A novel approach to fireball modeling: The observable and the calculated

Eleanor Kate SANSOM1*, Philip BLAND1, Jonathan PAXMAN2, and Martin TOWNER1

1Department of Applied Geology, Curtin University, GPO Box U1987, Perth, Western Australia 6845, Australia
2Department of Mechanical Engineering, Curtin University, GPO Box U1987, Perth, Western Australia 6845, Australia
*Corresponding author. E-mail: eleanor.sansom@postgrad.curtin.edu.au
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Abstract—Estimating the mass of a meteoroid passing through the Earth’s atmosphere is essential to determining potential meteorite fall positions. High-resolution fireball images from dedicated camera networks provide the position and timing for fireball bright flight trajectories. There are two established mass determination methods: the photometric and the dynamic. A new approach is proposed, based on the dynamic method. A dynamic optimization initially constrains unknown meteoroid characteristics which are then used in a parametric model for an extended Kalman filter. The extended Kalman filter estimates the position, velocity, and mass of the meteoroid body throughout its flight, and quantitatively models uncertainties. Uncertainties have not previously been modeled so explicitly and are essential for determining fall distributions for potential meteorites. This two-step method aims to automate the process of mass determination for application to any trajectory data set and has been applied to observations of the Bunburra Rockhole fireball. The new method naturally handles noisy raw data. Initial and terminal bright flight mass results are consistent with other works based on the established photometric method and cosmic ray analysis. A full analysis of fragmentation and the variability in the heat-transfer coefficient will be explored in future versions of the model.

INTRODUCTION

The full potential of meteorite analysis for providing valuable insights about protoplanetary disk formation cannot be reached without first constraining their origins in the solar system. As with terrestrial rocks, without context (outcrop) information, our understanding of the record that meteorites contain will only ever be partial. The recording of fireball phenomena permits the reconstruction of orbits, as well as determines possible meteorite fall locations to enable the recovery of fresh meteorites whose unique geological record can be fully exploited. This objective has been the driver for a number of dedicated fireball camera network projects dating back to the late 1950s (Ceplecha 1961) and has led to the recovery of multiple meteorites, including two by the Desert Fireball Network (DFN) in Australia during its trial phase (Towner et al. 2011; Spurný et al. 2012).

Over the next few months, the DFN will establish over 50 new Automated Desert Fireball Observatories (ADFOs), with all sky digital cameras, to expand its coverage to an area in excess of 2 million km². This will make it the largest fireball network in history, and with >100 TB of data being generated per year, automated systems of data analysis will be needed. The calculation of terminal bright flight mass will form part of the DFN’s automated work-flow from fireball detection and triangulation through to dark flight and climate modeling for fall calculations.

Once the light of the fireball goes out, there is usually no way of tracking any remaining fragments to the ground. To model this dark flight, and determine any potential fall positions, the terminal bright flight mass must be ascertained. An automated method of analyzing the bright flight data to extract this information is required and previous methods were investigated for suitability. The two previous approaches to analyzing image data for mass determination are: the photometric method and the dynamic method.

The photometric method relates the luminosity of a fireball to the proportion of kinetic energy that is lost...
due to ablation, as a method for obtaining masses (Ceplecha et al. 1998). It uses the luminosity of the fireball to determine the incoming “photometric” mass, and a corresponding luminous efficiency parameter as a proxy for mass loss. To apply this method, a high-resolution light curve of a fireball needs to be acquired. This can be obtained by the addition of a photoelectric photometer to a fireball observatory (Spurný et al. 2012). Not only is this an expensive piece of equipment in itself but also requires additional power supplies, which are limited in the remote locations of the DFN observatories.

Although advancements have been made to the photometric method, including fragmentation as well as dynamical aspects (Ceplecha and ReVelle 2005), it ultimately still requires qualitative comparisons of trajectories with the light curve and manual inputs of fragmentation information (Ceplecha and ReVelle 2005). These qualitative judgments make this method manually intensive and remove the ability to create fully reproducible data.

The dynamic method uses equations of flight through the atmosphere to calculate mass from deceleration (Whipple 1952). In the past, this approach was limited by the accuracy of measurements that could be interpreted from photographic plates (Ceplecha 1961; McCrosky et al. 1971). Ceplecha et al. (1993) used dynamic equations to determine the change in velocity and mass of a meteoroid during its trajectory, along with timings of single fragmentation events. However, the authors were unable to calculate initial masses and therefore relied on initial photometric masses. Considering mass loss is relative, this means the terminal mass is based on this photometric entry mass which may be unreliable (Brykina and Stulov 2012).

