

Science and Mathematics Education Centre

**Using Productive Pedagogies as a framework for Promoting the
Quality Teaching of Omani Mathematics Teachers**

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Doctor of Philosophy

Of

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DECLARATION

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Signature:

A handwritten signature in black ink, appearing to be 'G. J. S.', written in a cursive style.

Date: 1 July 2013

ABSTRACT

Enhancing the quality teaching has become a current global concern of current reforms in mathematics teaching points to the role of teachers and their pedagogies in supporting all learners to achieve intellectual and social outcomes. Productive Pedagogies provides a comprehensive and multi-dimensional framework with potential for enhancing the quality of teaching and the quality of teacher education.

The focus of this study is to investigate the introduction of Productive Pedagogies to a group of Omani mathematics teachers as a framework for quality teaching and reflection by teachers on their practice. In particular, this study aimed to investigate the development of teachers' understanding and their ability to implement the framework in their mathematics classrooms. It also aimed to determine the benefits and challenges experienced by those teachers and to explore students' perceptions on the change in pedagogy in their mathematics classrooms. The appropriateness of implementing the framework in the educational system of Sultanate of Oman was also examined.

This study is a qualitative research informed by grounded theory methodology. It took place during semester two for the academic year 2011/2012. For addressing these aims, three phases in the research (preparation, implementation and dissemination) were designed. In the preparation phase, the Productive Pedagogies framework was introduced to the participating teachers within a five day professional development program. The actual implementation in mathematics classrooms was followed and explored in the implementation phase during six cycles of 2 weeks. The dissemination phase offered opportunities for teachers in their school- groupings to make presentations about their experiences at a school-based professional development activity attended by other teachers in the region.

The study revealed that, overwhelmingly, mathematics teachers developed new understanding and ability to implement Productive Pedagogies in their mathematics teaching within three stages: *preparation*, *developing* and *consolidation*. In the *preparation* stage, the teachers prepared for their learning during their participation in the professional development program in collaboration with each other and the

researcher. The second stage of *developing* showed that teachers were gaining some confidence and an understanding of Productive Pedagogies and ability to introduce them in their teaching with diminishing support. During the third stage of *consolidation*, the teachers worked more autonomously and were able to defend their decisions using the dimensions of the framework. During their experience of the implementation, the teachers' practices showed an improvement to supporting their role as facilitators of students' learning. They identified the value of Productive Pedagogies as a guide that guides their preparation of mathematics lessons and informs the underlying basis of future development in their teaching practices. However, they pointed to some of the challenges that limited their best efforts to implement Productive Pedagogies. Moreover, the participants raised some theoretical questions around the meaning of the pedagogical element of *cultural knowledge* (an element of working and valuing difference dimension) to the Omani educational context. Reflecting on the main findings, this study makes some recommendations and suggestions for future research in teaching education and school professional development.

In the name of Allah, the Most Beneficent, the most Merciful

DEDICATION

This thesis is dedicated to my
lovely parents who have supported me all the way since the beginning of my studies;
dear husband who has never left my side and has been a great source of motivation
and inspiration;
wonderful children, Bayan, Alwarith, Roaa and the little lovely one, Alhawari who
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CHAPTER ONE

INTRODUCTION

1.1 Statement of the Problem

Enhancing quality teaching to respond to the needs of today's classrooms and communities' complexities has become a current global concern that points to the crucial role of teachers in reforming educational practices (Goodwin, 2010). Current educational reforms are based on the crucial role that teacher education and school professional development play in enhancing the quality of teaching and learning to support learners to be more productive and function in a globalized society (Chinnappan, 2008; Gore, Griffiths & Ladwig, 2004; Hayes, Mills, Christie & Lingard, 2006). As a result, the current call of supporting the quality of teaching deserves an investigation into the means to make the quality of teaching a reality. There is a demand for more conceptual, coherent and flexible enactments of teaching rather than new instructional methods (Goodwin, 2010; Hollins, 2011). There is also a need for best teacher preparation programs that support teacher learning and teaching (Goodwin, 2010; Grossman & McDonald, 2008; Hollins, 2011). Furthermore, there is a necessity to experience the new ideas of the theoretical knowledge that are central to the quality of learning and teaching in professional communities (Goodwin, 2010; Hollins, 2011). Carlgren, Handal and Vaage (1994) had reiterated that teaching practices face the problem of lacking in quality and effectiveness. The authors argued that strengthening pre-service teacher education programs to improve the quality of teaching and student learning is one way to deal with the problem of "poor quality of teaching". Equally important, schools should establish high quality programs for in-service teachers in order to enhance their current teaching performances. Gallimore and Stigler (2003) claimed that the quality of teaching that is needed to produce expected student outcomes depends on effective teacher professional development programs that are relevant to the real context of classrooms.

Understanding of what quality teaching looks like in the specific area of school mathematics teaching in order to produce the desirable outcomes for all students is a

challenge (Walshaw & Anthony, 2008). Zevenbergen, Niesche, Grootenboer and Boaler (2008) suggested that new forms of classroom pedagogy are needed to enable success in learning school mathematics. The development of new perspectives of the pedagogical practices in mathematics classrooms are a result of many current demands. First, the idea that mathematics knowledge is socially shared and that learning should be represented as a social practice in diverse classrooms has resulted in extensive demands on improving the pedagogical practices in mathematics classrooms (Boaler, 2000). Secondly, the process of moving pedagogical practice from its traditional view of transmitting mathematics knowledge that is value-free to the pedagogical practice that intellectually and culturally values mathematics knowledge opens a space for examining and improving the background that frames the mathematical practices in the classroom (Lerman, 2000). Thirdly, the reality that many mathematics teachers struggle to find pedagogical strategies designed to move away from the models of transforming teaching (Lerman, 2000; Rodrigues, 2004) promotes a rethinking of providing teachers with guidance for their practice (Jaworski, 2006; Loucks-Horsley, Love, Stiles, Mundry & Hewson, 2003).

Recent studies in the research literature point to the failure of embedding continuing professional development into the culture of schools and the absence of effective preparation programs in mathematics education as a continuing concern in many countries (Chinnappan, 2006, Fields, 2006; Gore et al., 2004; Hayes et al., 2006). Hollins (2011) had commented that recognising and providing professional learning support and practices to construct an understanding of how to facilitate learning in complex classroom contexts is a challenge for teacher preparation for quality teaching. As mentioned by Loucks-Horsley et al. (2003), teacher professional development is influenced by many critical factors that may add challenges to designing effective mathematics professional development programs. They have identified the constraints of time, of access to equitable professional development opportunities, the development of sustainable professional communities among teachers and generating public support in improving mathematics teaching and learning. Despite all the challenges, one of the valuable theoretical positions from which to view reform in mathematics teaching is the notion that teaching is an ongoing learning process within school community (Henniger, 2004; Jaworski, 2006). As a result of this position, teacher professional development has become the

driving force for introducing and sustaining quality pedagogy and an avenue for sharing ideas, reflecting and collaborating on work amongst teachers in the school community (Borko, Jacobs, Eiteljorg & Pittman, 2008). The idea of school as a learning community focuses on its role of providing ongoing teacher professional learning and collaboration to engender teachers' professional development, improve classroom practices and foster students' learning and engagement (Ailwood & Follers, 2002; Borko et al., 2008).

Although quality pedagogy is not defined as a particular teaching method, the core of quality lies in the interaction between the notions of intellectual depth, relevance and supportiveness (Hayes et al., 2006). Productive Pedagogies has received extensive acceptance amongst the research community of Australia and internationally as it promotes the vision of high quality intellectual and social outcomes for all students (Mills et al., 2009). The framework of Productive Pedagogies argues that "good social outcomes are more likely to be achieved by classroom practices that are intellectually demanding, connected to the students' worlds beyond schools and socially relevant" (Hayes et al., 2006, p. 37). Productive Pedagogies can be used as a professional development program for teachers who desire to achieve a significant change in their classroom practices from their preparation stage of the lesson to the final evaluation stage (Gore et al., 2004). Chinnappan (2008) mentioned that recent research work about mathematics pedagogy, mathematics teachers and mathematics educators supports the traits of Productive Pedagogies. Chinnappan argued that mathematics teachers could draw on the various dimensions of Productive Pedagogies to evaluate their classroom practices, construct a sequence of mathematics learning experiences, utilise the practices that could motivate learners to extend their mathematical construction in different directions and to provide a holistic meaning of the nature of mathematical knowledge.

This study is an attempt to enhance the quality teaching of Omani mathematics teachers by introducing Productive Pedagogies to a group of in-service mathematics teachers and then investigating their development of understanding and their ability to implement it in their classrooms. This study also aims to investigate students' perceptions of the change in pedagogy in their mathematics classrooms and to examine the appropriateness of implementing the Productive Pedagogies framework in the educational system of Oman.

1.2 Productive Pedagogies Framework

The Productive Pedagogies framework is a recent attempt to research and reform pedagogy in Australian schools that was developed by the Queensland School Reform Longitudinal Study (QSRLS). This initiative was built upon the work of Authentic Pedagogy research by Newman and his colleagues in extending the emphasis on quality and diversity of classroom pedagogy as a basis for improving students' intellectual and social outcomes (Hayes et al., 2006; Lingard, Hayes & Mills, 2003; Sellar & Cormack, 2009). The concept "Productive" is an indication of the production of learning outcomes in the classrooms (Hayes et al., 2006, p. 21). The term "Pedagogy" reflects the integration between the different aspects of teaching as science, art and practice (McLeod & Reynolds, 2007, p. 44). It refers to "the central expression of humanity in general and the professional identities and practices for teachers in particular" (Hayes et al. 2006, p. 21; Lingard, 2005, p. 172). The concept of "Pedagogies" reflects the description of the range of various texts on classroom practices rather than the many examples of teaching strategies (Hayes et al., 2006, p. 77).

The Productive Pedagogies framework emphasises the centrality of teachers in improving the academic and social outcomes of all students. It provides a useful lens to analyse and examine the richness, complexity and other aspects of classroom experiences. While the concept of Productive Pedagogies focuses on the role of teachers and pedagogies to provide quality classroom practices, it emphasises that the responsibility of the quality must be communal, involving teachers, school administrators, education organizations and local communities (Lingard et al., 2003). The Productive Pedagogies framework has been used in pre-service and in-service teacher education programs as a comprehensive framework and multidimensional construct for quality teaching model and as a means for teachers to reflect on their pedagogical practices, to inform the design of quality learning experiences and to enter into dialogue with the community of teachers about issues related to teaching and learning (Atweh, 2007a; Education Queensland, 2010b; Gore et al., 2004).

Productive Pedagogies dimensions

The dimensions of the Productive Pedagogies framework, namely, intellectual quality, connectedness, supportive classroom environment, and working and valuing

difference (Hayes et al., 2006), express the meaning and value of what “quality teaching” might look like and provide a descriptive language to support and engage teachers with sustained professional dialogue about their practices and performances in order to provide “critical friends’ comments” (Atweh, 2007a, p. 13; Aveling & Hatchell, 2007; Education Queensland, 2010b; Hayes et al., 2006). These four dimensions can provide teachers with a snapshot of their classroom practices that should be present to ensure that the intellectual and social outcomes of all students are improved (Hayes et al., 2006).

Intellectual quality dimension

The focus on high intellectual quality is necessary for all students to perform well academically across the curriculum. The intellectual quality dimension of the Productive Pedagogies framework stresses the importance of providing students with intellectually challenging work including engaging them in higher order thinking operations as well as sustained conversational dialogue among students, and between teacher and students to negotiate understanding of subject matter. The Productive Pedagogies framework argues that achieving high intellectual quality also includes an understanding of knowledge as being socially constructed, establishing relatively complex connections to the central learning concepts, demonstrating a deep understanding of those concepts and promoting high levels of talk and writing within classroom practices (Education Queensland, 2010a, pp. 3-9).

Connectedness dimension

Connected pedagogical approaches make positive difference to students’ attitudes, participation and achievement (Zyngier, 2008). In the Productive Pedagogies framework, the connectedness dimension aims to ensure that students are presented with practical, real, or hypothetical problems that have value and meaning beyond the instructional context and that make a connection to the students’ background knowledge and experience, the other subject areas and the wider social context in which students live (Education Queensland, 2010a, pp. 10-14).

Supportive classroom environment dimension

The supportive classroom environment dimension is based on the understanding that a focus on high intellectual quality and connectedness will not be a sufficient

condition for improved student outcomes, especially for students from disadvantaged backgrounds (Education Queensland, 2010a, p. 8). The dimension of supportive classroom environment emphasizes the importance of supporting students by conveying high expectations to them, applying student-centred learning activities, demonstrating self-regulation and academic engagement and providing students with frequent and detailed statements about their performance (Education Queensland, 2010a, pp. 15-19).

Working and valuing difference dimension

Within the diverse literature on teaching and learning, the emphasis is on recognising and valuing a range of cultures and social groups to help create a sense of community and identity. The dimension of working and valuing difference provides an insight that different cultures are equally valued in all curriculum knowledge, content and form. It encourages the attempts made to ensure that all individuals and groups have rights and responsibilities (Education Queensland, 2010a, pp. 20-24). Under the notion of this dimension, the style of teaching that is principally narrative is encouraged including the use of personal stories, biographies, historical accounts, and literary and cultural texts (Education Queensland, 2010a, p. 17).

1.3 The Context of the Study

This study is located within the context of Basic Education in Omani schools. The Sultanate of Oman is an Asian country which is located in the south-eastern quarter of the Arabian Peninsula and covers a total land area of 309,500 km². (Ministry of Information (Oman), 2011). Oman's educational program expanded from only three formal schools in the whole country in 1970 to 1,300 in 2008 providing education from grades 1 to 12 for over 600,000 students, 48% of whom are female. There are over 43,000 teachers, of whom 89% are Omanis (Ministry of Education (Oman), 2012, p. 39).

In the new educational reform in Oman, particularly the reforms after 1998, the "Basic Education System" (Cycle One (grades 1-4) and cycle Two (grades 1-10) was introduced to replace the former education system, "the general education system" (elementary, preparatory and the first grade of secondary education). It is designed to provide a unified grade 1–10 program for all school-age children and provides

adoption of new objectives for the education system, changes in curriculum content and textbooks, changes in student assessment and changes in the structure of the school system and expectations to achieve better quality outcomes (Ministry of Education (Oman), 2004a, pp. 15- 17)

The calls for quality of teaching

The Sultanate of Oman as any other Arab country in the Middle East, is confronted with the challenges of globalization, expansion of technological information, sustainable economic transformation, and the development of human skills. Therefore, the Ministry of Education in Oman, in the light of the recommendations of the “Vision of Oman’s Economy 2020”, embarked on developing the quality of education in order to support the development of the desired new modern society (Issan & Gomaa, 2010; Ministry of Education (Oman), 2004a, p. 9). The modernization of the Omani society imposes new dimensions on education which require developing the ability of learners to interact rationally with contemporary universal culture and to use scientific critical thinking and modern technology (Ministry of Education (Oman), 2004b). As a result, the role of teachers is expected to shift from teacher-centred approaches to becoming student-centred through a variety of quality teaching and learning approaches (Ministry of Education (Oman), 2008).

Oman is now shifting priorities from educational inputs to a greater emphasis on educational outcomes by providing education opportunities and enhancing service delivery and by focusing on education quality as measured by student learning outcomes (Ministry of Education (Oman), 2012, p. 63). The Ministry of Education in Oman has participated in many international studies to evaluate students’ achievement in mathematics and science such as (TIMSS 2007, TIMSS 2011). In addition, Oman has participated in international studies about evaluating teacher education programs such as the Teacher Education and Development Study in Mathematics (TEDS-M 2008) that emphasizes the central importance of teacher knowledge to quality learning and aims to investigate the development of the main characteristics of teacher education institutions and their programs. On the results achieved by the Sultanate of Oman in the international study TIMSS (2007, 2011) Omani students achieved below the international average in mathematics, science

and literacy. Thus, the Ministry of Education highlights the need of further studies and research regarding the low students' outcomes in mathematics and science (Ministry of Education (Oman), 2012, p. 66). Regarding the results of The Teacher Education and Development Study in Mathematics (TEDS-M 2008), the study recommends that it is necessary to combine theoretical knowledge with practical teaching experience in teacher education (Hsieh et al., 2011).

Moreover, the report of *Education in Oman: The Drive for Quality* (Ministry of Education (Oman), 2012) that was launched in October 2012 and was jointly prepared by the World Bank and Oman's Ministry of Education highlighted that the key challenge facing education in Oman is the improvement of the quality of student learning outcomes and that, enhancing quality should be the Government's main priority in education. The report suggested that "some indications that teacher quality may be weakened by poor pedagogical skills are: (1) practical skills of teaching are allocated a relatively small proportion of the time in teacher training courses, (2) faculty in universities often have limited teaching experience themselves, (3) new teacher induction tends to focus on procedures rather than pedagogical skills, and (4) experienced teachers are often withdrawn from the classroom to serve in supervisory or administrative positions" (p. 139). The report suggested the necessity of building a culture of high standards and developing the capacity of teachers in their teaching practices. The report concluded by referring to the problem of quality education in Oman stating that "Of all the factors affecting student learning that can potentially be influenced by public policy, teaching quality is the single most important one. An effective teaching force with strong pedagogical skills is the key to increasing education quality. Oman has a large teaching force with sufficient qualifications, but new teachers' skills are not developed adequately in pre-service teacher training due to the lack of emphasis on pedagogical skills and the limited use of practical training. Currently, "teacher trainers do not have sufficient classroom experience or practical teaching skills. In-service training should be more practical and responsive to teachers' needs. In school, teachers' work needs to be refocused with more time devoted to teaching and less to reporting and administrative requirements" (p. 25). In the report of *Education in Oman: The Drive for Quality*, (p.139), the following recommendations for policy implications towards quality teaching and learning are reinforced:

- While there is little systematic information on teaching quality, there are indications that more could be done to develop classroom teaching
- Despite the lack of direct evidence from inside classrooms, the combination of high enrolment, high staffing levels and unsatisfactory learning outcomes suggest that pedagogy quality could be strengthened
- Future teacher recruitment should be needs-based and classroom focused
- There is a need to refocus the system on quality, in particular teaching quality. Improving pedagogy quality is likely to involve sustained engagement with teachers, allowing them to put ideas into practice and take ownership of the process

1.4 Research Aims

The overall aim of this study is to investigate the introduction of Productive Pedagogies to a group of Omani mathematics teachers as a framework for quality teaching and reflection by teachers on their practice. In particular, specific research aims were formulated as follows:

- To examine the value of Productive Pedagogies framework on enhancing teaching quality of Omani mathematics teachers. To achieve this aim, this study aims to investigate the:
 - development of Omani mathematics teachers' understanding and ability of implementing Productive Pedagogies;
 - The benefits of implementing Productive Pedagogies by mathematics teachers and
 - challenges experienced by mathematics teachers in the implementation of Productive Pedagogies;
- To examine the appropriateness of implementing Productive Pedagogies framework in the educational system of Oman and
- To investigate students' perceptions of the change in pedagogy in their mathematics classrooms.

1.5 The Significance of the Study

This research is significant for four main reasons. Firstly, this study is the first study that was undertaken in the Omani context for enhancing the teaching quality of teachers in general and specifically for school mathematics teachers by applying the Productive Pedagogies framework. The study offers a new perspective for mathematics educators and supervisors to improve their plans for the enhancement of teacher professional development from a specific and more effective perspective. Offering a new applicable model from a research base may form a productive basis for any future improvement in learning and teaching mathematics in Oman.

Secondly, the objective of this study is in line with the current focus of the Ministry of Education in Oman towards improving the quality of education and the relevance of teaching and learning by focusing on teachers' pedagogy.

Thirdly, before this study was conducted in Oman, studies had already been implemented in several countries around the world that aimed to enhance the quality of teaching by applying the Productive Pedagogies framework. However, few of the studies investigated students' perceptions that resulted from a change in classroom pedagogy. Giving voice to students in this study to express their perceptions on the change in pedagogy in their mathematics classrooms refocuses the attention in the in the process of educational reform on the students.

Finally, the proposed research is a personally rewarding learning experience for me in my capacity as a supervisor of school mathematics teachers in the country. It gives me an opportunity to introduce new specific and research supported professional training programs to the teachers whom I will be working with in the future.

1.6 Overview of Study

Two groups of six Omani mathematics women teachers who taught in Cycle Two Schools (grades 5-12) in the academic year 2011/2012 volunteered to participate in this program. Four teachers taught grade 10 (student age 13 years old) and the other two teachers taught grade 7 (student age 16 years old). A total of 30 students from all the case study classes were interviewed. The 30 students comprised twenty students

(five students from each classroom) from grade 10 classes and ten students from grade 7 classes.

The research design consisted of three phases: preparation, implementation and dissemination. Phase one (preparation) began at the end of semester one when a five day professional development program was conducted by the researcher to introduce the Productive Pedagogies framework to the participating teachers. Phase two (implementation) began at the beginning of semester two. In phase two, the participating teachers began to apply Productive Pedagogies in their mathematics classrooms. The researcher followed the actual implementation in their mathematics classrooms during six cycles (each cycle last about two school weeks). During these cycles, the implementation of Productive Pedagogies, and the development of teachers' understanding and their ability to apply Productive Pedagogies were followed and explored by classroom observations by the researcher and the participating teachers. The researcher and the teachers coded the teaching practices of the observed mathematics lessons using the Queensland Classroom Observation Coding Manual. In addition, group discussions and lesson analyses were conducted regularly after the classroom observations. Furthermore, some interviews with individual teachers were conducted during the implementation cycles while other interviews took place at the end of semester two. Moreover, focus groups were conducted with students at the beginning and at the end of semester two. Phase three (dissemination) offered opportunities for teachers in their school groupings to make presentations about their experiences at school-based professional development activities attended by other teachers in the region.

1.7 Overview of the Thesis

Chapter one is an introduction to the study. It provides a statement of the problem that was researched and the significance of the study. It highlights the research aims and provides an overall review of the background of the study and the structure of the thesis.

Chapter two is the literature review that provides the background for investigating the research objectives and outlines the theoretical framework of this study. This review includes discussions about the issues related to quality learning and teaching. It also provides an overview of specific theories and perspectives on reconstructing

teaching and learning in mathematics. It also discusses the research literature about Productive Pedagogies as a framework of quality teaching and reflection.

Chapter three presents the research methodology employed in this study and focuses on the various aspects about how the research was conducted. It includes outlining of the research aims, reviewing of the research methodology and context. It also explains the research design that was used to investigate the research objectives. It provides discussion of the quality of data and offers a consideration of some ethical issues related to this research. Data analysis is also explained within this chapter.

Chapter four presents the data analysis based on the research aims. The data analysis explains the development of teachers' understanding and their ability in implementing Productive Pedagogies in their mathematics teaching. The chapter also presents data regarding the main benefits of the implementation on teachers' practices and students' learning and engagement and the key challenges that limited teachers' best efforts to introduce the Productive Pedagogies framework in their teaching practices. It also includes a presentation of data concerning students' perceptions about teaching practices in their mathematics classrooms.

Chapter five discusses the findings, draws out the main achievements and explains the results of the specific research aims. This includes a presentation of the limitations of this study and sets out the implications of the findings. It also includes suggestions of directions for future research based on the main overall ideas of the discussions and reflections that are provided in this study.

CHAPTER TWO

LITERATURE REVIEW

The purpose of the present study was to investigate the introduction of Productive Pedagogies to a group of Omani mathematics teachers as a framework for quality teaching and reflection by teachers on their practice. In specific, the research aimed to examine the value of Productive Pedagogies framework on enhancing teaching quality of Omani mathematics teachers. It also aimed to examine the appropriateness of implementing Productive Pedagogies framework in the educational system of Oman and to investigate students' perceptions of the change in pedagogy in their mathematics classrooms. The following review of literature related to this study is organized into nine main sections. Section 2.1 discusses the issue of quality teaching for quality learning. Section 2.2 reviews the professional development of teachers. Section 2.3 discusses the role of specific educational theories and perspectives on reconstructing teaching and learning mathematics. It also discusses the social turn in mathematics education. Section 2.4 reviews the vision of Authentic Pedagogy in teacher education. Section 2.5 discusses the base research and the dimensions of Productive Pedagogies. The research literature about Productive Pedagogies as a framework of quality mathematics teaching will be reviewed in Section 2.6. Section 2.7 illustrates the application of Productive Pedagogies in teacher development. Section 2.8 identifies some challenges of applying Productive Pedagogies. The main issues provided by reviewing the research literature that are related to this study, are summarized in Section 2.9.

2.1 Quality Teaching for Quality Learning

The current view of teaching as complex, demanding and uncertain (Loucks-Horsley et al., 2003) and its purpose to facilitate quality students' learning (McLeod & Reynolds, 2007) call for continuous emphasis on quality teaching. Current reforms point to the role of teachers and their pedagogies in supporting all learners to be more productive and to be able to function in a globalized society (Chinnappan, 2008; Hayes et al., 2006). Teaching is a complex, dynamic and demanding process of creating and organizing learning experiences through an active learning environment

that provides opportunities for all students to achieve quality outcomes (Loucks-Horsley et al., 2003). Henniger (2004, p. 5) mentioned that teaching is “the science” that requires deep knowledge and clear understanding, “the art” that continually requires rethink, reframe and reconfigure, and “the practice that requires considerable skills and efforts”. In considering the contexts in which mathematics becomes entrenched, the practices of the quality school mathematics teaching require a careful consideration of understanding mathematics learning as aspects of participation in social practices and as designs of language, power and effort (Boaler & Greeno, 2000; Zevenbergen, 2000). “Teaching mathematics well is complex endeavour” since it involves knowing and understanding mathematics as a discipline of deep and flexible knowledge, learners’ needs and challenges and pedagogy as a structure of decisions and strategies (National Council of Teachers of Mathematics, 2000, p. 17).

Understanding the quality of student learning is a key element of creating access to quality teaching (Henniger, 2004). The quality of students’ school learning shapes the development of the lifelong knowledge, skills and understanding that help learners to operate effectively in their society (McLeod & Reynolds, 2007). In fact, supporting all students demands high expectations of students’ learning and providing opportunities that support learning challenging content (Loucks-Horsley et al., 2003). Quality learning is also enhanced by ensuring that all students receive educational equity and opportunity for learning. McLeod and Reynolds (2007) claimed that providing active, purposeful and productive learning experiences involves supporting students to take responsibility and control over their own learning, structuring social situations where learners apply real challenging experiences and facilitate learning for diverse abilities, perspectives and backgrounds. The principles and standards for school mathematics in the United States mentioned that high quality mathematics education requires addressing “equity high mathematics expectations and strong support for all students” (National Council of Teachers of Mathematics, 2000, p. 11).

The purpose of teaching to facilitate students’ learning has confirmed the significance of quality pedagogies on students’ performance and the contributions of schools and classrooms as unique social contexts to offer quality learning opportunities (Lingard, 2008). It should be noted that the renewed emphasis on the

quality of pedagogy stems from its role to provide students with appropriate productive learning and teaching experiences (Wilson & Klein, 2000). Quality pedagogy that describes the teaching practices within classrooms is one of the main factors that support achieving intellectual and social outcomes (McLeod & Reynolds, 2007). Thus, the continuous concern for the quality of students' learning of school mathematics has placed an increased emphasis in recent thinking about the quality of classroom pedagogy as an important factor to make a difference in learning and teaching mathematics. The principles and standards for school mathematics in the United States has mentioned that achieving high standards of school mathematics learning requires an effective mathematics teaching, a coherent and focused mathematics curriculum, an active learning and assessment and an enhanced technology (National Council of Teachers of Mathematics, 2000).

Recent emphasis on quality pedagogy shifts the effective pedagogical discourse to extend the valuable insights of teaching practices within social communities (Lerman, 2000). Henniger (2004) argued that the kind of social forces add challenges on teachers today by increasing the level of expectations on teachers by different communities and by changing the role of teachers in response to the social forces. As Gore et al. (2004) mentioned, achieving high intellectual and social outcomes needs considerations on the importance of learning from community practices and the aspects of learning context, knowledge construction, caring and competence.

As quality pedagogy is the core of any discussion on quality teaching (Hayes et al., 2006; Henniger, 2004; McLeod & Reynolds, 2007), improving the quality of pedagogy in schools continues to be the central focus of educational reforms in many countries. There is a strong movement growing in many countries such as the United Kingdom, United States of America, and Australia, for example, demanding quality standards of pedagogy in order to overcome the lack of intellectual and social outcomes. In England, the recent interests and concerns of teachers' practices has been the development of "metacognitive skills" in which teachers respond to the learning and teaching task primarily by focusing on the intellectual learning outcomes. In the United States, there was a clear interest about the quality of pedagogy from the 1970s to the 1990s by considering the basic characteristics of good quality pedagogy such as active teaching that is based on active learning that can be achieved through cooperative learning activities, independent research

assignments and the use of different learning resources and methods. However, Gamoran, Marks and Newmann (1995) indicated the average performance of American students did not match the desirable levels of outcomes because the practices based on active learning may lead down a deceptive path where student participation in activities can become the main objective, regardless of the intellectual quality of students' work. In response to the interest in the sociology of education in Australia, recent attempts to reform pedagogy in Australian schools aim to improve classroom practices and organisational procedures that make a difference on the intellectual and social outcomes of students (Hayes et al., 2006; Lingard, 2005).

Within the context of this study, according to the results of the national report of *Education in Oman: The Drive for Quality* jointly prepared by the Ministry of Education in Oman and the World Bank, “the key challenges facing the education sector is to improve the quality of student learning outcomes and to raise awareness of enhancing the quality of teaching” (Ministry of Education (Oman), 2012, p. 23). According to the most recent reports from the Trends in International Mathematics and Science Study (TIMSS), Omani students in grade 8 have scored a maximum of 372 points since 2007 that is classified as low level of achievement. Based on the description of the international mathematics benchmarks of TIMSS, a score below 400 describes the students as having limited knowledge of whole numbers and decimals, operations and basic grades. Specifically, there was a significant gap between the average mathematics achievement of Omani students and the international average in the content domains of Number (363), Algebra (391) and Geometry (387) and particularly on in the cognitive domain of knowing (368), applying (372) and reasoning (397) (Mullis, Martin, & Foy, 2008, p. 121). Al kharusi’s study (2011) showed that while the Basic Education system in Oman encourages applying student-centred activities, many Omani teachers experience lack of ability in using more interactive and engaging teaching and learning activities. The Ministry of Education (Oman) in its national report of *Education in Oman: The Drive for Quality* suggests a number of qualitative improvements within two areas. The first area is related to learning. It involves (1) providing a high quality atmosphere to achieve high students’ outcomes, (2) focusing on clear learning targets, and (3) introducing effective evaluation to address under-achievement

problems. The second area of improvement is related to developing high quality teaching and providing in-service support to help teachers improve their teaching practices (Ministry of Education (Oman), 2012).

2.2 Professional Development of Teachers

Considering the importance of investigating the quality of students' intellectual and social performances and the complexity of classroom contexts, preparing effective teachers is significant but not a simple task (Good, 1990). Teaching is a very complex profession involving lifelong learning starting from the own experiences of teachers when they were learners in elementary to high school classrooms, during teacher education programs, through the first years of school teaching and then in continuing professional development (Henniger, 2004). Loucks-Horsley et al. (2003) claimed that to embrace the complexity of teaching as a cycle of preparation, participation and reflection, teachers need opportunities for professional development that develop their pedagogical knowledge and examine their practices. As a result, the focus in teachers' professionalism has moved steadily toward greater recent emphasis on teaching as requiring strong intellectual abilities, involving complex social interactions, requiring extensive specialized preparation, and providing essential support for social life (Henniger, 2004).

There is widespread agreement that teacher professional development experiences should emphasized conducting of high quality professional development that aims to improve students' outcomes (Gallimore & Stigler, 2003; Loucks-Horsley et al., 2003). Fishman, Marx, Best and Tal (2003) argued that it is important to build teachers' professional development programs that aim to improve classroom practices and students' outcomes. However, Loucks-Horsley and Matsumoto (1999) argued that the belief that teacher professional development programs are not valuable unless they are targeted toward improving students' intellectual and social outcomes, neglects other critical outcomes such as improvement of teachers' knowledge and skills, change in classroom and school culture and development of teachers' identities and beliefs. Loucks-Horsley and Matsumoto suggested that preparing teachers to teach based on more challenging standards that help all students regardless of their background to achieve high quality learning calls for more and better teacher professional development programs. Loucks-Horsley et al.

(2003) indicated that the professional development programs that aim to improve students' learning should also set goals for improving teacher practices and teacher learning.

School educational reforms consider teachers' professional development as one of the central elements in effective school reforms. Andrews and Lewis (2002) emphasized the role of effective school professional development programs as creating new understanding of teacher's work that is centred on classroom actions. Lingard, Mills and Hayes (2000) suggested that schools can create a teacher professional learning community that plays a significant role in effective teaching and students' learning. Moreover, Carlgren et al. (1994) indicated that schools have to improve the quality of teachers' professional development as well as teachers' work conditions including provision of time, and resources for teaching and learning and the use of technology. For example, improving a school's capacity to build professional development programs that are centred on an ongoing teachers' learning is recognised by offering effective professional content, time, funding arrangements, supportive policies and effective leadership (Lingard et al., 2003). As mentioned by Lingard et al. (2000), effective professional development programs for teachers should reflect some significant features and requirements such as:

- Construct useful connections for teachers by building on their current pedagogies and their prior content knowledge to add new knowledge
- Develop new understandings to change their teaching practices
- Provide an active engagement with experts to improve participants' practices.
- Offer an active engagement for the participants by identifying continuous teaching improvements and introducing new teaching and learning experiences, programs and frameworks
- Base teaching on students' learning data and
- Offer sufficient time, resources and support for teachers to reflect on their work experiences

Effective teacher professional development is useful when it is grounded in the classroom context, aligned in the content that teachers required to teach and develop in response to specific problems of teaching practice (Gallimore & Stigler, 2003). Loucks-Horsley and Matsumoto (1999) argued that teacher professional

development needs a sound foundation to enable teacher gain deep understanding of the major ideas of content knowledge that teachers teach knowledge of students, their ideas, their different ways of learning, their cultural backgrounds and the knowledge of the influence of the classroom context. McLeod and Reynolds (2007) indicated that when teachers reflect on their teaching practices they need to consider all elements of classroom environment that impact on students' learning particularly the social relationships and actions that occurred between the teacher and students and between students during instruction.

Understanding the continuous the process of professional change and growth and the conditions of support are important to facilitate teacher professional development. Effective professional development that has a meaningful and positive impact on teacher learning and supports improvement in classroom practice should be the focus of professional growth over time by offering high-quality professional development experiences. Guskey (2002) claimed that many teacher professional development programs focus on changing teachers' beliefs about specific aspects of teaching practices whereas they fail to consider the process of teacher change. He indicated that for effective teacher professional growth, it is essential to recognise that learning new knowledge requires both time and effort, supporting initial learning that is based on changing teachers' beliefs and attitudes and then ensuring that teachers continue to conduct reflections on their experiences in their classroom practices. Clarke and Hollingsworth (2002) argued that in-service teacher professional growth programs need to bring about a change in teachers' beliefs, attitudes and knowledge, a change in their classroom practices and a change in student learning outcomes. They emphasized that these changes in the translations of the new knowledge into actions and the process of reflection on the actions that have been experienced need to be continuously investigated.

Effective teacher professional development requires sustained observations and questioning (Gallimore & Stigler, 2003). Teachers' effective learning experiences should provide different kinds of opportunities to learn such as examining practice, conducting collaborative work, attending conferences and workshops (Loucks-Horsley & Matsumoto, 1999). Garet, Porter, Desimone, Birman and Yoon (2001) examined the relationship between characteristics of professional development that have been identified in a wide range of professional development activities and self-

reported changes in mathematics teachers' knowledge, skills and classroom practices. The survey examined 1,027 self-reports of mathematics and science teachers that participated in different professional development activities including workshops and conferences that took place outside schools or classrooms. Other forms of professional development activities that were examined were study groups, professional networks and peer coaching that were tied to ongoing classroom practices. The study showed that the forms of professional development that were based on sustained and intensive activities were more likely to have a positive impact on best teaching practice. The study emphasized the importance of offering opportunities for enhancing teachers' knowledge and skills, encouraging teachers' communication, active learning and for integrating their knowledge and skills into daily classroom practices.

Offering opportunity for teachers to take the responsibility to question the effectiveness of their teaching practices and meet their students' needs has played a significant role in the improvement of teachers' practices. Moore (2005) argued that teacher preparation and school teacher professional development programs need to help pre-service and in-service teachers to critically assess their own learning experiences. In the context of designing professional development for teachers of mathematics, Loucks-Horsley et al. (2003) indicated that currently there is a growing emphasis on professional development that engages mathematics teachers in opportunities to examine their practices and that of their peers through critical reflection in order to develop new knowledge required for the improvement of their teaching practices. Schön (1983) argued that reflection in action is seemed to be the core practice that enables the professional to become a researcher in the practical context. Schön mentioned that the idea of reflective practice has many significant implications for the professional's knowledge, to the community of practice and to research-based practice. He indicated that "reflection in action" is where a professional can respond to the current moment of action and construct new a theory of the unique action in this moment based on their previous knowledge about similar situations as well as on the new understanding of the uniqueness of the new action. McLeod and Reynolds (2007) indicated that the teacher as a reflective practitioner in action should respond to the instant challenges associated with constant changes in everyday teaching practices within the classroom context without affecting the

continuity of students' learning. In addition, the new professional knowledge that teachers construct is based partly on the knowledge that is formed in their previous teaching experiences and as well as to their responses to the new classroom actions. As showed by Henniger (2004), the reflective process on teaching practices involves providing opportunities for thoughts of previous experience, beliefs and expectations. Schön argued that the idea of reflective practice results in deeper understandings of the teaching profession through providing opportunities for reflection, of understanding the realities of schooling and of decision making. As well as reflection is a continues process of improvement, teachers need also to reflect on their teaching practices to evaluate their actual teaching practices and to understand the main factors that influence their teaching in order to engender improvements in their future teaching practices. Loucks-Horsley and Matsumoto (1999) indicated that offering opportunities of analysis and reflection are central to teacher learning and in the improvement in their teaching practices. Moreover, Henniger (2004) showed that the reflective process on teaching practices involves informing classroom practice and developing the aspects of reflection as well as researching, talking, discussion and presenting to others. It also involves applying the understanding and consideration of experiences to inform decision making in the classroom. Gallimore and Stigler (2003, p. 29) emphasized that for "practitioner-generated knowledge" to have professional value it must be made public and be presented to the members of the same profession and open for discussion, alteration and improvement.

In the context of school mathematics teaching, teacher professional development should prepare teachers to mathematically challenge their students to learn about socioeconomic and political issues and to make intelligent decisions about who can achieve and participate in mathematics in a highly diverse society (Kitchen, 2005; Leonard & Dantley, 2005; Moore, 2005). Gore (2001) argued that it is a requirement for teachers to be skilled and well-informed to be able to address and understand the social issues of inequality and diversity. Kitchen (2005) emphasized that mathematics teachers need support to explicitly connect the teaching and learning of mathematics with their students' socioeconomic realisms. For example, preparing mathematics teachers to teach for social justice needs a change in their beliefs during effective ongoing school professional development (Gonzalez, 2009). Further,

Gonzalez emphasized the need to investigate school teachers' understanding of mathematics for social justice in their mathematics classrooms.

Within the context of this study, the Ministry of Education in Oman believes that successful quality educational outcomes are dependent upon a well-skilled, well-informed and highly supportive teaching force (Ministry of Education (Oman), 2012). Issan and Gomaa, (2010) emphasized that the education system in Oman should continue monitoring changes in the development of the curriculum, assessments and in-service professional development in order to investigate and develop the individual needs of the teachers. The Ministry of Education in its policy report of *Education in Oman: The Drive for Quality* showed that in-service teacher professional development programs that were conducted to promote quality teaching faced many challenges. The report pointed that one of the obstacles that faces the implementation of the Basic System is that the new professional training courses are still theory based. In addition, the report pointed to the fact that training programs that are organised for the teachers were designed to be conducted for only short duration and concentrated on the theoretical knowledge with limited time for application. The research showed that school teacher professional development programs under the Basic Education System lacked efficiency and quality. Al kharusi (2011) indicated that these programs consisted of limited in-service training courses that were aligned with the changes in curriculum and teaching methods. The Ministry of Education in its policy report of *Education in Oman: The Drive for Quality* also showed that the other challenges were that those programs were theoretical, lacked emphasis on practical methods and effective communication skills. The research showed that there is a demand for evaluating and improving the quality of in-service training programs to cope with innovations in methods of teaching that enhances active learning (Al-Hajri, 2010). Al Barwani (2002) emphasized that both in-service and pre-service teacher training programs in Oman have to be developed to in keeping with the new requirements of the teacher's role in preparing learners to be active members of the society.

Overall, the importance of investigating the quality of teaching stems from the critical influence of the quality of teaching to achieve high-quality of students' learning and from the recent emphasis of the specific role of pedagogy to achieve higher standards of intellectual and social outcomes. That emphasis highlights

offering effective opportunities for teacher professional development. At various times, the importance of questioning the essentials and the requirements of good teaching are considered and influenced by many different theories, perspectives and frameworks of education. The following sections introduce the role of some learning and teaching theories, perspectives and pedagogical frameworks on the quality of teaching and learning experiences.

2.3 Learning and Teaching Theories and Perspectives

For over hundred years of educational research and theory, attempts were made to define what good teaching and learning should be. Currently, some of the important theories have been given considerable emphasis in schools in many countries around the world and have received central importance for those working within the classrooms are constructivism and social deconstructionism. Constructivism as a theory of learning has become the dominant view in education in the 1980s and 1990s that has been a significant influential force in shaping education reforms. The theory of constructivism in learning, that is based on the premise that students are active, self-motivated and are able to construct their own meaning and understanding of knowledge, draws attention to the need to question the impact of this theory on teaching practices within the classroom environment (Henniger, 2004; Westwood, 2008). “The historical roots of social constructivist theory, that emerged in the 1920s from Max Scheler (1980) and was developed by Karl Mannheim (1991)”, are based on the beliefs that knowledge is a part of society, learners are agents of societal growth and development, with teaching as models of democratic actions and classrooms as learning environments that model democratic ideals (Henniger, 2004, p. 262; Hruby, 2001). Under the perspective of social constructionism, schools are one of best agents that play a role in helping society to understand, value and work with the differences to create an equitable society. The social constructivist perception of students are based on the beliefs that teaching and learning should be directed to provide students with needed information and skills that enable them to be part of their society and to be capable of engendering successful change in their society (Henniger, 2004).

2.3.1 Constructivist perspective of learning

The constructivist theory of learning has resulted in significant change in the thinking of the design of effective learning experiences by emphasizing learning as a process of knowledge construction. Viewed through constructivism, learning is based on the belief that knowledge is constructed by the learner rather than directly transmitted by the teacher. This premise encourages learning to be an active process for searching meaning and constructing understanding (Clements & Battista, 1990; Muijs & Reynolds, 2011). Constructivism theory of learning that draws from Jean Piaget's work (1977) asserted the idea of learning as an active construction of meaning (Gray, 1997; Muijs & Reynolds, 2011). Piaget's theory which provides part of the foundation for constructivist learning, argued the importance of the active role of the learner, the learning process to be learner directed and the alteration of teaching approaches to be adapted to the cognitive development of the learner. Piaget identified four stages in cognitive development according to age: sensorimotor stage, pre-operational stage, concrete operational stage (elementary and early adolescence), and formal operational stage (adolescence and adulthood). Piaget supposed that a learner's biological development drives the movement from one cognitive stage to the next (Huitt & Hummel, 2003).

Recent thinking about quality learning is based on the idea that learning as a personal experience is supported by offering opportunities for rich and meaningful experiences of active engagement, social interactions and diverse experiences (Loucks-Horsley et al., 2003). As argued by Von Glasersfeld (1989), since human's experiences always include social interaction, it is of fundamental importance to investigate education by taking into account that learning for understanding needs social interaction and an active learning context. McLeod and Reynolds (2007) claimed that planning for quality learning needs to value the cognitive learning that shapes the way children think and learn, the social and emotional learning that forms the ways students interact and the physical learning that shapes the interaction of students with the learning environment.

As mentioned by Brophy (2002), learning as developing social interpretations enhanced structuring understanding through discussion, dialogue or discourse in school and classroom settings. Von Glasersfeld (1995) argued that learning requires

conceptual development through social interactions of reflection, verbalization, self-regulation, conversation and abstraction. The notion of social constructivism understands learning as a procedure through which learners construct their knowledge, it is also assumed to be a social process that reflects cultural and social contexts which learners develop from their cognitive growing into the surrounding intellectual life (Brophy, 2002; Matthews, 2003; Von Glasersfeld, 1990). Based on social constructivist beliefs learning as an active knowledge construction has a number of significances:

- Learning is authentic and an active process of understandings, reflection and metacognition
- Learning is a search of meaning around big ideas and explorations
- Learning is a base of human development theories
- Learning will take place through different forms of social interactions (Muijs & Reynolds, 2011, p. 79)

2.3.2 Constructivist perspective of teaching

The constructivist perspective of learning, that is based on the premise that students are active, self-motivated and are able to construct their own meaning and understanding of knowledge, draws attention to the need to question the impact of this perspective on teaching practices in the classroom environment (Henniger, 2004; Westwood, 2008). While constructivism is primarily a theory of learning rather than a theory of teaching, its beliefs bring into focus a variety of productive teachers' and learners' roles that have led to a variety of new teaching methods and strategies (Muijs & Reynolds, 2011; Westwood, 2008). Matthews (2003) indicated that the key notion of constructivist teaching is the importance of adapting teaching methods to the new student learning style. Applying constructivist beliefs to the issue of teaching rejects the assumption that the teacher can simply transmit the information directly and assume that understanding will result (Confrey, 1990). The idea of knowledge construction is based on the perspective that learning is a personal and active process which is built on the learners' existing knowledge. Due to the new constructivist beliefs about learning, the role of a teacher has been redefined through knowledge construction (Westwood, 2008). Under the notion of constructivist-based teaching, the role of teacher changes from taking the full responsibility and decision

making for managing students' learning to be a facilitator of learning and a guide of students' learning efforts (Matthews, 2003). According to the change of the role of the teacher based on constructivist beliefs, Brophy (2002) indicated that teachers have to be able to prepare a learning environment of meaningful experiences by posing questions, promoting reflection, conducting sustained classroom communication and by focusing on eliciting students' thinking.

One of the main challenges that faces applying constructivist teaching is the translation of learning theory into methods of teaching (Von Glasersfeld, 1995). Nuthall (2002) indicated that implementing social constructivist teaching requires developing classroom activities that focus on students' discussing their shared experiences in small groups, developing observation procedures and ensuring practice involving critical content. Brophy (2002) indicated that as constructivist teaching practices assume the motivation to learn, teachers should provide students with ongoing learning experiences that allow them to be active, capable of thinking and reasoning, self-regulating and motivated.

