Department of Electrical and Computer Engineering

Transceiver Optimization for Interference MIMO Relay Communication Systems

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Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Signature: ....

Date: ....

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Abstract

With the considerable growth of number of mobile subscriptions worldwide, the demand for wireless communication technologies that can provide higher data rates is increasing. This demand is also driving researchers to develop new ways to address capacity challenges and explore new network topologies that offer features and functions. Multiple-input multiple-output (MIMO) is one of the technologies has been developed not only to mitigate multipath fading but also turn it into a benefit for users. However, in long-range communication where the path attenuation of wireless channels becomes significant, the relay nodes are necessary to efficiently compensate the loss without increasing the transmit power. By incorporating relay nodes in a MIMO system, the network coverage and reliability can be significantly improved. In additional, the relay nodes can be installed in places where obstacles affect single-hop communications to mitigate shadowing.

In this thesis, we focus on interference MIMO relay systems. We first propose the iterative algorithms to jointly optimize the source, relay, and receiver matrices subjecting to the individual power constraints at the source and the relay nodes. Based on the fact that the mean-squared error (MSE) of the signal waveform estimation at the receivers is closely related to the raw bit-error-rate (BER), the minimal MSE (MMSE) is chosen as the design criterion. The direct paths between the source and destination nodes are taken into account as they provide valuable spatial diversity. The proposed algorithms outperform the existing techniques in terms of both MSE and BER. Next, to reduce the complexity of optimization problem, we investigate a simplified relay matrix design through modifying the transmission power constraint at the relay node. The modified relay optimization problem has a closed-form solution.

Then we study the transceiver design for two-way interference MIMO relay systems. Based on the simplified relay matrix design for one-way interference MIMO system that we proposed, we develop an advanced algorithm for two-way relay networks. The simulation results show that the proposed algorithm has a slightly worse performance than the existing works in terms of the system MSE and BER. However, the computational complexity of the simplified algorithm is significantly reduced for interference MIMO relay systems with a large number of user pairs.

Author's Note

Parts of this thesis and concepts from it have been previously published in the following journal and/or conference papers.

Journal Papers

1. K. X. Nguyen, Y. Rong, and S. Nordholm, "Joint source and relay matrices optimization for interference MIMO relay systems", *EURASIP Journal on Wireless Commun. Network.*, 2015: 73.

2. K. X. Nguyen, Y. Rong, and S. Nordholm, "MMSE-based transceiver design algorithms for interference MIMO relay systems", *IEEE Trans. Wireless Commun.*, vol. 14, no. 11, pp. 6414-6424, Nov 2015.

3. K. X. Nguyen, Y. Rong, and S. Nordholm, "Simplified MMSE precoding design in interference two-way MIMO relay systems", *IEEE Signal Process. Lett.*, vol. 23, no. 2, pp. 262-266, Feb. 2016.

Conference Papers

1. K. X. Nguyen, Y. Rong, and Z. He, "A frequency domain equalizer for amplify-and-forward underwater acoustic relay communication systems", *Proc. 9th Int. Conf. Inform. Commun. Signal Process. (ICICS)*, Tainan, Taiwan, Dec. 10-13, 2013.

2. K. X. Nguyen, Y. Rong, "Joint source and relay matrices optimization for interference MIMO relay systems", *Proc. Int. Symposium Inf. Theory Its*

Applications (ISITA'2014), Melbourne, Australia, Oct. 26-29, 2014, pp. 640-644.

3. K. X. Nguyen, Y. Rong, and S. Nordholm, "Transceiver optimization for interference MIMO relay systems using the structure of relay matrix", *Proc. 24th Wireless and Optical Commun. Conf. (WOCC'2015)*, Taipei, Taiwan, Oct. 23-24, 2015, pp. 29-33.

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List of Acronyms

AF	amplify-and-forward
AWGN	additive white Gaussian noise
BER	bit-error-rate
CCI	co-channel interference
CDMA	code division multiple access
CF	compress-and-forward
CSI	channel state information
DF	decode-and-forward
DL	direct links
FDMA	frequency division multiple access
IA	interference alignment
i.i.d.	independent and identically distributed
MIMO	multiple-input multiple-output
MMSE	minimal mean-squared error
MSE	mean-squared error
ProBaSeMO	projection based separation of multiple operators
PSD	positive semi-definite
QCQP	quadratically constrained quadratic programming

QoS quality-of-service

QPSK quadrature phase-shift keying

SI self-interference

SINR signal-to-interference-plus-noise ratio

SMSE sum mean-squared error

SNR signal-to-noise ratio

SVD singular value decomposition

TDMA time division multiple access

TLM total leakage minimization

WSMSE weighted sum mean-squared error

ZF zero-forcing

Chapter 1

Introduction

The aim of this thesis is to design advanced algorithms for interference multiple-input multiple-output (MIMO) relay communication systems. In this introductory chapter, we briefly present a necessary background on interference MIMO relay communication systems and overview the contributions of this thesis.

1.1 Overview of MIMO Communication Systems

Communication theories show that the degree-of-freedom of the communication system grows approximately linear with the number of equipped antennas [1]. Thus equipping the transceivers with multiple antennas leads to an increase in the channel capacity, especially in a scattering environment [1–5]. On the other hand, wireless transmission in a rich scattering environment suffers the multipath fading which reduces the performance of the communication system. In order to overcome this, the MIMO system is introduced not only to mitigate multipath fading, but also turn it into a benefit for the user [1]. By equipping multiple transmit and receive antennas, the MIMO technology offers several advantages compared to existing single-input single-output systems such as array gain, diversity gain, spatial multiplexing gain, and interference reduction [2, 6].

- The array gain is the improvement in signal-to-noise ratio (SNR) at the receiver. This is the result of a coherent combining effect of multiple transmitting and receiving antennas. The array gain requires perfect channel knowledge in either the transmitter or receiver and depends on the number of transmit and receive antennas.

- The diversity gain is the improvement in the reliability of the system and can be achieved by transmitting the same signal across multiple independent fading channels. The receiver receives multiple independent replicas of the same signal and the probability that at least one of the received signals is not suffered from deep fade increases. In effect, it improves the quality and reliability of the entire system.

- The spatial multiplexing gain is the linear increase in the achievable data rate. It relies on transmitting independent data streams through independent spatial channels. Under suitable channel conditions, for example rich scattering channels, multiple independent data streams can be transmitted within the same allocated bandwidth and the data streams can be separated at the receiver. Furthermore, the capacity of the system scales linearly with the number of established data streams.

- Interference reduction is achievable through exploiting the spatial difference between the desired signal and the cochannel signal. The knowledge of the desired signal's channel is required while it is not necessary to have knowledge of the interference channels. When interference reduction is implemented at the transmitter, it helps to minimize the co-channel interference (CCI) while transmitting the signal to the desired user.

Unfortunately, diversity gain and spatial multiplexing gain have benefits from the spatial degrees of freedom in different ways which makes it impossible to exploit all the features of MIMO technology simultaneously [6]. The choice of gain to implement within a particular MIMO system depends on the aim to improve the data rate or the reliability of the system. Such a trade-off is very useful in practical MIMO communication systems.

1.2 MIMO Relay Communication Systems

In long-range communication where the path attenuation of wireless channels becomes significant, the relay nodes are necessary to efficiently compensate the loss

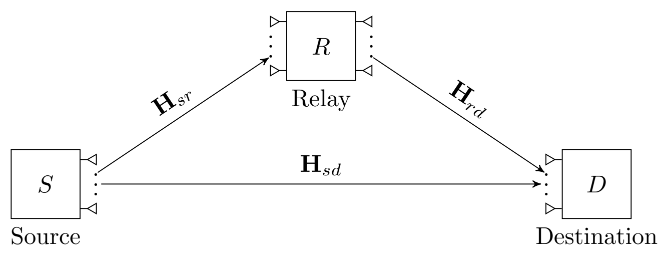

Figure 1.1: A three-node two-hop communication system

without increasing the transmit power. By incorporating relay nodes in a MIMO system, the network coverage and reliability can be significantly improved [7–26]. In additional, the relay nodes can be installed in places where obstacles affect single-hop communications to mitigate shadowing. Furthermore, when taking the direct paths into account, the provision of independent propagation paths can efficiently combat the multipath fading [7, 8].

Figure 1.1 illustrates the diagram of a three-node two-hop MIMO communication system. Here, $\mathbf{H}_{sr}$, $\mathbf{H}_{sd}$, and $\mathbf{H}_{rd}$ are the source-relay, source-destination, and relay-destination channels respectively. In a MIMO relay system, communication between source nodes and destination nodes can be assisted by single or multiple relays equipped with multiple antennas. The relays can either decode-and-forward (DF), amplify-and-forward (AF), or compress-and-forward (CF) the relayed signals. In the AF scheme, the received signals are simply amplified (including a possible linear transformation) through the relay precoding matrices before being forwarded to the destination nodes. Therefore, in general the AF strategy has lower complexity and shorter processing delay than the DF and CF strategy.

1.3 Interference MIMO Relay Communication Systems

In wireless communication networks, multiple transmitters share the same radio resources and each receiver not only receives data sent from the paired transmitter but also observes CCI from the other transmitters in the network. The CCI is being identified as one of the main impairments that limit the throughput in wireless communication networks. Thus, developing schemes that mitigate CCI has become critical.

Traditionally, the CCI has been dealt with by several approaches such as decoding the interference, using orthogonal multiplexing schemes, and treating the interference as noise.

- Decoding the interference is used when the interference is much stronger than the signal and it can be decoded along with the desired signal. This approach is less common in practice due the complexity of the receiver [34] and the extension of the results to more than two users is not straightforward
- When the interference is weak, introducing structure into the interference signals is not useful thus the interference is treated as noise.
- Orthogonal multiplexing is the practical approach to mitigate the interference which is as strong as signals. In frequency division multiple access (FDMA), the system bandwidth is divided among the transmitters while transmitters take turns transmitting data in time division multiple access (TDMA). One of the widely used scheme is the code division multiple access (CDMA). However, such interference avoidance solutions has become fairly inefficient as they cannot achieve the full degrees of freedom available in the channel.

Recently, a new approach in interference management, called interference alignment (IA), was first considered in [20]. IA is a cooperative interference management strategy that maximizes interference-free space for the desired signal. The transmitters design their transmitted signals such that the interference received at the receiver is aligned in only a portion of the signalling space.

Figure 1.2 shows the diagram of an interference MIMO relay system where K source-destination pairs $\{\mathbf{S}_i - \mathbf{D}_i,\ i = 1..K\}$ communicate simultaneously with the aid of L distributed relay nodes $\{\mathbf{R}_j,\ j = 1..L\}$. The source, destination, and relay nodes are equipped with multiple antennas.

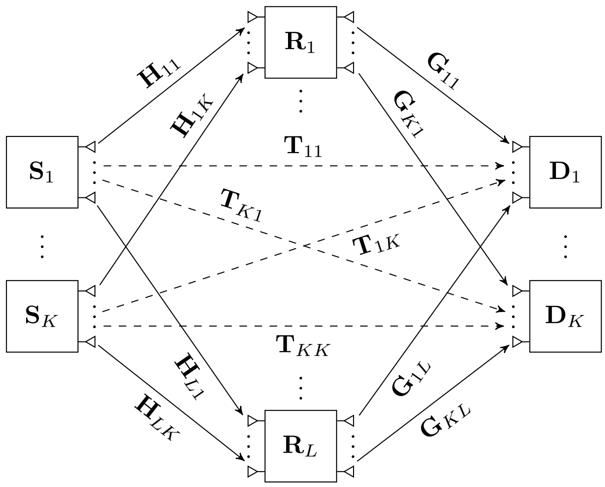

Figure 1.2: An interference MIMO relay communication system

1.4 Two-Way MIMO Relay Communication Systems

Most of the relay networks are assumed to work in half-duplex mode as it can avoid interference at the relay nodes [27]. Thus, in a typical AF two-way MIMO relay system, the communication between two source nodes is accomplished in four time slots: from source node 1 to the relay node, from the relay node to source node 2, from source node 2 to the relay node and from the relay node to source node 1. By using the idea of analog network coding [28], the two-way relaying protocol in which, the two source nodes exchange their information in two time slots without using extra channel resources, has been studied recently to overcome the loss in terms of spectral efficiency in half-duplex systems. In the first time slot, the relays receive data from the two source nodes simultaneously, and in the second phase, the relays re-transmit the received signal to both source nodes. Since each source node knows its own transmitted data, the self-interference (SI) in the transmitted signal can be cancelled.

Figure 1.3 shows the diagram of a two-hop interference MIMO relay communication system where K user pairs $\{\mathbf{S}_i - \mathbf{D}_i,\ i = 1..K\}$, distributed on two different sites, communicate with the aid of a single relay $\mathbf{R}$. The direct links (DL) between site 1 and site 2 user pair are ignored as they undergo much larger path attenuation compared with the links via the relay.

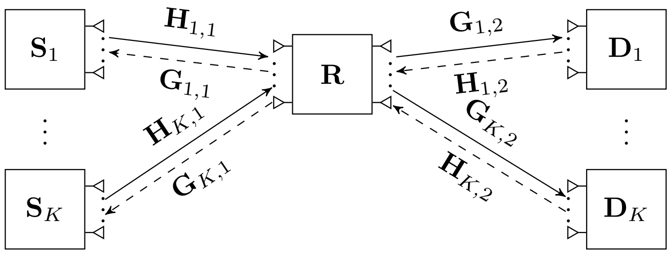

Figure 1.3: A multiuser two-hop interference MIMO relay communication system

1.5 Thesis Overview and Contributions

In this thesis, advanced transceiver designs for MIMO communication systems are presented and studied. In Chapter 2, we investigate the iterative algorithms for an AF interference MIMO relay communication system. Chapter 3 studies a robust design for the case in which a single relay node is used. By modifying the transmission power constraint at the relay node, the computational complexity of optimizing the relay precoding matrix is significantly reduced. In Chapter 4, we propose an iterative transceiver design algorithm for a two-way interference MIMO relay communication system where multiple user pairs communicate simultaneously with the aid of single relay node.

It is clear that the average BER across users is dominated by the highest MSE [30, 31]. The minimization of the MSE results in reduction of the intersymbol interference and indirectly reduces the BER. Thus, MSE is chosen as the precoders design criterion.

Throughout the chapters of this thesis, the proposed algorithms are carried out at a central controlling unit, which can be any node in the system. We assume that the controlling unit has knowledge of the global channel state information (CSI). After the convergence of the algorithms, the controlling unit sends the information on the optimal source, relay, and receiver matrices to corresponding nodes.

Chapter 2: Joint Source Relay Optimization

In this chapter, we develop two iterative algorithms to solve the highly nonconvex joint source, relay and receiver optimization problem for interference MIMO relay communication systems. The direct source-destination links are taken into account for the design of the transceivers. The minimal mean-squared error (MMSE) of the signal waveform estimation at the destination nodes is chosen as the design criterion to optimize the transceiver matrices for interference suppression. In the first algorithm, we iteratively optimize the source, relay, and receiver matrices. To reduce the per-iteration complexity, in second algorithm, we develop an iterative algorithm where each source and relay matrix is optimized individually by fixing all other matrices. Simulation results demonstrate that the proposed algorithms outperform the existing techniques in terms of the system mean-squared error (MSE) and bit-error-rate (BER).

Chapter 2 is based on the journal publication:

- K. X. Nguyen, Y. Rong, and S. Nordholm, "Joint source and relay matrices optimization for interference MIMO relay systems", *EURASIP Journal on Wireless Commun. Network.*, 2015: 73.

and two conference publications:

- K. X. Nguyen, Y. Rong, and Z. He, "A frequency domain equalizer for amplify-and-forward underwater acoustic relay communication systems", *Proc. 9th Int. Conf. Inform. Commun. Signal Process. (ICICS)*, Tainan, Taiwan, Dec. 10-13, 2013.

- K. X. Nguyen, Y. Rong, "Joint source and relay matrices optimization for interference MIMO relay systems", *Proc. Int. Symposium Inf. Theory Its Applications (ISITA'2014)*, Melbourne, Australia, Oct. 26-29, 2014, pp. 640-644.

Chapter 3: Simplified Transceiver Design for Interference MIMO Relay Systems

Complexity is a major concern in the practical implementation of MIMO systems. Signal processing algorithms typically improve the performance of the system at the cost of computational complexity. Several works have focused on achieving sub-optimal performance in order to significantly reduce the computational complexity. Thus, in this chapter, we investigate an interference MIMO relay communication system where multiple transmitter-receiver pairs communicate simultaneously with the aid of a relay node. Two iterative algorithms are proposed to exploit the performance and complexity trade-off.

Chapter 3 is based on the journal publication:

- K. X. Nguyen, Y. Rong, and S. Nordholm, "MMSE-based transceiver design algorithms for interference MIMO relay systems", *IEEE Trans. Wireless Commun.*, vol. 14, no. 11, pp. 6414-6424, Nov 2015.

and the conference publication:

- K. X. Nguyen, Y. Rong, and S. Nordholm, "Transceiver optimization for interference MIMO relay systems using the structure of relay matrix", *Proc. 24th Wireless and Optical Commun. Conf. (WOCC'2015)*, Taipei, Taiwan, Oct. 23-24, 2015, pp. 29-33.

Chapter 4: Transceiver Design for Two-Way MIMO Relay Systems

In this chapter, we investigate the transceiver design for interference two-way MIMO relay systems where multiple two-way links communicate simultaneously with the aid of a single relay node. We derive the optimal structure of the relay precoding matrix. By modifying the power constraint at the relay node, we propose a novel relay precoding matrix optimization algorithm with a closed-form solution. The proposed iterative transceiver design algorithm converges faster and has a lower computational complexity than existing algorithms particularly for interference MIMO relay systems with a large number of user pairs, with only small performance degradation.

Chapter 4 is based on the journal publication:

- K. X. Nguyen, Y. Rong, and S. Nordholm, "Simplified MMSE precoding mesign in interference two-way MIMO relay systems", *IEEE Signal Process. Lett.*, vol. 23, no. 2, pp. 262-266, Feb. 2016.

1.6 Notation

The notations used in this thesis are as follows: Scalars are denoted with lower or upper case normal letters, vectors are denoted with bold-faced lower case letters, and matrices are denoted with bold-faced upper case letters. Superscripts $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^{-1}$ denote matrix transpose, conjugate transpose, and inverse, respectively, $tr()$ stands for matrix trace, $vec()$ stacks columns of a matrix on top of each other into a single vector, $bd()$ denotes a block-diagonal matrix, $\otimes$ represents the Kronecker product, $E[\ ]$ denotes the statistical expectation, and $\mathbf{I}_n$ stands for the $n \times n$ identity matrix. Note that the scope of any variable in each chapter is limited to that particular chapter.

Chapter 2

Joint Source Relay Optimization

In this chapter, we investigate the transceiver design for linear non-regenerative interference MIMO relay communication systems when the direct links between the source and destination nodes are taken into consideration. The MMSE of the signal waveform estimation at the destination nodes is chosen as the design criterion to optimize the source, relay, and receiver matrices for interference suppression. As the joint source, relay, and receiver optimization problem is non-convex with matrix variables, a globally optimal solution is computationally intractable to obtain [29]. After a review of existing works in Section 2.1, the system model is introduced in Section 2.2. We propose two iterative algorithms in Section 2.3 to provide computationally efficient solutions to the original problem through solving convex subproblems. These two algorithms provide efficient performance-complexity tradeoff. Simulation results in Section 2.4 demonstrate that the proposed algorithms converge quickly after a few iterations and significantly outperform existing scheme in terms of the system BER. Conclusions are drawn in Section 2.5. The detailed proofs of (2.15) and (2.23) are given in Appendix 2.A and Appendix 2.B respectively.

2.1 Overview of Existing Techniques

Relay-aided MIMO communication technology has attracted great research interest recently [10, 16]. By incorporating relay nodes in a MIMO system, the network coverage and reliability can be significantly improved. In a MIMO relay system, communication between source nodes and destination nodes can be assisted by single or multiple relays equipped with multiple antennas. The relays can either DF or AF the relayed signals [17]. In the AF scheme, the received signals are simply amplified (including a possible linear transformation) through the relay precoding matrices before being forwarded to the destination nodes. Therefore, in general, the AF strategy has lower complexity and shorter processing delay than the DF strategy.

For single-user two-hop MIMO communication systems with a single relay node, the optimal source and relay precoding matrices have been developed in [18]. For a single-user two-hop MIMO relay system with multiple parallel relay nodes, the design of relay precoding matrices has been studied in [19]. Recent progress on the optimization of AF MIMO relay systems has been summarized in the tutorial of [16].

For MIMO interference channel, the idea of IA [20] was developed for interference suppression by arranging the desired signal and interference into appropriated signal spaces. The idea of IA has been applied in interference MIMO relay systems in [21, 22]. However, there is still no general solution for IA as a number of conditions must be met. One main reason is that the number of dimensions required for IA is very large and it depends on the number of independent fading channels. This leads to high computational complexity and infeasibility in practical systems. In [23], an iterative algorithm has been proposed to optimize the source beamforming vector and the relay precoding matrices to minimize the total source and relay transmit power such that a minimum signal-to-interference-plus-noise ratio (SINR) threshold is maintained at each receiver. Three iterative transceiver design algorithms to minimize either the matrix-weighted sum mean-squared error (SMSE) or the total leakage have been developed in [24]. However, the works in [21, 24] did not take the direct source-destination links into consideration.

The direct links between the source and destination nodes provide valuable spatial diversity to the MIMO relay system and should not be ignored. In this chapter, we investigate the transceiver design for AF interference MIMO relay communication systems where multiple source nodes transmit information simultaneously to the destination nodes with the aid of multiple relay nodes, and each node is equipped with multiple antennas. The direct source-destination links are taken into account for the design of the transceivers. We aim at optimizing the source, relay, and receiver matrices to suppress the interference and minimize the SMSE of the signal waveform estimation at the destination nodes, subjecting to transmission power constraints at the source and relay nodes. The SMSE criterion is chosen as it provides a good trade-off between performance and complexity. Since the joint source, relay, and receiver optimization problem is nonconvex with matrix variables, a globally optimal solution is computationally intractable to obtain. We propose two iterative algorithms to provide computationally efficient solutions to the original problem through solving convex subproblems. In each iteration of the first algorithm, we first optimize all receiver matrices based on the source and relay matrices from the previous iteration. Then, we optimize all relay matrices using the receiver matrices in this iteration and the source matrices from the previous iteration. Finally, the source matrices are updated.

In the second algorithm, the receiver matrices are optimized in the same way as the first algorithm. However, in contrast to the first algorithm, each source and relay matrix is optimized individually by fixing all other matrices. We show that both proposed algorithms converge. Comparing the two proposed algorithms, the first algorithm has a better MSE and BER performance, while the second algorithm has a smaller per-iteration computational complexity. Such performance-complexity trade-off is very useful for practical MIMO relay communication systems. Simulation results demonstrate that the proposed algorithms outperform the existing technique in terms of the system MSE and BER.

We assume that similar to [24], the two proposed algorithms are carried out at a central controlling unit, which can be any node in the system. The controlling unit has knowledge of the global CSI. After the convergence of the algorithms, the controlling unit sends the information on the optimal source, relay, and receiver matrices to corresponding nodes.

2.2 System Model and Problem Formulation

We consider a two-hop interference MIMO relay communication system where K source-destination pairs communicate simultaneously with the aid of a network of L-distributed relay nodes as shown in Fig. 2.1. The kth source node and the kth destination node are equipped with N_{sk} and N_{dk} antennas, respectively, $k = 1, \cdots, K$, and the number of antennas at the lth relay node is N_{rl}, $l = 1, \cdots, L$.

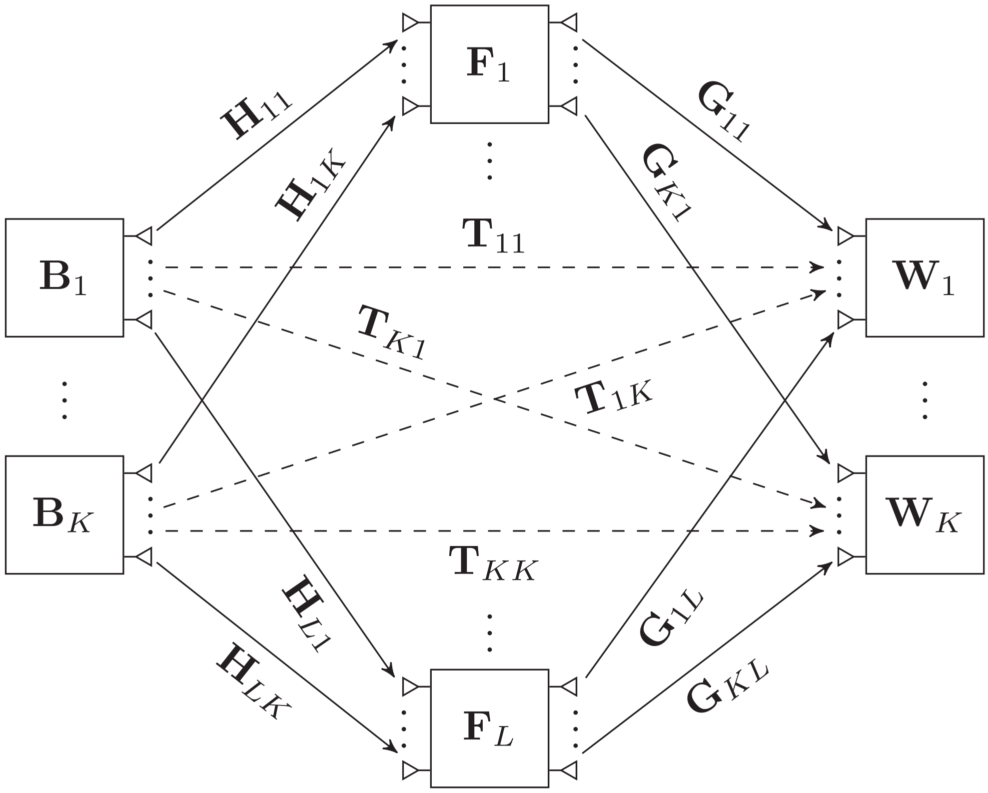

Figure 2.1: Block diagram of an interference MIMO relay system with multiple relay nodes.

Using half duplex relay nodes, the communication between source and destination pairs is completed in two time slots. At the first time slot, the kth source node transmits an $N_{sk} \times 1$ signal vector

$$\mathbf{x}_{sk} = \mathbf{B}_k \mathbf{s}_k, \quad k = 1, \cdots, K \tag{2.1}$$

to the relay nodes and the destination nodes, where $\mathbf{s}_k$ is the $d \times 1$ information-carrying symbol vector and $\mathbf{B}_k$ is the $N_{sk} \times d$ source precoding matrix. The received signal vectors at the lth relay node and the kth destination node are given by

$$\mathbf{y}_{rl} = \sum_{k=1}^{K} \mathbf{H}_{lk}\mathbf{x}_{sk} + \mathbf{v}_{rl}, \quad l = 1, \cdots, L \tag{2.2}$$

$$\mathbf{y}_{d1k} = \sum_{m=1}^{K} \mathbf{T}_{km}\mathbf{x}_{sm} + \mathbf{v}_{d1k}, \quad k = 1, \cdots, K \tag{2.3}$$

where $\mathbf{H}_{lk}$ is the $N_{rl} \times N_{sk}$ MIMO fading channel matrix between the kth source node and the lth relay node, $\mathbf{T}_{km}$ is the $N_{dk} \times N_{sm}$ MIMO fading channel matrix between the mth source node and the kth destination node, $\mathbf{v}_{rl}$ is the $N_{rl} \times 1$ additive white Gaussian noise (AWGN) vector at the lth relay node with zero mean and covariance matrix $E[\mathbf{v}_{rl}\mathbf{v}_{rl}^{H}] = \sigma_{rl}^2\mathbf{I}_{N_{rl}}$, $l = 1, \cdots, L$, and $\mathbf{v}_{d1k}$ is the $N_{dk} \times 1$ AWGN vector at the kth destination node at the first time slot with zero mean and covariance matrix $E[\mathbf{v}_{d1k}\mathbf{v}_{d1k}^{H}] = \sigma_{dk}^2\mathbf{I}_{N_{dk}}$, $k = 1, \cdots, K$.

During the second time slot, the received signal vector at the lth relay node is amplified with the $N_{rl} \times N_{rl}$ precoding matrix $\mathbf{F}_l$ as

$$\mathbf{x}_{rl} = \mathbf{F}_l\mathbf{y}_{rl}, \quad l = 1, \cdots, L. \tag{2.4}$$

The precoded signal vector $\mathbf{x}_{rl}$ is forwarded to the destination nodes. The received signal vector at the kth destination node is given by

$$\mathbf{y}_{d2k} = \sum_{l=1}^{L} \mathbf{G}_{kl}\mathbf{x}_{rl} + \mathbf{v}_{d2k}, \quad k = 1, \cdots, K \tag{2.5}$$

where $\mathbf{G}_{kl}$ is the $N_{dk} \times N_{rl}$ MIMO channel matrix between the lth relay node and the kth destination node and $\mathbf{v}_{d2k}$ is the $N_{dk} \times 1$ AWGN vector at the kth destination node at the second time slot with zero mean and covariance matrix $E[\mathbf{v}_{d2k}\ \mathbf{v}_{d2k}^{H}] = \sigma_{dk}^2\mathbf{I}_{N_{dk}}$, $k = 1, \cdots, K$.

From (2.1)-(2.5), the signal vector received at the kth destination node over two consecutive time slots is

$$\mathbf{y}_k = \begin{bmatrix} \mathbf{y}_{d2k} \\ \mathbf{y}_{d1k} \end{bmatrix} = \sum_{m=1}^{K} \begin{bmatrix} \sum\limits_{l=1}^{L} \mathbf{G}_{kl}\mathbf{F}_l\ \mathbf{H}_{lm} \\ \mathbf{T}_{km} \end{bmatrix} \mathbf{B}_m\mathbf{s}_m + \begin{bmatrix} \bar{\mathbf{v}}_{dk} \\ \mathbf{v}_{d1k} \end{bmatrix} \tag{2.6}$$

where $\bar{\mathbf{v}}_{dk} = \sum_{l=1}^{L} \mathbf{G}_{kl}\mathbf{F}_l\mathbf{v}_{rl} + \mathbf{v}_{d2k}$ is the total noise vector at the kth destination node at the second time slot.

