

**School of Economics and Finance**

**Applied Analysis of Labour and Financial Markets  
Using Time Series Methods**

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## ABSTRACT

The development of time series techniques associated with non stationary data, such as the testing for unit roots and cointegration has presented the applied worker with new challenges in the applied analysis of economic problems.

This thesis uses some of these methods to consider a number of questions in the area of labour and financial markets.

In particular the thesis considers the application of these methods to two general questions, the specification of the aggregate wage equation in Australia and the efficiency of the Australian stock market. More specifically the thesis focuses on the time series properties of variables commonly used in specifications of the wage equation and then tests them for cointegration. In the financial economics area the thesis tests for the gains to portfolio diversification from the perspective of an Australian investor and the applicability of the present value model of stock prices to the Australian stock market.

## Acknowledgements

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I would like to thank Professors J.L Ford and M. Miller, of Birmingham University and Warwick University (UK), respectively, for giving me my first jobs in academia, as their research assistant, and for making those jobs so rewarding and stimulating. They encouraged me to pursue an academic career.

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I appear to have “worn out” a number of supervisors in the course of this thesis. Thanks to Prof. P. Dawkins who made me get on with this and gave me the feeling that I really did have to get on and do it. Peter was my supervisor for the first three years before leaving for Melbourne University.

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Last, but of course, not least, thanks to my family, my wife Carol and our four children, all of who have, as always, been a source of love and inspiration. In particular, to my wife Carol, I dedicate this thesis.

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## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1 Introduction**

The core of this thesis contains four pieces of applied time series analysis, two in the area of labour economics, two in the area of financial economics. The common denominator to the work is the time series techniques used to carry out the applied analysis.

The thesis is in three sections. Section A deals with the background to the econometric tests used in the applied work, Section B contains the applied analysis in the labour market area and Section B the applied analysis in the area of financial economics. The content of these three section is described in more detail below in section 1.2. Section 1.3 outlines the structure of the thesis.

#### **1.2 Background and motivation**

##### **Section A**

The first part of this thesis, Section A, deals with the econometric background to the time series methods used. The applied work in this thesis deals with the estimation, and interpretation, of relationships between non stationary time series. As a result a large amount of use is made of the estimation techniques associated with the cointegration framework, error correction models (ECM's) and vector autoregressive models (VAR's). Section A, (Chapters 2, 3 and 4),



reviews, at a fairly general level, some of the econometrics of such techniques. The motivation for this is simply to provide the background to the empirical work which forms the core of this thesis and not to provide a comprehensive survey of the literature, indeed such an exercise would be beyond the scope of this thesis. Thus only a subset of the available techniques, those used in the empirical work in later chapters, is covered.

## **Section B**

Section B contains two chapters (Chapters 5 and 6) which focus on a particular applied problem in the area of labour economics, the estimation of a wage equation for Australia. The motivation for this work comes, in part, from earlier research, by the author in this area, Lewis and MacDonald (1993). Empirical investigations into the specification of the wage equation in Australia have either not used cointegration techniques or have found no evidence of cointegration between the variables. Chapters 5 and 6 re-examine the time series properties of data frequently used in the specification of a wage equation and consider whether a particular subset of that data cointegrates. Thus Chapter 5 begins by considering the time series properties of the variables commonly used in this area using the tests for a unit root described in Section A. This is an interesting question of itself. It is well known that the some of the more familiar test of the null of a unit root, such as the Dickey-Fuller tests (Dickey and Fuller 1979, 1981) suffer from problems of low power. As a result there have been numerous attempts to devise alternative testing methods which improve the power of unit root tests. Three of these tests, the KPSS test (detailed in Kwiatkowski, Phillips, Schmidt and Shin, 1992) and the DF-GLS test (detailed in Elliot, Rothenberg and Stock, 1992) and a test for a unit root against the alternative of stationarity about a breaking trend are discussed in Chapter 3 of Section A. Applications of these alternative tests have led to a reconsideration of the assumption that most macroeconomic

time series contain a unit root. It is therefore of interest to see if alternative tests provide any clear conclusions regarding the order of integration of Australian labour market data.

Having reconsidered the unit root hypothesis the applied work moves on to consider the question of cointegration between a sub set of the data considered. There have been few studies, in the Australian context, which have tested for cointegration amongst the wage equation variables, Watts and Mitchell (1990), being a notable exception. One reason for this, possibly, is that the variables do not in fact appear to cointegrate. Watts and Mitchell (1990) for example found no evidence of cointegration. This thesis considers one possible explanation for this, and that is that the cointegrating relationship between the variables considered underwent a regime shift in the mid 1970s, and that standard tests of cointegration fail to reject the null (of no cointegration) because of this. There are a number of reasons for believing that that such a regime shift might well be important in the context of Australian labour market data, the 1970's were a period of substantial change, the election of a Labour government and the beginnings of the move towards centralised wage bargaining which culminated in the 1980's with the famous Accord. Chapter 6 uses a recently developed test for cointegration which allows for a regime test, Gregory and Hansen (1996). The results show that a regime shift does play an important role in the Australian data, and that non allowance for it affects tests for cointegration in this area.

### **Section C**

The final section of this thesis (Chapters 7, 8 and 9) contains work in the area of financial economics. The common theme here is the testing of efficiency in the context of Australian stock market data.

Chapters 7 and 8 consider a data set which includes the stock market indices from 16 of the worlds major economies. The basic aim is to consider whether there are gains to be made from international portfolio diversification (from an Australian perspective). Chapter 7 reproduces, in large part, an already published paper by the author, Allen and MacDonald (1995) which tests for cointegration between sets of the stock market indices under consideration. The results, in common with other work in this area, suggest that there do exist cointegrating vectors between some of the markets considered. As well as having implications for the gains from diversification in the long run this finding also has implications for the weak form efficiency assumption. Chapter 8 considers this point by estimating simple vector error correction models, using the cointegrating vectors, and looks at the question of causality between the markets which are found to cointegrate.

Chapter 9 considers the efficiency of the Australian stock market more directly, by testing the restrictions implied by the present value model of stock prices on data for the Australian market. The framework used to test the model is that described in Campbell and Shiller (1987). This model has been much used in the literature and recently applied to data from the UK markets by MacDonald (1994), MacDonald and Power (1995) and Mills (1993, 1995). Chapter 9 looks at the implications of the model and the requirements, in terms of the time series properties of the data, for it to be valid. After testing the model and finding, in common with applications to other economies, that the restrictions implied by the model are rejected using Australian data, the chapter goes on to considers the relationship between stock prices and dividends. Recent work by MacDonald and Powers (1995) for the UK, using data up to the market crash in 1987, suggested that there was no evidence of cointegration between prices and dividends. Their solution was to add further variables, in particular, the firms earning retention ratio, in order to establish a cointegrating vector. Having done this they estimated a simple vector error

correction model which performed satisfactorily. In the Australian context the work in Chapter 9 finds that there is no need to include extra variables, that prices and dividends do in fact cointegrate over the period up to the crash and a simple forecasting model using only these two variables does as well as that of MacDonald and Powers.

### **1.3 Structure of the thesis.**

As noted above Section A deals with the econometric tests used in the applied work and consists of three chapters. Chapter 2 reviews, at a very simple level, the concept of stationarity and in particular develops the idea that a time series may be difference stationary, or as it is commonly referred to, has a unit root. Chapter 3 takes up the issue of whether a time series contains a unit root in the context of applied tests for a unit root in observed data. Three testing methods are described, each based on a different approach to testing the hypothesis of stationarity. Chapter 4, the final chapter in Section A, looks at tests for cointegration. Three tests are considered, two of which are standard in the literature and a third, recently developed test, which allows for a regime shift in the cointegrating vector.

Section B contains two chapters. Chapter 5 uses the unit root tests, described in Chapter 3, to test the unit root hypothesis on a range of data from the Australian labour market. The aim is to see if the different tests yield the same conclusions regarding the order of integration of the data, or whether there is some doubt about the unit root hypothesis. Chapter 6 considers two simple hypothesis related to the specification of the wage equation using Australian data. In particular the idea that a regime shift in the data is important is considered.

Section C contains three chapters. The first, Chapter 7 considers the gains to be made from international portfolio diversification using data from a range of countries stock markets. Chapter 8 extends the work in Chapter 7, by attempting to model the relationships between a subset of the indices which were found, in Chapter 7, to cointegrate. Finally Chapter 9 looks at the stock market from the perspective of the present value model and tests the restrictions implied by this model on the data. A simple model of stock prices, based only on dividends is also considered and estimated.

Finally Chapter 10 provides a conclusion to the thesis and summarises the major findings.

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Finally Chapter 10 provides a conclusion to the thesis and summarises the major findings.

## CHAPTER 2

### STATIONARY AND NON STATIONARY TIME SERIES

#### **2.1 Introduction**

This chapter briefly reviews the concepts of stationary and non stationary time series in the context of simple autoregressive models<sup>1</sup>. Consideration of non stationarity in this context demonstrates the possibility that a time series may only become stationary after first differencing. This idea, that a time series may require first differencing before it is stationary, has become a very important one in the area of applied economics and econometrics and has become widely known as testing for a unit root. The practical problem of testing for a unit root in observed time series is taken up in Chapter 3.

Whilst a more formal definition of stationarity and non stationarity is given below, it is worth considering, briefly and very simply, why the issue of non stationarity in time series is so important. Firstly a distinction can be made between series which are trend stationary and series which are difference stationary. Thus, a series with a deterministic time trend is non stationary and requires de-trending to make it stationary (or is often referred to as a trend stationary process (TSP)). This thesis is more concerned with time series which are “difference stationary” and require differencing to make them stationary. The most obvious example is the simple random walk model:

$$y_t = y_{t-1} + \varepsilon_t$$

such that  $\Delta y_t = \varepsilon_t$

and where  $\varepsilon_t$  is assumed to be stationary. Thus the first difference of a random walk is stationary and the process is often referred to as a difference stationary

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<sup>1</sup> The results presented here, which are standard in the literature, are drawn from a number of sources including, Anderson (1971), Spanos (1986) and Lutkepohl (1993).

process (DSP) and, in this case, is said to be integrated order 1, often written as  $I(1)$ . Thus when reference is made to non stationarity in what follows it is related to integrated processes unless otherwise stated. The problem for econometricians has been to devise tests which would allow the distinction to be made between a TSP and a DSP. Such tests are discussed below in Chapter 3 which examines a range of tests which attempt to answer this question.

First it is necessary to establish why it is important to determine whether a series is a TSP or a DSP. Consider a driftless random walk such as the model above. If the data were assumed to be a TSP the researcher might well attempt to make it stationary using a regression on time. Clearly this is incorrect, but it would probably be found that the fitted model had a reasonable  $R^2$  and would often reject the null of no dependence on time. Nelson and Kang (1981) for example demonstrate such results. It would also be the case that the de-trended series (the residuals) showed less true stochastic variance than the original series. A second problem would result from regressing one random walk process on another, completely unrelated, random walk, where again the researcher would probably find evidence of a significant relationship. This is the so called spurious regression (Granger and Newbold, 1974) or nonsense regression (Yule, 1926) problem. Essentially what this means is that the unwary researcher who runs OLS regressions on unrelated non stationary time series will often find that standard tests suggest a relationship between the variables when non is in fact present. Such spurious correlations would also appear in regressions including deterministic trends. Thus the traditional practice of either including time as a regressor, or de-trending the variables by regressing them against time and using the residuals has been found to be flawed.

The basic point to be made here is that the unwary researcher could draw false conclusions from regressions if she or he were unaware of the nature of the

time series properties of the data. Four simple possibilities arise:- assume two data series are under consideration :

1. The data are stationary in which case classical regression results apply.
2. The data are integrated of different orders. Here regression results will be meaningless.
3. The data both contain unit roots, ie both are integrated,  $I(1)$ , process. If the regression residuals contain a unit root then we have the spurious regression problem.
4. The data both contain a unit root but the regression residuals do not.

Cases 2 and 3 form the “problem” areas. It is possible to carry out sensible modelling with such variables but the point to be made here is that in order to do so requires knowledge of the problem, to know the time series properties of the data. Case 4 is a different matter. If by forming a linear combination of two  $I(1)$  series, for example by using linear regression, then it would be expected that the resultant series, the residuals, would also be  $I(1)$ . If they are not, if for example the residuals are stationary,  $I(0)$ , then the two series are said to cointegrate. Modelling strategies associated with this type of outcome are dealt with in more detail in Chapter 4.

The basic point, therefore, is that it is important that applied work be carried out with full knowledge of the time series properties of the data being considered and the tests described in Chapters 3 and 4 are those used in this thesis to examine these properties. Before doing this, however, a very brief definition of stationarity is required and that forms the content of the next section.

## 2.2 Stationarity

Let  $y_t$  be a univariate time series, weak stationarity requires the first two moments of the series to be independent of time, thus the mean, variance and covariance must be independent of time (see Harvey, 1981a):

$$E(y_t) = \mu \quad (2.1a)$$

$$E[(y_t - \mu)^2] = \text{Var}(y_t) = \lambda(0) \quad (2.1b)$$

$$E[(y_t - \mu)(y_{t-\tau} - \mu)] = \text{Cov}(y_t, y_{t-\tau}) = \lambda(\tau) \quad (2.1c)$$

So that the mean (2.1a) and variance (2.1b) are both constants and the covariance (2.1c) depends only on the gap between the two periods and not the actual time at which the covariance is evaluated.

Consider now a very simple model for  $y_t$ , the first order univariate autoregressive (AR(1)) process:

$$y_t = \rho y_{t-1} + \varepsilon_t \quad t=0,1,2,\dots \quad (2.2)$$

where  $\varepsilon_t$  is assumed to be stationary with

$$E(\varepsilon_t) = 0 \quad (2.3a)$$

$$\text{Var}(\varepsilon_t) = E(\varepsilon_t^2) = \sigma^2 \quad (2.3b)$$

$$\text{Cov}(\varepsilon_t, \varepsilon_{t-\tau}) = 0 \text{ for all } \tau \neq 0 \quad (2.3c)$$

These conditions make  $\varepsilon_t$  a special case of a stationary process referred to as white noise or a purely random variable.

It is useful to consider the conditions under which  $y_t$  will be a stationary process. It can be shown that the stationarity condition is satisfied only if the value of  $\rho$  is less than 1 in absolute value. When  $\rho$  is equal to 1 the process is said to have a unit root and is not stationary. The distinction between a stationary AR process and one with a unit root has become the focus of applied research in recent years and it is useful to note some of the distinctions between the two types of process.

Firstly for the unit root AR process the variance,  $\text{var}(y_t)$ , becomes a function of time and increases without bound as  $t$  tends to infinity.

Secondly the autocovariance function defined as:

$$\text{Corr}(y_t, y_{t-\tau}) = \frac{\text{cov}(y_t, y_{t-\tau})}{\sqrt{\text{var}(y_t) \text{var}(y_{t-\tau})}} = \rho^\tau \sqrt{\left(\frac{1-\rho^{2(t-\tau)}}{1-\rho^{2(t)}}\right)} \quad (2.4)$$

tends, for the stationary AR process, to fall in value as the value of  $\tau$  increases but remains constant at 1 for the unit root process.

Thirdly, for the stationary AR process the effects of shocks will die out over time whereas for the unit root process they last forever.

Clearly the problem for the economist is to distinguish between processes which have a unit root and those which are stationary. As noted by Hendry (1986) for situations in which the AR process is stationary but the value of  $\rho$  is close to one this distinction will be a difficult one to make using the finite size data sets of the type usually found in economics. The aim of the next chapter is to review some of the more popular tests available which are used in this thesis.

### 2.3 Conclusion

This chapter has reviewed at a very general level the concept of stationarity in the context of autoregressive time series. The main interest of this thesis is the testing for non stationarity in the form of a unit root in applied macroeconomic time series and the next chapter reviews a number of the more popular tests which have been used in applied work.



## CHAPTER 3

### TESTING FOR UNIT ROOTS

#### **3.1 Introduction**

Since the seminal work by Nelson and Plosser (1982), in which a number of US time series were tested for stationarity, there has been a veritable explosion of research in the area of testing for non stationarity in the form of a unit root in observed economic time series. The aim of this chapter is to provide the background to the tests used in the thesis; it is not intended to survey the whole range of tests available. Thus, a variety of techniques, including testing for unit roots in seasonally unadjusted data, fractional unit roots and non parametric testing methods, are not covered here. The survey article by Campbell and Perron (1991) provides an excellent coverage of a wide range of unit root tests and provides useful practical help on testing procedures. Banerjee, Dolado, Galbraith and Hendry (1993) also provide a comprehensive survey of the testing framework.

Section 3.2 provides an introduction to the problem and looks at the most commonly used test for a unit root, the (Augmented) Dickey Fuller ((A)DF henceforth) test. This section also outlines a testing strategy for the applied economist using the (A)DF test.

Section 3.3 looks, briefly, at two other tests used in this thesis. The first of these, detailed in Kwiatkowski, Phillips, Schmidt and Shin (1992) (and henceforth referred to as the KPSS test), essentially reverses the null hypothesis involved in the unit root test, and tests the null of stationarity against the alternative of a unit root. The second test, described in Elliott, Rothenberg and Stock (1992), (and henceforth denoted the DF-GLS test) is a variant of the standard (A)DF test which involves prior de-trending of the data

under a local alternative to the unit root null. Elliot *et al* (1992) report significant power gains for tests devised under such a method.

Section 3.4 considers another alternative which has become popular in the literature (Perron, 1989, Banerjee, Lumsdaine and Stock, 1992) namely the testing of the unit root hypothesis against an alternative hypothesis that the series is stationary around a breaking trend.

Finally Section 3.5 provides a conclusion.

### 3.2 Testing for a unit root - The Dickey Fuller test

The most popular tests for a unit root in economic time series are those described in Dickey and Fuller (1979,1981). This section describes those tests in some detail, paying particular attention to the problems the researcher faces in using the tests in applied work.

In the AR(1) case testing for a unit root essentially comes down to the condition that the autoregressive parameter is unity, thus:

$$y_t = y_{t-1} + \varepsilon_t \tag{3.1}$$

where  $\varepsilon_t$  is iid( $0, \sigma^2_\varepsilon$ ) and  $y_0 = 0$

and so  $\Delta y_t = \varepsilon_t$  ie the first difference of the AR(1) process in the presence of a unit root, is stationary. It is common to refer to the AR(1) process with a unit root as being integrated order 1, denoted I(1), implying that first differencing will ensure stationarity. For the AR(2) process there is the possibility of two unit roots. If the AR(2) process has a single unit root then it too has the characteristics of a random walk and be I(1). If the AR(2) has two unit roots then the process is I(2) and requires second differencing to ensure stationarity.

Generally, a series which requires differencing  $n$  times before becoming stationary is referred to as being integrated order  $n$  or  $I(n)$ .

Begin by considering a test for a unit coefficient in the regression equation:

$$y_t = \alpha y_{t-1} + \varepsilon_t \quad (3.2)$$

where  $\varepsilon_t$  is  $iid(0, \sigma^2_\varepsilon)$  and  $y_0 = 0$ .

The natural temptation would be to estimate (3.2) by OLS and use a  $t$  statistic to test the null that  $\alpha = 1$ . However, under the null that  $\alpha = 1$ , the series  $y_t$  is a non-stationary process with a variance increasing as  $t$  increases. Under such circumstances the  $t$  statistic for testing  $\alpha = 1$  does not have its normal distribution asymptotically

The distribution of the test statistic for testing the null that  $\alpha = 1$  is obtained by monte carlo simulation and is neither symmetric nor asymptotically normal. The best known results from such simulations are those found in Fuller (1976) and Dickey and Fuller (1979,1981) which contain a number of sets of tabulated critical values for testing the null of a unit root. They assume that the data generation process (DGP henceforth) is that of a random walk with zero drift and suggests three regression models in which the null of a unit root can be tested. The critical values of the test statistic depend upon which of these models is estimated. The three models are:

$$y_t = \alpha y_{t-1} + \varepsilon_t \quad (3.3a)$$

$$y_t = \mu + \alpha y_{t-1} + \varepsilon_t \quad (3.3b)$$

$$y_t = \mu + \beta t + \alpha y_{t-1} + \varepsilon_t \quad (3.3c)$$

where in each case the null hypothesis that  $\alpha = 1$  can be tested using the  $t$  ratio from the estimated co-efficient in equations (3.3a-c), remembering of course that the critical values for the test will be non standard. Following Dickey and

Fuller (1979), the statistics for testing the null that  $\alpha = 1$  in equations (3.3a), (3.3b) and (3.3c) are termed the  $\tau$ ,  $\tau_\mu$  and  $\tau_\tau$  statistics respectively. In a later paper, Dickey and Fuller (1981), three further test statistics,  $\phi_1$ ,  $\phi_2$  and  $\phi_3$  are described. These test joint restrictions on the parameters of equations (3.3b) and (3.3c) and are constructed as F tests, but once again do not have their usual shapes.  $\phi_1$  tests the null  $(\mu, \alpha) = (0, 1)$  in equation (3.3b),  $\phi_2$  tests the null  $(\mu, \beta, \alpha) = (0, 0, 1)$  in (3.3c) and  $\phi_3$  tests the null  $(\mu, \beta, \alpha) = (\mu, 0, 1)$  in (3.3c) and critical values can be found in Dickey and Fuller (1981).

So, the first problem for the researcher in this area, is to ensure that s/he is working with the correct set of critical values, and, whilst these tabulated values are widely available, there is a further problem here. The tables in Dickey and Fuller (1979,1981) provide critical values for a range of sample sizes for the above-mentioned statistics, however, these are based on the asymptotic distribution of the test statistic, and are available for discrete numbers of observations which frequently do not match the number used by the researcher. This is an important point which is currently receiving more attention in the literature. One solution to this problem has come from a number of papers (MacKinnon, 1991, Cheung and Lai, 1993, 1995a, 1995b) which report estimated response surfaces for the critical values for the more commonly used tests. These allow the researcher to calculate sample size specific critical values and, where appropriate, correct for degrees of freedom due to differing lag structures in the regression model. Where possible the tests in this thesis make use of these response surface equations to calculate critical values and the basic results of some of the papers mentioned above are examined in Appendix A1.

The next problem also relates to the critical values of the tests and is the question of the similarity<sup>2</sup> of the test under differing assumptions regarding the

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<sup>2</sup> A test is similar if the test statistic has the same distribution under differing assumptions regarding the inclusion of nuisance parameters.

DGP. If the DGP is as specified in (3.2), then it would be valid to use any of the three regression models above to test the unit root hypothesis, and they would have the tabulated critical values reported in Fuller (1976). If however the DGP includes extra parameters such as an unknown starting value, a constant term or a constant and a trend, then the three models above may not yield similar tests and the critical values reported would not be applicable. Banerjee Dolado, Galbraith and Hendry (1994) summarise a number of results in this area (Kiviet and Phillips, 1992, Evans and Savin 1981, 1984, Nankervis and Savin, 1985, 1987, Bhargava 1986, West, 1988). They conclude that for a test to have the Dickey-Fuller tabulated critical values the regression model used must contain at least one more parameter than the DGP. Thus, for example, if the DGP is a random walk with non zero drift,  $y_t = \mu + y_{t-1} + \varepsilon_t$ , then tests based on regression model (3.3b) would not have the property of similarity and the critical values of such tests would not correspond to those given in Fuller (1976). However tests based on regression model (3.3c) would be similar, since this model contains an extra regressor, the trend term, and thus the Fuller tables and critical values would be valid. Results in West (1988) show that in exactly parameterised cases, where the DGP and the regression model contain the same terms, the distribution of the test statistic is asymptotically normal. Thus, in the case above (with unknown starting value and constant in the DGP), model (3.3b) would be exactly parameterised and, thus, the distribution of the test statistic would be asymptotically normal. However, Hylleberg and Mizon (1989), and Banerjee et al (1994) present simulation evidence which suggests that the critical values depend upon the relative magnitude of the drift term and the stochastic trend. The tendency for the distribution to be asymptotically normal depends upon the drift term dominating the stochastic trend. Thus, if the model under consideration contains a random walk and significant drift, the problem is whether the drift term dominates the stochastic trend in terms of the variance of the process. Banerjee et al (1994) present (in their Table 6.3) critical values for the t test of

the null of a unit root based on the model  $y_t = \mu + \alpha y_{t-1} + \varepsilon_t$  under the assumption that the DGP is a random walk with non zero drift ( $\mu$ ) for a variety of values of the ratio  $\mu/\text{var}(\varepsilon_t)$ .

These points can be summarised as follows:

- Test statistics from regression models are typically non standard and have to be obtained via monte carlo methods. This means that the researcher must ensure s/he has the appropriate critical values.
- Under certain conditions regarding the DGP and the regression model used, the test statistics may in fact be asymptotically normal. However the tendency of the test statistic to be normally distributed depends, (mainly), upon the relative magnitude of the stochastic trends driving the data.

The next question to be considered is which of the three regression models above (3.3a - 3.3c) should actually be used to test the unit root hypothesis. One conclusion which might be drawn from the above discussion is that testing the unit root should begin with the most general specification of the regression model, including constant and time. Even if the DGP is a random walk with drift, the Fuller tables are valid as the test statistic has the non standard Dickey-Fuller distribution<sup>3</sup>. Whilst this is true it must be weighed against the fact that tests for a unit root, where the null is that the series has a unit root, have low power in general and the inclusion of extraneous regressors in the testing model reduce power even further. An excellent survey of this problem can be found in Campbell and Perron (1991) who describe a number of “rules” which researchers should follow when dealing with non stationary data. In this context for example, their “rule 4” notes that the omission of deterministic

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<sup>3</sup> And for most macroeconomic data one might be prepared to preclude the possibility that the data were generated as a random walk with trend since, in logarithms, this would imply an ever increasing rate of growth (see Nelson and Plosser, 1982).

variables from the regression model can lead to the power of the test going to zero as sample size increases (an inconsistent test) if the deterministic variable is growing at least as fast as any of the elements in the regression model.

So, the researcher has to choose which of the regression models is appropriate to test the null hypothesis of a unit root. This depends upon the underlying DGP which, typically, he or she will be unaware of. The appropriate tests, and their critical values, for the presence of deterministic variables is influenced by the presence or absence of a unit root and vice versa. In section 3.2.1 a testing strategy, using the statistics described above, is described which helps the researcher make these decisions.

The next complication is introduced by the fact that the critical values tabulated in Dickey and Fuller (1979,1981) are based on the assumption of white noise error terms in the above equations (3.3a-3.3c). Frequently, in applied work, this will not be the case. Tests based on the regression equations (3.3a - 3.3c) are referred to as Dickey-Fuller (DF) tests. In the situation where the residuals in the DF equation are autocorrelated one solution is to use an 'augmented' Dickey-Fuller (ADF) test based on regression models (3.4a-c)<sup>4</sup>.

$$\Delta y_t = \alpha y_{t-1} + \sum_{i=1}^k c_i \Delta y_{t-i} + u_t \quad (3.4a)$$

$$\Delta y_t = \mu + \alpha y_{t-1} + \sum_{i=1}^k c_i \Delta y_{t-i} + u_t \quad (3.4b)$$

$$\Delta y_t = \mu + \beta t + \alpha y_{t-1} + \sum_{i=1}^k c_i \Delta y_{t-i} + u_t \quad (3.4c)$$

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<sup>4</sup> Where  $y_t$  has been subtracted from both sides in order to put the regressions in the form typically used. The test statistics from these regressions have the same critical values as the DF tests.

The idea behind these models is to use lagged changes in the dependent variable to correct for residual autocorrelation and leave white residuals<sup>5</sup>. The validity of this procedure follows from noting that the AR(p) process

$$y_t = \mu + a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t-3} + \dots + a_p y_{t-p} + \varepsilon_t \quad (3.5)$$

can be written as

$$\Delta y_t = \mu + \alpha y_{t-1} + \sum_{i=2}^p \beta_i \Delta y_{t-i+1} + \varepsilon_t \quad (3.6)$$

$$\text{where } \alpha = -\left(1 - \sum_{i=1}^p a_i\right) \text{ and } \beta_i = \sum_{j=1}^p a_j$$

Thus the test comes down once again to whether  $\alpha = 0$  in the regression (3.6), in which case the equation is entirely in differences and the series has a unit root. So, the presence of serial correlation in the residuals of the regression model leads then, to a further problem for the researcher, namely, what is the appropriate lag length in the ADF test?

Overall then, there are two broad areas to be dealt with when testing the unit root hypothesis in the (A)DF framework.

The first set of questions relate to the critical values of the test statistics calculated. As noted these are non normal and have been obtained via Monte Carlo simulation. Even though these critical values are widely available in tabulated form the researcher will often want a critical value specific to her/his model and sample size. Short of carrying out the required Monte Carlo experiments one solution to this has been the estimation of response surface equations for a range of the more popular tests. These provide the researcher with a simple mechanism for the calculation of the required critical values.

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<sup>5</sup> Note that an alternative is to use the non-parametric method of correction for autocorrelation suggested by Phillips and Perron (Phillips, 1987, Perron, 1988 and Phillips and Perron, 1988) which, essentially, modifies the test statistic after estimation to correct for estimated residual autocorrelation rather than "allowing" for residual correlation in the regression equation.



The second set of questions relates to the choice of the regression model in which to test the unit root null. There are two aspects to this decision. The first relates to the choice of deterministic regressors, (constant and trend), to include in the regression model. The second choice which must be made is the lag length in the (A)DF version of the test when there is evidence of serial correlation in the residuals.

The next section, 3.2.1, outlines a testing sequence which provides guidelines for the researcher in making the decision on this latter set of questions. Once the regression model has been chosen, the appropriate critical values can be applied.

### **3.2.1 A sequential testing procedure**

Researchers have to choose which model to base testing on and, typically, in the absence of knowledge of the underlying DGP, such testing will be data based. As a result a number of testing procedures have been suggested in the literature (see Dolado, Jenkinson and Sosvilla-Rivero, 1990, Campbell and Perron, 1991 and Holden and Perman, 1994 for good surveys).

Consider, first, the order of the ADF test, where it is unlikely that the researcher will *know* the order of the autoregressive process under consideration. In practice some procedure must be adopted to select the lag order. In the empirical work which follows the following method, suggested in Campbell and Perron (1991), is adopted. Firstly, an upper bound to the ADF order is chosen, testing then proceeds from general to specific using either information criteria measures or simply testing the significance of the last included lag (stopping when the coefficient on the last included lag is significant). In all cases, tests for residual autocorrelation are carried out. Generally, where data limitations are not a problem it is considered safer to

allow for a generous lag length in the ADF. This recommendation is based on the loss of efficiency due to increased lags being a lesser problem than autocorrelation which invalidates the test statistics. Work by Said and Dickey (1984) also demonstrates that the above tests are applicable to more general ARMA(p,q) processes provided that the lag length in the ADF test is sufficiently long<sup>6</sup>.

Once the lag order required in the (A)DF is established the testing sequence moves to a consideration of which regression model to apply. The testing procedure outlined here is adapted from a number of sources including Campbell and Perron (1991), Dolado, Jenkinson and Sosvilla-Rivero (1990), Enders (1995) and Holden and Perman (1994).

**Step 1:** Begin testing with the most general model, based on equation (3.4c) This allows the unit root null to be tested against an alternative of trend stationarity<sup>7</sup>. Test the unit root null using the  $\phi_3$  test statistic which tests the null  $(\mu, \beta, \alpha) = (\mu, 0, 0)$  (remembering that 3.4c is 3.3c re-written, subtracting  $y_t$  from both sides, so the unit root test is now based on a test of  $\alpha = 0$ , rather than  $\alpha = 1$ ) using the critical values tabulated by Dickey and Fuller (1981). If the null hypothesis is rejected at this stage, proceed to step 2. If the null hypothesis cannot be rejected proceed to step 3.

**Step 2:** A rejection of the null hypothesis at this stage could be for one of three reasons. Either the trend term  $\beta$  is non zero or the coefficient  $\alpha$  is non zero (implying that there is no unit root) or both  $\alpha$  and  $\beta$

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<sup>6</sup> They demonstrate that the ADF technique is valid so long as the length of the autoregression used increases at the rate  $T^{1/3}$  as the sample size,  $T$ , increases.

<sup>7</sup> This model allows a consistent test of the null of a unit root against a trend stationary alternative under which the unit root null ensures the trend term is zero and the test has the property of similarity thanks to the additional regressor. If the trend term in the DGP is not zero the test is not similar and the Dickey Fuller distribution is invalid and the test critical values tend to asymptotic normal. If the trend is not zero and the DGP is a unit root with non zero trend then the series will have a non linear time trend.

are non zero (implying that the process is trend stationary). In this case implement the procedure suggested by Holden and Perman (1994).

On the *assumption* that the trend term is non zero then the test for a unit root in regression model (3.4c) is not similar and the distribution of the critical values for the null of a unit root is asymptotically normal. Holden and Perman, therefore, suggest testing the unit root using the t statistic (on the estimated parameter  $\alpha$ ) from regression model (3.4c) using the normal distribution. Clearly two outcomes are possible here:

**a) Do not reject unit root null.**

If rejection of the null of a unit root using standard normal critical values is not possible then, even if the trend term  $\beta$  is zero, it would not be possible to reject the unit root since, in that case, the distribution would follow the non standard Dickey Fuller distribution and the critical values would be smaller (larger in absolute terms since they are negative). At this stage the results imply that  $\alpha=0$  (the series has a unit root) and that the rejection, via the  $\phi_3$  statistic, is due to  $\beta \neq 0$ . This implies the series has a unit root and a deterministic trend in its stationary component<sup>8</sup>.

**b) Reject unit root null.**

If, however, it is possible to reject the unit root using the standard normal distribution then at this stage then the series is either stationary around a deterministic trend or stationary with no deterministic trend and standard tests can be used to ascertain this<sup>9</sup>.

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<sup>8</sup> This seems unlikely since, as most of the macroeconomic data used in applied work is in logarithms and this would imply an ever increasing or decreasing rate of change (Nelson and Plosser, 1982).

<sup>9</sup> Remembering that the  $\phi_3$  test has already rejected the null of a unit root with zero trend  $(\alpha, \beta) = (0, 0)$ .

**Step 3:** Non rejection of the unit root null could stem from the lack of power of the test and in particular it could be that there are extraneous deterministic regressors in the model, namely the trend. The question is, therefore, whether testing should move to regression model (3.4b). As pointed out by Perron (1988), whilst such a test carries greater power it should only be used when the DGP is a random walk with zero drift. If the DGP is a random walk with non zero constant and arbitrary starting value  $y_0$  such as  $y_t = \mu + y_{t-1} + u_t$  then under the null of a unit root the process can be written as  $y_t = y_0 + \mu t + \sum_{i=1}^t u_i$ . As Perron (1988) shows, more formally, if a model with only a constant is estimated, the constant term will become a drift term which can only occur when the autoregressive parameter is set at one. In the presence of non zero drift, testing should not move to regression model (3.4b). In practice therefore, if the  $\phi_3$  test cannot reject the null of a unit root, estimate the  $\phi_2$  test statistic which tests the null  $(\mu, \beta, \alpha) = (0, 0, 0)$ . Given that the  $\phi_3$  statistic has not rejected the null that  $\beta$  and  $\alpha$  are jointly zero then this can be interpreted as implying that the series has a unit root in which case the  $\phi_2$  test can be interpreted as a test for non zero drift. If this test cannot reject the null (of zero drift) then move to step 4. If the  $\phi_2$  test rejects the null, categorise the series as a random walk with non zero drift.

**Step 4:** Carry out tests of the unit root using regression model (3.4b) and test using the  $\tau_\mu$  and  $\phi_1$  test statistics which will have the Dickey Fuller distribution. If the  $\tau_\mu$  and  $\Phi_1$  tests cannot reject the unit root categorise the variable as random walk with zero drift.

In most practical applications testing will stop here with there being no need to proceed to tests based on model (3.4a) which would require the time series to

be a zero mean driftless random walk, which is unlikely in most applied work. Tests based on model (3.4a) do, however, find applications when testing for a unit root in the residuals from estimated regressions, however, in such cases, the critical values do not have the distribution tabulated in Dickey and Fuller (1979, 1981). This is a question which will be taken up in the next chapter in the context of residual based tests for cointegration.

The above testing strategy is the one applied in the empirical applications in the rest of this thesis, and allows the testing of the unit root null in the context of an observed time series,  $y_t$ . However, the possibility that  $y_t$  is  $I(2)$  or higher also needs to be considered. If the unit root null can be rejected, using the above testing framework, when applied to  $y_t$  in its (log) level form then  $y_t$  is categorised as  $I(0)$ . If the null cannot be rejected the series is differenced and re-tested, using the above framework<sup>10</sup>, for a unit root. If the null can now be rejected the series is categorised as  $I(1)$ . If not, take the second difference and re-test, categorising as  $I(2)$  if the unit root is now rejected and so on<sup>11</sup>.

This then describes the testing framework used when applying standard (A)DF tests to applied economic data. Whilst the (A)DF test is extremely popular in applied research, due largely to its, relative, simplicity and the fact that most econometric packages carry routines which perform the calculations (and calculate the critical vales), it does have its drawbacks. One of the most frequently quoted of these is its low power. There are two, broad , reasons for this. The first is that the tests use the criteria of classical statistical inference in judging the null hypothesis. The null hypothesis is that of a unit root, and unless there is overwhelming evidence to the contrary, that null will not be

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<sup>10</sup> So essentially the regression will take the form  $\Delta\Delta y_t = \mu + \alpha\Delta y_{t-1} + \sum_{i=1}^{i=k} \Delta\Delta y_{t-i} + e_t$ .

<sup>11</sup> The AR(2) process can be written as  $p(L)y_t = e_t$ , where  $e_t$  is white noise. This can be re-written as  $(1 - \alpha_1 L)(1 - L)y_t = e_t$  when one of the roots is one. Re-writing:  $(1 - \alpha_1 L)\Delta y_t = \Delta y_t - \alpha_1 \Delta y_{t-1} = e_t$  so that if one root is one the first difference is a stationary AR process. If both roots are one then  $(1 - L)(1 - L)y_t = (1 - L)^2 y_t = \Delta^2 y_t = e_t$  and the second difference is stationary.

overturned. The second is a more general problem. The hypothesis being tested is that the root is exactly one. This defines the presence of a unit root in the time series and its non stationarity. If the root is close, but not quite one, say 0.95 then the series will in fact be stationary and, obviously, does not have a unit root. Clearly, distinguishing between roots of one and roots close to one is likely to lead to low power in such tests.

In other chapters of this thesis alternative tests of the unit root null, which attempt to address this problem, are used and the next section looks at these tests in more detail.

### **3.3 Two alternative tests**

This section describes two alternatives, to the (A)DF tests described above, for testing for the presence of a unit root in an observed time series. Essentially, each test approaches the problem from a different perspective. The stimulus to all of the tests is the low power of the standard (A)DF test. The first test described, the DF-GLS test, achieves an increase in power by a prior detrending (de-meaning) of the data based on a specific local alternative to the unit root hypothesis. The second test, the KPSS test, attempts to solve the problem by reversing the null hypothesis so that the test tests the null of stationarity against the alternative of a unit root.

#### **(i) The DF-GLS test**

The first alternative to the (A)DF is the test proposed by Elliot, Rothenberg and Stock (1992) (ERS henceforth), the DF-GLS (so called as it is a Generalised Least Squares variant of the familiar DF test) test. In their paper,

ERS derive the asymptotic power envelope for tests of the unit root hypothesis. They consider a model of the form

$$\begin{aligned} y_t &= d_t + u_t \\ u_t &= \alpha u_{t-1} + v_t \end{aligned} \tag{3.9}$$

where  $d_t$  consists of known deterministic components. In deriving the power envelope ERS consider a sequence of tests of the null of a unit root ( $\alpha = 1$ ) against a sequence of local alternatives such that  $\bar{\alpha} = 1 + \bar{c} / T$  where  $T$  is the sample size and  $\bar{c}$  is a constant against which the power function is indexed. Thus, in the case for  $c = -10$  in a sample size of say 100 the alternative hypothesis considered is  $\alpha = 0.9$ .

Having derived the power envelope for tests of the unit root hypothesis, ERS consider test statistics which approximate this power envelope. One simple test which they find does so, is a version of the (A)DF test in which the data is locally detrended or demeaned using an alternative hypothesis based on a particular choice of  $\bar{c}$  prior to running the (A)DF type test. They term these tests the DF-GLS <sup>$\mu$</sup>  (in the case of demeaned data) and the DF-GLS <sup>$\tau$</sup>  test (detrended data). Monte Carlo simulations carried out by the authors suggest that both tests make substantial power gains over existing tests for a unit root.

Whilst, as noted by the authors, the asymptotic representations of the tests are complex, they are simple to calculate. Both tests require initial demeaning or detrending of the data and a vital parameter in this is the choice of  $\bar{c}$ . ERS recommend two values for this parameter,  $-7$  when using the DF-GLS <sup>$\mu$</sup>  test and  $-13.5$  for the DF-GLS <sup>$\tau$</sup>  test<sup>12</sup>.

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<sup>12</sup> These values are somewhat arbitrary and are based upon values of  $c$  which lead to tangency of the power function to the power envelope at a power of 50%.

The tests are conducted as follows. Given a time series  $y_t$ , the unit root test is carried out in a standard (A)DF framework using regressions of the form

$$\Delta y_t^\tau = a_o y_{t-1}^\tau + \sum_{j=1}^k \Delta y_{t-j}^\tau + u_t \quad (3.10)$$

for the DF-GLS $^\tau$  test for which critical values (under the assumption that  $k=0$ ) are provided in ERS Table 1, and

$$\Delta y_t^\mu = a_o y_{t-1}^\mu + \sum_{j=1}^k \Delta y_{t-j}^\mu + u_t \quad (3.11)$$

for the DF-GLS $^\mu$  test for which critical values are given in the first panel of Table 8.5.2 of Fuller (1976)<sup>13</sup>.

$y_t^\tau$  and  $y_t^\mu$  are obtained using the equations

$$y_t^\tau = y_t - \hat{\beta}_0 - \hat{\beta}_1 t \quad \text{and} \quad y_t^\mu = y_t - \hat{\beta}_0$$

where the  $\hat{\beta}$  are estimated coefficients from regressions. This involves, firstly constructing the variables  $\tilde{y} = (y_1, (1 - \bar{\alpha}L)y_2, \dots, (1 - \bar{\alpha}L)y_T)$  and  $\tilde{z} = (z_1, (1 - \bar{\alpha}L)z_2, \dots, (1 - \bar{\alpha}L)z_T)$  where  $z$  is a constant and trend  $\{1, t\}$  for the DF-GLS $^\tau$  and simply a constant  $\{1\}$  for the DF-GLS $^\mu$  test and  $L$  is the lag operator.  $\bar{\alpha}$  is determined by the constant  $\bar{c}$  and is given by  $\bar{\alpha} = 1 + \bar{c}/T$  which, as noted above, takes the value -13.5 and -7 in the detrended and demeaned cases, respectively. The  $\hat{\beta}$  are then given by the coefficients in a regression of  $\tilde{y}$  on  $\tilde{z}$ .

Finally the researcher has the problem of determining the lag order used in equations (3.10) and (3.11) and this is carried out using the same methods as described in the previous section.

In their paper, ERS (1992) note that there are substantial power gains for using these variants of the standard (A)DF test and it will be interesting to see,



in the applied work in later chapters, whether the results from these tests lead to differing conclusions regarding the unit root hypothesis in observed economic time series.

(ii) The KPSS test

The second alternative unit root test used is the one described in Kwiatkowski, Phillips, Schmidt and Shin (1992) (the KPSS test) They do not solve the power problem, but simply turn it on its head by making stationarity the null hypothesis and thus shifting the ‘burden of proof’ in the other direction, only rejecting the null of stationarity when there is strong evidence against it. Tests utilising this idea have been developed by a number of other authors including Park (1990), Park and Choi (1988), Rudebusch (1990) and Kahn and Ogaki (1992).

The concept behind their test is relatively simple. They consider a time series which is generated as the sum of a deterministic trend, a random walk and a stationary error term:

$$y_t = \gamma t + r_t + \varepsilon_t \quad (3.12)$$

the random walk component  $r_t$  is

$$r_t = r_{t-1} + \omega_t \quad (3.13)$$

where  $\varepsilon_t$  and  $\omega_t$  are iid  $N(0, \sigma_\varepsilon^2)$  and iid  $(0, \sigma_\omega^2)$  terms and  $r_0$  is assumed fixed. Harvey (1989) deals with models of this form and notes that the random walk component will become deterministic as the term  $\sigma_\omega^2$  gets smaller. Thus the

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<sup>13</sup> ERS show that for the demeaned case the test statistic has the same asymptotic distribution as the Dickey-Fuller t statistic for the no deterministic variables case (non constant or trend), so their tables of critical values can be used.

KPSS test for stationarity is based upon the null that  $\sigma_{\varepsilon}^2 = 0$  and hence that  $\Delta r_t = 0$  so that the process  $y_t$  will be trend stationary (the initial fixed value  $r_0$  will give  $r_t$  the role of a constant term). Drawing on the model of Nabeya and Tanaka (1988) they derive an LM test statistic to test this null hypothesis.

To test the null hypothesis of trend stationarity using this method the series,  $y_t$ , is regressed on a constant and trend and the residuals,  $u_t$  retrieved. If  $\hat{\sigma}_\varepsilon^2$  is the estimated error variance of the regression then the test statistic is given by:

$$LM = \frac{\sum_1^T S_t^2}{\hat{\sigma}_\varepsilon^2} \quad (3.14)$$

$$\text{where } S_t = \sum_{i=1}^t u_i$$

the assumption about the error term  $\varepsilon_t$  can be relaxed, in particular to allow for serial correlation, using the results of Phillips (1987), Phillips and Perron (1988) and Newey and West (1987). This method corrects the derived test statistic for autocorrelation, not by including extra regressors in the regression model, as in the ADF test, but rather, by accounting for the autocorrelation by correction to the test statistic after estimation. Indeed, using this method, Phillips (1987) and Phillips and Perron (1988) suggest variants of the standard ADF tests described above, frequently referred to as the  $Z_\tau$ ,  $Z_{\tau_\mu}$  and  $Z_{\tau_\tau}$  tests which are the equivalent of the  $\tau$ ,  $\tau_\mu$  and  $\tau_\tau$  statistics described above.

Define the estimator,  $s^2(l)$  of the long run variance  $\sigma^2$  to be:

$$s^2(l) = T^{-1} \sum_{t=1}^T u_t^2 + 2T^{-1} \sum_{s=1}^l w(k,l) \sum_{t=s+1}^T u_t u_{t-s} \quad (3.15)$$

where  $w(k,l)$  is a weighting function which guarantees the nonnegativity of  $s^2(l)$  as in Newey and West (1987) and is given by  $w(k,l) = 1 - k/(l + 1)$ . Using this estimate of the long run variance in place of  $\hat{\sigma}_\varepsilon^2$  Kwiatkowski et al (1992) derive two test statistics, denoted  $\hat{\eta}_\mu$  and  $\hat{\eta}_\tau$ , the first based on a regression of  $y_t$  on a constant only, the second based on a regression of  $y_t$  on a constant and trend. Thus the respective null hypothesis for the two test statistics are level stationarity and trend stationarity and the tests are given by

$$\hat{\eta}_\mu = T^{-2} \sum_{t=1}^T S_t^2 / s^2(l) \quad (3.16a)$$

$$\hat{\eta}_\tau = T^{-2} \sum_{t=1}^T S_t^2 / s^2(l) \quad (3.16b)$$

Kwiatkowski et al (1992) derive asymptotic distributions and critical values for the test statistics, given in their Table 1. They also carry out an extensive set of Monte Carlo simulations to test the size and power characteristics of the tests statistic. Their simulations reveal a clear trade off between size and power. In the presence of autocorrelated errors, small sample size and low values for  $l$  lead to considerable size distortions in the test statistic. On the other hand higher values of  $l$  lead to lower power. As a result, in empirical applications of this test the statistics are calculated for a range of values of  $l$ .

Once again interest will centre, in the empirical applications, on whether this test, with its reversed null, yields different results regarding the order of integration of the data.

### 3.4 Testing for a unit root in the presence of a structural break

One further type of unit root test carried out in this thesis involves the testing of the unit root null hypothesis against an alternative which allows the series to

be stationary around a breaking, deterministic trend. This type of test has been considered in the literature in a number of papers (Banerjee, Lumsdaine and Stock, 1992, Perron, 1989, 1990a, 1990b, Perron and Vogelsang, 1992, Rappoport and Reichlin, 1989, Zivot and Andrews, 1992). Essentially these papers point out that the unit root tests described above are based on the null of a unit root and, an alternative hypothesis of stationarity around, at best, a deterministic trend. If, in fact, the series is generated by a process which is stationary around a deterministic, but, breaking trend function then unit root tests based on standard A/DF procedures will be biased in favour of non rejection of the unit root null. Perron (1989, pg 1370) notes

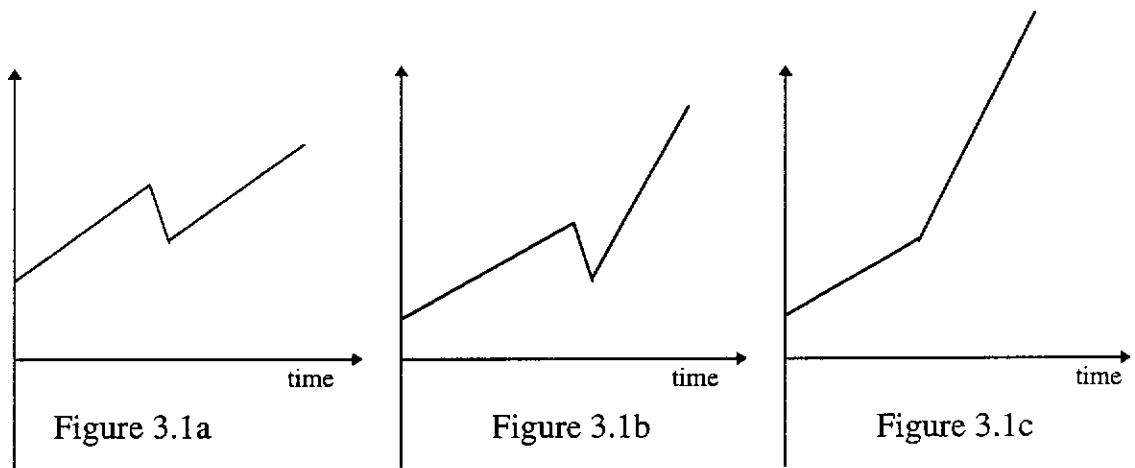
"...if the magnitude of the shift is significant, one could hardly reject the unit root hypothesis even if the series is that of a trend (albeit with a break) with i.i.d. disturbances. In particular, one would conclude that shocks have permanent effects. Here, the shocks clearly have no permanent effects, only the one time shift in the trend function is permanent"

The types of breaks considered can best be illustrated by figures 3.1(a-c) below which illustrate the effects of the exogenous event on the deterministic trend. Three basic types of trend breaks can be considered:

1. firstly a model in which the trend function had a one off shift in mean, a crash (Figure 3.1a and referred to as model (a) below)
2. secondly one with a change in the rate of growth and in the mean occurring at the same time (Figure 3.1b, referred to as model (b))
3. finally one with only a change in growth rate (Figure 3.1c, model (c)).