While constructivism as a theory of learning may reflect no direct presentation of effective teaching principles and methods, some of the research literature suggests several implications for practice. Proulx (2006) highlighted some of the following potential implications for constructivist teaching from the constructivist learning perspective:

- Recognising the prior knowledge and experiences of the learner and using these to build new meaningful understanding
- Giving attention to the language of communication between students and the teacher
- Being aware that while constructivist beliefs assert the own construction of knowledge, teachers and learners can be involved together in meaningful construction of knowledge
- Using learners' mistakes and difficulties as sources of learning and adaptations
- Taking into account learners' active learning to construct pedagogy

Since the role of the teacher in the classroom is shifted from transmitting to primarily guiding, the teaching approaches have also changed from teacher-centred to learner-centred approaches. Westwood (2008) explained the difference between these two

teaching approaches as teacher-centred approach encouraging the effective direct transmission of information and skills from the teacher to the learner while learner-centred methods are aligned with the constructivist theory of teaching that concerns the importance of the merits of knowledge construction to foster deeper conceptual understanding for students and focus on the active role of the learner in developing their own knowledge and skills. Muijs and Reynolds (2011) suggested that effective constructivist teaching is based on a number of common learner-centred teaching strategies such as connecting newly learned ideas to students' prior knowledge, modelling, scaffolding, coaching, collaboration, reflection, exploration and problem solving activities. They indicated that some form of teachers' guidance and direction need to be part of the learner-centred teaching approach. For example, during scaffolding and coaching the teacher may give assistance and support to achieve learning tasks by asking questions, offering resources and suggesting tasks.

Constructivism and mathematics teaching

The main ideas underpinning constructivist learning theories continue to influence mathematics and science education (Joldersma, 2011; Westwood, 2008). Simon (1995) indicated that constructivist theory has been prominent in research on mathematics learning in the 90s and has provided a basis for recent empirical and theoretical work in mathematics education. Applying constructivism in mathematics classroom highlights the importance of building a classroom environment that encourages students' learning and highlights new visions of the roles of mathematics teachers.

Mathematics, as any other knowledge, is a language of everyday experiences and human action involving the reflective process. It can be constructed effectively in models of reality that are formed in learning environments of social conventions and interactions (Goldin, 1990). Treagust, Duit and Fraser (1996) argued that learners are expected to be able to use their basic scientific knowledge to construct science and mathematics conceptions from their own activities. Cobb (1994) claimed that constructed mathematics knowledge is influenced by the active individual development of the learners and the social and cultural interactions in mathematics practices. Moreover, Clements and Battista (1990) indicated that mathematics

knowledge is actively created by reflecting on physical and mental action experiences and personal mathematics ideas are shaped through social interaction.

Constructivist theory has significant implications for mathematics teaching (Confrey, 1990; Tobin & ImWold, 1992). Burton (1992) claimed that constructivist views in mathematics value the notions of enquiry, prediction and discovery instead of theorems and proofs, and shifts the role of the teacher from the teacher's dissemination of mathematics facts to the learner's exploration of unknowns. Constructivist mathematics teachers need to understand the content of mathematics that is taught in order to value the different ways that students use to construct mathematics knowledge and to evaluate the different solutions that they arrive at (Mildren, 1992). Tobin and Imwold (1992) argued that the role of mathematics teachers in the constructivist mathematics classroom is assumed to monitor students' learning and challenge students thinking in a productive direction. Constructivism commits the teacher to encourage the ability of students to build more powerful constructions that reflect students' understanding of mathematics knowledge. The teacher can build on students' conceptual constructions by considering that all actions, thoughts and ideas that students display to make sense to them (Von Glasersfeld, 1995).

Constructivism draws the environment of learning from a classroom of students to a community of active learners (Muijs & Reynolds, 2011; Westwood, 2008). In constructivist classrooms, the learners are immersed in learning experiences that encourage the review of action, imagination, communication and reflection. Under the conception of constructivism, the classroom environment is characterised to be learner-centred with active instruction. It encourages social interaction such as collaboration and exchanging ideas. It is democratic as it emphasizes shared responsibility and decision making about learning and classroom environment learners and offers opportunists to ask and contribute. The constructivist mathematics classroom is interactive as it encourages authentic dialogue between the teacher and among students themselves (Gray, 1997). Von Glasersfeld (1995) stated that constructivism encourages the learning environment to reflect forms of motivation, satisfaction and greater pleasure.

However, constructivism in teaching mathematics is criticized for being “relativistic”. This critique of being relativistic argues that when constructing their own knowledge learners do not investigate the validity and the equality of learning products (Confrey, 1990; Von Glasersfeld, 1995). Westwood (2008) indicated that most daily learning processes that construct mathematics knowledge through individual discovery and experience can construct misleading conceptions as well as accurate conceptions from the learner’s personal knowledge construction. Even though, constructing mathematics knowledge does not mean building mathematical structures but it reflects the idea that knowledge is an individual unique experience and a part of the learner’s context. Constructing own meaning of knowledge does not deny the absolute reality of knowledge but it gives meaning to students’ experiential world. Moreover, constructivism assumes that students’ construction of knowledge is shaped by different social influences. Von Glasersfeld (1993) argued that because the constructive process is subject to differences in individual social influences such as language and decision making of resources and methods, it is assumed to be a product of making sense of knowledge in the learner’s world and it is a vision of building new understandings from individual experiences of conceptual learning organization.

Constructivism and teacher learning

Cobb, Wood and Yackel (1990) argued that the classroom environment is not only considered as a learning environment for students but also for mathematics teachers. Constructivism emphasizes the view of the teacher as a learner. Tobin and Imwold (1992) indicated that constructivism can be used as a tool for critical reflection that encourages teachers to give personal meanings to their teaching experiences. It can also be used as a reflective tool to help teachers to design and evaluate learning activities and implementations that adapt to the learners’ needs. Teachers can draw on constructivist beliefs in predicting the appropriate learning experiences to improve quality learning in such classroom situations.

Changing teachers’ beliefs and practices can be improved through questioning their current practices and analysing their students’ performances in whole classroom settings or in small group interpretations. Cobb et al. (1990) indicated that reflecting on the analysis of students’ mathematical learning where pedagogy was broadly

compatible with constructivist perspectives of cognitive and social interactions helps teachers to reconstruct their teaching approaches. Confrey's study (1990) that aimed to construct a model of teaching practices, "The reflective Practitioner", was committed to constructivist beliefs that focused on teacher-student classroom interactions, concluded that teachers' reflection is the bootstrap for the construction of mathematical ideas by encountering problematic situations and reflecting on students' actions that are used to deal with these situations.

Although social constructivism opened the door to the importance of the classroom as a social practice, it did not seem to consider the wider issues of social and political surrounds and the question of power in mathematics education. The following section will elaborate on the involvement of social norms in the learning and teaching mathematics.

2.3.3 Social turn in mathematics education

Over recent years, there has been growth in social elements involved in learning and teaching mathematics (Atweh, 2007a; Gates & Jorgensen, 2009; Kitchen, 2005; Lerman, 2000). Two themes are brought out as important in the social turn in mathematics education and play an importance influence on pedagogy. One of them considers the teaching and learning of mathematics from a social perspective that understands students' backgrounds, aims of critical mathematics teaching and the question of power. The other them is to calls for introducing social aspects into mathematics teaching.

Understanding teaching and learning mathematics from a social perspective

Understanding the social turn of teaching and learning school mathematics for the individuals highlights the importance of investigating the construction of mathematics knowledge, classroom culture, school structures, socialization processes, and teachers' professional development (Gates & Jorgensen, 2009; Gutiérrez, 2012). Mathematics as a set of knowledge and a set of social practices is socially constructed and mediated by institutional, historical and social norms (Kitchen, 2005; Lerman, 2000). Gutiérrez (2012) argued that the production of mathematics knowledge reflects the nature of the society in which it is created and it brings with it the power of the social interactions. Thus, the construction of school

mathematics knowledge requires mathematics teaching to consider the impact of the social context of mathematics classrooms (Cooper, 2007).

Research into student learning in mathematics has documented the important role of constructing new knowledge from learners' experience and prior knowledge (National Council of Teachers of Mathematics, 2000). McLeod and Reynolds (2007) argued that learning is influenced by not only previous learning content but also by the actions that formed their previous learning. To support quality learning, teachers need to consider the influence of learner's prior knowledge when designing new learning experiences. Loucks-Horsley et al. (2003) argued that learner's prior knowledge of what they already know, expect and believe of themselves as learners influence their learning.

The social interactions of the classroom are of major importance in the construction of the social context in classrooms (Atweh, Bleicher & Cooper, 1998). Learners enter classrooms with exclusive sets of cultural influences, life involvements, prior knowledge, attitudes and behaviours. Atweh (2007b) claimed that the increasing diversity in mathematics classrooms raises serious social justice issues such as participation and achievement gaps, recognizing the contribution of the different groups to mathematics and the consideration of quality teaching and learning practices that are valued and working with differences between the different groups. Therefore, schools and classroom as part of larger social world must be socially supportive and intellectually demanding (Gutiérrez, 2012; Hayes et al., 2006). Gutstein (2003) emphasized that creating a classroom environment in which meaningful social issues are discussed helps students develop as conscious managers of change.

Gutiérrez (2012) claimed that the main goals of critical mathematics are developing a political awareness within students to identify the position of the learner in society and to motivate them to use mathematics to express themselves and to act effectively on their mathematics knowledge. Students need to understand that the power of school mathematics ideas can be used to enable students become active citizens who use their democratic access to shape the future of their society (Malloy, 2002). As mentioned by Gutiérrez (2012), learning and teaching practices within mathematics classrooms require constructing the identities of individuals and enhancing the power

of doing mathematics successfully. Atweh (2007b) suggested that school mathematics teaching needs to consider the role of mathematics for reforming aspects of students' world and to build their teaching practices upon the justification of mathematics as developing skills and abilities that are useful for preparation for the future. He emphasized that the usefulness of mathematics can be demonstrated through real world activities that encourage students to engage critically with their social world as well as developing mathematics concepts and skills.

The call for introducing social aspects into mathematics teaching and learning

Malloy (2002) indicated that the benefits of introducing social issues such as social justice and equity in mathematics education are illustrated in at least three ways: raising the opportunities to learn for all students, understanding the power of the different applications of mathematics and improving the ability to apply mathematics to solve problems of social issues. The principles and standards for school mathematics emphasized that the vision of investigating equity in school mathematics education requires understanding that mathematics must be learned by all students regardless of their personal and background characteristics, demands high expectations of learning mathematics and provides access of equitable, supportive and intellectual mathematics programs (National Council of Teachers of Mathematics, 2000). Luke (2010) suggested that schools and educational systems need to consider the issues of equity and social justice by developing new strategies for closing the gap between different groups of students in classroom settings.

The “social turn” in mathematics education includes concerns about social issues such as justice, equity, diversity and participation (Atweh, 2007a). The calls for introducing social issues into mathematics teaching and learning reshape mathematics pedagogy to include social relevance and cultural responsiveness for improving mathematics achievement for all students. Research has indicated that diverse mathematics classrooms may bring a variety of ways of understanding, investigating, talking as well as different values and attitudes that demand teachers to learn to implement more culturally comprehensive and socially relevant pedagogies (Moore, 2005). The increasing diversity in mathematics classrooms raises serious social justice issues such as participation and achievement gaps, recognition of the contribution of the different groups to mathematics and the consideration of quality

teaching and learning practices that involve valuing and working with differences between the different groups (Atweh, 2007b).

Understanding the issue of introducing social aspects into mathematics teaching and learning draws on the understanding the issues of social justice, equity and diversity (Garii & Appova 2012; Gates & Jorgensen, 2009). School mathematics pedagogy can contribute effectively to the ability of students to function as effective citizens in the world. Lingard (2005) argued that the quality of pedagogies is an important way to achieving socially just outcomes from schooling. Mathematics teachers in schools play a significant impact on developing learners as active citizens (Gates & Jorgensen, 2009) by helping students develop abilities of questioning and analysing the conditions they live in and the political issues that shape their life in order to develop social and cultural identities (Gutiérrez, 2012). Teaching for social justice, diversity and citizenship has raised current interest in mathematics education (Banks, 2004). Approaching mathematics through a social justice context seems to be a growing interest in recent years. Gonzalez (2009) argued that teaching mathematics for social justice requires an access to high quality of mathematics pedagogy for all students especially those from disadvantaged backgrounds. It involves building upon students' experiences from disadvantaged backgrounds, using mathematics as a critical tool to examine social and political issues and offering equal distributions of society's opportunities and resources among all its members. Gonzalez emphasized that teaching for social justice involves preparing teachers to become increasingly aware of the social realities such as the differences among their students, different student learning approaches, different abilities and needs and different students' own cultures and understandings.

2.3.4 The shift towards pedagogical frameworks

Although the previous learning and teaching theories and the social turn have provided mathematics educators with useful perspectives about mathematics learning that have the potential to inform changes in classroom mathematics teaching, it seems that most of them concentrate on theoretical models and do not offer a particular vision of classroom pedagogy (Simon, 1995). Theoretical frameworks for interpreting social constructivist beliefs and social roots began to appear in the mathematics literature toward the end of the 1980s (Lerman, 2000). Treagust et al.

(1996) and Confrey and Kazak (2006) highlighted the need for more consistent empirical implications of constructivism to be incorporated in mathematics education by utilising more specific and elaborate pedagogical frameworks to apply constructivist ideas in mathematics classroom practices. Gore et al. (2004) indicated that while there are many initiatives in the long history of reforms in teacher education and teaching, there is still more need for more descriptive and analytical models of successful teaching practices to ensuring both better education programs and teacher school professional development that provide deep understanding of the characteristics of learning environments that support both learning experiences and values diversity. Over time, it became clear that the recent aspirations around quality pedagogy that stem from the recent awareness of students' learning and the continuous emphasis on quality teaching and the view of teaching as a demanding profession needed to focus on pedagogy through models of quality pedagogical standards and actions (Gore et al., 2004, Hayes et al., 2006; Lingard, 2005).

One of the recent attempts for reconstructing pedagogy based on providing more complex set of pedagogical standards as a basis for researching and improving teaching and learning practices is the Australian model of Productive Pedagogies (Gore et al., 2004). While most teacher education programs aim to model good classroom practices, the strength of the Productive Pedagogies framework is on centring the importance of pedagogy and the requirements of teachers to be reflective and professional. The Queensland School Reform Longitudinal Study (QSRLS) research team who studied Productive Pedagogies argued that the absence of the pedagogies that are productive is a social injustice issue. Under this argument, the importance of teachers and their pedagogies was supported to enhance intellectual educational outcomes. They suggested that intellectual social support of classroom experiences will be effective to improve students' outcomes if they value and reach all students especially disadvantaged students. These socioeconomically disadvantaged students need to be aware in their minds and beliefs that learning has some meaning to their public and personal lives (Hayes et al., 2006).

The Productive Pedagogies framework is based on the research model of school restructuring "Authentic Pedagogy" that was imported and developed by the Queensland School Reform Longitudinal Study (QSRLS) (Hayes et al., 2006). The QSRLS research team built upon the emphasis on intellectual outcomes developed

by Newmann and colleagues (Newmann & Wehlage, 1995) and added an emphasis on socially equitable outcomes as well as academic outcomes. Productive Pedagogies was developed taking into account Australia's, and specifically Queensland's educational context and by drawing on diverse literatures on social learning, curriculum theories, classrooms and schools studies, and an inclusive framework of teaching practices to achieve intellectual and social students' outcomes (Hayes et al., 2006; Lingard, 2005; Mills & Goos, 2007). In the following sections, the vision and the research on Authentic Pedagogy will be discussed followed by a discussion of Productive Pedagogy as a framework for quality teaching and for teacher professional development.

2.4 Authentic Pedagogy: Vision and Research

Some recent attempts for reconstructing pedagogy to include more on the social perspective from a constructivist view have been central to recent pedagogical reform movements (Muijs & Reynolds, 2011; Roelofs & Terwel, 1999). In particular, Authentic Pedagogy is a pedagogical vision that offers a conception of a classroom instruction and assessment tasks that are significant, meaningful, valuable, and intellectually demanding. Using the conception of authentic academic achievement, Newmann and his colleagues proposed standards of Authentic Pedagogy and authentic academic performance. The term "authentic" is used to distinguish learning from "achievement" by adding the element of meaningful purpose to learning that students should engage with. Authentic learning is based on constructing students' meaning and understanding of knowledge using disciplined inquiry and on building performances that have value beyond success in schools (Newmann, Marks & Gamoran, 1996, p. 282). Mims (2003) indicated that if learning is authentic, then students should be engaged in learning tasks that offer opportunities for higher order thinking operations and for making valuable connections to the world. Thus, achieving authentic learning requires applying pedagogical approaches that offer opportunities for centring classroom learning experiences on authentic tasks (Mims, 2003; Newmann et al., 1996; Newmann & Wehlage, 1995).

An authentic academic achievement vision reflects both similarities to and different points of view from constructivist ideas. Both of them share the assumption of

learning as knowledge construction rather than reproduction of specific information and content (King, Newmann & Carmichael, 2009; Newmann et al., 1996; Roelofs & Terwel, 1999). Newmann et al. (1996) indicated that authentic academic achievement occurs in an environment of constructivist learning with high intellectual quality. They argued that authentic constructing of knowledge involves applying and manipulating of students' prior knowledge to develop the new learning knowledge that has value beyond school.

While the specific standards for Authentic Pedagogy and student academic performances are consistent with the constructivist view of knowledge construction in which both of them encourage the view of the learner as a meaning making person, authentic students' performance goes further to reach the criteria of analysis and elaborately written communication using substantial knowledge from reliable fields. The standards of authentic academic performance encourage higher order thinking with mathematics and social studies content by drawing more emphasis on students' abilities of analysing, assuming, and synthesizing in order to reach meaningful conclusions. Using these standards, students' performance should demonstrate an understanding of the main learning ideas that have applications in their civic life (Newmann et al., 1996).

The vision of Authentic Pedagogy

In 1983 American educators and reformers had become increasingly concerned about American students' learning. Common worries rose through students producing shallow and weak intellectually work. They assumed that one of the main reasons was students spending much time absorbing and reproducing knowledge without constructing meanings and understandings outside the school (Newmann et al., 1996). The national Commission on Excellence in Education highlighted the need to "restructure" American schools. Hence, the Centre on Organisation and Restructuring of Schools (CORS), supported by the U.S. Department of Education and Wisconsin Centre for Education Research, conducted studies to develop new structural tools to be used for "restructuring" American schools (Newmann & Wehlage, 1995, p. 1).

Fred Newmann and his colleagues argued that restructuring schools should focus on pedagogy and on their empirical justification of "authentic" to be rooted in the

concern of consistent active learning and for enhancement of students' intellectual quality outcomes (Hayes et al., 2006; Newmann & Wehlage, 1995). The context for successful school restructuring focused on four "circles of support": student learning, Authentic Pedagogy, school organization capacity and external support. CORS then developed, between 1991 and 1994, a particular vision of high quality intellectual work called authentic academic achievement that aimed to enhance the intellectual quality of students learning. It also developed specific teaching standards to gauge the intellectual quality of classroom instruction and assessment tasks in a vision called Authentic Pedagogy (Newmann et al., 1996, p. 288).

The concept of authenticity is used to reflect the criteria of intellectual work: "constructed knowledge", "disciplined inquiry" and "value beyond school" (Newmann et al., 1996 p. 283). Authentic learning as a process of knowledge construction involves enhancing the skills of organizing, analysing, interpreting and evaluating or synthesizing of students' prior knowledge to solve new problems that cannot be solved by routine approaches. This knowledge construction must be guided by disciplined inquiry. Authentic achievement, as Newmann et al. (1996) claimed, must be grounded in student's prior knowledge base of facts, concepts, language and theories that are necessary to conduct rigorous inquiry. This knowledge base must occur in deep understanding rather than superficial understanding in order to be expressed through complex forms of communication tools such as verbal, written, visual, nuances and details. Moreover, developing learning produces discourse and performances that are authentic and require reflecting on students' personal values and social significance beyond school.

While learning is supposed to meet these key criteria of constructed knowledge, disciplined inquiry and value beyond school, these criteria of intellectual academic work also established the standards of Authentic Pedagogy. Pedagogy is defined as the combination of daily teaching practices and assessment tasks used by the teacher (Newmann & Wehlage, 1995). Newmann et al. (1996) indicated that Authentic Pedagogy emphasizes the idea of active learning that requires deep understandings of knowledge bases and effective explicit demonstration of those understandings. It promotes the application of learning experiences that encourage students' achievements that have some purposes beyond school. Using the standards of Authentic Pedagogy, classroom instruction should involve students in higher order

thinking, engage them in extended conversational exchanges, explore and produce complex understanding and deep knowledge and make connections to life beyond the classroom. The standards of assessment tasks were also developed to examine the ability of students to construct knowledge, show deep understanding through disciplined enquiry and express an elaborately written communication. Table 2.1 summarizes the standards for Authentic Pedagogy and student academic performances as illustrated by Newmann et al. (1996, pp. 288-289).

Table 2.1

Standards for authentic classroom instructions and assessment tasks

Standards for Authentic Pedagogy and student academic performance	
1. Authentic Pedagogy: classroom instruction	
<i>Standards</i>	<i>Explanation</i>
Higher- order thinking	Instruction involves students in manipulating information and ideas by synthesizing, generalizing, explaining hypothesizing, or arriving at conclusions that produce new meanings and understandings for them.
Substantive conversation	Students engage in extended conversational exchanges with the teacher and/or their peers about subject matter in a way that builds an improved and shared understanding of a ideas or topics
Deep knowledge	Instruction addresses central ideas of a topic or discipline with enough thoroughness to explore connections and relationships and to produce relatively complex understandings.
Connection to the world beyond the classroom	Students make connections between substantive knowledge and either public problems or personal experiences.
2. Authentic Pedagogy: assessment tasks	
Organization of information	The task asks students to organize, synthesize, interpret, explain, or evaluate complex information in addressing a concept, problem, or issue.
Consideration of alternatives	The task asks students to consider alternative solutions, strategies, perspectives, or points of view as they address a concept, problem, or issue.
Disciplinary content	The task asks students to show understanding and/or use of ideas, theories, or perspectives considered central to an academic or professional discipline.
Disciplinary process	The task asks students to use methods of inquiry, research or communication characteristic of an academic or professional discipline.
Elaborate written communication	The task asks students to elaborate their understanding, explanations, or conclusions through extended writing.
Problem connected to the world	The task asks students to address a concept, problem, or issue that is similar to one that they have encountered or are likely to encounter in life beyond the classroom.

Research on Authentic Pedagogy

Research studies on observing the extent of Authentic Pedagogy standards in classroom teaching practices demonstrate that the typical pedagogy provided in schools is far from meeting authentic standards of instruction (Ladwig, Smith, Gore, Amosa & Griffiths, 2007; Newmann et al., 1996; Roelofs & Terwel, 1999). Roelofs

and Terwel (1999) examined the extent of Authentic Pedagogy standards in secondary Dutch classrooms of grades 1-3 after implementing the school Dutch national curriculum. The research did not find any real extent of the standards of Authentic Pedagogy. Ladwig et al. (2007) analysed the relation between pedagogy and students' achievement using the data of the NSW public school study (Systemic Implications of Pedagogy and Achievement) that used authentic task measures that are adopted from Newmann's work on Authentic Pedagogy. This study indicated that high levels of authentic task scores were scarce and the intellectual quality as proposed by Authentic Pedagogy is a challenge for teachers. It also indicated the variation of Authentic Pedagogy within schools and grade levels. These findings are consistent the US study on Authentic Pedagogy (CORS) that examined the teaching practices in 504 lessons in mathematics and social studies in grades four and five in elementary schools, grades seven and eight in middle schools and grades nine and ten in high schools from twenty-four restructured schools across the United States. This study found a variation in the delivery of Authentic Pedagogy between American schools and between individual teachers. It also found that pedagogy within the observed classrooms were rarely rated at the levels of Authentic Pedagogy. While the research on Authentic Pedagogy found that some critical pedagogy were quite rare in classroom practices, it is interesting to note that at least some of the teaching practices may improve toward higher quality by utilising Authentic Pedagogy standards (Newmann et al., 1996).

Newmann et al. (1996) found that students' performance would benefit if teaching practices are student-centred and if they explicitly use authentic pedagogical standards. The team of CORS researchers found that while the teachers and students made substantial progress toward meeting Authentic Pedagogy that seems to increase students' performance in all primary, middle and high grades in mathematics and science, gaps in authentic performance between students of different backgrounds were found. They also found that the effect of Authentic Pedagogy on authentic academic performance is stable in social studies but it is not in mathematics. This effect was high on the performance of students from elementary and high schools but it was lower for students in middle schools. They suggested that authentic achievement should be promoted among all students regardless of their backgrounds. Roelofs and Terwel' study (1999) examined the impact of Authentic Pedagogy

standards on secondary Dutch classrooms in grades 1-3. Dutch teachers and their students have different views regarding the aspects of teaching practices. The study found that while teachers indicated that they often practised certain features of Authentic Pedagogy; their students were less positive about their teachers' pedagogy.

Research on Authentic Pedagogy also highlights the importance of school as a professional community for teachers (Newmann et al., 1996; Newmann & Wehlage, 1995; Roelofs & Terwel, 1999). Newmann et al.' study (1996) on American restructuring of schools suggested that successful restructured schools should be organized to function as professional communities and they have to be provided with financial, critical and political support in order to enhance their organizational capacity to deliver Authentic Pedagogy. Using Authentic Pedagogy standards, Louis and Marks (1998) examined the impact of the school professional community on the intellectual quality of student performance. They indicated that schools' professional community for teachers that are associated with Authentic Pedagogy have positive impact on students' authentic performance. Manning, Sisserson, Joliffe, Buenrostro and Jackson (2008) evaluated their experience of conducting professional development experiences for the teachers and leaders from 23 Chicago small schools based on authentic intellectual achievement standards. They emphasized the importance and the challenge of extending professional development experiences that reflect authentic standards to the entire Chicago small schools. Roelofs and Terwel (1999) indicated that implementing Authentic Pedagogy stresses the need for supportive at school level to demand the change in the teacher's role, the change of teaching strategies and basic skills of classroom communication.

The main findings of the research on Authentic Pedagogy indicated quite a rare implementation of the standards of authentic standards in classroom practices. QSRLS Productive Pedagogies represent a refinement and expansion of standards of Authentic Pedagogy to maximise teacher's effects in respect of both knowledge production and of teacher and student identity. As mentioned by Braden (2004, p. 21) in comparison with Authentic Pedagogy, Productive Pedagogies "emphasize and expand instructional features, decrease emphasis on precise quantitative measures of student work and teacher work, increase emphasis on and provide more explicit formation of how cultural aspect influence personal significance/value beyond school and emphasize policy and implementation over description and research". The

Productive Pedagogies framework is not only considered to be a framework that offered standards of quality pedagogy but also a framework for reflection that can be used for effective teacher professional development programs (Education Queensland, 2010b).

2.5 Productive Pedagogies: Research Base and Dimensions

Productive Pedagogies was studied by the Queensland School Reform Longitudinal Study (QSRLS), under the umbrella of the “New Basics Projects” (Education Queensland, 2010b). The New Basic projects were undertaken by Education Queensland to align curriculum, pedagogy and assessment to provide integrated approaches to public school reform. They present new ways for enhancing students’ academic and social growth and preparing them for an ever changing society (Atweh & Bland, 2005). The New Basics Projects consist of three frameworks: New Basics, Rich Tasks and Productive Pedagogies as illustrated in *Figure 2.1* below (Education Queensland, 2010b).

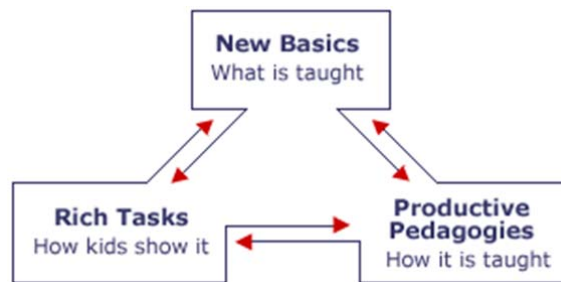


Figure 2.1 Frameworks of the New Basics Project. Retrieved from education.qld.gov.au/corporate/new-basics

The Productive Pedagogies study, that was commissioned by Education Queensland and undertaken by a group of University of Queensland-based researchers, began in 1997. At the end of the first year of QSRLS, the construction of a multi-dimensional framework of twenty Productive Pedagogies was formed, to reflect the basis of teaching practices that attempt to investigate improvement of academic and social outcomes for all students. The four dimensions of Productive Pedagogy were initially named: intellectual quality, supportive classroom environment, relevance, and recognition the difference. A classroom observational manual was also developed and used to capture the occurrence and regularity of some of classroom processes and events (Education Queensland, 2010a; Mills & Goos, 2007).

Over the following three years from 1998 to 2000, the research team conducted about 1000 formal observations from twenty four schools selected by “taking into account the basis of status for school reform and other factors such as location, demography and size. In this study, English, Science, Mathematics and Social Science classes in years six, eight, and eleven were observed by the researchers for looking at most effective productive practices for academic and social outcomes”. Extensive interviews with teachers and principals were also conducted to recognise the issues of schools’ support and policies. Classroom observations and interviews were combined by an analysis on samples of student work and assessment tasks (Hayes et al., 2006, p. 14; Lingard, 2005).

Responding to the need to assess the reality of the differences in Australian school society and the ability to deal with classroom diversity the term “recognition of difference” dimension changed to be “working and valuing difference” dimension. Productive Pedagogies research also expanded the meanings of “relevance” to “connectedness” dimension in order to provide students with social and cultural influences (Hayes et al., 2006, p. 19). The dimensions of Productive Pedagogies framework, namely, intellectual quality, supportive classroom environment, connectedness, and working and valuing difference, considered to express the meaning and the value of what “quality teaching” might look like and provide a descriptive language to support and engage teachers with sustained professional dialogue about their practices and performances (Atweh, 2007a; Aveling & Hatchell, 2007; Education Queensland, 2010b; Hayes et al., 2006).

The twenty Productive Pedagogies under the four dimensions are constructed in the Productive Pedagogies Classroom Reflection Manual, as a guide from Queensland Education, to provide an index of quality teaching and students’ learning and to be used to help teachers to reflect on their classroom practices and generating professional development dialogue. It could also be used to assist designing curriculum and learning experiences and help making intelligent decisions about individual students’ needs (Education Queensland, 2010b). The *Productive Pedagogies Classroom Observation Manual* offers explanations given for each dimension including examples and continuum of practice for each Productive Pedagogy scored on a five-point scale from 1 to 5 to indicate the range from low to high occurrence of Productive Pedagogies in classroom practices by considering

evidence seen during the specified teaching period (Education Queensland, 2010a; Mills & Goos, 2007). Table 2.2 showed the twenty Productive Pedagogies distributed in the four dimensions: intellectual quality, supportive classroom environment, connectedness and working and valuing difference. The pedagogies within each dimension are provided with key questions. The key questions reflect the main requirements of investigating the pedagogies and help teachers in building accurate perceptions about the overall requirements (Education Queensland, 2010a). Each dimension will be elaborated in turn in the following sections.

Table 2.2

Productive Pedagogies and their key questions

Dimension	Productive Pedagogies	Key questions
Intellectual quality	<i>Higher order thinking</i> <i>Knowledge as problematic</i> <i>Substantive conversation</i> <i>Depth of knowledge</i> <i>Depth of students' understanding</i> <i>Metalanguage</i>	Are students using higher-order thinking operations within a critical framework? Are students critically examining texts, ideas and knowledge? Does classroom talk lead to sustained conversational dialogue between students, and between teacher and students, to create or negotiate understanding of subject matter? Does the lesson cover operational fields in any depth, detail or level of specificity? Do the work and responses of the students demonstrate a deep understanding of concepts or ideas? Are aspects of language, grammar and technical vocabulary being given prominence?
Supportive classroom environment	<i>Student direction</i> <i>Social support</i> <i>Academic Engagement</i> <i>Explicit quality performance</i> <i>Criteria</i> <i>Student self-regulation</i>	Do students determine specific activities or outcomes of the lesson? Is the classroom characterised by an atmosphere of mutual respect and support between teacher and students, and among students? Are students engaged and on-task activities during the lesson? Are the criteria for judging the range of student performance made explicit? Is the direction of student behaviour implicit and self-regulatory?
Connectedness	<i>Knowledge integration</i> <i>Background knowledge</i> <i>Connectedness to the world beyond the classroom</i> <i>Problem based curriculum</i>	Does the lesson integrate a range of subject areas? Are links with students' background knowledge made explicit? Is the lesson, activity or task connected to competencies or concerns beyond the classroom? Is there a focus on identifying and solving intellectual and/or real-world problems?
Working and valuing difference	<i>Group identities</i> <i>Active citizenship</i> <i>Narrative</i> <i>Representative participation</i> <i>Cultural knowledge</i>	Does the teaching build a sense of community and identity? Are attempts made to encourage active citizenship within the classroom? Is the style of teaching principally narrative or is it expository? Are deliberate attempts made to ensure that students from diverse backgrounds are actively engaged in learning? Are non-dominant cultures valued?

Intellectual quality dimension

Hayes, et al. (2006) argued that teachers with high efficiency are inclined to concentrate on intellectual practices, maintain high academic standards, monitor students' engagement, and develop a warm, supportive classroom environment. The overall findings of QSRLS were consistent with authentic achievement standards that stressed the importance of intellectual quality in schooling as it might be a key uniting point for an advanced change and reform support (Dashwood, 2004). Marks, Secada and Doane (1996) argued that developing higher intellectual quality in students learning and professional practice is a strong theme in recent thinking on school reform. Newmann et al. (1996) claimed that teaching the basic knowledge should be in ways that promote the production of more multifaceted intellectual challenges. They indicated that high intellectual work should be based on: constructing of knowledge, gaining in depth understanding and valuing personal, social and cultural discourse and products. Dufficy (2005, p. 30) indicated that intellectual challenge in classrooms comes when we engage students with intellectually challenging tasks that have the following features:

- Connecting students' prior knowledge with unknown knowledge
- Involving problem solving, reflection, critical thinking and sharing ideas
- Assisting in-depth understanding and examination of ideas
- Allowing students' self- regulating and responsibility
- Encouraging motivation and competency

As the derive for intellectual quality outcomes, teaching for promoting higher order thinking is a crucial support for effective teaching and students learning. Lewis and Smith (1993) argued that teaching higher order thinking is important for all learners and it has to be carefully intertwined in classrooms. They suggest that the failure to investigate higher order skills may be the source of major learning difficulties. Zohar, Degani and Vaaknin (2001) studied teachers' knowledge and beliefs regarding low achieving students in the context of higher order thinking. The study showed that 45% of the 40 teachers that taught in two different schools (middle and high schools) believed that higher order thinking is unsuitable for low achieving students and that instruction of higher order thinking is appropriate only for high achieving students. Zohar et al.' study highlighted the need to change teachers'

beliefs that view students' learning as related to the academic level. They suggested working with teachers on developing their instructional practices to teach thinking for all students especially low achieving students.

McLeod and Reynolds (2007) argued that intellectual learning is improved when the classroom offers opportunities to the learners to talk constructively, to discuss and argue debatable issues and to question for understanding. Muijs and Reynolds (2011) emphasized the importance of effective classroom interactions to promote students' thinking and self-confidence. They indicated that substantive classroom discussions promote students' involvement and engagement and in helping students to develop deep understanding by allowing them to share their ideas and thinking during interactions between students and teachers and between students.

Language socialization in the mathematics classroom is one of the current concerns in mathematics education regarding the connection between classroom conversations about mathematics learning and teaching practice. While there is an agreement from the mathematics education reforms of the importance of classroom conversations, identifying the effective classroom communication practices is not clear for many teachers (Hicks, 1998). Chronaki and Christiansen (2005) argued that along with exploring the role of language in mathematics classroom practices, the patterns of social interaction in classroom interactions become more problematic in pedagogical practice. Rittenhouse (1998) suggested that using language should go beyond understanding mathematical technical terms and operations and the mechanisms of reading and writing to understand the effective mathematics communication in different contexts. Productive Pedagogies framework offers conceptions of the important features of substantive classroom conversations include: intellectual substance, dialogue, sustained exchange and logical extension and synthesis (Education Queensland, 2010b). Under the conception of Productive Pedagogies framework, the conversation reflects the dialogue and logical extension and synthesis features when classroom participants share ideas that promote improvement of cooperative understanding of a specific topic. The feature of intellectual substance moves the classroom's conversation from the recitation of information, facts, definitions and experiences to critical reasoning and applying. Classroom conversations can be extended to apply successive interchanges. Productive

Pedagogies also highlights providing high levels of talk and writing in classroom interactions (Education Queensland, 2010a).

Defining the intellectual quality of productive contributions to the relevant social and cultural norms within Productive Pedagogies supports the aim of teaching for social justice. The idea that intellectual learning is a social issue stresses the importance of the social context on effective teaching and learning mathematics knowledge process (Atweh et al., 1998; Gonzalez, 2009). Atweh (2007a) indicated that in traditional mathematics education reforms, intellectual quality often reflects the stance of learning mathematics as involving abstract concepts, symbolic language and standard efficient algorithms and proofs that are modelled in physical and economic problems. However, there is a recent increasing acknowledgment to view intellectual quality as a demand for social and cultural influences in order to develop diverse meanings and understandings of the generalised abstractions of mathematical concepts and skills. Atweh et al. (1998) argued that an interactive classroom learning environment that provides opportunities for reflection and engagement encourages constructing personal and cultural meanings of mathematics knowledge.

Gonzalez (2009) emphasized the importance of accessing high quality mathematics pedagogy for all students in order to use mathematics as a critical tool for understanding students' social life. He argued that teaching mathematics for social justice involves building upon the intellectual experiences of students from different backgrounds to value and examine social environments. Mathematics educators and teachers have invested considerable effort in developing teaching practices that aim to encourage the construction of mathematics' understanding and knowledge (Chinnappan, 2008). The research in the field of mathematics education continues to consider broader ideas of classroom pedagogies that consider the importance of the social issues to construct mathematics knowledge within classroom settings (Gutiérrez, 2012). The principles and standards for school mathematics that was produced by the National Council of Teachers of Mathematics (2000) in the United States emphasized the responsibility of the teacher for creating intellectual an environment where serious mathematics thinking and understanding are the norm. They indicated that effective teaching requires engaging and challenging students intellectually by providing meaningful mathematical tasks and opportunities for reflection and analysis.

Applying the elements of intellectual qualities that were developed in the Productive Pedagogies framework helped pre-service teachers to develop their own meaning and understanding. Alsharif (2011) studied the teaching practices of pre-service Saudi mathematics teachers who applied the elements of intellectual quality dimension that were developed in the Productive Pedagogies framework during their pre-service field experience. Alsharif's study showed that pre-service teachers attempted to apply the pedagogical elements of intellectual quality dimension in their teaching practice by challenging their students to construct knowledge to help their students to gain deeper understanding of the content discussed and to make the lesson more enjoyable. However, the results showed that the lack of experience of pre-service teachers and the insufficient teaching time that was allocated for one lesson limited their efforts to reach high levels of quality intellectual pedagogies.

Based on Productive Pedagogies framework, the dimension of intellectual quality aimed to help students to become producers of knowledge and be able to coherently communicate ideas, concepts, explanations, understandings and arguments. It includes six pedagogical elements: *higher order thinking*, *substantive conversations*, *depth of knowledge*, *depth of students' understanding*, *knowledge as problematic* and *metalanguage* (Education Queensland, 2010a) as explained in Table 2.3.

Table 2.3

The pedagogical elements of the intellectual quality dimension

<i>Higher order thinking</i>	Requires manipulating information and ideas through the processes of synthesis, generalization, explanation and analysis in order to help students to become producers of knowledge, problem solvers and sensitive to what is not openly expressed.
<i>Substantive conversations</i>	Involves a coherent shared understanding between teacher and student. This item encourages the teacher to evaluate the extent and the quality of talking to learn and to understand.
<i>Depth of knowledge</i>	Occurs when relatively complex connections are established to central concepts.
<i>Depth of students' understanding</i>	Occurs when relatively complex understanding of the main concepts is developed, new knowledge, ideas and problems are discovered, explanations and conclusions are constructed
<i>Knowledge as problematic</i>	Involves an understanding of knowledge not as fixed body of information, but rather as being constructed and subject to political and cultural influences.
<i>Metalanguage</i>	Emphasizes the importance of the specific technical vocabulary and words, of the meaning structures and text structures and of explicating high levels of talk about talk and writing.

Supportive classroom environment dimension

Most of the studies on classroom environments have identified the importance of a supportive classroom environment on encouraging students' engagement and learning. Recent research studies support the positive relation between effective academic achievement and academic motivation in that students can learn better in a positively motivated classroom environment (Dorman, 2001). Dart et al. (1999) found that developing students' meaningful learning can be promoted by creating learning environments that are safe and supportive and should be characterized by helpful relations and increasing students' learning responsibility. The key principles on motivation and emotion in classroom learning environments state that students are more motivated when they value the subject they learn, experience positive emotion of pleasure and pride and when they develop self-regulated learning (Boekaerts, 2010). Muijs and Reynolds (2011) identified the following main aspects of a warm, supportive classroom environment:

- Understanding of the environment in which students' views, opinions, emotional and social needs are valued
- Supporting competitive and cooperative environment
- Encouraging students' enthusiasm and participation
- Providing an appropriate and quick performance feedback
- Building effective social interrelations between teacher and students that are warm and friendly
- Enhancing students' contribution to the lesson

The principles and standards for school mathematics emphasize that teachers should establish and encourage a positive learning environment to learning mathematics that encourages students to think, discuss their ideas, and ask and solve problems (National Council of Teachers of Mathematics, 2000). Productive Pedagogies offer snapshots to help teachers to design experiences that move toward mathematical goals in a supportive classroom environment. Supportive classroom environment dimension in the Productive Pedagogies framework aims to ensure that students should engage seriously in their academic study (Hayes et al., 2006).

Research shows that such activities and teaching practices offer a supportive learning environment that explicit the criteria for judging student performances and encouraging them to take risks without fear of frustration from the teacher or other students. Alsharif (2011) indicated that mathematics teachers showed their ability to implement the pedagogies of *social support* and *academic engagement* at a good level in their teaching practices which raising the level of supportiveness of their mathematics classroom. This is consistent with the Productive Pedagogies research (QSRLS) that indicated that teachers are better at producing a supportive classroom environment (Gore et al., 2004).

The supportive classroom environment dimension aims to ensure that students engage seriously in their academic study. Such activities and teaching practices should explicit the criteria for judging student performances and encouraging them to take risks without fear of being “put down” by the teacher or other students. This dimension is a good aspect of successful classrooms as it stresses the importance of a socially supportive environment that must also be intellectually demanding (Education Queensland, 2010a, p. 15). It includes the pedagogies of: *academic engagement*, *student self-regulation*, *student direction of activities*, *social support* and *explicit quality performance criteria* (Education Queensland, 2010a) as described in Table 2.4.

Table 2.4

The pedagogical elements of supportive classroom environment dimension

<i>Academic engagement</i>	Is evident when students are attentive, deeply involved in pursuing the substance of the lesson.
<i>Student self-regulation</i>	Students demonstrate self-regulation when they show high implicit control on their behaviour or movement and dispositions.
<i>Student direction of activities</i>	Promote the importance to determine specific activities or outcomes of the lesson by the students. These activities are student-centred that may involve group work, individual research or investigative projects. They may be either independent or dependent on teacher regulations and characterised by high expectations.
<i>Social support</i>	Describes a classroom environment of mutual high respect and support among all members of the class. The climate of this environment encourages all members to learn important knowledge and skills and to try conscientiously to master challenging academic work.
<i>Explicit quality performance criteria</i>	Involves the specification in detailed, quality and exact outcomes and criteria for students' performance at different stages in lessons.

Connectedness dimension

Connectedness has been promoted as a valuable pedagogic strategy since the early twentieth century (Zyngier, 2003). As argued by Zyngier (2003, p. 43), such pedagogy can make a real difference in the productive learning of students when it falls between community needs and private actions in the classroom through connectedness. The QSRLS highlighted the importance of providing students with intellectually challenging tasks that are connected to their lives taking into account the difference in needs and ways of learning in a supportive equitable environment. Connectedness is characterised by challenging and connecting the new knowledge with the learner's background knowledge and in a larger social context. Students become more effective learners when they engage in learning experiences that have value and clear sense of purpose (Hayes et al., 2006).

Morony and Stocks (2005) indicated that making connections of mathematics to the real world, to other aspects of mathematics and to other school subjects and in students' abilities and needs can lead to motivation and engagement. Mathematics educational reforms and policy guidelines suggest that students should learn how to recognise and use connections among mathematics ideas and connect them with contexts outside of mathematics. The principles and standards for school mathematics establish the importance of the conceptual understanding of mathematics knowledge on building new mathematical knowledge from personal prior experiences, intuitions and from formal knowledge that is taught in mathematics classrooms (National Council of Teachers of Mathematics, 2000).

Zyngier (2005) claimed that pedagogical practices that connect students to their personal, cultural world are often ignored. White and Mitchelmore (2004) argued that mathematics teaching should be based on building new mathematics knowledge based on an existing knowledge and applying it in ways that have meaning beyond the classroom. As argued by Chinnappan (2008), mathematics pedagogy can be characterised as having a high quality of organized mathematical connectedness by developing multiple representations and complex relations with other mathematical experiences in meaningful contexts. This argument suggests the role of Productive Pedagogies to draw the way to investigate this connectedness for numeracy by examining real and practical issues beyond the classroom. However, many students

did not consider the value of learning mathematics because “mathematics pedagogy is disconnected from the implication of background knowledge and connectedness” (White & Mitchelmore, 2004, p. 595). Alsharif’s (2011) showed that in investigating the ability of Saudi pre-service mathematics teachers to introduce Productive Pedagogies to their teaching practices, the teachers have limited implication of connectedness and most of their attempts to connect the mathematics lesson content beyond the classroom were artificial and meaningless. Alsharif suggested that the lack of introducing the pedagogies that encourage connectedness is related to teachers’ previous learning that focused on abstract approaches. He indicated that following the mathematics textbooks that omitted real life activities and the restricted discussions between teachers of the same subjects or different subjects around preparing learning and teaching that has rich meaning, limited their ability to connect mathematical knowledge to the real world outside the classroom.

In the notion of Productive Pedagogies the connectedness dimension describes the “extent and value of students’ engagement with their prior knowledge, knowledge from multiple areas and with issues or problems in the larger social context within which students live” (Education Queensland, 2010a, p. 10). It includes the pedagogical elements: *knowledge integration*, *background knowledge*, *connectedness to the world beyond classroom* and *problem-based curriculum* as illustrated in Table 2.5.

Table 2.5

The pedagogical elements of the connectedness dimension

<i>Knowledge integration</i>	Describes the teaching practices, content and activities that achieve significant integration of knowledge from different school areas. Knowledge integration occurs when the boundaries of subject areas are not recognisable.
<i>Background Knowledge</i>	Occurs when teaching practices offer opportunities to the students to make connections between their background knowledge and the new content, skills and competencies. Students’ background knowledge may include their background community, school knowledge and cultural knowledge, sources and experiences.
<i>Connectedness to the world beyond the classroom</i>	Describes lesson topics, teaching practices and activities that help students to study recognise and explore the connection between their personal experiences and real-world public and global problems in order to create a personal value meaning that is significant for the knowledge.
<i>Problem-based curriculum</i>	Occurs when students are presented with small problems or large problems that are real and practical. Substantial knowledge construction, creativity and engagement are required over a number of lessons.

Working and valuing difference dimension

The current issues of equity, social justice and diversity in the classroom present some unique demands on schools and teachers to work with this diversity to maximise intellectual and social outcomes. Atweh and Seah (2008) claimed that supporting the learners to react positively in their private and social life requires providing them with opportunities to learn about equity and helping them to contribute to the ideological, cultural and political development of society. Classroom experiences should offer equal opportunities for all students to learn and be involved regardless of their gender, ethnicity, race and socioeconomic backgrounds (Campbell & Langrall, 1993). Some of the strategies that mathematics teachers can use to promote equity and achievement in diverse classrooms are described by Kitchen (2005). He indicated that one of strategies is making personal connections with students in order to make observations and justifications about incorporating equitable learning opportunities that promote higher order mathematics thinking and encourages a coherent shared understanding. Other strategies that teachers can use are creating mathematical social and political connections. These strategies require the teachers to evaluate the values that are implicit in the mathematics content that motivate reflections among students and build their pedagogical approaches.