Due to their simplicity, linear receivers are used at the destination nodes to retrieve the transmitted signals. Thus, the estimated signal vector at the kth destination node can be written as

$$\hat{\mathbf{s}}_k = \mathbf{W}_k^H \mathbf{y}_k, \quad k = 1, \cdots, K \tag{2.7}$$

where $\mathbf{W}_k = [\mathbf{W}_{k2}^T, \mathbf{W}_{k1}^T]^T$ is the receiver weight matrix, and $\mathbf{W}_{k1}$ and $\mathbf{W}_{k2}$ are the $N_{dk} \times d$ receiver weight matrices for the direct link and the relay link, respectively. In (2.6) and (2.7), we have

$$\begin{aligned}
\hat{\mathbf{s}}_k &= [\mathbf{W}_{k2}^H \ \ \mathbf{W}_{k1}^H] \begin{bmatrix} \mathbf{y}_{d2k} \\ \mathbf{y}_{d1k} \end{bmatrix} \\
&= \underbrace{\left(\mathbf{W}_{k2}^H \sum_{l=1}^{L} \mathbf{G}_{kl} \mathbf{F}_l \mathbf{H}_{lk} + \mathbf{W}_{k1}^H \mathbf{T}_{kk} \right) \mathbf{B}_k \mathbf{s}_k}_{\text{desired signal}} \\
&\quad + \underbrace{\sum_{m=1, m\neq k}^{K} \left(\mathbf{W}_{k2}^H \sum_{l=1}^{L} \mathbf{G}_{kl} \ \mathbf{F}_l \mathbf{H}_{lm} + \mathbf{W}_{k1}^H \mathbf{T}_{km} \right) \mathbf{B}_m \mathbf{s}_m}_{\text{interference}} \\
&\quad + \underbrace{\mathbf{W}_{k2}^H \bar{\mathbf{v}}_{dk} + \mathbf{W}_{k1}^H \mathbf{v}_{d1k}}_{\text{noise}}.
\end{aligned} \tag{2.8}$$

In (2.1) and (2.4), the transmission power constraints at the source and relay nodes can be written as

$$tr\left(\mathbf{B}_k \mathbf{B}_k^H\right) \leq P_{sk}, \quad k = 1, \cdots, K \tag{2.9}$$

$$tr\left(\mathbf{F}_l E\left[\mathbf{y}_{rl} \mathbf{y}_{rl}^H\right] \mathbf{F}_l^H\right) \leq P_{rl}, \quad l = 1, \cdots, L \tag{2.10}$$

where P_{sk} and P_{rl} denote the power budget at the kth source node and the lth relay node, respectively, and $E\left[\mathbf{y}_{rl}\mathbf{y}_{rl}^H\right] = \sum_{m=1}^{K} \mathbf{H}_{lm} \ \mathbf{B}_m \mathbf{B}_m^H \mathbf{H}_{lm}^H + \sigma_{rl}^2 \mathbf{I}_{N_{rl}}$ is the covariance matrix of the received signal vector at the lth relay node.

In this chapter, we aim at optimizing the source precoding matrices $\{\mathbf{B}_k\} = \{\mathbf{B}_k, k = 1, \cdots, K\}$, the relay precoding matrices $\{\mathbf{F}_l\} = \{\mathbf{F}_l, \, l = 1, \cdots, L\}$, and the receiver weight matrices $\{\mathbf{W}_k\} = \{\mathbf{W}_k, \, k = 1, \cdots, K\}$, to minimize the sum-MSE of the signal waveform estimation at the destination nodes under transmission power constraints at the source and relay nodes. We would like to mention that MMSE is a sensible design criterion based on the links of MSE to other performance measures in MIMO systems such as mutual information and SINR [18, 25].

From (2.8), the MSE of the kth source-destination pair can be calculated as

$$\begin{aligned}\mathrm{MSE}_k &= tr\Big(E\Big[(\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^H\Big]\Big)\\ &= tr\Big((\mathbf{W}_k^H\tilde{\mathbf{H}}_{kk} - \mathbf{I}_d)(\mathbf{W}_k^H\tilde{\mathbf{H}}_{kk} - \mathbf{I}_d)^H\\ &\quad +\mathbf{W}_k^H\mathbf{C}_k\mathbf{W}_k + \mathbf{W}_k^H\mathbf{\Xi}_k\mathbf{W}_k\Big),\ k = 1,\cdots,K \qquad (2.11)\end{aligned}$$

where $\tilde{\mathbf{H}}_{km}$ is the equivalent MIMO channel matrix from the mth source node to the kth destination node, $\mathbf{C}_k = E\Big\{\big[\bar{\mathbf{v}}_{dk}^T,\ \ \mathbf{v}_{d1k}^T\big]^T\big[\bar{\mathbf{v}}_{dk}^H,\ \mathbf{v}_{d1k}^H\big]\Big\}$ and $\mathbf{\Xi}_k$ are the covariance matrices of the equivalent noise and the interference at the kth destination node, respectively. For $k, m = 1,\cdots,K$, they are given respectively as

$$\tilde{\mathbf{H}}_{km} = \begin{bmatrix}\sum_{l=1}^{L}\mathbf{G}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lm}\\ \mathbf{T}_{km}\mathbf{B}_m\end{bmatrix}, \quad \mathbf{\Xi}_k = \sum_{m=1,m\neq k}^{K}\tilde{\mathbf{H}}_{km}\tilde{\mathbf{H}}_{km}^H$$

$$\mathbf{C}_k = \begin{bmatrix}\sum_{l=1}^{L}\sigma_{rl}^2\mathbf{G}_{kl}\mathbf{F}_l\mathbf{F}_l^H\mathbf{G}_{kl}^H + \sigma_{dk}^2\mathbf{I}_{N_{dk}} & \mathbf{0}\\ \mathbf{0} & \sigma_{dk}^2\mathbf{I}_{N_{dk}}\end{bmatrix}$$

where $\bar{\mathbf{H}}_{lm} = \mathbf{H}_{lm}\mathbf{B}_m$ is the equivalent MIMO channel matrix between the mth source node and the lth relay node.

From (2.9)-(2.11), the optimal source, relay, and receiver matrix design problem can be written as

$$\min_{\{\mathbf{W}_k\},\{\mathbf{B}_k\},\{\mathbf{F}_l\}} \quad \sum_{k=1}^{K}\mathrm{MSE}_k \qquad (2.12a)$$

$$s.t. \quad tr\big(\mathbf{B}_k\mathbf{B}_k^H\big) \le P_{sk},\ k=1,\cdots,K \qquad (2.12b)$$

$$tr\big(\mathbf{F}_l E[\mathbf{y}_{rl}\mathbf{y}_{rl}^H]\mathbf{F}_l^H\big) \le P_{rl},\ l=1,\cdots,L. \qquad (2.12c)$$

2.3 Proposed Source, Relay, and Receiver Matrix Design Algorithms

The problem (2.12) is highly nonconvex with matrix variables, and a globally optimal solution is intractable to obtain. In this section, we propose two block coordinate descent algorithms to solve the problem (2.12) by optimizing $\{\mathbf{W}_k\}$, $\{\mathbf{B}_k\}$, and $\{\mathbf{F}_l\}$ in an alternating way through solving convex subproblems.

2.3.1 Proposed Algorithm 2.1

In each iteration of this algorithm, we first optimize $\{\mathbf{W}_k\}$ based on $\{\mathbf{B}_k\}$ and $\{\mathbf{F}_l\}$ from the previous iteration. Then, we optimize all relay matrices based on $\{\mathbf{W}_k\}$ from the current iteration and $\{\mathbf{B}_k\}$ from the previous iteration. Finally, we optimize all source matrices using $\{\mathbf{W}_k\}$ and $\{\mathbf{F}_l\}$ from the current iteration.

It can be seen from (2.11) that $\mathbf{W}_k$ only affects MSE_k. Thus, with given $\{\mathbf{F}_l\}$ and $\{\mathbf{B}_k\}$, the optimal linear receiver matrix which minimizes MSE_k in (2.11) is the solution to the following unconstrained optimization problem

$$\min_{\mathbf{W}_k} \quad \text{MSE}_k. \tag{2.13}$$

The solution to the problem (2.13) is the well-known MMSE receiver [36] given by

$$\mathbf{W}_k \;=\; (\tilde{\mathbf{H}}_{kk}\tilde{\mathbf{H}}_{kk}^H + \mathbf{C}_k + \mathbf{\Xi}_k)^{-1}\tilde{\mathbf{H}}_{kk}, \quad k = 1, \cdots, K. \tag{2.14}$$

Let us introduce $\mathbf{f}_l = \text{vec}(\mathbf{F}_l)$, $l = 1, \cdots, L$. With given receiver matrices $\{\mathbf{W}_k\}$ and source precoding matrices $\{\mathbf{B}_k\}$, the sum-MSE $\text{SMSE} = \sum_{k=1}^{K} \text{MSE}_k$ can be rewritten as a function of $\mathbf{f} = [\mathbf{f}_1^T,\ \mathbf{f}_2^T,\ \cdots, \mathbf{f}_L^T]^T$ as

$$\begin{aligned}\psi_1(\mathbf{f}) \;=\; & \sum_{k=1}^{K}\Big[(\mathbf{O}_{kk}\mathbf{f} - \mathbf{o}_k)^H(\mathbf{O}_{kk}\mathbf{f} - \mathbf{o}_k) + \mathbf{f}^H\mathbf{Q}_k\mathbf{f} \\ & + \sum_{m\neq k}^{K}(\mathbf{O}_{km}\mathbf{f} - \mathbf{q}_{km})^H(\mathbf{O}_{km}\mathbf{f} - \mathbf{q}_{km})\Big] + t_1 \end{aligned} \tag{2.15}$$

where $t_1 = \sum_{k=1}^{K}\sigma_{dk}^2 tr(\mathbf{W}_k^H\mathbf{W}_k)$ is independent of $\mathbf{f}$ and for $k, m = 1, \cdots, K$,

$l = 1, \cdots, L$

$$\mathbf{O}_{km} = [\mathbf{O}_{k,1,m},\ \mathbf{O}_{k,2,m}, \cdots,\ \mathbf{O}_{k,L,m}] \tag{2.16}$$

$$\mathbf{Q}_k = bd\,(\mathbf{Q}_{k1},\ \mathbf{Q}_{k2},\ \cdots,\ \mathbf{Q}_{kL}) \tag{2.17}$$

$$\mathbf{o}_k = \text{vec}(\mathbf{I}_d - \bar{\mathbf{T}}_{kk}), \quad \mathbf{q}_{km} = -\text{vec}(\ \bar{\mathbf{T}}_{km}) \tag{2.18}$$

$$\mathbf{O}_{k,l,m} = \bar{\mathbf{H}}_{lm}^T \otimes \bar{\mathbf{G}}_{kl}, \quad \mathbf{Q}_{kl} = \sigma_{rl}^2 \mathbf{I}_{N_{rl}} \otimes (\bar{\mathbf{G}}_{kl}^H \bar{\mathbf{G}}_{kl}). \tag{2.19}$$

Here, $\bar{\mathbf{G}}_{kl} = \mathbf{W}_{k2}^H \mathbf{G}_{kl}$ is the equivalent MIMO channel matrix between the lth relay node and the kth destination node and $\bar{\mathbf{T}}_{km} = \mathbf{W}_{k1}^H \mathbf{T}_{km} \mathbf{B}_m$ is the equivalent direct link MIMO channel matrix between the mth source node and the kth destination node. The detailed proof of (2.15) is given in Appendix 2.A.

By introducing

$$\mathbf{D}_{ll} = \left(\sum_{m=1}^{K} \mathbf{H}_{lm} \mathbf{B}_m \mathbf{B}_m^H\ \mathbf{H}_{lm}^H + \sigma_{rl}^2 \mathbf{I}_{N_r} \right)^T \otimes \mathbf{I}_{N_{rl}},\ l = 1, \cdots, L \tag{2.20}$$

and $\bar{\mathbf{D}}_l = bd\,(\mathbf{D}_{l1},\ \mathbf{D}_{l2},\ \cdots, \mathbf{D}_{lL})$, where $\mathbf{D}_{lj} = \mathbf{0}$, $l \neq j$, the relay transmit power constraints in (2.10) can be rewritten as

$$\mathbf{f}^H \bar{\mathbf{D}}_l \mathbf{f} \leq P_{rl}, \quad l = 1, \cdots, L. \tag{2.21}$$

Using (2.15) and (2.21), the relay matrix optimization problem can be written as

$$\min_{\mathbf{f}} \quad \psi_1(\mathbf{f}) \tag{2.22a}$$

$$s.t. \quad \mathbf{f}^H \bar{\mathbf{D}}_l \mathbf{f} \leq P_{rl},\ \ l = 1, \cdots, L. \tag{2.22b}$$

The optimization problem (2.22) is a quadratically constrained quadratic programming (QCQP) problem [37]. From (2.19), we can see that $\mathbf{Q}_{kl}$, $k = 1, \cdots, K$, $l = 1, \cdots, L$ are positive semi-definite (PSD) matrices, and thus from (2.17), $\mathbf{Q}_k$, $k = 1, \cdots, K$ are PSD matrices. Moreover, it can be seen from (2.20) that $\mathbf{D}_{ll}$, $l = 1, \cdots, L$ are PSD matrices, and thus, $\bar{\mathbf{D}}_l$, $l = 1, \cdots, L$ are PSD matrices. Therefore, the QCQP problem (2.22) is convex and can be efficiently solved by the interior-point method [37]. In particular, the problem (2.22) can be solved by the CVX MATLAB toolbox for disciplined convex programming [38].

Let us introduce $\mathbf{b}_k = \text{vec}(\mathbf{B}_k)$, $k = 1, \cdots, K$. With given receiver matrices $\{\ \mathbf{W}_k\}$ and relay matrices $\{\mathbf{F}_l\}$, the sum-MSE can be rewritten as a function of

$\mathbf{b} = [\mathbf{b}_1^T,\ \mathbf{b}_2^T,\ \cdots, \mathbf{b}_K^T]^T$ as

$$\Phi_1(\mathbf{b}) = \sum_{k=1}^{K} \Big(\bar{\mathbf{S}}_k\mathbf{b} - \mathrm{vec}(\mathbf{I}_d)\Big)^H \Big(\bar{\mathbf{S}}_k\mathbf{b} - \mathrm{vec}(\mathbf{I}_d)\Big) + \mathbf{b}^H\mathbf{U}\mathbf{b} + t_2. \tag{2.23}$$

where $t_2 = \sum_{k=1}^{K} tr(\mathbf{W}_k^H\mathbf{C}_k\mathbf{W}_k)$ can be ignored in the optimization process as it is independent of $\mathbf{b}$ and

$$\mathbf{U} = bd(\mathbf{U}_1, \mathbf{U}_2, \cdots, \mathbf{U}_K) \tag{2.24}$$

$$\bar{\mathbf{S}}_k = [\mathbf{S}_{k1}, \mathbf{S}_{k2}, \cdots, \mathbf{S}_{kK}] \tag{2.25}$$

$$\mathbf{S}_{kk} = \mathbf{I}_d \otimes \bar{\mathbf{P}}_{kk}, \quad \mathbf{S}_{ki} = \mathbf{0}, i \neq k \tag{2.26}$$

$$\mathbf{U}_k = \mathbf{I}_d \otimes \left(\sum_{m=1, m\neq k}^{K} \bar{\mathbf{P}}_{mk}^H\bar{\mathbf{P}}_{mk} \right). \tag{2.27}$$

Here, $\bar{\mathbf{P}}_{mk} = \mathbf{W}_{m2}^H \sum_{l=1}^{L} \bar{\mathbf{G}}_{ml}\mathbf{F}_l\mathbf{H}_{lk} + \mathbf{W}_{m1}^H\mathbf{T}_{mk}$. The detailed proof of (2.23) is given in Appendix 2.B.

Let us introduce $\mathbf{E}_{ij} = \mathbf{I}_d \otimes \left(\mathbf{H}_{ij}^H\mathbf{F}_i^H\ \mathbf{F}_i\mathbf{H}_{ij}\right)$, $\mathbf{E}_l = bd\left(\mathbf{E}_{l1}, \mathbf{E}_{l2}, \cdots,\ \mathbf{E}_{lK}\right)$, $\bar{\mathbf{E}}_i = bd\left(\bar{\mathbf{E}}_{i1}, \bar{\mathbf{E}}_{i2}, \cdots, \bar{\mathbf{E}}_{iK}\right)$, where $\bar{\mathbf{E}}_{ii} = \mathbf{I}_{dN_s}$ and $\bar{\mathbf{E}}_{ij} = \mathbf{0}, i \neq j$. The optimal $\mathbf{b}$ can be obtained by solving the following problem

$$\min_{\mathbf{b}} \quad \Phi_1(\mathbf{b}) \tag{2.28a}$$

$$s.t. \quad \mathbf{b}^H\bar{\mathbf{E}}_m\mathbf{b} \leq P_{sm}, \quad m = 1, \cdots, K \tag{2.28b}$$

$$\mathbf{b}^H\mathbf{E}_l\mathbf{b} \leq P_{rl} - \sigma_{rl}^2 tr(\mathbf{F}_l\mathbf{F}_l^H), \quad l = 1, \cdots, L. \tag{2.28c}$$

From (2.27), we can see that $\mathbf{U}_k$, $k = 1, \cdots, K$ are PSD matrices, and thus from (2.24), $\mathbf{U}$ is PSD. Moreover, it can be seen that $\bar{\mathbf{E}}_m$, $m = 1, \cdots, K$ and $\mathbf{E}_l$, $l = 1, \cdots, L$ are PSD matrices. Therefore, the problem (2.28) is a convex QCQP problem and can be solved by the CVX MATLAB toolbox [38] for disciplined convex programming.

The steps of applying the proposed Algorithm 2.1 to optimize $\{\mathbf{B}_k\}$, $\{\mathbf{F}_l\}$, and $\{\mathbf{W}_k\}$ are summarized in Table 2.1, where the superscript (n) denotes the variable at the nth iteration, and ε is a small positive number up to which convergence is acceptable. Since all subproblems (2.13), (2.22), and (2.28) are convex, the solution to each subproblem is optimal. Thus, the value of the objective function (2.12a) monotonically decreases after each iteration. Moreover, the value of (2.12a) is lower bounded by at least zero. Therefore, the proposed Algorithm 2.1

Table 2.1: Procedure of solving the problem (2.12) by the proposed Algorithm 2.1

1. Initialize the Algorithm with $\{\mathbf{F}_l^{(0)}\}$ and $\{\mathbf{B}_k^{(0)}\}$ satisfying (2.9) and (2.10); Set $n = 0$.
2. Obtain $\{\mathbf{W}_k^{(n+1)}\}$ based on (2.14) with fixed $\{\ \mathbf{F}_l^{(n)}\}$ and $\{\mathbf{B}_k^{(n)}\}$.
3. Update $\{\mathbf{F}_l^{(n+1)}\}$ through solving the problem (2.22) with given $\{\mathbf{B}_k^{(n)}\}$ and $\{\ \mathbf{W}_k^{(n+1)}\}$.
4. Update $\{\mathbf{B}_k^{(n+1)}\}$ by solving the problem (2.28) with fixed $\{\mathbf{F}_l^{(n+1)}\}$ and $\{\mathbf{W}_k^{(n+1)}\}$.
5. If $\text{SMSE}^{(n)} - \text{SMSE}^{(n+1)} \leq \varepsilon$, then end.
   Otherwise, let $n := n + 1$ and go to step 2.

is guaranteed to converge. Adopting the results from [39], the detailed proof of convergence of the proposed Algorithm 2.1 is given in Appendix 2.C.

2.3.2 Proposed Algorithm 2.2

In the proposed Algorithm 2.1, all source precoding matrices are optimized together through $\mathbf{b}$, while all relay precoding matrices are updated together through $\mathbf{f}$. Since the dimensions of $\mathbf{b}$ and $\mathbf{f}$ are $\sum_{k=1}^{K} N_{sk}d$ and $\sum_{l=1}^{L} N_{rl}^2$, respectively, the computational complexity of solving the QCQP problems (2.22) and (2.28) using the interior point method [40] is $\mathcal{O}\big((\sum_{k=1}^{K} N_{sk}d)^3\big)$ and $\mathcal{O}\big((\sum_{l=1}^{L} N_{rl}^2)^3\big)$, respectively. Therefore, the computational complexity at each iteration of the proposed Algorithm 2.1 is $\mathcal{O}\big((\sum_{k=1}^{K} N_{sk}d)^3 + (\sum_{l=1}^{L} N_{rl}^2)^3\big)$, which can be very high for interference MIMO relay systems with a large K and L. To reduce the per-iteration complexity, in this subsection, we develop an iterative algorithm where each source and relay matrix is optimized individually by fixing all other matrices.

Adopting notations from proposed Algorithm 2.1, with given receiver matrices $\{\mathbf{W}_k\}$, source precoding matrices $\{\mathbf{B}_k\}$, and relay precoding matrices $\mathbf{F}_j$, $j =$

$1, \cdots, L$, $j \neq l$, the sum-MSE can be rewritten as a function of $\mathbf{F}_l$ as

$$\begin{aligned}\text{SMSE} &= \sum_{k=1}^{K} tr\Bigg[(\bar{\mathbf{G}}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lk} - \mathbf{A}_{kl})(\bar{\mathbf{G}}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lk} - \mathbf{A}_{kl})^H \\ &+ \sum_{l=1}^{L} \sigma_{rl}^2 \bar{\mathbf{G}}_{kl}\mathbf{F}_l\ \mathbf{F}_l^H \bar{\mathbf{G}}_{kl}^H + \sum_{m=1,m\neq k}^{K} (\bar{\mathbf{G}}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lm} - \mathbf{D}_{k,l,m}) \\ &\times (\bar{\mathbf{G}}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lm} - \mathbf{D}_{k,l,m})^H\Bigg] + t_1 \end{aligned} \tag{2.29}$$

where for $k, m = 1, \cdots, K$, $l = 1, \cdots, L$

$$\mathbf{A}_{kl} = \mathbf{I}_d - \sum_{j=1,j\neq l}^{L} \bar{\mathbf{G}}_{kj}\mathbf{F}_j\bar{\mathbf{H}}_{jk} - \mathbf{T}_{kk} \tag{2.30}$$

$$\mathbf{D}_{k,l,m} = -\sum_{j=1,j\neq l}^{L} \bar{\mathbf{G}}_{kj}\mathbf{F}_j\bar{\mathbf{H}}_{jm} - \mathbf{T}_{km}.$$

Using the identities in (2.37)-(2.39), the SMSE in (2.29) can be written as

$$\begin{aligned}\psi_2(\mathbf{f}_l) = \sum_{k=1}^{K} \big[&(\mathbf{O}_{k,l,k}\mathbf{f}_l - \mathbf{a}_{kl})^H(\mathbf{O}_{k,l,k}\mathbf{f}_l - \mathbf{a}_{kl}) + \mathbf{f}_l^H\mathbf{Q}_{kl}\mathbf{f}_l + r_{kl} \\ &+ \sum_{m=1,m\neq k}^{K} (\mathbf{O}_{k,l,m}\mathbf{f}_l - \mathbf{d}_{k,l,m})^H(\mathbf{O}_{k,l,m}\mathbf{f}_l - \mathbf{d}_{k,l,m})\big] \end{aligned} \tag{2.31}$$

where for $k, m = 1, \cdots, K$, $l = 1, \cdots, L$

$$\begin{aligned}\mathbf{a}_{kl} &= \text{vec}(\mathbf{A}_{kl}) \\ r_{kl} &= tr\left(\sum_{j=1,j\neq l}^{L} \sigma_{rj}^2 \bar{\mathbf{G}}_{kj}\mathbf{F}_j\ \mathbf{F}_j^H\bar{\mathbf{G}}_{kj}^H + \sigma_{dk}^2\mathbf{W}_k^H\mathbf{W}_k\right) \\ \mathbf{d}_{k,l,m} &= \text{vec}\left(\mathbf{D}_{k,l,m}\right).\end{aligned}$$

Note that since the terms r_{kl} in (2.31) are independent of $\mathbf{f}_l$, they can be ignored when optimizing $\mathbf{f}_l$. The relay transmit power constraint in (2.10) can be rewritten as

$$\mathbf{f}_l^H\mathbf{D}_{ll}\mathbf{f}_l \leq P_{rl}. \tag{2.32}$$

Based on (2.31) and (2.32), the optimal $\mathbf{f}_l$ can be obtained by solving the following problem for each $l = 1, \cdots, L$

$$\min_{\mathbf{f}_l} \psi_2(\mathbf{f}_l) \qquad s.t.\ \ \mathbf{f}_l^H\mathbf{D}_l\mathbf{f}_l \leq P_{rl}. \tag{2.33}$$

The problem (2.33) is a QCQP problem and can be solved effectively using the CVX toolbox.

With given receiver matrices $\{\mathbf{W}_k\}$, relay precoding matrices $\{\mathbf{F}_l\}$, and source precoding matrices $\mathbf{B}_j,\ j = 1, \cdots, K,\ j \neq k$, the SMSE can be rewritten as a function of $\mathbf{b}_k$ as

$$\Phi_2(\mathbf{b}_k) \;=\; (\mathbf{S}_{kk}\mathbf{b}_k - \mathrm{vec}(\mathbf{I}_d))^H(\mathbf{S}_{kk}\mathbf{b}_k - \mathrm{vec}(\mathbf{I}_d)) + \mathbf{b}_k^H\mathbf{U}_k\mathbf{b}_k + z_k$$

where

$$z_k \;=\; \sum_{m=1,m\neq k}^{K}\Big[(\mathbf{S}_{mm}\mathbf{b}_m - \mathrm{vec}(\mathbf{I}_d))^H(\mathbf{S}_{mm}\mathbf{b}_m - \mathrm{vec}(\mathbf{I}_d)) + \mathbf{b}_m^H\mathbf{U}_m\mathbf{b}_m\Big] + t_2$$
$$k = 1, \cdots, K.$$

By introducing $c_{lk} = \sigma_{rl}^2 tr(\mathbf{F}_l\mathbf{F}_l^H) + \sum_{j=1,j\neq k}^{K} \mathbf{b}_j^H\mathbf{E}_{lj}\mathbf{b}_j,\ k = 1, \cdots, K,\ l = 1, \cdots, L$, the optimal $\mathbf{b}_k$ can be obtained by solving the following problem for each $k = 1, \cdots, K$

$$\min_{\mathbf{b}_k} \quad \Phi_2(\mathbf{b}_k) \tag{2.34a}$$
$$s.t. \quad \mathbf{b}_k^H\mathbf{b}_k \leq P_{sk} \tag{2.34b}$$
$$\mathbf{b}_k^H\mathbf{E}_{lk}\mathbf{b}_k \leq P_{rl} - c_{lk}, \quad l = 1, \cdots, L. \tag{2.34c}$$

The problem (2.34) is a QCQP problem and can be solved by the CVX MATLAB toolbox [38] for disciplined convex programming. The steps of using the proposed Algorithm 2.2 to optimize $\{\mathbf{B}_k\}$, $\{\mathbf{F}_l\}$, and $\{\mathbf{W}_k\}$ are summarized in Table 2.2. Similar to the analysis used to the proposed Algorithm 2.1, since all subproblems (2.13), (2.33), and (2.34) are convex, the solution to each subproblem is optimal. Thus, the value of the objective function (2.12a) monotonically decreases after each iteration. Moreover, the value of (2.12a) is lower bounded by at least zero. Therefore, the convergence of the proposed Algorithm 2.2 follows directly from this observation.

Since the dimensions of $\mathbf{b}_k$ and $\mathbf{f}_l$ are $N_{sk}d$ and N_{rl}^2, respectively, the computational complexity of solving the QCQP problems (2.22) and (2.34) is $\mathcal{O}\big((N_{sk}d)^3\big)$ and $\mathcal{O}\big(N_{rl}^6\big)$, respectively. Thus, the computational complexity at each iteration of the proposed Algorithm 2.2 is $\mathcal{O}\big(\sum_{k=1}^{K}(N_{sk}d)^3 + \sum_{l=1}^{L}N_{rl}^6\big)$, which is lower than the per-iteration computational complexity of the proposed Algorithm 2.1. However, we will see through numerical simulations that the proposed Algorithm 2.1 has a better MSE and BER performance than that of the proposed Algorithm 2.2.

Table 2.2: Procedure of solving the problem (2.12) by the proposed Algorithm 2.2

1. Initialize the Algorithm with $\{\mathbf{F}_l^{(0)}\}$ and $\{\mathbf{B}_k^{(0)}\}$ satisfying (2.9) and (2.10); Set $n = 0$.
2. Obtain $\{\mathbf{W}_k^{(n+1)}\}$ based on (2.14) with fixed $\{\ \mathbf{F}_l^{(n)}\}$ and $\{\mathbf{B}_k^{(n)}\}$.
3. For $l = 1, \cdots, L$, update $\mathbf{F}_l^{(n+1)}$ through solving the problem (2.33) with given $\{\mathbf{B}_k^{(n)}\}$, $\{\mathbf{W}_k^{(n+1)}\}$, and $\mathbf{F}_j^{(n)}$, $j = 1, \cdots, L$, $j \neq l$.
4. For $k = 1, \cdots, K$, update $\mathbf{B}_k^{(n+1)}$ by solving the problem (2.34) with fixed $\{\mathbf{F}_l^{(n+1)}\}$, $\{\ \mathbf{W}_k^{(n+1)}\}$, and $\mathbf{B}_j^{(n)}$, $j = 1, \cdots, K$, $j \neq k$.
5. If $\mathrm{SMSE}^{(n)} - \mathrm{SMSE}^{(n+1)} \leq \varepsilon$, then end.
   Otherwise, let $n := n + 1$ and go to step 2.

