

School of Electrical Engineering and Computing
Department of Electrical and Computer Engineering

Design of Waveform Set for
Multiuser Ultra-Wideband Communications

Mohd Amaluddin Bin Yusoff

This thesis is presented for the degree of
Doctor of Philosophy
of
Curtin University

October 2016

Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Signature:

Date:

To

Ma, Abah, Norhuda, Zikry, Syafiq and Laila

Acknowledgements

Even until the time of writing this last piece of the thesis, I still could not identify the real reason for this PhD journey. However, I realize I am always surrounded by the ones who matter the most to me. I feel they are part of the reasons for this PhD. I am therefore eternally grateful to my parents Ma (Al-fatihah) and Abah, my beloved wife Norhuda, and my children Zikry, Syafiq and Laila for the inspiration and strength throughout this journey.

I would like to express my heartily gratitude to my supervisor Associate Prof Dr Zhuquan Zang. I felt humbled with his immense knowledge and experience. He patiently reviewed every detail of research outputs. His unique style of supervision has allowed me to ‘play’ with the topics fascinating to me.

I would also like to extend my gratitude to all members of my thesis committee; the Chairperson Associate Professor Dr Chua Han Bing, my co-supervisor Dr Weiyong Yan, and ex-Chairperson Professor Joan Gribble for their constant support and necessary guidance.

I am definitely thankful to Dr Lenin Gopal for constant email updates and continuous reminders. Sincerely, his PhD completion greatly amplifies my energy and determination.

I would like to thank Curtin University for initial financial funding and suitable environment for me to grow as a researcher, lecturer and life-long learner. I would also like to appreciate my current employer, Shell for all the opportunities and career path which indirectly influences this journey favorably.

Lastly and most importantly, all praise is due to Allah.

Abstract

One of the major focuses of current research is to develop new schemes which can be used to maximize multi-access communication system capacity and increase user bandwidth utilization efficiency. To maximize system capacity and bandwidth utilization efficiency for multiuser and UWB communication systems, a set of mutually orthogonal waveforms with low correlations and prescribed spectral properties are highly desirable. Recent release of spectral mask for UWB communications has further highlighted the need for a new systematic approach to design waveforms with maximum transmitted energy while satisfying the mask constraints. In this thesis the design of analogue waveform sets based on suitably chosen Hermite Rodriguez basis functions is investigated. The Hermite Rodriguez functions have been recently applied and proven to be useful in other areas such as radar, biomedical and image processing applications.

First, analogue waveform set design is formulated as a constrained nonlinear optimization problem to minimize stopband energy and shape the magnitude spectral functions to a fixed desired spectral shape. To minimize interferences in multiuser systems, autocorrelation and crosscorrelation constraints are incorporated into the optimization problem. The optimization problem is solved sequentially with the crosscorrelation constraints treated as linear inequality constraints. The proposed approach provides a new alternative to design signature waveforms and proves the suitability of the Hermite Rodriguez basis functions to design signals with finite time support.

Next, a sequential quadratic semi-infinite programming approach is proposed to design a set of spectrally efficient orthogonal UWB waveforms. Nonlinear spectral and crosscorrelation constraints formulated into the optimization problem are linearized respectively by using the real rotation theorem and by solving the quadratic semi-infinite program sequentially. Since recently released spectral mask limits the power spectral density (PSD) of a UWB

waveform to a very low level, it is of a paramount importance for the UWB waveforms to have maximum energy while satisfying the given spectrum mask constraint. The proposed approach therefore directly maximizes the energy of transmitted waveforms with no phase angle constraints imposed on the waveforms. The proposed approach produces high number of waveforms with high spectral efficiency. The flexibility of the proposed approach to suppress multiple narrowband interferences at multiple frequencies is further demonstrated through a numerical example.

Then, the spectral efficiency of the designed waveform is further improved by incorporating the power spectral density of a commonly used basic pulse into the design problem formulated as a semi-infinite quadratic optimization problem. The resulting nonlinear piece-wise continuous bounds are converted into linear constraints using the real rotation theorem. The objective function is formulated as direct maximization of a quadratic function of filter coefficients which eliminates the spectral factorization step commonly used in FIR filter design via semi-definite programming. Optimal upper bounds for each basic pulse are implemented as constraints in the optimization problem and therefore high spectral utilization is achieved especially when a suboptimal basic pulse is used.

Finally, Löwdin orthogonalization method is used in this research to generate an orthogonal set of spectrally efficient waveforms from a finite set of equally spaced shifts of an optimal waveform. The optimal waveform is designed based on Hermite Rodriguez basis functions via a semi-infinite quadratic programming approach. The Löwdin method generates a set of overlapping waveforms with minimal energy distortion with respect to the optimal waveform. This set of undistorted overlapping waveforms is desirable for orthogonal overlapping pulse position modulation (PPM) UWB systems. A numerical example is presented to illustrate the ability of this 2-step approach to produce many overlapping orthogonal waveforms while complying with the FCC spectral mask.

Author's Note

Parts of this thesis and concepts from it have been previously published in the following journal and/or conference papers.

Journal Papers

- [1] M. A. Yusoff, Z. Zang, S. Abeysekera, "Design of Spectrally Efficient UWB Waveforms using Hermite Rodriguez Functions", *IET Commun.*, 2012.
- [2] M. A. Yusoff, "Application of Hermite-Rodriguez Functions to Pulse Shaping Analog Filter Design", *Inter. Journal of Computer Science and Engineering*, Vol. 2, No. 2, Article 18, 2008.

Conference Papers

- [1] M. A. Yusoff, Z. Zang, "Quadratic Programming Approach for UWB Pulse Shaping Filter Design with Nonconstant Upper Bounds", *15th Int. Conf. on Advanced Communications Technology (ICACT 2013)*, Pyeongchang, Korea, Jan 2013.
- [2] M. A. Yusoff, Z. Zang, "UWB Waveform Set Design using Löwdin's Orthogonalization with Hermite Rodriguez Functions", *IET Int. Conf. on Wireless Communications and Applications (ICWCA 2012)*, Kuala Lumpur, October 2012.
- [3] M. A. Yusoff, Z. Zang, "Design of UWB Waveforms for Narrowband Interferences Suppression", *IEEE Asia Pacific Conference on Circuits and Systems*, Kuala Lumpur, Dec 2010.
- [4] M. A. Yusoff, Z. Zang, S. Abeysekera, "Spectrally Efficient Waveform Set Design for UWB Wireless Communications", *7th Inter. Conf. on Information, Communications and Signal Processing*, Macau, Dec 2009.

- [5] M. A. Yusoff, Z. Zang, "Waveform Set Design using Hermite-Rodriguez Functions for CDMA Communications", *Inter. Conf. on Signal Acquisition and Processing*, Kuala Lumpur, April 2009.

- [6] M. A. Yusoff, Z. Zang, S. Abeysekera, "UWB pulse shaping filter using Hermite-Rodriguez functions", *5th IASTED Asian Conf. on Comm. Systems and Networks*, Langkawi, April 2008.

Contents

Author's Note	vii
List of Figures	xi
List of Tables	xv
1 Introduction	1
1.1 Waveform Set Design	1
1.2 UWB Waveform Set Design	3
1.3 Thesis Objectives	4
1.4 Hermite-Rodriguez Functions	5
1.5 Thesis Overview and Contributions	7
2 Design of Waveform Set for CDMA Communications	11
2.1 Introduction	11
2.2 Problem Formulation and Solution Methods	13
2.3 An Illustrative Numerical Example	18
2.4 Chapter Summary	21
3 Design of Spectrally Efficient UWB Waveforms	23
3.1 Introduction	24
3.2 Hermite-Rodriguez Linear FM Signals	25
3.3 Waveform Set Design	26
3.4 Illustrative Numerical Examples	30
3.4.1 Design of 8 FCC-compliant waveforms	30
3.4.2 Waveform Set Design with NBI suppression	36

3.5	Chapter Summary	38
4	Design of UWB Pulse Shaping Filter with Nonconstant Upper Bounds	39
4.1	Introduction	39
4.2	Pulse Shaping Problem	40
4.3	Illustrative Design Examples	44
4.4	Chapter Summary	50
5	Design of UWB Waveform Set Using Löwdin's Orthogonalization	51
5.1	Introduction	51
5.2	Optimal Waveform Design	52
5.3	Waveform Orthogonalization	54
5.4	Design Examples	55
5.5	Chapter Summary	62
6	Conclusions and Future Work	63
6.1	Concluding Remarks	63
6.2	Future Works	65
A	Matlab code	67
	Bibliography	73

List of Figures

1.1	Block diagram of a multiuser UWB communication system. The research focuses on the design of filter sets.	4
1.2	Hermite-Rodriguez functions of order zero to four, $\lambda = 1$	6
1.3	Magnitude of Fourier transform of Hermite Rodriguez functions of order zero to four, $\lambda = 1$	7
2.1	Block diagram of a multiuser communication system.	13
2.2	Magnitude spectral responses of the 4 designed waveforms ($\alpha = 0.5$). . .	16
2.3	Impulse responses of the 4 designed waveforms ($\alpha = 0.5$).	17
2.4	Autocorrelation of the designed waveforms ($\alpha = 0.5$). Low correlation values were produced at $t = 3k$ where $k = \pm 1, \pm 2$	17
2.5	Crosscorrelation of the designed waveforms ($\alpha = 0.5$). Low correlation values were produced at $t = 3k$ where $k = 0, \pm 1, \pm 2$	18
2.6	Magnitude spectral responses of the 4 designed waveforms ($\alpha = 0.25$). .	19
2.7	Impulse responses of the 4 designed waveforms ($\alpha = 0.25$).	20
2.8	Autocorrelation of the designed waveforms ($\alpha = 0.25$). Low correlation values were produced at $t = 3k$ where $k = \pm 1, \pm 2$	20
2.9	Crosscorrelation of the designed waveforms ($\alpha = 0.25$). Low correlation values were produced at $t = 3k$ where $k = 0, \pm 1, \pm 2$	21
3.1	Plot of the 8 designed waveforms $g_i(t)$ ($N = 21$).	31
3.2	Plot of power spectrum of the 8 designed waveforms $g_i(t)$ and FCC mask ($N = 21$).	32
3.3	Plot of autocorrelations of $g_0(t)$ and $g_1(t)$, and their crosscorrelation ($N = 21$). The crosscorrelation between $g_0(t)$ and $g_1(t)$ is low at $t = 0$	33
3.4	Plot of η vs index of the waveforms being designed for $N = 16, 21, 26$. .	34

3.5	Plot of η vs λ for $N = 21$ ($M = 1$). The best $\lambda = 1.15 \times 10^{-10}$	34
3.6	Plot of η vs filter order, N for $\lambda_{best} = 1.15 \times 10^{-10}$ ($M = 1$). The η increases as N increases.	35
3.7	Plot of the designed waveforms $g_0(t)$ and $g_1(t)$ ($M = 2, N = 21$).	35
3.8	Plot of power spectrum of the 2 designed waveforms $g(t)$ ($M = 2, N = 21$). The waveforms suppress NBI at 5.2 GHz and 8.1 GHz, and maximize energy for the entire spectrum.	36
3.9	Plot of autocorrelation and crosscorrelation. The crosscorrelation at $t = 0$ is essentially zero.	37
4.1	The PSD of the Gaussian Monocycle for various τ s. The PSD with $\tau = 0.087$ ns occupies the main bandwidth 3.1 to 10.6 GHz.	45
4.2	The constraint $S(f)/ Q(f) ^2$ for $\tau = 0.087$ ns. Note the difference with $\Gamma(f)$ implemented in [38] using the Fourier series.	48
4.3	The constraint $S(f)/ Q(f) ^2$ for $\tau = 0.15$ ns. Note the difference with the bounds in Fig. 4.2.	48
4.4	Plot of PSD for $\tau = 0.087$ ns ($N = 20$). Both PSDs (HR and FIR) occupy the main bandwidth.	49
4.5	Plot of PSD for $\tau = 0.15$ ns. The PSDs still occupy the main bandwidth even with the less optimal basic pulse.	49
5.1	The optimal waveform $p(t)$ designed in the first step. Löwdin orthogonalization method will then transform $p(t)$ to an orthogonal set of pulses.	55
5.2	The PSD of the optimal waveform $p(t)$ designed in the first step, $\eta = 0.8033$	56
5.3	The set of 5 overlapping orthogonal waveforms, $\{p_k^o\}_{k=-2}^2$ generated using Löwdin's method based on optimal $p(t)$	56
5.4	The set of 7 overlapping orthogonal waveforms, $\{p_k^o\}_{k=-3}^3$ generated using Löwdin's method based on optimal $p(t)$	57
5.5	The set of 15 overlapping orthogonal waveforms, $\{p_k^o\}_{k=-7}^7$ generated using Löwdin's method based on optimal $p(t)$	57
5.6	The PSD of the 5 orthogonal waveforms plotted in Fig. 5.3 ($M = 2$).	58
5.7	The PSD of the 7 orthogonal waveforms plotted in Fig. 5.4 ($M = 3$).	58
5.8	The PSD of the 15 orthogonal waveforms plotted in Fig. 5.5 ($M = 7$).	59
5.9	The crosscorrelations of one of the generated waveform, p_{-1}^o with the other 4 waveforms in the set ($M = 2$). It is essentially zero at $t = 0$	59

-
- 5.10 The crosscorrelations of one of the generated waveform, p_{-1}^o with the other 6 waveforms in the set ($M = 3$). It is essentially zero at $t = 0$. . . 60
- 5.11 The crosscorrelations of one of the generated waveform, p_{-1}^o with the other 14 waveforms in the set ($M = 7$). It is sufficiently small at $t = 0$. . 61

List of Tables

3.1	Spectral Efficiency, η of the waveforms in Fig. 3.1	30
3.2	Crosscorrelation between g_i and g_j (1.0×10^{-3})	30
4.1	Spectral Efficiency, η for $q(t)$ with $\tau = 0.087$ ns.	44
4.2	Spectral Efficiency, η for $q(t)$ with $\tau = 0.15$ ns.	44
4.3	Spectral Efficiency, η for $q(t)$ with $\tau = 0$ ns.	46
4.4	Spectral Efficiency, η for $q(t)$ with $\tau = 0.2$ ns.	46
4.5	Spectral Efficiency, η for $q(t)$ with $\tau = 0.25$ ns.	46
4.6	Comparison of the number of parameters.	47
5.1	Spectral Efficiency, η for $M = 2$	60
5.2	Spectral Efficiency, η for $M = 3$	60
5.3	Spectral Efficiency, η for $M = 7$	61

Chapter 1

Introduction

In recent years, rapid advances in high data-rate communication services have resulted in strong interest in high speed and cost-effective multiuser communications. Driven by increasing user demand, one of the major focuses of current research is to develop new schemes which can be used to maximize multiple access communication system capacity and increase user bandwidth utilization efficiency. In this project the design of analogue waveform sets which can be used for bandwidth efficient multiuser and UWB communications is investigated. In this introductory chapter, necessary background of the waveform set design using orthogonal Hermite Rodriguez basis functions is briefly presented, and an overview of thesis contributions is described in the following section.

1.1 Waveform Set Design

In this thesis, the design of orthogonal analogue waveform sets which can be used for bandwidth efficient multiuser communications is investigated. In such a design, combinations of both correlation constraints and spectral shaping requirements are taken into consideration. This type of problems arises in high data-rate wireless multiuser communications system design [1–15].

The design of bandwidth efficient orthogonal signal sets is an important element in modern multidimensional signaling and multiuser communication system design [1–4, 6, 15–17]. Essentially, all multiple access techniques require that the messages corresponding to different data streams or different users be separated in some fashion so that they do not interfere with one another. This is usually accomplished by making

the messages orthogonal to one another in signal space. Orthogonality of two or more signals can be accomplished in several ways.

In time division multiple access (TDMA) or frequency division multiple access (FDMA) communications, orthogonality between different messages is achieved by assigning non-overlapping time slots or frequency bands to different users or data streams. In code division multiple access (CDMA) communications, messages from multiple users can be transmitted at the same time and in the same frequency band. Orthogonality between different messages is achieved by assigning different “signature waveforms” or “codes” to different users. Each transmitter sends its message by modulation its own signature waveform as in single-user communication system.

A good set of orthogonal waveforms should possess low values of crosscorrelation which is important because it greatly facilitates the design of high performance receiver/detector [9, 14–16, 18, 19] and also because of the need to increase the number of simultaneous access users while reducing multi-access interference (MAI) in CDMA communications systems [15].

Besides correlation values, prescribed spectral properties are also important in the design of practically viable signal sets in order to maximize user bandwidth utilization efficiency [2–4, 15, 16]. Due to the correlations (time domain) and spectral (frequency domain) constraints which were incorporated in the problem formulation, existing techniques for a single filter design [6, 7, 20, 21] are not directly applicable to solve the design problems investigated in this project. Specifically, design requirements for low values crosscorrelation result in nonlinear and non-convex constraints, they were thus transformed into standard forms and solved using standard optimization tools in this thesis.

Unlike digital finite impulse response (FIR) filter, analogue filter design needs to guarantee that the designed filters are physically realizable and easy to implement. In this thesis, the analogue filter set design problem was transformed into an optimal parameter selection problem using a set of suitably chosen basis functions i.e., the orthogonal Hermite-Rodriguez functions (see [22–25] and the references therein). As a result, various optimization techniques were employed in the design of filter sets which resulted in a systematic approach to deal with the design procedure.

1.2 UWB Waveform Set Design

Signal sets with prescribed spectral properties are very important in the design of practically viable signal sets in order to maximize user bandwidth utilization efficiency [2–4, 15, 16]. This is especially important in ultra-wideband (UWB) applications where the signals use extremely broad bandwidth for transmission by sharing the same frequency spectrum with other existing systems. To avoid interfering with other systems, designed signals must meet spectral constraints imposed by regulatory bodies such as the Federal Communications Commission (FCC) in the United States [26]. Recently, the signal design or pulse shaping filter in UWB communications systems have been actively investigated in the literature [5, 7, 8, 10, 11, 27–50].

In multiuser communication systems and high data rate applications, there is a need to have a large number of waveforms that are mutually orthogonal. It has been shown that higher capacity can be achieved if each user uses a signal set that consists of signals that are orthogonal to those used by other users [51–54]. Previous works have mainly focused on the design of a single waveform or a small set of waveforms with some constraints on the phase of the waveforms [5, 30–32, 46, 55]. Since large number of waveforms is highly desirable, effective design approach has been developed in this thesis to design waveform sets while satisfying relevant time and frequency domains constraints. Based on Löwdin orthogonalization method [56–58], the approach has been extended further to design orthogonal overlapping waveforms which are useful for orthogonal overlapping pulse position modulation (OOPPM) UWB systems.

In UWB systems, although the power spectral density (PSD) of the designed waveforms are very low, they can still affect the performance of narrowband systems operating in the same bandwidth [33–36, 45, 59, 60]. Therefore the coexistence and compatibility of designed waveforms are critical design issues. In this thesis, these issues have been incorporated as design constraints to mitigate multiple narrowband interferences.

Significant amount of research based on finite impulse response (FIR) filter has been carried out to design high spectrally efficient UWB waveforms [5, 37–40]. To improve the efficiency, the characteristic of the basic pulse used in a UWB system had been incorporated into the design and formulated as a semi-definite programming problem [38] which resulted in a piece-wise non-constant upper bound based on the regulatory bound imposed by the FCC [26]. To further improve the efficiency, generic upper bound for any basic pulse has been developed in this thesis which resulted in higher efficiency even for the less optimal basic pulse.

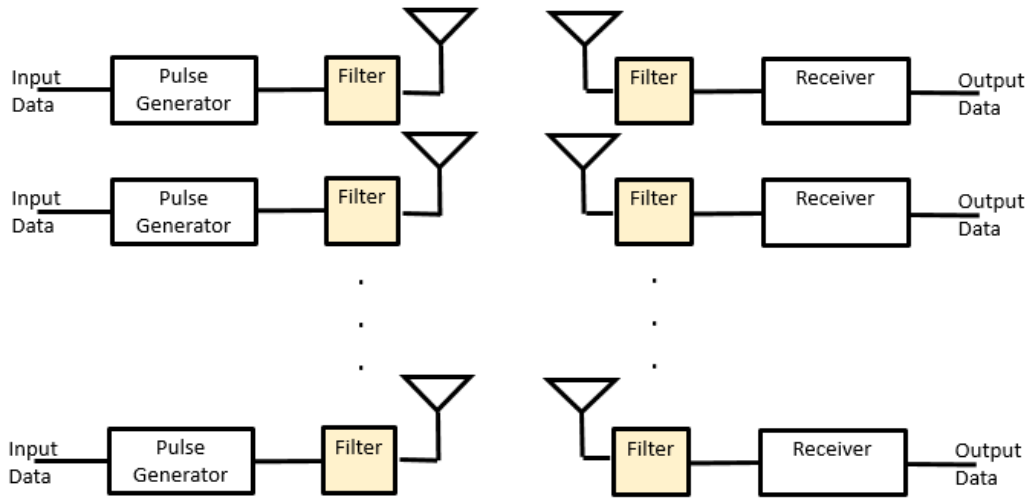


Figure 1.1: Block diagram of a multiuser UWB communication system. The research focuses on the design of filter sets.

1.3 Thesis Objectives

The objective of this research is to develop new schemes which can be used to maximize multiple access communication system capacity and improve user bandwidth utilization efficiency. The research focuses on the design of filter sets for a multiuser UWB communication system as shown in Figure 1.1. Specifically, the objectives of this research are to:

- design bandwidth efficient analogue waveform sets for multiuser and UWB communications.
- design analogue filter sets based on suitably chosen orthogonal Hermite-Rodriguez basis functions.
- develop practically relevant problem formulations and optimization methods for solving the analogue waveform set design problems.
- evaluate and validate the effectiveness of the designed analogue waveform sets using numerical analysis and computer simulations.

1.4 Hermite-Rodriguez Functions

Since Hermite-Rodriguez functions are used throughout the thesis, their relevant background material is summarized in this section. For more details, see [61] and the references therein.

The continuous Hermite Rodriguez function is defined as

$$\omega_{\lambda,k}(t) = \frac{1}{\sqrt{2^k k!}} H_k\left(\frac{t}{\lambda}\right) \frac{1}{\sqrt{\pi\lambda}} e^{-t^2/\lambda^2} \quad k \in [0, \infty) \quad (1.1)$$

where k is the order of the function, λ a scaling parameter and $H_k(t)$ the Hermite polynomials defined recursively as

$$H_k(t) = \begin{cases} 1 & k = 0 \\ 2t & k = 1 \\ 2tH_{k-1}(t) - 2(k-1)H_{k-2}(t) & k \geq 2. \end{cases} \quad (1.2)$$

The set of functions $\{\omega_{\lambda,k}(t)\}$ constitutes an orthogonal basis with respect to the weighted inner product

$$\langle \omega_{\lambda,j}, \omega_{\lambda,k} \rangle = \sqrt{\pi\lambda} \int_{-\infty}^{+\infty} \omega_{\lambda,j}(t) \omega_{\lambda,k}(t) e^{-t^2/\lambda^2} dt. \quad (1.3)$$

This expression is zero for $j \neq k$. Consequently, a signal $g(t)$ can be expanded into an HR series as

$$g(t) = \sum_{k=0}^{\infty} \alpha_{\lambda,k} \omega_{\lambda,k}(t) \quad (1.4)$$

where

$$\alpha_{\lambda,k} = \langle g, \omega_{\lambda,k} \rangle = \frac{1}{\sqrt{2^k k!}} \int_{-\infty}^{+\infty} g(t) H_k\left(\frac{t}{\lambda}\right) dt. \quad (1.5)$$

Fig. 1.1 shows the Hermite-Rodriguez functions of orders zero to four for $\lambda = 1$ and Fig. 1.2 shows their Fourier transforms. Note that the duration of the signals, $T \approx 6\lambda$.

