Joint Source and Relay Matrices Optimization for Interference MIMO Relay Systems

Khoa Xuan Nguyen
Dept. Electrical & Computer Engineering
Curtin University
Bentley, WA 6102, Australia
Email: xuan.khoa.nguyen@curtin.edu.au

Yue Rong
Dept. Electrical & Computer Engineering
Curtin University
Bentley, WA 6102, Australia
Email: y.rong@curtin.edu.au

***Abstract*—In this paper, we investigate the transceiver design for an amplify-and-forward interference multiple-input multiple-output (MIMO) relay communication system. The minimum mean-squared error (MMSE) of the signal waveform estimation is chosen as the design criterion to optimize the source, relay, and receiver matrices for interference suppression. An iterative algorithm is proposed to solve the nonconvex source, relay, and receiver optimization problem. Simulation results demonstrate that the proposed algorithm outperforms the existing technique in terms of both MSE and bit-error-rate.**

***Index Terms*—Interference channel, MIMO relay, MSE.**

I. Introduction

Relay aided multiple-input multiple-output (MIMO) communication technology has attracted great research interest recently [1]-[2]. By incorporating relay nodes in a MIMO system, the network coverage and reliability can be significantly improved. In a MIMO relay system, communication between source nodes and destination nodes can be assisted by single or multiple relays equipped with multiple antennas. The relays can either decode-and-forward (DF) or amplify-and-forward (AF) the relayed signals [3]. In the AF scheme, the received signals are simply amplified (including a possible linear transformation) through the relay precoding matrices before being forwarded to the destination nodes. Therefore, in general the AF strategy has lower complexity and shorter processing delay than the DF strategy.

For single-user two-hop MIMO communication systems with a single relay node, the optimal source and relay precoding matrices have been developed in [4]. For a single-user two-hop MIMO relay system with multiple parallel relay nodes, the design of relay precoding matrices has been studied in [5]. Recent progress on the optimization of AF MIMO relay systems has been summarized in the tutorial of [2].

For MIMO interference channel, the idea of interference alignment (IA) [6] was developed for interference suppression by arranging desired signal and interference into appropriated signal space. The idea of IA has been applied in interference MIMO relay system in [7]-[8]. However, there is still no general solution for IA as a number of conditions must be met. One main reason is that the number of dimensions required for IA is very large and it depends on the number of independent fading channels. This leads to high computational complexity and infeasibility in practical systems. In [9], an iterative algorithm has been proposed to optimize the source beamforming vector and the relay precoding matrices to maximize the signal-to-interference-noise (SINR) at the destination nodes.

In this paper, we investigate the transceiver design for an AF interference MIMO relay communication system where multiple source nodes transmit information simultaneously to the destination nodes with the aid of multiple relay nodes, and each node is equipped with multiple antennas. We aim at optimizing the source, relay, and receiver matrices to suppress the interference and minimize the mean-squared error (MSE) of the signal waveform estimation at the destination nodes, subjecting to transmission power conditions at source and relay nodes. Since the original optimization problem is nonconvex with matrix variables, we propose an iterative algorithm. In each iteration of the proposed algorithm, we first optimize all relay matrices based on the source and receiver matrices from the previous iteration. Then we optimize all source matrices using the relay matrices in this iteration and the receiver matrices from the previous iteration. Finally, the receiver matrices are updated. Simulation results demonstrate that the proposed algorithm outperforms the existing technique in terms of both MSE and bit-error-rate.

Throughout this paper, scalars are denoted with lower or upper case normal letters, vectors are denoted with bold-faced lower case letters, and matrices are denoted with bold-faced upper case letters. Superscripts T, H, and -1 denote transpose, conjugate transpose and inverse, respectively, $tr()$ stands for the trace of a matrix, $vec()$ stacks columns of a matrix on top of each other into a single vector, $bd()$ denotes a block-diagonal matrix, $\otimes$ represents the Kronecker product, $E[\,]$ denotes the statistical expectation, and $\mathbf{I}_n$ stands for the $n \times n$ identity matrix.

II. System Model And Problem Formulation

We consider a two-hop interference MIMO relay communication system where K source-destination pairs communicate simultaneously with the aid of a network of L distributed relay nodes as shown in Fig.1. Similar to [9], we ignore the direct links between source and destination nodes as they undergo much larger path attenuation compared with the links via relays. The kth source node and the kth destination node are

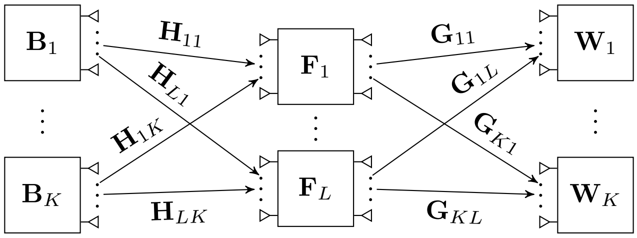

Fig. 1: Block diagram of an interference MIMO relay system.

equipped with N_{sk} and N_{dk} antennas, respectively, and the number of antennas at the lth relay node is N_{rl}.

