 4-3 Use of Differentiated Mathematics Instruction in Lesson A1

#	Differentiated Mathematics Instruction	Examples of evidence	Proportion of lesson time (%)
1	Content differentiation	Teacher associated the topic to students' environment by asking them to indicate the objects in the classroom that had angles.	1.70
2	Process differentiation	Teacher gave open-ended questions that encouraged students to think creatively.	17.48
3	Product differentiation	Teacher gave more time to the group that needed it to complete their work.	0.26
4	Assessment differentiation	Teacher provided a rubric for the assessment.	-
5	Learning environment differentiation	Teacher encouraged students to appreciate fellow students who answered the question correctly by giving applause. Teacher recognised students who did their job properly.	9.06
		Total	28.50

4.2.3 Lesson Study Group Meeting 2 in School A (LSGM-A2)

In LSGM-A2, the Lesson Study Group of School A (LSG-School A) reviewed Lesson A1 and generated Lesson Plan A2 as shown in Figure 4-14. The lesson plan has been translated from the original version (see Appendix 10).

Lesson Plan A2

Subject: Mathematics

Year/Semester: 4/1

Time allocation: 70 minutes

Topic: Angle

1.1 Core Competence

Understand factual knowledge by observing (hear, see, read) and ask based on curiosity about him/herself, God's creatures and activities, and the objects he/she encountered at home, school, and playground.

1.2 Basic Competencies

- Presented acute and obtuse angles in 2-dimensional geometric shapes.

1.3 Indicators

1. Design school or house building by paying attention to acute, obtuse, and right angles.
2. Determine acute, obtuse, and right angles in a two-dimensional geometric shape.

1.4 Objectives

1. Students can determine the types of angles of a house and the school's building.
2. Students can determine acute, obtuse, and right angle in a two-dimensional geometric shape.

1.5 Subject Matter

Angles

1.6 Strategies

Differentiated Mathematics Instruction:

- Relevant to students' daily life.
- Various activities that encourage creative thinking.
- Flexible in grouping.
- Flexible submission of the task.

1.7 Teaching Steps

- Initial activities
  - Teacher and student discuss the homework from the previous lesson.
  - Teach the topic.
- Main activities
  - Teacher groups the students.
  - Students are asked to go out of the classroom to observe buildings around the school.
  - After observation, students are asked to draw the building that they saw.
  - Students determine the types of angle in their picture.
  - Teacher explains the observation results.

- Teacher distributes worksheets.
- Each group completes the task.
- Teacher assists students to complete the task.
- Students present the discussion results.
- Other students give responses.

• Closing activities

- Teacher concludes the lesson.
- Homework.

1.8 References

- Curriculum 2013
- Teacher and student text book
- Mathematics IVA, Erlangga

1.9 Media

Houses around the school, school building

1.10 Tool

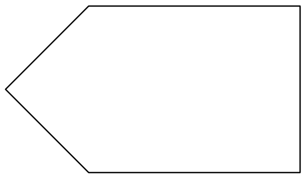
Two-dimensional geometric shapes

1.11 Assessment

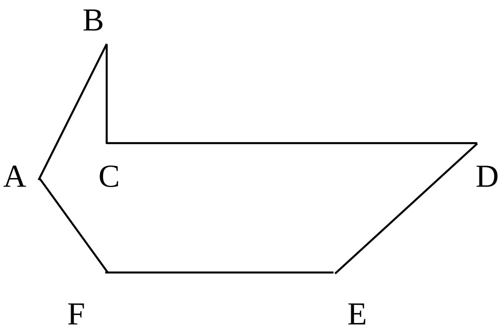
1. Worksheet
2. Written test

1.12 Questions

1. Determine the number of right angles on the picture below.



2. Name each of the angles in the picture below.



3. Draw one 2 dimensional geometric shape that has two obtuse and acute angles.

Figure 4-14 Lesson Plan A2

Mathematical Pedagogical Content Knowledge in LSGM-A2

Observer A2 noticed that when the students worked in groups, Teacher A moved from one group to another to assist them when needed. However, she also sat on her chair to work on another task (Figure 4-15), for quite a long period, 8.5 minutes in total. She did not pay attention to the students during that time, so did not know whether the students worked or not or whether they experienced any difficulties.

Figure 4-15 Teacher doing something else while students are working in groups

Observer A3 commented that by using Lesson Study the teachers could learn the new curriculum content for 2013 together. Teachers at the school received the curriculum just one month before these study and so they were not familiar with it. They used the curriculum for 2013 when they created the lesson plans during this study. This demonstrates that Lesson Study can be a forum for teachers to discuss new information and knowledge that is relevant to their teaching.

Differentiated Mathematics Instruction in LSGM-A2

The teachers in LSG-School A reviewed Lesson A1. Feedback for Teacher A included observations about the answers in unison that created a noisy classroom environment. Observer A3 suggested that one student should answer the question.

Another comment came from Observer A2, who said that the teacher had not yet differentiated the level of difficulty of the task. Consequently, Observer A2 noticed that one group finished much earlier than the others. As there were no challenging questions for students who finished earlier than others, that group had nothing to do. Therefore, the students were noisy, as they talked and walked around the classroom.

After LSG-School A reviewed Lesson A1, the teachers discussed the creation of the lesson plan for Lesson A2. The topic was still about angles, but LSG-School A planned to create an interesting activity. Students would be asked to leave the classroom and observe the school building and the houses surrounding the school. The groups of students would then draw the building they observed and classify the angles on the picture. The teachers also planned to use geometric shapes from cartoons to teach about acute, obtuse and right angles. These models would be used to introduce the naming of vertices, using capital letters at each vertex. These activities are indicators of process differentiation.

4.2.4 Lesson A2: Acute, right and obtuse angles

Mathematical Pedagogical Content Knowledge in Lesson A2

Figure 4-16 presents results obtained from analysis of Teacher A's Mathematical Pedagogical Content Knowledge in Lesson A2. Teacher A's knowledge of mathematics teaching strategies was articulated during 37.99% of the lesson time. For example, Teacher A changed the number of students in each group from the earlier lesson. In Lesson A1 she grouped students in groups of six but in Lesson A2 she grouped them in groups of three. Before she grouped the students, she asked this question: "Today, I will put you into six groups. You are eighteen students in this class. So, how many students are in one group?" She used this strategy to encourage the students to divide.

The results also show that Teacher A's mathematical content knowledge was practiced for 17.93% of Lesson A2. For instance, when Teacher A helped students classify angles in the pictures they had drawn and found that one group had difficulties, she prompted them: "Right angle? Right angle is like this one. Look, this is more than ninety degrees. If this is a right angle the line must be like this." She motivated students to find the correct answer by asking them "So, what is the name of this angle?"

The knowledge of students of Teacher A was detected 6.93% of the time in Lesson A2. One example was that, when Teacher A realised that most students were still having difficulties classifying the angles that they drew, she moved from group to group to help them. She also helped them to count the number of each type of angle.

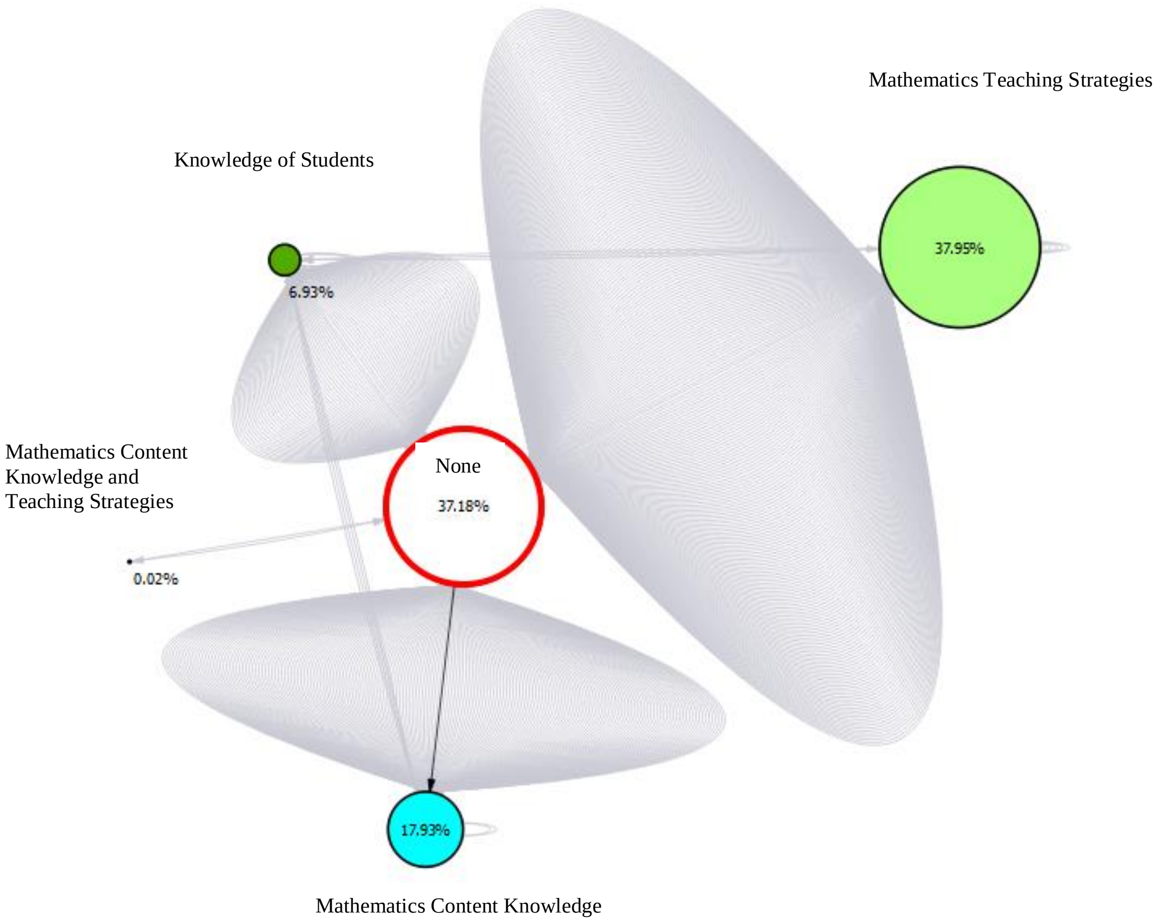


Figure 4-16 Mathematical Pedagogical Content Knowledge in Lesson A2

There was a significant amount (62.85%) of total practice of Mathematical Pedagogical Content Knowledge in Lesson A2 (see Table 4-4). There was, however, no evidence of the teacher's curriculum knowledge in this lesson.

Table 4-4 Mathematical Pedagogical Content Knowledge shown in Lesson A2

#	Mathematical Pedagogical Content Knowledge	Examples of evidence	Proportion of lesson time (%)
1	Mathematics content knowledge	The teacher explained how to draw an angle using a protractor to the students.	17.93
2a	Mathematical pedagogical knowledge: Knowledge of students	The teacher underlined students’ understanding about an angle.	6.93
2b	Mathematical pedagogical knowledge: Mathematics teaching strategies	The teacher asked students to work in groups, so they could help each other.	37.99
2c	Mathematical pedagogical knowledge: Curriculum knowledge	-	0
		Total	62.85

Figure 4-17 Use of visual aids in Lesson A2

Figure 4-17 shows Teacher A using geometric shapes made from different coloured cardboard to make them more interesting. However, she did not clean the blackboard before attaching the shapes, making identification of vertices more difficult in some cases.

In Figure 4-18, Teacher A has written the homework that was planned on the blackboard, but the homework question was not written in Lesson Plan A2. Teacher A drew a geometric shape, named the vertexes and wrote three questions about the shape: the number of angles is … angle(s); the type of angles are …; and the names of the angles are … The homework questions are on the right side of the blackboard in the figure.

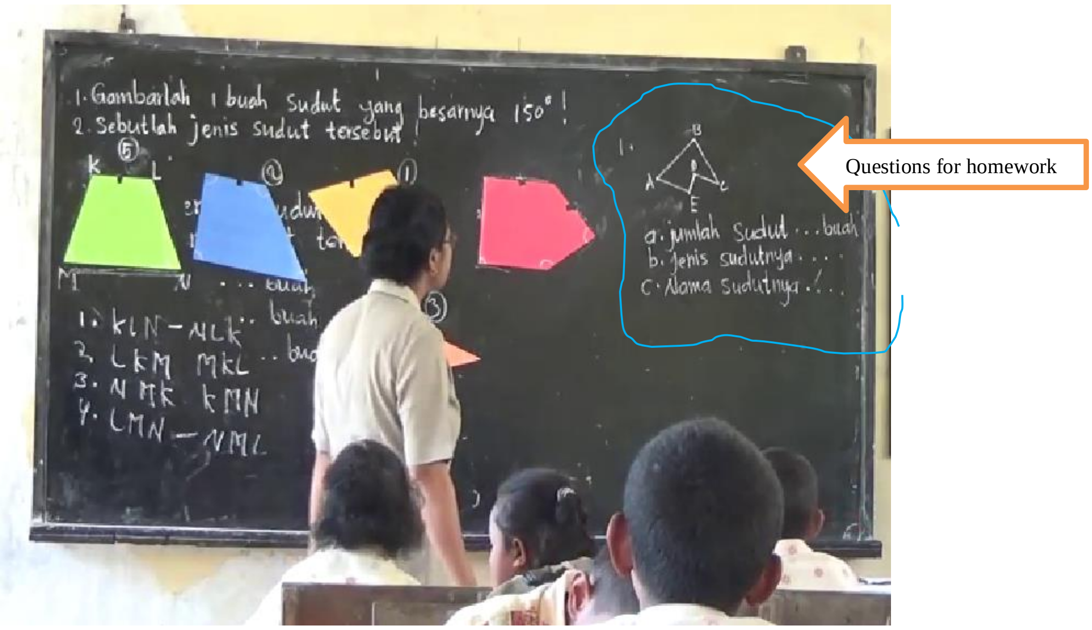


Figure 4-18 Homework question for Lesson A2

Differentiated Mathematics Instruction in Lesson A2

The total amount of Differentiated Mathematics Instruction in Lesson A2 increased to 33.39%. Figure 4-19 shows the proportions of each type: process differentiation was 28.83%, assessment differentiation was 2.75%, product differentiation was 1.19%, and learning environment differentiation was 0.62%. However, the traditional teaching approach was still dominant, with 66.61% of teaching time devoted to tasks of equal complexity. Examples of each type of differentiation in Lesson A2 are given in Table 4-5.

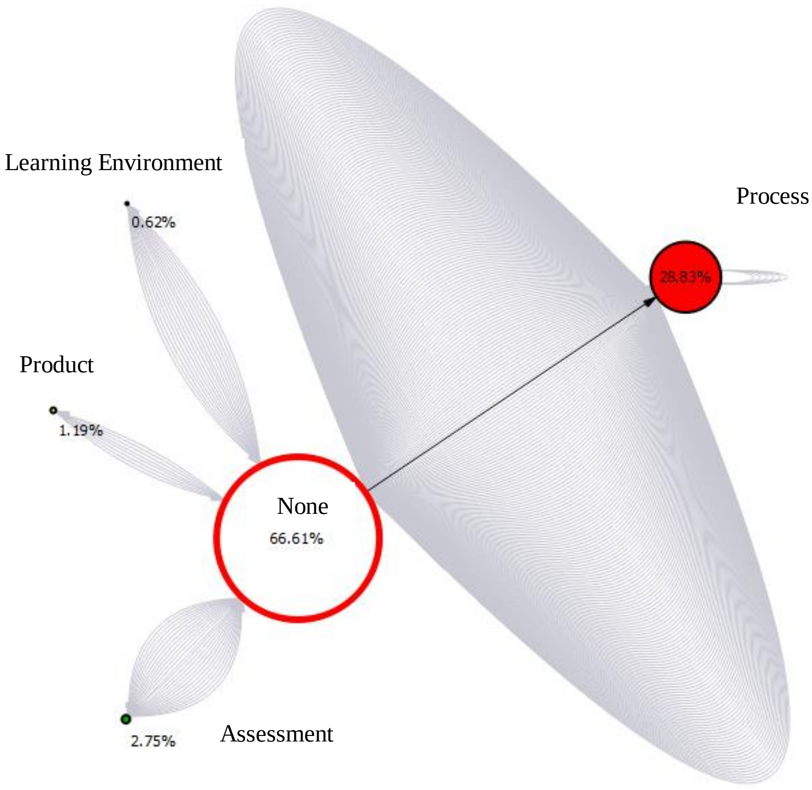


Figure 4-19 Differentiated Mathematics Instruction in Lesson A2

Lesson A2 was more varied that Lesson A1with respect to the process differentiation. Teacher A divided the students into groups of three and instructed them to go outside in order to identify angles in the surrounding buildings (see Figure 4-20). Each group was told to observe a different building, and the students were required to note the shape of the roof, the walls of the building, and one side of the building. The students then returned to the classroom and drew what they observed in two dimensions. From their drawings, they classified the angles as acute, obtuse or right angles. These activities required students to think critically, demonstrate an understanding of the different types of angles, and determine the type of the angles in their drawings. This activity also illustrated Teacher A’s performance in fostering students to do research. Students observed a building, then drew it and classified the angles they found from their drawings.

Figure 4-20 Teacher points to the building being observed by one group

In terms of product differentiation, Teacher A allowed the students to express their understanding in various ways. For example, when she asked the students to draw an angle of 120^0, the students drew angles facing in different directions (see Figure 4-21).

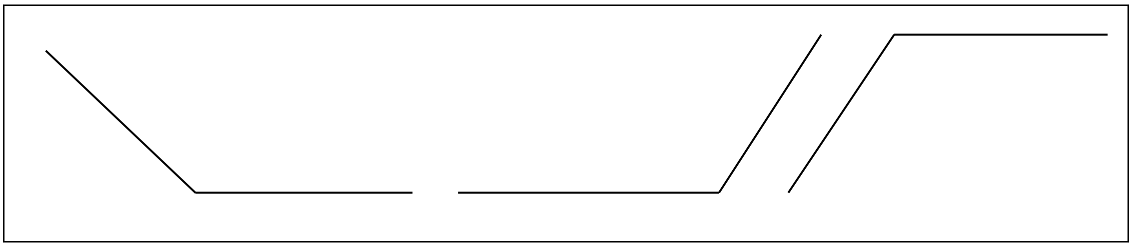


Figure 4-21 Angles of 120^0

In this second lesson, Teacher A attempted to allow the students to check their own homework. Allowing her students to evaluate their own progress meant that she applied assessment differentiation.

Teacher A consistently showed her appreciation when students answered. When a group of student answered incorrectly; for example, when students classified angles from their drawings, she helped them find the correct answer.

The use of Differentiated Mathematics Instruction in Lesson A2 is summarised in Table 4-5, with examples of each type.

Table 4-5 Use of Differentiated Mathematics Instruction in Lesson A2

#	Differentiated Mathematics Instruction	Examples of evidence	Proportion of lesson time (%)
1	Content of differentiation	-	0
2	Process of differentiation	Various activities: the teacher asked the students to observe buildings outside of the classroom then draw the buildings. Encouraged critical thinking: the teacher asked the students to classify the angles. They are acute, obtuse or right angles.	28.83
3	Product of differentiation	The teacher allowed the students to draw the angles in different ways.	1.19
4	Assessment of differentiation	Allowed students to evaluate their own progress: the teacher asked the students to check the angles that they drew at home.	2.75
5	Learning environment of differentiation	The teacher appreciated students' answers.	0.62
		Total	33.39

4.2.5 Lesson Study Group Meeting 3 in School A (LSGM-A3)

Mathematical Pedagogical Content Knowledge LSGM-A3

Observer A3 suggested that Teacher A pay more attention to time allocation because she taught over the time allocated for Lesson A2. Observer A1 noted that students could not concentrate for this length of time because they needed a break to eat their breakfast. School A has a rule that the first break is for students to eat their breakfast (students have to bring their own meal) because it is difficult for students there to have their breakfast before they go to school. School starts at 7.30 am and most parents do not provide breakfast for their children at home. As a result, most schoolchildren do not eat until lunchtime. Through introducing this regulation, School A wanted parents to take responsibility for providing breakfast for their children.

However, Observer A1 also noted Teacher A had altered her teaching strategy, by asking questions of individual students. She did not ask students to answer together during Lesson A2. This indicates that the teacher changed her practice after the review of Lesson A1.

Observer A2 suggested that Teacher A write the topic on the blackboard to help students understand what they had learnt that day. Observer A3 also noted that the indicators could be written on the blackboard; this would provide guidance for students about what they should expect to achieve at the end of the lesson. Observer A2 advised the teacher to clean the blackboard before attaching the geometric shapes, and recommended that the teacher introduce the sign for an angle. For example, when a student said “angle KLM”, Teacher A could write on the blackboard “∠ KLM”.

Differentiated Mathematics Instruction LSGM-A3

Observers A1–A3 noticed that Teacher A tried to connect the concepts of right, acute and obtuse angles to objects encountered by the students in their daily lives in the outside activity in Lesson A2. Observer A1 added that it was more efficient to group students in a group of three in Lesson A2 because, in a smaller group, every student had an opportunity to express their ideas. Observer 1 said that this also made it easier for Teacher A to appraise each student. This form of grouping was compared with Lesson A1 when Teacher A grouped students in groups of six.

4.2.6 Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in School A

Table 4-6 shows the indicators for Mathematical Pedagogical Content Knowledge that were achieved in Lessons A1 and A2. Teacher A demonstrated knowledge in most areas of Mathematical Pedagogical Content Knowledge. She demonstrated understanding of concepts of angle; acute, obtuse, and right angles; and experience in using mathematical equipment (the protractor).

In mathematical pedagogical knowledge, Teacher A also demonstrated her knowledge of students and different mathematics teaching strategies. For example, she used her knowledge of the students' backgrounds to group them. She wanted heterogeneous grouping in both lessons because she wanted the more able students to help others in their group. She also showed ability to manage students in the classroom, handling disruptive students effectively.

Teacher A's curriculum knowledge could not be observed during the lessons, but the objectives of mathematics education were mentioned in the Lesson Plans, as the Core and Base Competencies, which are derived from the national curriculum.

Table 4-6 does not show any evidence of connecting the topic to previous and later topics in the curriculum. For example, when Teacher A used geometric shapes in Lesson A2, she could have made such connections by naming each shape. However, she did not state the names of the shape; for instance, triangle, trapezium or parallelogram. Instead, every time she asked students to count angles or classify the angles of each shape, she simply referred to "this shape" or "that shape" while pointing to the shapes. For instance, she asked: "How many angles are in that shape?" or "How many acute angles are in this shape?" It is important to articulate the name of the shapes to remind students about them and make them realise that there is a connection to topics that have been taught previously.

Table 4-6 Mathematical Pedagogical Content Knowledge Indicators in Lessons A1 and A2

Type of Mathematical Knowledge	Indicators		Applied	
			Lesson A1	Lesson A2
Mathematical content knowledge	Mathematical conceptual knowledge		√	√
	Mathematical procedural knowledge		√	√
Mathematical pedagogical knowledge	Knowledge of students'	Background	√	√
		Understanding	√	√
	Mathematics teaching strategies	Classroom management	√	√
		Assessment	√	√
	Curriculum knowledge	Mathematics educational goals		
		Correlation to previous and later topic		
		Link to other subject		

The indicators of Differentiated Mathematics Instruction achieved in Lessons A1 and A2 in School A are summarised in Table 4-7.

Table 4-7 Differentiated Mathematics Instruction Indicators in Lessons A1 and A2

Type of Differentiation	Indicators	Cycle 1 Lesson A1		Cycle 2 Lesson A2	
		Planned	Applied	Planned	Applied
Content	Varied levels of content	√			
	Relevant to students' daily lives	√	√	√	√
Process	Flexible grouping	√	√	√	√
	Encourages creative thinking	√	√	√	√
	Encourages critical thinking		√		√
	Various activities, such as open-ended questions	√	√	√	√
	Encourages research			√	√
Product	Varied levels of task difficulty				
	Freedom in choosing the task				
	Flexible in expressing their understanding				√
	Flexible submission		√		√
Assessment	Ongoing				
	Provides rubric				
	Gives feedback on students' work				
	Allows students to evaluate their own progress				√
Learning Environment	Some learning centres in the classroom				
	Teacher values each student		√		√
	Encourages students to respect each other		√		√
	Encourages students to be responsible				

Table 4-7 is revealing in several ways. First, Teacher A applied differentiation that was planned in the Lesson Study Meeting and Lesson Plan; for instance, by grouping the students flexibly, using objects that are close to their daily lives, and encouraging them to think creatively. Second, there was differentiation that was planned but that did not occur in the lessons; for instance, creating various levels of content difficulty, encouraging students to think critically, and allowing them to submit tasks flexibly. Third, there was differentiation that was not planned but that Teacher A implemented; for instance, she valued each student, and encouraged students to respect each other. Finally, some forms of differentiation were neither planned nor applied; for instance, encouraging research, allowing freedom in choosing the task, and creating learning centres in the classroom. These issues are discussed further in Chapter 5.

Figure 4-22 shows the combinations of Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction as they occurred in Lesson A1. Figure 4-23 shows the same combinations for Lesson A2. From these graphs it can be seen that the teacher's Mathematical Pedagogical Content Knowledge occurred in each type of mathematics differentiation. For example, in Lesson A1 it was the teacher's knowledge of mathematics teaching strategies that inspired process differentiation most frequently (13.32%). The teacher used her ability in managing the classroom to create various activities when introducing the concept of an angle. She also captured students' interest by asking them to observe real objects in the classroom.

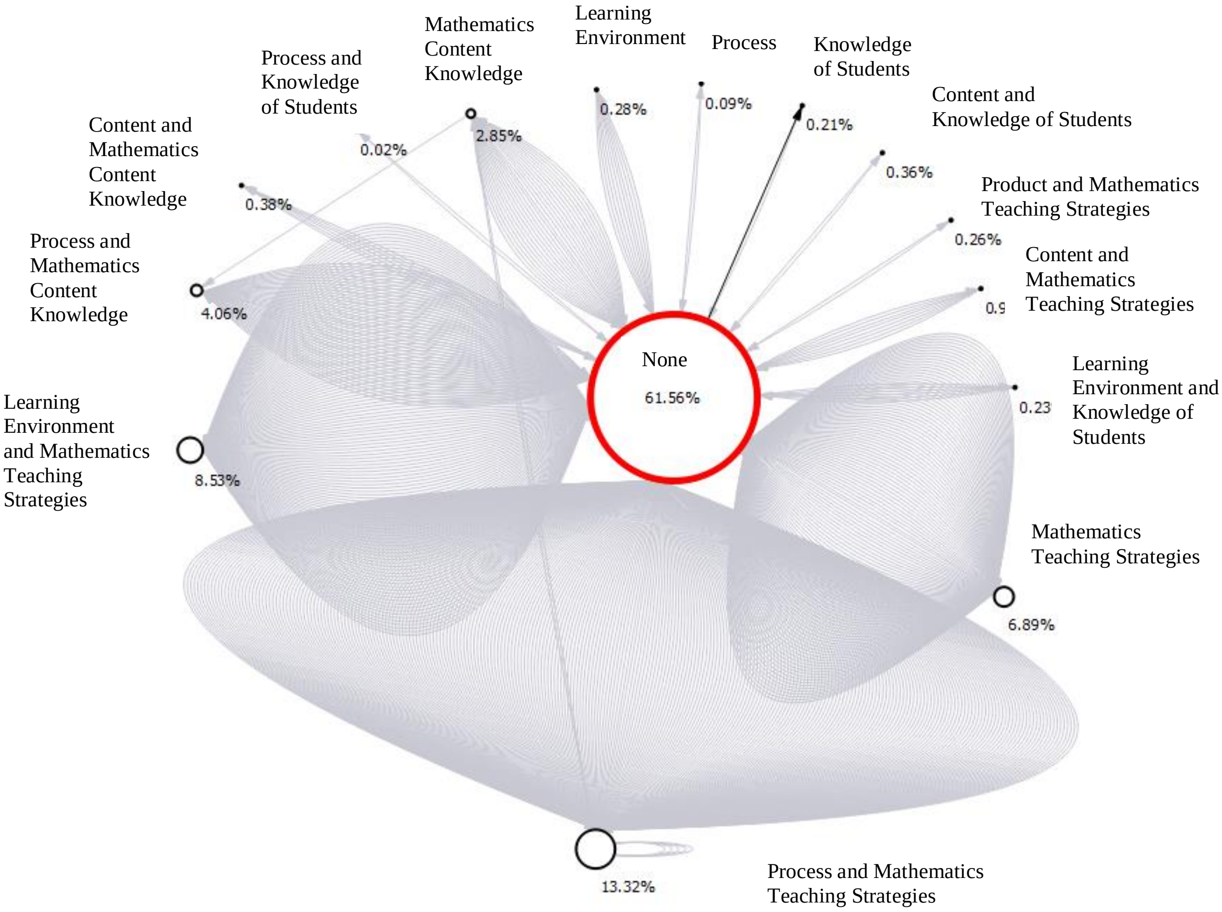


Figure 4-22 Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in School A: Lesson A1

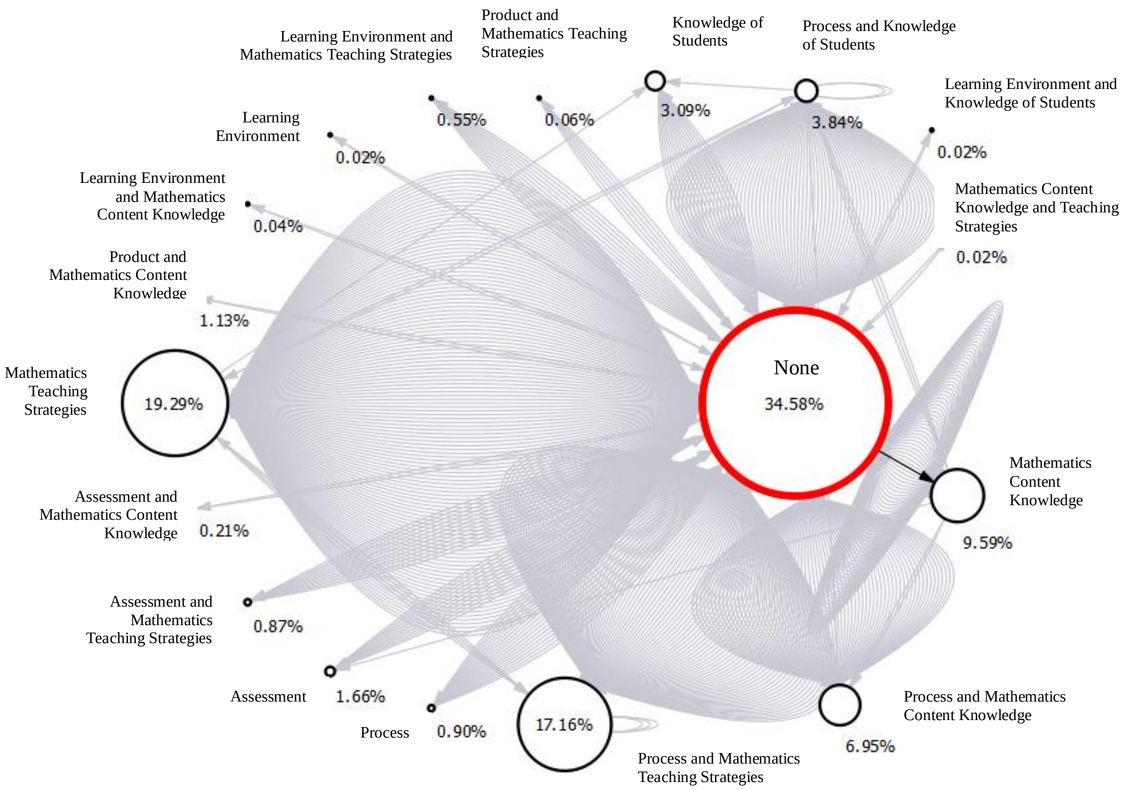


Figure 4-23 Mathematical Pedagogical Content Knowledge Differentiated Mathematics Instruction in School A: Lesson A2

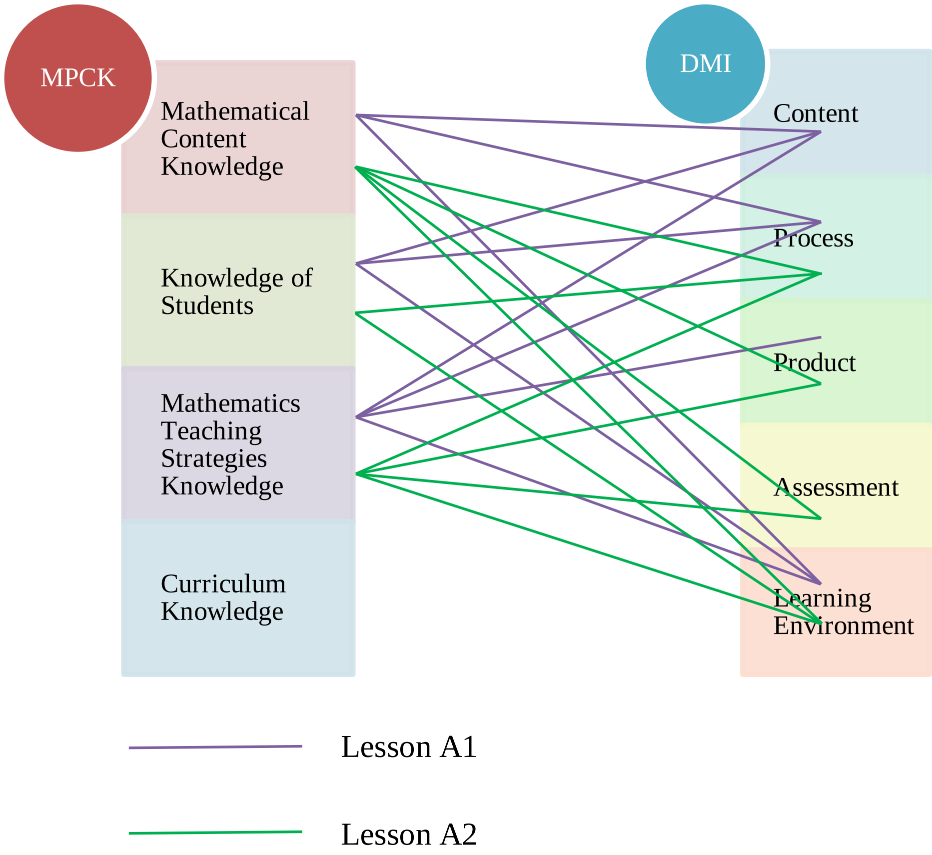


Figure 4-24 Relationship between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in School A

Figure 4-24 shows the relationship between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in School A for Lesson A1 (purple line) and Lesson A2 (green line). In this figure, it can be seen that Teacher A's mathematical content knowledge helped her to differentiate content, process and learning environment in Lesson A1 and to differentiate process, product, assessment and learning environment in Lesson A2. Teacher A's knowledge of students in Lesson A1 helped her differentiate the content, process and learning environment; in Lesson A2, this knowledge helped her differentiate process and learning environment.