The choice of which model is most appropriate is left to the researcher and based on observation of the series in question.

Perron (1989) carries out a simulation exercise to demonstrate that the standard Dickey-Fuller type test for a unit root will be biased against rejection of the null of a unit root when the series under question contains a break in its deterministic trend function. Taking one of the simplest examples used by Perron (Perron, 1994) demonstrates the point.



Suppose a time series is generated using the following DGP

$$y_t = \mu_1 DU_t + e_t \quad (t=1, \dots, T) \quad (3.17)$$

where  $DU = 1$  if  $t > T_b$  and 0 otherwise and  $e_t \sim \text{IID. } N(0,1)$ ,  $T=100$  and  $T_b$  (the time of the break) = 50.

This series will have mean 0 up to  $T_b$  and mean  $\mu_1$  thereafter and is stationary since  $e_t$  is assumed IID. Using the artificially generated data Perron then uses an equation such as:

$$y_t = \mu + \beta t + \alpha y_{t-1} + \sum_{i=1}^k c_i \Delta y_{t-i} + u_t \quad (3.18)$$

to carry out standard Dickey-Fuller tests on the data. By carrying out a large number of replications of this experiment Perron demonstrates that the

estimated value of  $\alpha$  is biased towards 1 as the size of the crash ( $\mu_1$ ) is increased and that the power of the Dickey - Fuller test is reduced. Even in this simple example, using 5,000 replications, Perron (1994) finds that the power of the Dickey Fuller test falls to zero when  $k = 0$  and  $\mu_1 = 25$ . As Perron notes, whilst these may seem to be extreme values the series under question has an IID noise component and the power of the tests will fall more rapidly, for much smaller shifts in the mean, when, as would be expected in practice, the error term is autocorrelated.

Whilst the basic point made is simple, the testing framework is complicated by the wide range of tests and test procedures this work has generated. In the initial paper (Perron, 1989) the dates of the crash were taken as being given (1929, the great crash and 1973 the oil price shock). By doing this, he generated a set of critical values for the tests based on the assumption of an assumed date for  $T_b$ . However, this assumption was criticised by an number of researchers (Banerjee, Lumsdaine and Stock, 1992, Zivot and Andrews, 1992, Christiano, 1992) who made the point that, essentially, Perron had biased his results by, as it were, "peeking" at the data and picking dates which would best support his thesis. Perron had chosen the dates, assumed they were given exogenously, and generated test statistics on that basis. These other authors argued that the break dates were in fact correlated with the data, and, quite correctly, argued that this affects the distribution of the test statistics. In effect the critical values used by Perron would be incorrect (too small), thus meaning that he would reject the null too frequently. The result was that all of these authors suggested that recursive techniques should be utilised to allow the data to determine the break point.

Thus, the first problem is that of selecting which model is most applicable and secondly selecting, using data based searches, the break point in the data. A third complication can now be introduced. The figures above (3.1a-c) make the

assumption that the break occurs instantaneously. This is frequently termed the additive outlier model (using the terminology of Box and Tiao (1975)). However it may well be more realistic, in practice, to assume that the change in the trend occurs gradually. This second case is termed the innovational outlier model and once again a different testing procedure and set of test statistics, according to which model is chosen, apply and, once again, the choice depends largely upon observation of the series in question.

Table 3.1 below summarises the models considered and the regression equations used to test the breaking trends model. Note that in the case of the additive outlier model two regression equations are estimated, the first detrends the data and the second tests the unit root null in an autoregression of the noise component (the residuals). For the innovation outlier models only one regression, which nests the null and alternative hypotheses, is required. As usual, testing of the unit root uses the t statistic on  $\alpha$  and once again critical values for the test statistics have to be derived from Monte Carlo simulation. Perron (1990a, 1990b, 1994), Banerjee, Lumsdaine and Stock (1992) and Zivot and Andrews (1992) provide such test statistics for a wide variety of models, each based on the approach taken in selecting lag orders in the autoregressions and the time of the break.

In the context of the above models two variables need to be determined, these are  $T_b$ , the point of the break and  $k$ , the order of the autoregressive parameter. In the empirical applications in later chapters the following method for determination of  $T_b$  and  $k$  is utilised. The regression equation was estimated recursively with each recursion moving the time of the break,  $T_b$ , one quarter further on through the sample. Following the suggestion in Banerjee, Lumsdaine and Stock (1992), the time of the break was allowed to vary through the range  $0.15T$  to  $0.85T$  where  $T$  is the number of observations in the

series, thus not allowing the break point to take values at the beginning or end of the sample.

**Table 3.1**

**Additive outlier models**

	Step 1 - detrend original series	Step 2 estimate autoregression
Model (a)	$y_t = \mu + \beta t + \gamma DU_t + \tilde{y}_t$	$\tilde{y}_t = \alpha \tilde{y}_{t-1} + \sum_{i=0}^k d_i D(T_b)_{t-i} + \sum_{j=1}^k a_j \Delta \tilde{y}_{t-j} + e_t$
Model (b)	$y_t = \mu + \beta t + \theta DU_t + \gamma DT_t^* + \tilde{y}_t$	$\tilde{y}_t = \alpha \tilde{y}_{t-1} + \sum_{i=0}^k d_i D(T_b)_{t-i} + \sum_{j=1}^k a_j \Delta \tilde{y}_{t-j} + e_t$
Model (c)	$y_t = \mu + \beta t + \gamma DT_t^* + \tilde{y}_t$	$\tilde{y}_t = \alpha \tilde{y}_{t-1} + \sum_{j=1}^k a_j \Delta \tilde{y}_{t-j} + e_t$

**Innovation outlier models**

Model (a)	$y_t = \mu + \beta t + \gamma DU_t + \delta D(T_b) + \alpha y_{t-1} + \sum_{i=1}^k a_i \Delta y_{t-i} + e_t$
Model (b)	$y_t = \mu + \beta t + \theta DU_t + \gamma DT_t^* + \delta D(T_b) + \alpha y_{t-1} + \sum_{i=1}^k a_i \Delta y_{t-i} + e_t$

where:

$T_b$  is the date of the break,  $DU_t = 1$  and  $Dt_t^* = (t - T_b)$  if  $t > T_b$  and 0 otherwise,  $D(T_b)_t = 1$  if  $t = T_b + 1$  and 0 otherwise.

For each  $T_b$  the regression was estimated with lags ranging from 0 to  $k$  where  $k$  was set at a maximum of 8. Then, for each  $T_b$  the lag order was determined



using a t tests on the last included lag.  $K$  was determined such that the coefficient on the last included lag was significant, but that on higher order lags was not significant. Information criteria, such as the Akaike Information Criteria were not used since in the empirical applications these tended to choose a low order for the lag parameter and the resultant residuals frequently exhibited serial correlation.

### **3.5 Conclusion**

The testing for unit roots in economic time series is a vital part of today's applied economists tool-kit. Whilst the evidence is still generally in favour of the hypothesis that many macroeconomic time series contain a unit root the original testing procedure has been shown to be biased against rejection of the null of a unit root. The low power of the original tests has spurred econometricians on to develop testing strategies which improve the power of the testing framework. To this end it seems advisable, where there is any doubt, to carry out a range of unit root tests. The above discussion has detailed the unit root tests used in this thesis. Whilst the tests statistics are often easy to construct they are plagued with pitfalls for the unwary applied economist, typically being non normally distributed, with asymptotically valid critical values, dependent on the role of deterministic variables and sensitive to residual autocorrelation. Despite these problems it is vital, as one of the first steps in applied work, to attempt to determine the time series properties of data before carrying out estimation. If data are stationary standard estimation procedures are valid, if not the researcher needs to consider the possibility of cointegrating relationships amongst the data and or stationary transformations of the data.

Testing for such cointegrating relationships between non stationary series is the subject of the next chapter.

## CHAPTER 4

### TESTING FOR CO-INTEGRATION.

#### 4.1 Introduction

The previous chapter might suggest that the presence of integrated variables makes the practice of econometric modelling more difficult. However, as noted by Campbell and Perron (1991), in the title of their survey paper, there are both “pitfalls” and “opportunities” for the researcher when dealing with non stationary data. Perhaps the main opportunities for the applied economist stem from the development of tests for cointegration. This chapter outlines the econometrics of the three main tests for cointegration which are used in the applied work in later chapters. Of these three, two are residual based tests, the Engle-Granger (1987) test and the Gregory and Hansen (1996) test. The third, associated with Johansen (1988, 1991) is a Maximum Likelihood estimator and is probably the most popular applied test in the literature today.

A formal definition of the term cointegration is required to understand the preceding paragraph, however, at this stage it can be noted that cointegration between time series implies that the series, which are assumed integrated of the same order,  $I(1)$ , for simplicity, move in a long run equilibrium<sup>14</sup> relationship such that deviations from this relationship are stationary or  $I(0)$ . Thus consider two  $I(1)$  variables  $x_t$  and  $y_t$  then cointegration between  $x$  and  $y$  would imply that there exist a linear combination  $x_t - \alpha y_t = u_t$  such that  $u_t$  is  $I(0)$ . This would be an “unusual” result in that it would be expected that the linear combination of two  $I(1)$  variables to result in an  $I(1)$  variable, or, more

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<sup>14</sup> Where the term equilibrium here implies that deviations from this relationship are bounded and stationary.

generally, that any linear combination of integrated variables will yield a series which is integrated at the highest order of the component series.

Attempting to model relationships between integrated data could lead to one of two “solutions”. Firstly eliminate the non stationarity by differencing the data at a sufficient level to make it stationary. Thus, if the data were  $I(1)$ , difference once, and model using standard techniques. This however presents a problem. Firstly, many economic hypothesis are phrased in terms of the levels of variables, and secondly, differencing loses any long run information that the data contains. Such long run information is often seen as essential in econometric models. Thus a second possible response, which may seem initially foolish, is to ignore the problems of integrated data, run regressions using levels of data in order to capture long run equilibrium relationships in the data and risk, as it were, the spurious regression problem. Indeed one of the most popular of the “econometric” models, developed by Sargan (1964), Hendry and Anderson (1977) and Davidson, Hendry, Srba and Yeo (1978) is the Error Correction Model (ECM) which used a mixture of levels variables to capture the long run equilibrium, and differenced variables to capture short run dynamics. Such models make good economic sense and yet, as noted above, make no econometric sense at first glance, but thanks to the concept of cointegration are a natural development. Thus to characterise the idea consider the factors influencing the rate of wage inflation. Clearly one factor will be the rate of price inflation, agents bargaining for a real wage will change nominal wages in the light of changes in prices. Thus:

$$\Delta w = f(\Delta p). \tag{4.1}$$

However agents may well also have a target real wage  $w/p$ . If the current real wage is below the target real wage this will exert additional upward pressure on wages, so:

$$\Delta w_t = f\{\Delta p_t, (w/p)_{t-1}\} \quad (4.2)$$

The problem here of course is that the model is mixing the orders of integration in our regression. If  $p$  and  $w$  are both  $I(1)$  then  $\Delta p$  and  $\Delta w$  will be  $I(0)$  whilst  $w/p$  will be  $I(1)$ . Thus the regression equation is unbalanced, the left hand side variable is  $I(0)$  whilst the right hand side is  $I(1)$ . However if, in the light of our simple description above,  $w$  and  $p$  are cointegrated such that  $w/p$  is  $I(0)$  then the regression equation based on the above formulation will be valid. Clearly a vital part of this modelling process would be ensuring that the linear combination of integrated variables used in the regression did indeed form a cointegrating set.

This is where the developments in the theory of and testing for cointegrated variables have provided opportunities for macroeconomists hoping to model economic time series. Cointegration helps us in a number of ways. Firstly tests have been developed which allow us to identify the linear combination, or combinations in the case of more than two variables, of a set of integrated variables which satisfy the condition for cointegration, i.e. produce a linear combination of the series which is of a lower order of integration. Typically, since *most* macroeconomic time series are  $I(1)$  this reduces to the idea that it is possible to identify linear combinations which are  $I(0)$  or stationary. Secondly the so called Granger Representation theorem (Engle and Granger, 1987) tells us that any set of cointegrated variables have a valid error correction representation, indeed they also have valid vector autoregressive and moving average representations all of which are isomorphic to one another. This has obvious advantages for econometric modellers. The answer to the question as to whether the ECM formulation is valid depends upon whether the linear combination of the levels variables cointegrates or not. Similarly if it is found

that a set of variables are cointegrated then there does exist a valid ECM formulation for these variables.<sup>15</sup>

A more formal definition of cointegration can be found in Engle and Granger (1987) and essentially they say the following. If  $y_t$  is a vector of variables which are individually  $I(d)$  then cointegration occurs when there exists a non zero vector  $\beta$  such that  $\beta'y_t \sim I(d-b)$  where  $d \geq b > 0$ . This definition allows for higher orders of integration of the variables and notes that cointegration does not necessarily mean that the resultant series is stationary. Thus for example a set of  $I(2)$  series may cointegrate to produce an  $I(1)$  series, this is frequently written as  $CI(2,1)$  where the first term indicates the order of integration and the second term the order of cointegration.

A helpful way of thinking about what is happening when a set of  $I(1)$  variables cointegrate can be found in Stock and Watson (1988a,b) who note that cointegrated variables must share one or more common stochastic trends. Thus for example two random walk processes will contain stochastic trends, the cointegrating vector (if it exists) is that linear combination of the random walks which cancels out the stochastic trend. Essentially the stochastic trend driving the processes must be the same after a simple linear transformation, this linear transformation is the cointegrating vector. In the case of  $n$  random walks then there are  $n$  stochastic trends driving these individual series and once again a cointegrating vector, if it exists, will be a linear combination of the random walks which purges the stochastic trend. Of course in this case there may be more than one linear combination of the trends which leads to their canceling out, hence there can exist, for any set of  $n$  variables  $(n-1)$  cointegrating vectors.

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<sup>15</sup> See Engle and Granger (1987) and Banerjee et al (1993) for an excellent discussion of these relationships.

The empirical work in this thesis uses two of the most popular methods of testing for cointegration between time series variables, the Engle Granger method and the Johansen method. The next two sections (4.2 and 4.3) briefly outline these two methods of estimating cointegrating vectors. Again the discussion below is not exhaustive<sup>16</sup> but rather simply covers the background to the techniques used in the thesis, excellent references for further reading would be Banerjee et al (1994), Dickey and Rossana (1994). Finally section 4.4 considers a recently developed, residual based test for cointegration in the presence of a single shift in the cointegrating vector, outlined in Gregory and Hansen (1996).

#### **4.2 The Engle Granger Two Step method**

We begin by looking at the Engle Granger (EG henceforth) two step method for identification of cointegration vectors. Despite its limitation this is still a popular method in the applied literature and has the advantage of simplicity in the estimation procedure. In particular the method allows estimation of the cointegrating vector without the necessity of modelling the dynamics until the error correction model is estimated, thus the first stage involves only linear OLS in the levels of the variables<sup>17</sup>.

Essentially the EG method can be explained by considering the time series properties of the data under consideration. Thus suppose that the appropriate unit root test procedures have been applied and the researcher has concluded that two sets of time series data  $x$  and  $y$  are integrated order 1 - are both  $I(1)$ . It should be the case that any linear combination of these data will also be  $I(1)$ .

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<sup>16</sup> Thus for example we do not cover the Stock and Watson (1988) method of identification of cointegration vectors which is also popular in applied work.

<sup>17</sup> This does however lead to a possible loss of power of the cointegration test outlined here. Kremers, Ericsson and Dolado (1992) note that the method of testing cointegration using a Dickey Fuller test on estimated residuals implies an implicit common factor restriction, which if invalid leads to a loss of power. This is one possible reason for often contradictory results on cointegration from tests using this method and tests using, say, Johansens (1988) method.

Thus a-priori it would be expected that  $z = x - \beta y$  would also be  $I(1)$ , if it is not, if it is a lower order of integration, say  $I(0)$ , then, in the light of the foregoing it would be the case that  $x$  and  $y$  are cointegrated ( $CI(1,1)$ ) with cointegrating parameter  $\beta$ .

This suggests an obvious estimation strategy:

- estimate the linear regression  $x_t = a_0 + \beta y_t + u_t$
- take the estimated residuals  $\hat{u}_t$  and carry out a DF or ADF test on them to establish their order of integration. This involves estimating a Dickey-Fuller test statistic of the form:  $\Delta \hat{u}_t = \alpha_0 \hat{u}_{t-1} + \sum_{i=1}^k \alpha_i \Delta \hat{u}_{t-i} + \varepsilon_t$  using the retrieved residuals from the static OLS regression.

If the results of this procedure suggest that the residuals are of a lower order of integration than the original series ( $x$  and  $y$ ) then infer that  $x$  and  $y$  cointegrate with  $\beta$  as the cointegrating parameter. This is the first step in the EG two step method. The  $t$  statistic for the test of the null of no cointegration is the normal  $t$  statistic on  $\alpha_0$  ( $H_0: \alpha_0=0$ ) in the above Dickey Fuller test. The null hypothesis, in the context of the unit root test, is that the residuals do contain a unit root. Non rejection thus implies that the residuals are  $I(1)$  and that  $x$  and  $y$  do not cointegrate. If it is possible, at normal significance levels, to reject the null then the implication is that the residuals are stationary ( $I(0)$ ) and that  $x$  and  $y$  cointegrate. The only departure from standard unit root testing in this framework is that the Dickey Fuller tables of critical values are not applicable since the series tested has been obtained from the linear regression, since the actual  $\hat{u}_t$  are not known (a-priori) only the estimate of  $\hat{u}_t$  is available<sup>18</sup>.

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<sup>18</sup> If the cointegrating vector parameters are known a-priori, perhaps given by theory, then the constructed residuals can be tested using standard unit root critical values.

Fortunately Engle and Yoo (1987) and MacKinnon (1991) provide test statistics.

Stage two of the EG method involves taking the residuals from the levels “cointegrating” regression and using these as the error correction term in the estimated ECM. In other words estimate:

$$\Delta x_t = \sum_{i=1}^n \delta \Delta x_{t-i} + \sum_{i=1}^n \delta \Delta y_{t-i} + \lambda (x_{t-1} - a_0 - \hat{\beta}_1 y_{t-1}) + \varepsilon_t \quad (4.3)$$

where  $x_{t-1} - a_0 - \hat{\beta}_1 y_{t-1} = \hat{u}_{t-1}$ .

Engle and Granger (1987) demonstrate in their “Theorem 2” that such a procedure is valid. Stock (1987) shows that when  $x$  and  $y$  cointegrate the estimated parameter  $\hat{\beta}_1$  converges to its true value more rapidly than normal estimators<sup>19</sup>. Granger and Engle use this result to show that the two step estimator of the error correction which they propose by simply using the estimator  $\hat{\beta}_1$  has the same limiting distribution as the Maximum Likelihood estimator using the true value and that least squares standard errors will be consistent estimators of the true standard errors. The near VAR in (4.3) contains only first difference terms, with the exception of the error correcting term, all variables are  $I(0)$  and efficient estimates can be obtained via standard OLS methods.

It is this simplicity which led to the enormous popularity of the EG two step method. However, as noted below there are difficulties associated with this method, and it is these which have led to the popularity of the second method of estimating cointegrating vectors which is used in this thesis - the Johansen method.



### 4.3 Johansen Estimation

There are a number of relatively unsatisfactory aspects of the Engle Granger two step method for estimating cointegrating vectors.

- Firstly the fact that it involves two steps, taking the residuals from the cointegrating vector and using them in an auxiliary regression, allows for the potential compounding of errors.
- Secondly, the procedure requires an arbitrary normalization and the results are often not invariant to the normalization in the finite samples applied economists normally deal with (and indeed it is potentially time consuming to test all of the alternative normalization's in a multiple variable setting).
- Thirdly the method only identifies a single cointegrating vector amongst the  $n$  variables and it is the case that for any  $n$  variables there are potentially  $(n-1)$  cointegrating vectors.
- Fourthly the method does not allow straightforward testing of hypothesis regarding the cointegrating vectors.
- Fifth small sample bias is likely to be a problem in sample sizes of the sort normally considered in applied work.
- Finally, and related to the above point. Whilst asymptotically it is true that the dynamics of the processes involved are  $I(0)$  and hence should have no effect on inference. In finite samples this is unlikely to be the case and some method of allowing for the effect of dynamics must be considered<sup>20</sup>.

Johansen (Johansen(1988, 1991) and Johansen and Juselius (1990)) estimation has the advantage of providing answers to all of the above problems as it

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<sup>19</sup> The so called "super consistency theorem"

<sup>20</sup> One method, not considered here, is the use of fully modified estimators, see Park and Phillips (1988), Phillips and Durlauf (1986), Phillips and Hansen (1990) and Phillips (1988, 1991) and Banerjee et al (1994) for an excellent survey.

allows estimation and testing of all the cointegrating vectors which exist among a set of time series and is based on the error correction representation of the VAR(p) model:

$$\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \mathbf{A}_3\mathbf{y}_{t-3} \dots + \mathbf{A}_p\mathbf{y}_{t-p} + \mathbf{u}_t \quad (4.4)$$

where  $\mathbf{y}$  is an  $(n \times 1)$  vector of  $I(1)$  variables. Subtracting  $\mathbf{y}_{t-1}$  from both sides gives:

$$\Delta\mathbf{y}_t = (\mathbf{A}_1 - \mathbf{I})\mathbf{y}_{t-1} + \mathbf{A}_2\mathbf{y}_{t-2} + \mathbf{A}_3\mathbf{y}_{t-3} + \dots + \mathbf{A}_p\mathbf{y}_{t-p} + \mathbf{u}_t$$

then, by adding and subtracting  $(\mathbf{A}_1 - \mathbf{I})\mathbf{y}_{t-2}$ , and re-arranging:

$$\Delta\mathbf{y}_t = (\mathbf{A}_1 - \mathbf{I})\Delta\mathbf{y}_{t-1} + (\mathbf{A}_2 + \mathbf{A}_1 - \mathbf{I})\mathbf{y}_{t-2} + \mathbf{A}_3\mathbf{y}_{t-3} + \dots + \mathbf{A}_p\mathbf{y}_{t-p} + \mathbf{u}_t \quad (4.5)$$

continuing this process finally yields:

$$\Delta\mathbf{y}_t = \sum_{i=1}^{p-1} \Pi_i \Delta\mathbf{y}_{t-i} + \Pi\mathbf{y}_{t-1} + \mathbf{u}_t \quad (4.6)$$

where

$$\Pi = -\left( \mathbf{I} - \sum_{i=1}^p \mathbf{A}_i \right)$$

$$\Pi_i = -\left( \mathbf{I} - \sum_{j=1}^i \mathbf{A}_j \right)$$

The Johansen procedure is based on maximum likelihood estimation of this equation and centres on consideration of the matrix  $\Pi$ . The right hand side of the equation will be stationary only if the components of  $\Pi y_{t-p}$  are stationary (the left hand side will be stationary for the case where  $y$  contains only I(1) variables). Three cases of interest arise: (i) if  $\Pi$  is of full rank,  $n$ , then all of the elements of  $y$  are stationary, (ii) if  $\text{rank } \Pi = 0$  then there are no combinations of the  $y$  which are stationary, i.e. there are no cointegrating vectors, (iii) if  $\text{rank } \Pi$  is  $r$  such that  $0 < r < n$  (where  $n$  = the number of variables in  $y$ ) then the  $y$  variables do cointegrate and there exist  $r$  cointegrating vectors. The rank order of the matrix  $\Pi$  can be found by an analysis of its characteristic roots. Thus, for a square ( $n \times n$ ) matrix  $\Pi$ , its determinant is equal to the product of its characteristic roots:

$$|\Pi| = \prod_{i=1}^n \lambda_i \text{ where the } \lambda_i \text{ are the roots of the characteristic equation.}$$

So the three cases considered above can be examined by testing for the number of non zero characteristic roots:

- For the matrix  $\Pi$  to be of full rank would require  $|\Pi| \neq 0$  and thus all of the characteristic roots will be non zero.
- For the matrix  $\Pi$  to be of zero rank would require all of the characteristic roots to equal 0.
- For the matrix  $\Pi$  to have rank  $r < n$  would require the matrix to have  $r$  characteristic roots which take non zero values and  $(n - r)$  characteristic roots which take the value zero.

Johansen identifies two test statistics which will, essentially test whether the estimated characteristic roots or eigenvalues of the matrix  $\Pi$  are significantly different from zero<sup>21</sup>, these are:

$$\begin{aligned}\lambda_{trace}(r) &= -T \sum_{i=r+1}^n \ln(1 - \hat{\lambda}_i) \\ \lambda_{max}(r, r+1) &= -T \ln(1 - \hat{\lambda}_{r+1})\end{aligned}\tag{4.7}$$

where the  $\lambda_i$  are the  $n$  distinct eigenvalues ordered by size, so  $\lambda_1$  is the largest eigenvalue and  $\lambda_n$  the smallest. The first test statistic,  $\lambda_{trace}$  tests the null of at least  $r$  cointegrating vectors against the general alternative that there are more cointegrating vectors (some of the eigenvalues  $\lambda_{r+1}, \lambda_{r+2} \dots \lambda_n$  are non zero) whilst the second test statistic,  $\lambda_{max}$  tests the null of at least  $r$  cointegrating vectors against the alternative of  $r+1$  cointegrating vectors.

Once the rank of  $\Pi$  has been determined then it is possible to use the eigenvectors associated with the largest  $r$  eigenvalues (which determined the cointegration rank of the matrix) to form the matrix of cointegrating vectors  $\beta$ . If for example in a 4 variable system we found, using the above test statistics, that  $r = 2$  (that there were 2 cointegrating vectors) then the eigenvectors associated with the 2 largest eigenvalues could be used to form the  $(4 \times 2)$  matrix  $\beta$ . Next form the matrix  $\alpha$  such that  $\alpha\beta' = \Pi$  with the interpretation that  $\beta$  is the matrix of cointegrating parameters and  $\alpha$  a matrix of speeds of adjustment<sup>22</sup>.

One complication in applying the above tests is the decision on how, if at all, any deterministic variables such as a constant should enter the estimation since

<sup>21</sup> Johansen and Juselius (1990) and Osterwald-Lenum (1992) also provide tabulated sets of critical values for the test statistics.

<sup>22</sup> This decomposition  $\Pi = \alpha\beta'$  is not unique but does define the space spanned by the cointegrating vectors. As such it will allow us to test whether restrictions on the parameters of interest lie within this space.

these can affect the asymptotic distribution of the test statistic for cointegration. Thus (4.6) could be extended to include a constant term:

$$\Delta y_t = \sum_{i=1}^{p-1} \Pi_i \Delta y_{t-i} + \Pi y_{t-i} + \mu + u_t \quad (4.8)$$

where  $\mu$  is a vector of 1's.

The presence of  $\mu$  in the model implies a linear trend in the data generation process, and it is frequently suggested that its inclusion should be based on an inspection of the series, to see if they have a tendency to increase or decrease, or could be based on the unit root testing - the finding that the variables were random walks with drift. Thus one possibility is to allow the constant term to enter the model unrestrictedly, it will imply a drift term in the non stationary part of the process and hence trending variables. However a second possibility is that a constant term enters only in the cointegrating relationships such that  $\mu = \alpha\beta_0$  so that (4.6) can be re-written as:

$$\Delta y_t = \sum_{i=1}^{p-1} \Pi_i \Delta y_{t-i} + \alpha(\beta' y_{t-i} + \beta_0) + u_t \quad (4.9)$$

Allowing the constant term to enter the model restrictedly (allowing it to only enter the Error Correction Mechanism via the cointegrating relationship) means that the variables will not be trended, it simply changes the mean value of the ECM. As a result estimation can be carried out under one of three assumptions regarding the constant term:

- a constant enters but it is restricted to the cointegration space, implying no linear trend in the variables
- a constant enters unrestrictedly and it imparts a linear trend to the variables
- a constant is excluded completely

Johansen and Juselius (1990), in a paper examining the demand for money using Finnish and Danish data, outline a simple test for how to enter the constant term in cointegration tests, Johansen (1991) succinctly summarizes these results and Osterwald-Lenum (1992) provides critical values under alternative assumptions. Essentially the test is based upon two estimates of the cointegrating rank, firstly allowing the constant to enter restricted to the cointegrating vector and secondly allowing the constant to enter unrestrictedly.

So, first the model is estimated with an unrestricted constant which yields estimates of the  $n$  eigenvalues, ordered by size  $\hat{\lambda}_1, \hat{\lambda}_2, \dots, \hat{\lambda}_n$ , next estimate the model allowing the constant to enter, restricting it to the cointegration space, and obtain a second set of eigenvalues ordered as  $\hat{\lambda}_1^*, \hat{\lambda}_2^*, \dots, \hat{\lambda}_n^*$ . Johansen (1991) shows that the test statistic:

$$-T \sum_{i=r+1}^n \left[ \ln(1 - \hat{\lambda}_i^*) - \ln(1 - \hat{\lambda}_i) \right] \quad (4.10)$$

has a  $\chi^2$  distribution with  $(n - r)$  degrees of freedom and that a significant value of the test statistic indicates that it is possible to reject the restriction that the constant enters only the cointegration space.

In the same way tests of restrictions on the  $\alpha$  and  $\beta$  matrices can also be carried out, again by estimating the eigenvalues without the restriction,  $\hat{\lambda}_1, \hat{\lambda}_2, \dots, \hat{\lambda}_n$  and with the restriction,  $\hat{\lambda}_1^*, \hat{\lambda}_2^*, \dots, \hat{\lambda}_n^*$  and setting up the test statistic as:

$$-T \sum_{i=1}^r \left[ \ln(1 - \hat{\lambda}_i^*) - \ln(1 - \hat{\lambda}_i) \right] \quad (4.11)$$

where, now,  $r$  is the number of cointegrating vectors. If the restriction is acceptable then once again the test statistic should be zero, a significant value indicating rejection of the restriction. Tests of this form allow the researcher to

test particular restrictions such as may be suggested by theory (including zero restrictions) on the parameters of the cointegrating vector.

In practice the implementation of the test procedure is relatively straight forward:

- Firstly pre-test the data. Typically, in macroeconomics the assumption will be that the data are  $I(1)$ . Useful information here will be whether the data appears to be random walk with or without drift since this will affect the specification of the model, i.e. whether the constant term should be allowed to enter unrestricted or restricted to the cointegration space.
- Second, establish the correct lag order for the VAR. This is achieved using a VAR in levels. Standard diagnostic testing of the VAR for residual autocorrelation and non normality of residuals should be carried out<sup>23</sup>.
- Estimate the model and test the rank of the matrix using the above methods<sup>24</sup>. Once the number of cointegrating vectors has been established their stability and interpretation can be tested using some of the tests described above. Thus tests on parameters or the validity of particular restrictions can be tested.
- The cointegrating vectors can now be used in a correctly specified Vector Error Correction Model in which causality testing can be carried out and a correctly specified structural model identified.

Johansen's method of testing for cointegrating vectors between time series is now, probably, the most popular of the tests used in applied work. It has the particular advantage of allowing tests of parameter restrictions to be carried

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<sup>23</sup> Cheung and Lai (1993) provide a useful survey of the robustness of the tests to lag length and non normalities.

<sup>24</sup> Most of the estimation in this thesis uses PC-GIVE and PC-FIML professional version 8.0 (Doornik and Hendry (1994)) however many other software packages such as Microfit (Peseran and Peseran (1991)) will also carry out the required estimation. For an excellent survey on the details of the estimation method see Dickey and Rosanna (1994).

out on the cointegrating vector, thus allowing the testing of economic hypothesis about the long run equilibrium relationships amongst the data.

#### **4.4 Testing for cointegration in the presence of a regime shift**

The final test used in this thesis allows testing for cointegration in the presence of a potential regime shift using the method described in Gregory and Hansen (1996). This test proves particularly useful in the applied work on labour markets in chapter 6.

Essentially, Gregory and Hansen (1996) consider the possibility that the cointegrating vector which exists between a set of integrated variables might be subject to a simple shift, either a mean shift or a one off change in the slope parameters. The concept of cointegration allows for the testing of whether there exists a linear combination of individually integrated series which is integrated of a lower order (usually  $I(1)$  series which become stationary on their linear combination). Gregory and Hansen's test allows consideration of this hypothesis in a framework where that linear combination is subject to a single change. The test is similar, in a multivariate context, to tests for a unit root in the presence of a break in the underlying deterministic trend<sup>25</sup> (see Perron, 1989, 1990a, 1990b, Banerjee, A., Lumsdaine, R.L. and Stock, J.H., 1992)

Applied economics suggests a number of examples where this might well be appropriate. The most obvious (and the one tested by Gregory and Hansen (1996)) is perhaps the relationship between the variables in the demand for money equation. Economic theory would suggest that the variables in the demand for money equation move in a long run equilibrium relationship. However it could easily be argued that this equilibrium relationship was

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<sup>25</sup> Lewis and MacDonald (1993) consider these tests in the context of a subset of variables commonly used in Australian wage equation specifications.



disturbed by the events of the 1970's financial deregulation process. Whilst the variables might be expected to remain in an equilibrium relationship it would clearly be possible that the parameters of this relationship changed as a result of the process of financial deregulation.

As noted in the introduction to this chapter, the tests suggested by Gregory and Hansen are residual based tests, and the test procedure is essentially the same as that described in the above discussion of the Engle Granger method of testing for cointegration. The only difference is that the cointegrating regression includes dummy variables to allow for the regime shift. Gregory and Hansen describe three basic models.

Model one is the standard model with no regime shift:

$$y_{1t} = \mu + \alpha y_{2t} + e_t \quad t= 1, \dots, n. \quad (4.12)$$

where  $y_2$  is a vector of  $m$  variables,  $\alpha$  is an, appropriately dimensioned, matrix of slope coefficients. For simplicity  $y_1$  and  $y_2$  are assumed to be  $I(1)$ .

Models two, three and four then include dummy variables to allow for shifts in the intercept,  $\mu$ , and slope,  $\alpha$ . Define the dummy variable:

$$\varphi_{t\tau} = \begin{cases} 0 & \text{if } t \leq [n \tau], \\ 1 & \text{if } t > [n \tau], \end{cases}$$

where  $\tau$  is an, unknown, parameter which refers to the relative timing of the structural change,  $n$  is the sample size and  $[ ]$  denotes the integer part. In practical applications recursive techniques are used to move the break point through the sample. Following Banerjee, Lumsdaine and Stock (1992) and Zivot and Andrews (1992), Gregory and Hansen suggest that the test statistic be calculated with the break point allowed to vary over the interval  $[0.15n]$ ,  $[0.85n]$ .

Model two allows for a simple level shift in the cointegrating relationship by allowing a change in the intercept. Thus, in this case, the cointegrating regression becomes:

$$y_{1t} = \mu_1 + \mu_2\phi_{t\tau} + \alpha y_{2t} + e_t \quad t = 1, \dots, n. \quad (4.13)$$

Model three is essentially the same but allows for a time trend and involves estimating the following cointegrating regression:

$$y_{1t} = \mu_1 + \mu_2\phi_{t\tau} + \beta t + \alpha y_{2t} + e_t \quad t = 1, \dots, n. \quad (4.14)$$

where  $t$  is a time trend.

Finally, model four allows a change in the slope vector, allowing the cointegrating relationship to rotate and shift parallel. Gregory and Hansen refer to this as the regime shift model:

$$y_{1t} = \mu_1 + \mu_2\phi_{t\tau} + \alpha y_{2t} + \alpha y_{2t}\phi_{t\tau} + e_t \quad t = 1, \dots, n. \quad (4.15)$$

In practical applications, one, or all three, of the above regression models is estimated and the residuals retrieved. Those residuals are then tested for a unit root, as in the Engle Granger method. In this case, however, the regression is run with the dummy variable moving through the sample (from  $[0.15n]$  to  $[0.85n]$ ) thus there will be  $[0.71n]$  sets of residuals and the same number of test statistics. Gregory and Hansen suggest the use of three test statistics to test the null of a unit root in the calculated residuals.

The first is simply the ADF test statistic. For each value of  $\tau$  the residuals from the cointegrating regression, denoted as  $\hat{e}_{t\tau}$ , are retrieved and an ADF test of the form:

$$\Delta \hat{e}_{t\tau} = \hat{e}_{t-1\tau} + \sum_{i=1}^k \Delta \hat{e}_{t-i\tau} + \varepsilon_t \quad (4.16)$$

is carried out. The statistic of interest will be the largest (negative) value of this test statistic.

The second two tests used by Gregory and Hansen are the  $Z_\alpha$  and  $Z_\tau$  tests suggested by Phillips (1987) and Phillips and Perron (1988). As noted in Chapter 3 these tests use an alternative method of correcting for residual autocorrelation in the unit root test based on a correction to the test statistic rather than inclusion of extra parameters in the regression equation. In the current context the two test statistics are calculated as follows.

After retrieving the estimated residuals  $\hat{e}_{t\tau}$  from the cointegrating regression the standard DF regression to test for a unit root is estimated:

$$\Delta \hat{e}_{t\tau} = \hat{\rho} \hat{e}_{t-1\tau} + \varepsilon_t \quad (4.17)$$

the  $Z_\alpha$  test is then calculated as:

$$\hat{Z}_\rho = T\hat{\rho} - (1/2)(S_{T\ell}^2 - S_\varepsilon^2) \left( T^{-1} \sum_{t=2}^T \hat{e}_{t-1\tau}^2 \right)^{-1} \quad (4.18)$$

where

$$S_\varepsilon^2 = T^{-1} \sum_{t=1}^T \hat{\varepsilon}_t^2, \quad S_{T\ell}^2 = T^{-1} \sum_{t=1}^T \hat{\varepsilon}_t^2 + 2T^{-1} \sum_{j=1}^{\ell} \omega_\ell(j) \sum_{t=j+1}^T \hat{\varepsilon}_t \hat{\varepsilon}_{t+j}$$

and where  $\omega_\ell(j) = 1 - j(\ell + 1)^{-1}$  and the test statistic is calculated for a variety of truncation lag parameters,  $\ell$ .

Using the same notation the  $Z_t$  test statistic is calculated as:

$$\hat{Z}_{t(\rho=0)} = \left( \sum_{t=2}^T \hat{e}_{t-1\tau}^2 \right)^{1/2} \hat{\rho} / S_{T\ell} - (1/2)(S_{T\ell}^2 - S_\varepsilon^2) \left[ S_{T\ell} \left( T^{-2} \sum_{t=2}^T \hat{e}_{t-1\tau}^2 \right)^{1/2} \right]^{-1} \quad (4.19)$$

Once again the statistic of interest will be the largest (negative) value obtained as the dummy variables are moved through the sample.

Gregory and Hansen (1996) provide critical values for the above test statistics for up to four regressors using results from extensive simulation exercises. In a comparison with standard ADF test they find that these new statistics are very useful in detecting cointegration in the presence of a simple regime shift. In the presence of a regime shift, as one would expect, the power of standard ADF type tests falls rapidly.

Whilst Gregory and Hansen warn against use of the test to answer questions about whether or not a regime shift has occurred, it is clear that if standard tests fail to reject the null of no cointegration, but these tests do reject the null, then this does imply that structural change is important in the cointegrating vector. In such a situation the researcher, using only standard tests, might, incorrectly, conclude that the set of variables in question do not cointegrate. The applied work in Chapter 6 provides a good example of the usefulness of this type of test.

#### **4.5 Conclusion**

The finding that many economic time series are best characterised as integrated, non stationary, processes, appears to pose problems for the researcher. Fortunately, the fact is, that many economic theories suggest that it would be expected that some combinations of these variables should move together in a long run equilibrium. So, for example, Purchasing Power Parity would suggest that, if  $e_t$  is the log exchange rate,  $p_t$  the log domestic price index and  $p_t^*$  the foreign price index, then whilst all three of them may well be non stationary, the linear combination should be stationary. This is where tests for cointegration have become useful. If the majority of macroeconomic time series are non stationary then it will be useful to have tests which identify linear combinations which are stationary, this is what tests for cointegration

attempt to do. In the context of the above example the researcher can simply test to see if the three variables do in fact cointegrate as a test of PPP.

Cointegration theory has also provided further useful tools for the researcher. A set of variables which cointegrate also have a valid error correction formulation. The variables must also exhibit some form of causality, and this can be tested in a correctly specified Vector Error Correction Model (VECM). So cointegration has become a vital stepping stone in the development of econometric models.

This chapter has briefly reviewed the econometrics of testing for cointegration using three different methods. The remaining chapters of this thesis put these tests to use, in a variety of applications, in the areas of finance and labour economics.

## CHAPTER 5

### TESTING FOR UNIT ROOTS IN AUSTRALIAN LABOUR MARKET DATA.

#### **5.1 Introduction**

This chapter and the next (Chapter 6) extends earlier work by the author regarding the time series properties of variables included in popular specifications of the Australian wage equation (Lewis and MacDonald, 1993). That earlier paper considered the time series properties of the key variables in four of the more popular specifications of the Australian wage equation, namely those of Murphy, (1989), Nif-88 (Simes and Richardson, 1987), Watts and Mitchell (1990) and Lewis and Kirby (1987,1988). The aim of this chapter is to re-test the unit root hypothesis using some of the more recently developed tests discussed in Chapter 3 on an extended and updated data set.

In section 5.2 the basic data are presented and three tests for a unit root carried out on it, the standard (A)DF test, the DF-GLS test the KPSS test. All of these tests are described in Chapter 3. The results are presented variable by variable and a decision made on the basis of these tests as to the time series properties of the variable. Section 5.3 reports the results of tests for a unit root in the presence of a break in the underlying trend using the test procedure described in Chapter 3. Section 5.4 provides a conclusion.

## 5.2 The data and unit root tests

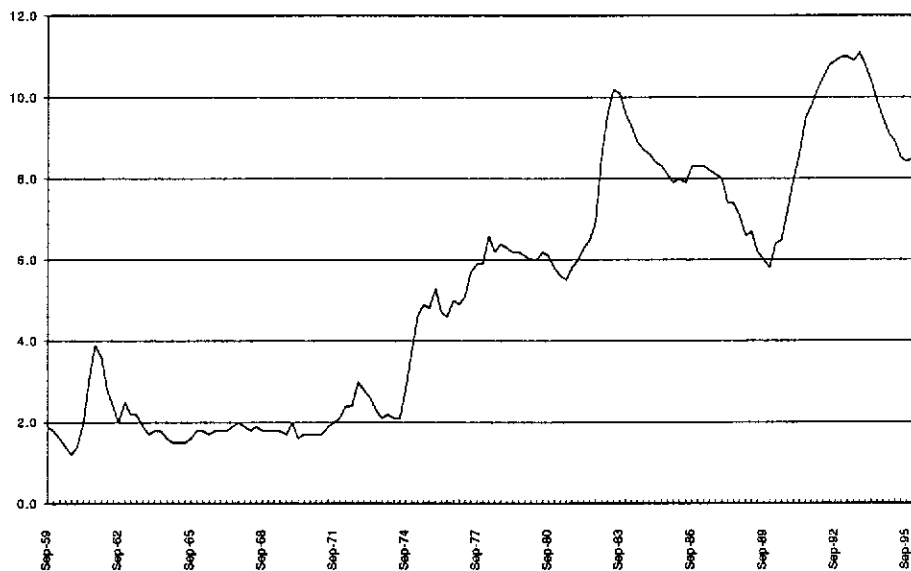
Step one was to run standard (A)DF tests. The procedure began with a regression model including constant and time trend. The first test statistic carried out was the  $\phi_3$  test. If this couldn't reject the unit root null then the  $\phi_2$  was run. Here a generous significance level was used to make the decision on drift. If it was possible to reject zero drift at the 10 per cent level then the decision on the order of integration was made on the basis of the  $\phi_3$  test and the (A)DF in the regression model with constant and trend. If the  $\phi_2$  test could not reject the null that the drift was zero the (A)DF based on a regression model with a constant only is also reported. Both the KPSS and DF-GLS tests were then carried out and, where applicable, information from the (A)DF tests was utilised in the determination of the regression specifications. If, in the level specification, the unit root null could not be rejected, the series was differenced and re-tested for a unit root using standard (A)DF tests once again. Since the strong prior was that the series would be at most  $I(1)$ , then, if rejection occurred here testing stopped, and the series was classified as  $I(1)$ . If the (A)DF could not reject a unit root in the first difference of the series (thus suggesting the series was  $I(2)$ ) then the KPSS and DF-GLS tests were also run on the first difference of the series.

In terms of the tests allowing for a structural break, observation of the data suggested that fitting the most general of the breaking trend models seemed the most appropriate. As a result the innovation outlier model (b) described in Chapter 3 (see Table 3.1 page 35) was used to test the null of a unit root against the alternative of stationarity around a breaking linear trend and the results from these tests are in section 5.3.

- **The unemployment rate**

Figure 5.1 plots the unemployment rate over the period. The (A)DF test statistics, reported in Table 5.1 suggest that the series cannot reject a unit root and has possibly zero drift, since  $\phi_2$  is not significant. For this series, given its nature, it seems sensible to accept that the variable is unlikely to have a

**Figure 5.1 Unemployment rate**



significant upward drift, or indeed trend, so testing is based upon the regression model with constant only. The results from the (A)DF tests are supported by those of the KPSS test (reported in Table 5.2), which rejects the null of stationarity at all lag lengths, and the DF-GLS test (reported in Table 5.3) which is unable to reject the null of a unit root. The first difference of the unemployment rate strongly rejects the null of a unit root using standard (A)DF tests and so the variable is categorised as  $I(1)$  with zero drift.

Due to the fact that the unemployment rate used is bounded between 0 and 100 (per cent) whereas the assumption made in the Dickey Fuller regression is that the errors are unbounded, the data was transformed using a logistic

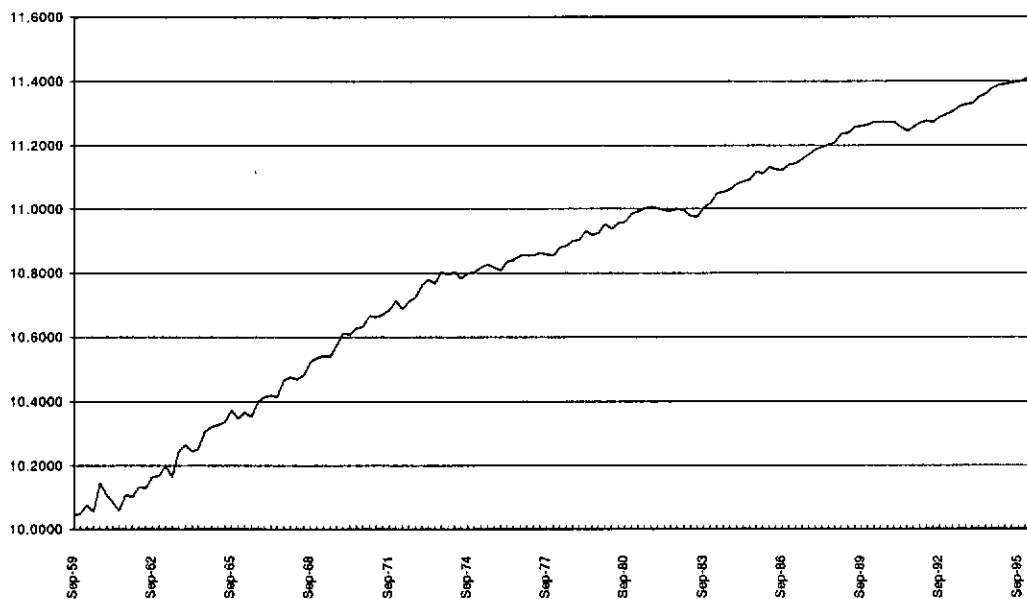


transformation such that  $TUR = \ln\left(\frac{UN}{1-UN}\right)$  where UN is the unemployment rate expressed as a fraction. As can be seen from the tables the results still suggest that the unit root null cannot be rejected.

- **Log of real GNP**

Here the  $\phi_3$  test (Table 5.1) yields a value of 7.69 which is significant at the 2.5 per cent level using the tables in Dickey and Fuller (1981). Using the test procedure suggested in Chapter 3, if we assume that the trend term is non zero, then the test statistic for the null of a unit root is asymptotically normal. In this case the t value is -2.48 which does suggest rejection of the unit root null.

**Figure 5.2 Log of real GNP**



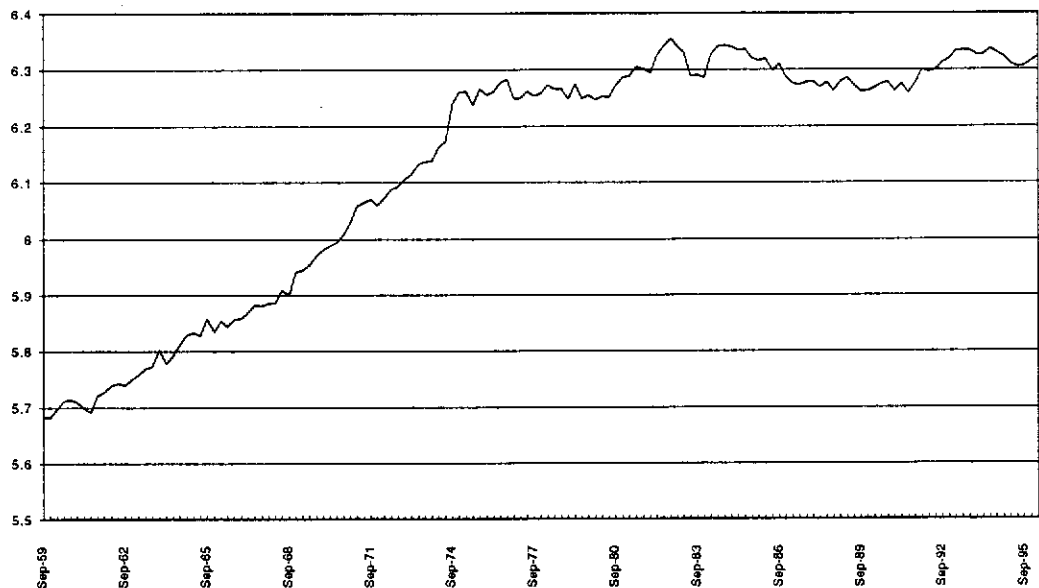
Since the t statistic on the trend term is 2.04 it could be concluded that the series is trend stationary. Note however that the standard ADF test in the regression model with constant and trend would not reject the unit root null using the non standard Dickey and Fuller critical values. Looking at the alternative tests, the KPSS test (Table 5.3) using the null of trend stationarity is able to reject the null hypothesis, suggesting the variable is  $I(1)$ . Similarly the

DF-GLS test (Table 5.3) using a constant and trend is unable to reject the null of a unit root. Thus the (A)DF test conflicts with the DF-GLS and KPSS tests. On taking first differences the series is able to reject a unit root using standard (A)DF tests. Overall the result from the tests are inconclusive and contradictory, however the assumption that real GNP is  $I(1)$  does not seem unreasonable.