Using half duplex relay nodes, the communication between source and destination pairs is completed in two time slots. At the first time slot, each source node transmits an $N_{sk} \times 1$ signal vector

$$\mathbf{x}_{sk} = \mathbf{B}_k \mathbf{s}_k, \quad k = 1, \cdots, K \tag{1}$$

to the relay nodes, where $\mathbf{s}_k$ is the $d \times 1$ information-carrying symbol vector and $\mathbf{B}_k$ is the $N_{sk} \times d$ source precoding matrix. The received signal vector at the lth relay node is given by

$$\mathbf{y}_{rl} = \sum_{k=1}^{K} \mathbf{H}_{lk} \mathbf{x}_{sk} + \mathbf{v}_{rl}, \quad l = 1, \cdots, L \tag{2}$$

where $\mathbf{H}_{lk}$ is the $N_{rl} \times N_{sk}$ MIMO channel matrix between the kth source node and the lth relay node and $\mathbf{v}_{rl}$ is the $N_{rl} \times 1$ additive white Gaussian noise (AWGN) vector at the lth relay node with zero mean and covariance matrix $E[\mathbf{v}_{rl}\mathbf{v}_{rl}^H] = \sigma_{rl}^2 \mathbf{I}_{N_{rl}}$, $l = 1, \cdots, L$.

The received signal vector at the lth relay node is amplified with the $N_{rl} \times N_{rl}$ precoding matrix $\mathbf{F}_l$ as

$$\mathbf{x}_{rl} = \mathbf{F}_l \mathbf{y}_{rl}, \quad l = 1, \cdots, L. \tag{3}$$

The precoded signal vector $\mathbf{x}_{rl}$ is forwarded to the destination nodes. The received signal vector at the kth destination node is given by

$$\mathbf{y}_{dk} = \sum_{l=1}^{L} \mathbf{G}_{kl} \mathbf{F}_l \mathbf{y}_{rl} + \mathbf{v}_{dk}, \quad k = 1, \cdots, K \tag{4}$$

where $\mathbf{G}_{kl}$ is the $N_{dk} \times N_{rl}$ MIMO channel matrix between the lth relay node and the kth destination node and $\mathbf{v}_{dk}$ is the $N_{dk} \times 1$ AWGN vector at the kth destination node with zero mean and covariance matrix $E[\mathbf{v}_{dk}\mathbf{v}_{dk}^H] = \sigma_{dk}^2 \mathbf{I}_{N_{dk}}$, $k = 1, \cdots, K$.

Due to their simplicity, linear receivers are used at the destination nodes to retrieve the transmitted signal. Thus, the estimated signal vector at the kth destination node can be written as

$$\hat{\mathbf{s}}_k = \mathbf{W}_k^H \mathbf{y}_{dk}, \quad k = 1, \cdots, K \tag{5}$$

where $\mathbf{W}_k$ is the $N_{dk} \times d$ receiver weight matrix. From (1)-(5), we have

$$\begin{aligned}\hat{\mathbf{s}}_k &= \mathbf{W}_k^H \left(\sum_{l=1}^{L} \mathbf{G}_{kl} \mathbf{F}_l \sum_{m=1}^{K} \mathbf{H}_{lm} \mathbf{B}_m \mathbf{s}_m + \bar{\mathbf{v}}_{dk} \right) \\ &= \underbrace{\mathbf{W}_k^H \sum_{l=1}^{L} \mathbf{G}_{kl} \mathbf{F}_l \mathbf{H}_{lk} \mathbf{B}_k \mathbf{s}_k}_{\text{desired signal}} \\ &\quad \underbrace{+ \mathbf{W}_k^H \sum_{l=1}^{L} \mathbf{G}_{kl} \mathbf{F}_l \sum_{m=1, m \neq k}^{K} \mathbf{H}_{lm} \mathbf{B}_m \mathbf{s}_m + \mathbf{W}_k^H \bar{\mathbf{v}}_{dk}}_{\text{interference plus noise}} \quad (6) \\ & \quad (7)\end{aligned}$$

where $\bar{\mathbf{v}}_{dk} \triangleq \sum_{l=1}^{L} \mathbf{G}_{kl} \mathbf{F}_l \mathbf{v}_{rl} + \mathbf{v}_{dk}$ is the total noise at the kth receiver.

