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Evaluation of ambiguity success rates based on multi-frequency GPS and Galileo

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Declaration:

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgement has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Balwinder Singh Arora
14th August, 2012

Dedicated to my parents

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Abstract

The precise positioning applications have long been carried out using dual frequency carrier phase and code observables from the Global Positioning System (GPS). The carrier phase observables are very precise in comparison to the code ones, the reason phase observables play an important role in precise geodetic applications. The carrier phase observables can have precision of about 3 millimeters. However the precision of the estimated parameter of interest, say the receiver position, depends upon the correct resolution of integer ambiguities present in the carrier phase observables. Significant contributions have been made in the last couple of decades towards integer ambiguity estimation to make precise positioning applications possible, using GPS carrier phase and code data from geodetic receivers.

Precise positioning applications have been successful in the past, but at the cost of time taken to correctly resolve the integer ambiguities. This delay in integer ambiguity estimation is caused due to the presence of various propagation and hardware related effects present in the observables of GPS or in that case, any other Global Navigation System. The propagation errors related to the atmosphere are significant for medium to long baseline lengths. Among the atmospheric errors, the ionosphere is found to have profound effect on the process of integer ambiguity estimation. With the aid of permanent reference networks, corrections for ionosphere could be interpolated and further transferred to the user with an aim to enhance users ambiguity resolution and fulfill the aim of an efficient and reliable precise positioning.

With the advancement of Global Navigation Satellite Systems (GNSS) several of the limiting factors which degrade users ambiguity resolution are seen to be met. The relatively poor precision of the code data in comparison to the phase data, is foreseen to improve for third GPS frequency, also called as GPS $L5$. Also most of the frequencies on Galileo system would have improved code precision.The ionosphere which has been a major blockade in fast integer ambiguity resolution,

for long baseline lengths, would also benefit in a multi-frequency, multi-GNSS scenario. Since a GNSS model, in which the ionosphere is considered unknown and estimated, gains strength with addition of a frequency. The addition of $L5$ on GPS and availability of up to four frequencies on Galileo system would strengthen the GNSS model which would be beneficial when ionosphere is parameterized for estimation. This study aims at understanding the above mentioned and other possible benefits of the future GPS and Galileo system.

The benefits that the future GPS and Galileo can bring to precise applications can be evaluated in terms of correct resolution of integer ambiguities present in the carrier phase data and further by understanding the contribution of the ambiguity resolution towards improvement of fixed-precision of the parameters of interest. The correct resolution of ambiguities was judged by computing the probability of correct integer bootstrap along with LAMBDA decorrelation method. The decorrelation of the ambiguity Variance Covariance matrix resulted the probability of Integer Bootstrap to correspond to lower bounds for the probability of Integer Least Square. The ambiguities were considered to be successfully resolved only after a minimum of 0.999 probability could be obtained from Integer Bootstrap. While all the ambiguities collectively contributed to give 0.999 Ambiguity Success Rate (ASR) it was termed as full Ambiguity Resolution (AR). In scenarios when full AR took large number of epochs to give 0.999 ASR, only a subset of ambiguities were fixed which met the 0.999 ASR criteria. This approach is known as Partial AR (PAR). PAR solution was accepted only when the resolved subset of ambiguities could contribute to give a minimum value of fixed-precision for the parameters of interest. Since this research involves future GPS and Galileo system, GNSS observables, real or simulated were not used. Instead simulations were done based on model assumptions, that is the functional and the stochastic model.

This research work focuses on understanding the benefits of multi-frequency GPS and Galileo to its core. This was done by planning multiple scenarios of GNSS frequencies, GNSS combinations, atmospheric considerations, latitudinal variations and baseline orientations. With the aid of this multiple scenario simulation, an estimate for time taken for successful AR and the fixed-precision of parameters of interest obtained after successful AR could be computed for a range of possible situations. When a multi-GNSS scenario consisting of future GPS and Galileo was considered, there have been challenges while a mathematical model for multi-GNSS was being formed. The design of the multi-GNSS mathematical model accounted for the Inter System Biases (ISB's) which surface while different

GNSS systems use the same reference satellite. While a rank defect between the ISB's and the ionosphere was detected, it was mitigated by choosing an appropriate S-Basis. To make the simulation software robust and realistic, accounting for setting and rising satellites and change of reference satellite was implemented. With the above considerations a multi-GNSS, multi-frequency simulation software was developed in MATLAB programming language. The results have been obtained based on assumption in the functional and stochastic models. In real practice unmodelled errors have an impact on ASR and time to fix the integer ambiguities to its correct solution due to multipath , insufficient knowledge of the stochastic model, etcetera.

Presented below are some of the important *findings of this study.*

The Geometry Free model does not gain strength with the addition of satellites. Since with addition of a satellite a receiver-satellite range is added to the unknowns. Also for a combined GPS and Galileo system, the Geometry Free model does not have a coupling parameter in the unknowns, say troposphere or receiver coordinates. Hence while the mathematical model is formed, from a single system to a combined system, the model does not gain strength. Hence a multi-GNSS constellation would not help to reduce the time-to-fix integer ambiguities for a Geometry Free model.

The permanent reference networks can benefit from an integrated GPS and Galileo system. The precision of the ionospheric estimates with a permanent network could reach 2cm instantaneously, almost any time of the day by using quadruple frequency $(L1(E1), L5(E5a), L2, E5b)$ GPS and Galileo combined system with the aid of PAR.

While the user aims at performing relative positioning using a permanent network, the benefits from a combined GPS and Galileo system are immense. For a user with low-end single frequency receiver, for short baseline lengths (<10Km), obtaining its receiver positions with 2cm precision for north- and east-components and 6cm precision for the up-component would be possible instantaneously using a combined GPS and Galileo. While the user is equipped with ionospheric corrections from the network, all the ambiguities could be resolved in a short time with a combined GPS and Galileo quadruple frequency system $(L1(E1), L5(E5a), L2, E5b)$. The findings from this simulation study shows that, while ionosphere corrections are given to the user, all the ambiguities could be successfully resolved (full AR) within 20 epochs (1 second sampling) by using quadruple frequency from an integrated GPS and Galileo system.

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Introduction

1.1 Background

The precise GPS applications require dual frequency GPS carrier phase and code observations. Carrier phase observables have precision of few millimeters (mm) which is much better than that of code observables by an order of 10^2 in case of geodetic receivers. However the carrier phase observables have the unknown integer ambiguities. Hence to exploit fully the precision of the phase observables, ambiguities need to be resolved to their correct integer values. Once the ambiguities are resolved, phase observations behave similar to very precise pseudo range observations. The parameters of interest can then be estimated from the set of ambiguity resolved phase and code observables.

The process of ambiguity resolution is found to be dependent on many factors such as, receiver-satellite geometry, measurement precision (of phase and code data), number of GNSS system frequencies and their separation, number of satellites tracked and on presence/absence of certain biases [1]. With the Next-Generation GNSS consisting of third GPS frequency ($L5$) and multi-frequency Galileo ($E1, E5a, E5b$ and $E6$) we have the increased number of frequencies and total number of satellites tracked by the receiver. Also, the precision of the code data for third frequency ($L5$) in GPS is expected to be improved. With the development of such magnitude, there is a promise which the multi-frequency, multi-GNSS hold towards faster resolution of integer ambiguities. Table 1.1 presents the GPS and Galileo frequencies considered for this research work.

	Frequency	**Wavelength**
Band	MHz	cm
L1, E1	1575.42	19.03
L2	1227.60	24.42
L5, E5a	1176.45	25.48
E5b	1207.14	24.85
E6	1278.75	23.44

Table 1.1: Future GNSS frequencies - GPS and Galileo

1.2 Objective

The objective of this research work is to evaluate the expected performance of ambiguity resolution for the Next-Generation GPS and Galileo (either as a stand-alone system or as an integrated one) and to understand its effect on precision of the non-ambiguity parameters (e.g., receiver coordinates, atmospheric delays).

Ambiguity Success Rate (ASR) gives a good evaluation of a particular ambiguity resolution strategy [2]. In addition, the convergence time taken to achieve a desired ASR (for example, 0.99 or 0.999 etc), is again a good measure of the ambiguity resolution strategy adopted. The desired ASR obtained using a single epoch of data is known as to the instantaneous ambiguity resolution. For a scenario which does not give satisfactory ASR, partial ambiguity resolution (PAR) can be used. PAR resolves only a subset of ambiguities satisfying the criteria of ASR (for instance, minimum required ASR should be 0.999). Also GPS and Galileo can be used together for certain scenarios which do not give satisfactory ASR.

The aim of AR is to utilise the precision of the phase data to estimate other parameters of interest, namely, ionosphere, troposphere, receiver coordinates and range. While full AR or PAR is performed, the precision of the parameters of interest after ambiguity fixing will however remain the final aim. The ASR and precision of the parameters of interest can be simulated for the future GNSS systems with the help of the functional and the stochastic model. This approach is further discussed in chapter 3 and chapters following thereafter.

The ambiguity resolution is to be evaluated for different GNSS models for single baseline scenario. Here Geometry Free and the Geometry Fixed GNSS models are considered before evaluating the ambiguity resolution for a Reference-Rover model. For each of which, time taken (convergence time) to achieve a desired ASR, will be evaluated. The three GNSS models, namely, Geometry Free, Geometry Fixed and Reference Rover will be described in detail in the following section.

1. Geometry Free model: In a Geometry Free model, the geometry is not considered, i.e. the satellite and the receiver position information is not used. Instead range between the satellite and the receiver is taken as an unknown parameter to be estimated. The Geometry Free model is therefore a rather weak model, although relatively simple for implementation.

2. Geometry Fixed model: The Geometry Fixed GNSS model can be considered as a special case of Geometry Based model in which the information on the receiver satellite geometry is completely known. Hence receiver or satellite coordinates need not be estimated anymore. A Geometry Fixed model is resembles a permanent station network, for example the CORS (Continuous Operating Reference Stations) networks.

3. Reference-Rover model: A Reference Rover model falls under the Geometry Based class of GNSS models. In a Reference Rover model, the coordinates of the rover receiver are estimated. Hence a Reference Rover model can be considered as weaker model than Geometry Fixed one.

Saying that Geometry Free is a weaker model, the combinations of model assumption resulting in instantaneous ASR for a Geometry Free model need not be simulated again for Geometry Fixed model. The observation equations for a Geometry Free and Geometry Based model will be discussed in detail in chapters 4 and 5, respectively.

While the different GNSS models are formed, there are different unknown parameters in the observables which need to be considered for parameterizations. The parameters in the observation equations, namely the troposphere and the ionosphere, can be assumed known (Fixed), unknown (Float) and weighted (for ionosphere only) depending on the baseline length. While the ionosphere is weighted and the other atmospheric unknown, the troposphere will be considered as unknown and parameterized. The ionosphere will be weighted using a linear weighting function of baseline length, the between receiver single difference ionosphere standard deviation considered is 0.68mm per Km, see [3], [4]. In case of Reference-Rover model, the coordinates of the reference stations are considered known while those of the rover is to be estimated. While a GNSS model is parameterized for various combinations of GNSS model and unknown parameters, ASR and convergence time will be analyzed and its effect on precision of baseline will be evaluated.

Since the research focuses on the multi-GNSS models, the evaluation of the success of ambiguity resolution and precision of unknown parameters for this research work is based on the simulation. Simulation will be done by considering the model assumptions, namely the design matrix of the underlying GNSS model and the precision of the measurements which forms the stochastic model. The simulations based on model assumptions are also referred to as "Planning computations"; such studies have been done by [5],[6],[7],[8],[9],[10],[11],[12] etc.

With an understanding that ASR is influenced by measurement precision and number of frequencies, these two contributing factors will be varied for each of the GNSS models, considering the developments with respect to multi-GNSS. The measurement precisions of the code and phase data for a user are dependent on the type of receiver and in any receiver they further vary with the elevation of the satellite [13]. High elevation satellites result is less noisy observables hence the observables resulting from high elevation satellites would have better measurement precision in comparison to low elevation satellites. This effect could be incorporated by choosing a suitable elevation dependent weighting function. Measurement precision for some of the high-end geodetic receivers is compared by [13] and for the miniaturized $L1$ receivers they are used by [14]. The values of measurement precision for Galileo can be found in [15]. In this research work, different combination of measurement precision will be used considering the high-end geodetic receivers to the low-end ones (e.g. u-blox) and also considering the improved code precision for GPS $L5$ frequency. Furthermore, the combination of the frequencies will be varied. For instance, in case of GPS-only models, dual and triple frequency combinations will be used to evaluate ambiguity resolution.

1.3 Outline of the thesis:

The research work is presented in detail in the commencing chapters. Chapter 2 presents literature study on ambiguity resolution with respect to multi-GNSS. The results obtained from multi-GNSS in terms of AR and positioning solutions (baseline estimations etc) are discussed.

Chapter 3 discusses the theory of integer ambiguity resolution with respect to the methods of ambiguity resolution and influence behind the choice of ambiguity estimator for this research work.

Chapter 4 describes the Geometry Free model approach to evaluate ambiguity resolution based on assumptions for atmospheric (ionosphere and troposphere), measurement precision values for different frequency combinations. PAR is performed for various combinations for which desired ASR is not achieved; its effect on non ambiguity parameters is evaluated. Also combination of both GPS and Galileo is considered for analysis in cases where the desired ASR is not achieved. Further the results are presented in form of graphs and tables and the behavior of ambiguity resolution and precision of non-ambiguity parameters is discussed.

Chapter 5 considers the results of ASR from chapter 3 and further evaluates ambiguity resolution of the Geometry Based model for different combinations of measurement precision, frequency and atmosphere. PAR and combination of GPS and Galileo have been considered for cases which do not produce satisfactory ASR. The performance of different combinations in terms of ASR and effect on precision of non ambiguity parameters is analyzed and presented in graphs and tables.

Chapter 6 presents the results and discussions for Reference-Rover model. The ASR, convergence time and improvement in precision of the baseline considering an ambiguity fixed solution will be evaluated for different scenarios.

The results obtained from different GNSS models and scenarios of measurement precision, atmosphere considerations, baseline lengths are summarized and concluded in chapter 7.

While this research work was being done, it was important to discuss the technical details behind the theory used, for example LAMBDA method of ambiguity resolution, Partial Ambiguity Resolution and satellite weighting function. An understanding of important trends and unexpected phenomenon in the results obtained was done through analysis of specific variables, for example effect of low elevation satellite on ASR, wobbling of the Double Difference precision curves (for ranges). These set of work is compiled and presented in the appendices.

Appendix A discusses the widely used Kronecker product and its properties.

Appendix B presents theory and design of function and stochastic model for Geometry Free model for simulation for ASR and fixed-precision of ranges.

Appendix C includes the discussion of the algorithm for ambiguity success rate based on LAMBDA. Both Full AR and Partial AR (PAR) algorithms are discussed.

Appendix D is dedicated to the discussion of unexpected phenomenon observed during simulations of ASR. Wobbling of the fixed-precision curves, sudden fall in the success rates due to incoming low elevation satellite and effect of low elevation satellite on ASR are discussed based on results. In addition to the the discussion of the above mentioned phenomenon, the exponential satellite elevation dependent weighting function is also discussed.

Appendix E presents all the figures by considering Geometry Free model, chapter 5, for simulation of ASR and evaluating the benefits of AR in terms of improvement in fixed-precision of ranges achieved.

Appendix F presents all the figures from chapter 5, Geometry Fixed model, for simulation of ASR and fixed-precision of ionosphere / troposphere.

Appendix G includes all the figures for simulation of ASR and fixed-precision of coordinates for Reference Rover chapter, chapter 6.

2

Literature Review

The precise positioning applications have long been carried out using the dual frequency GPS, carrier phase and code observables. The carrier phase observables technically can have the precision of up to a few millimeters. However the precision of the position depends upon the accurate estimations of unknown integer ambiguities present in the GPS phase observables. On successful fixing of ambiguities, the precision of the parameters of interest can be computed. The literature review focuses on discussion of multi-GNSS ambiguity fixing methods and results obtained. The comparison of different methods of computing Ambiguity Success Rates are discussed, whenever available. Also findings related to the improvement of precision of parameters of interest are discussed.

2.1 Unknowns in the GNSS observables

The biases/errors/unknowns present in GNSS observables are parameterized in the GNSS models based upon the availability of a-priori information about the biases. While speaking of one of the unknown parameters, the atmosphere, which encompasses both the troposphere and the ionosphere can have various considerations. The ionosphere can be considered unknown (float), with a-priori information of ionosphere (weighted), or when it is insignificant, can be considered known (fixed). While speaking of the tropospheric error, for Geometry Free model the troposphere is lumped with the ranges. For a Geometry Based model it can either be considered known (fixed) or unknown (float).

The relative atmospheric errors, both the troposphere and ionosphere are significant for medium and long baselines, say > 10 kilometers. In literature, the sensitivity of relative ionosphere to baseline length is reported as 4 millimeters(mm) per 10 kilometers (Km)(un-differenced standard deviation at zenith) [12]. Another literature gives a relation between un-differenced ionosphere standard deviation to the baseline length as σ_I $(mm) = 0.68 \times$ baseline length (Km)

[2], which means 6.8 mm per 10 Km. In addition to this, unlike ionosphere, troposphere can be significant to height variations, even for short baselines. For troposphere float scenarios, a single value of troposphere, per receiver, for all satellites, at zenith can be parameterized. This can be achieved with the aid of troposphere mapping functions, which maps the slant troposphere delays to zenith. [16] used Ifadis mapping function [17] for simulating the ASR using model assumptions for troposphere float scenarios.

For a Geometry Free model receiver positions are not parameterized, instead ranges are considered. One of the important parameter for GNSS community is the position, that is the GNSS receiver coordinates. In a Geometry Fixed model, the positions of the receiver and the satellite as considered known and hence held fixed. Whereas for a Reference-Rover model, the coordinates for the reference receiver are known, while for the rover they are unknown, baseline components are hence estimated for a Reference-Rover model. The other unknown parameters in the Geometry Based class of models can be the ionosphere and the troposphere. They are parameterized based on their a-priori information as discussed above. The detailed parameterizations of the Geometry Free, Geometry Fixed and Reference-Rover models are discussed in the commencing chapters.

2.2 Ambiguity resolution

The ambiguities are unknown integer cycles present in the carrier phase data. Its magnitude compared to the errors can be immensely large (for example, from one wavelength to many). Hence its estimation is important when one deals with precise positioning applications using carrier phase data. There has been significant research done on ambiguity resolution in order to exploit the precise carrier phase data efficiently. It has been understood that AR depends on receiver-satellite geometry, measurement precision of phase and code observables, number of GNSS system frequencies and their separation, number of satellites tracked and on presence/absence of biases [1]. The ambiguities do not have a temporal variation, unless there is loss of lock of the signal or due to cycle slips, the integer ambiguities are set to a new value. Hence again integer ambiguities need to be carefully mitigated. With the current dual frequency GPS, the AR for long baselines takes time to converge to an accurate solution.

AR resolution is important since it can significantly improve the precision of the non-ambiguity parameters [18]. Considering the importance of AR, chapter 3 is dedicated to AR in which the theory behind probability of AR, gain etcetera is discussed. While speaking of AR, the process since ambiguities are parameterized for estimation till the time the precision of the parameters of interest is improved through ambiguity resolution is discussed in detail in [19]. The four step procedure until the fixed ambiguities are used to improve the precision of the parameters of interest consist of firstly estimating the ambiguity-float solution, then further mapping the real valued ambiguity to its integer value which is called as integer solution constitutes the second part. Further a decision has to be made to accept the integer ambiguity solution which forms the third step. Finally by utilizing the ambiguity fixed solution for estimating the parameters of interest serves the goal of ambiguity resolution and also completes this four step procedure. The integer ambiguities are validated by defining the Integer Apertures (IA) which have an acceptance region and a rejection region, see [19]. There are various methods proposed for the acceptance tests, the controlled failure of fixed failure test [20], also ratio test, F-ratio test (F for Fisher), projector test, difference test which fall under class of IA estimators are discussed in [21]. The likelihood method and artificial nesting methods can be found in [22]. Acceptance test plays an important role while one estimates integer ambiguities and uses the solution to estimate the parameters of interest using real data. In this research work, all evaluation of AR is based on ASR and not on acceptance tests, since real data is not used for simulations in this study.

In this research, two approaches towards correct integer ambiguity resolution are adopted by which fixed precision of the non-ambiguity parameters is computed. Firstly, all the ambiguities are fixed to their integer values and are checked as to whether the desired ASR (0.999 ASR) is achieved instantaneously. If not so, the convergence time is then calculated. Secondly, only a subset of ambiguities which converge to a predefined value of ASR (for example 0.999 ASR) are only fixed. Moreover, the time taken for partial ambiguity fixing can be calculated along with the precision of the other parameters corresponding to partial ambiguity fixed solution. The second approach of ambiguity fixing discussed above is termed as Partial Ambiguity Resolution (PAR). Below some of the findings related to PAR are presented.

2.2.1 Partial Ambiguity Resolution (PAR)

Sometimes the full AR takes time to converge to a desired ASR, in such cases PAR can be performed. In PAR only a subset of ambiguities which meets the laid criteria (0.999 ASR) are resolved. Resolving only a subset of the ambiguities implies that not all of the carrier phase data will exhibit the property of precise pseudo ranges but only a subset that is resolved successfully. The improvement in precision of parameters of interest (ranges, ionosphere, rover receiver coordinates etcetera) as an effect of PAR will therefore always be smaller than it would have been in case of full ambiguity resolution. In fact, the precision improvement could even be so small, that the float solution reaches the same level of precision nearly as fast. In that case PAR would not be effective enough. For deciding whether PAR makes sense, an evaluation needs to be done based on the precision of the parameters of interest obtained by partial fixing of the ambiguities [23]. The improvement in precision of the parameters of interest, as a result of ambiguity-float and -fixed solutions will be discussed in the commencing section.

2.3 Gain

Gain, the improvement in precision of the non-ambiguity parameters, is dependent on the precision of the measurements (carrier phase and code). Gain becomes less pronounced, the better the code precision becomes [7]. Gain of non-ambiguity parameters increases with greater frequency separation for any GNSS. While performing PAR, greater improvement in the precision of the parameters of interest can be obtained by using lower frequencies having higher separation [7].

2.4 The promise of multi-GNSS

The GNSS in discussion for this research work are GPS and Galileo. Speaking about the future GPS, the modernized GPS will have and additional third frequency, denoted as $L5$, along with the current two. The precision of the code observables on third GPS frequency is expected to have an improved precision. This is important to note since the AR depends on the precision of the measurements, the results are better with better precision values. The measurement precision for different GNSS receivers based on the current and the future GNSS systems can be found in [13]; [14]; [15]; [24].

The future GNSS will have more satellites available for the user as compared to the current GPS only. As an effect of increase in number of frequencies, there will be significant increase in the available GNSS observables. This can be useful for AR, since the AR is affected by frequency separation. The user can thus benefit from selective data processing. Also with additional frequencies, the user will have additional data from the same satellite, this means a stronger data processing model.

The effect of multi-GNSS on AR and on gain can be analyzed by simulation studies based on model assumptions or by simulating observables itself. Such studies have been done by [5]; [6]; [8]; [7, 9]; [13]; [25]; [26]; [27]; [28]; [29]; [15]; [30]; [14]; [11, 12]; [31], [32],[33]. Presented below are some of the findings of the mentioned research. The findings for each of the mentioned sub-sections below relate directly to the ASR.

Measurement precision

Since the success of the ambiguity resolution depends on the measurement precision among various other factors, the improvement in the measurement precision values, especially the precision of the code data will have a significant effect on the AR process. In the future GNSS systems, the new $L5$ signal on GPS and the multi-frequency Galileo are expected to have better code precision.

The measurement precision values for different GNSS receivers can be found in literature. Un-differenced measurement precision for some of the high-end geodetic receivers at zenith are compared by [13]. The worst values for standard deviation of measurements for carrier phase varies from 0.3 to 3.9 millimeters (mm) and for code they lie between 22 centimeters (cm) to 5.57 meters (m) for different high-end GPS receivers [13]. The standard deviation of measurements for miniaturized $L1$ GPS receiver as assumed by Odijk were 5 mm and 1 m for phase and code respectively [14]. The standard deviation for different frequencies of Galileo system were found to lie between 7 to 9 mm and 9 mm to 6 cm for phase and code, respectively [15]. In one of the research work, Colomnia et. al (2012), see [24], presented the measurement precision values for pseudo ranges on E1 CBOC (Composite Binary Offset Carrier) and E5 AltBOC (Alternate BOC) signals. The precision for Galileo under absence of any canopy and under presence of dense canopy were found to be in the range 0.25 to 2.00 m on Galileo $E1$ frequency and 0.02 to 0.08 m on Galileo $E5$. the pseudo range precision of

$E5$ seems to be promising with the best precision estimated to be around 2cm. The precision of code for Galileo which is understood to be improved significantly brings with itself great deal of promise in terms of reduced time to fix for the integer ambiguities.

Further, some of the findings for future GNSS are discussed in the commencing section.

Simulation of ASR using GPS and Galileo

Below are presented some of the results obtained for ASR using multi-GNSS form various literatures. Since the research work discussed below considers multi-GNSS, simulated observables or simulations based on "Planning Computations" have been used. The discussion of the literature is broadly divided in two parts, firstly the results for Geometry Free model are discussed followed by discussion of Geometry Based results.

Chen et. al. (2004), see [26], gave the simulation for ambiguity Dilution of Precision (ADOP) based success rates and effect on relative ionosphere network corrections for GPS and Galileo. ADOP is given by the VC matrix of the float ambiguities as, $|Q_{\hat{x}_I}|^{1/2n}$, n being the total number of ambiguities, refer [34]. The ADOP was simulated for Geometry Free model using GPS and Galileo for different baselines, based on the relation of ionosphere standard deviation to baseline length (10ppm). It was not clearly mentioned whether observables were simulated, nonetheless, phrases like, '*estimated positions were compared with truth values*' hint on using simulated observables. It was further highlighted that, while modeling the ionosphere over the reference network, currently there is a limit to the baseline length. This is due to accommodation of interpolation errors due to modeling of ionosphere. Currently dual frequency tolerates maximum of 8 cm which comes to 88 % of NRTK interpolation errors (88 Km baseline). By using triple frequency, 16 cm of total error corresponding to 97% of NRTK interpolation error can be tolerated for same baseline. Hence with addition of multi-frequency data, the density of the reference stations can be reduced to maintain the performance of NRTK. Chen assumed measurement precision of phase to be 0.01 cycles (say, 2 mm for 0.2m wavelength), and for code the precision on all frequencies considered was 5cm except GPS $L1$ frequency. The code on GPS $L1$ was assumed to have a precision of 15cm. Multipath was assumed to have a precision of 21cm and ionosphere 10ppm (10cm per 10 km baseline). The ADOP based instantaneous success rates were computed, for dual and triple frequency GPS and Galileo, same are given below, see Table 2.1 (the values are interpreted from the figures).

	GPS		Galileo	
Baseline length (Km)	DF	TF	DF	TF
1	0.7488	0.999992	0.71816	0.999984
5	0.0667	0.97488	0.066745	0.9683772
10	0.0022	0.84151	0.002299	0.80047
15	-	0.683772	-	0.68011
20	-	0.450459	-	0.44664
25	-	0.25010	-	0.24993
30	-	0.205671	-	0.20384
40	-	0.16823	-	0.168236

Table 2.1: ADOP based instantaneous success rates for dual (DF) and triple frequency (TF) GPS and Galileo, based on Double-Difference Geometry Free model, as presented by Chen et. al. (2004). Please note, in the above table the ASR presented is in probability (range 0 and 1), it is converted from 'nines', since in the paper it was presented in 'nines'.

Feng et. al. (2005), see [35], evaluated performance of future GPS with three frequencies, $L1, L2, L5$ and Galileo with four frequencies, $E1, E5a, E5b, E6$. Ambiguity resolution performance was assessed using TCAR method of AR using a Geometry Free model. The probabilty of sucess of AR was calculated based on probability of integer rounding. With TCAR, for both GPS and Galileo, the Extra-Wide-Lane ambiguities $L2 - L5$ and $E5a - E5b$ could be resolved instantaneously with 100% success. The other combinations of Wide-Lane ambiguities could be resolved with 75 to 90% success rate in single epoch. The time taken for estimation of ionosphere with desired accuracy is also analyzed. The ionosphere uncertainty could be analyzed by forming difference of DD Extra-Wide-Lane (EWL) and DD Wide-Lane (WL) combinations for phase observables. Further the phase smoothing was performed for the above formed measurement to reduce their noise using the difference of DD phase observables on $L1/E1$ and $L2/E5b$ for GPS/Galileo. While forming linear combinations of observables, there is a resultant ionosphere noise factor for both GPS and Galileo in the Wide-Lane observation equations given as 83.1 and 50.3 respectively. The estimation of ionosphere with a 2.1cm error could be done with triple frequency GPS and Galileo, the results for the same are presented in Table 2.2, see below

GPS $\Phi1, \Phi2, \Phi5$		Galileo $\Phi1, \Phi5b, \Phi6$		GPS code	Galileo code
$\sigma_\Phi = 5mm$	$\sigma_\Phi = 3mm$	$\sigma_\Phi = 5mm$	$\sigma_\Phi = 3mm$	$\sigma_P = 60cm$	$\sigma_P = 40cm$
600	360	200	60	4000	1800

Table 2.2: Number of epochs taken for 100% ASR to obtain first order ionosphere delay estimation uncertainty of 2.1cm based on the difference of DD EWL and DD WL phase and code combinations for GPS and Galileo. The above formed difference for the measurements are further smoothed with carrier phase smoothing using DD $L1/E1$ and DD $L2/E5b$ phase data for GPS/Galileo, see Feng et. al. (2005).

Further the estimated DD EWL and DD WL ambiguities along with the estimated DD ionospheric bias is back substituted in the DD corresponding observation equations to estimate the uncertainty in the ranges. The positioning performance is then presented in form of the desired DD range accuracy. The results for number of epochs taken in order to obtain a DD range precision of 2.5cm are presented Table 2.3, see below

GPS $\Phi1, \Phi2, P1, P2$		GPS $\Phi1, \Phi2, \Phi5$		Galileo $\Phi1, \Phi5b, \Phi6$	
$\sigma_P = 30cm$,$\sigma_\Phi = 5mm$	$\sigma_P = 20cm$,$\sigma_\Phi = 3mm$	$\sigma_\Phi = 5mm$	$\sigma_\Phi = 3mm$	$\sigma_\Phi = 5mm$	$\sigma_\Phi = 3mm$
8000	2700	750	200	300	77

Table 2.3: Number of epochs taken for 100% ASR to obtain range estimation uncertainty of 2.5cm based on the DD WL phase and code combinations for GPS and Galileo. The DD WL ambiguity and DD ionosphere uncertainty is back substituted in DD WL observations to get the uncertainty in the range. After back substitution, the DD WL observables are smoothed with carrier phase smoothing using DD $L1/E1$ and DD $L2/E5b$ phase data for GPS/Galileo, see Feng et. al. (2005).

Further the literature based on a *combined Geometry Free and Geometry Based model* are presented, see below.

Sauer et. al. (2004), see [27], simulated GNSS observables for GPS and Galileo for various baselines (45), for 1200 epochs of data and evaluated AR based on FAMCAR (Factorized Multi-Carrier Ambiguity Resolution). FAMCAR technique of AR uses different combination of observables (phase only, phase and code together) considering different GNSS models, namely, Geometry Free and Geometry Based. By these multi-combination of observables, different error components (position, ambiguities, ionosphere and multipath) are estimated by applying an independent filter for each error component, for more details, see [36]. While simulating the observables, different error components like, troposphere, ionosphere and multipath were included. The measurement precision of phase in presence of multipath lied between 0.5 and 1mm and was 0.4mm in absence of multipath. For code, in presence of multipath, the precision was 22 and 20 cm for GPS and Galileo, without multipath the precision was, 7 and 5cm for GPS and Galileo respectively. The analysis presented was mean time to fix (TTF) for GPS (DF and TF) and Galileo (DF, TF, QF) in presence of multipath, see Table 2.4, (the values presented in the Table 2.4 are interpreted from the figures given in [27]).

	GPS		Galileo		
Baseline length (Km)	DF	TF	DF	TF	QF
5	5	4	4	2	1
20	25	18	28	20	18
35	55	47	50	32	25
57	97	65	90	52	40

Table 2.4: FAMCAR based results for mean TTF (sec) for dual (DF) and triple frequency (TF) GPS and Galileo, as a function of baseline length, in presence of multipath, presented by Sauer et. al. (2004)

It is important to note that the values of mean TTF reduces as number of carrier frequencies for any GNSS are increased.

Schlotzer & Martin (2005), see [29], gave the comparison between TCAR(Three Frequency Ambiguity Resolution) and ITCAR (Integrated Three Frequency Ambiguity Resolution) methods of AR for Galileo and future GPS. TCAR was based on Integer Rounding whereas ITCAR was based on Integer Bootstrap method of ambiguity fixing methods. However it was not clear from the research work, if LAMBDA de-correlation is applied to ITCAR before Integer Bootstrapping. Further, TCAR uses the geometry-free model whereas ITCAR uses geometry-free model for fixing ambiguities and after AR, uses the AR solution in geometry-based model. GNSS observables were simulated in TCAR simulator, effects of atmosphere and multipath were incorporated. The measurement precision of phase for GPS carrier frequencies $L1, L2, L5$ was assumed to be 3, 4 and 4mm, for Galileo carrier frequencies $E1, E5a, E5b, E6$, the phase precision was assumed to be 3, 4, 4 and 4mm respectively. The value of code precision could not be identified from the research work.

The observables were simulated for both GPS and Galileo carrier frequencies, three frequencies for GPS $(L1, L2, L5)$ and four frequencies for Galileo $(E1, E5a, E5b, E6)$, using TCAR simulator. Additional phase was simulated on $E5$ for Galileo. The TCAR and ITCAR uses the Extra-Wide Lane (EWL) and Wide Lane (WL) (differenced observables) for AR. The results presented in the table below have differenced observables L5-L2 (EWL) and L5-L1 (WL) for GPS and E5a-E6 (EWL) and E5a-L1 (WL) for Galileo, see Table 2.5.

	GPS (TF)		Galileo (TF)	
Baseline length (Km)	TCAR	ITCAR	TCAR	ITCAR
5	1.0	1.0	1.0	1.0
10	0.78	0.999	0.91	0.999
15	0.45	0.90	0.6	0.90
20	0.17	0.82	0.29	0.82
30	0.06	0.59	0.11	0.60
40	0.03	0.44	0.05	0.47
50	0.01	0.29	0.03	0.33

Table 2.5: Instantaneous success rates for triple frequency GPS only and Galileo only, based on TCAR and ITCAR, as presented by Schlotzer & Martin (2005)

In the following literature review, simulation based on *Geometry Based model* are presented.

Zhang et al. (2003), see [25] evaluated AR success based on Cascading Integer Resolution (CIR) scheme. However CIR was modified to use LAMBDA method of AR, generally CIR makes use of Integer Rounding for AR, see [37]. Also in CIR generally Geometry-Free model is used, here Geometry-Based model was used instead. AR was calculated for GPS and Galileo triple frequency by using the simulated observables incorporating effects of errors for the atmosphere, orbit and multipath. The values of measurement precision considered were 0.36m for code on $L1$, 0.04m for code on $L5$, 0.1m for code on $E1$, 0.045m for code on $E5a$, whereas the phase was kept at 0.003 cycles (say, 0.6 mm for 0.2m wavelength). ASR was simulated for different baselines, each baseline had a unique upper limit for number of epochs. For baseline lengths 1, 10, 20, 30, 40 and 50 Km the maximum number of epochs allotted were 300, 600, 1500, 1800, 2100 and 2500 respectively. Mean TTF (Time To Fix) and instantaneous AR success rates were evaluated, see Table 2.6 (the values presented in the table are interpreted from the figures given in [25]). The mean TTF denotes the average time the software needs to fix all the ambiguities correctly for the chosen mathematical model for ambiguity resolution. It is important conclusion that the mean TTF reduces

	GPS (TF)						Galileo (TF)						GPS (TF)+Galileo (TF)					
Baseline length (Km)	1	10	20	30	40	50	1	10	20	30	40	50	1	10	20	30	40	50
Mean TTF (epochs)	1	1	3	9	19	71	1	1	2	5	25	36	1	1	1	4	5	20
Instantaneous ASR (%)	70	62	18	4	2	< 1	65	56	18	1	1	< 1	100	100	47	11	4	< 1

Table 2.6: Results based on CIR (modified for LAMBDA method of AR), Geometry Based model. Presented are mean TTF (epochs, 1 epoch=1 second) and instantaneous ASR for triple frequency (TF) GPS and Galileo, stand-alone and together, as given in Zhang et al. (2003)

significantly for a combined GPS and Galileo system. For baselines upto 20 Km, the mean TTF was found to be 1 epoch.

Milbert (2005), see [28], presented instantaneous ASR for GPS and Galileo together. Single, dual and triple frequency combinations for combined GPS and Galileo with 6 satellites from each GNSS system were considered for simulation of ASR. Observation equations were parameterized for a Geometry-based (GB) model with double-differenced (DD) ambiguities, reference coordinates held fixed, partial derivatives of rover were estimated, ASR were computed by integer bootstrap after decorrelating the ambiguities by LAMBDA method. It was not mentioned specifically whether real data was used, but phrases like 'simulated success rates' and 'hypothetical satellite orbits' indicate that ASR was computed by model assumptions, that is float Variance-Covariance (VC) matrices of the ambiguities. Instantaneous ASR was simulated for short-baselines where atmospheric errors are neglected, with ionosphere unknown, ionosphere weighted, both ionosphere and troposphere unknown, and finally for both ionosphere and troposphere weighted scenarios. For each scenario, while simulating instantaneous ASR, the measurement precision of code was varied between 5mm and 1 meter, precision of phase was held fixed to 3mm. The results for all the scenarios are presented in the Table 2.7

GNSS model assumptions	Maximum σ_P @ 100% ASR (meters)	Minimum ASR	
		ASR (%)	σ_P (meters)
SF, short baseline	0.42	99.92	1
DF, short baseline	0.35	97.55	1
TF, short baseline	0.59	99.957	1
DF, ionosphere unknown, troposphere fixed	0.03	0.00	0.37
TF, ionosphere unknown, troposphere fixed	0.04	10	1
TF, ionosphere weighted, troposphere fixed	0.03	96.6	1
TF, ionosphere weighted, troposphere unknown	0.02	62	1
TF, ionosphere & troposphere (Reference receiver) weighted	0.02	87	1
TF, ionosphere & troposphere weighted	0.03	91.5	1

Table 2.7: Instantaneous ASR values based on Integer Bootstrap success rates and LAMBDA decorrelation, for different GNSS model assumptions, as presented by Milbert (2005). Measurement precision of phase was held fixed at 3mm, precision of code was varied between 5mm and 1m. SF indicates single frequency, DF (dual frequency) and TF (triple frequency)

ÓKeefe et al. (2006), see [30], evaluated triple frequency GPS and Galileo, stand-alone and together, by simulating ASR using LAMBDA method for baseline lengths of 30, 40 and 50 Kilometers for Southern Alberta, Canada. It was not bluntly specified in the research work if Integer-Least squares (ILS) was used to estimate integer ambiguities, but with simulated observables and LAMBDA method, it will be assumed that ILS would have been considered. Further, the ambiguity candidates were validated by ratio test. The code data on $L1$ and $E1$ frequencies was not considered for simulations, rest all the observables were simulated on GPS $L1, L2, L5$ and Galileo $E1, E5a, E5b$. The precision for the other parameters were considered as, for Double Difference (DD) troposphere-0.2cm/10Km, DD ionosphere-3cm/10Km, DD orbital error-0.1cm/10Km, DD multipath-0.14cm ($L5/E5a/E5b$) and 0.75cm ($L2$), for code data the thermal noise was considered to be 0.05m for ($L5$), 0.29m for ($L2$), 0.06m for ($E5a, E5b$) and for phase observables, thermal noise for all carrier frequencies was set to 0.003 cycles (0.6mm for 0.2m of carrier wavelength). Thermal noise causes error in the measurement of range, it can be assumed to be the measurement precision. Ionosphere was also modeled temporally with random walk with variation of $1cm^2$ per epoch for 50 Km baseline. The ASR results with the above setup are given in Table 2.8, see below, (the values presented in the table are interpreted from the figures given in [30].

	GPS (TF)			Galileo (TF)			GPS(TF)+Galileo(TF)		
Baseline length (Km)	30	40	50	30	40	50	30	40	50
Percentage of correct fixing of Ambiguities	100	100	98	100	98	100	100	100	100
Epochs taken for correct fixing of ambiguities (1 epoch = 1 second)	540	710	900	580	600	840	10	40	100
Estimated 3D position errors (cm)	22.4	9.5	14.3	1.1	1.1	1.1	0.8	0.8	0.9

Table 2.8: ASR and TTF for correct fixing of ambiguities for triple frequency (TF) GPS, Galileo, stand-alone and together, for baseline lengths 30 to 50Km, as presented by ÓKeefe et al. (2006)

Verhagen et al. (2007), see [11], simulated ASR for Geometry Based model by computing probability of integer bootstrap after LAMBDA decorrelation. In this study the troposphere parameters are estimated, whereas the ionosphere is weighted (for 15 Km and 100 Km baseline). Measurement precision values for GPS and Galileo considered, corresponded to high-end geodetic receivers. The measurement precision for dual frequency GPS and Galileo system of $L1$ and $L5$ carriers were 20 and 10 cm for code on $L1$ and $L5$ respectively and 1.3 and 1 mm for phase on $L1$ and $L5$ respectively. Instantaneous ASR was simulated, as a function of GNSS system, in first case only GPS, then only Galileo and then finally with both the systems were considered. This was done for 15 and 100 Km baseline, at 0^0, -30^0 and 70^0 latitude locations. Instantaneous ASR values were simulated for the whole day and mean instantaneous ASR values were presented. Table 2.9 presents the results for the above simulation, (the values presented in the table are interpreted from the figures given in [11]).

	GPS (DF)						Galileo (DF)						GPS (DF)+Galileo (DF)					
Baseline length (Km)	15			100			15			100			15			100		
Latitude Locations	$0°$	$70°N$	$30°S$	$0°$	$70°N$	$30°S$	$0°$	$70°N$	$30°S$	$0°$	$70°N$	$30°S$	$0°$	$70°N$	$30°S$	$0°$	$70°N$	$30°S$
Mean Instantaneous ASR (%)	100	100	100	35	40	25	100	100	100	57	60	32	100	100	100	100	100	100
	GPS (TF)						Galileo (TF)						GPS (TF)+Galileo (TF)					
Baseline length (Km)	15			100			15			100			15			100		
Latitude Locations	$0°$	$70°N$	$30°S$	$0°$	$70°N$	$30°S$	$0°$	$70°N$	$30°S$	$0°$	$70°N$	$30°S$	$0°$	$70°N$	$30°S$	$0°$	$70°N$	$30°S$
Mean Instantaneous ASR (%)	100	100	100	53	56	39	100	100	100	88	88	65	100	100	100	100	100	100

Table 2.9: Mean Instantaneous ASR for dual (DF) and triple (TF) frequency GNSS - GPS, Galileo, stand-alone and together, for 15 and 100 Km baseline, at 0^0, -30^0 and 70^0 latitudes, as presented by Verhagen et al. (2007)

The results were found to be better for Galileo only as compared to GPS only, and better for TF system than for DF system. A combined GPS and Galileo, TF system can give instantaneous AR even for 100 Km baseline, which is very promising.

The instantaneous ASR results for Galileo were better than for GPS, the reason highlighted in this research work was the number of satellites in view. The number of satellites were more for Galileo as compared to GPS system. Further, the instantaneous ASR was analyzed for Galileo only for $L1$ and $L5$, 100 Km baseline, at 0^0, -30^0 and 70^0 latitude locations, as a function of number of satellites. Refer Table 2.10.

	Galileo (DF)																	
Latitude Locations	$0°$					$70°N$							$30°S$					
No. of satellites	7	8	9	10	11	7	8	9	10	11	12	13	6	7	8	9	10	11
Mean Instantaneous ASR (%)	15	38	63	84	94	12	38	63	82	93	98	100	4	14	37	62	85	92

Table 2.10: Mean Instantaneous ASR for dual frequency (DF) Galileo, for 100 Km baseline, at 0^0, -30^0 and 70^0 latitude, as a function of satellite number, given by Verhagen et al. (2007)

ASR not only increases as number of satellites are increased, but also the range of values ASR can take for any given number of satellites increases as number of satellites are increased.

Ji et al. (2007), see [31], evaluated ASR using quadruple frequency Galileo for a 860 m baseline using simulations based on LAMBDA and Cascading Ambiguity Resolution (CAR) method for AR. CAR generally uses IR for ambiguity fixing, and further success rates are based on ADOP. On the other hand LAMBDA makes uses of decorrelation of the ambiguities for computing the ambiguity success rates. For LAMBDA, the ASR values were simulated using sharp lower bound for ILS, by using probability of Integer Bootstrap. Galileo observables were simulated, Gaussian noise was assumed for both code and carrier phase measurements. Tropospheric and ionospheric delays were modeled based on Hopfield and Klobuchar models respectively. a 24 hour observable data set was simulated with 1 epoch interval. The observables were parameterized by Geometry-Based (GB) GNSS model. For different measurement precision values of phase (3, 6, 12 mm) and code (50 cm, 1 m, 1.5m, 2 m), precision ASR (0.999 ASR probability was desired) was evaluated for single epoch and multi epoch. Results given by Ji are presented in Table 2.11.

Galileo only (QF)								
	CAR				LAMBDA			
Phase precision	code precision (meters)							
3 mm	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
Instantaneous ASR (%)	100	100	92.9	79.9	100	100	100	100
TTF for 0.999 ASR (epochs)	1	1	2	2	1	1	1	1
Phase precision	code precision (meters)							
6 mm	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
Instantaneous ASR (%)	100	100	65.42	37.6	100	100	100	99.58
TTF for 0.999 ASR (epochs)	1	1	2	3	1	1	1	2
Phase precision	code precision (meters)							
12 mm	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
Instantaneous ASR (%)	0	0	0	0	0	0	0	0
TTF for 0.999 ASR (epochs)	4	4	4	4	3	3	4	4

Table 2.11: Instantaneous ASR and TTF for Galileo, quadruple frequency (QF) for different values of measurement precision, presented by Ji et al. (2007)

It can be seen from the results that LAMBDA gives higher values of ASR than CAR. The precision of the code seems to be quite high, 50 cms to 2 m, still LAMBDA is able to give good instantaneous ASR values. The instantaneous ASR for phase precision of 12mm for all the code precision values are given as 0 (zero), it is not clear as to how zero would be ASR and what could be the possible reason for such low ASR values.

Santos et. al. (2008), see [32], in his research work described in brief the work done by Laval University under the project Geomatics for Informed Decision (GEOIDE) network. The simulations for improvement in ambiguity resolution with a combined GPS and Galileo systems was done for Geometry Based model with coordinates unknown and ionosphere weighted scenario. While forming the model, zenith ionospheric delays were parameterized, additionally clock and phase biases were also present indicating about the use of a un-differenced or zero-differenced type of parameterization. The standard deviations considered for phase and code measurements were 3mm and 30cm. With single frequency, ionosphere known scenario, a combined GPS and Galileo system was able to give full AR. The dual frequency combined GPS and Galileo system was redundant enough to resolve ambiguities for an ionospheric weighted model. The triple frequency combined GPS and Galileo system performed the best, instantaneous

100% of ambiguities were resolved for ionospheric standard deviation of upto 0.45m. With a stand alone GPS only, Galileo only, triple frequency system, instantaneous ASR of 100% was possible up to 0.18 and 0.23m ionospheric standard deviation for GPS and Galileo respectively. However, the method for simulating AR success rate was not mentioned and the reason to use a single value of standard deviation for the measurement for all the frequencies on GPS and Galileo was not clear. Since the GPS $L5$ and future Galileo frequencies promise better measurement precision.

ÓKeefe et. al. (2009), see [33], used the stand alone GPS and Galileo, as well as the combined GPS and Galileo together for orbit determination of Low Earth Orbiting (LEO) satellites. For the same single, dual and triple frequency combinations were formed for Geometry Based model. The success of ambiguity resolution was evaluated using two approaches, Cascade Ambiguity Resolution (CAR) and LAMBDA method. Integer Bootstrap was used as a lower bound along with LAMBDA decorrelation to compute the ASR. While the combined GPS and Galileo system were used together, different reference satellite for each system is understood to be considered. With CAR method $E1/E2$ ambiguities were formed for single frequency, Wide-Lane (WL) with $E1 - E2$ and $E1/E2$ ambiguities were formed with dual frequency, Extra-Wide-Lane (EWL) with $E5a - E5b$, WL and $E1/E2$ were formed for triple frequency system. It was concluded that the magnitude of the gain was less for ASR when a third frequency was added as compared to the magnitude of gain obtained during the transition from single frequency GNSS to dual frequency GNSS. Further it was concluded that LAMBDA method of AR is superior in comparison with CAR. LAMBDA performed the best for single frequency combined GPS and Galileo system.

Feng et. al. (2009), see [38], presented an analysis for three carrier ambiguity resolution for GPS and Galileo with TCAR and LAMBDA method of integer ambiguity estimation. Extra-Wide-Lane, Wide-Lane combinations were formed for both GPS and Galileo and ambiguity success rates were computed for integer rounding method with TCAR. LAMBDA method generally uses integer least square for integer ambiguity estimation from GNSS observables, real or simulated. In case GNSS observables were not involved in this research, ambiguity success rate could be computed with integer bootstrap for LAMBDA. Instantaneous AR success rates were compared for TCAR and LAMBDA. The LAMBDA method of AR was found to give better instantaneous ambiguity success rates than

TCAR. It would have been interesting to further see the comparison of TCAR and LAMBDA in terms of epochs taken to obtain 100% ambiguity success rate.

Analysis of literature review

Zhang et. al. [25] and Sauer et. al. [27] gave results for GPS and Galileo, triple frequency, with respect to TTF for ambiguities. For 20 Km baseline, the mean TTF were 3 sec (seconds) and 2 sec for GPS and Galileo as given by [25], where as the mean TTF was found to be 18 sec and 20 sec for GPS and Galileo, as given by [27]. It is noted that [25] used CIR for AR which he modified by implementing LAMBDA method for AR, whereas [27] used FAMCAR. Also, [25] considered optimistic values of measurement precisions, say 0.04m for code and 0.6mm for phase data, whereas measurement precision values from [27] were 22 and 20 cm for code for GPS and Galileo, and 7 and 5cm for phase for GPS and Galileo respectively.

Zhang et. al. (2003) [25] and ÓKeefe et. al. (2006) [30] gave mean TTF for triple frequency GPS and Galileo for 30, 40 and 50 Km baseline. The mean TTF for 30, 40 and 50 Km baseline were 9, 19, 71 and 5, 25, 36 for GPS and Galileo respectively as given by [25]. Whereas, mean TTF values as given by [30] for 30, 40 and 50 Km baseline were 540, 710, 900 and 580, 600, 840 for GPS and Galileo respectively. For the combined system, three carrier frequencies in each GNSS, the mean TTF for 30, 40 and 50 Km baseline were 4, 5, 20 as given by [25] and 10, 40, 100 as given by [30]. Is is noted that [25] used CIR with LAMBDA for AR whereas [30] used integer-bootstrap based on LAMBDA decorrelation. Additionally [30] did not include code data on $L1$ and $E1$ frequencies. [25] considered optimistic values of measurement precisions, say 0.04m for code and 0.6mm for phase data, whereas measurement precision values from [30] were not given, instead thermal noise values were presented. For code data the thermal noise was considered to be 0.05m for ($L5$), 0.29m for ($L2$), 0.06m for ($E5a, E5b$) and for phase observables, thermal noise for all carrier frequencies was set to 0.003 cycles (0.6mm for 0.2m of carrier wavelength). ÓKeefe considered more realistic values of measurement precision, for example code precision on $L2$ was 0.29m, whereas Zhang considered 0.04m code precision on all carriers. While both the researchers used simulated observables, LAMBDA method for ambiguity estimation and geometry-based model, the possible reason for difference in TTF for the two research works could be the measurement precision values. results presented by ÓKeefe should be considered to be more realistic as compared to Zhang.

Zhang et. al. [25] present instantaneous ASR for triple frequency (TF), GPS and Galileo, stand-alone, for various baselines, here we compare the 10 and 20 Km baseline results form [25] with 15 Km baseline results of instantaneous ASR from Verhagen et. al. (2007) [11]. The instantaneous ASR values for GPS were given as 62% and 18% for 10 and 20 Km baseline and for Galileo they were 56% and 18% for 10 and 20 Km baseline, as given by [25] for geometry based model, incorporating the effects of atmosphere, orbit and multipath. Whereas, the instantaneous ASR values, as given by [11], were 100% for 15 Km baseline for both GPS and Galileo, for geometry based model by without taking into affects the atmosphere, orbit or multipath. The measurement precision values as considered by [25] were 0.04m for code and 0.6mm for phase data, whereas [11] considered code precision to be 10 and 20 cms at $L1$ and $L5$ respectively and for phase, the precision considered was 1.3mm and 1mm at $L1$ and $L5$ respectively. Also, [25] used CIR for AR which he modified by implementing LAMBDA method for AR, whereas [11] used sharp lower-bounds of probability of Integer Bootstrap based on LAMBDA decorrelation. Zhang simulated the observables for computation of ASR, whereas the ASR from Verhagen were based on model assumptions which is one of the major differences in the approach of the two researchers.

The integer ambiguities could be resolved quicker by forming Wide-Lane combinations than by resolving the ambiguities independently. Such combinations are adopted by various ambiguity resolution algorithms like CIR, CAR. With the advance of GNSS systems, by availability of multi GNSS frequencies, in addition to Wide-Lane, Extra-Wide-Lane combinations could be formed. Multi carrier ambiguity resolution algorithms like TCAR make use of such combinations. Several literatures presenting multi-frequency ambiguity resolution make use of Wide-Lane and Extra-Wide-Lane combinations for AR, see [29], [25], [31], [33], [35], [38]. The ambiguity resolution by forming Extra-Wide-Lane combination is found to be quickest in comparison to Wide-Lane combination. Literatures working with Extra-Wide-Lane combination have report instantaneous AR for Extra-Wide-Lane ambiguities, see [35]. The ambiguity resolution method, Least squares AMBiguity Decorrelation Adjustment (LAMBDA) is found to form the most optimal linear combination of ambiguities while decorrelating the ambiguity Variance Covariance matrix, see [33]. Further discussions on the process of ambiguity resolution can be found in more detail in chapter 3.

3 Theory of Ambiguity Resolution

The ambiguities are the unknown integer cycles present in the GNSS carrier phase data. These are the initial unaccounted carrier phase cycles in the receiver and are present as a fixed bias throughout in the carrier phase data, unless the receiver faces a loss-of-lock or a cycle slip while logging the data. Since the magnitude of these ambiguities can range from a few centimeters (cm) to many meters (m), i.e. $n\lambda$, where $n \in \{0, \pm1, \pm2, \pm3, \cdots\}$ is an integer, it is important to mitigate then in-order to utilize the precise carrier phase data effectively.

The commonly used ambiguity techniques include the LAMBDA method and the Wide-Lane / Narrow-Lane techniques. The LAMBDA method proposed by [39] is based on the most optimal Least square estimation technique, that is widely used today. The LAMBDA method uses the Z-transformation to decorrelate the ambiguities, which causes reduction in size of the ambiguity search space by preserving volume, resulting in faster and efficient AR. A detailed explanation of the Z-transformation can be found in the commencing section. On the other hand, Wide-Lane and Narrow-Lane combinations focus on forming the linear combinations of the observables resulting in a wider net-wavelength (for example $E5a - E5b$ would give a net-wavelength of about 9.7684) and a narrower net-wavelength (for example $E1 + E2$). Since in Wide-Lane and Narrow-Lane approaches linear combinations of observables are formed, there is an increase in the measurement noise which degrades the quality of the observables, see [35]. LAMBDA does not pre-decide the combinations instead the most optimal combinations are chosen by the algorithm while the Variance-Covariance matrix of the ambiguities is decorrelated. Hence the linear combinations are not formed in the observation space, instead combinations are formed at the parameter space. This avoids any loss of information by forming linear combination of observables as well as any degradation of the measurement precision. Several studies have been successfully carried using LAMBDA method for multi-frequency AR [28], [30], [11], [31], [33]. Forming optimal combinations on the fly by LAMBDA method further increases its flexibility to apply it to multi-frequency scenarios. Studies showed that LAMBDA method was found to be an efficient method for AR as compared to generic Wide-Lane / Narrow-Lane techniques, [40], [31], [33].

3.1 Theory of AR and improvement in precision of parameters of interest

The mathematical expectation and dispersion for GPS observables is given as below.

$$E(y) = A_I x_I + A_{II} x_{II} \tag{3.1}$$

The above is also referred to as a functional model. In equation (3.1), E is the expectation operator, where y is the vector of GPS observables (carrier phase and code), A_I and A_{II} are the design matrices of ambiguities (non-temporal variation) and ionosphere, ranges (temporally varying parameters) and x_I and x_{II} are the unknowns, namely, ambiguities (non-temporal variation) and ionosphere, ranges (temporally varying) to be estimated. The ambiguities, x_I, are integer in nature, however they have to be estimated as integers from the existing real-value, can be denoted as $x_I \in \mathbb{Z}$. Where as the non-ambiguity parameters belong to a real space, can be denoted as $x_{II} \in \mathbb{R}$.

$$D(y) = Q_{yy} \tag{3.2}$$

The above is also called as a stochastic model. In equation (3.2), D is the mathematical dispersion, Q_{yy} is the measurement precision matrix of phase and code observables.

By using the information in the design matrices A and B and the precision Q_{yy}, the precision of the unknowns $Q_{\hat{x}_I}$ can be computed. Since $Q_{\hat{x}_I}$ partially depends on the design matrix A_{II}, its value will be different for Geometry Free and Geometry Based models. Here $Q_{\hat{x}}$ represents the variance-covariance matrix of the float solution with estimated unknowns $\hat{x}_I$ and $\hat{x}_{II}$, it is to be noted that here $\hat{x}_I$ represents non-integer ambiguities. The following is the expression for the estimated unknowns and its variance-covariance matrix for the float solution.

$$\hat{x} = \begin{pmatrix} \hat{x}_I \\ \hat{x}_{II} \end{pmatrix}, \quad Q_{\hat{x}} = \begin{pmatrix} Q_{\hat{x}_I} & Q_{\hat{x}_I \hat{x}_{II}} \\ Q_{\hat{x}_{II} \hat{x}_I} & Q_{\hat{x}_{II}} \end{pmatrix} \tag{3.3}$$

In the above equation we can see that the covariance exist between ambiguities and non-ambiguity parameters. Also there exist correlation between the ambiguities, denoted by sub-matrix $Q_{\hat{x}_I}$. Here we use the LAMBDA method to de-correlate the ambiguities. The ambiguity VC (Variance-Covariance) matrix is de-correlated by Z-transformation by using Z^T matrix defined as below.

$$\hat{z} = Z^T \hat{x}_I, \quad Q_{\hat{z}} = Z^T Q_{\hat{x}_I} Z \tag{3.4}$$

After obtaining an almost de-correlated $Q_{\hat{z}}$ matrix, the original ambiguities are fixed to integer values.

$$\check{x}_I = Z^{-T} \check{z} \tag{3.5}$$

where $\check{x}_I$ and $\check{z}$ corresponds to integer valued ambiguities (fixed-ambiguities). These integer ambiguities can be used to obtain the fixed solution for non-ambiguity parameters.

$$\check{x}_{II} = \hat{x}_{II} - Q_{\hat{x}_{II}\hat{x}_I} Q_{\hat{x}_I}^{-1} (\hat{x}_I - \check{x}_I) \tag{3.6}$$

The precision of the non-ambiguity parameters can be expected to improve, applying the propagation law to the above equation, we get

$$Q_{\check{x}_{II}} = Q_{\hat{x}_{II}} - Q_{\hat{x}_{II}\hat{x}_I} Q_{\hat{x}_I}^{-1} Q_{\hat{x}_I\hat{x}_{II}} \tag{3.7}$$

The probability distribution of the fixed solution can be given as,

$$\begin{aligned} P(\check{x}_I = z) &= \int\limits_{S_z} p_{\hat{x}_I}(y) dy \\ p_{\check{x}_{II}} &= \sum_{z \in \mathbb{Z}} p_{\hat{x}_{II}|\hat{x}_I}(x|y = z) P(\check{x}_I = z) \end{aligned} \tag{3.8}$$

where $S_z = \{y \in \mathbb{R} | z = \arg\min_{u \in \mathbb{Z}} \|y - u\|^2_{Q_{\hat{x}_I}}\}$, y can be said to be the original observations (ambiguities), u corresponds to the fixed real valued solution of the original ambiguities (y).

Once the ambiguities are de-correlated as shown in equation (3.5), an transformation is carried and the real-valued ambiguities are mapped to the integer space. This an be done by choosing an appropriate integer transformation method. The Integer rounding (IR) is the simplest of all the methods. We also have Integer Bootstrapping (IB) and Integer Least Squares (ILS). However the probabilities obtained using different methods may differ. The discussion for obtaining Ambiguity Success Rates (ASR) using each of the methods, IR, IB and ILS is presented in the coming section.

3.2 Types of ambiguity estimation techniques

Once we transform the float ambiguities $\hat{x}_I$ to de-correlated $\hat{z}$ by de-correlating the $Q_{\hat{x}_I}$ to form $Q_{\hat{z}}$, we can use the de-correlated real-valued ambiguities $\hat{z}$ and transform them into integer values. This can be done by using IR, IB or ILS ambiguity transformation.

1. **IR:** Integer Rounding is one of the simplest methods to round the ambiguities, the transformation can be shown as below

$$\check{z}_{IR} = ([\hat{z}_1], \cdots, [\hat{z}_m]) \tag{3.9}$$

   where, $[\cdot]$ means rounding to the nearest integer, m indicate the number of real ambiguities, $\check{z}_{IR}$ is the integer ambiguity on which m real ambiguities are mapped.

   Also we can see, that IR does not consider the correlation between the ambiguities. If an expression for probability for IR has to be given when ambiguities are correlated, it would correspond to a lower bound of probability. The expression for lower bound of probability of IR can be given as below,

$$P(\check{x}_{I_{IR}} = x_I) \geqslant \prod_{i=1}^{n} 2\Phi(\frac{1}{2\sigma_{\hat{x}_{I_i}}}) - 1 \tag{3.10}$$

   On the other hand, the ambiguities can be de-correlated, see LAMBDA method of ambiguity de-correlation [39]. The probability of integer rounding for m de-correlated ambiguities can be given in terms of an upper bound of probability of IB [2],

$$P(\check{x}_{I_{IR}} = x_I) \leqslant \prod_{i=1}^{n} 2\Phi(\frac{1}{2\sigma_{\hat{z}_{i|I}}}) - 1 \tag{3.11}$$

   where $\hat{z}_{i|I}$ denotes the conditional variance of the i^{th} ambiguity, to be explained in commencing section (IB), and Φ denotes the standard normal distribution, which can be given as

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}z^2)dz \tag{3.12}$$

2. **IB:** Integer Bootstrap though uses the IR, however it takes the correlation between the ambiguities into account. The bootstrapped estimator follows from a sequential Least Square adjustment and it is computed as follows. If m ambiguities are available, we start with the first ambiguity $\widehat{z}_1$, and round its value to the nearest integer. The remaining float ambiguities are corrected by virtue of their correlation with the first fixed ambiguity. Then the second, but now corrected, real-valued ambiguity estimate is rounded to its nearest integer and all remaining $(m-2)$ ambiguities are then again corrected, but now by virtue of their correlation with the second ambiguity. This process is continued until all ambiguities are considered. The components of the bootstrapped estimator $\check{z}_{IB}$ are given as

$$\begin{aligned} \check{z}_{IB,1} &= [\widehat{z}_1] \\ \check{z}_{IB,1} = [\widehat{z}_{2|1}] &= [\widehat{z}_2 - \sigma_{\widehat{z}_2\widehat{z}_1}\sigma\widehat{z}_1^{-2}(\widehat{z}_1 - \check{z}_{IB,1}) \\ &\vdots \\ \check{z}_{IB,m} = [\widehat{z}_{m|M}] = [\widehat{z}_m - \sum_{i=1}^{m-1} &\sigma_{\widehat{z}_m\widehat{z}_{i|I}}\sigma\widehat{z}_{i|I}^{-2}(\widehat{z}_{i|I} - \check{z}_{IB,i}) \end{aligned} \tag{3.13}$$

where $\widehat{z}_{i|I}$ denotes the i^{th} Least Square ambiguity obtained by conditional rounding of previous $I = \{1, \cdots, i-1\}$ sequentially rounded ambiguities.

The integer bootstrap probability for n ambiguities as found in [2], can be given as

$$P(\check{x}_{I_{IB}} = x_I) = \prod_{i=1}^{n} 2\Phi(\frac{1}{2\sigma_{\widehat{z}_{i|I}}}) - 1 \tag{3.14}$$

3. **ILS:** The Integer Least Squares method is the most rigorous integer method, which maps the integers from their float values by searching the optimal integer grid points over the m-dimensional hyper-ellipsoid defined by the variance-covariance matrix $Q_{\widehat{z}\widehat{z}}$ and a positive constant χ^2 with center $\widehat{z}$

$$(z - \widehat{z})^T Q_{\widehat{z}\widehat{z}}^{-1} (z - \widehat{z}) \leqslant \chi^2, \text{ with } z \in \mathbb{Z}^m \tag{3.15}$$

The integer grid point z inside hyper-ellipsoid that derives the minimum value of function $\|z - \widehat{z}\|^2_{Q_{\hat{z}\hat{z}}}$ is the optimal solution $\check{z}$

The probability of Integer Least Squares for n ambiguities can be given in terms of a lower bound of probability of IB [2],

$$P(\check{x}_{I_{ILS}} = x_I) \geqslant \prod_{i=1}^{n} 2\Phi(\frac{1}{2\sigma_{\hat{z}_{i|I}}}) - 1 \tag{3.16}$$

3.2.1 Easy bounds for ASR

Since in this research work, the focus is on simulation of ASR using model assumptions, the evaluation of ASR can be done by bounds which are easy to compute. It is well known about the relation between the probabilities of successful AR obtained through IR, IB and ILS, which is given as $P(\check{x}_{I_{IR}} = x_I) < P(\check{x}_{I_{IB}} = x_I) < P(\check{x}_{I_{ILS}} = x_I)$. Hence $P(\check{x}_{I_{IB}} = x_I)$ can be used as an upper-bound for IR and a lower bound probability for ILS. Since the P_{IB} is used as a lower bound for P_{ILS}, it is important that the simulated results replicate results as real as possible. For the same it is important that the ambiguities are *de-correlated* while computing the lower bound probability by IB. The de-correlated ambiguities (of the float-solution, VC matrix) will result in a *"sharp"* lower bound probability for ILS. The ASR computed and analyzed in this research work are obtained using probability of IB, as given in equation (3.14).

3.3 Partial Ambiguity Resolution

In cases where full AR take time to converge to a fixed solution, PAR should be considered. However the PAR makes sense only if the fixed solution is better than the float solution. That is the precision of non-ambiguity parameters obtained from the float solution and fixed solution (PAR) can be compared and then it can be decided whether it is sensible to accept the fixed solution (PAR). The time taken for resolution of all of the ambiguities (Full AR) can be termed as convergence time (CT). CT depends on the strength of the model, the number of unknowns subtracted from the number of observations. For any GNSS model, satisfactory ASR using dual frequency GPS mostly takes time to converge. With increasing the number of satellites by addition of Galileo system, there is a promise for the improvement of ASR. None the less, for this research work, in cases where satisfactory ASR takes time to converge for GPS only and Galileo only, PAR will be performed.

3.3.1 Method of implementing PAR

If PAR is desired to be performed, one first defines an minimum value of ASR based on ones application at hand. Then PAR starts with initially choosing the most precise, de-correlated ambiguity and ASR is computed. If the ASR is found to be higher than the defined value (say 0.9, 0.999 etc.), then the second best ambiguity is added to the first one and PAR is performed. LAMBDA method performs a de-correlation on the ambiguities and further uses ILS for ambiguity fixing, however in this research we use LAMBDA de-correlation and IB for ambiguity fixing. Further, on adding the second ambiguity if the ASR doesn't drop below the cutoff, a third ambiguity is added. This is done until ASR drops below desired cutoff, then the previous value of ASR and ambiguities corresponding to that ASR are considered for further computations (for gain of non-ambiguity parameters).

The Variance Covariance (VC) matrix for the float ambiguities can be mathematically expressed, say for n ambiguities the precision can be given by $Q_{\widehat{x_I}}$. While performing the LAMBDA decorrelation on the ambiguity VC matrix $Q_{\widehat{x_I}}$, and further fixing of the ambiguities, only a subset of ambiguities say p are fixed which fulfill the minimum success rate criteria of 0.999. The design matrix corresponding to the ambiguities fixed by PAR is denoted as $Z_{x_I = \check{x}_I}$, can be used to obtain the fixed precision of non-ambiguity parameters. The precision of the ambiguity fixed solution based on PAR can be given as follows

$$\underbrace{Q_{\hat{z}_I}}_{p\times p} = \underbrace{Z_{x_I=\check{x}_I}}_{p\times n}^{T} \underbrace{Q_{\widehat{x_I}}}_{n\times n} \underbrace{Z_{x_I=\check{x}_I}}_{n\times p} \tag{3.17}$$

Further, the improvement in precision of the parameters of interest can be computed from the VC matrix of the ambiguity fixed solution from equation (3.17), as follows

$$Q_{\check{x}_{II}} = Q_{\hat{x}_{II}} - Q_{\hat{z}_{II}\hat{z}_I} Q_{\hat{z}_I}^{-1} Q_{\hat{z}_I\hat{z}_{II}}$$

where $Q_{\hat{z}_I\hat{z}_{II}} = Z_{x_I=\check{x}_I}^T Q_{\hat{x}_I\hat{x}_{II}}$ and $Q_{\hat{z}_{II}\hat{z}_I} = Q_{\hat{z}_I\hat{z}_{II}}^T$.

Since with PAR, only a subset of ambiguities are fixed, the improvement in precision of the parameters of interest is dependent on the percentage of the ambiguities fixed, out of the total number of ambiguities.

3.4 Model Assumptions for the ASR Computation

The computation of ASR and calculation of improving in precision of (ambiguity fixed) non ambiguity parameters is done by simulations based on model assumptions. The model assumptions, namely the design matrix and the measurement precision matrix differ for different model types, say Geometry Free model with ionosphere known will have different model assumptions as compared to Geometry Free model, ionosphere unknown. Firstly, we discuss the basic Geometry Free model model assumption. Secondly, we discuss Geometry Free model for different ionosphere types in dedicated sections, following here after. We start our discussion by recalling the mathematical expectation and dispersion again.

$$E(y) = A_I x_I + A_{II} x_{II}$$

$$D(y) = Q_{yy}$$

In the above equations, A_I and A_{II} are the design matrices (functional model) and Q_{yy} is the measurement precision matrix (stochastic model). Further we explain in detail the formation of each of the mentioned models.

3.4.1 Functional model

The formation of design matrices for dual frequency, ionosphere fixed, Geometry Free GNSS model is explained in detail in Appendix B. Further under each subsection, namely, ionosphere fixed, float and weighted, the general form of design matrix for m satellites, j frequencies and k epochs are presented.

3.4.2 Stochastic model

The stochastic model represents the variability of all the observables, say carrier phase and code for every GPS frequency. Also when the errors in the GNSS observables, like ionosphere, are significant enough and on availability of corrections, can be a-priori defined in the GNSS model with its precision. The precision of the corrections are parameterized in the stochastic model (say as in ionosphere weighted case). Now coming back to the measurement precision, the values of measurement precision considered here are influenced and selected for low-end GPS receivers, high-end receivers and future $L5$ frequency (especially the code precision) for GPS.

Table 3.1 shows the values chosen for phase and code (un-differenced) standard deviations (in meters) for GPS. Further measurement precision values for Galileo are presented. In this research work, there are two cases under which the standard deviation values of the measurements are selected. In the first case the measurement precision values are varied and ASR and gain is computed, then in the second case, the measurement precision is kept fixed to a value and number of satellites or the baseline length is varied. Listed below are the numerical values used in both the above mentioned cases.

(1) **Measurement precision values for future GPS**

(i) Measurement precision is varied, satellite number held fix (6 satellites).

Below are the values of measurement precision for Phase (Φ) and code (P).

Sr. No	**Low-end receivers**		**High-end receivers**			
	$\Phi1$	$P1$	$\Phi1$, $\Phi2$	$P1, P2$	$\Phi5$	$P5$
1	0.003	0.25	0.002	0.1	0.002	0.05
2	0.003	0.5	0.002	0.15	0.002	0.1
3	0.003	0.75	0.002	0.2	0.002	0.125
4	0.003	1	0.003	0.25	0.002	0.15
5	0.003	1.25	0.003	0.3	0.002	0.175
6	0.003	1.5	0.003	0.35	0.002	0.2

Table 3.1: Un-differenced standard deviation values at zenith (meters) for GPS (6 scenarios)

The following scenarios of single, dual and triple frequencies are used for simulations.

(a) Low-end receivers

Single frequency GPS - Measurement precision values $\Phi1$ and $P1$ given in Table 3.1 are used.

(b) High-end receivers

Dual frequency - Measurement precision values $\Phi1$, $\Phi2$ and $P1$, $P2$ are used.

Triple frequency - Measurement precision values $\Phi1$, $\Phi2$, $\Phi5$ and $P1$, $P2$, $P5$ are used.

(ii) Measurement precision held fix (see table below for values)

The measurement precision value is held fixed, instead, baseline length or number of satellites are varied.

When the baseline length is varied, the values chosen for the same are 1 to 1000 Kms (6 scenarios), see ionosphere weighted section.

In other scenario, the satellite number is varied from a minimum of 2 to a maximum of 6 (maximum number of satellites consistently available during the whole time period (24 hours)). Satellite number is varied in steps of 1 (6 scenarios). the measurement precision is held fixed as given in Table 3.2. Similar to above, three different cases of single, dual and triple frequencies GPS are considered for simulations, see Appendix D.

Low-end receivers		**High-end receivers**			
$\Phi1$	$P1$	$\Phi1$, $\Phi2$	$P1, P2$	$\Phi5$	$P5$
0.003	0.5	0.003	0.25	0.002	0.15

Table 3.2: Un-differenced standard deviation values at zenith (meters) for GPS (1 scenario)

(2) **Measurement precision values for Galileo**

(i) Measurement precision is varied, satellite number held fix (6 satellites).

Measurement precision is varied, see Table 3.3, and satellite number is held fixed to 6. This is done for four different cases

Sr. No	Low-end receivers		High-end receivers			
	$\Phi 1$	$P1$	$\Phi 1$	$P1$	$\Phi 5a, \Phi 5b, \Phi 6$	$P5a, P5b, P6$
1	0.003	0.25	0.002	0.1	0.002	0.05
2	0.003	0.5	0.002	0.15	0.002	0.1
3	0.003	0.75	0.002	0.2	0.002	0.125
4	0.003	1	0.003	0.25	0.002	0.15
5	0.003	1.25	0.003	0.3	0.002	0.175
6	0.003	1.5	0.003	0.35	0.002	0.2

Table 3.3: Un-differenced standard deviation values at zenith (meters) for Galileo (6 scenarios)

(a) Low-end receivers

Single frequency Galileo - Measurement precision values $\Phi 1$ and $P1$ given in Table 3.3 are used.

(b) High-end receivers

Dual frequency - Measurement precision values $\Phi 1$, $\Phi 5a$ and $P1$, $P5a$ are used.

Triple frequency - Measurement precision values $\Phi 1$, $\Phi 5a$, $\Phi 5b$ and $P1$, $P5a$, $P5b$ are used.

Quadruple frequency - Measurement precision values $\Phi 1$, $\Phi 5a$, $\Phi 5b$, $\Phi 6$ and $P1$, $P5a$, $P5b$, $P6$ are used.

(ii) Measurement precision held fix (see table below for values)

The measurement precision value is held fixed, instead, baseline length or number of satellites are varied.

When the baseline length is varied, the values chosen for the same are 1 to 1000 Kms (6 scenarios), see ionosphere weighted section.

In other scenario, the satellite number is varied from a minimum of 2 to a maximum of 6 (6 scenarios), see Appendix D.

Using the above values of undifferenced measurement precision, the matrix

Low-end receivers		High-end receivers			
$\Phi 1$	$P1$	$\Phi 1$	$P1$	$\Phi 5a, \Phi 5b, \Phi 6$	$P5a, P5b, P6$
0.003	0.5	0.003	0.25	0.002	0.15

Table 3.4: Un-differenced standard deviation values at zenith (meters) for Galileo (1 scenario)

Q_{yy} consisting of DD values of measurement precisions, weighted for each satellite (based on satellite elevation) is computed. The exponential elevation weighting is applied, see Figure 3.1 The detailed description of formation of Q_{yy} can be found in Appendix B. The weighting function is as given as below (see also Appendix B for detailed description of formation of Q_{yy} using weighting function).

$$W = 2 * \sigma_i^2 * (\text{ diag } (1 + a * \exp(-\varepsilon/\varepsilon_0)))^2$$

where 2 is for double differenced observations, a is an amplification angle depending on observation type and frequency, here it is kept same for all observations and frequency to 10, ε is elevation of satellite, ε_0 is a coefficient for the scale value of the elevation error, see [41]. In this study ε_0 is set to 10 degrees. See Appendix D for detailed presentation of the weighting function. Further, σ_i is given as

$$\sigma_i = \begin{bmatrix} \sigma_{\Phi_1^1} \\ \vdots \\ \sigma_{\Phi_j^m} \\ \sigma_{P_1^1} \\ \vdots \\ \sigma_{P_j^m} \end{bmatrix}$$

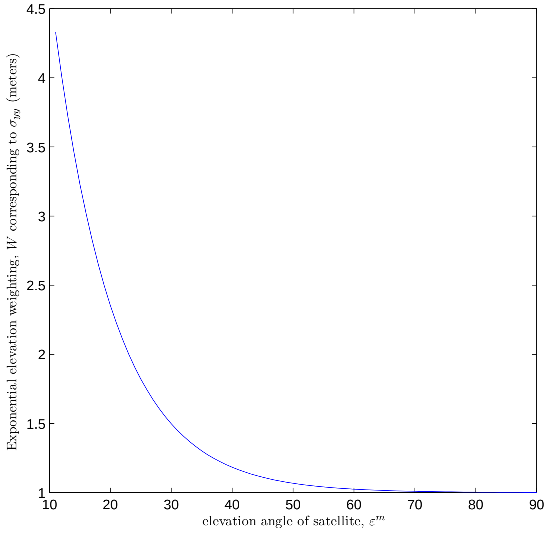

Figure 3.1: Undifferenced satellite elevation weighting (Exponential function)

Satellite elevation weighting over long distances

The satellite elevation is computed individually for each of the stations of the baseline for ionosphere float (baseline considered is 1000 km) and ionosphere weighted (for baselines from 1 Km and up to 1000 Km) scenarios. This is done by recomputing the coordinates of the second receiver forming the baseline with respect to the first receiver. While doing so, the second receiver is considered to be located at he same latitude as the first receiver, only longitudinal variation is considered for varying baseline length. For each of the receivers, GNSS satellites overhead are simulated from the almanacs and elevation of satellites are computed. The common satellites between each of the receiver are then selected and satellite elevation weighting function is further applied. This may be computed as follows.

$$\begin{aligned} W_{el_1}^{-1} &= \sigma_i^2 * \left(\text{diag } \left(1 + a * \exp(-\varepsilon_1^{1,\cdots,m}/\varepsilon_0)\right)\right)^2 \\ W_{el_2}^{-1} &= \sigma_i^2 * \left(\text{diag } \left(1 + a * \exp(-\varepsilon_2^{1,\cdots,m}/\varepsilon_0)\right)\right)^2 \\ W_{el}^{-1} &= W_{el_1}^{-1} + W_{el_2}^{-1} \end{aligned} \qquad (3.18)$$

where $W_1, W_2, \varepsilon_1, \varepsilon_2$ are the weighting functions and satellite elevations for receiver 1 and 2 respectively, the elevation for each receiver, $\varepsilon_1, \varepsilon_2$, are vectors having elevation of 1 to m satellites, a and ε_0 are the coefficients set to the values 10. The functions *diag* is a matlab function indicating a diagonal matrix. The elevation weighting function can be elaborated to express the *diag* function as below. The elevation weighting function for single GPS frequency $L1$ for receiver 1 and satellites $1, \cdots, m$ is given as below

$$W_{el_1}^{-1} = \begin{bmatrix} (W_{\phi(L1)}^{1})^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & (W_{\phi(L1)}^{m})^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & (W_{P(L1)}^{1})^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & (W_{P(L1)}^{m})^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} \sigma^2_{\phi(L1)} * \left(1 + a_{\phi(L1)} \exp(-\frac{\varepsilon^1}{\varepsilon^0})\right)^2 & \dots & 0 \quad 0 & & 0 \\ \vdots & & & & \vdots \\ 0 & & \ddots & & 0 \\ \vdots & & & & \vdots \\ 0 & & 0 \quad 0 & \dots & \sigma^2_{P(L1)} * \left(1 + a_{\phi(L1)} \exp(-\frac{\varepsilon^1}{\varepsilon^0})\right)^2 \end{bmatrix}$$

For multi-frequency case, the measurement precision of phase and code can be given in a vector form as $\sigma^2_{1,\cdots,j} = \left[\sigma^2_{\phi_1}, \cdots, \sigma^2_{\phi_j}, \sigma^2_{P_1}, \cdots, \sigma^2_{P_j}\right]^T$, the satellite elevation weighing function can now be expressed as follows

$$W_{el_{1,j}}^{-1} = \mathrm{diag}(\sigma^2_{1,\cdots,j}) \otimes \mathrm{diag}(\left(1 + a * \exp(-\varepsilon_1^{1,\cdots,m}/\varepsilon_0)\right)^2) \tag{3.19}$$

The detailed explanation of exponential satellite elevation weight is explained in Appendix D.

Elevations are interpreted in skyplot figures for GPS, Galileo and both the constellations together in the commencing section. For ionosphere weighted scenario, the standard deviation of the ionosphere is computed, depending upon the baseline length, considering that the ionosphere would vary at 6.8 mm per 10 Km (between receiver single difference ionosphere standard deviation, σ_I).

Skyplots of GNSS constellations

The skyplots of GNSS constellations, namely GPS, Galileo and GPS+Galileo together are given in Figure 3.2. The selected latitude locations are 0°, −30° and −60° latitude and longitude is kept fixed at 115°. For all the skyplots, the cutoff angle chosen is 10°. GPS constellation satellite are numbered 1 to 31 (31 satellites), whereas Galileo constellation satellites are given numbers 51 to 70 (30 satellites). The orbit information for each of the GNSS systems is extracted from YUMA almanacs for GPS [42] and Galileo [43], throughout this research work.

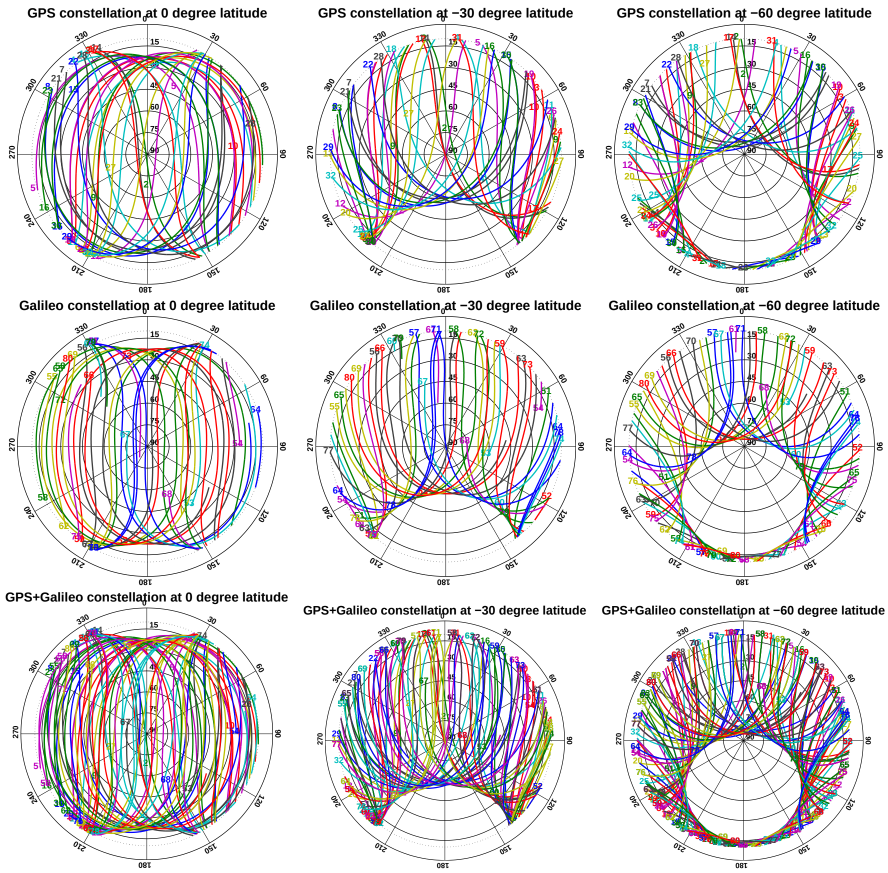

Figure 3.2: Sky plot of the satellites at 0°, −30° and −60° degree latitude, 150° longitude, for GPS, Galileo and both GPS+Galileo systems for 13-05-2011 from 00:00:00 to 23:59:30 hrs UTC

Stochastic model and result comparisons for different ionosphere scenarios

In ionosphere weighted model, only one value of phase and code precision σ_Φ, σ_P is considered, refer Table 3.2, and σ_I is varied based on the baseline length. The value of measurement precision, σ_Φ, σ_P, for ionosphere weighted scenario given in Table 3.2. The value of σ_Φ, σ_P which is considered for ionosphere weighted scenario also appears for ionosphere fixed and float scenarios, see Table 3.1 (highlighted in grey color). This overlapping measurement precision values would be used to compare the results, ASR and gain etcetera, between these three different ionosphere scenarios.

The Ambiguity Success Rate(ASR) for Geometry Free model is evaluated for only GPS system for single, dual, triple and quadruple frequency combinations. Measurement precision for $L5$ is considered only while evaluating ASR for triple frequency GPS.

The formation of the stochastic model for dual frequency, ionosphere fixed Geometry Free GNSS model is explained in detail in Appendix B. Hence for Geometry Free model in chapter 4, there is no further discussion regarding the formation of Q_{yy}. The computation of ASR and gain are discussed in Geometry Free model chapter 4. Further under each sub-section, namely, ionosphere fixed, float and weighted, the general form of stochastic model for m satellites, j frequencies and k epochs are presented.

The stochastic models for Geometry Based and Reference Rover model can be found in individual chapters, chapter 5 and 6, respectively.

4
Geometry Free model

The Geometry Free model, as the name suggests, does not consider the geometry between the receiver and the satellite. Instead of using satellite and receiver coordinate information and estimating receiver baseline / coordinates, the ranges between receiver and satellite are estimated. The ranges include the tropospheric delays lumped in it for a Geometry Free model.

The research presented in the current report is based on **GPS ($L1, L2$ and $L5$), Galileo ($E1, E5a, E5b, E6$) system data frequencies, standalone and together for Geometry Free model**. Ambiguity Success Rate (ASR), precision for the fixed (ambiguity-fixed) and float (ambiguity-float) solution are presented.

The results for ASR and precision of the ranges are computed while considering all the ambiguities present, this is also called as Full Ambiguity Resolution (Full AR). Further, research on Partial Ambiguity Resolution (PAR) is done, results based on PAR for ASR and precision of the ranges for GPS, Galileo and both the systems together (GPS+Galileo) are also presented.

In this research an attempt is made to understand as to how different combinations of frequencies, measurement precisions, ionosphere information (known, unknown, weighted) etcetera, affect the ASR and hence the precision of the estimated non ambiguity parameters (say range for a Geometry Free model). The troposphere is not parameterized separately for Geometry Free model, it is lumped with the ranges.

This research is based on evaluating the Ambiguity Success Rates and its effect on precision of the estimated parameters for the future GPS and Galileo by simulation of model assumptions, since real observables for future GNSS are unavailable. In the following section, Geometry Free model is discussed by considering the functional (design matrix) and the stochastic models (measurement precision matrix) which are here referred to as model assumptions. Further, by considering different scenarios of the atmosphere, namely the ionosphere, the functional and stochastic model is discussed for each of the ionosphere types, namely, known, unknown and weighted.

4.1 Geometry Free model design:

The Geometry Free model observables for double-differenced (DD) phase and code can be given as below.

$$\Phi^{1-s}_{(1-r),j} = \rho^{1-s}_{(1-r)} - \mu_j I^{1-s}_{(1-r)} + \lambda_j N^{1-s}_{(1-r),j} + \epsilon^{1-s}_{(1-r),j} \tag{4.1}$$

where, subscripts r, j and s indicate receiver, frequency number and the satellite respectively. The subscripts 1 indicate that satellite 1 is chosen as the reference satellite and same for the receiver. Φ, ρ, λ are the DD phase observations, range (geometric range and troposphere lumped together) and wavelength respectively. μ is $(f_1/f_j)^2$, f_1 is GPS frequency on $L1$ and f_j is j^{th} GPS frequency, ϵ is the error associated with the phase measurements.

Similarly for code, the DD observation equation can be given as

$$P^{1-s}_{(1-r),j} = \rho^{1-s}_{(1-r)} - \mu_j I^{1-s}_{(1-r)} + e^{1-s}_{(1-r),j} \tag{4.2}$$

where $P^{1-s}_{(1-r),j}$ is the DD code GPS data and $e^{1-s}_{(1-r),j}$ indicates the error of the code measurements.

4.1.1 Full ambiguity resolution

Computation of ASR and gain:

The computation of ASR and Gain is based on simulations which can be done by having information of the GNSS model chosen and measurement precision. Based on this information the errors in the GNSS observables are parameterized in a design matrix (*functional model*) and measurement precision matrix (*stochastic model*).

To begin with we recall that for the functional model the design matrix is computed at every epoch for the *temporal* varying (atmosphere / ionosphere, ranges) and the *non-temporal* parameters like the ambiguities (on the assumption that there are no cycle slips or loss of lock for the ambiguities).

For the stochastic model, as explained in Appendix B above, the satellite having the maximum elevation is chosen as a reference satellite (until it sets, it remains as a reference satellite), further considering the elevation weights for each satellite the final measurement precision matrix Q_{yy} is formed, see equation (B.9). The stochastic model too is computed at every epoch based on the elevation of the satellite etcetera.

For any GNSS model under consideration, say Geometry Free model, Ionosphere known scenario, the ASR is computed as explained below. The simulation explained below runs over from first to the k^{th} epoch.

The approach is similar to batch processing , the normal equations, denoted as $\mathcal{N}n$, are computed for every epoch and further stacked in $\mathcal{N}$. To formulate the computation of the normal matrix, the design matrix A, and the measurement precision matrix are used, they are given as,

$$\begin{aligned} A &= [A_I, A_{II}] \\ \mathcal{N}n &= A^T Q_{yy}^{-1} A \\ \mathcal{N}_k &= \mathcal{N}_{k-1} + \mathcal{N}n \end{aligned}$$

The above matrices, A_I, A_{II}, Q_{yy}, $\mathcal{N}n$ etcetera represent current epoch, whereas, $\mathcal{N}_k$ represent the stacked normal equation for k epochs.

The float solution, corresponding to the estimated ambiguity in non-integer form, has the Variance-Covariance of the non-ambiguity parameters, denoted as $Q_{\hat{x}\hat{x}}$ is given as

$$\boxed{Q_{\hat{x}\hat{x}} = \left(A^T Q_{yy}^{-1} A\right)^{-1} = \mathcal{N}_k^{-1} \qquad (4.3)}$$

Further the VC matrices for the ambiguity only, non-ambiguity only and covariance between the ambiguity and non-ambiguity parameters, are extracted from $Q_{\hat{x}\hat{x}}$, since.

$$\boxed{Q_{\hat{x}\hat{x}} = \begin{bmatrix} Q_{\hat{x}_I\hat{x}_I} & Q_{\hat{x}_I\hat{x}_{II}} \\ Q_{\hat{x}_{II}\hat{x}_I} & Q_{\hat{x}_{II}\hat{x}_{II}} \end{bmatrix}}$$

where $Q_{\hat{x}_I\hat{x}_I}$ correspond to variance matrix of ambiguity parameters, $Q_{\hat{x}_{II}\hat{x}_{II}}$ for non-ambiguity parameters and $Q_{\hat{x}_I\hat{x}_{II}}$ correspond to covariance between ambiguity and non-ambiguity parameters.

The VC matrix of ambiguities are de-correlated using the *decorr.m*, a Matlab software routine developed by Peter Joosten, Delft University and further modified by Bofeng Li, GNSS Research Group, Curtin University. The de-correlation routine returns the conditional variances of the ambiguities which are used to compute Integer Bootstrap ambiguity success probabilities (ASR) as given in chapter 3, equation (3.14). The detailed explanation of the computation of ASR using the LAMBDA software routines can be found in [44] and [45], it is also explained in Appendix C.

Further the precision the the temporal varying parameters (range) corresponding to ambiguity-fixed solution needs to be computed, it is denoted as $Q_{\check{x}_{II}}$. Firstly we recall the expression for computing $Q_{\check{x}_{II}}$ in chapter 3, equation (3.7).

$$Q_{\check{x}_{II}} = Q_{\hat{x}_{II}} - Q_{\hat{x}_{II}\hat{x}_I} Q_{\hat{x}_I}^{-1} Q_{\hat{x}_I\hat{x}_{II}}$$

By substituting the corresponding terms in the above equation, $Q_{\hat{x}_{II}}$ from equation (B.11), $Q_{\hat{x}_{II}\hat{x}_I} = {Q_{\hat{x}_I\hat{x}_{II}}}^T$, $Q_{\hat{x}_I\hat{x}_{II}}$ is as given in equation (B.14) and $Q_{\hat{x}_I}$ is as given in equation (B.10), equations given in Appendix B, $Q_{\check{x}_{II}}$ can be computed.

After desired ASR is achieved, the corresponding solution for $Q_{\check{x}_{II}}$ can be used to compute the gain of the other unknowns (ranges). Since the elevation weighting of the satellites is considered for simulation, the values of variance in the $Q_{\check{x}_{II}}$ for each satellite will be different. Hence the diagonal elements matrices $Q_{\check{x}_{II}}$ and $Q_{\hat{x}_{II}}$ are averaged over all the satellites, $(m-1)$ for every epoch.

$$\begin{aligned}
\overline{Q}_{\check{x}_{II}(i)} &= \frac{\sum\limits_{s=1}^{m-1} Q_{\check{x}_{II}}(s,s)(i)}{(m-1)} \\
\overline{Q}_{\hat{x}_{II}(i)} &= \frac{\sum\limits_{s=1}^{m-1} Q_{\hat{x}_{II}}(s,s)(i)}{(m-1)}
\end{aligned}$$

4.1.2 Partial ambiguity resolution

The VC matrices used in PAR are give as below.

The VC matrices, namely, $Q_{\hat{x}_I}, Q_{\hat{x}_{II}}$ and $Q_{\hat{x}_I\hat{x}_{II}}$ are extracted from $Q_{\hat{x}\hat{x}}$.

Further, after partially fixing the ambiguities using the LAMBDA software routines (for detailed explanation, see Appendix C, we get the de-correlated Z matrix ,which contains the information of the fixed ambiguities, for more information on design matrix of de-correlated ambiguities, Z, refer equation (3.5) in chapter 3.

The matrices $Q_{\hat{z}_I}$ and $Q_{\hat{z}_I\hat{z}_{II}}$ are computed after fixing the ambiguities as follows.

$$\left.\begin{aligned} Q_{\hat{z}_I} &= Z^T_{x_I=\check{x}_I} Q_{\hat{x}_I} Z_{x_I=\check{x}_I} \\ Q_{\hat{z}_I\hat{z}_{II}} &= Z^T_{x_I=\check{x}_I} Q_{\hat{x}_I\hat{x}_{II}} \end{aligned}\right\} \tag{4.4}$$

$$Q_{\check{x}_{II}} = Q_{\hat{x}_{II}} - Q_{\hat{z}_{II}\hat{z}_I} Q^{-1}_{\hat{z}_I} Q_{\hat{z}_I\hat{z}_{II}}$$

The average values for $Q_{\hat{x}_{II}}$ and $Q_{\check{x}_{II}}$, which correspond to the precision of the ranges for ambiguity-float and -fixed solutions are computed. For any i^{th} epoch, the average precision can be computed considering only ambiguities that are fixed, hence incase of PAR, only partial ambiguities are considered and in case of full AR, all ambiguities are considered. If p out of the $m-1$ ambiguities are fixed, than, the average precision of the ranges can be computed as follows

$$\begin{aligned} \overline{Q}_{\check{x}_{II}(i)} &= \frac{\sum_{s=1}^{p} Q_{\check{x}_{II}}(s,s)(i)}{p} \\ \overline{Q}_{\hat{x}_{II}(i)} &= \frac{\sum_{s=1}^{p} Q_{\hat{x}_{II}}(s,s)(i)}{p} \end{aligned}$$

Computation of gain:

The computation of gain is done using the above average values for float and fix solution variance. The gain or the improvement in precision is given as below.

$$\text{Gain}(i) = \frac{\overline{Q}_{\hat{x}_{II}(i)}}{\overline{Q}_{\check{x}_{II}(i)}} \tag{4.5}$$

4.1.3 Discussion of specific algorithms used in software

In this software while simulating the ASR and improvement in precision of the non-ambiguity parameters for ambiguity-fixed solution, the number of satellites are as available. The only constraint present is the cut-off angle of 10° for satellite to be accepted for simulations. Hence there are times that a satellite rises, sets as per the cut-off angle during the chosen day. Also there is a change in the reference satellite. To counter these effects, changes are applied at the level of normal equations, both at the stacked, $\mathcal{N}_{k-1}$, and the current epoch, $\mathcal{N}n$, normal equation.

(1) **Accounting for the new risen satellite**

The routine begins by identifying the rising satellite. For the same two variables are used which have information about the list of satellites (excludes the reference satellite) for the current and the previous epoch. The variables are `satc` and `satn` for the previous and the current epoch respectively.

(a) Identify the rising satellite

The Matlab code which reads as follows identifies the rising satellite
`[newsat, IA] = setdiff(satn, satc);`
where the output variables `newsat` has the PRN number of the satellite and `IA` has the order of the rising satellite as in the list `satn` corresponding to current epoch. For example

$$\texttt{satn} = \begin{bmatrix} 4 \\ 7 \\ 8 \\ 9 \end{bmatrix}; \texttt{satc} = \begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix}; \texttt{newsat} = 7; \texttt{IA} = 2$$

(b) Arrange in the order of new satellite last

A variable `ord` is generated which as the whole list of satellites ranking from `1` to `n`, `n` being the last satellite, then the rank of the new satellite is set to last. In the above example, `ord` will correspond to `[1;3;4;2]`, the rank of the new satellite, `2`, is pushed to last.

(c) Update the normal equation for the current epoch, $\mathcal{N}n$

Further the normal equation of the current epoch are updated as per the new order. Firstly as per the ambiguities, the variable `ord` is updated as below

`ref=ord; list=[1:1:n]` two variables are defined, `n` is the total number of satellites (single differenced)
`for i= 2:1:nf`, where `nf` are the number of frequencies chosen for the GNSS system
`ord = [ord ; pos+list(i-1)]`
`end`

Hence for dual frequency, with the above example the variable `ord` looks like `ord=[1;3;4;2;5;7;8;6]`.

Further ranges are added in the `ord` variable, considering ionosphere-fixed for a simple case, now the variable `ord` looks as, `ord=[1;3;4;2;5;7;8;6;9;11;12;10]`.

The normal matrix of the current epoch $\mathcal{N}n$ is updated as below
`temp=`$\mathcal{N}n$`(:,ord)`, $\mathcal{N}n$ `=temp(ord,:)`

(d) Insert zeros in the stacked normal equations, $\mathcal{N}_{k-1}$, for the new satellite If earlier, the normal equation matrix with only three pair of satellites, for dual frequency case, is updated as shown below

$$\mathcal{N}_{k-1} = \begin{bmatrix} \underbrace{Ambiguities_{\Phi_1}}_{3\times3} & \underbrace{Ambiguities_{\Phi_2\Phi_1}}_{3\times3} & \underbrace{RangesAmbiguities_{\Phi_1}}_{3\times3} \\ \underbrace{Ambiguities_{\Phi_1\Phi_2}}_{3\times3} & \underbrace{Ambiguities_{\Phi_2}}_{3\times3} & \underbrace{RangesAmbiguities_{\Phi_2}}_{3\times3} \\ \underbrace{Ambiguities_{\Phi_1}Ranges}_{3\times3} & \underbrace{Ambiguities_{\Phi_2}Ranges}_{3\times3} & \underbrace{Ranges}_{3\times3} \end{bmatrix}$$

$$\mathcal{N}_{k-1_{updated}} = \begin{bmatrix} \underbrace{Ambiguities_{\Phi_1}}_{3\times3} & 0 & \underbrace{Ambiguities_{\Phi_2\Phi_1}}_{3\times3} & 0 & \underbrace{RangesAmbiguities_{\Phi_1}}_{3\times3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \underbrace{Ambiguities_{\Phi_1\Phi_2}}_{3\times3} & 0 & \underbrace{Ambiguities_{\Phi_2}}_{3\times3} & 0 & \underbrace{RangesAmbiguities_{\Phi_2}}_{3\times3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \underbrace{Ambiguities_{\Phi_1}Ranges}_{3\times3} & 0 & \underbrace{Ambiguities_{\Phi_2}Ranges}_{3\times3} & 0 & \underbrace{Ranges}_{3\times3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(2) **Accounting for the set satellite** Changes are made to the normal equations $\mathcal{N}_{k-1}$, that is, the stacked normal equations until the previous epoch $k-1$. The order of the set satellite is identified from the variable *satc* and

satn, having information of the order of the satellites of the previous and the current epoch (excluding the reference satellite), as follows.

(a) Identify the order os the set satellite and create the order in which it appears in the normal equations as unknowns for ambiguities/ranges etcetera
The function *setdiff* is used to identify the order of the set satellite, as follows

```
[mssat, IA] = setdiff(satc, satn);
```

where variable `mssat` has the PRN number of the set satellite and `IA` has the order of the set satellite, as it appeared in variable `satc`. To give an example, for Ionosphere known, Geometry Free model with dual frequency GPS, we have

$$\texttt{satn} = \begin{bmatrix} 5 \\ 6 \\ 9 \end{bmatrix}; \texttt{satc} = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 9 \end{bmatrix}; \texttt{mssat} = 7; \texttt{IA} = 3$$

Further, the order in which the unknowns appear as per the set satellite is formed as follows.

```
pos = IA;
if model==1
for i = 2:1:(nxa/nsn)
pos = [pos, IA+nsc*(i-1)];
end
end
```

where `nxa` is total number of ambiguities and unknowns, in the above mentioned example, `nxa=9`, since number of satellites in current epoch are `3`, hence with dual frequency we have `6` ambiguities and `3` ranges. Whereas in the previous epoch, the number of satellites were `4`, hence the total number of unknowns were `12`. `nsn` is the total number of satellites (excluding the reference satellite) in current epoch, that is `3`.

As per the above example, we will have the variable `pos` equal to `[3, 7, 11]`.

(b) Update the normal equations of the previous epoch $\mathcal{N}_{k-1}$ - Gaussian elimination

```
N12 = 𝒩_{k-1}(:,pos); N22 = N12(pos,:); N12(pos,:)=[];
𝒩_{k-1}(:,pos)=[]; 𝒩_{k-1}(pos,:)=[];
𝒩_{k-1} = 𝒩_{k-1} - N12*inv(N22)*N12^T;
```

(c) Update variables `satc` and `nsc`, remove the entry for the set satellite. The variable `nsc` has the total number of satellites in the previous epoch.

(3) **Accounting for change of reference satellite** In theory, for the $k-1$ epoch (before the introduction of new reference satellite), the expectation was given as, $E\{y\} = Ax$. With the introduction of a new reference satellite, on the parameter side, the differencing of every parameter with respect to satellite changes, this change can be shown in terms of a translational matrix T. The expectation can now be given as, $E\{y\} = (AT^{-1})(Tx)$, a corresponding counter change is introduced on the design matrix side too, the new design matrix is AT^{-1}. The normal equations, denoted by $\mathcal{N}$ were earlier given as,

$$\mathcal{N}_{k-1} = A^T Q_{yy}^{-1} A \text{ and } \underbrace{A^T Q_{yy}^{-1} A}_{\mathcal{N}_{k-1}} \widehat{x} = A^T Q_{yy}^{-1} y$$

for the new reference satellite, the stacked normal equations can be updated as as,

$$\mathcal{N}_{k-1_{updated}} = T^{-T} A^T Q_{yy}^{-1} A T^{-1} \text{ and } \underbrace{T^{-T} A^T Q_{yy}^{-1} A T^{-1}}_{\mathcal{N}_{k-1_{updated}}} T\widehat{x} = T^{-T} A^T Q_{yy}^{-1} y.$$

(a) A translation matrix is created, it has the indicators -1 for the new chosen reference satellite. To give an example of the satellites in the previous epoch and current epoch and reference satellite in previous and current epoch, the translational matrix is formed as, see below.

$$\texttt{all satc} = \begin{bmatrix} 1 \\ 5 \\ 6 \\ 9 \end{bmatrix}; a_{\widehat{x_c}} = \begin{bmatrix} a_{15} \\ a_{16} \\ a_{19} \end{bmatrix}; \texttt{all satn} = \begin{bmatrix} 5 \\ 6 \\ 9 \end{bmatrix}; a_{\widehat{x_n}} = \begin{bmatrix} a_{65} \\ a_{69} \end{bmatrix}$$

Where, $a_{\widehat{x_c}}$ are double differenced estimable parameter vector for previous epoch, subscripts $1, 5, 6, 9$ indicate satellite PRN numbers, satellite 1 is the reference satellite. $a_{\widehat{x_n}}$ are double differenced estimable parameter vector for current epoch, satellite 6 is the reference satellite. Now a transformation has to be designed, such that $a_{\widehat{x_c}} = a_{\widehat{x_n}}$, it is shown as under

$$\underbrace{\begin{bmatrix} a_{65} \\ a_{61} \\ a_{69} \end{bmatrix}}_{a_{\widehat{x_n}}} = \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{T_t} \underbrace{\begin{bmatrix} a_{15} \\ a_{16} \\ a_{19} \end{bmatrix}}_{a_{\widehat{x_c}}}; \texttt{ that is; } a_{\widehat{x_n}} = T_t a_{\widehat{x_c}}$$

While performing the above step, information for for satellite 1, in the form a_{61}, which was present in the last epoch is also retained. In the above example, T_t is for $(m-1)$ ambiguities, phase only, it can be formed for all the phase and code observables, that is, for all the ambiguities and other unknowns like ranges. This transformation is indicated by the symbol T, it is split into T_a for ambiguity and T_b for non ambiguity parameters such that $T = \texttt{blkdiag}(T_a, T_b)$.

Further the stacked normal equations for $k-1$ epochs, $\mathcal{N}_{k-1}$, is operated with T, as shown below.

$$\mathcal{N}_{k-1_{updated}} = T^{-T} \mathcal{N}_{k-1} T^{-1}$$

(b) In second step, the parameters for the old reference satellite, in above example it is satellite number 1, are removed from the updated $\mathcal{N}_{k-1_{updated}}$ normal equations by a similar procedure as explained in the above section, "Accounting for a set satellite".

4.2 Simulation considerations

In this research, ASR is computed for all the ambiguities (Full ambiguity resolution) and corresponding gain is evaluated. This is done first for GPS only, then for Galileo only and finally for both GPS and Galileo. For each of the mentioned combinations (see list below), ASR and gain will be simulated for Geometry Free model, Geometry Based with coordinates known is referred as Geometry Fixed model and Geometry Based with rover coordinates unknown can be referred as Reference-Rover model. Further for each models, different atmosphere considerations (Ionosphere known, Ionosphere unknown, Ionosphere weighted, Troposphere known, Troposphere unknown, etc.) will be incorporated. The approach for simulation of ASR and gain for each of the mentioned GNSS systems is explained below.

(1) **Full AR - GPS only, Galileo only / PAR - GPS only, Galileo only**

(i) Low end receivers - Single frequency, high values of measurement precision, see Table 3.1

(ii) High end receivers - Dual frequency case, using $L1$ and $L2$ GPS frequencies, measurement precision given in Table 3.1

(iii) High end, future GPS receivers - Triple frequency, uses $L1, L2$ and $L5$ GPS data frequencies, measurement precision values given in Table 3.1

For the simulations stated below, the satellite orbit information for GPS is obtained from the YUMA almanac available on the following internet URL(Uniform Resource Locator): *http://www.navcen.uscg.gov/ ?pageName=gpsAlmanacs*

The ASR is simulated for May 13 2011, 00:00:00 to 23:59:30 UTC (hh:mm:ss), considering the constant maximum satellites in view available for the mentioned period over selected locations at 0°, −30° and −60°, the longitude was held fixed to 150° E for all the three latitude locations.

Now for all the three receiver types, for different ionosphere scenarios the following simulations were made.

(I) Varying the measurement precision as shown in Table 3.1 ASR and gain is simulated for k epochs, the number of satellites are as available.

(II) Varying the measurement precision, hourly instantaneous ASR is simulated, number of satellites are as available.

When measurement precision is held fixed, either the number of satellites are varied (from 2 to 6), or the baseline length is varied. The effect of satellite variation was done inorder to underhand the effect of the same on Geometry Free model ambiguity resolution. The results are presented and further discussed in Appendix D.

(III) Varying the number of satellites, starting with 2 satellites which are randomly selected, without considering the effect of combination of different satellites, measurement precision was held fixed to one value as shown in Table 3.2. The ASR and gain was simulated for k epochs. ASR was further simulated for 3, 4, 5, and 6 satellites each for k epochs.

(IV) Apart from the above two simulations, to consider the effect of satellite combination, the number of satellites were held fixed to two, the first satellite was taken as reference and the second satellite was varied (between different PRN's available for a particular session). This was only done for Geometry Free model, Ionosphere known scenario to understand the effect on ASR and gain. This experiment is elaborately discussed along with the findings, in Appendix D.

(V) For ionosphere weighted model, the baseline length is varied from 1 to 1000 Km. The measurement precision is held fixed. The effect of baseline length on Geometry Free model ambiguity resolution is discussed in ionosphere weighted section of this chapter.

Further, for simulating the ASR and gain by considering model assumptions in the form of design matrix and measurement precision matrix, the DD form of observables are considered. In this work a simple DD model consisting of a single baseline is considered.

4.3 Geometry Free model, Ionosphere known scenario

The atmosphere, namely the troposphere and the ionosphere contribute to the error in the estimated positions for any GNSS system. In a Geometry Free model, the troposphere is lumped to the ranges, whereas the ionosphere has to be accounted and parameterized. The atmosphere, here we speak of the ionosphere, for a DD approach is relevant for baselines long enough that can hold significant relative ionosphere bias. This is discussed in detail in the **Literature review**, chapter 2. Experience from GNSS studies is revealed in the literature, it is understood that, generally baselines less than 10 kilometers do not hold significant relative atmospheric bias [46], such baselines are referred to as short-baselines in GPS terminology. The word "significant" is very application specific, different applications aim at different values of precision. Now we discuss the functional model when ionosphere is insignificant enough and is considered fixed.

Geometry Free model, ionosphere known scenario		
Functional model		**Stochastic model**
Non-temporal parameters	Temporal parameters	
$A_{I(i)} = \begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes I_{m-1}$	$A_{II(i)} = \begin{pmatrix} e_f \\ e_f \end{pmatrix} \otimes I_{m-1}$	$Q_{y(i)} = \begin{bmatrix} Q_\Phi & \\ & Q_P \end{bmatrix} \otimes Q_{DD}(i)$
		where $Q_{DD}(i) = (D_m^T W_i^{-1} D_m)$
Redundancy (for GPS only, Galileo only, GPS+Galileo (common frequency $L1(E1), L5(E5a)$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $f * (m-1)$	Ranges: $k * (m-1)$	$2fk * (m-1)$
Redundancy of ionosphere known scenario		$(2fk - f - k) * (m-1)$
For GPS/Galileo only: $f = (1, \cdots, j)$, for GPS+Galileo: $f = (1, \cdots, j_c)$, j_c is the common/overlapping frequency,		
and $m_{jc} = (m_{GPS} + m_{Gal})$		
Redundancy (for GPS+Galileo (quadruple frequency $L1(E1), L5(E5a), L2, E5b$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)$	Ranges: $k * (m_{jc}-1)$	$2k * [2 * (m_{jc}-1) + (m_{GPS}-1) + (m_{Gal}-1)]$
Redundancy of quadruple frequency ionosphere known scenario		$(3k-2)(m_{jc}-1)+(2k-1)*[(m_{GPS}-1)+(m_{Gal}-1)]$
$f = 1, 2$ for GPS+Gaileo $L1(E1), L5(E5a)$, m_{jc} are the total number of satellites for combined GPS+Galileo system with GPS as reference satellite		

Table 4.1: Double-differenced Design matrix and VC matrix for Geometry Free model, ionosphere known scenario

In Table 4.1, subscript f indicate the total number of frequencies, $f = 1, \cdots, j$, D_m^T is the single differenced satellite design matrix is known as the difference operator for the satellites, W_i is the satellite elevation weight matrix for i^{th} epoch, see Appendix B, equation (B.5).

and $\Lambda = \text{diag}\ \underbrace{(\lambda_1, \cdots, \lambda_j)}_{j \times j}$

While considering the Ionosphere known scenario, the ionosphere term being considered as known, is not present in the design matrix in equation (B.2). Even though the model has zero redundancy for single frequency phase and code data, none the less ASR can be simulated for all single, dual, and triple frequency types. The following table present the results when measurement precision is varied for Geometry Free model, Ionosphere known scenario.

4.3.1 GPS only - full ambiguity resolution, Ionosphere known scenario

The following table present the results when measurement precision is varied for Geometry Free model, Ionosphere known scenario. The simulation is done at an interval of 5 seconds (17280 epochs in 24 hours) for the selected day (13 May 2011). The table below shows the instantaneous ASR, number of epochs for 0.999 ASR and gain at 0.999 ASR for different values of measurement precisions.

Note: All the values of fixed-precision of ranges marked as 0.000 indicate the values less than 1mm.

In the above table, Table 4.2, the highlighted values in green color will be used for comparing the convergence of Ionosphere known scenario to Ionosphere weighted scenario, refer Table 4.12(rows highlighted in green).

Further the Figures, 4.1, E.2 and E.3 in Appendix E give results for ASR and precision of float and fixed ambiguity states (for all latitude locations) for SF, DF and TF respectively. The point when 0.999 ASR is reached is marked with red dotted line and number of epochs taken for the same are marked on x-axis, corresponding value of fixed-precision of ranges are given on the y-axis (in red color). For all the values of measurement precision which did not converge to 0.999 ASR, the best value of ASR reached is marked with red-dotted line, the best value ASR obtained is given along with the number of epochs.

From Table 4.2 and Figures 4.1 to E.3 the analysis is summarized as below.

Phase	Code	Phase	Code	Instantaneous ASR			Epochs for 0.999 ASR			$\sigma_{\breve{\rho}}$(meters)@ 0.999 ASR		
(meters)				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
Φ	P	Φ	P	Low-end receivers, Single Frequency								
L1												
0.003	0.25	-	-	0.000	0.000	0.000	2581	1191	13230	0.000	0.000	0.000
							0.999	0.999	0.999			
0.003	0.50	-	-	0.000	0.000	0.000	17230	1613	15751	0.030	0.033	0.032
							0.985	0.983	0.986			
0.003	0.75	-	-	0.000	0.000	0.000	17230	1613	15751	0.044	0.049	0.048
							0.829	0.799	0.798			
0.003	1.00	-	-	0.000	0.000	0.000	17230	2470	15751	0.059	0.067	0.064
							0.543	0.563	0.486			
0.003	1.25	-	-	0.000	0.000	0.000	17230	2470	15751	0.074	0.084	0.080
							0.296	0.375	0.256			
0.003	1.50	-	-	0.000	0.000	0.000	3564	2470	15751	0.091	0.101	0.096
							0.150	0.237	0.131			
Φ	P	Φ	P	High-end receivers, Dual Frequency								
L1,L2												
0.002	0.10	-	-	0.436	0.838	0.498	10	4	8	0.002	0.002	0.002
0.002	0.15	-	-	0.248	0.662	0.263	14	6	11	0.001	0.002	0.002
0.002	0.20	-	-	0.139	0.506	0.134	19	8	15	0.001	0.001	0.001
0.003	0.25	-	-	0.020	0.193	0.015	33	14	27	0.001	0.002	0.001
0.003	0.30	-	-	0.009	0.127	0.006	40	17	34	0.001	0.001	0.001
0.003	0.35	-	-	0.004	0.085	0.003	46	20	40	0.001	0.001	0.001
Φ	P	Φ	P	High-end receivers, Triple Frequency								
L1,L2		L5										
0.002	0.10	0.002	0.05	0.831	0.991	0.901	4	2	4	0.002	0.002	0.002
0.002	0.15	0.002	0.1	0.652	0.934	0.725	8	3	6	0.001	0.002	0.002
0.002	0.20	0.002	0.125	0.588	0.901	0.651	9	4	7	0.001	0.002	0.002
0.003	0.25	0.002	0.15	0.318	0.718	0.340	15	6	12	0.001	0.002	0.001
0.003	0.30	0.002	0.175	0.282	0.680	0.296	17	7	14	0.001	0.002	0.001
0.003	0.35	0.002	0.2	0.257	0.652	0.267	18	7	14	0.001	0.002	0.001

Table 4.2: **GPS only**, **Full AR**, Geometry Free model - Ionosphere Fix scenario, ASR and gain values - **measurement precision is varied**, number of satellites are as available. For single frequency, most of the scenarios could not give 0.999 ASR within 17280 epochs, hence maximum ASR obtained is given below the number of epochs for each scenario for single frequency.

In our analysis firstly we will highlight the general conclusions, applicable to all the Geometry Free model scenarios. The conclusions drawn from (1) to (4) are general in nature.

General conclusions:

(1) The ASR for all single frequency (SF), dual frequency (SF) and triple frequency (TF) is found to be directly related to the precision of the measurements. Refer Figures 4.1 to E.3 for SF, DF and TF respectively. ASR is high for better values of measurement precision and vice versa. Also for any value of measurement precision at any given epoch (k), $ASR_{TF} > ASR_{DF} > ASR_{SF}$.

(2) The improvement in precision was highest for the first epoch (of the order 10^2 meters) and decreased further as the epochs increased. Refer Figures 4.1 to E.3 for SF, DF and TF respectively. Since the error in precision propagates by $1/k$ (k being the epoch number) for the computed variance matrices $Q_{\hat{x}_I}$ and $Q_{\hat{x}_{II}}$. Hence as the number of epochs increase, the precision of float parameters (unknowns) gets better.

(3) Analysis of the precision of non-ambiguity parameters (range) for *ambiguity-float* ($\sigma_{\hat{x}_{II}}$) and *ambiguity-fixed* ($\sigma_{\check{x}_{II}}$) solutions is can be done from Figures 4.1 to E.3 (for SF, DF and TF respectively). The precision of the float solution has a temporal variation (gets better with increasing number of epochs), where as the fixed solution precision has no temporal variation, single valued. the variation observed in the fixed solution in the figures is due to the fact that the precision values are averaged for all the $m-1$ satellites to obtain a single value of precision. The precision for range, obtained from the fixed solution is of the order 10^{-3} meters.

Explanation of unique behavior of parameters observed in the solution:

Wobbling of fixed and float precision curves: The wobbling of the precision curves is found to have two causes. Firstly, the precision presented is the average value of precision over $m-1$ satellites. Secondly, as the epochs progress, certain satellites are replaced. If a certain satellite which just comes in, has a low elevation, there is a significant change in the measurement precision VC matrix, Q_{yy} due to elevation dependent weighting, see Appendix D. Hence the wobbling is observed in the precision plots.

Latitudinal variation of success rate While we have a look at the latitude effect on ASR, for SF (refer Figure 4.1), any significant conclusion cannot be drawn for SF-ASR. For DF and TF (refer Figures E.2, E.3), the time taken for successful ASR (ASR = 0.999) for all the values of code precision at $0°$, $-30°$ and $-60°$, refer Table 4.2 show that for DF or TF, more epochs are taken for 0.999 ASR at $0°$ than at $-30°$ and $-60°$ almost similar number of epochs are required for 0.999 ASR, to be more precise, epoch taken for 0.999 ASR at $-60°$ are more than at $-30°$. This can be explained as follows. At the equator the number of satellites available are more in number and every satellite is visible for larger time period, as we go towards the poles, the situation is almost the opposite, refer Figure D.7 in Appendix D. At the equator the low elevation satellites are at both the ends, towards north and towards south of the equator, at -30 degrees, the low elevation satellites are at only the north of the station, and at -60 degrees, the low elevation satellites exist at both, north and south of the station. In this simulation the satellite elevation weighting is incorporated resulting in a different value of the measurement precision matrix Q_{yy} for different latitudes, see Figure 3.1. The difference in values of ASR with respect to the latitude locations is due to the elevation of satellites.

Below, some of the deductions are made for Geometry Free model, Ionosphere known case for most general values of measurement precisions.

Conclusions for Geometry Free model, Ionosphere known, full ambiguity resolution scenario

1. The best results for Geometry Free model, Ionosphere known case for ASR were for triple frequency with $\sigma_{\Phi_{L1,L2,L5}}$=2mm, $\sigma_{P_{L1,L2}}$=10cm, $\sigma_{P_{L5}}$=5cm. The ASR reached 0.999 in 2 epochs.

2. For single frequency GPS, with $\sigma_{\Phi_{L1}}$=3mm, $\sigma_{P_{L1}}$=25cm corresponding to low-end GPS receivers, to reach 0.999 ASR, it took 2581, 1191 and 13230 epochs at $0°, -30°$ and $-60°$ respectively, giving fixed-precision of DD ranges better than 1mm (indicated by 0.000 in Table 4.2). All the other values of measurement precision for single frequency did not converge to give 0.999 ASR.

3. For dual frequency, with $\sigma_{\Phi_{L1,L2}}$=2mm, $\sigma_{P_{L1,L2}}$=20cm corresponding to high-end GPS receivers, it took 19, 8 and 15 epochs at $0°, -30°$ and

$-60°$ respectively to reach 0.999 ASR.

4. For triple frequency, with $\sigma_{\Phi_{L1,L2,L5}}$=2mm, $\sigma_{P_{L1,L2}}$=20cm, $\sigma_{P_{L5}}$=12.5cm corresponding to high-end GPS receivers, it took 9, 4 and 7 epochs at $0°, -30°$ and $-60°$ respectively to reach 0.999 ASR.

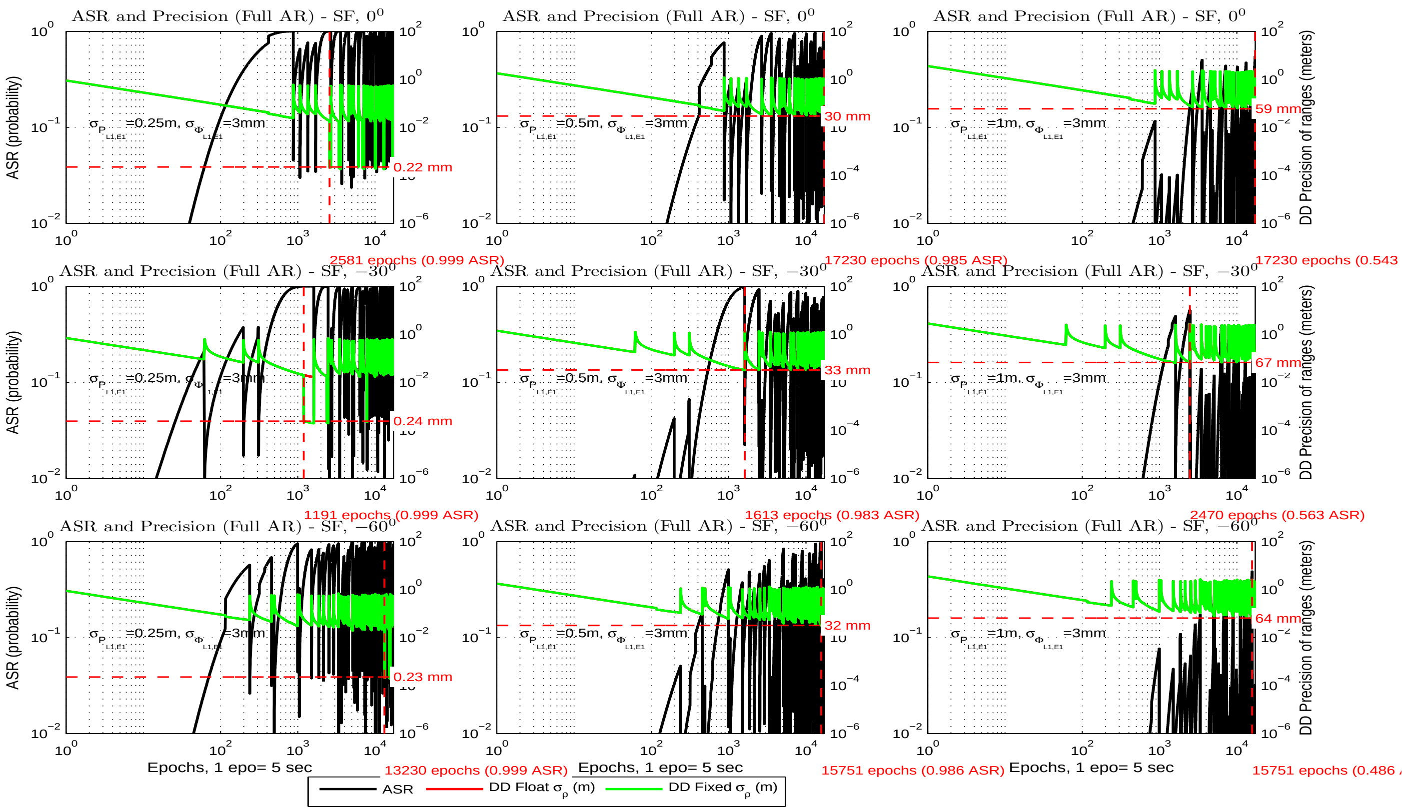

Figure 4.1: **GPS only**, **Full AR**, Single frequency, Geometry Free model, Ionosphere Fix scenario **(measurement precision is varied, number of satellites are as available)-** ASR and precision of float and fixed ambiguities are analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

4.3.2 GPS only - partial ambiguity resolution, Ionosphere known scenario

PAR, as discussed in chapter 2, aims at resolving a subset of ambiguities which suffice the minimum ASR requirements of the user. Since only a subset of ambiguities are resolved, it is important to understand as to how this can have an effect on the precision of the fixed-solution.

In PAR, the variance of the float ambiguities is used along with the user specified lower limit of ASR. The evaluation of the PAR for any specified value of desired ASR is dependent on the precision of the float ambiguities, as is the case in Full AR. The precision of the float ambiguities in-turn are influenced by,

1. Measurement precision of the phase and the code data
2. Variation in the measurement precision matrix Q_y due to addition of the weights of the low-elevation satellites (satellite elevation weighting).

After fixing of ambiguities by PAR, the computation of the precision of the ranges is done based only on the subset of ambiguities that are fixed. If none of the ambiguities are fixed, the precision of the fixed solution is same as the precision of the float solution. Hence if a small subset of ambiguities are fixed, it may happen that the precision of the fixed solution (say for ranges) is not better enough, especially if one is looking at precise positioning applications (a general remark).

Firstly, we analyze the ASR and gain with PAR, in terms of answering the question if full AR was better than PAR? Of course, this question will be evaluated by analyzing the precision of the ambiguity-fixed solution as obtained in full AR with PAR. Below are the results presented for number of epochs taken and fixed-precision of the ranges, for full AR and PAR, refer Table 4.3.

Figures, 4.2, E.5 and E.6 in Appendix E give results for partial AR and precision of float and fixed ambiguity states (for all latitude locations) for SF, DF and TF respectively. The plot with blue line indicate percentage of ambiguities fixed with respect to number of epochs, the percentage of ambiguities fixed are written along the blue line plot. The point when 100% of ambiguities are fixed with PAR is marked with red dotted line and number of epochs taken for the same are marked on x-axis, corresponding value of fixed-precision of ranges are given on the y-axis (in red color). For all the values of measurement precision which did not converge to fix all the ambiguities (100%), the maximum percentage of ambiguities fixed is marked with red-dotted line, the maximum ambiguities fixed is given along with the number of epochs.

		Full AR - minimum desired ASR = 0.999						Partial AR @ 0.999, criteria - similar or better value of fixed-precision								
Phase	Code	Epochs taken			$\sigma_{\check{p}}$(meters)			$\sigma_{\check{p}}$(meters)			Epochs taken			Ambiguities fixed(%)		
(meters)		0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
$\Phi 1$	$P1$	Low-end receivers, Single Frequency														
0.003	0.25	2581	1191	13230	0.000	0.000	0.000	0.000	0.000	0.000	2582	1192	13230	100	100	100
		0.999	0.999	0.999												
0.003	0.50	17230	1613	15751	0.030	0.033	0.032	0.030	0.033	0.031	2606	1365	5128	56	14	57
		0.985	0.983	0.986												
0.003	0.75	17230	1613	15751	0.044	0.049	0.048	0.044	0.049	0.048	17230	1613	15654	0	0	0
		0.829	0.799	0.798												
0.003	1.00	17230	2470	15751	0.059	0.067	0.064	0.059	0.067	0.064	17230	1562	15654	0	0	0
		0.543	0.563	0.486												
0.003	1.25	17230	2470	15751	0.074	0.084	0.080	0.074	0.084	0.080	17230	1562	15654	0	0	0
		0.296	0.375	0.256												
0.003	1.50	3564	2470	15751	0.091	0.101	0.096	0.091	0.101	0.096	3564	1562	15654	0	0	0
		0.150	0.237	0.131												
$\Phi 1$,$\Phi 2$	$P1$,$P2$	High-end receivers, Dual Frequency														
0.002	0.10	10	4	8	0.002	0.002	0.002	0.002	0.002	0.002	11	4	9	100	100	100
0.002	0.15	14	6	11	0.001	0.002	0.002	0.001	0.002	0.002	14	6	11	100	100	100
0.002	0.20	19	8	15	0.001	0.001	0.001	0.001	0.001	0.001	19	8	15	100	100	100
0.003	0.25	33	14	27	0.001	0.002	0.001	0.001	0.002	0.001	34	15	28	100	100	100
0.003	0.30	40	17	34	0.001	0.001	0.001	0.001	0.001	0.001	40	18	34	100	100	100
0.003	0.35	46	20	40	0.001	0.001	0.001	0.001	0.001	0.001	47	21	40	100	100	100
$\Phi 5$	$P5$	High-end receivers, Triple Frequency														
0.002	0.10	4	2	4	0.002	0.002	0.002	0.002	0.002	0.002	5	2	5	100	100	100
0.002	0.15	8	3	6	0.001	0.002	0.002	0.001	0.002	0.002	9	4	6	100	100	100
0.002	0.20	9	4	7	0.001	0.002	0.002	0.001	0.002	0.001	9	4	8	100	100	100
0.003	0.25	15	6	12	0.001	0.002	0.001	0.001	0.002	0.001	15	6	13	100	100	100
0.003	0.30	17	7	14	0.001	0.002	0.001	0.001	0.001	0.001	18	8	15	100	100	100
0.003	0.35	18	7	14	0.001	0.002	0.001	0.001	0.001	0.001	19	8	15	100	100	100

Table 4.3: GPS only, **PAR**, Geometry Free model, Ionosphere known scenario, ASR and fixed-precision for range values - **measurement precision is varied**, number of satellites are as available. A comparison of results obtained from Full AR with the ones obtained from PAR. For single frequency, most of the scenarios could not give 0.999 ASR within 17280 epochs, hence maximum ASR obtained is given below the number of epochs for each scenario for single frequency.

It is important to note that PAR involves step-wise, systematic fixing of ambiguities. After every fix, the remaining ambiguities are de-correlated and conditional variance for each of the ambiguity is recomputed again. Hence thought the similar integer-bootstrap success rate is used in full AR and PAR, it will be interesting to understand how does this step-wise ambiguity fixing and de-correlation at every step have an effect on fixed-precision of the ranges. The following is the analysis based on the results presented in Table 4.3.

1. One of the significant conclusions while performing partial AR could be seen for single frequency for the measurement precision of 3mm of phase and all values of code of 0.5m and more. With full AR, 0.999 ASR could not be obtained within the stipulated 17280 epochs, but with PAR, 56%, 14% and 57% of ambiguities were fixed for code precision of 0.5m at $0°$, $-30°$ and $-60°$ to give a fixed-precision of ranges similar to full AR (around 3cm fixed-precision). In full AR, it took 17230, 1613 and 15751 epochs to give ASR of 0.985, 0.983 and 0.986 at $0°$, $-30°$ and $-60°$, with PAR, it took 2606, 1365 and 5128 epochs to give similar precision by fixing 56%, 14% and 57% of ambiguities.

2. To obtain the same value of fixed-precision with PAR, as in full AR, could not give exceptional results in case of ionosphere-fixed scenario. At all times, the same number of epochs are required to obtain a fixed-precision by PAR as in full-AR, see dual and triple frequency results in Table 4.3. The ionosphere-fixed Geometry Free model is a stronger geometry-free model, with only ranges being unknown. As the ambiguities are fixed, the precision of the phase propagates directly to the precision of the estimated ranges. For a weak geometry-free model, when ionosphere is considered unknown (float) or even weighted, it would be interesting to evaluate PAR.

In the ionosphere-fixed case, the best value of precision for fixed solution for ranges lied around 1 mm. Such high precision for ranges is not necessarily required. It will be interesting to see whether for a particular lower value of precision (other than 1mm), can be obtained quicker with PAR (with respect to number of epochs). For example if a range precision of 2cm is desired, what is the percentage of ambiguities required to be resolved for the same. Here, the value 2cm precision of range is discussed, since in traditional NRTK, a user can get position estimates of 2cm or better. Hence 2cm is considered as a minimum criteria of precision of the estimated ranges / receiver coordinates etc. The following is the analysis of PAR for obtaining a fixed-precision of ranges of 2cm.

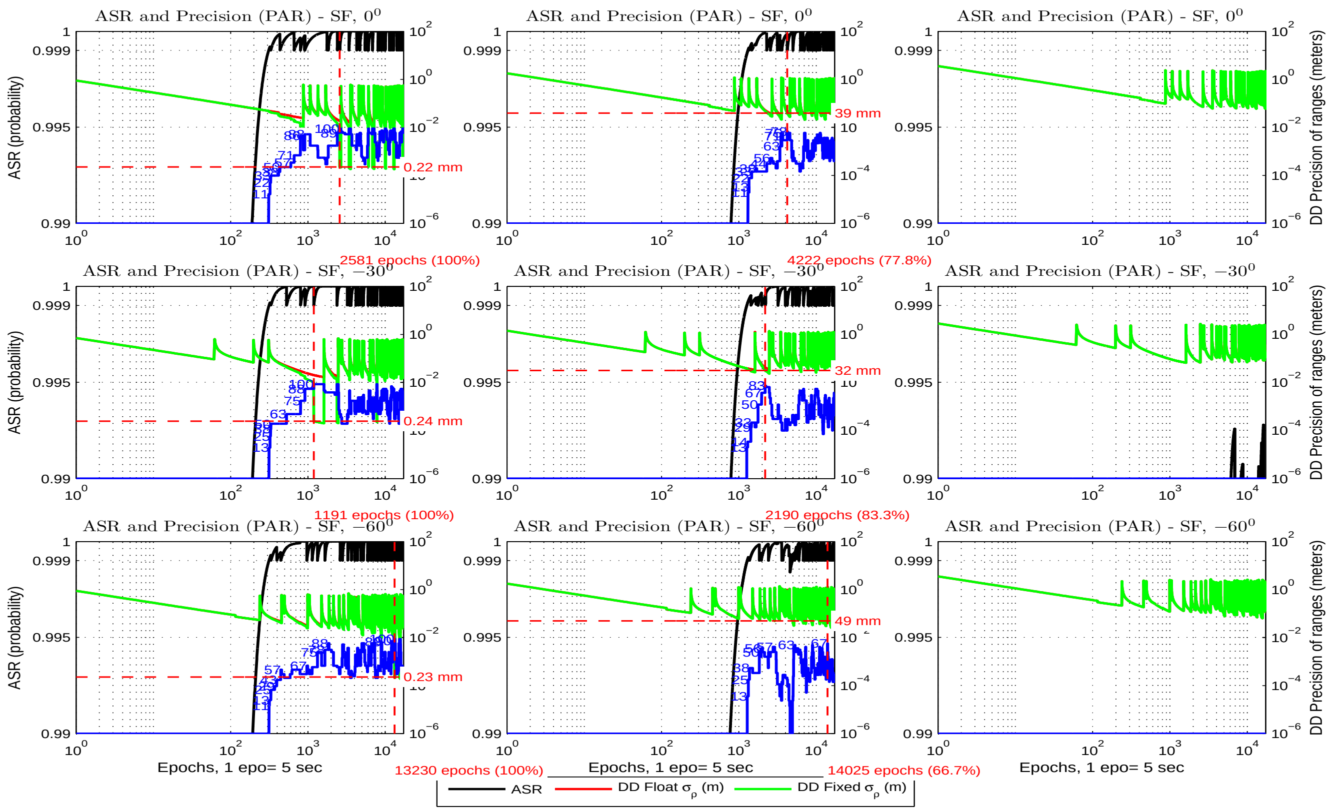

Figure 4.2: **GPS only**, Single frequency, **PAR**, Ionosphere known scenario, **(measurement precision is varied, number of satellites are as available)-** ASR and precision of float and fixed ambiguities are analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis).

4.3.3 GPS only - partial ambiguity resolution to obtain fixed-precision of 2 cm, Ionosphere known scenario

In Table 4.3, while evaluation PAR for a Geometry Free model, Ionosphere known scenario, the fixed-precision of the ranges obtained lied in the range 1mm to 10cm. For precise positioning applications, while estimating coordinates, one aims at an accuracy of the estimation to lie around 2cm, for the positions. Hence, when we see the fixed-precision of ranges to lie around 1mm, a question arises to evaluate PAR in order to obtain the fixed-precision of ranges to lie, say around 2cm with a shorter time to achieve a less ambitious value of fixed-precision. In Table 4.4, a fixed-precision of ranges of 2cm is aimed and the corresponding ambiguities fixed by PAR to achieve the same are presented and analyzed.

Note: In Table 4.4, some values of fixed-precision are higher than the desired 2cm, since the desired 2cm fixed-precision could not be obtained with a weak single frequency model for certain measurement precision scenarios. For all such scenarios the best possible value of fixed-precision obtained is presented.

The Table 4.4 with PAR, fixed-precision of 2cm for receiver-satellite ranges could be obtained for two scenarios of single frequency and all scenarios for dual and triple frequency GPS. For single frequency with full AR, for code precision of 25cm, the number of epochs lied around 2582, 1192 and 13230 by fixing 100% of ambiguities, refer Table 4.3. Now, refer Table 4.4, for the same value of measurement precision, the epochs lied around 664, 836 and 4976 and ambiguities resolved for the same were 71, 75 and 78%. Results for dual and triple frequency did not give exceptional results with PAR s compared to full AR in terms of number of epochs. The reason being the drastic fall of the fixed-precision could be achieved only when 100% of ambiguities were fixed. The fixed-precision from the order of 10^{-1}m falls to 10^{-2}m, that is from tens of cm to a few mm.

Below, some of the deductions are made for Geometry Free model, Ionosphere known, PAR for most general values of measurement precisions.

Conclusions for Geometry Free model, Ionosphere known, partial ambiguity resolution scenario

1. For single frequency PAR, with the measurement precision of $\sigma_{\Phi_{L1}}$=3mm, $\sigma_{P_{L1}}$=0.5m, corresponding to low-end GPS receivers, the full AR could not converge to 0.999 ASR within the stipulated 17280

		Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for range								
Phase	Code (meters)	$\sigma_{\check{\rho}}$(meters) 0°	−30°	−60°	Epochs taken 0°	−30°	−60°	Ambiguities fixed(%) 0°	−30°	−60°
$\Phi 1$	$P1$	Low-end receivers, Single Frequency								
0.003	0.25	0.018	0.020	0.020	664	836	4976	71	75	78
0.003	0.5	0.020	0.023	0.025	17199	2470	15667	63	80	43
0.003	0.75	0.044	0.049	0.048	17230	6386	15667	0	14	0
0.003	1.0	0.059	0.066	0.064	17230	1613	15667	0	0	0
0.003	1.25	0.074	0.082	0.080	17230	1613	15667	0	0	0
0.003	1.5	0.089	0.099	0.096	17230	1613	15667	0	0	0
$\Phi 1$,$\Phi 2$	$P1$,$P2$	High-end receivers, Dual Frequency								
0.002	0.1	0.002	0.002	0.002	10	4	8	100	100	100
0.002	0.15	0.001	0.002	0.002	14	6	11	100	100	100
0.002	0.2	0.001	0.001	0.001	19	8	15	100	100	100
0.003	0.25	0.001	0.002	0.001	33	14	27	100	100	100
0.003	0.3	0.001	0.001	0.001	40	17	34	100	100	100
0.003	0.35	0.001	0.001	0.001	46	20	40	100	100	100
$\Phi 5$	$P5$	High-end receivers, Triple Frequency								
0.002	0.05	0.002	0.002	0.002	4	2	4	100	100	100
0.002	0.1	0.001	0.002	0.002	8	3	6	100	100	100
0.002	0.125	0.001	0.002	0.002	9	4	7	100	100	100
0.002	0.15	0.001	0.002	0.001	15	6	12	100	100	100
0.002	0.175	0.001	0.002	0.001	17	7	14	100	100	100
0.002	0.2	0.001	0.002	0.001	18	7	14	100	100	100

Table 4.4: GPS only, **PAR**, Geometry Free model - Ionosphere known scenario, ASR and number of epochs for obtaining 2cm fixed-precision of ranges - **measurement precision is varied**, number of satellites are as available.

epochs for all latitude locations. PAR was able to fix 56, 14 and 57% of ambiguities for the same value of measurement precision in 2606, 1365 and 5128 epochs at 0°, −30° and −60°.

2. While evaluating how PAR performed to give a fixed-precision of ranges of 2cm, results for single frequency for measurement precision of $\sigma_{\Phi_{L1}}$=3mm, $\sigma_{P_{L1}}$=25cm, the number of epochs lied around 2582, 1192 and 13230 by fixing 100% of ambiguities, refer Table 4.3. Now, refer Table 4.4, for the same value of measurement precision, the epochs lied around 664, 836 and 4976 and ambiguities resolved for the same were 71, 75 and 78% giving fixed-precision of 2cm.

3. For dual and triple frequencies (corresponding to high-end GPS receivers), partial ambiguities could not be fixed to obtain similar fixed-precision as full AR. Also, the evaluation of PAR to give fixed-precision of 2cm could not give any significant results for dual and triple frequencies.

4.3.4 GPS only, full ambiguity resolution, hourly batches for simulating ASR, Ionosphere known scenario

The analysis of success rate is done at the beginning of every hour throughout the day, inorder to see the effect of satellite constellation on ambiguity resolution. In this analysis, the ambiguity success rate is computed by accumulating the epochs (3600 epochs, 1 epoch = 1 second) for 24 batches, starting at 00 UTC hour to 23:59:59 UTC hour. The ambiguities and other unknowns are reinitialized at the start of every hour. It is to be realized which hour gives the best ASR and even worst ASR values for any hour during the day, since this would be immensely useful for planning a GNSS campaign for a user. Following is the description of computation of success rate for every hour (24 batches).

Simulation and analysis of ASR is done for every hour at $0°, -30°$ and $-60°$ degree latitude, for all the frequency types, single, dual and triple and all six measurement precision scenarios, see Table 4.5. While doing so, at every hour, all the satellites above 10 degree elevation were used for simulating ASR. The values of measurement precision that were used to simulate the ASR for SF, DF and TF can be found in Table 3.1. In our analysis presented in Table 4.5, for 0.999 ASR (0.999 ASR, if obtained in 3600 epochs is considered else the best value of ASR obtained is considered), minimum epochs and maximum epochs (among the 24 batches) are presented for each value of measurement precision. Figure 4.3 presents the ASR, and corresponding values of double differenced fixed and float precision (of ranges) for each of the 24 batches, along with the number of epochs taken to converge to 0.999 / best value of ASR for the chosen value of measurement precision.

The following is the analysis based on Table 4.5 and Figure 4.3.

(1) In Figure 4.3, for SF, it can be noted that certain time periods which give better convergence for ASR than other hours during the day. It can hence be suggested to the user to carry out a campaign for SF based on the results. The best convergence for ASR can be noted, it took 3600 (00-01 hours UTC, batch 1), 2845 (10-11 hours UTC, batch 11) and 2732 (21-22 hours UTC, batch 22) giving 0.999, 0.999 and 1.0 ASR at $0°$, $-30°$ and $-60°$ latitudes respectively. This is for the chosen value of measurement precision,with σ_Φ=3mm and σ_P=50cm.

(2) It is important to note that for all SF, DF and TF, the best ASR results

vary for different latitude locations. For DF with measurement precision of σ_Φ=3mm and σ_P=25cm, at 0° the the fastest convergence of ASR (to 0.999) is obtained for batch 13 (11 epochs), where as at −30° it is obtained for batch 2 (11 epochs). At −60° degree latitude, the fastest convergence of ASR (to 0.999) is obtained for batch 8 (6 epochs).

(3) For TF with measurement precision of σ_Φ=3mm and σ_P=25cm for L_1, L_2 and σ_Φ=2mm and σ_P=15cm for L_5 the fastest convergence of ASR (to 0.999) at 0° is seen for batch 13 (5 epochs), at −30° it takes 5 epochs (batch 2) and at −60° it takes 3 epochs (batch 8).

(4) While analyzing the best and the worst scenarios, for single frequency since it does not converge to 0.999 ASR most of the times, it is excluded from this analysis. For dual frequency, only batches 1, 2 and 23 take less than 15 epochs to converge to 0.999 ASR at −30°, for worst scenarios, batches 3, 6, 12, 16, 17, 19 take more than 30 epochs. Similar analysis can be made for other latitude locations.

(5) For triple frequency, at −30°, batches 1, 2 and 23 take less than 6 epochs to converge to 0.999 ASR. Batches 3, 6, 12, 16, 17, 19 take more than 12 epochs.

				ASR			Minimum epochs required for 0.999 ASR (or maximum ASR)						Maximum epochs required for 0.999 ASR (or maximum ASR)					
Phase	Code	Phase	Code	(0.999 or maximum obtained)			Epochs taken			$\sigma_{\hat{\rho}}$(mm)			Epochs taken(corresponding hour batch)			$\sigma_{\hat{\rho}}$(mm)		
(meters)				0^0	-30^0	-60^0	0^0	-30^0	-60^0	0^0	-30^0	-60^0	0^0	-30^0	-60^0	0^0	-30^0	-60^0
Φ	P	Φ	P	Low-end receivers, Single Frequency														
L1																		
0.003	0.25	-	-	0.999	0.999	0.999	1036(12-13)	815(3-4)	752(17-18)	0.279	0.270	0.278	3250(19-20)	3380(0-1)	3460(7-8)	0.198	0.192	0.191
0.003	0.50	-	-	0.999	0.999	1.000	3600(0-1)	2845(10-11)	2732(21-22)	0.144	0.150	0.143	3600(0-1)	2894(1-2)	2732(21-22)	0.144	0.143	0.143
0.003	0.75	-	-	0.944	0.687	0.985	3600(0-1)	3600(0-1)	3151(21-22)	0.144	0.178	0.135	3600(0-1)	3600(0-1)	3151(21-22)	0.144	0.178	0.135
0.003	1.00	-	-	0.765	0.446	0.888	3600(0-1)	3600(0-1)	3151(21-22)	0.144	0.178	0.135	3600(0-1)	3600(0-1)	3151(21-22)	0.144	0.178	0.135
0.003	1.25	-	-	0.535	0.264	0.698	3600(0-1)	3600(0-1)	3151(21-22)	0.144	0.178	0.135	3600(0-1)	3600(0-1)	3151(21-22)	0.144	0.178	0.135
0.003	1.50	-	-	0.339	0.146	0.491	3600(0-1)	3600(0-1)	3151(21-22)	0.144	0.178	0.135	3600(0-1)	3600(0-1)	3151(21-22)	0.144	0.178	0.135
Φ	P	Φ	P	High-end receivers, Dual Frequency														
L1,L2																		
0.002	0.10	-	-	0.999	0.999	1.000	3(12-13)	3(1-2)	2(7-8)	2.445	2.207	2.362	10(0-1)	10(5-6)	10(6-7)	1.595	1.773	1.755
0.002	0.15	-	-	1.000	1.000	1.000	5(12-13)	5(1-2)	3(7-8)	1.894	1.710	1.929	15(7-8)	14(2-3)	14(6-7)	1.507	1.342	1.483
0.002	0.20	-	-	0.999	0.999	1.000	6(12-13)	6(1-2)	4(7-8)	1.729	1.561	1.670	20(7-8)	19(2-3)	19(6-7)	1.305	1.153	1.274
0.003	0.25	-	-	0.999	0.999	0.999	11(12-13)	11(1-2)	6(7-8)	1.916	1.729	2.046	36(7-8)	34(2-3)	35(6-7)	1.458	1.296	1.408
0.003	0.30	-	-	0.999	0.999	1.000	14(12-13)	14(1-2)	8(7-8)	1.698	1.532	1.772	44(7-8)	42(2-3)	43(14-15)	1.319	1.167	1.252
0.003	0.35	-	-	0.999	0.999	0.999	16(12-13)	16(1-2)	9(7-8)	1.588	1.433	1.670	52(7-8)	49(2-3)	50(14-15)	1.213	1.082	1.160
Φ	P	Φ	P	High-end receivers, Triple Frequency														
L1,L2		L5																
0.002	0.10	0.002	0.05	1.000	1.000	1.000	2(1-2)	2(0-1)	1(7-8)	2.391	2.323	2.726	5(7-8)	4(2-3)	5(14-15)	2.131	2.046	2.007
0.002	0.15	0.002	0.1	0.999	0.999	1.000	3(1-2)	3(0-1)	2(7-8)	1.953	1.897	1.928	8(0-1)	8(2-3)	8(6-7)	1.456	1.448	1.602
0.002	0.20	0.002	0.125	0.999	0.999	1.000	3(12-13)	3(1-2)	2(7-8)	1.997	1.802	1.929	10(7-8)	9(2-3)	9(5-6)	1.507	1.366	1.406
0.003	0.25	0.002	0.15	0.999	0.999	1.000	5(12-13)	5(1-2)	3(7-8)	1.949	1.759	1.984	16(0-1)	16(5-6)	16(6-7)	1.297	1.443	1.428
0.003	0.30	0.002	0.175	1.000	1.000	0.999	6(12-13)	6(1-2)	3(7-8)	1.779	1.606	1.984	18(7-8)	17(2-3)	18(14-15)	1.415	1.254	1.331
0.003	0.35	0.002	0.2	0.999	0.999	1.000	6(12-13)	6(1-2)	4(7-8)	1.779	1.606	1.719	19(7-8)	19(5-6)	19(6-7)	1.378	1.324	1.311

Table 4.5: GPS only, **Full AR**, Geometry Free model, Ionosphere known scenario, ASR and fixed-precision for range values for 24 batches of one hour (each batch of 3600 epochs, 1 second interval) - **measurement precision varied**, number of satellites are as available. The rows shaded in gray represent values of measurement precision for which simulated parameters are presented graphically in the following subsection.

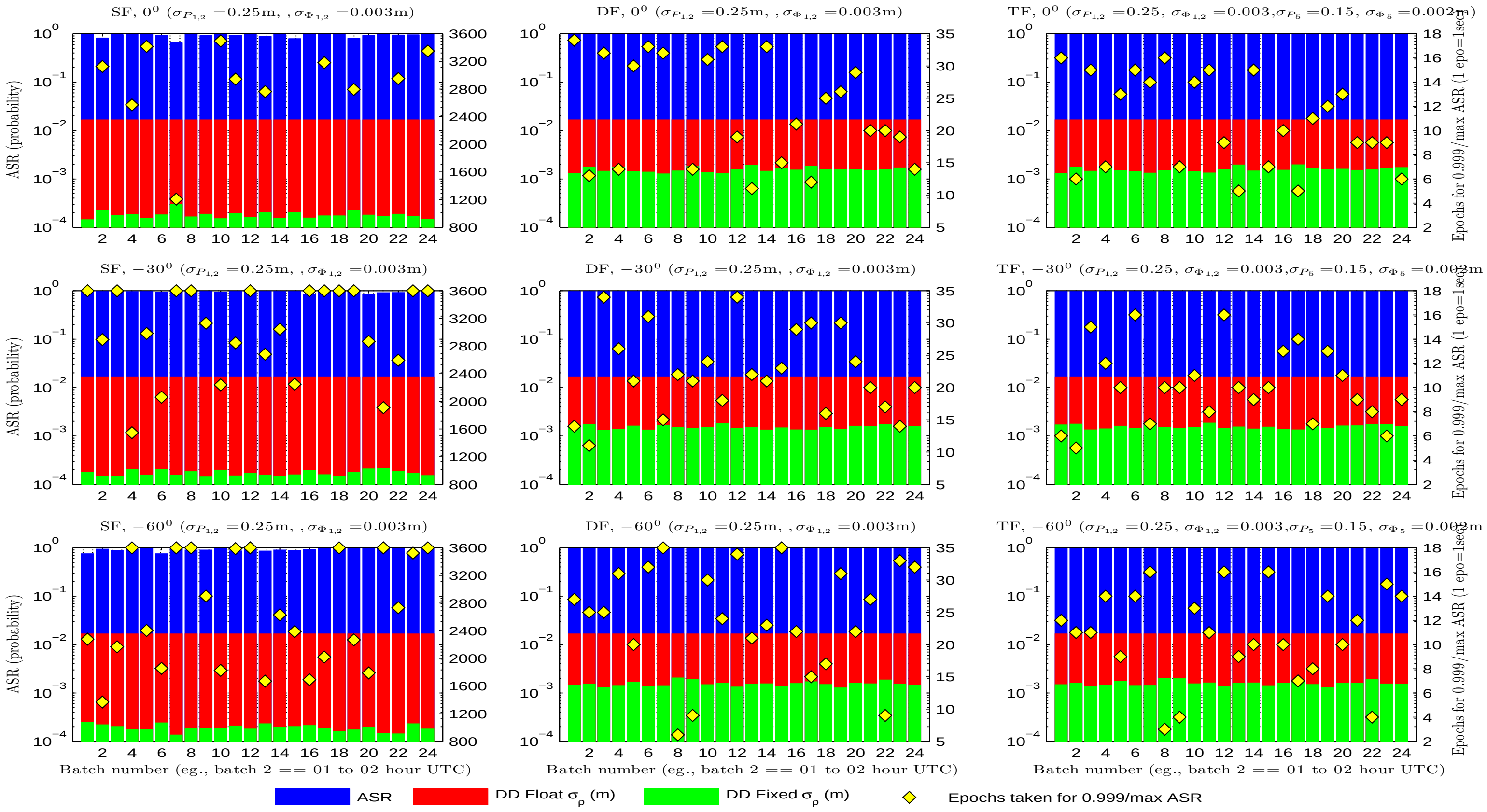

Figure 4.3: **GPS only**, Single, Dual and Triple Frequencies, Geometry Free model, Ionosphere known scenario, **Instantaneous ASR** analysis when **(measurement precision is varied, number of satellites are as available(not held fixed))** at $0°, -30°$ and $-60°$ degree latitude

4.4 Geometry Free model, Ionosphere unknown scenario

In GPS terminology, if the ionosphere is significant enough (here, in relative sense, for a baseline) and without any knowledge of its mathematical value, it is considered as unknown. When ionosphere is termed as unknown, it is required to be estimated along with other unknowns, such an ionosphere variable is termed as Ionosphere unknown in an GNSS model. Again the term significant is very application specific. When we speak of relative positioning, a general remark made is that for baselines larger than 10 kilometers, the ionosphere is significant for precise relative positioning applications. Under such a condition, one needs a-priori information of the ionosphere else ionosphere needs to be estimated. For more information regarding the relation of baseline length to the ionospheric delay, refer the chapter 2, **Literature review**. The Table 4.6 given below gives the closed forms of design matrices and stochastic model VC matrix for Ionosphere unknown scenario. Also, redundancy of Ionosphere unknown scenario is discussed.

where

$$\mu_f = \begin{bmatrix} \mu_{(1)} \\ \vdots \\ \mu_{(j)} \end{bmatrix} = \begin{bmatrix} \frac{\nu_1^2}{\nu_1^2} \\ \vdots \\ \frac{\nu_1^2}{\nu_j^2} \end{bmatrix} \tag{4.6}$$

and

where subscript f indicate the total number of frequencies, $f = 1, \cdots, j$, ν_1 is the frequency of GPS signal on L_1, D_m^T is the single differenced satellite design matrix is known as the difference operator for the satellites, W_i is the satellite elevation weight matrix for i^{th} epoch, see Appendix B, equation (B.5), $\Lambda =$ diag $\underbrace{(\lambda_1, \cdots, \lambda_j)}_{j \times j}$.

4.4.1 GPS only - full ambiguity resolution, Ionosphere unknown scenario

The un-differenced measurement precision at zenith can take the earlier 6 defined values, see Table 3.1. The Ionosphere unknown scenario, ASR simulation is done at an interval of 5 seconds (17280 epochs in 24 hours). Table 4.7 presents

Geometry Free model, ionosphere unknown scenario		
Functional model		**Stochastic model**
Non-temporal parameters	Temporal parameters	
$A_{I(i)} = \left[\begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes I_{m-1} \quad \begin{pmatrix} e_f \\ e_f \end{pmatrix} \otimes G_{m-1} \right]$	$A_{II(i)} = \left[\begin{pmatrix} -\mu_f \\ \mu_f \end{pmatrix} \otimes I_{m-1} \quad \begin{pmatrix} e_f \\ e_f \end{pmatrix} \otimes I_{m-1} \right]$	$Q_{y(i)} = \begin{bmatrix} Q_\Phi & \\ & Q_P \end{bmatrix} \otimes Q_{DD}(i)$ where $Q_{DD}(i) = D_m^T W_i D_m$
Redundancy (for GPS only, Galileo only, GPS+Galileo (common frequency $L1(E1), L5(E5a)$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $f * (m-1)$	Ranges: $k * (m-1)$ Ionosphere: $k * (m-1)$	$2fk * (m-1)$
Redundancy of Geometry Free model, ionosphere unknown scenario		$(2fk - f - 2k) * (m-1)$
For GPS/Galileo only: $f = (1, \cdots, j)$, for GPS+Galileo: $f = (1, \cdots, j_c)$, j_c is the common/overlapping frequency, and $m_{jc} = (m_{GPS} + m_{Gal})$		
Redundancy (for GPS+Galileo (quadruple frequency $L1(E1), L5(E5a), L2, E5b$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)$	Ranges: $k*(m_{jc}-1)$ Ionosphere: $k*(m_{jc}-1)$	$2k*[2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)]$
Redundancy of quadruple frequency ionosphere unknown scenario		$(2k-2)(m_{jc}-1)+(2k-1)((m_{GPS}-1)+(m_{Gal}-1))$
$f = 1, 2$ for GPS+Gaileo $L1(E1), L5(E5a)$, m_{jc} are the total number of satellites for combined GPS+Galileo system with GPS as reference satellite		

Table 4.6: Double-differenced Design matrix and VC matrix for Geometry Free model, ionosphere unknown scenario

the simulated results for dual and triple frequency GPS system, Geometry Free model-Ionosphere unknown scenario for ASR and gain at three latitude locations $0°$, $-30°$ and $-60°$.

Phase	Code	Phase	Code	Instantaneous ASR			Epochs for 0.999 ASR			$\sigma_{\hat{\rho}}$(meters)@ 0.999 ASR		
	(meters)			$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$
Φ	P	Φ	P	High-end receivers, Dual Frequency								
L1,L2												
0.002	0.10	-	-	0.000	0.000	0.000	15568	1899	15751	0.018	0.018	0.019
							0.960	0.990	0.984			
0.002	0.15	-	-	0.000	0.000	0.000	15568	1899	15751	0.027	0.028	0.029
							0.788	0.875	0.796			
0.002	0.20	-	-	0.000	0.000	0.000	15568	1899	15751	0.036	0.037	0.039
							0.539	0.633	0.485			
0.003	0.25	-	-	0.000	0.000	0.000	15568	1899	15751	0.045	0.046	0.048
							0.316	0.401	0.256			
0.003	0.30	-	-	0.000	0.000	0.000	15568	1899	15751	0.054	0.055	0.058
							0.171	0.242	0.131			
0.003	0.35	-	-	0.000	0.000	0.000	15568	1899	15751	0.063	0.064	0.068
							0.090	0.146	0.068			
Φ	P	Φ	P	High-end receivers, Triple Frequency								
L1,L2		L5										
0.002	0.10	0.002	0.05	0.000	0.000	0.000	719	1283	6957	0.001	0.000	0.000
0.002	0.15	0.002	0.1	0.000	0.000	0.000	2859	1560	13375	0.000	0.000	0.000
0.002	0.20	0.002	0.125	0.000	0.000	0.000	10756	1648	13452	0.000	0.000	0.000
0.003	0.25	0.002	0.15	0.000	0.000	0.000	10811	1899	15751	0.036	0.037	0.039
							0.990	0.999	0.998			
0.003	0.30	0.002	0.175	0.000	0.000	0.000	10811	1899	15751	0.043	0.044	0.046
							0.984	0.998	0.996			
0.003	0.35	0.002	0.2	0.000	0.000	0.000	10811	1899	15751	0.050	0.051	0.054
							0.979	0.996	0.994			

Table 4.7: GPS only, **Full AR**, Geometry Free model - Ionosphere unknown scenario, ASR and gain values - **measurement precision is varied**, number of satellites are as available. For dual frequency, most of the scenarios could not give 0.999 ASR within 17280 epochs, hence maximum ASR obtained is given below the number of epochs for each scenario for dual frequency.

In the above table certain values of measurement precision are highlighted in red color, the corresponding values of measurement precision in Table 4.12 in Ionosphere weighted section will indicate convergence of Ionosphere unknown and Ionosphere weighted ASR results. Figures 4.4 and E.8 in Appendix E give ASR, fixed and float precision of the range (in meters) for DF and TF at $0°$, $-30°$

and $-60°$ degree latitude. For Figures 4.4 and E.8, the point when 0.999 ASR is reached is marked with red dotted line and number of epochs taken for the same are marked on x-axis, corresponding value of fixed-precision of ranges are given on the y-axis (in red color). For all the values of measurement precision which did not converge to 0.999 ASR, the best value of ASR reached is marked with red-dotted line, the best value ASR obtained is given along with the number of epochs. Analysis based on Table 4.7 and Figures 4.4, E.8 conclude the following

1. The ASR takes time to converge for I-float model as compared to I-fixed model assumption. Since Ionosphere unknown type Geometry Free model considers ionosphere unknown, making the GNSS model weaker. With ionosphere to be estimated as an additional parameter, ambiguities take comparatively longer time as compared to Ionosphere known scenario. Consider TF ASR for Ionosphere known, which took a minimum of 4 epochs, for Ionosphere unknown it takes a minimum of 719 epochs (1 epoch is 5 seconds).

2. The Ionosphere unknown scenario gives very poor results for dual frequency GPS, for triple frequency GPS the results aren't convincing either, PAR and use of GPS + Galileo is very important to consider for Ionosphere unknown type condition. It is clearly seen from Table 4.7 for dual frequency, 0.999 ASR is not achieved, the lower value of ASR achieved is given under number of epochs taken. For dual frequency the best value of ASR obtained is given, for code precision of 10cm, the ASR values obtained are 0.960, 0.99 and 0.984 at $0°$, $-30°$ and $-60°$ latitude which took 15568, 1899 and 15751 epochs.

3. For triple frequency, it took a minimum of 719 epochs for code precision of 5cm for $l5$ at $0°$ latitude.

Below, some of the deductions are made for Geometry Free model, Ionosphere unknown, full ambiguity resolution case for most general values of measurement precisions.

Conclusions for Geometry Free, Ionosphere unknown, full ambiguity resolution scenario

1. For dual frequency, with $\sigma_{\Phi_{L1,L2}}$=2mm, $\sigma_{P_{L1,L2}}$=20cm corresponding to high-end GPS receivers, it took 10753, 8845 and 10017 epochs at $0°, -30°$ and $-60°$ respectively to reach 0.999 ASR.

2. For triple frequency, with $\sigma_{\Phi_{L1,L2,L5}}$=2mm, $\sigma_{P_{L1,L2}}$=20cm, $\sigma_{P_{L5}}$=12.5cm corresponding to high-end GPS receivers, it took 1472, 860 and 1020 epochs at $0°, -30°$ and $-60°$ respectively to reach 0.999 ASR.

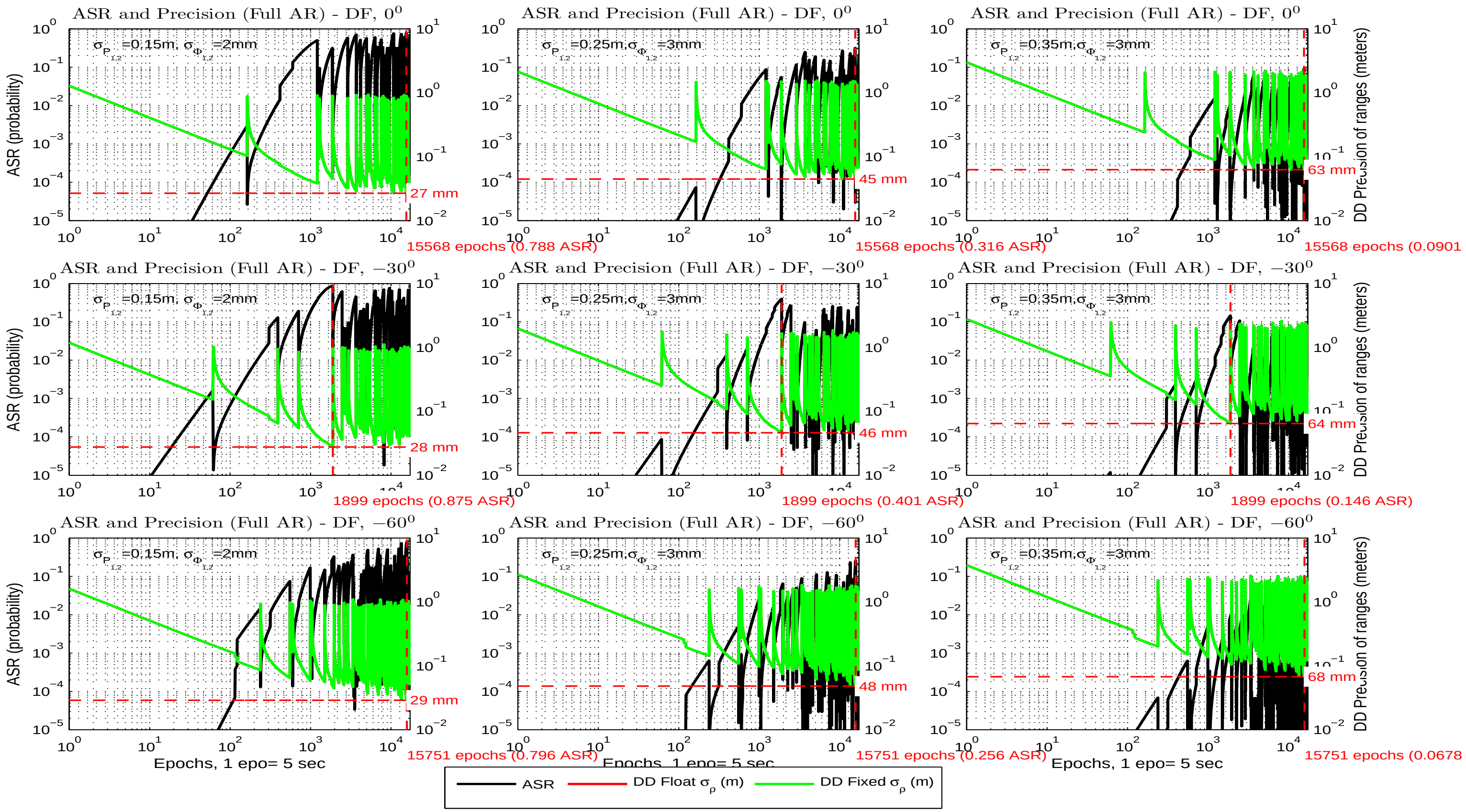

Figure 4.4: **GPS only**, **Full AR**, Dual frequency, Geometry Free model, Ionosphere unknown scenario, **(measurement precision is varied, number of satellites are as available)-** ASR and precision of float and fixed ambiguities are analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

4.4.2 GPS only, full ambiguity resolution, hourly batches for simulating ASR, Ionosphere unknown scenario

The analysis of success rate is done at the beginning of every hour throughout the day, inorder to see the effect of satellite constellation on ambiguity resolution. In this analysis, the ambiguity success rate is computed by accumulating the epochs (3600 epochs, 1 epoch = 1 second) for 24 batches, starting at 00 UTC hour to 23:59:59 UTC hour. The ambiguities and other unknowns are reinitialized at the start of every hour. It is to be realized which hour gives the best ASR and even worst ASR values for any hour during the day, since this would be immensely useful for planning a GNSS campaign for a user. Following is the description of computation of success rate for every hour (24 batches).

During this simulation, all the satellites above 10° elevation were used for simulating ASR, for all of the selected latitude locations, see Figure 4.5 for DF and TF. Also to note that, for each DF and TF, different values of measurement precision are used to simulate the ASR as given in Table 3.1. The results presented in Table 4.8 is based on 0.999 ASR (0.999 ASR, if obtained in 3600 epochs is considered else the best value of ASR obtained is considered), for which minimum epochs and then maximum epochs (among the 24 batches) are presented for each value of measurement precision. Figure 4.5 gives the ASR, and corresponding values of double differenced fixed and float precision (of ranges) for 24 batches, along with the number of epochs taken to converge to 0.999 / best value of ASR.

The following is the analysis based on Table 4.8 and Figure 4.5.

(1) It is important to note that for all DF and TF, the best ASR results vary for different latitude locations. For DF with measurement precision of σ_Φ=3mm and σ_P=25cm, at 0° the the fastest convergence of ASR (to the maximum value possible) is obtained for batch 1 (3600 epochs, 0.474 ASR), where as at −30° it is obtained for batch 23 (2981 epochs, 0.746 ASR). At −60° degree latitude, the fastest convergence of ASR is obtained for batch 22 (3151 epochs).

(2) For TF with measurement precision of σ_Φ=3mm and σ_P=25cm for L_1, L_2 and σ_Φ=2mm and σ_P=15cm for L_5 the fastest convergence of ASR (to 0.999) at 0° is seen for batch 15 (2485 epochs), at −30° it takes 1121 epochs (batch 3) and at −60° it takes 2210 epochs (batch 10).

(3) While analyzing the best and the worst scenarios, for dual frequency, since

it does not converge to 0.999 ASR most of the times, it is excluded from this analysis. For triple frequency, at $-30°$, batches 3 and 12 take less than 1200 epochs to converge to 0.999 ASR. Batches 8, 15, 16, and 21 do not converge to 0.999 ASR.

Phase	Code	Phase	Code	ASR (0.999 or maximum obtained)			Minimum epochs required for 0.999 ASR (or maximum ASR) Epochs taken			Minimum epochs required for 0.999 ASR (or maximum ASR) $\sigma_{\hat{\rho}}$(mm)			Maximum epochs required for 0.999 ASR (or maximum ASR) Epochs taken(corresponding hour batch)			Maximum epochs required for 0.999 ASR (or maximum ASR) $\sigma_{\hat{\rho}}$(mm)		
(meters)				0^0	-30^0	-60^0	0^0	-30^0	-60^0	0^0	-30^0	-60^0	0^0	-30^0	-60^0	0^0	-30^0	-60^0
Φ L1,L2	P L1,L2	Φ	P	High-end receivers, Dual Frequency														
0.002	0.10	-	-	0.996	0.999	1.000	3600(0-1)	1766(2-3)	2280(21-22)	0.297	0.320	0.304	3600(0-1)	3269(12-13)	2280(21-22)	0.297	0.266	0.304
0.002	0.15	-	-	0.906	0.990	0.987	3600(0-1)	2981(22-23)	3151(21-22)	0.299	0.267	0.266	3600(0-1)	2981(22-23)	3151(21-22)	0.299	0.267	0.266
0.002	0.20	-	-	0.700	0.912	0.892	3600(0-1)	2981(22-23)	3151(21-22)	0.300	0.267	0.267	3600(0-1)	2981(22-23)	3151(21-22)	0.300	0.267	0.267
0.003	0.25	-	-	0.474	0.746	0.703	3600(0-1)	2981(22-23)	3151(21-22)	0.449	0.400	0.400	3600(0-1)	2981(22-23)	3151(21-22)	0.449	0.400	0.400
0.003	0.30	-	-	0.295	0.554	0.496	3600(0-1)	2981(22-23)	3151(21-22)	0.449	0.401	0.400	3600(0-1)	2981(22-23)	3151(21-22)	0.449	0.401	0.400
0.003	0.35	-	-	0.176	0.391	0.329	3600(0-1)	2981(22-23)	3151(21-22)	0.450	0.401	0.401	3600(0-1)	2981(22-23)	3151(21-22)	0.450	0.401	0.401
Φ L1,L2	P L1,L2	Φ L5	P L5	High-end receivers, Triple Frequency														
0.002	0.10	0.002	0.05	0.999	0.999	0.999	571(20-21)	279(11-12)	350(7-8)	0.579	0.699	0.626	2383(9-10)	2830(18-19)	3190(5-6)	0.378	0.402	0.355
0.002	0.15	0.002	0.1	0.999	0.999	0.999	1082(20-21)	548(2-3)	783(17-18)	0.427	0.490	0.453	3519(9-10)	3483(0-1)	3415(8-9)	0.292	0.281	0.295
0.002	0.20	0.002	0.125	0.999	0.999	0.999	1283(20-21)	663(2-3)	933(17-18)	0.393	0.447	0.417	3095(17-18)	3324(6-7)	3526(20-21)	0.313	0.300	0.265
0.003	0.25	0.002	0.15	0.999	0.999	0.999	2485(14-15)	1121(2-3)	2210(9-10)	0.403	0.488	0.405	3331(0-1)	3365(4-5)	2522(18-19)	0.381	0.360	0.369
0.003	0.30	0.002	0.175	0.999	0.999	1.000	2758(14-15)	1258(2-3)	2280(21-22)	0.382	0.462	0.372	3558(3-4)	3584(4-5)	2522(18-19)	0.374	0.348	0.370
0.003	0.35	0.002	0.2	0.999	0.999	1.000	2971(14-15)	1366(2-3)	2280(21-22)	0.368	0.445	0.372	3166(23-24)	3297(10-11)	2523(18-19)	0.356	0.370	0.370

Table 4.8: GPS only, **Full AR**, Geometry Free model, Ionosphere unknown scenario, ASR and fixed-precision for range values for 24 batches of one hour (each batch of 3600 epochs, 1 second interval) - **measurement precision varied**, number of satellites are as available. The rows shaded in gray represent values of measurement precision for which simulated parameters are presented graphically in the following subsection.

4.4.3 GPS only - Partial Ambiguity Resolution (PAR), Ionosphere unknown scenario

The results obtained from partial fixing of ambiguities for Geometry Free model, Ionosphere unknown scenario, different values of measurement precision, at $0°$, $-30°$ and $-60°$ latitude locations are presented in Figures 4.6 and E.10 for dual and triple frequencies, see Appendix E. In the figures, blue line indicate percentage of ambiguities fixed with respect to number of epochs, the percentage of ambiguities fixed are written along the blue line plot. The point when 100% of ambiguities are fixed with PAR is marked with red dotted line and number of epochs taken for the same are marked on x-axis, corresponding value of fixed-precision of ranges are given on the y-axis (in red color). For all the values of measurement precision which did not converge to fix all the ambiguities (100%), the maximum percentage of ambiguities fixed is marked with red-dotted line, the maximum ambiguities fixed is given along with the number of epochs.Table 4.9 gives a comparison between Full AR and PAR by evaluating the precision of the ranges (ambiguity-fixed). The values of fixed-precision of ranges obtained with full AR are set as a criteria for PAR, it is to see how many ambiguities are needed to be fixed to get a same value of fixed-precision. Refer Table 4.9 for further discussion.

1. Results for dual frequency did not converge to give 0.999 ASR in case of full AR. The values of fixed-precision obtained with full AR were taken as reference values. PAR was used to achieve a smiliar fixed-precision as in full AR. In case of full AR, for code precision of 10cm, the best values of ASR obtained were 0.96, 0.99 and 0.984 which gave fixed-precision of 1.8, 1.8 and 1.9cm at $0°$, $-30°$ and $-60°$, the epochs required were 15568, 1899 and 15751. For similar latitude and code precision, PAR takes 2699, 1540 and 5132 epochs by fixing 78, 58 and 79 % of ambiguities to give a similar fixed-precision. For all other values of measurement precision too, PAR fixes only 50% of ambiguities, with a reduced number of epochs taken for the same.

2. For triple frequency system, for the first three values of measurement precision, full AR converged to 0.999, for the same values of measurement precision, PAR fixed 100% of ambiguities to get a similar value of fixed-precision as in full AR. For the last three values of measurement precision, which have higher values of uncertainty, full AR did not converge to 0.999

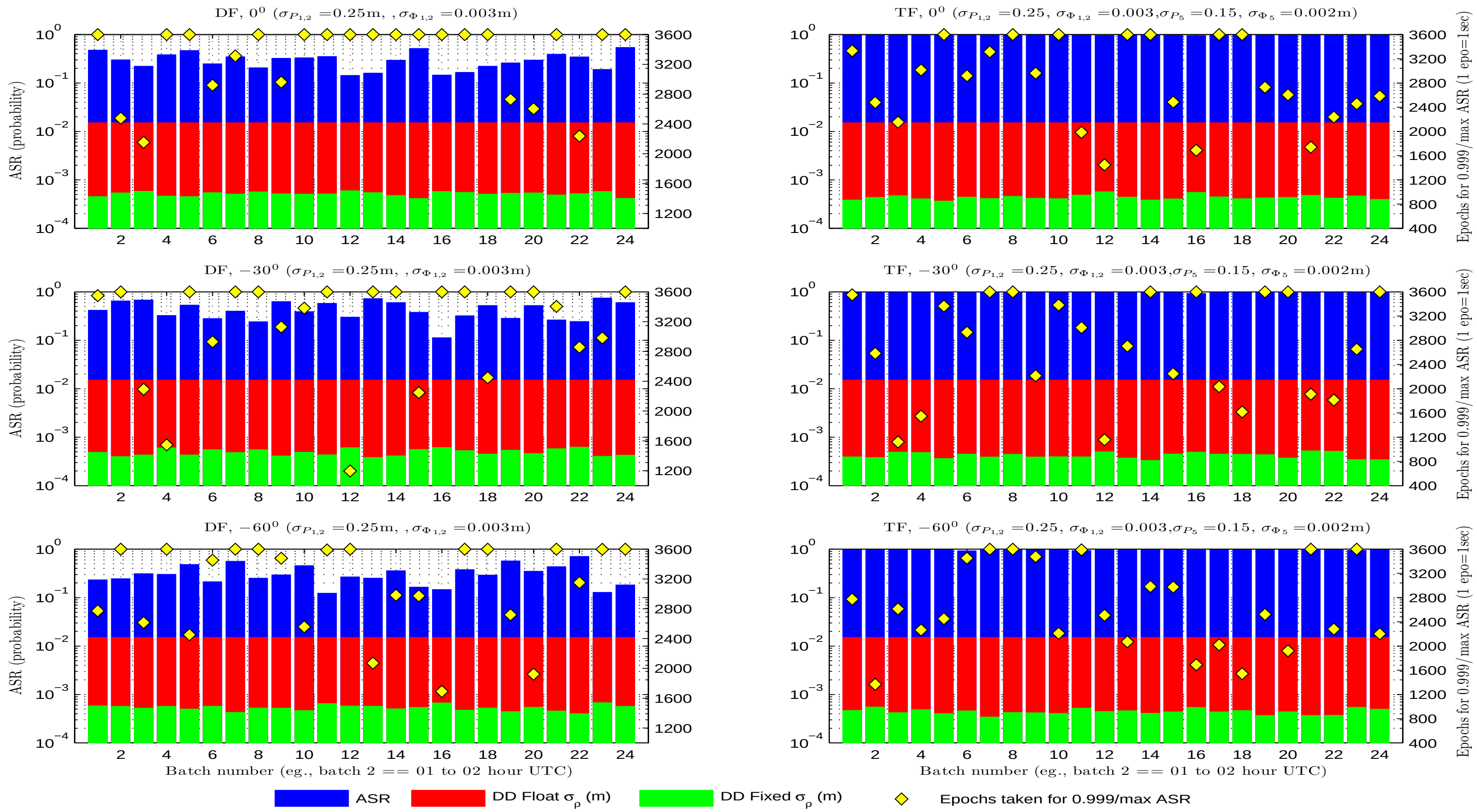

Figure 4.5: **GPS only**, **Full AR**, Dual and Triple Frequencies, Geometry Free model, Ionosphere unknown scenario, **Instantaneous ASR** and Gain analysis when **(measurement precision is varied, number of satellites are as available(not held fixed))** at $0°$, $-30°$ and $-60°$ degree latitude

Phase	Code	Full AR - minimum desired ASR = 0.999						Partial AR @ 0.999, criteria - similar or better value of fixed-precision								
		Epochs taken			$\sigma_{\check{\rho}}$(meters)			$\sigma_{\check{\rho}}$(meters)			Epochs taken			Ambiguities fixed(%)		
(meters)		0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
$\Phi1,\Phi2$	$P1,P2$	High-end receivers, Dual Frequency														
0.002	0.10	15568	1899	15751	0.018	0.018	0.019	0.018	0.018	0.019	2699	1540	5132	78	58	79
		0.960	0.990	0.984												
0.002	0.15	15568	1899	15751	0.027	0.028	0.029	0.027	0.028	0.029	10623	1755	13463	50	50	50
		0.788	0.875	0.796												
0.002	0.20	15568	1899	15751	0.036	0.037	0.039	0.036	0.037	0.039	10623	1754	13463	50	50	50
		0.539	0.633	0.485												
0.003	0.25	15568	1899	15751	0.045	0.046	0.048	0.045	0.046	0.048	10623	1754	13463	50	50	50
		0.316	0.401	0.256												
0.003	0.30	15568	1899	15751	0.054	0.055	0.058	0.054	0.055	0.058	10623	1754	13463	50	50	50
		0.171	0.242	0.131												
0.003	0.35	15568	1899	15751	0.063	0.064	0.068	0.063	0.064	0.068	10558	1754	13463	50	50	50
		0.090	0.146	0.068												
$\Phi5$	$P5$	High-end receivers, Triple Frequency														
0.002	0.05	719	1283	6957	0.001	0.000	0.000	0.001	0.000	0.000	719	1283	6958	100	100	100
0.002	0.1	2859	1560	13375	0.000	0.000	0.000	0.000	0.000	0.000	2859	1561	13375	100	100	100
0.002	0.125	10756	1648	13452	0.000	0.000	0.000	0.000	0.000	0.000	10756	1648	13453	100	100	100
0.002	0.15	10811	1899	15751	0.036	0.037	0.039	0.036	0.037	0.039	454	346	472	67	67	67
		0.990	0.999	0.998												
0.002	0.175	10811	1899	15751	0.043	0.044	0.046	0.043	0.043	0.046	408	305	416	67	67	67
		0.984	0.998	0.996												
0.002	0.2	10811	1899	15751	0.050	0.051	0.054	0.050	0.051	0.054	348	273	194	67	67	67
		0.979	0.996	0.994												

Table 4.9: GPS only, **PAR**, Geometry Free model, Ionosphere unknown scenario, ASR and fixed-precision for range values - **measurement precision varied**, number of satellites are as available. A comparison of results obtained from Full AR with the ones obtained from PAR.

within the stipulated epochs. PAR took 67% of ambiguities to get fixed to give a similar measurement precision as in full AR. The number of epochs were significantly reduced from over 10,000 to just between 194 to 472.

PAR definitely promises the results of similar magnitude (in terms of fixed-precision) with less number of epochs required to do the same, as compared to full AR. In Ionosphere unknown section, fixed-precision of ranges at 2cm is be evaluated using PAR (as done for Ionosphere known scenario), in the commencing section.

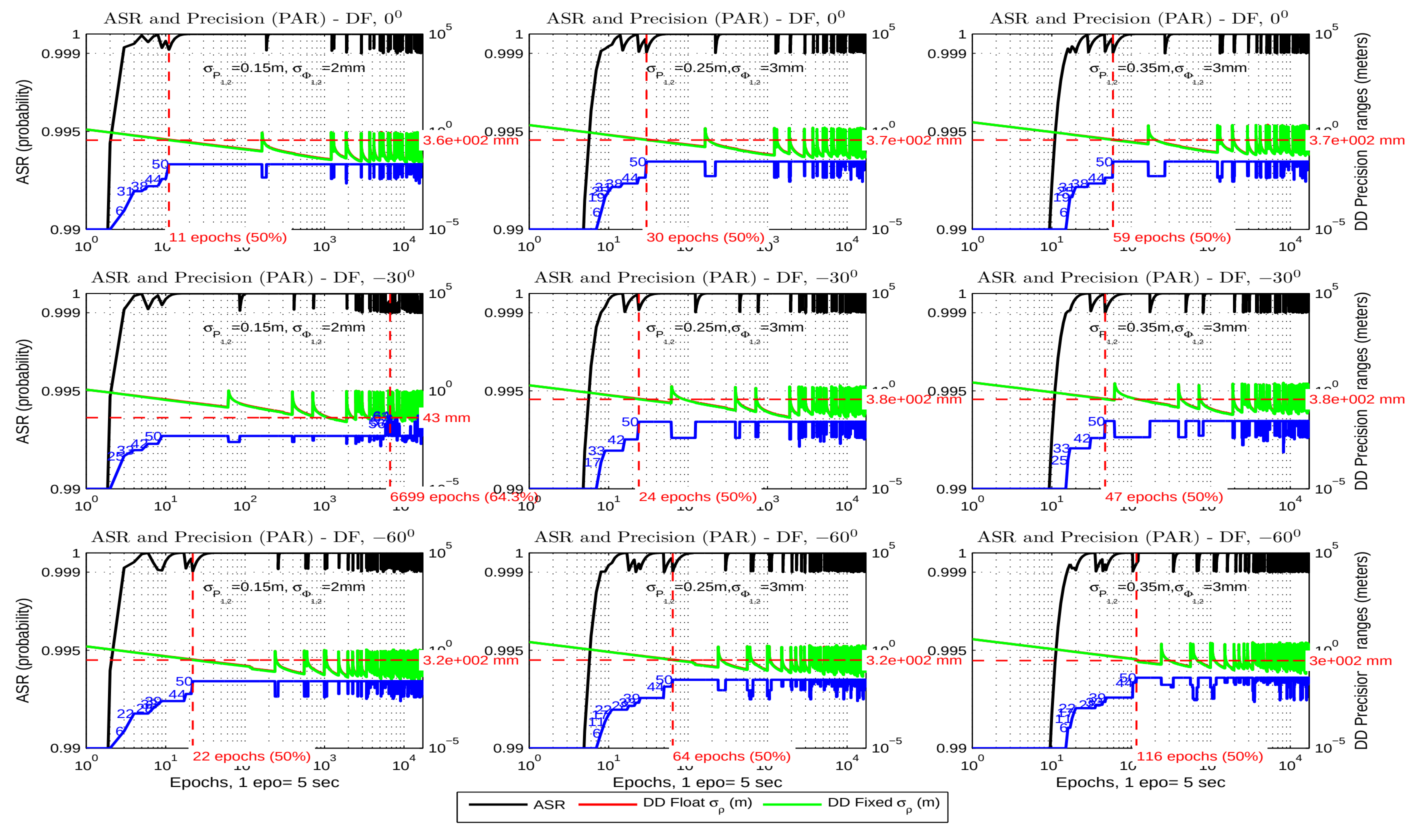

Figure 4.6: **GPS only**, Dual frequency, **PAR**, Ionosphere unknown scenario, **(measurement precision is varied, number of satellites are as available)-** ASR and precision of float and fixed ambiguities are analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis).

4.4.4 GPS only - partial ambiguity resolution to obtain fixed-precision of 2 cm, Ionosphere unknown scenario

In Table 4.9, while evaluation PAR for a Geometry Free model, Ionosphere unknown scenario, the fixed-precision of the ranges obtained lied in the range 1mm or less and 6.8cm. In Table 4.10, a fixed-precision of ranges of 20mm or 2cm is aimed and the corresponding ambiguities fixed by PAR to achieve the same are presented and analyzed.

		Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for range								
Phase	Code	$\sigma_{\check{p}}$(meters)			Epochs taken			Ambiguities fixed(%)		
(meters)		0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
Φ1,Φ2	$P1$,$P2$	High-end receivers, Dual Frequency								
0.002	0.1	0.020	0.020	0.020	2603	1381	5096	72	57	79
0.002	0.15	0.025	0.026	0.027	15568	1899	15751	50	50	50
0.002	0.2	0.033	0.034	0.036	15568	1899	15751	50	50	50
0.003	0.25	0.042	0.043	0.045	15568	1899	15751	50	50	50
0.003	0.3	0.050	0.052	0.054	15568	1899	15751	50	50	50
0.003	0.35	0.059	0.060	0.063	15568	1899	15751	50	50	50
Φ5	$P5$	High-end receivers, Triple Frequency								
0.002	0.05	0.020	0.020	0.020	315	250	208	85	86	76
0.002	0.1	0.020	0.020	0.020	490	609	550	83	90	81
0.002	0.125	0.020	0.020	0.020	559	653	1482	79	90	88
0.002	0.15	0.019	0.020	0.020	892	1101	1955	76	88	88
0.002	0.175	0.018	0.020	0.020	1010	1166	5000	81	79	90
0.002	0.2	0.018	0.020	0.020	1096	1194	5043	81	79	86

Table 4.10: GPS only, **PAR**, Geometry Free model - Ionosphere unknown scenario, ASR and number of epochs for obtaining 2cm fixed-precision of ranges - **measurement precision is varied**, number of satellites are as available.