- **Log of real wages**

Figure 5.3 plots the real wage series. Standard (A)DF tests cannot reject the unit root and the  $\phi_2$  test suggests non zero drift. This result is supported by the KPSS and DF-GLS tests. The series rejects a unit root upon first differencing using standard (A)DF tests and so is categorised as  $I(1)$  with non zero drift.

**Figure 5.3** Log of real wages

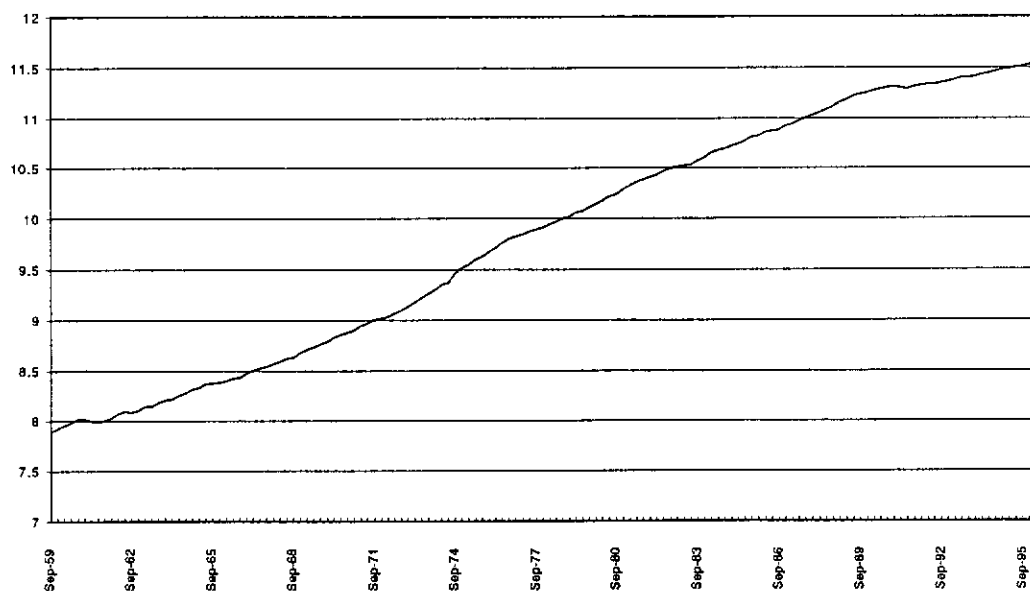


- **Log of nominal GNP**

None of the standard (A)DF tests can reject a unit root in the log level of the series. Given that the  $\phi_3$  and the  $\phi_2$  statistics are not significant there is

evidence that the series may have zero drift, however the ADF test statistics from regression models including constant and trend and constant only are both insignificant and cannot reject the null of a unit root in the series. Observation of the plot in Figure 5.4 is, on the other hand, informally suggestive of drift in the series. As a result the KPSS and DF-GLS tests are carried out using both a constant and a constant and trend in the regression model. Both the DF-GLS and the KPSS tests are supportive of the hypothesis that nominal GNP contains a unit root in levels. Using standard (A)DF tests the series is unable to reject a unit root in the first difference, but does reject on second differencing. Thus the conclusion from these tests would be that nominal GNP is an  $I(2)$  variable. However the DF-GLS test does reject the unit root null at the 5% significance level on first differencing whilst, at lags 6 and 8 the KPSS test is unable to reject the null of stationarity on the first difference of the series. Thus these two tests provide some evidence that nominal GNP is  $I(1)$ . Thus there is a conflict between the tests and they lead to no conclusive evidence on the unit root hypothesis. Whilst there is some evidence here that nominal GNP is  $I(2)$  it would seem reasonable to categorise it as  $I(1)$ .

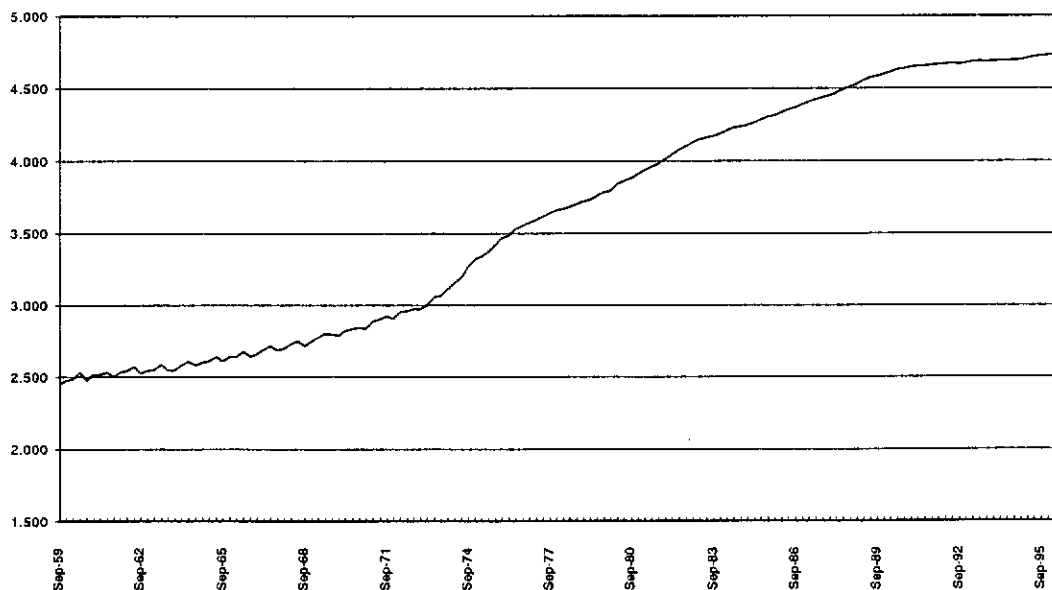
**Figure 5.4 Log of Nominal GNP**



- **Log of the implicit GNP deflator**

The  $\phi_3$  and  $\phi_2$  tests cannot reject a unit root and suggest possible zero drift, in contrast once again to the informal evidence from the time series plot of the series. However the (A)DF tests using either regression model (constant and trend or constant only) cannot reject the unit root null, a result supported by both the DF-GLS and KPSS tests. On first differencing the series cannot reject a unit root using standard (A)DF tests but can reject on second differencing, thus suggesting that the series is  $I(2)$ . Once again however, evidence from the DF-GLS test and the KPSS test suggest that the series is in fact  $I(1)$ . The DF-GLS test can reject a unit root in the first difference at the 5% level and the KPSS cannot reject the null of stationarity, in the first difference of the series, at the 5% level at lags 4,6 and 8. Thus, like nominal GNP the results are inconclusive. Whilst the deflator clearly has a single unit root there is some doubt over the presence of two unit roots and again the assumption that the variable is  $I(1)$  does not seem unreasonable.

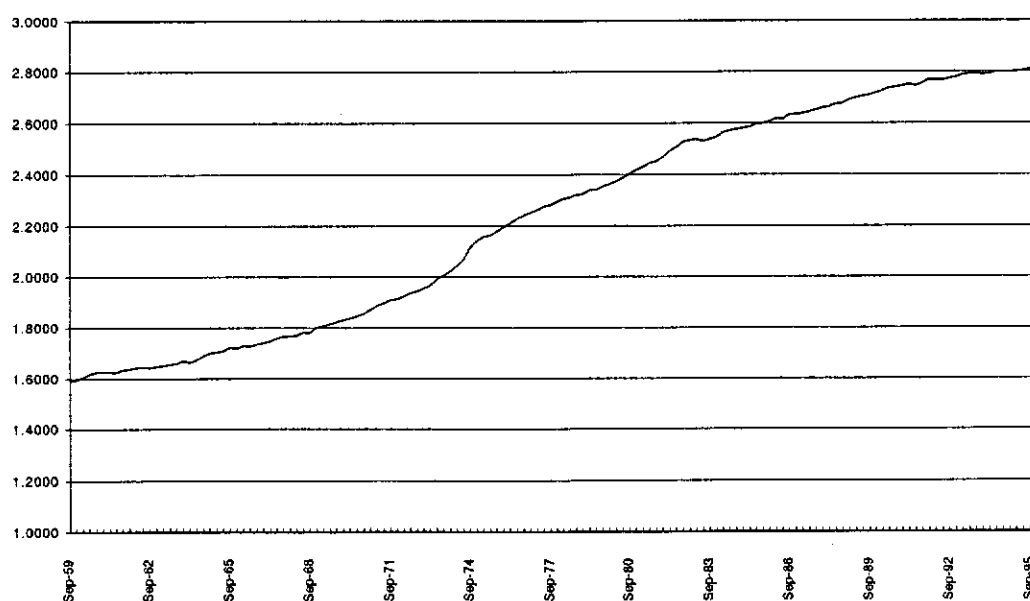
**Figure 5.5 Log of the Implicit GNP Deflator**



- **Log of Nominal Wages**

The nominal wage series tells a similar story to the last two. The (A)DF tests cannot reject a unit root in levels. In first differences the series cannot reject a unit root at the 5 per cent level (but can using a 10 per cent significance level) but can on second differencing, once again suggesting the series may be  $I(2)$ . In this case the evidence against two unit roots is not quite as strong. The DF-GLS can only reject the unit root in the first difference of the series at the 10 per cent level, whilst the KPSS cannot reject the null of stationarity at lags 4, 6 and 8. Thus again the evidence is mixed. The assumption that the series is  $I(1)$  finds some support but given the doubt there may be some support for the notion that real wages should be the variable used in the modelling stage. Indeed this was a conclusion reached in a recent paper by Caporale (1996) who reports similar findings for wages and prices using data on the UK, namely evidence that wages and prices are  $I(2)$  and thus chooses to model the aggregate wage equation using real wages.

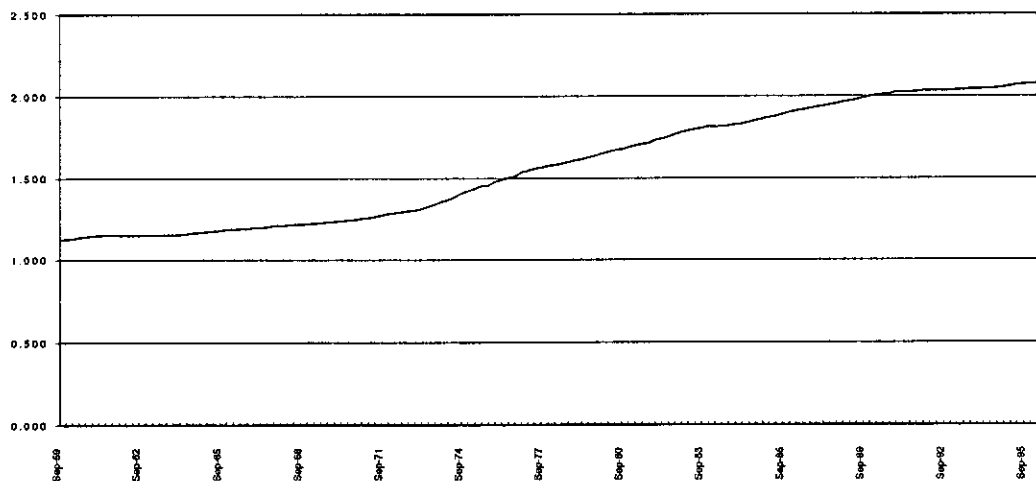
**Figure 5.6 Log of Nominal Wages**



- **Log of the CPI**

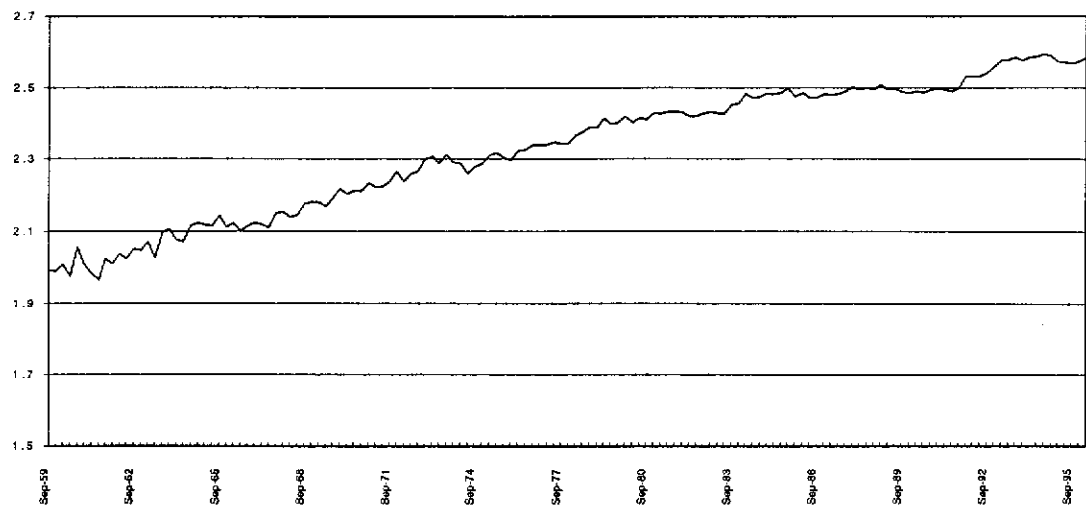
The story is repeated for the CPI series which cannot reject the unit root null in levels or first differences using (A)DF tests but can on second differencing, suggesting an I(2) series. The DF-GLS cannot reject a unit root in levels or first differences at the 5 per cent significance level but can reject a unit root in first differences at the 10 per cent significance level. At the 5 per cent level the KPSS can reject stationarity in levels but not in the first difference at lags of 6 and 8. Thus again the evidence is clearly mixed.

**Figure 5.7 Log of the CPI**



- **The log of Productivity**

Here the results are fairly clear cut. In log levels the series cannot reject a unit root using standard (A)DF tests, the  $\phi_2$  test suggests rejection of the zero trend null, and this is supported by evidence from the time series plot. The null of stationarity in levels can be rejected by the KPSS test whilst the null of a unit root cannot be rejected by the DF-GLS test. On first differencing standard (A)DF tests strongly reject the unit root null. Thus the simple conclusion here is that the series is I(1) with non zero drift.

**Figure 5.8 Log of Productivity****Table 5.1 The (A)DF results**

Levels	$\Phi_3$	$\Phi_2$	(A)DF	Critical values 10%, 5%	Auto ( $\chi^2(4)$ )
UR (c & t)	5.79*	4.09	-3.39* (2)	-3.14, -3.43	4.4 [.36]
UR (c)			-1.24 (4)	-2.56, -2.86	1.7 [.79]
TUR	4.90	3.58	-3.10 (2)	-3.14, -3.43	7.7 [.10]
TUR			-1.46 (4)	-2.56, -2.86	8.0 [.09]
LRGNP (c & t)	7.69**	11.3**	-2.48 (7)	-3.11, -3.40	4.2 [.39]
LRWAG (c & t)	2.74	4.51*	-0.76 (2)	-3.14, -3.43	2.9 [.57]
LGNP (c & t)	2.53	3.40	-0.79 (7)	-3.11, -3.40	4.6 [.33]
LGNP (c)			-2.17 (7)	-2.55, -2.85	4.7 [.32]
LGNPDEF (c & t)	3.65	3.06	-2.61 (5)	-3.12, -3.41	6.1 [.19]
LGNPDEF (c)			-1.08 (5)	-2.56, -2.86	6.8 [.15]
LWAG (c & t)	1.14	2.57	-1.38 (5)	-3.12, -3.41	5.0 [.29]
LWAG (c)			-0.79 (5)	-2.56, -2.86	4.9 [.29]
LPCPI (c & t)	3.89	3.44	-2.78 (4)	-3.12, -3.42	5.0 [.29]
LPCPI (c)			-0.52 (4)	-2.56, -2.86	5.7 [.23]
LPROD (c & t)	4.41	9.22**	-1.98 (7)	-3.11, -3.40	2.9 [.56]
LLABF(c & t)	2.48	6.04**	-1.65 (8)	-3.10, -3.39	6.2 [.19]
LBEN (c & t)	4.32	3.21	-2.57 (4)	-3.13, -3.42	6.4 [.17]
LBEN (c)			-2.95** (4)	-2.56, -2.86	6.2 [.19]
LTU (c & t)	5.02	3.63	-3.17* (4)	-3.13, -3.42	1.3 [.87]

LTU (c)			-1.19 (4)	-2.56, -2.86	2.3 [.68]
LWEDGE (c & t)	4.87	3.68	-1.52 (4)	-3.12, -3.42	4.7 [.33]
LWEDGE (c)			-2.97** (4)	-2.56, -2.86	3.9 [.43]
LLABF			-3.80** (7)	-2.55, -2.85	5.7 [.23]
LBEN			-3.33** (3)	-2.57, -2.87	3.4 [.49]
LTU			-3.48** (3)	-2.57, -2.87	3.4 [.50]
LWEDGE			-4.98** (3)	-2.57, -2.87	3.0 [.56]

First Differences	Regression model included constant only.	(A)DF		
UR		-5.82** (3)	-2.57, -2.87	1.6 [.80]
TUR		-7.60** (3)	-2.57, -2.87	6.1 [.19]
LRGNP		-3.52** (7)	-2.55, -2.85	6.2 [.18]
LRWAG		-7.21** (1)	-2.58, -2.88	1.9 [.74]
LGNP		-2.45 (7)	-2.55, -2.85	6.3 [.18]
LGNPDEF		-2.42 (6)	-2.55, -2.85	2.5 [.65]
LWAG		-2.77* (4)	-2.56, -2.86	4.4 [.36]
LPCPI		-1.94 (3)	-2.57, -2.87	5.7 [.22]
LPROD		-5.54** (6)	-2.55, -2.85	4.2 [.38]
<b>Second Differences</b>				
LGNP		-9.4** (5)	-2.56, -2.86	4.9 [.30]
LGNPDEF		-5.3** (5)	-2.56, -2.86	3.5 [.47]
LWAG		-8.4** (5)	-2.56, -2.86	1.2 [.87]
LCPI		-11.1** (2)	-2.57, -2.87	5.8 [.21]
LPOP		-5.6** (4)	-2.56, -2.86	6.3 [.18]

The critical values for the  $\Phi_3$  test at the 5 and 10 per cent levels are, approximately, 6.49 and 5.47 respectively (source: Table VI of Dickey and Fuller (1981))

The critical values for the  $\Phi_2$  test at the 5 and 10 per cent levels are approximately 4.88 and 4.16 respectively (source: Table V of Dickey and Fuller (1981))

Critical values for the (A)DF vary by sample size and lag length and are calculated using response surface in Cheung and Lai (1995a).

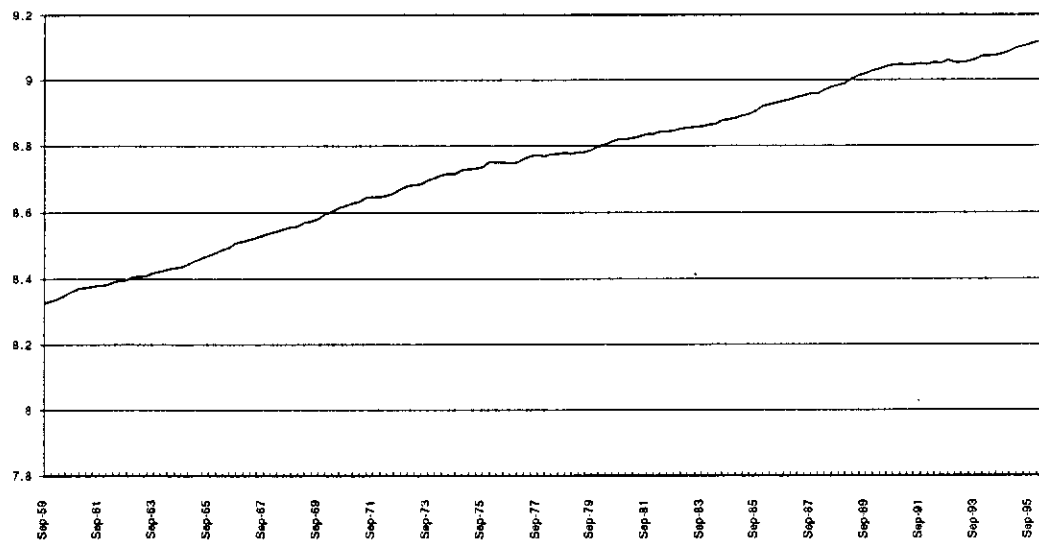
\*\* indicates significant at the 5 per cent level, \* at the 10 per cent level.



- **Log of the labour force**

Here, whilst the  $\phi_3$  test does not reject the  $\phi_2$  does, suggesting, as does the plot of the series, non zero drift. The standard ADF test does not reject the unit root in levels, but does on first differencing. This suggests the series is I(1) a conclusion supported by the evidence from the KPSS and DF-GLS tests. The log of the labour force is thus categorised as I(1).

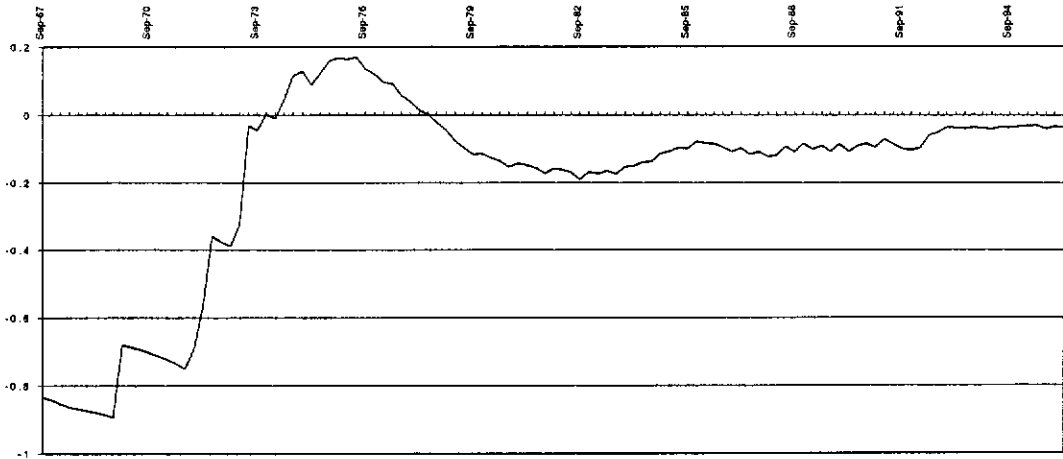
**Figure 5.10** Log of the labour force



- **Log of unemployment benefits**

With neither the  $\phi_3$  nor the  $\phi_2$  test significant the ADF test based on the regression model with constant only appears most appropriate and this rejects the unit root in BEN at the 5 per cent level. The KPSS, however, rejects the null of stationarity at all lag lengths and the DF-GLS test statistics are not significant. On first differencing the (A)DF test is able to reject a unit root. Thus the tests are inconclusive with their being some evidence that the series is I(0). Overall it might be reasonable to assume that the series is I(1) on the basis of the KPSS and DF-GLS test.

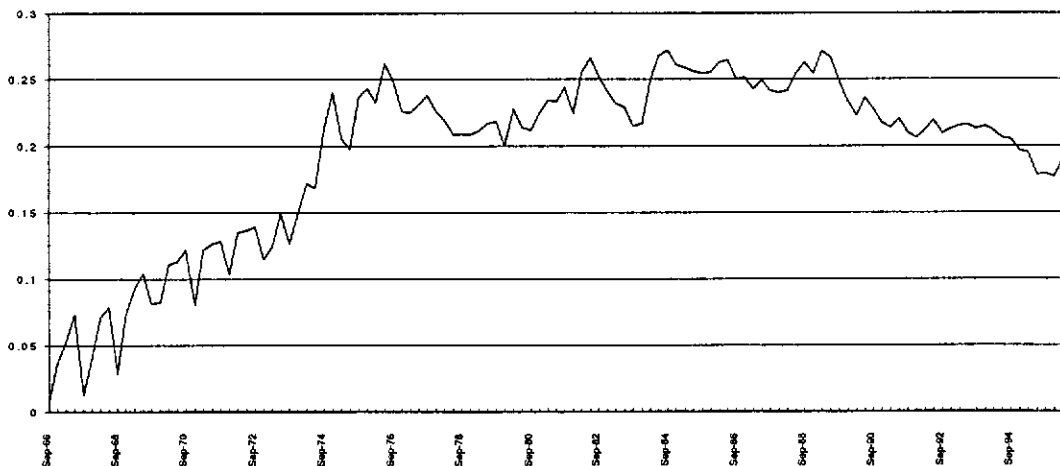
**Figure 5.11** Log of unemployment benefits



- **Log of the tax wedge**

The results here are essentially the same as for unemployment benefits. Neither the  $\phi_3$  nor the  $\phi_2$  test is significant and the ADF test based on the regression model with a constant rejects the unit root in WEDGE at the 5 per cent level. Once again however the KPSS test rejects the null of stationarity at all lag lengths and the DF-GLS test statistics are not significant. On first differencing the (A)DF test rejects the unit root. Thus, again, the tests are inconclusive with there being some evidence that the series is  $I(0)$  and some evidence, from the alternative tests, that it is  $I(1)$ .

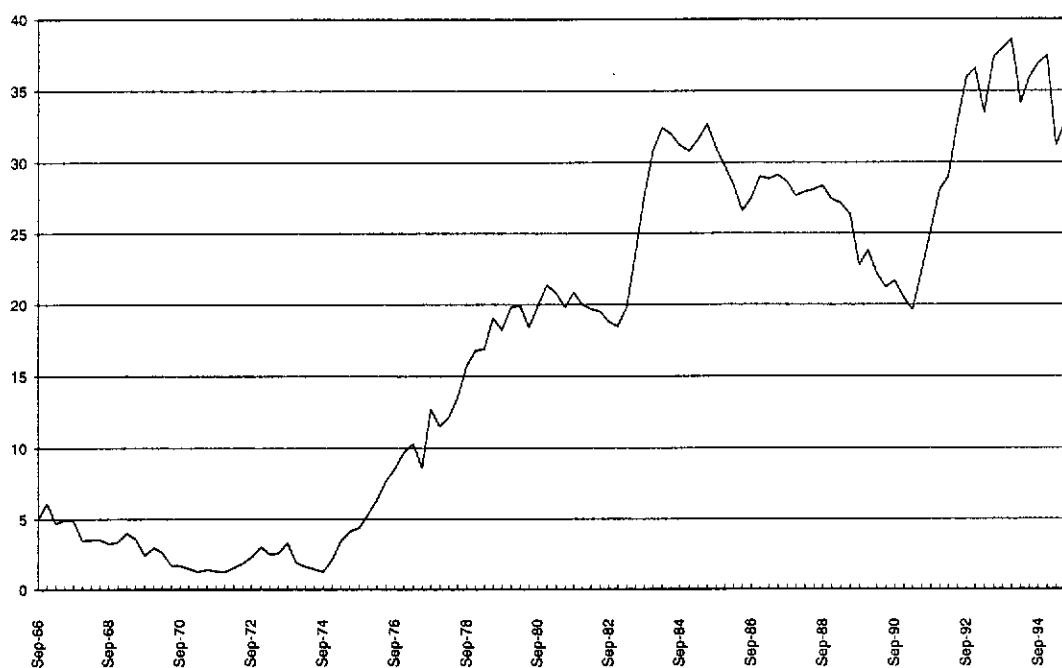
**Figure 5.12** Log of the tax wedge



- **Log of ratio of long term to total unemployment**

Finally long term unemployment, which is expressed as a ratio of total unemployment is examined. Neither  $\phi_2$  and  $\phi_3$  are significant so zero drift is suggested. The ADF with constant only cannot reject a unit root but the regression with constant and time in does reject the unit root at the 10 per cent level. The DF-GLS test suggests non rejection of the unit root. The KPSS test with constant only suggests rejection of the null of stationarity but the test with constant and trend suggests that stationarity is not rejected at higher than the 10 per cent level for lags 6 and 8. Standard (A)DF tests reject the unit root on first differencing. Thus once again there is some evidence of conflict between the tests.

**Figure 5.13 Log of the ratio of long term to total unemployment.**



**Table 5.2    The DF-GLS results**

<b>Levels</b>	<b>(A)DF-GLS test statistic</b>	<b>AUTO (<math>\chi^2(4)</math>)</b>	<b>10%, 5% critical values</b>
UR (c)	-0.33 (4)	1.9 [.76]	-1.73, -2.04
TUR(c)			
LRGNP (c & t)	-1.04 (8)	6.6 [.16]	-2.59, -2.87
LRWAG (c & t)	-0.50 (2)	1.9 [.76]	-2.66, -2.95
LGNP (c & t)	-1.91 (7)	5.3 [.26]	-2.61, -2.89
LGNP (c)	-0.71 (7)	4.9 [.30]	-1.71, -2.02
LGNPDEF (c & t)	-2.39 (5)	5.9 [.21]	-2.63, -2.92
LGNPDEF (c)	-0.33 (5)	6.7 [.15]	-1.73, -2.04
LWAG (c & t)	-1.46 (5)	5.3 [.26]	-2.63, -2.92
LWAG (c)	0.54 (5)	5.4 [.25]	-1.73, -2.04
LPCPI (c & t)	-1.94 (4)	5.6 [.23]	-2.64, -2.93
LPCPI (c)	0.05 (4)	5.7 [.22]	-1.73, -2.04
LPROD (c & t)	-1.37 (7)	3.9 [.42]	-2.61, -2.89
LPOP (c & t)	-0.32 (6)	5.6 [.23]	-2.62, -2.90
LPOP (c)	0.37 (6)	6.3 [.17]	-1.72, -2.02
LLABF(c & t)	-1.15 (8)	5.6 [.23]	-2.59, -2.87
LBEN (c & t)	-1.59 (4)	4.0 [.40]	-2.66, -2.95
LBEN (c)	-0.62 (4)	3.7 [.45]	-1.75, -2.06
LTU (c)	-0.59 (4)	2.6 [.64]	-1.75, -2.06
LWEDGE(c & t)	-0.50 (4)	2.8 [.60]	-2.66, -2.95
LWEDGE (c)	-0.18 (4)	2.7 [.61]	-1.75, -2.06
<b>First Differences</b>			
LGNP (c)	-2.16** (6)	6.6 [.17]	-1.72, -2.03
LGNPDEF (c)	-2.07** (6)	2.6 [.62]	-1.72, -2.03
LWAG (c)	-1.86* (5)	5.9 [.21]	-1.73, -2.04
LPCPI (c)	-1.93* (4)	2.6 [.63]	-1.73, -2.04
LPOP (c)	-0.53 (7)	1.8 [.77]	-1.71, -2.02

Critical values calculated using response surface in Cheung and Lai (1995b).

\*\* indicates significant at the 5 per cent level, \* at the 10 per cent level.

**TABLE 5.3 The KPSS test results**

KPSS TEST	Truncation lag parameter			
	$\ell=2$	$\ell=4$	$\ell=6$	$\ell=8$
LUR(c)	4.381**	2.674**	1.946**	1.543**
TUR(c)				
LRGNP(c & t)	1.034**	0.635**	0.464**	0.369**
LRW(c & t)	1.186**	0.725**	0.527**	0.417**
LGNP (c & t)	0.679**	0.419**	0.309**	0.249**
LGNP (c)	5.022**	3.049**	2.203**	1.734**
LGNPDEF (c & t)	0.571**	0.351**	0.258**	0.207**
LGNPDEF (c)	5.008**	3.035**	2.190**	1.721**
LWAG(c & t)	0.656**	0.403**	0.296**	0.237**
LWAG(c)	5.007**	3.037**	2.193**	1.724**
LPCPI(c)	4.999**	3.031**	2.187**	1.719**
LPCPI(c & t)	0.640**	0.393**	0.288**	0.230**
LPROD(c & t)	0.968**	0.615**	0.461**	0.374**
LPOP(c & t)	0.787**	0.492**	0.366**	0.297**
LPOP(c)	4.373**	2.669**	1.938**	1.533**
LLABF (c & t)	0.785**	0.484**	0.355**	0.285**
LBEN(c & t)	0.555**	0.343**	0.253**	0.204**
LBEN(c)	1.470**	0.911**	0.675**	0.546**
LTU(c & t)	0.287**	0.183**	0.140*	0.118
LTU(c)	3.520**	2.150**	1.570**	1.25**
LWEDGE(c & t)	0.871**	0.547**	0.408**	0.331**
LWEDGE(c)	2.390**	1.480**	1.090**	0.871**
<b>First Differences</b>				
LGNP (c)	0.739**	0.557*	0.449*	0.378*
LGNPDEF (c)	0.612**	0.451*	0.351*	0.295
LWAG (c)	0.639**	0.476**	0.376*	0.315
LPCPI (c)	0.910**	0.587**	0.445*	0.364*
LPOP (c)	2.440**	1.580**	1.190**	0.970**

Critical values: Constant only: 10% = 0.347, 5% = 0.463

Constant and trend: 10% = 0.119, 5% = 0.146

\*\* indicates significant at the 5 per cent level, \* at the 10 per cent level.

### 5.3 Testing for a unit root against a breaking trend alternative.

It seems possible, from the above time series plots, that a number of the variables have in fact undergone changes in mean/trend at some point in their data history. Under such circumstances, tests which fail to allow for a change in the underlying trend function will be biased in favour of the unit root hypothesis and, as a result, it would seem sensible to re test the unit root hypothesis allowing for this possibility. As noted above the innovation outlier model (b), described in Chapter 3, was used to test the hypothesis that the series are in fact stationary around a breaking trend.

Table 5.4 reports the results. As noted in Chapter 3 the date of the break was selected by recursively estimating the regression equation:

$$y_t = \mu + \beta t + \theta DU + \gamma DT^* + \delta D(T_b) + \alpha y_{t-1} + \sum_{i=1}^k a_i \Delta y_{t-i} + u_t$$

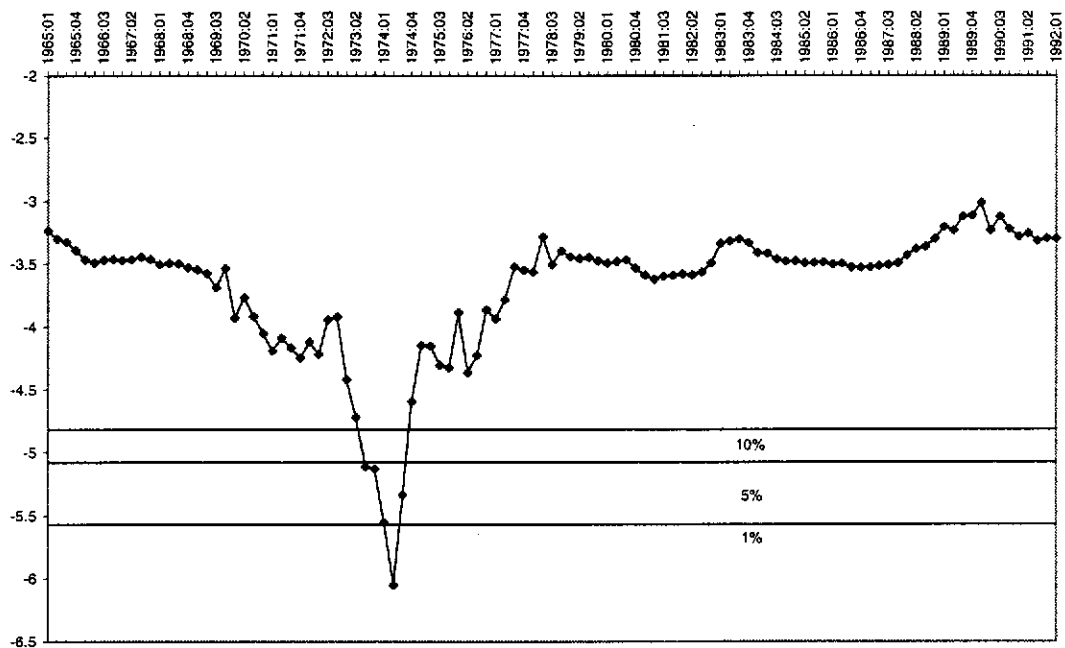
where:  $T_b$  is the date of the break,  $DU_t = 1$  and  $Dt^*_t = (t - T_b)$  if  $t > T_b$  and 0 otherwise,  $D(T_b)_t = 1$  if  $t = T_b + 1$  and 0 otherwise.

The date of the break ( $T_b$ ) moves through the sample one quarter at a time. Following the suggestion of Banerjee *et al* 1992  $T_b$  was varied from 0.15T to 0.85T where T is the number of observations in the sample. The lag length, k, was selected by allowing k to vary from 8 to zero at each recursion and then selecting k using a simple general to specific method, reducing the lag length until the coefficient on the last included lag was significant at the 5 per cent level.

The results are reported in Table 5.4, with two exceptions, it is not possible to reject the unit root hypothesis even when allowance is made for a break in the trend function. The two, clear exceptions are real GNP and the transformed unemployment rate variable, both of which can reject at a high level of significance. Indeed the transformed unemployment rate variable provides a

good example of how important the use of recursive methods are in carrying out this type of test. Figure 5.14 plots the recursively estimated t statistic for testing the null of a unit root. As can be seen in this particular case the test statistic has a minimum value at 1974 quarter 2, and the test statistic is only significant at better than the 10 per cent level from 1973 quarter 4 to 1974 quarter 4. Thus selecting the break point without using this technique could easily have led to non rejection of the unit root null.

Figure 5.14 Recursively estimated t statistic testing null of a unit root, TUR



Although the results of these tests show that the majority of the series still cannot reject a unit root further tests do suggest a solution to some of the more ambiguous results in the previous section. In the tests carried out in section 5.1 above there was some question as to whether four of the series, nominal GNP, nominal wages, the CPI and the GNP deflator were all  $I(2)$ . Plots of the first differences of these series, shown below in Figures 5.15 - 5.18 show that the first differences of the series appear to have undergone changes in mean/trend. As a result the above testing procedure was applied to the first differenced series and the results are reported in Table 5.5. As can be seen all four series

now strongly reject the null of a unit root. Thus it would appear that the series are in fact I(1) with a break and that this was causing the spurious I(2) finding.

Table 5.4 Unit root tests allowing for a breaking trend: levels data.

	$T_b$	$k$	$\beta$	$\theta$	$\gamma$	$t(\alpha=1)$	$t(\text{lag})$	LM(4)
LBEN	72Q4	4	0.006 (2.8)	0.055 (1.7)	-0.007 (-2.9)	-4.0	3.8	5.08 [.28]
LLABF	75Q3	8	0.0008 (3.3)	-0.004 (-2.8)	-0.0002 (-2.6)	-3.4	-2.2	1.75 [.78]
LGNP	86Q2	2	0.002 (3.8)	0.002 (0.4)	-0.001 (-4.2)	-3.6	2.1	7.39 [.11]
LGNPDEF	87Q4	7	0.0007 (2.8)	-0.001 (-0.23)	-0.0006 (-2.0)	-2.7	-2.7	2.8 [.60]
LPCPI	84Q2	4	0.0002 (3.9)	0.002 (1.5)	-0.0001 (-2.9)	-3.7	2.0	3.5 [.48]
LPROD	77Q3	8	0.002 (4.4)	0.012 (2.4)	-0.001 (-4.0)	-4.6	1.9	7.3 [.12]
LRGNP	73Q1	4	0.006 (5.8)	-0.016 (-3.1)	-0.002 (-5.8)	-5.6***	4.8	7.5 [0.11]
LRWAG	73Q3	7	0.002 (4.5)	0.024 (3.1)	-0.002 (-4.6)	-4.7	3.4	4.4 [.35]
LTU	82Q3	4	0.57 (3.4)	1.35 (2.1)	-0.51 (-2.6)	-4.2	3.5	0.7 [.95]
LWAG	72Q3	5	0.0004 (3.7)	0.012 (3.8)	-0.0002 (-2.6)	-2.9	2.0	6.6 [.16]
LPOP	86Q3	8	0.0001 (2.4)	0.0001 (0.6)	0.00001 (-2.4)	-2.5	2.0	2.3 [.68]
UR	74Q1	2	-0.001 (-0.2)	0.424 (3.4)	0.007 (2.04)	-4.7	2.7	3.5 [.48]
TUR	74Q2	2	-0.000 (-0.0)	0.21 (5.2)	0.002 (2.0)	-6.1***	2.1	5.8 [.21]
LWEDGE	73Q2	4	0.0005 (1.0)	0.023 (2.9)	-0.0007 (-1.3)	-3.9	2.5	0.6 [.97]

Figures in ( ) parenthesis are t statistics, figures in [ ] parenthesis are p values for the relevant test statistic. Based on the regression:

$$y_t = \mu + \beta t + \theta DU + \gamma DT^* + \delta D(T_b) + \alpha y_{t-1} + \sum_{i=1}^k a_i \Delta y_{t-i} + u_t$$

- Indicates significant at 10 per cent, \*\* at 5 per cent and \*\*\* at 2.5 per cent.
- $t(\alpha=1)$  is the t statistic testing the null that  $\alpha=1$  in the above regression.
- $t(\text{lag})$  is the t statistic on the last included lag of  $\Delta y$  in the above regression.
- LM(4) is an LM test for residual serial correlation.
- $T_b$  is the date of the break.

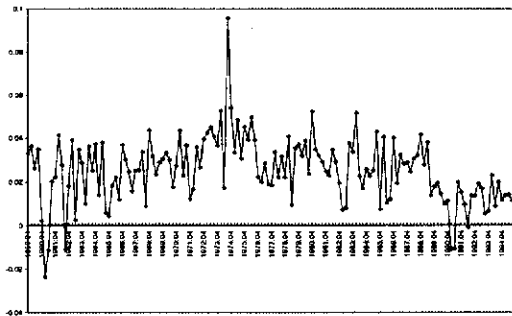
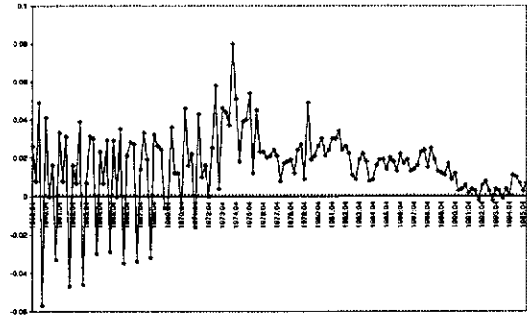
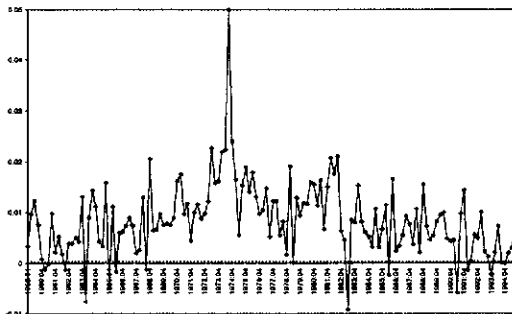
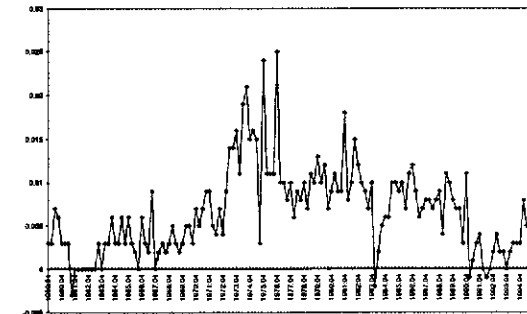


Table 5.5 Unit root tests allowing for a breaking trend: first differences.

	$T_b$	$k$	$\beta$	$\theta$	$\gamma$	$t(\alpha=1)$	$t(\text{lag})$	LM(4)
LWAG	72Q3	7	0.0002 (2.2)	0.007 (2.8)	-0.0004 (-3.5)	-5.1	2.7	6.0 [.20]
LGNP	75Q1	0	0.0004 (4.2)	-0.004 (-1.0)	-0.0007 (-5.4)	-10.9	-	7.2 [.12]
LPCPI	72Q4	6	0.0001 (2.7)	0.008 (4.8)	-0.0002 (-4.5)	-6.7	2.6	0.18 [.99]
LGNPDEF	72Q4	6	0.0003 (2.0)	0.023 (4.1)	-0.0007 (-3.8)	-5.8	3.9	6.3 [.18]

Figures in ( ) parenthesis are t statistics, figures in [ ] parenthesis are p values for the relevant test statistic.

- Indicates significant at 10 per cent, \*\* at 5 per cent and \*\*\* at 2.5 per cent.
- $t(\alpha=1)$  is the t statistic testing the null that  $\alpha=1$  in the above regression.
- $t(\text{lag})$  is the t statistic on the last included lag of  $\Delta y$  in the above regression.
- LM(4) is an LM test for residual serial correlation.
- $T_b$  is the date of the break.

Figure 5.15 First difference of GNPFigure 5.16 First difference of GNP deflatorFigure 5.17 First difference of wagesFigure 5.16 First difference of CPI

## 5.4 Conclusion

The results above, whilst contradictory and inconclusive in some cases, do suggest that it would be reasonable, in the empirical work in the next chapter,

to assume that most of the series under consideration are non stationary  $I(1)$  processes. Some caveats and exceptions to that statement must however be considered

Firstly, the results using standard tests suggested that some of the nominal variables, wages, GNP, CPI and the GNP deflator, are  $I(2)$ . However when tested using the breaking trend model the results suggest that the variables are in fact  $I(1)$  with a structural break.

Secondly the log of real GNP and the logistically transformed unemployment rate variable both appear to be  $I(0)$  when allowance is made for a structural break.

Both of these results suggest that the cointegration modelling in the next section should be carried out bearing in mind that a number of the series do seem to have been affected by the events of the early to mid 1970's.

Finally there is some evidence, from standard (A)DF tests that three of the series, the log of unemployment benefits, the log of the wedge and the ratio of long term to total unemployment, can reject the unit root in levels. These tests are contradicted to some extent by results from the KPSS and DF-GLS tests and thus the results are inconclusive.

The applied analysis in the next chapter concentrates on three variables, the real wage, productivity and unemployment (expressed as a percentage). The tests in this chapter suggest that all three of these variables should be considered as  $I(1)$ .

## CHAPTER 6

### TESTING FOR COINTEGRATION AMONGST AUSTRALIAN WAGE EQUATION VARIABLES

#### **6.1 Introduction**

In a broad context this chapter considers the question, posed in a recent paper by Darby and Wren-Lewis (1993) for the UK, of whether there exists a cointegrating vector for Australian wages. This is a much discussed question in the literature, and section 6.2 below considers, briefly, the more popular models of wage equations and the variables used in such models, whilst section 6.3 considers specific Australian applications.

However, the focus of the main part of the chapter, section 6.4, is more specific. The discussion below, in sections 6.2 and 6.3 leads to the consideration of whether cointegration exists between real wages, productivity and unemployment. If it does it has potential implications for identification of wage equations. However the main focus in section 6.4, which centres around these three variables, is the consideration of whether the move away from free collective bargaining and towards a more centralised system of wage determination (culminating in Australia in the famous Accord) has led to a shift in the cointegrating vector between the variables. This leads to a consideration of whether standard methods of testing for cointegration lead to different results from methods which allow for a regime shift in the cointegrating vector. In order to examine this question, cointegration will be tested for using three techniques, standard residual based Engle Granger

methods, Johansen estimation and a recently developed, residual based test, which allows for a regime shift, suggested by Gregory and Hansen (1996). The background to all three of these methods was discussed in Chapter 4. The results suggest that these three variables do cointegrate, but only once allowance is made for a regime shift in early 1974.

Overall the findings in this chapter suggest that a recently developed test for cointegration in the presence of a regime shift (Gregory and Hansen, 1996) is particularly useful in examining the interrelationships between labour market variables in Australia. The results also support the notion that the move to centralised wage bargaining has increased the role of unemployment and lowered the role of productivity in the wage determination process. This is in line with the idea that the Accord and other centralised systems of wage fixing gives greater voice to the unemployed in the wage determination process.

## 6.2 Background

The majority of recent work on the empirical modelling of the wage equation draws, for its theoretical background, on a series of papers by Layard and Nickell (see for example Nickell and Andrews, 1983, Layard and Nickell, 1985, 1986, 1990, Layard, Nickell and Jackman, 1991). This model, with many variants, has been the basis for a wide range of empirical studies in a number of countries (for example in the UK see Layard and Nickell *op. cit.*; for Spain see Dolado, Malo de Molina and Zabalza, 1986, for 19 OECD countries see Bean, Layard and Nickell 1986, for Australia see Huay and Groenewold, 1992). While there are a wide number of variants on the Layard-Nickell model the basic framework is as follows. Firms and unions engage in some form of wage setting mechanism (varying from a competitive market to some form of collective bargaining between firms and unions), firms aim to maximise profits and are usually assumed to have the right to set prices and

employment levels. Unions typically have a utility function which depends upon the value of employment in the firm and the alternatives (such as income from unemployment benefits). Thus the wage outcome is the result of the bargaining strengths of the firms and unions and factors such as productivity, the level of demand and the probability of finding alternative employment. Once the firm has set prices, demand will determine the level of employment and thus the level of unemployment.