In the following, $g(t)$ is represented as a truncated Hermite Rodriguez series using N terms in the expansion. Thus, the *Hermite Rodriguez filter* of order N is defined as

$$g(t) = \sum_{k=0}^{N-1} x_k \omega_{\lambda,k}(t) = \mathbf{x}^T \mathbf{w}_{\lambda}(t) \quad (1.6)$$

where

$$\begin{aligned} \mathbf{x} &= [x_0, x_1, \dots, x_{N-1}]^T, \\ \mathbf{w}_{\lambda}(t) &= [\omega_{\lambda,0}(t), \omega_{\lambda,1}(t), \dots, \omega_{\lambda,N-1}(t)]^T, \end{aligned}$$

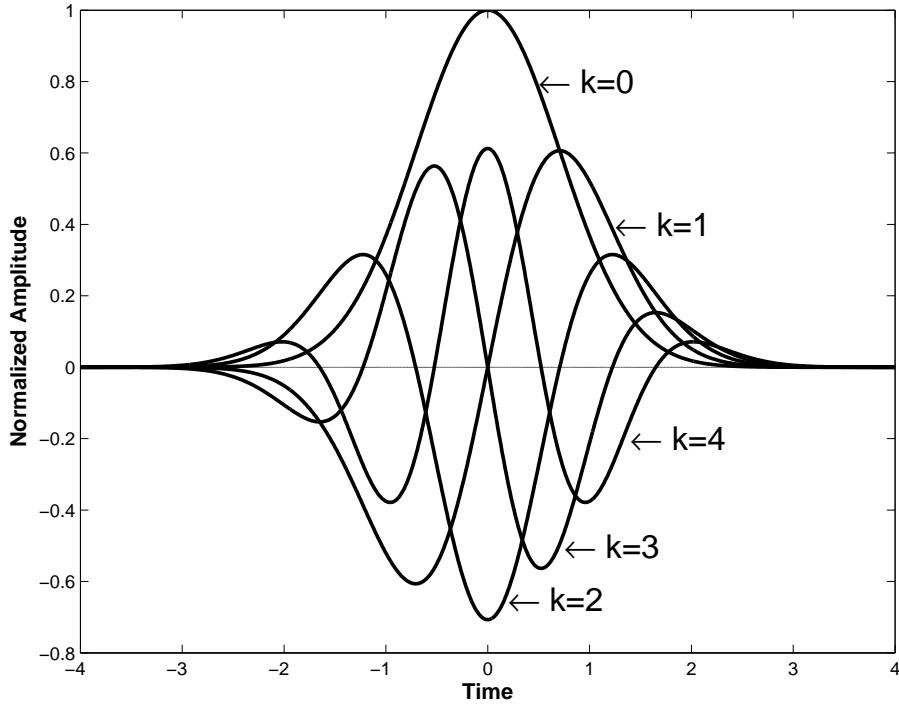


Figure 1.2: Hermite-Rodriguez functions of order zero to four, $\lambda = 1$.

x_k is the k^{th} filter coefficient and $\omega_{\lambda,k}(t)$ is the k^{th} order HR function given by (1.1).

The Fourier transform of $g(t)$ is given by

$$G(f) = \int_{-\infty}^{+\infty} g(t) e^{-j2\pi ft} dt = \sum_{k=0}^{N-1} x_k W_{\lambda,k}(f)$$

where $W_{\lambda,k}(f)$ is the Fourier transform of the Hermite-Rodriguez function given by

$$\mathcal{F}\{\omega_{\lambda,k}(t)\} = W_{\lambda,k}(f) = \frac{(-j)^k}{\sqrt{2^k k!}} (2\pi f \lambda)^k e^{-(2\pi f \lambda/2)^2}. \quad (1.7)$$

Let $\mathbf{W}(f) = [W_{\lambda,0}(f), W_{\lambda,1}(f), \dots, W_{\lambda,N-1}(f)]^T$. Then $G(f)$ can be written as $G(f) = \mathbf{x}^T \mathbf{W}(f)$ and the power spectral density of $g(t)$ can be expressed as

$$\Psi_g(f) = |G(f)|^2 = \mathbf{x}^T \Phi(f) \mathbf{x}$$

where $\Phi(f) = \mathbf{W}(f) \mathbf{W}^T(f)$ is an N by N non-negative definite matrix.

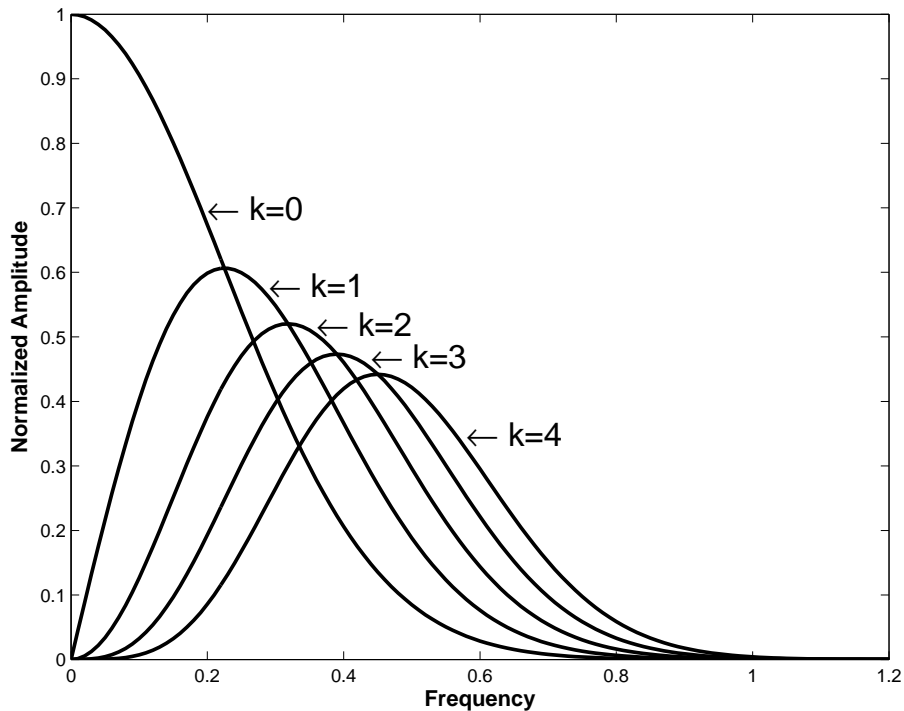


Figure 1.3: Magnitude of Fourier transform of Hermite Rodriguez functions of order zero to four, $\lambda = 1$.

1.5 Thesis Overview and Contributions

The design of bandwidth efficient orthogonal waveform sets is an important element in modern multidimensional signaling and multiuser communication system design [1–4, 6, 15–17]. In general, waveform design problems with time and frequency domains constraints are highly nonlinear and nonconvex. These problems become more challenging and complex considering the need to have as many orthogonal waveforms as possible. Recently digital waveform set design problem has been formulated as constrained nonlinear optimization problem with the objective function to minimize the stopband energy of the designed waveforms while meeting desired magnitude spectral and correlation constraints [62, 63]. The waveform set design problem is investigated in this thesis using orthogonal Hermite Rodriguez basis functions.

A recent approach to design UWB waveform which was based on the second-order cone programming (SOCP) was able to produce many highly spectrally efficient UWB waveforms but it did not directly maximize the energy of each waveform and nonlinear phase constraint was imposed on the waveforms [55]. The approach proposed in

this thesis directly maximizes the energy without any phase constraints and it is quite flexible to incorporate additional constraints to mitigate narrowband interferences. The approach has been extended further to design orthogonal overlapping waveforms by using Löwdin orthogonalization method [56–58].

Spectral efficiency of UWB waveforms have been further improved by incorporating the power spectral density of a commonly used basic pulse and formulating the design problem into linear matrix inequalities (LMI) framework [38]. The approach developed optimal non-constant upper bounds for a specific Gaussian pulse but sub-optimal for other basic pulses. In contrast, different bounds for different basic pulses could easily be implemented as constraints in the optimization problem formulated in this thesis.

In this thesis, systematic approaches for the waveform set design problem based on orthogonal Hermite Rodriguez basis functions are presented. One of the major contributions of this thesis is applying the Hermite Rodriguez basis functions to this design problem and then transforming the nonlinear and nonconvex problems into suitable forms which can be efficiently solved using standard optimization tools.

In Chapter 2, the waveform set design problem using orthogonal Hermite Rodriguez basis functions with spectral and correlations constraints suitable for multiuser communication systems is investigated. Chapter 3 proposes a sequential quadratic semi-infinite programming approach to design a set of spectrally efficient orthogonal UWB waveforms. The efficiency of the designed waveform is further improved in Chapter 4 by incorporating non-constant upper bounds in the optimization problem formulation. Chapter 5 applied Löwdin orthogonalization method to transform the optimal designed waveform to an orthogonal overlapping set of waveforms with minimal energy distortion with respect to the optimal waveform. Chapter 6 summarizes the thesis and highlights some interesting future works.

Chapter 2: Design of Waveform Set for CDMA Communications

In this chapter, the waveform set design for CDMA communications using orthogonal Hermite- Rodriguez basis functions is investigated. The waveform set design problem is formulated as a constrained \mathcal{L}_2 space optimization problem. For bandwidth efficiency, all the waveforms in the set are constrained to have approximately the same desired spectral shapes in a prescribed passband. To minimize intersymbol interference and co-channel interference, all the waveforms in the set are constrained to have low values of autocorrelation and crosscorrelation. Numerical results are presented to illustrate the

effectiveness of Hermite-Rodriguez functions and the usefulness of the design method to solve the waveform set design problem.

Chapter 2 is based on the following conference publication:

- M. A. Yusoff, Z. Zang, “Waveform Set Design using Hermite-Rodriguez Functions for CDMA Communications”, *Inter. Conf. on Signal Acquisition and Processing*, Kuala Lumpur, April 2009.

Chapter 3: Design of Spectrally Efficient UWB Waveforms

Spectral efficiency is a major requirement in ultra-wideband (UWB) communication systems because of very low power spectral density (PSD) regulations imposed on the transmitted signals. Besides, large number of orthogonal waveforms is highly desirable in multiuser systems. In this chapter, the design of a set of spectrally efficient orthogonal waveforms is investigated. The design of UWB waveforms based on orthogonal Hermite-Rodriguez basis functions and linear frequency modulation (LFM) is proposed herein. The design is formulated as a quadratic semi-infinite programming problem with spectral and orthogonality constraints. Numerical examples are presented to illustrate the effectiveness of Hermite-Rodriguez based waveforms with the proposed optimization approach. Comparison with a recent result shows that the proposed approach compares favourably with those reported in the literatures in terms of the number of waveforms can be produced and the achievable spectral efficiency of the designed waveforms. An example to mitigate multiple narrowband interferences is also presented to illustrate the flexibility of the proposed optimization approach.

Chapter 3 is based on the journal publication:

- M. A. Yusoff, Z. Zang, S. Abeysekera, “Design of Spectrally Efficient UWB Waveforms using Hermite Rodriguez Functions”, *IET Commun.*, 2012.

and two conference publications:

- M. A. Yusoff, Z. Zang, “Design of UWB Waveforms for Narrowband Interferences Suppression”, *IEEE Asia Pacific Conference on Circuits and Systems*, Kuala Lumpur, Dec 2010.
- M. A. Yusoff, Z. Zang, S. Abeysekera, “Spectrally Efficient Waveform Set Design for UWB Wireless Communications”, *7th Inter. Conf. on Information, Communications and Signal Processing*, Macau, Dec 2009.

Chapter 4: Design of UWB Pulse Shaping Filter with Nonconstant Upper Bounds

In UWB pulse shaping filter design, FIR filter formulated as a semi-definite programming problem has been widely used. Recently, piece-wise continuous bounds has been incorporated into the FIR filter design to avoid power loss due to the constant spectral mask indirectly assumed in the design. In this chapter, the pulse shaping filter design is formulated as a semi-infinite quadratic optimization problem using Hermite-Rodriguez basis functions. Computationally effective approach to solving the optimization problem is proposed. Through numerical design examples, the spectral utilization of the approach is compared with those FIR-based pulse shaping filter designs. It is demonstrated that the proposed approach is easy to implement and achieves better spectral utilization than a recent result when a suboptimal basic pulse is used.

Chapter 4 is based on the following conference publication:

- M. A. Yusoff, Z. Zang, “Quadratic Programming Approach for UWB Pulse Shaping Filter Design with Nonconstant Upper Bounds”, *15th Int. Conf. on Advanced Communications Technology (ICACT 2013)*, Pyeongchang, Korea, Jan 2013.

Chapter 5: Design of UWB Waveform Set using Löwdin’s Orthogonalization

In this chapter, Löwdin orthogonalization method is used to transform a finite set of equally spaced shifts of an optimal waveform to an orthogonal set of spectrally efficient waveform. The optimal waveform was first designed based on Hermite Rodriguez basis functions via a semi-infinite quadratic programming approach. This 2-step approach is able to produce many overlapping orthogonal waveforms with high spectral efficiency.

Chapter 5 is based on the following conference publication:

- M. A. Yusoff, Z. Zang, “UWB Waveform Set Design using Löwdin’s Orthogonalization with Hermite Rodriguez Functions”, *IET Int. Conf. on Wireless Communications and Applications (ICWCA 2012)*, Kuala Lumpur, October 2012.

Chapter 2

Design of Waveform Set for CDMA Communications

In this chapter, the design of waveform sets is formulated as an optimization problem with time and frequency domain constraints. In the time domain, the waveforms are constrained to have low values of autocorrelation and crosscorrelation. In the frequency domain, the waveforms are constrained to have approximately the same spectral shapes. The effectiveness of Hermite-Rodriguez functions to solve the design problem is illustrated through numerical examples.

The chapter is organized as follows. Section 2.2 outlines the waveform set design problem formulation and solution methods. In Section 2.3 a numerical example of the waveform set design is presented. Section 2.4 concludes the chapter.

2.1 Introduction

The design of bandwidth efficient orthogonal signal sets is an important element in modern multidimensional signaling and multiuser communication system design [4, 15–17]. Essentially, all multiple access techniques require that the messages corresponding to different data streams or different users be separated in some fashion so that they do not interfere with one another. This is usually accomplished by making the messages orthogonal to one another in the signal space. In code division multiple access (CDMA) communications, orthogonality between different messages is achieved by assigning “signature waveforms” or “codes” to different users [15, 16]. The orthogonality

property of the set of waveforms is important because it greatly facilitates the design of high performance receiver/detector for the successful decoding of the transmitted user messages [15, 16].

A good set of orthogonal waveforms should at least possess the following correlation properties:

- For each signal in the set, its autocorrelation should be small at all non-zero translates of multiple symbol intervals.
- For any two signals in the set, their crosscorrelation should be small for all translates of multiple symbol intervals.

The first property is crucial for synchronization and controlling inter-symbol interference (ISI). The second property is important for combating the adverse effect of cochannel interference (CCI).

Spectral properties are also very important in the design of practically viable orthogonal waveform set. This is important especially in high data rate multiuser communications such as the new generation (3G and 4G) wireless mobile CDMA communications, where user demand for more bandwidth increases rapidly and spectrum resources are limited and expensive. Efficient spectral shaping often uses signature waveforms that occupy more than one symbol interval [64, 65]. However, extending the signature waveforms in time domain will shrink the frequency domain characteristics of the signal and lead to overlapping signatures of successive symbols. The design of the signature waveforms should ensure no ISI between consecutive symbols, as well as no crosstalk between symbols in each dimension. Mathematically, these design requirements need to be formulated as non-convex correlation constraints.

Recently, a new approach for the design of digital waveform has been proposed in [62, 63] which can be used as transmitter signature waveforms in bandwidth efficient multidimensional signaling and multiuser communications. In the approach, the digital waveform set design problem has been formulated as constrained nonlinear optimization problem in which the performance criterion is to minimize the stopband energy of the designed digital waveforms and to shape the magnitude spectral function of the overall system impulse response (combined transmitter, channel and receiver) to a fixed spectral shape of a prescribed Nyquist pulse.

In this chapter, the waveform set design problem is investigated by using orthogonal Hermite Rodriguez (HR) basis functions. It shall be demonstrated that the use of HR

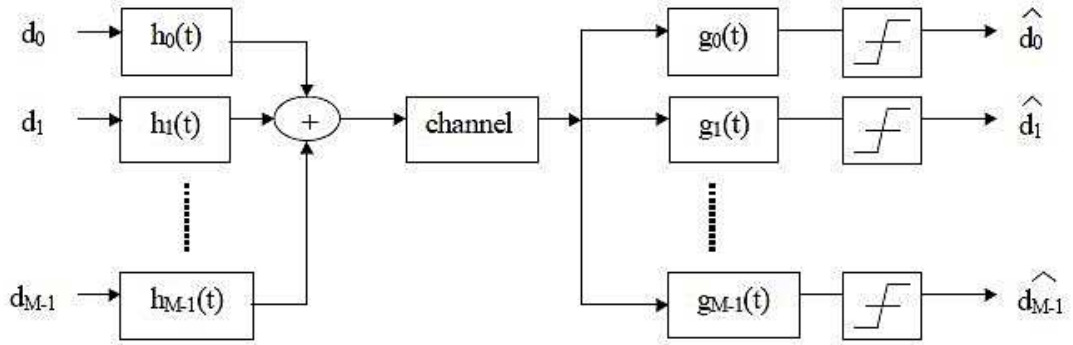


Figure 2.1: Block diagram of a multiuser communication system.

functions provides a very efficient alternative in waveform set design problem. These basis functions are derived as the product of Hermite polynomials and a Gaussian function. Due to Gaussian weighting function, the HR functions have proven to be useful in many signal processing applications such as in the design of radar signals, biomedical signals, electromagnetic signals, and image processing [61, 66–69].

The Hermite-Rodriquez series expansion is useful because of the following properties and advantages:

- The HR basis functions are orthogonal,
- HR polynomials can easily be computed through a recurrence relation,
- Any signal can be represented to a high degree of accuracy by using a sufficient number of terms in the expansion, and particularly important
- HR expansion is well suited for the design of pulse shaping filter with finite time support [61, 66].

2.2 Problem Formulation and Solution Methods

Consider a multi-user communication system in Figure 2.1 where M users simultaneously transmit their information data through a shared communication channel. The linearly combined signal at the input of the channel can be written as

$$S(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{M-1} d_n[k] h_n(t - kT)$$

where $d_n[k]$ are the information symbols of the n -th user, T is the symbol interval and $h_n(t)$ is the signature waveform of the n -th user. If the channel is flat, a bank of matched filters with impulse response $g_n(t) = h(-t)$ can be used at the receiver provided the overall transfer function satisfies the general Nyquist criterion [16]. The design of user-signature waveform $h_n(t)$ can be generalized to the design of a set of filters with prescribed time and frequency domain properties [62, 63]. We refer to such a set of filters as a set of signature waveforms.

Now the waveform set design problem can be posed as follows:

Problem (P₁): Find a set of M waveforms

$$\mathcal{H}_M = \{H_0(e^{j\omega}), H_1(e^{j\omega}), \dots, H_{M-1}(e^{j\omega})\}$$

which solves the following constrained optimization problem

$$\min_{\mathbf{x}_n} \max_n \max_{\omega_p \in \Omega_p} \left\{ \left| |H_n(e^{j\omega_p})|^2 - |H_d(\omega_p)|^2 \right| + \gamma \frac{1}{2\pi} \int_{\Omega_s} |H_n(e^{j\omega})|^2 d\omega \right\} \quad (2.1)$$

for $0 \leq n \leq M - 1$, subject to autocorrelation constraint

$$|R_{ac}(kT)| = \left| \int_{t=-\infty}^{+\infty} h_n(t) h_n^*(t + kT) dt \right| \leq \epsilon \quad (2.2)$$

for $0 \leq n \leq M - 1$, $k = 1, 2, \dots$ and crosscorrelation constraint

$$|R_{cc}(kT)| = \left| \int_{t=-\infty}^{+\infty} h_m(t) h_n^*(t + kT) dt \right| \leq \epsilon \quad (2.3)$$

for $m \neq n$, $0 \leq m, n \leq M - 1$, $k = 0, \pm 1, \pm 2, \dots$, where

$$H_n(e^{j\omega_p}) = \sum_{k=0}^{N-1} x_k W_{\lambda,k}(e^{j\omega_p})$$

is the frequency response function of the designed transmitter filter represented by the HR filter (1.6),

$$h_n(t) = \mathbf{x}^T \mathbf{w}_\lambda(t)$$

and $W_{\lambda,k}(e^{j\omega_p})$ is the Fourier transform of the Hermite-Rodriguez function given by (1.7). $H_d(\omega)$ is the desired frequency response spectral function, Ω_p and Ω_s are, respectively, the sets of passband and stopband frequencies and γ is a weighting constant chosen by the designer. Eqs. (2.2) and (2.3) can be simplified as

$$R(kT) = \mathbf{x}^T \mathbf{G}(kT) \mathbf{x}$$

where

$$\mathbf{G}(kT) = \int_{t=-\infty}^{+\infty} \mathbf{w}_\lambda(t) \mathbf{w}_\lambda^T(t + kT) dt. \quad (2.4)$$

The objective function is to minimize the passband error of the overall transfer function and the stop band energy leakage. The first part of the objective function in (2.1) is also used to shape the overall transfer function of all the filters to a desired spectral shape as specified by $|H_d(\omega_p)|$ to ensure bandwidth efficiency. $|H_d(\omega_p)|$ is chosen to be the spectral function of a Nyquist waveform. As a result, when matched filter receiver is used the overall impulse response between the transmitter and the receiver (when the channel is flat) will be approximately a Nyquist waveform. The constraints in (2.2) and (2.3) specify the need to have low auto and cross correlations of the signature waveforms in a set.

Assuming the channel is flat and matched filter receivers are used, the magnitude of the overall transfer function can be written as

$$|H_n(\omega)|^2 = \mathbf{x}_n^T \mathbf{Q}(\omega) \mathbf{x}_n \quad (2.5)$$

where $\mathbf{x}_n = [x_n(0), x_n(1), \dots, x_n(N-1)]$ and

$$\mathbf{Q}(\omega) = W_{\lambda,k}(\omega) W_{\lambda,k}^T(\omega).$$

Then the error function of the overall magnitude response in the passband can be written as

$$\varepsilon_1 = |\mathbf{x}_n^T \mathbf{Q}(\omega_p) \mathbf{x}_n - |H_d(\omega_p)|^2| \quad (2.6)$$

where ω_p is a passband frequency and the mean-squared stopband energy error of the transmitter filter can be written as

$$\varepsilon_2 = \frac{1}{2} \mathbf{x}_n^T \mathbf{H}_f \mathbf{x}_n \quad (2.7)$$

where \mathbf{H}_f is an N by N constant matrix defined as

$$\mathbf{H}_f = \frac{1}{\pi} \int_{\Omega_s} \mathbf{Q}(\omega) d\omega. \quad (2.8)$$

Instead of solving problem (P₁) for all the $H_n(e^{j\omega})$ simultaneously, one suboptimal approach is to solve the optimization problem (P₁) sequentially, that is, first problem (P₁) is solved for $H_0(e^{j\omega})$ subject to the magnitude spectrum and autocorrelation constraints. Using $H_0(e^{j\omega})$, the optimization problem (P₁) is then solved for $H_1(e^{j\omega})$

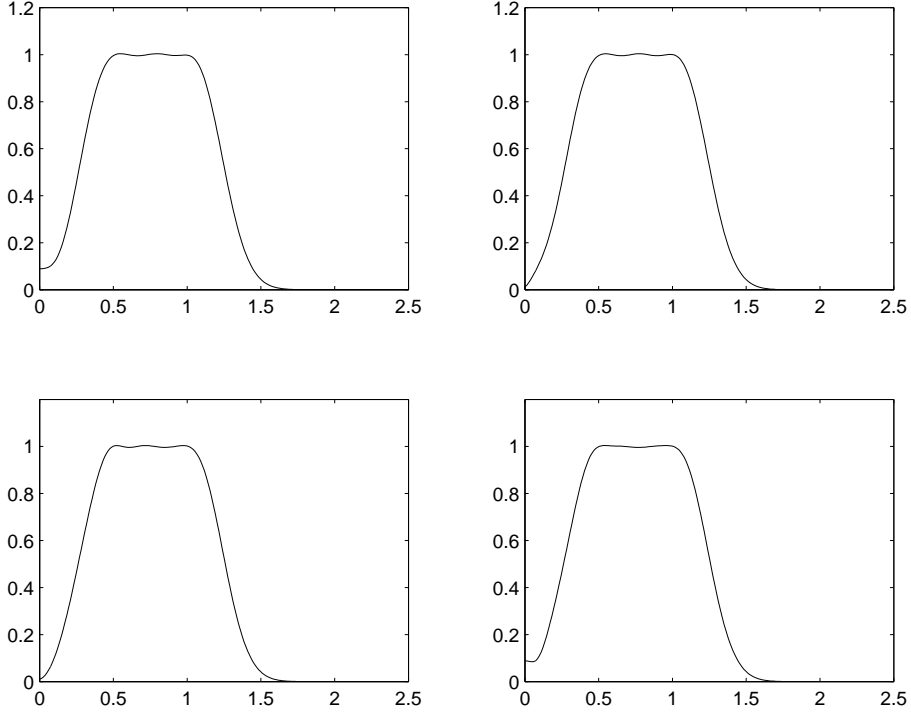


Figure 2.2: Magnitude spectral responses of the 4 designed waveforms ($\alpha = 0.5$).

subject to magnitude spectrum, autocorrelation and crosscorrelation constraints etc. In this approach, the crosscorrelation constraint can be treated as a linear constraint.