Note: In Table 4.4, some of the values of fixed-precision are more than the desired 2cm, the reason being the value of 2cm is not possible with certain measurement precision scenarios. For all such values the best value of fixed-precision obtained is written.

The results presented in Table 4.10 are analyzed below.

1. For dual frequency, only for the first value, which is the best value of measurement precision (with code precision as 10cm), fixed-precision was less than 2cm. Hence while evaluating PAR, only first measurement precision scenario showed improvement with respect to reduced number of epochs. It took 2603, 1381 and 5096 epochs to give a fixed-precision of 2cm, while resolving full ambiguities, it gave a fixed-precision of around 1.9cm in 15568, 1899 and 15751 epochs. For all other values of measurement precision scenarios, fixed-precision of 2cm could not be obtained.

2. For triple frequency, the fixed-precision of 2cm was obtained in a reduced time for all measurement precision scenarios, since it fixed only 76 to 90% of ambiguities to do the same.

3. In all, PAR is a useful tool when one desires accuracies of an order of 2cm, since at all times, whether in dual or in triple frequency, not even a single time 100% of ambiguities were fixed.

Below, some of the deductions are made for Geometry Free model, Ionosphere unknown scenario, partial ambiguity resolution case for most general values of measurement precisions.

Conclusions for Geometry Free model, Ionosphere unknown scenario, partial ambiguity resolution scenario

1. For dual frequency, with $\sigma_{\Phi_{L1,L2}}$=2mm, $\sigma_{P_{L1,L2}}$=20cm corresponding to high-end GPS receivers, it took 10623, 1754 and 13463 epochs by fixing 50% of ambiguities at $0°, -30°$ and $-60°$ respectively to reach 0.999 ASR for the same fixed-precision as in full AR (for full AR, it took 15568, 1899 and 15751 epochs).

2. For triple frequency, with $\sigma_{\Phi_{L1,L2,L5}}$=2mm, $\sigma_{P_{L1,L2}}$=20cm, $\sigma_{P_{L5}}$=12.5cm the results were similar to full AR. $\sigma_{\Phi_{L1,L2}}$=2mm, $\sigma_{\Phi_{L5}}$=3mm, $\sigma_{P_{L1,L2}}$=25cm and above, $\sigma_{P_{L5}}$=1.5cm and above, full AR did not converge to 0.999 within the stipulated epochs. PAR took 67% of ambiguities to get fixed to give a similar measurement precision as in full AR. The number of epochs were significantly reduced from over 10,000 to just between 194 to 472.

3. While PAR was evaluated to obtain a fixed-precision of 2cm, it performed significantly well for triple frequency Ionosphere unknown scenario. PAR fixed only 76 to 90% of ambiguities, with reduced number of epochs as compared to full AR to achieve the same.

4.5 Geometry Free model, Ionosphere weighted scenario

The ionosphere weighted (Ionosphere weighted), Geometry Free model is based on the assumption that ionosphere is known a-priori with a certain precision. The precision of the ionosphere, here we write in terms of undifferenced ionosphere precision at zenith denoted as, σ_I is converted to its DD counterpart by multiplying it by 2. The DD ionosphere VC matrix is formed, denoted as Q_I, see Table 4.11, which is then added to the measurement precision matrix Q_{yy}, which is then used to estimate the other unknowns (ranges etc.). It is to be noted that since ionosphere is not estimated, the design matrix for non-temporal parameters, denoted as A_{II} in Table 4.11, is similar to Ionosphere known scenario. Refer Table 4.11 for closed form of the functional and stochastic models for ionosphere weighed scenario, also the redundancy of the ionosphere weighted model is presented in Table 4.11.

In the commencing section, simulation results for ASR and fixed-precision of ranges is presented different baselines, ranging from 1 to 1000 Km.

4.5.1 GPS only, Full ASR analysis for selective values of baselines for k epochs, Ionosphere weighted scenario

An analysis of ASR and fixed-precision of ranges is presented below for different values of baselines, see Table 4.12 and Figures 4.7 to E.13 in Appendix E. The undifferenced precision of the ionosphere at zenith, denoted as σ_I is taken to be a fixed function of baseline length, $\sigma_I = 0.68$ mm per Km, refer chapter 2. Baseline length is varied from 1 to 1000 Km.

In Table 4.12 results for single, dual and triple frequency system are presented, the measurement precision for each of the frequency combination correspond to low-end (single frequency) high-end (dual, and triple frequencies), and hence measurement precision values are chosen accordingly and fixed to one value, see Table 3.2. The number of satellites are as available. Only the baseline length is varied in order to understand the effect of ionosphere weighting on ASR and fixed-precision. In Table 4.12, under each single, dual and triple frequency results, green (Ionosphere known scenario, from Table 4.2) and red (Ionosphere unknown scenario, from Table 4.7) results are presented for similar values of measurement precision, so that one can compare how the ionosphere-fixed and -float scenarios

Geometry Free model, ionosphere weighted scenario		
Functional model		**Stochastic model**
Non-temporal parameters	Temporal parameters	
$A_{I(i)} = \left[\begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes I_{m-1} \quad \begin{pmatrix} e_f \\ e_f \end{pmatrix} \otimes G_{m-1} \right]$	$A_{II(i)} = \left[\begin{pmatrix} -\mu_f \\ \mu_f \end{pmatrix} \otimes I_{m-1} \quad \begin{pmatrix} e_f \\ e_f \end{pmatrix} \otimes I_{m-1} \right]$	$Q_{y(i)} = \begin{bmatrix} Q_\Phi & \\ & Q_P \end{bmatrix} \otimes Q_{DD}(i)$ where $Q_{DD}(i) = (D_m^T W_i^{-1} D_m)$ $Q_I(i) = CI \otimes Q_{DD}(i)$ $Q_{y,I}(i) = Q_y(i) + Q_I(i)$
Redundancy (for GPS only, Galileo only, GPS+Galileo (common frequency $L1(E1), L5(E5a)$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $f * (m-1)$	Ranges: $k * (m-1)$ Ionosphere $= k * (m-1)$	$2fk * (m-1)$ Pseudo observations (Ionosphere) $= k * (m-1)$
Redundancy of ionosphere weighted scenario		$(2fk - f - k) * (m-1)$
For GPS/Galileo only: $f = (1, \cdots, j)$, for GPS+Galileo: $f = (1, \cdots, j_c)$, j_c is the common/overlapping frequency and $m_{jc} = (m_{GPS} + m_{Gal})$		
Redundancy (for GPS+Galileo (quadruple frequency $L1(E1), L5(E5a), L2, E5b$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)$	Ranges: $k * (m_{jc} - 1)$ Ionosphere$=k * (m_{jc} - 1)$	$2k * [2 * (m_{jc} - 1) + (m_{GPS} - 1) + (m_{Gal} - 1)]$ Pseudo observations (Ionosphere) $=k * (m_{jc} - 1)$
Redundancy of quadruple frequency ionosphere weighted scenario		$(3k - 2)(m_{jc} - 1) + (2k - 1) * [(m_{GPS} - 1) + (m_{Gal} - 1)]$
$f = 1, 2$ for GPS+Gaileo $L1(E1), L5(E5a)$, m_{jc} are the total number of satellites for combined GPS+Galileo system with GPS as reference satellite		

Table 4.11: Double-differenced Design matrix and VC matrix for Geometry Free model, ionosphere weighted scenario

converge to ionosphere weighted scenario.

Note: All the values of fixed-precision of ranges marked as 0.000 indicate the values less than 1mm.

Based on the results from Table 4.12 and Figures 4.7 to E.13 for single, dual and triple frequency GPS systems, following analysis is drawn

1. In Table 4.12, the ionosphere-weighted scenario with baseline length 1 Km corresponds to Ionosphere known scenario (green rows), in terms of instantaneous ASR and number of epochs required to converge to 0.999 ASR. For dual frequency the instantaneous ASR with ionosphere weighted scenario at 0°, −30° and −60° is 0.20, 0.192 and 0.015, with Ionosphere known scenario the results are 0.20, 0.193 and 0.15, hence very similar. The number of epochs for 0.999 ASR with ionosphere weighted at 0°, −30° and −60° are 33, 14, 27, at 1 Km baseline, with Ionosphere known scenario they are 33, 14, 27 hence again very similar results. Similar observations can be made for single and triple frequency systems.

2. For single frequency, for baselines upto 250 Km, the number of epochs taken for 0.999 ASR increase marginally, say by less than 10 % at 0° latitude. For dual and triple frequency systems, at 100 Km the epochs taken increase by 350 to 400%. Hence one can understand that for dual and triple frequency systems the increase in number of epochs is more abrupt as compared to single frequency. This is due to the imprecise code measurements on single frequency (50cm), with further addition of low values of ionosphere weights (of the order of 0.68 mm per Km), do not matter much(at least up to 250 Km), whereas for precise measurements (high end geodetic receivers), the effect of ionosphere can be observed right from 10 Km.

3. For geodetic receivers, the baseline length from 10 up to 1000 Km are significant in terms of AR. For dual frequency system, 0.999 ASR was achieved for baselines upto 250 Km, and not for 500 and 100 Km baseline. The epochs taken for 0.999 ASR range between 15 and 1748 for 10 to 250 Km baseline at −30° and for triple frequency they range between 6 and 801 epochs for 10 and 250 Km baseline, for 500 and 1000 Km baseline, epochs taken were 1412 and 1829 at −30°. It will be interesting to evaluate the precise multi-frequency Galileo system for similar baselines and combination of GNSS systems, apart from using PAR.

4. The results for Ionosphere unknown solution are based on ionosphere un-

Baseline length	Instantaneous ASR (probability)			Epochs for 0.999 ASR			$\sigma_{\hat{\rho}}$ (meters) @ 0.999 ASR		
(Km)	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
	Low-end receivers, Single Frequency						$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m		
1	0.000	0.000	0.000	17231	1613	15751	0.030	0.033	0.032
							0.986	0.983	0.986
10	0.000	0.000	0.000	17232	1615	15751	0.030	0.033	0.032
							0.985	0.983	0.986
100	0.000	0.000	0.000	17249	1639	15751	0.030	0.033	0.032
							0.984	0.982	0.984
250	0.000	0.000	0.000	17275	1678	15751	0.030	0.034	0.033
							0.973	0.971	0.970
500	0.000	0.000	0.000	13360	1748	15751	0.033	0.036	0.036
							0.910	0.893	0.879
1000	0.000	0.000	0.000	10811	1899	15751	0.040	0.045	0.045
							0.592	0.489	0.423
I-Fix	0.000	0.000	0.000	17230	1613	15751	0.030	0.033	0.032
							0.985	0.983	0.986
	High-end receivers, Dual Frequency						$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m		
1	0.020	0.192	0.015	33	14	27	0.001	0.002	0.001
10	0.011	0.124	0.008	34	15	28	0.003	0.004	0.003
100	0.000	0.000	0.000	122	146	121	0.003	0.003	0.003
250	0.000	0.000	0.000	695	843	975	0.001	0.001	0.001
500	0.000	0.000	0.000	7464	1748	15751	0.019	0.023	0.022
							0.998	0.992	0.992
1000	0.000	0.000	0.000	10811	1899	15751	0.028	0.032	0.033
							0.865	0.803	0.735
I-Fix	0.020	0.193	0.015	33	14	27	0.001	0.002	0.001
I-Float	0.000	0.000	0.000	15568	1899	15751	0.045	0.046	0.048
							0.316	0.401	0.256
	High-end receivers, Triple Frequency						$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m		
1	0.318	0.718	0.340	15	6	12	0.001	0.002	0.002
10	0.284	0.671	0.295	15	6	12	0.004	0.006	0.004
100	0.000	0.000	0.000	114	138	113	0.002	0.002	0.002
250	0.000	0.000	0.000	492	801	934	0.001	0.001	0.001
500	0.000	0.000	0.000	2721	1412	13398	0.001	0.001	0.001
1000	0.000	0.000	0.000	13506	1829	15743	0.027	0.000	0.000
							0.998	0.999	0.999
I-Fix	0.318	0.718	0.340	15	6	12	0.001	0.002	0.001
I-Float	0.000	0.000	0.000	10811	1899	15751	0.036	0.037	0.039
							0.990	0.999	0.998

Table 4.12: GPS only, **Full AR**, Geometry Free model, Ionosphere Weighted scenario, ASR and fixed-precision of ranges(meters)are presented - measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, σ_I = 0.68mm per km.

known and cannot be compared with ionosphere weighted scenario (ionosphere known with certain precision). But definitely, one can easily understand that ionosphere weighted is better, even at 1000 Km baseline length, as compared to ionosphere unknown scenario.

Conclusions for Geometry Free model, ionosphere weighted, full ambiguity resolution scenario

1. For baseline lengths 1 and 10 Km, the performance is similar in terms of ambiguity resolution, that is the number of epochs taken to resolve ambiguities are almost similar. The values of fixed-precision are imprecise for 10Km as compared with 1Km, accompanied by a slight fall in instantaneous ASR for 10 Km baseline, especially for dual and triple frequencies. The added ionosphere weight is more significant in dual and triple frequencies since precise measurements are used, as compared to single frequency. $0°, -30°$ and $-60°$ respectively.

2. For baseline lengths of 250, 500 and 1000 Km, for dual and triple frequencies, with $\sigma_{\Phi_{L1,L2}}$=3mm, $\sigma_{P_{L1,L2}}$=25cm, $\sigma_{\Phi_{L5}}$=2mm, $\sigma_{P_{L5}}$=15cm corresponding to high-end GPS receivers, the relative positioning results for σ_I=0.68mm per Km (un-differenced, at Zenith), instantaneous ASR of 0.999 is not achieved for any baseline length.

3. For dual frequency, it takes 843 epochs, a fixed-precision of 1mm is obtained for 250 Km baseline at $-30°$ latitude. For 500 and 1000 Km baseline length 0.999 ASR is not achieved.

4. For triple frequency, it takes 801, 1412 and 1829 epochs which gives a fixed-precision of 1mm for 250, 500 and 1000 Km at $-30°$ latitude.

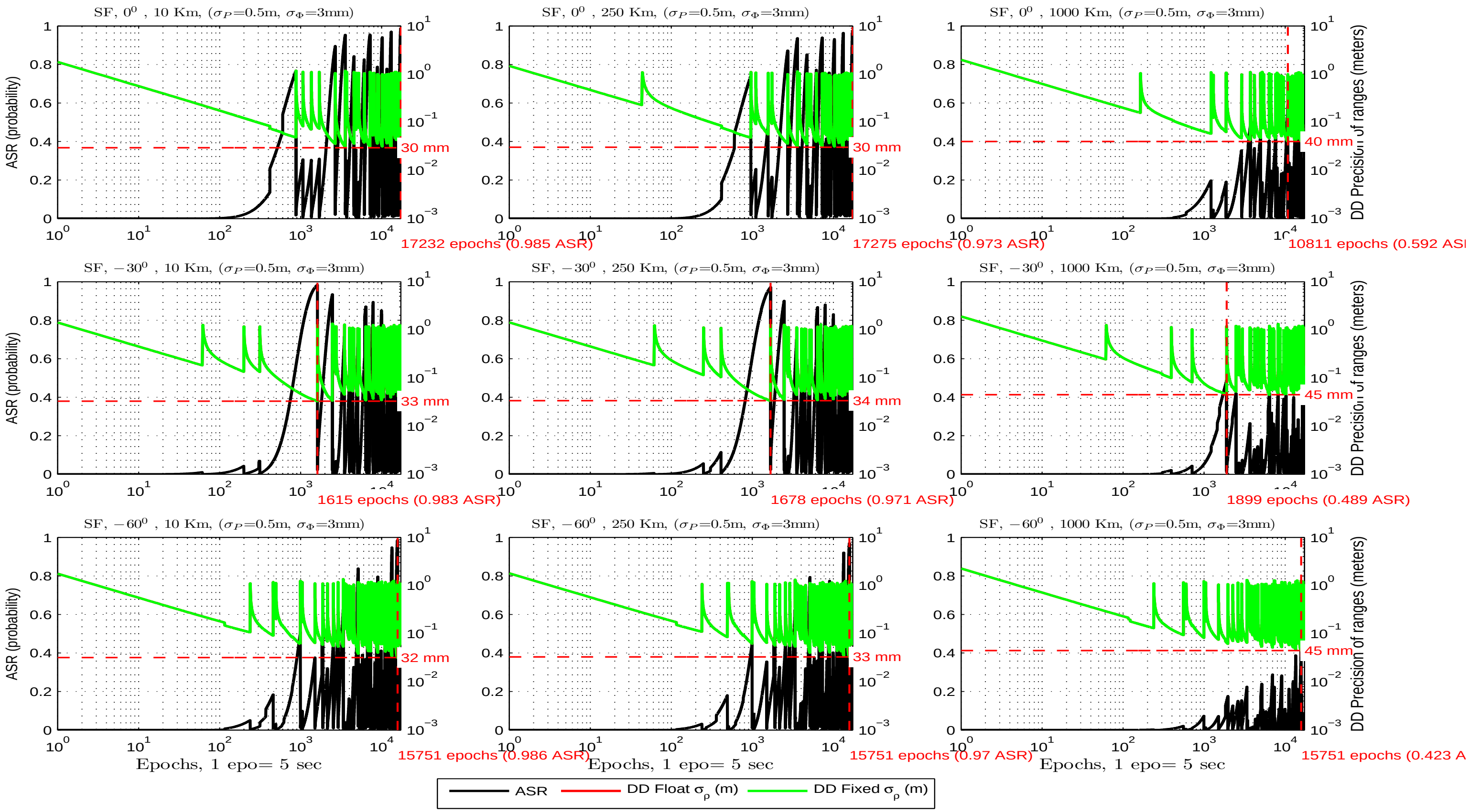

Figure 4.7: **GPS only**, **Full AR**, Single frequency, Geometry Free model, Ionosphere Weighted scenario, (measurement precision and satellite number is held fixed), **baseline length is varied from 1 Km to 1000 Km**- ASR and Gain analysis at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

4.5.2 GPS only, Partial AR analysis for selective values of baselines for k epochs, Ionosphere weighted scenario

The Ionosphere Weighted scenario, partial ambiguity resolution based, ASR and precision of ranges for ambiguity-fixed solution results are presented in Figures 4.8 to E.16 for single, dual and triple frequencies of GPS, see Appendix E. The Table 4.13 shows an comparison for precision of ranges corresponding to ambiguity-fixed solution (at 0.999 ASR), for full AR and partial AR.

The analysis of the results based on Table 4.13 and Figures 4.8 to E.16 is presented below

Conclusions for Geometry Free model, ionosphere weighted, partial ambiguity resolution scenario

1. One of the conclusions for single frequency GPS ($\sigma_{\Phi_{L1}}$=3mm, $\sigma_{P_{L1}}$=50cm corresponding to low-end GPS receivers) that can be drawn looking at partial AR is that for baseline length of 1000 Km, it does not make sense to fix ambiguities. Since the precision obtained is a function of number of epochs only (that is, it corresponds to ambiguity-float solution). For baseline lengths 1 to 250 Kms, the number of epochs are drastically reduced. At 0° latitude, it took 2607 to 2705 with PAR to give fixed-precision of around 3cm, with full it took 17231 to 17275 epochs. It is important to note that 0.999 ASR could not be achieved for any baseline length with single frequency, full AR.

2. Instantaneous ASR of 0.999 or more could not be obtained with GPS only, partial ambiguity resolution for single/dual or triple frequency for any baseline length between 1 and 1000 Km.

3. For baseline length of 1000 Km, for dual and triple frequencies, with $\sigma_{\Phi_{L1,L2}}$ =3mm, $\sigma_{P_{L1,L2}}$=25cm, $\sigma_{\Phi_{L5}}$=2mm, $\sigma_{P_{L5}}$=15cm corresponding to high-end GPS receivers, the relative positioning results for σ_I=0.68mm per Km (un-differenced, at Zenith) for partial AR are convincing. For dual frequency, for baseline lengths 1 to 250 Km, PAR performs similar to full Ar in terms of number of epochs taken to achieve fixed-precision as in full AR. For baseline lengths 500 and 1000 Km, 0.999 ASR is not achieved in full AR. PAR fixes 50 to 67% of ambiguities and it takes

	Full AR - minimum desired ASR = 0.999						Partial AR @ 0.999, criteria - similar or better value of fixed-precision								
Baseline length	Epochs taken			$\sigma_{\hat{\rho}}$(meters)			$\sigma_{\hat{\rho}}$(meters)			Epochs taken			Ambiguities fixed(%)		
(Km)	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
	Low-end receivers, Single Frequency									$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m					
1	17231	1613	15751	0.030	0.033	0.032	0.029	0.033	0.031	2607	1365	5128	56	14	57
	0.986	0.983	0.986												
10	17232	1615	15751	0.030	0.033	0.032	0.030	0.033	0.031	2608	1367	5128	56	14	57
	0.985	0.983	0.986												
100	17249	1639	15751	0.030	0.033	0.032	0.030	0.033	0.032	2625	1389	13259	56	14	38
	0.984	0.982	0.984												
250	17275	1678	15751	0.030	0.034	0.033	0.030	0.034	0.033	2715	1645	13415	44	0	14
	0.973	0.971	0.970												
500	13360	1748	15751	0.033	0.036	0.036	0.033	0.036	0.036	7270	1748	15751	13	0	0
	0.910	0.893	0.879												
1000	10811	1899	15751	0.040	0.045	0.045	0.040	0.045	0.045	10811	1899	15751	0	0	0
	0.592	0.489	0.423												
I-Fix	17230	1613	15751	0.030	0.033	0.032	0.030	0.033	0.031	2606	1365	5128	56	14	57
	0.985	0.983	0.986												
	High-end receivers, Dual Frequency									$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m					
1	33	14	27	0.001	0.002	0.001	0.001	0.002	0.001	33	15	28	100	100	100
10	34	15	28	0.003	0.004	0.003	0.003	0.004	0.003	34	15	29	100	100	100
100	122	146	121	0.003	0.003	0.003	0.003	0.003	0.003	122	147	121	100	100	100
250	695	843	975	0.001	0.001	0.001	0.001	0.001	0.001	696	844	976	100	100	100
500	7464	1748	15751	0.019	0.023	0.022	0.017	0.023	0.022	2284	1249	1918	67	50	50
	0.998	0.992	0.992												
1000	10811	1899	15751	0.028	0.032	0.033	0.028	0.032	0.033	3624	1647	13277	50	50	50
	0.865	0.803	0.735												
I-Fix	33	14	27	0.001	0.002	0.001	0.001	0.002	0.001	34	15	28	100	100	100
I-Float	15568	1899	15751	0.045	0.046	0.048	0.045	0.046	0.048	10623	1754	13463	50	50	50
	0.316	0.401	0.256												
	High-end receivers, Triple Frequency									$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m					
1	15	6	12	0.001	0.002	0.002	0.001	0.002	0.002	16	7	13	100	100	100
10	15	6	12	0.004	0.006	0.004	0.004	0.006	0.004	16	6	13	100	100	100
100	114	138	113	0.002	0.002	0.002	0.002	0.002	0.002	114	138	114	100	100	100
250	492	801	934	0.001	0.001	0.001	0.001	0.001	0.001	493	801	934	100	100	100
500	2721	1412	13398	0.001	0.001	0.001	0.001	0.001	0.001	2721	1412	13398	100	100	100
1000	13506	1829	15743	0.027	0.000	0.000	0.027	0.000	0.000	587	1829	15744	67	100	100
	0.998	0.999	0.999												
I-Fix	15	6	12	0.001	0.002	0.001	0.001	0.002	0.001	15	6	13	100	100	100
I-Float	10811	1899	15751	0.036	0.037	0.039	0.036	0.037	0.039	454	346	472	67	67	67
	0.990	0.999	0.998												

Table 4.13: GPS only, **PAR**, Geometry Free model, Ionosphere Weighted scenario, ASR and fixed-precision of ranges (meters) are presented - measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, σ_I = 0.68 mm per km.

1249 and 1647 for 500 and 1000 Km baseline at $-30°$, with full AR it took 1748 and 1899 epochs.

4. For triple frequency, PAR performs similar to full AR in terms of number of epochs taken for baseline length of 1 to 500 Km. For baseline length of 1000 Km at $0°$ latitude, full AR did not converge to give 0.999 ASR within the stipulated epochs. With PAR, it took 587 epochs for 1000 Km baseline at $0°$ latitude by fixing 67% of ambiguities, with full AR it took 13506 epochs. For all other latitude locations at 1000 Km baseline, PAR performed similar to full AR.

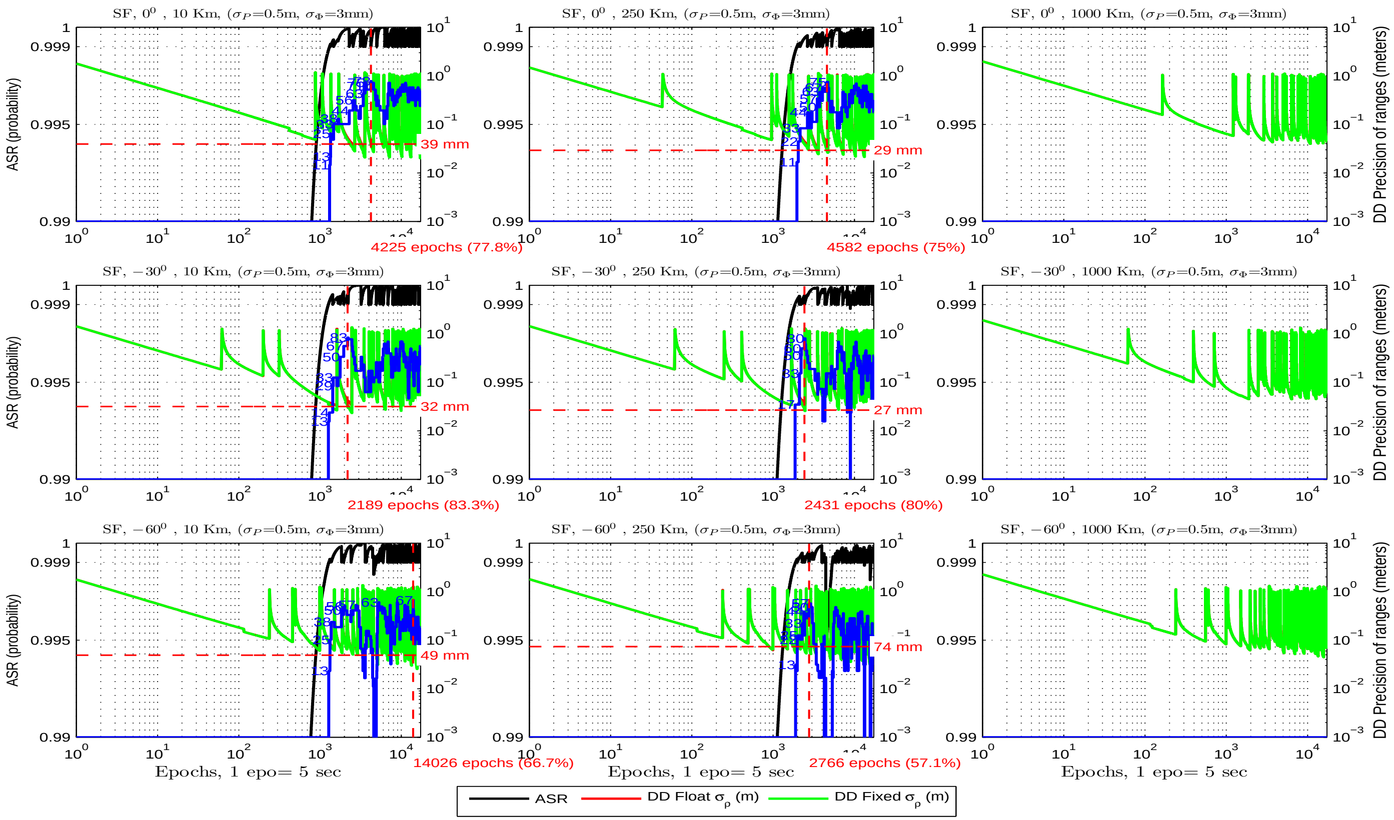

Figure 4.8: **GPS only**, **PAR**, Single frequency, Geometry Free model, Ionosphere Weighted scenario, (measurement precision and satellite number is held fixed), **baseline length is varied from 1 Km to 1000 Km**- ASR and Gain analysis at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis).

4.5.3 GPS only - partial ambiguity resolution to obtain fixed-precision of 2 cm, Ionosphere weighted scenario

In Table 4.13, while evaluation PAR for a Geometry Free model, Ionosphere known scenario, the fixed-precision of the ranges obtained lied in the range 1mm or less and 6.8cm. In Table 4.14, a fixed-precision of ranges of 20mm or 2cm is aimed and the corresponding ambiguities fixed by PAR to achieve the same are presented and analyzed.

Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for range									
Baseline length (Km)	$\sigma_{\check{p}}$(meters) 0°	−30°	−60°	Epochs taken 0°	−30°	−60°	Ambiguities fixed(%) 0°	−30°	−60°
Low-end receivers, Single Frequency						$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m			
1	0.020	0.023	0.025	17200	2470	15667	63	80	43
10	0.020	0.023	0.025	17202	2470	15666	63	80	43
100	0.020	0.024	0.026	17236	2470	15659	63	80	43
250	0.021	0.025	0.031	13266	2470	15646	67	80	14
500	0.028	0.034	0.036	13276	6386	15751	33	14	0
1000	0.040	0.045	0.045	10811	6386	15751	0	0	0
High-end receivers, Dual Frequency						$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m			
1	0.001	0.002	0.001	33	14	27	100	100	100
10	0.003	0.004	0.003	34	15	28	100	100	100
100	0.020	0.008	0.020	86	136	82	50	93	50
250	0.020	0.020	0.020	438	561	476	50	50	50
500	0.017	0.020	0.020	2284	1554	2458	67	50	69
1000	0.025	0.028	0.030	13506	6386	15751	50	50	50
High-end receivers, Triple Frequency						$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m			
1	0.001	0.002	0.002	15	6	12	100	100	100
10	0.004	0.006	0.004	15	6	12	100	100	100
100	0.020	0.020	0.020	79	94	78	67	67	67
250	0.020	0.020	0.020	325	386	384	67	67	67
500	0.020	0.020	0.020	646	705	850	67	83	83
1000	0.018	0.020	0.019	829	992	1835	76	88	88

Table 4.14: GPS only, **PAR**, Geometry Free model - Ionosphere weighted model, ASR and number of epochs for obtaining 2cm fixed-precision of ranges - **measurement precision is varied**, number of satellites are as available.

The analysis for PAR to obtain a fixed-precision of 2cm, based on the values presented in Table 4.14 is given below.

1. For single frequency, for baseline lengths 1 to 100 Km, fixed-precision of 2cm is obtained only at 0° latitude, for all other latitude locations and baseline lengths of 250 Km and above, fixed-precision is much higher than 2cm. For scenarios where 2cm fixed-precision is obtained, there is fall in number of epochs taken by a modest 30 epoch from the original 17231 to 17200 for baseline length 1Km, 0° latitude. Reduction of number of epochs of similar magnitude is observed for baseline lengths 10 an 100 Km, 0° latitude.

2. For dual frequency, for baseline lengths 1 to 500 Km, 2cm fixed-precision is obtained with a fall in number of epochs taken as compared to full AR (of course in full AR, the fixed-precision obtained was better at times). With full AR, it took 14 to 1748 epochs at −30° latitude which gave a fixed-precision of 1mm to 2.3cm (here ASR was less than 0.999), with PAR to obtain 2cm fixed-precision, it took 14 to 1554 epochs by fixing between 50 and 93% of ambiguities.

3. For triple frequency, 2cm precision for DD ranges was obtained for baseline all baseline lengths, the number of epochs reduced significantly for baseline lengths of 100 to 1000 Km as compared to full AR (with full AR, better precision was obtained at times, than 2cm). With full AR, it took 6 to 1829 epochs for 1 and 1000 Km baseline length at 0° latitude, with PAR to obtain fixed-precision of 2cm, it took, 6 to 992 epochs by fixing between 67 and 88% of ambiguities.

4. It can be concluded, without any doubt that PAR performs exceptionally well for long baselines, Geometry Free model, ionosphere weighted scenario, especially when one aims for obtaining a modest value of fixed-precision, say 2cm.

4.5.4 GPS only, full ambiguity resolution, hourly batches for simulating ASR, Ionosphere weighted scenario

The analysis of ambiguity resolution is done at the beginning of every hour throughout the day inorder to see the effect of satellite constellation on success rate. The ambiguity success rate is computed by accumulating the epochs (3600 epochs, 1 epoch = 1 second) for 24 batches, starting at 00 UTC hour and ending at 23:59:59 UTC hour. The ambiguities and other unknowns are reinitialized at the start of every hour.

The ASR was simulated for ionosphere weighted scenario, for different values of baseline lengths, ranging from 1 Km to 1000 Km. For doing so, all the satellites in view were considered for simulation, the criteria for selection being that the elevation angle of any satellite should be more than 10°. The values of measurement precision considered for each single, dual and triple frequency are as given in Table 3.2 and are also found at the header of every frequency section in Table 4.15. Also in Table 4.15, for 0.999 ASR (0.999 ASR, if obtained in 3600 epochs is considered else the best value of ASR obtained is considered), minimum epochs and maximum epochs (among the 24 batches) are presented for each value of measurement precision. Figures 4.9 and 4.10 presents the ASR, and corresponding values of double differenced fixed and float precision (of ranges) for each of the 24 batches, along with the number of epochs taken to converge to 0.999 / best value of ASR for the 250 and 1000 Kms baseline length respectively. The chosen values of baseline are close to the current baseline distance for networks in current and future scenarios.

The following is the analysis based on Table 4.15 and Figures 4.9 and 4.10.

(1) For single frequency, 0.999 ASR is achieved within stipulated 3600 epochs for different batches during the day upto 250 Km baseline at $-30°$ and $-60°$ latitude. The number of epochs required for 0.999 ASR for 250 Km baseline were 3009(2-3) and 2629(22-23) at $-30°$ and $-60°$ latitude. for 1000 Km baseline, maximum ASR achieved was 0.783, 0.831 and 0.8 for $0°, -30°$ and $-60°$ latitude respectively.

(2) For dual frequency, 0.999 ASR could be achieved for all latitude location upto 500 Km baseline length. For 250 Km baseline it took 649(2-3), 689(4-5) and 690(2-3) epochs at $0°, -30°$ and $-60°$ latitude respectively. For 1000 Km baseline length, maximum ASR of 0.967, 0.981 and 0.974 could be achieved

at $0°, -30°$ and $-60°$ latitude respectively.

(3) For triple frequency, 0.999 ASR could be achieved at all latitude locations for all baseline lengths. For 250 Km baseline length, it took 474(2-3), 449(4-5) and 438(8-9) and for 1000 Km baseline length, it took 1686(21-22), 997(3-4), 1315(18-19) epochs at $0°, -30°$ and $-60°$ latitude respectively.

(4) It is important to analyze and pick the batches which took minimum number of epochs during the day at different latitude locations. At $0°$, the best batch would be number 1 and 2, however batches 3, 11, 20, 22 can be avoided. At $-30°$ the best batch would be 4, batches 1, 8, 12, 13, 16, and 19 can be avoided, and at $-60°$ batches 2 and 22 can be picked and 3, 6, 8, 9, 11 and 23 can be avoided.

Baseline length (Kilometers)	ASR (0.999 or maximum obtained)			Minimum epochs required for 0.999 ASR (or maximum ASR): Epochs taken			$\sigma_{\hat{\rho}}$(mm)			Maximum epochs required for 0.999 ASR (or maximum ASR): Epochs taken(corresponding hour batch)			$\sigma_{\hat{\rho}}$(mm)		
	0^0	-30^0	-60^0	0^0	-30^0	-60^0	0^0	-30^0	-60^0	0^0	-30^0	-60^0	0^0	-30^0	-60^0
Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m								
1	0.999	0.999	1.000	3600(1-2)	2845(11-12)	2732(22-23)	0.146	0.152	0.145	3600(1-2)	2894(2-3)	2732(22-23)	0.146	0.145	0.145
10	0.999	0.999	1.000	3600(1-2)	2851(11-12)	2728(22-23)	0.270	0.297	0.298	3600(1-2)	2892(2-3)	2728(22-23)	0.270	0.292	0.298
100	0.998	0.999	1.000	3600(1-2)	2868(2-3)	2692(22-23)	2.220	2.479	2.558	3600(1-2)	2941(11-12)	2692(22-23)	2.220	2.454	2.558
250	0.997	0.999	0.999	3600(1-2)	3009(2-3)	2629(22-23)	4.992	5.392	5.755	3600(1-2)	3243(11-12)	2629(22-23)	4.992	5.249	5.755
500	0.986	0.996	0.994	3600(1-2)	3600(2-3)	3151(22-23)	7.899	7.624	8.086	3600(1-2)	3600(2-3)	3151(22-23)	7.899	7.624	8.086
1000	0.783	0.831	0.800	3600(1-2)	3600(2-3)	3151(22-23)	10.560	9.736	10.077	3600(1-2)	3600(2-3)	3151(22-23)	10.560	9.736	10.077
High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m								
1	0.999	0.999	0.999	11(13-14)	11(2-3)	6(8-9)	1.991	1.811	2.172	36(8-9)	35(12-13)	35(7-8)	1.488	1.454	1.440
10	0.999	0.999	0.999	12(13-14)	12(2-3)	7(8-9)	4.505	4.294	5.343	37(8-9)	36(12-13)	36(7-8)	2.783	2.749	2.751
100	0.999	0.999	0.999	120(2-3)	120(1-2)	120(3-4)	2.338	2.253	2.396	166(1-2)	150(12-13)	123(23-24)	2.693	3.043	3.167
250	0.999	0.999	0.999	694(2-3)	689(4-5)	690(2-3)	0.991	0.864	0.978	1071(20-21)	1101(10-11)	1351(11-12)	1.247	1.152	1.309
500	0.999	0.999	0.999	2300(5-6)	2232(15-16)	2268(19-20)	0.592	0.491	0.511	3566(23-24)	3556(16-17)	3554(17-18)	0.527	0.574	0.487
1000	0.967	0.981	0.974	3600(1-2)	3600(2-3)	3151(22-23)	0.449	0.395	0.400	3600(1-2)	3600(2-3)	3151(22-23)	0.449	0.395	0.400
High-end receivers, Triple Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m, $\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m								
1	0.999	0.999	1.000	5(13-14)	5(2-3)	3(8-9)	2.168	1.999	2.331	16(1-2)	16(6-7)	16(7-8)	1.401	1.537	1.523
10	0.999	0.999	1.000	5(13-14)	5(2-3)	3(8-9)	6.720	6.333	7.671	16(1-2)	16(6-7)	16(7-8)	3.777	3.937	3.928
100	0.999	0.999	0.999	112(2-3)	111(1-2)	111(8-9)	1.969	1.906	1.604	154(1-2)	138(12-13)	114(4-5)	2.359	2.668	2.461
250	0.999	0.999	0.999	474(2-3)	449(4-5)	438(8-9)	0.965	0.862	0.811	619(1-2)	715(1-2)	816(6-7)	1.072	1.067	1.051
500	0.999	1.000	0.999	1033(21-22)	890(3-4)	925(18-19)	0.607	0.546	0.595	2737(6-7)	3230(16-17)	3266(8-9)	0.551	0.519	0.489
1000	0.999	0.999	0.999	1686(21-22)	997(3-4)	1315(18-19)	0.486	0.517	0.500	3051(14-15)	3534(7-8)	3470(7-8)	0.425	0.396	0.356

Table 4.15: GPS only, **Full AR**, Geometry Free model, Ionosphere Weighted scenario, ASR and fixed-precision for range values for 24 batches of one hour (each batch of 3600 epochs, 1 second interval) - **measurement precision varied**, number of satellites are as available. The rows shaded in gray represent values of measurement precision for which simulated parameters are presented graphically in the following subsection.

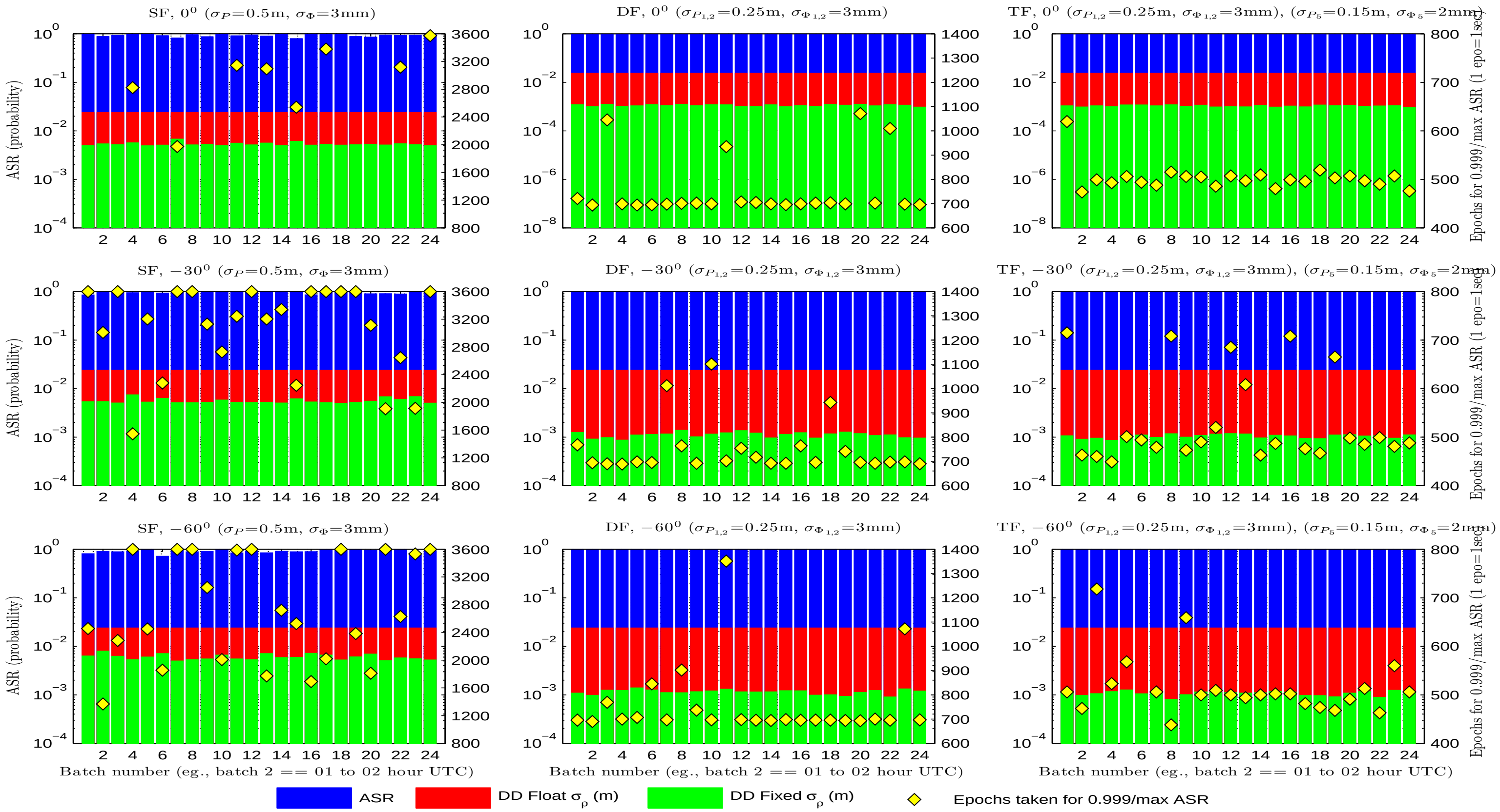

Figure 4.9: **GPS only**, **Hourly ASR** analysis, Single, Dual and Triple Frequencies, Geometry Free model, Ionosphere Weighted scenario, (measurement precision is held fixed, number of satellites are as available), **baseline length = 250 Km-** ASR and fixed-precision (ranges) analysis at $0°, -30°$ and $-60°$ degree latitude

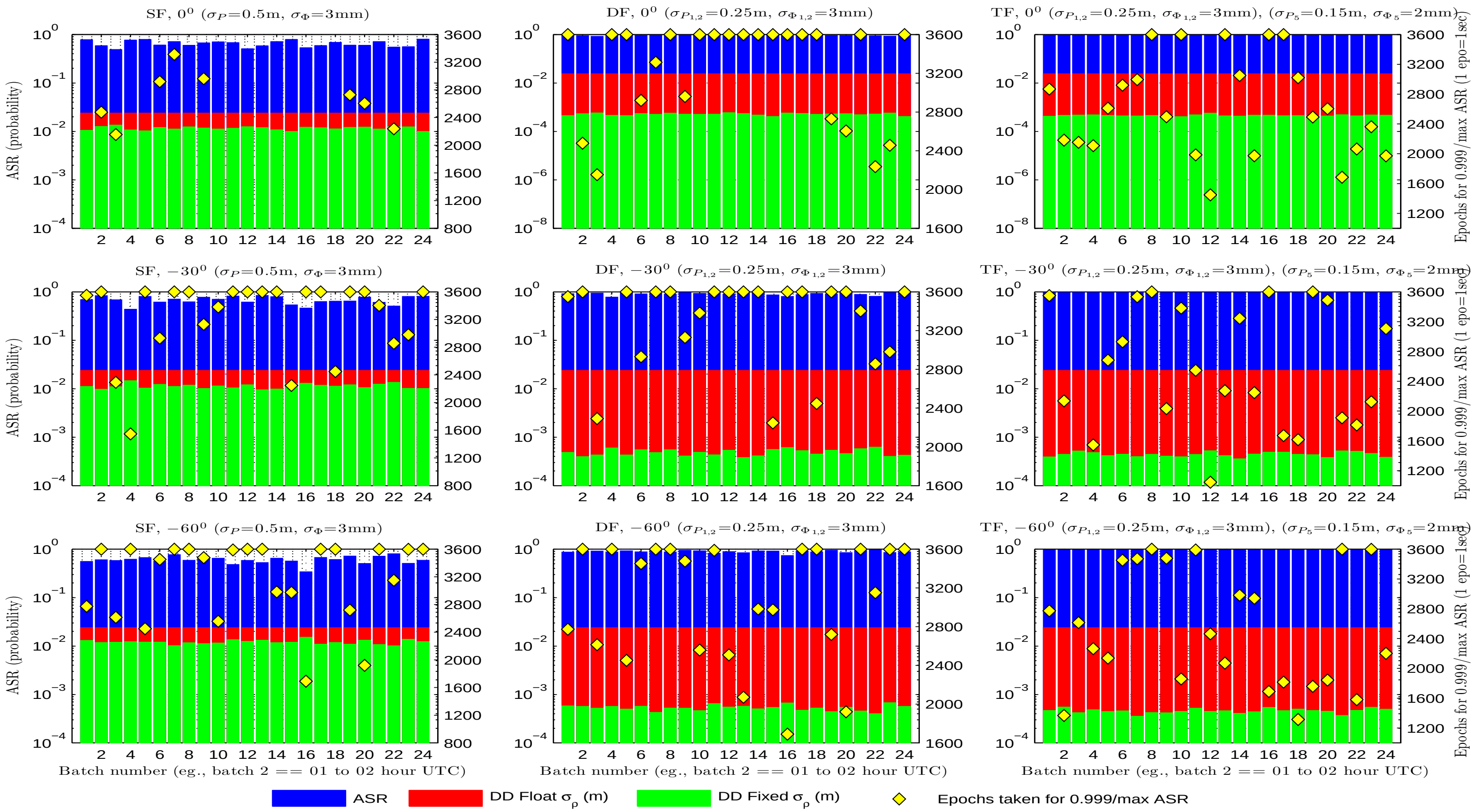

Figure 4.10: **GPS only**, **Hourly ASR** analysis, Single, Dual and Triple Frequencies, Geometry Free model, Ionosphere Weighted scenario, (measurement precision is held fixed, number of satellites are as available), **baseline length = 1000 Km-** ASR and fixed-precision (ranges) analysis at $0°, -30°$ and $-60°$ degree latitude

5

Geometry Fixed model

5.1 Introduction

The Geometry Based (GB) model considers the geometry between the receiver and the satellite by incorporating the information of the receiver and the satellite coordinates. The GNSS observables for a geometry based model for a single epoch can be given as below,

$$\Phi_{1r,j}^{1s} = -\mu_j I_{1r}^{1s} + \psi^{1s} T_{1r} + \lambda_j N_{1r,j}^{1s} + \epsilon_{1r,j}^{1s} \tag{5.1}$$

and similarly for code, we have

$$P_{1r,j}^{1s} = \mu_j I_{1r}^{1s} + \psi^{1s} T_{1r} + e_{1r,j}^{1s} \tag{5.2}$$

where $\Phi_{1r,j}^{1s}$ and $P_{1r,j}^{1s}$ is the DD phase and code GPS/Galileo observables, subscripts r, j and s indicate receiver, frequency number and the satellite respectively, and the subscripts 1 indicate that satellite 1 is chosen as the reference satellite and same applicable to the receiver. Φ, λ are the DD phase observations and wavelength respectively, μ is $(\nu_1/\nu_j)^2$, ν_1 is GPS/Galileo frequency on $L1$ and ν_j is GPS/Galileo frequency on L_j, T_{1r} is the tropospheric delay in Zenith and ψ^{1s} is the tropospheric delay mapping function which maps the zenith delay to the slant (satellite) direction using the satellite elevation angle, $\epsilon_{1r,j}^{1s}$ and $e_{1r,j}^{1s}$ are the error associated with the phase and code measurements respectively.

In this analysis, receiver is considered to be static, hence the receiver coordinates do not vary with time. However, there are two scenarios considered for receiver coordinate information, in one the receiver position is known, in other, they are estimated. For satellite coordinates, even though the position of the satellites change in time, their information is considered to be known (YUMA almanacs are used for satellite orbits).

5.1.1 Double differenced equations for the combined GPS and Galileo system

For a combined GPS and Galileo system, with GPS satellite as the reference, after forming double differences, the GPS and Galileo observables can be reduced to the form

$$\left.\begin{array}{rcl} & \text{For GPS system} & \\ \Phi^{1_G,s_G}_{1r,j} & = & -\mu_j I^{1_G,s_G}_{1r} + \psi^{1_G,s_G} T_{1r} + \lambda_j N^{1_G,s_G}_{1r,j} + \epsilon^{1_G,s_G}_{1r,j} \\ P^{1_G,s_G}_{1r,j} & = & \mu_j I^{1_G,s_G}_{1r} + \psi^{1s} T_{1r} + e^{1_G,s_G}_{1r,j} \end{array}\right\} \tag{5.3}$$

$$\left.\begin{array}{rcl} & \text{For Galileo system} & \\ \Phi^{1_G,s_E}_{1r,j} & = & -\mu_j I^{1_G,s_E}_{1r} + \psi^{1_G,s_E} T_{1r} + \lambda_j N^{1_E,s_E}_{1r,j} + \epsilon^{1_G,s_E}_{1r,j} \\ P^{1_G,s_E}_{1r,j} & = & \mu_j I^{1_G,s_E}_{1r} + \psi^{1_G,s_E} T_{1r} + e^{1_G,s_E}_{1r,j} \end{array}\right\} \tag{5.4}$$

The notions used in the above equation ar explained earlier. Further, the functional and stochastic models can be defined as below,

$$\begin{aligned} E(\Phi^{1_G})(G) &= A^{1_G}_{I}(G)x^{1_G}_{I}(G) + A^{1_G}_{II}(G)x^{1_G}_{II}(G) \\ E(P^{1_G})(G) &= A^{1_G}_{II}(G)x^{1_G}_{II}(G) \\ E(\Phi^{1_G})(E) &= A^{1_G}_{I}(E)x^{1_G}_{I}(E) + A^{1_G}_{II}(E)x^{1_G}_{II}(E) \\ E(P^{1_G})(E) &= A^{1_G}_{II}(E)x^{1_G}_{II}(E) \end{aligned}$$

$$
\begin{aligned}
E\begin{pmatrix}
\Phi^{1_G}_{L1(E1)} = \begin{cases}\Phi^{1_G}_{L1}\\ \Phi^{1_G}_{E1}\end{cases}\\
\Phi^{1_G}_{L5(E5a)} = \begin{cases}\Phi^{1_G}_{L5}\\ \Phi^{1_G}_{E5a}\end{cases}\\
P^{1_G}_{L1(E1)} = \begin{cases}P^{1_G}_{L1}\\ P^{1_G}_{E1}\end{cases}\\
P^{1_G}_{L5(E5a)} = \begin{cases}P^{1_G}_{L5}\\ P^{1_G}_{E5a}\end{cases}
\end{pmatrix}
&=
\begin{bmatrix}
A^{1_G}_{I_{L1}} & 0 & 0 & 0 & A^{1_G}_{I_{L1}}(\mathcal{G}) & A^{1_G}_{II_{L1}} & \\
0 & A^{1_E}_{I_{E1}} & 0 & 0 & A^{1_G}_{I_{E1}}(\mathcal{G}) & & A^{1_G}_{II_{E1}}\\
0 & 0 & A^{1_G}_{I_{L5}} & 0 & A^{1_G}_{I_{L5}}(\mathcal{G}) & A^{1_G}_{II_{L5}} & \\
0 & 0 & 0 & A^{1_E}_{I_{E5a}} & A^{1_G}_{I_{E5a}}(\mathcal{G}) & & A^{1_G}_{II_{E5a}}\\
0 & 0 & 0 & 0 & A^{1_G}_{I_{L1}}(\mathcal{G}) & A^{1_G}_{II_{L1}} & \\
0 & 0 & 0 & 0 & A^{1_G}_{I_{E1}}(\mathcal{G}) & & A^{1_G}_{II_{E1}}\\
0 & 0 & 0 & 0 & A^{1_G}_{I_{L5}}(\mathcal{G}) & A^{1_G}_{II_{L5}} & \\
0 & 0 & 0 & 0 & A^{1_G}_{I_{E5a}}(\mathcal{G}) & & A^{1_G}_{II_{E5a}}
\end{bmatrix}
\cdot
\begin{bmatrix}
x^{1_G}_{I_{L1}}\\ x^{1_E}_{I_{E1}}\\ x^{1_G}_{I_{L5}}\\ x^{1_E}_{I_{E5a}}\\ x^{1_G}_{I}(\mathcal{G})\\ x^{1_G}_{II}(G)\\ x^{1_G}_{II}(E)
\end{bmatrix}\\
D\begin{pmatrix}
\Phi^{1_G}_{L1}\\ \Phi^{1_G}_{E1}\\ \Phi^{1_G}_{L5}\\ \Phi^{1_G}_{E5a}\\ P^{1_G}_{L1}\\ P^{1_G}_{E1}\\ P^{1_G}_{L5}\\ P^{1_G}_{E5a}
\end{pmatrix}
&=
\begin{bmatrix}
C_{\Phi^{1_G}_{L1}} & C_{\Phi^{1_G}_{L1,E1}} & 0 & 0 & 0 & 0 & 0 & 0\\
C_{\Phi^{1_G}_{E1,L1}} & C_{\Phi^{1_G}_{E1}} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & C_{\Phi^{1_G}_{L5}} & C_{\Phi^{1_G}_{L5,E5a}} & 0 & 0 & 0 & 0\\
0 & 0 & C_{\Phi^{1_G}_{E5a,L5}} & C_{\Phi^{1_G}_{E5a}} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & C_{P^{1_G}_{L1}} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{E1}} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{L5}} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{E5a}}
\end{bmatrix}
\end{aligned}
\tag{5.5}
$$

In equation (5.5), $E(\Phi^{1_G})(G)$ and $E(\Phi^{1_G})(E)$ correspond to the expectation of the phase data for GPS (indicated by (G)) and Galileo (indicated by (E)) with GPS as reference satellite indicated with superscripts $()^{1_G}$, similarly expectation for the code data is given, A_I and A_{II} are the design matrices corresponding to non-temporal (ambiguities, receiver coordinates, troposphere, ISB's) and temporal parameters (ionosphere), $x_{I_{L1(E1)}} = [x^{1_G}_{I_{L1}}, x^{1_E}_{I_{E1}}]^T = [N^{1_G}_{1r,L1}, N^{1_E}_{1r,E1}]^T$, corresponding to a combined system of GPS+Galileo for common frequency $L1(E1)$ with GPS satellite as reference satellite and $x_{I_{L5(E5a)}} = [x^{1_G}_{I_{L5}}, x^{1_E}_{I_{E5a}}]^T = ([N^{1_G}_{1r,L5}, N^{1_E}_{1r,E5a}])^T$ for common frequency $L5(E5a)$, are the double differenced ambiguity parameters, the non-temporal parameters, apart from the ambiguities are formed in the design matrix $A^{1_G}_{I_j}(\mathcal{G})$, $\mathcal{G}$ indicates geometry parameters, namely the mapping function for the troposphere, $x^{1_G}_{I}(\mathcal{G}) = [T_{1r}]$ are the geometry parameters, that is the troposphere for both GPS and Galileo (with GPS as reference satellite), $x^{1_G}_{II} = [x^{1_G}_{II}(G), x^{1_G}_{II}(E)]^T = [I^{1_G,s_G}_{1r}, I^{1_G,s_E}_{1r}]^T$ has the temporal varying parameter, namely the double differenced ionosphere for both GPS and Galileo (with GPS as reference satellite).

The above combined GPS and Galileo system is possible to be implemented while considering common frequencies. In this research the common frequency system is applicable to a combined single frequency system consisting of $L1(E1)$ GPS and Galileo frequency and dual frequency system of $L1(E1), L5(E5a)$ combined GPS and Galileo system. For quadruple frequency case consisting of additional $L2, E5b$ frequencies of GPS and Galileo, a different parameterization is to be considered which is applicable to a combined system of four frequencies (since $L2, E5b$ are considered independent two frequencies) with observables on $L2$ having GPS satellite as a reference satellite and for $E5b$ Galileo satellite would be chosen as the reference satellite.

5.1.2 Parameterization of combined GPS and Galileo quadruple frequency system

The DD observation equations for the two common frequencies, namely, $L1(E1)$, $L5(E5a)$ for a combined GPS+Galileo system are discussed in the earlier section in equation (5.4). It should be noted that the common frequencies of Galileo and GPS will have a **GPS satellite as a reference satellite**, the uncommon GPS frequency $L2$ will have the same GPS satellite as reference as for common frequency system, and the Galileo uncommon frequency $E5b$ will have a Galileo satellite as a reference satellite. The DD observation equations of the remaining two uncommon frequencies for GPS ($L2$) and Galileo ($E5b$) are given below,

$$\left.\begin{array}{lcl}
\Phi_{1r,L2}^{Gs_G} & = & -\mu_{L2} I_{1r}^{1_G,s_G} + \psi^{1_G,s_G} T_{1r} + \lambda_{L2} N_{1r,L2}^{1_G,s_G} + \epsilon_{1r,L2}^{1_G,s_G} \\
P_{1r,L2}^{1_G,s_G} & = & \mu_{L2} I_{1r}^{1_G,s_G} + \psi^{1s} T_{1r} + e_{1r,L2}^{1_G,s_G} \\
\Phi_{1r,E5b}^{1_E,s_E} & = & -\mu_{E5b} I_{1r}^{1_E,s_E} + \psi^{1_E,s_E} T_{1r} + \lambda_{E5b} N_{1r,E5b}^{1_E,s_E} + \epsilon_{1r,E5b}^{E,s_E} \\
P_{1r,E5b}^{1_E,s_E} & = & \mu_{E5b} I_{1r}^{1_E,s_E} + \psi^{1_E,s_E} T_{1r} + e_{1r,E5b}^{1_E,s_E}
\end{array}\right\} \quad (5.6)$$

The notations used in the above equation, equation (5.6) have been already explained earlier while describing the two common frequencies of GPS+Galileo combined system. Here, as expressed earlier, GPS $L2$ frequency has a GPS reference satellite and Galileo $E5b$ frequency has a Galileo reference satellite.

In this section the parameterization is expressed in a basic form of the expectation and dispersion, to begin with consider equation (5.5) for the combined GPS and Galileo overlapping frequency case. To express this four frequency model, and index for common frequency, j_c will be used. The four frequencies can be represented as $L1(E1), L5(E5a), L2, E5b$ from now on in further notations, with the common frequencies shown in the brackets. The expectation and dispersion can now be given as follows.

$$
E\left(\begin{array}{l}
\Phi^{1_G}_{L1(E1)} = \begin{cases}\Phi^{1_G}_{L1}\\ \Phi^{1_G}_{E1}\end{cases}\\
\Phi^{1_G}_{L5(E5a)} = \begin{cases}\Phi^{1_G}_{L5}\\ \Phi^{1_G}_{E5a}\end{cases}\\
\Phi^{1_G}_{L2}\\
\Phi^{1_E}_{E5b}\\
P^{1_G}_{L1(E1)} = \begin{cases}P^{1_G}_{L1}\\ P^{1_G}_{E1}\end{cases}\\
P^{1_G}_{L5(E5a)} = \begin{cases}P^{1_G}_{L5}\\ P^{1_G}_{E5a}\end{cases}\\
P^{1_G}_{L2}\\
P^{1_E}_{E5b}
\end{array}\right)
=
\begin{bmatrix}
A^{1_G}_{I_{L1}} & 0 & 0 & 0 & 0 & 0 & A^{1_G}_{I}(\mathcal{G}) & A^{1_G}_{II_{L1}} & \\
0 & A^{1_E}_{I_{E1}} & 0 & 0 & 0 & 0 & A^{1_G}_{I}(\mathcal{G}) & & A^{1_G}_{II_{E1}}\\
0 & 0 & A^{1_G}_{I_{L5}} & 0 & 0 & 0 & A^{1_G}_{I}(\mathcal{G}) & A^{1_G}_{II_{L5}} & \\
0 & 0 & 0 & A^{1_E}_{I_{E5a}} & 0 & 0 & A^{1_G}_{I}(\mathcal{G}) & & A^{1_G}_{II_{E5a}}\\
0 & 0 & 0 & 0 & A^{1_G}_{I_{L2}} & 0 & A^{1_G}_{I}(\mathcal{G}) & A^{1_G}_{II_{L2}} & 0\\
0 & 0 & 0 & 0 & 0 & A^{1_E}_{I_{E5b}} & A^{1_E}_{I}(\mathcal{G}) & 0 & A^{1_E}_{II_{E5b}}\\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_G}_{I}(\mathcal{G}) & A^{1_G}_{II_{L1}} & \\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_G}_{I}(\mathcal{G}) & & A^{1_G}_{II_{E1}}\\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_G}_{I}(\mathcal{G}) & A^{1_G}_{II_{L5}} & \\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_G}_{I}(\mathcal{G}) & & A^{1_G}_{II_{E5a}}\\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_G}_{I}(\mathcal{G}) & A^{1_G}_{II_{L2}} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_E}_{I}(\mathcal{G}) & 0 & A^{1_E}_{II_{E5b}}
\end{bmatrix}
\cdot
\begin{bmatrix}
x^{1_G}_{I_{L1}}\\ x^{1_E}_{I_{E1}}\\ x^{1_G}_{I_{L5}}\\ x^{1_E}_{I_{E5a}}\\ x^{1_G}_{I_{L2}}\\ x^{1_E}_{I_{E5b}}\\ x_I(\mathcal{G})\\ x_{II}(G)\\ x_{II}(E)
\end{bmatrix}
$$

$$
D\begin{pmatrix}
\Phi^{1_G}_{L1}\\ \Phi^{1_G}_{E1}\\ \Phi^{1_G}_{L5}\\ \Phi^{1_G}_{E5a}\\ \Phi^{1_G}_{L2}\\ \Phi^{1_E}_{E5b}\\ P^{1_G}_{L1}\\ P^{1_G}_{E1}\\ P^{1_G}_{L5}\\ P^{1_G}_{E5a}\\ P^{1_G}_{L2}\\ P^{1_E}_{E5b}
\end{pmatrix}
=
\begin{bmatrix}
C_{\Phi^{1_G}_{L1}} & C_{\Phi^{1_G}_{L1,E1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
C_{\Phi^{1_G}_{E1,L1}} & C_{\Phi^{1_G}_{E1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & C_{\Phi^{1_G}_{L5}} & C_{\Phi^{1_G}_{L5,E5a}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & C_{\Phi^{1_G}_{E5a,L5}} & C_{\Phi^{1_G}_{E5a}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & C_{\Phi^{1_G}_{L2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & C_{\Phi^{E}_{E5b}} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{L1}} & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{E1}} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{L5}} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{E5a}} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{L2}} & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{E}_{E5b}}
\end{bmatrix}
\tag{5.7}
$$

In the above equation, equation (5.7), the new observables introduced are $\Phi^{1_G}_{L2}(G), \Phi^{1_E}_{E5b}(E)$ for phase and $P^{1_G}_{L2}(G), P^{1_E}_{E5b}(E)$ for code, here, the symbol in the bracket (E) or (G) indicate to which GNSS system the observable belongs and the superscript $()^{1_G}$ or $()^{1_E}$ indicates to which system the chosen reference satellite belongs, G stands for GPS and E stands for Galileo, and the subscript with unknowns $x^{1_G}_{I_{L2}}, x^{1_E}_{I_{E5a}}$ as ambiguities on $L2, E5a$ parameterized in the functional model within the design matrices $A^{1_G}_{I_{L2}}$ for GPS and $A^{1_E}_{I_{E5b}}$ for Galileo. In the above functional and stochastic models whenever the index j_c is used, it represents frequencies $L1$ for GPS and $E1$ for Galileo, single frequency case and $L1, L5$ for GPS and $E1, E5a$ for Galileo in a dual frequency case and of course in the quadruple frequency case (which in fact has four frequency parameterization) has the two common frequencies and additionally $L2, E5b$ frequencies.

5.2 Computation of normal equation and ambiguity-float solution

The simulation of ambiguity success rate and ambiguity-float and -fixed-precision of the other unknowns (ionosphere etcetera) is done by using the functional and the stochastic model. The functional and stochastic models are formed as explained in the above section. Further, by using the functional and stochastic models, normal equations can be formed and the VC-matrix of the ambiguity-float solution can be computed. The computation of the normal equations and float VC-matrix is explained below.

5.2.1 Forming the normal equations

The computation of ASR and Gain is based on simulations which can be done by having information of the GNSS model chosen and measurement precision. Based on this information the errors in the GNSS observables are parameterized in a design matrix (*functional model*) and measurement precision matrix (*stochastic model*).

To begin with we recall that for the functional model the design matrix is computed at every epoch for the *temporal* varying (ionosphere, troposphere) and the *non-temporal* parameters like the ambiguities (on the assumption that there are no cycle slips or loss of lock for the ambiguities).

For the stochastic model, as explained in Appendix B above, the satellite having the maximum elevation is chosen as a reference satellite (until it sets, it remains as a reference satellite), further considering the elevation weights for each satellite the final measurement precision matrix Q_{yy} is formed, see equation (B.9). The stochastic model too is computed at every epoch based on the reference satellite chosen and the elevation of all the satellites.

Further to derive an expression for computation of the float solution for ambiguities and troposphere (temporal) and ionosphere (non-temporal) say for atmosphere float scenario, we begin the discussion by considering a simple two epoch case. The expectation for the two epochs case can be given as,

$$E\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} A_I(1) & A_{II}(1) & 0 \\ A_I(2) & 0 & A_{II}(2) \end{bmatrix} \cdot \begin{bmatrix} x_I \\ x_{II}(1) \\ x_{II}(2) \end{bmatrix}, \quad D\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{bmatrix} Q_{y_1} & 0 \\ 0 & Q_{y_2} \end{bmatrix}$$

where A_I and A_{II} is the design matrix for non-temporal and temporal varying parameters, Q_{y_1} and Q_{y_1} represent the stochastic model for observations y_1 and y_2.

In the above equation, only two phase observations are used in the model deliberately in order to enhance simplification and ease in understanding of the parameterization of temporal and non temporal parameters. Further the normal equations for the above case can be computed as below,

$$N = \begin{bmatrix} A_I^T(1) & A_I^T(2) \\ A_{II}^T(1) & 0 \\ 0 & A_{II}^T(2) \end{bmatrix} \begin{bmatrix} Q_{y_1}^{-1} & 0 \\ 0 & Q_{y_2}^{-1} \end{bmatrix} \begin{bmatrix} A_I(1) & A_{II}(1) & 0 \\ A_I(2) & 0 & A_{II}(2) \end{bmatrix}$$

$$N = \begin{bmatrix} A_I^T(1)Q_{y_1}^{-1}A_I(1) + A_I^T(2)Q_{y_2}^{-1}A_I(2) & A_I^T(1)Q_{y_1}^{-1}A_{II}(1) & A_I^T(2)Q_{y_2}^{-1}A_{II}(2) \\ A_{II}^T(1)Q_{y_1}^{-1}A_I(1) & A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1) & 0 \\ A_{II}^T(2)Q_{y_2}^{-1}A_I(2) & 0 & A_{II}^T(2)Q_{y_2}^{-1}A_{II}(2) \end{bmatrix}$$

A general expression for the normal equations for k epochs can be given as below,

$$N_k = \begin{bmatrix} \sum_{i=1}^{k} A_I^T Q_{y_i}^{-1} A_I & A_I^T(1)Q_{y_1}^{-1}A_{II}(1) & \cdots & A_I^T(i)Q_{y_i}^{-1}A_{II}(i) \\ A_{II}^T(1)Q_{y_1}^{-1}A_I(1) & A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_{II}^T(i)Q_{y_i}^{-1}A_I(i) & 0 & \cdots & A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i) \end{bmatrix} \quad (5.8)$$

In the above normal equation N_k, $k = 1, \cdots, i$, where k are the total number of epochs. The above normal equation consists of four parts the temporal, non temporal and two parts of covariances between the temporal and non-temporal parameters. They are explicitly given below.

$$N_{A_{I_k}} = \sum_{i=1}^{k} A_I^T Q_{y_i}^{-1} A_I \quad \Leftrightarrow \text{ will be referred as } \mathbf{A}$$

$$N_{A_{I_k}A_{II_k}} = \begin{bmatrix} A_I^T(1)Q_{y_1}^{-1}A_{II}(1) & \cdots & A_I^T(i)Q_{y_i}^{-1}A_{II}(i) \end{bmatrix} \quad \Leftrightarrow \text{ will be referred as } \mathbf{B}$$

$$N_{A_{II_k}} = \begin{bmatrix} A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i) \end{bmatrix} \quad \Leftrightarrow \text{ will be referred as } \mathbf{D}$$

In the above, $\mathbf{C}$ is not explicitly defined but it should be noted that $\mathbf{C} = \mathbf{B}^T$. In the above equation it can be seen that the temporal varying parameters cause an increase in the dimension of the normal equations. It can be a computation burden to accommodate all the temporal parameters, especially when large number of epochs is considered for simulation. The temporal varying parameters can be alternatively reduced in the normal equations and the problem of increasing normal equation can be overcome. The reduction of normal equation is explained in the sections coming ahead.

For a GNSS model when the atmosphere is considered unknown, the strength of the GNSS model is less as compared to atmosphere known case. With a-priori information of the atmosphere, for example ionosphere, being available, it can be introduced in the GNSS model as weights. This definitely makes the GNSS model stronger and relatively better performance is expected (in terms of time taken for correct ambiguity fixing), as compared to the atmosphere float scenario. The weights can be introduced in the GNSS model by different approaches, two of them are discussed in the coming section.

5.2.2 Introducing ionosphere weight in a GNSS model

The ionosphere is weighted with respect to the baseline length using the relation that the standard deviation of the ionosphere varies as 6.8mm per 10 Km (SD (Single Difference) standard deviation). The SD standard deviation is converted to the un-difference counterpart by dividing it by 2, as per the law of error propagation. The un-differenced standard deviation is converted to its variance nd further operated with DD satellite-receiver elevation weight matrix, as explained below. The elevation dependency of the satellite is accounted by generating a elevation weight matrix for all the satellites considered for simulation by the relation given below.

$$
\begin{aligned}
W_{el_1}^{-1} &= \left(\sigma_i * \text{ diag } \left(1 + a * \exp(-\varepsilon_1^{1,\cdots,m}/\varepsilon_0^{1,\cdots,m})\right)\right)^2 \\
W_{el_2}^{-1} &= \left(\sigma_i * \text{ diag } \left(1 + a * \exp(-\varepsilon_2^{1,\cdots,m}/\varepsilon_0^{1,\cdots,m})\right)\right)^2 \\
W_{el}^{-1} &= W_{el_1}^{-1} + W_{el_2}^{-1}
\end{aligned}
\tag{5.9}
$$

where W_1^{-1}, W_2^{-1}, $\varepsilon_1^{1,\cdots,m}$, $\varepsilon_2^{1,\cdots,m}$ are the weighting functions and satellite elevations for receiver 1 and 2 respectively, for satellites 1 to m. The coefficients a and ε_0 are set to 10. The detailed explanation of exponential satellite elevation weight is explained in Appendix D.

Further, the computed weight is converted to the elevation dependent double differenced variance matrix, Q_{DD} by the following transformation

$$Q_{DD} = D^T \cdot (2 \cdot W_{el_1}^{-1}) \cdot D$$

where, D^T is the satellite design matrix. In the above equation, $D^T \cdot D$ form between satellite single difference and $(2 \cdot W_{el_1}^{-1})$ from between receiver single difference. While the baseline length is relatively short only elevation weight matrix of receiver 1 can be considered. Incase of the receivers forming the baseline are separated enough and have significantly difference elevation angles, then the above equation can be written as below,

$$Q_{DD} = D^T \cdot ({W_{el_1}}^{-1} + {W_{el_2}}^{-1}) \cdot D \tag{5.10}$$

where ${W_{el_1}}^{-1}$ and ${W_{el_2}}^{-1}$ correspond to elevation weight matrices of receiver 1 and 2 respectively.

The satellite design matrix D^T contains information of all the satellites in the column which defines their relation to other satellites, that is it has m columns, the observation for the reference satellite is exempted in D^T hence it has $m-1$ rows. For example for 4 satellites, with satellite 3 as reference satellite, D^T will have the form given below.

$$D^T = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \tag{5.11}$$

It should be noted that the factor $D^T \cdot D$ corresponds to between satellite single difference matrix. To from a DD stochastic model for the ionosphere, additionally between receiver single difference information in from of ionosphere variance can be used. The between receiver single difference ionosphere variance is computed from the relation $\sigma_I^2 \cdot BL$, where BL is the baseline length in kilometers. The ionosphere is assumed to vary as $(0.00068)^2$ per kilometer, that is $\sigma_I^2 = (0.00068)^2$. Using the information ionosphere VC matrix cam be computed and further implemented as shown below.

The ionosphere weighted model can be implemented in two ways, firstly eliminating the ionosphere a priori from observations, hence adding the ionosphere variance, Q_I, to the stochastic model, Q_y, see [47]. Secondly, the VC matrix for the ionosphere can be used to compute the ionosphere weight matrix,$W_I = Q_I^{-1}$, The computed ionosphere weight matrix can be added to the ionosphere part of the normal equations. Both the approaches are discussed in detail, see below

(1) **Eliminating ionosphere using ionosphere a-priori information:**

If the ionosphere is known a-priori with the information of its variance, it can be eliminated a-priori from the GNSS model. The expectation and dispersion for the GNSS model when ionosphere is eliminated can be given as below,

$$\begin{array}{lcl} E(y_i - A_{II_i} x_{II_i}) & = & A_{I_i} x_{I_i} \\ D(y_i) & = & Q_{y_i} + Q_{I_i} \end{array}$$

where, Q_{I_i} is the ionosphere VC matrix computes as below,

$$Q_{I_i} = \frac{\sigma_I^2 \cdot BL}{2} \cdot (CI \otimes Q_{DD}) \tag{5.12}$$

where σ_I^2 is the single differenced ionosphere expressed as a function of baseline length BL, it is divided by 2 to convert it to its un-differenced form, $\otimes$ is the Kronecker product, Q_{DD} is explained in above section and CI is the coefficient of the ionosphere given as below.

$$CI = \begin{bmatrix} -\mu_f \\ \mu_f \end{bmatrix} \cdot \begin{bmatrix} -\mu_f & \mu_f \end{bmatrix} \text{ and } \mu_f = \begin{bmatrix} \mu_{(1)} \\ \vdots \\ \mu_{(j)} \end{bmatrix} = \begin{bmatrix} \frac{\nu_1^2}{\nu_1^2} \\ \vdots \\ \frac{\nu_1^2}{\nu_j^2} \end{bmatrix} \tag{5.13}$$

(2) **Adding ionosphere weight to the normal equations:**

Here ionosphere is parameterized as unknowns in the design matrix A_{II_i}, i corresponds to the current epoch. The normal equations are computed for the current epochs i for all the parameters, say, ambiguities, ionosphere etcetera. Further, the ionosphere weight, that is $W_I = Q_I^{-1}$ is added to the ionosphere part of the normal equation for the current epoch, given as below, (see equation (5.8), term for the ionosphere corresponding to $A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i) = N_{A_{II_i}}$ refers to normal equation of the ionosphere for the current epoch)

$$N_{A_{II_{j(IW)}}} = N_{A_{II_i}} + W_{I_i} \tag{5.14}$$

where W_{I_i} is the ionosphere weight matrix given as $W_{I_i} = Q_{I_i}^{-1}$, Q_{I_i} is the VC matrix for ionosphere computed as below

$$Q_I = \frac{\sigma_I^2 \cdot BL}{2} \cdot Q_{DD}$$

Further, the normal equations are reduced for the ionosphere for ionosphere weighted model. The reduction of normal equations will be discussed in detail in the coming section, below is given an general expression for reduced normal equations for ionosphere weighted model.

$$N_{A_I|A_{II_{(IW)}}} = N_{A_{I_i}} - N_{A_{II_i}A_{I_i}} \cdot \Big(N_{A_{II_i}} + W_{I_i}\Big)^{-1} \cdot N_{A_{I_i}A_{II_i}} \tag{5.15}$$

In the above expression, equation (5.15), $N_{A_I|A_{II_{(IW)}}}$ correspond to the reduced normal equation for ionosphere weighted model, $N_{A_{I_i}}$ correspond to the non-temporal part of the normal equation (say ambiguities, troposphere, coordinates, Inter System Biases etcetera) for epochs i, $N_{A_{II_i}}$ correspond to temporal varying ionosphere for epochs i, $N_{A_{II_i}A_{I_i}}$ are the normal equations for temporal and non-temporal varying parameters.

Comparison of ionosphere-weighted and -float scenarios:

Since the weight added to the normal equations for the ionosphere is inversely proportional to the standard deviation of the ionosphere (here it is assumed to be spatially related to the baseline length with a linear relation, varying as 6.8mm per 10 Km), as the standard deviation increases, the ionosphere weight W_{I_i} tends to zero, the normal equations then are equivalent to ionosphere float scenario, given as below.

$$N_{A_I|A_{II}} = N_{A_{I_i}} - N_{A_{II_i}A_{I_i}} \cdot \left(N_{A_{II_i}}\right)^{-1} \cdot N_{A_{I_i}A_{II_i}}$$

It can be noted that, since W_{I_i} is positive definite, $\left(N_{A_{II_i}} + W_{I_i}\right)^{-1} \leqslant \left(N_{A_{II_i}}\right)^{-1}$, that is, $N_{A_I|A_{II_{(IW)}}} \geqslant N_{A_I|A_{II}}$ and hence $Q_{\widehat{x_I}\widehat{x_I}(IW)} = \left(N_{A_I|A_{II_{(IW)}}}\right)^{-1}$ would have better precision than $Q_{\widehat{x_I}\widehat{x_I}}$ (Ionosphere float scenario). This implies that the performance of ionosphere weighted model would be better that the performance of ionosphere float model.

Proposed ionosphere weighting scheme:

In this research the first approach for ionosphere weighting is used, while ionosphere is eliminated a-priori and its weight added to the VC matrix of the observations. Since in this approach the ionosphere need not be parameterized in the GNSS model, whereas in the second approach (adding weight to normal equation), ionosphere is parameterized and further reduced. However, it should be emphasized that both the mentioned ionosphere weighting approaches are found to be equivalent, the reason for choosing the first approach is due to ease of parameterization while ISB's are estimated.

The ionosphere weight VC matrix given by equation (5.12) is applicable for standalone GNSS system and for integrated GNSS system having overlapping frequencies. For the integrated GNSS system having additional frequencies from the overlapping frequencies, the ionosphere VC matrix can be derived as below.

5.2.3 Ionosphere Variance Covariance matrix for integrated GPS and Galileo system

While a integrated GPS and Galileo system is considered with two overlapping frequencies, $L1(E1), L5(E5a)$) having GPS as the reference satellite and independent frequencies $L2$ and $E5b$ having GPS and Galileo as the reference satellite while speaking of the ionosphere. Considering the difference of reference satellites the stochastic model for the ionosphere needs special consideration, for ionosphere weighted scenario. To begin with the derivation of ionosphere VC matrix, the expectation of the DD observables for the common frequency jc for two overlapping frequencies $L1(E1), L5(E5a)$) and for the independent frequencies $L2$ and $E5b$ is given as below

$$\begin{aligned}
E(y_{jc}) &= \mu_{jc} I_{1r}^{1_G,s} + \cdots \\
E(y_{L2}) &= \mu_{L2} I_{1r}^{1_G,s_G} + \cdots \\
E(y_{E5b}) &= \mu_{E5b} I_{1r}^{1_E,s_E} + \cdots
\end{aligned}$$

where, $E(y_{jc})$ gives the expectation for the ionosphere for the common frequency, $E(y_{L2})$ and $E(y_{E5b})$ for the independent frequencies $L2$ and $E5b$, 1_G indicate GPS satellite as a reference satellite and 1_E indicate Galileo satellite as reference satellite, s_G indicates all the GPS satellites excluding the GPS reference satellite, s_E indicates all Galileo satellites excluding the Galileo reference satellite s indicates GPS plus Galileo satellites excluding the GPS reference satellite.

The unknown x_I for the ionosphere can be given as

$$x_I = \begin{bmatrix} I_{m-1} \\ I_{m_G-1} \\ I_{m_E-1} \end{bmatrix}$$

m, m_G and m_E indicate total number of GPS+Galileo, GPS only and Galileo only satellites respectively. The definition of the ionosphere coefficient can be found in equation (5.13).

The ionospheric delay which is one ionosphere per satellite, is made independent of the frequency by using the coefficient μ can be interrelated for the integrated and independent frequencies. The ionospheric delays on $L2$ and $E5b$ can be related to the ionospheric delay on overlapping frequencies $L1(E1), L5(E5a))$. This relation can be defined with the help of a transformation matrix. To begin with definition of the transformation matrix, initially the relation for ionosphere on $E5b$ with Galileo as reference satellite has to be defined with respect to GPS as reference satellite. This can be done from the following linear property of ionospheric delay on $E5b$.

$$I_{1r}^{1_E,s_E} = I_{1r}^{1_G,s_E} - I_{1r}^{1_G,1_E}$$

Further the transformation matrices for $L2$ and $E5b$ can be given as follows

$$\begin{aligned}
\underbrace{T_1}_{[(m_G-1)\times(m-1)]} &= [I_{m_G-1} \;\; 0_{(m_G-1)\times(m_E)}] \\
\underbrace{T_2}_{[(m_E-1)\times(m-1)]} &= \underbrace{D_E^T}_{[(m_E-1)\times m_E]} \;\; [0_{(m_E)\times(m_G-1)} \;\; I_{m_E}]
\end{aligned}$$

where T_1 and T_2 are the transformation matrices for ionosphere on $L2$ and $E5b$ respectively. It defines their relation with the ionosphere on common frequency. D_E^T is the satellite design matrix for Galileo system, given as follows

$$D_E^T = [-e_{(m_E-1)\times 1} \;\; I_{(m_E-1)}]$$

The column corresponding to the Galileo reference satellite have the value -1. In general, the structure of D_E^T is similar to D given in equation (5.11).

The design matrix for forming the ionosphere VC matrix can now be defined as below

$$A_I = \begin{bmatrix} (\mu_{jc} \otimes D^T)^T \\ (\mu_{L2} \otimes T_1 D^T)^T \\ (\mu_{E5b} \otimes T_2 D^T)^T \end{bmatrix}$$

where μ_{jc} is the coefficient for ionosphere for two overlapping frequencies $L1(E1)$, $L5(E5a)$), for two phase and two code observables, it has dimensions of 4×4, μ_{L2} has dimensions 2×2 for GPS frequency $L2$ and μ_{E5b} for GPS frequency $E5b$ with dimensions 2×2, D^T is the satellite design matrix for combined GPS and Galileo system as given earlier in equation (B.9).

Finally the VC matrix for the combined GPS and Galileo multi-frequency (overlapping and independent frequencies) can be given as below

$$Q_I = \begin{bmatrix} A_I W_{el}^{-1} A_I^T & -A_I W_{el}^{-1} A_I^T \\ A_I W_{el}^{-1} - A_I^T & A_I W_{el}^{-1} A_I^T \end{bmatrix} \tag{5.16}$$

where W_{el}^{-1} is the satellite elevation weight matrix for integrated GPS and Galileo system given earlier in equation (5.9).

5.2.4 Reduction of normal equations

The normal equations are reduced, for ionosphere parameters (atmosphere float scenario) since they are temporal varying (at every epoch). The reduction is done based on the technique explained in Teunissen et.al.(2004), Appendix A, page 313. The VC matrices can be derived using the normal equation for the current epoch as given in equation (5.8), the derivations are presented below

$$\mathcal{N}_k = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$$

Then the inverse of the normal equation can be given in a decomposed form, as derived in Teunissen et.al.(2004), Appendix A, page 313, is given below,

$$\mathcal{N}_k^{-1} = \begin{bmatrix} (\mathbf{A}-\mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1} & -(\mathbf{A}-\mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1}\mathbf{B}\mathbf{D}^{-1} \\ -\mathbf{D}^{-1}\mathbf{C}(\mathbf{A}-\mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1} & \mathbf{D}^{-1}+\mathbf{D}^{-1}\mathbf{C}(\mathbf{A}-\mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1}\mathbf{B}\mathbf{D}^{-1} \end{bmatrix} \tag{5.17}$$

then,

$$\mathcal{N}_{\mathbf{A}|\mathbf{D}_k} = \mathbf{A} - \mathbf{B}\cdot\mathbf{D}^{-1}\cdot\mathbf{C} \tag{5.18}$$

The above equation, equation (5.18) represents the reduced normal equation generated using the stacked normal equations. Further computation of the float VC-matrices can be done based on the reduced normal equations, explained as below.

5.2.5 Computation of float VC-matrices

The float solution, corresponding to the ambiguities denoted as $Q_{\hat{a}\hat{a}}$, can be extracted from the VC matrix of the non-temporal parameters ($Q_{\widehat{x_I}\widehat{x_I}}$) for any epoch k as shown below

$$\begin{aligned} Q_{\widehat{x_I}\widehat{x_I}}(k) &= (\mathcal{N}_{\mathbf{A}|\mathbf{D}_k})^{-1} = (\mathbf{A}-\mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1} \\ &= \Bigg\{ \sum_{i=1}^{k} A_I Q_{y_i}^{-1} A_I - \end{aligned}$$

$$\begin{bmatrix} A_I^T(1)Q_{y_1}^{-1}A_{II}(1) & \cdots & A_I^T(i)Q_{y_i}^{-1}A_{II}(i) \end{bmatrix} \cdot \begin{bmatrix} (A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1))^{-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & (A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i))^{-1} \end{bmatrix}$$

$$\cdot \begin{bmatrix} A_{II}^T(1)Q_{y_1}^{-1}A_I(1) \\ \vdots \\ A_{II}^T(i)Q_{y_i}^{-1}A_I(i) \end{bmatrix} \Bigg\}^{-1}$$

In general, at epoch k, $Q_{\widehat{x_I}\widehat{x_I}}(k)$ can be obtained with the relation

$$Q_{\widehat{x_I}\widehat{x_I}}(k) = \left[\sum_{i=1}^{k}\left(A_{I_i}^T Q_{y_i}^{-1} A_{I_i} - A_{I_i}^T Q_{y_i}^{-1} A_{II_i}(A_{II_i}^T Q_{y_i}^{-1} A_{II_i})^{-1} A_{II_i}^T Q_{y_i}^{-1} A_{I_i}\right)\right]^{-1} \tag{5.19}$$

The above expression for $Q_{\widehat{x_I}\widehat{x_I}}(k)$ for epochs k can be generated using the reduced normal equations for any epoch j, and then stacking the reduced normal equations over k epochs. Further the VC matrix for ambiguities can be extracted from $Q_{\widehat{x_I}\widehat{x_I}}(k)$, see below an example of receiver coordinates fixed scenario,

$$Q_{\widehat{x_I}\widehat{x_I}}(k) \quad = \quad \begin{bmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{T}} \\ Q_{\hat{T}\hat{a}} & Q_{\hat{T}\hat{T}} \end{bmatrix}$$

where $Q_{\hat{a}\hat{a}}$ is the float VC matrix for ambiguities and $Q_{\hat{T}\hat{T}}$ for troposphere.

In the above expression, the non temporal parameters are stacked under the summation, for example, $\mathcal{N}_{A_I}$ and the temporal varying parameters are as computed by least squares adjustment for the current epoch (they vary with epochs, hence the current epoch parameters are used), for example $\mathcal{N}_{A_{II_k}A_{II_k}}$

$$\begin{aligned}
Q_{\widehat{x_{II}}\widehat{x_I}}(k) \quad &= \quad -\mathbf{D^{-1}C}(\mathbf{A}-\mathbf{BD^{-1}C})^{-1} \\
&= -\begin{bmatrix} (A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1))^{-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & (A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i))^{-1} \end{bmatrix} \cdot \begin{bmatrix} A_{II}^T(1)Q_{y_1}^{-1}A_I(1) \\ \vdots \\ A_{II}^T(i)Q_{y_i}^{-1}A_I(i) \end{bmatrix} \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \\
&= \quad -\begin{bmatrix} \left(A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1)\right)^{-1} \cdot A_{II}^T(1)Q_{y_1}^{-1}A_I(1) \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \\ \vdots \\ \left(A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i)\right)^{-1} \cdot A_{II}^T(i)Q_{y_i}^{-1}A_I(i) \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \end{bmatrix}
\end{aligned}$$

In general, at epoch k, $Q_{\widehat{x_{II}}\widehat{x_I}}(k)$ can be obtained with the relation

$$Q_{\widehat{x_{II}}\widehat{x_I}}(k) = -\left(A_{II_i}^T Q_{y_i}^{-1} A_{II_i}\right)^{-1} \cdot A_{II_i}^T Q_{y_i}^{-1} A_{I_i} \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \tag{5.20}$$

$$Q_{\widehat{x_{II}}\widehat{x_I}}(k) \quad = \quad \begin{bmatrix} Q_{\hat{I}\hat{a}} \\ Q_{\hat{I}\hat{T}} \end{bmatrix}$$

where $Q_{\hat{I}\hat{a}}$ is the float covariance matrix for ionosphere and ambiguities and $Q_{\hat{I}\hat{T}}$ is the covariance matrix for ionosphere and troposphere (receiver coordinates are assumed fixed in above example).

$$Q_{\widehat{x_{II}}\widehat{x_{II}}}(k) = \mathbf{D^{-1}} + \mathbf{D^{-1}C}(\mathbf{A} - \mathbf{BD^{-1}C})^{-1}\mathbf{BD^{-1}}$$

$\mathbf{D^{-1}C}(\mathbf{A} - \mathbf{BD^{-1}C})^{-1}$ is derived in the above expression for $Q_{\widehat{x_{II}}\widehat{x_I}}(k)$, substituting in $Q_{\widehat{x_{II}}\widehat{x_{II}}}(k)$

$$= \mathbf{D^{-1}} + \begin{bmatrix} \left(A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1)\right)^{-1} \cdot A_I^T(1)Q_{y_1}^{-1}A_{II}(1) \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \\ \vdots \\ \left(A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i)\right)^{-1} \cdot A_I^T(i)Q_{y_i}^{-1}A_{II}(i) \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \end{bmatrix} \cdot BD^{-1}$$

$$= \mathbf{D^{-1}} + \begin{bmatrix} \left(A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1)\right)^{-1} \cdot A_I^T(1)Q_{y_1}^{-1}A_{II}(1) \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \\ \vdots \\ \left(A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i)\right)^{-1} \cdot A_I^T(i)Q_{y_i}^{-1}A_{II}(i) \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \end{bmatrix} \cdot \begin{bmatrix} A_I^T(1)Q_{y_1}^{-1}A_{II}(1) \cdot \left(A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1)\right)^{-1} \\ \vdots \\ A_I^T(i)Q_{y_i}^{-1}A_{II}(i) \cdot \left(A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i)\right)^{-1} \end{bmatrix}^T$$

$$= \mathbf{D^{-1}} + \begin{bmatrix} \left(A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1)\right)^{-1} \cdot A_I^T(1)Q_{y_1}^{-1}A_{II}(1) \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \cdot A_I^T(1)Q_{y_1}^{-1}A_{II}(1) \cdot \left(A_{II}^T(1)Q_{y_1}^{-1}A_{II}(1)\right)^{-1} \\ \vdots \\ \left(A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i)\right)^{-1} \cdot A_I^T(i)Q_{y_i}^{-1}A_{II}(i) \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \cdot A_I^T(i)Q_{y_i}^{-1}A_{II}(i) \cdot \left(A_{II}^T(i)Q_{y_i}^{-1}A_{II}(i)\right)^{-1} \end{bmatrix}$$

In general, at epoch k, $Q_{\widehat{x_{II}}\widehat{x_{II}}}(k)$ can be obtained with the relation

$$\boxed{\begin{aligned} Q_{\widehat{x_{II}}\widehat{x_{II}}}(k) = {} & \left(A_{II_i}^T Q_{y_i}^{-1} A_{II_i}\right)^{-1} + \\ & \left(A_{II_i}^T Q_{y_i}^{-1} A_{II_i}\right)^{-1} \cdot A_{I_i}^T Q_{y_i}^{-1} A_{II_i} \cdot Q_{\widehat{x_I}\widehat{x_I}}(k) \cdot A_{I_i}^T Q_{y_i}^{-1} A_{II_i} \cdot \left(A_{II_i}^T Q_{y_i}^{-1} A_{II_i}\right)^{-1} \end{aligned}} \quad (5.21)$$

The above expression can also be called as float VC matrix of the ionosphere, that is $Q_{\widehat{x_{II}}\widehat{x_{II}}}$ can be referred as $Q_{\hat{b}}$.

5.3 Computation of Ambiguity Success Rates (ASR)

The VC matrix of ambiguities are de-correlated using the *decorr.m*, a Matlab software routine developed by Peter Joosten, Delft University and further modified by Bofeng Li, GNSS Research Group, Curtin University. The de-correlation routine returns the conditional variances of the ambiguities which are used to compute Integer Bootstrap probabilities, see chapter 3, equation (3.14). The definition of the desired probability is the criteria of judging the Ambiguity Success Rate (ASR). The criteria for accepting that the ambiguity is correctly fixed is that it gives at least 0.999 probability. for full AR, all the ambiguities should collectively give at least 0.999 probability of Integer Bootstrap and for Partial AR (PAR), each ambiguity should give at least 0.999 probability. The detailed explanation of the computation of ASR using the LAMBDA software routines can be found in [44] and [45], it is also explained in Appendix C. While the ambiguities are accepted to be correctly fixed, the precision of the parameters of interest, namely the receiver-satellite ranges, ionosphere, receiver coordinates etcetera is aimed to be improved from the fixing of ambiguities.

The precision the the temporal varying parameters (say ionosphere) corresponding to ambiguity-fixed solution needs to be computed, it is denoted as $Q_{\check{b}}$. It can be computed by using the $Q_{\hat{b}}$ from equation (5.21), $Q_{\hat{b}\hat{a}}$ is the covariance between the ionosphere and the ambiguities as found in equation (5.20) under $Q_{\hat{I}\hat{a}}$, the expression for $Q_{\hat{a}}$ is derived in equation (5.19)

5.3.1 Full ambiguity resolution - computation of fixed-precision of parameters of interest

After fixing the ambiguities using LAMBDA routine, the the de-correlated Z matrix ,which contains the information of the fixed ambiguities, (for more information on design matrix of de-correlated ambiguities, Z, refer equation (3.5) in chapter 3) is used to compute the matrices $Q_{\hat{z}_I}$ and $Q_{\hat{z}_I\hat{z}_{II}}$ as follows.

$$\boxed{\begin{aligned} &\left.\begin{aligned} Q_{\hat{z}_I} &= Z^T_{x_I=\check{x}_I} Q_{\hat{a}} Z_{x_I=\check{x}_I} \\ Q_{\hat{z}_I\hat{z}_{II}} &= Z^T_{x_I=\check{x}_I} Q_{\hat{b}\hat{a}}{}^T \end{aligned}\right\} \qquad (5.22) \\ &Q_{\check{b}} = Q_{\hat{b}} - Q_{\hat{z}_{II}\hat{z}_I} Q^{-1}_{\hat{z}_I} Q_{\hat{z}_I\hat{z}_{II}} \end{aligned}}$$

As the criteria for successful ambiguity resolution, that is 0.999 ASR is met, the corresponding solution for $Q_{\check{b}}$ can be used to compute the gain of the other unknowns (atmospheric parameters). Since the elevation weighting of the satellites is considered for simulation, the values of variance in the $Q_{\check{b}}$ for each satellite will be different. Hence a volume based geometric average of the VC matrices $Q_{\check{b}}$ and $Q_{\hat{b}}$ is computed for every epoch by $|Q_{\hat{b}\hat{b}}|^{1/2n}$ and $|Q_{\check{b}\check{b}}|^{1/2n}$ for the fixed and float case, where n is the number of non-ambiguity parameters in the VC matrix.

5.3.2 Partial ambiguity resolution - computation of fixed-precision of parameters of interest

The VC matrices, namely, $Q_{\hat{a}}, Q_{\hat{b}}$ and $Q_{\hat{a}\hat{b}}$ are computed as explained in the above section

Further, after partially fixing the ambiguities using the LAMBDA software routines (for detailed explanation, see Appendix C, we get the de-correlated Z matrix ,which contains the information of the fixed ambiguities, for more information on design matrix of de-correlated ambiguities, Z, refer equation (3.5) in chapter 3.

The matrices $Q_{\hat{z}_I}$ and $Q_{\hat{z}_I\hat{z}_{II}}$ are computed after fixing the ambiguities as follows.

$$\left.\begin{aligned} Q_{\hat{z}_I} &= Z^T_{x_I=\check{x}_I} Q_{\hat{a}} Z_{x_I=\check{x}_I} \\ Q_{\hat{z}_I\hat{z}_{II}} &= Z^T_{x_I=\check{x}_I} {Q_{\hat{b}\hat{a}}}^T \end{aligned}\right\}$$

$$Q_{\check{b}} = Q_{\hat{b}} - Q_{\hat{z}_{II}\hat{z}_I} Q_{\hat{z}_I}^{-1} Q_{\hat{z}_I\hat{z}_{II}}$$

If p out of the $m-1$ ambiguities are fixed, than, the geometric precision of can be computed from the VC matrices $Q_{\check{b}}$ and $Q_{\hat{b}}$ is computed for every epoch by $|Q_{\hat{b}\hat{b}}|^{1/2n}$ and $|Q_{\check{b}\check{b}}|^{1/2n}$ for the fixed and float case, where n is the number of non-ambiguity parameters in the VC matrix.

5.3.3 Computation of gain:

The computation of gain is done using the VC matrices for float and fixed solution. The gain or the improvement in precision is given as below.

$$\text{Gain} = \frac{|Q_{\widehat{b}\widehat{b}}|^{1/2n}}{|Q_{\check{b}\check{b}}|^{1/2n}} \tag{5.23}$$

5.4 Simulation considerations

The simulations will be carried out for the following scenarios

(1) Receiver coordinates consideration:

   (a) Known - Permanent network station model (eg, CORS network), here baseline is considered instead of network

(2) Atmosphere consideration

   (a) Fixed (both troposphere and ionosphere are considered known), applicable for short baseline lengths (in this research, baseline lengths $<$ 10 Km are considered short baselines, the relative atmosphere can be assumed to be insignificant)
   For atmosphere known case, baseline length of 1 Kilometer is considered. Measurement precision is varied for 6 different sets of phase and code, see Table 3.1. Simulation is done for 3 latitude locations, $0°, -30°$ and $-60°$.

   (b) Float (both troposphere and ionosphere are considered as unknown), applicable for medium and long baseline lengths ($>$ 10 Km)
   The baseline length of 250 Kilometers will be considered. Measurement precision is varied for 6 different sets of phase and code, see Table 3.1. Simulation is done for 3 latitude locations, $0°, -30°$ and $-60°$. While the troposphere is parameterized in the design matrix, it is re-initialized after every 2 hours in order to have scenario similar to estimating a new tropospheric delay every 2 hours.

   (c) Weighted (ionosphere will be weighted, whereas, the troposphere is considered to be unknown)
   The baseline length is varied between 1 and 1000 Kilometers is considered. Measurement precision is held fixed to one value for both phase and code, see Table 3.2. Simulation is done for 3 latitude locations, $0°, -30°$ and $-60°$. Troposphere is re-initialized after every 2 hours.

(3) Other considerations

(a) Latitude locations: simulations will be carried for three latitude locations 0°, −30° and −60° latitudes. Longitude remains fixed at one end (say receiver number 1) at 115° longitude, at the other end (say for receiver number 2), variation is done if east-west oriented baseline is chosen.

(b) Baseline orientations: Baseline length of 250 Km is considered for atmosphere float scenario. Two different orientations of baseline ar considered, namely, east-west (E-W) oriented baseline (only longitudinal orientation is performed) and north-south (N-S) (only latitudinal variation is performed) oriented baseline.

(c) Frequency combinations: For GPS only system, three frequency combination, namely single ($L1$), dual ($L1$ and $L2$) and triple ($L1, L2$ and $L5$) are considered. For Galileo only, four combinations, namely, single ($E1$), dual ($E1$ and $E5a$), triple ($E1, E5a$ and $E5b$) and quadruple ($E1, E5a, E5b$ and $E6$) are considered. For a combined GPS and Galileo system three combination are considered, namely single ($L1(E1)$), dual ($L1(E1)$, $L5(E5a)$) and quadruple ($L1(E1)$, $L5(E5a)$, $L2$, $E5b$). Triple frequency combination is not considered on the understanding that the high-end receivers will have options to either lock two frequencies of each GPS and Galileo or three frequencies of each GPS and Galileo, bringing to the frequency combination to a dual (two overlapping frequencies) or a quadruple frequency combination (two overlapping and two non-overlapping frequencies).

For the above 6 combination of scenarios, all the measurement precision values (6 different values as shown in Table 3.1) will be considered. All the satellites above 10° elevation are considered for simulation. The three different receiver types, low-end single frequency, high-end dual frequency, high-end triple frequency and high-end quadruple frequency will be considered. The definition of the type of receiver is based on the value of measurement precision chosen (see Tables 3.1 and 3.2). The interval between the epochs is considered to be 1 second (sampling interval).

The estimation of the ambiguities is based on the stacking of the normal equations among epochs. Both full AR and partial AR (PAR) are simulated for Geometry Based model for all the above mentioned scenarios.

5.5 Receiver coordinates known, atmosphere known scenario

The atmosphere is considered to be insignificant for relative positioning when short baselines ($<$ 10 Km) are considered. Such a scenario is termed as atmosphere fixed while describing the parameterization of the GNSS observables. Both ionosphere and troposphere are parameterized for a geometry based model, unlike geometry free model, where the troposphere is lumped with the ranges. In this scenario, the receiver coordinates are considered to be known with a high accuracy, they are assumed to be fixed (for example, coordinates of CORS stations). In this scenario, when the receiver coordinates are known and fixed, the GNSS model can be referred as Geometry Fixed model. The baseline length is considered as 1 Kilometre. Satellite coordinates are assumed to be known and are generated from YUMA almanacs. Presented below is the functional and stochastic models, see Table 5.1. The redundancy for Geometry based model, coordinates known, atmosphere known scenario, is presented in Table 5.1 and Figure 5.1, see below.

Geometry Based model, coordinates known, atmosphere known scenario		
Functional model		**Stochastic model**
Non-temporal parameters	Temporal parameters	
$A_{I(i)} = \begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes I_{m-1}$	-	$Q_{y(i)} = \begin{bmatrix} Q_\Phi & \\ & Q_P \end{bmatrix} \otimes Q_{DD}(i)$
		where $Q_{DD}(i) = (D_m^T W_i^{-1} D_m)$
Redundancy (for GPS only, Galileo only, GPS+Galileo (common frequency $L1(E1), L5(E5a)$**))**		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $f * (m-1)$	-	$k * 2f * (m-1)$
Redundancy of coordinates known, atmosphere known model		$(2k-1) * f(m-1)$
For GPS/Galileo only: $f = (1, \cdots, j)$, for GPS+Galileo: $f = (1, \cdots, j_c)$, j_c is the common/overlapping frequency, and $m_{jc} = (m_{GPS} + m_{Gal})$		
Redundancy (for GPS+Galileo (quadruple frequency $L1(E1), L5(E5a), L2, E5b$**))**		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $2 * (m_{jc} - 1) + (m_{GPS} - 1) + (m_{Gal} - 1)$	-	$4 * k * (m_{jc} - 1) + 2 * k((m_{GPS} - 1) + (m_{Gal} - 1))$
Redundancy of coordinates known, atmosphere known, quadruple frequency model		$(2k-1)\,(2 * (m_{jc} - 1) + (m_{GPS} - 1) + (m_{Gal} - 1))$
$f = 1, 2$ for GPS+Gaileo $L1(E1), L5(E5a)$, m_{jc} are the total number of satellites for combined GPS+Galileo system with GPS as reference satellite		

Table 5.1: Double-differenced Design matrix and VC matrix for coordinates known, atmosphere known scenario

In Table 5.1, since there are only ambiguities as unknowns, the space under non-ambiguity parameters is intentionally left blank. It is to be noted that, since the simulations is done for a baseline, receiver subscript is ignored.

In Table 5.1, subscript f indicates the total number of frequencies, $f = 1, \cdots, j$, $D_m^T = [-e_{m-1}, I_{m-1}]$; is the satellite design matrix defined earlier as D^T, W_i is the satellite elevation weight matrix for i^{th} epoch, see Appendix B, equation (B.5), and $\Lambda = \text{diag}\ \underbrace{(\lambda_1, \cdots, \lambda_j)}_{j \times j}$

The redundancy is also represented graphically, see Figure 5.1. In Figure 5.1, redundancy for single system (see top row, say for GPS or Galileo) and combined GPS and Galileo system (bottom row) is presented, for single, dual, triple and quadruple frequencies, single epoch (top left) and multi epoch (top right). For single epoch (left hand side) a minimum of 2 satellites are needed so that the GNSS model is redundant enough. For single epoch, two system (bottom row, right hand side), the redundancy of dual frequency is seen to be more than the quadruple system. This is because of the fact that the combined system with one GPS reference satellite has 2 observations more at the cost of one additional unknown (ambiguity).

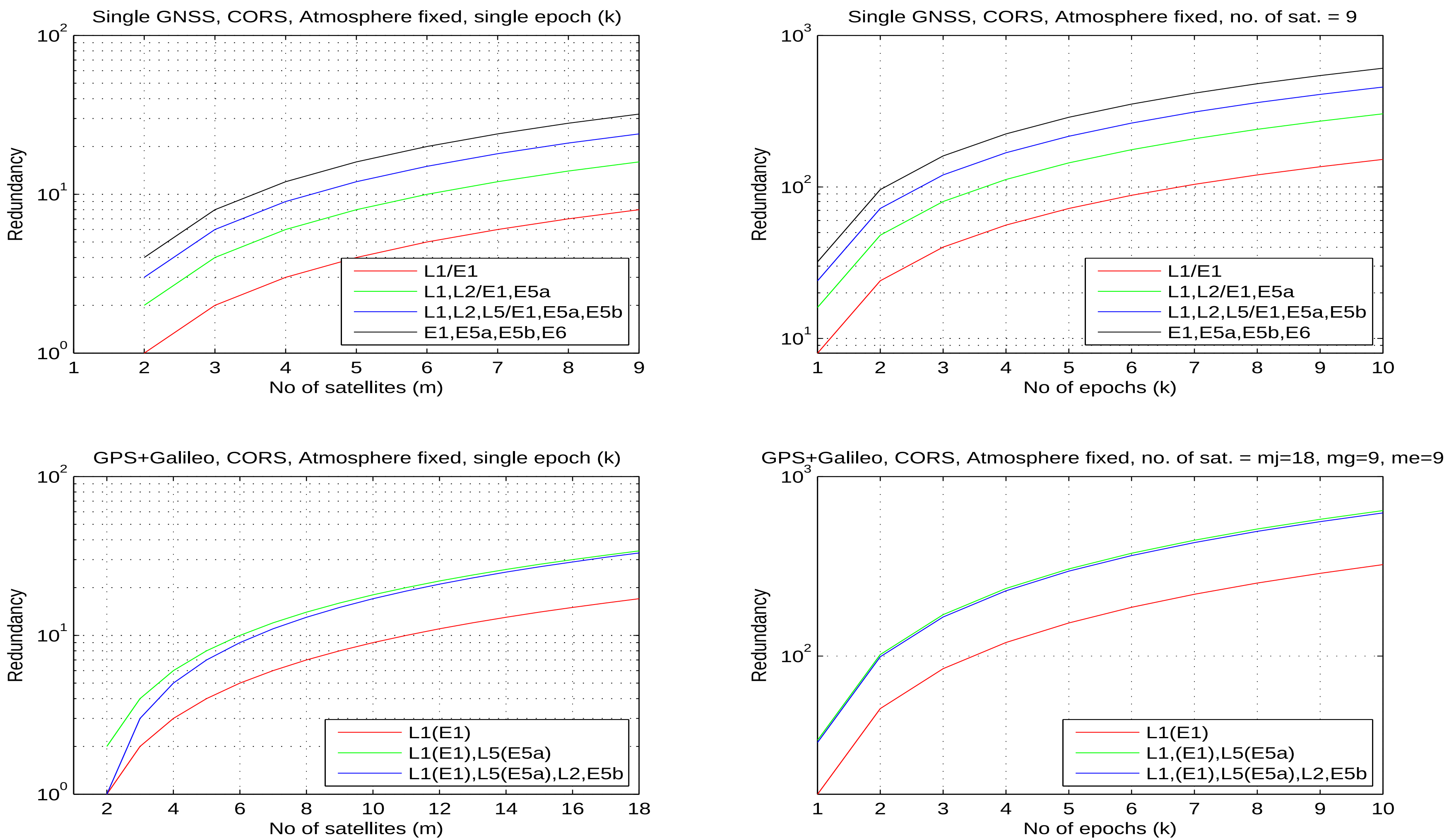

Figure 5.1: Redundancy plot of **GPS only**, **Galileo only** (top two plots) and **GPS + Galileo** (bottom two plots) for single epoch (left hand side plots) and multi-epoch (right hand side plots), **Geometry based** model, coordinates known, atmosphere known scenario, single, dual, triple and quadruple frequencies

5.5.1 GPS only - Full and Partial Ambiguity Resolution, atmosphere known scenario

For short baseline lengths (atmosphere known scenario), the geometry fixed model is strong enough with only ambiguities as unknowns. It is understood that only ambiguity fixing is not in interest when all other unknowns are fixed (in relative sense, double differenced model), since the aim of using the carrier phase data is to utilize the precision of the phase data for estimating other unknowns. None-the-less, for this scenario the number of epochs taken to fix the ambiguities are analyzed in order to see if this strong model can give instantaneous ambiguity results. Table 5.2 presents the number of epochs taken for 0.999 ASR.

In Table 5.2, for single frequency, instantaneous ambiguity resolution can be achieved for all the three latitude locations. All the ambiguities converged to 0.999 ASR with 1 epoch. This was possible to be achieved, since as per the assumptions made for this Geometry Fixed model, the standard deviation of the float solution is defined by the ratio precision of the phase observables and the wavelength of the GPS/Galileo frequency, that is $Q_{\hat{x}\hat{x}} = Q_{\hat{a}\hat{a}} = \sigma_{\Phi}^2/\lambda_j^2$. With such a high float precision, all the ambiguities could

Further, for this scenario, results for GPS dual and triple frequency, Galileo only and GPS and Galileo combined system are not presented. Since as discussed earlier the float ambiguities are precise enough and hence instantaneous ASR would be achieved. It should also be noted that Partial AR (PAR) will not be evaluated for this scenario. From Table 5.2 it can be seen that for all the scenarios of single, dual and triple frequency, instantaneous ASR of 0.999 is achieved.

Phase	Code	Phase	Code	Full AR - minimum desired ASR = 0.999		
				Epochs taken		
	(meters)			0°	−30°	−60°
Φ	P	Φ	P	Low-end receivers, Single Frequency		
L1						
0.003	0.25	-	-	1	1	1
0.003	0.50	-	-	1	1	1
0.003	0.75	-	-	1	1	1
0.003	1.00	-	-	1	1	1
0.003	1.25	-	-	1	1	1
0.003	1.50	-	-	1	1	1
Φ	P	Φ	P	High-end receivers, Dual Frequency		
L1,L2						
0.002	0.10	-	-	1	1	1
0.002	0.15	-	-	1	1	1
0.002	0.20	-	-	1	1	1
0.003	0.25	-	-	1	1	1
0.003	0.30	-	-	1	1	1
0.003	0.35	-	-	1	1	1
Φ	P	Φ	P	High-end receivers, Triple Frequency		
L1,L2		L5				
0.002	0.10	0.002	0.05	1	1	1
0.002	0.15	0.002	0.1	1	1	1
0.002	0.20	0.002	0.125	1	1	1
0.003	0.25	0.002	0.15	1	1	1
0.003	0.30	0.002	0.175	1	1	1
0.003	0.35	0.002	0.2	1	1	1

Table 5.2: **GPS only**, **Geometry based** model, coordinates known, atmosphere known scenario, Full AR analysis, number of epochs taken for 0.999 ASR is presented - **measurement precision varied**, number of satellites as available.

5.6 Receiver coordinates known, atmosphere unknown scenario

For baseline when the atmosphere is said to be significant for relative positioning applications, it is no longer termed as a short baseline. Generally, baseline lengths of longer than 10 kilometres are said to hold significant relative atmosphere bias, however it should be noted that the term significant is application specific. For a Geometry Fixed model, both ionosphere and troposphere being unknowns are parameterized in the design matrix for estimation. As per the current assumption, the coordinates are considered to be known with a high accuracy, they are assumed to be fixed. The baseline length considered here is 250 Kilometres. Satellite coordinates are assumed to be known and are generated from YUMA almanacs. Presented below is the functional and stochastic models, see Table 5.3. The redundancy for Geometry based model, coordinates known, atmosphere unknown, is presented in Table 5.3 and Figure 5.2, see below.

Geometry based model, coordinates known, atmosphere unknown scenario		
	Functional model	**Stochastic model**
Non-temporal parameters	Temporal parameters	
$A_{I(i)} = \left[\begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes I_{m-1} \quad \begin{pmatrix} e_f \\ e_f \end{pmatrix} \otimes G_{m-1} \right]$	$A_{II(i)} = \begin{pmatrix} -\mu_f \\ \mu_f \end{pmatrix} \otimes I_{m-1}$	$Q_{y(i)} = \begin{bmatrix} Q_\Phi & \\ & Q_P \end{bmatrix} \otimes Q_{DD}(i)$ where $Q_{DD}(i) = D_m^T W_i D_m$
Redundancy (for GPS only, Galileo only, GPS+Galileo (common frequency $L1(E1), L5(E5a)$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $f * (m-1)$ Troposphere: $= 1$	Ionosphere: $k * (m-1)$	$k * 2f * (m-1)$
Redundancy of Geometry based model, coordinates known, atmosphere unknown scenario		$\{(2kf - f - 1) * (m-1)\} - 1$
For GPS/Galileo only: $f = (1, \cdots, j)$, for GPS+Galileo: $f = (1, \cdots, j_c)$, j_c is the common/overlapping frequency, and $m_{jc} = (m_{GPS} + m_{Gal})$		
Redundancy (for GPS+Galileo (quadruple frequency $L1(E1), L5(E5a), L2, E5b$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)$ Troposphere: $= 1$	Ionosphere: $k * (m_{jc} - 1)$	$2k * (2 * (m_{jc} - 1) + (m_{GPS} - 1) + (m_{Gal} - 1)$
Redundancy of quadruple frequency, coordinates known, atmosphere unknown scenario		$(4k - 3)(m_{jc} - 1) + (2k - 1)[(m_{GPS} - 1) + (m_{Gal} - 1)] - 1$
$f = 1, 2$ for GPS+Gaileo $L1(E1), L5(E5a)$, m_{jc} are the total number of satellites for combined GPS+Galileo system with GPS as reference satellite		

Table 5.3: Double-differenced Design matrix and VC matrix for Geometry Based model, coordinates known, atmosphere unknown scenario

where G_{m-1} has the tropospheric mapping function, it is given as below,

$$G_{m-1} = \begin{bmatrix} \psi^{1-2} \\ \vdots \\ \psi^{1-m} \end{bmatrix}$$

In Table 5.3, ψ is the DD tropospheric mapping function. In this study a cosine mapping function is used defined as, $\psi = 1/\cos(z)$, where z is the zenith angle of the satellite. D_m^T is the single differenced satellite design matrix is known as the difference operator for the satellites, W_i is the satellite elevation weight matrix and $\Lambda = \text{diag}\,\underbrace{(\lambda_1, \cdots, \lambda_j)}_{j\times j}$.

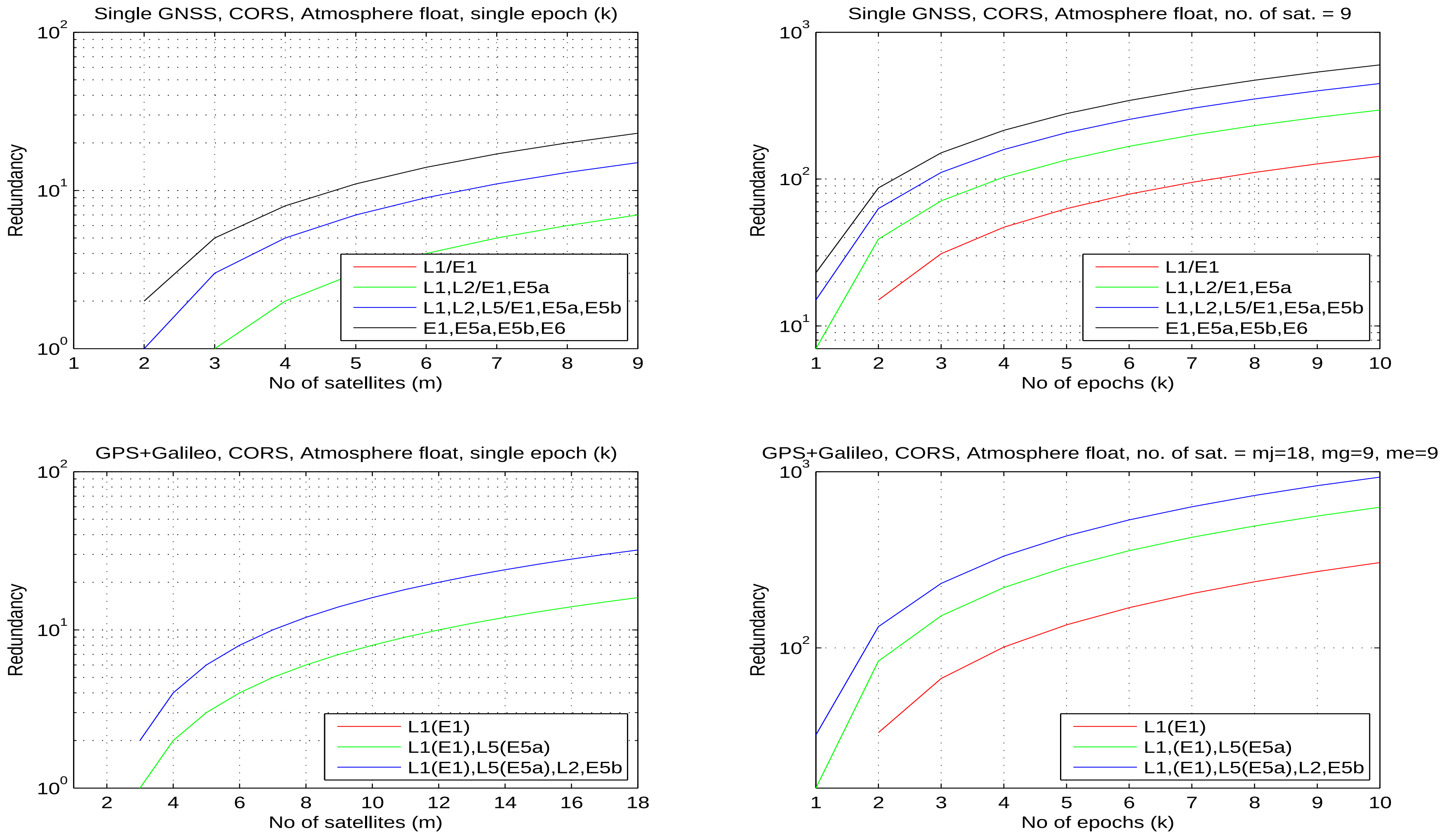

Figure 5.2: Redundancy plot of **GPS only**, **Galileo only** (top two plots) and **GPS + Galileo** (bottom two plots) for single epoch (left hand side plots) and multi-epoch (right hand side plots), **Geometry Based** model, coordinates known, atmosphere unknown, single, dual, triple and quadruple frequencies

5.6.1 Hourly batches, Full Ambiguity Resolution, atmosphere unknown scenario

Initially, ASR is simulated for hourly batches, throughout the day 24 hourly batches are formed starting from 00 UTC and ending at 2359 UTC. The interval between the epochs is taken as 1 second, making the total number of epochs to be 3600 in each batch. It is to see the average behaviour for achieving 0.999 ASR during different times of the day. The results for single, dual, triple and quadruple (Galileo only) frequency, GPS, Galileo and with both systems together is presented in the Figures F.1 to F.3 in Appendix F. The results for average number of epochs taken for each of the GNSS system are also presented in Table 5.4, see below.

Analysis of the averaged 24 hourly batches from Table 5.4 and Figures F.1 to F.3 is presented below.

GPS only

Dual frequency:

1. With GPS only, the average number of epochs (for 0.999 ASR) lie between 11 and 62 with dual frequency for different measurement precision scenarios for E-W baseline and for N-S baseline the average number of epochs lie between 10 and 58. It can be noted that with E-W baseline (250 Km baseline), the worst performance is for 0° latitude and same is the case for N-S baseline.

Triple frequency:

2. For triple frequency, the average number of epochs lie between 6 and 15 for E-W baseline and 5 to 14 epochs for the N-S baseline.

East-West and North-South oriented baseline comparison:

3. GPS only takes relatively less number of epochs at 0° latitude and more epochs at −60° latitude for both E-W and N-S baseline.

Phase	Code	Phase	Code	Avg. no. of Epochs taken 0°	−30°	−60°	Avg. no. of satellites 0°	−30°	−60°	Avg no. of Low-elevation Sat.⩾ 10°, ⩽ 30° 0°	−30°	−60°	Avg. no. of Epochs taken 0°	−30°	−60°	Average no. of satellites 0°	−30°	−60°	Avg. no. of Low-elevation Sat.⩾ 10°, ⩽ 30° 0°	−30°	−60°
Hourly, Full AR - minimum desired ASR = 0.999																					
(meters)				**East-West Baseline - 250Km**									**North-South Baseline - 250Km**								
GPS only																					
Φ L1,L2	P L1,L2	Φ	P	High-end receivers, Dual Frequency																	
0.002	0.10	-	-	11	11	12	10	8	9	5	3	5	10	11	12	9	8	9	5	3	4
0.002	0.15	-	-	15	16	18	10	8	9	5	3	5	15	16	17	9	8	9	5	3	4
0.002	0.20	-	-	20	21	24	10	8	9	5	3	5	20	21	22	9	8	9	5	3	4
0.003	0.25	-	-	36	38	43	10	8	9	5	3	5	35	40	41	9	8	9	5	3	4
0.003	0.30	-	-	43	45	54	10	8	9	5	3	5	41	47	51	9	8	9	5	3	4
0.003	0.35	-	-	50	52	62	10	8	9	5	3	5	47	55	58	9	8	9	5	3	4
Φ L1,L2	P L1,L2	Φ L5	P L5	High-end receivers, Triple Frequency																	
0.002	0.10	0.002	0.05	6	6	6	10	8	9	5	3	5	5	6	6	9	8	9	5	3	4
0.002	0.15	0.002	0.1	6	7	7	10	8	9	5	3	5	6	7	7	9	8	9	5	3	4
0.002	0.20	0.002	0.125	7	7	7	10	8	9	5	3	5	6	7	7	9	8	9	5	3	4
0.003	0.25	0.002	0.15	12	14	14	10	8	9	5	3	5	12	14	14	9	8	9	5	3	4
0.003	0.30	0.002	0.175	12	14	14	10	8	9	5	3	5	12	14	14	9	8	9	5	3	4
0.003	0.35	0.002	0.2	13	14	15	10	8	9	5	3	5	12	14	14	9	8	9	5	3	4
Galileo only																					
Φ E1	P E1	Φ E5a	P E5a	High-end receivers, Dual Frequency																	
0.002	0.10	0.002	0.05	6	6	6	10	8	10	5	3	4	6	7	7	10	9	9	5	3	4
0.002	0.15	0.002	0.1	14	13	16	10	8	10	5	3	4	14	14	16	10	9	9	5	3	4
0.002	0.20	0.002	0.125	21	19	25	10	8	10	5	3	4	22	21	25	10	9	9	5	3	4
0.003	0.25	0.002	0.15	31	28	35	10	8	10	5	3	4	31	30	36	10	9	9	5	3	4
0.003	0.30	0.002	0.175	45	38	48	10	8	10	5	3	4	44	41	49	10	9	9	5	3	4
0.003	0.35	0.002	0.2	61	49	66	10	8	10	5	3	4	57	53	67	10	9	9	5	3	4
Φ E1	P E1	Φ E5a,E5b	P E5a,E5b	High-end receivers, Triple Frequency																	
0.002	0.10	0.002	0.05	5	6	6	10	8	10	5	3	4	6	6	6	10	9	9	5	3	4
0.002	0.15	0.002	0.1	6	6	7	10	8	10	5	3	4	6	7	7	10	9	9	5	3	4
0.002	0.20	0.002	0.125	6	7	7	10	8	10	5	3	4	6	7	7	10	9	9	5	3	4
0.003	0.25	0.002	0.15	10	11	12	10	8	10	5	3	4	10	12	12	10	9	9	5	3	4
0.003	0.30	0.002	0.175	10	11	12	10	8	10	5	3	4	11	12	12	10	9	9	5	3	4
0.003	0.35	0.002	0.2	10	11	12	10	8	10	5	3	4	11	12	12	10	9	9	5	3	4
Φ E1	P E1	Φ E5a,E5b,E6	P E5a,E5b,E6	High-end receivers, Quadruple Frequency																	
0.002	0.10	0.002	0.05	5	5	5	10	8	10	5	3	4	5	6	6	10	9	9	5	3	4
0.002	0.15	0.002	0.1	5	6	6	10	8	10	5	3	4	6	6	6	10	9	9	5	3	4
0.002	0.20	0.002	0.125	6	6	6	10	8	10	5	3	4	6	7	6	10	9	9	5	3	4
0.003	0.25	0.002	0.15	10	11	11	10	8	10	5	3	4	10	12	11	10	9	9	5	3	4
0.003	0.30	0.002	0.175	10	11	11	10	8	10	5	3	4	10	12	12	10	9	9	5	3	4
0.003	0.35	0.002	0.2	10	11	11	10	8	10	5	3	4	10	12	12	10	9	9	5	3	4
GPS + Galileo																					
Φ L1(E1)	P L1(E1)	Φ L5(E5a)	P L5(E5a)	High-end receivers, Dual Frequency																	
0.002	0.10	0.002	0.05	6	6	7	19	17	19	10	6	9	6	6	7	19	17	18	10	7	8
0.002	0.15	0.002	0.1	17	16	20	19	17	19	10	6	9	16	18	19	19	17	18	10	7	8
0.002	0.20	0.002	0.125	26	25	32	19	17	19	10	6	9	26	27	29	19	17	18	10	7	8
0.003	0.25	0.002	0.15	37	36	45	19	17	19	10	6	9	37	39	44	19	17	18	10	7	8
0.003	0.30	0.002	0.175	54	49	64	19	17	19	10	6	9	52	53	62	19	17	18	10	7	8
0.003	0.35	0.002	0.2	73	65	81	19	17	19	10	6	9	67	68	83	19	17	19	10	7	8
Φ L1(E1),L2,E5b	P L1(E1),L2,E5b	Φ L5(E5a)	P L5(E5a)	High-end receivers, Quadruple Frequency																	
0.002	0.10	0.002	0.05	4	4	5	19	17	19	10	6	9	4	5	5	19	17	18	10	7	8
0.002	0.15	0.002	0.1	5	5	6	19	17	19	10	6	9	5	5	6	19	17	18	10	7	8
0.002	0.20	0.002	0.125	6	6	6	19	17	19	10	6	9	5	6	6	19	17	18	10	7	8
0.003	0.25	0.002	0.15	9	9	10	19	17	19	10	6	9	9	9	10	19	17	18	10	7	8
0.003	0.30	0.002	0.175	9	9	10	19	17	19	10	6	9	9	9	10	19	17	18	10	7	8
0.003	0.35	0.002	0.2	9	9	10	19	17	19	10	6	9	9	9	10	19	17	18	10	7	8

Table 5.4: **GPS only, Galileo only, GPS + Galileo** GNSS systems, **Geometry Based** model, coordinates known, atmosphere unknown scenario, **Full AR**, Average number of epochs, average number of satellites and average number of low elevation satellites over 24 batches (1 batch = 1 hour, 3600 epochs, 1 second interval) are presented - **measurement precision varied**, number of satellites as available.

Galileo only

Dual frequency:

4. For dual frequency Galileo, the average number of epochs taken lie between 6 and 66 for E-W baseline and 6 and 67 for N-S baseline. Galileo only outperforms GPS only with dual frequency for the first two (better) values of measurement precision, see Table 5.4. For the further four scenarios of relatively poor measurement precision, the performance is similar of poorer than GPS, for similar scenarios. The better performance of Galileo is credited to its improved measurement precision in comparison to GPS.

Triple frequency:

5. With triple frequency Galileo, the average number of epochs over 24 batches to obtain 0.999 ASR lied between 5 and 12 for E-W baseline and 6 to 12 for N-S baseline. The performance is better in comparison to GPS only which is an effect of better measurement precision for Galileo system in comparison to GPS system.

Quadruple frequency:

6. The quadruple frequency Galileo marginally performs better than the triple frequency Galileo, say for some scenarios, the quadruple system takes an epochs less than the triple frequency system. The improvement in performance is due to the addition of a frequency.

East-West and North-South oriented baseline comparison:

7. Galileo only takes relatively less number of epochs at $0°$ and $-30°$ (prominent in case of dual frequency) latitude and more epochs at $-60°$ latitude for E-W baseline in comparison to N-S baseline.

GPS + Galileo

Dual frequency:

8. For a combined GPS and Galileo system, it takes an average number of epochs between 6 and 81 for E-W baseline and 6 to 83 for N-S baseline for different scenarios of measurement precision and latitude location. Also it can be noted that, for a scenario when the GPS only and Galileo only models are redundant enough, the combined GPS and Galileo dual frequency system performs similar to standalone GPS or Galileo. This is due to the fact that for ionosphere float model, the GNSS model does not gain strength as number of satellites are increased. Since there is no information added to the model related to ionosphere, see [48].

Quadruple frequency:

9. It takes and average number of epochs between 4 and 10 for E-W baseline and N-S baseline, both. The quadruple GPS+Galileo system outperforms the quadruple Galileo only system. This is expected when ionosphere float model is under consideration, the strength of the GNSS ionosphere float model increases as number of frequencies are increased, see [48]. Since one ionosphere parameter is estimated for all the frequencies, ionosphere information is added to the model when additional frequency is added. The GPS+Galileo model on the other hand has more number of satellites as compared to Galileo only, it has hence an edge over Galileo only.

East-West and North-South oriented baseline comparison:

10. The combined GPS and Galileo system takes relatively less number of epochs at $0°$ and $-30°$ latitude and more epochs at $-60°$ latitude for E-W baseline in comparison to N-S baseline.

5.6.2 GPS only - Full and Partial Ambiguity Resolution, atmosphere unknown scenario

The results for GPS system, ionosphere float Geometry based model, coordinates known, atmosphere unknown scenario are presented below. Figures F.5 and F.6 give full ambiguity resolution results and Figures F.7 and F.8 present the partial ambiguity results for dual and triple frequency GPS respectively, see Appendix F. Figure 6.10 presents the number of satellites for E-W oriented baseline (250 Km) and N-S baseline (250 Km). Table 5.5 presents the full ambiguity resolution results in terms of number of epochs taken for 0.999 ASR and corresponding ionosphere precision (for ambiguity-fixed and -float solution) along with the gain obtained for the ionosphere for ambiguity-fixed solution is presented.

The following is the analysis based on Table 5.5 and Figures 6.10 to 5.5.

General remarks:

(1) It can be noted that for atmosphere unknown scenario (when ionosphere and troposphere both parameterized for estimation), instantaneous ASR (Full AR) of 0.999 is not achieved for Geometry fixed model.

(2) There is a significant difference in number of epochs taken for 0.999 ASR for E-W and N-S oriented baseline for dual frequency system. The triple frequency system having a better measurement precision than dual frequency system takes nearly same number of epochs for E-W and N-S oriented baseline. For just a few scenarios the N-S baseline takes an epoch more than the E-W baseline.

(3) The change in the number of epochs taken for E-W and N-S baselines is due to the number of common satellites seen at two baselines, see Figure 6.10. Further more, it can be noted that the number of epochs taken for 0.999 full AR are less for a certain latitude location, as discussed in the earlier section. For both E-W and N-S baseline the number of epochs taken for 0.999 full AR are less at $0°$ latitude and more at $-60°$ latitude locations.

Time to regain 0.999 ASR:

				Full AR - minimum desired ASR = 0.999											
Phase	Code	Phase	Code	Epochs taken			$\sigma_{\check{I}}$(meters)			$\sigma_{\hat{I}}$(meters)			Gain=$\sigma_{\hat{I}}/\sigma_{\check{I}}$		
	(meters)			0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km															
Φ L1,L2	P	Φ	P	High-end receivers, Dual Frequency											
0.002	0.10	-	-	9	10	17	0.003	0.003	0.004	0.064	0.059	0.057	19.03	17.84	13.73
0.002	0.15	-	-	12	15	25	0.003	0.003	0.004	0.083	0.072	0.070	24.99	22.12	17.06
0.002	0.20	-	-	16	19	34	0.003	0.003	0.004	0.096	0.085	0.080	29.14	26.32	19.47
0.003	0.25	-	-	29	36	63	0.005	0.005	0.006	0.089	0.077	0.073	18.35	16.12	11.90
0.003	0.30	-	-	35	43	75	0.005	0.005	0.006	0.097	0.084	0.079	20.06	17.60	12.90
0.003	0.35	-	-	40	51	88	0.005	0.005	0.006	0.105	0.089	0.083	21.83	18.68	13.65
Φ L1,L2	P	Φ L5	P	High-end receivers, Triple Frequency											
0.002	0.10	0.002	0.05	6	5	9	0.003	0.002	0.003	0.039	0.041	0.038	15.41	16.42	12.37
0.002	0.15	0.002	0.1	6	6	10	0.003	0.002	0.003	0.070	0.068	0.066	27.75	27.30	21.30
0.002	0.20	0.002	0.125	6	6	10	0.003	0.002	0.003	0.090	0.087	0.084	35.53	34.94	27.24
0.003	0.25	0.002	0.15	11	11	19	0.003	0.003	0.004	0.081	0.078	0.075	27.28	26.76	20.40
0.003	0.30	0.002	0.175	11	12	20	0.003	0.003	0.004	0.095	0.088	0.085	32.13	30.26	23.41
0.003	0.35	0.002	0.2	11	12	20	0.003	0.003	0.004	0.109	0.101	0.098	36.97	34.80	26.90
North-South Baseline - 250Km															
Φ L1,L2	P	Φ	P	High-end receivers, Dual Frequency											
0.002	0.10	-	-	9	8	15	0.003	0.003	0.003	0.064	0.065	0.052	19.03	19.76	15.04
0.002	0.15	-	-	12	12	22	0.003	0.003	0.003	0.084	0.080	0.064	24.99	24.55	18.73
0.002	0.20	-	-	16	16	29	0.003	0.003	0.003	0.096	0.092	0.074	29.14	28.56	21.63
0.003	0.25	-	-	30	30	84	0.005	0.005	0.006	0.088	0.084	0.059	18.06	17.64	10.49
0.003	0.30	-	-	35	36	95	0.005	0.005	0.006	0.097	0.091	0.064	20.06	19.28	11.51
0.003	0.35	-	-	41	42	106	0.005	0.005	0.006	0.104	0.098	0.069	21.57	20.71	12.42
Φ L1,L2	P	Φ L5	P	High-end receivers, Triple Frequency											
0.002	0.10	0.002	0.05	5	5	9	0.003	0.002	0.003	0.043	0.041	0.033	16.71	16.42	12.84
0.002	0.15	0.002	0.1	6	5	9	0.003	0.002	0.003	0.070	0.074	0.060	27.75	29.65	23.10
0.002	0.20	0.002	0.125	6	5	10	0.003	0.002	0.003	0.090	0.094	0.073	35.53	37.95	28.17
0.003	0.25	0.002	0.15	11	9	19	0.003	0.003	0.003	0.081	0.086	0.064	27.28	29.33	21.12
0.003	0.30	0.002	0.175	11	10	19	0.003	0.003	0.003	0.096	0.096	0.076	32.13	32.91	24.82
0.003	0.35	0.002	0.2	11	10	19	0.003	0.003	0.003	0.110	0.110	0.087	36.97	37.85	28.47

Table 5.5: **GPS only**, **Geometry Based** model, coordinates known, atmosphere unknown scenario, **Full AR** analysis, ASR and ambiguity-fixed and -float precision for ionosphere are presented - **measurement precision varied**, number of satellites as available.

(4) It can be seen at certain times after initial 0.999 ASR is achieved, the gained criteria is lost which is due to an incoming low elevation satellite. The time taken to regain the criteria of 0.999 ASR is the lowest for better measurement precision. For example see Figure 5.4, the first row represents three values of measurement precision for 0° latitude. The width of the spike in the green curve (fixed-precision of ionosphere curve) is the least for better measurement precision (first/third row, first column plot) and the width is larger for the worst measurement precision (first/third row, last column plot).

(5) The same phenomenon can be seen in partial ambiguity resolution figure, see Figure 5.5. The incoming new satellite does not qualify for 0.999 ASR criteria; hence there is a drop in % of ambiguities fixed, from 100% to 89%.

For East-West baseline:

(6) For dual frequency, at $-30°$ latitude with the measurement precision of σ_Φ=3mm on $L1, L2$ and 2mm on $L5$ and σ_P=25cm on $L1, L2$ and 15cm on $L5$, it took 36 epochs for dual frequency GPS whereas for triple frequency 11 epochs were needed for 0.999 ASR (Full AR).

For North-South baseline:

(7) For dual frequency, at $-30°$ latitude with the measurement precision of σ_Φ=3mm on $L1, L2$ and 2mm on $L5$ and σ_P=25cm on $L1, L2$ and 15cm on $L5$, it took 30 epochs for dual frequency GPS whereas for triple frequency 9 epochs were needed for 0.999 ASR (Full AR).

Considering the fixed-precision of ionosphere to lie around 3mm in most of the cases when all the ambiguities were fixed, PAR will be evaluated for obtaining a fixed-precision of 2cm for troposphere in the coming section.

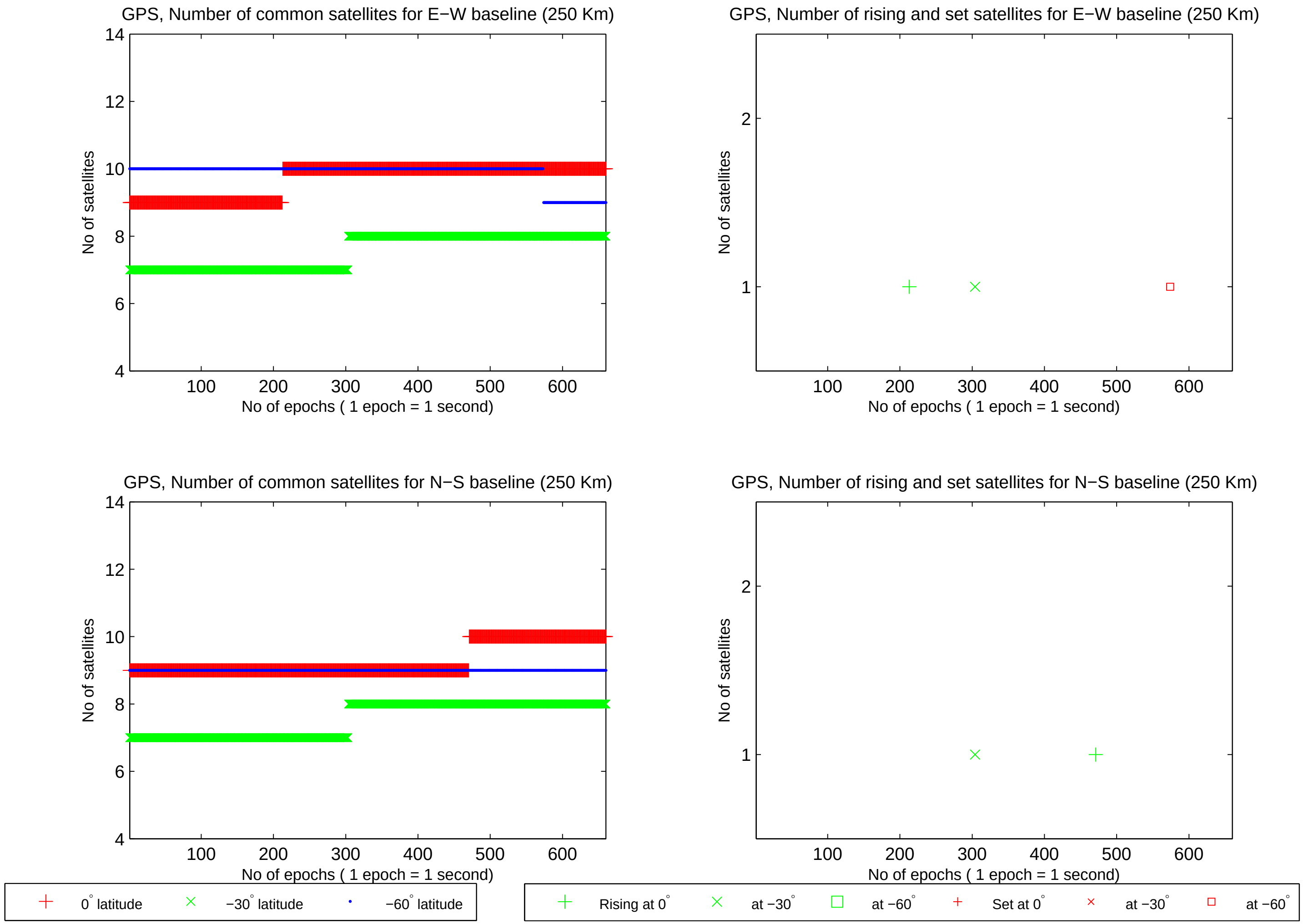

Figure 5.3: Number of satellites for **GPS only**, 250 Km East-West and North-South baseline at $0°, -30°$ and $-60°$ degree latitude.

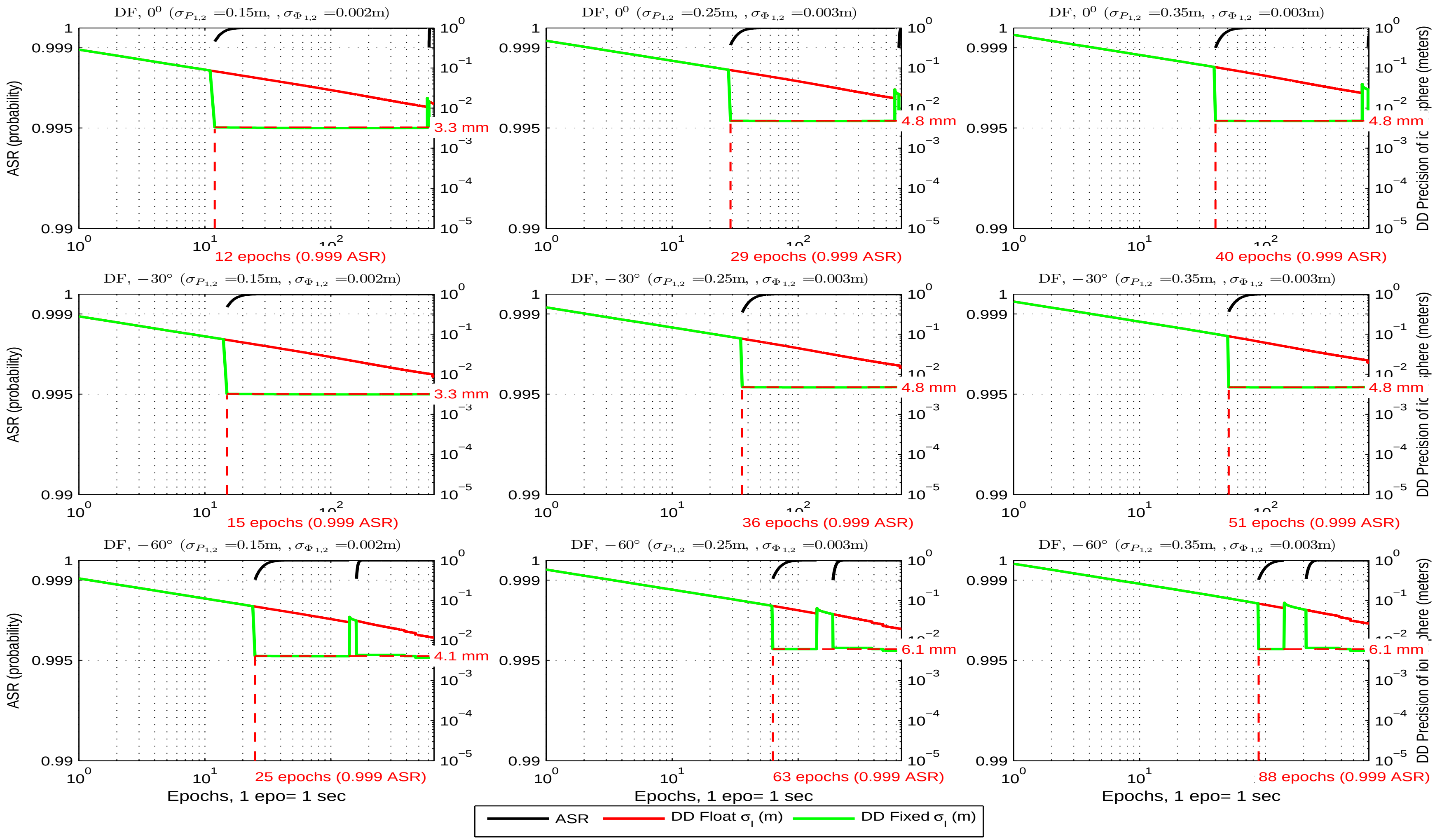

Figure 5.4: **GPS only**, Dual frequency, **Full AR**, **Geometry Based** model, coordinates known, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

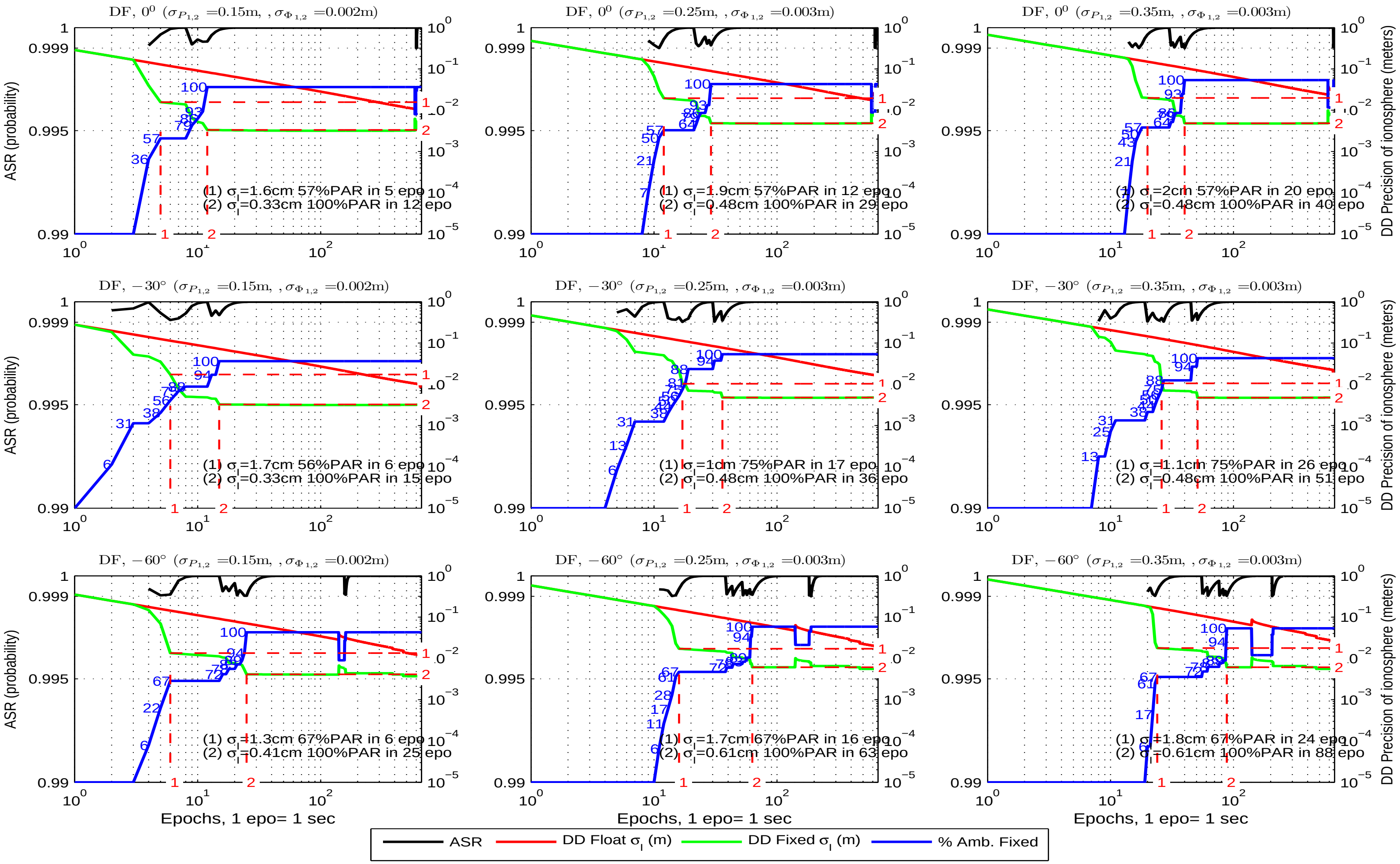

Figure 5.5: **GPS only**, Dual frequency, **Partial AR**, **Geometry Based** model, coordinates known, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

5.6.3 GPS only, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, atmosphere unknown scenario

In this section, it is to be analyzed if by partially fixing the ambiguities, does the solution converge faster to a modestly chosen fixed-precision of 2cm for ionosphere. Table 5.6 presents the results in terms of fixed-precision to be equal to or better than 2cm and corresponding epochs taken and ambiguities fixed are noted.

Referring Table 5.6, for all the scenarios of measurement precision (out of 18 scenarios for each dual and triple frequency) less than 100% of ambiguities were fixed to give a fixed-precision of 4mm to 2cm.

General remarks::

(1) A fixed-precision of 2cm for ionosphere could not be achieved instantaneously with GPS dual or triple frequency system, for E-W and for N-S baseline both.

(2) For dual frequency system, the E-W baseline took less number of epochs to give a fixed-precision of 2cm as compared to N-S baseline, except for $-30°$ latitude. At $-30°$ latitude the N-S baseline took relatively less number of epochs by partially fixing relatively less ambiguities. It can be noted that at $-30°$ latitude, both E-W and N-S baseline had similar number of common satellites for the time period under consideration.

(3) For triple frequency GPS, the N-S baseline outperformed the E-W baseline at $-60°$ latitude for a few scenarios of measurement precision. It should be noted that for all those scenarios, for the E-W baseline relatively lower percentage of ambiguities were fixed (78, 67 and 67%) than for N-S baseline (76, 76, 71%), see Table 5.6. Also it can be seen from Figure 6.10 that for the selected time period, the N-S baseline had less number of satellites (9) than the E-W baseline (10). Hence with higher elevation resulting in relatively stronger stochastic model for N-S baseline, the results proved to be better than E-W baseline. For all the

				Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for ionosphere														
Phase	Code (meters)	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{I}}$(meters)			$\sigma_{\hat{I}}$(meters)			Gain=$\sigma_{\hat{I}}/\sigma_{\check{I}}$		
				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km																		
Φ L1,L2	P	Φ	P	High-end receivers, Dual Frequency														
0.002	0.10	-	-	4	4	5	57	56	67	0.014	0.017	0.012	0.096	0.093	0.105	6.88	5.51	8.59
0.002	0.15	-	-	5	6	6	57	56	67	0.016	0.017	0.013	0.128	0.114	0.144	8.23	6.62	10.67
0.002	0.20	-	-	7	10	9	57	81	67	0.016	0.007	0.014	0.145	0.118	0.156	8.97	16.27	11.46
0.003	0.25	-	-	12	17	16	57	75	67	0.019	0.010	0.017	0.138	0.113	0.146	7.10	10.92	8.53
0.003	0.30	-	-	15	21	19	57	63	67	0.020	0.019	0.018	0.148	0.121	0.161	7.45	6.26	9.12
0.003	0.35	-	-	18	26	24	57	75	67	0.020	0.011	0.018	0.158	0.127	0.166	7.76	12.04	9.36
Φ L1,L2	P	Φ L5	P	High-end receivers, Triple Frequency														
0.002	0.10	0.002	0.05	2	2	3	86	71	81	0.006	0.014	0.007	0.067	0.065	0.066	11.05	4.53	9.17
0.002	0.15	0.002	0.1	3	2	3	81	67	78	0.007	0.019	0.009	0.099	0.117	0.120	14.68	6.26	13.11
0.002	0.20	0.002	0.125	3	2	3	81	67	67	0.007	0.019	0.019	0.127	0.150	0.154	18.54	7.71	8.12
0.003	0.25	0.002	0.15	6	4	6	81	67	78	0.007	0.019	0.010	0.109	0.129	0.133	15.03	6.65	13.21
0.003	0.30	0.002	0.175	6	4	6	81	67	67	0.007	0.020	0.020	0.129	0.152	0.156	17.57	7.67	7.96
0.003	0.35	0.002	0.2	6	4	6	76	67	67	0.013	0.020	0.020	0.148	0.175	0.180	11.39	8.70	9.04
North-South Baseline - 250Km																		
Φ L1,L2	P	Φ	P	High-end receivers, Dual Frequency														
0.002	0.10	-	-	4	4	4	64	63	64	0.011	0.013	0.015	0.096	0.092	0.101	8.75	6.88	6.91
0.002	0.15	-	-	5	5	6	57	56	64	0.016	0.018	0.016	0.129	0.124	0.124	8.23	6.97	7.71
0.002	0.20	-	-	7	7	8	57	56	64	0.016	0.018	0.017	0.146	0.139	0.143	8.97	7.64	8.47
0.003	0.25	-	-	12	14	16	57	63	71	0.020	0.020	0.018	0.139	0.123	0.126	7.10	6.08	6.82
0.003	0.30	-	-	15	18	21	57	63	79	0.020	0.020	0.013	0.149	0.130	0.131	7.45	6.45	10.33
0.003	0.35	-	-	19	21	24	57	63	79	0.020	0.020	0.013	0.154	0.140	0.142	7.65	6.88	11.01
Φ L1,L2	P	Φ L5	P	High-end receivers, Triple Frequency														
0.002	0.10	0.002	0.05	2	2	2	86	75	67	0.006	0.011	0.020	0.067	0.064	0.071	11.04	5.80	3.60
0.002	0.15	0.002	0.1	3	2	3	81	71	81	0.007	0.014	0.009	0.100	0.116	0.104	14.67	8.18	11.82
0.002	0.20	0.002	0.125	3	2	3	71	71	81	0.012	0.015	0.009	0.128	0.149	0.134	10.84	10.16	14.77
0.003	0.25	0.002	0.15	6	4	5	81	71	76	0.007	0.015	0.013	0.110	0.128	0.126	15.02	8.52	9.76
0.003	0.30	0.002	0.175	6	4	5	76	71	76	0.009	0.015	0.013	0.130	0.151	0.149	14.43	9.86	11.32
0.003	0.35	0.002	0.2	6	4	5	71	67	71	0.016	0.018	0.017	0.149	0.174	0.171	9.56	9.50	10.16

Table 5.6: **GPS only**, **PAR**, **Geometry Based** model, coordinates known, atmosphere unknown scenario. Presented is the evaluation of ASR and number of epochs for obtaining 2cm fixed-precision of ionosphere - **measurement precision is varied**, number of satellites are as available.

remaining scenarios of 0° and −30° it performed similar to E-W baseline in terms of number of epochs taken for obtaining a fixed-precision of 2cm for ionosphere.

For East-West baseline:

(4) For dual frequency, at −30° latitude with the measurement precision of σ_Φ=3mm on $L1, L2$ and 2mm on $L5$ and σ_P=25cm on $L1, L2$ and 15cm on $L5$, it took 17 epochs by fixing 75% of ambiguities by PAR and for triple frequency, it took 4 epochs by fixing 67% of ambiguities.

For North-South baseline:

(5) For dual frequency, at −30° latitude with the measurement precision of σ_Φ=3mm on $L1, L2$ and 2mm on $L5$ and σ_P=25cm on $L1, L2$ and 15cm on $L5$, it took 14 epochs by fixing 63% of ambiguities and for triple frequency, it took 4 epochs by fixing 71% of ambiguities.

5.6.4 Galileo only - Full and Partial Ambiguity Resolution, atmosphere unknown scenario

Galileo system with four frequencies, namely $E1, E5a, E5b$ and $E6$ is evaluated for atmosphere unknown scenario with coordinates considered to be fixed. Galileo system which promises an improved precision of code for $E5a, E5b, E6$ will be tested in terms of its performance to give better or even instantaneous ambiguity resolution as compared to GPS for Geometry Fixed, atmosphere unknown scenario. The results for ionosphere float Geometry Fixed model for Galileo system of dual, triple and quadruple frequency combinations are presented below. Figures F.10 to F.12 give full ambiguity resolution results and Figures F.13 to F.15 present the partial ambiguity results, see Appendix F. Figure 6.13 presents the number of satellites for E-W oriented baseline (250 Km) and N-S baseline (250 Km) for Galileo system. Table 5.7 presents the full ambiguity resolution results in terms of number of epochs taken for 0.999 ASR and corresponding ionosphere precision (for ambiguity fixed solution and float solution) along with the gain achieved for the fixed solution.

The following is the analysis based on Table 5.7 and Figures 6.13 to 5.8.

General remarks:

(1) It can be noted that with Galileo system, Geometry Fixed model, atmosphere unknown scenario, instantaneous ASR (Full AR) of 0.999 is not achieved.

(2) Galileo only performed better at $-60°$ latitude, excluding the last scenario of measurement precision. At $-60°$ latitude, there was a rising satellite at epoch 25 which caused a perturbation in the stochastic model as seen in Figure 5.7 (see last row). As the measurement precision worsen the time to fix again all the ambiguities is increased, as discussed earlier. The worst measurement precision is hence affected by this perturbation, which causes it eventually to take more than 100 epochs to successfully fix all the ambiguities.

Comparison with GPS only:

				Full AR - minimum desired ASR = 0.999											
Phase	Code	Phase	Code	Epochs taken			$\sigma_{\check{I}}$(meters)			$\sigma_{\hat{I}}$(meters)			Gain=$\sigma_{\hat{I}}/\sigma_{\check{I}}$		
(meters)				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km															
Φ	P	Φ	P	High-end receivers, Dual Frequency											
E1		E5a													
0.002	0.10	0.002	0.05	7	9	4	0.004	0.003	0.003	0.044	0.036	0.047	11.91	10.46	15.23
0.002	0.15	0.002	0.1	19	22	9	0.004	0.003	0.003	0.050	0.044	0.060	14.14	12.98	19.87
0.002	0.20	0.002	0.125	30	35	14	0.004	0.003	0.003	0.051	0.044	0.061	14.33	13.00	20.36
0.003	0.25	0.002	0.15	43	50	21	0.004	0.004	0.003	0.051	0.044	0.060	13.47	12.19	18.72
0.003	0.30	0.002	0.175	59	68	28	0.004	0.004	0.003	0.051	0.044	0.061	13.49	12.08	19.07
0.003	0.35	0.002	0.2	77	88	122	0.004	0.004	0.003	0.051	0.043	0.035	13.49	11.94	10.16
Φ	P	Φ	P	High-end receivers, Triple Frequency											
E1		E5a,E5b													
0.002	0.10	0.002	0.05	7	9	3	0.003	0.003	0.003	0.035	0.029	0.043	11.61	10.19	16.87
0.002	0.15	0.002	0.1	7	9	4	0.003	0.003	0.003	0.067	0.055	0.072	22.26	19.58	28.47
0.002	0.20	0.002	0.125	7	10	4	0.003	0.003	0.003	0.084	0.066	0.091	28.05	23.46	35.86
0.003	0.25	0.002	0.15	13	17	6	0.003	0.003	0.003	0.075	0.061	0.090	23.92	20.80	33.72
0.003	0.30	0.002	0.175	13	17	6	0.003	0.003	0.003	0.087	0.072	0.105	27.99	24.32	39.46
0.003	0.35	0.002	0.2	13	17	6	0.003	0.003	0.003	0.100	0.082	0.120	32.06	27.82	45.19
Φ	P	Φ	P	High-end receivers, Quadruple Frequency											
E1		E5a,E5b,E6													
0.002	0.10	0.002	0.05	6	8	3	0.003	0.002	0.002	0.031	0.025	0.036	12.34	10.66	16.68
0.002	0.15	0.002	0.1	7	9	3	0.003	0.002	0.002	0.056	0.046	0.069	22.12	19.44	32.19
0.002	0.20	0.002	0.125	7	9	4	0.003	0.002	0.002	0.070	0.058	0.076	27.84	24.45	35.52
0.003	0.25	0.002	0.15	12	16	6	0.003	0.002	0.002	0.065	0.052	0.074	24.89	21.47	33.74
0.003	0.30	0.002	0.175	12	16	6	0.003	0.002	0.002	0.076	0.061	0.087	29.11	25.09	39.47
0.003	0.35	0.002	0.2	13	16	6	0.003	0.002	0.002	0.083	0.070	0.100	32.13	28.70	45.18
North-South Baseline - 250Km															
Φ	P	Φ	P	High-end receivers, Dual Frequency											
E1		E5a													
0.002	0.10	0.002	0.05	5	12	4	0.003	0.003	0.003	0.049	0.029	0.048	14.11	9.28	15.24
0.002	0.15	0.002	0.1	12	21	10	0.003	0.003	0.003	0.060	0.042	0.057	17.84	13.46	18.91
0.002	0.20	0.002	0.125	19	29	16	0.003	0.003	0.003	0.061	0.045	0.057	18.16	14.48	19.10
0.003	0.25	0.002	0.15	28	42	23	0.004	0.003	0.003	0.060	0.045	0.058	16.85	13.50	17.92
0.003	0.30	0.002	0.175	37	55	31	0.004	0.003	0.003	0.061	0.045	0.058	17.26	13.68	18.15
0.003	0.35	0.002	0.2	49	71	112	0.004	0.003	0.003	0.061	0.045	0.037	17.21	13.57	10.69
Φ	P	Φ	P	High-end receivers, Triple Frequency											
E1		E5a,E5b													
0.002	0.10	0.002	0.05	5	8	4	0.003	0.003	0.003	0.039	0.029	0.038	13.74	10.92	14.79
0.002	0.15	0.002	0.1	5	10	4	0.003	0.003	0.003	0.075	0.049	0.073	26.30	18.89	28.47
0.002	0.20	0.002	0.125	5	11	4	0.003	0.003	0.003	0.095	0.059	0.092	33.16	22.75	35.86
0.003	0.25	0.002	0.15	9	16	7	0.003	0.003	0.003	0.085	0.059	0.084	28.73	21.70	31.42
0.003	0.30	0.002	0.175	9	17	7	0.003	0.003	0.003	0.100	0.067	0.098	33.63	24.66	36.77
0.003	0.35	0.002	0.2	9	17	7	0.003	0.003	0.003	0.115	0.076	0.112	38.53	28.22	42.11
Φ	P	Φ	P	High-end receivers, Quadruple Frequency											
E1		E5a,E5b,E6													
0.002	0.10	0.002	0.05	4	7	3	0.002	0.002	0.002	0.036	0.025	0.036	15.05	11.51	16.68
0.002	0.15	0.002	0.1	5	9	4	0.002	0.002	0.002	0.063	0.043	0.061	26.13	19.71	28.23
0.002	0.20	0.002	0.125	5	10	4	0.002	0.002	0.002	0.079	0.052	0.076	32.91	23.62	35.52
0.003	0.25	0.002	0.15	8	16	6	0.002	0.002	0.002	0.076	0.049	0.075	30.37	21.79	33.75
0.003	0.30	0.002	0.175	8	16	6	0.002	0.002	0.002	0.088	0.057	0.088	35.53	25.46	39.47
0.003	0.35	0.002	0.2	8	16	6	0.002	0.002	0.002	0.101	0.066	0.101	40.69	29.12	45.19

Table 5.7: **Galileo only**, **Geometry Based** model, coordinates known, atmosphere unknown scenario, **Full AR** analysis, ASR and ambiguity-fixed and -float precision for ionosphere are presented - **measurement precision varied**, number of satellites as available.

(3) Galileo only out performed GPS only for dual and triple frequency. For dual frequency GPS took between 9 to 88 and 9 to 106 epochs for E-W and N-S baselines, whereas Galileo only took between 4 to 122 and 4 to 112 epochs (the last scenario of measurement precision is a special case as discussed earlier). Triple frequency GPS took between 5 to 20 and 5 to 19 for E-W and N-S oriented baselines whereas Galileo only took 3 to 17 and 4 to 17 epochs for similar scenario.

(4) While GPS only gave the best results at $0°$ and $-30°$ latitude, this is basically due to availability of large number of high elevation satellites and longer satellite tracks, on the other hand, Galileo only gave better results at $-60°$ latitude (excluding the worst value of measurement precision, that is the last scenario of measurement precision). While worst performance is evaluated, GPS only performed poor at $-60°$ latitude whereas Galileo only performed poor at $-30°$ latitude, for the time period under which simulation was performed.

(5) In general the quadruple Galileo outperformed triple frequency Galileo for some scenarios, hence the quadruple frequency Galileo outperforms GPS too. The reason for better performance of a GNSS system as the number of frequencies are increased is related to an increase in the ionosphere information resulting in strengthening of GNSS model, as discussed earlier.

For East-West baseline:

(6) For dual frequency with $E1, E5a$ frequencies, triple with $E1, E5a, E5b$ and quadruple with $E1, E5a, E5b, E6$ frequencies, having the undifferenced measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b, E6$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b, E6$. It took minimum of 50 epochs for dual frequency, whereas for triple and quadruple frequency, a minimum of 17 and 16 epochs were needed for 0.999 ASR (Full AR).

For North-South baseline:

(7) For dual, triple and quadruple frequency, at $-30°$ latitude, a minimum of 42, 16 and 16 epochs were needed for 0.999 ASR (Full AR).

Considering the fixed-precision of ionosphere to lie around 3mm in most of the cases when all the ambiguities are fixed, PAR will be evaluated for obtaining a fixed-precision of 2cm for ionosphere in the coming section.

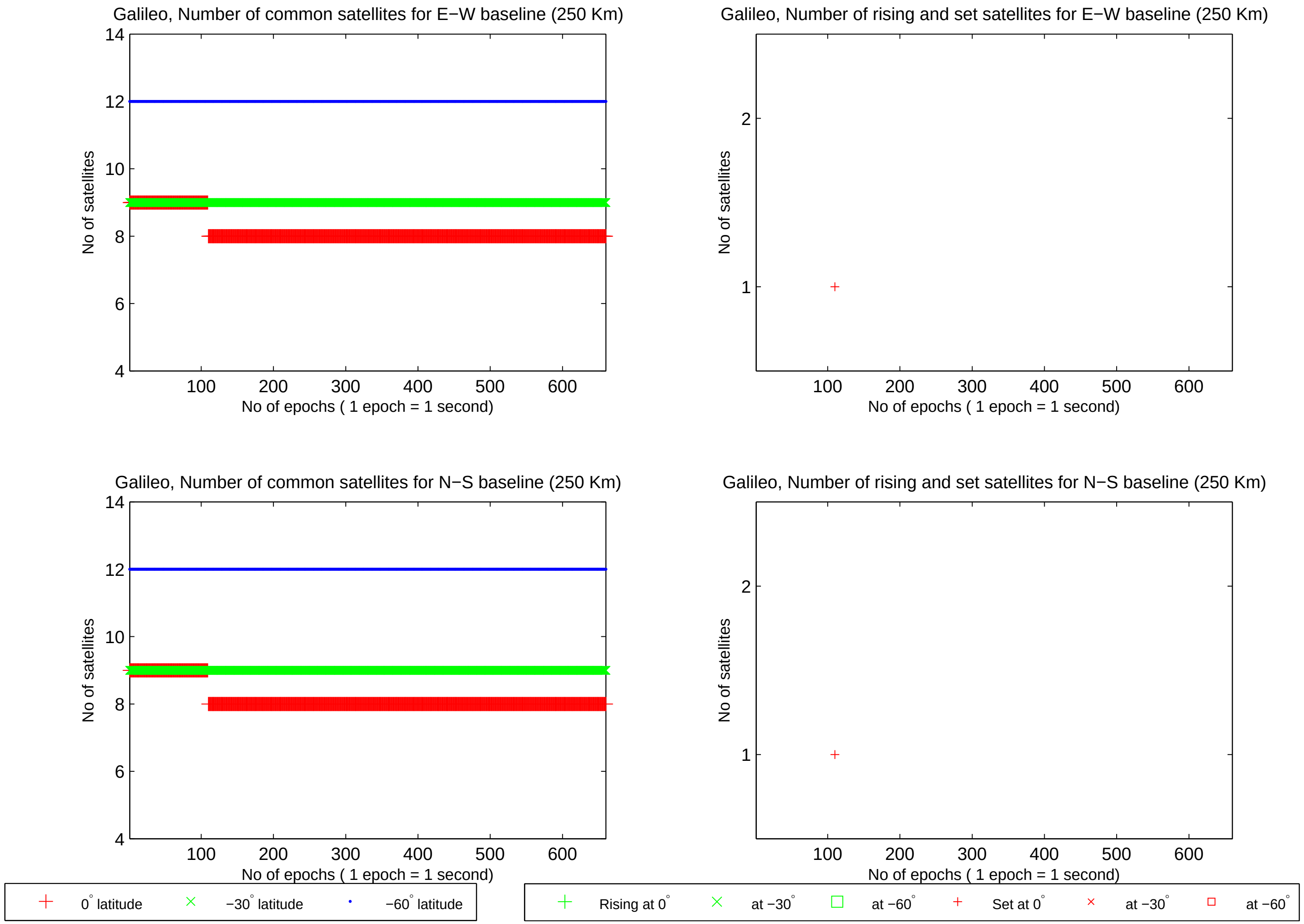

Figure 5.6: Number of satellites for **Galileo only**, 250 Km East-West and North-South baseline at $0°$, $-30°$ and $-60°$ degree latitude.

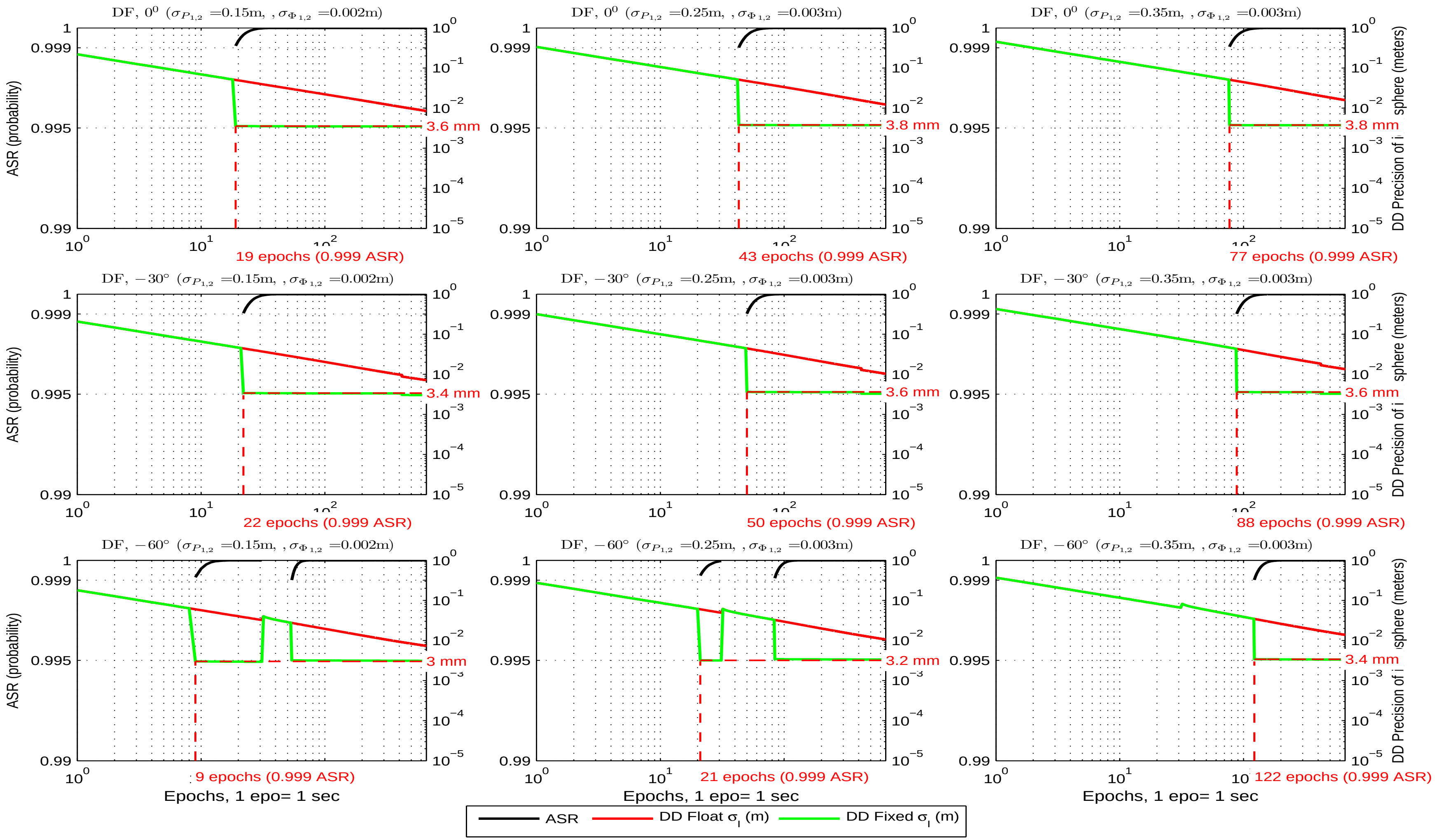

Figure 5.7: **Galileo only**, Dual frequency, **Full AR**, **Geometry Based** model, coordinates known, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

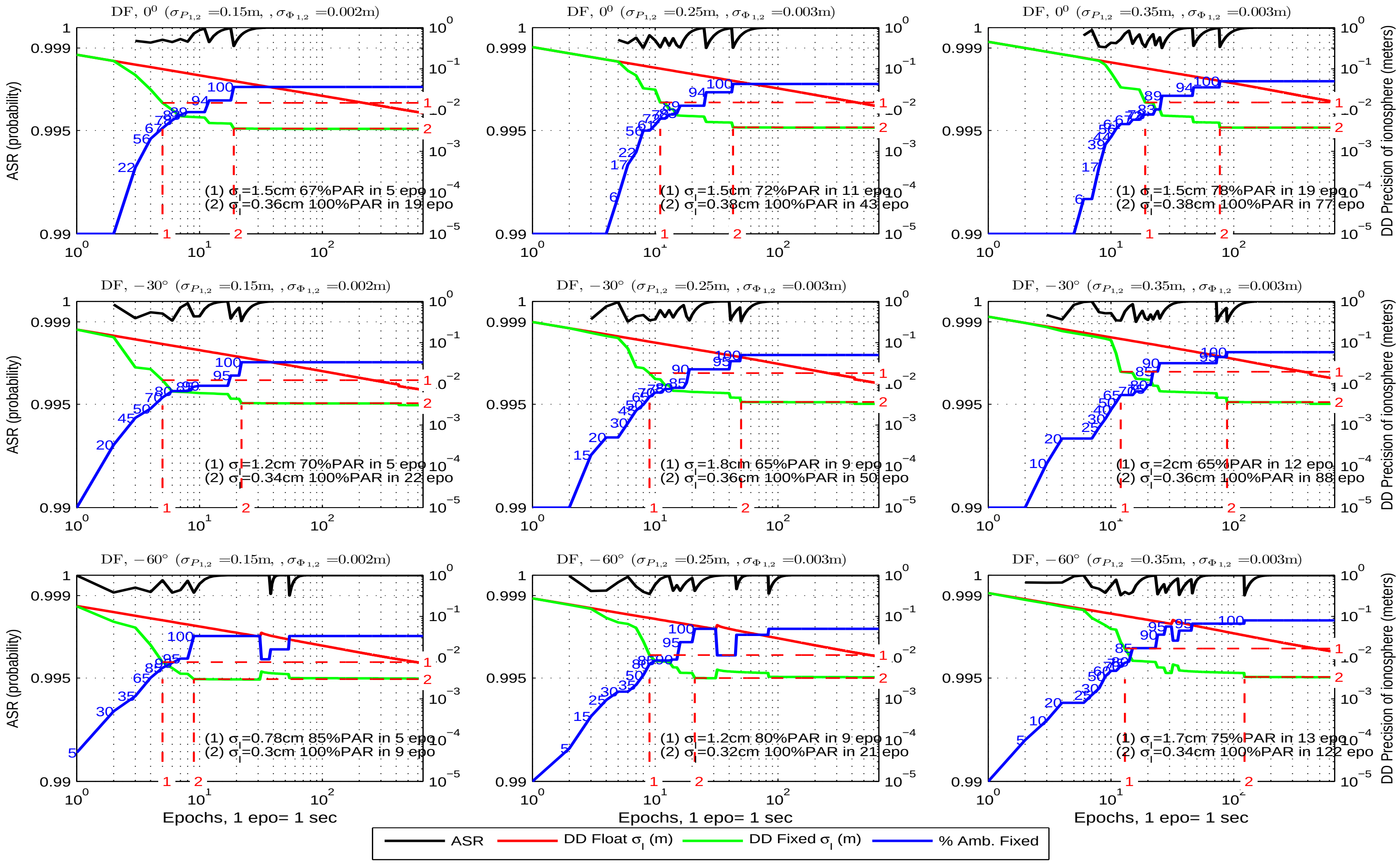

Figure 5.8: **Galileo only**, Dual frequency, **Partial AR**, **Geometry Based** model, coordinates known, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

5.6.5 Galileo only, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, atmosphere unknown scenario

In this section, it is to be analyzed if partially fixing the ambiguities help to converge to a solution faster when a modest fixed-precision of 2cm for ionosphere is aimed for. Table 5.8 presents the results in terms of fixed-precision to be equal to or better than 2cm and corresponding epochs taken and ambiguities fixed are noted.

The following is the analysis based on Table 5.8.

General remarks:

(1) Results with Galileo only to obtain a fixed-precision of 2cm for ionosphere proved to be promising as compared to GPS only. fixed-precision of 2cm could be achieved instantaneously for 1 scenarios of triple, and 3 scenarios of quadruple frequency for East-West baseline with Galileo system. For North-South baseline, similar results were obtained.

(2) The partial ambiguities of less than 100% were fixed for all the scenarios of dual, triple quadruple frequencies each to obtain a fixed-precision of 2cm for ionosphere.

Comparison with GPS only:

(3) While with GPS, instantaneous fixed-precision of 2cm could not be achieved, with Galileo similar instantaneous fixed-precision of 2cm could be achieved for 1 and 3 scenarios of triple and quadruple frequency. For N-S baseline too Galileo outperformed GPS.

For East-West baseline:

(4) At $-30°$ latitude with the measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b, E6$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b, E6$, with dual, triple and quadruple frequencies PAR fixed 65, 70 and 80% of ambiguities by taking 9, 3 and 2 epochs respectively to reach a

				Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for ionosphere														
Phase	Code	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{I}}$(meters)			$\sigma_{\hat{I}}$(meters)			Gain=$\sigma_{\hat{I}}/\sigma_{\check{I}}$		
(meters)				$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$
East-West Baseline - 250Km																		
Φ E1	P	Φ E5a	P	High-end receivers, Dual Frequency														
0.002	0.10	0.002	0.05	3	2	2	72	60	60	0.008	0.014	0.013	0.067	0.077	0.067	7.94	5.34	5.12
0.002	0.15	0.002	0.1	5	5	4	67	70	65	0.015	0.012	0.020	0.098	0.093	0.090	6.59	7.54	4.45
0.002	0.20	0.002	0.125	8	6	6	78	65	85	0.014	0.018	0.008	0.098	0.107	0.093	6.79	6.01	11.52
0.003	0.25	0.002	0.15	11	9	9	72	65	80	0.015	0.018	0.012	0.101	0.105	0.092	6.59	5.77	7.90
0.003	0.30	0.002	0.175	15	10	11	72	65	80	0.015	0.019	0.012	0.102	0.117	0.097	6.65	6.10	8.22
0.003	0.35	0.002	0.2	19	12	13	78	65	75	0.015	0.020	0.017	0.103	0.123	0.102	6.78	6.26	6.13
Φ E1	P	Φ E5a,E5b	P	High-end receivers, Triple Frequency														
0.002	0.10	0.002	0.05	2	1	2	74	73	87	0.014	0.014	0.006	0.065	0.086	0.053	4.52	6.07	9.45
0.002	0.15	0.002	0.1	3	2	2	89	73	83	0.006	0.012	0.007	0.102	0.118	0.102	18.25	9.52	13.67
0.002	0.20	0.002	0.125	3	2	2	89	70	83	0.006	0.014	0.008	0.129	0.148	0.129	22.96	10.76	17.14
0.003	0.25	0.002	0.15	4	3	3	74	70	80	0.014	0.014	0.010	0.134	0.146	0.127	9.41	10.37	13.09
0.003	0.30	0.002	0.175	4	3	3	70	67	80	0.018	0.016	0.010	0.157	0.171	0.148	8.94	10.51	15.28
0.003	0.35	0.002	0.2	4	3	3	70	67	80	0.018	0.017	0.010	0.180	0.196	0.170	10.09	11.71	17.47
Φ E1	P	Φ E5a,E5b,E6	P	High-end receivers, Quadruple Frequency														
0.002	0.10	0.002	0.05	2	1	1	86	80	80	0.008	0.013	0.015	0.054	0.071	0.062	6.71	5.70	4.08
0.002	0.15	0.002	0.1	2	1	2	78	78	88	0.016	0.018	0.007	0.104	0.138	0.085	6.63	7.87	12.83
0.002	0.20	0.002	0.125	2	2	2	78	80	88	0.016	0.011	0.007	0.131	0.123	0.107	8.24	10.92	16.05
0.003	0.25	0.002	0.15	4	2	2	83	80	78	0.011	0.013	0.020	0.112	0.149	0.129	10.44	11.52	6.59
0.003	0.30	0.002	0.175	4	2	2	83	80	78	0.011	0.013	0.020	0.131	0.174	0.151	12.18	13.39	7.67
0.003	0.35	0.002	0.2	4	2	2	83	80	78	0.011	0.013	0.020	0.150	0.199	0.173	13.91	15.27	8.75
North-South Baseline - 250Km																		
Φ E1	P	Φ E5a	P	High-end receivers, Dual Frequency														
0.002	0.10	0.002	0.05	3	2	2	63	67	60	0.013	0.011	0.013	0.063	0.072	0.067	4.95	6.27	5.15
0.002	0.15	0.002	0.1	6	3	4	81	56	75	0.011	0.020	0.011	0.085	0.111	0.091	8.02	5.51	7.99
0.002	0.20	0.002	0.125	7	5	6	75	67	85	0.017	0.019	0.008	0.100	0.109	0.094	6.01	5.70	11.51
0.003	0.25	0.002	0.15	12	7	9	75	67	80	0.016	0.020	0.012	0.092	0.111	0.092	5.59	5.51	7.90
0.003	0.30	0.002	0.175	13	10	11	75	67	80	0.017	0.020	0.012	0.104	0.109	0.098	5.95	5.55	8.22
0.003	0.35	0.002	0.2	16	12	13	75	67	75	0.018	0.020	0.017	0.107	0.114	0.103	6.08	5.74	6.13
Φ E1	P	Φ E5a,E5b	P	High-end receivers, Triple Frequency														
0.002	0.10	0.002	0.05	2	1	2	71	78	87	0.017	0.011	0.006	0.062	0.080	0.053	3.59	7.06	9.46
0.002	0.15	0.002	0.1	2	2	2	71	78	83	0.018	0.010	0.007	0.119	0.110	0.103	6.52	10.99	13.74
0.002	0.20	0.002	0.125	2	2	2	71	74	83	0.018	0.011	0.008	0.150	0.138	0.130	8.15	12.33	17.23
0.003	0.25	0.002	0.15	3	2	3	71	74	80	0.019	0.013	0.010	0.148	0.167	0.128	7.65	12.82	13.28
0.003	0.30	0.002	0.175	3	2	3	71	74	77	0.019	0.013	0.012	0.173	0.195	0.150	8.92	14.71	12.79
0.003	0.35	0.002	0.2	4	2	3	71	74	77	0.017	0.014	0.012	0.172	0.224	0.171	9.98	16.57	14.62
Φ E1	P	Φ E5a,E5b,E6	P	High-end receivers, Quadruple Frequency														
0.002	0.10	0.002	0.05	2	1	1	84	83	78	0.009	0.010	0.019	0.051	0.067	0.062	5.92	6.66	3.24
0.002	0.15	0.002	0.1	2	1	2	78	83	88	0.017	0.011	0.007	0.099	0.129	0.086	5.86	11.96	12.90
0.002	0.20	0.002	0.125	2	2	2	78	86	88	0.017	0.007	0.007	0.125	0.115	0.108	7.30	15.96	16.13
0.003	0.25	0.002	0.15	3	2	2	78	83	78	0.018	0.010	0.020	0.123	0.139	0.130	6.76	13.42	6.59
0.003	0.30	0.002	0.175	3	2	2	78	83	78	0.018	0.010	0.020	0.144	0.163	0.152	7.87	15.62	7.67
0.003	0.35	0.002	0.2	3	2	2	78	83	78	0.018	0.010	0.020	0.165	0.186	0.174	8.98	17.82	8.75

Table 5.8: **Galileo only**, **PAR** analysis, **Geometry Based** model, coordinates known, atmosphere unknown scenario. Evaluation of ASR and number of epochs for obtaining 2cm fixed-precision of ionosphere - **measurement precision is varied**, number of satellites are as available.

fixed-precision of 2m or better for the ionosphere.

For North-South baseline:

(5) At $-30°$ latitude with the measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b, E6$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b, E6$, for dual frequency, it took 7 epochs by fixing 67% of ambiguities. For triple and quadruple frequencies, PAR fixed only 74% and 83% of the ambiguities by taking 2 epoch for each triple and quadruple frequency Galileo.

5.6.6 GPS + Galileo - Full and Partial Ambiguity Resolution, atmosphere unknown scenario

The ASR for the combined GPS and Galileo system are simulated for Geometry Based model, coordinates known, atmosphere unknown scenario, with coordinates assumed to be known, hence fixed is simulated along with the fixed-precision of the ionosphere and other unknowns. The atmosphere, both ionosphere and troposphere are assumed to be unknown and are parameterized in the design matrix of the functional model. Table 5.9 and Figures F.17 to F.20 in Appendix F present the results full and partial AR for dual and quadruple frequency, for combined GPS and Galileo for 250 Km baseline in East-West direction and Figures F.21 and F.24 present results for dual and quadruple frequency system for the North-South baseline, see Appendix F. Figure 6.16 presents the number of satellites for E-W oriented baseline (250 Km) and N-S baseline (250 Km) for the combined GPS+Galileo system.

Analysis based on Figures 6.16 and Table 5.9 is given below.

General remarks:

(1) Instantaneous ambiguity resolution could not be achieved for any scenario of measurement precision for a combined GPS and Galileo system, using dual and quadruple frequency both.

Comparison with GPS only and Galileo only systems:

(2) Instantaneous ASR of 0.999 could not be achieved with both GPS and Galileo standalone systems for any scenario of measurement precision. Nonetheless it could be noted that Galileo only outperforms GPS only for number of epochs taken for 0.999 ASR.