Such performance-complexity trade-off is very useful for practical interference MIMO relay communication systems.

2.4 Numerical Examples

In this section, we illustrate the performance of the proposed algorithms through numerical simulations. All channel matrices have independent and identically distributed (i.i.d.) complex Gaussian entries with zero mean and unit variance. The noises are i.i.d. Gaussian with zero mean and unit variance. Unless explicitly mentioned, the quadrature phase-shift keying (QPSK) constellations are used to modulate the source symbols. For the sake of simplicity, we set $d = 2$ and assume that all nodes have three antennas, i.e., $N_{sk} = N_{dk} = N_{rl} = 3$, $k = 1, \cdots, K$, $l = 1, \cdots, L$, all source nodes have the same power budget as $P_{sk} = 15$dB, $k = 1, \cdots, K$, and all relay nodes have the same power budget as $P_{rl} = P$, $l = 1, \cdots, L$.

For all simulation examples, the simulation results are averaged over 10^5 independent channel realizations. Unless explicitly mentioned, we assume that there are $K = 4$ source-destination pairs and $L = 5$ relay nodes in the interference MIMO relay system. The proposed algorithms are initialized at $\mathbf{F}_l^{(0)} = \sqrt{P_{rl}/tr\big(\sum_{k=1}^{K} \bar{\mathbf{H}}_{lk}\bar{\mathbf{H}}_{lk}^H + \mathbf{I}_{N_{rl}}\big)}\mathbf{I}_{N_{rl}}$, $l = 1, \cdots, L$, and $\mathbf{B}_k^{(0)} = \sqrt{P_{sk}/N_{sk}}\mathbf{I}_{N_{sk}}$,

$k = 1, \cdots, K$. We would like to mention that when the matrix weight is identity matrix, the performance of the matrix-weighted sum-MSE minimization (WSMSE) algorithm without power control in [24] is similar to the proposed Algorithm 2.2 without considering the direct links.

In the first example, we study the performance of the proposed algorithms at different number of iterations. We also compare the performance of the algorithms when the direct links are ignored. Moreover, the performance of the total leakage minimization (TLM) algorithm in [24] is included as a benchmark. Fig. 2.2 shows the MSE performance of the proposed algorithms versus P at different number of iterations for the first source-destination pair ($k = 1$). It can be seen from Fig. 2.2 that both proposed algorithms perform better than the TLM algorithm when the direct links are ignored. The performance of both proposed algorithm is significantly improved when the direct links are taken into account. For both proposed algorithms, the MSE reduces with increasing number of iterations. Moreover, it can be observed that after ten iterations, the decreasing of the MSE is small. Thus, we suggest that only ten iterations need to be carried out in practice to achieve a good performance-complexity trade-off. It can also be seen from Fig. 2.2 that both proposed algorithms have almost the same MSE performance at convergence.

For this example, the average BER of all source-destination pairs yielded by both proposed algorithms versus P at different number of iterations is shown in Fig. 2.3. It can be clearly seen that the proposed algorithms with direct links yield much smaller BER than the case when the direct links are ignored, especially at high P level. We can also observe from Fig. 2.3 that the proposed Algorithm 2.1 has a slightly better BER performance than the proposed Algorithm 2.2. It can also be seen from Fig. 2.3 that when the direct links are ignored, the proposed algorithms perform better than the TLM algorithm.

In the second example, we study the performance of the proposed algorithms with different number of relay nodes. Fig. 2.4 shows the MSE performance of the proposed Algorithm 2.1 versus P with $L = 5$ and $L = 10$. It can be seen that by doubling the number of relay nodes, a power gain of 10 dB is obtained at the MSE of 0.2.

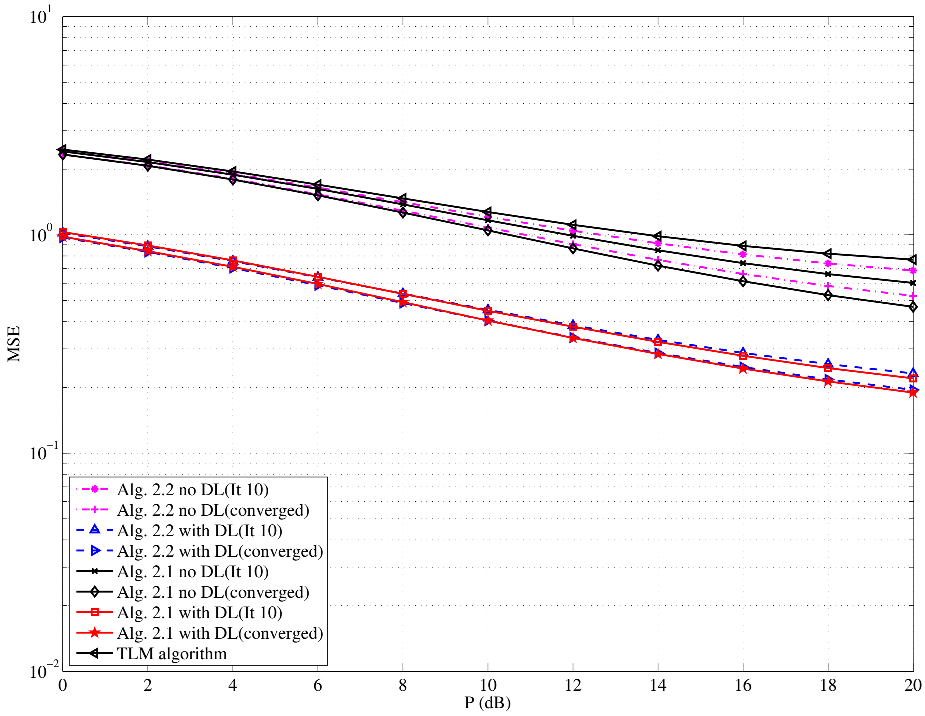

Figure 2.2: Example 2.1: MSE versus P at different number of iterations.

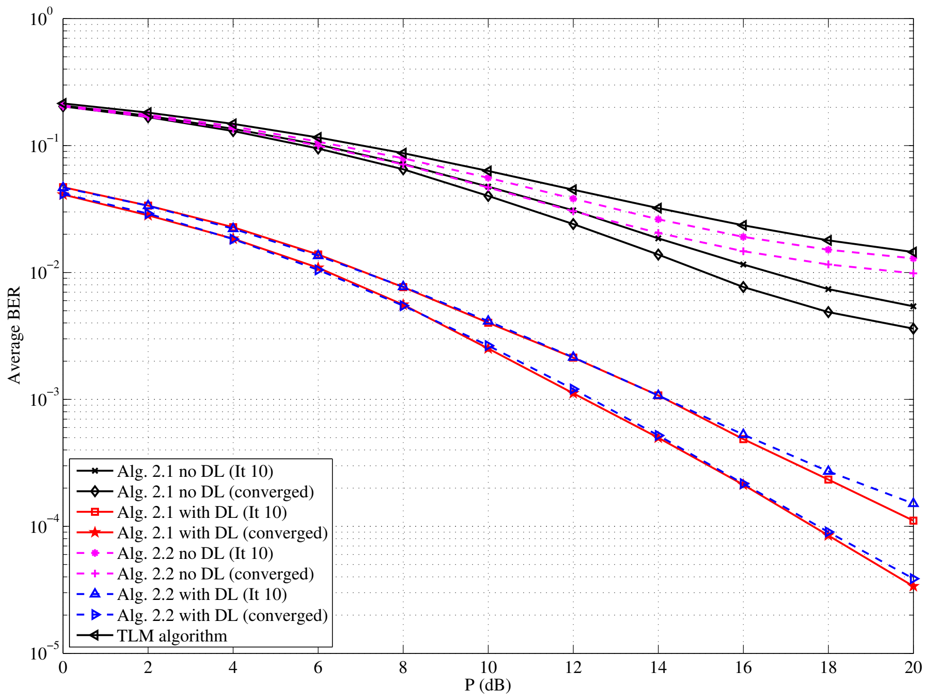

Figure 2.3: Example 2.1: BER versus P at different number of iterations.

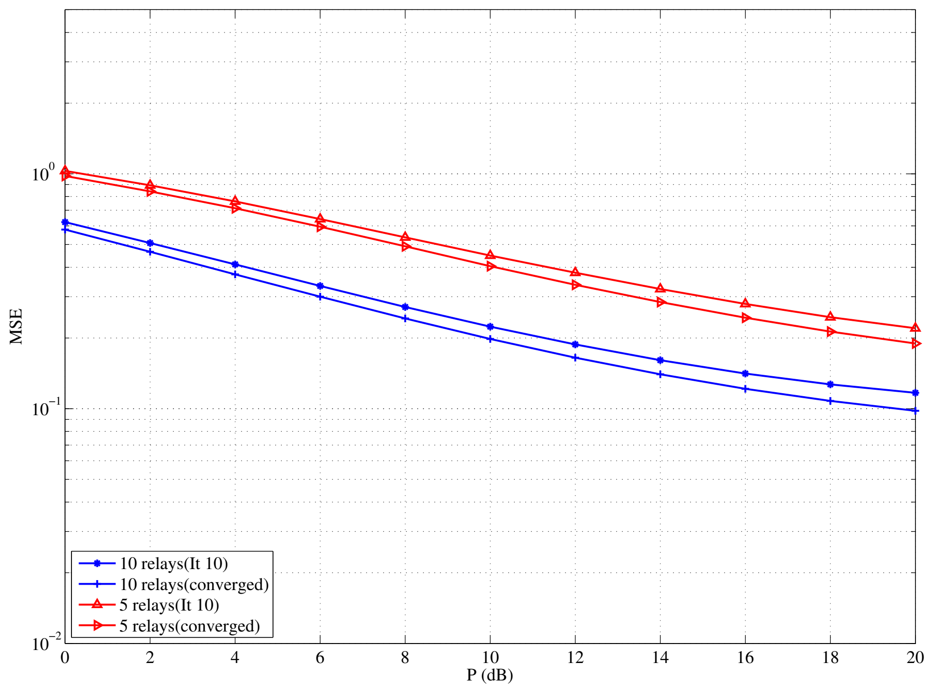

Figure 2.4: Example 2.2: MSE versus P for different L.

For this example, the BER performance of the proposed Algorithm 2.1 with $L = 5$ and $L = 10$ is illustrated in Fig. 2.5. It can be seen that by increasing the number of relay nodes, the system spatial diversity is increased, and thus, a better BER performance is achieved. In particular, we observe that an 8 dB gain is obtained at the BER of 10^{-3} by increasing L from 5 to 10. It is worth to notice that in the case that $L = 5$ relays are used, running the proposed algorithm until converged can significantly improve the performance by 2 dB at BER of 10^{-3}.

In the next example, we study the performance of the proposed algorithms with different number of source-destination pairs. Fig. 2.6 shows the BER performance of both proposed algorithms versus P. Moreover, the BER of both algorithms using the 16QAM modulation scheme is also illustrated in Fig. 2.6. As expected, the system BER is increased when higher order constellations are used. We can also observe from Fig. 2.6 that with a smaller number of source-destination pairs, the number of interference channels decreases which yields a better BER performance. Interestingly, the BER difference between the two proposed algorithm becomes bigger when $K = 3$.

In the last example, we study the performance of the proposed algorithms on

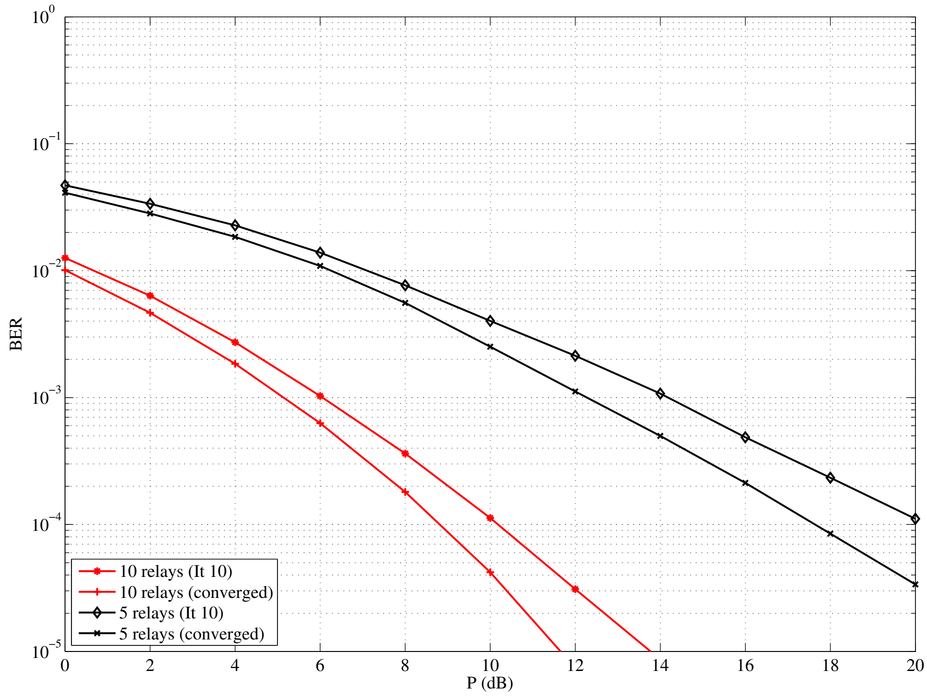

Figure 2.5: Example 2.2: BER versus P for different L.

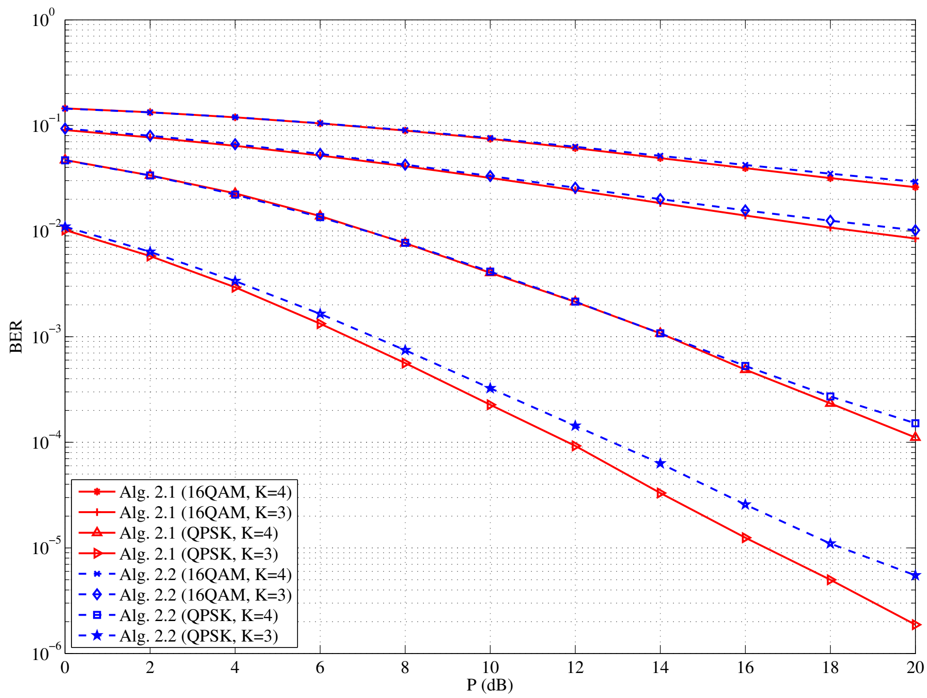

Figure 2.6: Example 2.3: BER versus P for different K.

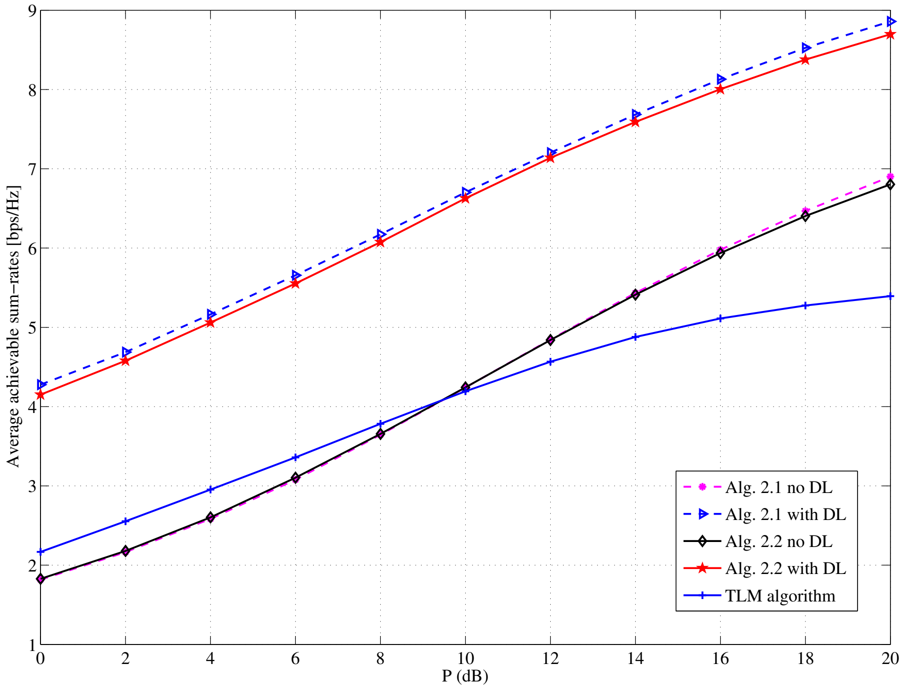

Figure 2.7: Example 2.4: Achievable end-to-end sum-rates.

the achievable end-to-end sum-rates of all source-destination pairs. The achievable end-to-end sum rates can be calculated as

$$\mathbf{SR} = \sum_{k=1}^{K} \log_2 \left(\det \left(\mathbf{I}_d + \tilde{\mathbf{H}}_{kk}^H \left(\mathbf{C}_k + \mathbf{\Xi}_k \right)^{-1} \tilde{\mathbf{H}}_{kk} \right) \right) \tag{2.35}$$

It can be seen from Fig. 2.7 that, as expected, with the direct links taken into account, both proposed algorithms achieve a higher sum-rate. Fig. 2.7 shows that the proposed Algorithm 2.1 yields slightly better rate than the proposed Algorithm 2.2.

2.5 Chapter Summary

In this chapter, we have investigated the transceiver design for interference MIMO relay systems with direct source-destination links based on the MMSE criterion. Two block coordinate descent algorithms have been developed to jointly optimize the source, relay, and receiver matrices under power constrains at each source node and relay node. Numerical simulation results show that the proposed algorithms converge quickly after a few iterations. The system MSE and BER performance can be significantly improved compared with the algorithms without considering the direct links. The proposed Algorithm 2.1 has a better MSE and BER performance than the proposed Algorithm 2.2 at a higher per-iteration computational complexity.

2.A Proof of (2.15)

From (2.11), we have

$$\begin{aligned}\text{SMSE} &= \sum_{k=1}^{K} tr\Bigg[\Big(\sum_{l=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lk}+\bar{\mathbf{T}}_{kk}-\mathbf{I}_d\Big)\\ &\quad\times\Big(\sum_{l=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lk}+\bar{\mathbf{T}}_{kk}-\mathbf{I}_d\Big)^H+\sum_{l=1}^{L}\sigma_{rl}^2\bar{\mathbf{G}}_{kl}\ \mathbf{F}_l\mathbf{F}_l^H\bar{\mathbf{G}}_{kl}^H\\ &\quad+\sigma_{dk}^2\mathbf{W}_{k2}^H\mathbf{W}_{k2}+\sigma_{dk}^2\ \mathbf{W}_{k1}^H\mathbf{W}_{k1}\\ &\quad+\sum_{m\neq k}^{K}\Big(\sum_{l=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lm}+\bar{\mathbf{T}}_{km}\Big)\Big(\sum_{l=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lm}+\bar{\mathbf{T}}_{km}\Big)^H\Bigg].\end{aligned} \tag{2.36}$$

Using the identities of [41]

$$tr(\mathbf{A}^T\mathbf{B}) = (\text{vec}(\mathbf{A}))^T\text{vec}(\mathbf{B}) \tag{2.37}$$

$$tr(\mathbf{A}^H\mathbf{BAC}) = (\text{vec}(\mathbf{A}))^H(\mathbf{C}^T\otimes\mathbf{B})\text{vec}(\mathbf{A}) \tag{2.38}$$

$$\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T\otimes\mathbf{A})\text{vec}(\mathbf{B}) \tag{2.39}$$

the SMSE ((2.36)) can be represented as a function of $\mathbf{f}_l$, $l=1,\cdots,L$, as

$$\begin{aligned}\text{SMSE} &= \sum_{k=1}^{K}\Bigg[\Big(\sum_{l=1}^{L}\mathbf{O}_{k,l,k}\mathbf{f}_l-\ \mathbf{o}_k\Big)^H\Big(\sum_{l=1}^{L}\mathbf{O}_{k,l,k}\mathbf{f}_l-\mathbf{o}_k\Big)\\ &\quad+\sum_{m\neq k}^{K}\Big(\sum_{l=1}^{L}\mathbf{O}_{k,l,m}\mathbf{f}_l-\ \mathbf{q}_{km}\Big)^H\Big(\sum_{l=1}^{L}\mathbf{O}_{k,l,m}\mathbf{f}_l-\ \mathbf{q}_{km}\Big)+\sum_{l=1}^{L}\mathbf{f}_l^H\mathbf{Q}_{kl}\mathbf{f}_l\Bigg]+t_1\\ &= \psi_1(\mathbf{f}).\end{aligned}$$

2.B Proof of (2.23)

From (2.11), we have

$$\begin{aligned}&\text{SMSE}\\&=\sum_{k=1}^{K} tr\Big[\Big(\bar{\mathbf{P}}_{kk}\mathbf{B}_k-\mathbf{I}_d\Big)\Big(\bar{\mathbf{P}}_{kk}\mathbf{B}_k-\mathbf{I}_d\Big)^H+\sum_{m=1,m\neq k}^{K}\bar{\mathbf{P}}_{km}\mathbf{B}_m\mathbf{B}_m^H\bar{\mathbf{P}}_{km}^H\Big]+t_2.\end{aligned} \tag{2.40}$$

Using the identities in (2.37)-(2.39), the SMSE function in (2.40) can be written as

$$\begin{aligned}&\text{SMSE}\\&=\sum_{k=1}^{K}\Big[(\mathbf{S}_{kk}\mathbf{b}_k-\text{vec}(\mathbf{I}_d))^H(\mathbf{S}_{kk}\mathbf{b}_k-\text{vec}(\mathbf{I}_d))+\sum_{m=1,m\neq k}^{K}\mathbf{b}_m^H\Big(\mathbf{I}_d\otimes\bar{\mathbf{P}}_{km}^H\bar{\mathbf{P}}_{km}\Big)\mathbf{b}_m\Big]+t_2\\&=\sum_{k=1}^{K}\Big[(\mathbf{S}_{kk}\mathbf{b}_k-\text{vec}(\mathbf{I}_d))^H(\mathbf{S}_{kk}\mathbf{b}_k-\text{vec}(\mathbf{I}_d))+\mathbf{b}_k^H\mathbf{U}_k\mathbf{b}_k\Big]+t_2\\&=\Phi_1(\mathbf{b}).\end{aligned}$$

2.C Proof of convergence of Proposed Algorithm 2.1

At the convergence point, since $\{\mathbf{F}_l^{(n)}\}$ and $\{\mathbf{B}_k^{(n)}\}$ are the optimal solution to the subproblems (2.22) and (2.28) respectively, we have

$$tr\left(\nabla_{\mathbf{W}}J\left(\mathbf{\Theta}^{(n)}\right)^T\left(\mathbf{W}-\mathbf{W}^{(n)}\right)\right)\geq 0 \tag{2.41}$$

$$tr\left(\nabla_{\mathbf{F}_l}J\left(\mathbf{\Theta}^{(n)}\right)^T\left(\mathbf{F}_l-\mathbf{F}_l^{(n)}\right)\right)\geq 0 \tag{2.42}$$

$$tr\left(\nabla_{\mathbf{B}_k}J\left(\mathbf{\Theta}^{(n)}\right)^T\left(\mathbf{B}_k-\mathbf{B}_k^{(n)}\right)\right)\geq 0 \tag{2.43}$$

where $\mathbf{W}\triangleq[\mathbf{W}_1,\cdots,\mathbf{W}_K]$, $\mathbf{W}^{(n)}\triangleq[\mathbf{W}_1^{(n)},\cdots,\mathbf{W}_K^{(n)}]$, $\mathbf{B}\triangleq[\mathbf{B}_1,\cdots,\mathbf{B}_K]$, $\mathbf{B}^{(n)}\triangleq[\mathbf{B}_1^{(n)},\cdots,\mathbf{B}_K^{(n)}]$, $\mathbf{F}\triangleq[\mathbf{F}_1,\cdots,\mathbf{F}_L]$, $\mathbf{F}^{(n)}\triangleq[\mathbf{F}_1^{(n)},\cdots,\mathbf{F}_L^{(n)}]$, $\mathbf{\Theta}^{(n)}\triangleq[\mathbf{W}^{(n)},\mathbf{B}^{(n)},\mathbf{F}^{(n)}]$ and $\nabla_{\mathbf{X}}J\left(\mathbf{\Theta}^{(n)}\right)$ denotes the gradient of the objective function (2.12a) along the direction of $\mathbf{X}\in\{\mathbf{W}^{(n)},\mathbf{B}^{(n)},\ \mathbf{F}^{(n)}\}$ at $\mathbf{\Theta}^{(n)}$. By summing up (2.41)-(2.43), we have $tr\left(\nabla J\left(\mathbf{\Theta}^{(n)}\right)^T\left(\mathbf{\Theta}-\mathbf{\Theta}^{(n)}\right)\right)\geq 0$, where $\nabla J\left(\mathbf{\Theta}^{(n)}\right)\triangleq[\nabla_{\mathbf{W}}J\left(\mathbf{\Theta}^{(n)}\right),\nabla_{\mathbf{F}}J\left(\mathbf{\Theta}^{(n)}\right),\nabla_{\mathbf{B}}J\left(\mathbf{\Theta}^{(n)}\right)]$, which shows that $\{\mathbf{F}_l\}$, $\{\mathbf{B}_k\}$ and $\{\mathbf{W}_k\}$ may either decrease or maintain but cannot increase the objective function (2.12a). Moreover, the objective function is lower bounded by at least zero. Therefore, the iterative algorithm converges to (at least) a stationary point of (2.12a) [44].

Chapter 3

Simplified Transceiver Design for Interference MIMO Relay Systems

In this chapter, we investigate the robust transceiver design for AF interference MIMO relay communication systems, where multiple transmitter-receiver pairs communicate simultaneously with the aid of a relay node. The aim is to minimize the MSE of the signal waveform estimation at the receivers subjecting to transmission power constraints at the transmitters and the relay node. As the transceiver optimization problem is nonconvex with matrix variables, the globally optimal solution is intractable to obtain. To overcome the challenge, we propose an iterative transceiver design algorithm where the transmitter, relay, and receiver matrices are optimized iteratively by exploiting the optimal structure of the relay precoding matrix. To reduce the computational complexity of optimizing the relay precoding matrix, we propose a simplified relay matrix design through modifying the transmission power constraint at the relay node. The modified relay optimization problem has a closed-form solution. The system model and problem formulation are introduced in Section 3.2. The proposed joint transmitter, relay, and receiver matrices design algorithms are presented in Section 3.3. In Section 3.4, we discuss the ability to extend the proposed algorithms to the more general scenarios such as imperfect CSI and multiple relay nodes. Simulation results are presented in Section 3.5 to demonstrate the performance of the proposed algorithms. Conclusions are drawn in Section 3.6.

3.1 Introduction

The transceiver design in Chapter 2 takes the direct links into account as they may provide valuable spatial diversity. In the scenarios where the direct links are blocked by obstacles or suffer from higher path loss (i.e. source-destination distance is much longer that source-relay or relay-destination distances) [23], they are much weaker than the source-relay or the relay-destination links. Thus, in this chapter, we consider an interference MIMO relay communication system where multiple transmitter-receiver pairs communicate simultaneously with the aid of a single relay node. The relay node has an important role as the direct links are omitted. The transmitters, receivers, and the relay node are equipped with multiple antennas. The CSI of all the source-relay and relay-destination links are assumed to be known at both the transmitters and receivers. Based on the fact that the raw BER is closely related to the MSE of the signal waveform estimation at the receivers, the MMSE is chosen as the design criterion.

Complexity is one of the criteria that mostly used for assessing the performance of a communication system. Complexity of an algorithm can be calculated by the number of instructions or the run time until the algorithm converges. Greater complexity can be equated with greater run time, and in most cases, it also implies greater accuracy or performance. Thus, the objective of new transceiver design is to obtain an overall complexity reduction while satisfying a certain performance requirement. In Chapter 2, we propose the transceiver design based on iterative approach to solve the highly nonconvex optimization problem. The proposed algorithms provide good performance in term of MSE and BER. However, they have high computational complexity. To reduce the complexity, in this chapter, we investigate an approach that exploits the structure of the relay precoding matrix. We propose two algorithms to significantly reduce the computational complexity compared to the algorithms in Chapter 2. By using the iterative approach, the nonconvex optimization problem is decoupled into convex subproblems which can be solved effectively. Furthermore, by modifying the power constraint at the relay node, the relay optimization problem has a closed-form solution.

The design of the two proposed algorithms follows the procedure of designing Algorithm 2.2. Different from Algorithm 2.2, the relay precoding matrix is designed based on its optimal structure which yields smaller computational complexity to optimize. Moreover, in the second algorithm, we propose a simplified relay matrix design through modifying the transmission power constraint at the relay node. The modified relay optimization problem is suboptimal, but it is convex and has a closed-form solution. Simulation results show that the simplified relay matrix design has a slightly worse performance than the optimal relay matrix in terms of the system MSE and BER. However, the computational complexity of the simplified algorithm is much smaller than that of the optimal relay design for interference MIMO relay systems with a large number of transmitter-receiver pairs.