The usual result is an equation in which real wages are affected by a variety of factors which can be conveniently written, as in Manning (1993):

$$(w - p)_t = -\gamma_1 u_t + \beta_{11} \mathbf{x}_{1t} + \beta_{12} \mathbf{x}_{2t} + v_{1t} \quad (6.1)$$

along with what is typically referred to as a pricing equation of the form:

$$u_t = \gamma_2 (w - p)_t + \beta_{21} \mathbf{x}_{1t} + v_{2t} \quad (6.2)$$

Where  $(w - p)$  can be considered as the real consumer wage and  $u$  unemployment. The vector  $\mathbf{x}_1$  contains productivity, the tax wedge<sup>1</sup> and employment so that equation (6.2) can also be written as an employment or demand for labour equation. The vector  $\mathbf{x}_2$  contains wage pressure variables such as union power and unemployment benefits. The result is a wage equation, in which the real wage is negatively influenced by unemployment, but positively related to productivity variables and wage push variables. As noted by Manning (1993) since equation (6.1) (the real wage equation) contains all of the variables in the system then it is not identified. In practice this identification problem is typically “overcome” by excluding some of the productivity variables included in (6.2) from the wage equation (6.1). However

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<sup>1</sup> Where the tax wedge contains variables which drive a wedge between real labour costs and real take home pay such as employer and income taxes and the ratio of producer prices to the CPI.

a significant role for productivity in the wage equation has been found in a large number of empirical studies and is frequently justified on the basis of the results of insider-outsider models such as Nickell and Wadhvani (1990). Layard, Nickell and Jackman (1991) argue that, even if the reduced form equations are a mixture of wage and price equations the estimates remain unaffected. Indeed this seems to have been the implicit approach taken in the wide range of empirical modelling carried out in this area. Manning (1993), however, in a discussion of this problem notes that while many empirical studies of the wage process may have found evidence of a positive relationship between real wages and productivity, this does not necessarily imply that the structural equation for the wage determination need necessarily contain a productivity variable. Observed positive correlations between real wages and productivity could simply arise from the labour demand schedule and be nothing to do with the process of wage determination. Indeed Manning goes on to argue that, in a static context, a reduced form unemployment relationship should be estimated. In a dynamic context he develops a wage equation which does not include productivity and looks rather like a Phillips curve specification.

This raises the problem of how applied analysis of the wage determination process can deal with this problem in practice. A number of "solutions" present themselves.

Firstly the problem can be ignored as suggested by Layard Nickell and Jackman (1991). A recent study of the Australian wage equation by Huay and Groenewald (1992) which is discussed below, essentially takes this approach, estimating a wage equation which includes both productivity and unemployment making no reference to the problem. Secondly the wage equation could be identified by exclusion restrictions. In the context of the Australian wage equation this appears, implicitly possibly, to be the approach taken in a number of cases. For example Flatau, Lewis and Rushton (1991)

and Watts and Mitchell (1990) exclude either productivity or unemployment in a number of their specifications of the wage equation. Finally, whilst it is not a solution to the problem, the use of Johansens method of estimating cointegrating vectors would allow the researcher to test for the existence of a separate wage equation. If wages, productivity and unemployment were cointegrated and the researcher then estimated a “wage equation” including these and other variables then Johansen estimation should reveal the existence of more than one cointegrating vectors. One of the vectors should contain only wages, productivity and unemployment, the other should contain some or all of the extra variables and could be interpreted as the wage equation.

The question of whether, in an Australian context, real wages, productivity and unemployment are cointegrated and thus there exists an identification problem for any wage equation including these variables is taken up below in an analysis of equation 6.2. Whilst this might be an interesting result it is not the main focus of the next section which aims to test whether the shift from free market wage determination to centralised bargaining of one form or another and the Accord has affected the relationship between real wages, productivity and unemployment in Australia.

### **6.3 Australian evidence**

The use of cointegration techniques to test for the presence of long run relationships among the variables in the Australian wage equation has been relatively limited. Earlier work, by Lewis and MacDonald (1993), concentrated on the time series properties of the variables involved, and did not consider, in any depth, cointegration between the variables. Watts and Mitchell (1990) in their survey of Australian wage inflation use the Engle Granger method of testing for cointegration using a wide variety of specifications of the wage equation. They test for cointegration using eleven different specifications of

the equation with both average weekly earnings and the real wage (average weekly earnings deflated by the Consumer Price Index) as dependant variables. Arguing that Australian unemployment experience is better characterised by hysteresis arguments (Mitchell, 1987) than a natural rate approach they do not include unemployment as one of their explanatory variables but rather use proxies for insider power such as capacity utilisation, overtime and average weekly hours. Overall, and as they note, their results are inconclusive. Although they find some evidence of cointegration between real wages and average weekly hours they prefer to use a model developed without using cointegration techniques.

Flatau, Lewis and Rushton (1991) in a paper examining the relationship between real wages and unemployment test for cointegration between unemployment and long term unemployment, as a test of the hysteresis argument, but do not test for cointegration among the variables which they include in their wage equation. Their test of the hysteresis argument is based on the observation by Cross, Hutchinson and Yeoward (1990) that, under the natural rate model, there will be a tendency to converge on a unique duration composition of total unemployment. Their results find support for the hysteresis argument in Australian data with results from Engle Granger tests of cointegration between unemployment and long term unemployment suggesting that there is no cointegrating vector between the two variables. Using the model of Lewis and Kirby (1987,1988) they argue that changes in the composition of unemployment are important in the context of wage determination in Australia. Thus, as the proportion of long term unemployed increase, this effectively reduces labour supply and thus raises real wages. Their estimated real (consumer) wage equation uses real GNP as exogenous demand side influences on the real wage, and population and unemployment benefits as exogenous supply side variables. They also include a variety of wedge variables and the long term unemployment rate as a test of the



hysteresis argument. They argue that the period of the Accord has lowered real wage growth, giving greater voice to outsiders in the wage bargaining process. This in turn has lowered the number of long term unemployed which itself increases supply and further reduces real wages.

Finally, in more recent work, Huay and Groenewald (1992) estimate a wage equation using the real wage and data in levels but once again do not test for a cointegrating relationship between the variables involved. The model used by Huay and Groenewald is based Layard and Nickell (1985, 1986) and results in a real wage equation in which real wages depend on expected demand conditions, technical progress, the capital/labour ratio, the unemployment rate (as a proxy for the ratio of employment to the labour force), price expectations and wage push factors. The final estimated equation (estimated in levels) finds a significant role for unemployment (with expected positive sign), productivity (proxied by a time trend), the replacement ratio and a number of the wedge variables.

One possible reason for this absence of (positive) results regarding cointegration between the wage equation variables in Australia is that events of the early 1970s led to a change in long run relationships between the variables considered. In this chapter, based upon the results in Chapter 5, the variables considered are taken as being  $I(1)$ , but allowance is made for a possible regime shift in the cointegration framework by making use of a simple test for cointegration suggested by Gregory and Hansen (1996). The justification for such a regime shift being of importance in the Australian labour market is suggested by a consideration of the events of the early 1970s which were, of course, the period of the first oil shock. 1974 also saw the election of a new Labour government in Australia and a dramatic rise in both wages and non labour costs, a rise which some blame for the rapid rise in unemployment over this. It was also the period when Australia began its move

towards a more centralised wage bargaining structure. Australia has, as is widely known, operated a system of some form of incomes policy since the mid 1970's, with the introduction of the first phase of wage indexation introduced in 1975 quarter 2 and the first phase of the Accord in 1983 quarter 3. The effects of this extensive period of incomes policy have been much debated in the literature (see Lewis and Spiers (1989), Chapman and Gruen (1989), Lewis and Kirby (1987) for surveys of this issue).

All of the above leads to the conclusion that, in the context of the Australian wage equation, it may well be important to consider possible regime shifts in applied analysis.

#### **6.4 Wages, productivity and unemployment**

The applied analysis begins by consideration of equation (6.2). On the basis of the above discussion, and dependent upon the assumptions made, it would be expected that there would be an equilibrium relationship between real wages, productivity and unemployment. This, potential relationship, is of interest for two reasons. First, it is important to identify any potential cointegrating vector between this subset of variables prior to looking for cointegration between the variables in equation (6.1), since, if such a cointegrating vector exists, estimated wage equations which include unemployment and productivity will not be identified.

A second reason, however, is that the relationship between these variables may be of interest of itself, particularly in the context of testing for cointegration in the presence of a regime shift. A recent paper by Alexander (1993) considers the relationship between real wages, productivity and unemployment in the UK in an attempt to test whether the transition from a period dominated by incomes policies in the UK to the deregulated labour market during the period

of the Thatcher government (and subsequent Conservative governments) makes any difference to the causal relationships between the variables. The simple argument is that the relationship between wages and productivity should be less strong under the period of income policy than under free market conditions whereas the opposite might be expected regarding unemployment. This is, in fact, the conclusion of Alexander's paper, which finds that there was a structural break in the relationship between the three variables in the late 1970's, the period when the Thatcher government was elected and moved to free up the labour market. This is clearly a question of some interest in the Australian market, where it would be possible to conjecture that the relationship between these variables has been affected by Australia's movement from a system of free collective bargaining to one of centralised bargaining. This process began in the 1970's with the movement towards a system of centralised bargaining and culminated in the Accord system of wage determination. Indeed the suggestion made by Flatau et al (1991), that the Accord has led to greater voice for the unemployed, would suggest that this would be the case. In particular it will be of interest to see if a cointegrating vector can be found over the whole period if allowance is made for the regime shift. One of the weakness of Alexander's methodology is that Johansen's method of testing for cointegration is used over the whole period, with no evidence of a cointegrating vector, however, one is assumed, so as to be able to apply Chow tests for a structural break to the estimated Vector Error Correction Model. Finding evidence of such a break, the sample is then split in two to identify the cointegrating vectors over the two sub periods. The use of Gregory and Hansen's (1996) method of testing for a structural break will provide a simple method of testing for cointegration allowing for a potential break in the series without having to split up the sample. This method should also allow the identification of the period of the shift in the relationship.

The data used is that considered in Chapter 5 and described more fully in Appendix A2. This section uses the log of productivity, PROD, the log of the consumer real wage, RWAG and the unemployment rate, UR. All data is available for a full, quarterly sample from 1959 quarter 3 to 1996 quarter 1. First, standard tests for cointegration were applied to the data, to see whether the null of cointegration could be rejected over the full sample. An OLS regression of the log of real wages (RWAG) on the log of productivity (PROD) and the unemployment rate (UR) produced the following, clearly spurious regression (t statistics in parenthesis):

147 observations used for estimation from 1959Q3 to 1996Q1

$$\text{RWAG} = 3.05 + 1.35\text{PROD} - 0.12\text{UN} \quad (6.3)$$

(21.2) (19.4)      (-2.9)

$$R^2 = 0.91 \quad \text{DW} = 0.095$$

A unit root test on the residuals, which are plotted in Figure 6.1 and clearly look non stationary, confirms that the null of no cointegration cannot be rejected with the ADF(1)<sup>2</sup> statistic having a value of -1.2, which is not significant using critical values from MacKinnon (1991).

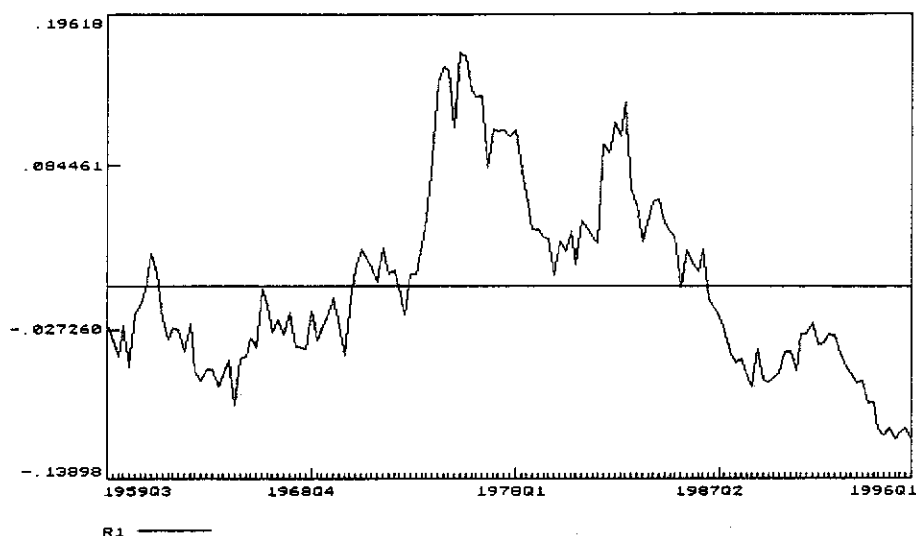
As a further test, Johansen's method of testing cointegration was applied to the data. A VAR in levels was estimated and a lag length of 5 was found to produce acceptable diagnostics<sup>3</sup>. Table 6.1 reports the results of the Johansen estimation using an unrestricted constant (to allow for non zero drift in the data) and Figure 6.2 plots the, recursively estimated, largest eigenvalue.

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<sup>2</sup> In this and later plots the order of the ADF was chosen based on the significance of the last included lag and the absence or autocorrelation in the residuals.

<sup>3</sup> Although there was evidence of non normality in the unemployment equation Cheung and Lai (1993) suggest that the Johansen test is relatively robust to non normalities.

**Figure 6.1** Residuals from ols regression of RWAG on PROD and UN



**Table 6.1** Tests for cointegration over full sample using Johansen estimation

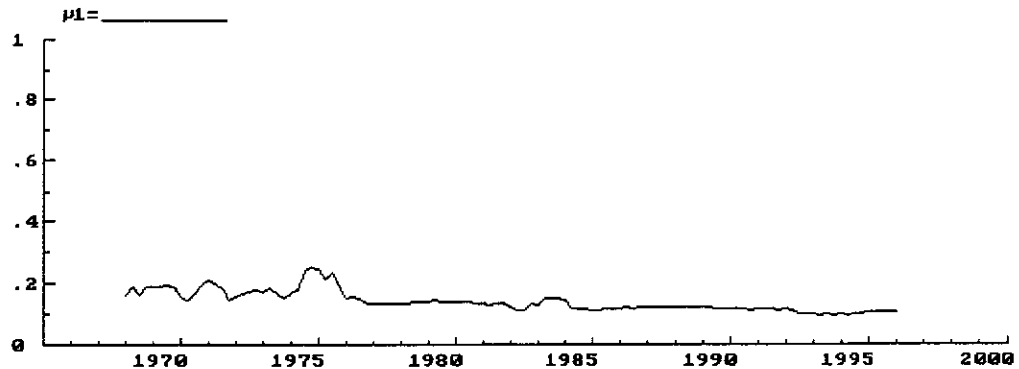
<u>VAR in levels - 5 lags, individual equation diagnostics</u>		
	AR1-5 F( 5,120)	Normality $\chi^2(2)$
RWAG equation	0.62652 [0.6798]	4.5113 [0.1048]
PROD equation	0.95739 [0.4469]	5.4655 [0.0650]
UN equation	1.678 [0.1450]	15.363 [0.0005] **

**Cointegration analysis 1961 (1) to 1996 (1)**

	Maximal Eigenvalue	95%	Trace	95%	
p=0	15.87	21.0	24.28	29.7	
p≤1	6.64	14.1	8.41	15.4	
p≤1	1.766	3.8	1.766	3.8	
Standardised $\beta$ eigenvector associated with largest eigenvalue			RWAG	PROD	UN
			1.0	-2.26	0.103

AR(1-5) is a test for residual autocorrelation, Normality a test for the normality of the residuals. Maximal eigenvalue and Trace are the tests for cointegration based on Johansens method and described in Chapter 4. All of the estimation in this Chapter was carried out using PC-FIML (part of PC-GIVE professional version 8, Doornik and Hendry, 1994), the mis-specification tests reported in this and other tables are those produced by PC-FIML and described in Doornik and Hendry (1994).

**Figure 6.2** Recursively estimated largest eigenvalue from full sample estimation



As can be seen, there is no evidence of a cointegrating vector, with both the trace and maximum eigenvalue tests below their 5 per cent critical values. It is also interesting to note, from Figure 6.2, the instability of the largest eigenvalue which probably reflects parameter instability, around 1974/1975 and then again around 1984. So, these results confirm that standard cointegration tests, applied to the full sample yield no evidence of a cointegrating vector between the three variables.

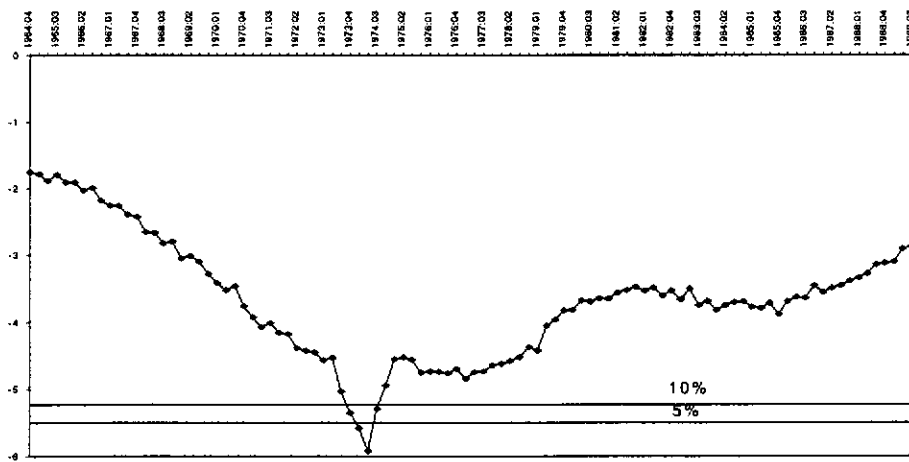
Turning now to the results of testing for cointegration allowing for a regime shift. As noted in chapter 4, Hansen and Gregory (1996) describe three models in which the hypothesis of cointegration with a regime shift can be tested. In the context of the current data their model 4 (described more fully in Chapter 4), which allows for a regime shift, where both the mean and slope of the cointegrating vector are allowed to shift, was used<sup>4</sup>. Figures 6.3, 6.4 and 6.5 show, respectively, the recursively estimated  $Z_{\alpha}$  and  $Z_{\tau}$  and (A)DF statistics (again described more fully in Chapter 4), along with both 5 percent and 10 percent critical values (from Table 1 of Hansen and Gregory, 1996).

<sup>4</sup> Tests using either of the other two models yielded insignificant test statistics.

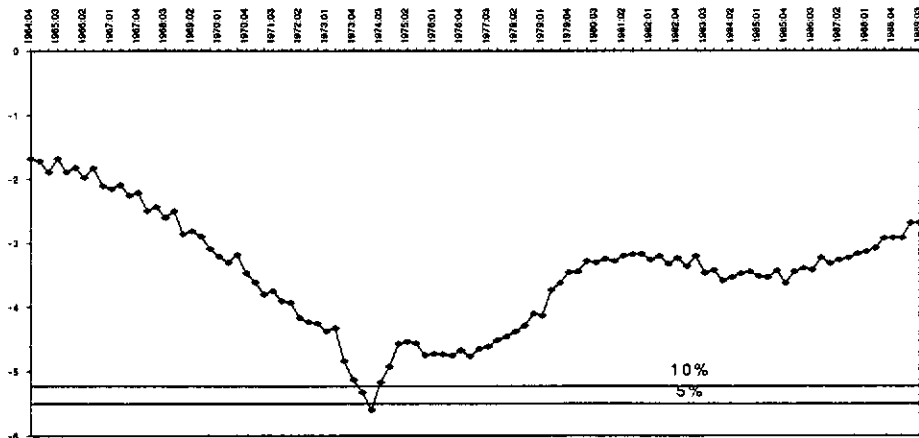
**Figure 6.3** Recursively estimated  $Z_{\alpha}$  test statistic for model 4.



**Figure 6.4** Recursively estimated  $Z_t$  test statistic for model 4.



**Figure 6.5** Recursively estimated ADF test statistic for model 4.



In each case there is a very pronounced minimum for the test statistics, and, in each case, this occurs in 1974 quarter 2. The results for the ADF test on the residuals from model 4 used a lag length of zero (a DF test) since the

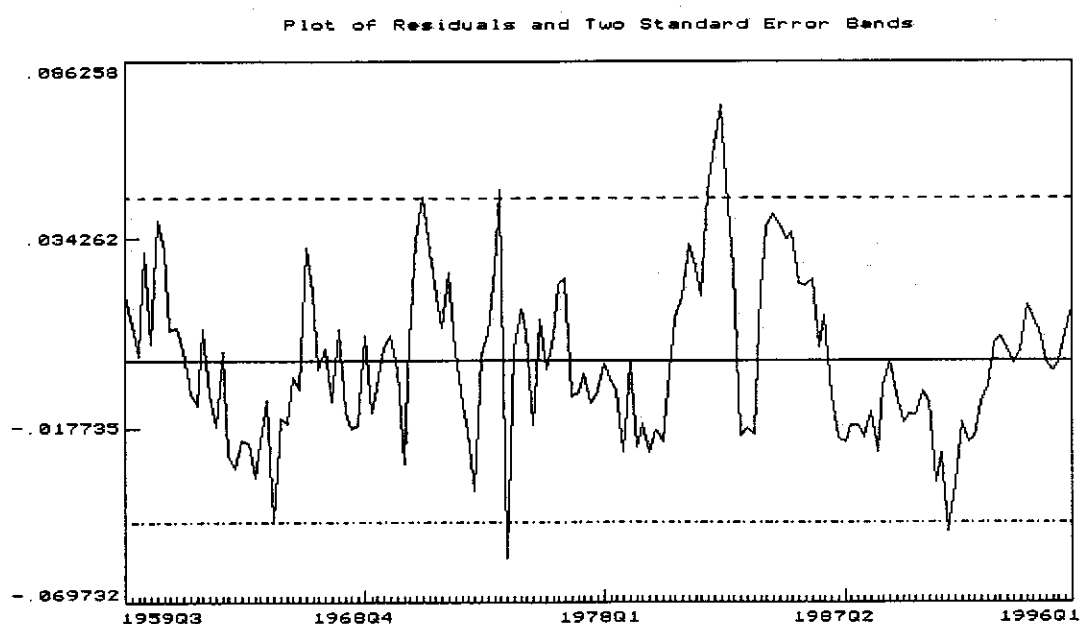
associated test of the unit root in the residuals showed no evidence of autocorrelation and higher order lags were insignificant. Table 6.2 below reports the estimated equation and the DF test and Figure 6.5 plots the residuals, which now appear stationary. As can be seen the DF statistic of -5.6 is significant at the 5 percent level using the critical values in Table 1 of Gregory and Hansen, suggesting that the three variables do cointegrate when allowance is made for a regime shift.

**Table 6.2** OLS estimation of Model 4 and associated cointegration test on residuals

Gregory and Hansens's Model 4 - estimated by OLS						
	Constant	DUM	PROD	UN	DPROD	DUN
RWAG	2.7 (38.5)	3.3 (24.2)	1.48 (46.2)	0.02 (3.0)	-1.4 (-22.4)	-0.01 (-1.5)
Cointegration test using retrieved residuals.						
	res(-1)	DW	AR(1-4) F(4,141)			
$\Delta$ res	-0.35 (-5.6)	2.14	0.49 [.74]			

Where DUM is a step dummy taking the value 0 from 1959(3) to 1974(1) and 1 elsewhere. DPROD = DUM x PROD. DUN = DUM x U. Res are the residuals from the estimated equation. Figures in ( ) are estimated t ratios, figures in [ ] are probability values for reported test statistics.

**Figure 6.6** Residuals from ols regression.





So, where as standard testing procedures revealed no evidence of a cointegrating vector, there is evidence of cointegration once allowance is made for a single regime shift in the early 1970s. It is also interesting to note that the regression results show that the coefficients on productivity and unemployment fell after the break point, with the coefficient on productivity falling by 1.4 from 1.48 to 0.08 and that on unemployment from 0.02 to 0.01<sup>5</sup>. Thus, in this context, Gregory and Hansen's method of testing for cointegration appears useful and suggests that, since the two types of tests yield different results, the regime shift hypothesis is a tenable one.

Given that the evidence from estimation using Gregory and Hansens method suggests that cointegration between the variables exists, but that the cointegrating vector has been subject to a regime shift, it seems valid to split the sample and test for cointegration in the sub samples. The tests described above suggest that the break point occurred in 1974 quarter 2. Splitting the sample at this point and testing for cointegration using data from 1959 quarter 3 up to 1974 quarter 1 and from 1974 quarter 3 suggested that the variables did not cointegrate over the two sub samples. However, it would seem likely that the shift in the cointegrating relationship did not occur instantaneously, but rather over a period of time around 1974. Models estimated using the data up to 1972 and from 1976 (allowing approximately two years either side of the break to allow for its effect on the variables) to the end of the sample did in fact reveal evidence of cointegration and so these are reported below. with tables 6.3 and 6.4 reporting the results of Johansen estimation<sup>6</sup>.

As can be seen from the results in Tables 6.3 and 6.4 there is now evidence of a single cointegrating vector over both sub samples. Before moving on to more

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<sup>5</sup> Simple ols regressions over the two sub samples confirm this.

<sup>6</sup> Over the sub samples a VAR of order 4 was found to yield acceptable diagnostics. Once again the constant was allowed to enter unrestricted.

formal testing of these cointegrating vectors a number of simple points can be made.

**Table 6.3** Test for cointegration using Johansen method, sub sample 1961quarter 1 to 1971quarter 4.

Cointegration analysis 1961 (1) to 1971 (4)

	<i>Maximal Eigenvalue</i>	95%	<i>Trace</i>	95%
p=0	32.33	21.0	42	29.7
p≤1	9.23	14.1	9.67	15.4
p≤2	0.44	3.8	0.44	3.8
Standardised $\beta$ eigenvector associated with largest eigenvalue		<b>RWAG</b>	<b>PROD</b>	<b>UN</b>
		1.0	-1.703	-0.0678

**Table 6.4** Test for cointegration using Johansen method, sub sample 1976 quarter 1 to 1996 quarter 1.

Cointegration analysis 1976 (1) to 1996 (1)

	<i>Maximal Eigenvalue</i>	95%	<i>Trace</i>	95%
p=0	40.22	21.0	48.86	29.7
p≤1	5.63	14.1	8.64	15.4
p≤2	3.01	3.8	3.01	3.8
Standardised $\beta$ eigenvector associated with largest eigenvalue		<b>RWAG</b>	<b>PROD</b>	<b>UN</b>
		1.0	0.740	-0.042

Firstly, normalising on real wages, the cointegrating vectors are,

$$1961(1) \text{ to } 1971(1) \text{ RWAG} = 1.70\text{PROD} + 0.07\text{UN}$$

$$1976(1) \text{ to } 1996(1) \text{ RWAG} = -0.74\text{PROD} + 0.04\text{UN}$$

As can be seen productivity has a positive coefficient in the cointegrating vector over the first sub sample (free collective bargaining) but negative over the second half of the sample. Whilst the sign on unemployment falls by half it

is positive in both parts of the sample, suggesting that these results might be best interpreted as demand for labour relationships.

Ordinary least squares estimation over the same two sub sample yielded different parameter estimates but told a similar story. The coefficients on both productivity and unemployment fall in the second sample with the coefficient on productivity (now remaining positive over both samples) being significant in the first sample and insignificant in the second and with the coefficient on unemployment being insignificant over the first sample and significant over the second.

$$1961(1) \text{ to } 1971(1) \quad RWAG = 2.76 + 1.45PROD + 0.02UN$$

$$\qquad\qquad\qquad (15.2) \quad (18.3) \qquad (1.61)$$

$$\qquad\qquad\qquad (15.3)$$

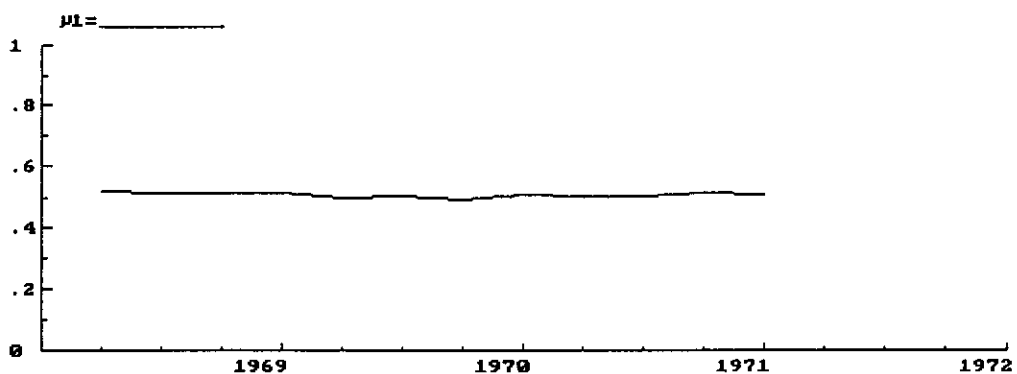
$$1976(1) \text{ to } 1996(1) \quad RWAG = 6.03 + 0.09PROD + 0.01UN$$

$$\qquad\qquad\qquad (46.6) \quad (1.5) \qquad (3.1)$$

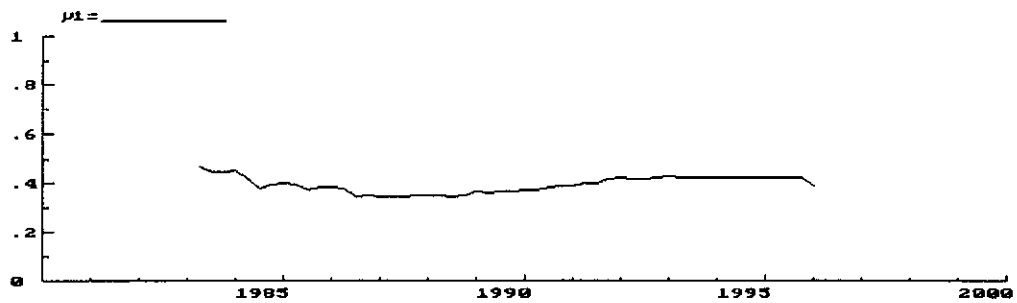
t statistics in parenthesis.

These results also confirm the suspicions, raised by the plot of the full sample recursive eigenvalue, regarding parameter instability. As can be seen from Figures 6.7 and 6.8 the recursive eigenvalues estimated over the two sub sample now show less evidence of instability.

**Figure 6.7 Largest, recursively estimated, eigenvalue, sample 1961 quarter1 to 1971 quarter 4**



**Figure 6.8** Largest, recursively estimated, eigenvalue, sample 1976 quarter1 to 1996 quarter 1.



Moving to more formal tests of the cointegrating vectors, it was noted in Chapter 4 that it is possible to construct tests of restrictions on the parameters of the cointegrating vector. Table 6.5 reports Likelihood Ratio (LR) tests of zero restrictions on firstly unemployment and then productivity in the estimated cointegrating vectors. As can be seen these all reject strongly, so both unemployment and productivity are required in the cointegrating vectors over both sample periods.

**Table 6.5** Tests of cointegrating vectors.

Sample	zero coefficient on Productivity		zero coefficient on unemployment	
1961(1) to 1971(4)	22.55	[0.00]	21.36	[0.00]
1976(1) to 1996(1)	27.88	[0.00]	34.23	[0.00]

Table 6.6 reports the dynamic results of estimation of the VECM model over the two sub samples. Since interest centres on the effects of productivity and unemployment on real wages, only the real wage equation is reported. Looking first at the sub sample up to the end of 1971, the VECM is reported with 3 lags, since 4 lags produced evidence of serial correlation in the  $\Delta$ wage equation. As can be seen the equation passes the usual range of diagnostics. A Wald test for the restriction that  $\Delta UN_{t-1} = \Delta UN_{t-2} = \Delta UN_{t-3} = 0$  produces a test

Table 6.6

Sample: 1961 quarter 2 to 1971 quarter 4		Sample: 1996 quarter 1 to 1996 quarter 1	
	Dependant variable $\Delta RWAG$		Dependant variable $\Delta RWAG$
$\Delta RWAG(-1)$	-0.19 (-1.0)		0.02 (0.2)
$\Delta RWAG(-2)$	0.30 (1.5)		0.27 (2.1)
$\Delta RWAG(-3)$	0.01 (0.1)		-0.00 (-0.0)
$\Delta RWAG(-4)$			0.04 (0.3)
$\Delta PROD(-1)$	-0.34 (-1.2)		-0.33 (-2.1)
$\Delta PROD(-2)$	-0.62 (-2.6)		-0.32 (-1.8)
$\Delta PROD(-3)$	-0.16 (-0.7)		-0.18 (-1.0)
$\Delta PROD(-4)$			-0.18 (-1.2)
$\Delta UN(-1)$	-0.01 (-1.2)		-0.02 (-3.5)
$\Delta UN(-2)$	-0.00 (-0.2)		-0.00 (-0.1)
$\Delta UN(-3)$	-0.01 (-1.1)		0.01 (0.9)
$\Delta UN(-4)$			0.01 (1.5)
$ECM(-1)$	-0.34 (-2.3)		-0.07 (-0.9)
$\sigma$	0.01		0.01
$AR(1-5)$	0.22		0.41
(F test)	[.95]		[.84]
Normality	1.91		2.13
$\chi^2(2)$	[.38]		[.34]
$ARCH(5)$	0.41		0.19
(F test)	[.84]		[.94]

$AR(1-5)$  is a test for residual autocorrelation, Normality a test for the normality of the residuals,  $ARCH$  a test for autoregressive conditional heteroscedasticity in the residuals. The misspecification tests reported in this table are those produced by PC-FIML and described in Doornik and Hendry (1994).

statistic of 2.32 which is distributed as  $\chi^2(3)$  and hence not significant at normal significance levels. Thus in the period, broadly identified as one of free collective bargaining, the dynamics of unemployment are not significantly different from zero in the dynamic wage equation. On the other hand a test of zero restrictions on  $\Delta\text{PROD}_{t-1}$ ,  $\Delta\text{PROD}_{t-2}$  and  $\Delta\text{PROD}_{t-3}$  produces a test statistic of 7.33 which again is distributed as  $\chi^2(3)$  and significant at the 6 per cent level. Thus the dynamics of productivity appear to be important over the sub sample. It should also be noted that the error correction term is significant over this sub sample, thus there is a potential role for unemployment in the period.

Looking now at the sub sample 1976 quarter 1 to 1996 quarter 1, a VECM with four lags produced acceptable diagnostics. A test of zero restrictions on  $\Delta\text{PROD}_{t-1}$ ,  $\Delta\text{PROD}_{t-2}$ ,  $\Delta\text{PROD}_{t-3}$  and  $\Delta\text{PROD}_{t-3}$  produces a test statistic of 5.87 which is distributed as  $\chi^2(4)$  and thus not significant at normal levels. Thus the null hypothesis that the lags of  $\Delta\text{PROD}$  are jointly zero cannot be rejected. On the other hand the Wald test for zero restrictions on the lags of  $\Delta\text{UN}$  evaluates to 14.94 and is thus significant at the 0.5 per cent level. So over the later half of the sample, which includes the period of the ACCORD, unemployment dynamics appear significant in the  $\Delta\text{wage}$  equation whereas productivity does not.

## **6.5 Conclusion**

The cointegration analysis carried out in this chapter has yielded some interesting results. Standard testing procedures (Engle/Granger and Johansen) reveal no evidence of a cointegrating vector between real wages, productivity and unemployment over the full sample of data available. However, recent tests which allow for a single regime shift which affects the relationship between the variables, suggest that there is evidence of cointegration with a regime shift

occurring in 1974. This concept is supported by evidence from standard cointegration tests on a split sample. As a result, if it is accepted that these three variables cointegrate then any wage equation which included productivity and unemployment suffers from the identification problem noted above. It is therefore clear that the events of the early 1970's, the oil price shock, and the move, in Australia, towards more centralised wage bargaining have had a significant effect on the variables considered here and researchers should be aware of the effects of this change when testing for cointegration between wage equation variables.

More significantly the results show that, just as found for the UK by Alexander (1993) the move from free market wage bargaining to centralised bargaining has affected the relative importance of productivity and unemployment in the wage process. Evidence from the cointegration results and from the resultant Vector Error Correction Models suggests that the dynamics of productivity have no effect on wage dynamics in the later sample, the period of more centralised bargaining. The dynamics of unemployment which appear to be insignificant over the first sample are significant in the second sample. These results strongly support the notion that as Australia moved from a period of free wage bargaining into one of centralised bargaining unemployment became more important in determining the wage outcome and productivity less so.

## CHAPTER 7

### THE LONG-RUN GAINS FROM INTERNATIONAL EQUITY DIVERSIFICATION<sup>1</sup>

#### **7.1 Introduction and background**

In this chapter the cointegration framework is used to analyse the benefits available from international equity diversification to Australian investors for the period 1970 to 1992 using monthly index data for 16 countries supplied by Morgan Stanley Capital International. Results from the standard Granger/Engle two step OLS procedure and the Johansen (1988) maximum likelihood procedure for testing for cointegration are compared. The finding is that, as in other recent work (Taylor and Tonks (1989) and Andrade, Clare and Thomas (1991)), the two techniques lead to different conclusions in certain cases. The results also suggest, as in Kasa (1992), that there is evidence of cointegration amongst a subset of the indices considered. A further finding of interest to the applied worker is that the results in the Johansen procedure are sensitive to the VAR specification and it would seem that Hall's (1991) warning regarding the reporting of tests from this procedure is valid.

The trend towards market deregulation and the greater integration of world capital markets has lead to an increased focus on the potential benefits available from the international diversification of investment portfolios. The formal analysis of the benefits of diversification was pioneered by Markowitz

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<sup>1</sup> This chapter reproduces, with minor editing, largely to avoid repetition of material in Chapters 3 and 4, Allen and MacDonald (1995), "The long run gains from international equity diversification: Australian evidence from cointegration tests".



(1952) and Tobin (1958). Grubel (1968) pioneered the application of this analysis to international markets and stimulated a series of further studies such as Levy and Sarnat (1970), Solnik (1974), and Lessard (1976). Work on international diversification from an Australian viewpoint was initiated by Watson and Dickinson (1981) followed by Mitchell, Wapnah and Izan (1988). Both studies confirmed the benefits of international diversification to Australian investors.

These early studies used ex-post analysis in which it is assumed that the required inputs, (expected returns, variances and covariances), estimated to form internationally diversified portfolios, are known with certainty. This approach ignores the fact that the weights utilised in forming the optimal portfolios may be subject to estimation risk. Variations in expected returns can result from the extension of the sample period and the addition of further observations. Jorion (1985, 1986) proposed the utilisation of the Bayes-Stein estimator to control for estimation risk and Eun and Resnick (1988) also applied it in a study of the gains from international diversification in a US context. Izan, Jalleh and Ong (1991) undertook an Australian study which controls for both estimation risk and foreign currency risk. They concluded that diversification strategies which include the controls dominate those that do not.

A feature of the above approaches is that the windows used to estimate the covariance structures of returns are usually relatively short (maximum length a few years). The Bayes-Stein approach suggests the shrinkage of past averages towards a common value. The evidence suggests that this reduces estimation risk in the case of the minimum variance portfolios but the grand mean may be sample specific and dependent on the sample window. This will not be such a problem for short holding periods but could be misleading if the national

indices utilised for the studies are trending together over time and the investor has a long-term investment horizon.

In this Chapter the cointegration framework is used to allow consideration of the longer term time series behaviour of various national market indices in a study of the benefits of international equity diversification. It is of particular relevance to institutions such as superannuation funds and life insurance companies who would wish to hold long-term investment portfolios and may be adopting a policy of passive diversification.

## **7.2 Testing for gains from portfolio diversification**

Modern portfolio theory demonstrates that the gains from international portfolio diversification are inversely connected to the correlations in security returns. If national equity markets have a long term tendency to trend together, the apparent gains from international diversification, as suggested by the previously mentioned studies, may overstate the case for the long term investor. The increasing tendency towards the globalization and deregulation of world capital markets may have reduced segmentation between the various national markets. This could have led to increased correlation between these markets. Grubel and Fadner (1971), Panton, Lessig and Joy (1976), plus Taylor and Tonks (1989) have reported a “high” degree of correlation between world equity markets.

Another set of studies, which have implications for portfolio analysis, analyse the extent to which common factors drive returns in international markets. Cho, Eun and Senbet (1986) report that there may be three or four factors driving equity returns in an international study of arbitrage pricing theory. Campbell and Hamao (1992) use monthly data from the US and Japanese markets in a study of the integration of the two markets using an observable

factor model and a single latent variable model. They report that similar variables, including the dividend-price ratio and interest rate variables, help to forecast excess returns in each country. They report some evidence of common movement in expected excess returns across the two countries which is indicative of integration of long-term capital markets. Further evidence is provided by Roll (1992), in a study using daily data from April 1988 through to March 1991. He reports that three separate influences appear to drive national indices in a sample of 24 countries: technical aspects of index construction, each country's industrial structure plus exchange rate behaviour. The relative composition of national industrial structures appears to have the strongest influence on market behaviour.

This Chapter reports the results of a study of sixteen of the world's financial markets: Australia, Austria, Belgium, Canada, France, Germany, Hong Kong, Italy, Japan, Norway, Singapore and Malaysia, Spain, Sweden, Switzerland, UK, and the USA. The study is conducted from the viewpoint of an Australian investor. It uses the monthly data taken from the accumulation indices in each of these 15 external markets which are converted into Australian dollar terms. The data was provided by Morgan Stanley Capital International (MSCI). The indices of market returns are constructed using market capitalisation's as the weights and the series adopted were accumulation indices with gross dividends reinvested. The companies in the MSCI indices represent approximately 60 per cent of the aggregate market capitalisation's of the stock exchanges included. The indices are constructed using a weighted arithmetic average and are adjusted for capitalisation changes. The indices with dividends reinvested provide an estimate of the total return that would be achieved by reinvesting one twelfth of the monthly dividend yield reported at every end month. The series with gross dividends reinvested takes into account actual dividends before withholding taxes but excludes special tax credits declared by the companies. The indices were converted into Australian dollar terms using

month end exchange rates. The MSCI series is particularly suited for this purposes because all of the indices are constructed on a consistent common basis. The degree of integration between these markets is examined using the techniques of cointegration analysis which has had an enormous impact on applied finance and economics in recent years.

The existence of cointegration between two financial markets suggests that in the long run their returns will be highly correlated, even though they may diverge in the short run. It thus implies that diversifying between them over the long run is not likely to lead to large benefits in risk reduction. Taylor and Tonks (1989) report cointegration between the UK and various international equity markets after 1979. However, Andrade, Clare and Thomas (1991) report a lack of cointegration in the same markets suggesting the potential for long run gains from international diversification. Kasa (1992) considers the five major markets of the US, Canada, UK, Japan and Germany and finds that there does exist a single common trend which drives the markets

A potential reason for the disagreement between these two UK studies is the difference in research methods adopted. Taylor and Tonks (1989) use the Engle and Granger (1987) two step procedure for testing for cointegration whilst Andrade et al., use the Johansen (1988) procedure. The latter, as noted in chapter 4 has the advantage of being more general since it allows the inclusion of deterministic variables (such as dummies etc) in the cointegrating vector. Furthermore, the Engle and Granger method involves an arbitrary normalising of the cointegrating vector on one of the variables, implicitly making therefore the assumption that that variable takes a non zero value. It also has the disadvantages of assuming that the cointegrating vector is unique. The Johansen procedure imposes no such priors and permits an estimation of the number of cointegrating vectors in the system.

If cointegration exists between two financial markets it suggests that one of the markets will help predict the other since a valid error correcting representation will exist. This is a violation of the weak form of the efficient market hypothesis. Keim and Stambough (1986) and Campbell and Hamao (1992) have reported results which imply that certain variables can be used to predict stock returns. Thus, tests for the existence of cointegration can also be interpreted as tests of market efficiency.

### **7.3 Unit Root tests on individual series**

The first step in the analysis is to test the data for the presence of unit roots. This was carried out using Dickey-Fuller type tests as detailed in Chapter 3. Tests began with a lag order of 12 and both information criteria measures (Final Prediction Error and Schwarz Criteria) and an F test of the significance of the last included lag (stopping when the coefficient on the last included lag was significant) were used to determine lag order. Where there was disagreement, tests at each suggested lag order were estimated.

Using a 5% critical value it was found that in fourteen out the sixteen cases the test results were robust to the lag order and showed no evidence of rejection of the unit root null, only in the case of Singapore/Malaysia was the lag order of any importance. Table 7.1 reports the results. For each series, tests based on the lag order selected using the above methods are reported, where the tests conflict results using the different lag lengths suggested are reported. Both the  $\phi(3)$  and the  $\phi(2)$  test as well as ADF tests are reported.

At the 10% significance level none of the series can reject the unit root in levels. On taking first differences all of the series strongly reject the unit root at all lags at the 5% level. Thus the tests appeared very robust to the lag length selection (and indeed the regression equation used). The conclusion is,

therefore, that the series are  $I(1)$ , giving support to the assumption of weak form efficiency in these markets.

#### **7.4 Bivariate Cointegration Tests - Johansen Method**

Having established that the series are  $I(1)$  it is possible to test for cointegration amongst these series.

Table 7.2 reports the results of tests for cointegration between Australia and each of the other countries (in bivariate portfolios) using Johansen estimation techniques. The equations are estimated using a variety of lag orders in the VAR. For each case an LM test for residual correlation in the VAR is also reported. A dummy variable was introduced for the October 1987 crash. A Bera-Jarque test, for the normality of the residuals, typically proved significant even after introducing the October '87 crash dummy but the residuals were symmetric around zero and the non normality was the result of a few outliers.

Since most of the series have an upward trend the model was estimated allowing the constant to enter unrestrictedly. As noted by Hall (1991) the choice of the lag order in the VAR is of some importance in the sense that too low an order is likely to lead to problems with serial correlation whereas too high an order could potentially lead to small sample problems. In tests on UK consumption and income data he finds that results from the Johansen test are sensitive to the lag length chosen and suggests that applied researchers either report test statistics for a range of lags or choose the lag length in the VAR using the minimum test statistic as a lag length selection criteria. As a result of this suggestion results are reported for a range of lags. However neither of these factors appeared to be a problem with this data set since there are a large number of observations and serial correlation does not seem to be a problem, even in low order VAR's.

**TABLE 7.1 Unit root tests on log levels of series.**

	Australia	Austria	Belgium	Canada	France	Germany	Hong Kong	Italy
Min	.0049	.0047	.0039	.0032	.0053	.0044	.0134	.0064
FPE	(0)	(1)	(1)	(0)	(1)	(7)	(1)	(11)
Min SC	-5.3	-5.3	-5.5	-5.7	-5.2	-5.4	-4.3	-5.0
	(0)	(1)	(0)	(0)	(0)	(0)	(0)	(0)
F test	(0)	(10)	(7)	(0)	(0)	(10)	(10)	(11)
$\Phi(3)$	5.39(0)	1.61(1)	1.47(0)	2.61(0)	2.77(0)	3.16(0)	3.92(0)	3.11(0)
		3.10(10)	1.52(1)		3.04(1)	3.47(7)	4.87(1)	3.06(11)
			1.67(7)			4.10(10)	4.87(10)	
$\Phi 2$	4.9(0)	3.06(1)	5.33*(0)	4.11(0)	4.21(0)	4.62(0)	4.71(0)	2.63(0)
		2.89(10)	4.55(1)		3.87(1)	3.74(7)	4.91(1)	2.26(11)
			3.04(7)			3.95(10)	5.28*(10)	
ADF	0.40(0)	-0.20(1)	0.47(0)	-0.41(0)	0.53(0)	0.19(0)	-0.71(0)	0.07(0)
		-0.62(10)	0.37(1)		0.41(1)	-0.45(7)	-0.75(1)	-0.80(11)
			0.11(7)			-0.57(10)	-0.69(10)	
	Japan	Norway	Singapore Malaysia	Spain	Sweden	Switzerland	UK	USA
Min	.0047	.0069	.0081	.0046	.0044	.0034	.0056	.0027
FPE	(0)	(4)	(1)	(8)	(1)	(0)	(2)	(0)
Min SC	-5.3	-4.9	-4.8	-5.3	-5.4	-5.6	-5.1	-5.9
	(0)	(1)	(1)	(0)	(0)	(0)	(0)	(0)
F test	(9)	(10)	(11)	(8)	(7)	(12)	(9)	(0)
$\Phi(3)$	0.70(0)	2.77(1)	5.15(1)	1.28(0)	2.91(0)	2.66(0)	3.26(0)	3.38(0)
	2.03(9)	2.52(4)	6.97*(11)	1.06(8)	2.77(1)	4.6(12)	2.90(2)	
		4.00(10)			2.49(7)		3.12(9)	
$\Phi 2$	4.53(0)	3.21(1)	4.96*(1)	2.28(0)	5.59*(0)	4.89*(0)	4.53(0)	5.54*(0)
	2.50(9)	3.15(4)	6.14*(11)	1.45(8)	4.89*(1)	4.32(12)	4.46(2)	
		3.47(10)			3.86(7)		3.65(9)	
ADF	-0.92(0)	-0.71(1)	-1.22(1)	0.33(0)	0.36(0)	0.07(0)	0.37(0)	0.77(0)
	-1.41(9)	-0.72(4)	-1.54(11)	-0.19(8)	0.22(1)	-0.67(12)	0.37(2)	
		-0.70(10)			-0.00(7)		0.10(9)	

Min FPE and Min SC refer to the minimum value of the Final Prediction Error and Schwarz criteria, used to determine the lag length. Figures in parenthesis below the test statistic indicate the lag length chosen in the ADF. F test is a test on the significance of the last included lag in the ADF and the figure, in parenthesis, indicates the lag length chosen.

The  $\phi_3$ ,  $\phi_2$  and ADF test are reported for each lag length selected by the different criteria, figures in parenthesis once again indicate lag length used in calculation of the test statistic.

The ADF test is calculated using a regression with constant and trend included.

A \* indicates significance at the 5% level.

As can be seen from Table 7.2 there is evidence of cointegration between Australia and: Canada, France, Germany, Hong Kong, Switzerland and the UK. In the case of the UK, Hong Kong and Canada the test statistics are significant at the 5% or higher level at all lags, Germany and Switzerland produce test statistics which are significant at the 10% or better level at all lags. The weakest case is that of France which produces test statistics which are significant at the 10% level or better at most lags, however lag 6 produces test statistics which are not significant.

These results are not easy to interpret. Certainly a case could be made for Australia's close relationship with the Canadian market due to the resource based nature of the two economies. A similar argument could extend to the UK due to the importance of the oil revenue component in the economy during the 80's. Traditional links and trading patterns with the UK, Canada and Hong Kong could also play a role, however these have diminished in importance in recent years. It is difficult to provide explanations for the results for Germany and Switzerland. Overall no simple intuitive explanation springs to mind and further work in this area will concentrate on examining these results more closely to provide explanations.