(3) While dual frequency systems are compared, namely, GPS only, Galileo only and GPS with Galileo together, the combined GPS and Galileo system does not perform extraordinarily, in fact it performs just as GPS only or Galileo only. For example, with dual frequency systems, GPS only at $-30°$ latitude took 3,3,4,5,9,10 epochs for 6 scenarios of measurement precision, Galileo only took 2,4,6,9,12,16 epochs and a combined GPS

				Full AR - minimum desired ASR = 0.999											
Phase	Code	Phase	Code	Epochs taken			$\sigma_{\check{I}}$(meters)			$\sigma_{\hat{I}}$(meters)			Gain=$\sigma_{\hat{I}}/\sigma_{\check{I}}$		
	(meters)			0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km															
Φ	P	Φ	P	High-end receivers, Dual Frequency											
L1(E1)		L5(E5a)													
0.002	0.10	0.002	0.05	7	8	8	0.003	0.003	0.003	0.035	0.032	0.033	11.21	10.44	10.41
0.002	0.15	0.002	0.1	19	22	25	0.003	0.003	0.003	0.041	0.037	0.036	13.26	12.25	11.46
0.002	0.20	0.002	0.125	30	35	67	0.003	0.003	0.003	0.041	0.037	0.029	13.37	12.25	8.92
0.003	0.25	0.002	0.15	43	50	84	0.003	0.003	0.003	0.041	0.037	0.031	12.55	11.50	8.90
0.003	0.30	0.002	0.175	58	68	102	0.003	0.003	0.003	0.041	0.037	0.033	12.64	11.47	9.35
0.003	0.35	0.002	0.2	76	88	125	0.003	0.003	0.003	0.041	0.037	0.033	12.61	11.44	9.55
Φ	P	Φ	P	High-end receivers, Quadruple Frequency											
L1(E1),L2,E5b		L5(E5a)													
0.002	0.10	0.002	0.05	5	5	7	0.002	0.002	0.002	0.026	0.025	0.022	12.36	12.30	10.47
0.002	0.15	0.002	0.1	5	6	8	0.002	0.002	0.002	0.050	0.045	0.041	24.20	22.05	19.19
0.002	0.20	0.002	0.125	6	7	9	0.002	0.002	0.002	0.058	0.052	0.048	27.82	25.68	22.75
0.003	0.25	0.002	0.15	9	10	14	0.002	0.002	0.002	0.057	0.052	0.047	26.47	25.01	21.24
0.003	0.30	0.002	0.175	9	10	14	0.002	0.002	0.002	0.066	0.061	0.054	30.93	29.22	24.82
0.003	0.35	0.002	0.2	9	11	14	0.002	0.002	0.002	0.076	0.067	0.062	35.39	31.91	28.39
North-South Baseline - 250Km															
Φ	P	Φ	P	High-end receivers, Dual Frequency											
L1(E1)		L5(E5a)													
0.002	0.10	0.002	0.05	5	6	7	0.003	0.003	0.003	0.040	0.035	0.032	13.21	12.02	11.17
0.002	0.15	0.002	0.1	13	18	18	0.003	0.003	0.003	0.047	0.039	0.039	16.01	13.56	13.55
0.002	0.20	0.002	0.125	20	28	29	0.003	0.003	0.003	0.048	0.039	0.038	16.37	13.74	13.49
0.003	0.25	0.002	0.15	29	40	85	0.003	0.003	0.003	0.048	0.039	0.030	15.29	12.91	8.96
0.003	0.30	0.002	0.175	39	54	103	0.003	0.003	0.003	0.049	0.039	0.031	15.46	12.96	9.38
0.003	0.35	0.002	0.2	51	70	123	0.003	0.003	0.003	0.049	0.039	0.032	15.48	12.94	9.69
Φ	P	Φ	P	High-end receivers, Quadruple Frequency											
L1(E1),L2,E5b		L5(E5a)													
0.002	0.10	0.002	0.05	3	5	5	0.002	0.002	0.002	0.032	0.024	0.024	15.86	12.35	12.32
0.002	0.15	0.002	0.1	4	5	6	0.002	0.002	0.002	0.055	0.047	0.042	27.05	24.17	22.09
0.002	0.20	0.002	0.125	4	5	6	0.002	0.002	0.002	0.069	0.059	0.053	33.98	30.35	27.73
0.003	0.25	0.002	0.15	7	8	10	0.002	0.002	0.002	0.063	0.056	0.049	30.00	27.97	25.08
0.003	0.30	0.002	0.175	7	9	11	0.002	0.002	0.002	0.074	0.062	0.055	35.06	30.87	27.97
0.003	0.35	0.002	0.2	7	9	11	0.002	0.002	0.002	0.084	0.071	0.063	40.12	35.32	32.00

Table 5.9: **GPS + Galileo**, **Geometry Based** model, coordinates known, atmosphere unknown scenario, **Full AR** analysis, ASR and ambiguity-fixed and -float precision for ionosphere are presented - **measurement precision varied**, number of satellites as available.

and Galileo system took 2,4,6,9,12,16 epochs. The explanation for such a behaviour is related to the unknowns in the model, especially the ionosphere. The GNSS ionosphere float model, with respect to ionosphere is not affected by increasing the number of satellites, since an equal number of ambiguities, ionosphere etcetera is added to the model, keeping the overall strength unchanged.

(4) A comparison for quadruple frequency between Galileo only and GPS, Galileo combined system shows that the combined system outperforms Galileo only for the N-S baseline length. At $0°$ and $-60°$ latitude for N-S baseline, Galileo only takes 2,2,2,4,4,4 epochs for 6 scenarios for each latitude, whereas the combined GPS, Galileo system takes 2,2,2,3,3,3 epochs only for each latitude location.

For East-West baseline:

(5) At $-30°$ latitude, it took 9 epochs for dual frequency measurement precision of 3mm on phase and 25cm on code on $L1(E1)$ and 2mm and 15cm on phase and code on $L5(E5a)$. For quadruple frequency $(L1(E1), L2, E5b$ and $L5(E5a))$, at $-30°$ latitude it took 3 epochs.

For North-South baseline:

(6) At $-30°$ latitude, it took 9 epochs for dual frequency measurement precision of 3mm on phase and 25cm on code on $L1(E1)$ and 2mm and 15cm on phase and code on $L5(E5a)$. For quadruple frequency $(L1(E1), L2, E5b$ and $L5(E5a))$, at $-30°$ latitude it took 3 epochs.

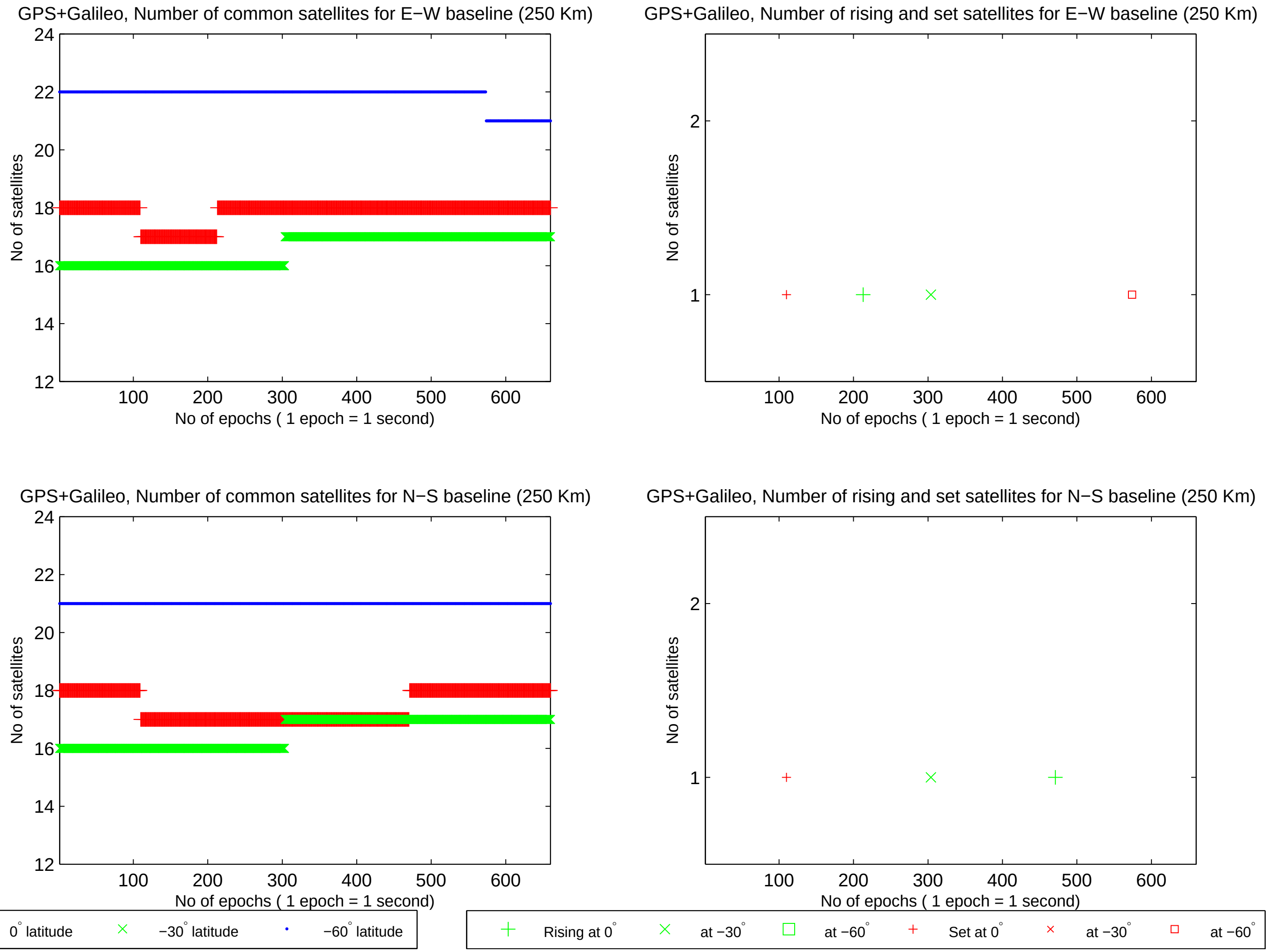

Figure 5.9: Number of satellites for **GPS + Galileo**, 250 Km East-West and North-South baseline at $0°, -30°$ and $-60°$ degree latitude.

5.6.7 GPS + Galileo, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, atmosphere unknown scenario

While resolving all the ambiguities, a fixed-precision of 2mm and better was obtained. In this analysis it is to examine if a fixed-precision of 2cm is obtained in reduced number of epochs and by partial fixing of ambiguities. The results are presented in Table 5.10, see below.

Analysis based on Table 5.8 is given below.

General remarks:

(1) For all the scenarios of measurement precision, less than 100% of ambiguities were fixed in order to obtain a fixed-precision of 2cm for ionosphere.

(2) The fixed-precision of 2cm for ionosphere could be achieved instantaneously for 9 scenarios of measurement precision for quadruple frequency for the E-W and N-S baseline both.

Comparison with GPS only and Galileo only:

(3) GPS only was not able to give instantaneous fixed-precision of 2cm any scenario of measurement precision with dual and triple frequency, Galileo only gave instantaneous fixed-precision of 2cm for 1 and 4 scenarios of measurement precision of triple and quadruple frequency respectively. With GPS and Galileo together, instantaneous 2cm fixed-precision could be achieved for 9 scenarios of measurement precision with quadruple frequency for E-W baseline and N-S baseline both.

For East-West baseline:

(4) At $-30°$ latitude, for dual frequency (measurement precision of 3mm on phase and 25cm on code on $L1, E1$ and 2mm and 15cm on phase and code on $L5, E5a$) and quadruple frequency (similar measurement precision), it took 6 and 2 epochs by fixing 66 and 77% of ambiguities, for dual an quadruple respectively, in order to give a fixed-precision of 2cm or better.

Phase	Code	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{I}}$(meters)			$\sigma_{\hat{I}}$(meters)			Gain=$\sigma_{\hat{I}}/\sigma_{\check{I}}$		
				Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for ionosphere														
(meters)				$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$
East-West Baseline - 250Km																		
Φ	P	Φ	P	High-end receivers, Dual Frequency														
L1(E1)		L5(E5a)																
0.002	0.10	0.002	0.05	2	2	2	62	79	75	0.011	0.006	0.007	0.065	0.063	0.065	6.18	10.83	9.62
0.002	0.15	0.002	0.1	3	3	4	65	68	75	0.019	0.015	0.010	0.102	0.099	0.089	5.43	6.47	8.93
0.002	0.20	0.002	0.125	4	5	5	65	68	68	0.020	0.015	0.017	0.112	0.097	0.101	5.69	6.44	5.86
0.003	0.25	0.002	0.15	6	6	8	65	66	70	0.020	0.020	0.015	0.110	0.107	0.096	5.47	5.46	6.50
0.003	0.30	0.002	0.175	8	8	10	68	66	68	0.020	0.020	0.018	0.112	0.108	0.101	5.53	5.51	5.67
0.003	0.35	0.002	0.2	11	11	14	71	68	73	0.020	0.019	0.015	0.109	0.106	0.097	5.49	5.48	6.61
Φ	P	Φ	P	High-end receivers, Quadruple Frequency														
L1(E1),L2,E5b		L5(E5a)																
0.002	0.10	0.002	0.05	1	1	1	78	79	83	0.007	0.006	0.006	0.057	0.056	0.059	8.13	9.90	10.43
0.002	0.15	0.002	0.1	1	1	1	68	73	73	0.011	0.008	0.009	0.113	0.110	0.115	9.89	13.24	12.71
0.002	0.20	0.002	0.125	1	1	1	64	68	66	0.014	0.013	0.012	0.141	0.138	0.145	10.40	10.35	11.82
0.003	0.25	0.002	0.15	2	2	2	72	77	80	0.009	0.007	0.007	0.120	0.117	0.123	12.90	17.50	17.70
0.003	0.30	0.002	0.175	2	2	2	70	77	78	0.010	0.007	0.007	0.141	0.137	0.144	14.23	20.01	19.24
0.003	0.35	0.002	0.2	2	2	2	70	75	73	0.010	0.008	0.009	0.161	0.156	0.165	15.94	19.85	19.37
North-South Baseline - 250Km																		
Φ	P	Φ	P	High-end receivers, Dual Frequency														
L1(E1)		L5(E5a)																
0.002	0.10	0.002	0.05	2	2	2	63	83	83	0.010	0.005	0.005	0.063	0.060	0.060	6.38	12.37	12.37
0.002	0.15	0.002	0.1	3	3	3	69	72	67	0.016	0.013	0.019	0.098	0.094	0.095	6.05	7.19	4.89
0.002	0.20	0.002	0.125	4	4	5	69	67	72	0.017	0.020	0.016	0.108	0.103	0.093	6.35	5.16	5.91
0.003	0.25	0.002	0.15	7	7	7	69	72	69	0.017	0.014	0.016	0.098	0.094	0.094	5.86	6.95	5.74
0.003	0.30	0.002	0.175	8	8	10	69	69	75	0.018	0.017	0.013	0.108	0.103	0.093	6.15	6.06	6.88
0.003	0.35	0.002	0.2	11	11	12	75	72	69	0.017	0.017	0.020	0.105	0.101	0.097	6.11	6.02	4.87
Φ	P	Φ	P	High-end receivers, Quadruple Frequency														
L1(E1),L2,E5b		L5(E5a)																
0.002	0.10	0.002	0.05	1	1	1	83	83	89	0.006	0.005	0.004	0.056	0.054	0.053	9.31	11.27	13.27
0.002	0.15	0.002	0.1	1	1	1	70	77	77	0.010	0.007	0.007	0.110	0.105	0.104	10.75	15.32	14.79
0.002	0.20	0.002	0.125	1	1	1	68	74	72	0.011	0.009	0.010	0.138	0.132	0.130	12.08	14.40	13.44
0.003	0.25	0.002	0.15	2	2	2	77	79	85	0.008	0.006	0.005	0.118	0.113	0.111	15.20	18.03	22.62
0.003	0.30	0.002	0.175	2	2	2	74	79	83	0.008	0.006	0.005	0.138	0.132	0.129	16.88	20.77	24.50
0.003	0.35	0.002	0.2	2	2	2	72	77	81	0.010	0.007	0.006	0.157	0.151	0.148	16.48	21.89	26.74

Table 5.10: **GPS + Galileo**, **PAR** analysis, **Geometry Based** model, coordinates known, atmosphere unknown scenario. Evaluation of ASR and number of epochs for obtaining 2cm fixed-precision of ionosphere - **measurement precision is varied**, number of satellites are as available.

For North-South baseline:

(5) At $-30°$ latitude, for dual frequency (measurement precision of 3mm on phase and 25cm on code on $L1, E1$ and 2mm and 15cm on phase and code on $L5, E5a$) and quadruple frequency, it took 7 and 2 epochs by PAR and 72 and 79% of ambiguities were fixed in order to give a fixed-precision of 2cm or better.

5.7 Receiver coordinates known, ionosphere weighted scenario

When the atmosphere is significant, there are additional unknowns in the model, namely the ionosphere and the troposphere which make the GNSS model weak. This results the process of ambiguity resolution to take longer convergence times which can make precise near-real time applications impossible. Among the atmosphere, the ionosphere is the dominant error source, with a-prior information of the ionosphere becoming available through ionosphere observations, the time to correctly fix the integer ambiguities can be reduced significantly. Ionosphere can be eliminated from the GNSS observables only if it is known with a good accuracy, otherwise imprecise ionosphere observations can affect the precision of the other unknowns. A better approach would consist of estimating the ionosphere, as well as using its a-priori information and using additional ionosphere observations, in this case, ionosphere weight is introduced in the stochastic model (as a function of baseline length), which provides additional constraint, hence a stronger model. The troposphere can still be considered unknown. As per the current assumption, the coordinates are considered to be known with high accuracy, they are assumed to be fixed. The baseline length is varied between 1 and 1000 Kilometres. Satellite coordinates are assumed to be known and are generated from YUMA almanacs. Presented below is the functional and stochastic models, see Table 5.11. The redundancy for Geometry Fixed, atmosphere known scenario is presented in Table 5.11 and Figure 5.10, see below.

where CI is given by equation (5.13), all the variables presented in Table 5.11 have been explained in the previous section.

Geometry based model, coordinates known, ionosphere weighted scenario		
Functional model		**Stochastic model**
Non-temporal parameters	Temporal parameters	
$A_{I(i)} = \begin{bmatrix} \begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes I_{m-1} & \begin{pmatrix} e_f \\ e_f \end{pmatrix} \otimes G_{m-1} \end{bmatrix}$	$A_{II(i)} = \begin{pmatrix} -\mu_f \\ \mu_f \end{pmatrix} \otimes I_{m-1}$	$Q_{y(i)} = \begin{bmatrix} Q_\Phi & \\ & Q_P \end{bmatrix} \otimes Q_{DD}(i)$ where $Q_{DD}(i) = (D_m^T W_i^{-1} D_m)$ $Q_I(i) = CI \otimes Q_{DD}(i)$ $Q_{y,I}(i) = Q_y(i) + Q_I(i)$
Redundancy (for GPS only, Galileo only, GPS+Galileo (common frequency $L1(E1), L5(E5a)$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $f * (m-1)$ Troposphere: $= 1$	Ionosphere: $k * (m-1)$	$k * 2f * (m-1)$ Pseudo observations (Ionosphere) $= k * (m-1)$
Redundancy of coordinates known, ionosphere weighted scenario		$[(2k-1) * f * (m-1)] - 1$
For GPS/Galileo only: $f = (1, \cdots, j)$, for GPS+Galileo: $f = (1, \cdots, j_c)$, j_c is the common/overlapping frequency and $m_{jc} = (m_{GPS} + m_{Gal})$		
Redundancy (for GPS+Galileo (quadruple frequency $L1(E1), L5(E5a), L2, E5b$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)$ Troposphere: $= 1$	Ionosphere$=k * (m_{jc}-1)$	$k*2*(2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1))$ Pseudo observations (Ionosphere) $=k * (m_{jc}-1)$
Redundancy of coordinates known, ionosphere weighted scenario		$(2k-1)[2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)]-1$
$f = 1, 2$ for GPS+Gaileo $L1(E1), L5(E5a)$, m_{jc} are the total number of satellites for combined GPS+Galileo system with GPS as reference satellite		

Table 5.11: Double-differenced Design matrix and VC matrix for Geometry based model, coordinates known, ionosphere weighted, troposphere float scenario

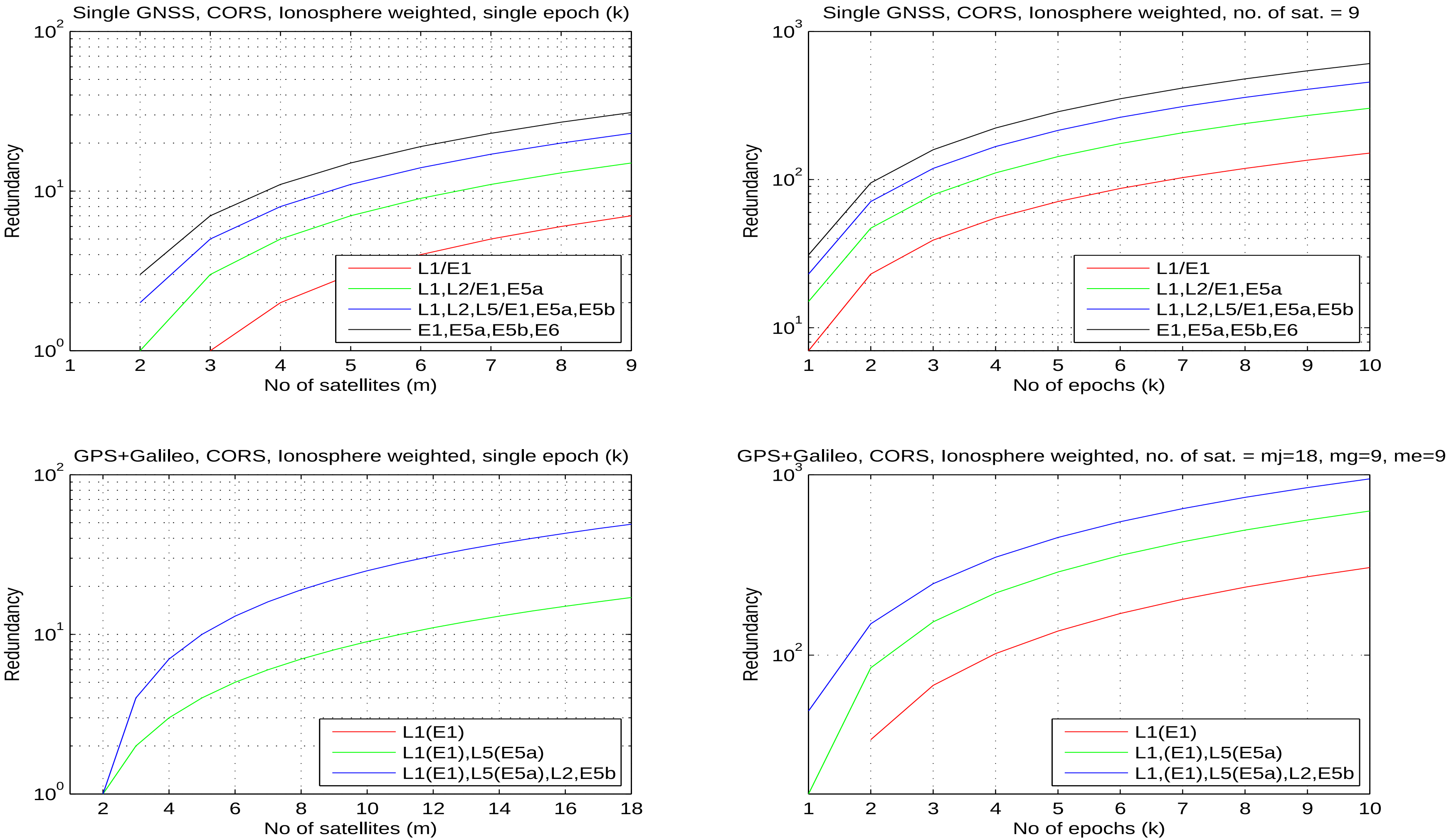

Figure 5.10: Redundancy plot of **GPS only**, **Galileo only** (top two plots) and **GPS + Galileo** (bottom two plots) for single epoch (left hand side plots) and multi-epoch (right hand side plots), **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario single, dual, triple and quadruple frequencies

5.7.1 Hourly batches, Full Ambiguity Resolution, ionosphere weighted scenario

Initially, ASR is simulated for hourly batches, in all 24 batches throughout the day are considered for simulation. The interval between the epochs is taken as 1 second. It is to see the average behaviour for achieving 0.999 ASR during different times of the day. The results for single, dual, triple and quadruple (Galileo only and GPS+Galileo) frequency, for GPS only, Galileo only and with both systems together is presented in the Figures F.26 to F.34, see Appendix F. The results for average number of epochs taken for each of the GNSS system are also presented in Table 5.12, see below.

The analysis based on Figures F.26 to F.31 and Table 5.12 is presented below.

General remarks

1. The number of epochs taken for 0.999 ASR increase as baseline length is increased. However, it should be noted that, for baseline lengths larger than 250 Km, the ionosphere weighted model is weak enough and is as good as ionosphere float model. The baseline length for which ionosphere weighted scenario becomes equivalent to ionosphere float scenario highly depends upon the strength of a GNSS model. The strength increases with increase in number of frequencies and also with improvement in measurement precision.

2. For larger baseline lengths, it can be seen that the number of epochs taken for 0.999 ASR decrease as the baseline length increases. For example, for Galileo system, triple and quadruple frequency case the number of epochs reduce for 0.999 ASR by one or more as baseline length increases. This is only due to the satellite geometry affecting the stochastic model. The stochastic model is influenced by the elevation dependent weighting function, which is used throughout this research. It can be seen that for 1000 Km baseline length, the total number of common satellites and the low elevation satellites reduce by one.

3. It can be noted that single frequency GNSS systems could not give 0.999 ASR for 1000 Km baseline length within the stipulated 3600 epochs. Henceforth for single frequency, 1000 Km baseline length will be excluded from further discussions in this section.

Baseline length (Km)	Hourly, Full AR - minimum desired ASR = 0.999 Average number of Epochs over 24 hourly batches																	
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
	Single Frequency			Dual Frequency			Triple Frequency			Quadruple Frequency			Average no. of satellites			Average no. of Low-elevation Sat.$\geqslant 10°, \leqslant 30°$		
East-West Baseline																		
GPS only																		
	$L1$			$L1, L2$			$L1, L2, L5$											
1	1	1	1	1	1	1	1	1	1	-	-	-	10	9	9	5	3	5
10	3	4	3	3	3	3	3	3	3	-	-	-	10	9	9	5	3	5
100	170	184	195	16	20	20	11	13	12	-	-	-	10	9	9	5	3	5
250	959	874	1113	24	27	30	11	13	13	-	-	-	9	8	9	5	3	5
500	2364	2182	2514	29	32	38	11	13	14	-	-	-	9	8	9	5	3	5
1000	> 3600	> 3600	> 3600	31	33	40	11	13	14	-	-	-	9	8	9	4	3	5
Galileo only																		
	$E1$			$E1, E5$			$E1, E5a, E5b$			$E1, E5a, E5b, E6$								
1	1	1	1	1	1	1	1	1	1	1	1	1	10	9	10	5	4	5
10	3	4	3	3	3	3	2	3	3	2	3	3	10	9	10	5	4	5
100	162	172	188	12	12	13	10	11	11	10	10	11	10	9	10	5	3	5
250	798	865	1017	22	20	25	10	11	12	10	10	11	10	8	10	5	3	5
500	2126	2045	2501	29	25	31	10	11	12	9	10	11	10	8	10	5	3	4
1000	> 3600	> 3600	> 3600	25	26	33	9	11	11	9	10	11	9	8	10	4	3	4
GPS + Galileo																		
	$L1(E1)$			$L1(E1), L5(E5)$						$L1(E1), L5(E5), L2, E5b$								
1	1	1	1	1	1	1	-	-	-	1	1	1	20	17	19	10	7	9
10	3	3	3	3	3	3	-	-	-	3	2	3	20	17	19	10	7	9
100	169	170	188	13	13	14	-	-	-	9	9	10	20	17	19	10	6	9
250	937	978	1536	26	26	33	-	-	-	9	9	10	19	17	19	10	6	9
500	2311	2560	2731	33	31	40	-	-	-	9	9	10	19	17	19	10	6	9
1000	> 3600	> 3600	> 3600	31	31	42	-	-	-	8	8	10	18	16	19	9	6	9
North-South Baseline																		
GPS only																		
	$L1$			$L1, L2$			$L1, L2, L5$											
1	1	1	1	1	1	1	1	1	1	-	-	-	10	9	9	5	3	5
10	3	3	3	3	3	3	3	3	3	-	-	-	10	9	9	5	3	5
100	166	186	191	16	19	19	10	12	12	-	-	-	9	9	9	5	3	4
250	894	922	1093	24	28	29	11	13	13	-	-	-	9	8	9	5	3	4
500	2305	2262	2354	28	30	31	11	12	12	-	-	-	9	8	9	5	3	4
1000	> 3600	> 3600	> 3600	31	30	30	11	11	12	-	-	-	9	8	8	4	3	3
Galileo only																		
	$E1$			$E1, E5$			$E1, E5a, E5b$			$E1, E5a, E5b, E6$								
1	1	1	1	1	1	1	1	1	1	1	1	1	10	9	10	5	4	5
10	3	3	3	3	3	3	2	3	3	2	3	3	10	9	10	5	4	5
100	151	167	194	12	12	13	10	11	12	10	11	11	10	9	10	5	3	4
250	828	838	1034	22	22	25	10	12	12	10	11	11	10	9	9	5	3	4
500	2371	1964	2142	27	27	30	10	11	12	10	11	11	10	9	9	5	3	4
1000	> 3600	> 3600	> 3600	27	24	27	10	11	11	9	10	11	9	8	8	4	3	3
GPS + Galileo																		
	$L1(E1)$			$L1(E1), L5(E5)$						$L1(E1), L5(E5), L2, E5b$								
1	1	1	1	1	1	1	-	-	-	1	1	1	20	17	19	10	7	9
10	3	3	3	3	3	3	-	-	-	2	2	3	20	17	19	10	7	9
100	157	172	193	12	13	14	-	-	-	9	9	10	19	17	19	10	7	9
250	1028	936	1424	27	27	30	-	-	-	9	9	10	19	17	18	10	7	8
500	2523	2501	2319	32	34	35	-	-	-	8	9	9	19	17	18	9	6	7
1000	> 3600	> 3600	> 3600	32	30	31	-	-	-	8	7	7	18	16	16	8	6	6

Table 5.12: **GPS only, Galileo only, GPS + Galileo**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario **Full AR**, Averaged number of epochs over 24 batches are presented - measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, SD precision $\sigma_I = 0.68$ mm per km.

4. It an also be noticed that there is a significant improvement in the number of epochs taken for 0.999 ASR as frequencies for any GNSS system are increased. The change is most significant when one compares single and dual frequency results and least significant on comparison with triple and quadruple frequency GNSS system results.

5. Instantaneous ASR could be achieved for all frequency combinations for all the GNSS systems for 1 Km baseline length. It has already been expressed earlier that with ionosphere fixed scenario, for CORS baseline, instantaneous ASR can be achieved. Hence forth 1 Km baseline results will not be discussed in this section.

GPS only

Single frequency:

6. For single frequency GPS, it took a minimum of 3 and a maximum of 2364 epochs on an average for baseline lengths lying between 10 and 500 Km for different scenarios of measurement precision and latitude location for E-W baseline. For N-S oriented baseline, it took an average number of epochs between 3 and 2354. Single frequency GPS performed better at $-30°$ latitude for E-W baseline and at $0°$ for N-S baseline.

Dual frequency:

7. The average number of epochs taken to obtain 0.999 ASR for baseline length between 10 and 1000 Km lied between 3 and 40 for dual frequency GPS for E-W baseline. For N-S baseline, it took an average number of epochs between 3 and 31 for similar scenario. Dual frequency GPS performed best at $0°$ latitude for E-W baseline and at $-30°$ latitude for N-S baseline.

Triple frequency:

8. For triple frequency, the average number of epochs lie between 3 and 14 for E-W baseline and 3 to 12 epochs for the N-S baseline for baseline lengths between 10 to 1000 Kms. Triple frequency performed better at $0°$ latitude for both E-W and N-S oriented baselines.

Galileo only

Single frequency:

9. With Galileo only, single frequency, for baseline lengths between 10 and 500 Km, it took on an average number of epochs between 3 and 2501 for E-W baseline and between 3 and 2371 for N-S oriented baseline. The best result was found to be for $-30°$ latitude for both E-W and N-S oriented baselines.

Dual frequency:

10. For dual frequency Galileo, for baseline lengths between 10 and 1000 Kms, the average number of epochs taken for 0.999 ASR lie between 3 and 33 for E-W baseline and 3 and 31 for N-S baseline. The best result was found to be for $-30°$ latitude for both E-W and N-S oriented baselines.

Triple frequency:

11. With triple frequency Galileo, for baseline lengths between 10 and 1000 Kms, the average number of epochs over 24 batches to obtain 0.999 ASR lied between 2 and 12 for E-W baseline and N-S baseline both. The best result was found to be for $0°$ latitude for both E-W and N-S oriented baselines.

Quadruple frequency:

12. The quadruple frequency Galileo marginally performs better than the triple frequency Galileo, say for some scenarios, the quadruple system takes an epochs less than the triple frequency system. The improvement in performance is due to the addition of a frequency.

Comparison with GPS only:

13. The performance is better in comparison to GPS only, for dual and triple frequencies. This is credited to the fact that Galileo has better

measurement precision in comparison to GPS system.

GPS + Galileo

Single frequency:

14. For a combined GPS and Galileo systems, single frequency, having almost double the number of satellites for a standalone single frequency system, however does not perform exceptionally as compared to standalone GPS or Galileo systems. This is due to the fact that there is no additional information added to the unknowns present in the GNSS model by addition of the satellites. Hence in spite of the fact that a combined GPS and Galileo model with single reference satellite is more redundant, the results in terms of epochs taken for correct fixing of ambiguities are not affected significantly. For baseline lengths between 10 and 500 Kms it takes between 3 to 2095 epochs for E-W baseline and 3 to 2113 epochs for N-S baseline length.

Dual frequency:

15. For a combined GPS and Galileo system, for baseline lengths between 10 and 1000 Kms, it takes an average number of epochs between 3 and 41 for E-W baseline and 3 to 35 for N-S baseline for different scenarios of latitude location. Also it can be noted that, for a scenario when the GPS only and Galileo only models are redundant enough, the combined GPS and Galileo dual frequency system performs similar to standalone GPS or Galileo. This is due to the fact that for ionosphere weighted model, the GNSS model does not gain strength as number of satellites are increased. Since there is no information added to the model related to ionosphere, see [48].

Quadruple frequency:

16. It takes and average number of epochs between 2 and 10 for E-W baseline and N-S baseline, both. The quadruple frequency system performs much better than the dual frequency GPS+Galileo system as

an effect of additional frequencies.

Comparison with GPS only and Galileo only

17. The dual frequency combined GPS and Galileo system does not perform better than the standalone GPS only or Galileo only. This is due to the fact that in a combined dual frequency system, the increase in number of satellites do not add any information to the model, and hence do not reduce the time taken for 0.999 ASR.

18. The quadruple frequency GPS and Galileo performs better than the quadruple frequency Galileo only. In general the quadruple frequency GPS+Galileo system performs the best of all the combinations considered.

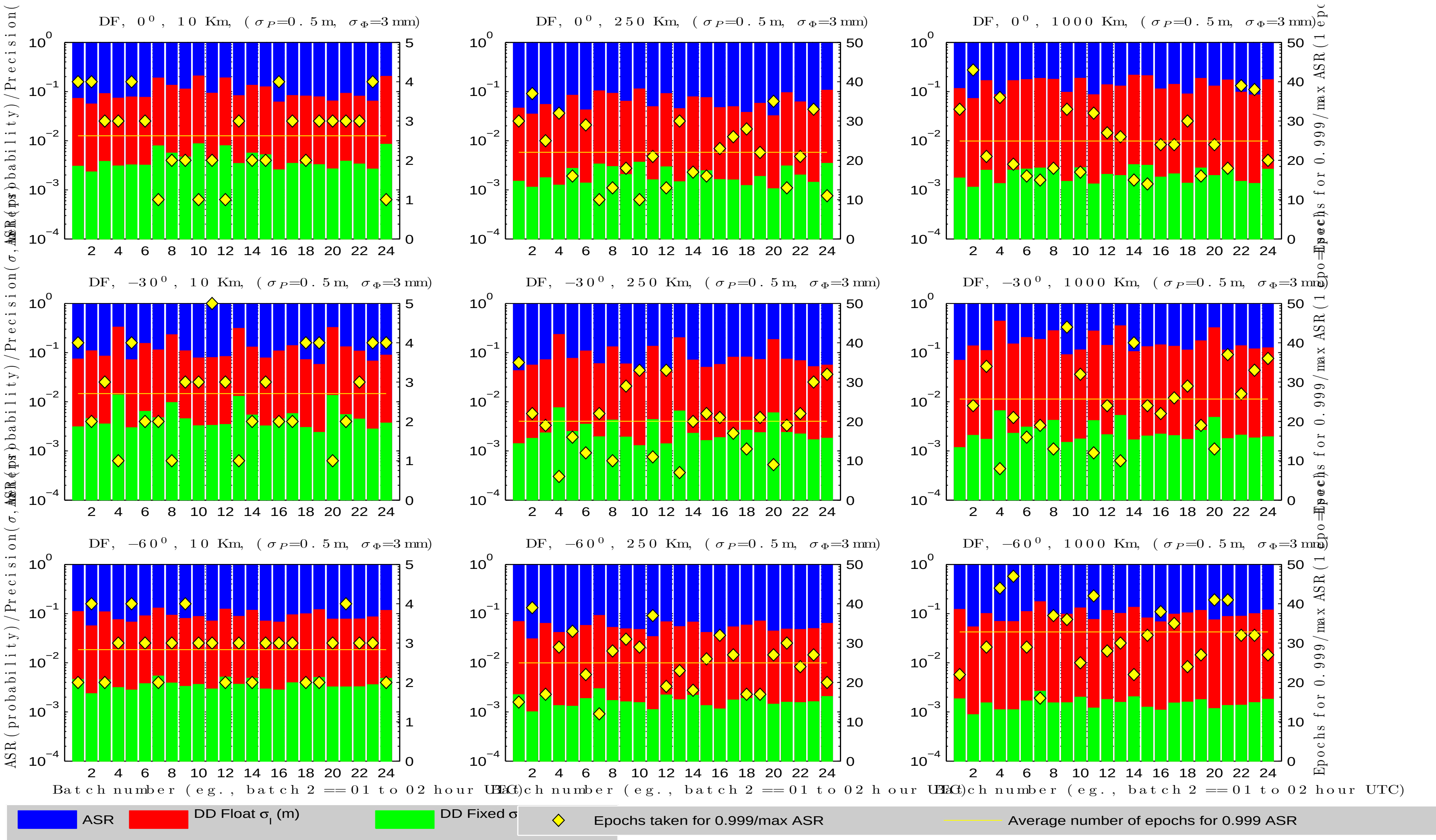

Figure 5.11: **Galileo only**, Ionosphere Weighted, **Geometry Fixed model**, **dual frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed-precision of ionosphere.

5.7.2 GPS only - Full and Partial Ambiguity Resolution, ionosphere weighted scenario

The results for ionosphere weighted, Geometry Fixed model considering GPS system frequencies are presented below. Figures F.36 to F.38 in Appendix F give full ambiguity resolution results and Figures F.39 to F.41, Appendix F present the partial ambiguity results for GPS single, dual and triple frequency respectively. Table 5.13 presents the full ambiguity resolution results in terms of number of epochs taken for 0.999 ASR and corresponding DD troposphere precision (for ambiguity fixed solution) is presented. The results for PAR are presented on the right hand side. PAR is evaluated based on the criteria to obtain the same value of fixed-precision of troposphere as in full AR. That is for same value of fixed-precision as in full AR it is to evaluate whether PAR is able to give similar results by fixing only a partial subset of the ambiguities with less number of epochs.

In this section, the simulations are done for E-W baseline for all the frequency combinations (single, dual and triple). For evaluating the ambiguity success rates for N-S baseline, simulations were not carried for single frequency considering its weak performance during E-W baseline.

The following is the analysis based on Table 5.13 and Figures 6.21 to 5.13.

General conclusions:

(1) Instantaneous ASR of 0.999 is achieved for baseline length of 1 Km for dual and triple frequency combinations and all the latitude locations. For single frequency GPS, instantaneous ASR of 0.999 was achieved at $-30°$ latitude only for two other latitude locations it took 2 epochs for 0.999 ASR.

(2) For ionosphere weighted model, as the baseline length increases, the number of epochs taken for 0.999 ASR increases, since σ_I is increasing with baseline length which adds up to the stochastic model. Further when the baseline is large enough so that the ionosphere weighted model is equivalent to ionosphere float model (say for baseline lengths above 250 Km), then the full AR is influenced by number of common satellites available and number of low elevation satellites. This behaviour is addressed in earlier section (averaged number of epochs over 24 batches

	Full AR - minimum desired ASR = 0.999											
	Epochs taken			$\sigma_{\check{T}}$(meters)			$\sigma_{\hat{T}}$(meters)			Gain=$\sigma_{\hat{T}}/\sigma_{\check{T}}$		
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline												
	Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m				
1	2	1	2	0.002	0.004	0.002	0.399	0.662	0.355	164.53	164.57	164.44
10	4	3	4	0.003	0.004	0.003	0.282	0.382	0.251	88.21	88.19	88.14
100	161	166	299	0.005	0.005	0.003	0.055	0.050	0.024	10.48	10.32	9.54
250	2026	644	1771	0.003	0.007	0.003	0.009	0.024	0.008	3.17	3.40	2.55
500	> 3600	1482	2830	-	0.008	0.005	-	0.020	0.008	-	2.65	1.80
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
	High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m				
1	1	1	1	0.002	0.003	0.002	0.200	0.234	0.178	79.84	79.84	79.84
10	4	2	4	0.003	0.005	0.003	0.100	0.166	0.089	31.91	31.91	31.90
100	16	16	28	0.003	0.003	0.002	0.066	0.062	0.036	21.29	21.28	21.23
500	24	33	55	0.003	0.002	0.001	0.100	0.075	0.045	38.19	37.80	36.83
1000	24	40	59	0.003	0.002	0.001	0.154	0.098	0.063	56.82	54.82	52.24
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	29	36	63	0.005	0.005	0.006	0.089	0.077	0.073	18.35	16.12	11.90
	High-end receivers, Triple Frequency							$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m				
1	1	1	1	0.002	0.002	0.002	0.129	0.151	0.115	69.84	69.84	69.84
10	4	2	4	0.003	0.005	0.003	0.065	0.107	0.058	21.43	21.43	21.43
100	11	10	18	0.003	0.003	0.002	0.058	0.057	0.033	19.21	19.21	19.20
250	11	11	19	0.003	0.003	0.002	0.092	0.085	0.050	29.99	29.98	29.91
500	10	12	19	0.003	0.003	0.002	0.156	0.126	0.079	47.42	47.33	47.11
1000	10	13	19	0.003	0.003	0.002	0.226	0.167	0.113	66.61	66.30	65.81
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	11	11	19	0.003	0.003	0.004	0.081	0.078	0.075	27.28	26.76	20.40
North-South Baseline												
	Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m				
1	2	1	2	0.002	0.004	0.002	0.399	0.662	0.355	164.53	164.57	164.44
10	4	3	4	0.003	0.004	0.003	0.282	0.382	0.251	88.21	88.19	88.14
100	162	151	291	0.005	0.005	0.003	0.054	0.053	0.029	10.48	10.38	8.90
250	1163	696	933	0.004	0.007	0.005	0.016	0.024	0.018	4.12	3.46	3.58
500	2353	1734	3021	0.005	0.007	0.005	0.011	0.019	0.009	2.44	2.65	1.88
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
	High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m				
1	1	1	1	0.002	0.003	0.002	0.200	0.234	0.178	79.84	79.84	79.84
10	4	2	4	0.003	0.005	0.003	0.100	0.166	0.089	31.91	31.91	31.90
100	16	15	24	0.003	0.003	0.003	0.066	0.064	0.055	21.29	21.29	21.19
250	20	20	68	0.003	0.003	0.001	0.075	0.070	0.035	26.47	26.45	25.08
500	25	23	13	0.003	0.002	0.005	0.099	0.095	0.194	38.19	38.12	38.20
1000	35	24	17	0.002	0.002	0.005	0.132	0.141	0.265	56.36	56.66	56.63
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	30	30	84	0.005	0.005	0.006	0.088	0.084	0.059	18.06	17.64	10.49
	High-end receivers, Triple Frequency							$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m				
1	1	1	1	0.002	0.002	0.002	0.129	0.151	0.115	69.84	69.84	69.84
10	4	2	4	0.003	0.005	0.003	0.065	0.107	0.058	21.43	21.43	21.43
100	11	10	16	0.003	0.003	0.003	0.058	0.058	0.049	19.21	19.21	19.18
250	11	9	18	0.003	0.003	0.002	0.092	0.096	0.072	29.99	29.99	29.83
500	10	8	6	0.003	0.003	0.006	0.157	0.162	0.287	47.42	47.42	47.41
1000	12	8	7	0.003	0.003	0.006	0.215	0.232	0.393	66.56	66.63	66.56
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	11	9	19	0.003	0.003	0.003	0.081	0.086	0.064	27.28	29.33	21.12

Table 5.13: **GPS only**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, **Full AR**, ASR and DD fixed-precision of troposphere (meters) are presented - measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, SD precision $\sigma_I = 0.68$ mm per km

simulation). For longer baselines, it is generally observed that the number of epochs taken for 0.999 ASR reduce. Same can be observed in GPS only simulation for a few scenarios, see Table 5.13.

Fall in Partial AR:

(3) It can be seen in the partial AR plots, incase of new incoming reference satellite when a perturbation is caused in the stochastic model, the percentage of ambiguities fixed by PAR drops. See Figure 5.14. Same can be observed in case of full AR from Figure 5.13.

Single frequency:

(4) It took between 1 and 2830 epochs for single frequency for baseline lengths of 1 to 500 Kms (all latitude location scenarios) for E-W baseline and between 1 to 3021 epochs for N-S baseline for a similar scenario. The best results were seen to be for $-30°$ latitude for both E-W and N-S baseline.

(5) For 250 Km baseline length, it took 2026, 644 and 1771 epochs at $0°$, $-30°$ and $-60°$ for 0.999 ASR with ionosphere weighted model, E-W baseline. For N-S baseline, it took 1163, 696 and 933 epochs for a similar scenario.

Dual frequency:

(6) Number of epochs for 0.999 ASR for baseline length of 10 to 1000 Km lied between 2 and 59 for E-W baseline and between 2 and 68 for N-S baseline. The best consistent results were seen to be at $0°$ for E-W baseline and at $-30°$ for N-S baseline.

(7) The number of epochs taken at 250 Km baseline length for 0.999 ASR were 21, 24, 43 for $0°$, $-30°$ and $-60°$ latitude for E-W baseline. AS compared to ionosphere float, 250 Km baseline length when it took 20, 20 and 68 epochs, the performance of ionosphere weighted is much better, as expected. Similar results can be observed for N-S baseline.

Triple frequency:

(8) For obtaining an ASR of 0.999, it took 2 to 19 epochs for 10 to 1000 Km baseline lengths, considering all the latitude locations for E-W baseline. For N-S baseline, it took between 2 to 18 epochs for a similar scenario. The best results could be seen at 0° for E-W baseline and at −30° for N-S baseline.

(9) The comparison of ionosphere weighted and float model showed that ionosphere weighted model was almost equivalent float counterpart. The weighted model took 11, 11, 19 epochs for 250 Km baseline at 0°, −30° and −60° latitude for E-W baseline, which is similar to the float model performance. For N-S baseline, for 250 Km baseline, it took 11, 9, 18 epochs which again is almost similar to ionosphere float model (11, 9, 19 epochs).

Considering the fixed-precision of troposphere to lie around 3mm in most of the cases when 100% ambiguities are fixed, PAR will be evaluated for obtaining a fixed-precision of 2cm for troposphere in the coming section.

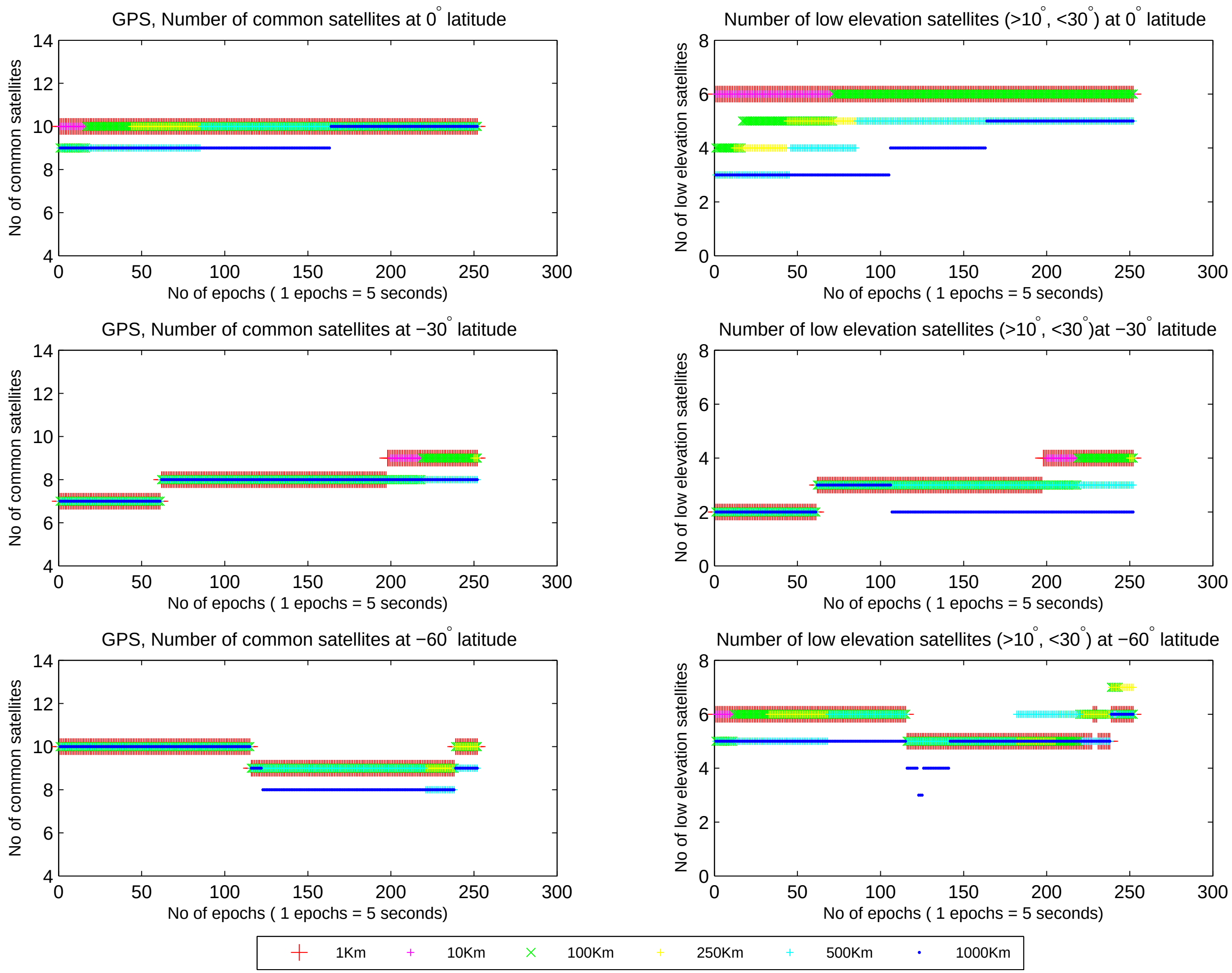

Figure 5.12: **GPS only**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, East-West oriented baseline.

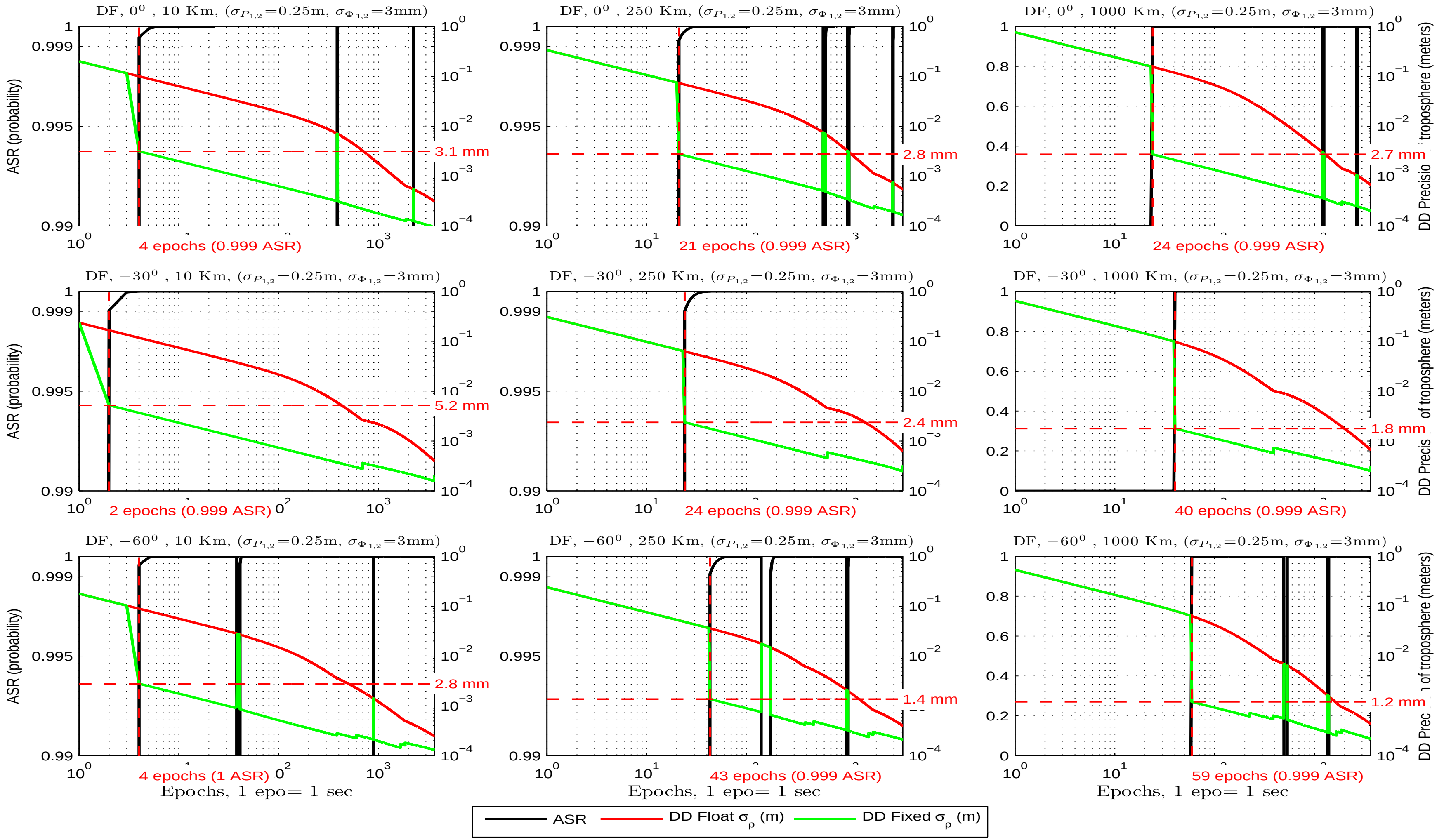

Figure 5.13: **GPS only**, Dual frequency, **Full AR**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision σ_I = 0.68 mm per km - ASR and DD fixed-precision of troposphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

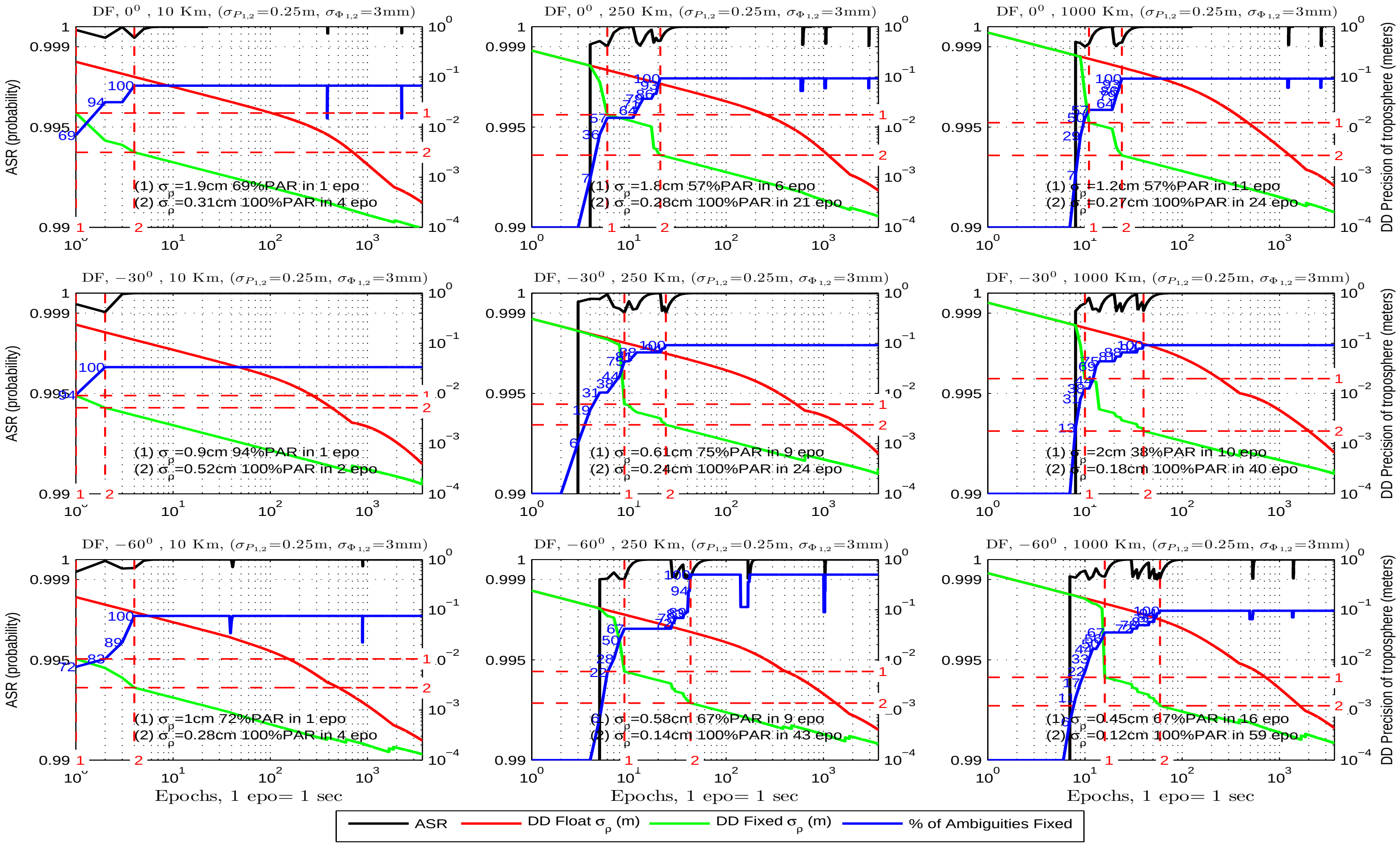

Figure 5.14: **GPS only**, Dual frequency, **Partial AR**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision σ_I = 0.68 mm per km - ASR and DD fixed-precision of troposphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision under (1), and for fixing 100% of ambiguities by PAR under (2), along with the fixed-precision obtained.

5.7.3 GPS only, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, ionosphere weighted scenario

In this section, it is to be analyzed whether the partially ambiguity fixing help in converging the solution faster to a modest fixed-precision of 2cm for troposphere. Table 5.14 presents the results in terms of fixed-precision to be equal to or better than 2cm and corresponding epochs taken and ambiguities fixed are noted.

The following is the analysis based on Table 5.14.

General conclusion:

(1) An instantaneous fixed-precision of 2cm could be obtained for 1 and 10 Km baseline length for all the scenarios of dual and triple frequencies at all latitude locations. With single frequency GPS, instantaneous fixed-precision of 2cm could only be obtained for 1 Km baseline length, for all scenarios of latitude locations.

(2) For all the remaining scenarios of baseline lengths above 10 Km, less than 100% of ambiguities were fixed to obtain a fixed-precision of 2cm for troposphere.

(3) For 1000 Km baseline length too, fixed-precision of 2cm could be achieved well within stipulated 3600 epochs.

(4) Ionosphere weighted scenario performs better than ionosphere float scenario for all the different cases considered in Table 5.14.

Single frequency:

(5) For single frequency, it took a minimum of 2 and a maximum of 1673 epochs for baseline lengths between 10 and 1000 Kms, E-W baseline. For N-S baseline, it took between 2 and 1381 epochs for a similar scenario. The best performance could be seen at $-60°$ latitude for both E-W and N-S baseline.

Dual frequency:

	Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for troposphere														
Baseline length	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{T}}$(meters)			$\sigma_{\hat{T}}$(meters)			Gain=$\sigma_{\hat{T}}/\sigma_{\check{T}}$		
(Km)	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline															
Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m								
1	1	1	1	88	100	78	0.004	0.004	0.004	0.565	0.662	0.502	132.61	164.57	115.44
10	2	3	2	62	100	67	0.020	0.004	0.008	0.399	0.382	0.355	20.40	88.19	45.39
100	113	52	52	71	62	56	0.020	0.017	0.015	0.066	0.091	0.070	3.23	5.29	4.57
250	402	251	184	57	62	40	0.020	0.017	0.020	0.035	0.042	0.037	1.72	2.44	1.84
500	705	654	501	57	57	25	0.020	0.019	0.020	0.029	0.028	0.025	1.40	1.46	1.23
1000	1239	1673	1196	38	57	12	0.019	0.017	0.018	0.026	0.025	0.021	1.34	1.46	1.14
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m								
1	1	1	1	100	100	100	0.002	0.003	0.002	0.200	0.234	0.178	79.84	79.84	79.84
10	1	1	1	69	94	72	0.019	0.009	0.010	0.200	0.234	0.178	10.50	25.93	17.07
100	5	6	6	71	81	67	0.018	0.007	0.007	0.118	0.101	0.077	6.51	13.62	10.98
250	6	9	9	57	75	67	0.018	0.006	0.006	0.137	0.102	0.079	7.79	16.76	13.64
500	9	13	12	57	75	67	0.014	0.005	0.005	0.164	0.122	0.100	11.58	24.00	19.69
1000	11	10	16	57	38	67	0.012	0.020	0.004	0.229	0.203	0.132	18.78	10.31	29.50
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	12	17	16	57	75	67	0.019	0.010	0.017	0.138	0.113	0.146	7.10	10.92	8.53
High-end receivers, Triple Frequency							$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m								
1	1	1	1	100	100	100	0.002	0.002	0.002	0.129	0.151	0.115	69.84	69.84	69.84
10	1	1	1	79	96	81	0.016	0.009	0.010	0.129	0.152	0.115	7.87	17.62	11.70
100	3	4	4	81	88	78	0.019	0.007	0.007	0.112	0.090	0.069	5.80	12.28	9.94
250	4	5	5	81	88	78	0.017	0.007	0.006	0.153	0.126	0.098	8.85	18.99	15.44
500	6	6	6	86	83	78	0.014	0.017	0.006	0.201	0.179	0.142	14.40	10.57	24.41
1000	7	6	8	90	83	78	0.012	0.020	0.005	0.270	0.247	0.177	22.92	12.39	34.53
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	6	4	6	81	67	78	0.007	0.019	0.010	0.109	0.129	0.133	15.03	6.65	13.21
North-South Baseline															
Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m								
1	1	1	1	88	100	78	0.004	0.004	0.004	0.565	0.662	0.502	132.61	164.57	115.43
10	2	2	2	62	88	67	0.020	0.007	0.008	0.399	0.468	0.355	20.40	69.83	45.30
100	112	49	56	71	62	62	0.020	0.018	0.015	0.066	0.095	0.087	3.24	5.26	5.82
250	407	229	198	50	62	57	0.020	0.018	0.020	0.034	0.045	0.044	1.69	2.43	2.18
500	674	631	599	50	50	43	0.020	0.020	0.020	0.027	0.029	0.033	1.33	1.41	1.60
1000	1381	1212	1294	38	57	29	0.020	0.020	0.020	0.023	0.028	0.027	1.11	1.37	1.31
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m								
1	1	1	1	100	100	100	0.002	0.003	0.002	0.200	0.234	0.178	79.84	79.84	79.84
10	1	1	1	69	94	72	0.019	0.009	0.010	0.200	0.234	0.178	10.49	25.94	17.04
100	5	5	6	71	75	86	0.018	0.010	0.007	0.118	0.111	0.110	6.47	10.69	15.47
250	7	9	10	57	69	86	0.017	0.007	0.006	0.127	0.104	0.107	7.64	15.87	18.94
500	9	10	13	57	62	100	0.015	0.008	0.005	0.165	0.145	0.194	11.20	19.29	38.20
1000	30	19	17	86	75	100	0.003	0.004	0.005	0.143	0.159	0.265	45.88	39.86	56.63
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	12	14	16	57	63	71	0.020	0.020	0.018	0.139	0.123	0.126	7.10	6.08	6.82
High-end receivers, Triple Frequency							$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m								
1	1	1	1	100	100	100	0.002	0.002	0.002	0.129	0.151	0.115	69.84	69.84	69.84
10	1	1	1	79	96	81	0.016	0.009	0.010	0.129	0.152	0.115	7.87	17.63	11.68
100	3	4	4	81	88	90	0.019	0.007	0.007	0.112	0.091	0.098	5.76	12.42	13.97
250	4	6	5	76	92	90	0.018	0.005	0.006	0.153	0.117	0.138	8.66	24.35	21.46
500	5	6	6	76	92	100	0.016	0.005	0.006	0.222	0.187	0.287	13.87	39.59	47.41
1000	10	7	7	90	92	100	0.007	0.005	0.006	0.236	0.248	0.393	35.83	46.73	66.56
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	6	4	5	81	71	76	0.007	0.015	0.013	0.110	0.128	0.126	15.02	8.52	9.76

Table 5.14: **GPS only**, **PAR**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, Partial ambiguities fixed (percentage) and number of epochs for obtaining 2cm fixed-precision of troposphere are presented, measurement precision is held fixed, **baseline length varied from 1 to 1000 Km**, SD precision σ_I = 0.68 mm per km, number of satellites are as available.

(6) For baseline lengths between 100 and 1000 Km, it took between 5 and 16 epochs considering all latitude locations for E-W oriented baseline. For N-S baseline, it took between 5 and 30 epochs. The best performance could be seen at $0°$ latitude for both E-W and N-S baseline for majority of baseline lengths (except 1000 Km baseline).

Triple frequency:

(7) For triple frequency, it took a minimum of 3 and a maximum of 8 epochs for baseline lengths between 10 and 1000 Km considering different latitude locations for E-W baseline. For N-S baseline, it took between 3 and 10 epochs for a similar scenario. A good performance could be seen at $-30°$ and $-60°$ latitude for both E-W and N-S baseline.

5.7.4 Galileo only - Full and Partial Ambiguity Resolution, ionosphere weighted scenario

The Galileo system with four frequencies $E1, E5a, E5b$ and $E6$ were used in combination to form single, dual, triple and quadruple system in order to simulate ambiguity success rates and fixed-precision of the troposphere. Ionosphere was considered a-priori known and hence weighted, the SD precision of ionosphere considered is $\sigma_I = 0.68$ mm per km. The results for Geometry fixed ionosphere weighted scenario are presented in Figures F.43 to F.46 for full ambiguity resolution and in Figures F.47 to F.50 for partial ambiguity resolution, see Appendix F. Table 5.15 gives a results for epochs taken in order to obtain 0.999 ASR for different scenarios of baseline length, frequency combination and latitude location along with the ambiguity-float and -fixed-precision of the troposphere.

The following is the analysis based on Table 5.15, see below.

General Conclusion:

(1) Instantaneous ASR of 0.999 could be achieved for baseline length of 1 Km for all the scenarios of latitude locations for dual, triple and quadruple frequency combinations. With single frequency, instantaneous ASR could not be achieved at $-30°$ latitude location only.

(2) It can be noted that the number of epochs required to obtain 0.999 ASR increases as baseline length increases. However, it is again stressed that, for long baseline when ionosphere weighted scenario is equivalent to ionosphere float case (say for baseline length of 500 and 1000 Kms), 0.999 ASR could be achieved quicker depending on the number of common satellites and number of low elevation satellites available. As an effect it could be noticed that for 1000 Km baseline the number of epochs taken for 0.999 Full ASR ar less than that for 500 Km baseline.

(3) At $-60°$ latitude, the results with Galileo are exceptional as compared to other latitude locations. see Table 5.15, dual, triple and quadruple frequency results. This is due to the fact that for the selected time period 0000 to 0100 UTC, Galileo performs exceptionally well for longer baselines, see Figure 5.11 (batch 1 corresponds to 0000 to 0100 UTC.

	Full AR - minimum desired ASR = 0.999											
	Epochs taken			$\sigma_{\check{T}}$(meters)			$\sigma_{\hat{T}}$(meters)			Gain=$\sigma_{\hat{T}}/\sigma_{\check{T}}$		
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline												
	Low-end receivers, Single Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.5m								
1	1	2	1	0.003	0.002	0.004	0.563	0.408	0.605	164.57	164.42	164.57
10	4	4	2	0.003	0.003	0.005	0.282	0.288	0.428	88.23	88.13	88.23
100	214	262	242	0.004	0.003	0.003	0.038	0.033	0.032	10.52	9.84	10.05
250	1079	948	1629	0.004	0.005	0.003	0.017	0.018	0.009	4.53	3.75	3.46
500	3151	1977	2807	0.003	0.005	0.003	0.008	0.013	0.008	2.61	2.45	2.31
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
	High-end receivers, Dual Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m					, $\sigma_{\Phi_{E5a}}$=0.002m , $\sigma_{P_{E5a}}$=0.15m			
1	1	1	1	0.002	0.002	0.002	0.145	0.148	0.156	70.62	70.62	70.62
10	4	4	2	0.003	0.003	0.005	0.073	0.074	0.110	24.02	24.02	24.02
100	16	19	7	0.002	0.002	0.003	0.042	0.040	0.068	20.82	20.79	20.82
250	30	35	15	0.001	0.001	0.002	0.046	0.043	0.069	30.84	30.49	30.85
500	36	45	19	0.001	0.001	0.002	0.066	0.057	0.095	47.53	45.47	47.56
1000	33	54	22	0.002	0.001	0.002	0.115	0.070	0.122	65.95	59.25	65.91
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	43	50	21	0.004	0.004	0.003	0.051	0.044	0.060	13.47	12.19	18.72
	High-end receivers, Triple Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m				, $\sigma_{\Phi_{E5a,E5b}}$=0.002m , $\sigma_{P_{E5a,E5b}}$=0.15m				
1	1	1	1	0.002	0.002	0.002	0.110	0.113	0.118	66.21	66.21	66.21
10	3	4	2	0.003	0.003	0.004	0.064	0.057	0.084	18.83	18.83	18.83
100	14	17	6	0.002	0.002	0.003	0.037	0.034	0.061	17.55	17.53	17.55
250	13	17	6	0.002	0.002	0.003	0.064	0.057	0.100	28.90	28.83	28.90
500	12	16	6	0.002	0.002	0.003	0.109	0.095	0.161	46.61	46.35	46.62
1000	10	20	7	0.003	0.002	0.003	0.204	0.125	0.211	65.85	64.97	65.84
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	13	17	6	0.003	0.003	0.003	0.075	0.061	0.090	23.92	20.80	33.72
	High-end receivers, Quadruple Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m			, $\sigma_{\Phi_{E5a,E5b,E6}}$=0.002m , $\sigma_{P_{E5a,E5b,E6}}$=0.15m					
1	1	1	1	0.001	0.002	0.002	0.092	0.094	0.099	62.14	62.14	62.14
10	3	4	2	0.003	0.003	0.004	0.053	0.047	0.070	15.91	15.91	15.91
100	13	16	5	0.002	0.002	0.004	0.035	0.032	0.060	16.41	16.40	16.41
250	12	16	6	0.002	0.002	0.003	0.066	0.058	0.098	29.16	29.10	29.16
500	11	16	6	0.002	0.002	0.003	0.114	0.095	0.161	47.82	47.53	47.82
1000	10	19	6	0.003	0.002	0.003	0.202	0.128	0.226	67.10	66.27	67.11
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	12	16	6	0.003	0.002	0.002	0.065	0.052	0.074	24.89	21.47	33.74
North-South Baseline												
	Low-end receivers, Single Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.5m								
1	1	2	1	0.003	0.002	0.004	0.563	0.408	0.605	164.57	164.42	164.57
10	4	4	2	0.003	0.003	0.005	0.281	0.288	0.427	88.23	88.13	88.23
100	241	233	231	0.003	0.004	0.003	0.035	0.037	0.033	10.50	8.46	10.12
250	728	848	1489	0.005	0.005	0.003	0.024	0.022	0.010	4.63	4.14	3.65
500	> 3600	1742	2581	-	0.006	0.003	-	0.017	0.008	-	2.65	2.38
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
	High-end receivers, Dual Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m					, $\sigma_{\Phi_{E5a}}$=0.002m , $\sigma_{P_{E5a}}$=0.15m			
1	1	1	1	0.002	0.002	0.002	0.145	0.148	0.156	70.62	70.62	70.62
10	4	4	2	0.003	0.003	0.005	0.072	0.074	0.110	24.02	24.02	24.02
100	18	19	7	0.002	0.002	0.003	0.039	0.040	0.068	20.82	20.79	20.82
250	19	29	16	0.002	0.002	0.002	0.066	0.059	0.066	30.86	30.60	30.85
500	30	33	24	0.002	0.002	0.002	0.079	0.089	0.082	47.59	46.56	47.45
1000	28	31	23	0.002	0.002	0.002	0.132	0.140	0.133	66.16	64.34	66.00
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	28	42	23	0.004	0.003	0.003	0.060	0.045	0.058	16.85	13.50	17.92
	High-end receivers, Triple Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m				, $\sigma_{\Phi_{E5a,E5b}}$=0.002m , $\sigma_{P_{E5a,E5b}}$=0.15m				
1	1	1	1	0.002	0.002	0.002	0.110	0.113	0.118	66.21	66.21	66.21
10	3	4	2	0.003	0.003	0.004	0.064	0.057	0.084	18.83	18.83	18.83
100	15	17	6	0.002	0.002	0.003	0.035	0.035	0.060	17.55	17.53	17.55
250	9	16	7	0.003	0.003	0.003	0.088	0.073	0.091	28.90	28.83	28.90
500	10	13	8	0.003	0.003	0.003	0.131	0.138	0.136	46.62	46.46	46.61
1000	9	10	7	0.003	0.004	0.004	0.227	0.245	0.236	65.87	65.67	65.86
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	9	16	7	0.003	0.003	0.003	0.085	0.059	0.084	28.73	21.70	31.42
	High-end receivers, Quadruple Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m			, $\sigma_{\Phi_{E5a,E5b,E6}}$=0.002m , $\sigma_{P_{E5a,E5b,E6}}$=0.15m					
1	1	1	1	0.001	0.002	0.002	0.092	0.094	0.099	62.14	62.14	62.14
10	3	4	2	0.003	0.003	0.004	0.053	0.047	0.070	15.91	15.91	15.91
100	15	17	6	0.002	0.002	0.003	0.032	0.031	0.055	16.41	16.40	16.41
250	8	15	6	0.003	0.003	0.003	0.092	0.074	0.097	29.16	29.10	29.16
500	10	13	7	0.003	0.003	0.003	0.131	0.138	0.145	47.82	47.65	47.81
1000	9	10	7	0.003	0.004	0.003	0.225	0.243	0.234	67.11	66.90	67.10
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	8	16	6	0.002	0.002	0.002	0.076	0.049	0.075	30.37	21.79	33.75

Table 5.15: **Galileo only**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, **Full AR** analysis, ASR and fixed-precision of troposphere (meters) are presented - measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, $\sigma_I = 0.68$ mm per km.

Single frequency:

(4) For baseline lengths between 1 to 500 Km, it took between 1 to 3151 epochs (**GPS only** took 1 to >3600 epochs) to obtain 0.999 full AR for E-W baseline length. For N-S baseline length, it took between 2 to >3600 (**GPS only** took 1 to 3021 epochs) epochs for similar scenarios. The performance for both E-W and N-S baseline was better at $-30°$ latitude location.

(5) At 250 Km baseline length, it took 1079, 948 and 1629 epochs (**GPS only** took 2026, 644 and 1771 epochs) for E-W baseline and 728, 848 and 1489 epochs for N-S baseline (**GPS only** took 1163, 696 and 933 epochs) at $0°, -30°$ and $-60°$ degree latitude respectively.

Dual frequency:

(6) With dual frequency, it took a minimum of 2 and a maximum of 54 epochs (**GPS only** took 2 to 59 epochs) for baseline lengths between 10 and 1000 Km for E-W baseline. For N-S baseline, it took between 2 to 33 epochs (**GPS only** took 2 to 68 epochs) for similar scenario. The dual frequency system seemed to favour $-60°$ latitude for both the E-W and N-S baseline.

(7) For 250 Km baseline length, it took 30, 35 and 15 epochs (**GPS only** took 21, 24 and 43 epochs) at $0°, -30°$ and $-60°$ degree latitude and for N-S baseline it took 19, 29 and 16 epochs (**GPS only** took 20, 20 and 68 epochs) for similar latitude locations.

Triple frequency:

(8) Triple frequency Galileo took between 2 and 20 epochs (**GPS only** took 2 to 19 epochs) for all latitude locations for baseline length of 10 to 1000 Kms for E-W baseline. For N-S baseline, it took between 2 and 17 epochs (**GPS only** took 2 to 18 epochs) for a similar scenario. Undoubtedly Galileo only favours $-60°$ latitude for the selected time period.

(9) For 250 Km baseline length, it took 13, 17 and 6 epochs (**GPS only** took 11, 11 and 19 epochs) at $0°, -30°$ and $-60°$ degree latitude for E-W

baseline. For N-S baseline, it took 9, 16 and 7 epochs (**GPS only** took 11, 9 and 18 epochs) for a similar scenario.

Quadruple frequency:

(10) In general quadruple frequency performed marginally better for some scenarios of baseline lengths by taking one epoch less for 0.999 ASR. For all other scenarios the performance of quadruple frequency is similar to triple frequency Galileo.

Comparison with GPS only:

(11) Comparison with GPS only shows that Galileo performs better, see above dual and triple frequency comparisons. The improved measurement precision of Galileo system plays an important part in the reduced time to achieve 0.999 ASR as compared to GPS only.

(12) Quadruple frequency Galileo out performs triple frequency Galileo, hence it can also be said to perform better that GPS system.

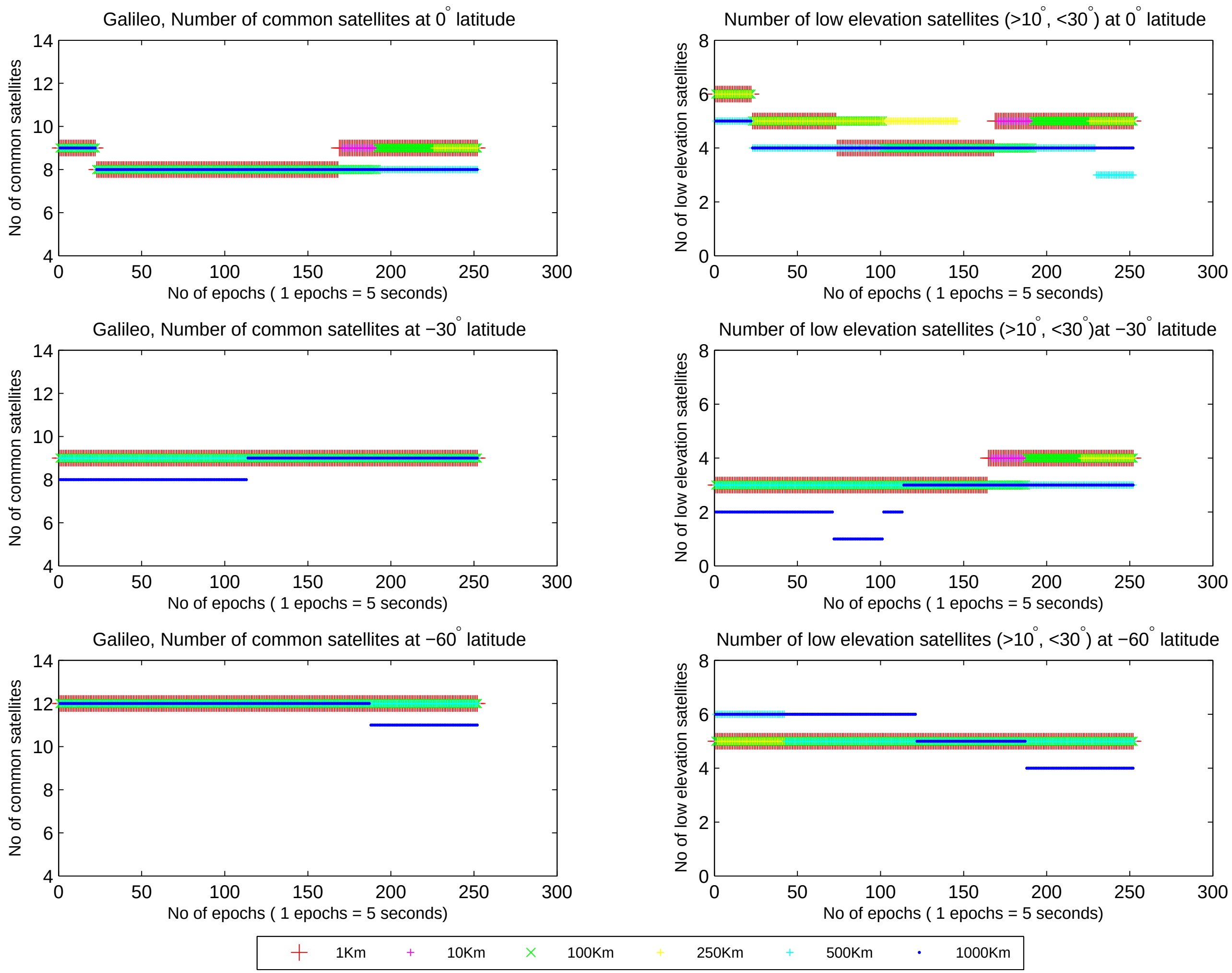

Figure 5.15: **Galileo only**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, East-West oriented baseline.

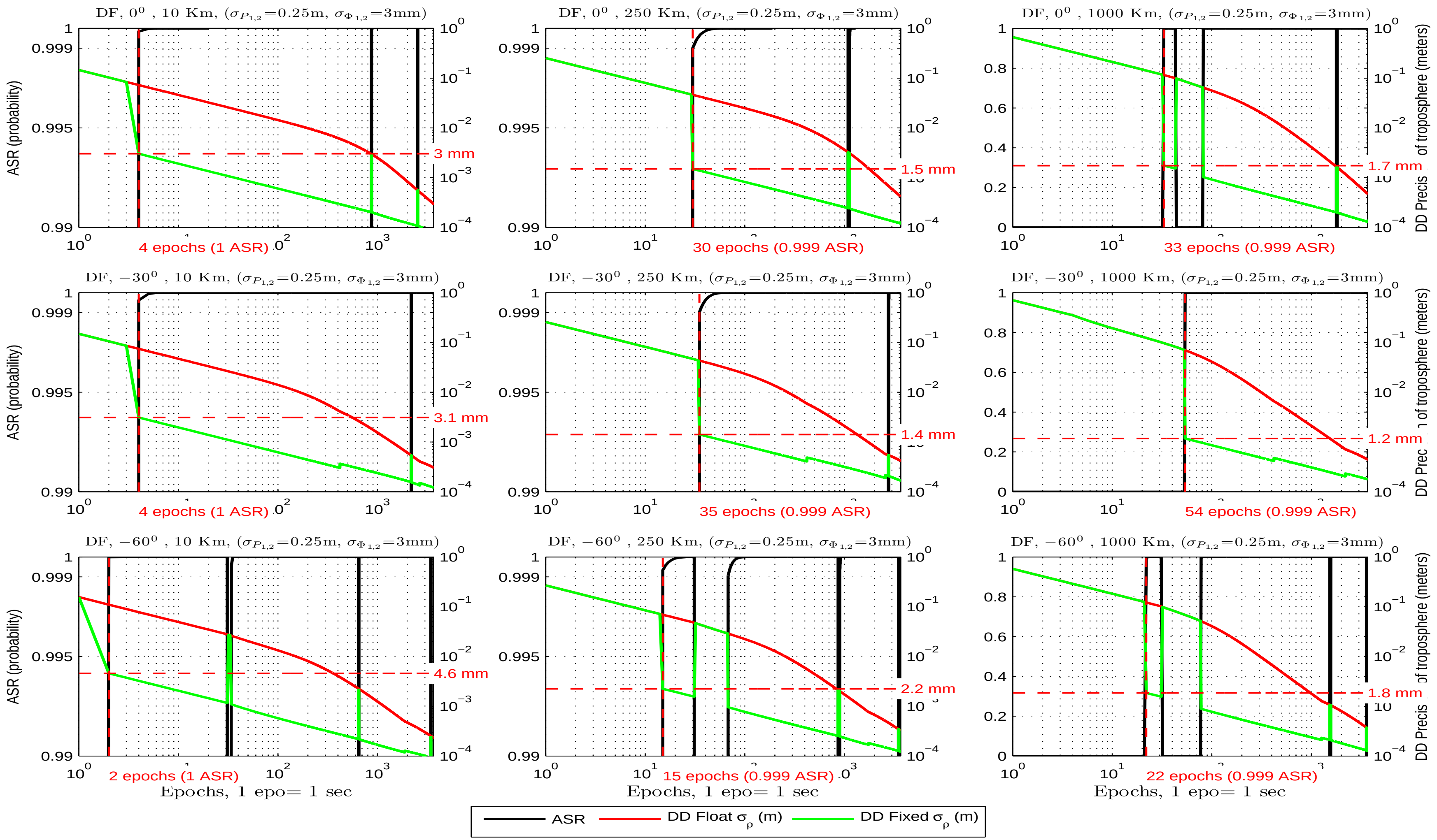

Figure 5.16: **Galileo only**, Dual frequency, **Full AR**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

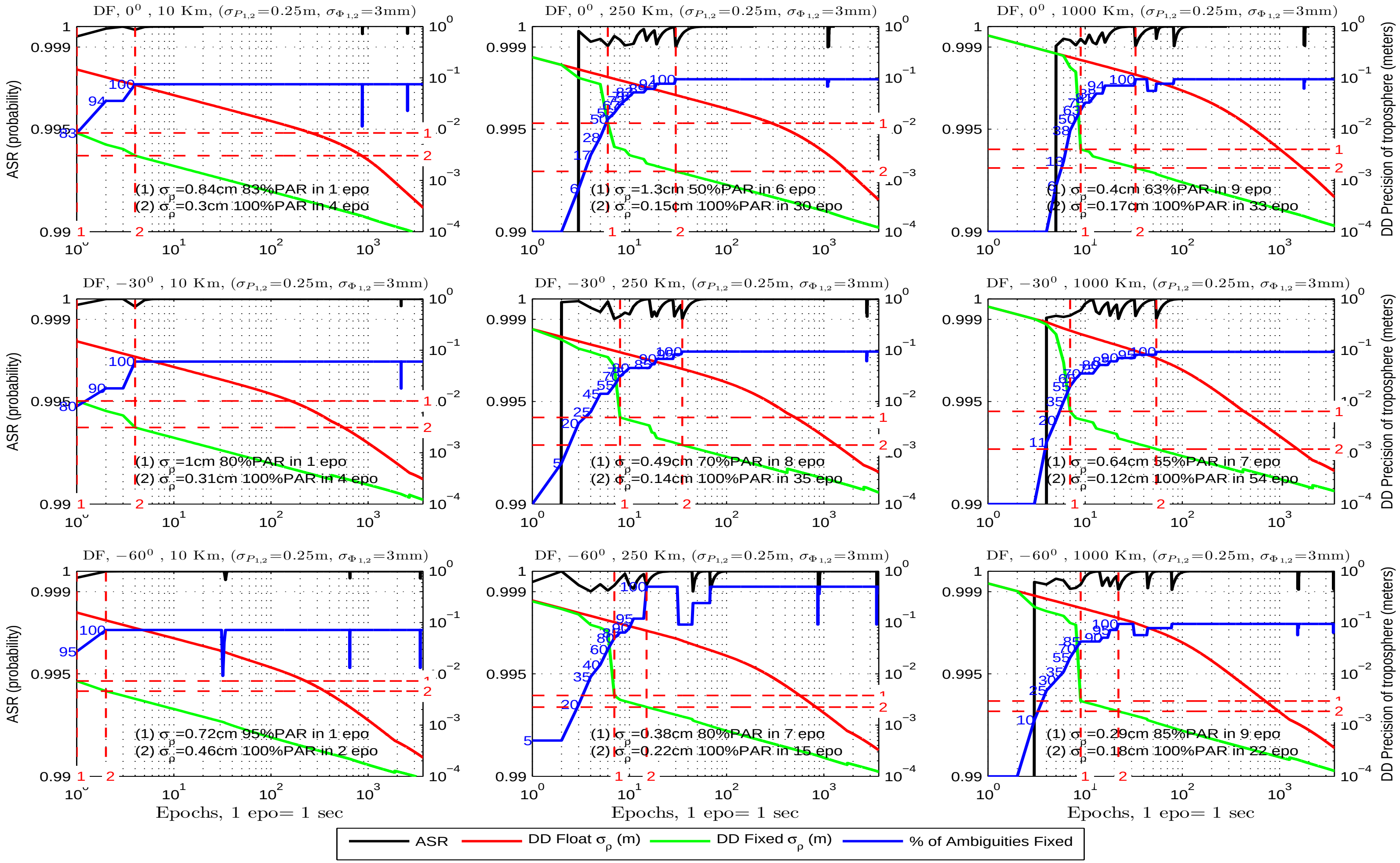

Figure 5.17: **Galileo only**, Dual frequency, **Partial AR**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision under(1), and for fixing 100% of ambiguities by PAR under (2), along with the fixed-precision obtained.

5.7.5 Galileo only, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, ionosphere weighted scenario

The evaluation of PAR with Galileo system is done in order to obtain 2cm fixed-precision of troposphere. The evaluation is based on number of epochs taken to fulfil the criteria laid. Table 5.16 presents the results of the simulation for single, dual, triple and quadruple Galileo system frequency combinations.

The analysis from the results presented in Table 5.16 is given below.

General remarks:

(1) Instantaneous fixed-precision of 2cm could be achieved for baseline lengths of 1 and 10 Km for all the scenarios of latitude locations for dual, triple and quadruple frequency Galileo. For single frequency Galileo, instantaneous fixed-precision of 2cm could be obtained only for 1 Km baseline length. A similar result was obtained with **GPS only** too.

(2) For all other scenarios of baseline lengths of 100 to 1000 Kms, for all frequency combinations, PAR fixed only a partial subset of ambiguities (less than 100%) to obtain a fixed-precision of 2cm.

(3) The results for ionosphere weighted scenarios in order to obtain a fixed-precision of 2cm for the troposphere were much better than for ionosphere float scenario. See Table 5.16, 250 Km baseline length.

Single frequency:

(4) For single frequency, it took a minimum of 2 and a maximum of 1321 epochs (**GPS only** took between 2 and 1673 epochs) for baseline lengths between 10 and 1000 Kms, E-W baseline. For N-S baseline, it took between 2 and 1486 epochs (**GPS only** took between 2 and 1381 epochs) for a similar scenario. The best performance could be seen at $-30°$ latitude for both E-W and N-S baseline.

Dual frequency:

	Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for troposphere														
Baseline length	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{T}}$(meters)			$\sigma_{\hat{T}}$(meters)			Gain=$\sigma_{\hat{T}}/\sigma_{\check{T}}$		
(Km)	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline															
Low-end receivers, Single Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.5m								
1	1	1	1	100	80	100	0.003	0.006	0.004	0.563	0.577	0.605	164.57	99.40	164.57
10	2	2	2	89	80	100	0.005	0.008	0.005	0.398	0.408	0.428	73.46	53.32	88.23
100	88	74	72	78	80	82	0.008	0.010	0.008	0.060	0.067	0.064	7.45	6.47	8.44
250	408	306	231	67	60	64	0.019	0.016	0.013	0.029	0.033	0.035	1.47	2.04	2.77
500	668	611	539	33	56	27	0.020	0.020	0.019	0.024	0.027	0.024	1.20	1.30	1.29
1000	1321	1148	976	22	44	18	0.020	0.020	0.020	0.022	0.024	0.022	1.08	1.17	1.06
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
High-end receivers, Dual Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a}}$=0.002m , $\sigma_{P_{E5a}}$=0.15m								
1	1	1	1	100	100	100	0.002	0.002	0.002	0.145	0.148	0.156	70.62	70.62	70.62
10	1	1	1	83	80	95	0.008	0.010	0.007	0.145	0.149	0.156	17.23	14.54	21.53
100	5	6	5	67	80	85	0.016	0.006	0.018	0.075	0.070	0.081	4.84	12.67	4.37
250	6	8	7	50	70	80	0.013	0.005	0.004	0.103	0.091	0.101	7.95	18.69	26.82
500	7	8	7	56	65	65	0.005	0.005	0.004	0.150	0.142	0.157	33.10	29.17	41.05
1000	9	7	9	62	55	85	0.004	0.006	0.003	0.223	0.240	0.192	55.23	37.42	65.62
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	11	9	9	72	65	80	0.015	0.018	0.012	0.101	0.105	0.092	6.59	5.77	7.90
High-end receivers, Triple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b}}$=0.002m , $\sigma_{P_{E5a,E5b}}$=0.15m								
1	1	1	1	100	100	100	0.002	0.002	0.002	0.110	0.113	0.118	66.21	66.21	66.21
10	1	1	1	93	93	97	0.008	0.009	0.007	0.110	0.113	0.118	13.98	12.57	16.86
100	3	4	4	78	87	90	0.020	0.010	0.006	0.080	0.071	0.074	4.03	7.18	11.49
250	4	6	5	81	93	97	0.018	0.005	0.004	0.116	0.096	0.109	6.40	17.76	25.36
500	5	6	5	93	93	97	0.005	0.005	0.004	0.170	0.156	0.177	33.01	29.08	40.79
1000	5	4	4	92	89	90	0.006	0.011	0.007	0.288	0.339	0.279	46.64	30.17	39.98
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	4	3	3	74	70	80	0.014	0.014	0.010	0.134	0.146	0.127	9.41	10.37	13.09
High-end receivers, Quadruple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b,E6}}$=0.002m , $\sigma_{P_{E5a,E5b,E6}}$=0.15m								
1	1	1	1	100	100	100	0.001	0.002	0.002	0.092	0.094	0.099	62.14	62.14	62.14
10	1	1	1	94	95	98	0.008	0.009	0.007	0.093	0.095	0.099	11.84	10.66	14.25
100	3	4	4	83	92	95	0.019	0.008	0.005	0.073	0.065	0.068	3.75	8.47	12.74
250	4	6	5	86	95	98	0.018	0.005	0.004	0.114	0.095	0.108	6.45	17.92	25.59
500	4	5	5	86	92	98	0.017	0.007	0.004	0.190	0.171	0.177	11.45	24.25	41.84
1000	5	4	4	97	92	92	0.005	0.011	0.007	0.286	0.337	0.277	56.78	30.74	40.74
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	4	2	2	83	80	78	0.011	0.013	0.020	0.112	0.149	0.129	10.44	11.52	6.59
North-South Baseline															
Low-end receivers, Single Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.5m								
1	1	1	1	100	80	100	0.003	0.006	0.004	0.563	0.577	0.605	164.57	99.39	164.57
10	2	2	2	89	80	100	0.005	0.008	0.005	0.398	0.408	0.427	73.44	53.27	88.23
100	91	74	62	78	80	64	0.008	0.010	0.011	0.058	0.067	0.070	7.44	6.43	6.41
250	455	420	238	62	89	64	0.020	0.010	0.012	0.031	0.035	0.034	1.50	3.54	2.84
500	814	877	557	38	56	27	0.020	0.020	0.019	0.024	0.027	0.024	1.17	1.31	1.30
1000	1373	1486	1063	38	56	17	0.020	0.020	0.020	0.024	0.026	0.021	1.18	1.28	1.05
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
High-end receivers, Dual Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a}}$=0.002m , $\sigma_{P_{E5a}}$=0.15m								
1	1	1	1	100	100	100	0.002	0.002	0.002	0.145	0.148	0.156	70.62	70.62	70.62
10	1	1	1	83	80	95	0.008	0.010	0.007	0.145	0.149	0.156	17.23	14.53	21.53
100	5	6	5	61	80	85	0.017	0.006	0.018	0.074	0.071	0.080	4.36	12.57	4.36
250	8	21	6	69	94	60	0.004	0.002	0.019	0.102	0.070	0.108	25.45	30.46	5.80
500	11	12	7	69	83	65	0.003	0.004	0.004	0.131	0.150	0.153	39.31	36.01	40.83
1000	16	7	13	86	44	94	0.003	0.008	0.003	0.176	0.304	0.179	65.67	40.41	66.12
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	12	7	9	75	67	80	0.016	0.020	0.012	0.092	0.111	0.092	5.59	5.51	7.90
High-end receivers, Triple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b}}$=0.002m , $\sigma_{P_{E5a,E5b}}$=0.15m								
1	1	1	1	100	100	100	0.002	0.002	0.002	0.110	0.113	0.118	66.21	66.21	66.21
10	1	1	1	93	93	97	0.008	0.009	0.007	0.110	0.113	0.118	13.98	12.56	16.86
100	3	4	4	78	87	90	0.020	0.014	0.006	0.079	0.071	0.074	3.89	4.93	11.56
250	4	9	5	79	96	97	0.019	0.005	0.004	0.132	0.098	0.108	6.78	21.43	25.25
500	5	7	5	83	96	97	0.016	0.005	0.004	0.185	0.188	0.172	11.81	35.54	40.57
1000	6	4	5	86	85	96	0.010	0.020	0.005	0.278	0.388	0.279	28.65	19.72	57.07
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	3	2	3	71	74	80	0.019	0.013	0.010	0.148	0.167	0.128	7.65	12.82	13.28
High-end receivers, Quadruple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b,E6}}$=0.002m , $\sigma_{P_{E5a,E5b,E6}}$=0.15m								
1	1	1	1	100	100	100	0.001	0.002	0.002	0.092	0.094	0.099	62.14	62.14	62.14
10	1	1	1	94	95	98	0.008	0.009	0.007	0.092	0.095	0.099	11.84	10.66	14.25
100	3	4	4	83	92	95	0.020	0.014	0.005	0.072	0.065	0.067	3.62	4.73	12.77
250	4	9	5	84	97	98	0.019	0.004	0.004	0.130	0.096	0.106	6.84	21.63	25.48
500	5	7	4	88	97	92	0.015	0.005	0.020	0.185	0.188	0.192	12.11	36.45	9.57
1000	6	4	5	89	92	97	0.009	0.013	0.005	0.276	0.386	0.277	29.19	29.37	58.14
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	3	2	2	78	83	78	0.018	0.010	0.020	0.123	0.139	0.130	6.76	13.42	6.59

Table 5.16: **Galileo only**, **PAR**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, Partial ambiguities fixed (percentage) and number of epochs for obtaining 2cm fixed-precision of troposphere are presented, measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km** in East-West direction, $\sigma_I = 0.68$ mm per km.

(5) For baseline lengths between 100 and 1000 Km, it took between 5 and 9 epochs (**GPS only** took between 5 and 16 epochs) considering all latitude locations for E-W oriented baseline. For N-S baseline, it took between 5 and 21 epochs (**GPS only** took between 5 and 30 epochs). Galileo only dual frequency performed well for $-30°$ and $-60°$ latitude.

Triple frequency:

(6) For triple frequency, it took a minimum of 3 and a maximum of 6 epochs (**GPS only** took between 3 and 8 epochs) for baseline lengths between 100 and 1000 Km considering different latitude locations for E-W baseline. For N-S baseline, it took between 3 and 9 epochs (**GPS only** took between 3 and 10 epochs) for a similar scenario. Galileo only triple frequency performed well for $-30°$ and $-60°$ latitude.

Quadruple frequency:

(7) The quadruple frequency Galileo system performed better than the triple frequency Galileo system. For some of the scenarios it took one epoch less to obtain a fixed-precision of 2cm as compared to triple frequency Galileo. For all other scenarios, the results for triple and quadruple frequency Galileo were exactly the same, see Table 5.16.

Comparison with GPS only:

(8) Galileo system performs better than GPS only for obtaining a fixed-precision of 2cm for troposphere, see above comparisons for dual and triple frequency Galileo and GPS.

(9) Quadruple frequency Galileo system performs better than triple frequency Galileo and GPS both.

5.7.6 GPS + Galileo - Full and Partial Ambiguity Resolution, ionosphere weighted scenario

A combined GPS and Galileo system is considered for simulation of ASR and fixed-precision of the troposphere. The frequency combinations of single $L1(E1)$, dual $L1(E1), L5(E5a)$ and quadruple $L1(E1), L5(E5a), L2, E5b$ frequency are considered for simulation with geometry fixed, ionosphere weighted model. Ionosphere is weighted as a function of baseline length with DD standard deviation corresponding to $\sigma_I = 0.68$ mm per km. The results for the simulation are presented in Table 5.17 and Figures 6.27 to 5.21 for full and partial ambiguity resolution.

The analysis from Table 5.17 and Figures 6.27 to 5.21 is presented below.

General remarks:

(1) Instantaneous 0.999 ASR could be achieved for 1 Km baseline lengths for all scenarios of latitude and frequency combinations. With GPS only and Galileo only, instantaneous ASR of 0.999 could not be achieved for single frequency at $-30°$ latitude location.

(2) It can be noted that the number of epochs required to obtain 0.999 ASR increases as baseline length increases. However, it is again stressed that, for long baseline when ionosphere weighted is equivalent to ionosphere float case, 0.999 ASR could be achieved quicker depending on the number of common satellites and number of low elevation satellites available. The number of low elevation satellites fall resulting in achieving a quicker 0.999 ASR for some scenarios of long baseline lengths (500 and/or 1000 Km).

Single frequency:

(3) For baseline lengths between 10 to 500 Km, it took between 3 to >3600 epochs (**GPS only** took 3 to >3600 epochs and **Galileo only** took 3 to 3151 epochs) to obtain 0.999 full AR for E-W baseline length. For N-S baseline length, it took between 3 to >3600 (**GPS only** took 3 to 3021 epochs and **Galileo only** took 3 to >3600 epochs) epochs for similar scenarios.

	Full AR - minimum desired ASR = 0.999											
	Epochs taken			$\sigma_{\check{T}}$(meters)			$\sigma_{\hat{T}}$(meters)			Gain=$\sigma_{\hat{T}}/\sigma_{\check{T}}$		
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline												
	Low-end receivers, Single Frequency				$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.5m							
1	1	1	1	0.002	0.003	0.002	0.396	0.429	0.385	164.57	164.57	164.57
10	4	3	4	0.002	0.003	0.002	0.198	0.248	0.192	88.22	88.19	88.16
100	184	202	265	0.003	0.003	0.002	0.032	0.029	0.020	10.51	10.17	9.84
250	1969	824	1738	0.002	0.004	0.002	0.007	0.015	0.006	3.62	3.72	2.99
500	> 3600	1819	2842	-	0.004	0.003	-	0.011	0.006	-	2.56	2.08
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
	High-end receivers, Dual Frequency				$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m							
1	1	1	1	0.001	0.002	0.001	0.102	0.110	0.099	70.62	70.62	70.62
10	4	3	3	0.002	0.003	0.002	0.051	0.064	0.057	24.02	24.02	24.02
100	15	16	18	0.002	0.002	0.001	0.033	0.032	0.027	20.82	20.80	20.80
250	30	35	68	0.001	0.001	0.001	0.035	0.032	0.019	30.82	30.54	29.85
500	36	45	77	0.001	0.001	0.001	0.051	0.042	0.026	47.40	45.74	43.33
1000	33	54	78	0.001	0.001	0.001	0.083	0.050	0.034	65.68	59.87	56.06
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	43	50	84	0.003	0.003	0.003	0.041	0.037	0.031	12.55	11.50	8.90
	High-end receivers, Quadruple Frequency				$\sigma_{\Phi_{L1(E1),L2,E5b}}$=0.003m , $\sigma_{P_{L1(E1),L2,E5b}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m							
1	1	1	1	0.001	0.001	0.001	0.078	0.084	0.075	67.54	66.16	65.97
10	3	3	3	0.001	0.002	0.002	0.045	0.049	0.044	37.15	20.30	19.64
100	9	12	14	0.001	0.002	0.001	0.034	0.031	0.025	45.85	20.28	19.29
250	9	13	15	0.001	0.001	0.001	0.045	0.048	0.040	60.73	33.34	31.81
500	8	12	14	0.001	0.001	0.001	0.057	0.076	0.066	70.75	52.17	50.26
1000	7	9	14	0.001	0.002	0.001	0.083	0.026	0.088	74.65	15.99	68.03
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	9	10	14	0.002	0.002	0.002	0.057	0.052	0.047	26.47	25.01	21.24
North-South Baseline												
	Low-end receivers, Single Frequency				$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.5m							
1	1	1	1	0.002	0.003	0.002	0.396	0.429	0.385	164.57	164.57	164.57
10	4	3	4	0.002	0.003	0.002	0.198	0.248	0.192	88.22	88.19	88.16
100	208	198	234	0.003	0.003	0.002	0.029	0.030	0.024	10.48	9.20	9.79
250	2308	725	1818	0.002	0.004	0.002	0.006	0.017	0.006	3.42	3.92	3.07
500	> 3600	1569	2898	-	0.005	0.003	-	0.013	0.006	-	2.73	2.16
1000	> 3600	3622	> 3600	-	0.005	-	-	0.010	-	-	2.07	-
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
	High-end receivers, Dual Frequency				$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m							
1	1	1	1	0.001	0.002	0.001	0.102	0.110	0.099	70.62	70.62	70.62
10	4	3	4	0.002	0.003	0.002	0.051	0.064	0.050	24.02	24.02	24.02
100	17	16	12	0.001	0.002	0.002	0.031	0.032	0.040	20.82	20.80	20.81
250	20	28	29	0.002	0.001	0.001	0.047	0.040	0.037	30.85	30.67	30.61
500	30	32	24	0.001	0.001	0.001	0.059	0.059	0.071	47.50	46.86	47.32
1000	32	30	23	0.001	0.001	0.002	0.089	0.089	0.113	65.76	64.97	65.67
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	29	40	85	0.003	0.003	0.003	0.048	0.039	0.030	15.29	12.91	8.96
	High-end receivers, Quadruple Frequency				$\sigma_{\Phi_{L1(E1),L2,E5b}}$=0.003m , $\sigma_{P_{L1(E1),L2,E5b}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m							
1	1	1	1	0.001	0.001	0.001	0.078	0.084	0.075	67.54	66.16	65.97
10	3	3	3	0.001	0.002	0.002	0.045	0.049	0.044	37.16	20.29	19.63
100	10	12	10	0.001	0.002	0.002	0.032	0.031	0.036	46.03	20.14	19.90
250	6	10	11	0.001	0.002	0.002	0.063	0.062	0.055	56.57	33.73	32.46
500	7	9	7	0.001	0.002	0.002	0.070	0.102	0.119	69.08	52.30	51.47
1000	8	8	6	0.001	0.002	0.003	0.082	0.148	0.187	74.60	69.22	68.96
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	7	8	10	0.002	0.002	0.002	0.063	0.056	0.049	30.00	27.97	25.08

Table 5.17: **GPS + Galileo**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, **Full AR** analysis, ASR and fixed-precision of troposphere (meters) are presented - measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, σ_I = 0.68 mm per km.

(4) At 250 Km baseline length, it took 1969, 824 and 1738 epochs (**GPS only** took 2026, 644 and 1771 epochs and **Galileo only** took 1079, 948 and 1629 epochs) for E-W baseline and 2308, 725 and 1818 epochs (**GPS only** took 1163, 696 and 933 epochs and **Galileo only** took 728, 848 and 1489 epochs) for N-S baseline length at $0°, -30°$ and $-60°$ degree latitude respectively.

Dual frequency:

(5) With dual frequency, it took between 3 and 78 epochs (**GPS only** took 2 to 59 epochs and **Galileo only** took 2 to 54 epochs) for baseline lengths between 10 and 1000 Km for E-W baseline. For N-S baseline, it took between 3 to 32 epochs (**GPS only** took 2 to 68 epochs and **Galileo only** took 2 to 33 epochs) for similar scenario.

(6) For 250 Km baseline length, it took 30, 35 and 68 epochs (**GPS only** took 21, 24 and 43 epochs and **Galileo only** took 30, 35 and 15 epochs) at $0°, -30°$ and $-60°$ degree latitude and for N-S baseline it took 20, 28 and 29 epochs (**GPS only** took 20, 20 and 68 epochs and **Galileo only** took 19, 29 and 16 epochs) for similar latitude locations.

Quadruple frequency:

(7) Quadruple frequency combined GPS and Galileo took between 3 and 15 epochs (**Galileo only** took 2 to 19 epochs) for all latitude locations for baseline length of 10 to 1000 Kms for E-W baseline. For N-S baseline, it took between 3 and 11 epochs (**Galileo only** took 2 to 17 epochs) for a similar scenario.

(8) For 250 Km baseline length, it took 9, 13 and 15 epochs (**Galileo only** took 12, 16 and 6 epochs) $0°, -30°$ and $-60°$ degree latitude for E-W baseline. For N-S baseline, it took 6, 10 and 11 epochs (**Galileo only** took 8, 15 and 6 epochs) for a similar scenario.

Comparison with GPS only and Galileo only:

(9) Comparison with GPS only and Galileo only for a single or dual frequency system does not promise exceptional outcomes, as seen from the above results. Since with the increase in the number of satellites only does not change the strength of the ionosphere weighted model.

(10) Comparison for quadruple frequency GPS+Galileo system with Galileo only is promising. The results for the combined system are better than the standalone Galileo only.

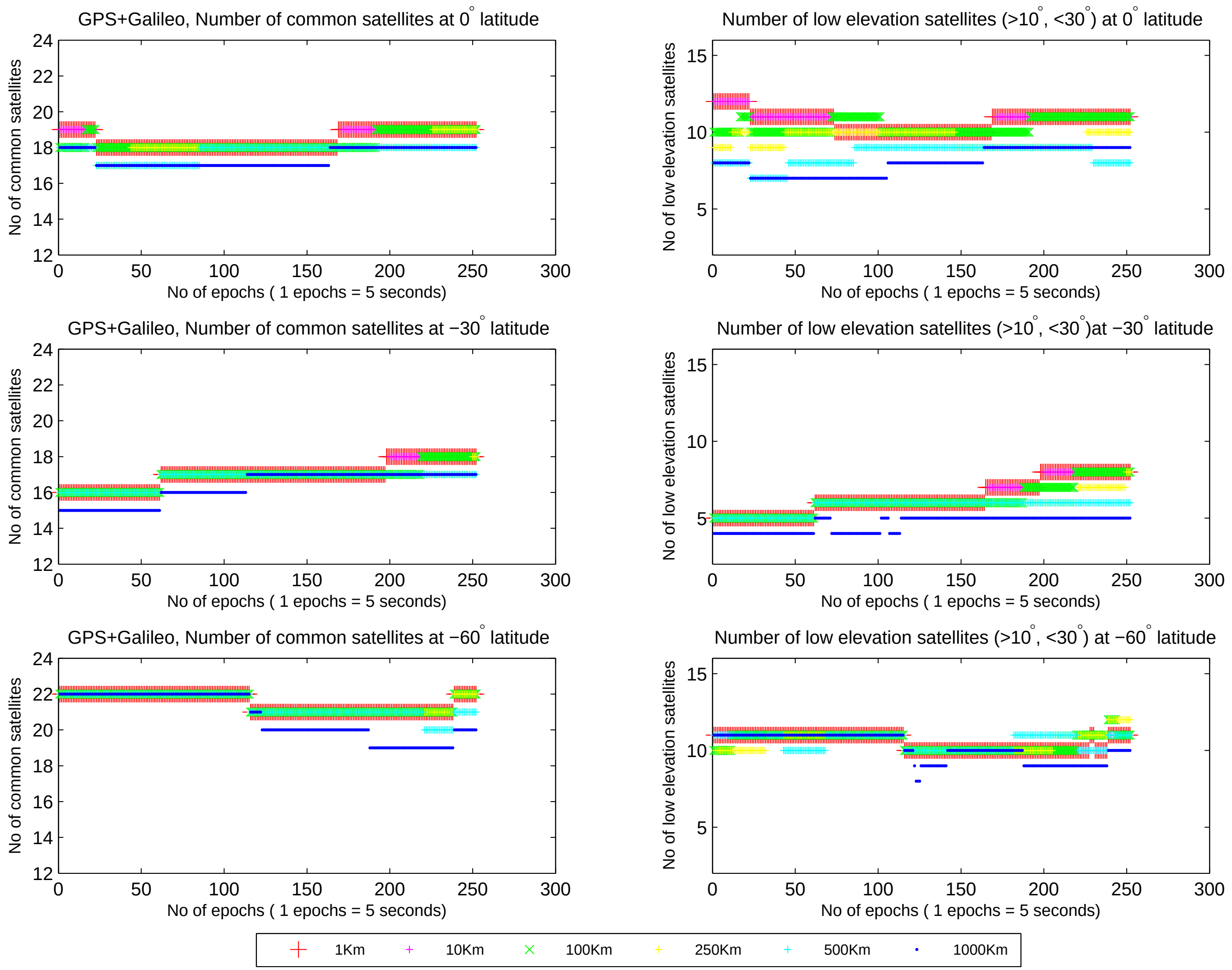

Figure 5.18: **GPS + Galileo**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, **East-West** oriented baseline

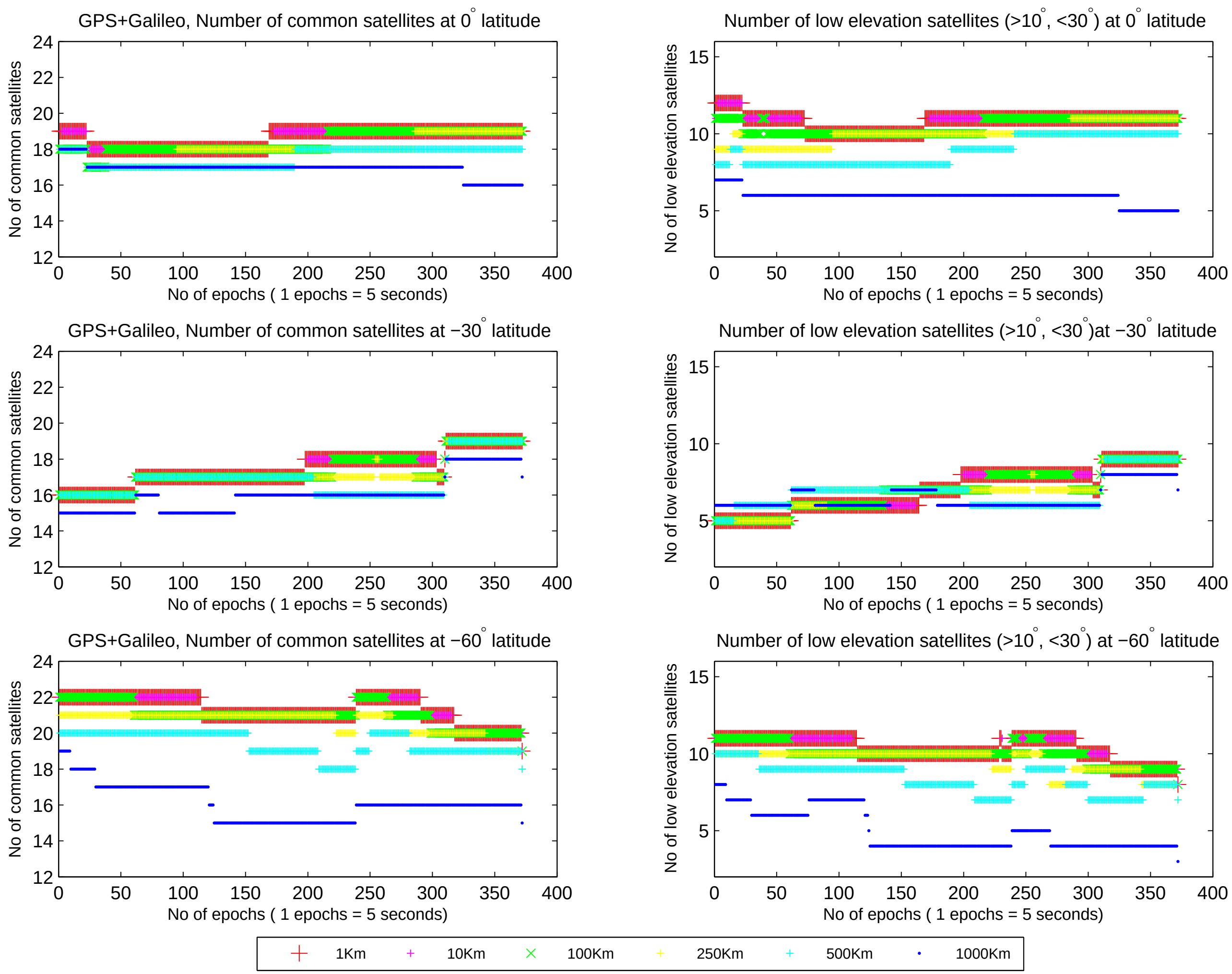

Figure 5.19: **GPS + Galileo**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, **North-South** oriented baseline

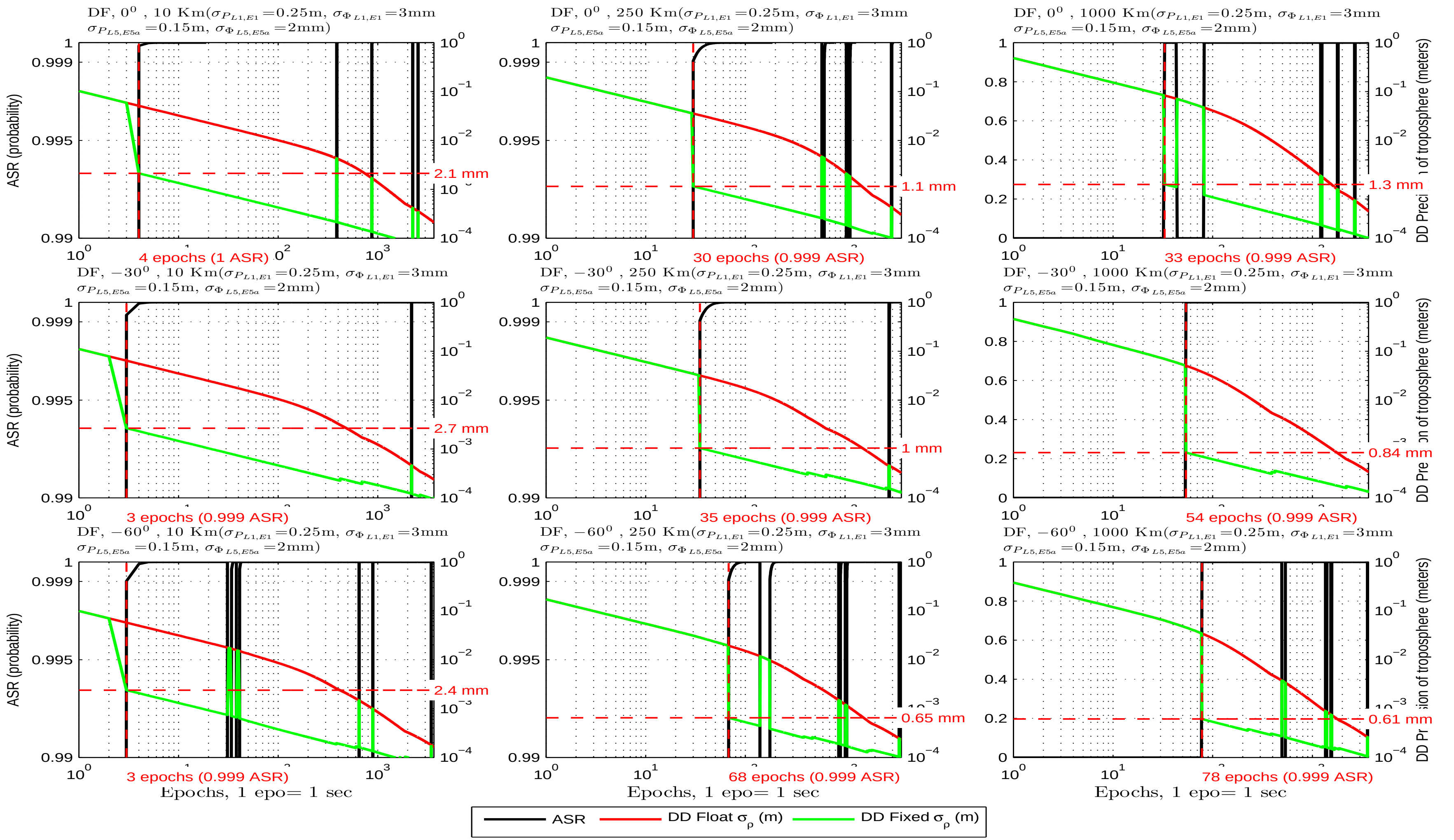

Figure 5.20: **GPS+Galileo**, Dual frequency, **Full AR**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of troposphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

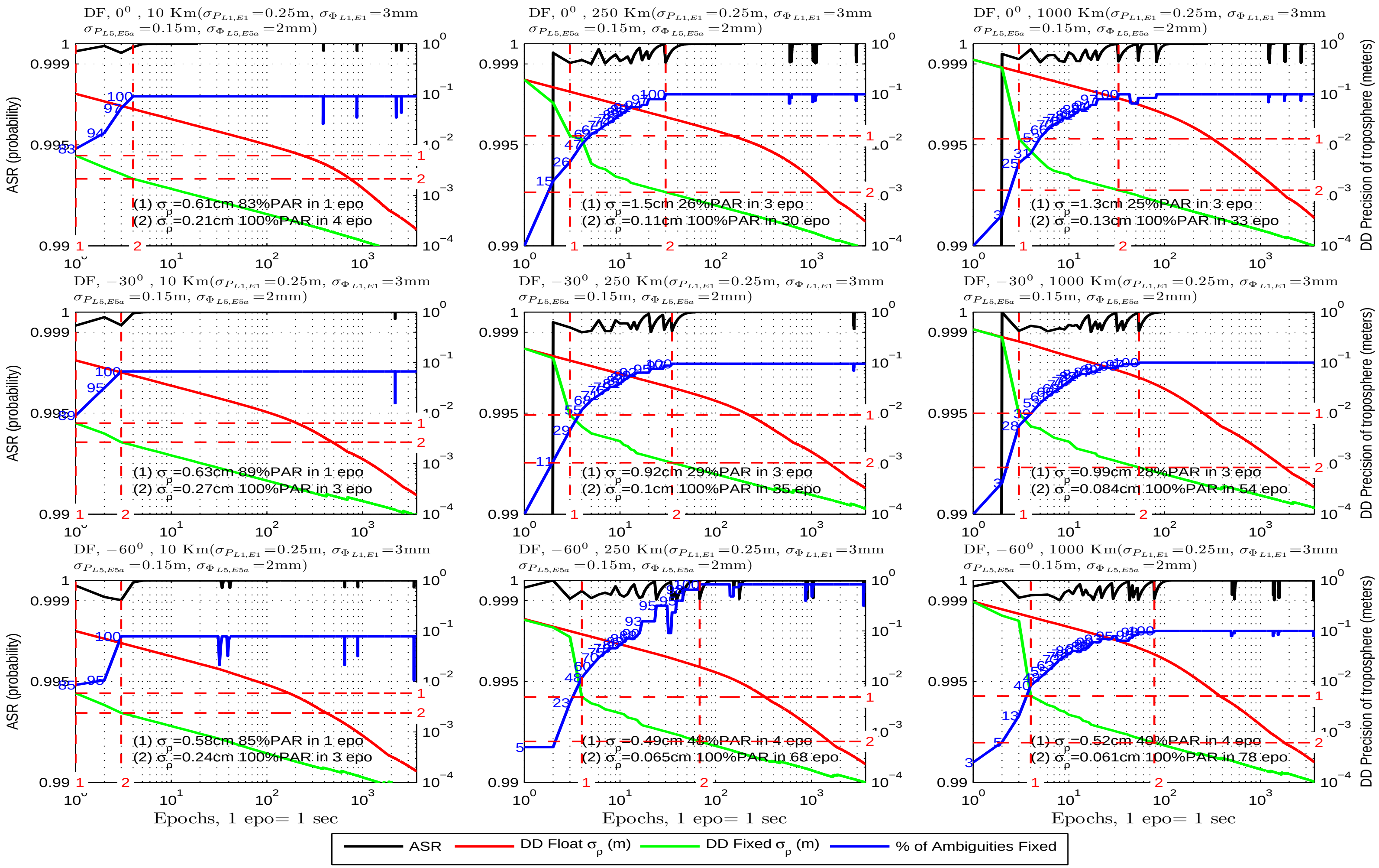

Figure 5.21: **GPS+Galileo**, Dual frequency, **Partial AR**, **Geometry based model**, coordinates known, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision σ_I = 0.68 mm per km - ASR and DD fixed-precision of troposphere (meters) are presented for 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

5.7.7 GPS + Galileo , Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, ionosphere weighted scenario

The combined GPS and Galileo system frequencies are used for simulation of PAR, in order to evaluate PAR in terms of number of epochs taken for obtaining a fixed-precision of troposphere of 2cm while considering a ionosphere weighted scenario. The results for the fixed-precision of the troposphere, number of epochs taken and percentage of ambiguities fixed is presented in Table 5.18.

General remarks:

(1) Instantaneous fixed-precision of 2cm could be achieved for baseline lengths of 1 and 10 Km for all the scenarios of frequency combinations and latitude locations. A similar result was obtained with **GPS only** too. Additionally for a combined GPS and Galileo system instantaneous fixed-precision of 2cm could be achieved with quadruple frequency for 100 Km baseline length.

(2) For all other scenarios of baseline lengths of 100 to 1000 Kms, for all frequency combinations, PAR fixed only a partial subset of ambiguities (less than 100%) to obtain a fixed-precision of 2cm.

(3) The results for ionosphere weighted scenarios in order to obtain a fixed-precision of 2cm for the troposphere are better than for ionosphere float scenario for dual frequency. With quadruple frequency the results for ionosphere float and weight scenarios are the same for 250 Km baseline length. See Table 5.16, comparison of 250 Km baseline length of ionosphere weighted with ionosphere float (shaded in pink colour).

Single frequency:

(4) For single frequency, it took a minimum of 31 and a maximum of 820 epochs (**GPS only** took between 33 and 1673 epochs and **Galileo only** took between 31 and 1321 epochs) for baseline lengths between 100 and 1000 Kms, E-W baseline. For N-S baseline, it took between 31 and 924 epochs (**GPS only** took between 33 and 1381 epochs and **Galileo only**

took between 31 and 1486 epochs) for a similar scenario.

Dual frequency:

(5) For baseline lengths between 100 and 1000 Km, it took between 2 and 4 epochs (**GPS only** took between 5 and 16 epochs and **Galileo only** took between 5 and 9 epochs) considering all latitude locations for E-W oriented baseline. For N-S baseline, it took between 2 and 4 epochs (**GPS only** took between 5 and 30 epochs and **Galileo only** took between 5 and 21 epochs).

Quadruple frequency:

(6) For quadruple frequency, it took a maximum of 2 epochs (**Galileo only** took between 2 and 4 epochs) for baseline lengths between 100 and 1000 Km considering different latitude locations for E-W baseline. For N-S baseline, it took 2 epochs only for all scenarios similar to E-W baseline (**Galileo only** took between 2 to 3 epochs).

Comparison with GPS only and Galileo only:

(7) A combined system of GPS and Galileo performed well when a fixed-precision of 2cm was aimed for. Since while aiming for fixed-precision of 2cm, PAR was used. A combined GPS and Galileo system having a large dimension for the ambiguities (almost twice in number as compared to standalone GNSS systems) brings with itself an opportunity of a more gradual PAR, than a sudden fix of a huge subset, hence intermediate fixed-precision levels could be obtained.

	Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for troposphere														
Baseline length (Km)	Epochs taken $0°$	$-30°$	$-60°$	Ambiguities fixed(%) $0°$	$-30°$	$-60°$	$\sigma_{\check{T}}$(meters) $0°$	$-30°$	$-60°$	$\sigma_{\hat{T}}$(meters) $0°$	$-30°$	$-60°$	Gain=$\sigma_{\hat{T}}/\sigma_{\check{T}}$ $0°$	$-30°$	$-60°$
East-West Baseline															
Low-end receivers, Single Frequency	$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.5m														
1	1	1	1	100	100	100	0.002	0.003	0.002	0.396	0.429	0.385	164.57	164.57	164.57
10	1	1	1	72	84	80	0.007	0.007	0.006	0.396	0.429	0.385	53.97	61.33	60.41
100	61	31	31	59	58	35	0.012	0.018	0.020	0.056	0.077	0.069	4.72	4.21	3.54
250	210	153	139	47	53	29	0.020	0.019	0.020	0.031	0.035	0.032	1.51	1.86	1.55
500	394	381	334	41	37	18	0.020	0.020	0.019	0.025	0.024	0.021	1.20	1.19	1.11
1000	820	817	645	24	29	10	0.020	0.020	0.020	0.022	0.022	0.020	1.09	1.06	1.00
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
High-end receivers, Dual Frequency	$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m														
1	1	1	1	100	100	100	0.001	0.002	0.001	0.102	0.110	0.099	70.62	70.62	70.62
10	1	1	1	83	89	85	0.006	0.006	0.006	0.102	0.111	0.099	16.61	17.45	16.98
100	2	3	3	47	63	65	0.018	0.008	0.006	0.092	0.074	0.066	5.00	9.55	11.03
250	3	3	4	26	29	48	0.015	0.009	0.005	0.112	0.110	0.086	7.44	11.98	17.42
500	3	3	4	24	29	42	0.015	0.009	0.005	0.177	0.169	0.133	12.15	18.95	26.60
1000	3	3	4	25	28	40	0.013	0.010	0.005	0.279	0.262	0.189	21.12	26.49	36.61
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	6	6	8	65	66	70	0.020	0.020	0.015	0.110	0.107	0.096	5.47	5.46	6.50
High-end receivers, Quadruple Frequency	$\sigma_{\Phi_{L1(E1),L2,E5b}}$=0.003m , $\sigma_{P_{L1(E1),L2,E5b}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m														
1	1	1	1	100	100	100	0.001	0.001	0.001	0.078	0.084	0.075	67.54	66.16	65.97
10	1	1	1	92	95	92	0.002	0.005	0.005	0.078	0.085	0.076	35.84	15.91	14.90
100	1	1	1	48	55	61	0.012	0.010	0.011	0.102	0.106	0.095	8.24	10.57	8.55
250	2	2	2	74	75	81	0.002	0.006	0.006	0.097	0.122	0.111	53.38	20.80	19.87
500	2	2	2	70	75	81	0.003	0.005	0.005	0.114	0.186	0.175	34.03	33.90	31.85
1000	2	2	2	79	77	80	0.002	0.005	0.005	0.155	0.254	0.235	70.69	50.20	44.41
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	2	2	2	72	77	80	0.009	0.007	0.007	0.120	0.117	0.123	12.90	17.50	17.70
North-South Baseline															
Low-end receivers, Single Frequency	$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.5m														
1	1	1	1	100	100	100	0.002	0.003	0.002	0.396	0.429	0.385	164.57	164.57	164.57
10	1	1	1	72	84	80	0.007	0.007	0.006	0.395	0.429	0.384	53.95	61.33	60.36
100	59	31	33	53	58	37	0.015	0.019	0.019	0.056	0.078	0.080	3.75	4.17	4.14
250	236	151	149	50	56	30	0.020	0.020	0.020	0.032	0.040	0.033	1.54	1.98	1.63
500	433	419	363	47	44	26	0.019	0.020	0.020	0.025	0.027	0.025	1.26	1.34	1.21
1000	924	918	805	35	29	20	0.020	0.020	0.019	0.022	0.023	0.022	1.07	1.14	1.12
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
High-end receivers, Dual Frequency	$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m														
1	1	1	1	100	100	100	0.001	0.002	0.001	0.102	0.110	0.099	70.62	70.62	70.62
10	1	1	1	83	89	85	0.006	0.006	0.006	0.102	0.111	0.099	16.61	17.45	16.96
100	2	3	3	47	61	72	0.019	0.009	0.006	0.091	0.074	0.079	4.86	8.08	13.24
250	3	4	4	25	53	56	0.016	0.005	0.004	0.122	0.107	0.102	7.71	19.54	22.69
500	3	3	4	25	31	53	0.016	0.010	0.005	0.188	0.197	0.175	11.86	20.50	38.55
1000	4	3	4	33	28	47	0.008	0.011	0.005	0.257	0.289	0.275	31.57	25.22	59.01
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	7	7	7	69	72	69	0.017	0.014	0.016	0.098	0.094	0.094	5.86	6.95	5.74
High-end receivers, Quadruple Frequency	$\sigma_{\Phi_{L1(E1),L2,E5b}}$=0.003m , $\sigma_{P_{L1(E1),L2,E5b}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m														
1	1	1	1	100	100	100	0.001	0.001	0.001	0.078	0.084	0.075	67.54	66.16	65.97
10	1	1	1	91	95	92	0.002	0.005	0.005	0.078	0.085	0.075	35.83	15.90	14.88
100	1	1	1	46	55	64	0.015	0.010	0.011	0.101	0.106	0.113	6.77	10.29	9.95
250	2	2	2	77	79	87	0.003	0.006	0.006	0.109	0.138	0.130	32.64	21.98	23.09
500	2	2	2	77	79	86	0.003	0.006	0.006	0.131	0.217	0.223	41.06	34.70	38.26
1000	2	2	2	73	77	87	0.005	0.006	0.006	0.165	0.297	0.324	31.18	47.58	53.93
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	2	2	2	77	79	85	0.008	0.006	0.005	0.118	0.113	0.111	15.20	18.03	22.62

Table 5.18: **GPS+Galileo**, **PAR**, **Geometry Based model**, coordinates known, ionosphere weighted, troposphere float scenario, Partial ambiguities fixed (percentage) and number of epochs for obtaining 2cm fixed-precision of troposphere are presented, measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, σ_I = 0.68 mm per km.

6 Reference Rover model

6.1 Introduction

For a Geometry Based model, when the coordinates of one or more receivers or one or more satellites is unknown then it can be called as a Geometry Float model. In the current scenario, the coordinates of one of the receivers forming the baseline are unknown. This corresponds to a Reference-Rover model. In the previous chapter, when the coordinates of both the receivers forming the baseline were known, it corresponded to a CORS scenario or permanent reference station scenario. This chapter is an extension of the predecessor Geometry Fixed model. The DD observation equations of the Reference Rover (RR) model for GPS and Galileo systems can be given as below.

$$\Phi_{1r,j}^{1s} = -u_r^{1s^T}\Delta r_{1r} - \mu_j I_{1r}^{1s} + \psi^{1s} T_{1r} + \lambda_j N_{1r,j}^{1s} + \epsilon_{1r,j}^{1s} \tag{6.1}$$

and similarly for code, we have

$$P_{1r,j}^{1s} = -u_r^{1s^T}\Delta r_{1r} + \mu_j I_{1r}^{1s} + \psi^{1s} T_{1r} + e_{1r,j}^{1s} \tag{6.2}$$

where $\Phi_{1r,j}^{1s}$ and $P_{1r,j}^{1s}$ are the linearised DD phase and code GPS/Galileo observables (observed minus computed values), subscripts r, j and s indicate receiver, frequency number and the satellite respectively, and the subscripts 1 indicate that satellite 1 is chosen as the reference satellite and same applicable to the receiver. Φ, λ are the DD phase observations and wavelength respectively, $u_r^{1s^T} = u_r^{s^T} - u_r^{1^T}$ $= \frac{r^s - r_r}{\|r_r^s\|} - \frac{r^1 - r_r}{\|r_r^1\|}$ is the DD unit vector between the receiver and the satellite, Δr_r is the increment in the receiver positions with time, μ is $(\nu_1/\nu_j)^2$, ν_1 is GPS/Galileo frequency on $L1$ and ν_j is GPS/Galileo frequency on L_j, T_{1r} is the tropospheric delay in Zenith and ψ^{1s} is the tropospheric delay mapping function which maps the zenith delay to the slant (satellite) direction using the satellite elevation angle, $\epsilon_{1r,j}^{1s}$ and $e_{1r,j}^{1s}$ are the error associated with the phase and code measurements respectively.

The above equations are applicable to GPS and Galileo standalone systems. However, when both the GNSS systems are considered together. For the overlapping two frequencies say $L1$ and $E1$ of GPS and Galileo respectively, will have a common reference satellite (in this case GPS satellite is the reference satellite). In such a case one needs to account for the Inter-system Biases (ISB's) [49]. The ISB's are explained in detail in the commencing section.

6.1.1 Inter System Biases in case of a combined GPS and Galileo system

GPS and Galileo satellites transmit at the individual time scales which are not synchronized. This leads to GPS Galileo Time offset or GGTO which needs to be accounted for when one considers a combined GPS and Galileo system [50], [51]. The un-differenced GPS and Galileo observables with respect to each systems reference times can be given as,

$$\left.\begin{array}{rcl} & \text{For GPS system} & \\ \Phi_{r,j}^{s} & = & -u_r^{sT}\Delta r_r + u_r^s \Delta r^s + c(dt_r^G + \delta_{r,j}^G) - \mu_j I_r^s + \psi_r^s T_r + \lambda_j N_{r,j}^s + \epsilon_{r,j}^s \\ P_{r,j}^{s} & = & -u_r^{sT}\Delta r_r + u_r^s \Delta r^s + c(dt_r^G + d_{r,j}^G) + \mu_j I_r^s + \psi_r^s T_r + e_{r,j}^s \\ & \text{For Galileo system} & \\ \Phi_{r,j}^{s} & = & -u_r^{sT}\Delta r_r + u_r^s \Delta r^s + c(dt_r^E + \delta_{r,j}^E - GGTO) - \mu_j I_r^s + \psi_r^s T_r + \lambda_j N_{r,j}^s + \epsilon_{r,j}^s \\ P_{r,j}^{s} & = & -u_r^{sT}\Delta r_r + u_r^s \Delta r^s + c(dt_r^E + d_{r,j}^E - GGTO) + \mu_j I_r^s + \psi_r^s T_r + e_{r,j}^s \end{array}\right\} \quad (6.3)$$

In the above equations, dt_r^G and dt_r^E are clock errors for GPS and Galileo respectively, subscripts G and E are for GPS and Galileo respectively, $\delta_{r,j}^G$ and $\delta_{r,j}^E$ are the hardware delays for GPS and Galileo on phase data and $d_{r,j}^G$ and $d_{r,j}^E$ are for code data, $GGTO$ is the time difference between two systems. Since we consider GPS as reference, $GGTO$ is applied to Galileo system.

When the GPS and Galileo observables are parameterized in an differenced format, say double differences (DD), GGTO which is associated with the clock delay terms disappears and the biases in the receiver hardware delays for the carrier phase and the code measurements can be seen. These biases are termed as Inter System Bias (ISB). Since GPS satellite is chosen as the reference satellite for a combined GPS and Galileo system, hence the ISB term appears at the Galileo

DD observable. This difference between the delay in the receiver that GPS and Galileo experiences or the ISB can be given in DD form as, $(\delta^{E}_{r,j} - \delta^{E}_{1,j}) - (\delta^{G}_{r,j} - \delta^{G}_{1,j}) = \delta^{E}_{1r,j} - \delta^{G}_{1r,j} = \delta^{GE}_{1r,j}$ for phase and $(d^{E}_{r,j} - d^{E}_{1,j}) - (d^{G}_{r,j} - d^{G}_{1,j}) = d^{E}_{1r,j} - d^{G}_{1r,j} = d^{GE}_{1r,j}$ for code data, see [49], where G and E are the GPS and Galileo satellites (as in the superscript), 1 is the reference station (as in the subscript). The magnitude of these biases were found to be 0.4mm and 14cm for phase and code respectively on same receivers and 3.9cm and 1.65m for phase and code on different receivers (Trimble-Septentrio receivers), see [52].

6.1.2 Double differenced equations for the combined GPS and Galileo system

For a combined GPS and Galileo system, with GPS satellite as the reference, after forming double differences, the GPS and Galileo observables can be reduced to the form

$$\left.\begin{array}{rcl} & \text{For GPS system} & \\ \Phi^{1_G,s_G}_{1r,j} &=& -u^{1_G,s_G\,T}_{r}\Delta r_{1r} - \mu_j I^{1_G,s_G}_{1r} + \psi^{1_G,s_G}T_{1r} + \lambda_j N^{1_G,s_G}_{1r,j} + \epsilon^{1_G,s_G}_{1r,j} \\ P^{1_G,s_G}_{1r,j} &=& -u^{1_G,s_G\,T}_{r}\Delta r_{1r} + \mu_j I^{1_G,s_G}_{1r} + \psi^{1_G,s_G}T_{1r} + e^{1_G,s_G}_{1r,j} \end{array}\right\} \quad (6.4)$$

$$\left.\begin{array}{rcl} & \text{For Galileo system} & \\ \Phi^{1_G,s_E}_{1r,j} &=& -u^{1_G,s_E\,T}_{r}\Delta r_{1r} + \underbrace{\left(c\delta^{GE}_{1r,j} + \lambda_j N^{1_G,1_E}_{1r,j}\right)}_{c\bar{\delta}^{GE}_{1r,j}} - \mu_j I^{1_G,s_E}_{1r} + \psi^{1_G,s_E}T_{1r} + \\ & & +\lambda_j N^{1_E,s_E}_{1r,j} + \epsilon^{1_G,s_E}_{1r,j} \\ P^{1_G,s_E}_{1r,j} &=& -u^{1_G,s_E\,T}_{r}\Delta r_{1r} + c d^{GE}_{1r,j} + \mu_j I^{1_G,s_E}_{1r} + \psi^{1_G,s_E}T_{1r} + e^{1_G,s_E}_{1r,j} \end{array}\right\} \quad (6.5)$$

In the above equations, $\delta^{GE}_{1r,j}$ and $d^{GE}_{1r,j}$ are the ISB terms for phase and code data on Galileo, the ISB on phase data can be seen to have been lumped with fraction of ambiguities for GPS and Galileo resulting in a new ISB on phase denoted as $\bar{\delta}^{GE}_{1r,j}$, as a result of lumping of one of the ambiguities on ISB, the remaining ambiguities to be estimated are given by $N^{1_G,s_E}_{1r,j} - N^{1_G,1_E}_{1r,j} = N^{1_E,s_E}_{1r,j}$, in the superscripts $(.)^{1_G,s_E}$, the term 1_G indicates the reference satellite to be GPS and s_E indicate the satellites for Galileo system, similarly s_G indicates satellites for GPS system (minus the GPS reference satellite). In the above equation, it can be noted that the satellite coordinates are not parameterized, whereas the receiver coordinates are. The satellite coordinates are assumed to be known whereas the receiver coordinates are estimated for the rover incase of Reference-Rover model, for CORS scenario (Geometry Fixed model) they are assumed to be known.

It should be noted that the common frequencies of Galileo and GPS will have a **GPS satellite as a reference satellite**, the uncommon GPS frequency $L2$ will have the same GPS satellite as reference as for common frequency system, and the Galileo uncommon frequency $E5b$ will have a Galileo satellite as a reference satellite.

Earlier, the rank deficiency between the ISB's and the ambiguities were mentioned, as found by [48]. [48] gave the rank deficiency while working on ionosphere weighted scenario. For GPS+Galileo combined system, in case of ionosphere float scenario, there exist an additional rank deficiency of *rank one* between the ISB's and ionospheric delays for the overlapping frequencies. The mitigation of the rank defect is explained below

6.1.3 Mitigation of rank defect between ISB and the ionosphere

For the common, dual and quadruple frequency GPS+Galileo system, there exist a rank deficiency between the ionosphere and the ISB for overlapping frequencies for the DD model in discussion. This is an additional rank deficiency, apart from the rank deficiency between the ambiguity and the ISB, which is mitigated as shown in equation (6.5), based on [52]. The mitigation of the rank deficiency can be done based on theory of S-Transformation as given by [53], [54].

Consider the unknowns, namely ISB's and ionosphere, from equation (6.5) on phase and code of Galileo observables as given below

$$\begin{aligned} E(\Phi_{1r,j}^{1_G,s_E}) &= -\mu_j I_{1r}^{1_G,s_E} + c\bar{\delta}_{1r,j}^{GE} + \cdots \\ E(P_{1r,j}^{1_G,s_E}) &= \mu_j I_{1r}^{1_G,s_E} + c d_{1r,j}^{GE} + \cdots \end{aligned}$$

with the functional model and the unknowns for a two common frequency GPS+Galileo system are given below

$$= \underbrace{\begin{bmatrix} -\mu_1 & 1 & 0 & 0 & 0 \\ -\mu_2 & 0 & 1 & 0 & 0 \\ \mu_1 & 0 & 0 & 1 & 0 \\ \mu_2 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathcal{A}} \cdot \underbrace{\begin{bmatrix} I_{1r}^{1_G,s_E} \\ \\ c\bar{\delta}_{1r,1}^{GE} \\ \\ c\bar{\delta}_{1r,2}^{GE} \\ \\ cd_{1r,1}^{GE} \\ \\ cd_{1r,2}^{GE} \end{bmatrix}}_{\beta}$$

In the above equation, subscripts 1 and 2 represent the common frequencies $L1(E1)$ and $L5(E5a)$ respectively.

Further the null-space $\mathcal{V}$ can be defined such that $\mathcal{A}(\beta+\mathcal{V}\alpha) = \mathcal{A}\beta$, $\mathcal{A}\cdot\mathcal{V}\alpha = 0$, given as below

$$\mathcal{V} = \begin{bmatrix} 1 \\ \mu_1 \\ \mu_2 \\ -\mu_1 \\ -\mu_2 \end{bmatrix} \cdot \alpha \;\; \text{, if } \alpha = -1/\mu_1, \text{ then, } \mathcal{V} = \begin{bmatrix} -1/\mu_1 \\ -1 \\ -\mu_2/\mu_1 \\ 1 \\ \mu_2/\mu_1 \end{bmatrix}$$

where α is the arbitrary chosen scalar, here it defines the frequency on which the ISB would be eliminated, for first common frequency, $\alpha = -1/\mu_1$. Further the minimum constraint γ can be defined by identifying the parameter to be fixed among unknowns β, in this case the code ISB on second common frequency is fixed. Hence the S-basis $(\mathcal{S}^{\perp})^T$ can be given as

$$(\mathcal{S}^{\perp})^T \cdot \beta \;=\; \gamma$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{1r}^{1_G,s_E} \\ c\bar{\delta}_{1r,1}^{GE} \\ c\bar{\delta}_{1r,2}^{GE} \\ cd_{1r,1}^{GE} \\ cd_{1r,2}^{GE} \end{bmatrix} \;=\; cd_{1r,2}^{GE}$$

Further, $\mathcal{S}$ can be defined such that the properties $(\mathcal{S}^{\perp})^T \cdot \mathcal{S} = 0$ and $[\mathcal{S} \cdot \mathcal{V}]^{-1}$

is square and invertible, hold true.

$$\mathcal{S} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then $\mathcal{A} \cdot \mathcal{S}$ gives the new full rank design matrix $\mathcal{A}'$ with the unknowns given by $\tilde{\beta}$, where $\tilde{\beta}$ can be found by the relation, $\tilde{\beta} = \left[I - \mathcal{V} \cdot \left(\mathcal{S}^{\perp} \cdot \mathcal{V} \right)^{-1} \cdot \mathcal{S}^{\perp} \right] \beta$, where I is an identity matrix of size $n \times n$, if β is a vector of size $n \times 1$. The full rank design matrix $\mathcal{A}'$ and the new vector of unknowns $\tilde{\beta}$ are given below

$$E(y') = \underbrace{\begin{bmatrix} -\mu_1 & 1 & 0 & 0 \\ -\mu_2 & 0 & 1 & 0 \\ \mu_1 & 0 & 0 & 1 \\ \mu_2 & 0 & 0 & 0 \end{bmatrix}}_{\mathcal{A}'} \cdot \underbrace{\begin{bmatrix} \overline{I}_{1r}^{1_G, s_E} \\ \\ c\overline{\overline{\delta}}_{1r,1}^{GE} \\ \\ c\overline{\overline{\delta}}_{1r,2}^{GE} \\ \\ c\overline{d}_{1r,2}^{GE} \end{bmatrix}}_{\tilde{\beta}}$$

where, $\overline{I}_{1r}^{1_G, s_E} = I_{1r}^{1_G, s_E} + \frac{1}{\mu_1} \cdot (cd_{1r,2}^{GE})$, $c\overline{\overline{\delta}}_{1r,1}^{GE} = c\overline{\delta}_{1r,1}^{GE} + cd_{1r,2}^{GE}$, $c\overline{\overline{\delta}}_{1r,2}^{GE} = c\overline{\delta}_{1r,2}^{GE} + \frac{\mu_2}{\mu_1} \cdot (cd_{1r,2}^{GE})$ and $c\overline{d}_{1r,1}^{GE} = cd_{1r,1}^{GE} - \frac{\mu_2}{\mu_1} \cdot (cd_{1r,2}^{GE})$

The updated DD observation equations for Galileo the combined GPS and Galileo system now read as,

$$\left.\begin{array}{rcl} \Phi_{1r,E1}^{1_G, s_E} & = & -u_r^{1_G, s_E T} \Delta r_{1r} + c\overline{\overline{\delta}}_{1r,E1}^{GE} - \mu_j \overline{I}_{1r}^{1_G, s_E} + \psi^{1_G, s_E} T_{1r} + \lambda_{E1} N_{1r,E1}^{1_E, s_E} + \epsilon_{1r,E1}^{1_G, s_E} \\ \\ \Phi_{1r,E5a}^{1_G, s_E} & = & -u_r^{1_G, s_E T} \Delta r_{1r} + c\overline{\overline{\delta}}_{1r,E5a}^{GE} - \mu_j \overline{I}_{1r}^{1_G, s_E} + \psi^{1_G, s_E} T_{1r} + \lambda_{E5a} N_{1r,E5a}^{1_E, s_E} + \epsilon_{1r,E5a}^{1_G, s_E} \\ \\ P_{1r,E1}^{1_G, s_E} & = & -u_r^{1_G, s_E T} \Delta r_{1r} + c\overline{d}_{1r,E1}^{GE} + \mu_j \overline{I}_{1r}^{1_G, s_E} + \psi^{1_G, s_E} T_{1r} + e_{1r,E1}^{1_G, s_E} \\ \\ P_{1r,E5a}^{1_G, s_E} & = & -u_r^{1_G, s_E T} \Delta r_{1r} + \mu_j \overline{I}_{1r}^{1_G, s_E} + \psi^{1_G, s_E} T_{1r} + e_{1r,E5a}^{1_G, s_E} \end{array}\right\} \quad (6.6)$$

The above formed S-basis is applicable for two common GPS, Galileo frequencies namely $L1(E1), L5(E5a)$ for both dual and quadruple frequency combination while simulations are carried for the combined GPS and Galileo system with ISB's as unknowns.

6.1.4 Mitigation of Inter System Biases

In this research work, there are two main scenarios under which parameterizations of ISB's can be realized. firstly, when CORS scenario is considered, CORS permanent stations are assumed to have same type of receivers; hence the DD ISB's would be insignificant, see [52]. Hence in this scenario the ISB's are not estimated as unknowns. Secondly when Reference-Rover model is considered, it is understood that both the reference and rover would not have same receiver types. Hence the ISB's are understood to be different for different receivers which would make the DD ISB's to be non-zero. The DD ISB's will hence be parameterized for estimation (only for Galileo observables which have overlapping frequencies with GPS). The functional and stochastic models for GPS and Galileo dual frequency (two overlapping frequencies, $L1(E1), L5(E5a)$ can be given as below.

$$
\begin{aligned}
E(\Phi^{1_G})(G) &= A_I^{1_G}(G)x_I^{1_G}(G) + A_{II}^{1_G}(G)x_{II}^{1_G}(G) \\
E(P^{1_G})(G) &= A_{II}^{1_G}(G)x_{II}^{1_G}(G) \\
E(\Phi^{1_G})(E) &= A_I^{1_G}(E)x_I^{1_G}(E) + A_{II}^{1_G}(E)x_{II}^{1_G}(E) \\
E(P^{1_G})(E) &= A_{II}^{1_G}(E)x_{II}^{1_G}(E)
\end{aligned}
$$

$$
E\begin{pmatrix}
\Phi^{1_G}_{L1(E1)} = \begin{cases} \Phi^{1_G}_{L1} \\ \Phi^{1_G}_{E1} \end{cases} \\
\Phi^{1_G}_{L5(E5a)} = \begin{cases} \Phi^{1_G}_{L5} \\ \Phi^{1_G}_{E5a} \end{cases} \\
P^{1_G}_{L1(E1)} = \begin{cases} P^{1_G}_{L1} \\ P^{1_G}_{E1} \end{cases} \\
P^{1_G}_{L5(E5a)} = \begin{cases} P^{1_G}_{L5} \\ P^{1_G}_{E5a} \end{cases}
\end{pmatrix}
=
\begin{bmatrix}
A^{1_G}_{I_{L1}} & 0 & 0 & 0 & A^{1_G}_{I_{L1}}(\mathcal{G}) & 0 & 0 & 0 & A^{1_G}_{II_{L1}} & \\
0 & A^{1_E}_{I_{E1}} & 0 & 0 & A^{1_G}_{I_{E1}}(\mathcal{G}) & A^{GE}_{I_{E1}}(\Phi_{ISB}) & 0 & 0 & & A^{1_G}_{II_{E1}} \\
0 & 0 & A^{1_G}_{I_{L5}} & 0 & A^{1_G}_{I_{L5}}(\mathcal{G}) & 0 & 0 & 0 & A^{1_G}_{II_{L5}} & \\
0 & 0 & 0 & A^{1_E}_{I_{E5a}} & A^{1_G}_{I_{E5a}}(\mathcal{G}) & 0 & A^{GE}_{I_{E5a}}(\Phi_{ISB}) & 0 & & A^{1_G}_{II_{E5a}} \\
0 & 0 & 0 & 0 & A^{1_G}_{I_{L1}}(\mathcal{G}) & 0 & 0 & 0 & A^{1_G}_{II_{L1}} & \\
0 & 0 & 0 & 0 & A^{1_G}_{I_{E1}}(\mathcal{G}) & 0 & 0 & 0 & & A^{1_G}_{II_{E1}} \\
0 & 0 & 0 & 0 & A^{1_G}_{I_{L5}}(\mathcal{G}) & 0 & 0 & A^{GE}_{I_{E1}}(P_{ISB}) & A^{1_G}_{II_{L5}} & \\
0 & 0 & 0 & 0 & A^{1_G}_{I_{E5a}}(\mathcal{G}) & 0 & 0 & 0 & & A^{1_G}_{II_{E5a}}
\end{bmatrix}
\cdot
\begin{bmatrix}
x^{1_G}_{I_{L1}} \\ x^{1_E}_{I_{E1}} \\ x^{1_G}_{I_{L5}} \\ x^{1_E}_{I_{E5a}} \\ x_I^{1_G}(\mathcal{G}) \\ x_{I_{E1}}(\Phi_{ISB}) \\ x_{I_{E5a}}(\Phi_{ISB}) \\ x_{I_{E5a}}(P_{ISB}) \\ x_{II}^{1_G}(G) \\ x_{II}^{1_G}(E)
\end{bmatrix}
\tag{6.7}
$$

$$
D\begin{pmatrix}
\Phi^{1_G}_{L1} \\ \Phi^{1_G}_{E1} \\ \Phi^{1_G}_{L5} \\ \Phi^{1_G}_{E5a} \\ P^{1_G}_{L1} \\ P^{1_G}_{E1} \\ P^{1_G}_{L5} \\ P^{1_G}_{E5a}
\end{pmatrix}
=
\begin{bmatrix}
C_{\Phi^{1_G}_{L1}} & C_{\Phi^{1_G}_{L1,E1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
C_{\Phi^{1_G}_{E1,L1}} & C_{\Phi^{1_G}_{E1}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{\Phi^{1_G}_{L5}} & C_{\Phi^{1_G}_{L5,E5a}} & 0 & 0 & 0 & 0 \\
0 & 0 & C_{\Phi^{1_G}_{E5a,L5}} & C_{\Phi^{1_G}_{E5a}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C_{P^{1_G}_{L1}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{E1}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{L5}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{E5a}}
\end{bmatrix}
$$

In equation (6.7), $E(\Phi^{1_G})(G)$ and $E(\Phi^{1_G})(E)$ correspond to the expectation of the phase data for GPS (indicated by (G)) and Galileo (indicated by (E)) with

GPS as reference satellite indicated with superscripts $()^{1_G}$, similarly expectation for the code data is given, A_I and A_{II} are the design matrices corresponding to non-temporal (ambiguities, receiver coordinates, troposphere, ISB's) and temporal parameters (ionosphere), $x_{I_{L1(E1)}} = [x_{I_{L1}}^{1_G}, x_{I_{E1}}^{1_E}]^T = [N_{1r,L1}^{1_G}, N_{1r,E1}^{1_E}]^T$, corresponding to a combined system of GPS+Galileo for common frequency $L1(E1)$ with GPS satellite as reference satellite and $x_{I_{L5(E5a)}} = [x_{I_{L5}}^{1_G}, x_{I_{E5a}}^{1_E}]^T = ([N_{1r,L5}^{1_G}, N_{1r,E5a}^{1_E}])^T$ for common frequency $L5(E5a)$, are the double differenced ambiguity parameters, the non-temporal parameters, apart from the ambiguities are formed in the design matrix $A_{I_j}^{1_G}(\mathcal{G})$, $\mathcal{G}$ indicates geometry parameters, namely the unit vector and mapping function for the troposphere, $x_I^{1_G}(\mathcal{G}) = [\Delta r_{1r}, T_{1r}]$ are the geometry parameters, that is, between-receiver single differenced receiver coordinate increments and troposphere for both GPS and Galileo (with GPS as reference satellite) and $x_{I,j}(\Phi_{ISB}) = \bar{\bar{\delta}}_{1r,j}^{GE}$ and $x_{I,j}(P_{ISB}) = \bar{d}_{1r,j}^{GE}$ are the Inter System Biases for phase data and for code data (for Galileo system, with GPS as reference satellite) partitioned in non-temporal design matrix A_I, $x_{II}^{1_G} = [x_{II}^{1_G}(G), x_{II}^{1_G}(E)]^T = [I_{1r}^{1_G,s_G}, \bar{I}_{1r}^{1_G,s_E}]^T$ has the temporal varying parameter, namely the double differenced ionosphere for both GPS and Galileo (with GPS as reference satellite).

The above combined GPS and Galileo system is possible to be implemented while considering common frequencies. In this research the common frequency system is applicable to a combined single frequency system consisting of $L1(E1)$ GPS and Galileo frequency and dual frequency system of $L1(E1), L5(E5a)$ combined GPS and Galileo system. For quadruple frequency case consisting of additional $L2, E5b$ frequencies of GPS and Galileo, a different parameterization is to be considered which is applicable to a combined system of four frequencies (since $L2, E5b$ are considered independent two frequencies) with observables on $L2$ having GPS satellite as a reference satellite and for $E5b$ Galileo satellite would be chosen as the reference satellite. It is also important to note that since $E5b$ frequency correspond to an independent Galileo system, the ISB's would not be parameterized for estimation with $E5b$ observables.

6.1.5 Parameterization of combined GPS and Galileo quadruple frequency system

The DD observation equations for the two common frequencies, namely, $L1(E1)$, $L5(E5a)$ for a combined GPS+Galileo system are discussed in the earlier section in equation (6.5). Further, before the expectation and dispersion of GPS+Galileo quadruple frequency are discussed, the DD observation equations of the remaining two uncommon frequencies are given below,

$$\left.\begin{array}{lcl}
\Phi_{1r,L2}^{Gs_G} & = & -u_r^{1_G,s_G T}\Delta r_{1r} - \mu_{L2} I_{1r}^{1_G,s_G} + \psi^{1_G,s_G} T_{1r} + \lambda_{L2} N_{1r,L2}^{1_G,s_G} + \epsilon_{1r,L2}^{1_G,s_G} \\
P_{1r,L2}^{1_G,s_G} & = & -u_r^{1_G,s_G T}\Delta r_{1r} + \mu_{L2} I_{1r}^{1_G,s_G} + \psi^{1_G,s_G} T_{1r} + e_{1r,L2}^{1_G,s_G} \\
\Phi_{1r,E5b}^{1_E,s_E} & = & -u_r^{1_E,s_E T}\Delta r_{1r} - \mu_{E5b} I_{1r}^{1_E,s_E} + \psi^{1_E,s_E} T_{1r} + \lambda_{E5b} N_{1r,E5b}^{1_E,s_E} + \epsilon_{1r,E5b}^{E,s_E} \\
P_{1r,E5b}^{1_E,s_E} & = & -u_r^{1_E,s_E T}\Delta r_{1r} + \mu_{E5b} I_{1r}^{1_E,s_E} + \psi^{1_E,s_E} T_{1r} + e_{1r,E5b}^{1_E,s_E}
\end{array}\right\} \quad (6.8)$$

The notations used in the above equation, equation (6.8) have been already explained earlier while describing the two common frequencies of GPS+Galileo combined system. Here, as expressed earlier, GPS $L2$ frequency has a GPS reference satellite and Galileo $E5b$ frequency has a Galileo reference satellite.

In this section the parameterization is expressed in a basic form of the expectation and dispersion, to begin with consider equation (6.7) for the combined GPS and Galileo overlapping frequency case. To express this four frequency model, and index for common frequency, j_c will be used. The four frequencies can be represented as $L1(E1), L5(E5a), L2, E5b$ from now on in further notations, with the common frequencies shown in the brackets. The expectation and dispersion can now be given as follows.

$$
E\begin{pmatrix}
\Phi^{1_G}_{L1(E1)} = \begin{cases} \Phi^{1_G}_{L1} \\ \Phi^{1_G}_{E1} \end{cases} \\
\\
\Phi^{1_G}_{L5(E5a)} = \begin{cases} \Phi^{1_G}_{L5} \\ \Phi^{1_G}_{E5a} \end{cases} \\
\\
\Phi^{1_G}_{L2} \\
\Phi^{1_E}_{E5b} \\
\\
P^{1_G}_{L1(E1)} = \begin{cases} P^{1_G}_{L1} \\ P^{1_G}_{E1} \end{cases} \\
\\
P^{1_G}_{L5(E5a)} = \begin{cases} P^{1_G}_{L5} \\ P^{1_G}_{E5a} \end{cases} \\
\\
P^{1_G}_{L2} \\
P^{1_E}_{E5b}
\end{pmatrix}
=
\begin{bmatrix}
A^{1_G}_{I_{L1}} & 0 & 0 & 0 & 0 & 0 & A^{1_G}_I(\mathcal{G}) & 0 & 0 & 0 & A^{1_G}_{II_{L1}} & & \\
0 & A^{1_E}_{I_{E1}} & 0 & 0 & 0 & 0 & A^{1_G}_I(\mathcal{G}) & A^{GE}_{I_{E1}}(\Phi_{ISB}) & 0 & 0 & & & A^{1_G}_{II_{E1}} \\
\\
0 & 0 & A^{1_G}_{I_{L5}} & 0 & 0 & 0 & A^{1_G}_I(\mathcal{G}) & 0 & 0 & 0 & A^{1_G}_{II_{L5}} & & \\
0 & 0 & 0 & A^{1_E}_{I_{E5a}} & 0 & 0 & A^{1_G}_I(\mathcal{G}) & 0 & A^{GE}_{I_{E5a}}(\Phi_{ISB}) & 0 & & & A^{1_G}_{II_{E5a}} \\
\\
0 & 0 & 0 & 0 & A^{1_G}_{I_{L2}} & 0 & A^{1_G}_I(\mathcal{G}) & 0 & 0 & 0 & A^{1_G}_{II_{L2}} & 0 & \\
0 & 0 & 0 & 0 & 0 & A^{1_E}_{I_{E5b}} & A^{1_E}_I(\mathcal{G}) & 0 & 0 & 0 & 0 & A^{1_E}_{II_{E5b}} & \\
\\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_G}_I(\mathcal{G}) & 0 & 0 & 0 & A^{1_G}_{II_{L1}} & & \\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_G}_I(\mathcal{G}) & 0 & 0 & 0 & & & A^{1_G}_{II_{E1}} \\
\\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_G}_I(\mathcal{G}) & 0 & 0 & A^{GE}_{I_{E1}}(P_{ISB}) & A^{1_G}_{II_{L5}} & & \\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_G}_I(\mathcal{G}) & 0 & 0 & 0 & & & A^{1_G}_{II_{E5a}} \\
\\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_G}_I(\mathcal{G}) & 0 & 0 & 0 & A^{1_G}_{II_{L2}} & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & A^{1_E}_I(\mathcal{G}) & 0 & 0 & 0 & 0 & A^{1_E}_{II_{E5b}} &
\end{bmatrix}
\cdot
\begin{bmatrix}
x^{1_G}_{I_{L1}} \\
x^{1_E}_{I_{E1}} \\
\\
x^{1_G}_{I_{L5}} \\
x^{1_E}_{I_{E5a}} \\
\\
x^{1_G}_{I_{L2}} \\
\\
x^{1_E}_{I_{E5b}} \\
\\
x_I(\mathcal{G}) \\
\\
x_{I_{E1}}(\Phi_{ISB}) \\
x_{I_{E5a}}(\Phi_{ISB}) \\
x_{I_{E5a}}(P_{ISB}) \\
\\
x_{II}(G) \\
\\
x_{II}(E)
\end{bmatrix}
\tag{6.9}
$$

$$
D\begin{pmatrix}
\Phi^{1_G}_{L1} \\
\Phi^{1_G}_{E1} \\
\\
\Phi^{1_G}_{L5} \\
\Phi^{1_G}_{E5a} \\
\\
\Phi^{1_G}_{L2} \\
\Phi^{1_E}_{E5b} \\
\\
P^{1_G}_{L1} \\
P^{1_G}_{E1} \\
\\
P^{1_G}_{L5} \\
P^{1_G}_{E5a} \\
\\
P^{1_G}_{L2} \\
P^{1_E}_{E5b}
\end{pmatrix}
=
\begin{bmatrix}
C_{\Phi^{1_G}_{L1}} & C_{\Phi^{1_G}_{L1,E1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
C_{\Phi^{1_G}_{E1,L1}} & C_{\Phi^{1_G}_{E1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\\
0 & 0 & C_{\Phi^{1_G}_{L5}} & C_{\Phi^{1_G}_{L5,E5a}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C_{\Phi^{1_G}_{E5a,L5}} & C_{\Phi^{1_G}_{E5a}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\\
0 & 0 & 0 & 0 & C_{\Phi^{1_G}_{L2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & C_{\Phi^{E}_{E5b}} & 0 & 0 & 0 & 0 & 0 & 0 \\
\\
0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{L1}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{E1}} & 0 & 0 & 0 & 0 \\
\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{L5}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{E5a}} & 0 & 0 \\
\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{1_G}_{L2}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_{P^{E}_{E5b}}
\end{bmatrix}
$$

In the above equation, equation (6.9), the new observables introduced are $\Phi^{1_G}_{L2}(G), \Phi^{1_E}_{E5b}(E)$ for phase and $P^{1_G}_{L2}(G), P^{1_E}_{E5b}(E)$ for code, here, the symbol in the bracket (E) or (G) indicate to which GNSS system the observable belongs and the superscript $()^{1_G}$ or $()^{1_E}$ indicates to which system the chosen reference satellite belongs, G stands for GPS and E stands for Galileo, and the subscript with unknowns $x^{1_G}_{I_{L2}}, x^{1_E}_{I_{E5a}}$ as ambiguities on $L2, E5a$ parameterized in the functional model within the design matrices $A^{1_G}_{I_{L2}}$ for GPS and $A^{1_E}_{I_{E5b}}$ for Galileo. In the above functional and stochastic models whenever the index j_c is used, it represents frequencies $L1$ for GPS and $E1$ for Galileo, single frequency case and $L1, L5$ for GPS and $E1, E5a$ for Galileo in a dual frequency case and of course in the quadruple frequency case (which in fact has four frequency parameterization) has the two common frequencies and additionally $L2, E5b$ frequencies.

6.2 Computation of normal equation and ambiguity-float solution

The computation of the ambiguity success rate and ambiguity float solution are explained in detail in chapter 5. The precision of the coordinates corresponding to ambiguity float solution and ambiguity-fixed solution (after successful fixing of ambiguities, criteria 0.999 Ambiguity Success Rate) can be used to compute the gain. The computation of gain is done by using the following expression.

6.2.1 Computation of gain:

The computation of gain is done for the the coordinate components using the ambiguity-float and -fixed-precision obtained from least square solution. The Earth Centered Earth Fixed (ECEF) coordinate components x, y and z are converted to the north, east and up, n, e and u components by using a rotation matrix as given in [55, p. 137]. The expression for the same is given below.

$$R = \begin{bmatrix} -\sin\theta\cos\phi & -\sin\theta\sin\phi & \cos\theta \\ -\sin\phi & \cos\phi & 0 \\ \cos\theta\cos\phi & \cos\theta\cos\phi & \sin\theta \end{bmatrix}$$

where, ϕ and θ are the longitude and latitude of the GNSS rover station forming the baseline. Further, the VC matrix of the x, y, z components is transformed to VC matrix of the n, e, u components using the following transformation.

$$\begin{aligned} Q_{\widehat{neu}} &= R \cdot Q_{\widehat{xyz}} \cdot R^T \\ Q_{\widetilde{neu}} &= R \cdot Q_{\widetilde{xyz}} \cdot R^T \end{aligned}$$

where $Q_{\widehat{neu}}$ and $Q_{\widetilde{neu}}$ correspond to the ambiguity float and fixed solution. The n, e and u components are use to evaluate PAR based on a predefined fixed-precision which is explained in detail in the following section.

The computation of gain is done using the VC matrices of the coordinates for the ambiguity-float and -fixed solution. The gain or the improvement in precision is given as below

$$\text{Gain}(i) = \frac{|Q_{\widehat{xyz}}|^{1/2m}}{|Q_{\widetilde{xyz}}|^{1/2m}} = \frac{|Q_{\widehat{neu}}|^{1/2m}}{|Q_{\widetilde{neu}}|^{1/2m}} \quad \text{since } |R| \cdot |R^T| = 1$$

where m is 3, corresponding to x, y and z components of the coordinates, $|\cdot|$ denotes the determinant.

6.2.2 Criteria for evaluating PAR:

The ambiguities fixed by PAR are the ones which comply to give an ASR of 0.999. However after each epoch, fixed-precision of the north, east and up components is monitored till a standard deviation of 2cm for north and east -components (σ_n and σ_e) and 6 cm for the up-component (σ_u) is achieved. The partial subset of ambiguities responsible for the ambiguity-fixed standard deviation are noted and the percentage of ambiguities fixed is computed and presented in the analysis.

6.3 Simulation considerations

The simulations will be carried out for the following scenarios

(1) Receiver coordinates consideration:

   (a) Coordinates for the rover receiver are unknown - Reference-Rover model

(2) Atmosphere consideration

   (a) Fixed (both troposphere and ionosphere are considered known), applicable for short baseline lengths (in this research, baseline lengths < 10 Km are considered short baselines, the relative atmosphere can be assumed to be insignificant)
   For atmosphere known case, baseline length of 1 Kilometer is considered. Measurement precision is varied for 6 different sets of phase and code, see Table 3.1. Simulation is done for 3 latitude locations, $0°, -30°$ and $-60°$.

   (b) Float (both troposphere and ionosphere are considered as unknown), applicable for medium and long baseline lengths (> 10 Km)
   The baseline length of 250 Kilometers will be considered. Measurement precision is varied for 6 different sets of phase and code, see Table 3.1. Simulation is done for 3 latitude locations, $0°, -30°$ and $-60°$. While the troposphere is parameterized in the design matrix, it is re-initialized after every 2 hours in order to have scenario similar to estimating a new tropospheric delay every 2 hours.

   (c) Weighted (ionosphere will be weighted, whereas, the troposphere is considered to be unknown)
   The baseline length is varied between 1 and 1000 Kilometers is considered. Measurement precision is held fixed to one value for both phase and

code, see Table 3.2. Simulation is done for 3 latitude locations, $0°$, $-30°$ and $-60°$. Troposphere is re-initialized after every 2 hours.

(3) Other considerations

(a) Latitude locations: simulations will be carried for three latitude locations $0°$, $-30°$ and $-60°$ latitudes. Longitude remains fixed at one end (say receiver number 1) at $115°$ longitude, at the other end (say for receiver number 2), variation is done if east-west oriented baseline is chosen.

(b) Baseline orientations: Baseline length of 250 Km is considered for atmosphere unknown scenario. Two different orientations of baseline ar considered, namely, east-west (E-W) oriented baseline (only longitudinal orientation is performed) and north-south (N-S) (only latitudinal variation is performed) oriented baseline.

(c) Frequency combinations: For GPS only system, three frequency combination, namely single ($L1$), dual ($L1$ and $L2$) and triple ($L1, L2$ and $L5$) are considered. For Galileo only, four combinations, namely, single ($E1$), dual ($E1$ and $E5a$), triple ($E1, E5a$ and $E5b$) and quadruple ($E1, E5a, E5b$ and $E6$) are considered. For a combined GPS and Galileo system three combination are considered, namely single ($L1(E1)$), dual ($L1(E1), L5(E5a)$) and quadruple ($L1(E1), L5(E5a)$, $L2$, $E5b$). Triple frequency combination is not considered on the understanding that the high-end receivers will have options to either lock two frequencies of each GPS and Galileo or three frequencies of each GPS and Galileo, bringing to the frequency combination to a dual (two overlapping frequencies) or a quadruple frequency combination (two overlapping and two non-overlapping frequencies).

For the above 6 combination of scenarios, all the measurement precision values (6 different values as shown in Table 3.1) will be considered. All the satellites above $10°$ elevation are considered for simulation. The three different receiver types, low-end single frequency, high-end dual frequency, high-end triple frequency and high-end quadruple frequency will be considered. The definition of the type of receiver is based on the value of measurement precision chosen (see Tables 3.1 and 3.2). The interval between the epochs is considered to be 1 second (sampling interval).

The estimation of the ambiguities is based on the stacking of the normal equations among epochs. Both full AR and PAR are simulated for Geometry

Based model for all the above mentioned scenarios.

6.4 Reference-Rover model, atmosphere known scenario

The atmosphere is considered to be insignificant for relative positioning when short baselines are considered. Such a scenario is termed as atmosphere fixed (known) while parameterization of the GNSS observables. For a Geometry Based model, both ionosphere and troposphere are parameterized for a geometry based model, unlike geometry free model, where the troposphere is lumped with the ranges. In this scenario, the receiver coordinates are considered to be unknown and are parameterized, satellite coordinates are assumed to be known and are generated from YUMA almanacs. The baseline length is considered as 1 Kilometre. All the different types of frequency combinations, namely single, dual, triple and quadruple will be considered. Presented below is the functional and stochastic models, see Table 6.1.

Reference-Rover model, atmosphere known scenario		
Functional model		**Stochastic model**
Non-temporal parameters	Temporal parameters	
$A_{I(i)} = \begin{bmatrix} \begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes I_{m-1} & \begin{pmatrix} e_f \\ e_f \end{pmatrix} \otimes G_{m-1} \end{bmatrix}$		$Q_{y(i)} = \begin{bmatrix} Q_\Phi & \\ & Q_P \end{bmatrix} \otimes Q_{DD}(i)$ where $Q_{DD}(i) = D_m^T W_i D_m$
Redundancy (for GPS only, Galileo only, GPS+Galileo (common frequency $L1(E1), L5(E5a)$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $f * (m-1)$		$k * 2f * (m-1)$
Coordinates: $3(n-1) = 3$		
Redundancy of Reference-Rover - atmosphere known scenario		$\{(2k-1) * f(m-1)\} - 3$
For GPS/Galileo only: $f = (1, \cdots, j)$, for GPS+Galileo: $f = (1, \cdots, j_c)$, j_c is the common/overlapping frequency, and $m_{jc} = (m_{GPS} + m_{Gal})$		
Redundancy (for GPS+Galileo (quadruple frequency $L1(E1), L5(E5a), L2, E5b$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)$		$4 * k * (m_{jc} - 1) + 2 * k((m_{GPS} - 1) + (m_{Gal} - 1))$
Coordinates: $3(n-1) = 3$		
Redundancy of Reference-Rover, atmosphere known scenario		$\{(2k-1)\,[2*(m_{jc}-1) + (m_{GPS}-1) + (m_{Gal}-1)]\} - 3$
$f = 2$ for GPS+Gaileo $L1(E1), L5(E5a)$ for atmosphere known model		
m_{jc} are the total number of satellites for combined GPS+Galileo system with GPS as reference satellite		

Table 6.1: Double-differenced Design matrix and VC matrix for Reference-Rover model, Atmosphere known scenario

In Table 6.1, G_{m-1} has the Line of Sight (LOS) vectors, given as,

$$G_{m-1} = \begin{bmatrix} -u^{1-2} \\ \vdots \\ -u^{1-m} \end{bmatrix}$$

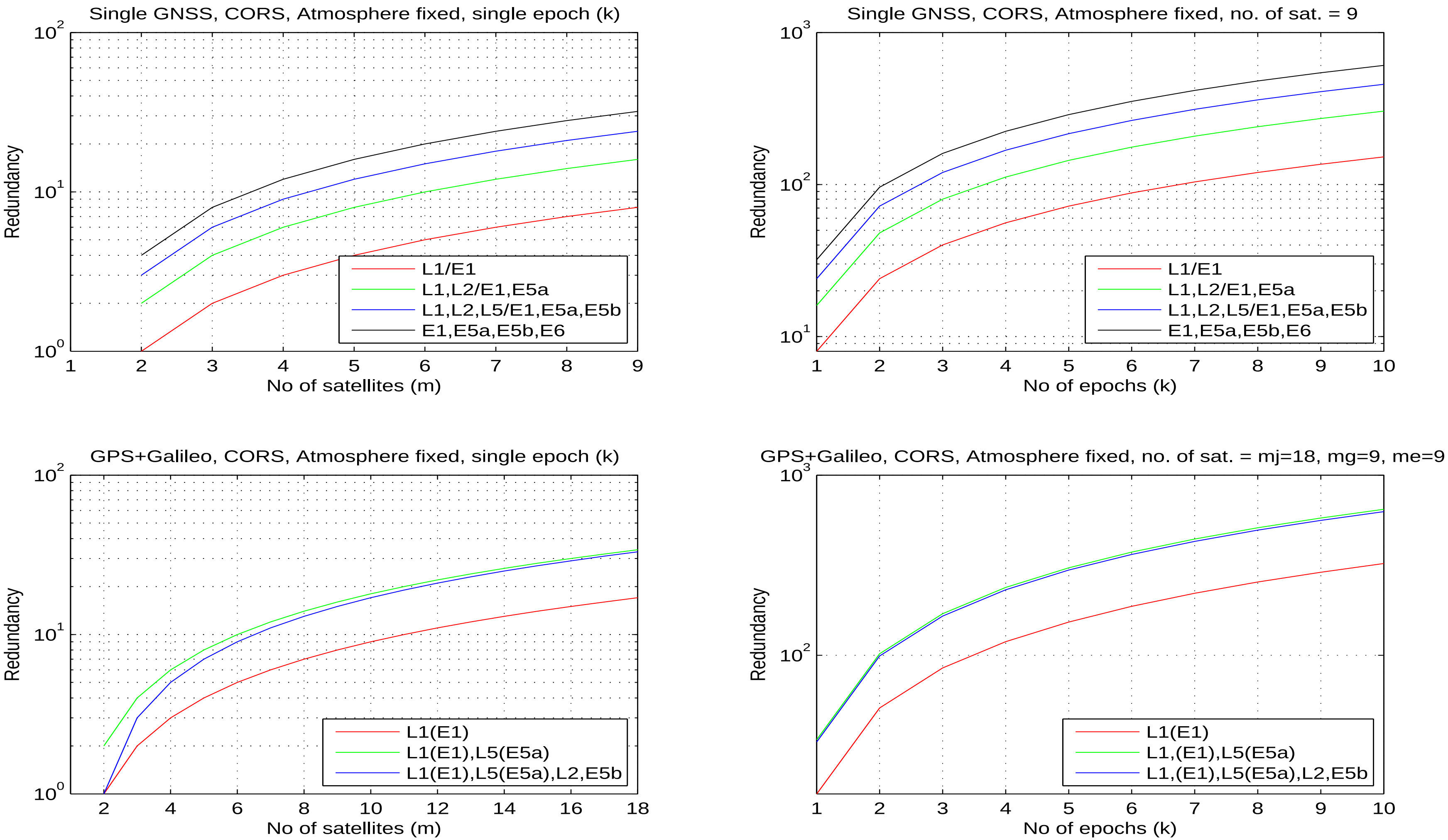

Figure 6.1: Redundancy plot of **GPS only**, **Galileo only** (top two plots) and **GPS + Galileo** (bottom two plots) for single epoch (left hand side plots) and multi-epoch (right hand side plots), atmosphere known scenario, **Geometry based model, Receiver coordinates for the rover are unknown**, single, dual, triple and quadruple frequencies

6.4.1 Hourly batches, Full Ambiguity Resolution, atmosphere known scenario

Initially, ASR is simulated for hourly batches, throughout the day 24 hourly batches are formed starting from 0000 UTC and ending at 2359 UTC. The interval between the epochs is taken as 1 second, making the total number of epochs to be 3600 in each batch. The number of epochs taken in order to achieve 0.999 ASR during different times of the day are simulated and their average behaviour is computed. The results for single, dual, triple (only for GPS and Galileo standalone systems) and quadruple (only for Galileo and GPS+Galileo system) frequency for GPS only, Galileo only and with both systems GPS and Galileo together are presented in the Figures G.1 to G.3, see Appendix G. The results for average number of epochs taken for each of the GNSS system are also presented in Table 6.2, see below.

Analysis of the averaged 24 hourly batches from Table 6.2 and Figures G.1 to G.3 (in Appendix G is presented below.

General conclusions:

1. It can be noted from Table 6.2 that for all the GNSS systems and their combinations considered, for frequency combinations higher than single, that is dual, tripe and quadruple, instantaneously the criteria to obtain 0.999 ASR is met. Hence for dual and higher frequency combinations, results for the worst measurement precision are only presented. Further in this section results of single frequency only will be discussed.

2. It can be noted that for both E-W and N-S baseline, the epochs taken for 0.999 ASR and number of common and low-elevation satellites remain same. The reason being, for atmosphere known scenario, baseline length of 1 Km is considered which leaves the above mentioned parameters with no variation what so ever for E-W and N-S baseline. Hence forth in this section only results for E-W baseline will be discussed.

GPS only

Hourly, Full AR - minimum desired ASR = 0.999

Phase	Code	Phase	Code	Average no. of Epochs taken			Average no. of satellites			Average no. of Low-elevation Sat.$\geqslant 10°, \leqslant 30°$			Average no. of Epochs taken			Average no. of satellites			Average no. of Low-elevation Sat.$> 10°, 30°$		
(meters)				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
				East-West Baseline - 1Km									**North-South Baseline - 1Km**								
GPS only																					
Φ	*P*	Φ	*P*	High-end receivers, Single Frequency																	
L1																					
0.003	0.25	-	-	5	6	6	10	9	10	5	3	5	5	6	6	10	9	10	5	3	5
0.003	0.50	-	-	9	11	10	10	9	10	5	3	5	9	11	10	10	9	10	5	3	5
0.003	0.75	-	-	11	15	13	10	9	10	5	3	5	11	15	13	10	9	10	5	3	5
0.003	1.00	-	-	13	19	15	10	9	10	5	3	5	13	19	15	10	9	10	5	3	5
0.003	1.25	-	-	15	21	17	10	9	10	5	3	5	15	21	17	10	9	10	5	3	5
0.003	1.50	-	-	16	23	19	10	9	10	5	3	5	16	23	19	10	9	10	5	3	5
Φ	*P*	Φ	*P*	High-end receivers, Dual Frequency																	
L1,L2																					
0.003	0.35	-	-	1	1	1	10	9	10	5	3	5	1	1	1	10	9	10	5	3	5
Φ	*P*	Φ	*P*	High-end receivers, Triple Frequency																	
L1,L2		L5																			
0.003	0.35	0.002	0.2	1	1	1	10	9	10	5	3	5	1	1	1	10	9	10	5	3	5
Galileo only																					
Φ	*P*	Φ	*P*	High-end receivers, Single Frequency																	
E1																					
0.003	0.25	-	-	5	10	6	10	9	10	5	4	5	5	10	6	10	9	10	5	4	5
0.003	0.50	-	-	7	17	10	10	9	10	5	4	5	7	17	10	10	9	10	5	4	5
0.003	0.75	-	-	9	21	13	10	9	10	5	4	5	9	21	13	10	9	10	5	4	5
0.003	1.00	-	-	11	24	15	10	9	10	5	4	5	11	24	15	10	9	10	5	4	5
0.003	1.25	-	-	12	26	17	10	9	10	5	4	5	12	26	17	10	9	10	5	4	5
0.003	1.50	-	-	13	27	18	10	9	10	5	4	5	13	27	18	10	9	10	5	4	5
Φ	*P*	Φ	*P*	High-end receivers, Dual Frequency																	
E1		E5a																			
0.003	0.35	0.002	0.2	1	1	1	10	9	10	5	4	5	1	1	1	10	9	10	5	4	5
Φ	*P*	Φ	*P*	High-end receivers, Triple Frequency																	
E1		E5a,E5b																			
0.003	0.35	0.002	0.2	1	1	1	10	9	10	5	4	5	1	1	1	10	9	10	5	4	5
Φ	*P*	Φ	*P*	High-end receivers, Quadruple Frequency																	
E1		E5a,E5b,E6																			
0.003	0.35	0.002	0.2	1	1	1	10	9	10	5	4	5	1	1	1	10	9	10	5	4	5
GPS + Galileo																					
Φ	*P*	Φ	*P*	High-end receivers, Single Frequency																	
L1(E1)																					
0.003	0.25	-	-	1	1	2	20	17	19	10	7	9	1	1	2	20	17	19	10	7	9
0.003	0.50	-	-	2	2	2	20	17	19	10	7	9	2	2	2	20	17	19	10	7	9
0.003	0.75	-	-	2	2	2	20	17	19	10	7	9	2	2	2	20	17	19	10	7	9
0.003	1.00	-	-	2	2	2	20	17	19	10	7	9	2	2	2	20	17	19	10	7	9
0.003	1.25	-	-	2	2	2	20	17	19	10	7	9	2	2	2	20	17	19	10	7	9
0.003	1.50	-	-	2	3	3	20	17	19	10	7	9	2	3	3	20	17	19	10	7	9
Φ	*P*	Φ	*P*	High-end receivers, Dual Frequency																	
L1(E1)		L5(E5a)																			
0.003	0.35	0.002	0.2	1	1	1	20	17	19	10	7	9	1	1	1	20	17	19	10	7	9
Φ	*P*	Φ	*P*	High-end receivers, Quadruple Frequency																	
L1(E1),L2,E5b		L5(E5a)																			
0.003	0.35	0.002	0.2	1	1	1	20	17	19	10	7	9	1	1	1	20	17	19	10	7	9

Table 6.2: **GPS only, Galileo only, GPS + Galileo** GNSS systems, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario **Full AR**, Average number of epochs, average number of satellites and average number of low elevation satellites over 24 batches (1 batch = 1 hour, 3600 epochs, 1 second interval) are presented - **measurement precision varied**, number of satellites as available.

3. With single frequency GPS, the average number of epochs (for 0.999 ASR) lie between 5 and 23 for different measurement precision scenarios for E-W baseline. It can be noted that with E-W baseline (1 Km baseline length), the worst performance is for $-30°$ latitude.

Galileo only

4. For single frequency Galileo, the average number of epochs taken lie between 5 and 27 for E-W baseline. Galileo only outperforms GPS only at $0°$ latitude for baseline lengths of 10 Km and above, see Table 6.2. Also it can be noted that at $-30°$ latitude, Galileo performs worst than GPS only. It can be noted that at $-30°$ Galileo has on an average 4 low elevation satellites, whereas GPS had 3. The average number of common satellites remains same for both the systems.

GPS + Galileo

5. For a combined GPS and Galileo system, it takes an average number of epochs between 1 (instantaneous, on two occasions) and 3 for E-W baseline for different scenarios of measurement precision and latitude location. In comparison to GPS only and Galileo only, the combined GPS and Galileo systems performs much better. It wont be a surprise if with PAR, instantaneous results could be obtained for single frequency GNSS systems for RR model, atmosphere known scenario when a fixed-precision of 2cm is aimed for.

Since for atmosphere known scenario, reference-rover model, only the single frequency simulations could not give instantaneous ASR of 0.999, for further analysis only single frequency results will be evaluated.

6.4.2 GPS only - Full and Partial Ambiguity Resolution, atmosphere known scenario

The results for GPS system, Geometry based model, coordinates of the rover being unknown, atmosphere (both ionosphere and troposphere) known scenario are presented below. Figure G.4 give full ambiguity resolution results and Figure G.7 present the partial ambiguity results for single frequency GPS, see Appendix G. Table 6.3 presents the full ambiguity resolution results in terms of number of epochs taken for 0.999 ASR and corresponding rover-receiver coordinate precision (for ambiguity-fixed and -float solution) along with the gain obtained for the rover-receiver coordinate for ambiguity-fixed solution is presented.