Difficulties with the dynamic method are also due to the unknown characteristics of the meteoroid such as density and shape that are required for the dynamic calculation. Work by Stulov et al. (1995) has enabled the application of an analytical solution by combining these unknown parameters into two dimensionless constants. This has been applied by Gritsevich (2008a, 2008b) to the Canadian MORP network data sets, as well as others that have led to meteorite recoveries. This provides good model fits to the data to which it was applied, but assumptions of these same meteoroid characteristics are required to quantify entry mass and subsequently terminal bright flight mass.

Given the limitations of established techniques and improvements to observation technologies, we chose to explore a new approach to the dynamic method. The use of an extended Kalman filter to incorporate the data into the model and provide error estimates was determined to be the most promising approach. An extended Kalman filter is a method of statistically optimizing estimates of an instantaneous state of nonlinear dynamic systems (Grewal and Andrews 1993). An accompanying covariance matrix allows the uncertainties in the state estimations to be determined and propagated. The Kalman filter estimates the bright flight states (distance traveled, mass, and velocity) based on a two-step process of “predict” and “update.” However, this method still requires values for meteoroid parameters to be estimated. To maximize confidence in chosen meteoroid parameters, rather than simply picking values, the Extended Kalman Filter is preceded by a dynamic optimization step. This stage is implemented to constrain the combinations of meteoroid characteristics that will permit a fit to the data. These parameters are then used to initialize a series of extended Kalman filters. To test the new method of mass determination, the data set of the Bunburra Rockhole meteorite fall is used as published by Spurný et al. (2012). This is the most complete fireball data set for which a meteorite has been recovered.

The objective of an automated method of mass determination requires an efficient method that will give sufficiently accurate results to determine a practical search area for likely meteorites. As this new approach is based entirely on the photographic data, this significantly reduces the cost of each ADFO unit as there is no requirement for a photoelectric photometer. The new approach to fireball modeling that we outline here will enable the terminal bright flight mass to be approximated from observable data in a fully automatable method, with uncertainties, to enable rapid recovery of meteorite samples which may provide invaluable data for cosmochemists (particularly when combined with orbital data).

**MODELING**

In the case of the DFN, ADFOs record high-resolution images throughout the night. Fireball observations made by multiple long-exposure cameras can be used to triangulate the position (latitude, longitude, and altitude) of the meteoroid during its flight. To acquire velocity information, however, requires some specialized modifications. Using a customized shutter within the camera lenses, the light path is interrupted at a known frequency (approximately 20 Hz in the ADFO systems). After calibration to remove the effects of lens distortion and triangulation, we have a series of position observations which underpins the subsequent modeling. Velocity may be calculated based on the change in these positions with time. The accuracy of the position observations
determines the accuracy of the velocity values and can cause high scatter in values as seen in the Bunburra Rockhole data set.

All models explored in this work are based on the dynamic equations that characterize the change in mass and velocity of a meteoroid during bright flight through the atmosphere (Baldwin and Sheaffer 1971):

\[
\frac{dv}{dt} = -\frac{1}{2} \frac{c_d \rho_a v^2 S}{m} + g \sin \gamma_e
\]

where \( m \) is the meteoroid mass (kg), \( v \) is the velocity (m s\(^{-1}\)), \( t \) is the time (s), \( c_d \) is the drag coefficient, \( \rho_a \) is the atmospheric density (kg m\(^{-3}\)), \( S \) is the cross sectional area of the body (m\(^2\)), \( g \) is the gravitational constant (m s\(^{-2}\)), \( \gamma_e \) is the entry angle of the meteoroid to the horizontal, \( H^* \) is the enthalpy of sublimation (J kg\(^{-1}\)), and \( c_h \) is the heat-transfer coefficient.

The position or length along the path of the trajectory, \( l \), is the primary observation extracted from the triangulated images. Its change with time is also included in all models and gives the velocity, i.e., \( \frac{dl}{dt} = v \).

A New Approach

The new approach to determining the terminal masses of meteoroids discussed in this paper is a two-step approach, based on the dynamic Equations 1–2. The initial step is a dynamic optimization which runs a global search for the combination of meteoroid characteristics (model parameters) and unknown initial states (initial mass, \( m_0 \) and initial velocity, \( v_0 \)) that provide a good fit to the observational data. The initial position, \( l_0 \), is also an initial state but as the length along the flight path is relative, we can set it to be 0 m (similar to Cephecha and ReVelle 2005). Errors associated with observational uncertainties in this postulation will be taken into account when the extended Kalman filter is initialized.

The second, main step, runs an extended Kalman filter which uses the unknown initial states and parameters from the dynamic optimization to estimate the states (position, \( l \); mass, \( m \); velocity, \( v \)) throughout the entire trajectory, including an explicit uncertainty model.