Many school mathematics teachers face challenges utilizing their pedagogical strategies for social justice outcomes (Garii & Appova, 2012; Kitchen, 2005). As mentioned by Lerman (2000), many mathematics teachers struggle to find ways to enable participation in social practices from the unique experience of each learner or group from different sociocultural experiences in the classroom. Atweh (2007b) indicated that the challenge that school mathematics teachers faced is to design learning and teaching activities that support students to be responsible members of society. Limited classroom practices that involve social justice issues, as Gutstein (2003) showed, are for two reasons. Firstly, the idea of questioning the norms and power within society are potentially problematic for teachers and students. Some school educational systems avoid discussing social issues and concentrate on encouraging the isolation of teaching from social justice issues. Second reason is related to the roots of teaching mathematics as objective knowledge rather than

socially-constructed that limits the teachers' efforts to incorporate social issues within their teaching practices (Gutstein, 2003; Kitchen, 2005).

Meeting the needs of all students in diverse classrooms is crucial for teachers. Zevenbergen et al. (2008) indicated that the increase of the diversity in classrooms required new forms of quality that enable the highest possibility for success for all students. Some teachers find difficulties in incorporating diverse learning approaches and cultural pedagogies to their teaching practices due to the lack of their teacher education programs in devoting attention to prepare teachers to teach for diversity (Luykx, Cuevas, Lambert & Lee, 2005). Dempsey and Arthur-Kelly (2007) indicated that at least two issues that are needed to enhance the teachers' effective pedagogies in diverse classroom are: teachers' reflecting on their practices and their flexibility to improve the efficiency of their pedagogies.

Mathematics education research has paid increasing attention to the need to challenge pre-service teachers to recognise and evaluate the cultural norms implicit in various mathematics contexts in which teaching and learning mathematics takes place. Kitchen (2005) suggested that pre-service mathematics teachers should be provided with opportunities to review and create learning experiences that are socially or culturally connected to their students. Dunn (2005) studied the pre-service teachers' pedagogies in mathematics lessons during their school field experience for eight weeks in order to investigate their abilities to reconceptualising mathematics for diverse students. This study suggested developing teacher education programs that are aimed at raising pre-service teachers to a greater awareness of social factors within mathematics classrooms. In the study of Garii and Appova (2012), the ability of eighteen pre-service mathematics teachers who teach grades 1-6 to use their pedagogical and mathematics knowledge to incorporate social justice into their mathematics lessons was examined. The results showed that the pre-service teachers shared the lesson plans that addressed social justice issues with their peers. While pre-service teachers understand the meaning of social justice in teaching mathematics, there is a need for greater encouragement on the actual incorporating of social issues in their pedagogical actions and curricular content. Moreover, Leonard and Dantley (2005) examined the effective pedagogies that were used to influence the attitudes and beliefs of 107 pre-service teachers who were enrolled in the mathematics methods course (MATH ED 141) about teaching diverse students

during the academic year 2000-2001. This study highlighted the importance of engaging pre-service teachers earlier in their teacher education courses with activities to construct new concepts about social-justice issues in order to critically reflect on their situations and to take transformative action in the future in real school mathematics teaching practices. Kitchen (2005) also studied the journals that were provided by pre-service teachers during their study of two-months in the university course in which teachers examined how to use statistics enhanced specific social issues from printed articles. Kitchen's study suggested the importance of preparing pre-service teachers to incorporate the real-world issues from real data in order to demonstrate to their students that they have a role to resolve political and social issues.

The framework of Productive Pedagogies argues the centrality of teachers in improving the academic and social outcomes of all students and to provide a useful lens to analyse and examine the richness, complexity and aspects of classroom experiences. Productive Pedagogies emphasized that intellectual social-support classroom experiences will be effective in improving students' outcomes if they value and reach all students especially disadvantaged students. These socio-economically disadvantaged students need to draw in their minds and beliefs that learning has some meaning to their public and personal lives. Lingard (2005) stressed the role of the Productive Pedagogies model using pedagogies of *narrative*, *inclusivity* and *active citizenship*. The research on applying Productive Pedagogies suggests the importance for teachers to understand the impact of social elements on students learning and to improve their pedagogies towards more valuing of the diversity of the classroom. Tanko (2012) applied the Productive Pedagogies framework to reflect critically on his teaching practices that consider the issues of social justice in his teaching of Practical Numeracy to the Diploma Foundation students in the United Arab Emirates. He indicated that teaching for social justice helps pre-service teachers to take responsibility for their learning outcomes and to identify the connectedness of mathematics to everyday actions and problems. Alsharif's (2011) also emphasized that need. It showed that during observation of the teaching practices of pre-service mathematics teachers in their field experience, it was clear that introducing social elements were omitted. Alsharif indicated that the limited understanding of teachers about some social and cultural issues appeared to

play a main role in the lack of the implementation of the pedagogies that recognise and value differences between students. The Queensland School Reform Longitudinal Study (QSRLS) also identified the need for more valuing of differences in pedagogies in order to improve productive performance and learning. They suggested giving more attention to connect students' work to their biographies and their world outside the classroom (Lingard et al., 2000).

In the Productive Pedagogies framework, working and valuing difference dimension is the most theoretically significant dimension that aims to ensure that students, especially from disadvantaged sociocultural backgrounds, can improve academic achievement, value a range of cultures, respect individuals and create positive and legitimate aspects about their classroom community. This dimension includes the pedagogical elements: *cultural knowledge*, *inclusivity*, *narrative*, *group identities in a learning community* and *active citizenship* (Education Queensland, 2010a) as illustrated in Table 2.6.

Table 2.6

The pedagogical elements of working and valuing difference dimension

<i>Cultural knowledge</i>	Involves curriculum knowledge, practices and ways of knowing in the classroom environment, valuing the social characteristics of different cultural groups such as gender, religion, age and economic states for all students. This happens through the inclusion, recognition and transmission of cultural knowledge.
<i>Inclusivity</i>	Describes classroom practices that support the diversity of the students, treats them as a homogeneous group and recognises the variations in their learning needs.
<i>Narrative</i>	Involves that lessons processes and lesson content consist of a linked sequence of events such as personal stories, historical issues and cultural texts.
<i>Group identities in a learning community</i>	Occurs when classroom presentations offer positive recognition of different group identities in order to build a strong sense of the classroom, school and wider community.
<i>Active citizenship</i>	Aims to ensure that all individuals and groups are not excluded from the classroom practices and the institution and they have rights and responsibilities.

The previous discussion about the theoretical framework of Productive Pedagogies emphasized that students should experience classrooms that are intellectually demanding connected to their lives and personal experiences, recognising the different needs and different ways of learning in socially supportive situations

(Hayes et al., 2006; Zyngier, 2005). The following section will explain the role of Productive Pedagogies as a framework of quality teaching, reflection and teacher professional development from the research base.

2.6 Productive Pedagogies for Improving Quality Mathematics Learning and Teaching

Gore, et al. (2004) and Hayes, et al. (2006) argued that Productive Pedagogies provide a feasible alternative framework for bringing greater coherence and more confident knowledge base to the role of school teachers. Hayes et al. (2006) indicated that Productive Pedagogies framework places more attention on quality demand by providing four simple intellectual dimensions to achieve high quality learning outcomes for all students. The framework of Productive Pedagogies is an example of a useful framework for teachers who want to provide a learning environment which is “both intellectually demanding and socially supportive, both academic rigorous and connected knowledge, and both proficient and meaningful learning” (Gore et al., 2004, p. 386).

Chinnappan (2008) argued that the Productive Pedagogies framework can enable mathematics teachers to think deeply about quality teaching mathematics and to draw on Productive Pedagogies dimensions to construct learning experiences that deal with the complex interpretations of mathematical ideas. He examined the characteristics of mathematical understanding for numeracy that learners could construct in order to participate in activities related to their global community. These characteristics were analysed by drawing on the theoretical perspective of the connectedness dimension of the Productive Pedagogies framework. Chinnappan suggested that organizing higher complex mathematical links and reinforcing meaningful and problematic contexts will help to build strong and deep understanding of mathematics concepts and conventions.

While the Productive Pedagogies framework appears to provide suitable approaches for supporting mathematics learning, the main concern is the way of translating Productive Pedagogies dimensions into real mathematics classroom actions. Chinnappan’s study (2006) argued that mathematics teaching pedagogies need to offer multiple opportunities to engage constructively and critically with mathematics concepts and ideas. Recent research work, about mathematics pedagogy,

mathematics teachers and mathematics educators that focuses on mathematics construction, supports the traits of Productive Pedagogies. Chinnappan' study aimed to ascertain the quality of teachers' pedagogies of a group of beginning primary teachers using a collaborative network online learning environment. The participations were expected to demonstrate a rich understanding of "multiplication" that is recognised as one of the central primary mathematics concepts. The results found that Productive Pedagogies enable mathematics teachers to construct and implement learning experiences that are culturally sensitive and comprehensive. Varying degrees of espousing and embracing Productive Pedagogies dimensions by participants are the main significant results. These results indicate that these teachers were flexible and could adapt to learning conditions if the situation demanded.

Cronin, Sarra, and Yellend (2002) drew on the Productive Pedagogies framework to evaluate the role of pedagogical practices in order to enhance positive outcomes in numeracy for young Australian Indigenous students. This study examined the childhood pedagogical practices of four teachers and their relation to their teaching style, methods, cultural values and numeracy development. The results found that Productive Pedagogies are relevant and applicable for teaching numeracy to Indigenous students. Lerman and Zevenbergen (2006) built on Productive Pedagogies to analyse 40 classroom lessons of middle school teachers who used ICT. The results indicated the poor use of most the Productive Pedagogies in teaching of mathematics specifically the pedagogies of the dimension of working and valuing difference such as *student direction*, *active citizenship* and *inclusivity*. Lerman and Zevenbergen's study suggested that explicit mathematics teachers' consciousness of different forms of pedagogy in different social groups may meet more equitable outcomes.

2.7 Application of Productive Pedagogies in Teacher Development

Productive Pedagogies framework has highest levels of agreement in recent teacher education reforms with close alignment of the intellectual quality dimension in the Productive Pedagogies framework with the "disciplinary preparation" concept in teacher development reforms and the alignment between working and valuing difference dimension in Productive Pedagogies with "multicultural competence" in teacher education reforms. Supportive classroom environment and connectedness

dimensions are closely related to teacher education frameworks that aim for good teaching in traditional and recent educational reforms (Gore, 2001). The Productive Pedagogies research found independent positive effects of the role of teachers in classrooms and the impact of their disciplinary knowledge and professional development on effective teaching practices. They suggested that improving students' outcomes requires valuing and developing significant relationships between teacher learning and students learning in real learning situations (Hayes et al., 2006; Lingard, 2005). Reviewing the literature, most research studies on implementing Productive Pedagogies in teacher development focused on exploring the value of Productive Pedagogies on teacher learning as a framework for reflection and for designing teacher professional programs, on investigating the impact of applying Productive Pedagogies on teaching improvement and students performance and on examining actual teachers' implementation experiences of Productive Pedagogies in classrooms.

2.7.1 Productive Pedagogies as a framework for teacher learning

In current reforms teacher professional development programs continue to play a significant strategy of educational reforms to develop teachers' pedagogical experiences through the ideas of critical reflections on personal teaching and on others' classroom practices. Gore et al. (2004) argued that the Productive Pedagogies framework can be used as a professional development program for teachers who desire to achieve a significant change in their classroom practices from their preparation stage of the lesson to the final evaluation stage. The Queensland School Reform Longitudinal Study (QSRLS) research of Productive Pedagogies recognised the consideration of teacher professional learning as a requirement for changing classroom pedagogies and to disseminate Productive Pedagogies across the school culture. It found that schools who reflected the most widespread practices of Productive Pedagogies focused on teacher development programs and they have substantial collaborations not just inside the school community but with other schools and educational communities (Lingard et al., 2003). Gore et al. (2004) studied the improvement of the overall performance of two groups of teachers after conducting a professional development program to introduce Productive Pedagogies. One of the groups consisted of 12 in-service teachers from a small rural primary school and the other group included 14 teachers from a secondary urban school.

Using a series of workshops and group discussions of participants' in observed classes, teachers developed an understanding of Productive Pedagogies dimensions and drew their planning of lessons and assessment tasks from it. Gore et al. found that teachers are able to improve their practices and they are able to produce higher levels of Productive Pedagogies. The results highlighted the impact of providing opportunities of professional collaborations on teachers' understanding and their ability to implement Productive Pedagogies in their teaching practices.

While Productive Pedagogies can be used as a framework for teacher learning within school a community, it can be also applied as a framework for pre-service teachers' training. Gore et al.'s study (2004) introduced the Productive Pedagogies framework to a group of pre-service teachers in their final year of their teacher preparation program. The concept of the Productive Pedagogies framework was introduced through a series of 18 hours of workshops. The teaching practices of the pre-service teachers were observed and coded using the QSLRS classroom observation manual. The participants highlighted the importance of utilizing Productive Pedagogies as a part of their fundamental teacher preparation programs. Alsharif's study (2011) aimed to investigate the Saudi pre-service teachers' engagements with the Productive Pedagogies framework and their ability to implement it in their teaching practices. A group of eighteen pre-service teachers in their final year in higher education were introduced to Productive Pedagogies in the unit of "Mathematics Teaching Methods". Data from focus groups, interviews, classroom observations and reflective journals emphasized the value of the Productive Pedagogies framework on pre-service teaching as it guides and organizes their planning and teaching practices. The pre-service teachers showed improvement in their teaching practices towards student-centred teaching and learning.

2.7.2 Productive Pedagogies as a framework for reflection on teaching practices

Hill (2002) argued that part of the required teacher professional development is the reflection on teaching practices using the Productive Pedagogies framework. The model of Productive Pedagogies offers a basis for teachers to reflect on their teaching practices. Lingard et al. (2000) argued that schools, as a teacher professional learning community, may offer opportunities for reflection on pedagogies by conducting substantive conversations, group work discussions and collaborative lesson

preparation. From the perspective of reflection a “critical friend”, Hill (2002) offered opportunities for a group of teachers in a professional development program that was designed utilising the Productive Pedagogies framework. In Hill’s study, Productive Pedagogies was introduced to the participants using strategies of exploring and examining participants’ own understanding of the framework by identifying the aspects of Productive Pedagogies in video-taped lessons. The reflective experience also included generating teaching strategies that were helpful in implementing Productive Pedagogies. Hill suggested that creating constructive dialogue about the Productive Pedagogies framework for teachers enables them to use the framework to review their teaching practices.

The Productive Pedagogies framework can be used as a metalanguage for developing teacher’s knowledge and understanding of teaching. Hayes (2003) claimed that the main influence of the Productive Pedagogies framework in the process of supporting the three message systems of schooling that are to assessing meaningful learning, informing curriculum reforms and developing quality pedagogy is its effect for supporting professional dialogue among teachers. Zyngier’s study (2005) studied the value of Productive Pedagogies for pre-service teachers as a metalanguage to describe and analyse their lesson observations. The pre-service teachers who participated in the study were engaged in powerful substantive conversations about their teaching practices. They were able to talk about their observations utilising the language of Productive Pedagogies. Zinger’s study suggested that Productive Pedagogies is suitable for all teaching styles and grade levels even for teaching infants. Teachers who participated in this study emphasized the value of using Productive Pedagogies as a metalanguage for analysing, evaluating and assisting their teaching practices for higher order thinking and helping teachers to engage with differences in their students.

Most of the research studies indicate that teachers identify the potential value of Productive Pedagogies for improving their teaching practices. Gore et al. (2004) identified the value of the Productive Pedagogies framework as a research tool for exploring and evaluating classroom practices, assessment tasks, and student performances that have positive impact upon academic and social outcomes of all students. Alsharif and Atweh (2010) indicated that by introducing a group of pre-service teachers in a teachers’ pre-service unit in mathematics education in Saudi

Arabia, teachers expressed very positive views about the potential of Productive Pedagogies as a valuable framework that guides and organizes their teaching practices. They mentioned that their teaching approaches improved towards being learner-centred. Gore et al.'s study (2004) also indicated that some of the pre-service teachers who apply Productive Pedagogies to their teaching practices during undertaking an elective subject on teaching viewed Productive Pedagogies as a valuable framework for guiding and organizing their teaching.

Despite the importance of Productive Pedagogies as a framework for teacher's planning, discussing and work evaluation and as an organizer of their classroom practices, the research on the actual implementation of Productive Pedagogies in classrooms show little evidence of an optimal application of Productive Pedagogies dimensions in teaching practices. Gore et al.'s study (2004) showed that while some of the pre-service teachers who applied Productive Pedagogies to their teaching practices recognized Productive Pedagogies as a valuable framework for guiding and organizing their teaching, others viewed Productive Pedagogies as additional to their normal lesson planning that has limited applications. Alsharif and Atweh's study (2010) indicated the classroom observations of the observed lessons of Saudi pre-service teachers in their field work indicated their classroom practices showed little evidence of pre-service teachers understanding and attempts to apply the connectedness dimension in the mathematics content as encouraged by the Productive Pedagogies framework. These findings are consistent with the findings from the Queensland School Reform Longitudinal Study (QSRLS) that showed low levels of the pedagogies of intellectual quality dimension, connectedness dimension and recognition of difference dimension.

While the research core of this study is related to the Productive Pedagogies of school teachers, it is important to take a general stance about "Productive Assessments" since the Productive Pedagogies framework argues about the purposeful correlation between Productive Pedagogy, productive assessments, and students' outcomes (Hayes et al., 2006). Productive Pedagogies research also argues the importance of productive assessment to inform classroom practices, to indicate and support individual student's learning and to show the success of schooling by providing measures of academic and social outcomes (Hayes et al., 2006; Lingard, 2005). The basic standards of productive assessment are shaped in a model of 18

elements distributed among the dimensions of Productive Pedagogies. By analysing the assessment tasks of students from grades 4, 6, and 8, the QSRLS findings indicated that in many ways such assessment tasks do not reflect students' acquisition of knowledge (Hayes et al., 2006). They found that most assessment tasks that were analysed especially in grades 6 and 8 did not explicit students experience beyond the classroom. They believed that assessment tasks are a basic element of quality classroom practices and it needs to be a basic part in teachers' professional dialogue (Hayes et al. 2006). For teacher educators, Aveling and Hatchell (2007) argued the value of Productive Pedagogies to examine their own pedagogies by analysing 257 students' responses to specific final examination questions relating to teaching strategies, stories and gender. Most of the students were pre-service teachers from diverse backgrounds. The analysis indicated the need to move from summative to productive assessment that utilises the four dimensions of Productive Pedagogies.

2.8 Challenges of Applying Productive Pedagogies

Regarding the factors that can limit teachers to introduce Productive Pedagogies in their classroom practices, the research indicated some of the problems that teachers faced. Lingard (2005) indicated that there were some structural factors that the QSRLS findings showed such as class and school sizes, testing policies, crowded curriculum, time constraints, work pressures on teachers and lack of effective professional development programs. He suggested that while these factors are more serious in some school systems than others, the challenge of these factors was evidenced in the implementation of Productive Pedagogies in schools. This was consistent with the findings of Newmann and his colleagues on Authentic Pedagogy that showed that all students will benefit if school systems support working conditions toward achieving quality pedagogies to enhance students' performance (Newmann et al., 1996). Moreover, Gore et al.'s study (2004) indicated that pre-service teachers cited time as a constraint that is linked to everyday teaching work beside their university work. This constraint as they noted limited their ability to plan and prepare to apply Productive Pedagogies in their classes. In his applying of Productive Pedagogies framework to reflect on his own teaching practices for pre-service teachers, Alsharif (2011) indicated that while Productive Pedagogies helped him to organize his teaching practices, he experienced the challenges of transforming from his teacher-centred approaches that he always applied to student-centred

teaching approaches as well as to connect the lesson to the world beyond the classroom in each class. Alsharif indicated that the change and improvement in applying Productive Pedagogies needs time, resources and continuous reflection.

Moreover, some limitations, questions and comments have to be considered in applying the Productive Pedagogies framework. Sellar and Cormack (2009, p. 125) indicated that while the Productive Pedagogies framework is centred on the importance of pedagogy, “it does not describe the actual pedagogical processes in detail, such as the actual process of pedagogy to generate deep knowledge and deep understanding”. This comment may indicate that the Productive Pedagogies framework focus in the direction of pedagogical outcomes rather than processes.

In addition, some research’s’ perspectives about the Productive Pedagogies framework indicate that it is too large and the most effective way to deal with it is by concentrating on one dimension. However, Hayes et al. (2006) mentioned that the argument of this framework is to investigate high quality of teaching and learning outcomes. This argument highlights the importance of all Productive Pedagogies dimensions and opens the door to more flexibility in implementing all of them in classrooms according to learning objectives, students’ needs and teachers’ decisions to reach the goal of high quality outcomes.

In relation to using the classroom coding instrument that was developed by Productive Pedagogies research, Hayes et al. (2006) indicated that one of the major limitations of using this instrument is its narrow coding range that may not sufficiently describe all pedagogical processes. As mentioned by Gore et al. (2004), some of the difficulties with coding the dimension of working and valuing difference are identified since it may have not enough explanations of the elements of this dimension such as the pedagogies of *inclusivity* and *active citizenship*. However, Hayes et al. (2006) claimed the main point of interest of the Productive Pedagogies framework is its concern about the quality rather than quantity of measurements. It is about reflection rather than coding. They suggested more explanations and examples of implementation are needed to support the concept of recognition of the difference dimension

Some other limitations include lack of teachers understanding of the framework, an insufficient focus on teachers’ pedagogical content and an absence of students’

voices that are related to Productive Pedagogies research (Hayes et.al, 2006; Mills & Goos, 2007). These issues support the importance of professional development programs for teachers to develop better understanding of Productive Pedagogies and also the use of combinations of surveys and interviews to picture all the integral components of classroom systems from different research methods.

2.9 Chapter Summary

This chapter has provided a review of the relevant literature for the present study. The quality of teaching that is necessary to achieve good quality of students' learning appears to matter; and it specifically matters for mathematics teaching and learning. Understanding students' learning requires understanding students' needs, responding to their needs and identifying the influence of the environment on students' learning. The research literature has shown that while there is strong responsibility that drives teachers' pedagogies to improve students' intellectual and social outcomes, there is a need to support teaching practices within the classroom. The complex and demanding teaching practices, the diversity of classroom context and the influence of educational philosophies and social perspectives reshape the responses that teachers should make to be facilitators of learning.

The review of literature shows that in order to improve the quality of teaching, there is a need to qualify pre-service and in-service education towards quality pedagogy. Preparing teachers to achieve quality teaching requires engaging them in quality professional development experiences. Offering opportunities for teachers to be involved in critical reflection on their own practices and on those of their peers can make a difference on teachers' professional growth and on students' learning.

The chapter also reviewed the influence of constructivism on shifting from the traditional idea of transmitting knowledge to students towards facilitating students to construct their own meaning and understanding through effective social reactions. While the social teaching of constructivist perspectives opened the door to add an emphasis on the role of classroom interactions to support the construction of mathematics knowledge, this emphasis was supported with many efforts to introduce the social norms in mathematics teaching and learning in order to prepare students to be agents of change in their society. The review of the literature pointed to some of the pedagogical models that aim to qualify teaching by considering the importance of

classroom pedagogies such as the standards of Authentic Pedagogy and the framework of Productive Pedagogies. The research literature demonstrates the benefit of utilising the Productive Pedagogies framework in classroom practices and the requirement to support teacher education priorities that focus more on teaching quality and processes that consider the importance of offering rich and meaningful learning and teaching experiences. This review also emphasizes the worth of Productive Pedagogies in qualifying school teachers' professionalism and pre-service teacher education. The review of literature also shows that using the Productive Pedagogies framework helps mathematics teachers to think more deeply about lesson aims, context tasks, and their teaching practices. Mathematics teachers could draw on the various dimensions of Productive Pedagogies to evaluate their classroom practices, construct a consequence of mathematics learning experiences and utilise their practices that could motivate learners to extend their mathematical construction in different directions. Mathematics teachers could also use the Productive Pedagogies framework to provide a holistic meaning of mathematical knowledge in order to develop citizens who are actively developing cultural and intellectual achievement.

CHAPTER THREE

RESEARCH METHDODOLOGY

The previous chapter on literature review provides the background for investigating the research objectives of this study. This chapter presents the research methodology employed by this study and focuses on the various aspects about how the research was conducted. This chapter is divided into eight main sections. Section 3.1 outlines the research's objective and aims. Section 3.2 reviews the research methodology. A vision around the research context is provided in section 3.3. Section 3.4 explains the research design by describing the research sample, data collection methods and research procedures that were used to investigate the research's objectives. Section 3.5 includes a discussion of the quality of data. Section 3.6 offers a consideration of some ethical issues related to this research. Data analysis is explained in section 3.7 and section 3.8 provides a summary of the chapter.

3.1 Research Aims

The overall aim of this study is to investigate the introduction of Productive Pedagogies to a group of Omani mathematics teachers as a framework for quality teaching and reflection by teachers on their practice. In particular, specific research aims were formulated as follows:

- To examine the value of Productive Pedagogies framework on enhancing teaching quality of Omani mathematics teachers. To achieve this aim, this study aims to investigate the:
 - development of Omani mathematics teachers' understanding and ability of implementing Productive Pedagogies;
 - The benefits of implementing Productive Pedagogies by mathematics teachers and
 - challenges experienced by mathematics teachers in the implementation of Productive Pedagogies;
- To examine the appropriateness of implementing Productive Pedagogies framework in the educational system of Oman and

- To investigate students' perceptions of the change in pedagogy in their mathematics classrooms.

3.2 Research Context

This research was located in the context of Omani schools in cycle two for grades (5-10). The Omani Educational context is traditionally Arabic and Islamic oriented. The Omani Educational Philosophy is derived from its own particular sociocultural heritage and values (Al-salmi, 1994). The educational system in Oman is new and it is under substantial development. The following sub-sections provide some details about the research context that are useful for understanding the procedures and methods that are used to attain the objectives of this study. They include information about the geographic location of the Sultanate of Oman and its educational system.

3.2.1 The geographic location of the Sultanate of Oman

The Sultanate of Oman is an Asian country which is located in Southwest Asia and occupies the south-eastern tip of the Arabian Peninsula. It is the third largest area among Arab countries with a total area of 309,500 km² (Ministry of Information (Oman), 2011). According to the final results of the 2010 census, the total population of Oman has gone up to 2,773,479. Oman is divided into ten Governorates (Muscat, Dhofar, Musandam, Al Buraymi, the North Batinah, the South Batinah, Ad Dakhliyah, Ad Dhahirah, Al Wusta and Al Sharqiyah). The research was conducted in the North Batinah Governorate which occupies an important location as a coastal strip between the sea and the mountains with an area of 12,500 km² and a total of population of 772,590. With its 483,582 inhabitants, according to the 2010 Census of Population, Housing and Establishments, the North Batinah Governorate has one of the largest population concentrations in the Sultanate after the Governorate of Muscat. It has six "Wilayas"; Sohar (the centre), Shinas, Liwa, Saham, Al Khabourah and Al Suwaiq (National Centre for Statistics and Information (Sultanate of Oman), 2011).



Figure 3.1 Map of Oman. Retrieved from liezel.8m.com

3.2.2 Educational system in the Sultanate of Oman

Education in Oman until the second half of the nineteenth century was offered only by the Muslim elders in the mosques and concentrated on teaching the fundamentals of reading, writing and arithmetic. The teachers were not professionally qualified educators (Al Ghafri, 2002).

An extraordinary turning point in the history of education was marked in July 1970 with the beginning of an inclusive social economic and political development, following the agreement to install His Higher Majesty Sultan Qaboos to power in the Sultanate of Oman (Al Ghafri, 2002). The spread of education shaped a priority in government policies. Despite the challenges of the lack of school buildings, teachers, textbooks, and other requirements from 1970 to 1975, the seeds of development were sown during that period. Oman's Educational program then expanded from only three formal schools in the whole country in 1970 to 1,052 public schools in 2007-2008 (Ministry of Education (Oman), 2008).

The Ministry of Education in Oman, in the light of the recommendations of the "vision of Oman's economy 2020", embarked on developing the quality of education

in order to achieve the development of a new modern society (Issan & Gomaa, 2010; Ministry of Education (Oman), 2004a, p. 20). To achieve this, a new Basic Education programme was developed in 1997 to gradually replace the General Education System which consisted of three levels (primary, preparatory and secondary).

The formal Basic Education System is organised into three Cycles: Basic Education Cycle One which covers grades 1 to 4 and ages 6 to 9, Basic Education Cycle Two covering grades 5 to 10 and ages 9 to 15, and Post Basic Education which is the final Cycle covering grades 11 and 12. Basic Education Cycle One classes contain mixed gender students and employs only female teachers and staff. The feminization of the teaching force and administrative staff of Cycle One schools is one of the main features of the Basic Education System (Ministry of Education (Oman), 2008). Rassekh (2004) indicated that the employment of only female teachers in Cycle One is because the Ministry of Education in Oman believe that female teachers are generally be more appropriate for and understanding of younger students than male teachers. Taking into consideration Omani cultural norms, males and females are segregated in most Cycle Two schools and all Post- Basic schools. This research was conducted within Cycle Two female schools (grades 5-10) and specifically in grades seven and ten.

The main aim of the Basic Education system is to prepare students for higher education, the labour market and to produce citizens who can live and work productively in a dynamic complex world (Ministry of Education (Oman), 2004b). Al Mushaifri (2006) indicated that the Basic Education System was developed to ensure that students' learning is relevant to their present and future needs as citizens and to prepare them to meet the demands of rapid social changes taking place in Oman. Rassekh (2004) noted that that the specific improvements within teaching and learning areas under the Basic education system include:

- Changes in curriculum content to adopt critical thinking and problem solving
- Improvement in teaching methods to concentrate on learning through active experiences
- Changes in student assessment from summative to formative assessment
- Improved teacher training

This research study is aligned with the above list of improvements by focusing on improving teaching in schools. Under the Basic Education programme, the goals of the Omani mathematical curriculum are for students to obtain important mathematical knowledge (including mathematical facts and experience in mathematical activities) necessary for daily life and practical use, to acquire the skills of problem solving, and to learn reasoning and mathematical thinking in order to solve problems in their daily life and in other school subjects. In addition, the mathematics curriculum aimed to sensitize Omani students to the connection between mathematics, nature and human society (Ministry of Education (Oman), 2011). The main objective of the mathematics curriculum in the formal Basic Education System for grades seven and ten is to enable students to develop the following skills (Ministry of Education (Oman), 2011):

- Perform mathematical operations in Algebra and Number Theory such as real numbers, equations, and functions and use them in real life applications
- Know and apply Euclidian Geometric theories such as triangular geometry and circular geometry
- Carry out simple research on financial issues using mathematics concepts and rules
- Know concepts about probability and use them in real life experiences.
- Understand the applications of probability and statistics in real life situations
- Understand and use triangulation ratio concepts in real life applications

The role of mathematics teachers in the classrooms has also changed in the Basic Education System to that of creator of interactive learning environment. Their teaching approaches are student-centred rather than teacher-centred (Ministry of Education (Oman), 2004a). The Ministry of Education, according to Al-Hajri (2010), expects teachers to provide students with the required tools for lifelong learning and to adapt their teaching practices to improve students' outcomes. Teachers are expected to use a variety of learning and teaching approaches to develop progressive learners who have the skills of critical, independent and higher order thinking. In particular, there is more emphasis in the Basic Education System on applying learning and teaching experiences that help students become independent learners,

demonstrate creativity, master scientific knowledge and communication technologies, and acquire the ability to interact rationally with contemporary worldwide culture (Ministry of Education (Oman), 2004a). The task of improving teaching methods and adopting student-centred classroom practices in order to raise the quality of students' learning outcomes is crucial to the success of the Basic Education System. As stated in the "Vision 2020 document" produced in 1995, the call to implement teaching methods and education practices that encourage learning by doing has been made by the Ministry of Education (Rassekh, 2004). The report *Education in Oman: The Drive for Quality* emphasized in its recommendations that establishing higher standards of learning outcomes requires focusing on the quality of learning and teaching experiences that students receive, on the skills that teachers should have to improve their teaching practices and on the support that teachers receive from their managers when support is needed (Ministry of Education (Oman), 2012). This research came into being to enhance the quality of teaching for mathematics teachers by introducing to them the concept of Productive Pedagogies that offer characteristics of effective pedagogy.

3.3 The Research Methodology

Qualitative research methodology informed by grounded theory was the foundation chosen to achieve the aims of this study. Qualitative research focuses on studying the quality of social relationships, situations, experiences or materials. Tuckman and Harper (2012) indicated that qualitative research displays an emphasis on how and why people are experiencing a naturalistic event that occurs in a specific context. Qualitative research in education is especially appropriate when we want to obtain detailed information and subjective understanding about human behaviours experiences that are shaped in natural contexts and settings and should be taken and studied as they are found and as a whole (Erickson, 1998; Sherman & Webb, 1988). Sherman and Webb (1988) argued that qualitative research has the aim of understanding and interpreting the meaning of the educational experience as nearly as possible in order to give a reasoned, significance and value judgment. Goetz and LeCompte (1984) noted that qualitative research outcomes are important for educational policy makers as they offer more accurate explanations and expectations about what schools, families and other organizations can do to direct and improve education. The nature of studying the quality of social experiences in their natural

context is appropriate for this study as this project aimed to study the experience of implementing Productive Pedagogies by Omani mathematics teachers in the natural setting of their mathematics classroom. Moreover, this study aimed to obtain an understanding of the development of teachers' knowledge, their understanding of, and ability to implement a set of pedagogical practices through observation, interaction and group activities.

Richer (1975) argued that relevant aspects of schooling such as ongoing classroom experiences and effects of teachers were best derived through a grounded theory approach. Hutchinson and Campus (1988) indicated that in education there is a current need for a data-based theory that explains the naturalistic settings of teachers, students and school administrations. Taber (2000) believed that grounded theory provides a sound methodology for educational researchers that enable them to offer rich data, meaningful insights and generalized accounts for curriculum planners and classroom teachers. These benefits encouraged utilizing this methodology in the current study for two reasons. Firstly, this study concentrated on researching classroom experiences by investigating the nature and the development of teaching practices through the introduction of the idea of quality teaching and reflection that is offered by the Productive Pedagogies framework. Secondly, studying the appropriateness of applying Productive Pedagogies in Omani educational contexts by analysing fieldwork data may provide useful insights for Omani educational policy makers and guide future improvements in the teaching of mathematics.

Grounded theory as a methodology was first introduced by Glaser and Strauss in 1967. It is designed to develop a well-integrated set of ideas that is delivered through a theoretical explanation of the social phenomena under study (Liamputtong, 2013). Grounded theory as a research methodology is considered to be a way of thinking about and theorizing (Corbin & Strauss, 1990). In this methodology, the emphasis is on theory development. The theory may be produced initially from the data or grounded if it is present. Following the research investigation, the theory then may be elaborated and modified (Strauss & Corbin, 1994). Qualitative research methodology informed by grounded theory is also appropriate for studying new phenomenon that has not been the subject of previous research. It is designed for exploration rather than confirmation of results.

The data collection procedures for a grounded theory involve interviews, field observations, and examination of documents, video tapes and other qualitative research sources. Although qualitative data are collected prior to beginning systematic analysis, data collection and analysis in grounded theory are interrelated processes. Corbin and Strauss (1990, p. 6) indicated that “the data analysis stage begins as the first bit of data is collected in order to enable the research process to capture all possible relevant aspects which then direct subsequent stages of data collection”. In grounded theory studies, the multiple perspectives of the participants regarding the patterns and processes of diverse actions and interactions must be systematically sought during the research inquiry (Bryman, 2004; Strauss & Corbin, 1994).

Grounded theory, however, has been criticised for the challenges it may pose to researchers (Allan, 2003; Jones & Alony, 2011; Mjøset, 2005). Liamputtong, (2013) indicated that coding process can be problematic for inexperienced researchers as it is a process that needs the development of coding categories from empirical data. Allan (2003) indicated that grounded theory is more demanding in data analysis than simple examination of the data. Glaser (1992, cited in Allan (2003)) believed that the microanalysis of data in grounded theory is time consuming and results in “over-conceptualising” of ideas. In order to overcome the difficulty of data analysis and use grounded theory effectively, Fernández and Lehmann (2005, p. 97) stated “the researcher has to be creative, be open to emerging evidence that may change the way the researcher thought, to trust emerging data without worrying about justification and to get deep in data analysis and discussion”.

In this research, some of the features offered by grounded theory are applied to exploring teachers’ experience and the development of their understanding and ability to implement Productive Pedagogies in mathematics classrooms. Taber (2000) noted that the emphasis of grounded theory is on the reality of actions and problematic situations. Strauss and Corbin (1994) believed that grounded theory methodology is used to discover the realities of the research participants’ experiences. In this study, to understand the actual implementation of Productive Pedagogies by mathematics teachers, there is a need for the researcher to be present in the classroom to observe the teaching practices as closely as possible. Moreover, one of the main features of using grounded theory is its appropriateness for socially

constructed experiences (Jones & Alony, 2011). The emphasis in this study is on the change and process of developing understanding and ability to implement Productive Pedagogies. The experience of implementing Productive Pedagogies continually evolves through the social interaction process occurring in group discussions and through reflection. Thus, the objective of this study was not only to describe the actual teaching practices within mathematics classrooms but to go beyond the meanings and reasons of the actions in order to develop substantive explanations. This research also utilized the feature of grounded theory that involves recurrent processes of data collection and data analysis which informed the review of teachers' practices during each fieldwork phase and the planning of further support for teachers for subsequent fieldwork phases. This is did not mean that the fieldwork phases were treated as separate studies. Rather, they required continual constructive reading and analysing so that more effective ways could be devised to encourage teachers to develop greater awareness of and more reflection on their teaching.

3.4 The Research Design

This section begins with an overview of the research design and then offers a description of the sampling method used to identify the participants and the data collection instruments. The different stages of data collection and the procedures followed are then explained in detail.

3.4.1 Overview of the research

There were three phases in the research design: preparation, implementation and dissemination. Phase one (preparation) began at the end of semester1 (16th January- 24th January, 2012) when a professional development program was conducted by the researcher to introduce the Productive Pedagogies framework, to a group of Omani mathematics teachers of grades 7 and 10. Six Omani mathematics teachers from two schools who participated in this study attended the professional development program for five days (25 hours). The details will follow below.

Phase two (implementation) began at the beginning of semester two on 20th February 2012 and continued till 1st May, 2012. In phase two, the teachers who participated in the research began to apply Productive Pedagogies in their mathematics classrooms. The researcher followed the actual implementation in mathematics classrooms during

six continual cycles (each cycle last about two school weeks). During these cycles, the implementation of Productive Pedagogies was observed and the development of teachers' understanding and their ability to apply Productive Pedagogies was followed and explored. In addition, during those cycles, group discussions on lesson analysis were conducted regularly after classroom observations.

During both Phase one and two, the teachers also had opportunities to devise learning experiences that utilized Productive Pedagogies. Some interviews with individual teachers were conducted during the implementation cycles while other interviews took place at the end of semester two. In addition, students' perceptions about their teachers' practices were examined using focus student groups' interviews that were conducted initially at the beginning of semester two and at approximately the end of semester two.

Phase three (dissemination) offered opportunities for teachers as school-groups to make a presentation about their experience of applying Productive Pedagogies in their mathematics classrooms at a school-based professional development activity attended by other teachers in the region. In each school-group, teachers worked together to present the benefits, challenges, actual classroom practices and significant events they experienced during the implementation of Productive Pedagogies in semester two. Mathematics teachers from other schools and mathematics educational supervisors attended an open day was allocated for the presentations.

3.4.2 Research participants

Sampling in qualitative research is a procedure that has a deep effect on the ultimate quality of the research. Coyne (1997) indicated that the researcher should find out the most useful and suitable method of sampling whatever the phenomenon under study. Creswell (2005) believed that sampling in qualitative research requires considering some important factors such as sample size, kinds of purposeful sampling and qualitative research methods in order to obtain enough and rich data. Johnson and Christensen (2008) pointed to purposeful sampling as one of the major characteristics of qualitative research design besides naturalistic inquiry, openness and flexibility to adapting inquiry.

Purposeful sampling is the better choice in studies that offer rich information and useful manifestations of the research issues of interest (Johnson & Christensen, 2008). Creswell (2005) pointed that purposeful sampling is the standard term used for qualitative sampling to choose participants and sites which are “information rich”. Marshall (1996) believed that the purposeful sample is the most productive sample for developing deep understanding of the human issues under study. As mentioned by Liamputtong (2013), the sample size in qualitative research should reflect the quality, flexibility and depth of the research context. Adequate sample size in qualitative research is determined by research purposes, target phenomenon and the nature of the research’s societal contexts (Luborsky & Rubinstein, 1995). Wallen and Fraenkel (2001) stated that the samples studied in qualitative studies are often small in order to provide deep understanding of a particular situation. In this study, the participant schools were chosen based on a mixture of convenience and purposeful motives of sampling. The researcher had pre-access and previous acquaintance with the schools and mathematics teachers. The researcher was a member of the mathematics educational fieldwork for approximately seven years as a mathematics schools supervisor and thus had previous direct contact with schools and most mathematics teachers. The two schools (Afra-Sea School and Ain-Coast School¹) were somewhat similar as they were the first schools in the region to adopt the new Omani Basic Education System and they were involved in the same educational projects under this system. The two schools are located in the same Governorate of the North Batinah. While the two schools are in two different Wilayas (districts) in the North Batinah Governorate (Saham and Al Suwaiq), they share the same characteristics of being on the coast of Gulf of Oman and the diversity of their citizens’ economic activities. Commerce, agriculture and fishing are the most important economic activities in the Saham Wilaya in which Afra-Sea School is located. The Al Suwaiq Wilaya, in which Ain-Coast School is located, is characterized by a diversity of economic activities consisting of commercial, industrial, agricultural and tourism activities. Predominantly, commercial and agricultural activities are the most important economic activities in this Wilaya. Both schools are socioeconomically fall in low and middle income. A large sector of

¹ All names of schools and teachers are pseudonyms to guard the confidentiality of the participants

people has traditionally been farmers and fishermen (Ministry of Information (Oman), 2011).

The researcher chose two schools rather than one in order to obtain some variation in the grade level that teachers taught and the length of their teaching experience (which was between three to twelve years), variation in the mathematics achievements within the grade levels and to avoid the impact of unexpected circumstances such as the transfer of teachers from their schools. Ain-Coast school was opened in 80's. The teachers from Ain-Coast School had more experience teaching the lower grade levels in Cycle two (Grades 5 and 7) and their length of teaching experience was four to five years. On the other hand, Afra-Sea School was opened in 70's at the beginning of the first formal Omani Educational program. The teachers from Afra-Sea School had more experience in teaching mathematics in different grade levels (grades 6, 7, 8, 9, 10) as well as teaching experience ranging from three to eleven years. All the Afra-Sea mathematics teachers who participated in this study had more experience in teaching the highest grade levels in Cycle two. They had experience in teaching grade 10 which is the transition grade to the Post Basic Education grades 11 and 12. According to the results of students' mathematics achievement in Ain-Coast School at the end of semester one, the overall average was 63.13 for grade 7 and 62.70 for grade 10. The mean average of students' mathematics achievement in Afra-Sea School was 87.31 for grade 7 and 71.17 for grade 10.

It was decided to keep the sample size of teachers small to facilitate in depth observations of and interviews on teachers' practices. Moreover, conducting group discussions with teachers required finding a common free time in the school time table and that can be done only when the number of teachers involved is small. Potential participants were informed initially via informal interviews. The researcher asked the schools mathematics supervisors to provide current information about the teachers, school administration, class sizes and learning resources. Using these details, the researcher then talked to the first (senior) mathematics teacher from each of the two schools to explain the purpose of conducting this study, the research's procedures and the role of participants. Time was given for these teachers to talk to their school's mathematics teachers. Two groups of six Omani mathematics female teachers who taught in Cycle Two Schools (grades 5-12) in the academic year 2011/2012 volunteered to participate. Four teachers from Afra-Sea Cycle Two

School volunteered to participate in this study. These four teachers always worked as a group and they taught the same grade level (grade 10). Two other teachers from Ain-Coast Cycle Two School agreed to participate as they also taught the same grade (grade 7) which would help them to share ideas and prepare learning and teaching activities.

The researcher asked the teachers to nominate five to six students who represented the range of their students in mathematics achievement, engagement and classroom participation. Up to five students from each case study classroom were purposefully chosen to maximise variation in the student sample for focus group interviews. Hence, a total of 30 students from all the case study classes were interviewed. The 30 students comprised ten students (five students from each classroom) from Ain-Coast Cycle Two School and twenty students from Afra-Sea School. Table 3.1 provides details of participant schools, teachers who participated in this study, their years of experience as mathematics teachers, the grade level they were teaching in this project and the class size.

Table 3.1

Participant schools and teachers

School (fictitious names)	Teacher (fictitious names)	Years of experience in teaching mathematics in these grade levels		Grade level	Class size
Ain-Coast Cycle Two	<ul style="list-style-type: none"> • Alabeer • Aljawa 	Five years	Grade 7 and 10	7	36
		Four years	Grade 5,7 and 9	7	35
Afra-Sea Cycle two	<ul style="list-style-type: none"> • Alnaeem • Sama • Alsalh • Alhuda 	Eleven years	Grade 10	10	35
		Three years	Grades 9 and 10	10	34
		Six years	Grades 8 and 10	10	36
		Three years	Grades 6,7,8,9 and 10	10	33

3.4.3 Data collection instruments

Wallen and Fraenkel (2001) indicated that while there are many different types of qualitative methodologies in educational research, most of them share some general characteristics such as: a) exploring and describing some phenomena, particularly social phenomena. b) The focus of inquiry is broadened or narrowed over time. c) An understanding the phenomena in context and d) the emphasis on ongoing and inductive data collection and analysis. The features of qualitative research suggest that the important objective of data collection methods in qualitative research is to

enable the researcher to capture the language and behaviours of the participants. Hence, typical data collection methods used in qualitative methods are observation of participants, in-depth interviews, focus group interviews and examination of real documents. In the current research, different kinds of qualitative data collection methods were applied such as observation of participants, teacher group interviews, individual teacher interviews, student focus group interviews, maintaining a researcher's diary and observations from teachers' presentations.

Participant observation

Bryman (2008) indicated that participant observation provides a better picture of social reality as the researcher is in close interaction with the participants over a period of time. Wallen and Fraenkel (2001) noted that observing participants in naturalistic situations gives a more accurate indication of what happened. Participant observation offers a detailed and rich description for in depth inquiry (Johnson & Christensen, 2008). Angrosino (2012) indicated that participant observation is a naturalistic observation that encourages the researcher to be an active member of the observed participants' group. In classroom observations in this study, the researcher worked with the participants as a group in analysing the observed lessons, preparing teaching and learning activities and improving their teaching practices.

Adler and Clark (2011) noted that observation is useful in following the changes and development in social situations by providing a real time view of human behaviour. Johnson and Christensen (2008) claimed that naturalistic observation is an important method of documenting behavioural patterns of people in real-world settings. The main strength of participant observation is its concern with understanding social issues and applying the findings to bring about change and behaviour development (Liamputtong 2013). However, the participants who are observed may change their behaviours and behave less naturally when they are being observed (Adler & Clark, 2011). Bryman (2008) stated that while the participants may tend to behave less naturally at the initial stages of participant observation, they can over time adjust to being observed and will behave more naturally.