To summarize, the following simplified waveform set design problem is obtained:

Problem (P₂)

$$\min_{\mathbf{x}_n} \max_n \max_{\omega_p \in \Omega_p} \left\{ |\mathbf{x}_n^T \mathbf{Q}(\omega_p) \mathbf{x}_n - |H_d(\omega_p)|^2| + \gamma \frac{1}{2} \mathbf{x}_n^T \mathbf{H}_f \mathbf{x}_n \right\} \quad (2.9)$$

subject to:

$$\begin{aligned} |R_{ac}(kT)| &= |\mathbf{x}_n^T \mathbf{G}(kT) \mathbf{x}_n| \leq \epsilon, \text{ for } k \neq 0 \\ |R_{cc}(kT)| &= |\mathbf{A}(n, kT) \mathbf{x}_n| \leq \epsilon, \text{ for } n = 0, 1, \dots, M-1 \end{aligned}$$

where $\mathbf{G}(kT)$ is defined by (2.4) and

$$\mathbf{A}(n, kT) = \mathbf{x}_m^T \mathbf{G}(kT)$$

where $m = 0, 1, \dots, n-1$.

The optimization problem (P₂) can be solved using general purpose optimization software packages such as Matlab and its Optimization Toolbox.

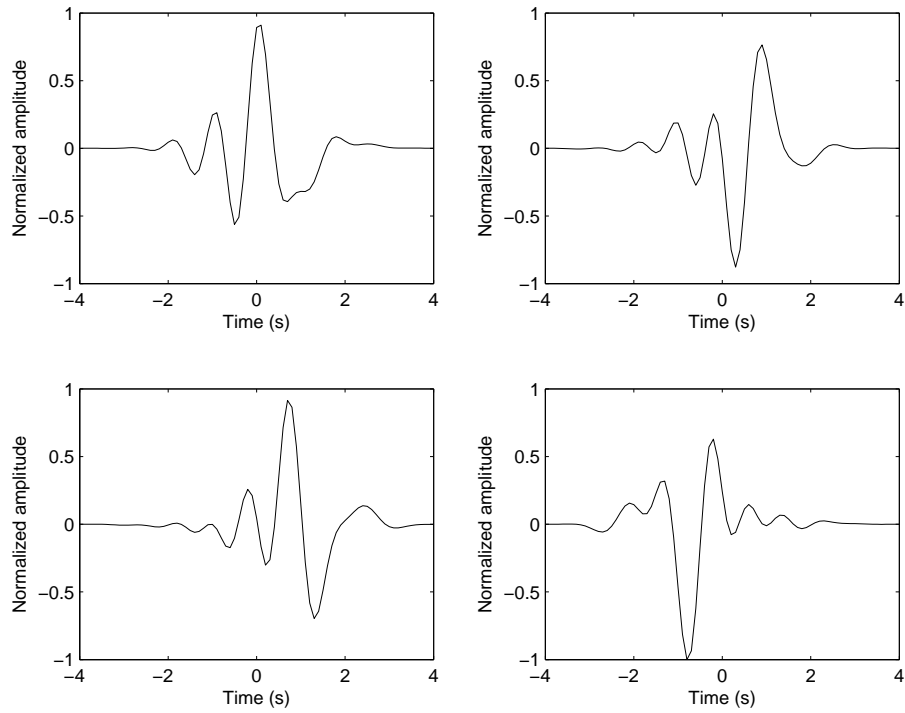


Figure 2.3: Impulse responses of the 4 designed waveforms ($\alpha = 0.5$).

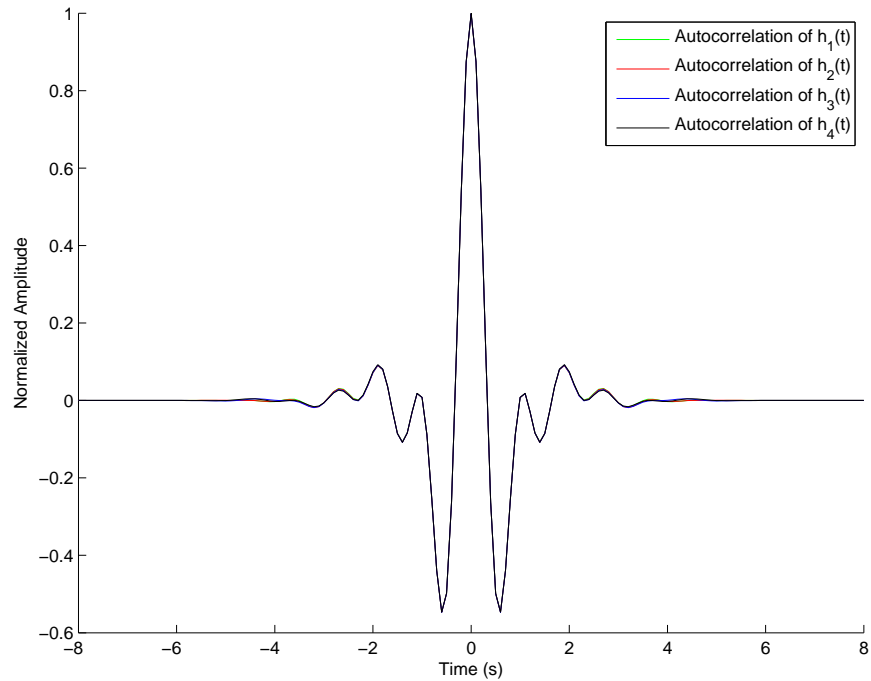


Figure 2.4: Autocorrelation of the designed waveforms ($\alpha = 0.5$). Low correlation values were produced at $t = 3k$ where $k = \pm 1, \pm 2$.

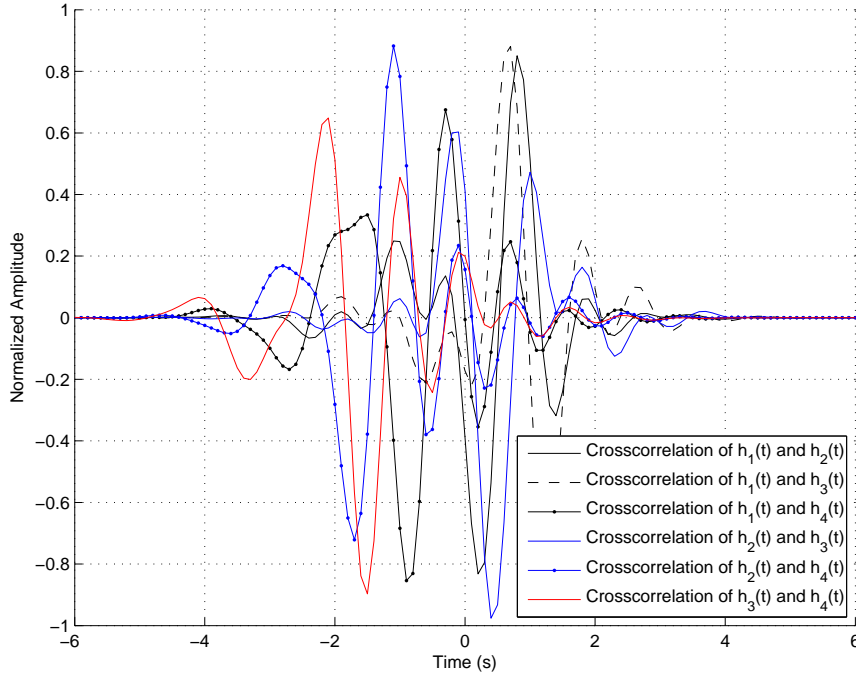


Figure 2.5: Crosscorrelation of the designed waveforms ($\alpha = 0.5$). Low correlation values were produced at $t = 3k$ where $k = 0, \pm 1, \pm 2$.

2.3 An Illustrative Numerical Example

In this section, a numerical example is presented to illustrate the effectiveness of Hermite Rodriguez function and the usefulness of the approach. The objective is to design a set of M waveforms which solves the nonlinear optimization (P₂). The desired overall frequency response $H_d(\omega)$ is chosen to be the spectral function of the time-domain modulated passband raised cosine pulse with roll-off factor, α .

- In this case $M = 4$, $N = 25$ and $\alpha = 0.5$. Fig. 2.2 shows the magnitude spectrum responses of the 4 waveforms. Fig. 2.3 shows the corresponding 4 orthogonal waveforms. The filter length, $N = 25$ is much lower compared to $N = 65$ in [63]. Figs. 2.4 and 2.5 show the autocorrelations and crosscorrelations for the designed waveforms respectively. It can be seen that low correlation values were produced at $t = 3k$ where $k = \pm 1, \pm 2$ for autocorrelations and $k = 0, \pm 1, \pm 2$ for crosscorrelations as constrained by Eqs. (2.2) and (2.3), respectively.
- In this case $M = 4$, $N = 29$ and $\alpha = 0.25$. Fig. 2.6 shows the magnitude spectrum responses of the 4 waveforms. Fig. 2.7 shows the corresponding 4 orthogonal

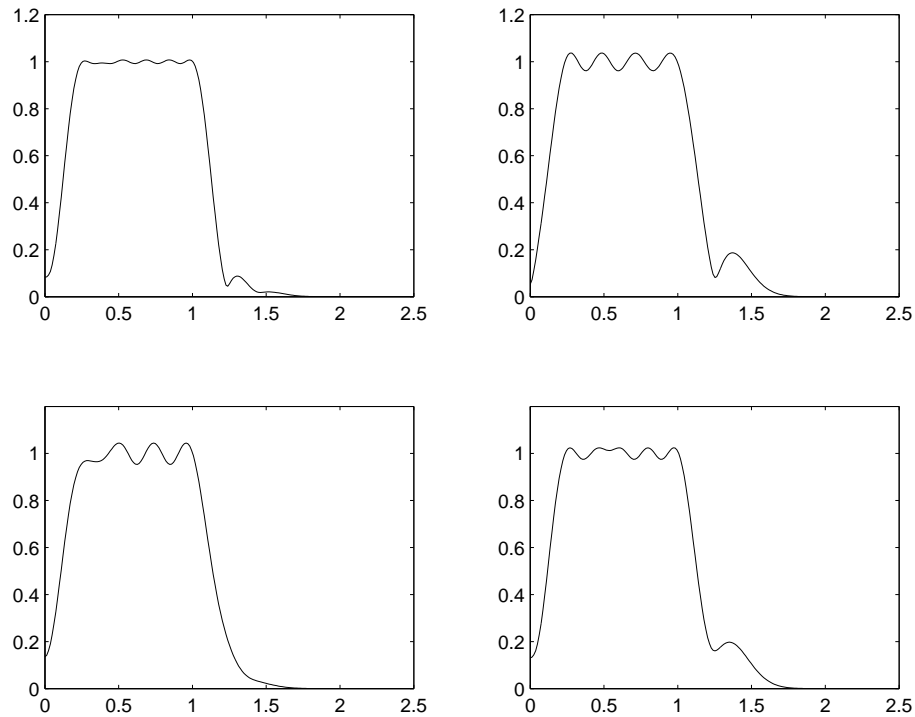


Figure 2.6: Magnitude spectral responses of the 4 designed waveforms ($\alpha = 0.25$).

waveforms. Fig. 2.8 and Fig. 2.9 show the autocorrelations and crosscorrelations for the designed waveforms respectively.

Note that all the waveforms occupy approximately the same frequency bandwidth and with very low stopband energy leakage.

The ability of Hermite Rodriguez functions to solve the nonlinear optimization (P_2) using less terms that is shorter filter order, N is due to the property of Hermite Rodriguez functions where it is well suited for signals with finite time support [61, 66]. Besides that the width of Hermite Rodriguez signals can also be adjusted using the scaling factor, λ . In this particular example, $\lambda = 1$.

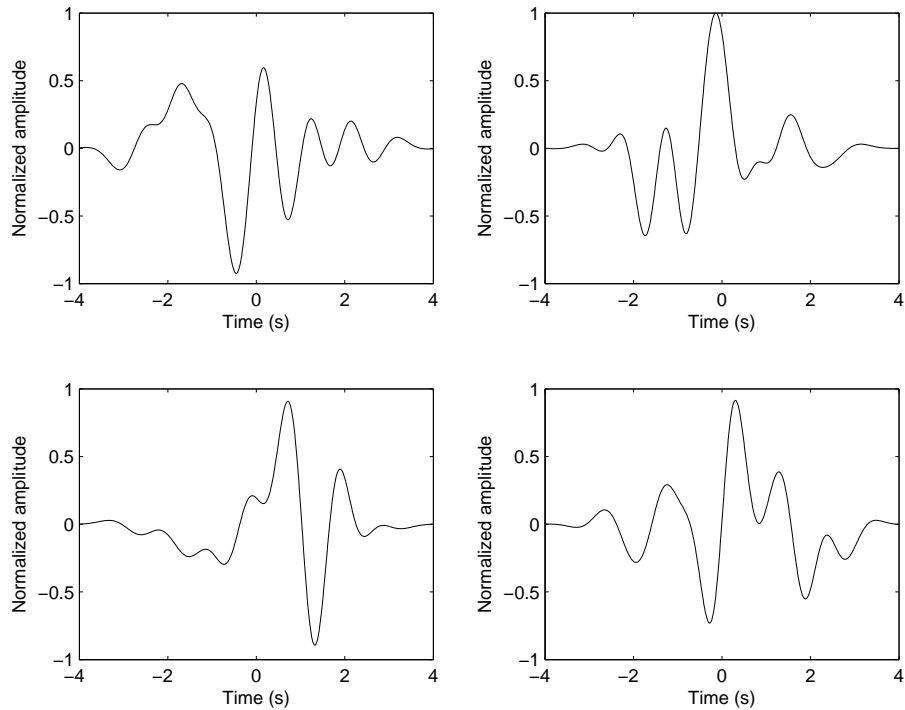


Figure 2.7: Impulse responses of the 4 designed waveforms ($\alpha = 0.25$).

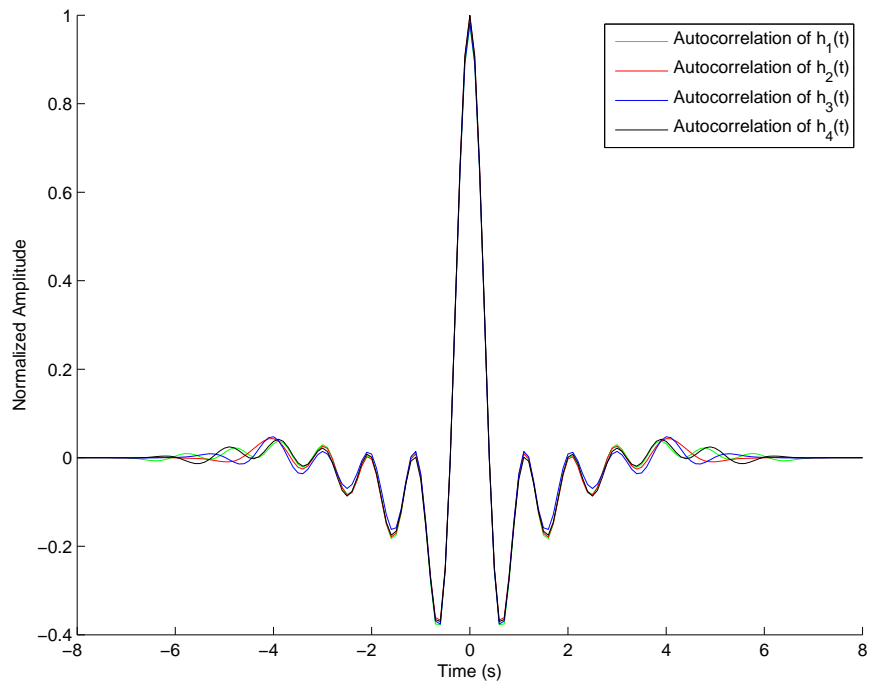


Figure 2.8: Autocorrelation of the designed waveforms ($\alpha = 0.25$). Low correlation values were produced at $t = 3k$ where $k = \pm 1, \pm 2$.

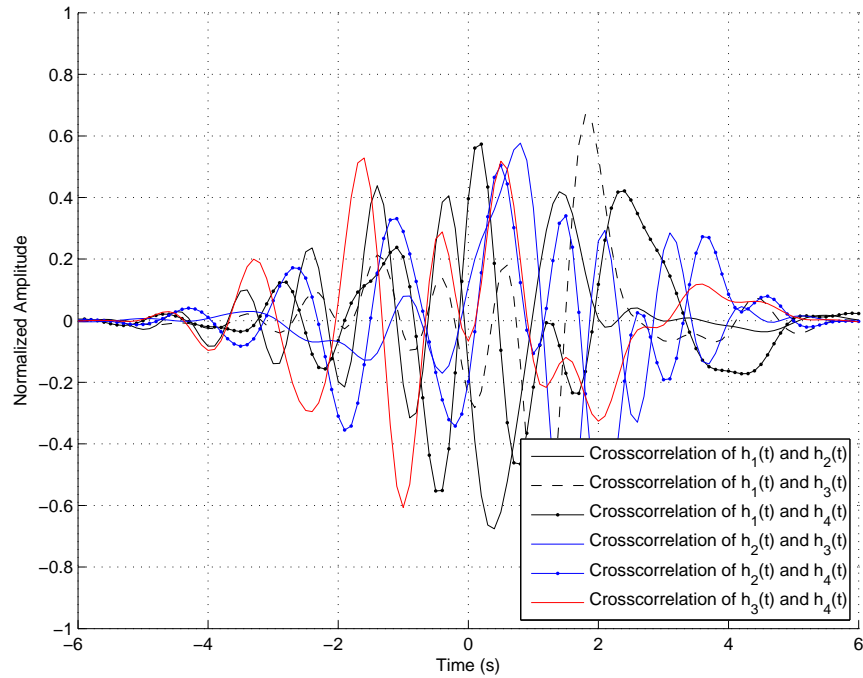


Figure 2.9: Crosscorrelation of the designed waveforms ($\alpha = 0.25$). Low correlation values were produced at $t = 3k$ where $k = 0, \pm 1, \pm 2$.

2.4 Chapter Summary

In this chapter orthogonal Hermite Rodriguez basis functions have been applied to waveform set design problem. The numerical studies have shown that the low order filter can be obtained due to property of the Hermite Rodriguez functions. Future research would include obtaining optimal λ and optimal filter order, N for this waveform set design.

Chapter 3

Design of Spectrally Efficient UWB Waveforms

In multiuser UWB communication systems, a set of spectrally efficient orthogonal waveforms is highly desirable. In this chapter, the waveform set design is formulated as a quadratic semi-infinite programming problem with spectral and orthogonality constraints using orthogonal Hermite-Rodriguez basis functions and linear frequency modulation (LFM).

The chapter is organized as follows. Section 3.2 describes HR-LFM signals. In Section 3.3, the waveform set design problem is formulated as a constrained maximization problem. Then, it is converted into a constrained semi-infinite quadratic programming (QP) problem. A solution method is then proposed. Section 3.3 is concluded with a step-by-step algorithm which can be used in combination with any computationally efficient software such as Matlab and its Optimization Toolbox to solve the proposed waveform set design problem. In Section 3.4, a numerical example of the design of multiple orthogonal pulses complying with the Federal Communications Commission (FCC) spectral mask is presented. The design is then compared with a recent work. An example of multiple Narrowband Interferences (NBI) suppression problem is also presented to illustrate the flexibility of the proposed approach. Section 3.5 concludes the chapter.

3.1 Introduction

In recent years, UWB communication systems have attracted great interest after the U.S. FCC released spectral masks in 2002 [26]. This mask constrains the power spectral density (PSD) of a UWB signal to a very low level so that UWB systems do not interfere with existing systems operating over the same frequency bands. In contrast, in order for the UWB systems to achieve reliable transmission, UWB signals have to utilize the given spectrum mask efficiently, that is to have maximum energy while satisfying the given spectrum mask constraint. Besides, in multiuser systems and high data rate applications, there is a need to have a large number of waveforms that are mutually orthogonal. Moreover, it has been shown that higher capacity can be achieved if each user uses a signal set that consists of signals being orthogonal to those used by other users [51].

Previous works on UWB waveform design have been based on a single waveform or a small set of waveforms with linear phase imposed on the design [5, 30–32, 55]. Recently, nonlinear phase waveform set design was investigated where an iterative optimization procedure based on the second-order cone programming (SOCP) has been proposed to design spectrally efficient orthogonal pulses [55]. The proposed approach in [55] was able to obtain a relatively large number of waveforms with improved spectral efficiency compared with those reported in [5, 30–32]. However, the method in [55] did not directly optimize the spectral efficiency and the nonlinear phase was imposed sub-optimally in an iterative fashion. In this chapter, spectral efficiency is optimized directly by maximizing the energy of each waveform in a prescribed passband while complying with the prescribed spectral mask constraint. Moreover, the proposed approach does not impose any constraints on the phase of the waveforms. The design of spectrally efficient orthogonal waveforms is formulated as a constrained quadratic semi-infinite optimization problem. The rotation theorem [21] is used to linearise the nonlinear magnitude constraints. The orthogonality constraints are solved sequentially as linear equality constraints. Numerical studies demonstrated that the proposed approach is able to produce more waveforms with better spectral efficiency than those obtained in [55].

Although the spectral mask limits the PSD of a UWB signal to a very low level, UWB systems can still greatly affect the performance of the existing narrowband systems operating in UWB band such as IEEE 802.11a WLAN [33]. Therefore the coexistence

and compatibility become a critical design issue. To address this issue, [34] used eigenvalue decomposition to effectively generate spectral null at the operating frequency of a narrowband system. Then, [35] extended the approach to handle multiple operating frequencies. In [36], Prolate spheroidal wave functions have been used as basis functions in designing a waveform which suppresses an NBI. To design a set of spectrally efficient waveforms with good suppression to multiple NBIs, an additional constraint can be easily introduced into the proposed optimization problem.

In the proposed design, Hermite Rodriguez (HR) functions and LFM are used, unlike [5, 31, 32, 55] where finite impulse response (FIR) filters have been used. HR functions with LFM signals were originally proposed in [66] to design a single spectrally efficient waveform. Hermite polynomials based functions have been used in waveform design for UWB communications. For examples, [51, 70] used the modified Hermite polynomials (MHP) and [69, 71–73] used Hermite functions, the same functions referred as HR functions in [24, 74]. In this chapter, LFM signal amplitude is represented as an HR series expansion. Furthermore, LFM signals are attractive as the generation of the signals and implementation of matched filters/correlators can be realized using completely passive, low-cost surface acoustic wave (SAW) chirp delay lines [75, 76].

3.2 Hermite-Rodriguez Linear FM Signals

A waveform $g(t)$, selected as an LFM signal is given by

$$g(t) = f(t)e^{j(2\pi\kappa t + \pi\beta t^2)}$$

where $f(t) = \mathbf{x}^T \mathbf{w}_\lambda(t)$ is an arbitrary envelope represented by the HR filter (1.6), $\beta = B/T$ is the frequency sweep rate, B is the bandwidth, T is the duration of the signal, and κ is the center frequency. Crosscorrelation between two LFM signals denoted by $g_n(t) = \mathbf{x}_n^T \mathbf{w}_\lambda(t)e^{j(2\pi\kappa t + \pi\beta t^2)}$ and $g_k(t) = \mathbf{x}_k^T \mathbf{w}_\lambda(t)e^{j(2\pi\kappa t + \pi\beta t^2)}$, is defined as

$$C_{k,n}(\tau) = \int_{-\infty}^{+\infty} g_k(t)g_n^*(t + \tau)dt \quad (3.1)$$

$$= \mathbf{x}_k^T \mathbf{R}(\tau) \mathbf{x}_n \quad (3.2)$$

where $\mathbf{R}(\tau)$ is an $N \times N$ matrix, and $\mathbf{x}_i = [x_0^{(i)}, x_1^{(i)}, \dots, x_{N-1}^{(i)}]^T$.