Phase	Code	Phase	Code	Full AR - minimum desired ASR = 0.999											
				Epochs taken			$\sigma_{\check{neu}}$(meters)			$\sigma_{\hat{neu}}$(meters)			Gain=$\sigma_{\hat{neu}}/\sigma_{\check{neu}}$		
(meters)				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 1Km															
Φ	P	Φ	P	High-end receivers, Single Frequency											
L1															
0.003	0.25	-	-	6	4	5	0.003	0.003	0.003	0.245	0.283	0.264	83.28	83.32	83.30
0.003	0.50	-	-	10	7	9	0.002	0.003	0.002	0.377	0.426	0.391	165.44	166.12	165.74
0.003	0.75	-	-	15	9	11	0.002	0.002	0.002	0.449	0.559	0.524	241.18	246.97	245.44
0.003	1.00	-	-	19	12	12	0.002	0.002	0.002	0.503	0.630	0.657	303.79	321.26	321.07
0.003	1.25	-	-	22	13	13	0.002	0.002	0.002	0.541	0.736	0.767	351.56	390.76	390.46
0.003	1.50	-	-	24	16	14	0.001	0.002	0.002	0.572	0.749	0.856	388.54	441.35	452.08
North-South Baseline - 1Km															
Φ	P	Φ	P	High-end receivers, Single Frequency											
L1															
0.003	0.25	-	-	6	4	5	0.003	0.003	0.003	0.245	0.283	0.264	83.28	83.32	83.30
0.003	0.50	-	-	10	7	9	0.002	0.003	0.002	0.377	0.426	0.391	165.44	166.12	165.74
0.003	0.75	-	-	15	9	11	0.002	0.002	0.002	0.449	0.559	0.524	241.18	246.97	245.44
0.003	1.00	-	-	19	12	12	0.002	0.002	0.002	0.503	0.630	0.657	303.79	321.26	321.07
0.003	1.25	-	-	22	13	13	0.002	0.002	0.002	0.541	0.736	0.767	351.56	390.76	390.46
0.003	1.50	-	-	24	16	14	0.001	0.002	0.002	0.572	0.749	0.856	388.54	441.35	452.08

Table 6.3: **GPS only, Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, **Full AR** analysis, ASR and ambiguity-fixed and -float precision for rover-receiver coordinate are presented - **measurement precision varied**, number of satellites as available.

The following is the analysis based on Table 6.3 and Figures 6.2 to 6.3.

General remarks:

(1) It can be noted that for Reference-Rover model, atmosphere known scenario (when ionosphere and troposphere are known), instantaneous ASR (Full AR) of 0.999 is not achieved.

(2) There is no difference in number of epochs taken for 0.999 ASR for E-W and N-S oriented baseline, since the baseline length considered for atmosphere known scenario is 1 Km. The number of common satellites and the elevation of satellites do not change significantly for 1 Km baseline for the time period considered (0000 to 0010 UTC) causing the results to be similar. Hence forth only E-W oriented baseline results will be discussed for atmosphere known scenario.

For East-West baseline:

(3) For single frequency GPS, it took between 4 and 24 epochs to obtain 0.999 ASR (Full AR) for different scenarios of measurement precision and baseline length. With measurement precision of measurement precision of 3mm on phase and 50cm on code on $L1$, it took 10, 7 and 9 epochs at $0°$, $-30°$ and $-60°$ latitude locations.

Considering the fixed-precision of rover-receiver coordinate to lie around 3mm to 1.3cm when all the ambiguities were fixed, PAR will be evaluated for obtaining a fixed-precision of σ_x, σ_y =2cm and σ_z=6cm for rover coordinates in the coming section.

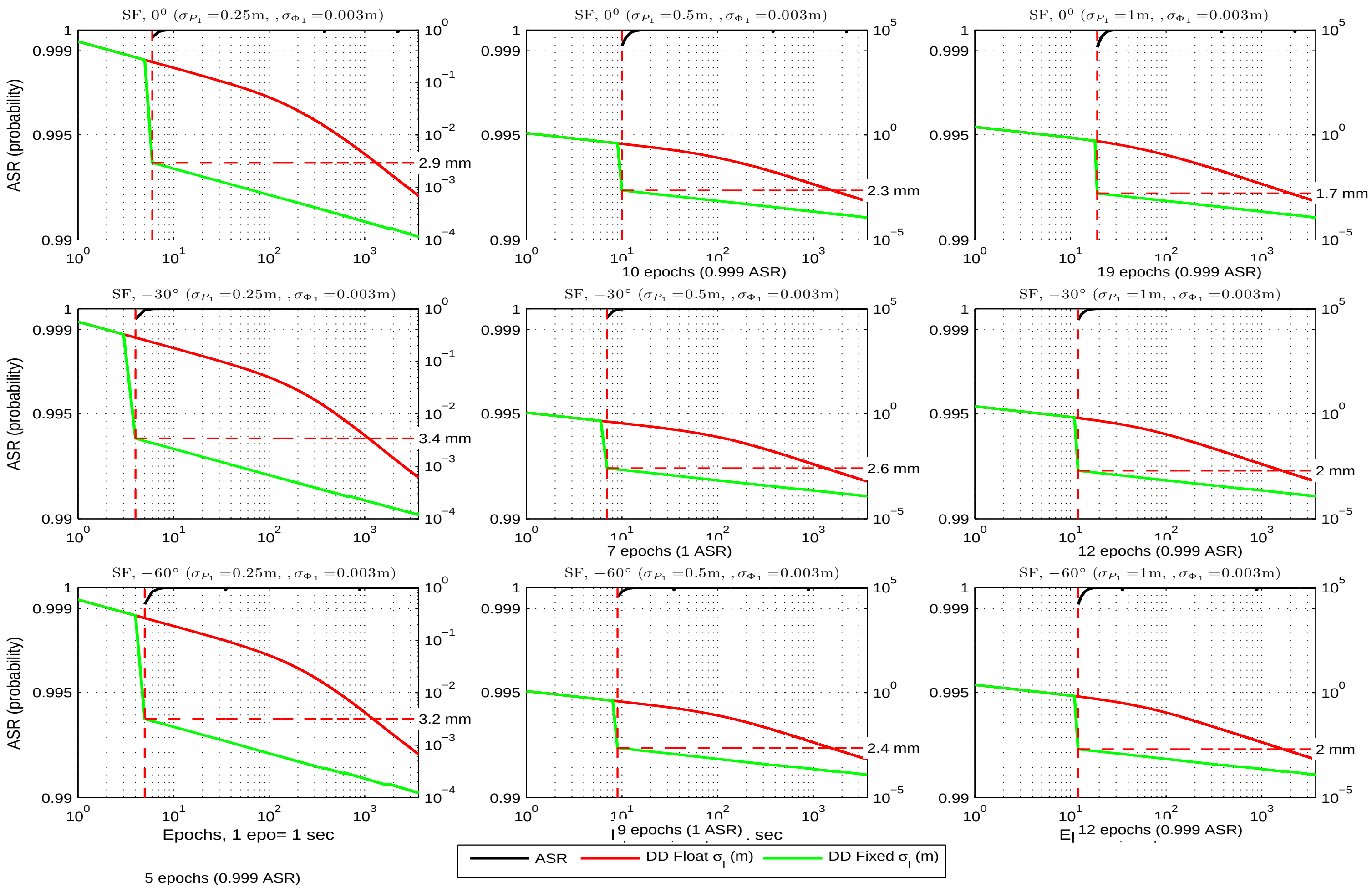

Figure 6.2: **GPS only**, Single frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**-ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

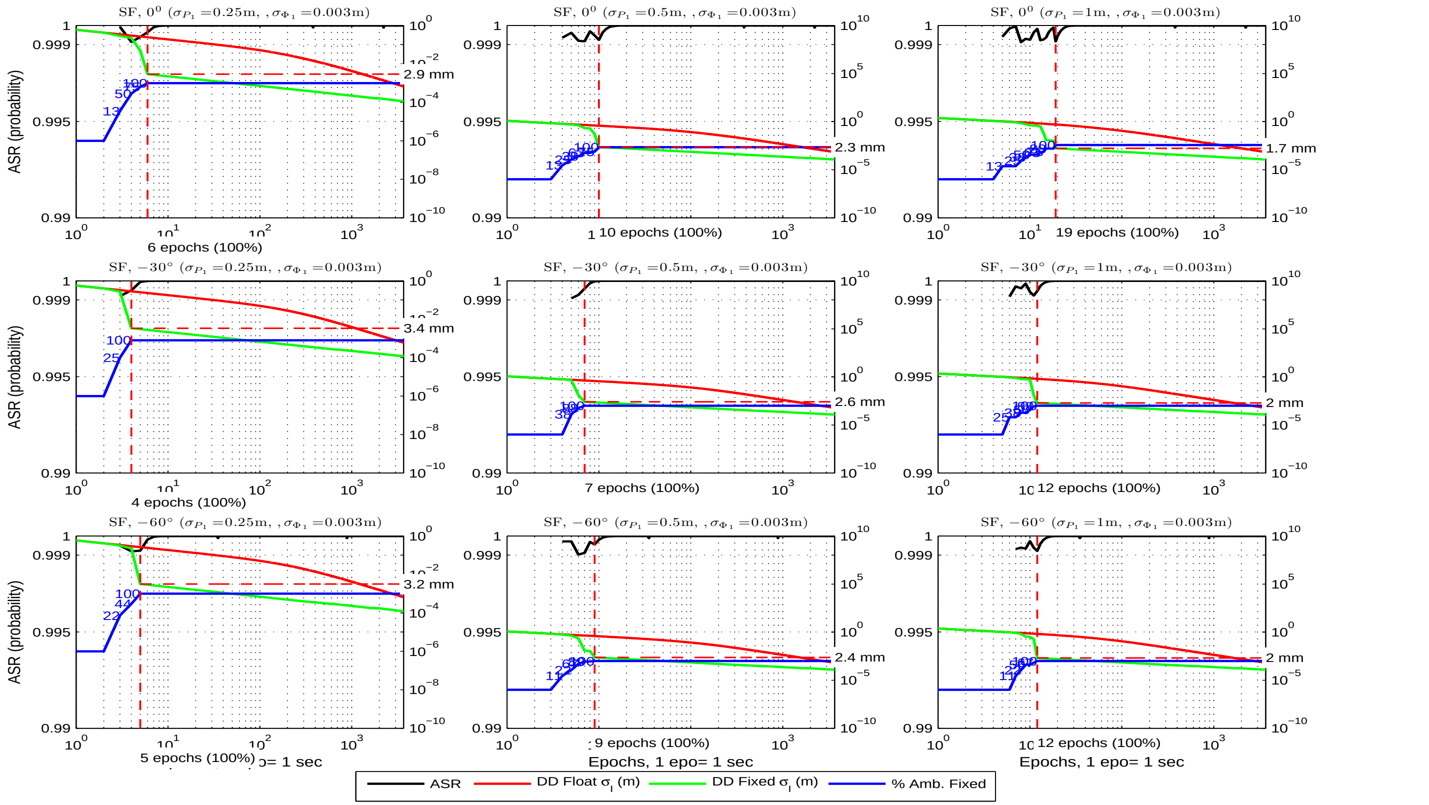

Figure 6.3: **GPS only**, Single frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

6.4.3 GPS only, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, atmosphere known scenario

In this section, it is to be analyzed if by partially fixing the ambiguities, does the solution converge faster to a modestly chosen fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for rover-receiver coordinate. Table 6.4 presents the results in terms of number of epochs taken for the criteria laid for fixed-precision along with the percentage of partial ambiguities fixed.

Partial AR @ 0.999 ASR criteria - σ_n, σ_e =2cm and σ_u=6cm fixed-precision for rover-receiver coordinate																		
Phase	Code	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\check{neu}}$		
	(meters)			0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 1Km																		
Φ	P	Φ	P	High-end receivers, Single Frequency														
L1																		
0.003	0.25	-	-	6	4	5	100	100	100	0.003	0.003	0.003	0.245	0.283	0.264	83.28	83.32	83.30
0.003	0.50	-	-	10	6	7	100	88	89	0.002	0.013	0.012	0.377	0.461	0.445	165.44	36.81	38.39
0.003	0.75	-	-	13	9	10	88	100	89	0.012	0.002	0.010	0.487	0.559	0.552	41.45	246.97	55.63
0.003	1.00	-	-	15	11	12	88	88	100	0.012	0.012	0.002	0.584	0.661	0.657	50.54	53.63	321.07
0.003	1.25	-	-	17	13	13	88	100	100	0.011	0.002	0.002	0.652	0.736	0.767	58.14	390.76	390.46
0.003	1.50	-	-	18	15	14	88	88	100	0.011	0.011	0.002	0.721	0.784	0.856	64.87	70.40	452.08

Table 6.4: **GPS only**, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario. Presented is the evaluation of ASR and number of epochs for obtaining σ_n, σ_e =2cm and σ_u=6cm fixed-precision of rover-receiver coordinate - **measurement precision is varied**, number of satellites are as available.

Referring Table 6.4, for all the scenarios of measurement precision (out of 18 scenarios for each dual and triple frequency) less than 100% of ambiguities were fixed to give a fixed-precision of 4mm to 2cm.

General remarks::

(1) A fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for rover-receiver coordinates could not be achieved instantaneously with single frequency GPS system.

For East-West baseline:

(2) For 9 out of 18 scenarios less than 100 % of ambiguities were fixed to obtain a fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for rover-receiver coordinates.

(3) For single frequency with measurement precision of 3mm on phase and 50cm on code on $L1$, it took 10, 6 and 7 epochs epochs by fixing 100, 88 and 89% of ambiguities by PAR at 0°, −30° and −60° latitude.

In the above analysis, a fixed-precision for the rover coordinate components with a predefined precision of σ_n, σ_e =2cm and σ_u=6cm was aimed with an aim to understand how would PAR facilitate high precision users. Further another analysis for PAR while a fixed-precision of 20 cm for σ_n, σ_e is presented below in order to understand as to how would PAR benefit users aiming at 20cm precision.

6.4.4 GPS only, Partial Ambiguity Resolution to obtain fixed-precision of 20 cm, atmosphere known scenario

The promise of PAR is yet again evaluated while a fixed-precision of σ_n, σ_e =20cm is aimed for rover-receiver coordinate. It can be noted that in this analysis only horizontal fixed-precision is analyzed, this is because of the fact that the up-component, σ_u, of the coordinates does not converge to a fixed-precision of such an order for majority of the cases until all the ambiguities are successfully fixed. Table 6.5 presents the results in terms of number of epochs taken for the criteria laid for fixed-precision along with the percentage of partial ambiguities fixed.

				Partial AR @ 0.999 ASR criteria - σ_n, σ_e =20cm fixed-precision for rover-receiver coordinate														
Phase	Code (meters)	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$		
				$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$
East-West Baseline - 1Km																		
Φ	P	Φ	P	High-end receivers, Single Frequency														
L1																		
0.003	0.25	-	-	5	4	4	75	100	44	0.050	0.003	0.227	0.269	0.283	0.295	5.42	83.32	1.30
0.003	0.50	-	-	10	6	7	100	88	89	0.002	0.013	0.012	0.377	0.461	0.445	165.44	36.81	38.39
0.003	0.75	-	-	13	9	10	88	100	89	0.012	0.002	0.010	0.487	0.559	0.552	41.45	246.97	55.63
0.003	1.00	-	-	15	11	12	88	88	100	0.012	0.012	0.002	0.584	0.661	0.657	50.54	53.63	321.07
0.003	1.25	-	-	17	13	13	88	100	100	0.011	0.002	0.002	0.652	0.736	0.767	58.14	390.76	390.46
0.003	1.50	-	-	18	15	14	88	88	100	0.011	0.011	0.002	0.721	0.784	0.856	64.87	70.40	452.08

Table 6.5: **GPS only**, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario. Presented is the evaluation of ASR and number of epochs taken for obtaining σ_n, σ_e =20cm fixed-precision of rover-receiver coordinate - **measurement precision is varied**, number of satellites are as available.

Referring Table 6.5, for all the scenarios of measurement precision (out of 18 scenarios for each dual and triple frequency) less than 100% of ambiguities were fixed to give a fixed-precision of 4mm to 2cm.

General remarks::

(1) A fixed-precision of σ_n, σ_e =20cm for horizontal components of the rover-receiver coordinates could not be achieved instantaneously with single frequency GPS system.

For East-West baseline:

(2) The results for 20cm fixed-precision for the horizontal coordinate components are compared to the 2cm fixed-precision results.

(3) For only 2 scenarios, the number of epochs taken for 20cm fixed-precision were less by one than 2cm fixed-precision criteria.

6.4.5 Galileo only - Full and Partial Ambiguity Resolution, atmosphere known scenario

Single frequency Galileo system with is evaluated for atmosphere known scenario, Reference-Rover model (coordinates of the rover are considered to be unknown). The results for atmosphere (both ionosphere and troposphere) known, Reference-Rover model for Galileo system, single frequency are presented below. Figure G.10 gives full ambiguity resolution results and Figure G.14 present the partial ambiguity results, see Appendix G. Table 6.6 presents the full ambiguity resolution results in terms of number of epochs taken for 0.999 ASR and corresponding rover-receiver coordinate precision (for ambiguity-fixed solution and -float solution) along with the gain achieved for the fixed solution.

				Full AR - minimum desired ASR = 0.999												
Phase	Code	Phase	Code	Epochs taken			$\sigma_{\check{neu}}$(meters)			$\sigma_{\hat{neu}}$(meters)			Gain=$\sigma_{\hat{neu}}/\sigma_{\check{neu}}$		
(meters)				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 1Km															
Φ	P	Φ	P	High-end receivers, Single Frequency											
E1															
0.003	0.25	-	-	4	3	3	0.004	0.585	0.531	0.300	0.585	0.531	83.32	1.00	1.00
0.003	0.50	-	-	6	4	4	0.003	1.170	1.061	0.489	1.170	1.061	166.28	1.00	1.00
0.003	0.75	-	-	9	6	4	0.002	1.755	1.592	0.593	1.755	1.592	247.06	1.00	1.00
0.003	1.00	-	-	10	6	5	0.002	2.340	2.123	0.740	2.340	2.123	325.02	1.00	1.00
0.003	1.25	-	-	11	7	5	0.002	2.925	2.653	0.865	2.925	2.653	398.09	1.00	1.00
0.003	1.50	-	-	12	8	6	0.002	3.509	3.184	0.966	3.509	3.184	464.67	1.00	1.00

Table 6.6: **Galileo only**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, **Full AR** analysis, ASR and ambiguity-fixed and -float precision for rover-receiver coordinate are presented - **measurement precision varied**, number of satellites as available.

The following is the analysis based on Table 6.6 and Figures 6.13 to 6.5.

General remarks:

(1) It can be noted that with Galileo system, Reference-Rover model, atmosphere known scenario, instantaneous ASR (Full AR) of 0.999 is not achieved.

For East-West baseline:

(2) With single frequency Galileo, it took between 3 and 12 epochs (**GPS only** took between 4 and 24 epochs) for different scenarios of measurement precision and latitude location. For single frequency (measurement precision of 3mm on phase and 50cm on code on $E1$), it took minimum of 6, 4 and 4 epochs (**GPS only** took 10, 7 and 9 epochs) at $0°$, $-30°$ and $-60°$ latitude for dual frequency for 0.999 ASR (Full AR).

Comparison with GPS only:

(3) It can be seen from the above presented comparison that Galileo performed better than GPS system.

Considering the fixed-precision of rover-receiver coordinates to lie around 3mm in most of the cases when all the ambiguities are fixed, PAR will be evaluated for obtaining a fixed-precision of 2cm for rover-receiver coordinates in the coming section.

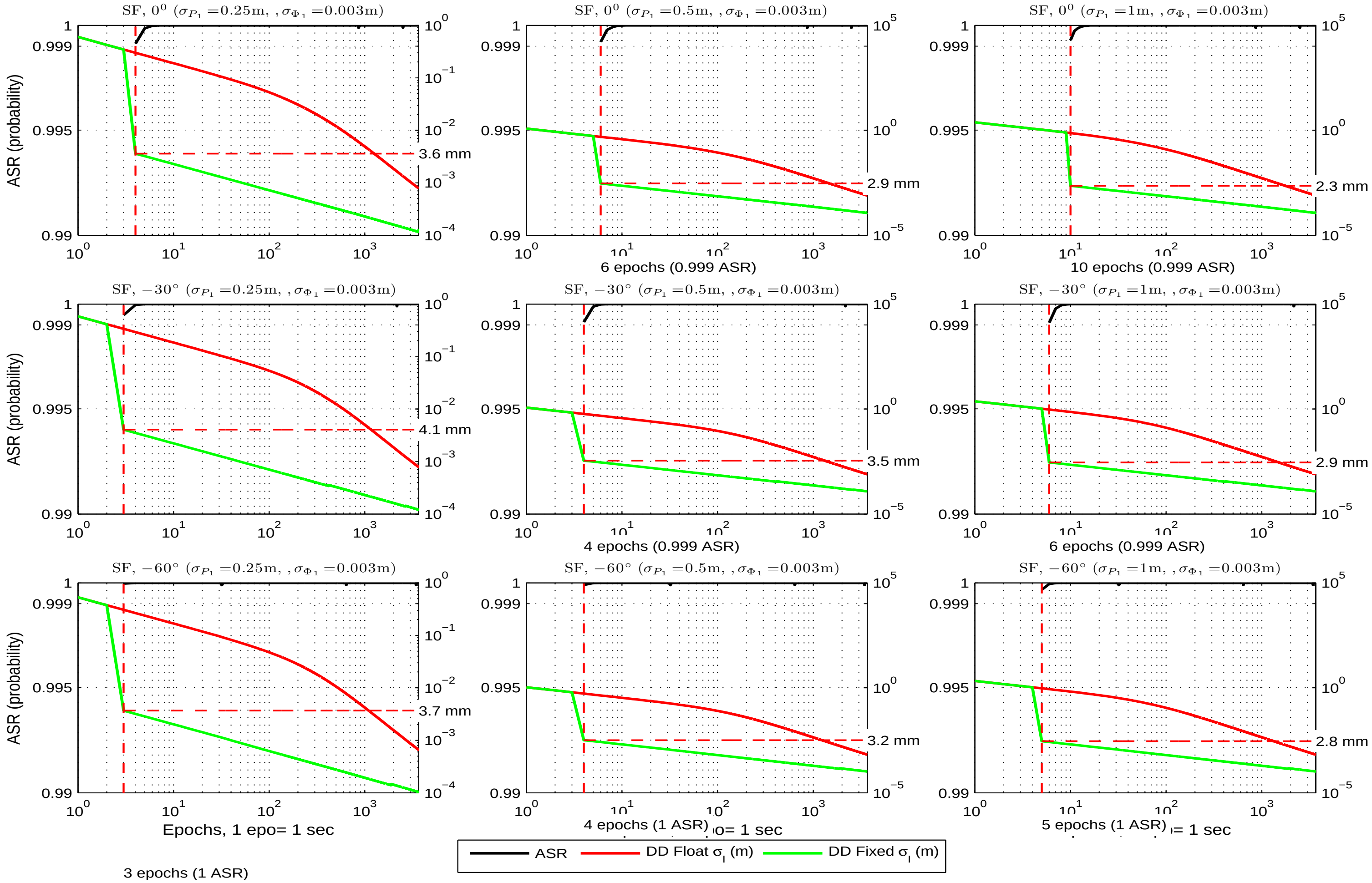

Figure 6.4: **Galileo only**, Single frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)-** ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

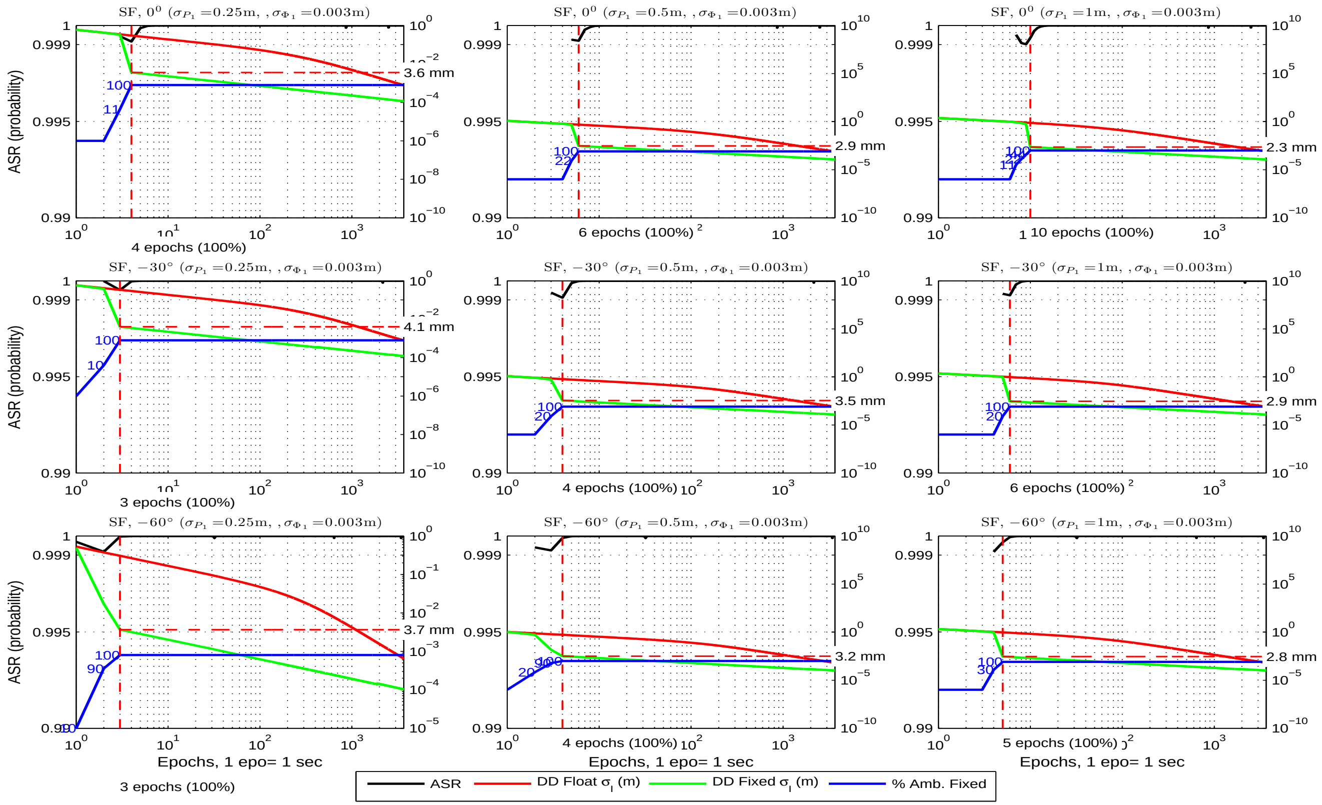

Figure 6.5: **Galileo only**, Single frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

Phase	Code	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$		
				Partial AR @ 0.999 ASR, criteria - σ_n, σ_e =2cm and σ_u=6cm fixed-precision for rover-receiver coordinates														
(meters)				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 1Km																		
Φ	P	Φ	P	High-end receivers, Single Frequency														
E1		E5a																
0.003	0.25	-	-	4	3	2	100	100	90	0.004	0.004	0.017	0.300	0.338	0.375	83.32	83.33	21.83
0.003	0.50	-	-	6	4	3	100	100	90	0.003	0.004	0.015	0.489	0.585	0.613	166.28	166.57	40.85
0.003	0.75	-	-	8	5	4	100	100	100	0.003	0.003	0.003	0.631	0.783	0.795	247.67	249.47	249.61
0.003	1.00	-	-	10	6	5	100	100	100	0.002	0.003	0.003	0.740	0.950	0.945	325.02	331.50	331.87
0.003	1.25	-	-	11	7	5	100	100	100	0.002	0.003	0.003	0.865	1.093	1.179	398.09	411.83	413.83
0.003	1.50	-	-	12	8	6	100	100	100	0.002	0.002	0.003	0.966	1.214	1.282	464.67	489.28	492.96

Table 6.7: **Galileo only**, **PAR** analysis, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario. Evaluation of ASR and number of epochs for obtaining σ_n, σ_e =2cm and σ_u=6cm fixed-precision of rover-receiver coordinates - **measurement precision is varied**, number of satellites are as available.

6.4.6 Galileo only, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, atmosphere known scenario

In this section, it is to be analyzed if partially fixing the ambiguities help to converge to a solution faster when a modest fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for rover-receiver coordinates is aimed for. Table 6.7 presents the results in terms of fixed-precision to be equal to or better than σ_n, σ_e =2cm and σ_u=6cm, corresponding epochs taken and ambiguities fixed are noted.

The following is the analysis based on Table 6.7 and Figure 6.5.

For East-West baseline:

(1) For 1 scenario out of the total 18 scenarios of measurement precision and latitude location less than 100% of ambiguities were fixed. Since it takes a maximum of 12 epochs for full AR, the fixed-precision as per the criteria laid could not be achieved. A sharp fall in fixed-precision can be seen in Figure 6.5 for partial ambiguity resolution.

(2) With single frequency Galileo, measurement precision of 3mm on phase and 50cm on code on $E1$, it took 6, 4 and 3 epochs to give a fixed-precision as per the criteria laid by partially fixing 100, 100 and

90% of ambiguities (**GPS only** took 10, 6 and 7 epochs epochs by fixing 100, 88 and 89% of ambiguities).

Comparison with GPS only:

(3) The performance for Galileo is better than GPS, even though partial ambiguity fixing was possible for 1 scenario of Galileo as compared to 9 scenarios of GPS.

In the above analysis, the partial ambiguities fixed and number of epochs taken to obtain a fixed-precision of 2cm was presented. The criteria set for the coordinate components of 2cm appeals more for precise positioning requirements. Hence further a fixed-precision of 20cm for the horizontal coordinate components is aimed for and corresponding ambiguities fixed and epochs taken are presented. The analysis for number of epochs required to obtain a fixe precision of 20cm for the horizontal coordinate components is presented below.

Phase	Code	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widecheck{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widecheck{neu}}$		
Partial AR @ 0.999 ASR criteria - σ_n, σ_e =20cm fixed-precision for rover-receiver coordinates																		
(meters)				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 1Km																		
Φ	P	Φ	P	High-end receivers, Single Frequency														
E1		E5a																
0.003	0.25	-	-	4	3	2	100	100	90	0.004	0.004	0.017	0.300	0.338	0.375	83.32	83.33	21.83
0.003	0.50	-	-	6	4	3	100	100	90	0.003	0.004	0.015	0.489	0.585	0.613	166.28	166.57	40.85
0.003	0.75	-	-	8	5	4	100	100	100	0.003	0.003	0.003	0.631	0.783	0.795	247.67	249.47	249.61
0.003	1.00	-	-	10	6	5	100	100	100	0.002	0.003	0.003	0.740	0.950	0.945	325.02	331.50	331.87
0.003	1.25	-	-	11	7	5	100	100	100	0.002	0.003	0.003	0.865	1.093	1.179	398.09	411.83	413.83
0.003	1.50	-	-	12	8	6	100	100	100	0.002	0.002	0.003	0.966	1.214	1.282	464.67	489.28	492.96

Table 6.8: **Galileo only**, **PAR** analysis, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario. Evaluation of ASR and number of epochs for obtaining σ_n, σ_e =20cm fixed-precision of rover-receiver coordinates - **measurement precision is varied**, number of satellites are as available.

6.4.7 Galileo only, Partial Ambiguity Resolution to obtain fixed-precision of 20 cm, atmosphere known scenario

The results and the discussions to obtain a fixed-precision of 20cm for the horizontal coordinate components σ_n, σ_e is presented below, see Table 6.8. Table 6.8 presents the results in terms of fixed-precision to be equal to or better than σ_n, σ_e =20cm corresponding epochs taken and ambiguities fixed are presented for discussion.

The analysis for obtaining fixed-precision of 20 cm is based on comparison with the criteria of obtaining fixed-precision of 2cm. The results obtained, see Table 6.8 are similar to the results obtained for obtaining a fixed-precision of 2cm. That is the intermediate levels of fixed-precision of 20cm up to 2cm could not be reached for Galileo only Reference Rover model, atmosphere fixed scenario.

6.4.8 GPS + Galileo - Full and Partial Ambiguity Resolution, atmosphere known scenario

The ASR for the combined GPS and Galileo system are simulated for Geometry Based model, coordinates of the rover are unknown, atmosphere known scenario, fixed-precision of the rover-receiver coordinates and other unknowns are simulated along with ASR. Table 6.9 and Figure G.18 and G.19 (see Appendix G) present the results of full and PAR for single frequency combined GPS and Galileo system for 1 Km baseline in E-W direction.

				Full AR - minimum desired ASR = 0.999											
Phase	Code	Phase	Code	Epochs taken			$\sigma_{\check{neu}}$(meters)			$\sigma_{\hat{neu}}$(meters)			Gain=$\sigma_{\hat{neu}}/\sigma_{\check{neu}}$		
(meters)				$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$	$0°$	$-30°$	$-60°$
East-West Baseline - 1Km															
Φ	P	Φ	P	High-end receivers, Single Frequency											
L1(E1)															
0.003	0.25	-	-	1	1	1	0.005	0.005	0.005	0.412	0.394	0.388	83.34	83.34	83.34
0.003	0.50	-	-	2	2	1	0.003	0.003	0.005	0.583	0.556	0.776	166.64	166.65	166.67
0.003	0.75	-	-	2	2	2	0.003	0.003	0.003	0.874	0.835	0.823	249.91	249.92	249.91
0.003	1.00	-	-	2	2	2	0.003	0.003	0.003	1.165	1.112	1.096	333.11	333.14	333.11
0.003	1.25	-	-	2	2	2	0.003	0.003	0.003	1.456	1.390	1.370	416.24	416.30	416.24
0.003	1.50	-	-	2	2	2	0.003	0.003	0.003	1.747	1.668	1.643	499.25	499.36	499.25

Table 6.9: **GPS + Galileo**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, **Full AR** analysis, ASR and ambiguity-fixed and -float precision for rover-receiver coordinates are presented - **measurement precision varied**, number of satellites as available.

Analysis based on Figures 6.16 and Table 6.9 is given below.

General remarks:

(1) Instantaneous ambiguity resolution could be achieved for 4 scenarios of measurement precision for single frequency, combined GPS and Galileo system.

For East-West baseline:

(2) At $-30°$ latitude with measurement precision of 3mm on phase and 50cm on code on $L1(E1)$, it took 2, 2 and 1 epochs (**GPS only** took 10,

7 and 9 epochs, **Galileo only** took 6,4 and 4 epochs) at 0°, −30° and −60° latitude .

Comparison with GPS only and Galileo only systems:

(3) The combined GPS and Galileo systems performs very well than the individual systems with instantaneous AR (0.999 ASR) for 4 scenarios and further took 2 epochs to achieve 0.999 ASR for the remaining 12 scenarios.

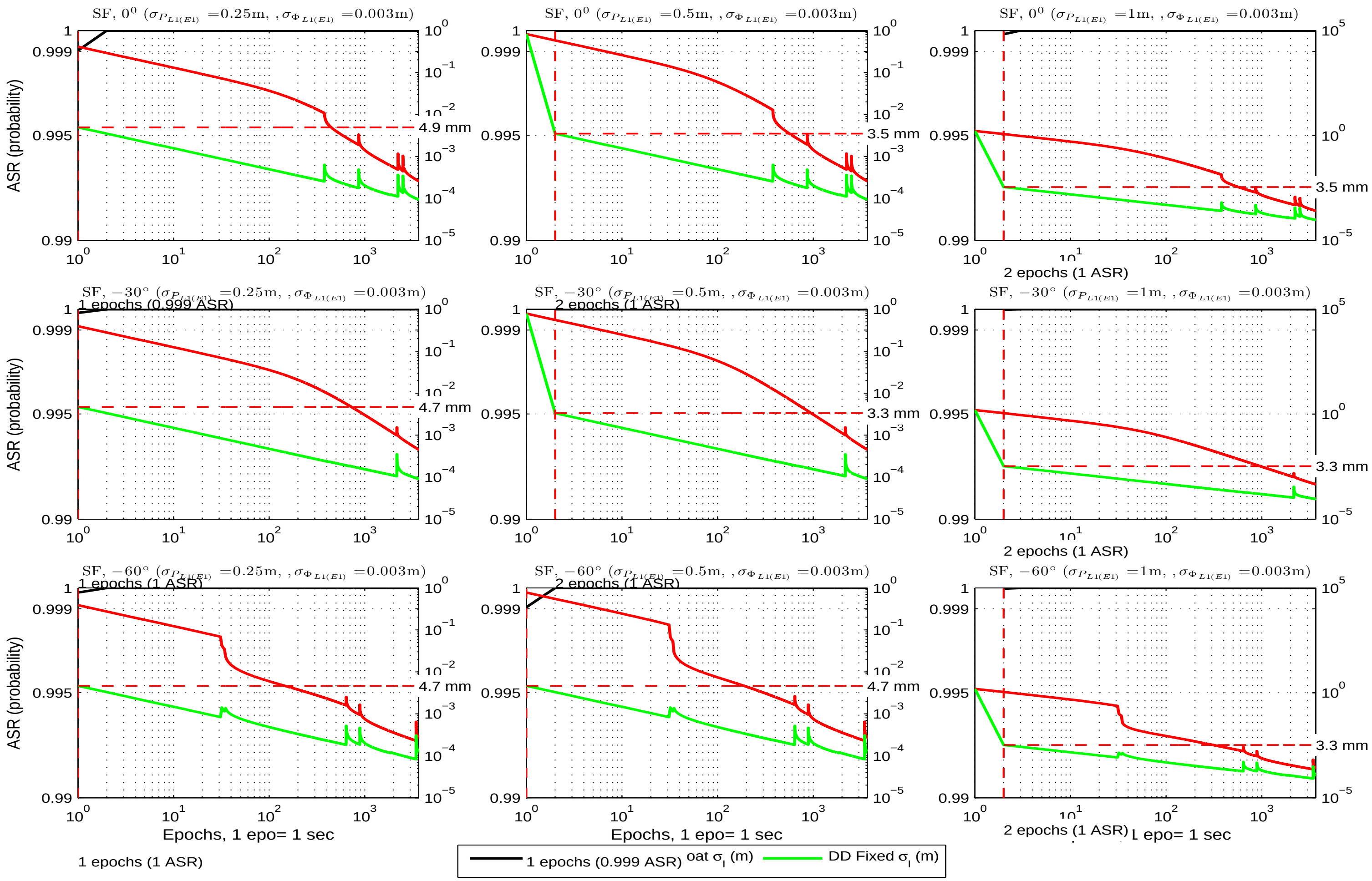

Figure 6.6: **GPS + Galileo**, Single frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

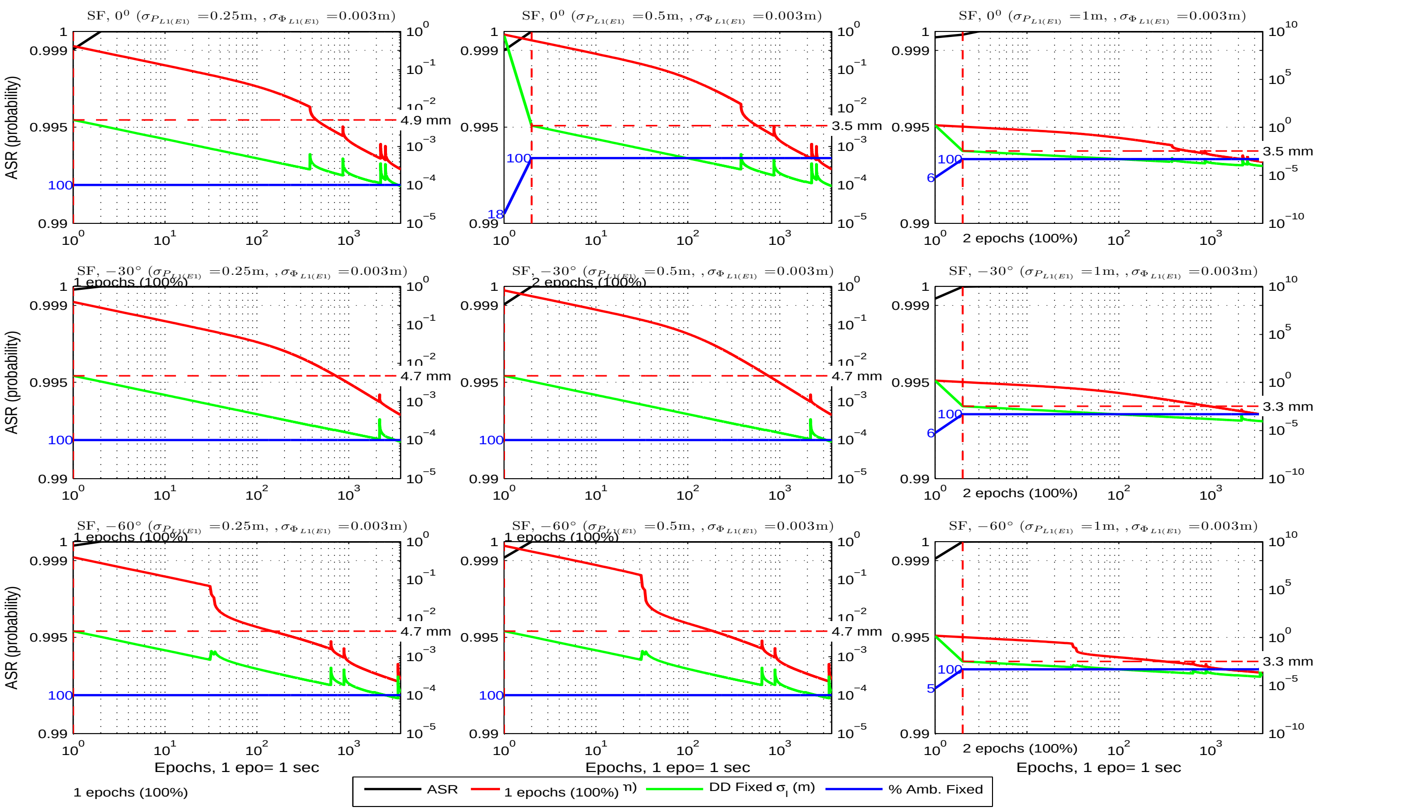

Figure 6.7: **GPS + Galileo**, Single frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

				Partial AR @ 0.999 ASR criteria - σ_n, σ_e =2cm and σ_u=6cm fixed-precision for rover-receiver coordinates														
Phase	Code (meters)	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$		
				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 1Km																		
Φ L1(E1)	P	Φ	P	High-end receivers, Single Frequency														
0.003	0.25	-	-	1	1	1	100	100	100	0.005	0.005	0.005	0.412	0.394	0.388	83.34	83.34	83.34
0.003	0.50	-	-	2	1	1	100	100	100	0.003	0.005	0.005	0.583	0.787	0.776	166.64	166.67	166.67
0.003	0.75	-	-	2	2	2	100	100	100	0.003	0.003	0.003	0.874	0.835	0.823	249.91	249.92	249.91
0.003	1.00	-	-	2	2	2	100	100	100	0.003	0.003	0.003	1.165	1.112	1.096	333.11	333.15	333.11
0.003	1.25	-	-	2	2	2	100	100	100	0.003	0.003	0.003	1.456	1.390	1.370	416.24	416.30	416.24
0.003	1.50	-	-	2	2	2	100	100	100	0.003	0.003	0.003	1.747	1.668	1.643	499.26	499.36	499.26

Table 6.10: **GPS + Galileo**, **PAR** analysis, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario. Evaluation of ASR and number of epochs for obtaining σ_n, σ_e =2cm and σ_u=6cm fixed-precision of rover-receiver coordinates - **measurement precision is varied**, number of satellites are as available.

6.4.9 GPS + Galileo, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, atmosphere known scenario

While resolving all the ambiguities, a fixed-precision of 5mm and better was obtained. In this analysis it is to examine if a fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for the rover coordinate x, y and z component could be obtained in reduced number of epochs and by partial fixing of ambiguities. The results are presented in Table 6.10, see below.

Analysis based on Table 6.7 is given below.

General remarks:

(1) For all the scenarios of measurement precision, 100% of ambiguities were fixed in order to obtain a fixed-precision of 2cm for rover-receiver coordinates.

Comparison with GPS only and Galileo only:

(2) The number of epochs taken for full AR (100% of ambiguities fixed in PAR) lie between 1 and 2. This performance of a combined GPS+Galileo system is much better than standalone GPS and Galileo systems.

Partial AR @ 0.999 ASR criteria - σ_n, σ_e =20cm fixed-precision for rover-receiver coordinates																			
Phase	Code	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$			
(meters)				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	
East-West Baseline - 1Km																			
Φ	P	Φ	P	High-end receivers, Single Frequency															
L1(E1)																			
0.003	0.25	-	-	1	1	1	100	100	100	0.005	0.005	0.005	0.412	0.394	0.388	83.34	83.34	83.34	
0.003	0.50	-	-	2	1	1	100	100	100	0.003	0.005	0.005	0.583	0.787	0.776	166.64	166.67	166.67	
0.003	0.75	-	-	2	2	2	100	100	100	0.003	0.003	0.003	0.874	0.835	0.823	249.91	249.92	249.91	
0.003	1.00	-	-	2	2	2	100	100	100	0.003	0.003	0.003	1.165	1.112	1.096	333.11	333.15	333.11	
0.003	1.25	-	-	2	2	2	100	100	100	0.003	0.003	0.003	1.456	1.390	1.370	416.24	416.30	416.24	
0.003	1.50	-	-	2	2	2	100	100	100	0.003	0.003	0.003	1.747	1.668	1.643	499.26	499.36	499.26	

Table 6.11: **GPS + Galileo**, **PAR** analysis, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario. Evaluation of ASR and number of epochs for obtaining σ_n, σ_e =20cm fixed-precision of rover-receiver coordinates - **measurement precision is varied**, number of satellites are as available.

Further the results for combined GPS and Galileo system in order to give a fixed-precision of 20cm for the horizontal components are presented. PAR is used and the number of epochs taken to fulfil the criteria are presented along with the percentage of ambiguities fixed.

6.4.10 GPS + Galileo, Partial Ambiguity Resolution to obtain fixed-precision of 20 cm, atmosphere known scenario

In this analysis it is to examine if a fixed-precision of σ_n, σ_e =20cm for the horizontal rover coordinate component could be obtained in reduced number of epochs and by partial fixing of ambiguities. The results are presented in Table 6.11, see below.

The results obtained from the criteria of20cm fixed-precision for the horizontal coordinate components for the rover receiver are compared with the earlier 2cm fixed-precision criteria. The statistics based on Table 6.8 in terms of partial ambiguities fixed and number of epochs taken are exactly similar to the number of epochs taken to obtain a fixed-precision of 2cm. It is concluded that the intermediate levels of fixed-precision could not be reached for GPS+Galileo combined system, Reference rover model, ionosphere fixed scenario.

6.5 Reference-Rover model, atmosphere unknown scenario

For baseline lengths which hold significant atmospheric bias for relative positioning applications are no longer termed as short baselines. In fact they are referred as medium (up to 250Km) and long (above 250Km) baselines. For a Geometry Based model, both ionosphere and troposphere are parameterized, unlike geometry free model where troposphere is lumped with the satellite-receiver ranges. When the atmosphere, namely, the ionosphere and troposphere are parameterized for estimation, such a scenario is termed as atmosphere float (unknown) in GNSS terms. In the current assumption, the receiver coordinates for the rover are considered to be unknown and are parameterized, satellite coordinates are assumed to be known and are computed using YUMA almanacs. The baseline length is considered as 250 Kilometres. All the different types of frequency combinations, namely single, dual, triple (for only GPS and Galileo standalone systems) and quadruple (only for Galileo and GPS+Galileo systems) will be considered. Presented below is the functional and stochastic models, see Table 6.12.

In Table 6.12, G_{m-1} has the Line of Sight (LOS) vectors and the tropospheric mapping function, it is given as below,

$$G_{m-1} = \begin{bmatrix} -u^{1-2} & \psi^{1-2} \\ \vdots & \vdots \\ -u^{1-m} & \psi^{1-m} \end{bmatrix}$$

Reference-Rover model, atmosphere unknown scenario		
Functional model		**Stochastic model**
Non-temporal parameters	Temporal parameters	
$A_{I(i)} = \left[\begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes I_{m-1} \quad \begin{pmatrix} e_f \\ e_f \end{pmatrix} \otimes G_{m-1} \right]$	$A_{II(i)} = \begin{pmatrix} -\mu_f \\ \mu_f \end{pmatrix} \otimes I_{m-1}$	$Q_{y(i)} = \begin{bmatrix} Q_\Phi & \\ & Q_P \end{bmatrix} \otimes Q_{DD}(i)$ where $Q_{DD}(i) = D_m^T W_i D_m$
Redundancy (for GPS only, Galileo only, GPS+Galileo (common frequency $L1(E1), L5(E5a)$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $f * (m-1)$ Troposphere: $= 1$ Coordinates: $3(n-1) = 3$	Ionosphere: $k * (m-1)$	$k * 2f * (m-1)$
Redundancy of Reference-Rover - atmosphere unknown scenario		$\{(2kf - f - 1) * (m-1)\} - 4$
For GPS/Galileo only: $f = (1, \cdots, j)$, for GPS+Galileo: $f = (1, \cdots, j_c)$, j_c is the common/overlapping frequency, and $m_{jc} = (m_{GPS} + m_{Gal})$		
Redundancy (for GPS+Galileo (quadruple frequency $L1(E1), L5(E5a), L2, E5b$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)$ Troposphere: $= 1$ Coordinates: $3(n-1) = 3$	Ionosphere: $k * (m_{jc} - 1)$	$2k * (2 * (m_{jc} - 1) + (m_{GPS} - 1) + (m_{Gal} - 1)$
Redundancy of quadruple frequency scenario		$(4k-3)(m_{jc}-1) + (2k-1)[(m_{GPS}-1) + (m_{Gal}-1)] - 4$
$f = 2$ for GPS+Gaileo $L1(E1), L5(E5a)$ m_{jc} are the total number of satellites for combined GPS+Galileo system with GPS as reference satellite		

Table 6.12: Double-differenced Design matrix and VC matrix for Geometry based model, Receiver coordinates for the rover are unknown, Atmosphere unknown scenario

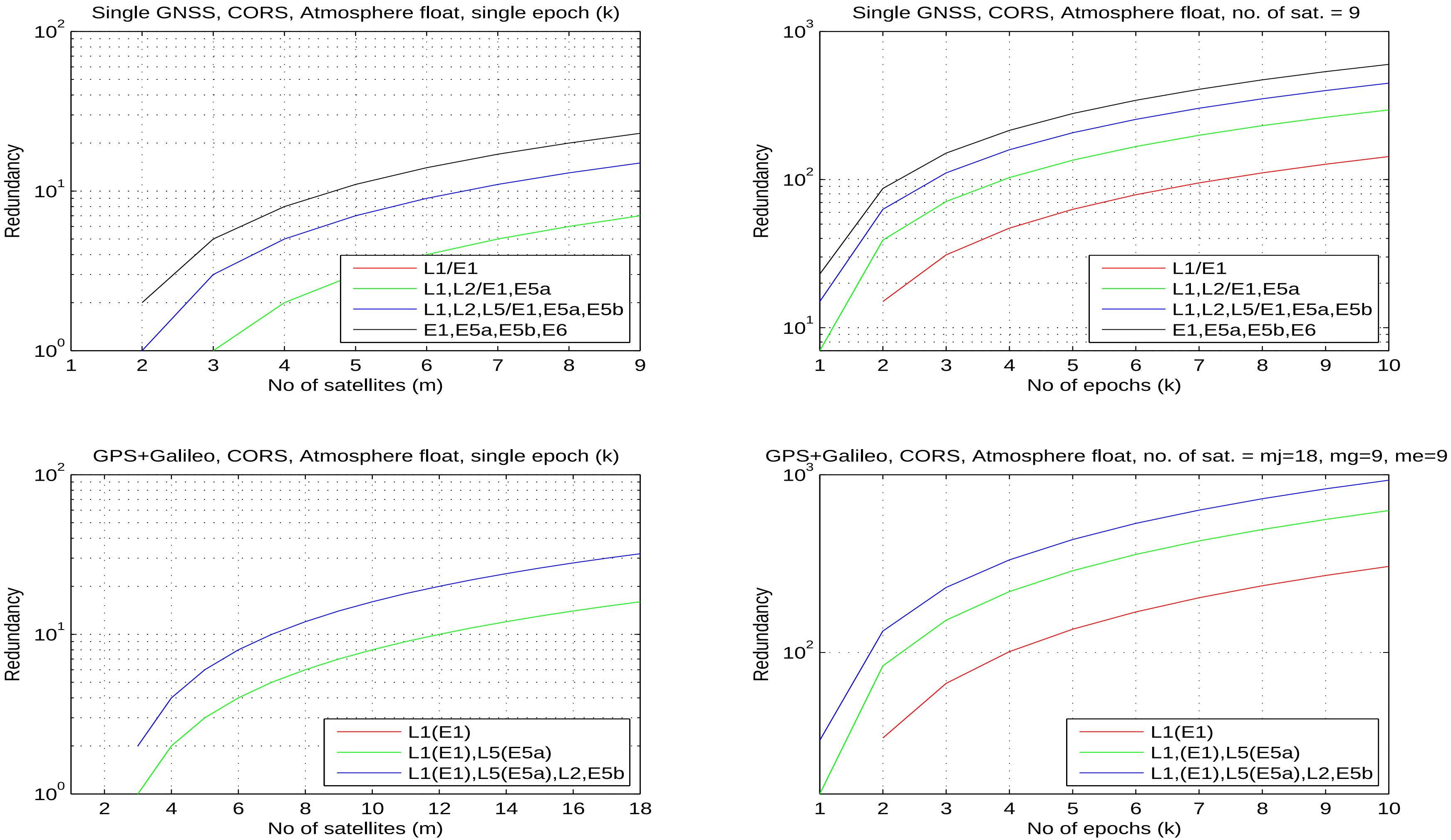

Figure 6.8: Redundancy plot of **GPS only**, **Galileo only** (top two plots) and **GPS + Galileo** (bottom two plots) for single epoch (left hand side plots) and multi-epoch (right hand side plots), atmosphere unknown scenario, **Geometry based model, Receiver coordinates for the rover are unknown**, single, dual, triple and quadruple frequencies

6.5.1 Hourly batches, Full Ambiguity Resolution, atmosphere unknown scenario

Initially, ASR is simulated for hourly batches, throughout the day 24 hourly batches are formed starting from 00 UTC and ending at 2359 UTC. The interval between the epochs is taken as 1 second, making the total number of epochs to be 3600 in each batch. The number of epochs taken in order to achieve 0.999 ASR during different times of the day are simulated and their average behaviour is computed. The results for single, dual, triple and quadruple (for Galileo and GPS+Galileo system) frequency for GPS only, Galileo only and with both systems GPS and Galileo together are presented in the Figures G.20 to G.22, see Appendix G. The results for average number of epochs taken for each of the GNSS system are also presented in Table 6.13, see below.

Analysis of the averaged 24 hourly batches from Table 6.13 and Figures G.20 to G.22 is presented below.

GPS only

Dual frequency:

1. With GPS only, the average number of epochs (for 0.999 ASR) lie between 92 and 280 with dual frequency for different measurement precision scenarios for E-W baseline and for N-S baseline the average number of epochs lie between 90 and 269. It can be noted that for both E-W and N-S baseline (250 Km baseline), the worst performance is for $-30°$ latitude.

Triple frequency:

2. For triple frequency, the average number of epochs lie between 35 and 145 for E-W baseline and 37 to 125 epochs for the N-S baseline. For triple frequency, the worst performance for both E-W and N-S baseline is at $-30°$ latitude.

East-West and North-South oriented baseline comparison:

				Hourly, Full AR - minimum desired ASR = 0.999																	
Phase	Code	Phase	Code	Average no. of Epochs taken			Average no. of satellites			Average no. of Low-elevation Sat.$\geqslant 10°, \leqslant 30°$			Average no. of Epochs taken			Average no. of satellites			Average no. of Low-elevation Sat.$> 10°, 30°$		
	(meters)			0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
				East-West Baseline - 250Km									**North-South Baseline - 250Km**								
				GPS only																	
Φ L1,L2	P	Φ	P	High-end receivers, Dual Frequency																	
0.002	0.10	-	-	93	129	92	10	9	10	5	3	5	90	124	112	9	8	9	5	3	4
0.002	0.15	-	-	117	152	113	10	9	10	5	3	5	113	148	135	9	8	9	5	3	4
0.002	0.20	-	-	131	165	129	10	9	10	5	3	5	127	161	148	9	8	9	5	3	4
0.003	0.25	-	-	205	260	209	10	9	10	5	3	5	210	249	232	9	8	9	5	3	4
0.003	0.30	-	-	216	271	219	10	9	10	5	3	5	219	260	242	9	8	9	5	3	4
0.003	0.35	-	-	227	280	229	10	9	10	5	3	5	228	269	249	9	8	9	5	3	4
Φ L1,L2	P	Φ L5	P	High-end receivers, Triple Frequency																	
0.002	0.10	0.002	0.05	35	60	39	10	9	10	5	3	5	37	59	50	9	8	9	5	3	4
0.002	0.15	0.002	0.1	51	81	54	10	9	10	5	3	5	50	79	67	9	8	9	5	3	4
0.002	0.20	0.002	0.125	56	87	58	10	9	10	5	3	5	54	84	72	9	8	9	5	3	4
0.003	0.25	0.002	0.15	97	137	98	10	9	10	5	3	5	96	131	118	9	8	9	5	3	4
0.003	0.30	0.002	0.175	101	141	102	10	9	10	5	3	5	100	135	122	9	8	9	5	3	4
0.003	0.35	0.002	0.2	104	145	104	10	9	10	5	3	5	103	137	125	9	8	9	5	3	4
				Galileo only																	
Φ E1	P	Φ E5a	P	High-end receivers, Dual Frequency																	
0.002	0.10	0.002	0.05	42	72	60	10	9	10	5	3	5	39	65	83	10	9	9	5	3	4
0.002	0.15	0.002	0.1	73	116	96	10	9	10	5	3	5	68	108	120	10	9	9	5	3	4
0.002	0.20	0.002	0.125	87	130	111	10	9	10	5	3	5	80	123	134	10	9	9	5	3	4
0.003	0.25	0.002	0.15	140	190	169	10	9	10	5	3	5	126	182	205	10	9	9	5	3	4
0.003	0.30	0.002	0.175	152	200	182	10	9	10	5	3	5	137	193	217	10	9	9	5	3	4
0.003	0.35	0.002	0.2	163	212	193	10	9	10	5	3	5	146	202	228	10	9	9	5	3	4
Φ E1	P	Φ E5a,E5b	P	High-end receivers, Triple Frequency																	
0.002	0.10	0.002	0.05	28	48	42	10	9	10	5	3	5	27	43	53	10	9	9	5	3	4
0.002	0.15	0.002	0.1	36	62	53	10	9	10	5	3	5	34	56	67	10	9	9	5	3	4
0.002	0.20	0.002	0.125	38	66	55	10	9	10	5	3	5	36	59	80	10	9	9	5	3	4
0.003	0.25	0.002	0.15	56	93	82	10	9	10	5	3	5	54	81	118	10	9	9	5	3	4
0.003	0.30	0.002	0.175	57	95	83	10	9	10	5	3	5	55	83	120	10	9	9	5	3	4
0.003	0.35	0.002	0.2	58	96	84	10	9	10	5	3	5	56	84	121	10	9	9	5	3	4
Φ E1	P	Φ E5a,E5b,E6	P	High-end receivers, Quadruple Frequency																	
0.002	0.10	0.002	0.05	24	41	37	10	9	10	5	3	5	23	38	45	10	9	9	5	3	4
0.002	0.15	0.002	0.1	33	57	48	10	9	10	5	3	5	31	52	62	10	9	9	5	3	4
0.002	0.20	0.002	0.125	35	61	51	10	9	10	5	3	5	33	55	65	10	9	9	5	3	4
0.003	0.25	0.002	0.15	53	89	78	10	9	10	5	3	5	52	78	113	10	9	9	5	3	4
0.003	0.30	0.002	0.175	55	91	79	10	9	10	5	3	5	53	80	115	10	9	9	5	3	4
0.003	0.35	0.002	0.2	55	92	81	10	9	10	5	3	5	54	81	116	10	9	9	5	3	4
				GPS + Galileo																	
Φ L1(E1)	P	Φ L5(E5a)	P	High-end receivers, Dual Frequency																	
0.002	0.10	0.002	0.05	9	11	10	19	17	19	10	6	9	9	11	10	19	17	18	10	7	8
0.002	0.15	0.002	0.1	18	18	21	19	17	19	10	6	9	17	19	20	19	17	18	10	7	8
0.002	0.20	0.002	0.125	26	26	33	19	17	19	10	6	9	27	28	30	19	17	18	10	7	8
0.003	0.25	0.002	0.15	39	39	46	19	17	19	10	6	9	39	41	45	19	17	18	10	7	8
0.003	0.30	0.002	0.175	56	50	65	19	17	19	10	6	9	53	54	63	19	17	18	10	7	8
0.003	0.35	0.002	0.2	74	67	83	19	17	19	10	6	9	68	70	84	19	17	19	10	7	8
Φ L1(E1),L2,E5b	P	Φ L5(E5a)	P	High-end receivers, Quadruple Frequency																	
0.002	0.10	0.002	0.05	5	6	6	19	17	19	10	6	9	5	6	5	19	17	18	10	7	8
0.002	0.15	0.002	0.1	7	7	7	19	17	19	10	6	9	7	7	7	19	17	18	10	7	8
0.002	0.20	0.002	0.125	9	9	9	19	17	19	10	6	9	9	9	9	19	17	18	10	7	8
0.003	0.25	0.002	0.15	14	14	13	19	17	19	10	6	9	14	15	13	19	17	18	10	7	8
0.003	0.30	0.002	0.175	18	17	17	19	17	19	10	6	9	18	18	17	19	17	18	10	7	8
0.003	0.35	0.002	0.2	23	22	21	19	17	19	10	6	9	23	22	21	19	17	18	10	7	8

Table 6.13: **GPS only, Galileo only, GPS + Galileo** GNSS systems, **Geometry based model, Receiver coordinates for the rover are unknown**, Atmosphere unknown scenario **Full AR**, Average number of epochs, average number of satellites and average number of low elevation satellites over 24 batches (1 batch = 1 hour, 3600 epochs, 1 second interval) are presented - **measurement precision varied**, number of satellites are as available.

3. GPS only takes relatively less number of epochs at $0°$ latitude and more epochs at $-60°$ latitude for both E-W and N-S baseline.

Galileo only

Dual frequency:

4. For dual frequency Galileo, the average number of epochs taken lie between 42 and 212 for E-W baseline and 39 and 228 for N-S baseline. Galileo only outperforms GPS only with dual frequency for all the scenarios of measurement precision, see Table 6.13. The better performance of Galileo is credited to its improved measurement precision in comparison to GPS. Galileo only performs the best at $0°$ latitude for both E-W and N-S baseline.

Triple frequency:

5. With triple frequency Galileo, the average number of epochs over 24 batches to obtain 0.999 ASR lie between 28 and 96 for E-W baseline and 27 to 121 for N-S baseline. The performance is better in comparison to GPS only which is an effect of better measurement precision for Galileo system in comparison to GPS system. For both E-W and N-S baseline, Galileo only performs best at $0°$ latitude.

Quadruple frequency:

6. The quadruple frequency Galileo takes between 24 and 92 for the E-W baseline and between 23 and 116 epoch for the N-S baseline. In comparison to Galileo only dual and triple frequency system, Galileo only quadruple frequency performs better for all the scenarios of measurement precision and latitude location. The improvement in performance is due to the addition of a frequency. Galileo only performs best at $0°$ latitude for both E-W and N-S baseline.

East-West and North-South oriented baseline comparison:

7. Galileo looks to be symmetric at the equator, since it almost takes equal number of epochs on an average for 0.999 ASR for both E-W and N-S baseline (250 Km baseline length). Galileo only takes relatively less number of epochs at $0°$ and $-30°$ (prominent in case of dual frequency) latitude and more epochs at $-60°$ latitude for E-W baseline in comparison to N-S baseline.

GPS + Galileo

Dual frequency:

8. For a combined GPS and Galileo system, it takes an average number of epochs between 9 and 83 for E-W baseline and 9 to 84 for N-S baseline for different scenarios of measurement precision and latitude location. The combined system performs much better than the standalone systems, unlike the case for CORS scenario when the combined system performed almost similar to standalone system models. Here due to presence of additional unknowns, that is the receiver coordinates, the standalone systems are not strong enough. For ionosphere float scenario, the receiver coordinates and troposphere, which the coupling parameters for both GNSS systems. With twice the number of satellites for a combined system as compared to standalone system, tremendous strength is gained for the coupling parameters making the combined GPS+Galileo model much stronger than the standalone GNSS system models.

Quadruple frequency:

9. It takes and average number of epochs between 5 and 23 for E-W baseline and N-S baseline, both. The quadruple GPS+Galileo system outperforms the dual GPS+Galileo and quadruple Galileo only system.

East-West and North-South oriented baseline comparison:

10. The combined GPS and Galileo dual frequency system takes relatively less number of epochs at $-30°$ latitude for E-W baseline and at $0°$ for N-S baseline. For quadruple frequency system, the difference cannot be

clearly observed, since it seems to perform almost similar at all latitude locations.

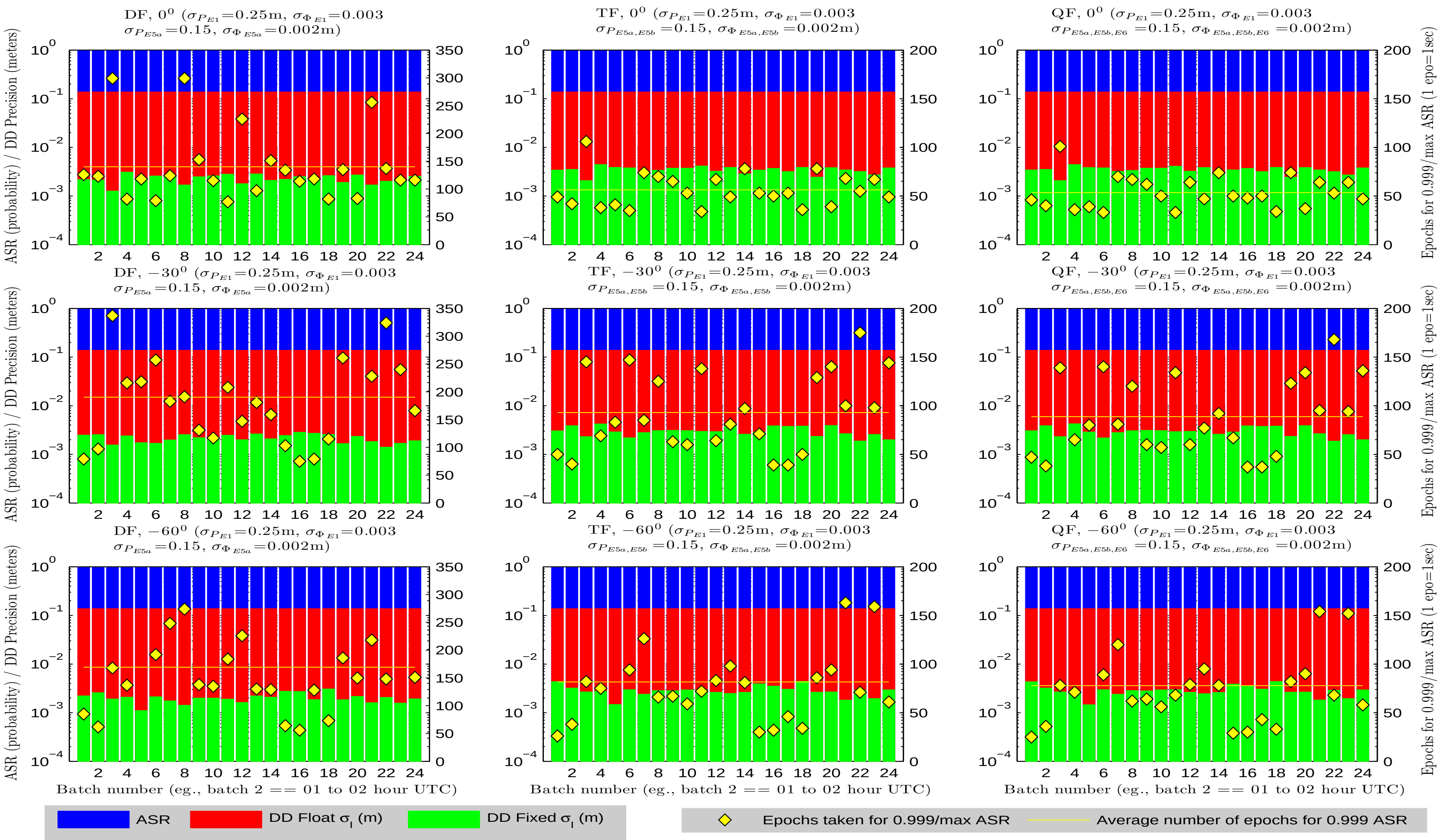

Figure 6.9: **Galileo only**, atmosphere unknown scenario, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual, triple and quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR and, float and fixed-precision of ionosphere.

6.5.2 GPS only - Full and Partial Ambiguity Resolution, atmosphere unknown scenario

The results for GPS system, ionosphere float Geometry based model, coordinates known, atmosphere unknown scenario are presented below. Figures G.23 and G.24 give full ambiguity resolution results and Figures G.25 and G.26 present the partial ambiguity results for dual and triple frequency GPS respectively, see Appendix G. Figure 6.10 presents the number of satellites for E-W oriented baseline (250 Km) and N-S baseline (250 Km). Table 6.14 presents the full ambiguity resolution results in terms of number of epochs taken for 0.999 ASR and corresponding rover-receiver coordinate precision (for ambiguity-fixed and -float solution) along with the gain obtained for the rover-receiver coordinate for ambiguity-fixed solution is presented.

The following is the analysis based on Table 6.14 and Figures 6.10 to 6.12.

General remarks:

(1) It can be noted that for atmosphere unknown scenario (when ionosphere and troposphere both parameterized for estimation), instantaneous ASR (Full AR) of 0.999 is not achieved for Geometry based model when receiver coordinates for the rover are unknown.

(2) There is a significant difference in number of epochs taken for 0.999 ASR for E-W and N-S oriented baseline for dual frequency system. This is due to the fact that for a 250 Km E-W and N-S oriented baseline, see Figure 6.10, the number of common satellites, low elevation satellites and in general the elevation of each of the satellite is different causing the results of epochs taken for 0.999 ASR to be different.

(3) Further more, it can be noted that the number of epochs taken for 0.999 full AR are less for a certain latitude location, as discussed in the earlier section. For both E-W and N-S baseline the number of epochs taken for 0.999 full AR are less at $-30°$ latitude than at other latitude locations.

Time to regain 0.999 ASR:

Phase	Code	Phase	Code	Full AR - minimum desired ASR = 0.999											
	(meters)			Epochs taken			$\sigma_{\check{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\check{neu}}$		
				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km															
Φ	P	Φ	P	High-end receivers, Dual Frequency											
L1,L2															
0.002	0.10	-	-	189	71	90	0.002	0.002	0.002	0.057	0.099	0.086	36.35	45.53	45.56
0.002	0.15	-	-	208	93	110	0.001	0.002	0.002	0.065	0.113	0.103	43.38	58.87	60.00
0.002	0.20	-	-	215	110	121	0.001	0.002	0.002	0.071	0.120	0.117	48.44	68.06	71.07
0.003	0.25	-	-	334	197	203	0.002	0.002	0.002	0.061	0.095	0.098	34.44	48.29	52.80
0.003	0.30	-	-	336	215	214	0.002	0.002	0.002	0.064	0.095	0.102	36.41	50.55	56.43
0.003	0.35	-	-	338	227	228	0.002	0.002	0.002	0.067	0.096	0.102	37.90	52.19	58.11
Φ	P	Φ	P	High-end receivers, Triple Frequency											
L1,L2		L5													
0.002	0.10	0.002	0.05	75	26	34	0.002	0.003	0.003	0.091	0.142	0.121	43.87	46.69	46.64
0.002	0.15	0.002	0.1	107	38	50	0.002	0.003	0.002	0.099	0.171	0.145	55.72	66.91	66.65
0.002	0.20	0.002	0.125	115	41	55	0.002	0.002	0.002	0.111	0.208	0.174	64.53	83.97	83.28
0.003	0.25	0.002	0.15	223	76	94	0.002	0.003	0.002	0.077	0.171	0.152	43.72	66.56	67.07
0.003	0.30	0.002	0.175	229	80	98	0.002	0.003	0.002	0.080	0.188	0.167	46.42	74.73	75.45
0.003	0.35	0.002	0.2	233	82	100	0.002	0.002	0.002	0.084	0.205	0.183	48.64	82.35	83.11
North-South Baseline - 250Km															
Φ	P	Φ	P	High-end receivers, Dual Frequency											
L1,L2															
0.002	0.10	-	-	122	70	124	0.002	0.002	0.002	0.082	0.102	0.069	42.01	45.81	42.64
0.002	0.15	-	-	160	92	148	0.002	0.002	0.001	0.085	0.116	0.082	49.23	59.31	55.02
0.002	0.20	-	-	178	106	156	0.002	0.002	0.001	0.088	0.126	0.094	53.93	69.21	64.53
0.003	0.25	-	-	266	173	249	0.002	0.002	0.002	0.080	0.109	0.083	39.67	51.28	44.25
0.003	0.30	-	-	275	182	258	0.002	0.002	0.002	0.082	0.115	0.086	41.63	55.26	46.79
0.003	0.35	-	-	282	190	266	0.002	0.002	0.002	0.084	0.118	0.088	43.11	58.11	48.35
Φ	P	Φ	P	High-end receivers, Triple Frequency											
L1,L2		L5													
0.002	0.10	0.002	0.05	43	26	48	0.003	0.003	0.002	0.128	0.144	0.102	46.15	46.74	45.35
0.002	0.15	0.002	0.1	63	36	67	0.002	0.003	0.002	0.149	0.180	0.121	63.87	67.51	63.28
0.002	0.20	0.002	0.125	70	40	75	0.002	0.003	0.002	0.172	0.215	0.141	77.00	84.68	77.73
0.003	0.25	0.002	0.15	124	71	119	0.002	0.003	0.002	0.138	0.184	0.127	57.99	68.00	62.83
0.003	0.30	0.002	0.175	129	75	125	0.002	0.003	0.002	0.147	0.201	0.138	63.03	76.46	69.80
0.003	0.35	0.002	0.2	132	77	129	0.002	0.003	0.002	0.156	0.220	0.147	67.39	84.37	75.72

Table 6.14: **GPS only**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, **Full AR** analysis, ASR and ambiguity-fixed and -float precision for rover-receiver coordinate are presented - **measurement precision varied**, number of satellites as available.

(4) It can be seen at certain times after initial 0.999 ASR is achieved, the gained criteria is lost which is due to an incoming low elevation satellite. The time taken to regain the criteria of 0.999 ASR is the lowest for better measurement precision. For example see Figure 6.11, the first row represents three values of measurement precision for 0° latitude. The width of the spike in the green curve (fixed-precision of rover-receiver coordinate curve) is the least for better measurement precision (first/third row, first column plot) and the width is larger for the worst measurement precision (first/third row, last column plot).

Dual Frequency:

(5) The number of epochs taken for 0.999 ASR for all scenarios of measurement precision and baseline length lied between 71 and 338 for E-W oriented baseline and between 70 and 282 for N-S oriented baseline.

(6) For dual frequency with the measurement precision of σ_Φ=3mm and σ_P=25cm on $L1, L2$, it took 334, 197 and 203 epochs at 0°, −30° and −60° latitude E-W oriented baseline and 266, 173 and 249 for N-S oriented baseline.

Triple Frequency:

(7) The number of epochs taken to obtain 0.999 ASR lied between 26 and 233 for E-W oriented baseline and between 26 and 132 for N-S oriented baseline.

(8) The epochs taken for 0.999 ASR for the measurement precision of σ_Φ=3mm on $L1, L2$ and 2mm on $L5$ and σ_P=25cm on $L1, L2$ and 15cm on $L5$ were 233, 76 and 94 for E-W oriented baseline and 124, 71 and 119 for N-S oriented baseline at 0°, −30° and −60° latitudes respectively.

Considering the fixed-precision of rover-receiver coordinate to lie around 3mm in most of the cases when all the ambiguities were fixed, PAR will be evaluated for obtaining a fixed-precision of 2cm for troposphere in the coming section.

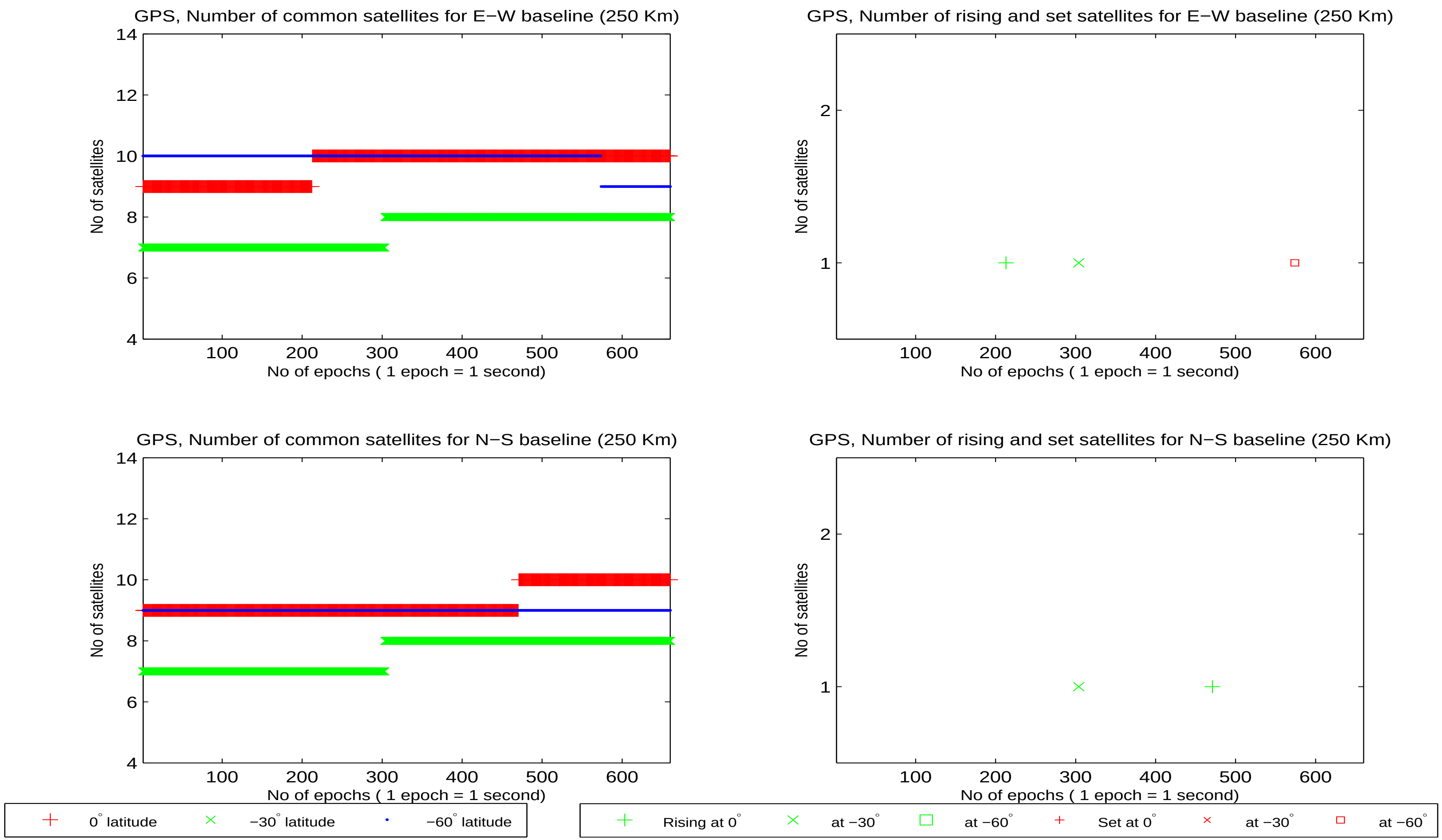

Figure 6.10: Number of satellites for **GPS only**, 250 Km East-West and North-South baseline at 0°, −30° and −60° degree latitude.

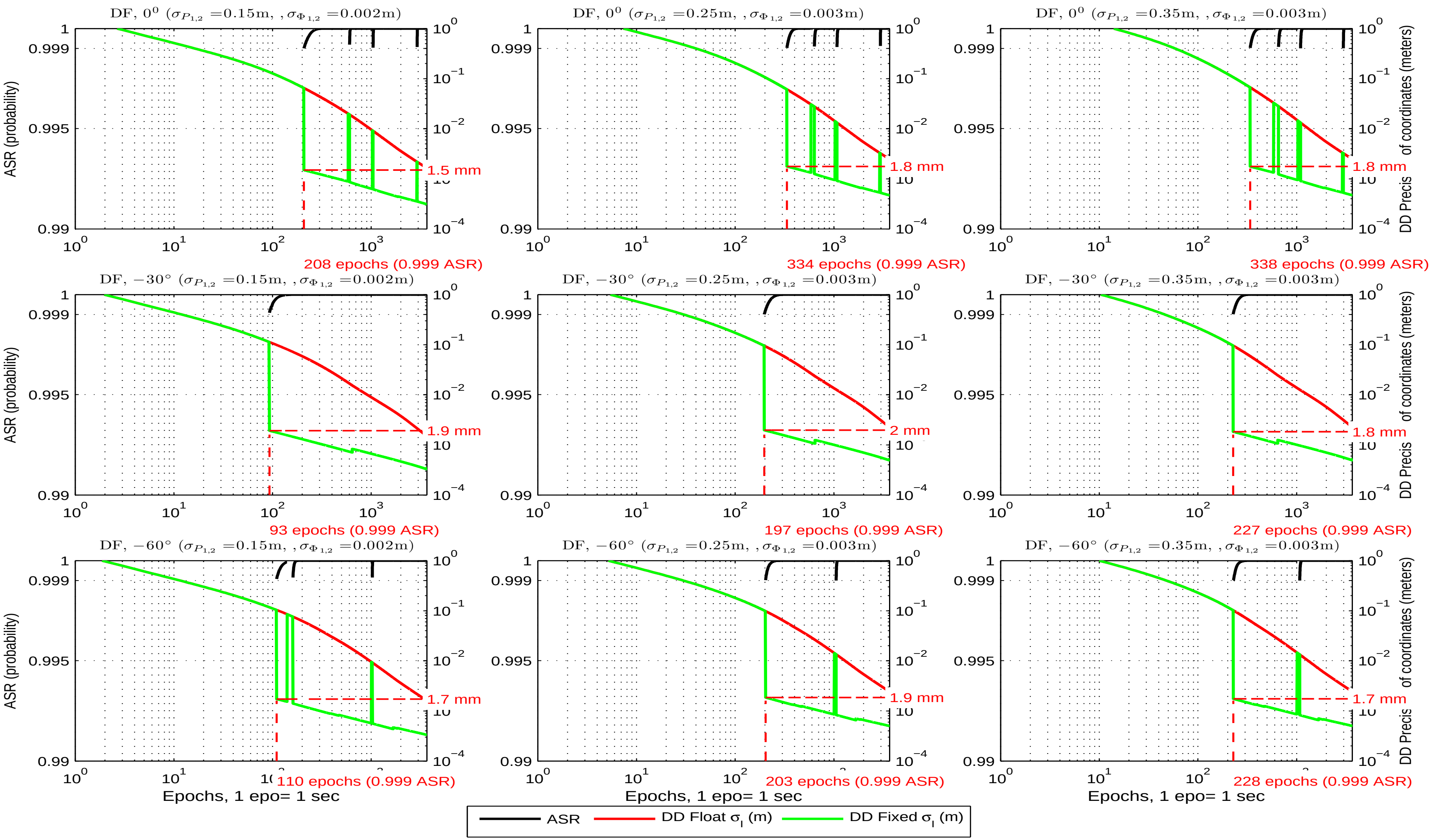

Figure 6.11: **GPS only**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

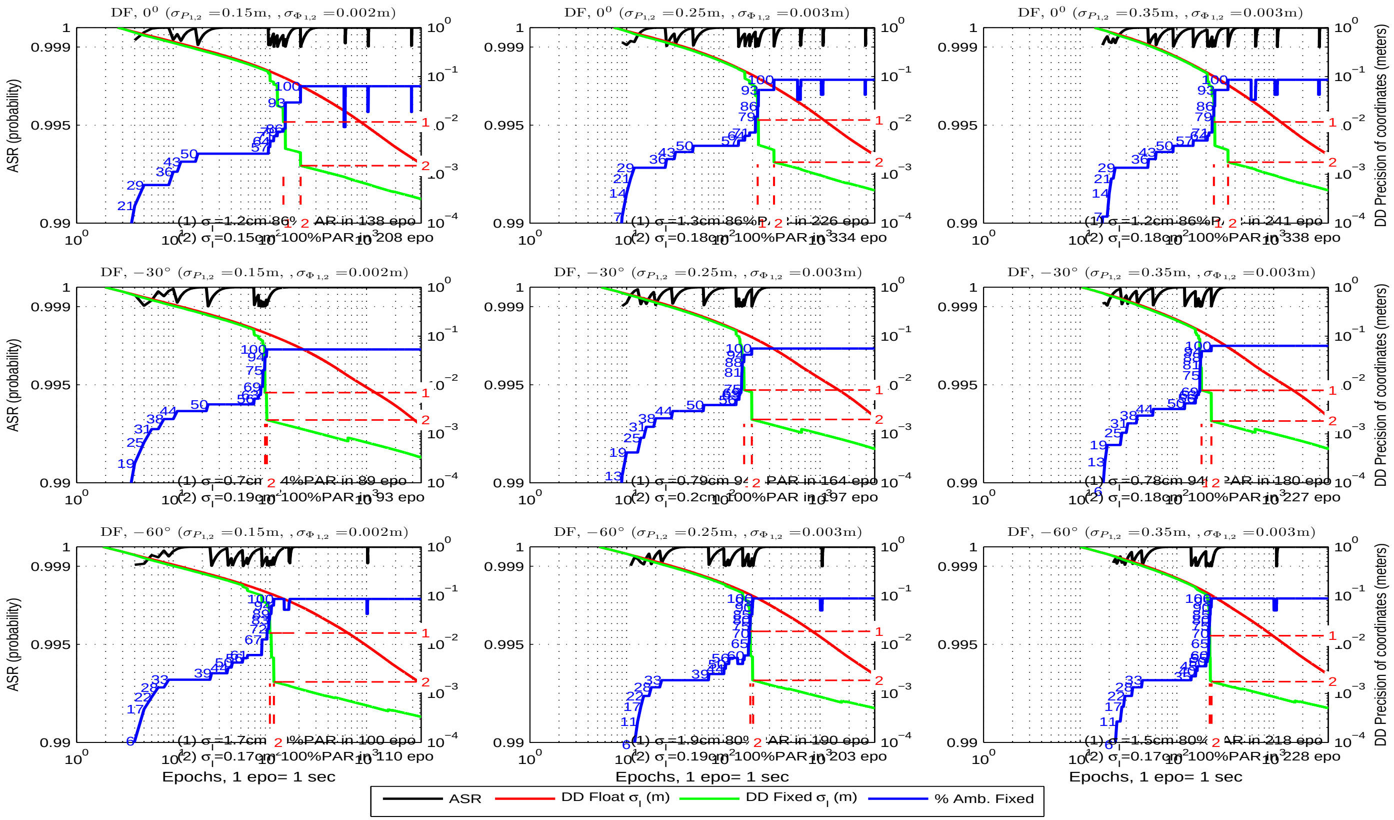

Figure 6.12: **GPS only**, Dual frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

6.5.3 GPS only, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, atmosphere unknown scenario

In this section, it is to be analyzed if by partially fixing the ambiguities, does the solution converge faster to a modestly chosen fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for rover-receiver coordinate components x, y and z. Table 6.15 presents the results in terms of fixed-precision to be equal to or better than the set values and corresponding epochs taken and ambiguities fixed are noted.

Referring Table 6.15, for all the scenarios of measurement precision (out of 18 scenarios for each dual and triple frequency) less than 100% of ambiguities were fixed to give a fixed-precision of coordinate components, desired to be equal or better than σ_n, σ_e =2cm and σ_u=6cm for the x, y and z component of the rover receiver coordinates.

General remarks::

(1) A fixed-precision of 2cm for rover-receiver coordinate could not be achieved instantaneously with GPS dual or triple frequency system, for E-W and for N-S baseline both.

(2) Partial ambiguity fixed a subset of ambiguities at most of the times (72 out of 70 occasions) to give a fixed-precision of coordinates as desired.

Dual Frequency:

(3) The number of epochs taken for 0.999 ASR for all scenarios of measurement precision and baseline length lied between 70 and 241 for E-W oriented baseline and between 69 and 273 for N-S oriented baseline.

(4) For dual frequency with the measurement precision of σ_Φ=3mm and σ_P=25cm on $L1, L2$, it took 224, 164 and 190 epochs by fixing 86, 94 and 80% of ambiguities by PAR at 0°, −30° and −60° latitude E-W oriented baseline and 258, 171 and 233 epochs by fixing 86, 94 and 86 % of ambiguities for N-S oriented baseline.

				Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for rover-receiver coordinate														
Phase	Code	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{neu}}$(meters)			$\sigma_{\hat{neu}}$(meters)			Gain=$\sigma_{\hat{neu}}/\sigma_{\check{neu}}$		
	(meters)			0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km																		
Φ L1,L2	P	Φ	P	High-end receivers, Dual Frequency														
0.002	0.10	-	-	112	70	74	86	94	89	0.010	0.007	0.019	0.087	0.100	0.098	8.95	13.78	5.07
0.002	0.15	-	-	138	89	100	86	94	89	0.012	0.007	0.017	0.096	0.117	0.111	8.20	16.71	6.44
0.002	0.20	-	-	154	102	116	86	88	89	0.012	0.020	0.016	0.101	0.128	0.121	8.77	6.43	7.58
0.003	0.25	-	-	226	164	190	86	94	80	0.013	0.008	0.019	0.094	0.113	0.105	7.28	14.36	5.61
0.003	0.30	-	-	234	172	201	86	94	80	0.012	0.008	0.016	0.098	0.120	0.109	8.25	15.25	6.87
0.003	0.35	-	-	241	180	218	86	94	80	0.012	0.008	0.015	0.100	0.123	0.107	8.53	15.86	7.03
Φ L1,L2	P	Φ L5	P	High-end receivers, Triple Frequency														
0.002	0.10	0.002	0.05	35	24	32	90	88	93	0.011	0.020	0.019	0.142	0.148	0.125	12.92	7.47	6.55
0.002	0.15	0.002	0.1	54	37	40	90	92	93	0.011	0.017	0.020	0.164	0.174	0.166	15.03	10.41	8.18
0.002	0.20	0.002	0.125	60	41	43	90	100	93	0.011	0.002	0.020	0.191	0.208	0.204	17.93	83.97	10.03
0.003	0.25	0.002	0.15	98	74	81	90	92	93	0.012	0.018	0.020	0.167	0.175	0.169	14.29	9.91	8.57
0.003	0.30	0.002	0.175	103	78	83	90	92	93	0.012	0.018	0.020	0.179	0.192	0.191	15.48	10.94	9.58
0.003	0.35	0.002	0.2	108	81	85	90	96	93	0.011	0.008	0.020	0.187	0.207	0.209	16.41	26.66	10.50
North-South Baseline - 250Km																		
Φ L1,L2	P	Φ	P	High-end receivers, Dual Frequency														
0.002	0.10	-	-	108	69	97	86	88	88	0.017	0.009	0.018	0.090	0.103	0.082	5.34	12.09	4.54
0.002	0.15	-	-	145	91	115	86	88	88	0.016	0.015	0.018	0.093	0.117	0.100	5.77	7.66	5.54
0.002	0.20	-	-	166	106	127	86	100	88	0.015	0.002	0.018	0.095	0.126	0.113	6.13	69.21	6.40
0.003	0.25	-	-	258	171	233	86	94	86	0.020	0.005	0.017	0.082	0.111	0.088	4.03	21.19	5.25
0.003	0.30	-	-	267	181	243	93	94	86	0.007	0.005	0.017	0.085	0.116	0.092	12.50	22.75	5.51
0.003	0.35	-	-	273	189	253	93	94	86	0.007	0.005	0.016	0.087	0.119	0.093	12.91	23.93	5.68
Φ L1,L2	P	Φ L5	P	High-end receivers, Triple Frequency														
0.002	0.10	0.002	0.05	38	24	39	95	92	83	0.007	0.011	0.018	0.137	0.150	0.116	19.78	14.14	6.35
0.002	0.15	0.002	0.1	58	34	55	90	92	88	0.020	0.017	0.018	0.158	0.187	0.139	7.84	10.75	7.92
0.002	0.20	0.002	0.125	63	38	64	90	92	92	0.020	0.017	0.018	0.186	0.223	0.158	9.25	13.28	8.95
0.003	0.25	0.002	0.15	115	67	103	90	92	92	0.018	0.010	0.009	0.147	0.192	0.142	8.28	19.28	16.10
0.003	0.30	0.002	0.175	120	70	108	90	92	92	0.018	0.010	0.009	0.158	0.213	0.156	8.89	21.60	17.86
0.003	0.35	0.002	0.2	123	72	112	90	92	92	0.018	0.010	0.009	0.167	0.232	0.167	9.42	23.73	19.37

Table 6.15: **GPS only**, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario. Presented is the evaluation of ASR and number of epochs for obtaining 2cm fixed-precision of rover-receiver coordinate - **measurement precision is varied**, number of satellites are as available.

Triple Frequency:

(5) The number of epochs taken to obtain 0.999 ASR lied between 24 and 108 for E-W oriented baseline and between 24 and 123 for N-S oriented baseline.

(6) The epochs taken for 0.999 ASR (measurement precision of σ_{Φ}=3mm on $L1, L2$ and 2mm on $L5$ and σ_P=25cm on $L1, L2$ and 15cm on $L5$) were 98, 74 and 81 by fixing partially 90, 92 and 93 % of ambiguities for E-W oriented baseline and 115, 67 and 103 epochs by fixing 90, 92 and 92 % of ambiguities by PAR for N-S oriented baseline at $0°$, $-30°$ and $-60°$ latitudes respectively.

Further the results and discussion for PAR inorder to obtain a fixed preciison of 20cm for the horizontal coordinate components is presented.

6.5.4 GPS only, Partial Ambiguity Resolution to obtain fixed-precision of 20 cm, atmosphere unknown scenario

In this section, it is to be analyzed if by partially fixing the ambiguities, does the solution converge faster to a modestly chosen fixed-precision of σ_n, σ_e =20cm for the horizontal rover-receiver coordinate components. Table 6.16 presents the results in terms of fixed-precision to be equal to or better than the set values and corresponding epochs taken and ambiguities fixed are noted.

Referring Table 6.16, for all the scenarios of measurement precision (out of 18 scenarios for each dual and triple frequency) less than 100% of ambiguities were fixed to give a fixed-precision of coordinate components, desired to be equal or better than σ_n, σ_e =20cm for the horizontal component of the rover receiver coordinates.

General remarks::

(1) A fixed-precision of 20cm for the horizontal rover-receiver coordinate could not be achieved instantaneously with GPS dual or triple frequency system, for E-W and for N-S baseline both.

(2) Less than 100% of the ambiguities were fixed at all times to give a fixed-precision of coordinates as predefined.

Dual Frequency:

(3) The number of epochs taken for 0.999 ASR for all scenarios of measurement precision and baseline length lied between 7 and 99 (for σ_n, σ_e =2cm it took between 70 and 241 epochs) for E-W oriented baseline and between 8 and 113 (for σ_n, σ_e =2cm it took between 69 and 273 epochs) for N-S oriented baseline.

(4) For dual frequency with the measurement precision of σ_Φ=3mm and σ_P=25cm on $L1, L2$, it took 51, 40 and 66 epochs (for σ_n, σ_e =2cm it took 224, 164 and 190 epochs) by fixing 50, 44 and 33% of ambiguities by PAR at 0°, −30° and −60° latitude E-W oriented baseline and 47,

				Partial AR @ 0.999 ASR criteria - 20 cm fixed-precision for horizontal rover-receiver coordinate components														
Phase	Code	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widecheck{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widecheck{neu}}$		
	(meters)			0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km																		
Φ	P	Φ	P	High-end receivers, Dual Frequency														
L1,L2																		
0.002	0.10	-	-	9	7	14	50	44	39	0.339	0.333	0.228	0.362	0.353	0.243	1.07	1.06	1.06
0.002	0.15	-	-	20	16	30	50	44	39	0.336	0.326	0.227	0.359	0.345	0.241	1.07	1.06	1.06
0.002	0.20	-	-	34	27	45	50	44	39	0.327	0.318	0.232	0.347	0.335	0.245	1.06	1.06	1.06
0.003	0.25	-	-	51	40	66	50	44	33	0.323	0.318	0.232	0.342	0.335	0.244	1.06	1.05	1.05
0.003	0.30	-	-	68	52	83	43	44	33	0.304	0.309	0.228	0.319	0.325	0.238	1.05	1.05	1.05
0.003	0.35	-	-	86	68	99	43	44	33	0.278	0.284	0.222	0.289	0.298	0.230	1.04	1.05	1.04
Φ	P	Φ	P	High-end receivers, Triple Frequency														
L1,L2		L5																
0.002	0.10	0.002	0.05	2	2	3	57	63	56	0.325	0.272	0.217	0.605	0.520	0.414	1.86	1.91	1.90
0.002	0.15	0.002	0.1	4	3	5	57	58	56	0.333	0.333	0.244	0.660	0.655	0.495	1.98	1.97	2.02
0.002	0.20	0.002	0.125	5	4	6	57	58	56	0.334	0.326	0.253	0.781	0.750	0.597	2.33	2.30	2.36
0.003	0.25	0.002	0.15	8	6	11	52	58	56	0.353	0.344	0.241	0.767	0.761	0.547	2.17	2.21	2.27
0.003	0.30	0.002	0.175	9	8	12	52	58	56	0.359	0.319	0.248	0.863	0.786	0.624	2.40	2.46	2.51
0.003	0.35	0.002	0.2	11	9	13	52	58	56	0.344	0.317	0.252	0.904	0.858	0.694	2.62	2.71	2.75
North-South Baseline - 250Km																		
Φ	P	Φ	P	High-end receivers, Dual Frequency														
L1,L2																		
0.002	0.10	-	-	8	8	14	43	44	43	0.365	0.316	0.246	0.388	0.335	0.260	1.06	1.06	1.06
0.002	0.15	-	-	18	16	31	50	44	43	0.359	0.330	0.234	0.383	0.350	0.247	1.07	1.06	1.06
0.002	0.20	-	-	31	27	49	43	44	38	0.350	0.323	0.223	0.371	0.341	0.235	1.06	1.06	1.05
0.003	0.25	-	-	47	40	73	43	44	38	0.346	0.323	0.218	0.365	0.341	0.229	1.05	1.05	1.05
0.003	0.30	-	-	65	52	95	43	44	38	0.319	0.315	0.206	0.334	0.331	0.216	1.05	1.05	1.05
0.003	0.35	-	-	83	66	113	43	44	38	0.290	0.297	0.199	0.302	0.311	0.207	1.04	1.05	1.04
Φ	P	Φ	P	High-end receivers, Triple Frequency														
L1,L2		L5																
0.002	0.10	0.002	0.05	2	2	3	57	63	62	0.329	0.273	0.240	0.611	0.527	0.445	1.86	1.93	1.85
0.002	0.15	0.002	0.1	4	3	5	57	58	62	0.337	0.334	0.270	0.666	0.665	0.531	1.98	1.99	1.97
0.002	0.20	0.002	0.125	5	4	7	57	58	62	0.339	0.325	0.257	0.788	0.761	0.591	2.32	2.34	2.30
0.003	0.25	0.002	0.15	7	7	11	52	58	62	0.381	0.318	0.265	0.828	0.714	0.584	2.17	2.25	2.21
0.003	0.30	0.002	0.175	9	8	12	52	58	62	0.362	0.318	0.271	0.871	0.797	0.663	2.41	2.50	2.45
0.003	0.35	0.002	0.2	11	9	13	52	54	62	0.347	0.353	0.274	0.912	0.871	0.734	2.63	2.47	2.68

Table 6.16: **GPS only**, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario. Presented is the evaluation of ASR and number of epochs for obtaining 20cm fixed-precision of rover-receiver coordinate - **measurement precision is varied**, number of satellites are as available.

40 and 73 epochs (for σ_n, σ_e =2cm it took 258, 171 and 233 epochs) by fixing 43, 44 and 38% of ambiguities for N-S oriented baseline.

Triple Frequency:

(5) The number of epochs taken to obtain 0.999 ASR lied between 2 and 13 (for σ_n, σ_e =2cm it took between 24 and 108 epochs) for E-W oriented baseline and between 2 and 13 (for σ_n, σ_e =2cm it took between 24 and 123 epochs) for N-S oriented baseline.

(6) The epochs taken for 0.999 ASR (measurement precision of σ_Φ=3mm on $L1, L2$ and 2mm on $L5$ and σ_P=25cm on $L1, L2$ and 15cm on $L5$) were 8, 6 and 11 epochs (for σ_n, σ_e =2cm it took 98, 74 and 81 epochs) by fixing partially 52, 58 and 56% of ambiguities for E-W oriented baseline and 7, 7 and 11 epochs (for σ_n, σ_e =2cm it took 115, 67 and 103 epochs) by fixing 52, 58 and 62 % of ambiguities by PAR for N-S oriented baseline at 0°, −30° and −60° latitudes respectively.

6.5.5 Galileo only - Full and Partial Ambiguity Resolution, atmosphere unknown scenario

Galileo system with four frequencies, namely $E1, E5a, E5b$ and $E6$ is evaluated for atmosphere unknown model with coordinates considered to be fixed. Galileo system which promises an improved precision of code for $E5a, E5b, E6$ will be tested in terms of its performance to give better or even instantaneous ambiguity resolution as compared to GPS for Geometry based model, atmosphere unknown scenario when the coordinates of the rover receiver are estimated. The results for for Galileo system of dual, triple and quadruple frequency combinations are presented below. Figures G.27 to G.29 give full ambiguity resolution results and Figures G.30 to G.32 present the partial ambiguity results, see Appendix G. Figure 6.13 presents the number of satellites for E-W oriented baseline (250 Km) and N-S baseline (250 Km) for Galileo system. Table 6.17 presents the full ambiguity resolution results in terms of number of epochs taken for 0.999 ASR and corresponding rover-receiver coordinate precision (for ambiguity fixed solution and float solution) along with the gain achieved for the fixed solution.

The following is the analysis based on Table 6.17 and Figures 6.13 to 6.15.

General remarks:

(1) It can be noted that with Galileo system, geometry based model, atmosphere and coordinates of the rover being unknown, instantaneous ASR (Full AR) of 0.999 is not achieved.

(2) Galileo only performed better at $-60°$ latitude with triple and quadruple frequency system. This is due to the fact that for the chosen simulation time period, 0000 to 0010 hours UTC, the performance of Galileo system at $-60°$ latitude is the best, see Figure 6.9.

Dual Frequency:

(3) The number of epochs taken for 0.999 ASR for all scenarios of measurement precision and baseline length lied between 18 and 158 (**GPS only** took 71 and 338) for E-W oriented baseline and between 19 and 166 (**GPS only** took 70 and 282) for N-S oriented baseline.

				Full AR - minimum desired ASR = 0.999											
Phase	Code	Phase	Code	Epochs taken			$\sigma_{\check{neu}}$(meters)			$\sigma_{\hat{neu}}$(meters)			Gain=$\sigma_{\hat{neu}}/\sigma_{\check{neu}}$		
(meters)				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km															
Φ	*P*	Φ	*P*	High-end receivers, Dual Frequency											
E1		E5a													
0.002	0.10	0.002	0.05	33	27	18	0.003	0.003	0.004	0.137	0.136	0.173	46.34	46.04	46.38
0.002	0.15	0.002	0.1	60	41	55	0.002	0.002	0.002	0.151	0.163	0.132	67.28	66.61	65.95
0.002	0.20	0.002	0.125	75	49	68	0.002	0.002	0.002	0.168	0.188	0.143	83.35	83.49	80.53
0.003	0.25	0.002	0.15	126	79	85	0.002	0.002	0.002	0.147	0.174	0.154	67.70	70.28	69.70
0.003	0.30	0.002	0.175	143	88	103	0.002	0.002	0.002	0.150	0.187	0.151	73.25	79.36	75.53
0.003	0.35	0.002	0.2	158	99	124	0.002	0.002	0.002	0.150	0.192	0.142	76.97	86.48	78.49
Φ	*P*	Φ	*P*	High-end receivers, Triple Frequency											
E1		E5a,E5b													
0.002	0.10	0.002	0.05	24	25	13	0.003	0.003	0.004	0.151	0.133	0.191	44.30	43.99	44.32
0.002	0.15	0.002	0.1	31	28	16	0.003	0.003	0.004	0.213	0.199	0.276	68.51	67.42	68.72
0.002	0.20	0.002	0.125	31	28	17	0.003	0.003	0.004	0.277	0.257	0.349	88.75	86.63	89.02
0.003	0.25	0.002	0.15	49	50	26	0.003	0.003	0.004	0.267	0.227	0.344	78.71	74.74	79.39
0.003	0.30	0.002	0.175	49	50	26	0.003	0.003	0.004	0.314	0.264	0.405	92.11	86.74	93.21
0.003	0.35	0.002	0.2	50	50	27	0.003	0.003	0.004	0.353	0.299	0.453	104.65	98.23	106.19
Φ	*P*	Φ	*P*	High-end receivers, Quadruple Frequency											
E1		E5a,E5b,E6													
0.002	0.10	0.002	0.05	21	22	12	0.003	0.003	0.004	0.159	0.139	0.195	45.86	45.58	45.86
0.002	0.15	0.002	0.1	28	25	15	0.003	0.003	0.004	0.220	0.207	0.279	69.88	68.94	70.01
0.002	0.20	0.002	0.125	29	26	16	0.003	0.003	0.004	0.281	0.262	0.353	90.53	88.55	90.76
0.003	0.25	0.002	0.15	46	47	25	0.003	0.003	0.004	0.272	0.231	0.345	79.52	75.71	80.02
0.003	0.30	0.002	0.175	47	48	25	0.003	0.003	0.004	0.315	0.266	0.406	92.95	87.67	93.97
0.003	0.35	0.002	0.2	48	48	26	0.003	0.003	0.004	0.356	0.302	0.455	105.69	99.32	107.10
North-South Baseline - 250Km															
Φ	*P*	Φ	*P*	High-end receivers, Dual Frequency											
E1		E5a													
0.002	0.10	0.002	0.05	42	35	19	0.003	0.003	0.004	0.137	0.127	0.169	46.00	45.50	46.36
0.002	0.15	0.002	0.1	76	49	52	0.002	0.002	0.002	0.145	0.156	0.139	64.31	64.71	66.35
0.002	0.20	0.002	0.125	91	57	64	0.002	0.002	0.002	0.160	0.180	0.152	76.98	80.23	81.67
0.003	0.25	0.002	0.15	144	93	79	0.002	0.002	0.002	0.141	0.164	0.165	61.21	66.53	71.09
0.003	0.30	0.002	0.175	155	102	95	0.002	0.002	0.002	0.147	0.176	0.164	66.01	74.55	77.67
0.003	0.35	0.002	0.2	166	109	114	0.002	0.002	0.002	0.149	0.186	0.156	69.27	81.52	81.33
Φ	*P*	Φ	*P*	High-end receivers, Triple Frequency											
E1		E5a,E5b													
0.002	0.10	0.002	0.05	31	32	15	0.003	0.003	0.004	0.150	0.125	0.179	44.13	43.59	44.29
0.002	0.15	0.002	0.1	38	35	17	0.003	0.003	0.004	0.215	0.187	0.269	67.69	66.04	68.65
0.002	0.20	0.002	0.125	39	36	17	0.003	0.003	0.004	0.274	0.236	0.351	86.85	83.84	89.05
0.003	0.25	0.002	0.15	63	62	29	0.003	0.003	0.004	0.256	0.209	0.325	75.35	71.31	78.87
0.003	0.30	0.002	0.175	64	63	29	0.003	0.003	0.004	0.293	0.239	0.382	86.74	81.97	92.42
0.003	0.35	0.002	0.2	64	63	29	0.003	0.003	0.004	0.330	0.270	0.437	97.43	92.27	105.48
Φ	*P*	Φ	*P*	High-end receivers, Quadruple Frequency											
E1		E5a,E5b,E6													
0.002	0.10	0.002	0.05	27	29	13	0.003	0.003	0.004	0.158	0.129	0.189	45.72	45.19	45.85
0.002	0.15	0.002	0.1	35	33	16	0.003	0.003	0.004	0.220	0.190	0.272	69.11	67.44	69.94
0.002	0.20	0.002	0.125	37	33	16	0.003	0.003	0.004	0.276	0.243	0.355	88.66	86.01	90.79
0.003	0.25	0.002	0.15	59	59	27	0.003	0.003	0.004	0.261	0.212	0.332	76.45	72.25	79.69
0.003	0.30	0.002	0.175	61	60	28	0.003	0.003	0.004	0.297	0.243	0.383	87.92	83.09	93.18
0.003	0.35	0.002	0.2	61	60	28	0.003	0.003	0.004	0.334	0.274	0.438	98.90	93.58	106.39

Table 6.17: **Galileo only**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, **Full AR** analysis, ASR and ambiguity-fixed and -float precision for rover-receiver coordinate are presented - **measurement precision varied**, number of satellites as available.

(4) For dual frequency with the measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a$ and σ_P=25cm on $E1$ and 15cm on $E5a$, it took 126, 79 and 85 epochs (**GPS only** took 334, 197 and 203 epochs) at 0°, −30° and −60° latitude E-W oriented baseline and 144, 93 and 85 (**GPS only** took 266, 173 and 249) for N-S oriented baseline.

Triple Frequency:

(5) The number of epochs taken to obtain 0.999 ASR lied between 13 and 50 (**GPS only** took 26 and 233) for E-W oriented baseline and between 15 and 64 (**GPS only** took 26 and 132) for N-S oriented baseline.

(6) The epochs taken for 0.999 ASR for the measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b$ were 49, 50 and 26 (**GPS only** took 233, 76 and 94) for E-W oriented baseline and 63, 62 and 29 (**GPS only** took 124, 71 and 119) for N-S oriented baseline at 0°, −30° and −60° latitudes respectively.

Quadruple Frequency:

(7) The number of epochs taken to obtain 0.999 ASR lied between 12 and 48 for E-W oriented baseline and between 13 and 61 for N-S oriented baseline.

(8) The epochs taken for 0.999 ASR for the measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b, E6$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b, E6$ were 46, 47 and 25 for E-W oriented baseline and 59, 59 and 27 for N-S oriented baseline at 0°, −30° and −60° latitudes respectively.

Comparison with GPS only:

(9) Galileo only out performed GPS only for dual and triple frequency. The better performance of Galileo system is as a result of its improved measurement precision as compared to GPS system.

(10) In general the quadruple Galileo outperformed triple frequency Galileo, hence the quadruple frequency Galileo outperforms GPS too. The reason

for better performance of a GNSS system as the number of frequencies are increased is related to an increase in the ionosphere information resulting in strengthening of GNSS model, as discussed earlier.

Considering the fixed-precision of rover-receiver coordinates to lie around 3mm in most of the cases when all the ambiguities are fixed, PAR will be evaluated for obtaining a fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for rover-receiver coordinate components (x, y and z) in the coming section.

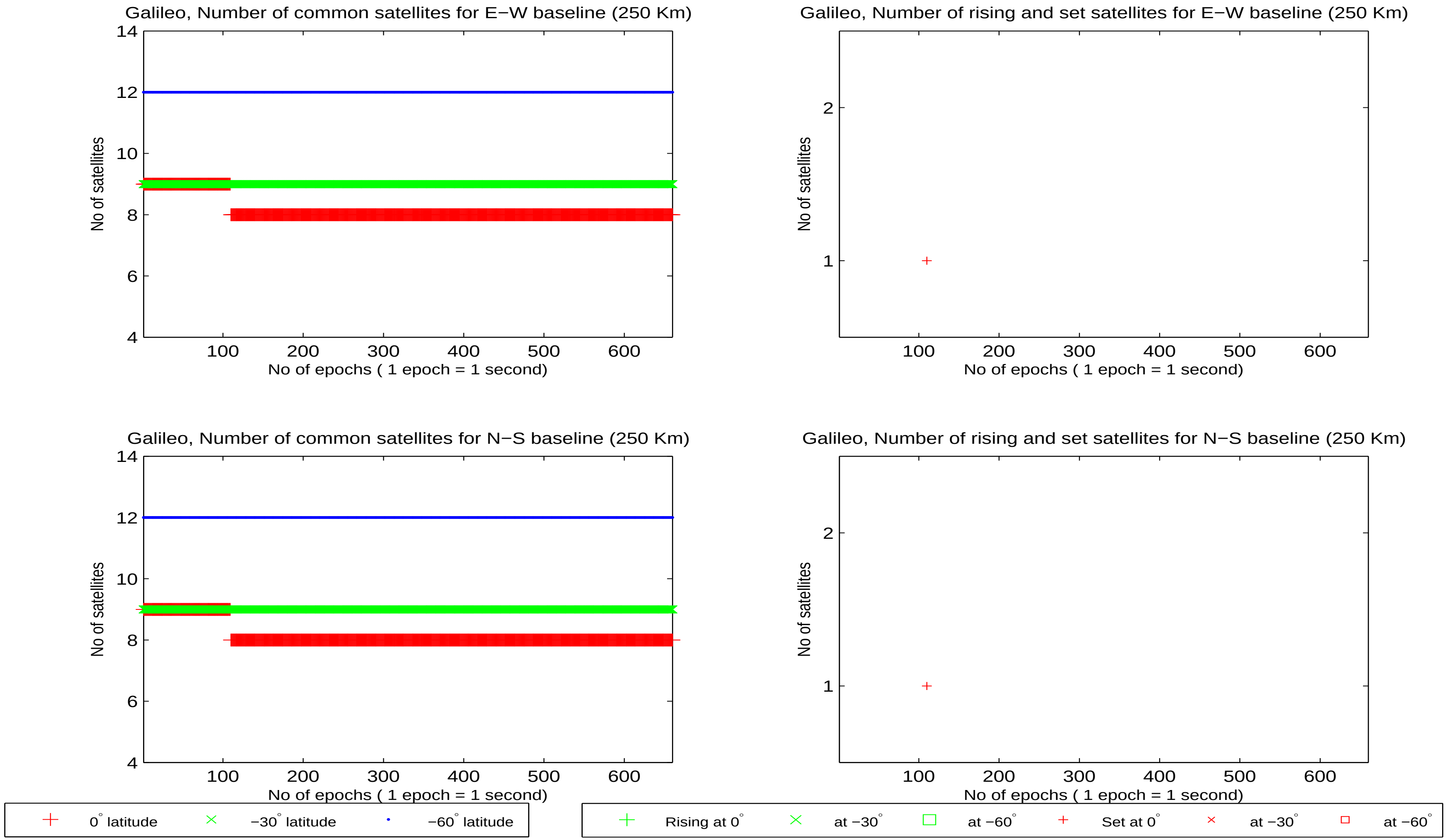

Figure 6.13: Number of satellites for **Galileo only**, 250 Km East-West and North-South baseline at $0°$, $-30°$ and $-60°$ degree latitude.

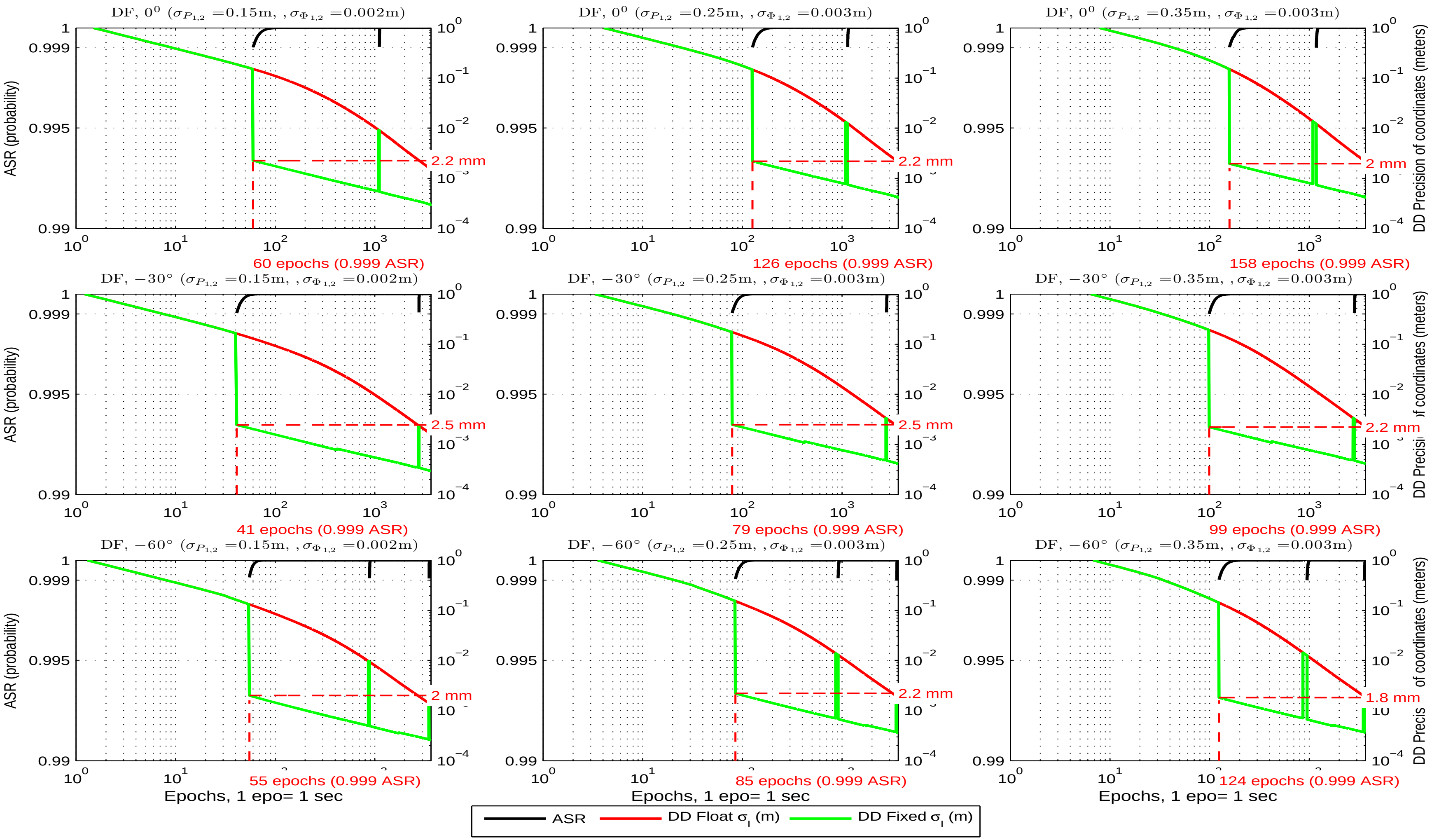

Figure 6.14: **Galileo only**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

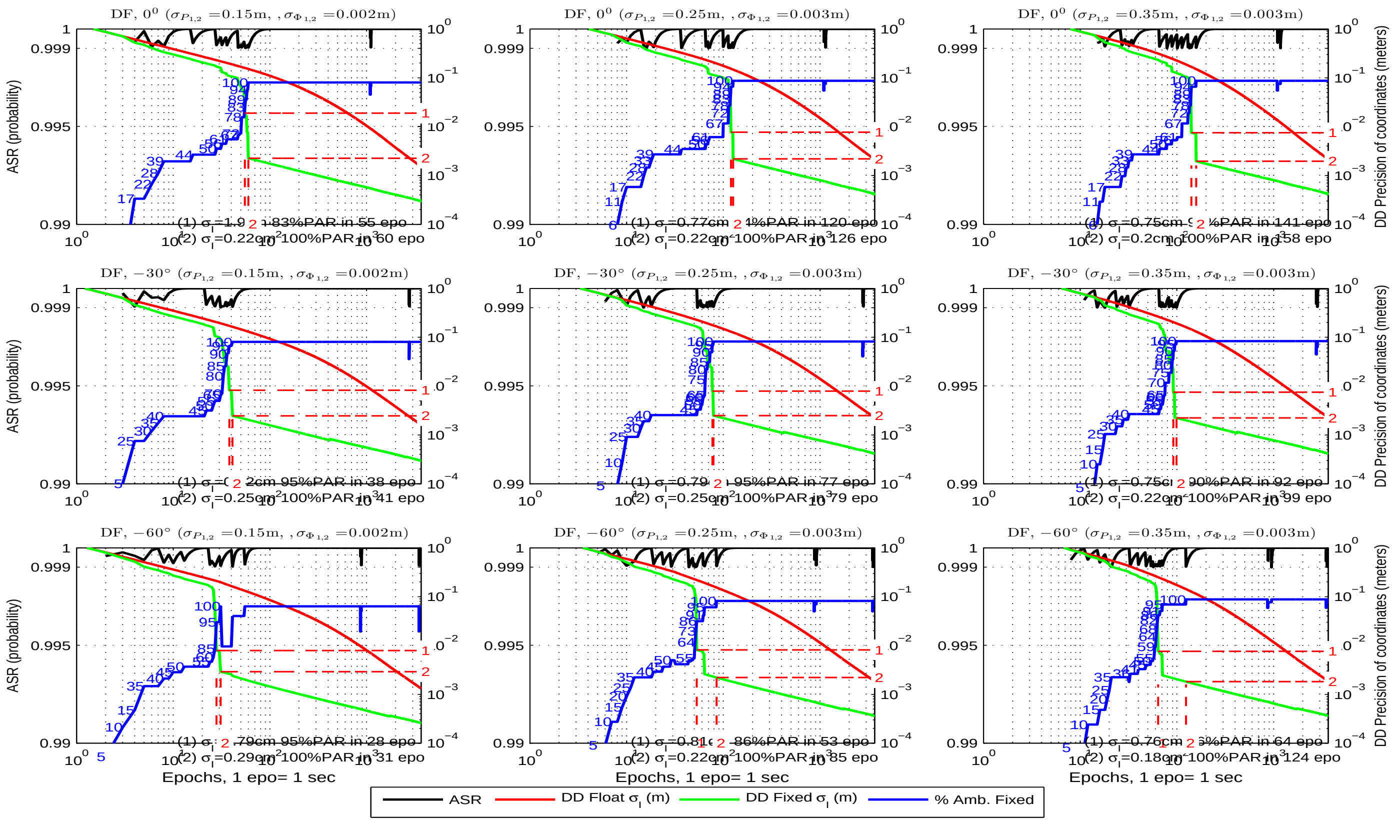

Figure 6.15: **Galileo only**, Dual frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

6.5.6 Galileo only, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, atmosphere unknown scenario

In this section, it is to be analyzed if partially fixing the ambiguities help to converge to a solution faster when a modest fixed-precision of σ_n, σ_e =2cm and σ_u=6cm is aimed for rover-receiver coordinate components x, y and z. Table 6.18 presents the results in terms of fixed-precision to be equal to or better than the predefined values and corresponding epochs taken and ambiguities fixed are noted.

The following is the analysis based on Table 6.18.

General remarks:

(1) Instantaneous predefined fixed-precision of the rover coordinate components could not be achieved with Galileo system, ionosphere float scenario.

(2) The partial ambiguities of less than 100% were fixed for almost all the scenarios of dual, triple and quadruple frequencies to obtain a predefined fixed-precision for rover-receiver coordinates.

Dual Frequency:

(3) The number of epochs taken for 0.999 ASR for all scenarios of measurement precision and baseline length lied between 17 and 141 (**GPS only** took 70 and 241) for E-W oriented baseline and between 16 and 165 (**GPS only** took 69 and 273) for N-S oriented baseline.

(4) For dual frequency with the measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a$ and σ_P=25cm on $E1$ and 15cm on $E5a$, it took 120, 77 and 53 epochs (**GPS only** took 224, 164 and 190 epochs) by fixing 96, 87 and 93% of ambiguities by PAR at 0°, −30° and −60° latitude E-W oriented baseline and 143, 90 and 54 epochs (**GPS only** took 258, 171 and 233 epochs) by fixing 94, 94 and 86 % of ambiguities for N-S oriented baseline.

Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for rover-receiver coordinates																		
Phase	Code (meters)	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{neu}}$(meters)			$\sigma_{\hat{neu}}$(meters)			Gain=$\sigma_{\hat{neu}}/\sigma_{\check{neu}}$		
				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km																		
Φ E1	P E1	Φ E5a	P E5a	High-end receivers, Dual Frequency														
0.002	0.10	0.002	0.05	32	21	17	89	85	95	0.013	0.019	0.009	0.140	0.155	0.178	10.53	8.25	19.93
0.002	0.15	0.002	0.1	55	38	28	83	95	95	0.019	0.008	0.008	0.159	0.171	0.209	8.40	20.79	26.57
0.002	0.20	0.002	0.125	68	46	32	89	95	86	0.016	0.008	0.008	0.179	0.196	0.250	11.10	25.57	33.01
0.003	0.25	0.002	0.15	120	77	53	94	95	86	0.008	0.008	0.008	0.153	0.177	0.219	19.93	22.47	26.98
0.003	0.30	0.002	0.175	131	86	59	94	95	86	0.008	0.008	0.008	0.162	0.190	0.235	21.29	24.88	30.15
0.003	0.35	0.002	0.2	141	92	64	94	90	86	0.008	0.008	0.008	0.167	0.203	0.250	22.25	27.04	32.93
Φ E1	P E1	Φ E5a,E5b	P E5a,E5b	High-end receivers, Triple Frequency														
0.002	0.10	0.002	0.05	22	15	13	93	90	100	0.017	0.008	0.004	0.158	0.172	0.191	9.47	21.33	44.32
0.002	0.15	0.002	0.1	29	20	15	96	93	90	0.007	0.007	0.017	0.220	0.238	0.286	31.72	34.02	16.39
0.002	0.20	0.002	0.125	30	20	16	96	90	97	0.007	0.019	0.009	0.282	0.310	0.360	40.99	16.61	39.68
0.003	0.25	0.002	0.15	45	29	25	96	87	93	0.008	0.017	0.012	0.281	0.314	0.351	37.14	18.81	29.12
0.003	0.30	0.002	0.175	46	30	25	96	90	90	0.008	0.008	0.018	0.326	0.362	0.414	43.42	44.26	22.49
0.003	0.35	0.002	0.2	46	30	26	89	87	97	0.018	0.017	0.007	0.372	0.414	0.464	21.25	24.99	70.20
Φ E1	P E1	Φ E5a,E5b,E6	P E5a,E5b,E6	High-end receivers, Quadruple Frequency														
0.002	0.10	0.002	0.05	18	13	11	92	92	100	0.018	0.008	0.004	0.172	0.182	0.204	9.54	22.23	45.88
0.002	0.15	0.002	0.1	27	18	14	97	95	95	0.007	0.007	0.012	0.224	0.246	0.289	32.35	34.75	24.59
0.002	0.20	0.002	0.125	28	19	15	97	95	98	0.007	0.007	0.009	0.287	0.312	0.365	41.80	44.72	40.44
0.003	0.25	0.002	0.15	43	28	24	97	92	95	0.008	0.008	0.012	0.282	0.314	0.352	37.45	38.22	29.35
0.003	0.30	0.002	0.175	44	29	24	97	92	92	0.007	0.008	0.018	0.328	0.362	0.416	43.79	44.59	22.65
0.003	0.35	0.002	0.2	45	29	23	97	90	90	0.007	0.016	0.019	0.370	0.414	0.489	49.82	25.15	25.30
North-South Baseline - 250Km																		
Φ E1	P E1	Φ E5a	P E5a	High-end receivers, Dual Frequency														
0.002	0.10	0.002	0.05	38	25	16	88	83	90	0.016	0.014	0.014	0.144	0.152	0.185	9.17	10.55	13.47
0.002	0.15	0.002	0.1	75	46	27	88	89	95	0.016	0.010	0.008	0.147	0.162	0.215	9.25	16.05	25.83
0.002	0.20	0.002	0.125	90	54	33	94	94	86	0.008	0.008	0.008	0.161	0.187	0.246	21.28	23.80	31.71
0.003	0.25	0.002	0.15	143	90	54	94	94	86	0.007	0.008	0.008	0.142	0.168	0.218	19.77	20.43	26.26
0.003	0.30	0.002	0.175	154	95	60	94	94	86	0.007	0.008	0.008	0.148	0.185	0.235	21.07	22.56	29.31
0.003	0.35	0.002	0.2	165	101	66	94	94	86	0.007	0.008	0.008	0.150	0.198	0.247	21.97	24.37	31.87
Φ E1	P E1	Φ E5a,E5b	P E5a,E5b	High-end receivers, Triple Frequency														
0.002	0.10	0.002	0.05	23	15	12	92	85	93	0.018	0.019	0.013	0.174	0.185	0.200	9.54	9.90	15.12
0.002	0.15	0.002	0.1	32	20	14	96	85	93	0.007	0.017	0.014	0.236	0.256	0.298	32.71	14.84	21.06
0.002	0.20	0.002	0.125	34	21	15	96	85	93	0.007	0.017	0.014	0.296	0.323	0.375	41.97	19.03	27.05
0.003	0.25	0.002	0.15	46	30	24	92	85	93	0.018	0.019	0.013	0.310	0.329	0.362	16.91	17.51	27.22
0.003	0.30	0.002	0.175	47	30	24	92	85	93	0.018	0.019	0.013	0.358	0.385	0.427	19.64	20.44	31.98
0.003	0.35	0.002	0.2	47	31	24	92	85	93	0.018	0.019	0.013	0.408	0.431	0.490	22.26	23.18	36.60
Φ E1	P E1	Φ E5a,E5b,E6	P E5a,E5b,E6	High-end receivers, Quadruple Frequency														
0.002	0.10	0.002	0.05	20	13	10	94	92	95	0.018	0.017	0.014	0.184	0.196	0.215	9.99	11.40	15.73
0.002	0.15	0.002	0.1	29	18	13	97	89	95	0.007	0.017	0.014	0.243	0.264	0.303	33.41	15.16	21.51
0.002	0.20	0.002	0.125	31	20	14	94	92	95	0.016	0.015	0.014	0.305	0.325	0.381	18.72	21.01	27.54
0.003	0.25	0.002	0.15	44	28	23	94	89	95	0.018	0.019	0.013	0.312	0.335	0.363	17.06	17.70	27.43
0.003	0.30	0.002	0.175	45	29	23	94	89	95	0.018	0.019	0.013	0.361	0.386	0.429	19.81	20.59	32.22
0.003	0.35	0.002	0.2	45	30	23	94	89	95	0.018	0.018	0.013	0.411	0.432	0.492	22.46	23.35	36.88

Table 6.18: **Galileo only**, **PAR** analysis, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario. Evaluation of ASR and number of epochs for obtaining 2cm fixed-precision of rover-receiver coordinates - **measurement precision is varied**, number of satellites are as available.

Triple Frequency:

(5) The number of epochs taken to obtain 0.999 ASR lied between 13 and 46 (**GPS only** took 24 and 108) for E-W oriented baseline and between 12 and 47 (**GPS only** took 24 and 123) for N-S oriented baseline.

(6) The epochs taken for 0.999 ASR (measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b$) were 45, 29 and 25 (**GPS only** took 98, 74 and 81 epochs) by fixing partially 96, 87 and 93 % of ambiguities for E-W oriented baseline and 46, 30 and 24 (**GPS only** took 115, 67 and 103) epochs by fixing 92, 85 and 93 % of ambiguities by PAR for N-S oriented baseline at 0°, $-30°$ and $-60°$ latitudes respectively.

Quadruple Frequency:

(7) The number of epochs taken to obtain 0.999 ASR lied between 11 and 45 for E-W oriented baseline and between 10 and 45 for N-S oriented baseline.

(8) The epochs taken for 0.999 ASR (measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b, E6$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b, E6$) were 45, 29 and 25 by fixing partially 96, 87 and 93 % of ambiguities for E-W oriented baseline and 46, 30 and 24 epochs by fixing 92, 85 and 93 % of ambiguities by PAR for N-S oriented baseline at 0°, $-30°$ and $-60°$ latitudes respectively.

Comparison with GPS only:

(9) Galileo system out performed GPS only by taking less number of epochs to achieve similar predefined criteria of fixed-precision of rover receiver coordinate components.

(10) The quadruple frequency Galileo performed better than the triple frequency Galileo and hence triple frequency GPS too.

Further results and discussion is done for number of epochs taken and ambiguities fixed in order to obtain a fixed-precision of 20cm for the horizontal coordinate components.

6.5.7 Galileo only, Partial Ambiguity Resolution to obtain fixed-precision of 20 cm, atmosphere unknown scenario

In this section, it is to be analyzed if partially fixing the ambiguities help to converge to a solution faster when a modest fixed-precision of σ_n, σ_e =20cm is aimed for horizontal rover-receiver coordinate components. Table 6.22 presents the results in terms of fixed-precision to be equal to or better than the predefined values and corresponding epochs taken and ambiguities fixed are noted.

The following is the analysis based on Table 6.22.

General remarks:

(1) Instantaneous predefined fixed-precision of 20cm for the horizontal coordinate components for rover receiver could be achieved for 2 scenarios of triple and quadruple frequency Galileo system, ionosphere float scenario.

(2) The partial ambiguities of less than 100% were fixed for all the scenarios of dual, triple and quadruple frequencies to obtain a predefined fixed-precision for rover-receiver coordinates.

Dual Frequency:

(3) The number of epochs taken for 0.999 ASR for all scenarios of measurement precision and baseline length lied between 3 and 46 (**GPS only** took 7 and 99) for E-W oriented baseline and between 3 and 47 (**GPS only** took 8 and 113) for N-S oriented baseline.

(4) For dual frequency with the measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a$ and σ_P=25cm on $E1$ and 15cm on $E5a$, it took 17, 18 and 27 epochs (**GPS only** took 51, 40 and 66 epochs) by fixing 28, 40 and 45% of ambiguities by PAR at 0°, −30° and −60° latitude E-W oriented baseline and 18, 16 and 28 epochs (**GPS only** took 47, 40 and 73 epochs) by fixing 31, 39 and 45% of ambiguities for N-S oriented

Phase	Code	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$		
	(meters)			0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
Partial AR @ 0.999 ASR — criteria - 20 cm fixed-precision for horizontal rover-receiver coordinate components																		
East-West Baseline - 250Km																		
Φ	P	Φ	P	High-end receivers, Dual Frequency														
E1		E5a																
0.002	0.10	0.002	0.05	3	3	4	39	40	50	0.302	0.270	0.222	0.458	0.414	0.369	1.52	1.53	1.66
0.002	0.15	0.002	0.1	8	8	12	39	40	45	0.335	0.299	0.247	0.432	0.390	0.327	1.29	1.30	1.32
0.002	0.20	0.002	0.125	12	12	19	33	40	45	0.350	0.309	0.246	0.466	0.418	0.338	1.33	1.35	1.37
0.003	0.25	0.002	0.15	17	18	27	28	40	45	0.363	0.305	0.248	0.487	0.421	0.348	1.34	1.38	1.40
0.003	0.30	0.002	0.175	23	25	36	28	40	41	0.365	0.297	0.242	0.496	0.417	0.338	1.36	1.40	1.40
0.003	0.35	0.002	0.2	30	32	46	28	40	41	0.365	0.292	0.232	0.498	0.414	0.323	1.36	1.42	1.39
Φ	P	Φ	P	High-end receivers, Triple Frequency														
E1		E5a,E5b																
0.002	0.10	0.002	0.05	1	1	2	59	60	67	0.371	0.329	0.210	0.741	0.669	0.488	2.00	2.04	2.32
0.002	0.15	0.002	0.1	2	2	3	59	60	67	0.341	0.302	0.214	0.848	0.765	0.644	2.48	2.54	3.00
0.002	0.20	0.002	0.125	2	2	3	56	53	67	0.369	0.326	0.224	1.113	1.004	0.845	3.02	3.08	3.78
0.003	0.25	0.002	0.15	3	3	4	56	57	67	0.333	0.293	0.211	1.125	1.015	0.905	3.38	3.46	4.29
0.003	0.30	0.002	0.175	4	3	4	56	53	67	0.297	0.303	0.214	1.160	1.209	1.078	3.91	3.99	5.04
0.003	0.35	0.002	0.2	5	3	4	56	53	63	0.271	0.309	0.276	1.204	1.402	1.250	4.44	4.54	4.53
Φ	P	Φ	P	High-end receivers, Quadruple Frequency														
E1		E5a,E5b,E6																
0.002	0.10	0.002	0.05	1	1	2	69	70	75	0.340	0.301	0.189	0.729	0.658	0.480	2.15	2.19	2.53
0.002	0.15	0.002	0.1	2	2	3	69	70	75	0.326	0.288	0.204	0.830	0.749	0.630	2.54	2.60	3.09
0.002	0.20	0.002	0.125	2	2	3	69	70	75	0.355	0.313	0.216	1.091	0.984	0.828	3.07	3.14	3.83
0.003	0.25	0.002	0.15	3	2	3	69	70	70	0.323	0.349	0.322	1.103	1.219	1.025	3.42	3.49	3.18
0.003	0.30	0.002	0.175	3	3	4	69	70	75	0.334	0.295	0.210	1.315	1.186	1.058	3.93	4.02	5.04
0.003	0.35	0.002	0.2	4	3	4	69	70	75	0.296	0.302	0.213	1.321	1.376	1.227	4.46	4.56	5.77
North-South Baseline - 250Km																		
Φ	P	Φ	P	High-end receivers, Dual Frequency														
E1		E5a																
0.002	0.10	0.002	0.05	3	3	4	44	44	45	0.337	0.290	0.234	0.519	0.445	0.371	1.54	1.53	1.59
0.002	0.15	0.002	0.1	8	7	13	38	39	45	0.379	0.351	0.239	0.489	0.448	0.316	1.29	1.27	1.32
0.002	0.20	0.002	0.125	13	11	20	38	39	45	0.377	0.355	0.242	0.506	0.469	0.330	1.34	1.32	1.37
0.003	0.25	0.002	0.15	18	16	28	31	39	45	0.393	0.356	0.246	0.534	0.481	0.343	1.36	1.35	1.40
0.003	0.30	0.002	0.175	25	22	37	31	39	41	0.389	0.350	0.240	0.535	0.479	0.335	1.38	1.37	1.39
0.003	0.35	0.002	0.2	32	29	47	31	39	41	0.390	0.339	0.231	0.537	0.468	0.321	1.38	1.38	1.39
Φ	P	Φ	P	High-end receivers, Triple Frequency														
E1		E5a,E5b																
0.002	0.10	0.002	0.05	2	1	2	63	63	67	0.286	0.353	0.211	0.593	0.719	0.490	2.07	2.04	2.32
0.002	0.15	0.002	0.1	2	2	3	63	63	67	0.365	0.322	0.215	0.959	0.822	0.647	2.63	2.55	3.00
0.002	0.20	0.002	0.125	3	3	3	63	63	67	0.316	0.282	0.225	1.028	0.881	0.850	3.25	3.13	3.78
0.003	0.25	0.002	0.15	3	3	4	63	63	67	0.346	0.310	0.212	1.273	1.090	0.910	3.68	3.52	4.29
0.003	0.30	0.002	0.175	4	4	4	63	63	67	0.306	0.276	0.215	1.313	1.124	1.084	4.29	4.08	5.04
0.003	0.35	0.002	0.2	5	4	4	63	59	63	0.278	0.281	0.313	1.362	1.304	1.258	4.90	4.64	4.02
Φ	P	Φ	P	High-end receivers, Quadruple Frequency														
E1		E5a,E5b,E6																
0.002	0.10	0.002	0.05	1	1	2	72	72	75	0.368	0.322	0.191	0.825	0.707	0.482	2.24	2.19	2.53
0.002	0.15	0.002	0.1	2	2	3	72	72	75	0.348	0.308	0.205	0.939	0.805	0.633	2.69	2.62	3.09
0.002	0.20	0.002	0.125	2	3	3	72	72	75	0.374	0.272	0.217	1.234	0.863	0.833	3.30	3.18	3.83
0.003	0.25	0.002	0.15	3	3	4	72	72	75	0.337	0.302	0.207	1.248	1.069	0.893	3.70	3.54	4.31
0.003	0.30	0.002	0.175	4	3	4	72	72	75	0.300	0.311	0.211	1.288	1.274	1.064	4.30	4.10	5.04
0.003	0.35	0.002	0.2	5	4	4	72	72	75	0.273	0.275	0.214	1.337	1.280	1.235	4.90	4.65	5.77

Table 6.19: **Galileo only**, **PAR** analysis, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario. Evaluation of ASR and number of epochs for obtaining 20cm fixed-precision for the horizontal rover-receiver coordinate components - **measurement precision is varied**, number of satellites are as available.

baseline.

Triple Frequency:

(5) The number of epochs taken to obtain 0.999 ASR lied between 1 and 5 (**GPS only** took 2 and 13) for E-W oriented baseline and between 1 and 5 (**GPS only** took 2 and 13) for N-S oriented baseline.

(6) The epochs taken for 0.999 ASR (measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b$) were 3, 3 and 4 (**GPS only** took 8, 6 and 11 epochs) by fixing partially 56, 57 and 67% of ambiguities for E-W oriented baseline and 3, 3 and 4 (**GPS only** took 7, 7 and 11 epochs) epochs by fixing 63, 63 and 67% of ambiguities by PAR for N-S oriented baseline at 0°, −30° and −60° latitudes respectively.

Quadruple Frequency:

(7) The number of epochs taken to obtain 0.999 ASR lied between 1 and 4 for both E-W and N-S oriented baseline.

(8) The epochs taken for 0.999 ASR (measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b, E6$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b, E6$) were 3, 2 and 3 by fixing partially 69, 70 and 70% of ambiguities for E-W oriented baseline and 3, 3 and 4 epochs by fixing 72, 72 and 75% of ambiguities by PAR for N-S oriented baseline at 0°, −30° and −60° latitudes respectively.

Comparison with GPS only:

(9) Galileo system out performed GPS only by taking less number of epochs to achieve similar predefined criteria of fixed-precision of rover receiver coordinate components.

(10) The quadruple frequency Galileo performed better than the triple frequency Galileo and hence triple frequency GPS too.

6.5.8 GPS + Galileo - Full and Partial Ambiguity Resolution, atmosphere unknown scenario

The ASR for the combined GPS and Galileo system are simulated for Geometry Based model, coordinates known, atmosphere unknown scenario, with coordinates assumed to be known, hence fixed is simulated along with the fixed-precision of the rover-receiver coordinates and other unknowns. The atmosphere, both ionosphere and troposphere are assumed to be unknown and are parameterized in the design matrix of the functional model. Table 6.20 and Figures G.33 to G.36 in Appendix G present the results full and PAR for dual and quadruple frequency, for combined GPS and Galileo for 250 Km baseline in East-West direction, see Appendix G. Figure 6.16 presents the number of satellites for E-W oriented baseline (250 Km) and N-S baseline (250 Km) for the combined GPS+Galileo system.

Analysis based on Figures 6.16 to 6.18 and Table 6.20 is given below.

General remarks:

(1) Instantaneous ambiguity resolution could not be achieved for any scenario of measurement precision for a combined GPS and Galileo system, using dual and quadruple frequency both.

Dual Frequency:

(2) The number of epochs taken for 0.999 ASR for all scenarios of measurement precision and baseline length lied between 9 and 126 (**GPS only** took 7 and 123 and **Galileo only** took 18 and 158) for E-W oriented baseline and between 9 and 124 (**GPS only** took 70 and 282 and **Galileo only** took 19 and 166) for N-S oriented baseline.

(3) For dual frequency with the measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a$ and σ_P=25cm on $E1$ and 15cm on $E5a$, it took 44, 51 and 84 epochs (**GPS only** took 334, 197 and 203 epochs and **Galileo only** took 126, 79 and 85) at $0°$, $-30°$ and $-60°$ latitude E-W oriented baseline and 32, 41 and 85 (**GPS only** took 266, 173 and 249 and **Galileo only** took 144, 93 and 85) for N-S oriented baseline.

				Full AR - minimum desired ASR = 0.999											
Phase	Code	Phase	Code	Epochs taken			$\sigma_{\widecheck{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widecheck{neu}}$		
	(meters)			0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km															
Φ	P	Φ	P	High-end receivers, Dual Frequency											
L1(E1)		L5(E5a)													
0.002	0.10	0.002	0.05	9	10	10	0.004	0.003	0.003	0.186	0.151	0.150	46.61	46.56	46.59
0.002	0.15	0.002	0.1	19	22	25	0.003	0.002	0.002	0.196	0.155	0.143	69.97	69.16	69.25
0.002	0.20	0.002	0.125	30	35	68	0.002	0.002	0.001	0.202	0.155	0.100	90.34	86.98	81.49
0.003	0.25	0.002	0.15	44	51	84	0.003	0.002	0.002	0.204	0.155	0.110	79.05	75.15	71.20
0.003	0.30	0.002	0.175	59	69	103	0.002	0.002	0.001	0.198	0.147	0.108	88.85	82.74	77.52
0.003	0.35	0.002	0.2	77	89	126	0.002	0.002	0.001	0.183	0.136	0.101	93.57	87.28	80.21
Φ	P	Φ	P	High-end receivers, Quadruple Frequency											
L1(E1),L2,E5b		L5(E5a)													
0.002	0.10	0.002	0.05	5	6	8	0.004	0.003	0.003	0.138	0.103	0.102	35.48	34.50	38.98
0.002	0.15	0.002	0.1	7	7	9	0.003	0.003	0.003	0.201	0.166	0.162	59.76	58.71	64.45
0.002	0.20	0.002	0.125	9	9	10	0.003	0.003	0.002	0.230	0.189	0.199	77.21	75.59	83.44
0.003	0.25	0.002	0.15	13	13	17	0.003	0.003	0.002	0.234	0.192	0.187	71.14	69.72	76.78
0.003	0.30	0.002	0.175	18	16	18	0.003	0.002	0.002	0.235	0.203	0.214	83.74	81.75	90.37
0.003	0.35	0.002	0.2	23	21	21	0.002	0.002	0.002	0.238	0.201	0.226	95.83	92.70	103.02
North-South Baseline - 250Km															
Φ	P	Φ	P	High-end receivers, Dual Frequency											
L1(E1)		L5(E5a)													
0.002	0.10	0.002	0.05	9	9	10	0.004	0.004	0.003	0.199	0.169	0.161	46.60	46.57	46.53
0.002	0.15	0.002	0.1	15	18	19	0.003	0.003	0.003	0.237	0.182	0.177	70.06	69.40	69.07
0.002	0.20	0.002	0.125	21	28	30	0.003	0.002	0.002	0.261	0.185	0.177	91.26	88.07	86.68
0.003	0.25	0.002	0.15	32	41	85	0.003	0.002	0.002	0.260	0.185	0.109	80.17	76.35	68.65
0.003	0.30	0.002	0.175	41	55	103	0.003	0.002	0.001	0.264	0.178	0.108	92.15	84.85	74.84
0.003	0.35	0.002	0.2	52	71	124	0.003	0.002	0.001	0.256	0.168	0.102	100.44	90.46	78.00
Φ	P	Φ	P	High-end receivers, Quadruple Frequency											
L1(E1),L2,E5b		L5(E5a)													
0.002	0.10	0.002	0.05	4	5	7	0.005	0.003	0.003	0.172	0.121	0.115	35.71	34.58	38.69
0.002	0.15	0.002	0.1	6	6	7	0.004	0.003	0.003	0.240	0.191	0.194	59.84	58.76	64.39
0.002	0.20	0.002	0.125	9	9	9	0.003	0.003	0.003	0.254	0.201	0.222	77.31	75.59	83.17
0.003	0.25	0.002	0.15	13	12	14	0.004	0.003	0.003	0.259	0.213	0.218	70.96	69.72	76.85
0.003	0.30	0.002	0.175	18	16	16	0.003	0.003	0.003	0.259	0.216	0.239	83.38	81.52	90.15
0.003	0.35	0.002	0.2	23	21	20	0.003	0.002	0.002	0.262	0.213	0.242	95.11	92.14	102.03

Table 6.20: **GPS + Galileo, Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, **Full AR** analysis, ASR and ambiguity-fixed and -float precision for rover-receiver coordinates are presented - **measurement precision varied**, number of satellites as available.

Quadruple Frequency:

(4) The number of epochs taken to obtain 0.999 ASR lied between 5 and 23 (**Galileo only** took 12 and 48) for E-W oriented baseline and between 4 and 23 (**Galileo only** took 13 and 61) for N-S oriented baseline.

(5) The epochs taken for 0.999 ASR for the measurement precision of σ_{Φ}=3mm on $E1$ and 2mm on $E5a, E5b, E6$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b, E6$ were 13, 13 and 17 (**Galileo only** took 46, 47 and 25) for E-W oriented baseline and 13, 12 and 14 (**Galileo only** took 59, 59 and 27) for N-S oriented baseline at $0°$, $-30°$ and $-60°$ latitudes respectively.

Comparison with GPS only and Galileo only systems:

(6) While dual frequency systems are compared, namely, GPS only, Galileo only and GPS+Galileo combined, the combined GPS and Galileo system performed much better than the individual GPS and Galileo systems. The comparisons are presented in the above analysis.

(7) A comparison for quadruple frequency between Galileo only and GPS, Galileo combined system shows that the combined system outperforms the standalone Galileo system. In general the quadruple frequency GPS+Galileo almost takes half the number of epochs or even less than that as compared to standalone Galileo. For a Reference-rover scenario when the coordinates of rover receiver are estimated the standalone system proves to be much weaker than the combined GPS+Galileo system.

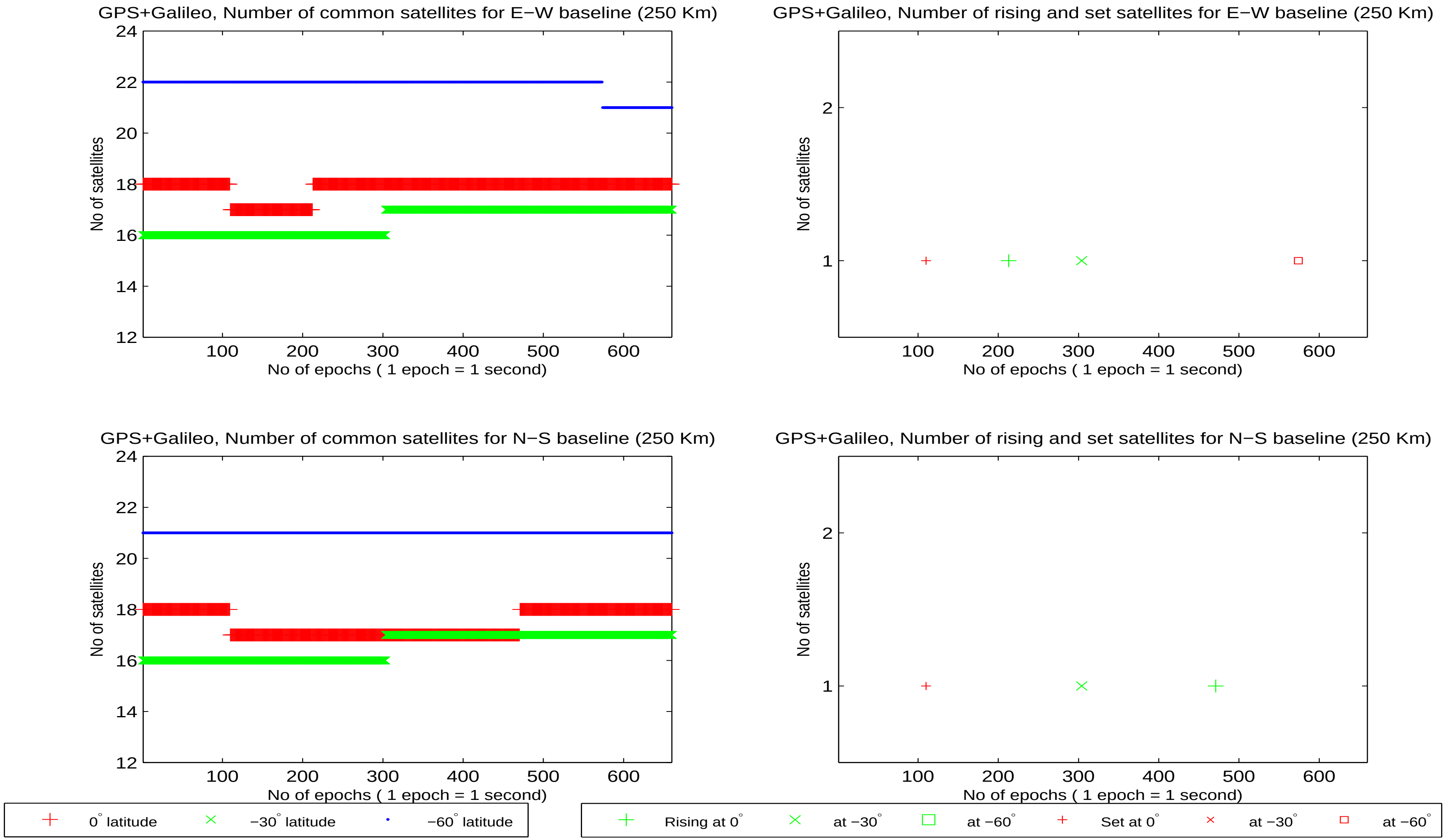

Figure 6.16: Number of satellites for **GPS + Galileo**, 250 Km East-West and North-South baseline at 0°, −30° and −60° degree latitude.

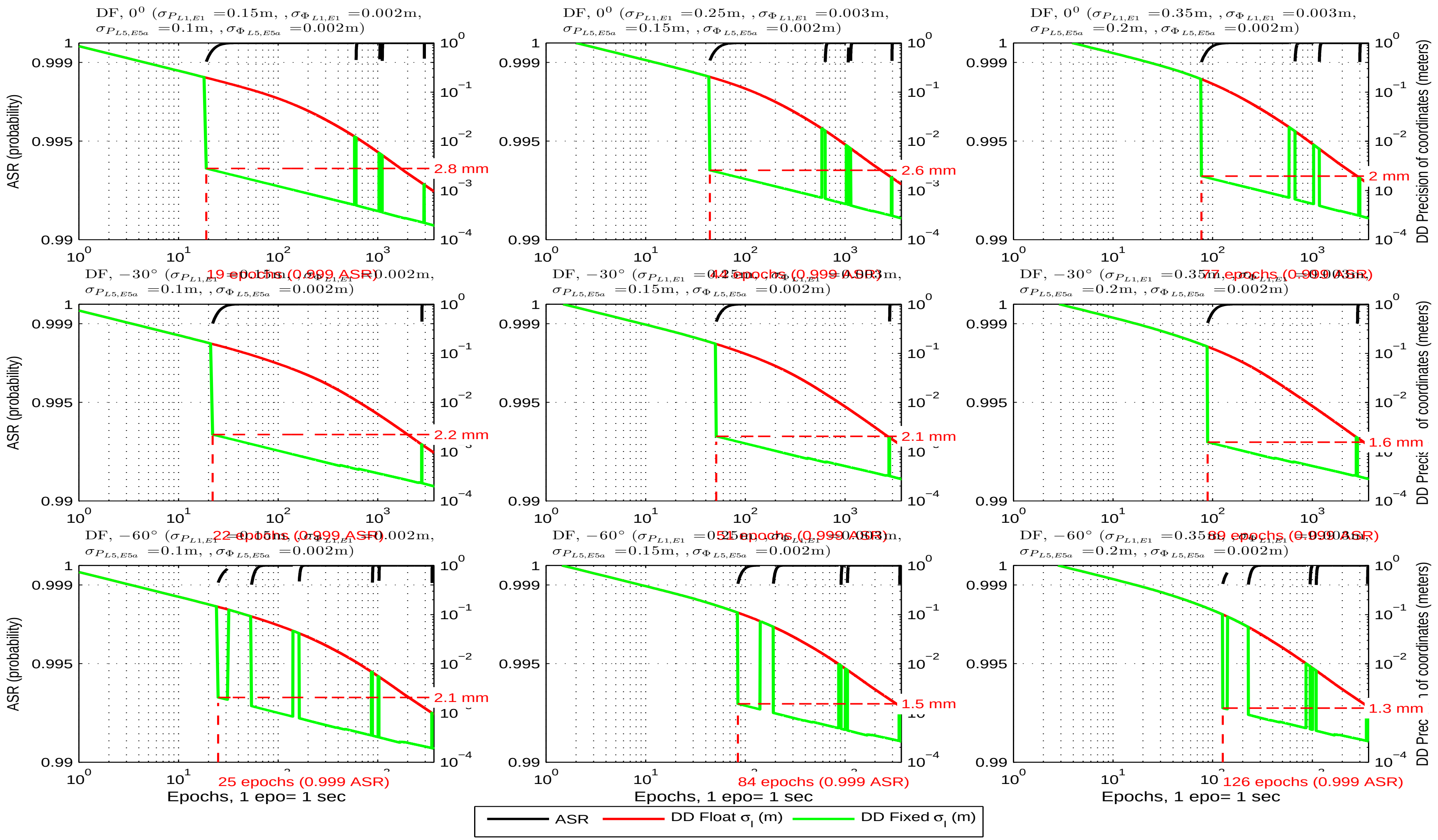

Figure 6.17: **GPS + Galileo**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

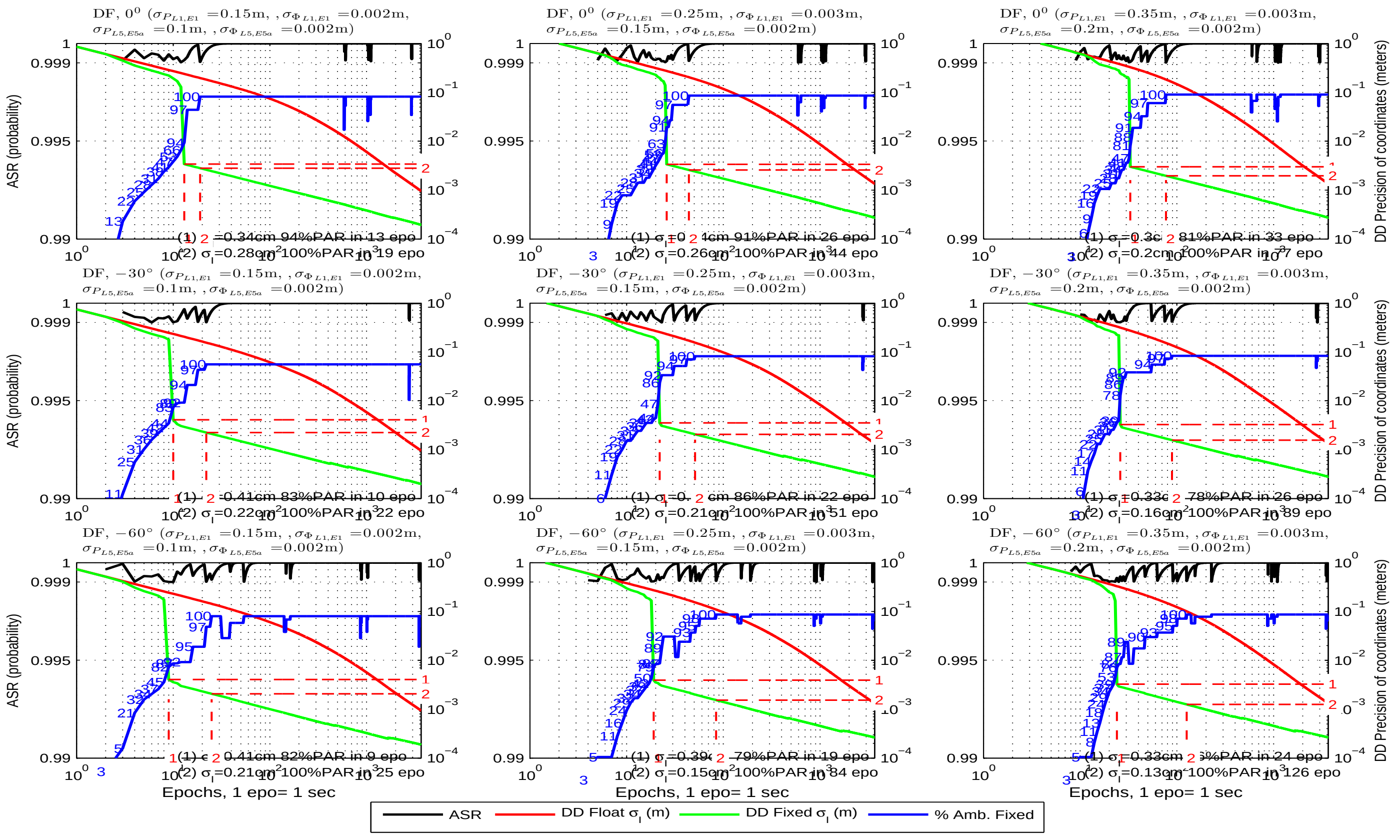

Figure 6.18: **GPS + Galileo**, Dual frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

6.5.9 GPS + Galileo, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, atmosphere unknown scenario

While resolving all the ambiguities, a fixed-precision of 5mm and better was obtained. In this analysis it is to examine if a fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for the x, y and z components of rover-receiver coordinates could be obtained in reduced number of epochs and by partial fixing of ambiguities. The results are presented in Table 6.21, see below.

Analysis based on Table 6.21 is given below.

General remarks:

(1) For all the scenarios of measurement precision, less than 100% of ambiguities were fixed in order to obtain a predefined fixed-precision for rover-receiver coordinates.

(2) The predefined fixed-precision for rover-receiver coordinates could not be achieved instantaneously for any of the scenarios.

Dual Frequency:

(3) The number of epochs taken for 0.999 ASR for all scenarios of measurement precision and baseline length lied between 6 and 33 (**GPS only** took 70 and 241 and **Galileo only** took 17 and 141) for E-W oriented baseline and between 7 and 35 (**GPS only** took 69 and 273 and **Galileo only** took 16 and 165) for N-S oriented baseline.

(4) For dual frequency with the measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a$ and σ_P=25cm on $E1$ and 15cm on $E5a$, it took 26, 22 and 19 epochs (**GPS only** took 224, 164 and 190 epochs and **Galileo only** took 120, 77 and 53 epochs) by fixing 91, 86 and 79% of ambiguities by PAR at 0°, −30° and −60° latitude E-W oriented baseline and 28, 22 and 19 epochs (**GPS only** took 258, 171 and 233 epochs and **Galileo only** took 143, 90 and 54 epochs) by fixing 93, 91 and 88% of ambiguities for N-S oriented baseline.

Quadruple Frequency:

Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for rover-receiver coordinates																		
Phase	Code (meters)	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\check{neu}}$		
				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km																		
Φ	P L1(E1)	Φ	P L5(E5a)	High-end receivers, Dual Frequency														
0.002	0.10	0.002	0.05	8	7	6	94	92	71	0.009	0.005	0.016	0.197	0.181	0.193	22.75	37.88	11.83
0.002	0.15	0.002	0.1	13	10	9	94	83	82	0.003	0.004	0.004	0.237	0.233	0.243	70.00	56.99	59.66
0.002	0.20	0.002	0.125	15	13	11	91	92	79	0.003	0.003	0.003	0.291	0.268	0.290	91.89	91.57	84.49
0.003	0.25	0.002	0.15	26	22	19	91	86	79	0.003	0.004	0.004	0.273	0.253	0.272	81.13	71.94	69.25
0.003	0.30	0.002	0.175	30	25	21	91	86	76	0.003	0.003	0.004	0.299	0.279	0.307	95.49	90.55	81.84
0.003	0.35	0.002	0.2	33	26	24	81	78	76	0.003	0.003	0.003	0.326	0.314	0.329	108.73	96.48	101.18
Φ	P L1(E1),L2,E5b	Φ	P L5(E5a)	High-end receivers, Quadruple Frequency														
0.002	0.10	0.002	0.05	2	2	2	92	91	89	0.007	0.005	0.006	0.219	0.179	0.204	33.10	32.97	34.78
0.002	0.15	0.002	0.1	3	3	2	88	87	81	0.006	0.005	0.006	0.307	0.254	0.344	55.26	53.93	54.32
0.002	0.20	0.002	0.125	4	3	3	92	83	88	0.005	0.005	0.005	0.345	0.328	0.366	72.37	60.28	76.00
0.003	0.25	0.002	0.15	5	5	4	79	85	86	0.007	0.005	0.005	0.379	0.312	0.390	56.99	60.19	70.96
0.003	0.30	0.002	0.175	6	5	4	90	83	68	0.005	0.006	0.020	0.409	0.369	0.462	77.92	65.57	23.53
0.003	0.35	0.002	0.2	7	6	5	92	85	88	0.005	0.005	0.005	0.438	0.388	0.478	89.63	75.53	97.23
North-South Baseline - 250Km																		
Φ	P L1(E1)	Φ	P L5(E5a)	High-end receivers, Dual Frequency														
0.002	0.10	0.002	0.05	9	7	7	100	97	97	0.004	0.004	0.005	0.199	0.192	0.193	46.60	43.27	42.71
0.002	0.15	0.002	0.1	14	11	9	97	94	88	0.004	0.003	0.004	0.245	0.234	0.261	69.97	69.82	64.01
0.002	0.20	0.002	0.125	16	13	11	93	94	85	0.003	0.003	0.004	0.302	0.283	0.311	91.61	91.41	83.68
0.003	0.25	0.002	0.15	28	22	19	93	91	88	0.003	0.004	0.004	0.280	0.267	0.290	80.49	74.62	73.66
0.003	0.30	0.002	0.175	32	24	21	93	88	82	0.003	0.003	0.012	0.307	0.300	0.324	94.22	87.59	26.73
0.003	0.35	0.002	0.2	35	27	24	87	88	85	0.003	0.003	0.004	0.333	0.322	0.344	106.79	106.76	97.87
Φ	P L1(E1),L2,E5b	Φ	P L5(E5a)	High-end receivers, Quadruple Frequency														
0.002	0.10	0.002	0.05	2	2	2	93	94	94	0.007	0.006	0.006	0.243	0.191	0.215	33.87	33.37	36.47
0.002	0.15	0.002	0.1	4	3	2	98	90	80	0.005	0.005	0.007	0.294	0.270	0.365	59.85	54.32	51.00
0.002	0.20	0.002	0.125	4	3	3	93	88	92	0.005	0.005	0.005	0.381	0.350	0.387	73.74	64.03	78.93
0.003	0.25	0.002	0.15	6	5	4	93	90	92	0.006	0.006	0.006	0.382	0.332	0.412	69.21	58.90	74.75
0.003	0.30	0.002	0.175	7	5	5	96	88	94	0.005	0.006	0.005	0.419	0.393	0.436	82.09	69.56	88.61
0.003	0.35	0.002	0.2	7	6	5	93	90	94	0.005	0.005	0.005	0.484	0.413	0.504	94.42	80.11	102.30

Table 6.21: **GPS + Galileo**, **PAR** analysis, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario. Evaluation of ASR and number of epochs for obtaining 2cm fixed-precision of rover-receiver coordinates - **measurement precision is varied**, number of satellites are as available.

(5) The number of epochs taken to obtain 0.999 ASR lied between 2 and 7 (**Galileo only** took 11 and 45) for E-W oriented baseline and between 2 and 7 (**Galileo only** took 10 and 45) too for N-S oriented baseline.

(6) The epochs taken for 0.999 ASR (measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b, E6$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b, E6$) were 5, 5 and 4 epochs (**Galileo only** took 45, 29 and 25 epochs) by fixing partially 79, 85 and 86% of ambiguities for E-W oriented baseline and 6, 5 and 4 epochs (**Galileo only** took 46, 30 and 24 epochs) by fixing 93, 90 and 92 % of ambiguities by PAR for N-S oriented baseline at 0°, −30° and −60° latitudes respectively.

Comparison with GPS only and Galileo only:

(7) The combined GPS and Galileo, dual frequency performed way better than the individual GPS only and Galileo only systems. Same is reflected in the comparisons presented above.

(8) The quadruple frequency system performed much better in itself by taking just 2 to 7 epochs for the predefined fixed-precision. While the quadruple frequency combined systems is compared to quadruple frequency Galileo only, again it can be noticed that the quadruple frequency system performs way better than the standalone system.

Further results and discussions for number of epochs taken and ambiguities fixed are presented in order to obtain a fixed-precision of 20cm for the horizontal coordinate components.

6.5.10 GPS + Galileo, Partial Ambiguity Resolution to obtain fixed-precision of 20 cm, atmosphere unknown scenario

While resolving all the ambiguities, a fixed-precision of 5mm and better was obtained. In this analysis it is to examine if a fixed-precision of σ_n, σ_e =20cm for the horizontal components of rover-receiver coordinates could be obtained in reduced number of epochs and by partial fixing of ambiguities. The results are presented in Table 6.22, see below.

Analysis based on Table 6.22 is given below.

General remarks:

(1) For all the scenarios of measurement precision, less than 100% of ambiguities were fixed in order to obtain a predefined fixed-precision for rover-receiver coordinates.

(2) The predefined fixed-precision for rover-receiver coordinates could be achieved instantaneously for 3 scenarios of quadruple frequency.

Dual Frequency:

(3) The number of epochs taken for 0.999 ASR for all scenarios of measurement precision and baseline length lied between 2 and 22 (**GPS only** took 7 and 99 and **Galileo only** took 3 and 46) for E-W oriented baseline and between 2 and 23 (**GPS only** took 8 and 113 and **Galileo only** took 3 and 47) for N-S oriented baseline.

(4) For dual frequency with the measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a$ and σ_P=25cm on $E1$ and 15cm on $E5a$, it took 10, 9 and 15 epochs (**GPS only** took 51, 40 and 66 epochs and **Galileo only** took 17, 18 and 27 epochs) by fixing 22, 22 and 37% of ambiguities by PAR at 0°, −30° and −60° latitude E-W oriented baseline and 9, 9 and 15 epochs (**GPS only** took 47, 40 and 73 epochs and **Galileo only** took 18, 16 and 28 epochs) by fixing 23, 26 and 41% of ambiguities for N-S oriented baseline.

Quadruple Frequency:

				Partial AR @ 0.999 ASR criteria - 20 cm fixed-precision for rover-receiver coordinates														
Phase	Code (meters)	Phase	Code	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$		
				0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline - 250Km																		
Φ	P	Φ	P	High-end receivers, Dual Frequency														
L1(E1)		L5(E5a)																
0.002	0.10	0.002	0.05	2	2	2	31	39	34	0.266	0.221	0.225	0.394	0.339	0.335	1.48	1.53	1.49
0.002	0.15	0.002	0.1	4	4	6	22	25	34	0.352	0.303	0.234	0.429	0.369	0.298	1.22	1.22	1.27
0.002	0.20	0.002	0.125	7	6	9	22	22	37	0.341	0.324	0.230	0.429	0.399	0.321	1.26	1.23	1.40
0.003	0.25	0.002	0.15	10	9	15	22	22	37	0.348	0.323	0.221	0.446	0.404	0.308	1.28	1.25	1.40
0.003	0.30	0.002	0.175	14	12	18	22	22	37	0.346	0.328	0.229	0.449	0.416	0.333	1.30	1.27	1.45
0.003	0.35	0.002	0.2	18	15	22	22	22	39	0.349	0.335	0.227	0.457	0.429	0.345	1.31	1.28	1.52
Φ	P	Φ	P	High-end receivers, Quadruple Frequency														
L1(E1),L2,E5b		L5(E5a)																
0.002	0.10	0.002	0.05	1	1	1	42	52	51	0.151	0.113	0.111	0.310	0.253	0.288	2.05	2.25	2.60
0.002	0.15	0.002	0.1	2	2	2	35	50	81	0.226	0.124	0.006	0.376	0.311	0.344	1.67	2.51	54.32
0.002	0.20	0.002	0.125	2	2	3	29	43	88	0.334	0.163	0.005	0.488	0.402	0.366	1.46	2.46	76.00
0.003	0.25	0.002	0.15	3	3	3	31	46	47	0.315	0.155	0.222	0.489	0.403	0.450	1.55	2.59	2.03
0.003	0.30	0.002	0.175	3	3	4	29	39	68	0.348	0.255	0.020	0.579	0.476	0.462	1.66	1.87	23.53
0.003	0.35	0.002	0.2	4	4	4	31	46	47	0.325	0.161	0.228	0.579	0.476	0.535	1.79	2.96	2.35
North-South Baseline - 250Km																		
Φ	P	Φ	P	High-end receivers, Dual Frequency														
L1(E1)		L5(E5a)																
0.002	0.10	0.002	0.05	2	2	2	30	38	38	0.285	0.234	0.246	0.423	0.359	0.361	1.48	1.53	1.47
0.002	0.15	0.002	0.1	4	4	6	23	26	38	0.373	0.317	0.254	0.460	0.391	0.321	1.23	1.23	1.26
0.002	0.20	0.002	0.125	6	6	10	23	24	44	0.389	0.340	0.215	0.497	0.421	0.326	1.28	1.24	1.51
0.003	0.25	0.002	0.15	9	9	15	23	26	41	0.387	0.329	0.235	0.505	0.427	0.329	1.30	1.30	1.40
0.003	0.30	0.002	0.175	12	12	19	23	24	41	0.394	0.344	0.231	0.521	0.439	0.343	1.32	1.28	1.49
0.003	0.35	0.002	0.2	16	16	23	23	24	50	0.390	0.339	0.157	0.520	0.436	0.353	1.33	1.29	2.26
Φ	P	Φ	P	High-end receivers, Quadruple Frequency														
L1(E1),L2,E5b		L5(E5a)																
0.002	0.10	0.002	0.05	1	1	1	44	51	53	0.172	0.119	0.092	0.343	0.270	0.304	2.00	2.27	3.31
0.002	0.15	0.002	0.1	2	2	2	38	51	80	0.239	0.145	0.007	0.416	0.331	0.365	1.74	2.29	51.00
0.002	0.20	0.002	0.125	2	2	3	31	43	92	0.358	0.178	0.005	0.540	0.429	0.387	1.51	2.41	78.93
0.003	0.25	0.002	0.15	3	3	3	33	47	47	0.338	0.167	0.237	0.541	0.429	0.476	1.60	2.57	2.00
0.003	0.30	0.002	0.175	3	3	4	31	39	71	0.372	0.278	0.034	0.641	0.508	0.488	1.72	1.83	14.55
0.003	0.35	0.002	0.2	4	4	4	33	47	47	0.346	0.172	0.268	0.641	0.507	0.564	1.85	2.95	2.10

Table 6.22: **GPS + Galileo**, **PAR** analysis, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario. Evaluation of ASR and number of epochs for obtaining 20cm fixed-precision of rover-receiver coordinates - **measurement precision is varied**, number of satellites are as available.

(5) The number of epochs taken to obtain 0.999 ASR lied between 1 and 4 (**Galileo only** took 1 and 4) for both E-W oriented and N-S oriented baseline.

(6) The epochs taken for 0.999 ASR (measurement precision of σ_Φ=3mm on $E1$ and 2mm on $E5a, E5b, E6$ and σ_P=25cm on $E1$ and 15cm on $E5a, E5b, E6$) were 3, 3 and 3 epochs (**Galileo only** took 3, 2 and 3 epochs) by fixing partially 31, 46 and 47% of ambiguities for E-W oriented baseline and 3, 3 and 3 epochs (**Galileo only** took 3, 3 and 4 epochs) by fixing 33, 47 and 47% of ambiguities by PAR for N-S oriented baseline at 0°, $-30°$ and $-60°$ latitudes respectively.

Comparison with GPS only and Galileo only:

(7) The combined GPS and Galileo, dual frequency performed way better than the individual GPS only and Galileo only systems.

6.6 Reference-Rover model, ionosphere weighted scenario

In this scenario, the receiver coordinates are considered to be unknown and are parameterized, satellite coordinates are assumed to be known and are generated from YUMA almanacs. When the atmosphere is unknown, both the troposphere and the ionosphere, it can take larger time for the ambiguities to converge to their correct integer solution. However, if the ionosphere is known a-priori and weighted, it can result in faster ambiguity resolution. Such an ionosphere weighting is used in this section. The baseline length which is directly related to the uncertainty in the ionosphere will be varied between 1 and 1000 Kilometers to vary the ionosphere weight. All the different types of frequency combinations, namely single, dual, triple and quadruple will be considered. Presented below is the functional and stochastic models, see Table 6.23.

In Table 6.23, G_{m-1} has the Line of Sight (LOS) vectors and the tropospheric mapping function, it is given as below,

$$G_{m-1} = \begin{bmatrix} -u^{1-2} & \psi^{1-2} \\ \vdots & \vdots \\ -u^{1-m} & \psi^{1-m} \end{bmatrix}$$

where, $-u^s$ is the DD (double-differenced) unit vector between the receiver and the satellite, since the receiver coordinates are to be estimated, the information for forming the unit vectors comes from the a-priori values of the receiver coordinates. ψ is the DD tropospheric mapping function. In this study a cosine mapping function is used defined as, $\psi = 1/\cos(z)$, where z is the zenith angle of the satellite. D_m^T is the single differenced satellite design matrix is known as the difference operator for the satellites, W_i is the satellite elevation weight matrix for i^{th} epoch, see Appendix B, equation (B.5), CI is given by equation (5.13) in chapter 5.

and $\Lambda = \text{diag}\ \underbrace{(\lambda_1, \cdots, \lambda_j)}_{j\times j}$

Reference-Rover model, ionosphere weighted scenario		
Functional model		**Stochastic model**
Non-temporal parameters	Temporal parameters	
$A_{I(i)} = \left[\begin{pmatrix} \Lambda \\ 0 \end{pmatrix} \otimes I_{m-1} \quad \begin{pmatrix} e_f \\ e_f \end{pmatrix} \otimes G_{m-1} \right]$	$A_{II(i)} = \begin{pmatrix} -\mu_f \\ \mu_f \end{pmatrix} \otimes I_{m-1}$	$Q_{y(i)} = \begin{bmatrix} Q_\Phi & \\ & Q_P \end{bmatrix} \otimes Q_{DD}(i)$ where $Q_{DD}(i) = (D_m^T W_i^{-1} D_m)$ $Q_I(i) = CI \otimes Q_{DD}(i)$ $Q_{y,I}(i) = Q_y(i) + Q_I(i)$
Redundancy (for GPS only, Galileo only, GPS+Galileo (common frequency $L1(E1), L5(E5a)$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $f*(m-1)$ Troposphere: $= 1$ Coordinates: $3(n-1) = 3$	Ionosphere: $k*(m-1)$	$k*2f*(m-1)$ Pseudo observations (Ionosphere) = $k*(m-1)$
Redundancy of Reference-Rover - ionosphere weighted model		$[(2k-1)*f*(m-1)]-4$
For GPS/Galileo only: $f = (1, \cdots, j)$, for GPS+Galileo: $f = (1, \cdots, j_c)$, j_c is the common/overlapping frequency and $m_{jc} = (m_{GPS} + m_{Gal})$		
Redundancy (for GPS+Galileo (quadruple frequency $L1(E1), L5(E5a), L2, E5b$))		
Non-temporal parameters	Temporal parameters	Observations
Ambiguities: $2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)$ Troposphere: $= 1$ Coordinates: $3(n-1) = 3$	Ionosphere$=k*(m_{jc}-1)$	$k*2*(2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1))$ Pseudo observations (Ionosphere) $=k*(m_{jc}-1)$
Redundancy of Reference-Rover - ionosphere weighted model		$(2k-1)[2*(m_{jc}-1)+(m_{GPS}-1)+(m_{Gal}-1)]-4$
$f = 1, 2$ for GPS+Gaileo $L1(E1), L5(E5a)$ m_{jc} are the total number of satellites for combined GPS+Galileo system with GPS as reference satellite		

Table 6.23: Double-differenced Design matrix and VC matrix for Geometry based model, Receiver coordinates for the rover are unknown, ionosphere weighted scenario

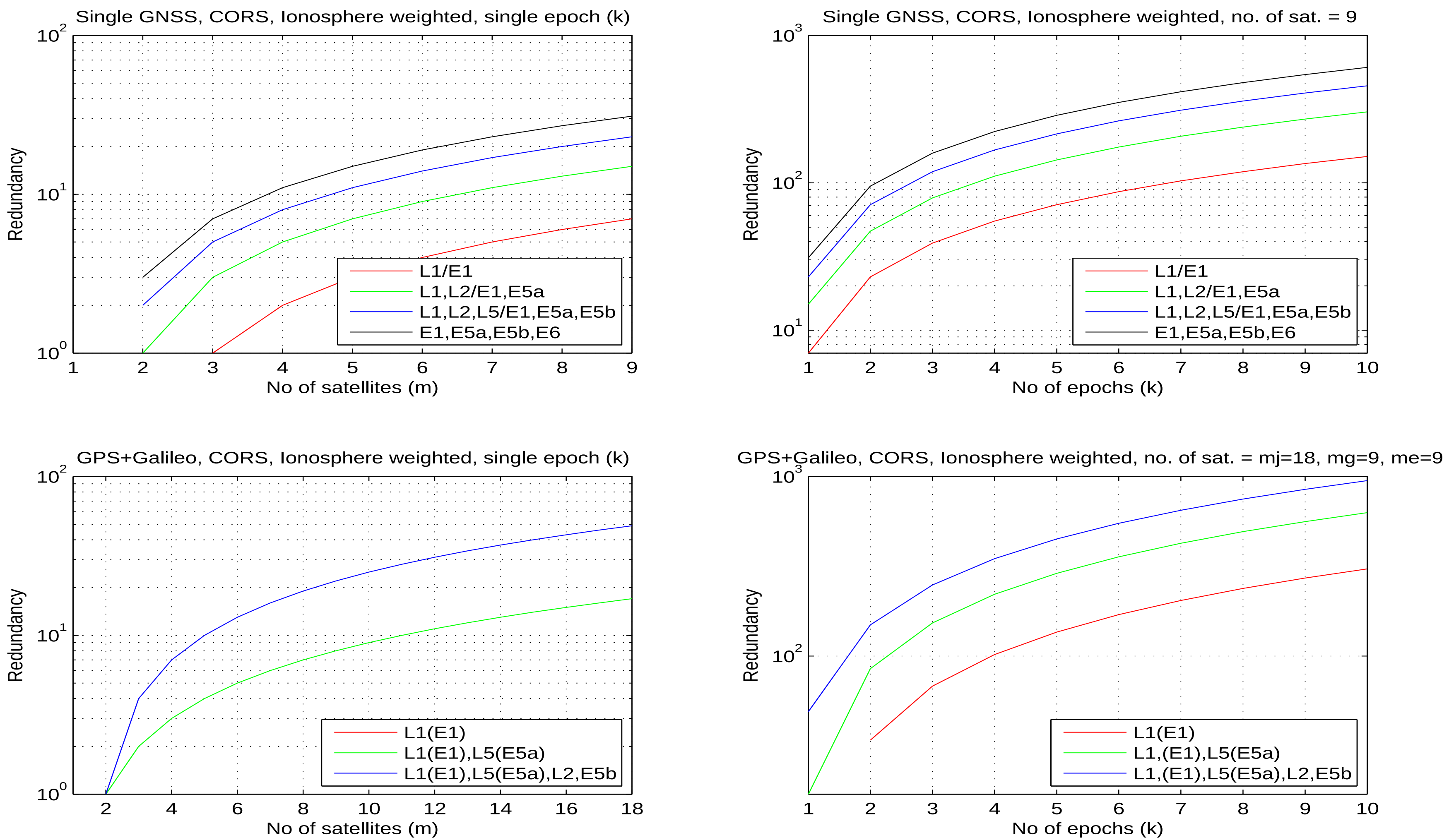

Figure 6.19: Redundancy plot of **GPS only**, **Galileo only** (top two plots) and **GPS + Galileo** (bottom two plots) for single epoch (left hand side plots) and multi-epoch (right hand side plots), Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, single, dual, triple and quadruple frequencies

6.6.1 Hourly batches, Full Ambiguity Resolution, ionosphere Weighted scenario

The ASR is simulated for hourly batches, in all 24 batches throughout the day are considered for simulation. The interval between the epochs is taken as 1 second, hence for each batch there are a maximum of 3600 epochs available. It is to understand the average behaviour for achieving 0.999 ASR during different times of the day. The results for single, dual, triple and quadruple (for Galileo and GPS+Galileo system) frequency for GPS only, Galileo only and with both systems GPS and Galileo together are presented in the Figures G.37 to G.46, see Appendix G. The results for average number of epochs taken for each of the GNSS system are also presented in Table 6.24, see below.

The analysis based on Figures G.37 to G.46 and Table 6.24 is presented below.

General remarks

1. The number of epochs taken for 0.999 ASR increase as baseline length is increased.
2. It can be noted that single frequency GNSS systems could not give 0.999 ASR for 1000 Km baseline length within the stipulated 3600 epochs. Henceforth for single frequency, 1000 Km baseline length will be excluded from further discussions in this section.
3. It an also be noticed that there is a significant improvement in the number of epochs taken for 0.999 ASR as frequencies for any GNSS system are increased. The strength increases with increase in number of frequencies and also with improvement in measurement precision.

GPS only

<u>Single frequency:</u>

4. For single frequency GPS, it took a minimum of 27 and a maximum of 2863 epochs on an average for baseline lengths lying between 1 and 500 Km for different scenarios of measurement precision and latitude

Baseline length (Km)	Hourly, Full AR - minimum desired ASR = 0.999 Average number of Epochs over 24 hourly batches																	
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
	Single Frequency			Dual Frequency			Triple Frequency			Quadruple Frequency			Average no. of satellites			Average no. of Low-elevation Sat. $\geqslant 10°, \leqslant 30°$		
East-West Baseline																		
GPS only																		
	$L1$			$L1, L2$			$L1, L2, L5$											
1	27	37	29	2	3	2	1	1	1	-	-	-	10	9	9	5	3	5
10	49	67	51	4	6	5	4	5	5	-	-	-	10	9	9	5	3	5
100	492	571	521	41	76	56	31	58	43	-	-	-	10	9	10	5	3	5
250	1463	1469	1837	90	136	99	62	97	67	-	-	-	10	8	10	5	3	5
500	2863	2568	2748	142	201	150	83	130	91	-	-	-	9	8	9	5	3	4
1000	> 3600	> 3600	> 3600	207	275	192	115	163	104	-	-	-	9	8	9	4	3	4
Galileo only																		
	$E1$			$E1, E5$			$E1, E5a, E5b$			$E1, E5a, E5b, E6$								
1	21	38	29	1	2	1	1	1	1	1	1	1	10	9	10	5	4	5
10	41	69	58	4	5	5	4	5	4	4	5	4	10	9	10	5	4	5
100	465	625	628	32	49	49	29	43	43	28	42	42	10	9	10	5	3	5
250	1701	1573	1932	66	108	93	45	72	68	43	69	65	10	9	10	5	3	5
500	2653	2643	3315	113	159	133	59	82	77	56	78	74	10	8	10	5	3	4
1000	> 3600	> 3600	> 3600	140	199	153	66	91	71	63	86	68	9	8	10	4	3	4
GPS + Galileo																		
	$L1(E1)$			$L1(E1), L5(E5)$						$L1(E1), L5(E5), L2, E5b$								
1	3	3	3	1	1	1	-	-	-	1	1	1	20	17	19	10	7	9
10	7	7	7	3	4	3	-	-	-	3	3	3	20	17	19	10	7	9
100	233	272	265	16	19	18	-	-	-	11	13	12	20	17	19	10	6	9
250	1247	1305	1586	27	27	33	-	-	-	11	13	12	19	17	19	10	6	9
500	2323	2728	3050	34	34	41	-	-	-	11	14	12	19	17	19	10	6	9
1000	> 3600	> 3600	> 3600	33	35	42	-	-	-	11	12	12	18	16	19	9	6	9
North-South Baseline																		
GPS only																		
	$L1$			$L1, L2$			$L1, L2, L5$											
1	27	37	29	2	3	2	1	1	1	-	-	-	10	9	9	5	3	5
10	48	68	52	4	7	5	4	5	4	-	-	-	10	9	9	5	3	4
100	498	553	514	43	76	57	32	58	42	-	-	-	10	9	9	5	3	4
250	1504	1438	1966	93	134	113	63	94	82	-	-	-	9	8	9	4	3	4
500	2961	2700	2823	144	196	181	84	123	113	-	-	-	9	8	9	5	3	4
1000	> 3600	> 3600	> 3600	217	292	291	112	166	168	-	-	-	9	8	8	4	3	3
Galileo only																		
	$E1$			$E1, E5$			$E1, E5a, E5b$			$E1, E5a, E5b, E6$								
1	21	38	29	1	2	1	1	1	1	1	1	1	10	9	10	5	4	5
10	41	69	58	4	6	5	4	5	4	4	5	4	10	9	10	5	4	5
100	471	625	666	30	45	52	27	40	47	26	39	45	10	9	10	5	3	4
250	1731	1517	2084	63	95	131	44	64	100	42	61	96	10	9	9	5	3	4
500	2752	2582	2829	105	146	166	59	77	98	56	74	94	10	9	9	5	3	4
1000	> 3600	> 3600	> 3600	149	184	217	70	88	118	67	84	113	9	8	8	4	3	3
GPS + Galileo																		
	$L1(E1)$			$L1(E1), L5(E5)$						$L1(E1), L5(E5), L2, E5b$								
1	3	3	3	1	1	1	-	-	-	1	1	1	20	17	19	10	7	9
10	7	7	7	3	4	3	-	-	-	3	3	3	20	17	19	10	7	9
100	225	312	269	16	19	17	-	-	-	11	12	12	19	17	19	10	7	9
250	1343	1411	1650	28	29	31	-	-	-	11	13	12	19	17	18	10	7	8
500	2931	2744	2692	33	35	36	-	-	-	11	13	12	19	17	18	9	6	7
1000	> 3600	> 3600	> 3600	34	36	37	-	-	-	11	13	14	18	16	16	8	6	6

Table 6.24: **GPS only, Galileo only, GPS + Galileo**, **Geometry based model, Receiver coordinates for the rover are unknown**, Ionosphere Weighted, Troposphere float scenario **Full AR**, Averaged number of epochs over 24 batches are presented - measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, SD precision $\sigma_I = 0.68$ mm per km.

location for E-W baseline. For N-S oriented baseline, it took an average number of epochs between 27 and 2961.