Classroom observation began at the beginning of semester two. Regular classroom observations were conducted during six fieldwork cycles (each cycle took two school weeks). Each teacher presented one lesson every two weeks and hence was observed

once every two weeks. Each fortnight, the researcher observed four lessons from Afra-Sea Cycle Two School and two lessons from Ain-Coast Cycle Two School. In Ain-Coast Cycle Two School where two teachers were involved, the researcher and one of the teachers observed together the teaching practices of the other teacher. In Afra-Sea Cycle Two School, where there were four teachers, each of the teachers was observed by the researcher and the other three teachers together. While that was the planned arrangement for conducting classroom observations, there was flexibility in varying the plan in Afra-Sea Cycle Two School to accommodate their timetables. The lessons to be observed were nominated by the participants. The observed lessons were also coded individually by the researcher and teachers using the Arabic version of *Productive Pedagogies Classroom Observation Manual* (Education Queensland, 2010a).

Productive Pedagogies Classroom Observation Manual

The *Productive Pedagogies Classroom Observation Manual* was developed in the Queensland's School Reform Longitudinal Study (QSRLS). Gore, Cooper and Williams (2005, p. 1) noted that Productive Pedagogies instruments "have now been widely tested and have generated substantial data of Productive Pedagogies, Productive Performance and Productive Assessment". The Classroom Observation Manual used in the current study gave explanations for each dimension of Productive Pedagogies framework and a key question for each element under each dimension. Using a scale of scores from 1 to 5, with 5 indicating that the pedagogical practice was present and sustained and 1 indicating that the practice was not observed, the observer coded the practices by considering the evidence seen during the classroom observation period. Observers asked the core question in relation to each element as a means of focusing on this aspect of the classroom, and then in response to this question, the observer allocated a score from 1 to 5 based on quantity or quality of the element present.

The researcher was granted permission to use the Arabic version of the *Productive Pedagogies Classroom Observation Manual* that was translated from English to Arabic by Alsharif (2011). Participants' teaching was observed and coded by the researcher and the participant's peer using the Arabic version of the Classroom Observation Manual. The Arabic version offered a useful explanation about the

dimensions of Productive Pedagogies and their twenty elements. It also offered examples from mathematics teaching experiences that utilized Productive Pedagogies. The classroom observations by the researcher and the participating teachers resulted in 35 coded observations. The Arabic version of the Classroom Observation Manual has not been validated for the Omani context since inter-rater reliability was not computed. The data from the coded classroom observations served only the purpose of triangulating qualitative classroom observations.

Teachers' interviews and discussions

Qualitative research is known for giving an opportunity for participants to talk, interact and express their feelings, ideas and views (Liamputtong, 2013). Interviewing is a key method of finding out participants' ideas, feelings, thoughts and intentions which cannot be observed (Wallen & Fraenkel, 2001). Mears (2012) argued that in-depth interviews are purposeful interactions that attempt to provide a depth of understanding and the significance of meaning of participants' experiences and thoughts.

Zhang and Wildemuth (2006) indicated that a significant advantage of using unstructured interviews is its potential to provide researchers with an in-depth understanding of a particular phenomenon within a particular research context. Unstructured interview as a data collection method is aligned with the constructivist view of social experiences. Zhang and Wildemuth emphasized that flexibility is one important feature of qualitative interviewing as the interviewer responds to points that seem worthy of follow-up or are important in explaining and understanding events, actions and patterns. DiCicco-Bloom and Crabtree (2006) noted that the unstructured interview is an important means of collecting qualitative observational data. During the unstructured interviews in this study, points of interest arising from the observational data provided a basis for further discussion with the participants.

Qualitative interviewing tends to be a flexible data collection method that enables researchers to obtain rich and detailed data (Bryman, 2008). Adler and Clark (2011) noted that group interviews are useful when the participant group is the unit of analysis as interviewing a group can yield rich information and disparate views of group members' experiences. Johnson and Christensen (2008) indicated that

informal conversational interviews centred on fieldwork observations help to increase the salience and significance of conversations.

In qualitative research, however, there is often a variation in the duration of interviews. Bryman (2008) suggested that the amount of interview time is determined by the significance of the interviewee's data. Another doubt about the effectiveness of interviews noted by Johnson and Christensen (2008) is that group conversational interviews may provide less systematic data and organization more structured data collection methods. Bryman (2008) suggested that audio-recording interviews will help to overcome the natural limitations of researcher's memories and notes, and permit repeated examination of the data. However, audio recording may also cause anxiety to the interviewee as well as requiring the availability of quality recording equipment.

In this study, classroom observations were followed by two kinds of teacher group discussions: discussion of lesson analysis and discussion related to the preparation of teaching and learning materials and activities. Teacher group discussions of lesson analysis were conducted to investigate teachers' understanding of Productive Pedagogies, their perceptions of its impact on their pedagogical practices and on their students, and their understanding of the issues related to the implementation of Productive Pedagogies principles and dimensions. Group discussions on lesson analysis, conducted regularly in each fieldwork cycle, were all audio-recorded. The discussions were conducted in an unstructured manner. A minimum of two teacher group discussions of lesson analysis were conducted in each data collection cycle (two fieldwork weeks) for each group. Each group interview for lesson analysis took from 40 minutes to one hour. Some of the points of interest from classroom observations pertaining to teachers' experiences in applying Productive Pedagogies in their mathematics classrooms formed the basis of teacher group discussions. These group discussions of lesson analysis yielded insights for further improvements in the teachers' implementation of Productive Pedagogies.

In addition to discussions on lesson analysis, during the six cycles of fieldwork, each participant's group had opportunities for planning teaching and learning activities that incorporated the features of Productive Pedagogies. The aim of these group planning sessions was to raise the teachers' ability to implement Productive

Pedagogies. The teachers and the researcher held at least one group planning discussion in each fieldwork cycle. Each group discussion took from 40 to one hour. Some of the discussions were recorded. These group discussions offered a significant source of data. When the teachers worked in groups, different points of common interest were raised for discussion.

In addition, a minimum of two individual interviews were conducted with individual teachers during and at the end of semester two to provide opportunities for dialogue between each teacher and the researcher on the teacher's personal experience in implementing Productive Pedagogies (their perception of the benefits and challenges, their actual teaching practices, need for support, and other issues). Each individual interview took around 40 minutes. All individual teacher interviews were recorded. Sample of questions that the participants were asked in the individual interviews, in order to open a discussion with the teachers around their experience in implementing Productive Pedagogies and their understanding of the pedagogical elements, were as follows:

- What are the things that are principal in your mind when you plan your lesson, prepare for learning activities or questions?
- What would be required to ensure that you taught in the most productive ways?
- Can you explain your attempt to apply Productive Pedagogies in your classroom (during preparation stage of lesson, inside the classroom)?
- When you observe the practices of your peers, what kinds of things in the classroom practices did you identify as either present or absent?
- How do you understand the relationship between the dimensions and the elements of Productive Pedagogies?
- How it is easy/ difficult to apply Productive Pedagogies in preparing mathematics lessons?
- Do you think that there are specific factors which will restrict or encourage using Productive Pedagogies?
- Do you have any suggestions about how to apply Productive Pedagogies in the Omani's schools in general and in mathematics classrooms in specific?

Students' focus group interviews

The interaction between participants in focus groups is useful as it encourages groups of peers to express their perspectives. Morgan and Krueger (1993) stated that the interactions in a focus group can provide an explicit evidence for exploring the range of participants' opinions and perceptions. Focus group interviews, as Frey and Fontana (1993) noted, can be used for exploratory purposes to stimulate new ideas, and identify new concepts and symbols for theorizing and expanding understanding of relevant social events and actions.

One of the major advantages of focus groups is that they offer the opportunity to observe the social interactions between participants who may offer many different individual opinions. In the view of Liamputtong (2013) the main advantages of using focus groups are: a) obtaining in-depth data on sensitive or hidden issues, b) encouraging interaction between participants, and c) building social networks. Gibbs (2012) indicated that an indisputable advantage of focus groups is their role in providing information about salient issues and their potential to bring about change through the new ideas that emerge during participants' dialogue. However, given that focus group interviews are social events involving interaction between research participants, some limitations must be kept in mind when using focus groups. According to Liamputtong (2013) one limitation that the data gathered from focus groups only indicate a range of perspectives but cannot provide information on the prevalence or otherwise of any perspective or reflect the personal experience of individual participants. Terence, Gerianne and Joseph (1993) argued that focus group interactions being communication events, they raise concerns about understanding the communication process and the quality of data generated. Halcomb et al. (2007, cited in Gibbs (2012) indicated that focus groups may affect participants' confidence as group discussions may produce conflicts that may cause problems in the interaction between group members.

To exploit the advantages of focus groups and ensure the quality of the data collected, Morgan and Krueger (1993) noted that consideration should be given to certain factors such as clarity of research purposes, appropriate choice of participants and environment, effective questions and careful data analysis. Bryman (2008) indicated that the sample size and composition of focus groups are influenced by the

research context which can be wide ranging in its diversity. A number of researchers have recommended a sample size of six to twelve participants in a focus group in order to obtain in-depth data and to facilitate social interactions among participants (McLafferty, 2004).

This study took into account student voice by conducting focus group interviews with students from the case study classrooms in order to determine students' perceptions on teaching practices in mathematics classrooms. Student focus group interviews were conducted in a manner that offered a flexible and non-threatening environment for students to express and share their views. Five to six students were selected from each case study classroom. Each group from each classroom was interviewed at the beginning and at the end of semester two. The focus group interviews provided an informal environment for students to talk and share their thoughts and feelings about studying mathematics and the impact of the teaching practices applied in their mathematics classrooms. Each student focus group interview took around one hour and was audio-taped.

The aim of the initial student focus group interviews conducted at the beginning of semester two was to encourage a free dialogue about students' previous mathematics learning experience, the expected role of mathematics teachers from a student's point of view and the importance of learning mathematics in their current and future life. Each group consisted of ten students from two case study classrooms.

The student focus group interviews at the end of the semester were conducted with the aim of exploring students' perception of the teaching practices they experienced in their mathematics classrooms during semester two. Most of the students who participated in the initial focus group interviews also participated in focus group interviews at the end of semester two. However, the researcher included additional other students to be interviewed at the end of the semester. The additional students were chosen because they had been observed by the researcher to display some notable change in their behaviour and level of mathematics learning or engagement in mathematics lessons during the semester.

Researcher's diary

Personal documents such as diaries, letters and autobiographies have been used in qualitative research to enable the researcher to interpret social developments by examining witnesses' accounts of an event or process (Bryman, 2004; McCulloch, 2012). Iida, Shrout, Laurenceau and Bolger (2012) noted that diaries contain detailed reports that capture actions, reflection or interaction on a daily basis over a specific period of time.

McCulloch (2012) emphasized that personal diaries are an important resource for research in education as they can provide more than one perspective on personal and public issues. Bolger, Davis and Rafaeli (2003) indicated that there are two fundamental advantages of using diary methods. First, diary methods provide an ongoing self-report of events in their natural setting. Second, they minimize the amount of time between an experience and the account of this experience. Liamputtong (2013) indicated that personal field notes do not only provide an account of events but also reflect the active process of constructing meanings and representations of social reality. However, Iida et al. (2012) stated that one challenge in using diary methods is the need to balance the length of accounts of events in relation to the duration of the diary period and the frequency of the events. Johnson and Christensen (2008) suggested that researchers' diaries be used as secondary data to corroborate evidence obtained by other data collection methods.

The researcher's diary in the current study recorded accounts of classroom observations and group discussions between the researcher and the participants. The diary provided a rich source of data information that helped the researcher to determine the participants' understanding of the Productive Pedagogies construct and inform strategies for strengthening their understanding.

In this research, the researcher's diary was a direct record of important events in the mathematics classrooms observed by the researcher to track the actual implementation of Productive Pedagogies. These diaries were analysed systematically by the researcher at the end of each fieldwork cycle to evaluate the success of the classroom practices so that support could be planned for subsequent stages of implementation of Productive Pedagogies.

Teachers' presentations

Loucks-Horsley et al. (2003) indicated that providing opportunities for participants to present their experiences and research findings to various audiences and to participate in discussing the implications of their findings for teaching and schools encourage teachers to go beyond what they learn from the research. Johnson and Christensen (2008) indicated that personal documents that are written, photographed and recoded to reflect personal experiences, thoughts and ideas can be used to strengthen the evidence found in a research study. In this study, the recorded presentations made by the teachers at the end of semester two provided a rich data resource for the researcher. Specifically, teacher talked about developments in their professional knowledge and their attempts to improve their teaching practices during the implementation of Productive Pedagogies contributed to the researcher's understanding of the teachers' experience in developing their new professional knowledge and also provided pointers to how their teaching practices might be improved.

The teachers who participated in this study presented their experience of applying Productive Pedagogies to mathematics teachers from other schools before the end of semester two. The teachers worked in groups to make their presentations. The presentations were conducted as part of a school professional development activity and generated much discussion and many questions from the audience. The school professional development activity lasted approximately five hours beginning at 10 am and ending at 2 pm. Firstly, the researcher provided a presentation on the Productive Pedagogies framework and the objectives of conducting this study. Then the teachers from each participating school offered a 30 to 45 minute oral presentation which was followed by an open discussion between the teachers and the audience. Parts of the presentations were recorded and the PowerPoint slides were collected.

3.4.4 Data collection procedures

When the researcher initially made an official agreement with the Omani Ministry of Education to carry out this study, the researcher explained to the school principals and teachers the aims of the study, the nature of the research procedures, the estimated time needed to complete the study and the expected role of the researcher

and the participants. The researcher first met with the schools' principals individually and explained to them the research objectives, the role of the school, teachers and students and the support sought from the school principal. The researcher then discussed with school principals and mathematics teachers the stages and procedures of data collection. Information and consent forms for school principals and teachers were distributed for their signature. After final agreement and consent forms were collected from teachers and school principals, preparations for training in Productive Pedagogies were discussed with the teachers and the training programme modified to fit teachers' circumstances. The conduct of the research followed three main phases: preparation, implementation and dissemination.

Phase one: preparation

The phase of preparation began during the period allocated for school professional development programmes in the school calendar between the two semesters. A professional development program to introduce the Productive Pedagogies framework was conducted by the researcher. The five day professional development program took the form of a series of seminars and workshops. One of the main challenges the researcher encountered during this phase had to change the timing of the professional development program. After the researcher and teachers had planned the schedule of the professional development program, some of the teachers were called to perform additional duties outside their schools. The professional development programs planned for delivery over five days had to be spread out over two weeks instead of one week.

Clarke and Hollingsworth (2002) emphasized that the notion of ongoing and life-long professional learning for teachers involves an improvement in teachers' knowledge, beliefs, classroom practices and students learning outcomes. Rodrigues (2004) claimed that teacher professional development is influenced by developments in technology, educational politics and pedagogy. Kreemer-Hayon, Vonk and Flessner (1993) argued that the current growth of knowledge in the area of teaching and the high expectations for teacher's roles and practices constitute an additional demand on teachers to function on a high professional level. In particular, Loucks-Horsley et al. (2003) indicated that the attainment of the desired state of teaching and learning

science and mathematics subjects involves an emphasis on effective teacher professional development programs.

Teachers' professional development programs offer opportunities for teachers to develop their pedagogical practices through critical reflection and rich experiences of practice that support them through the complexity of learning and teaching (Loucks-Horsley et al., 2003). Clarke (1994) argued that professional development programs encourage teachers to reflect on their current practice. He claimed that these programs offer opportunities for teachers to work with, observe, or receive feedback from peers in their teaching. Supovitz and Turner (2000) argued that high quality teacher professional development can produce quality teaching practices in classrooms which can improve students' achievement.

A professional development program was conducted at the first phase of this study to introduce the Productive Pedagogies framework to the participating Omani mathematics teachers. Loucks-Horsley et al. (2003) indicated that designing professional development programs for improving teaching practices and for developing a set of understanding, knowledge and skills involves a process of thoughtful, conscious and decision making. The design of the professional development program in this study followed the process mapped out in *Figure 3.2*. It consisted of four main stages: set aims, plan, apply and reflect.

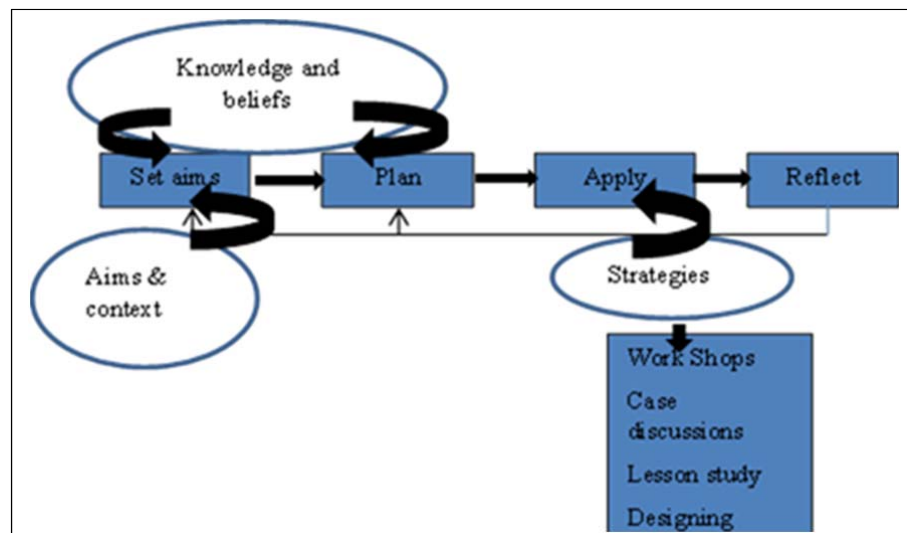


Figure 3.2 Professional development programme design. (Loucks-Horsley et al., 2003, p. 6)

Professional development programme

Loucks-Horsley et al. (2003) argued that specific targeted aims are the driving force behind effective professional development programs. The aims of the teachers' professional development program stem from the research objective of introducing the Productive Pedagogies framework to a group of six Omani mathematics teachers who participated in this study. The specific aims of the professional development programme were to:

- Develop an understanding of the Productive Pedagogies framework and its role in reflection on teaching
- Develop an understanding of the model of the classroom observation scoring manual that was developed to code classroom practice in the QSRLS
- Design curriculum and learning experiences, activities, and assessment tasks that may be used to implement the different elements of the framework to maximise the pedagogical elements of Productive Pedagogies framework under the four dimensions of intellectual quality, connectedness, supportive classroom environment and working and valuing difference
- Generate a professional dialogue to discuss classroom practices

Professional development programme plan

Supovitiz and Turner (2000) emphasized that effective teacher professional programs that aim to improve teaching practices need sufficient time, rich and sustained development activities, supportive procedures and curriculum materials. Loucks-Horsley et al. (2003) believed that planning a professional development program should take into consideration the requirements of sufficient time, identification of new issues in teaching and learning, relevance of activities to teachers' situations and requirements, and willingness to engage in a continuous process of improvement. Ailwood and Follers (2002) suggested that teacher professional learning should be characterised by shared standards and values, reflective discourse, collaboration, improvement of practice and an emphasis on student outcomes. The researcher's review of effective models of professional development of science and mathematics teachers guided the design of the professional development program in this study.

The following five general principles, from Loucks-Horsley et al. (2003), informed the design of the professional development program in this study:

- Building new knowledge on teachers' existing ideas, beliefs and experiences
- Providing opportunities for teachers to construct their new pedagogical experiences
- Providing opportunities for teachers for collaboration, discussion, reflection and implementation
- Engaging teachers in a continuous process of improvement such as developing new understanding and making changes in their practice

In this study, the professional development program took the form of a series of workshops (seminars and tutorials) amounting to a total of 25 hours (5 hours per day) and was facilitated by the researcher at the end of semester one in the period (7th of January- 18th of January, 2012) allocated for professional programs in schools. In the sessions of the professional development program, participants developed an understanding of the Productive Pedagogies framework and the Classroom Observation Coding Manual in order that they could use the manual to code their practices and their peers' practices in semester two. The participants had opportunities to plan lessons, activities, and assessment tasks to realise the features of the pedagogical elements under its four dimensions. The content of this planning was based on the mathematics topics for grades seven and ten that the teachers were teaching in semester two. Table 3.2 presents the content and activities that were featured in the teacher professional development program.

Table 3.2
Plan for professional development program

Professional development program's topic	Content of topic	Group activities
Quality teaching Productive Pedagogies framework Productive Pedagogies Classroom Observation' Manual and scale. Preparing teaching activities	The main characteristics of good teaching/ Good teachers (<i>approximate time: 45 minutes</i>). Obstacles facing teachers to raise the quality of their teaching practices (<i>approximate time: 60 minutes</i>). The four dimensions and the 20 elements (<i>approximate time: 45-60 minutes for each dimension</i>). The practice of reflection (<i>approximate time: 30 minutes</i>) Nature of the scale, how it is organized (<i>approximate time: 30 minutes</i>). How to use it for coding classroom practices (<i>approximate time: 30 minutes</i>). Preparing teaching and learning activities to deliver the content of unit one of the mathematics curriculums of grades seven and ten	<i>Group discussions:</i> Group members come together to discuss new concepts, common interests or analyse examples from classroom practices that apply Productive Pedagogies (<i>approximate time: one hour and 40 minutes per day</i>). <i>*Study group:</i> Group members come together to plan and prepare learning activities (<i>approximate time: one hour and 45 minutes per day</i>).

Phase two: implementation

This phase of fieldwork began at the beginning of semester two (18/2/2012). Data collection was conducted employing two main methods: classroom observations of mathematics lessons and audio taped interviews about participants' experience of learning and applying Productive Pedagogies and about students' perceptions on the implementation of Productive Pedagogies in their mathematics classrooms.

At the beginning of this phase, the researcher and teachers drew up a tentative timetable of classroom observations and group discussions for the initial cycles of data collection. The initial timetable was subsequently modified before each cycle to accommodate the continual changes in the teachers' teaching schedules. The researcher had to reorganize the scheduling of data collection a few times to avoid

conflict with unexpected circumstances arising in the schools that made the teachers unavailable. As the classroom observations and group discussions preceded, the other data collection methods of teacher and student interviews were also implemented.

Phase three: dissemination

This phase began before the end of semester two to offer opportunities for participants to present to mathematics teachers from other schools the participants' views and thoughts on their experience of applying Productive Pedagogies. The presentations were made as part of a school-based professional development activity attended by the two groups of teachers participating in the study, the researcher, mathematics teachers from other schools including Cycle Two Schools and schools with grades 11 and 12, and mathematics education supervisors. The researcher and teacher participants talked about their experience of introducing Productive Pedagogies into mathematics teaching. The presentations were followed by useful discussions on the benefits and challenges of implementing Productive Pedagogies. An overview of the study's collection procedures in the three phases is given Table 3.3 below.

Table 3.3
Overview of data collection procedures

Data collection's period	Data collection procedure	Aims of procedure
Semester one (7/1/2011-18/1/2011)	Professional development program	To introduce the Product Pedagogies framework to the participants
Semester two Data collection (cycle one) 21-22/2/2012	Initial focus group interviews (Grade7) Initial focus group interviews (Grade10)	To explore students' initial perception of studying mathematics
Cycle two (25/2/2012-7/3/2012)	Classroom observation; group discussions on lesson analysis and preparing teaching activities	To investigate teachers' understanding of Productive Pedagogies.
Cycle three (10/3/2012-21/3/2012)	Classroom observation; group discussions on lesson analysis and preparing teaching activities	To identify teachers' perceptions of its impact on their pedagogical practices and on their students.
Cycle four (24/3/2012-4/4/2012)	Classroom observation; group discussions on lesson analysis and preparing teaching activities	
Cycle five (8/4/2012-18/4/2012)	Classroom observation; group discussions on lesson analysis and preparing teaching activities Individual teacher -interviews	To generate discussion on issues related to their understanding and implementation of Productive Pedagogies.
Cycle six (22/4/2012-2/5/2012)	Classroom observation; group discussions on lesson analysis and preparing teaching activities. Final student focus group interviews (grade 7 and 10) Individual teacher interviews Teachers' presentation on their experience of applying Productive Pedagogies (school-based professional development activity).	To investigate students' perceptions of their teachers' practices

There were three stages in the implementation of the phase of dissemination: preparation, presentation, and open discussion.

Preparation

The researcher and each school's group of participants met to plan the specific arrangements for a school professional development day aimed at sharing with teachers from other schools the participants' experience of implementing Productive Pedagogies. Both school groups were given the freedom to choose the content and the approach that they would use in their presentation. The time, place and the main purpose of the school professional activity were decided by the researcher. In a meeting with the teachers from each school two weeks before the professional development day, the researcher explained the purpose of conducting the school-based professional development activity and suggested the main points that may be of interest to the audience, such as the benefits the teachers found in applying Productive Pedagogies, the challenges they faced and their experience of classroom observations, lesson analysis and lesson preparation. The attendance of teachers from

other schools was organised by the researcher through the mathematics education supervisors whose responsibilities include informing schools of professional development activities.

Presentations

The researcher and the teachers gave three presentations. Presentation one was introduced by the researcher and was aimed at providing an overview of the Productive Pedagogies framework. This presentation, which lasted 30 minutes, also provided general information about the main objectives of this study and the research procedures.

Presentation two was a group effort by grade 7 teachers from Ain-Coast Cycle Two School. They talked about their experience of implementing Productive Pedagogies: their initial reactions towards its implementation, the benefits for teachers and students, the challenges they encountered in applying the approach the support they needed, and the conditions conducive to more widespread use of Productive Pedagogies.

Presentation three was made by a group of four grade 10 teachers from Afra-Sea Cycle Two School. This presentation began with one of the teachers giving a summary of their experience implementing Productive Pedagogies, tracing the development of their journey from initial attempt to the more advanced stages. She also talked about the specific instance of applying Productive Pedagogies to the content of the grade 10 mathematics curriculum, which included taking into account the knowledge and interests of students in that grade. She illustrated her talk with examples from the group's teaching practices in mathematics classrooms that utilized Productive Pedagogies. Then each teacher talked about her personal experience, sharing her views on the benefits and challenges of Productive Pedagogies, and illustrating points with an example from her teaching practices. In particular the teachers talked about the following main points:

- The comparison between their teaching experiences prior to their use of Productive Pedagogies and their experience teaching after their acquaintance with Productive Pedagogies

- The change in their teaching approach and relationships with other teachers in their group
- The problem of stagnation in their students' progress in learning mathematics and how a change occurred during the semester they implemented Productive Pedagogies
- Student engagement with classroom practices, especially in the case of low achieving students
- The teachers' reaction to applying Productive Pedagogies at the beginning of the study and later at its completion

Open discussion

Discussion on issues raised by the presentations was followed by a general open discussion between the research participants and the audience. The open discussion highlighted specifically the following questions:

- How does the use of Productive Pedagogies make a difference to the teaching of mathematics and to students' engagement?
- What is the difference between the conventional way of teaching of mathematics, as applied in the teachers' previous teaching experience, and teaching mathematics with the Productive Pedagogies.
- What were the main obstacles? How can they be overcome?
- Is there interest in applying Productive Pedagogies outside this study? Is the school environment ready for implementing Productive Pedagogies? What are the requirements?

The main benefit of the open discussion phase was the interaction generated between the research participants and the audience during discussion of the above questions. Detailed information about this stage and the activities of the school-based professional development activity are provided in the following Table 3.4.

Table 3.4

Activities in the dissemination phase

Preparation	<p><i>Time of meeting:</i> 10 am – 10:30 am</p> <p><i>Venue:</i> Ain-Coast Cycle two School, Learning Resources Centre.</p> <p><i>Attendees:</i> 12 mathematics teachers (some of them in their first year of teaching from six schools and two education supervisors).</p>
Presentations	<p><i>Time of event:</i> 10:30 am – 12:30 pm</p> <p>Three power point presentations.</p> <p><i>Presentation one</i> by the researcher; 30 minutes</p> <p><i>Presentation two</i> by Grade 7 teacher participants from school 1 (Ain-Coast Cycle two School); 30-45 minutes.</p> <p><i>Presentation three:</i> by Grade 10 teacher participants from school 2 (Afra-Sea Cycle two School); 45 minutes</p>
Open Discussion	<p><i>Time of event:</i> 12:30 pm – 1:30 pm</p> <p>Discussion took the form of response to questions and comments.</p>

3.5 Data Analysis Methods

Qualitative data analysis in grounded theory starts at the moment of initial interaction with the issue under study and continues through the research activities and data collection processes (Johnson & Christensen, 2008). Strauss and Corbin (1990) stated that “the data analysis of grounded theory is a process of providing the grounding, building the density, and developing the sensitivity and integration needed to generate a rich, tightly woven, explanatory theory that closely approximates the reality it represents” (p. 57). While the traditional coding process in grounded theory was aimed at examining events and actions, the current view is that coding data is for the purpose of Uncovering patterns in participants’ experience (Liamputtong, 2013). Strauss and Corbin (1994) emphasized that making comparisons and asking various questions about the phenomenon under study are two processes central to all coding procedures. There are three stages in the coding process in grounded theory-open coding, axial coding and selective coding. Table 3.5 provides details of these stages as explained by (Strauss & Corbin, 1990, p. 61).

Table 3.5
Coding stages in grounded theory

Coding Data are broken down, conceptualized and put back together in new ways
Coding stage one: Open Coding The process of breaking down data, examining, comparing, conceptualizing, and categorizing
Coding stage two: Axial Coding A set of procedures by which connections are made between categories of data .
Coding stage three: Selective Coding The process of selecting the core category, systematically relating it to other categories, validating those relationships and filling in categories that need further refinement and development.

3.5.1 Analytical procedures in coding process

The data from classroom observations, teacher group interviews, student focus group interviews and researcher’s diaries were transcribed from audio materials and paper field notes to produce electronic word documents. Walsh (2003) indicated that NVivo is a useful tool that allows more freedom for researchers to link, organize and compare patterns within and across data documents. Bandara (2006) noted that as a qualitative data analysis tool NVivo is a comprehensive tool for managing research data. NVivo as a computer program for qualitative data analysis has many advantages. For example, it allows the researcher to import and code textual data, edit the text; recover, evaluate and recode coded data; and search texts for groups of words to look for patterns (Walsh, 2003).

Using the NVivo program, transcriptions from word documents were entered into NVivo’s internal sources. The word documents were written in Arabic as Arabic was the language of the research context. Within the NVivo Program, the data coding was recorded both Arabic and English. Following the coding procedure set out in Strauss and Corbin (1994), the researcher employed four analytical procedures in NVivo: conceptualizing the data, categorizing, developing interrelations between categories and subcategories, and building a descriptive overview of the data.

Conceptualizing the data

In conceptualising the data, each part of an observation, sentence, note and paragraph was given a concept name that represented some aspect of the experience of applying Productive Pedagogies in mathematics teaching or reflected any of the issues relevant to this study. Conceptualisation of the data was guided by the following questions:

- What is this idea, action or concept and what is its source (from classroom practices, discussion, teachers previous experience, teacher belief, etc.?)
- What does it represent?
- What is the relation between this item of and other similar items?

Categorizing

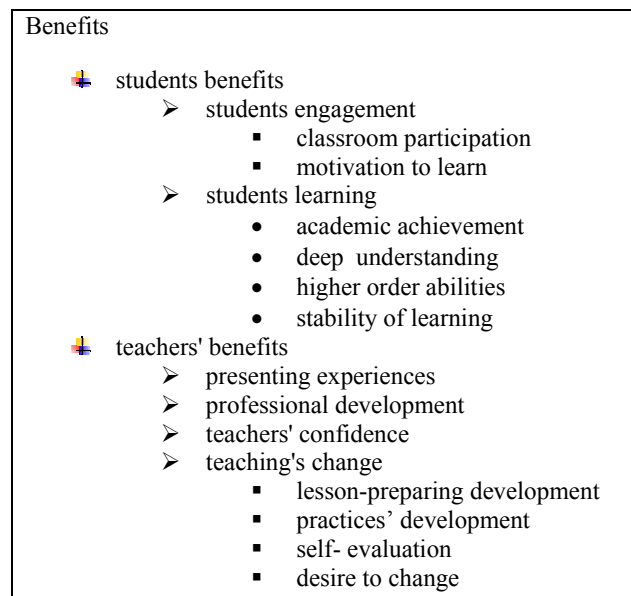
The concepts identified in the conceptualising procedure were grouped into categories according to common properties and dimensions. Concepts, categories and subcategories were written as code notes (a type of memo) in the NVivo Program.

Developing interrelations between categories and subcategories

This procedure enabled the researcher to think systematically about the data and identify relationships between categories of concepts that explain the motivation underlying teachers’ actions, understand students’ and teachers’ perceptions and obtain a clearer picture of the real situation in which the Productive Pedagogies approach was implemented. The relationships represent the development of action and interaction in the research context (the specific case study-classroom conditions and the school environment). In the process of identifying relationships the range of data categories began to be narrowed. Table 3.6 shows an example of one of the main categories and its subcategories as illustrated in coding nodes.

Table 3.6

Category and sub-categories of benefits



Building a descriptive overview

The results of the processes of conceptualizing, categorizing and developing interrelations were integrated to produce a descriptive overview of the journey undertaken by the researcher and the teacher participants in applying Productive Pedagogies in mathematics teaching. Aspects of their experience with Productive Pedagogies were identified and conclusions drawn from the data.

3.5.2 Quantitative data analysis

The quantitative data obtained from the 35 coded classroom observations were analysed using the descriptive statistics procedures in the Statistical Package for the Social Sciences program (IBM SPSS Statistics, version 21). According to the Classroom Observation Coding Manual, the classroom observations were coded for four dimensions: intellectual quality, connectedness, supportive classroom environment, and working and valuing difference. Twenty pedagogical elements under these four dimensions were scored from 1 to 5. The descriptions of the scores from 1-5 on each pedagogical element, which is provided by the Classroom Observation Coding Manual, are used to constitute the minimum criteria for each element. For example, the descriptions of the scores from 1-5 for the pedagogical element *knowledge as problematic* is explained in the following Table 3.7.

Table 3.7

The descriptions of the scores from 1-5 to constitute the minimum criteria for knowledge as problematic

1 =	No knowledge as problematic. All knowledge is presented in an uncritical fashion.
2 =	Some knowledge seen as problematic - but interpretations linked/reducible to given body of facts.
3 =	Approximately half knowledge seen as problematic. Multiple interpretations recognised as variations on a stable theme.
4 =	Explicit valuation of multiple interpretations and constructions of information, presented as having equal status, and being equally accommodated and accepted by others.
5 =	All knowledge as problematic. Knowledge is seen as socially constructed, with conflicting implications and social functions producing resolution and/or conflict.

The dimensions and the twenty elements were treated as variables in the variable sheet in SPSS. The scores assigned to the twenty elements under the four dimensions were entered into the SPSS data sheet for statistical analysis. Due to the fact that the sample size was small, no statistical testing was conducted. The quantitative data were used to show the development of the use of Productive Pedagogies by teachers during the different cycles of implementation. That data were presented in graphs in order to show improvements in teaching practices during the period of classroom observation.

3.6 Quality of Data

Assessment of the quality of qualitative enquiry is essential to ensure the “rigour” or “trustworthiness” of the research (Liamputtong, 2013, p. 24). Guba and Lincoln (1989) had developed criteria for evaluating the quality of data obtained in social qualitative research. The criteria include credibility, transferability, conformability and dependability. These criteria, according to Liamputtong (2013), represent the equivalent of the conventional principles of validity and reliability associated with quantitative research. The credibility criterion is equivalent to internal validity, transferability to external validity, conformability to the conventional criterion of objectivity and dependability to reliability.

3.6.1 Credibility and authenticity

Wallen and Fraenkel (2001) suggested that qualitative researchers have to study phenomena objectively by spending considerable time in the field, collecting data from a variety of perspectives, using multiple instruments and working with a variety of arrangements. Liamputtong (2013) indicated that credibility in qualitative research is achieved when participants are selected purposively and when the multiple realities experienced by the participants are recorded as precisely and adequately as possible. Qualitative research validity can be promoted through different strategies such as triangulation, prolonged fieldwork, participants’ feedback, peer review and pattern matching (Johnson & Christensen, 2008). Strategies such as triangulation, sustaining fieldwork over a period of time, and the use of participants’ feedback were applied in this study to meet for supporting the criterion of credibility.

Triangulation

Triangulation is one of the strategies used to promote the validity of qualitative research by cross-checking information and conclusions from multiple data sources, through different research methods, and applying multiple theories and perspectives (Johnson & Christensen, 2008). Liangputtong (2013) stated that “the most powerful tool for strengthening credibility in qualitative research is triangulation” (p. 30). Biesta (2012) indicated that triangulation is a strategy of looking for convergence and corroboration of results from different data collection methods in order to enhance the strength and validity of research outcomes. For example, using mixed methods and sources of data is a means of triangulation as data obtained by one method or from one source are complemented by or elaborated by data from the other methods and sources. In this study, different research methods were used to investigate the implementation of Productive Pedagogies such as classroom observations, group and individual interviews and personal documents. To enhance validity, the researcher conducted multiple classroom observations, group interviews and individual interviews. One other effective strategy for ensuring validity is investigator triangulation which involves the use of multiple observers across multiple investigations (Johnson & Christensen, 2008). In this study, the researcher and the participating teachers were co-observers of lessons. Furthermore, group discussion of results of classroom observation threw up different ideas and perceptions, thus providing additional sources of data.

Sustained engagement

Wallen and Fraenkel (2001) claimed that consistence in observing and interviewing research participants over a period of time is an important factor in achieving reliability. Johnson and Christensen (2008) stated that extended fieldwork is an important strategy to provide for both discovery and validation. Liangputtong (2013) stated that lengthy engagement in fieldwork helps to reduce the bias in research and allows a trusting relationship to develop between researcher and participants. The researcher in the current study had a long period of engagement with the teachers as she spent fourteen weeks with them and had met many times to work with groups of them. During the study the researcher met the teachers of each school five to six times every two weeks (two classroom observations and three group discussions).

Persistent observation

A sufficient number of observations is important as it adds to the depth and quality of data made possible by sustained engagement over time (Guba & Lincoln, 1989). The researcher observed each teacher in a classroom visit regularly in all the six fieldwork phases during semester two. The researcher also met the participants for group discussions twice every two weeks, with each group discussion lasting from 40 to 60 minutes.

Participants' feedback

Participants' feedback is one of the strategies that promote qualitative research validity (Johnson & Christensen, 2008). In this study, participant feedback was obtained through the group activities which provided the participants with the opportunity to express their views on the research procedures and outcomes. Also, the discussion of the coding scores between the researcher and the teachers during the group discussion of lesson analysis offered opportunities for the teachers to express their understanding and provide explanations.

3.6.2 Transferability

Transferability is the degree to which study's findings can inform and be applied to similar individuals, groups or settings (Liamputtong, 2013). Research methodologists have long been interested in the issue of generalizability of qualitative research by which the findings from a study can be generalized to the entire population. Generalizability requires a large study sample to match the study population and to ensure comparability of demographic characteristics. Qualitative research design does not attempt to generalize findings but to explore a situation and obtain rich information. Unlike generalizability, transferability requires the processes of the study to be accessible and the results to be presented descriptively. Meeting transferability norms does not involve the making of wide claims or a large sample size, but involves the establishment of connections between the elements of a study and the experience of those in comparable situations. In many qualitative research situations, a small sample size may be more advantageous for the research situation than a large sample. Other strategies used to ensure transferability include thick description of the methodology, research context, data instruments and research

processes (Johnson & Christensen, 2008). This study involved a purposeful small sample. Thick descriptions of the research context and close analysis of teachers' experience of applying Productive Pedagogies in mathematics lessons rendered the results transferable to some extent to other mathematics teaching and learning situations. Mathematics teachers and education decision makers may benefit from the thick descriptions of the research design, data collection processes and participants' actions.

3.6.3 Dependability

Dependability involves a review of the research methodology, data collection methods and the research processes (Liamputtong, 2013). Some of the research strategies used in this study to ensure consistency was triangulation and prolonged engagement and field work. Moreover, tape recording of most classroom events, interviews and focus group discussions enabled the researcher to reflect critically on the research process and helped to ensure that the findings are valid and grounded within the theory.

3.6.4 Conformability

Liamputtong (2013) indicated that conformability is related to avoiding the influence of the researcher's bias and interests on the research findings and interpretation of the findings. The researcher in the current study, although an insider of the Omani educational system and a long serving school mathematics supervisor was able to maintain a level of objectivity as she was not directly involved in the schools.

3.7 Ethical Issues

Orb, Eisenhauer and Wynaden (2001) noted that any research that involves the participation of people requires awareness of ethical issues. Two fundamental principles of research ethics are securing participants' agreement and protection of confidentiality (Angrosino, 2012). Adler and Clark (2011) indicated that the main ethical principles in research conducted in social contexts are voluntary participation and informed consent. Wallen and Fraenkel (2001) emphasized that the identity of all participants in qualitative research should always be protected and care should be taken to ensure confidentiality of participants' information. Tuckman and Harper (2012) noted that participants involved in qualitative research have the right to

informed consent, privacy, confidentiality, and the option of remaining anonymous. Informed consent, confidentiality, minimization of risk, and principles of justice were observed in this study. The ethical approval of research with low risk was approved by Human Research Ethics Committee (The protocol approval number was SMEC-96-11, see Appendix 1)

Informed consent

Embarking on involves a negotiation with the research participants about the nature of the research and the treatment procedures in the process of obtaining informed consent (Seymour & Ingleton, 1999). Orb et al. (2001, p. 95) indicated that the “effort to secure participants’ consent is considered to be a negotiation that enables participants to exercise their right to volunteer or refuse participation in the research”. Before participation in the study, potential participants should be informed that they are not obliged to participate and can withdraw at any time, and they should be provided with accurate information about the research purpose and data collection methods (Adler & Clark, 2011). In this study, three types of information sheets and consent forms were provided to the school principals, mathematics teachers and their students before they began participation. The information sheets and consent forms described the research aims, data collection procedures and the specific role of the participants. They also explained the risks, benefits and assured participants of confidentiality (see documents in Appendix 2).

Confidentiality

Liamputtong (2013) defined confidentiality as protection of participants’ identity by not recording their names and not revealing their personal details. Tuckman and Harper (2012) indicated that all contributors in human research have the right to be assured that their individual identities not be featured in the research. For example, teachers and students in school studies may be concerned that the research data could be used for evaluating their and performance (Tuckman & Harper, 2012). In this study, the researcher took care to maintain confidentiality of participants’ identity and personal details by assuring the participating teachers, principals and students that the data they provided in the course of the research would be kept separate from their personal details, and only the researcher and the thesis committee would have access to the data.

Risks

The teachers in this study were able to create a sense of trust among them and act as a learning community. While the procedures of this study involved extra time and effort and exposed their teaching to other teachers and the researcher, they received support and benefited in some ways. The benefit of working in groups for reflection, lesson analysis and preparation of teaching and learning activities was that they were able to share their ideas and experiences and mutually support each other. The researcher also contributed support for teachers by providing alternative solutions to the challenges they faced and encouraging flexibility in their implementation of Productive Pedagogies. In addition, risk was reduced by allowing teachers the freedom to choose the lessons to be observed and the time of the lessons with the researcher helping to coordinate the schedule of classroom observations and group discussions.

Justice

The principle of justice refers to the recognition of vulnerability of the participants and their contributions to the study (Orb et al., 2001). In this study all the schools and mathematics teachers involved volunteered to participate. The researcher informed the participants orally in informal interviews and in the written consent form that their involvement in the research was entirely voluntary. They had the right to withdraw at any stage without it affecting their rights or the researcher's responsibilities.

3.8 Chapter Summary

This chapter has described the research methodology of this study. The sample consisted of six Omani mathematics teachers who taught grades seven and ten during the academic year (2011-2012). The research design consisted of three important phases:

- Phase one (preparation): the introduction of the Productive Pedagogies framework to the teachers in a professional development program
- Phase two (implementation): the implementation of the Productive Pedagogies framework by the mathematics teachers during semester two

- Phase three (dissemination): Teachers' presentation on their experience of Productive Pedagogies at a school professional development event

This chapter has explained in detail the procedure of conducting the professional development program and the different stages of data collection. Several data collection methods were used including classroom observation, group and individual interviews with teachers, focus student group interviews and analysis of personal documents.

The method of data analysis and coding based on the principles of grounded theory were explained and justified in this chapter. Coding of the data was carried out following different analytical procedures in order to achieve a more accurate and in depth analysis. These procedures began with conceptualizing the data during open coding and ended with building a general descriptive overview.

In addition to the above, this chapter discussed validity considerations and ethical issues.

CHAPTER FOUR

DATA ANALYSIS

The overall aim of this study is to investigate the introduction of Productive Pedagogies to a group of Omani mathematics teachers. In particular, the purpose of this research is to investigate the development of Omani school mathematics teachers' understanding and their ability to implement Productive Pedagogies framework in their mathematics classrooms. It also aimed to determine the benefits and challenges experienced by those teachers and to explore students' perceptions towards the change in pedagogy in their mathematics classrooms. The study also aimed to examine the appropriateness of implementing the framework in the educational system of Oman.

In this research, three phases were designed to investigate those research aims: preparation, implementation and dissemination. Phase one (preparation) began at the end of semester one attempting to introduce Productive Pedagogies framework to a group of six Omani mathematics teachers from two schools who attended a professional development program for five days. Phase two (implementation) began at the beginning of semester two in which the teachers started to apply Productive Pedagogies. Within this phase, the researcher participated in the actual implementation during six consecutive cycles (each cycle took around two school-weeks). The third phase of dissemination offered opportunities for teachers as school-groups to present their experience of applying Productive Pedagogies to other mathematics teachers from the region in a school-based professional development day at the end of semester two.

The data in this study were collected from qualitative and quantitative sources. Qualitative data collection occurred at multiple main sources: researcher's diary, field notes of classroom observations and group discussions, teachers' interviews and students' focus group interviews. The quantitative data were collected from 35 classroom observations of mathematics lessons that were coded using the *Productive Pedagogies Classroom Observation Manual*.

In this chapter, the data analysis is presented in five main sections. Section 4.1 presents data explained the development of teachers' understanding and ability of implementing Productive Pedagogies in their mathematics teaching. Section 4.2 presents data regarding the main benefits of the implementation on teachers' practices and students' learning and engagement. Section 4.3 presents data indicating the key challenges that limited teachers' best efforts to introduce Productive Pedagogies framework to their teaching practices. Section 4.4 presents data concerning students' perceptions towards teaching practices in their mathematics classrooms. Section 4.5 presents data summarising the main results.

4.1 Development of Teachers' Understanding and Ability of Implementing Productive Pedagogies

The overall data from classroom observations, group discussions and interviews indicated that mathematics teachers developed new understanding and ability to implement Productive Pedagogies in their mathematics teaching. In particular, the data indicated that in comparison with teacher's initial implementation, teachers' beliefs, understanding and their ability of implementation were developed.

By reflection on the grounded theory analysis, the data drew three stages of teachers' development of their understanding and ability of implementing Productive Pedagogies: *preparation*, *developing* and *consolidation*. In the *preparation* stage, the teachers prepared for their learning during their participation in the professional development program and within their initial attempts of implementation in the first cycle (the first two school-weeks). The attempts of implementation within this stage were limited due to the influence of teachers' previous teaching beliefs and practices and by their initial reaction towards applying Productive Pedagogies. The second stage of *developing* showed that teachers were developing an understanding of Productive Pedagogies and ability to introduce them in their teaching. In particular, during the *developing* stage, specific needs to support each Productive Pedagogies dimension were identified. Thus, the researcher's efforts were directed to meet those needs during group discussions of lesson analysis and during those discussions to prepare common teaching and learning materials. These efforts formed the basis for developing teachers' ability to apply Productive Pedagogies in their mathematics classrooms that are illustrated in the third stage of *consolidation*. The following

Figure 4.1 explains the sequences of the stages in the development that will be discussed in the following sections.