The Fourier transform of $g(t)$ is given by

$$G(f) = \int_{-\infty}^{+\infty} g(t)e^{-j2\pi ft}dt = \sum_{k=0}^{N-1} x_k W_{\lambda,k}(f)$$

where x_k is the filter coefficient of the envelope $f(t)$, and $W_{\lambda,k}(f)$ is the Fourier transform of the LFM Hermite-Rodriguez function given by

$$W_{\lambda,k}(f) = \int_{-\infty}^{+\infty} \omega_{\lambda,k}(t) e^{j(2\pi\kappa t + \pi\beta t^2)} e^{-j2\pi ft} dt.$$

A closed form expression of $W_{\lambda,k}(f)$ is obtained from [66] as

$$W_{\lambda,k}(f) = m H_k(f') \exp\left(-\frac{\pi^2 \lambda^2 (f - \kappa)^2}{1 - j\pi\beta\lambda^2}\right) \quad (3.3)$$

where

$$\begin{aligned} m &= \frac{(-j)^k (j\pi\beta\lambda^2)^{k/2} e^{j(k+1)\varphi/2}}{\sqrt{2^k k!} (1 + \pi^2 \beta^2 \lambda^4)^{(k+1)/4}} \\ f' &= \sqrt{\frac{\pi}{2\beta}} \frac{(f - \kappa)(1 - j) e^{j\varphi/2}}{(1 + \pi^2 \beta^2 \lambda^4)^{1/4}} \\ \varphi &= \tan^{-1}(\pi\beta\lambda^2). \end{aligned}$$

Let $\mathbf{W}(f) = [W_{\lambda,0}(f), W_{\lambda,1}(f), \dots, W_{\lambda,N-1}(f)]^T$. Then $G(f)$ can be written as

$$G(f) = \mathbf{x}^T \mathbf{W}(f) \quad (3.4)$$

and the power spectral density of the transmitted waveform $g(t)$ can be expressed as

$$\Psi(f) = |G(f)|^2 = \mathbf{x}^T \Phi(f) \mathbf{x} \quad (3.5)$$

where $\Phi(f) = \mathbf{W}(f) \mathbf{W}^H(f)$ is an N by N non-negative definite matrix.

3.3 Waveform Set Design

In waveform set design, the objective is to design M orthogonal waveforms which maximizes the power of the waveform in a prescribed passband while forcing the waveform to satisfy a prescribed mask constraints. The waveforms are given by

$$g_i(t) = \mathbf{x}_i^T \mathbf{w}_\lambda(t) e^{j(2\pi\kappa t + \pi\beta t^2)}, \quad i = 0, 1, \dots, M - 1 \quad (3.6)$$

where \mathbf{x}_i is the filter coefficients for each waveform. The optimal waveform $g_i(t)$ is defined as the waveform which maximizes the power in the passband while satisfying prescribed spectral mask constraints. Similar to [5, 30, 55], orthogonality is defined as zero crosscorrelation between any two signals for zero time shift *i.e.*, $C_{i,n}(0) = 0$ where $C_{i,n}(\tau)$ is given by (3.2).

Let Ω_p denote the sets of passband frequencies and $S(f)$ the prescribed spectral masks. Then the optimal waveform set design problem can be formulated as the following constrained optimization problem

$$\max \int_{\Omega_p} |G_i(f)|^2 df \quad (3.7)$$

subject to

$$|G_i(f)| \leq S(f), \forall f, \quad (3.8)$$

$$C_{i,n}(0) = 0, \forall n \neq i. \quad (3.9)$$

Using (3.4), the spectral mask constrained waveform set design problem (3.7) and (3.8) can be converted to the following simplified nonlinear optimization problem

$$\max \mathbf{x}_i^T Q \mathbf{x}_i \quad (3.10)$$

subject to

$$|G_i(f)| \leq S(f), \forall f, \quad (3.11)$$

where $Q = \int_{\Omega_p} \Phi(f) df$ is a positive definite matrix. Note that (3.10) and (3.11) define a constrained nonlinear optimization problem, where the objective function is quadratic but the constraints in general are nonlinear and non-convex.

First, the linearization of the inequality constraints (3.11) is considered. For any complex number $z = \xi + j\eta$, it is known from the rotation theorem [21] that

$$|z| = \max_{0 \leq \theta < 2\pi} \Re\{ze^{j\theta}\}.$$

Using the rotation theorem, the magnitude inequality constraint (3.11) can be written as

$$\mathbf{a}^T(f, \theta) \mathbf{x}_i \leq S(f), \forall \theta \in [0, 2\pi), \forall f \quad (3.12)$$

where

$$\mathbf{a}^T(f, \theta) \mathbf{x}_i = \Re\{G_i(f)e^{j\theta}\}, \quad (3.13)$$

and

$$\mathbf{a}(f, \theta) = \Re\{\mathbf{W}(f)e^{j\theta}\}. \quad (3.14)$$

Note that the constraint (3.12) is linear but the number of constraints increases due to the introduction of the new variable θ . As can be seen that (3.10) and (3.12) specify a semi-infinite QP problem where the number of variables to be optimized is finite

but the number of inequality constraints is infinite (depend on θ and f). In numerical optimization, either the parameter θ can be discretized or the approach in [77, 78] can be adopted.

Remarks:

1. In order to satisfy the orthogonality constraint (3.9), the waveforms may be designed sequentially. The orthogonality of the waveform being designed at the current sequence step with the set of waveforms designed previously is ensured by the equality constraint (3.9). For example, in the first step ($i = 0$), (3.10) and (3.12) are solved. Then, in the second step ($i = 1$), (3.10), (3.12) and (3.9) with $n = 0$ are solved.
2. In the design of any particular waveform $g_i(t)$, if there is no time or frequency domain constraints, the coefficients x_k computed using (1.5) will be the optimal \mathcal{L}_2 space solution. However, in the waveform set design considered in this chapter, frequency domain constraints are needed in order to force the PSD of the waveforms to lie within a prescribed spectral mask and time domain constraints are needed to ensure the orthogonality of the waveform set. In this case, the coefficients x_k are necessarily the solution of some well-formulated constrained optimization problem.
3. Note that the optimization problem (3.10), (3.12) and (3.9) is non-convex due to the fact that the convex objective function is to be maximized [79]. However, since all the constraints are linear, the convex set defined by the constraints is a convex polyhedron. This means that the global maximum is achieved at the extreme points of the polyhedron [79]. For a bounded convex polyhedron, all its finite number of extreme points can be easily found by solving a set of linear equations. Alternatively, since the objective function is quadratic and all the constraints are linear, the first order Karush-Kuhn-Tucker conditions of the constrained optimization problem are linear and can be solved in a straightforward way [79].

In summary, the following algorithm can be used to solve the waveform set design problem (3.9), (3.10), and (3.12). First, fix M , N , λ , β , and κ . Next, compute $C_{i,n}(\tau)$ in (3.9), Q in (3.10), $\mathbf{a}(f, \theta)$ in (3.14), and $S(f)$. Then, proceed with the following steps.

0. for $i = 0$; design $g_0(t)$ *i.e.*, solve the constrained optimization problem as defined by (3.10) and (3.12) for \mathbf{x}_0 .

1. for $i = 1, n = 0$; design $g_1(t)$ *i.e.*, solve the constrained optimization problem as defined by Eqs. (3.9), (3.10), and (3.12) for \mathbf{x}_1 , with \mathbf{x}_0 obtained from step 0.
2. for $i = k, n = 0, 1, \dots, k - 1$; design $g_k(t)$ *i.e.*, solve the constrained optimization problem as defined by Eqs. (3.9), (3.10), and (3.12) for \mathbf{x}_k , with $\mathbf{x}_n, n = 0, 1, \dots, k - 1$ obtained from step 0 to $k - 1$.

Repeat the above step until $k = M - 1$.

Note that, in each step, unlike the iterative procedure in [55] where the nonlinear phase was adjusted heuristically, the semi-infinite QP problem is solved to obtain an optimal waveform in the sense of having maximum energy and orthogonal to all waveforms designed in previous steps.

Table 3.1: Spectral Efficiency, η of the waveforms in Fig. 3.1

Waveform Index	1	2	3	4	5	6	7	8
η	0.7634	0.7663	0.7368	0.6320	0.7428	0.5963	0.5960	0.4654

Table 3.2: Crosscorrelation between g_i and g_j (1.0×10^{-3})

j	0	1	2	3	4	5	6	7
$i = 0$	–	-0.0750	0.0581	0.0437	0.0576	-0.0651	-0.0355	-0.1318
$i = 1$	-0.0750	–	0.0517	0.0386	0.0511	-0.0580	-0.0309	-0.1162
$i = 2$	0.0581	0.0517	–	-0.0368	-0.0519	0.0591	0.0344	0.1101
$i = 3$	0.0437	0.0386	-0.0368	–	-0.0330	0.0356	0.0153	0.0674
$i = 4$	0.0576	0.0511	-0.0519	-0.0330	–	0.0488	0.0256	0.1156
$i = 5$	-0.0651	-0.0580	0.0591	0.0356	0.0488	–	-0.0295	-0.1336
$i = 6$	-0.0355	-0.0309	0.0344	0.0153	0.0256	-0.0295	–	-0.0819
$i = 7$	-0.1318	-0.1162	0.1101	0.0674	0.1156	-0.1336	-0.0819	–

3.4 Illustrative Numerical Examples

3.4.1 Design of 8 FCC-compliant waveforms

To illustrate the effectiveness of the proposed approach, a numerical example dealing with the design of 8 waveforms $g_i(t)$ satisfying FCC spectral mask is studied in this section. The FCC spectral mask [26] is given by

$$|S_{FCC}(f)|^2 = \begin{cases} -41.3 \text{ dB} & 0 \leq f < 0.96 \text{ GHz} \\ -75.3 \text{ dB} & 0.96 \text{ GHz} \leq f < 1.61 \text{ GHz} \\ -53.3 \text{ dB} & 1.61 \text{ GHz} \leq f < 1.99 \text{ GHz} \\ -51.3 \text{ dB} & 1.99 \text{ GHz} \leq f < 3.1 \text{ GHz} \\ -41.3 \text{ dB} & 3.1 \text{ GHz} \leq f < 10.6 \text{ GHz} \\ -51.3 \text{ dB} & 10.6 \text{ GHz} \leq f < 20 \text{ GHz} \\ 0, & f \geq 20 \text{ GHz}. \end{cases}$$

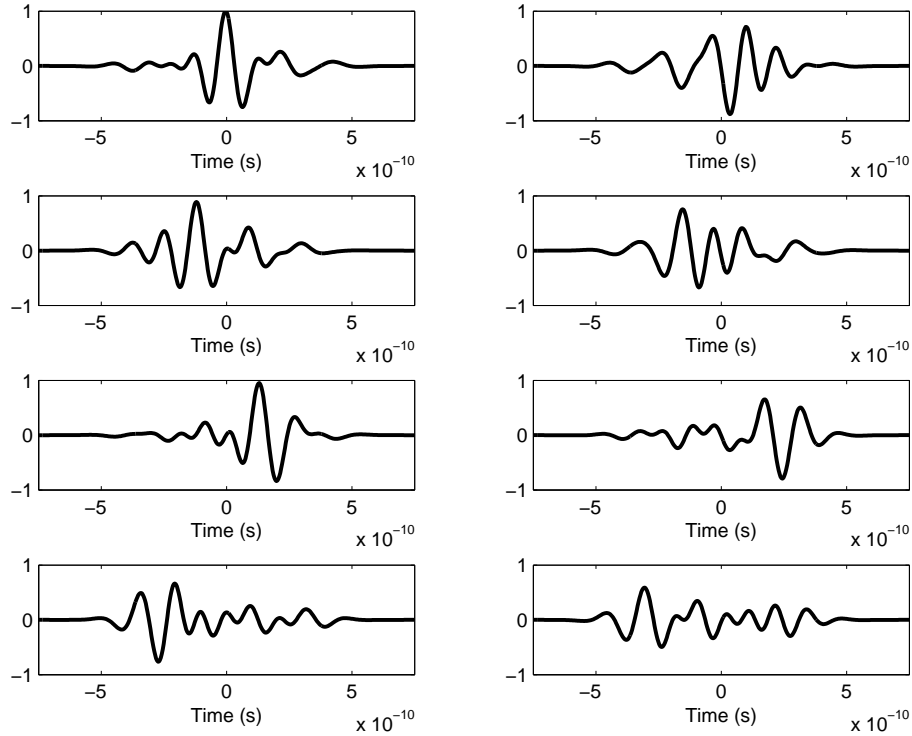


Figure 3.1: Plot of the 8 designed waveforms $g_i(t)$ ($N = 21$).

Similar to [5] and [55], $S(f)$ used in the constraint (3.12) is a stricter version of $S_{FCC}(f)$ given by

$$|S(f)|^2 = \begin{cases} -41.3 \text{ dB} & 3.1 \text{ GHz} \leq f \leq 10.6 \text{ GHz} \\ -81.3 \text{ dB} & 0 \leq f < 3.1 \text{ GHz} \\ -56.3 \text{ dB} & f > 10.6 \text{ GHz}. \end{cases} \quad (3.15)$$

The objective is to design 8 orthogonal waveforms $g_i(t)$ which maximize the power in the pass band (3.1 GHz to 10.6 GHz) while forcing the waveforms $g_i(t)$ to satisfy the mask constraints for the entire spectrum range (0 to 16 GHz). Similar to [5, 32, 55], the power utilization efficiency, η is defined as

$$\eta = \frac{\int_{3.1}^{10.6} |G_i(f)|^2 df}{\int_{3.1}^{10.6} |S(f)|^2 df}. \quad (3.16)$$

In this particular example, N was first set to 21. Since the waveforms are to occupy a finite spectrum range *i.e.*, to have essentially zero magnitude after a certain maximum frequency and it is known that λ is inversely proportional to the maximum frequency (see Fig. 1.2), λ was set to $\frac{2}{16\text{GHz}} = 1.25 \times 10^{-10}$ and $\beta = 1 \times 10^{18}$. Since the main focus is on the solution of optimization problem *i.e.*, filter coefficients, the center frequency of the LFM signal, κ was set to 0.

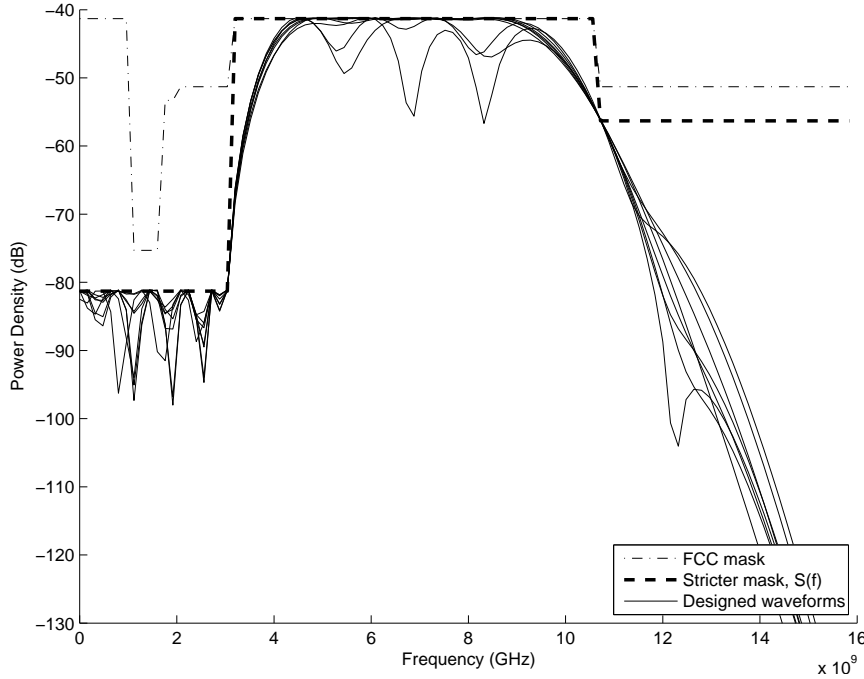


Figure 3.2: Plot of power spectrum of the 8 designed waveforms $g_i(t)$ and FCC mask ($N = 21$).

The constrained optimization problem is then solved using Matlab and its Optimization Toolbox¹. The 8 designed waveforms are plotted in Fig. 3.1 and their power spectrum are plotted in Fig. 3.2. It is clear that the designed waveforms $g_i(t)$ utilize the pass band (3.1 to 10.6 GHz) and also comply with FCC regulations for the entire spectrum range. The η for these waveforms are shown in Table 3.1². Fig. 3.3 shows autocorrelations of $g_0(t)$ and $g_1(t)$, and their crosscorrelation. It can be seen that the crosscorrelation between $g_0(t)$ and $g_1(t)$ is low at $t = 0$ as constrained by (3.9). The crosscorrelations for all possible pairs of the 8 designed waveforms are also low as shown in Table 3.2. It is evidenced that the proposed sequential semi-infinite QP approach is not only able to utilize the pass band but also guarantee the orthogonality of the waveforms.

To compare the effectiveness of the proposed approach with the approach in [55], similar to Fig. 11 in [55], Fig. 3.4 shows the plot of the spectrum utilization, η vs the index of the waveform being designed for filter order, $N = 16, 21, 26$ ($\lambda = 1.25 \times 10^{-10}$,

¹The main MATLAB files are included in the Appendix A.

²As waveform index increases, it is harder to satisfy the orthogonality constraints in the sequential optimization problem, and thus the efficiency generally decreases.

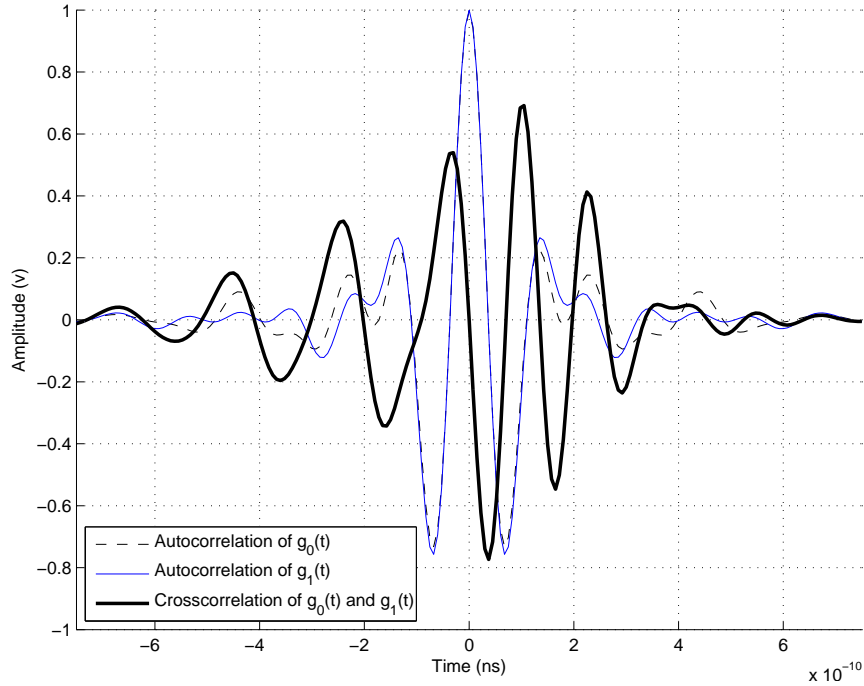


Figure 3.3: Plot of autocorrelations of $g_0(t)$ and $g_1(t)$, and their crosscorrelation ($N = 21$). The crosscorrelation between $g_0(t)$ and $g_1(t)$ is low at $t = 0$.

$\beta = 1 \times 10^{18}$). For $N = 21$, the proposed approach produced 4 waveforms with η higher than 0.7 and for $N = 26$, there are 5 waveforms with η higher than 0.8. It is clear that the proposed approach was able to design more waveforms with higher efficiency than those obtained in [55].

The effect of λ on the designed waveforms was also studied *i.e.*, by computing the power utilization efficiency, η of the designed waveforms for a certain range of λ . Fig. 3.5 shows the plot of power utilization efficiency, η vs λ for $M = 1$, $N = 21$, $\beta = 1 \times 10^{18}$, and $\kappa = 0$. It can be seen that the best λ within the range of investigation is $\lambda_{best} = 1.15 \times 10^{-10}$ which is reasonably close to the value that was selected for the numerical study ($\lambda = 1.25 \times 10^{-10}$). Fig. 3.6 shows the plot of power utilization efficiency, η vs filter order, N for $\lambda_{best} = 1.15 \times 10^{-10}$, $\beta = 1 \times 10^{18}$, $M = 1$, and $\kappa = 0$. It can be seen that the efficiency increases as filter order increases.

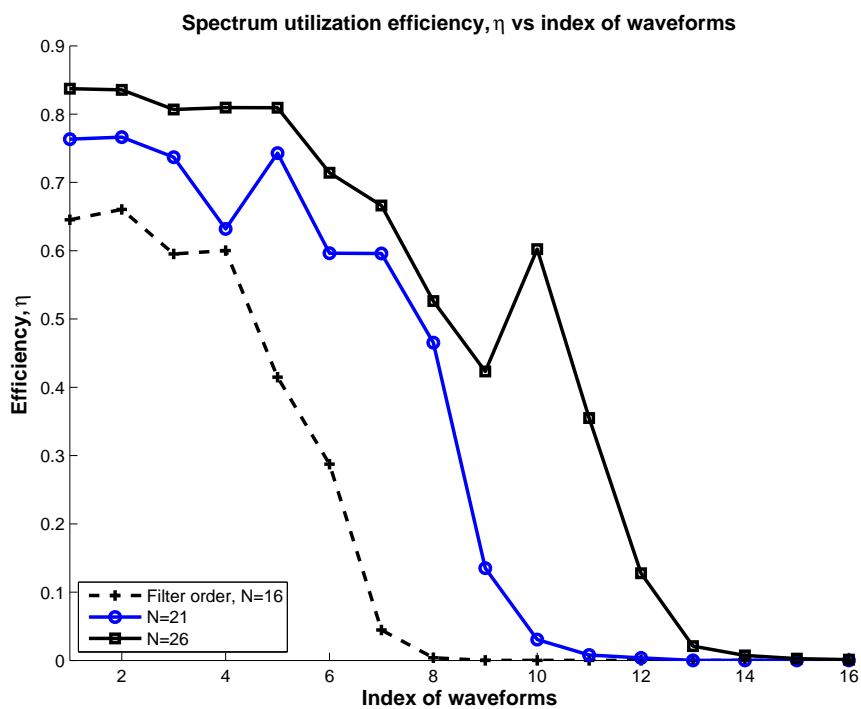


Figure 3.4: Plot of η vs index of the waveforms being designed for $N = 16, 21, 26$.

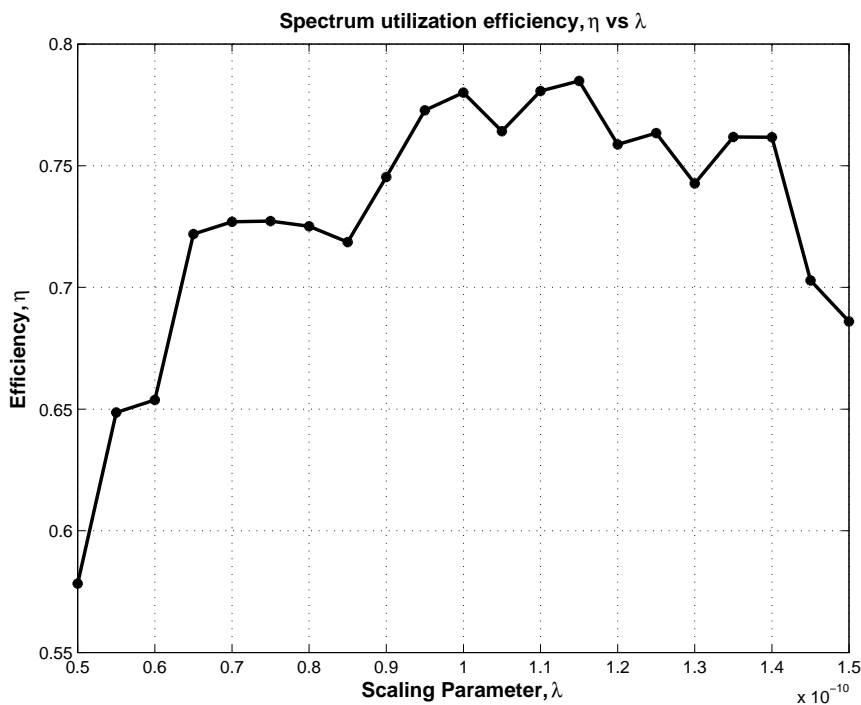


Figure 3.5: Plot of η vs λ for $N = 21$ ($M = 1$). The best $\lambda = 1.15 \times 10^{-10}$.