The cross sectional area, \( S \), in the dynamic Equations 1–2, is dependent on the amount of mass lost due to ablation and may be defined as a function of the mass, meteoroid density, \( \rho_m \), and shape parameter, \( A \) (a cross sectional area to volume ratio) (equation 3.5; Bronshten 1983)

\[
S = A \left( \frac{m}{\rho_m} \right)^{\frac{2}{3}}
\]  

The change in cross sectional area can be written in terms of the shape change parameter, \( \mu \) (Equation 4) (Bronshten 1983).

\[
S = S_0 \left( \frac{m}{m_0} \right)^{\mu}
\]

\( S_0 \) and \( m_0 \) are the initial cross sectional area and initial mass respectively.

By writing Equation 3 in terms of initial parameters only we can combine it with Equation 4 to give

\[
S = A_0 \frac{m_0}{\rho_m} \left[ \frac{2}{3} - \mu \right] m^\mu
\]

Substituting Equation 5 into Equations 1–2 allows the dependent variable \( S \) to be removed from the dynamic equations. The modeling of meteoroid states during bright flight will therefore be based on the following differential equations.

\[
\frac{dv}{dt} = -\frac{1}{2} \frac{c_d \rho_a A_0}{\rho_m} \frac{2}{3} \frac{m_0}{m} \left( \frac{2}{3} - \mu \right) v^2 m^{(\mu - 1)} + g \sin \gamma_e
\]

\[
\frac{dm}{dt} = -\frac{1}{2} \frac{c_h \rho_a A_0}{H^* \rho_m} \frac{2}{3} \frac{m_0}{m} \left( \frac{2}{3} - \mu \right) v^2 m^{\mu}
\]

Constants Used in All Model Stages

Although the unknown parameters \( \mu \) and \( \frac{d\mu}{dt} \) in Equations 6–7 are variable, they are approximated as constant for both the dynamic optimization and EKF models, along with the remaining unknown initial parameters, \( m_0 \), \( v_0 \), and \( \frac{d\mu}{dt} \) (which will hereby be referred to as the shape-density parameter). This has been the typical assumption in previous works also (Bronshten 1983; Gritsevich 2008b).

The shape change parameter, \( \mu \), has a range from 0, being no rotation, to 2/3, indicating that rotation is rapid enough for uniform ablation to occur across the entire surface area. It is typically assumed that \( \mu \) has a value of 2/3 (Bronshten 1983) and as the dynamic equations are highly sensitive to the value of \( \mu \), this
value is also used in our current model and will not be optimized further at this stage. Note that this removes \( m_0 \) as a coefficient from Equations 6–7, although \( m_0 \) is still present in the optimization as the initial value for mass.

**Atmospheric Properties**

The NRLMSISE-00 empirical atmosphere model was used to calculate values of atmospheric densities and pressures (Picone et al. 2002). This enables values for temperature, pressure, density, speed of sound, and dynamic viscosity of the atmosphere to be determined as accurately as possible.

**Drag Coefficient**

The drag coefficient, \( c_d \), can be calculated throughout the trajectory based on a set of fluid dynamic parameters. ReVelle (1976) discusses the dependence of the Reynolds number and flow regime on the drag coefficient, but does not include a criterion for when the Mach regime is no longer hypersonic. This is unlikely to happen during fireball phenomena but is included here for completeness.

The Knudsen number (\( Kn \)) (Equation 8) can be used to determine the flow regime of the flight path and is the ratio of the mean free path length to the object length. \( Kn \) may be written as a function of the calculable Mach (\( Ma \)) and Reynolds (\( Re \)) numbers (Hayes and Probstein 1959; Truitt 1959) and the ratio of specific heats \( \gamma \), which for dry air at atmospheric temperatures is taken to be 1.40.

\[
Kn = \frac{Ma}{Re} \times \sqrt{\frac{\gamma \pi}{2}} \tag{8}
\]

Values of \( Kn \geq 10 \) indicate free molecular flow, \( 10 < Kn < 0.1 \) a transitional flow regime, and \( Kn \leq 0.1 \) continuum flow (ReVelle 1976). Within the continuum flow regime, the Mach regime defined by the Mach number needs to be taken into consideration. Only when below a \( Ma \) of 1.1 is \( Re \) used to directly calculate the drag coefficient. For values below the critical \( Re \) associated with drag reattachment (\( Re \sim 2e5 \)) (Schlichting et al. 2000), Equation 11 from Haider and Levenspiel (1989) is used, although it is expected that bright flight values of \( c_d \) will remain in the hypersonic regime. Determining the values of \( c_d \) for different regimes and turbulence are outlined in Table 1.

For the Bunburra Rockhole data set, the meteoroid remains in the hypersonic regime for the duration of bright flight. In this version of the model, for simplicity, we will assume a hypersonic drag coefficient corresponding to that of a circular cylinder.