In Lesson A1, Teacher A's mathematics teaching strategies knowledge assisted her in all areas of differentiation except assessment. Likewise, in Lesson A2, this knowledge assisted Teacher A in all areas except content differentiation. Interestingly, there is no evidence to show that Teacher A's curriculum knowledge could help her to differentiate in any of the indicators for Differentiated Mathematics Instruction indicators in either Lesson A1 or A2.

Table 4-8 Mathematical Pedagogical Content Knowledge (MPCK) and Differentiated Mathematics Instruction (DMI) in School A

MPCK \ DMI		Content (LA1, LA2) (%)	Process (LA1, LA2) (%)	Product (LA1, LA2) (%)	Assessment (LA1, LA2) (%)	Learning Environment (LA1, LA2) (%)
Mathematical content knowledge: Conceptual and Procedural		(0.38, 0)	(4.06, 6.95)	(0, 1.13)	(0, 0.21)	(0.28, 0.04)
Mathematical pedagogical knowledge	Knowledge of Students	(0.36, 0)	(0.02, 3.84)	-	-	(0.23, 0.02)
	Mathematics Teaching Strategies Knowledge	(0.96, 0)	(13.32, 17.16)	(0.26, 0.06)	(0, 0.87)	(8.53, 0.59)
	Curriculum Knowledge	-	-	-	-	-

(LA1, LA2) = (Percentage for Lesson A1, percentage for Lesson A2)

The results in Table 4-8 indicate that the combination of mathematics teaching strategies knowledge and process differentiation reached the highest percentage of any of the combinations: in Lesson A1 it was 13.32%, and in Lesson A2 it was 17.16%. In one example of this, Teacher A used a strategy to introduce an angle by moving from the real object to an abstract formulation of the concept as a mathematical diagram. She used real objects like tables, the blackboard, the school building and other buildings around the school, then geometric shapes from cardboard, then pictures of various angles. This process is about strategy and the teacher varied the activities in the learning process. That is one of the indicators of process differentiation and the meaning of mathematics teaching strategies knowledge.

The combination of mathematics teaching strategies knowledge and learning environment differentiation occurred in 8.53% of Lesson A1, but appears to have happened very little in Lesson A2 (0.59%). In this case, Teacher A managed the class in such a way that every student paid attention when a fellow student was asked to work at the blackboard, for example, measuring an angle by using a protractor. Teacher A then asked students to praise their friend’s work by giving applause. This showed that Teacher A valued the student. Managing the classroom effectively is an indicator of the mathematics teaching strategies' knowledge. Valuing student contribution is an indicator of learning environment differentiation.

The combination of mathematics content knowledge and process differentiation occurred 4.06% of the time in Lesson A1 and 6.95% of the time in Lesson A2. In one example of this, Teacher A underlined the meaning of obtuse angles, then challenged students to show that the obtuse angles they classified were correct. Demonstration of understanding of a concept is referred to as mathematics conceptual knowledge, which is part of Mathematical Pedagogical Content Knowledge. The way in which Teacher A challenged students to show that angles were obtuse encouraged their creative thinking, which is another indicator of Mathematical Pedagogical Content Knowledge.

The results for School A showed that a teacher's Mathematical Pedagogical Content Knowledge (other than curriculum knowledge) can help them differentiate their mathematics instruction. Each part of Mathematical Pedagogical Content Knowledge can be differentiated. For example, a teacher's knowledge of their students can help them differentiate the content, process and learning environment of the topic and lesson.

4.3 School B: Overview of Lessons B1 and B2

In School B, the topics taught were the commutative law (Lesson B1) and the associative law (Lesson B2). At the start of Lesson B1, Teacher B asked students to count in order, beginning at the first row at the front and ending with the students at the back of the classroom. She then used these numbers as examples to introduce the commutative (Lesson B1) and associative laws (Lesson B2) of addition and multiplication. After Teacher B assisted students to formulate the commutative law, she asked them to work individually on questions about commutative properties of addition and multiplication. At the end of the lesson, she assigned homework (see Lesson Plan in Figure 4-25).

Teacher B began Lesson B2 by asking the students about their homework. Afterwards, she assisted them to find the formula for association law. To give the students practice in using and understanding the associative law, Teacher B organised them into groups of four. After the students finished the group work, Teacher B led a discussion to check the results of each question. At the end of Lesson B2, Teacher B assigned homework (see Lesson Plan in Figure 4-25).

4.3.1 Lesson Study Group Meeting 1 in School B (LSGM-B1)

In LSGM-B1, the Lesson Study Group of School B (LSG-School B) consisting of Teacher B and two observers, Observer B1 and B2, discussed and created Lesson Plan B1. The aim of Lesson Plan B1 was to introduce the commutative law of addition and multiplication. In LSG-School B the discussion focused on the indicators and how to deliver commutative law to the students. Then, the group discussed the types of activities appropriate to support the teaching of the topic. They also discussed the time allocation for each activity. The standard and basic competencies were taken from the national curriculum. Figure 4-25 is an English-language version of Lesson Plan B1, which has been translated from the original version (see Appendix 10).

Lesson Plan B1

School : School B
Subject : Mathematics
Year/Semester : IV/1

1.1 Standard Competence

Understand and use the characteristics of computation in solving problems.

1.2 Basic Competence

Identify basic number computation.

1.3 Objectives

Students are able to:

1. Recognise the characteristics of the commutative law.
2. Solve commutative law problems.

1.4 Topic

Commutative law

1.5 Methods

1. Discovery
2. Question and answer
3. Practice
4. Assignment

1.6 Steps

A. Initial Activities/Apperception (5 minutes)
   1. Counting from 1 to 10 while stamping the legs and swinging the arms (odd numbers put their arms down and even numbers put their arms at the front of their bodies). Do this twice.
   2. Inform students of the topic.

B. Main Activities (55 minutes)
   1. Teacher writes addition and multiplication questions on the blackboard. (10 minutes)
   2. Students solve the questions. (10 minutes)
   3. Give students opportunity to ask about anything that they have not understood yet.
   4. Students solve problems with different level of difficulty individually. (20 minutes)

Questions:

Fill in the box with the correct number.

1. 5 + 7 = $\boxed{}$ + 5
2. 3 x 6 = 6 x $\boxed{}$
3. $\boxed{}$ x 5 = 5 x 2
4. 12 + $\boxed{}$ = 10 + 12
5. 14 + 27 = $\boxed{}$ + 14
6. 15 + 68 = 68 + $\boxed{}$
7. 43 + $\boxed{}$ = 69 + 43
8. 345 + $\boxed{}$ = 220 + 345
9. $\boxed{}$ + 195 = 195 + 210
10. 8 x 10 = $\boxed{}$ x 8

5. Check and give mark. (10 minutes)

C. Closing Activities

1. Conclude the lesson.
2. Sing a song, “Di sini senang”, together and dance.
3. Give homework.

1.7 Questions:

Solve this problem:

1. 12 x 10 = $\boxed{}$ x 12
2. 9 x 7 = 7 x $\boxed{}$
3. 20 x 35 = 35 x $\boxed{}$
4. $\boxed{}$ x 345 = 345 + 212
5. 200 + 300 = $\boxed{}$ + 200
6. 8 x 15 = 15 x $\boxed{}$
7. 782 + $\boxed{}$ = 321 + 782
8. 537 + 277 = 277 + $\boxed{}$
9. 13 x 9 = $\boxed{}$ x 13
10. 610 + $\boxed{}$ = 59 + 610

1.8 Reference

- Curriculum book

- Mathematics book for Year 4

1.9 Assessment

a. Process

The rubric is attached (see Figure 4-31).

b. Written

a. Assignment

b. Homework

Nangaroro, 06-08-2014

Principal	Year 4 Mathematics Teacher
Signature	Signature

Figure 4-25 Lesson Plan B1

Mathematical Pedagogical Content Knowledge in LSGM-B1

In terms of Mathematical Pedagogical Content Knowledge, at the start of LSGM-B1, Teacher B planned to teach commutative and associative law for addition and multiplication in one lesson. The teachers discussed the possibility of this plan. They thought about students' ability and whether they would be able to learn the two laws at the same time. They also thought about whether there would be enough time to teach both laws and give sufficient practice. Observer B1 suggested that they focus on commutative law only in Lesson B1, and teach associative law in Lesson B2.

Differentiated Mathematics Instruction in Lesson Study Meeting 1 in School B

LSG-School B focused on content differentiation, in particular, using diverse levels of difficulty for the task, as recorded in Lesson Plan B1 (see Figure 4-25). However, there were limited ways in which levels of content difficulty could be applied. For example, the teachers created a set of questions with different levels of difficulty but these questions were to be given to all students as what they usually do, this is regardless of their ability.

4.3.2 Lesson B1: Commutative law

Mathematical Pedagogical Content Knowledge in Lesson B1

Figure 4-27 shows the analysis of Mathematical Pedagogical Content Knowledge in Lesson B1. These results indicate that mathematics teaching strategies knowledge was used most frequently, for 15.61% of the lesson time. For example, Teacher B's ability to manage the classroom was demonstrated at the beginning of the lesson, when she asked the students to count one by one in sequence, then used these numbers as examples of addition and multiplication to teach the commutative law. She used this strategy to engage students.

Mathematics content knowledge was displayed by Teacher B for 5.20% of the lesson time. An example of this knowledge was observed when the teacher facilitated students to discover the formula of commutative law. Below is an extract from her teaching the concept of commutative law.

Teacher B: What number did you say...student two?

Student B2: Two.

(Teacher B wrote 2 on the blackboard then pointed to another student.)

Student B4: Four.

(Teacher B wrote 4 on the blackboard.)

Teacher B: Now, we link those two numbers with one symbol of operation.

(Teacher B wrote + between 2 and 4 on the blackboard)

Teacher B: How to read this?

Whole class: Two plus four.

Teacher B: If I put this symbol...now, how to read this?

Whole class: Equals.

Teacher B: Say from the beginning.

Whole class: Two plus four equals...

Teacher B: Student B2, please say it again.

Student B2: Two plus four equals...

Teacher B: Good. Two plus four equals...?

Whole class: Six

Teacher B: Two plus four equals six. Now, please consider.

(Teacher B wrote 4 + 2 on the blackboard.)

Teacher B: You

Student B15: Four plus two equals six.

Teacher B: Look...on the top row...two plus four equals six. On the bottom row...four plus two equal six. The sum of two and four is six...and the sum of four and two is also six. So, two plus four equals...?

Whole class: Four plus two.

Teacher B then asked two other students to write their numbers on the blackboard and they repeated the same steps together. After that, Teacher B concluded with a statement of the law (see Figure 4-26).

Teacher B: This is commutative law and the formula is a plus b equals ...?

Whole class: b plus a.

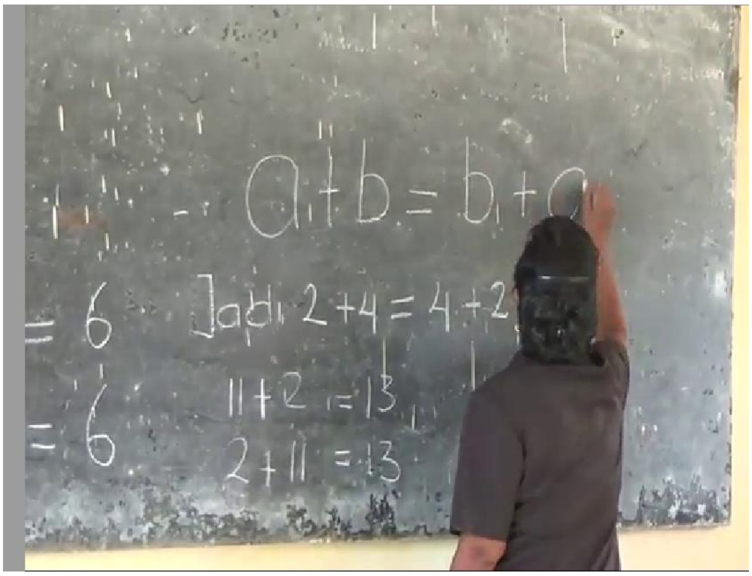


Figure 4-26 Teacher B wrote the commutative law formula

Next, using the same strategy, she assisted students to discover that the commutative law also applies to multiplication. To do this, she picked students five and seven for the first example and asked all students to multiply five times seven and seven times five. For the second example, she chose students ten and three and asked all students to multiply ten times three and three times ten. Then, because the product of five times seven equals seven times five and ten times three equal three times ten, Teacher B concluded that the commutative law is applicable for multiplication. She wrote the formula $a \times b = b \times a$ on the blackboard.

There was no significant practice in curriculum knowledge, which accounted for only 0.25% of the lesson time (see Figure 4-27). The single example of this occurred when Teacher B related the commutative law to a multiplication topic students had learnt previously. Teacher B asked the students to remember and learn multiplication again to help them solve commutative problems in multiplication.

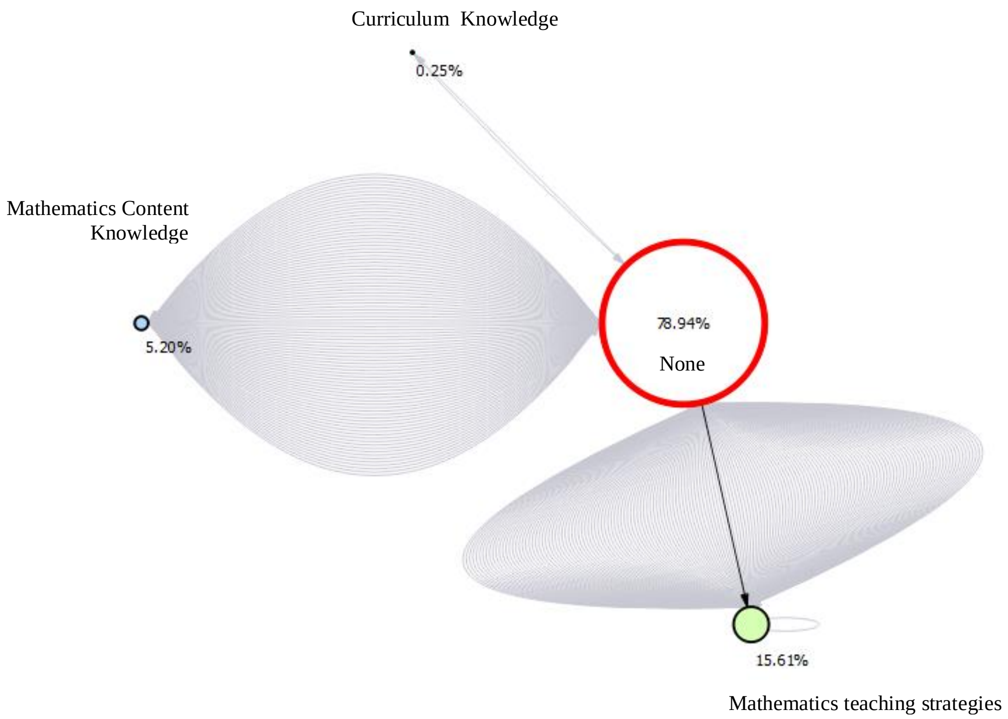


Figure 4-27 Mathematical Pedagogical Content Knowledge in Lesson B1

Table 4-9 shows that there was no evidence demonstrating Teacher B's knowledge of the students' background and understanding. The total amount (21.06%) of Mathematical Pedagogical Content Knowledge was derived from the teacher's knowledge of mathematics content, mathematics teaching strategies and curriculum knowledge.

Table 4-9 Mathematical Pedagogical Content Knowledge shown in Lesson B1

#	Mathematical Pedagogical Content Knowledge	Examples of evidence	Proportion of lesson time (%)
1	Mathematics content knowledge	The teacher demonstrated the commutative law in addition and multiplication, then stated the formulae.	5.20
2a	Mathematical pedagogical knowledge: Knowledge of students	-	0
2b	Mathematical pedagogical knowledge: Mathematics teaching strategies	The teacher asked the students to count from one to the end of the students, then she used the students' numbers as examples of addition and multiplication to demonstrate the commutative law.	15.61
2c	Mathematical pedagogical knowledge: Curriculum knowledge	The teacher linked the topic, commutative law, to the previous topic about addition and multiplication.	0.25
		Total	21.06

Differentiated Mathematics Instruction in Lesson B1

There was no significant differentiated instruction in Lesson B1, as shown in Figure 4-27, which shows the relative percentages of Differentiated Mathematics Instruction in Lesson B1. Differentiated instruction occurred for only 0.89% of the lesson, when Teacher B tried to engage students in the activity by asking them to count one by one and used these numbers as examples for discovering commutative law together. Teacher B then attempted to use other numbers to demonstrate the concept in different ways. For example, Teacher B asked two volunteers to come forward and write their numbers on the blackboard. The numbers were then used as another example of the commutative law of addition. In this way, the Teacher B used a variety of activities to teach commutative law to the students.

After facilitating students to discover the commutative law in addition, Teacher B continued to use the students' number to introduce the commutative law in multiplication, as the following extract:

Teacher B: What is the number that you were saying?

Student B5: Five.

(Teacher B wrote 5 on the blackboard then pointed to another student.)

Teacher B: and...you?

Student B7: Seven.

(Teacher B wrote 7 on the blackboard.)

Teacher B: Now, we will test whether commutative law is applicable for multiplication or not.

(The teacher wrote x between 5 and 7 on the blackboard.)

Teacher B: Five times seven. Together.

Whole class: Five times seven.

Teacher B: Now, think...how much is five times seven?

Whole class: Thirty five.

Teacher B: Thirty five. Five times seven equals...?

Whole class: Thirty five.

Teacher B: Now...seven times five?

Whole class: Thirty five.

Teacher B: Pardon?

Whole class: Thirty five

Teacher B: Thirty five too. Five times seven equals thirty five, seven times five equals thirty five. So...?

Whole class: Five times seven equal seven times five.

Teacher B: So, five times seven equal seven times five.

Teacher B:	Now, we will examine with other numbers. We used five and seven. Now, try other numbers. You …?
Students B10:	Ten.
Teacher B:	You?
Student B3:	Three.
Teacher B:	Ten times three …?
Whole class:	Thirty.
Teacher B:	Read the equation completely.
Whole Class:	Ten times three equals thirty.
Teacher B:	Then …? (She wrote 3 x 10 on the blackboard.)
Whole Class:	Three times ten equals thirty.
Teacher B:	Ten times three is thirty, three times ten is thirty.The results are the same. So?
Whole Class:	So, ten times three equal three times ten.
Teacher B:	So, ten times three equal three times ten. It means the commutative law is also applicable for multiplication. The formula is a times b equal b times a. (She wrote $a \times b = b \times a$ on the blackboard.)

From this extract, it is evident that Teacher B asked the students to read and answer together, like a choir. This made it difficult, if not impossible, to determine which students were active and which students were not, even though the majority seemed to answer correctly.

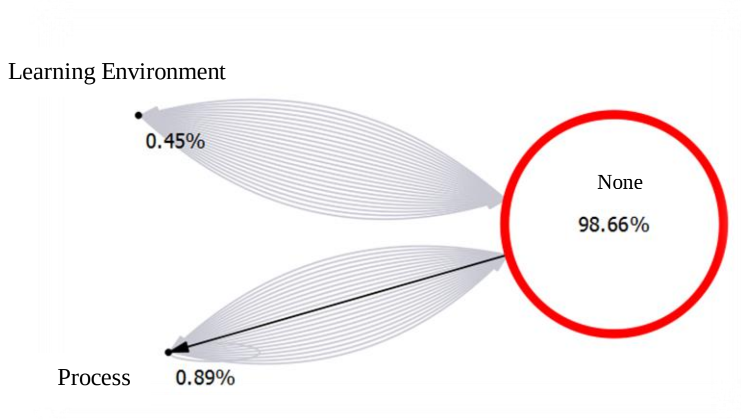


Figure 4-28 Differentiated Mathematics Instruction in Lesson B1

Differentiation of the learning environment occurred for 0.45% of the lesson (see Figure 4-28 above), when the teacher asked the class to clap their hands if a student answered a question correctly. Another example occurred when a student wrote his number on the blackboard as required, but wrote it too small. Teacher B said "Good...but please make it a little bit bigger...well done.". Teacher B spoke nicely and this reflected her respect for the student.

Table 4-10 shows differentiation occurred in Lesson B2 for a total of 1.34% of the lesson, with differentiation in only two areas, process and learning environment.

Table 4-10 The use of Differentiated Mathematics Instruction in Lesson B1

#	Differentiated Mathematics Instruction	Examples of evidence	Proportion of lesson time (%)
1	Content of differentiation	-	-
2	Process of differentiation	Teacher used various activities such as asking students to count one by one, then used the students' numbers as examples for showing the commutative law in addition and multiplication.	0.89
3	Product of differentiation	-	-
4	Assessment of differentiation	From the lesson plan, it can be seen that the teacher provided a rubric for the lessons.	-
5	Learning environment of differentiation	Teacher encouraged students to appreciate their friends who answered the question correctly by applauding.	0.45
		Total	1.34

In Lesson Plan B1 (see Figure 4-25), the LSG-School B planned to differentiate levels of task difficulty. They developed ten questions with different levels of difficulty with the premise that students could work on different number of questions, but Teacher B gave all ten questions to all the students and required them to complete them all. This differed from the understanding of differentiation, which would have meant that every student could complete different questions, depending on their ability.

4.3.3 Lesson Study Group Meeting 2 in School B (LSGM-B2)

In LSGM-B2, the teachers reviewed Lesson B1 and used this input to create Lesson Plan B2 as shown in Figure 4-29, which has been translated from the original version (see Appendix 11).

Lesson Plan B2

School: School B

Subject: Mathematics

Year/Semester: IV/1

2.1 Standard Competence

Understand and use the characteristics of computation in solving problems.

2.2 Basic Competence

Identify basic number computation.

2.3 Objectives

Students are able to:

1. Explain the characteristics of associative law.
2. Write problems according to associative law.

2.4 Topic

Associative law

2.5 Methods

1. Questions and answers
2. Practice
3. Discussion
4. Assignment

2.6 Steps

A. Initial Activities/Apperception (5 minutes)

- Questions and answers about subtraction and division, and check two students' writing.
- Submit homework.
- Sing a song "Aku senang aku bahagia".
- Inform students about the lesson topic.

B. Main Activities (55 minutes)

- Explain computation with associative law.

  Example:

  1. (3 + 5) + 2   3 + (5 + 2)
     = … + 2   = 3 + …
     = …   = …

     The Formula is: $a + (b + c) = (a + b) + c$

     So, (3 + 5) + … = 3 + (… + 2)

2. 4 + (3 + 5) (4 + 3) + 5
= 4 + … = … + 5
= … = …
So, 4 + (… + …) = (… + …) + 5
The formula is: *a x (b x c) = (a x b) x c*

3. 2 x (4 x 5) (2 x 4) x 5
= 2 x … = … + 5
= … = …
So, 2 x (… + …) = (2 + …) x 5

- Students write the answers to the questions that are written on the blackboard.

Example: (2 + 3) + 1 2 + (3 + 1)
= … + 1 = 2 + …
= … = …
So, (2 + …) + 1 = (2 + 3) + …

- Group students in groups of four to work on a worksheet.

Questions:

1. (44 + 334) + 66 44 + (334 + 66)
= … + 66 = 44 + …
= … = …
So, (44 + …) + 66 = … + (334 + 66)

2. 121 + (112 + 122) (121+ 112) + 122
= 121 + … = … + 122
= … = …
So, (121 + 112) + … = 121 + (… + 122)

3. 8 x (10 x 5) (8 x 10) x 5
= 8 x … = … x 5

= … = …

So, 8 x (… x 5) = (8 x 10) x …

4. (5 x 9) x 4 5 x (9 x 4)

= … x 4 = 5 x …

= … = …

So, (… x 9) x 4 = 5 x (… x 4)

5. 4 x (8 x 2) (4 x 8) x 2

= 4 x … = … x 2

= … = …

So, 4 x (8 x …) = … x (8 x 2)

- Teacher checks and marks students' work.

2.7 Closing Activities (10 minutes)

1. Conclude the lesson.
2. Give homework.

2.8 Questions:

1. (1,521 + 2,755) + 4,325 = 1,521 + (2,755 + …)
2. (2,145 + 3,256) + 7,189 = 2,145 + (3,256 + …)
3. (5,190 + 3,700) + 6,792 = 5,190 + (… + 6,792)
4. (6,051 + 2,162) + 4,001 = … + (2,162 + 4,001)
5. (9,275 + 4,320) + 6,526 = 9,275 + (… + 6,526)
6. (59 x 20) x 32 = 59 x (20 x …)
7. (45 x 21) x 48 = 45 x (21 x …)
8. (35 x 43) x 72 = 35 x (… x 72)
9. (15 x 12) x 7 = 15 x (… x …)
10. (142 x 26) x 5 = 142 x (… x …)

2.9 Media and Reference

- Curriculum book
- Mathematics book for Year 4

- Worksheet

2.10 Assessment

Process

The rubric is attached. (see Figure 4-31)

Nangaroro, 07-08-2014

Principal — Mathematics Teacher Year 4

Figure 4-29 Lesson Plan B2

Mathematical Pedagogical Content Knowledge in LSGM-B2

In the LSGM-B2, Observer B1 said that Teacher B showed recognition of students' understanding through questions and answers in Lesson B1. Observer B2 noticed that Teacher B did not use any aids when talking about addition in commutative law and speculated that the reason may have been that Teacher B knew that the students already understood addition. Another point made was that Teacher B could motivate students. Teacher B also reflected on her teaching. She realised that she had undertaken an activity in the classroom that was not in the Lesson Plan B1, when she asked the students to count in order and used their numbers as examples to demonstrate the commutative law.

After reviewing Lesson B1, the teachers in LSG-School B planned Lesson B2. The teachers discussed assessment, and planned to do ongoing assessment. This meant that student achievement would be measured not only by the test but also by tasks, homework and the students' understanding during group discussion.

Differentiated Mathematics Instruction in LSGM-B2

When reviewing Lesson B1, Observer B1 said that Teacher B differentiated the product by giving tasks of different levels of difficulty of the tasks. While it was correct that there were questions of different levels of difficulty, but all questions were given to all students and all students had to complete all the questions. This does not reflect differentiation, however. In differentiation, teachers create questions of varying levels of difficulty and each student works with different questions depending on their ability. Observer B2 commented on the way students answered in chorus, which made it difficult to determine which students were able to answer correctly and which students were not. On the observation form, Observer B2 also indicated that Teacher B paid less attention to the low ability students, and did not assist them students when they did questions that they did not understand yet.

LSG-School B then discussed how to differentiate the process in Lesson B2. Besides discussing how the topic would be taught, they discussed the appropriate number of students in each group. They also discussed whether homogeneous or heterogeneous group would be more suitable, and considered students' ability in the lesson. Teacher B said that she would like to group students based on their ability: one group of four more able students, two groups of four average students, and one group of four less able students. She would also like to create three different levels of difficulty in questions. However, Observer B1 suggested putting students in heterogeneous groups. Each group would contain one able student, two average students and one less able student. This would give the able students a chance to assist their friends in the same group.

4.3.4 Lesson B2: Associative law

Mathematical Pedagogical Content Knowledge in Lesson B2

In Lesson B2, Teacher B demonstrated mathematics content knowledge (8.87%), mathematics teaching strategies knowledge (22.12%), and curriculum knowledge (0.33%). Figure 4-30 shows the practice of Mathematical Pedagogical Content Knowledge in Lesson B2 and is quite different from the previous analysis diagrams (see Figure 4-7, Figure 4-8, Figure 4-16, Figure 4-19 and Figure 4-26). In this figure, the mathematics teaching strategies knowledge circle is larger than the mathematics content knowledge circle. This means that Teacher B demonstrated more mathematics teaching strategies than mathematics content knowledge. However, the transitions to and from mathematics content knowledge was more frequent than that of mathematics teaching strategies, as indicated by the relative number of transition lines in Figure 4-30. This happened because Teacher B's practice of teaching strategies knowledge occured for a long time in each application; thus, the total time was more than that of mathematics content knowledge, which only occurred for a short time of time in each application.

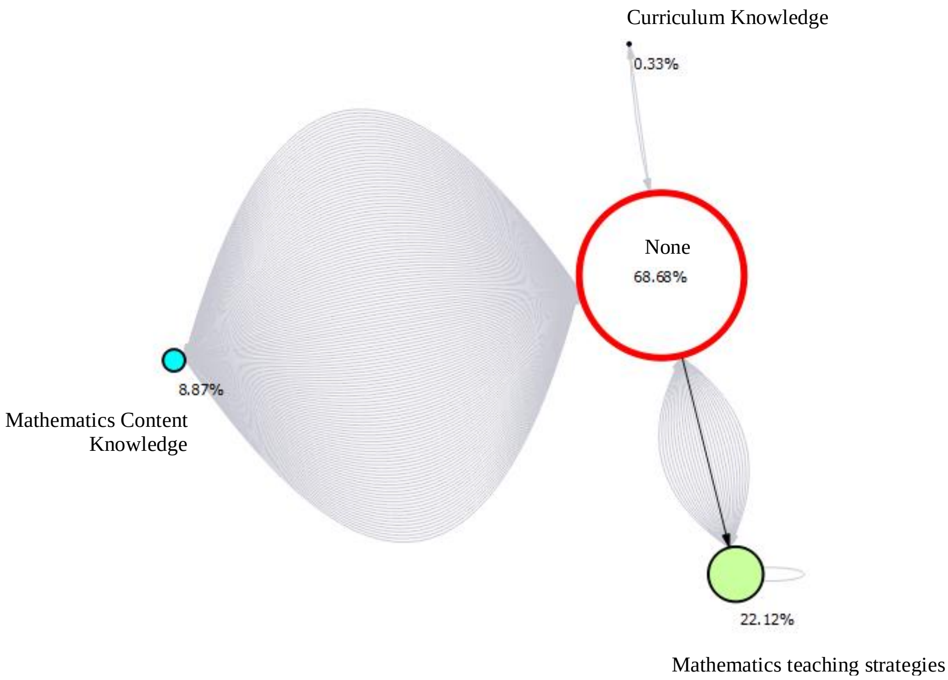


Figure 4-30 Mathematical Pedagogical Content Knowledge in Lesson B2

Table 4-8 lists each form of Mathematics Pedagogical Content Knowledge in Lesson B2. Teacher B did not demonstrate her knowledge of students in Lesson B2.

In relation to mathematics content knowledge, before Teacher B continued teaching associative law, she reminded students about the formula of commutative law that they had learnt previously, by asking them questions:

Teacher B: What is the formula for commutative law?

Whole class: *a* plus *b* equal *b* plus *a*.

Teacher B: *a* plus *b* equal *b* plus *a*. That is for addition. How about in multiplication?

Whole class: *a* times *b* equal *b* times *a*.

Teacher B also demonstrated mathematics content knowledge when she helped students discover whether commutative law is applicable for subtraction or not. The following extract provides evidence of this.

Teacher B: Like this...how much is five take away three?

Whole class: Two

Teacher B: How much is three take away five?

Whole class: It can’t.

Teacher B: Why? Student B10.

Student B10: Three is less than five.

Teacher B: Three is less than five. Because three cannot be taken away five...it means the result is not the same. So, in subtraction the commutative law is not applicable.

Table 4-11 Mathematical Pedagogical Content Knowledge shown in Lesson 2 in School B

#	Mathematical Pedagogical Content Knowledge	Examples of evidence	Proportion of lesson time (%)
1	Mathematics Content Knowledge	The teacher demonstrated the associative law in addition and multiplication then stated the formulae.	8.87
2a	Mathematical pedagogical knowledge: Knowledge of students	-	0
2b	Mathematical pedagogical knowledge: Mathematics teaching strategies	The teacher grouped the students to work on associative law.	22.12
2c	Mathematical pedagogical knowledge: Curriculum knowledge	The teacher linked the associative law to the previous concept of addition of three numbers if there is a bracket.	0.33
		Total	31.32

Strong evidence of mathematics teaching strategies knowledge was found when Teacher B grouped students heterogeneously. She put students in groups of four composed of one most able student, two students of average ability, and one student below average ability. Teacher B determined the group composition before the lesson. While students worked in groups, Teacher B moved from group to group to see if anyone needed assistance. She also observed and evaluated students on two indicators: students are able to recognise the characteristics of associative law, and students can solve problems according to associative law (see Figure 4-31). The rubric showed Teacher B's ability in mathematics teaching strategies in the classroom management and assessment indicators.

On-going Assessment B2

Number	Students' name	Recognise the characteristics of associative law			Solve problems according to associative law			Test score
		A	B	C	A	B	C	
1	Student B1							
2	Student B2							
3	Student B3							

A = very good

B = satisfactory

C = unsatisfactory/need improvement/low achievement

Figure 4-31 Rubric of Lesson B2

In addition, Teacher B showed her curriculum knowledge by linking the topic of associative law to a previous topic. She said "In year three we learnt that in addition, if there is addition within brackets...we solve the addition within the brackets first." Through her reminding the students of this earlier topic, they were able to solve the associative law questions more easily.

Differentiated Mathematics Instruction in Lesson B2

At the beginning of Lesson B2, Teacher B asked the students whether they had done their homework from the previous lesson. She asked students to check whether commutative law is applicable to subtraction and division. However, Teacher B did not give clear instructions. She asked:

"We have proved that commutative law is applicable in multiplication and addition. You try to prove in subtraction and division by using the available numbers. Does it work or not, please try at home. We did *a* plus *b* and *b* plus *a*. Prove it, whether it works for subtraction and division or not. If it works, it means that commutative law also applicable for other operations, not only for multiplication and addition. Can you do at home?"

She tried to let students work independently and creatively, but no students did the homework, perhaps because they did not know how to prove that the communicative law works for subtraction. In Lesson B2, Teacher B gave more precise instructions about proving this. She asked students to count five and take away three and then count three and take away five. She thus facilitated students to prove that the commutative law does not work in subtraction.

Teacher B then shifted the topic to associative law. The little differentiation that did occur in Lesson B2 was in process (2.84%) and learning environment (0.09%). This was slightly more than in Lesson B1.

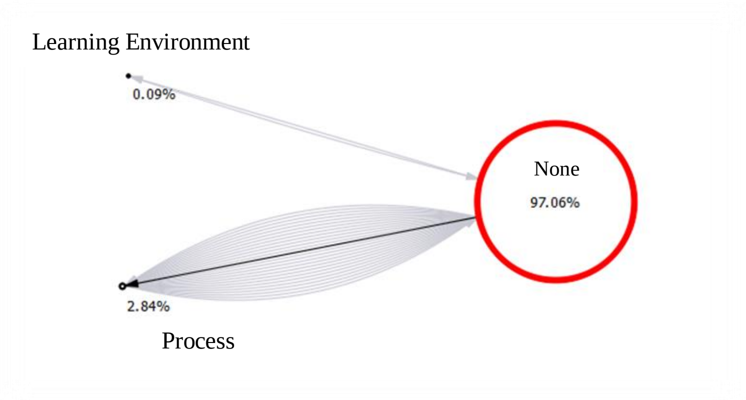


Figure 4-32 Differentiated Mathematics Instruction in Lesson B2

Teacher B differentiated the process by encouraging the students to think critically. For example, she demonstrated associative law in addition, then asked:

"They all have three, they have five, they have two. On the second row, here is three, but here eight. Then, here is seven, but here two. Then, the result is the same, ten. So, I want to ask you, what is the difference? Where is the difference?" (see Figure 4-33).