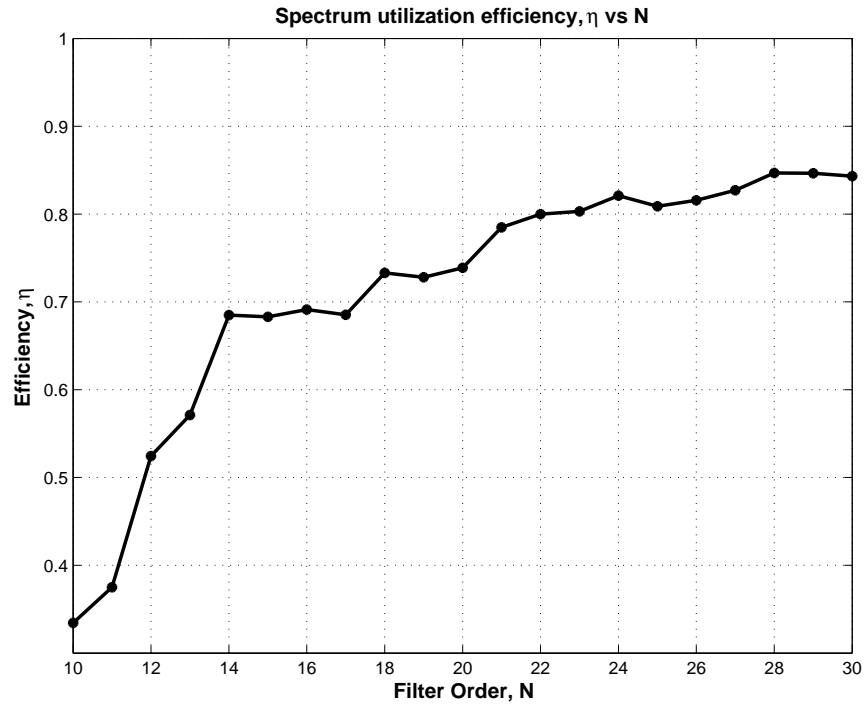


Figure 3.6: Plot of η vs filter order, N for $\lambda_{best} = 1.15 \times 10^{-10}$ ($M = 1$). The η increases as N increases.

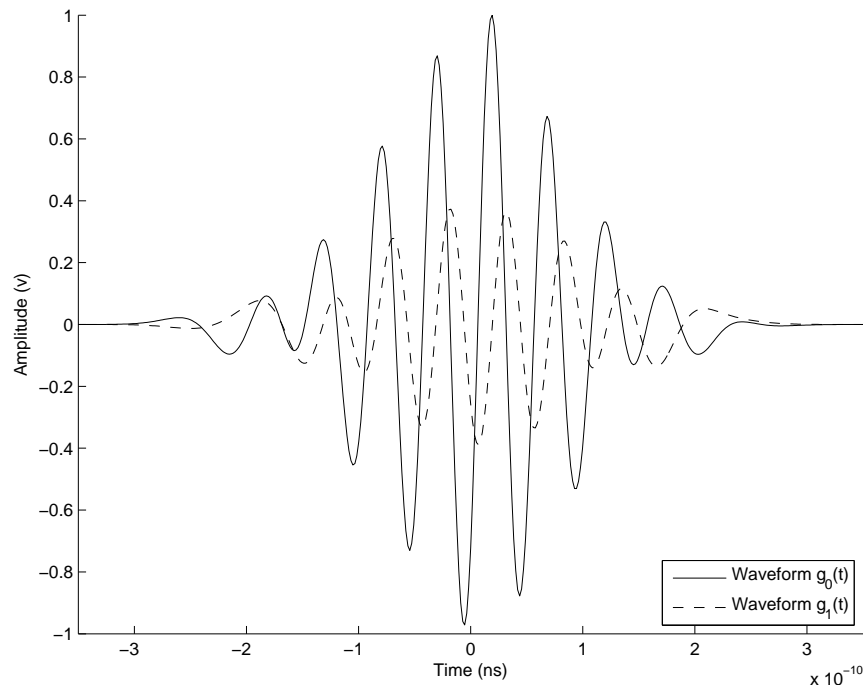


Figure 3.7: Plot of the designed waveforms $g_0(t)$ and $g_1(t)$ ($M = 2$, $N = 21$).

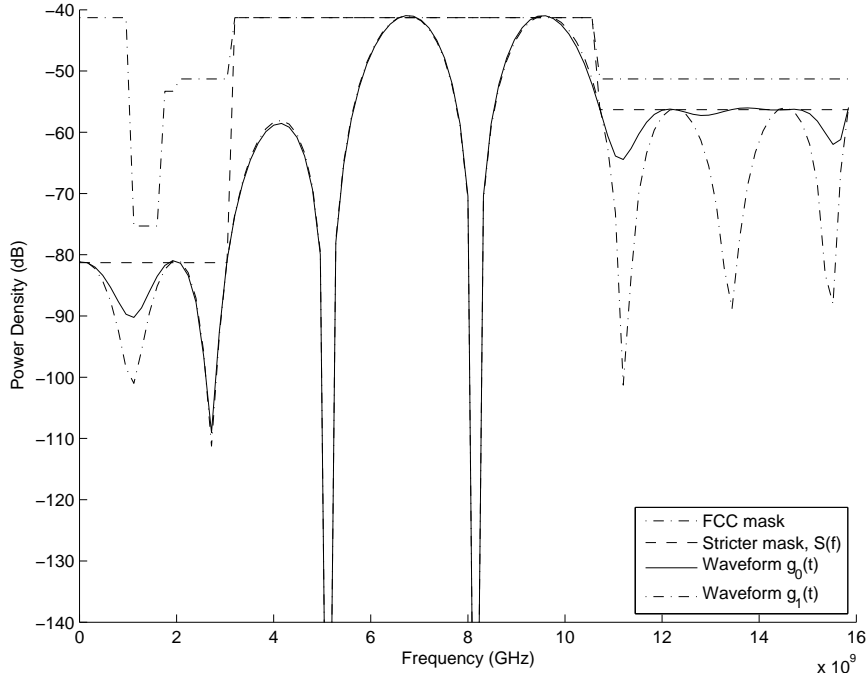


Figure 3.8: Plot of power spectrum of the 2 designed waveforms $g(t)$ ($M = 2$, $N = 21$). The waveforms suppress NBI at 5.2 GHz and 8.1 GHz, and maximize energy for the entire spectrum.

3.4.2 Waveform Set Design with NBI suppression

To illustrate the flexibility of the proposed approach, a numerical example dealing with the design of UWB waveforms with NBI suppression is presented in this section. To suppress interferences at certain frequencies, an additional constraint

$$|G(f)| = 0 \forall f \in \mathcal{N}, \quad (3.17)$$

where \mathcal{N} is the set of interfering frequencies is introduced to the optimization problem (3.10), (3.12) and (3.9). Practically, constraint (3.17) can be replaced by

$$|G(f)| \leq \delta \forall f \in \mathcal{N}, \quad (3.18)$$

where δ is a practically small constant³. Similar to (3.11) where it was rewritten into (3.12), (3.18) can also be linearized using the rotation theorem [21].

In this example, the objective is to design 2 orthogonal waveforms $g(t)$ ($M = 2$) which maximize the power in the pass band (3.1 GHz to 10.6 GHz) while suppressing

³ $\delta = 0.00001$ (−100 dB) was used in the numerical example.

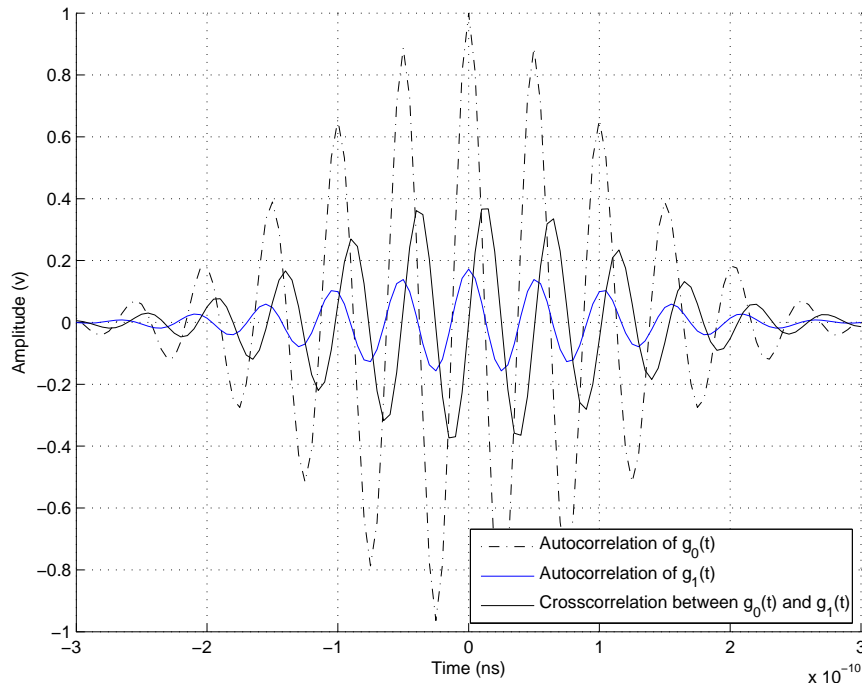


Figure 3.9: Plot of autocorrelation and crosscorrelation. The crosscorrelation at $t = 0$ is essentially zero.

NBI at 5.2 GHz and 8.1 GHz and forcing the waveforms $g(t)$ to satisfy the mask constraints for the entire spectrum (0 to 16 GHz). Using 21 terms ($N = 21$) and with $\lambda = 6 \times 10^{-11}$, $\beta = 1 \times 10^{18}$, $\kappa = 0$, the constrained optimization problem is then solved using Matlab and its Optimization Toolbox. The designed waveforms $g(t)$ are plotted in Fig. 3.7 and their power spectrum is plotted in Fig. 3.8. It is clear that the designed waveforms $g(t)$ suppress NBI at 5.2 GHz and 8.1 GHz and utilize the pass band (3.1 to 10.6 GHz) while complying with FCC regulations for the entire spectrum. Fig. 3.9 shows autocorrelations of $g_0(t)$ and $g_1(t)$, and their crosscorrelation, and it can be seen that the crosscorrelation at $t = 0$ is essentially zero.

3.5 Chapter Summary

In this chapter, linear FM and orthogonal Hermite Rodriguez basis functions have been applied to design a set of orthogonal waveforms for UWB communication systems. Using the rotation theorem, the nonlinear spectral constraints have been converted into linear constraints with the addition of a new variable. The crosscorrelation constraints have also been linearised by solving the quadratic semi-infinite programming sequentially. Numerical examples have demonstrated that the effectiveness of the proposed design approach compares favourably with the recent work in [55] in terms of the number of waveforms can be produced and the achievable spectral efficiency of the designed waveforms. This is due to the suitability of Hermite-Rodriguez functions for designing signals with finite time support and the computational effectiveness of the proposed optimization approach. The proposed approach is also flexible to handle additional constraints. Future work includes constraining the crosscorrelation to be low at multiple time shifts and a new formulation to include synchronisation jitter. Hardware implementation of Hermite Rodriguez functions will also be considered besides investigating for a more systematic approach in computing the optimal λ for a certain application.

Chapter 4

Design of UWB Pulse Shaping Filter with Nonconstant Upper Bounds

In this chapter, UWB pulse shaping filter design is formulated as a semi-infinite quadratic optimization problem with piece-wise continuous bounds incorporated in the design. The proposed approach is compared with the widely used finite impulse response (FIR) filter designed via semi-definite programming.

The chapter is organized as follows. Section 4.2 describes the pulse shaping problem with the nonconstant upper bounds. The solution method is then proposed. Design examples will be presented in Section 4.3. Lastly, Section 4.4 concludes the chapter.

4.1 Introduction

In UWB systems, unmodulated basic pulse is employed in the transmission and the PSD of the basic pulse determines the PSD of the overall UWB signal. Given the importance of the basic pulse, a lot of research have been done by using FIR prefilters to shape the basic pulses [5, 37–40]. However, designing the FIR filter coefficients to maximize power while satisfying the spectral mask is not trivial. The traditional FIR design algorithm has been used in [37] but it did not directly maximize the transmit power. In order to directly maximize the power, FIR filter design based on semidefinite programming has been used [5, 38–40] where the constant FCC spectral constraint was cast into

linear matrix inequalities (LMI) framework in their convex optimization formulation [5, 39, 40].

In the most recent work [38], the pulse shaping filter design using FIR filters is improved by incorporating the basic pulse which is commonly generated by analog pulse generators. Incorporating basic pulse in the design leads to piece-wise continuous upper bounds instead of constant spectral mask constraint. Using modified Fourier series expansion to approximate the piece-wise continuous bounds, these bounds were cast in the LMI framework. This approach has achieved high spectral utilization even when a suboptimal basic pulse was used [38].

Maximizing signal power leads to maximizing quadratic function of filter coefficients. To cast this maximizing problem into convex optimization problem, autocorrelation of the filter coefficients was used in [5, 38–40]. Thus, de-correlation or spectral factorization has to be carried out in order to obtain the designed filter coefficients. This step of the design can be numerically problematic and computationally expensive.

In this chapter the applicability of Hermite-Rodriguez (HR) filter as the pulse shaping filter or transmit filter to shape the basic pulses in the UWB pulse shaping filter design is investigated. Unlike [38], the pulse shaping filter design problem is directly formulated as a maximization of a quadratic function of the filter coefficients subject to linearizable inequality constraints. Similar to [38], the basic pulse is also incorporated in the proposed formulation. The resulting non-constant bounds are implemented and linearized using the rotation theorem [21]. Through both theoretical argument and numerical design examples, it is demonstrated that the proposed design is computationally efficient to implement and the design result is either comparable to or better than those reported in [38] especially when the suboptimal basic pulse is incorporated into the design. Thus, the main contribution of this chapter is direct maximization of power incorporating non-constant bounds without using spectral factorization.

4.2 Pulse Shaping Problem

Generally, the basic pulses $p(t)$ used in UWB systems not only do not optimally exploit the allowable spectrum but also do not comply with some specific constraints such as the FCC spectral mask. An example of a basic pulse $p(t)$ is the Gaussian Monocycle [38, 80]. HR based transmit filters will then be used to adapt the basic pulses to a certain spectral constraints. Let $q(t)$ denote the pulses created by the analog pulse

generators. Then, following the standard system model [38, 80], $p(t)$ can be expressed as

$$p(t) = q(t) * h(t) = \int_{-\infty}^{+\infty} q(\tau)h(t - \tau)d\tau \quad (4.1)$$

where

$$h(t) = \sum_{k=0}^{N-1} x_k \omega_{\lambda,k}(t)$$

is the transmit filter represented by (1.6) with x_k will be the design parameters. The PSD of $p(t)$ is given by

$$|P(f)|^2 = |H(f)|^2|Q(f)|^2.$$

Now, let Ω_p denote the sets of passband frequencies and let $S(f)$ denote the spectral masks. Then the optimal pulse shaping problem can be formulated as the following constrained optimization problem

$$\max_{\mathbf{x}} \int_{\Omega_p} |P(f)|^2 df \quad (4.2)$$

subject to

$$|P(f)|^2 \leq S(f), \forall f \quad (4.3)$$

i.e., maximizing the power while satisfying the given spectral mask constraint.

The pulse shaping problem (4.2) and (4.3) can be converted to the following simplified non-linear optimization problem

$$\max_{\mathbf{x}} \mathbf{x}^T A \mathbf{x} \quad (4.4)$$

subject to

$$|H(f)| = |\mathbf{x}^T \mathbf{W}(f)| \leq \frac{\sqrt{S(f)}}{|Q(f)|}, \forall f, \quad (4.5)$$

where

$$A = \int_{\Omega_p} \Phi(f) |Q(f)|^2 df \quad (4.6)$$

is a positive definite matrix. Note that (4.4) and (4.5) define a constrained non-linear optimization problem, where the objective function is quadratic but the constraint in general is non-linear and non-convex.

Next, the linearization of the inequality constraints (4.5) is considered. For any complex number $z = \xi + j\eta$, it is known from the rotation theorem [21] that

$$|z| = \max_{0 \leq \theta < 2\pi} \Re\{ze^{j\theta}\}.$$

Using the rotation theorem, the magnitude inequality constraint (4.5) can be written as

$$\max_{0 \leq \theta < 2\pi} \Re\{\mathbf{x}^T \mathbf{W}(f) e^{j\theta}\} \leq \frac{\sqrt{S(f)}}{|Q(f)|}. \quad (4.7)$$

Note that the constraint (4.7) is linear but the number of constraints increases due to the introduction of the new parameter θ . As can be seen that (4.4) and (4.7) specify a semi-infinite quadratic programming problem where the number of variables to be optimized is finite but the number of inequality constraints is infinite (depend on θ and f). In numerical optimization, either the parameter θ can be discretized or the approach in [77, 78] can be adopted.

For simplicity, the continuous variable θ is discretized and constraint (4.7) is considered over a discrete set $\{\theta_k\}_{k=1}^{2p}$ with $\theta_k = \pi(k-1)/p$, where $p \geq 2$ is a sufficiently large integer. Furthermore, $M_n^p(f)$ is defined as

$$M_n^p(f) = \max_{1 \leq k \leq 2p} \Re\{\mathbf{x}^T \mathbf{W}(f) e^{j\theta_k}\}. \quad (4.8)$$

It can be proved [81, 82] that

$$M_n^p(f) \leq \max_{0 \leq \theta \leq 2\pi} \Re\{\mathbf{x}^T \mathbf{W}(f) e^{j\theta}\} \leq M_n^p(f) \sec\left(\frac{\pi}{2p}\right). \quad (4.9)$$

Note that $\sec(\pi/(2p)) \rightarrow 1$ (as $p \rightarrow \infty$). This means that for a sufficiently large integer p , M_n^p gives a fairly good estimate of $H(f)$. In fact, for $p = 8$, $\sec(\pi/(2p)) = 1.020$. Hence, instead of considering constraint (4.7), the following strengthened inequality constraint shall be considered over discrete sets $\{f_l\}_{l=1}^L$ and $\{\theta_k\}_{k=1}^{2p}$

$$\max_{1 \leq k \leq 2p} \Re\{\mathbf{x}^T \mathbf{W}(f_l) e^{j\theta_k}\} \leq \frac{\sqrt{S(f_l)}}{|Q(f_l)| \sec\left(\frac{\pi}{2p}\right)}. \quad (4.10)$$

Constraint (4.10) is equivalent to the following set of linear inequality constraints

$$B_l \mathbf{x} \leq \mathbf{c}_l \quad (4.11)$$

where, for each l , B_l is a $2p$ by N matrix with

$$\mathbf{b}^T(f_l, \theta_k) = \Re\{\mathbf{W}^T(f_l) e^{j\theta_k}\}$$

as its k th row, and \mathbf{c}_l is a $2p$ -dimensional vector with all its entries

$$c_l = \frac{\sqrt{S(f_l)}}{|Q(f_l)| \sec\left(\frac{\pi}{2p}\right)}.$$

In short, the pulse shaping problem can be summarized into the following simplified quadratic optimization problem: Given the discrete sets $\{\theta_k\}_{k=1}^{2p}$ and $\{f_l\}_{l=1}^L$, find \mathbf{x}^* which solve the following optimization problem

$$\max_{\mathbf{x}} \mathbf{x}^T \mathbf{A} \mathbf{x} \quad (4.12)$$

subject to

$$B_l \mathbf{x} \leq \mathbf{c}_l \quad (4.13)$$

for $1 \leq l \leq L$. Note that \mathbf{A} is a positive definite matrix. The solution of the optimization problem (4.12) and (4.13) is straightforward and can be carried out, for example, using simple MATLAB programs.

Table 4.1: Spectral Efficiency, η for $q(t)$ with $\tau = 0.087$ ns.

$(\tau = 0.087 \text{ ns})$ N	10	20	30
HR (QSIP)	0.4227	0.7850	0.8388
FIR (QSIP)	0.6966	0.8079	0.8172
FIR (SDP) [38]	0.7135	0.8230	-

Table 4.2: Spectral Efficiency, η for $q(t)$ with $\tau = 0.15$ ns.

$(\tau = 0.15 \text{ ns})$ N	10	20	30
HR (QSIP)	0.3665	0.7672	0.8431
FIR (QSIP)	0.6189	0.8164	0.8485
FIR (SDP) [38]	0.5588	0.6518	-

4.3 Illustrative Design Examples

As an example, a pulse shaping filter $h(t)$ as a transmit filter is designed *i.e.*, (4.12) and (4.13) are solved for the filter coefficients. For this particular design, firstly, λ of the HR filter is fixed. Since the overall signal is to occupy a finite spectrum range *i.e.*, to have essentially zero magnitude outside a certain maximum frequency and it is known that λ is inversely proportional to the maximum frequency, λ is set to $\frac{2}{16\text{GHz}} = 1.25 \times 10^{-10}$. Besides the HR filter, an FIR filter is also designed. It can be expressed as

$$H(f) = \sum_{k=0}^{N-1} x_k e^{j2\pi kTf}$$

where $T = 0.0333$ ns.

The basic pulse $q(t)$ is the Gaussian monocycle whose PSD is given as $|Q(f)|^2 \propto f^2 e^{-\pi(\tau f)^2}$ and shown in Fig 4.1 for various τ s. Similar to [38], the passband is $\Omega_p \in [0, 15]$ GHz. A in (4.12) is then computed using (4.6). The spectral mask used is the FCC indoor limit [26]. B_l and c_l in (4.13) are computed with $L = 200$ and $p = 8$. The constrained optimization problem (4.12) and (4.13) is then solved using Matlab and its Optimization Toolbox.

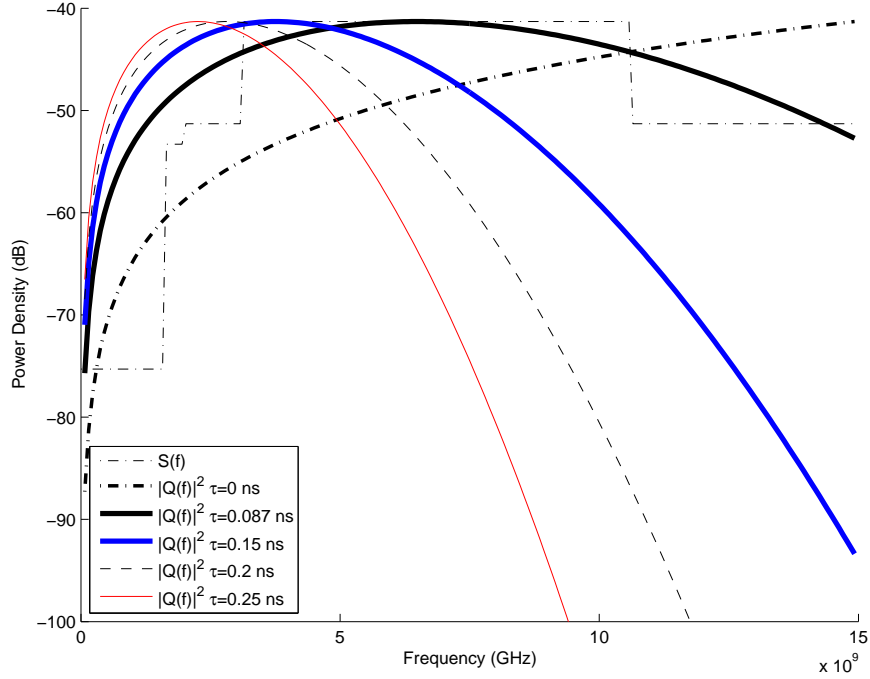


Figure 4.1: The PSD of the Gaussian Monocycle for various τ s. The PSD with $\tau = 0.087$ ns occupies the main bandwidth 3.1 to 10.6 GHz.

To compare with the previous work [38], the power utilization efficiency, η is similarly defined as

$$\eta = \frac{\int_{\Omega_p} |P(f)|^2 df}{\int_{\Omega_p} S(f) df}. \quad (4.14)$$

Design examples with $q(t)$ for two values of τ s are studied i.e., the optimal $\tau = 0.087$ ns and the less optimal $\tau = 0.15$ ns. The constraint $S(f)/|Q(f)|^2$ for these two basic pulses are plotted in Fig. 4.2 and Fig. 4.3 respectively.

- Design with $\tau = 0.087$ ns

The PSD of the pulses designed using both HR and FIR filters are plotted in Fig. 4.4. The spectral efficiency, η for different N s are tabulated in Table 4.1. The η from Table II of [38] is also tabulated for easy comparison. It can be seen that the results reported in [38] is slightly better than the results of the proposed design. However, it is important to note that a relatively small number of p is used in the discretization¹.

¹ $p = 8$ was used. Discretization points increase linearly with p whereas total element of semidefinite matrices in [38] increase quadratically with filter order N .

Table 4.3: Spectral Efficiency, η for $q(t)$ with $\tau = 0$ ns.