In all other cases there are either no significant test statistics (Austria, Belgium, Italy, Spain, Sweden) or significant test statistics at only one or two lags (Japan, USA). Thus these pairwise portfolios are interpreted as showing no evidence of cointegration.

The implications of these results are that there are no gains to the Australian investor from pairwise portfolio diversification in the 5 countries France, Germany, Hong Kong Switzerland, the UK and Canada. However it must be noted that this result must be taken in context. The finding of a cointegrating



vector between the two series demonstrates that over the sample the two series have moved together in an equilibrium relationship. The term equilibrium in the cointegration literature simply means that the two series have maintained a constant relationship to each other which has been maintained throughout the sample. It does not mean that there have not been sub periods during which the two indices have moved apart. Also since cointegration implies causality these markets must be seen as violating weak form efficiency since one of the markets can help forecast the other. This is an issue taken up in the next chapter. In all of the other cases the rejection of cointegration between the pairs of series implies that there are sufficient long run differences between the markets for an Australian investor to gain by portfolio diversification in : Austria, Belgium, Italy, Japan, Norway, Singapore and Malaysia, Spain, Sweden, and the USA.

### **7.5 Bivariate Cointegration Tests - Engle-Granger Method**

As a further test on the robustness of the results tests for cointegration between the pairs which the Johansen estimation showed to be cointegrated were carried out using the more familiar Granger Engle two step procedure.

As can be seen from Table 7.3, there is clear evidence of cointegration between the Australian market and those of the Hong Kong, UK and Canada. In the other cases the evidence is less clear cut with France able to reject the null of non cointegration at the 10% level. For Switzerland and Germany the applied researcher would be unable to reject the null of no cointegration at the 10% level. Thus these examples provide a source of disagreement between the two techniques since both Switzerland and Germany showed reasonably strong and robust evidence of cointegration in the Johansen framework. Thus that the fact that Andrade, Clare and Thomas (1991) and Taylor and Tonks (1989) came to different conclusions regarding the cointegration between the UK and

**TABLE 7.2** Testing for cointegration in bivariate portfolios using the method of Johansen

		Austria	Belgium	Canada	France	Germany	Hong Kong	Italy	Japan
k (length of VAR)									
8	ME	7.66	8.02	21.92**	15.44**	19.42**	16.05**	7.42	14.40*
	T	8.54	8.82	21.92**	16.03*	19.46**	17.7*	8.83	14.67
	LM ( $\chi^2(12)$ )	11.8	10.2	12.7	13.7	10.3	16.4	10.1	8.9
7	ME	6.77	7.64	20.38**	13.46*	17.26**	16.05**	7.80	13.33*
	T	8.07	8.62	20.39**	13.91	12.27**	17.6*	9.20	13.54
	LM ( $\chi^2(12)$ )	9.4	10.0	9.7	11.8	7.7	13.6	9.7	10.0
6	ME	6.20	6.83	16.33**	11.98	16.05**	15.04**	6.66	11.39
	T	7.27	7.74	16.33**	12.20	16.05**	16.44**	7.88	11.75
	LM ( $\chi^2(12)$ )	9.1	9.1	8.9	11.3	8.1	13.9	8.6	7.4
5	ME	6.05	6.85	16.37**	13.21*	15.95**	15.17**	8.21	11.36
	T	7.08	7.73	16.41**	13.69	15.97**	16.41*	9.25	11.86
	LM ( $\chi^2(12)$ )	7.8	10.0	9.1	11.1	7.7	7.25	7.8	7.3
4	ME	7.63	9.70	20.95**	15.58**	18.18**	16.73**	7.55	12.53
	T	8.68	10.31	21.04**	15.96*	18.24**	18.43**	8.75	13.11
	LM ( $\chi^2(12)$ )	10.9	15.4	10.0	12.2	8.2	9.1	8.4	8.1
3	ME	7.18	10.21	21.38**	15.65**	16.90**	16.79**	8.62	12.22
	T	8.42	10.87	21.53**	16.31*	16.92**	18.76**	9.67	12.63
	LM ( $\chi^2(12)$ )	8.8	11.3	7.9	11.2	6.0	8.8	7.6	6.6
2	ME	6.03	9.29	19.83**	14.74*	15.72**	17.57**	8.22	11.91
	T	7.09	9.76	19.93**	15.29	15.76**	19.43**	9.39	12.37
	LM ( $\chi^2(12)$ )	8.7	10.8	9.9	11.4	6.9	8.2	8.1	7.6
1	ME	6.46	12.01	23.86**	19.57**	20.28**	17.38**	8.53	13.33*
	T	7.72	12.53	23.90**	20.10**	20.29**	19.55**	10.22	13.69
	LM ( $\chi^2(12)$ )	9.3	11.6	8.8	12.1	7.3	8.5	8.7	8.2

TABLE 7.2 (continued)

		Norway	Singapore and Malaysia	Spain	Sweden	Switzerland	UK	USA
k (length of VAR)								
8	ME	13.69*	14.4*	5.45	9.17	21.62**	23.96**	10.80
	T	15.04*	15.1	6.13	9.62	21.65**	24.32**	11.30
	LM ( $\chi^2(12)$ )	13.4	15.0	8.6	13.7	11.0	13.6	18.0
7	ME	13.26*	12.3	5.95	8.94	20.21**	20.42**	10.74
	T	14.48	13.0	6.51	9.42	20.22**	20.76**	11.27
	LM ( $\chi^2(12)$ )	10.8	11.3	8.1	11.3	9.2	10.6	14.0
6	ME	12.13	10.5	6.82	8.82	18.58**	23.76**	10.53
	T	13.20	11.3	7.42	9.40	18.61**	24.05**	11.01
	LM ( $\chi^2(12)$ )	10.4	10.9	8.0	10.8	9.8	10.7	12.8
5	ME	11.99	10.7	6.74	8.42	17.58**	23.76**	10.54
	T	13.11	11.3	7.22	8.96	17.66*	24.03**	11.14
	LM ( $\chi^2(12)$ )	9.7	8.0	5.9	10.3	9.5	10.9	11.6
4	ME	14.18*	11.2	6.79	8.80	21.11**	26.11**	13.67*
	T	15.54	11.7	7.27	9.29	21.26**	26.32**	14.34
	LM ( $\chi^2(12)$ )	11.4	10.1	7.6	11.4	9.7	12.4	15.2
3	ME	13.53	10.8	7.44	8.79	19.67**	23.92**	14.38*
	T	15.01	11.5	7.80	9.12	19.77**	24.27**	14.95
	LM ( $\chi^2(12)$ )	9.2	8.4	7.0	9.5	6.8	8.7	12.2
2	ME	12.05	10.6	6.50	8.02	16.98**	21.37**	13.63*
	T	13.32	11.3	6.99	8.24	17.15*	21.72**	14.15
	LM ( $\chi^2(12)$ )	9.0	9.2	7.2	9.8	7.8	9.1	12.8
1	ME	12.08	10.3	7.16	10.66	23.54**	25.00**	17.86**
	T	13.51	11.0	7.81	10.89	23.65**	25.35**	18.57**
	LM ( $\chi^2(12)$ )	9.4	8.7	8.4	10.8	7.5	9.3	12.6

All variables in log levels.

\* Indicates significant at the 10% level, \*\* at the 5% level.

LM( $\chi^2(12)$ ) is a test for serial correlation in the VAR.

ME and T are, respectively, the maximal eigenvalue and trace test proposed by Johansen (1988,1989) and detailed in Chapter 4. The results presented are for the test of the null of 0 cointegrating vectors. Since there are only two variables tests of the null of 1 against an alternative of two would be tests of stationarity of the data (note however that the tests did show non rejection of the null of 1 against the alternative of two in any of the cases thus supporting the assumption that the data are I(1)).

other markets is reflected in this study when the two techniques used by the different papers are applied to the same data set.

Given the weakness of the Johansen results for France (insignificant test statistics at lag 6) and the fact that the ADF results in Table 7.3 suggest that it can only reject the null of no cointegration at the 10% level cointegration between the Australian market and the French market is rejected. Thus Hall's (1991) warning regarding the sensitivity of the Johansen test and the need to establish robust results appears to be valid in this case.

In the cases of Germany and Switzerland, given the relative strength and robustness of the Johansen estimation results, it was concluded that these markets do cointegrate with the Australian market and that the conflict with the result above, in table 7.3, lies with the power of the test procedure in the Granger/Engle methodology.

## **7.6 Multivariate Cointegration Tests**

One potential problem with the results thus far is that they only consider bi-variate portfolios and clearly the Australian investor would be likely to consider a wider portfolio in making investment decisions. The response to this is twofold. In the first instance the bi-variate results do yield useful information in the consideration of wider portfolios as they demonstrate which series are moving together in the long run. Thus the results suggest a portfolio including Australia, the UK, Hong Kong and Canada would make little sense since the Australian market moves in a cointegrating relationship with the UK Hong Kong and Canadian markets.

Secondly and from a more practical point of view given the nature of the data set the arbitrary selection of a portfolio and the carrying out of cointegration

**TABLE 7.3 Testing for cointegration - the Granger-Engle method**

	Austria	Belgium	Canada	France	Germany	Hong Kong	Italy	Japan
DF	-2.30	-2.70	-4.0*	-3.5*	-2.7	-3.93*	-2.14	-2.68
ADF(1)	-2.28	-2.51	-3.6*	-3.3	-2.5	-3.93*	-2.09	-2.47
ADF(2)	-2.35	-2.55	-3.5*	-3.3	-2.5	-3.83*	-1.94	-2.43
ADF(3)	-2.43	-2.52	-3.5*	-3.2	-2.4	-3.75*	-1.96	-2.37
ADF(4)	-2.34	-2.43	-3.4*	-3.1	-2.7	-3.45*	-2.18	-2.31
ADF(8)	-2.17	-2.37	-3.5*	-3.0	-2.9	-3.35	-1.92	-2.38
ADF(12)	-2.27	-2.29	-2.8	-3.1	-3.1	-3.20	-2.19	-2.47

	Norway	Singa- pore and Malaysia	Spain	Sweden	Switzer- land	UK	USA
DF	-3.17	-2.43	-1.23	-2.67	-2.9	-4.2*	-3.41*
ADF(1)	-3.16	-2.43	-1.22	-2.53	-2.7	-4.1*	-3.2
ADF(2)	-3.38*	-2.37	-1.20	-2.58	-2.7	-4.1*	-3.09
ADF(3)	-3.44*	-2.38	-1.22	-2.67	-2.7	-4.0*	-3.11
ADF(4)	-3.14	-2.42	-1.37	-2.78	-2.9	-4.2*	-3.02
ADF(8)	-3.11	-2.37	-1.22	-2.78	-2.8	-4.3*	-3.15
ADF(12)	-2.90	-2.39	-1.32	-2.85	-2.9	-4.7*	-3.22

Unit root tests on residuals from OLS regression of Australian index on constant and each other market index.

5% significance level: -3.36.

\* indicates significant at the 5% level.

tests might lead to mistaken inferences being drawn. To illustrate the point, three simple portfolios, which a portfolio manager might select, were considered and tested for cointegration. The first portfolio (portfolio A) included Australia, Japan and the US, the three major markets from the perspective of an Australian investor. The second portfolio (B) included Australia, Japan, the US and the UK whilst the third (C) portfolio included Australia, the UK, the US and Canada.

The results, in table 4.9, show that there is no evidence of cointegration in the case of portfolio A suggesting gains for the Australian investor from a three country diversification. Portfolios B and C did show evidence of cointegration with the tests suggesting a single cointegrating vector in portfolio B and two cointegrating vectors in portfolio C. The estimates of  $\beta$ , the cointegrating parameters for portfolio B, based on the largest eigenvalue (normalised on Australia) were:

$$\text{AUS} = .949\text{UK} - .276\text{US} + .069\text{JAP}.$$

For Portfolio C the two vectors were:

$$\text{AUS} = 1.074\text{UK} - 0.027\text{CANADA} - 0.326\text{US}$$

and

$$\text{AUS} = -0.675\text{UK} + 1.516\text{CANADA} + 0.546\text{US}$$

**TABLE 7.4 Johansen cointegration test results for three sample portfolios**

null hypothesis	Portfolio A	Portfolio B	Portfolio C	
r=0	16.17(21.07)	30.91(27.14)	28.08(27.14)	
r=1	7.66(14.9)	10.96(21.07)	22.37(21.07)	
r=2	0.45(8.18)	8.55(14.9)	5.86(14.9)	
r=3		0.58(8.18)	0.04(8.18)	

Tests based on maximum eigenvalue of stochastic matrix  $\Pi$ .

Figures in parenthesis are 95% critical values.

Thus both sets of results might be interpreted to imply that there were no gains from portfolio diversification in these portfolios. However for portfolio B a Likelihood Ratio test for the exclusion of JAP and US (ie imposing zero restrictions on their parameters) yielded  $LR(\chi^2(2)) = 1.97$  which is not significant suggesting that the restriction cannot be rejected and that the vector was simply picking up the cointegration between Australia and the UK. A similar test for a zero restriction on the UK yielded  $LR(\chi^2(1)) = 14.65$  which is clearly significant and hence rejects the restriction. This results can be interpreted as follows. The Johansen estimation technique shows no evidence of cointegration between the market set of Australia, Japan and the US. When the UK market is added to the portfolio, there is evidence of cointegration, however the cointegrating vector is really picking up the bivariate cointegration between the Australian and UK markets, giving zero weights to the US and Japanese markets.

In the case of portfolio C tests based on the hypothesis that the two cointegrating vectors were subject to zero restrictions on the parameters yielded the result that zero restrictions on Australia, Canada and the UK could be rejected but a zero restriction on the US ( $LR(\chi^2(2)) = 3.61$ ) could not be rejected. Thus drawing the conclusion that portfolio diversification would not be profitable in this case would be invalid, the US shows no evidence of cointegration with the Australian market and the test is picking up cointegration between Australia and Canada and the UK.

Thus, the point is made that finding that there exists a set of say  $n$  variables which yields a cointegrating relationship does not imply that a subset  $n-k$  of these variable will also cointegrate. In the case of portfolio diversification where the technique is simply used to test for long run relationships in data and no theory is imposed on the relationships, then simple observation of the result that a number of variables cointegrate may suggest that there are no gains from portfolio diversification. However the results above show that the dropping of one or more of the variables may lead to a rejection of cointegration and hence

imply gains from diversification. Thus analysis of more extensive portfolios and the drawing of conclusions regarding portfolio diversification must be carried out with great care.

## 7.7 Conclusions

These results suggest that for most pairwise portfolios there exist potential long run portfolio diversification gains to the Australian investor in the sense that there is no evidence of cointegrating relationships. Three clear exceptions to this appear to be the pairwise portfolios of Australia and Canada Australia and the UK and Australia and Hong Kong which show robust evidence of cointegration over the sample period, hence indicating that diversification in these two cases would not be profitable and that the markets are weakly inefficient. The Johansen estimation results also suggest that the Australian market is cointegrated with the markets of Germany and Switzerland. The results also show that there does exist the potential for conflicting results from the traditional Granger/Engle two step cointegrating methodology and the more recently developed Johansen estimation procedure with the possible source of conflict being the power of the unit root tests in the two step procedure.

In one other case, that of France, cointegration is rejected and Hall's (1991) warning that the Johansen estimation results should be tested for robustness by estimating cointegrating regressions over a range of lag lengths in the VAR, or indeed using the minimum test statistic VAR length, is one that the applied researcher take note of. The results demonstrate the need to test the cointegrating regressions over a range of lags using the Johansen procedure as the results in some cases did seem to be sensitive to the lag chosen.



Finally, in the consideration of portfolios containing a range of assets, care needs to be taken in drawing conclusions regarding the benefits or otherwise of portfolio diversification using cointegration techniques. One possible procedure would be to use the information from bivariate portfolios in building larger portfolios and carry out tests for zero restrictions at each stage.

## CHAPTER 8

### COMMON TRENDS AND A SIMPLE ERROR CORRECTION MODEL OF INTERNATIONAL STOCK MARKET DATA

#### **8.1 Introduction**

The results in the previous chapter suggested that there were some portfolios of the stock price indices considered which cointegrated. This finding has a number of implications, and this chapter considers two of them.

Firstly, following Kasa (1992) the common trends in stock prices are explicitly modelled in order to show how well these trends do in describing the basic data. These common trends are derived as the orthogonal complement of the cointegrating vectors. Whilst no explicit model is assumed here, these common trends can be thought of as the underlying growth components which drive the series. In the context of the stock market these can be thought of as the expected earnings and dividends of the market, itself probably driven by the underlying growth in the economy. Overall these common trends are found, as Kasa (1992) found for the markets he considered, to do a very good job in explaining the overall trends in the markets considered here.

Secondly, using those portfolios which exhibited evidence of cointegration in the previous chapter, the behavior of those stock price indices is modelled using a Vector Error Correction Model (VECM) to test for causal relationships between the markets. As is well known the existence of cointegration implies the existence of some form of causal (in the sense of Granger, 1969) link

between the series. In this context causality must be considered in a correctly specified VECM.

The results of the previous chapter are drawn on in that only a subset of the indices considered there are considered in this chapter. In particular the indices which showed most robust evidence of cointegration, namely those of Australia, Hong Kong, the UK, and Canada.

The previous chapter used monthly data, from December 1969 to April 1992, and required high lag orders in order to ensure as near to random errors as possible and to capture any long term mean reversion in stock prices. In this chapter the equivalent quarterly data series for the indices, taken from the same source, was used, allowing a more parsimonious modelling of the dynamics of stock prices. Thus the data set used here runs from 1971 quarter two to 1992 quarter one.

## **8.2 Identifying the common trends in the indices.**

Since the data set used in this chapter is quarterly, it seems worthwhile to re-test for the presence of a cointegrating vector in the portfolios considered. Table 8.1 reports both diagnostics on the levels VAR, and the results of Johansen estimation for the bivariate portfolios considered.

Lag orders in the VARs were selected to ensure that both the single equations and system under consideration passed the full range of diagnostics in order to be consistent with the assumption of non autocorrelated errors.

As can be seen from the table there is still strong evidence of the existence of cointegrating vectors between Australia and Canada, Australia and the UK and

**TABLE 8.1 Cointegration - bivariate portfolios**

	Australia and UK	Australia and Canada	Australia and Hong Kong
Lag Order	6	5	6
<i>Single Equation Diagnostics</i>			
<b>Australia</b>			
Error	0.9079	1.1056	0.8492
Auto-correlation (AR 1 - 5)	[.48]	[.37]	[.52]
Normality	3.3543 [.19]	3.0619 [.22]	1.9189 [.38]
Heteroscedasticity.	1.6591 [.07]	0.6742 [.83]	0.7137 [.81]
<b>Other Country</b>			
Error	0.8155	1.1212	0.9201
Auto-correlation (AR 1 - 5)	[.54]	[.36]	[.47]
Normality	3.1671 [.21]	1.5109 [.47]	1.2232 [.54]
Heteroscedasticity.	0.9948 [.49]	0.7851 [.72]	0.7735 [.75]
<i>System Diagnostics</i>			
Error	1.0178	0.8840	1.0153
Auto-correlation (AR 1 - 5)	[.45]	[.61]	[.45]
Normality	6.027 [.20]	2.2596 [.69]	4.3661 [.36]
Heteroscedasticity.	0.9504 [.59]	0.6609 [.97]	0.6378 [.98]
<b>Cointegration results</b>			
<i>Maximal Eigenvalue</i>			
Ho rank = 0	22.44*** <i>(14.1, 12.1)</i>	15.32** <i>(14.1, 12.1)</i>	22.4*** <i>(14.1, 12.1)</i>
Ho rank ≤ 1	2.07 <i>(3.8, 2.7)</i>	0.23 <i>(3.8, 2.7)</i>	0.002 <i>(3.8, 2.7)</i>
<i>Trace</i>			
Ho rank = 0	24.51*** <i>(15.4, 13.3)</i>	15.55** <i>(15.4, 13.3)</i>	22.4*** <i>(15.4, 13.3)</i>
Ho rank ≤ 1	2.07 <i>(3.8, 2.7)</i>	0.23 <i>(3.8, 2.7)</i>	0.02 <i>(3.8, 2.7)</i>
Normalised $\beta$ coefficient (Australia = -1)	-0.80	-1.12	-0.84

Figures in *italic* in parenthesis below cointegration test statistics are 95 per cent and 90 per cent critical values for cointegration tests. \*\*\* indicates significant at 1per cent, \*\* at 5 per cent and \* at 10 per cent

Estimation sample 1971Q2 - 1992Q1.

Figures in  $\square$  below test statistics are p values.

Each regression included a constant and two impulse dummy variables, one for 1974 quarter 3 and the second for 1987 quarter 4, both of which were allowed to entered the estimation unrestrictedly.

Estimation carried out using PC-FIML (part of PC-GIVE professional version 8, Doornik and Hendry, 1994), the mis-specification tests reported are those produced by PC-FIML and described in Doornik and Hendry (1994).

between Australia and Hong Kong. Given these results cointegration between the three pairwise portfolios using quarterly data is accepted<sup>1</sup>.

In order to construct the common trends it is important that the values found for the coefficients of the cointegrating vector ( $\beta'$ ) are relatively robust to the VAR specification. This indeed appears to be the case. The above results are presented for the most parsimonious VAR specification which passed a range of diagnostic tests. Lag orders from 6 to 1 were also estimated and the cointegration results were robust to the VAR order and the parameter estimates of the cointegrating vector changed very little (Hall 1986 advises reporting results for a range of VAR lag order specifications). For example, taking the bivariate portfolio of Australia and Canada, the (normalised) coefficient on Canada in the single cointegrating vector ranged between -1.131 and -1.21. Results for the other portfolios were similar.

In the case of the pairwise portfolios, since there are two variables ( $n = 2$ ) and one cointegrating vector ( $r = 1$ ), there is a single common stochastic trend driving the two markets ( $n - r = 1$ ). For portfolios containing more markets the number of common stochastic trends is given by  $n - r$ , the number of variables minus the number of cointegrating vectors. Below the particular method of extracting the common trends in the data, which follows the method of Kasa (1992), is outlined.

As detailed in Chapter 4, Johansens method of estimating the number of cointegrating vectors begins with the VAR(k) model:

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<sup>1</sup> For all of the portfolios considered in this chapter the estimated cointegrating vector along with the recursively estimated eigenvalues and the maximum eigenvalue test on which inference regarding cointegration was based are presented in diagram form in Appendix A3 as figures A3.1 to A3.7.

$$X_t = \mu + A_1 X_{t-1} + \dots + A_k X_{t-k} + \varepsilon_t \quad (8.1)$$

where  $X$  is an  $n \times 1$  vector of known  $I(1)$  variables. This can be re-written as:

$$\Delta X_t = \mu + \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + \varepsilon_t$$

where

$$\Gamma_i = -(I - A_1 - \dots - A_i) \quad i = 1, \dots, k-1 \quad (8.2)$$

$$\Pi = -(I - A_1 - \dots - A_k)$$

As noted in Chapter 4,  $\Pi$ , is the long run impact matrix and the inference regarding cointegration is based around the rank of this matrix. If cointegration does occur then the rank of  $\Pi$  will be  $n - r$  where  $r$  is the number of cointegrating vectors. The long run impact matrix can be factorised as  $\Pi = \alpha\beta'$  where  $\alpha$  and  $\beta$  are  $n \times r$  matrices such that the columns of  $\beta$  contain the cointegrating vectors.

Kasa's (1992) method for decomposing the series into a stochastic trend and stationary component uses the idea that the  $r$  independent cointegrating vectors in  $\beta$  span a closed sub-space of  $\mathbb{R}^n$ , call it  $M$  and that  $X$  can be decomposed into the sum of its orthogonal projections onto  $M$  and  $M_\perp$  (where  $M_\perp$  denotes the orthogonal complement of  $M$ <sup>2</sup>). Thus taking a simple example to illustrate, suppose for the four markets there were three cointegrating vectors, thus  $n = 4$  and  $r = 3$ . This implies that there is a single common trend driving the four markets. The  $\beta'$  matrix will take the form:

<sup>2</sup> If  $M$  is  $m \times n$  with  $m > n$  and  $\text{rank}(M) = n$  then the orthogonal complement of  $M$ , called  $M_\perp$  is the  $m \times (m - n)$  matrix such that  $M'M_\perp = 0$  with  $\text{rank } M_\perp = m - n$ .  $M_\perp$  spans the null space of  $M$ .

$$\beta' = \begin{bmatrix} a_{11} & \cdot & a_{13} \\ a_{21} & \cdot & \cdot \\ a_{31} & \cdot & \cdot \\ a_{41} & \cdot & a_{43} \end{bmatrix}$$

where the columns contain the coefficients of the cointegrating vector and the matrix spans  $\mathbb{R}^3$ , a subspace of  $\mathbb{R}^4$ . Kasa then suggests finding the orthogonal complement of this subspace,  $\beta_{\perp}$ , which will, in this example be  $4 \times 1$ .  $X_t$  can then be decomposed as:

$$X_t = \beta(\beta'\beta)^{-1}\beta'X_t + \beta_{\perp}(\beta'_{\perp}\beta_{\perp})\beta_{\perp}X_t, \quad (8.3)$$

where  $\beta(\beta'\beta)^{-1}\beta'$  and  $\beta_{\perp}(\beta'_{\perp}\beta_{\perp})\beta_{\perp}$  are the projection operators and the second term in the above equation will represent the common trend.

In common with other methods of decomposition of series into common trend and stationary component (Stock and Watson, 1988b, Beveridge and Nelson, 1981) there is an identification problem associated with the decomposition. Whilst the projection operators are invariant to the normalisation, additional restrictions must be imposed to separate the trend component into a vector of factor loadings and the common trend. Kasa's method is to define  $(\beta'_{\perp}\beta_{\perp})^{-1}\beta_{\perp}$  as the common trend and to normalise  $\beta_{\perp}$  so that the elements of  $(\beta'_{\perp}\beta_{\perp})^{-1}\beta_{\perp}$  sum to one. This then gives the common trend as a weighted average of the markets, and  $\beta_{\perp}$  represents the factor loadings, telling us how important the common trend is to each market.<sup>3</sup>

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<sup>3</sup> Kasa notes that this method will produce a common trend which will not necessarily be a pure random walk, since it will contain any short run dynamics which are orthogonal to the cointegrating relationships.

Below the common trends driving the portfolios listed above are estimated. A simple example, the bivariate portfolio containing {Australia, UK}, is used to illustrate the method.

Firstly, Johansen estimation suggested the pairwise portfolio cointegrates. Thus the markets are driven by a single common trend (since  $n = 2$  there is at most one cointegrating vector and one common trend).

Results from the Johansen estimation yielded the following estimates of the  $\beta$  matrix.

$$\beta = \begin{bmatrix} 0.43975 \\ -0.35591 \end{bmatrix} \quad \text{where we have} \quad \begin{pmatrix} \text{Australia} \\ \text{UK} \end{pmatrix}$$

The orthogonal matrix  $\beta_{\perp}$  can now be constructed as:

$$\beta_{\perp} = \begin{bmatrix} 0.8848 \\ 1.0932 \end{bmatrix} \quad \text{such that} \quad (\beta_{\perp}' \beta_{\perp}) \beta_{\perp}' = (0.44733, 0.5527)$$

where the loading vector has been re-normalised so that the elements of  $(\beta_{\perp}' \beta_{\perp}) \beta_{\perp}'$  sum to 1.

Thus the common trend is:

$$CT_1 = (0.44733 \times \text{Australia}) + (0.5527 \times \text{UK})$$

The loading vector can then be used to calculate the trend for each market. Figures 8.1 and 8.2 plot the trend and the indices for these two markets. As in



Kasa (1992) the common trend (plus any orthogonal short term dynamics) does an excellent job in tracking the actual price movements in the two markets.

Repeating the process for the two other bivariate portfolios:

For Australia and Canada

$$CT_2 = (-0.5283 \times \text{Australia}) - (0.4717 \times \text{Canada})$$

with loading factors -1.0532 and -0.94038

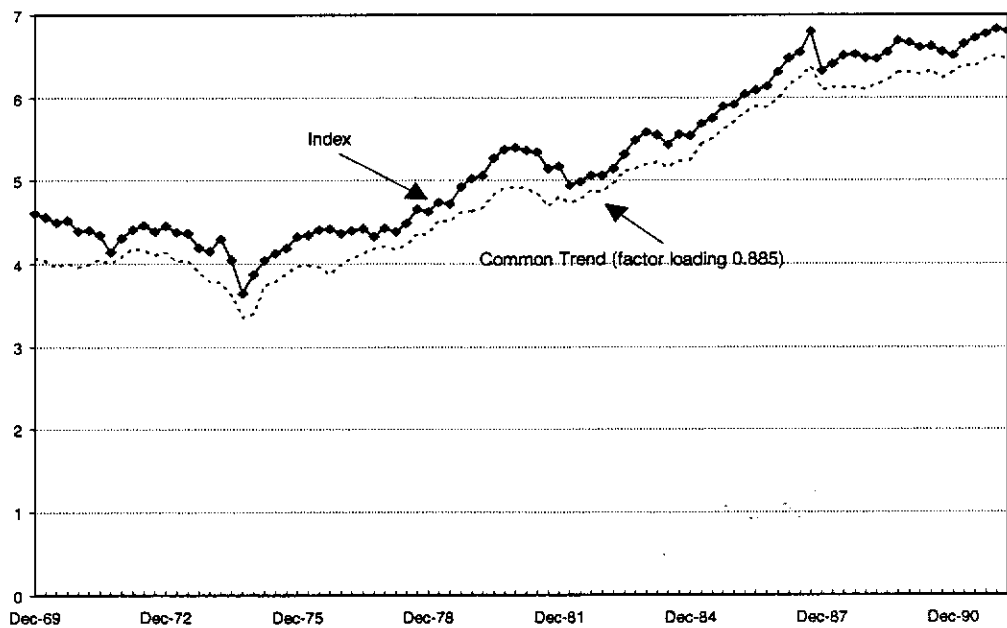
For Australia and Hong Kong

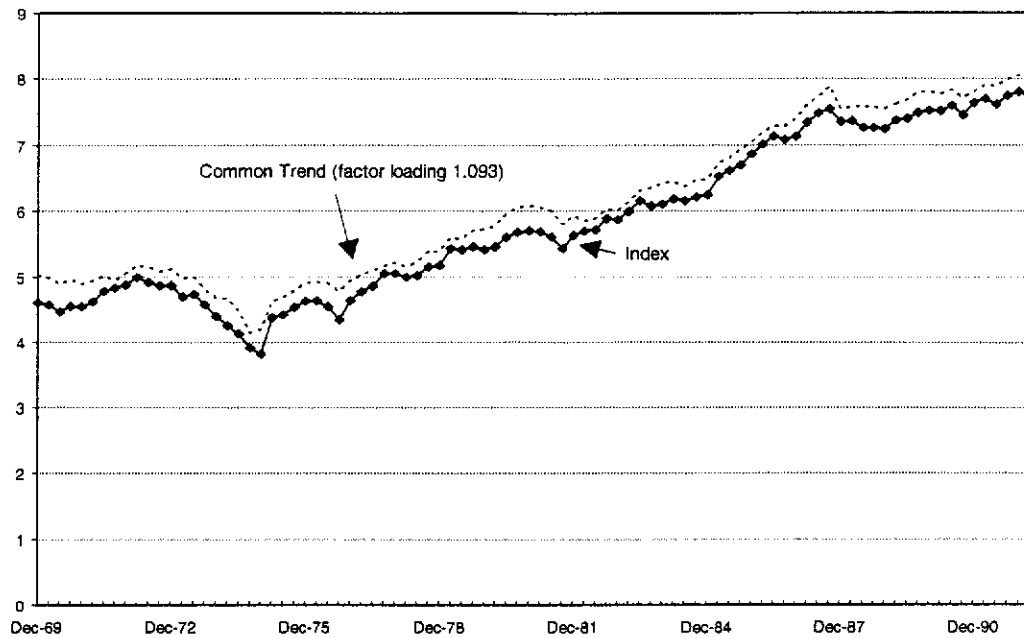
$$CT_3 = (0.4565 \times \text{Australia}) + (0.5435 \times \text{Hong Kong})$$

with loading factors 0.90619 and 1.07879.

These series are plotted as Figures A3.8 and A3.9 in Appendix A2.

**FIGURE 8.1** Australian stock price index and calculated common trend



**FIGURE 8.2 UK stock price index and calculated common trend**

Moving on now to the portfolios involving more than just two indices. Table 8.2 looks at the question of cointegration between portfolios involving three of the four selected countries and a portfolio involving all four countries<sup>4</sup>. As can be seen the two portfolios (Australia, Hong Kong, UK) and (Australia, Hong Kong and Canada) show evidence of two cointegrating vectors and thus one common trend. The portfolio (Australia, Hong Kong, UK) and the portfolio involving all four variables show evidence of only one and two cointegrating vectors respectively and thus have two common trends driving the system.

As a result the above methodology was used to identify the common trend for the first two portfolios only. The resulting Common Trends are given by:

$$CT_4 = (0.3244 \times \text{Australia}) + (0.3863 \times \text{Hong Kong}) + (0.2893 \times \text{Canada})$$

with loading factors 0.9594, 1.1425 and 0.85523

<sup>4</sup> Diagnostics on the system of equations only are presented to save space, however all individual equations in the system passed the usual diagnostics.

TABLE 8.2 **Cointegration - multivariate portfolios**

	Australia Kong and UK	Hong Kong and Canada	Australia Hong Kong and UK	Canada	Australia, Canada, Hong Kong and UK
<b>Lag Order</b>	5	5	5	5	5
<b>System</b>					
<b>Diagnostics</b>					
Error	0.7788	0.9689	0.8919		1.2423
Auto-correlation	[.83]	[.53]	[.66]		[.12]
Normality	9.88 [.13]	4.0468 [.67]	4.0501 [0.67]		6.8508 [.55]
Heteroscedasticity	.5645 [.99]	0.54413 [.99]	0.6918 [.99]		0.568 [.99]
<b>Cointegration results</b>					
<b>Maximal Eigenvalue</b>					
Ho rank = 0	30.3*** <i>(21.0, 18.6)</i>	28.1*** <i>(21.0, 18.6)</i>	24.0** <i>(21.0, 18.6)</i>		34.7*** <i>(27.1, 24.7)</i>
Ho rank ≤ 1	13.7* <i>(14.1, 12.1)</i>	13.7* <i>(14.1, 12.1)</i>	8.9 <i>(14.1, 12.1)</i>		18.5 <i>(21.0, 18.6)</i>
Ho rank ≤ 2	1.5 <i>(3.8, 2.7)</i>	0.2 <i>(3.8, 2.7)</i>	0.27 <i>(3.8, 2.7)</i>		9.9 <i>(14.1, 12.1)</i>
Ho rank ≤ 3					0.1 <i>(3.8, 2.7)</i>
<b>Trace</b>					
Ho rank = 0	45.5*** <i>(29.7, 26.8)</i>	42.0*** <i>(29.7, 26.8)</i>	33.22** <i>(29.7, 26.8)</i>		63.1*** <i>(47.2, 44.0)</i>
Ho rank ≤ 1	15.2* <i>(15.4, 13.3)</i>	13.9* <i>(15.4, 13.3)</i>	9.21 <i>(15.4, 13.3)</i>		28.4* <i>(29.7, 26.8)</i>
Ho rank ≤ 2	1.5 <i>(3.8, 2.7)</i>	0.2 <i>(3.8, 2.7)</i>	0.27 <i>(3.8, 2.7)</i>		9.9 <i>(15.4, 13.3)</i>
Ho rank ≤ 3					0.1 <i>(3.8, 2.7)</i>
<b>Normalised β coefficients</b>					
	(1, -0.5678, -0.2494) (aus, uk, hk)	(1, -0.455, -0.514) (aus, hk, canada)	(1, -0.2795, -0.6011) (aus, canada, uk)		(1, -0.26, -0.36, -0.3) (aus, hk, canada, uk)
	(-0.5878, 1, -0.5386) (aus, uk, hk)	(1.914, 1, -3.483) (aus, hk, canada)			(2.30, 1, -0.27, -2.60) (aus, hk, canada, uk)

Figures in *italic* in parenthesis below cointegration test statistics are 95 per cent and 90 per cent critical values for cointegration tests. \*\*\* indicates significant at 1per cent, \*\* at 5 per cent and \* at 10 per cent

Estimation sample 1971Q2 - 1992Q1.

Figures in [ ] below test statistics are p values.

Each regression included a constant and two impulse dummy variables, one for 1974 quarter 3 and the second for 1987 quarter 4, both of which were allowed to entered the estimation unrestrictedly.

Estimation carried out using PC-FIML (part of PC-GIVE professional version 8, Doornik and Hendry, 1994), the mis-specification tests reported are those produced by PC-FIML and described in Doornik and Hendry (1994).

and

$$CT_3 = (0.2912 \times \text{Australia}) + (0.3494 \times \text{Hong Kong}) + (0.3594 \times \text{UK})$$

with loading factors 0.8665, 1.0398 and 1.0694

These series are plotted as Figures A3.10 and A3.11 in Appendix A3. Table 8.3 summarises these results, showing the  $R^2$  of simple ols regressions of the index against the calculated common trends. As can be seen these are all in the range of 0.96 to 0.99 indicating that the common trends explain a large proportion of the variation in the indices.

**Table 8.3**      **$R^2$  for regressions of index against common trend**

	<i>Australia</i>	<i>Hong Kong</i>	<i>Canada</i>	<i>UK</i>
<u>Portfolio</u>				
(Australia, Hong Kong)	$R^2=0.96$	$R^2=0.98$		
(Australia, UK)	$R^2=0.99$			$R^2=0.99$
(Australia, Canada)	$R^2=0.99$		$R^2=0.99$	
(Australia, Hong Kong, UK)	$R^2=0.97$	$R^2=0.97$		$R^2=0.99$
(Australia, Hong Kong, Canada)	$R^2=0.97$	$R^2=0.98$	$R^2=0.98$	

Overall the results show that the estimated common trends from these cointegrating vectors do an extremely good job in explaining the general movement in the indices.

### **8.3**     **Testing for Causality and Estimating the Error Correction Models**

In this final section two questions which arise directly out of the results on cointegration are considered. Firstly since sub-sets of the data cointegrate then, it must be the case that there exists Granger causality in at least one direction amongst the data. Secondly, and related, the Granger Representation Theorem

tells us that there must exist a valid Error Correction Model representation for the data<sup>5</sup>. Both questions are of direct interest in the context of market efficiency and portfolio diversification. Cointegration implies that the data have a valid error correction representation, implying that information on one market can be used to help us forecast the other market, either directly through the short run dynamics or through the error correcting term. However it could be the case that market A helps predict market B but that market B contains no information on the movements of market A, in other words it could be that causality is uni-directional. Such information will come from the causality tests carried out below.

The causality tests carried out below are simple extensions of the normal Granger causality tests, extended to allow for the presence of a cointegrating vector and can be illustrated by a simple example.

Consider the bivariate portfolio {Australia, UK}, and denote Australia as variable  $x_t$  and the UK as variable  $y_t$ . Evidence from the previous sections suggests that there exists a single cointegrating vector between  $x_t$  and  $y_t$ , thus also implying that the variables share a single common trend. If  $x_t$  and  $y_t$  had not been found to cointegrate (but were both I(1)) then a traditional VAR in first differences would have been appropriate. This would have involved the estimation of:

$$\Delta x_t = \sum_{i=1}^{i=n} \Delta x_{t-i} + \sum_{j=1}^{j=k} \Delta y_{t-j} + \varepsilon_t \quad (8.4)$$

$$\Delta y_t = \sum_{i=1}^{i=n} \Delta x_{t-i} + \sum_{j=1}^{j=k} \Delta y_{t-j} + \varepsilon_t \quad (8.5)$$

and causality testing would have proceeded in the normal way, based upon statistical tests of the significance of the lagged  $\Delta y$ 's in the  $\Delta x$  equation and the

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<sup>5</sup> See Chapter 4 for the relevant background material on this subject.

lagged  $\Delta x$ 's in the  $\Delta y$  equation. The finding that  $x$  and  $y$  are cointegrated, however, implies that the above VAR is mis-specified and that the cointegrating vectors must be included in the VAR, yielding a Vector Error Correction Model (VECM). In this case there is only one cointegrating vector, denoting the error correction term derived from this vector as  $z$  then the VECM can be written as:

$$\Delta x_t = \sum_{i=1}^{i=n} \Delta x_{t-i} + \sum_{j=1}^{j=k} \Delta y_{t-j} - \alpha_x z_{t-1} + \varepsilon_t \quad (8.6)$$

$$\Delta y_t = \sum_{i=1}^{i=n} \Delta x_{t-i} + \sum_{j=1}^{j=k} \Delta y_{t-j} + \alpha_y z_{t-1} + \varepsilon_t \quad (8.7)$$

where the coefficients,  $\alpha_x$  and  $\alpha_y$  can be interpreted as speed of adjustment factors, measuring how quickly each market responds to deviations from the long run equilibrating relationship. If the cointegrating vector between  $x$  and  $y$  were denoted  $(1, -\beta)$  then the error correcting term  $z_{t-1}$  is equivalent to  $(x_{t-1} - \beta y_{t-1})$ . So, in equation (8.6) if the error correcting term is positive then this implies a positive deviation from the equilibrium relationship, hence  $\Delta x$  should be negative - allowing  $x$  to fall back to equilibrium. In equation (8.7) the same disequilibrium should cause  $y$  to rise ie  $\Delta y$  will be positive. This is reflected in the different signs given to the error correcting terms. Of course it is possible for one of the error correcting terms to have a zero coefficient. This would imply that whilst the two markets move in an equilibrium relationship one of the markets does all of the re-adjustment. Thus if above  $\alpha_x = 0$ , then if a positive disequilibrium arises then it is market  $y$  which does all of the adjusting back to equilibrium. Of course, given the Granger Representation Theorem, which tells us that cointegration and error correction are equivalent representations, it is not possible for both  $\alpha_y = \alpha_x = 0$  since this would imply a normal VAR in differences and the two variables would not be cointegrated. Thus in one sense, the tests carried out below provide a check on the

cointegrating results. It is expected that the error correction term will have a coefficient significantly different from zero in at least one of the equations of the VAR model.

The VECM representation thus has obvious implications for causality testing. As well as testing for zero restrictions on lagged first difference terms in the model, the coefficient on the lagged error correction term(s) must also be tested, to see if it can reject a zero restriction. If the lagged first differences of say,  $x$  in the  $\Delta y$  equation, cannot reject a test that they are jointly zero this implies that there is no short run causality from  $x$  to  $y$ . In other words the lagged changes in  $x$  have no predictive power on the change in  $y$  above its own lags. If a test for a zero restriction on the lagged error correction term in the  $\Delta y$  equation cannot reject then this implies that the error correction does not play a role in the equation and thus there is no long run causal link, in this case and as noted above  $y$  does not correct for any deviation from the equilibrium relationship.

Below the VECM's for the portfolios which exhibited evidence of cointegration are estimated. Causality tests are then carried out using the full sample of data. Firstly then, consider the bivariate portfolio {Australia, UK}. Equations 8.8 and 8.9 below report the estimated VECM equations. 3 lags were included in the VECM and, as can be seen, this produces a satisfactory set of diagnostics, two dummy variables, the first in 1975 quarter 1 and the second in 1987 quarter 4 were included to correct for non normalities due to significant outliers.

## RESULTS FOR BIVARIATE PORTFOLIO - {AUSTRALIA, UK}

$$\begin{aligned} \Delta UK = & -0.014 + 0.154\Delta UK_{t-1} + 0.137\Delta UK_{t-2} + 0.209\Delta UK_{t-3} - 0.033\Delta AUS_{t-1} \\ & (-0.2) \quad (1.2) \quad (1.1) \quad (1.7) \quad (-0.2) \\ & + 0.094\Delta AUS_{t-2} - 0.022\Delta AUS_{t-3} + 0.646D75Q1 - 0.278D87Q4 + 0.045Z_{t-1} \quad (8.8) \\ & (0.7) \quad (-0.2) \quad (4.9) \quad (-2.3) \quad (0.4) \end{aligned}$$

$$\begin{aligned} R^2 &= 0.33, DW = 1.92, \sigma = 0.11 \\ AR\ 1-5\ F(5,66) &= 0.76[.58] \\ ARCH4\ F(4,63) &= 1.13[.35] \\ NORM\ \chi^2(2) &= 1.97[.37] \\ HETRO\ F(16,54) &= 1.68[0.08] \end{aligned}$$

$$\text{Wald test (Ho: } \Delta AUS_{t-1} = \Delta AUS_{t-2} = \Delta AUS_{t-3} = 0) F(3,71) = 0.29 [.83]$$

$$\text{Wald test (Ho: } Z_{t-1} = 0) F(1,71) = 0.14 [.71]$$

$$\begin{aligned} \Delta AUS = & 0.203 - 0.072\Delta UK_{t-1} + 0.019\Delta UK_{t-2} + 0.019\Delta UK_{t-3} + 0.063\Delta AUS_{t-1} \\ & (3.4) \quad (-0.6) \quad (0.2) \quad (0.2) \quad (0.5) \\ & + 0.027\Delta AUS_{t-2} + 0.071\Delta AUS_{t-3} - 0.375D74Q3 - 0.49D87Q4 - 0.30Z_{t-1} \quad (8.9) \\ & (0.3) \quad (0.8) \quad (-3.6) \quad (-4.8) \quad (-2.9) \end{aligned}$$

$$\begin{aligned} R^2 &= 0.47, DW = 1.78, \sigma = 0.097 \\ AR\ 1-5\ F(5,66) &= 1.64[.16] \\ ARCH4\ F(4,63) &= 0.59[.67] \\ NORM\ \chi^2(2) &= 0.94[.62] \\ HETRO\ F(16,54) &= 1.16[0.32] \end{aligned}$$

$$\text{Wald test (Ho: } \Delta UK_{t-1} = \Delta UK_{t-2} = \Delta UK_{t-3} = 0) F(3,71) = 0.21 [.89]$$

$$\text{Wald test (Ho: } Z_{t-1} = 0) F(1,71) = 8.32 [.005]$$

In the reported equations, figures in ( ) parenthesis below estimated coefficients are t statistics,  $Z_{t-1}$  is the error correcting term, derived from the cointegrating vector. Estimation carried out using PC-FIML professional version 8, Doornik and Hendry, (1994), and the mis-specification tests, for residual autocorrelation (AR), hetroskedasticity (HETRO), normality (NORM) and autoregressive conditional hetroskedasticity (ARCH) reported are those produced by the package and described in Doornik and Hendry (1994), figures in [ ] parenthesis are probability values for the reported test statistics.



## RESULTS FOR BIVARIATE PORTFOLIO - {AUSTRALIA, HONG KONG}

$$\begin{aligned}
 \Delta HK = & 0.307 + 0.001\Delta HK_{t-1} - 0.009\Delta HK_{t-2} + 0.097\Delta HK_{t-3} + 0.04\Delta HK_{t-4} \\
 & (4.0) \quad (0.0) \quad \quad \quad (-0.1) \quad \quad \quad (0.9) \quad \quad \quad (0.4) \\
 & + 0.17\Delta AUS_{t-1} + 0.061\Delta AUS_{t-2} + 0.092\Delta AUS_{t-3} + 0.115\Delta AUS_{t-4} \\
 & \quad \quad (0.8) \quad \quad \quad (0.3) \quad \quad \quad (0.5) \quad \quad \quad (0.6) \\
 & - 0.676D87Q4 + 0.447Z_{t-1} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (8.11) \\
 & \quad \quad (-3.3) \quad \quad \quad (3.7)
 \end{aligned}$$

$$R^2 = 0.31, DW = 1.97, \sigma = 0.20.$$

$$AR\ 1-5\ F(5,67) = 0.66[.65]$$

$$ARCH4\ F(4,64) = 1.82[.13],$$

$$NORM\ \chi^2(2) = 0.97[.62]$$

$$HETRO\ F(19,52) = 1.29[0.22]$$

$$Wald\ test\ (H_0: \Delta AUS_{t-1} = \Delta AUS_{t-2} = \Delta AUS_{t-3} = \Delta AUS_{t-4} = 0)\ F(4,72) = 0.31\ [.87]$$

$$Wald\ test\ (H_0: Z_{t-1} = 0)\ F(1,72) = 13.7\ [.0004]$$

$$\begin{aligned}
 \Delta AUS = & 0.025 - 0.037\Delta HK_{t-1} + 0.055\Delta HK_{t-2} - 0.089\Delta HK_{t-3} + 0.028\Delta HK_{t-4} \\
 & (0.6) \quad (-0.6) \quad \quad \quad (0.9) \quad \quad \quad (-1.4) \quad \quad \quad (0.4) \\
 & + 0.061\Delta AUS_{t-1} - 0.023\Delta AUS_{t-2} + 0.15\Delta AUS_{t-3} + 0.012\Delta AUS_{t-4} \\
 & \quad \quad (0.5) \quad \quad \quad (-0.2) \quad \quad \quad (1.4) \quad \quad \quad (0.1) \\
 & - 0.409D74Q3 - 0.556D87Q4 - 0.02Z_{t-1} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (8.12) \\
 & \quad \quad (-3.4) \quad \quad \quad (-4.9) \quad \quad \quad (-0.3)
 \end{aligned}$$

$$R^2 = 0.38, DW = 1.75, \sigma = 0.11.$$

$$AR\ 1-5\ F(5,66) = 1.38[.24]$$

$$ARCH4\ F(4,63) = 2.46[.06],$$

$$NORM\ \chi^2(2) = 3.94[.14]$$

$$HETRO\ F(20,50) = 1.27[0.24].$$

$$Wald\ test\ (H_0: \Delta HK_{t-1} = \Delta HK_{t-2} = \Delta HK_{t-3} = \Delta HK_{t-4} = 0)\ F(4,71) = 0.93\ [.45]$$

$$Wald\ test\ (H_0: Z_{t-1} = 0)\ F(1,71) = 0.1\ [.75]$$

## RESULTS FOR BIVARIATE PORTFOLIO - {AUSTRALIA, CANADA}

$$\begin{aligned}
 \Delta\text{CAN} = & -0.015 + 0.064\Delta\text{CAN}_{t-1} + 0.012\Delta\text{CAN}_{t-2} - 0.016\Delta\text{CAN}_{t-3} - 0.373\Delta\text{CAN}_{t-4} \\
 & \quad (-0.1) \quad (0.4) \quad \quad (0.1) \quad \quad (-0.1) \quad \quad (-2.8) \\
 & + 0.032\Delta\text{AUS}_{t-1} + 0.0056\Delta\text{AUS}_{t-2} + 0.103\Delta\text{AUS}_{t-3} + 0.068\Delta\text{AUS}_{t-4} \\
 & \quad (0.3) \quad \quad (0.5) \quad \quad (1.0) \quad \quad (0.7) \\
 & - 0.289\text{D87Q4} - 0.038\text{Z}_{t-1} \\
 & \quad (-2.9) \quad \quad (-0.4)
 \end{aligned} \tag{8.14}$$

$R^2 = 0.20$ ,  $\text{DW} = 1.90$ ,  $\sigma = 0.09$ .

