Analysis of the Polarization Properties of Dual Polarized Inverted Vee Dipole Antennas over a Ground Plane

Rene Baelemans*†, Adrian Sutinjo* and Peter Hall*
*International Centre for Radio Astronomy Research/
Curtin Institute of Radio Astronomy (ICRAR/CIRA)
Perth, Western Australia

Bart Smolders†
†Department of Electrical Engineering
Eindhoven University of Technology
Eindhoven, The Netherlands

***Abstract*—This paper presents the derivation of an explicit closed-form expression of dual-polarized inverted-vee dipole antenna behavior based upon electromagnetic theory and physical explanations. The expression is used to determine the intrinsic cross-polarization ratio (IXR) as function of the droop angle, position of the sky-vector, the height above a ground plane and frequency. The expression is verified using full-wave simulations with a Method-of-Moments solver, and shows excellent agreement with simulations. It explains the increase observed in IXR if an infinite perfect electric conductor ground plane is deployed.**

I. INTRODUCTION

A high polarization purity is necessary in the next generation low frequency radio telescopes to achieve important science goals. The dual-polarized inverted-vee dipole antenna, and antenna designs based upon this antenna, is the most elementary and commonly used design in phased array radio telescopes [1],[2],[3]. Several design reasons have been given as to why inverted-vee antennas should be used over normal flat dipole antennas but to the best of the authors' knowledge these inferences are all based upon simulation software, and a physical explanation of these phenomena is yet to be made [4]. It is however useful to understand in physical terms the effects of the droop angle, and other design parameters of inverted-vee antennas, on its polarization properties in terms of the intrinsic cross-polarization ratio (IXR). The IXR [5] is a fundamental figure-of-merit for assessing the polarimetric performance of antenna (systems). It provides an upper-bound estimate of the polarimetric error after full polarimetric calibration which is independent of the chosen coordinate system. The IXR is based upon the Jones matrix ($\boldsymbol{J}$) which describes the polarimetric response of a system through $\boldsymbol{V} = \boldsymbol{J}\boldsymbol{E}$, where $\boldsymbol{V}$ indicates the measured voltage vector and $\boldsymbol{E}$ the polarization vector of the source. Having determined the Jones matrix of a system, the IXR is calculated via

$$\mathrm{IXR}(\boldsymbol{J}) = \left(\frac{\mathrm{cond}(\boldsymbol{J})+1}{\mathrm{cond}(\boldsymbol{J})-1}\right)^2 \tag{1}$$

where $\mathrm{cond}(\boldsymbol{J})$ denotes the condition number of the Jones matrix. The condition number of the Jones matrix is a measure of the ability to determine the inverse Jones matrix and hence the ability to recover the polarization vector from the measured voltage vector, assuming a perfect characterization of the antenna. From the definition follows that a high IXR implies that a good recovery of the polarization is feasible.

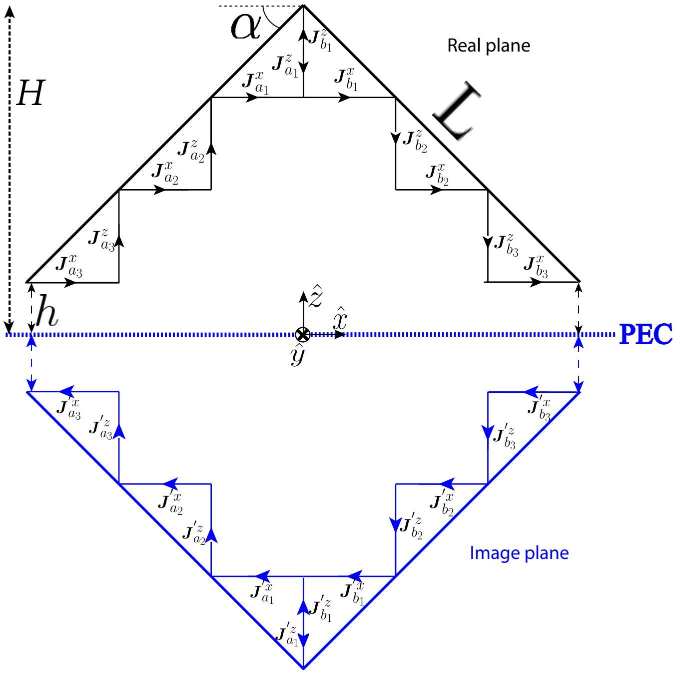

Fig. 1: One of the two pair of orthogonal legs of a dual-polarized inverted-vee dipole antenna modeled by a three-stage Hertzian dipole staircase approximation. In free space (black) and above a infinite PEC ground plane (black+blue).