$(\tau = 0 \text{ ns})$ N	10	20	30
HR (QSIP)	0.4429	0.7667	0.8413
FIR (QSIP)	0.6811	0.8047	0.8030

Table 4.4: Spectral Efficiency, η for $q(t)$ with $\tau = 0.2$ ns.

$(\tau = 0.2 \text{ ns})$ N	10	20	30
HR (QSIP)	0.2705	0.6306	0.7683
FIR (QSIP)	0.6117	0.7900	0.8292

- Design with $\tau = 0.15$ ns

When a suboptimal basic pulse *i.e.*, the pulse with $\tau = 0.15$ ns is used, a much better spectral efficiency is achieved. The PSD of the pulses using both HR and FIR filters for $N = 20, 30$ are plotted in Fig. 4.5. Comparing this with Fig. 9 of [38], it is clear that the designed pulses utilized the allowable spectrum more efficiently. The η for the designed pulses as well as from Table III of [38] are tabulated in Table 4.2.

The proposed approach is more straightforward than the approach in [38] where the piece-wise continuous constraints were implemented heuristically due to the limitations of the Fourier series. Due to high derivative at discontinuities, the constraint $S(f)/|Q(f)|^2$ was manually limited in [38] when the values were much higher than the

Table 4.5: Spectral Efficiency, η for $q(t)$ with $\tau = 0.25$ ns.

$(\tau = 0.25 \text{ ns})$ N	10	20	30
HR (QSIP)	0.1940	0.5905	0.8086
FIR (QSIP)	0.5118	0.7443	0.7980

Table 4.6: Comparison of the number of parameters.

L	200	250	300	350
$2pL, (p = 8)$	3200	4000	4800	5600
N	20	25	30	35
$(i + 1)N^2 + i(N - 1)^2, (i = 5)$	4205	6630	9605	13130
% Difference	31	66	100	134

smallest value in an interval. The nonconstant bounds implemented using the approach in [38] were plotted and denoted as $\Gamma(f)$ in Fig. 4.2. Since these bounds were constructed based on $\tau = 0.087$ ns, they are optimal only when the basic pulse with $\tau = 0.087$ is used. However when the basic pulse with $\tau = 0.15$ ns is used, the designed waveform will not have maximum allowable energy throughout the spectrum. In the proposed approach, different bounds for the different basic pulses can be easily implemented. This is because the characteristic of the basic pulse i.e., the PSD has been directly incorporated into the problem formulation, see Section 4.2.

Tables 4.3, 4.4, and 4.5 show the spectral efficiency η for $\tau = 0$ ns, $\tau = 0.2$ ns, and $\tau = 0.25$ ns, respectively. This clearly illustrate the flexibility of the proposed approach to implement different bounds for different basic pulses which is the main reason of why the proposed approach achieved much better results even for the less optimal pulse.

To get a sense of the size of the parameters involved in solving (4.12) and (4.13), the total elements of semidefinite matrices in [38] will be compared. Approximately, there are $2pL$ parameters in the proposed approach and $(i + 1)N^2 + i(N - 1)^2$ variables in [38] where i is the number of bounds and N is the filter order. Table 4.6 shows that there are 3200 parameters with $p = 8$ and $L = 200$, whereas 4205 variables in [38] with $N = 20$ and $i = 5$. It can be seen that discretization points in the proposed approach increase linearly with p whereas total element of semidefinite matrices in [38] increase quadratically with filter order N .

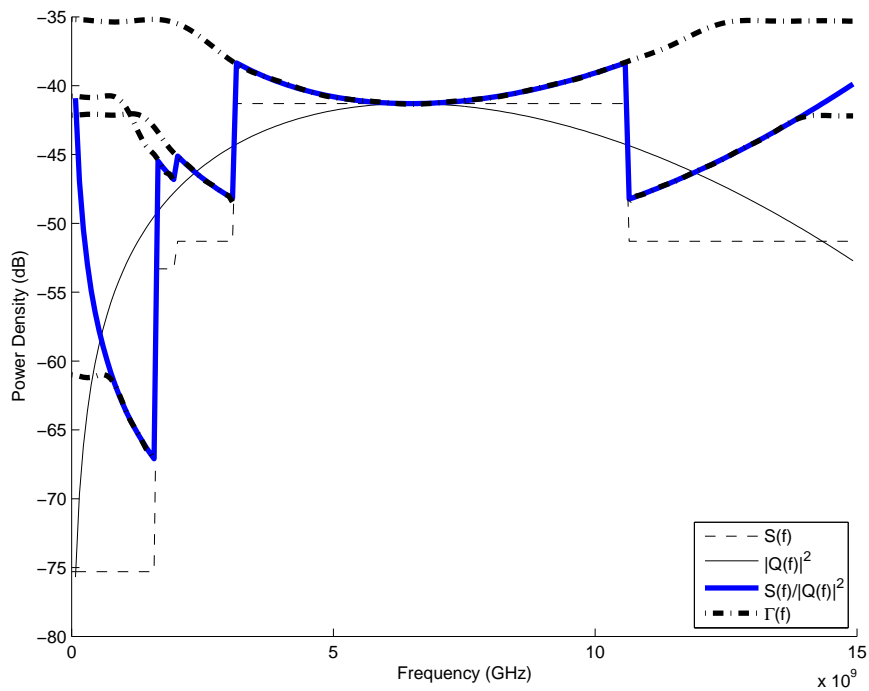


Figure 4.2: The constraint $S(f)/|Q(f)|^2$ for $\tau = 0.087$ ns. Note the difference with $\Gamma(f)$ implemented in [38] using the Fourier series.

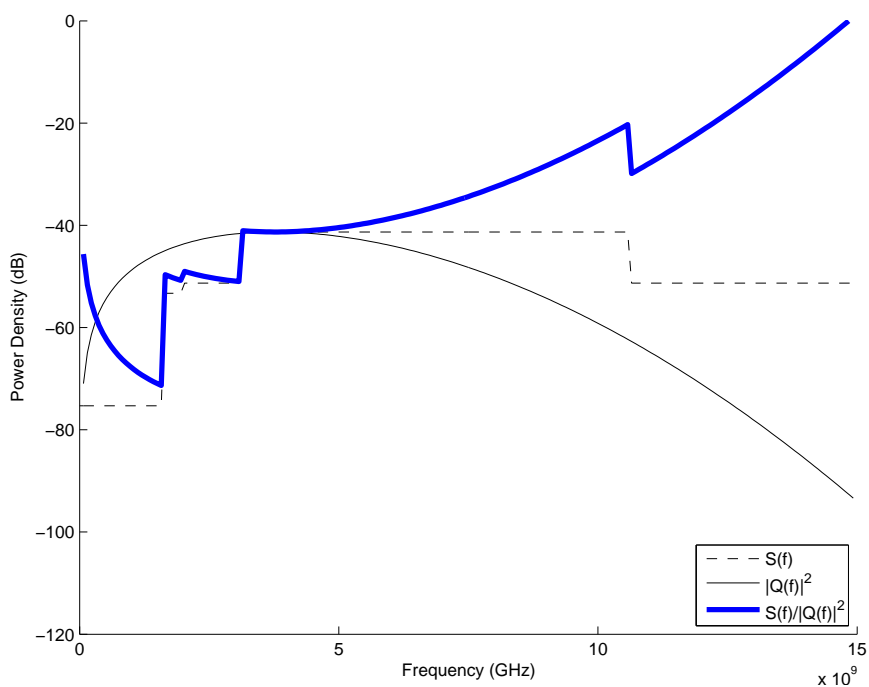


Figure 4.3: The constraint $S(f)/|Q(f)|^2$ for $\tau = 0.15$ ns. Note the difference with the bounds in Fig. 4.2.

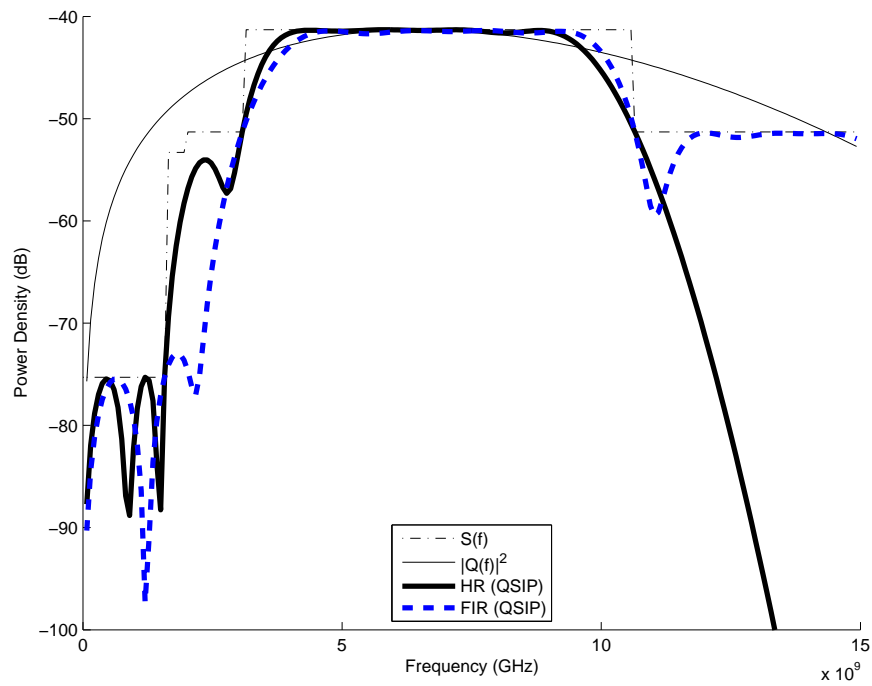


Figure 4.4: Plot of PSD for $\tau = 0.087$ ns ($N = 20$). Both PSDs (HR and FIR) occupy the main bandwidth.

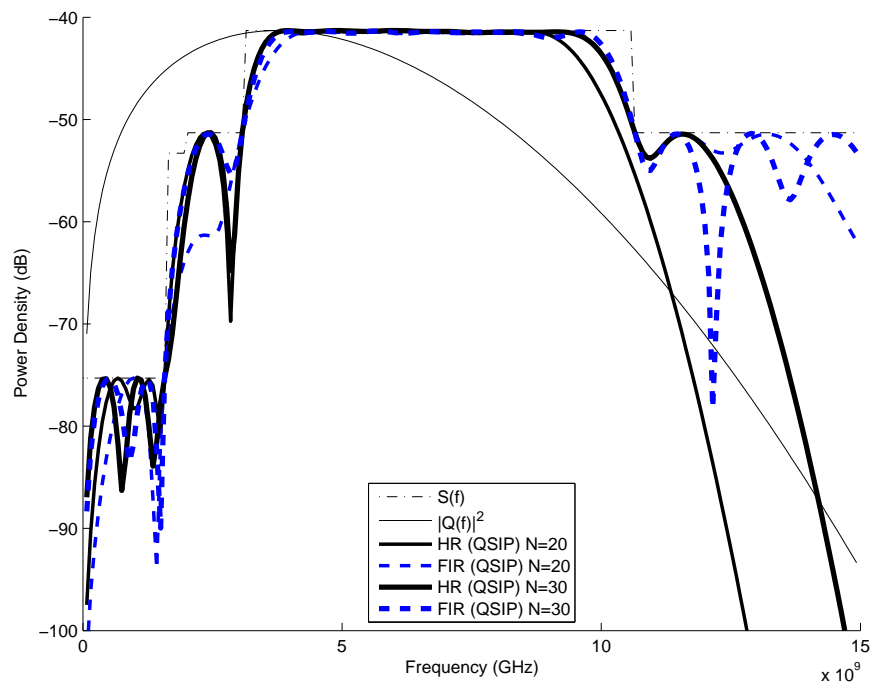


Figure 4.5: Plot of PSD for $\tau = 0.15$ ns. The PSDs still occupy the main bandwidth even with the less optimal basic pulse.

4.4 Chapter Summary

In this chapter orthogonal Hermite Rodriguez basis functions have been applied to design a pulse shaping filter for UWB communication systems. The design problem is directly formulated to maximize the transmit power in terms of filter coefficients, thus avoiding the spectral factorization step. The basic pulse is incorporated into the design and using the rotation theorem, the resulting nonlinear piece-wise continuous bounds have been converted into linear constraints with the addition of a new parameter. Design examples using both FIR and HR filter have demonstrated that the proposed design approach is more straightforward and achieves comparable spectral utilization with the recent work in [38]. When a suboptimal basic pulse is used, better spectral utilization is achieved.

Chapter 5

Design of UWB Waveform Set Using Löwdin's Orthogonalization

In this chapter, Löwdin orthogonalization method is used to produce overlapping orthogonal waveforms with high spectral efficiency by transforming a finite set of equally spaced shifts of an optimal waveform to an orthogonal set of spectrally efficient waveforms.

The chapter is organized as follows. In Section 5.2, the single waveform design problem is formulated as a constrained maximization problem. Then, it is converted into a constrained semi-infinite quadratic programming (QP) problem. In Section 5.3, the designed waveform from optimization step in Section 5.2 is used in Löwdin orthogonalization method to generate shift orthogonal waveform set. In Section 5.4, a numerical example of the design of orthogonal pulses complying with the FCC spectral mask is presented. Section 5.5 concludes the chapter.

5.1 Introduction

In multiuser communication systems and high data rate applications, there is a need to have a large number of waveforms that are mutually orthogonal. It has been shown that higher capacity can be achieved if each user uses a signal set that consists of signals being orthogonal to those used by other users [51–54]. This chapter aims to propose a method to design a large number of spectrally efficient orthogonal waveforms for UWB communication systems.

There are essentially two main steps in the proposed approach. First, spectral efficiency of a single UWB waveform is optimised directly by maximising the energy of the waveform in a prescribed passband while complying with the prescribed spectral mask constraint. The design of a spectrally efficient waveform is formulated as a constrained non-linear maximization problem where the objective function is quadratic but the constraints are in general non-linear. The rotation theorem [21] is then used to linearise the non-linear magnitude constraints. This optimization formulation did not impose any constraints on the phase of the waveform.

To design the single UWB waveform, Hermite-Rodriguez (HR) functions will be used as the basis functions. The HR basis functions are attractive because a signal can be represented to a high degree of accuracy by using just a few terms in the HR series expansion, and more importantly HR function is well suited for the design of signal with finite time support [61].

In the second step, Löwdin orthogonalization [56, 57] method is used to transform a finite set of equally spaced shifts of the optimal pulse (designed in the first step) to an orthogonal set of pulses. This method minimizes energy distortion with respect to the optimal pulse, thus producing orthogonal waveforms occupying the same spectrum. Besides, the generated orthogonal pulses are not distorted unlike those produced by the sequential approach in [53, 83]. Distorted pulses can not be used directly for pulse position modulation (PPM) in UWB systems [53, 84].

The main contribution of this chapter is designing orthogonal overlapping waveforms suitable for orthogonal overlapping PPM (OOPPM) using Löwdin orthogonalization method based on strictly time-limited pulses.

5.2 Optimal Waveform Design

HR filter given by (1.6) is used to represent the basic pulse commonly used in a UWB communication system. Thus, UWB basic pulse $p(t)$ can be expressed as

$$p(t) = \sum_{k=0}^{N-1} x_k \omega_{\lambda,k}(t) \quad (5.1)$$

where x_k will be the design parameters. The PSD of $p(t)$ is

$$|P(f)|^2 = |G(f)|^2 = \mathbf{x}^T \Phi(f) \mathbf{x}$$

where $\Phi(f) = \mathbf{W}(f) \mathbf{W}^T(f)$.

Now, let Ω_p denote the sets of passband frequencies and let $S(f)$ denote the spectral masks. Then the optimal waveform design problem can be formulated as the following constrained optimization problem

$$\max_{\mathbf{x}} \int_{\Omega_p} |P(f)|^2 df \quad (5.2)$$

subject to

$$|P(f)|^2 \leq S(f), \quad \forall f \quad (5.3)$$

i.e., maximizing the signal power while satisfying the given spectral mask constraint.

The waveform design problem (5.2) and (5.3) is formulated as direct maximization of a quadratic function of the filter coefficients subject to linearizable inequality constraints. Our approach does not use the autocorrelation of the filter coefficients unlike the convex optimization approach, where spectral factorization is required to obtain the designed filter coefficients [5, 38–40]. To proceed, the waveform design problem (5.2) and (5.3) is converted to the following simplified non-linear optimization problem

$$\max_{\mathbf{x}} \mathbf{x}^T A \mathbf{x} \quad (5.4)$$

subject to

$$|P(f)| = |\mathbf{x}^T \mathbf{W}(f)| \leq \sqrt{S(f)}, \quad \forall f, \quad (5.5)$$

where

$$A = \int_{\Omega_p} \Phi(f) df \quad (5.6)$$

is a positive definite matrix. Note that (5.4) and (5.5) define a constrained non-linear optimization problem, where the objective function is quadratic but the constraint in general is non-linear and non-convex.

Next, the magnitude inequality constraint (5.5) is linearized using the rotation theorem [21] as

$$\max_{0 \leq \theta < 2\pi} \Re\{\mathbf{x}^T \mathbf{W}(f) e^{j\theta}\} \leq \sqrt{S(f)}. \quad (5.7)$$

Note that the number of constraints in (5.7) increases due to the introduction of the new parameter θ . As can be seen that (5.4) and (5.7) specify a semi-infinite quadratic programming problem where the number of variables to be optimized is finite but the number of inequality constraints is infinite (depend on θ and f). In this chapter, θ and f were discretized and the optimization problem (5.4) and (5.7) is solved using a simple MATLAB program.

5.3 Waveform Orthogonalization

Löwdin orthogonalization method is applied to the optimal waveform designed from the solution of the semi-infinite quadratic programming problem described in Section 5.2. Consider a finite set of $2M + 1$ equally spaced translates (shifts) of the optimal waveform $p(t)$ denoted as

$$\{p(\cdot - kT)\}_{k=-M}^M.$$

Then, Löwdin orthogonalization method [56, 57] is applied to transform this set of shifts to an orthonormal basis

$$\{p_k^o\}_{k=-M}^M$$

such that the average energy distortion

$$\sum_{k=-M}^M \|p(\cdot - kT) - p_k^o\|_2^2$$

is minimized. The Löwdin transform is given by [56, 85]

$$p_n^o = \sum_{m=-M}^M [G^{-\frac{1}{2}}]_{nm} p(\cdot - mT) \quad (5.8)$$

where $G^{-\frac{1}{2}}$ denotes the inverse square root of $2M + 1 \times 2M + 1$ dimensional matrix with the elements given by

$$[G]_{nm} = \int_{-\infty}^{+\infty} p(t - mT) p^*(t - nT) dt. \quad (5.9)$$

The Löwdin orthogonalization method can be well approximated using the Zak transform [86] allowing for an efficient implementation using the discrete Fourier transform [58, 84, 87].

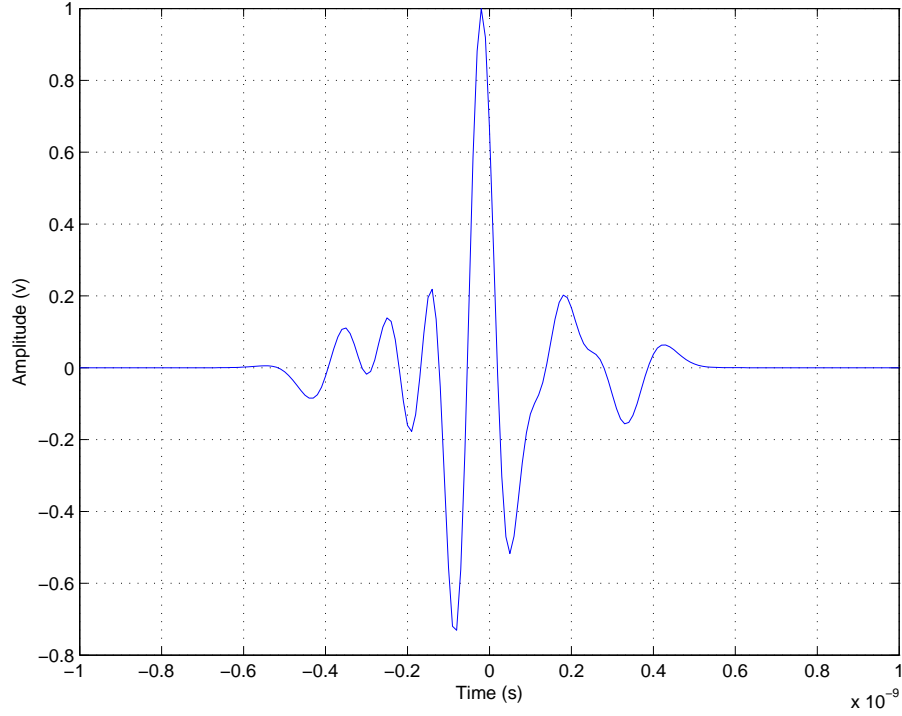


Figure 5.1: The optimal waveform $p(t)$ designed in the first step. Löwdin orthogonalization method will then transform $p(t)$ to an orthogonal set of pulses.

5.4 Design Examples

To design optimal $p(t)$ with $\lambda = 1.25 \times 10^{-10}$ and $N = 20$, optimization problem (5.4) and (5.7) is solved. The plots of $p(t)$ and its PSD are plotted in Fig. 5.1 and 5.2 respectively. Power utilization efficiency, η defined in 4.14 for optimal $p(t)$ is 0.8033.

Fig. 5.3, 5.4 and 5.5 show the set of orthogonal waveforms generated using the Löwdin transform (5.8) for $M = 2$, $M = 3$ and $M = 7$ respectively. It is important to note that the waveforms generated occupy 3 times of the original waveform duration independent of the number of waveforms generated. We can see from Fig. 5.1 that the duration of optimal waveform $p(t)$ is approximately 2ns and therefore total duration of generated waveforms is approximately 6ns which can be clearly seen from Fig. 5.3, 5.4 and 5.5. This is important as it is able to design many overlapping orthogonal signals.

Since Löwdin orthogonalization method minimizes average energy distortion with respect to the optimal $p(t)$, the waveforms generated by the orthogonalization method may not satisfy the FCC spectral mask. Thereafter the generated waveforms were adjusted to ensure their PSD are equal or below the spectral mask. This adjustment is

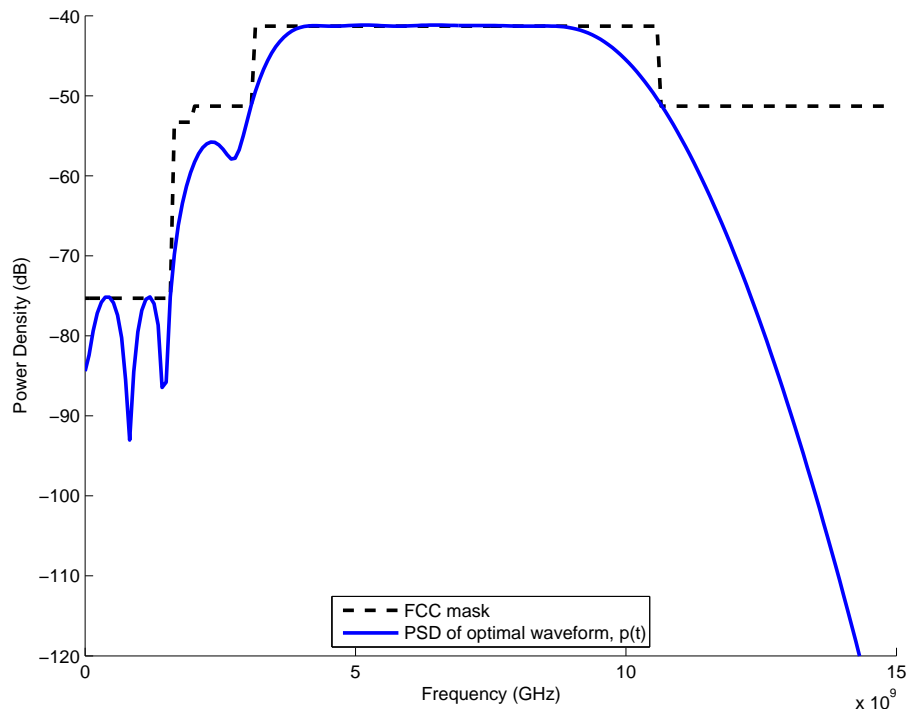


Figure 5.2: The PSD of the optimal waveform $p(t)$ designed in the first step, $\eta = 0.8033$.