AR 1-5  $F(5,67) = 0.91$  [.48]

ARCH4  $F(4,64) = 0.41$  [.80]

NORM  $\chi^2(2) = 1.64$  [.44]

HETRO  $F(16,52) = 0.73$  [0.76]

Wald test ( $H_0: \Delta\text{AUS}_{t-1} = \Delta\text{AUS}_{t-2} = \Delta\text{AUS}_{t-3} = \Delta\text{AUS}_{t-4} = 0$ )  $F(4,72) = 0.35$  [.84]

Wald test ( $H_0: \text{Z}_{t-1} = 0$ )  $F(1,72) = 0.18$  [.67]

$$\begin{aligned}
 \Delta\text{AUS} = & -0.35 + 0.013\Delta\text{CAN}_{t-1} + 0.216\Delta\text{CAN}_{t-2} - 0.108\Delta\text{CAN}_{t-3} - 0.161\Delta\text{CAN}_{t-4} \\
 & \quad (-2.9) \quad (0.1) \quad \quad (1.4) \quad \quad (-0.7) \quad \quad (-1.1) \\
 & - 0.052\Delta\text{AUS}_{t-1} - 0.044\Delta\text{AUS}_{t-2} + 0.12\Delta\text{AUS}_{t-3} + 0.048\Delta\text{AUS}_{t-4} \\
 & \quad (-0.4) \quad \quad (-0.4) \quad \quad (1.2) \quad \quad (0.5)
 \end{aligned} \tag{8.15}$$

$$\begin{aligned}
 & - 0.429\text{D74Q3} - 0.437\text{D87Q4} - 0.30\text{Z}_{t-1} \\
 & \quad (-4.1) \quad \quad (-4.2) \quad \quad (-3.2)
 \end{aligned}$$

$R^2 = 0.51$ ,  $\text{DW} = 1.79$ ,  $\sigma = 0.10$ .

AR 1-5  $F(5,66) = 1.13$  [.35]

ARCH4  $F(4,63) = 1.46$  [.22]

NORM  $\chi^2(2) = 3.06$  [.21]

HETRO  $F(20,50) = 0.57$  [0.92].

Wald test ( $H_0: \Delta\text{CAN}_{t-1} = \Delta\text{CAN}_{t-2} = \Delta\text{CAN}_{t-3} = \Delta\text{CAN}_{t-4} = 0$ )  $F(4,71) = 1.18$  [.33]

Wald test ( $H_0: \text{Z}_{t-1} = 0$ )  $F(1,71) = 10.4$  [.002]

The results show no evidence of causality, either long run (through the error correcting term) or short run from Australia to the UK, the Wald test (reported below the equations and used to test zero restrictions on parameters of interest) cannot reject either the zero restrictions on the lagged difference of AUS in the  $\Delta UK$  equation nor the zero restrictions on the lagged error correcting term ( $Z_{t-1}$ ). On the other hand whilst the zero restrictions on the lagged differences of the UK, in the  $\Delta AUS$  equation cannot be rejected the zero restriction on the error correcting term in this equation is, strongly, rejected. Thus the conclusion from these tests is that, whilst the short run dynamics of each market are of no value in forecasting the dynamics of the other market, there is evidence of long run causality in one direction, from the UK to Australia. This implies that over the long run the Australian market moves to correct for any deviation from its equilibrium relationship with the UK market, the UK market on the other hand is completely independent of the Australian market.

This pattern of causality is repeated for the other two bivariate portfolios estimated {Australia, Hong Kong } and {Australia, Canada}. The results of the causality tests are reported below. Summarising these, it is found that for the portfolio {Australia, Hong Kong} there is long run causality from Australia to Hong Kong but no evidence of causality, long or short run, from Hong Kong to Australia, whilst for the portfolio {Australia, Canada} there is evidence of long run causality from Canada to Australia and no evidence of causality in the opposite direction.

Turning now to the larger portfolios, Table 8.4 reports the results of the causality tests<sup>6</sup>. The table reports the results of Wald tests of zero restrictions on the parameters of interest. Consider the first row of the table. This reports the results of a test of zero restrictions on the dynamics of the Hong Kong market (in the column headed  $\Delta HK$ ) in the dynamic equation for Australia,

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<sup>6</sup> The estimated equations are not presented in order to save space.

showing that the zero restrictions can only be rejected at a significance level of 41 per cent. Similarly the entry under the column  $\Delta UK$  shows that the dynamics of the UK market are not significant in the  $\Delta AUS$  equation, whilst the entry under the column ECM, shows that a test for a zero restriction on the error correcting term is significant at the 1 per cent or better level. As can be seen the results reflect the same essential pattern as the bivariate portfolios above. In each portfolio the only strong evidence of causality is long run causality, the lagged first differences of the other markets are typically insignificant in each equation. Also in the same way the markets of Australia and Hong Kong show strongest evidence of inefficiency in that it is in these markets that the error correcting term is significant at the 5per cent or higher level.

This result is mirrored in the second trivariate portfolio {Australia, Hong Kong and Canada} with the error correcting term once again significant in both the Hong Kong and the Australia equation but not the Canadian equation. In the final trivariate portfolio {Australia, UK and Canada) there is strong evidence of long run causality from the error correcting term to the Australian market and, at the 5per cent level, to the Canadian market. Finally in the four variable portfolio the error correcting term is significant at the 1per cent level in both the Australia and Hong Kong equation and, albeit at a significance level of 7per cent, in the Canadian equation.

**TABLE 8.4 Causality tests for three and four country portfolios**

	$\Delta$ AUS	$\Delta$ CAN	$\Delta$ HK	$\Delta$ UK	ECM	AR(1-5)	ARCH4	NOR	HET
<b>Portfolio {<math>\Delta</math>AUS, <math>\Delta</math>UK, <math>\Delta</math>HK}</b>									
$\Delta$ AUS EQN		-	1.0	1.4	6.2**	[0.2]	[0.9]	[0.6]	[0.7]
			[0.41]	[0.25]	[0.00]				
$\Delta$ UK EQN	0.4	-	0.3		0.05	[0.6]	[0.4]	[0.4]	[0.4]
	[0.82]		[0.88]		[0.95]				
$\Delta$ HK EQN	0.1	-		0.2	6.2**	[0.4]	[0.4]	[0.6]	[0.7]
	[0.99]			[0.92]	[0.00]				
<b>Portfolio {<math>\Delta</math>AUS, <math>\Delta</math>HK, <math>\Delta</math>CAN}</b>									
$\Delta$ AUS EQN		0.8	0.6	-	4.6*	[0.1]	[0.2]	[0.3]	[0.8]
		[0.49]	[0.62]		[0.01]				
$\Delta$ HK EQN	0.9	2.2		-	9.3**	[0.9]	[0.4]	[0.8]	[0.1]
	[0.42]	[0.08]			[0.00]				
$\Delta$ CAN EQN	0.1		1.4	-	0.7	[0.1]	[0.6]	[0.4]	[0.4]
	[0.97]		[0.24]		[0.50]				
<b>Portfolio {<math>\Delta</math>AUS, <math>\Delta</math>UK, <math>\Delta</math>CAN}</b>									
$\Delta$ AUS EQN		0.9	-	0.8	14.7**	[0.3]	[0.8]	[0.3]	[0.9]
		[0.46]		[0.50]	[0.00]				
$\Delta$ UK EQN	0.4	1.1	-		0.1	[0.9]	[0.3]	[0.7]	[0.6]
	[0.83]	[0.37]			[0.70]				
$\Delta$ CAN EQN	0.4		-	0.6	4.2*	[0.3]	[0.9]	[0.7]	[0.9]
	[0.84]			[0.63]	[0.05]				
<b>Portfolio {<math>\Delta</math>AUS, <math>\Delta</math>CAN, <math>\Delta</math>UK, <math>\Delta</math>HK}</b>									
$\Delta$ AUS EQN		1.1	0.7	0.6	5.0**	[0.2]	[0.9]	[0.8]	[0.6]
		[0.35]	[0.57]	[0.63]	[0.01]				
$\Delta$ UK EQN	0.4	1.1	0.4		0.5	[0.9]	[0.3]	[0.9]	[0.7]
	[0.81]	[0.36]	[0.79]		[0.64]				
$\Delta$ CAN EQN	0.4		1.2	0.6	2.8	[0.6]	[0.9]	[0.8]	[0.8]
	[0.83]		[0.32]	[0.67]	[0.07]				
$\Delta$ HK EQN	1.1	2.5*		0.7	7.1**	[0.4]	[0.9]	[0.9]	[0.7]
	[0.38]	[0.05]		[0.57]	[0.00]				

\*\*Indicates significance at the 1 per cent level, \* at the 5per cent level.

AR(1-5), ARCH4, NOR and HET are tests for residual autocorrelation, Autoregressive Conditional Heteroskedasticity, Normality of residuals and Heteroskedasticity, note that only p values for the test statistics are reported. Estimation carried out using PC-FIML (part of PC-GIVE professional version 8, Doornik and Hendry, 1994), the mis-specification tests reported are those produced by PC-FIML and described in Doornik and Hendry (1994).

The only evidence of any short run causality between any of the markets is from Canada to Hong Kong where, in the portfolios {Australia, Hong Kong, Canada} and {Australia, Hong Kong, Canada, UK} the lagged differences of Canada are significantly different from zero at the 8 per cent and 5 per cent levels respectively.

Overall the following conclusions are drawn from the above results. Firstly, whilst the UK market plays a role in a number of the cointegrating vectors there is no evidence that any of the other markets Granger cause the UK market in terms of either long run or short run causality. This result is not difficult to interpret, whilst cointegration implies that the long run levels of these stock price indices are tied together in an equilibrium relationship it does not imply that they are perfectly correlated in the short run. In the short run these indices may move in opposite directions, so long as these movements are bounded. It thus does not mean that shorter run gains from diversification may not be possible. Indeed this seems to be the best interpretation of the results, whilst the short run dynamics of these indices appear largely unrelated there is evidence that in the long run a number of them trend together. The causality results tell us that the Australian and Hong Kong markets tend to correct any divergence from the more established markets of the UK and Canada in the longer run. In the case of the UK, the most established of the markets considered, it shows no evidence of being Granger caused, either in the short run or the long run, by any of the other markets considered. This thus implies that it is the other markets in the portfolio with the UK that carry the burden of adjustment to equilibrium, the UK market is exogenous.

## 8.4 Conclusion

This chapter has considered two simple extensions of the material in Chapter 7. Firstly, since there is evidence that some sets of the indices under consideration cointegrate, this implies that the markets are driven by common stochastic trends. The first half of this chapter utilised Kasa's (1992) method of estimating these common trends for a sub set of the portfolios where there was a single common trend. As Kasa (1992) found for data on the US and related markets, these common trends do a good job in explaining the general movements in the markets. The second extension of the work in Chapter 7 considered, in the context of a vector error correction model the causal links between the portfolios which were cointegrated. The general finding from these VECM's is that there is little evidence of short run causality between the markets. There is however evidence of long run causality in the sense that the error correcting terms are typically significant in some of the equations (as was to be expected since the portfolios showed evidence of cointegration). In the bivariate portfolios of {Australia, UK}, {Australia, Hong Kong} and {Australia, Canada} the error correcting term was only significant in the dynamic equation for Australia, Hong Kong and Australia respectively. Since these are bivariate portfolios this implies that only one of the markets makes the adjustments to the long run equilibrium relationship between the markets. So in the {Australia, UK} case it is the Australian market which does all of the adjusting to disequilibrium. This result carried over to broader portfolios where typically one market, the UK showed no evidence of being affected by any of the other markets and not making adjustments to the long run equilibrium between the markets.

## CHAPTER 9

### TESTING THE PRESENT VALUE MODEL USING AUSTRALIAN STOCK MARKET DATA

#### **9.1 Introduction**

This chapter looks at the efficiency and rationality of the Australian stock market using the familiar present value model as described in Campbell and Shiller (1987). This is a much tested model in the literature with a number of recent papers such as Mills (1993, 1995) and MacDonald (1994,1995) using the framework described below in the context of UK data. The application of recursive estimation techniques used here highlights some of the problems of using such a framework in the context of the Australian market.

The modelling framework used by Campbell and Shiller (1987) has the advantage, in terms of applied work, of leading, under the right conditions regarding the data properties, to a simple to estimate VAR in which the rational expectations restrictions of the present value model can be directly tested. In common with most other papers in the area the present value model is rejected by Australian data at a high level of significance. The fragility of some of the key assumptions regarding the time series properties of the data are identified as being a potential source of this failure.

This chapter draws motivation from an earlier paper by the author, Allen, Black and MacDonald (1996). This paper tested the present value model for Australia using annual data over a smaller sample. The material in this chapter



extends the econometric analysis, uses a longer, quarterly, data sample and goes into greater detail regarding the specification of the model.

## 9.2 Testing the efficiency of the Australian stock market

This section uses the familiar present value model of stock prices to test the efficiency of the Australian stock market. The popular and much tested model of Campbell and Shiller (1987) is used to motivate the testing framework. The requirements made of the variables, in terms of their time series properties, for this testing framework to be valid are noted below. In particular the evidence for cointegration between prices and dividends, an essential pre-requisite for the development of the model is found to be a somewhat fragile assumption in the context of the Australian series. In this context it should be noted that many other researchers, Mills (1993), MacDonald (1994) and indeed Campbell and Shiller (1987) themselves found only weak evidence that prices and dividends were indeed cointegrated. The present value model is tested for both a full sample (1959 - 1994 quarterly) and for a sub sample which was selected to give the model the best chance of success. However the formal tests of the present value restrictions imposed on the estimated VAR are strongly rejected, even when, as in the sub sample, the evidence of cointegration is strong.

## 9.3 Testing the Present Value model in the context of the Australian Stock Market.

The first order expectational difference equation used as the basis for testing the present value model arises simply out of the arbitrage equation. Investors equate the required rate of return,  $r$ , assumed to be constant, to the return from the stock market where  $p$  is the stock price and  $d$  the dividend. Thus<sup>1</sup>:

---

<sup>1</sup> Equation (9.1) derives from  $p_t = \frac{E[p_{t+1}|I_t] + d_t}{1+r}$  where  $r$  is the required rate of return. This simply says that current price is the discounted expected future price plus dividend.

$$r = \frac{E[p_{t+1}|I_t] - p_t}{p_t} + \frac{d_t}{p_t} \quad (9.1)$$

or:

$$p_t = \alpha E[p_{t+1}|I_t] + \alpha d_t \quad (9.2)$$

where  $\alpha = \frac{1}{1+r}$  and thus for non zero (or negative)  $r$ ,  $\alpha < 1$ .

To solve equation (9.2), assume that agents form rational expectations and then simply solve recursively and apply the law of iterated expectations. Thus:

$$p_{t+1} = \alpha E[p_{t+2}|I_{t+1}] + \alpha d_{t+1} \quad (9.3)$$

taking expectations at time  $t$  gives:

$$E[p_{t+1}|I_t] = \alpha E[E(p_{t+2}|I_{t+1})|I_t] + \alpha E[d_{t+1}|I_t] \quad (9.4)$$

and using the law of iterated expectations:

$$E[p_{t+1}|I_t] = \alpha E[p_{t+2}|I_t] + \alpha E[d_{t+1}|I_t] \quad (9.5)$$

substituting this in (9.2):

$$p_t = \alpha^2 E[p_{t+2}|I_t] + \alpha^2 E[d_{t+1}|I_t] + \alpha d_t \quad (9.6)$$

repeating and solving up to time  $T$ :

$$p_t = \sum_{i=0}^{T-t} \alpha^{i+1} E[d_{t+i}|I_t] + \alpha^{T-t+1} E[p_{t+T+1}|I_t] \quad (9.7)$$

The first term will converge as  $T \rightarrow \infty$  provided that dividends do not grow faster than the interest rate. A particular solution, the no bubbles solution, assumes that:

$$\lim_{T \rightarrow \infty} \alpha^{T+1} E[p_{t+T+1} | I_t] = 0 \quad (9.8)$$

yielding:

$$p_t = \sum_{i=0}^{\infty} \alpha^{i+1} E[d_{t+i} | I_t] \quad (9.9)$$

Equation (9.9) forms the basis for tests of the present value model (Campbell and Shiller, 1987, MacDonald, 1994, Mills, 1993). It is important therefore to note that rejection of the model to be tested could stem from the failure of any one of the assumptions made in deriving equation (9.9), namely, the assumption of the rationality of expectations, the ruling out of bubbles and the assumption of a constant discount rate.

Subtracting  $\theta d_t$  from both sides of equation (9.9)<sup>2</sup> and where  $\theta$  is defined as

$\frac{\alpha}{1-\alpha}$  gives:

$$p_t - \theta d_t = \sum_{i=0}^{\infty} \alpha^{i+1} E[(d_{t+i} - d_t) | I_t]. \quad (9.10)$$

Further re-arrangement<sup>3</sup> yields:

$$p_t - \theta d_t = \theta \sum_{i=1}^{\infty} \alpha^i E[\Delta d_{t+i} | I_t] \quad (9.11)$$

Campbell and Shiller term  $p_t - \theta d_t$  the spread,  $S_t$ . Equation (9.11) is important in the way it is used to set up a testable model and some of the implications of equation (9.11) and the assumptions required about the variables  $p$  and  $d$  which are required to make the model operational should be stressed.

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<sup>2</sup> and noting that  $\sum_{i=0}^{\infty} \alpha^{i+1} = \frac{\alpha}{1-\alpha}$

<sup>3</sup> and noting that  $d_{t+i} - d_t = \Delta d_{t+1} + \dots + \Delta d_{t+i}$

Firstly equation (9.11) defines a variable  $p_t - \theta d_t = S_t$ , the constructed spread, dependant upon an estimate of  $\theta$  which is in some sense the correct one. The theoretical spread, call it  $S_t^*$ , can also be derived from (9.11) such that  $S_t = E S_t^*$  is some weighted average of expected future changes in the discount rate<sup>4</sup>.

Now, in an empirical context where  $p$  and  $d$  are considered to be time series then, if it happens that  $p$  and  $d$  are  $I(1)$  series, which is assumed here for convenience, equation (9.11) implies that they will be cointegrated with a cointegrating vector  $(1 - \theta)$ . If  $p$  and  $d$  turn out to be  $I(1)$  but empirically not cointegrated then this casts doubt on the whole framework of the present value model used here. However continuing on the assumption that they are  $I(1)$  and that they do cointegrate then the cointegrating regression between  $p$  and  $d$  will provide a superconsistent estimate of  $\theta$  (Stock 1987) hence allowing construction of the spread variable. Note also that under these conditions the constructed spread will be a stationary  $I(0)$  variable.

Having constructed the spread variable Campbell and Shiller demonstrate how the restrictions implied by the model can be tested in the framework of a bivariate VAR model, using the first difference of dividends and the constructed spread variable. It should also be noted that such a formulation avoids potential mis-specification problems associated with a VAR model involving  $\Delta p$  and  $\Delta d$  if  $p$  and  $d$  are in fact cointegrated.

Thus, set up the bivariate VAR model:

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<sup>4</sup>  $S_t^* = \theta \sum_{i=1}^{\infty} \alpha^i [\Delta d_{t+i} | I_t]$

$$\begin{bmatrix} \Delta d_t \\ S_t \end{bmatrix} = \begin{bmatrix} \delta(L) & \beta(L) \\ \lambda(L) & \rho(L) \end{bmatrix} \begin{bmatrix} \Delta d_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \quad (9.12)$$

which can be expressed in companion form as:

$$\begin{bmatrix} \Delta d_t \\ \cdot \\ \cdot \\ \Delta d_{t-p+1} \\ S_t \\ \cdot \\ \cdot \\ S_{t-p+1} \end{bmatrix} = \begin{bmatrix} \delta_1 & \cdot & \cdot & \delta_p & \beta_1 & \cdot & \cdot & \beta_p \\ 1 & & & & & & & \\ & 1 & & & & & & \\ & & 1 & & & & & \\ \lambda_1 & \cdot & \cdot & \lambda_p & \rho_1 & \cdot & \cdot & \rho_p \\ & & & & 1 & & & \\ & & & & & 1 & & \\ & & & & & & 1 & \\ & & & & & & & 1 \end{bmatrix} \begin{bmatrix} \Delta d_{t-1} \\ \cdot \\ \cdot \\ \Delta d_{t-p} \\ S_{t-1} \\ \cdot \\ \cdot \\ S_{t-p} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ 0 \\ 0 \\ 0 \\ u_{2t} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (9.13)$$

where blank elements are zero.

Using obvious notation re-write this as:

$$\mathbf{z}_t = \mathbf{A}\mathbf{z}_{t-1} + \mathbf{v}_t \quad (9.14)$$

The present value model expressed in the form of equation (9.9) expresses prices as a function of expected future dividends. Restricting the information set to be the current and lagged values of  $\Delta d$  and  $S$  and denoting it as  $\mathbf{H}_t$  then the expectation of future values of  $\mathbf{z}_t$  conditional on  $\mathbf{H}_t$  can be written as:

$$E(\mathbf{z}_{t+i} | \mathbf{H}_t) = \mathbf{A}^i \mathbf{z}_t \quad (9.15)$$

This equation can then be used to re-write equation (9.11), the spread equation, as:

$$\mathbf{g}'\mathbf{z}_t = \theta \sum_{i=1}^{\infty} \alpha^i \mathbf{h}'\mathbf{A}^i \mathbf{z}_t \quad (9.16)$$

where  $\mathbf{g}'$  and  $\mathbf{h}'$  are selection vectors<sup>5</sup>. Re-arranging and evaluating the convergent sum yields:

$$\mathbf{g}' = \theta \sum_{i=1}^{\infty} \alpha^i \mathbf{h}'\mathbf{A}^i = \frac{\theta \mathbf{h}'\alpha \mathbf{A}}{(\mathbf{I} - \alpha \mathbf{A})} = \theta \mathbf{h}'\alpha \mathbf{A}(\mathbf{I} - \alpha \mathbf{A})^{-1} \quad (9.17)$$

for this to hold non trivially the following 2p restrictions must hold:

$$\mathbf{g}' - \theta \mathbf{h}'\alpha \mathbf{A}(\mathbf{I} - \alpha \mathbf{A})^{-1} = 0 \quad (9.18)$$

which can be written as the following set of linear restrictions for given  $\alpha$  and  $\theta$ :

$$\begin{aligned} \rho_1 &= \alpha^{-1} - \theta\beta_1 \\ \rho_i + \theta\beta_i &= 0 \text{ for } i = 2, \dots, p \\ \lambda_i + \theta\delta_i &= 0 \text{ for } i = 1, \dots, p \end{aligned} \quad (9.19)$$

These restrictions are made interpretable by consideration the origin of the present value model in the form of equation (9.2) ( $p_t = \alpha E[p_{t+1}|I_t] + \alpha d_t$ ) which says that the current price is the discounted expected future price plus discounted dividends. Looking at this in the context of the restricted information set  $\mathbf{H}_{t-1}$  then:

$$E(p_t - \frac{p_{t-1}}{\alpha} + d_{t-1} | \mathbf{H}_{t-1}) = 0 \quad (9.20)$$

---

<sup>5</sup>  $\mathbf{g}'$  is a row vector of dimension 2p with zeroes everywhere except the p+1 th element, and  $\mathbf{h}'$  is similarly a 2p vector of zeroes with a 1 as its first element.

implying that the innovation in the stock price is essentially unpredictable given past prices and dividends. Using the spread equation ( $S_t = p_t - \theta d_t$ ) to substitute out for  $p_t$  and  $p_{t-1}$  yields:

$$E(S_t - \frac{S_{t-1}}{\alpha} + \theta \Delta d_t | \mathbf{H}_{t-1}) = 0 \quad (9.21)$$

Re-writing this in the form of the VAR model:

$$\rho_1 S_{t-1} + \theta \beta_1 S_{t-1} - \alpha^{-1} S_{t-1} + \sum_{i=2}^p (\rho_i + \theta \beta_i) S_{t-i} + \sum_{i=1}^p (\lambda_i + \theta \delta_i) \Delta d_{t-i} \quad (9.22)$$

which, recalling the restrictions imposed on the VAR, is zero under the restrictions imposed by (9.19). So the restrictions show that the innovation in the stock price should be unpredictable given the history of  $S$  and  $\Delta d$ . In the empirical applications below these cross equation restrictions are tested in the bivariate VAR model and form the basic test of the present value model.

However other tests of the model can be considered, as in Campbell and Shiller. Firstly a weak implication of the model is that  $S_t$  should Granger cause  $\Delta d_t$ , and this is tested the context of the equation for  $\Delta d$  in the VAR model (9.13) by simply testing that all of the lagged  $S$  in the  $\Delta d$  equation have zero coefficients ie:  $\beta_1 = \beta_2 = \dots = \beta_p = 0$ .

Finally following Campbell and Shiller and in the light of earlier work on the present value model the VAR model is used to construct an estimate of the theoretical spread  $S^*$ .

$$S_t^* = \theta \sum_{i=1}^{\infty} \alpha^i [\Delta d_{t+i} | I_t] = \theta \mathbf{h}' \alpha \mathbf{A} (\mathbf{I} - \alpha \mathbf{A})^{-1} \mathbf{z}_t \quad (9.23)$$

If the present value model holds true then  $\text{var}(S_t) = \text{var}(S_t^*)$ . Once again the estimated VAR model can be used to construct the theoretical spread ( $S_t^*$ ) and the cointegrating relationship between p and d to construct the spread variable  $S_t$ .

#### **9.4 Unit roots, Cointegration and Testing the Present Value Model**

The testing frame work described requires three steps. Firstly the order of integration of the data has to be established. Secondly, and if valid, prices and dividends have to be tested for cointegration. Thirdly an appropriately specified VAR model including both the spread and the change in dividends can be estimated. Note, of course, that stage three depends upon the finding that prices and dividends are indeed cointegrated.

The data used in this study comprises real quarterly stock prices and associated dividends of the Australian All Ordinaries index over a sample period of 1959(3) to 1994(2)<sup>6</sup>. The dividend data was constructed using quarterly averages of the all ordinaries index dividend yield series. First month of quarter data for the stock price index were used. The use of beginning of quarter stock prices and quarter averages for dividends follows MacDonald (1994) who notes that such a choice is in the spirit of much of the literature on the efficient market hypothesis which assumes that whilst prices are known at the beginning of the period dividends are assumed to accrue at the end of the period. Later, in the testing of the present value model, and again following MacDonald (1994) this assumption is relaxed by using the lag of dividends in the model.

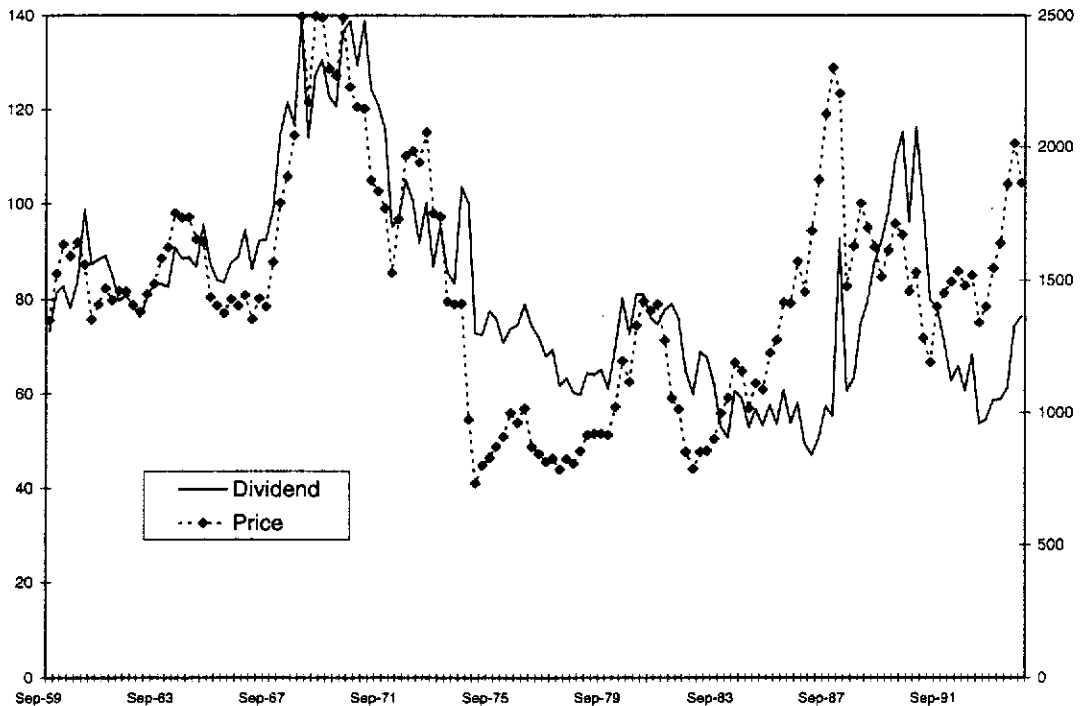
Figure 9.1 below plots the constructed price and dividend series. Below, a more formal testing of the relationship between prices and dividends is carried



out, however the graph can be used to infer some of the major characteristics of the data. As can be seen prices and dividends do appear reasonably positively correlated over the period. The simple correlation over the full sample is 0.633.

It is also fairly evident from the graph that the period post 1984/5 shows far weaker correlation between the data. Around this period the stock market began its rapid upward climb prior to the October 1987 crash. It is also clear that dividends did not rise along with the price index. Indeed they stayed fairly flat until around 1988 when they quickly peaked, fell, and rose rapidly to a

**FIGURE 9.1 Australian prices and dividends time series plot**



NB left hand scale - dividends  
right hand scale - price index

second higher peak in 1989/90 before falling back to their 1985 levels in about 1992. These changes are reflected in the simple correlations. Using data from 1959 quarter 3 to 1984 quarter 4 the correlation is 0.881 whilst for the period

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<sup>6</sup> See appendix A4 for the sources of the data.

1985 quarter 1 to 1994 quarter 2 the correlation is as low as 0.075. Clearly it would appear that the close relationship between prices and dividends has been broken in the post 1984 data, this is one of the aspects of the model tested more formally below.

Testing begins with consideration of the unit root hypothesis. Despite strong priors that both series would be  $I(1)$  a range of unit root tests were carried out in order to confirm this. The tests are described in more detail in Chapter 3. Firstly standard DF and ADF tests were used on the levels of the series and then on the difference of the series. The results are in Table 9.1a For each series testing began using an ADF with 4 lags, the lags were reduced until an F test on the last included lag rejected the zero restriction.  $\Phi_3$  tests (to test the

**TABLE 9.1** Unit root tests on individual series

Table 9.1a			
	lag length	LM test for residual autocorrelation	DF ADF
<b>OLS, levels</b>			
<i>prices</i>	0	5.3 (0.26)	-1.99
<i>dividends</i>	3	6.6 (0.16)	-1.99
<b>OLS first difference</b>			
<i>prices</i>	0	3.4 (0.49)	-11.3*
<i>dividends</i>	2	5.3 (0.26)	-6.56*
Table 9.1b			
			DF-GLS
<b>DF-GLS<sup>†</sup></b>			
<i>prices</i>	0	5.1 (.27)	-2.02
<i>dividends</i>	3	6.5 (.16)	-2.01
<b>DF-GLS<sup>‡</sup></b>			
<i>prices</i>	0	5.1 (.27)	-1.97
<i>dividends</i>	3	6.4 (.17)	-1.91

\* indicates significant at the 5% level

5% critical value for the DF-GLS<sup>†</sup> test = -2.95 and for the DF-GLS<sup>‡</sup> test = -2.05, estimated using Cheung and Lai (1995b) response surface equation.

unit root null against a trend stationary alternative) and then  $\Phi_2$  tests were initially calculated. For both prices and dividends the  $\Phi_3$  test could not reject the unit root and the non significance of the  $\Phi_2$  test was taken to indicate that

the series had zero drift and so the unit root test was carried out in a regression with constant only. As can be seen from Table 9.1a the levels could not reject a unit root but the first difference of the series could, thus suggesting the series be categorised as  $I(1)$ , random walks with zero drift. Further confirmation of the unit root null comes from the DF-GLS tests which were carried out on the levels of the series. Both the  $DF-GLS^T$  and  $DF-GLS^H$  tests (described more fully in Chapter 3) could not reject a unit root in the levels of the series and are reported in Table 9.1b.

Having established that the two series are  $I(1)$  testing can now move on to the question of cointegration between prices and dividends. This is a vital step in the development of the model since it is required in order for the VAR model to be correctly specified. The cointegrating relationship will be used to construct the spread variable ( $S_t$ ) which will be included in the VAR along with the first difference of dividends ( $\Delta d_t$ ). Since dividends are  $I(1)$  then their first difference will be  $I(0)$ . Unless there is evidence of cointegration between  $p$  (which is  $I(1)$ ) and  $d$  (which is  $I(1)$ ) then the spread variable will be  $I(1)$  by construction and the VAR will be unbalanced in the sense that it will contain an  $I(1)$  and an  $I(0)$  variable.

Given the vital nature of this pre-requisite it is surprising that many researchers have found only weak evidence of cointegration between  $p$  and  $d$  and have yet continued with the testing. Thus Campbell and Shiller (1987) in their paper found only weak evidence of cointegration and yet proceeded to set up the VAR on the basis that the spread variable was stationary. MacDonald (1994) found, using the UK ordinary share index, that he could not reject the null of no cointegration over the full sample of his data (1947-1987) and had to restrict analysis to a sub-sample (1960-1987) which showed evidence of cointegration at the 10% significance level. Both these papers used the familiar Engle-Granger two step estimation procedure. Using monthly data for the UK Financial Times Actuaries All Share Index from 1965 to 1990 Mills (1993)

finds evidence of cointegration only at the 12.5% significance level using Engle-Granger estimation but claims that stronger evidence (better than 1%) in favour of cointegration comes from using Johansen's Maximum Likelihood estimation procedure. However it appears that Mills has used the incorrect line from the relevant tables so that in fact he also finds only weak evidence of cointegration<sup>7</sup>. Overall then the evidence is mixed with relatively weak rejections of the null of no cointegration.

Testing, using the data considered here, began with standard Engle-Granger tests for cointegration. Thus Table 9.2 reports the results from running a "cointegrating regression" between prices and dividends and carrying out a unit root test on the retrieved residuals. The following cointegrating regression was estimated

$$p_t = a_0 + a_1 d_t + u_t \quad (9.26)$$

and the estimated residuals,  $\hat{u}$  were retrieved. These residual were then tested for a unit root using the familiar DF/ADF regression:

$$\Delta \hat{u}_t = \beta_0 \hat{u}_{t-1} + \sum_{i=1}^k \beta_1 \Delta \hat{u}_{t-i} + \varepsilon_t \quad (9.27)$$

The t statistic for the test of the null of no cointegration is the normal t statistic on  $\beta_0$  ( $H_0: \beta_0=0$ ) in the above regression. Essentially this method of testing for cointegration relies upon the testing of the null of a unit root in the estimated

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<sup>7</sup> Mill's test statistic (including an unrestricted constant in the VAR) is reported as 10.31. This is presumably the test statistic for the null of zero cointegrating vectors against the alternative of 1 (in a bivariate VAR the next test, of a null of 1 cointegrating vector against an alternative of 2 would imply stationarity of the variables if the null was rejected). Mills compares the test statistic reported to a 2.5% significance level of 5.33, this is however the test statistic under the null of 1 cointegrating vector, the correct test statistic is in fact 15.81 implying the model does not reject the null of no cointegration.

residuals. However since the regression uses the residuals from an OLS regression to provide estimates of the parameter  $a_1$ , the cointegration test is not the same as a standard test for a unit root in an observed time series and so the tabulated critical values from Fuller (1976) cannot be used. Fortunately MacKinnon (1991) has provided, on the basis of an extensive simulation exercise, a set of response surfaces for the critical values required for a variety of models. Essentially MacKinnon allows for a cointegrating equation with constant and trend, with constant only or with no constant and no trend and an ADF with no constant or trend and for a number of variables in the cointegrating relationship (note that the same can be achieved using a cointegrating regression with no constant or trend and then adding constant and or trend to the ADF equation - see Banerjee et al (1994) for a discussion).

In this case the model has two variables and includes a constant but no trend. Critical values can be obtained using the equations:

$$1\% \text{ critical value} = -3.9001 - 10.534(T^{-1}) - 30.03(T^{-2})$$

$$5\% \text{ critical value} = -3.3377 - 5.967(T^{-1}) - 8.98(T^{-2})$$

$$10\% \text{ critical value} = -3.0462 - 4.069(T^{-1}) - 5.73(T^{-2})$$

(where T is the sample size, see MacKinnon for details)

**Table 9.2** Engle-Granger test for cointegration - full sample results

		10% value	Critical value	5% value	critical	Coefficient on dividends ( $\theta$ )	LM(4) test for residual correlation in ADF
ADF(3)	-3.415	-3.08		-3.38		12.3752	5.76 [.22]

Critical values from MacKinnon (1991) response surface estimate equation for 2 variables constant, no trend and 136 observations.

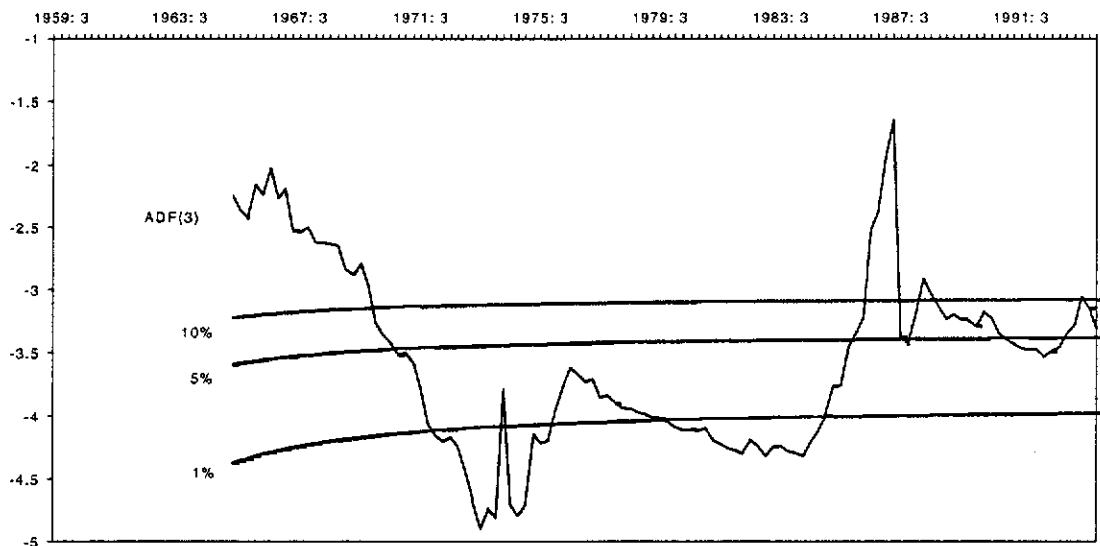
The ADF(3) is the ADF test statistic carried out on the residuals from the regression of price on constant and dividends.

Figure in [ ] parenthesis is test statistic p value.

As can be seen the result is encouraging in that the null of no cointegration can be rejected at the 5% significance level. However recursive estimation of the cointegrating equation suggests that the relationship between prices and dividends was significantly affected by the events of the mid 1980's.

Figure 9.2 below plots the t statistic used to test the null of no cointegration for a recursively estimated cointegrating regression of the Engle-Granger type for sample sizes between 25 observations (1959Q3 to 1965Q3) up to the full sample of 140 observations (1959Q3 to 1994Q2).

**FIGURE 9.2 Recursively estimated t statistic testing the null of no cointegration and critical values**



Critical values were calculated for the sample sizes as  $T$  evolves and are also plotted in Figure 9.2 (the plot shows the ADF(3) t statistic as it was found that 3 lags were necessary in the ADF test to correct for residual autocorrelation). As can be seen the statistics suggest that there is strong evidence of cointegration up to around the mid 1980's with the t statistic being consistently significant at better than the 5% significance level and occasionally at the 1% level. The events of the mid 1980's and onwards, the rapidly rising price index

and the flat dividend series, severely distort the picture and if the sample had finished in 1987, at the time of the crash there would have been no evidence of cointegration. One obvious interpretation of the full sample results is simply that the events of the mid 1980's led to a breakdown in the relationship between prices and dividends.

Since there is some evidence of instability in the relationship between prices and dividends it would seem appropriate to test for cointegration using the method suggested by Gregory and Hansen (1996). Tests suggested that their model four, which allows for a regime shift is the most appropriate<sup>8</sup> and Table 9.3 reports the results from the ADF test. As can be seen the test rejects the null of no cointegration at the 5 per cent level. However, since the conventional ADF test also rejects the null it is not possible to infer from this result that there is a structural break in the relationship<sup>9</sup>. Indeed analysis of the regression results suggests that the inclusion of such a regime shift in the model is inappropriate in this context. As can be seen from the estimated coefficients, the coefficient on dividends is 19.36 up to the point of the break (which was identified as 1984 quarter 3) but then falls significantly to approximately 1.53. This would imply that the real interest rate (which can simply be calculated as  $1/\theta$ ) rose from approximately 5.1 per cent to 65.4 per cent which is clearly implausible in this context.

Whilst these results must be viewed as non informative on the issue of a break in the relationship, the fact that the technique identified 1984 as the year of the shift re-enforces the belief that the relationship between prices and dividends was affected by the events of this period which followed Australia's move to a floating exchange rate (1983) and marked the beginning of a period of rapid financial deregulation.

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<sup>8</sup> Use of the other models suggested by Gregory and Hansen yielded no evidence of cointegration.

<sup>9</sup> See Gregory and Hansen's (1996) illustrative example.

**Table 9.3 OLS estimation of Model 4 and associated cointegration test on residuals**

<b>Gregory and Hansens's Model 4 - estimated by OLS 1959(3) to 1994(2)</b>				
	Constant	Dum	Divs	Divs*Dum
Price	-257.9 (-2.6)	1725.7 (9.7)	19.36 (17.2)	-17.8 (-7.8)
<b>Cointegration test using retrieved residuals.</b>				
	res(-1)	DW	AR(1-4) F(4,128)	
$\Delta$ res	-0.31 (-5.12)	2.01	0.14 [.97]	

Where Dum is a step dummy taking the value 0 from 1959(3) to 1984(3) and 1 elsewhere. Figures in ( ) are estimated t ratios, figures in [ ] are probability values for reported test statistics. Three lags of  $\Delta$ res were used in estimated ADF test on the residuals.

As a result, and since the question of cointegration between prices and dividends is so essential in the context of the testing procedure, it was decided to carry out tests on the sub sample of data pre and post 1984. Table 9.4 reports the results of testing for cointegration over two samples, 1959 quarter 3 to 1984 quarter 4 and 1985 quarter 1 to 1994 quarter 2.

**Table 9.4 Sub sample Engle-Granger test for cointegration**

Sample	ADF lag order	Test Statistic	10% critical value	5% critical value	1% critical value	Coefficient on dividends ( $\theta$ )	LM(4) test for residual correlation in ADF
59(3) to 84(4)	3	-4.21	-3.09	-3.40	-4.01	19.18	3.08 [0.55]
85(1) to 94(2)	0	-2.44	-3.16	-3.50	-4.20	1.05	2.75 [0.60]

Critical values from MacKinnon (1991) response surface estimate equation for 2 variables constant, no trend.

Figure in [ ] parenthesis is test statistic p value.

As can be seen, over the first sub sample up to the end of 1984, the null of no cointegration between prices and dividends can now be rejected at the 1 per cent level. The coefficient on dividends is 19.18 which implies a real interest rate of 5.21 per cent. Over the second sub sample from 1985 quarter 1 to 1994 quarter 2 there is no evidence of cointegration between prices and dividends



and, as suggested by the Gregory Hansen results above, the coefficient on dividends falls dramatically to 1.05 implying a real rate of return of 95 per cent. This is clearly an implausible result.

The above results suggest that it would be sensible to restrict analysis of the present value model using data up to the end of 1984 only. However, one potential objection to the above results is that they have been derived using the Engle-Granger method to estimate the cointegrating relationship and, as was noted in Chapter 4, it might be sensible to re-test the cointegrating relationship using the method of Johansen (1988) to see if the results are robust to the estimation method.

The first step in the Johansen method is the pre testing of the VAR for prices and dividends to establish the lag order. The VAR was initially estimated with eight lags and then sequentially reduced down to one lag. At each stage a likelihood ratio test for the exclusion of the last lag was carried out. The likelihood ratio test statistic is calculated as :  $(T - c) \times ( \ln |\Sigma_R| - \ln |\Sigma_U| )$  where  $|\Sigma_R|$  and  $|\Sigma_U|$  are the determinants of the estimated covariance matrices of the residuals from the restricted and unrestricted VAR, and  $(T - c)$  is the small sample correction suggested by Sims (1980) where  $T$  is the number of observations used and  $c$  is the number of estimated parameters in the unrestricted system.

Given that research (Cheung and Lai, 1993) appears to suggest that the Johansen test is sensitive to under parameterisation of the VAR but not to over parameterisation a liberal approach to the significance of lags in the VAR was taken. As can be seen from table 9.5 the Likelihood Ratio test can reject the restriction of the VAR from a VAR(3) to a VAR(2) at the 10% critical value and the restriction from VAR(2) to VAR(1) at a very high significance level.

On the basis of these results the VAR was estimated with 3 lags. Table 9.5 also reports tests on the normality and autocorrelation of the VAR system and the individual equations in the VAR. As can be seen the individual equations show no evidence of significant non normalities or autocorrelation. These results are confirmed by the graphical analysis of the residuals from the VAR which are reproduced as Figure 9.3. As can be seen the correlogram supports the notion that residuals appear to be tolerably white.

**Table 9.5 Test on the two variable VAR system**

<b>Tests on the lag order</b>							
<b>Likelihood Ratio Tests</b>							
Lag restriction	8 to 7	7 to 6	6 to 5	5 to 4	4 to 3	3 to 2	2 to 1
LR test	4.8	3.3	5.6	1.9	5.1	7.8*	70.4**
<b>Diagnostic tests</b>							
<b>Error Autocorrelation</b>							
	prices	dividends					
LM test (F version)							
AR( 1 - 5)	0.80 [0.55]	1.25 [0.29]					
Residual	0.71	2.10					
Normality	[0.70]	[0.34]					

Estimation carried out using PC-FIML (part of PC-GIVE professional version 8, Doornik and Hendry, 1994), the mis-specification tests reported are those produced by PC-FIML and described in Doornik and Hendry (1994).

The LR test is distributed as  $\chi^2$ , since each reduction in lag order of the VAR by 1 leads to a fall in the number of estimated parameters by 4 the test statistic is  $\chi^2(4)$  which has a 10 per cent critical value of 7.8.

Figures below test statistics in [ ] are p values.

\* indicates significance at 10 per cent level, \*\* at the 5% level.

**Figure 9.3** Correlograms for residuals from VAR for Price and Dividends respectively

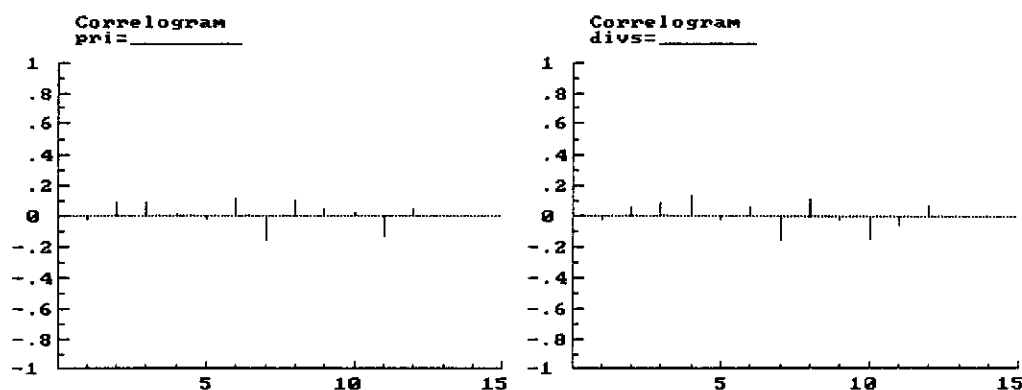


Table 9.6 reports the results from the Johansen estimation using a VAR lag length of 3. As can be seen, even without adjustment for lag length and sample size there is no evidence of cointegration between prices and dividends at normal significance levels. Since the unit root tests suggested that prices and dividends were random walks, with possibly zero drift, the cointegrating vector was estimated firstly with the constant term entered unrestricted (allowing non zero drift in the random walks) and then with the constant entered restricted to the ECM. The estimated eigenvalues were approximately the same, being 0.07758 and 0.02535 when the constant entered unrestricted and 0.07845 and 0.2535 when the constant entered the model restricted to the ECM. The test statistic described in Chapter 4 to test the restriction is:

$$-T \sum_{i=r+1}^n \left[ \ln(1 - \hat{\lambda}_i^*) - \ln(1 - \hat{\lambda}_i) \right]$$

which, under the assumption that there are zero cointegrating vectors evaluates to a test statistic of 0.127 which is distributed as  $\chi^2(2)$  and hence is not significant. As a result the model was estimated allowing the constant to enter restricted to the cointegrating vector.

**Table 9.6** Cointegration analysis full sample results

Eigenvalues		Constant entered restricted.							
0.0784521									
0.025356	Maximal eigenvalue test				Trace test				
Ho:rank=p	-Tlog(1- $\mu$ )	using T-nm	90%	95%	-T $\sum$ log(1- $\mu$ )	using T-nm	90%	95%	
p=0	10.78	10.29	13.8	15.7	14.17	13.53	17.9	20.0	
p $\leq$ 1	3.39	3.24	7.5	9.2	3.39	3.24	7.5	9.2	
Standardised $\beta'$ eigenvectors									
price	dividends	constant							
1.0	-14.89	-259.4							
0.0095	1.0	-95.98							

Critical values from Osterwald-Lenum.