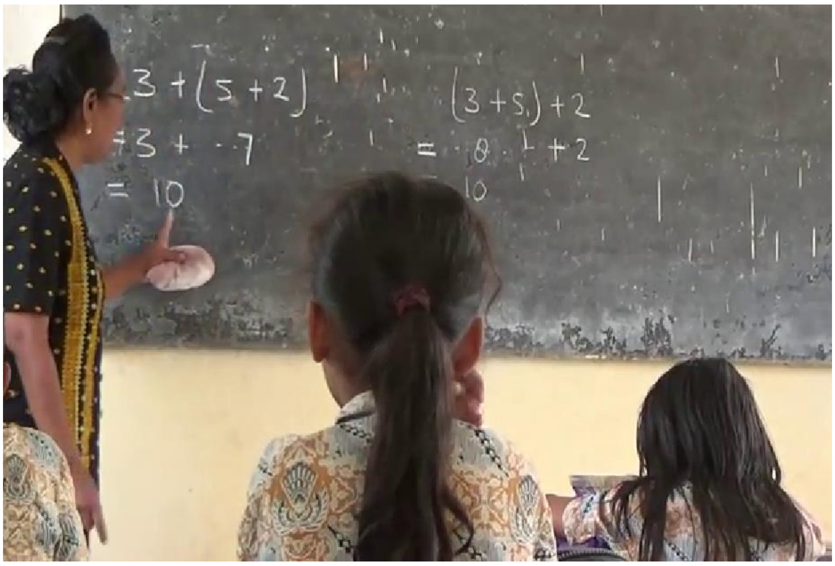


Figure 4-33 Teacher B proved associative law in addition

Differentiation in the learning environment happened when Teacher B valued students who answered correctly. She said “Good” or asked other students to praise their friends by applauding.

Table 4-12 Use of Differentiated Mathematics Instruction in Lesson 2 in School B

#	Differentiated Mathematics Instruction	Examples of evidence	Proportion of lesson time (%)
1	Content of differentiation	-	-
2	Process of differentiation	Teacher gave open-ended questions that encouraged students to think creatively; for example the teacher asked about the difference between 3 + (5 + 2) and (3 + 5) + 2.	2.84
3	Product of differentiation	-	-
4	Assessment of differentiation	-	-
5	Learning environment of differentiation	Teacher encouraged students to appreciate their friends who answered the question correctly by giving applause. Teacher appreciates students who did their job properly.	0.09
		Total	2.93

4.3.5 Lesson Study Group Meeting 3 in School B (LSGM-B3)

Mathematical Pedagogical Content Knowledge in LSGM-B3

In this Lesson Study Group Meeting, Observer B2 said that generally, Teacher B's practice was good, but her teaching needed improvement in some areas. For example, in classroom management, Teacher B took a lot of time in apperception, nearly twice the time allocated in Lesson Plan B2. Observer B2 also recommended advised that Teacher B write more detail about time allocation on the lesson plan, especially in the main activity. In Lesson Plan B2, 55 minutes was allocated for the main activity, but there is allocation of time to explain the topic for group discussion or for the class discussion. Observer B2 then said that Teacher B used mainly lecturing in delivering the associative law topic, and advised her to use a discovery method and other aids in her teaching for this topic.

Observer B1 noted that Teacher B explained associative law in addition but not in multiplication. However, addition and multiplication questions were among the tasks for group discussion.

Differentiated Mathematics Instruction in LSGM-B3

Observer B2 said that Teacher B grouped students properly but did not notice that not all students were actively involved in the group discussion. In addition, she said that Teacher B needs to value each student by giving attention and assistance to the low ability students. For example, using students' names will make them feel appreciated. Observer B2 advised Teacher B to let the students discover the associative law: when the students are given the opportunity to work independently, they will work together and help each other. Observer B2 said that this kind of strategy would encourage students to think creatively. On the other hand, she noticed that Teacher B had refined the way she asked students to answer her questions; by pointing to individual students to answer a question, she reduced the amount of answering done in chorus.

4.3.6 Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in School B

Table 4-10 shows the indicators for Mathematical Pedagogical Content Knowledge that were achieved in Lessons B1 and B2. Teacher B demonstrated knowledge in most areas of Mathematical Pedagogical Content Knowledge. In terms of mathematical content knowledge, she demonstrated her understanding of the concepts of the commutative and associative laws to addition and multiplication, and of the inapplicability of the commutative law to subtraction. Teacher B's performance also demonstrated her mathematical procedural knowledge, when she demonstrated the procedure to prove that both laws apply to addition and multiplication.

In mathematical pedagogical knowledge, Teacher B demonstrated her knowledge of mathematics teaching strategies. In terms of classroom management, for example, when she asked students to count in sequence, sometimes she asked students to say their number but sometimes she asked them to come and write their number on the blackboard. In terms of assessment knowledge, Teacher B observed and noted student ability. She used a rubric, as shown in Figure 4-31.

Teacher B demonstrated her curriculum knowledge in Lessons Plan B1 and B2 in the core and base competencies, which are derived from the national curriculum. She also demonstrated knowledge of the curriculum by linking the commutative and associative laws to a previous topic of addition and multiplication.

Table 4-13 Mathematical Pedagogical Content Knowledge indicators in Lessons B1 and B2

Type of Mathematical Knowledge	Indicators		Applied	
			Lesson B1	Lesson B2
Mathematical content knowledge	Mathematical conceptual knowledge		√	√
	Mathematical procedural knowledge		√	√
Mathematical pedagogical knowledge	Knowledge of students'	Background		
		Understanding		
	Mathematics teaching strategies	Classroom management	√	√
		Assessment	√	√
	Curriculum knowledge	Mathematics educational goals		
		Connection to previous and later topic	√	√
		Link to other subjects		

However, Teacher B did not demonstrate knowledge such as linking of the commutative and associative topics to other subjects. Although her knowledge of curriculum, especially in mathematics educational goals, was not evident in the two lessons, these goals were written in the Lesson Plans.

Table 4-14 summarises the indicators of Differentiated Mathematics Instruction and Mathematical Pedagogical Content Knowledge achieved in Lessons B1 and B2.

Table 4-14 Differentiated Mathematics Instruction indicators in Lessons B1 and B2

Type of Differentiation	Indicators	Cycle 1 Lesson B1		Cycle 2 Lesson B2	
		Planned	Applied	Planned	Applied
Content	Various levels of content				
	Relevant to students’ daily live				
Process	Flexible grouping				√
	Encourages creative thinking				
	Encourages critical thinking				√
	Various activities, such as open-ended questions	√	√	√	√
	Encourages research				
Product	Various levels of task difficulties				
	Freedom in choosing the task				
	Flexible submission				
Assessment	Ongoing	√	√	√	√
	Provides rubric	√	√	√	√
	Gives feedback on students’ work				
	Allows students to evaluate their own progress				
Learning Environment	Some learning centres in the classroom				
	Teacher values each student		√		√
	Encourages students to respect others		√		√
	Encourages students to be responsible				

Table 4-14 is revealing in several ways. First, Teacher B applied differentiation that was planned in the Lesson Study Meeting and Lesson Plan; for instance, by varying activities in teaching the laws in order to motivate and activate the students. She asked students to count in sequence then used the numbers as examples in proving the laws in each lesson. This required students to remember their number and concentrate so that they could say their number if Teacher B pointed to them.

Second, there was differentiation that was not planned but that Teacher B implemented; for instance, she encouraged student to think critically. When she proved the commutative law is not applicable in subtraction, she asked students to give reason why three take away five cannot be solved. Third, some forms of differentiation were neither planned nor applied; for instance, varying levels of content difficulty, flexible submission, and creating learning centres in the classroom. These issues are discussed further in Chapter 5.

Figure 4-34 shows the combinations of Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction as they occurred in Lesson B1. Figure 4-35 shows the same combinations for Lesson B2. From these graphs, it can be seen that the teacher's Mathematical Pedagogical Content Knowledge occurred with each type of mathematics differentiation.

There were three intersections in Lesson B1 (see Figure 4-34). First, in mathematics teaching strategies and process differentiation (0.77%), Teacher B demonstrated her ability in managing the classroom with varied activity in order to maintain attention. Second, in mathematics teaching strategies and learning environment differentiation (0.32%), Teacher B managed the classroom by valuing each student who answered a question. Third, in mathematics content knowledge and process differentiation (0.30%), Teacher B demonstrated her knowledge of the commutative law concept and the procedure proving that commutative law is applicable in addition and multiplication. She also differentiated the process of teaching by using open-ended question.

In Lesson B2, three combinations of Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction occurred. Teacher B demonstrated her mathematics content knowledge and process differentiation (1.33%), by, for example, encouraging the students to think critically while proving that the commutative law is not applicable to subtraction. In mathematics teaching strategies and process differentiation (0.33%), Teacher B used grouping strategies to train students in associative law questions, by grouping students in mixed ability groups. Teacher B demonstrated her mathematics content knowledge and learning environment differentiation (0.09%) when she underlined a student's answer about a concept of addition and valued the student by giving praise.

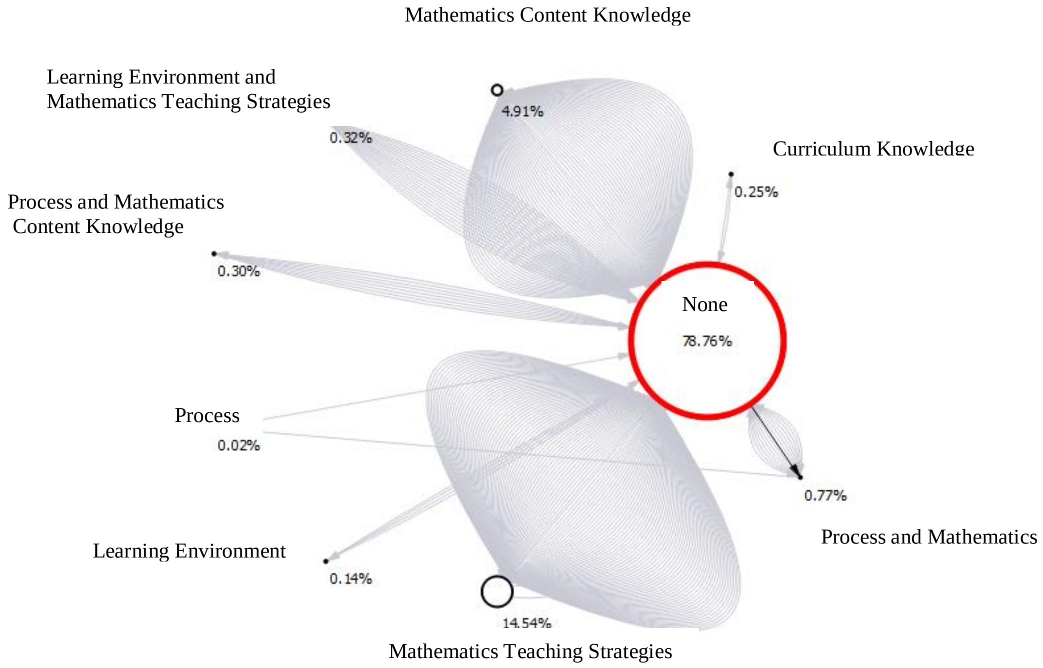


Figure 4-34 Differentiated Mathematics Instruction and Mathematical Pedagogical Content Knowledge in School B: Lesson B1

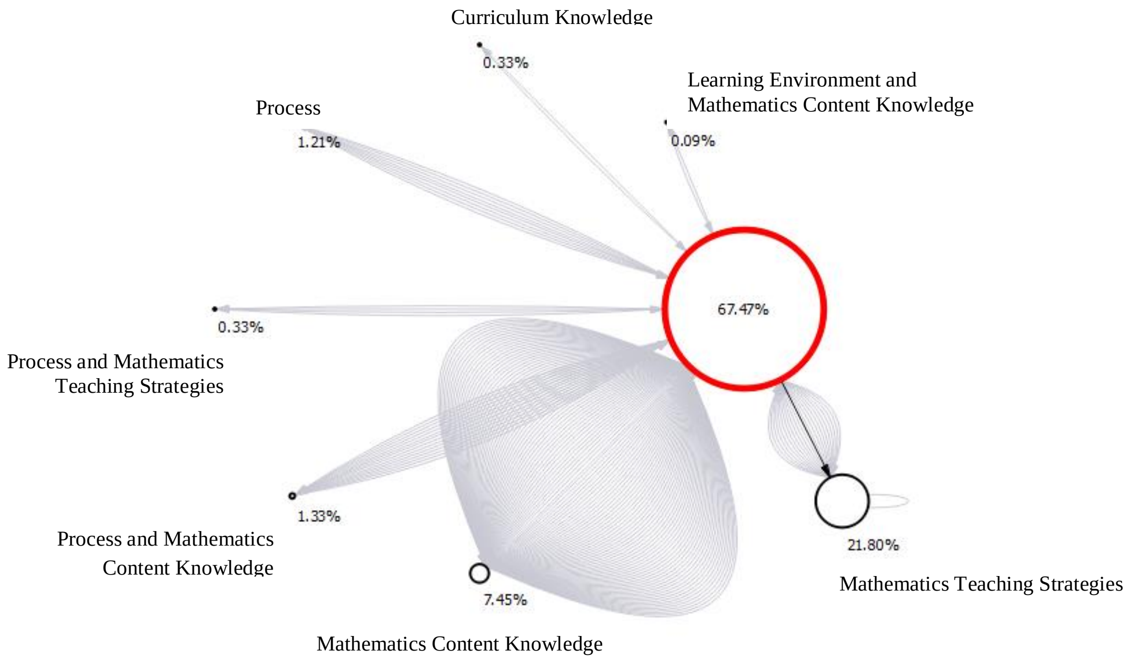


Figure 4-35 Differentiated Mathematics Instruction and Mathematical Pedagogical Content Knowledge in School B: Lesson B2

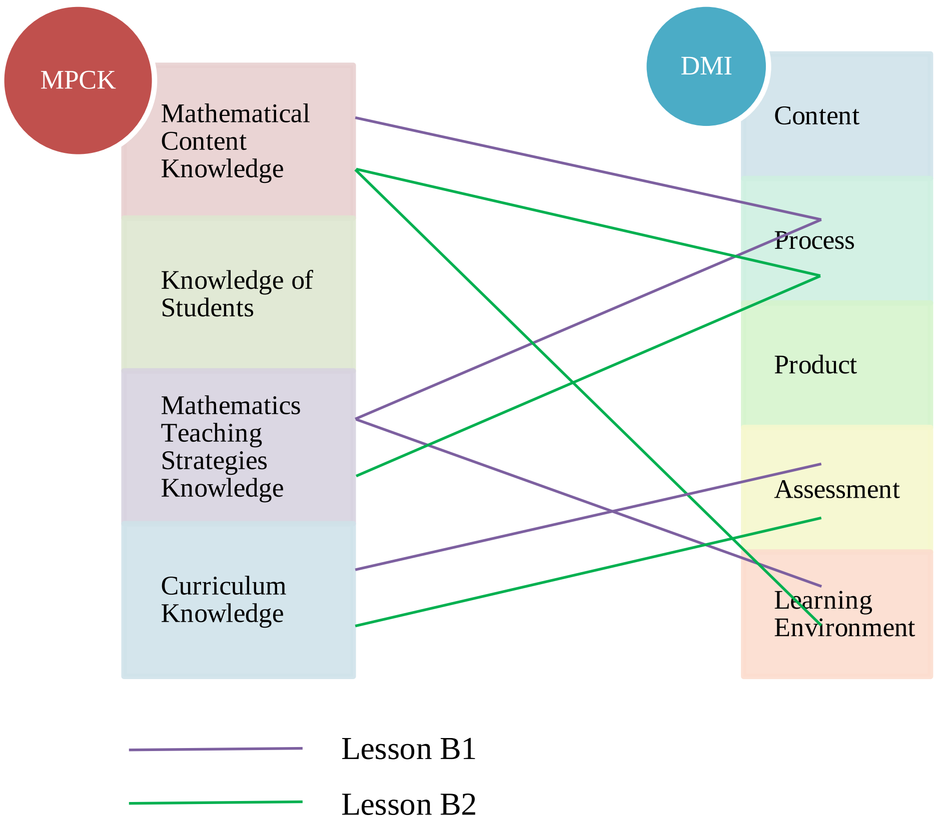


Figure 4-36 Relationship between Mathematical Pedagogical Content Knowledge (MPCK) and Differentiated Mathematics Instruction (DMI) in School B

Figure 4-36 shows the connections between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction that occurred in Lesson B1 (purple line) and Lesson B2 (green line). In this figure, it can be seen that Teacher B's mathematical content knowledge helped her to differentiate process in Lesson B1 and process and learning environment in Lesson B2. There was no evidence that Teacher B's knowledge of students helped her differentiate mathematics instruction in either lesson. Teacher B's mathematics teaching strategies knowledge helped her differentiate the process and learning environment in Lesson B1 and the process only in Lesson B2. Interestingly, in both lessons, Teacher B's curriculum knowledge aided her in differentiating assessment.

Table 4-15 shows the percentages for all combinations in Lessons B1 and B2. Teacher B's use of mathematics content knowledge when differentiating process increased from 0.30% in Lesson B1 to 1.33% in Lesson B2. Her use of mathematics content knowledge when differentiating learning environment changed from none in Lesson B1 to 0.09% in Lesson B2. However, in mathematics teaching strategies knowledge, Teacher B's performance declined from 0.77% in Lesson B1 to 0.33% Lesson B2 for process differentiation, and from 0.32% in Lesson B1 to none in Lesson B2 for learning differentiation.

Table 4-15 Mathematical Pedagogical Content Knowledge (MPCK) and Differentiated Mathematics Instruction (DMI) in School B

MPCK \ DMI		Content (LB1, LB2) (%)	Process (LB1, LB2) (%)	Product (LB1, LB2) (%)	Assess-ment (LB1, LB2) (%)	Learning Environment (LB1, LB2) (%)
Mathematical content knowledge: Conceptual and Procedural		-	(0.30, 1.33)	-	-	(0, 0.09)
Mathematical pedagogical knowledge	Knowledge of Students	-	-	-	-	-
	Mathematics Teaching Strategies Knowledge	-	(0.77, 0.33)	-	-	(0.32, 0)
	Curriculum Knowledge	-	-	-	-	-

(LB1, LB2) = (Percentage for Lesson B1, percentage for Lesson B2)

4.4 The Digital Mixed-Methods Research Design in Lesson Study

Mathematics lessons in two schools in Nagekeo were video recorded to study the lessons in detail. After the videos were imported to the Multimodal Analysis Video software and the classroom interactions were transcribed, the lessons were analysed based on the Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction categories outlined in Section 1.6, Conceptual Framework (page 14). The software facilities enabled analysis of teachers' knowledge (for example, see Figure 4-37(a)) and their performance in differentiating mathematics instructions (for example, see Figure 4-37(b)). The relationships between these two practices were also investigated (for example, see Figure 4-37(c)), as discussed in this chapter.

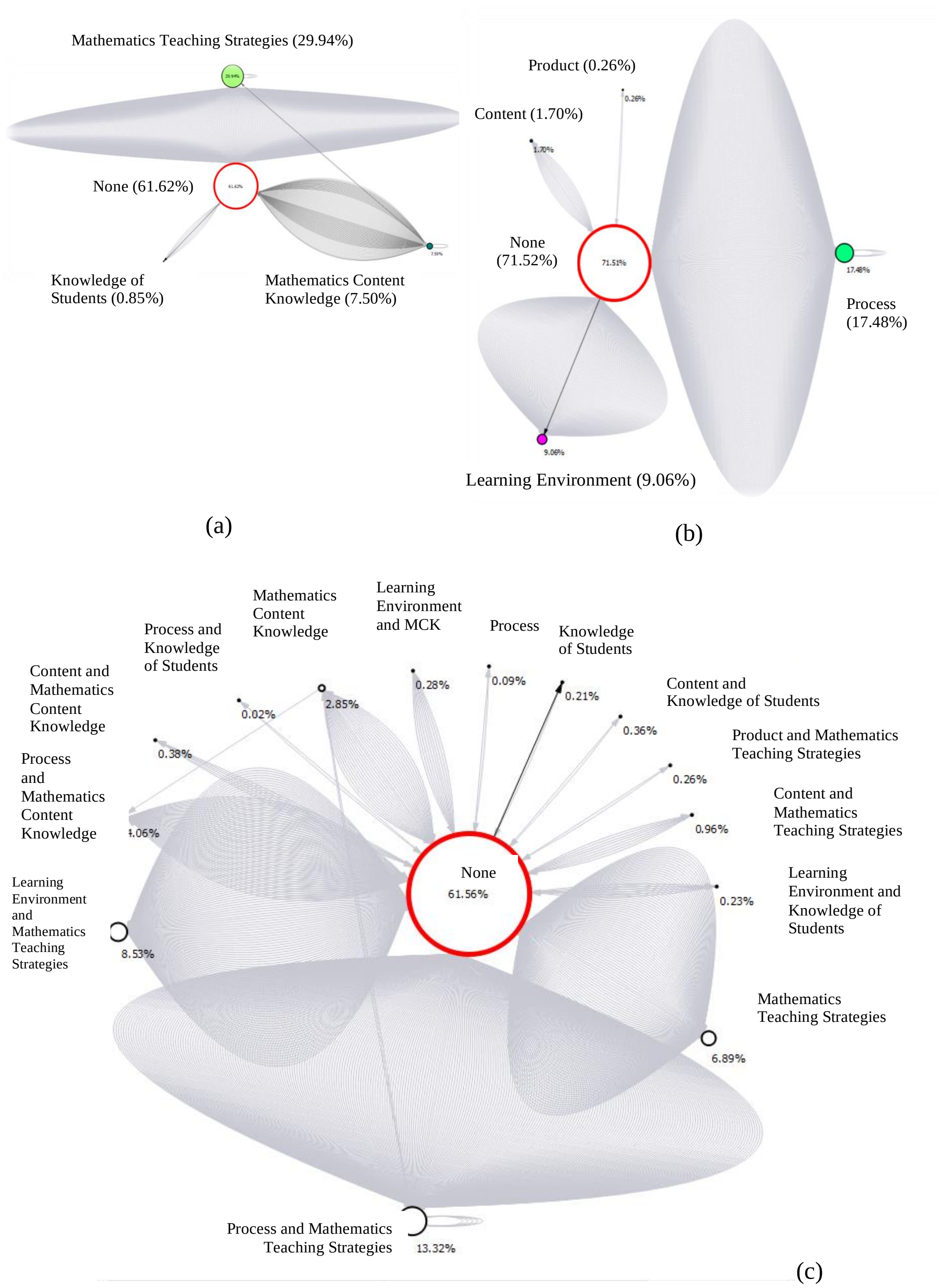


Figure 4-37 Mathematical Pedagogical Content Knowledge (a); Differentiated Mathematics Instruction (b); connection between the two (c) in Lesson A1

The Multimodal Analysis Video software transforms qualitative data (in this case, teachers' performance in mathematics lessons) into quantitative data that is displayed as a percentage of total lesson time for each data category. For example, Figure 4-37(a) shows that Teacher A displayed knowledge in mathematics teaching strategies for 29.94% of the total teaching time in Lesson A1. Figure 4-37(b) shows Teacher A differentiated the learning environment for 9.06% of Lesson A1, and Figure 4-37(c) shows that Teacher A demonstrated knowledge in mathematics teaching strategies to differentiate the learning environment for 8.53% of the lesson. Figure 4-38 show all connections between mathematics teaching strategies knowledge and differentiation of the learning environment by Teacher A in Lesson A1.

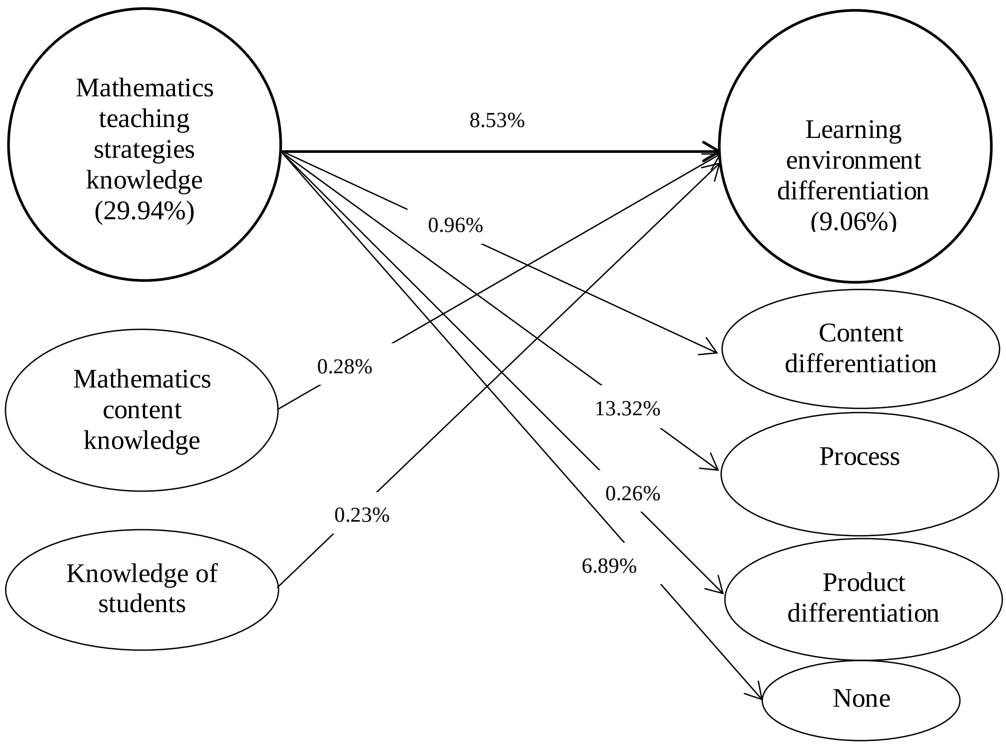


Figure 4-38 Connections between mathematics teaching strategies knowledge and learning environment differentiation in Lesson A1

From the data in Figure 4-38, it can be said that in Lesson A1, Teacher A's knowledge of mathematics teaching strategies (29.94%) helped her to differentiate not only the learning environment (8.53%) but also other factors (content differentiation 0.96%, process differentiation 13.32%, product differentiation 0.26%). Teacher A even showed her knowledge in mathematics teaching strategies without differentiation (6.89%). The learning environment differentiation (9.06%) performed by Teacher A in Lesson A1 was influenced by her knowledge of mathematics teaching strategies (8.53%), mathematics content knowledge (0.28%) and students (0.23%).

From this example, it is evident that a digital mixed methods approach, here undertaken using the Multimodal Analysis Video software, enables study of the impact of Lesson Study on teachers' knowledge and skills in differentiating mathematics instruction. The previous sections in this chapter have explored the results of each aspect of Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction during the Lesson Study, and the connections in both concepts. The results were interpreted from the quantitative data extracted from analysis of the qualitative data (the videos) by the software. The qualitative data that was collected in the study and the quantitative data that was generated from the software support each other in addressing the research questions in this study.

4.5 Conclusion

These findings suggest that Lesson Study may provide an opportunity for teachers in Schools A and B to discuss not only the topic lesson but also the strategies for teaching mathematics and the new ideas about differentiation that they learnt at the beginning of this project. Teachers in both schools had the opportunity to build understanding and improve their knowledge and teaching skills in the Lesson Study Group Meetings. Teacher A and B received positive feedback in these meetings, which aimed to improve them as professional teachers. Also in these meetings, several teachers worked together to create and refine Lesson Plans, providing a chance for them to develop closer relationships with their colleagues.

The data presented in this chapter indicates that teachers' Mathematical Pedagogical Content Knowledge may contribute to the differentiation of mathematics instruction, even though, in some cases, Mathematical Pedagogical Content Knowledge happened without differentiation or vice versa.

The Multimodal Analysis Video software was able to expand the research design in this study, permitting the relationship between Mathematical Pedagogical Content Knowledge and the differentiation of mathematics instruction to be closely examined. The digital mixed methods research approach was also beneficial in facilitating the interpretation of the study results.

Chapter 5 – Discussion

5.1 Introduction

The primary research question in this study was: What impact does Lesson Study have on teachers' knowledge and skills in differentiating primary school mathematics instruction?

From this main research question, the objectives of this study are:

1. To describe the processes involved in the implementation of Lesson Study.
2. To investigate the influence of Lesson Study on primary school teachers' mathematics pedagogy.
3. To investigate the influence of Lesson Study on primary school teachers' curriculum differentiation in mathematics.
4. To investigate the connections between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction during Lesson Study.
5. To identify the key characteristics of Lesson Study as a form of professional development.

The secondary research question in this study was: What is the nature of a digital mixed methods research design for investigating the relations between teachers' Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction during Lesson Study?

Based on this question, an objective of this study is to develop a digital mixed methods approach for studying the impact of Lesson Study on teachers' knowledge and skills in differentiating mathematics instruction.

This chapter presents a comprehensive investigation of the key research results presented in Chapter 4. Following this introduction, the processes involved in the implementation of Lesson Study are described and the influence of Lesson Study on primary school teachers' mathematics pedagogy is investigated. The influence of Lesson Study on teachers' curriculum differentiation in mathematics is then explored and the potential of the key characteristics of Lesson Study as a form of professional development are discussed. Finally, this chapter ends with a summary of conclusions drawn.

5.2 The Process of Implementing Lesson Study

Lesson Study, involving the process of "preparation, actual class, and class review" (Baba, 2007, p. 2), is a way to "study curriculum and formulate goals, plan, conduct research, and reflect" to improve the quality of teaching and learning (Lewis et al., 2006). In this study, the Lesson Study process involved planning, teaching, observing, reviewing, and refining classroom lessons as discussed in the following sections.

5.2.1 Planning

In the Lesson Study meetings, planning included selecting the topic, setting the goals, designing teaching steps, creating the task, and planning assessment. The teachers created lesson plans together. They discussed each strategy for teaching topics and proposed the aids or media that seemed appropriate. For instance, in School A, one group member advised using a special wood protractor for teaching about measuring an angle in the geometry lesson. In School B, the teachers discussed whether the topic to be taught should be commutative law only or both commutative and associative laws.

In this step, during the Lesson Study meetings, the teachers generated Lesson Plans A1, A2, B1, and B2 (see Appendices 9, 10, 11 and 12). These findings support an idea by Lewis (2002), which defined the first stage of the Lesson Study cycle as goal setting and planning. Goal setting is not mentioned explicitly in this study, as it was included in the planning step.

5.2.2 Teaching

In the teaching section of this study, teachers implemented the lesson plans they created in the Lesson Study meetings. For example, Teacher A implemented the activity in Lesson Plan A2. In that Lesson Plan, Lesson Study Group A planned to take students out of the classroom to observe the buildings around the school in order to draw and examine different kinds of angles. Teacher B also implemented what was planned in the Lesson Plans. For instance, she taught the associative law as documented in Lesson Plan B2.

However, there was evidence that the teachers sometimes deviated from their lesson plans. For instance, Teacher A gave homework questions to the students, but the questions were not planned in the meeting or written in the Lesson Plan. It seems possible that in these cases an idea emerged suddenly while the teacher was teaching. It is not possible to plan every single act of teaching, and everything that has been planned is not always taught. However, if the questions for homework in this study had been planned in advance, the outcomes would have been improved because the questions would have been set for students to practice what they had learnt on that day. The teaching step is "the opportunity to test the lesson plan" (Doig & Groves, 2011, p. 81). Through the application of a Lesson Plan, teachers will recognise which sections of the plan work properly and which parts need to be improved.

5.2.3 Observing

In this study, the observers made notes with a focus on the teacher's actions and the impact on students' learning, using the observation form in Appendix 7. On this observation form, observers from both schools reported their observations about teacher practice and student responses to the teaching. They then recorded what they perceived in the next step, reviewing that stage in the Lesson Study meeting. For example, Observer A3 observed that Teacher A gave the students opportunities to express their responses, but sometimes the students answered together and this created a noisy environment. In another example, Observer B3 reported that Teacher B paid attention to the groups of students as a whole when they were doing the assignments but did not help individual students with low ability.

Sam, White, and Mon (2005) report that the observing stage in Lesson Study helps teachers identify their obstacles in managing the classroom. Although the students initially felt that they were being examined, once it was explained to them they understood that the observation activity was for development of the lesson and to assist them in their study (Sam, White, & Mon, 2005).

In this study, the observers focused only on the teachers' teaching and sat at the back of the classroom, behind the students, to avoid disrupting the students' concentration. In another study by Doig and Groves (2011), observers focused on both teachers' teaching and students' learning, moving from one desk to another in order to look more closely at the students' work. Another difference was the number of the observers used. In Doig and Groves' (2011) study, from the photo they provided, there were around eight observers. This study involved three observers only in School A and two observers in School B, to avoid having many adults present in the classroom. There were also similarities between this study and the study by Doig and Groves (2011). The observers in both studies did not interact with or assist students and teachers during the lesson as the Lesson Study was focused on the teachers' practice. Again, in both studies the observation activity aimed to survey and record the impact of the application of the Lesson Plans.

5.2.4 Reviewing

Reviewing happened in the Lesson Study meeting after the lessons. In this study, the observers appraised the teaching process based on their observation notes. For example, in Lesson Study Meeting 2 in School A, the observers were critical of Teacher A because her lesson took more than the predetermined time and extended into recess. In another example, in Lesson Study Meeting 3 in School B, Observer B2 complimented Teacher B about grouping the students differently in Lessons B1 and B2. The observers identified the positive aspects of the teaching and those aspects that needed to be improved. They did not discuss the teachers' personality, and this is in line with Doig and Groves' (2011) study.

The observers in this study felt free to comment on the teachers' teaching positively and to identify those things that needed to be refined. They offered feedback in a helpful way so that it did not appear to negatively impact on the teachers' feelings, even though some observers were younger with less teaching experience than the other teachers. These findings are in contrast to Rock and Wilson's (2005) study, which found that observers considered it challenging to review teachers who were older and had more teaching experience.

5.2.5 Refining

After reviewing the lesson in the Lesson Study meetings, group members planned the next lesson, taking into account the points they had discussed in order to improve the Lesson Plan and teaching. These findings are consistent with those of other studies, and suggest that the Lesson Study benefits from the members of the Lesson Study group refining the lesson (Cerbin, 2011). As an example, in the first lesson in School A where Teacher A divided the students into groups of six, the observers noticed that the group size was quite big and influenced the effectiveness of students' engagement in the group discussion. In the Lesson Study meeting, the observers suggested changing the number of students within one group for the next lesson. Teacher A accepted this suggestion and divided the students into groups of three.