From (1) and (3), the transmission power constraints at the source and relay nodes can be written as

$$tr\big(\mathbf{B}_k \mathbf{B}_k^H\big) \le P_{sk}, \quad k = 1, \cdots, K \tag{8}$$

$$tr\big(\mathbf{F}_l E\big[\mathbf{y}_{rl}\mathbf{y}_{rl}^H\big] \mathbf{F}_l^H\big) \le P_{rl}, \quad l = 1, \cdots, L \tag{9}$$

where P_{sk} and P_{rl} denote the power budget at the kth source node and the lth relay node, respectively, and $E\big[\mathbf{y}_{rl}\mathbf{y}_{rl}^H\big] = \sum_{m=1}^{K} \mathbf{H}_{lm}\mathbf{B}_m\mathbf{B}_m^H\mathbf{H}_{lm}^H + \sigma_{rl}^2 \mathbf{I}_{N_{rl}}$ is the covariance matrix of the received signal vector at the lth relay node.

In this paper, we aim at optimizing the source precoding matrices $\{\mathbf{B}_k\} \triangleq \{\mathbf{B}_k, k = 1, \cdots, K\}$, the relay precoding matrices $\{\mathbf{F}_l\} \triangleq \{\mathbf{F}_l, l = 1, \cdots, L\}$, and the receiver weight matrices $\{\mathbf{W}_k\} \triangleq \{\mathbf{W}_k, k = 1, \cdots, K\}$, to minimize the sum-MSE of the signal waveform estimation at the destination nodes under transmission power constraints at the source and relay nodes. We would like to mention that minimal MSE (MMSE) is a sensible design criterion based on the links of MSE to other performance measures in MIMO systems such as mutual information and SINR [4], [10].

From (7), the MSE of the kth source-destination pair can be calculated as

$$\begin{aligned}\mathrm{MSE}_k &= tr\Big(E\Big[(\hat{\mathbf{s}}_k - \mathbf{s}_k)(\hat{\mathbf{s}}_k - \mathbf{s}_k)^H\Big]\Big) \\ &= tr\Big((\mathbf{W}_k^H \tilde{\mathbf{H}}_k - \mathbf{I}_d)(\mathbf{W}_k^H \tilde{\mathbf{H}}_k - \mathbf{I}_d)^H \\ &\quad + \mathbf{W}_k^H \mathbf{C}_k \mathbf{W}_k + \mathbf{W}_k^H \boldsymbol{\Xi}_k \mathbf{W}_k\Big), \ k = 1, \cdots, K \end{aligned} \tag{10}$$

where $\tilde{\mathbf{H}}_k$ is the equivalent MIMO channel matrix of the kth source-destination pair, $\mathbf{C}_k = E\big[\bar{\mathbf{v}}_{dk}\bar{\mathbf{v}}_{dk}^H\big]$ and $\boldsymbol{\Xi}_k$ are the covariance matrices of the equivalent noise and the interference at the kth pair, respectively. They are given respectively as

$$\tilde{\mathbf{H}}_k = \sum_{l=1}^{L} \mathbf{G}_{kl} \mathbf{F}_l \bar{\mathbf{H}}_{lk}, \quad k = 1, \cdots, K$$

$$\begin{aligned}\mathbf{C}_k &= E\left[\left(\sum_{l=1}^{L} \mathbf{G}_{kl}\mathbf{F}_l\mathbf{v}_{rl} + \mathbf{v}_{dk}\right)\left(\sum_{l=1}^{L} \mathbf{G}_{kl}\mathbf{F}_l\mathbf{v}_{rl} + \mathbf{v}_{dk}\right)^H\right] \\ &= \sum_{l=1}^{L} \sigma_{rl}^2 \mathbf{G}_{kl}\mathbf{F}_l\mathbf{F}_l^H\mathbf{G}_{kl}^H + \sigma_{dk}^2 \mathbf{I}_{N_{dk}}, \quad k = 1, \cdots, K\end{aligned}$$

$$\boldsymbol{\Xi}_k = \sum_{l=1}^{L}\sum_{n=1}^{L} \mathbf{G}_{kl}\mathbf{F}_l\boldsymbol{\Xi}_{k,l,n}\mathbf{F}_n^H\mathbf{G}_{kn}^H, \quad k = 1, \cdots, K$$

where $\bar{\mathbf{H}}_{lk} \triangleq \mathbf{H}_{lk}\mathbf{B}_k$ is the equivalent MIMO channel matrix between the kth source node and the lth relay node and $\boldsymbol{\Xi}_{k,l,n} \triangleq \sum_{m=1,m\neq k}^{K} \bar{\mathbf{H}}_{lm}\bar{\mathbf{H}}_{nm}^H$, $k = 1, \cdots, K$, $l, n = 1, \cdots, L$.

From (8)-(10), the optimal source, relay, and receiver matrices design problem can be written as

$$\min_{\{\mathbf{W}_k\},\{\mathbf{B}_k\},\{\mathbf{F}_l\}} \sum_{k=1}^{K} \mathrm{MSE}_k \tag{11}$$

$$s.t.\ tr\left(\mathbf{B}_k\mathbf{B}_k^H\right) \leq P_{sk},\ k=1,\cdots,K \tag{12}$$

$$tr\left(\mathbf{F}_l E[\mathbf{y}_{rl}\mathbf{y}_{rl}^H]\mathbf{F}_l^H\right) \leq P_{rl},\ l=1,\cdots,L. \tag{13}$$

III. Proposed Source, Relay, and Receiver Matrices Design Algorithm

The problem (11)-(13) is highly nonconvex with matrix variables, and a globally optimal solution is intractable to obtain. In this section, we propose an iterative algorithm to solve the problem (11)-(13) by optimizing $\{\mathbf{W}_k\}$, $\{\mathbf{B}_k\}$, and $\{\mathbf{F}_l\}$ in an alternating way.