**Dynamic Optimization**

The dynamic optimization based on Equations 6–7 aims to approximate values for \( \frac{\Delta m}{m_0} \) and \( \frac{\Delta p}{p_0} \), as well as an entry mass, \( m_0 \), and velocity, \( v_0 \). This is performed by assigning assumed values to these parameters within given ranges and the constrained optimization then searches millions of combinations to determine the set of parameters that best fit the position data and return the lowest cost. The cost function used is the sum of the squared errors between the modeled and the observed position data. Costs are normalized to the lowest value, showing 1.0 to be the best fit, to allow comparisons between different parameter sets. As there are multiple unknown parameters, there is a large degree of freedom in the number of plausible combinations. The models that produce cost values $>0.98$ (best 2%) are selected for consideration in the following stage of the mass determination method.

The parameter constraints used are shown in Table 2. Ranges for \( P_{m_0} \) are given as assumed preatmospheric meteorite density ranges for typical meteorites. \( A_{sphere} = 1.21 \) although it is expected that \( A \) values should typically be in the range of 2–4 (Zhdan et al. 2007). The shape parameter may also be less than that of a sphere depending on which axis is oriented in the direction of the trajectory. The lower and upper bounds for \( A_0 \) are chosen as realistic ranges. \( \frac{\Delta p}{p_0} \) is given a wide range so that the average value of this variable throughout bright flight is determined.

**Extended Kalman Filter**

An extended Kalman filter (EKF) is a method of statistically optimizing estimates of state variables for nonlinear dynamic systems (Grewal and Andrews 1993). For bolide bright flight path analysis, the state vector, \( x_k \), is the instantaneous representation of the state at a time \( k \), and is written in terms of the variables’ distance along the bright flight path (\( l \)), mass (\( m \)), and velocity (\( v \)) (Equation 9).

\[
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}_k = \begin{pmatrix} l_k \\ m_k \\ v_k \end{pmatrix} \tag{9}
\]

The state vector at \( t_0 \) is initialized as

\[
x_0 = \begin{pmatrix} 0 \\ m_0 \\ v_0 \end{pmatrix} \tag{10}
\]

\( x_k \) can be determined using the nonlinear state equations:
Predicting Future States

The prediction step uses all previous data to derive a suitable state estimate, $\hat{x}_k$: 

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1}|k-1, k)$$

(13)

$$\dot{P}_{k|k-1} = F_k P_{k|k-1} F_k^T + Q_k$$

(14)

$F_k$ is the state transition matrix (Equation 15). $P_k$ is the covariance matrix for the state estimate and is a primary motivation for using an EKF. The diagonal elements can be read to give an indication of the variance for distance, mass, and velocity.

$$F_k = \begin{bmatrix}
\frac{\partial f}{\partial \hat{x}} & \frac{\partial f}{\partial \dot{\hat{x}}} & \frac{\partial f}{\partial \ddot{\hat{x}}} \\
\frac{\partial f}{\partial \dot{\hat{x}}} & \frac{\partial f}{\partial \ddot{\hat{x}}} & \frac{\partial f}{\partial \hat{v}} \\
\frac{\partial f}{\partial \ddot{\hat{x}}} & \frac{\partial f}{\partial \hat{v}} & \frac{\partial f}{\partial \dot{\hat{v}}} \\
\end{bmatrix} \bigg|_{\hat{x}=\hat{x}_k}$$

(15)

where:

$$l_{k+1} = f_{l}(l_k, m_k, v_k, t_k) = \dot{l}_k + \frac{dl_k}{dt} \Delta t = l_k + v_k \Delta t$$

(16)

$$m_{k+1} = f_{m}(l_k, m_k, v_k, t_k) = m_k + \frac{dm_k}{dt} \Delta t$$

(17)

$$v_{k+1} = f_{v}(l_k, m_k, v_k, t_k) = v_k + \frac{dv_k}{dt} \Delta t$$

(18)

And:

$$k_v = \frac{1}{2} c_d \rho_a A_0 \left( \frac{2}{3} \gamma_v \right) , k_m = \frac{1}{2} c_d \rho_a A_0 \left( \frac{2}{3} \gamma_m \right)$$

(19)

where $c_d$ is the drag coefficient for spheres (Bronshten 1983; ReVelle 1976; Masson et al. 1960), $c_d = 4 \pi^2 / 3$ for circular cylinders (Truit 1959), $c_d = 2 \pi / 3$ for tiles and bricks (Zhdan et al. 2007), and $A$ is the cross-sectional area of the object (see Haider and Levenspiel 1989).
Measurement Updates