5.3 The Influence of Lesson Study on Primary School Teachers' Mathematics Pedagogy

Teachers' Mathematical Pedagogical Content Knowledge consists of mathematical content knowledge, mathematical pedagogical knowledge and context knowledge (see Figure 2.1 page 22). Investigation of the influence of Lesson Study on each aspect of Mathematical Pedagogical Content Knowledge in this study is discussed in this section. Like Yamnitzky (2010) who found that Lesson Study can influence primary school teachers' Mathematical Pedagogical Content Knowledge, this Lesson Study process also showed its potential to improve the teachers' knowledge, despite the limited time in which it was formally undertaken.

5.3.1 The Influence of Lesson Study on Mathematics Content Knowledge

This study confirms that in Lesson Study groups meetings, teachers can share their content knowledge of the topic being taught. They can also refine other teachers' knowledge. For instance, when a member of Lesson Study Group B shared her misconception about the angle of a straight line, another member corrected the concept. This finding is similar to other studies in which Lesson Study influenced teachers' understanding of mathematics concepts (Yamnitzky, 2010). It seems that the teachers in this study benefitted from Lesson Study with regard to improving their mathematics content knowledge. This finding is aligned with other studies which are conducted by Lewis, Jennifer, Fischman, Riggs, and Wasserman (2013), and Tepylo and Moss (2011). In another Lesson Study project, Sam, White, and Mon (2005) also found that the observation and meeting steps in Lesson Study improved and enriched teachers' mathematics content knowledge.

5.3.2 The Influence of Lesson Study on Mathematical Pedagogical Knowledge

In Lesson Study Group Meeting A1, one teacher described a strategy to teach the concept of angles to the students. The teachers in this group then decided to use real objects inside the classroom to introduce the concept of the angle of an object. Teacher A then explained the kinds of angles in the real objects: acute, right and obtuse angles. Next, the teachers in Lesson Study Group A agreed to use pictures of a range of angles in order to build student understanding. Thus, the study participants in School A tried to assist students to learn about angles from real material objects before shifting to their abstract representations. This finding corroborates previous research in this field which links Lesson Study and teachers' mathematical pedagogical knowledge. Sam et al. (2005) and Shimahara (1998) found that Lesson Study may develop teachers' pedagogical knowledge, especially by helping them to recognise students' understanding of the mathematics being taught (Yamnitzky, 2010) and how their students learn (Dudley, 2013).

5.3.3 The Influence of Lesson Study on Context Knowledge

It is somewhat surprising that there was little evidence in the Lesson Study Meetings of teachers' context knowledge in either school. It is possible that this was due to the short duration of this study. However, Lesson Study meetings were beneficial to the teachers, because this study was conducted early in 2014, the year in which a new curriculum, Curriculum 2013, from the Indonesian government had just been introduced to the two schools in the study. In the Lesson Study Meetings, the teachers were able to discuss implementation of the new curriculum and recognise that its content was different from the previous curriculum. This study confirms that Lesson Study can be a significant tool for exploring alterations to the national curriculum (Lewis & Tsuchida, 1999), which can be classified as external factors in relation to teachers' contextual knowledge (Barnett & Hodson, 2001).

5.4 The Influence of Lesson Study on Curriculum Differentiation in Mathematics

In this study, evidence of differentiated mathematics instruction was found in both lessons from each school. Surprisingly, the teachers continued to teach conventionally for the most part, but did differentiate in some ways. For example, they grouped students flexibly, related the topic to students' daily lives, and valued each student. Even though, they were not familiar with the term "differentiated instruction", it seems that teachers in this study already differentiated in some areas. However, through their Lesson Study activities, the teachers developed a deeper understanding of differentiated instruction, as discussed in this section.

In what follows, the influence of Lesson Study on each aspect of differentiated mathematics instruction as found in this study is discussed and compared with the findings of other studies.

5.4.1 The Influence of Lesson Study on Differentiating Content

Based on the literature review in this study, the intention was for teachers to differentiate the level of content taught based on students' abilities, interests, and readiness, and to make connections with their daily lives. The findings of this study do not fully support previous research (for example, Rock and Wilson (2005)). The study in this thesis found that the teachers had difficulty in differentiating level of content based on students' abilities, interests, and readiness. They still taught content with the same level of difficulty to all students. This is different from the outcomes of a study in an elementary school in North Carolina, which found that Lesson Study impacted on teachers' performance in differentiating their mathematics instruction based on students' ability (Rock & Wilson, 2005). The teachers in Rock and Wilson's study (2005) taught the same topic to all students but with different levels of difficulty.

The current study found that the teachers tried to connect the topics with students' daily lives. For example, Teacher A used objects in the classroom to introduce angles to the students. She also asked students to observe buildings around the school and then classify the angles on the buildings.

5.4.2 The Influence of Lesson Study on Differentiating Process

Two indicators of process differentiation are encouraging students' creative thinking and encouraging their critical thinking. There was some evidence in this study that teachers encouraged students' creative and critical thinking in the lessons. For example, Teacher A asked the students to define the meaning of an angle after they saw angles on the objects in the classroom. Adams (2013) found in her study that there was no difference in frequency between the control group and the experimental group that used Lesson Study, in encouraging students' creative and critical thinking.

5.4.3 The Influence of Lesson Study on Differentiating Product

Differentiation of product in the lesson can be seen in the ways in which teachers vary the level of task difficulty, give students freedom in choosing a task, allow them to express their understanding in different ways, and allow them to submit their tasks at different times. In this study, the teachers tried to differentiate product by allowing students to express their understanding differently and giving extra time to students who needed it to complete their task. For instance, when Teacher A asked students to draw an angle of a certain degree, she let students draw the angle facing in different ways, as long as the degree was correct. The teachers in Rock and Wilson's (2005) study showed a similar improvement in differentiating instruction of assignments when using Lesson Study.

5.4.4 The Influence of Lesson Study on Differentiating Assessment

The traits of assessment differentiation are ongoing assessment, using rubrics, giving feedback on students' work, and allowing students to evaluate their own progress. In this study, the teachers applied ongoing assessment, created rubrics (see Figure 4-31) and allowed students to appraise their own work, but there was no evidence that the teachers gave feedback on the students' work. For instance, in School B, Teacher B noticed and assessed students' progress using the rubric created in the Lesson Study Group Meeting, though the rubrics need to be improved with clearer criteria for each category. Teacher B tested the students' knowledge during the group discussion. She also marked students' assignments individually. If Lesson Study continues to be used to develop teachers' skills and understanding, their skill of creating and doing assessment would be expected to grow. This is similar to the findings of another single Lesson Study cycle study that found improvement in teachers' development in differentiating the instruction including their assessment (Hocket, 2010). In Hocket's (2010) study, the four teachers also learnt how to create, adapt and analyse the assessment. However, Hocket's study used only one Lesson Study cycle and the teachers needed more training in designing differentiated assessment.

5.4.5 The Influence of Lesson Study on Differentiating the Learning Environment

The characteristics of learning environment differentiation are related to learning stations (different areas with different levels of task from the topic being taught), teachers valuing each student, and encouraging students to respect each other and to be responsible. In this study, there was no evidence of learning stations being created in the classroom. However, the teachers showed that they valued each student and encouraged students to respect each other. For example, both Teachers A and B praised students who answered correctly, and encouraged students who still needed help solving problems. As outlined by Tomlinson and Moon (2013), the learning environment is not only about the physical classroom but also the atmosphere in which the learning process happens. Therefore, the practice of Teachers A and B in valuing their students was part of differentiating the learning environment, even though they still needed to improve their ways of differentiating the classroom environment by, for example, creating learning centres.

5.5 The Influence of Teachers' Mathematical Pedagogical Content Knowledge in Differentiating Mathematics Instruction

Very little was found in the literature on the influence of teachers' Mathematical Pedagogical Content Knowledge on differentiating mathematics instruction. This study attempted to uncover in some detail whether teachers' Mathematical Pedagogical Content Knowledge had an impact on the way they differentiated mathematics instruction. The result of this study showed some evidence, though not in all areas of differentiated instruction. For example, there was no evidence showing that teachers' knowledge of curriculum contributed to them differentiating the learning environment.

Table 5-1 shows the connections between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction in this study. The aspects of teachers' knowledge that helped them to differentiate the mathematics instruction in the lessons during this study were mathematical content knowledge, mathematical pedagogical knowledge, and context knowledge.

Table 5-1 Connection between Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction

DMI / MPCK		Content Differentiation	Process Differentiation	Product Differentiation	Assessment Differentiation	Learning Environment Differentiation
Mathematical content knowledge: Conceptual and Procedural			• Critical and creative thinking encouraged • Varied activities		• Ongoing • Rubric provided	
Mathematical pedagogical knowledge	Knowledge of Students: students' background and understanding		• Varied activities	• Flexible submission		• Teachers value each student
	Mathematics Teaching Strategies Knowledge: classroom management and assessment	• Relevant to students' daily lives	• Flexible grouping • Varied activities	• Flexible submission		• Students encouraged to respect each other • Students encouraged to be responsible
	Curriculum Knowledge: mathematics educational goals, correlation to previous and later topic, and link to other subjects	• Relevant to students' daily lives	• Varied activities • Critical thinking encouraged • Creative thinking encouraged • Research encouraged		• Ongoing Rubric provided	
Context Knowledge: Regulation from Central and Local Government and Community						

Teachers' mathematical content knowledge, both conceptual and procedural, may help them to create questions that require students to think critically and creatively. For example, after asking the students to show angles of some objects in the classroom, Teacher A showed them a circular object and ask whether the circle has an angle or not; this required the students to examine the object and think about it critically (Scriven & Paul, 2011). Teacher A had knowledge of the concept of angles and understood which objects have angles and which ones have no angles.

There was also evidence in this study that teachers' mathematical content knowledge meant that they were open to the students' giving correct answers in varying ways. For example, when Teacher A required students to draw a certain angle, she accepted answers that were drawn facing in different directions, as long as the degree of angle was correct. For example, some students drew the angle facing right, some drew it facing up and some facing down.

Mathematical content knowledge supported teachers in creating various activities that related to the topic. For example, in this study, the teachers thought about what questions were appropriate to the topic. Having mathematical content knowledge does not mean that teachers can automatically create various activities in their lessons, but to generate such activities they do need to understand the topic properly, so that the activities will support the students' understanding of the mathematical concepts. For example, Teacher A asked students to observe and draw one side of a building. This activity required that the teacher understand the mathematical content to be able to decide which objects observed by the students were relevant within the context of this lesson.

As mentioned in the literature review, mathematical pedagogical knowledge consists of knowledge of students, mathematics teaching strategies' knowledge, and curriculum knowledge. Knowledge of students requires teachers to know about their students' backgrounds and understandings. This study found two aspects of knowledge of students used to differentiating the process and learning environment. The teachers used their knowledge of the students to create activities that were appropriate to the students' level of thinking and the topic being taught.

In product differentiation, the teachers allowed students to submit their work at different times. The teachers understood that their students had different levels of ability and various speeds of working. However, the teachers in this study did not prepare extra questions for the students who finished early, which would have challenged them and kept them occupied. In addition, by understanding their students, the teachers showed their respect for students' responses. For example, Teacher B asked her students to count one by one and used their numbers to make examples of mathematics operations for showing the commutative and associative laws. Teacher B understood that using one- and two-digit numbers were suitable for her students in Year 4. She also valued the students who needed help in solving the questions.

Two aspects of knowledge of mathematics teaching strategies can be viewed through classroom management and assessment. In this study, the teachers' knowledge enabled them to differentiate content, process, product and learning environment. Although the results showed that differentiation was not evident in all indicators of each aspect, the teachers did demonstrate differentiation using their knowledge of mathematics teaching strategies in some areas. In content differentiation, for example, Teacher B applied a strategy of relating the topic to objects that were close to students' daily lives, by asking them to identify objects in the classroom that had angles. This strategy was also useful in managing the classroom because it required students to observe and think about which objects in their classroom were answers to the teacher's questions.

Teachers' knowledge about managing the classroom in this study also supported them to create activities. The teachers thought about what kind of activities supported the topic and how to handle the students; for example, whether they might work in a group or individually. The teachers' knowledge of managing the classroom then enabled them to allow the students to submit their tasks when they finished their work; in so doing they also valued each student. The teachers knew that every student is an individual and that they needed to be respected, even if they did not answer questions correctly. Some students needed extra time to finish their work and/or more guidance to answer the questions.

Knowledge of the curriculum, and the way in which content of mathematics educational goals connect to previous and future topics and link to other subjects, was demonstrated in this study. This knowledge influenced teachers to differentiate content, process and assessment. For example, curriculum knowledge led Teacher A to relate the topic of angles to geometric shapes, a topic that the students had studied previously. She created an activity that required students to analyse some geometric shapes and categorise the angles of each shape. This knowledge also helped the teachers to undertake ongoing assessment. Teachers A and B in this study not only used a single assessment at the end of the lesson or after the topic was finished, but also assessed students' progress through their activities in group work. The teachers also tried to create rubrics, though these need improvement.

In this study, the evidence of context knowledge found was in the Lesson Study Meetings, where the teachers discussed the implementation of the new national curriculum. It was, however, difficult to find evidence of the teachers' knowledge of central and local government regulations, or their knowledge of the community, in this short-term study. A longer study and other instruments and strategies may be required in order to recognise the influence of context knowledge on differentiating mathematics instruction in lessons.

5.6 Potential Key Characteristics of Lesson Study as a Form of Professional Development

Various studies have identified characteristics of Lesson Study. For instance, Murata (2011, p. 10) listed five characteristics of Lesson Study as "centered around teachers' interests, student focused, has a research lesson, a reflective process, and collaborative". However, Murata (2011) said that other researchers may modify the characteristics based on their different backgrounds. In this study, from the Lesson Study cycles of two schools, it was found that, as a form of professional development, Lesson Study produced a number of potentially noteworthy characteristics in teachers, as described in this section. As this was a short term and limited study, caution must be applied, as some of these findings may not be transferable to longer Lesson Study cycles or in different settings. The noteworthy characteristics of Lesson Study were found in this study were cooperation, effectiveness, independence, tolerance, reflection, refinement, dedication and continuity.

5.6.1 Cooperation

The Lesson Study Meeting showed that the teachers in the Lesson Study Group worked together to create lesson plans. Evidence of cooperation was clearly seen in the Lesson Study Groups, where teachers discussed the strategies that they would use to teach the topics.

The Lesson Study Groups from Schools A and B had three meetings each. In the first couple of meetings each group produced two Lesson Plans. The Lesson Study Group from School A produced Lesson Plans A1 and A2 (see Appendix 9 for the Indonesian versions; see Chapter 4, Figure 4-6 on page 75 and Figure 4-14 on page 91 for the English versions). The Lesson Study Group from School B generated Lesson Plans B1 and B2 (see Appendix 10 for the Indonesian versions; see Chapter 4, Figure 4-25 on page 111 and Figure 4-29 on page 122 for the English versions).

This finding is consistent with previous research, which found that Lesson Study can build cooperation among teachers in planning, teaching, and reflecting (Subadi et al., 2013). Also, as a team, the teachers in the Lesson Study groups shared responsibilities, where one teacher taught and other teachers observed, as Dudley (2011) also found. Lesson Study is intended to be a way for teachers to work together to improve teaching quality (Cajkler, Wood, Norton, & Pedder, 2014; Lewis & Tsuchida, 1999).

5.6.2 Effectiveness

The teachers developed their professional practice at the school without leaving their students or their school. They maintained their normal teaching duties; the professional development was ‘in-house’ and Tanaka (2007) named this as “in-school training" (p. 150). In this study, the teachers came from two different schools and they did the Lesson Study activities in their own school.

These Lesson Study cycles were based on the content that the teachers were already teaching. This meant that teachers could improve their teaching based on regular practice. For example, in the Lesson Study Meeting A2 in School A, Observer A2 commented that Teacher A was busy with her own work and did not pay attention to students' group activities for quite a long time in Lesson A1. Teacher A took notice of this input and refined in her approach in Lesson A2. In the Lesson Study Meeting B2 in School B, Teacher B was observed asking the students to answer and read her questions together in unison. After feedback from the observers, she changed the strategy. She asked individual students to answer her questions. That is, teachers refined their teaching based on their review of the previous lessons. Borko (2004) also found that professional development that was based on teaching practice was more effective than managed programs like workshops.

One question that needs to be asked, however, is whether or not teachers can improve their knowledge and skills with a more appropriate or different approach without a qualified facilitator. To meet this need, the Lesson Study group can invite an outside expert (Fernandez & Yoshida, 2012) to attend in the Lesson Study meeting in order to facilitate their problem solving and develop their knowledge and skills. Actually, the experts could be a school supervisor, a competent teacher from another school, or a proficient lecturer from a university. Thus, the teachers will get a facilitator who is appropriate to their needs in order to achieve their aims and support them professionally. However, the Lesson Study groups in this study did not invite an outside expert in this case, given the short duration of the study.

5.6.3 Independence

The teachers in the Lesson Study groups managed their own time to meet. They also decided whether they were able to solve problems themselves or whether they needed a facilitator. However, the Lesson Study group activities still needed supervision from the principal to make sure that the process was run correctly. The principals of School A and B were also needed to verify that the lesson plans were suitable to the curriculum as a whole and that they were consistent with the guide from the Ministry of Education and Culture in Indonesia (Kemendikbud, 2013). However, the teachers managed the Lesson Study activities independently, including their meeting times and the topics discussed. In Japan, Shimahara (1998) also found that professional development based on teachers' initiative was more likely to develop their teaching skills. Lesson Study was run by the teachers as the participants in professional development, but was not steered by a facilitator or experts. A facilitator was invited to attend to encourage the teachers' initiative to improve their professional development in the areas that they needed.

5.6.4 Tolerance

The teachers in this study showed tolerance when their colleagues, as observers, criticised them. They realised that the observers saw teaching deficiencies that they themselves did not see. For instance, when it was suggested to Teacher A to avoid "choir" answers in Lesson Study Meeting A2, she accepted the advice happily and then tried to refine the strategy by asking the students one by one in Lesson A2. In another example, when Teacher B was told that she did not give clear instructions for the homework to demonstrate the commutative law in subtraction and division, she received the feedback and indicated she appreciated her colleagues' candour. In the Lesson Study Group Meeting B2, Teacher B said that she accepted the comments from the observers.

The teachers realised that their belief could be incorrect (Spiegel, 2012); that is why they needed advice from others. This tolerance helped them to refine their teaching practice. It is also important to note that observers need to consider ethical issues when giving feedback to a teacher. This point was stressed in the Lesson Study session at the beginning of this study in each school. The feedback should focus on the teacher's performance that relates to the teaching process and avoid commenting on the teacher's personality. Teachers can learn to observe and give feedback to each other objectively.

5.6.5 Reflection

In this study, teachers not only received feedback from the observers in the Lesson Study group during the meeting, they also reflected together on their teaching. For instance, in Lesson Study Meeting A3, Teacher A reflected on her teaching, saying that the indicators were achieved and the process steps ran as planned in Lesson Study Meeting A2 and written in Lesson Plan A2. She realised that she still needed to improve her teaching about measuring an angle using a protractor. In another example, in Lesson Study Meeting B2, Teacher B reflected that she used a strategy that was not planned in Lesson Plan B1. She thought that it was a good idea to ask the students to count one by one and then use their numbers for examples to demonstrate the commutative law, even though this strategy was not written in the Lesson Plan.

Reflection is an important way to improve professionalism in teaching (Clarke, 2000; Jaeger, 2013; Lewis & Tsuchida, 1999). Clarke (2000) maintains that reflection may help teachers construct their understanding based on their teaching experience. Moreover, Sam et al. (2005) found that self-reflection could help teachers revise their own teaching inaccuracies. As Jaeger (2013) claims: “there is almost universal agreement that effective teachers reflect regularly and deeply on their practice” (2013, p. 98).

5.6.6 Refinement

In the schools in this study, both teachers A and B demonstrated that they refined their teaching according to suggestions from the observers in the Lesson Study meetings. For example, in Lesson Study meetings A2 and B2, each of them was advised to avoid chorused answers from the students; they did so in Lessons A2 and B2, by pointing to a student with mentioned student’s name when they wanted a response. The findings in this study seem consistent with other research, which found that Lesson Study can improve teaching practice (Lewis & Tsuchida, 1999). Yoshida (2012) also found that Lesson Study improved teachers’ learning.

5.6.7 Dedication

In this study, Lesson Study meetings took place both in school hours and outside school hours. The participant teachers attended all the sessions required, showing commitment to the process. The participants were involved in three Lesson Study meetings and two lessons. In addition, when one or more members acted as an observer, the observer sometimes used their free time to attend and observe their colleague in the classroom. This meant that the teachers from Schools A and B who attended the lesson as an observer, did so because they were not scheduled to teach at that time. Generally, they would use their free time to prepare for other lessons or mark student work. In a Lesson Study project in Malaysia, the time factor was a barrier and caused some meetings to be deferred or even stopped (Sam et al., 2005).

The primary school teachers in Schools A and B in this study had a full schedule. Most of them were class teachers who taught more than one subject. The teachers who were involved in this study only had free time when the students had physical education and religion classes, because these subjects are usually taught by a specific teacher. In England, time constraints posed a challenge for schools with a heavy timetable when arranging Lesson Study activities (Cajkler et al., 2014). However, in this study the teachers demonstrated their desire for better lesson plans and teaching, despite the need for extra time. They seemed to be dedicated to improving their professional practice, even though the Lesson Study process in this study was only of two cycles. These present findings are consistent with Smith's (2008) research, which found that teachers were dedicated to the Lesson Study process and produced quality work.

5.6.8 Continuity

Even though the Lesson Study in this research was conducted in only two cycles, analysis of the Lesson Study activities revealed that this form of professional development could potentially run throughout the school year. This study ran Lesson Study meeting 1, Lesson 1, Lesson Study meeting 2, Lesson 2, and Lesson Study meeting 3 for each school. The teachers can continue these cycles of activities. This is consistent with the findings of Dudley (2013) that teachers can learn continually in

Lesson Study. Teachers can refine their teaching after discussion in Lesson Study meetings. Furthermore, in a longer-term teacher-training program conducted by Lewis, Fischman, Riggs and Wasseman (2013), the researchers reported that Lesson Study could improve teachers' teaching in mathematics on an ongoing basis.

5.7 Digital Mixed-Methods Approach for Studying the Impact of Lesson Study on Teachers' Knowledge and Skills in Differentiating Mathematics Instruction

Mixed methods studies can be time consuming as both quantitative and qualitative data is required (Fraenkel & Wallen, 2009). The mixed methods in this study were not concerned with synthesising qualitative and quantitative data, but rather with modifying qualitative data using digital software (Multimodal Analysis Video). The qualitative analysis was transformed the data into quantitative data (O'Halloran et al., 2016) to help interpret the results of the study.

"Mixed methods potentially offer depth of qualitative understanding with the reach of quantitative techniques" (Fielding, 2012, p. 124). The quantitative data generated from the software validated the interpretation of the qualitative data. Some of the data sources in this study (video recordings, Lesson Plans, and classroom observations) were used as triangulation for validity. Moreover, the interpretation of results from the state machine of the Multimodal Analysis Video software was supported by screen shots of the videos for close study of the various relationships.

Fielding (2012) states that mixed methods assist the researcher when investigating concepts. This point was supported by this study, which explored the concepts of Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction, and the connections between the two concepts. Fraenkel, Wallen, and Hyun (2012) write that mixed methods assist in discovering the connections between two concepts.

5.8 Conclusion

This chapter discussed the findings of this study with reference to each research question and connected them to relevant earlier research. This study demonstrated that the Lesson Study processes of planning, teaching, observing, reviewing, and refining can be used effectively to improve classroom teaching practices.

This study found that Lesson Study influenced teachers' Mathematical Pedagogical Content Knowledge, as they were able to share their difficulties with others and complement each other's knowledge. Some instances demonstrated the influence of Lesson Study on teachers' curriculum differentiation in mathematics. Through the Lesson Study activities in this study, the teachers learnt and tried to apply differentiation of content, process, product, assessment, and learning environment. Although not all indicators of differentiation were employed, there were some changes in their teaching.

This study also found that teachers' Mathematical Pedagogical Content Knowledge might affect their capacity for differentiating mathematics instruction. As there is no previous research in this field, this study is a starting point for further research to enrich knowledge in this area. A longer and more comprehensive study would uncover more of the impact of teachers' Mathematical Pedagogical Content Knowledge in differentiating mathematics instruction.

The results of this study found evidence in the teachers of the characteristics of Lesson Study in terms of cooperation, effectiveness, independence, tolerance, reflection, refinement, dedication and continuity. The presence of these characteristics in this study supports the notion that Lesson Study might be an effective professional development tool to improve teaching practices.

Finally, the digital mixed methods used in this study benefit education research by using new technology, namely, Multimodal Analysis Video software. The quantitative data generated from the qualitative analysis provided meaningful interpretation.

Chapter 6 – Conclusions

6.1 Introduction

This chapter presents a summary of this study, which investigated the impact of Lesson Study on primary school teachers' knowledge of differentiating mathematics instruction in two primary schools in Nagekeo District, Flores Island, Indonesia. Following this, the implications and suggestions for future research are discussed. Finally, this chapter addresses the limitations of the study.

Concern for improving teacher quality has led to the provision of a great deal of professional development over the last few decades, particularly in the area of mathematics education. Various types of professional development have been introduced and implemented, but many have been erratic, which has not necessarily led to improvement in teaching practice. However, since the late nineteenth century, Japan has used Lesson Study to develop teacher professionalism (Isoda, 2007). Lesson Study has since been used in countries such as Philippines, Kenya, America, and Indonesia. In Indonesia, Lesson Study came into use in the early 2000s (Tatang, 2012).

The framework of this study predicted that the influence of Lesson Study on teachers' knowledge in teaching mathematics would result in differentiation that could be seen in the teaching practice in lessons. The Lesson Study model of professional development offered processes to enable the teachers, as participants, to challenge their proficiency to develop and use Mathematical Pedagogical Content Knowledge in order to differentiate their instruction. The impact of Lesson Study during the course of the study, demonstrated connection between teachers' Mathematical Pedagogical Content Knowledge and the Differentiated Mathematics Instruction undertaken in the classroom.

6.2 Lesson Study as a Professional Development Model to Change Primary School Teachers' Knowledge in Differentiating Mathematics Instruction

Lesson Study was chosen as a professional development model for teachers so that they could continue their school-based duties. The teachers had their own knowledge when they came in this study. They were introduced to differentiated mathematics instruction as a possible answer to the diversity of students in a regular classroom.

This study was designed to see if Lesson Study could facilitate changes in teacher knowledge in differentiating mathematics instruction. In particular, this study set out to describe the processes involved in the implementation of Lesson Study; investigate the influence of Lesson Study on primary school teachers' mathematics pedagogy; investigate the influence of Lesson Study on primary school teachers' curriculum differentiation in mathematics; and identify the potential key characteristics of Lesson Study as a form of professional development. In order to address these questions, a digital mixed methods approach was developed, in order to study classroom interactions.

Data was gathered from two Lessons and three Lesson Study Meetings from two schools. The activities from the Lessons and Lesson Study Meetings were video recorded and analysed using Multimodal Analysis Video. Teachers' Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction were explored based on data generated from the video recordings.

6.2.1 Lesson Study Steps in this Study

The study was carried out at two primary schools in Nangaroro sub-district, Nagekeo District, Flores Island, Indonesia at different times but with the same sequence of activities. Participating teachers at each school first attended in professional development sessions that presented Differentiated Mathematics Instruction and Lesson Study. Each school created one Lesson Study Group (LSG), School A with four teachers (LSG-A) and School B with three teachers (LSG-B). The Lesson Study cycles in this study are shown in Figure 6-1.

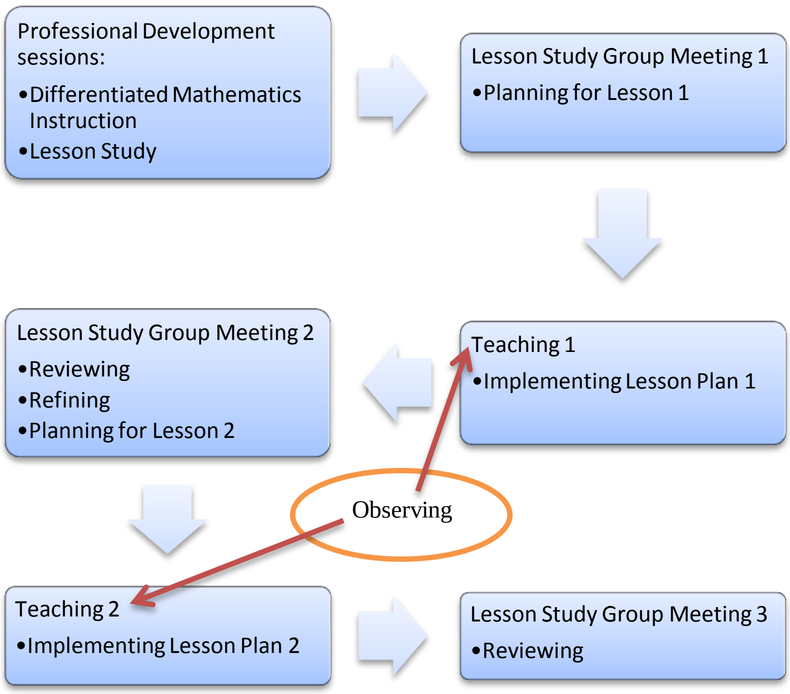


Figure 6-1 Lesson Study Cycles in this study

6.2.2 Lesson Study and Mathematical Pedagogical Content Knowledge

The results of this investigation showed that Lesson Study influenced primary school teachers' mathematics pedagogy: their mathematics content knowledge, their mathematical pedagogical knowledge, and their context knowledge. Through discussion in during the Lesson Study meetings, the teachers could share knowledge such as their understanding of a mathematics concept that was to be taught. For example, when one teacher had a misconception, other teachers were able to explain why this was a misconception. This was very important as a teacher's misconception may lead to students having the same misconception.

The Lesson Study model also enabled the teachers to discuss ways in which a topic could be taught; for instance, resources needed, teaching aids that could be used, and organisation of how to student activities during lessons. Additionally, the process helped teachers to build their context knowledge; for instance, when the central government instructed teachers to use new curriculum, they could discuss this.

6.2.3 Lesson Study and Differentiated Mathematics Instruction

The results of this investigation showed that Lesson Study influenced the teachers' curriculum differentiation in mathematics including differentiation of content, processes, products, assessment, and learning environment in lessons. The teachers' practice in this study illustrated their efforts to differentiate mathematics instruction in their lessons. For example, they connected topics being studied with students' daily lives, grouped students flexibly, employed a rubric for assessment, and valued each student by complimenting them or asking their classmates to clap their hands to show appreciation.

Although areas of differentiation they applied still need to be developed; for example, how to create an effective rubric, the teachers' efforts showed that they grasped the basic principles of differentiation. Changing teachers' pedagogy takes time and this includes differentiation; therefore, it is unrealistic to expect large changes to occur during two cycles of Lesson Study experience. Wormeli (2003) stated that it may take three or more years for teachers to be able to practice differentiation effectively; it requires commitment to improve knowledge and skills and to become proficient in differentiating mathematics instruction. When teachers work together to develop professionally, it may motivate them more than if they do it alone. Lesson Study offers an opportunity for teachers to improve throughout their career, providing they are prepared to take the necessary time required.

6.2.4 Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction

Although the findings of this study did not show connection of all aspects of teachers' Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction, there were some evidence that demonstrated the connection among them. For example, teachers' knowledge of mathematics teaching strategies, especially in classroom management, helped them to differentiate process by grouping the students flexibly.

6.2.5 Potential Characteristics of Lesson Study

One finding to emerge from this study is the identification of key characteristics of Lesson Study as a form of professional development. It was found that the characteristics of Lesson Study were cooperation, effectiveness, independence, tolerance, reflection, refinement, dedication, and continuity. These characteristics emerged even though this study was conducted over a relatively short period, but had the Lesson Study process continued, the characteristics would be embedded in this kind of professional development.

The findings of this study, especially on the influence of Lesson Study on teachers' knowledge in differentiating mathematics instruction, will enrich research in this area, given the limited amount of work that has been undertaken so far. Apart from Rock and Wilson's (2005) and Hocket's (2010) studies, it was hard to find other research that explored the impact of Lesson Study on teachers' knowledge in differentiating mathematics instruction, even though Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction have been used in various areas around the world.

This study's findings support those of Rock and Wilson (2005), that teachers' knowledge of differentiating mathematics instruction can be extended through Lesson Study. Hocket's (2010) study also found that, through Lesson Study, teachers can develop their understanding and skills in differentiating instruction.

In general, these findings suggest that Lesson Study poses opportunities for teachers to discuss not only lesson topics but also strategies for teaching mathematics. It is possible for teachers to learn from their colleagues in Lesson Study meetings. The Lesson Study meetings in both schools in this study showed that Lesson Study provided opportunities for the teachers to discuss both topics and strategy. This will improve teachers' Mathematical Pedagogical Content Knowledge, which can influence them to differentiate mathematics instruction.

6.2.6 Digital Mixed Methods Research Design

Multimodal Analysis Video, a new software package developed by Kay O'Halloran and her colleagues, was used in this study to facilitate data conversion. The qualitative data collected was transformed into quantitative data using the software (O'Halloran et al., 2016). The state transition diagrams permitted patterns in Mathematical Pedagogical Content Knowledge, Differentiated Mathematics Instruction and the relationships between these two classroom practices to be studied. Interpretation of the data was augmented with the Lesson Plans and classroom observation that were generated from the Lesson Study activities during the study. Screenshots from the data analysis were also used to strengthen the interpretation of the data analysis.

6.3 Contributions to the Field of Mathematics Education

This study contributes to the field of education through the use of another kind of professional development, namely Lesson Study. The study has helped introduce a type of professional development that can be undertaken while teachers continue their regular teaching, and assisted teachers to realise the benefits of working collaboratively with colleagues.

In addition, this study contributes evidence to the field of mathematics education about expanding teachers' knowledge of mathematical pedagogical content to differentiate their instruction. In order to meet the needs of students in regular classrooms containing a diversity of students, teachers need knowledge not only of mathematics content but also of teaching mathematics and the context that may influence the teaching. This study has shown that teachers' knowledge of mathematics teaching strategies and curriculum has helped them differentiate lesson content by, for example, connecting the topic to students' daily lives.

This study contributes significantly in the areas of Lesson Study, Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction. There are a numerous studies of those three areas. However, the connection among these areas is not yet developed. This study provides an initial investigation and reference into the connection of Mathematical Pedagogical Content Knowledge and Differentiated Mathematics Instruction.

A further contribution of this study is the development and use of a digital mixed methods design for analysis of interactions in classroom lessons. This will assist educational studies in systematic analysis and interpretation of data.

6.4 Implications

The findings of this research have implications for future developments in Lesson Study as an in-house professional development procedure. The effective use of Lesson Study may have the following implications for education institutions and educators.

First, Lesson Study may build strong relationships among teachers. As a team, teachers in a Lesson Study group have the opportunity to discuss lesson plans, teaching strategies, problems that may arise during the lesson, and the use of teaching aids. When they review the teaching in the meeting, they can also learn from others' feedback and advice. This will help teachers refine and improve their performance.