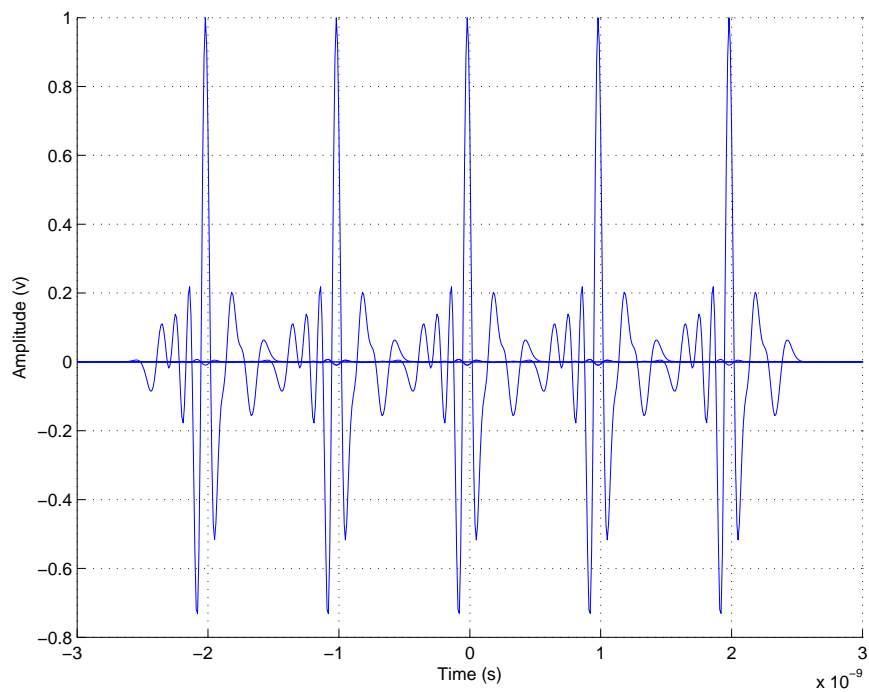


Figure 5.3: The set of 5 overlapping orthogonal waveforms, $\{p_k^o\}_{k=-2}^2$ generated using Löwdin's method based on optimal $p(t)$.

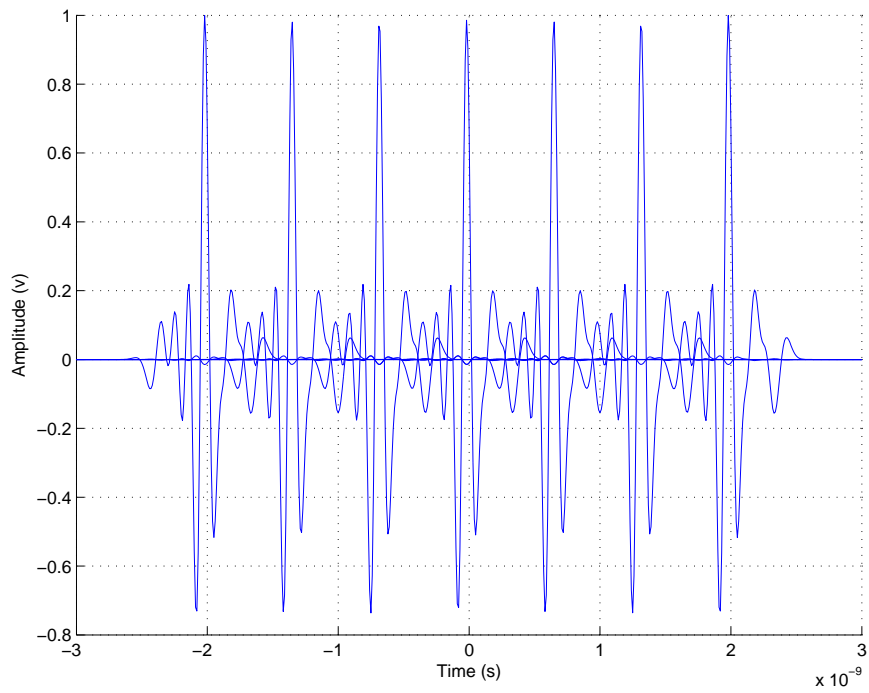


Figure 5.4: The set of 7 overlapping orthogonal waveforms, $\{p_k^o\}_{k=-3}^3$ generated using Löwdin's method based on optimal $p(t)$.

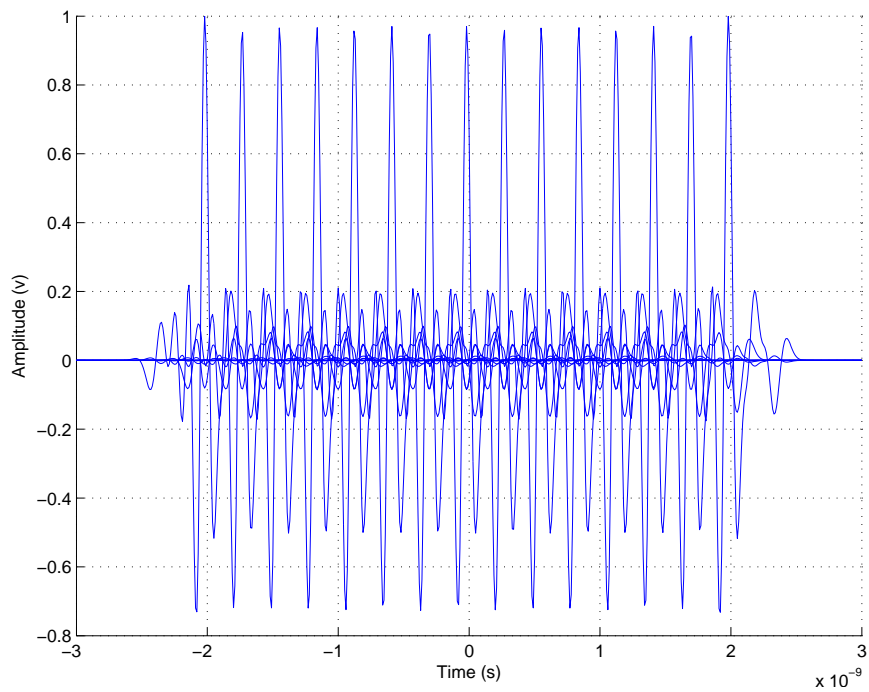


Figure 5.5: The set of 15 overlapping orthogonal waveforms, $\{p_k^o\}_{k=-7}^7$ generated using Löwdin's method based on optimal $p(t)$.

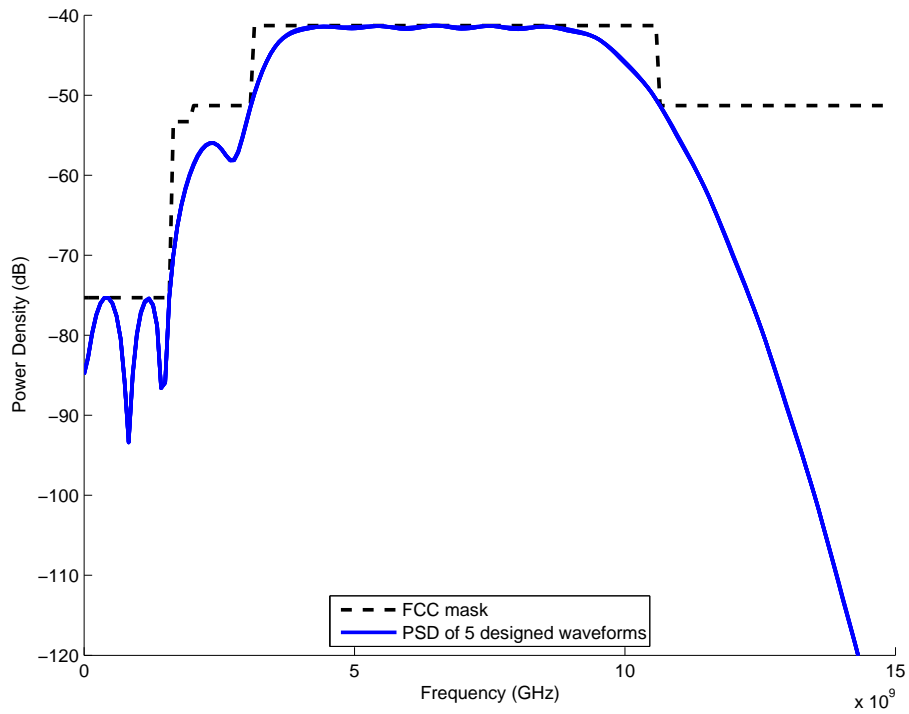


Figure 5.6: The PSD of the 5 orthogonal waveforms plotted in Fig. 5.3 ($M = 2$).

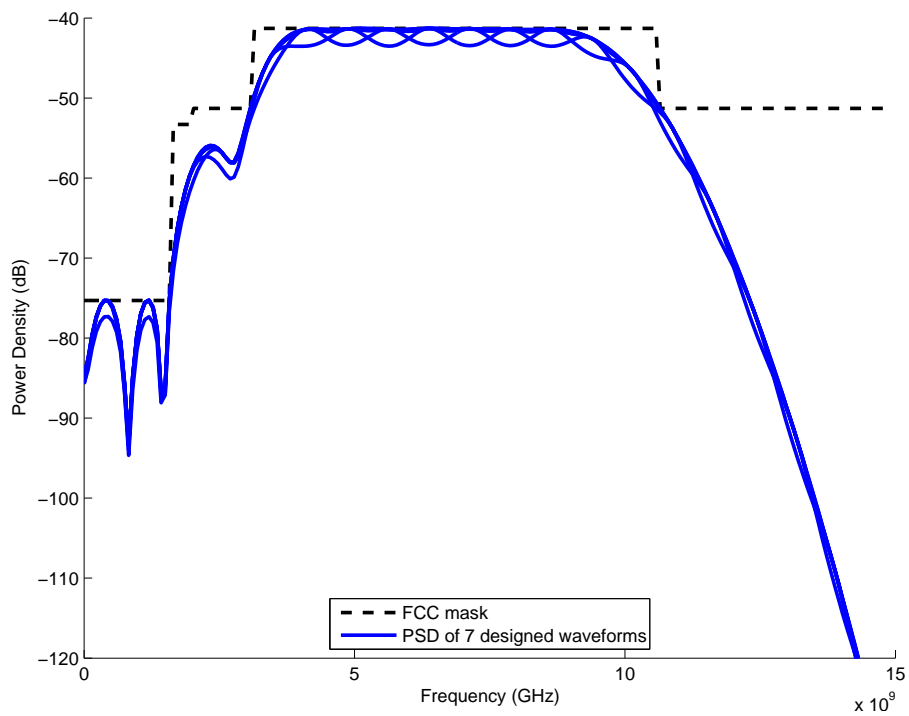


Figure 5.7: The PSD of the 7 orthogonal waveforms plotted in Fig. 5.4 ($M = 3$).

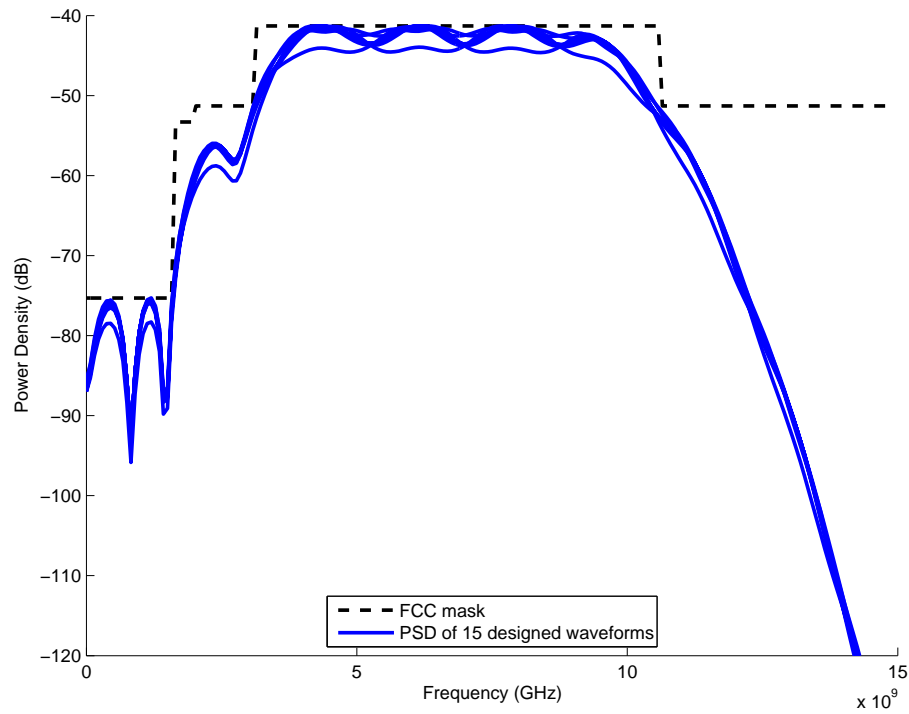


Figure 5.8: The PSD of the 15 orthogonal waveforms plotted in Fig. 5.5 ($M = 7$).

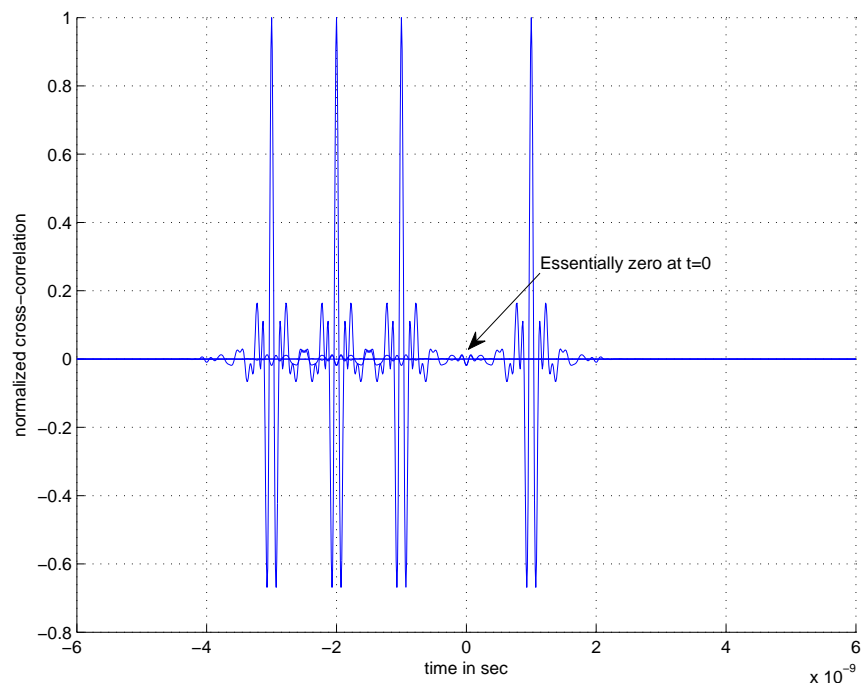


Figure 5.9: The crosscorrelations of one of the generated waveform, p_{-1}^o with the other 4 waveforms in the set ($M = 2$). It is essentially zero at $t = 0$.

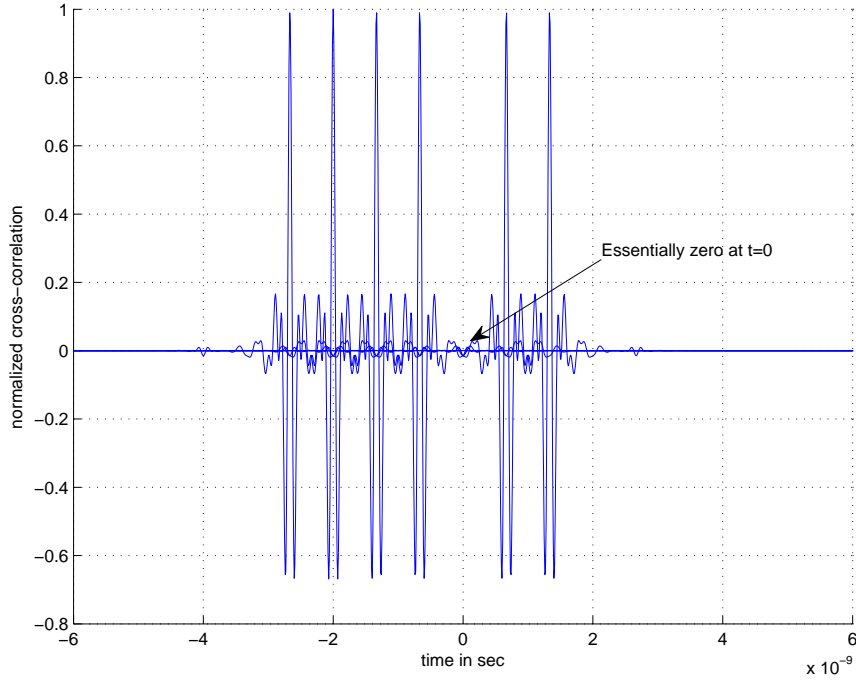


Figure 5.10: The crosscorrelations of one of the generated waveform, p_{-1}^o with the other 6 waveforms in the set ($M = 3$). It is essentially zero at $t = 0$.

Table 5.1: Spectral Efficiency, η for $M = 2$

Waveform Index	1	2	3	4	5
η	0.7641	0.7501	0.7500	0.7501	0.7641

Table 5.2: Spectral Efficiency, η for $M = 3$

Waveform Index	1	2	3	4	5	6	7
η	0.6204	0.7775	0.7589	0.7569	0.7536	0.7776	0.6103

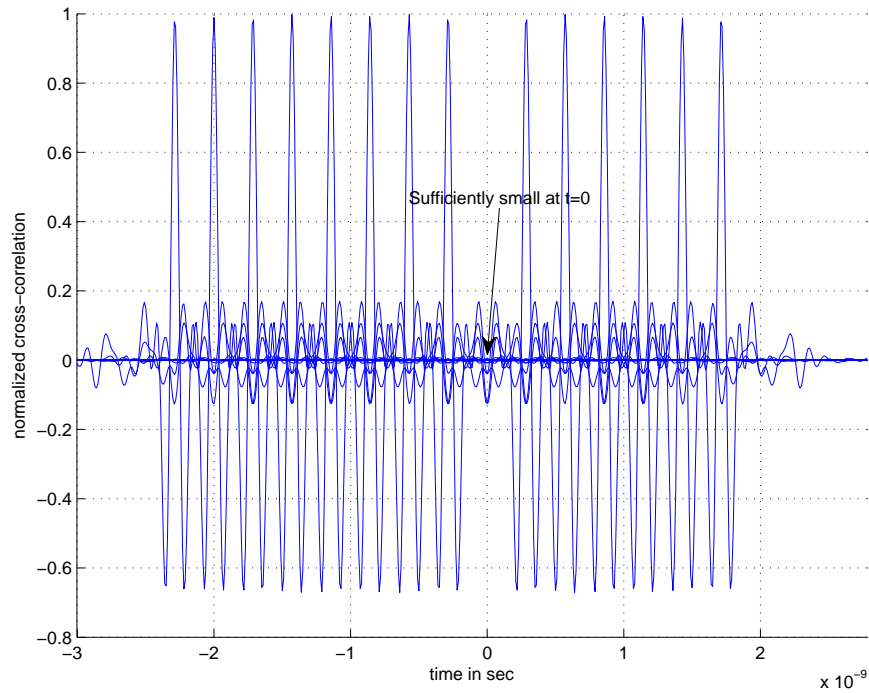


Figure 5.11: The crosscorrelations of one of the generated waveform, p_{-1}^o with the other 14 waveforms in the set ($M = 7$). It is sufficiently small at $t = 0$.

Table 5.3: Spectral Efficiency, η for $M = 7$

Waveform Index	1	2	3	4	5	6	7	8
η	0.5221	0.6822	0.6047	0.6249	0.6360	0.6367	0.6384	0.6387
Waveform Index	9	10	11	12	13	14	15	
η	0.6383	0.6373	0.6300	0.6189	0.5989	0.6761	0.5542	

similar to the approaches in [58, 87]. The PSD of the orthogonal waveforms for $M = 2$, $M = 3$ and $M = 7$ are plotted in Fig. 5.6, 5.7 and 5.8 respectively. It can be seen clearly that the waveforms occupy the same spectrum. The spectral efficiency, η for $M = 2$, $M = 3$ and $M = 7$ are tabulated in Table 5.1, 5.2 and 5.3 respectively.

Fig. 5.9, 5.10 and 5.11 shows the crosscorrelations of a waveform with the rest of the waveforms in the set for $M = 2$, $M = 3$ and $M = 7$ respectively.

5.5 Chapter Summary

In this chapter, orthogonal Hermite Rodriguez basis functions have been applied to design a basic pulse for UWB systems. The pulse design problem was formulated as direct maximization of quadratic function of filter coefficients, thus avoiding the spectral factorization step normally used in convex optimization approaches. The rotation theorem has been used to convert the nonlinear and nonconvex constraints into linear constraints with the addition of a new parameter. The designed optimal pulse is then used in Löwdin orthogonalization method to generate a set of orthogonal and overlapping spectrally efficient waveforms. Design examples have demonstrated that the proposed approach is able to produce many overlapping orthogonal waveforms which efficiently utilize a given spectral mask.

Chapter 6

Conclusions and Future Work

Driven by increasing user demand for high speed communication services, one of the major focuses of current research is to develop new schemes which can be used to maximize multiple access communication system capacity and increase bandwidth utilization efficiency. In the thesis, the design of analogue waveform sets with time and frequency domains constraints for multiuser and UWB communications has been investigated. First, analogue waveform sets with correlations and spectral constraints have been designed based on Hermite Rodriguez basis functions. Next, spectrally efficient waveform sets with orthogonality constraint for UWB communications have been designed. Then, the spectral efficiency of the UWB waveform has been further improved by incorporating PSD of a basic pulse into the design. Finally, Löwdin orthogonalization method has been applied to generate a sufficient number of overlapping orthogonal UWB waveforms with high spectral efficiency.

6.1 Concluding Remarks

In the thesis, the design of analogue waveform sets based on orthogonal Hermite-Rodriguez basis functions has been investigated. New design approaches have been developed to maximize multiple access capacity and bandwidth efficiency for multiuser aided UWB communications. The design was formulated into optimization problems and various optimization methods were used throughout the thesis. The performance of the designed waveforms has been evaluated and compared with recent results in the literature.

In Chapter 2, the waveform set design problem has been formulated as min-max optimization problem to minimize stopband energy while satisfying spectral and correlations constraints. The optimization problem has been solved sequentially with crosscorrelations constraints treated as linear constraints. The numerical studies presented have shown that the low order Hermite Rodriguez filter can be obtained due to its suitability to design signals with finite time support.

In Chapter 3, the design of a set of spectrally efficient orthogonal waveforms has been formulated as a quadratic semi-infinite programming problem with spectral and crosscorrelation constraints. The energy of each waveform in the set has been directly maximized. The nonlinear spectral and crosscorrelation constraints have been linearized respectively by using the rotation theorem and solving the quadratic semi-infinite program sequentially. The proposed approach is flexible to incorporate additional constraints to suppress narrowband interferences. Illustrative numerical examples have shown that the proposed approach was able to produce a relatively high number of waveforms with high spectral efficiency.

Then in Chapter 4, the design of spectrally efficient UWB waveform has been further improved by incorporating power spectral density of a basic pulse into a semi-infinite quadratic optimization problem. The resulting nonlinear piece-wise continuous bounds have been converted into linear constraints using the rotation theorem. The design problem has been formulated in terms of filter coefficients to directly maximize the transmit power which then eliminated the spectral factorization step commonly used in FIR filter based semi-definite programming problem using autocorrelation functions. It has been demonstrated through numerical examples that the proposed approach is relatively easy to implement and achieves high spectral utilization even when a suboptimal basic pulse is used.

Finally in Chapter 5, Löwdin orthogonalization method has been used to transform a finite set of equally spaced shifts of an optimal waveform to an orthogonal set of spectrally efficient waveforms. First, the optimal waveform has been designed via a semi-infinite quadratic programming problem formulated as direct maximization of quadratic function of filter coefficients. Then, Löwdin orthogonalization method has been applied to the optimal waveform. Through numerical examples, it has been shown that the generated waveforms occupy 3 times of the original waveform duration independent of the number of waveforms in the set, thus demonstrated its ability to produce many *overlapping* orthogonal waveforms while efficiently utilize a given spectral mask.

6.2 Future Works

In this thesis, a few advanced signal processing and optimization approaches for waveform set design based on orthogonal Hermite Rodriguez basis functions have been developed. It would be interesting to apply the waveforms generated in this thesis to a UWB communication system model and assess their bit error rate (BER) performance. Throughout the thesis, various types of constraints such as autocorrelations and cross-correlations of a set of waveforms have been formulated and included into various optimization problems. To extend this research work further, other practical constraints such as synchronization sensitivity, channel uncertainty and analog/digital conversion (ADC) requirements could also be incorporated to design more robust waveforms.