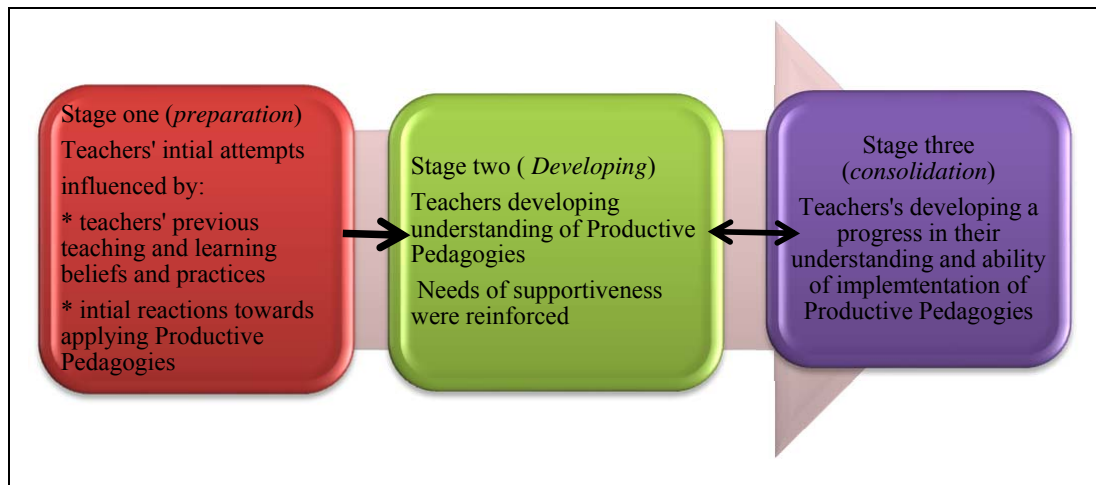


Figure 4.1 Stages of implementing Productive Pedagogies during the observation time

4.1.1 Stage one (*preparation*): the grounding of teachers' understanding and ability of implementation

The general data from the stage of *preparation* indicated that within all dimensions, teachers' initial efforts of implementation were slow. In particular, the initial teachers' attempts to introduce Productive Pedagogies to their teaching were limited as the teachers were influenced by their previous teaching experiences and by their initial understanding and reactions towards applying Productive Pedagogies. However, this stage formed the grounding base of teachers' understanding and ability of implementation in the advanced stages of development. Within this section, it was useful to identify teachers' views about their previous teaching experiences and the main initial teachers' reactions that influenced their initial attempts of applying Productive Pedagogies. Some of the lessons that showed the initial teachers efforts of implementing during the stage of *preparation* will also be illustrated and compared with the teaching practices from the following stages of *developing* and *consolidation*.

Teachers' views about their previous teaching experience

During the professional development program and before applying of Productive Pedagogies, mathematics teachers expressed specific views regarding their teaching experiences during in-service years. In particular, they indicated specific reactions

regarding their learning and teaching beliefs and their previous teaching approaches. The data also identified specific initial reactions towards Productive Pedagogies.

Previous teaching beliefs and approaches

Drawing on data from professional development programs that were conducted before semester two, teachers believed that one of the goals was to build upon their teaching practices of preparing students for formal tests. According to that teaching belief, teachers focused on completing the teaching curriculum content and targeted their attention and efforts to finish practicing all the examples and exercises in the mathematics textbook. Thus, teachers noted that the goal of teaching all the examples and activities with students affected the quality of teaching practices and led to a lack of teaching methods that encouraged motivation and engagement. One of teachers noted that their previous teaching lacks emotional learning and connectedness. She said:

In fact, my previous actual teaching practices lacked the emotional-learning standards and missed the connectedness between mathematics as knowledge and its applications and importance in a student's life. (Teacher's interview, 1/4/2012)

In addition, one of the points of interest in the participants' teaching experiences was the kind of planning that teachers knew and applied in their previous teaching. It was a short-range planning which means that mathematics teachers planned day by day or lesson by lesson and relied on the order of the formal mathematics textbook. In fact, their actual teaching practices of mathematics lessons relied mainly on mathematics activities, examples and problems that were included in the formal mathematics textbooks. That kind of planning may not help them to investigate the integration within mathematics content and between mathematics contexts and objectives.

Within mathematics classrooms, teachers played the main role of teaching and decision-making and students responded and followed their instructions. For example, teachers' actual instruction depended on asking direct questions and receiving short answers. One of the teachers pointed to the direct teaching approach that she used. She commented:

I used one simple teaching style, which is direct teaching method that is based on asking and receiving answers because it was easier by virtue of time constraints. (Group discussion, 11/1/2012)

Moreover, the data indicated that the previous teaching practices of the participants using simple and direct teaching strategies. In particular, teachers noted that their previous teaching experiences of teaching mathematics lessons were based on using direct instruction. Teachers indicated that their presenting of mathematics content started with the parts and then moved to the whole. Most specifically, teachers taught their mathematics lessons by starting with a review and then introducing the main mathematics concepts directly by showing illustrative examples following them by directed exercises and finally providing students with some exercises and mathematics problems to solve. One of the teachers indicated that her teaching approaches were based on transmitting mathematics knowledge without challenging students' understanding. A teacher said:

The previous ways that I used to introduce mathematical concepts and to teach mathematics problems and ideas did not challenge and support students to develop their own understanding. (Group discussion, 16/1/2012)

While the teachers also noted that even they used some good approaches such as narrative style, collaborative groups and e-learning, those strategies were used just to add some kind of change in their classroom situations. One of the teachers commented on her teaching practices and pointed to the gap between her teaching strategies and what Productive Pedagogies targeted. She said:

In my previous teaching practices, I used some of the elements of Productive Pedagogies such as narrative, but without scientific basis. For example, I introduced stories in my mathematics classroom, but my use of this method was not in a scientific way and not in the form of any connection to community and identity. I used it randomly in a few typical lessons and not in all or most of the lessons. (Group discussion, 4/3/2012)

Drawing on a previous comment, the implementation of the narrative style in teachers' practices was by introducing some short background stories during teaching mathematics lessons. However, those stories were not related to what they

taught in the mathematics classroom but were aimed to reduce the boredom of the mathematics lesson. Moreover, some of the group-learning activities did not encourage interactive learning between students. Even in those activities that were designed for groups, sometimes students worked individually or one student completed the activity while the others watched what he did. Regarding applying e-learning approaches in teaching mathematics, that approach involved conducting the lesson in the computer-lab while students followed the content of the lesson on the computer without using any programs that were designed specifically to learn mathematics.

Regarding teachers' beliefs about the idea of connectedness in learning and teaching mathematics, they considered that their idea of connection in mathematics involved recovering the basic body of mathematics information for each new mathematics concept or idea. The following comment from group discussion during a professional development program provided an indication of teachers' beliefs about knowledge about connectedness:

Our previous ideas about connectedness in teaching mathematics were to connect the new mathematics knowledge that they taught with students' basic mathematics knowledge that they already had. (Group discussion, 12/01/2012)

The above belief showed clearly that teachers' investigation of the connection of mathematics knowledge is by connecting the new knowledge with the basic abstract mathematics concepts. While this conception is true in some way, it may not have been in accordance with the intended meaning and the aims of knowledge connections such as its value beyond the classroom and its integration within other subject knowledge and student's personal knowledge. Regarding teacher's effort to recall the mathematics knowledge that students have learned in the previous lessons within the same grade or previous grades, most teachers did that at the beginning of each lesson. This kind of recalling of the essential mathematics information related to what teachers learned during their pre-service education or within the first in-service years in which mathematics education supervisors of teachers emphasized the recalling of the basics for introducing any new mathematics concept. Some of the teachers, as they noted, recalled the essential information of the lesson by asking

their students few questions about the previous lessons or went quickly over what students learned in the previous grades. Others noted that even the students did not understand what they learned in the previous grades, however, teachers had no time to go over previous material, as they have to continue their teaching in order to cover the curriculum of this grade.

To sum up, the overall teachers' previous teaching practices concentrated on using teacher-centred approaches in which the teacher has the main responsibility for managing students' learning and leading them through textbook learning activities that require following step-by-step methods. The teachers have believed that their concern is on encouraging their students practicing to practice what has been transmitted to them in order to prepare them for the final exams.

Initial teachers' reactions toward Productive Pedagogies

During professional development sessions, mathematics teachers indicated some positive reactions towards the Productive Pedagogies framework. However, they also questioned the relevance of some elements of Productive Pedagogies to the Omani context such as the *cultural knowledge* element.

Teachers showed positive reactions towards introducing Productive Pedagogies in their teaching practices for two main reasons. Firstly, Productive Pedagogies can help them to develop their plans of teaching mathematics lessons. They noted that:

Keeping the Productive Pedagogies dimensions in their minds during the preparation of the mathematics lessons can help teachers to avoid missing out anything in their teaching practices. (Group discussion, 18/1/2012)

Secondly, teachers developed new knowledge about effective teaching practices and the standards that they have to consider while preparing the mathematics lessons, as well as while teaching and evaluating students' learning. They referred to the role of the elements of Productive Pedagogies dimensions that can be used as reference to judge and improve their teaching practices. While some of these elements are new to them, by using and applying them, teachers indicated that they developed new broad meanings of specific elements that they did not previously possess. For example, such meanings that teachers developed helped them to break away from their narrow thinking of the idea of connectedness as recovering only the basic body of

mathematics information. Those new meanings developed towards the broader value of connectedness that covers deep understandings of the application of this dimension in teaching mathematics. The following comment of one of the teachers showed an indication of a new understanding of *background knowledge* element. She noted:

We developed a new understanding of background knowledge element as it was an element that aimed to include the basic knowledge of mathematics concepts and rules but it extended to include students' personal experiences and knowledge of the community. (Group discussion, 27/2/2012)

Moreover, teachers indicated that Productive Pedagogies could help them to begin thinking about the appropriate teaching practices that have positive effects on students' learning and engagement. One of the teachers pointed to the value of Productive Pedagogies as it directed their attention to improve their practices towards achieving best outcomes on students learning and engagement. She talked about her initial reactions when she recognized the aims of applying Productive Pedagogies, and said:

Some of the elements of Productive Pedagogies struck me to tell myself that if I applied and could pay attention to these elements, the change will be positive for the students' learning and engagement. We need those ideas, which direct teachers to apply these pedagogies and to realize their importance. Teachers need some new structured ideas that encourage them to improve their teaching practices. (Teachers' interviews, 1/4/2012)

The previous comment indicated that teachers desired to improve their teaching practices as well as they needed support towards achieving that improvement. While the previous comments showed that teachers indicated some positive initial reactions towards the Productive Pedagogies framework, they showed that they did not reflect all the dimensions of Productive Pedagogies in their teaching or were aware of their importance. For example, their ideas around some of them were different from what the framework targeted. One of the teacher's comments pointed clearly to that point of view. She said:

We do not have complete knowledge of all the dimensions of Productive Pedagogies and our knowledge of them is entirely different from what the framework targets. (Group discussion, 16/1/2012)

In particular, teachers expressed that the meaning or relevance of some elements was questioned such as *cultural knowledge* element and *metalanguage* element. For example, teachers indicated that their beliefs about learning mathematics did not give attention to the elements that recognize social constructions or value multiple cultures. In particular, teachers pointed out that the working and valuing difference dimension was missing from their previous teaching practices and was out of their attention. When this dimension was brought to the attention of the participants during the professional development sessions, teachers indicated that the main differences between students were the differences in their learning. They indicated some question marks about some of the elements included in this dimension. Specifically, teachers questioned the appropriateness of applying the element of *cultural knowledge* in the Omani educational environment in general and in their mathematics classrooms in particular. That was because the element of *cultural knowledge* in Productive Pedagogies framework aims, as teachers understood it, to value non-dominant *cultural knowledge* that requires the presence of more than one cultural group and takes into account the differences of gender, race, economic status and other cultural and social factors. Thus, according to the conception of this element, teachers indicated that they did not believe that the *cultural knowledge* element has a real application in the Omani context for the following reasons.

First, Oman, as they indicated, is a homogeneous culture. The homogeneity of Omani cultural occurs in many aspects. For example, nearly all Omanis are Arabic and Muslim. The Omani national identity has evolved from its predominant Arab language and Islamic religion. Regarding the participating classes, all students are Omani, Muslim and from the same region.

Secondly, in teachers' minds gender was not relevant in their context because most of the middle-grade schools of boys and girls were separate. Moreover, concerning economic status, teachers noted that all Omani students received free learning and were provided with free textbooks. The government in specific affordable and stable

prizes for all Omanis fixed writing books and school instruments that students bought.

In addition, regarding the differences based on economic grade level, many debatable points were also raised during some group discussions about the significant gaps in economic status between students of low academic achievement and high academic achievement. Some teachers pointed out the necessity of taking into account the disparity in economic level between the students as a factor that affected students' motivation to learn and hence affected their academic achievement. One of the teachers indicated the extent of differences of economic status among students by stating that:

Economic disparity between students of the school is clear as in some areas the economic level is below the overall average. That affected the level of students' achievement. From my experience over many years, we as well as school administration took this fact about those students in the classroom, but we cannot solve this problem. (Group discussion, 16/1/2012)

On the other hand, other teachers played down the impact of the different economic level factors as most of the students are from similar economic levels. They mentioned that students from low economic levels received aid from the government, but most of the participants pointed to the influence of social situations such as the cases of divorce, orphan hood, and ongoing family problems. They indicated that those situations were the most negative influential factors on their students' motivation to study. They indicated that most of the low achieving students have social problems at home rather than background social economic situations or factors.

The previous comments about applying *cultural knowledge* element raised many debatable points about the appropriateness of introducing this element to the Omani context. Regarding teachers' views about the element of *metalanguage* (an element of the intellectual quality dimension), they expressed the view that the idea of commenting on students' talk and writing tasks such as correcting students' mistakes of using specific Arabic vocabularies and grammar in mathematics classrooms was a new conception that they previously did not give any attention to. Their attention

focused on the consolidation of the language of mathematics; the language of numbers and mathematical symbols and terminology.

To sum up, teachers' experiences and reactions that were expressed indicated that those experiences shaped teachers' first attempts of application of Productive Pedagogies. Teachers' previous experiences and beliefs limited their initial efforts and slowed the translation from their traditional teaching to the practical implementation as will be discussed below. However, those experiences motivated them to introduce Productive Pedagogies in their teaching practices. The following data indicated that teachers showed a positive development in their initial attempts at implementing Productive Pedagogies by illustrated lessons from the stage of *preparation* with comparisons with lessons from stages of *developing* and *consolidation*.

Initial attempts at implementation of Productive Pedagogies

The data from classroom observations and group discussions indicated that the teaching practices within mathematics classrooms showed limited abilities of teachers to in the implementation of Productive Pedagogies during the initial stages. Most of their teaching practices were influenced by their previous teaching experiences that were based on transmitting mathematics knowledge. They were influenced by the heavy reliance on using direct questions and receiving short answers. Their limited understanding of some elements of Productive Pedagogies also influenced them. The following example of one of the observed lessons showed an introduction to the concept of 'quadratic function' by presenting it in association with the survival of natural ants and the movement of a swarm of birds. That lesson aimed to introduce the concept of quadratic function for students in grade 10. The teacher tried to apply the *connectedness to the world beyond the classroom* element.

Grade 10, school technology Centre, the lesson of “Function” , 29/2/2012

The teacher began her lesson by introducing a video clip that illustrates the movement of ants in water during a flood to save them as they moved in interdependent groups. The teacher and students expressed their views about the causes for the movement of the ants in that way, as well as how they were able to breathe as some of ants moved in groups under water.



There was also a discussion around the value of connection and collaboration between ants as one group.

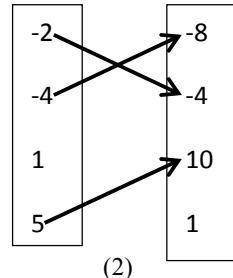
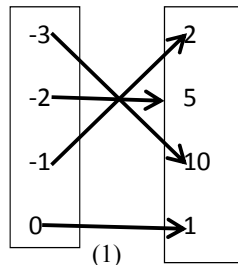
Then the teacher asked students to draw geometrical graphs for the ants’ movement in water. Some of the students drew their shapes on the whiteboard.

The following classroom discussion at this point was as follows:

The teacher: what is the name of the shape that illustrated the ants’ movement?

Student: Curve

The teacher: This curve indicates the “function”, but if each curve is a curve of a function; we have to know what the features of “function” is. First, you studied the concept of “relation” in grade nine and as we knew that, there is such a relation between the set of elements in one domain and other elements in the opposite domain. Let us look at the following examples. How do you compare both of them?



Student: all the elements in the first domain are related to the elements in the opposite domain while in the second diagram not all of them have a relationship with other elements.

The teacher: Yes, when all elements have relations with the elements in the opposite domain, this relation can be called a function. What other relations that you have studied are used in our life?

Students expressed different ideas such as the mathematical relation between human length and weight and between pressure and weight. Then the teacher showed her students a part of a video clip of the “movement of a swarm of birds in the air”.



She asked students to draw the shape of that movement and make comparisons to the movement of ants. Discussion around students’ drawings and comparisons led to the suggestion of “the movement of birds’ as an anti-example of a function (Researcher’s diary, 29/02/2012).

The above classroom practices that aimed to introduce the mathematical concept function indicated a superficial understanding of the mathematics in many aspects of life or artificial examples of actual application of mathematics concepts in the world. The teacher tried to point to the connection between mathematical concepts and shape of the ants' movement. However, the example of the movement of ants in a flood and the movement of a swarm of birds is misleading as they are not really graphs of functions. For example, those shapes did not reflect that there was a relation between two sets of elements. Regarding forming the shape of ants' movement, each ant holds the ant that is above it and under it. There is nothing specific about the relation between the two sets of elements. Moreover, there was a danger of developing inappropriate conceptions about function especially about the important aspect of function that is based on understanding that each element has a unique value that depends on the formula of the relation. In other words, the confusion was also to concentrate on the shape rather than what it represented as a relationship between two variables, which was not a good connection with the real world.

The teacher gave her students different opportunities and different ways to express their ideas by using different approaches such as: drawing the movement, explaining the drawing of the graphs and writing the main observed characteristics. However, the classroom discussion based on asking questions and receiving short answers may reflect the influence of their previous teaching practices that involved transmitting knowledge during direct classroom discussion between the teacher and students. The teaching practices of that lesson indicated that the classroom discussion did not developed well to meet the actual meaning of function as well as it did not reflect a substantive conversation between the teacher and students. For example, when the discussion showed that the curve was a shape of function, there was a need here for more comments from the teacher to avoid misunderstanding of the concept, as the shape of the curve does not mean that the graph is for a function. The classroom discussion could have been directed towards understanding the actual mathematical meanings of function as a relation between a set of elements that are defined as inputs and a set of allowable outputs with the property that each element of input is related to exactly one output. In addition, maybe there was a need to connect the new concept of function to the basic concepts that students learned about relation, co-

domain and domain in order to understand the similarities and differences between them and to construct cumulative meaning of the concept. Moreover, that classroom discussion that was built on comparing between the mathematical relation and function may need a focus on the deep knowledge that formed this concept in order to develop deep understanding of function as a new concept.

Within the previous classroom practices, the teacher also tried to develop her students' values by emphasizing the value of cooperation between ants, as the value of cooperation is one of values that students need to develop during their learning of any subject. Maybe this value developed if students worked together in groups to experience cooperative work such as drawing graphs of different functions and making comparisons between the graphs that illustrated functions and those that illustrated non-functions.

While those teaching practices were not really reflecting good applications of the element of *connectedness to the world beyond the classroom*, they formed the basis for improving the implementation of Productive Pedagogies in the advanced stages. Maybe the following example of the lesson of "function and vertical line test" is a good example of some improvement in implementation. The previous and following lessons were conducted in the same cycle of implementing the previous lesson "function" and within the content of the same mathematics unit of grade 10. The following classroom practices indicated the teacher's attempts to investigate connectedness in her teaching practices. It also showed some improvement on classroom conversations by giving students opportunities to make comparisons and construct explanations.

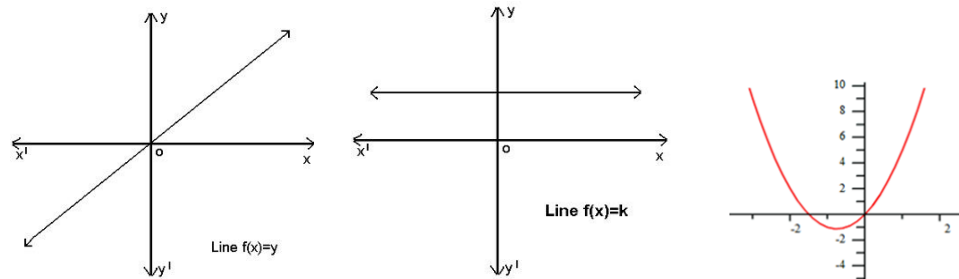
Function and Vertical Line Test, Grade 10

The teacher first discussed the question: “Do we need this ‘quadratic function’ knowledge in our life?”

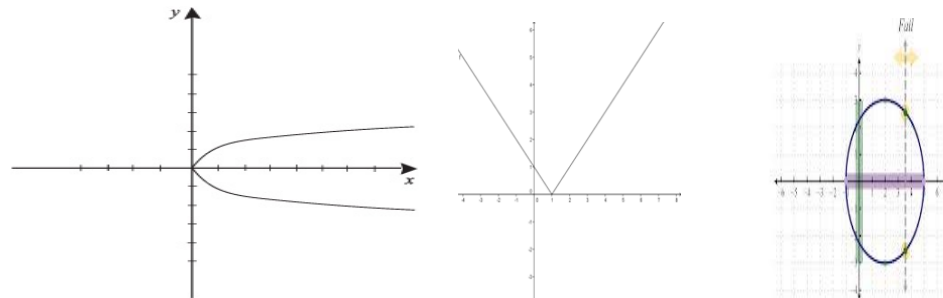
There were many different answers from students. Most of them were about some examples of shapes that illustrated linear and quadratic functions found in our real life. The students gave many examples of shapes that relate the idea of quadratic function to shape such as many objects that found in schools, roads and homes.

Teacher emphasized what students said and indicated that engineers need to understand quadratic equations or functions to design many beautiful profiles in our schools, kitchens and our rooms. The pan, cups and spoons in the kitchen and the umbrella in the school yard are examples of that are designed using quadratic expressions.

The teacher then drew on the whiteboard some graphs that illustrated linear and quadratic functions and asked their students to draw their own figures that illustrated the shape of linear and quadratic functions.



Some of the students drew the following examples:



Based on those graphs that students drew, the teacher asked all students to draw a vertical line that cut each graph that was drawn and write what they found, and then the teacher discussed with students their observations and concentrated on the previous examples. The teacher-students discussion about students’ observations and comments were as follows:

Teacher: Are these graphs of functions?

Student 1: it looks like graphs of functions.

Teacher: How do you know?

Student 2: The graphs are relations between x and y . They are linear or semi-circular shape

Teacher: Is it enough to be a function? Is any relation between x and y with these shapes a function? When you draw a vertical line, what are the differences between the graphs in group one and those of group two?

Student 3: both groups have point of intersections between the graph and the vertical line.

Teacher: How many points of intersection for each value of x are there in the two groups? (the teacher took the comments from more than one student). Most students identified that in graphs of group one “each value of x has a unique value” while in group two “each value of x has more than one value and that means that graphs in group two do not illustrated functions”.

The teacher then indicated that those graphs illustrated examples of mathematical relations that will have their own concepts and will be studied later in year 11 and twelve. The teacher indicated also to the importance of the vertical line to easily and quickly test whether or not the graphs were graphs of functions. The teacher advised students that they could place the vertical line on the graph. If the graph was a function, the graph should cross the vertical line only once and there is no restriction where on the graph you place the vertical line (Researcher’s diary, 03/03/2012).

The above teaching practices indicated that the teacher attempted to reach the idea of connectedness in teaching the use of a vertical line to test the graph of a function. The teacher pointed to the importance of learning “function” for students by illustrating some examples from real life of the application to help engineers to design many things. Those examples may not illustrate better connection with the world because the examples that students illustrated concentrated on the shape of the curve in the objects that they found such as cups. While the teacher indicated that understanding the idea behind designing those objects is by understanding the idea of function, there was however a need from the teacher to point to a model of a specific real situation that illustrated function as a rate of change between two variables.

Moreover, the teacher indicated that this lesson will form the basis for other mathematics lessons in grade 10 and even in grades eleven and twelve. That suggestion may also reflect the teachers’ attempt to add a real value to her lesson by emphasizing its importance to understand other mathematics lessons in the following grade level. In addition, the teacher emphasized the importance of the vertical test approach, as it is an easy and quick way to test the graph of function.

Regarding classroom conversation, offering opportunities for students to compare two groups of different graphs that were drawn by students themselves and expressing their ideas may help them to build their own knowledge and understanding of the concept of function. While the classroom discussion was based on questions and short answers, the questions encouraged students to explain what they observed. Maybe, the teacher could have challenged their students to examine the connections between the equation of the function and the graph as they already had learned in semester one how to get the slope of the graph and determine the equation.

To sum up, while the previous teaching practices of that lesson showed some attempts to apply the idea of connectedness in teaching geometrical concepts, they highlighted the teachers’ need to understand the meaning of the real application of mathematics in life. In comparison with the previous lesson at the initial implementation during the stage of *preparation*, the classroom practices of this lesson indicated that there was some improvement on directing the classroom

discussions toward more opportunities of classroom conversations that were based on students' observations.

Another example from the initial implementation from a teacher who taught grade 7 illustrated how the teacher introduced the concept "algebraic expression". The following practices showed limited understanding of some of the elements such as *narrative*.

The value of algebraic expression, Grade 7, 4/3/2012

The teacher told the students the following short story.

"There are two cats have done a good job for their father and the father wants to give them a reward in return for their good job that they did. He gave each one an enclosed small pocket of money. Both of the two cats did not know how much money was in the pocket but both of the pockets contained the same amount of money. Then the father gave the first cat an extra 2 Rials while gave the other cat three Rials". (Rial is the Omani currency)

Then the discussion between the teacher and students was as follows:

Teacher: How many cats were in the story?

Student 1: Two

Teacher: How many pockets did each cat receive?

Student 2: One

Teacher: If we do not know the amount of money in the pockets, how can we express that mathematically?

Student 3: x

Teacher: Is it only x or any other symbol?

Student 4: y , z or any symbol

Teacher: How can represent the amount of money that the two cats have?

Student 5: Cat one has $x + 2$ and cat two has $x + 3$

Teacher: Yes, if I supposed that $x = 3$, how much did each cat have?

Student 6: Cat one has 5 and cat two 6

Teacher: If we put the value of x in the algebraic expression, we can get the value of the algebraic expression.

Then teacher then asked students to get the value of $3n + 5$ when $n = 1$ and $n = 2$ (Researcher's diary, 04/03/2012).

The above practices showed that the teacher tried to model the problem of the two cats that was illustrated in the short story. However, it is still artificial and unlikely to be related to real life. During the group discussion of the lesson analysis, the teacher noted that she attempted to apply the narrative approach in her teaching of the lesson "the value of algebraic expression". The students were familiar with what happened in the story but the teacher added only the last part. However, using the short story as an example of applying *narrative* element indicated a shallow understanding of this element, as the good application of narrative style required an important target, a link of sequence of events and a link to some personal and cultural events that were not reflected in the lesson.

Moreover, the classroom discussion concentrated on transmitting information and leading students to give short right answers to the teacher's questions. During the

teachers' group discussion, which was conducted after the class to analyze this lesson and improve the teaching practices, the teacher talked about her teaching practices and how she introduced the Productive Pedagogies to that lesson and said:

I did not apply the elements of knowledge as problematic and higher order thinking as the lesson was just about getting the value of algebraic expression by substituting the value of a variable. I began from the simplest part and continued to the whole part. I used narrative style and I used the same story for many lessons. I think the students were engaging and participating well. I supported them with positive words. (Group discussion, 04/03/2012)

Regarding the previous teacher's comments, it is promising that the teacher has kept in mind most of the dimensions of Productive Pedagogies while evaluating her lesson. However, some of her indications of the use of the elements of Productive Pedagogies indicated a limited understanding of them and of the importance of applying them. The evaluation of her teaching practices based on her efforts to support her students by encouraging their class participation. However, answering simple questions and directing students to give the right answer may not reflect good indications of real engagement and participation. The classroom discussion did not convey good conversation between the teacher and students and between students. It was influenced by the teacher's previous teaching experiences that encouraged following the teacher's questions. Moreover, the idea of *narrative* in this lesson did not convey understanding of the importance of the structure of events that includes personal stories, historical accounts or biographies, as the content of the story did not reflect a real life context. It represented the teacher's previous beliefs about using short stories to add some enjoyable events in the mathematical class regardless or not if it was relevant to students' lives.

The following example from a mathematics classroom may show an improvement in the teaching practices of the same teacher who taught the previous lesson. The lesson conducted in the advanced stages of implementation after one week of conducting the previous lesson with the same class in grade 7.

Introducing the concept of polynomial, Grade 7, 14/3/2012

The teacher began her lesson by asking short questions about some mathematical concepts that students learned in the previous grades. She received short answers to these questions from students. Those questions were about the concepts of constants, variables, fractions and roots. The main questions were:

Teacher: What can we call $\frac{a}{b}$ if a, b are numbers or variables?

Student 1: Fraction

Teacher: What are the limitations for the values of a and b to be $\frac{a}{b}$ fraction?

Student 2: $b \neq \text{zero}$

Teacher: Can you read it? Give me examples of a fraction? (Students gave different examples of fractions).

Teacher: What can we call: $x + \frac{2}{3}$,

Student 3: not a fraction

Teacher: Why?

Student 3: It consists of a variable added to a fraction

Student 4: We know that x is a variable and $\frac{2}{3}$ is a constant; this is a mixture of variables and constants.

Teacher: What is the operation between the variable x and the constant $\frac{2}{3}$?

Student 5: Addition.

Teacher: Can we put x as a fraction?

Student 6: Yes, $x = \frac{x}{1}$

Teacher: What about $\frac{1}{x}$?

Student 7: It is a fraction

Student 8: It is a fraction divided by a variable. It is different from $\frac{x}{1}$

Teacher: $\frac{x}{1}$ and x are algebraic expressions of one variable. More about them will be discussed today.

The teacher then wrote the following two groups and asked students to work in groups to make comparisons between them:

Group one
$4x$
$6.4n - \frac{2}{7}n^2 - \pi n^5$
$2x^2 - 5x + \sqrt{3}x^2$

Group two
$\frac{4}{x^2}$
$\frac{2}{n} - 3n^2 + 2$
$2\sqrt{x} - 3x + 1$

Each group talked about their observations and the teacher wrote their comments on the whiteboard and discussed with all students the common and different comments between the groups. The discussion then concentrated on the ideas of dividing by a variable or the exponent of $(1/2)$. The teacher said that group one can be given a new name "polynomials". Then the teacher asked students to think about the following questions if we called group one polynomials".

- Based on the main characteristics of the polynomial that you found, can you give a definition of the polynomial? What we can we call the polynomial if it consisted of one term, two, three, or more?
- Can you order the polynomials according to the degree of the variables?
- What is the meaning of poly?
- Is constant a kind of polynomial or not? And why?
- Can you give me examples of polynomials and non-polynomials and why they are not?

All these questions were discussed within group activities. Discussing the previous questions helped students and the teacher to reach the conclusion of what a polynomial is. Polynomial comes from 'poly-' meaning 'many' and 'nomial', in this case meaning 'term'. So, it says "many terms", and they are special names for polynomials with 1 or 2 (monomial and binomial). At the end of this period, the teacher gave exercises about polynomials and asked students to work individually to:

- Recognise the polynomials from non-polynomials.
- Modify non-polynomials to be polynomials.
- Identify the degree of a polynomial.

While the researcher observed students doing their exercises, the answers provided by students reflected their understanding of the concept of polynomial and their ability to apply their knowledge in forming their own examples of polynomials. From the beginning of the lesson, I found students participated well in classroom discussion and within-group activities (Researcher's diary, 14/03/2012).

The teaching practices showed some good implementation of *substantive conversation* element. In particular, the classroom discussion within those classroom practices took different forms of substantive conversations. The first form is the direct conversation between the teacher and the whole class about the basic information that they learned in the previous grades that were important to introducing the concept of polynomial. For example, the teacher asked students about the concepts of constant, variable, fraction and roots. Recalling these concepts are important for learning the concept of polynomials. While that discussion based mostly on asking direct questions, some of the questions aimed to give examples, explain answers and give explanations by asking questions like ‘Why it was not a function?’, ‘Can you give me examples?’. The second form was the classroom discussion between students within group-activities that required making comparisons to identify the main characteristics of polynomials. For example, the teacher asked students to work together to compare polynomials and non-polynomials. Within this group activity, the students discussed six different examples that offered opportunities for sharing ideas, giving explanations and constructing other examples. Each group talked about their comments orally to the other groups. Expressing their ideas orally to the whole class required common decision and obvious common ideas that have to be constructed during their social interactions. For example, displaying to students a definition of polynomial, critiquing the given examples of polynomials, illustrating their own examples of polynomials, all those actions required a common decision, collaborative work and critical explanations. These forms of classroom discussions may reflect a good attempt of the teacher to apply the *substantive conversation* element in her teaching practices. These forms also showed a difference in the classroom practices compared to their previous teaching experiences that was based on direct classroom discussion that encouraged asking direct questions and receiving short answers. The role of the teacher in classroom discussion was also changed from having the main role of presenting, to sharing that role with students and giving them the opportunities to talk, discuss, explain and express their ideas.

Moreover, the teaching practices in the previous lesson indicated that the teacher attempted to apply *depth of students’ understanding* element. Regarding helping students to develop deep understanding of polynomials, the students constructed their

own meaning of polynomials and explained their own definition of the concept based on their observations and discussion during group conversations. Also, modifying the non-polynomials to become polynomials required understanding of the features of polynomials. Moreover, constructing students own examples of polynomials needs recalling their knowledge about the previous lessons on algebraic expressions and then considering the limitations of the expressions to be polynomials. For example, when the teacher asked students to distinguish polynomials from non-polynomials, they could not do so without recalling the limitation of algebraic expressions to be defined as polynomials. Moreover, modifying the non-polynomials to be polynomials, as the teacher asked, required transformation of their understanding of the limitations of polynomials to construct new forms of polynomials. That helped them to develop deep understanding of the concept and thus they were able to apply this understanding in doing their individual exercises.

Overall, the initial teachers' attempts at implementing Productive Pedagogies showed that their attempts were influenced by the teachers' previous beliefs on teaching that was based on transmitting information and using direct teaching approaches. Those attempts were in some aspects weak and reflected a need for more explanation of Productive Pedagogies. All their initial attempts at implementation encouraged the need for supporting teachers to improve their understanding and ability of implementing Productive Pedagogies in their mathematics classrooms. However, change needs time and support. Thus, the advanced stages of *developing* and *consolidation* indicated some improvement in application as was illustrated in the comparisons within this section and that will be discussed in the following sections.

4.1.2 Stage Two (*developing*): the development of teachers' understanding of Productive Pedagogies

The previous comparison between the attempts of teachers' implementation of Productive Pedagogies in the stage of *preparation* and in their implementation in the stages of *developing* and *consolidation* showed that there were some indications of a development in teachers' understanding of the pedagogies as well as in their ability to apply them in their mathematics classrooms. However, shallow understanding of the elements of *connectedness to the world beyond the classroom* and *narrative* was

illustrated in the previous comparison. In particular, in the previous examples of teaching practices, teachers demonstrated rather shallow understandings of the *narrative* element by using short stories that did not reflect the real-world context of the students and the real application of mathematics in the real world.

Within this stage of *developing*, teachers' experiences of reflection in classroom observations and the group discussions of lesson analysis as well as the group discussion of preparing learning and teaching experiences helped them to develop their understanding of the Productive Pedagogies. The following group discussion that was conducted to analyze the observed lessons and to prepare learning and teaching experiences that utilized Productive Pedagogies also showed the development of teachers' understanding of Productive Pedagogies. However, those group discussions also pointed to superficial understanding of some of the elements such as *narrative* and *connectedness to the world beyond the classroom*. One of the group discussions that were conducted to analyze the lesson on algebraic expression that was taught during the stage of *preparation* indicated limited understanding of using the *narrative* element and the possible application of *knowledge of problematic* element. The following group discussion reflected not just limited understanding of these elements but limited support from the group to the teacher, as there was a need to demonstrate more examples and explanations on the use of Productive Pedagogies.

Group discussion of lesson analysis “Algebraic Expression”, Ain-Coast Cycle Two, Grade 7, 4/3/2012

The researcher: Can you talk about your teaching practices in this lesson? How do you prepare for teaching it?

Alabeer: First, I want to ensure that my students know what the meaning of “variable” is, because if they understand “variable”, they can understand algebraic expression. I used narrative style by telling a story of two cats that have two pockets of money of unknown value.

The researcher: while students liked the story and they participated well, maybe if we can use this story to introduce the concept of algebraic expression, not just the variable or improve it to be more meaningful and useful. For example, if we give both cats extra specific amount of money and asked students to identify how much they have. The pockets will be expressed by a variable and the added money with a constant and the operation of addition will give a good representation of simple algebraic expression.

Aljawa: We can also give students opportunities to predict the amount of money in the pockets.

Aljawa: Regarding students’ engagement, I think students participated very well.

Researcher: Maybe a few of them as I observed were writing, or talking; we need to engage all students to participate in the lesson.

Researcher: What is the possible application of *knowledge as problematic* element in this lesson?

Alabeer: In this lesson, this element was not applied; each lesson has its own feature that identifies what the elements of Productive Pedagogies dimensions are appropriate to use.

The researcher: If students constructed their own meaning of algebraic expression within real life situations, it may help them to recognize the importance of algebraic expressions in their life. For example, when people go shopping, they did many algebraic expressions in their minds, or for calculating money or choosing the best and lowest price. Maybe discussing some real applications is useful. For example, if any student buys three types of clothes x , y , n with different values (2, 2, 4 for each); or if she wants to buy one of them or choose the best one. According to money, she has (20 Rial); she will construct her algebraic expression and calculate their values. This means that students will use the algebraic expression $20 - 3x + 2y + 4n$ and according to the value of the variable, they can compute the best and least value for her. Maybe we need to explain more about *knowledge as problematic* element, using narrative style.

Alabeer: I think only three students may arrive at the answer to this problem, may be one not three. I think my students need more practice in *higher order thinking*.

Aljawa: In this lesson, I like supporting students’ answers; it is a good idea to create a supportive classroom. It is good for individuals and for groups; as each group is able to do good and progressive work; the group can climb the success ladder that is drawn on the board and solve problems that are more complex. It is a kind of competition that encourages students to work together.

Alabeer: My students really like it (Group discussion, 04/03/2012).

The data from the previous group discussion indicated that teachers developed some improvement in their ability to evaluate their teaching practices and the teaching practices of their peers using the language of Productive Pedagogies. However, the previous conversation indicated that some of the pedagogies needed deeper understanding such as *narrative* element and *connectedness to the world beyond the classroom* element. In particular, one of indications of an artificial understanding of *narrative* element was when the teacher talked about her using the story-problem of two-cats as an example of her attempts at applying this element. Within this example, the problem situation was not experientially real to the students. It was important to make the story relevant to the student’s lives in order to force them to make the connections needed to be rich and meaningful. In addition, the previous group discussion showed that teachers tried to illustrate some examples that helped to

develop those pedagogies such as using some of the ideas of algebraic expressions in some situations in shopping and for counting and estimating values of different objects to choose the best value. However, the improvement of teaching practices that make learning mathematics more applicable to the real world needs to be increased. There is a need to support teachers to develop a range of strategies that address the connectedness and *narrative* elements.

The following group discussions may show a significant discussion about the use of *group identity* element and the aspects of applying connectedness in the lesson. The following group discussion during cycle three of implementation (after approximately five-school weeks), was conducted in the same day as that the group observed the lesson of “trigonometric ratios” in grade 10-class.

Part of Group discussion of lesson analysis, Afra-Sea Cycle Two School 20/3/2012

Three teachers and the researcher sat together to discuss their observations about the observed lesson “trigonometric ratios”

Alnaeem: at the beginning of the lesson, I think it is good that Alhuda (*the teacher who taught this lesson*) asked her students about what students had learned in the previous lesson. It is important to remind the students about “standard position of an angle”, “The positive and negative measurement of standard angle” and about “reference angles”. All this information is important for introducing trigonometric ratios”.

Sama: we have to ensure that students understand the previous mathematical knowledge that is basic for introducing new concepts.

Alnaeem: I think that the teacher also challenged the students as she asked them to determine the height of a flag using the simple tools of measurement.

The researcher: while students found the measurements challenging, their responses to the problem was good. They tried to give different ideas about the best way for measuring the height and at the same time this encouraged their desire to learn something new and easy to find the solution to the problem.

Sama: Yes, that was what the teacher indicated when she said that ‘today you will study new mathematical concepts and rules that help you to solve this problem’. That showed the importance of the lesson and encouraged students’ to find out and learn.

Sama: Also what I like is when the teacher talked about Abu Abdallah Mohammad ibn Jabir al-Battani. She indicated that he was one of the famous Arab and Muslim observers and a leader in geometry. Maybe this reflected the use of *group identities* element as the teaching practices build a sense of the role of Arabs and Muslims in mathematics.

The researcher: Yes,, also the teacher’s emphasis about the role of Battani in producing a number of trigonometric relationships such as $\tan\theta = \frac{\sin\theta}{\cos\theta}$ is important to value the roles of Arab and Islamic scientists in developing mathematics knowledge.

Teacher 1: about the relation of $\tan\theta = \frac{\sin\theta}{\cos\theta}$, the teacher taught the students that $\tan\theta = \frac{\text{the length of the opposite side of the angle}}{\text{the length of the beside side}}$. Another observation that caught my attention

was when one of the students said that $\tan\theta = \frac{\sin\theta}{\cos\theta}$. It is good that one of the students observed this relation before the teacher indicated that, but the teacher did not comment on the student’s observation. I think it is good to show the importance of this observation.

Alhuda (who conducted the lesson): Maybe it was a good suggestion from the student, but I don’t want to jump to this relationship.

The researcher: maybe it is good to not jump but I think it is good to give a positive comment and say that we could discuss this conclusion later (Group discussion, 20/03/2012).

Drawing on the previous teachers' conversations, teachers reflection on their observations of the teaching practice improved as their discussion was not only concentrated on the teaching actions but it went beyond the issues related on the requirements of effective students' learning. For example, they discussed the importance of recalling the previous knowledge of standard position of an angle in order to help students to make good connections between the current knowledge of trigonometric ratios that they have to learn and their previous knowledge that they already possessed. Moreover, one aspect that may indicate an improvement in teachers' reflection was once teachers illustrated some examples from the teaching practices that they observed and recognized their benefits and shortcomings. Some of the examples were their discussion about the role of Al-Batatani and their comments about one of the students' observations and answers. That kind of discussion may help teachers to building upon in their future attempts for implementation.

Regarding supporting teachers to improve their understanding of Productive Pedagogies as well as their ability to introduce them to their teaching, the following dialogue between the teachers of grade 7 aimed to provide some examples of the real application of transformational geometry in real life situations. Some types of transformational geometry were a part of the unit that teachers taught while that group discussion was conducted.

**Group discussions for preparing teaching and learning experiences, Ain-Coast Cycle two
School, 7/4/2012**

The researcher: I think in some of the mathematics teaching practices of some teachers, transformational geometry is usually taught by drawing plan coordinates and using a pen to translate points or shapes. I think it may be difficult for students to see how this can be in any way relevant to real world applications.

The researcher: If students understand the value of using the geometrical idea behind translation in planning, mapping or reconstructing buildings that will be great. An example could be, planning property developments or relocating the buildings by charting their movement across a space and representing them using a standard shape for a building such as restaurants, stations, or others by applying transformational rules to make it move places across the map.

Alabeer: I think we can talk about the importance of translation rules for solving some problems in our real life.

The researcher: Can we give examples of that?

Alabeer: consider one of the serious problems that we have in our region, "the Garbage collectors". The garbage needs to be re-located to other places, and if we can use the maps of the region, we can change their places. It is easy to find other places and how we translate them by determining the units of translation that are needed for that and the direction of translations. That can help to create another map based on transformation geometry rules.

AllJawa, I think that planning and mapping may be suitable for introducing the concept of "translation" and for showing the importance of its application in real life situations, especially for engineers (Group discussion, 07/04/2012).

The previous group conversation was targeted to discuss the specific issue related to the question “how students see how transformational geometry can be in any way relevant to the real world applications”. Within this group discussion, the teachers attempted to illustrate some of the examples that gave indication to the value of transformational geometry such as the use of the geometrical idea behind translation in planning, mapping, relocating or reconstructing buildings.

To sum up, previous examples of group discussions, which were illustrated within this stage of *developing*, reflected that teachers’ understanding of Productive Pedagogies gained some improvement. Within those group discussions, teachers reflected on the observed teaching practices as well as they attempted to prepare learning and teaching practices that utilized Productive Pedagogies. In their reflection on the observed classroom practices, they analysed the classroom practices, made comments and illustrated with examples. While that reflection indicated teachers’ use of the language of the Productive Pedagogies in their analysis and discussion, it highlighted their need for continuous support on understanding *narrative* element and connectedness dimension. In their preparation of common learning and teaching experiences, teachers shared some ideas about the idea of connection in learning and teaching mathematics which was of an artificial understanding that limited their ability of implementing the element of *connectedness to the world beyond classroom*. In particular, they shared their ideas around the real value of using the geometrical idea behind translation in planning, mapping or reconstructing buildings. Building upon the development on teachers’ attempts at implementing Productive Pedagogies through the stage of *preparation* and the stage of *developing* as well as understanding the needs of more support in understanding specific elements during those previous stages which formed the base of the following stage of development, the data showed some progress in teachers’ understanding and ability of implementing Productive Pedagogies in their mathematics classrooms within the stage of *consolidation* as illustrated in the following section.

4.1.3 Stage Three (*consolidation*): the progress in teachers' understanding and ability in implementing Productive Pedagogies

The data collected from classroom observations and group discussions showed indications of a development in applying Productive Pedagogies by mathematics teachers. During this advanced stage of applying Productive Pedagogies to their teaching practices, the teachers pointed to their progress in their ability to implement Productive Pedagogies. The following comment from one of the teachers pointed to the development of her implementation by comparing the beginning of her application with her improvement in her ability of implementation the dimensions of Productive Pedagogies with continuous application. She said:

Generally, the beginning of the application was simple and difficult, but now I can manage my lesson comfortably based on Productive Pedagogies. My first attempts were very simple but with continued use, I acquired more understanding of the dimensions and the mechanism of their effective application. Therefore, I tried to apply all the dimensions in one lesson. (Teacher's interview, 20/4/2012).

The previous comment showed that there was such an increase in teachers' confidence to implement Productive Pedagogies. The teacher pointed to her progress in her ability to apply Productive Pedagogies by her attempts to apply all the dimensions in one lesson. That progress in her attempts indicated that teachers' desire to change had improved towards better application. It reflected that teachers recognized the importance of all Productive Pedagogies dimensions in their teaching and confirmed the effectiveness of all Productive Pedagogies in their mathematics classrooms.