Second, Lesson Study will improve teachers professionally. By reviewing and refining what they do, teachers will develop professionally by improving their knowledge and skills. In a Lesson Study group, teachers can learn from their colleagues and share their understanding and their difficulties (Ahearn, 2011). Teachers may also come to understand student needs better. By understanding students' needs, the teachers will be able to differentiate the instruction in their classroom. This curriculum differentiation in mathematics may help students to achieve their goals of learning, because students will improve their achievement based on their ability.

Third, Lesson Study cycles will develop teachers' skills in research. The steps in the Lesson Study are a form of classroom action research in which teachers are planning, implementing, observing, reviewing, and refining in a systematic manner.

Fourth, Lesson Study may become an effective tool for professional development. Lesson Study is different to other professional development approaches. In Lesson Study, teachers do not need to leave their students and spend time and resources on externally-provided professional development activities. Lesson Study has potential as a long-term continuous professional learning strategy. These factors also make it easier for principals and education supervisors to support and supervise change in teachers' performance. However, adopting Lesson Study requires strong commitment among teachers and between teachers, principals and education supervisors.

Finally, the main implication from the findings of this study is the connection between teachers' Mathematical Pedagogical Content Knowledge and their ability to differentiate mathematics instruction in the regular classroom. This knowledge can assist teachers to understand their students and classroom circumstances, and then create the most appropriate lesson plan and teaching strategies in order to meet students' needs.

6.5 Limitations and Suggestions for Future Research

The study had several limitations.

The most obvious limitation was the duration of the research, which was conducted in a relatively short time (two weeks), so it was not possible to test whether Lesson Study would consistently happen over a longer period. Smit and Humpert (2012) suggest that it takes more than two years to alter teachers' traditions in delivering lessons. Therefore, it is suggested that a longer-term study is needed to monitor the deeper impact of Lesson Study on teachers' knowledge in differentiating mathematics instruction. One advantage of a long term of study would be the ability to capture the change in teachers' knowledge that might not be recognised during a relatively short-term study. This study however, provides a starting point for research into Lesson Study and Differentiated Mathematics Instruction to be developed further with an extended timeframe and more cycles.

The study was limited by the presence of the researcher, which may have influenced the natural behaviour of the participants. Some behaviours may have arisen because they knew that they were being observed by the researcher (the Hawthorne effect, (Sedgwick, 2012)). The teachers may have returned to their accustomed behaviour of teaching after the departure of the researcher. Further research could consider a strategy that could monitor teachers’ performance naturally, to see if the changes that teachers made during the research period continue to be applied consistently.

Another limitation was that there was no pre-study before the intervention was conducted, so it is not possible to determine whether what happened occurred because of Lesson Study or whether the teachers already worked that way. For example, when the teachers valued each student in the lessons during this study, did this also happen before the study? Is it possible that teachers did differentiation before the intervention but did not identify it as such?

This study was limited by no involvement of an outside expert. This research was designed to let the teachers, as participants, share their knowledge and understanding without input from the researcher. The researcher limited herself to involvement in the Lesson Study Meeting discussions. However, during the study, it was realised that the attendance of an expert was needed to help the teachers understand and apply Lesson Study and Differentiated Mathematics Instruction more effectively and at a deeper level. Chokshi and Fernandez (2004) also found in the US Lesson Study experience that experts from outside the school were needed. They said that an outside expert could assist teachers by providing new knowledge and advice. This was especially so in this study, because the topics of Lesson Study and Differentiated Mathematics Instruction were new to the teachers.

Therefore, it is suggested that the researcher could act as an outside expert in the Lesson Study Meetings. This may help teachers to overcome their difficulties and give them experience of interacting with an expert from a university nearby. This would build a closer relationship with universities. In the Lesson Study Group Meeting, teachers needed not only to discuss and share knowledge and problems with colleagues, but also to obtain knowledge or skills that none of them possessed: an expert from a university could provide this.

6.6 Summary

From the findings of this study, Lesson Study seems to provide opportunities to improve teachers' Mathematical Pedagogical Content Knowledge in differentiating mathematics instruction.

It is important for teachers to develop their professionalism continuously in order to help students achieve their learning goals. Teachers are the main asset in students' education (Ugbe, Bessong, & Agah, 2010). When teachers differentiate instruction, it means that they focus on the students' need. Although it requires a lot of time and energy, Lesson Study has the potential to build a habit of learning together while teachers continue to carry out their main duties as a teacher. Lesson Study can also encourage teachers to learn and improve their research ability.

The mixed methods research design for transforming qualitative analysis of classroom interactions into quantitative data is a significant step forward for research in education. This design model permitted patterns of teacher and student interactions to be studied in close detail, combined with the teacher observations and Lesson Plans produced during the Lesson Study cycles.

Further research that replicates and enlarges this study is required. Although more research is needed on the significance of the influence of Lesson Study on teacher knowledge in differentiating mathematics instruction, the findings to date suggest that teachers' mathematical pedagogical content knowledge may be the most important factor in enabling them to differentiate mathematics instruction to address students' learning needs in mathematics.

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Appendices

Appendix 1: Lesson Plan example of integers number

Model of Differentiated Mathematics Instruction for Regular Classrooms in Flores, Indonesia

Overview

Subject:	Mathematics
Topic:	Integers
Year/Semester:	4/1

1.1 Core Competence:

Understand factual knowledge by observing (listening, seeing, and reading) and ask questions based on curiosity about themselves, God's creatures and activities, and the objects they encounter at home and at school.

1.2 Basic Competence:

1. Understand the patterns of addition and subtraction of integers using concrete objects and the number line
2. Understand the concept of negative numbers using concrete objects and the number line

1.3 Indicators:

Students will be able to

1) Explain the meaning of positive and negative numbers in everyday life	Lesson 1
2) State the opposite of the numbers	
3) Express using 'greater' and 'less' than	
4) Order integers	
5) Calculate addition with integers	Lesson 2
a. Positive plus positive	

b. Negative plus positive	
c. Positive plus negative	Lesson 3
d. Negative plus negative	
6) Calculate subtraction with integers	Lesson 4
a. Positive take away positive	
b. Positive take away negative	
c. Negative take away positive	Lesson 5
d. Negative take away negative	
7) Solve the everyday life problems associated with the operation of integers	Lesson 1 - 5

1.4 Time

10 x 35 minutes (note: each lesson equal 2 x 35 minutes)

1.5 Strategy

Differentiated Mathematics Instruction: differentiate the content, process, and learning environment based on students' readiness. Use flexible grouping and open-ended questions.

1.6 Lesson 1 (2 x 35'): Introduce positive and negative numbers

1.6.1 Preparation

- Create well picture to introduce negative and positive numbers.
- Provide a box with number cards from -10 to 10.
- Compare and order integers worksheet

1.6.2 Indicators

Students will be able to

1) Explain the meaning of positive and negative numbers in everyday life

2) State the opposite of the numbers
3) Express using 'greater' and 'less' than
4) Order integers

1.6.3 Students' Prior knowledge

Numbers, greater and less than

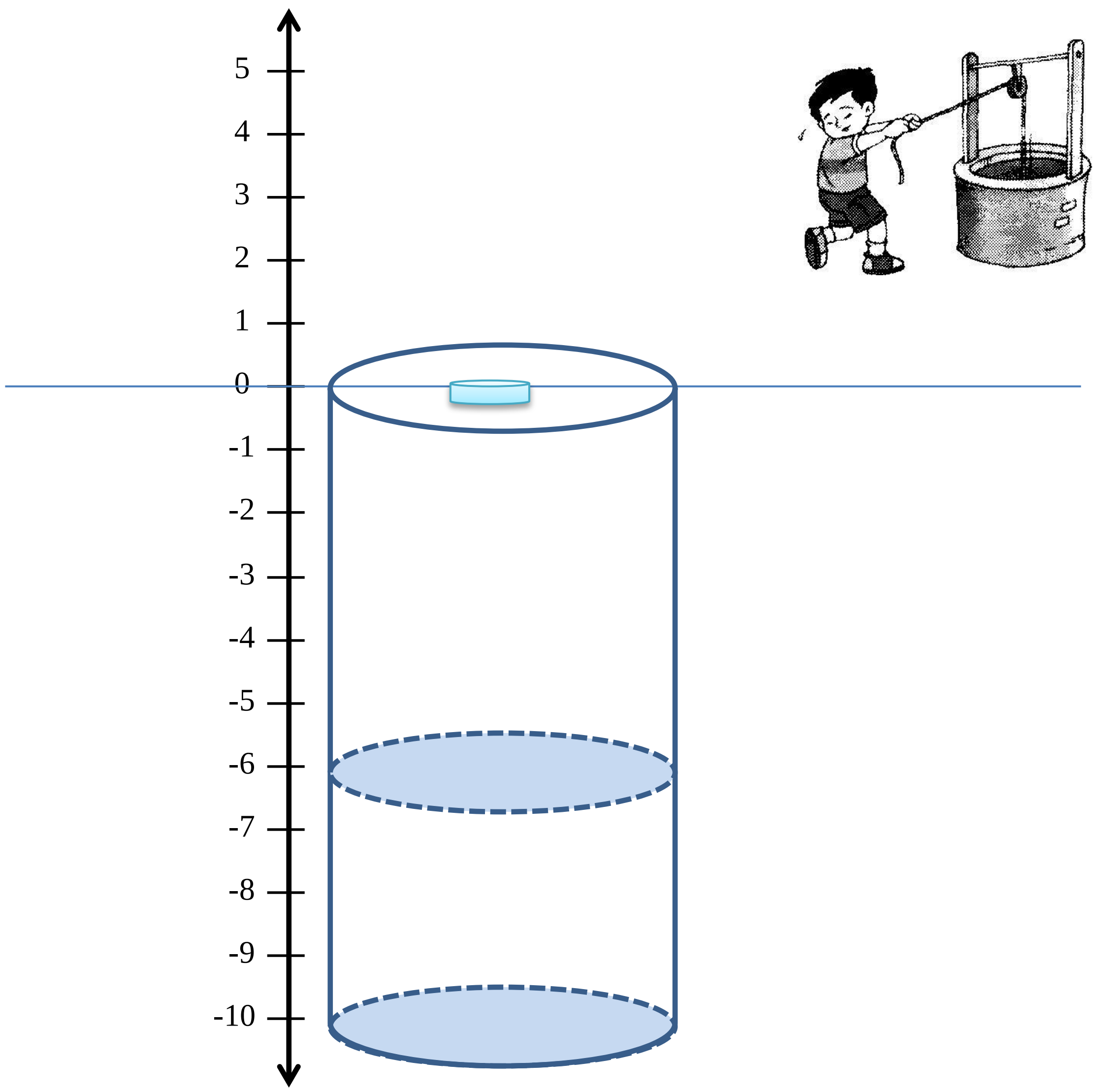
1.6.4 Actual time

A. Initial activities (5')

Beginning with introductory sentences about well

B. Main activities (55')

1) Use well picture with positive and negative numbers on it to build understanding about positive and negative numbers and the opposite numbers. (10')



For example:

i. If we pull the bucket up eight metres above the zero level, we can write it as 8 (*say:* eight or positive eight).

ii. If we lower the bucket down 8 metres below the zero level, we can write -8 (*say*: negative eight).

Then, -8 is called the opposite of 8 and vice versa.

2) Still using the well, build an understanding about comparing integers. (10’)

If we pull the bucket up, this will go higher and higher and the number shown will become bigger. But if we lower the bucket down, we get smaller numbers. For example, if we have two numbers 3 and -5, from the number line above, the position of -5 is below 3, this means that 3 is bigger than -5 and can be written as 3 > -5 (*say:* positive three is greater than negative five), or we can say that -5 is smaller than 3 and can be written as -5 < 3 (*say:* or negative five is less than positive 3).

3) Games (10')
   - ✓ Ask five students to come in front of the class.
   - ✓ Ask them to take one card from the box provided.
   - ✓ Ask them to look the number on their own and friends' card.
   - ✓ Ask them to stand in order from the least to the greatest numbers and from the left to the right class.
   - ✓ Discuss what they do with the whole students.
   - ✓ Do this activity once more time with eight different students.

4) Discussion (25')
   - ✓ Discuss with the whole class about the use of positive and negative numbers in everyday life for example, temperature, earning/saving money and spending/lending money, the height of a mountain (above sea level) and the depth of the sea (below sea level), and the height of a house or tree and the depth of a well.
   - ✓ Student work in pairs, ask them to discuss the questions in Worksheet 1.
   - ✓ Teachers move from one to another group to help students who need assistance.

C. Closing activities (10')

1) Teacher gives homework
   1. Please fill the dots below with "<" or ">":
      i. 12 … 8
      ii. -2 … 5

iii. -8 … 3

iv. 8 … -9

v. 10 … -6

vi. -7 … -1

vii. 0 … -3

viii. -4 … -15

2. State the opposite of each number below:
   i. The opposite of 5 is …
   ii. The opposite of -9 is …
   iii. The opposite of 0 is …
3. Sort the following numbers from the smallest to the biggest correctly: 3, 0, 2, -8, 5, -1, 7, -4, -6, and 9.
4. Sort the following numbers from the biggest to the smallest correctly: 8, -7, 4, -2, 6, 1, -5, -9, -3, and 0.

1.6.5 Assessment – ongoing assessment: observation and portfolios

1. Observe students while they work in their group and use this rubric:

#	Student name	Explain positive and negative numbers in everyday life			Ability in using greater and less than signs			Order integers		
		Very good	Fair	Need assistance	Very good	Fair	Need assistance	Very good	Fair	Need assistance
1	Student 1									
2	Student 2									

2. Collect students' work as their portfolios

1.7 Lesson 2 (2 x 35') Addition with integers (positive plus positive and negative plus positive)

1.7.1 Outcomes

Students will be able to

1) Calculate addition with integers
   a. Positive plus positive

b. Negative plus positive

2) Solve the everyday life problems associated with the operation of integers

1.7.2 Prior knowledge

- Addition with positive numbers
- Positive and negative numbers

1.7.3 Preparation

Worksheet 2: Addition with integers (positive plus positive and negative plus positive)

1.7.4 Actual time

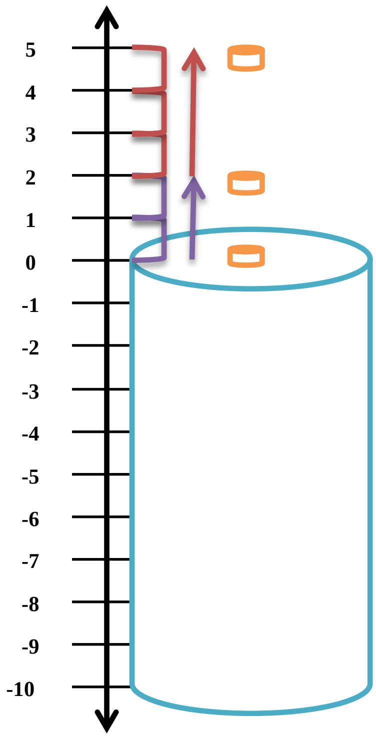
A. Initial activities (10')

1) Remind students about positive and negative numbers.
2) Re-discuss the use of positive and negative numbers in real life. *(for example, temperature, earning/saving money and spending/lending money, the height of a mountain (above sea level) and the depth of the sea (below sea level), the height of a house or tree and the depth of a well, ...)*

B. Main activities (50')

1) Give examples about the movement of a bucket that represent positive plus positive and negative plus positive. (10')

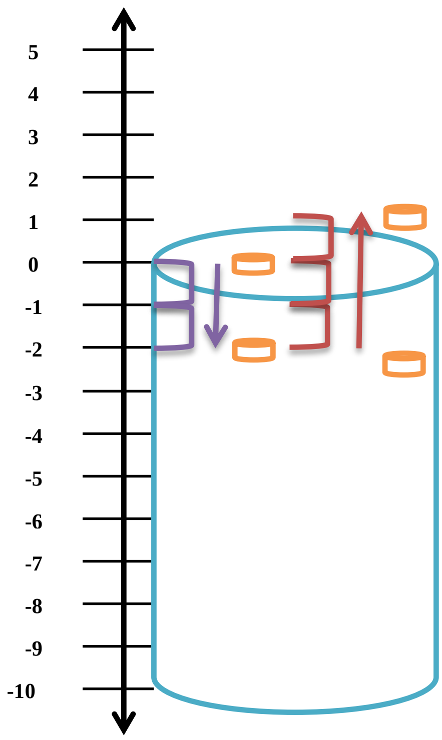
   Example 1: Positive plus positive



Cicilia pulls the bucket up two metres above the zero level then continues to pull it up three metres. What is the position of her bucket now?

i. Pulls the bucket up from zero to two.
ii. Then continues to pull the bucket up as long as three steps from 2.
iii. The bucket ends on five.
iv. Now, the position of Cicilia's bucket is on five metres above the zero level.
v. The right symbolic representation for this problem is 2 + 3 = 5

Example 2: Negative plus positive



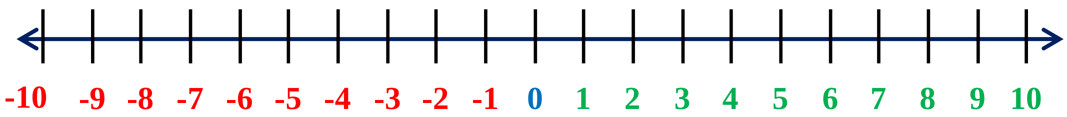
Oscar lowers the bucket down two metres below the zero level then continues to pulls it up as far as three metres. What is the position of the bucket now?

i. Lowers the bucket down from zero to negative two.
ii. Then continues to pull the bucket up as long as three steps from -2.
iii. The bucket ends on 1.
iv. Now, the position of Oscar's bucket is on one metre above the zero level.
v. The right symbolic representation for this problem is (-2) + 3 = 1

2) Put students in heterogeneous groups of three to discuss the problems in the worksheet 2 (Part A) provided. Each group of three will have students

of high, middle and low ability based on their understanding in lesson 1. Teachers move from one to another group to help students who need assistance.(15’)

3) Discuss with the whole class about vertical number line. (10’)
   After students understand positive and negative numbers from the problems (well) above, alter the vertical number line beside the well to become a horizontal number line with integers along it.



   Solve the problem below and represent it using mathematics symbol:

   1. Markus walks six steps from the zero point to the right then continues to walk two steps to the right. What is the position of Markus now?
   2. A frog stands on the zero point of a number line. Then he jumps four units to the left and then jumps seven units to the right. What is the position of the frog now?

4) Then ask the students to complete worksheet 2 Part B. (15’)

C. Closing activities (10’)
   1) Teacher gives homework
      I. Solve the problem below using horizontal number line and represent it using mathematics symbol:
         1. Grace runs eight metres from the zero point to the right then continues to walk seven metres to the right. What is the position of Grace now?
         2. Antonia walks eight metres from the zero point to the left then walk back nine metres. What is the position of Antonia now?

3. Daniel walks ten metres from the zero point to the left then walk back seven metres. What is the position of Daniel now?
4. I stand on the zero point. Tina stands 3 steps to the right of me and Ansel stands 5 steps to the left of me. How many steps is the distance between them?

II. Solve the addition below:

1. 8 + 6 = …
2. 12 + 9 = …
3. (-5) + 9 = …
4. (-8) + 6 = …
5. (-15) + 25 = …
6. (-14) + 19 = …
7. (-21) + 32 = …
8. (-28) + 26 = …
9. (-22) + 17 = …
10. (- 9) + … = 1
11. (-5) + … = -2
12. … + 8 = 6

1.7.5 Assessment

1. Observe students while they work in their group and use this rubric:

No	Student's name	Calculate addition ((+) + (+) and (-) + (+)) with integers		
		Very good	Fair	Need assistance
1	Student 1			
2	Student 2			

2. Collect students' work as their portfolios

1.8 Lesson 3 (2 x 35'): Addition with integers (positive plus negative and negative plus negative)

1.8.1 Outcomes

Students will be able to

1) Calculate addition with integers
    a. Positive plus negative
    b. Negative plus negative
2) Solve the everyday life problems associated with the operation of integers

1.8.2 Prior knowledge

- Addition with positive numbers
- Positive and negative numbers

1.8.3 Preparation

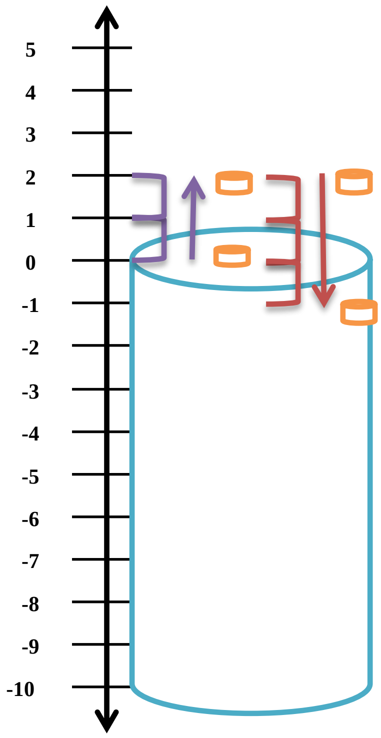
- Worksheet 3: Addition with integers (positive plus negative and negative plus negative)

1.8.4 Actual time

A. Initial activities (10')
    1) Remind students about addition with integers: positive plus positive and negative plus positive

B. Main activities (50')
    1) Discuss the homework in lesson 2. (15')
    2) Give examples about the movement of a bucket that represent positive plus negative and negative plus negative. (10')

Example 1: Positive plus negative



Cyntia pulls the bucket up two metres above the zero level then continues to lower it down three metres. What is the position of her bucket now?

i. Pulls the bucket up from zero to two.

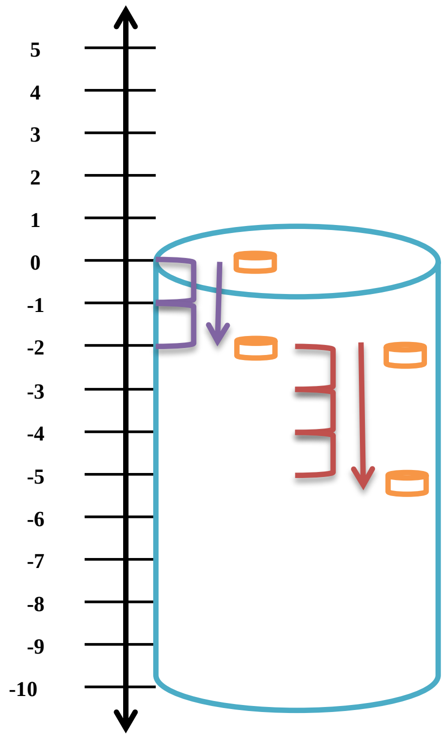
ii. Then continues to lower the bucket down as long as three steps from 2.

iii. The bucket ends on negative one.

iv. Now, the position of Cyntia's bucket is on one metre below the zero level.

v. The right symbolic representation for this problem is 2 + (-3) = -1

Example 2: Negative plus negative



Mario lowers the bucket down two metres below the zero level then continues to lower it down as far as three metres. What is the position of the bucket now?

i. Lowers the bucket down from zero to negative two.
ii. Then continues to lower the bucket down as long as three steps from -2.
iii. The bucket ends on negative five.
iv. Now, the position of Mario's bucket is on five metres below the zero level.
v. The right symbolic representation for this problem is (-2) + (-3) = -5

3) Put students in heterogeneous groups of three to discuss the problems in the worksheet 3 provided. Each group of three will have students of high, middle and low ability based on their understanding in the previous lesson. Teachers move from one to another group to help students who need assistance. (25')

C. Closing activities (10')

1) Teacher gives homework

1. 5 + (-7) = …
2. 9 + (-4) = …
3. 8 + (-8) = …
4. 15 + (-9) = …
5. 12 + (-23) = …
6. (-3) + (-5) = …
7. (-9) + (-6) = …

8. (-8) + (-8) = …
9. (-28) + (-16) = …
10. (-17) + (-35) = …
11. Write as many additions as you can between positive and negative numbers that the result is seven.
12. Write as many additions as you can between positive and negative numbers that the result is negative six.

2) Teacher gives assignment. Students may submit by the end of lesson 6.

Complete the table below

+	-8	6	-9	5	-3	12	-15	…
7	-1							28
-4								
10								
-16								
…			-7					
-25								
27								
…							-34	

1.8.5 Assessment

1. Observe students while they work in their group and use this rubric:

No	Student's name	Calculate addition ((+) + (-) and (-) + (-)) with integers		
		Very good	Fair	Need assistance
1	Student 1			
2	Student 2			

2. Collect students' work as their portfolios

1.9 Lesson 4 (2 x 35’): Subtraction with integers (positive take away positive and negative take away positive)

1.9.1 Outcomes

Students will be able to

1) Calculate subtraction with integers
    a. Positive take away positive
    b. Negative take away positive
2) Solve the everyday life problems associated with the operation of integers

1.9.2 Prior knowledge

- Subtraction with positive numbers

1.9.3 Preparation

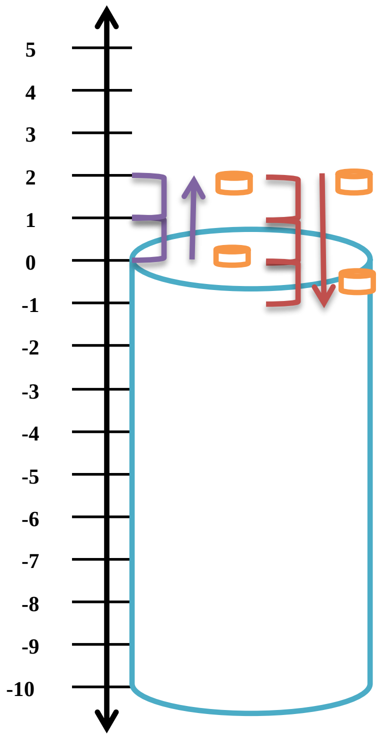
Worksheet 4: Subtraction with integers (positive take away positive and negative take away positive)

1.9.4 Actual time

A. Initial activities (10’)
    1) Discuss the homework from the previous lesson (10’).

B. Main activities (50’)
    1) Remind students about subtraction with positive numbers. (5’)
    2) Discuss one or two problem if a smaller number is subtracted by a bigger number. Give examples about the movement of a bucket that represent positive take away positive and negative take away positive. (15’)

Example 1: Positive take away positive

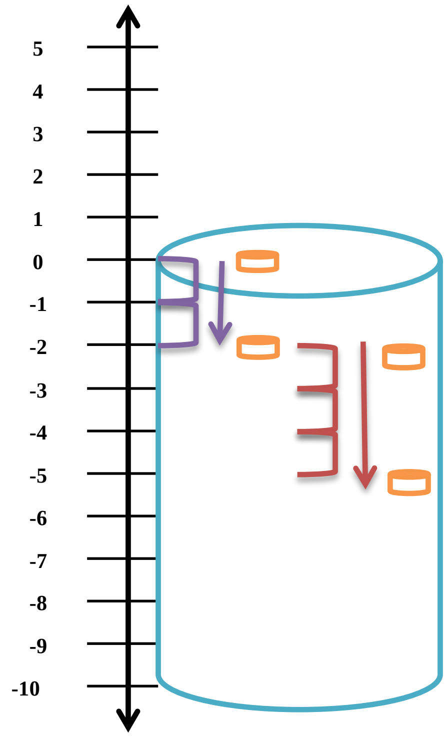


Doni pulls the bucket up two metres above the zero level but the bucket suddenly slides down for three metres. What is the position of her bucket now?

i. Pulls the bucket up from zero to two.
ii. Then the bucket slides down as long as three steps from 2.
iii. The bucket ends on negative one.
iv. Now, the position of Cicilia's bucket is on one metre below the zero level.
v. The right symbolic representation for this problem is 2 - 3 = -1
vi. Remember lesson 3 example 1: 2 + (-3) = -1

So, 2 + (-3) = 2 – 3

Example 2: Negative take away positive



Mona lowers the bucket down two metres below the zero level and the bucket suddenly continues to slide down for three metres. What is the position of the bucket now?

i. Lowers the bucket down from zero to negative two.

ii. Then the bucket slides down as long as three steps from -2.

iii. The bucket ends on negative five.

iv. Now, the position of Mona's bucket is on five metres below the zero level.

v. The right symbolic representation for this problem is $(-2) - 3 = -5$

vi. Remember lesson 3 example 2: $(-2) + (-3) = -5$

vii. So, $(-2) + (-3) = (-2) - 3$

3) Put students in heterogeneous groups of three to discuss the problems in the worksheet 4 provided. Each group of three will have students of high, middle and low ability based on their understanding in the previous lesson. Teachers move from one to another group to help students who need assistance. (30')

C. Closing activities (10')

1) Teacher gives homework

1. 25 - 7 = …
2. 9 - 14 = …
3. 8 - 28 = …
4. 15 - 29 = …
5. 23 - 38 = …
6. (-3) - 5 = …
7. (-9) - 7 = …
8. (-8) - 8 = …
9. (-28) - 16 = …
10. (-17) - 39 = …
11. ….. – 9 = 5
12. ….. – 12 = -5
13. 6 - ….. = - 10
14. (-8) - … = - 16
15. ….. – 9 = - 12

1.9.5 Assessment

1. Observe students while they work in their group and use this rubric:

No	Student's name	Calculate subtraction ((+) – (+) and (-) – (+)) with integers		
		Very good	Fair	Need assistance
1	Student 1			
2	Student 2			

2. Collect students' work as their portfolios

1.10 Lesson 5 (2 x 35’): Subtraction with integers (positive take away negative and negative take away negative)

1.10.1 Outcomes

Students will be able to

1) Calculate subtraction with integers
    a. Positive take away negative
    b. Negative take away negative
2) Solve the everyday life problems associated with the operation of integers

1.10.2 Prior knowledge

- Addition and subtraction with integers

1.10.3 Preparation

- Worksheet 5: Subtraction with integers (positive take away negative and negative take away negative)

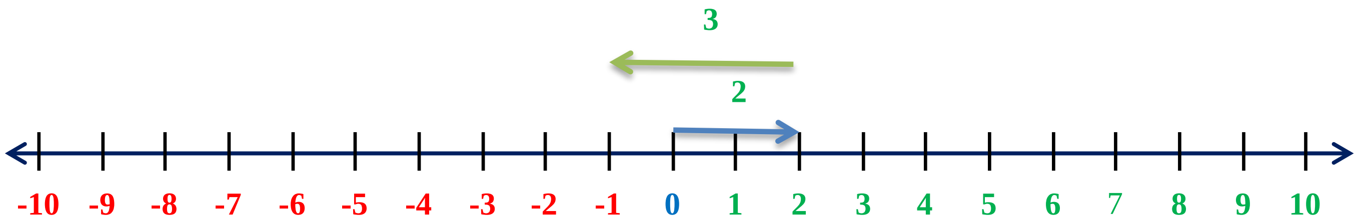
1.10.4 Actual time

A. Initial activities (10’)
    1) Discuss homework from lesson 4.

B. Main activities (50’)
    1) Discuss other subtraction (positive take away negative and negative take away negative). Give examples about positive take away negative and negative take away negative by using horizontal number line. (20’)

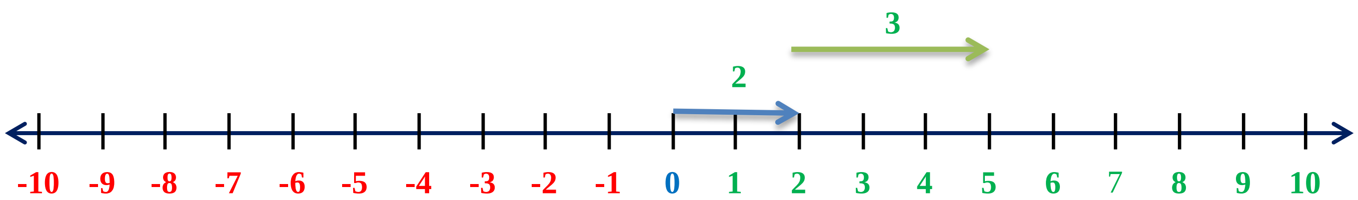
Example 1: positive take away negative

i. Remind student about positive take away positive using horizontal number line, for example 2 -3 = -1



ii. Now, 2 – (-3) = …

iii. If we do take away 3, the arrow go to the left for three steps (see i). So, if we do take away -3, the arrow go to the opposite way. It means the arrow go to the right for three steps.



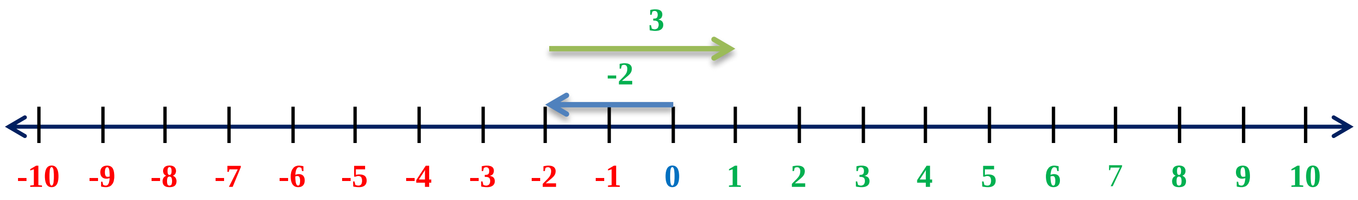
iv. 2 – (-3) = 5.

v. Remember example 1 in lesson 2: 2 + 3 = 5 and now, 2 – (-3) = 5. So, 2 – (-3) = 2 + 3

Example 2:

i. (-2) – (-3) = …

ii. The same as in example 1 above, take away negative means the arrow go to the right as long as three steps.



iii. (-2) – (-3) = 1

iv. Remember in example 2 lesson 2: (-2) + 3 = 1 and now (-2) – (-3) = 1. So, (-2) – (-3) = (-2) + 3.

2) Put students in heterogeneous groups of three to discuss the problems in the worksheet 5 provided. Each group of three will have students of high, middle and low ability based on their understanding in the previous lesson. Teachers move from one to another group to help students who need assistance. (30')

C. Closing activities (10')

1) Teacher gives homework.

1. Do the subtraction below

   1.32 – (-19) = …

   2.(-9) – (-8) = …

   3.16 – (-21) = …

   4.(-7) – (-8) = …

   5.(-21) – (-5) = …

   6.… - (- 9) = - 3

   7.(- …) – (-8) = 5

   8.(- …) - … = - 6

2. I stand on the zero point. Tina stands 3 steps to the right of me and Ansel stands 5 steps to the left of me. How many steps is the distance between them?
3. A frog stands on the zero point of a number line. Then he jumps four units to the left and then jumps six units to the right. What is the position of the frog now?
4. One day, the temperature in Ende is 35°C and in New York it is -6°C. How many degrees differences are there between those two cities?
5. One day, the temperature on the top of Merbabu Mountain at midday is -1°C and -8°C at midnight. Has the temperature decreased or increased? How much is the change?