The measurement update step follows an observation $z_k$ (Equation 20), which for ADFO observations is only the distance data $l_k = (x_1)_k$. $H_k$ provides a relationship between the state of the dynamic system and the measureable observations, simply put, $H_k x_k = l_k$. $n_k$ is the measurement noise with a mean of zero and covariance $R_k$ (Equation 21). $R_k$, therefore, accounts for errors between measured position and true position due to aspects such as camera calibration, triangulation, camera resolution etc.

\[
\begin{align*}
  z_k &= H_k x_k + n_k \\
  R_k &= E[n_k n_k'] = \sigma_{zz}^2 
\end{align*}
\]  

The predicted measurement can be made using the output of Equation 13

\[
\hat{z}_k = H_k \hat{x}_{k|k-1}
\]

The residual difference between $z_k$ and $\hat{z}_k$ is $y_k$ (Equation 23). $S_k$ (Equation 24) projects the system uncertainty into the measurement space and includes uncertainties in the model up to $t_{k-1}$, as well as the noise covariance of the current measurement. The optimum Kalman gain, $K_k$ (Equation 25) is used to update the state ($\hat{x}_k$) and covariance matrices ($P_k$) (Equations 26–27)

\[
\begin{align*}
  y_k &= z_k - \hat{z}_k \\
  S_k &= H_k P_{k|k-1} H_k^T + R_k \\
  K_k &= P_{k|k-1} H_k^T S_k^{-1} \\
  \hat{x}_k &= \hat{x}_{k|k-1} + K_k y_k \\
  P_k &= (I - K_k H_k) P_{k|k-1}
\end{align*}
\]

The square root of the diagonal elements of $P_k$ is plotted as error bars so that the evolution of state uncertainty with time can be visualized in a meaningful way.

Fireball Applications

For the nonlinear bolide dynamical equations, $\hat{x}_{k+1|k}$ is calculated by solving the nonlinear
Equations 6–7 between $t_k+1$ and $t_k$. $P_k$, however, is solved using the linearized state transition matrix, $F_k$ (Equation 28). The linearization of $F_k$ approximates to:

$$
F_k = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 - k_m v_0^2 m_0 \left( \frac{\mu}{m_0} \right) \Delta t & -k_m v_0^2 m_0 \left( \frac{\mu}{m_0} \right) \Delta t & 0
\end{pmatrix}
$$

(28)

The errors associated with this linearization are included in the process covariance matrix, $Q_k$, along with the uncertainties in the model due to unmodeled factors such as atmospheric disturbances and uncertainties in the atmospheric model used. The value of $Q_k$ encapsulates these model uncertainties and is specific to the individual data set being analyzed.

$P_0$ is initialized at $t_0$ as a function of initial data uncertainty (Equation 29). As the length along the flight path is relative to the initial point, there is no model error in $\sigma_0$ being zero (error in observation of positions is accounted for in $R_k$). The initial mass covariance is given as 0.5 times the initial mass determined by the dynamical optimization. Distance error and timing information give uncertainties of up to $\pm 1500$ m s$^{-1}$ for velocity.

$$
P_0 = \begin{bmatrix}
\sigma_{h0}^2 & 0 & 0 & 0 \\
0 & \sigma_{m0}^2 & 0 & 0 \\
0 & 0 & \sigma_{m0}^2 & 0 \\
0 & 0 & 0 & \left( 1500 \text{ km s}^{-1} \right)^2
\end{bmatrix}
$$

(29)

The initial errors are large but $P_k$ is updated throughout the iterative estimation, giving a concrete representation of the evolution of the confidence of the state estimate, incorporating the uncertainties defined by the process noise covariance, $Q_k$ (Equation 12), and the measurement noise covariance, $R_k$ (Equation 21). The measurement noise covariance for the bolide motion is set to be $(100 \text{ m})^2$ and is dependent on camera resolution, the angle of the fireball with respect to the camera, and calibration of lens distortion.

**Smoothing Problem**

More generally, we can apply a smoothing estimator to our fireball data sets, as we will always have the observations from the entire trajectory available when the estimation is performed. A filtering estimator, such as described above, uses only past data (and hence is suitable for real-time estimation), whereas a smoothing estimator uses all data (future and past) to generate an optimal state estimate. The Rauch–Tung–Striebel (RTS) smoothing algorithm is implemented using the method described by Sarkka (2008). The resulting state estimate values for the trajectory are improved, along with their uncertainties.

**RESULTS**

The most complete data set available to test this method is that of Bunburra Rockhole, published by Spurný et al. (2012), which contains 113 data points with time, length of segment, and altitude information. As this data resulted in a recovered meteorite, constraints are available on final mass (Spurný et al., 2012), and cosmic ray exposure rates (Welten et al. 2012) provide an estimate of initial body diameter.