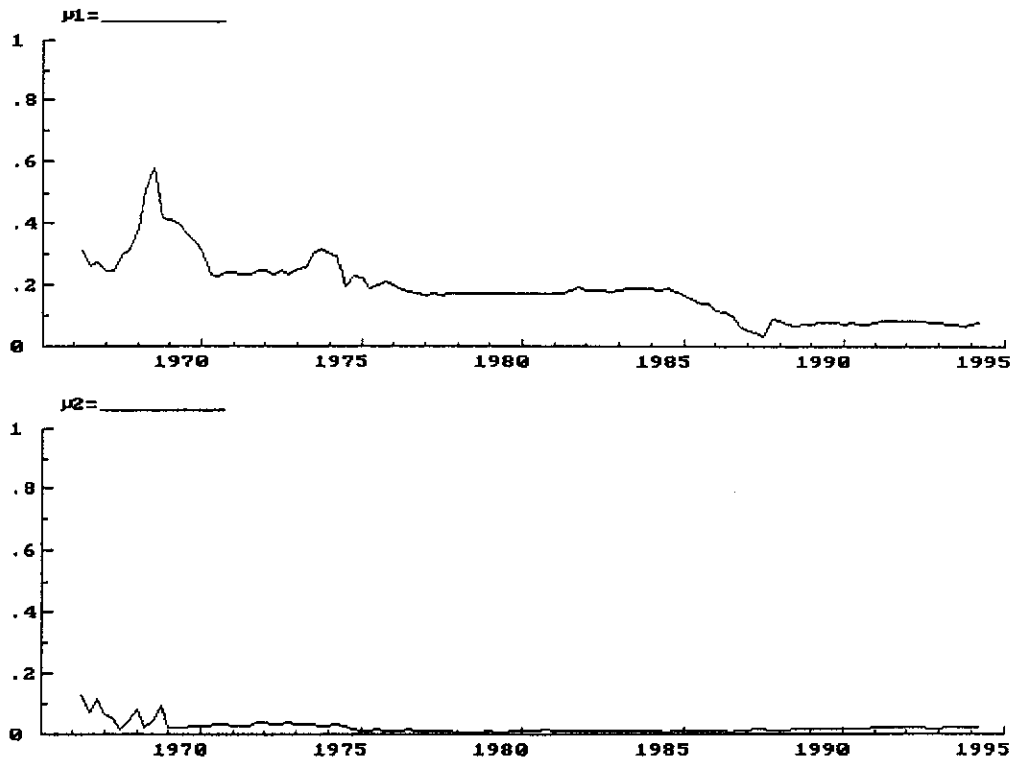
Test statistics under the column headed "using T - nm" are corrected for degrees of freedom using the method suggested by Reimers (1992).

As can be seen neither the maximal eigenvalue nor the trace test is significant at the 10 per cent level, even before adjustment for degrees of freedom. The coefficient on dividends in the first cointegrating vector is 14.9 which is similar to that obtained from the Engle Granger estimation.

Figure 9.4 plots the two recursively estimated eigenvalues<sup>10</sup> and Figure 9.5 plots the first cointegrating vector. As can be seen the first eigenvalue is relatively stable in value over the period 1975 - 1985 (with a value of around 0.2) but then falls significantly in the period leading up to the crash and stays relatively low, just below 0.1, to the end of the sample. The second eigenvalue is close to zero for most of the sample. Thus the Johansen estimation results are similar to those from the ols estimation and suggest a break in the relationship between prices and dividends around 1984/5. The cointegrating vector plotted in figure 9.6 confirms the apparent lack of cointegration and looks non stationary.

<sup>10</sup> The model estimation sample was 1961 quarter three to 1994 quarter three and nineteen observations were used to initialise the recursive estimation.

**Figure 9.4** Recursively estimated eigenvalues



**Figure 9.5** Cointegrating vector from full sample estimation

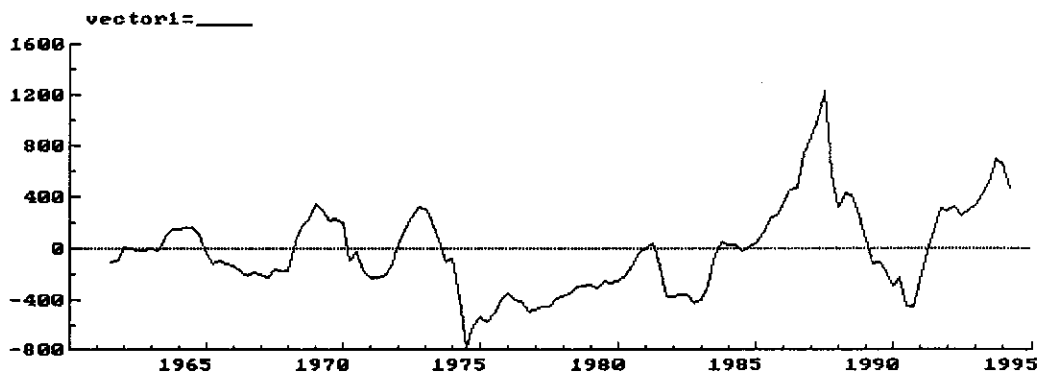


Table 9.7 reports the results of the Johansen estimation over the sub sample, up to the end of 1984. The VAR was once again estimated with a lag length of three and the constant was restricted to the cointegrating vector<sup>11</sup>. The results now show strong evidence of a single cointegrating vector, with the maximal eigenvalue test significant at the 5 per cent level (both unadjusted and adjusted for degrees of freedom) and the trace test at the 5 per cent level (unadjusted)

<sup>11</sup> The likelihood ratio test for the restriction is 0.313 which is distributed as  $\chi^2(1)$  and hence is not significant.

and 10 per cent level (adjusted). Figure 9.6 plots the cointegrating vector which looks reasonably stationary. It is also interesting to note that both the Engle-Granger method and the Johansen estimation yield a very similar estimate of the value of  $\theta$ , 19.18 from the OLS estimation and 19.62 from the Johansen estimation. Finally, and as a further test of the validity of the full sample estimates, the sub sample Johansen estimation was carried out with the parameters of the  $\beta'$  matrix restricted to their full sample values (price = 1.0, dividends = -14.84). The likelihood ratio test for the restriction was  $\chi^2(2) = 7.49$  which rejects the restriction at the 2 per cent level.

**Table 9.7**     **Cointegration analysis from estimation up to 1984 quarter four.**

Eigenvalues    Constant entered restricted. Sample 1960 q2 to 1984 q4								
0.174026								
0.0127592	Maximal eigenvalue test				Trace test			
Ho:rank=p	-Tlog(1- $\mu$ )	using T-nm	90%	95%	-T $\sum$ log(1- $\mu$ )	using T-nm	90%	95%
	18.93**	17.78**	13.8	15.7	20.2**	18.98*	17.9	20.0
	1.27	1.19	7.5	9.2	1.27	1.19	7.5	9.2
Standardised $\beta'$ eigenvectors								
price	dividends	constant						
1.0	-19.62	286.5						
-0.007	1.0	-67.7						

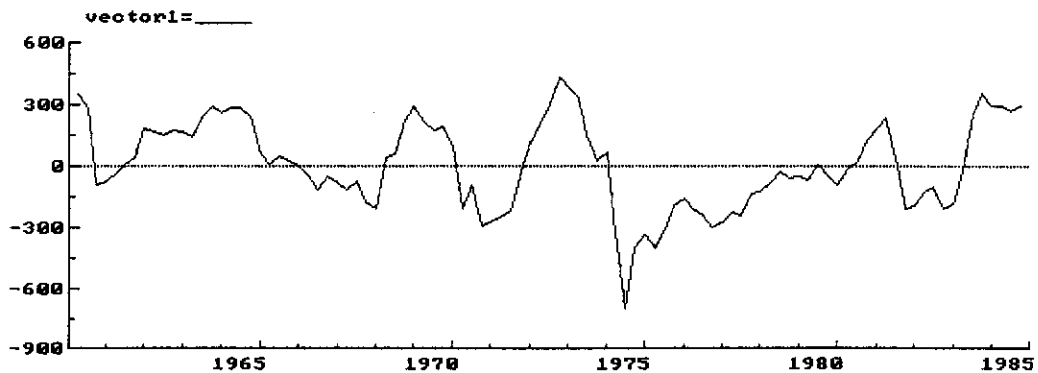
Critical values from Osterwald-Lenum.

Test statistics under the column headed "using T -nm" are corrected for degrees of freedom using the method suggested by Reimers (1992).

\*\* indicates significant at 5 per cent \* at 10% level

These results confirm the finding above using ols techniques that it would seem sensible to test the present value model on a sub sample of data, up to 1984 quarter 4, where the evidence of cointegration is strong and where the results make sense in terms of the implied rate of return in the market.

**Figure 9.6 Cointegrating vectors from estimation up to 1984 quarter four.**



One final potential problem with the above specification of the model is that the price data used is beginning of quarter and the dividend data is quarter average, thus as noted by MacDonald (1993) whilst this specification might be consistent with the efficient market model it is reasonable to also reconstruct the spread variables as  $S_t = p_t - d_{t-1}$ , this conforms with the model used by Campbell and Shiller in their original paper.

Thus in what follow the spread variable is constructed, using the parameter estimate from the Johansen estimation over the sub sample, as  $s_t = p_t - 19.62d_t$  and  $s_t = p_t - 19.62d_{t-1}$ .

Having constructed the spread variables, the next task is to estimate the two variable VAR system containing  $S_t$  and  $\Delta d_t$ <sup>12</sup>. The systems were initially estimated with eight lags in each variable and then the lag order was sequentially reduced (only symmetric lag structures were considered). Table 9.8 below reports various diagnostics on the two systems considered. In order

<sup>12</sup> The bivariate VAR model will be for  $S_t$  and  $\Delta d_t$  in the case where spread is constructed as  $S_t = p_t - \theta d_t$  and for  $S_t$  and  $\Delta d_{t-1}$  in the case where spread is constructed as  $S_t = p_t - \theta d_{t-1}$ .

**Table 9.8** **Lag orders for the two variable VAR's**

Spread constructed as:	Sample	Likelihood ratio test statistic	Test for error autocorrelation in individual equations	
			$\Delta$ Dividends	Spread
			AR (1- 5)	AR (1- 5)
$p_t - 19.62d_t$	61q4-84q4	37.6* (2)	0.41 [0.84]	2.02 [0.08]
$p_t - 19.6d_{t-1}$	62q1-84q4	9.9* (3)	1.95 [0.10]	1.88 [0.11]

Estimation carried out using PC-FIML (part of PC-GIVE professional version 8, Doornik and Hendry, 1994), the mis-specification tests reported are those produced by PC-FIML and described in Doornik and Hendry (1994).

to save space only the significant Likelihood Ratio test statistic for lag reduction is reported along with the lag length suggested. The results of imposing the restrictions suggested by the present value model on the VAR's are reported below in Table 9.9. As can be seen the Wald test (final column headed  $\chi^2$ ) of the present value model is strongly rejected in both cases. Some support for the present value model does, however, come from the simple Granger causality tests carried out on the VAR, where it is found that in each case lagged spreads have predictive power on  $\Delta d_t$ , a prediction of the present value model.

Overall the results in this section suggest that, in common with other research in this area using US and UK data sets, the VAR fails the formal restrictions imposed on it by the present value model using Australian data. This result holds for a sub sample where the evidence of cointegration between prices and dividends was very strong. To the extent that this framework provides a valid test of the present value the evidence suggests this model is rejected by the data. This formal rejection of the model could stem from any one of the underlying assumptions in its formulation such as the absence of speculative bubbles or the constancy of the discount rate. On the other hand the less formal tests do provide some evidence in favour of the present value formulation.



Thus the fact that there is strong evidence that lagged spreads Granger cause the change in dividends which again would be a prediction of the model.

**TABLE 9.9** Tests on the bivariate VAR model

Sample period	Spread constructed as:	$R^2(\Delta d)$	$R^2(S)$	F test on lagged spread in $\Delta$ dividend equation	F test on lagged $\Delta$ dividends in spread equation	$\chi^2$
61q4-84q4	$p_t - 19.62d_t$	0.20	0.78	5.6 [0.01]	6.9 [0.00]	41.8
62q1-84q4	$p_t - 19.6d_{t-1}$	0.58	0.39	51.3 [0.00]	3.9 [0.02]	46.3

**F** is an F test on zero restrictions on lagged  $S$  in the  $\Delta d$  equation and is thus the Granger causality test carried out here.

The  $R^2$  are from individual equation OLS estimation.

The final column  $\chi^2$  test is a test of the cross equation restrictions implied by the present value model, the 5% significance level for  $\chi^2(4)$  is 9.5 and for  $\chi^2(6)$  is 12.6, only the last equation involves six restrictions.

Figures in [ ] are probability values for the respective test statistics.

One further result of interest in this section is the finding that the cointegrating relationship between prices and dividends is one which is questionable over the whole sample. Once again this finding is common in previous studies such as Campbell and Shiller (1987), Mills (1993) and MacDonald (1994), who as was done above, shortened the sample to obtain a rejection of the null of no cointegration at acceptable significance levels. Whilst the Engle-Granger method finds evidence of a cointegrating vector at the 5 per cent significance level, the Johansen method finds no evidence of cointegration at the 10 per cent level over the whole sample. Both methods confirm the existence of a single cointegrating vector over the sub sample up to the 1984. The results are suggest that the cointegrating relationship between prices and dividends broke down over the period of the mid 1980's. One possible approach to this problem is to consider whether other fundamentals (other than expected dividends) are important in determining stock prices. This is the approach taken recently by MacDonald and Powers (1995) who augment the basic

model by adding a term involving retained earnings. Thus they argue that it is earnings, not just dividends which affect the stock price. Whilst noting that Modigliani and Miller (1958) argue that earnings adds no extra information to the model they suggest that retained earnings may significantly affect the investment growth opportunities of the firm. Whilst conceptually it is difficult to see how earnings could contain extra information, not contained in the dividend yield, MacDonald and Power find far stronger evidence of cointegration, using the US Standard and Poor's index when they include retained earnings in the cointegrating vector. The derived short term forecasting model for the change in stock prices does a good job in forecasting over the period 1976 to 1987, passing a range of diagnostic including a Chow test for predictive failure once the crash (1987) was taken out of the model.

The final section of this chapter considers a simple Vector Error Correction Model for forecasting stock prices based only on dividends. Whilst it would be interesting to consider other fundamentals, such as earnings, to see if they improve the cointegration result full sample a consistent earnings series was not available at the time of writing. It is also the case that the model above, involving only prices and dividends, shows a strong cointegrating relationship up to 1984 quarter four, so it is worthwhile investigating the forecasting performance of a VECM based on this data.

### **9.5 Estimation of a Vector Error Correction Model for prices and dividends**

The cointegrating vector estimated over the sub sample ending in 1984 quarter 4 was used as the error correction term in the vector error correction model (VECM) discussed below. As noted in Chapter 4, the Engle-Granger representation theorem implies that if  $p$  and  $d$  are cointegrated then there exists a valid dynamic error correction representation of the data. In this case, since

attention focuses on the stock price the following equation (9.35) was estimated using ols.

$$\Delta p_t = \alpha + \sum_{i=1}^n \Delta p_{t-i} + \sum_{i=1}^n \Delta d_{t-i} + \rho z_{t-1} + u_t \quad (9.35)$$

Sequential reduction of the model led to equation (9.36)<sup>13</sup>.

Sample: 1961 (2) to 1984 (4)

$$\begin{aligned} \Delta \text{pri} = & +0.2817 & +0.8944 \Delta \text{pri}_{t-1} & +0.2584 \Delta \text{pri}_{t-2} \\ & (10.17) & (0.1199) & (0.0763) \\ & +0.283 \Delta \text{pri}_{t-6} & -16.74 \Delta \text{divs}_{t-1} & -0.1225 z_{t-1} \\ & (0.07833) & (1.878) & (0.05466) \\ & -381.3 (i1968q4) & & \\ & (106.3) & & \end{aligned} \quad (9.36)$$

Standard errors are in parenthesis

$R^2 = 0.537113$   $F(6, 88) = 17.018$  [0.0000]  $\sigma = 6.7\%$ <sup>14</sup>  $DW = 1.94$

AR 1- 5F( 5, 83) = 0.54753 [0.7397]

ARCH 4 F( 4, 80) = 1.5285 [0.2018]

Normality  $\text{Chi}^2(2) = 0.93923$  [0.6252]

Hetro  $F(12, 75) = 0.62905$  [0.8110]

RESET  $F( 1, 87) = 0.032216$  [0.8580]

Estimation carried out using PC-FIML (part of PC-GIVE professional version 8, Doornik and Hendry, 1994), the mis-specification tests reported are those produced by PC-FIML and described in Doornik and Hendry (1994).

where AR is an LM test for residual autocorrelation , ARCH an LM test for autoregressive conditional hetroscedasticity, Normality a test for the

<sup>13</sup> The fourth quarter of 1968 saw a substantial one period fall in both prices and dividends. In estimation without a dummy variable this quarter produced a significant residual and so an impulse dummy variable was included in the regression.

<sup>14</sup> The standard error is expressed as a percent of the mean of the original level of the dependant variable, prices.

distribution of the residuals based on their skew and kurtosis compared to the normal distribution, Heteroscedasticity is Whites (1980) test for heteroscedasticity and RESET is Ramsey's (1969) test of regression specification.

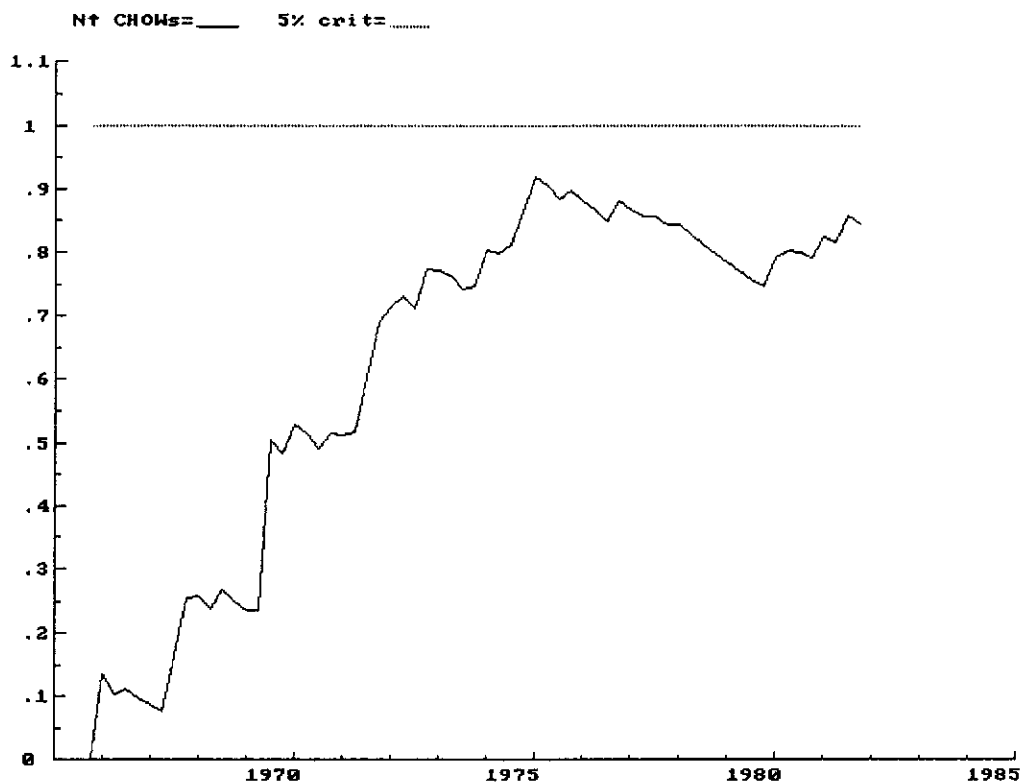
As can be seen the regression passes a range of diagnostic tests. The lagged error correction term is correctly signed and significant at slightly less than the 3 per cent level, however the coefficient is relatively small suggesting that prices revert to the equilibrium path relatively slowly.

One of the questions posed by the above analysis was the stability of the relationship between prices and dividends. In order to examine this question more closely the VECM was estimated recursively and  $N\uparrow$  Chow tests were calculated. These test the model based on estimation over the initial period (in this case the first three years of the sample were used to initialise the model) against a model estimated over the increasing recursion period and are calculated as:

$$\frac{(RSS_t - RSS_{M-1})(M - k - 1)}{RSS_{M-1}(t - M + 1)}$$

where  $M$  is the number of observations used to initialise the estimation. Thus as the recursive estimation proceeds the forecast period increases, hence the description as an  $N\uparrow$  Chow test. Figure 9.7 below plots the statistic scaled by the relevant critical value (in this case the 5 per cent significance level) from the  $F$  distribution to correct for degrees of freedom. Thus the critical value becomes a straight line at the value unity.

**Figure 9.7** Recursively estimated  $N\uparrow$  Chow test for equation (9.37)



Finally the proof of any model such as this lies in its ability to forecast. The above model (equation 9.36) was re-estimated using the sample up to 1984 quarter 4 and 12 observations (3 years) were reserved for forecasting purposes. The regression is reported below as equation 9.37. As can be seen the model performs tolerably well. Two tests of the constancy of parameters are presented. The first is a test of parameter constancy over the forecast period and is calculated as:

$$\xi_1 = \sum_{t=T+1}^{T+H} \frac{e_t^2}{\hat{\sigma}_u^2} \sim \chi^2(H) \text{ under the null.}$$

where estimation is carried out up to observation T and the model forecasts over the next H periods e is the one step forecast error and  $\hat{\sigma}_u^2$  is the estimated residual variance. Here the null is that there is no structural change in the

parameters between the sample and forecast periods. The second test is the more familiar Chow test of parameter constancy over the sample period. As can be seen the model passes both tests as well as the other usual diagnostics.

Figure 9.8 plots the actual and fitted whilst Figure 9.9 plots the actual and forecasts values, as can be seen, whilst the model doesn't pick up all of the turning points it doesn't do too badly.

Sample: 1961 (2) to 1984 (4) less 12 forecasts

The forecast period is: 1982 (1) to 1984 (4)

$$\begin{aligned}
 \Delta pri_t = & -0.3946 & +0.9895 \Delta pri_{t-1} & +0.2756 \Delta pri_{t-2} \\
 & (10.6) & (0.12) & (0.08) \\
 & +0.2847 \Delta pri_{t-6} & -18.51 \Delta divs_{t-1} & -0.1328 z_{t-1} \\
 & (0.08) & (1.99) & (0.05) \\
 & -386.2 (i1968p4) & & (9.37) \\
 & (104.4) & & 
 \end{aligned}$$

$R^2 = 0.597$   $F(6, 76) = 18.74$  [0.0000]  $\sigma = 6.7\%$   $DW = 1.90$

AR 1- 5F( 5, 71) = 0.77 [0.57]

ARCH 4 F( 4, 68) = 1.99 [0.11]

Normality  $\chi^2(2)$  = 0.58 [0.75]

Hetro F(12, 63) = 0.63 [0.81]

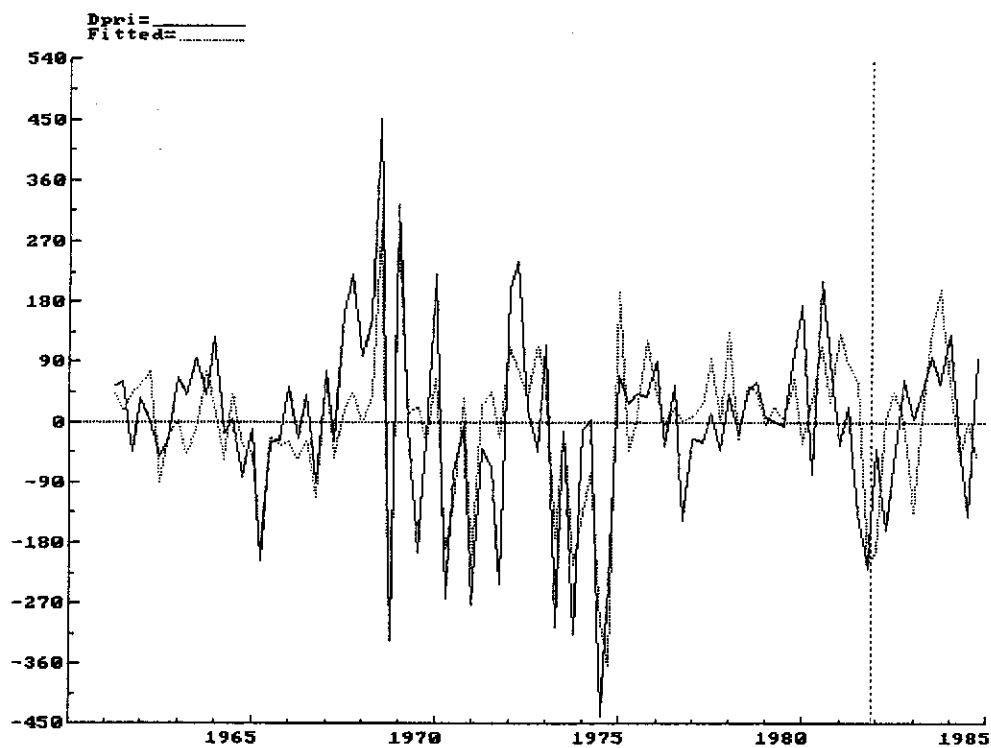
RESET F( 1, 79) = 0.00 [0.98.]

ests of parameter constancy over: 1982 (1) to 1984 (4)

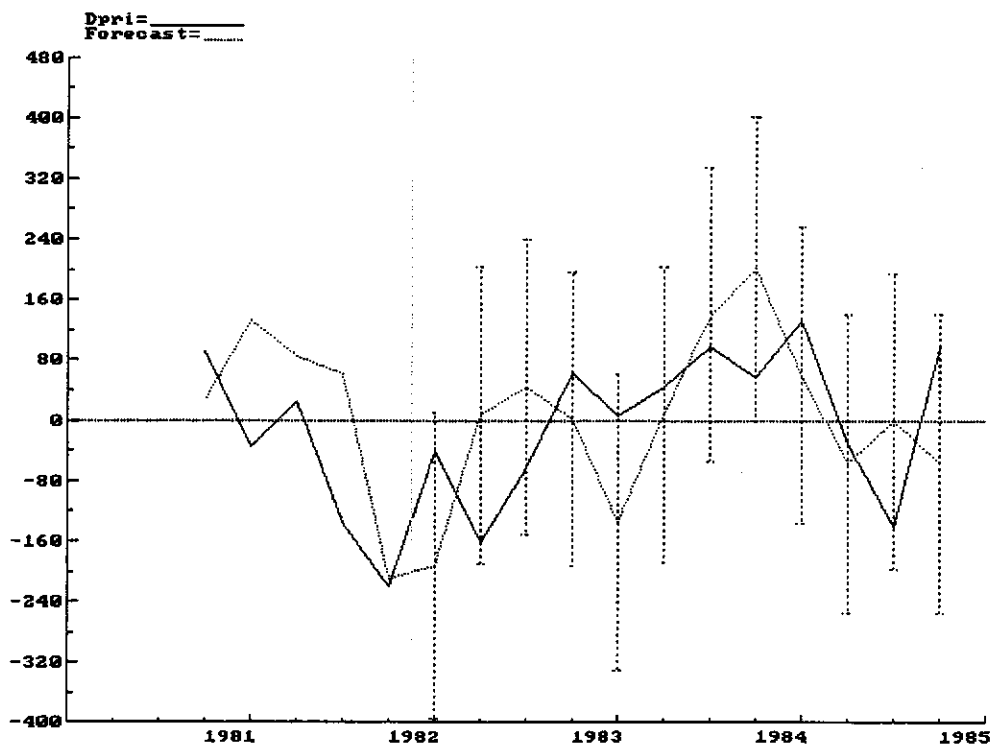
$\xi_1 \sim \chi^2(12)$  = 17.3 [0.14]

Chow F(12, 76) = 1.35 [0.21]

**Figure 9.8** Actual and fitted for equation (9.37)



**Figure 9.9** One step ahead forecasts and associated  $\pm 2$  standard error bands for equation (9.37)



Overall the conclusion is that, for Australia at least, it is possible to estimate a simple short run dynamic equation for stock prices which includes only lagged information on prices and dividends along with an error correction term which performs well up to the end of 1984.

## 9.6 Conclusion

This chapter has tested the present value model popularised by Campbell and Shiller (1987) against Australian data. In common with other work in the area for other countries the formal restrictions implied by the model are rejected when using Australian stock price and dividend data.

The results in this chapter also suggest that, again in common with other work, one of the vital prerequisite of the model, the existence of cointegration between prices and dividends is for the whole sample, a somewhat questionable assumption. However, cointegration between prices and dividends is found in a sample which runs up to 1984, prior to the 1987 stock market crash. In contrast to recent work by MacDonald and Power (1995) which found that, using pre crash data for the UK, prices and dividends did not cointegrate and that additional variables (the retention ratio) were required to establish a cointegrating vector this chapter finds that, using pre crash data there is strong evidence of cointegration between prices and dividends. The resultant cointegrating vector is used to construct a simple VECM which performs as well as that of MacDonald and Power in terms of its ability to forecast.



## CHAPTER 10

### CONCLUSIONS

#### 10.1 Introduction

This chapter very briefly summarises the main findings of the thesis. It is in two sections, section 10.2 considers the results from the applied work in the area of labour markets which forms the basis of Section B of the thesis. Section 10.3 considers the results from Section C of the thesis, the applied work in the area of financial economics.

#### 10.2 The applied analysis of labour markets.

The applied work in chapters 5 and 6 considered, firstly, the time series properties of a range of variables commonly used in time series work in the area, and secondly, the question of cointegration between sub sets of the variables commonly associated with the specification of a wage equation.

In particular, Chapter 5 used three tests for the presence of a unit root in observed time series. Whilst the tests did provide a conclusive answer in some cases, they were contradictory in others. Overall it was concluded that, for the most part, the assumption that the series considered were  $I(1)$  was reasonable<sup>1</sup> and that they could thus be considered, in Chapter 6 for cointegration.

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<sup>1</sup> One exception being the log of population which all three tests suggested was  $I(2)$ .

Chapter 6 looked at the question of cointegration between variables commonly associated with applied research on the specification of the wage equation. One of the most interesting findings of this chapter was that the recently developed test for cointegration in the presence of a regime shift (Gregory and Hansen, 1996) proved very useful in detecting cointegration when the other two methods, residual based Engle Granger tests and Johansen estimation failed to do so. Whilst standard tests for cointegration failed to detect a cointegrating vector when the Gregory and Hansen method suggested that there had been a regime shift in the cointegrating vector, Johansen estimation did find evidence of cointegration when the cointegrating vector appeared to undergo a simple mean shift. Overall the results from this chapter suggest that Gregory and Hansen's method of testing for cointegration is a very useful one which should be added to the tool-kit of the applied economist. It also suggests that further research on the power of Johansens method of testing for cointegration in the presence of different types of regime shift might prove useful.

A number of other results emerged from the work in this chapter. Firstly, the results suggest that real wages, productivity and unemployment are cointegrated over the sample period considered once allowance is made for a regime shift in the cointegrating vector. This result has implications for the identification of wage equations which include productivity and unemployment. Secondly, in common with recent work by Alexander (1993), there is evidence from the results in this chapter that Australia's move away from free collective bargaining to a system of centralised wage determination has affected the relationship between these three variables, with unemployment becoming more important in the determination of wages and productivity less so.

### **10.3 The applied analysis of financial markets.**

Chapters 7 and 8 looked at the gains from portfolio diversification, from an Australian perspective, using data on a wide range of international stock market indices. The basic results from the applied work in these chapters suggested that there was some evidence of cointegration between sub sets of the indices considered, suggesting therefore that the gains of diversification for long term investors is limited. In Chapter 8 the relationships between the markets which did cointegrate was analysed in more detail. Using a method suggested by Kasa (1993) it was found that the common trends driving these markets did a very good job in explaining their long run movements. Using a simple vector error correction formulation of the cointegrated markets it was found that, in a number of the portfolios one of the markets, the UK was independent of the other markets. Thus whilst the UK market entered a number of the cointegrating vectors there was never any evidence of causality, either from lagged dynamics or the error correction term from the other markets to the UK. On the other hand the error correcting term was frequently significant in the equations for other markets. In the bivariate portfolio's examined the finding was that it was one of the two markets which did all of the adjusting back to long run equilibrium.

Finally in Chapter 9 the present value model of stock prices was tested using Australian stock market price and dividend data. The finding was that the restrictions implied by the model were strongly rejected by the data. However, it was found that the cointegrating relationship between prices and dividend had been significantly affected by the events of the stock market crash in 1987. Using a sample which ended just prior to the crash strong evidence of cointegration between the two variables was found and a simple vector error correction model was found to do a reasonable job of forecasting the dynamics of stock prices. This result is in contrast to recent work on the UK market by

MacDonald and Power who found that a further variable, firms earning retention ratio, had to be added to the cointegrating relationship to provide a satisfactory outcome.

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## **Appendix A1**

### **Sources of critical values for unit root and cointegration test statistics.**

Chapters 3 and 4 have outlined the tests for unit roots and cointegration used in this thesis. As noted there, the distributions of these tests are asymptotically non normal, and have critical values which have to be obtained via monte carlo simulation. It is also the case that many of the critical values of these statistics are not invariant to nuisance parameters, such as a constant and or time trend, or the order of lags used to correct for residual correlation (in (A)DF tests for example). This places limitations on the applied worker who will frequently have a sample size which does not exactly tally with tabulated critical values and who will use varying lag orders to correct for serial correlation, knowing that both have the potential to affect the size of critical values for the test statistics used, particularly in small samples. Fortunately a large amount of use has been made of the method of estimating response surface equations for critical values in this area of applied econometrics<sup>2</sup>. The following sub sections review the applications of such methods, looking first in section A1.1 at the (A)DF test and its critical values, then, in section A1.2, at the DF-GLS test and finally in section A1.3 at the critical values for the Johansen cointegration test. Where response surface equations are not available, reference is made at the appropriate places in the text to the sources of critical values.

#### **A1.1 The (A)DF test**

The (A)DF test finds two uses in applied economics. The first is in the testing for a unit root in univariate time series, the second is testing residuals from a cointegrating regression for a unit root to test the null of no cointegration in the Granger Engle two step method. Tabulated critical values can be found in a number of sources including Fuller (1976) and Dickey and Fuller (1979,1981),



Engle and Yoo (1987) and Phillips and Ouliaris (1990). There are two problems with these tabulated values. The first is the obvious one that they provide values for a limited set of discrete sample sizes, the second is that they are all calculated on the basis that the lag order in the (A)DF test (the lagged first difference terms on the right hand side) is zero thus limiting consideration to the AR(1) model. Asymptotically the (A)DF test for the AR(k) model has the same critical values, however in finite samples of the type used in applied macroeconomics the critical values could be biased by ignoring dependence on lag order. Two papers address these problems and can be used to calculate critical values.

The first is MacKinnon (1991) who provides response surface equations for critical values of cointegrating tests using the (A)DF framework for the most commonly used models and a variety of number of variables. The critical values can be calculated using the parameters provided by MacKinnon in the surface response equation which takes the general form:

$$C(p) = \phi_{\infty} + \phi_1^{N-1} + \phi_2^{N-2} \quad (\text{A1.1})$$

where  $N$  is the sample size and the parameters  $\phi_{\infty}$ ,  $\phi_1$ ,  $\phi_2$  are provided by MacKinnon for the differing models (constant with no trend, constant and trend etc) and for a variety of number of variables,  $n$ , in the cointegrating relationship ( $n=1$  corresponds to testing for a unit root in a time series). Once again these critical values are given for the AR(1) model only.

The second source of critical values is Cheung and Lai (1995a) who provide a surface response equation which allows for control variables of both sample size ( $N$ ) and lag order ( $k$ ) in the (A)DF test for a unit root<sup>3</sup>. Their analysis suggests that for sample sizes of the type commonly used in applied work the lag order can affect the critical values and hence it is best to use lag adjusted critical values in empirical work. Their response surface equation takes the general form:

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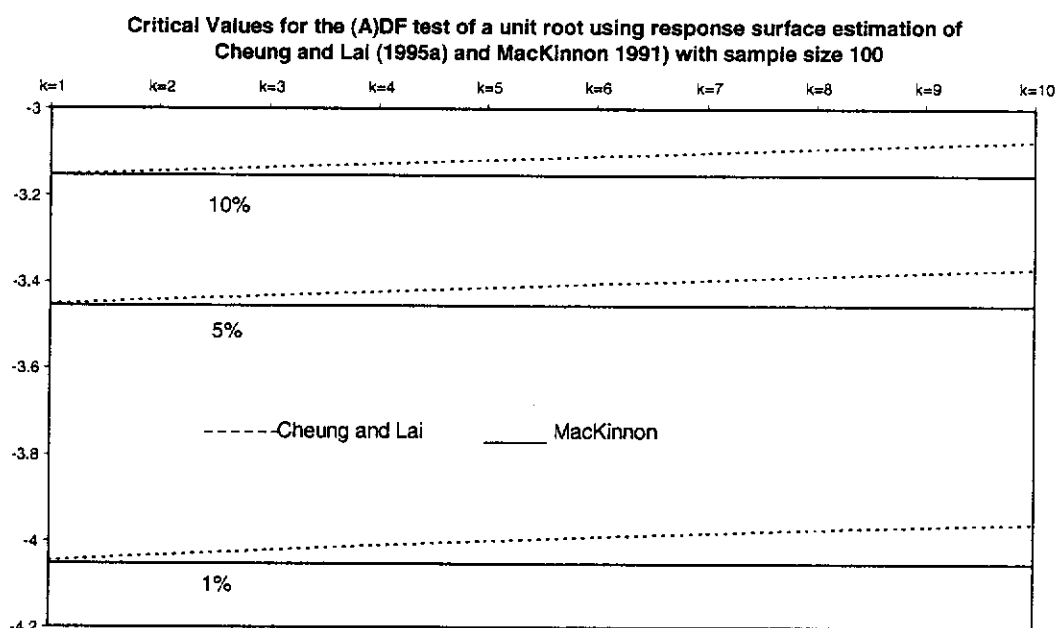
<sup>2</sup> See Myers, Khuri and Carter (1989) for a survey of this area

<sup>3</sup> Their response surface equation cannot be used to calculate critical values for the (A)DF test of residuals from a cointegrating regression.

$$CR_{N,k} = \tau_0 + \sum_{i=1}^2 \tau_i (1/T)^i + \sum_{j=1}^2 \phi_j [(k-1)/T]^j + \varepsilon_{N,k} \quad (\text{A1.2})$$

where  $N$  is sample size,  $k$  is the lag order and  $T = N-k$ .

The plot below illustrates the effects of considering lag order by plotting the 10%, 5% and 1% critical values for an (A)DF test using the two response surface equations. As can be seen in a sample size of 100 as the lag order in the (A)DF test increases the critical values for the test fall (in absolute value)<sup>4</sup>.



Although the differences in critical values may be small in absolute terms, given the low power of the (A)DF test to reject a false null, it would seem of value to adjust the critical values for lag order, not doing so clearly contributes to the probability of not rejecting the unit root null.

## **A1.2 The DF-GLS test**

<sup>4</sup> Note the scaling of the axis. In this example, at the 1% level and for  $k=4$  the critical value from MacKinnon is  $-4.052$  and from Cheung and Lai  $-4.009$ , a difference in absolute terms of  $0.043$ .

The DF-GLS test has also been the subject of a monte carlo study by Cheung and Lai (1995b). Once again they provide an estimated response surface equation which allows the calculation of critical values for the DF-GLS test for any particular sample size and values of  $k$ , the lag order. In their (1995b) paper Cheung and Lai note that whilst Elliot, Rothenberg and Stock (1992) provide estimated critical values of their test statistic for a variety of sample sizes these are all based on  $k = 0$  in the same way that standard ADF tests use the same critical values as the DF test statistic as noted above. Thus the applied worker only has one set of critical values irrespective of the lag order used in practice. Their results suggest that the critical values are affected by the size of  $k$  and that not accounting for this can affect the results of the testing procedure. In their Table 1 they report 4 regression equations which estimate the parameters of the response surface equation and can be used to calculate critical values. Which one to use depends upon the critical value required (5% or 10%) and whether or not the regression included just a constant or a constant and time. The response surface equation is of the following general form (their equation (4)):

$$CR_{T,p} = \tau_0 + \sum_{i=1}^r \tau_i (1/T)^i + \sum_{j=1}^s \phi_j (p/T)^j + \varepsilon_{T,p} \quad (\text{A1.3})$$

where  $T$  is the sample size,  $p$  the lag length and  $\tau$  and  $\phi$  are estimated parameters. Again where applicable in the applied work in later chapters this response surface equation is used to calculate the critical values for the DF-GLS tests.

### **A1.3 The Johansen test**

Finally tabulated critical values for the trace and maximal eigenvalue test for testing the number of cointegrating vectors using the Johansen method, described in chapter 4, are available from a number of sources including Johansen (1988), Johansen and Juselius (1990) and Osterwald-Lenum (1992). The critical values for these tests depend upon how linear trends and seasonal

dummies are included in the estimation. Thus, for example, the tables in Johansen and Juselius (1990) are in three sections, one corresponding to the case where a constant is entered unrestrictedly, corresponding to a drift term, one where the constant is included but is restricted to the cointegrating vector, and one where no constant is included. Osterwald-Lenum (1992) extends these tables to allow for a greater number of variables in the cointegrating relationship (up to 11) and provides new tables which allow for the inclusion of a greater range of deterministic variables. The Osterwald-Lenum paper is particularly useful since it provides a set of tabulated value in indexed tables and a set of decision criteria which guide the applied worker to the correct table.

The tabulated critical values discussed thus far are all based on asymptotic approximations and a number of papers have dealt with the effect of finite sample sizes on these critical values. Reinsel and Ahn (1988) and Reimers (1992) suggest the use of a scaling factor (based on sample size, number of variables in the cointegrating relationship and the lag order in the VAR). Reimers scaling factor  $(T - nk)/T$  where  $T$  is sample size,  $k$  the lag order and  $n$  the number of variables adjusts the test statistic which can then be compared to asymptotic critical values from the appropriate source<sup>5</sup>. Cheung and Lai (1993) examine the problem using response surface methods and find that the Reinsel-Ahn method of correcting for finite sample effects is in fact not as efficient as using their method. They suggest an adjustment to the critical values rather than the test statistic using a scaling factor:

$$CR_T/CR_\infty = SF = T/(T - nk) \quad (A1.4)$$

where  $CR_T$  is the critical value for sample size  $T$ , and  $CR_\infty$  the asymptotic critical value and  $SF$  the scaling factor. They estimate a response surface equation of the form:

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<sup>5</sup> This is the method of correction for degrees of freedom used in PC-FIML 8.0 Doornik and Hendry (1994) which was the estimation package used for most of the econometric work in this thesis.

$$CR_{Tj}/CR_{\infty} = \beta_0 + \beta_1 SF_j + \text{errors} \quad (\text{A1.5})$$

and note that the Reinsel-Ahn correction implies  $\beta_0 = 0$ ,  $\beta_1 = 1$  whereas the asymptotic restrictions on the scaling factor simply require  $\beta_0 + \beta_1 = 1$  (ensuring that as  $T \rightarrow \infty$  then  $CR_{Tj}/CR_{\infty} \rightarrow 1$ ). Table 1 of their paper provides parameter estimates for  $\beta_0$  and  $\beta_1$  for both the trace and maximal eigenvalue test at the 5% and 10% levels. Their results suggest that the Reinsel-Ahn restriction is not supported by the data and that their response surface equations provide a more efficient method of correcting for finite sample bias. This bias leads to over rejection of the no cointegrating hypothesis too often in the Johansen framework and is an increasing function of  $n$  and  $k$ . Other results in the Cheung and Lai (1993) paper suggest that the Johansen procedure is biased in favour of finding cointegration too often when the lag length of the VAR is too short, and they suggest that standard tests for lag length selection help avoid such bias. They also find that the trace test is more robust to non-normalities than the maximal eigenvalue test.

## Appendix A2

### DATA SOURCES FOR THE APPLIED ANALYSIS OF LABOUR MARKETS<sup>6</sup>

- **Unemployment rate.**

Total unemployment rate, per cent, seasonally adjusted. Source NIF10s model database, identifier: VNEQ.AN\_RNU.

- **Nominal GNP.**

Gross non farm product at factor cost, \$million, seasonally adjusted. Source NIF10s model database, identifier: VNEQ.AC\_GNF\$.

- **GNP deflator.**

Gross non farm product (expenditure based) implicit price deflator, seasonally adjusted. Source TSS database.

- **Nominal wages.**

Average earning of non farm wage and salary earners, \$ per week, seasonally adjusted. Source NIF10s model database, identifier: VNEQ.AC\_WAR\$.

- **Consumer Price Index.**

Consumer price index, weighted average of the eight capital cities, 1989/90=1. Source NIF10s model database, identifier: VNEQ.UI90\_PCPI.

- **Population.**

Civilian population, aged 15 years and over, persons, 000's. Source NIF10s model database, identifier: VNEQ.UN\_NAP.

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<sup>6</sup> Data from the NIF10s model database and the TSS database were taken from DX database supplied by Econdata Pty Ltd.

- **Labour Force.**

Total civilian labour force, persons, 000's, seasonally adjusted. Source NIF10s model database, identifier: VNEQ.AN\_NLF.

- **Real GNP**

Nominal GNP deflated by the GNP deflator.

- **Real wage.**

Nominal wage deflated by the CPI.

- **Productivity.**

Calculated as  $\frac{\text{REAL GNP}}{\text{EMP}}$  where

**EMP** is the non farm civilian wage and salary earners, persons, 000's, seasonally adjusted. Source NIF10s model database, identifier: VNEQ.AN\_NNC.

- **Ratio of long term to total unemployment**

This is the ratio of long term unemployment to total unemployment (seasonally adjusted) where long term unemployment is defined as those unemployed over 52 weeks. Source Australian Bureau of Statistics catalogue nos. 6203, 6204.

I am grateful to Associate professor P. Lewis of Murdoch University who supplied this data.

- **Unemployment benefits.**

Weighted average quarterly unemployment benefit rate, \$000's. Source NIF10s model database, identifier: VNEQ.UK81\_TUBR.

- **The tax wedge**

This is the wedge between real labour costs and real take home pay defined as:

$$\log(\text{wedge}) = \log \frac{W(1+t_e)}{P} - \log \frac{W(1-t_y)}{P_c} = \log(1+t_e) + \log \frac{P_c}{P} - \log(1-t_y)$$

where  $P_c$  is the consumer price index,  $P$  the GNP deflator,  $t_e$  the employers tax rate and  $t_y$  the income tax rate.