1.10.5 Assessment

1. Observe students while they work in their group and use this rubric:

No	Student's name	Calculate subtraction ((+) – (-) and (-) – (-)) with integers		
		Very good	Fair	Need assistance
1	Student 1			
2	Student 2			

2. Collect students' work as their portfolios

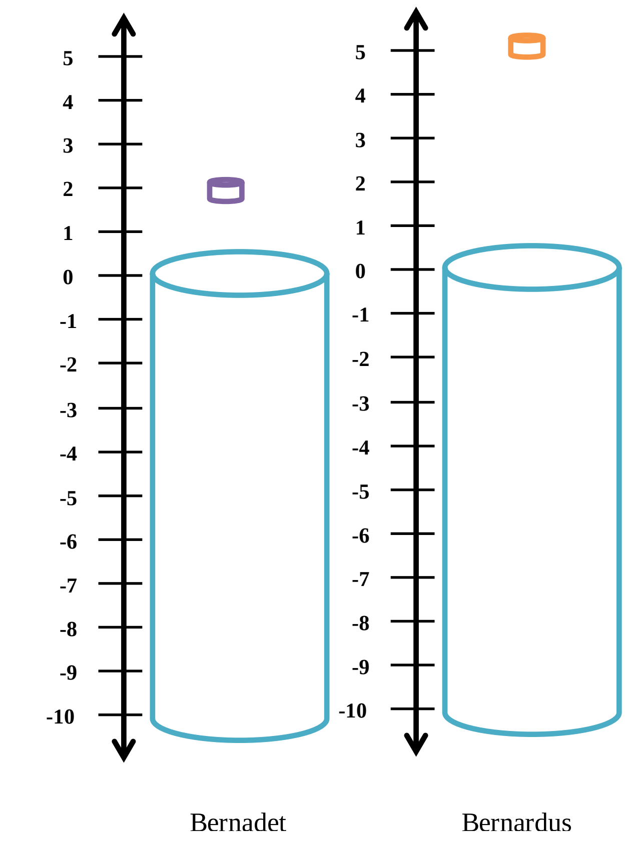
Appendix 2: Example of Worksheet 1 – Compare and Order Integers

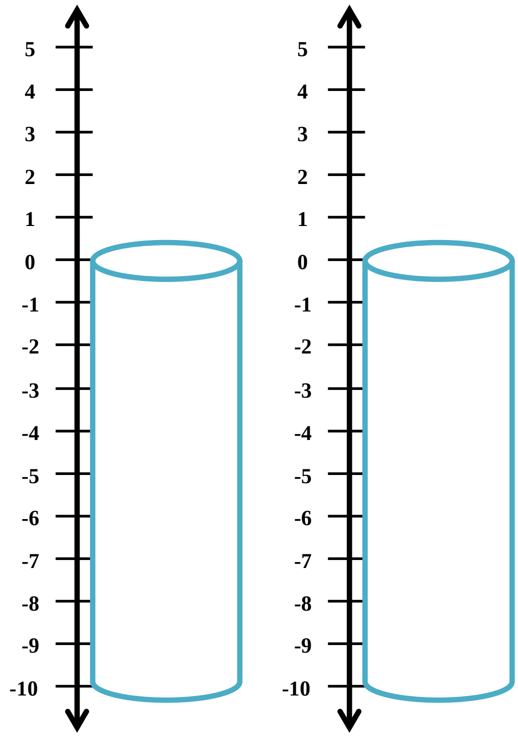
Name: ____

Part A

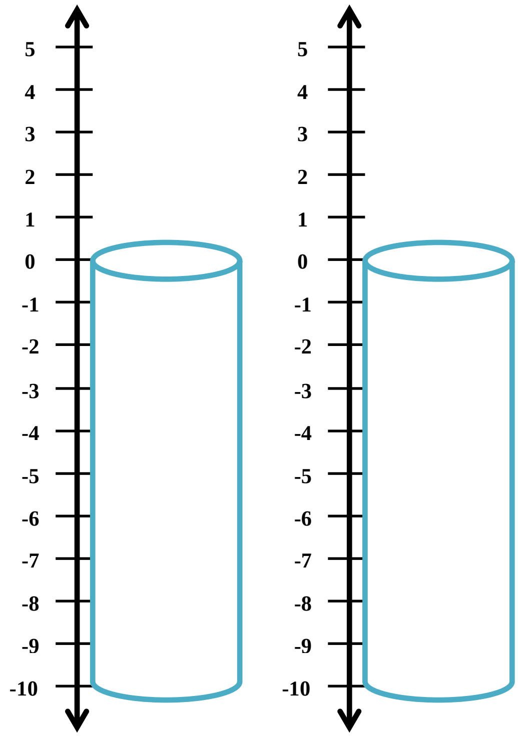
Instruction:

1. Read each question carefully.
2. Use the picture on the left side to assist you to solve the problem.

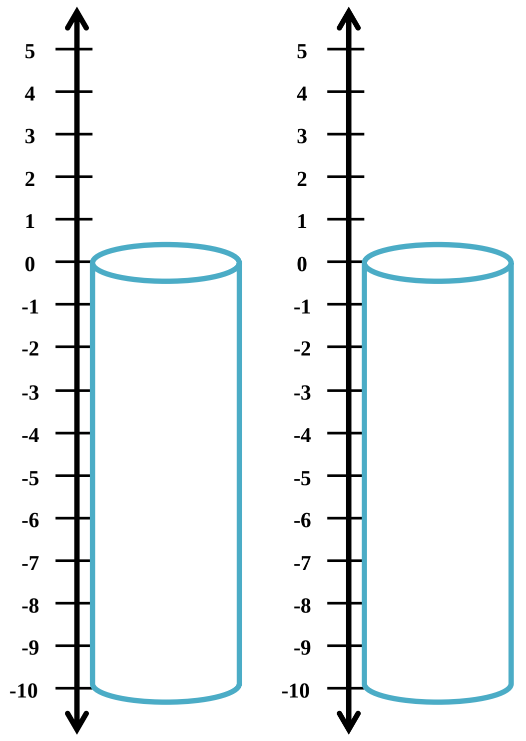
	1) Bernadet pulls her bucket up two metres above the zero level and Bernardus pulls his bucket up five metres above the zero level. Draw the final right position of each bucket. Whose bucket is higher?



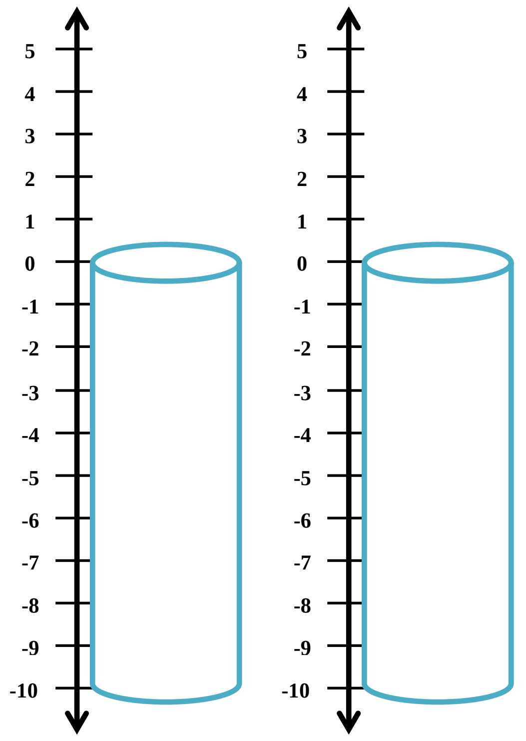
2) Anton pulls his bucket up three metres above the zero level and Ani lowers her bucket down 8 metres below the zero level. Draw the right position of each bucket. Whose bucket is higher?



3) Cintia lowers her bucket down eight metres below the zero level while Martin pulls his bucket up five metres above the zero level. Draw the right position of each bucket. Whose bucket is higher and whose bucket is lower?



4) Maya lowers her bucket down three metres while Martin lowers his bucket down nine metres below the zero level. Draw the right position of each bucket. Whose bucket is higher and whose bucket is lower?



5) Ester lowers her bucket down five metres below the zero level while Edmon pulls his bucket up five metres above the zero level. Draw the right position of each bucket. Whose bucket is higher and whose bucket is lower? What is the distance between their bucket?

Part B

5) Please fill the dots below with “<” or “>”:

   i. 3 … 8
   ii. 9 … 5
   iii. -2 … 3
   iv. 8 … -9
   v. -10 … 6
   vi. 7 … -1
   vii. 0 … -3
   viii. 4 … 0

6) Sort the following numbers from the smallest to the biggest correctly: 8, -5, 1, -7, 4, -3, 0, -2, 6, and -9.
7) Sort the following numbers from the biggest to the smallest correctly: 2, -6, -1, 5, -8, 7, -4, 9, 3, and 0.

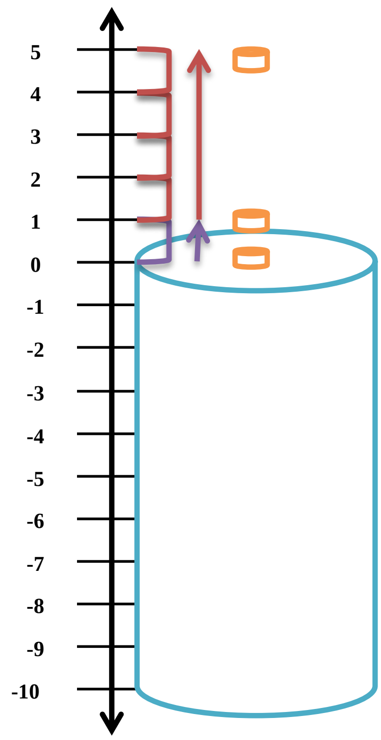
Appendix 3: Example of Worksheet 2 – Addition with integers number (positive plus positive and negative plus positive)

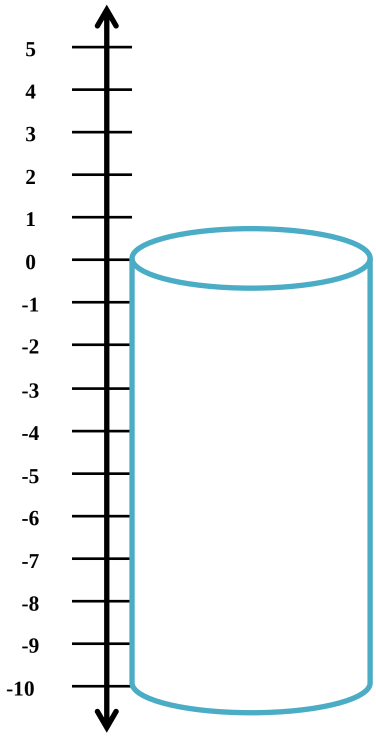
Name : ____

Part A

Instruction:

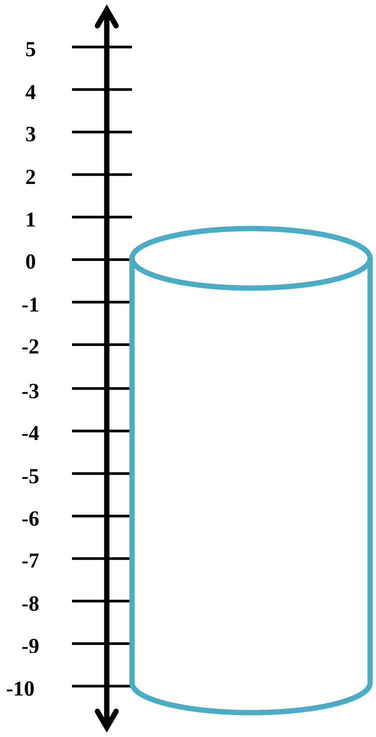
1. Read each question carefully.
2. Use the picture on the left side to assist you to solve the problem.

	1) Febi pulls the bucket up one metre above the zero level then continues to pull up four metres. a. Draw the movement of the bucket. b. What is the position of Febi's bucket now? c. The right symbolic representation for this problem is ….. + ….. = …..



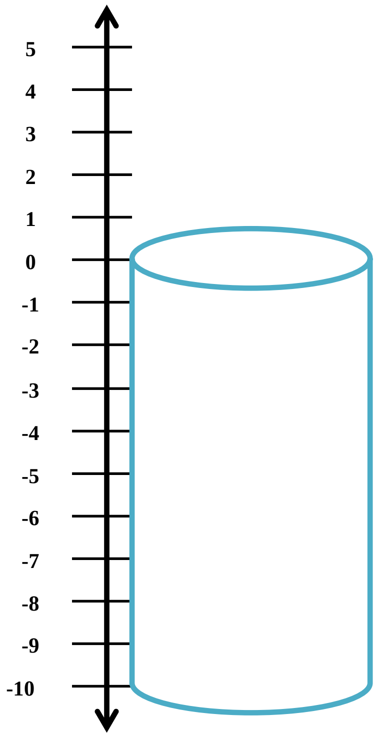
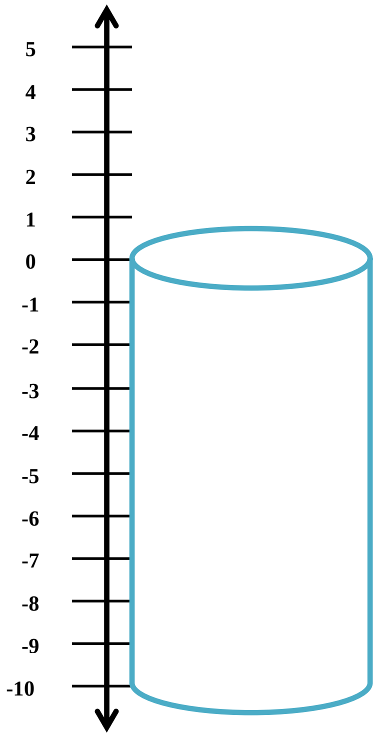
2) Ester pulls the bucket up three metres above the zero level then continues to pull it up one metre.
   a. Draw the movement of the bucket.
   b. What is the position of the bucket now?
   c. The right symbolic representation for this problem is

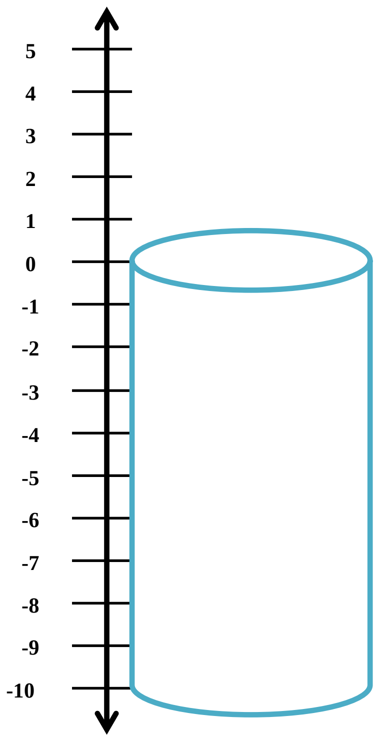
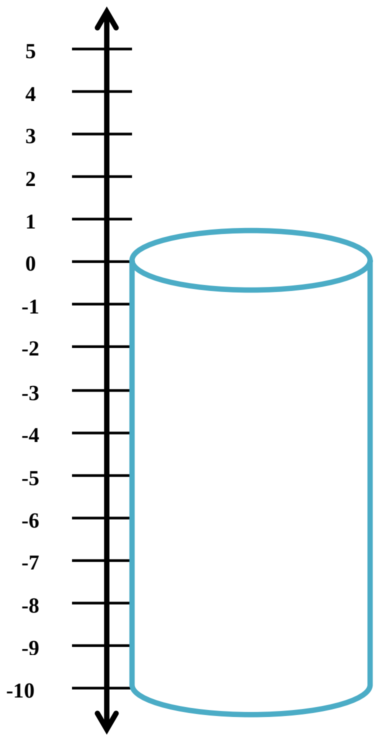
      ….. + ….. = …..



3) Andi pulls the bucket up two metres above the zero level then continues to pull it up two metres.
   a. Draw the movement of the bucket.
   b. What is the position of the bucket now?
   c. The right symbolic representation for this problem is

      ….. + ….. = …..

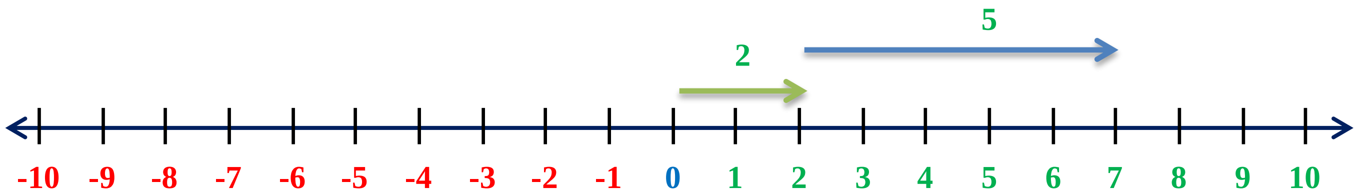
	4) Martin lowers the bucket down five metres below the zero level then continues to pulls it up as far as eight metres. a. Draw the movement of the bucket. b. What is the position of the bucket now? c. The right symbolic representation for this problem is (- …..) + ….. = …..
	5) Hendra lowers the bucket down seven metres below the zero level and then pulls it up as far as nine metres. a. Draw the movement of the bucket. b. What is the position of his bucket now? c. The right symbolic representation for this problem is ….. + ….. = …..

	6) Juni lowers her bucket down six metres below the zero level then pulls her bucket up as long as six metres. a. Draw the movement of the bucket. b. What is the position of her bucket now? c. The right symbolic representation for this problem is ….. + ….. = …..
	7) Sandro lowers his bucket down ten metres below the zero level then pulls it up three metres. a. Draw the movement of the bucket. b. What is the position of his bucket now? c. The right symbolic representation for this problem is ….. + ….. = …..

Part B

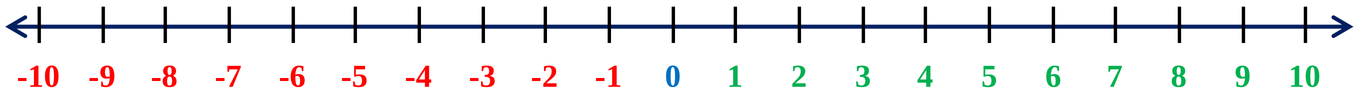
Use horizontal number line to solve the addition below.

Example 1:

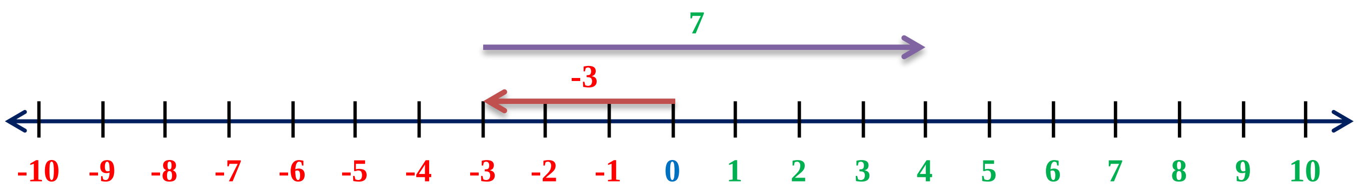


+ 5 = …

1) 3 + 7 = …

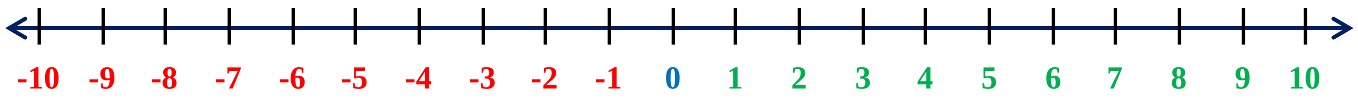


Example 2:

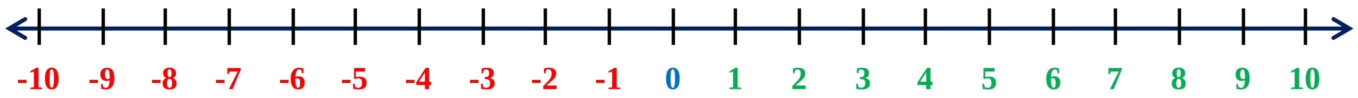


(-3) + 7 = …

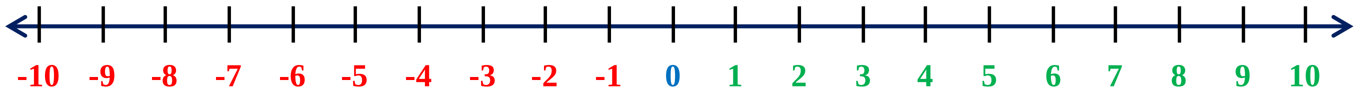
2) (-5) + 8 = …



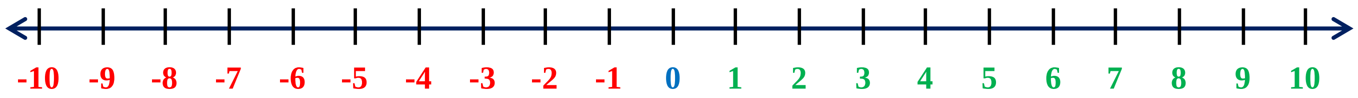
3) (-8) + 3 = …



4) (-7) + (7) = …



5) (-9) + 0 = …



Part C

1) 9 + 7 = …
2) … + 8 = 12
3) (-2) + 9 = …
4) (-8) + 15 = …
5) (-9) + 6 = …
6) (-17) + 9 = …
7) (-8) + 8 = …
8) (-1) + … = 8
9) (-9) + … = -5
10) … + … = -3

Part D (not compulsory, only for the group who have finished part C)

1) Fill the dots below with the right numbers. The number on the first row is the result of addition between the numbers in the first and second column.

8	
5	3
…..	7
…..	9
-3	…..
-…..	…..

-5	
-5	0
…..	7
-9	…..
…..	12
-…..	…..

2) Make one addition table like number one above, and then ask your pair to complete your table. After finish, check the result.

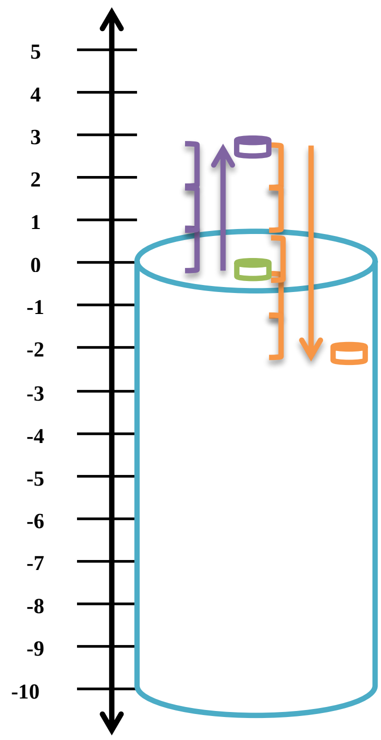
Appendix 4: Example of Worksheet 3 – Addition with integers (positive plus negative and negative plus negative)

Name: ______________________________________

Part A

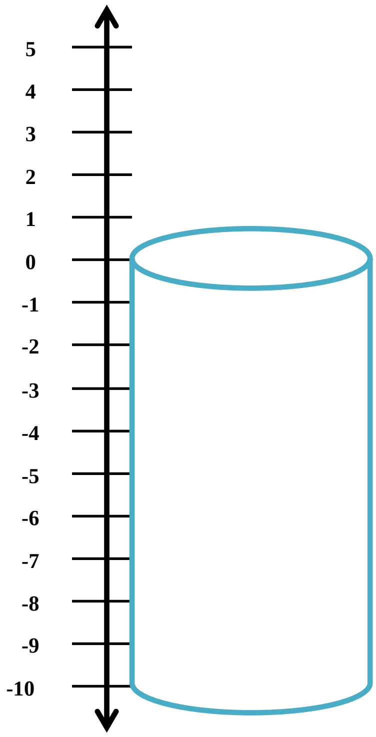
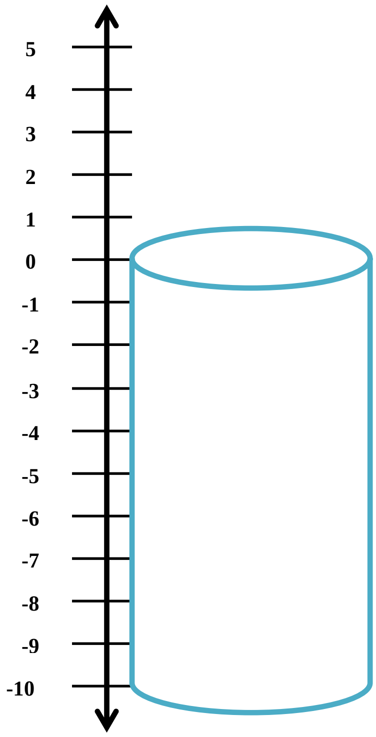
Instruction:

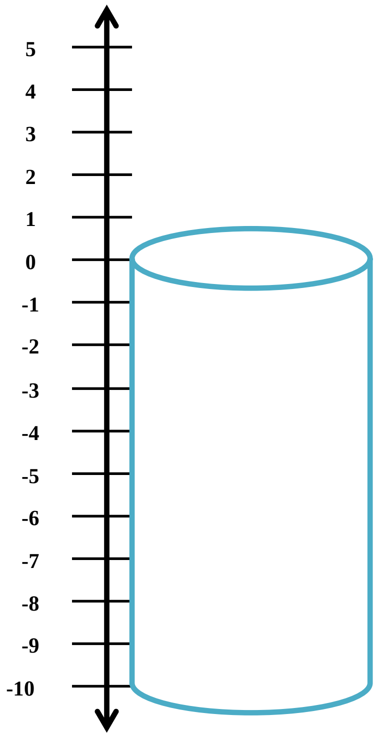
1. Read each question carefully.
2. Use the picture on the left side to assist you to solve the problem.



1) Febi pulls the bucket up three metres above the zero level then to lower it down five metres.
   a. Draw the movement of the bucket.
   b. What is the position of Febi's bucket now?
   c. The right symbolic representation for this problem is:

      ….. + (- …..) = …..

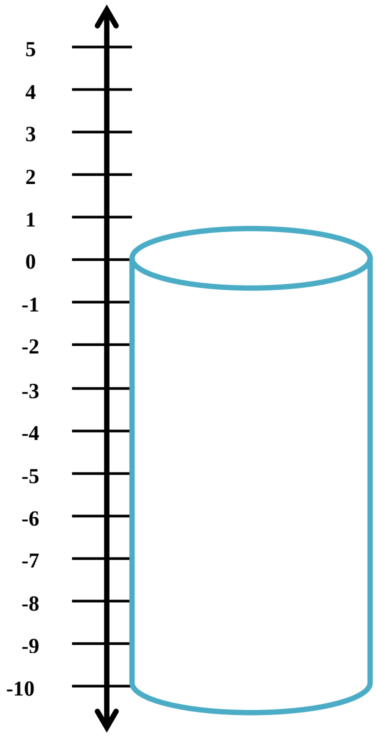
	2) Ester pulls the bucket up two metres above the zero level then continues to lower it down seven metres. a. Draw the movement of the bucket. b. What is the position of the bucket now? c. The right symbolic representation for this problem is: ….. + (- …..) = …..
	3) Andi pulls his bucket five metres above the zero level and then lowers it down three metres. a. Draw the movement of the bucket. b. What is the position of the bucket now? c. The right symbolic representation for this problem is: ….. + ….. = …..



4) Martin pulls the bucket up five metres above the zero level then lowers it down five metres.

   a. Draw the movement of the bucket.
   b. What is the position of the bucket now?
   c. The right symbolic representation for this problem is:

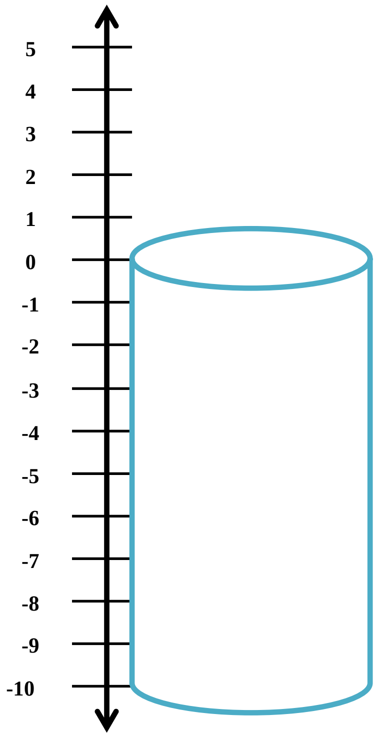
      ….. + ….. = …..



5) Hendra pulls the bucket up five metres above the zero level then he lowers it down ten metres.

   a. Draw the movement of the bucket.
   b. What is the position of the bucket now?
   c. The right symbolic representation for this problem is:

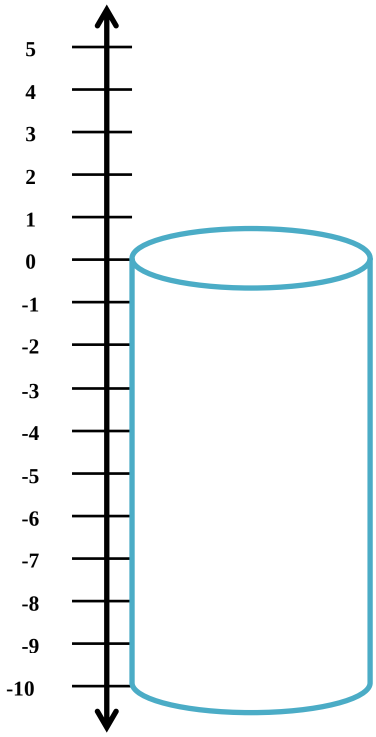
      ….. + ….. = …..



6) Juni lowers her bucket down five metres below the zero level then continues to lower it down three metres.

   a. Draw the movement of the bucket.
   b. What is the position of her bucket now?
   c. The right symbolic representation for this problem is:

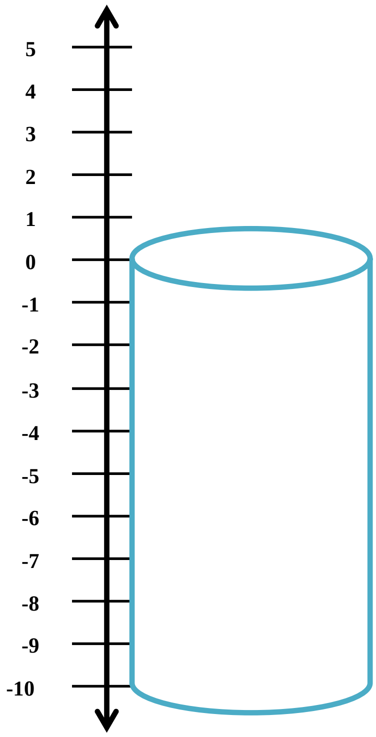
      (- …..) + (- …..) = …..

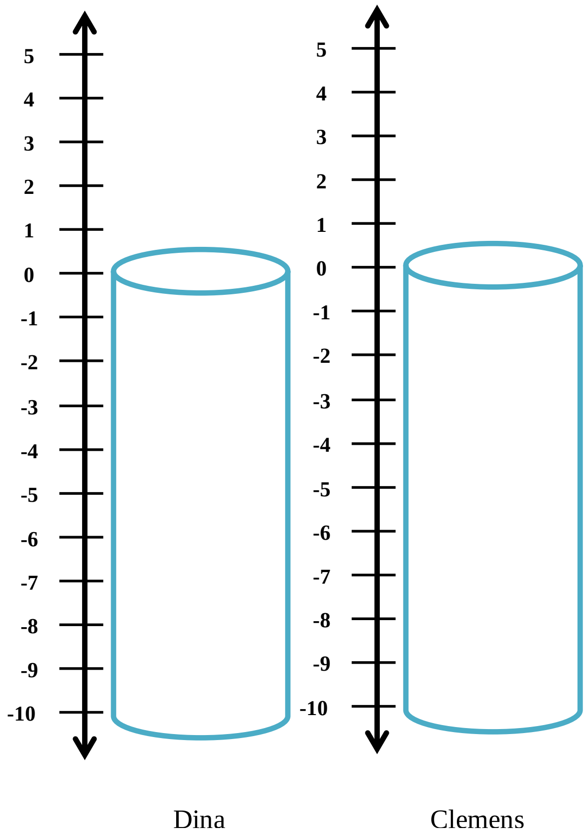


7) Sandro lowers his bucket down two metres below the zero level then his bucket slides down again another seven metres.

   a. Draw the movement of the bucket.
   b. What is the position of his bucket now?
   c. The right symbolic representation for this problem is:

      (- …..) + (- …..) = …..

	8) Wati lowers her bucket down four metres below the zero level then she lowers it down four metres. a. Draw the movement of the bucket. b. What is the position of her bucket now? c. The right symbolic representation for this problem is: ….. + ….. = …..



9) Dina pulls her bucket up three metres above the zero level then lowers it down eight metres while Clemens lowers his bucket down two metres below the zero level and then lowers it down another eight metres.

a. Draw the movement of their buckets.

b. The right symbolic representation for Dina's bucket is:

….. + …. = …..

c. The right symbolic representation for Clemens' bucket is:

….. + ….. = …..

d. Whose bucket is deepest?

e. The right symbolic representation for this problem is:

….. + ….. < / > ….. + …..

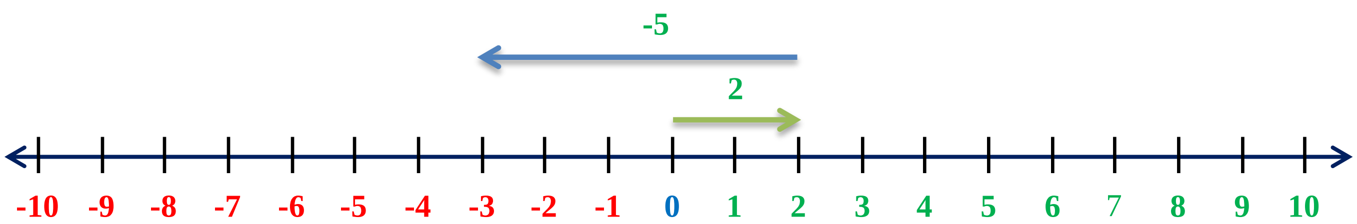
….. < / > ….

Part B

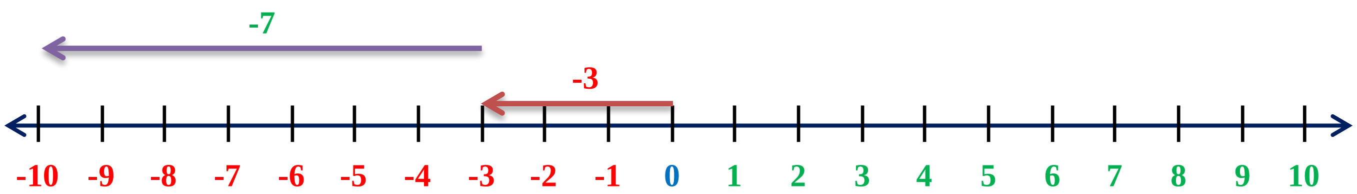
Use horizontal number line to solve the addition below.