The participants developed an understanding of Productive Pedagogies framework in general and of the pedagogical elements of the dimensions. Their initial understanding depended on considering the framework as a combination of teaching methods or strategies that should be practiced by the teacher in the classroom. In addition, they understood initially that the dimensions are separate and all the pedagogical elements should be occurring in one lesson and in each classroom situation. During the stages of the development, the participants developed an understanding of the concept of Productive Pedagogies as a framework of quality

teaching and reflection. The engagement of the participants with sustained professional dialogue about their practices and performances using the language of Productive Pedagogies helped them to develop an understanding of Productive Pedagogies as a framework that can assist them to reflect on their classroom practices. It can provide an index of quality teaching and students' learning. Even though teachers found difficulties in implementing some of the pedagogical elements, they developed an understanding of them and their importance on teaching improvement. For example, teachers' initial understanding of the pedagogical elements of *connectedness to the world beyond classroom* (an element of connectedness dimension) was focused on the idea of connecting mathematics concepts and idea with the fundamental concepts, rules and algorithms that students studied in their previous grades. The idea of connectedness was developed to understand the importance on providing students with learning experiences that teach them how to recognise and use connections among mathematics ideas and connect them to students' personal and cultural world. Regarding the pedagogical element of *student self-regulation* (an element of supportive classroom environment dimension) teachers' initial thinking considered that achieving this element depends on the role of teacher's ability to regulate students' behaviours. They developed an understanding that it should be based on students' ability to demonstrate high implicit control on their behaviours or movements through their engagement to the lesson. Moreover, the overall idea of differences in the dimension of working and valuing difference was developed. Initially, teachers understood the differences between students as only the difference in their abilities and academic achievement. They learned that the idea of working and valuing difference dimension is about providing all students with opportunities to learn how to participate positively in social practices and to be responsible members of society. One of the teachers summarized her experience of applying Productive Pedagogies and pointed to the development of her understanding and ability of implementing in her following comments:

The beginning of the application of Productive Pedagogies was difficult and not in-depth because of the translation from the theoretical to practical ways. Gradually I gained more understanding about the dimensions and the actual

application in mathematics classroom resulted in a change in my students' motivation, participation and learning. (Teacher's interview, 30/4/2012)

Building on the previous comment, the actual understanding and implementing of Productive Pedagogies developed during the continuing of application. The teachers pointed to the difficulty of their translation from the conceptual knowledge about Productive Pedagogies dimensions to the actual introduction of the dimensions in their teaching practices. That difficulty limited their progress during their initial attempts of implementation. With more opportunities and time for reflecting on their classroom practices, more evaluating of teaching practices and more group discussions conducted during the application, some progress at implementing Productive Pedagogies was observed by the researcher. The following presentation of data will explain the actual implementation of Productive Pedagogies in this stage of consolidation. Within the discussion of the implementation of each dimension of Productive Pedagogies, examples from mathematics classrooms that showed teachers' ability of implementation are illustrated.

However, even this stage of *consolidation* was an advanced stage of improvement; teachers did not reach a stage of perfection in using Productive Pedagogies. There is always the effect of past assumptions, beliefs, and teaching practices. In particular, there was no evidence from classroom observations of an application of some of the important elements such as *knowledge as problematic*, an element of intellectual quality dimension and *active citizenship*, an element of working and valuing difference dimension. Some of the other elements such as *narrative* and *metalanguage* were limited by teachers' previous teaching beliefs and experiences as illustrated in the following sections

Implementation of intellectual quality dimension in mathematics classrooms

The data from classroom observations in the stage of *consolidation* indicated that teachers attempted to apply some of the elements of intellectual quality dimension such as *higher order thinking* and *substantive conversation* elements. However, there is no evidence of an application of some of the important elements such as *knowledge as problematic*. One of the teachers of grade 10 commented that while she identified the importance of *knowledge as problematic* element, she pointed to the difficulty of applying *knowledge as problematic* and she said:

I knew I am supposed to presenting mathematics knowledge as problematic, but I did not know how to achieve that in my mathematics teaching practices. (Teacher's interview, 1/04/2012)

Regarding the application of intellectual quality dimension within teachers' mathematics teaching practices, the qualitative data from classroom observations showed that teachers attempted to apply some activities that aimed to deepen students' understanding of the main mathematics ideas that teachers taught. In the following example of one of the observed lessons, the teacher of grade 10 attempted to implement *higher order thinking* element. The teacher applied group-activities of problem solving in the lesson of "trigonometric ratios" in order to develop higher order thinking operations of analyzing, measuring, synthesizing, making connections and drawing conclusions.

Application on "The angles of Elevation and Depressions", Grade 10, 01/04/2012

The teacher divided the class into five groups, she gave each group two sheets of exercises and asked them to choose one sheet and solve the problem as a group activity.

The activity was about word problems that showed the actual implications of the angles of elevation and depression in real life situations.

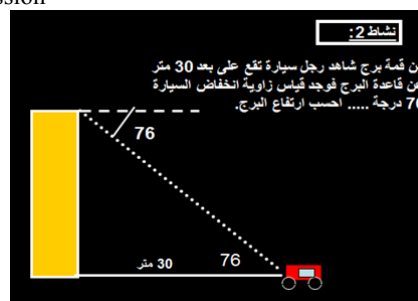
The students were asked to draw a diagram that explained the problem, analyse the information given, choose the best way of solution, check the answer and explain if there are other ways of solving the problems beside the way that was used by the group.

Activity one was about "a person who wanted to measure the height of the Mosque "the place for praying for Muslims" and he stood about 25m from the base of the building".

Activity two was about "a person who stood on the top of a tower observing the road below. He knew the height of the tower based on the angle of depression".



Activity one



Activity two

The groups worked together, I observed the interaction of group-members and how they solved the problem: most of them successfully drew the diagram, identified the main given information and the requirements. However, some of the groups experienced some difficulty in choosing the way to solve the problem at the beginning but they progressed well after short discussion with the teacher when she moved around to observe their work.

After the time for group activity had finished, the teacher asked every two groups that had chosen the same problem to talk together about their findings and then talk to the whole class. One of the groups used more than one way for solving the problem. The groups showed understanding of the concepts of elevation and depression angles, ability to apply Pythagorean Theorem and triangular ratios and complementary angles to solve the problems.

The students worked together, one drew, one calculated, others read, all discussed and contributed to solving the problem (Researcher's diary, 01/04/2012).

The previous classroom practices indicated an attempt from the teacher to apply *higher order thinking* element and showed an emphasis by the teacher to improve the ability of students in problem solving. *Higher order thinking* element was demonstrated in the previous lesson by offering opportunities for students for translating the word problem into arithmetic and algebraic formula. For many students, the transformation of mathematical word problems may be challenging as it required some good skills such as linguistic and literacy abilities in which students had to translate the normal language of the word problem to the mathematical symbolism. Students also have to transform their understanding of the problem by modelling it in a geometrical shape. That kind of transformation required different effective skills such as the abilities of analysing of the problem, recalling of their previous knowledge, identifying the relation between given information, what they needed to solve the problem and the abilities of drawing an illustrated graph that demonstrated all previous skills. To do that, the students need to know the meaning of the mathematics concepts that were involved in the problem, to recall their previous knowledge about trigonometric ratios and right-angled theorems and to make significant relations between the word problem and the graph that illustrated it. All these actions were done by students' interactions while they worked as groups.

Moreover, the researcher's diary noticed that performing problem solving within group activities helped students to construct their knowledge, understand and be able to solve the problem during social interactions. The students worked together to solve the problems but they challenged each other as they had to display more than one way of solving the problem as well as to check the results which needed more examination of the problem and the solution. These practices helped students to develop their abilities for analysing, synthesizing and concluding. In addition, when the teacher gave the groups opportunity to choose the problem that they wanted to solve, this opportunity encouraged them to develop an understanding of both problems in order to make a good decision. The students demonstrated good understanding of the angles of elevation and depression as well as good performing of the problem solving strategies. While I observed students' group work, I asked one of the groups about the possible ways to know that they reached the right solution of the problem that they had worked on it. One of the students said that "*When we got back from the solution to the given problem, maybe that helped us to*

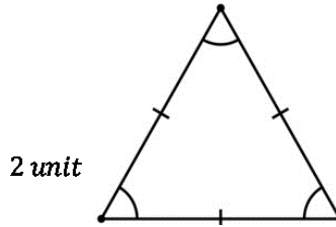
identify the correctness of our solution". While another student indicated that "*If we solve this problem by another way or using another rule and gave the same solution, that will be great*". This group of students presented to the class two possible and correct ways to solve the problem (Researcher's diary, 1/04/2012).

The previous scenario from the mathematics classroom showed a continuous conversation between students as one group and between the groups that chose the same problem as well as between students and the teacher. First of all, the students discussed which problem they would choose. That required reading of both problems, understanding the requirements and then made the decision based on their understanding. Then, students worked together to display a geometrical diagram that illustrated the given information. While one of the students drew the graph, the other students observed and modified the diagram. In addition, the discussion between the two groups that chose the same problem offered opportunities to share different ideas not just between the students of one group but with other groups. That broadened the circle of classroom discussion and students' conversations to involve a larger number of students and many forms of ideas and reactions. As I observed, the students were engaged and participated well during that lesson. In addition, finding the solutions needed concentration and sharing of ideas. While thinking of other possible ways required mostly individual thinking, discussing the new ideas and developing them required students' interactions. All the actions from reading, drawing, finding the solution and searching for other possibilities required substantive conversations between students. However, while the teacher gave one of the groups who displayed another possible idea to find the solution the opportunity to express their idea, the teacher only focused on the final step of solution that reflected the final result. Supporting students' construction of new ideas of solutions required more attention from the teacher.

The following example may also be considered as a good example of the kind of teaching practices that reflected applying the element *depth of students' understanding* by helping students to develop deep understanding of the mathematical ideas. In the following mathematics lesson, the relationships between the triangular ratios of special angles were investigated by students themselves during continuous practice of identifying the main ideas, suggesting many possibilities and testing them to reach good conclusions.

Triangular Ratios of special Angles, Grade 10, 27/3/2012

The teacher asked her students some questions that were related to what they had studied in the previous lesson. In particular she asked about trigonometric ratios and reciprocal ratios. The teacher drew a 60-60-60 triangle (an equilateral triangle), with sides having a length of two units.



Then she asked students to complete the following table without using calculators and used what they studied in the previous lessons

α	30°	60°	90°	45°	0°
$\sin \alpha$					
$\cos \alpha$					
$\tan \alpha$					

To complete this table, the students

- Drew the drop of a vertical bisector from the top angle down to the bottom side to form right-angled triangles.
- Used the rules of symmetry and Pythagorean Theorem to determine the length of the sides of the right-angled triangle and based on the rules of triangular ratios, they completed the first three columns.
- Most of students found it challenging to get the triangular ratios of $45^\circ, 0^\circ$
- Some of the students (not all) drew an isosceles right-angled triangle by taking the sides of 1 unit length and using the Theorem of Pythagoras, they get the triangular ratio of 45° .
- To get the triangular ratio of 0° , some of them used the basis of the unit circle.

Then the students completed the table on the board and discussed with the teacher what they found. The teacher asked students to work in groups to identify the relations between triangular ratios based on the table. Many important relations were recognized and discussed. Then the teacher discussed the importance of these triangular ratios of special angles as knowing the relationships of the angles or ratios of sides of these special right triangles allows anyone to quickly compute various lengths in geometric problems without using more complicated methods. Moreover, there is no need to memorize them; they can easily get them by drawing or comparing (Researcher's diary, 27/03/2012).

In the previous classroom scenario, some of the teaching practices indicated the attempts to offer opportunities for students to discover new relationships between triangular ratios of special angles. For example, the teacher asked her students to use the equilateral triangle of 2-units-side long to get the triangular ratios of special angles without using calculators to get the ratios of angles. Some of the students drew a vertical bisector from the top angle down to the bottom side to form right-angled triangles. That indicated their understanding of the requirements of triangular ratios. Also, when students decided to choose another way to get the ratios of 0° and 45° such as drawing unit circle, they produced new ways of solving problems. These learning experiences helped them to develop deep understanding of the main ideas of special angles and the relationships between them. While not all students produced all relationships, most of them showed an understanding of the

main ideas of triangular ratios as well as in developing their skills to apply these ideas to investigate new knowledge of triangular ratios of special angles.

One of the other elements of intellectual quality dimension that was implemented by mathematics teachers is *substantive conversation* element. The data showed that classroom discussion changed from asking questions and receiving specific answers to exchanging ideas and constructing shared meanings. Within the observed mathematics classroom, some of the teaching practices reflected good substantive conversations between students and the teacher and between students themselves. One good application of *substantive conversation* element is illustrated in the following classroom discussion.

The lesson of ‘Equation’, Grade 7, 21/3/2012

The students were divided into five groups. The lesson began with a short conversation between two students; the first one had two closed-bags that contained an unknown number of small cubes and the other one had forty cubes that are the same as the cubes in the first bag. Both students did not know how many cubes there were in the two bags.

Student 1: I think my two bags together are heavier than your forty small cubes.

Student 2: I do not think so. My 40 cubes are heavier. Put them in both your hands.

Student 1: Still, I think the tow bags are heavier.

Student 2: No, they are equal.

Student1: How are you are sure that they are equal?

The teacher: Can we then ask your classmates to help you to find other ways for measuring the equality of the objects or any instruments that they knew for comparing the weights of objects rather than using your hands to decide if the two objects are equal or not. Can you discuss together and help your classmate?

Each group of students discussed the best way they experienced in order to help the student make a decision. The teacher asked each group to talk about their suggestion to the class. The teacher wrote the names of the groups on the whiteboard and its comments beside each name. (the groups in mathematics classrooms are given specific names by the teacher)

Group one decided to use electronic balance and weigh the two bags together, then weigh the forty cubes and then compared them.

Group two used the spring balance.

Group three used the graduated instrument that uses differences of water height to weigh the objectives.

Group four and five chose two-pan balances.

The teacher: According to these given suggestions, which do you think is the best way for measurement in this situation?

Student 3: I think the two-pan balance is the best as it is easy, accurate and is not just for comparisons but can used to make the two objects equal by adding or subtracting.

Teacher (to students 1 & 2): Can you come in front of the class and use the two-pan balance?

The two students used the two-pan balance and found that the two bags weighed less than the forty cubes.

The teacher: Can you balance them by adding other small cubes?

They added two more small cubes to balance them.

The teacher: If we assumed that the number of each bag is x , can you express the equality within this situation?

Student 4: The equality can be expressed in the form $2x + 2 = 40$

The teacher: $2x + 2 = 40$ is an equation, it expressed the equality, it consists of two equal expressions, the equality of this mathematical statement can be expressed by the symbol ‘=’

The teacher then gave each group one cube, pencils, other objects and two-pan balance and asked them to create their own equations and write them down.

As I observed, it was enjoyable to do this activity as students tried to reach the balance between the objects by adding, subtracting and did that many times with different objects. They also wrote many different formulations of many equations. In most students’ responses, the teacher supported them positively.

The previous teaching and learning practices showed different kinds of classroom conversations. The first type of conversation was the dialogue between two students who explained to the class their problem comparing between two things. This discussion formed the basis upon which the concept of equation was built. The other type of conversation was the discussion between students within groups to make a decision about the best way for measuring objects and for explaining the characteristics for each scale of measurement. Moreover, the talk within groups was intellectual as students aimed to make distinctions and form their own expressions of equations. The problem of the two students about identifying the equality and the

best way to weigh objects indicated the teacher's attempts to apply *problem-based curriculum* element (an element of connectedness dimension). That problem did not require a specific correct solution but a discussion of different ways of measuring required students' previous experiences as well as classroom discussion to construct the definition of equation. The teaching practices within the lesson of equation were based on the problem of the two students as all classroom discussion was built upon the problem until the discussion reached the definition of equation. In addition, the group activity that was presented at the end of the lesson was based on the experiences that students developed during their discussion of that problem.

To sum up, the previous examples from classroom practices indicated that mathematics teachers attempted to apply the elements of intellectual quality dimensions such as *substantive conversation*, *higher order thinking* and *depth of students' understanding*. Some of the classroom practices indicated efforts by teachers to apply group activities that helped students to share their ideas, discuss different ways of solutions and express their findings to their classmates. In addition, the classroom conversations improved from the direct discussion that was based on direct questions towards substantive conversation between students and between students and the teacher. Moreover, some of the group activities of solving word mathematical problems may have some benefits in encouraging students to transform their understanding of the main mathematical concepts into different mathematical models such as formulas, graphs and through different solutions of the problem. All the above results of implementing the intellectual quality dimension by the participants showed some good examples from classroom practices that supported teachers' efforts of applying the elements that reflected intellectual quality outcomes.

Implementation of connectedness dimension in mathematics classrooms

Within mathematics subject that teachers taught, the participants made some attempts to apply the connectedness dimension in their mathematics classrooms. For example, the teachers of grade 10 tried to achieve the integration within multiple areas of mathematics by encouraging their students to apply and use their knowledge of "triangular ratios of special angles" in the unit of "trigonometric function" to solve problems of the followed mathematics unit of "vectors". That kind of integration

helped students to realize the importance of all areas of mathematics within this grade or in the following grades.

The data also showed indications from mathematics classrooms of connecting mathematics with other school subjects such as Islamic education, Geography and Science by showing its importance and applications in these subjects. The following example in one of the observed lessons indicated the attempts of one grade 7-teacher who applied some of the elements of the connectedness dimension. In her teaching of “Time’s Zones”, a connection between the lesson and students’ personal experiences were displayed during a discussion of travelling experiences. The connection also improved when students applied their Geography knowledge of latitude and longitude as well as their mathematical knowledge about time in-group activities.

Time’s Zones lesson, mathematics classroom, grade 7, 21/03/2011

The teacher discussed with her students their experiences of travelling. She opened the door of interactive interchanges around the countries students visited, the reasons for their travelling and the benefits and difficulties of travelling. Drawing on students’ personal experiences, the teacher discussed the problem of two students who were friends but in different countries and they did not know how to use time zones to calculate the time differences between different countries. The teacher discussed with her students some of the information they had studied in geography on latitude and longitude. The students referred to their knowledge about North and South Poles and Prime Meridian that helped to identify the location of any country. Using the map of the world, students identified the countries that they visited and determined the number of time zones between them. In groups, students used the Atlas to determine the time difference between different countries based on the latitude and longitude (Researcher’ diary, 21/03/2012).

The previous teaching practices indicated that the teacher attempted to apply the knowledge integration element. In that part of the lesson, students used their background knowledge of Geography such as their information of longitude and latitude, their skills in using the Atlas and their information about countries. Students’ previous experiences in using the Atlas helped them to identify different locations in the world map using their latitude and longitude and to pinpoint to the specific location of their country and other countries. This information that students learned in Geography was essential to study the lesson of time zones. Knowledge integration between mathematics and geography can be more explicit if there was collaboration between the two teachers who taught the two subjects for planning that lesson. That might enhance students’ comprehension and understanding of the new knowledge that they have not learned before and to recognize the non-boundaries between school subjects. Within the idea of collaboration between school teachers, teachers who participated in this study noted that:

We found difficulty to prepare common learning and teaching experiences that explicitly involved the interrelation of different subject areas as the idea of offering collaborative opportunities between teachers of school subjects to prepare common teaching and learning experiences were not in our interest and something that we did not experience before. (Group discussion, 21/03/2012)

While the nature of the lesson required retrieval of that kind of information, the direction of classroom discussions towards applying that information in a mathematics lesson motivated students to learn about time zones. For example, providing opportunities for students to talk and tell short stories about their experiences on travelling helped the teacher to explore what prior knowledge students had about time zones. During class discussion about the problem of the two friends, from the short stories that students told, the teacher identified that students had recognized the importance of learning about time zones. Students recognized that as many people move rapidly across great distances, time zones became more important and critical.

The *problem-based curriculum*, as an element of connectedness dimension, identified in the following example in which students presented private problems that students may have encountered in their daily life. The students explained their problem to their peers who tried to help them find a solution. That problem formed the basis on which classroom discussion was directed to introduce the characteristics of concave and convex polygons through the following teaching and learning practices. The following example showed some attempts at implementing *problem-based curriculum* element as well as *connectedness to the world beyond the classroom* element and *substantive conversation* element (an element of intellectual quality dimension).

“Concave and convex polygons” lesson, mathematics classroom, grade 7, 07/04/2012

polygon	Line of segment	Angles (type and measurements)	diagonals	Other features
Star				
regular hexagonal polygon				

Two of the students provided a dialogue between them to the whole class. The main idea of the dialogue was to explain a problem that faced one of the students when choosing the best area for a swimming pole that will be built in her home. The student asked her classmates to help her to choose between a polygon that was shaped like a star and the other one shaped in the form of a regular hexagonal polygon. She placed a sheet of paper with a regular hexagonal polygon and a star drawn on it.



The dialogue between the two students was as follows:

Student 1: My father wanted to build a swimming pool for our new house and I have to choose which shape it took, a polygon in the form of a star or a regular hexagonal polygon.

Student 2: It is nice to build a new swimming pool, I am happy that you will choose the shape; I think the star form is a beautiful and new shape. I do not know. The hexagon could be bigger.

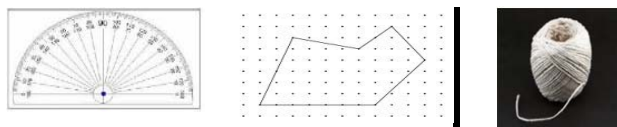
Student 1: Do you think that if a specific area is allocated for the swimming pool, the hexagon shape will be bigger than the star shape?

Student 2: Maybe it seems wider! I am not sure. Can we take other views from our classmates?

Then the teacher asked students to discuss within groups for the best shape for the swimming pool and give reasons for their choice.

Each group talked to the class about the best shape that could be used to build the swimming pool and why they chose it; few of the groups agreed with the star shape and most of them choose a regular hexagonal polygon. The teacher wrote the common reasons from students’ responses on the whiteboard by drawing two columns. The students discussed the features of safety, widening, boundaries, vertices using their daily language. They used mathematical language such as the area, the sides of polygon and angles. During this discussion, the teacher asked students if they like “the sport of swimming” and “what the importance of this sport was for people”.

Then the teacher gave each group a grid-board of equal distance grids, string, and protractor and asked them to follow the instructions in the group-activity sheet



- 1- Use the string and grid-board to construct both star and regular hexagonal polygon.
- 2- Use the string to form the line segment between vertices or diagonals and write your observations in the table.
- 3- What do you notice about the internal angles of both shapes? What are the similarities and differences between them?
- 4- Construct other polygons and write your comments. Can you give names for both according to their different similarities?
- 5- Where do we find these kinds of polygons? How we can use them?

Students’ responses and reasons produced many characteristics and differences about concave and convex polygons that they did not know before.

During the group activity, the students participated well, they constructed, measured, wrote their observations and discussed together. The teacher followed the group activities and supported the students. The students identified the differences between concave and convex polygons and recognized the beauty of these shapes from their applications and the extent of these polygons in nature, in geology, in bees’ homes and in structuring traditional and modern buildings (Researcher’s diary, 7/4/2012).

The previous teaching practices showed that the teacher attempted to apply *problem-based curriculum* element by introducing her lesson by posing a small problem that illustrated the difficulty that one of the students found when deciding on the choice of the best shape for a swimming pool. That personal problem that she faced did not need any specific solution. It needed different students' views and ideas. According to students' responses, their discussion was around the issues of bigger area, the role of vertices to offer the safety for the swimmer and the beauty of the shape. That best solution of that personal problem involved recalling of the previous students' knowledge about regular polygons. Moreover, reaching the best choice of the swimming pool required identifying the main features of concave and convex polygons. It also required making significant distinctions between them according to the angles, line of segment and diagonals. Those requirements formed the remarkable parts of the lesson. Looking at the social context that was illustrated in that personal problem, most houses in Oman in the past had a swimming pool that was located in the garden and some of the modern ones have swimming pools. While that may be connecting to some aspects of students' lives, it is not common to build a star or hexagonal polygon pool. As that problem was made by teacher, she tried to connect it to the lesson even though some of the realities were missed. Moreover, the teacher attempted to add some value to the lesson by offering opportunities for the students to regain the value and meaning of the lesson beyond the instructional context by identifying the beauty and the usefulness of polygons in nature and for constructing beautiful buildings. While those attempts investigated some of the value of connectedness, they reflected some of the potential for teachers' needs to prepare rich activities that maximized the connection of mathematics to its application in real life. That shapes of polygons were not realistic, however, did not seem to bother the students. They still found it useful to conceptualize the shapes as three-dimensional. Hence, they were still useful for them to make connections with the real world and to achieve the purpose of the teacher.

There was also some good reflection of *substantive conversation* within the previous classroom discussion. For example, the students' interaction in group-activities and the expression of their views and observations to the whole class dominated class discussions. In particular, the engagement within the classroom activity that was targeted to choose the best shape for building the swimming pool and the significant

reasons behind their decision were illustrated through enthusiastic discussion and questioning. To make a common decision is to build social interactions between students. When the teacher asked her students to use the string and the grid-board to construct both the star and regular hexagonal polygon and write their comments on the main features of both polygons that helped them to build effective social interactions and conversations. Within their searching about the features, they needed to listen to all different students' views, choosing the best, correct views, and then expressing their final findings. The students during all those actions were involved in much discussion, sharing and targeted conversations. In addition, while students were working in that group activity, the teacher supported them by following their progress in doing their work and by answering some of their questions.

One of the striking features that I observed of that lesson was the enthusiastic way in which students were engaged in group-activities. They respected all different views, contributed effectively and provided useful comments. Moreover, the students were invited to freely express their ideas and make relevant suggestions to help their classmate to find the best choice and best solution to her confusion, which illustrated students supporting and encouraging each other and the teacher as a support and guide to students' thinking. In addition, working within the group activity that aimed to recognize the different features of the concave and convex polygons and identifying the main distinctions between them may help students to share their ideas. Students applied different roles of constructing shapes, writing their observations, discussing the differences and summarizing the main findings. All those activities of constructing shapes using the grid-map, making comments, making decisions and expressing their findings orally may offer opportunities for continuous and different social interactions between students to take place. They may provide students with different roles and different ways of learning (Researcher's diary, 07/04/2012).

The above overall qualitative results showed that teaching practices exhibited some degree of connectedness by providing students with opportunities to make connections between mathematics and other subjects such as geography and in real life applications. The previous classroom teaching and learning experiences indicated that teachers developed some good ability to introduce the connectedness dimension to their teaching mathematics during the advanced stages of implementation. Within

this stage, teachers attempted to add value to their lessons by offering many opportunities for substantive conversations within classroom discussion and by posing some semi-real problems. However, teachers' needs for preparing learning and teaching activities that reflected real life applications of mathematics were highlighted within this stage.

Implementation of supportive classroom environment dimension in mathematics classrooms

The teachers felt that the idea behind supportive classroom environment was very close to their traditional way of teaching. Most of the elements of supportive classroom environment, as they indicated, were significantly applied by mathematics teachers because those elements were a part of their normal teaching. One of the teachers said:

This dimension depends on the teacher's personality such as teacher's ability to control her class, her effective ability to assess students' work and her excellence in dealing with her students. From the beginning, I feel that I am empowered to achieve this dimension. (Teacher's interview, 30/04/2012)

The previous comment indicated that the dimension of supportive classroom environment was familiar to the teachers as it depended on their' previous experiences with assessing students and engaging them in their work. However, it may indicate a limited previous understanding of the idea of engagement as it is related to explicit higher degree of class management of students' behaviors. The word "control" demonstrates a traditional understanding of teachers' role to be in control in the classroom that relied on teachers' responsibility to reduce disruptive behaviors and manage students learning. In Productive Pedagogies, the control should been given to students by offering opportunities for self-directed management of learning. From the initial classrooms that were observed, some good indication of the elements of *social support, academic engagement, self-regularity*, were evident as discussed below.

An atmosphere of mutual respect between teachers and their students characterized the mathematics classroom environment that I observed. Teachers developed good expectations towards students' achievement and engagement especially the low

achieving students. Developing good expectations during their introduction of Productive Pedagogies was demonstrated when they attempted to offer different opportunities for interactions in their mathematics classrooms. Some of the activities that teachers implemented helped students of different abilities to contribute. For example, the activities of measuring, drawing and observing within the previously discussed lessons may help students take different roles and different possible ways to learn. The lessons also gave students opportunities to work individually which may help the teacher to support the individual difficulties of her students. One of the teachers pointed to the change in students' participation especially the students that did not participated well previously. She said:

We have a group of students who did not speak a single word in the classroom. It does not mean that the teacher does not bother about this category of students. Repeatedly we tried to give them the opportunity to participate. Their participation changed positively, maybe due to changing the my style of teaching or due to the feeling of the students that mathematics is important or to the change the quality of the classroom activities such as using exploratory activities and exercises sometimes outside the classroom. (Teacher's interview, 01/04/2012)

The previous comment indicated that there was such a change of students' participation in their mathematics lessons especially those students that did not engage previously during mathematics lessons. While the comment indicated different possible reasons for the change, the reasons pointed to the change in the classroom environment towards supportiveness. It also indicated a change of the idea of the supportiveness of the learning environment from the management of students' behavior as the initial teachers' responses towards supporting students' learning and provided them with a classroom environment that encouraged their engagement and participation. For example, in most of the previous examples of lessons that were discussed within this stage of *consolidation*, students showed enthusiasm for their work by contributing and engaging effectively in their group activities. In the observed classrooms the students also tried to do their best to complete their class activities. In the group tasks, they contributed and helped peers. The students developed a serious respect for their peers' views. The opportunity to implement group activates in the mathematics classroom was one of the experiences that

encouraged students' motivation and engagement and for the teacher to construct a social supportive environment. I observed students during these activities sharing their ideas. In particular, the data from classroom observations indicated that the mathematics classroom reflected a supportive classroom environment when students were involved in challenging group activities that encouraged them to work as a group and exchange their views in order to reach conclusions or identify features or characteristics. For example, in studying "triangular ratios of special angles" as discussed in the previous sections, the teacher challenged her students. She asked them to discover new relationships between triangular ratios of special angles using their previous knowledge of triangular ratios in the current mathematical unit and by recalling their previous knowledge about triangles, angles and the theorem of Pythagoras that they had previously learned in grades eight and nine.

In addition, the learning environment was supportive as the data indicated that when the students were engaged in the lessons they learned. In particular, the activities that required group work offered opportunities for students to express their ideas to their peers and to the group of students that they worked with as well as the whole class. The teacher supported her students by giving them the freedom to respect others' views as well as to critique and comment on peers' observations and conclusions. For example, when the teachers taught the lesson of "polynomials", the lesson of "concave and convex polygons" and the lesson of "equation", the students experienced many social interactions. In these lessons, discussed in the previous sections, students had opportunities to talk to their whole class and to express their findings about their work as a group. They talked about their findings of the differences between polynomials and non-polynomials and the main characteristics that they found between concave and convex polygons. They expressed their ideas about the best measurement scale and the best way to reach the equality and then they added comments on their group findings. Students engaged and participated during classroom discussion using dialogue and intellectual exchanges. Using short stories in mathematics classrooms also helped to make the lessons more enjoyable. Moreover, the students were supported when they conducted individual activities that examined their understanding and reflected their needs of supportiveness from the teacher. For example, while the students learned about polynomials as illustrated previously, the teacher gave exercises about polynomials and asked students to work

individually to recognise the polynomials from non-polynomials, modify non-polynomials to be polynomials and to identify the degree of a polynomial. While students did their individual exercises, the teacher helped them by following their actions, identified their needs and supported their progress.

Regarding self-regularity of the students within the observed classroom, the issues of teachers' hints, direct instruction and punishments to regulate students' behaviors were not observed during my classroom visits. Most of the observed mathematics lessons proceeded without substantial interruptions. Few minor teachers' comments on student's behaviors or movements were made during the observed lessons in grade 7. Initially I felt that students' self-regularity was due to our presence as new visitors but then I observed them engaging seriously with good implicit control.

Within all those observed practices, most of students were engaging, participating and enjoying the experiences of working together and reaching common conclusions. The teachers attempted to provide their students with opportunities for participating in classroom learning activities, classroom discussions and classroom interrelations.

Implementation of working and valuing difference dimension in mathematics classrooms

The data indicated that the teachers' ability of implementing working and valuing difference dimension developed during the observation time. Some of the teachers' comments indicated that they gradually elaborated some meanings of the elements of *active citizenship* and *narrative* when they presenting mathematics knowledge that they thought as problematic. One of the teachers asserted that and commented:

Active citizenship element is the first element that comes to my mind, as it may be a side missing in my teaching practices; I used the knowledge as problematic and the narrative style. With these elements, the students had become productive. They thought of themselves as someone of value and importance to the community. (Teacher's interview, 01/04/2012).

Drawing on the previous comment, while the teachers noted that their implementing the working and valuing difference dimension in their mathematics classroom was modest in their initial attempts of implementation, the application of some elements under this dimension such *cultural knowledge* and *active citizenship* was limited as

there was no evidence of the application within observed lessons. In particular, teachers' ability to implement the *cultural knowledge* element was limited because of the influence of their initial reactions, which questioned the relevance of the element to the Omani educational context as discussed in the previous sections. Applying *Active citizenship* element was influenced by teachers' difficulty to introduce that element in their mathematics lessons. While most teachers and their students indicated the usefulness of *narrative* element in their mathematics classrooms, their attempts within the previous stages of *preparation* and *developing* were artificial. Teachers attempted to apply *narrative* element in their mathematics lessons in this stage of consolidation, however, the application of the *narrative* as in the previous stages did not reach a stage of perfection. For example, in some of the observed lessons during the stage of *consolidation*, some of the mathematics lessons processes and content were narrative. However, the effective application of *narrative* element in teaching mathematics content is limited. This limitation indicated teachers' superficial understanding of *narrative* or their insufficient experiences to apply the narrative style in their mathematics lessons. This limitation may be related to the influence of teachers' previous experiences that limited the application of the narrative approach in short stories that added more enjoyment to the mathematics lesson. The limited implementation highlighted teachers' needs for more improvement in their application of the narrative approach in teaching mathematics. Teachers in this stage attempted to apply the narrative style by using some historical stories about deep religious Islamic values such as the values of giving and honesty. The teachers tried to introduce some historical stories about mathematical scientists and especially about Islamic and Arab mathematicians such as the role of Abu Abdallah Mohammad ibn Jabir Al-Battani as one of the famous Arab and Muslim observers and a leader in discovering some geometrical rules such as the geometrical ratio $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Discussing the role of these mathematicians in teaching the unit of trigonometric ratios may encourage students' motivation to learn the content of the unit.

The following mathematics classroom practices in one of the observed lesson, was an example of a teacher's attempt to introduce the concepts of the angles of elevation and depression using a story from Islamic history.

Mathematics classroom, grade10, “elevation angles and depression” lesson

The lesson was conducted in the school lab of learning resources. The students stayed in groups. The following practices were a part of the lesson that aimed to introduce the new concepts “the angle of elevation and depression”

The teacher began her lesson using the narrative method. She told the students a story about “a young man from the beginning of the Islamic age. That young man was picking an apple from a farm without asking permission, but he felt guilty and remorseful for what he did. Repeatedly he apologized to the owner of the farm, but the farm owner refused to forgive him”. The following discussion was between the teacher and students:

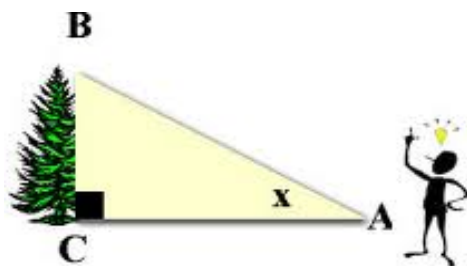
The teacher: what have you learned from the story?

Student1: People have to be honest and not take anything without permission

Student 2: The story taught us that we have to be straightforward and admit our mistakes and correct them.

Student 3: We know that Islam tell us that, but the main thing is the application of these values.

While the teacher told that story, one of the students drew an illustrated picture about it on the blackboard.



The dialogue between the teacher and her students continued with discussion on the possible and different ways of picking that apple from the high tree. Then the teacher asked the students to use their own language to give names to define the young man and the apple. The students defined the young man as the observer of the apple; the apple is the observed object, the line between the observer’s eyes and the observed object is the line of sight. Students concluded that the observer’s line and horizontal line formed an angle. They also identified that the angle differed according to the position of the observer, if he is on the tree or under it. This led to the geometric concept of the angles of elevation and depression. The teacher then asked her students to give examples from their daily practices and from the classroom position about the application and the extent of the elevation angles and depression. There was an atmosphere of enthusiasm and interaction; the students showed deep understanding of the basic concepts, embodiment of the religious and ethical values and connectivity between Arabic and Islamic identity and a commemoration of the prophet of Muslims (Researcher diary, 01/04/2012).

The previous observed lesson processes were introduced using a historical story that reflected some indications to Islamic and Arabic values. The lesson was based on a story from Islamic history that indicated many values such as truth, honesty and trust. The students identified these values during the discussion between them and the teacher while she was telling the story. That story was also beneficial as it motivated and engaged students. However, the story did not reflect a deeper understanding of *narrative* element. Still, the application of that element is influenced by the teacher’s previous assumptions that were based on using stories to make the mathematics lesson more enjoyable.

While the story is not related directly to mathematics learning, the teacher used it to draw a diagram that illustrated its events and actions and applied the geometrical

diagram to introduce the geometrical concepts of the line of sight, and the angles of elevation and depression without direct references to those new concepts. The students drew on the whiteboard the full diagram of the events of the story. Within that classroom practices that I observed, the students were active and engaged. Most students contributed and participated. In addition, the classroom discussion around the story indicated the teacher's attempts to identify the value of honesty that the story reflected. When the teacher asked her students "what have you learned from the lesson", she gave their students the opportunity to recognize the potential values that related to their Islamic religion. As teachers noted

The actual previous teaching practices omitted any discussion around cultural values, the values of honesty, collaboration and other values not discussed in our usual teaching practices in mathematics classrooms. The consideration was about the abstracted mathematics knowledge. (group discussion, 01/04/2012)

Above all, the results indicated that the overall application of Productive Pedagogies by teachers in their mathematics classrooms was developed and associated in one way or another with the nature of mathematical knowledge that was taught and the development in the convictions and capacity of teachers. The implementing of Productive Pedagogies by the mathematics teachers was based on the feedback that was received from classroom observations and from the activities that were prepared in the group discussions. The teachers developed a confidence and a capacity to apply Productive Pedagogies whenever they encountered new teaching and learning situations or had been associated with group-work with their peers. While teachers' attempts at implementing Productive Pedagogies indicated some good improvements, some of the elements such as *narrative* and *connectedness with the world beyond classroom* highlighted the teachers' need for further support in this regards. The teachers noted that with continuing applying Productive Pedagogies, their future application would improve further. One of the teachers commented:

I expect that if I applied Productive Pedagogies in the coming year, my performance will be better and more organized. Looking back to my memories of teaching that lesson of "Standard Angle", I touched on new and beautiful ideas. (Teacher's interview, 01/04/2012)

While the previous comment indicated teachers' hope of better future implementation of Productive Pedagogies, it may indicate that they developed the confidence and desire to continue introducing Productive Pedagogies in their mathematics lessons. The previous comments also showed that teachers referred the value of the Productive Pedagogies framework as illustrated in the following sections.

While the qualitative data from classroom observations, group discussions and interviews identified the development of teachers' understanding and ability to apply Productive Pedagogies in their mathematics classrooms during the previous stages of preparation, improved and consolidated. The quantitative data also recognized that there was development of teachers' ability from the results of coded classroom observations as shown in the following section.

4.1.4 Quantitative evidence on the development of implementing Productive Pedagogies during observation time

The researcher and the teachers coded the teaching practices of the observed mathematics lessons using the Arabic version of Classroom Observation Coding Manual. According to the Classroom Observation Coding Manual, the classroom observations were coded under four dimensions: intellectual quality, connectedness, supportive classroom environment and working and valuing difference. The twenty elements under these four dimensions were scored from 1 to 5. A total of 35 coded classroom observations were completed by the researcher and the teachers. One of the limitations of using quantitative data in this analysis was identified when the teachers were generous in their coding as they displayed high level scores to their peers in general. Another limitation was identified when all the coding observations of the researcher and teachers were analysed together, not separately, as they observed the same lessons at the same time. Those scores were not the main source of data. The Classroom Observation Coding instrument was not used to measure the actual implementation. It was used to generate a discussion and in that case if there was any difference of opinions, it was good to open a discussion. In addition, the instrument gave an indication to support triangulate of the data and the inter-coder reliability is not calculated. Moreover, the Arabic version of the Classroom Observation Coding Manual was not validated to the Omani context as no attempts

had been made to ensure inter-reliability. Thus, the quantitative data were used to triangulate other qualitative observational resources. The quantitative data provided supplemental data in order to provide a general indication of teachers' application during the whole observation time.

The quantitative results of the coded teachers' classroom observations recognized that development for all Productive Pedagogies dimensions through the observation times of the stages of preparation, developing and consolidation. *Figure 4.2* shows that applying intellectual quality dimension and supportive classroom environment dimension at the initial attempts in the stage of *preparation* scored more than the other two dimensions. However, at that stage of *preparation*, the data indicated that the implementation of working and valuing difference dimension was applied less the other three dimensions. While all dimensions varied in their average scores of implementations in mathematics classrooms at the early stage of the *preparation* and *developing*, they had evolved to reach a similar average score in the stage of *consolidation*.

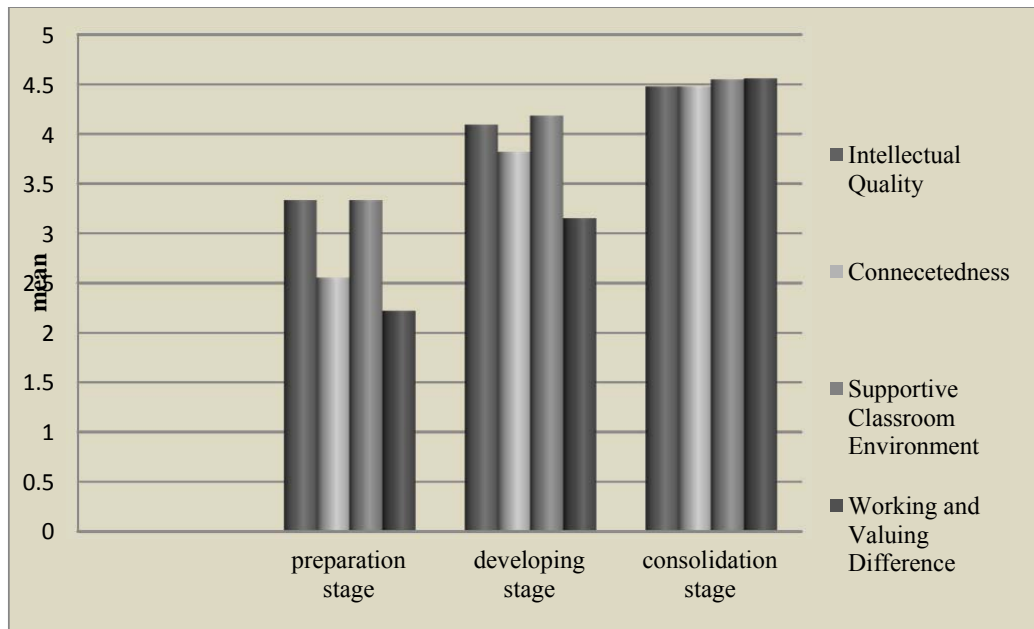


Figure 4.2 Development of implementing Productive Pedagogies dimensions through phases of participants' implementation

The quantitative data of coded classroom observations also recognized the development of Productive Pedagogies elements under the dimension of intellectual quality. This dimension indicated a rising level of implementation. *Figure 4.3* shows the development of implementing the elements of intellectual quality dimension

during the different stages of applying Productive Pedagogies by mathematics teachers. All the elements of intellectual quality dimension were developed in the actual implementation during the observation time of semester two. While all of them differ at the initial observed efforts of the stage of *preparation*, all of them developed and reached good levels of implementation in the stages of *developing* and *consolidation*.

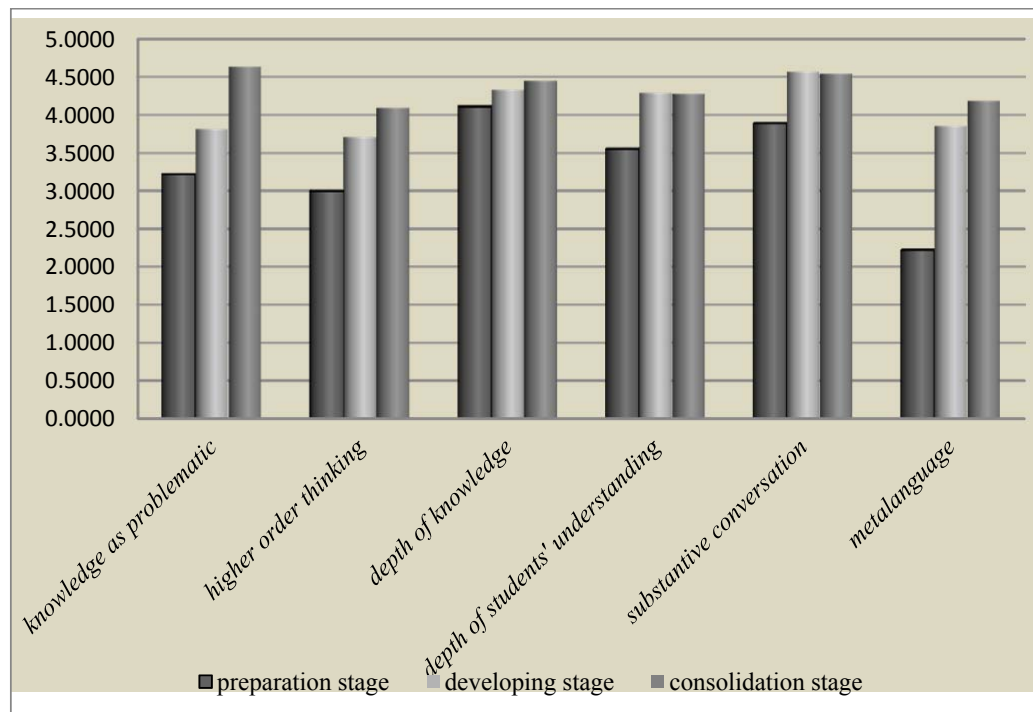


Figure 4.3 Teachers' scores on the implementations of the elements of intellectual quality dimension during the different stages of the application

The quantitative data of coded classroom observations also recognized the development of Productive Pedagogies under the dimension of connectedness dimension. The pedagogical elements of this dimension indicated a growing level of implementation. Figure 4.4 shows the development of implementing the elements of connectedness dimension during the different stages of applying Productive Pedagogies by mathematics teachers. Within this dimension, the elements displayed good levels of implementation as the coded classroom observations indicated in the following graph.