Dual frequency:

5. The average number of epochs taken to obtain 0.999 ASR for baseline length between 1 and 1000 Km lied between 2 and 275 for dual frequency GPS for E-W baseline. For N-S baseline, it took an average number of epochs between 2 and 292 for similar scenario.

Triple frequency:

6. Triple frequency GPS gave instantaneous ASR of 0.999 for 1 Km baseline at all latitude locations for Reference-rover scenario.
7. For triple frequency, the average number of epochs lie between 4 and 163 for E-W baseline and 4 to 168 epochs for the N-S baseline for baseline lengths between 10 to 1000 Kms.

Galileo only

Single frequency:

8. With Galileo only, single frequency, for baseline lengths between 1 and 500 Km, it took on an average number of epochs between 21 and 3315 (**GPS only** took between 27 and 2863 epochs) for E-W baseline and between 21 and 2829 (**GPS only** took between 27 and 2961 epochs) for N-S oriented baseline.

Dual frequency:

9. Dual frequency Galileo was able to give instantaneous ASR of 0.999 for baseline length of 1 Km at 0° and −60° latitude. It can be noted that dual frequency GPS could not give instantaneous ASR for any scenario. The better performance of Galileo system is credited to the relatively better measurement precision of Galileo system as compared to GPS system.

10. For dual frequency Galileo, for baseline lengths between 1 and 1000 Kms, the average number of epochs taken for 0.999 ASR lie between 1 and 199 (**GPS only** took between 2 and 275 epochs) for E-W baseline and 1 and 217 (**GPS only** took between 2 and 292 epochs) for N-S baseline.

Triple frequency:

11. Triple frequency Galileo was able to give instantaneous ASR of 0.999 for 1 Km baseline at all latitude locations for Reference-rover scenario.

12. With triple frequency Galileo, for baseline lengths between 10 and 1000 Kms, the average number of epochs over 24 batches to obtain 0.999 ASR lied between 4 and 97 (**GPS only** took between 4 and 163 epochs) for E-W baseline between 4 and 118 (**GPS only** took between 4 and 168 epochs) for N-S baseline both.

Quadruple frequency:

13. Quadruple frequency Galileo took an average number of epochs between 4 and 86 for E-W baseline for baseline length between 10 and 1000 Km. For N-S baseline it took between 4 and 113 epochs for a similar scenario. The quadruple frequency system performed better than the triple frequency Galileo system as an effect of the strength gained due to increase in frequency.

Comparison with GPS only:

14. The performance is better in comparison to GPS only, for dual and triple frequencies. This is credited to the fact that Galileo has better measurement precision in comparison to GPS system.

GPS + Galileo

Single frequency:

15. For a combined GPS and Galileo system, for baseline lengths between 1 and 500 Kms it took between 3 to 3050 epochs (**GPS only** took between 27 and 2863 epochs and **Galileo only** took between 21 and 3315 epochs) for E-W baseline and 3 to 2931 epochs (**GPS only** took between 27 and 2961 epochs and **Galileo only** took between 21 and 2829 epochs) for N-S baseline length.

Dual frequency:

16. With dual frequency GPS and Galileo combined system, instantaneous ASR of 0.999 was achieved for baseline length of 1 Km at $0°$ and $-60°$ latitude.

17. For a combined GPS and Galileo system, for baseline lengths between 1 and 1000 Kms, it takes an average number of epochs between 1 and 42 (**GPS only** took between 2 and 275 epochs and **Galileo only** took between 1 and 199 epochs) for E-W baseline and 1 to 37 (**GPS only** took between 2 and 292 epochs and **Galileo only** took between 1 and 217 epochs) for N-S baseline for different scenarios of latitude location.

Quadruple frequency:

18. Instantaneous ASR of 0.999 was achieved for baseline length of 1 Km for all the three latitude locations for both E-W and N-S oriented baselines.

19. It takes and average number of epochs between 3 and 14 epochs (**Galileo only** took between 4 and 86, and 4 and 113 epochs for E-W and N-S baseline respectively) for E-W baseline and N-S baseline, both.

Comparison with GPS only and Galileo only

20. The dual frequency combined GPS and Galileo system does not perform better than the standalone GPS only or Galileo only. This is due to the fact that in a combined dual frequency system, the increase in number of satellites do not add any information to the model, and hence do not reduce the time taken for 0.999 ASR.

21. The quadruple frequency GPS+Galileo performs better than the quadruple frequency Galileo only and triple frequency GPS only etcetera.

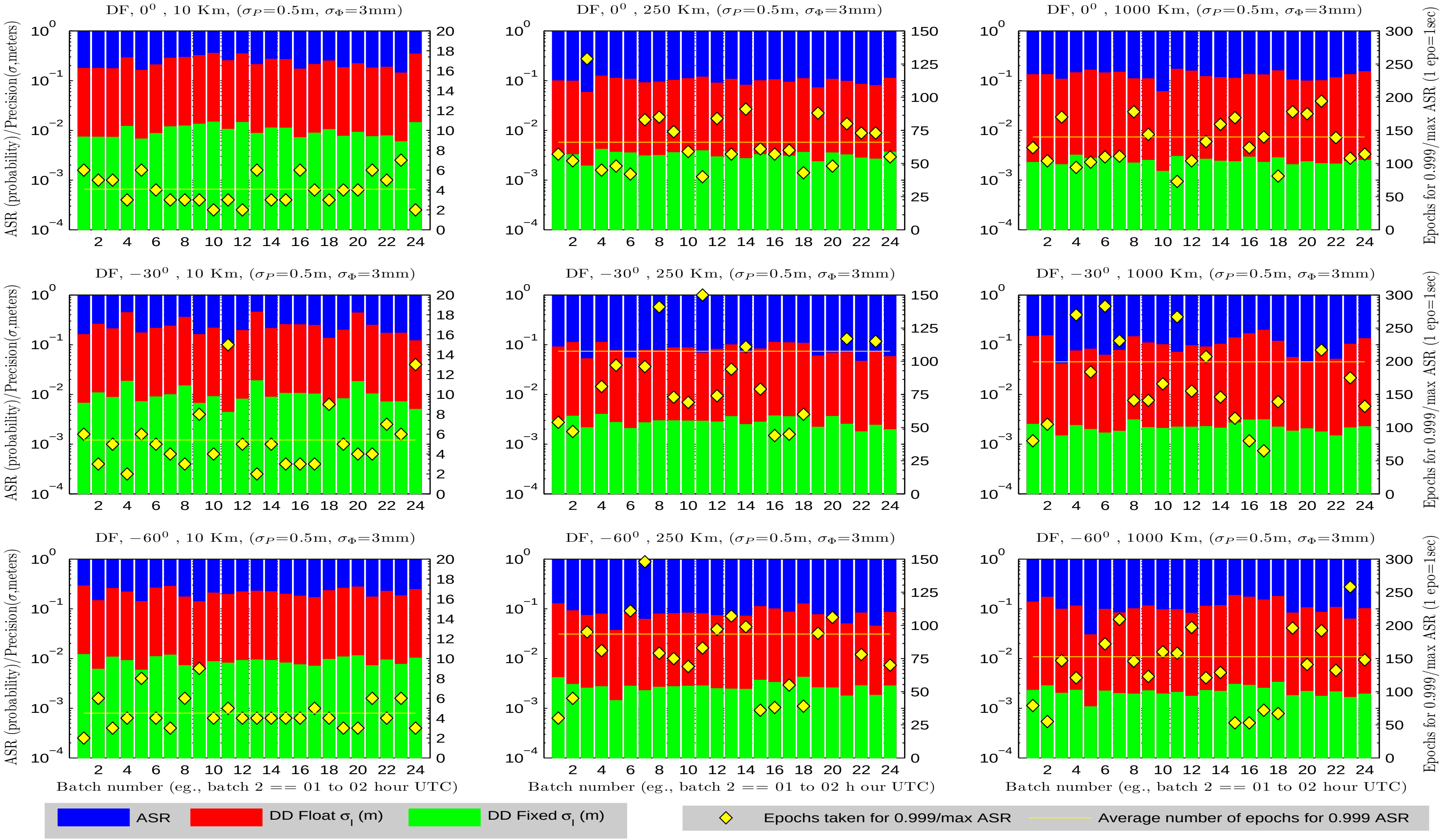

Figure 6.20: **Galileo only**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed-precision of ionosphere.

6.6.2 GPS only - Full and Partial Ambiguity Resolution, ionosphere Weighted scenario

The results for ionosphere weighted, Geometry Fixed model considering GPS system frequencies are presented below. Figures G.47 to G.49 in Appendix G give full ambiguity resolution results and Figures G.50 to G.52, Appendix G present the partial ambiguity results for GPS single, dual and triple frequency respectively. Table 6.25 presents the full ambiguity resolution results in terms of number of epochs taken for 0.999 ASR and corresponding DD rover-receiver coordinates precision (for ambiguity fixed solution) is presented. The results for PAR are presented on the right hand side. PAR is evaluated based on the criteria to obtain the same value of fixed-precision of rover-receiver coordinates as in full AR. That is for same value of fixed-precision as in full AR it is to evaluate whether PAR is able to give similar results by fixing only a partial subset of the ambiguities with less number of epochs.

In this section, the simulations are done for E-W baseline for all the frequency combinations possible with GPS (single, dual and triple).

The following is the analysis based on Table 6.25 and Figures 6.21 to 6.22.

General conclusions:

(1) Instantaneous ASR of 0.999 is achieved for baseline length of 1 Km for triple frequency GPS only, at all the latitude locations.

(2) For ionosphere weighted model, as the baseline length increases, the number of epochs taken for 0.999 ASR increases, since σ_I is increasing with baseline length which adds up to the stochastic model. Further when the baseline is large enough so that the ionosphere weighted model is equivalent to ionosphere float model (say for baseline lengths above 250 Km), then the full AR is influenced by number of common satellites available and number of low elevation satellites. This behaviour is addressed in earlier section (averaged number of epochs over 24 batches simulation). For longer baselines, it is generally observed that the number of epochs taken for 0.999 ASR reduce. Same can be observed in GPS only simulation for a few scenarios, see Table 6.25.

	Full AR - minimum desired ASR = 0.999											
	Epochs taken			$\sigma_{\widecheck{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widecheck{neu}}$		
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline												
	Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m				
1	28	21	23	0.002	0.002	0.002	0.280	0.308	0.294	150.88	148.92	151.42
10	52	39	44	0.003	0.003	0.003	0.205	0.226	0.210	80.95	79.90	81.41
100	719	542	606	0.006	0.006	0.006	0.050	0.054	0.052	8.68	8.76	8.87
250	2272	749	1968	0.007	0.014	0.008	0.026	0.053	0.028	3.49	3.79	3.55
500	> 3600	2387	3103	-	0.012	0.010	-	0.032	0.026	-	2.59	2.45
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	10	7	9	0.002	0.003	0.002	0.377	0.426	0.391	165.44	166.12	165.74
	High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m				
1	2	2	2	0.005	0.005	0.005	0.404	0.391	0.383	79.84	79.83	79.83
10	7	5	6	0.007	0.008	0.007	0.216	0.247	0.221	31.90	31.90	31.90
100	52	39	65	0.004	0.004	0.003	0.094	0.093	0.070	21.12	21.14	21.00
250	204	65	88	0.002	0.003	0.003	0.051	0.088	0.074	22.67	25.68	25.57
500	165	113	135	0.003	0.003	0.002	0.080	0.087	0.079	31.47	33.33	33.89
1000	213	163	171	0.002	0.002	0.002	0.084	0.091	0.092	37.10	40.76	44.29
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	334	197	203	0.002	0.002	0.002	0.061	0.095	0.098	34.44	48.29	52.80
	High-end receivers, Triple Frequency							$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m				
1	1	1	1	0.005	0.005	0.005	0.369	0.358	0.351	69.84	69.84	69.84
10	7	4	5	0.007	0.008	0.007	0.140	0.179	0.157	21.43	21.43	21.43
100	47	26	45	0.004	0.004	0.003	0.073	0.084	0.062	19.11	19.16	19.11
250	140	43	58	0.002	0.003	0.003	0.060	0.101	0.085	27.02	29.46	29.38
500	94	67	75	0.003	0.003	0.003	0.115	0.120	0.112	42.23	43.38	44.26
1000	106	74	84	0.003	0.003	0.002	0.137	0.151	0.141	52.96	56.29	58.75
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	223	76	94	0.002	0.003	0.002	0.077	0.171	0.152	43.72	66.56	67.07
North-South Baseline												
	Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m				
1	28	22	23	0.002	0.002	0.002	0.280	0.299	0.294	150.88	147.88	151.42
10	52	38	42	0.003	0.003	0.003	0.205	0.230	0.216	80.95	80.21	82.02
100	752	442	598	0.006	0.007	0.006	0.048	0.063	0.055	8.42	9.14	8.52
250	1347	779	2091	0.010	0.014	0.008	0.039	0.052	0.028	3.87	3.84	3.60
500	> 3600	2544	3317	-	0.012	0.010	-	0.030	0.025	-	2.54	2.44
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	10	7	9	0.002	0.003	0.002	0.377	0.426	0.391	165.44	166.12	165.74
	High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m				
1	2	2	2	0.005	0.005	0.005	0.404	0.391	0.383	79.84	79.83	79.83
10	7	5	6	0.007	0.008	0.007	0.216	0.247	0.221	31.90	31.90	31.90
100	47	34	93	0.005	0.005	0.003	0.099	0.101	0.058	21.15	21.18	20.60
250	108	66	135	0.003	0.003	0.002	0.079	0.089	0.057	25.12	25.70	24.29
500	194	110	175	0.002	0.003	0.002	0.072	0.093	0.072	30.15	33.84	31.72
1000	232	153	226	0.002	0.002	0.002	0.081	0.103	0.079	36.67	43.03	38.85
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	266	173	249	0.002	0.002	0.002	0.080	0.109	0.083	39.67	51.28	44.25
	High-end receivers, Triple Frequency							$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m				
1	1	1	1	0.005	0.005	0.005	0.369	0.358	0.351	69.84	69.84	69.84
10	7	4	5	0.007	0.008	0.007	0.140	0.179	0.157	21.43	21.43	21.43
100	41	24	64	0.004	0.005	0.003	0.078	0.088	0.053	19.13	19.17	18.95
250	73	45	89	0.003	0.003	0.002	0.090	0.100	0.067	29.04	29.44	28.39
500	119	65	117	0.002	0.003	0.002	0.099	0.126	0.092	40.33	43.93	41.23
1000	119	71	150	0.002	0.003	0.002	0.129	0.166	0.101	52.21	58.57	49.89
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	124	71	119	0.002	0.003	0.002	0.138	0.184	0.127	57.99	68.00	62.83

Table 6.25: **GPS only**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **Full AR**, ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented - measurement precision is held fix, number of satellites are as available, **baseline length varied**

Fall in Partial AR:

(3) It can be seen in the PAR plots, incase of new incoming reference satellite when a perturbation is caused in the stochastic model, the percentage of ambiguities fixed by PAR drops. See Figure 6.23. Same can be observed in case of full AR from Figure 6.22.

Single frequency:

(4) To achieve 0.999 ASR, it took between 21 and >3600 epochs for single frequency for baseline lengths of 1 to 500 Kms (all latitude location scenarios) for E-W baseline and between 22 to >3600 epochs for N-S baseline for a similar scenario.

(5) For 250 Km baseline length, it took 2272, 749 and 1968 epochs at 0°, −30° and −60° for 0.999 ASR with ionosphere weighted model, E-W baseline. For N-S baseline, it took 1347, 779 and 2091 epochs for a similar scenario.

Dual frequency:

(6) Number of epochs for 0.999 ASR for baseline length of 1 to 1000 Km lied between 2 and 213 for E-W baseline and between 2 and 232 for N-S baseline.

(7) The number of epochs taken at 250 Km baseline length for 0.999 ASR were 204, 65 and 88 for 0°, −30° and −60° latitude for E-W baseline. AS compared to ionosphere float, 250 Km baseline length when it took 29, 36 and 63 epochs, the performance of ionosphere weighted is much better, as expected. For N-S baseline, it took 108, 66 and 135 epochs for a similar scenario.

Triple frequency:

(8) Instantaneous ASR of 0.999 could be achieved for baseline length of 1 Km for all latitude locations with triple frequency GPS.

(9) For obtaining an ASR of 0.999, it took between 4 to 106 epochs for 10 to 1000 Km baseline lengths, considering all the latitude locations for E-W

baseline. For N-S baseline, it took between 4 to 150 epochs for a similar scenario. The best results could be seen at $0°$ for E-W baseline and at $-30°$ for N-S baseline.

(10) At $0°$, $-30°$ and $-60°$ latitude for E-W oriented 250 Km baseline, it took 140, 43 and 58 epochs respectively, and for 250 Km N-S oriented baseline, it took 73, 45 and 89 epochs for a similar scenario.

Considering the fixed-precision of rover-receiver coordinates to lie around 3mm in most of the cases when 100% ambiguities are fixed, PAR will be evaluated for obtaining a fixed-precision of 2cm for x and y components and 6cm for z component for rover-receiver coordinates in the coming section.

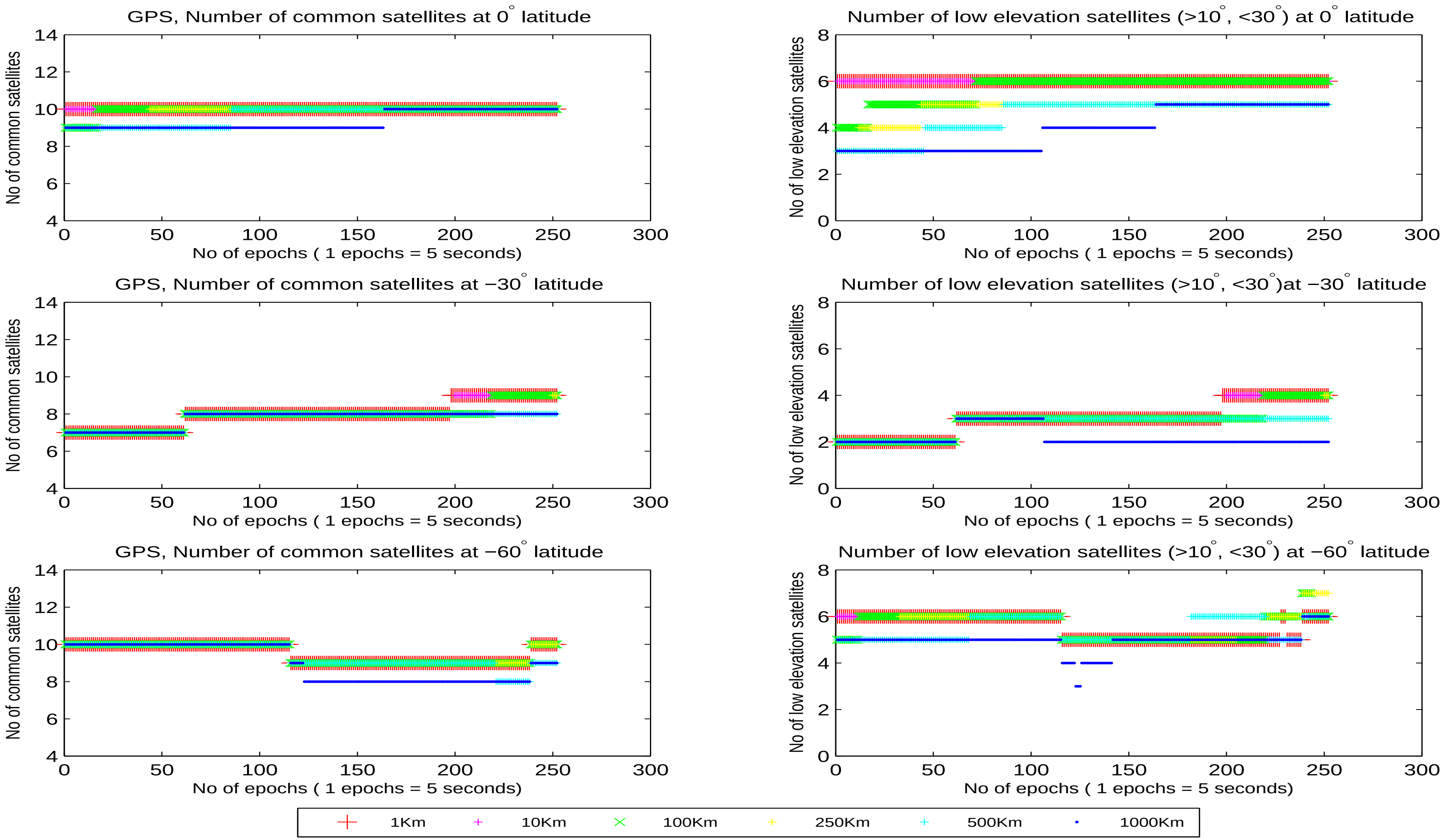

Figure 6.21: **GPS only**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, East-West oriented baseline.

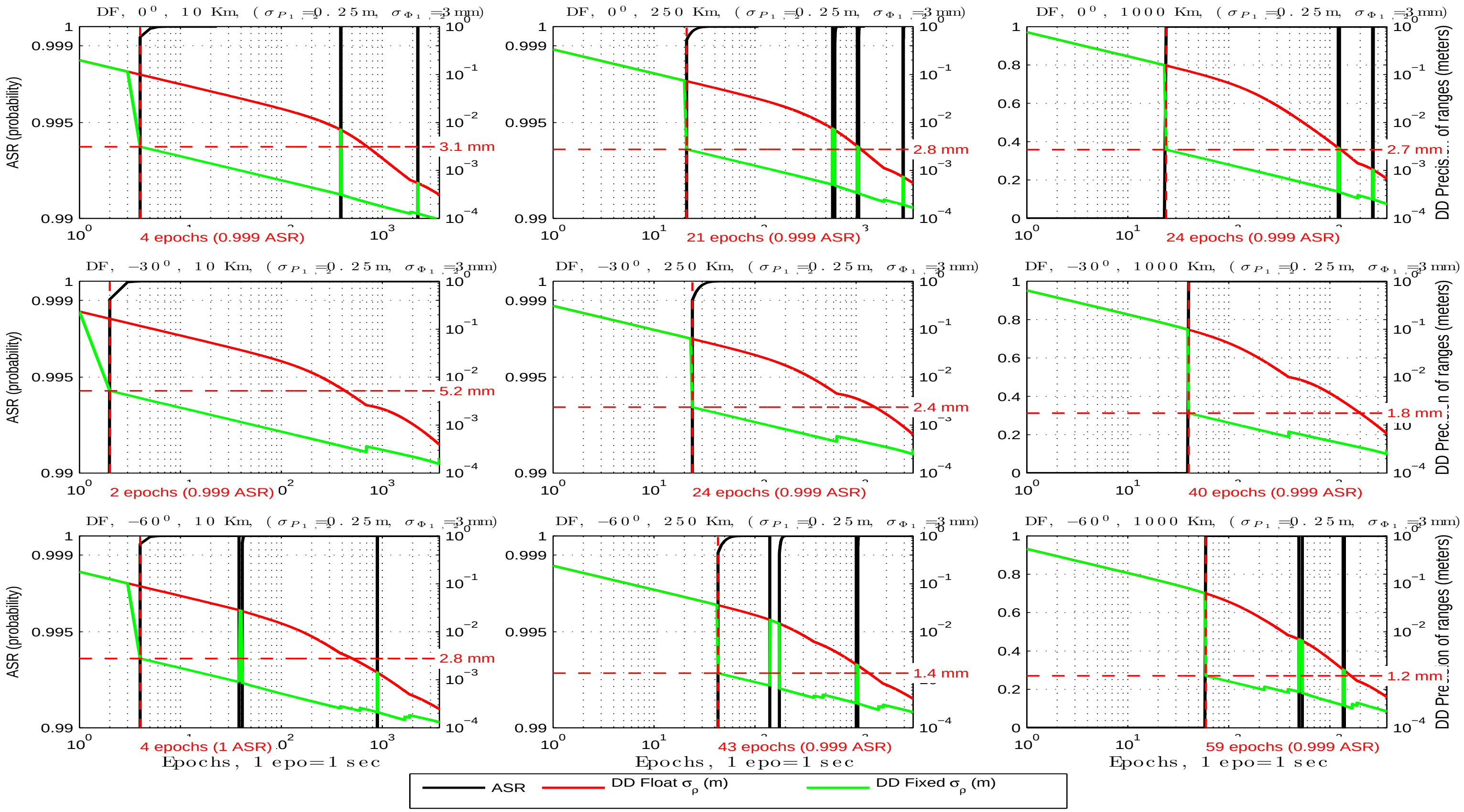

Figure 6.22: **GPS only**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision σ_I = 0.68 mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

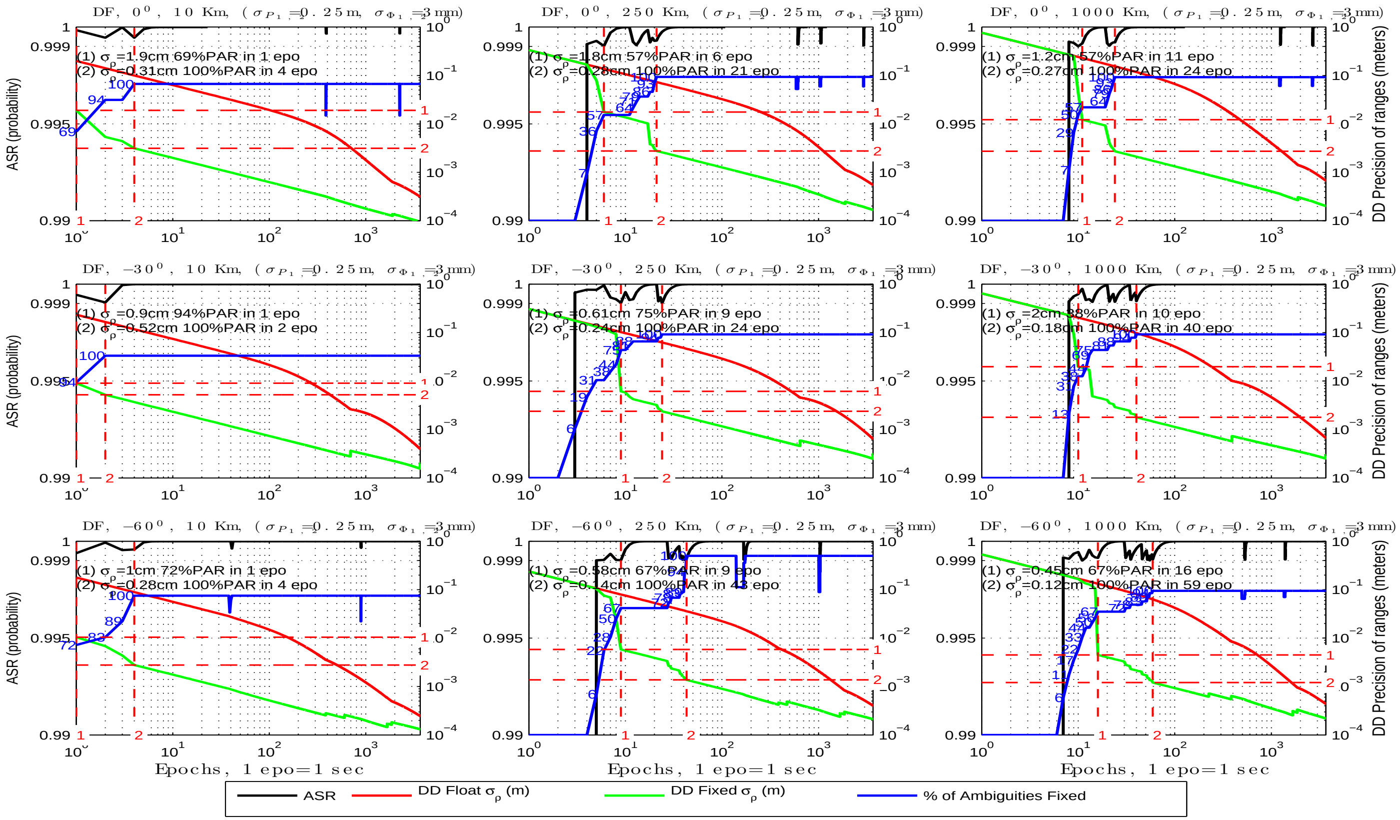

Figure 6.23: **GPS only**, Dual frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

6.6.3 GPS only, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, ionosphere Weighted scenario

In this section, it is to be analyzed whether the partially ambiguity fixing help in converging the solution faster to a modest DD fixed-precision of 2cm for σ_n, σ_e and 6cm for σ_u for rover-receiver coordinates. Table 6.26 presents the results in terms of fixed-precision to be equal to or better than the laid criteria and corresponding epochs taken and ambiguities fixed are noted.

The following is the analysis based on Table 6.26.

General conclusion:

(1) An instantaneous fixed-precision as per the laid criteria (σ_n, σ_e =2 cm σ_u=6cm) could be obtained for 1 Km baseline length for triple frequency GPS only at all latitude locations. Triple frequency GPS was able to give instantaneous ASR of 0.999 for 1 Km baseline length, as presented in earlier section.

(2) For all the remaining scenarios of baseline lengths above 1 Km, less than 100% of ambiguities were fixed to obtain a fixed-precision as per the laid criteria (σ_n, σ_e =2 cm σ_u=6cm) for rover-receiver coordinates.

(3) For 1000 Km baseline length, fixed-precision (σ_n, σ_e =2 cm σ_u=6cm) could be achieved only at $-30°$ latitude within stipulated 3600 epochs.

(4) Ionosphere weighted scenario performs better than ionosphere float scenario for all the different cases considered in Table 6.26.

Single frequency:

(5) For single frequency, it took a minimum of 21 and a maximum of >3660 epochs for baseline lengths between 1 and 1000 Kms, for both E-W and N-S baseline. The best performance could be seen at $-30°$ latitude for both E-W and N-S baseline.

Dual frequency:

	Partial AR @ 0.999 ASR criteria - σ_n, σ_e =2 cm σ_u=6cm, for fixed-precision of rover-receiver coordinates														
Baseline length (Km)	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widehat{neu}}$(meters)			$\sigma_{\check{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\check{neu}}$		
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline															
Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m								
1	28	21	23	100	100	100	0.002	0.002	0.002	0.280	0.308	0.294	150.88	148.92	151.42
10	51	39	43	88	100	80	0.009	0.003	0.009	0.208	0.226	0.214	23.96	79.90	24.48
100	475	300	366	67	75	50	0.011	0.019	0.017	0.071	0.082	0.070	6.45	4.34	4.08
250	1058	671	912	33	71	63	0.020	0.018	0.020	0.047	0.056	0.047	2.31	3.16	2.38
500	1828	1391	1801	33	71	50	0.020	0.019	0.018	0.037	0.045	0.035	1.81	2.33	1.99
1000	3660	2858	3660	13	57	29	0.027	0.020	0.023	0.031	0.036	0.029	1.12	1.82	1.26
I-Fix(1Km)	10	6	7	100	88	89	0.002	0.013	0.012	0.377	0.461	0.445	165.44	36.81	38.39
High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m								
1	2	2	2	100	100	100	0.005	0.005	0.005	0.404	0.391	0.383	79.84	79.83	79.83
10	3	2	3	94	94	83	0.011	0.014	0.012	0.330	0.391	0.313	28.96	28.15	27.07
100	25	21	28	79	75	72	0.020	0.020	0.016	0.136	0.128	0.108	6.70	6.40	6.59
250	73	57	74	86	75	83	0.013	0.017	0.016	0.098	0.095	0.081	7.65	5.46	4.97
500	155	100	118	79	88	89	0.018	0.012	0.019	0.084	0.095	0.086	4.57	8.03	4.45
1000	209	159	166	86	88	89	0.016	0.019	0.017	0.085	0.092	0.094	5.51	4.80	5.58
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	226	164	190	86	94	80	0.013	0.008	0.019	0.094	0.113	0.105	7.28	14.36	5.61
High-end receivers, Triple Frequency							$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m								
1	1	1	1	100	100	100	0.005	0.005	0.005	0.369	0.358	0.351	69.84	69.84	69.84
10	2	2	2	92	96	89	0.014	0.013	0.013	0.262	0.253	0.249	18.73	19.14	18.76
100	20	17	22	86	83	81	0.020	0.020	0.017	0.112	0.104	0.090	5.50	5.19	5.43
250	53	38	48	90	83	89	0.012	0.018	0.017	0.106	0.108	0.094	8.56	6.07	5.46
500	85	54	66	90	92	93	0.011	0.013	0.020	0.123	0.137	0.121	11.16	10.64	5.98
1000	99	70	78	90	92	93	0.015	0.018	0.019	0.145	0.157	0.148	9.97	8.77	7.83
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	98	74	81	90	92	93	0.012	0.018	0.020	0.167	0.175	0.169	14.29	9.91	8.57
North-South Baseline															
Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m								
1	28	21	23	100	100	100	0.002	0.002	0.002	0.280	0.308	0.294	150.88	148.93	151.42
10	51	38	42	88	100	90	0.005	0.003	0.009	0.208	0.230	0.216	46.16	80.21	23.97
100	481	315	395	75	75	71	0.011	0.018	0.017	0.071	0.080	0.071	6.43	4.36	4.21
250	1010	669	938	38	63	71	0.020	0.017	0.020	0.049	0.056	0.049	2.39	3.23	2.49
500	1811	1396	1826	38	71	50	0.020	0.019	0.018	0.038	0.045	0.037	1.86	2.42	2.02
1000	3660	3060	3660	13	83	43	0.028	0.016	0.024	0.031	0.034	0.031	1.11	2.19	1.30
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m								
1	2	2	2	100	100	100	0.005	0.005	0.005	0.404	0.391	0.383	79.84	79.83	79.83
10	3	2	3	94	94	83	0.011	0.014	0.012	0.330	0.391	0.313	28.96	28.17	27.07
100	26	22	33	79	75	79	0.020	0.020	0.017	0.134	0.126	0.108	6.70	6.29	6.43
250	81	58	82	79	88	69	0.015	0.011	0.019	0.093	0.096	0.077	6.25	8.44	4.09
500	155	99	155	86	88	79	0.018	0.011	0.018	0.086	0.099	0.078	4.76	8.92	4.47
1000	220	147	215	86	94	79	0.013	0.008	0.018	0.085	0.106	0.083	6.77	13.29	4.56
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	258	171	233	86	94	86	0.020	0.005	0.017	0.082	0.111	0.088	4.03	21.19	5.25
High-end receivers, Triple Frequency							$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m								
1	1	1	1	100	100	100	0.005	0.005	0.005	0.369	0.358	0.351	69.84	69.84	69.84
10	2	2	2	92	96	89	0.014	0.013	0.013	0.262	0.254	0.248	18.73	19.15	18.76
100	20	18	25	86	88	86	0.020	0.015	0.017	0.112	0.102	0.091	5.51	7.00	5.32
250	58	39	56	86	92	88	0.015	0.011	0.019	0.102	0.108	0.088	6.90	9.42	4.59
500	81	53	87	90	92	86	0.020	0.012	0.019	0.129	0.143	0.115	6.33	11.82	5.90
1000	103	69	102	90	92	86	0.016	0.017	0.020	0.145	0.169	0.139	8.82	9.79	6.80
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	115	67	103	90	92	92	0.018	0.010	0.009	0.147	0.192	0.142	8.28	19.28	16.10

Table 6.26: **GPS only**, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, Partial ambiguities fixed (percentage) and number of epochs for obtaining 2cm fixed-precision of rover-receiver coordinates are presented, measurement precision is held fixed, **baseline length varied from 1 to 1000 Km**, SD precision σ_I = 0.68 mm per km, number of satellites are as available.

(6) For baseline lengths between 1 and 1000 Km, it took between 2 and 209 epochs considering all latitude locations for E-W oriented baseline. For N-S baseline, it took between 2 and 220 epochs for a similar scenario.

Triple frequency:

(7) For triple frequency, it took between 2 and 99 epochs for baseline lengths between 10 and 1000 Km considering different latitude locations for E-W baseline. For N-S baseline, it took between 2 and 103 epochs for a similar scenario. The best performance could be seen at $-30°$ latitude for both E-W and N-S baseline.

The results and discussion for number of epochs taken and ambiguities fixed by PAR in order to obtain a fixed-precision of 20cm for the horizontal coordinate components in presented below.

6.6.4 GPS only, Partial Ambiguity Resolution to obtain fixed-precision of 20 cm, ionosphere Weighted scenario

In this section, it is to be analyzed whether the partially ambiguity fixing help in converging the solution faster to DD fixed-precision of 20cm for σ_n, σ_e for the horizontal rover-receiver coordinate components. Table 6.27 presents the results in terms of fixed-precision to be equal to or better than the laid criteria and corresponding epochs taken and ambiguities fixed are noted.

The following is the analysis based on Table 6.27.

General conclusion:

(1) An instantaneous fixed-precision as per the laid criteria (σ_n, σ_e =20cm) could be obtained for 1 and 10 Km baseline length for triple frequency GPS only at all latitude locations. Triple frequency GPS was able to give instantaneous ASR of 0.999, Full AR, for 1 Km baseline length, as presented in earlier section.

	Partial AR @ 0.999 ASR criteria - σ_n, σ_e =20 cm, for fixed-precision of rover-receiver coordinates														
Baseline length	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$		
(Km)	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline															
Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m								
1	21	19	20	13	13	56	0.324	0.304	0.110	0.334	0.329	0.321	1.03	1.08	2.92
10	22	22	31	0	0	33	0.338	0.320	0.198	0.338	0.320	0.262	1.00	1.00	1.32
100	27	23	47	0	0	0	0.352	0.327	0.224	0.352	0.327	0.224	1.00	1.00	1.00
250	31	24	48	0	0	0	0.337	0.329	0.227	0.337	0.329	0.227	1.00	1.00	1.00
500	41	28	54	0	0	0	0.318	0.331	0.231	0.318	0.331	0.231	1.00	1.00	1.00
1000	84	46	79	0	0	0	0.282	0.329	0.239	0.282	0.329	0.239	1.00	1.00	1.00
High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m								
1	2	1	2	100	38	100	0.005	0.317	0.005	0.404	0.553	0.383	79.84	1.74	79.83
10	2	2	3	44	94	83	0.076	0.014	0.012	0.404	0.391	0.313	5.33	28.15	27.07
100	4	4	6	7	13	17	0.294	0.214	0.208	0.342	0.294	0.235	1.16	1.37	1.13
250	7	5	9	7	0	6	0.312	0.330	0.221	0.325	0.330	0.240	1.04	1.00	1.08
500	11	9	16	29	19	28	0.325	0.329	0.223	0.377	0.358	0.260	1.16	1.09	1.17
1000	36	21	34	43	38	33	0.281	0.324	0.240	0.307	0.352	0.262	1.09	1.08	1.09
High-end receivers, Triple Frequency							$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m								
1	1	1	1	100	100	100	0.005	0.005	0.005	0.369	0.358	0.351	69.84	69.84	69.84
10	1	1	1	58	83	56	0.046	0.024	0.086	0.370	0.358	0.352	8.03	14.81	4.09
100	2	1	2	52	33	44	0.180	0.329	0.178	0.354	0.430	0.298	1.97	1.31	1.67
250	3	3	4	52	54	56	0.304	0.269	0.212	0.454	0.390	0.329	1.49	1.45	1.55
500	6	5	7	57	58	56	0.310	0.292	0.233	0.511	0.481	0.394	1.65	1.65	1.69
1000	10	6	9	67	58	56	0.257	0.326	0.247	0.563	0.629	0.488	2.19	1.93	1.98
North-South Baseline															
Low-end receivers, Single Frequency							$\sigma_{\Phi_{L1}}$=0.003m , $\sigma_{P_{L1}}$=0.5m								
1	21	19	20	13	13	44	0.324	0.303	0.160	0.334	0.329	0.321	1.03	1.09	2.00
10	22	22	30	0	0	33	0.338	0.320	0.201	0.338	0.320	0.267	1.00	1.00	1.33
100	26	23	45	0	0	0	0.360	0.329	0.239	0.360	0.329	0.239	1.00	1.00	1.00
250	27	25	46	0	0	0	0.365	0.327	0.241	0.365	0.327	0.241	1.00	1.00	1.00
500	31	30	61	0	0	0	0.371	0.328	0.249	0.371	0.328	0.249	1.00	1.00	1.00
1000	48	53	91	0	0	0	0.377	0.317	0.255	0.377	0.317	0.255	1.00	1.00	1.00
High-end receivers, Dual Frequency							$\sigma_{\Phi_{L1,L2}}$=0.003m , $\sigma_{P_{L1,L2}}$=0.25m								
1	2	1	2	100	38	100	0.005	0.317	0.005	0.404	0.553	0.383	79.84	1.74	79.83
10	2	2	3	44	94	83	0.076	0.014	0.012	0.404	0.391	0.313	5.34	28.17	27.07
100	4	4	6	7	6	14	0.295	0.260	0.241	0.343	0.296	0.254	1.17	1.14	1.06
250	6	6	9	7	6	14	0.339	0.287	0.219	0.354	0.306	0.258	1.04	1.06	1.18
500	10	10	17	21	19	42	0.364	0.319	0.304	0.401	0.349	0.357	1.10	1.09	1.17
1000	22	24	41	36	31	43	0.373	0.317	0.278	0.404	0.341	0.301	1.08	1.08	1.08
High-end receivers, Triple Frequency							$\sigma_{\Phi_{L5}}$=0.002m , $\sigma_{P_{L5}}$=0.15m								
1	1	1	1	100	100	100	0.005	0.005	0.005	0.369	0.358	0.351	69.84	69.84	69.84
10	1	1	1	58	83	56	0.046	0.024	0.086	0.370	0.359	0.351	8.04	14.84	4.09
100	2	1	2	52	33	48	0.181	0.331	0.196	0.355	0.433	0.323	1.97	1.31	1.65
250	3	3	4	52	54	62	0.307	0.271	0.233	0.458	0.396	0.354	1.49	1.46	1.52
500	5	4	8	57	54	67	0.344	0.350	0.290	0.567	0.552	0.525	1.65	1.58	1.81
1000	7	7	10	57	54	67	0.352	0.324	0.314	0.680	0.603	0.684	1.93	1.86	2.18

Table 6.27: **GPS only**, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, Partial ambiguities fixed (percentage) and number of epochs for obtaining 20cm fixed-precision for the horizontal rover-receiver coordinates are presented, measurement precision is held fixed, **baseline length varied from 1 to 1000 Km**, SD precision σ_I = 0.68 mm per km, number of satellites are as available.

(2) Dual frequency GPS was able to give instantaneous fixed-precision of 20cm at $-30°$ latitude by partial fixing less than 100% of ambiguities.

(3) For all the remaining scenarios of baseline lengths above 1 Km, less than 100% of ambiguities were fixed to obtain a fixed-precision as per the laid criteria (σ_n, σ_e =2 cm σ_u=6cm) for rover-receiver coordinates.

(4) With dual frequency for baseline length 10 Km and above, ambiguity fixing was not needed to obtain a fixed-precision of 20cm for the horizontal coordinate components.

Single frequency:

(5) For single frequency, it took between 21 and 79 epochs for baseline lengths between 1 and 1000 Kms, for both E-W and between 29 and 91 epochs for N-S baseline.

Dual frequency:

(6) For baseline lengths between 1 and 1000 Km, it took between 1 and 36 epochs considering all latitude locations for E-W oriented baseline. For N-S baseline, it took between 1 and 41 epochs for a similar scenario.

Triple frequency:

(7) For triple frequency, it took between 1 and 10 epochs for baseline lengths between 100 and 1000 Km considering different latitude locations for both E-W baseline and N-S baseline.

6.6.5 Galileo only - Full and Partial Ambiguity Resolution, ionosphere Weighted scenario

The Galileo system with four frequencies $E1, E5a, E5b$ and $E6$ were used in combination to form single, dual, triple and quadruple system in order to simulate ambiguity success rates and fixed-precision of the rover-receiver coordinates. Ionosphere was considered a-priori known and hence weighted, the un-differenced precision of ionosphere considered is $\sigma_I = 0.68$ mm per km. The results for Geometry fixed ionosphere weighted scenario are presented in Figures G.53 to G.56 for full ambiguity resolution and in Figures G.57 to G.60 for partial ambiguity resolution, see Appendix G. Table 6.28 gives a results for epochs taken in order to obtain 0.999 ASR for different scenarios of baseline length, frequency combination and latitude location along with the ambiguity-float and -fixed-precision of the rover coordinates.

The following is the analysis based on Table 6.28 and 6.29, see below.

General Conclusion:

(1) Instantaneous ASR of 0.999 could be achieved for baseline length of 1 Km for dual, triple and quadruple frequency Galileo for all the scenarios of latitude locations.

(2) It can be noted that the number of epochs required to obtain 0.999 ASR increases as baseline length increases. However, it is again stressed that, for long baseline when ionosphere weighted is equivalent to ionosphere float case, 0.999 ASR could be achieved quicker depending on the number of common satellites and number of low elevation satellites available. The number of low elevation satellites fall (especially for 500 Km baseline) causing to achieve quicker 0.999 ASR as compared to 1000 Km baseline length.

(3) At $-60°$ latitude, the results with Galileo are exceptional as compared to other latitude locations. see Table 6.28, dual, triple and quadruple frequency results. This is due to the fact that for the selected time period 0000 to 0100 UTC, Galileo performs exceptionally well for longer baselines, see Figure 6.20 (batch 1 corresponds to 0000 to 0100 UTC.

	Full AR - minimum desired ASR = 0.999											
	Epochs taken			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$		
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline												
	Low-end receivers, Single Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.5m								
1	21	11	9	0.002	0.003	0.003	0.356	0.448	0.520	159.37	160.63	161.73
10	38	24	18	0.003	0.004	0.004	0.265	0.301	0.367	85.59	85.45	86.49
100	495	573	429	0.007	0.007	0.006	0.072	0.057	0.060	9.84	8.72	9.35
250	2040	1270	1708	0.008	0.010	0.007	0.031	0.041	0.028	3.64	3.95	3.75
500	3467	2299	2946	0.010	0.012	0.010	0.024	0.033	0.024	2.31	2.62	2.44
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	6	4	4	0.003	1.170	1.061	0.489	1.170	1.061	166.28	1.00	1.00
	High-end receivers, Dual Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m					, $\sigma_{\Phi_{E5a}}$=0.002m , $\sigma_{P_{E5a}}$=0.15m			
1	1	1	1	0.006	0.006	0.006	0.433	0.391	0.409	70.62	70.62	70.62
10	6	6	2	0.007	0.007	0.012	0.177	0.160	0.289	24.02	24.02	24.02
100	32	56	19	0.004	0.003	0.005	0.089	0.060	0.108	20.80	20.60	20.80
250	57	54	30	0.003	0.003	0.004	0.099	0.090	0.127	30.62	30.22	30.68
500	85	61	78	0.003	0.003	0.002	0.122	0.128	0.103	45.83	45.05	44.64
1000	124	80	79	0.002	0.003	0.002	0.132	0.147	0.137	58.25	58.37	59.54
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	126	79	85	0.002	0.002	0.002	0.147	0.174	0.154	67.70	70.28	69.70
	High-end receivers, Triple Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m				, $\sigma_{\Phi_{E5a,E5b}}$=0.002m , $\sigma_{P_{E5a,E5b}}$=0.15m				
1	1	1	1	0.005	0.004	0.005	0.328	0.297	0.310	66.21	66.21	66.21
10	6	6	2	0.007	0.006	0.012	0.134	0.122	0.220	18.83	18.83	18.83
100	29	51	17	0.004	0.003	0.005	0.077	0.052	0.094	17.54	17.43	17.54
250	38	49	23	0.004	0.003	0.005	0.111	0.087	0.133	28.81	28.44	28.80
500	50	39	25	0.003	0.003	0.004	0.155	0.157	0.202	45.97	45.41	46.16
1000	57	39	27	0.003	0.004	0.004	0.214	0.222	0.269	63.49	63.08	64.51
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	49	50	26	0.003	0.003	0.004	0.267	0.227	0.344	78.71	74.74	79.39
	High-end receivers, Quadruple Frequency			$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m , $\sigma_{\Phi_{E5a,E5b,E6}}$=0.002m , $\sigma_{P_{E5a,E5b,E6}}$=0.15m								
1	1	1	1	0.004	0.004	0.004	0.275	0.249	0.260	62.14	62.14	62.14
10	6	6	2	0.007	0.006	0.012	0.113	0.102	0.185	15.91	15.90	15.91
100	28	49	16	0.004	0.003	0.005	0.071	0.048	0.088	16.40	16.32	16.40
250	37	46	22	0.004	0.003	0.005	0.111	0.089	0.133	29.07	28.74	29.07
500	48	37	24	0.003	0.003	0.004	0.159	0.161	0.206	47.18	46.64	47.37
1000	56	37	25	0.003	0.004	0.004	0.215	0.227	0.278	64.67	64.45	65.87
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	46	47	25	0.003	0.003	0.004	0.272	0.231	0.345	79.52	75.71	80.02

Table 6.28: **Galileo only**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **Full AR** analysis, ASR and fixed-precision of rover-receiver coordinates (meters) are presented - measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, σ_I = 0.68 mm per km.

	Full AR - minimum desired ASR = 0.999											
	Epochs taken			$\sigma_{\check{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\check{neu}}$		
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
North-South Baseline												
	Low-end receivers, Single Frequency				$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.5m							
1	21	11	9	0.002	0.003	0.003	0.356	0.448	0.520	159.37	160.63	161.73
10	39	24	18	0.003	0.004	0.004	0.261	0.301	0.367	85.47	85.46	86.49
100	563	559	410	0.007	0.007	0.007	0.066	0.059	0.062	9.65	8.86	9.43
250	2440	1202	1607	0.008	0.011	0.008	0.026	0.044	0.030	3.29	4.00	3.82
500	> 3600	2009	2778	-	0.014	0.010	-	0.038	0.025	-	2.64	2.46
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	6	4	4	0.003	1.170	1.061	0.489	1.170	1.061	166.28	1.00	1.00
	High-end receivers, Dual Frequency				$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m					, $\sigma_{\Phi_{E5a}}$=0.002m , $\sigma_{P_{E5a}}$=0.15m		
1	1	1	1	0.006	0.006	0.006	0.433	0.391	0.409	70.62	70.62	70.62
10	6	6	2	0.007	0.007	0.012	0.177	0.160	0.289	24.02	24.02	24.02
100	37	64	19	0.004	0.003	0.005	0.082	0.056	0.108	20.79	20.55	20.80
250	70	67	31	0.003	0.003	0.004	0.100	0.086	0.125	30.36	29.74	30.67
500	104	71	69	0.003	0.003	0.003	0.119	0.128	0.115	44.35	43.83	45.35
1000	213	79	71	0.002	0.003	0.003	0.091	0.170	0.167	43.00	56.33	60.75
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	144	93	79	0.002	0.002	0.002	0.141	0.164	0.165	61.21	66.53	71.09
	High-end receivers, Triple Frequency				$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m					, $\sigma_{\Phi_{E5a,E5b}}$=0.002m , $\sigma_{P_{E5a,E5b}}$=0.15m		
1	1	1	1	0.005	0.004	0.005	0.328	0.297	0.310	66.21	66.21	66.21
10	6	6	2	0.007	0.006	0.012	0.134	0.122	0.220	18.83	18.83	18.83
100	34	59	17	0.004	0.003	0.005	0.071	0.048	0.094	17.53	17.40	17.54
250	52	61	26	0.004	0.003	0.004	0.107	0.083	0.125	28.67	28.09	28.78
500	58	48	44	0.004	0.003	0.003	0.160	0.154	0.145	45.54	44.60	45.63
1000	87	40	30	0.003	0.004	0.005	0.187	0.256	0.289	58.04	61.84	64.17
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	63	62	29	0.003	0.003	0.004	0.256	0.209	0.325	75.35	71.31	78.87
	High-end receivers, Quadruple Frequency				$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m		, $\sigma_{\Phi_{E5a,E5b,E6}}$=0.002m , $\sigma_{P_{E5a,E5b,E6}}$=0.15m					
1	1	1	1	0.004	0.004	0.004	0.275	0.249	0.260	62.14	62.14	62.14
10	6	6	2	0.007	0.006	0.012	0.113	0.102	0.185	15.91	15.90	15.91
100	33	56	17	0.004	0.003	0.005	0.065	0.045	0.086	16.40	16.30	16.40
250	49	58	25	0.004	0.003	0.004	0.108	0.084	0.126	28.95	28.40	29.05
500	56	46	30	0.003	0.003	0.004	0.162	0.158	0.186	46.73	45.81	47.14
1000	83	38	29	0.003	0.004	0.004	0.191	0.261	0.292	59.49	63.22	65.43
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	59	59	27	0.003	0.003	0.004	0.261	0.212	0.332	76.45	72.25	79.69

Table 6.29: continued from Table 6.28 ... **Galileo only**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **Full AR** analysis

Single frequency:

(4) For baseline lengths between 1 to 500 Km, it took between 9 to 3467 epochs (**GPS only** took 21 and >3600 epochs) to obtain 0.999 full AR for E-W baseline length. For N-S baseline length, it took between 9 to >3600 epochs (**GPS only** took 22 and >3600 epochs) for similar scenarios.

(5) At 250 Km baseline length, it took 2040, 1270 and 1708 epochs (**GPS only** took 2272, 749 and 1968 epochs) for E-W baseline and 2440, 1202 and 1607 epochs for N-S baseline (**GPS only** took 1347, 779 and 2091 epochs) at $0°, -30°$ and $-60°$ degree latitude respectively.

Dual frequency:

(6) With dual frequency, it took a minimum of 2 and a maximum of 124 epochs (**GPS only** took 5 to 213 epochs) for baseline lengths between 10 and 1000 Km for E-W baseline. For N-S baseline, it took between 2 to 213 epochs (**GPS only** took 5 to 232 epochs) for similar scenario. The dual frequency system seemed to favour $-60°$ latitude for both the E-W and N-S baseline.

(7) For 250 Km baseline length, it took 57, 54 and 30 epochs (**GPS only** took 29, 36 and 63 epochs) at $0°, -30°$ and $-60°$ degree latitude and for N-S baseline it took 70, 67 and 31 epochs (**GPS only** took 108, 66 and 135 epochs) for similar latitude locations.

Triple frequency:

(8) Triple frequency Galileo took between 2 and 57 epochs (**GPS only** took 4 to 106 epochs) for all latitude locations for baseline length of 10 to 1000 Kms for E-W baseline. For N-S baseline, it took between 2 and 87 epochs (**GPS only** took 4 to 150 epochs) for a similar scenario. Undoubtedly Galileo only favours $-60°$ latitude for the selected time period.

(9) For 250 Km baseline length, it took 38, 49 and 23 epochs (**GPS only** took 140, 43 and 58 epochs) at $0°, -30°$ and $-60°$ latitude for E-W baseline. For N-S baseline, it took 52, 61 and 26 epochs (**GPS only**

took 73, 45 and 89 epochs) for a similar scenario.

Quadruple frequency:

(10) In general quadruple frequency performed marginally better for some scenarios of baseline lengths by taking one to two epoch less for 0.999 ASR. For all other scenarios the performance of quadruple frequency is similar to triple frequency Galileo.

(11) with quadruple frequency Galileo, it took between 2 and 56 epochs afor E-W oriented baseline and between 2 and 83 epochs for N-S oriented baseline.

(12) At 250 Km baseline length, it took 37, 46 and 22 epochs at $0°$, $-30°$ and $-60°$ latitude for E-W oriented baseline and 49, 58 and 25 epochs for N-S oriented baseline for similar latitude locations.

Comparison with GPS only:

(13) Comparison with GPS only shows that Galileo performs better, see above dual and triple frequency comparisons. The improved measurement precision of Galileo system plays an important part in the reduced time to achieve 0.999 ASR as compared to GPS only.

(14) Quadruple frequency Galileo out performs triple frequency Galileo, hence it can also be said to perform better that GPS system.

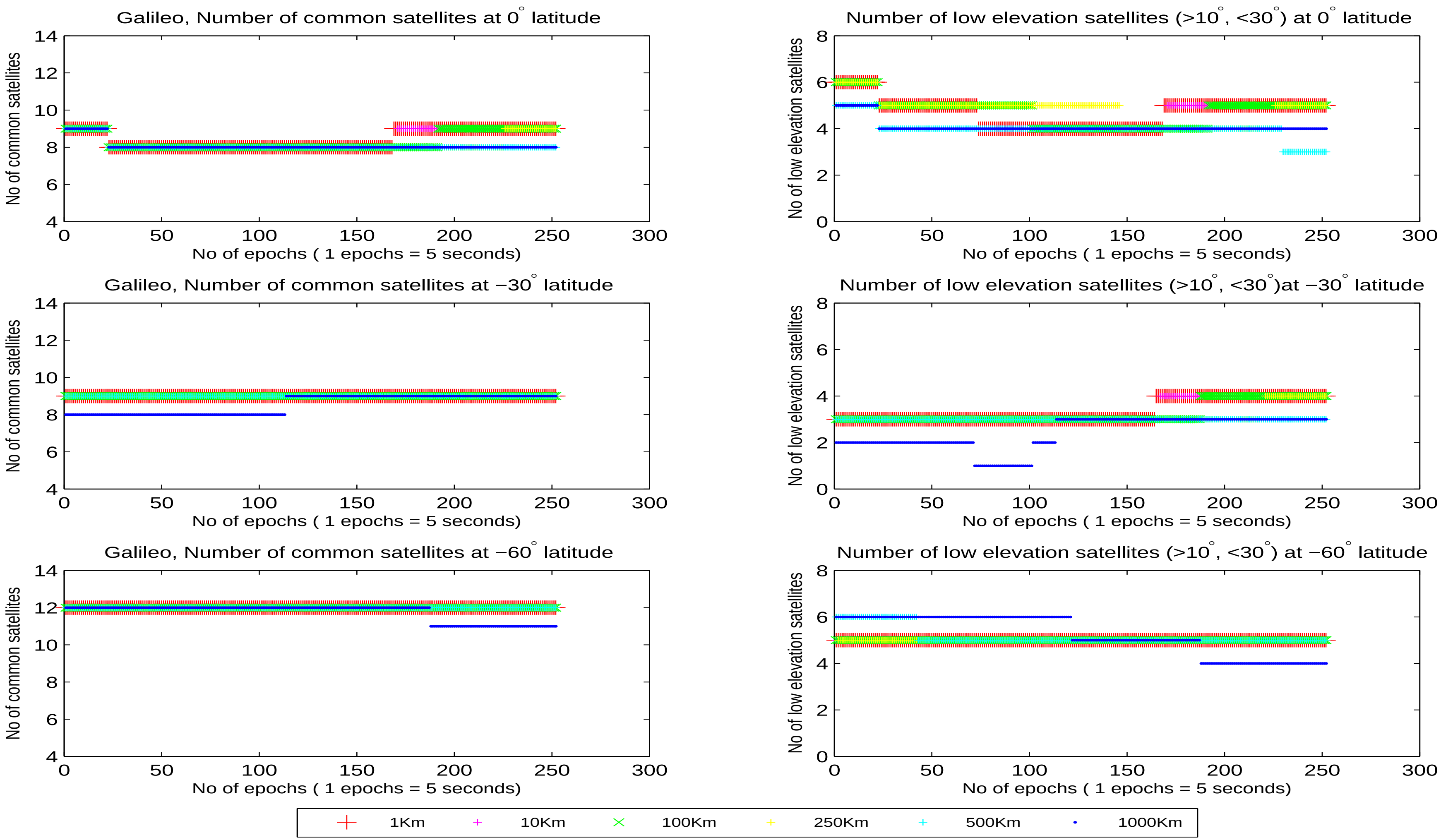

Figure 6.24: **Galileo only**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, East-West oriented baseline.

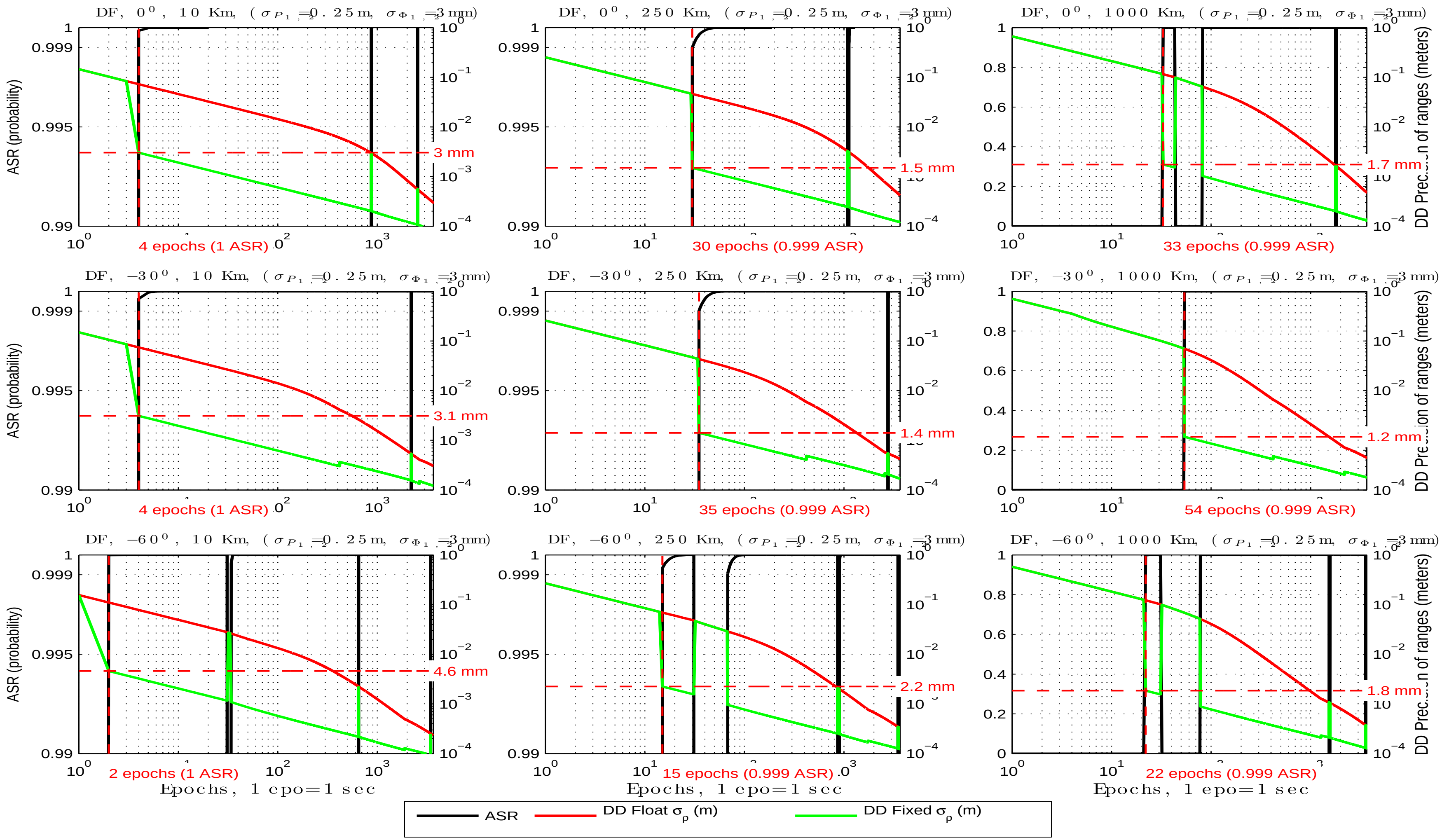

Figure 6.25: **Galileo only**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

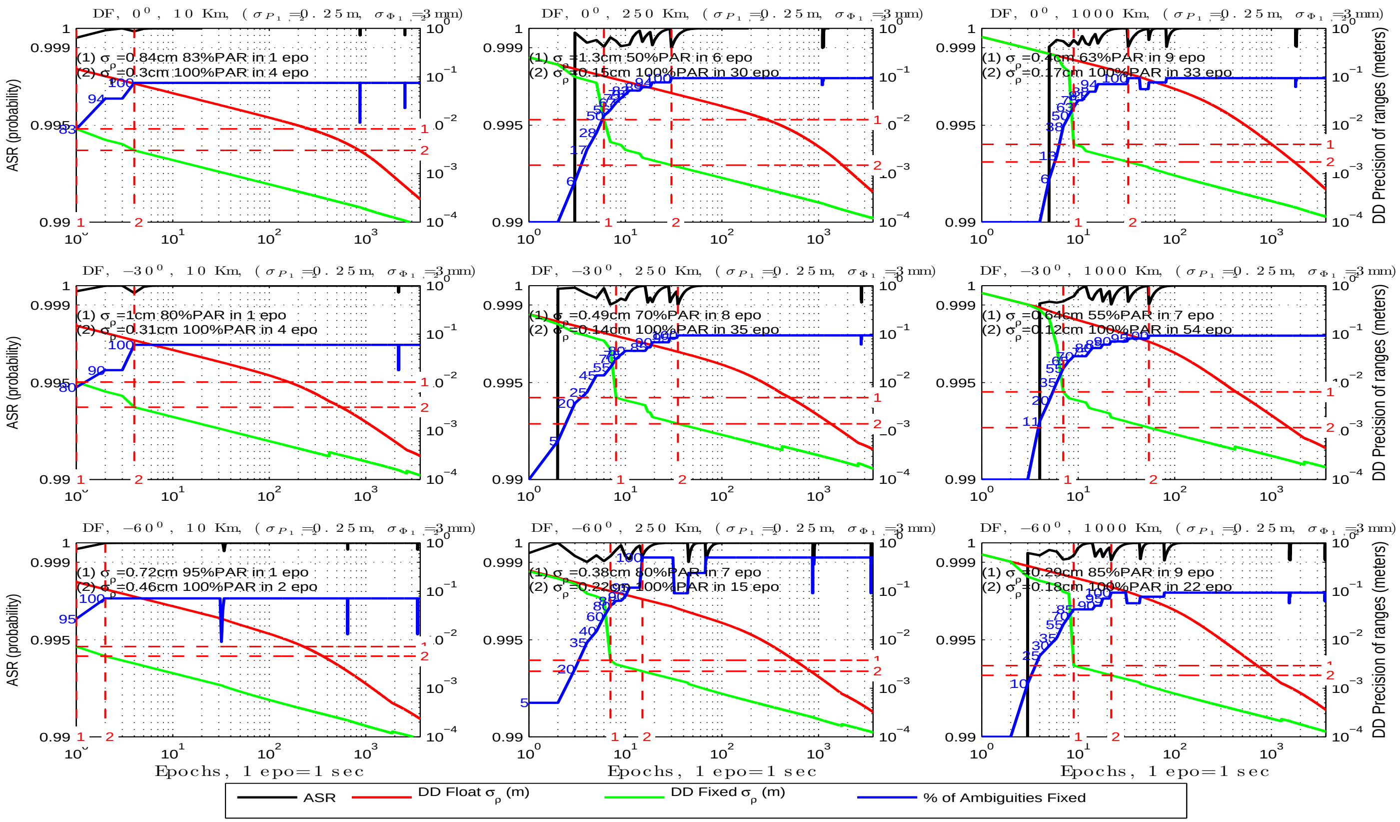

Figure 6.26: **Galileo only**, Dual frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

6.6.6 Galileo only, Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, ionosphere Weighted scenario

The evaluation of PAR with Galileo system is done in order to obtain fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for the rover-receiver coordinates. The evaluation is based on number of epochs taken to fulfil the criteria laid. Table 6.30 presents the results of the simulation for single, dual, triple and quadruple Galileo system frequency combinations.

The analysis from the results presented in Table 6.30 is given below.

General remarks:

(1) Instantaneous fixed-precision of σ_n, σ_e =2cm and σ_u=6cm could be achieved for baseline lengths of 1 Km for dual, triple and quadruple frequency Galileo, for all the scenarios of latitude locations. With **GPS only** instantaneous fixed-precision for the laid criteria was obtained for triple frequency only.

(2) For all other scenarios of baseline lengths of 10 to 1000 Kms, for all frequency combinations, PAR fixed only a partial subset of ambiguities (less than 100%) to obtain a fixed-precision as specified (σ_n, σ_e =2cm and σ_u=6cm).

(3) The results for ionosphere weighted scenarios in order to obtain a fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for the rover-receiver coordinates were much better than for ionosphere float scenario. See Table 6.30, 250 Km baseline length.

Single frequency:

(4) For single frequency, it took a minimum of 8 and a maximum of >3600 epochs (**GPS only** took between 21 and >3600 epochs) for baseline lengths between 1 and 1000 Kms for both E-W and N-S oriented baseline (**GPS only** took between 21 and >3600 epochs). The best performance could be seen at $-60°$ latitude for both E-W and N-S baseline.

	Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for rover-receiver coordinates														
Baseline length	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$		
(Km)	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline															
Low-end receivers, Single Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.5m								
1	20	11	8	89	100	90	0.010	0.003	0.014	0.365	0.448	0.554	35.94	160.63	38.42
10	36	22	17	89	80	90	0.012	0.017	0.012	0.273	0.316	0.378	22.49	18.98	32.68
100	353	252	249	67	70	82	0.020	0.017	0.012	0.088	0.093	0.084	4.29	5.43	6.90
250	916	734	731	78	89	73	0.016	0.015	0.015	0.056	0.057	0.050	3.41	3.74	3.25
500	2035	1533	1444	60	56	58	0.019	0.018	0.017	0.038	0.042	0.037	2.04	2.31	2.22
1000	3660	3476	3044	22	50	36	0.022	0.016	0.020	0.030	0.033	0.030	1.33	1.99	1.48
I-Fix(1Km)	6	4	3	100	100	90	0.003	0.004	0.015	0.489	0.585	0.613	166.28	166.57	40.85
High-end receivers, Dual Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a}}$=0.002m , $\sigma_{P_{E5a}}$=0.15m								
1	1	1	1	100	100	100	0.006	0.006	0.006	0.433	0.391	0.409	70.62	70.62	70.62
10	2	2	2	89	90	100	0.015	0.014	0.012	0.306	0.277	0.289	19.91	20.44	24.02
100	20	18	16	72	70	85	0.015	0.020	0.015	0.112	0.107	0.118	7.58	5.23	7.82
250	54	39	28	78	75	85	0.019	0.015	0.017	0.102	0.107	0.131	5.41	7.13	7.51
500	80	56	40	89	85	86	0.013	0.011	0.009	0.126	0.134	0.158	9.70	12.34	18.13
1000	115	78	47	89	95	82	0.014	0.008	0.020	0.139	0.149	0.195	10.08	19.41	9.87
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	120	77	53	94	95	86	0.008	0.008	0.008	0.153	0.177	0.219	19.93	22.47	26.98
High-end receivers, Triple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b}}$=0.002m , $\sigma_{P_{E5a,E5b}}$=0.15m								
1	1	1	1	100	100	100	0.005	0.004	0.005	0.328	0.297	0.310	66.21	66.21	66.21
10	2	2	2	93	93	100	0.015	0.013	0.012	0.233	0.211	0.220	15.54	15.96	18.83
100	16	14	14	89	93	90	0.013	0.007	0.015	0.103	0.100	0.104	7.83	13.33	6.69
250	31	22	20	89	87	90	0.011	0.018	0.014	0.123	0.132	0.142	11.44	7.36	9.97
500	38	27	23	93	87	90	0.010	0.017	0.013	0.179	0.191	0.211	18.43	11.23	15.98
1000	52	34	25	89	90	87	0.015	0.015	0.019	0.227	0.240	0.280	14.86	16.33	14.75
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	45	29	25	96	87	93	0.008	0.017	0.012	0.281	0.314	0.351	37.14	18.81	29.12
High-end receivers, Quadruple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b,E6}}$=0.002m , $\sigma_{P_{E5a,E5b,E6}}$=0.15m								
1	1	1	1	100	100	100	0.004	0.004	0.004	0.275	0.249	0.260	62.14	62.14	62.14
10	2	2	2	94	95	100	0.015	0.013	0.012	0.195	0.177	0.185	13.18	13.53	15.91
100	16	14	13	92	95	93	0.013	0.007	0.016	0.094	0.091	0.098	7.29	12.45	6.22
250	29	21	19	89	90	93	0.020	0.018	0.014	0.125	0.132	0.144	6.13	7.41	10.05
500	36	26	22	94	90	93	0.010	0.017	0.013	0.184	0.194	0.216	18.90	11.50	16.39
1000	49	32	24	92	93	90	0.015	0.015	0.019	0.235	0.247	0.284	15.20	16.68	15.04
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	43	28	24	97	92	95	0.008	0.008	0.012	0.282	0.314	0.352	37.45	38.22	29.35

Table 6.30: **Galileo only**, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, Partial ambiguities fixed (percentage) and number of epochs for obtaining 2cm fixed-precision of rover-receiver coordinates are presented, measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km** in East-West direction, σ_I = 0.68 mm per km.

	Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for rover-receiver coordinates														
Baseline length	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{neu}}$(meters)			$\sigma_{\hat{neu}}$(meters)			Gain=$\sigma_{\hat{neu}}/\sigma_{\check{neu}}$		
(Km)	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
North-South Baseline															
Low-end receivers, Single Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.5m								
1	20	11	8	89	100	90	0.009	0.003	0.014	0.365	0.448	0.554	38.67	160.63	38.41
10	36	22	17	89	80	90	0.012	0.017	0.012	0.273	0.316	0.378	22.50	19.00	32.62
100	355	253	252	67	67	82	0.020	0.019	0.012	0.087	0.094	0.084	4.26	4.84	6.92
250	1034	766	744	70	89	67	0.016	0.015	0.015	0.056	0.058	0.050	3.42	3.76	3.22
500	2215	1533	1538	40	56	58	0.020	0.019	0.017	0.037	0.045	0.036	1.80	2.34	2.17
1000	3660	3477	2866	29	50	36	0.023	0.019	0.020	0.033	0.035	0.032	1.40	1.80	1.60
I-Fix(1Km)	6	4	3	100	100	90	0.003	0.004	0.015	0.489	0.585	0.613	166.28	166.57	40.85
High-end receivers, Dual Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a}}$=0.002m , $\sigma_{P_{E5a}}$=0.15m								
1	1	1	1	100	100	100	0.006	0.006	0.006	0.433	0.391	0.409	70.62	70.62	70.62
10	2	2	2	89	90	100	0.015	0.014	0.012	0.306	0.277	0.289	19.91	20.44	24.02
100	21	19	15	78	70	85	0.012	0.020	0.016	0.109	0.105	0.122	8.81	5.13	7.69
250	54	42	27	88	78	90	0.018	0.020	0.013	0.115	0.111	0.135	6.50	5.47	10.40
500	102	64	41	88	83	82	0.015	0.019	0.014	0.121	0.136	0.159	7.82	7.06	11.54
1000	198	79	63	86	100	90	0.018	0.003	0.007	0.098	0.170	0.182	5.49	56.33	24.22
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	143	90	54	94	94	86	0.007	0.008	0.008	0.142	0.168	0.218	19.77	20.43	26.26
High-end receivers, Triple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b}}$=0.002m , $\sigma_{P_{E5a,E5b}}$=0.15m								
1	1	1	1	100	100	100	0.005	0.004	0.005	0.328	0.297	0.310	66.21	66.21	66.21
10	2	2	2	93	93	100	0.015	0.013	0.012	0.233	0.211	0.220	15.54	15.96	18.83
100	17	14	14	89	93	90	0.013	0.008	0.016	0.100	0.100	0.104	7.83	13.24	6.59
250	34	24	20	92	89	93	0.020	0.018	0.014	0.133	0.135	0.143	6.65	7.57	10.12
500	46	29	22	96	85	93	0.008	0.020	0.014	0.181	0.203	0.219	22.92	9.95	15.88
1000	72	35	26	95	89	89	0.009	0.018	0.015	0.213	0.277	0.313	22.96	15.28	21.20
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	46	30	24	92	85	93	0.018	0.019	0.013	0.310	0.329	0.362	16.91	17.51	27.22
High-end receivers, Quadruple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b,E6}}$=0.002m , $\sigma_{P_{E5a,E5b,E6}}$=0.15m								
1	1	1	1	100	100	100	0.004	0.004	0.004	0.275	0.249	0.260	62.14	62.14	62.14
10	2	2	2	94	95	100	0.015	0.013	0.012	0.195	0.177	0.185	13.17	13.53	15.91
100	16	14	13	92	95	93	0.013	0.007	0.016	0.094	0.091	0.098	7.28	12.37	6.12
250	33	23	19	94	92	95	0.020	0.018	0.014	0.133	0.136	0.144	6.69	7.61	10.20
500	43	28	21	94	89	95	0.018	0.020	0.014	0.187	0.207	0.224	10.65	10.19	16.29
1000	69	34	25	96	92	92	0.009	0.018	0.015	0.217	0.279	0.317	23.46	15.57	21.61
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	44	28	23	94	89	95	0.018	0.019	0.013	0.312	0.335	0.363	17.06	17.70	27.43

Table 6.31: continued from Table 6.30... **Galileo only**, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario

Dual frequency:

(5) For baseline lengths between 10 and 1000 Km, it took between 2 and 115 epochs (**GPS only** took between 2 and 209 epochs) considering all latitude locations for E-W oriented baseline. For N-S baseline, it took between 2 and 198 epochs (**GPS only** took between 2 and 220 epochs). Galileo only dual frequency performed well for $-30°$ and $-60°$ latitude.

Triple frequency:

(6) For triple frequency, it took a minimum of 2 and a maximum of 52 epochs (**GPS only** took between 2 and 99 epochs) for baseline lengths between 10 and 1000 Km considering different latitude locations for E-W baseline. For N-S baseline, it took between 2 and 72 epochs (**GPS only** took between 2 and 103 epochs) for a similar scenario. Galileo only triple frequency performed well for $-60°$ latitude.

Quadruple frequency:

(7) The quadruple frequency Galileo system performed better than the triple frequency Galileo system. for some of the scenario the number of epochs taken to obtain a fixed-precision of σ_n, σ_e =2cm and σ_u=6cm were better by 1 to 3 epochs as compared to triple frequency. For all other scenarios, the results for triple and quadruple were exactly the same, see Table 6.30.

(8) With quadruple frequency Galileo, it took between 2 and 49 epochs for E-W oriented baseline and between 2 and 69 epochs for N-S oriented baseline.

Comparison with GPS only:

(9) Galileo system performs better than GPS only for obtaining a fixed-precision of σ_n, σ_e =2cm and σ_u=6cm for rover-receiver coordinates, see above comparisons for dual and triple frequency Galileo and GPS.

(10) Quadruple frequency Galileo system performs better than triple frequency Galileo and GPS both.

The results and discussion for number of epochs taken and ambiguities fixed by PAR in order to obtain a fixed-precision of 20cm for the horizontal coordinate components in presented below.

6.6.7 Galileo only, Partial Ambiguity Resolution to obtain fixed-precision of 20 cm, ionosphere Weighted scenario

The evaluation of PAR with Galileo system is done in order to obtain fixed-precision of σ_n, σ_e =20cm for the rover-receiver coordinates. The evaluation is based on number of epochs taken to fulfil the criteria laid. Table 6.30 presents the results of the simulation for single, dual, triple and quadruple Galileo system frequency combinations.

The analysis from the results presented in Table 6.32 is given below.

General remarks:

(1) Instantaneous fixed-precision of σ_n, σ_e =20cm could be achieved for baseline lengths of 1 Km for dual, triple and quadruple frequency Galileo, for all the scenarios of latitude locations. 100% of ambiguities were fixed for 1Km baseline, dual, triple and quadruple frequency which indicated full AR instantaneously. With **GPS only** instantaneous fixed-precision for the laid criteria was obtained for triple frequency only.

(2) Instantaneous fixed-precision of 20cm for horizontal coordinate components could be achieved for 10 and 100 Km baseline length with triple and quadruple frequency Galileo. In this case less than 100% of ambiguities were fixed.

(3) For all other scenarios of baseline lengths, for all frequency combinations, PAR fixed only a partial subset of ambiguities (less than 100%) to obtain a fixed-precision as specified (σ_n, σ_e =20cm).

Single frequency:

(4) For single frequency, it took between 8 and 71 epochs (**GPS only** took between 21 and 79 epochs) for baseline lengths between 1 and 1000

	Partial AR @ 0.999 ASR criteria - 20 cm fixed-precision for rover-receiver coordinates																	
Baseline length	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\check{neu}}$					
(Km)	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°			
East-West Baseline																		
Low-end receivers, Single Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.5m											
1	15	11	8	44	100	90	0.284	0.003	0.014	0.427	0.448	0.554	1.51	160.63	38.42			
10	21	19	16	11	30	70	0.341	0.286	0.090	0.364	0.342	0.391	1.07	1.20	4.36			
100	23	24	39	0	0	0	0.353	0.312	0.241	0.353	0.312	0.241	1.00	1.00	1.00			
250	24	25	40	0	0	0	0.355	0.314	0.243	0.355	0.314	0.243	1.00	1.00	1.00			
500	31	29	46	0	0	0	0.339	0.316	0.239	0.339	0.316	0.239	1.00	1.00	1.00			
1000	71	44	70	0	0	0	0.294	0.327	0.236	0.294	0.327	0.236	1.00	1.00	1.00			
High-end receivers, Dual Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a}}$=0.002m , $\sigma_{P_{E5a}}$=0.15m											
1	1	1	1	100	100	100	0.006	0.006	0.006	0.433	0.391	0.409	70.62	70.62	70.62			
10	2	2	1	89	90	35	0.015	0.014	0.109	0.306	0.277	0.409	19.91	20.44	3.75			
100	2	3	4	0	5	20	0.355	0.252	0.186	0.355	0.262	0.236	1.00	1.04	1.27			
250	5	5	8	0	5	30	0.336	0.299	0.192	0.336	0.303	0.247	1.00	1.01	1.29			
500	9	9	14	17	25	35	0.335	0.289	0.230	0.389	0.351	0.286	1.16	1.21	1.24			
1000	22	13	23	44	30	40	0.286	0.325	0.234	0.381	0.421	0.304	1.33	1.29	1.30			
High-end receivers, Triple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b}}$=0.002m , $\sigma_{P_{E5a,E5b}}$=0.15m											
1	1	1	1	100	100	100	0.005	0.004	0.005	0.328	0.297	0.310	66.21	66.21	66.21			
10	1	1	1	70	77	90	0.035	0.025	0.023	0.329	0.298	0.311	9.34	12.10	13.80			
100	1	1	1	52	53	60	0.231	0.204	0.206	0.413	0.373	0.388	1.79	1.83	1.88			
250	2	2	2	56	53	67	0.279	0.249	0.231	0.485	0.438	0.451	1.74	1.75	1.95			
500	2	2	3	56	53	67	0.352	0.313	0.224	0.786	0.710	0.589	2.24	2.27	2.64			
1000	4	3	4	63	63	67	0.278	0.291	0.203	0.866	0.930	0.713	3.11	3.19	3.52			
High-end receivers, Quadruple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b,E6}}$=0.002m , $\sigma_{P_{E5a,E5b,E6}}$=0.15m											
1	1	1	1	100	100	100	0.004	0.004	0.004	0.275	0.249	0.260	62.14	62.14	62.14			
10	1	1	1	78	83	93	0.035	0.024	0.022	0.276	0.250	0.261	7.98	10.29	11.73			
100	1	1	1	67	68	70	0.219	0.195	0.205	0.377	0.341	0.354	1.72	1.75	1.73			
250	2	2	2	69	70	75	0.273	0.243	0.228	0.477	0.430	0.443	1.75	1.77	1.94			
500	2	2	3	69	70	75	0.344	0.306	0.219	0.786	0.710	0.589	2.29	2.32	2.69			
1000	4	3	4	72	72	75	0.272	0.284	0.198	0.860	0.923	0.708	3.17	3.25	3.58			

Table 6.32: **Galileo only**, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, Partial ambiguities fixed (percentage) and number of epochs for obtaining 20cm fixed-precision of rover-receiver coordinates are presented, measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km** in East-West direction, σ_I = 0.68 mm per km.

	Partial AR @ 0.999 ASR criteria - 20 cm fixed-precision for rover-receiver coordinates																	
Baseline length	Epochs taken			Ambiguities fixed(%)			$\sigma_{\check{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\check{neu}}$					
(Km)	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°			
North-South Baseline																		
Low-end receivers, Single Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.5m											
1	15	11	8	44	100	90	0.281	0.003	0.014	0.427	0.448	0.554	1.52	160.63	38.41			
10	22	19	15	0	40	50	0.355	0.265	0.226	0.355	0.342	0.404	1.00	1.29	1.79			
100	23	23	39	0	0	0	0.352	0.320	0.242	0.352	0.320	0.242	1.00	1.00	1.00			
250	27	23	42	0	0	0	0.379	0.352	0.238	0.379	0.352	0.238	1.00	1.00	1.00			
500	34	24	50	0	0	0	0.362	0.386	0.233	0.362	0.386	0.233	1.00	1.00	1.00			
1000	61	57	93	0	0	0	0.380	0.342	0.230	0.380	0.342	0.230	1.00	1.00	1.00			
High-end receivers, Dual Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a}}$=0.002m , $\sigma_{P_{E5a}}$=0.15m											
1	1	1	1	100	100	100	0.006	0.006	0.006	0.433	0.391	0.409	70.62	70.62	70.62			
10	2	2	1	89	90	35	0.015	0.014	0.109	0.306	0.277	0.409	19.91	20.44	3.75			
100	3	2	4	0	5	20	0.289	0.310	0.186	0.289	0.322	0.236	1.00	1.04	1.27			
250	6	5	8	13	6	30	0.304	0.322	0.195	0.347	0.326	0.248	1.14	1.01	1.27			
500	10	8	15	19	17	35	0.344	0.374	0.227	0.412	0.413	0.279	1.20	1.10	1.23			
1000	16	17	27	29	39	44	0.386	0.325	0.241	0.509	0.427	0.318	1.32	1.31	1.32			
High-end receivers, Triple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b}}$=0.002m , $\sigma_{P_{E5a,E5b}}$=0.15m											
1	1	1	1	100	100	100	0.005	0.004	0.005	0.328	0.297	0.310	66.21	66.21	66.21			
10	1	1	1	70	77	90	0.035	0.025	0.023	0.329	0.298	0.311	9.34	12.10	13.79			
100	1	1	1	52	53	57	0.230	0.205	0.212	0.411	0.375	0.389	1.79	1.83	1.84			
250	2	2	2	63	59	63	0.307	0.268	0.245	0.549	0.470	0.453	1.79	1.75	1.85			
500	2	3	3	58	63	63	0.374	0.281	0.244	0.878	0.642	0.597	2.35	2.29	2.45			
1000	4	5	5	62	63	67	0.322	0.247	0.206	0.986	0.768	0.727	3.06	3.10	3.52			
High-end receivers, Quadruple Frequency							$\sigma_{\Phi_{E1}}$=0.003m , $\sigma_{P_{E1}}$=0.25m $\sigma_{\Phi_{E5a,E5b,E6}}$=0.002m , $\sigma_{P_{E5a,E5b,E6}}$=0.15m											
1	1	1	1	100	100	100	0.004	0.004	0.004	0.275	0.249	0.260	62.14	62.14	62.14			
10	1	1	1	78	83	93	0.035	0.024	0.022	0.276	0.250	0.261	7.98	10.29	11.72			
100	1	1	1	67	68	70	0.219	0.194	0.206	0.375	0.342	0.355	1.72	1.76	1.72			
250	2	2	2	72	72	73	0.303	0.262	0.242	0.540	0.463	0.446	1.78	1.77	1.85			
500	2	3	3	72	72	73	0.366	0.275	0.239	0.878	0.642	0.597	2.40	2.34	2.50			
1000	4	4	5	75	72	75	0.273	0.270	0.202	0.979	0.853	0.722	3.58	3.16	3.58			

Table 6.33: continued from Table 6.30... **Galileo only**, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario

Kms for E-W oriented baseline and between 8 and 93 epochs (**GPS only** took between 29 and 91 epochs) for N-S oriented baseline. The best performance could be seen at $-60°$ latitude for both E-W and N-S baseline.

Dual frequency:

(5) For baseline lengths between 10 and 1000 Km, it took between 1 and 23 epochs (**GPS only** took between 2 and 36 epochs) considering all latitude locations for E-W oriented baseline. For N-S baseline, it took between 1 and 27 epochs (**GPS only** took between 2 and 41 epochs). Galileo only dual frequency performed well for $-30°$ and $-60°$ latitude.

Triple frequency:

(6) For triple frequency, it took between 2 and 4 epochs (**GPS only** took between 3 and 10 epochs) for baseline lengths between 250 and 1000 Km considering different latitude locations for E-W baseline. For N-S baseline, it took between 2 and 5 epochs (**GPS only** took between 3 and 10 epochs) for a similar scenario. Galileo only triple frequency performed well for $-60°$ latitude.

Quadruple frequency:

(7) The quadruple frequency Galileo system performed similar to the triple frequency Galileo taking same number of epochs to meet the predefined criteria.

Comparison with GPS only:

(8) Galileo system performs better than GPS only for obtaining a fixed-precision of σ_n, σ_e =20cm for rover-receiver horizontal coordinate components, see above comparisons for dual and triple frequency Galileo and GPS.

6.6.8 GPS + Galileo - Full and Partial Ambiguity Resolution, ionosphere Weighted scenario

A combined GPS and Galileo system is considered for simulation of ASR and fixed-precision of the rover-receiver coordinates. The frequency combinations of single $L1(E1)$, dual $L1(E1), L5(E5a)$ and quadruple $L1(E1), L5(E5a), L2, E5b$ frequency are considered for simulation using a geometry based model, coordinates of rover-receiver being unknown, ionosphere weighted scenario. Ionosphere is weighted as a function of baseline length with SD standard deviation corresponding to $\sigma_I = 0.68$mm per km. The results for the simulation are presented in Table 6.34 and Figures 6.27 and 6.28 given below, and Figures G.61 to G.66 in Appendix G for full and partial ambiguity resolution.

The analysis from Table 6.34 and Figures 6.27 to 6.30 is presented below.

General remarks:

(1) Instantaneous 0.999 ASR could be achieved for 1 Km baseline lengths for dual and quadruple frequency GPS+Galileo, at all latitude locations.

(2) It can be noted that for long baseline when ionosphere weighted is equivalent to ionosphere float case, 0.999 ASR could be achieved quicker depending on the number of common satellites and number of low elevation satellites available. The number of low elevation satellites fall resulting in achieving a quicker 0.999 ASR as compared to ionosphere weighted scenario (250 Km baseline and less).

Single frequency:

(3) For baseline lengths between 1 to 500 Km, it took between 2 to >3600 epochs (**GPS only** took 21 and >3600 epochs and **Galileo only** took 9 to 3467 epochs) to obtain 0.999 ASR, full AR for E-W baseline length. For N-S baseline length, it took between 2 to >3600 (**GPS only** took 21 and >3600 epochs and **Galileo only** took 9 and >3600 epochs) epochs for similar scenarios.

(4) At 250 Km baseline length, it took 2047, 1086 and 1794 epochs (**GPS only** took 2272, 749 and 1968 epochs and **Galileo only** took 2040, 1270

	Full AR - minimum desired ASR = 0.999											
	Epochs taken			$\sigma_{\widehat{neu}}$(meters)			$\sigma_{\widecheck{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widecheck{neu}}$		
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline												
	Low-end receivers, Single Frequency			$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.5m								
1	3	2	2	0.004	0.004	0.004	0.650	0.718	0.713	164.45	164.46	164.49
10	7	6	5	0.005	0.005	0.005	0.425	0.414	0.451	88.13	88.05	88.14
100	267	295	327	0.007	0.006	0.005	0.071	0.056	0.050	10.30	9.99	9.96
250	2047	1086	1794	0.006	0.007	0.005	0.020	0.030	0.020	3.62	4.04	3.68
500	> 3600	2039	2921	-	0.009	0.007	-	0.024	0.018	-	2.68	2.47
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
	High-end receivers, Dual Frequency			$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m								
1	1	1	1	0.004	0.004	0.004	0.290	0.261	0.260	70.62	70.62	70.62
10	5	4	4	0.005	0.005	0.005	0.130	0.131	0.130	24.02	24.02	24.02
100	19	20	21	0.004	0.003	0.003	0.081	0.068	0.066	20.81	20.80	20.80
250	31	36	68	0.003	0.002	0.002	0.094	0.075	0.052	30.78	30.59	30.23
500	36	46	77	0.003	0.002	0.002	0.135	0.101	0.073	47.25	46.10	45.12
1000	34	55	78	0.003	0.002	0.002	0.205	0.126	0.098	65.14	61.42	60.69
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	44	51	84	0.003	0.002	0.002	0.204	0.155	0.110	79.05	75.15	71.20
	High-end receivers, Quadruple Frequency			$\sigma_{\Phi_{L1(E1),L2,E5b}}$=0.003m , $\sigma_{P_{L1(E1),L2,E5b}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m								
1	1	1	1	0.003	0.003	0.003	0.220	0.198	0.197	66.73	66.35	66.67
10	3	4	4	0.005	0.005	0.004	0.127	0.100	0.099	27.04	21.82	23.40
100	9	18	18	0.003	0.003	0.002	0.094	0.059	0.057	29.83	21.40	23.13
250	9	18	19	0.003	0.003	0.002	0.139	0.093	0.087	43.71	33.87	35.76
500	9	18	19	0.003	0.003	0.002	0.193	0.141	0.127	59.78	50.73	52.17
1000	8	12	18	0.004	0.004	0.002	0.267	0.058	0.169	71.16	16.52	67.75
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	13	13	17	0.003	0.003	0.002	0.234	0.192	0.187	71.14	69.72	76.78
North-South Baseline												
	Low-end receivers, Single Frequency			$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.5m								
1	3	2	2	0.004	0.004	0.004	0.650	0.718	0.713	164.45	164.46	164.49
10	7	6	5	0.005	0.005	0.005	0.425	0.414	0.451	88.13	88.05	88.14
100	315	258	330	0.006	0.006	0.005	0.065	0.061	0.051	10.19	9.74	9.40
250	2397	963	1864	0.005	0.008	0.005	0.018	0.033	0.020	3.40	4.14	3.69
500	> 3600	1776	2973	-	0.010	0.007	-	0.027	0.018	-	2.76	2.47
1000	> 3600	> 3600	> 3600	-	-	-	-	-	-	-	-	-
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
	High-end receivers, Dual Frequency			$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m								
1	1	1	1	0.004	0.004	0.004	0.290	0.261	0.260	70.62	70.62	70.62
10	5	4	4	0.005	0.005	0.005	0.130	0.131	0.130	24.02	24.02	24.02
100	22	21	18	0.004	0.003	0.004	0.075	0.066	0.077	20.81	20.80	20.79
250	21	29	30	0.004	0.003	0.003	0.123	0.088	0.087	30.82	30.65	30.58
500	31	33	25	0.003	0.003	0.003	0.156	0.129	0.161	47.30	46.77	47.27
1000	35	31	24	0.003	0.003	0.004	0.214	0.193	0.249	64.98	64.78	65.34
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	32	41	85	0.003	0.002	0.002	0.260	0.185	0.109	80.17	76.35	68.65
	High-end receivers, Quadruple Frequency			$\sigma_{\Phi_{L1(E1),L2,E5b}}$=0.003m , $\sigma_{P_{L1(E1),L2,E5b}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m								
1	1	1	1	0.003	0.003	0.003	0.220	0.198	0.197	66.73	66.35	66.67
10	3	4	4	0.005	0.005	0.004	0.127	0.100	0.099	27.05	21.82	23.40
100	10	18	15	0.003	0.003	0.003	0.089	0.059	0.068	29.74	21.37	23.06
250	8	16	17	0.004	0.003	0.003	0.163	0.105	0.099	42.40	33.73	35.56
500	9	14	11	0.004	0.003	0.004	0.215	0.174	0.192	58.17	50.67	52.09
1000	10	11	12	0.005	0.004	0.004	0.322	0.278	0.264	68.60	67.48	67.71
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-
I-Float(250Km)	13	12	14	0.004	0.003	0.003	0.259	0.213	0.218	70.96	69.72	76.85

Table 6.34: **GPS + Galileo, Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **Full AR** analysis, ASR and fixed-precision of rover-receiver coordinates (meters) are presented - measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, σ_I = 0.68 mm per km.

and 1708 epochs) for E-W baseline and 2397, 963 and 1864 epochs (**GPS only** took 1347, 779 and 2091 epochs and **Galileo only** took 2440, 1202 and 1607 epochs) for N-S baseline length at $0°, -30°$ and $-60°$ degree latitude respectively.

Dual frequency:

(5) With dual frequency, it took between 4 and 78 epochs (**GPS only** took 2 to 213 epochs and **Galileo only** took 2 to 124 epochs) for baseline lengths between 10 and 1000 Km for E-W baseline. For N-S baseline, it took between 4 to 35 epochs (**GPS only** took 4 to 232 epochs and **Galileo only** took 2 to 213 epochs) for similar scenario.

(6) For 250 Km baseline length, it took 31, 36 and 68 epochs (**GPS only** took 29, 36 and 63 epochs and **Galileo only** took 57, 54 and 30 epochs) at $0°, -30°$ and $-60°$ degree latitude and for N-S baseline it took 21, 29 and 30 epochs (**GPS only** took 108, 66 and 135 epochs and **Galileo only** took 70, 67 and 31 epochs) for similar latitude locations.

Quadruple frequency:

(7) Quadruple frequency combined GPS and Galileo took between 3 and 19 epochs (**Galileo only** took 2 to 56 epochs) for all latitude locations for baseline length of 10 to 1000 Kms for E-W baseline. For N-S baseline, it took between 3 and 17 epochs (**Galileo only** took 2 to 83 epochs) for a similar scenario.

(8) For 250 Km baseline length, it took 9, 18 and 19 epochs (**Galileo only** took 37, 46 and 22 epochs) $0°, -30°$ and $-60°$ degree latitude for E-W baseline. For N-S baseline, it took 8, 16 and 17 epochs (**Galileo only** took 49, 58 and 25 epochs) for a similar scenario.

Comparison with GPS only and Galileo only:

(9) A combined system performs much better than the individual systems (GPS only and Galileo only) for all scenarios of frequency combinations, baseline lengths and latitude locations.

6.6.9 GPS + Galileo , Partial Ambiguity Resolution to obtain fixed-precision of 2 cm, ionosphere Weighted scenario

The combined GPS and Galileo system frequencies are used for simulation of PAR, in order to evaluate PAR in terms of number of epochs taken for obtaining a fixed-precision of rover-receiver coordinates of 2cm while considering a ionosphere weighted scenario. The results for the fixed-precision of the rover-receiver coordinates, number of epochs taken and percentage of ambiguities fixed is presented in Table 6.35.

General remarks:

(1) Instantaneous fixed-precision of 2cm could be achieved for baseline lengths of 1 and 10 Km for dual and quadruple frequency for all latitude locations.

(2) For almost all the scenarios of baseline lengths and frequency combinations, PAR fixed only a partial subset of ambiguities (less than 100%) to obtain a fixed-precision of 2cm (except one scenario of quadruple frequency, 1000 Km N-S baseline).

(3) The results for ionosphere weighted scenarios in order to obtain a fixed-precision of 2cm for the rover-receiver coordinates were much better than for ionosphere float scenario. See Table 6.30, comparison of 250 Km baseline length of ionosphere weighted with ionosphere float (shaded in pink colour).

Single frequency:

(4) For single frequency, it took a minimum of 2 and a maximum of >3600 epochs (**GPS only** took between 21 and >3600 epochs and **Galileo only** took 8 to >3600 epochs) for baseline lengths between 1 and 1000 Kms for both E-W and N-S oriented baseline took between 2 and >3600 (**GPS only** took between 21 and >3600 epochs and **Galileo only** took epochs). The best performance could be seen at $-60°$ latitude for both E-W and N-S baseline.

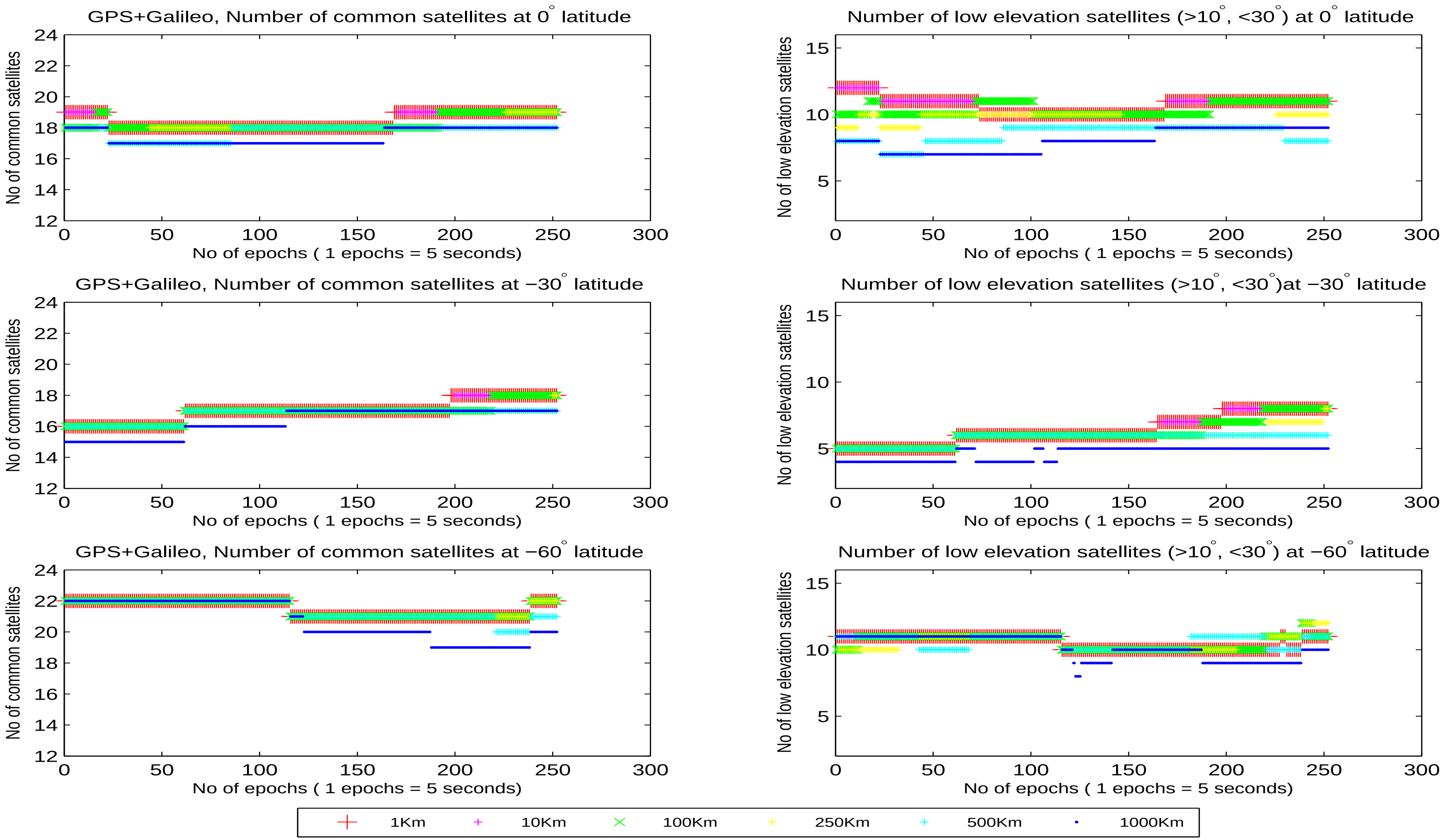

Figure 6.27: **GPS + Galileo**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, **East-West** oriented baseline.

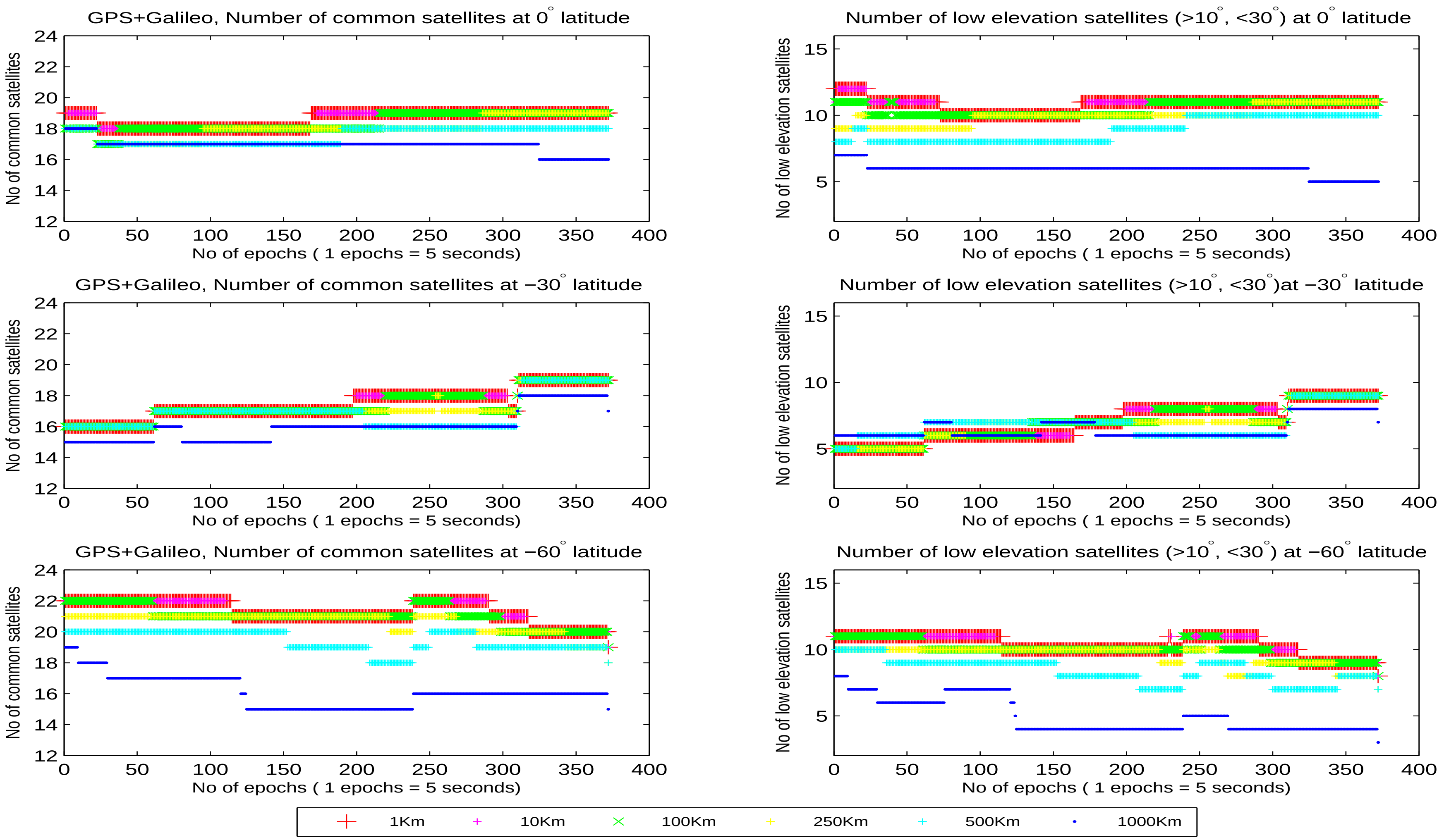

Figure 6.28: **GPS + Galileo**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, **North-South** oriented baseline

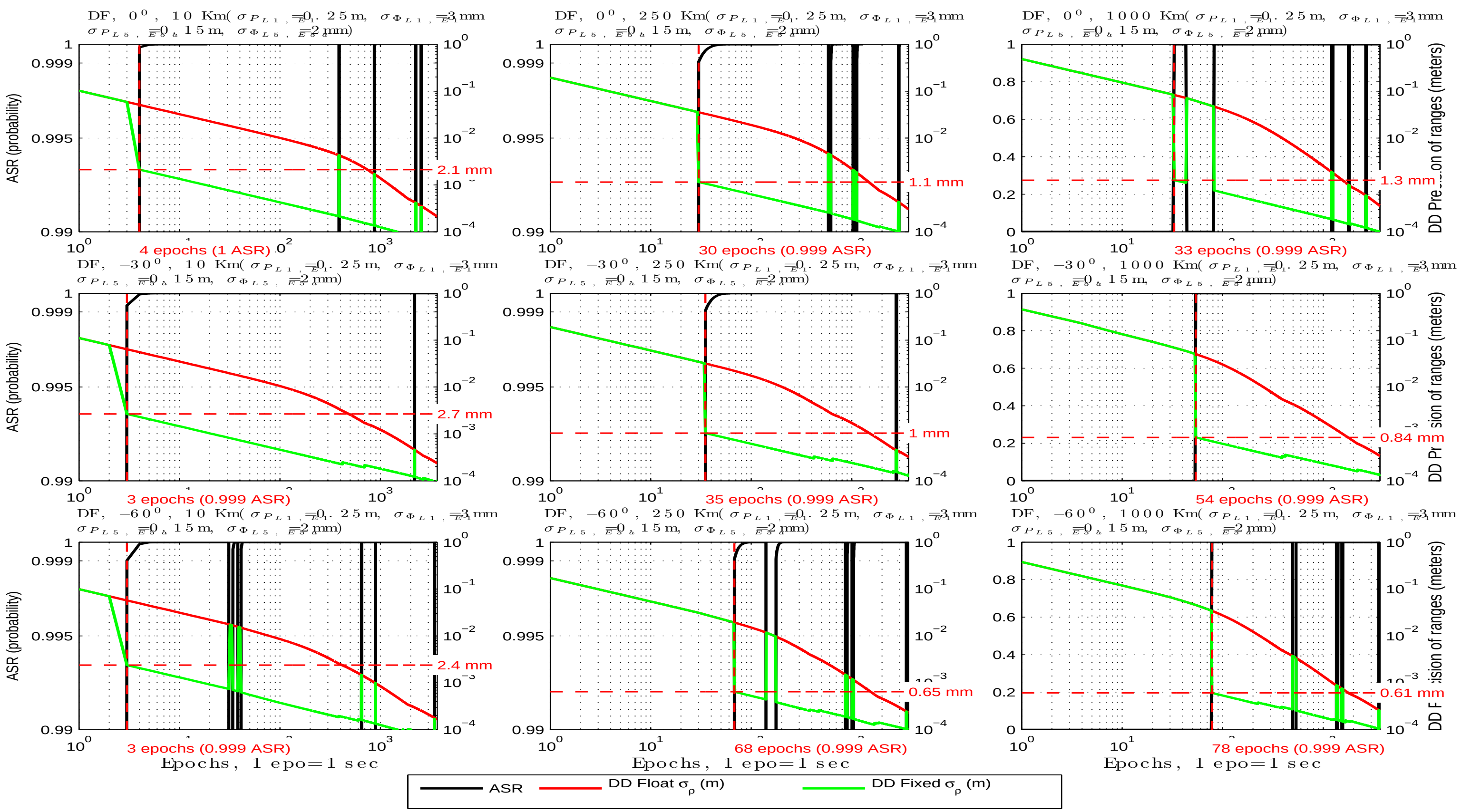

Figure 6.29: **GPS+Galileo**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

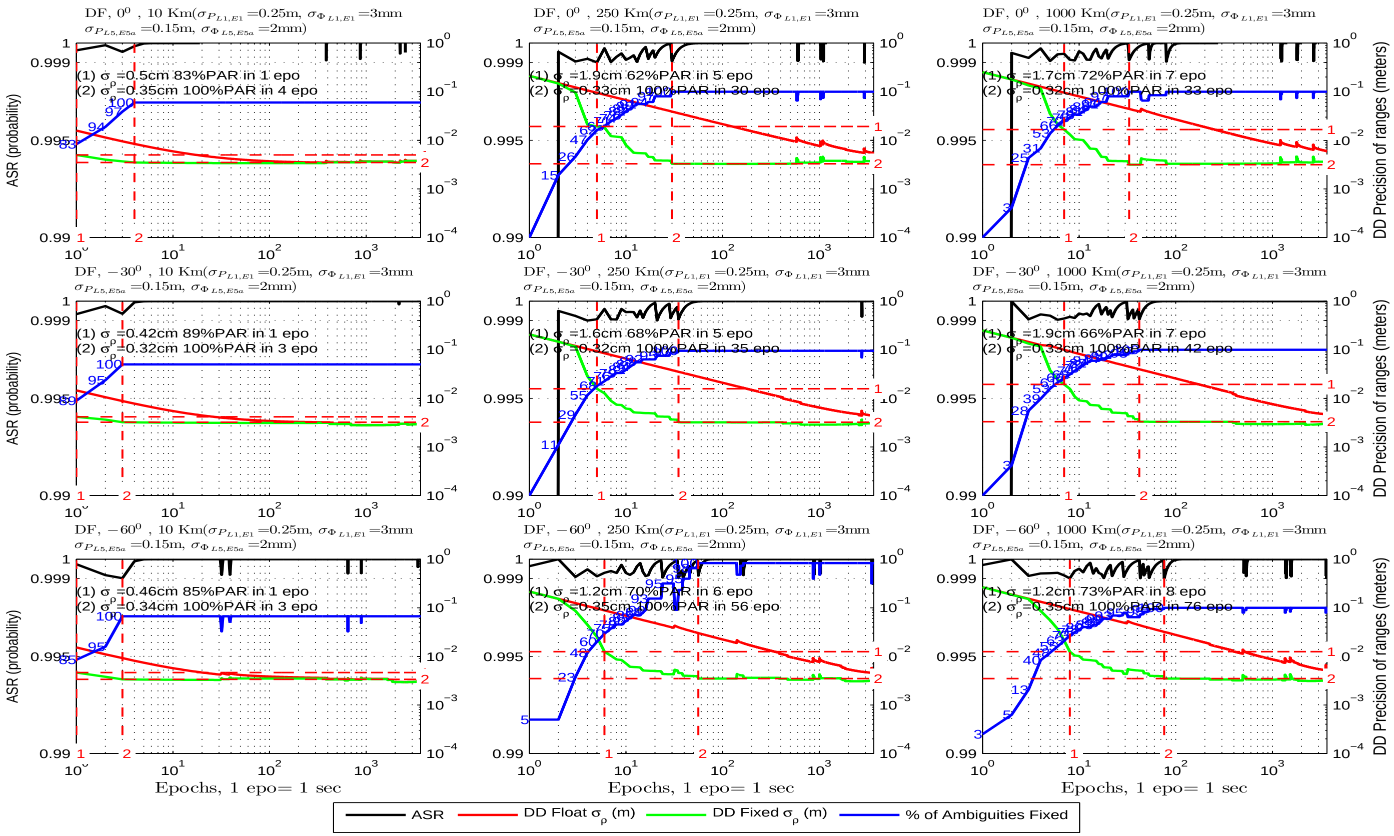

Figure 6.30: **GPS+Galileo**, Dual frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD (Single Difference) precision σ_I = 0.68 mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed-precision and for fixing 100% of ambiguities by PAR, along with the fixed-precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

Dual frequency:

(5) For baseline lengths between 10 and 1000 Km, it took between 4 and 78 epochs (**GPS only** took between 5 and 209 epochs and **Galileo only** took 2 and 115 epochs) considering all latitude locations for E-W oriented baseline. For N-S baseline, it took between 4 and 35 epochs (**GPS only** took between 5 and 220 epochs and **Galileo only** took 2 to 198 epochs). Galileo only dual frequency performed well for $-30°$ and $-60°$ latitude.

Quadruple frequency:

(6) With quadruple frequency Galileo, it took between 4 and 19 epochs (**Galileo only** took 2 to 49 epochs) for E-W oriented baseline and between 3 and 18 epochs (**Galileo only** took 2 to 69 epochs) for N-S oriented baseline.

Comparison with GPS only and Galileo only:

(7) A combined system of GPS and Galileo outperformed standalone GPS and Galileo, in both dual and quadruple frequency combinations when a predefined fixed-precision was aimed for.

The results and discussion for number of epochs taken and ambiguities fixed by PAR in order to obtain a fixed-precision of 20cm for the horizontal coordinate components in presented below.

	Partial AR @ 0.999 ASR criteria - 2 cm fixed-precision for rover-receiver coordinates														
Baseline length (Km)	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$		
	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline															
Low-end receivers, Single Frequency						$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.5m									
1	3	2	2	100	100	100	0.004	0.004	0.004	0.650	0.718	0.713	164.45	164.46	164.49
10	7	5	5	100	89	100	0.005	0.006	0.005	0.425	0.454	0.451	88.13	78.09	88.14
100	147	96	94	81	61	57	0.011	0.017	0.016	0.097	0.104	0.100	8.66	6.22	6.20
250	501	328	354	63	44	57	0.015	0.019	0.016	0.054	0.057	0.051	3.56	3.03	3.20
500	1070	815	1027	35	44	53	0.020	0.019	0.017	0.039	0.040	0.033	1.89	2.05	1.87
1000	2114	1961	1995	29	38	22	0.019	0.020	0.019	0.029	0.031	0.028	1.51	1.52	1.46
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
High-end receivers, Dual Frequency						$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m									
1	1	1	1	100	100	100	0.004	0.004	0.004	0.290	0.261	0.260	70.62	70.62	70.62
10	1	1	1	76	83	76	0.016	0.014	0.015	0.290	0.262	0.260	18.20	19.36	16.91
100	8	7	7	72	83	74	0.017	0.007	0.008	0.125	0.115	0.114	7.31	16.84	13.67
250	15	11	11	84	78	68	0.009	0.005	0.007	0.136	0.137	0.135	15.13	25.04	20.13
500	21	16	15	91	81	76	0.004	0.005	0.004	0.178	0.176	0.179	47.42	39.01	40.64
1000	25	19	17	97	78	76	0.004	0.004	0.004	0.242	0.231	0.235	65.74	55.61	57.33
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	26	22	19	91	86	79	0.003	0.004	0.004	0.273	0.253	0.272	81.13	71.94	69.25
High-end receivers, Quadruple Frequency						$\sigma_{\Phi_{L1(E1),L2,E5b}}$=0.003m , $\sigma_{P_{L1(E1),L2,E5b}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m									
1	1	1	1	100	100	100	0.003	0.003	0.003	0.220	0.198	0.197	66.73	66.35	66.67
10	1	1	1	90	93	86	0.008	0.011	0.011	0.221	0.199	0.198	26.48	18.41	17.42
100	5	4	5	96	81	82	0.004	0.017	0.008	0.126	0.124	0.109	29.32	7.34	13.53
250	7	7	7	98	94	86	0.004	0.006	0.006	0.158	0.150	0.143	43.49	27.21	23.19
500	8	8	8	98	94	86	0.003	0.005	0.006	0.204	0.213	0.196	59.47	41.25	33.93
1000	8	5	8	100	93	86	0.004	0.007	0.006	0.267	0.099	0.256	71.16	14.98	44.60
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	5	5	4	79	85	86	0.007	0.005	0.005	0.379	0.312	0.390	56.99	60.19	70.96
North-South Baseline															
Low-end receivers, Single Frequency						$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.5m									
1	3	2	2	100	100	100	0.004	0.004	0.004	0.650	0.718	0.713	164.45	164.46	164.49
10	7	5	5	100	89	100	0.005	0.006	0.005	0.425	0.454	0.451	88.13	78.05	88.14
100	148	98	96	81	72	63	0.011	0.015	0.016	0.097	0.103	0.103	8.59	6.93	6.26
250	497	338	368	56	53	67	0.016	0.019	0.017	0.056	0.059	0.054	3.48	3.20	3.17
500	1033	870	979	35	44	42	0.020	0.020	0.019	0.041	0.040	0.036	2.00	2.06	1.92
1000	2168	2105	2185	33	44	33	0.020	0.016	0.019	0.033	0.031	0.028	1.66	1.93	1.52
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
High-end receivers, Dual Frequency						$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m									
1	1	1	1	100	100	100	0.004	0.004	0.004	0.290	0.261	0.260	70.62	70.62	70.62
10	1	1	1	76	83	76	0.016	0.014	0.015	0.290	0.262	0.260	18.19	19.36	16.89
100	8	7	7	69	83	79	0.018	0.007	0.010	0.125	0.115	0.123	6.80	16.87	12.90
250	16	12	12	90	85	85	0.009	0.005	0.005	0.141	0.138	0.139	16.10	26.57	26.08
500	24	17	15	93	88	84	0.004	0.004	0.012	0.178	0.183	0.209	47.34	44.39	17.73
1000	31	20	20	93	91	97	0.004	0.004	0.004	0.228	0.244	0.275	65.22	65.70	65.52
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	28	22	19	93	91	88	0.003	0.004	0.004	0.280	0.267	0.290	80.49	74.62	73.66
High-end receivers, Quadruple Frequency						$\sigma_{\Phi_{L1(E1),L2,E5b}}$=0.003m , $\sigma_{P_{L1(E1),L2,E5b}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m									
1	1	1	1	100	100	100	0.003	0.003	0.003	0.220	0.198	0.197	66.73	66.35	66.67
10	1	1	1	90	93	86	0.008	0.011	0.011	0.221	0.199	0.198	26.49	18.41	17.41
100	5	4	5	94	81	88	0.004	0.017	0.008	0.126	0.124	0.118	29.14	7.39	14.33
250	7	7	6	98	96	82	0.004	0.006	0.019	0.174	0.159	0.167	40.87	28.61	8.68
500	8	8	7	98	96	85	0.004	0.005	0.019	0.228	0.230	0.242	56.23	44.18	12.96
1000	10	9	8	100	98	91	0.005	0.005	0.018	0.322	0.308	0.324	68.60	64.11	17.96
I-Fix(1Km)	1	1	1	-	-	-	-	-	-	-	-	-	-	-	-
I-Float(250Km)	6	5	4	93	90	92	0.006	0.006	0.006	0.382	0.332	0.412	69.21	58.90	74.75

Table 6.35: **GPS+Galileo**, **PAR**, Geometry Fixed, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, Partial ambiguities fixed (percentage) and number of epochs for obtaining 2cm fixed-precision of rover-receiver coordinates are presented, measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, σ_I = 0.68 mm per Km.

6.6.10 GPS + Galileo , Partial Ambiguity Resolution to obtain fixed-precision of 20 cm, ionosphere Weighted scenario

The combined GPS and Galileo system frequencies are used for simulation of PAR, in order to evaluate PAR in terms of number of epochs taken for obtaining a fixed-precision of rover-receiver coordinates of 20cm while considering a ionosphere weighted scenario. The results for the fixed-precision of the rover-receiver coordinates, number of epochs taken and percentage of ambiguities fixed is presented in Table 6.36.

General remarks:

(1) Instantaneous fixed-precision of 20cm could be achieved for baseline lengths of 1 Km by fixing 100% of ambiguities for dual and quadruple frequency for all latitude locations.

(2) Instantaneous fixed-precision was also achieved for baseline length of 10 Km for dual fervency and 10 and 100 Km for quadruple frequency by fixing less than 100% of ambiguities.

(3) For all the remaining scenarios of baseline lengths and frequency combinations, PAR fixed only a partial subset of ambiguities (less than 100%) to obtain a fixed-precision of 20cm.

Single frequency:

(4) For single frequency, it took between 2 and 42 epochs (**GPS only** took between 21 and 79 epochs and **Galileo only** took 8 to 71 epochs) for baseline lengths between 1 and 1000 Kms for both E-W and N-S oriented baseline took between 2 and >3600 (**GPS only** took between 29 and 91 epochs and **Galileo only** took between 8 and 93 epochs). The best performance could be seen at $-60°$ latitude for both E-W and N-S baseline.

Dual frequency:

(5) For baseline lengths between 10 and 1000 Km, it took between 1 and 12 epochs (**GPS only** took between 2 and 36 epochs and **Galileo only** took 1 and 23 epochs) considering all latitude locations for E-W oriented baseline. For N-S baseline, it took between 1 and 12 epochs (**GPS only** took between 2 and 41 epochs and **Galileo only** took 1 to 27 epochs). Galileo only dual frequency performed well for $-30°$ and $-60°$ latitude.

Quadruple frequency:

(6) With quadruple frequency Galileo for baseline length between 250 and 1000Km, it took between 1 and 3 epochs (**Galileo only** took 2 to 4 epochs) for E-W oriented baseline and between 1 and 3 epochs (**Galileo only** took 2 to 5 epochs) for N-S oriented baseline.

Comparison with GPS only and Galileo only:

(7) A combined system of GPS and Galileo outperformed standalone GPS and Galileo, in both dual and quadruple frequency combinations when a predefined fixed-precision was aimed for.

	Partial AR @ 0.999 ASR criteria - 20 cm fixed-precision for rover-receiver coordinates														
Baseline length	Epochs taken			Ambiguities fixed(%)			$\sigma_{\widetilde{neu}}$(meters)			$\sigma_{\widehat{neu}}$(meters)			Gain=$\sigma_{\widehat{neu}}/\sigma_{\widetilde{neu}}$		
(Km)	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°	0°	−30°	−60°
East-West Baseline															
Low-end receivers, Single Frequency	$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.5m														
1	3	2	2	100	100	100	0.004	0.004	0.004	0.650	0.718	0.713	164.45	164.46	164.49
10	7	5	5	100	89	100	0.005	0.006	0.005	0.425	0.454	0.451	88.13	78.09	88.14
100	12	10	19	0	0	0	0.343	0.323	0.232	0.343	0.323	0.232	1.00	1.00	1.00
250	13	10	20	0	0	0	0.339	0.332	0.232	0.339	0.332	0.232	1.00	1.00	1.00
500	17	12	23	0	0	0	0.321	0.330	0.234	0.321	0.330	0.234	1.00	1.00	1.00
1000	42	19	34	0	0	0	0.273	0.337	0.240	0.273	0.337	0.240	1.00	1.00	1.00
High-end receivers, Dual Frequency	$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m														
1	1	1	1	100	100	100	0.004	0.004	0.004	0.290	0.261	0.260	70.62	70.62	70.62
10	1	1	1	76	83	76	0.016	0.014	0.015	0.290	0.262	0.260	18.20	19.36	16.91
100	2	1	2	3	0	5	0.239	0.304	0.189	0.250	0.304	0.213	1.04	1.00	1.12
250	3	2	4	0	0	5	0.304	0.321	0.215	0.304	0.321	0.224	1.00	1.00	1.04
500	6	5	8	9	6	18	0.307	0.299	0.211	0.334	0.316	0.245	1.09	1.06	1.16
1000	12	8	11	30	19	32	0.273	0.306	0.228	0.353	0.364	0.293	1.29	1.19	1.29
High-end receivers, Quadruple Frequency	$\sigma_{\Phi_{L1(E1),L2,E5b}}$=0.003m , $\sigma_{P_{L1(E1),L2,E5b}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m														
1	1	1	1	100	100	100	0.003	0.003	0.003	0.220	0.198	0.197	66.73	66.35	66.67
10	1	1	1	90	93	86	0.008	0.011	0.011	0.221	0.199	0.198	26.48	18.41	17.42
100	1	1	1	40	50	51	0.172	0.141	0.137	0.282	0.248	0.244	1.64	1.76	1.78
250	2	1	2	42	44	61	0.201	0.270	0.157	0.295	0.397	0.268	1.47	1.47	1.70
500	2	2	2	42	56	60	0.245	0.216	0.196	0.409	0.426	0.392	1.67	1.97	2.01
1000	3	2	3	49	53	61	0.215	0.270	0.167	0.436	0.612	0.419	2.03	2.27	2.51
North-South Baseline															
Low-end receivers, Single Frequency	$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.5m														
1	3	2	2	100	100	100	0.004	0.004	0.004	0.650	0.718	0.713	164.45	164.46	164.49
10	7	5	5	100	89	100	0.005	0.006	0.005	0.425	0.454	0.451	88.13	78.05	88.14
100	12	10	19	0	0	0	0.342	0.324	0.252	0.342	0.324	0.252	1.00	1.00	1.00
250	12	11	20	0	0	0	0.378	0.335	0.250	0.378	0.335	0.250	1.00	1.00	1.00
500	14	13	24	0	0	0	0.379	0.340	0.268	0.379	0.340	0.268	1.00	1.00	1.00
1000	23	22	40	0	0	0	0.391	0.341	0.270	0.391	0.341	0.270	1.00	1.00	1.00
High-end receivers, Dual Frequency	$\sigma_{\Phi_{L1(E1)}}$=0.003m , $\sigma_{P_{L1(E1)}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m														
1	1	1	1	100	100	100	0.004	0.004	0.004	0.290	0.261	0.260	70.62	70.62	70.62
10	1	1	1	76	83	76	0.016	0.014	0.015	0.290	0.262	0.260	18.19	19.36	16.89
100	1	1	2	0	0	6	0.352	0.304	0.204	0.352	0.304	0.231	1.00	1.00	1.13
250	3	2	4	0	0	6	0.326	0.339	0.232	0.326	0.339	0.242	1.00	1.00	1.04
500	6	5	8	10	6	22	0.327	0.320	0.246	0.358	0.339	0.287	1.10	1.06	1.17
1000	8	8	12	21	18	40	0.376	0.326	0.281	0.458	0.390	0.358	1.22	1.20	1.28
High-end receivers, Quadruple Frequency	$\sigma_{\Phi_{L1(E1),L2,E5b}}$=0.003m , $\sigma_{P_{L1(E1),L2,E5b}}$=0.25m, $\sigma_{\Phi_{L5(E5a)}}$=0.002m , $\sigma_{P_{L5(E5a)}}$=0.15m														
1	1	1	1	100	100	100	0.003	0.003	0.003	0.220	0.198	0.197	66.73	66.35	66.67
10	1	1	1	90	93	86	0.008	0.011	0.011	0.221	0.199	0.198	26.49	18.41	17.41
100	1	1	1	40	50	53	0.172	0.142	0.148	0.281	0.249	0.264	1.64	1.76	1.78
250	2	1	2	44	47	63	0.220	0.283	0.171	0.326	0.420	0.289	1.48	1.48	1.69
500	2	2	2	44	57	60	0.268	0.231	0.236	0.456	0.461	0.452	1.70	2.00	1.92
1000	2	2	3	43	55	64	0.331	0.258	0.216	0.723	0.655	0.531	2.18	2.54	2.46

Table 6.36: **GPS+Galileo**, **PAR**, Geometry Fixed, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, Partial ambiguities fixed (percentage) and number of epochs for obtaining 20cm fixed-precision of rover-receiver coordinates are presented, measurement precision is held fix, number of satellites are as available, **baseline length varied from 1 to 1000 Km**, σ_I = 0.68 mm per km.

7
Conclusions

With the availability of multi-frequency, multi-GNSS data in the near future from modernized Global Positioning System (GPS), Galileo system, and other global navigation systems, an efficient and quicker solution towards positioning or other applications is highly expected. The positioning solution or any parameter of interest to be estimated from GNSS can be evaluated in terms of its precision obtained. While precise GNSS applications are considered, the use of the precise carrier phase data is important. The precision of the carrier phase data could be achieved only after correctly fixing the integer ambiguities present in the phase data. The evaluation of the benefits of future GPS and Galileo are based on two main evaluation criteria, firstly the evaluation of correctly fixing the integer ambiguity and secondly evaluation of the precision of the parameters of interest with the aid of Ambiguity Resolution (AR). The correct fixing of ambiguities is given by Ambiguity Success Rate (ASR) which is based on the probability of Integer Bootstrap (IB) method along with LAMBDA decorrelation of the ambiguity Variance-Covariance (VC) matrix. The probability of Integer Bootstrap (IB) equipped with LAMBDA decorrelation corresponds to a sharp lower bound of probability of Integer Least Squares. The minimum criteria laid for correct resolution of ambiguities is to obtain 0.999 probability from IB (corresponding to 99.9%). While all the ambiguities together contributed to give 0.999 ASR, it was termed as full AR. In certain cases while it took time to achieve full AR, a subset of ambiguities could be fixed which satisfied the criteria of 0.999 ASR and is termed as Partial AR (PAR). The success of PAR was judged while the subset of ambiguities fixed by PAR contributed to achieve an predefined fixed precision of parameters of interest (for example, up to 2cm standard deviation for ionosphere). This research work is based on simulations using model assumptions.

The objective of this research work is to evaluate the expected performance of ambiguity resolution for the Next-Generation Global Navigation Satellite Systems (GNSS), namely the modernized GPS and

Galileo (either as a stand-alone system or as an integrated one) and to understand its effect on precision of the parameters of interest (e.g., receiver coordinates or ionosphere).

In this research, simulations were done for current and modernized GPS and the Galileo system, in both the standalone mode and the integrated one. For each system various frequency combinations, single, dual, triple and quadruple, were considered for simulations. It should be noted that with GPS single ($L1$), dual ($L1, L2$) and triple ($L1, L2, L5$) frequency combinations could be formed. Galileo had additional quadruple ($E1, E5a, E5b, E6$) frequency combination. With combined GPS and Galileo system, single $L1(E1)$, dual $L1(E1), L5(E5a)$ and quadruple $L1(E1), L5(E5a), L2, E5b$ frequency combinations could be formed. The quadruple frequency combination had two overlapping $L1(E1), L5(E5a)$ and two independent $L2, E5b$ frequencies. The design matrix for the overlapping frequencies formed considered a common GPS reference satellite and for the individual frequencies had a reference satellite from their individual systems. Some of the important deductions obtained based on this multi-frequency future GPS and Galileo simulation study are presented as follows.

The simulations were carried out for three major GNSS models, the Geometry Free, Geometry Fixed and Reference Rover model. The discussion of the results for Geometry Free model is as follows

7.1 Results and Discussions

7.1.1 Geometry Free model

The summary for the simulation of full and partial Ambiguity Success Rates and precision of the parameters of interest resulting from fixing of ambiguities is presented in Table 7.1.

In Table 7.1, the results are representative of a certain time period, that is 0000 to 2359 UTC and not the averaged values. This representation is to help the reader to compare full AR and PAR results. The averaged results for Geometry Based model with rover coordinates unknown can be found in Tables 4.5, 4.8 and 4.15.

It should be noted that for Geometry Free model, the troposphere could not

					Full AR - minimum desired ASR = 0.999		Partial AR @ 0.999, criteria - 2cm fixed-precision	
	Phase Φ L1/L2	Code P L1/L2	Phase Φ L5	Code P L5	Epochs taken	$\sigma_{\hat{\rho}}$(meters)	Epochs taken	Ambiguities fixed(%)
Ionosphere known scenario (1 Km baseline)								
SF	0.003	0.50	-	-	>17280	-	2470	80
DF	0.003	0.25	-	-	14	0.002	14	100
TF	0.003	0.25	0.002	0.15	6	0.002	6	100
Ionosphere unknown scenario (250 Km baseline)								
DF	0.003	0.25	-	-	>43200	-	1899	50
TF	0.003	0.25	0.002	0.15	18996	0.037	1899	50
Ionosphere weighted scenario (250 Km baseline)								
SF	0.003	0.50	-	-	>17280	-	2470	80
DF	0.003	0.25	-	-	8431	0.001	561	50
TF	0.003	0.25	0.002	0.15	8011	0.001	386	67
SF=Single Frequency, DF=Dual Frequency, TF=Triple Frequency								

Table 7.1: Summary of Full AR and PAR for GPS only, Geometry Free model, single, dual and triple frequency for different ionosphere considerations. Simulations were done for the time period 0000 to 2359 UTC at three latitude locations 0°, −30° and −60°. Above results are for −30° latitude and 115° longitude.

be parameterized as a separate parameter, instead they were lumped with the ranges. Presented below are the important conclusions for Geometry Free model.

1. The Geometry Free model does not gain strength with the addition of satellites, using GPS and Galileo in combination is not seen to improve the time taken for correctly resolving ambiguities.

2. It can further be noted that for standalone GPS or Galileo systems, the Geometry Free model being a weaker model as compared to Geometry Based model, takes immensely large number of epochs for medium to long baselines.

3. For the combined GPS and Galileo system, the Geometry Free model does not gain strength as compared to the stand alone systems, unlike Geometry Based models. This is due to the fact that the Geometry Based models have troposphere as the coupling parameter between the two GNSS systems, since it remains constant for 2 hours. Also when the receiver coordinates are estimated, the coordinates add up to another set of coupling parameters. On the other hand, Geometry Free model does not have any coupling parameters. The ambiguity resolution or positioning solution is not seen to improve from standalone GPS to standalone Galileo, and same is the case

when combined GPS and Galileo are used. Hence for Geometry Free model results obtained for standalone GPS system are only discussed.

Further, the findings from the simulation for Geometry Fixed model are discussed.

7.1.2 Geometry Fixed model

The summary of the results for Geometry Fixed model are presented in Table 7.2.

	Phase Φ L1/L2/E1	Code P	Phase Φ L5/E5a/E5b	Code P	GPS Epochs	GPS σ_I(meters)	Galileo Epochs	Galileo σ_I(meters)	GPS+Galileo Epochs	GPS+Galileo σ_I(meters)	GPS Epochs	GPS A.F.(%)	Galileo Epochs	Galileo A.F.(%)	GPS+Galileo Epochs	GPS+Galileo A.F.(%)
Atmosphere unknown scenario (250 Km baseline)																
					Full AR - minimum desired ASR = 0.999						Partial AR @ 0.999, criteria - 2cm fixed-precision for ionosphere					
DF	0.003	0.25	-	-	36	0.005	50	0.004	50	0.003	17	75	9	65	6	66
TF	0.003	0.25	0.002	0.15	11	0.003	17	0.003	-	-	14	63	3	70	-	-
QF	0.003	0.25	0.002	0.15	-	-	16	0.002	10	0.002	-	-	2	80	2	77
Ionosphere weighted scenario (250 Km baseline)																
					Full AR - minimum desired ASR = 0.999						Partial AR @ 0.999, criteria - 2cm fixed-precision for troposphere					
SF	0.003	0.25	-	-	644	0.007	948	0.005	824	0.004	251	62	306	60	153	53
DF	0.003	0.25	0.002	0.15	24	0.002	35	0.001	35	0.001	9	75	8	70	3	29
TF	0.003	0.25	0.002	0.15	11	0.003	17	0.002	-	-	5	88	6	93	-	-
QF	0.003	0.25	0.002	0.15	-	-	16	0.002	13	0.001	-	-	6	95	2	75
SF=Single Frequency, DF=Dual Frequency, TF=Triple Frequency, QF=Quadruple Frequency, A.F.=Ambiguities Fixed																

Table 7.2: Summary of Full AR and PAR for Geometry Fixed model, single, dual, triple and quadruple frequency combinations for GPS only, Galileo only, and GPS+Galileo system combinations, for atmosphere unknown (both troposphere and ionosphere) and ionosphere weighted (troposphere unknown) considerations. Simulations were done for the time period 0000 to 0100 UTC at three latitude locations $0°$, $-30°$ and $-60°$. Above results are for $-30°$ latitude and $115°$ longitude.

In Table 7.2, the results are representative of a certain time period, that is 0000 to 0100 UTC and not the averaged values. This representation is to help the reader to compare full AR and PAR results. The averaged results for Geometry Based model with rover coordinates unknown can be found in Tables 5.4 and 5.12.

While the atmosphere is unknown, the ionosphere is parameterized in the design matrix and its VC matrix for the ambiguity-float and -fixed case is computed. Hence the evaluation is based on fixed-precision of ionosphere while PAR is used. Whereas for ionosphere weighted scenario, the ionosphere is assumed to be known and a scenario while ionosphere could be eliminated from the observables is considered. Hence the ionosphere is not parameterized for ionosphere weighted model. While evaluating PAR to obtain a fixed precision of 2cm, fixed-precision of troposphere is presented.

Simulations for *Geometry Fixed model* (Geometry Based model, receiver position known), which resembles a permanent reference network were done for all three types of GNSS considered for this research work, namely, GPS standalone, Galileo standalone and GPS and Galileo integrated system. The Geometry Fixed model is also referred to as Continuously Operating Reference Stations (CORS) scenario. The results from Geometry Fixed model are presented below.

1. With atmosphere float scenario is considered, the results with a combined GPS and Galileo system are promising. The results presented in Table 7.2 for ambiguity-fixed-precision of ionosphere of 2cm with Partial Ambiguity Resolution (PAR) are discussed. While with standalone GPS only and Galileo only, when fixed-precision of 2cm for ionosphere is aimed, it takes relatively longer time in comparison to an integrated GPS and Galileo system. Considering the combined GPS and Galileo, the quadruple frequency system is able to give a 2cm ambiguity-fixed-precision of ionosphere instantaneously for most of scenarios of varying measurement precision. For the remaining scenarios of varying measurement precision it just takes 2 epochs to achieve 2cm fixed-precision for ionosphere. This is applicable to both E-W and N-S oriented baselines (250 Km baseline length).

2. In case ionosphere weighted scenario, the quadruple frequency combined GPS and Galileo system $(L1(E1), L5(E5a), L2, E5b)$ is able to give a 2cm fixed-precision of troposphere up to 1000 Km baseline length in just 2 epochs.

3. The results presented in Table 7.2 for ionosphere unknown scenario do not show improvement for a integrated GPS and Galileo system for dual frequency case. While the number of frequencies remain the same, the unknown ionosphere does not gain strength just by the addition of satellites. Also to note that the common parameter while one goes from single GNSS system to an integrated GNSS system is troposphere. The common parameters form a coupling between the systems, and it can be said that for CORS scenario there is a weak coupling while one considers and integrated GNSS system.

The simulation study shows that with an integrated GPS and Galileo system CORS network would benefit. Since corrections for ionosphere could be generated almost instantaneously with a precision of 2cm using quadruple frequency GPS and Galileo combined system.

Further simulations for *Reference Rover* GNSS model (Geometry Based model, rover receiver's position are unknown) are discussed as follows.

7.1.3 Reference Rover model

The summary of the results for Reference Rover model are presented in Table 7.3.

					Full AR - minimum desired ASR = 0.999						Partial AR @ 0.999, criteria - 2cm fixed-precision					
	Phase	Code	Phase	Code	GPS		Galileo		GPS+Galileo		GPS		Galileo		GPS+Galileo	
	Φ	P	Φ	P	Epochs	$\sigma_{\widetilde{xyz}}$(meters)	Epochs	$\sigma_{\widetilde{xyz}}$(meters)	Epochs	$\sigma_{\widetilde{xyz}}$(meters)	Epochs	A.F.(%)	Epochs	A.F.(%)	Epochs	A.F.(%)
	L1/L2/E1		L5/E5a/E5b													
Atmosphere known scenario (1 Km baseline)																
SF	0.003	0.50	-	-	7	0.003	4	1.170	2	0.003	6	88	4	100	1	100
Atmosphere unknown scenario (250 Km baseline)																
DF	0.003	0.25	-	-	197	0.002	79	0.002	51	0.002	164	94	77	95	22	86
TF	0.003	0.25	0.002	0.15	76	0.003	50	0.003	-	-	74	92	29	87	-	-
QF	0.003	0.25	0.002	0.15	-	-	47	0.003	13	0.003	-	-	28	92	5	85
Ionosphere weighted scenario (250 Km baseline)																
SF	0.003	0.25	-	-	749	0.014	1270	0.010	1086	0.007	671	71	734	89	328	44
DF	0.003	0.25	0.002	0.15	65	0.003	54	0.003	36	0.002	57	75	39	75	11	78
TF	0.003	0.25	0.002	0.15	43	0.003	49	0.003	-	-	38	83	22	87	-	-
QF	0.003	0.25	0.002	0.15	-	-	46	0.003	18	0.003	-	-	21	90	7	94
SF=Single Frequency, DF=Dual Frequency, TF=Triple Frequency, QF=Quadruple Frequency, A.F.=Ambiguities Fixed																

Table 7.3: Summary of Full AR and PAR for Geometry Based model, rover receiver coordinates unknown, single, dual, triple and quadruple frequency combinations for GPS only, Galileo only, and GPS+Galileo system combinations, for atmosphere known and unknown (both troposphere and ionosphere) and ionosphere weighted (troposphere unknown) considerations. Simulations were done for the time period 0000 to 0100 UTC at three latitude locations 0°, $-30°$ and $-60°$. Above results are for $-30°$ latitude and 115° longitude.

In Table 7.3, the results are representative of a certain time period, that is 0000 to 0100 UTC and not the averaged values. This representation is to help the reader to compare full AR and PAR results. The averaged results for Geometry Based model with rover coordinates unknown can be found in Tables 6.2, 6.13 and 6.24.

Some of the findings for Reference Rover model are presented as follows.

(1) For short baselines, when the Double Difference (DD) atmosphere is insignificant for the rover with respect to a reference receiver, the standalone single

frequency GPS and Galileo systems take relatively long time to obtain 0.999 ASR (full AR) in comparison to an integrated GPS Galileo system, see Table 7.3 atmosphere known, single frequency results. GPS only takes 5 to 24 epochs for varying values of measurement precision, see Table 6.3 in chapter 6, Galileo only takes between 3 to 12 epochs, see Table 6.6 in chapter 6. The integrated GPS and Galileo system gives instantaneous 0.999 ASR for full AR for some scenarios of varying values of measurement precision. For the remaining values of measurement precision the integrated system takes only 2 epochs (2 seconds), see Table 7.3 atmosphere known, single frequency results.

(2) Simulations have been done for Reference-Rover model, with ionosphere and troposphere both as unknowns for 250 Km baseline length using quadruple frequency GPS and Galileo combined system. The coordinates of the rover receiver could be obtained in just 2 to 7 epochs (2 to 7 seconds), see Table 6.21 in chapter 6, for varying measurement precision values to meet a predefined criteria for fixed-precision (σ_n, σ_e =2cm and σ_u=6cm) with the aid of Partial Ambiguity Resolution (PAR).

(3) When ionosphere is weighted, the full AR for the rover is possible to be achieved within the range 3 to 19 epochs (3 to 19 seconds), see Table 6.34 in chapter 6, for baseline lengths of 10 to 1000 Kms with quadruple frequency GPS+Galileo system.

(4) For ionosphere weighted scenario, when a predefined fixed-precision for the coordinates (σ_n, σ_e =2cm and σ_u=6cm) is aimed, it just takes between 1 and 10 epochs (1 to 10 seconds) with a quadruple frequency combined GPS+Galileo system with the aid of PAR.

(5) With a combined GPS and Galileo quadruple frequency system, PAR is able to give a fixed-precision of 20cm for the horizontal coordinate components in just 1 to 3 epochs for baseline length of 1 to 1000 Km for ionosphere weighted scenario, see Table 6.36 in chapter 6. For atmosphere float scenario, it just takes between 1 to 4 epochs to obtain a fixed-precision of 20cm for 250 Km baseline length for various scenarios of measurement precision with PAR using quadruple frequency GPS+Galileo system, see Table 6.21 in chapter 6.

(6) Also it can be noted that for the integrated GPS and Galileo system, the performance of the integrated system is better that the individual GNSS

systems in terms of number of epochs taken for full AR. As discussed earlier about the coupling, for Reference Rover model, there exist a strong coupling for the integrated system with common parameters being troposphere and coordinates. Reference Rover model is seen to benefit significantly with the integrated GNSS systems.

The above are some of the important findings based on this research work. The results found for the integrated GPS and Galileo system further motivates towards understanding the usefulness that the Next Generation GNSS systems, GPS, Galileo along with the Russian GLObal Navigation Satellite System (GLONASS), the Chinese Compass would bring to the user community. With the availability of regional navigation and augmentation systems like United States of America's Wide Area Augmentation System (WAAS), European Union's European Geostationary Navigation Overlay Service (EGNOS), Japan's MTSAT Satellite Augmentation System (MSAS) and Quasi Zenith Satellite System (QZSS), Chinese BeiDou and India's Indian Region Navigation Satellite System (IRNSS) and GPS Aided Geo Augmented Navigation (GAGAN), the benefits will only get richer. To reap the richness of Next Generation GNSS, research towards development of mathematical theory and computational algorithms which make possible to accommodate multi-GNSS data in an efficient manner has to be realised.

7.2 Scope of future developments

The evaluation of the benefits Next-Generation GNSS holds was evaluated in this research by integrating GPS and Galileo. The dual frequency integrated GPS and Galileo had the overlapping frequencies $L1(E1)$ and $L5(E5a)$ with a common GPS reference satellite which performed exceptionally well for Reference Rover scenario in comparison to the stand alone GPS or Galileo. With the $B2$ of Compass overlapping with the $E5b$ of Galileo, it would be interesting to evaluate the performance in terms of the time taken to achieve a predefined ambiguity-fixed precision of rover receiver coordinates.

For Galileo system the frequencies considered were $E1, E5a, E5b$ and $E6$. Simulations can be done further by incorporating $E5$ frequency of Galileo to the above mentioned frequencies. Since the GNSS model gains strength by addition of frequencies while ionosphere is estimated, its effect on time taken to obtain a desired ambiguity-fixed precision ionosphere can be evaluated for Galileo only scenario.

In this research, simulation were done for a single baseline. For Geometry Fixed model which corresponds to CORS scenario, simulations can be done by extending the number of stations which would correspond to regional CORS network.

In this study, simulations were done while considering three latitude locations, $0°$, $-30°$ and $-60°$ latitude and $115°$ longitude with baseline length of up to 1000 Km. By selecting further distributed locations for Australia, similar simulation studies can be done.

Appendices

A
Kronecker product

The Kronecker product for matrices A and B of size 3 x 3 can be given as ,

$$\begin{bmatrix} A \otimes B \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \otimes \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

Where $A \otimes B$ an be further elaborated as,

$$\begin{bmatrix} A_{11} \cdot B_{11} & A_{11} \cdot B_{12} & A_{11} \cdot B_{13} & \ldots & A_{13} \cdot B_{11} & A_{13} \cdot B_{12} & A_{13} \cdot B_{13} \\ A_{11} \cdot B_{21} & A_{11} \cdot B_{22} & A_{11} \cdot B_{23} & \ldots & A_{13} \cdot B_{21} & A_{13} \cdot B_{22} & A_{13} \cdot B_{23} \\ A_{11} \cdot B_{31} & A_{11} \cdot B_{32} & A_{11} \cdot B_{33} & \ldots & A_{13} \cdot B_{31} & A_{13} \cdot B_{32} & A_{13} \cdot B_{33} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{31} \cdot B_{11} & A_{31} \cdot B_{12} & A_{31} \cdot B_{13} & \ldots & A_{33} \cdot B_{11} & A_{33} \cdot B_{12} & A_{33} \cdot B_{13} \\ A_{31} \cdot B_{21} & A_{31} \cdot B_{22} & A_{31} \cdot B_{23} & \ldots & A_{33} \cdot B_{21} & A_{33} \cdot B_{22} & A_{33} \cdot B_{23} \\ A_{31} \cdot B_{31} & A_{31} \cdot B_{32} & A_{431} \cdot B_{33} & \ldots & A_{33} \cdot B_{31} & A_{33} \cdot B_{32} & A_{33} \cdot B_{33} \end{bmatrix} \tag{A.1}$$

The following are the properties of Kronecker product for two matrices A (m x n) and B (p x q) [56].

$$(A + C) \otimes B = A \otimes B + C \otimes B \tag{A.2}$$

$$A \otimes (B \otimes C) = (A \otimes B) \otimes C \tag{A.3}$$

$$(A \otimes B)(C \otimes D) = (AC) \otimes (BD) \tag{A.4}$$

$$(A \otimes B)^* = A^* \otimes B^* \tag{A.5}$$

$$|A \otimes B| = (|A|)^m (|B|)^n \tag{A.6}$$

$$(A \otimes B)^{-1} = A^{-1} \otimes B^{-1} \tag{A.7}$$

In the last two equations A and B are square m x m and n x n matrices respectively.

B

Geometry Free GNSS model derivations

B.1 Dual Frequency, functional and stochastic models

The design matrix for a dual frequency (DF), ionosphere known (Fixed), DD GPS observables can be given as below (ignoring the subscripts r, j and s, instead we use k for k^{th} epoch, (1) for 1^{st} frequency and m for number of satellites).

Functional model:

The functional model for Geometry Free, DF, ionosphere known GPS observables (carrier phase and code) can be given as below. The model below represents the single difference model (with respect to the receiver), since here baseline is considered, subscript to indicate single difference with respect to the receiver is deliberately avoided to make things look simpler.

$$\underbrace{\begin{bmatrix} \Phi_{(1)} \\ \Phi_{(2)} \\ P_{(1)} \\ P_{(2)} \end{bmatrix}}_{E(y)} = \underbrace{\begin{bmatrix} \lambda_{(1)} & 0 & 1 \\ 0 & \lambda_{(2)} & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}}_{[A_I, A_{II}]} \cdot \underbrace{\begin{bmatrix} N_{(1)} \\ N_{(2)} \\ \rho \end{bmatrix}}_{[x_I; x_{II}]} \tag{B.1}$$

The first term on the right hand side are the design matrices $[A_I, A_{II}]$ both lumped together.

Further for m satellites over head we can write the above design matrix as

$$
\begin{bmatrix} \Phi^1_{(1)} \\ \vdots \\ \Phi^{m-1}_{(1)} \\ \Phi^1_{(2)} \\ \vdots \\ \Phi^{m-1}_{(2)} \\ P^1_{(1)} \\ \vdots \\ P^{m-1}_{(1)} \\ P^1_{(2)} \\ \vdots \\ P^{m-1}_{(2)} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} \lambda_{(1)} & 0 \\ 0 & \lambda_{(2)} \end{pmatrix} \otimes I_{m-1} & e_2 \otimes I_{m-1} \\ \\ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \otimes I_{m-1} & e_2 \otimes I_{m-1} \end{bmatrix} . \begin{bmatrix} N_{(1)^1} \\ \vdots \\ N_{(1)^{m-1}} \\ N_{(2)^1} \\ \vdots \\ N_{(2)^{m-1}} \\ \rho^1 \\ \vdots \\ \rho^{m-1} \end{bmatrix} \tag{B.2}
$$

Where, I_{m-1} indicate an identity matrix of sizes $(m-1)$, e_{np}, e_{nc} both are column matrices of each element equal to 1, length depends on np and nc, i.e, number of phase and code observables respectively. the symbol $\otimes$ indicate the Kronecker's product (*Kron*) (for more information on Kronecker's product see Appendix A).

Further for k epochs, we define an identity matrix, I_k and a column matrix e_k and *Kron* it with different elements (ambiguity, ranges) of above design matrix as shown below.

$$\begin{bmatrix} \Phi^1_{(1)1} \\ \vdots \\ \Phi^{m-1}_{(1)1} \\ \vdots \\ \Phi^{m-1}_{(1)k} \\ \Phi^1_{(2)1} \\ \vdots \\ \Phi^{m-1}_{(2)1} \\ \vdots \\ \Phi^{m-1}_{(2)k} \\ P^1_{(1)} \\ \vdots \\ P^{m-1}_{(1)} \\ P^1_{(2)} \\ \vdots \\ P^{m-1}_{(2)} \end{bmatrix} = \begin{bmatrix} e_k \otimes \begin{pmatrix} \lambda_{(1)} & 0 \\ 0 & \lambda_{(2)} \end{pmatrix} \otimes I_{m-1} & I_k \otimes (e_2 \otimes I_{m-1}) \\ e_k \otimes \left(\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \otimes I_{m-1} \right) & I_k \otimes (e_2 \otimes I_{m-1}) \end{bmatrix} \cdot \begin{bmatrix} N^1_{(1)1} \\ \vdots \\ N^{m-1}_{(1)1} \\ \vdots \\ N^{m-1}_{(1)k} \\ N^1_{(2)1} \\ \vdots \\ N^{m-1}_{(2)1} \\ \vdots \\ N^{m-1}_{(2)k} \\ \rho^1_1 \\ \vdots \\ \rho^{m-1}_1 \\ \vdots \\ \rho^1_k \\ \vdots \\ \rho^{m-1}_k \end{bmatrix} \tag{B.3}$$

The ambiguities do not vary with epochs unless there is a cycle slip or loss of lock due to some reason, whereas the ionosphere has temporal variability. This is the reason an identity matrix is used for ionosphere since it has to be estimated for every epoch and column matrix for ambiguities since it does not vary with every epoch.

Stochastic model:

For DF, ionosphere fixed case, the stochastic model can be explained as below. We first define a column matrix of standard deviation of phase and code used.

$$\sigma_y = \begin{bmatrix} \sigma_{\phi(1)} \\ \sigma_{\phi(2)} \\ \sigma_{P(1)} \\ \sigma_{P(2)} \end{bmatrix} \tag{B.4}$$

Now for all the satellites $(m-1)$ we weigh each of the phase and code with respect to the elevation as given in Visual manual [16]. For phase, we can weigh the standard deviations as given below.

$$
\begin{aligned}
W^{-1} &= \begin{bmatrix}
(W^1_{\phi(1)})^{-1} & 0 & 0 & 0 & 0 & 0 \\
0 & \ddots & 0 & 0 & 0 & 0 \\
0 & 0 & (W^m_{\phi(1)})^{-1} & 0 & 0 & 0 \\
0 & 0 & 0 & (W^1_{\phi(2)})^{-1} & 0 & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & 0 & (W^m_{\phi(2)})^{-1}
\end{bmatrix} \\
&= \begin{bmatrix}
\left(\sigma_{\phi(1)}(1 + a_{\phi(1)}exp(-\frac{\varepsilon^1}{\varepsilon^0}))\right)^2 & \dots & 0 & 0 & 0 \\
\vdots & & & & \vdots \\
0 & & \ddots & & 0 \\
\vdots & & & & \vdots \\
0 & & 0 & 0 \quad \dots & \left(\sigma_{\phi(2)}(1 + a_{\phi(2)}exp(-\frac{\varepsilon^m}{\varepsilon^0}))\right)^2
\end{bmatrix}
\end{aligned}
\tag{B.5}
$$

where, $W^{-1}_{\phi(j)}$ is the elevation compensated variance of ϕ at frequency j, $a_{\phi(j)}$ is amplification angle dependent on observation type (ϕ or P) and frequency (j) and is set to 10 for all observable types, ε^m is the elevation of the satellite m (in degrees), ε^0 is the is the scale value of the elevation error and is set to 10 degrees.

Further the weight matrix W for phase can be given as below

$$
\left.\begin{aligned}
(W^{1,..,m}_{\phi(1)})^{-1} &= diag((W^1_{\phi(1)})^{-1}, \cdots, (W^m_{\phi(1)})^{-1}) \\
(W^{1,..,m}_{\phi(2)})^{-1} &= diag((W^1_{\phi(2)})^{-1}, \cdots, (W^m_{\phi(2)})^{-1})
\end{aligned}\right\}
\tag{B.6}
$$

Similarly the weight matrix for code can be derived as shown below

$$
\left.\begin{aligned}
(W^{1,..,m}_{P(1)})^{-1} &= diag((W^1_{P(1)})^{-1}, \cdots, (W^m_{P(1)})^{-1}) \\
(W^{1,..,m}_{P(2)})^{-1} &= diag((W^1_{P(2)})^{-1}, \cdots, (W^m_{P(2)})^{-1})
\end{aligned}\right\}
\tag{B.7}
$$

We further write the single difference design matrix, D, for satellites, for the same the satellite having the maximum elevation angle is chosen as reference satellite. The design matrix D has the dimensions as $m \times (m-1)$. For $m = 4$, taking satellite 3 as reference satellite, the design matrix D^T would look as below.

$$
D^T = \begin{bmatrix}
1 & 0 & -1 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & -1 & 1
\end{bmatrix}
\tag{B.8}
$$

Finally, we write the variance matrix for dual frequency observables as below

$$Q_{yy} = 2\begin{bmatrix} D^T\cdot(W^{1,..,m}_{\phi(1)})^{-1}\cdot D & 0 & 0 & 0 \\ 0 & D^T\cdot(W^{1,..,m}_{\phi(2)})^{-1}\cdot D & 0 & 0 \\ 0 & 0 & D^T\cdot(W^{1,..,m}_{P(1)})^{-1}\cdot D & 0 \\ 0 & 0 & 0 & D^T\cdot(W^{1,..,m}_{P(2)})^{-1}\cdot D \end{bmatrix} \tag{B.9}$$

Considering 4 satellites, and dual frequency (one code and one phase observable), the above matrix Q_{yy} will have the dimensions of 6×6 or $j(m-1)\times j(m-1)$. In the above matrix, $D^T\cdot D$ gives the between satellite single difference. The elevation weight matrix given in the above equation $(W^{1,..,m}_{\phi(1)})^{-1}$ is for one receiver, incase the baseline length is relatively short, for example 1Km baseline length for atmosphere fixed scenario. In such a case the elevation weight matrix is undifferenced hence it is multiplied by factor 2 in order to change it to its single difference form. While the receivers are sufficiently separated, then the elevation weight matrix is considered for each receiver. The above expression for any GNSS observables would then be given as $D^T\cdot[(W^{1,..,m}_1)^{-1}+(W^{1,..,m}_2)^{-1}]\cdot D$. The weight matrixes correspond to receiver 1 and 2 indicated by W_1 and W_2, together they form between receiver single difference weight matrices.

B.2 Geometry Free model, variances for float and fixed solutions

We have defined $\overline{A}_I$, by reducing the normal equations. For the same a square and full-rank matrix is used which can eliminate the temporal parameters $\hat{x}_{II}$, refer [57]

$$\overline{A}_I = P^{\perp}_{A_{II}}A_I$$

where,

$$P^{\perp}_{A_{II}} = I - A_{II}(A^T_{II}Q^{-1}_{yy}A_{II})^{-1}A^T_{II}Q^{-1}_{yy}$$

The system of reduced normal equations read as,

$$\begin{bmatrix} \overline{A}^T_I Q^{-1}_{yy}\overline{A}_I & 0 \\ A^T_{II}Q^{-1}_{yy}A_I & A^T_{II}Q^{-1}_{yy}A_{II} \end{bmatrix}\begin{bmatrix} \hat{x}_I \\ \hat{x}_{II} \end{bmatrix} = \begin{bmatrix} \overline{A}^T_I Q^{-1}_{yy}y \\ A^T_{II}Q^{-1}_{yy}y \end{bmatrix}$$

Further $Q_{\hat{x}_I}$ can be given as,

$$Q_{\hat{x}_I} = \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} = \left(\sum_{i=1}^{k} A_I^T(i) Q_{yy}^{-1}(i) P_{A_{II}}^{\perp}(i) A_I(i)\right)^{-1} \tag{B.10}$$

Further $Q_{\hat{x}_{II}}$ is derived from $\hat{x}_{II}$,

$$\begin{aligned}
\hat{x}_{II} &= \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} (y - A_I \hat{x}_I) \\
&= \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} \left(I - A_I \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T Q_{yy}^{-1}\right) y \\
&= \left\{\left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} - \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} A_I \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T Q_{yy}^{-1}\right\} y \\
&= \left\{\left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} - \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_I \underbrace{\left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1}}_{Q_{\hat{x}_I}} \overline{A}_I^T Q_{yy}^{-1}\right\} A_{II}^T Q_{yy}^{-1} y \\
&= \left\{\left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} - \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \left(A_I Q_{\hat{x}_I} \overline{A}_I^T\right) Q_{yy}^{-1}\right\} A_{II}^T Q_{yy}^{-1} y
\end{aligned}$$

$$\begin{aligned}
Q_{\hat{x}_{II}} &= \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} + \\
&+ \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} \left(A_I Q_{\hat{x}_I} A_I^T\right) Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1}
\end{aligned} \tag{B.11}$$

Computation of $Q_{\hat{x}_I \hat{x}_{II}}$

It has been discussed that $Q_{\hat{x}_I \hat{x}_{II}}$ can be derived from $\hat{x}_I$ and $\hat{x}_{II}$. $\hat{x}_I$ and $\hat{x}_{II}$ can be given as,

$$\hat{x}_I = \underbrace{\left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T Q_{yy}^{-1}}_{A_{\hat{x}_I}} y \tag{B.12}$$

$$
\begin{aligned}
\hat{x}_{II} &= \underbrace{\left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} \left(I - A_I \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T Q_{yy}^{-1}\right)}_{A_{\hat{x}_{II}}} y \\
&= \Big\{ \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} - \\
&- \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} A_I \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T Q_{yy}^{-1} \Big\} y
\end{aligned}
\tag{B.13}
$$

and

$$
Q_{\hat{x}_I \hat{x}_{II}} = A_{\hat{x}_I} Q_{yy} A_{\hat{x}_{II}}^T
$$

$$
\begin{aligned}
Q_{\hat{x}_I \hat{x}_{II}} &= \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T Q_{yy}^{-1} Q_{yy} \\
&\left(\left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} - \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} A_I \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T Q_{yy}^{-1} \right)^T \\
&= \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T \\
&\left(Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} - Q_{yy}^{-1} \overline{A}_I \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} A_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \right) \\
&= \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} - \\
&- \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T Q_{yy}^{-1} \overline{A}_I \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} A_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \\
&= \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \overline{A}_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} - \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} A_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \\
&= \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \left(\overline{A}_I^T - A_I^T\right) Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \\
&= \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} \left(P_{A_{II}}^T A_I^T - A_I^T\right) Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \\
&= \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} P_{A_{II}}^T A_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} - \left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} A_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \\
&= -\left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} A_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1}
\end{aligned}
$$

$$
\boxed{Q_{\hat{x}_I \hat{x}_{II}} = -\left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} A_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1}} \tag{B.14}
$$

Further, $Q_{\check{x}_{II}}$ can be derived as below. To begin with, we use the following mathematical formulation for $Q_{\check{x}_{II}}$

$$Q_{\check{x}_{II}} = Q_{\hat{x}_{II}} - Q_{\hat{x}_{II}\hat{x}_I} Q_{\hat{x}_I}^{-1} Q_{\hat{x}_I\hat{x}_{II}}$$

Substituting the corresponding terms for $Q_{\hat{x}_{II}}$, $Q_{\hat{x}_{II}\hat{x}_I} = Q_{\hat{x}_I\hat{x}_{II}}{}^T$, and $Q_{\hat{x}_I}$,

$$\begin{aligned}
Q_{\check{x}_{II}} &= \underbrace{\left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} + \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} \left(A_I Q_{\hat{x}_I} A_I^T\right) Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1}}_{Q_{\hat{x}_{II}}} - \\
&\quad - \underbrace{\left\{ -\left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} A_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \right\}^T}_{Q_{\hat{x}_{II}\hat{x}_I}} \underbrace{\left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)}_{(Q_{\hat{x}_I})^{-1}} \\
&\quad \underbrace{\left(-\left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1} A_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \right)}_{Q_{\hat{x}_I\hat{x}_{II}}} \\
&= \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} + \\
&\quad + \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} \left(A_I Q_{\hat{x}_I} A_I^T\right) Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} - \\
&\quad - \left\{ \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} A_I \underbrace{\left(\overline{A}_I^T Q_{yy}^{-1} \overline{A}_I\right)^{-1}}_{Q_{\hat{x}_I}} \right\} A_I^T Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \\
&= \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} + \\
&\quad + \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} \left(A_I Q_{\hat{x}_I} A_I^T\right) Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} - \\
&\quad - \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} A_{II}^T Q_{yy}^{-1} \left(A_I Q_{\hat{x}_I} A_I^T\right) Q_{yy}^{-1} A_{II} \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1} \\
&= \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1}
\end{aligned}$$

$$\boxed{Q_{\check{x}_{II}} = \left(A_{II}^T Q_{yy}^{-1} A_{II}\right)^{-1}}$$

C

Ambiguity Success Rates based on LAMBDA

The simulations of ASR (Ambiguity Success Rate) are based on the LAMBDA method incorporating sequential conditional least squares estimation and decorrelation of the ambiguities and further computing the probability of Integer Bootstrap. LAMBDA stands for (Least-squares AMBiguity Decorrelation Adjustment), a method widely used to estimate integer ambiguities, is also applicable to diverse geodesy applications [58], has been developed by [39]. The correct resolution integer ambiguities, as discussed in chapter 3, is done by computing the lower bounds for Integer Least-squares. This is done by de-correlating the float ambiguities VC matrix using LAMBDA and further computing the probability of Integer Bootstrap, this corresponds to sharp lower bounds for Integer Least Squares.

The computation of ASR, as explained above, is done by using the software routines developed in Matlab by [44] and further modified by [45]. To give a general idea of the Matlab routines, [44] implemented the computation of integer ambiguities by LAMBDA method, by sequential conditional least squares estimation and then de-correlation the float ambiguity VC matrix and further computation of fixed solution and ASR for Integer Least Squares (lower bound of ASR-Integer Bootstrap), for more details refer [44]. [45] made the computing efficient by implementing a shrinking search-space for estimating integer ambiguities by LAMBDA method. In the recent upgrade of LAMBDA software routines, ASR-Integer Bootstrap can be computed, hence it further facilitates the user to consider ASR-Integer Bootstrap as upper and lower bounds for ASR-Integer Rounding and ASR-Integer Least Squares respectively. Further, partial integer ambiguities can now be estimated as compared to earlier when only all the ambiguities were used for ambiguity resolution and ASR computation, refer [45]. This research work is based on computation of sharp lower-bounds for Integer Least

Squares (by computing ASR for Integer Bootstrap) is done in two ways, first by considering all the ambiguities (Full AR) and then by considering a subset of ambiguities (PAR), same will be discussed in the commencing sections.

C.1 Full ambiguity resolution

Since this research work is based on simulation of ASR, the VC matrix of the float-ambiguity variance is only used as an input to simulate the ASR, by the above Matlab routines. The procedure of full AR is explained in the following steps.

(1) Sequential conditional least squares estimation:

The VC matrix,$Q_{\widehat{x_I}}$, is first de-composed into L and D by LDL transformation, L is the inverse of the lower triangular matrix, corresponding to the original VC matrix, and D has the conditional variances of the i^{th} ambiguity over all $I, I = \{i+1, \cdots m\}$ remaining ambiguities, out of total m ambiguities.

$$Q_{\hat{a}} = L^{-1}D^{-1}L^{-T}$$

(2) Decorrelation of the decomposed ambiguities: Further L is de-correlated and corresponding D is recomputed and updated. The transposed inverse matrix Z^{-T} keeps track of the changes, $Z = I_m$, I is the identity matrix of size m, m total number of ambiguities.

Since all the ambiguities are fixed, the final Z^{-T} of size $m \times m$, is used to recompute the $Q_{\widehat{x_I}}$ matrix

$$Q_{\widehat{x_I}} \quad = \quad Z^T Q_{\widehat{x_I}} Z$$

(3) Computation of ASR for Integer Bootstrap (corresponding to lower bound of ASR-Integer Least Squares) using the decorrelated-conditional variance of ambiguities, given in D, see chapter 3, equation (3.14), $D_i \equiv \sigma_{i|I}$

C.2 Partial ambiguity resolution

The computation of a subset of ambiguities which together can give the desired ASR can be done by using the Partial ambiguity resolution (PAR) software routine *psearch.m*. The VC matrix of the float ambiguities along with the user defined ASR is given as the input to *psearch.m*. In this research work the desired ASR is 0.999 used in PAR computation. The computation steps for PAR are given as below.

Steps 1 to 5 given below are repeated for all i, $i = 1, \cdots, m$ unless an ambiguity is found which has an ASR such that when added to the ASR of previous ambiguities, reduces the net ASR from 0.999, i.e, ASR_i is rejected and no further computations are done if, $ASR_{(1,\cdots,i-1)}/ASR_i < 0.999$, where $ASR_{(1,\cdots,i-1)} = 1/ASR_1/\cdots/ASR_{i-1}$.

(1) Sequential conditional least squares estimation or the LDL decomposition, (definition of LDL is given in the previous section) (for the first time, LDL decomposition is done for all the m ambiguities, subsequently as the subset of PAR increases (variances of selected ambiguities give the desired ASR).

(2) Decorrelation of the decomposed ambiguities, see previous section, Z^T corresponds to design matrix for decorrelated ambiguities.

(3) Selecting the minimum variance ambiguity and computing the ASR. If ASR > 0.999 then next step is executed, else the computed ASR's value is returned and no further computation is performed.

(4) Since an ambiguity is selected, which can give an desired ASR, for further computation the variance, covariance of the selected ambiguity is not used. Z^T and $Q_{\widehat{x_I}}$ are updated (they both get reduced in size, the new size will be $(m - i \times m - i)$.

(5) For the ambiguity used in PAR, say p out of m ambiguities are fixed, the corresponding row and column of each fixed ambiguity in the design matrix Z^T are stored in a new design matrix Z_p^T.

(6) Using the updated Z^T and $Q_{\widehat{x_I}}$, repeat steps 1 to 5, until $i - m = 1$. When $i - m = 1$ go to step 3.

(7) Finally, the ASR (partial ambiguity resolution) and design matrix Z_p^T is returned to the main routine.

D
Experiments - Research and analysis

D.1 Wobbling of the curves in the gain plots:

The wobbling of the gain curves is seen in Figures 4.1, E.2, E.3, etc. Lets us take the first Figure 4.1, at $0^0 latitude$ the wobbling occurs for the first time at epoch number 360, corresponding to 00:30:00 UTC GPS time. Lets us observe the sky plots just before 360th epoch and after epoch number 360, see Figure D.1 (left), satellite vehicle (SV) 4 disappears and another SV 15 appears at very low elevation. Similarly at 01:30:00 UTC second wobble can seen in Figure 4.1 at $0^0 latitude$, looking at the sky plot in Figure D.1 (right), SV 10 disappears and SV 21 appears at low elevation. The wobbling is observed to be related to the change in the measurement precision matrix due to inclusion of low elevation satellites (since elevation weighting of satellites is incorporated).

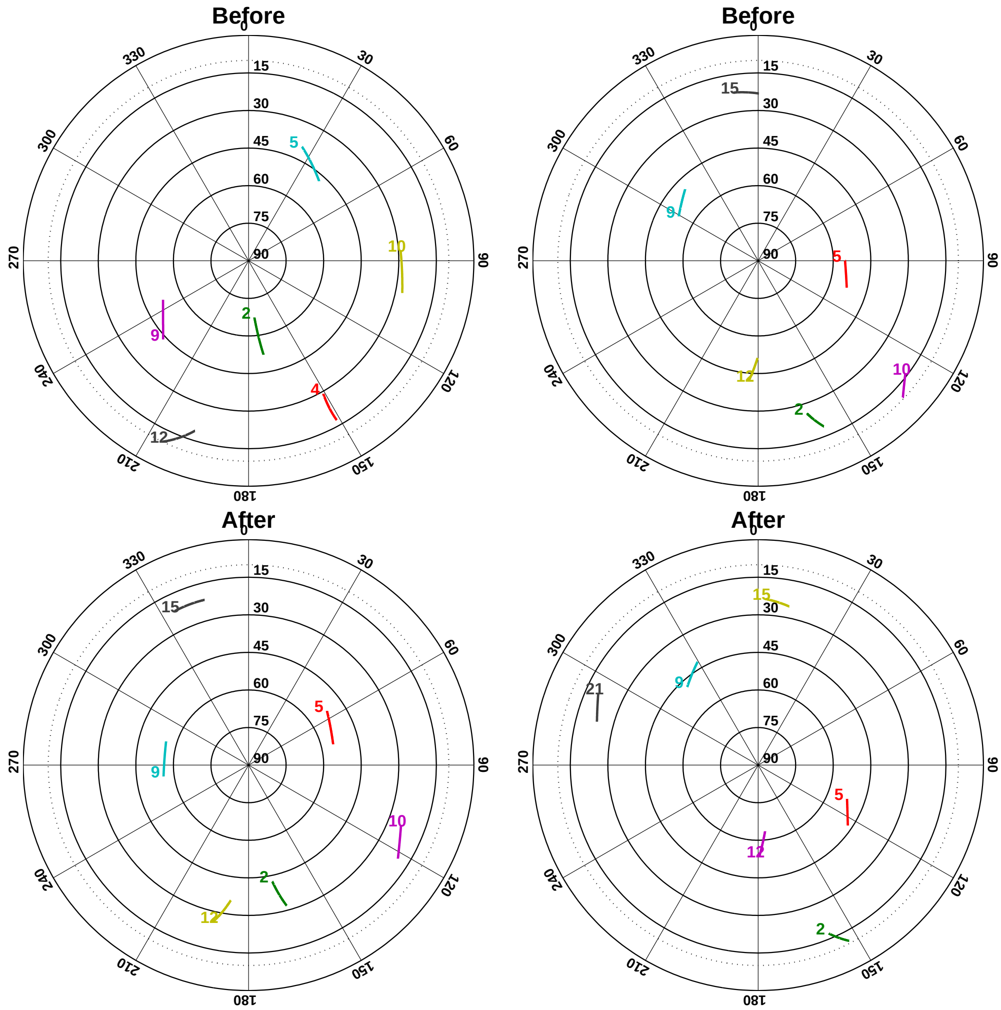

Figure D.1: Sky plot of the satellites at 0^0 degree latitude, before (top) and after (bottom) at transition times 00:30:00 (left) and 01:30:00 (right) hrs UTC

D.2 Analysis of ASR and Gain when satellite combination is varied

The simulation of ASR and gain is done for different satellite combinations. Here the measurement precision is held fixed (refer Table 3.2) and the total number of satellites is two. This analysis is done for DF and TF cases only considering the redundancy of the GNSS model.

The analysis is made for, at 0^0 and -30^0 latitudes, refer Figures D.2 and D.3 respectively, (considering the consistency of availability of satellites, a fixed set of satellites should be available for atleast one hour, any time of the day) for 13 May 2011. The period of the day is selected such that the selected set of satellites remain available throughout the selected duration. The epochs chosen for 0^0 latitude were epoch number 855 to 1350 with 10 second interval corresponding to GPS time 02:22:30 to 03:43:00. At 0^0 latitude the interval chosen was 00:26:30 to 01:47:00 (epoch number 154 to 650). Out of the selected set of 9 satellites, the first satellite is chosen as the reference satellite, the second satellite is varied among the remaining available 8 satellites, making different combinations of two satellites at a time.

Analysis from the Figures D.2 and D.3 for 0^0 degree -30^0 degree latitude respectively show

1. ASR for some satellite combinations were quiet higher instantaneously (0.8 or higher) whereas for certain combination of satellites, the ASR was significantly low. ASR for Satellite Vehicle (SV) combinations 4,17 and 4,25 indicated by solid-black and yellow curves in Figure D.2 were around 0.2 and 0.4 for 1^{st} epoch. The SV numbers 17 and 25 had low elevations, but so did other SV's, 25(yellow), 20(green), 14(red) and 26(dotted-black). One important thing can be noted that the SV's under scanner, SV 25 had almost a constant value of elevation throughout the chosen time period. The third worst instantaneous ASR of SV 20(green) (ASR of approx 0.7) also has almost constant value of elevation, but slightly higher that SV 25. For the other SV under analysis, SV 17 has the elevation values exactly in opposite correlation to the reference SV (SV 4). Infact they intersect at the center of the selected time period. It will be interesting to analyze the SV-elevation correlation and its effect on ASR.

2. For Figure D.3, the SV's 28 (dotted-black), 24 (magenta) and 4(blue) give

the lowest three instantaneous ASR of approximately 0.2, 0.3, 0.5 respectively. The elevation of the reference satellite decreases throughout the chosen time period and for the whole time period only four satellites have their elevation rising, the three SV's we mentioned, i.e., 28, 24 and 4 and the fourth SV which is not mentioned earlier is SV 11. SV 11, inspite of having negative elevation correlation with the reference SV, has very high instantaneous ASR of 0.9. To conclude, it can be said that, there is a possibility of dependency of ASR with the elevation correlation of the reference satellite, which needs to be analyzed in near future.

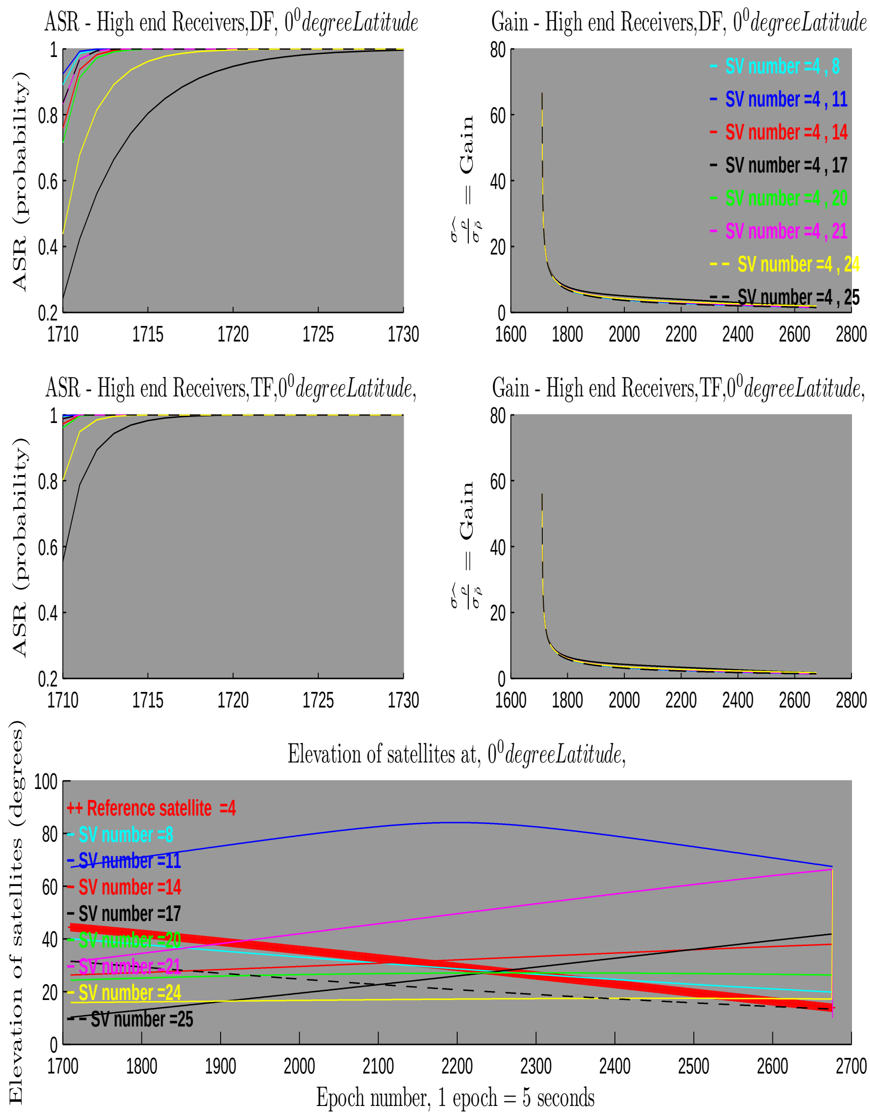

Figure D.2: ASR and Gain analysis, Ionosphere Fix **(satellite combination is varied, number of satellites=2, measurement precision is held fixed)** at 0^0 degree latitude for dual and triple GPS frequency

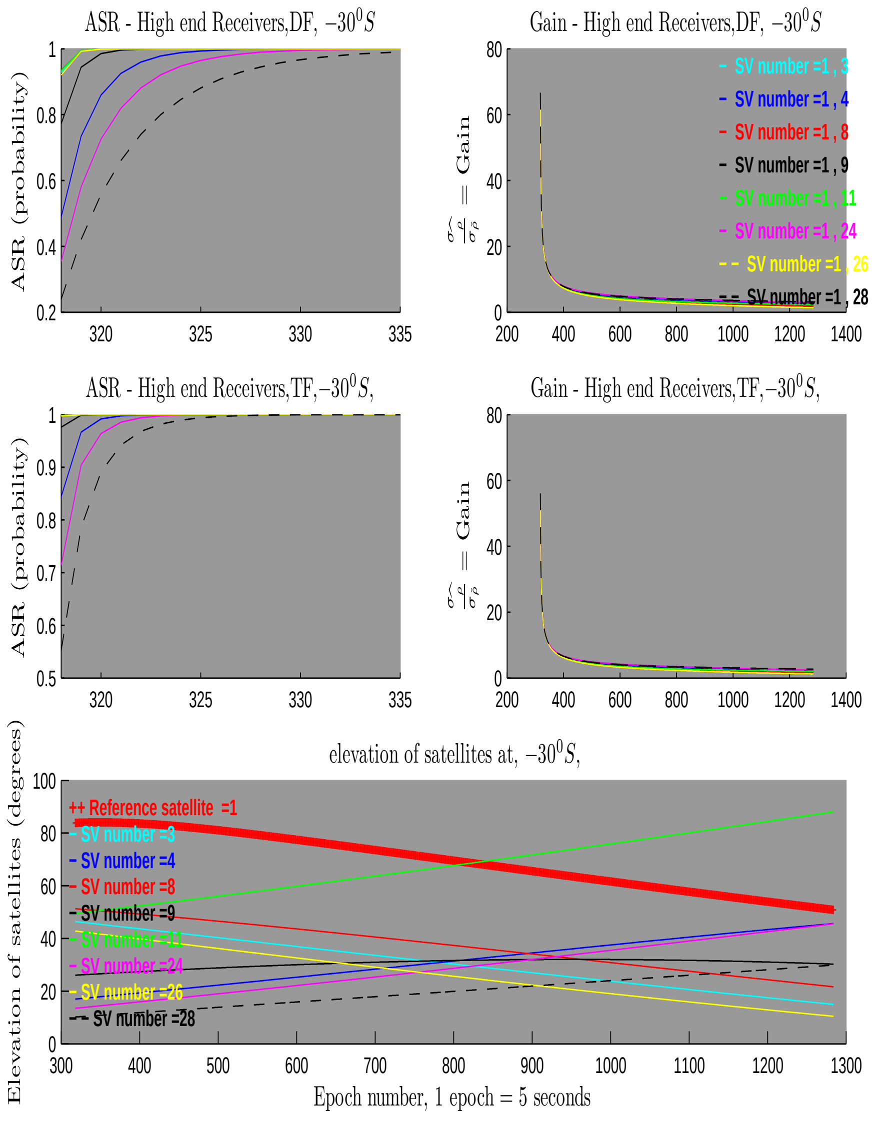

Figure D.3: ASR and Gain analysis, Ionosphere Fix **(satellite combination is varied, number of satellites=2, measurement precision is held fixed)** at 30^0 degree latitude for dual and triple GPS frequency

D.3 Relation of elevation of satellite and ASR

The variation of ASR with elevation angle is shown in Figure D.4, the top row gives the ASR for DF and TF for high elevation satellites, the middle row represents ASR for high and low elevation satellites, the bottom row gives the ASR curve for low elevation satellites.

Since the elevation dependent weighting is considered, the elevation of the satellites have a significant affect on the measurement precision matrix, and hence on the ASR. The low elevation satellites carry high weights in the measurement precision matrix resulting in more number of epochs for 0.999 ASR. For high elevation satellites, it takes 2 epochs for DF (blue line-ASR plot) to achieve 0.999 ASR, which increases to 10 epochs and 14 epochs for one high-one low (middle row) and both low elevation satellites (bottom row) considerations. Similar analysis can be made for TF (red line-ASR plot), but the results being better than for DF, considering the precise $L5$ code used in measurement precision matrix.