Example:

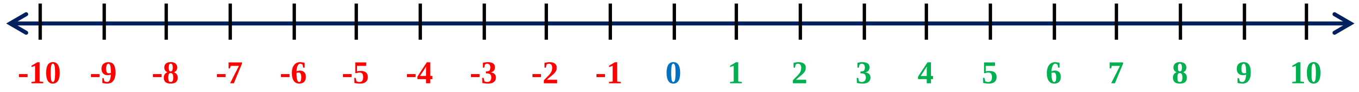
2 + (-5) = …



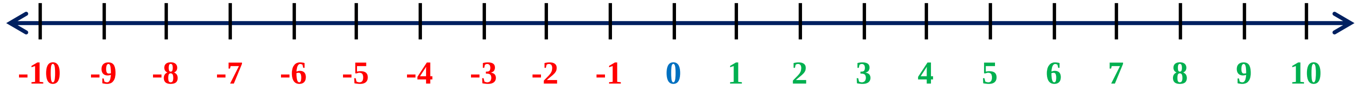
(-3) + (-7) = …



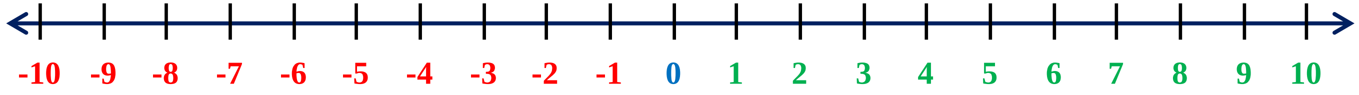
6) 3 + (-7) = …


-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

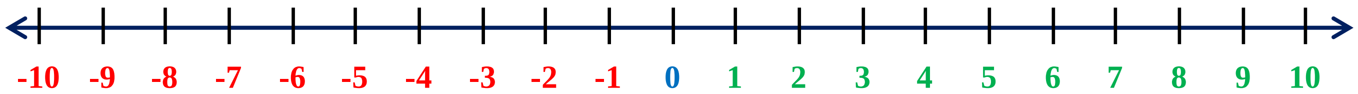
7) 10 + (-8) = …


-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

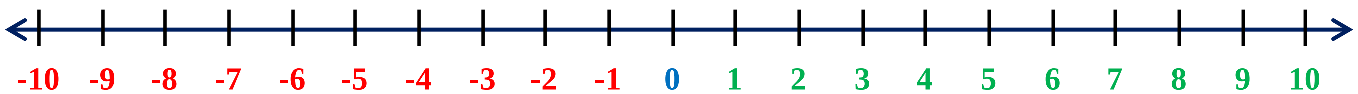
8) (-8) + (-2) = …


-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

9) (-3) + (-3) = …



10) (-1) + (-6) = …



Part C

1) 7 + (-2) = …
2) 17 + (-9) = …
3) 8 + (-10) = …
4) 6 + (-12) = …
5) (-2) + (-6) = …
6) (-12) + (-29) = …
7) (-9) + (-2) = …
8) (-14) + (-10) = …
9) … + (-5) = -7
10) (-3) + … = -12

Part D (not compulsory, only for the group who have finished part C)

1) Fill the dots below with the right numbers. The number on the first row is the result of addition between the numbers in the first and second column.

8	
10	-2
…..	-4
15	…..
26	…..
…..	- …..

-9	
-6	-3
…..	-7
-9	…..
…..	-12
…..	-…..

2) Make one addition table like number one above, and then ask your pair to complete your table. After finish, check the result.

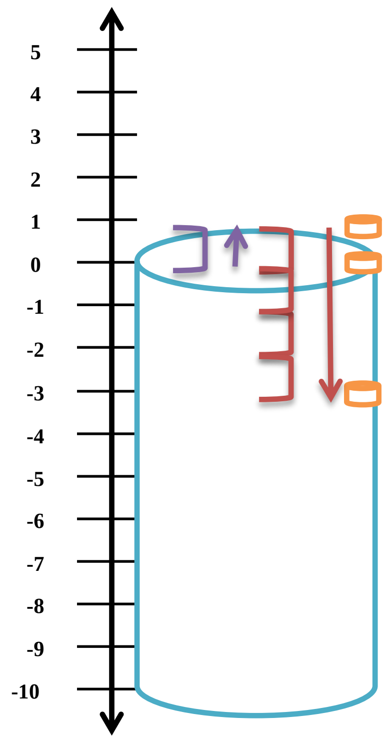
Appendix 5: Example of Worksheet 4: Subtraction with integers (positive take away positive and negative take away positive)

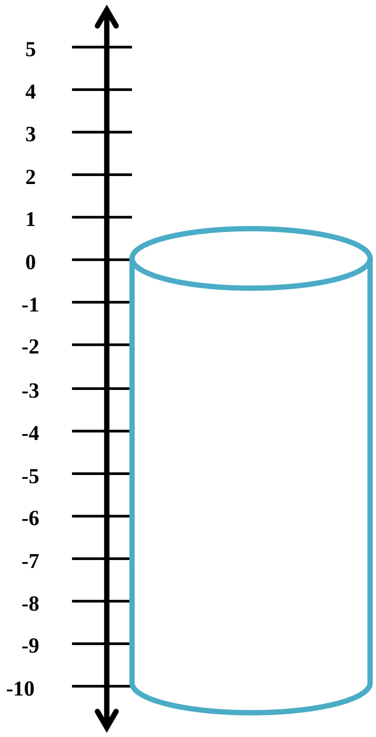
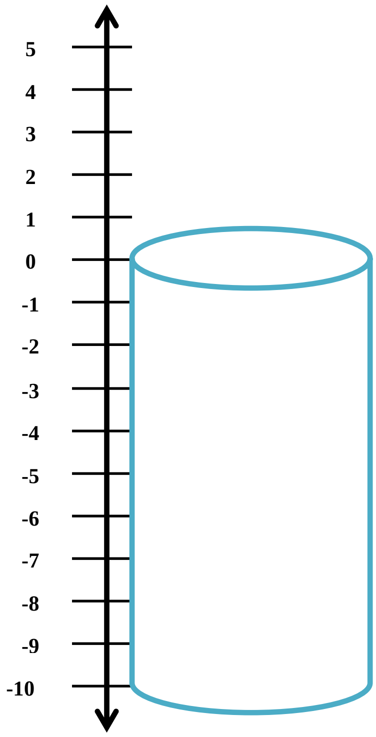
Name: ____

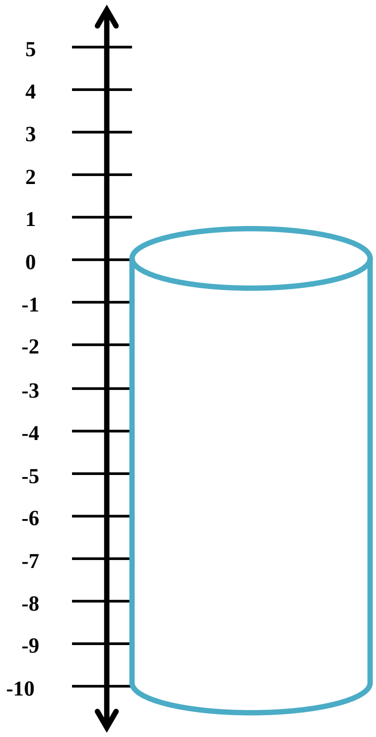
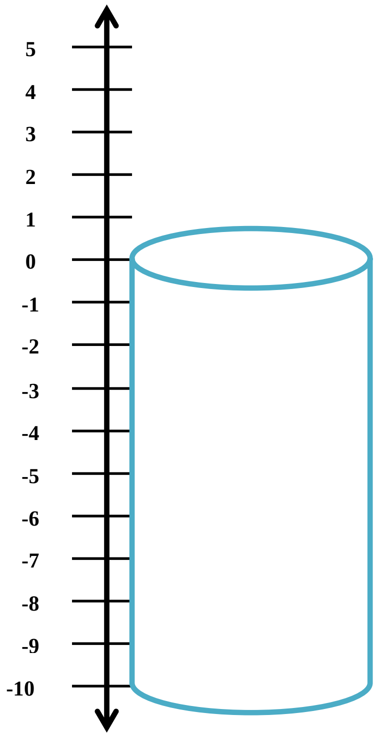
Part A

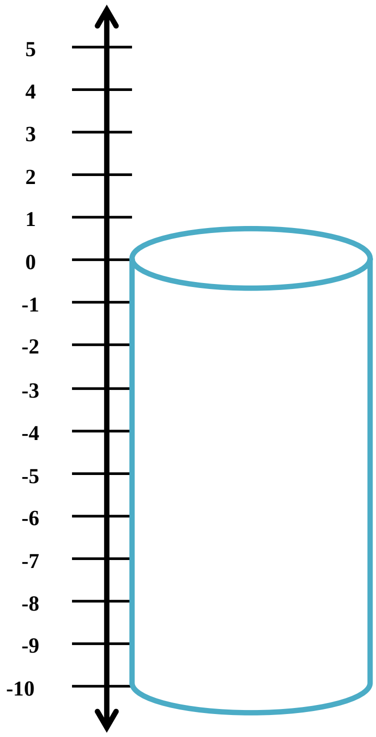
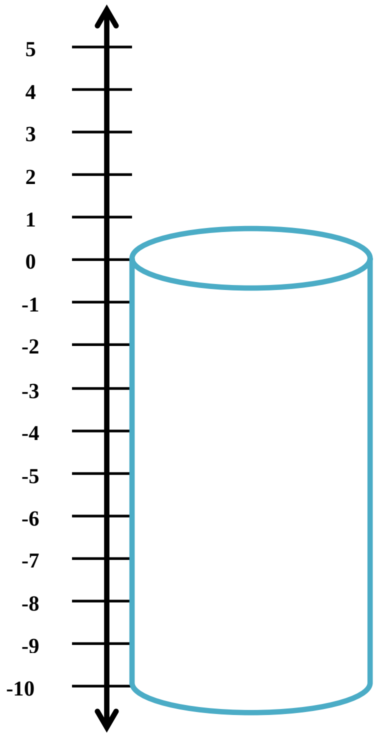
Instruction:

1. Read each question carefully.
2. Use the picture on the left side to assist you to solve the problem.

	1) Lani pulls the bucket up one metre above the zero level but the bucket suddenly slides down for four metres. a. Draw the movement of the bucket. b. What is the position of Lani's bucket now? c. The right symbolic representation for this problem is 1 - 4 = …..

	2) Martin pulls the bucket up five metres above the zero level but the bucket suddenly slides down for four metres. a. Draw the movement of the bucket. b. What is the position of the bucket now? c. The right symbolic representation for this problem is ….. - ….. = …..
	3) Andi pulls the bucket up two metres above the zero level but the bucket suddenly slides down for eight metres. a. Draw the movement of the bucket. b. What is the position of the bucket now? c. The right symbolic representation for this problem is ….. - ….. = …..

<table>
<tr>
<td>
</td>
<td>4) Carlos pulls the bucket up three metres above the zero level but the bucket suddenly slides down for seven metres.
a. Draw the movement of the bucket.
b. What is the position of the bucket now?
c. The right symbolic representation for this problem is
….. - ….. = …..</td>
</tr>
<tr>
<td>
</td>
<td>5) Hendra lowers the bucket down one metre below the zero level and the bucket suddenly continues to slide down for four metres.
a. Draw the movement of the bucket.
b. What is the position of his bucket now?
c. The right symbolic representation for this problem is
(-…..) - ….. = …..</td>
</tr>
</table>

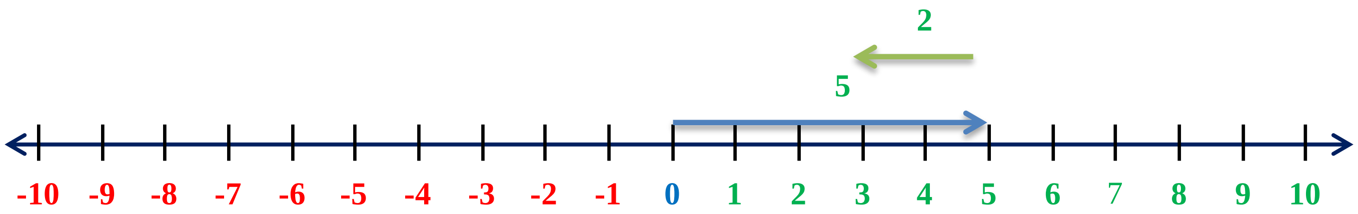
	6) Juni lowers her bucket down three metres below the zero level and the bucket suddenly continues to slide down for six metres. a. Draw the movement of the bucket. b. What is the position of her bucket now? c. The right symbolic representation for this problem is ….. - ….. = …..
	7) Sandro lowers his bucket down eight metres below the zero level and the bucket suddenly continues to slide down for two metres. a. Draw the movement of the bucket. b. What is the position of his bucket now? c. The right symbolic representation for this problem is ….. - ….. = …..

Part B

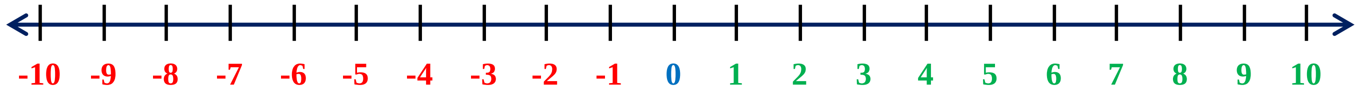
Use horizontal number line to solve the subtraction below.

Example 1:

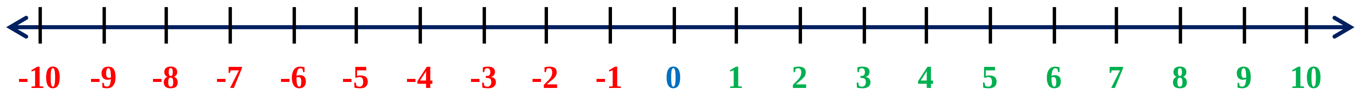
5 - 2 = …



1) 8 - 3 = …

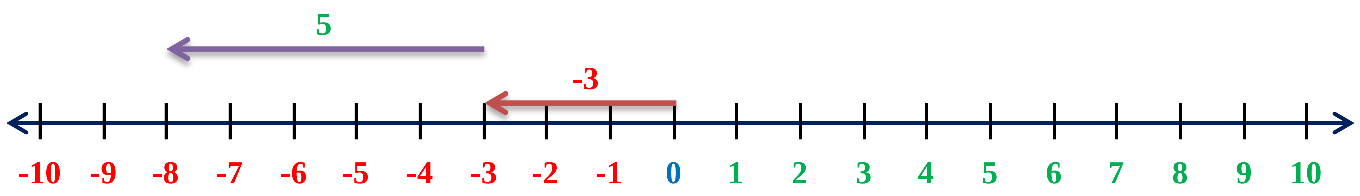


2) 5 - 9 = …

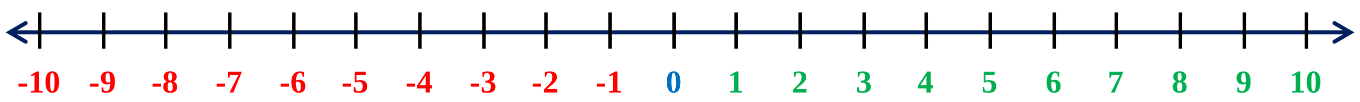


Example 2:

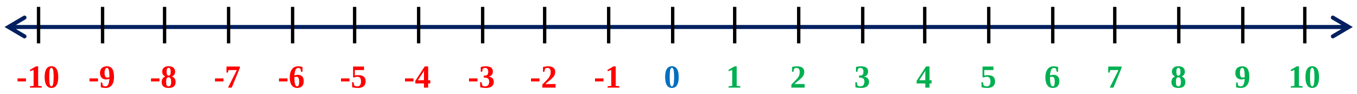
(-3) - 5 = …



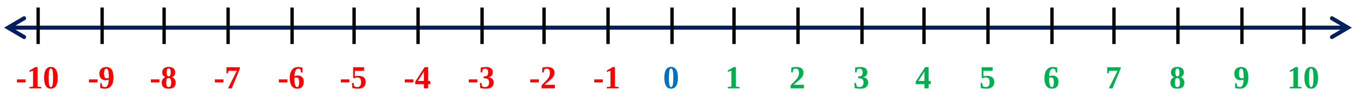
3) (-2) - 7 = …



4) (-7) - (1) = …



5) (-9) - 0 = …



Part C

Calculate each subtraction below:

1) 15 – 8 = …
2) 8 – 9 = …
3) 2 – 10 = …
4) 0 – 5 = …
5) 12 – 15 = …
6) 23 – 35 = …
7) (-3) – 5 = …
8) (-5) – 7 = …
9) (-10) – 2 = …
10) (-17) – 38 = …

Part D (not compulsory, only for the group who have finished part C)

1) Fill the dots below with the right numbers. The number on the first row is the result of subtraction between the numbers in the first and second column.

8	
10	2
…..	4
17	…..
28	…..
…..	…..

-9	
-6	3
…..	7
-9	…..
…..	12
-…..	…..

2) Make one subtraction table like number one above, and then ask your pair to complete your table. After finish, check the result.

Appendix 6: Example of Worksheet 5: Subtraction with integer numbers (positive take away negative and negative take away negative)

Name: ____

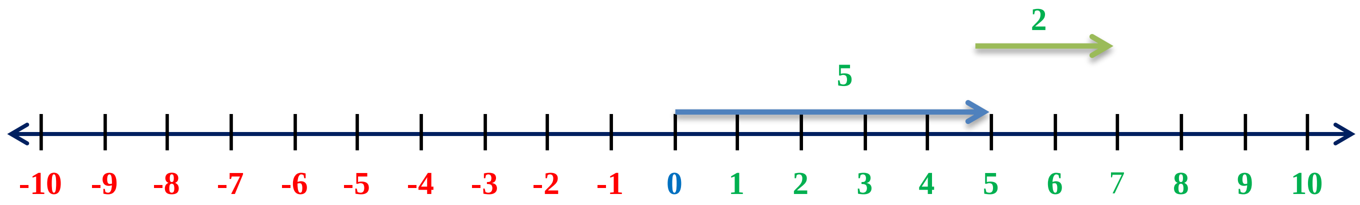
Part A

Instruction:

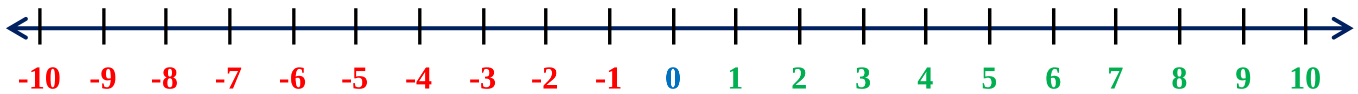
1. Read each question carefully.
2. Use horizontal number line to solve the subtraction below.

Example 1:

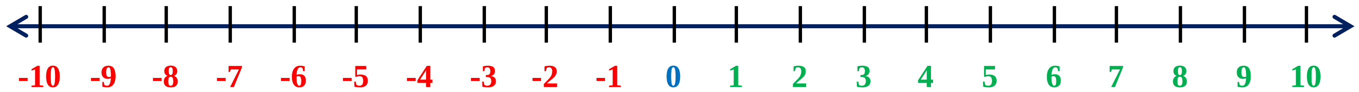
5 – (-2) = …



1) 7 – (-3) = …

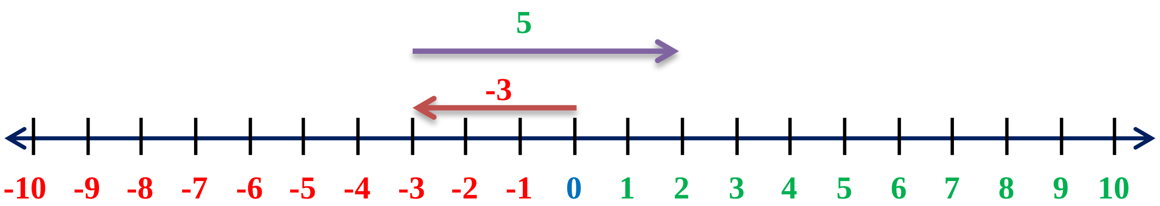


2) 3 – (-6) = …

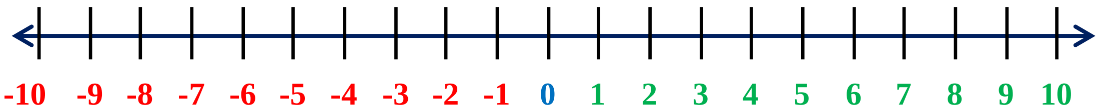


Example 2:

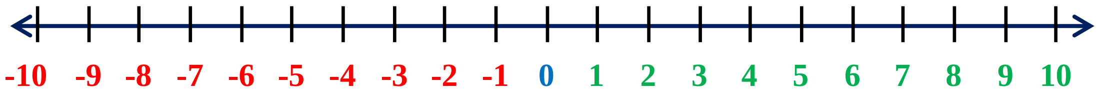
(-3) – (-5) = …



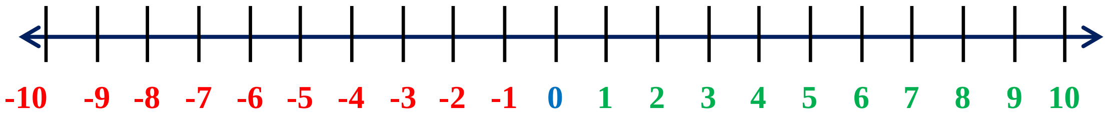
3) (-2) – (-7) = …



4) (-7) - (-1) = …



5) (-9) – (-9) = …



Part B

Calculate each subtraction below:

1) 5 – (-8) = …
2) 8 – (-9) = …
3) 12 – (-10) = …
4) 0 – (-15) = …
5) (-2) – (-10) = …
6) (-11) – (-15) = …
7) (-23) – (-5) = …
8) (-35) – (-17) = …
9) (-10) – (-22) = …
10) (-27) – (-18) = …

Part C (not compulsory, only for the group who have finished part B)

1) Fill the dots below with the right numbers. The number on the first row is the result of subtraction between the numbers in the first and second column.

8	
6	-2
…..	-4
-7	…..
	-9
-…..	- …..

-9	
-10	-1
-12	…..
-9	…..
…..	-12
-…..	-…..

2) Make one subtraction table like number one above, and then ask your pair to complete your table. After finish, check the result.

Appendix 7: Lesson Study: Class observation form

Teaching and Learning Process – Mathematics Differentiated Instruction in Lesson Study

Observation Form

Date of observation lesson	: ………………	Teacher	: …………………………...
Class	: ……………….	Observer	: …………………………...
Start time	: ……………….	Finish time	: ……………………………
Topic	…………………………………………………………..........		
Goals of the lesson	……………………………………………………………….. ………………………………………………………………… ………………………………………………………………… ………………………………………………………………… …………………………………………………………………		
Classroom	………………………………………………………………..		

arrangement (desks and chairs setting)	……………………………………………………………………. …………………………………………………………………….

Time	Teacher	Students	Other Notes

Resources used	…………………………………………………………………………
IT used	…………………………………………………………………………
Teaching aids used	…………………………………………………………………………

Observer, …………………………………

Appendix 8: Lesson Study: Class observation guidelines

1. Observer sits on the back of the classroom or another space which is suitable to observe the teaching and learning process.
2. Do not interfere the teaching and learning process for example talk to the teacher or students, walking during the lesson, or block students' view.
3. Observer may also make a note on the lesson plan to help them at the review session in Lesson Study Group Meeting.
4. Observer need to complete the observation form during the lesson. Note the teacher practice and students' response during the lesson.
    a. Does the teacher teach as in the lesson plan?
    b. Which part(s) of differentiated instruction is(are) used in this lesson? How does it work? What are the responses of the students?
        i. Content
            1. Various level of content
            2. Relevant to students' daily lives
        ii. Process
            1. Flexible grouping
            2. Various activities
        iii. Product
            1. Various level of task difficulties
            2. Flexible submission
        iv. Assessment
            1. Ongoing
            2. Provide rubric
        v. Learning environment
            1. Some learning centre in the classroom
            2. Teachers value each student
    c. Note teacher's question and students' answer and vice versa.
    d. Note students' behaviour.

Appendix 9: Lesson Plan A1 (School A-Indonesian version)

RENCANA PELAKSANAAN PEMBELAJARAN.
(RPP).

Mata pelajaran : Matematika
Kelas / Semester : IV / I.
Alokasi waktu : 2 x 35 menit
Topik : Sudut.

Kompetensi inti

Memahami pengertian faktual dengan cara mengamati dan menanya berdasarkan rasa ingin tahu tentang dirinya, makluk ciptaan Tuhan dan kegiatannya, dan benda - benda yg dijumpainya di rumah, di sekolah, dan tempat bermain.

Kompetensi Dasar

3.12 Mengenal sudut siku - siku, melalui pengamatan dan membandingkannya dengan sudut yg berbeda

4.13. Mempresentasikan sudut lancip dan sudut tumpul dalam bangun datar.

I. Indikator

1. Membedakan jenis sudut lancip, tumpul, dan siku - siku.
2. Mengukur besar sudut dengan menggunakan busur
3. Mendeskripsikan bentuk - bentuk sudut.

II. Tujuan pembelajaran.

1. Siswa dapat membedakan jenis - jenis sudut (siku - siku, lancip dan tumpul).
2. Siswa dapat mengukur besar sudut dengan menggunakan busur
3. Siswa dapat mendeskripsikan bentuk - bentuk sudut.

III. Materi pokok

Sudut.

VI Strategis : Pembedaan pembelajaran matematika :
- Bervariasi berdasarkan kemampuan, ketertarikan, dan kesiapan siswa.
- Sesuai kehidupan sehari - hari
- variasi kegiatan, mendorong berpikir kreatif dan kritis
- Fleksibel dalam pengelompokan
- Fleksibel mengenai waktu mengumpulkan tugas
- Menyediakan rubrik penilaian.

V Model Pembelajaran.

VII Langkah - langkah pembelajaran.
- kegiatan awal. (10 menit)
A. Apersepsi
- Siswa diminta ~~dintuk~~ untuk mengamati benda - benda yang ada dalam kelas.
B. Informasi materi
- Guru menyampaikan materi yg akan diajarkan.

- Kegiatan Inti (50 menit)
- Siswa diminta menjelaskan pengertian tentang sudut.
- Guru memaparkan jenis - jenis sudut (gambar jenis - jenis sudut).
- Guru memberikan pertanyaan - pertanyaan penuntun.
- Siswa dibagi dalam beberapa kelompok
- Siswa diminta untuk menggambar sebuah sudut sesuai ukuran yg ditentukan.
- Guru memperbaiki cara mengukur sudut dgn menggunakan busur yang tepat.
- Siswa dapat membedakan jenis - jenis sudut.
- Siswa dapat mengelompokan jenis - jenis sudut.

- Kegiatan Akhir (10 menit)
  - Kesimpulan akhir dari guru.
  - Tugas rumah. (PR).

  Soal.
  1. Gambarlah sudut siku-siku, lancip, dan sudut tumpul.
  2. Gambarlah sebuah sudut dengan besarnya 120°.

VII Sumber/alat belajar.
- Kurikulum 2013
  Buku Guru dan buku murid.

VIII Penilaian
LKS (Terlampir).

IX Penilaian yang berkesinambungan: Pengamatan dan portofolio. (Terlampir).

(2)

VISION

Appendix 10: Lesson Plan A2 (School A-Indonesian version)

Date

Rencana Pelaksanaan Pembelajaran II

Nama Sekolah : SDI Madambake.
Mata Pelajaran : Matematika.
Kelas / Semester : IV / I.
Topik : Sudut.

I. Kompetensi Inti :
Memahami pengetahuan faktual dengan cara mengamati dan menanya berdasarkan rasa ingin tahu tentang dirinya mahluk ciptaan Tuhan dan kegiatannya, dan benda-benda yang dijumpainya di rumah, di sekolah dan tempat bermain.

II. Kompetensi Dasar :
- Mempresentasikan sudut lancip dan sudut tumpul dalam bangun datar.

III. Indikator : Mendesain rumah sekolah dengan memperhatikan sudut lancip, tumpul dan siku-siku.
: Menentukan sudut lancip, tumpul dan siku-siku dalam sebuah bangun datar.

IV. Tujuan pembelajaran :
1. Siswa dapat menentukan jenis-jenis sudut yang terdapat pada rumah sekolah.
2. Siswa dapat menentukan sudut lancip, tumpul, siku-siku dalam sebuah bangun datar.

V. Materi : Sudut.
Strategi : Pembedaan Pembelajaran matematika :
1. Sesuai kehidupan harian siswa.
2. Variasi kegiatan, mendorong berpikir kreatif.
3. Fleksibel dlm mengelompokkan.
4. Fleksibel mengenai waktu.

VI. Langkah - langkah Pembelajaran.
⇒ Kegiatan Awal :
1. Guru dan murid menyelesaikan tugas rumah.
2. Menyampaikan Materi yang akan diajarkan.
⇒ Kegiatan Inti :
1. Guru membagi siswa dalam kelompok.
2. Siswa diminta untuk keluar kelas guna mengamati gedung-gedung ada di sekitarnya.

(KIKY)

3. Setelah mengamati siswa di minta untuk menggambar bentuk gedung yg diamati.
4. Siswa menentukan sudut - sudut yang ada dalam gambar.
5. Guru menjelaskan tentang hasil pengamatan.
6. Guru membagikan LKS.
7. Setiap kelompok menyelesaikan tugas yang diberikan siswa.
8. Guru membimbing siswa dalam menyelesaikan tugas kelompok.
9. Siswa mempresentasikan hasil diskusi.
10. Siswa lain menanggapi.

⇒ Kegiatan Akhir.
1. Kesimpulan Guru
2. Pemberian Tugas Rumah.

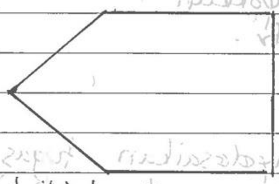
VII Sumber belajar :
- Kurikulum
- Buku murid dan Buku guru. K.13
- Buku matematika Erlangga kls IV A.

VIII Media :
- Rumah, Gedung sekolah.

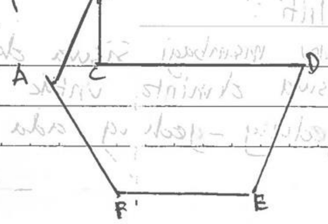
IX. Alat :
- Bangun datar.

* Penilaian :
1. LKS (Terlampir).
2. Tes Tertulis.

Soal :
I. Jawablah pertanyaan di bawah ini!
1. Tentukan banyak sudut siku² pada gambar di bawah ini!



2. Sebutlah nama - nama sudut yg ada pada gambar di bawah ini!



Date

3. Gambarlah sebuah bangun datar yang terdiri dari 2 sudut tumpul dan 2 sudut lancip.

Appendix 11: Lesson Plan B1 (School B-Indonesian version)

Date

Rencana Pelaksanaan Pembelajaran

Sekolah : SDK Nangaroro
Mata Pelajaran : Matematika
Kelas / Semester : IV/1

A. Standar Kompetensi :
Memahami dan menggunakan sifat-sifat operasi hitung bilangan dalam pemecahan masalah.

B. Kompetensi Dasar :
Mengidentifikasi sifat-sifat operasi hitung.

C. Tujuan Pembelajaran
Peserta didik dapat :
1. Mengenal sifat pertukaran /komutatif.
2. Menyelesaikan soal sesuai sifat pertukaran atau komutatif.

D. Materi Ajar :
Sifat pertukaran atau komutatif.

E. Metode Pembelajaran :
1. Penemuan
2. Tanya jawab
3. Latihan
4. Tugas.

F Langkah-langkah Pembelajaran :
A. Kegiatan Awal / Apersepsi (5 menit).
1. Membilang bilangan dari 1 - 10 sambil membanting kaki dan mengayunkan tangan (bilangan ganjil tangan lurus kebawah sedangkan bilangan genap kedua tangan kedepan untuk 2 kali perhitungan).
2. Informasi Awal tentang materi yang akan diajarkan.

B. Kegiatan inti (55 menit).
1. Guru menuliskan soal penjumlahan dan perkalian dipapan tulis (10 menit)
2. Anak menyelesaikan soal yang ditulis guru di papan tulis (10 menit).

Date

3. Bertanya jawab tentang hal-hal yang belum di ketahui anak.
4. Secara individu anak menyelesaikan soal yang diberikan guru dengan tingkat kesulitan yang berbeda (20 menit).

Soal

Selesaikan

1. $5 + 7 = \boxed{} + 5$
2. $3 \times 6 = 6 \times \boxed{}$
3. $\boxed{} \times 5 = 5 \times 2$
4. $12 + \boxed{} = 10 + 12$
5. $14 + 27 = \boxed{} + 14$
6. $15 + 68 = 68 + \boxed{}$
7. $43 + \boxed{} = 69 + 43$
8. $345 + \boxed{} = 220 + 345$
9. $\boxed{} + 195 = 195 + 210$
10. $8 \times 10 = \boxed{} \times 8$

5. Memeriksa dan memberi nilai (10 menit).

C. Kegiatan Penutup:
1. Memberikan peneguhan tentang materi yang barusan di terapkan.
2. Menyanyikan lagu disini senang secara bersama dengan gerak.
3. Memberikan soal-soal PR.

Soal

Selesaikanlah.

1. $12 \times 10 = \boxed{} \times 12$
2. $9 \times 7 = 7 \times \boxed{}$
3. $20 \times 35 = 35 \times \boxed{}$
4. $\boxed{} + 345 = 345 + 212$
5. $200 + 300 = \boxed{} + 200$
6. $8 \times 15 = 15 \times \boxed{}$
7. $782 + \boxed{} = 321 + 782$
8. $537 + 277 = 277 + \boxed{}$
9. $13 \times 9 = \boxed{} \times 13$
10. $610 + \boxed{} = 59 + 610$

KIKY

Date

G. Alat dan Bahan Ajar :
-. Buku Kurikulum
-. Buku Matematika Kelas IV

H. Penilaian
a. Proses
Lembaran Penilaian terlampir.
b. Tertulis
a. Tugas
b. PR.

Mengetahui
Kepala Sekolah

= Maldetnidis Eju =
Nip 19620210 198607 2 001

Nangaroro, 06-08-2014
Guru Matematika Kelas IV

= Rosalia Ito =
Nip: 19570919 197803 2013

KIKY

Appendix 12: Lesson Plan B2 (School B-Indonesian version)

Date

Rencana Pelaksanaan Pembelajaran

Sekolah : SDK Nangaroro
Mata Pelajaran : Matematika
Kelas / Semester : IV/1

A. Standar Kompetensi :
Memahami dan menggunakan sifat-sifat operasi hitung bilangan dalam pemecahan masalah.

B. Kompetensi Dasar :
Mengidentifikasi sifat-sifat operasi hitung.

C. Tujuan Pembelajaran :
Peserta didik dapat :
1. Menjelaskan sifat pengelompokan / Asosiatif
2. Menuliskan soal sesuai dengan sifat pengelompokan atau Asosiatif.