In each iteration of this algorithm, we first optimize $\{\mathbf{W}_k\}$ based on $\{\mathbf{B}_k\}$ and $\{\mathbf{F}_l\}$ from the previous iteration. Then we optimize all relay matrices based on $\{\mathbf{W}_k\}$ from the current iteration and $\{\mathbf{B}_k\}$ from the previous iteration. Finally, we optimize all source matrices using $\{\mathbf{W}_k\}$ and $\{\mathbf{F}_l\}$ from the current iteration.

It can be seen from (10) the $\mathbf{W}_k$ only affects MSE_k. Thus, with given $\{\mathbf{F}_l\}$ and $\{\mathbf{B}_k\}$, the optimal linear receiver matrix which minimizes MSE_k in (10) is the well-known MMSE receiver [11] given by

$$\mathbf{W}_k = (\tilde{\mathbf{H}}_k\tilde{\mathbf{H}}_k^H + \mathbf{C}_k + \boldsymbol{\Xi}_k)^{-1}\tilde{\mathbf{H}}_k, \ \ k = 1, \cdots, K. \tag{14}$$

With given receiver matrices $\{\mathbf{W}_k\}$ and source precoding matrices $\{\mathbf{B}_k\}$, the sum-MSE $\mathrm{SMSE} = \sum_{k=1}^{K}\mathrm{MSE}_k$ can be rewritten as a function of $\{\mathbf{F}_l\}$ as

$$\begin{aligned}\mathrm{SMSE} = &\sum_{k=1}^{K} tr\Big(\Big(\sum_{l=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lk} - \mathbf{I}_d\Big)\Big(\sum_{l=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\bar{\mathbf{H}}_{lk} - \mathbf{I}_d\Big)^H \\ &+ \sum_{l=1}^{L}\sigma_{rl}^2\bar{\mathbf{G}}_{kl}\mathbf{F}_l\mathbf{F}_l^H\bar{\mathbf{G}}_{kl}^H + \sigma_{dk}^2\mathbf{W}_k^H\mathbf{W}_k \\ &+ \sum_{l=1}^{L}\sum_{n=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\boldsymbol{\Xi}_{k,l,n}\mathbf{F}_n^H\bar{\mathbf{G}}_{kn}^H\Big)\end{aligned} \tag{15}$$

where $\bar{\mathbf{G}}_{kl} \triangleq \mathbf{W}_k^H\mathbf{G}_{kl}$ is the equivalent MIMO channel matrix between the lth relay node and the kth destination node.

Using the identities of [12]

$$tr(\mathbf{A}^T\mathbf{B}) = (vec(\mathbf{A}))^T vec(\mathbf{B}) \tag{16}$$

$$tr(\mathbf{A}^H\mathbf{B}\mathbf{A}\mathbf{C}) = (vec(\mathbf{A}))^H(\mathbf{C}^T\otimes\mathbf{B})vec(\mathbf{A}) \tag{17}$$

$$vec(\mathbf{A}\mathbf{B}\mathbf{C}) = (\mathbf{C}^T\otimes\mathbf{A})vec(\mathbf{B}) \tag{18}$$

the SMSE (15) can be represented as a function of $\mathbf{f}_l \triangleq vec(\mathbf{F}_l)$, $l = 1, \cdots, L$, as

$$\begin{aligned}&\mathrm{SMSE} \\ &= \sum_{k=1}^{K}\Bigg[\Big(\sum_{l=1}^{L}\mathbf{O}_{kl}\mathbf{f}_l - vec(\mathbf{I}_d)\Big)^H\Big(\sum_{l=1}^{L}\mathbf{O}_{kl}\mathbf{f}_l - vec(\mathbf{I}_d)\Big) \\ &\quad + \sum_{l=1}^{L}\mathbf{f}_l^H\mathbf{Q}_{kl}\mathbf{f}_l + \sum_{l=1}^{L}\sum_{n=1}^{L}\mathbf{f}_n^H(\hat{\boldsymbol{\Xi}}_{k,l,n}^T\otimes\hat{\mathbf{G}}_{kn}^H\hat{\mathbf{G}}_{kl})\mathbf{f}_l\Bigg] + t_1\end{aligned} \tag{19}$$

where $t_1 \triangleq \sum_{k=1}^{K}\sigma_{dk}^2 tr(\mathbf{W}_k^H\mathbf{W}_k)$ is independent of $\mathbf{f}_l$, $l = 1, \cdots, L$, and for $k = 1, \cdots, K$, $l = 1, \cdots, L$

$$\mathbf{O}_{kl} = \bar{\mathbf{H}}_{lk}^T\otimes\bar{\mathbf{G}}_{kl}, \ \ \mathbf{Q}_{kl} = \sigma_{rl}^2\mathbf{I}_{N_{rl}}\otimes(\bar{\mathbf{G}}_{kl}^H\bar{\mathbf{G}}_{kl}). \tag{20}$$