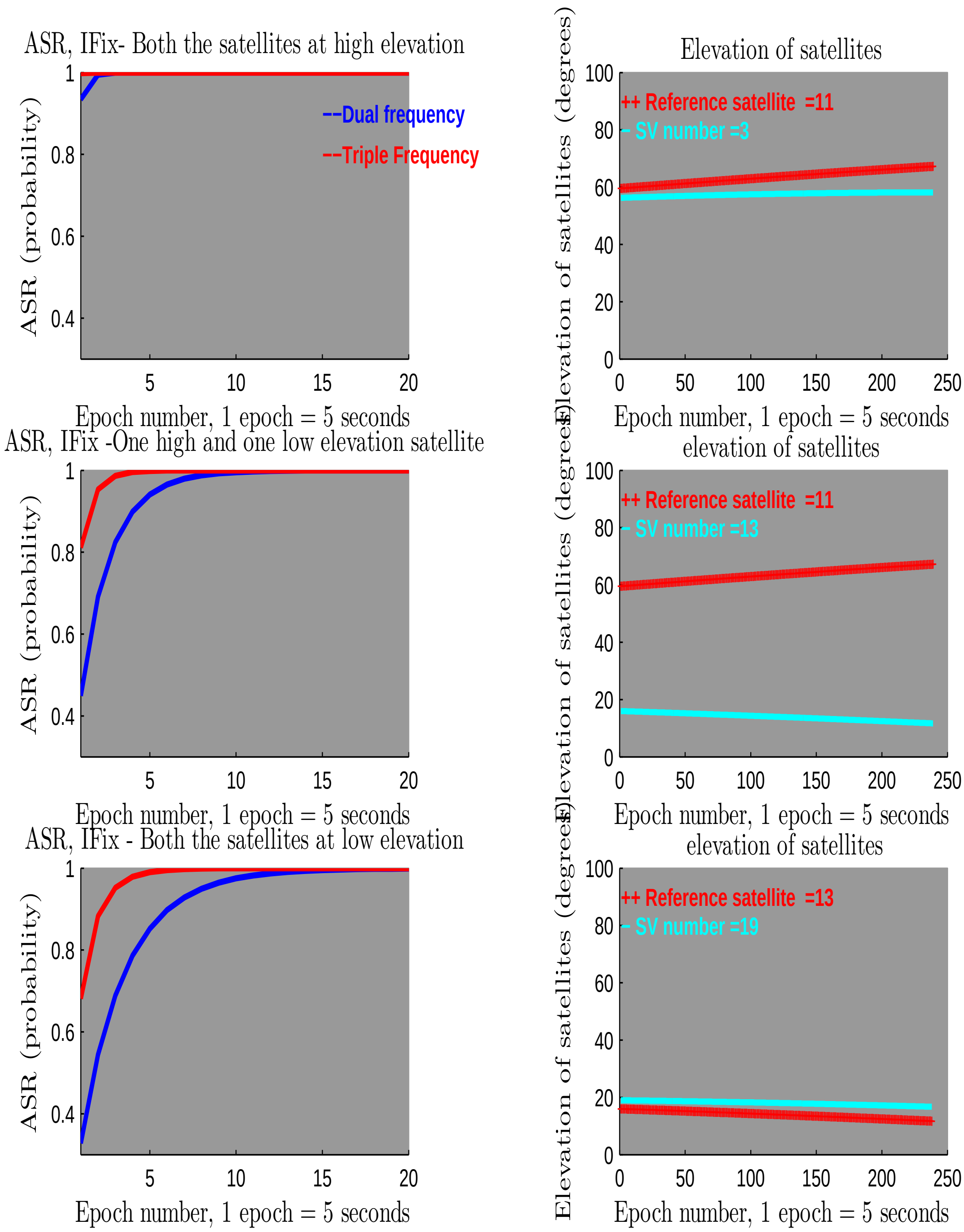

Figure D.4: ASR analysis, Ionosphere Fix **(satellite elevation angle is varied, number of satellites=2, measurement precision is held fixed)** for dual and triple GPS frequency

D.4 Elevation weighting for satellites

The effect of elevation can be incorporated in a GNSS stochastic model by using a elevation weighting function. The importance of using an adequate stochastic model, which incorporates the different effects on the GNSS observables (elevation-weighting, other correlations etc.) has been highlighted by [59]. In fact it has been mentioned that precise positioning can be carried effectively if adequate information of the GNSS stochastic model is known [59]. Further, the incorporated variance in the stochastic model, say of satellite elevation weighting, depends on the type of receiver used. [41] gave an exponential function for satellite elevation weighting and for the same function gave different values for receivers used, Trimble SST and Rogue. the exponential weighting function was given as,

$$a_0 + a_1 \exp(-\varepsilon/\varepsilon_0) \tag{D.1}$$

[41] used two receivers Trimble SST and Rogue. The C/A (Coarse Acquisition) code data is less precise on Trimble SST and values were found to be around 1.5m at zenith and 5m at 15 degree elevation [41]. Whereas, JPL's Rogue gave precise code measurement, between zenith and 45 degree elevations, the code precision was between 10 and 20 cms and at 15 degrees the uncertainty reached to 1.2 m. While giving the values of a_0 and a_1, in the equation (D.4), the precision of the measurement (code) was included [41], we rewrite the equation for our ease of understanding.

$$\sigma_P \cdot a_0 + \sigma_P \cdot a_1 \exp(-\varepsilon/\varepsilon_0) \tag{D.2}$$

where σ_P is the un differenced code precision. Based on the precision of the code measurement, the constants $\sigma_P \cdot a_0$ had values 1.4 and 0.08 meters for Trimble SST and Rogue and $\sigma_P \cdot a_1$ was 8.0 and 4.5 meters for Trimble SST and Rogue respectively. ε is the elevation of the satellite and ε_0 is the scale value of the elevation error. The values of ε_0 were chosen as 10 and 20 degrees for Trimble SST and Rogue respectively. If we look at the above values carefully, we can get values of coefficients a_0, a_1 themselves.

$$\begin{aligned}
\sigma_{P_{Trimble}} \cdot a_0 + \sigma_{P_{Trimble}} \cdot a_1 \cdot \exp(-\varepsilon/\varepsilon_0) &= 1.4 + 8.0 \cdot \exp(-\varepsilon/\varepsilon_0) \\
\Rightarrow \sigma_{P_{Trimble}} = 1.4 \quad a_0 = 1 \quad a_1 = 5.71 & \\
\sigma_{P_{Rogue}} \cdot a_0 + \sigma_{P_{Rogue}} \cdot a_1 \cdot \exp(-\varepsilon/\varepsilon_0) &= 0.08 + 4.5 \cdot \exp(-\varepsilon/\varepsilon_0) \\
\Rightarrow \sigma_{P_{Rogue}} = 0.08 \quad a_0 = 1 \quad a_1 = 56.25 &
\end{aligned}$$

In one of the internal research works of GNSS research group, Dr Nandakumaran Nadarajah computed values for elevation weighting function coefficients for μ-blox receivers. The initial results gave the following values

$$\begin{aligned}
\sigma_P \cdot a_0 + \sigma_P \cdot a_1 \cdot \exp(-\varepsilon/\varepsilon_0) &= 0.7 + 1.4 \cdot \exp(-\varepsilon/\varepsilon_0) \\
\Rightarrow \sigma_P = 0.7 \quad a_0 = 1 \quad a_1 = 2 & \\
\sigma_\Phi \cdot a_0 + \sigma_\Phi \cdot a_1 \cdot \exp(-\varepsilon/\varepsilon_0) &= 0.002 + 0.004 \cdot \exp(-\varepsilon/\varepsilon_0) \\
\Rightarrow \sigma_\Phi = 0.002 \quad a_0 = 1 \quad a_1 = 2 &
\end{aligned}$$

In another research work by [11] the values of a_0, a_1 and ε_0 were chosen as 1, 10 and 10 respectively. Similar values for the coefficients are considered for this research work.

Below are the plot of DD phase and code variance with satellite elevation weighting (exponential functions). In the figure below undifferenced phase and code precision is 3 mm and 10 cm. The values of coefficient a_1 are varied in the figure below between 1 to 10, whereas a_0 and ε_0 are taken as 1 and 10 respectively. Since 10 degree is the cutoff chosen for this research work, the results presented below are for elevations more than 10 degrees.

The DD phase precision which is originally $2 * 0.003$, i.e., 6mm, at 10 degree elevation, for $a_1 = 10$, the value of phase precision reaches 2.81 cms.

The DD value of code precision is $2 * 10$, i.e, 20 cm. At 10 degree elevation, for $a_1 = 10$, the value of code precision reaches 93.58 cms.

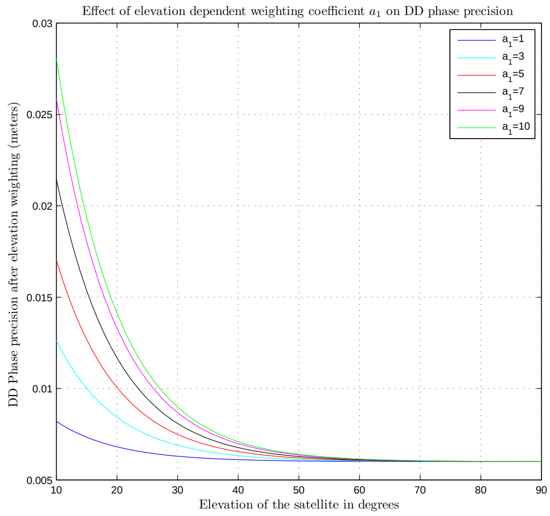

Figure D.5: Effect of satellite elevation weighting (exponential) coefficient $a1$ on DD phase precision

Geometry Free- IFix- Analysis of ASR and Gain when satellite number is varied

Further when number of satellites are varied instead of measurement precision, satellite number is varied from 2 to 6, see sky plots for different satellite numbers for al the three latitude locations in Figure, D.7. The table below shows the instantaneous ASR, number of epochs for 0.999 ASR and gain at 0.999 ASR for different values of satellite numbers, single value of measurement precision for SF, DF and TF.

Further from the Figures, D.8, D.9, D.10 give results for ASR and gain (for all latitude locations) for SF, DF and TF respectively.

From Table D.1 and Figures D.8 to D.10 the analysis is summarized as below.

(1) Now when we consider ASR and gain with respect to variation in number of satellites, refer Figures D.8, D.9, D.10 (for SF, DF and TF respectively), ASR drops as number of satellites are increased. Since the success rate is computed by Integer Bootstrap, the probability of successful AR (ASR) is the product of ASR of each satellite (computed by variance of ambiguity for each satellite conditional to variance of all other satellites). ASR lies between 0 and 1, ambiguity of any satellite which is not fixed correctly causes drop in the final ASR. As the number of satellites increase this fall in the final ASR is clearly seen.

Phase (meters)	Code (meters)	No of Sat.	Instantaneous ASR 0^0	-30^0	-60^0	Epochs for 0.999 ASR 0^0	-30^0	-60^0	Gain @ 0.999 ASR 0^0	-30^0	-60^0
$\Phi1$	$P1$		Low-end receivers, Single Frequency								
0.003	0.5	2	0.058	0.074	0.065	3310	2593	1877	4.734	4.168	2.036
0.003	0.5	3	0.004	0.006	0.004	2945	2818	2535	4.093	4.097	4.088
0.003	0.5	4	0	0	0	2739	2177	2449	2.729	3.003	4.112
0.003	0.5	5	0	0	0	2611	2105	2326	3.607	2.385	4.023
0.003	0.5	6	0	0	0	2670	1929	2549	2.213	4.286	3.986
$\Phi1$,$\Phi2$	$P1$,$P2$		High-end receivers, Dual Frequency								
0.003	0.25	2	0.824	0.937	0.886	4	3	4	33.26	38.50	33.40
0.003	0.25	3	0.783	0.912	0.829	4	3	4	33.26	38.50	33.40
0.003	0.25	4	0.766	0.586	0.830	4	7	3	33.26	25.21	38.54
0.003	0.25	5	0.677	0.565	0.393	4	7	11	33.26	25.21	20.23
0.003	0.25	6	0.164	0.292	0.353	24	10	11	13.56	21.09	20.23
$\Phi5$	$P5$		High-end receivers, Triple Frequency								
0.002	0.15	2	0.986	0.988	0.994	2	1	2	39.65	56.11	39.70
0.002	0.15	3	0.988	0.998	0.993	2	1	2	39.65	56.11	39.70
0.002	0.15	4	0.990	0.932	0.995	2	3	2	39.65	32.41	39.70
0.002	0.15	5	0.984	0.933	0.828	2	3	5	39.65	32.41	25.16
0.002	0.15	6	0.548	0.808	0.826	10	4	5	17.63	28.06	25.16

Table D.1: Geometry Free model, Ionosphere Fix scenario, ASR and gain values - **satellite number varied**

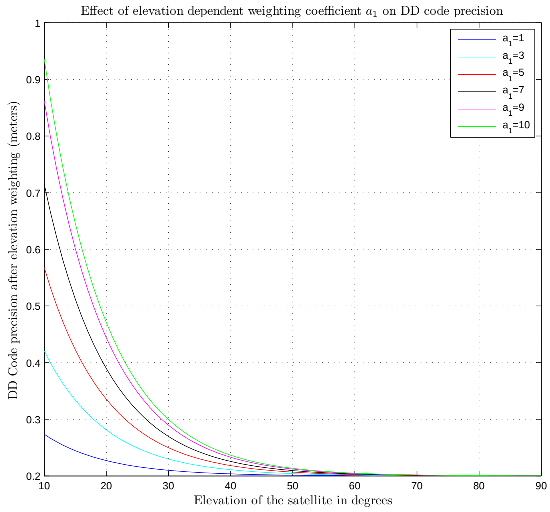

Figure D.6: Effect of satellite elevation weighting (exponential) coefficient $a1$ on DD code precision

(2) The effect on gain with respect to increase in number of satellites is insignificant. Refer Figures D.8, D.9, D.10. Since gain is dependent on measurement precision, in this analysis the measurement precision is held to a fixed value, see Table 3.2, hence gain does not have any effect apart from the fact that it decreases as the number of epochs increase.

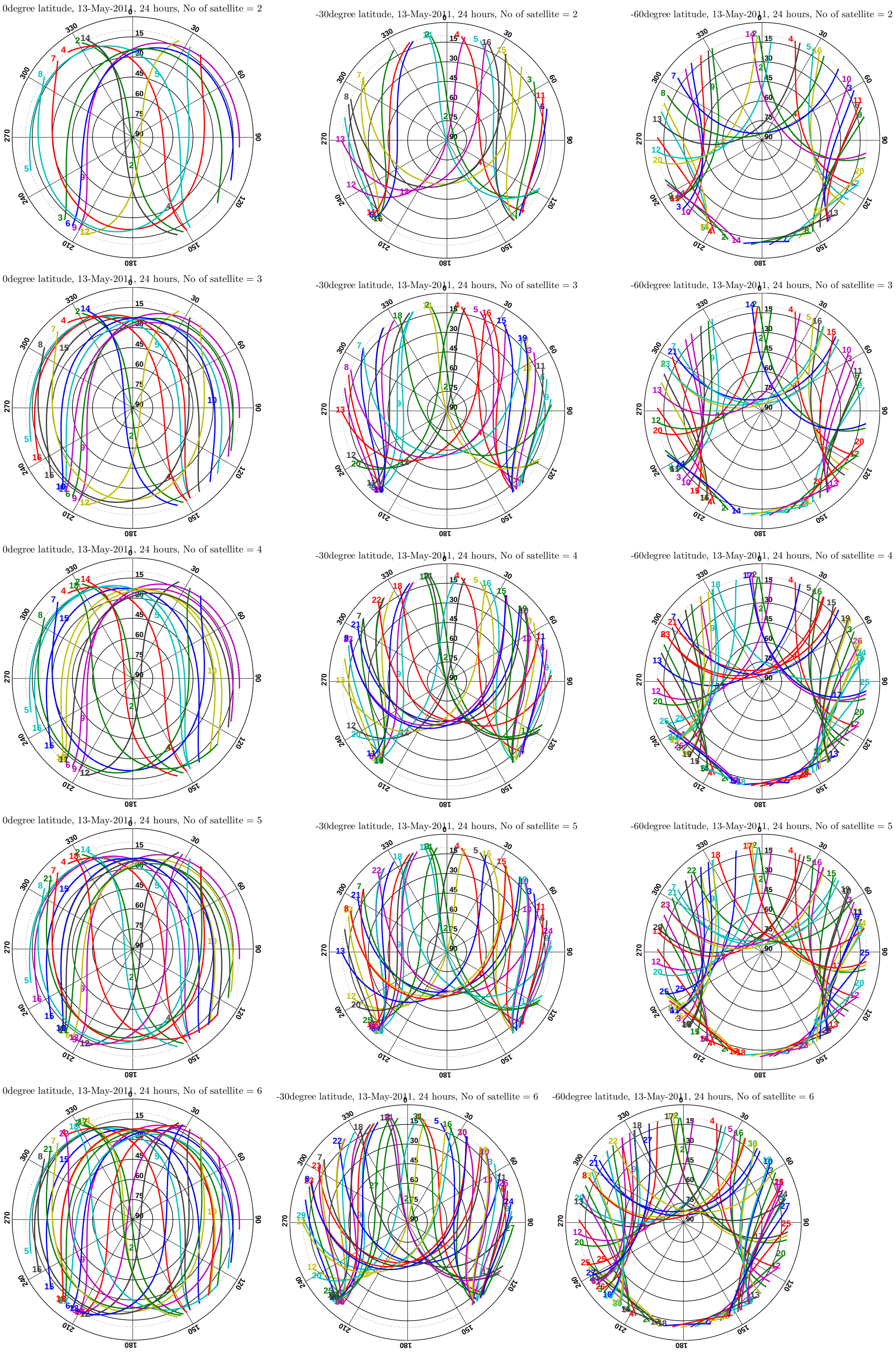

Figure D.7: Number of Satellite variation from 2 to 6 (top to bottom) at $0^0, -30^0$ and -60^0 degree latitude (left to right)

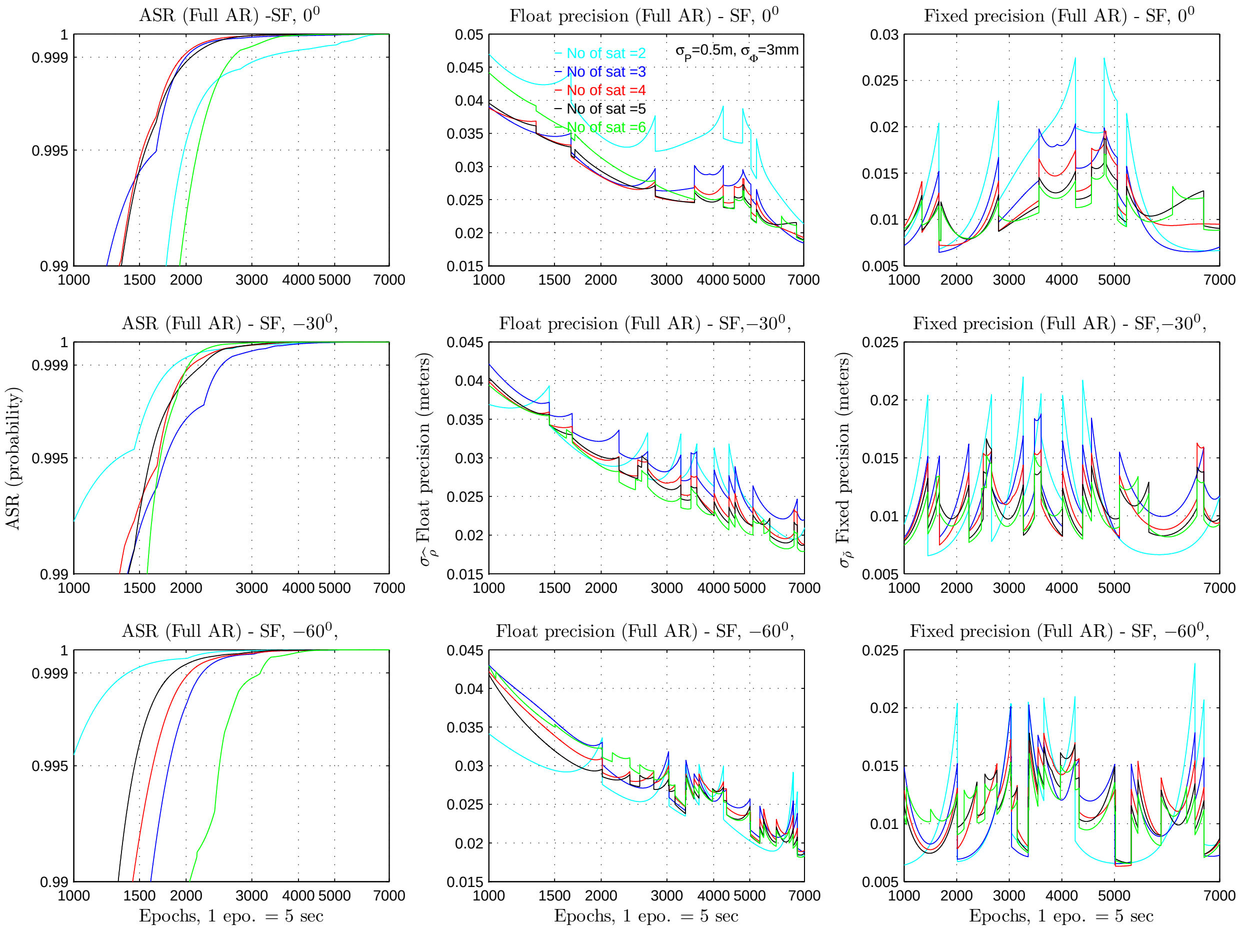

Figure D.8: **GPS only**, Single frequency, Geometry Free model, Ionosphere Fix scenario, **(satellite number is varied, measurement precision is held fixed)**- ASR and precision of float and fixed ambiguities are analyzed at 0^0, -30^0 and -60^0 degree latitude

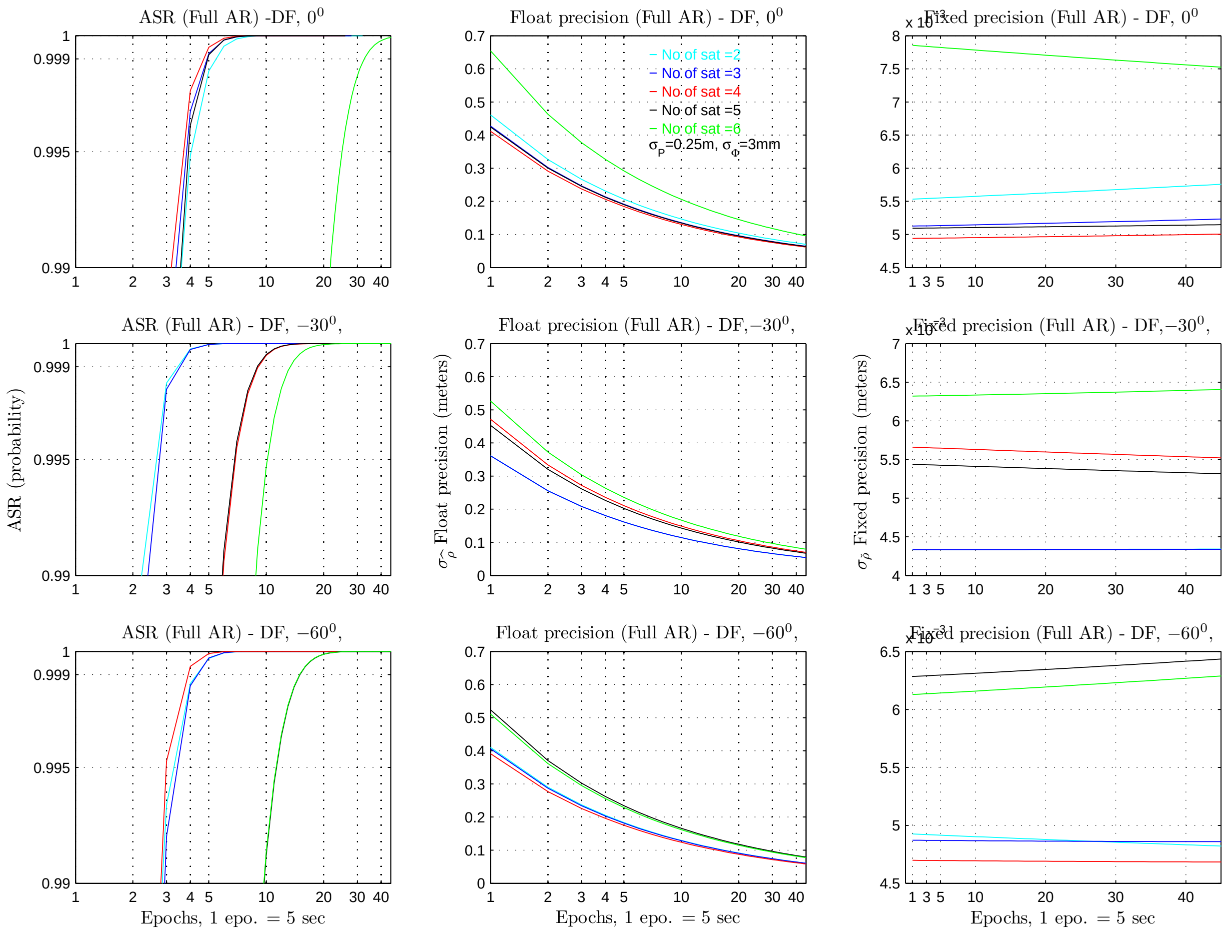

Figure D.9: **GPS only**, Dual frequency, Geometry Free model, Ionosphere Fix scenario, **(satellite number is varied, measurement precision is held fixed)**- ASR and precision of float and fixed ambiguities are analyzed at 0^0, -30^0 and -60^0 degree latitude

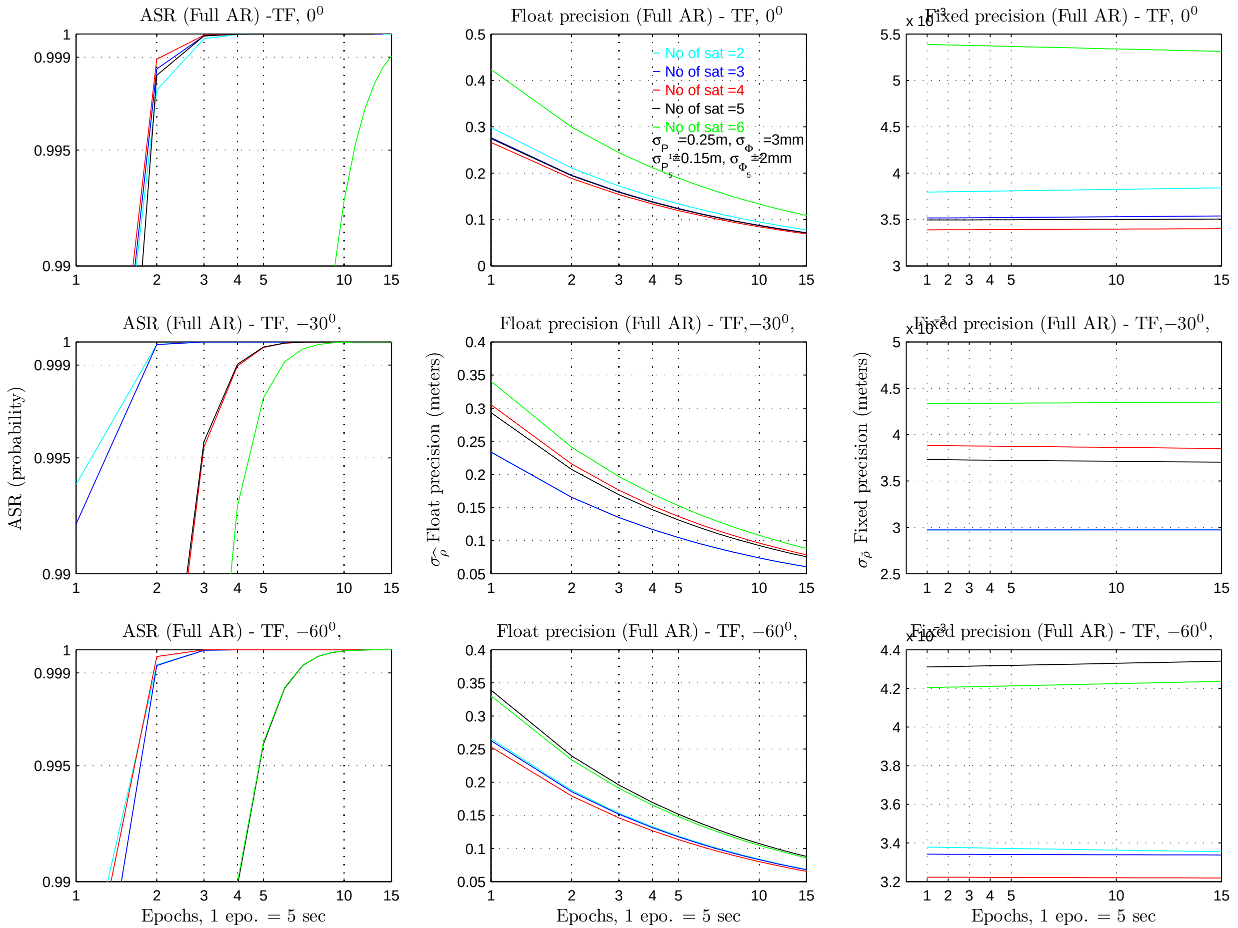

Figure D.10: **GPS only**, Triple frequency, Geometry Free model, Ionosphere Fix scenario, **(satellite number is varied, measurement precision is held fixed)**- ASR and precision of float and fixed ambiguities are analyzed at $0^0, -30^0$ and -60^0 degree latitude

E

Figures for Geometry Free model

In this appendix, the figures for Geometry Free model, when the rover coordinates are unknown are presented for three atmospheric scenarios, atmosphere known, atmosphere unknown and ionosphere weighted with troposphere unknown. For each atmosphere scenario, figures show Ambiguity Success Rate (ASR) and Double Difference (DD) precision of the between receiver-satellite ranges for ambiguity-fixed and float solution. The DD precision for the ranges is computed based on a volumetric, ADOP style expression given as $|Q_{\hat{\rho}}|^{1/2n}$ for the ambiguity float solution and $|Q_{\check{\rho}}|^{1/2n}$ for ambiguity fixed solution, $n = m - 1$ are the total number of satellites minus one. Different GNSS system considerations and combination are used, namely GPS only, Galileo only and GPS and Galileo together. Also different frequency combinations, namely, single, dual, triple and quadruple are considered.

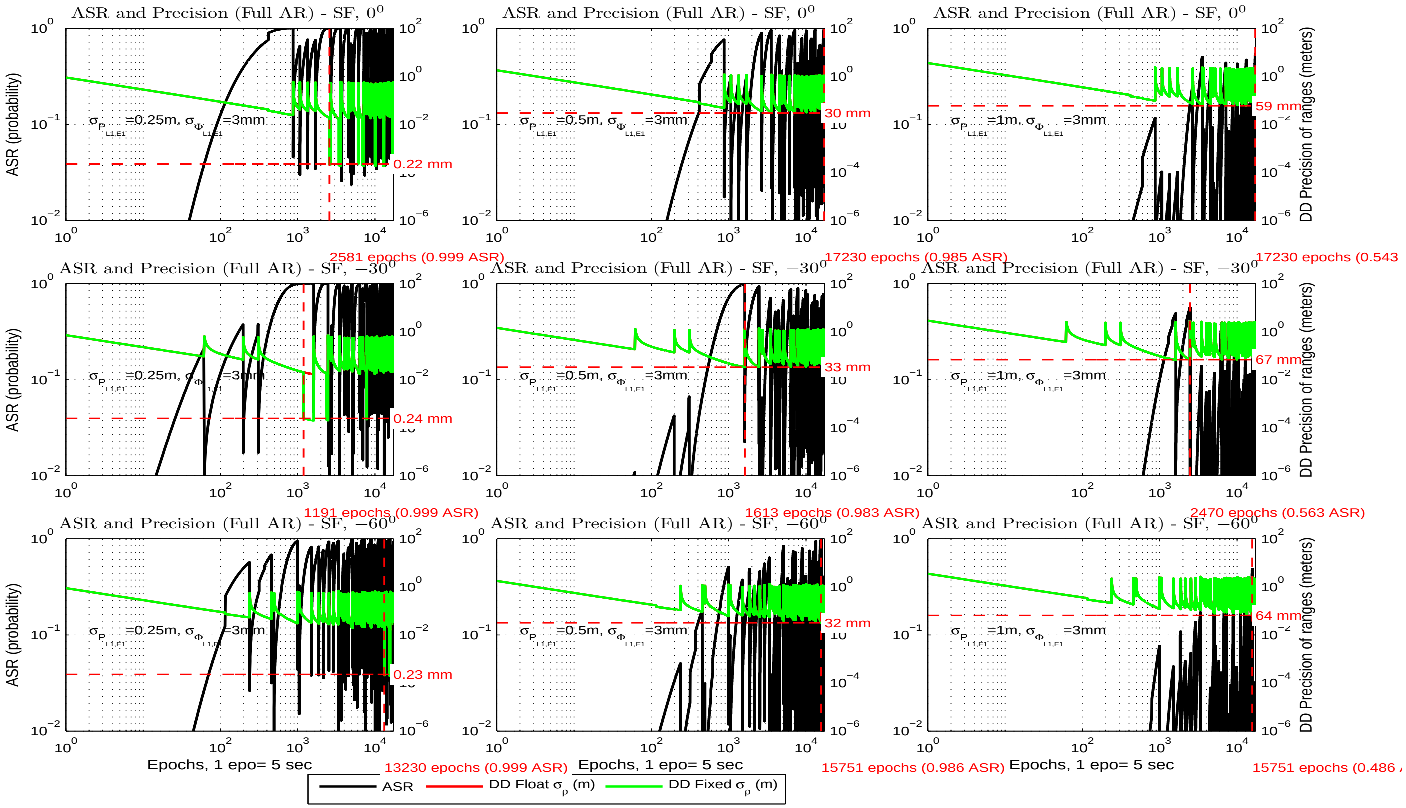

Figure E.1: **GPS only**, **Full AR**, Single frequency, Geometry Free model, Ionosphere Fix scenario **(measurement precision is varied, number of satellites are as available)-** ASR and precision of float and fixed ambiguities are analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

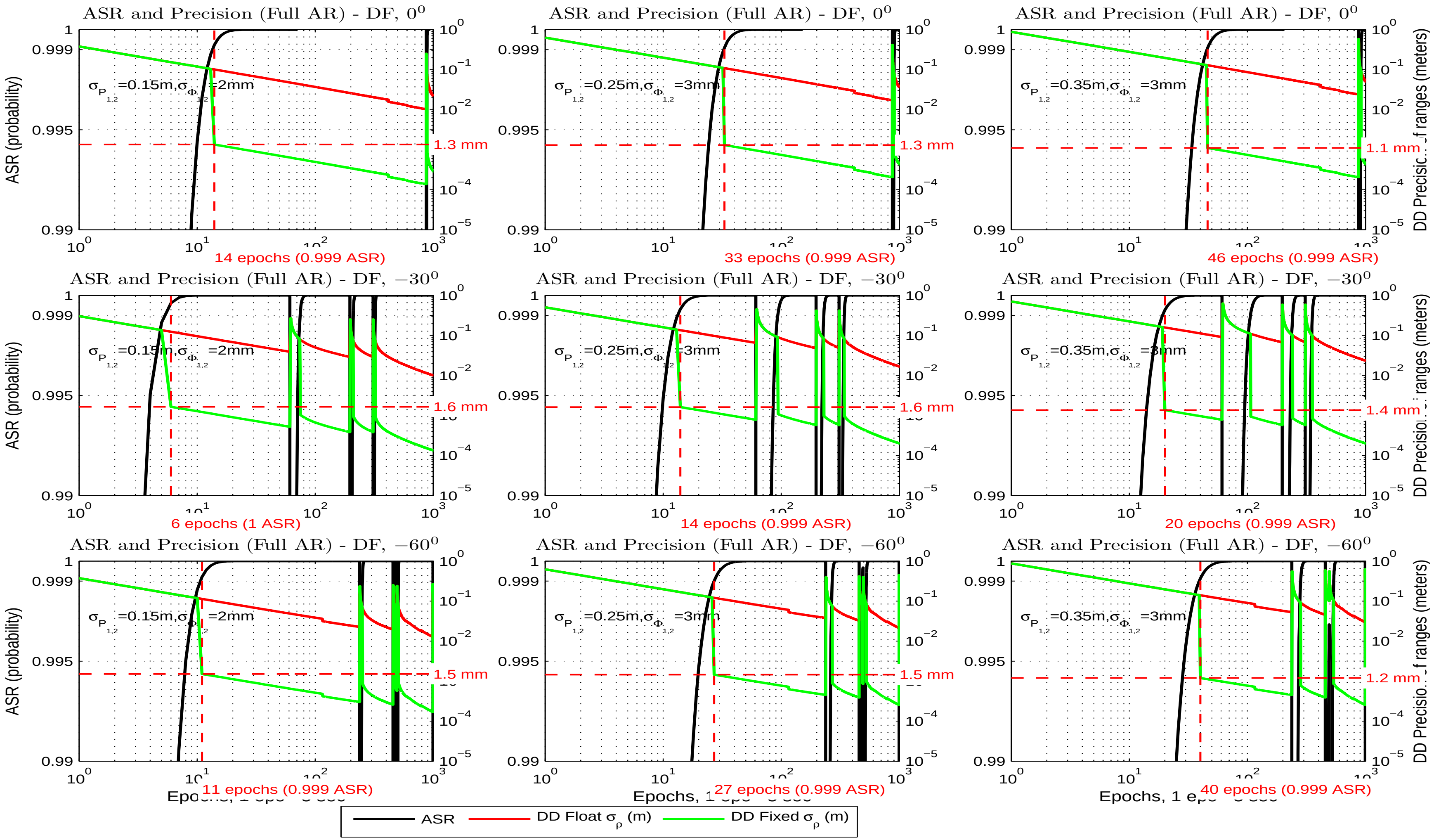

Figure E.2: **GPS only**, **Full AR**, Dual frequency, Geometry Free model, Ionosphere Fix scenario **(measurement precision is varied, number of satellites are as available)**- ASR and precision of float and fixed ambiguities are analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

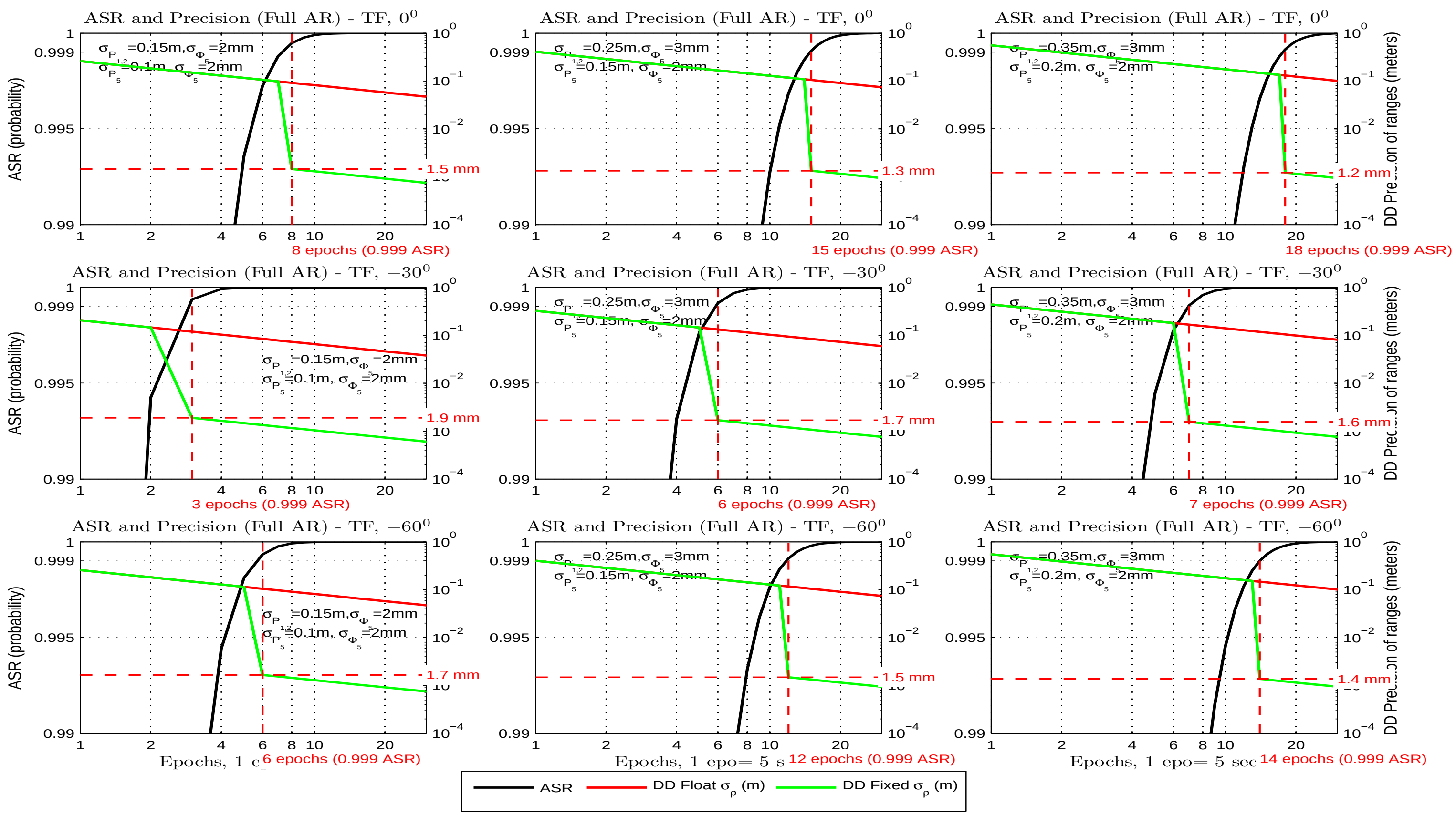

Figure E.3: **GPS only**, **Full AR**, Triple frequency, Geometry Free model, Ionosphere Fix scenario **(measurement precision is varied, number of satellites are as available)-** ASR and precision of float and fixed ambiguities are analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

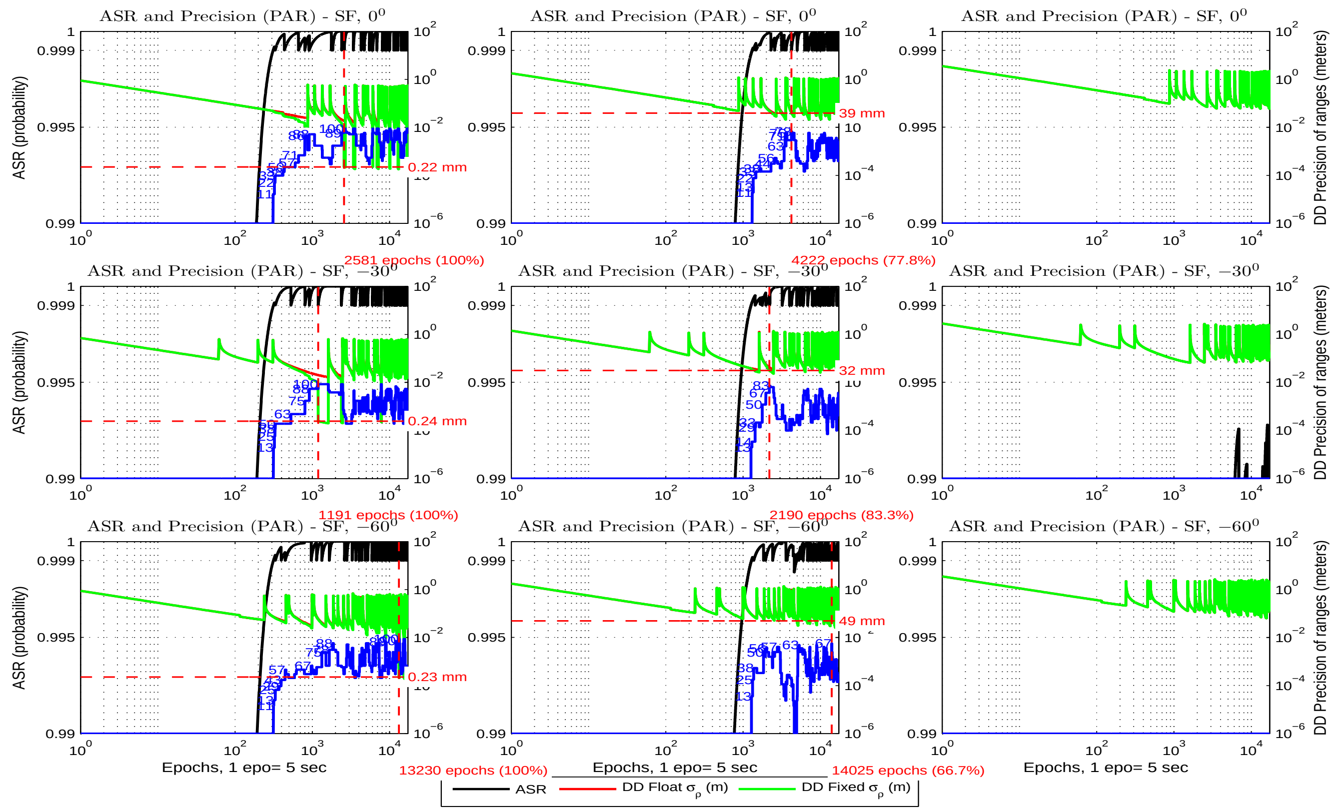

Figure E.4: **GPS only**, Single frequency, **PAR**, Ionosphere known scenario, **(measurement precision is varied, number of satellites are as available)-** ASR and precision of float and fixed ambiguities are analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis).

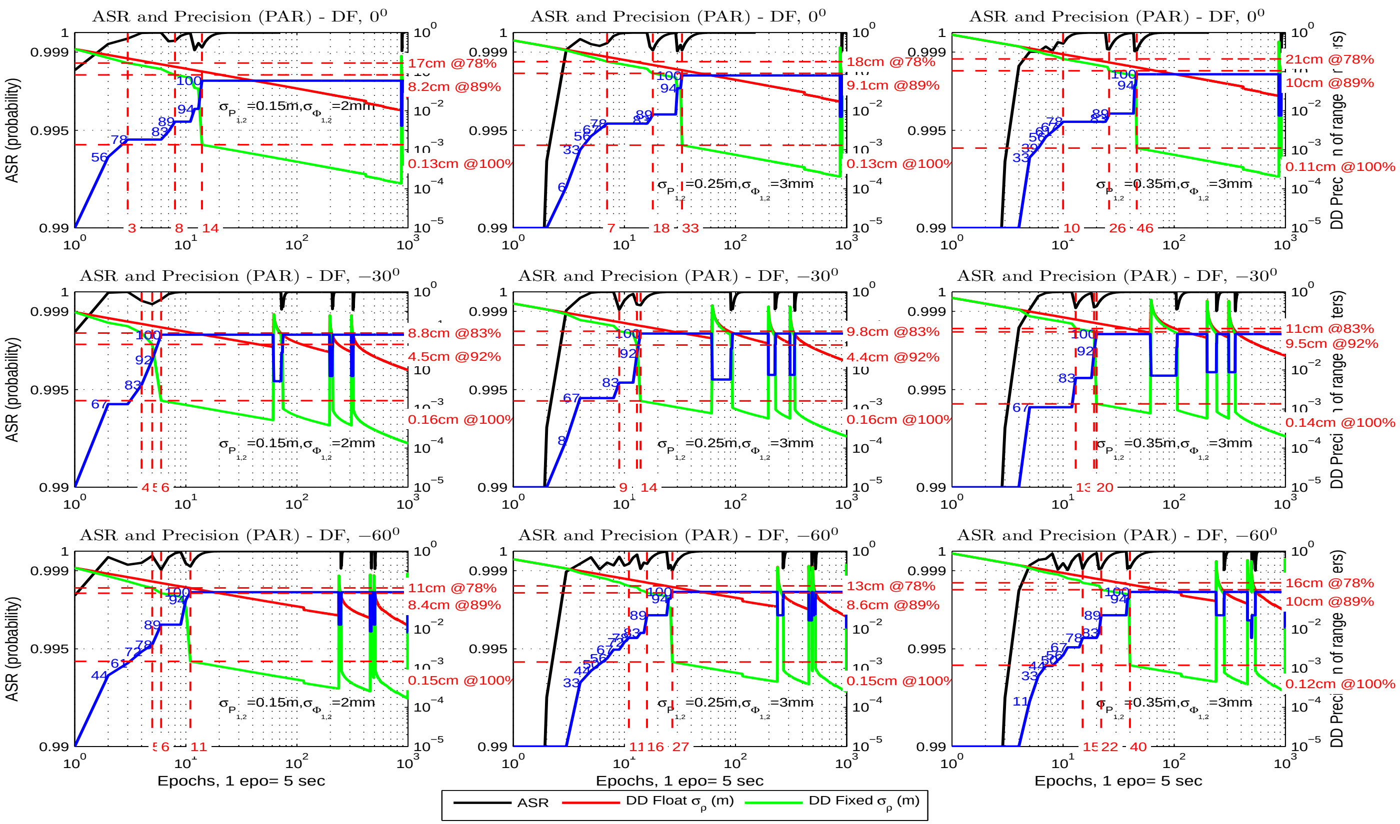

Figure E.5: **GPS only**, Dual frequency, **PAR**, Ionosphere known scenario, **(measurement precision is varied, number of satellites are as available)**- ASR and precision of float and fixed ambiguities are analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis). Additionally, two more values of around 80% and 90% of ambiguities fixed by PAR are indicated.

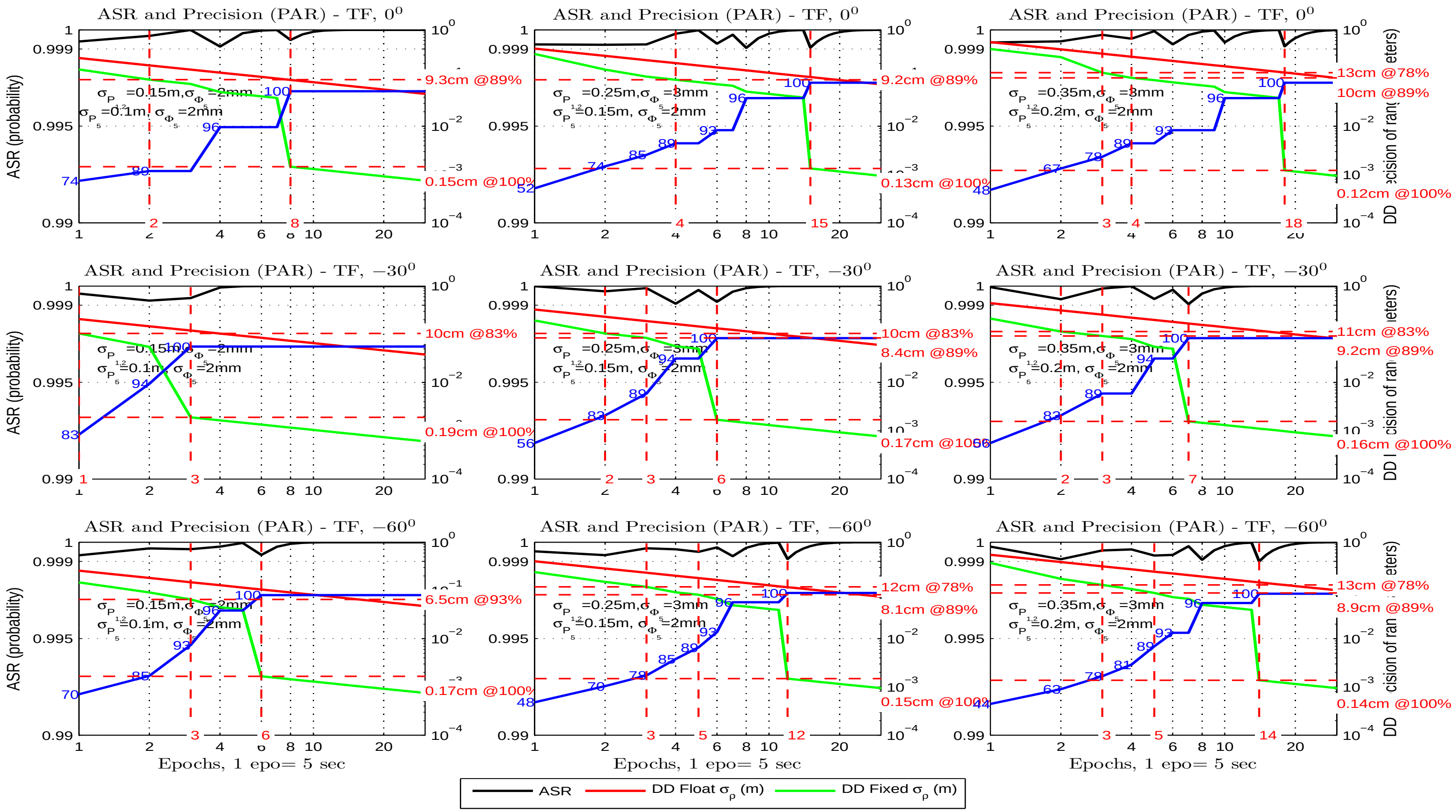

Figure E.6: **GPS only**, Triple frequency, **PAR**, Ionosphere known scenario, **(measurement precision is varied, number of satellites are as available)-** ASR and precision of float and fixed ambiguities are analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis). Additionally, two more values of around 80% and 90% of ambiguities fixed by PAR are indicated.

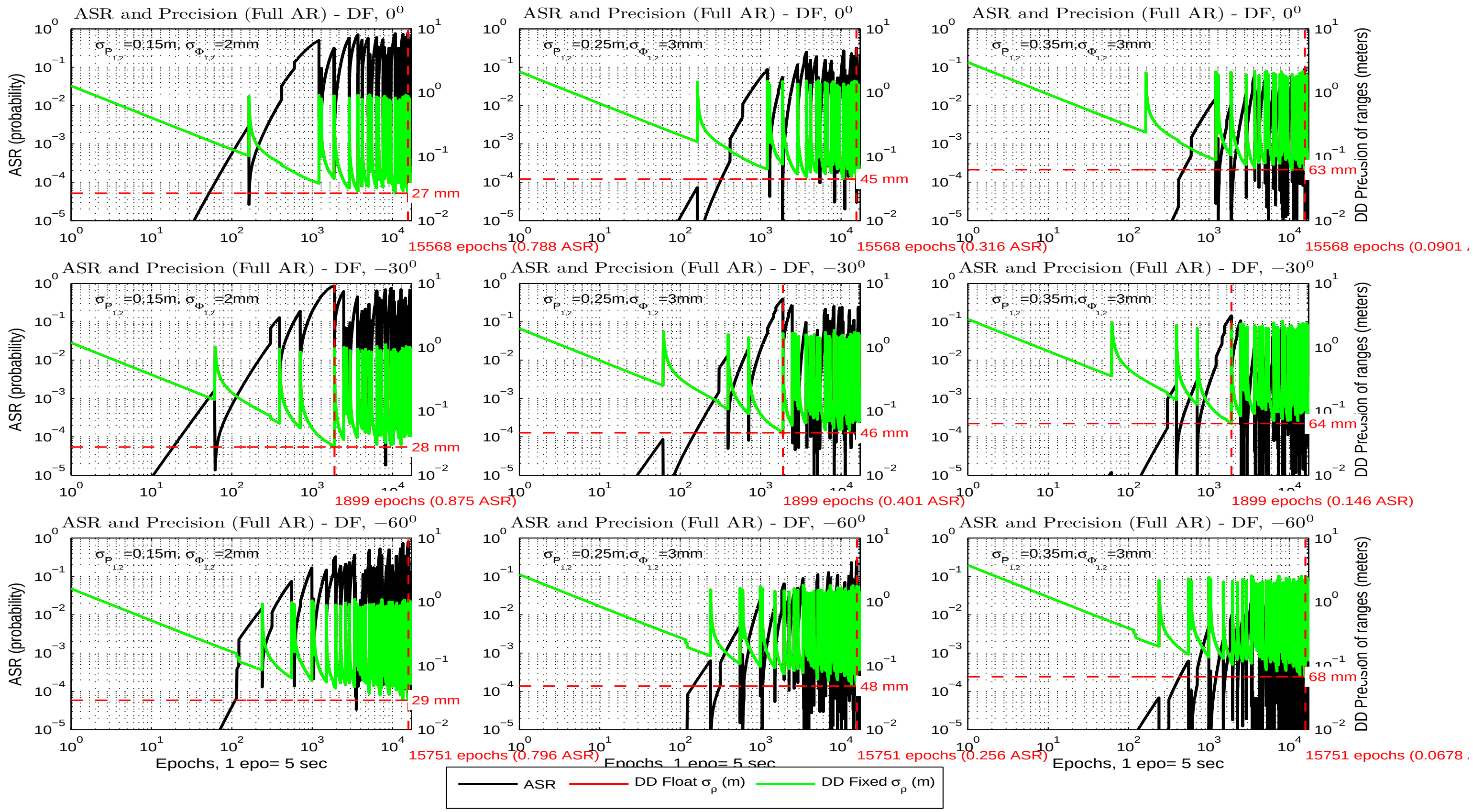

Figure E.7: **GPS only**, **Full AR**, Dual frequency, Geometry Free model, Ionosphere unknown scenario, **(measurement precision is varied, number of satellites are as available)**- ASR and precision of float and fixed ambiguities are analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

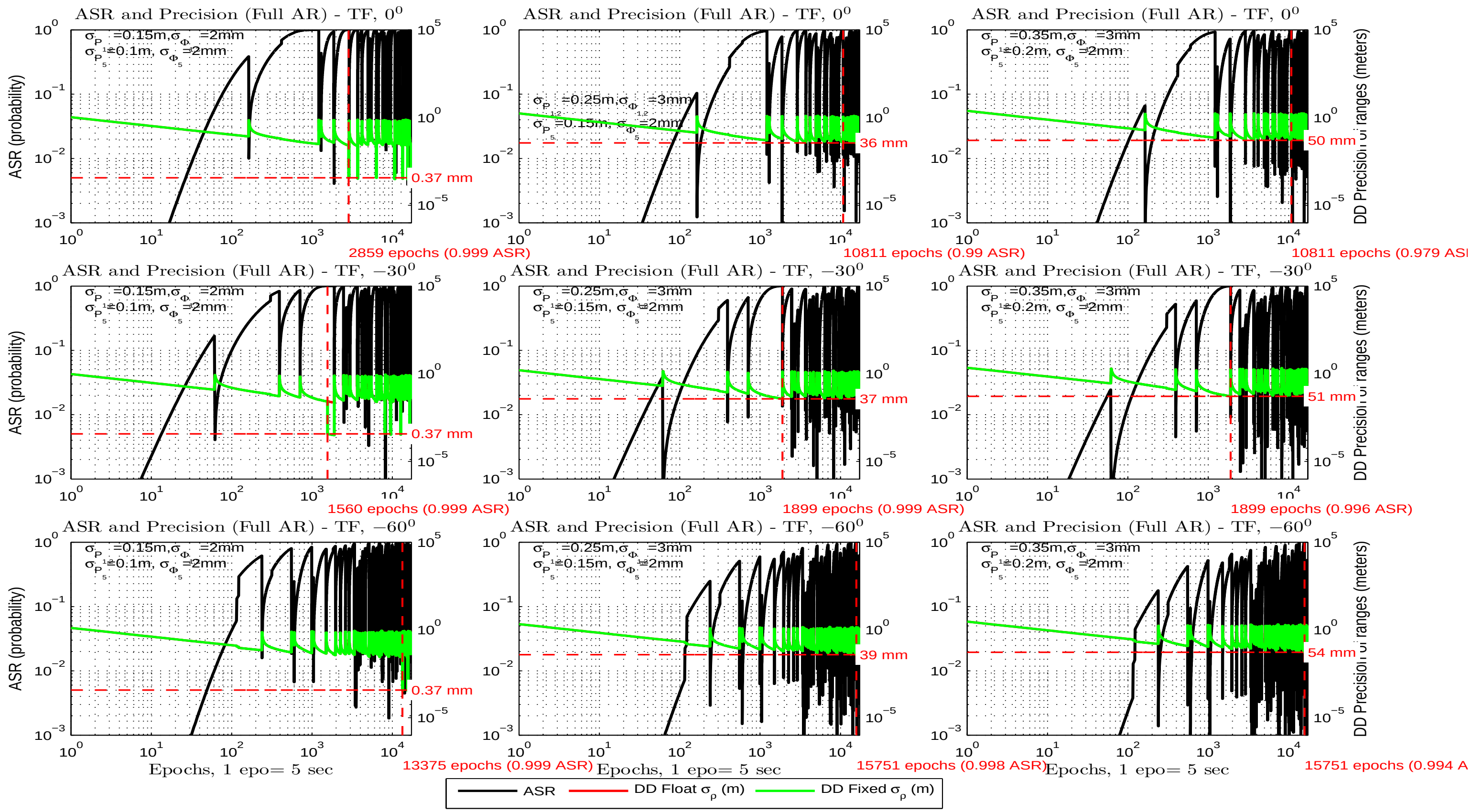

Figure E.8: **GPS only**, **Full AR**, Triple frequency, Geometry Free model, Ionosphere unknown scenario, **(measurement precision is varied, number of satellites are as available)-** ASR and precision of float and fixed ambiguities are analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

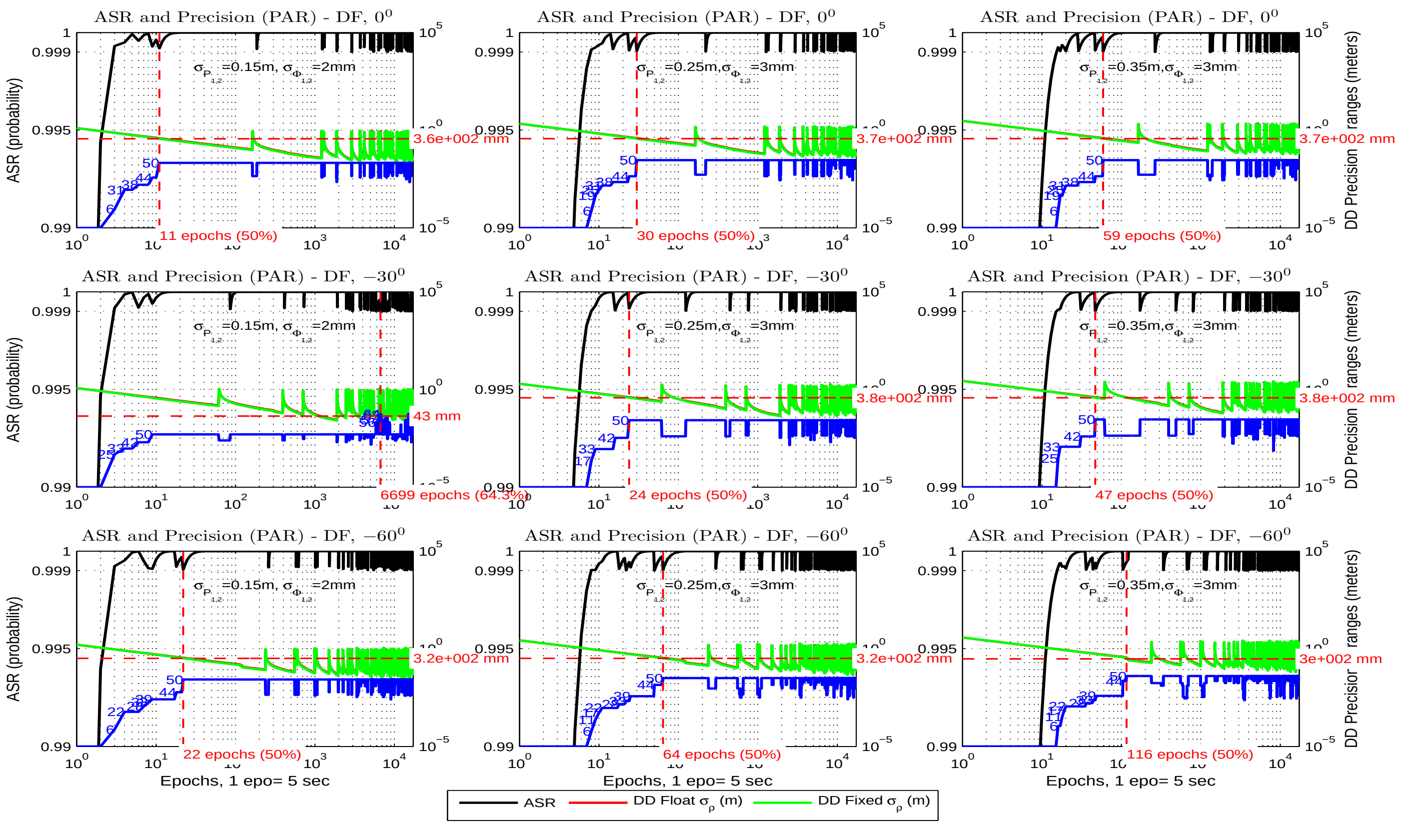

Figure E.9: **GPS only**, Dual frequency, **PAR**, Ionosphere unknown scenario, **(measurement precision is varied, number of satellites are as available)**- ASR and precision of float and fixed ambiguities are analyzed at $0°$, $-30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis).

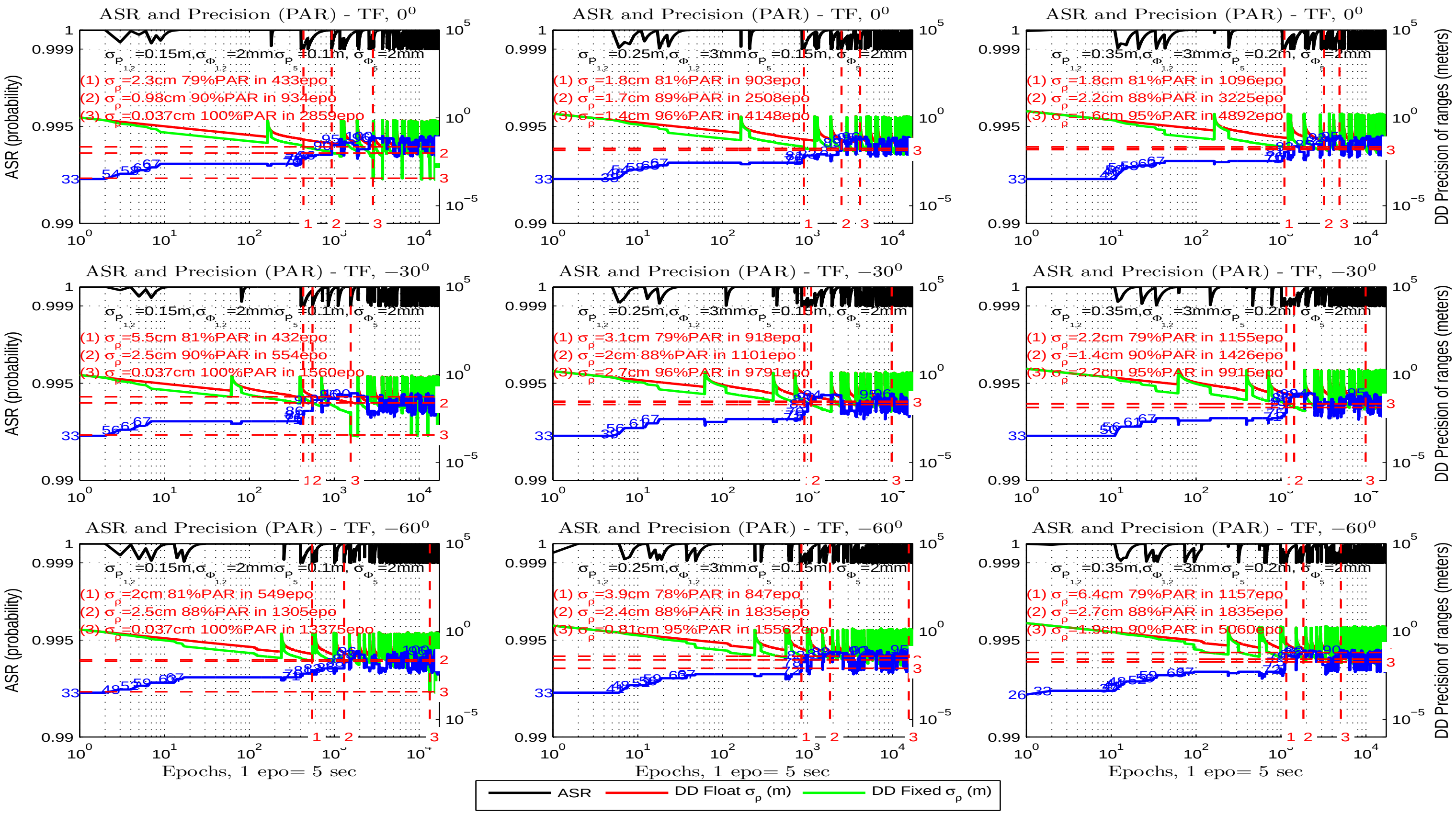

Figure E.10: **GPS only**, Triple frequency, **PAR**, Ionosphere unknown scenario, **(measurement precision is varied, number of satellites are as available)-** ASR and precision of float and fixed ambiguities are analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis). Additionally, two more values of around 80% and 90% of ambiguities fixed by PAR are indicated.

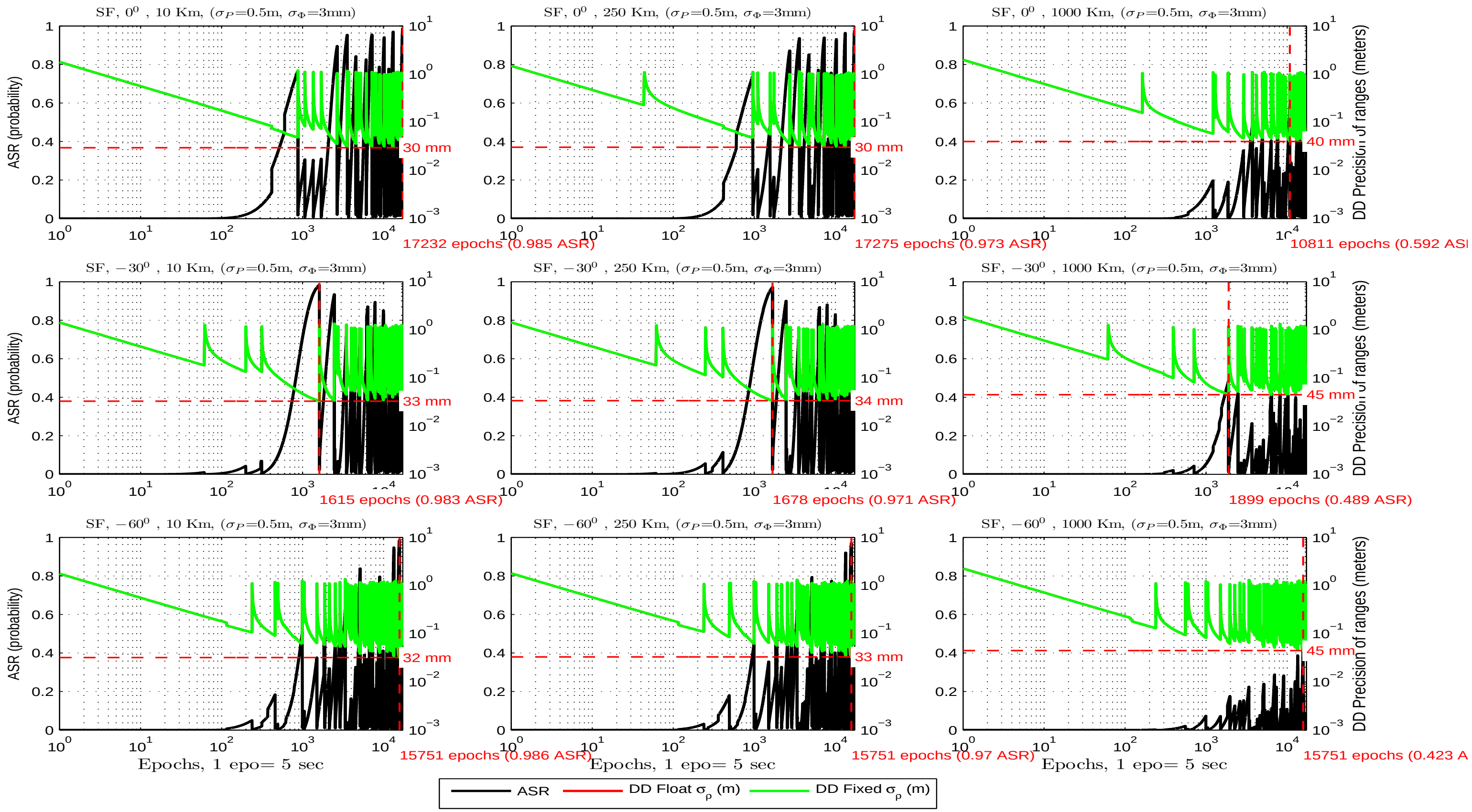

Figure E.11: **GPS only**, **Full AR**, Single frequency, Geometry Free model, Ionosphere Weighted scenario, (measurement precision and satellite number is held fixed), **baseline length is varied from 1 Km to 1000 Km**- ASR and Gain analysis at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

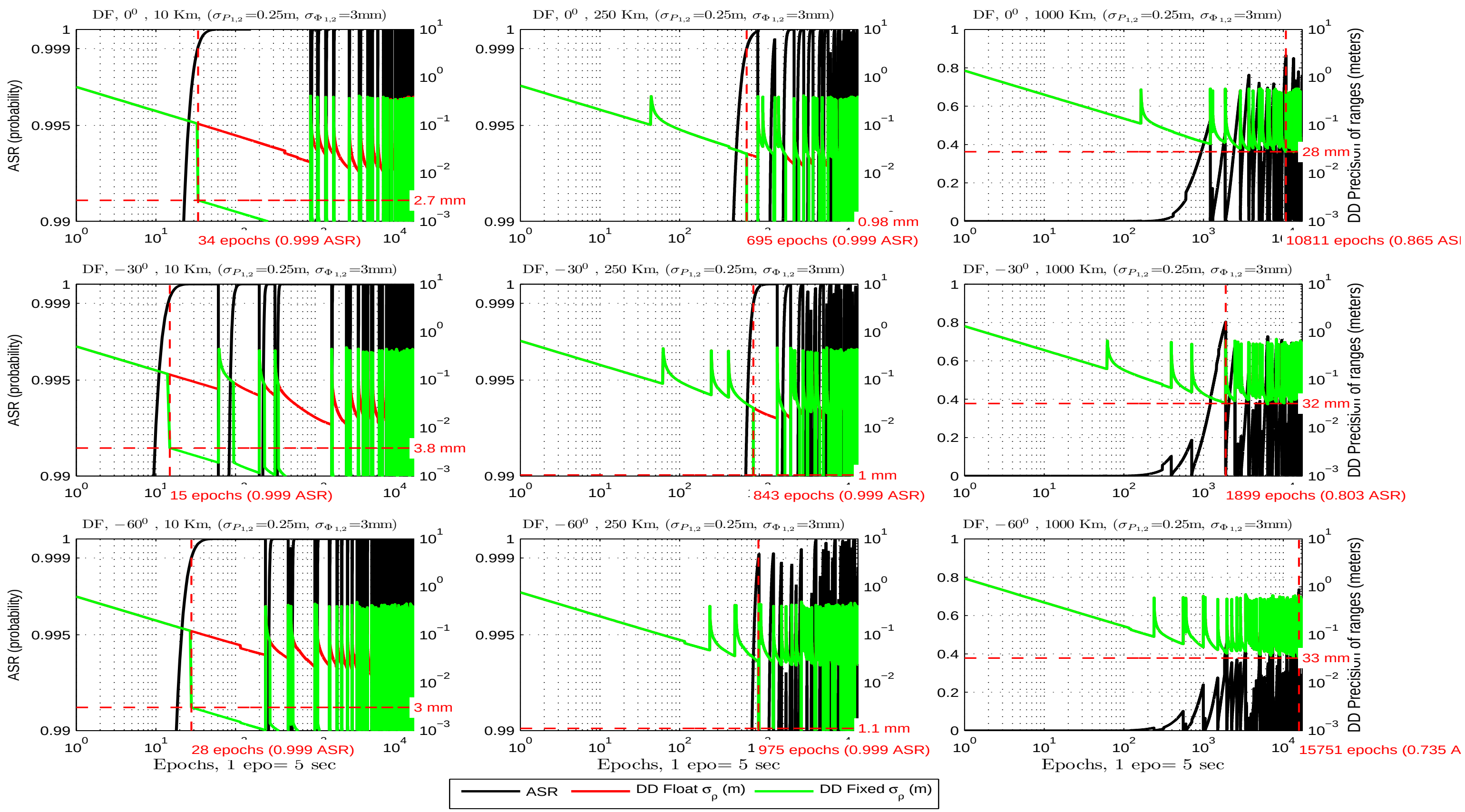

Figure E.12: **GPS only**, **Full AR**, Dual frequency, Geometry Free model, Ionosphere Weighted scenario, (measurement precision and satellite number is held fixed), **baseline length is varied from 1 Km to 1000 Km**- ASR and Gain analysis at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

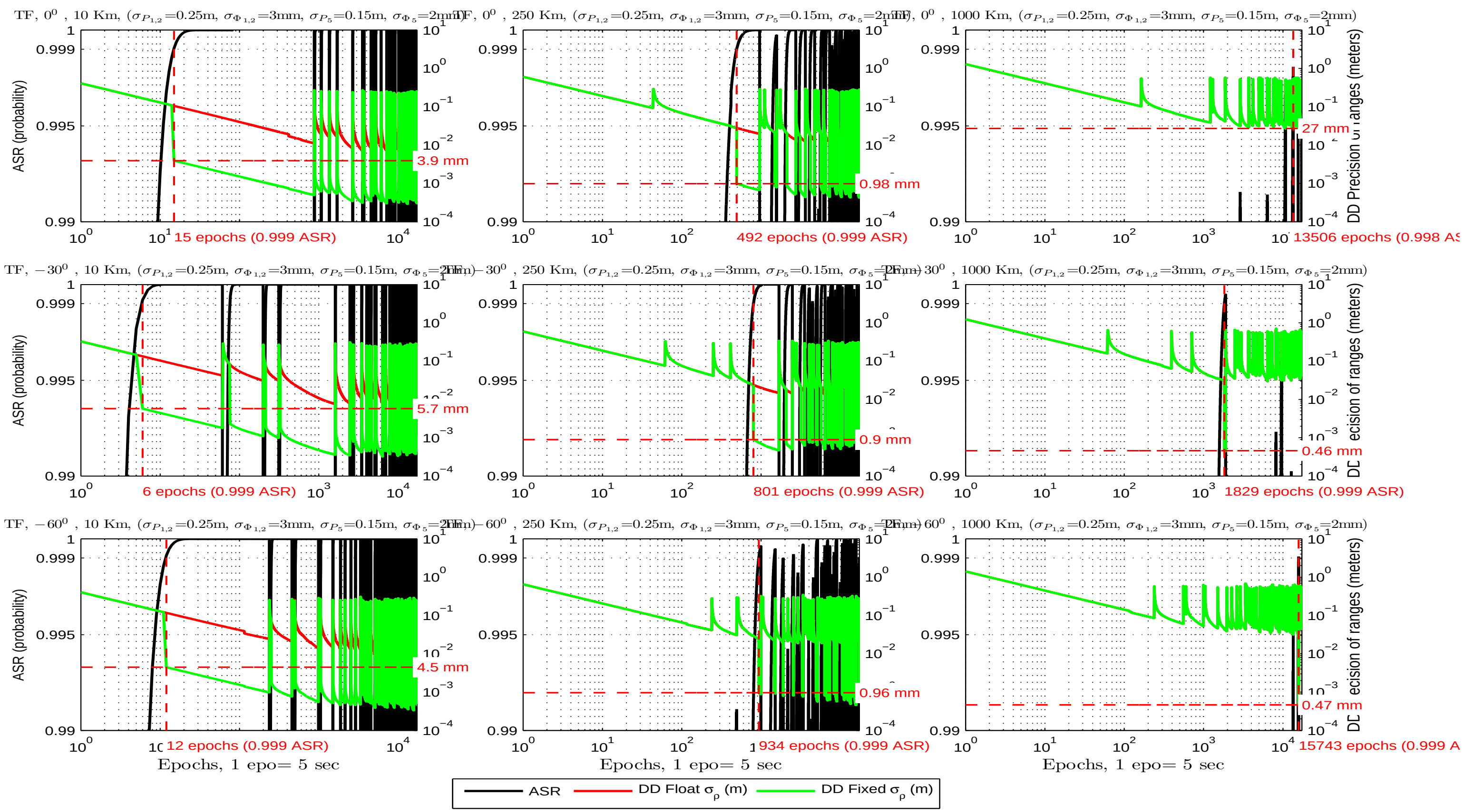

Figure E.13: **GPS only**, **Full AR**, Triple frequency, Geometry Free model, Ionosphere Weighted scenario, (measurement precision and satellite number is held fixed), **baseline length is varied from 1 Km to 1000 Km**- ASR and Gain analysis at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 0.999 ASR is reached and corresponding value of DD fixed-precision of ranges obtained (on y-axis). If 0.999 ASR is not achieved, maximum ASR achieved is marked.

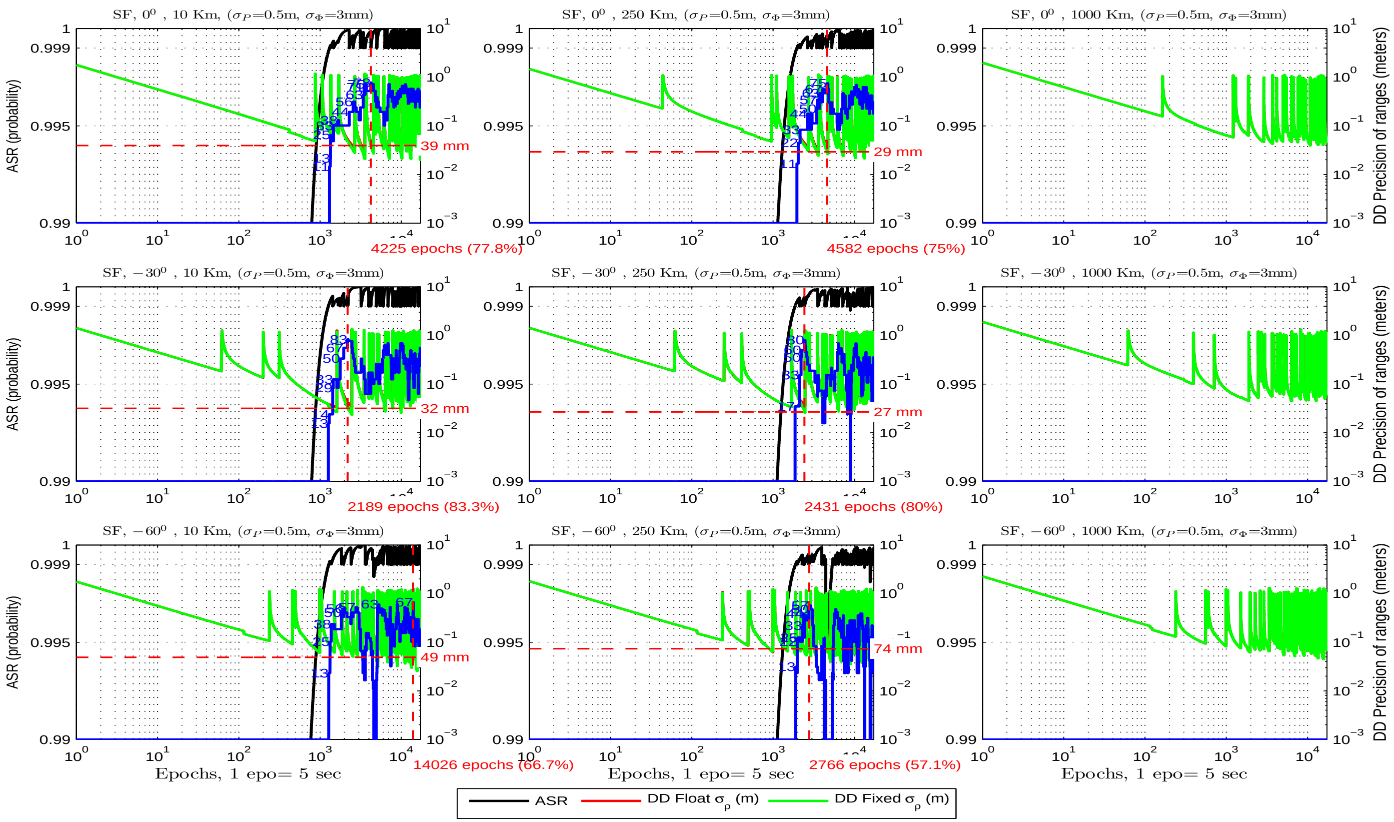

Figure E.14: **GPS only**, **PAR**, Single frequency, Geometry Free model, Ionosphere Weighted scenario, (measurement precision and satellite number is held fixed), **baseline length is varied from 1 Km to 1000 Km**- ASR and Gain analysis at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis).

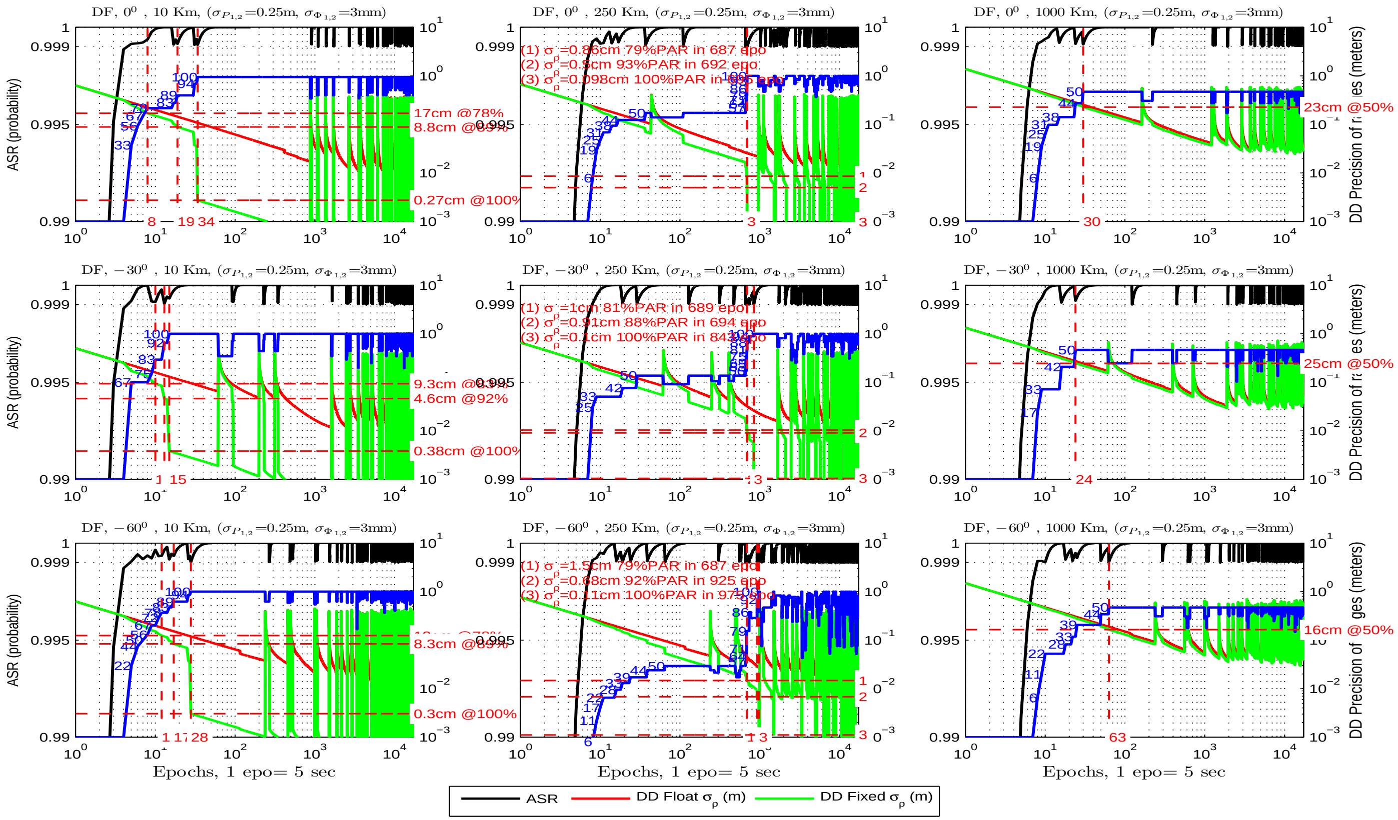

Figure E.15: **GPS only**, **PAR**, Dual frequency, Geometry Free model, Ionosphere Weighted scenario, (measurement precision and satellite number is held fixed), **baseline length is varied from 1 Km to 1000 Km**- ASR and Gain analysis at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis). Additionally, two more values of around 80% and 90% of ambiguities fixed by PAR are indicated.

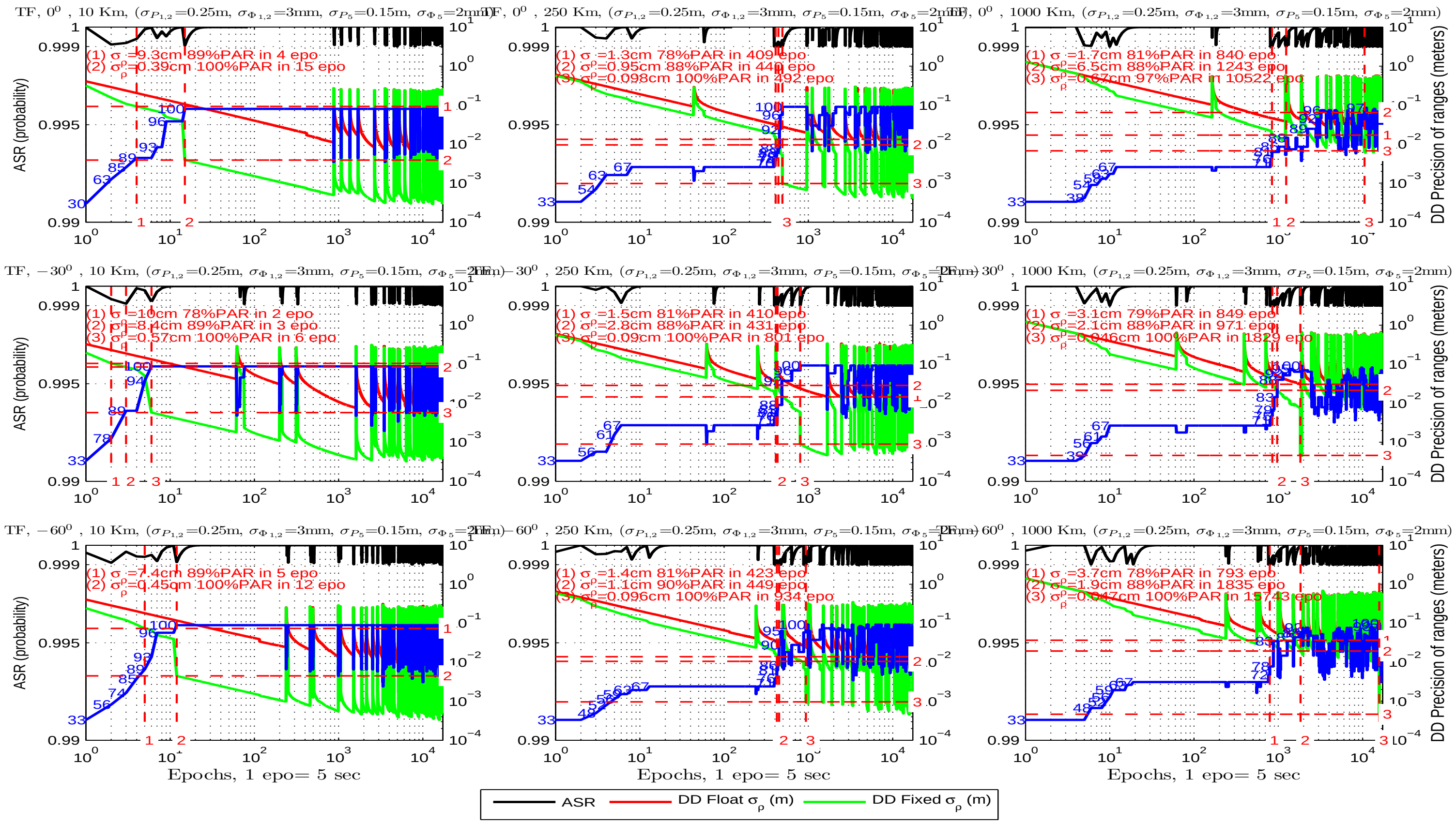

Figure E.16: **GPS only**, **PAR**, Triple frequency, Geometry Free model, Ionosphere Weighted scenario, (measurement precision and satellite number is held fixed), **baseline length is varied from 1 Km to 1000 Km**- ASR and Gain analysis at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for 100% of ambiguities to be fixed in PAR and corresponding value of DD fixed-precision of ranges obtained (on y-axis). Additionally, two more values of around 80% and 90% of ambiguities fixed by PAR are indicated.

F

Figures for Geometry Fixed model

In this appendix, the figures for Geometry Based model, with coordinates being known are presented for three atmospheric scenarios, atmosphere known, atmosphere unknown and ionosphere weighted with troposphere unknown. For each atmosphere scenario, figures show Ambiguity Success Rate (ASR) and Double Difference (DD) precision of the ionosphere for ambiguity-fixed and float solution. The DD precision for the ionosphere is computed based on a volumetric, ADOP style expression given as $|Q_{\widehat{b}\widehat{b}}|^{1/2n}$ for the ambiguity float solution and $|Q_{\widetilde{b}\widetilde{b}}|^{1/2n}$ for ambiguity fixed solution, n is equal to total number of satellites minus one. Different GNSS system considerations and combination are used, namely GPS only, Galileo only and GPS and Galileo together. Also different frequency combinations, namely, single, dual, triple and quadruple are considered. For ionosphere weighted scenario, the precision of the troposphere is presented in the figures instead of precision for ionosphere due to the ionosphere weighted model considered for simulations.

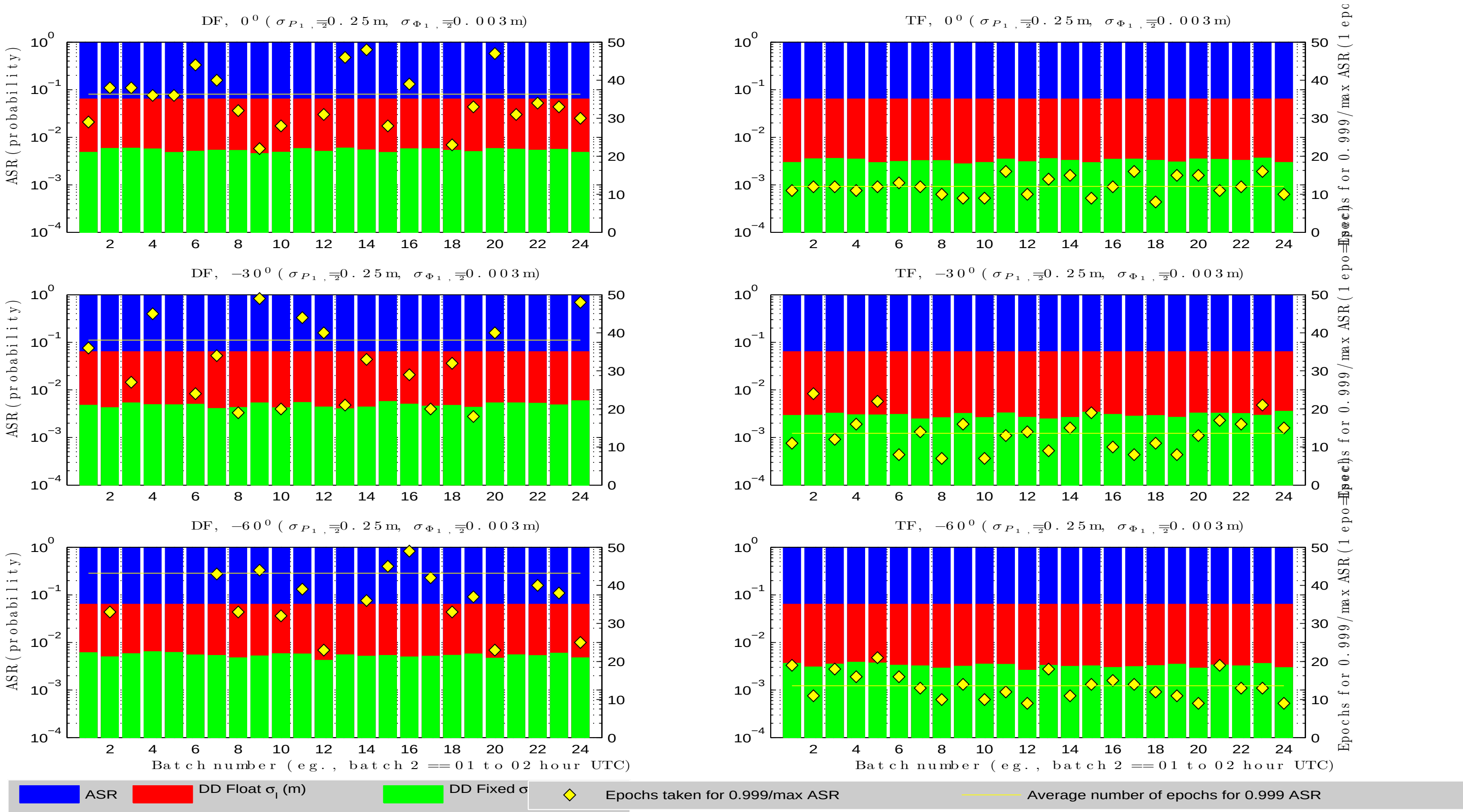

Figure F.1: **GPS only**, Atmosphere float scenario, **Geometry Fixed model**, **dual and triple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR and, float and fixed precision of ionosphere.

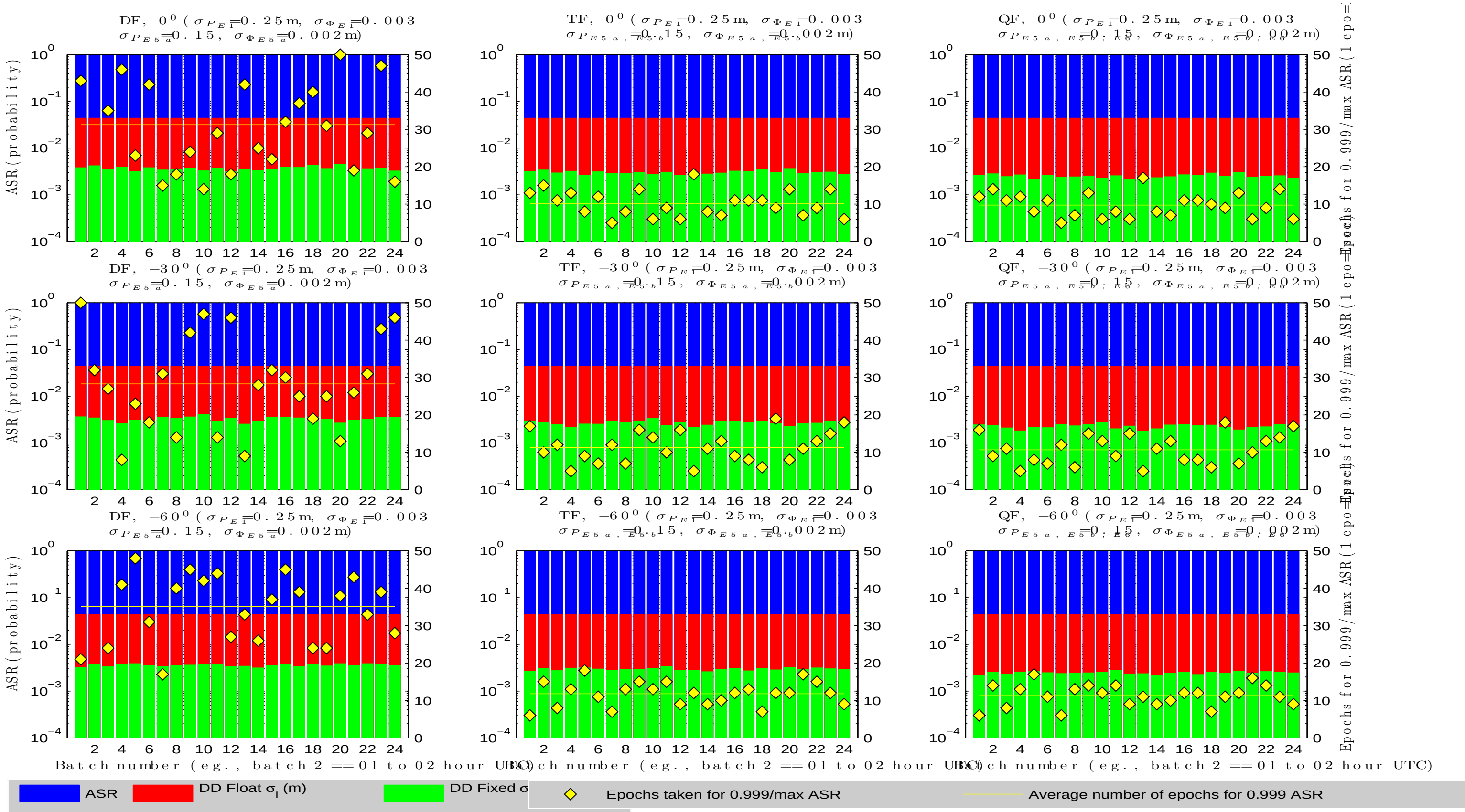

Figure F.2: **Galileo only**, Atmosphere float scenario, **Geometry Fixed model**, **dual, triple and quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR and, float and fixed precision of ionosphere.

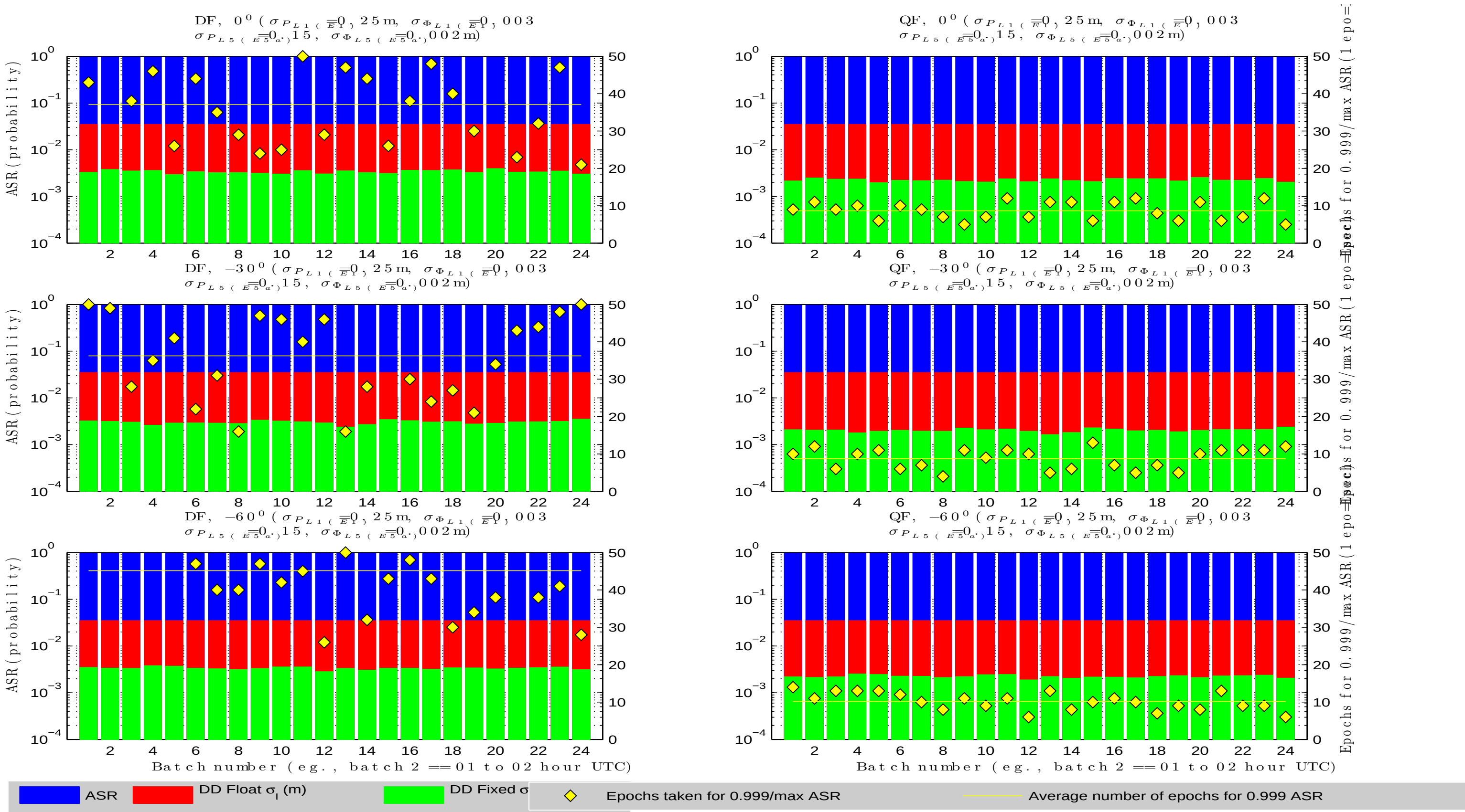

Figure F.3: **GPS + Galileo**, Atmosphere float scenario, **Geometry Fixed model**, **dual and quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR and, float and fixed precision of ionosphere.

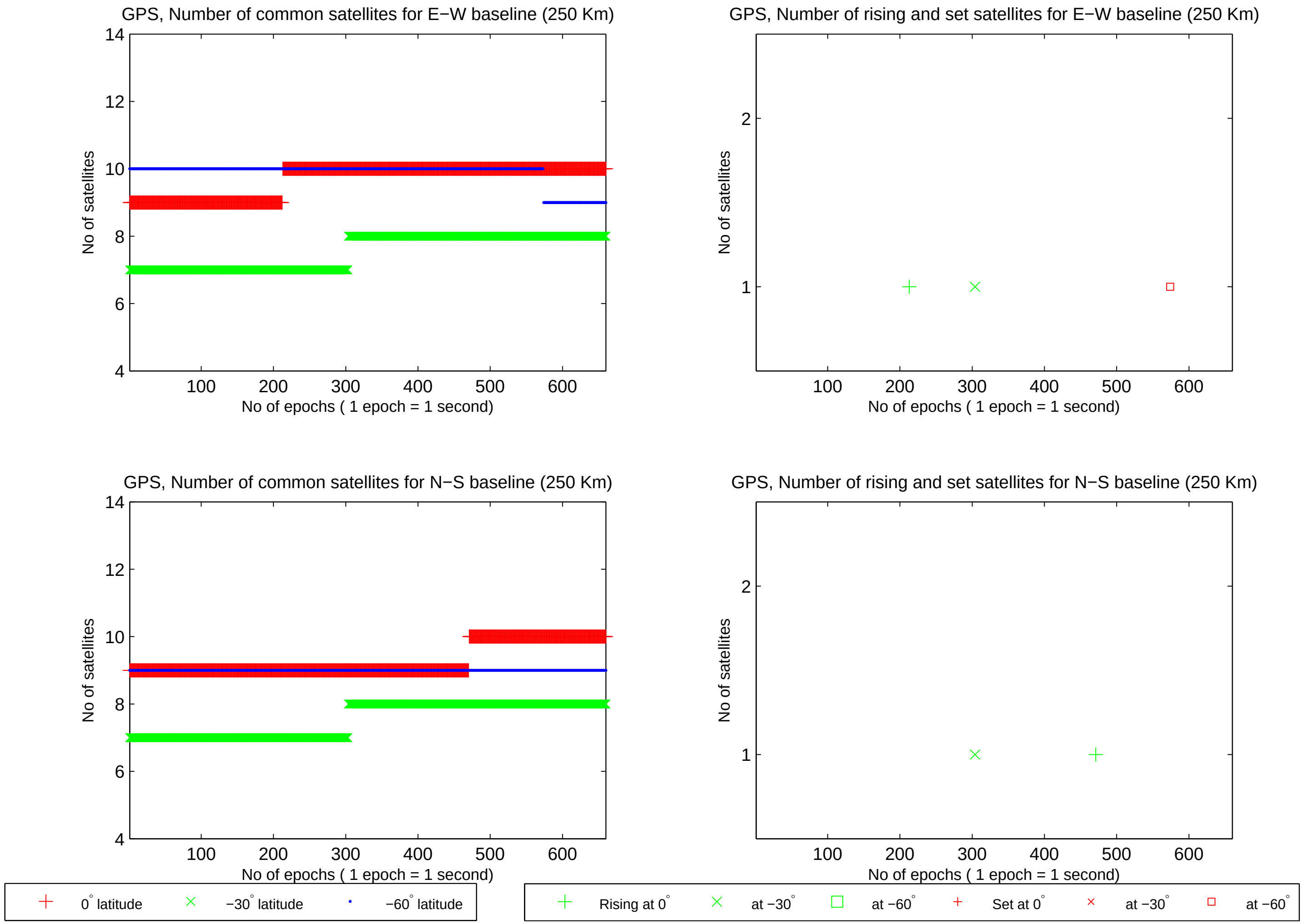

Figure F.4: Number of satellites for **GPS only**, 250 Km East-West and North-South baseline at $0°, -30°$ and $-60°$ degree latitude.

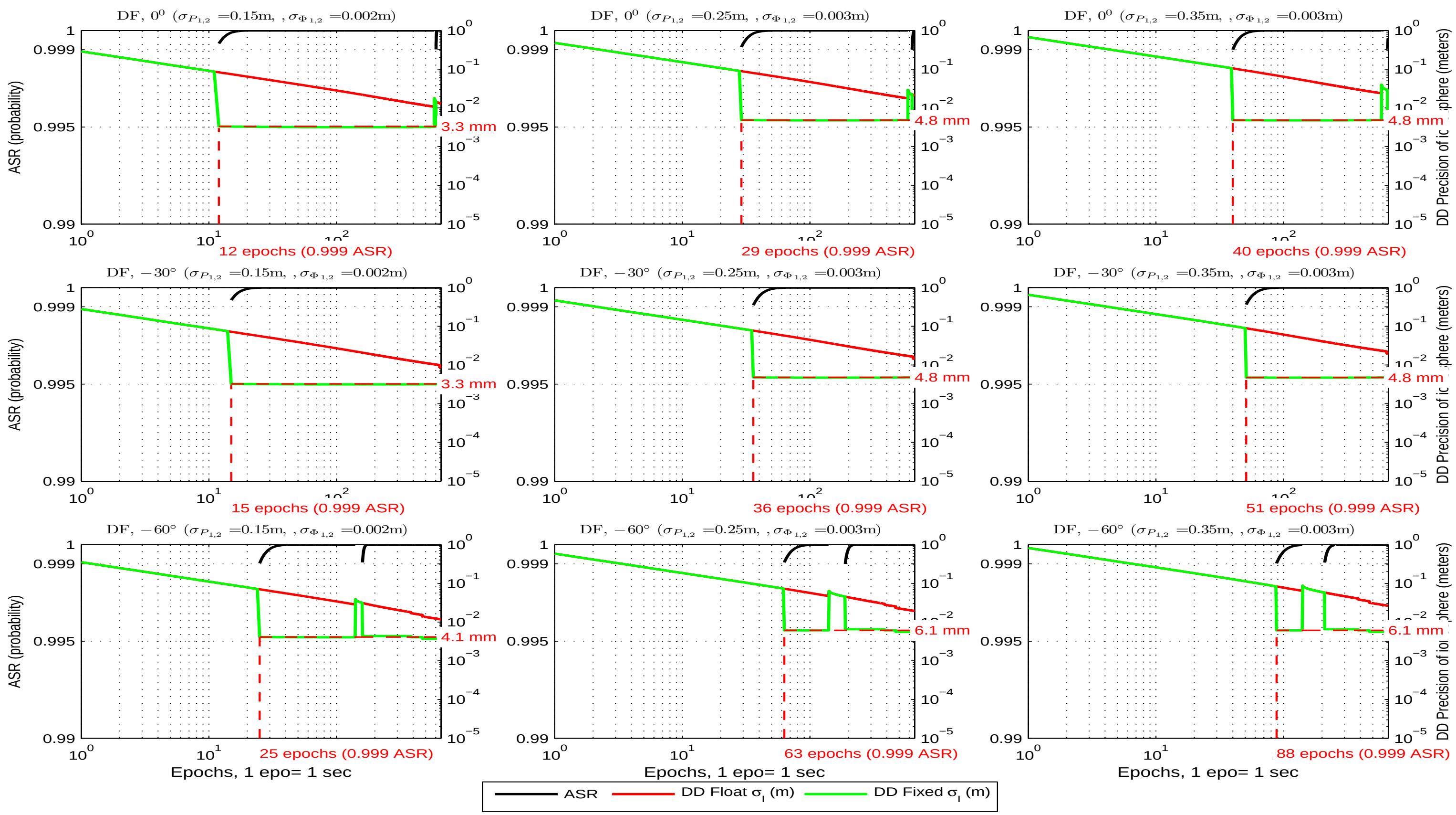

Figure F.5: **GPS only**, Dual frequency, **Full AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

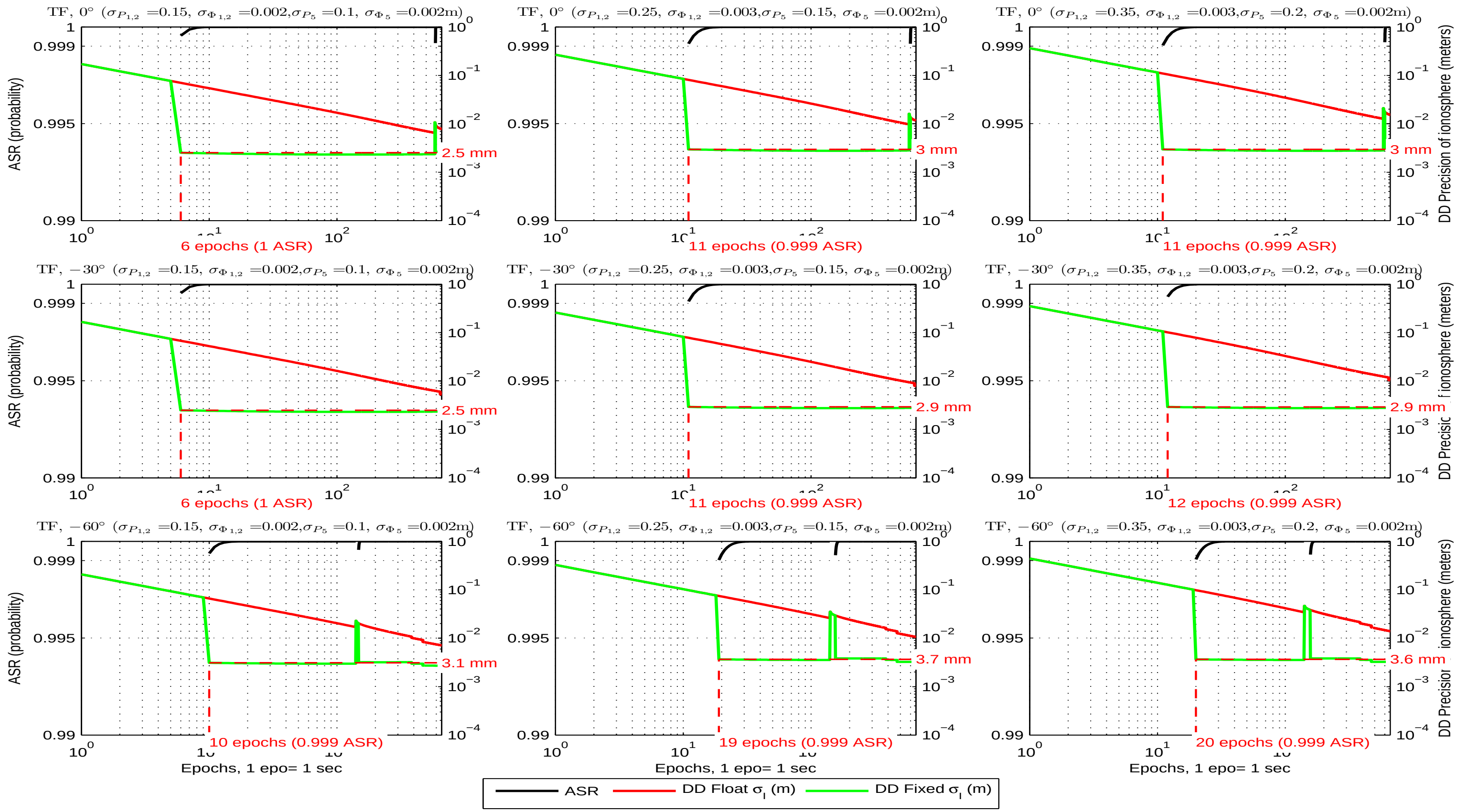

Figure F.6: **GPS only**, Triple frequency, **Full AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

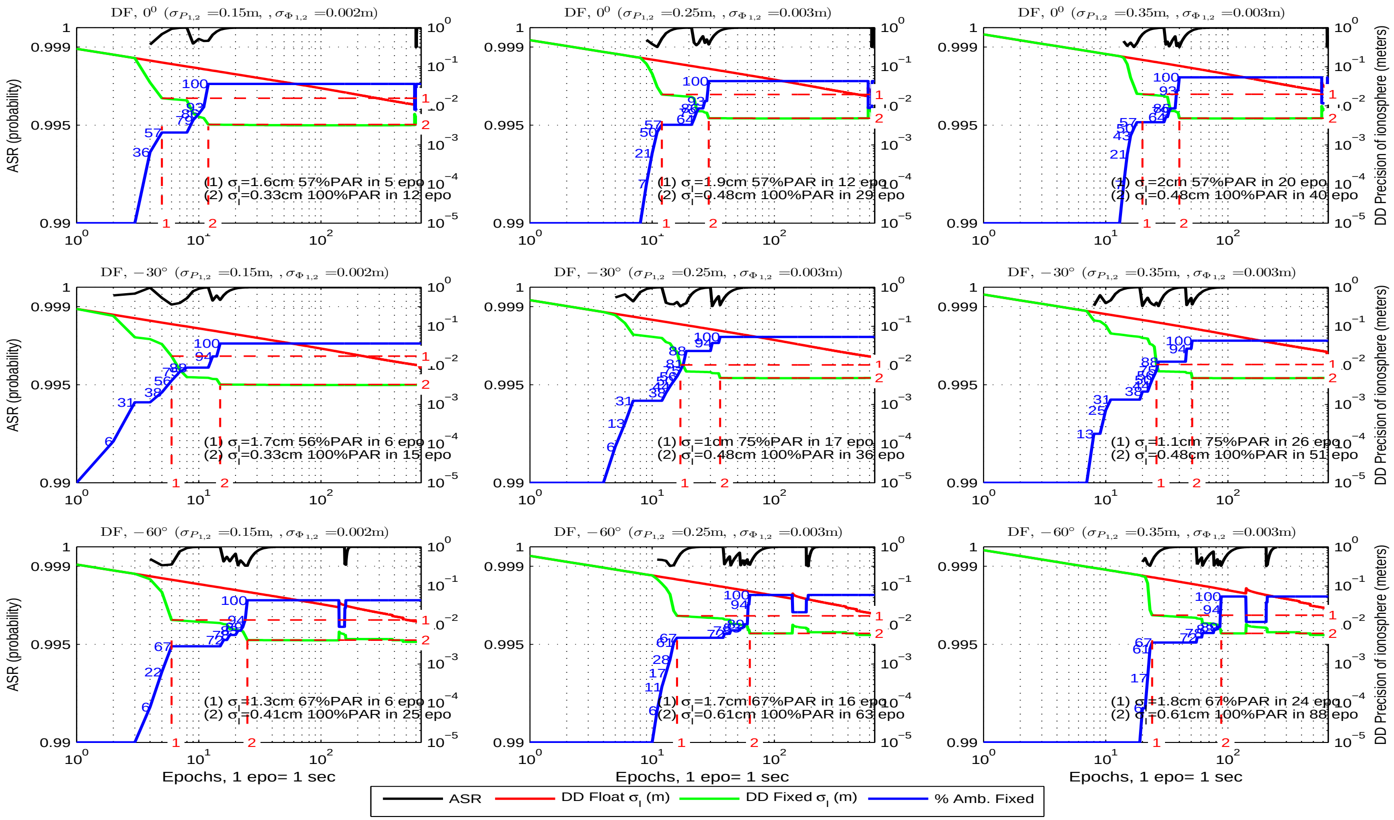

Figure F.7: **GPS only**, Dual frequency, **Partial AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

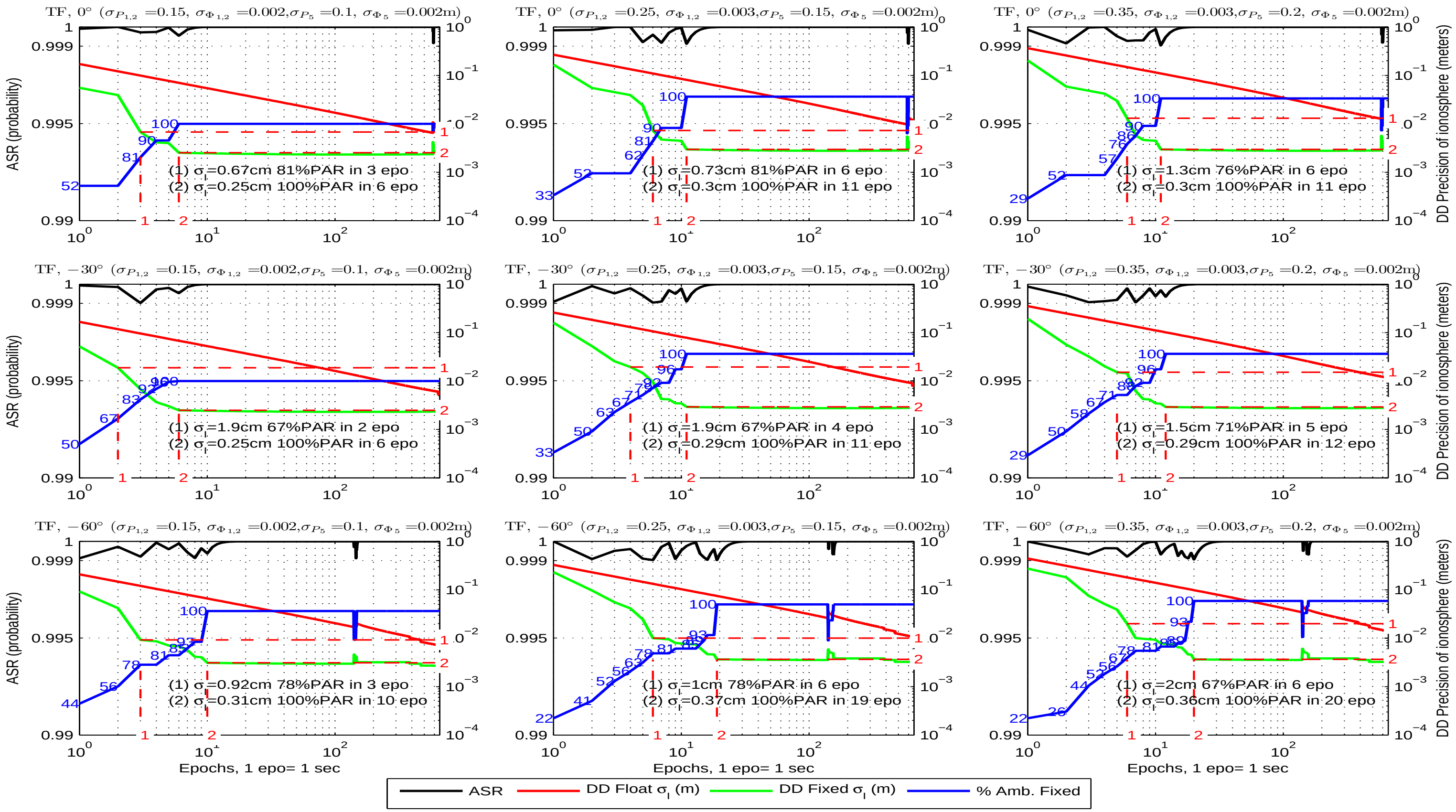

Figure F.8: **GPS only**, Triple frequency, **Partial AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

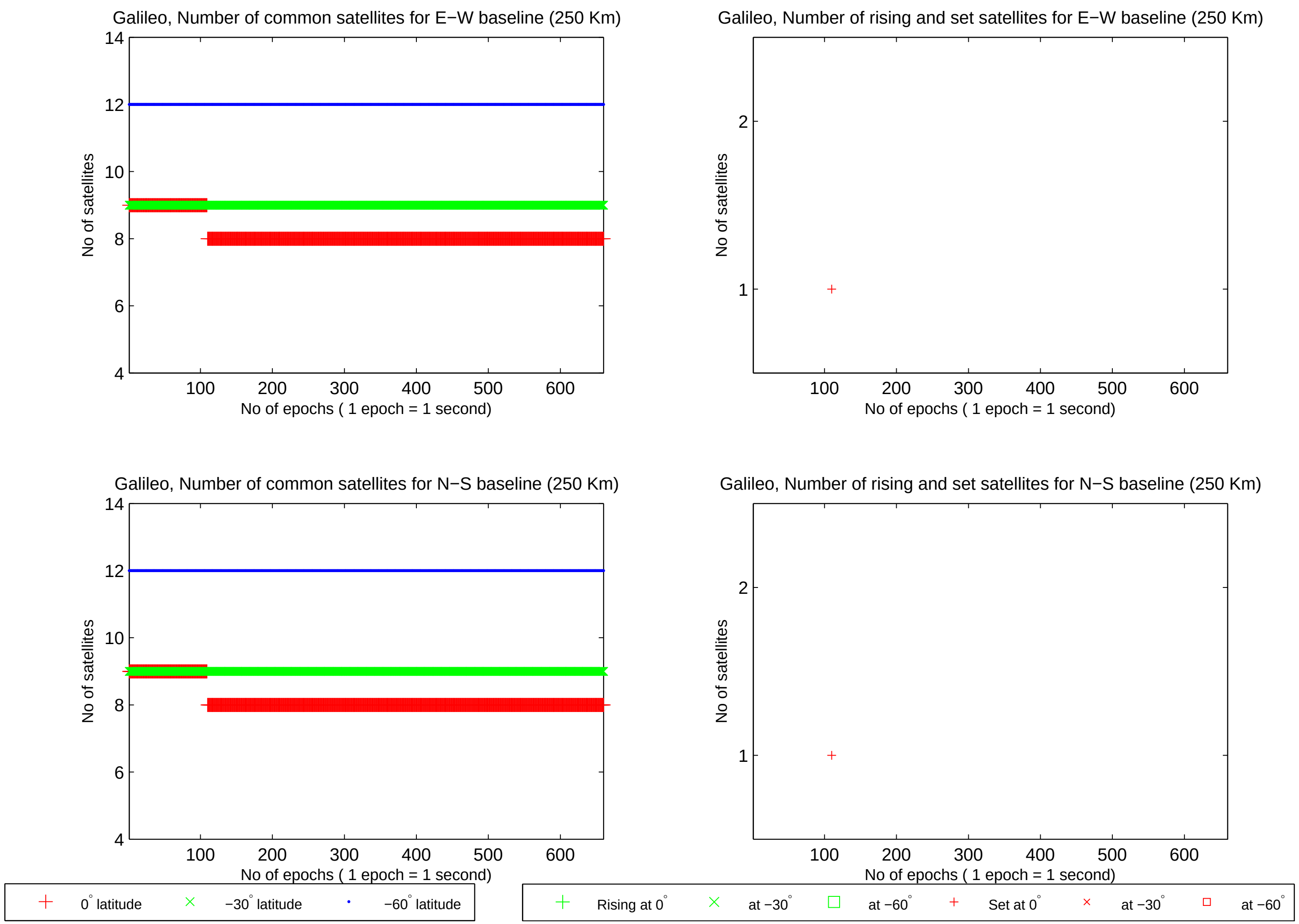

Figure F.9: Number of satellites for **Galileo only**, 250 Km East-West and North-South baseline at 0°, −30° and −60° degree latitude.

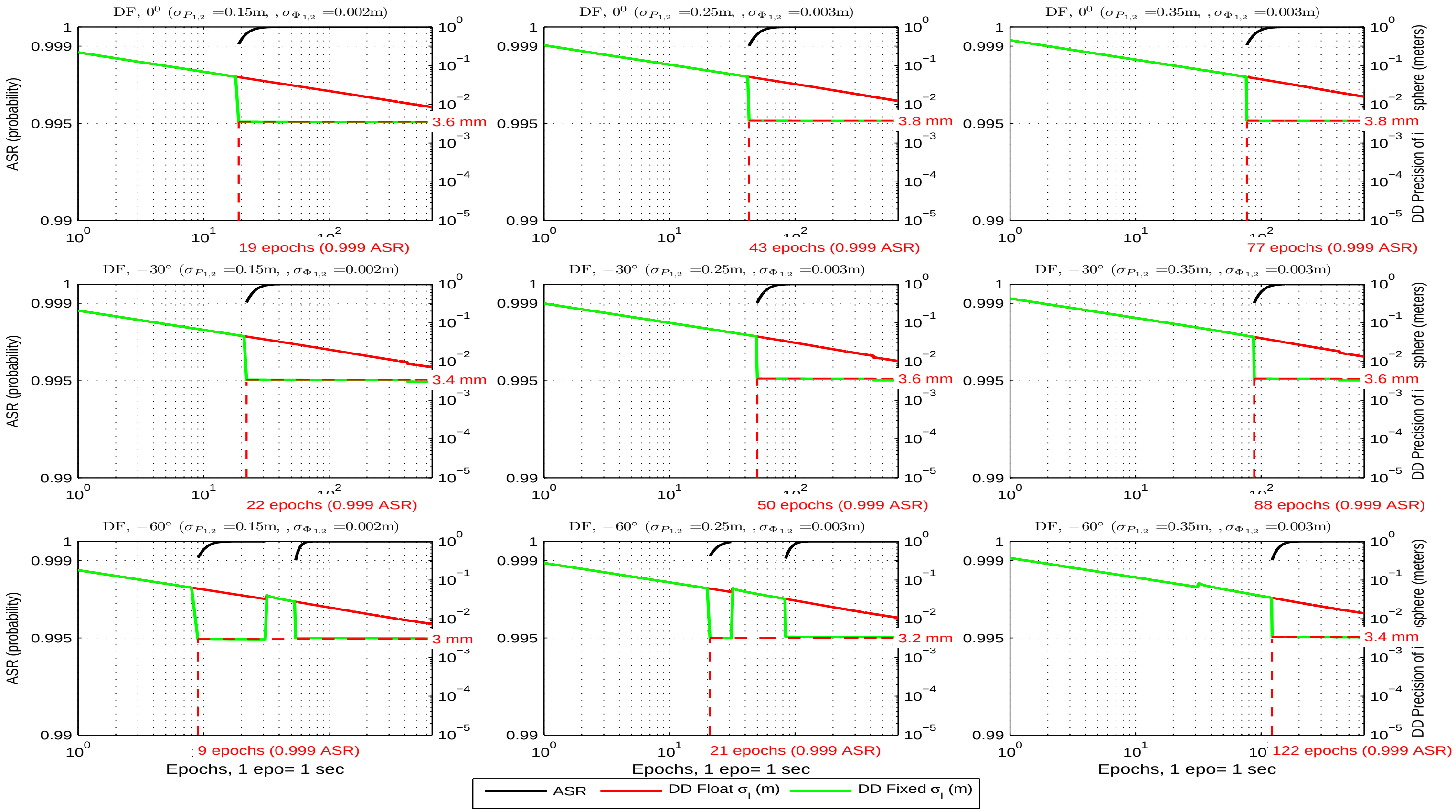

Figure F.10: **Galileo only**, Dual frequency, **Full AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

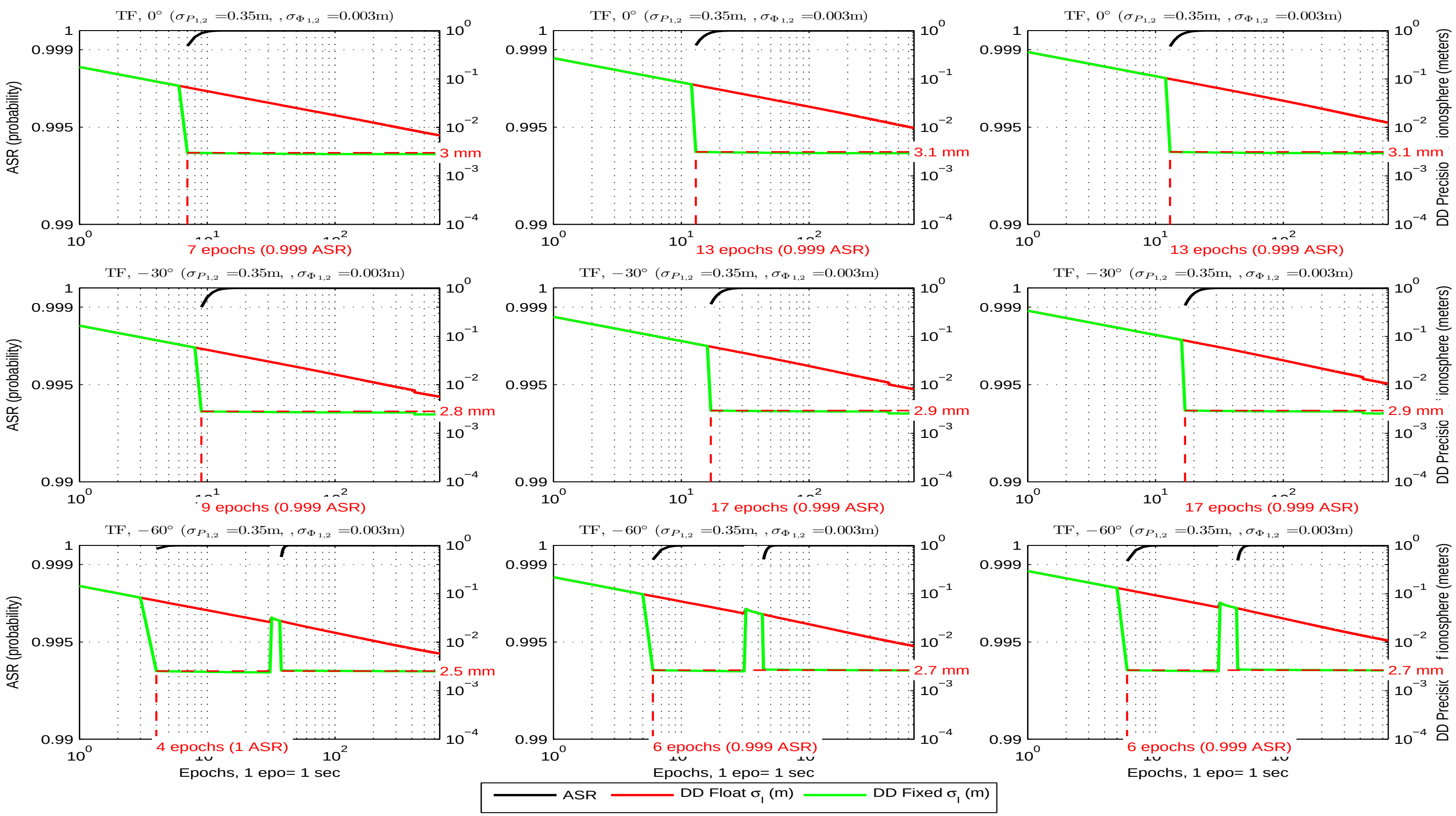

Figure F.11: **Galileo only**, Triple frequency, **Full AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

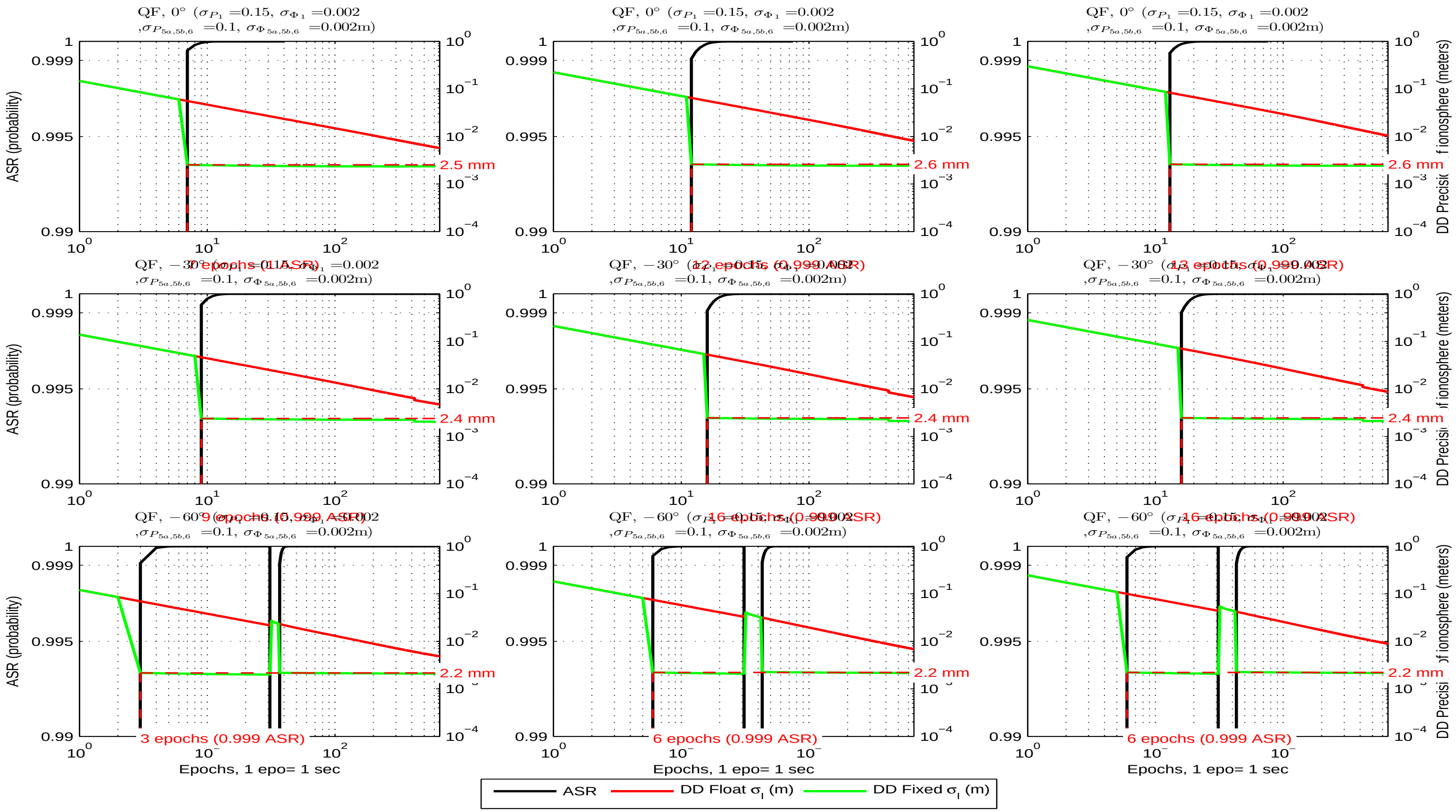

Figure F.12: **Galileo only**, Quadruple frequency, **Full AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

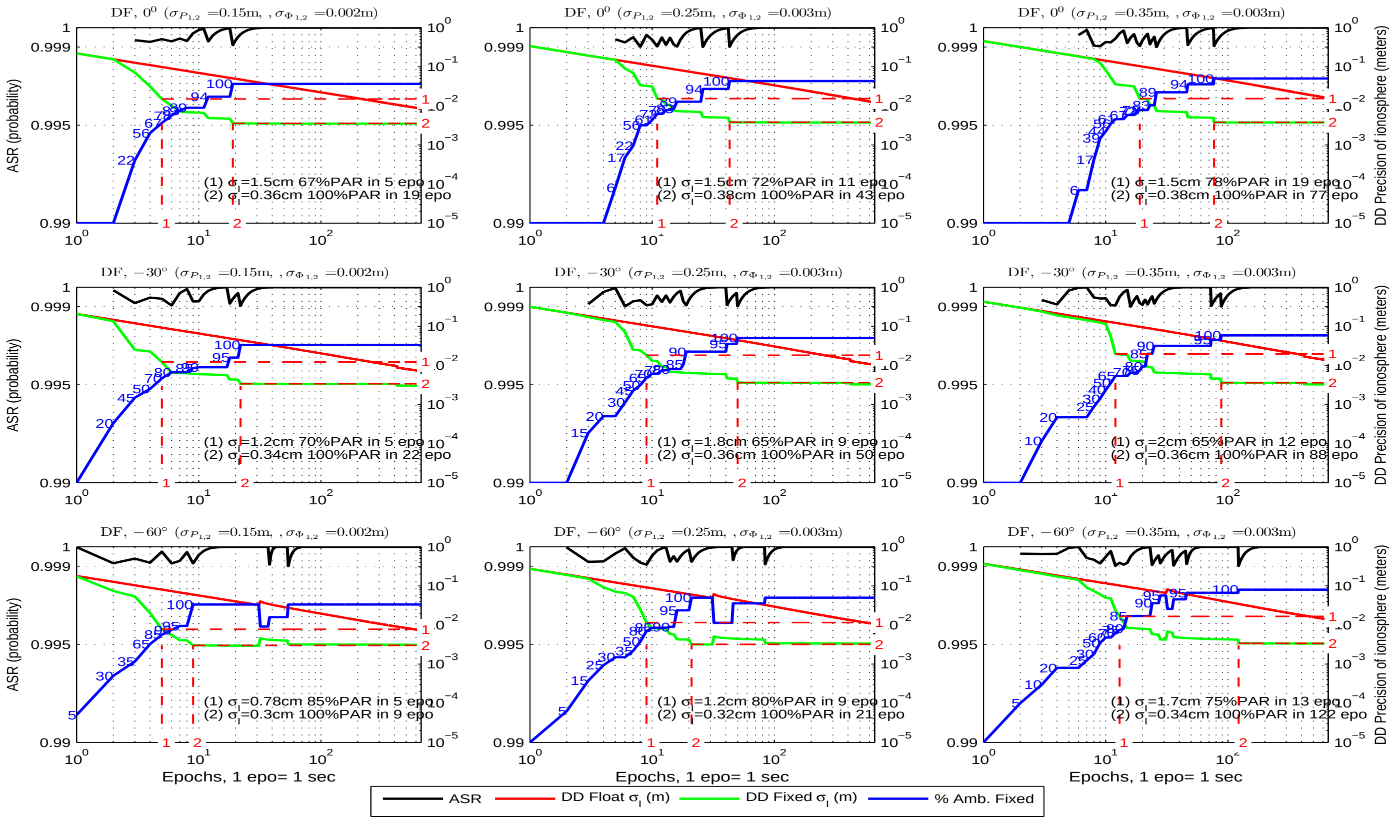

Figure F.13: **Galileo only**, Dual frequency, **Partial AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

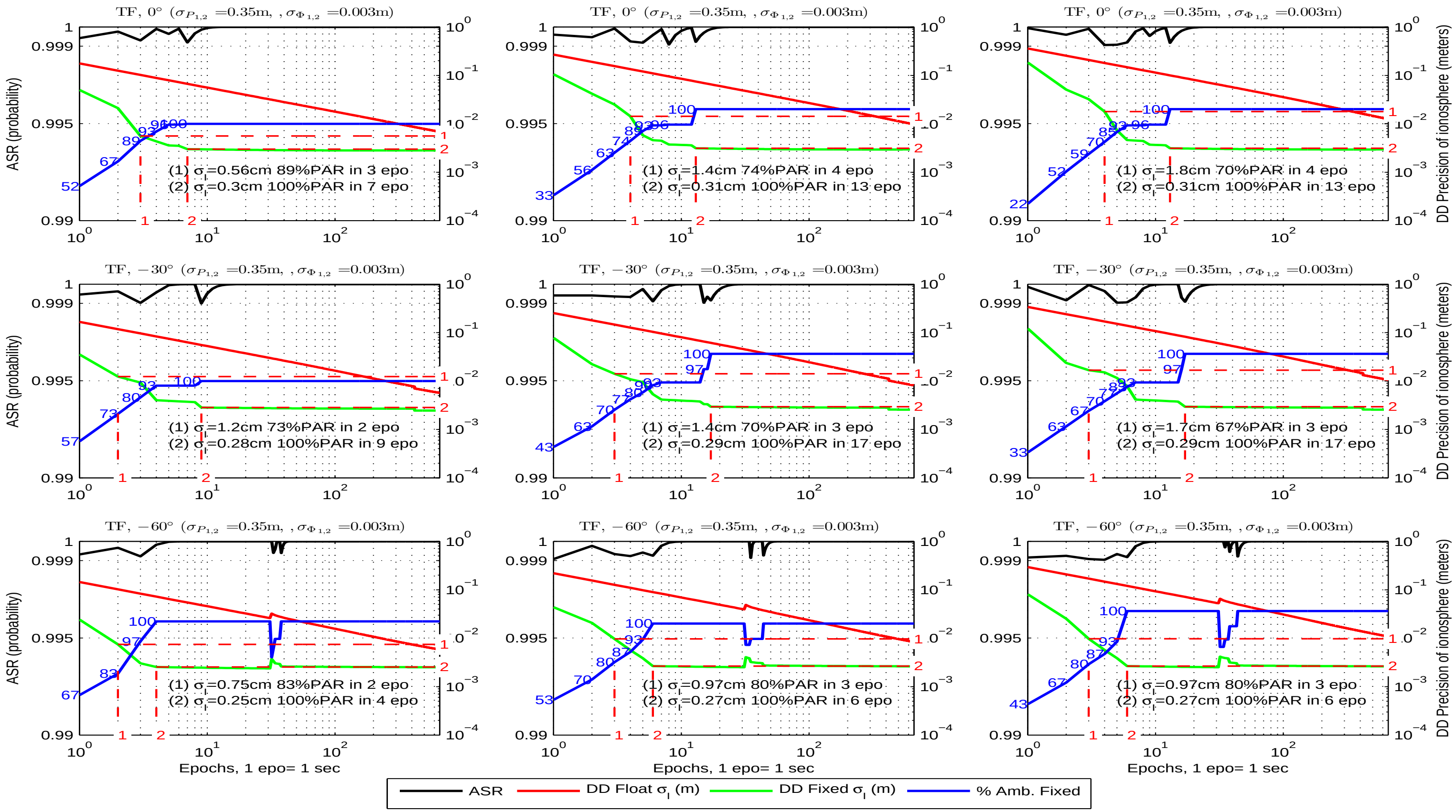

Figure F.14: **Galileo only**, Triple frequency, **Partial AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

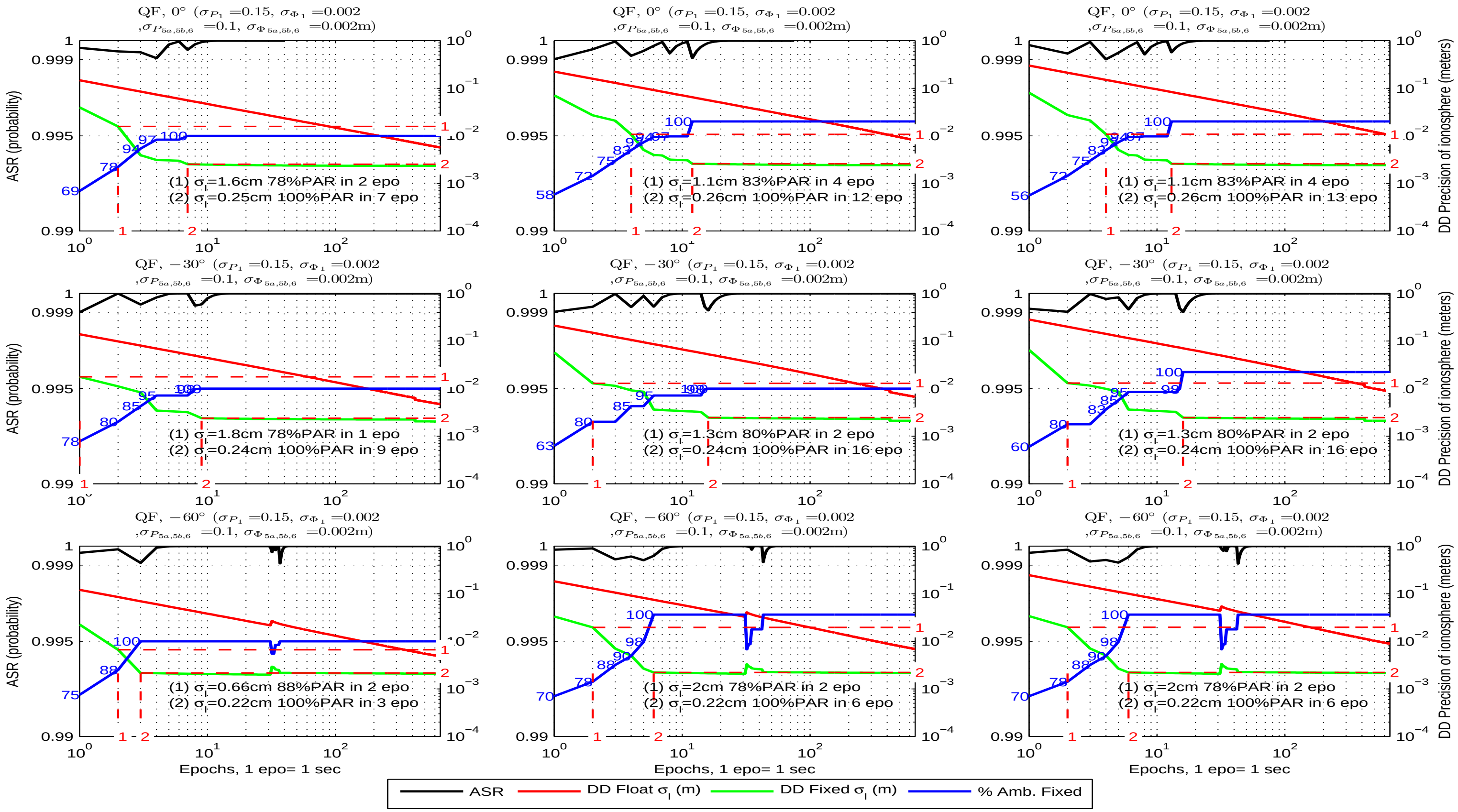

Figure F.15: **Galileo only**, Quadruple frequency, **Partial AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

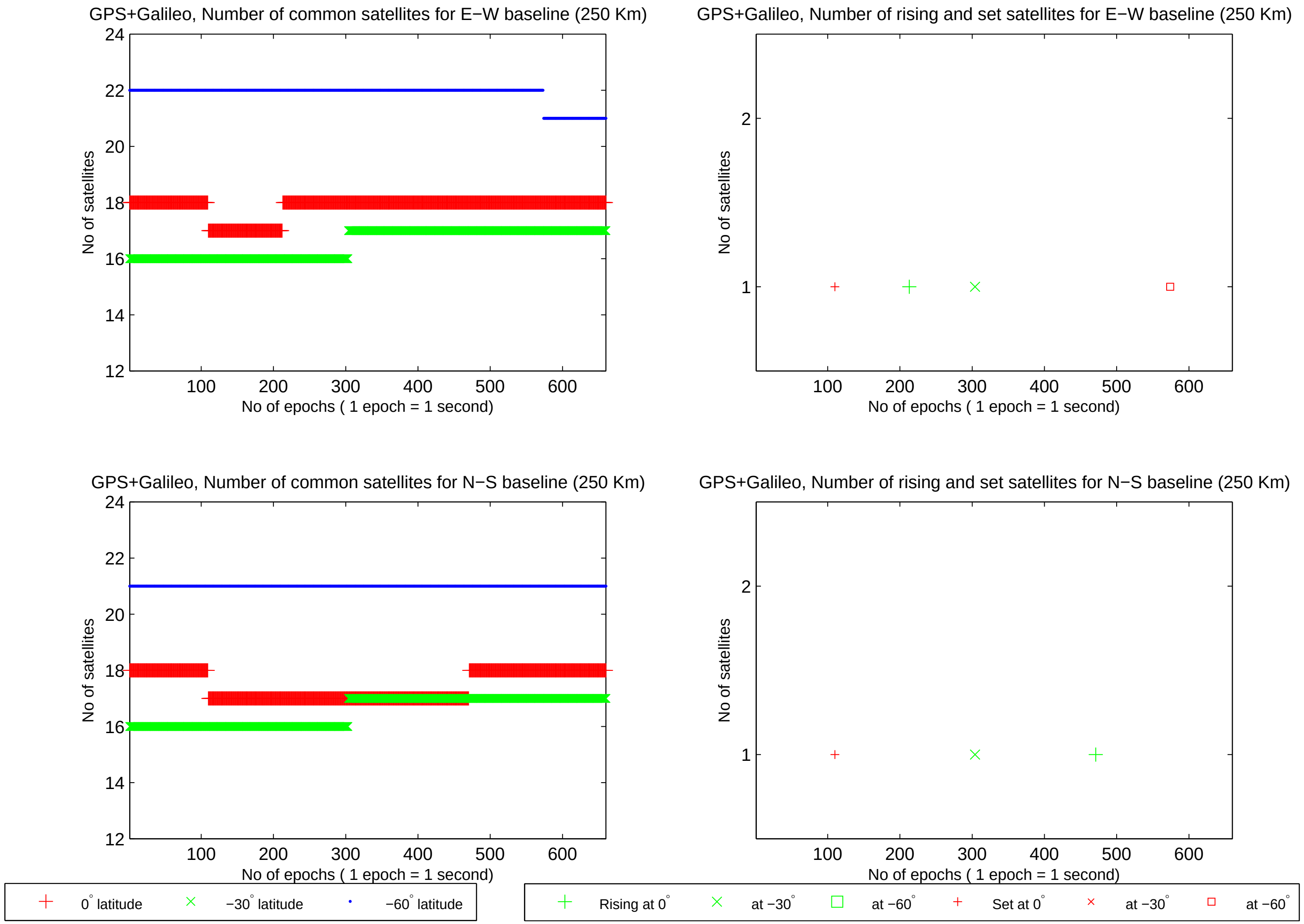

Figure F.16: Number of satellites for **GPS + Galileo**, 250 Km East-West and North-South baseline at 0°, −30° and −60° degree latitude.

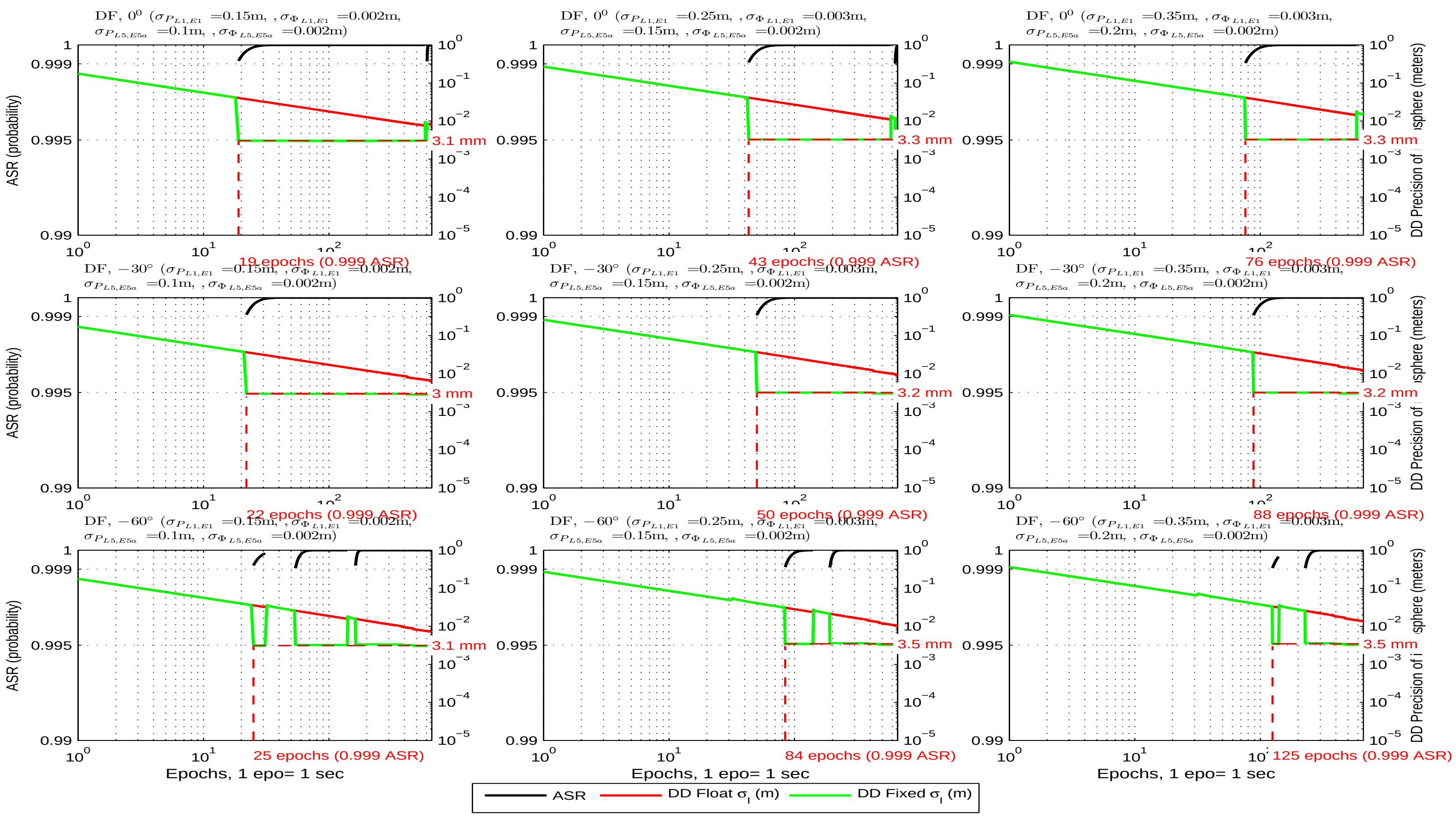

Figure F.17: **GPS + Galileo**, Dual frequency, **Full AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

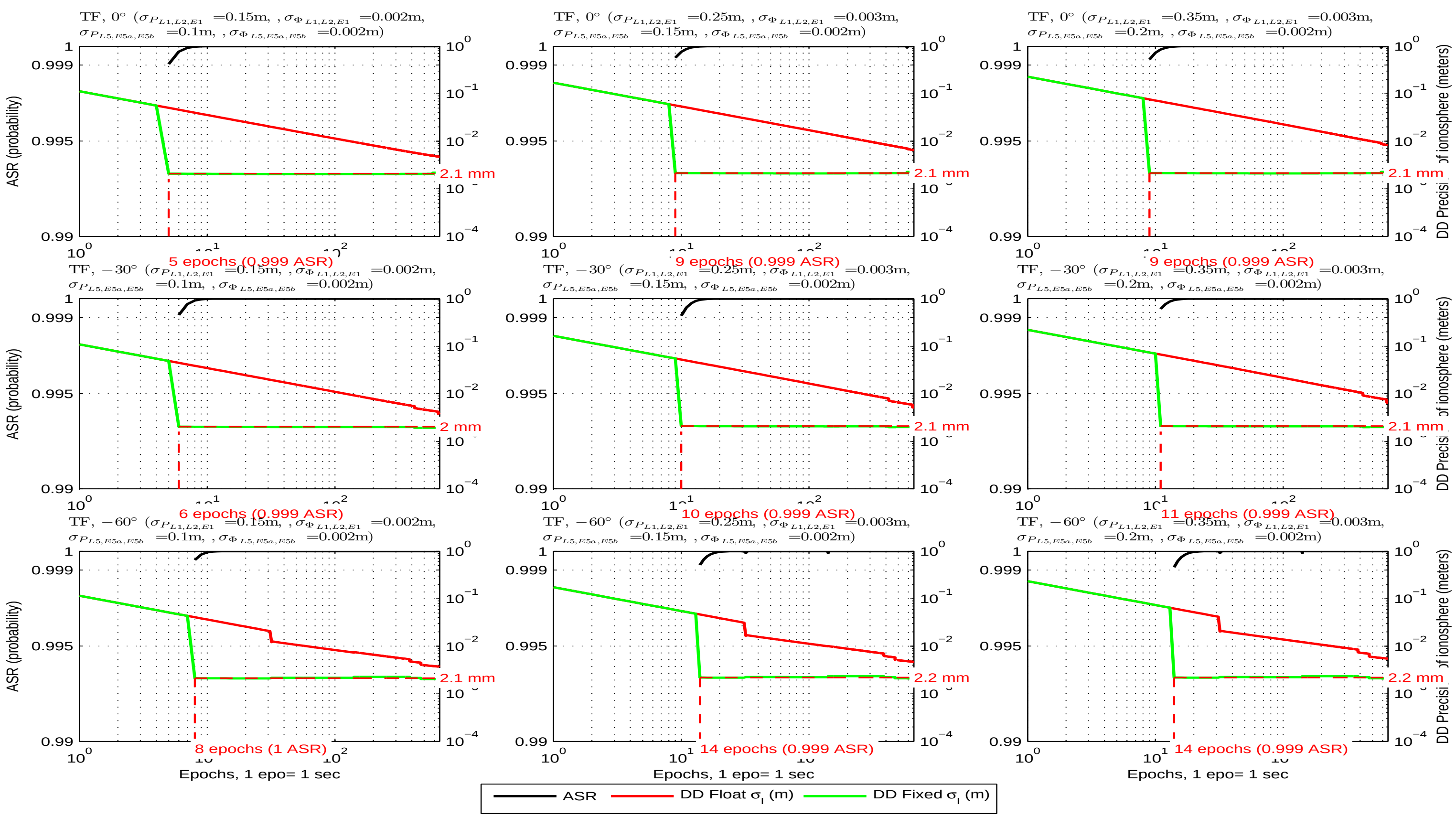

Figure F.18: **GPS + Galileo**, Quadruple frequency, **Full AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

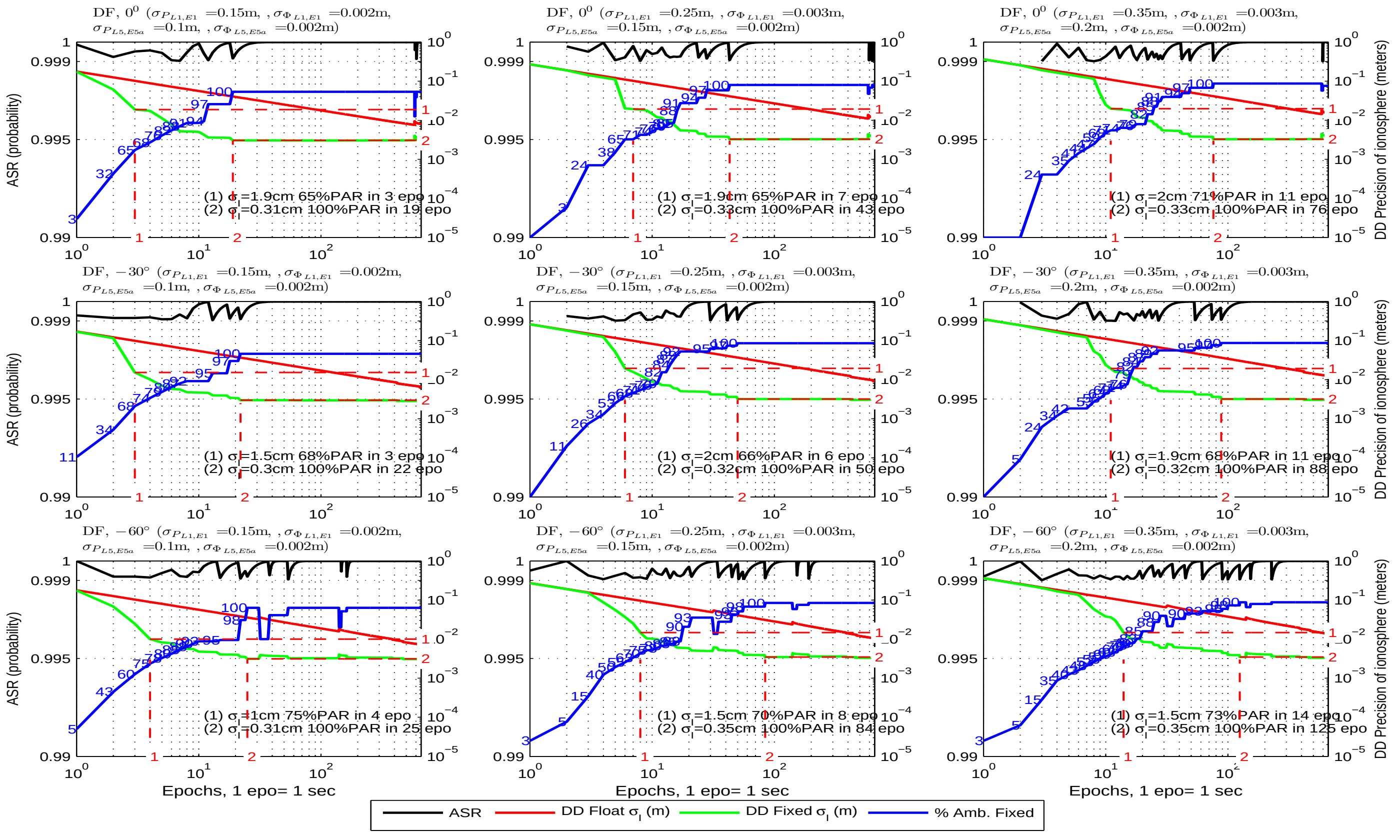

Figure F.19: **GPS + Galileo**, Dual frequency, **Partial AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)-** ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

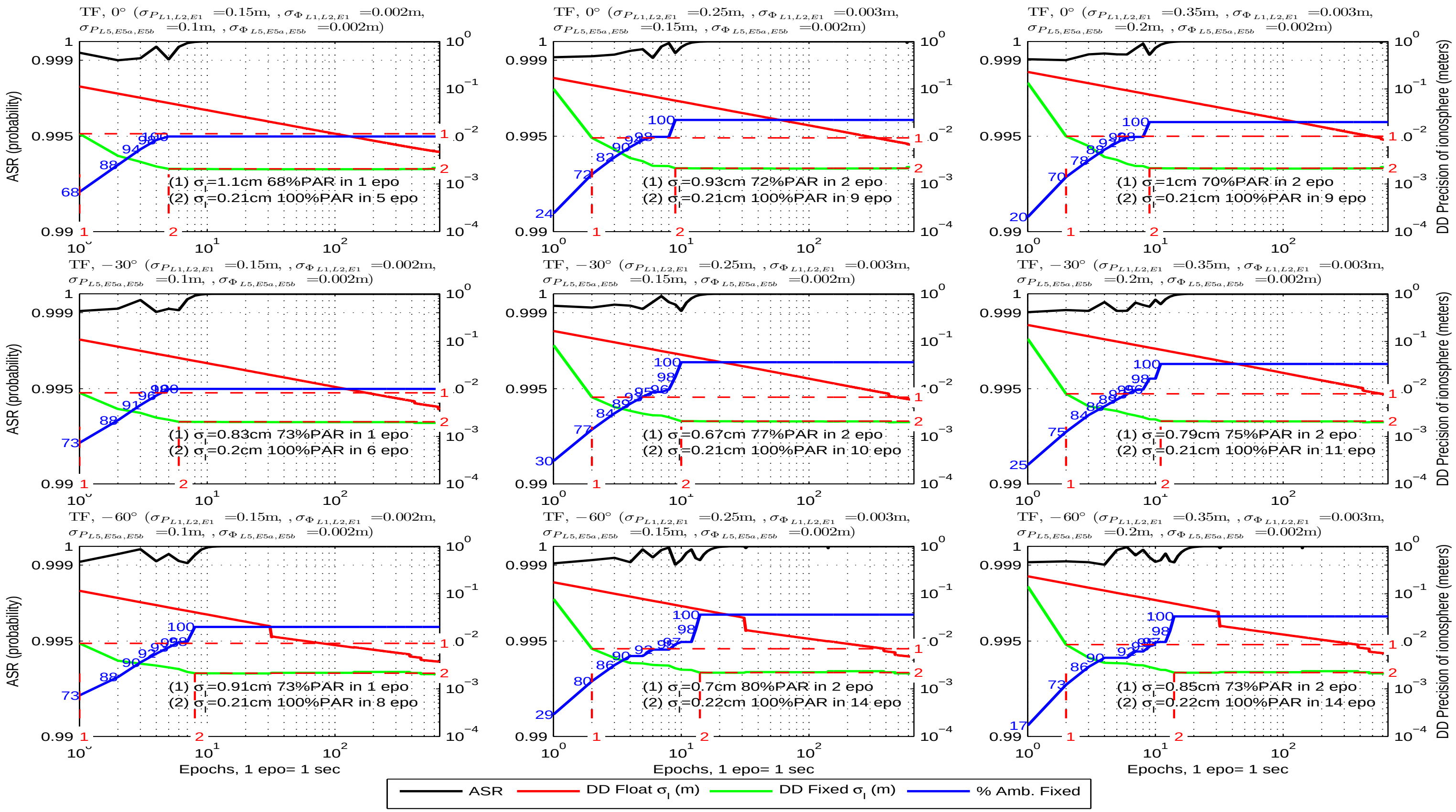

Figure F.20: **GPS + Galileo**, Quadruple frequency, **Partial AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

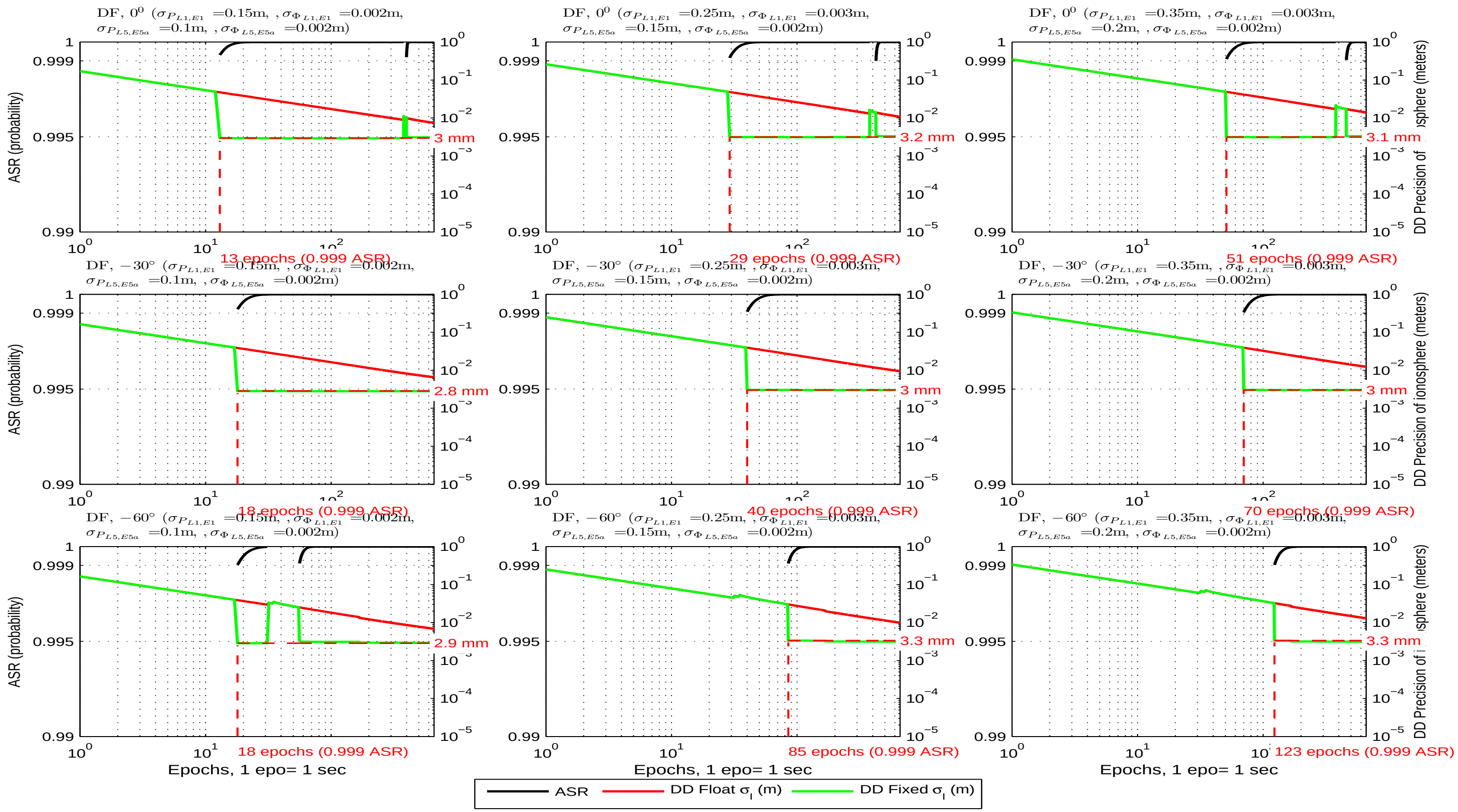

Figure F.21: **GPS + Galileo**, Dual frequency, **Full AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km North-South baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

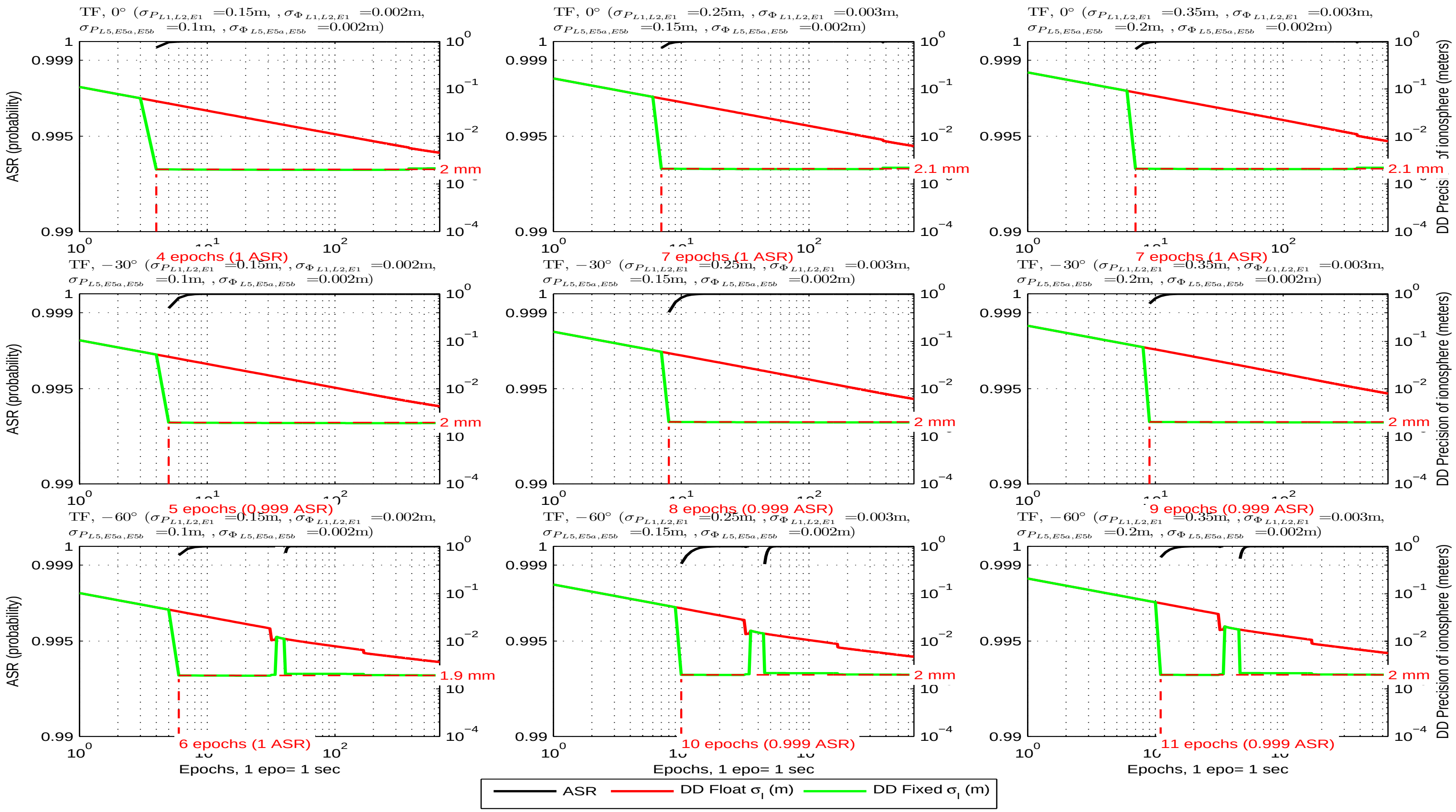

Figure F.22: **GPS + Galileo**, Quadruple frequency, **Full AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km North-South baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

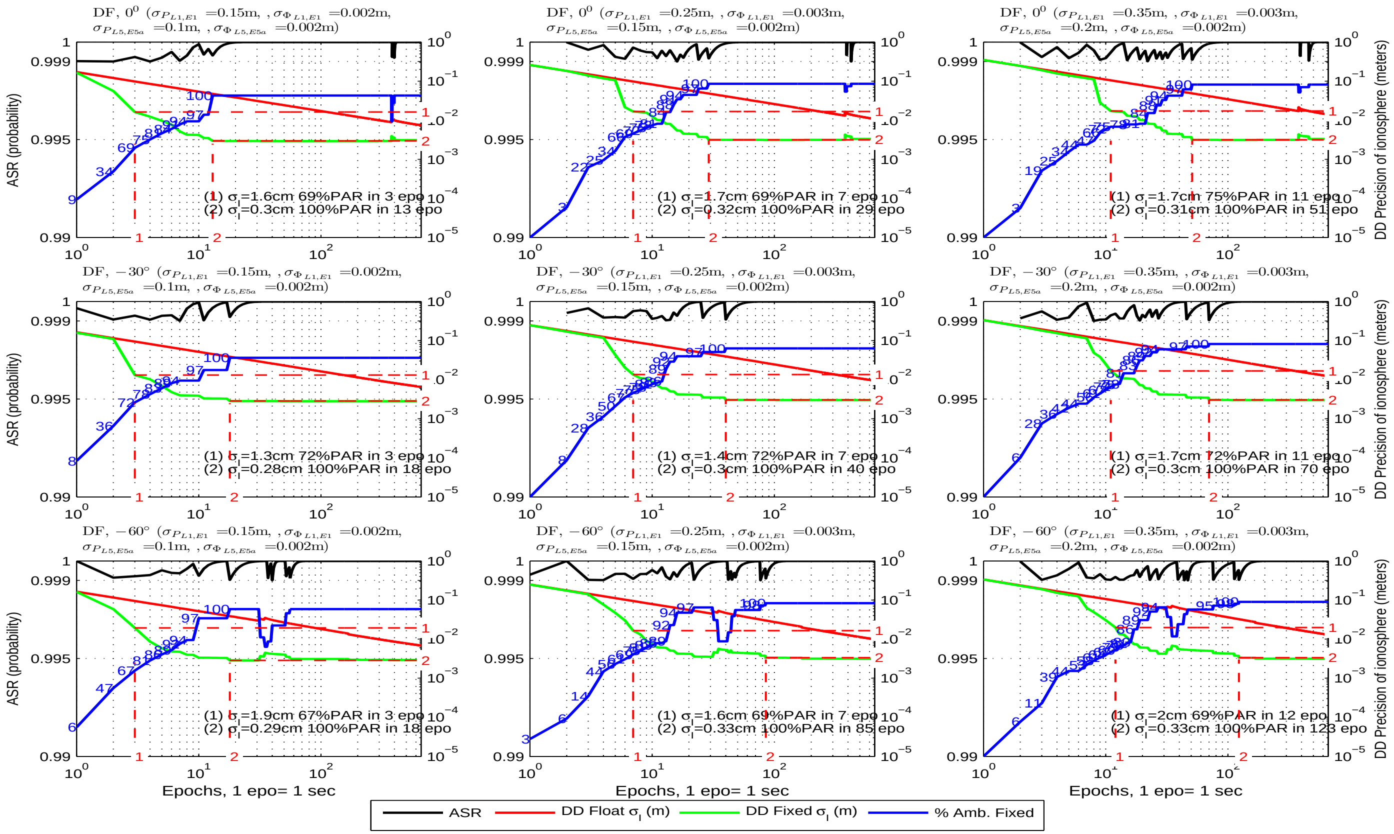

Figure F.23: **GPS + Galileo**, Dual frequency, **Partial AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km North-South baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°$, $-30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

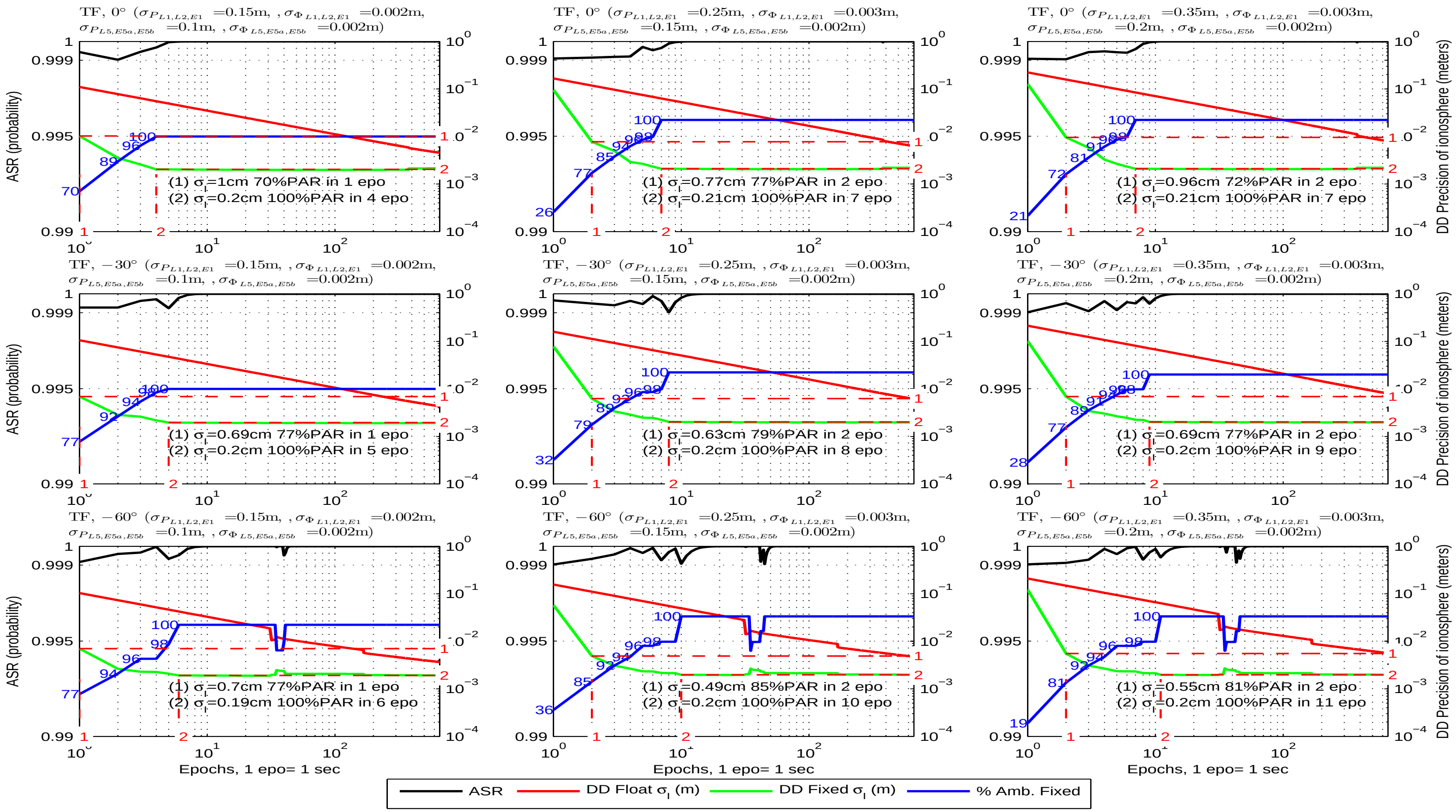

Figure F.24: **GPS + Galileo**, Quadruple frequency, **Partial AR**, **Geometry Fixed model**, Atmosphere float scenario, 250 Km North-South baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

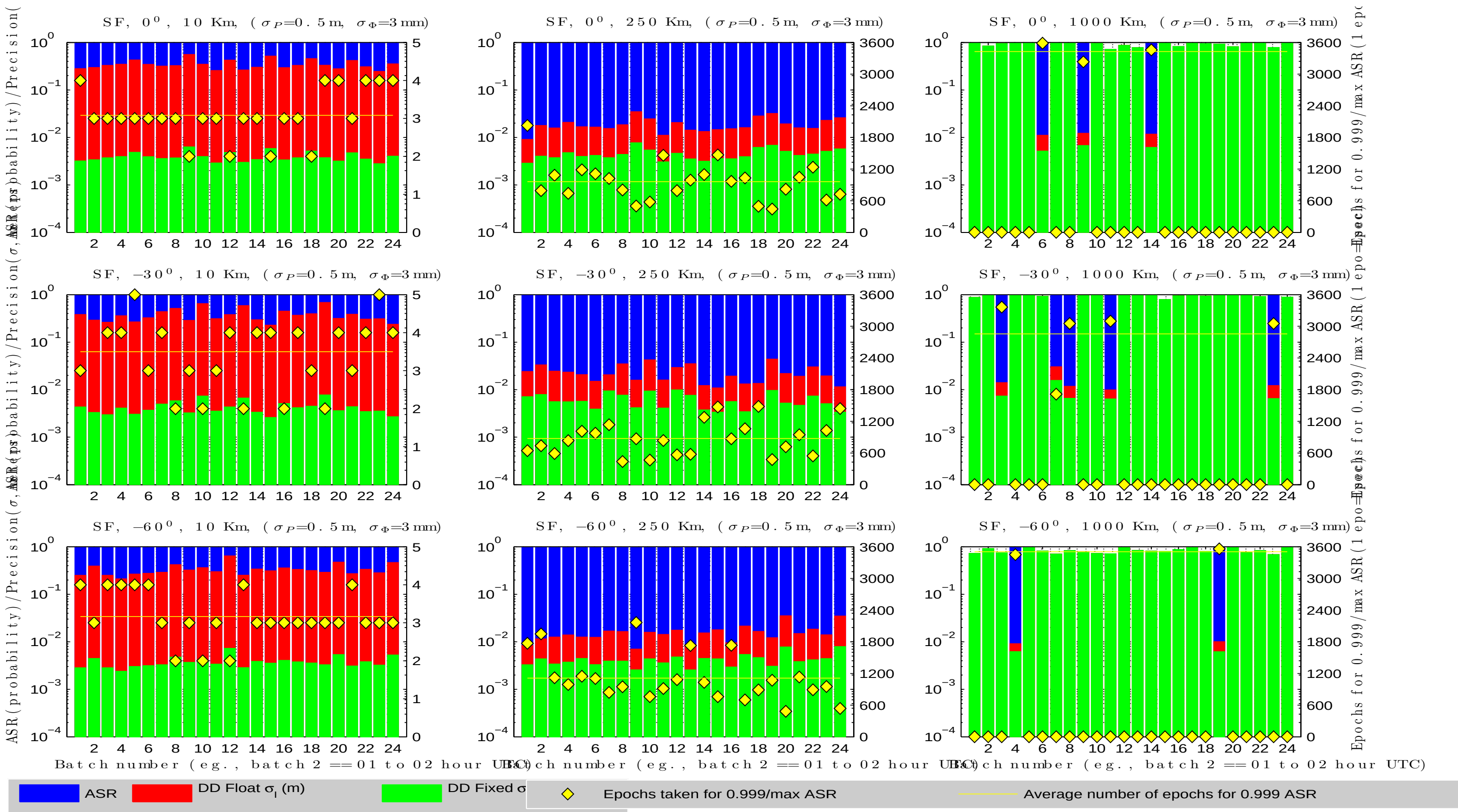

Figure F.25: **GPS only**, Ionosphere Weighted, **Geometry Fixed model**, **single frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

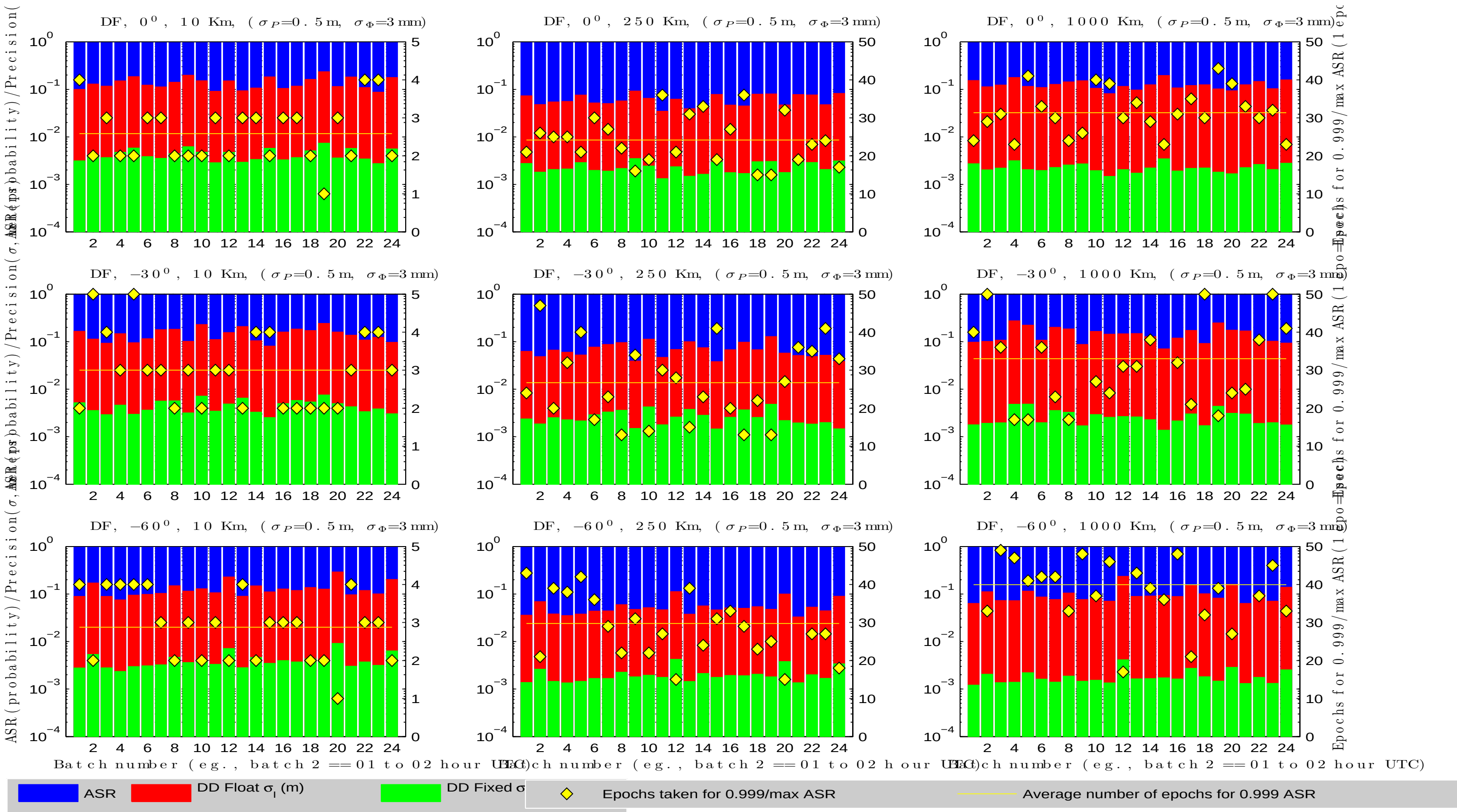

Figure F.26: **GPS only**, Ionosphere Weighted, **Geometry Fixed model**, **dual frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