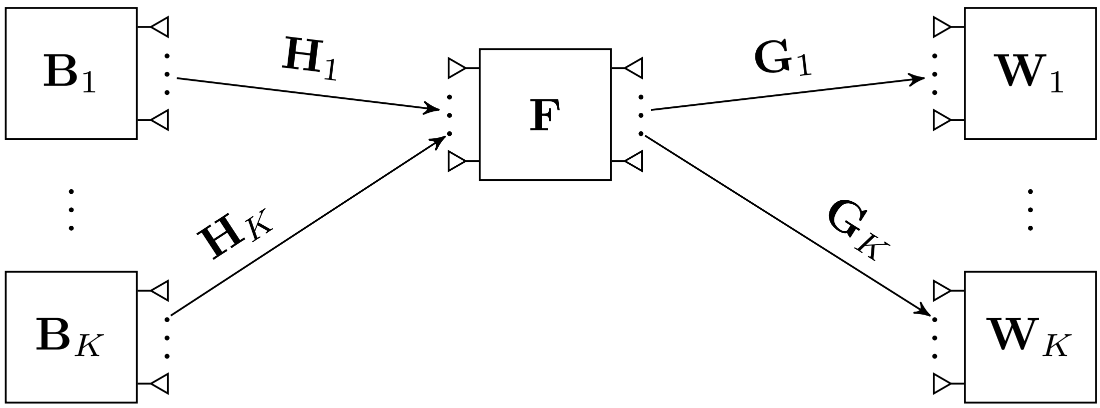

Figure 3.1: Block diagram of an interference MIMO relay system with a single relay node.

3.2 Interference MIMO Relay Systems Model

We consider a two-hop interference MIMO relay communication system where K transmitter-receiver pairs communicate simultaneously with the aid of a single relay node as shown in Fig. 3.1. The direct links between transmitters and receivers are ignored as they undergo much larger path attenuation compared with the links via the relay node [23]. The kth transmitter and receiver are equipped with N_{sk} and N_{dk} antennas, respectively, and the number of antennas at the relay node is N_r.

We assume that the relay node works in the half-duplex mode so the communication between transmitter-receiver pairs is completed in two time slots. In the first time slot, the kth transmitter encodes the $d \times 1$ information-carrying symbol vector $\mathbf{s}_k$ with the $N_{sk} \times d$ transmitter precoding matrix $\mathbf{B}_k$ before transmitting the $N_{sk} \times 1$ precoded signal vector

$$\mathbf{x}_{sk} = \mathbf{B}_k \mathbf{s}_k, \quad k = 1, \cdots, K \tag{3.1}$$

to the relay node. The received signal vector at the relay node is given by

$$\mathbf{y}_r = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{B}_k \mathbf{s}_k + \mathbf{n}_r \tag{3.2}$$

where $\mathbf{H}_k$ is the $N_r \times N_{sk}$ MIMO channel matrix between the kth transmitter and the relay node, $\mathbf{n}_r$ is the $N_r \times 1$ AWGN vector at the relay node with zero mean and covariance matrix $E\left[\mathbf{n}_r \mathbf{n}_r^H\right] = \sigma_r^2 \mathbf{I}_{N_r}$.

In the second time slot, the relay node amplifies the received signal vector with the $N_r \times N_r$ precoding matrix $\mathbf{F}$ as

$$\mathbf{x}_r = \mathbf{F} \mathbf{y}_r. \tag{3.3}$$

The precoded signal vector $\mathbf{x}_r$ is forwarded to the receivers. The received signal vector at the kth receiver is given by

$$\mathbf{y}_{dk} = \mathbf{G}_k \mathbf{x}_r + \mathbf{n}_{dk}, \quad k = 1, \cdots, K \tag{3.4}$$

where $\mathbf{G}_k$ is the $N_{dk} \times N_r$ MIMO channel matrix between the relay node and the kth receiver, $\mathbf{n}_{dk}$ is the $N_{dk} \times 1$ AWGN vector at the kth receiver with zero mean and covariance matrix $E\left[\mathbf{n}_{dk} \mathbf{n}_{dk}^H\right] = \sigma_{dk}^2 \mathbf{I}_{N_{dk}}$.

Due to their simplicity, linear receivers are used to retrieve the transmitted signals, and we have $d \leq N_r$ and $d \leq N_{dk}$, $k = 1, \cdots, K$. The estimated signal vector at the kth receiver can be written as

$$\hat{\mathbf{s}}_k = \mathbf{W}_k^H \mathbf{y}_{dk}, \qquad k = 1, \ldots, K \tag{3.5}$$

where $\mathbf{W}_k$ is the $N_{dk} \times d$ receiver weight matrix. Using (3.2)-(3.4), the estimated signal vector in (3.5) becomes

$$\begin{aligned}\hat{\mathbf{s}}_k &= \mathbf{W}_k^H \left(\mathbf{G}_k \mathbf{F} \sum_{m=1}^{K} \mathbf{H}_m \mathbf{B}_m \mathbf{s}_m + \bar{\mathbf{n}}_{dk} \right) \\ &= \underbrace{\mathbf{W}_k^H \mathbf{G}_k \mathbf{F} \mathbf{H}_k \mathbf{B}_k \mathbf{s}_k}_{\text{desired signal}} + \underbrace{\mathbf{W}_k^H \mathbf{G}_k \mathbf{F} \sum_{m=1, m\neq k}^{K} \mathbf{H}_m \mathbf{B}_m \mathbf{s}_m + \mathbf{W}_k^H \bar{\mathbf{n}}_{dk}}_{\text{interference plus noise}} \end{aligned} \tag{3.6}$$

where $\bar{\mathbf{n}}_{dk} \triangleq \mathbf{G}_k \mathbf{F} \mathbf{n}_r + \mathbf{n}_{dk}$ is the total noise vector at the kth receiver.

The signal vectors sent by transmitters and the signal vector forwarded from the relay node must satisfy the following transmission power constraints

$$tr\left(\mathbf{B}_k E\left[\mathbf{s}_k \mathbf{s}_k^H\right] \mathbf{B}_k^H\right) \leq P_{sk}, \quad k = 1, \cdots, K \tag{3.7}$$

$$tr\left(\mathbf{F} E\left[\mathbf{y}_r \mathbf{y}_r^H\right] \mathbf{F}^H\right) \leq P_r \tag{3.8}$$

where P_{sk} and P_r denote the power budget at the kth transmitter and the relay node, respectively, $E\left[\mathbf{s}_k \mathbf{s}_k^H\right] = \mathbf{I}_d$ is the covariance matrix of the information-carrying symbol vector at the kth transmitter, and the covariance matrix of the received signal vector at the relay node $E\left[\mathbf{y}_r \mathbf{y}_r^H\right] = \sum_{m=1}^{K} \mathbf{H}_m \mathbf{B}_m \mathbf{B}_m^H \mathbf{H}_m^H + \sigma_r^2 \mathbf{I}_{N_r}$.

From (3.6), the MSE of estimating $\mathbf{s}_k$ can be calculated as

$$\begin{aligned} \text{MSE}_k &= tr\left(E\left[(\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^H\right]\right) \\ &= tr\left((\mathbf{W}_k^H \tilde{\mathbf{H}}_k - \mathbf{I}_d)(\mathbf{W}_k^H \tilde{\mathbf{H}}_k - \mathbf{I}_d)^H + \mathbf{W}_k^H \mathbf{C}_{nk} \mathbf{W}_k + \mathbf{W}_k^H \boldsymbol{\Xi}_k \mathbf{W}_k\right) \\ & \quad k = 1, \cdots, K \end{aligned} \tag{3.9}$$

where $\tilde{\mathbf{H}}_k$ is the equivalent MIMO channel matrix of the kth transmitter-receiver pair, $\mathbf{C}_{nk} = E\left[\bar{\mathbf{n}}_{dk} \bar{\mathbf{n}}_{dk}^H\right]$ is the covariance matrix of the equivalent noise, and $\boldsymbol{\Xi}_k$ is the covariance matrix of interference at the kth receiver. They are given respectively as

$$\begin{aligned} \tilde{\mathbf{H}}_k &= \mathbf{G}_k \mathbf{F} \bar{\mathbf{H}}_k, \quad k = 1, \cdots, K \\ \mathbf{C}_{nk} &= E\left[(\mathbf{G}_k \mathbf{F} \mathbf{n}_r + \mathbf{n}_{dk})(\mathbf{G}_k \mathbf{F} \mathbf{n}_r + \mathbf{n}_{dk})^H\right] \\ &= \sigma_r^2 \mathbf{G}_k \mathbf{F} \mathbf{F}^H \mathbf{G}_k^H + \sigma_{dk}^2 \mathbf{I}_{N_{dk}}, \quad k = 1, \cdots, K \\ \boldsymbol{\Xi}_k &= \mathbf{G}_k \mathbf{F} \sum_{m=1, m\neq k}^{K} \bar{\mathbf{H}}_m \bar{\mathbf{H}}_m^H \mathbf{F}^H \mathbf{G}_k^H, \quad k = 1, \cdots, K \end{aligned}$$

where $\bar{\mathbf{H}}_k \triangleq \mathbf{H}_k\mathbf{B}_k$ is the equivalent MIMO channel matrix between the kth transmitter and the relay node.

The aim of this chapter is to optimize the transmitter precoding matrices $\{\mathbf{B}_k\} \triangleq \{\mathbf{B}_k, k = 1, \cdots, K\}$, the relay precoding matrix $\mathbf{F}$, and the receiver weight matrices $\{\mathbf{W}_k\} \triangleq \{\mathbf{W}_k,\ k = 1, \cdots, K\}$, to minimize the sum-MSE of the signal waveform estimation at the receivers under transmission power constraints at the transmitters and the relay node. From (3.7)-(3.9), the optimal transmitter, relay, and receiver matrices design problem can be written as

$$\min_{\{\mathbf{W}_k\},\{\mathbf{B}_k\},\mathbf{F}} \quad \sum_{k=1}^{K} \mathrm{MSE}_k \tag{3.10a}$$

$$\text{s.t.} \quad tr\left(\mathbf{B}_k\mathbf{B}_k^H\right) \leq P_{sk},\ k=1,\cdots,K \tag{3.10b}$$

$$tr\left(\mathbf{F}E[\mathbf{y}_r\mathbf{y}_r^H]\mathbf{F}^H\right) \leq P_r. \tag{3.10c}$$

3.3 Proposed Transmitter, Relay, and Receiver Matrices Design Algorithms

The problem (3.10) is highly nonconvex with matrix variables, and a globally optimal solution is intractable to obtain. To overcome this challenge, in this section, we propose two iterative algorithms to solve the problem (3.10) by optimizing $\{\mathbf{W}_k\}$, $\{\mathbf{B}_k\}$, and $\mathbf{F}$ in an alternating way through solving convex subproblems.

3.3.1 Proposed Tri-Step Algorithm

In each iteration of this algorithm, we first optimize $\{\mathbf{W}_k\}$ based on $\{\mathbf{B}_k\}$ and $\mathbf{F}$ from the previous iteration. Then by using the optimized receiver matrices $\{\mathbf{W}_k\}$ and the transmitter matrices $\{\mathbf{B}_k\}$ from the previous iteration, we optimize the relay matrix $\mathbf{F}$. Finally, we optimize the transmitter matrices $\{\mathbf{B}_k\}$ based on $\{\mathbf{W}_k\}$ and $\mathbf{F}$ obtained from the current iteration.

It can be seen from (3.7) and (3.8) that the power constraints are independent of $\{\mathbf{W}_k\}$. Thus, with given relay matrix and transmitter matrices, the optimal linear receiver matrix which minimizes MSE in (3.9) is the well-known MMSE receiver [36]

$$\mathbf{W}_k = \left(\mathbf{L}_k\mathbf{L}_k^H + \mathbf{C}_{nk} + \mathbf{\Xi}_k\right)^{-1}\mathbf{L}_k, \quad k = 1, \cdots, K. \tag{3.11}$$

With given transmitter matrices $\{\mathbf{B}_k\}$ and receiver matrices $\{\mathbf{W}_k\}$ obtained in (3.11), the sum-MSE SMSE $= \sum_{k=1}^{K} \mathrm{MSE}_k$ can be rewritten as a function of $\mathbf{F}$ as

$$\begin{aligned}
\mathrm{SMSE} \;=\; & \sum_{k=1}^{K} tr\Big(\Big(\mathbf{W}_k^H\mathbf{G}_k\mathbf{F}\bar{\mathbf{H}}_k - \mathbf{I}_d\Big)\Big(\mathbf{W}_k^H\mathbf{G}_k\mathbf{F}\bar{\mathbf{H}}_k - \mathbf{I}_d\Big)^H \\
& + \sigma_r^2\mathbf{W}_k^H\mathbf{G}_k\mathbf{F}\mathbf{F}^H\mathbf{G}_k^H\mathbf{W}_k + \sigma_{dk}^2\mathbf{W}_k^H\mathbf{W}_k \\
& + \mathbf{W}_k^H\mathbf{G}_k\mathbf{F}\sum_{m=1, m\neq k}^{K}\bar{\mathbf{H}}_m\bar{\mathbf{H}}_m^H\mathbf{F}^H\mathbf{G}_k^H\mathbf{W}_k\Big).
\end{aligned} \tag{3.12}$$

Let us introduce

$$\mathbf{H} = [\mathbf{H}_1\mathbf{B}_1, \ldots, \mathbf{H}_K\mathbf{B}_K] = \mathbf{U}_h\mathbf{\Lambda}_h\mathbf{V}_h^H \tag{3.13}$$

$$\mathbf{G} = [\mathbf{G}_1^T, \ldots, \mathbf{G}_K^T]^T = \mathbf{U}_g\mathbf{\Lambda}_g\mathbf{V}_g^H \tag{3.14}$$

as the singular-value decomposition (SVD) of the equivalent transmitters-relay channel $\mathbf{H}$ and the equivalent relay-receivers channel $\mathbf{G}$. The dimensions of $\mathbf{U}_h$, $\mathbf{\Lambda}_h$, $\mathbf{V}_h$ are $N_r \times L_1$, $L_1 \times L_1$, $Kd \times L_1$, respectively and the dimensions of $\mathbf{U}_g$, $\mathbf{\Lambda}_g$, $\mathbf{V}_g$ are $\bar{N}_d \times L_2$, $L_2 \times L_2$, $N_r \times L_2$, respectively, where $\bar{N}_d \triangleq \sum_{k=1}^{K} N_{dk}$, $L_1 \triangleq \min(Kd, N_r)$, and $L_2 \triangleq \min(\bar{N}_d, N_r)$.

It can be shown similar to [47] that the optimal structure of the relay precoding matrix $\mathbf{F}$ is

$$\mathbf{F} = \mathbf{V}_g\mathbf{A}\mathbf{U}_h^H \tag{3.15}$$

where $\mathbf{A}$ is an $L_2 \times L_1$ matrix. It can be seen from (3.15) that we only need to optimize $\mathbf{A}$ in order to optimize $\mathbf{F}$. Since the dimension of $\mathbf{A}$ is smaller than or equal to that of $\mathbf{F}$, optimizing $\mathbf{A}$ may have a smaller computational complexity than directly optimizing $\mathbf{F}$.

From (3.13) and (3.14), we have

$$\mathbf{H}_k\mathbf{B}_k = \mathbf{U}_h\mathbf{\Lambda}_h\mathbf{V}_{h,k}^H, \; \mathbf{G}_k = \mathbf{U}_{g,k}\mathbf{\Lambda}_g\mathbf{V}_g^H, \quad k = 1, \cdots, K \tag{3.16}$$

where $\mathbf{V}_{h,k}$ contains the $((k-1)d+1)$-th to the kd-th rows of $\mathbf{V}_h$, and $\mathbf{U}_{g,k}$ contains the $(\sum_{i=1}^{k-1} N_{di} + 1)$-th to the $(\sum_{i=1}^{k} N_{di})$-th rows of $\mathbf{U}_g$, that is, $\mathbf{V}_h =$

$[\mathbf{V}_{h,1}^T, \ldots, \mathbf{V}_{h,K}^T]^T$, $\mathbf{U}_g = [\mathbf{U}_{g,1}^T, \ldots, \mathbf{U}_{g,K}^T]^T$. Note that $\mathbf{V}_{h,k}$ and $\mathbf{U}_{g,k}$ have dimensions of $d \times L_1$ and $N_{dk} \times L_2$, respectively. By substituting (3.15) into (3.16), we obtain that for $k = 1, \cdots, K$

$$\mathbf{G}_k\mathbf{F}\bar{\mathbf{H}}_k = \mathbf{U}_{g,k}\boldsymbol{\Lambda}_g\mathbf{A}\boldsymbol{\Lambda}_h\mathbf{V}_{h,k}^H \tag{3.17}$$

$$\mathbf{G}_k\mathbf{F}\mathbf{F}^H\mathbf{G}_k^H = \mathbf{U}_{g,k}\boldsymbol{\Lambda}_g\mathbf{A}\mathbf{A}^H\boldsymbol{\Lambda}_g\mathbf{U}_{g,k}^H \tag{3.18}$$

$$\mathbf{G}_k\mathbf{F}\sum_{m=1,m\neq k}^{K}\bar{\mathbf{H}}_m\bar{\mathbf{H}}_m^H\mathbf{F}^H\mathbf{G}_k^H = \mathbf{U}_{g,k}\boldsymbol{\Lambda}_g\mathbf{A}\sum_{m=1,m\neq k}^{K}\boldsymbol{\Lambda}_h\mathbf{V}_{h,m}^H\mathbf{V}_{h,m}\boldsymbol{\Lambda}_h\mathbf{A}^H\boldsymbol{\Lambda}_g\mathbf{U}_{g,k}^H \tag{3.19}$$

Using (3.17)-(3.19), the SMSE in (3.12) becomes

$$\begin{aligned}\text{SMSE} \;=\; & \sum_{k=1}^{K} tr\Big(\Big(\mathbf{W}_k^H\mathbf{U}_{g,k}\boldsymbol{\Lambda}_g\mathbf{A}\boldsymbol{\Lambda}_h\mathbf{V}_{h,k}^H - \mathbf{I}_d\Big)\Big(\mathbf{W}_k^H\mathbf{U}_{g,k}\boldsymbol{\Lambda}_g\mathbf{A}\boldsymbol{\Lambda}_h\mathbf{V}_{h,k}^H - \mathbf{I}_d\Big)^H \\ & + \mathbf{W}_k^H\mathbf{U}_{g,k}\boldsymbol{\Lambda}_g\mathbf{A}\sum_{m=1,m\neq k}^{K}\boldsymbol{\Lambda}_h\mathbf{V}_{h,m}^H\mathbf{V}_{h,m}\boldsymbol{\Lambda}_h\mathbf{A}^H\boldsymbol{\Lambda}_g\mathbf{U}_{g,k}^H\mathbf{W}_k \\ & + \sigma_r^2\mathbf{W}_k^H\mathbf{U}_{g,k}\boldsymbol{\Lambda}_g\mathbf{A}\mathbf{A}^H\boldsymbol{\Lambda}_g\mathbf{U}_{g,k}^H\mathbf{W}_k + \sigma_{dk}^2\mathbf{W}_k^H\mathbf{W}_k\Big)\end{aligned} \tag{3.20}$$

Using the identities of [41]

$$tr(\mathbf{A}^T\mathbf{B}) = (vec(\mathbf{A}))^T vec(\mathbf{B}) \tag{3.21}$$

$$tr(\mathbf{A}^H\mathbf{B}\mathbf{A}\mathbf{C}) = (vec(\mathbf{A}))^H(\mathbf{C}^T \otimes \mathbf{B})vec(\mathbf{A}) \tag{3.22}$$

$$vec(\mathbf{A}\mathbf{B}\mathbf{C}) = (\mathbf{C}^T \otimes \mathbf{A})vec(\mathbf{B}) \tag{3.23}$$

the SMSE (3.20) can be represented as a function of $\mathbf{a} \triangleq vec(\mathbf{A})$ as

$$\text{SMSE} = \sum_{k=1}^{K}\Big[\Big(\mathbf{O}_k\mathbf{a} - vec(\mathbf{I}_d)\Big)^H\Big(\mathbf{O}_k\mathbf{a} - vec(\mathbf{I}_d)\Big) + \mathbf{a}^H\mathbf{Q}_k\mathbf{a} + \mathbf{a}^H\mathbf{S}_k\mathbf{a}\Big] + t_1 \tag{3.24}$$

where $t_1 \triangleq \sum_{k=1}^{K}\sigma_{dk}^2 tr(\mathbf{W}_k^H\mathbf{W}_k)$ does not depend on $\mathbf{a}$, and for $k = 1, \cdots, K$

$$\begin{aligned}\mathbf{O}_k &= \big(\boldsymbol{\Lambda}_h\mathbf{V}_{h,k}^H\big)^T \otimes (\mathbf{W}_k^H\mathbf{U}_{g,k}\boldsymbol{\Lambda}_g) \\ \mathbf{Q}_k &= \sigma_r^2\mathbf{I}_{L_1} \otimes (\boldsymbol{\Lambda}_g\mathbf{U}_{g,k}^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{U}_{g,k}\boldsymbol{\Lambda}_g) \\ \mathbf{S}_k &= \left(\sum_{m=1,m\neq k}^{K}\boldsymbol{\Lambda}_h\mathbf{V}_{h,m}^H\mathbf{V}_{h,m}\boldsymbol{\Lambda}_h\right)^T \otimes (\boldsymbol{\Lambda}_g\mathbf{U}_{g,k}^H\mathbf{W}_k\mathbf{W}_k^H\mathbf{U}_{g,k}\boldsymbol{\Lambda}_g)\end{aligned}$$

From (3.15), the relay node transmission power constraint (3.8) can be written as

$$tr(\mathbf{F}E\left[\mathbf{y}_r\mathbf{y}_r^H\right]\mathbf{F}^H) = tr(\mathbf{A}(\boldsymbol{\Lambda}_h^2 + \sigma_r^2\mathbf{I}_{L_1})\mathbf{A}^H) \tag{3.25}$$

By introducing $\mathbf{D} = (\mathbf{\Lambda}_h^2 + \sigma_r^2 \mathbf{I}_{L_1}) \otimes \mathbf{I}_{L_2}$, (3.25) can be rewritten as

$$\mathbf{a}^H \mathbf{D} \mathbf{a} \leq P_r \tag{3.26}$$

From (3.24) and (3.26), the relay matrix optimization problem can be written as

$$\min_{\mathbf{a}} \quad \text{SMSE} \tag{3.27a}$$

$$\text{s.t.} \quad \mathbf{a}^H \mathbf{D} \mathbf{a} \leq P_r \tag{3.27b}$$

The problem (3.27) is a QCQP problem [37], which is a convex optimization problem and can be efficiently solved by the interior-point method [37]. The problem (3.27) can be solved by the CVX MATLAB toolbox for disciplined convex programming [38].

With given receiver matrices $\{\mathbf{W}_k\}$ and the relay matrix $\mathbf{F}$, the sum-MSE can be rewritten as a function of $\{\mathbf{B}_k\}$ as

$$\begin{aligned} \text{SMSE} \;\; = \;\; & \sum_{k=1}^{K} tr\Big(\Big(\bar{\mathbf{G}}_k \mathbf{F} \mathbf{H}_k \mathbf{B}_k - \mathbf{I}_d\Big)\Big(\bar{\mathbf{G}}_k \mathbf{F} \mathbf{H}_k \mathbf{B}_k - \mathbf{I}_d\Big)^H \\ & + \bar{\mathbf{G}}_k \mathbf{F} \sum_{m=1, m\neq k}^{K} \mathbf{H}_m \mathbf{B}_m \mathbf{B}_m^H \mathbf{H}_m^H \mathbf{F}^H \bar{\mathbf{G}}_k^H \Big) + t_2 \end{aligned} \tag{3.28}$$

where $\bar{\mathbf{G}}_k = \mathbf{W}_k^H \mathbf{G}_k$ and $t_2 \triangleq \sum_{k=1}^{K} tr(\mathbf{W}_k^H \mathbf{C}_{nk} \mathbf{W}_k)$ can be ignored in the optimization process as it does not depend on $\{\mathbf{B}_k\}$.

Using the identities in (3.21)-(3.23), the SMSE function in (3.28) can be written as a function of $\mathbf{b}_k \triangleq vec(\mathbf{B}_k)$ as

$$\begin{aligned} \text{SMSE} = & \sum_{k=1}^{K} \Big[(\mathbf{S}_k \mathbf{b}_k - vec(\mathbf{I}_d))^H (\mathbf{S}_k \mathbf{b}_k - vec(\mathbf{I}_d)) \\ & + \sum_{m=1, m\neq k}^{K} \mathbf{b}_m^H \Big(\mathbf{I}_d \otimes \mathbf{H}_m^H \mathbf{F}^H \bar{\mathbf{G}}_k^H \bar{\mathbf{G}}_k \mathbf{F} \mathbf{H}_m\Big) \mathbf{b}_m \Big] + t_2 \\ = & \sum_{k=1}^{K} \Big[\Big(\mathbf{S}_k \mathbf{b}_k - vec(\mathbf{I}_d)\Big)^H \Big(\mathbf{S}_k \mathbf{b}_k - vec(\mathbf{I}_d)\Big) + \mathbf{b}_k^H \mathbf{T}_k \mathbf{b}_k \Big] + t_2 \end{aligned} \tag{3.29}$$

where for $k = 1, \cdots, K$

$$\begin{aligned} \mathbf{S}_k \;&\triangleq\; \mathbf{I}_d \otimes (\bar{\mathbf{G}}_k \mathbf{F} \mathbf{H}_k) \\ \mathbf{T}_k \;&\triangleq\; \mathbf{I}_d \otimes \sum_{m=1, m\neq k}^{K} \mathbf{H}_k^H \mathbf{F}^H \bar{\mathbf{G}}_m^H \bar{\mathbf{G}}_m \mathbf{F} \mathbf{H}_k \end{aligned}$$

By introducing $\mathbf{T} \triangleq bd(\mathbf{T}_1, \cdots, \mathbf{T}_K)$ and $\bar{\mathbf{S}}_k \triangleq [\mathbf{S}_{k1}, \cdots, \mathbf{S}_{kK}]$, where $\mathbf{S}_{kk} = \mathbf{S}_k$ and $\mathbf{S}_{ki} = \mathbf{0}$, $i \neq k$, the SMSE function (3.29) can be written as a function of $\mathbf{b} = [\mathbf{b}_1^T,\ \mathbf{b}_2^T,\ \cdots, \mathbf{b}_K^T]^T$ as

$$\Phi_1(\mathbf{b}) = \sum_{k=1}^{K} \Big(\bar{\mathbf{S}}_k\mathbf{b} - vec(\mathbf{I}_d)\Big)^H \Big(\bar{\mathbf{S}}_k\mathbf{b} - vec(\mathbf{I}_d)\Big) + \mathbf{b}^H\mathbf{T}\mathbf{b}. \tag{3.30}$$

Let us introduce $\mathbf{E}_j = \mathbf{I}_d \otimes \big(\mathbf{H}_j^H\mathbf{F}^H\mathbf{F}\mathbf{H}_j\big)$, $\mathbf{E} = bd\,(\mathbf{E}_1, \mathbf{E}_2, \cdots, \mathbf{E}_K)$, $\bar{\mathbf{E}}_i = bd\,\big(\bar{\mathbf{E}}_{i1}, \bar{\mathbf{E}}_{i2}, \cdots, \bar{\mathbf{E}}_{iK}\big)$, where $\bar{\mathbf{E}}_{ii} = \mathbf{I}_{dN_s}$ and $\bar{\mathbf{E}}_{ij} = \mathbf{0}, i \neq j$. The optimal $\mathbf{b}$ can be obtained by solving the following problem

$$\min_{\mathbf{b}} \quad \Phi_1(\mathbf{b}) \tag{3.31a}$$

$$\text{s.t.} \quad \mathbf{b}^H\bar{\mathbf{E}}_k\mathbf{b} \leq P_{sk}, \qquad k = 1, \cdots, K \tag{3.31b}$$

$$\mathbf{b}^H\mathbf{E}\mathbf{b} \leq P_r - \sigma_r^2 tr(\mathbf{F}\mathbf{F}^H) \tag{3.31c}$$

The problem (3.31) is a QCQP problem and can be solved by the CVX MATLAB toolbox [38] for disciplined convex programming.

The steps of applying the proposed tri-step algorithm to optimize $\{\mathbf{B}_k\}$, $\mathbf{F}$, and $\{\mathbf{W}_k\}$ are summarized in Table 3.1, where the superscript (n) denotes the variable at the nth iteration, and ε is a small positive number up to which convergence is acceptable. By adopting the proof for convergence of the proposed Algorithm 2.1 in 2.A, since all subproblems (3.11), (3.27), and (3.31) are convex, the iterative algorithm converges to (at least) a stationary point of (3.10a).

Now we analyze the computational complexity of the proposed tri-step algorithm assuming $Kd \leq N_r$ (i.e., $L_1 = Kd$) and $\bar{N}_d \leq N_r$ (i.e., $L_2 = \bar{N}_d$). Since the dimension of $\mathbf{b}$ is $\sum_{k=1}^{K} N_{sk}d$ and the dimension of $\mathbf{a}$ is $\bar{N}_dKd = \sum_{k=1}^{K} N_{dk}Kd$, the computational complexity of solving the QCQP problems (3.27) and (3.31) using the interior point method [40] is $\mathcal{O}\big((\sum_{k=1}^{K} N_{dk}Kd)^3\big)$ and $\mathcal{O}\big((K+1)^{\frac{1}{2}}(\sum_{k=1}^{K} N_{sk}d)^3\big)$, respectively. Therefore, the computational complexity at each iteration of the proposed tri-step algorithm is $\mathcal{O}\big((\sum_{k=1}^{K} N_{dk}Kd)^3 + (K+1)^{\frac{1}{2}}(\sum_{k=1}^{K} N_{sk}d)^3\big)$. It can be seen that the per-iteration computational complexity of the tri-step algorithm can be very high for interference MIMO relay systems with a large number of users K, and in this case, the complexity is dominated by the relay matrix optimization.