For $k = 1, \cdots, K$, let us introduce

$$\begin{aligned}\mathbf{O}_k &= [\mathbf{O}_{k1},\ \mathbf{O}_{k2}, \cdots,\ \mathbf{O}_{kL}] \\ \mathbf{Q}_k &= bd\left(\mathbf{Q}_{k1},\ \mathbf{Q}_{k2},\ \cdots,\ \mathbf{Q}_{kL}\right) \\ \mathbf{U}_{k,ln} &= \boldsymbol{\Xi}_{k,l,n}^T\otimes(\bar{\mathbf{G}}_{kn}^H\bar{\mathbf{G}}_{kl}) \\ \mathbf{U}_k &= \begin{bmatrix}\mathbf{U}_{k,11} & \cdots & \mathbf{U}_{k,1L} \\ \vdots & \ddots & \vdots \\ \mathbf{U}_{k,L1} & \cdots & \mathbf{U}_{k,LL}\end{bmatrix}.\end{aligned}$$

Then the SMSE function (19) can be written as a function of $\mathbf{f} = [\mathbf{f}_1^T,\ \mathbf{f}_2^T,\ \cdots, \mathbf{f}_L^T]^T$ as

$$\begin{aligned}\psi_1(\mathbf{f}) = &\sum_{k=1}^{K}\Big[(\mathbf{O}_k\mathbf{f} - vec(\mathbf{I}_d))^H(\mathbf{O}_k\mathbf{f} - vec(\mathbf{I}_d)) \\ &+ \mathbf{f}^H\mathbf{Q}_k\mathbf{f} + \mathbf{f}^H\mathbf{U}_k\mathbf{f}\Big] + t_1.\end{aligned} \tag{21}$$

By introducing

$$\mathbf{D}_l = \left(\sum_{m=1}^{K}\mathbf{H}_{lm}\mathbf{B}_m\mathbf{B}_m^H\mathbf{H}_{lm}^H + \sigma_{rl}^2\mathbf{I}_{N_r}\right)^T\otimes\mathbf{I}_{N_{rl}},\ l = 1, \cdots, L$$

and $\bar{\mathbf{D}}_l = bd\left(\mathbf{D}_{l1},\ \mathbf{D}_{l2},\ \cdots, \mathbf{D}_{lL}\right)$, where $\mathbf{D}_{ll} = \mathbf{D}_l$ and $\mathbf{D}_{lj} = \mathbf{0}$, $l \neq j$, the relay transmit power constraint in (9) can be rewritten as

$$\mathbf{f}^H\bar{\mathbf{D}}_l\mathbf{f} \leq P_{rl}, \quad l = 1, \cdots, L. \tag{22}$$

From (21) and (22), the relay matrices optimization problem can be written as

$$\min_{\mathbf{f}}\ \psi_1(\mathbf{f}) \tag{23}$$

$$s.t.\ \mathbf{f}^H\bar{\mathbf{D}}_l\mathbf{f} \leq P_{rl},\ \ l = 1, \cdots, L. \tag{24}$$

The problem (23)-(24) is a quadratically constrained quadratic programming (QCQP) problem [13], which is a convex optimization problem and can be efficiently solved by the interior-point method [13]. The problem (23)-(24) can be solved by the CVX MATLAB toolbox for disciplined convex programming [14].

With given receiver matrices $\{\mathbf{W}_k\}$ and relay matrices $\{\mathbf{F}_l\}$, the sum-MSE can be rewritten as a function of $\{\mathbf{B}_k\}$ as

$$\begin{aligned}&\text{SMSE}\\&=\sum_{k=1}^{K}tr\Big(\Big(\sum_{l=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\mathbf{H}_{lk}\mathbf{B}_k-\mathbf{I}_d\Big)\Big(\sum_{l=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\mathbf{H}_{lk}\mathbf{B}_k-\mathbf{I}_d\Big)^H\\&\quad+\sum_{l=1}^{L}\sum_{n=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\sum_{m=1,m\neq k}^{K}\mathbf{H}_{lm}\mathbf{B}_m\mathbf{B}_m^H\mathbf{H}_{lm}^H\mathbf{F}_n^H\bar{\mathbf{G}}_{kl}^H\Big)+t_2 \quad (25)\end{aligned}$$

where $t_2 \triangleq \sum_{k=1}^{K} tr(\mathbf{W}_k^H\mathbf{C}_k\mathbf{W}_k)$ can be ignored in the optimization process as it is independent of $\{\mathbf{B}_k\}$.