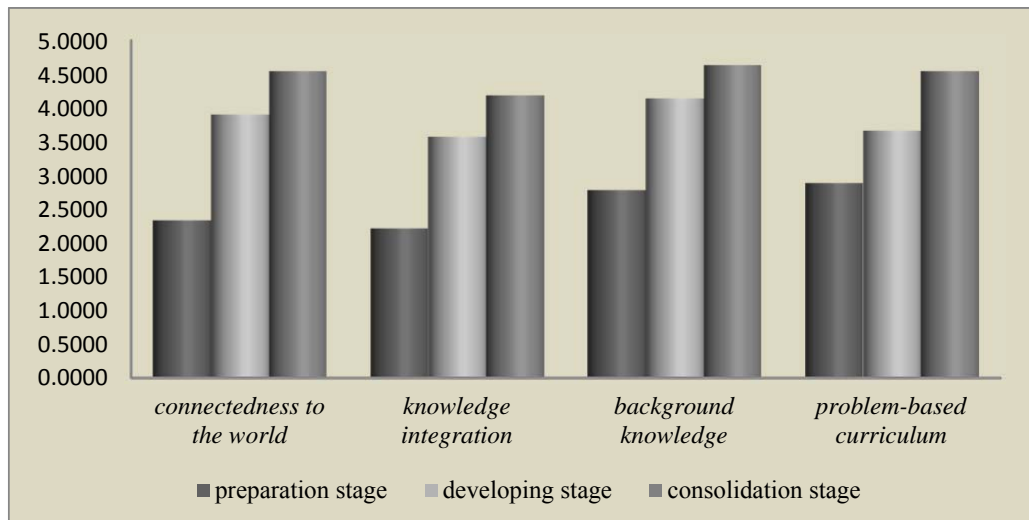


Figure 4.4 Teachers' scores on the implementations of the elements of connectedness dimension during the different stages of the application

In detail, the above graph showed that most of the elements under the connectedness dimension scored high levels specifically in the stages of consolidation. The data within the graph indicated that background knowledge scored high levels while knowledge integration scored less. That was consistent with the previous discussion around teachers' attempts at implementing Productive Pedagogies that indicated limited efforts from teachers of school subjects to develop common learning and teaching experiences that investigated the integration between the subjects.

Under the dimension of supportive classroom environment, the elements scored an increasing level of implementation. Figure 4.5 shows the development of implementing the pedagogical elements of supportive classroom environment dimension during the different stages of applying Productive Pedagogies by mathematics teachers. From the initial stages of implementation, the elements under supportive classroom environment reflected good application in mathematics classrooms.

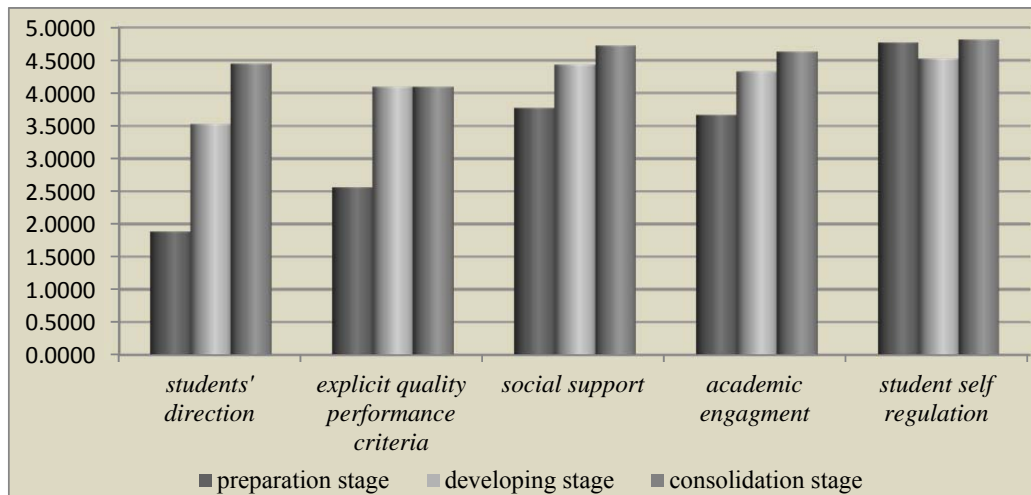


Figure 4.5 Teachers' scores on the implementations of the elements of supportive classroom environment during the different stages of the application

The quantitative data of coded classroom observations also recognized the development of Productive Pedagogies under the dimension of working and valuing difference dimension. While the implementation in the initial stages scored less than all other dimensions, the elements under this dimension indicated a growing level of implementation during the time of observation. Figure 4.6 shows the development of implementing the pedagogical elements of the working and valuing difference dimension during the different stages of applying Productive Pedagogies by mathematics teachers.

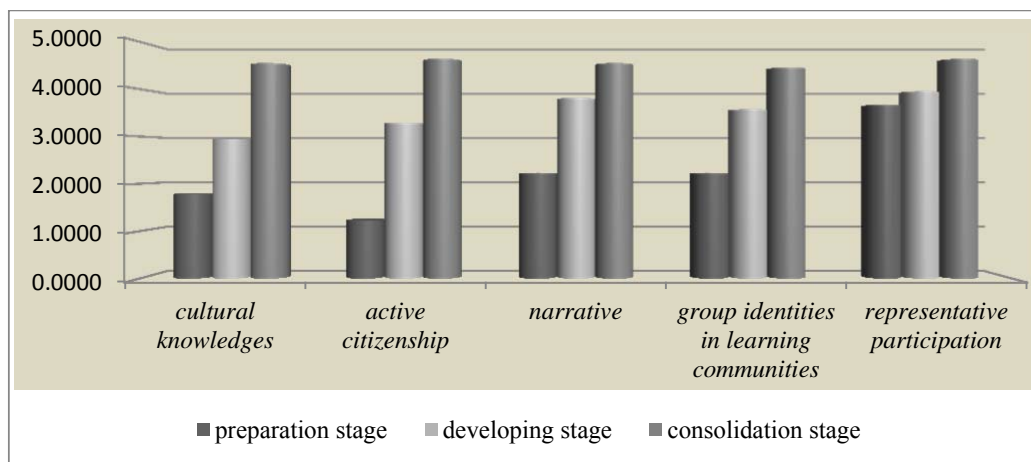


Figure 4.6 Teachers' scores on the implementations of the elements of working and valuing difference dimension during the different stages of the application

Above all, the qualitative and quantitative data indicated that the main value that mathematics teachers gained in the application of Productive Pedagogies was that

teachers developed new understanding and ability to introduce Productive Pedagogies in their mathematics classrooms. The following section showed other benefits of this application.

4.2 The benefits of Implementing Productive Pedagogies by Mathematics Teachers

The previous section showed that teachers developed new understanding and ability to implement Productive Pedagogies in their mathematics teaching. While the main benefit of applying Productive Pedagogies was the development of teachers' understanding and ability of implementing, teachers also recognized five other benefits that reflected the effectiveness of the framework and its positive impact on their teaching practices and professionalism. Firstly, Productive Pedagogies was an effective framework that changed teachers' traditional learning and teaching beliefs and improved their teaching strategies. Secondly, teachers recognized the value of implementing the framework in their students' learning and engagement. Lastly, applying Productive Pedagogies offers opportunities for teachers to build their pedagogical knowledge and to examine their practice.

4.2.1 Change in teaching beliefs and strategies

The data identified that many of teachers' previous knowledge, views and experiences about their teaching practices had changed. Introducing the Productive Pedagogies framework to teachers' practices in their mathematics classrooms helped them, as they commented, to develop new beliefs and strategies about the introduction of mathematics knowledge in their classes. In particular, teachers noted that their recognizing of the connection between mathematics knowledge and students' personal experiences and the actual contemporary issues was one of the important additions to their knowledge and practices. One of the teachers of grade 10 commented that Productive Pedagogies helped her to find new ways and characteristics of presenting mathematics different to what she previously possessed. She said:

Productive Pedagogies helped me to find out, more accurate characterization of knowledge presentation, which is different from my prior knowledge. (Teacher's interview, 24/04/2012)

Drawing on the previous comment, engaging teachers with new knowledge and understanding of Productive Pedagogies elements helped them to move from their new understanding to actual change in practice. In particular, participants' previous teaching methods that focused on employing mathematical rules and algorithms through repetitive routines changed to focus on transforming deep mathematical understandings and meanings through discovering and solving real problems. One of the teachers commented on her improvement on teaching strategies when she noted:

For me as a teacher there is a diversification in my previous teaching practices; previously I was using routine and conventional repeated methods and when any change in my teaching occurred, it occurred within narrow limits; but now, based on Productive Pedagogies, I began by comfortably and flexibly changing my teaching practices and I feel whenever I went to the mathematics class, I would submit something different and I have something new, and I feel confident that I can change in the way of teaching even during class. (Teacher's interview, 30/4/2012)

Teachers were being able to produce new teaching strategies. The previous comparison from one of the teachers regarding her previous and present teaching practices indicated that change is a process that takes time, structure and support. Teachers also noted that they tried to apply new effective teaching approaches. From the classroom observation, there were attempts from teachers to apply teaching strategies that centered on the learner. Strategies such as problem solving, collaborative teaching approaches and narrative approaches offered opportunities for students to experience effective social interactions, knowledge construction and enjoyable classroom discussion. The previous discussion about the actual implementation of Productive Pedagogies in mathematics classrooms illustrated teachers' attempts to apply effectively those strategies. For example, performing problem solving group activities in the lesson of "the angle of elevation and depression" by solving some examples of real life problems helped students to improve their skills of transforming, analyzing and reasoning. Moreover, applying collaborative activities to discover the trigonometric ratios of special angles and the specific relations between them encouraged students to work together to reach reasonable conclusions by sharing their ideas, critiquing their effectiveness and improving them towards discovering new relationships. Within the lessons of

‘equation’ and ‘elevation angles and depressions’, the teacher attempted to use some narrative approaches such as personal stories and historical stories.

Moreover, teachers gained new ways to improve classroom discussion. For example, different forms of classroom conversations were applied as illustrated in the previous discussions. The teachers tried to offer opportunities for talking and discussing during group activities. They also offered opportunities to make different interactions within the groups of students and with the teacher. For example, the lesson of introducing the concept of polynomial, as discussed in a previous section, reflected different forms of classroom conversation such as substantive conversation between the teacher and students and between students themselves, dialogue between different groups of students and an expression of group work findings to the whole classroom. This change was significant as the classroom discussion improved from the type that was based on asking questions and receiving short answers to the discussion that was based on performing different conversations.

One of the teachers mentioned that she become more interested in new teaching ideas that encouraged students’ motivation and helped them to be a part of their society in their mathematics classrooms. She said:

I have become more open and broader in my thinking; I have become more open to new ideas that I have applied more effectively in my teaching approaches. (Teacher’s interview, 24/4/2012)

The previous comment provides a good indication of the effectiveness of Productive Pedagogies on teachers’ thinking and teaching strategies. Regarding the improvement on teachers’ strategies, the participants also highlighted the positive impact of Productive Pedagogies to fill the gap in their teaching practices. Particularly they pointed to the connectedness dimension as this dimension was previously missing in their teaching practices. One of the teachers said:

The connectedness dimension was a part that was missing in our previous teaching of mathematics. (Group discussion, 27/03/2012)

Another teacher indicated she began to apply the connectedness dimension and directed her planning of mathematics lesson towards *problem-based curriculum* element. She said:

During preparation of my mathematics lessons, I found myself first starting to thinking of applying the connectedness dimension, where if I pose to my students a problem from their lives, I can comfortably complete my lesson building on this problem. (Group discussion, 21/03/2012)

Another teacher talked about her experience of applying the connectedness dimension and said:

After understanding the correct application of the connectedness dimension, which was based on investigating the link with reality issues like students' experiences and social problems, I applied many ideas in my mathematics lessons. (Teacher's interview, 30/4/2012)

The previous comments showed that teachers could present and clarify their understanding of the elements of Productive Pedagogies dimensions using their own ideas and experiences to learn more about the practice and to develop new ideas that focus on their practice. The previous comments also indicated teachers' attempts to apply the elements of connectedness dimension in their mathematics lessons. The observed lessons showed that teachers attempted to apply the element of problem-based curriculum by posing personal problems that had no correct solution and required continued consideration beyond the lesson. For example, in one of the observed lessons, the personal problem of two students to identify the equality between two unknown closed pockets and forty small cubes formed the basis of the lesson of equations in grade 7.

4.2.2 Positive effects on students' engagement and learning

The teachers also expressed positive views about the impact of Productive Pedagogies on students' learning and engagement. The data indicated that teachers identified an improvement in students' participation and interactions. Also they indicated that their applying of Productive Pedagogies helped them to deal with students' lack of motivation and to make a difference on students' learning of mathematics.

Improvement on students' participation and interactions

Teachers recognized the effectiveness of the Productive Pedagogies framework and its positive impact on students' interaction and participation in mathematics lessons. In most of the observed lessons, most students were engaging and contributing. Most students engaged especially when classroom discussions were conducted as group activities. While these activities helped the teachers to encourage considerable interactions and subsequent interchanges, they also helped to introduce some of the main mathematics concepts that were taught.

Teachers also mentioned that their applying the element of connectedness in their mathematics lessons showed a significant and direct change on students' participation and interactions. The connectedness dimension met with great interest by teachers and a clear application in the mathematics classes that were observed. This interest, as teachers indicated, came from their conviction of the importance and usefulness of this dimension in promoting students' motivation to study mathematics. One of the teachers used the role of connectedness in improving students' participation in mathematics classroom. She said:

Connectedness dimension is dominant in my thinking where I feel that it raises the overall students' enthusiasm and participation, and if progress is made in this dimension, all the other dimensions can be achieved. (Teacher's interview, 24/4/2012)

Moreover, while all of teachers indicated that during their previous teaching experiences, they always suffered from their students' lack of interest to participate; all of them expressed happiness with the development of the interaction of their students especially the low achieving students. For example, one of the teachers commented that applying substantive conversation element helped her students to move from their limited circle of dialogue that was based of displaying questions and receiving answers to a broad and meaningful dialogue. She said

What concerns me is the motivation of my students and their desire to learn, what Productive Pedagogies changed; the participation of my students improved, and the students who did not participate at all, beginning to feel a change in mathematics lessons became more engaged because they had

found much to talk about during mathematics class. (Presentation, 25/04/2012)

The previous concern of one of the teachers about her students' learning and engagement highlighted the significant desire to analyze and change the conceptual barriers that were likely to hinder their students' learning. Thus, teachers identified that effective teaching involves continually assessing of students motivation, participation and engagement. The data indicated that teachers had acquired new and better experiences in dealing with students' lack of motivation. In fact, all teachers who participated noted that most of their students faced problems of weakness in the essential mathematics knowledge as well as some weakness in literacy and numeracy skills. Although these problems were deep-rooted and complex causes, the largest part of their problem, as teachers noted, was due to students' lack of motivation to learn mathematics and to the instability of the impact of learning mathematics on building students' knowledge. One of the teachers commented that her applying of Productive Pedagogies helped her to deal with students' weakness by changing her teaching strategies and by distributing her attention to all students. She said

Productive Pedagogies helped me to deal with the weakness of my students. That was the biggest problem that I faced. Productive Pedagogies was an effective addition to my teaching. Now I use different teaching strategies and I distribute my attention to all students. (Teacher's interview, 01/04/2012)

The previous comment pointed to the change in their teaching strategies to reach all students especially those who were identified as less achieving students. While the concept of students' weakness reflected many issues related to students' learning, teachers used that concept to identify low achieving students. From their viewpoint, the low achieving students in mathematics were those who did not achieve good marks in the final mathematics exams in semester one or in the short quizzes in semester two. They also identified the students who did not participate well in class activities (group or individual activities) as less achieving students. That reflected their traditional teaching beliefs that relied on the aim of achieving success in the final exam. It also reflected their limited conceptions about students' performance and learning that was based on students' examination results.

The teachers also noted that improving the motivation of learning by applying Productive Pedagogies dimensions such as the pedagogical elements of connectedness was one of the points of interest that showed an improvement in students' learning of the basic mathematics concepts, which may help to reduce the impact of these problems in later grades. One of the teachers indicated the positive impact of classroom substantive conversation and discovery activities on low achieving students. She said

For students of low achievement, their participation has changed in mathematics class for the better, especially in situations where different points of students' views have been discussed and especially in the activities of inductive discovery. (Teacher's interview, 1/4/2012)

While the previous comments showed the teacher's concern about the weakness of her students' learning, it was an indication to the teachers' efforts to make learning achievable by giving her attention to all students, which is one of the main arguments for the best application of Productive Pedagogies.

The data also identified an improvement in teachers' expectations towards low achieving students. Teachers' initial attitudes reflected that most of them did not display high expectations for all students. Their frustration was because of the presence of samples of low achieving students who did not even want to talk in mathematics class. For example, they noted that they were not sure that all students could learn important and challenging knowledge and skills. These initial attitudes came from the frustration of teachers regarding the results of their students which were low in semester one. One of the teachers pointed to her disappointment from her student's results in semester one and noted the need for a solution:

We were disappointed from the results of the first semester. Hence, applying Productive Pedagogies was timely as we were looking for an opportunity to change our teaching methods. (Group discussion, 01/04/2012)

As mentioned in the previous comment, while there was a direct indication from the teacher that the view of good teaching reflected the good standards of students' achievements in the national assessments, there was also an indication that teachers were aware of a rethinking of their teaching practices. Teachers noted that the change

in their teaching practices towards applying Productive Pedagogies helped them to give those students who disengaged in mathematics classrooms many opportunities of classroom participations and interactions. The following comment of one of the teachers explained the significant change in her students' participation based on her persistence in applying Productive Pedagogies.

In my classrooms, I recognized examples of students who were surprises of change. Imagine that a student who did not speak and did not participate engaged well, I observed the student who was walking during class and playing during mathematics lessons has become more integrated and interactive, and the student who was always afraid to express her opinion began to talk about her views and ideas comfortably. The main reason was my insistence on applying Productive Pedagogies. I feel comfortable because this framework is useful and productive and helps students to produce their own ideas and to provide their knowledge in their own way. A huge jump occurred in the motivation of those students. (Teacher's interview, 1/4/2012)

Based on the data from the previous content and from classroom observation, the teachers tried to give low achieving students opportunities to express their thoughts about some issues that they did not need correct answers and helped them to talk about some personal experiences. They also gave them significant roles in collaborative group tasks. The attitudes changed for the better among students who were observed during semester two and there was a significant change in their motivation to learn. However, they still needed more assistance in their academic achievement as their mathematics background achievements were low. The teachers identified that while a change in students' views towards the difficulty of mathematics had improved, students' problems in their mathematical basic knowledge still concerned them. One of them noted:

Regarding our students, I think they benefited as their views about mathematics as a difficult subject and the mathematics lessons as boring sessions changed for the better. However, their basic mathematics abilities were still a problem for them and for us. (Teachers' interview, 1/4/2012)

Improvement on students learning

The students' learning achieved good attention from the teachers as they indicated in the professional development day in which teachers summarized their experiences of implementing Productive Pedagogies in their mathematics classroom. They expressed the following comments:

- Provided students with learning activities that improved their abilities of reasoning, analyzing and concluding positively influenced by their learning of mathematics
- Recognized the value and the beauty of mathematics knowledge from the real applications of mathematics concepts and rules helped students to make good connections with what they had learned
- Supported learning through interaction among students the ideas of mathematics (Presentation, 25/4/2012)

The data indicated that using some of the higher order thinking activities such as inductive and deductive activities was one of the teaching experiences that had some benefits on students' engagement and better understanding of the mathematics knowledge that were taught. The data also showed that students could produce good learning conceptions and ideas by providing them with good higher order activities and effective classroom discussions that value their ideas. In one of the lessons that I observed, the teacher implemented activities that challenged her students by conducting activities that allowed them to determine a variety of characteristics and reasons about "polynomials". Then she asked her students to give different examples of "polynomials". In the first lesson of introducing this concept, students produced different examples and anti-examples of polynomials and gave many explanations for why anti-examples did not represent polynomials. Some of them corrected the anti-examples to meet the definition of polynomials. These activities helped students to synthesize, explain and reach conclusions (Researcher's diary, 14/03/2012)

Moreover, data from classroom observations also indicated that teachers implemented some good higher order thinking activities in their classrooms. For example, one of the teachers of grade 10 when teaching the lesson on "trigonometric functions" implemented different deductive activities outside the classroom by dividing students into four groups to investigate the mathematical relations between

trigonometric ratios. Another teacher from grade 7 also used these kinds of activities to help students discover new characteristics of convex and concave polygons. In these activities, students were receptive to different ideas and ways of problem solving.

Applying Productive Pedagogies also helped teachers to make a difference in their students' mathematics academic achievement. Teachers identified that Productive Pedagogies had the effect of reducing the negative impact on the instability of learning. They noted that their students could not remember what they had learned. They always tried to repeat what they were taught. One of the teachers noted that applying the higher order element and connectedness elements helped students to remember the mathematics knowledge that they had learnt. She commented:

Our experience as mathematics teachers with Productive Pedagogies served us a lot. As teachers; after many years of teaching and after we applied Productive Pedagogies felt that the problem that confronted our students in their study of mathematics, which is the instability of the impact of learning, can be solved by changing our teaching towards applying Productive Pedagogies. (Presentation, 25/4/2012)

The teachers also indicated that there was an improvement in students' results in the short assessments in semester two. One of the teachers said:

My students demonstrated good understanding and achievement; there was an improvement in my students' results in classroom quizzes and short assignments. (Group discussion, 20/04/2012)

Another teacher commented that her students' learning improved especially among the low achieving students. She said:

When I changed my teaching style, I felt a change in my students. They tried to construct new personal meaning in the concepts that they were taught. The students that I consider always as a low achieving changed to be at better level. (Teacher's interview, 1/4/2012)

Drawing on the previous comments, the teachers recognized that teaching practices should target understanding the reasons that created instability in learning. Learning

needs to be situated and supported in meaningful and relevant learning experiences. Learning also needed support through social interactions. Students acquire new knowledge by constructing it from real experiences and interactions. The following classroom practices that in parts encouraged students' conversations and supported them to make comments on their peer observations during supportive classroom discussions, was illustrated in the following classroom discussion.

Angle of elevation and the angle of depression, Grade 10, (a part of the lesson)

The lesson was conducted in the Learning Resources Lab, the students sitting in groups; the teacher sometimes used PowerPoint. The following is a part of the lesson that aimed to introduce angle of elevation and the angle of depression.

The teacher asked her students to watch the story about two friends Sali and Reami and write their observations about the geometrical shapes that are illustrated in the pictures such as the angles, vectors, etc.

The students observed the actions of the story and wrote their comments about the geometrical diagrams that were drawn.

Then the teacher asked their students to talk about what they observed and understood from the story.

قررت ريمي في يوم من الأيام أن تخرج للزّهة مع أختها سالي.

صرخت سالي باغتي صوتها: "ريمي أين انت أنا في انتظارك" نظرت ريمي إلى الأسفل، فكانت زاوية انخفاض.

زاوية انخفاض

تأخرت ريمي عن الموعد المحدد، فانتظرتها سالي أمام باب المنزل حتى تخرج، وكان نظرها باتجاه الباب.

بينما نظرت سالي إلى الأعلى فكانت زاوية ارتفاع.

زاوية ارتفاع

The discussion between students and the teacher are as follows:

Student 1: all the diagrams show that there is a line drawn from Sali's eyes towards Reami.

Student 2: But in some of the diagrams, the line is drawn from Reami's eyes towards Sali

Teacher: What is the difference between the two kinds of diagrams?

Student 3: When Reami observed Sali walking in the street from her room, the line is drawn from Reami's eyes down to Sali, while when Sali saw her friend, she looked up and the line is drawn from Sali's eyes to Reami.

Teacher: Good, so we have something or an object which is observed and a line drawn between the observers' eyes which and the object; also we have two lines of observation up and down. What else have you found?

Student 4: Angles.

Teacher: What is the meaning of an angle (to student4?)

Student 4: Two lines connected together to form an angle. One of them is a line from the object to the observer's eyes.

Teacher: Where are they connected?

Student 5: Two lines or vectors connected at their vertex

Student 6: What is the other line that formed the angles?

Teacher: Can anyone show me the angles and their components (as in the pictures), How many kinds of angles and what are the two vectors?

Student 7: Horizontal line, line from the object to the observer's eyes; they both formed the angle when Reami looked up and when Sali looked down.

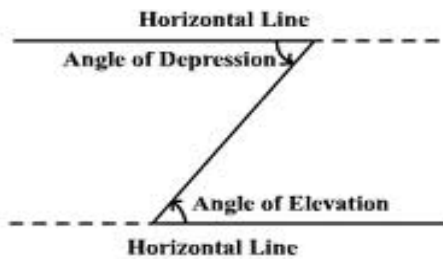
Teacher: We can give these components names: The line from the object to the observer's eyes is the line of sight, the other line is a horizontal line and the angle is called the angle of elevation. What is the other angle?

Student 8: It is when the person who observes is up and looks down; it is also between the horizontal and the line of sight of the observer.

Student 9: According to this situation, we can call it the angle of depression.

The teacher then asked students to give examples illustrating the real application of the angles of depression and elevation in their lives. The students demonstrated good examples that showed their understanding of the main concepts and their ability to recognise them in their surroundings.

Then the teacher discussed with their students the relationship between the angles of elevation and depression that were illustrated in the previous diagrams. One of the students drew a horizontal line that is parallel to the horizontal line in order to get the relationship between the angles. Most of the students recognized that both of angles are equalled based on their knowledge of mathematics in grade nine about alternate interior and exterior angles (Researcher's diary, 01/04/2012).



The previous discussion showed collective conversations towards understanding the concept of the angles of elevation and depression. The discussion between students 1 and 2, for example, showed that students listened carefully to their peers and evaluated what they said by adding comments to their observations. The overall discussion gradually developed towards the best understanding of the main concepts. Maybe it is good for students if the teacher helped them to display their own examples of the angle of elevation and depression from their classroom or school.

To sum up, the overall data indicated that teachers identified the value of implementing Productive Pedagogies as it has a positive impact on the improvement of learning and teaching beliefs and strategies. They also emphasized the impact of applying Productive Pedagogies in their students' engagement and learning. While teachers expressed positive reactions towards introducing Productive Pedagogies in their teaching practices, they indicated some key challenges that faced their implementation and limited their best efforts. Section 4.3 will present data that explain these challenges.

4.2.3 Opportunities for teachers' professionalization

The data indicated that introducing Productive pedagogies by mathematics teachers in their teaching offered opportunities for effective professional development experiences from many aspects. First, teachers' experiences through their implementing of Productive Pedagogies provided opportunities for active engagement, discussion, and reflection to challenge their teaching practices and construct new ideas for improvement. The teachers developed an ability to use the Productive Pedagogies vision as a common language to plan, prepare learning and teaching experiences and to analyze, evaluate and improve their own teaching practices and the teaching experiences of their peers. Second, teachers emphasized the positive value of Productive Pedagogies as a guiding framework for their preparation of their lessons. Third, there was the development of teachers' willingness and confidence to change their teaching.

Teachers pointed to the value of the Productive Pedagogies framework as a framework that helped them to reflect critically on their teaching practices. The data indicated that teachers were able to use the language of Productive Pedagogies to plan, reflect and defend their teaching practices. Teachers noted that they gained many benefits from conducting classroom observations and group discussions to analyze and discuss their observations of their own and peers' practices and to prepare teaching and learning materials. One of the teachers pointed to the role of Productive Pedagogies as a language of a dialogue between teachers, on improving their teaching approaches. She said:

My teaching approaches improved according to the feedback that I received from other teachers that implemented Productive Pedagogies. Our analysis about the lessons that we observed gave me new directions to better implementation of Productive Pedagogies in the mathematics content that we taught. (Teachers' interview, 1/4/2012)

The previous comment provides an indication that teachers shared and learned together to develop a new range of practices that be in agreement with new understanding of Productive Pedagogies. Another teacher also indicated that her ability to analyze her teaching practices building on Productive Pedagogies improved. She commented:

I can discuss any lesson process and practices by referring to Productive Pedagogies. That is because the elements of Productive Pedagogies dimensions cover all teaching aspects that were supposed to be included in teaching. (Teacher's interview, 1/4/2012)

Drawing on the previous teacher's comments, the Productive Pedagogies framework provides a well-defined image of the effective teaching that teachers need in their pedagogical knowledge and experiences. Moreover, providing opportunities for teachers to collaborate with their peers and other experts to improve their teaching practices helped effective classroom learning and teaching. Teachers noted that group discussions of lesson analysis and for preparing learning experience helped them to exchange their ideas and to identify the strengths and the points of development on their teaching strategies. Furthermore, the Productive Pedagogies framework, as they noted, helped them to develop their ability of reflection by providing them with a scientific background as a reference for them. One of the teachers noted that the opportunity of observing, analyzing and preparing helped her to overcome the shortcomings in her teaching. She said:

I benefited from classroom observations as they gave me opportunities to develop new ideas of applying Productive Pedagogies elements that may be absent from my mind and I tried to re-apply them in my mathematics classroom and developed them to be meaningful. I benefited from my peers' experiences and addressed the deficiencies in my teaching practices. (Teacher's interview, 24/4/2012)

The previous view of the teacher indicated that one of the main requirements for effective transformative learning experiences is engaging teachers in a continuous process with opportunities of improvement such as observation, discussion and sharing ideas. The following part from one of the group discussions showed the teachers' sharing of some of their experiences of teaching mathematics.

Part from a group discussion of preparing learning and teaching experiences, Afra-Sea Cycle Two School, 20/3/2012

Alhuda: I challenged my students when I asked them to get the height of the flag that was positioned in the schoolyard using their simple rulers and geometrical tools. The problem: "From a point 5 m from the base of a flagpole, determine the height of the flag". When I posed this problem to the students, their desire to know the best way helped to recognise the importance of trigonometry in solving real problems or simplifying them. They understood that the flag was positioned to the ground in right-angle and they could draw a simple graph of a right-angled triangle but how to get the angle was the most challenging point. That gave me opportunity to introduce the value of the angle of elevation to solve this problem, which is the objective of the lesson.

Sama: That may be a good example for *connectedness*; I also found this problem helped students to identify the value of mathematics beyond the classroom. By discussing the problem, to find the accurate position that enables the pilot of the airplane to help the person who is stand on a rock in the Wadi. This application showed also the value and the meaning of using the angles of elevation and depression by determining the height of a mountain or the depth of a pool and a distant object, that were very difficult to measure directly.

(Wadi: is a kind of small fluid of deep moving water that falls from the mountains towards peoples' houses; it is one of the natural events during rainfall in Oman that sometimes kills people).

Moreover, teachers' confidence about applying Productive Pedagogies in their mathematics reflected their ability to share new experiences of application in discussions. The two groups of teachers who participated in this study presented their experiences to other mathematics teachers from the participants' schools and from other public schools. Teachers were able to open useful communication with other members of the teaching profession by involving in discussions about the value of applying Productive Pedagogies in their mathematics teaching, as well as the challenges and the actual process of improvement in their teaching practices. Teachers noted that their experience of presenting and talking about their teaching practices was a new experience for them. Most of the teachers noted that the idea of talking about their teaching experiences and the actual exchange of ideas about their teaching practices during the professional development day was a new experience that offered new opportunities for them to understand their experiences and share their knowledge and ideas in groups.

In addition, teachers highlighted the value of the Productive Pedagogies framework as a guiding framework for their thoughts, preparation, practice and evaluation. It provided them, as they noted, with a complete overview and better understanding of their classroom situations. The teachers noted that the preparation and teaching of their mathematics lessons became more organized, based on comprehensive and systematic elements, covered many effective teaching principles and created an interactive learning atmosphere in their mathematics classroom.

The data indicated that teachers drew on Productive Pedagogies to construct their lesson plans. One of the teachers explained her new experience of preparing mathematics lessons based on utilizing Productive Pedagogies, and said:

When I start preparing my mathematics lessons, I recall in my memory the dimensions of Productive Pedagogies and build the lesson's plan upon them; the first dimension that emerges in my memory is the connectedness dimension. (Group discussion, 21/03/2012)

The previous comment strengthened the framework as a guide to the process of planning which is the initial and important step for teachers to begin their teaching practices confidently. The results of data collection also indicated a significant change in participants' willingness to develop their teaching practices and their ability to cause this change in the reality of teaching mathematics in their mathematics classrooms. One of the teachers considered the Productive Pedagogies to be an essential part in her future teaching. She commented:

I will turn to applying the Productive Pedagogies framework consistently because I realized its feasibility and confirmed the usefulness of it and I feel comfortable in teaching more than in the past; we and our students have benefited greatly. (Teacher's interview, 1/4/2012)

Teacher's confidence, as the previous comment showed, related in one way or another on their ability to recognize the real usefulness of the framework from their actual classroom experiences. The data also identified that teachers felt confident in teaching their mathematics lessons and thus that added an urgent desire to develop their teaching practices. One of the teachers asserted that comment and said:

I have created a sense of certainty that I could use Productive Pedagogies in each lesson and I can link each lesson to the reality of the world. Myself as a first teacher to the group of school mathematics teachers, I found my peers produced new ideas and methods in their teaching practices. (Teacher's interview, 24/04/2012)

The willingness to change also extended to reach other school mathematics teachers that did not participate in this study. The teachers specified that their experience in applying Productive Pedagogies encouraged a group of other school mathematics

teachers who did not participated in this study to apply some of Productive Pedagogies in their mathematics lessons. In the observed classrooms that I visited as a researcher, these teachers attended to the observed lessons in order to know more about Productive Pedagogies.

While teachers expressed positive reactions about applying Productive Pedagogies in their mathematics teaching, they identified the benefits of improvement on teaching beliefs and strategies and recognized the mechanism of improving their professional knowledge; they also identified some of the challenges that limited their best efforts to introduce Productive Pedagogies in their teaching as discussed in the following section.

4. 3 Challenges in the implementation of Productive Pedagogies

Drawing on data from classroom observations of the participants' teaching and group discussions as well as teachers' interviews about their experience of applying Productive Pedagogies, some of the challenges teachers faced that hindered their best efforts to apply Productive Pedagogies in their mathematics classrooms had been identified. The challenges differ in their causes, time of appearance within the implementation phases and in their strength and persistence.

The results indicated that some of challenging factors occurred only with the initial attempts of applying Productive Pedagogies to mathematics teaching and then declined in later efforts. The data also shows that some of the challenges were associated with the actual teachers' understanding and applying of the Productive Pedagogies framework. Other challenges were associated with contexts factors that influenced teachers' professional development.

4.3.1 Challenges associated teachers' use of the framework

In their attempts to understand the Productive Pedagogies framework and to apply it in the classroom, the teachers encountered some difficulties. Two sources of difficulties were identified. First, the teachers not only had to come to an understanding of the new terminology of the framework but also operate with their colleagues in the project in a novel way. Secondly, and more specifically, certain elements of the framework were less implemented than others. Not only the teachers were not familiar with these particular elements and their impact on mathematics

classes, the literature in mathematics education is often silent on these particular elements. These difficulties will be discussed in turn below.

Productive Pedagogies as a new experience

Teachers noted that at the initial attempts of implementation they did not have sufficient experience and skills to design rich activities and learning experiences that emphasize the role of centring mathematics learning around the student. While there were many good attempts from the teachers to apply Productive Pedagogies, much support was needed during the different stages of implementation. For example, in their initial attempts of implementing, teachers found difficulty to prepare rich tasks and learning experiences which targeted to make mathematics explicit, to go beyond students' engagement and to transfer students learning of mathematics knowledge and skills outside the mathematics classroom. The observed classrooms that were discussed in the previous sections indicated the difficulty to apply activities that showed the relevance of mathematics to real life. Some of the observed lessons indicated to the superficial connection of mathematics content that teachers taught and real life applications. The following comment from one of the teachers indicated the difficulty of designing high order thinking activities:

With the beginning of the implementation, I found difficulty in designing classroom activities that encourage higher order thinking such as discovery activities that encourage students' abilities of deduction and induction and their abilities to discover, analyse, and conclude. (Teacher's interview, 24/4/2012)

Based on this difficulty, teachers also indicated that during the first attempts of implementation, they used extra effort and time to prepare learning experiences that adopted Productive Pedagogies. They noted that while some of the Productive Pedagogies were known by them, their previous knowledge and understanding of those concepts differed in their value, uses and purposes from what Productive Pedagogies target. Hence, their applying of Productive Pedagogies involved spending time and effort from them to understand the possible applications of Productive Pedagogies in their mathematics classrooms. They also noted that change towards learner-centred approaches required many opportunities for group discussions and reflections. The arrangement of their time table and the short free

time for most of them limited those opportunities. Teachers noted that their experience in applying Productive Pedagogies and their experience of reflection on their teaching practices were new experiences for them. Regarding the Productive Pedagogies framework, many teachers saw that they have many elements and they are many similarities between them which may lead to some misunderstandings and misconceptions when applying them for the first time. They indicated that the large number of elements may reduce teachers' desire to implement Productive Pedagogies. One of the teachers commented on the difficulty of experiencing new conceptions of pedagogies and pointed to the many elements of Productive Pedagogies. She emphasized that it may become easier with more experience and time. She said:

The biggest difficulty I face in the application is the experience of the new theoretical framework and the twenty elements in the dimensions. When a teacher who attempts for the first time finds that there are twenty elements, what comes to her mind, is that the mechanism of the application will be difficult. (Teacher's interview, 1/4/2012)

The previous comment indicated that developing new pedagogical content needs providing sufficient time and opportunities for teachers to discuss, read about and make sense of what they experienced.

The data also showed that teachers indicated that they previously did not collaborate with peers and other experts to improve their practices. Teachers mentioned that their experience of observing and coding classroom practices was also a new experience for them. During their previous in-service years, they did not experience transmitting and receiving feedback from their peers. Teachers also noted the difficulty they faced when coding their teaching practices and peers' practices using the *Productive Pedagogies classroom observation manual*. They noted that this manual contains many statements of coding and there are many similarities between these statements. They also noted that they spent time during the initial classroom observations to choose the best evaluated score according to the principles of the Productive Pedagogies framework. One of the teachers commented on her previous experience of coding classroom lessons before applying Productive Pedagogies and said:

My previous experience in evaluating my peer classroom practices that I visited was the use of a descriptive form from the Ministry of Education and contained approximately 18 items. We wrote our quantitative evaluation based on our previous knowledge and experiences and without being subject to any scientific references or frameworks, I chose the teacher who had experience or helpful teaching ideas to visit in order to gain some benefit. But after that visit, we did not sit as a group to discuss the observed teaching practices; it was not something familiar to us and not even the visited teacher asked me about any feedback or comments to share. (Teacher's interview, 24/4/2012)

The above comments indicated that teachers experienced a quantitative approach of classroom observation. The reflection of the observing and analysing the classroom situations was missing from their practices as teachers. Thus, they felt that the idea of using Productive Pedagogies as a framework for reflection was new and they spent time and effort to achieve the main benefits.

Limitations of the actual application of Productive Pedagogies

The failure to develop a working understanding of some of Productive Pedagogies elements such as *metalanguage* and *cultural knowledge* had reduced the teachers' ability to apply them in their teaching practices. Through group discussions, signs of lack of deep understanding of those elements began to loom on the horizon of the discussions to give an important signal of a need to deepen and clarify the concepts of those elements. So the group discussion efforts were then directed towards helping teachers to overcome that difficulty through discussing and preparing some practical ideas which applied those elements within their current mathematics content that was taught.

Regarding implementing the *metalanguage* element as an element of intellectual quality dimension, the data showed that the teachers were not sure of the importance of its application in their mathematics lessons. This view related to their initial teaching tradition that was focused on mathematics language, not on the aspects of written and spoken language. Ongoing targeted discussions between the researcher and the participants and between the participants themselves were conducted to demonstrate the participants of the usefulness of *metalanguage* element. One of the

difficulties here is that the mathematics education literature and professional development around the world in general tends not to focus on students' talk and writing, emphasizing the quality of written and spoken texts in mathematics lessons and identifying the role of specific technical vocabulary and words in mathematics lessons. Few attempts by the participants in clarifying some aspects of Arabic language such as correcting verbal and writing mistakes without providing students with ongoing and frequent direct assistance were observed. They also tried to use students' daily vocabulary to give alternative terms for some of the mathematical concepts they taught. In one of the group discussions, the teachers mentioned limited application of *metalinguage* element in their classroom as illustrated below.

Part from group discussion of lesson analysis, lesson: trigonometric ratios, grade 10, Afra Bint Obaid Cycle two schools

Alhuda (who conducted the lesson): Now I tried to remind myself to comment on students' use of language. I corrected the mistakes of one of the students when she wrote on the whiteboard.
The researcher: When you corrected the wrong spelling of this girl, what did she say? She said that correcting our writing is not important in mathematics class; this is a mathematics class not Arabic. What do you think the reasons for that belief?
Alhuda: Maybe, we have not previously paid attention to correcting the mistakes of students' writing or reading in mathematics lessons.
Sama: Just correcting their mistakes in using mathematics language?
Alhuda: Actually, the correct use of mathematics language did not occur to me as being important.
The researcher: I think we can change these beliefs if we recognise the integration between teaching and the type of language that is used; the teacher can stop for some moments and explain on some aspect of language that she found in classroom practices.

The previous short dialogue in a group discussion did not only show the limited use of *metalinguage* element but the limited understanding of the element and its value. From the initial teachers' reactions towards applying Productive Pedagogies in the stage of *preparation* and during the improvement of teachers' application of Productive Pedagogies in the advanced stages of *developing* and *consolidation*, the implementations of *metalinguage* element was limited. Moreover, the data indicated that applying *cultural knowledge* element oscillated between participants' misunderstanding of the requirements of investigating *cultural knowledge* element and their views about the relevance of that element to the Omani context as discussed in the previous sections.

4.3.2 Challenges associated with contextual factors

The data indicated that some factors that related to the school context such as time constraints and a crowded mathematics curriculum were widely cited by teachers that

limited their efforts to explicitly plan and prepare to apply Productive Pedagogies in their teaching. Teachers also identified other challenges such as class size, lack of encouragement and poor quality of school professional development programs.

Time constraints

The data indicated that time constraints were linked to everyday pressure of teaching and other school workload. Teachers noted that they did not have enough time to reflect on their teaching. Actually, the school timetable and the heavy school workload such as teaching, preparing exams, entering students' results and analysing them did not help teachers to meet, discuss and share their experiences. In particular, each teacher taught 21 mathematics periods in five school days as well as she had to take full responsibility for one main school activity. Moreover, teachers had to fill the absence of any mathematics teacher. During the application part of the program, some of the teachers took around 28 periods because of carrying the workload of absent teachers. Hence, at times it was difficult to arrange the school timetable for conducting classroom observations and group discussions. One of the teachers commented on time constraints that related to the heavy school and teaching workload and said:

We were looking for every opportunity to discuss and prepare learning activities and to exchange our teaching ideas, especially at the beginning of applying Productive Pedagogies. However, those meetings and group discussions for exchanging teaching ideas became less frequent because of pressures of work and the lack of available opportunities to meet. (Presentation, 25/04/2012)

The previous comment pointed to the influence of the school as a community of supporting ongoing teachers' learning. Effective teachers' professional development as a part of school norms and culture needs support from the school community in order to meet teachers' needs for growth and learning.

The data also showed that the instability of the decision by the Ministry of Education to confirm the dates of the end of the terms and of exams was one of the important obstacles that teachers faced during their implementation of Productive Pedagogies. During semester two and after one month those dates changed. That instability added

much pressure on teachers and did not help them to organize their programs. The time constraints did not give teachers the flexibility and desire to change their practices and to apply new ideas.

Crowded curriculum

Teachers highlighted the issue of crowded mathematics curriculum content. The teachers noted that they saw that mathematics curriculum was weak in three main aspects. First, the content that had to be taught to students every semester was seen as extensive. Second, there was no connection between the content of the mathematics textbooks and the basic knowledge that students had learnt in previous grades. Third, the limited time allocated to teach the curriculum and students' literacy and numeracy weaknesses made it difficult to complete the crowded curriculum. They noted that teaching time is eroded by the current nonessential workload that teachers undertake and did not allow for more teaching time. Moreover, time on task is limited by the relatively short school year. They pointed to the practices that related to public examinations and result in shortening the school year.

Teachers also commented that the impact of the crowded curriculum increased with the lack of connection of the mathematics to real life situations and not using mathematics in social and human contexts. They noted that while the content illustrates some word problems from real life but it still orbits around abstract and decontextualized mathematics knowledge. One of the teachers said:

Mathematics curriculum is crowded as it contains intensive mathematical knowledge which cannot be covered in the allocated teaching time and with using Productive Pedagogies. The efficient way is re-examine the curriculum content and the distribution of lessons within grade levels. (Group discussion, 17/01/2012)

In addition, as teachers and the school system aimed to prepare their students for the final exam, they concentrated on finishing teaching the crowded content in a short time. A tradition of past examinations, that allow success through memorization, dictated pedagogical practices and thus teachers tended to teach to the test. This practice did not help students to understand the important mathematics knowledge in depth and detail as required by one of the main objectives of Productive Pedagogies.

This issue of crowded curriculum with insufficient time had been referred to by students as well who indicated that they had not received an adequate training in mathematics exercises because of the limited time available for studying mathematics.

Some of other difficulties that were discussed below faced by the teachers during the application which were associated with the usual school reality were the large class sizes, insufficient learning and teaching resources, lack of promotion opportunities for teachers and lack of effective school professional development programs.

Large class size

Largest class size with students of different achieving abilities was one of the challenges that teachers noted. In fact, the class contained around 35 students in average. In particular, teachers highlighted the difficulty to take care to all those students and to reach the objectives of “representative participation” element in such a short time. Also, the students did not learn to take responsibility for their learning and that added more pressure on teachers. One of the teachers commented on the effect of large class size and said:

Best application of Productive Pedagogies requires all students having opportunities to participate and be engaged. However, following-up 35 students with different abilities and learning skills creates an extra burden and difficulty for the teacher. Maybe if the class size were reduced to 15 that would be better. (Teacher’s interview, 30/04/2012)

Another teacher also identified the negative impact of class size and pointed to the difficulty of following students in group-work activities. The following comments also indicated the benefits of reducing class size. She said:

I faced more difficulty to monitor the performance of each student individually in group-activities. With a smaller number of students in class groups, I think I will have better idea about the learning needs of each student and about their levels of thinking, weaknesses and strength. (Teacher’s interview, 30/04/2012)

The data also showed that the difficulty of big class size increased with weakness in students' basic mathematics knowledge and with the gaps in learning abilities. For example, teachers noted that one of the difficulties that confronted their students' learning was students' weakness in literacy and numeracy skills that were important skills in their learning of mathematics. Hence, with a big class size, finding out the common mistakes and working to overcome them, was essential but time consuming and had an overload effect on teachers. The new mathematics concepts and skills that were taught were hampered by the superficial fundamental mathematical knowledge of the students. One of the teachers pointed to this issue and said:

Poor students' basics of reading and writing is one of the main difficulties that we faced; sometimes, even if student discovered and understood the information that they learn, the issues of weakness in reading and writing becomes the cause of the delay in this understanding and discovery.
(Presentation, 25/04/2012)

Drawing on the previous comments, the ongoing problems of students' weakness in the basics of numeracy and literacy that teachers faced in the middle-schools highlighted the importance of providing the learners with quality teaching and high expectations in the early stages of their learning mathematics and other subjects.

Lack of encouragement from school system

Teachers indicated that lack of encouragement from school principals and from teachers of other school subjects for the teachers who used good practice and who developed expectations and systems to change their practice limited teachers' desire to change in the future. In addition, teachers pointed to the lack of electronic resources, as well as advanced educational books and geometric tools which are essential in teaching and learning mathematics. However, while providing teachers with learning and teaching references and resources is important, the unavailability of some of the resources such as geometric tools may not reflect significant reasons that prevented teachers from implementing good teaching.