II. ANALYSIS & RESULTS

Two cases are analyzed, the dual-polarized inverted-vee dipole antenna in free space, and the same antenna over an infinite PEC ground plane. The Jones matrix of the antenna in free space is derived by using a Hertzian dipole staircase approximation of the legs of the antenna, shown in Fig. 1 for one of the two orthogonal pair of legs. The coordinate system is chosen such that the Hertzian dipoles are directed in the three primary cartesian directions only. The image principle is applied to each Hertzian dipole for the analysis of the antenna in the infinite PEC ground plane case, creating an equivalent system of the original antenna over a perfect ground plane. The polarization properties of a source can be characterized by defining two orthogonal electric field vectors in the plane transverse to the propagation direction of the radiation. The response of each Hertzian dipole to these two electric field vectors is easily found by basic EM theory. The Jones matrix of a single stage, consisting of eight Hertzian dipoles in the free space case and sixteen Hertzian dipoles in the ground plane case, is derived from this response. The Jones matrix of the entire antenna is then found by taking the summation over the Jones matrices of each stage, assuming a sinusoidal current distribution over the legs of the antenna. Taking the limit to infinity for the number of stages reduces the approximation error. A closed form solution of the Jones matrix as a function of the droop angle of the antenna, position of the source in the sky and the ratio of the frequency of the radiation and the resonance frequency of the antenna, as well as the height above the ground plane, is hence derived. Calculation of the polarization properties of the antenna in terms of the IXR using 1 is now possible. The IXR of the dual-polarized inverted-vee antenna shows a strong dependence on the zenith angle and only a weak dependence on the azimuthal angle, with minima at $\phi = 0^o, 90^o, 180^o$ and 270^o. Fig. 2 shows the IXR as function of the zenith angle, where $\theta = 0^o$ is the zenith point, for three different droop angles, all at the resonance frequency. It was found that a flat dipole pair, droop angle of $\alpha = 0^o$, is independent to any ground effects when it comes to its polarization properties. Deploying an infinite PEC ground plane beneath the antenna does not only show an increase but even a secondary maximum in the IXR contour. Increasing the droop angle of the antenna, given by α, significantly improves the IXR. The results have been verified by full-wave simulations using a Method-of-Moments solver in FEKO [6], and show near perfect agreement with simulation results. The models have been used to understand and explain the polarization properties in terms of the IXR of two antenna designs (to be) used in phased-array radio-telescopes, the Low-Frequency Array Low Band Antenna (LOFAR-LBA) [1] and the Square Kilometre Array Log-periodic antenna (SKALA) [7].

III. Conclusion

Explicit closed-form expressions describing the polarization properties of dual-polarized inverted-vee dipole antennas in free space and above an infinite PEC ground plane have been derived. These expressions are verified to high accuracy using simulations. The closed-form expressions give a good understanding of the underlying physical phenomena which influence the IXR of inverted-vee antennas, such as the droop angle of the antenna, the height of the antenna, position of the sky-vector, and the frequency of the radiation. Using these closed-form expressions and their resulting IXR one can justify and understand the necessity of ground planes and large droop angles in radio astronomy applications based upon dual-polarized inverted-vee dipole antennas. The derived closed-form expressions can be used as a first design step for antennas, since they give both a better understanding and near-instant results relative to full wave EM simulations.

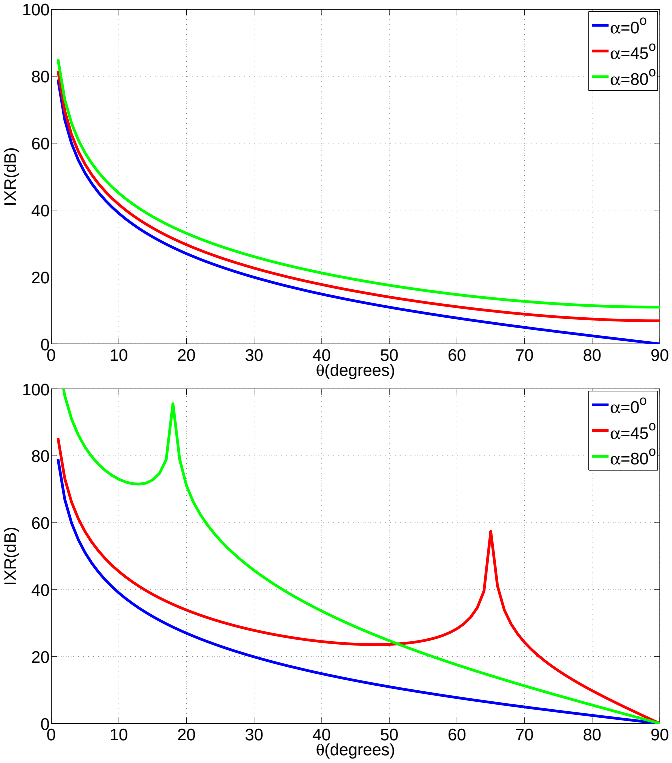

Fig. 2: IXR (dB) of a dual-polarized inverted-vee dipole antenna. As function of the zenith angle (θ) for an azimuthal angle of $\phi = 0^o$ and at the resonance frequency, for three different droop angles (α). Top graph: IXR in free space. Bottom graph: IXR over a infinite ground plane, antenna tip placed at a $\lambda/4$ above the ground plane.

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