Using the identities in (16)-(18), the SMSE function in (25) can be written as

$$\begin{aligned}&\text{SMSE}\\&=\sum_{k=1}^{K}\Big[(\mathbf{S}_k\mathbf{b}_k-vec(\mathbf{I}_d))^H(\mathbf{S}_k\mathbf{b}_k-vec(\mathbf{I}_d))\\&\quad+\sum_{m=1,m\neq k}^{K}\mathbf{b}_m^H\Big(\mathbf{I}_d\otimes\sum_{l=1}^{L}\sum_{n=1}^{L}\mathbf{H}_{nm}^H\mathbf{F}_n^H\bar{\mathbf{G}}_{kn}^H\bar{\mathbf{G}}_{kl}\mathbf{F}_l\mathbf{H}_{lm}\Big)\mathbf{b}_m\Big]+t_2\\&=\sum_{k=1}^{K}\Big[(\mathbf{S}_k\mathbf{b}_k-vec(\mathbf{I}_d))^H(\mathbf{S}_k\mathbf{b}_k-vec(\mathbf{I}_d))+\mathbf{b}_k^H\mathbf{T}_k\mathbf{b}_k\Big]+t_2 \quad (26)\end{aligned}$$

where for $k=1,\cdots,K$

$$\mathbf{S}_k \triangleq \mathbf{I}_d\otimes\sum_{l=1}^{L}\bar{\mathbf{G}}_{kl}\mathbf{F}_l\mathbf{H}_{lk}$$

$$\mathbf{T}_k \triangleq \mathbf{I}_d\otimes\sum_{m=1,m\neq k}^{K}\sum_{l=1}^{L}\sum_{n=1}^{L}\mathbf{H}_{nk}^H\mathbf{F}_n^H\bar{\mathbf{G}}_{mn}^H\bar{\mathbf{G}}_{ml}\mathbf{F}_l\mathbf{H}_{lk}.$$

By introducing $\mathbf{T} \triangleq bd(\mathbf{T}_1,\mathbf{T}_2,\cdots,\mathbf{T}_K)$ and $\bar{\mathbf{S}}_k \triangleq [\mathbf{S}_{k1},\mathbf{S}_{k2},\cdots,\mathbf{S}_{kK}]$, where $\mathbf{S}_{kk}=\mathbf{S}_k$ and $\mathbf{S}_{ki}=\mathbf{0}$, $i\neq k$, the SMSE function (26) can be written as a function of $\mathbf{b}=[\mathbf{b}_1^T,\ \mathbf{b}_2^T,\ \cdots,\mathbf{b}_K^T]^T$ as

$$\Phi_1(\mathbf{b})=\sum_{k=1}^{K}\Big(\bar{\mathbf{S}}_k\mathbf{b}-vec(\mathbf{I}_d)\Big)^H\Big(\bar{\mathbf{S}}_k\mathbf{b}-vec(\mathbf{I}_d)\Big)+\mathbf{b}^H\mathbf{T}\mathbf{b}+t_2. \quad (27)$$

Let us introduce $\mathbf{E}_{ij}=\mathbf{I}_d\otimes\big(\mathbf{H}_{ij}^H\mathbf{F}_i^H\mathbf{F}_i\mathbf{H}_{ij}\big)$, $\mathbf{E}_l=bd\left(\mathbf{E}_{l1},\mathbf{E}_{l2},\cdots,\mathbf{E}_{lK}\right)$, $\bar{\mathbf{E}}_i=bd\left(\bar{\mathbf{E}}_{i1},\bar{\mathbf{E}}_{i2},\cdots,\bar{\mathbf{E}}_{iK}\right)$, where $\bar{\mathbf{E}}_{ii}=\mathbf{I}_{dN_s}$ and $\bar{\mathbf{E}}_{ij}=\mathbf{0}, i\neq j$. The optimal $\mathbf{b}$ can be obtained by solving the following problem

$$\min_{\mathbf{b}}\ \Phi_1(\mathbf{b}) \quad (28)$$

$$s.t.\ \mathbf{b}^H\bar{\mathbf{E}}_m\mathbf{b}\leq P_{sm},\ \ m=1,\cdots,K \quad (29)$$

$$\mathbf{b}^H\mathbf{E}_l\mathbf{b}\leq P_{rl}-\sigma_{rl}^2tr(\mathbf{F}_l\mathbf{F}_l^H),\ \ l=1,\cdots,L. \quad (30)$$

The problem (28)-(30) is a QCQP problem and can be solved by the CVX MATLAB toolbox [14] for disciplined convex programming.

The steps of applying the proposed iterative algorithm to optimize $\{\mathbf{B}_k\}$, $\{\mathbf{F}_l\}$, and $\{\mathbf{W}_k\}$ are summarized in Table I, where the superscript (n) denotes the variable at the nth iteration, and ε is a small positive number up to which convergence is acceptable. Since all subproblems (11), (23)-(24), and (28)-(30) are convex, the solution to each subproblem is optimal. Thus, the value of the objective function (11) decreases after each iteration. Moreover, the objective function is lower bounded by at least zero. Therefore, the iterative algorithm converges to (at least) a locally optimal solution.