Table 3.1: Procedure of solving the problem (3.10) by the proposed tri-step algorithm.

1. Initialize the algorithm with $\mathbf{F}^{(0)}$ and $\{\mathbf{B}_k^{(0)}\}$ satisfying (3.7) and (3.8); Set $n = 0$.
2. Obtain $\{\mathbf{W}_k^{(n+1)}\}$ based on (3.11) with fixed $\mathbf{F}^{(n)}$ and $\{\mathbf{B}_k^{(n)}\}$.
3. Update $\mathbf{A}$ through solving the problem (3.27) with given $\{\mathbf{B}_k^{(n)}\}$ and $\{\mathbf{W}_k^{(n+1)}\}$.
4. Update $\mathbf{F}^{(n+1)}$ based on (3.15) from the optimal $\mathbf{A}$.
5. Update $\{\mathbf{B}_k^{(n+1)}\}$ by solving the problem (3.31) with fixed $\mathbf{F}^{(n+1)}$ and $\{\mathbf{W}_k^{(n+1)}\}$.
6. If $\text{MSE}^{(n)} - \text{MSE}^{(n+1)} \leq \varepsilon$, then end.
   Otherwise, let $n := n + 1$ and go to Step 2.

3.3.2 Simplified Relay Matrix Design

To reduce the computational complexity of optimizing the relay matrix, in this subsection, we develop a simplified relay matrix design algorithm by modifying the power constraint at the relay node, which enables the relay optimization problem to be decomposed into convex subproblems with closed-form solutions.

Substituting the MMSE receiver in (3.11) to (3.20) and using (3.17)-(3.19), the SMSE can be rewritten as

$$\begin{aligned}
&\text{SMSE} \\
&= \sum_{k=1}^{K} tr\Big(\mathbf{I}_d - \bar{\mathbf{H}}_k^H \mathbf{F}^H \mathbf{G}_k^H (\mathbf{G}_k \mathbf{F} \bar{\mathbf{H}}_k \bar{\mathbf{H}}_k^H \mathbf{F}^H \mathbf{G}_k^H + \mathbf{C}_{nk} + \mathbf{\Xi}_k)^{-1} \mathbf{G}_k \mathbf{F} \bar{\mathbf{H}}_k\Big) \\
&= \sum_{k=1}^{K} tr\Big(\mathbf{I}_d - (\mathbf{U}_{g,k} \mathbf{\Lambda}_g \mathbf{A} \mathbf{\Lambda}_h \mathbf{V}_{h,k}^H)^H \Big(\sum_{m=1}^{K} \mathbf{U}_{g,m} \mathbf{\Lambda}_g \mathbf{A} \mathbf{\Lambda}_h \mathbf{V}_{h,m}^H (\mathbf{U}_{g,m} \mathbf{\Lambda}_g \mathbf{A} \mathbf{\Lambda}_h \mathbf{V}_{h,m}^H)^H \\
&\quad + \sigma_r^2 \mathbf{U}_{g,k} \mathbf{\Lambda}_g \mathbf{A} \mathbf{A}^H \mathbf{\Lambda}_g \mathbf{U}_{g,k}^H + \sigma_{dk}^2 \mathbf{I}_{N_{dk}}\Big)^{-1} \mathbf{U}_{g,k} \mathbf{\Lambda}_g \mathbf{A} \mathbf{\Lambda}_h \mathbf{V}_{h,k}^H\Big).
\end{aligned} \tag{3.32}$$

Let us introduce

$$\mathbf{\Lambda}_g \mathbf{A} = \mathbf{U}_g^H \mathbf{C} = \sum_{k=1}^{K} \mathbf{U}_{g,k}^H \mathbf{C}_k \tag{3.33}$$

where $\mathbf{C} = [\mathbf{C}_1^T, \mathbf{C}_1^T, \cdots, \mathbf{C}_K^T]^T$ and $\mathbf{C}_k$ is an $N_{dk} \times L_1$ matrix. Since $\mathbf{U}_g$ is a unitary matrix, for any $\mathbf{A}$, we have $\mathbf{C} = \mathbf{U}_g \mathbf{\Lambda}_g \mathbf{A}$. Thus, instead of optimizing $\mathbf{A}$, we can optimize $\{\mathbf{C}_k\} \triangleq \{\mathbf{C}_1, \cdots, \mathbf{C}_K\}$.

Using (3.33), the optimal $\mathbf{F}$ in (3.15) is

$$\mathbf{F} = \mathbf{V}_g \mathbf{\Lambda}_g^{-1} \mathbf{U}_g^H \mathbf{C} \mathbf{U}_h^H. \tag{3.34}$$

By substituting (3.33) back into (3.32), we obtained the SMSE as a function of $\{\mathbf{C}_k\}$ as

$$\text{SMSE} = \sum_{k=1}^{K} \psi_k(\mathbf{C}_k) \tag{3.35}$$

where

$$\begin{aligned} \psi_k(\mathbf{C}_k) \;\; = \;\; & tr\Big(\mathbf{I}_d - \mathbf{V}_{h,k} \mathbf{\Lambda}_h \mathbf{C}_k^H \Big(\sum_{m=1}^{K} \mathbf{C}_k \mathbf{\Lambda}_h \mathbf{V}_{h,m}^H \mathbf{V}_{h,m} \mathbf{\Lambda}_h \mathbf{C}_k^H \\ & + \sigma_r^2 \mathbf{C}_k \mathbf{C}_k^H + \sigma_{dk}^2 \mathbf{I}_{N_{dk}} \Big)^{-1} \mathbf{C}_k \mathbf{\Lambda}_h \mathbf{V}_{h,k}^H \Big). \end{aligned} \tag{3.36}$$

Interestingly, it can be seen from (3.35) and (3.36) that the MSE of the kth transmitter-receiver pair ψ_k is a function of $\mathbf{C}_k$ only. In other words, the objective function is decomposed in terms of the optimization variable.

From (3.33), the transmission power constraint at the relay node (3.25) can be written as

$$tr(\mathbf{A}(\mathbf{\Lambda}_h^2 + \sigma_r^2 \mathbf{I}_{L_1})\mathbf{A}^H) = tr(\mathbf{C}^H \mathbf{\Pi} \mathbf{C} \mathbf{\Psi}) \leq P_r \tag{3.37}$$

where $\mathbf{\Pi} = \mathbf{U}_g \mathbf{\Lambda}_g^{-2} \mathbf{U}_g^H$ and $\mathbf{\Psi} = \mathbf{\Lambda}_h^2 + \sigma_r^2 \mathbf{I}_{L_1}$. It can be seen from (3.37) that $\mathbf{C}_k$, $k = 1, \cdots, K$, are coupled through the power constraint. We propose to modify the power constraint (3.37) by applying the inequality of $tr(\mathbf{AB}) \leq tr(\mathbf{A})tr(\mathbf{B})$. The transmit power at the relay node becomes

$$tr(\mathbf{C}^H \mathbf{\Pi} \mathbf{C} \mathbf{\Psi}) \leq tr(\mathbf{C} \mathbf{\Psi} \mathbf{C}^H) tr(\mathbf{\Pi}). \tag{3.38}$$

Then the power constraint in (3.37) is modified to be

$$\sum_{k=1}^{K} tr(\mathbf{C}_k \mathbf{\Psi} \mathbf{C}_k^H) \leq P_r / tr(\mathbf{\Lambda}_g^{-2}). \tag{3.39}$$

In fact, (3.39) imposes a stricter transmission power constraint at the relay node, i.e., if (3.39) holds, the original power constraint (3.37) is also satisfied.

Based on (3.36) and (3.39), the modified relay matrix optimization problem can be written as

$$\min_{\{\mathbf{C}_k\}} \quad \sum_{k=1}^{K} \psi_k(\mathbf{C}_k) \tag{3.40a}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} tr(\mathbf{C}_k \mathbf{\Psi} \mathbf{C}_k^H) \leq \bar{P}_r \tag{3.40b}$$

where $\bar{P}_r = P_r/tr(\mathbf{\Lambda}_g^{-2})$ is the modified power budget at the relay node. We can see from (3.40a) and (3.40b) that the relay matrix optimization problem can be decomposed into K subproblems where the kth subproblem is to optimize $\mathbf{C}_k$ as

$$\min_{\mathbf{C}_k} \quad \psi_k(\mathbf{C}_k) \tag{3.41a}$$

$$\text{s.t.} \quad tr(\mathbf{C}_k\mathbf{\Psi}\mathbf{C}_k^H) \leq P_{rk}. \tag{3.41b}$$

Here $P_{rk} \geq 0$, $k = 1, \cdots, K$, and $\sum_{k=1}^{K} P_{rk} = \bar{P}_r$. Interestingly, by adopting the derivation from [18], in the following, we show that the problem (3.41) can be viewed as the MMSE-based relay matrix optimization problem for a single-user two-hop MIMO relay system, which is convex and has a closed-form solution.

Let us introduce the following matrices for $k = 1, \cdots, K$

$$\mathbf{J}_{rk} = \sum_{m=1, m\neq k}^{K} \mathbf{\Lambda}_h \mathbf{V}_{h,m}^H \mathbf{V}_{h,m} \mathbf{\Lambda}_h + \sigma_r^2 \mathbf{I}_{L_1} \tag{3.42}$$

$$\mathbf{X}_k = \mathbf{J}_{rk}^{-\frac{1}{2}} \mathbf{\Lambda}_h \mathbf{V}_{h,k}^H \tag{3.43}$$

$$\mathbf{Y}_k = \mathbf{C}_k \mathbf{J}_{rk}^{\frac{1}{2}} \tag{3.44}$$

where the dimensions of $\mathbf{X}_k$ and $\mathbf{Y}_k$ are $L_1 \times d$ and $N_{dk} \times L_1$, respectively. The MSE for the kth transmitter-receiver pair becomes

$$\begin{aligned} f_k(\mathbf{Y}_k) &= tr\Big(\mathbf{I}_d - \mathbf{X}_k^H\mathbf{Y}_k^H\Big(\mathbf{Y}_k\mathbf{X}_k\mathbf{X}_k^H\mathbf{Y}_k^H + \mathbf{Y}_k\mathbf{Y}_k^H + \sigma_{dk}^2\mathbf{I}_{N_{dk}}\Big)^{-1}\mathbf{Y}_k\mathbf{X}_k\Big) \\ &= tr((\mathbf{I}_d + \mathbf{X}_k^H\mathbf{Y}_k^H(\mathbf{Y}_k\mathbf{Y}_k^H + \sigma_{dk}^2\mathbf{I}_{N_{dk}})^{-1}\mathbf{Y}_k\mathbf{X}_k)^{-1}) \end{aligned} \tag{3.45}$$

and the power constraint (3.41b) becomes

$$tr(\mathbf{Y}_k(\mathbf{X}_k\mathbf{X}_k^H + \mathbf{I}_{L_1})\mathbf{Y}_k^H) \leq P_{rk}. \tag{3.46}$$

Using (3.45) and (3.46), the problem (3.41) can be equivalently rewritten as

$$\min_{\mathbf{Y}_k} \quad f_k(\mathbf{Y}_k) \tag{3.47a}$$

$$\text{s.t.} \quad tr(\mathbf{Y}_k(\mathbf{X}_k\mathbf{X}_k^H + \mathbf{I}_{L_1})\mathbf{Y}_k^H) \leq P_{rk}. \tag{3.47b}$$

The problem (3.47) is the MMSE-based relay matrix optimization problem for a single-user two-hop MIMO relay system [18], [42] with the first hop channel $\mathbf{X}_k$, the relay matrix $\mathbf{Y}_k$ and the second hop channel $\mathbf{I}_{N_{dk}}$. It can be shown similar to [18], [42] that the optimal structure of $\mathbf{Y}_k$ is

$$\mathbf{Y}_k = \big[\mathbf{I}_d, \ \mathbf{0}_{d\times(N_{dk}-d)}\big]^T \mathbf{\Lambda}_{y,k}\mathbf{U}_{x,k}^H \tag{3.48}$$

where $\mathbf{X}_k = \mathbf{U}_{x,k}\mathbf{\Lambda}_{x,k}\mathbf{V}_{x,k}^H$ is the SVD of $\mathbf{X}_k$, and the dimensions of $\mathbf{U}_{x,k}$, $\mathbf{\Lambda}_{x,k}$ and $\mathbf{V}_{x,k}$ are $L_1 \times d$, $d \times d$, and $d \times d$, respectively.

By substituting (3.48) back into (3.40a) and (3.40b), the relay matrix optimization problem becomes

$$\min_{\{\mathbf{\Lambda}_{y,k}\}} \quad \sum_{k=1}^{K} tr\Big((\mathbf{I}_d + \mathbf{\Lambda}_{x,k}^2(\mathbf{I}_d + \sigma_{dk}^2\mathbf{\Lambda}_{y,k}^{-2})^{-1})^{-1}\Big) \tag{3.49a}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} tr(\mathbf{\Lambda}_{y,k}^2(\mathbf{\Lambda}_{x,k}^2 + \mathbf{I}_d)) \leq \bar{P}_r \tag{3.49b}$$

where $\{\mathbf{\Lambda}_{y,k}\} \triangleq \{\mathbf{\Lambda}_{y,1}, \cdots, \mathbf{\Lambda}_{y,K}\}$. The problem (3.49) can be equivalently rewritten as the following problem with scalar variables

$$\min_{\{\lambda_{y,k,i}\}} \quad \sum_{k=1}^{K}\sum_{i=1}^{d}\left(1 + \frac{\lambda_{x,k,i}^2\lambda_{y,k,i}^2}{\lambda_{y,k,i}^2 + \sigma_{dk}^2}\right)^{-1} \tag{3.50a}$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\sum_{i=1}^{d}\lambda_{y,k,i}^2(\lambda_{x,k,i}^2 + 1) \leq \bar{P}_r \tag{3.50b}$$

$$\lambda_{y,k,i} \geq 0,\ k = 1, \cdots, K,\ i = 1, \cdots, d \tag{3.50c}$$

where $\lambda_{x,k,i}$ and $\lambda_{y,k,i}$, $i = 1, \cdots, d$, are the ith diagonal element of $\mathbf{\Lambda}_{x,k}$ and $\mathbf{\Lambda}_{y,k}$, respectively, and $\{\lambda_{y,k,i}\} \triangleq \{\lambda_{y,1,1}, \cdots, \lambda_{y,K,d}\}$.

The problem (3.50) has the well-known water-filling solution and is given by

$$\lambda_{y,k,i} = \sqrt{\frac{1}{\lambda_{x,k,i}^2 + 1}\left[\sqrt{\frac{\sigma_{dk}^2\lambda_{x,k,i}^2}{(\lambda_{x,k,i}^2 + 1)\beta}} - \sigma_{dk}^2\right]^{\dagger}}$$
$$k = 1, \cdots, K,\ i = 1, \cdots, d \tag{3.51}$$

where $[x]^{\dagger} \triangleq \max(x, 0)$, and $\beta > 0$ is the solution to the following equation

$$\sum_{k=1}^{K}\sum_{i=1}^{d}\left[\sqrt{\frac{\sigma_{dk}^2\lambda_{x,k,i}^2}{(\lambda_{x,k,i}^2 + 1)\beta}} - \sigma_{dk}^2\right]^{\dagger} = \bar{P}_r. \tag{3.52}$$

As the left-hand side of (3.52) is a non-increasing function of β, it can be efficiently solved by the bisection method [37]. Finally, the relay precoding matrix can be obtaining from (3.34), (3.44), (3.48), and (3.51).

The transmitter matrices $\{\mathbf{B}_k\}$ and receiver matrices $\{\mathbf{W}_k\}$ can be optimized through (3.31) and (3.11), respectively. The steps of applying the simplified relay matrix design to solve the transceiver optimization problem are summarized in

Table 3.2: Procedure of solving the problem (3.10) through the simplified relay matrix design.

1. Initialize the algorithm with $\mathbf{F}^{(0)}$ and $\{\mathbf{B}_k^{(0)}\}$ satisfying (3.7) and (3.8); Set $n = 0$.
2. Obtain $\{\mathbf{W}_k^{(n+1)}\}$ based on (3.11) with fixed $\mathbf{F}^{(n)}$ and $\{\mathbf{B}_k^{(n)}\}$.
3. Solve the problems (3.50) with given $\{\mathbf{B}_k^{(n)}\}$ to find $\{\lambda_{y,k,i}\}$ and update $\mathbf{F}^{(n+1)}$ through (3.34), (3.44), (3.48), and (3.51).
4. Update $\{\mathbf{B}_k^{(n+1)}\}$ by solving the problem (3.31) with fixed $\mathbf{F}^{(n+1)}$ and $\{\mathbf{W}_k^{(n+1)}\}$.
5. If $\mathrm{MSE}^{(n)} - \mathrm{MSE}^{(n+1)} \leq \varepsilon$, then end.
   Otherwise, let $n := n + 1$ and go to Step 2.

Table 3.2. Since the dimension of $\{\lambda_{y,k,i}\}$ is Kd, the computational complexity of solving the problem (3.50) is $\mathcal{O}(Kd)$. When $L_1 = Kd$ (as in the complexity analysis in Section 3.3.1), the SVD of $\mathbf{X}_k$ has a complexity order of $\mathcal{O}(Kd^3)$. Therefore, the complexity of the simplified relay matrix design is $\mathcal{O}(K^2d^3)$, which is much lower than the computational complexity of the relay matrix design in the previous subsection. However, we will see through numerical simulations that the proposed algorithm in Table 3.1 has a better MSE and BER performance than the algorithm in Table 3.2. Such performance-complexity tradeoff is very useful for practical interference MIMO relay communication systems.

3.4 Extension of The Proposed Algorithms

3.4.1 Interference MIMO Relay Systems with CSI Mismatch

In case of CSI mismatch, we assumed that the source-relay channel matrix $\mathbf{H}_k$ is estimated at the relay and the relay-destination channel matrix is estimated at the destination. Each channel can be modelled as a channel estimate and its estimation error covariance.

$$\mathbf{H}_k = \hat{\mathbf{H}}_k + \mathbf{\Theta}_{h,k}^{\frac{1}{2}} \mathbf{H}_{w,k} \mathbf{\Phi}_{h,k}^{\frac{T}{2}}, \; k=1,\cdots,K \tag{3.53}$$

$$\mathbf{G}_k = \hat{\mathbf{G}}_k + \mathbf{\Theta}_{g,k}^{\frac{1}{2}} \mathbf{G}_{w,k} \mathbf{\Phi}_{g,k}^{\frac{T}{2}}, \; k=1,\cdots,K \tag{3.54}$$

where $\hat{\mathbf{H}}_k$, $\hat{\mathbf{G}}_k$ are the estimated channel matrices, $\mathbf{\Theta}_{h,k}$ and $\mathbf{\Phi}_{h,k}$ denote the covariance matrix of channel estimation error seen from transmitter side and receiver side, respectively. The matrix $\mathbf{H}_{w,k}$ and $\mathbf{G}_{w,k}$ are the Gaussian random matrix with i.i.d. zero mean and unit variance entries and are the unknown part in the CSI mismatch. The dimensions of $\mathbf{\Theta}_{h,k}$ is $N_{sk} \times N_{sk}$, $\mathbf{\Theta}_{g,k}$ and $\mathbf{\Phi}_{h,k}$ have a dimension of $N_r \times N_r$, while $\mathbf{\Phi}_{g,k}$ is a $N_{dk} \times N_{dk}$ matrix. Thus, the true channel matrices can be modelled as the well-known Gaussian-Kronecker model

$$\mathbf{H}_k \sim CN\left(\hat{\mathbf{H}}_k, \mathbf{\Theta}_{h,k} \otimes \mathbf{\Phi}_{h,k}\right), \; k=1,\cdots,K \tag{3.55}$$

$$\mathbf{G}_k \sim CN\left(\hat{\mathbf{G}}_k, \mathbf{\Theta}_{g,k} \otimes \mathbf{\Phi}_{g,k}\right), \; k=1,\cdots,K \tag{3.56}$$

As the exact CSI is unknown, in the following, we show that both proposed algorithms can be extended to design statistically robust transceivers. It is shown in [43] that for $\mathbf{H} \sim CN\left(\hat{\mathbf{H}}, \mathbf{\Theta} \otimes \mathbf{\Phi}\right)$, there is

$$E_H[\mathbf{H}\mathbf{X}\mathbf{H}^H] = \hat{\mathbf{H}}\mathbf{X}\hat{\mathbf{H}}^H + tr(\mathbf{X}\mathbf{\Theta}^T)\mathbf{\Phi} \tag{3.57}$$

where $E_H[.]$ stands for the expectation with respect to the H random matrix H. Considering the CSI mismatch (3.53), (3.54) and using (3.57), we have for $m,k = 1,\cdots,K$

$$E_{G,H}\left[\mathbf{L}_k\right] = \hat{\mathbf{G}}_k \mathbf{F} \hat{\mathbf{H}}_k \mathbf{B}_k \triangleq \hat{\mathbf{L}}_k \tag{3.58}$$

$$\begin{aligned} E_{G,H}\left[\mathbf{G}_k \mathbf{F} \bar{\mathbf{H}}_m \bar{\mathbf{H}}_m^H \mathbf{F}^H \mathbf{G}_k^H\right] &= E_G\left[\mathbf{G}_k \mathbf{F}\left(\hat{\mathbf{H}}_m \mathbf{B}_m \mathbf{B}_m^H \hat{\mathbf{H}}_m^H + \alpha_m \mathbf{\Phi}_{h,m}\right) \mathbf{F}^H \mathbf{G}_k^H\right] \\ &= \hat{\mathbf{G}}_k \mathbf{F}\left(\hat{\mathbf{H}}_m \mathbf{B}_m \mathbf{B}_m^H \hat{\mathbf{H}}_m^H + \alpha_m \mathbf{\Phi}_{h,m}\right) \mathbf{F}^H \hat{\mathbf{G}}_k^H + \beta_{m,k} \mathbf{\Phi}_{g,k} \end{aligned} \tag{3.59}$$

$$\sigma_r^2 E_G\left[\mathbf{G}_k \mathbf{F} \mathbf{F}^H \mathbf{G}_k^H\right] = \sigma_r^2 \hat{\mathbf{G}}_k \mathbf{F} \mathbf{F}^H \hat{\mathbf{G}}_k^H + \gamma_k \mathbf{\Phi}_{g,k} \tag{3.60}$$

where for $m,k = 1,\cdots,K$

$$\begin{aligned} \alpha_m &= tr(\mathbf{B}_m \mathbf{B}_m^H \mathbf{\Theta}_{h,m}^T) \\ \beta_{m,k} &= tr\left(\mathbf{F}\left(\hat{\mathbf{H}}_m \mathbf{B}_m \mathbf{B}_m^H \hat{\mathbf{H}}_m^H + \alpha_m \mathbf{\Phi}_{h,m}\right) \mathbf{F}^H \mathbf{\Theta}_{g,k}^T\right) \\ \gamma_k &= tr(\mathbf{F}\mathbf{F}^H \mathbf{\Theta}_{g,k}^T) \end{aligned}$$

Using (3.58)-(3.60), the statistical expectation of the sum-MSE in (3.12) with respect to $\mathbf{H}_k$ and $\mathbf{G}_k$, $k = 1, , K$, can be calculated as

$$E_{G,H}[\text{SMSE}] = \sum_{k=1}^{K} tr\Big((\mathbf{W}_k^H\hat{\mathbf{L}}_k - \mathbf{I}_d)(\mathbf{W}_k^H\hat{\mathbf{L}}_k - \mathbf{I}_d)^H + \mathbf{W}_k^H\big(\hat{\mathbf{C}}_{n,k} + \hat{\mathbf{\Xi}}_k + \gamma_k\mathbf{\Phi}_{g,k}$$
$$+ \sum_{m=1}^{K}(\alpha_m\hat{\mathbf{G}}_k\mathbf{F}\mathbf{\Phi}_{h,m}\mathbf{F}^H\hat{\mathbf{G}}_k^H + \beta_{m,k}\mathbf{\Phi}_{g,k})\big)\mathbf{W}_k\Big)$$

where for $m, k = 1, \cdots, K$

$$\hat{\mathbf{C}}_{n,k} = \sigma_r^2\hat{\mathbf{G}}_k\mathbf{F}\mathbf{F}^H\hat{\mathbf{G}}_k^H + \sigma_{dk}^2\mathbf{I}_{N_{dk}}$$
$$\hat{\mathbf{\Xi}}_k = \hat{\mathbf{G}}_k\mathbf{F}\sum_{m=1,m\neq k}^{K}\hat{\mathbf{H}}_m\mathbf{B}_m\mathbf{B}_m^H\hat{\mathbf{H}}_m^H\mathbf{F}^H\hat{\mathbf{G}}_k^H$$

By introducing

$$\mathbf{P}_1 \triangleq \sum_{m=1}^{K}\alpha_{1,m}\Phi_{h,m} + \sigma_r^2\mathbf{I}_{N_r} \tag{3.61}$$

$$\mathbf{P}_{2,k} \triangleq \sum_{m=1}^{K}\beta_{m,k}\Phi_{g,k} + \sigma_{dk}^2\mathbf{I}_{N_{dk}} + \alpha_{2,k}\Phi_{g,k}, \ k = 1, \cdots, K \tag{3.62}$$

we can rewrite (3.61) as

$$E_{G,H}[\text{SMSE}]$$
$$= \sum_{k=1}^{K} tr\Big((\mathbf{W}_k^H\hat{\mathbf{L}}_k - \mathbf{I}_d)(\mathbf{W}_k^H\hat{\mathbf{L}}_k - \mathbf{I}_d)^H + \mathbf{W}_k^H(\hat{\mathbf{G}}_k\mathbf{F}\mathbf{P}_1\mathbf{F}^H\hat{\mathbf{G}}_k^H + \hat{\mathbf{\Xi}}_k + \mathbf{P}_{2,k})\mathbf{W}_k\Big) \tag{3.63}$$

Let us introduce

$$\tilde{\mathbf{W}}_k^H \triangleq \mathbf{W}_k^H\mathbf{P}_{2,k}^{\frac{1}{2}}, \ \tilde{\mathbf{D}}_k \triangleq \mathbf{P}_i^{-\frac{1}{2}}\hat{\mathbf{H}}_k, \ \tilde{\mathbf{E}}_k \triangleq \mathbf{P}_{2,k}^{-\frac{1}{2}}\hat{\mathbf{G}}_k, \ \tilde{\mathbf{F}} \triangleq \mathbf{F}\mathbf{P}_1^{\frac{1}{2}} \tag{3.64}$$

we can rewrite (3.63) as

$$E_{G,H}[\text{SMSE}] = \sum_{k=1}^{K} tr\Big((\tilde{\mathbf{W}}_k^H\tilde{\mathbf{L}}_k - \mathbf{I}_d)(\tilde{\mathbf{W}}_k^H\tilde{\mathbf{L}}_k - \mathbf{I}_d)^H + \tilde{\mathbf{W}}_k^H(\tilde{\mathbf{C}}_{n,k} + \tilde{\mathbf{\Xi}}_k)\tilde{\mathbf{W}}_k\Big) \tag{3.65}$$

where for $m, k = 1, \cdots, K$

$$\tilde{\mathbf{L}}_k = \tilde{\mathbf{G}}_k\tilde{\mathbf{F}}\tilde{\mathbf{H}}_k\mathbf{B}_k$$
$$\tilde{\mathbf{C}}_{n,k} = \tilde{\mathbf{G}}_k\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H\tilde{\mathbf{G}}_k^H + \mathbf{I}_{N_{dk}}$$
$$\tilde{\mathbf{\Xi}}_k = \sum_{m=1,m\neq k}^{K}\tilde{\mathbf{G}}_k\tilde{\mathbf{F}}\tilde{\mathbf{H}}_k\mathbf{B}_k\mathbf{B}_k^H\tilde{\mathbf{H}}_k^H\tilde{\mathbf{F}}^H\tilde{\mathbf{G}}_k^H$$

In the case of CSI mismatch, the power constraint in (3.10c) becomes

$$
\begin{aligned}
tr\big(\mathbf{F}E[\mathbf{y}_r\mathbf{y}_r^H]\mathbf{F}^H\big) &= tr\big(\mathbf{F}(\sum_{m=1}^{K}(\hat{\mathbf{H}}_m\mathbf{B}_m\mathbf{B}_m^H\hat{\mathbf{H}}_m^H + \alpha_m\mathbf{\Phi}_{h,m}) + \sigma_r^2\mathbf{I}_{N_r})\mathbf{F}^H\big)\\
&= tr\big(\tilde{\mathbf{F}}(\sum_{m=1}^{K}\tilde{\mathbf{H}}_m\mathbf{B}_m\mathbf{B}_m^H\tilde{\mathbf{H}}_m^H + \sigma_r^2\mathbf{I}_{N_r})\tilde{\mathbf{F}}^H\big) \qquad (3.66)
\end{aligned}
$$

Using (3.65) and (3.66), the statistically robust transmitter, relay, and receiver matrices design problem for interference MIMO relay systems under CSI mismatch can be equivalently written as

$$
\min_{\{\tilde{\mathbf{W}}_k\},\{\mathbf{B}_k\},\tilde{\mathbf{F}}} \quad E_{G,H}\,[\mathrm{SMSE}] \qquad (3.67a)
$$

$$
\text{s.t.} \quad tr\big(\mathbf{B}_k\mathbf{B}_k^H\big) \le P_{sk},\ k=1,\cdots,K \qquad (3.67b)
$$

$$
tr\big(\tilde{\mathbf{F}}(\sum_{m=1}^{K}(\tilde{\mathbf{H}}_m\mathbf{B}_m\mathbf{B}_m^H\tilde{\mathbf{H}}_m^H + \mathbf{I}_{N_r})\tilde{\mathbf{F}}^H\big) \le P_r \qquad (3.67c)
$$

where $\{\tilde{\mathbf{W}}_k\} \triangleq \{\tilde{\mathbf{W}}_1,\cdots,\tilde{\mathbf{W}}_K\}$. By comparing the problem (3.67) with the problem (3.10), it can be seen that the problem (3.67) is in fact a transmitter optimization problem for an "equivalent" interference MIMO relay system where the transmitter-relay and relay-receiver channels are $\tilde{\mathbf{H}}_k$ and $\tilde{\mathbf{G}}_k$, $k = 1,\cdots,K$, respectively, the relay precoding matrix is $\tilde{\mathbf{F}}$, and the transmitter and receiver matrices are $\mathbf{B}_k$ and $\tilde{\mathbf{W}}_k,\ k = 1,\cdots,K$, respectively. Therefore, both proposed algorithms can be applied to solve the problem (3.67).