D. Materi Ajar :
Sifat pengelompokan atau Asosiatif.

E. Metode Pembelajaran :
1. Tanya jawab
2. Latihan
3. Diskusi
4. Tugas

F. Langkah-Langkah Pembelajaran
A. Kegiatan Awal (5 menit)
- Tanya jawab tentang tugas pengurangan dan pembagian serta periksa tulisan dari dua anak.
- Kumpul PR
- Nyanyi lagu Aku senang Aku bahagia.
- Informasi materi yang akan disampaikan.

B. Kegiatan inti : (55 menit)
- Memberi penjelasan tentang pengerjaan hitung dengan sifat pengelompokan atau Asosiatif. (15 menit)
Contoh : 1. (3+5)+2 = ...+2 = ...
3 + (5+2) = 3 + ... = ...

KIKY

Date Rumusnya : a + (b + c) = (a + b) + c

Jadi, (3 + 5) + ... = 3 + (... + 2)

2. 4 + (3 + 5) | (4 + 3) + 5
= 4 + ... | = ... + 5
= .... | = ...

Jadi, 4 + (... + ...) = (... + ...) + 5

Rumusnya: a × (b × c) = (a × b) × c

3. 2 × (4 × 5) | (2 × 4) × 5
= 2 × .... | = ... + 5
= ... | = ...

Jadi, 2 × (... + ...) = (2 + ...) × 5

- Anak menulis jawaban terhadap soal yang ditulis di papan tulis.

Contoh : (2 + 3) + 1 | 2 + (3 + 1)
= ... + 1 | = 2 + ...
= ... | = ...

Jadi, (2 + ...) + 1 = (2 + 3) + ...

- Anak dibagi 4 kelompok untuk menyelesaikan LKS.

Soal

1. (44 + 334) + 66 | 44 + (334 + 66)
= ... + 66 | = 44 + ...
= ... | = ...

Jadi, (44 + ...) + 66 = ... + (334 + 66).

2. 121 + (112 + 122) | (121 + 112) + 122
= 121 + ... | = ... + 122
= ... | = ...

Jadi, (121 + 112) + ... = 121 + (... + 122)

3. 8 × (10 × 5) | (8 × 10) × 5
= 8 × .... | = ... × 5
= ... | = ...

Jadi, 8 × (... × 5) = (8 × 10) × ...

4. (5 × 9) × 4 | 5 × (9 × 4)
= ... × 4 | = 5 × ...
= ... | = ...

Jadi, (... × 9) × 4 = 5 × (... × 4)

5. 4 × (8 × 2) | (4 × 8) × 2
= 4 × ... | = ... × 2
= ... | = ...

Jadi, 4 × (8 × ...) = ... × (8 × 2).

Date

-. Guru memeriksa dan menilai Lembaran kerja Siswa.

C. Kegiatan Penutup (10 menit)

1. Memberikan peneguhan tentang materi yang sudah di sampaikan.
2. Memberikan Soal PR.

Soal.

1. (1.521 + 2.755) + 4.325 = 1.521 + (2.755 + ...)
2. (2.145 + 3.256) + 7.189 = 2.145 + (3.256 + ...)
3. (5.190 + 3.700) + 6.792 = 5.190 + (... + 6.792)
4. (6.051 + 2.162) + 4.001 = ... + (2.162 + 4.001)
5. (9.275 + 4.320) + 6.526 = 9.275 + (... + 6.526)
6. (59 x 20) x 32 = 59 x (20 x ...)
7. (45 x 21) x 48 = 45 x (21 x ...)
8. (35 x 43) x 72 = 35 x (... x 72)
9. (15 x 12) x 7 = 15 x (... x ...)
10. (142 x 26) x 5 = 142 x (... x ...)

G. Alat dan Sumber bahan :

- Kurikulum
- Buku Matematika jilid IV
- LKS

H. Penilaian :

Proses

Lembaran penilaian proses terlampir dibelakang.

Mengetahui
Kepala Sekolah

= Waldetrudis Eju =
NIP 19620210 1986 07 2001

Nangaroro, 07-08-2014
Guru Matematika Kelas IV

= Rosalia Ito =
NIP: 19570919 197803 2013

Appendix 13a: Approval Request Letter for Policy Maker in Nagekeo District (English version)

APPROVAL REQUEST TO CONDUCT RESEARCH IN NAGEKEO DISTRICT

The Head of Department of Education Office

Nagekeo District

East Nusa Tenggara Province

INDONESIA

Dear Sir/Madam,

My name is Mariana Harsono. I am currently completing research for my Doctor of Philosophy (PhD) in the School of Education at Curtin University, Perth, Western Australia.

I have selected Nagekeo District for possible involvement in conducting my research. My research is entitled “The impact of Lesson Study on primary school teachers’ knowledge and skills in differentiating primary school mathematics instruction: A digital mixed methods approach”. The purpose of the research is to investigate the impact of Lesson Study, a well-regarded teacher professional development program, on teachers’ knowledge and skills in differentiating primary school mathematics instruction. Furthermore, the aims are to identify the key characteristics of Lesson Study as a form of professional development, to describe the processes involved in the implementation of Lesson Study, to investigate the influence of it on Indonesian primary school teachers’ mathematics pedagogy and on teachers’ curriculum differentiation in mathematics. To achieve this purpose and the aims of the research, I wish to collect data from selected teachers through observations, interviews, documents, video recording, and voice recording.

Therefore, I am seeking your permission to allow teachers in your district to participate in the research. Each participant will be asked to provide written consent before any data collection commences. They will also be informed that their participation is voluntary, that they may withdraw from the study at any time, and that they can decline to answer specific questions. All data collected will be treated with confidentiality and the anonymity of those participating will be protected at all times.

If you approve of this research study I would be grateful. Please do not hesitate to contact me if you have any questions and/or if you require further information.

Yours faithfully,

Mariana Harsono

Curtin University Postgraduate Student

Mobile: +61 424377188

Email address: mariana.harsono@postgrad.curtin.edu.au

Appendix 13b: Approval request Letter for Policy Maker in Nagekeo District (Indonesian version)

Perihal: Permohonan melakukan penelitian di Kabupaten Nagekeo

Kepada yang terhormat,

Kepala Dinas Pendidikan

Kabupaten Nagekeo

Propinsi Nusa Tenggara Timur

Indonesia

Dengan hormat,

Nama saya Mariana Harsono. Saat ini saya sedang menempuh pendidikan dan melakukan penelitian untuk meraih gelar Doctor of Philosophy (Ph. D.) di School of Education, Curtin University, Perth, Australia Barat.

Saya memilih Kabupaten Nagekeo sebagai tempat untuk melakukan penelitian saya. Penelitian saya berjudul "Dampak *Lesson Study* pada pengetahuan dan keterampilan guru sekolah dasar dalam membedakan instruksi pembelajaran matematika: Pendekatan metode campuran digital". Maksud dari penelitian ini untuk menyelidiki dampak dari *Lesson Study*, yang merupakan salah satu bentuk dari pengembangan professional, pada pengetahuan, dan keterampilan guru dalam membedakan pembelajaran matematika di sekolah dasar. Selanjutnya, tujuan dari penelitian ini adalah untuk mengidentifikasi ciri-ciri dari *Lesson Study* sebagai suatu bentuk dari peningkatan professional guru, untuk menjelaskan proses dalam mengimplementasikan *Lesson Study*, serta untuk menyelidiki pengaruh *Lesson Study* terhadap pedagogi matematika guru dan pembedaan pembelajaran matematika. Untuk mencapai maksud dan tujuan tersebut, saya akan mengumpulkan data melalui observasi, wawancara, dokumen, rekaman video dan suara.

Untuk itu, saya mohon ijin agar beberapa guru di Kabupaten Nagekeo dapat berpartisipasi dalam penelitian ini. Setiap peserta akan diminta untuk memberikan persetujuan tertulis sebalum pengumpulan data dilakukan. Mereka juga akan diberitahukan bahwa keterlibatan ini adalah sukarela, mereka dapat menarik diri kapanpun dan diperkenankan untuk tidak menjawab pertanyaan. Semua data yang dikumpulkan akan disimpan secara rahasia dan tanpa nama.

Sebelumnya, saya menyampaikan rasa terima kasih yang dalam atas perkenan untuk melakukan penelitian di Kabupaten Nagekeo. Saya dapat dihubungi melalui telepon atau email jika ada pertanyaan sehubungan dengan penelitian ini.

Hormat saya,

Mariana Harsono

Mahasiswa Doktoral Curtin University

Nomor HP: +61 424377188

Alamat email: mariana.harsono@postgrad.curtin.edu.au

Appendix 14a: Approval Request Letter for School Principals (English version)

The Principal

__________________ Primary School

Nagekeo District

East Nusa Tenggara Province

INDONESIA

Dear Sir/Madam,

My name is Mariana Harsono. I am currently completing research for my Doctor of Philosophy (Ph. D.) in the School of Education at Curtin University, Perth, Western Australia.

I have selected Nagekeo District for possible involvement in conducting my research. My research is entitled “The impact of Lesson Study on primary school teachers’ knowledge and skills in differentiating primary school mathematics instruction: A digital mixed methods approach”. The purpose of the research is to investigate the impact of Lesson Study, a well-regarded teacher professional development program, on teachers’ knowledge and skills in differentiating primary school mathematics instruction. Furthermore, the aims are to identify the key characteristics of Lesson Study as a form of professional development, to describe the processes involved in the implementation of Lesson Study, to investigate the influence of it on Indonesian primary school teachers’ mathematics pedagogy and on teachers’ curriculum differentiation in mathematics. To achieve this purpose and the aims of the research, I wish to collect data from selected teachers through observations, interviews, documents, video recording, and voice recording.

Therefore, I am seeking your permission to allow teachers in your school to participate in the research. Each participant will be asked to provide written consent before any data collection commences. They will also be informed that their participation is voluntary, that they may withdraw from the study at any time, and that they can decline to answer specific questions. All data collected will be treated with confidentiality and the anonymity of those participating will be protected at all times.

If you approve of this research study I would be grateful. Please do not hesitate to contact me if you have any questions and/or if you require further information.

Yours faithfully,

Mariana Harsono

Curtin University Postgraduate Student

Mobile : +61 424377188

Email address: mariana.harsono@postgrad.curtin.edu.au

Appendix 14b: Approval Request Letter for School Principals (Indonesian version)

Perihal: Permohonan melakukan penelitian di Sekolah Dasar …

Kepada yang terhormat,

Kepala Sekolah Dasar ……….

Kabupaten Nagekeo

Propinsi Nusa Tenggara Timur

Indonesia

Dengan hormat,

Nama saya Mariana Harsono. Saat ini saya sedang menempuh pendidikan dan melakukan penelitian untuk meraih gelar Doctor of Philosophy (PhD) di School of Education, Curtin University, Perth, Australia Barat.

Saya memilih Sekolah Dasar … di Kabupaten Nagekeo sebagai tempat untuk melakukan penelitian saya. Penelitian saya berjudul "Dampak *Lesson Study* pada pengetahuan dan keterampilan guru sekolah dasar dalam membedakan instruksi pembelajaran matematika: Pendekatan metode campuran digital". Maksud dari penelitian ini untuk menyelidiki dampak dari *Lesson Study*, yang merupakan salah satu bentuk dari pengembangan professional, pada pengetahuan, pemahaman, keahlian dan penghargaan guru dalam membedakan pembelajaran matematika di sekolah dasar. Selanjutnya, tujuan dari penelitian ini adalah untuk mengidentifikasi ciri-ciri dari *Lesson Study* sebagai suatu bentuk dari peningkatan professional guru, untuk menjelaskan proses dalam mengimplementasikan *Lesson Study,* serta untuk menyelidiki pengaruh *Lesson Study* terhadap pedagogi matematika guru dan pembedaan pembelajaran matematika. Untuk mencapai maksud dan tujuan tersebut, saya akan mengumpulkan data melalui observasi, wawancara, dokumen, rekaman video dan suara.

Untuk itu, saya mohon ijin agar beberapa guru di Sekolah Dasar … dapat berpartisipasi dalam penelitian ini. Setiap peserta akan diminta untuk memberikan persetujuan tertulis sebalum pengumpulan data dilakukan. Mereka juga akan diberitahukan bahwa keterlibatan ini adalah sukarela, mereka dapat menarik diri kapanpun dan diperkenankan untuk tidak menjawab pertanyaan. Semua data yang dikumpulkan akan disimpan secara rahasia dan tanpa nama.

Sebelumnya, saya menyampaikan rasa terima kasih yang dalam atas perkenan untuk melakukan penelitian di Sekolah Dasar …, Kabupaten Nagekeo. Saya dapat dihubungi melaui telepon atau email jika ada pertanyaan sehubungan dengan penelitian ini.

Hormat saya,

Mariana Harsono

Mahasiswa Doktoral Curtin University

HP: +61 424377188

E-mail: mariana.harsono@postgrad.curtin.edu.au

Appendix 15a: Participant Information Sheet (English version)

School of Education
Curtin University

The impact of Lesson Study on primary school teachers' knowledge and skills in differentiating primary school mathematics instruction: A digital mixed methods approach

Dear Teacher,

My name is Mariana Harsono. While I am a lecturer in Teacher Training and Education Faculty at Flores University I am currently on leave to undertake my Doctor of Philosophy (Ph.D.) in the School of Education at Curtin University, Perth, Western Australia.

1.1 The purpose and the aims of the research

The purpose of the research is to investigate the impact of Lesson Study on teachers' knowledge and skills in differentiating primary school mathematics instruction. Furthermore, the aims are (i) to identify the key characteristics of Lesson Study as a form of professional development, (ii) to describe the processes involved in its implementation, and (iii) to investigate its influence on Indonesian primary school teachers' mathematics pedagogy and curriculum differentiation in mathematics. To achieve the purpose and the aims of the research, data will be collected through observations, interviews, documents, video recording, and voice recording from teachers in two selected primary schools.

1.2 Benefits of participating in the study

The benefits in participating in this study are: (i) you will enhance your understanding about differentiated mathematics instruction; (ii) you will be empowered to initiate, plan and manage your own professional learning through Lesson Study; (iii) you will be more able to recognise students' needs more effectively; (iv) you will receive on-going expert support during this period to assist you to use the Lesson Study program.

1.3 Your role

If you choose to participate, you will be involved in a Lesson Study group within your school. This group will meet regularly over one semester to design, implement, observe, reflect and refine lessons based on mathematics differentiation. During this time you might volunteer to teach your own class using the lesson plan designed by the group, or you could be an observer, observing the volunteer teacher as part of the Lesson Study process. Moreover, you will be required to write a journal entry every week about the process you are undertaking and send it to the researcher by email or facsimile.

1.4 Participant requirements

Participants will need to spend one hour per week participating in Lesson Study Group Meetings and maybe another hour per week, if the participant is chosen as an observer to participate in observing their colleague's teaching. You will also need to spend an approximately a half hour per week writing up your journal entry.

1.5 Confidentiality and security

Participation in this research is purely voluntary, and participants may refuse to participate at any time without consequence or prejudice. All personal information and details gathered during the study will be utilised anonymously. Only the researcher will know the real names of the participants in this study. All data collected will be stored in a lockable cabinet in Dr Kay O'Halloran's office, Curtin University and all electronic data will be backed up.

1.6 Contact details

If you have any further questions or concerns about this study, please contact me directly or my supervisors whose details follow:

Name: Mariana Harsono
Mobile: +61 424 377 188
E-mail: mariana.harsono@postgrad.curtin.edu.au

Supervisors

Name:	Dr Kay O'Halloran	Name : Dr Rachel Sheffield
Phone:	+61 8 92662182	Phone: +61 8 92662163
E-mail:	kay.ohalloran@curtin.edu.au	E-mail: rachel.sheffield@curtin.edu.au

Secretary of the Human Research Ethics Committee

Phone: +61 8 92669223
E-mail: hrec@curtin.edu.au
Mail address: C/-Office of Research and Development, Curtin University of Technology, GPO Box U1987, Perth 6845

If you decide to take part in this study, you should keep this information sheet sign the accompanying consent form and return it to me.

Thank you very much for considering participating in this study. Your involvement will be greatly appreciated.

Sincerely,

Mariana Harsono

Curtin University Postgraduate Student

Student ID 15738273

Appendix 15b: Participant Information Sheet (Indonesian version)

School of Education
Curtin University

Dampak Lesson Study pada pengetahuan dan keterampilan guru sekolah dasar dalam membedakan instruksi pembelajaran matematika: Pendekatan metode campuran digital

Yang terhormat Guru peserta penelitian,

Nama saya Mariana Harsono. Saya seorang dosen di Fakultas Keguruan dan Ilmu Pendidikan, Universitas Flores dan saat ini saya sedang menempuh pendidikan dan melakukan penelitian untuk meraih gelar Doctor of Philosophy (Ph. D.) di School of Education, Curtin University, Perth, Australia Barat.

1.1 Tujuan penelitian

Maksud dari penelitian ini untuk menyelidiki dampak dari *Lesson Study*, yang merupakan salah satu bentuk dari pengembangan professional, pada pengetahuan dan keahlian guru dalam membedakan instruksi matematika di sekolah dasar. Selanjutnya, tujuan dari penelitian ini adalah untuk mengidentifikasi ciri-ciri dari *Lesson Study* sebagai suatu bentuk dari peningkatan professional guru, untuk menjelaskan proses dalam mengimplementasikan *Lesson Study*, serta untuk menyelidiki pengaruh *Lesson Study* terhadap pedagogi matematika guru dan pembedaan pembelajaran matematika. Untuk mencapai maksud dan tujuan tersebut, saya akan mengumpulkan data melalui observasi, wawancara, dokumen, rekaman video dan suara dari para guru peserta penelitian.

1.2 Manfaat berpartisipasi dalam penelitian ini

Manfaat berpartisipasi dalam penelitian ini adalah: (i) Anda akan meningkatkan pemahaman tentang pembedaan pembelajaran matematika, (ii) Anda akan diberdayakan untuk memulai, merencanakan dan mengelola pembelajaran professional Anda melalui *Lesson Study*, (iii)

Anda akan dapat mengetahui kemampuan siswa secara lebih efektif, (iv) Anda akan menerima dukungan dari ahli dalam menggunakan *Lesson Study* selama masa penelitian ini.

1.3 Peran Anda

Jika Anda memilih untuk berpartisipasi, Anda akan terlibat dalam kelompok *Lesson Study* di sekolah Anda. Kelompok ini akan bertemu secara teratur selama satu semester untuk merancang, melaksanakan, mengamati, merefleksi dan memperbaiki pelajaran berdasarkan pembedaan pembelajaran matematika. Selama masa penelitian ini mungkin Anda akan menjadi sukarelawan untuk mengajar di kelas Anda sendiri dengan menggunakan rencana pembelajaran yang dirancang oleh kelompok, atau Anda bisa menjadi pengamat, mengamati guru relawan sebagai bagian dari proses Lesson Study. Selain itu, Anda akan diminta untuk menulis jurnal setiap minggu tentang proses yang telah Anda lakukan dan mengirimkannya kepada peneliti melalui email atau faksimili.

1.4 Persyaratan peserta

Peserta akan menghabiskan satu jam per minggu untuk berpartisipasi dalam pertemuan kelompok Lesson Study dan mungkin satu jam per minggu lagi, jika peserta terpilih sebagai pengamat untuk berpartisipasi dalam mengamati guru sukarelawan mengajar. Anda juga akan membutuhkan waktu sekitar setengah jam per minggu untuk menulis jurnal Anda.

1.5 Kerahasiaan dan keamanan

Partisipasi dalam penelitian ini adalah murni sukarela, dan peserta dapat menolak untuk berpartisipasi pada setiap saat tanpa konsekuensi atau prasangka. Semua informasi pribadi dan rincian yang dikumpulkan selama penelitian akan menggunakan anonim. Hanya peneliti yang mengetahui nama sebenarnya dari peserta dalam penelitian ini. Semua data yang dikumpulkan akan disimpan dalam lemari terkunci di kantor Dr Kay O'Halloran, Curtin University dan semua data elektronik akan memiliki cadangan.

1.6 Kontak Personil

Jika Anda memiliki pertanyaan lebih lanjut atau masalah tentang studi ini, silahkan hubungi saya langsung atau pembimbing saya dengan data sebagai berikut:

Nama: Mariana Harsono

HP: +61 424 377 188

E-mail: mariana.harsono@postgrad.curtin.edu

Pembimbing

Nama: Dr Kay O'Halloran	Nama: Dr Rachel Sheffield
Telepon: +61 8 92662182	Telepon: +61 8 92662163
E-mail: kay.ohalloran@curtin.edu.au	E-mail: rachel.sheffield@curtin.edu.au

Sekretaris the Human Research Ethics Committee

Telepon:	+61 8 92669223
E-mail:	hrec@curtin.edu.au
Alamat:	C/-Office of Research and Development, Curtin University of Technology, GPO Box U1987, Perth 6845

Jika Anda memutuskan untuk mengambil bagian dalam penelitian ini, Anda dapat menyimpan lembar informasi ini, menandatangani formulir persetujuan dan mengembalikan ke saya.

Terima kasih banyak atas kesediaan Anda berpartisipasi dalam penelitian ini.

Hormat saya,

Mariana Harsono

Mahasiswa Doktoral Curtin University

Nomor mahasiswa: 15738273

Appendix 16a: Participant Consent Form (English version)

Title of Project: The impact of Lesson Study on primary school teachers' knowledge and skills in differentiating primary school mathematics instruction: A digital mixed methods approach.

Researcher: Mariana Harsono

I have been informed of and understand the purpose of the study. I have been given an opportunity to ask questions. I understand I can withdraw at any time without prejudice. Any information which might potentially identify me will not be used in published materials. I agree to participate in this study as outlined to me.

Name of participant: ____

Signature: ____

Date: ____

Appendix 16b: Participant Consent Form (Indonesian version)

SURAT PERSETUJUAN PARTISIPAN

Judul Penelitian: Dampak *Lesson Study* pada pengetahuan dan keterampilan guru sekolah dasar dalam membedakan instruksi pembelajaran matematika: Pendekatan metode campuran digital.

Peneliti: Mariana Harsono

Saya telah diberitahu dan memahami tujuan penelitian. Saya telah diberi kesempatan untuk mengajukan pertanyaan. Saya mengerti bahwa saya dapat menarik setiap saat tanpa prasangka. Setiap informasi yang berhubungan dengan identitas pribadi tidak akan digunakan dalam penelitian ini. Saya setuju untuk berpartisipasi dalam penelitian ini sebagaimana dijelaskan kepada saya.

Nama peserta: ____

Tanda tangan: ____

Tanggal: ____

Appendix 17a: Participant Permission for Audio-visual recording (English version)

Title of Project: The impact of Lesson Study on primary school teachers' knowledge and skills in differentiating primary school mathematics instruction: A digital mixed methods approach.

I give Mariana Harsono permission to audio-visually record me in the Lesson Study Group Meetings and classroom teaching. This audio-visual recording will be utilised as a part of her research project at Curtin University. The audio-visual recording will be held confidentially and stored for five years after completion of the study at which time it will be destroyed.

I understand that I can withdraw my participation in this study at any time without consequences or prejudice.

Participant's signature _____________________________

Date ____________________

For more information concerning the recording or any other matter, please contact:

Researcher

Name:	Mariana Harsono
Mobile:	+61 424 377 188
E-mail:	mariana.harsono@postgrad.curtin.edu.au

Supervisors

Name:	Dr Kay O'Halloran	Name: Dr Rachel Sheffield
Phone:	+61 8 9266 2182	Phone: +61 8 9266 2163
E-mail:	kay.ohalloran@curtin.edu.au	E-mail : rachel.sheffield@curtin.edu.au

Appendix 17b: Participant Permission for Audio-visual recording (Indonesian version)

Surat Persetujuan Peserta untuk Direkam dengan Video

Judul Penelitian: Dampak *Lesson Study* pada pengetahuan dan keterampilan guru sekolah dasar dalam membedakan instruksi pembelajaran matematika: Pendekatan metode campuran digital.

Saya memberi ijin kepada Mariana Harsono untuk merekam saya dalam pertemuan kelompok Lesson Study dan pembelajaran di kelas. Rekaman video ini akan digunakan sebagai bagian dari penelitian di Curtin University. Rekaman video ini akan disimpan secara rahasia selama lima tahun setelah penelitian ini selesai dan akan dihancurkan.

Saya memahami bahwa saya dapat menarik diri dari keikutsertaan dalam penelitian ini setiap saat tanpa konsekuensi ataupun prasangka.

Tanda tangan peserta: ________________________________

Tanggal____________________

Untuk informasi lebih lanjut mengenai perekaman dan masalah lainnya, silahkan menghubungi:

Peneliti:

Nama: Mariana Harsono
HP: +61 424 377 188
E-mail: mariana.harsono@postgrad.curtin.edu.au

Pembimbing:

Nama:	Dr Kay O'Halloran	Nama:	Dr Rachel Sheffield
Telepon:	+61 8 9266 2182	Telepon:	+61 8 9266 2163
E-mail:	kay.ohalloran@curtin.edu.au	E-mail:	rachel.sheffield@curtin.edu.au

Appendix 18a: Parent/Guardian Information Sheet (English version)

School of Education
Curtin University

The impact of Lesson Study on primary school teachers' knowledge and skills in differentiating primary school mathematics instruction: A digital mixed methods approach

Dear Parent/Guardian,

My name is Mariana Harsono. While I am a lecturer in Teacher Training and Education Faculty at Flores University I am currently on leave to undertake my Doctor of Philosophy (Ph.D.) in the School of Education at Curtin University, Perth, Western Australia.

Your child's teacher has volunteered to participate in a study which explores how mathematics instruction in the classroom can be adapted to suit the needs of different children. This is called 'differentiated learning' and will be applied to one topic in mathematics, a total of three individual lessons.

To record the teachers' use of the various teaching strategies I will be placing a video camera in the classroom to record the lessons. The focus is on the actions of the teacher in the context of differentiated learning instruction.

The lessons cover the mathematics curriculum and do not disrupt the normal running of the classroom. Your child's participation in this study is purely voluntary, and they may refuse to participate at any time without consequence or prejudice. All personal information and details gathered during the study will be used anonymously. Only the researcher will know the real names of the participants in this study. All data collected will be stored in a lockable cabinet in Dr Kay O'Halloran's office, Curtin University and all electronic data will be backed up.

If you have any further questions or concerns about this study, please contact either your child's teacher, me directly or my supervisors whose details follow:

Name:	Mariana Harsono
Mobile:	+61 424 377 188
E-mail:	mariana.harsono@postgrad.curtin.edu.au

Supervisors

Name:	Dr Kay O'Halloran	Name: Dr Rachel Sheffield
Phone:	+61 8 92662182	Phone: +61 8 92662163
E-mail:	kay.ohalloran@curtin.edu.au	E-mail: rachel.sheffield@curtin.edu.au

Secretary of the Human Research Ethics Committee

Phone:	+61 8 92669223
E-mail:	hrec@curtin.edu.au
Mail:	Office of Research and Development, Curtin University of Technology, GPO Box U1987, Perth 6845

If you decide to let your child take part in this study, you should keep this information sheet and sign the attached consent form and return it to your child's teacher.

Thank you for considering your child's participation in this study.

With kind regards,

Mariana Harsono

Curtin University Postgraduate Student

Student ID 15738273

Appendix 18b: Parent / Guardian Information Sheet (Indonesian version)

Lembaran Informasi Orangtua/Wali Siswa
School of Education
Curtin University

Dampak Lesson Study pada pengetahuan dan keterampilan guru sekolah dasar dalam membedakan instruksi pembelajaran matematika: Pendekatan metode campuran digital

Yang terhormat orangtua/wali siswa,

Nama saya Mariana Harsono. Saya seorang dosen di Fakultas Keguruan dan Ilmu Pendidikan, Universitas Flores dan saat ini saya sedang menempuh pendidikan dan melakukan penelitian untuk meraih gelar Doctor of Philosophy (Ph. D.) di School of Education, Curtin University, Perth, Australia Barat.

Guru anak Anda telah bersedia untuk berpartisipasi dalam sebuah studi yang mengeksplorasi bagaimana pembelajaran matematika di kelas dapat disesuaikan dengan kebutuhan anak yang berbeda. Ini disebut 'pembelajaran yang berbeda' dan akan diterapkan pada satu topik dalam pelajaran matematika, sebanyak tiga kali proses pembelajaran.

Untuk merekam penggunaan dari berbagai strategi pengajaran oleh guru, saya akan menempatkan kamera video di dalam kelas untuk merekam pelajaran. Fokusnya adalah pada tindakan guru dalam konteks instruksi pembelajaran yang berbeda.

Pelajaran yang akan diberikan sesuai dengan kurikulum matematika yang berlaku dan tidak mengganggu jalannya proses pembelajaran di kelas. Partisipasi anak Anda dalam penelitian ini adalah murni sukarela, dan mereka dapat menolak untuk berpartisipasi setiap saat tanpa konsekuensi atau prasangka. Semua informasi pribadi dan rincian data yang dikumpulkan selama penelitian akan menggunakan anonim. Hanya peneliti yang akan mengetahui nama sebenarnya dari peserta dalam penelitian ini. Semua data yang dikumpulkan akan disimpan dalam lemari terkunci di kantor Dr Kay O'Halloran, Curtin University dan semua data elektronik akan memiliki cadangan.

Jika Anda memiliki pertanyaan lebih lanjut atau masalah tentang studi ini, silahkan hubungi guru anak Anda, saya atau pembimbing saya dengan data sebagai berikut:

Nama: Mariana Harsono
HP: +61 424 377 188
E-mail: mariana.harsono@postgrad.curtin.edu.au

Pembimbing

Nama: Dr Kay O'Halloran
Telepon: +61 8 92662182
E-mail: kay.ohalloran@curtin.edu.au

Nama: Dr Rachel Sheffield
Telepon: +61 8 92662163
E-mail: rachel.sheffield@curtin.edu.au

Sekretaris the Human Research Ethics Committee

Telepon: +61 8 92669223
E-mail: hrec@curtin.edu.au
Alamat: C/-Office of Research and Development, Curtin University of Technology, GPO Box U1987, Perth 6845

Jika Anda mengijinkan anak Anda untuk turut mengambil bagian dalam penelitian ini, Anda dapat menyimpan lembar informasi ini, menandatangani formulir persetujuan dan mengembalikan ke guru anak Anda.

Terima kasih banyak untuk mempertimbangkan keikutsertaan anak Anda dalam penelitian ini.

Hormat saya,

Mariana Harsono

Mahasiswa Doktoral Curtin University

Student ID 15738273

Appendix 19a: Parent/Guardian Consent Form (English version)

Title of Project: The impact of Lesson Study on primary school teachers' knowledge and skills in differentiating primary school mathematics instruction: A digital mixed methods approach.

I have been informed of and understand the purpose of the study.

1. I understand the purpose and procedures of the study.
2. I understand that the video recording process will record teacher practices and classroom activities.
3. I understand that my child's involvement is voluntary and that I can withdraw them at any time without prejudice.
4. I understand that any information which might potentially identify my child will not be used in published materials
5. I understand that all information will be securely stored for 5 years before being destroyed.
6. I have been given opportunity to ask questions.

Parent Signature: ___________________________

Parent's name (printed): ____________________________________

Dated: ____/_____/_______

For more information concerning the recording or any other matter, please contact:

Researcher

Name: Mariana Harsono
Mobile: +61 424 377 188
E-mail: mariana.harsono@postgrad.curtin.edu.au

Supervisors

Name: Dr Kay O'Halloran
Phone: +61 8 9266 2182
E-mail: kay.ohalloran@curtin.edu.au

Name: Dr Rachel Sheffield
Phone: +61 8 9266 2163
E-mail: rachel.sheffield@curtin.edu.au

Appendix 19b: Parent / Guardian Consent Form (Indonesian version)

Formulir Persetujuan Orangtua atau Wali

Judul Penelitian: Dampak *Lesson Study* pada pengetahuan dan keterampilan guru sekolah dasar dalam membedakan instruksi pembelajaran matematika: Pendekatan metode campuran digital.

Saya telah diberitahu dan memahami maksud penelitian ini.

1. Saya memahami prosedur dan tujuan penelitian ini.
2. Saya memahami bahwa proses perekaman video hanya akan merekam cara guru mengajar dan aktivitas di kelas.
3. Saya memahami bahwa keterlibatan anak saya adalah sukarela dan saya dapat menarik anak saya setiap saat tanpa prasangka.
4. Saya memahami bahwa setiap informasi yang mengarah pada identitas pribadi tidak akan digunakan dalam publikasi laporan penelitian ini.
5. Saya menyadari bahwa semua informasi akan disimpan secara aman selama 5 tahun setelah selesai studi dan akan dihancurkan.
6. Saya telah diberi kesempatan untuk bertanya.

Tanda tangan orangtua atau wali: ___________________________

Nama orangtua atau wali: ___________________________________

Tanggal: ____/_____/_______

Untuk informasi lebih lanjut mengenai perekaman atau masalah lainnya, silahkan menghubungi guru anak Anda atau:

Peneliti:

Nama: Mariana Harsono
HP: +61 424 377 188
E-mail: mariana.harsono@postgrad.curtin.edu.au

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Appendix 20a: Consent Form (English version)

Dear Student,

My name is Mariana Harsono. I am a lecturer in Teacher Training and Education Faculty at Flores University and at the moment I am studying at Curtin University in Australia. I want to work with your teacher in your classroom to look how he/she teaches mathematics.

I will be filming your mathematics lesson focusing on your teacher.

I would really appreciate you being part of my study and that may mean you appear in the video whilst you are working on your mathematics in class.

If you want to be part of this study, please write your name below and tick the first box. If you do not want to participate, that is not a problem. Please write your name and tick the second box. There is no penalty if you do not want to participate in the study.

□ I want to be part of the study

□ I do not want to be part of the study

Name ___________________________________

Date ___________________________________

Appendix 20b: Student Consent Form (Indonesian version)

Formulir Persetujuan

Siswa yang terkasih,

Nama saya Mariana Harsono. Saya seorang dosen di Fakultas Keguruan dan Ilmu Pendidikan di Universitas Flores dan saat ini saya belajar di Curtin University di Australia. Saya ingin bekerja dengan guru Anda untuk melihat bagaimana ia mengajar matematika.

Saya akan merekam pelajaran matematika yang berlangsung dan berfokus pada guru Anda.

Saya akan sangat menghargai keterlibatan Anda dalam studi saya dan itu mungkin berarti Anda muncul dalam video saat sedang bekerja pada pelajaran matematika di kelas.

Jika Anda ingin ikut serta dalam studi ini, silahkan tulis nama Anda di bawah ini dan beri tanda centang pada kotak pertama. Jika Anda tidak bersedia, itu tidak masalah. Silahkan tulis nama Anda dan beri tanda centang pada kotak kedua. Tidak ada hukuman jika tidak ingin berpartisipasi dalam studi ini. Terima kasih.

□ Saya ingin berpartisipasi

□ Saya tidak ingin berpartisipasi

Name ____________________________________

Date ____________________________________