**Dynamic Optimization**

The dynamic optimization method described earlier, is applied to the data set using the constraints on parameters given in Table 2. Five parameter sets produce a fit with cost values $>0.98$ (Table 3). The initial masses range from 27.65 to 30.12 kg (Fig. 2) but the final masses converge to values of $\sim 2.4$ kg.

Figure 2 allows a visual comparison of these model outputs to the raw data. The parameter sets defined in Table 3 are used to initialize a set of Kalman filters that will take the data itself into consideration to determine a final mass.

**Kalman Filter**

The Extended Kalman Filter runs separately on each set of parameters resulting from the dynamical optimization stage. The final states of each model setup are given in Table 4.

The change in state values during the iterative EKF process are graphed against time with covariance plotted as approximate error bars (Fig. 3). The uncertainties are high initially. Mass uncertainties are only constrained by the data through the link to velocity with the dynamic equations and therefore remain high while the iterative process determines a value.

After running the forward EKF, the Rauch–Tung–Striebel smoothing algorithm is run (Fig. 4). The outcome of smoothing produces an initial entry mass of $30.20 \pm 6.53$ kg.

**Checking Results Using the Dimensionless Coefficient Method**

As a comparison, we also analyzed the Bunburra Rockhole data set using the approach based on Stulov et al. (1995) and applied by Gritsevich (2008b). In this
method, the dynamic Equations 1–2 are modified by normalizing the values of mass, velocity, and altitude \((h)\) to the entry mass, entry velocity, and the scale height of the homogeneous atmosphere \((h_0 = 7160 \text{ m})\), respectively. A set of dimensionless parameters (ballistic coefficient, \(\alpha\) [26], and mass loss parameter, \(\beta\) [31]) are substituted to remove the need of unknown individual variables.

\[
\alpha = \frac{c_d \rho_0 h_0 A_0 m_0^{-1/3}}{2 \rho m_0^{2/3} \sin \gamma}
\]  
\[
\beta = (1 - \mu) \frac{c_d h_0^2}{2 c_d H^2}
\]  

Table 3. Top five best fit parameter sets resulting from dynamical optimization.

<table>
<thead>
<tr>
<th>Normalized sum of square differences to position</th>
<th>(m_0) (kg)</th>
<th>(v_0) (m s(^{-1}))</th>
<th>(\frac{a^2}{m_0}) (kg m(^{-3}))(^{-2/3})</th>
<th>(\frac{\alpha}{\rho_0}) ((\times 10^{-8} \text{ J kg}^{-1}))</th>
<th>(l_f) (m)</th>
<th>(m_f) (kg)</th>
<th>(v_f) (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00000</td>
<td>30.12</td>
<td>13198</td>
<td>0.009511</td>
<td>4.82</td>
<td>60071.8</td>
<td>2.36</td>
<td>6109</td>
</tr>
<tr>
<td>0.99859</td>
<td>30.95</td>
<td>13203</td>
<td>0.009689</td>
<td>4.76</td>
<td>60042.4</td>
<td>2.50</td>
<td>6100</td>
</tr>
<tr>
<td>0.98862</td>
<td>29.82</td>
<td>13203</td>
<td>0.009545</td>
<td>4.68</td>
<td>60061.8</td>
<td>2.51</td>
<td>6124</td>
</tr>
<tr>
<td>0.98344</td>
<td>28.64</td>
<td>13204</td>
<td>0.009466</td>
<td>4.66</td>
<td>60057.6</td>
<td>2.44</td>
<td>6125</td>
</tr>
<tr>
<td>0.98108</td>
<td>27.65</td>
<td>13205</td>
<td>0.009394</td>
<td>4.64</td>
<td>60052.7</td>
<td>2.38</td>
<td>6126</td>
</tr>
</tbody>
</table>

Fig. 2. Top left: position data subtracted from modeled position for models with parameters given in Table 3. Red curve is model that gives the lowest normalized sum of square differences (initial mass of 30.12). Dotted line is one standard deviation (70.14 m). Top right: shows associated change in mass for corresponding model parameters with costs >0.98. Bottom left: derivative of mass with time for models. Bottom right: comparison of models (red curves) to calculated velocity (blue points).

Table 4. Final states \((x_f, m_f, v_f)\) for parameter sets from dynamic optimization corresponding to the following initial masses.