3.4.2 Multiple Relays MIMO Communication System

The proposed tri-step algorithm can be easily extended to interference MIMO relay systems with multiple relay nodes. Let us consider a system with L relay nodes, where $\mathbf{F}_l$ denotes the precoding matrix at the lth relay node, $\mathbf{H}_{lk}$ and $\mathbf{G}_{kl}$ are the channel matrices from the kth transmitter to the lth relay, and from the lth relay to the kth receiver, respectively. Let us introduce the following SVDs for $l = 1,\cdots,L$

$$
\begin{aligned}
&[\mathbf{H}_{l1}\mathbf{B}_1,\ \ldots,\ \mathbf{H}_{lK}\mathbf{B}_K] = \mathbf{U}_{h,l}\mathbf{\Lambda}_{h,l}\mathbf{V}_{h,l}^H\\
&[\mathbf{G}_{1l}^T,\ \ldots,\ \mathbf{G}_{Kl}^T]^T = \mathbf{U}_{g,l}\mathbf{\Lambda}_{g,l}\mathbf{V}_{g,l}^H
\end{aligned}
$$

Similar to (3.15), it can be shown the optimal structure of relay precoding matrix $\mathbf{F}_l$ is

$$\mathbf{F}_l = \mathbf{V}_{g,l}\mathbf{A}_l\mathbf{U}_{h,l}^H, \quad l = 1, \cdots, L \tag{3.68}$$

Similar to the procedure in Table 3.1, in each iteration of the tri-step algorithm, we first update $\mathbf{W}_k$ with given $\mathbf{B}_k$ and $\{\mathbf{F}_l\} \triangleq \{\mathbf{F}_1, \cdots, \mathbf{F}_L\}$. Then we update each relay matrix $\mathbf{F}_l$ based on its optimal structure (3.68) with fixed $\mathbf{W}_k$, $\mathbf{B}_k$, and other relay matrices $\mathbf{F}_m$, $m = 1, \cdots, L$, $m \neq l$. Finally, we optimize $\mathbf{B}_k$ with given $\mathbf{W}_k$ and $\mathbf{F}_l$. On the other hand, the proposed simplified relay matrix design cannot be straightforwardly extended to multi-relay systems. Similar to (3.34), the optimal $\mathbf{F}_l$ in (3.68) can be written as

$$\mathbf{F}_l = \mathbf{V}_{g,l}\mathbf{\Lambda}_{g,l}^{-1}\mathbf{U}_{g,l}^H\mathbf{C}_l\mathbf{U}_{h,l}^H. \tag{3.69}$$

where $\mathbf{C}_l = [\mathbf{C}_{l,1}^T, \cdots, \mathbf{C}_{l,K}^T]^T$. It can be shown that the MSE of the kth transmitter-receiver pair is a function of $\mathbf{C}_{1,k}$, $\mathbf{C}_{2,k}, \cdots,$ $\mathbf{C}_{L,k}$. However, the power constraint at the lth relay node is a function of $\mathbf{C}_{l,1}$, $\mathbf{C}_{l,2}, \cdots,$ $\mathbf{C}_{l,K}$. Thus, unlike a single-relay system, the optimal relay matrix design problem in multi-relay systems cannot be easily decomposed into K subproblems with closed-form solutions, due to the couplings among all $\mathbf{C}_{l,k}$. Developing a simplified relay matrices design algorithm for an interference MIMO relay system with multiple relay nodes is an interesting future research topic.

3.5 Numerical Examples

In this section, we study the performance of the proposed joint transceiver matrices design algorithms for interference MIMO relay systems in Table 3.1 (Algorithm 3.1) and Table 3.2 (Algorithm 3.2) through numerical simulations. We consider an interference MIMO relay system with $d = 3$, where all transmitters and receivers have the same number of antennas, i.e., $N_{sk} = N_{dk} = 4$, $k = 1, \cdots, K$, and the relay node has $N_r = 20$ antennas. We also assume that all transmitters have the same power budget of $P_{sk} = 20$dB, $k = 1, \cdots, K$. All channel matrices have i.i.d. complex Gaussian entries with zero mean and unit variance, and all noises are i.i.d. Gaussian with zero mean and unit variance. The QPSK

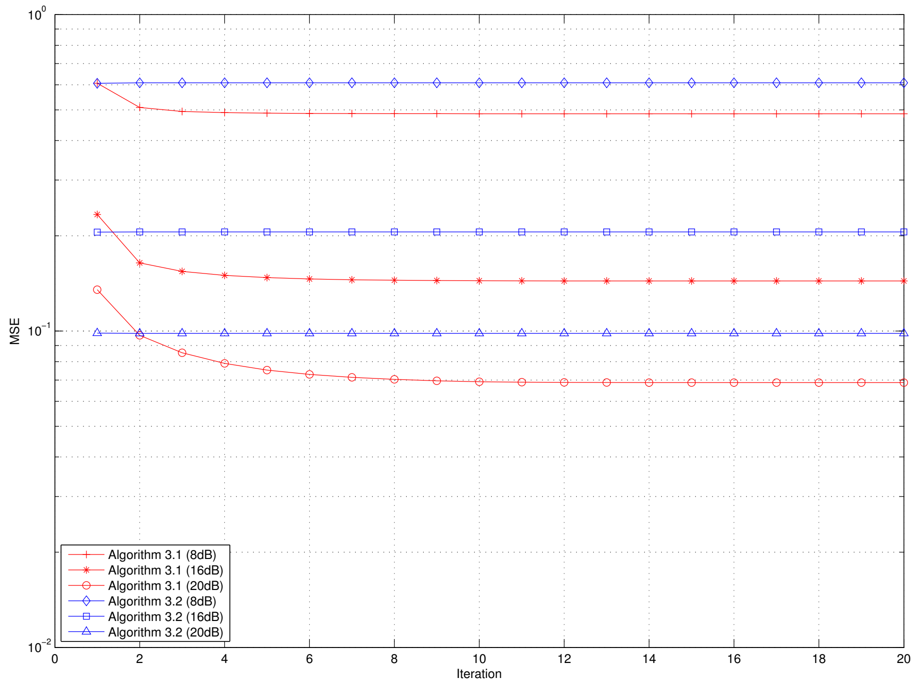

Figure 3.2: Example 3.1: MSE versus the number of iterations, $K = 2$.

constellations are used to modulate the source symbols. All simulation results are averaged over 5×10^5 independent channel realizations. Both proposed algorithms are initialized with $\mathbf{F}^{(0)} = \sqrt{P_r/tr\big(\sum_{k=1}^{K} P_{sk}\mathbf{H}_k\mathbf{H}_k^H/N_{sk} + \mathbf{I}_{N_r}\big)}\mathbf{I}_{N_r}$ and $\mathbf{B}_k^{(0)} = \sqrt{P_{sk}/N_{sk}}\mathbf{I}_{N_{sk}}$, $k = 1, \cdots, K$. As a benchmark, the performance of the proposed algorithms is compared with the joint power control and transceiver-relay beamforming (TxRxBF) algorithm developed in [23] and the TLM algorithm developed in [24].

In the first numerical example, we study the convergence speed of the proposed algorithms. Fig. 3.2 and Fig. 3.3 show the performance of the two proposed algorithms versus different number of iterations. We also observe that the proposed Algorithm 3.1 converges around 10 iterations. In fact, the decreasing of the MSE and the BER are negligible after the five iterations. Thus, we suggest that only 5 iterations are needed for the proposed Algorithm 3.1 to achieve a good performance. The simulation results show that the conditions for convergence of the proposed Algorithm 3.2, step 5 in Table 3.2, is typically met with two iterations. By imposing the stricter condition on the power constraints in (3.39), the search for $\mathbf{F}$ is now limited in a stricter space. Thus, the optimal $\mathbf{F}$ after the first

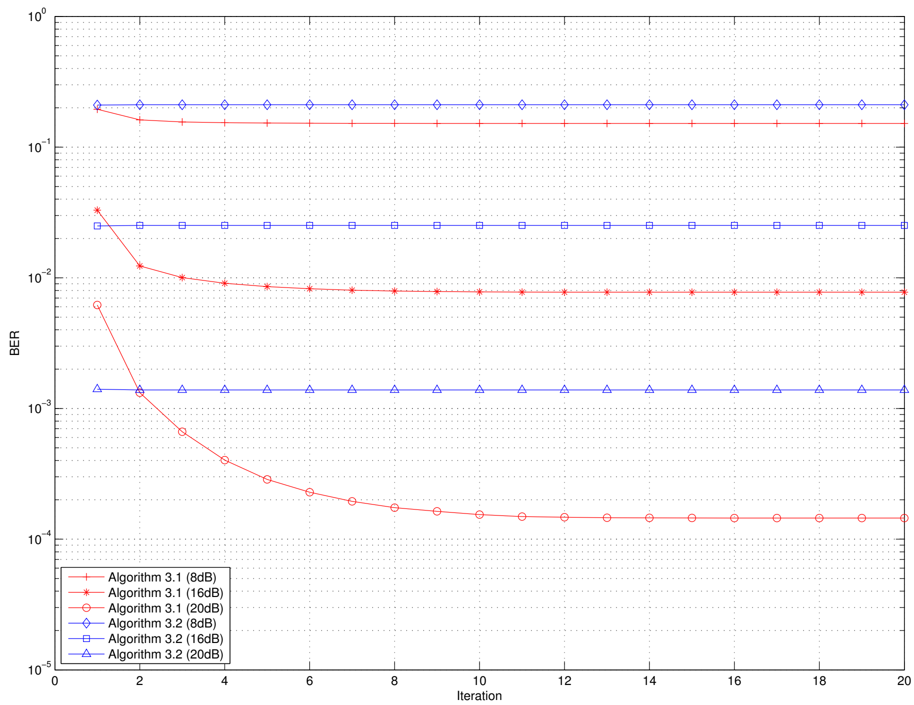

Figure 3.3: Example 3.1: BER versus the number of iterations, $K = 2$.

iteration is typically closer to the final optimal solution compare to the proposed Algorithm 3.1.

In the second numerical example, we compare the performance of the two proposed algorithms together with the TLM algorithms. Fig. 3.4 shows the normalized SMSE performance of the three algorithms tested versus P_r with $K = 2$. It can be seen that both proposed algorithms outperform the TLM algorithm throughout the whole P_r range. While the proposed Algorithm 3.1 has a better MSE performance than the proposed Algorithm 3.2 at convergence, the latter algorithm has a lower computational complexity.

For this example, the BER of all transmitter-receiver pairs versus P_r yielded by the three algorithms is shown in Fig. 3.5. It can be seen that both proposed algorithms yield smaller BER than the TLM algorithm over the whole P_r range. Moreover, when it converges, the proposed Algorithm 3.1 has a better BER performance than the proposed Algorithm 3.2 at a higher computational complexity. It also shows that both transmitter-receiver pairs achieve almost identical BER, indicating that both proposed algorithms are fair to all links.

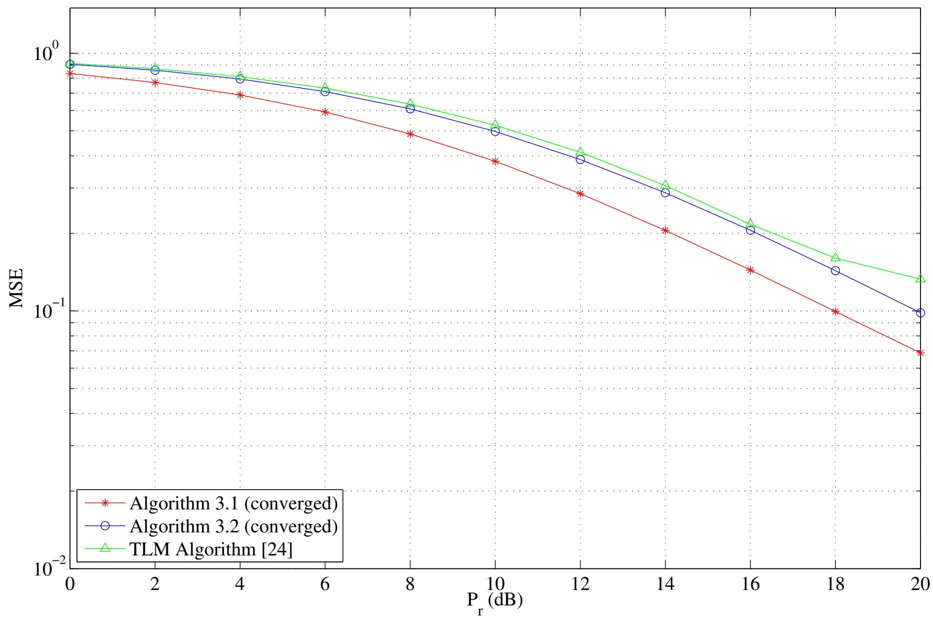

Figure 3.4: Example 3.2: MSE versus P_r at convergence, $K = 2$.

In the third numerical example, we study the performance of the proposed algorithms with different number of transmitter-receiver pairs K. The normalized SMSE performance of both proposed algorithms versus P_r is shown in Fig. 3.6 for $K = 2, 3, 4$. As expected, for both algorithms, the MSE increases with K. Moreover, the proposed Algorithm 3.1 has better MSE performance than the proposed Algorithm 3.2 for all K values.

For the fourth numerical example, we study the effect of CSI mismatch on the performance of the two proposed algorithms. In our simulation, we assume the channel estimation error at the receiver side is uncorrelated, i.e., $\boldsymbol{\Theta}_{h,k} = \sigma_e^2 \mathbf{I}_{N_{sk}}$ and $\boldsymbol{\Theta}_{g,k} = \sigma_e^2 \mathbf{I}_{N_r}$ when measures the variance of the channel estimation error. The covariance matrix of the channel estimation error at the receiver side is set as

$$\boldsymbol{\Phi}_{h,k} = \begin{bmatrix} 1 & \phi_h & \phi_h^2 & \phi_h^3 \\ \phi_h & 1 & \phi_h & \phi_h^2 \\ \phi_h^2 & \phi_h & 1 & \phi_h \\ \phi_h^3 & \phi_h^2 & \phi_h & 1 \end{bmatrix} \boldsymbol{\Phi}_{g,k} = \begin{bmatrix} 1 & \phi_g & \phi_g^2 & \phi_g^3 \\ \phi_g & 1 & \phi_g & \phi_g^2 \\ \phi_g^2 & \phi_g & 1 & \phi_g \\ \phi_g^3 & \phi_g^2 & \phi_g & 1 \end{bmatrix}$$

where we choose $\phi_h = \phi_g = 0.45$ in the simulation. Fig. 3.7 shows the performance of the two proposed algorithms under at $\sigma_e^2 = 0.01$ and 0.001 respectively.

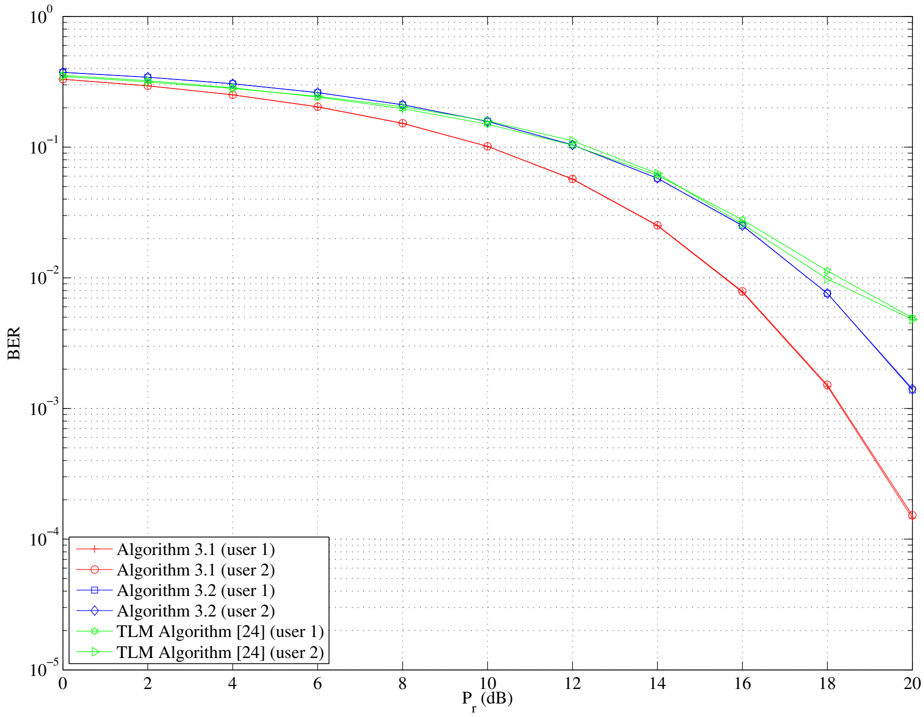

Figure 3.5: Example 3.2: BER versus P_r for each transmitter-receiver pair, $K = 2$.

As the TxRxBF algorithm in [23] considered an interference MIMO relay system with $d = 1$, thus in this final example, we compare the performance of the two proposed algorithms with the TxRxBF algorithm and the TLM algorithm. Fig. 3.8 shows that the proposed Algorithm 3.1 has a better performance than the TxRxBF algorithm while it outperforms the proposed Algorithm 3.2. The computational complexity of the TxRxBF algorithm is higher than the proposed Algorithm 3.2 and lower than the proposed Algorithm 3.1. Thus, the simulation result confirms such performance-complexity tradeoff.

3.6 Chapter Summary

In this chapter, we have presented two algorithms for jointly optimizing the transmitter, relay, and receiver matrices of interference MIMO relay systems. In particular, the optimal structure of the relay precoding matrix has been derived to reduce the computational complexity. Moreover, by modifying the power constraint at the relay node, a simplified relay matrix design has been proposed which

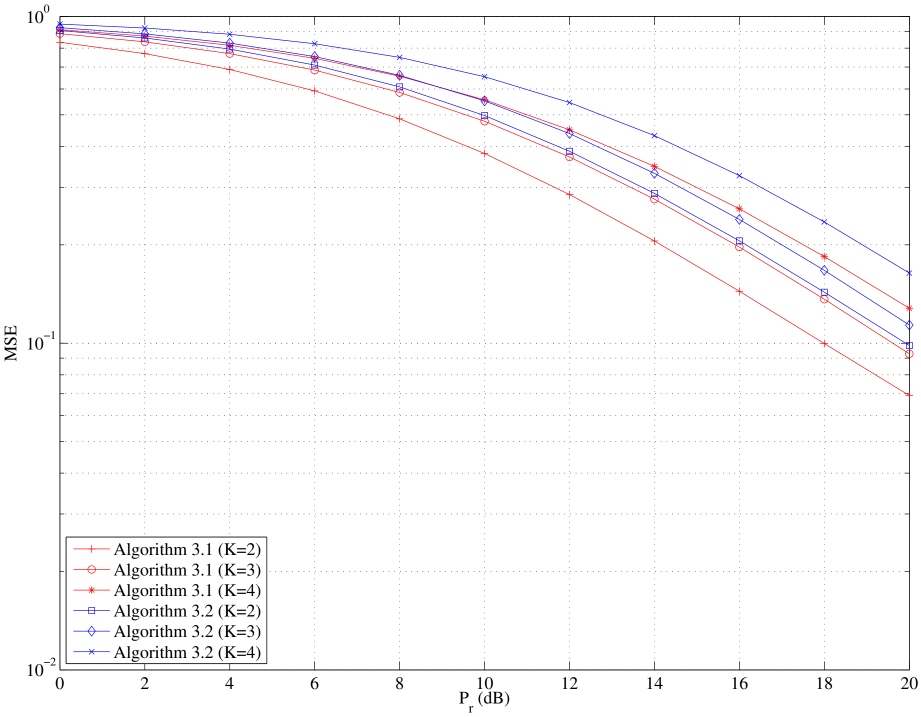

Figure 3.6: Example 3.3: MSE versus P_r for different K.

has a closed-form solution. Numerical simulation results show that the proposed algorithms converge quickly after a few iterations. The proposed Algorithm 3.1 has a better MSE and BER performance than the proposed Algorithm 3.2 at a higher computational complexity.

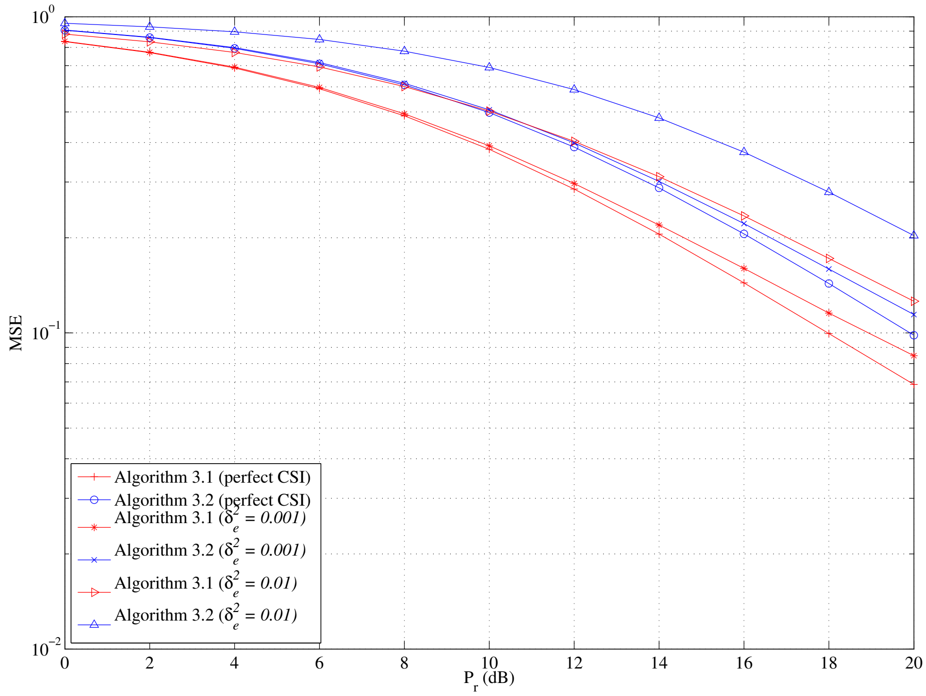

Figure 3.7: Example 3.4: Effect of CSI mismatch on the proposed algorithms.

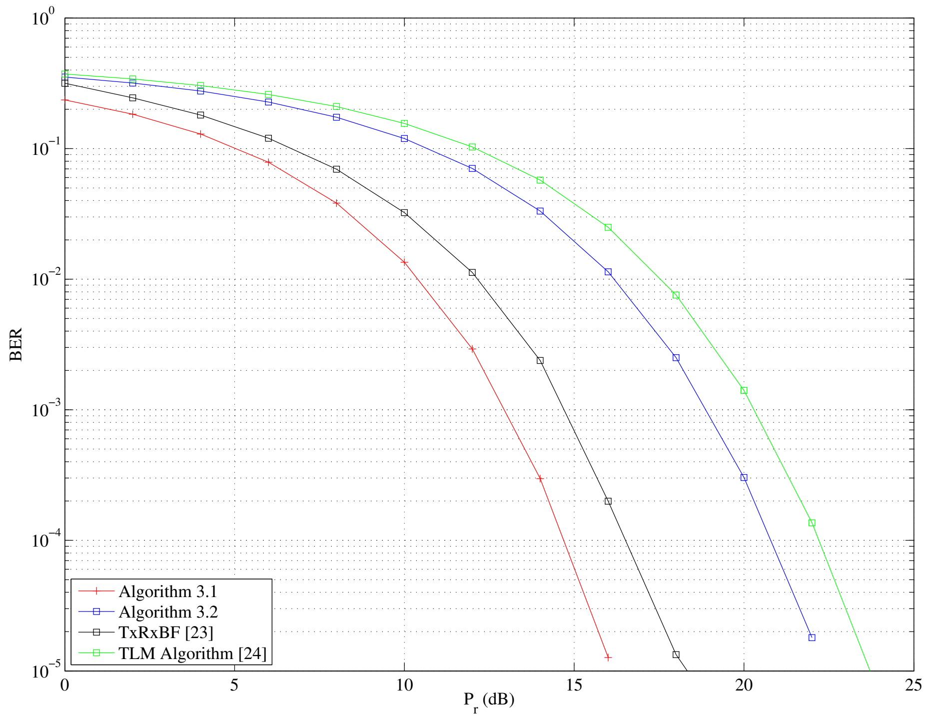

Figure 3.8: Example 3.5: MSE versus P_r for different algorithms.

Chapter 4

Interference Two-Way MIMO Relay Systems

In this chapter, the two-way interference MIMO relay systems are investigated. After a review of existing works in Section 4.1, the system model and problem formulation are introduced in Section 4.2. The transceivers design algorithm is developed in Sections 4.3. Simulation results are presented in Section 4.4 to demonstrate the performance of the proposed algorithm. Conclusions are drawn in Section 4.5.

4.1 Introduction

Most of the relay networks are assumed to work in half-duplex mode as it can conserve bandwidth and avoid interference at the relay nodes [27]. Thus, in an typical AF two-way MIMO relay system, the communication between two source nodes is accomplished in four time slots: from source node 1 to the relay node, from the relay node to source node 2, from source node 2 to the relay node and from the relay node to source node 1. By using the idea of analogue network coding [28], the two-way relaying protocol in which, the two source nodes exchange their information in two time slots without using extra channel resources, has been studied recently to overcome the loss in terms of spectral efficiency in half-duplex systems. In the first time slot, the relays receive data from the two source nodes simultaneously, and in the second phase, the relays re-transmit the received signal to both source nodes. Since the received signal at each source node contains the data from the other node and its own transmitted data, the SI in the transmitted signal can be cancelled.

For single user two-way AF MIMO single relay systems, the optimal design of source and relay matrices have been developed in [48] to maximize the achievable weighted sum rate. An iterative algorithm based on the convex QCQP problem were introduced in [49]. A unified framework has been developed in [50] to optimize the source and relay matrices for a broad class of frequently used objective functions such as the MMSE, the MMI, and the minimax MSE. The impact of quality-of-service (QoS) constraints on two-way MIMO relay systems has been studied in [51]. For a single-user two-hop MIMO relay system with multiple parallel relay nodes, an iterative algorithms using the gradient descent algorithm has been studied in [52]. The works in [53]-[54] designed the optimal transceiver processing matrices based on both the zero-forcing (ZF) and MMSE criteria for interference two-way MIMO relay systems; however, the work only consider the case of single relay node. In [55], the projection based separation of multiple operators (ProBaSeMO) decouples the system into multiple independent single-user two-way MIMO relay subsystems before enhancing the performance of each subsystem individually. In [56], a general multi-user multi-cell relay network has been investigated. The algorithms proposed in [55] and [56] adopt the block diagonal technique to align the desired signal of a user in the null space of the combined channels of all the other users. Maximal sum-rate is the chosen design criterion.

In this chapter, we investigate the simplified relay matrix design for a two-way interference MIMO relay system where multiple user pairs communicate simultaneously with the aid of single relay node. The source nodes and the relay node are equipped with multiple antennas. The aim of this chapter is to optimize the source and relay matrices to suppress the interference and minimize the SMSE of the signal waveform estimation at the receivers, subjecting to transmission power constraints at transmitters and the relay node. We propose an iterative transceiver design algorithm to solve the subproblems as the transceiver optimization problem is nonconvex with matrix variables. Compared with existing work such as [53]-[54], this chapter exploits the optimal structure of the relay matrix to significantly reduce the computational complexity. Furthermore, the optimization problem has a closed-form solution. The simplified relay matrix design cannot be found in existing works on transceiver design for interference MIMO relay systems [48]-[56]

In each iteration of this algorithm, we first update the receiver matrices based on the transmitter and relay matrices from the previous iteration. Then based on the transmitter matrices from the previous iteration and the receiver matrices in this iteration, we modified the transmission power constraint at the relay and obtain the closed form solution of relay matrix. Finally, the transmitter matrices are updated with the optimal relay matrix and receiver matrices obtain in this iteration. The MSE and BER simulation results show that the proposed algorithm has a slightly worse performance than the existing works in terms of the system MSE and BER. However, the computational complexity of the simplified algorithm is significant reduced for interference MIMO relay systems with a large number of user pairs.

4.2 System Model and Problem Formulation

We investigate a multi-user two-hop interference MIMO relay communication system where K user pairs, distributed on two different sites, communicate with the aid of a single relay as shown in Fig. 4.1. For simplicity, the direct links between site 1 and site 2 user pair are ignored as they undergo much larger path attenuation compared with the links via the relay. Each kth node at site 1 and site 2 is equipped with $N_{k,1}$ and $N_{k,2}$ antennas, respectively, and the number of antennas at the relay node is N_r.