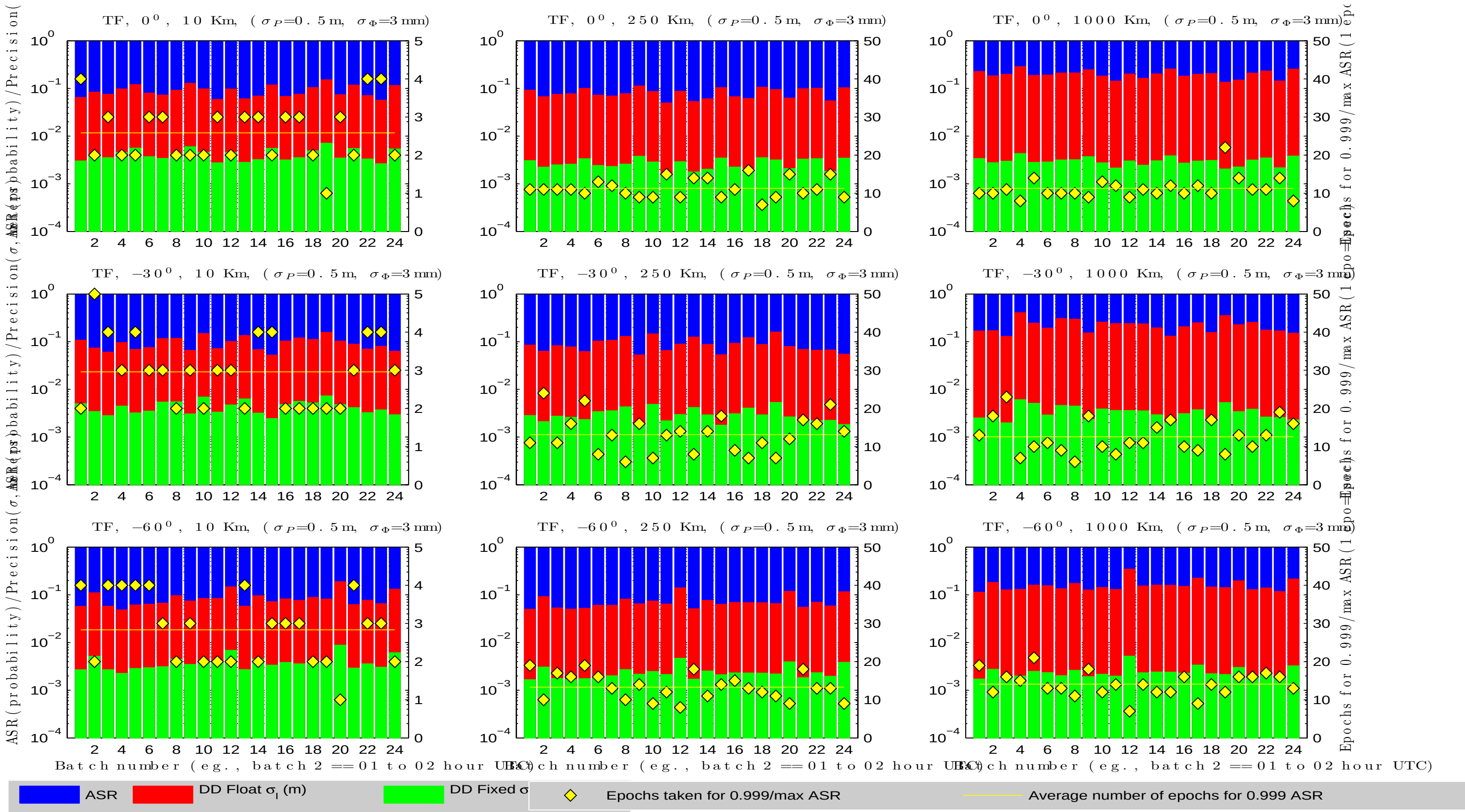

Figure F.27: **GPS only**, Ionosphere Weighted, **Geometry Fixed model**, **triple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

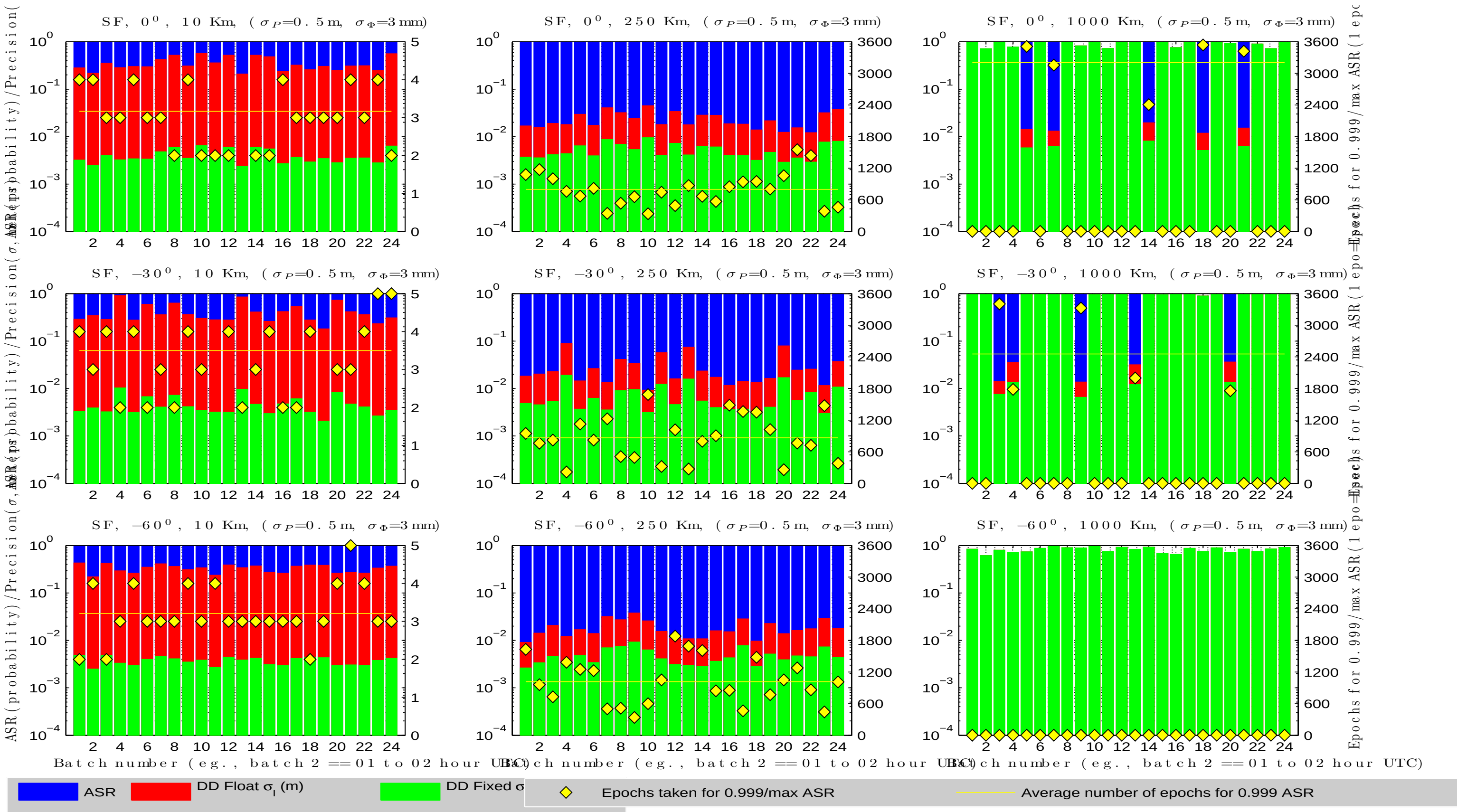

Figure F.28: **Galileo only**, Ionosphere Weighted, **Geometry Fixed model**, **single frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

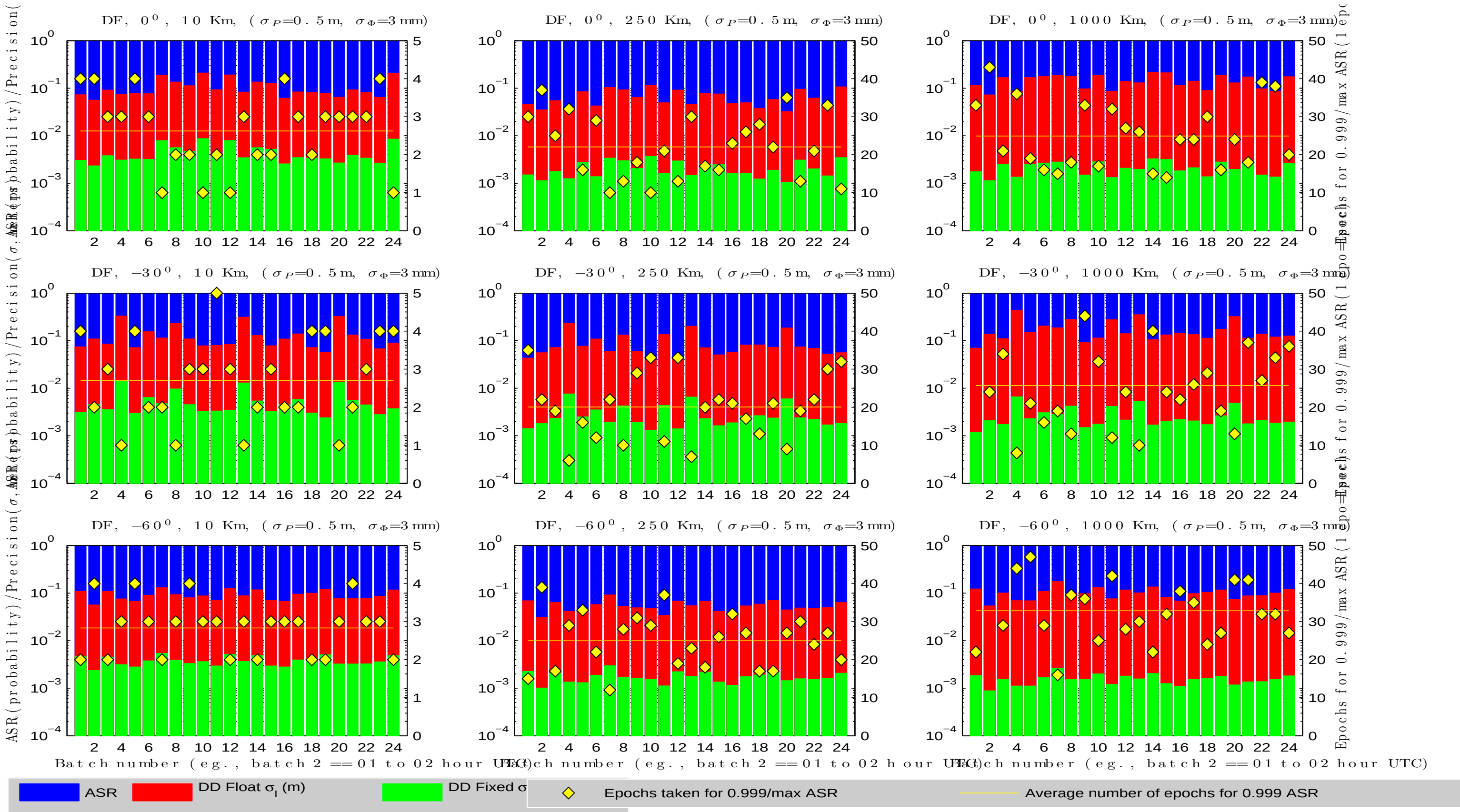

Figure F.29: **Galileo only**, Ionosphere Weighted, **Geometry Fixed model**, **dual frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

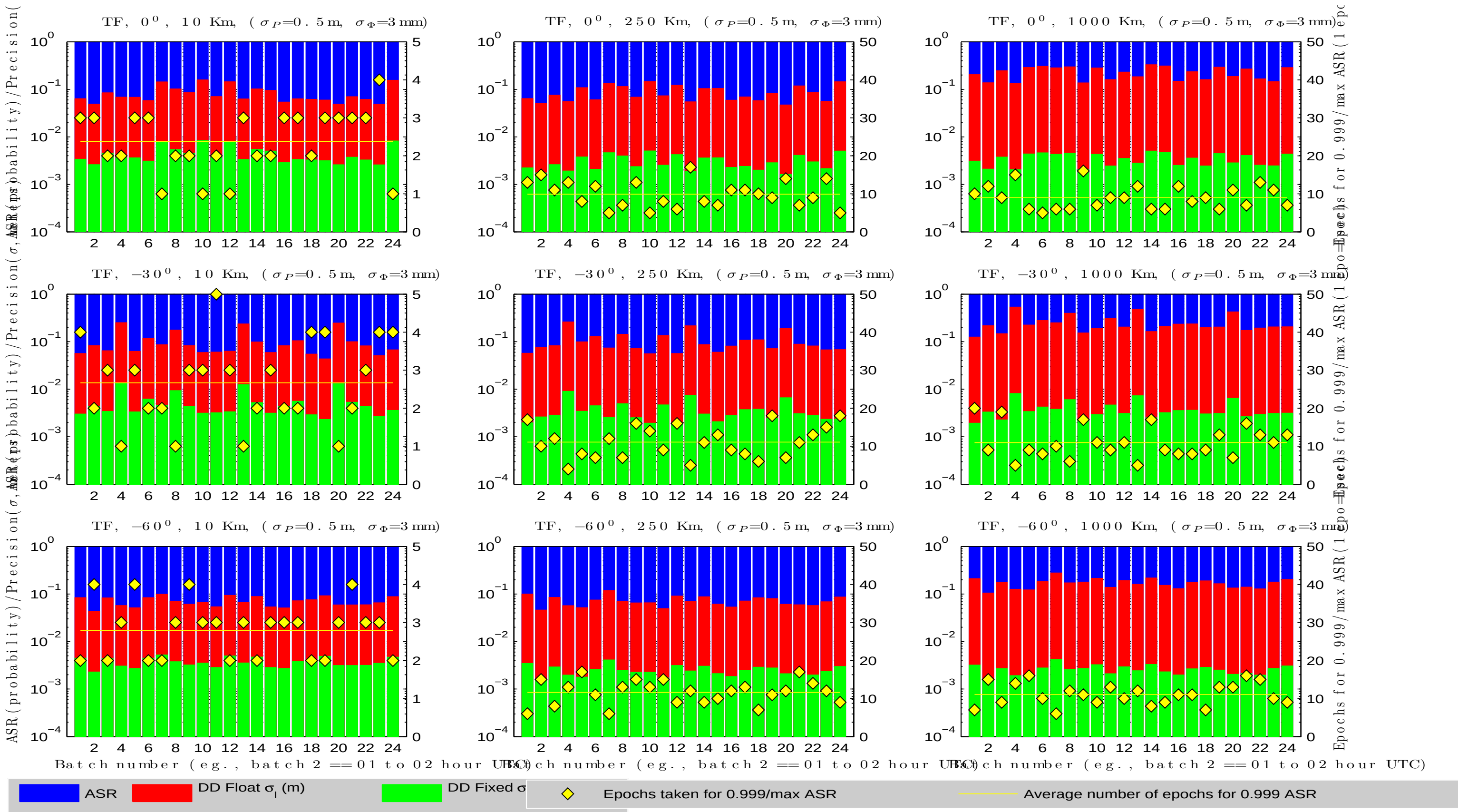

Figure F.30: **Galileo only**, Ionosphere Weighted, **Geometry Fixed model**, **triple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

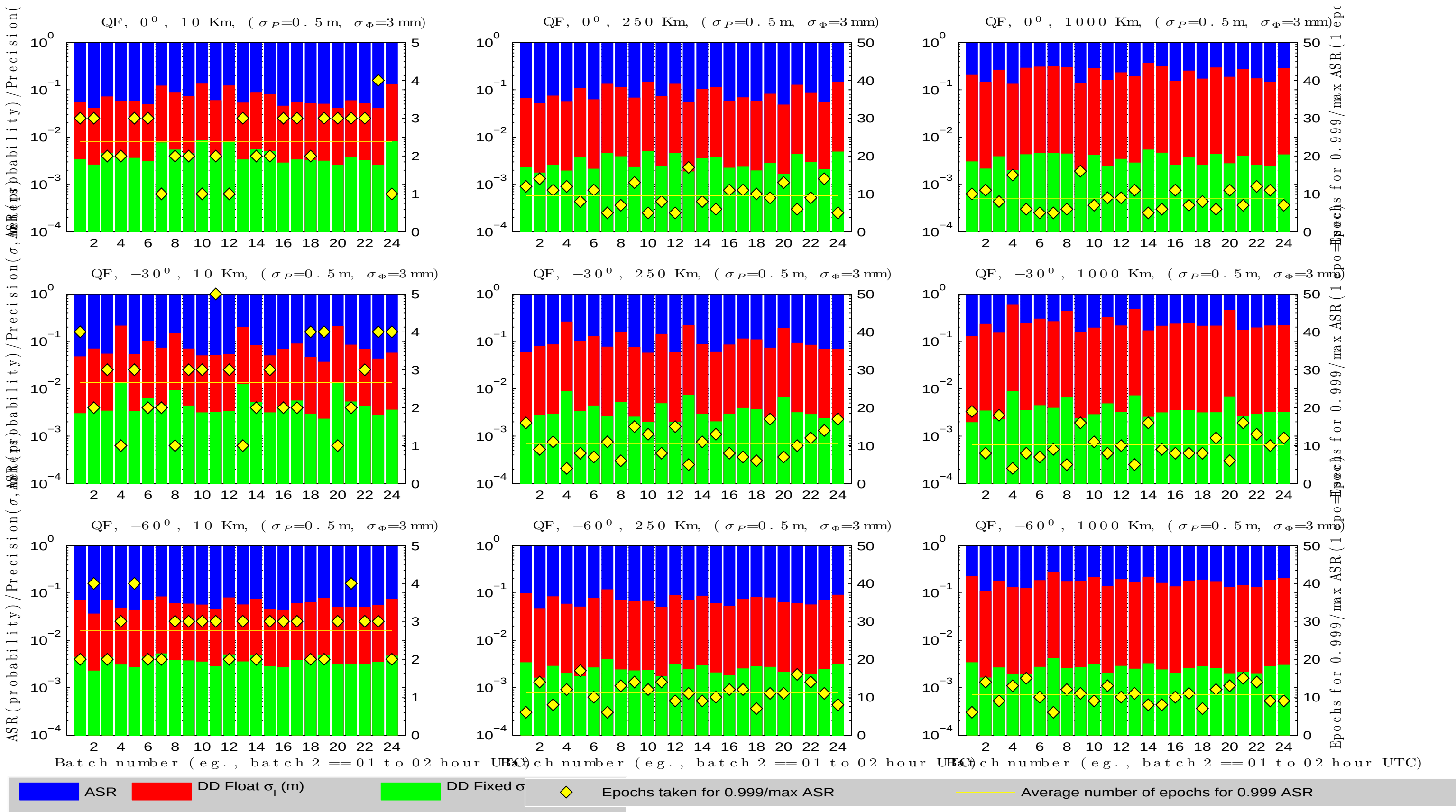

Figure F.31: **Galileo only**, Ionosphere Weighted, **Geometry Fixed model**, **quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

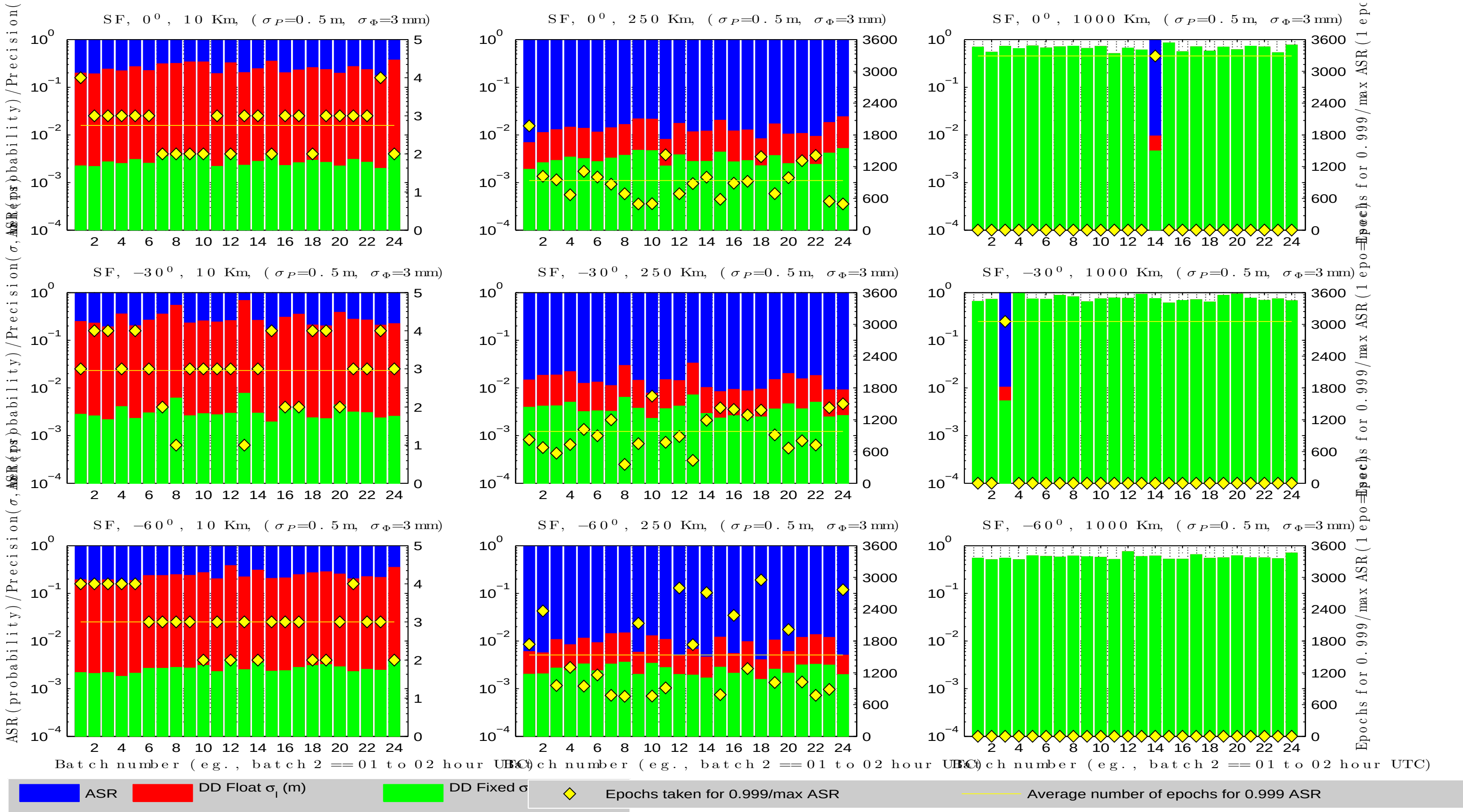

Figure F.32: **GPS + Galileo**, Ionosphere Weighted, **Geometry Fixed model**, **single frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

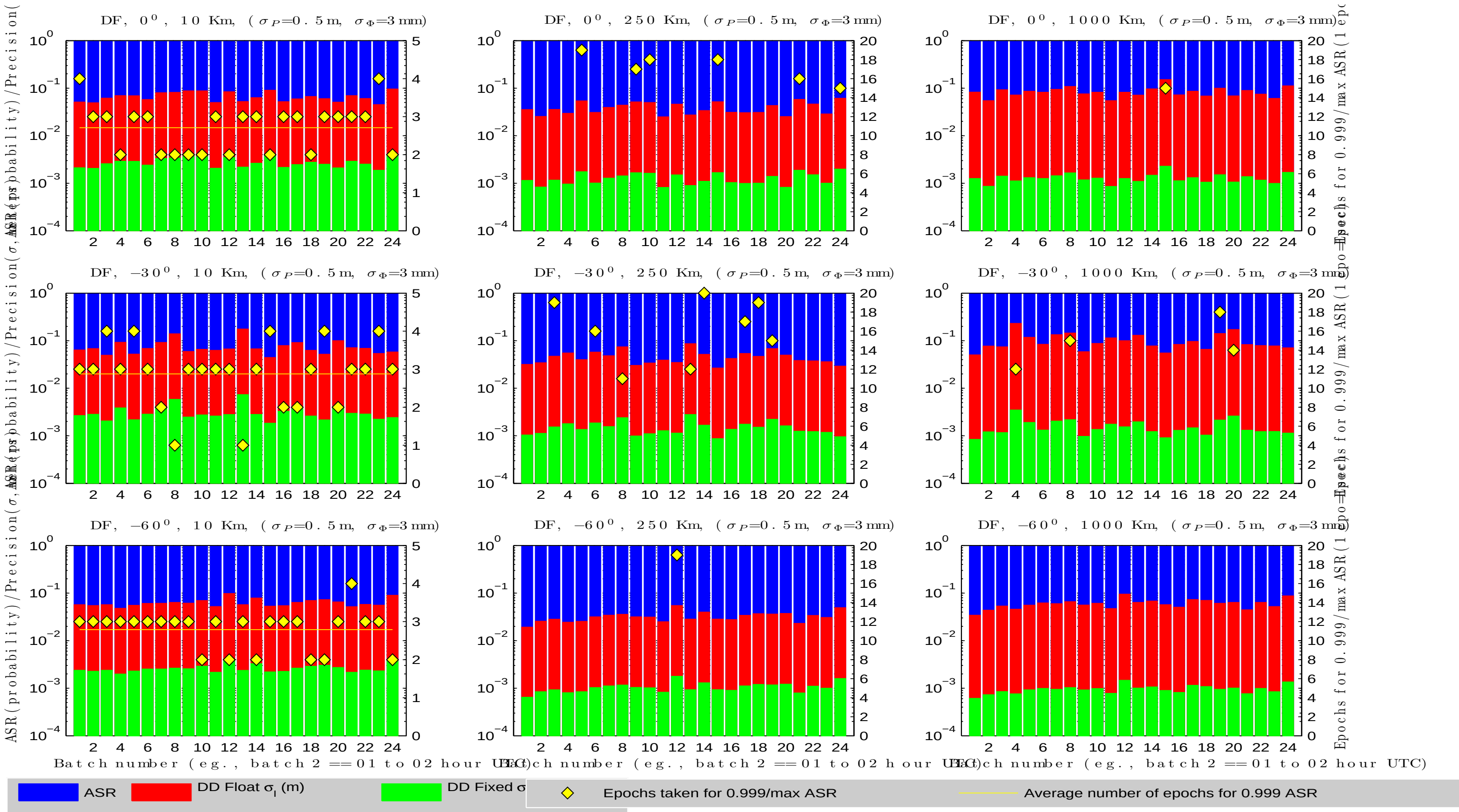

Figure F.33: **GPS + Galileo**, Ionosphere Weighted, **Geometry Fixed model**, **dual frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

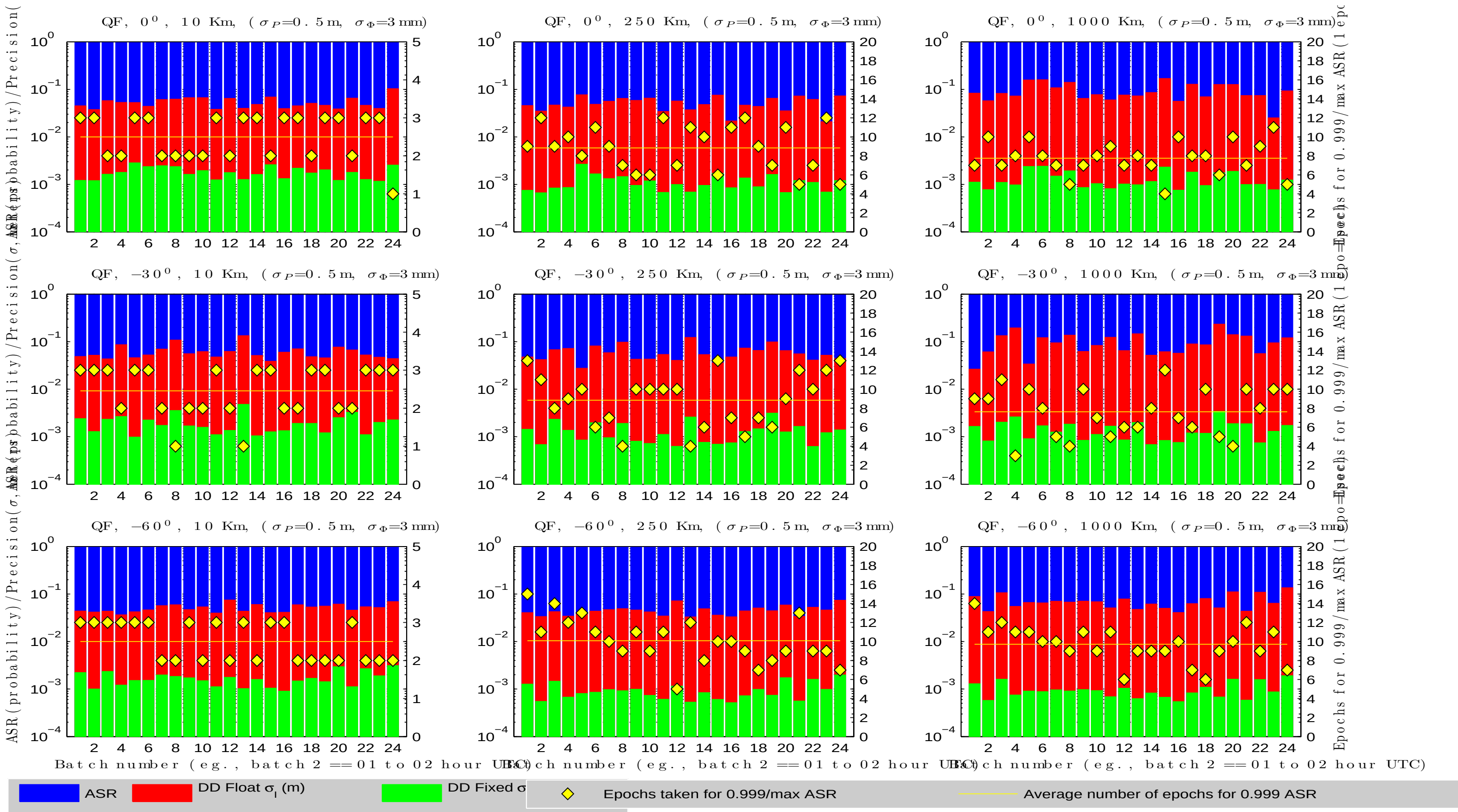

Figure F.34: **GPS + Galileo**, Ionosphere Weighted, **Geometry Fixed model**, **quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

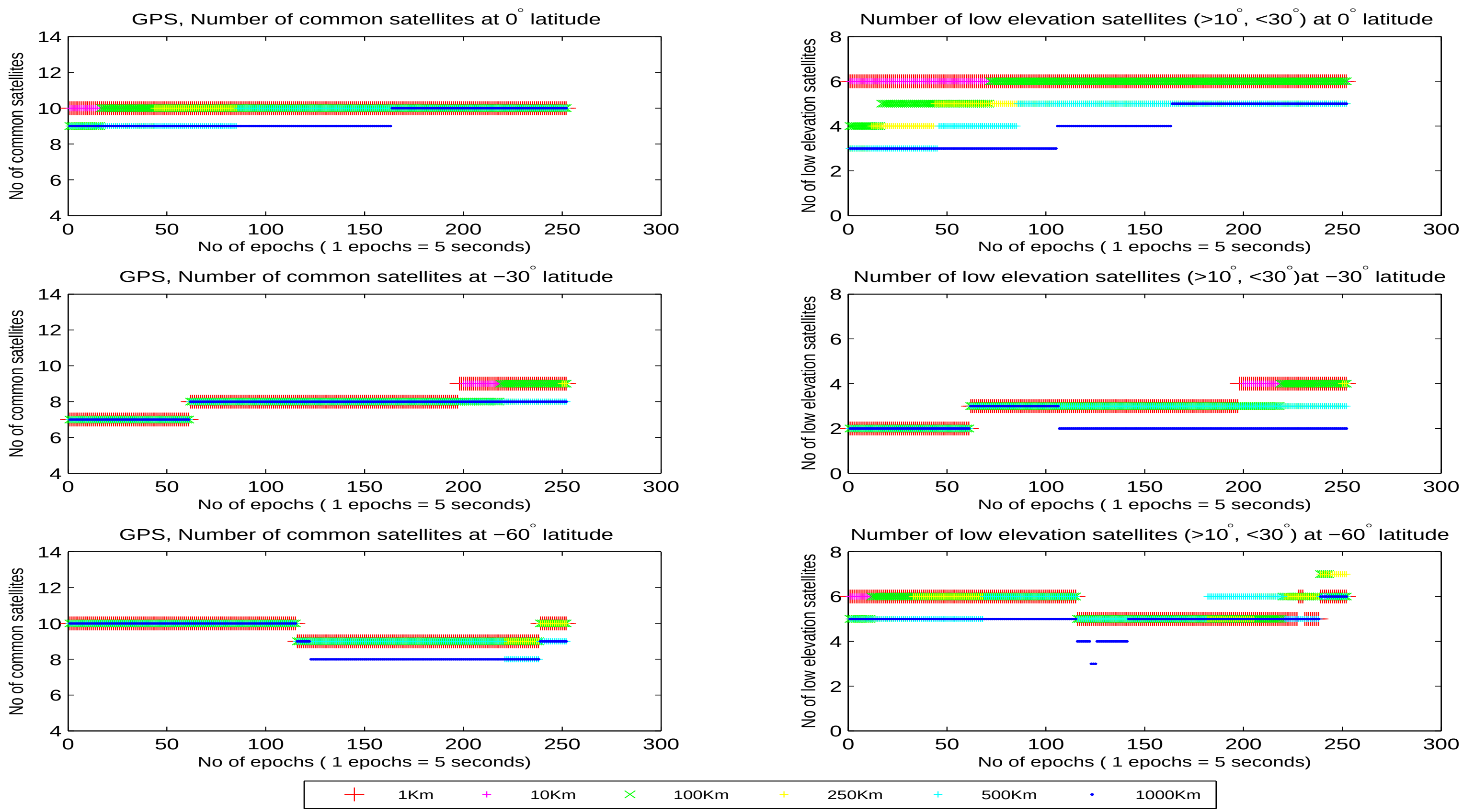

Figure F.35: **GPS only**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, East-West oriented baseline.

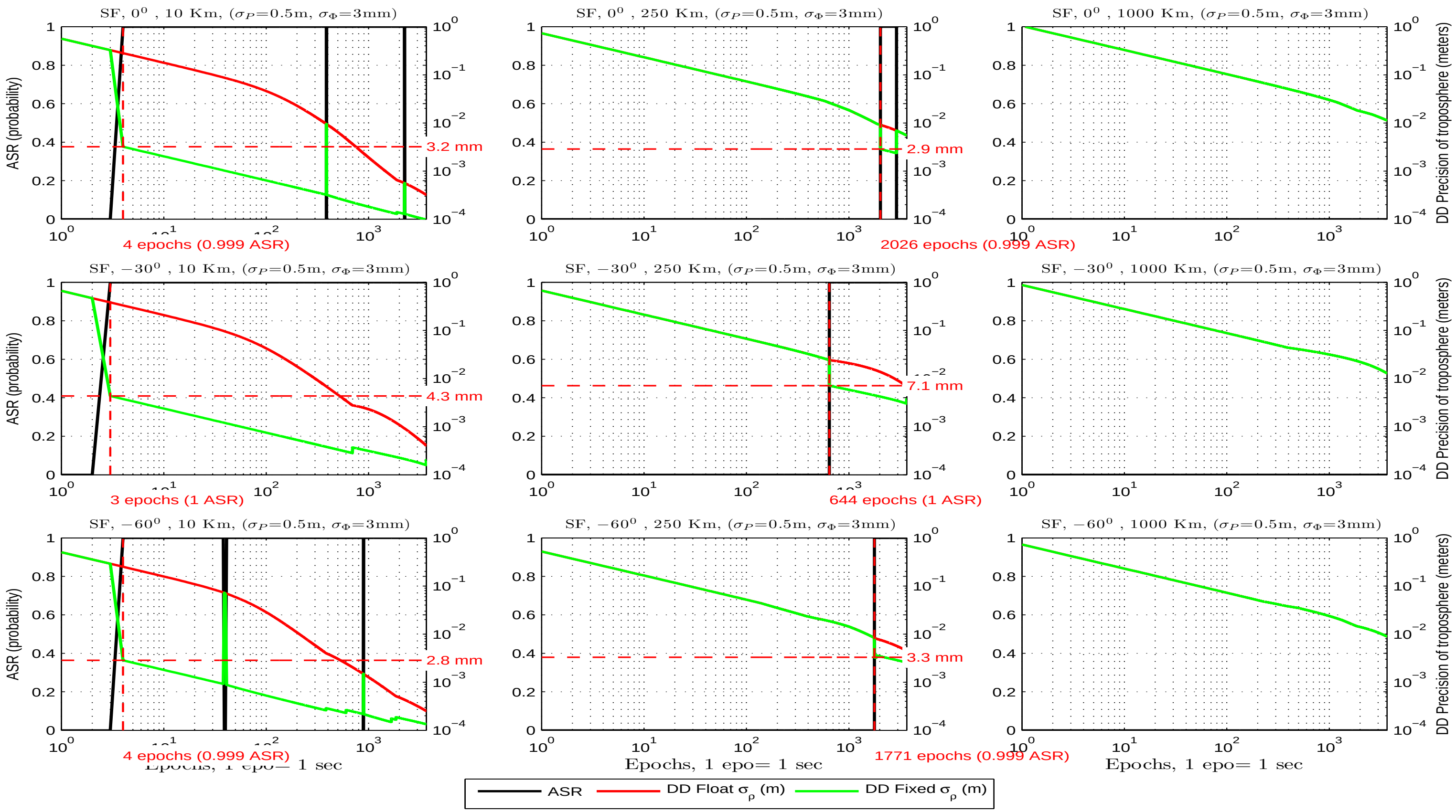

Figure F.36: **GPS only**, Single frequency, **Full AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced σ_I = 0.68 mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

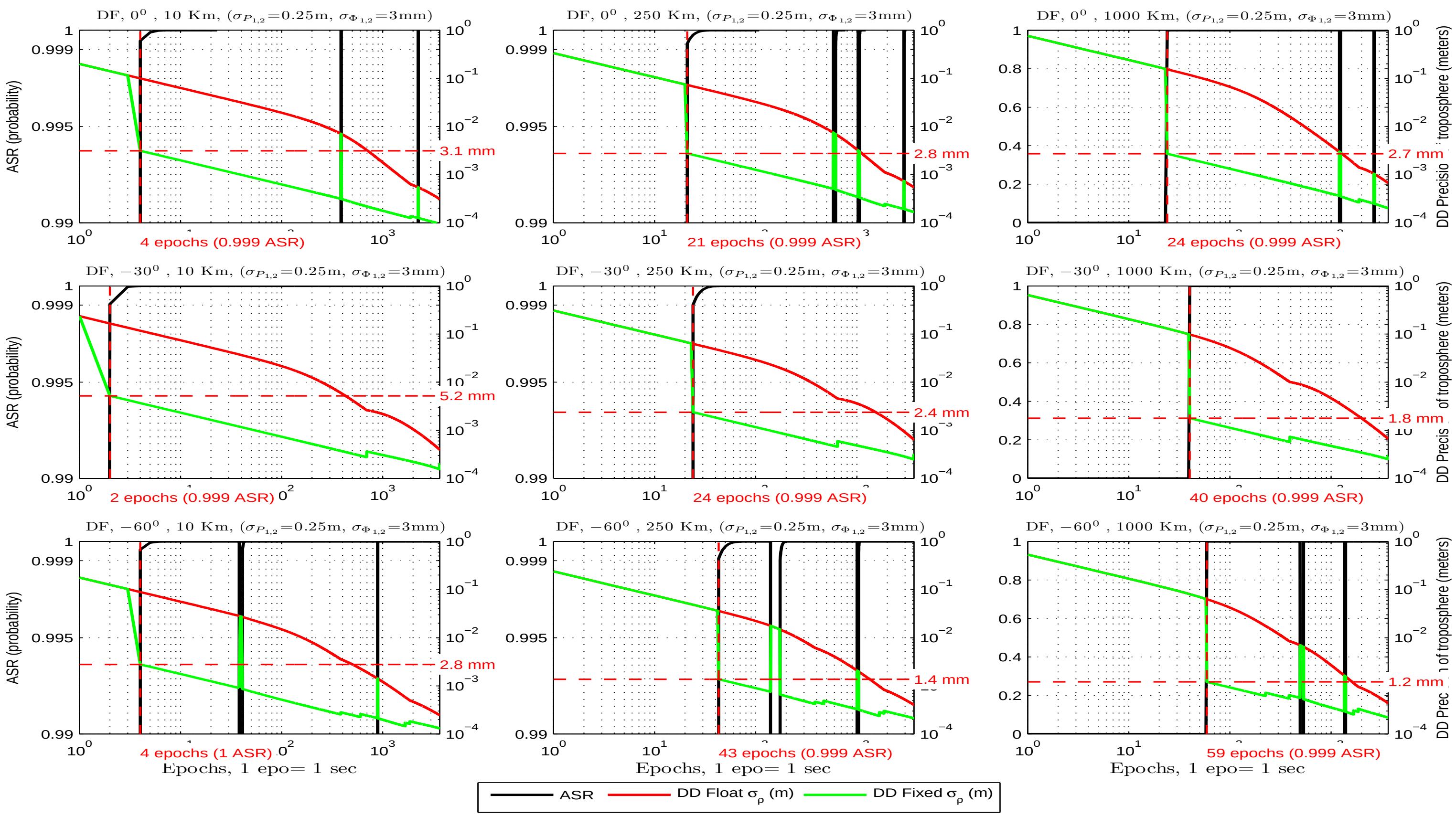

Figure F.37: **GPS only**, Dual frequency, **Full AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced σ_I = 0.68 mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

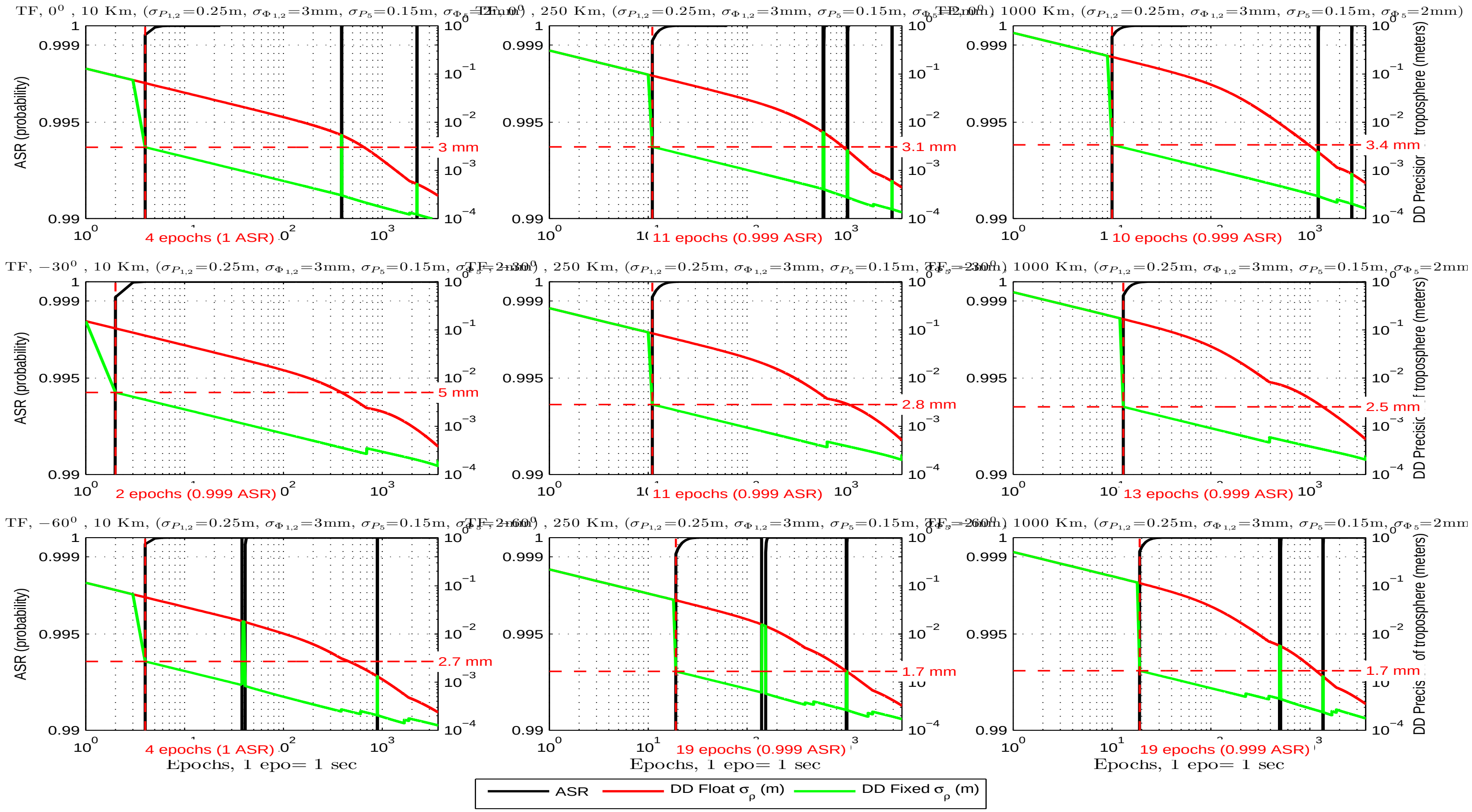

Figure F.38: **GPS only**, Triple frequency, **Full AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

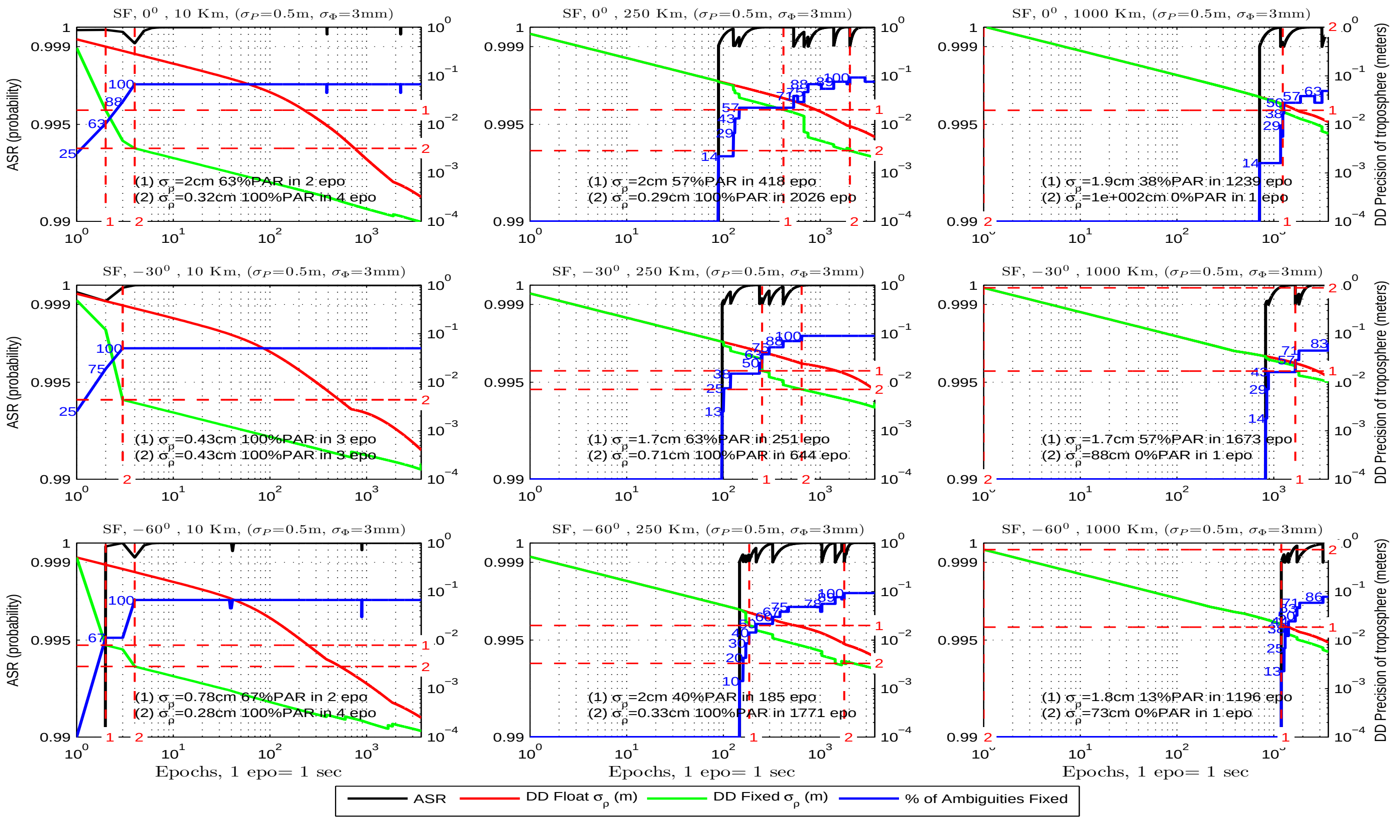

Figure F.39: **GPS only**, Single frequency, **Partial AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

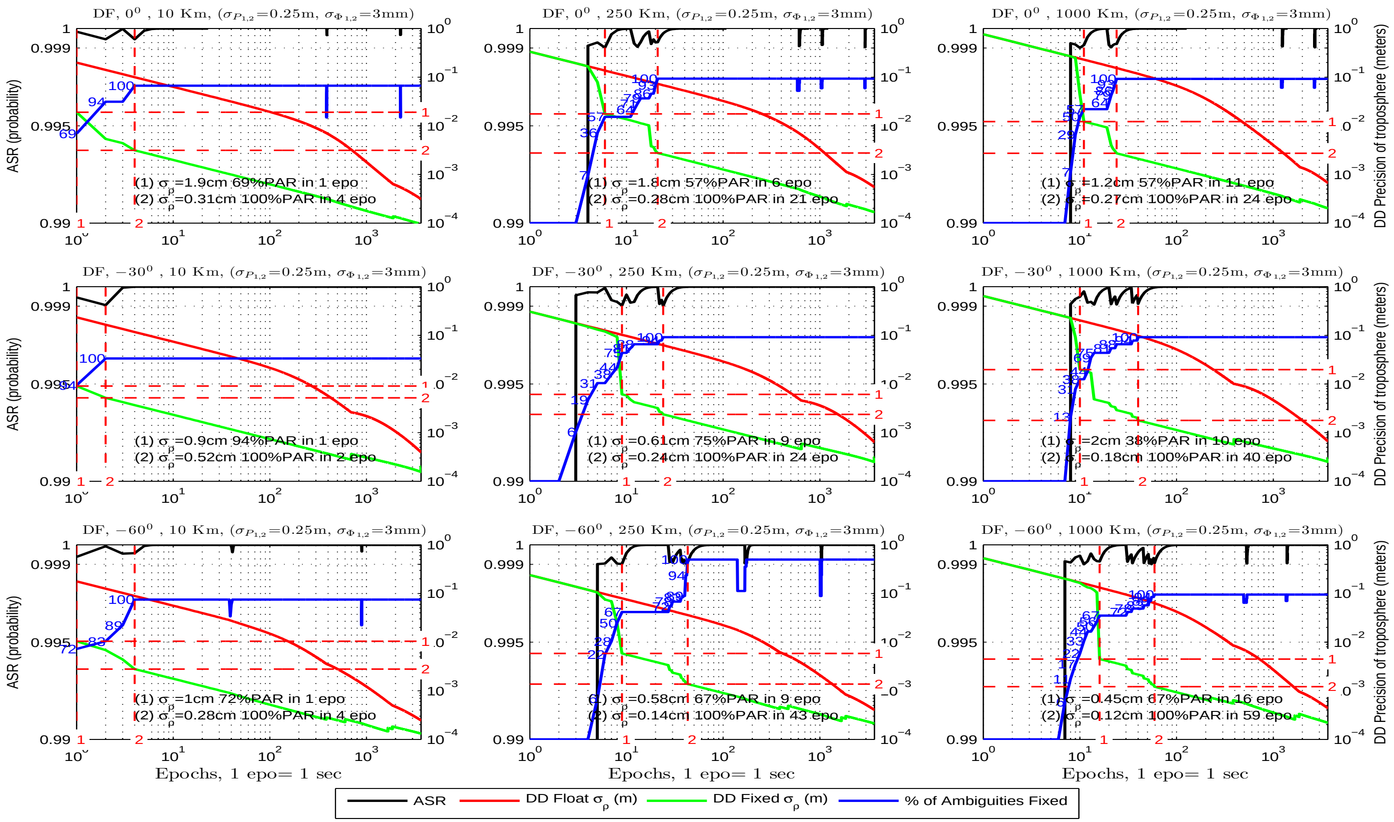

Figure F.40: **GPS only**, Dual frequency, **Partial AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced σ_I = 0.68 mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

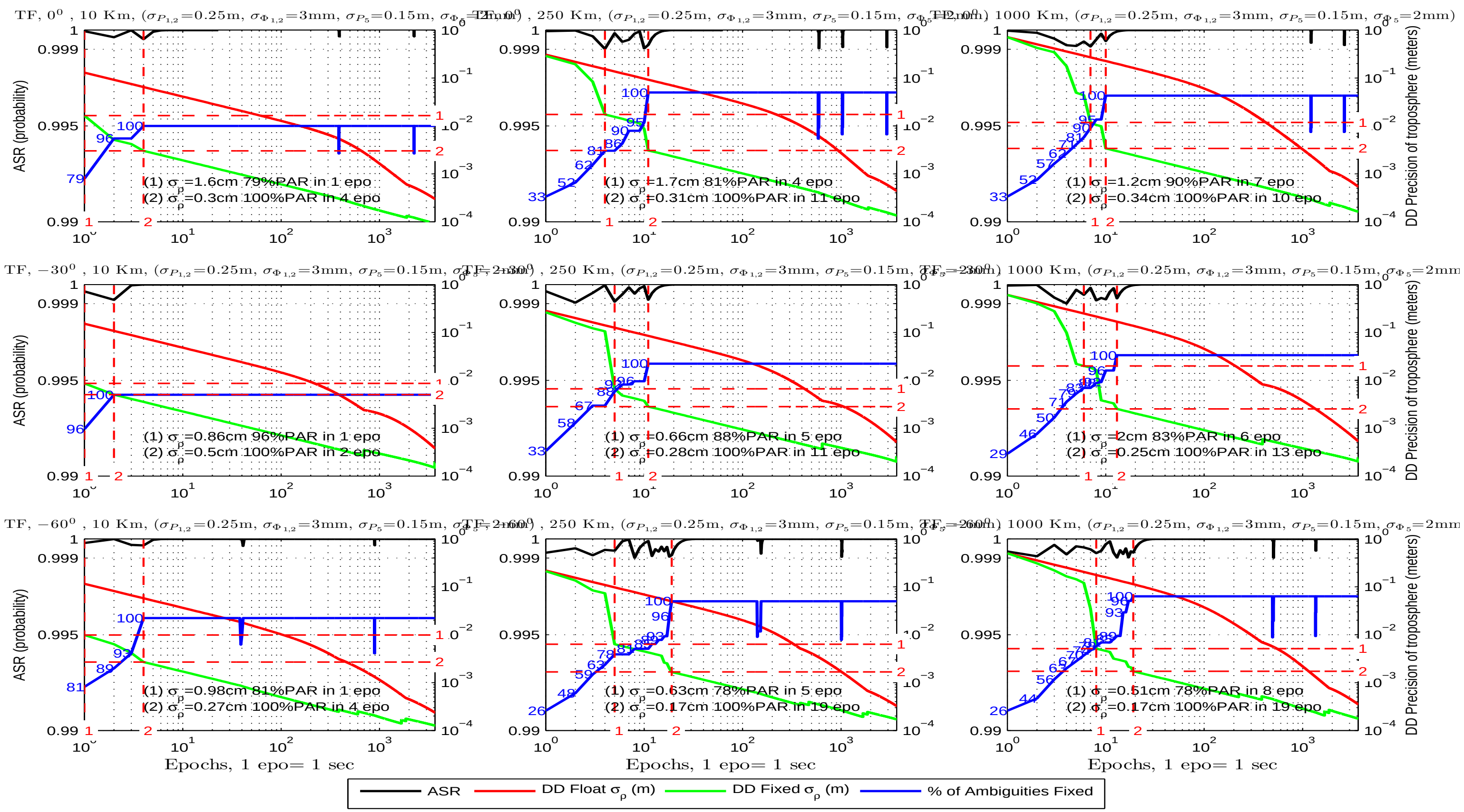

Figure F.41: **GPS only**, Triple frequency, **Partial AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

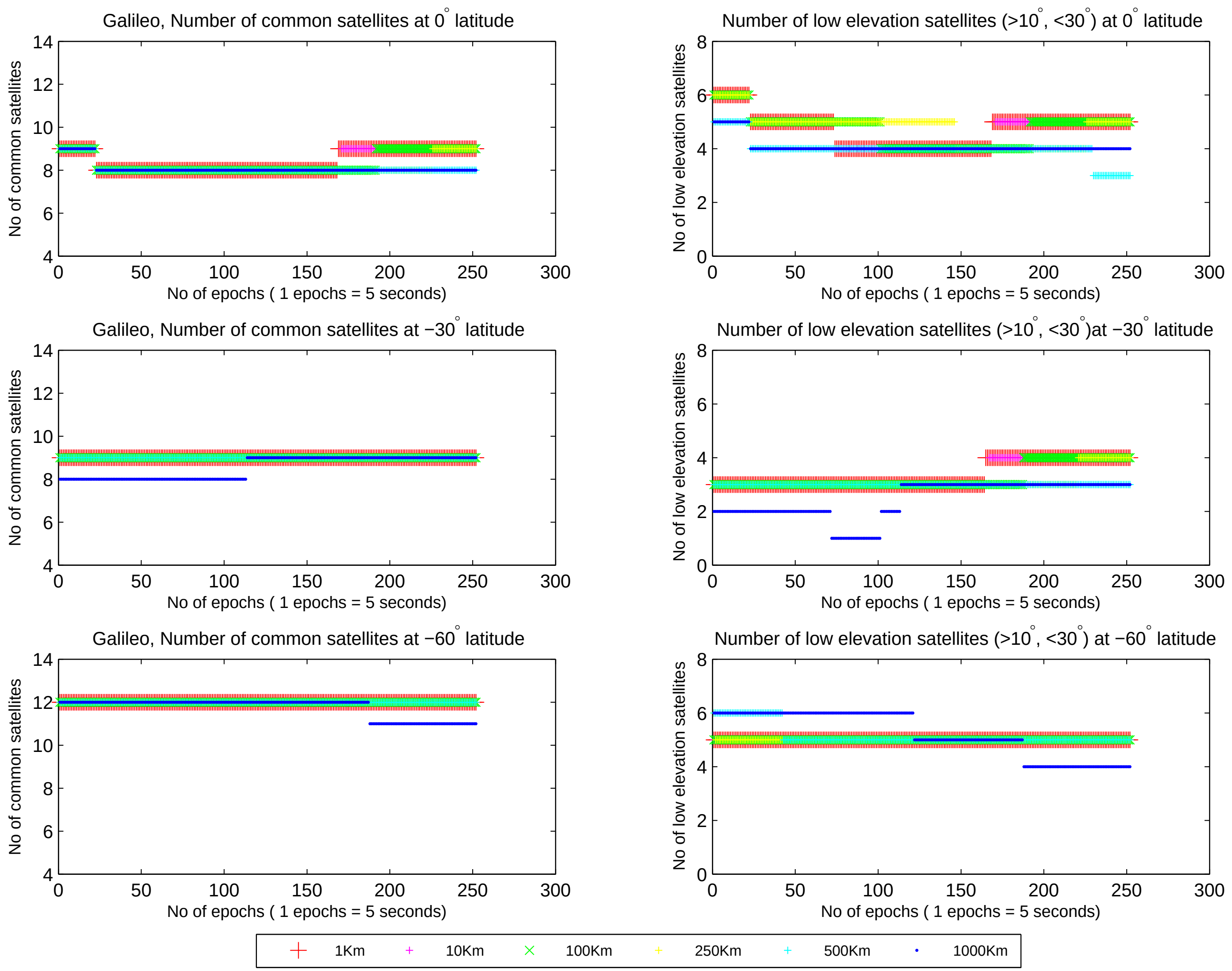

Figure F.42: **Galileo only**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, East-West oriented baseline.

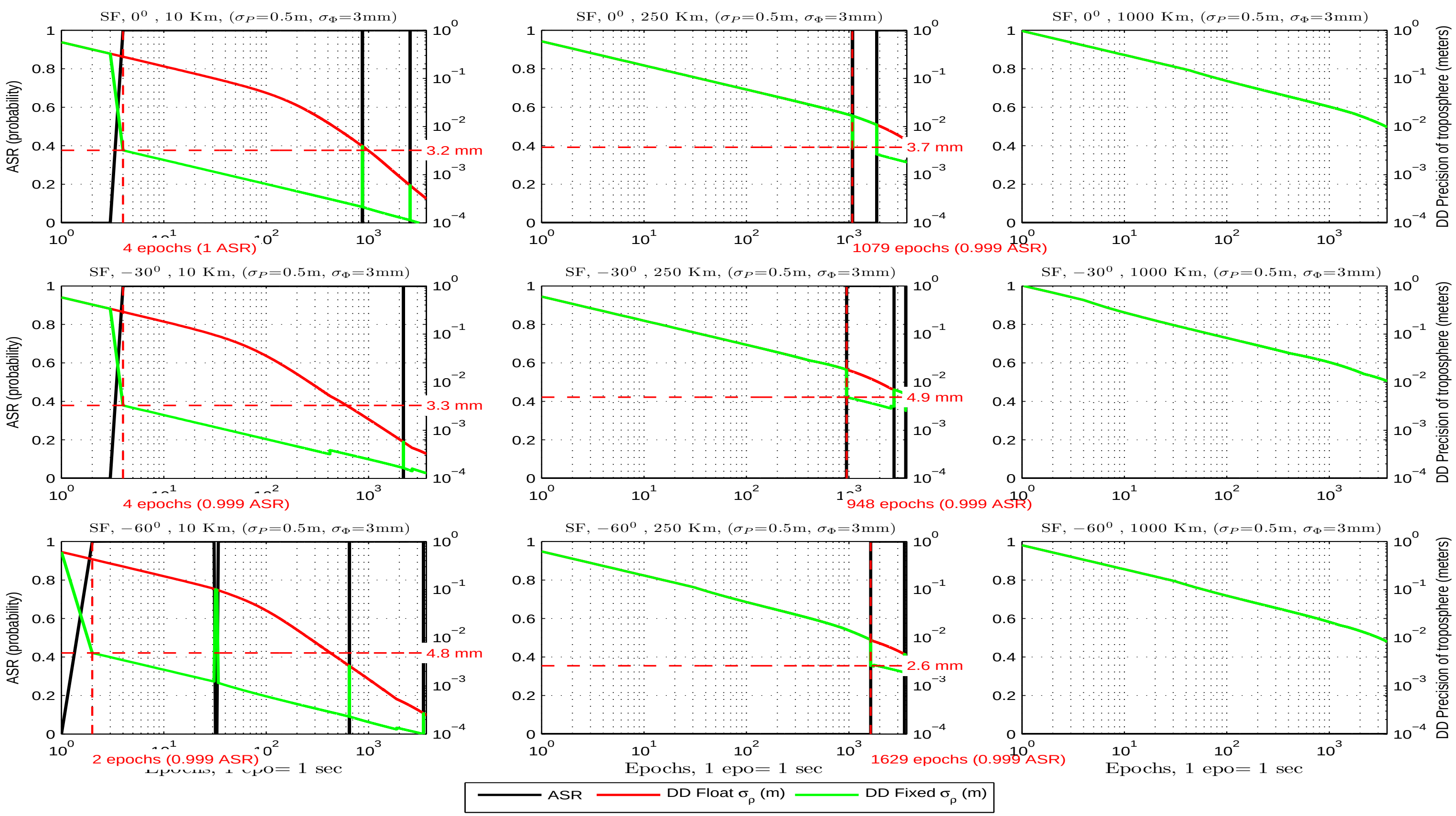

Figure F.43: **Galileo only**, Single frequency, **Full AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

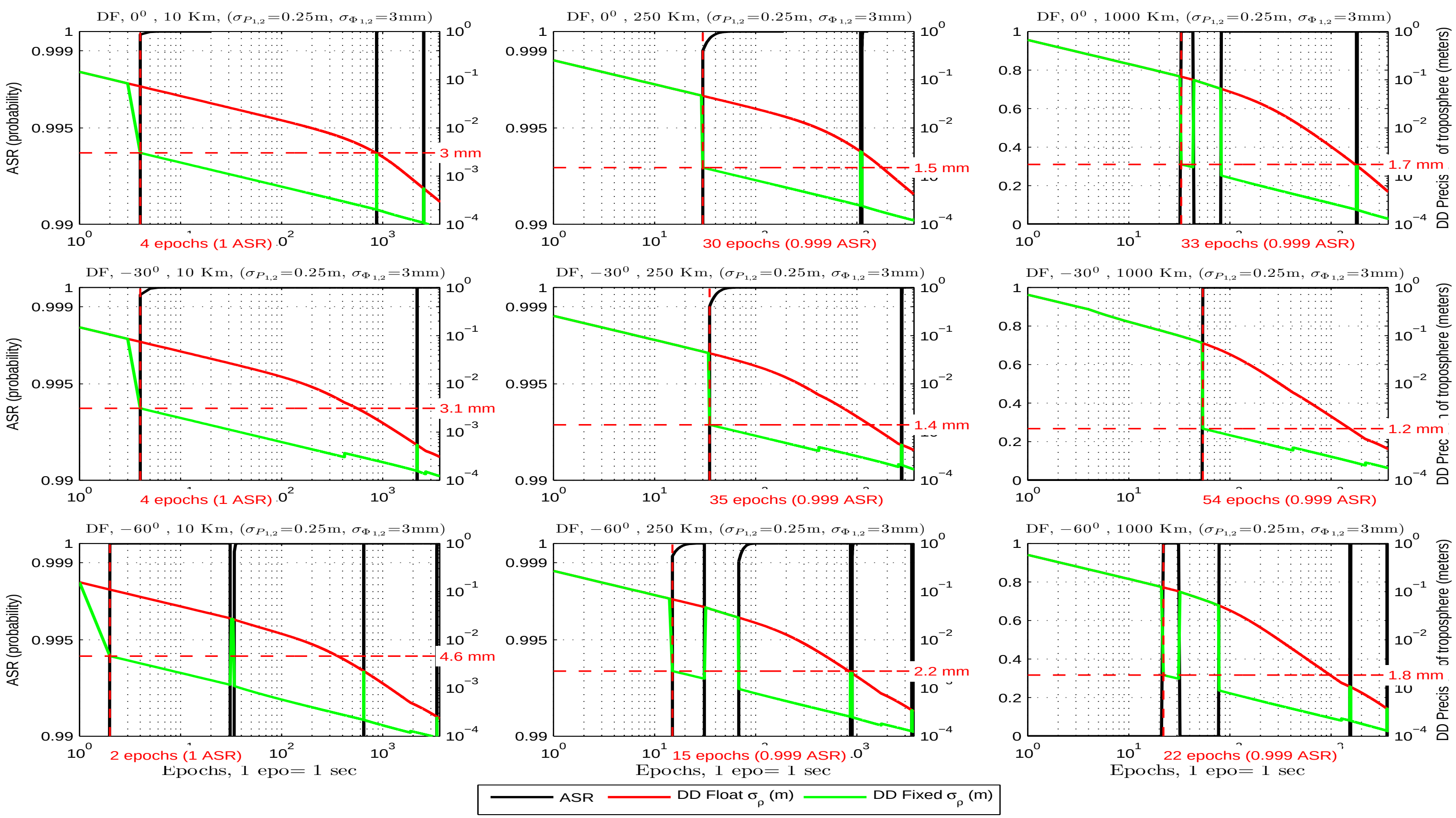

Figure F.44: **Galileo only**, Dual frequency, **Full AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

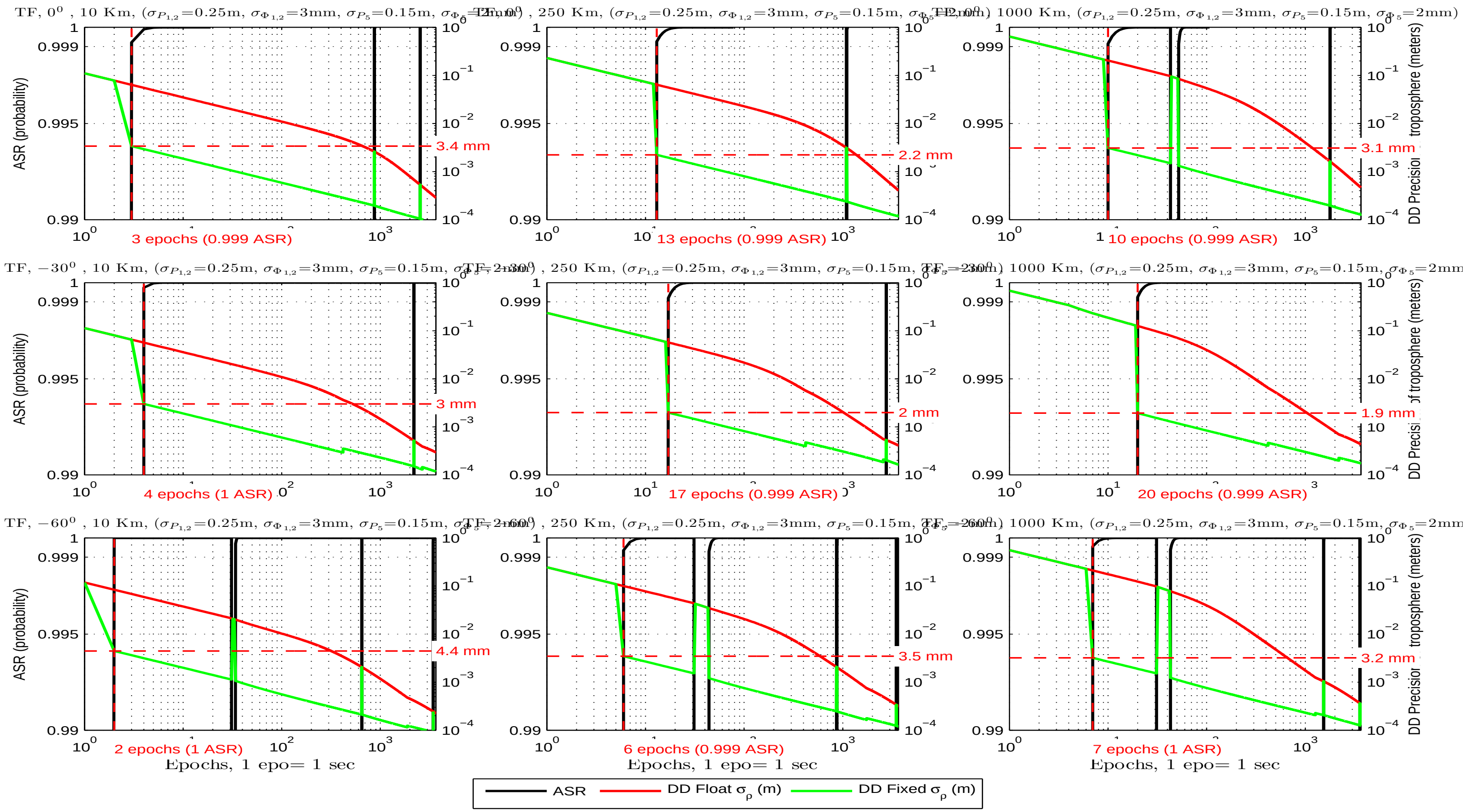

Figure F.45: **Galileo only**, Triple frequency, **Full AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°$, $-30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

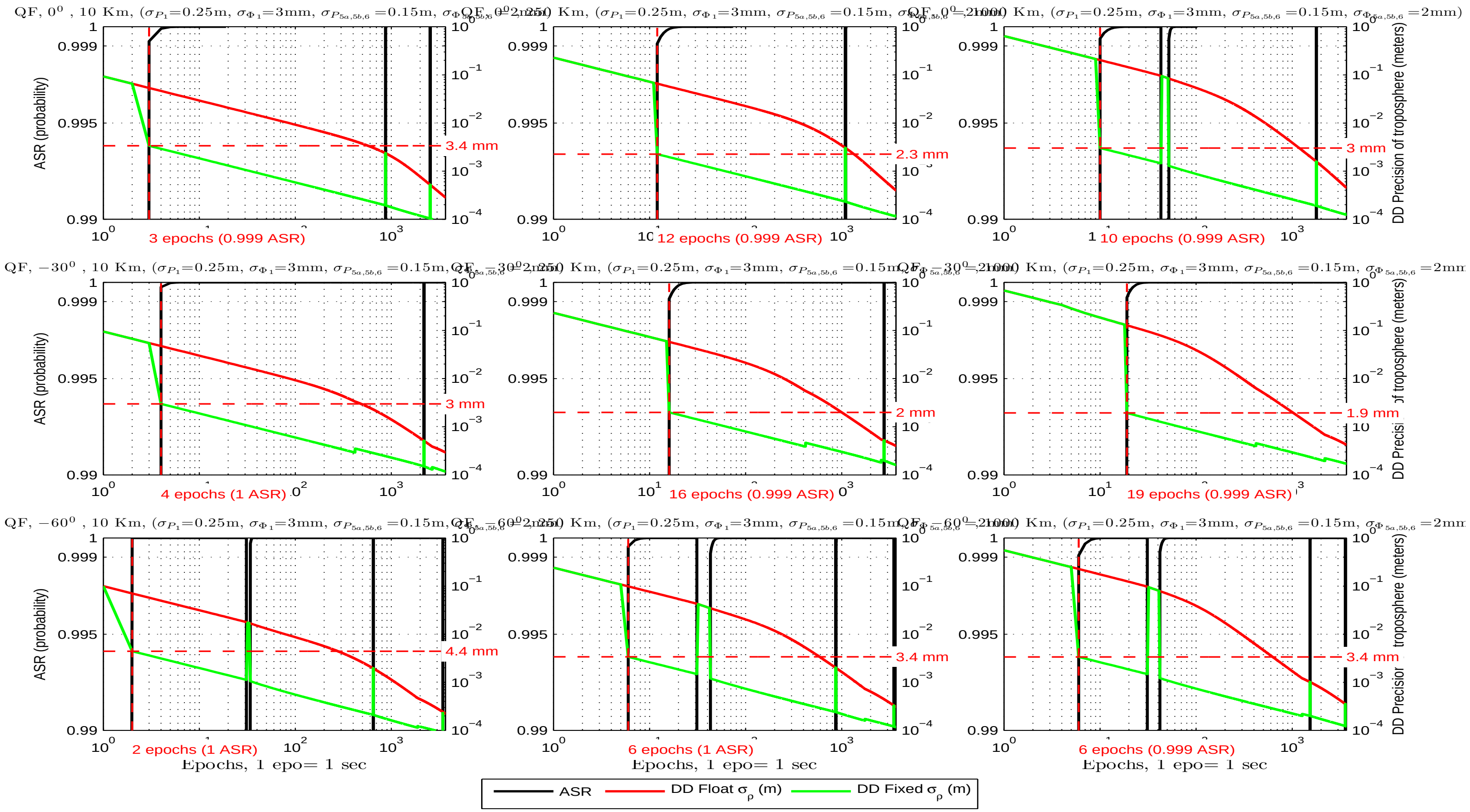

Figure F.46: **Galileo only**, Quadruple frequency, **Full AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

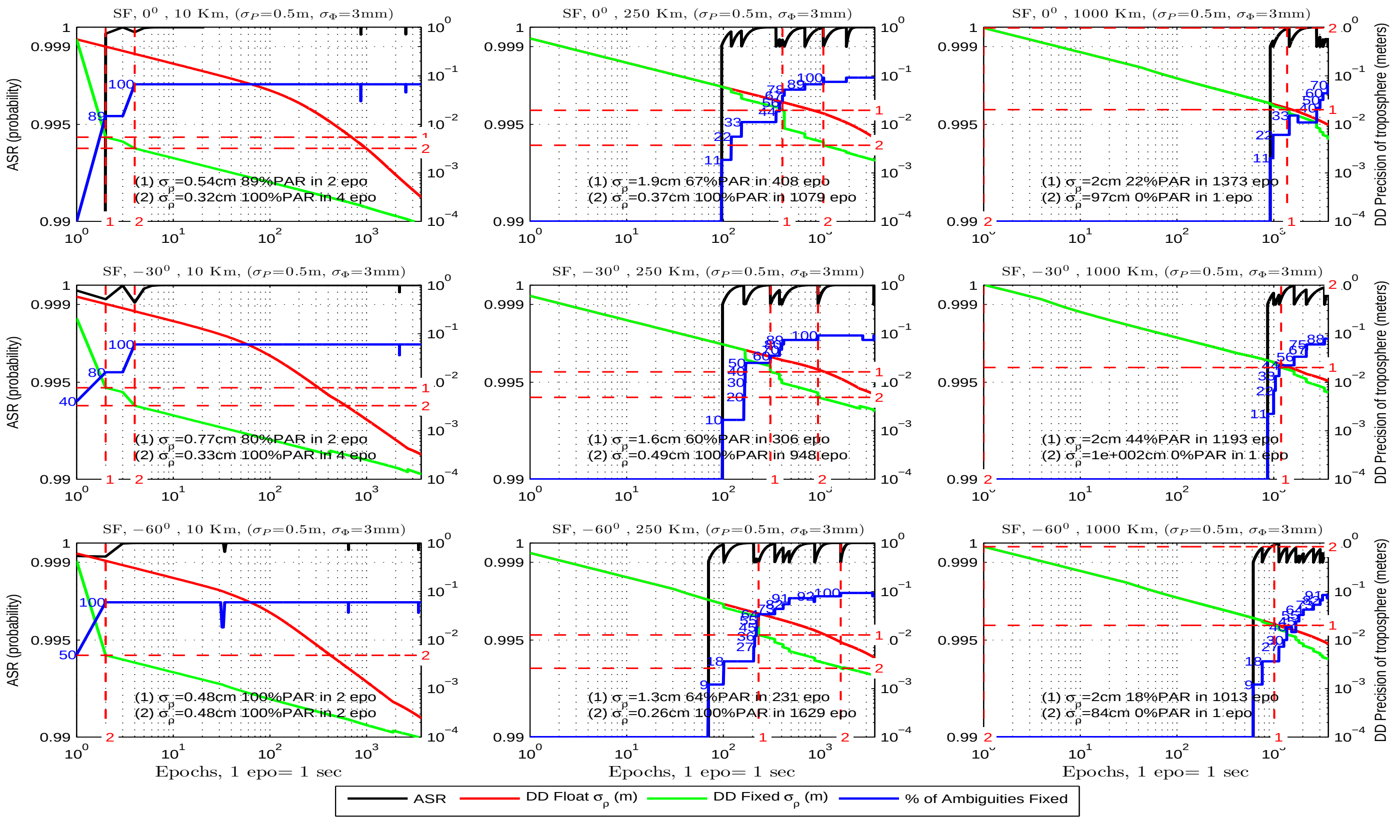

Figure F.47: **Galileo only**, Single frequency, **Partial AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

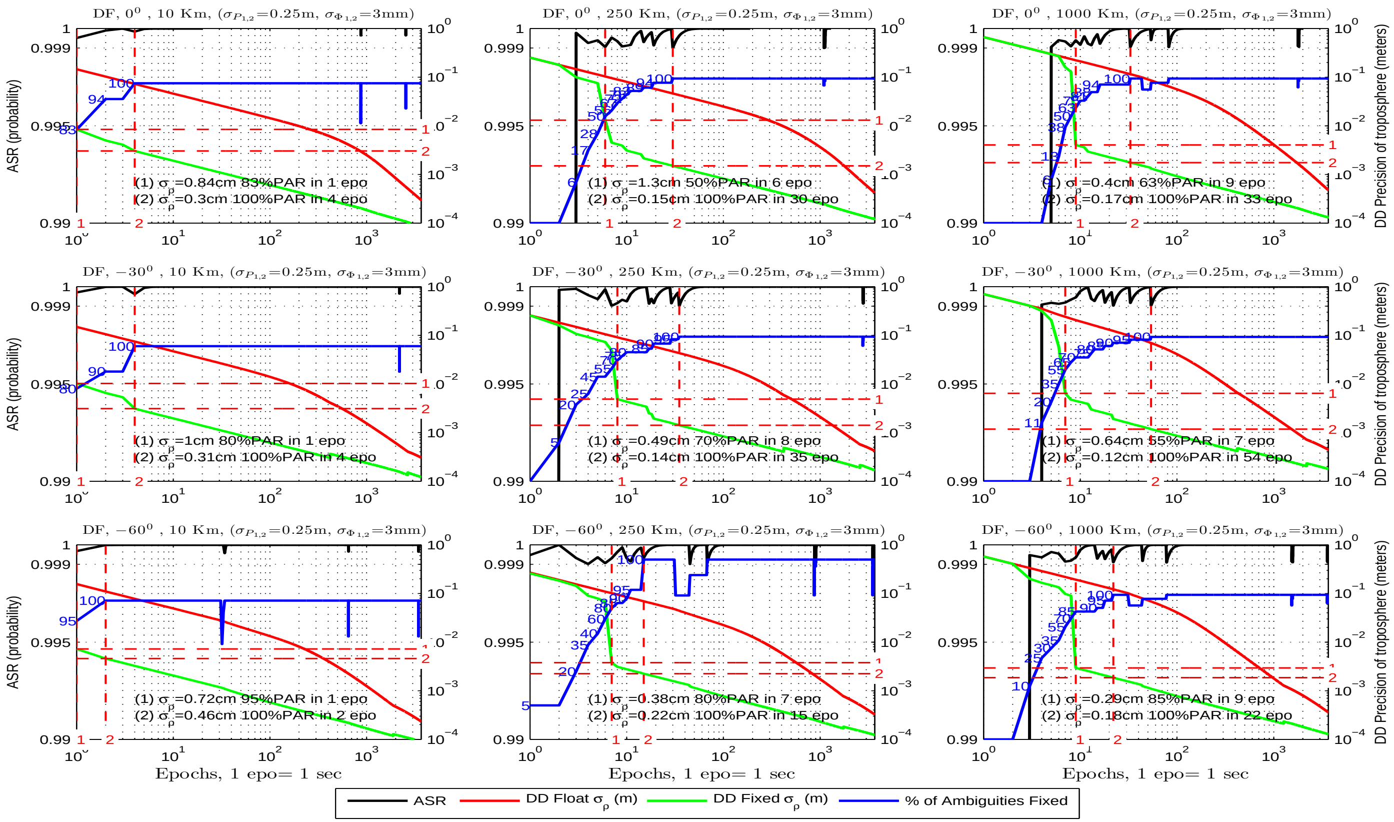

Figure F.48: **Galileo only**, Dual frequency, **Partial AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

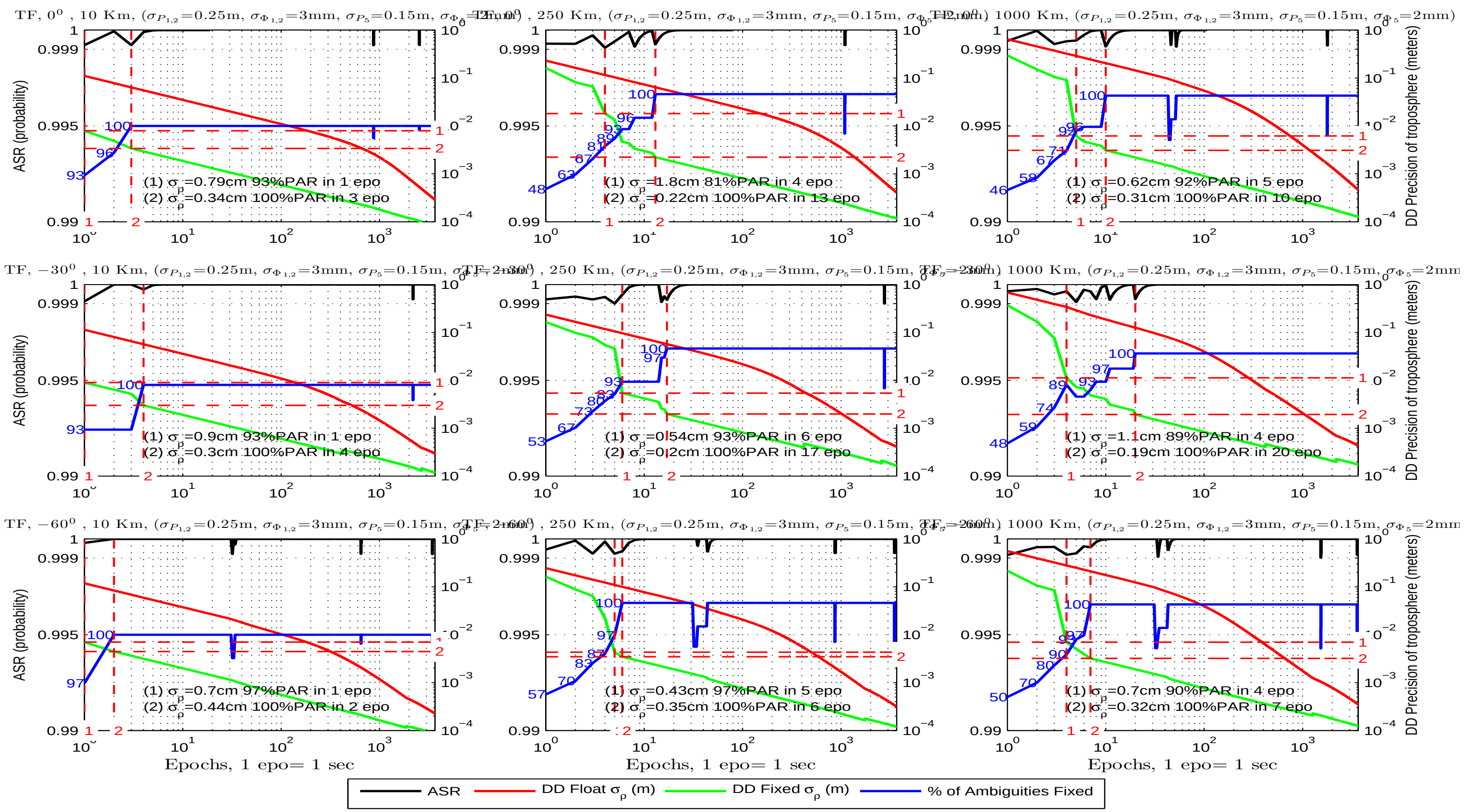

Figure F.49: **Galileo only**, Triple frequency, **Partial AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

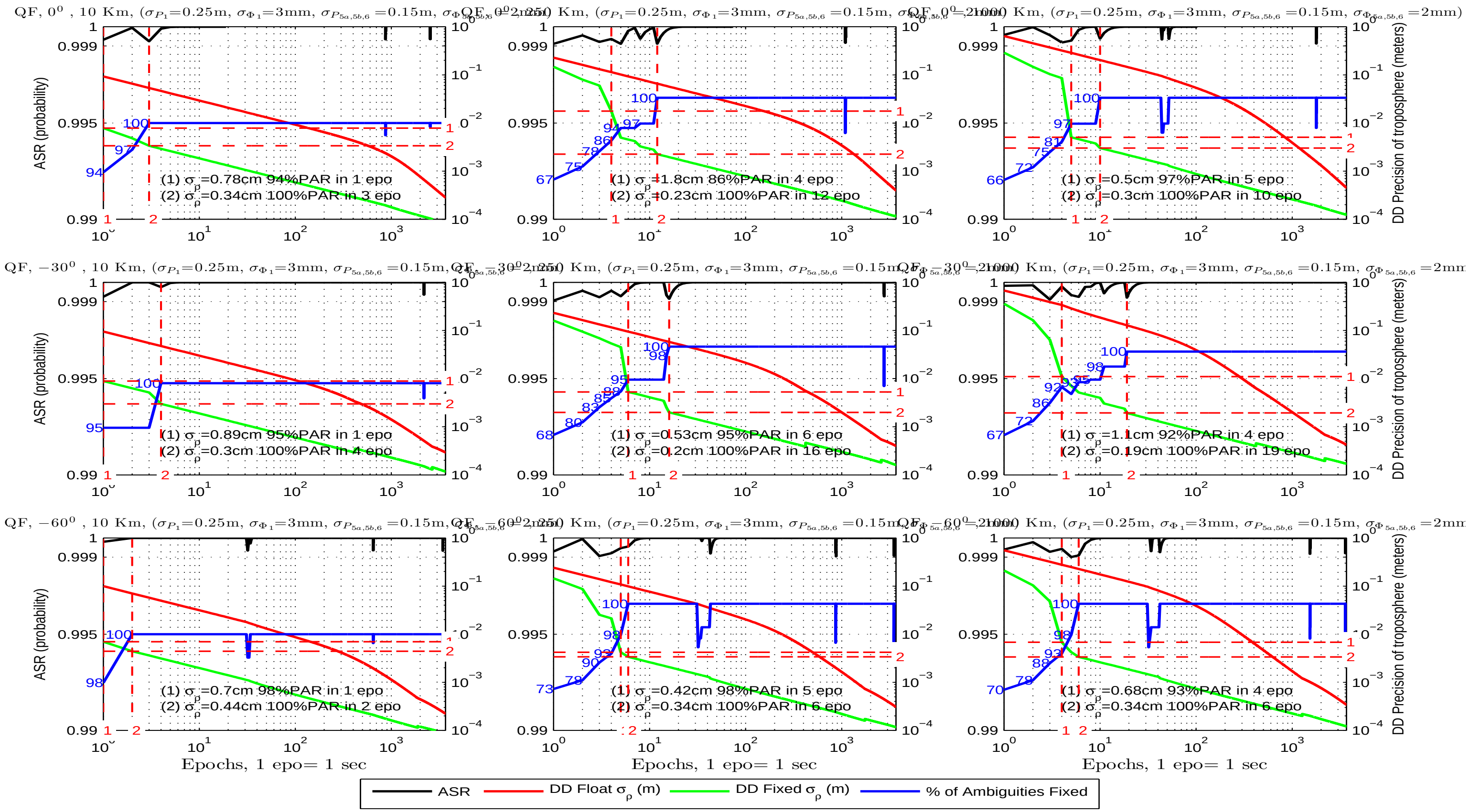

Figure F.50: **Galileo only**, Quadruple frequency, **Partial AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

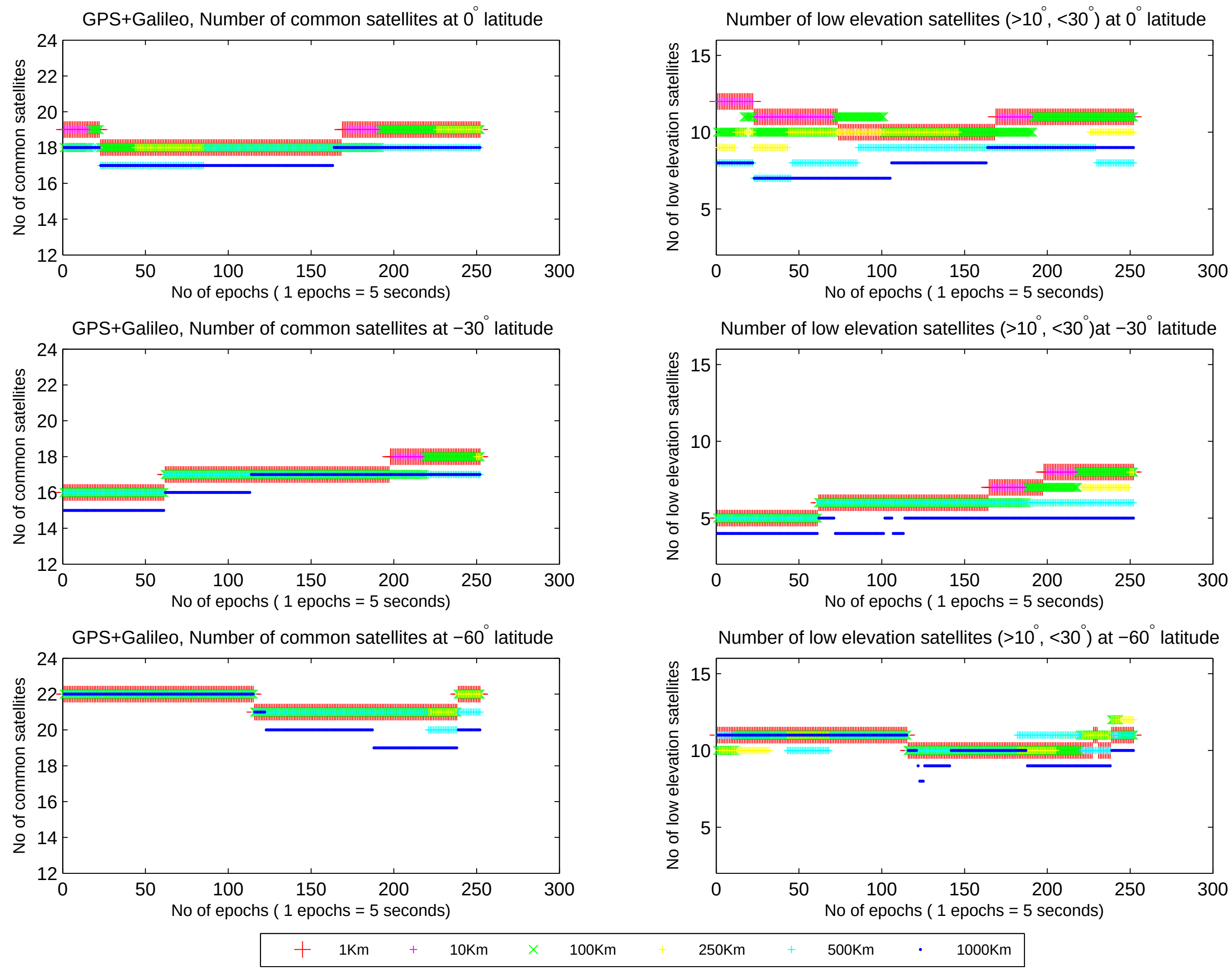

Figure F.51: **GPS + Galileo**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, **East-West** oriented baseline.

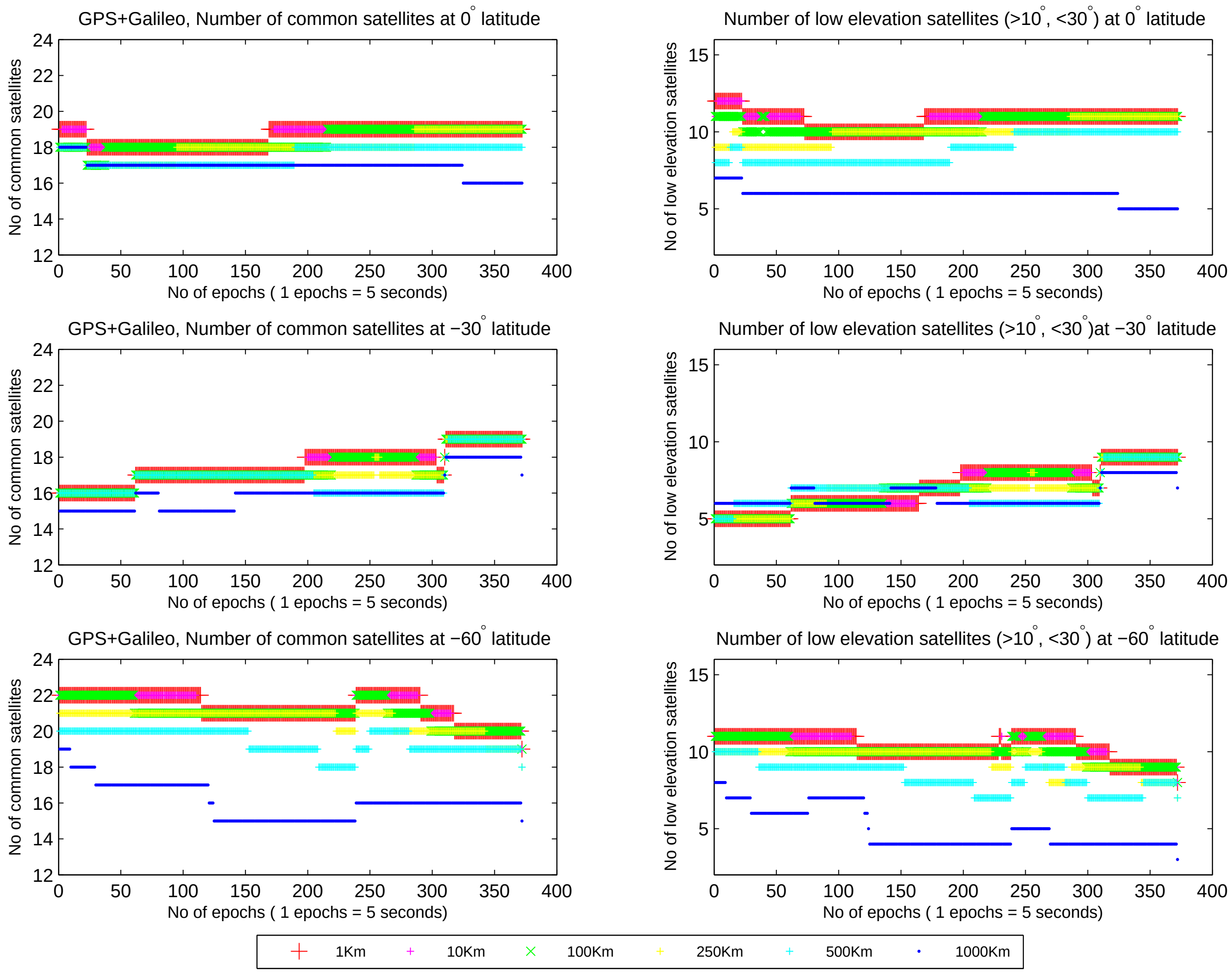

Figure F.52: **GPS + Galileo**, total number of common satellites (left side) and low elevation satellites (between 10° and 30° elevation) among the common satellites (right side), for different baseline lengths, at 0°, −30° and −60° degree latitude, **North-South** oriented baseline.

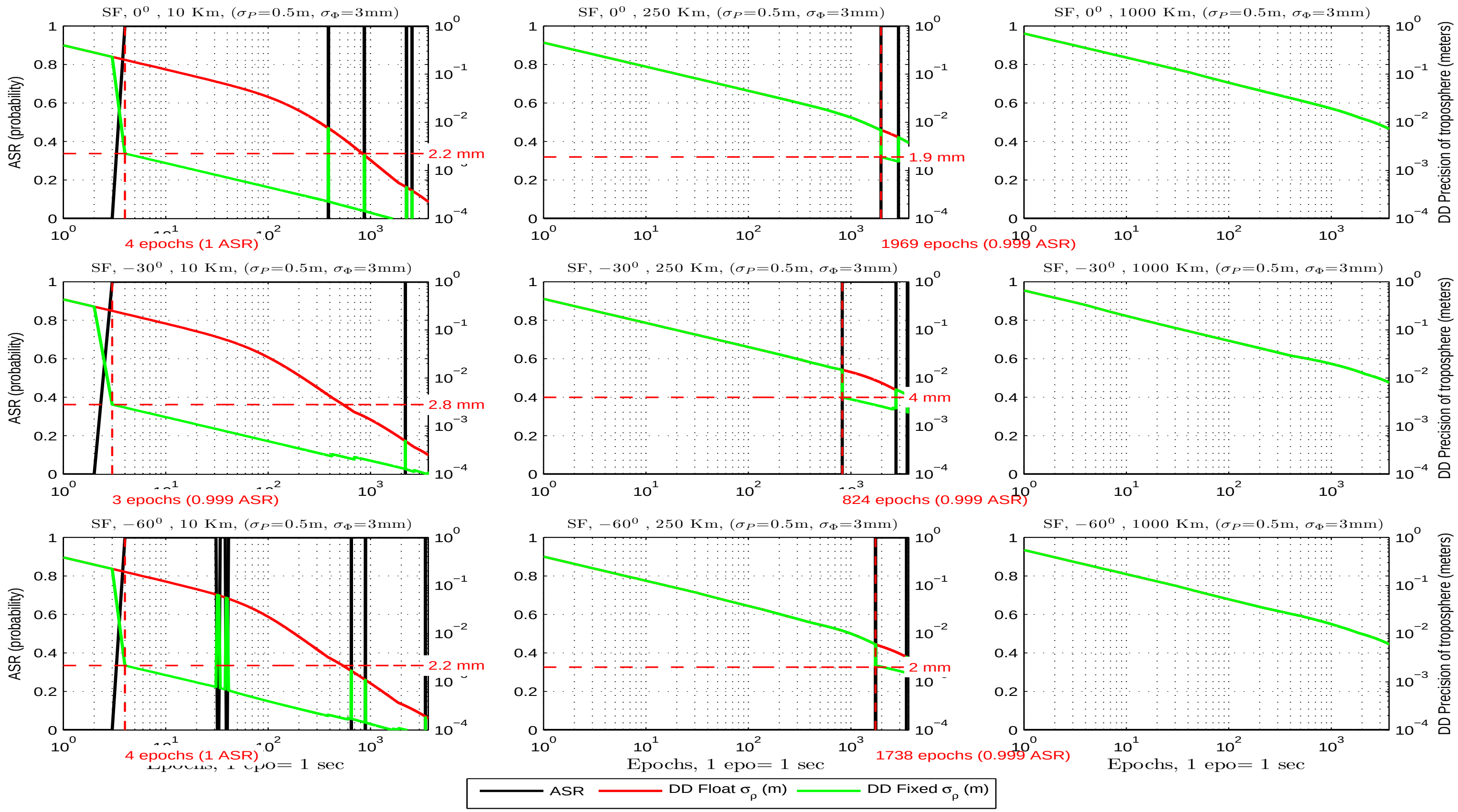

Figure F.53: **GPS+Galileo**, Single frequency, **Full AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced σ_I = 0.68 mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

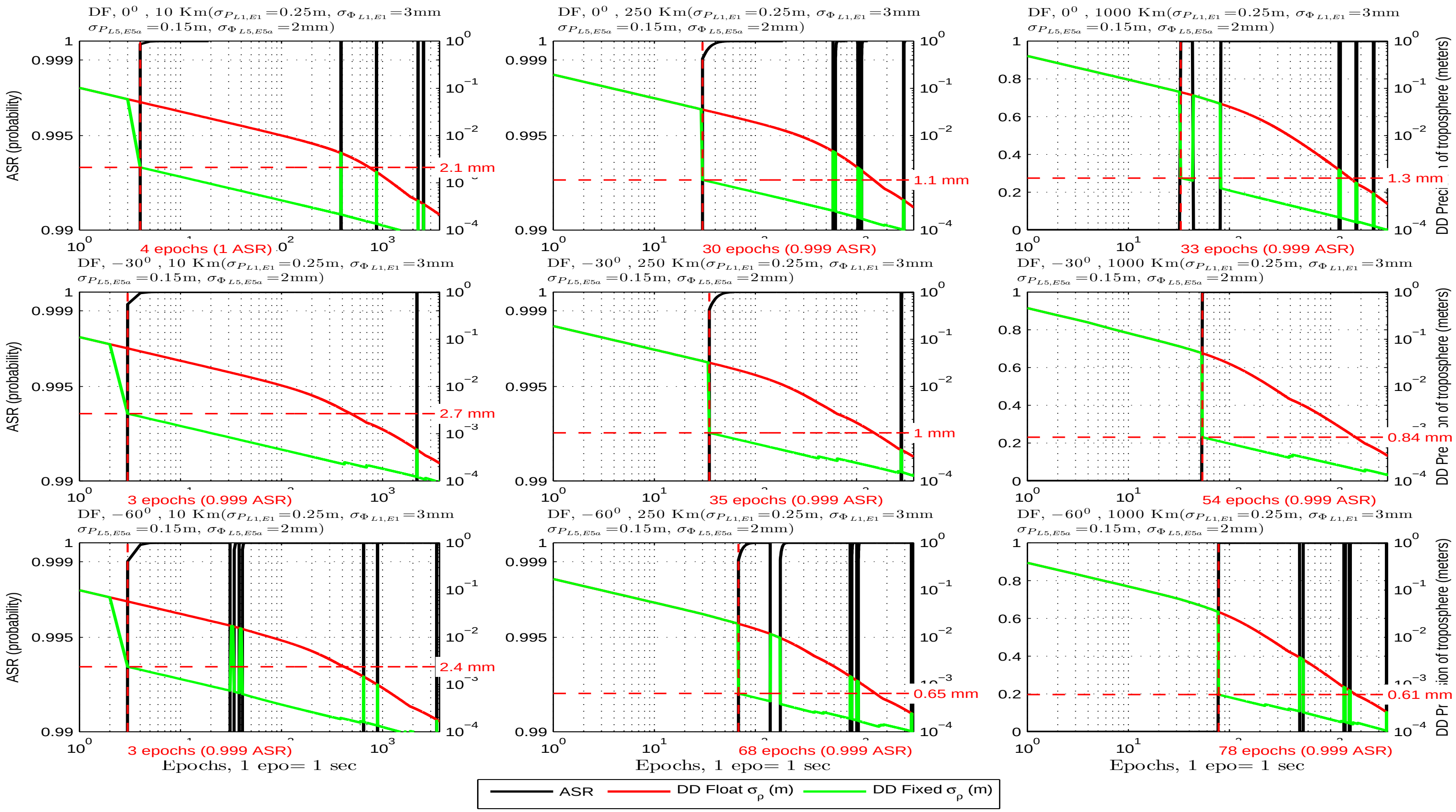

Figure F.54: **GPS+Galileo**, Dual frequency, **Full AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced σ_I = 0.68 mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

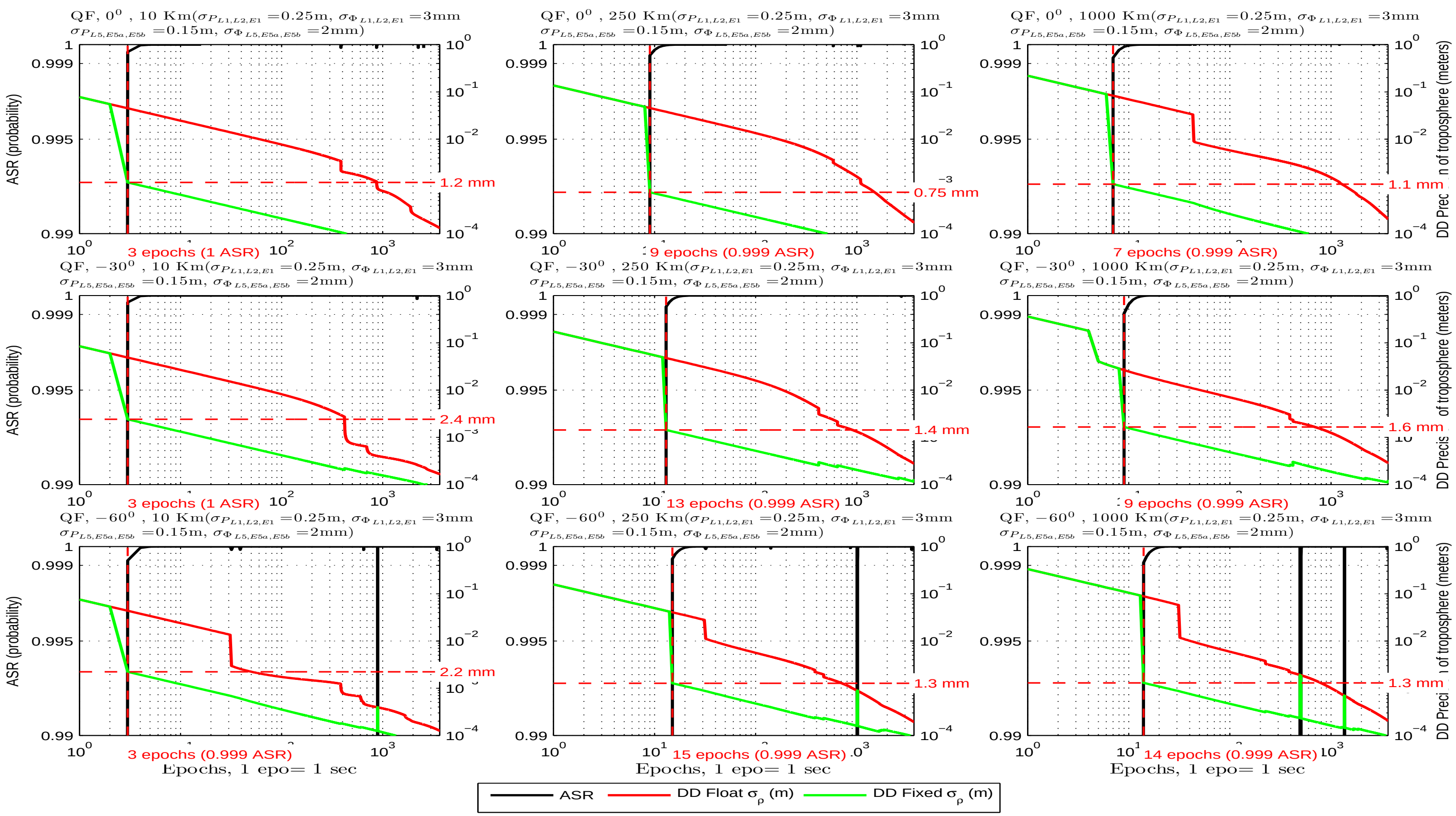

Figure F.55: **GPS+Galileo**, quadruple frequency, **Full AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

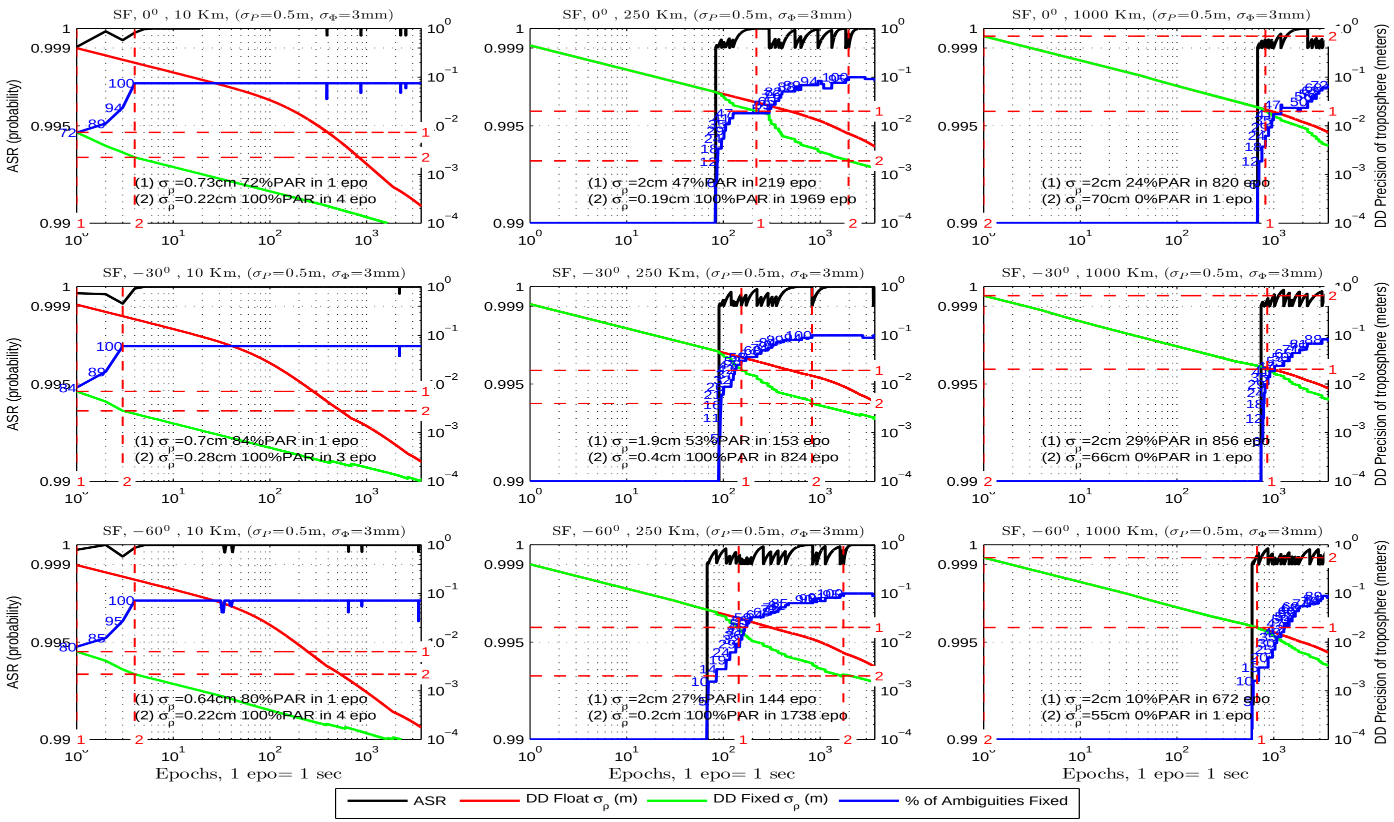

Figure F.56: **GPS+Galileo**, Single frequency, **Partial AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

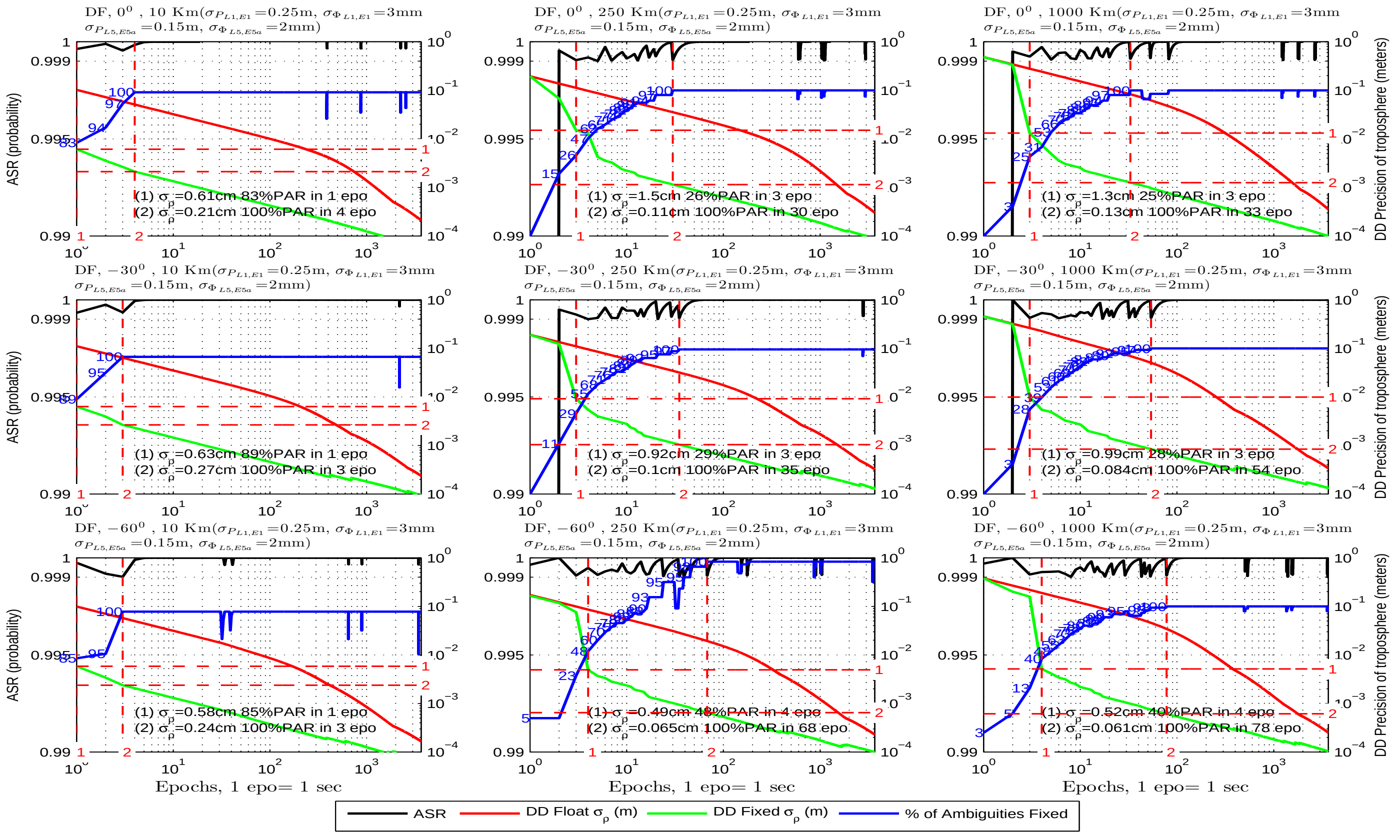

Figure F.57: **GPS+Galileo**, Dual frequency, **Partial AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

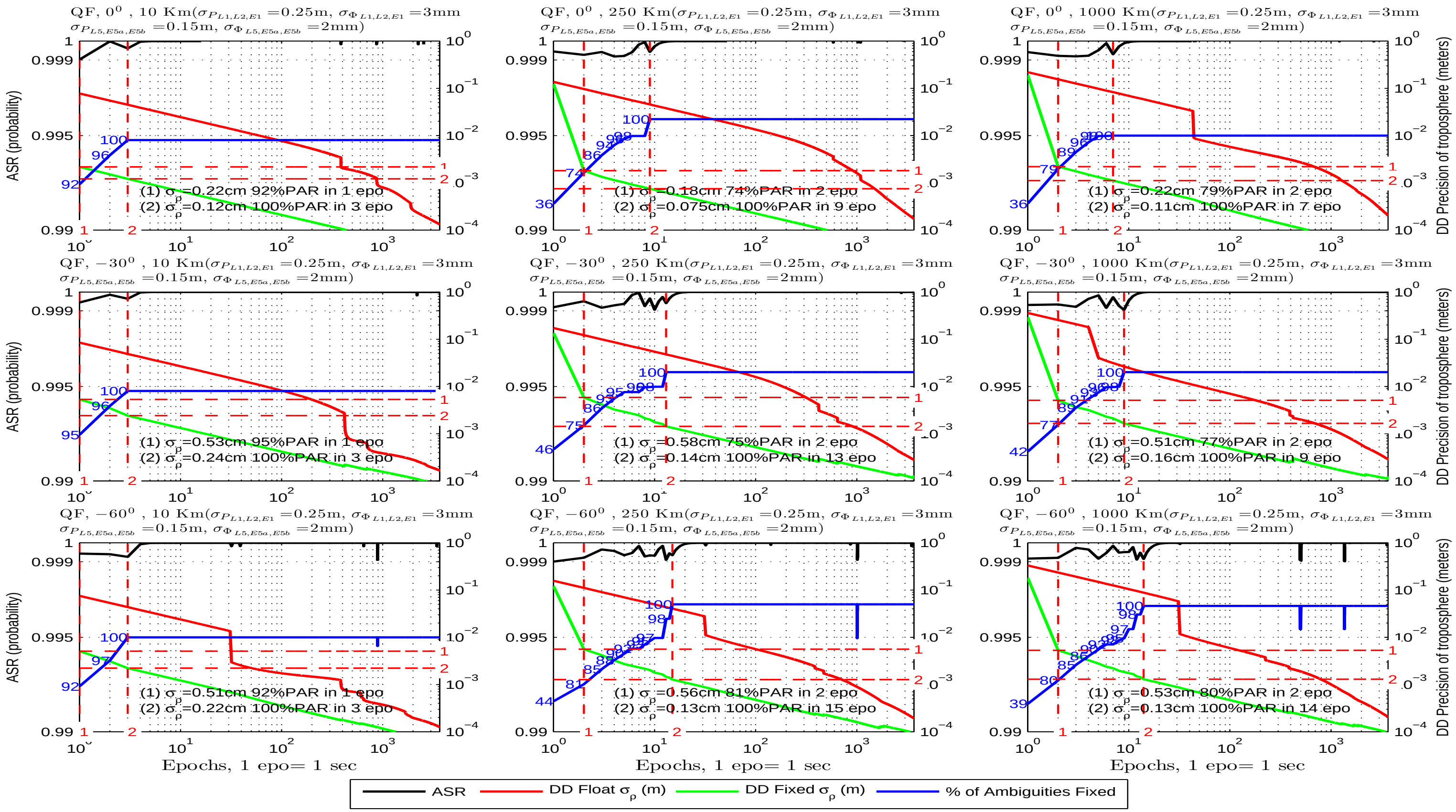

Figure F.58: **GPS+Galileo**, quadruple frequency, **Partial AR**, Ionosphere weighted, troposphere float, **Geometry Fixed model**, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, un-differenced $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of ionosphere (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

G
Figures for Reference Rover model

In this appendix, the figures for Geometry Based model, when the rover coordinates are unknown are presented for three atmospheric scenarios, atmosphere known, atmosphere unknown and ionosphere weighted with troposphere unknown. For each atmosphere scenario, figures show Ambiguity Success Rate (ASR) and Double Difference (DD) precision of the coordinates for ambiguity-fixed and float solution. The DD precision for the coordinates is computed based on a volumetric, ADOP style expression given as $|Q_{\widehat{xyz}}|^{1/2n}$ for the ambiguity float solution and $|Q_{\widetilde{xyz}}|^{1/2n}$ for ambiguity fixed solution, n =3 for three coordinate components. Different GNSS system considerations and combination are used, namely GPS only, Galileo only and GPS and Galileo together. Also different frequency combinations, namely, single, dual, triple and quadruple are considered.

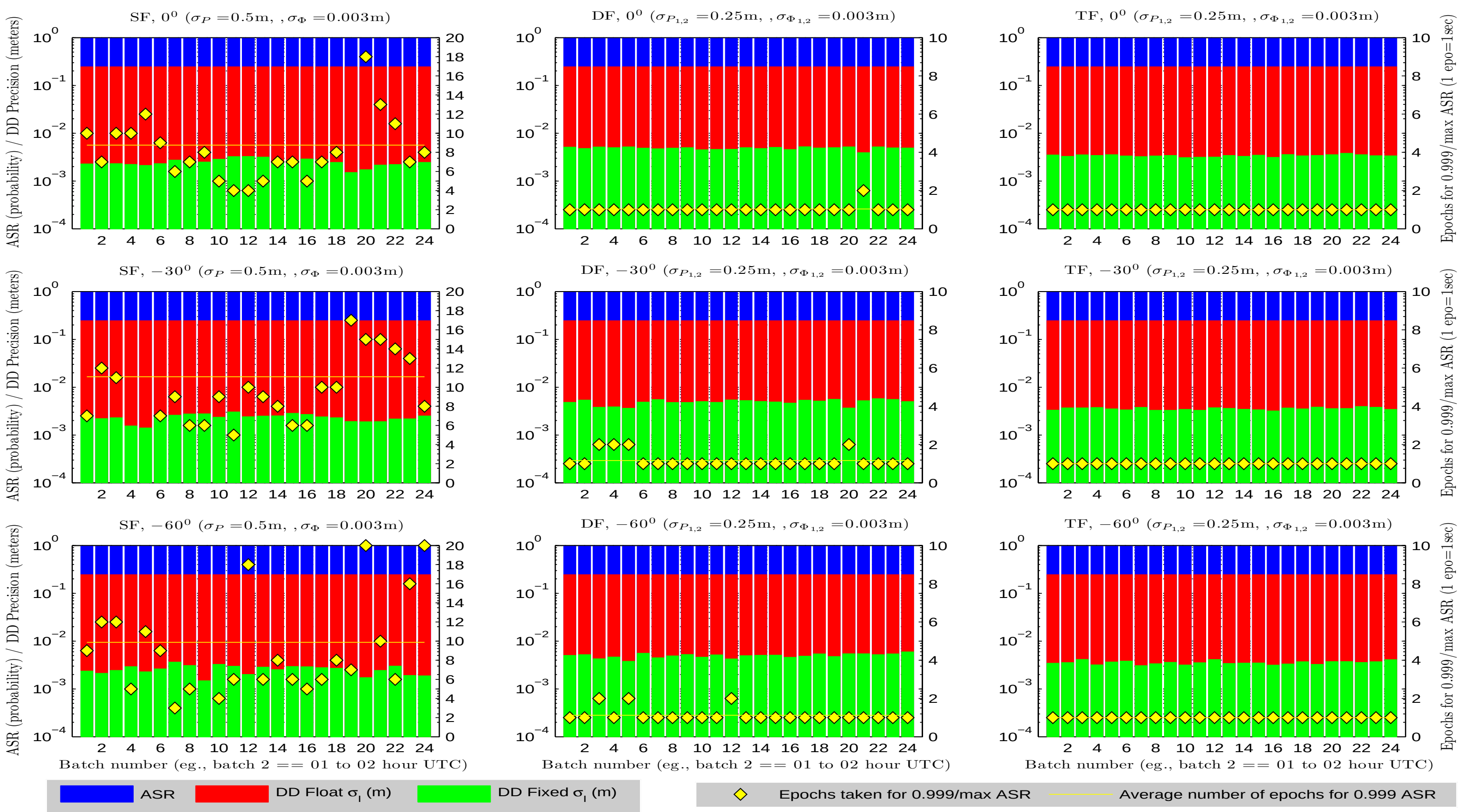

Figure G.1: **GPS only**, Atmosphere known scenario, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual and triple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR and, float and fixed precision of ionosphere.

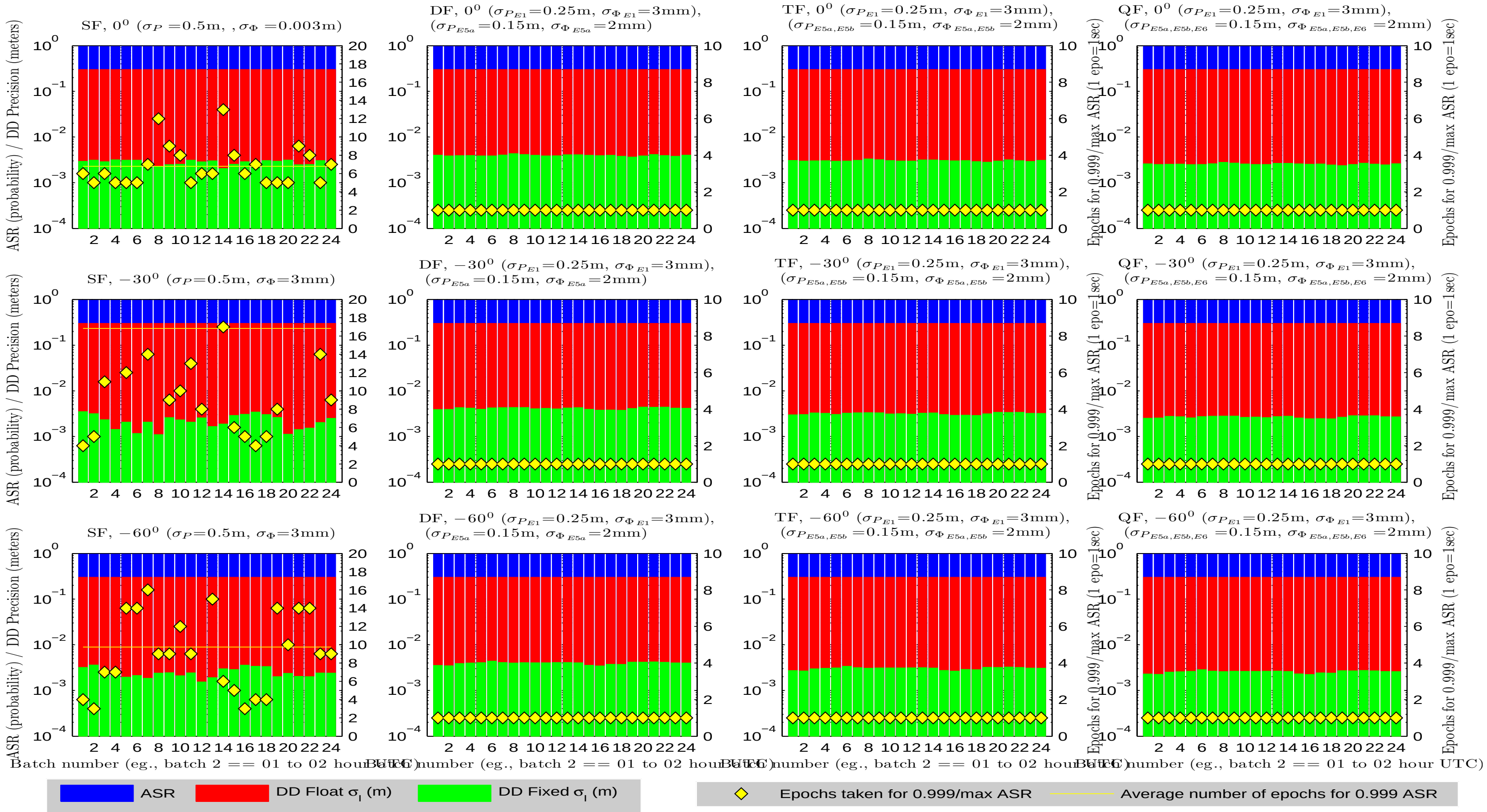

Figure G.2: **Galileo only**, Atmosphere known scenario, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual, triple and quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR and, float and fixed precision of ionosphere.

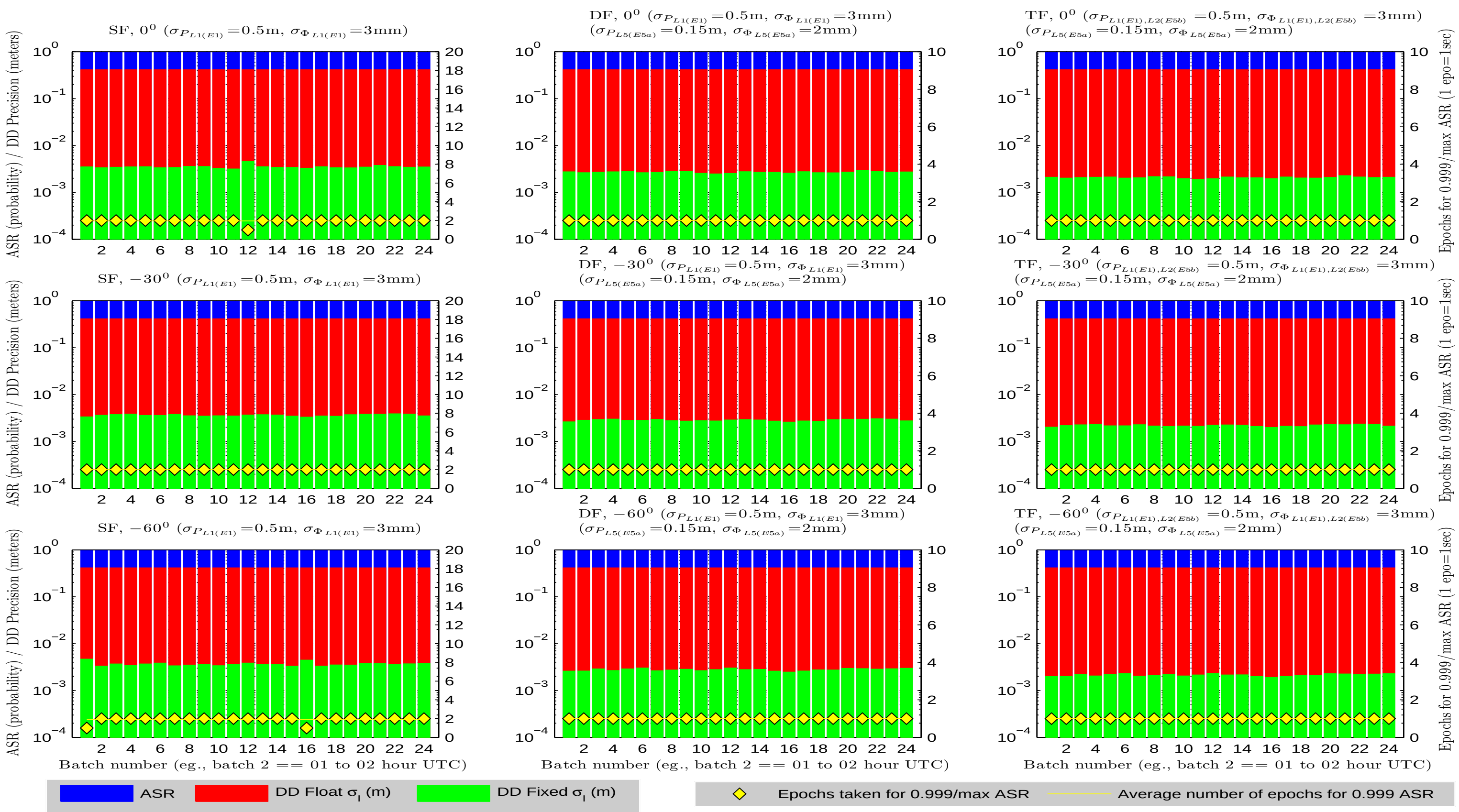

Figure G.3: **GPS + Galileo**, Atmosphere known scenario, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual and quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR and, float and fixed precision of ionosphere.

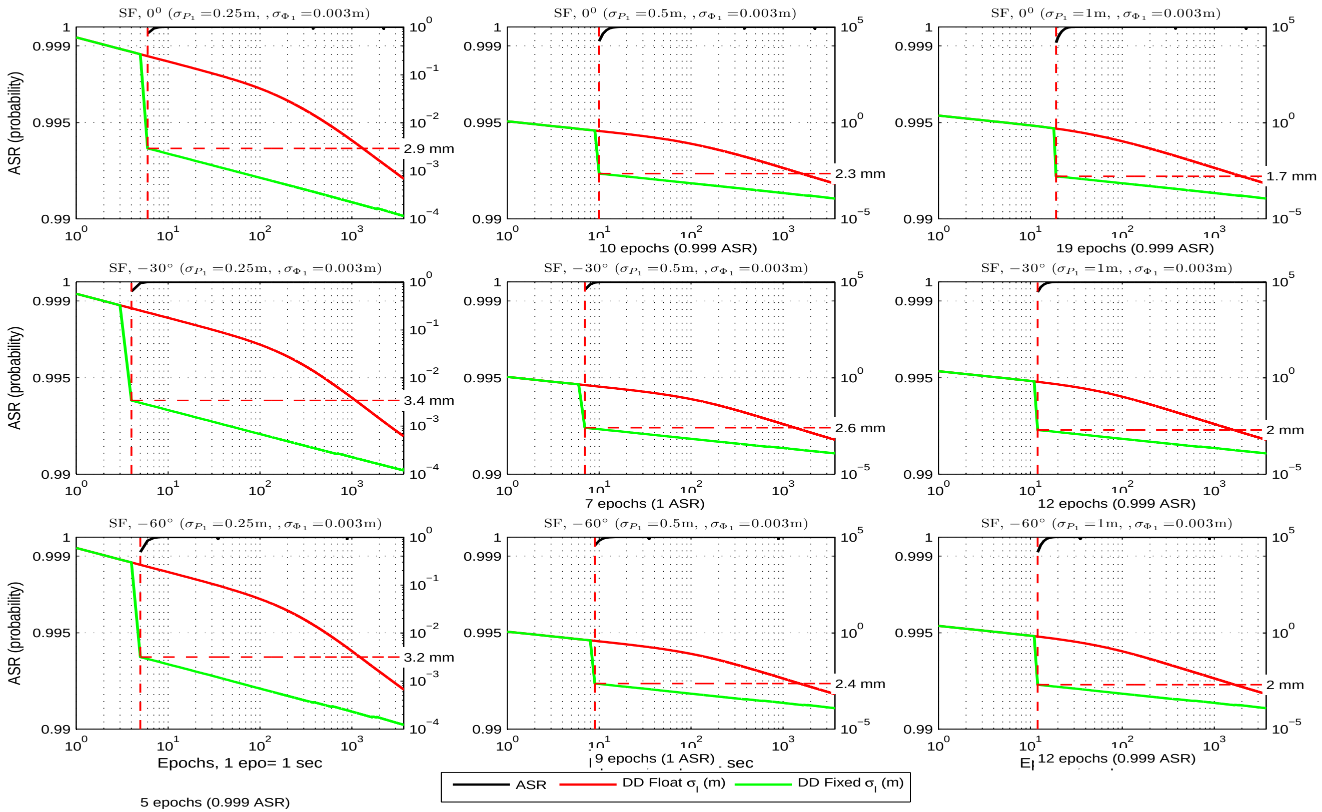

Figure G.4: **GPS only**, Single frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

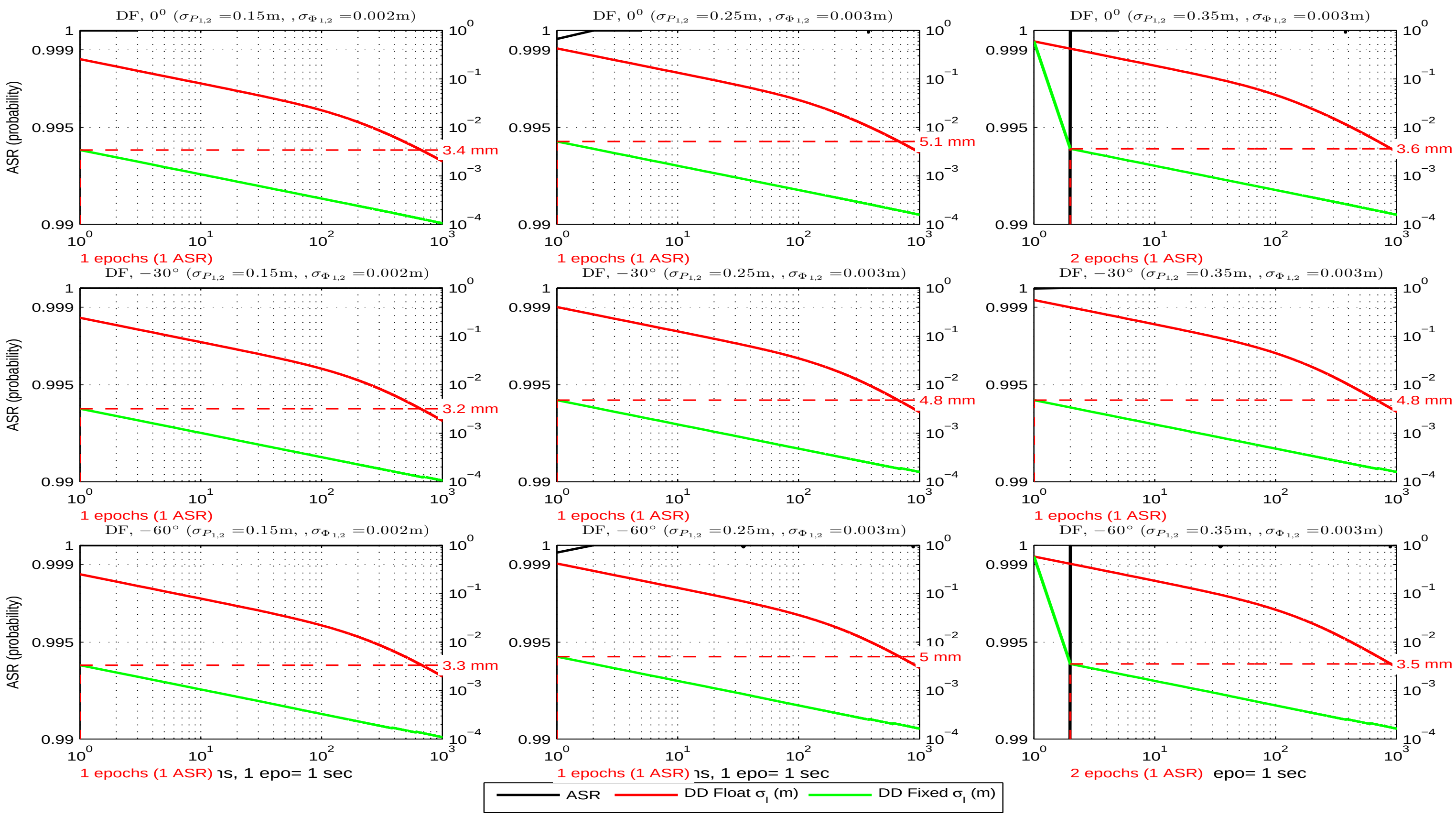

Figure G.5: **GPS only**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**-ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

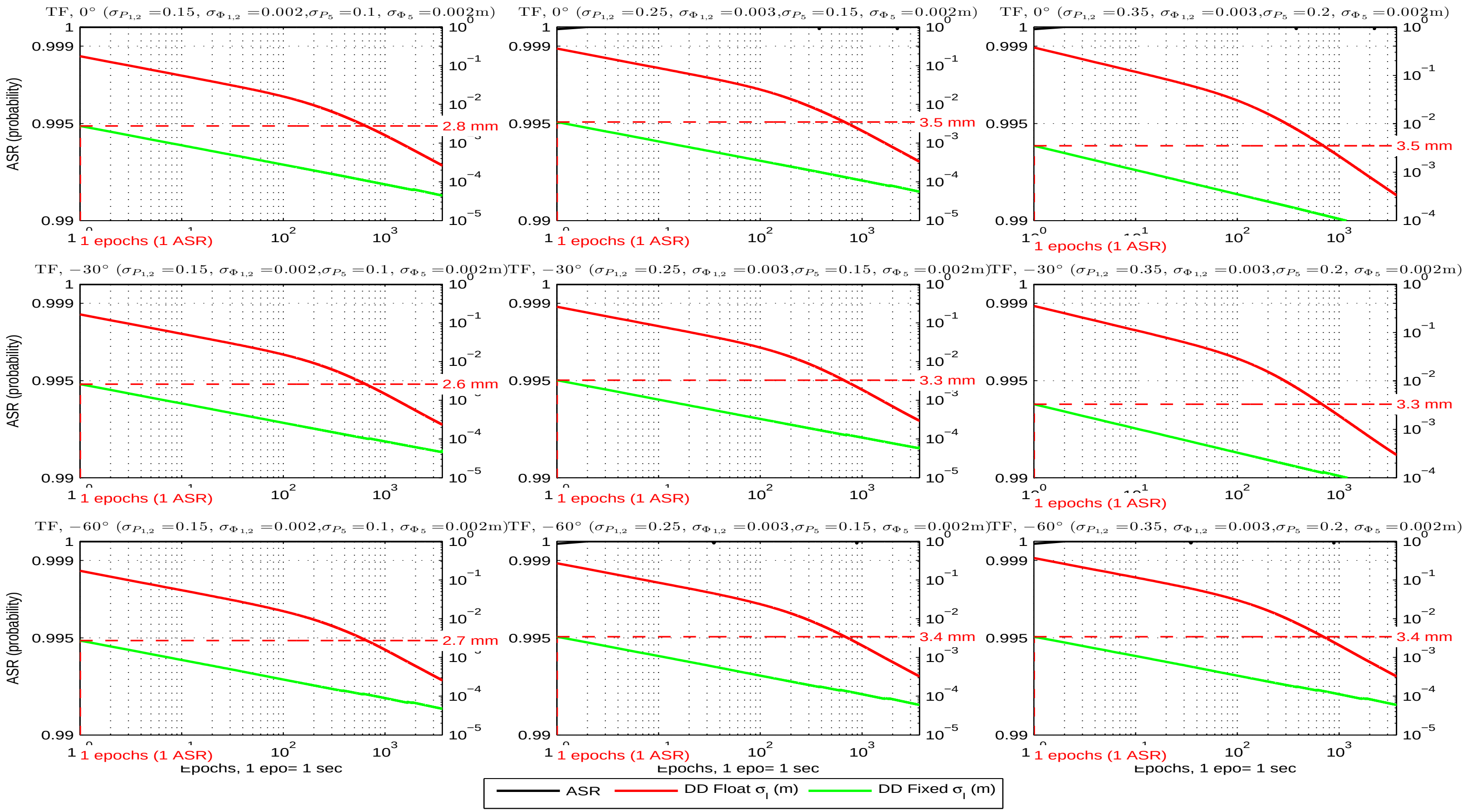

Figure G.6: **GPS only**, Triple frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

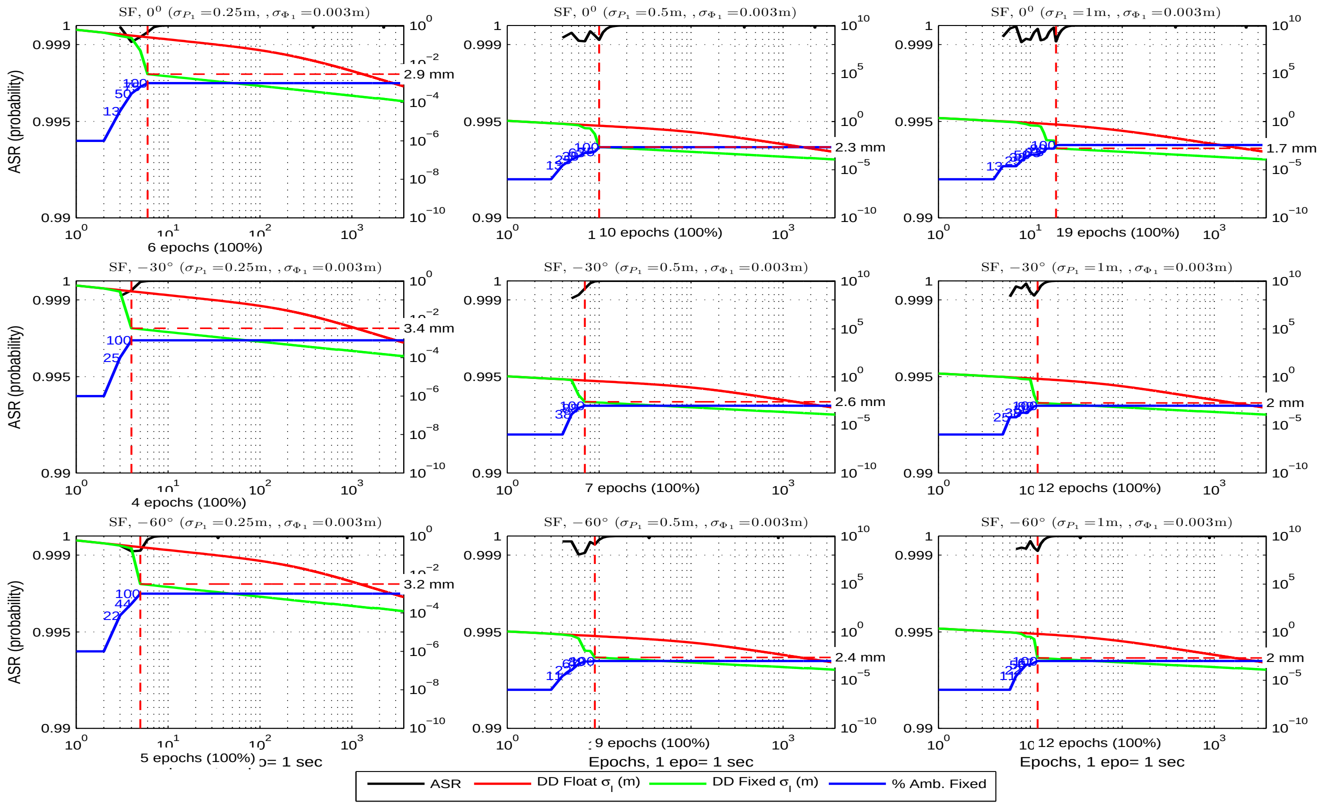

Figure G.7: **GPS only**, Single frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

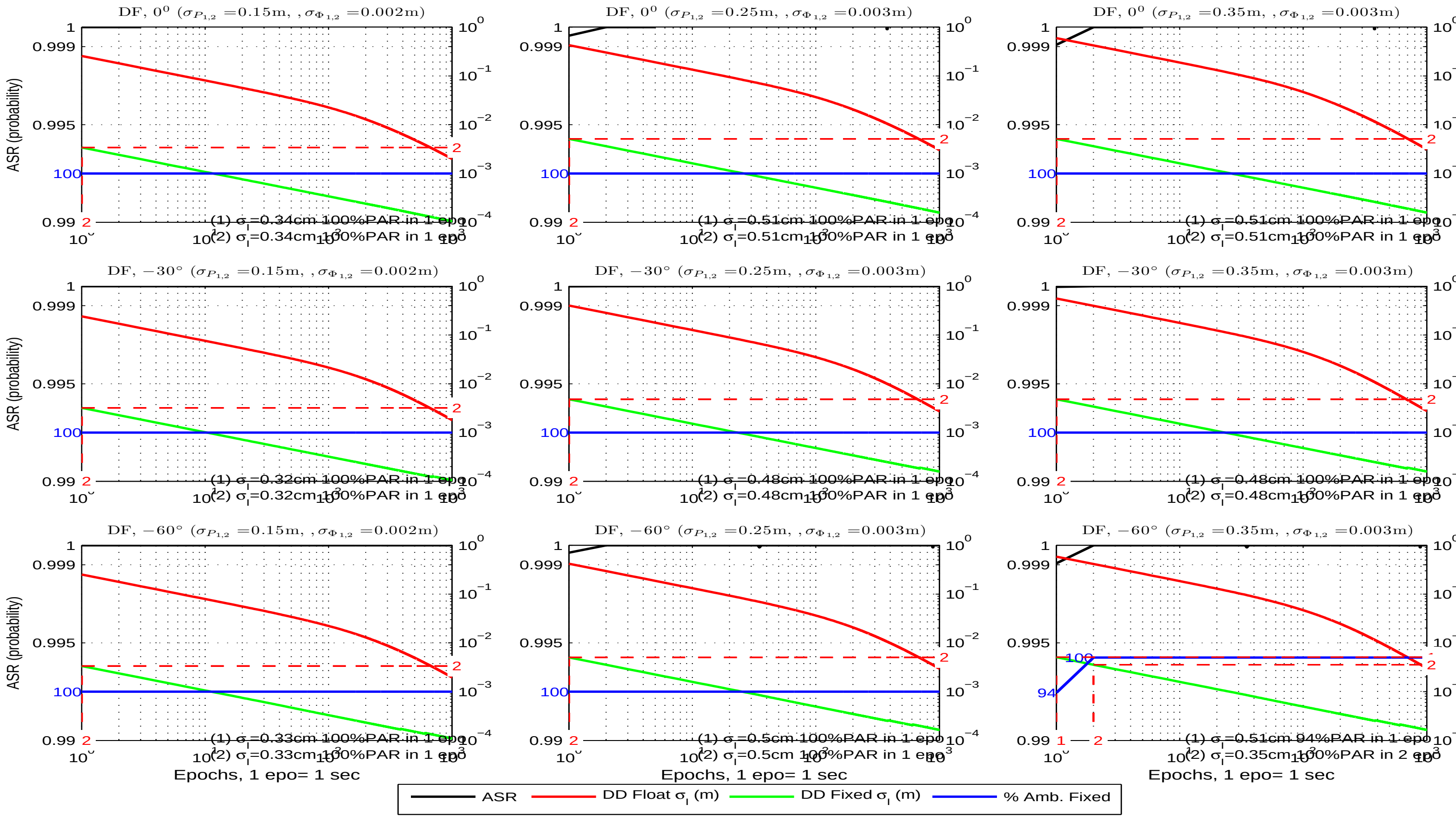

Figure G.8: **GPS only**, Dual frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

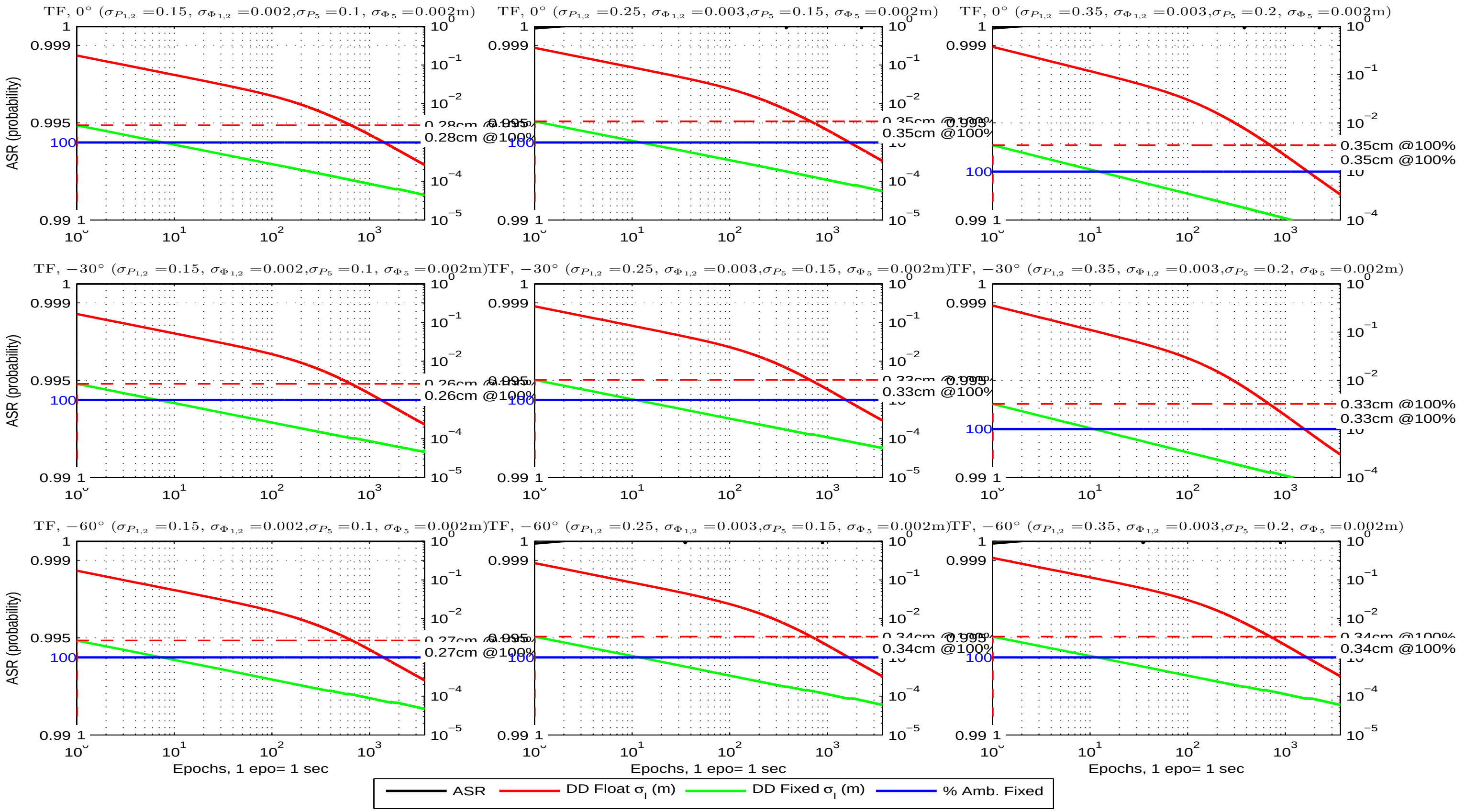

Figure G.9: **GPS only**, Triple frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°$, $-30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

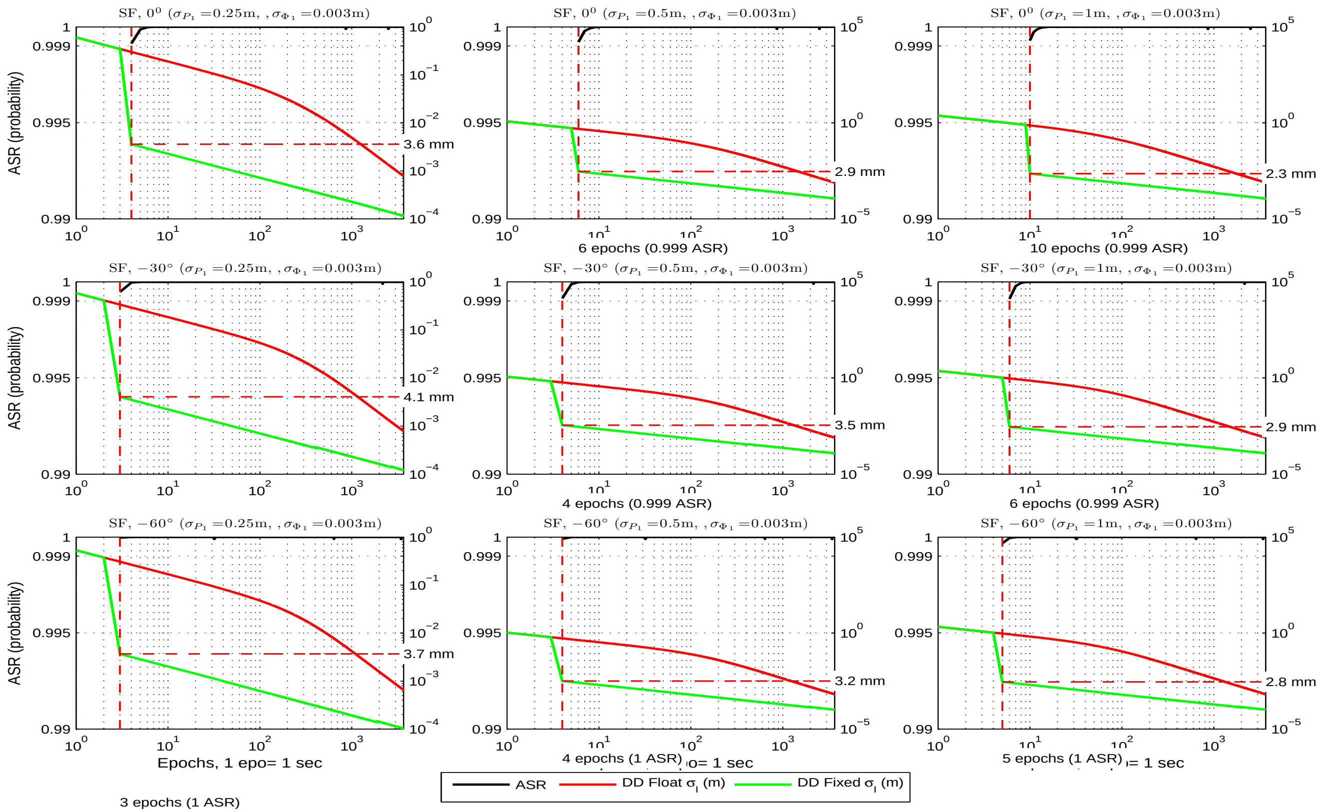

Figure G.10: **Galileo only**, Single frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

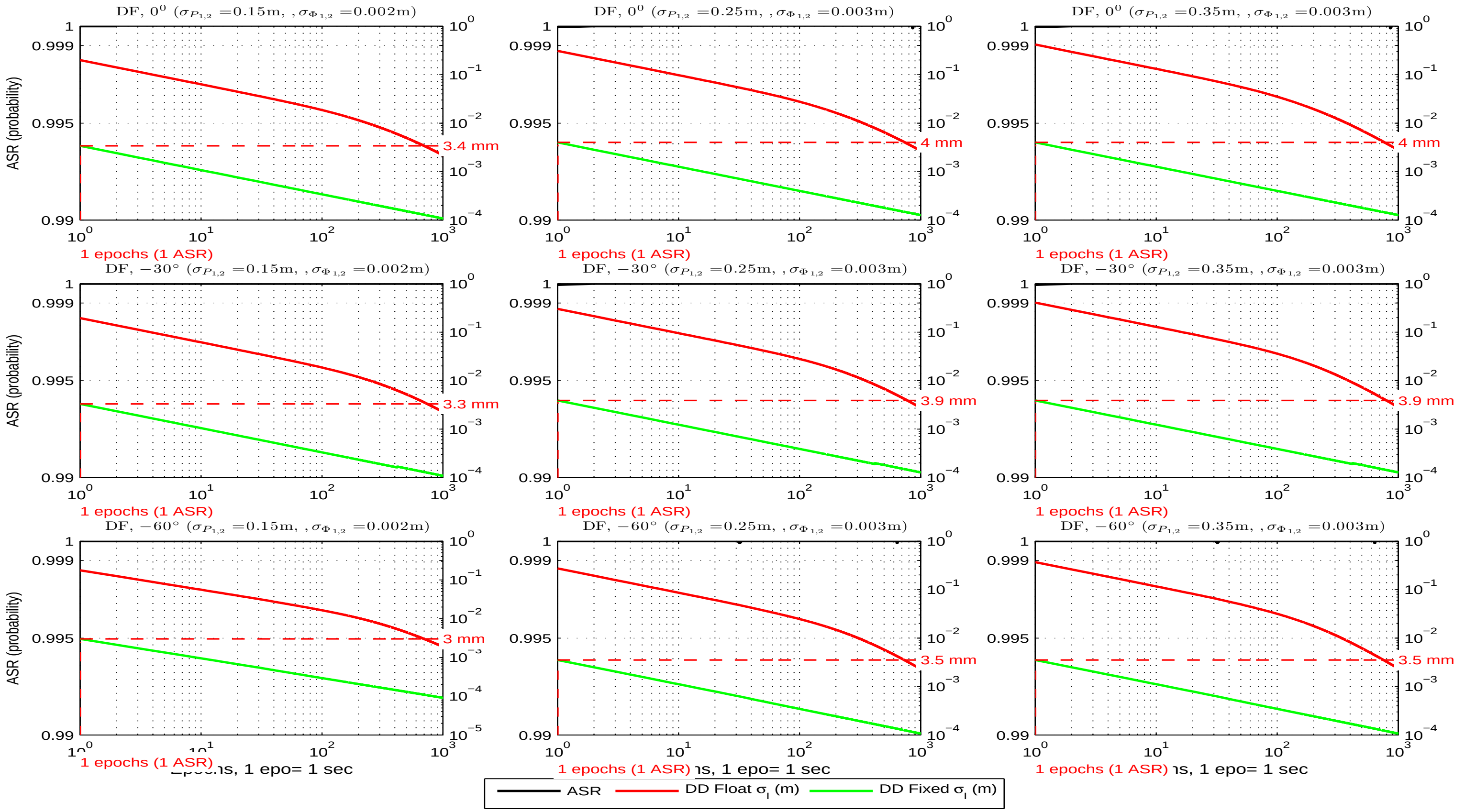

Figure G.11: **Galileo only**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

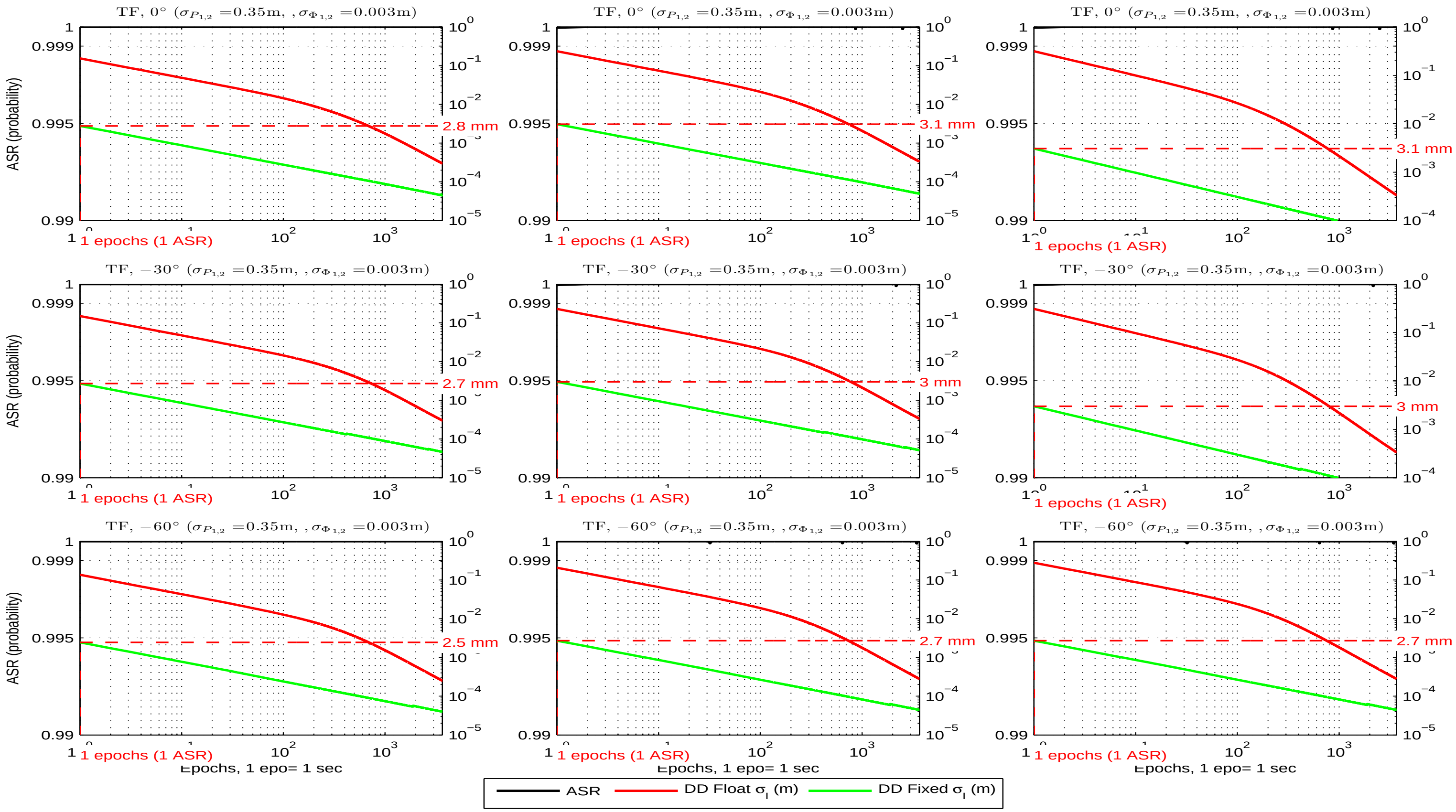

Figure G.12: **Galileo only**, Triple frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

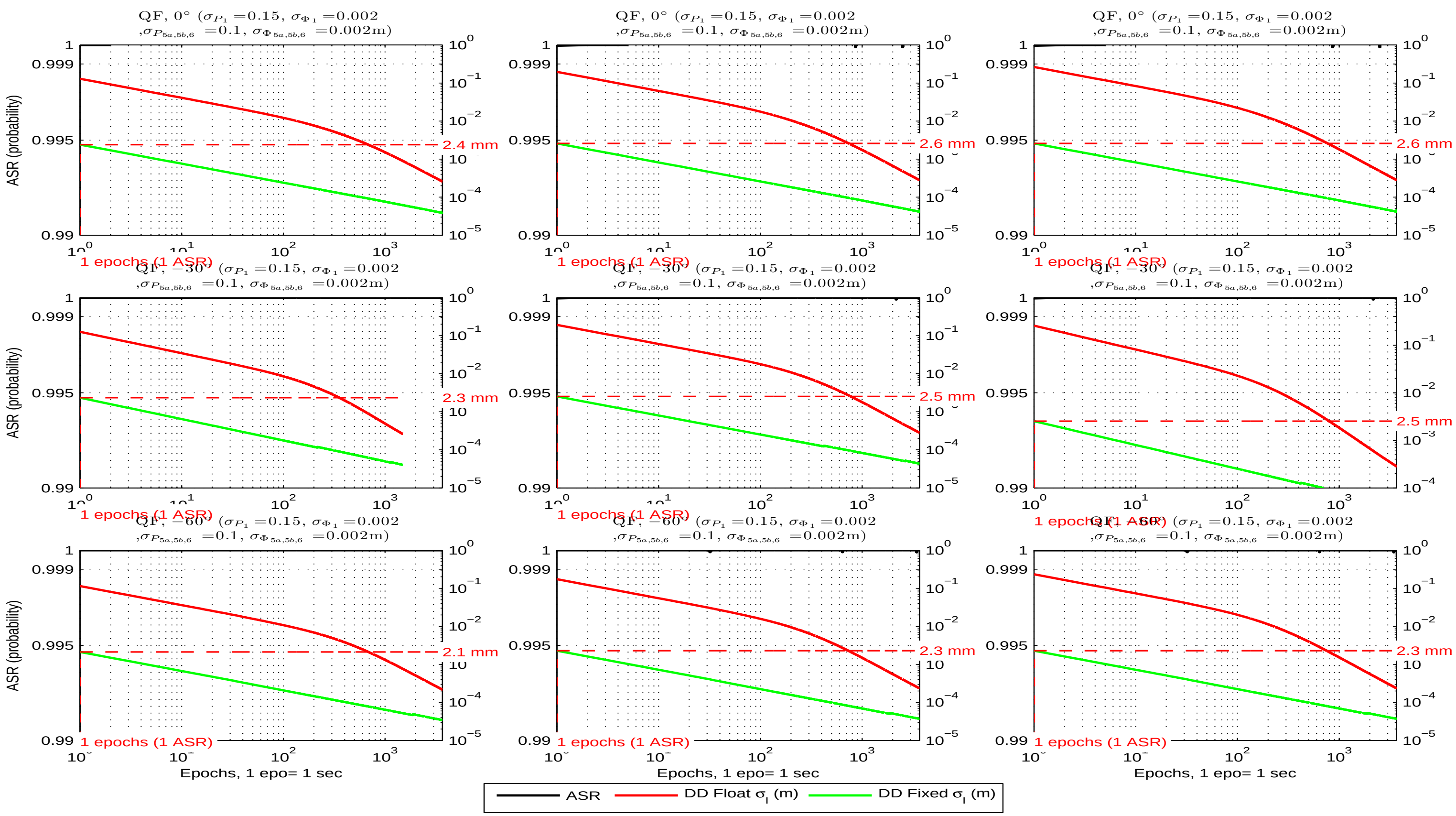

Figure G.13: **Galileo only**, Quadruple frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

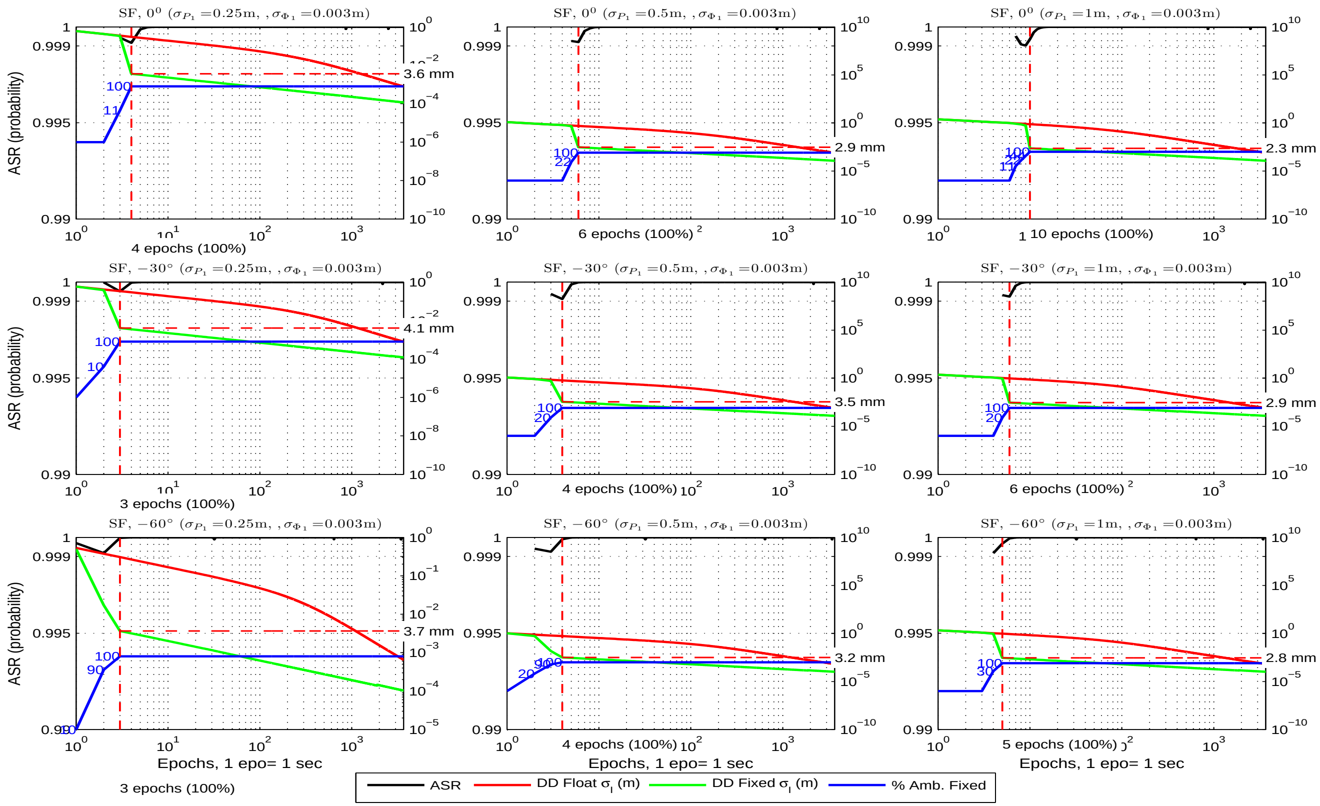

Figure G.14: **Galileo only**, Single frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

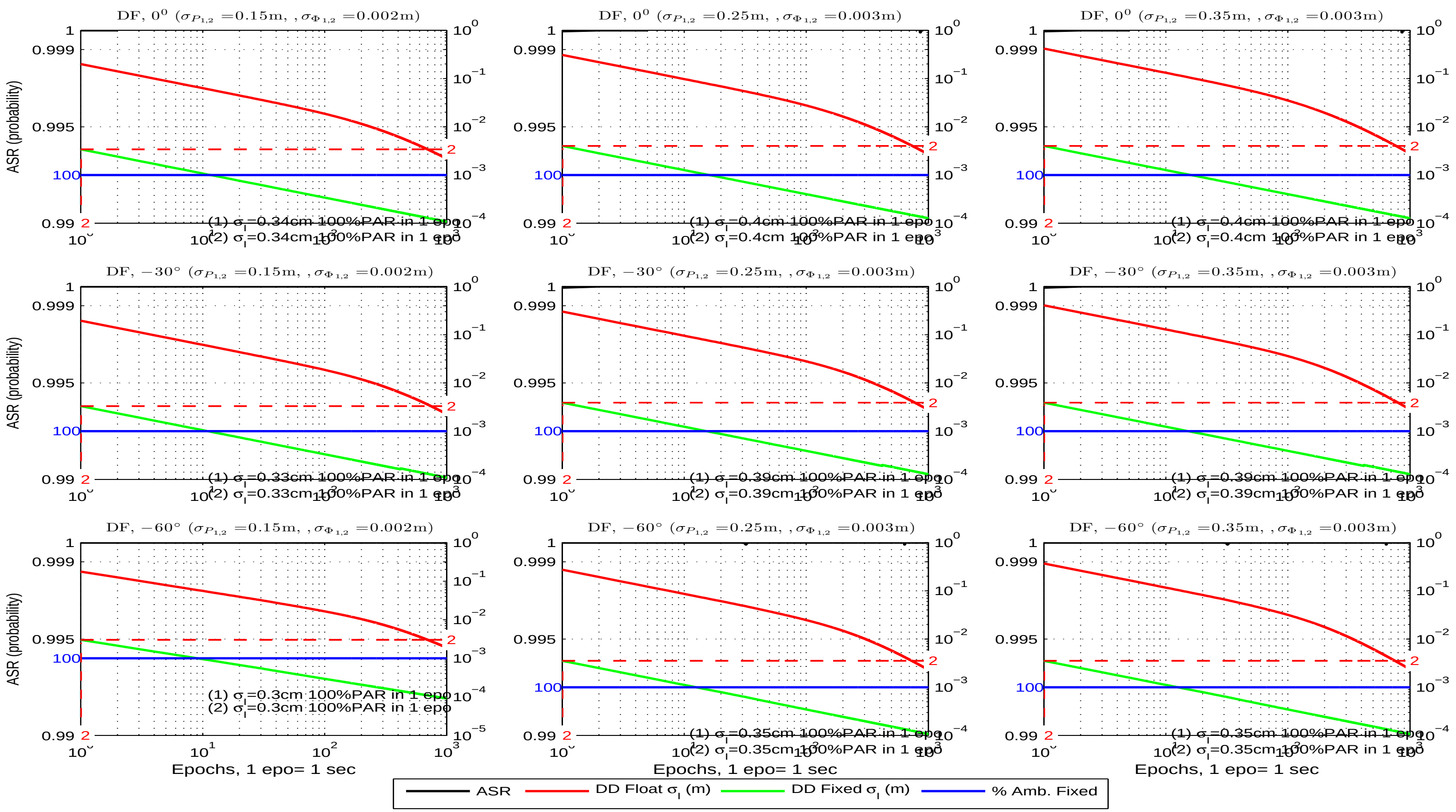

Figure G.15: **Galileo only**, Dual frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

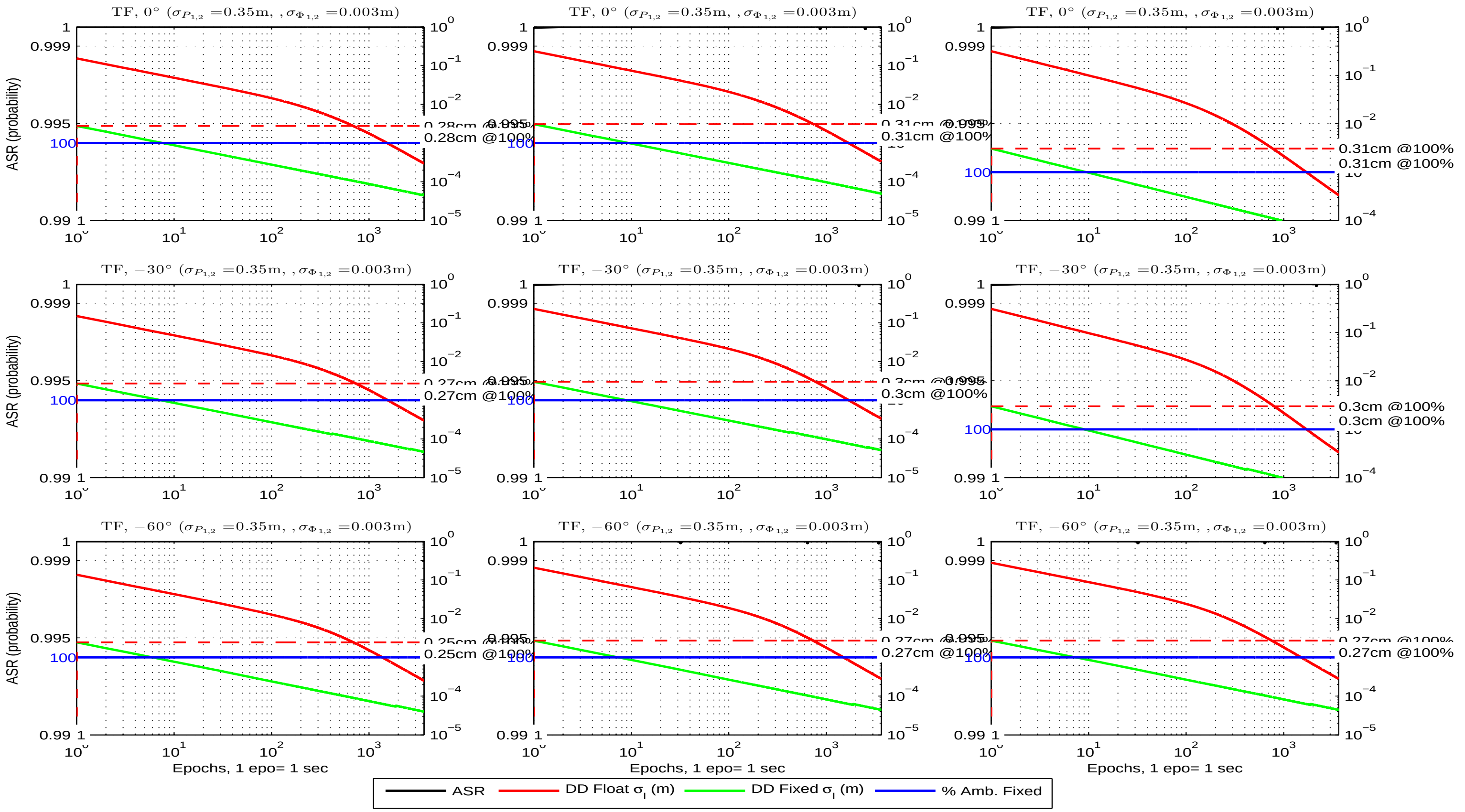

Figure G.16: **Galileo only**, Triple frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

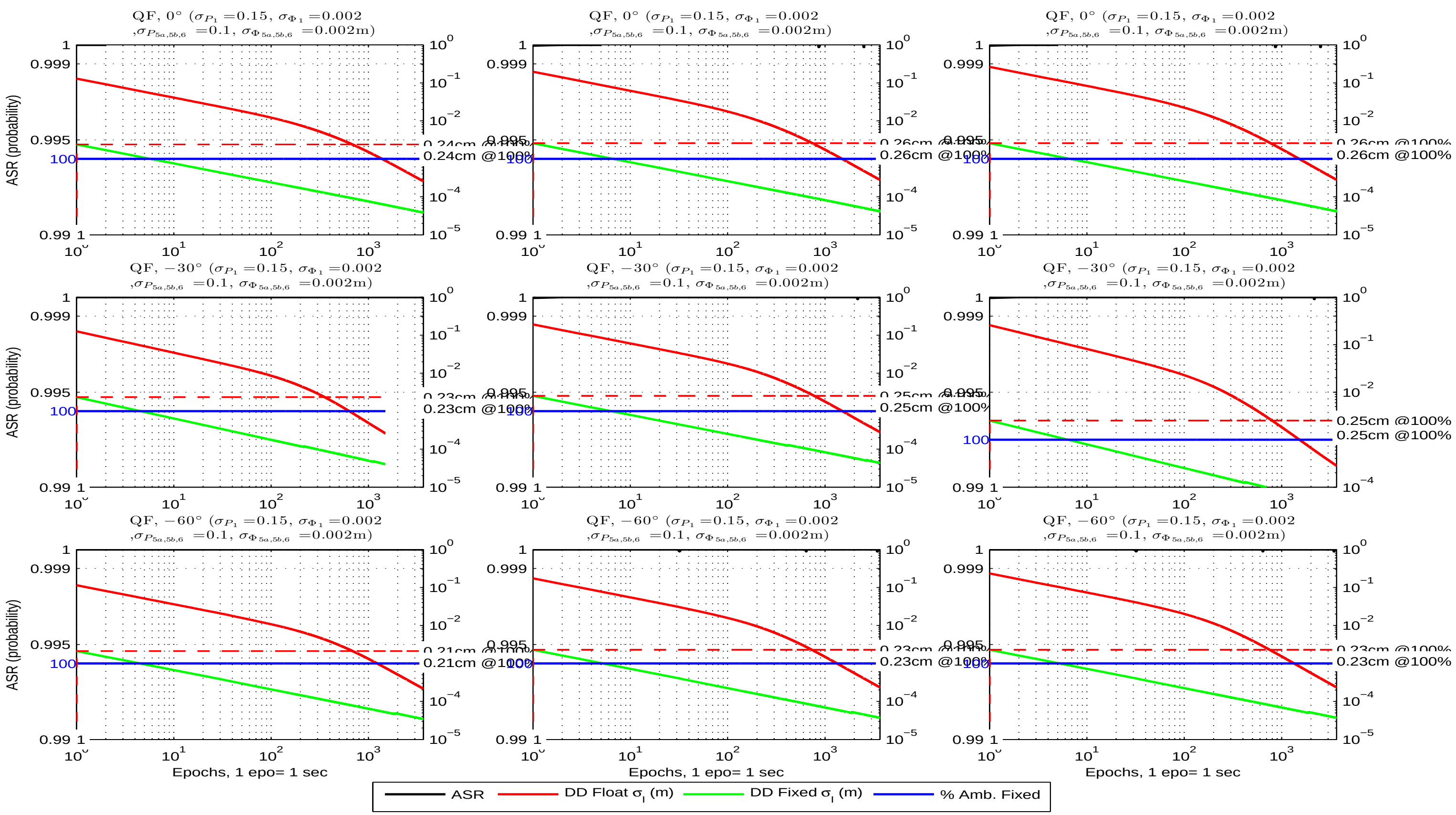

Figure G.17: **Galileo only**, Quadruple frequency, **PAR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

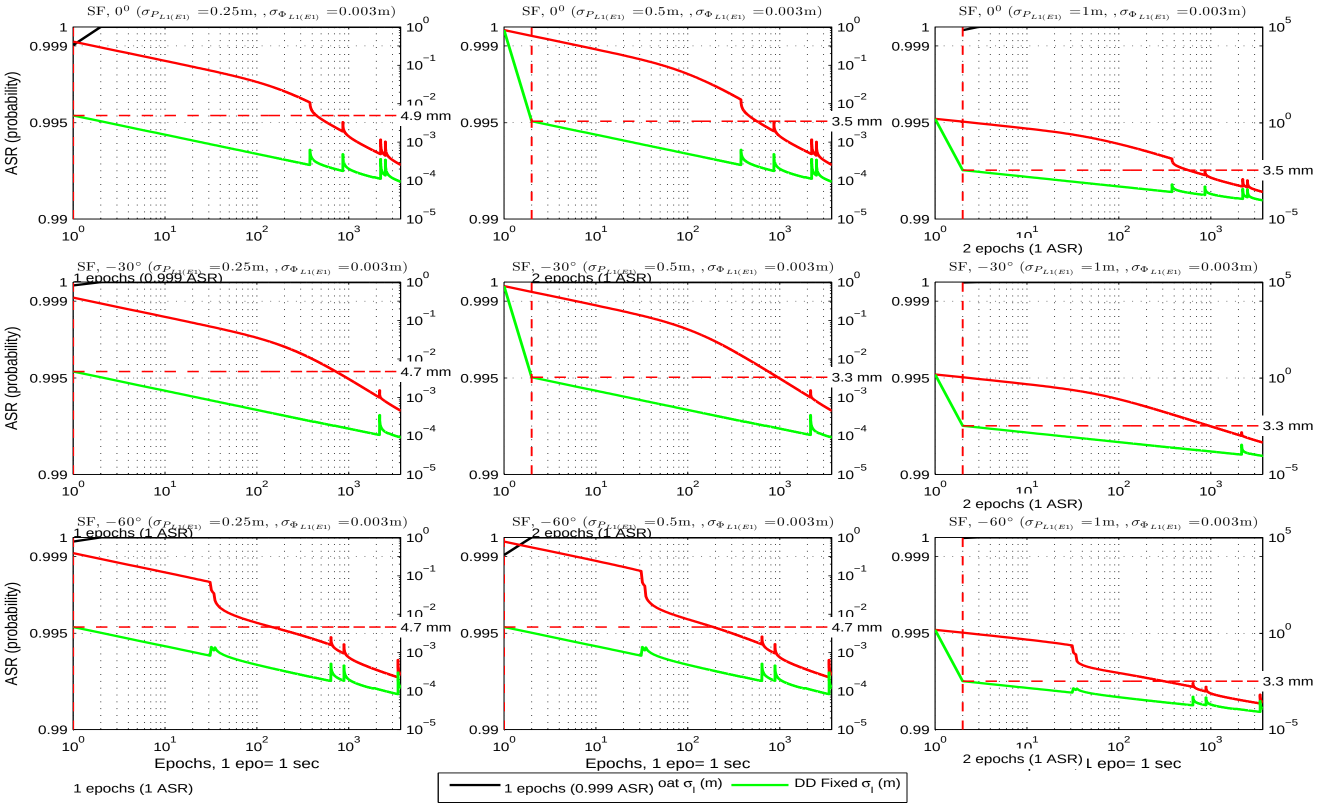

Figure G.18: **GPS + Galileo**, Single frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

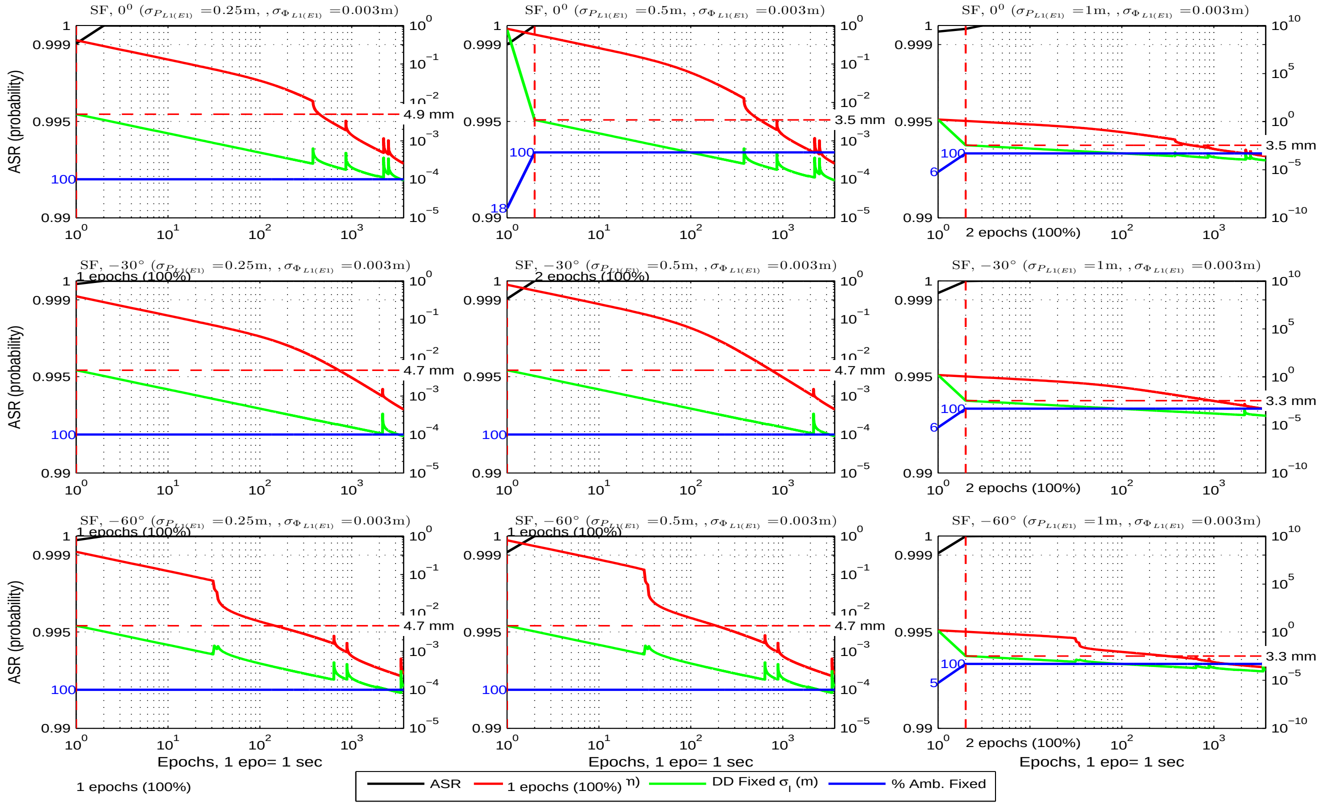

Figure G.19: **GPS + Galileo**, Single frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere known scenario, 1 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

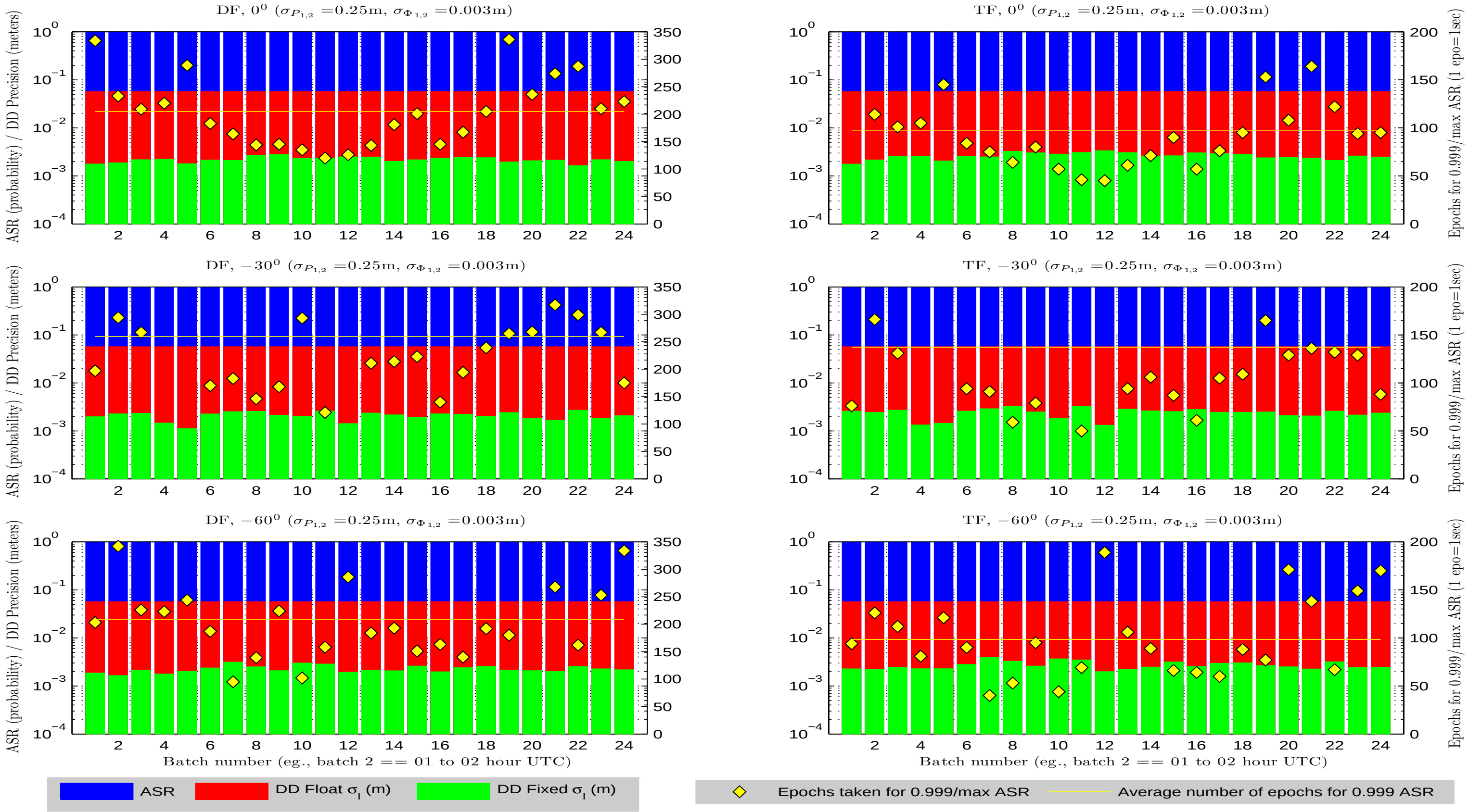

Figure G.20: **GPS only**, Atmosphere float, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual and triple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR and, float and fixed precision of ionosphere.