<table>
<thead>
<tr>
<th>(m_0) (kg)</th>
<th>(x_f) (m)</th>
<th>(m_f) (kg)</th>
<th>(v_f) (m s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.12</td>
<td>60032 ± 62</td>
<td>2.30 ± 1.63</td>
<td>6052 ± 241</td>
</tr>
<tr>
<td>30.95</td>
<td>60032 ± 62</td>
<td>2.47 ± 1.67</td>
<td>6057 ± 236</td>
</tr>
<tr>
<td>29.82</td>
<td>60033 ± 62</td>
<td>2.40 ± 1.67</td>
<td>6061 ± 238</td>
</tr>
<tr>
<td>28.64</td>
<td>60033 ± 62</td>
<td>2.35 ± 1.66</td>
<td>6062 ± 240</td>
</tr>
<tr>
<td>27.65</td>
<td>60033 ± 62</td>
<td>2.29 ± 1.64</td>
<td>6062 ± 242</td>
</tr>
</tbody>
</table>
where $\rho_0$ is the atmospheric density near the surface and $\gamma$ is the trajectory entry angle.

The $Q_4$ method of least-squares minimization defined by Gritsevich (2008b) is used to create a fit of the Kulakov and Stulov (1992) Equation 32 to the Bunburra Rockhole data set.

$$y = \ln \alpha - \ln(-\ln V) + 0.83\beta(1 - V)$$

where $y = -\ln \rho_0$ and $V = \frac{v}{v_0}$.

The isothermal atmosphere approximation is used to derive Equation 32: $\rho_\infty = e^{-y}$, making it difficult to implement a more accurate atmosphere model.

Although this method has proved successful on previous fireball data sets (Gritsevich 2008a), these are limited to fewer than 20 velocity points with an average based smoothing applied (Ceplecha 1961). The value of $v_0$ that is used to normalize all velocity values is simply the initial velocity. For the Bunburra Rockhole data set, the 113 data points show high scatter and the velocity range within the first half a second has a range of over 3500 m s$^{-1}$. It was found that the noise in the raw data could not be accommodated by this method without pretreating the data, making it rather unsuitable for use in an automated data pipeline where large noisy data sets need to be processed.

Smoothing the data using a five-point moving average, and using the average initial velocity from Table 3, 13,200 m s$^{-1}$, and a value of 2/3 for the shape change parameter allows a result to be calculated as a comparison to the new method. This gives $\alpha = 25.23$ and $\beta = 1.53$ (Fig. 5). The equation for the ballistic coefficient (Equation 30) allows an initial mass to be calculated. By assuming values of the shape-density parameter from the dynamic optimization, and a constant drag coefficient of 1.3, an approximate value for $m_0$ is determined to be 84.92 kg.

When used in the following Equation 33, along with a value of 2/3 for rotation, a final mass of 1.90 kg results.

$$m_f = m_0e^{-\left(\frac{1}{1.3} - \frac{1}{v_0}\right)}$$

(equation 6 [Gritsevich 2008b])
It is difficult to assess the error in this case, and the ranges in initial and final masses are harder to obtain. The amount of scatter in the velocity data is significant and a change in initial velocity used by 1% can result in initial masses varying by $30$ kg and final masses to be $2$ kg.

**DISCUSSION**

**Determining Model Parameters**

The dynamical optimization of the Bunburra Rockhole data set returned a large number of parameter sets with cost values $>0.9$, although only five with $>0.98$, all of which show relatively similar starting masses. The ranged (27.65–31.12 kg) initial masses converge (Fig. 2) to give very similar final mass values (Table 3). As the final masses are needed for determining any potential fall positions, it is more important that these values be limited. It should be remembered that the dynamic optimization is estimating appropriate meteoroid parameters to use as inputs in our main model (EKF step) based on this specific fireball data set. Previous works have assumed “typical,” or average meteoroid parameter values, without the link to the data from the event in question (e.g., densities by Borovička et al. [1998, 2013] and McCrosky et al. [1971]; shape density coefficient used by Ceplecha and ReVelle [2005] and Spurný et al. [2012]). We believe that this is an advantage of our approach. This step gives us greater confidence in the estimates to be used in the EKF step, especially considering the similarities in meteoroid characteristics of the top results (Table 3).

The shape parameter and preatmospheric meteoroid density cannot be uniquely identified in this model. The values of $A_0$ and $q_{m_0}$ in Table 3 could correspond to a spherical object ($A_0 = 1.21$) with a preatmospheric meteoroid density of $\sim 1400$ kg m$^{-3}$, a circular cylinder with a cross sectional diameter to length ratio of roughly 1:1 and $q_{m_0} \sim 2700$ kg m$^{-3}$, or even a 3:2:1 triaxial ellipsoid (as suggested by Zhdan et al. 2007) with $q_{m_0} \sim 3500$ kg m$^{-3}$.

A unique solution is not needed for finding any potential meteorites and any fragments found will be able to resolve these two parameters.