TABLE I: Procedure of solving the problem (11)-(13) by the Proposed Algorithm 1.

1) Initialize the algorithm with $\{\mathbf{F}_l^{(0)}\}$ and $\{\mathbf{B}_k^{(0)}\}$ satisfying (8) and (9); Set $n=0$.
2) Obtain $\{\mathbf{W}_k^{(n+1)}\}$ based on (14) with fixed $\{\mathbf{F}_l^{(n)}\}$ and $\{\mathbf{B}_k^{(n)}\}$.
3) Update $\{\mathbf{F}_l^{(n+1)}\}$ through solving the problem (23)-(24) with given $\{\mathbf{B}_k^{(n)}\}$ and $\{\mathbf{W}_k^{(n+1)}\}$.
4) Update $\{\mathbf{B}_k^{(n+1)}\}$ by solving the problem (28)-(30) with fixed $\{\mathbf{F}_l^{(n+1)}\}$ and $\{\mathbf{W}_k^{(n+1)}\}$.
5) If $\text{SMSE}^{(n)}-\text{SMSE}^{(n+1)}\leq\varepsilon$, then end. Otherwise, let $n:=n+1$ and go to Step 2.

IV. Numerical Examples

In this section, we illustrate the performance of the proposed algorithm through numerical simulations. All channel matrices have independent and identically distributed (i.i.d.) complex Gaussian entries with zero-mean and unit variance. The noises are i.i.d. Gaussian with zero mean and unit variance. The QPSK constellations are used to modulate the source symbols. For the sake of simplicity, we assume that all nodes have three antennas, i.e., $N_{sk}=N_{dk}=N_{rl}=3$, $k=1,\cdots,K$, $l=1,\cdots,L$, all source nodes have the same power budget as $P_{sk}=P_s=15$dB, $k=1,\cdots,K$, and all relay nodes have the same power budget as $P_{rl}=P$, $l=1,\cdots,L$. For all simulation examples, there are $K=3$ source-destination pairs, and the simulation results are averaged over 10^5 independent channel realizations. Unless explicitly mentioned, we assume that there are $L=5$ relay nodes in the interference MIMO relay system. As a benchmark, we compare the performance of the proposed algorithm with the naive AF algorithm with the source and relay precoding matrices are scaled identity matrices.

In the first example, we study the performance of the proposed algorithm at different number of iterations. Fig. 2 shows the BER performance of the proposed algorithm at different number of iterations for the first source-destination pair ($k=1$). It can be seen from Fig. 2 that the proposed algorithm yields a much smaller BER than the naive AF algorithm, since the source and relay precoding matrices are not optimized in the naive AF algorithm. It can also be observed from Fig. 2 that the system BER reduces with increasing number of iterations. During simulations, we observed that after 20 iterations, the decreasing of the SMSE objective function is negligible. Thus, we suggest that only 20 iterations are needed to achieve good performance.

For this example, the BER of each source-destination pair versus P at 10 iterations is shown in Fig. 3. It can be seen

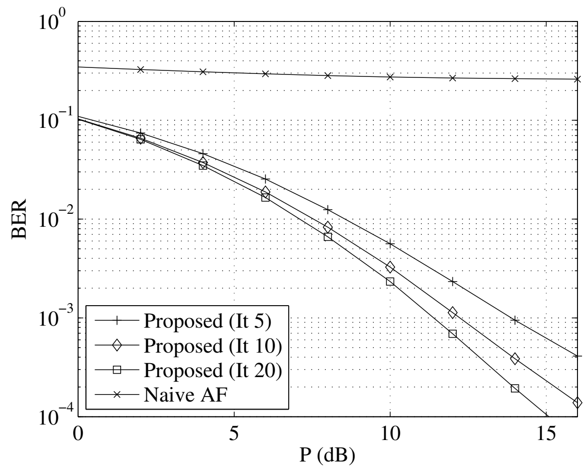

Fig. 2: Example 1: BER versus P at different number of iterations.

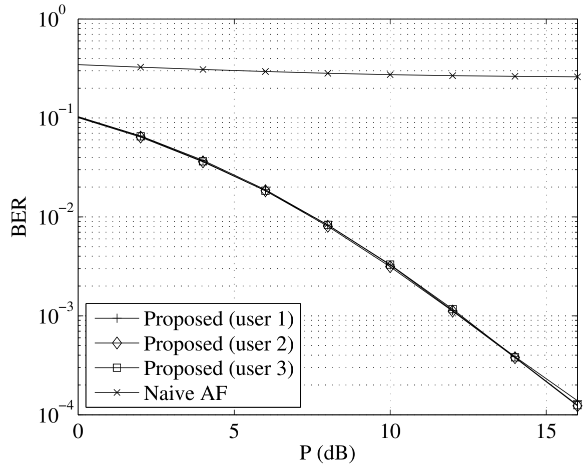

Fig. 3: Example 1: BER versus P for each source-destination pair.

that all three source-destination pairs achieve almost identical BER, indicating that the proposed algorithm is fair to all links.