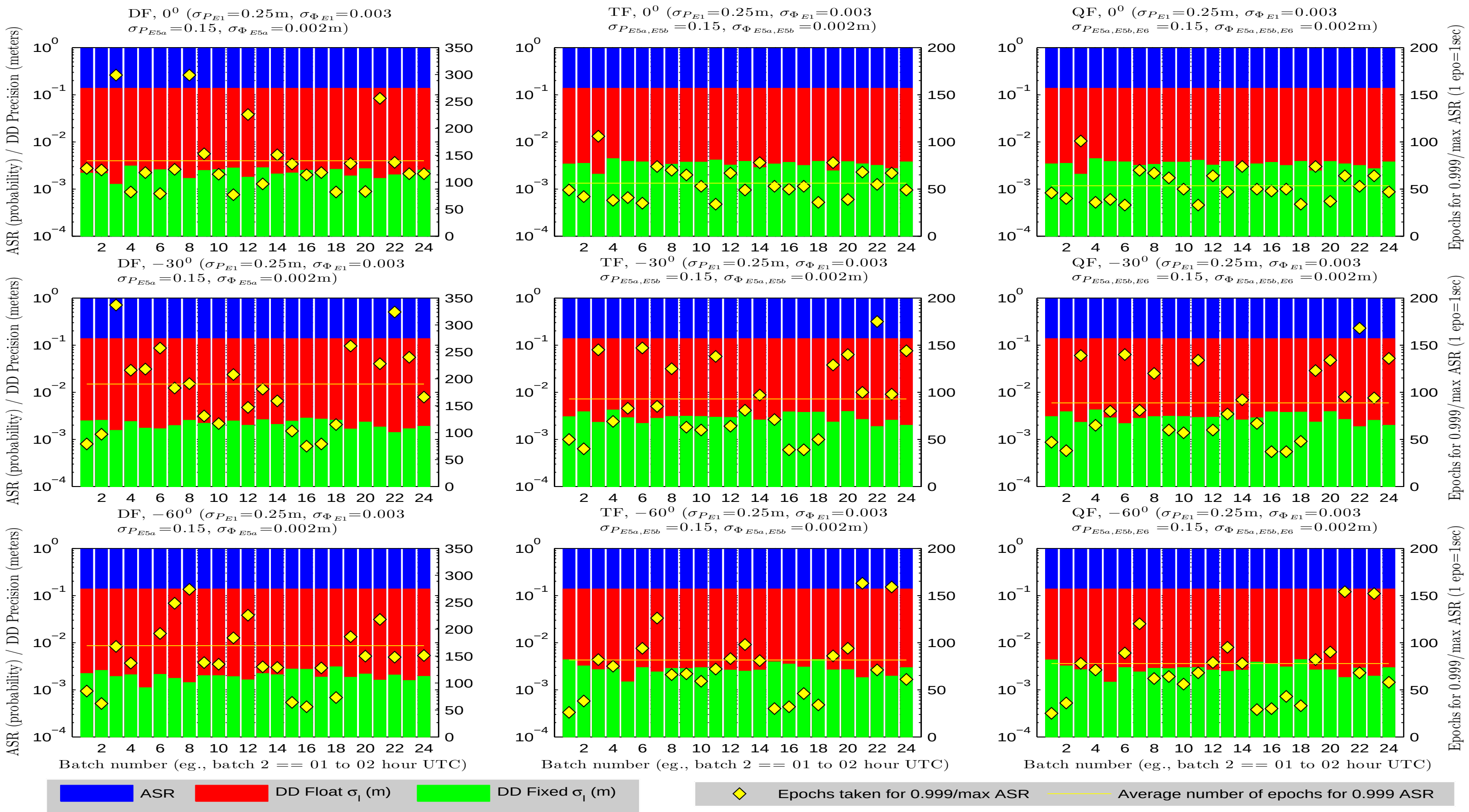

Figure G.21: **Galileo only**, Atmosphere float, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual, triple and quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR and, float and fixed precision of ionosphere.

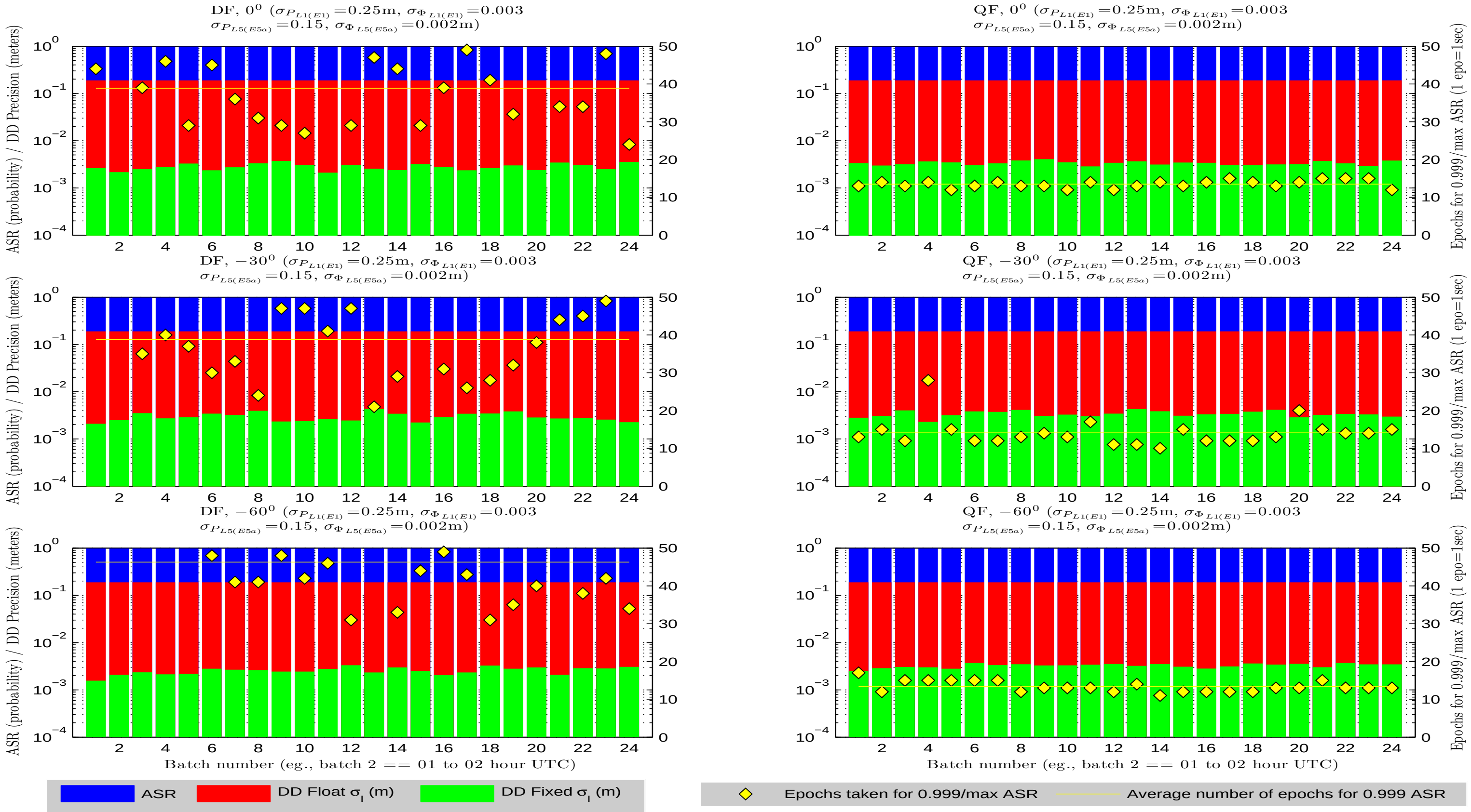

Figure G.22: **GPS + Galileo**, Atmosphere float, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual and quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR and, float and fixed precision of ionosphere.

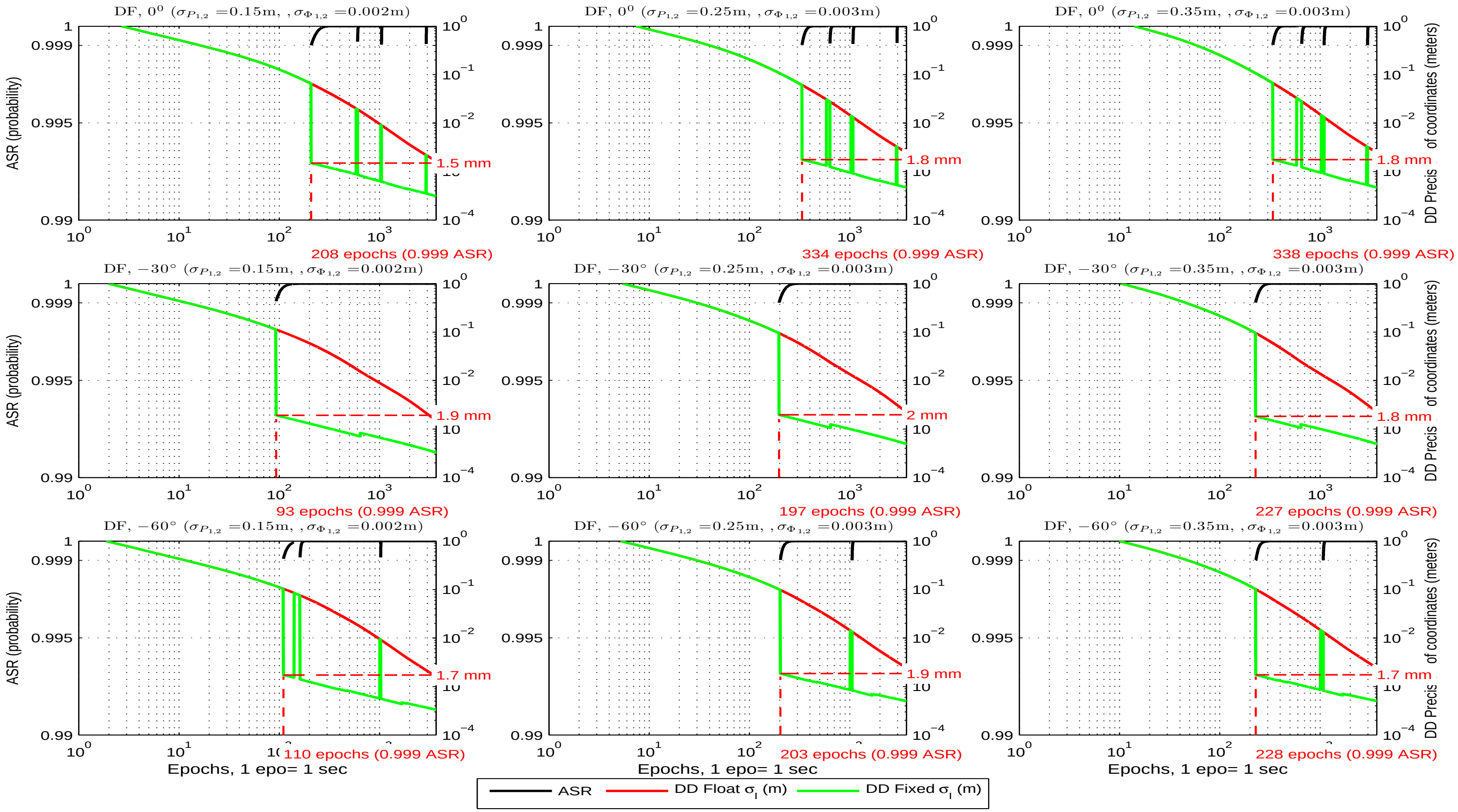

Figure G.23: **GPS only**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

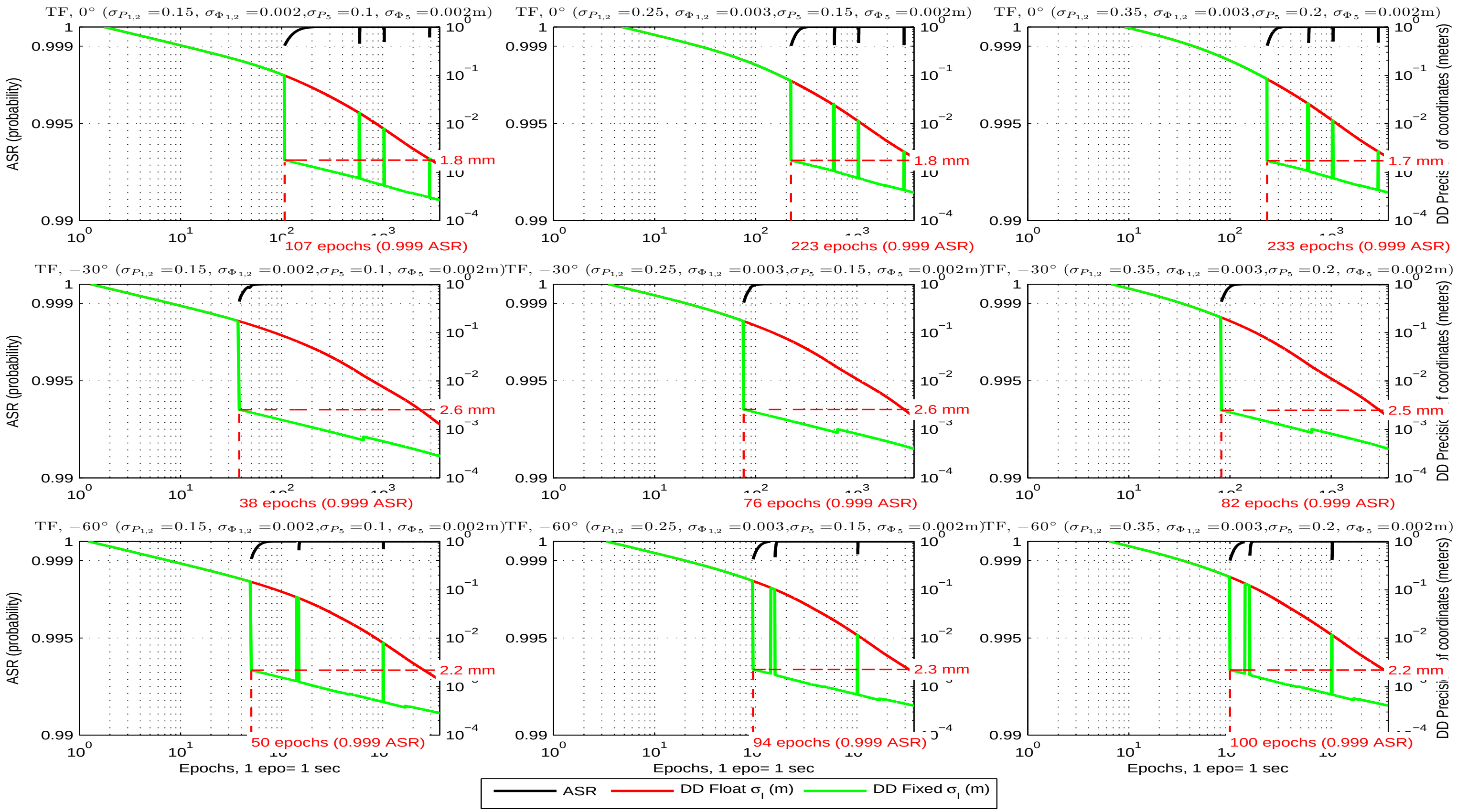

Figure G.24: **GPS only**, Triple frequency, **Full AR**, **Geometry Based** model, receiver coordinates unknown, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

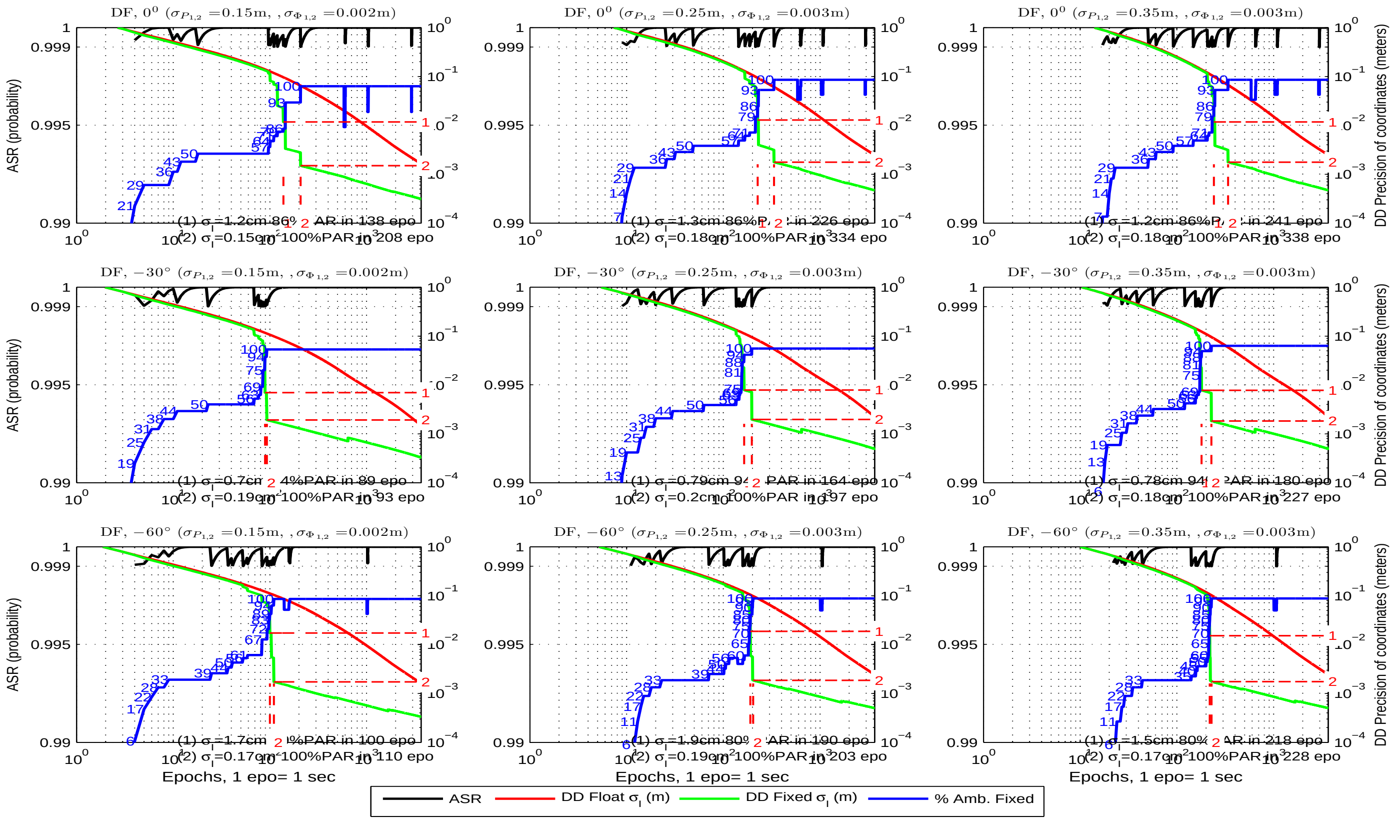

Figure G.25: **GPS only**, Dual frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

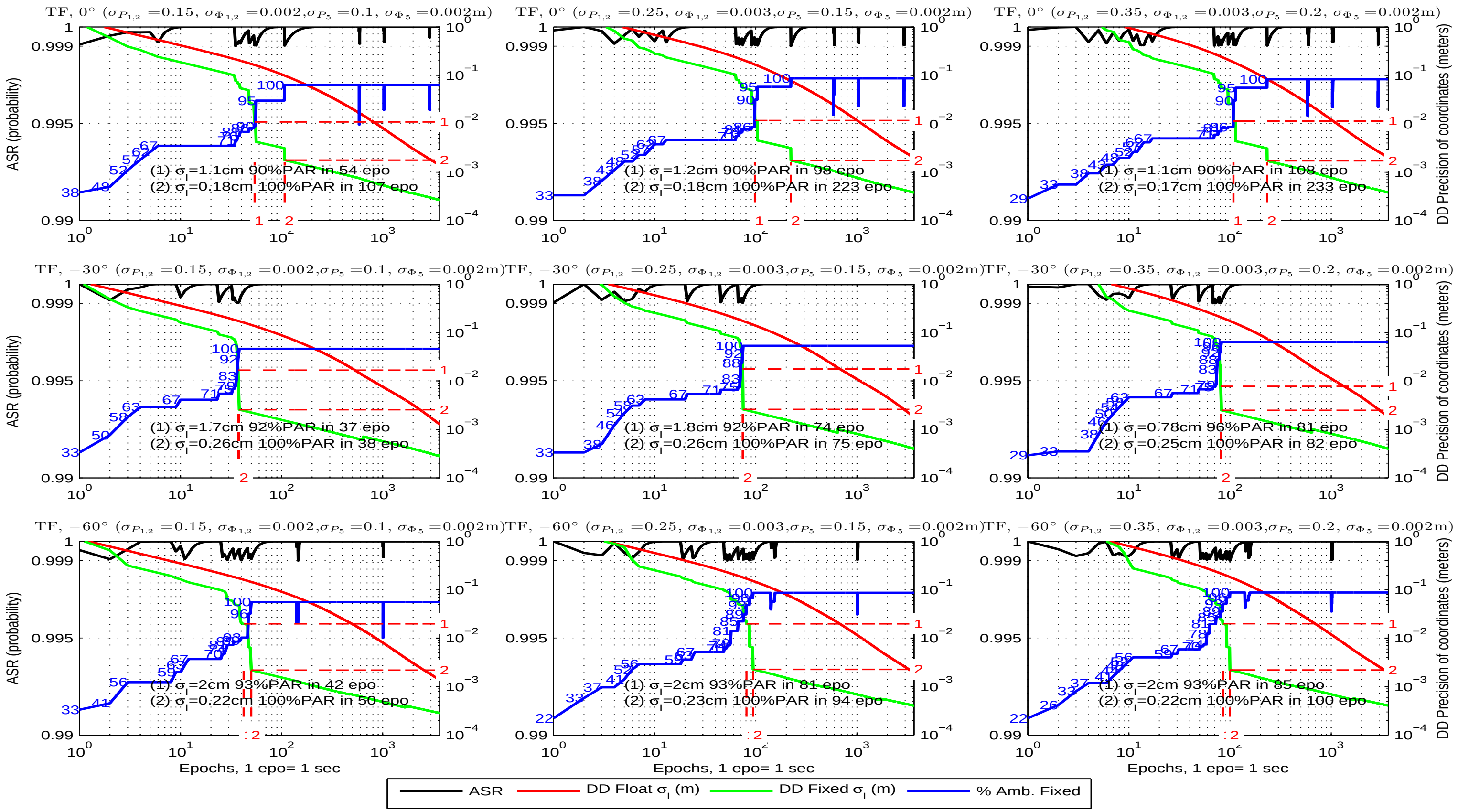

Figure G.26: **GPS only**, Triple frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°$, $-30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

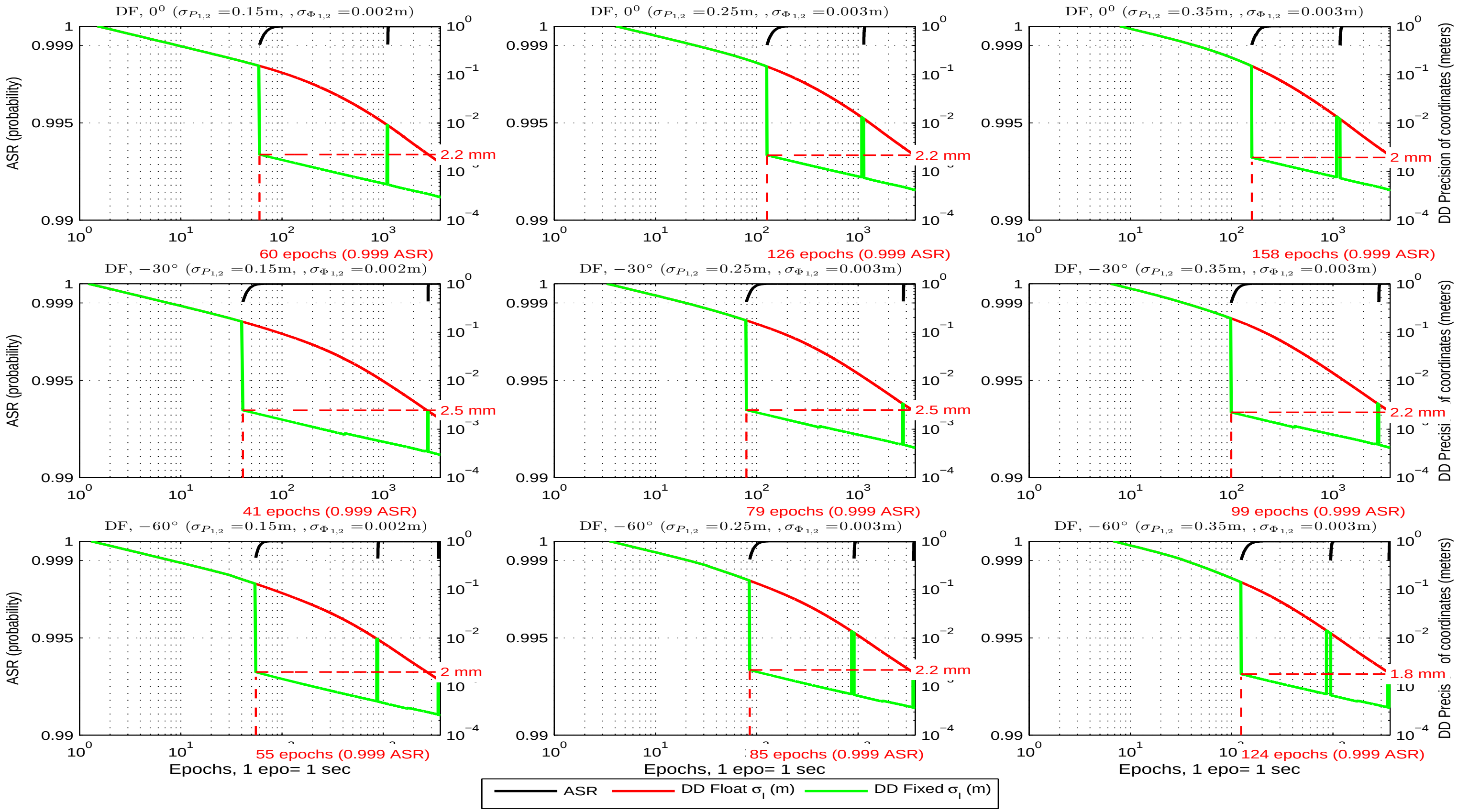

Figure G.27: **Galileo only**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

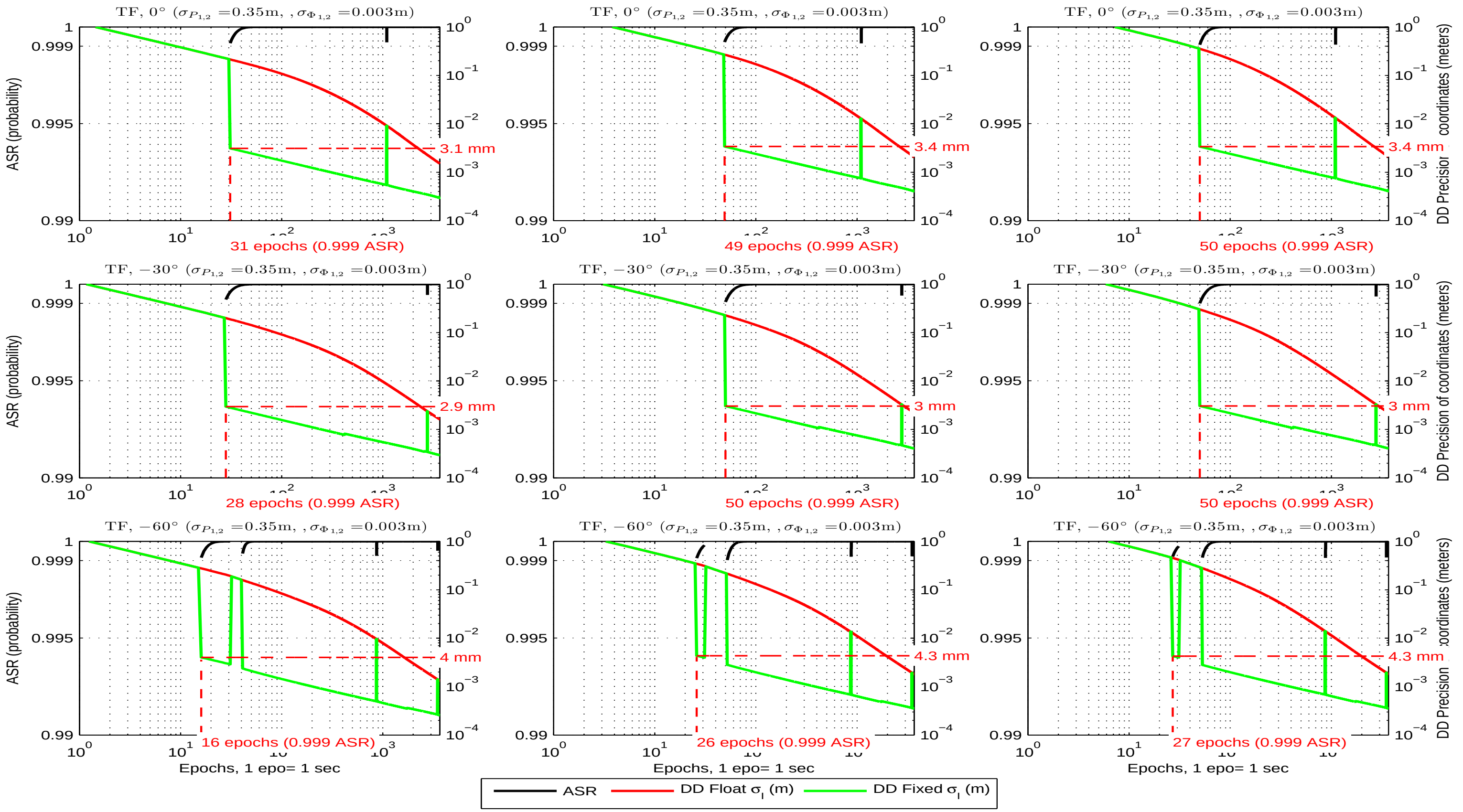

Figure G.28: **Galileo only**, Triple frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

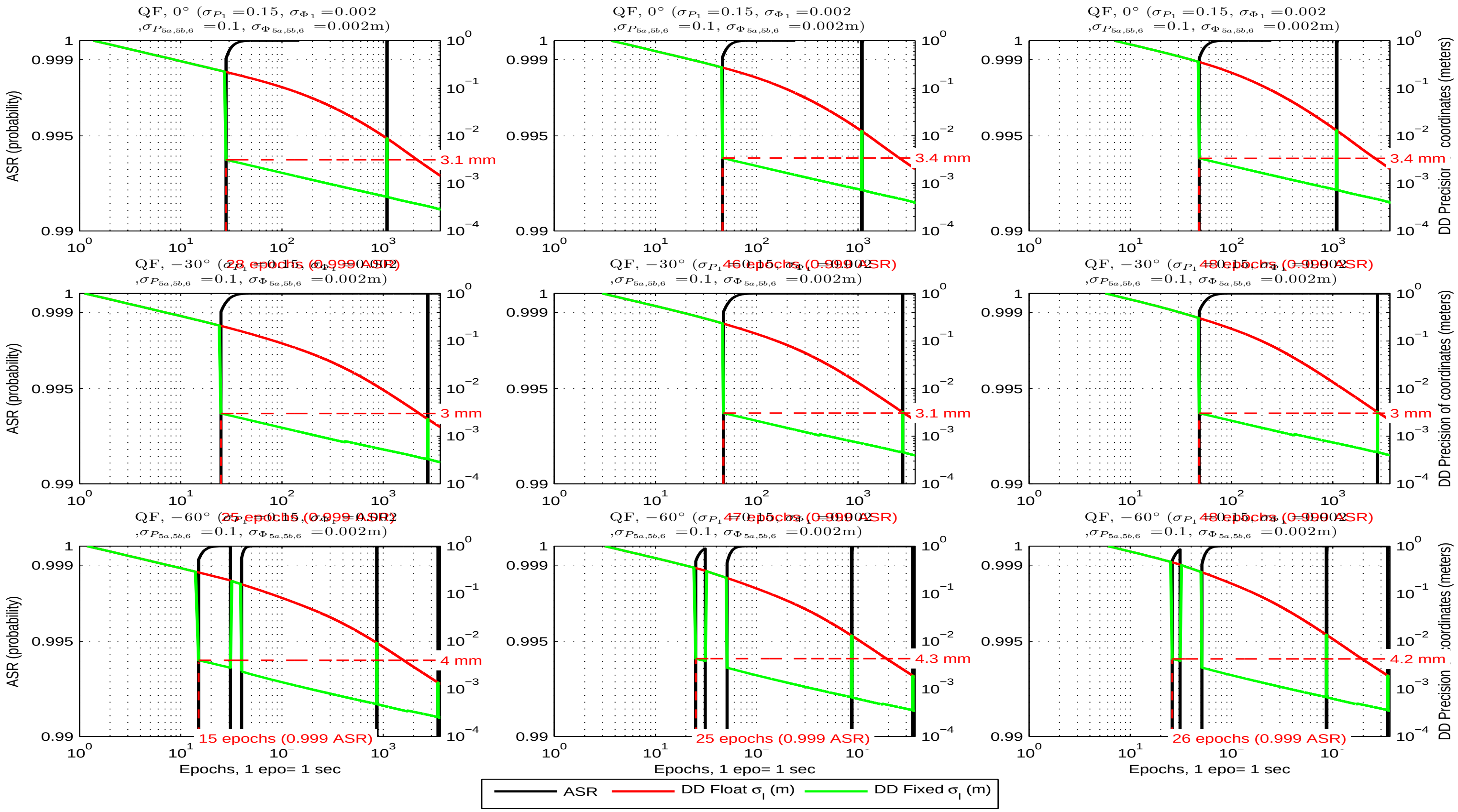

Figure G.29: **Galileo only**, Quadruple frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

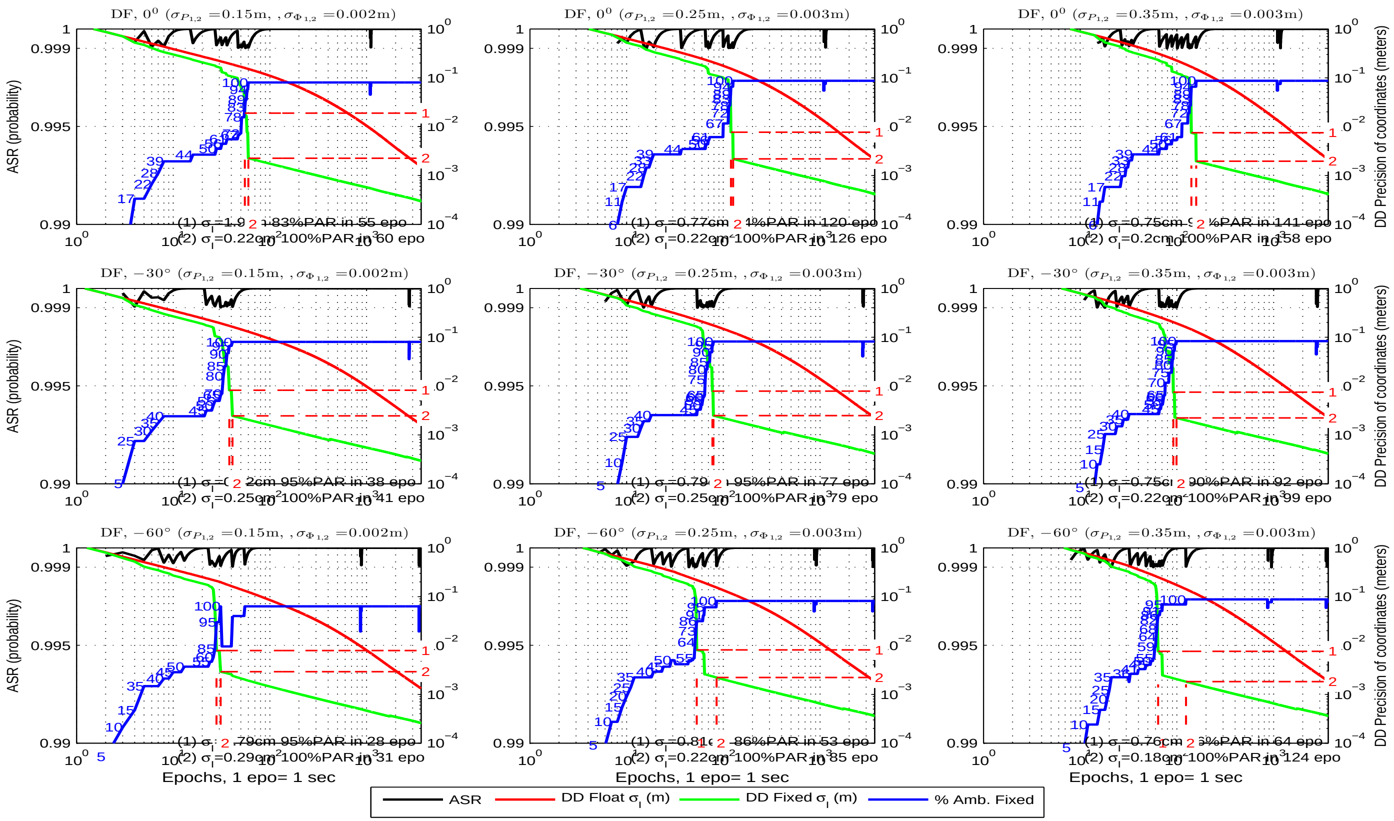

Figure G.30: **Galileo only**, Dual frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

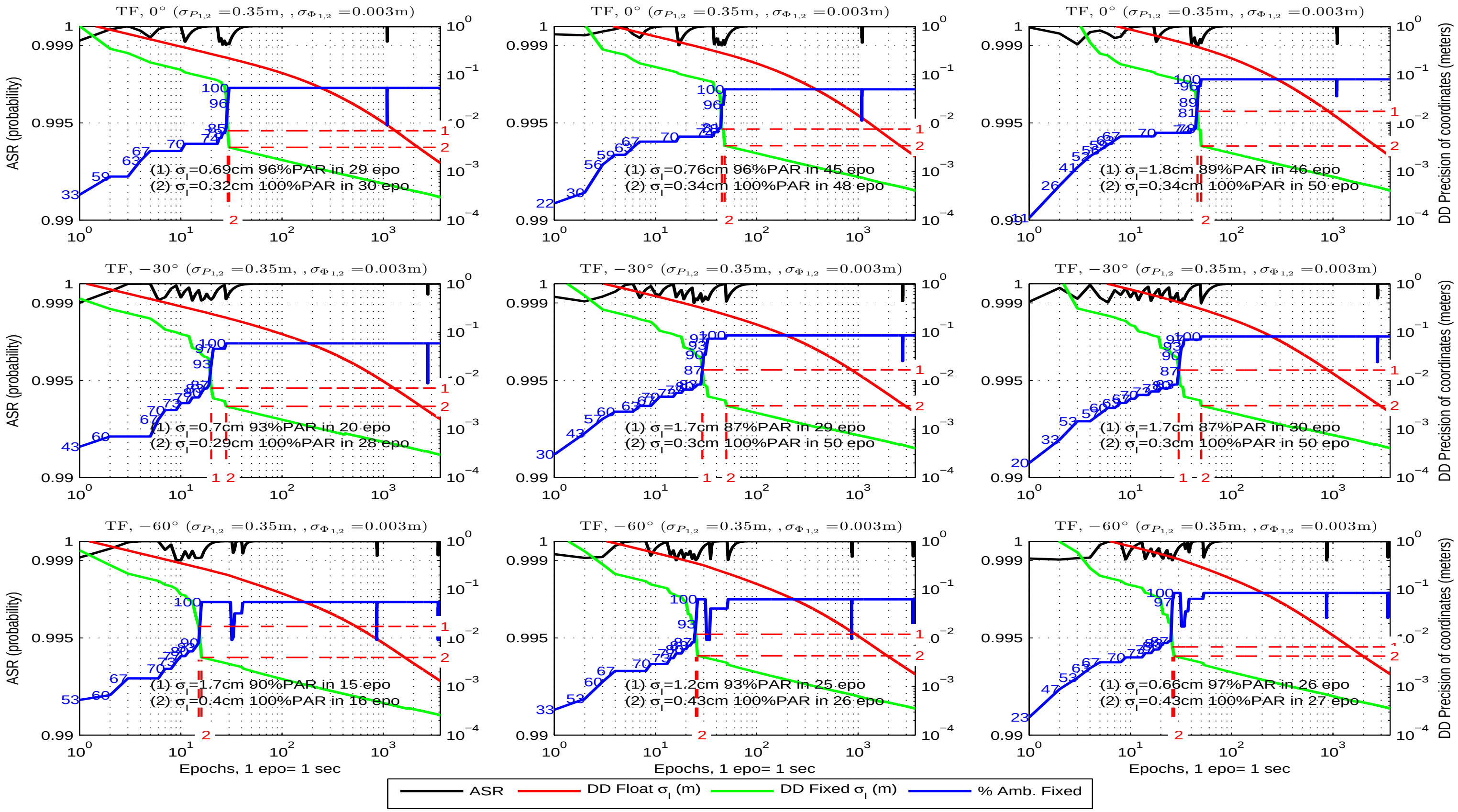

Figure G.31: **Galileo only**, Triple frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

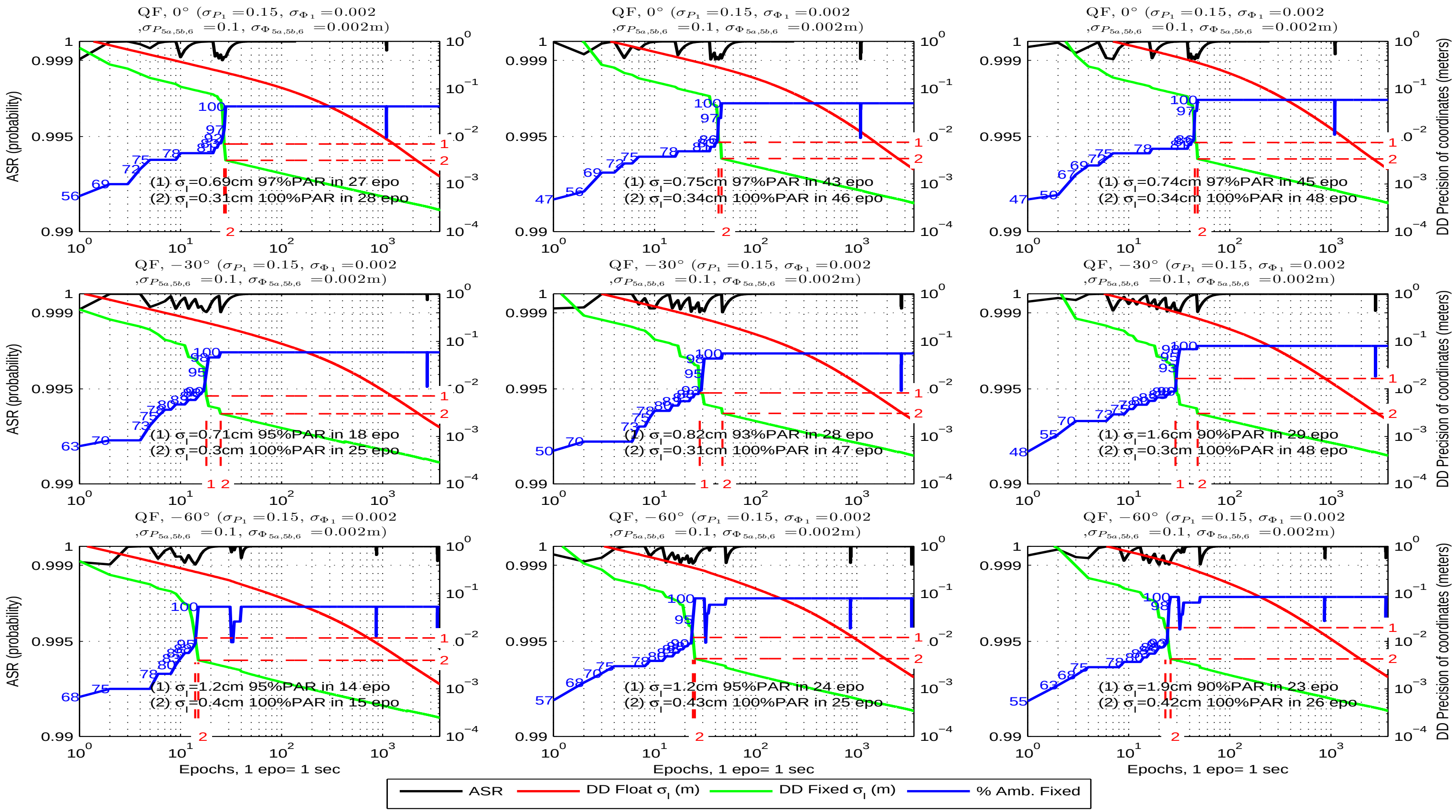

Figure G.32: **Galileo only**, Quadruple frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

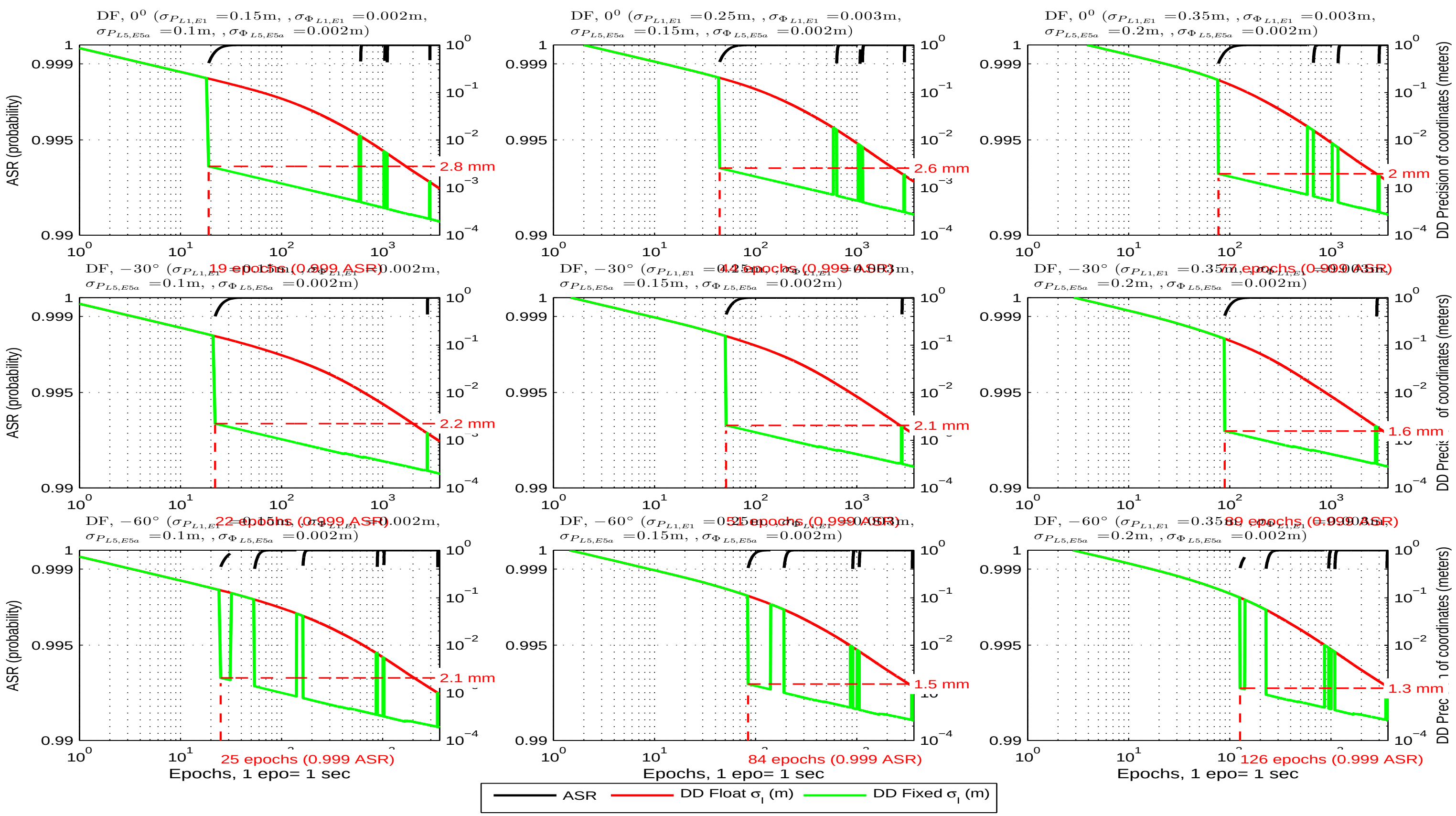

Figure G.33: **GPS + Galileo**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

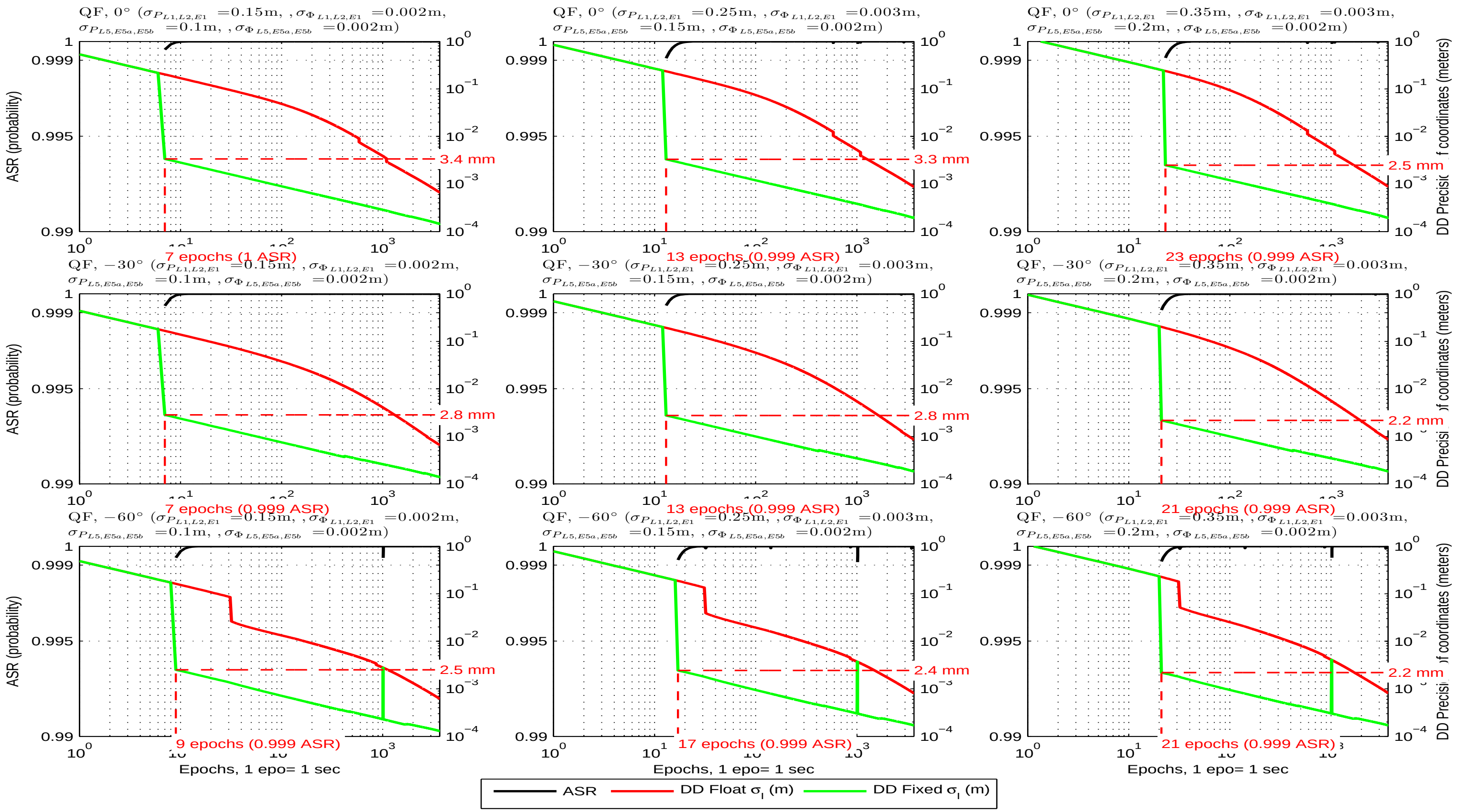

Figure G.34: **GPS + Galileo**, Quadruple frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

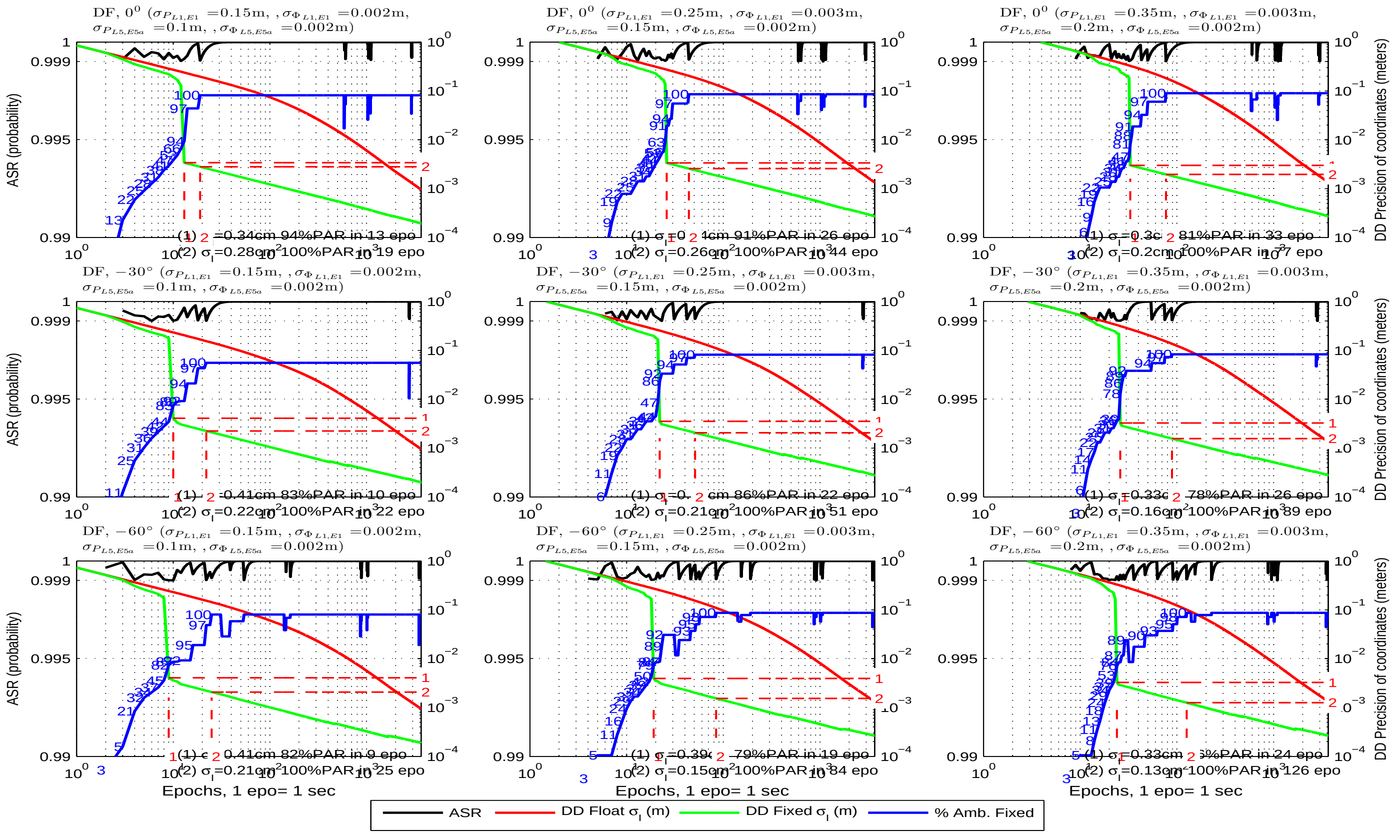

Figure G.35: **GPS + Galileo**, Dual frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red color cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

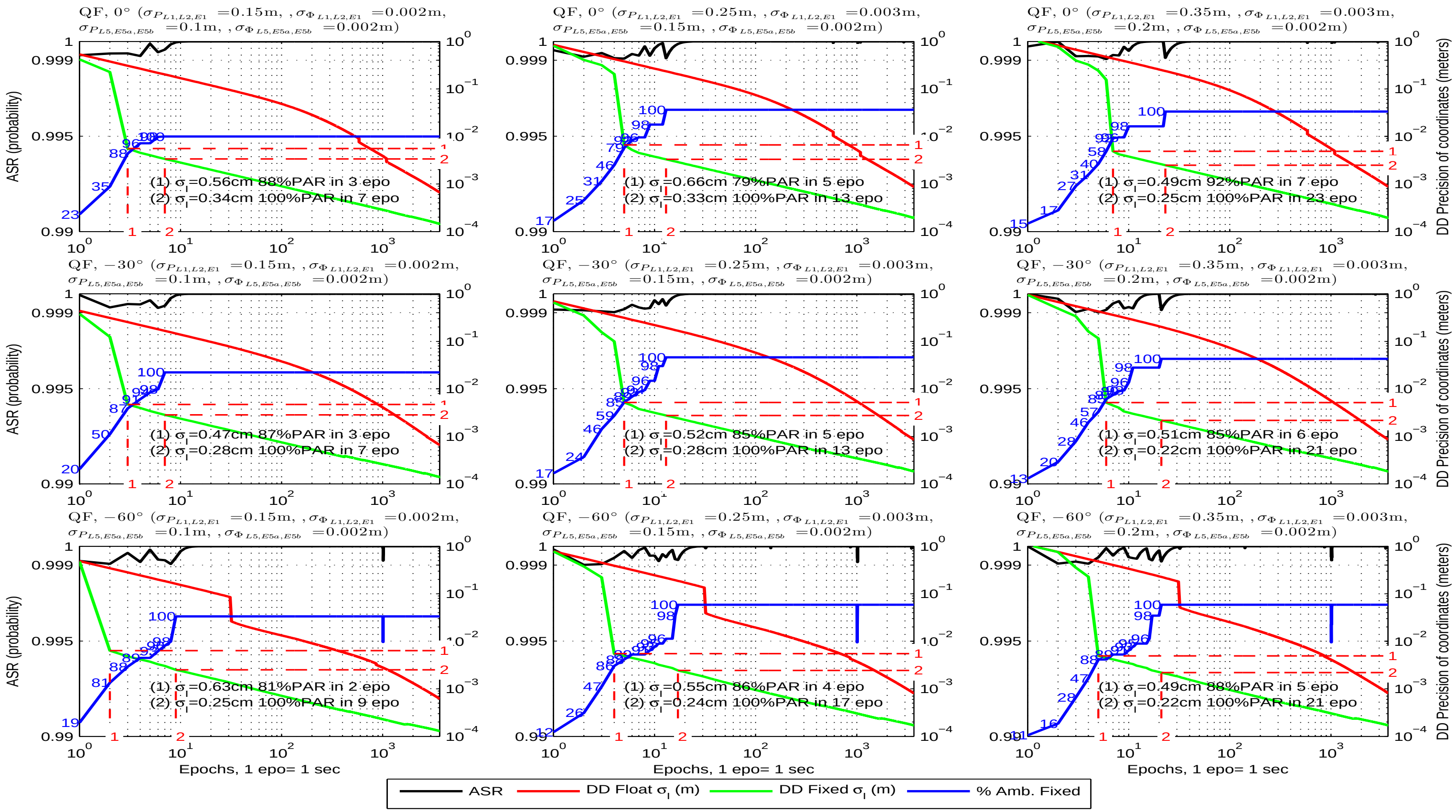

Figure G.36: **GPS + Galileo**, Quadruple frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, atmosphere unknown scenario, 250 Km East-West baseline, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

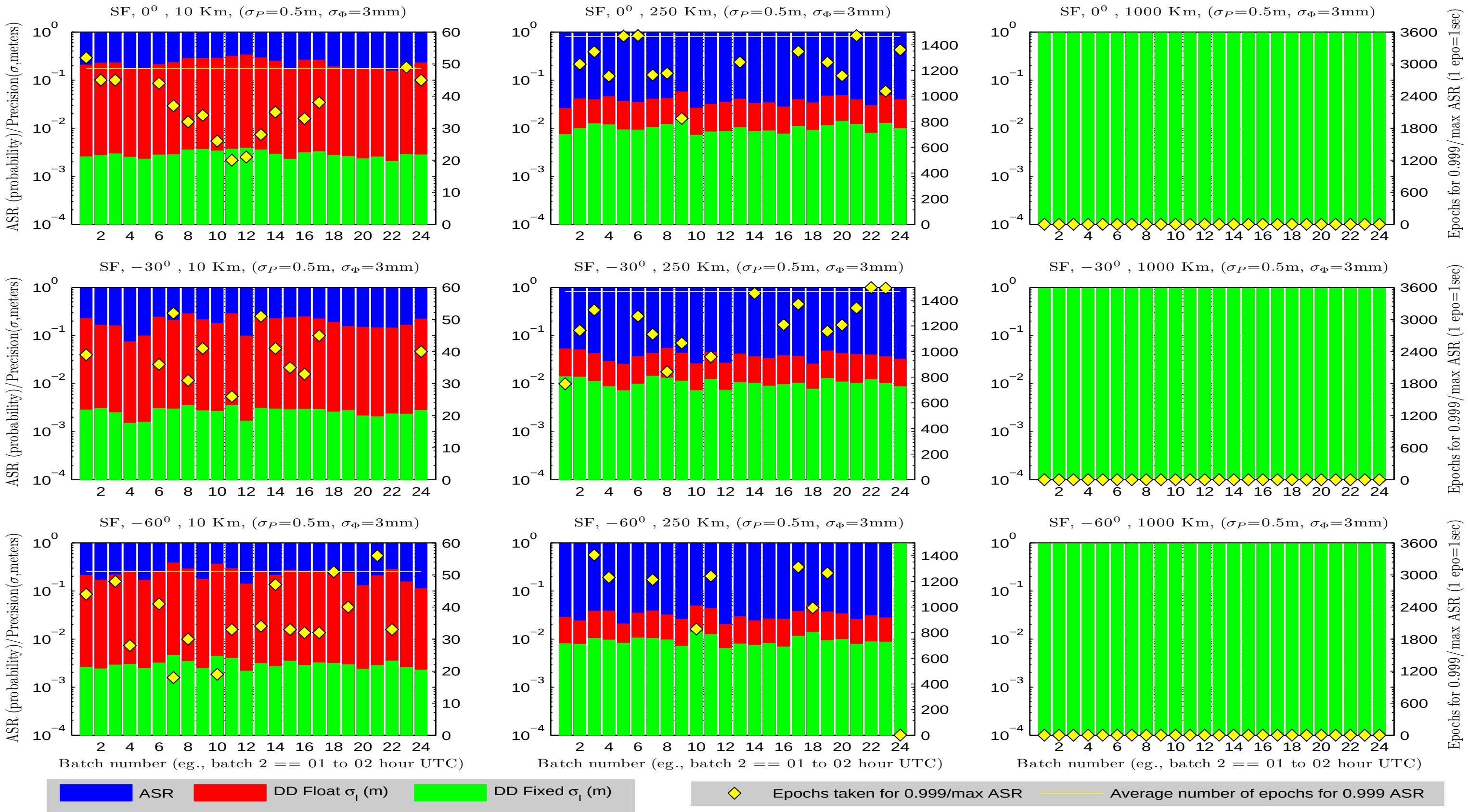

Figure G.37: **GPS only**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **single frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

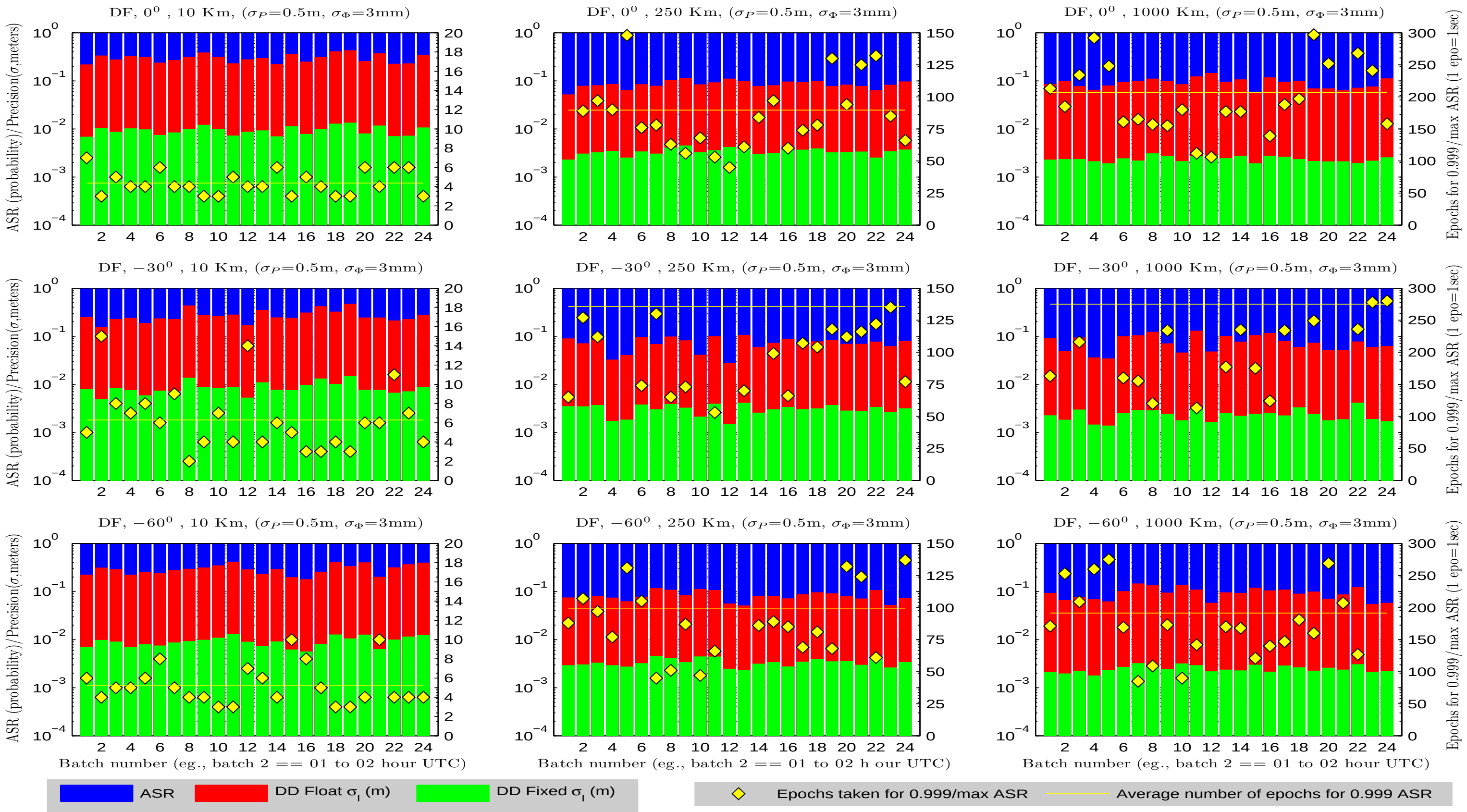

Figure G.38: **GPS only**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

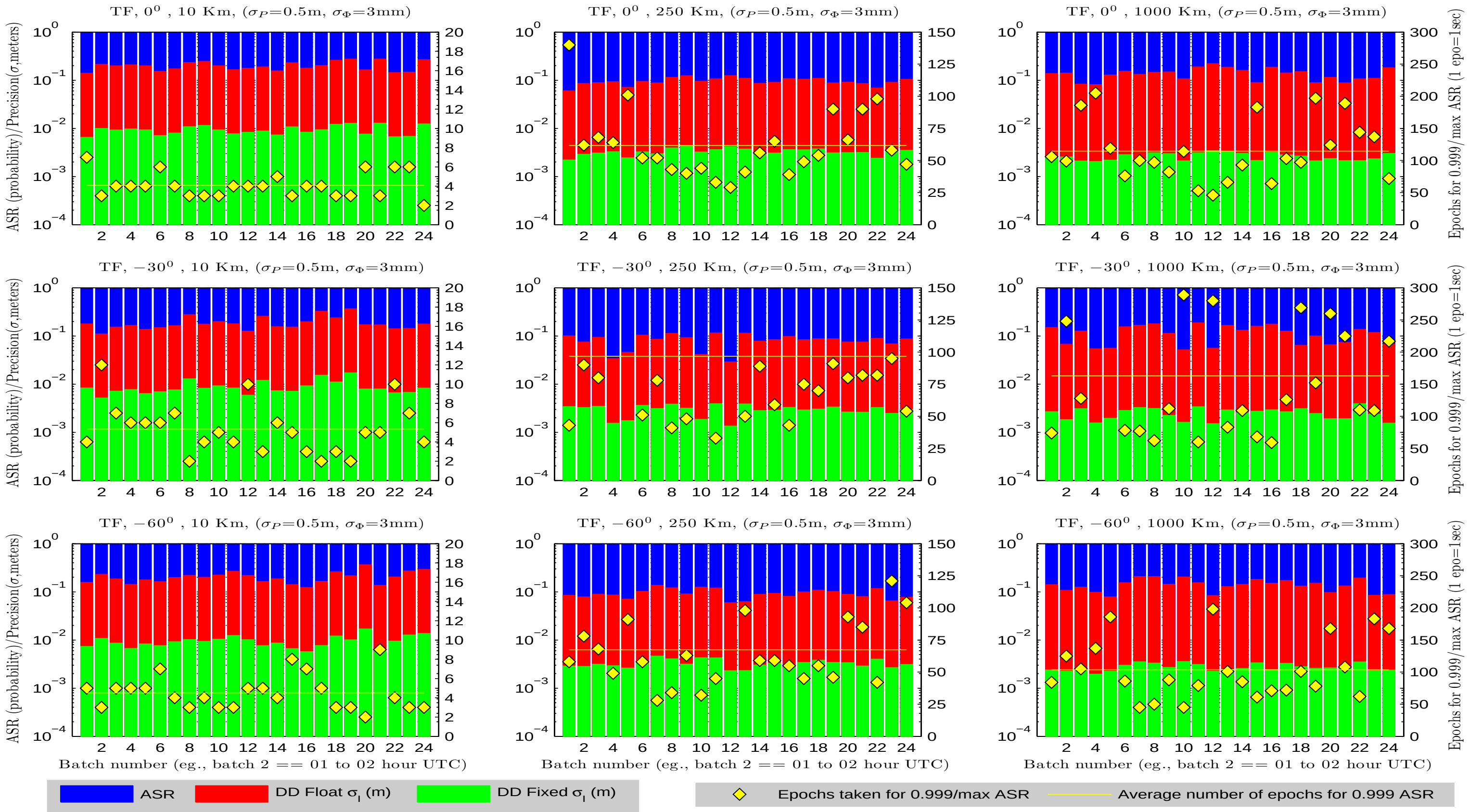

Figure G.39: **GPS only**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **triple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

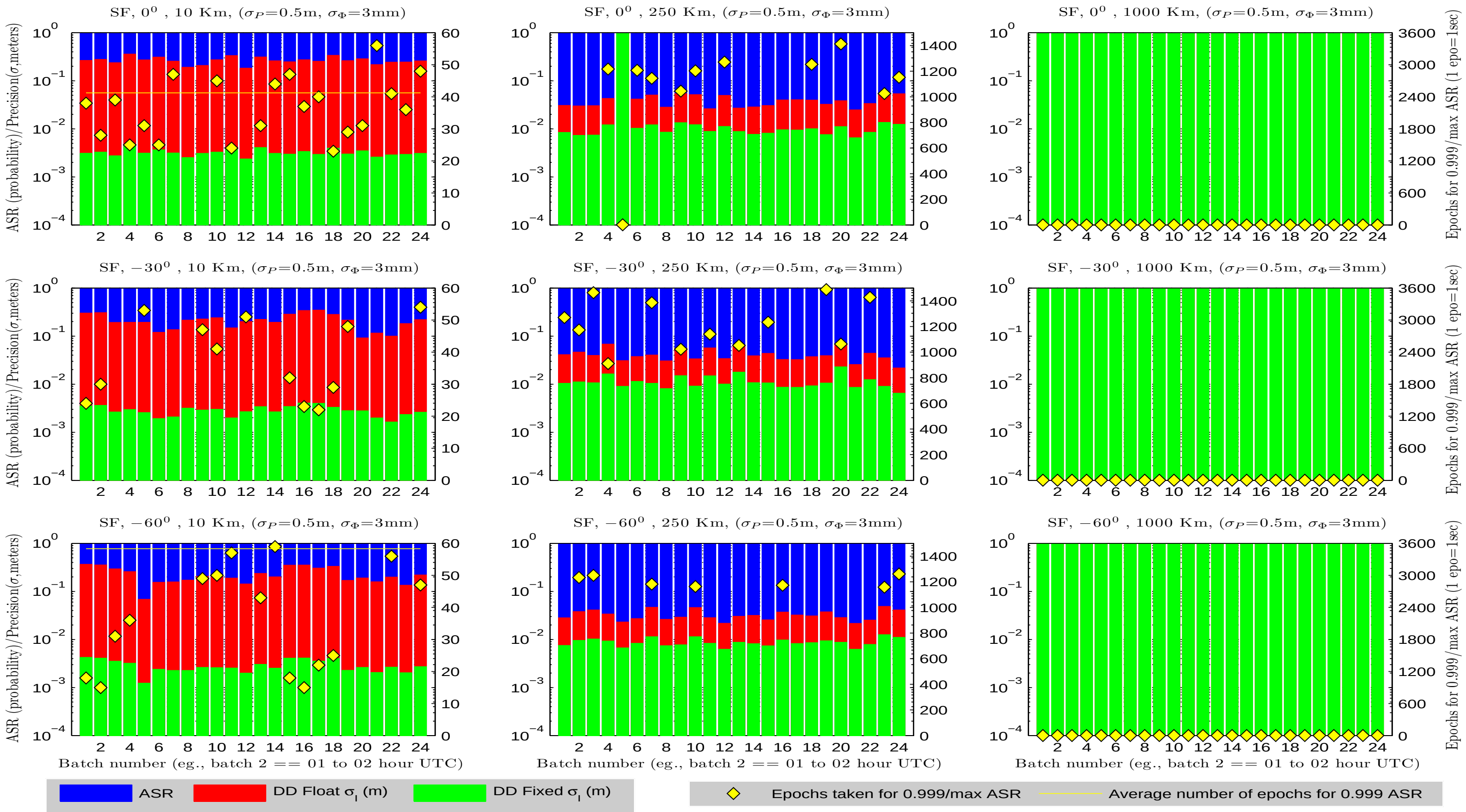

Figure G.40: **Galileo only**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **single frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

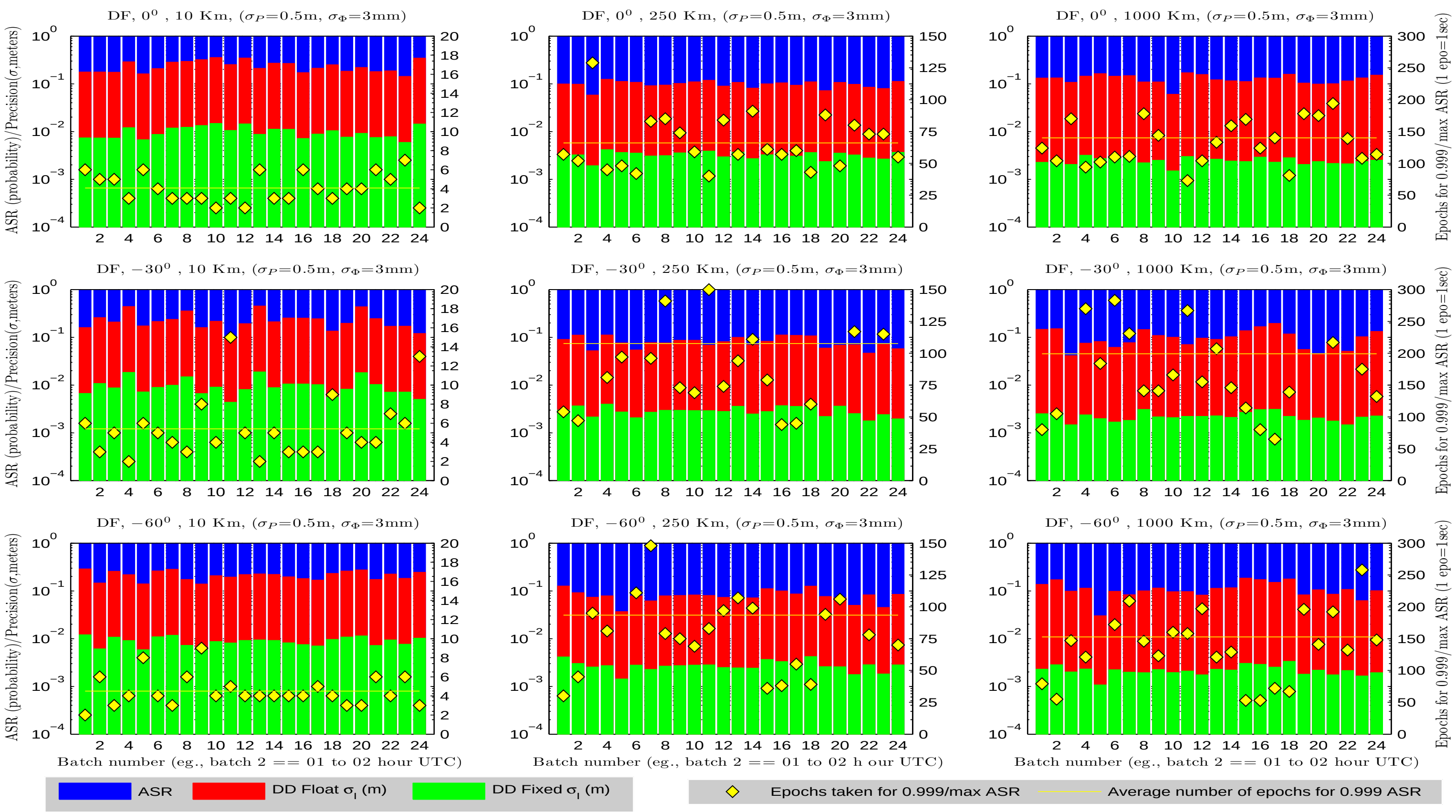

Figure G.41: **Galileo only**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

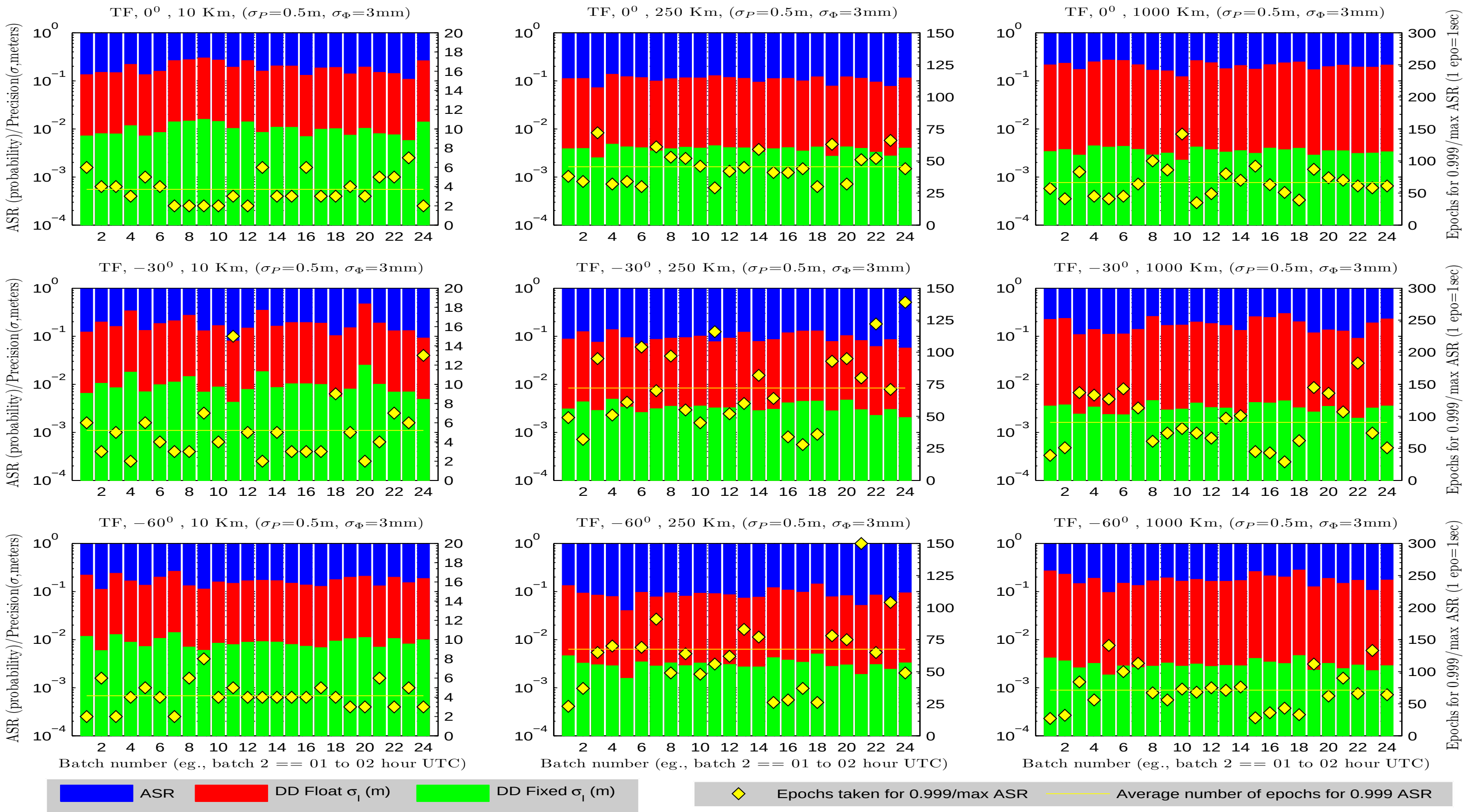

Figure G.42: **Galileo only**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **triple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

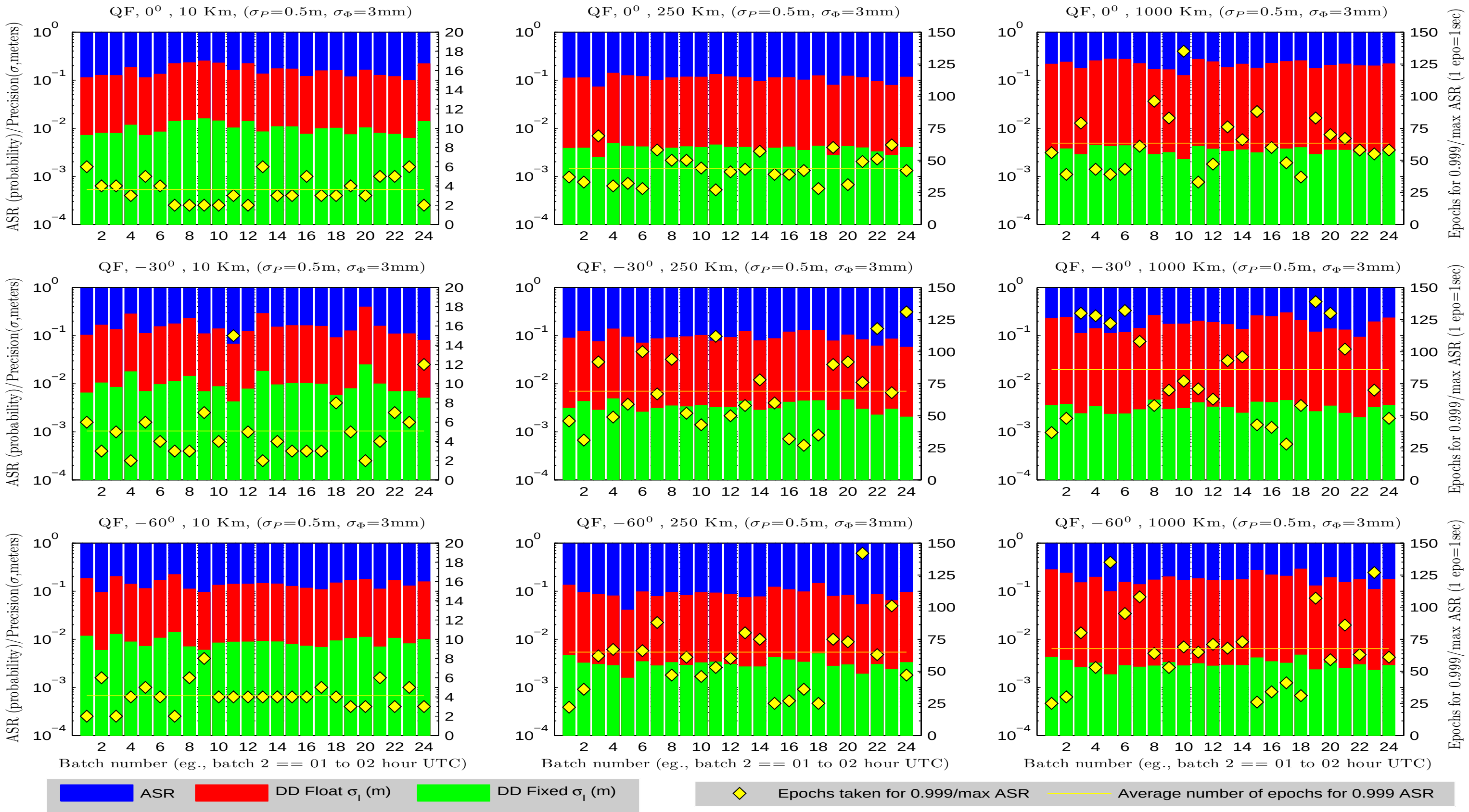

Figure G.43: **Galileo only**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

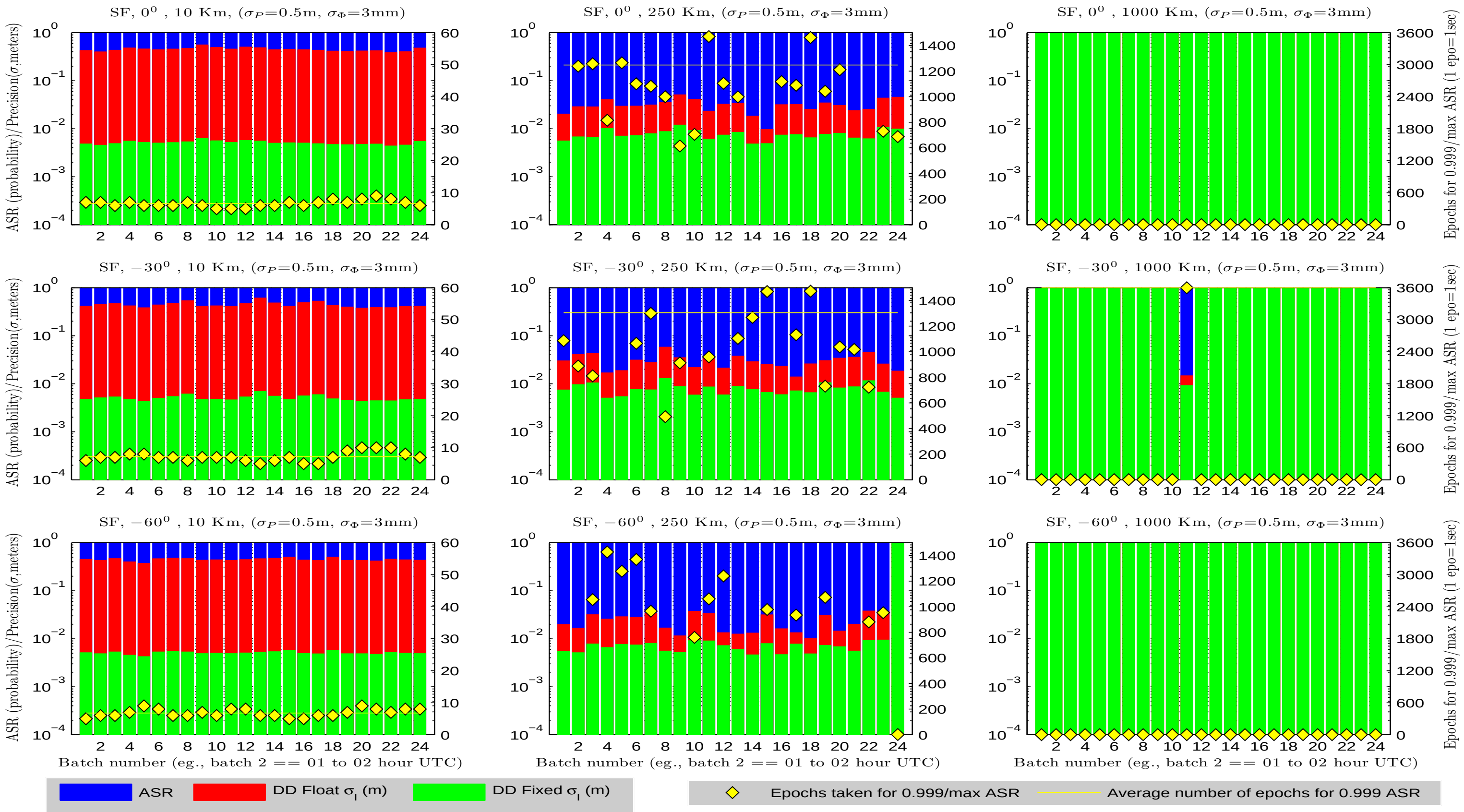

Figure G.44: **GPS + Galileo**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **single frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

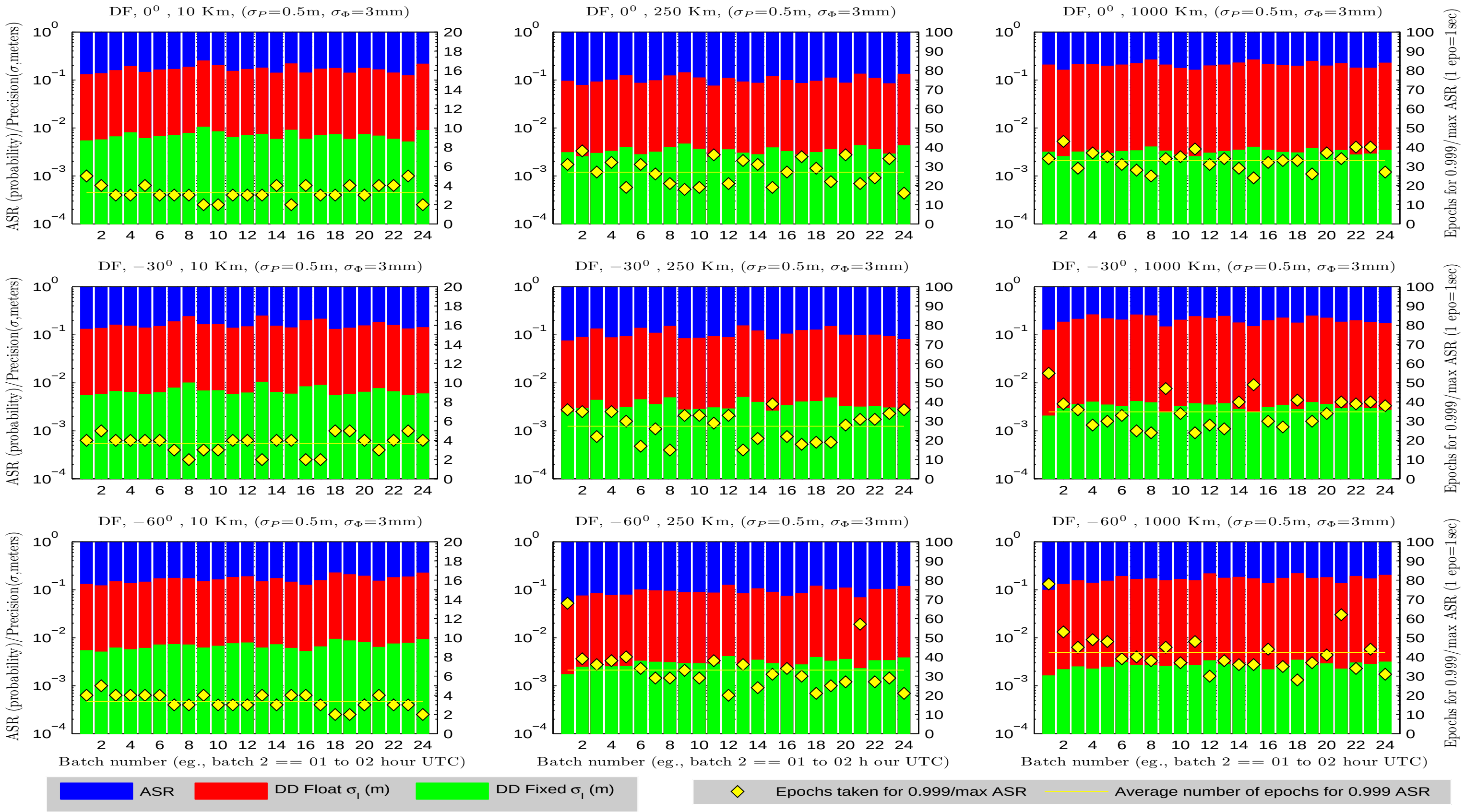

Figure G.45: **GPS + Galileo**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **dual frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

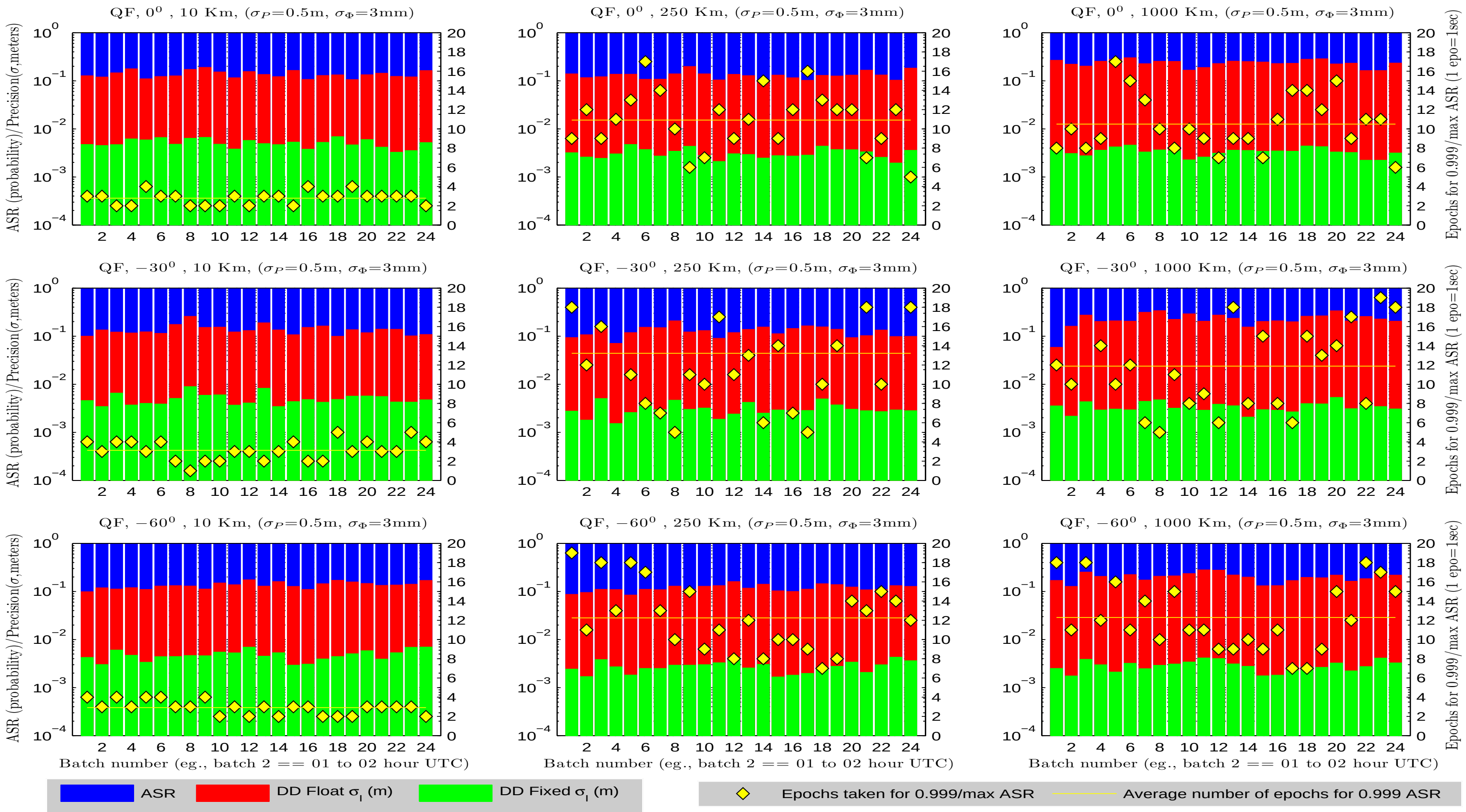

Figure G.46: **GPS + Galileo**, Ionosphere Weighted, **Geometry based model, Receiver coordinates for the rover are unknown**, **quadruple frequency**, Number of epochs taken for 0.999 ASR for each batch (each batch of 1 hour and 3600 epochs, 1 epoch = 1 second) during the day are presented along with ASR obtained, float and fixed precision of ionosphere.

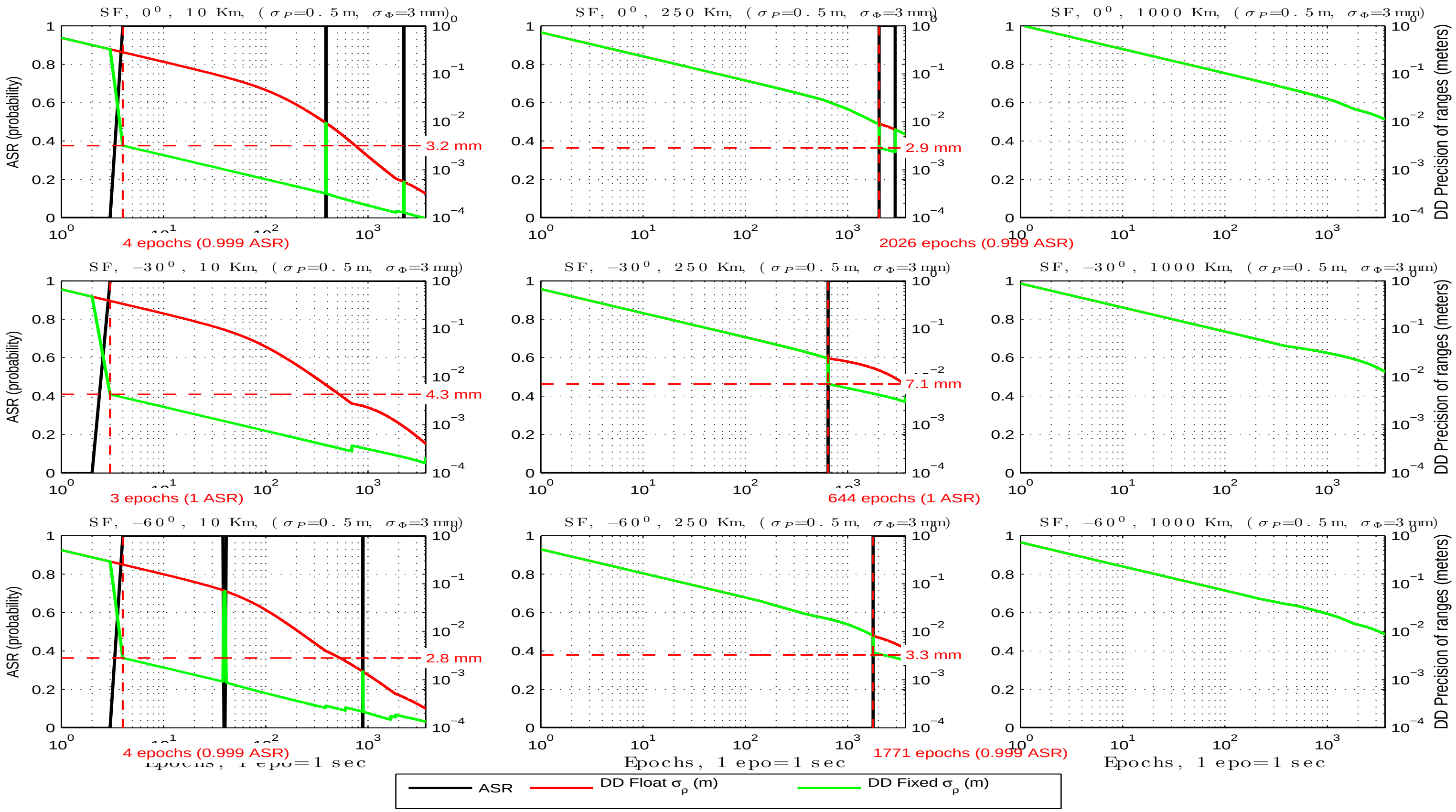

Figure G.47: **GPS only**, Single frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision σ_I = 0.68 mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

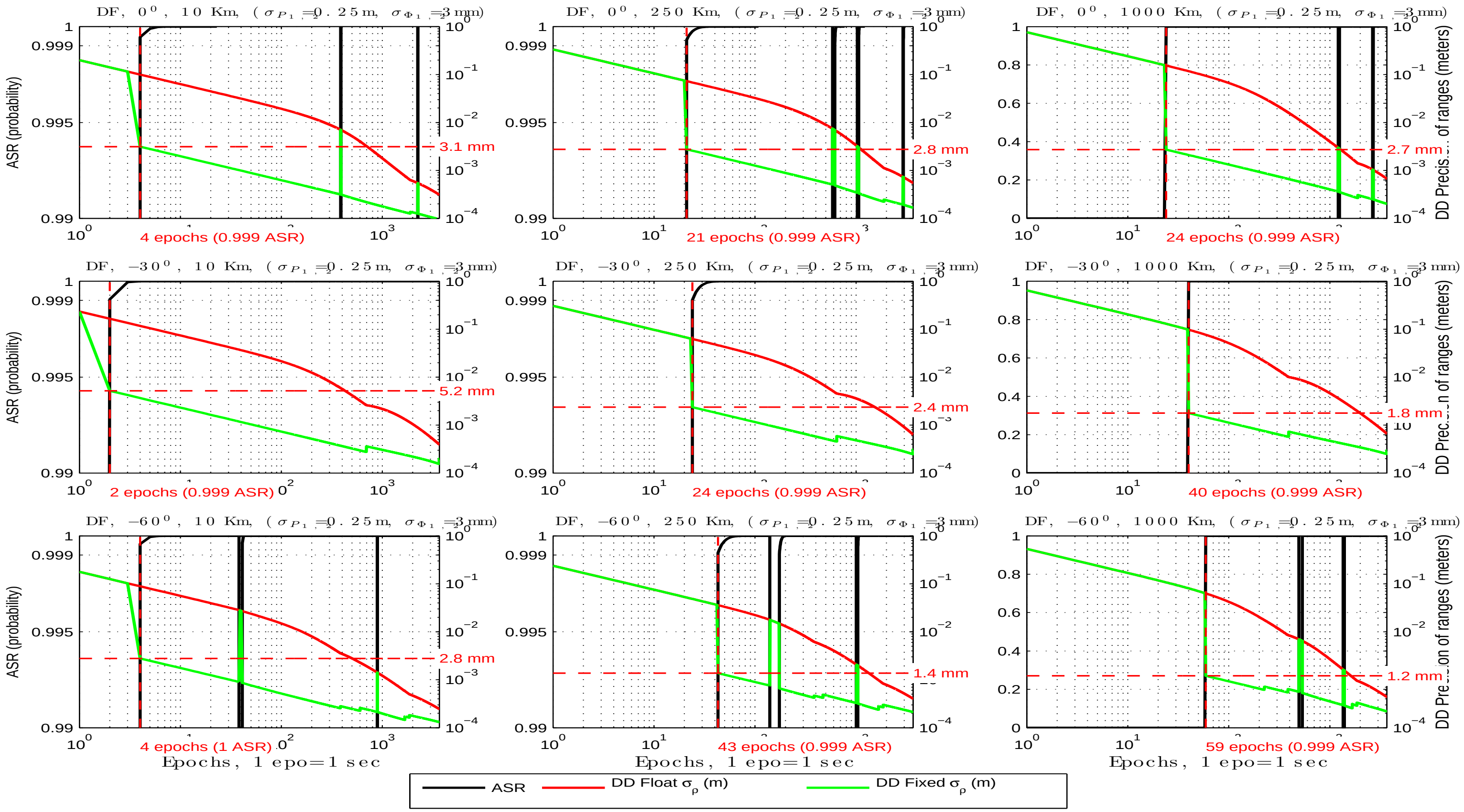

Figure G.48: **GPS only**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision σ_I = 0.68 mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

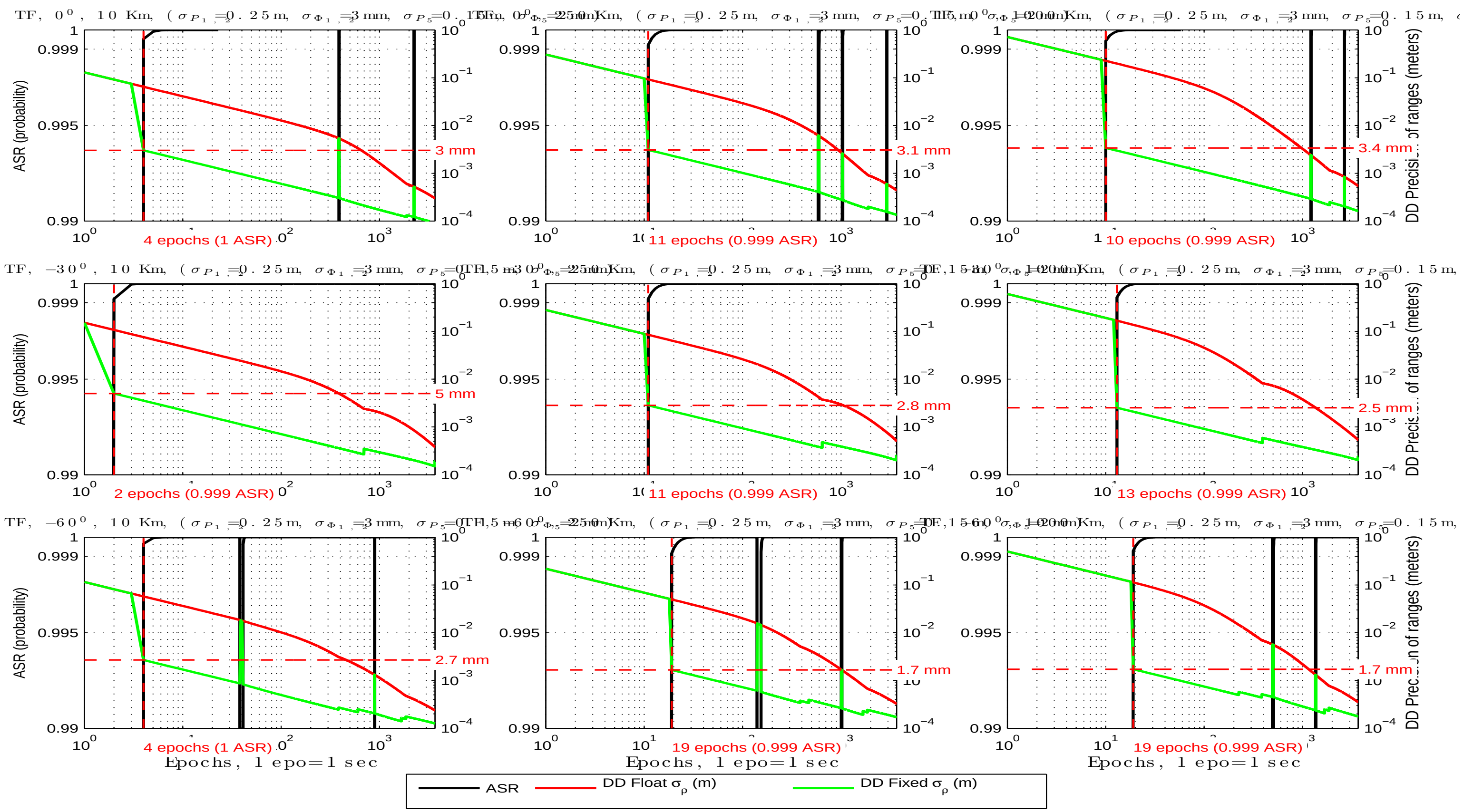

Figure G.49: **GPS only**, Triple frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision σ_I = 0.68 mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

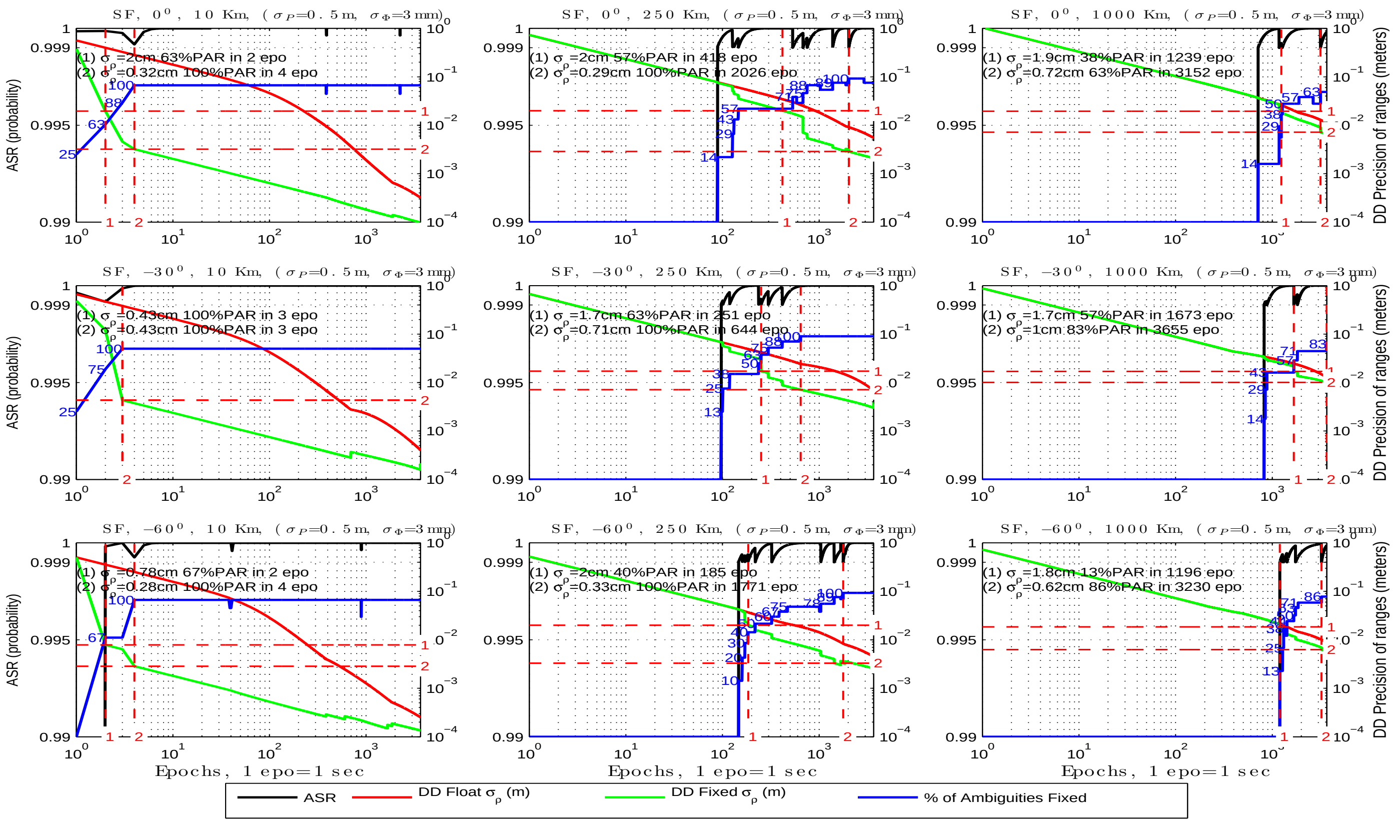

Figure G.50: **GPS only**, Single frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

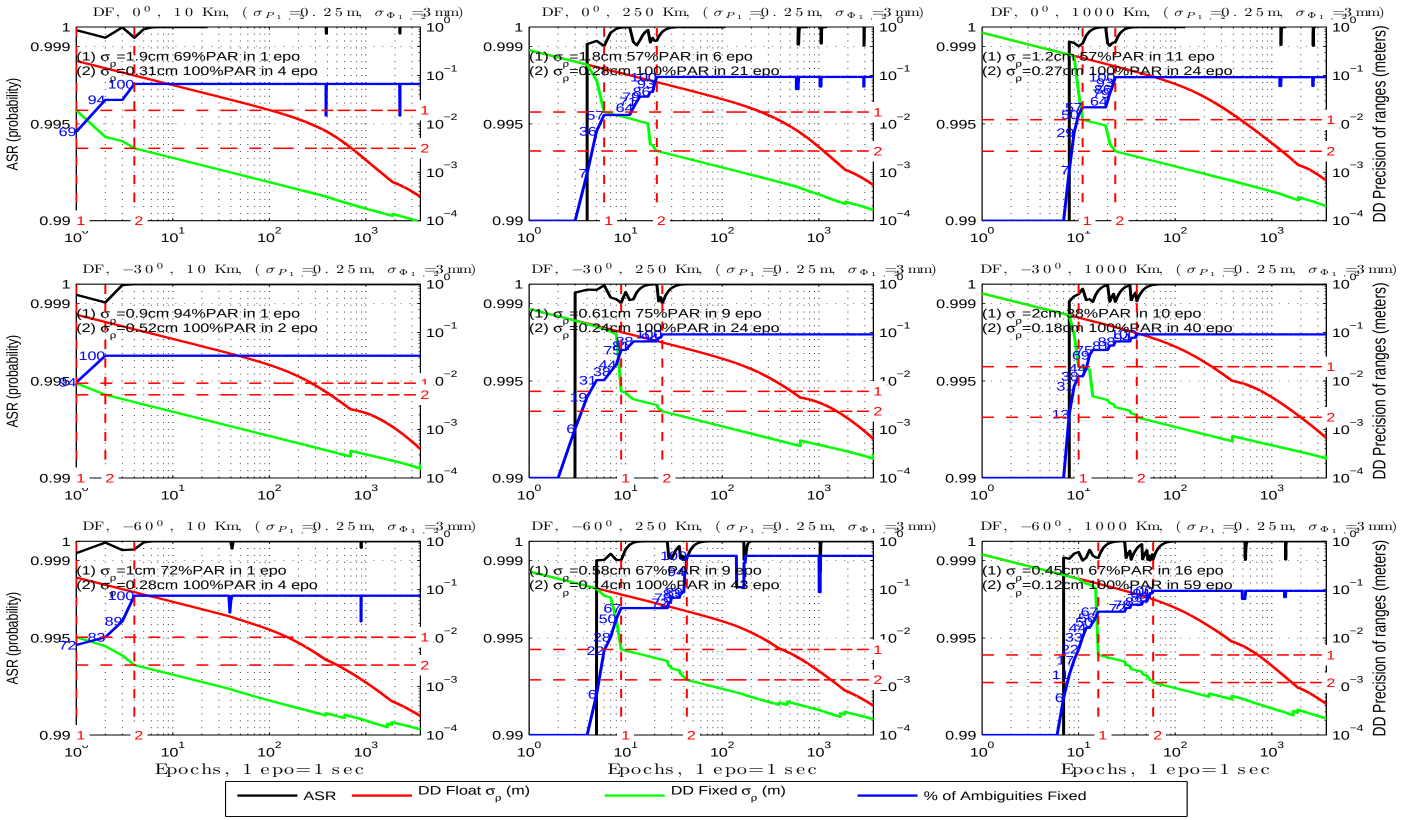

Figure G.51: **GPS only**, Dual frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°$, $-30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

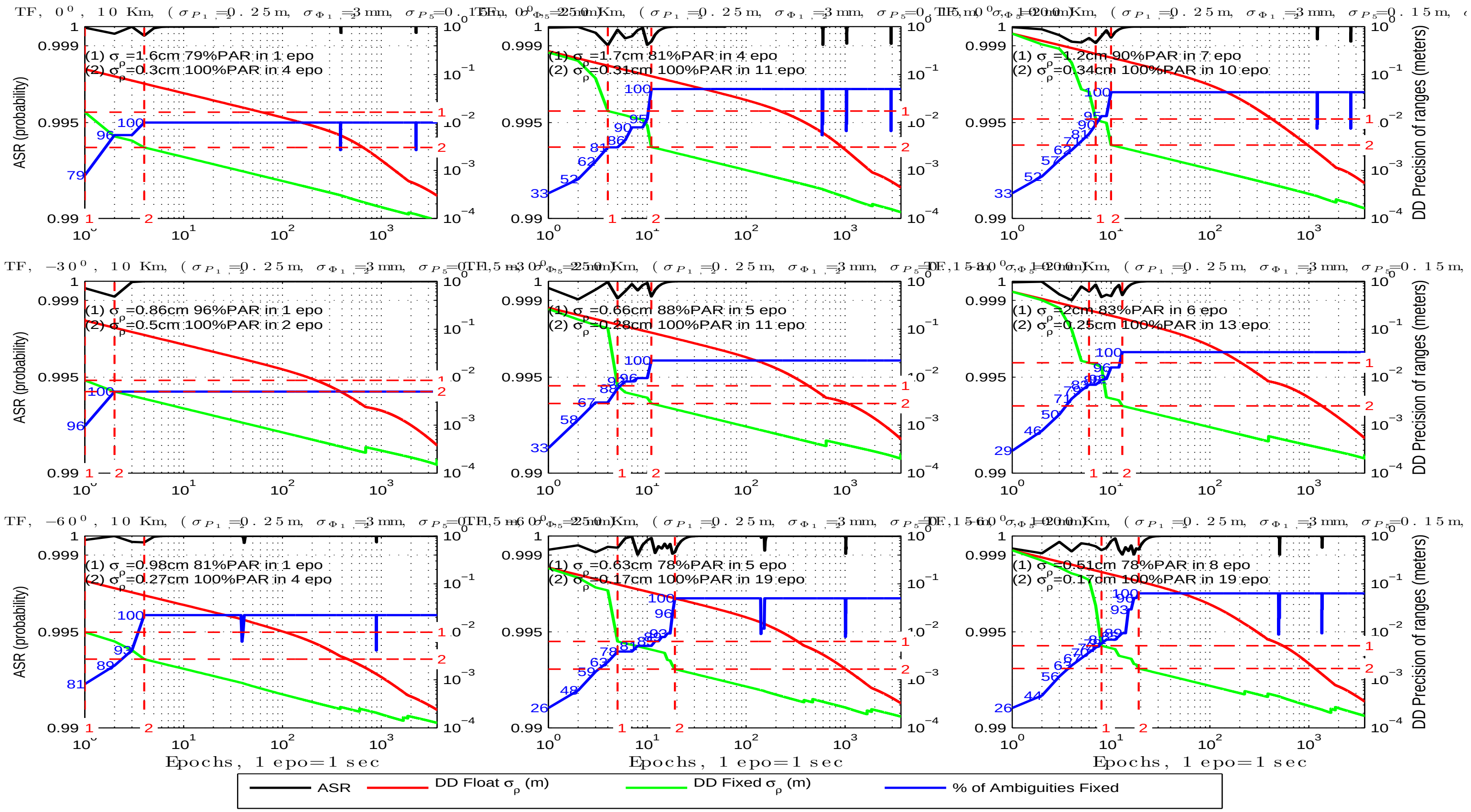

Figure G.52: **GPS only**, Triple frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

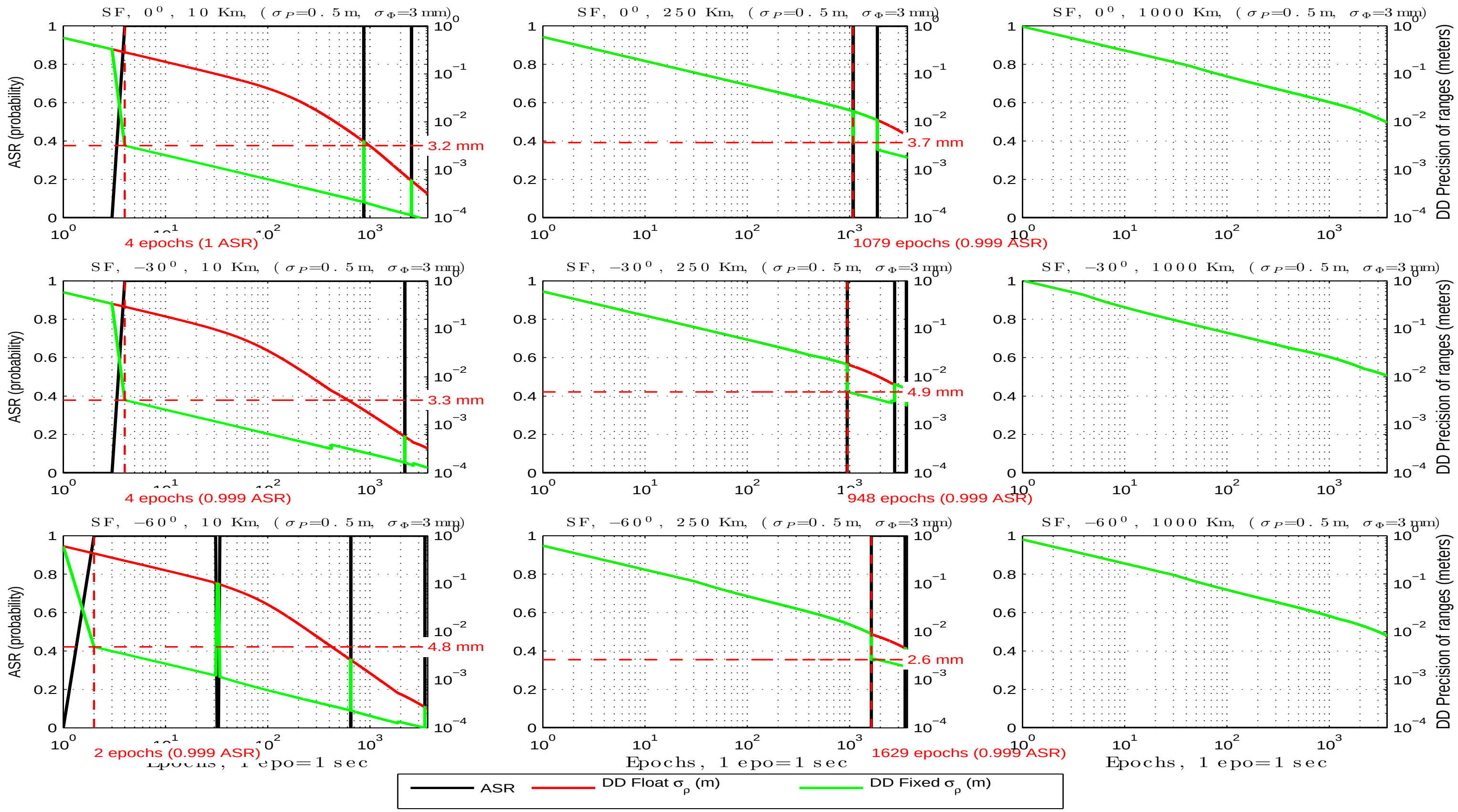

Figure G.53: **Galileo only**, Single frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

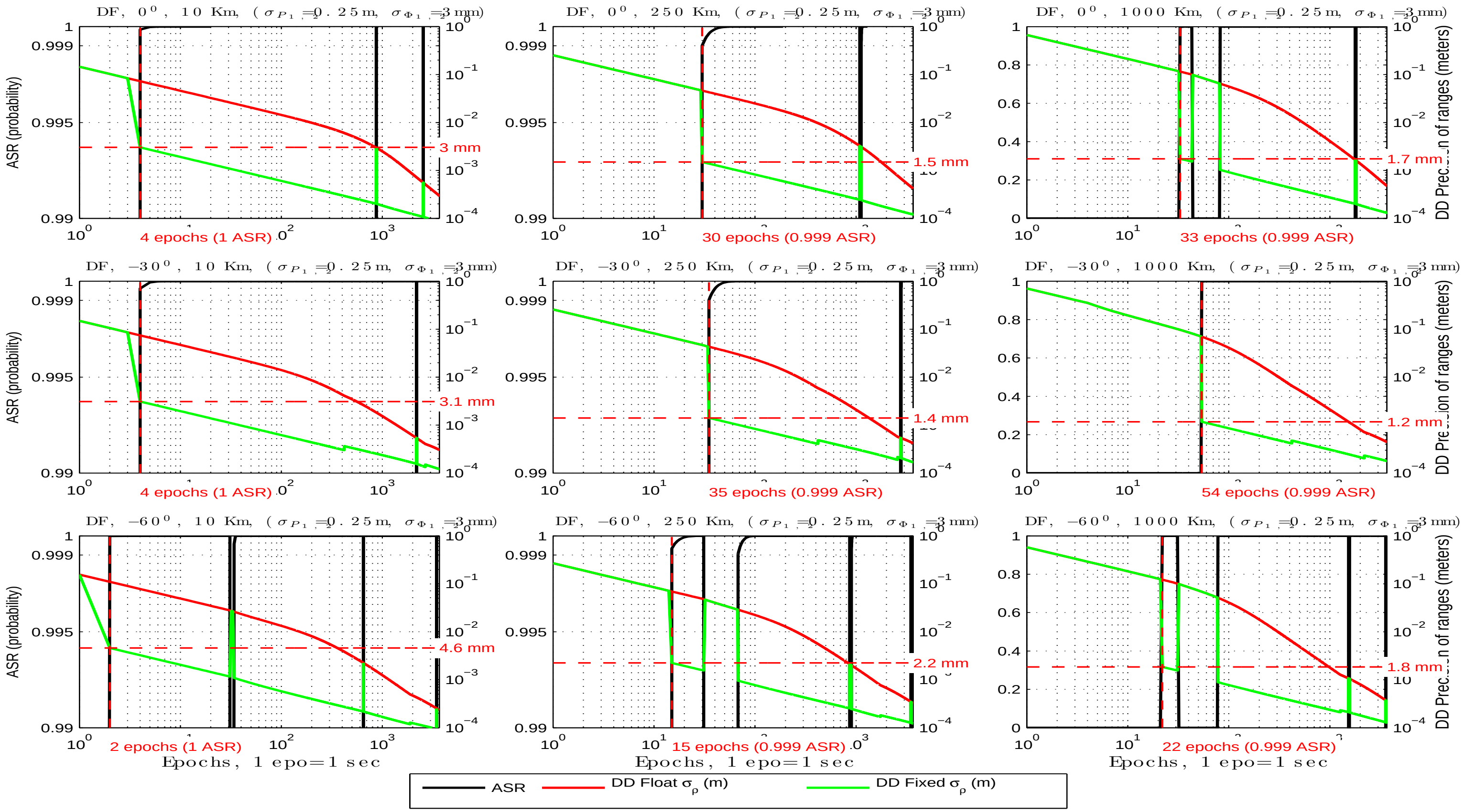

Figure G.54: **Galileo only**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

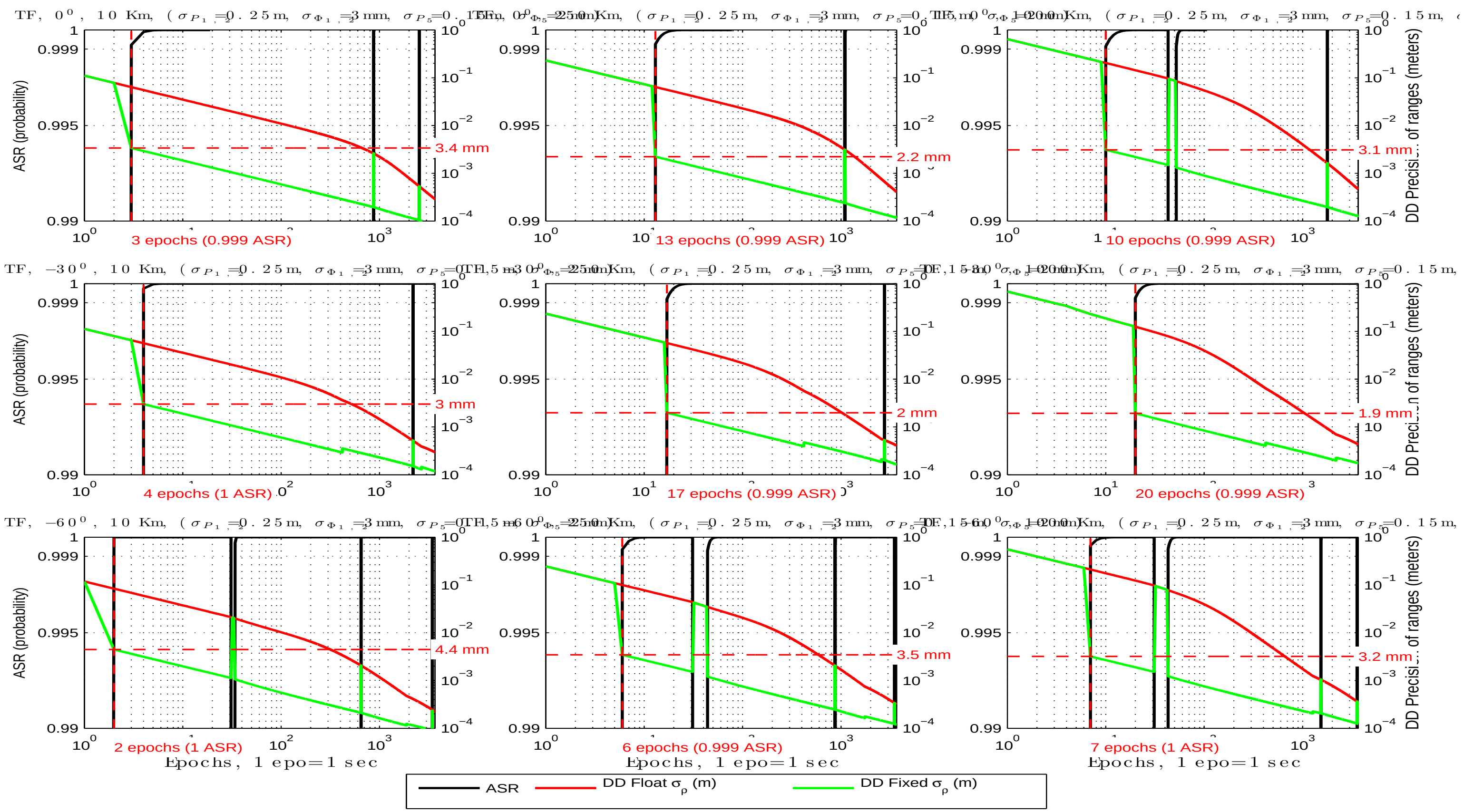

Figure G.55: **Galileo only**, Triple frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°$, $-30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

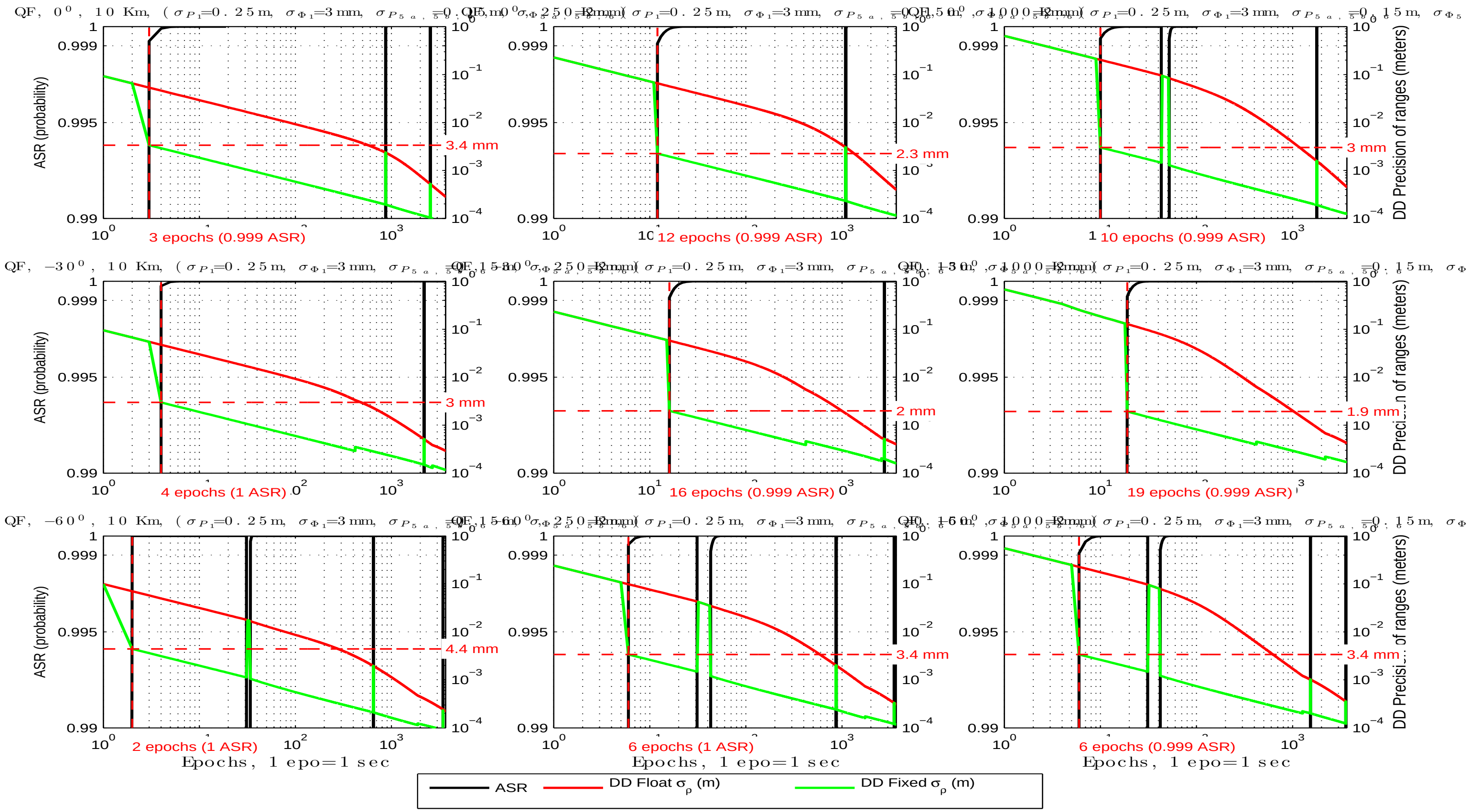

Figure G.56: **Galileo only**, Quadruple frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

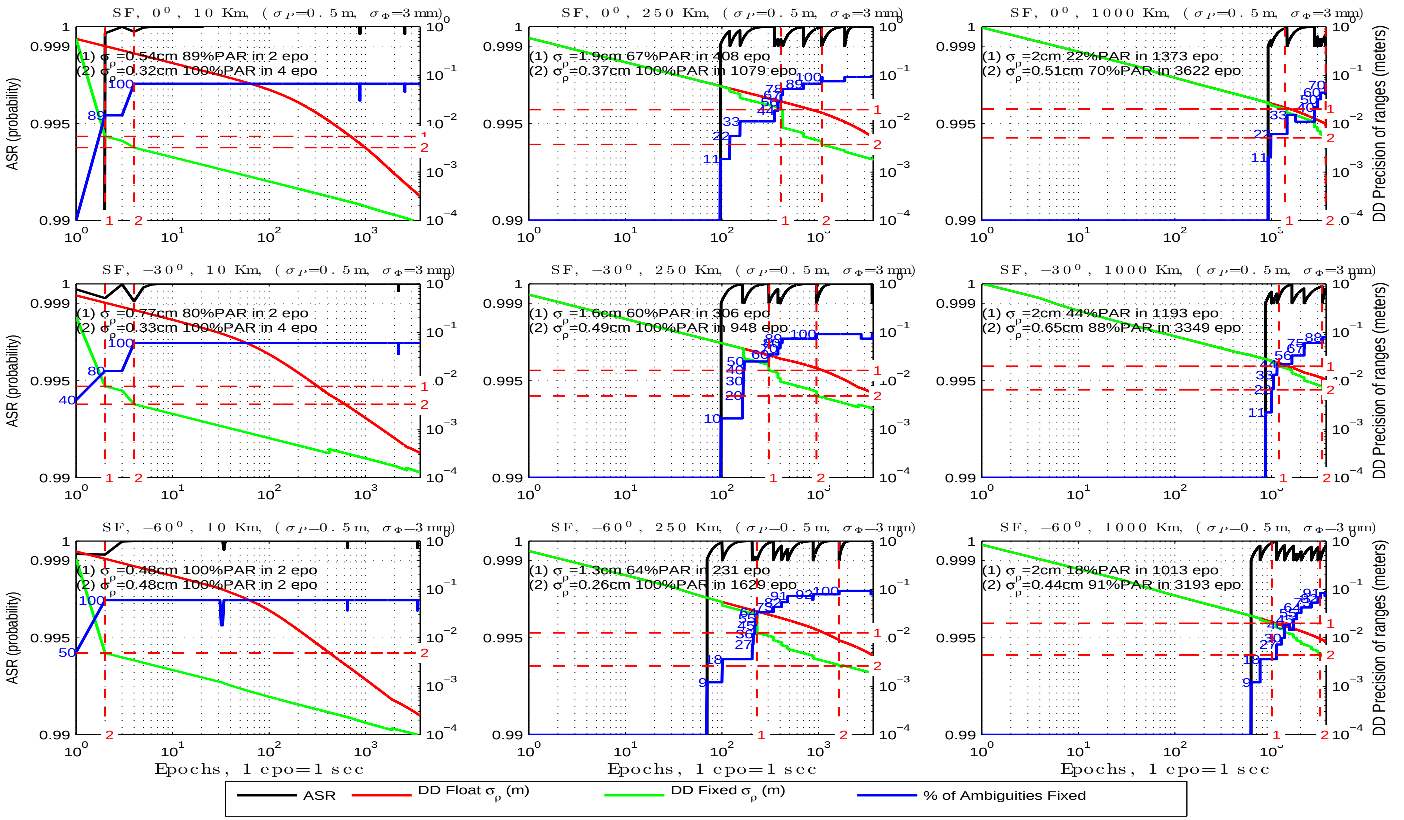

Figure G.57: **Galileo only**, Single frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

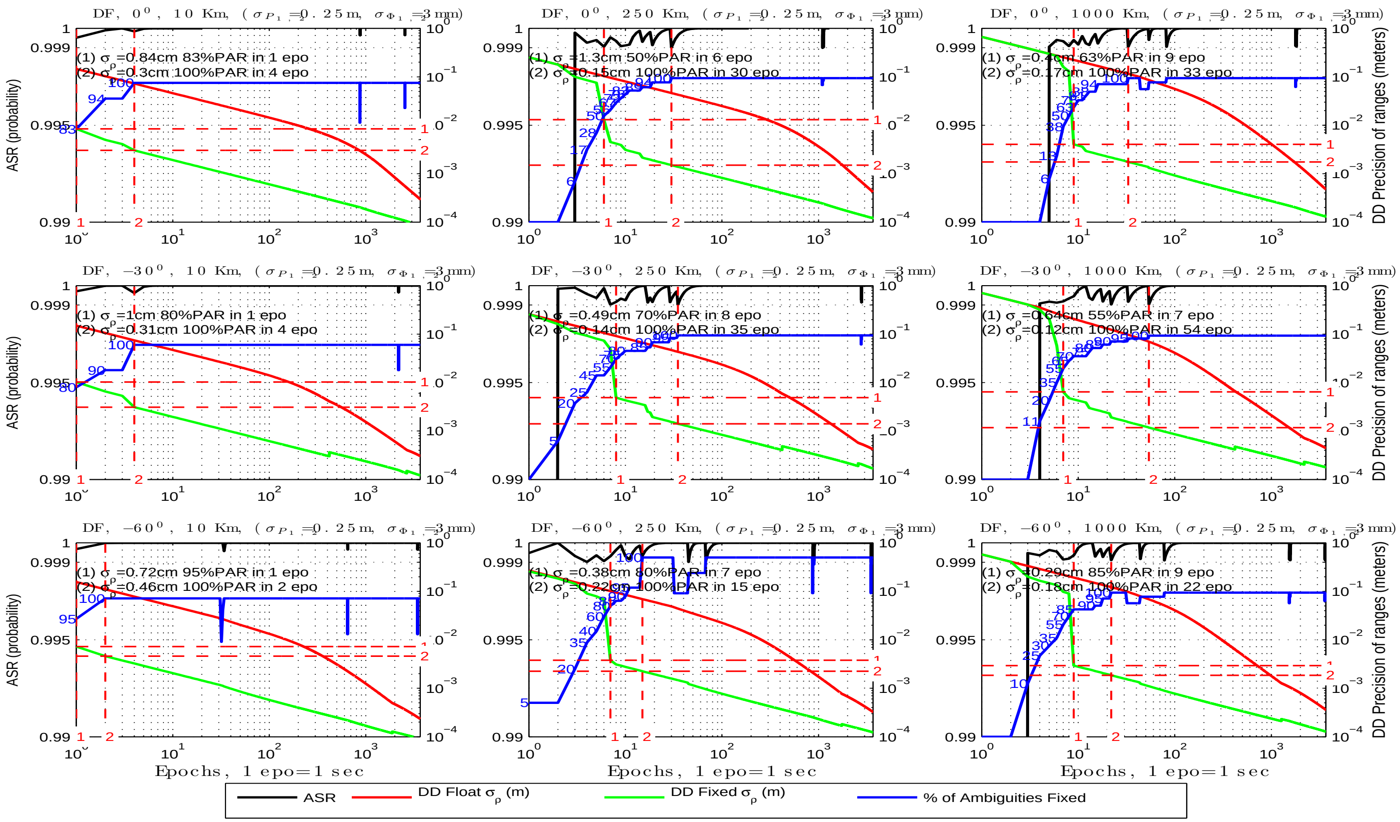

Figure G.58: **Galileo only**, Dual frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

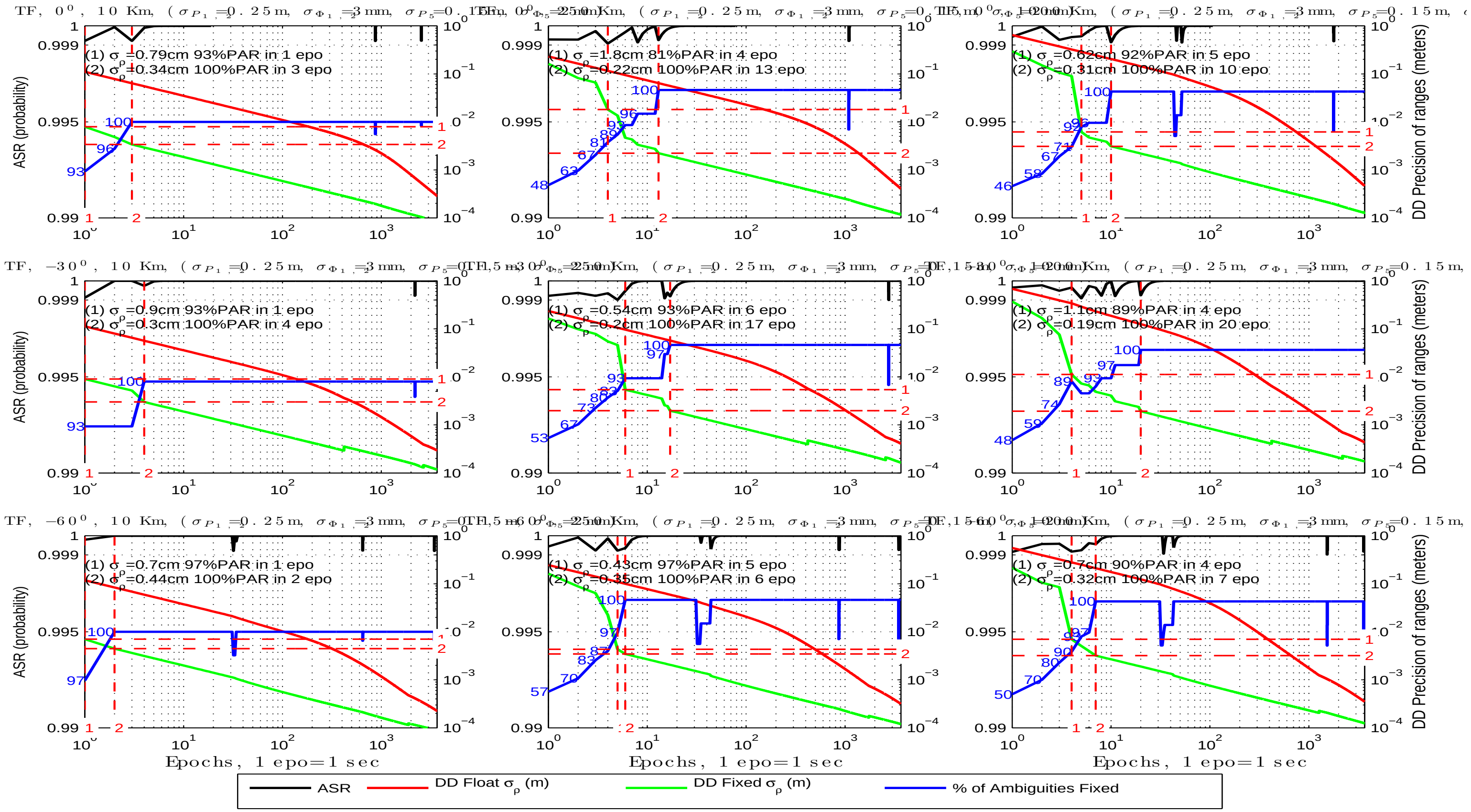

Figure G.59: **Galileo only**, Triple frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at 0°, −30° and −60° degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

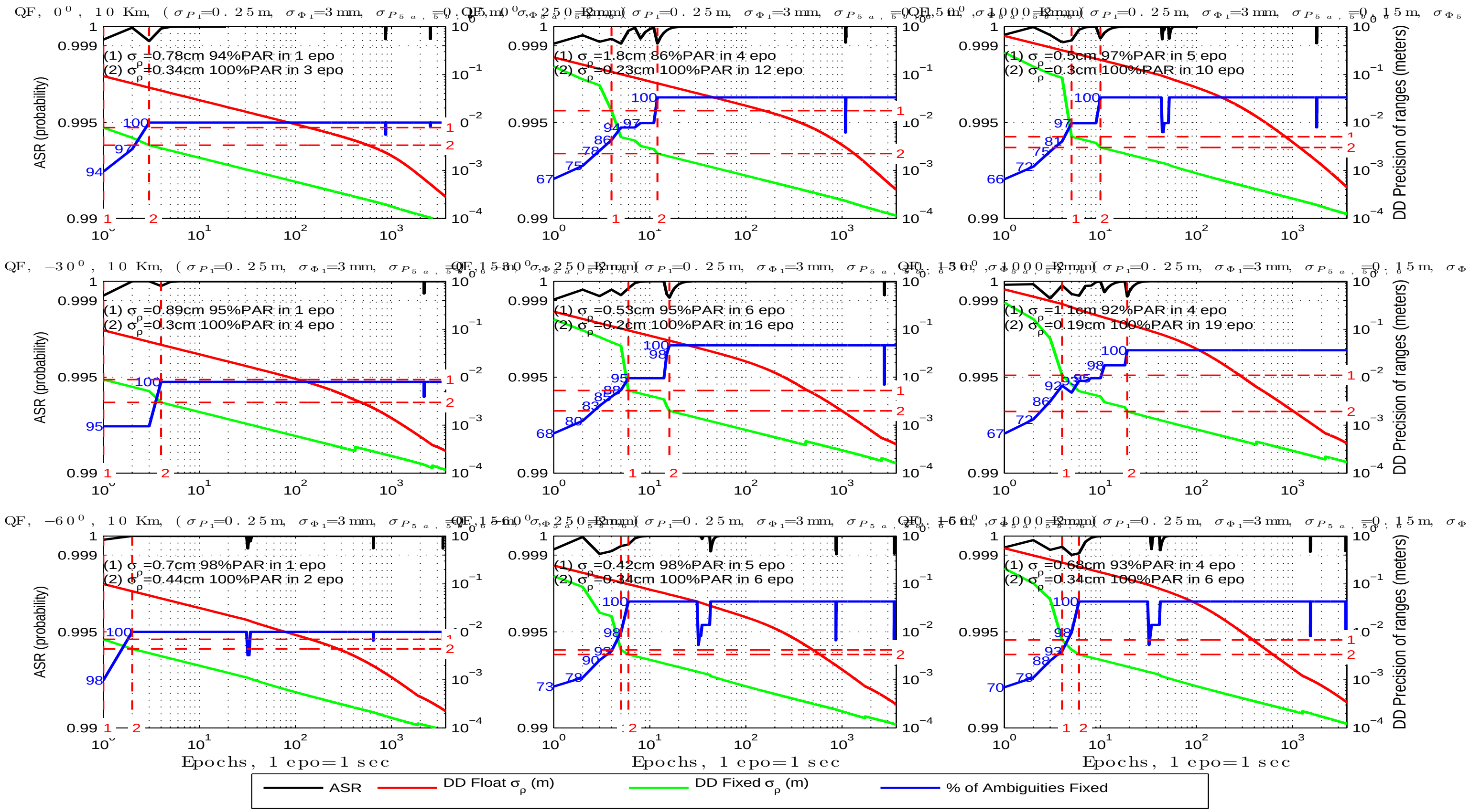

Figure G.60: **Galileo only**, Quadruple frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**- ASR is analyzed at $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

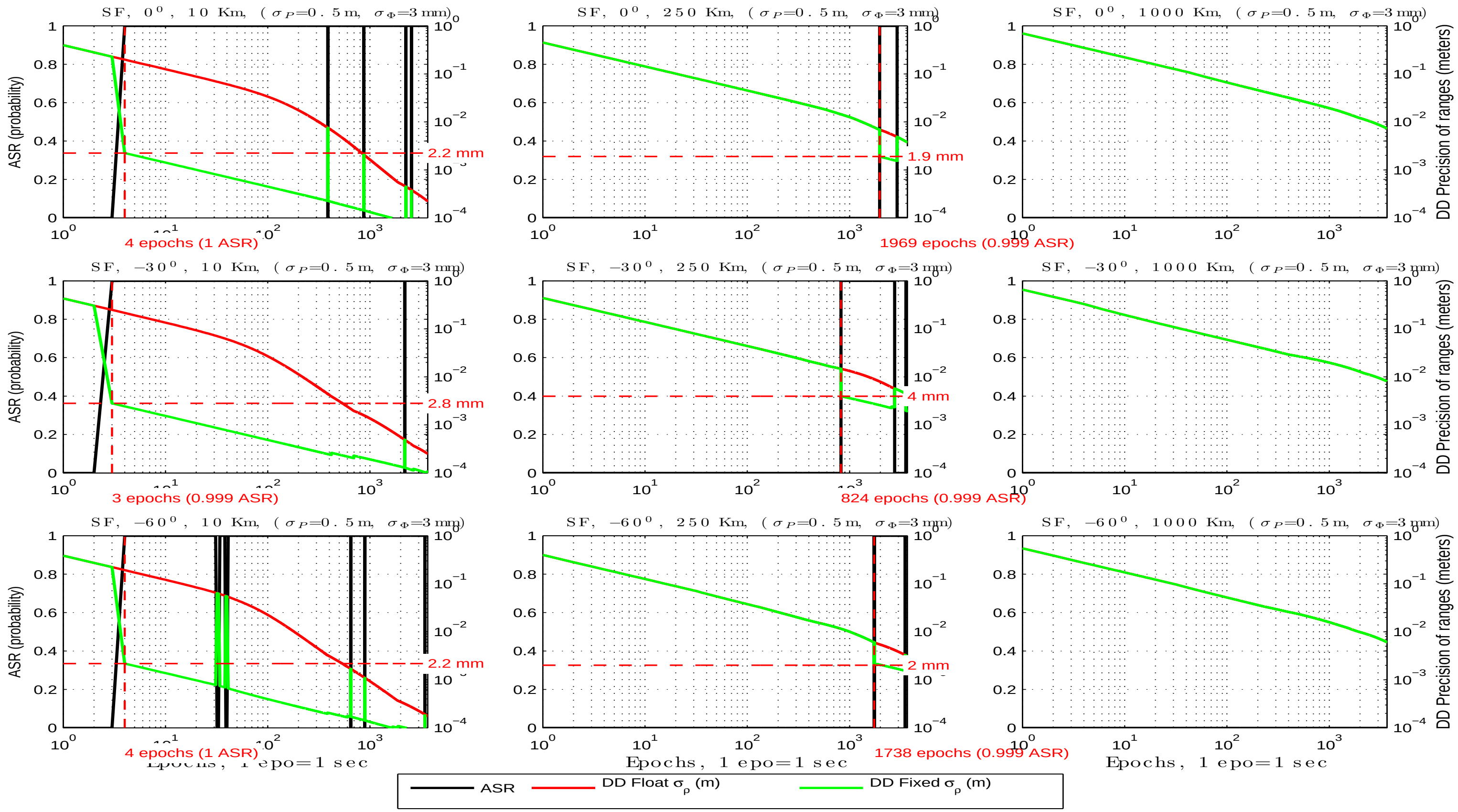

Figure G.61: **GPS+Galileo**, Single frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

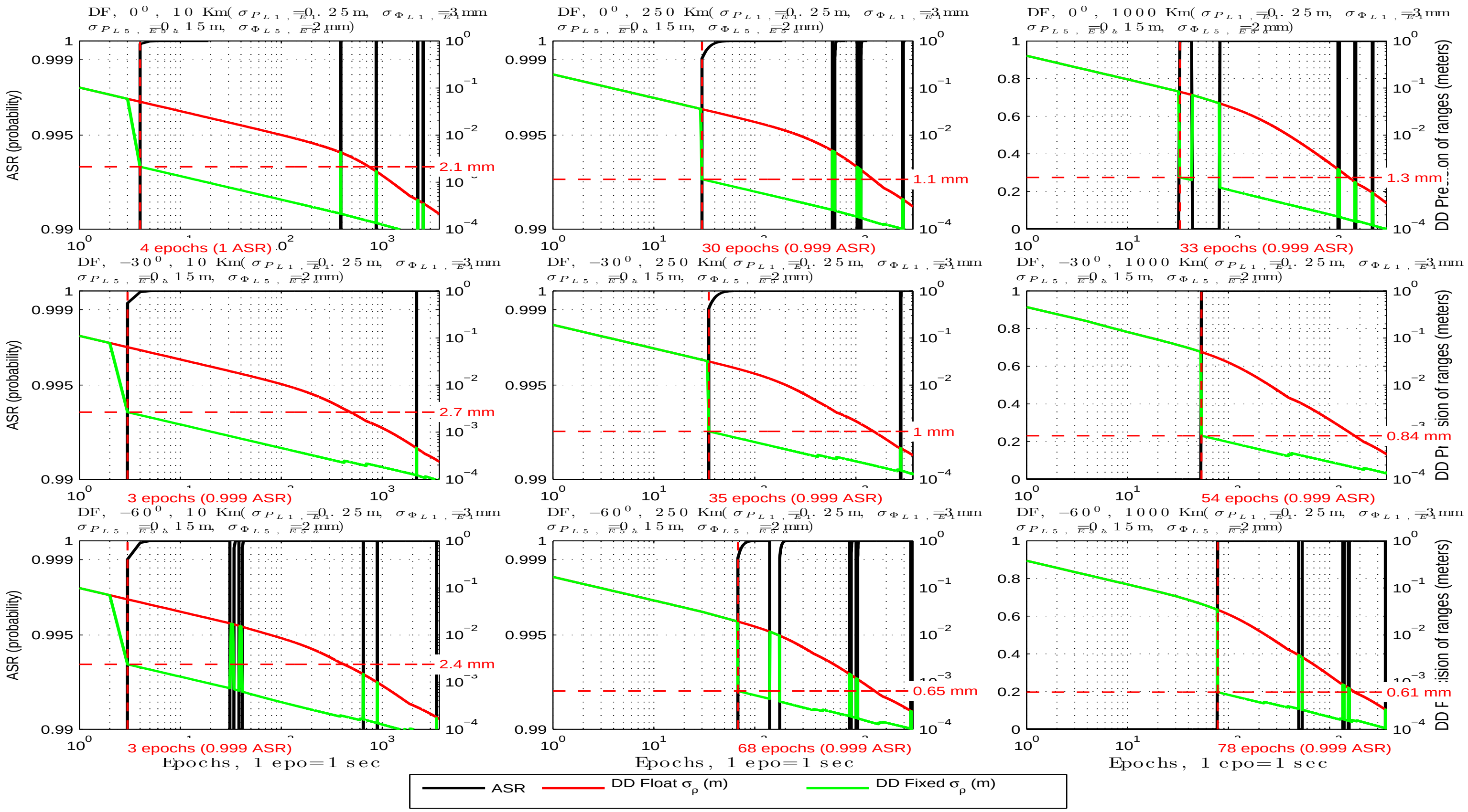

Figure G.62: **GPS+Galileo**, Dual frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

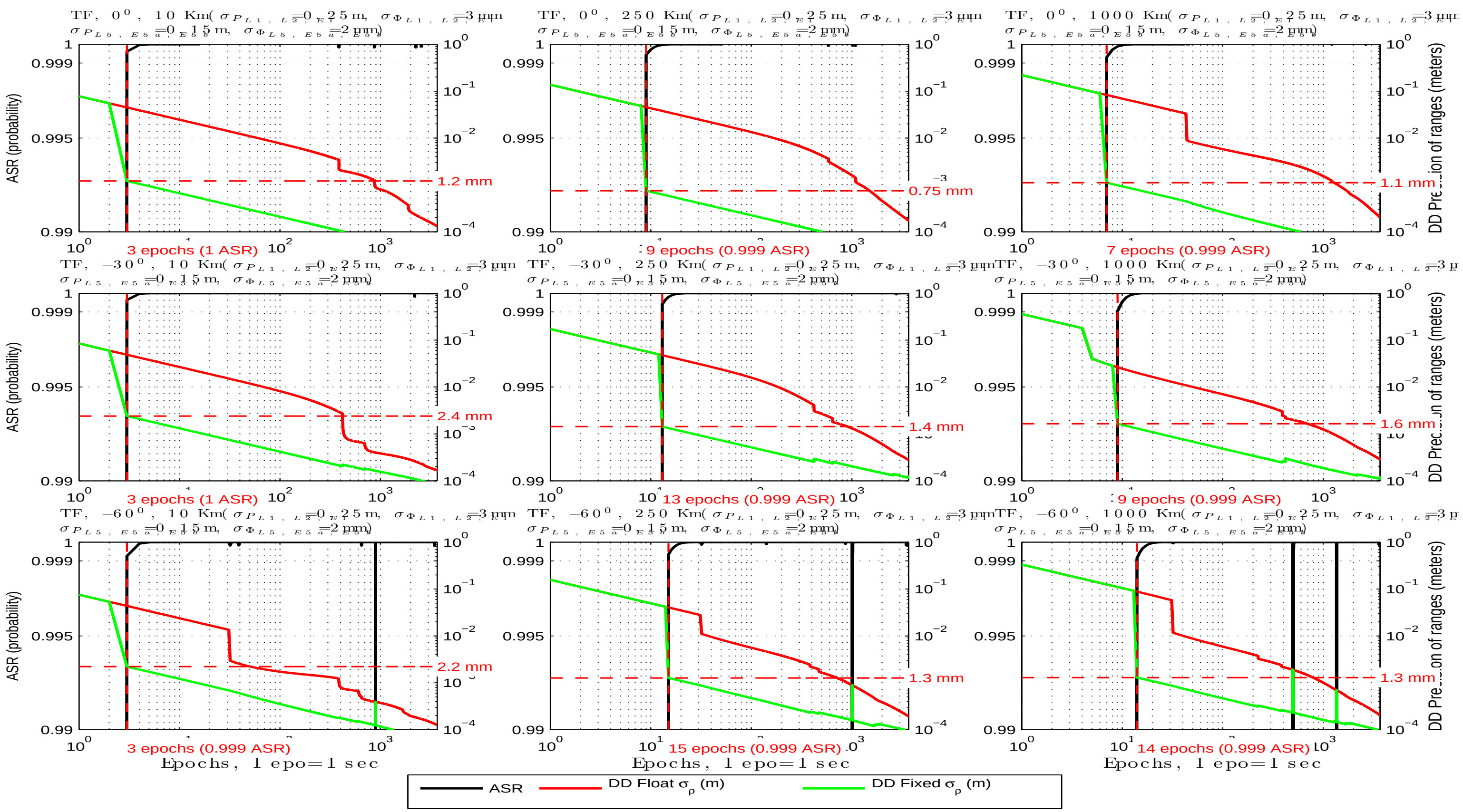

Figure G.63: **GPS+Galileo**, quadruple frequency, **Full AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for all the ambiguities to be fixed by full AR.

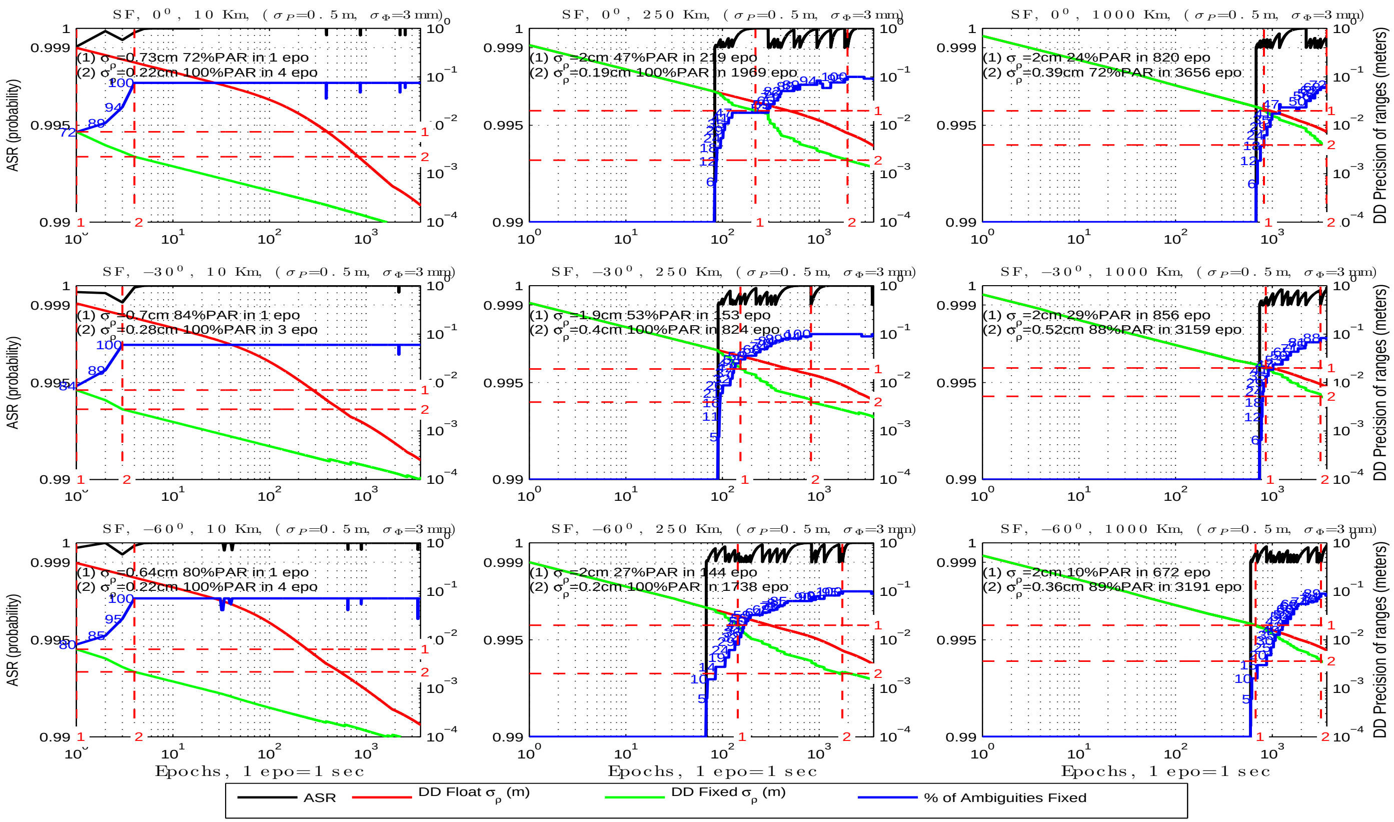

Figure G.64: **GPS+Galileo**, Single frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision $\sigma_I = 0.68$ mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

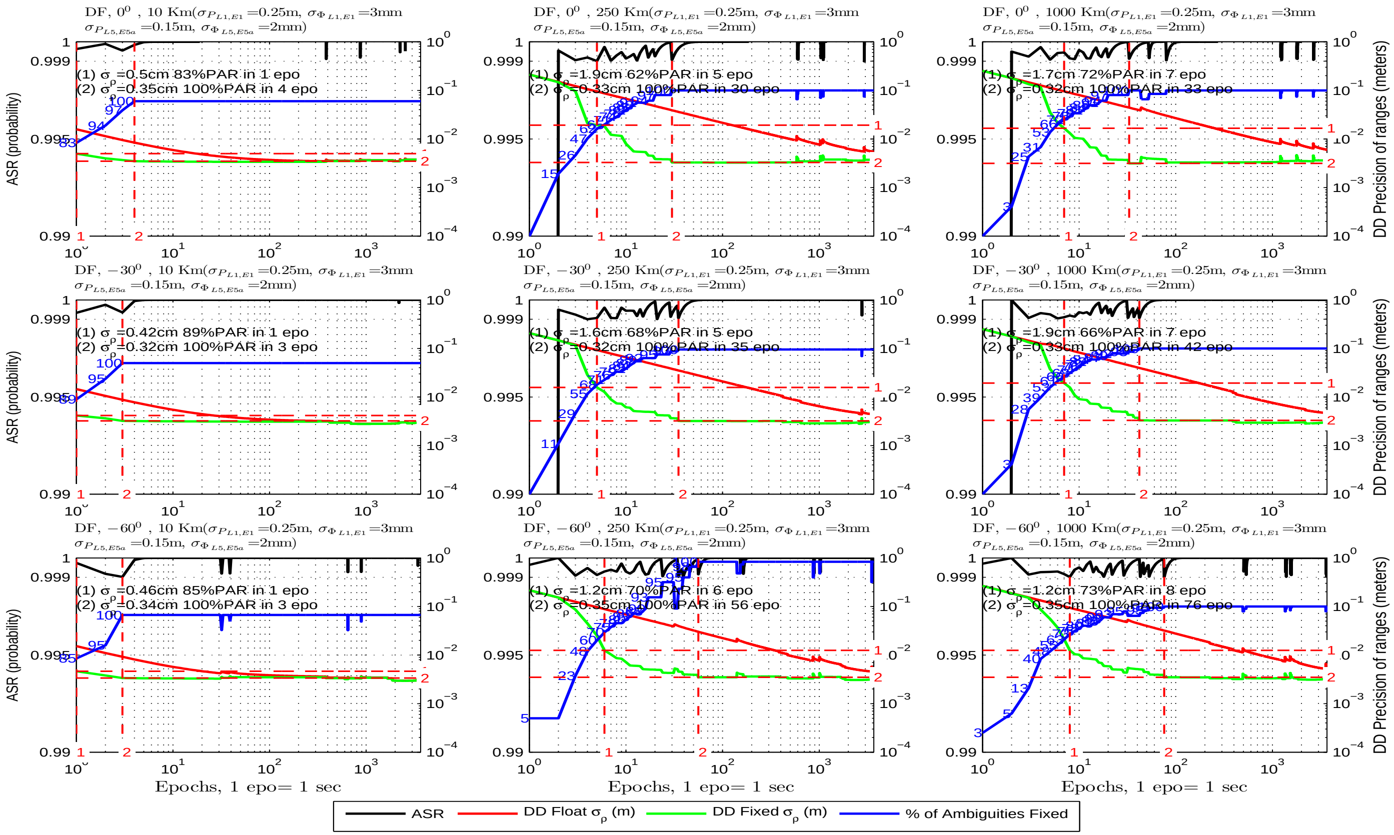

Figure G.65: **GPS+Galileo**, Dual frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD (Single Difference) precision σ_I = 0.68 mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

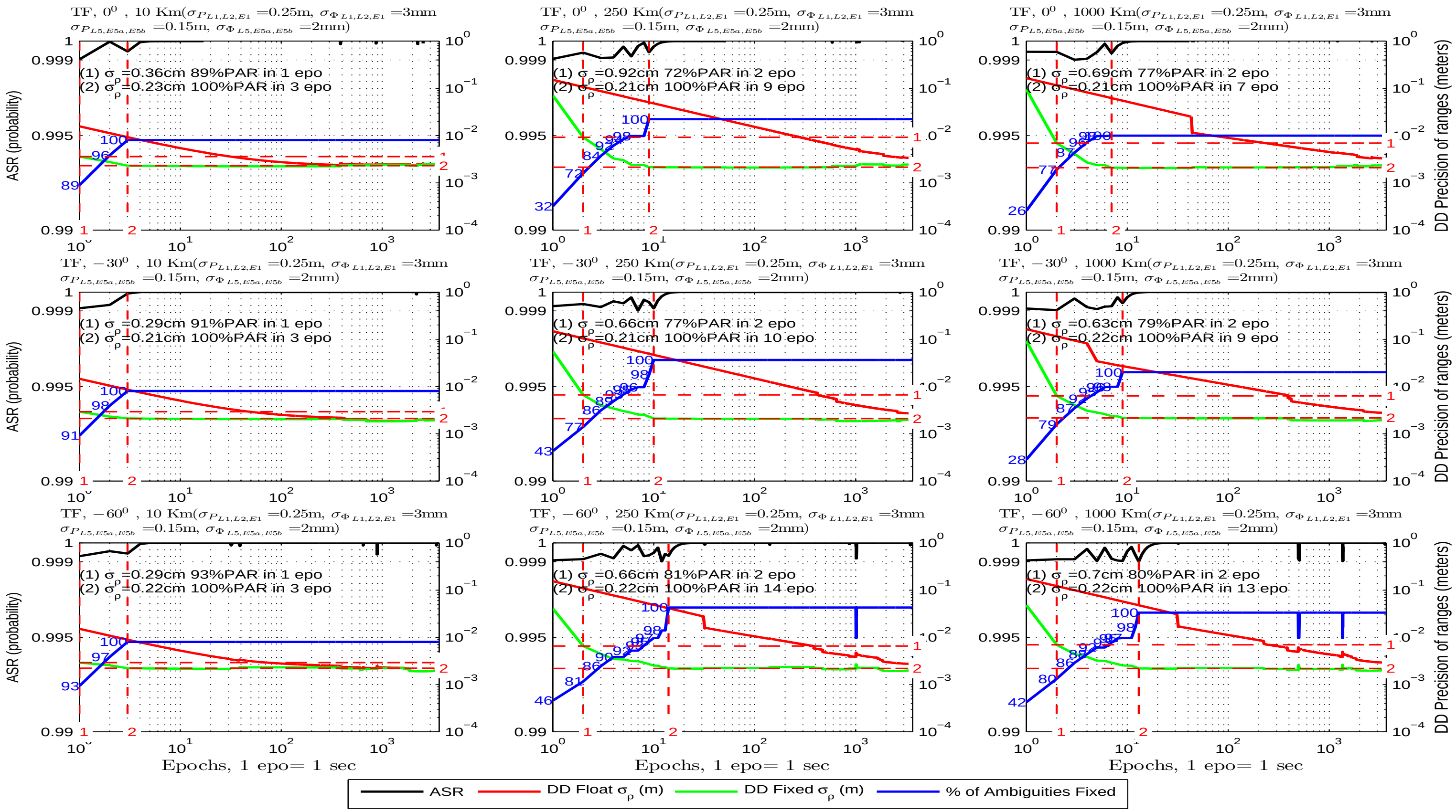

Figure G.66: **GPS+Galileo**, Quadruple frequency, **Partial AR**, **Geometry based model, Receiver coordinates for the rover are unknown**, ionosphere weighted, troposphere float scenario, **(measurement precision is varied, number of satellites are as available)**, **baseline length varied from 1 to 1000 Km** in East-West direction, SD precision σ_I = 0.68 mm per km - ASR and DD fixed-precision of rover-receiver coordinates (meters) are presented for $0°, -30°$ and $-60°$ degree latitude. The red colour cross-hair indicates the number of epochs (on x-axis) taken for obtaining 2cm fixed precision and for fixing 100% of ambiguities by PAR, along with the fixed precision obtained. These values are marked as 1 and 2 (on both x- and y-axis) and the corresponding number inside the figure presents its statistics.

Bibliography

[1] Teunissen, P.J.G., “A course on Integer Ambiguity Estimation With The LAMBDA Method.” Department of Spatial Sciences, Curtin University, Bentley campus, Australia, 2010.

[2] Teunissen, P.J.G. and Odijk, D. and Joosten, P., “A probabilistic evaluation of correct GPS ambiguity resolution,” *In Proceedings of ION GNSS, Nashville, USA*, 1998.

[3] Bock, Y. and Gourevitch, S.A. and Counselman III, C.C. and King, R.W. and Abbot, R.I., “Interferometric analysis of GPS phase observations,” *Manuscripta geodaetica*, vol. 11, pp. 282–288, 1986.

[4] Odijk, D., “Treatment of ionospheric effect on GPS Positioning,” *Delft Computing centre, Faculty of Geodetic Engineering, Delft University of Technology*, 1997.

[5] Jonkman, N.F. and Teunissen, P.J.G. and Joosten, P. and Odijk, D., “GNSS long baseline ambiguity resolution: Impact of a third navigation frequency,” *In: Schwarz K.P. (ed.), Geodesy Beyond 2000 - The Challenges of the First Decade. International Association of Geodesy Symposia, Springer-Verlag, Berlin Heidelberg*, vol. 121, pp. 349–354, 1999.

[6] de Jonge, P. and Teunissen, P.J.G. and Jonkman, N.F. and Joosten, P., “The Distributional Dependence Of The Range On Triple Frequency GPS Ambiguity Resolution,” *In Proceedings of ION National Technical Meeting, Anaheim, USA*, pp. 605–612, 2000.

[7] Teunissen, P.J.G. and Joosten, P. and de Jong, C.D., “The Distributional Dependence Of The Range On Triple Frequency GPS Ambiguity Resolution,” *In Proceedings of ION National Technical Meeting*, 2000.

[8] Teunissen, P.J.G. and Joosten, P. and Jonkman, N.F., “Evaluation of long range GNSS ambiguity resolution,” *Proc. of the 8th European CGSIC IISC Meeting, 2-3 December, 1999, Prague, Czech Republik*, pp. 171–177, 2000.

[9] Teunissen, P. J. G., “Galileo and ambiguity resolution,” in *In: There is more than Geometry, H. Aalders, E. Fendel (Eds.), VBK Editorial*, vol. ISBN 90-806917-1-2, pp. 279–286, 2002.

[10] Odijk, D. and Teunissen, P.J.G., "ADOP in closed form for a hierarchy of multi-frequency single-baseline GNSS models," *J. of Geodesy*, vol. doi:10.1007/s00190-007-0197-2, 2008.

[11] Verhagen, S. and Teunissen, P.J.G. and Odijk, D., "Carrier-phase ambiguity success rates for integrated GPS-Galileo satellite navigation," *Proc. Space, Aeronautical and Navigational Electronics Symposium SANE2007, Japan*, vol. 107(2), pp. 139–144, 2007.

[12] Verhagen, S. and Odijk, D. and Teunissen, P.J.G. and Huisman, L., "Performance improvement with low-cost multi-GNSS receivers," *Satellite Navigation Technologies and European Workshop on GNSS Signals and Signal Processing (NAVITEC), 2010 5th ESA Workshop on*, vol. doi:10.1109/NAVITEC.2010.5708015, pp. 1 –8, 2010.

[13] Bona, P. and Tiberius, C., "An experimental comparison of noise characteristics of seven high-end dual frequency GPS receiver-sets," *Proc. of the IEEE Position Location and Navigation Symposium, San Diego, California*, pp. 237–244, 2000.

[14] Odijk, D. and Traugott, J. and Sachs, G. and Montenbruck, O. and Tiberius, C., "Two Approaches to Precise Kinematic GPS Positioning with miniaturized L1 Receivers," *In Proceedings of ION National Technical Meeting, Fort Worth, Tex, USA*, vol. 3, pp. 827–838, 2007.

[15] Simsky, A. and Sleewaegen, J.-M. and Hollreiser, M. and Crisci, M., "Performance assessment of Galileo ranging signals transmitted by GSTB-V2 satellites," *In Proceedings of ION National Technical Meeting, Fort Worth, Tex, USA*, vol. 3, p. 15471559, 2006.

[16] Verhagen, S., "Visualization of GNSS-related design parameters; Manual for the Matlab user interface VISUAL," *Delft University of Technology, The Netherlands*, 2006.

[17] Ifadis, I. M., "The excess propagation path of radio waves: Study of the influence of the atmospheric parameters on its elevation depedence," *Survey Review*, vol. 31, p. 289298, 1992.

[18] Teunissen, P.J.G. and Joosten, P. and Jonkman, N.F., "Evaluation of long range GNSS ambiguity resolution," *Presented at the 8th European CGSIC Meeting, Prague*, 1999.

[19] Teunissen, P.J.G. and Verhagen, S., "Integer Aperture Estimation," *Inside GNSS*, 2011.

[20] Teunissen, P.J.G., "GNSS ambiguity resolution with optimally controlled failure-rate," *Artificial Satellites*, vol. 40, no. 4, pp. 219–227, 2005.

[21] Verhagen, S., *The GNSS integer ambiguities: estimation and validation.* PhD thesis, Delft University of Technology, Delft, The Netherlands, 2005.

[22] Wang, J. and Stewart, MP and Tsakiri, M., "A discrimination test procedure for ambiguity resolution on-the-fly," *Journal of Geodesy*, vol. 72, no. 11, pp. 644–653, 1998.

[23] Teunissen, P.J.G. and Joosten, P. and de Jong, C.D., "Frequency selection and ambiguity resolution," *Galileo's World, 2(2)*, pp. 38–43, 2000.

[24] Colomina, I. and Miranda, C. and Eulàlia Parés, M. and Andreotti, M. and Hill, C. and da Silva, P.F. and Silva, J.S. and Peres, T. and Galera Monico, J.F. and Camargo, P.O. and others, "Galileo's Surveying Potential-E5 Pseudorange Precision," *GPS World*, vol. 23, no. 3, p. 18, 2012.

[25] Zhang, W. and Cannon, M.E. and Julien, O. and Alves, P., "Investigation of combined GPS/Galileo cascading ambiguity resolution schemes," *In Proceedings of ION GNSS*, 2003.

[26] Chen, X. and Vollath, U. and Landau, H. and Sauer, K., "Will GALILEO/Modernized GPS obsolete network RTK?," *In Proceedings of ION GNSS*, 2004.

[27] Sauer, K. and Vollath, U. and Amarillo, F., "Three and Four Carriers for Reliable Ambiguity Resolution," *European GNSS 2004 Conference, Rotterdam, The Netherlands*, 2004.

[28] Milbert, D., "Influence of pseudorange accuracy on phase ambiguity resolution in various GPS modernization scenarios," *Navigation*, vol. 52(1), pp. 29–38, 2005.

[29] Schlotzer, S. and Martin, S., "Performance study of multi carrier ambiguity resolution techniques for Galileo and modernized GPS," *In Proceedings of ION GNSS*, 2005.

[30] O'Keefe, K. and Julien, O. and Cannon, M.E. and Lachapelle, G., "Availability, accuracy, reliability and Carrier-phase ambiguity resolution with Galileo and GPS," *Acta Astronautica*, vol. 58, pp. 422–434, 2006.

[31] Ji, S. and Chen, W. and Zhao, C. and Ding, X. and Chen, Y., "Single Epoch Ambiguity Resolution for Galileo with the CAR and LAMBDA Methods," *GPS Solutions*, vol. 11, p. 259268, 2007.

[32] Santos, M.C. and Langley, R.B. and Leandro, R.F. and Pagiatakis, S. and Bisnath, S. and Santerre, R. and Cocard, M. and El-Rabbany, A. and Landry, R. and Dragert, H. and others, "Next-Generation Algorithms for Navigation, Geodesy and˜ Earth Sciences Under Modernized Global Navigation Satellite Systems (GNSS)," *Observing our Changing Earth*, pp. 817–824, 2008.

[33] O'Keefe, K. and Petovello, M. and Cao, W. and Lachapelle, G. and E. Guyader, "Comparing Multicarrier Ambiguity Resolution Methods for Geometry-Based GPS and Galileo Relative Positioning and Their Application to Low Earth Orbiting Satellite Attitude Determination," *International Journal of Navigation and Observation*, vol. vol. 2009, Article ID 592073, 2009.

[34] Teunissen, P.J.G. and Odijk, D., "Ambiguity dilution of precision: Definition, properties and application," *In Proceedings of ION GNSS*, pp. 891–899, 1997.

[35] Feng, Y. and Rizos, C., "Three carrier approaches for future global, regional and local GNSS positioning services: concepts and performance perspectives," in *Proceedings of the 18th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS 05), Long Beach, California, USA, September 2005*, pp. 2277–2287, 2005.

[36] Vollath, U., "The factorized multi-carrier ambiguity resolution (FAMCAR) approach for efficient carrier-phase ambiguity estimation," *Proceedings of ION GNSS 2004*, 2004.

[37] Jung, J., "High Integrity Carrier Phase Navigation for Future LAAS using Multiple Civilian GPS Signals," *Proceedings of ION GPS-99, The Institute of Navigation, Sept. 14-17*, pp. 14–17, 1999.

[38] Feng, Y. and Rizos, C., "Network-based geometry-free three carrier ambiguity resolution and phase bias calibration," *GPS solutions*, vol. 13, no. 1, pp. 43–56, 2009.

[39] Teunissen, P.J.G., "Least-squares estimation of the integer GPS ambiguities," *In: Section IV Theory and Methodology IAG General Meeting*, 1993.

[40] Teunissen, P.J.G. and Joosten, P. and Jonkman, N.F., "GNSS three carrier phase ambiguity resolution using the LAMBDA method," *Proc. GNSS 1999*, pp. 367–372, 1999.

[41] Euler, H. and Goad, C., "On optimal filtering of GPS dual frequency observations without using orbit information," *Bull Geod*, vol. 65, pp. 130–143, 1991.

[42] "GPS almanacs." http://www.navcen.uscg.gov/?pageName=gpsAlmanacs.

[43] *European GNSS (Galileo) open service: signal in space: interface control document.* Publications Office of European Union, http://ec.europa.eu/enterprise/policies/satnav/galileo/files/galileo-os-si 2010.

[44] de Jonge, P. and Tiberius, C.C.J.M., "The Lambda method for integer ambiguity estimation: implementation aspects," *LGR Publication No. 12*, pp. 1–49, 1996.

[45] Li, B., "The implementation of new LAMBDA software package: versatile options," *Technical document (internal), GNSS research group, Curtin University, Bentley campus, Australia*, 2010.

[46] Teunissen, P. J. G. and Kleusberg, A., *GPS for geodesy.* Springer, Berlin, 1998.

[47] Teunissen, P. J. G., "The ionosphere-weighted GPS baseline precision in canonical form," *Journal of Geodesy*, vol. 72, no. 2, pp. 107–111, 1998.

[48] Odijk, D. and Teunissen, P. J. G., "A theoretical study on the bottlenecks of GPS phase ambiguity resolution in a CORS RTK Network," *Journal of Geodetic Science*, vol. 1, no. 2, pp. 143–153, 2011.

[49] Odijk, D. and Teunissen, P. J. G. and L. Huisman, "First results of mixed GPS+GIOVE single-frequency RTK

in Australia," *Journal of Spatial Science*, vol. 57:1, 3-18, doi:http://dx.doi.org/10.1080/14498596.2012.679247, 2012.

[50] Bonhoure, B. and Boulanger, C. and Legenne, J., "GPS–GIOVE Mixed PVT Experimentation," *ION-GNSS-2009*, 2009.

[51] Hahn, J.H. and Powers, E.D., "Implementation of the GPS to Galileo time offset (GGTO)," in *Frequency Control Symposium and Exposition, 2005. Proceedings of the 2005 IEEE International*, pp. 5–pp, IEEE, 2005.

[52] Odijk, D. and Teunissen, P. J. G. and B. Zhang, "Single-frequency integer ambiguity resolution enabled precise point positioning," *Journal of Surveying Engineering*, vol. doi:http://dx.doi.org/10.1061/(ASCE)SU.1943-5428.0000085, 2012.

[53] Baarda, W., "S-transformations and criterion matrices," *Second Edition, Netherlands Geodetic Commission, Rijkscommissie voor Geodesie*, vol. 5, no. 1, 1981 http://www.ncg.knaw.nl/Publicaties/Geodesy/pdf/18Baarda.pdf.

[54] Teunissen, P.J.G., "Generalized inverses, adjustment, the datum problem and S-transformations," *Lecture notes International School of Geodesy, Erice, Italy, April*, 1984.

[55] Misra, P. and P. Enge, *Global Positioning System: Signals, Measurements, and Performance.* Ganga-Jamuna Press, P. O. Box 692, Lincoln, MA 01773, Second ed., 2006.

[56] Rao, C. R., "Linear statistical inference and its applications, 2nd Edition," *Wiley*, 1973.

[57] Odijk, D., *Fast Precise GPS Positioning in the Presence of Ionospheric Delays.* PhD thesis, Delft University of Technology, Delft, The Netherlands, 2003.

[58] Joosten, P. and Tiberius, C.C.J.M., "Lambda: FAQ's," *GPS Solutions*, vol. doi:10.1007/s10291-002-0004-8, 2002.

[59] Li, B. F. and Shen, Y. Z. and Xu, P. L., "Assessment of stochastic models for GPS measurements with different types of receivers," *Chinese Sci. Bull.*, vol. 53(20), pp. 3219–3225, 2008.