In this chapter, the relay node is assumed to work in the half-duplex mode so the communication between the user pairs is completed in two time slots. In the first time slot, the kth nodes at site $i = 1, 2$, encodes the symbol vector $\mathbf{s}_{k,i}$ with the source precoding matrices $\mathbf{B}_{k,i}$ before transmitting the $N_{k,i} \times 1$ precoded signal vector

$$\mathbf{x}_{k,i} = \mathbf{B}_{k,i}\mathbf{s}_{k,i}, \quad k = 1, \ldots, K, \quad i = 1, 2 \tag{4.1}$$

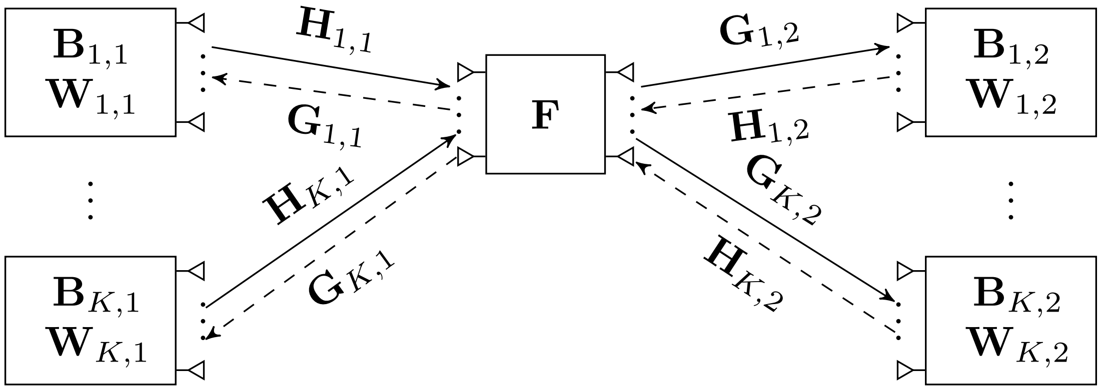

Figure 4.1: Block diagram of an interference two-way MIMO relay system.

to the relay nodes. The information-carrying vector $\mathbf{s}_{k,i}$ and the source precoding matrix $\mathbf{B}_{k,i}$ have the size of $d \times 1$ and $N_{k,i} \times d$, respectively. The received signal vector at the relay node is given by

$$\mathbf{y}_r = \sum_{k=1}^{K} \mathbf{H}_{k,1}\mathbf{B}_{k,1}\mathbf{s}_{k,1} + \sum_{k=1}^{K} \mathbf{H}_{k,2}\mathbf{B}_{k,2}\mathbf{s}_{k,2} + \mathbf{n}_r \tag{4.2}$$

where $\mathbf{H}_{k,i}$ is the $N_r \times N_{k,i}$ up-link MIMO channel matrix between the kth node at site i and the relay node, $\mathbf{n}_r$ is the $N_r \times 1$ AWGN vector at the relay node with zero mean and covariance matrix $E\left[\mathbf{n}_r\mathbf{n}_r^H\right] = \sigma_r^2\mathbf{I}_{N_r}$.

In the second time slot, the relay amplifies the received signal with the $N_r \times N_r$ precoding matrix $\mathbf{F}$ as

$$\mathbf{x}_r = \mathbf{F}\mathbf{y}_r. \tag{4.3}$$

The precoded signal vector $\mathbf{x}_r$ is broadcast back to the nodes at site $i = 1, 2$. The received signal vector at the kth node of site i is given by

$$\mathbf{y}_{k,i} = \mathbf{G}_{k,i}\mathbf{F}\mathbf{y}_r + \mathbf{n}_{k,i}, \quad k = 1, \ldots, K, \quad i = 1, 2 \tag{4.4}$$

where $\mathbf{G}_{k,i}$ is the $N_{k,i} \times N_r$ down-link MIMO channel matrix between the relay node and the kth node at site i, $\mathbf{n}_{k,i}$ is the $N_{k,i} \times 1$ AWGN vector at the kth destination node with zero mean and covariance matrix $E\left[\mathbf{n}_{k,i}\mathbf{n}_{k,i}^H\right] = \sigma_k^2\mathbf{I}_{N_{k,i}}$.

In this chapter, linear receivers are used to retrieve the transmitted signal. The estimated signal vector at the kth node of site i can be written as

$$\bar{\mathbf{s}}_{k,i} \;=\; \mathbf{W}_{k,i}^H\mathbf{y}_{k,i} \quad k = 1, \ldots, K, \quad i = 1, 2 \tag{4.5}$$

where $\mathbf{W}_k$ is a $N_k \times d$ receiver matrix at the kth node of site i. Using (4.4), the estimated signal in (4.5) becomes:

$$
\begin{aligned}
\bar{\mathbf{s}}_{k,i} = {} & \underbrace{\mathbf{W}_{k,i}^H \mathbf{G}_{k,i} \mathbf{F} \mathbf{H}_{k,\bar{i}} \mathbf{B}_{k,\bar{i}} \mathbf{s}_{k,\bar{i}}}_{desired\ signal} \\
& \underbrace{+ \mathbf{W}_{k,i}^H \mathbf{G}_{k,i} \mathbf{F} \sum_{m \neq k}^{K} \mathbf{H}_{m,\bar{i}} \mathbf{B}_{m,\bar{i}} \mathbf{s}_{m,\bar{i}} + \mathbf{W}_{k,i}^H \mathbf{G}_{k,i} \mathbf{F} \sum_{m=1}^{K} \mathbf{H}_{m,i} \mathbf{B}_{m,i} \mathbf{s}_{m,i}}_{interference} \\
& + \underbrace{\mathbf{W}_{k,i}^H \bar{\mathbf{n}}_{k,i}}_{noise,} . \qquad k = 1, \ldots, K, \quad i = 1, 2
\end{aligned} \tag{4.6}
$$

where $\bar{\mathbf{n}}_{k,i} \triangleq \mathbf{G}_{k,i} \mathbf{F} \mathbf{n}_r + \mathbf{n}_{k,i}$ is the total noise at the kth node of the ith site. Each of kth node at site i has knowledge of its own transmitted signal vector thus the SI in (4.6) can be easily cancelled. The estimated signal vector at the kth node of the ith site becomes:

$$
\begin{aligned}
\hat{\mathbf{s}}_{k,i} = {} & \mathbf{W}_{k,i}^H \mathbf{G}_{k,i} \mathbf{F} \mathbf{H}_{k,\bar{i}} \mathbf{B}_{k,\bar{i}} \mathbf{s}_{k,\bar{i}} + \mathbf{W}_{k,i}^H \mathbf{G}_{k,i} \mathbf{F} \sum_{m \neq k}^{K} \mathbf{H}_{m,\bar{i}} \mathbf{B}_{m,\bar{i}} \mathbf{s}_{m,\bar{i}} \\
& + \mathbf{W}_{k,i}^H \mathbf{G}_{k,i} \mathbf{F} \sum_{m \neq k}^{K} \mathbf{H}_{m,i} \mathbf{B}_{m,i} \mathbf{s}_{m,i} + \mathbf{W}_{k,i}^H \bar{\mathbf{n}}_{dk,i}, \quad k = 1, \ldots, K, \quad i = 1, 2
\end{aligned} \tag{4.7}
$$

At the source nodes and the relay node, the signal vectors transmitted from each node must satisfy the transmission power constraints as

$$
tr\left(\mathbf{F} E\left[\mathbf{y}_r \mathbf{y}_r^H\right] \mathbf{F}^H\right) \leq P_r \tag{4.8}
$$

$$
tr\left(\mathbf{B}_{k,i} E\left[\mathbf{s}_{k,i} \mathbf{s}_{k,i}^H\right] \mathbf{B}_{k,i}^H\right) \leq P_{k,i} \tag{4.9}
$$

$$
k = 1, \ldots, K, \; i = 1, 2.
$$

where $P_{k,i}$ and P_r denote the power budget at the kth node of site i and the relay node, respectively, and $E\left[\mathbf{s}_{k,i} \mathbf{s}_{k,i}^H\right] = \mathbf{I}_d$ is the covariance matrix of the information-carrying symbol vector at the kth node of site i, and $E\left[\mathbf{y}_r \mathbf{y}_r^H\right] = \sum_{i=1}^{2} \sum_{k=1}^{K} \mathbf{H}_{k,i} \mathbf{B}_{k,i} \mathbf{B}_{k,i}^H \mathbf{H}_{k,i}^H + \sigma_r^2 \mathbf{I}_{N_r}$ is the covariance matrix of the received signal vector at the relay node.

The aim of this chapter is to optimize the precoding matrices $\{\mathbf{B}_{k,i}\} \triangleq \{\mathbf{B}_{k,i}, k = 1, \cdots, K; i = 1, 2\}$, the relay precoding matrix $\mathbf{F}$, and the receiver matrices $\{\mathbf{W}_{k,i}\} \triangleq \{\mathbf{W}_{k,i},\, k = 1, \cdots, K; i = 1, 2\}$, to minimize the sum-MSE of the signal waveform estimation at the receivers under transmission power constraints at the source and relay nodes.

The MSE of the signal waveform estimation at the kth node of site i can be calculated as

$$\begin{aligned}
&\mathrm{MSE}_{k,i} \\
&= tr(E\left[\left(\hat{\mathbf{s}}_{k,i} - \mathbf{s}_{k,\bar{i}}\right)\left(\hat{\mathbf{s}}_{k,i} - \mathbf{s}_{k,\bar{i}}\right)^H\right]) \\
&= tr((\mathbf{W}_{k,i}^H \mathbf{L}_{k,i} - \mathbf{I}_d)(\mathbf{W}_{k,i}^H \mathbf{L}_{k,i} - \mathbf{I}_d)^H + \mathbf{W}_{k,i}^H \mathbf{C}_{nk,i} \mathbf{W}_{k,i} + \mathbf{W}_{k,i}^H \mathbf{\Xi}_{k,i} \mathbf{W}_{k,i}) \quad (4.10) \\
&\quad k = 1, \ldots, K, \quad i = 1, 2
\end{aligned}$$

where $\mathbf{L}_{k,i}$ is the equivalent MIMO channel matrix of the kth site 1 - site 2 user pair, $\mathbf{C}_{nk,i} = E\left[\bar{\mathbf{n}}_{k,i}\bar{\mathbf{n}}_{k,i}^H\right]$ is the covariance matrix of the equivalent noise, and $\mathbf{\Xi}_{k,i}$ is the covariance matrix of interference at the kth node of site i. They are given respectively as

$$\begin{aligned}
\mathbf{L}_{k,i} &= \mathbf{G}_{k,i}\mathbf{F}\bar{\mathbf{H}}_{k,\bar{i}} && (4.11) \\
\mathbf{C}_{nk,i} &= E\left[(\mathbf{G}_{k,i}\mathbf{F}\mathbf{n}_r + \mathbf{n}_{k,i})(\mathbf{G}_{k,i}\mathbf{F}\mathbf{n}_r + \mathbf{n}_{k,i})^H\right] \\
&= \sigma_r^2 \mathbf{G}_{k,i}\mathbf{F}\mathbf{F}^H\mathbf{G}_{k,i}^H + \sigma_{k,i}^2 \mathbf{I}_{N_k} && (4.12) \\
\mathbf{\Xi}_{k,i} &= \mathbf{G}_{k,i}\mathbf{F}\sum_{j=1}^{2}\sum_{m\neq k}^{K} \bar{\mathbf{H}}_{m,j}\bar{\mathbf{H}}_{m,j}^H \mathbf{F}^H \mathbf{G}_{k,i}^H && (4.13) \\
& k = 1, \ldots, K, \quad i = 1, 2
\end{aligned}$$

where $\bar{\mathbf{H}}_{k,i} = \mathbf{H}_{k,i}\mathbf{B}_{k,i}$ is the equivalent MIMO channel matrix between the kth source node of site i and the relay node.

From (4.8)-(4.10), the optimal source, relay, and receiver matrices design problem can be written as

$$\begin{aligned}
\min_{\{\mathbf{W}_{k,i}\},\{\mathbf{B}_{k,i}\},\mathbf{F}} \quad & \sum_{i=1}^{2}\sum_{k=1}^{K} \mathrm{MSE}_{k,i} && (4.14a) \\
s.t. \quad & tr\left(\mathbf{B}_{k,i}\mathbf{B}_{k,i}^H\right) \leq P_{k,i} && (4.14b) \\
& tr\left(\mathbf{F}E[\mathbf{y}_r\mathbf{y}_r^H]\mathbf{F}^H\right) \leq P_r. && (4.14c) \\
& k = 1, \ldots, K, \quad i = 1, 2
\end{aligned}$$

4.3 Proposed Algorithm 4

The problem (4.14) is highly nonconvex with matrix variables, and a globally optimal solution is intractable to obtain. In this section, we propose an algorithm that iteratively design the optimal receiver matrices $\mathbf{W}_{k,i}$, the source matrices $\mathbf{B}_{k,i}$ and the relay matrix $\mathbf{F}$, such that the sum MSE is minimized with the power constraints in (4.8) and (4.9). Furthermore, the power constraint at the relay node (4.8) is modified to obtain the closed-form solution for the relay matrix $\mathbf{F}$. In each iteration, we optimize $\{\mathbf{W}_{k,i}\}$ based on $\{\mathbf{B}_{k,i}\}$ and $\mathbf{F}$ from the previous iteration. Then using the optimized receiver matrices $\{\mathbf{W}_{k,i}\}$ and the source matrices $\mathbf{B}_{k,i}$ from previous iteration, we optimize the relay matrix $\mathbf{F}$. Finally, we optimize source matrices based on $\{\mathbf{W}_{k,i}\}$ and $\mathbf{F}$ obtained from this iteration.

From (4.8) and (4.9), the power constraint is independent of $\mathbf{W}_{k,i}$. Thus, with the given relay matrix and source matrices, the optimal linear receiver matrix which minimizes MSE in (4.10) is the well-known MMSE receiver:

$$\mathbf{W}_{k,i} = \left(\mathbf{L}_{k,i}\mathbf{L}_{k,i}^H + \mathbf{C}_{nk,i} + \mathbf{\Xi}_{k,i}\right)^{-1}\mathbf{L}_{k,i} \tag{4.15}$$

Substitute the MMSE receiver obtained in ((4.15)) to (4.10), the sum-MSE $\text{SMSE} = \sum_{i=1}^{2}\sum_{k=1}^{K}\text{MSE}_{k,i}$ can be rewritten as a function of $\mathbf{F}$ as

$$\begin{aligned}
\text{SMSE} &= \sum_{i=1}^{2}\sum_{k=1}^{K} tr(\mathbf{I}_d - \mathbf{L}_{k,i}^H\mathbf{W}_{k,i}) \\
&= \sum_{i=1}^{2}\sum_{k=1}^{K} tr\left(\mathbf{I}_d - \mathbf{L}_{k,i}^H\left(\mathbf{L}_{k,i}\mathbf{L}_{k,i}^H + \mathbf{N}_{k,i} + \mathbf{\Xi}_{k,i}\right)^{-1}\mathbf{L}_{k,i}\right) \\
&= \sum_{i=1}^{2}\sum_{k=1}^{K} tr\Big[\mathbf{I}_d - \bar{\mathbf{H}}_{k,\bar{i}}^H\mathbf{F}^H\mathbf{G}_{k,i}^H(\mathbf{G}_{k,i}\mathbf{F}\bar{\mathbf{H}}_{k,\bar{i}}\bar{\mathbf{H}}_{k,\bar{i}}^H\mathbf{F}^H\mathbf{G}_{k,i}^H \\
&\quad +\mathbf{C}_{nk,i} + \mathbf{\Xi}_{k,i})^{-1}\mathbf{G}_{k,i}\mathbf{F}\bar{\mathbf{H}}_{k,\bar{i}}\Big]
\end{aligned} \tag{4.16}$$

Let us denote

$$\mathbf{H} = [\bar{\mathbf{H}}_{1,2}, \ldots, \bar{\mathbf{H}}_{K,2}, \bar{\mathbf{H}}_{1,1}, \ldots, \bar{\mathbf{H}}_{K,1}] = \mathbf{U}_h\mathbf{\Lambda}_h\mathbf{V}_h^H \tag{4.17}$$

$$\mathbf{G} = [\mathbf{G}_{1,1}^T, \ldots, \mathbf{G}_{K,1}^T, \mathbf{G}_{1,2}^T, \ldots, \mathbf{G}_{K,2}^T]^T = \mathbf{U}_g\mathbf{\Lambda}_g\mathbf{V}_g^H \tag{4.18}$$

as the SVD of the equivalent relay-nodes channel $\mathbf{H}$ and the equivalent nodes-relay channel $\mathbf{G}$. The dimensions of $\mathbf{U}_h$, $\mathbf{\Lambda}_h$, $\mathbf{V}_h$ are $N_r \times L_1$, $L_1 \times L_1$, $L_1 \times L_1$, respectively and the dimension of $\mathbf{U}_g$, $\mathbf{\Lambda}_g$, $\mathbf{V}_g$ are $L_2 \times L_2$, $L_2 \times L_2$, $N_r \times L_2$, respectively, where $L_1 = 2Kd$ and $L_2 = \sum_{i=1}^{2}\sum_{k=1}^{K} N_{k,i}$. We can derive the following

$$\begin{aligned} \mathbf{H}_{k,i}\mathbf{B}_{k,i} &= \mathbf{U}_h\mathbf{\Lambda}_h\mathbf{V}_{hk,i}^H & (4.19)\\ \mathbf{G}_{k,i} &= \mathbf{U}_{gk,i}\mathbf{\Lambda}_g\mathbf{V}_g^H & (4.20) \end{aligned}$$

where $\mathbf{V}_{hk,i}$ contains the $((k-1)d+1+(2-i)L_1)$th to $(kd+(2-i)L_1)$th rows from $\mathbf{n}_h$, and $\mathbf{U}_{gk,i}$ contains the $(\sum_{m=1}^{k-1} N_{m,i}+1+(i-1)\sum_{m=1}^{K} N_{m,\bar{i}})$th to $(\sum_{m=1}^{k} N_{m,i}+(i-1)\sum_{m=1}^{K} N_{m,\bar{i}})$th rows from $\mathbf{U}_g$ that $\mathbf{V}_h = [\mathbf{V}_{h1,2}^T, \ldots, \mathbf{V}_{hK,2}^T, \mathbf{V}_{h1,1}^T, \ldots, \mathbf{V}_{hK,1}^T]^T$, $\mathbf{U}_g = [\mathbf{U}_{g1,1}^T, \ldots, \mathbf{U}_{gK,1}^T, \mathbf{U}_{g1,2}^T, \ldots, \mathbf{U}_{gK,2}^T]^T$. We should note that $\mathbf{n}_{hk,i}$ and $\mathbf{U}_{gk,i}$ have dimension of $d \times L_1$, $N_{k,i} \times L_2$, respectively.

Adopt the proof from [47], the optimal structure for relays matrix $\mathbf{F}$ is

$$\mathbf{F} = \mathbf{V}_g\mathbf{A}\mathbf{U}_h^H \qquad (4.21)$$

where $\mathbf{A}$ is a $L_2 \times L_1$ arbitrary matrix. By substituting (4.21) into (4.19) and (4.20), we obtain the following

$$\begin{aligned} \mathbf{G}_{k,i}\mathbf{F}\bar{\mathbf{H}}_{k,\bar{i}} &= \mathbf{U}_{gk,i}\mathbf{\Lambda}_g\mathbf{A}\mathbf{\Lambda}_h\mathbf{V}_{hk,\bar{i}}^H & (4.22)\\ \mathbf{G}_{k,i}\mathbf{F}\mathbf{F}^H\mathbf{G}_{k,i}^H &= \mathbf{U}_{gk,i}\mathbf{\Lambda}_g\mathbf{A}\mathbf{A}^H\mathbf{\Lambda}_g\mathbf{U}_{gk,i}^H & (4.23)\\ \mathbf{\Xi}_{k,i} &= \mathbf{U}_{gk,i}\mathbf{\Lambda}_g\mathbf{A}\sum_{j=1}^{2}\sum_{m\neq k}^{K}\mathbf{\Lambda}_h\mathbf{V}_{hm,j}^H\mathbf{V}_{hm,j}\mathbf{\Lambda}_h\mathbf{A}^H\mathbf{\Lambda}_g\mathbf{U}_{gk,i}^H & (4.24) \end{aligned}$$

The SMSE in (4.16) becomes

$$\begin{aligned} &\text{SMSE}\\ &= \sum_{i=1}^{2}\sum_{k=1}^{K} tr\Big[\mathbf{I}_d - \mathbf{V}_{hk,\bar{i}}\mathbf{\Lambda}_h\mathbf{A}^H\mathbf{\Lambda}_g\mathbf{U}_{gk,i}^H(\mathbf{U}_{gk,i}\mathbf{\Lambda}_g\mathbf{A}\mathbf{\Lambda}_h\mathbf{V}_{hk,\bar{i}}^H\mathbf{V}_{hk,\bar{i}}\mathbf{\Lambda}_h\mathbf{A}^H\mathbf{\Lambda}_g\mathbf{U}_{gk,i}^H\\ &\quad +\sigma_r^2\mathbf{U}_{gk,i}\mathbf{\Lambda}_g\mathbf{A}\mathbf{A}^H\mathbf{\Lambda}_g^H\mathbf{U}_{gk,i}^H + \sigma_k^2\mathbf{I}_{N_{k,i}}\\ &\quad +\mathbf{U}_{gk,i}\mathbf{\Lambda}_g\mathbf{A}\sum_{j=1}^{2}\sum_{m\neq k}^{K}\mathbf{\Lambda}_h\mathbf{V}_{hm,j}^H\mathbf{V}_{hm,j}\mathbf{\Lambda}_h\mathbf{A}^H\mathbf{\Lambda}_g\mathbf{U}_{gk,i}^H)^{-1}\mathbf{U}_{gk,i}\mathbf{\Lambda}_g\mathbf{A}\mathbf{\Lambda}_h\mathbf{V}_{hk,\bar{i}}^H\Big] \quad (4.25) \end{aligned}$$

From (4.25), the SMSE can not be decomposed into subproblems because of the complicating variable matrix $\mathbf{A}$. Let us introduce

$$\mathbf{\Lambda}_g\mathbf{A} = \mathbf{U}_g^H\mathbf{C} = \sum_{i=1}^{2}\sum_{k=1}^{K}\mathbf{U}_{gk,i}^H\mathbf{C}_{k,i} \qquad (4.26)$$

where $\mathbf{C} = [\mathbf{C}_{1,1}^T, \ldots, \mathbf{C}_{K,1}^T, \mathbf{C}_{1,2}^T, \ldots, \mathbf{C}_{K,2}^T]^T$ and $\mathbf{C}_{k,i}$ is an $N_{k,i} \times L_1$ matrix. Since $\mathbf{U}_g$ is an unitary matrix, for any matrix $\mathbf{A}$, we have $\mathbf{C} \triangleq \mathbf{U}_g \mathbf{\Lambda}_g \mathbf{A}$. Thus, instead of optimizing $\mathbf{A}$, we can optimize $\{\mathbf{C}\} = \{\mathbf{C}_{1,1}, \ldots, \mathbf{C}_{K,1}, \mathbf{C}_{1,2}, \ldots, \mathbf{C}_{K,2}\}$. The relay matrix $\mathbf{F}$ in (4.21) becomes

$$\mathbf{F} = \mathbf{V}_g \mathbf{\Lambda}_g^{-1} \mathbf{U}_g^H \mathbf{C} \mathbf{U}_h^H. \tag{4.27}$$

By substituting (4.27) into (4.25) we obtain the SMSE as a function of $\mathbf{C}_{k,i}$ as

$$\text{SMSE} = \sum_{i=1}^{2} \sum_{k=1}^{K} \psi_{k,i}(\mathbf{C}_{k,i}). \tag{4.28}$$

where

$$\begin{aligned}\psi_{k,i}(\mathbf{C}_{k,i}) = tr\Big[&\mathbf{I}_d - \mathbf{V}_{hk,\bar{i}} \mathbf{\Lambda}_h \mathbf{C}_{k,i}^H \Big(\mathbf{C}_{k,i} \mathbf{\Lambda}_h \mathbf{V}_{hm,\bar{i}}^H \mathbf{V}_{hm,\bar{i}} \mathbf{\Lambda}_h \mathbf{C}_{k,i}^H + \sigma_r^2 \mathbf{C}_{k,i} \mathbf{C}_{k,i}^H \\ &+ \mathbf{C}_{k,i} \sum_{j=1}^{2} \sum_{m \neq k}^{K} \mathbf{\Lambda}_h \mathbf{V}_{hm,j}^H \mathbf{V}_{hm,j} \mathbf{\Lambda}_h \mathbf{C}_{k,i}^H + \sigma_{k,i}^2 \mathbf{I}_{N_{k,i}}\Big)^{-1} \mathbf{C}_{k,i} \mathbf{\Lambda}_h \mathbf{V}_{hk,\bar{i}}^H\Big]\end{aligned} \tag{4.29}$$

It can be seen from (4.28) and (4.29) that the MSE of received signal transmitted from the kth node of site $\bar{i}$ to the kth node of site i is a function of $\mathbf{C}_{k,i}$ only. In other words, the objective function SMSE is decomposed in terms of the optimization variable.

From (4.26), the transmit power at the relay in (4.8) can be rewritten as

$$\begin{aligned} tr(\mathbf{F} E[\mathbf{y}_r \mathbf{y}_r^H] \mathbf{F}^H) &= tr(\mathbf{A}(\sum_{i=1}^{2} \sum_{k=1}^{K} \mathbf{\Lambda}_h \mathbf{V}_{hk,i}^H \mathbf{V}_{hk,i} \mathbf{\Lambda}_h + \sigma_r^2 \mathbf{I}_{L_1}) \mathbf{A}^H) \\ &= tr(\mathbf{\Lambda}_g^{-1} \mathbf{U}_g^H \mathbf{C} (\mathbf{\Lambda}_h^2 + \sigma_r^2 \mathbf{I}_{L_1}) \mathbf{C}^H \mathbf{U}_g \mathbf{\Lambda}_g^{-1}) \\ &= tr\left(\mathbf{C}^H \mathbf{U}_g \mathbf{\Lambda}_g^{-2} \mathbf{U}_g^H \mathbf{C} \left(\mathbf{\Lambda}_h^2 + \sigma_r^2 \mathbf{I}_{L_1}\right)\right) \\ &= tr(\mathbf{C}^H \mathbf{\Pi} \mathbf{C} \mathbf{\Psi}) \le P_r. \end{aligned} \tag{4.30}$$

where $\mathbf{\Pi} = \mathbf{U}_g \mathbf{\Lambda}_g^{-2} \mathbf{U}_g^H$, $\mathbf{\Psi} = \mathbf{\Lambda}_h^2 + \sigma_r^2 \mathbf{I}_{L_1}$. It can be seen from (4.30) that $\mathbf{C}_{k,i}, k = 1, \ldots, K, i = 1, 2$ are coupled through the power constraint. We propose to modify the power constraint (4.30) by applying the inequality of $tr(\mathbf{AB}) \le tr(\mathbf{A}) tr(\mathbf{B})$. The transmit power at the relay node becomes

$$tr\left\{\mathbf{C}^H \mathbf{\Pi} \mathbf{C} \mathbf{\Psi}\right\} \le tr\left\{\mathbf{C} \mathbf{\Psi} \mathbf{C}^H\right\} tr\left\{\mathbf{\Pi}\right\}. \tag{4.31}$$

Then the power constraint in (4.30) is modified to be

$$\sum_{j=1}^{2} \sum_{k=1}^{K} tr\left\{\mathbf{C}_{k,i} \mathbf{\Psi} {\mathbf{C}_{k,i}}^H\right\} \le P_r / tr(\mathbf{\Lambda}_g^{-2}). \tag{4.32}$$

In fact, (4.32) imposes a stricter transmission power constraint at the relay node, i.e., if (4.32) holds, the original power constraint (4.30) is also satisfied.

Based on (4.28) and (4.32), the modified relay matrix optimization problem can be written as

$$\min_{\{\mathbf{C}_{k,i}\}} \quad \sum_{i=1}^{2}\sum_{k=1}^{K} \psi_{k,i}(\mathbf{C}_{k,i}) \tag{4.33a}$$

$$s.t. \quad \sum_{j=1}^{2}\sum_{k=1}^{K} tr\left\{\mathbf{C}_{k,i}\boldsymbol{\Psi}\mathbf{C}_{k,i}{}^{H}\right\} \leq \bar{P}_r \tag{4.33b}$$

where $\bar{P}_r = P_r/tr(\boldsymbol{\Lambda}_g^{-2})$ is the modified power budget at the relay. We can see from (4.33a) and (4.33b) that the relay matrix optimization problem can be decomposed into $2K$ subproblems where the (k,i)th subproblem is to optimize $\mathbf{C}_{k,i}$ as

$$\min_{\mathbf{C}_{k,i}} \quad \psi_{k,i}(\mathbf{C}_{k,i}) \tag{4.34a}$$

$$s.t. \quad tr\left\{\mathbf{C}_{k,i}\Psi\mathbf{C}_{k,i}^{H}\right\} \leq P_{rk,i} \tag{4.34b}$$

where $P_{rk,i} \geq 0, k = 1, \ldots, K; i = 1, 2$ and $\sum\limits_{i=1}^{2}\sum\limits_{k=1}^{K} P_{rk,i} = P_r/tr(\boldsymbol{\Lambda}_g^{-2})$.