- **Employers tax rate**

This is calculated as  $\frac{TPR\$ + FBT\$}{YWN\$}$  where:

**TPR\$** is total payroll tax payments (\$million, seasonally adjusted), source NIF10s-model database.

**FBT\$** is fringe benefits tax payments (seasonally adjusted), source Australian Bureau of Statistics cat. No. 5206.

**YWN\$** is wages, salaries and supplements, non farm (\$ million, seasonally adjusted), source NIF10s-model data base.

- **Income tax rate**

This is calculated as  $\frac{TPAY\$ - TREF\$}{YWN\$}$  where

**TPAY\$** is Gross PAYE instalments (\$million, seasonally adjusted), source NIF10s-model database.

**TREF\$** is refunds on PAYE instalments (\$million, seasonally adjusted), source NIF10s model database.



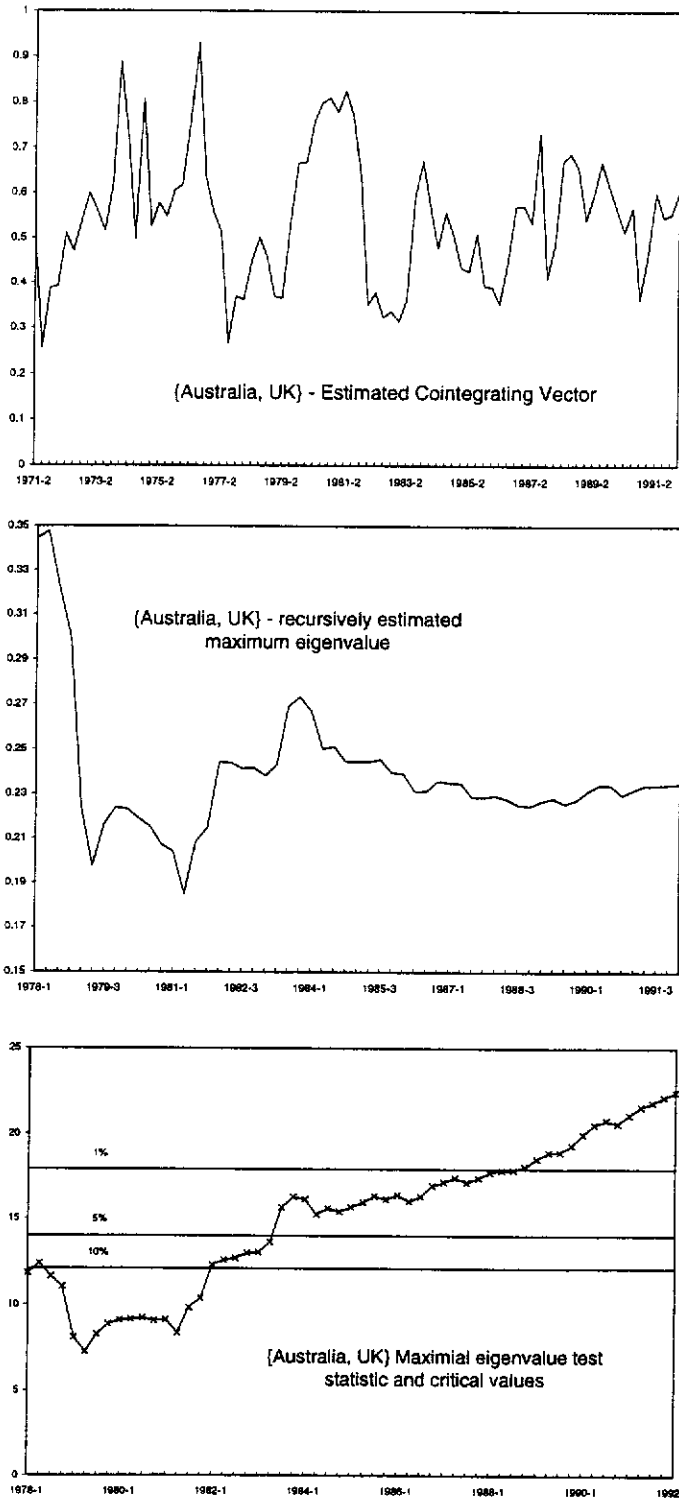
## Appendix A3

### PLOTS FROM CHAPTER 8

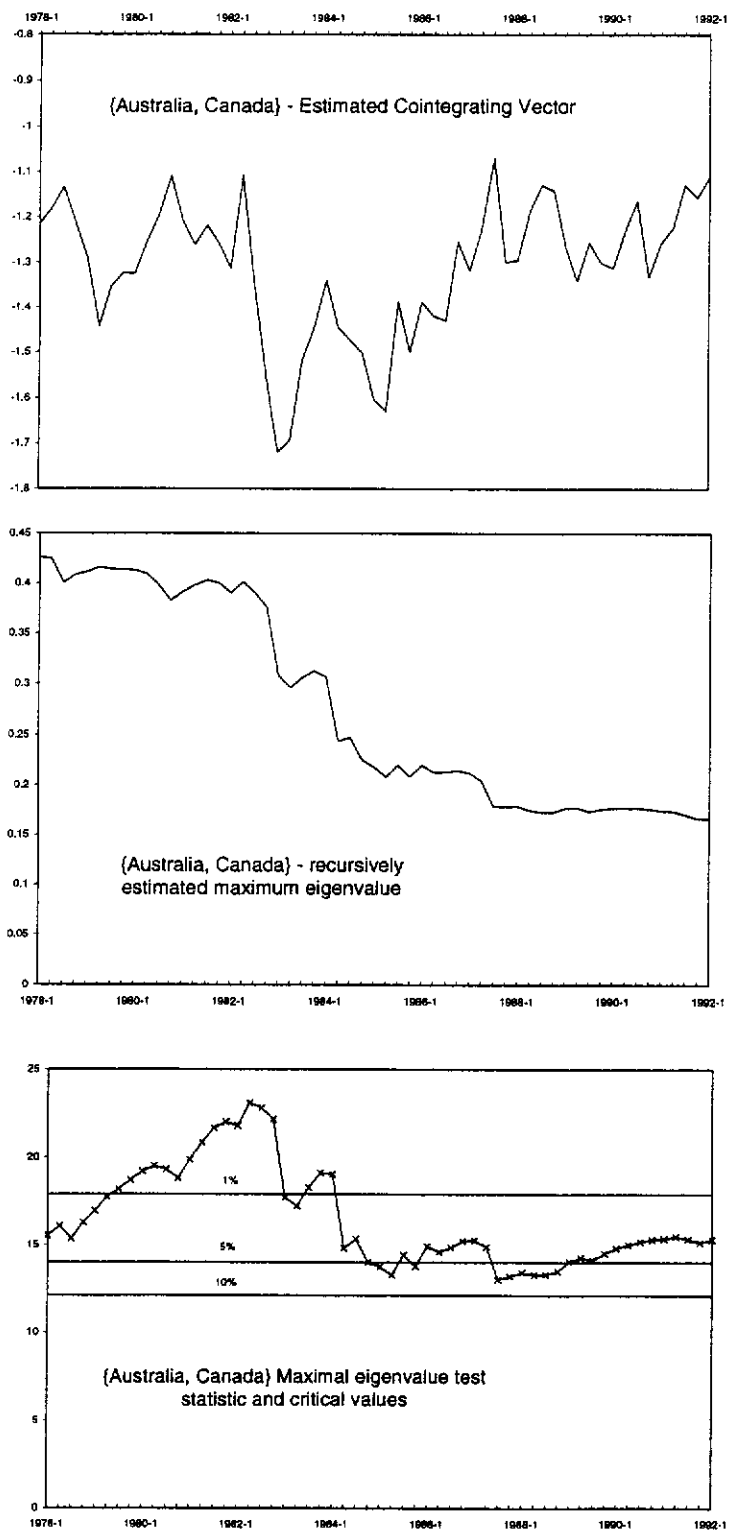
The first section of this Appendix contains plots of the estimated cointegrating vectors, the recursively estimated eigenvalues and the test statistic for the null of zero cointegrating vectors for the portfolios estimated in Chapter 8.

The second section contains plots of the estimated common trends, again from Chapter 8.

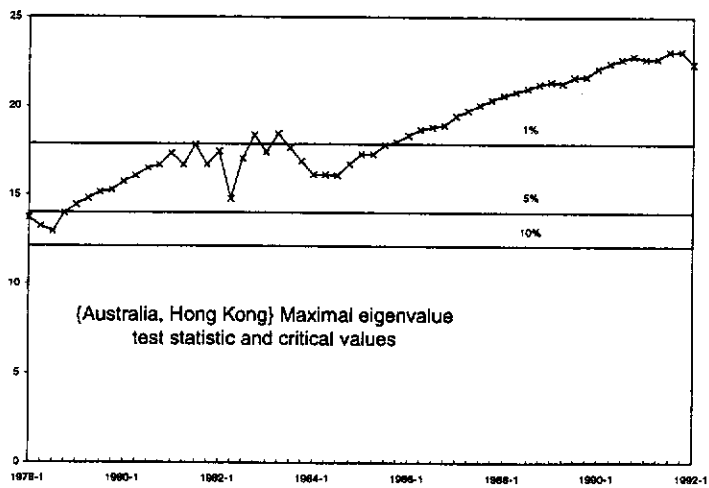
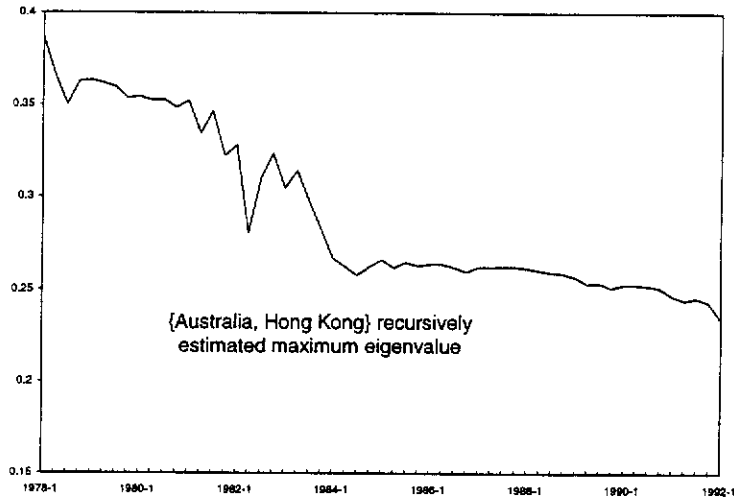
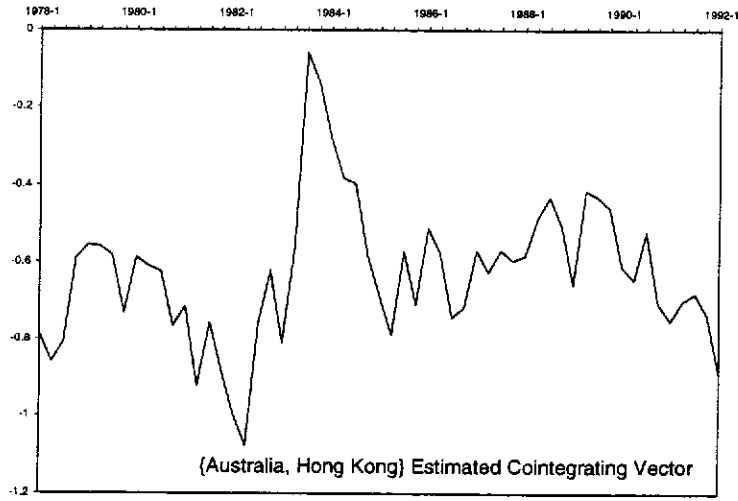
**Figure A3.1 Portfolio: {Australia, UK}**



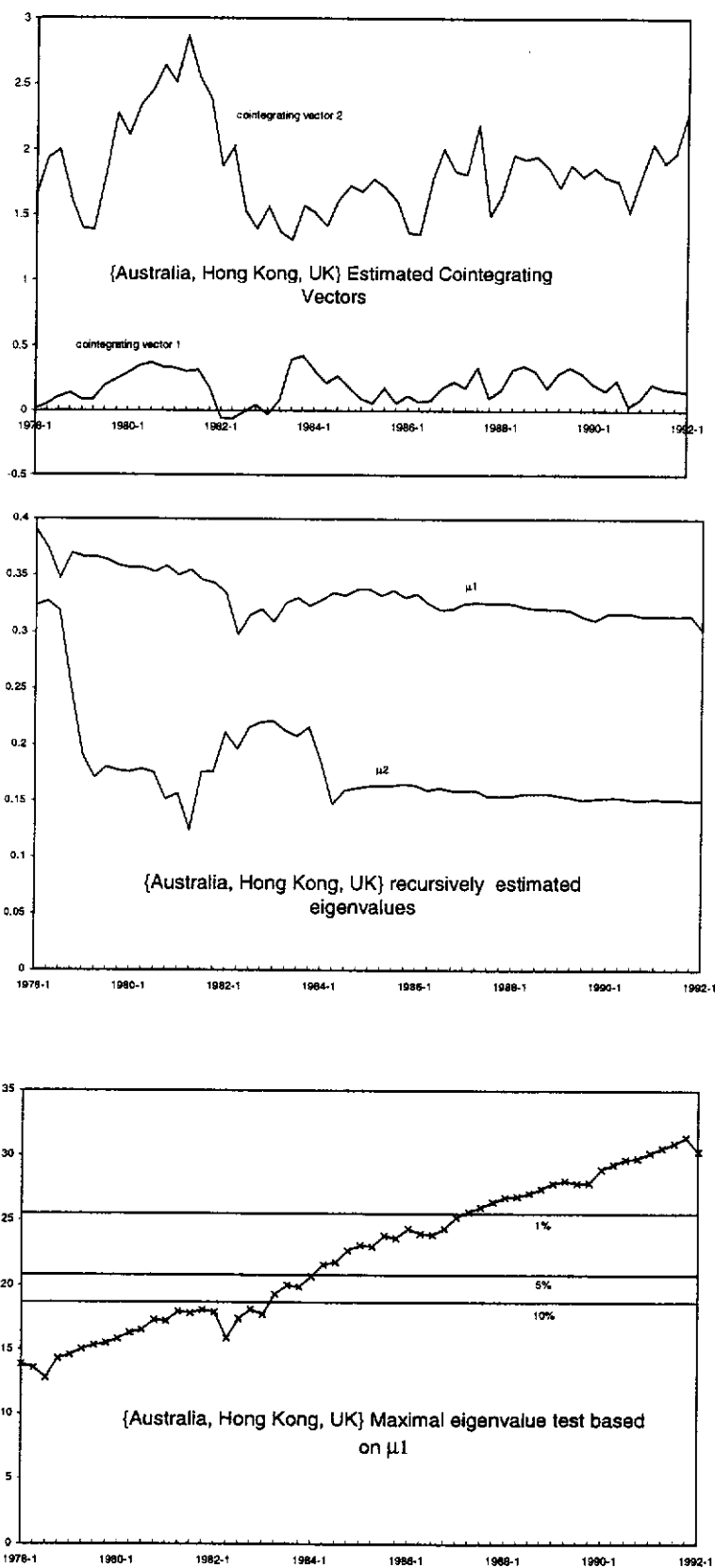
**Figure A3.2 Portfolio: {Australia, Canada}**

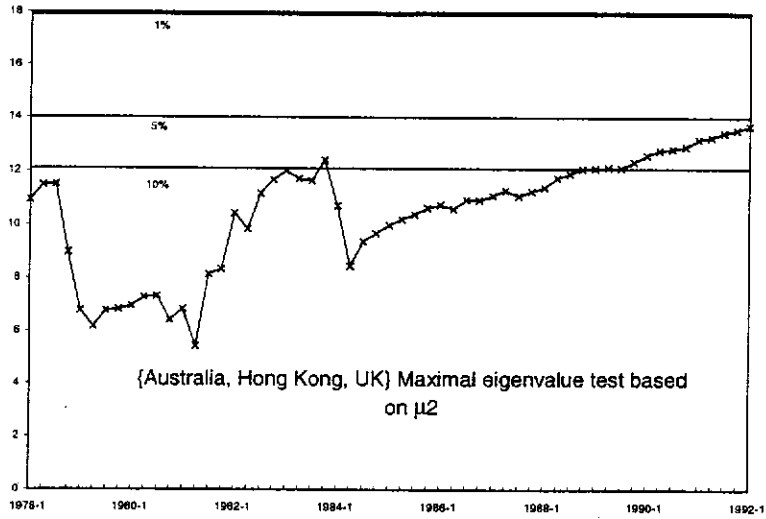


**Figure A3.3 Portfolio: {Australia, Hong Kong}**

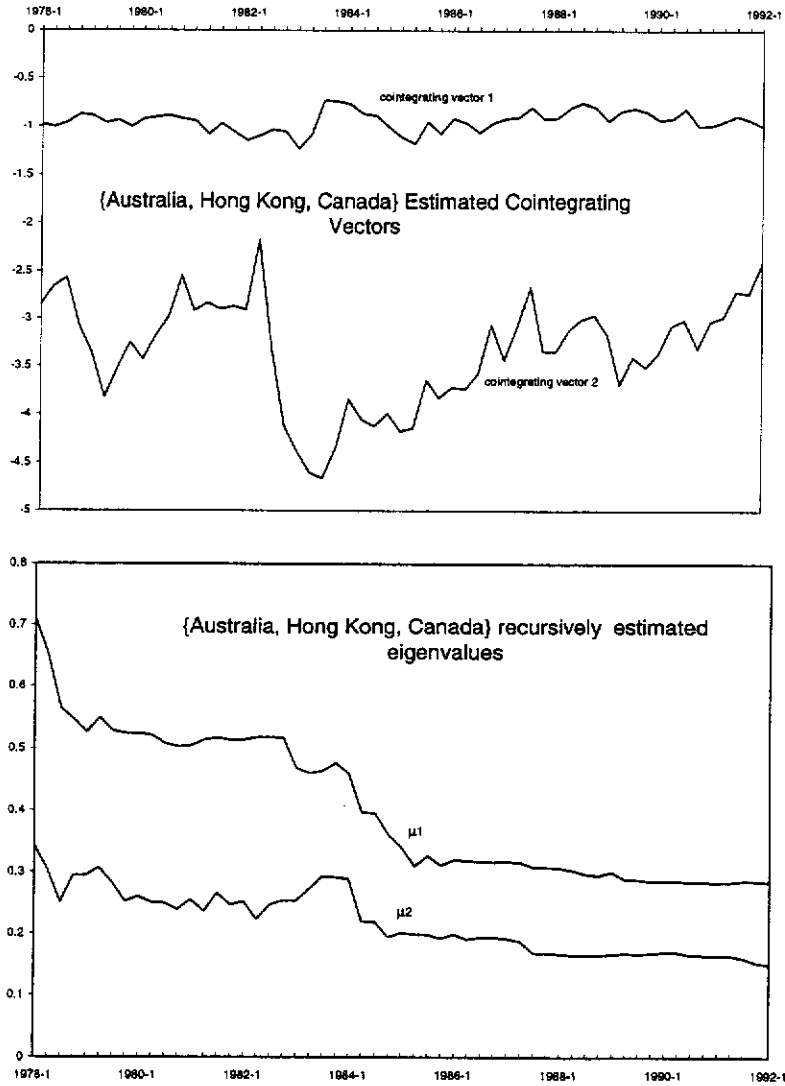


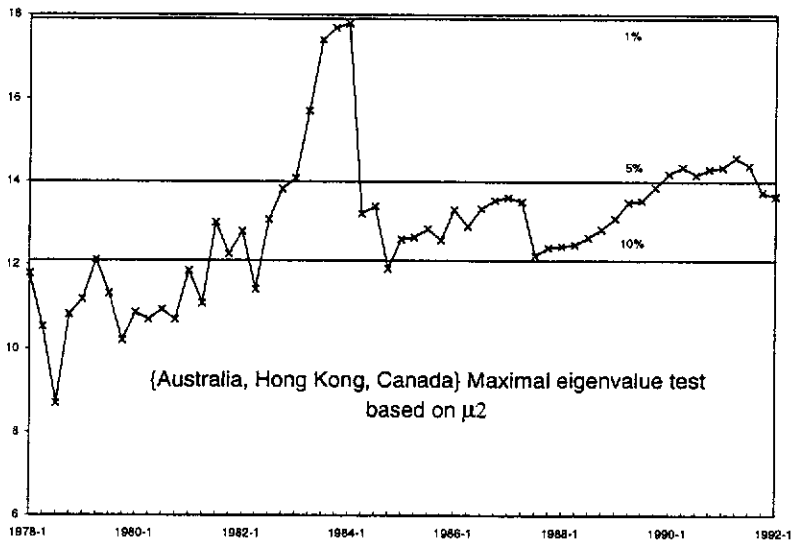
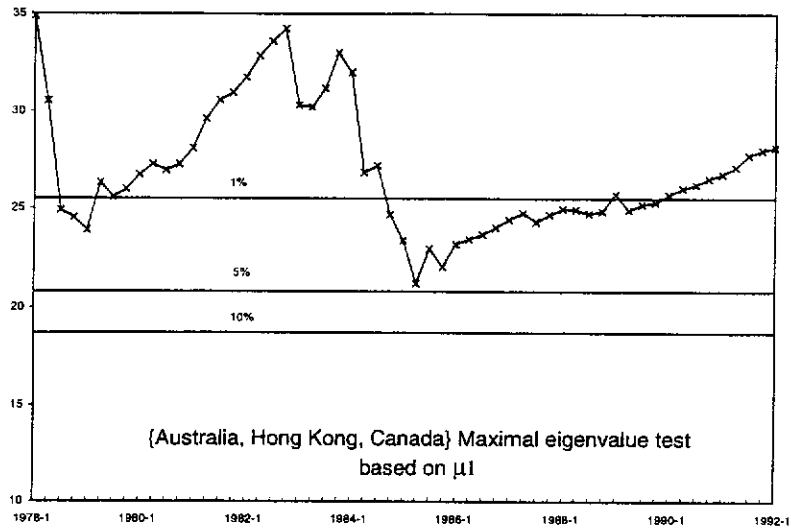
**Figure A3.4 Portfolio: {Australia, Hong Kong, UK}**



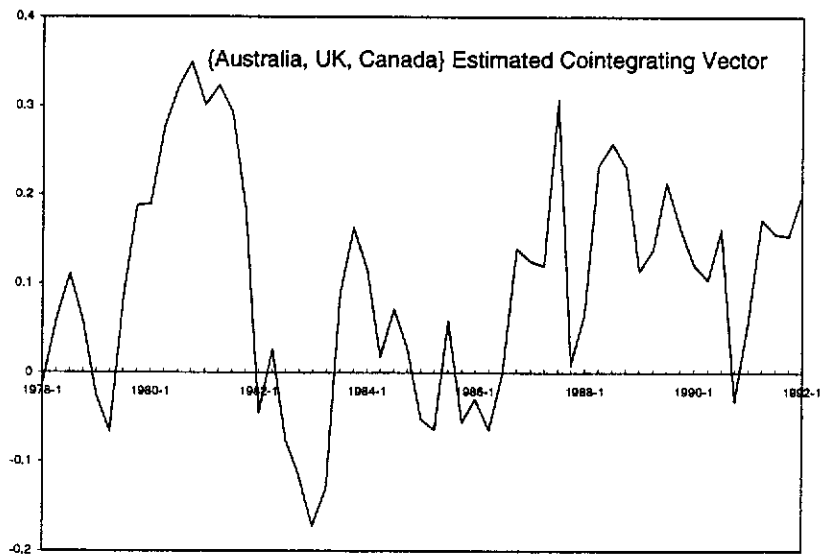


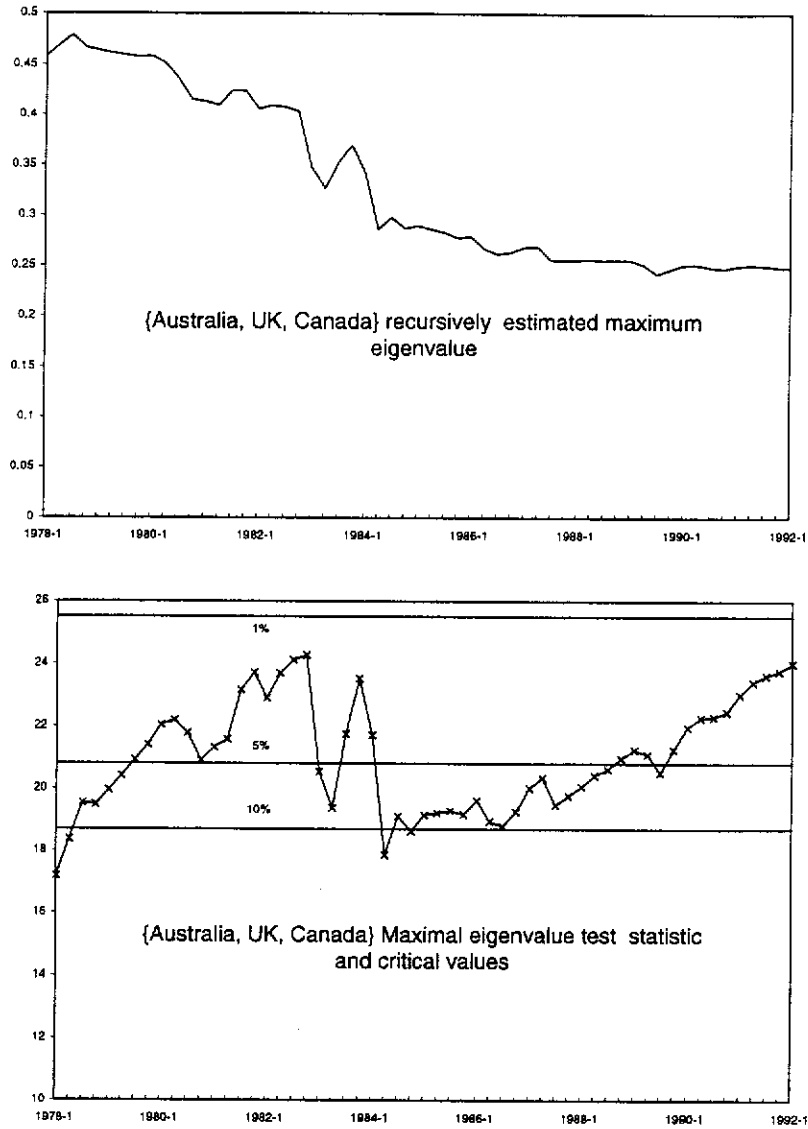
**Figure A3.5 Portfolio: {Australia, Hong Kong, Canada}**



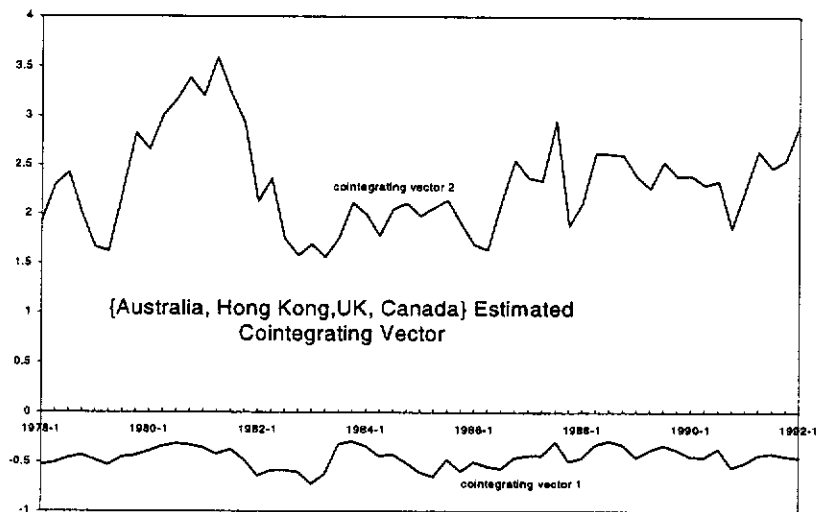


**Figure A3.6** Portfolio: {Australia, UK, Canada}

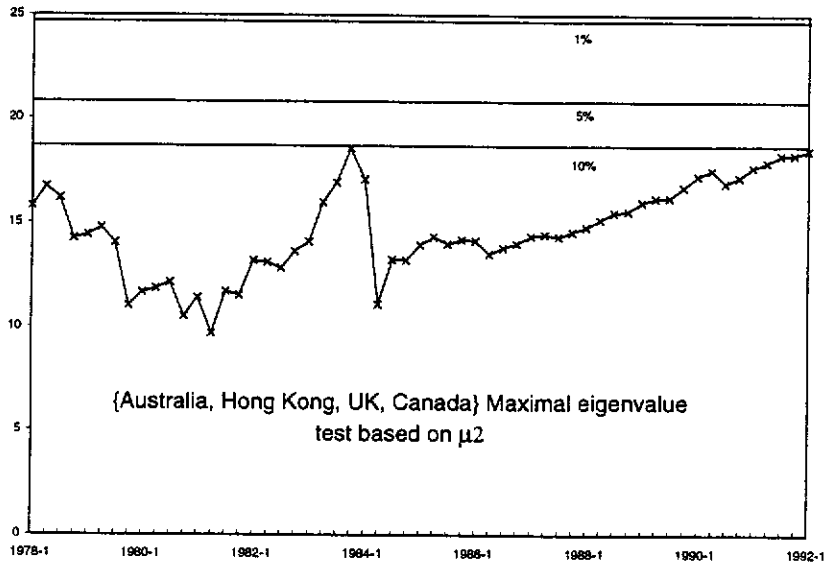
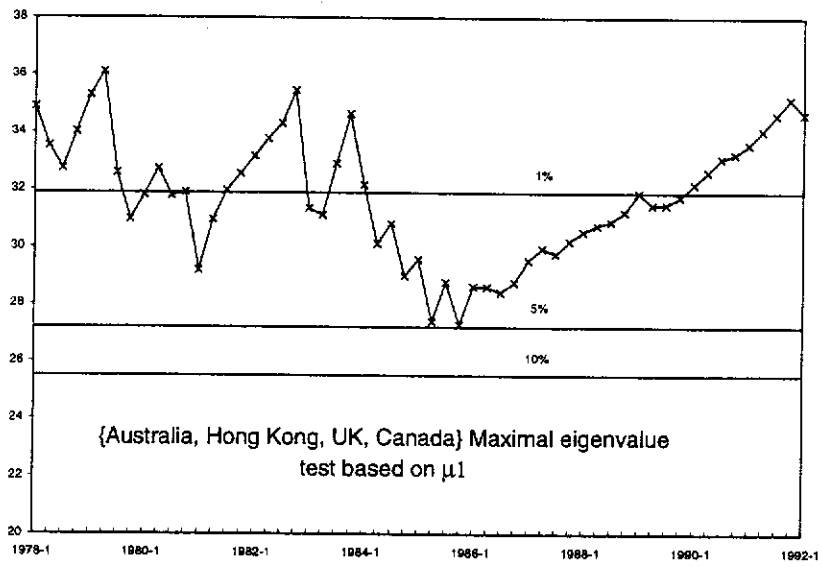
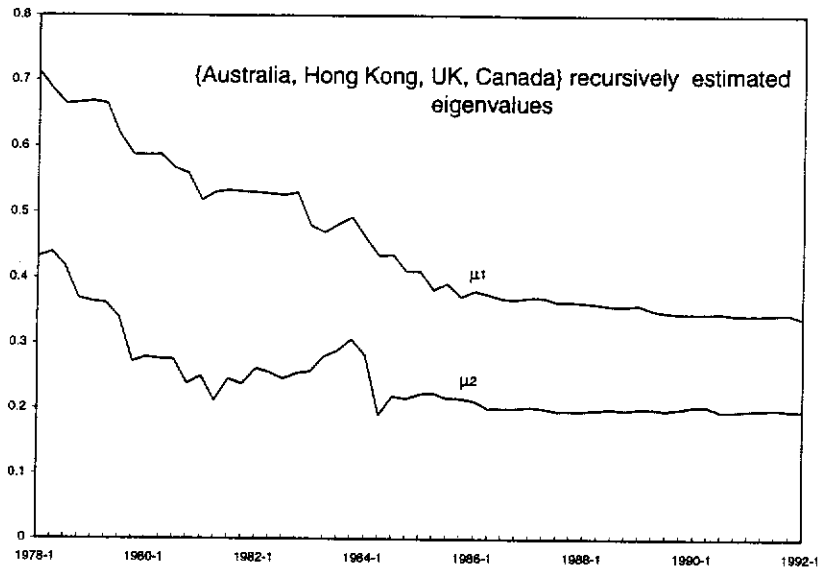




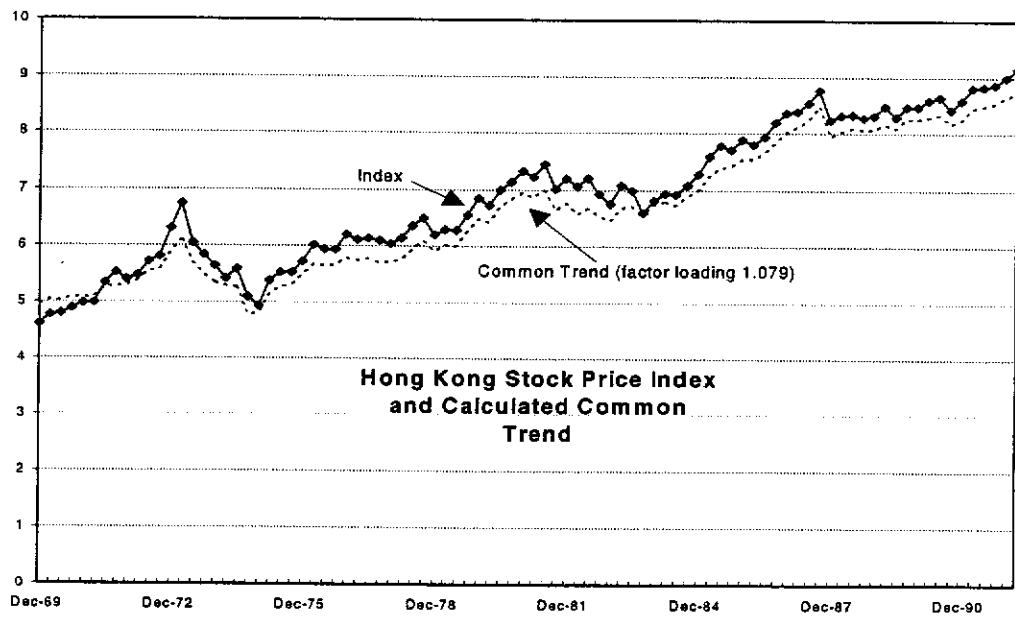
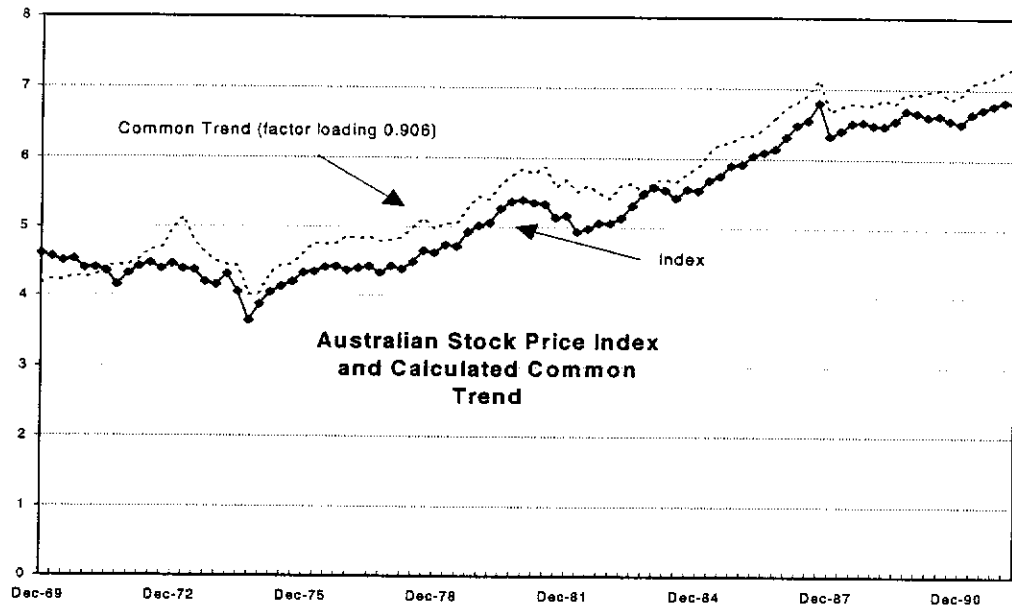
**Figure A3.7** Portfolio: {Australia, Hong Kong, UK, Canada}



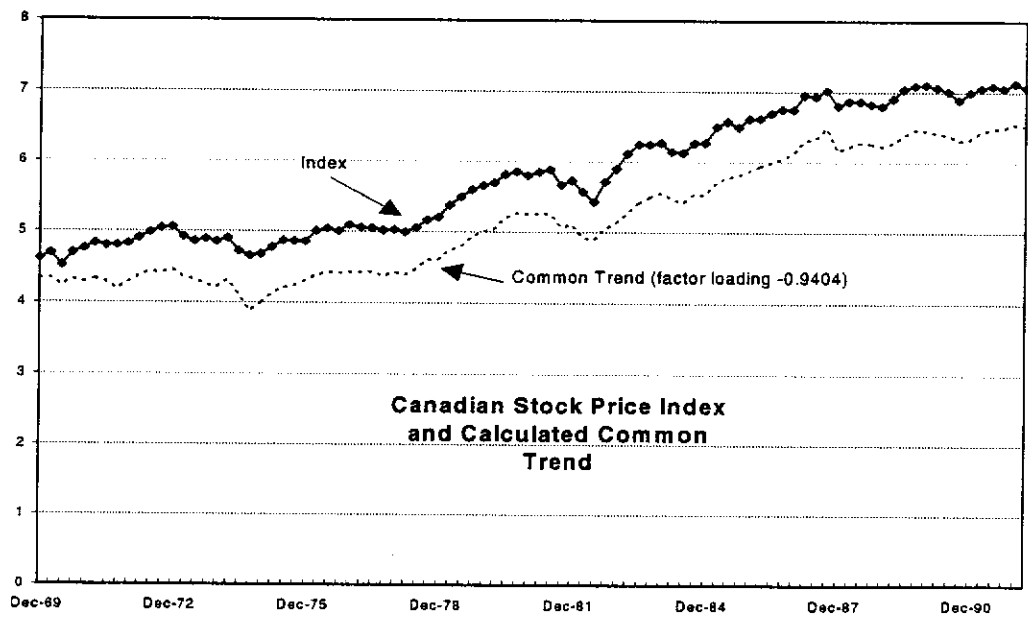
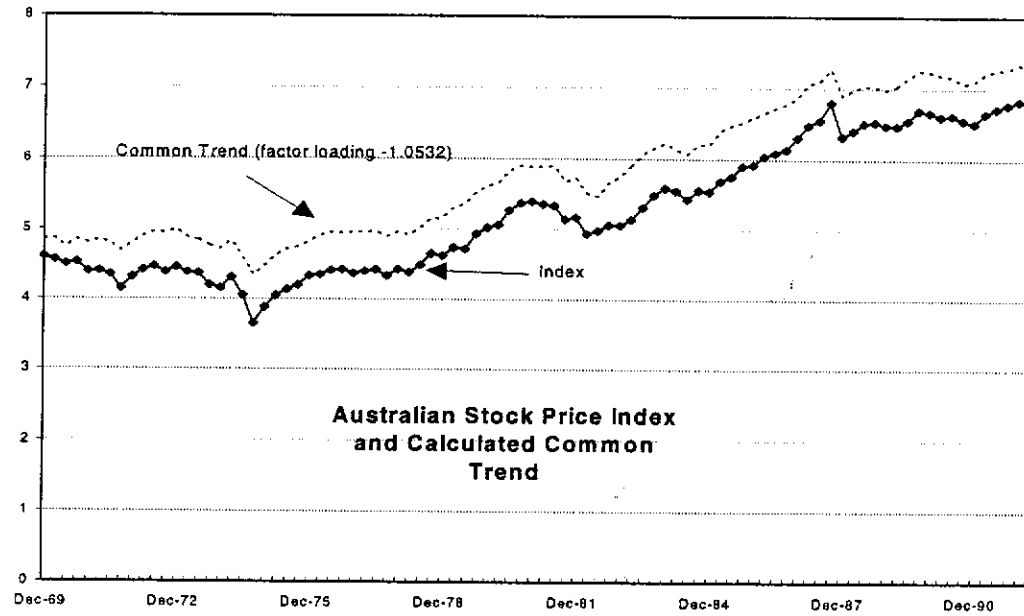




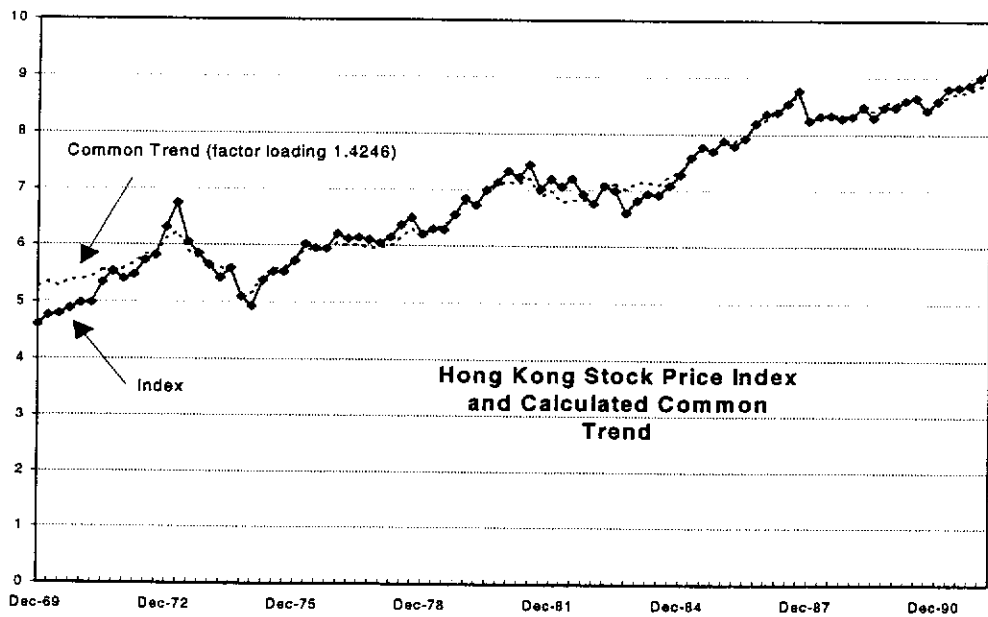
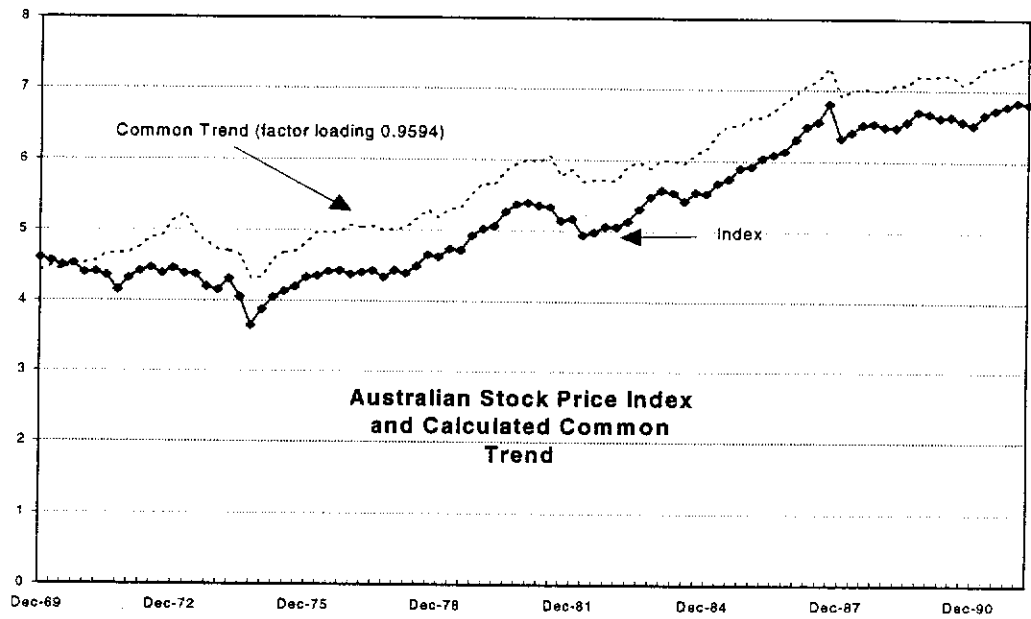
**Figure A3.8 Indices and common trend, bivariate portfolio {Australia, Hong Kong}**

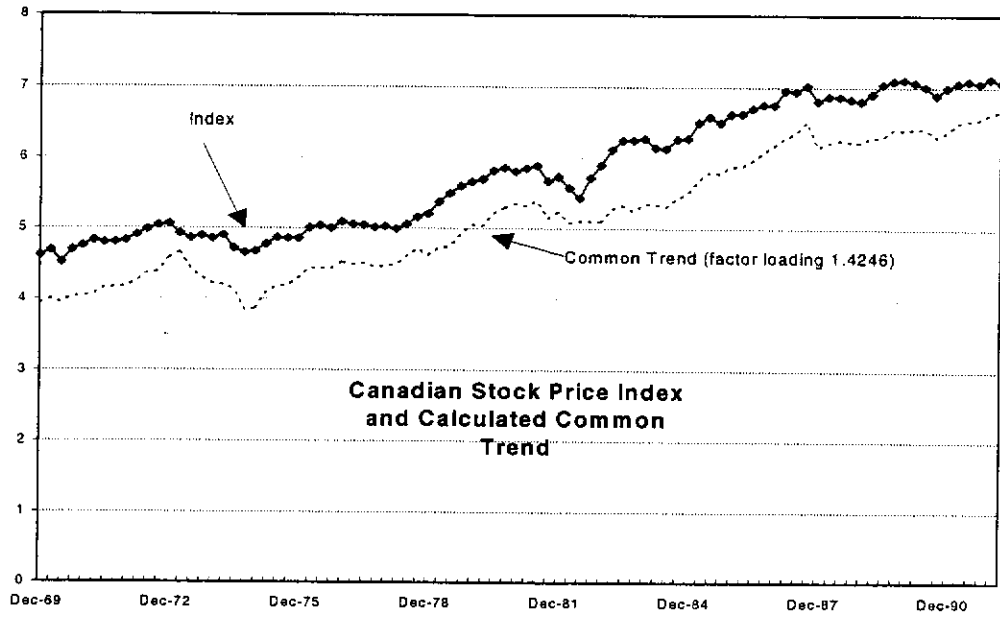


**Figure A3.9 Indices and common trend, bivariate portfolio {Australia, Canada}**

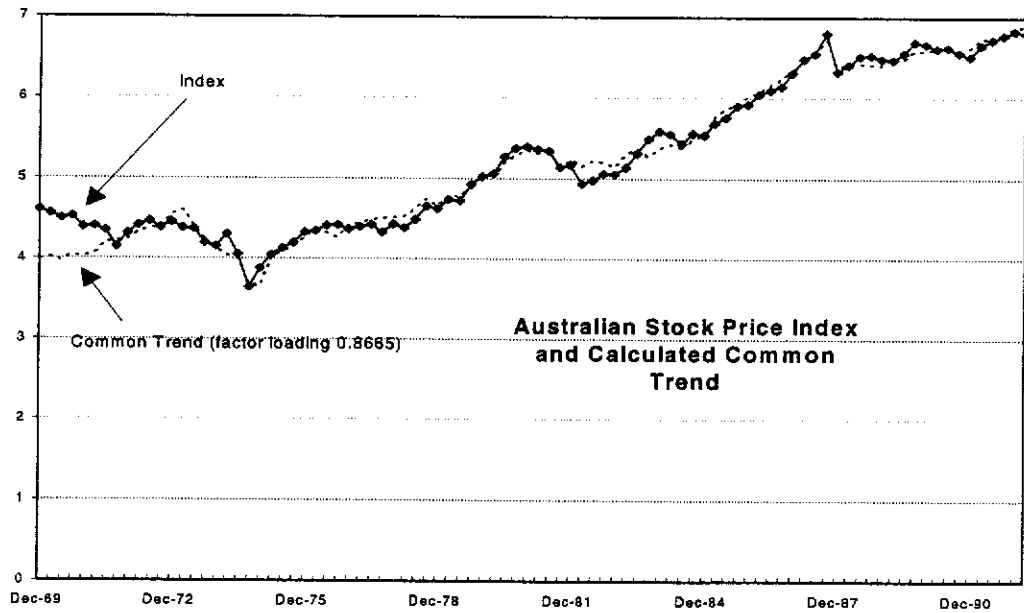


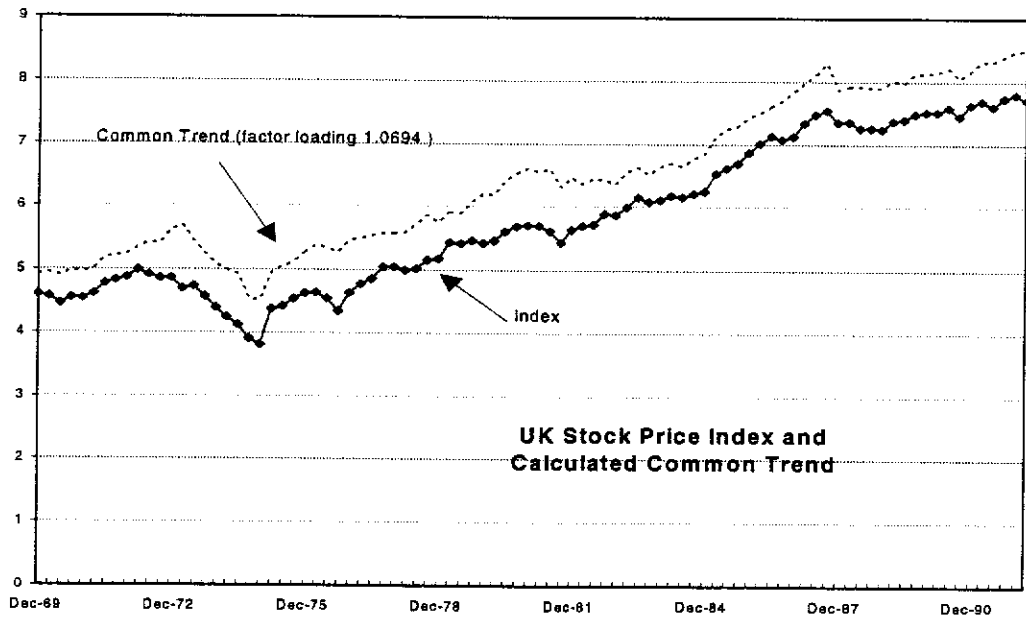
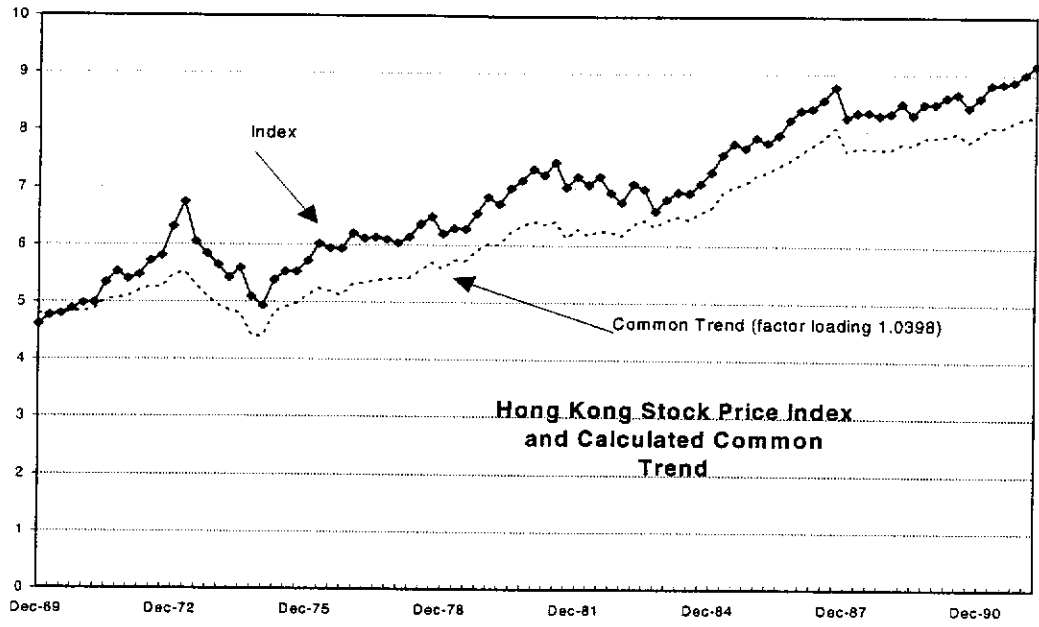
**Figure A3.10** Indices and common trend, trivariate portfolio  
{Australia, Hong Kong, Canada}





**Figure A3.11** Indices and common trend, trivariate portfolio  
{Australia, Hong Kong, UK}





## Appendix A4

### DATA SOURCES FOR THE APPLIED ANALYSIS OF FINANCIAL MARKETS

The data used in Chapter 9 were as follows.

- Monthly all ordinaries share price index. The share price index was to quarterly by taking the first month of quarters figure. data were taken from the DX data, Reserve Bank of Australia, database, identifier: FSMASXSPAQ.
- Quarterly data on dividend yield. Again this data was taken from the DX database, NIF-10 model database, identifier UNEQ\_UN\_RDV and is consistent with the unweighted dividend yield series originally published in the Australian stock Exchange Journal. The data is the average for the three months for the quarter.
- CPI, average of eight capital cities, quarterly data from DX database, NIF-10 model database, identifier UNEQ\_UI90\_PCPI, with 1989/90 = 100.
- The dividend yield and the share price index were used to construct the dividend series used in the paper and all data were converted to real using the CPI series.

NB beginning of months share price index and monthly average dividend yield were initially used as best representing the assumption of an efficient market. However, as noted in the text the spread variable is reconstructed to be prices minus dividends lagged one period in order to relax this assumption.