Knowing the Bunburra Rockhole bulk meteorite density to be $2700$ kg m$^{-3}$ (Spurný et al. 2012) enables us to approximate the meteoroid shape, $A \sim 1.85$. This corresponds to a circular cylinder with a cross sectional...
diameter to length ratio of roughly 1:1, or a 3:2:1.5 axial ellipsoid. If the value of $A$ and $q_m$ were to remain constant, these values of $A_0$ and $q_{m0}$ with the given initial mass corresponds approximately to a cross sectional area of 0.092 m$^2$. Figure 7.2 in Stulov et al. (1995) shows a distribution of values of $H^*$ for bolides, resulting in an overwhelming majority with entry masses $>1$ kg, having values close to $2 \times 10^6$ J kg$^{-1}$. If this value is assumed for $H^*$, values for $c_h$ can be approximated (Table 5).

The initial mass determined by Spurný et al. (2012) using both the methods described by both Ceplecha et al. (1998) and Ceplecha and ReVelle (2005) is $22.0 \pm 1.3$ kg. Cosmic ray exposure rates were analyzed for the Bunburra Rockhole meteorite; however, the pre-entry radius was determined to be larger than a radius corresponding to a mass of 22 kg (Welten et al. 2012). By performing a reverse extended Kalman filter, the entry mass is determined to be closer to 30.20 $\pm$ 6.53 kg. This corresponds to a pre-entry radius of around 17.1 cm. This is close to the 13–17 cm range determined by (Welten et al. 2012).

Although fragmentation is not yet explicitly handled using this method, the data reflects both effects of ablation and fragmentation. The process noise $Q_k$ in the EKF model handles some degree of unexpected mass change, allowing these variations to be incorporated in the final mass estimates.

Furthermore, sudden increases in the magnitude of the state variance matrix $P_k$ can give an indication that a fragmentation event may have occurred, along with...
examining the change in mass with time (Figs. 3 and 4). It is noticeable from both Figs. 3 and 4 that there are peaks of maximum mass loss at around 3.133 and 3.845 s as well as at 4.415 s in Fig. 3. It is likely that these correspond to fragmentation events. These times correspond to altitudes of 41.31, 37.16, and 34.18 km, respectively, allowing a comparison to fig. 13 in Spurný et al. (2012) which shows significant changes of mass at 37.8 and 35.85 km altitude. The significant mass loss event seen in fig. 13 in Spurný et al. (2012) at 54.9 km (corresponding to 1.0 s) is not evident, although it is well within the large error bracket given at this time. Future work will aim to capture this fragmentation information in a coherent and consistent way.

The scatter in the Bunburra Rockhole data set presented difficulties when initially attempting to use the method outlined by Gritsevich (2008a). After smoothing the data and using the initial parameters determined by the dynamic optimization, final values are similar to those determined using this new method. The dependence on an initial velocity for normalization makes it very sensitive to initial scatter and there is no constraint on the errors this or the smoothing may cause. The EKF method avoids these dependences.

**CONCLUSION**

The method outlined here provides a consistent and detailed approach to characterizing meteoroids without the need for brightness data as they pass through the atmosphere. In addition, it provides a rigorous way of propagating uncertainties in trajectory states (position, mass, and velocity), something that previous approaches have not explicitly described.

A dynamic optimization determines the optimum parameters for the meteoroid flight such as the shape-density parameter and initial mass. An extended Kalman filter then includes observation and dynamic uncertainty models, which are valuable in understanding the errors in the model states, and which can adapt to fragmentation events or other unexpected dynamic changes. The initial (30.20 ± 6.53 kg) and final masses (2.30 ± 1.63 kg) calculated for the Bunburra Rockhole data set is within the range of previously published values by Spurný et al. (2012) (22.0 ± 1.3 and 1.1 kg, respectively) and corresponds with cosmic ray exposure studies (Welten et al. 2012) to constrain preatmospheric radius and mass. Although the method used by Gritsevich (2008b) was re-created using the meteoroid characteristics determined by dynamic optimization, the sensitivity of this method to (widely varying) data for initial entry velocity translates to a range of estimates for entry and terminal masses. As the errors are not quantified, the confidence in mass calculations using this method—crucial for automating our data flow and constraining search areas—cannot be constrained.

The two-step approach outlined in this paper is an automated method which will allow the DFN to reduce data for every observed fireball, rather than only selecting high value or unusual cases. For the subset that involve a meteorite fall, this approach will calculate multiple fall positions with comprehensive error values to allow for efficient recovery searches. Work still needs to be carried out on integrating the variability in the heat-transfer coefficient. The assumption in this method that it remains constant throughout the trajectory is a simplification. The identification and analysis of fragmentation events also needs to be incorporated in a more coherent and consistent manner.

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**REFERENCES**


A novel approach to fireball modelling