Let us introduce the following matrices for $k = 1, \ldots, K$

$$\mathbf{J}_{rk} = \sum_{j=1}^{2}\sum_{m=1,m\neq k}^{K} \boldsymbol{\Lambda}_h \mathbf{V}_{hm,j}^{H}\mathbf{V}_{hm,j}\boldsymbol{\Lambda}_h + \sigma_r^2\mathbf{I}_{L_1} \tag{4.35}$$

$$\mathbf{X}_{k,i} = \mathbf{J}_{rk}^{-\frac{1}{2}}\boldsymbol{\Lambda}_h\mathbf{V}_{hk,\bar{i}}^{H} \tag{4.36}$$

$$\mathbf{Y}_{k,i} = \mathbf{C}_{k,i}\mathbf{J}_{rk}^{\frac{1}{2}} \tag{4.37}$$

where the dimensions of $\mathbf{X}_{k,i}$ and $\mathbf{Y}_{k,i}$ are $L_1 \times d$ and $N_{k,i} \times L_1$ respectively. The MSE at the kth node on site i becomes

$$\begin{aligned} &f_{k,i}(\mathbf{Y}_{k,i}) \\ &= tr\Big(\mathbf{I}_d - \mathbf{X}_{k,i}^{H}\mathbf{Y}_{k,i}^{H}\Big(\mathbf{Y}_{k,i}\mathbf{X}_{k,i}\mathbf{X}_{k,i}^{H}\mathbf{Y}_{k,i}^{H} + \mathbf{Y}_{k,i}\mathbf{Y}_{k,i}^{H} + \sigma_{k,i}^2\mathbf{I}_{N_{k,i}}\Big)^{-1}\mathbf{Y}_{k,i}\mathbf{X}_{k,i}\Big) \end{aligned} \tag{4.38}$$

and the power constraint at the relay in (4.34b) becomes

$$tr\left\{\mathbf{Y}_{k,i}\left(\mathbf{X}_{k,i}\mathbf{X}_{k,i}^{H} + \mathbf{I}_{L_1}\right)\mathbf{Y}_{k,i}^{H}\right\} \leq P_{rk,i} \tag{4.39}$$

Using (4.38)-(4.39), the problem (4.34) can be equivalently rewritten as

$$\min_{\mathbf{Y}_{k,i}} \quad f_{k,i}(\mathbf{Y}_{k,i}) \tag{4.40a}$$

$$s.t. \quad tr\left\{\mathbf{Y}_{k,i}\left(\mathbf{X}_{k,i}\mathbf{X}_{k,i}^{H} + \mathbf{I}_{L_1}\right)\mathbf{Y}_{k,i}^{H}\right\} \leq P_{rk,i} \tag{4.40b}$$

The problem (4.40) is the MMSE-based relay matrix optimization problem for a single-user two-hop MIMO relay system [18] with the first hop channel $\mathbf{X}_{k,i}$, the relay matrix $\mathbf{Y}_{k,i}$ and the second hop channel $\mathbf{I}_{N_{k,i}}$. It can be shown similar to [18] that the optimal structure of $\mathbf{Y}_k$ is

$$\begin{aligned} \mathbf{Y}_{k,i} &= \left[\mathbf{I}_d,\ 0_{d\times(N_{k,i}-d)}\right]^T \mathbf{\Lambda}_{yk,i}\mathbf{U}^H_{xk,i} \\ & \qquad k=1,\cdots,K,\ i=1,2 \end{aligned} \tag{4.41}$$

where $\mathbf{X}_{k,i} = \mathbf{U}_{xk,i}\mathbf{\Lambda}_{xk,i}\mathbf{V}^H_{xk,i}$ is the SVD of $\mathbf{X}_{k,i}$, $\mathbf{Y}_{k,i} = \mathbf{U}_{yk,i}\mathbf{\Lambda}_{yk,i}\mathbf{V}^H_{yk,i}$ is the SVD of $\mathbf{Y}_{k,i}$, and the dimensions of $\mathbf{U}_{xk,i}$, $\mathbf{\Lambda}_{xk,i}$, $\mathbf{V}_{xk,i}$, $\mathbf{U}_{yk,i}$, $\mathbf{\Lambda}_{yk,i}$, $\mathbf{V}_{yk,i}$ are $L_1\times d$, $d\times d$, $d\times d$, $N_{k,i}\times d$, $d\times d$, $d\times L_1$, respectively.

By substituting (4.41) back into the problem in (4.33a) and (4.33b), the relay matrix optimization problem becomes

$$\min_{\{\mathbf{\Lambda}_{yk,i}\}} \quad \sum_{i=1}^{2}\sum_{k=1}^{K} tr\left(\mathbf{I}_d + \mathbf{\Lambda}^2_{xk,i}\left(\mathbf{I}_d + \sigma^2_{k,i}\mathbf{\Lambda}^{-2}_{yk,i}\right)^{-1}\right)^{-1} \tag{4.42a}$$

$$s.t. \quad \sum_{i=1}^{2}\sum_{k=1}^{K} tr\left\{\mathbf{\Lambda}^2_{yk,i}\left(\mathbf{\Lambda}^2_{xk,i} + \mathbf{I}\right)\right\} \le \bar{P}_r \tag{4.42b}$$

where $\{\mathbf{\Lambda}_{yk,i}\} \triangleq \{\mathbf{\Lambda}_{y1,1},\ldots,\mathbf{\Lambda}_{yK,1},\ldots,\mathbf{\Lambda}_{y1,2},\ldots,\mathbf{\Lambda}_{yK,2}\}$. The problem (4.42) can be equivalently rewritten as the following problem with scalar variables

$$\min_{\{\lambda_{yk,i,j}\}} \quad \sum_{i=1}^{2}\sum_{k=1}^{K}\sum_{j=1}^{d}\left\{1 + \frac{\lambda^2_{xk,i,j}\lambda^2_{yk,i,j}+1}{\lambda^2_{yk,i,j}+\sigma^2_{k,i}}\right\}^{-1} \tag{4.43a}$$

$$s.t. \quad \sum_{i=1}^{2}\sum_{k=1}^{K} tr\left\{\lambda^2_{yk,i}\left(\lambda^2_{xk,i}+1\right)\right\} \le \bar{P}_r \tag{4.43b}$$

$$\lambda_{yk,i,j} \le 0, k=1,\ldots,K, i=1,2, j=1,\ldots,d \tag{4.43c}$$

where $\lambda_{xk,i,j}$ and $\lambda_{yk,i,j}$, $j=1,\ldots,d$, are the jth diagonal element of $\mathbf{\Lambda}_{xk,i}$ and $\mathbf{\Lambda}_{yk,i}$, respectively, and $\{\lambda_{xk,i,j}\} \triangleq \{\lambda_{x1,1,1},\ldots,\lambda_{xK,2,d}\}$.

The problem (4.43) has the well-known water-filling solution and is given by

$$\begin{aligned} \lambda_{yk,i,j} &= \sqrt{\frac{1}{\lambda^2_{xk,i,j}+1}\left[\sqrt{\frac{\sigma^2_{k,i}\lambda^2_{xk,i,j}}{(\lambda^2_{xk,i,j}+1)\beta}} - \sigma^2_{k,i}\right]^{\dagger}} \\ &= k=1,\ldots,K,\ i=1,2,\ j=1,\ldots,d \end{aligned} \tag{4.44}$$

where $[x]^{\dagger} \triangleq max(x,0)$, and $\beta > 0$ is the solution to the following equation

$$\sum_{i=1}^{2}\sum_{k=1}^{K}\sum_{j=1}^{d}\left[\sqrt{\frac{\sigma^2_{k,i}\lambda^2_{xk,i,j}}{(\lambda^2_{xk,i,j}+1)\beta}} - \sigma^2_{k,i}\right]^{\dagger} = \bar{P}_r \tag{4.45}$$

As the left-hand side of (4.44) is a non-increasing function of β, it can be efficiently solved by the bisection method [37]. Finally, the relay precoding matrix can be obtaining from (4.27), (4.37), (4.41) and (4.44).

With given receiver matrices $\{\mathbf{W}_{k,i}\}$ and relay matrices $\mathbf{F}$, the sum-MSE can be rewritten as a function of $\{\mathbf{B}_{k,i}\}$ as

$$\begin{aligned}&\text{SMSE}\\ &=\sum_{i=1}^{2}\sum_{k=1}^{K} tr\Big(\Big(\bar{\mathbf{G}}_{k,i}\mathbf{H}_{k,\bar{i}}\mathbf{B}_{k,\bar{i}}-\mathbf{I}_d\Big)\Big(\bar{\mathbf{G}}_{k,i}\mathbf{H}_{k,\bar{i}}\mathbf{B}_{k,\bar{i}}-\mathbf{I}_d\Big)^H\\ &\quad+\bar{\mathbf{G}}_{k,i}\sum_{j=1}^{2}\sum_{m\neq k}^{K}\mathbf{H}_{m,j}\mathbf{B}_{m,j}\mathbf{B}_{m,j}^H\mathbf{H}_{m,j}^H\bar{\mathbf{G}}_{k,i}^H\Big)+t_2\end{aligned}\tag{4.46}$$

where $\bar{\mathbf{G}}_{k,i}=\mathbf{W}_{k,i}^H\mathbf{G}_{k,i}\mathbf{F}$ and $t_2 \triangleq \sum_{i=1}^{2}\sum_{k=1}^{K} tr(\mathbf{W}_{k,i}^H\mathbf{C}_{nk,i}\mathbf{W}_{k,i})$ can be ignored in the optimization progress as it does not depend on $\{\mathbf{B}_{k,i}\}$.

Using the identities of [41]

$$tr(\mathbf{A}^T\mathbf{B}) = (vec(\mathbf{A}))^T vec(\mathbf{B}) \tag{4.47}$$

$$tr(\mathbf{A}^H\mathbf{B}\mathbf{A}\mathbf{C}) = (vec(\mathbf{A}))^H(\mathbf{C}^T\otimes\mathbf{B})vec(\mathbf{A}) \tag{4.48}$$

$$vec(\mathbf{A}\mathbf{B}\mathbf{C}) = (\mathbf{C}^T\otimes\mathbf{A})vec(\mathbf{B}) \tag{4.49}$$

the SMSE function in (4.46) can be written as a function of $\mathbf{b}_{k,i}\triangleq vec(\mathbf{B}_{k,i})$, as

$$\begin{aligned}&\text{SMSE}\\ &=\sum_{i=1}^{2}\sum_{k=1}^{K}\Big[\big(\mathbf{S}_{k,i}\mathbf{b}_{k,\bar{i}}-vec(\mathbf{I}_d)\big)^H\big(\mathbf{S}_{k,i}\mathbf{b}_{k,\bar{i}}-vec(\mathbf{I}_d)\big)\\ &\quad+\sum_{j=1}^{2}\sum_{m\neq k}^{K}\mathbf{b}_{m,j}^H\Big(\mathbf{I}_d\otimes\mathbf{H}_{m,j}^H\bar{\mathbf{G}}_{k,i}^H\bar{\mathbf{G}}_{k,i}\mathbf{H}_{m,j}\Big)\mathbf{b}_{m,j}\Big]+t_2\\ &=\sum_{i=1}^{2}\sum_{k=1}^{K}\Big[\big(\mathbf{S}_{k,\bar{i}}\mathbf{b}_{k,i}-vec(\mathbf{I}_d)\big)^H\big(\mathbf{S}_{k,\bar{i}}\mathbf{b}_{k,i}-vec(\mathbf{I}_d)\big)+\mathbf{b}_{k,i}^H\sum_{j=1}^{2}\mathbf{T}_{k,j,i}\mathbf{b}_{k,i}^H\Big]+t_2\end{aligned}\tag{4.50}$$

where for $k=1,\cdots,K$

$$\begin{aligned}\mathbf{S}_{k,i} &\triangleq \mathbf{I}_d\otimes\bar{\mathbf{G}}_{k,i}\mathbf{H}_{k,\bar{i}}\\ \mathbf{T}_{k,j,i} &\triangleq \mathbf{I}_d\otimes\sum_{m=1,m\neq k}^{K}\mathbf{H}_{k,i}^H\bar{\mathbf{G}}_{m,j}^H\bar{\mathbf{G}}_{m,j}\mathbf{H}_{k,i}.\end{aligned}$$

By introducing $\mathbf{T}_i\triangleq bd(\sum_{j=1}^{2}\mathbf{T}_{1,j,i},\cdots,\sum_{j=1}^{2}\mathbf{T}_{K,j,i})$ and $\bar{\mathbf{S}}_{k,i}\triangleq[\mathbf{S}_{k1,i},\cdots,\mathbf{S}_{kK,i}]$, where $\mathbf{S}_{kk,i}=\mathbf{S}_{k,i}$ and $\mathbf{S}_{kj,i}=\mathbf{0}$, $j\neq k$, the SMSE function (4.50) can be written

Table 4.1: Procedure of solving the problem (4.14) by the proposed Algorithm 4.

1. Initialize the algorithm with $\mathbf{F}^{(0)}$ and $\{\mathbf{B}_{k,i}^{(0)}\}$ satisfying (4.8) and (4.9); Set $n = 0$.
2. Obtain $\{\mathbf{W}_{k,i}^{(n+1)}\}$ based on (4.15) with fixed $\mathbf{F}^{(n)}$ and $\{\mathbf{B}_{k,i}^{(n)}\}$.
3. Solve the problems (4.42) with given $\{\mathbf{B}_{k,i}^{(n)}\}$ to find $\{\lambda_{y,k,i}\}$ and update $\mathbf{F}^{(n+1)}$ through (4.44), (4.41), (4.37) and (4.27).
4. Update $\{\mathbf{B}_{k,i}^{(n+1)}\}$ by solving the problem (4.51) with fixed $\mathbf{F}^{(n+1)}$ and $\{\mathbf{W}_{k,i}^{(n+1)}\}$.
5. If $\mathrm{MSE}^{(n)} - \mathrm{MSE}^{(n+1)} \leq \varepsilon$, then end.
   Otherwise, let $n := n + 1$ and go to Step 2.

as a function of $\mathbf{b}_i = [\mathbf{b}_{1,i}^T,\ \mathbf{b}_{2,i}^T,\ \cdots, \mathbf{b}_{K,i}^T]^T$, as

$$\sum_{i=1}^{2} \Phi_i(\mathbf{b}_i) = \sum_{i=1}^{2}\left[\sum_{k=1}^{K}\Big(\bar{\mathbf{S}}_{k,\bar{i}}\mathbf{b}_i - vec(\mathbf{I}_d)\Big)^H\Big(\bar{\mathbf{S}}_{k,\bar{i}}\mathbf{b}_i - vec(\mathbf{I}_d)\Big) + \mathbf{b}_i^H\mathbf{T}_i\mathbf{b}_i\right].$$

Let us introduce $\mathbf{E}_{k,i} = \mathbf{I}_d \otimes \left(\mathbf{H}_{k,i}^H\mathbf{F}^H\mathbf{F}\mathbf{H}_{k,i}\right)$, $\mathbf{E}_i = bd\left(\mathbf{E}_{1,i}, \ldots, \mathbf{E}_{K,i}\right)$, $\bar{\mathbf{E}}_{k,i} = bd\left(\bar{\mathbf{E}}_{k1,i}, \ldots, \bar{\mathbf{E}}_{kK,i}\right)$, where $\bar{\mathbf{E}}_{kk,i} = \mathbf{I}_{dN_{k,i}}$ and $\bar{\mathbf{E}}_{kj,i} = \mathbf{0}, k \neq j$. The optimal $\{\mathbf{b}_i\} \triangleq \{\mathbf{b}_i,\ i = 1, 2\}$ can be obtained by solving the following problem

$$\min_{\{\mathbf{b}_i\}} \quad \sum_{i=1}^{2} \Phi_i(\mathbf{b}_i) \tag{4.51a}$$

$$s.t. \quad \mathbf{b}_i^H\bar{\mathbf{E}}_{k,i}\mathbf{b}_i \leq P_{k,i} \tag{4.51b}$$

$$\mathbf{b}_i^H\mathbf{E}_i\mathbf{b}_i \leq P_r - \sigma_r^2 tr(\mathbf{F}\mathbf{F}^H). \tag{4.51c}$$

$$k = 1, \cdots, K,\ i = 1, 2$$

The problem (4.51) is a QCQP problem and can be solved by the CVX MATLAB toolbox [38] for disciplined convex programming.

The steps of applying the simplified relay matrix design to solve the transceiver optimization problem are summarized in Table 4.1. Since the dimension of $\{\lambda_{y,k,i}\}$ is $2Kd$, the computational complexity of solving the problem (4.43) is $\mathcal{O}(Kd)$. When $L_1 = Kd$, the SVD of $\mathbf{X}_{k,i}$ has a complexity order of $\mathcal{O}(Kd^3)$. Therefore, the complexity of the simplified relay matrix design is $\mathcal{O}(K^2d^3)$, which is much lower than the computational complexity of the existing algorithms. However, we will see through numerical simulations that the proposed algorithm 4 in Table 4.1 has a slightly worse MSE and BER performance than the existing algorithms. Such performance-complexity tradeoff is very useful for practical interference MIMO relay communication systems.

4.4 Numerical Examples

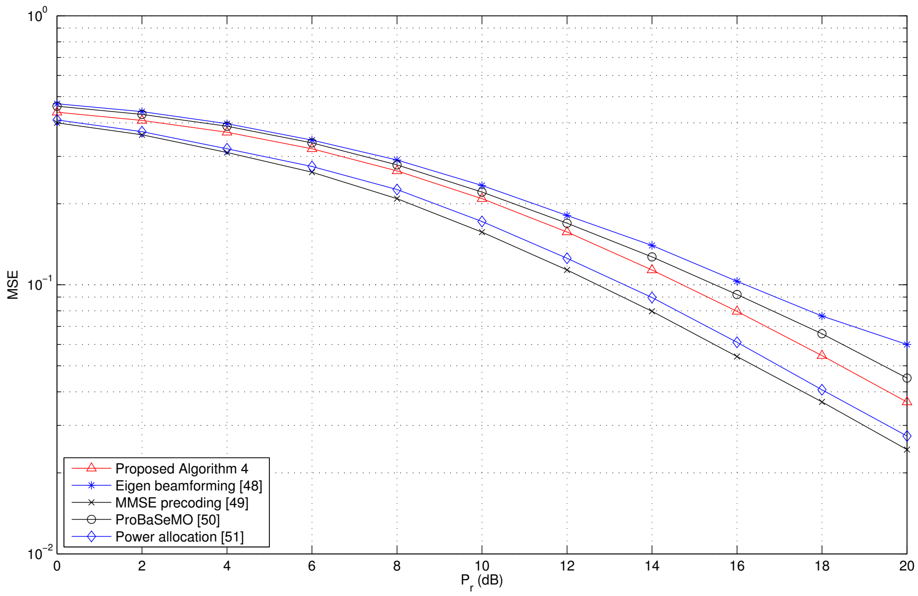

Figure 4.2: Example 4.1: MSE versus P_r, $K = 2$

In this section, we study the performance of the proposed algorithm 4 to jointly optimize the transceiver matrices through numerical simulations. We consider an interference MIMO relay system with $d = 1$, where all transmitters and receivers have the same number of antennas, i.e., $N_{k,1} = N_{k,2} = 2$; and the relay node has $N_r = 10$ antennas. We also assume that all source nodes have the same power budget as $P_{k,1} = P_{k,2} = 15$dB, $k = 1, \cdots, K$, and the relay node has the power budget as P_r. As the power constraints in (4.51b) and (4.51c) are inequalities, it is possible to scale up the actual power used by the proposed algorithm to met the power constraints with equalities. However, scaling up the power also increases the impact of the interference. In the following simulations, we skip the scaling up process. All channel matrices have i.i.d. complex Gaussian entries with zero-mean and unit variance. The noises are i.i.d. Gaussian with zero mean and unit variance. The QPSK constellations are used to modulate the source symbols. In all simulation examples, there are $K = 2$ source-destination pairs, and the simulation results are averaged over 5×10^5 independent channel realizations. The proposed algorithms are initialized with $\mathbf{F}^{(0)} = \sqrt{P_r/tr(\sum_{i=1}^{2}\sum_{k=1}^{K}\mathbf{H}_{k,i}\mathbf{B}_{k,i}\mathbf{B}_{k,i}^H\mathbf{H}_{k,i}^H + \sigma_r^2\mathbf{I}_{N_r})}\mathbf{I}_{N_r}$ and $\{\mathbf{B}_{k,i}^{(0)} = \sqrt{P_{k,i}/N_{k,i}}\mathbf{I}_{N_{k,i}}\}$. As a benchmark, the performance of the proposed algorithm is compared with the MMSE relay precoding algorithms in [53] and [54].

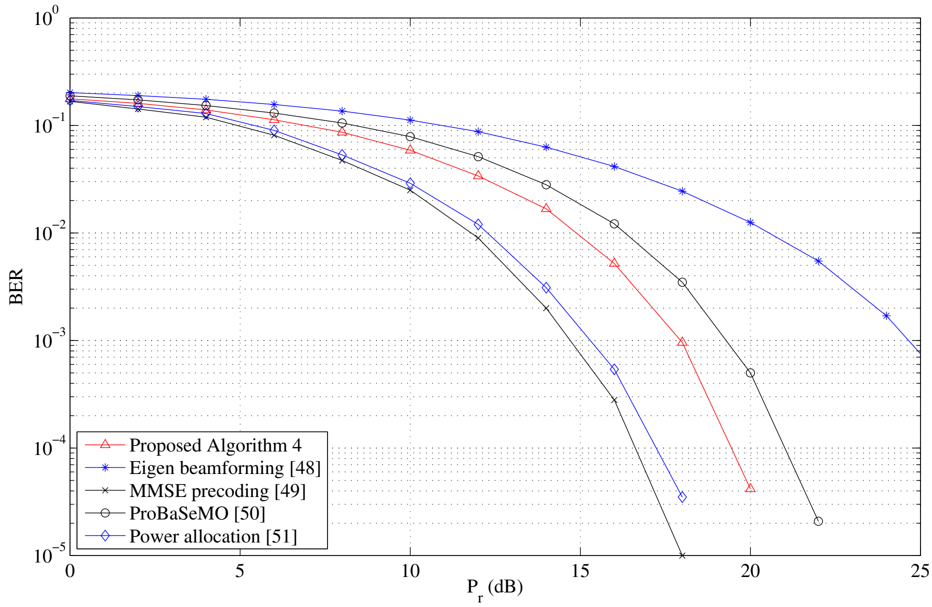

Figure 4.3: Example 4.1: BER versus P_r, $K = 2$

In the first example, we compare the performance of the proposed algorithm with the transceiver design algorithms in [53] -[56] for a MIMO relay system with $K = 2$ two-way link pairs. For a fair comparison with [53], we set $d = 1$. Fig. 4.4 and Fig. 4.3 show the normalized SMSE and the BER performance of the five algorithms tested versus P_r. It can be seen that while the proposed algorithm outperforms the eigen-beamforming algorithm in [53] and the ProBaSeMO in [55], the MMSE precoding algorithm in [54] and the power allocation algorithm in [56] outperform the proposed algorithm. However, the computational complexity of the algorithms in [54] and [56] is higher than the proposed algorithm. Such performance-complexity tradeoff is very useful for practical interference two-way MIMO relay communication systems.

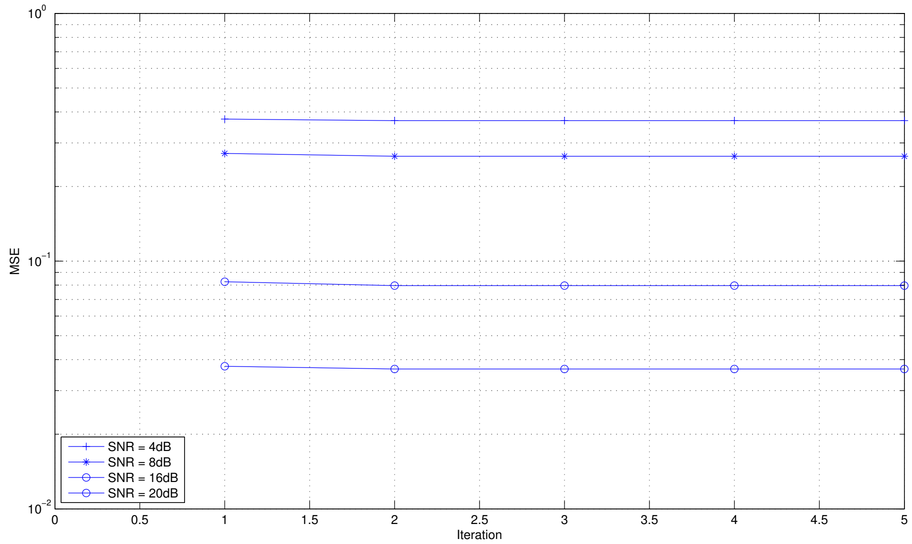

Figure 4.4: Example 4.2: MSE versus the number of iterations, $K = 2$.

In the second example, we study the convergence speed of the proposed algorithm. Fig. 4.4 and Fig. 4.5 show respectively the normalized SMSE and BER performance of the proposed algorithm versus the number of iterations at various levels of P_r with $K = 2$. The simulation results show that the conditions for convergence of the proposed Algorithm 4, step 5 in Table 4.1, is typically met with two iterations. By imposing the stricter condition on the power constraints in (4.32), the search for $\mathbf{F}$ is now limited in a stricter space. Thus, the optimal $\mathbf{F}$ after the first iteration is typically closer to the final optimal solution. It can be seen that at all P_r levels, the proposed algorithm converges within two iterations.

4.5 Chapter Summary

In this chapter, we have presented an approach for jointly optimizing source, relay and receiver matrices of an interference two-way MIMO relay system. Compared to other iterative algorithms, the proposed algorithm derives the optimal structure of the relay precoding matrix to reduce the computational complexity. Furthermore, the power constraint at the relay node is modified which decoupled the highly nonconvex relay optimization problem into the convex sub-problems. Interestingly, each of the simplified sub-problems has a closed-form solution. Numerical simulation results show that the algorithm converges quickly after a few

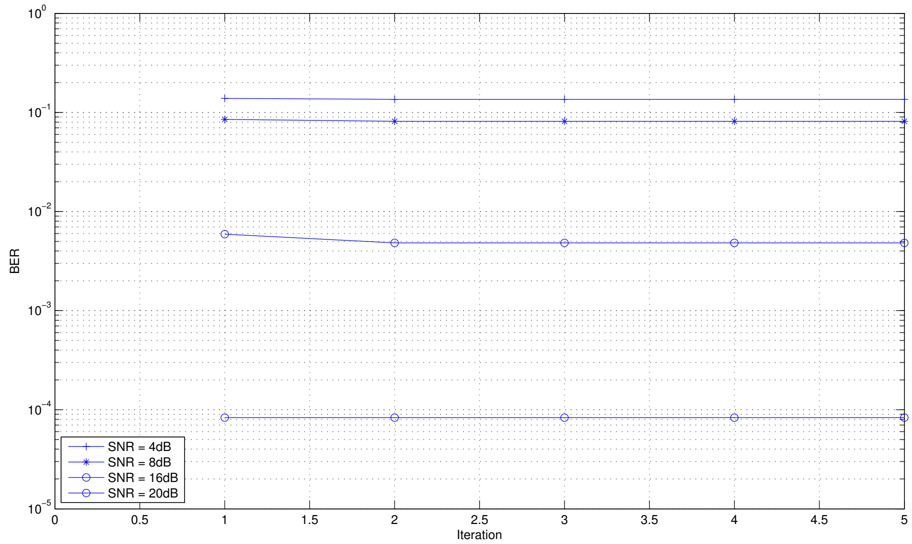

Figure 4.5: Example 4.2: BER versus the number of iterations, $K = 2$.

iterations. Moreover, it shows that all users can achieve performance fairness among users.

Chapter 5

Conclusions and Future Work

Interference MIMO offers several advantages over single user MIMO communication. This thesis aims at designing and studying advanced algorithms for interference MIMO communication systems.

5.1 Concluding Remarks

Advanced transceiver designs for MIMO communication systems have been investigated. In Chapter 2, we focus on the interference MIMO relay systems where the direct links are sufficiently strong and taken into account. We develop two iterative algorithms to solve the highly nonconvex joint source, relay and receiver optimization problem for interference MIMO relay communication systems. The MMSE of the signal waveform estimation at the destination nodes is chosen as the design criterion to optimize the transceiver matrices for interference suppression. The simulation results show that the proposed algorithms have much better MSE and BER performance compared with the algorithms without considering the direct links. Besides, for the slight lost of performance, the proposed Algorithm 2.2 has a lower per-iteration computational complexity than the proposed Algorithm 2.1.

Considering complexity as a main design criterion, in Chapter 3, we investigate the scenarios where the direct links of the interference MIMO relay systems are weak enough and can be ignored. The two proposed algorithms in this chapter, the tri-step algorithm and the simplified relay matrix design, follow the similar design procedure of the Algorithm 2.2. However, instead of optimizing the full size matrix, we exploit the optimal structure of the relay matrix which significantly reduces the computational complexity. Besides, we modify the transmission power constraint at the relay node in the simplified matrix design which further reduces the complexity. The simulation results show the simplified matrix design has a slightly worse performance than the tri-step algorithm. However, in a network that has a large number of transmitter-receiver pairs, the computational complexity of the simplified algorithm is much lower. We also investigate the possibility to extend the two proposed algorithms to the more general scenarios such as imperfect CSI and multiple relay nodes. Unfortunately, unlike the tri-step algorithm, to the best of our knowledge, the simplified algorithm can not be extended to the case of multiple relays.

Finally, we extend the simplified relay matrix design to the AF two-way interference MIMO relay systems. Compared with the existing iterative algorithms, the proposed algorithm derives the optimal structure of the relay precoding matrix. Furthermore, the highly nonconvex relay optimization problem is decoupled into convex sub-problems by modifying the power constraint at the relay node. The simulation results show that the proposed algorithm achieves significant reduction in computational complexity in the trade of system performance.

5.2 Future Works

In this thesis, we have developed several advanced signal processing algorithms for interference MIMO relay systems. In Chapter 2, two iterative algorithms are proposed to jointly optimize source, relay and receiver matrices. The complexity of iterative algorithms is higher than closed-form solutions, thus closed-form solution to the problem can be an interesting future work.

In Chapter 3 and Chapter 4, it has been shown that the simplified relay matrix designs significantly reduce the complexity of the system while keeping the performance at a reasonable level. Moreover, the algorithms also have closed-form solution, which has comparatively lower complexity than iterative algorithms. However, the proposed algorithms are limited to the scenario that a single relay node is in operation. Thus, extending the algorithms for the system with multiple relays still remains open as a challenging problem.

It has been shown in Chapter 2 that the direct links between the source and destination nodes provide valuable spatial diversity and should not be ignored. Thus, considering the direct links for the two-way interference MIMO relay systems in Chapter 4 will also be interesting to investigate.

The finally yet importantly, it is practical to investigate the robust solution against imperfect CSI for each problem investigated in the thesis.

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