Application of the Hermite Rodriguez basis functions in engineering problems could also be explored further. In this thesis, the filter order N and the scaling factor of Hermite Rodriguez functions λ have been chosen manually and heuristically based on simulation experience. It is interesting to research for a systematic approach to find the optimal filter order and optimal scaling factor for a given filter design problem. This approach could possibly be based on a well formulated optimization problem. Furthermore, the issue of finding the λ that minimizes the mean squared error (MSE) of an N^{th} -order Hermite series expansion is still open [61].

Another obvious research direction to pursue is hardware implementation of Hermite Rodriguez filter suitable for UWB communication systems. We have actually made an attempt to implement the Hermite Rodriguez basis functions on an FPGA board but this is a prohibitively resource-constrained approach [88].

Appendix A

Matlab code

`runsetup.m` and `runopt.m` are the main files used to implement the design of spectrally efficient UWB waveforms in Chapter 3.

```
1 % file name: runsetup.m
2 % 31/1/08 - Added comment.
3 %           - Added sqrt of the mask in order to be absolutely correct with
4 %           the paper. This is because the FCC mask is in PSD.
5 %=====
6
7 close all;
8 clear all;
9 pause;
10
11 M = 8;           % No of filter
12 N = 21;         % Order of the filter
13 max_flag = 1;   % 1 = maximize, else minimize.
14
15 % LFM Parameters
16 B = 1e18;
17 L = 2/16e9;
18 K = 0;
19
20 % Simulation Parameters
21 f_start = 0;
```

```

22 f_end = 16e9;
23 f_low = 3.1e9;
24 f_high = 10.6e9;
25 f_point = 100;
26 f_step = (f_end - f_start)/f_point;
27
28 w_point = 16;
29 w_step = 2*pi/w_point;
30
31 maxIteration = 50000;
32 maxFuncEval = 300000;
33
34 %=====
35 % Compute the Fourier Transform of LFM Hermite Rodriguez ft
36 f = f_start:f_step:f_end-f_step;
37 for n=1:N,
38     Z(n,:) = FTLMHR(B,L,K,n-1,f); %Fourier Transform of LFM HR ft.
39 end
40
41 %=====
42 % Compute the LFM Hermite Rodriguez ft
43 Tmax = 6*L;
44 dt = Tmax/100;
45 t = -Tmax:dt:Tmax;
46
47 tmp = exp(j*(2*pi*K*t+pi*B*t.*t))*exp(j*(2*pi*K*t+pi*B*t.*t))';
48 for n=1:N,
49     znt(n,:) = HR(L,n-1,t); %Hermite Rodriguez ft
50 end
51 xcorr = tmp*znt*znt'*dt;
52 clear tmp;
53
54 %=====
55 % Compute Q to be used in objective function.
56 % The objective is to minimize the power over the STOP band.
57 Δ = 0;
58 if max_flag == 1

```



```
59     Q = calQmax(Z, f_step, f_low, f_high) + Δ*eye(N);
60 else
61     Q = calQ(Z, f_step, f_low, f_high) + Δ*eye(N);
62 end
63
64 %=====
65 % Creating necessary inputs for constraint function.
66 [maskup, maskdn, maskst]=mask(f); % maskup is (1 x Nf)
67 tmp=maskst'*ones(1, w_point); % tmp is (Nf x 1)(1 x Nt) = (Nf x Nt)
68 maskst=sqrt(tmp(:)); % vectorize the matrix
69 % (IMPORTANT! column by column)
70 tmp=maskdn'*ones(1, w_point);
71 maskdn=sqrt(tmp(:));
72 clear tmp;
73
74 w = 0:w_step:2*pi-w_step;
75 for n=1:N,
76     for i=1:length(f),
77         tmp(n, i, :) = real(Z(n, i)*exp(j*w));
78     end
79     aup(n, :) = tmp(n, :);
80 end
81
82 for i=1:length(f),
83     adn(i, :, :) = real(Z(:, i))*real(Z(:, i))' + imag(Z(:, i))*imag(Z(:, i))';
84 end
85
86 save all.mat;
```

```
1 % file name: runopt.m
2 %=====
3
4 close all;
5 clear all;
6 pause;
7 load all.mat;
8
9 %=====
10 x0 = 10*ones(N,1); % initial point
11 %load xdata6.mat;
12 %load xdata16.mat;
13 xx = zeros(M,N);
14
15 %=====
16 %Parameters relevant to quadprog
17 Hf = 2*Q;
18 ff = zeros(N,1);
19 Aqp = aup';
20 bqp = maskst;
21 Aeq = xx*xcorr;
22 beq = zeros(M,1);
23
24 %=====
25 for ii = 1:M,
26
27     repeat = 'Y';
28     while(strcmp(repeat,'Y') || strcmp(repeat,'y'))
29
30         if max_flag == 1
31             disp('Maximizing ...')
32             x = quadprog(-Hf, ff, Aqp, bqp, Aeq, beq);
33         else
34             disp('Minimizing ...')
35             x = quadprog(Hf, ff, Aqp, bqp, Aeq, beq);
36         end
37     end
```

```
38     %x0 = 10*ones(N,1);    %initialize randomly OR
39     x0 = x;                %use previous solution
40     xx(ii,:) = x;
41     Aeq = xx*xcorr;
42
43     %repeat = input('Do you want to repeat? Y/N: ', 's');
44     repeat = 'n';
45
46     end
47
48 end
49
50 disp('Run printing script ...')
51 printing
```


Bibliography

- [1] A. Kapur, M. K. Varanasi, and C. T. Mullis, “Signal design for bandwidth efficient multiple access under asymptotic effective energy constraints,” *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 2837–2847, August 2007.
- [2] H. H. Chen, C. H. Cai, C. Y. Chao, and Y. H. Lin, “Synthesis of pulse shaping waveforms for spectral efficient digital modulations: some practical approaches,” *European Trans. Telecommunications*, vol. 17, pp. 99–110, 2006.
- [3] T. Wang, W. Y. Yan, and Z. Zang, “Optimal design of spectrum constrained analog signal sets with correlation analysis,” *IEEE Trans. Circuits and Systems I*, August 2006.
- [4] G. Chandran and J. S. Jaffe, “Signal set design with constrained amplitude spectrum and specified time-bandwidth product,” *IEEE Trans. Commun.*, vol. 44, no. 6, pp. 725–732, 1996.
- [5] X. Wu, Z. Tian, T. Davidson, and G. B. Giannakis, “Optimal waveform design for UWB radios,” *IEEE Trans. Signal Process.*, vol. 54, (6), pp. 2009–2021, 2006.
- [6] T. N. Davidson, Z. Q. Luo, and K. M. Wong, “Design of orthogonal pulse shapes for communications via semidefinite programming,” *IEEE Trans. on Signal Processing*, vol. 48, no. 5, pp. 1433–1445, 2000.
- [7] W. Lee, S. Kunaruttanapruk, and S. Jitapunkul, “Optimum pulse shape design for UWB systems with timing jitter,” *IEICE Trans. Commun.*, vol. E91-B, no. 3, March 2008.
- [8] M. Wang, S. Yanga, and S. Wua, “A GA-based UWB pulse waveform design method,” *Digital Signal Processing*, vol. 18, no. 1, pp. 65–74, 2008.

-
- [9] L. Hanzo, "Interference-mitigating waveform design for next-generation wireless systems," in *IET Forum on Radar, Sonar and Navigation Network*, November 2006.
- [10] S. K. Jayaweera, "Signal design for non-coherent UWB with PPM modulation in fading channels," in *Proc. IEEE Wireless Communications and Networking Conference*, vol. 2, pp. 770-775, 2005.
- [11] F. Ramirez-Mireles, "Signal design for Ultra-Wide-Band communications in dense multipath," *IEEE Trans. Vehicular Tech.*, vol. 51, no. 6, pp. 1517-1521, 2002.
- [12] W. Gao, J. H. Cho, and J. S. Lehnert, "Chip waveform design for DS/SSMA systems with aperiodic random spreading sequences," *IEEE Trans. Wireless Commun.*, vol. 1, pp. 37-45, January 2002.
- [13] J. H. Cho and J. S. Lehnert, "An optimal signal design for band-limited asynchronous DS-CDMA communications," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1172-1185, May 2002.
- [14] C. Ding and J. Yin, "Signal sets from functions with optimum nonlinearity," *IEEE Trans. Commun.*, vol. 55, no. 5, pp. 936-940, May 2007.
- [15] S. Verdu, *Multiuser Detection*. Cambridge University Press, 1998.
- [16] E. A. Lee and D. G. Messerschmitt, *Digital Communication (2nd Ed)*. Boston: Kluwer Academic Publishers, 1994.
- [17] H. Sari, "A generalization of multidimensional modulation," in *Proc of IEEE ICC*, pp. 683-687, 1995.
- [18] J. Ren and T. Li, "Design of pseudo-random spreading sequences for CDMA systems," in *Proc. Global Telecommunications Conference (GLOBECOM)*, vol. 2, pp. 775-779, 2004.
- [19] H. H. Nguyen, "An improved design of chip waveforms for band-limited DS-CDMA systems," *IEEE Trans. Vehicular Tech.*, vol. 53, no. 5, pp. 1379-1386, September 2004.
- [20] L. J. Karam and J. H. McClellan, "Complex chebyshev approximation for FIR filter design," *IEEE Trans. Circuits Syst. II*, vol. 42, no. 3, pp. 207-216, 1995.

-
- [21] T. W. Parks and C. S. Burrus, *Digital Filter Design*. New York: Wiley, 1987.
- [22] Z. Zang, A. Cantoni, and K. L. Teo, "Continuous-time envelope constrained filter design via Laguerre filters and hinf optimization methods," *IEEE Trans. Signal Processing*, vol. 46, pp. 2601–2610, October 1998.
- [23] B. Vo, Z. Zang, A. Cantoni, and K. L. Teo, "Continuous-time envelope constrained filter design via orthonormal filters," in *IEE Proc.: Vision, Image, Signal Processing*, vol. 142, pp. 389–394, 1995.
- [24] M. A. Yusoff, Z. Zang, and S. S. Abeysekera, "UWB pulse shaping filter design using hermite rodriguez functions," in *Proc. 5th IASTED Asian Conf on Comm Systems and Networks*, Langkawi, Malaysia, pp. 158–162, April 2008.
- [25] M. A. Yusoff, "Application of hermite-rodriguez functions to pulse shaping analog filter design," *Inter. Journal of Computer Science and Engineering*, vol. 2, no. 2, (18), 2008.
- [26] *Federal Communications Commission*, Revision of Part 15 of the Commission's Rules Regarding Ultra-Wideband Transmission Systems (ET Docket), 2002.
- [27] J. A. N. da Silva and M. L. R. de Campos, "Spectrally efficient UWB pulse shaping with application on orthogonal PSM," *IEEE Trans. on Communications*, vol. 55, no. 2, pp. 313–322, 2007.
- [28] Y. Wang, X. Dong, and I. J. Fair, "Spectrum shaping and NBI suppression in UWB communications," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1944–1952, May 2007.
- [29] X. L. Wu, X. J. Sha, and N. T. Zhang, "Modified hermite function based pulse shaping algorithm for ultra-wideband communications," in *IEEE Radio and Wireless Symposium*, pp. 395–398, Jan. 2007.
- [30] B. Parr, B. Cho, K. Wallace, and Z. Ding, "A novel ultrawideband pulse design algorithm," *IEEE Commun. Lett.*, vol. 7, (5), pp. 219–221, 2003.
- [31] X. L. Luo, L. Q. Yang, and G. B. Giannakis, "Designing optimal pulse-shapers for ultra-wideband radios," in *Proc. IEEE Conf. on Ultra Wideband Systems and Tech*, pp. 349–353, 2003.

-
- [32] X. Wu, Z. Tian, T. N. Davidson, and G. B. Giannakis, "Optimal waveform design for UWB radios," in *Proc. Int Conf. on Acoustics Speech and Signal Processing*, 2004, pp. 521-524.
- [33] J. Bellorado, S. Ghassemzadeh, L. Greenstein, T. Sveinsson, and V. Tarokh, "Co-existence of ultra-wideband systems with IEEE 802.11a wireless LANs," in *Proc. IEEE GLOBECOM*, 2003, vol. 1, pp. 410-414.
- [34] Z. Luo, H. Gao, Y. Liu, and J. Gao, "A new UWB pulse design method for narrowband interference suppression," in *Proc. IEEE GLOBECOM*, 2004, vol. 6, pp. 3488-3492.
- [35] L. Xu and C. Wen, "A new UWB pulse design method for multiple narrow-band interference and wide-band interference mitigation," in *Fourth International Conference on Communications and Networking, ChinaCOM*, 2009, pp. 1-5.
- [36] W. Changguo, J. Gangxing, and Z. Hongbo, "SSA realization for spectrum shaping and NBI suppression in cognitive UWB radios," in *International Conference on Microwave and Millimeter Wave Technology, ICMMT*, 2008, pp. 1454-1457.
- [37] X. Luo, L. Yang, and G. B. Giannakis, "Designing optimal pulse-shapers for UWB radios," *J. Commun. Netw.*, vol. 5, no. 4, pp. 344-353, December 2003.
- [38] C. R. Berger, M. Eisenacher, S. Zhou, and F. K. Jondral, "Improving the UWB pulseshaper design using nonconstant upper bounds in semidefinite programming," *IEEE J. Selected Topics in Signal Processing*, vol. 1, no. 3, pp. 396-404, October 2007.
- [39] S.-P. Wu, S. Boyd, and L. Vandenberghe, "FIR filter design via semidefinite programming and spectral factorization," in *Proc. 35th IEEE Conf. Decision and Control*, Dec 1996, vol. 1, pp. 271-276.
- [40] T. N. Davidson, Z.-Q. Luo, and J. F. Sturm, "Linear matrix inequality formulation of spectral mask constraints with applications to FIR filter design," *IEEE Trans. Signal Process.*, vol. 50, no. 11, pp. 2702-2715, Nov. 2002.
- [41] B. Hu and N. C. Beaulieu, "Pulse shapes for ultrawideband communication systems," *IEEE Transactions on Wireless Communications*, vol. 4, no. 4, pp. 1789-1797, July 2005.

- [42] N. C. Beaulieu and B. Hu, "A pulse design paradigm for ultra-wideband communication systems," *IEEE Transactions on Wireless Communications*, vol. 5, no. 6, pp. 1274–1278, June 2006.
- [43] X. Liu, A. B. Premkumar, and A. S. Madhukumar, "Pulse shaping functions for UWB systems," *IEEE Transactions on Wireless Communications*, vol. 7, no. 5, pp. 1512–1516, May 2008.
- [44] M. G. D. Benedetto and B. R. Vojcic, "Ultra wide band wireless communications: A tutorial," *Journal of Communications and Networks*, vol. 5, no. 4, pp. 290–302, Dec 2003.
- [45] L. Emmanuel and X. N. Fernando, "Wavelet-based spectral shaping of UWB radio signal for multisystem coexistence," *Computers and Electrical Engineering*, vol. 36, no. 2, pp. 261 – 268, 2010, wireless ad hoc, Sensor and Mesh Networks. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0045790609000433>
- [46] Z. Bai, J. Liu, and H. H. Chen, "Design of ultra-wideband pulses based on spectrum shifted gaussian waveforms," *IET Communications*, vol. 7, no. 6, pp. 512–520, April 2013.
- [47] M. B. Menon, A. Gopakumar, and N. V. Iqbal, "A hybrid approach for UWB pulse shaping," in *Electronics and Communication Systems (ICECS), 2015 2nd International Conference on*, Feb 2015, pp. 373–377.
- [48] I. E. Lager, A. T. de Hoop, and T. Kikkawa, "Model pulses in the TD design of UWB wireless signal transfer systems," in *The 8th European Conference on Antennas and Propagation (EuCAP 2014)*, April 2014, pp. 3080–3084.
- [49] N. Rajesh and S. Pavan, "Programmable analog pulse shaping for ultra-wideband applications," in *2015 IEEE International Symposium on Circuits and Systems (ISCAS)*, May 2015, pp. 461–464.
- [50] V. V. Kumar, A. V., and M. Meenakshi, "Wavelet-based gaussian impulse generation and optimization for UWB communication," in *2015 Fifth International Conference on Advances in Computing and Communications (ICACC)*, Sept 2015, pp. 214–218.

-
- [51] L. Michael, M. Ghavami, and R. Kohno, "Multiple pulse generator for ultra-wideband communication using hermite polynomial based orthogonal pulses," in *Proc. IEEE Conf. Ultra Wideband Systems and Technologies*, 2002, pp. 47-51.
- [52] M. K. Simon, S. M. Hinedi, and W. C. Lindsey, *Digital Communication Techniques: Signal Design and Detection*. Prentice Hall, 1994.
- [53] Z. Tian, T. N. Davidson, X. Luo, X. Wu, and G. B. Giannakis, *Ultra Wideband Wireless Communication: Ultra Wideband Pulse Shaper Design*. Wiley, 2006.
- [54] M. Z. Win and R. A. Scholtz, "Impulse radio: how it works," *IEEE Commun. Letter*, vol. 2, no. 2, pp. 36-38, Feb. 1998.
- [55] I. Dotlic and R. Kohno, "Design of the family of orthogonal and spectrally efficient UWB waveforms," *IEEE J. Selected Topics in Signal Processing*, vol. 1, pp. 21-30, 2007.
- [56] P. O. Löwdin, "On the nonorthogonality problem connected with the use of atomic wave functions in the theory of molecules and crystals," *J. Chem. Phys.*, vol. 18, pp. 367-370, 1950.
- [57] ———, "On the nonorthogonality problem," *Advances in Quantum Chemistry*, vol. 5, pp. 185-199, 1970.
- [58] P. Walk and P. Jung, "Approximation of Löwdin orthogonalization to a spectrally efficient orthogonal overlapping PPM design for UWB impulse radio," *Signal Processing*, vol. 92, pp. 649-666, 2012.
- [59] N. C. Beaulieu and D. J. Young, "Statistical and intuitive nonlinear signal processing designs for multiuser interference mitigation in UWB systems," in *Information Science, Signal Processing and their Applications (ISSPA), 2012 11th International Conference on*, July 2012, pp. 243-252.
- [60] S. Gao, Z. Bai, P. Dong, and K. Kwak, "Adaptive non-linear chirp UWB waveform design for multiple narrowband interferences suppression," in *Communications and Information Technologies (ISCIT), 2014 14th International Symposium on*, Sept 2014, pp. 508-512.

- [61] L. R. Lo Conte, R. Merletti, and G. V. Sandri, "Hermite expansions of compact support waveforms: applications to myoelectric signals," *IEEE Trans. on Biomedical Engineering*, vol. 41, (12), pp. 1147–1159, 1994.
- [62] Z. Zang and S. Nordholm, "Design of ODMA digital waveforms using non-convex optimization methods," *Annals of Operations Research*, vol. 133, pp. 319–330, 2005.
- [63] —, "Orthogonal digital waveform set for multidimensional signaling and multiuser communications with matched filter receivers," in *10th Asia Pacific Conf. on Commun. and 5th Int. Symposium on Multi-Dimensional Mobile Commun.*, China, 2004.
- [64] A. F. Shalash and K. K. Parhi, "Multidimensional carrierless AM/PM systems for digital subscriber loop," *IEEE Trans. Commun.*, vol. 47, no. 11, pp. 1655–1667, 1999.
- [65] J. J. Werner, "Tutorial on carrierless AM/PM, part I and II," *Contribution to ANSI X3T9.5 TP/PMD Working Group*, 1992 and 1993.
- [66] S. S. Abeysekera and Z. Zang, "Wideband sonar waveform design using linear FM signals and hermite-rodriguez functions," in *Proc. IEEE OCEANS'06 Asia Pacific*, Singapore, May, 2006.
- [67] M. R. Mackenzie and A. K. Tieu, "Hermite neural network correlation and application," *IEEE Trans. on Signal Processing*, vol. 51, (12), pp. 3210–3219, 2003.
- [68] —, "Gaussian filters and filter synthesis using a hermite/laguerre neural network," *IEEE Trans. on Neural Networks*, vol. 15, (1), pp. 206–214, January 2004.
- [69] J. A. Ney Da Silva and M. L. R. De Campos, "Spectrally efficient UWB pulse shaping with application on orthogonal PSM," *IEEE Trans. on Communications*, vol. 55, (2), pp. 313–322, February 2007.
- [70] M. Ghavami, L. B. Michael, and R. Kohno, "Hermite function based orthogonal pulses for ultra wideband communications," in *Proc. Wireless Personal Multimedia Communications (WPMC'01)*, Alborg, Denmark, Sept 2001, pp. 437-440.
- [71] J. A. Ney Da Silva and M. L. R. De Campos, "Orthogonal pulse shape modulation for impulse radio," in *Proc. IEEE International Telecommunications Symposium (ITS'02)*, Natal, Brazil, Sept 2002, pp. 911-921.

-
- [72] ———, “Performance comparison of binary and quaternary UWB modulation schemes,” in *Proc. IEEE Global Telecommunications Conference (GLOBECOM'03)*, San Francisco, USA, December 2003.
- [73] A. C. Gordillo and R. Kohno, “Design of spectrally efficient hermite pulses for PSM UWB communications,” *IEICE Trans. Fundamentals*, vol. E91, (8), pp. 2016–2024, August 2008.
- [74] M. A. Yusoff, Z. Zang, and S. S. Abeysekera, “Spectrally efficient waveform set design for UWB wireless communications,” in *Proc 7th Inter Conf. on Information Comm. and Signal Processing*, Macau, Dec 2009.
- [75] R. Brocato, E. Heller, J. Wendt, J. Blaich, G. Wouters, E. Gurule, G. Omdahl, and D. Palmer, “UWB communication using SAW correlators,” in *Proc. IEEE RAWCON*, Sept 2004, pp. 267-270.
- [76] A. Springer, W. Gugler, M. Huemer, R. Koller, and R. Weigel, “A wireless spread-spectrum communication system using SAW chirped delay lines,” *IEEE Trans. Microw. Theory Tech*, vol. 49, pp. 754–760, April 2001.
- [77] S. Nordebo and Z. Zang, “Semi-infinite linear programming: a unified approach to digital filter design with time and frequency domain specifications,” *IEEE Transactions on Circuit and Systems II*, vol. 46, (6), pp. 765–775, 1999.
- [78] S. Nordebo, Z. Zang, and I. Claesson, “A semi-infinite quadratic programming algorithm with applications to array pattern synthesis,” *IEEE Trans. Circuits and Systems II: Analog and Digital Signal Processing*, vol. 48, (3), pp. 225–232, 2001.
- [79] D. G. Luenberger, *Linear and Nonlinear Programming*. Massachusetts: Addison-Wesley, 1984.
- [80] L. Q. Yang and G. B. Giannakis, “Ultra-wideband communications: An idea whose time has come,” *IEEE Signal Process. Mag*, vol. 21, no. 6, pp. 26–54, November 2004.
- [81] R. L. Streit and A. H. Nuttall, “A general chebyshev complex function approximation procedure and an application to beamforming,” *Journal of the Acoustical Society of America*, vol. 72, no. 1, pp. 181–190, 1982.

- [82] ———, “A note on the semi-infinite programming approach to complex approximation,” *Mathematics of Computation*, vol. 40, no. 162, pp. 599–605, 1983.
- [83] X. Wu, Z. Tian, T. N. Davidson, and G. B. Giannakis, “Orthogonal waveform design for UWB radios,” in *Proc. of the IEEE Signal Processing Workshop on Advances in Wireless Communications*, 2004, pp. 11–14.
- [84] P. Walk and P. Jung, “Löwdin’s approach for orthogonal pulses for UWB impulse radio,” *IEEE Workshop on Signal Processing Advances in Wireless Communications*, 2010.
- [85] P. Jung and G. Wunder, “The WSSUS pulse design problem in multicarrier transmission,” *IEEE Trans. on Communications*, vol. 55, no. 10, pp. 1918–1928, Oct. 2007.
- [86] A. J. E. M. Janssen, “The Zak transform: A signal transform for sampled time-continuous signals,” *Philips J. Res.*, vol. 43, pp. 23–69, 1988.
- [87] P. Walk, P. Jung, and J. Timmermann, “Löwdin transform on FCC optimized UWB pulses,” in *IEEE WCNC*, 2010.
- [88] A. Thangarajah and M. A. Yusoff, “FPGA implementation of Hermite-Rodriguez functions for UWB systems,” in *IEEE TENCON*, Bali, Nov 2011.

Every reasonable effort has been made to acknowledge the owners of copyright material. I would be pleased to hear from any copyright owner who has been omitted or incorrectly acknowledged.