In the second example, we study the performance of the proposed algorithm with different number of relay nodes. Fig. 4 shows the BER performance of the proposed algorithm at 10 iterations with $L = 5$ and $L = 10$. It can be seen that by increasing the number of relay nodes, the system spatial diversity is increased, and thus, a better BER performance is achieved. In particular, we observe that a 10dB gain is obtained at the BER of 10^{-3} by increasing L from 5 to 10.

V. Conclusion

We have investigated transceiver design for interference MIMO relay systems based on the MMSE criterion. An iterative algorithm has been developed to jointly optimize the source, relay, and receiver matrices under power constrains at each source node and relay node. Numerical simulation results show that this algorithm converges quickly after a few iterations and has better BER performance than the existing technique.

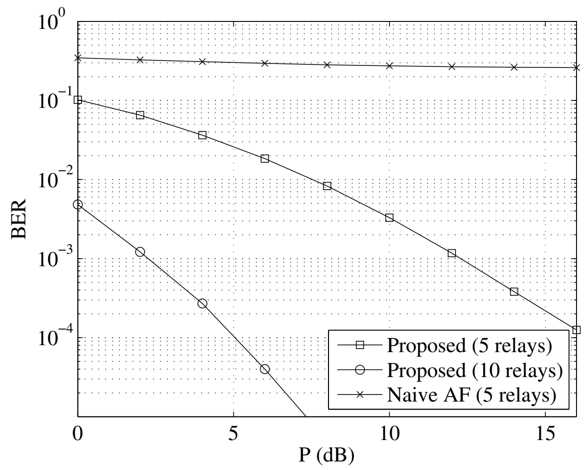

Fig. 4: Example 2: BER versus P for different L.

Acknowledgment

This research was supported under the Australian Research Council's Discovery Projects funding scheme (DP140102131).

References

[1] Y. Fan and J. Thompson, "MIMO configurations for relay channels: Theory and practice," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1774-1786, May 2007.

[2] L. Sanguinetti, A. A. D'Amico, and Y. Rong, "A tutorial on the optimization of amplify-and-forward MIMO relay systems," *IEEE J. Selet. Areas Commun.*, vol. 30, pp. 1331-1346, Sep. 2012.

[3] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, pp. 3037-3063, Sep. 2005.

[4] Y. Rong, X. Tang, and Y. Hua, "A unified framework for optimizing linear nonregenerative multicarrier MIMO relay communication systems," *IEEE Trans. Signal Process.*, vol. 57, pp. 4837-4851, Dec. 2009.

[5] C. Zhao and B. Champagne, "Joint design of multiple non-regenerative MIMO relaying matrices with power constraints," *IEEE Trans. Signal Process.*, vol. 61, pp. 4861-4873, Oct. 2013.

[6] M. Maddah-Ali, A. Motahari, and A. Khandani, "Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis," *IEEE Trans. Inf. Theory*, vol. 54, pp. 3457-3470, Aug. 2008.

[7] B. Nourani, S. Motahari, and A. Khandani, "Relay-aided interference alignment for the quasi-static X channel," in *Proc. IEEE Int. Symposium Inf. Theory*, Seoul, Korea, Jun. 28-Jul. 3, 2009, pp. 1764-1768.

[8] X. Wang, Y.-P. Zhang, P. Zhang, and X. Ren, "Relay-aided interference alignment for MIMO cellular networks," in *Proc. IEEE Int. Symposium Inf. Theory*, Cambridge, MA, Jul. 1-6, 2012, pp. 2641-2645.

[9] M. Khandaker and Y. Rong, "Interference MIMO relay channel: Joint power control and transceiver-relay beamforming," *IEEE Trans. Signal Process.* vol. 60, pp. 6509-6518, Dec. 2012.

[10] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: A unified framework for convex optimization," *IEEE Trans. Signal Process.*, vol. 51, pp. 2381-2401, Sep. 2003.

[11] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Englewood Cilffs, NJ: Prentice Hall, 1993.

[12] J. W. Brewer, "Kronecker products and matrix calculus in system theory," *IEEE Trans. Circuits Syst.*, vol. 25, pp. 772-781, Sep. 1978.

[13] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U. K.: Cambridge University Press, 2004.

[14] M. Grant and S. Boyd, "Cvx: Matlab software for disciplined convex programming (webpage and software)," [Online]. Available: http://cvxr.com/cvx, Apr. 2010.

[15] Y. Nesterov and A. Nemirovski, *Interior Point Polynomial Algorithms in Convex Programming*. Philadelphia, PA: SIAM, 1994.