Regarding school teachers support to apply Productive Pedagogies, the participating teachers faced difficulty in encouraging teachers of other school subjects to recognize and collaborate investigating the element of *knowledge integration* in order to connect mathematics knowledge with school subjects and connect them with

the mathematics knowledge that was inherent in other subjects. For example, while teaching the lesson of time zones that is related to the content of geography subject, the participating teachers that taught grade 7 found difficulty to prepare common learning activities with geography teachers who taught the same class. The school teachers did not experience the idea of integration between school subjects and the practices of teachers' peer-activities. This state of affairs may necessitate the need to spread the value of *knowledge integration* between school subject teachers. Encouraging and supporting teachers to establish and participate in teacher-team activities designed to improve curriculum knowledge integration or in pedagogical approaches can be useful for improving student learning.

Moreover, the teachers indicated that they could not go beyond the limits of the specified mathematics textbooks because of the limitations caused by the formal tests and principals' traditional views about the importance of completing most of the content of the textbooks. One of the teachers indicated that applying Productive Pedagogies needs a re-examination of the mathematics curriculum content and the provision of more trust, support and time to teachers. She said:

Re-examining the curriculum to give freedom to the teacher and students to produce educational activities and lessons that serve mathematics teaching are the main factors that will help teachers in the application of Productive Pedagogies. In the second semester, I have a tremendous amount of four units of mathematics teaching. If I want to develop my teaching practices to produce quality teaching, I need time, tools, confidence, and curriculum assistance. (Teacher's interview, 01/04/2012)

The previous comment summarized the main challenges that teachers faced during the implementing of Productive Pedagogies in order to improve their classroom practices. The comments pointed to the idea of improving mathematics classroom practices that required understanding of the challenges of the curriculum being taught, the physical arrangement and sources of the learning environment, the assessment system as well as the quality of pedagogies.

Teachers conveyed the idea of Productive Pedagogies to principals, decision-makers, and educational supervisors in the Ministry of Education in order to make available

all the circumstances and conditions for applying the framework in schools. One of the teachers said:

It was important to spread the message about Productive Pedagogies to decision-makers in the Ministry of Education in order for them to provide the necessary conditions for its implementation. (Presentation, 25/04/2012)

Another teacher pointed to the importance of spreading the message about the Productive Pedagogies framework within schools by stressing the main benefits that school teachers will gain if they introduced the framework in their teaching practices. She said:

My experience with Productive Pedagogies is positive, useful and beautiful. This framework has to be advertised publically. Some may say that teachers already applied these methods normally, but the reality shows that if we applied the ideas correctly, we may realize its importance and usefulness. We suggest that those who direct teacher professional development programs should utilize the dimensions of Productive Pedagogies. (Teacher's interview, 01/04/2012)

The previous comments pointed to the influence of the school and the Ministry of Education in supporting teachers' professional development programs. For example, identifying teaching standards and frameworks that have produced improvements in increasing students' learning and engagement need support from the communities that have these responsibilities.

Poor quality of school professional development programs

The teachers highlighted the issue of the poor quality and support of professional development programs that were provided for teachers as they did not help them to gain new and significant professional knowledge. They noted that the Productive Pedagogies framework provides significant guidance that helps teachers to build upon their teaching practices. However, this framework requires understanding of the pedagogical background and capabilities that teachers should have in order to benefit from Productive Pedagogies, such as having the ability to prepare *higher order thinking* activities and to design learning experiences in mathematics support the connectedness idea of mathematics as knowledge. They indicated that their pre- and

in-service professional development programs did not prepare them for designing these types of teaching and learning experiences. One of the teachers commented on the type of professional development programs that she received during in-service years and she said:

I attended just two professional training and workshops during my four in-service years', most of them were dominated by theoretical ideas. (Teacher's interview, 01/04/2012)

Other teacher who has more than four years teaching experiences said:

I attended workshops in the analysis of tests at the beginning of employment but I did not attend any training programs for in-service on teaching methods. (Teacher's interview, 01/04/2012)

The previous teachers' reactions towards the quality of pre-service and in-service professional development programs indicated the challenges in constructing strong bridges between theory and practice. Thus, the issue of professional development for mathematics teachers requires teachers to experience the theories and then examine and evaluate their effectiveness. Based on these challenges that limited teachers' best efforts to apply the Productive Pedagogies framework, teachers recognized the need for support to help them to better implement the framework. The following section presents data indicating the need for support as suggested by teachers.

Teachers suggested some areas of support that schools and educational systems could provide in order to help school teaches benefit from Productive Pedagogies as a framework of quality teaching and for reflection. These suggestions highlight the importance of conducting quality professional development programs and the spread of the idea of the Productive Pedagogies framework in Omani schools.

Teachers highlighted that the main reason for the gap in their knowledge and their ability to prepare learning experiences that adopt Productive Pedagogies was related to the lack of professional development programs that were offered to teachers before and during their service. They also raised the issue of the quality of some of those programs which were theoretical and shallow. Hence, the teachers pointed to the need for quality professional development programs that introduce Productive Pedagogies framework for prospective teachers in university or teaching colleges.

They commented that they needed in-service programs that taught them to analyze classroom situations and to provide feedback based on Productive Pedagogies through realistic classrooms situations. One of the teachers commented on their requirements as teachers for best application of Productive Pedagogies and said:

I as a teacher would like to develop my teaching practices; I need time, tools, confidence, and curriculum assistance. The decision-makers in the Ministry of Education are supposed to disseminate the Productive Pedagogies framework by creating appropriate conditions for the application. (Teacher's interview, 01/04/2012)

The teachers also suggested that they needed more professional development programs that helped them to design learning experiences that utilized higher order thinking beyond normal instruction. In response to the needs of teachers, some of the group discussions then focused on preparing activities within the content of the current mathematics unit that teachers taught. We during group work involving the participants and the researcher, tried to prepare samples of inductive and deductive types of activities in order to develop the participants' capacities to prepare rich tasks and experiences that could help students to synthesize, analyze and reach conclusions. However, teachers also emphasized that applying Productive Pedagogies in mathematics lessons needed extensive quality workshops with sufficient time as their knowledge and understanding of intellectual quality dimensions needed scientific knowledge support.

To sum up, the previous overall challenges and needs of support indicated that some of those challenges were mentioned by the teachers before introducing Productive Pedagogies framework to their teaching practices. Others were highlighted during the actual implementation. Moreover, some of them were related to teachers' previous teaching experiences while others related to new experiences during implementation. Moreover, the challenges associated with daily school situations and the educational system may be overcome by teachers, while others needed formal decisions related to the Omani educational system. Based on teachers' experience of implementing Productive Pedagogies, teachers expressed general suggestions to investigate the best implementation in Omani schools in general and for mathematics teachers in particular.

4.4 Students' Perceptions of the Change in Pedagogy in their Mathematics Classrooms

Data in this project was obtained about the initial student attitudes towards: learning mathematics, the role of mathematics teachers and the opportunities for participating that they experienced in mathematics lessons in semester one and in their previous grades. Those initial attitudes were investigated during the first week of semester two. Five students from each participating classroom were interviewed within focus groups. The data also examined students' perceptions towards the teaching practices in their mathematics lessons in semester two. The students' perceptions were re-examined at the end of semester two in focus group interviews. Students' reactions indicted an improvement in their initial attitudes towards learning mathematics. They also pointed to the significant change in their teachers' practices. They pointed directly to some examples of applying Productive Pedagogies in their mathematics lessons.

4.4.1 Negative initial attitudes towards learning mathematics

The data from initial focus group interviews at the beginning of semester two indicated that students had negative attitudes toward studying mathematics. In particular, they felt difficulty in studying and reviewing mathematics. The students indicated several reasons that made mathematics difficult for them. Firstly, it was difficult as mathematics lessons overlapped and they could not identify the differences between them. Secondly, the students also noted that the difficulty of mathematics was related to their inability to solve mathematics problems in the final exams.

Difficulty of mathematics subject

Regarding the difficulty of the overlapping of mathematics lessons that students found while learning mathematics, that difficulty may indicate that students did not recognize the relationship between different areas of mathematics content. The following comments indicated students attitudes towards learning some of mathematics lessons such as probability and statistics that they found them difficulty and not important. One of the students said:

Probability lessons are difficult and I do not know their importance. (Focus groups' interviews, Grade 7, 20/02/2012)

Another comment from one of the students indicated her difficulty in learning mathematics. She said:

I feel that mathematics is difficult for me. Statistics lessons are difficult for me. Sometimes my mind busy and I see that the teacher makes a great effort to keep us attentive in class. (Focus groups' interviews, Grade 7, 20/02/2012)

Drawing on the previous comments, students did not identify the importance of some mathematics lessons that they learned in the previous grades. While the content of probability and statistics has many real life applications that encouraged students to recognize their relevance to their lives, the students did not recognize their meaning and value. That highlighted students' needs to recognize and use valuable connections of mathematics ideas and with contexts outside the classroom. The previous comments also showed that students were convinced that their teacher has the main role to create an interactive learning environment. The following responses from students also pointed to the basic role of the teacher on students' attitudes towards the difficulty of learning mathematics. They highlighted the main role of the teaching practices and behaviors that have influenced students' learning of mathematics.

I do not like mathematics class when I do not understand the lesson. I think that the teacher is the main reason for our ability to understand mathematics. (Focus groups interviews, Grade 10, 22/02/2012)

I think that the teacher has a big role in our understanding of mathematics and I hope, this year I can get good marks. During the past year, I encountered difficulties in understanding mathematics due to the teacher presenting information in a manner that lacked respect for the student. (Focus groups' interviews, Grade 10, 22/02/2012)

In the previous students' comments, there were indications that some students found difficulty in understanding some topics in mathematics. As the students were convinced that the teacher played the main role and had the major responsibility on their learning, they also blamed the teacher for any failure to understand the subject.

All these perceptions reflected the heavy dependence of students on their mathematics teachers. Students did not take responsibility for their own learning.

Moreover, the data from initial focus group interviews indicated that while some students identified the importance of mathematics in their lives, however, that awareness seems to be superficial and limited. These reactions emphasized students' previous comments about the importance of studying mathematics for their entering into higher educational institutions. For example, their consciousness of the importance of mathematics stems from their views of mathematics as a subject of counting or as an opportunity for them to enter universities and then get a job. When the students were asked about their views of the importance of mathematics, the following responses were identified. The following comments from focus group interviews asserted that idea. One of the students in grade 7 said:

Some topics in mathematics were not as important; sometimes when I study the rules and laws of mathematics, I do not feel their importance in life. I just study it because success in mathematics means that there is good opportunity for me to enter the university. (Focus groups' interviews, Grade 10, 22/02/2012)

Another student who expressed her views about the importance of mathematics as success in mathematics meaning completing higher education said:

I think mathematics is important because it is a chance to enter university. Sometimes it used in financial accounts. (Focus groups' interviews, 21/02/2012)

Another student from grade 10 commented:

Is mathematics we are studying from grade one to grade twelve, important for studying in university? Sometimes I feel that some mathematics lessons are not important such as probability. I think that any student can remember only the information that is used in daily life. (Focus groups' interviews, Grade 10, 21/02/2012)

Another student said:

Some math lessons are important because they are related to reality, such as mathematics information gathering, while some of the lessons do not have real life applications, for example Algebra. (Focus groups' interviews, Grade 10, 21/02/2012)

Students' previous reactions towards the importance of mathematics as a school subject indicated they did not recognize the actual value and meaning of mathematics beyond the classroom. This view gave an indication of the cumulative impact of those beliefs from the early stages that have influenced students' participation and engagement in mathematics lessons.

The students also pointed to the role of the teacher to make mathematics lessons less difficult. Within one of the focus group interviews, students' comments asserted the idea of difficulty that related to the overlapping lesson. One of the students expressed her feeling by saying:

I feel that mathematics is difficult and not easy and it depends on the teacher's methods to simplify it.... similar and overlapping mathematics lessons are difficult but the lessons that are different in their content are easy to understand. (Focus groups' interviews, Grade 7, 20/02/2012)

Another student commented:

The lesson is difficult if there are overlaps between the mathematics processes or topics. (Focus groups' interviews, Grade 7, 20/02/2012)

The previous comments indicated students' dependence on the teacher to transmit the mathematics knowledge and to simplify the subject. That may indicate centering the teaching practices of the teacher in mathematics lessons. The students relied on the mathematics teacher to take full responsibility for their learning.

Students' initial views towards classroom participation

Regarding students' participation in mathematics lessons, some students indicated that they have good opportunities to participate but most of them indicated that they did not. The following comments present the reactions of most of students towards

their classroom participation. One of the students indicated that teacher focused on specific students to participate in mathematics lessons. She said:

In the first semester, the teacher chose specific students to participate and sometimes I did not participate in a mathematics class for more than two days. (Focus groups' interviews, 20/02/2012)

The following comments from students pointed to the negative impact of focusing on specific students:

I feel a sense of boredom and oppression when I cannot participate. Among thirty students, the teacher focuses on just two students. Why have I to take care of my learning of mathematics if I do not have opportunity to participate? (Focus groups' interviews, Grade 7, 20/02/2012)

Another student said:

Sometimes the teacher does not allow me to participate for a week. The problem is not about participation but about discrimination and justice. (Focus groups' interviews, Group seven, 20/02/2012)

Another student noted that she did not participate in the class because she was afraid that her answers may be wrong. She said:

I did not participate when I was not sure about the answer. If I answered wrong, the teacher would be angry. (Focus groups' interviews, 21/02/2012)

Another student commented:

Focus on students' correct answers shakes my confidence. (Focus groups' interviews, Group ten, 21/02/2012)

The previous comments pointed to the type of classroom discussion that was based on asking questions and receiving answers. The students' responses pointed to answering questions as an opportunity for them to participate in mathematics lessons. Thus, they had a feeling of unfairness when they did not have the opportunities for answering the teacher's questions. That may reflect on the limited opportunities for classroom discussion and substantive conversations in the mathematics classrooms. It may also indicate the teachers' emphasis on asking questions and receiving

answers to build the mathematics classroom interactions. That pointed directly to teachers' previous teaching practices that were based on transmitting knowledge and developing direct classroom discussion by asking questions directed at particular students and receiving short answers from students.

Students' anxiety in mathematics exams

The following comment pointed to students' fear of mathematics tests. It indicated that the difficulty of mathematics tests stems from students' inability to reuse what they have learnt in mathematics lessons to solve problems during mathematics tests.

I feel that mathematics is difficult; I wish there were no tests. I found difficulty to remember what I studied. We always try to prepare for the test by resolving the mathematical problems on our own. (Focus groups' interviews, Group ten, 22/02/2012)

Regarding students' conceptions about their anxiety of exams, some of the issues should be considered. First, the teaching and learning experiences in mathematics lessons may target success in final exams. That was consistent with teachers' views about their previous teaching practices that relied on completing exercises in mathematics textbooks to prepare students for the final exams. In addition, that might relate to students' concern and to their parents' expectations towards the success in the final exams to provide a better academic future for students. These expectations may reflect that success in the exams is the only way from students and parents' views for providing opportunities for students to enter higher education institutions, especially for girls. Moreover, the fear of exams may indicate the emphasis by teachers on applying a particular type of assessment to evaluate students' performance. It may indicate that students did not have any idea about the different evaluation approaches that mathematic teachers might use to evaluate their performance such as observation, short quizzes, projects and other class activities. . However, their reactions pointed to other factors rather than their abilities in understanding of mathematics. They pointed to the impact of the content of mathematics within the textbooks, the role of the mathematics teacher, the impact of their experiences in studying specific mathematics content such as probability and statistics and to their positive attitude to the mathematics final exams.

Drawing on the previous perceptions and comments, students felt not confident in learning mathematics, due to unfairness, as they did not have equal opportunities of participation in the mathematics lessons and freedom to express their answers. Their awareness of the importance of learning mathematics was related to their concern of their academic future. They did not recognize the value of mathematics in their personal and cultural life.

4.4.2 Students' change in attitudes

The students were re-interviewed after more than two months at the conclusion of the study. Some change in their attitudes toward mathematics and their awareness of the importance of mathematics in their lives were identified. In particular, the students expressed positive reactions towards their participation in mathematics lessons. In particular, the students enjoyed the classroom practices that introduced short personal stories and stories that reflected historical experiences. In addition, students pointed to some specific ideas that reflected the teachers' implementation of some elements of Productive Pedagogies.

Positive reactions towards participation in mathematics classrooms

The classroom observations showed students' engagement not only to mathematics but beyond mathematics subject. Students' awareness of the aspects of beauty in mathematics and their recognition of the extent and the importance of mathematics in their lives were recognized when they interact positively in collaborative activities in classroom and outside it as discussed in the previous sections. Some of the classroom activities, as discussed previously, showed students' attempts to find more than one solution and one way to solve mathematics problems. Students' interactions were developed during the observed time. Focus-group interviews also showed students' desire to give more attention and to work hardly to study mathematics because of their recognition of its important for their academic achievement and to their future.

While the data from classroom observations and teachers' interviews showed an improvement in students' participation and interactions in mathematics lessons, students' perceptions also indicated an improvement. The following comment from one of the students indicated her conception towards class participation and the learning environment atmosphere in semester two:

The whole class was involved. The atmosphere of the mathematics class was exciting and competitive (Focus groups' interviews, Grade 7, 24/04/2012).

The previous comment may also reflect a supportive learning environment that was both competitive and encouraging. The previous comment indicated that students felt happy and excited as they were provided with opportunities for the whole class to participate. When I interviewed them, I recognized the positive feelings especially when I compared with the initial interviews with the same students. The students pointed directly to the change in their participation in mathematics lessons in semester two compared with semester one. One of the students said:

I feel I have been involved during this semester more than in previous years as I participated with my peers in many group activities (Focus groups' interviews, Grade 10, 24/04/2012)

Another student commented on the benefits of changing the mathematics teacher on students' participation. She said:

The new teaching ways that the teacher used helped me to participate in the lessons whereas I had not participated in a mathematics class before. (Focus groups' interviews, Grade 10, 24/04/2012)

Students demonstrated positive reactions when they compared the limited opportunities of classroom participation in semester one or in their previous experiences with the positive change in their participation in semester two. Drawing on the previous comment, students indicated positive reactions towards their participation in mathematics lessons. Offering opportunities for them, as they noted, to participate in group activities may reflect the positive impact of interacting with their peers to learn mathematics. Students felt that they enjoyed their participation in class group activities. Such attitude might indicate students' desire to take responsibility for their own learning. Their perceptions may also indicate the potential that they were willing to perform different roles within these class activities.

While the previous students' perceptions indicated a change in students' participation in mathematics lessons by providing them with opportunities to interact with their peers through group activities, the following perceptions pointed to such

opportunities of classroom communication offered for students within their classroom practices by discussing some problems or telling stories. One of the students who were interviewed commented:

The problem that was discussed in our mathematics class was the problem of cake division between mother, rabbit and fox, the story of the apple, the problem of accidents and building bridges, and the problem of linking regions of the Sultanate of Oman through different streets. All those problems gave us opportunities to express our views and our opinions. (Focus groups' interviews, Grade 10, 18/04/2012)

Drawing on the previous comments, students identified some classroom discussions about problems that have impact in the Omani context. They also pointed to the classroom discussion that reflected free opportunities of expression of their views, ideas and feelings. Providing students with learning experiences that were a part of their daily life experiences might give them new views about the type of mathematics lessons that go beyond the direct discussion that was based on abstract mathematics ideas to an open discussion that encouraged expressing different views and reactions.

Enjoyable classroom practices

The previous comments about students' perceptions expressed the positive reactions about mathematics classroom participation that students experienced in semester two. The classroom observations showed students' engagement not only to mathematics but beyond mathematics subject. Students' awareness of the aspects of beauty in mathematics and their recognition of the extent and the importance of mathematics in their lives were recognized when they interact positively in collaborative activities in classroom and outside it as discussed in the previous sections. Some of the classroom activities, as discussed previously, showed students' attempts to find more than one solution and one way to solve mathematics problems. Students' interactions were developed during the observed time. Focus-group interviews also showed students' desire to give more attention and to work hardly to study mathematics because of their recognition of its important for their academic achievement and to their future. Students also expressed positive reactions towards some of the classroom practices that were enjoyable. In particular, they pointed to introducing stories in mathematics classroom practices. The previous comments

pointed to the use of some enjoyable stories that the teacher told in mathematics lessons. One of the students from grade 10 narrated some examples of mathematics lessons that introduced personal stories and historical stories. She said:

using the story of cake division in studying division in the polynomial lesson and the story of the apple in the lesson on elevation and depression angles tells us a lot as well as connecting us to what we will study in Geometry in the future. (Focus groups' interviews, Grade 7, 24/04/2012)

The previous comments pointed to some of the classroom practices that aimed to use personal stories in geometrical concepts such as the angle of elevation and depression. The previous discussion about teachers' implementation of the elements of Productive Pedagogies dimensions within the previous sections showed that students found the teaching practices enjoyable. That may reflect students' desire to be introduced to mathematics lessons in ways that are more enjoyable. The following comments indicated classroom practices that were narrative in nature from students' views. The students were familiar with the term *narrative* approach or style, as they had learned about it in the subject of Arabic. Their ideas about narrative style were based on using stories from personal experiences or historical events. Their responses about introducing stories in mathematics lessons indicated that the presenting of stories in teaching mathematics content was new and were surprised that this was part of the mathematics lesson. The students identified the impact of these stories for remembering what they had learned in mathematics lessons and for participating in mathematics lessons. One of the student's comments from grade 10 highlighted these ideas:

The methods that the teacher used such as the narrative style and the link with life helped me to participate in mathematics lessons, when I had not participated before. I feel that I participated during this semester more than in previous years. (Focus groups interviews, Grade 7, 24/04/2012)

The previous comment showed a change in the learning and teaching practices that they did not previously experience. While students knew about some of the aspects of narrative style in teaching by introducing stories as they had experienced in other school subjects, they were surprised that those aspects could be applied in mathematics lessons. This feeling reflected students' positive reactions towards the

change in teaching styles of mathematics lessons which reflected some kind of enjoyment. The following comments from one of the students from grade 7 asserted their identification of the change and the positive reaction towards introducing stories that provided opportunities for interactive discussion. She said:

There is a dramatic change because the lessons have become more enjoyable. When I compared with previous years, the teacher did not allow me to participate. When using stories such as the story of rain in mathematics lessons and discussing problems such as a problem of the coastline (Omani Project of Coastline) and linking them to the lessons of equation and the geometrical translations, I felt that we had become more responsive (Focus groups' interviews, Grade 10,18/04/2012)

Indication of some elements of Productive Pedagogies dimensions

While the previous students' perceptions showed a change in classroom practices, some of their reactions reflected indications of the classroom practices that applied elements of Productive Pedagogies dimensions. Students pointed directly to the idea of connectedness and the classroom activities that applied *higher order thinking* element. One of the students expressed her comparisons between semester one and semester two by indicating teacher's efforts to connect mathematics lessons to their life experiences from grade 10. She said:

In the first semester, I did not feel there was any relation between our lives and the mathematics lessons; in this second semester, the lessons are related and have value and importance to life. While studying quadratic functions, we discussed the problems of congestion and car accidents. We also discussed how to utilize mathematical functions to build bridges. (Focus groups' interviews, Grade 7, 24/04/2012)

The previous comment indicated that student recognized the extent of mathematics in their lives through the real applications of the mathematics concept of function. It also indicated that the content of the lesson that was taught at the beginning of the semester two was still in her mind. The student through her feeling of its importance remembered the lesson. The focus group interviews with students also showed that students sensed the application of the connectedness dimension in their mathematics

lessons. Some of them indicated the words that pointed directly to the connectedness dimension. In one of the focus group interviews, one of the students said:

In mathematics lessons, the teacher associated mathematics with information in science subject, geography and from Islamic education such as applying mathematics operations to determine the amount of “Zakat” {Zakat: is an Islamic principle of Muslims: the annual contribution to poor people} (Focus groups’ interviews, Grade 10, 18/04/2012)

The previous comment indicated that students identified some of the teaching practices that aimed to indicate the relation between mathematics and other school subjects such as science, geography and Islamic studies. That may help students to identify the value of learning mathematics in learning other school subjects. When students identified the importance of mathematics, their motivation to learn mathematics may improve. The students were able to illustrate some examples from mathematics lessons that used some information from other subjects. One of the students who were interviewed in a focus group at the end of semester two talked about learning trigonometric ratios in grade 10 using their knowledge of geography. She said:

Learning trigonometric ratios in mathematics lessons by using the map of Africa in geography, which was in the form of a right-angled triangle, helped us to provide different ideas to solve trigonometric problems. (Focus group interviews, Grade 10, 18/04/2012)

The previous comments pointed to the lessons that have application in the other school subjects that they experienced in their mathematics lessons. Students’ perceptions also highlighted the importance of making connection to what they learned in mathematics lessons to what they experienced in real life situations. One of the students from grade 10 noted that connectedness helped them to remember what they learnt while abstract concepts did not.

If we go back to the abstract concepts and the directed examples and exercises, we will go back to the technique of boredom. Abstract concepts may be useful in clarifying and understanding of mathematics facts but

usefulness of these facts can only be achieved when they are linked with reality. (Focus groups' interviews, Grade 10, 18/04/2012)

Drawing on the previous students' perceptions, they expressed positive reactions towards learning mathematics as they were enjoying classroom practices and identifying the value of learning mathematics. That may have an impact on their learning and engagement. The data also indicated that students' perceptions about their teachers' practices were an indication of using *higher order thinking* operations. Some of students also indicated that they applied these kinds of activities, which are meaningful and enjoyable, as they noted, for the first time. One student noted:

We have implemented activities outside mathematics classroom to measure the angle of the flagpole; it was a beautiful experience, as we measured by ourselves and we discussed different views of whether it was possible to measure the angle, which school flag formed with the yard without the use of measurement tools. It was the first experience for us in taking measurements outside the classroom; we love discovery. For me I am feeling better as I progressed from the very good level to excellent level in short tests. (Focus groups' interviews, Grade 10, 18/04/2012)

The previous comment indicated the new and enjoyable learning activities that were conducted outside classroom. Offering opportunities for students to take the responsibility for their learning may help their engagement in mathematics lessons. The students may be engaged when they applied different techniques in group-activities such as measurement, discussion and decision-making. The student also indicated the change in her performance in class assessments. In the same focus group, students commented on the benefits of applying learning activities that helped them to construct their own meanings of the mathematics ideas that they learned:

Now we can discover and we can deduce mathematical rules and definitions by ourselves, none that we have studied were forgotten because it linked to our life experiences. (Focus groups' interviews, Grade 7, 24/04/2012)

To sum up, the overall data of students' perceptions showed that students noticed a change in their teachers' practices in mathematics lessons. In particular, students' perceptions showed indications of a change in classroom participation and classroom

discussions that they experienced. Students' previous perceptions also pointed to their willingness and desire to perform different significant roles in classroom activities and discussions. Some of the students' comments pointed directly to teachers' implementations of the elements of Productive Pedagogies.

4.5 Summary of Results

The overall results indicated that Omani mathematics teachers who participated in this study identified some significant benefits that were gained through applying the Productive Pedagogies framework. Developing new knowledge and understanding of Productive Pedagogies was important to teachers. The emerging new knowledge and understanding of Productive Pedagogies was developed gradually during semester two. Teachers' advanced attempts to applying Productive Pedagogies in their mathematics lessons developed significantly compared to their initial attempts. The improvement of teachers' understanding and ability to implement Productive Pedagogies developed during the three stages of improvement: *preparation*, *developing* and *consolidation*. The teachers grounded their understanding and ability of implementation by preparing their lessons during their participation in the professional development program and during their initial attempts of implementation in their classrooms. Teachers then developed some good understanding of Productive Pedagogies during the stage of developing and then they were able to identify some progress in their attempts at implementing Productive Pedagogies during the stage of consolidation. In their mathematics classroom, they illustrated some good attempts at implementing Productive Pedagogies.

The data analysis also indicated that teachers showed positive views towards the value of Productive Pedagogies in their mathematics lessons. Developing new efficient teaching and learning experiences and encouraging students' learning and engagement were key benefits that were acquired during the implementation of Productive Pedagogies. Providing opportunities for teachers' professionalization was the main outcome of their experience of applying Productive Pedagogies in their mathematics lessons and in their conducting of group discussions of lesson analysis and for preparing common teaching and learning materials.

While the data indicated that teachers identified the value of Productive Pedagogies, the results also identified some key challenging factors that limited teachers' best

efforts to implement Productive Pedagogies. These challenges related to teachers' initial understanding and their ability to apply Productive Pedagogies. These challenges differed in their occurrence, strength and their impact during the actual implementation of the Productive Pedagogies framework. Other challenges related to the context factors that influenced teachers' professional development such as time constraints, crowded mathematic curriculum and the lack of quality of the general school professional development programs. Some of these challenges can be overcome while others needed more support and formal decisions. Based on these challenges, teachers suggested some recommendations for better application of Productive Pedagogies such as offering opportunities for the professional development programs that utilised the conceptions of the framework. They also highlighted the importance of spreading the idea of Productive Pedagogies in the Omani educational context.

The data in this chapter also considered students' voice about the teaching and learning practices in their mathematics lessons in semester two. The overall students' perceptions indicated that they were engaged in some useful activities that encouraged their classroom practices and strengthened their motivation to learn mathematics. Students' perceptions pointed to aspects of applying the elements of Productive Pedagogies.

CHAPTER FIVE

DISCUSSION AND CONCLUSIONS

Enhancing quality pedagogy has become a focus of educational reforms in both school teaching and teacher education (Gore et al., 2004; Hayes et al., 2006). In the light of this focus, valuing the quality of teachers' pedagogical practices, their knowledge and ongoing learning to improve students' outcomes has become a key consideration in reforming educational practices (Lingard et al., 2003). In particular, supporting teachers to improve their teaching practices and to ensure a positive impact on students' learning and the school community has become "a key strategy in science and mathematics educational reform movements" (Loucks-Horsley et al., 2003, p. 44). Accordingly, teacher professional development offers opportunities for teachers to build an understanding of quality classroom learning and teaching practices to enhance their pedagogical knowledge and skills and to examine their teaching practices critically (Loucks-Horsley et al., 2003).

Improving the quality of educational outcomes is one of the challenges facing the Ministry of Education in the Sultanate of Oman (Ministry of Education (Oman), 2012, p. 233). The Ministry of Education, in its national report entitled *Education in Oman: The Drive for Quality* (2012, p. 25) pointed to the lack of emphasis in pre-service teacher education courses on improving their pedagogical skills as well as the limited practical training provided to pre-service teachers. Equally important, the in-service teacher professional programs were deemed to be theoretical and did not align with teachers' needs. Therefore, the Ministry of Education highlighted the importance of giving appropriate priority for enhancing the pedagogical skills and the practices of teachers within teacher education and school professional development programs.

Productive Pedagogies provides a comprehensive and multi-dimensional framework with potential for enhancing the quality of teaching and the quality of teacher education (Gore et al., 2004). It can serve as a teacher professional development program that assists teachers to improve intellectual and social outcomes of students (Education Queensland, 2010a). The main objective for conducting this study was

for promoting quality teaching of Omani mathematics teachers using Productive Pedagogies as a framework to engender quality teaching and promote teachers' reflection on their practice. The framework was introduced to a group of teachers and its value on enhancing their teaching quality was examined. In particular, this study aimed firstly to investigate the development of teachers' understanding and their ability of implementing Productive Pedagogies. Secondly, the obstacles they experienced that limited teachers' ability to implement the framework were studied. Thirdly, students' perceptions on the change in pedagogy in their mathematics classrooms were investigated. Finally, the appropriateness of implementing the framework in the educational system in Oman was examined.

For addressing these aims, three phases in the research (preparation, implementation and dissemination) were designed. A teacher professional development program was conducted in the preparation phase to introduce Productive Pedagogies to the participants. Six Omani mathematics teachers who taught grades 7 and 10 from two schools who participated in this study attended the professional development program for five days (25 hours). In the implementation phase, the applying of Productive Pedagogies framework by teachers was explored. In this phase, classroom observations and group discussions on lesson analysis and collaborative preparation were conducted regularly during six continuous cycles (each cycle lasting about two school weeks). In the dissemination phase, opportunities were offered to the participating teachers in their school-groups to make presentations about their experience of applying Productive Pedagogies at a school-based professional development activity attended by other teachers in the region. In these professional development sessions, the researcher and the teachers from each participating school made 30 to 45 minute oral presentations that were followed by an open discussion between the teachers and the audience.

In the research reported here, qualitative and quantitative data collection methods were implemented. Different kinds of qualitative methods were used such as observation of participants, teacher group interviews, individual teacher interviews, student focus group interviews, maintaining a researcher's diary and observations of teachers' presentations on their experiences. This research utilized the feature of grounded theory that involved recurrent processes of data collection and data analysis. Four analytical procedures of qualitative data were employed using NVivo:

conceptualizing the data, categorizing, developing interrelations between categories and subcategories, and building a descriptive overview of the data. The quantitative data were obtained from 35 coded classroom observations by the researcher and the participating teachers using the *Productive Pedagogies Classroom Observation Manual* that was developed in the Queensland's School Reform Longitudinal Study (QSRLS). The data were analysed using the descriptive statistics procedures in the Statistical Package for the Social Sciences program (SPSS).

The findings of this study will be discussed in this chapter within four sections. Section 5.1 discusses the findings related to the specific research aims. In this section, even though the headings are rose from the research questions, the sub-headings highlight the new findings from the study. Section 5.2 discusses limitations of this study and section 5.3 sets out the implications of the findings and suggests directions for future research. Section 5.4 presents the overall conclusion of the chapter.

5.1 Discussion of the Major Research Findings

This section discusses the findings of this study that aimed to investigate the introduction of Productive Pedagogies to a group of Omani mathematics teachers as a framework for quality teaching and reflection by teachers on their practice. Based on the particular research questions posited earlier, the following themes are discussed:

- The development of understanding of, and ability in, implementing Productive Pedagogies;
- The benefits of implementing Productive Pedagogies by mathematics teachers;
- The challenges faced in the implementation of Productive Pedagogies;
- The appropriateness of implementing Productive Pedagogies framework in the educational system of Oman and
- Students' perceptions on the change in pedagogy in their mathematics classrooms.

5.1.1 The development of understanding of, and ability in, implementing Productive Pedagogies

Teachers' learning as a process of growth and change is a gradual process (Loucks-Horsley et al., 2003). In the work with the teachers who were involved in this study, there was evidence of teachers' development in their understanding and ability to implement Productive Pedagogies in their mathematics classrooms. The development of teachers' understanding and ability of implementation in their teaching practices reflected their attempts to identify the direction of the potential change, to plan for the change and to evaluate their implementation. The teachers recognized the value of Productive Pedagogies as a guide to planning for teaching, as a framework for enhancing the quality of teaching and learning practices and as a language of reflection. Teachers' attempts in applying the framework, as the findings illustrated in the previous chapter, reflected their desire to introduce the framework in their teaching. They also reflected their recognition of its positive effect on enhancing their teaching practices. The development of their understanding and ability of implementation was represented in the continuing stages of learning and growth: *preparation*, *developing* and *consolidation*. These stages by which teachers developed their new knowledge reflected the conditions that supported and promoted that development and the efforts from the researcher and the participating teachers to facilitate teachers' understanding and their ability of implementation. They reflected how teachers reconstructed their beliefs and incorporated the new knowledge of Productive Pedagogies to their existing teaching beliefs and practices towards achieving quality teaching practices. Loucks-Horsley et al. (2003) argued that teachers' learning occurs in a process of change in which new ideas are incorporated into existing ideas to construct new knowledge. Guskey (2002) indicated that learning to use new sustained practices should be seen as a process not as an event. It involves continued support that allows teachers to be engaged in the gradual process of implementation requiring persistent and stimulating efforts to bring about changes.

In the initial stage of learning (*preparation*), teachers critically reflected on the new knowledge introduced by the framework and compared it within their existing teaching knowledge, their beliefs and practices. One of the ways of facilitating teachers learning during this stage depended on investigating teachers' previous teaching experiences, perceptions and values that helped them becoming aware of

practices that were effective. In the first sessions of the professional development program that was conducted by the researcher to introduce the framework, the discussions between the participating teachers and the researcher concentrated on: the obstacles that teachers faced in their previous years of teaching, their beliefs about the best practices in mathematics teaching and learning, their expectations about students learning and engagement. During these sessions, the framework was used as a resource of the ideas that direct teachers' attention towards the practices in classrooms, to evaluate their teaching and to envisage improvements in their teaching practices. Avalos (2010) showed that teacher professional growth is a complex process that requires restructuring beliefs and building new understandings.

Then, the teachers' journey was supported by their attempts of translating the new knowledge into actions of preparing, decision-making, practicing, reflecting, and sharing their experiences with others. During the stage of *developing*, the translation and practicing of the new knowledge was supported through their work in groups for analysing their reflections and for co-planning of teaching strategies. That helped the practicing teachers to increase their understanding and to improve their skills to adapt to the changes in their teaching practices. As mentioned by Loucks-Horsley et al. (2003), teacher learning, individually and cooperatively, is a demanding process that is a part of their intellectual and emotional involvement. Teacher learning also requires them to have the capacity and willingness to challenge their present teaching practices and develop appropriate alternatives for improvement or change. During this stage of development, teachers were encouraged to reflect on their teaching practices in order to achieve their desire to change by introducing Productive Pedagogies in their mathematics teaching. Including teachers in classroom observations and group discussions of lesson analysis provided opportunities for them to learn from others and to share their ideas and reflections. Loucks-Horsley et al. (2003) argued that building teachers' knowledge requires opportunities for case discussions and professional dialogue. During this stage of development, the four dimensions of Productive Pedagogies have provided support to teachers to question if and how their lessons have demonstrated the pedagogical elements of these dimensions that promote high quality outcomes. In their reflection on the observed classroom practices, teachers analysed the classroom practices and made comments with illustrations. Such reflections indicated teachers' use of the language of

Productive Pedagogies in their analysis and discussion. Reflections on their observations of the teaching practice improved as their discussion was not only concentrated on the teaching actions but also went beyond the issues related to the requirements of effective students' learning.

Finally, teachers' change in their beliefs and desires during the first stage of *preparation* and their attempts to adapt their teaching practices to the Productive Pedagogies dimensions during the stage of *developing* assisted in improving their ability of implementation. In the dissemination phase, teachers' presentation of their experiences in implementing Productive Pedagogies in their mathematics classroom to mathematics teachers from other schools encouraged the latter to question their beliefs and to examine the quality of their teaching practices. Gallimore and Stigler (2003, p. 29) suggested that "teachers defining, presenting and communicating their new experiences can in fact support a change in the teaching community" by the sharing of teachers' experiences with other teachers with the purpose of engendering discussion, refutation and improvement of their new experiences.

In Summary, the use of Productive Pedagogies as a framework for developing teachers' understanding to engendering improvement in teaching practices was a gradual and demanding process. During the different stages of development, the participating teachers were continuously involved in activities to facilitate their knowing, understanding, preparing, trying, evaluating and sharing. In order to enable teachers to implement their new experiences, they need to be supported by providing them with opportunities to examine their practice critically and to develop their new knowledge. As mentioned by Loucks-Horsley et al. (2003), in offering professional support for change and growth of mathematics teachers, there is need to consider the expert knowledge base of the participants and the process of change in which the new knowledge is constructed by providing diverse feedback through direct observations, supportive interactions and expert input.

5.1.2 The benefits of implementing Productive Pedagogies by mathematics teachers

Three main benefits of implementing Productive Pedagogies by mathematics teachers were identified in this research to be 1) an improvement in teaching practices in which participating teachers become facilitators of learning, 2) the use of

dimensions of Productive Pedagogies as a guide for quality teaching, and 3) teachers' engagement in professional development opportunities. Each of these areas is discussed in more details in the sections that follow.

Improvement in teaching practices

One of the major aspects of the change in teachers' practices is the change from traditional teaching practices that focused on transmitting mathematics knowledge to the practices that supported the role of teacher as a facilitator of students' learning. In this facilitator role, teachers: a) recognised the necessity of students having a significant role in their own learning, b) applied student-centred approaches, and c) developed positive expectations for their students' learning. The findings indicated teachers' attempts to apply teaching methods that encouraged constructing knowledge through active engagement. Regardless of the limited attempts to implement some of the pedagogical elements of Productive Pedagogies by the mathematics teachers that will be discussed below, the findings indicated teachers' attempts to encourage students to provide explanations, make generalizations and to discuss interrelationships among mathematics concepts by applying the elements of intellectual quality dimension such as *higher order thinking* and *substantive conversations*. The participating teachers tended to focus on learning activities that placed much more responsibility on the students and on the learning situations and activities that required students' collaboration and developed students' ability in problem solving. The findings indicated efforts by teachers to conduct group activities that helped students to share their ideas, discuss different ways of solutions and explain their findings to the rest of the class. Productive Pedagogies enabled the teachers to place a greater level of emphasis on the provision of learning opportunities that encouraged students to extend their mathematical constructions and understanding. While these opportunities were related to the kinds of outcomes that quality teaching and learning aim to produce, practicing these opportunities in the classroom was the important outcome. Alsharif (2011) showed that the Productive Pedagogies framework could assist mathematics teachers to achieve the shift from the traditional teacher-centred approaches to more student-centred learning methods.

Mathematics teachers also attempted to provide these learning opportunities in a supportive learning environment that were both effective and motivational. These attempts have encouraged teachers to rethink the appropriate contexts for quality classroom activities and tasks that were centred on the learners. The findings indicated that teachers recognized the effectiveness of the Productive Pedagogies framework and its positive impact on students' interaction and participation in mathematics lessons. In most of the observed lessons, most students were engaged and contributed especially when classroom discussions were conducted as group activities. One of the teachers pointed to the change in her teaching practices towards student-centred approaches and said:

Previously we did not pay enough attention to deepen higher levels of thinking among students and to consider that the student is the focus of our teaching. We started giving more interest to the role of the student to discover, explain and draw conclusions about some of the mathematical facts and ideas. (Teachers' interview, 24/04/2012)

Teachers' initial attitudes towards the students who had disengaged themselves in mathematics classrooms have changed. The findings identified an improvement in teachers' expectations towards low achieving students. They attempted to help these students by providing them with learning activities that offered many opportunities for classroom participation and interaction. Strategies such as problem solving, collaborative teaching approaches and narrative approaches offered opportunities for students to experience effective social interactions, knowledge construction and enjoyable classroom discussion. Chinnappan (2008, p. 185) stated that "in classrooms where teachers support students to talk, the higher level of input from learners during their critical evaluation of mathematical concepts would help them reflect and reconstruct new understandings". Klem and Connell (2004) suggested that supportive learning environments in which teachers' expectations towards students are high, strong and reasonable are more likely to result in students' engagement in their lessons at school.

The findings also showed that applying the connectedness dimension met with great interest by teachers as their application of the pedagogical elements of this dimension in their mathematics lessons resulted in a significant and direct change in students'

participation and interactions as well as in improving their motivation. The findings showed that the teaching practices of the teachers exhibited some degree of connectedness by providing students with opportunities to make connections between mathematics and other subjects such as geography and to add value to the lessons by posing some real world problems. Researchers suggested that students' engagement and learning of mathematics can be improved when teachers engage in teaching and learning practices that focus on "conceptualizing and creating meaning and relevance" (Singh, Granville & Dika, 2002, p. 330). Muijs and Reynolds (2011) proposed that teachers who believe in the importance of learning to be interactive and relevant, encourages students to: a) use realistic problems, b) process a lot of their prior knowledge and c) explore more efficient strategies to achieve better understanding.

Guiding teachers' preparation

There was evidence from the findings that mathematics teachers valued Productive Pedagogies as a framework that guides their preparation of mathematics lessons and informs the underlying basis of future development in their teaching practices. Teachers' use of Productive Pedagogies to guide their decisions about the aims, learning activities and teaching strategies of their lessons helped them to focus explicitly on the potential of the four core dimensions of their teaching practices. These findings are consistent with the findings of other research studies that examined the value of Productive Pedagogies on enhancing teachers' practices. Alsharif (2011) pointed to the value of Productive Pedagogies as an overall guide for successful practices from lesson planning to implementation by pre-service mathematics teachers. Gore et al. (2004) cited the value of the dimensions of Productive Pedagogies as a guide for teachers' preparation of quality teaching. Chinnappan (2008) emphasised that Productive Pedagogies do provide a useful framework to guide teachers' thinking about the different ways that enrich the quality of learners' understanding of mathematics in the classroom and beyond. In this study, the participating teachers used the Productive Pedagogies dimensions to guide their preparation of whole mathematics units individually or as in groups. They built on Productive Pedagogies dimensions to plan for the lessons for each mathematics unit before teaching it. Teachers working in groups to plan teaching activities in different mathematics units such as Algebra and Geometry helped them

to bear in mind the dimensions of Productive Pedagogies and recognise the possible ways of implementing them in different content areas.

Offering opportunities for professional growth

Providing the teachers with opportunities to reflect on their mathematics lessons and to examine students' engagement was another benefit identified in this study. This is in line with the professionalization of teachers (Atweh, 2004). In this study, applying Productive Pedagogies as a new learning experience for mathematics teachers and a new practice for finding meaning of quality pedagogical actions displayed a continuous process of engagement, discussion and reflection. To develop improvement in implementation, teachers used directed observations and conversations about the practices of their classroom teaching that aimed to apply the dimensions of Productive Pedagogies. Offering opportunities for collaboration and reflective teaching was one of the new experiences that helped teachers to work collaboratively in examining, planning and preparing teaching activities to strengthen their ability of implementation. Loucks-Horsley et al. (2003) claimed that developing new knowledge of mathematics teachers and investigating their professional growth require engaging teachers in the continuous process of engagement, discussion and reflection.

While the Productive Pedagogies framework involves a unique attention to the quality of classroom practices, it also has potential for use as a language by mathematics teachers with which to construct productive conversations about their teaching practices. When teachers have opportunities to challenge, analyse and re-examine their teaching and learning thoughts and practices, they were able to reflect on their teaching practices. The research literature supports the value of the framework to provide opportunities for reflection and as a common language of professional dialogue. Zyngier (2005) indicated that one of strengths of Productive Pedagogies is its competence as a language for teachers in analysing, discussing and improving their teaching practices. Atweh (2004) suggested that Productive Pedagogies can be used as a teacher professional development considering its ability for providing critical feedback for improvement and its language for conducting substantive conversations for promoting quality teaching and learning. As stated by Chinnappan (2008), teachers could utilise the dimensions of the framework to

evaluate the learning experiences offered in the mathematics classroom to provide them with richer ideas and strategies that help to engender higher levels of understanding of mathematics among learners.

5.1.3 Challenges experienced by mathematics teachers in the implementation of Productive Pedagogies

The findings of this research showed that teachers' ability to implement Productive Pedagogies involved some challenges. One of the challenges is related to teachers' limited implementation of some of the pedagogical elements of Productive Pedagogies. Teachers' limited experience in the implementation of the pedagogical elements was well illustrated in the dimensions of working and valuing difference, and in particular, the elements of *active citizenship*, *narrative* and *cultural knowledge*. Another challenge for teachers arises from the influence of their previous traditions of teaching practices and to the kinds of teaching strategies that they designed to maximize the aspects of Productive Pedagogies framework in their mathematics classes, in particular, the elements of *metalanguage* and *connectedness to the world beyond the classroom*. Lastly, contextual challenges that are associated with questioning the appropriateness of the framework to the Omani educational context and with the inadequate support in school communities are also identified as a challenge to implementation. These challenges will be discussed in turn.

Challenges in implementing the pedagogical elements of working and valuing difference dimension

Perhaps of the four dimensions of Productive Pedagogies, mathematics teachers in the study found it easiest to implement the supportive classroom environment dimension. This finding is consistent with the Gore et al.' study (2004) that demonstrated that teachers are better in producing a supportive classroom environment than they are at producing the pedagogical elements of the other dimensions of intellectual quality, connectedness and working and valuing difference. The pedagogical elements of this dimension were the closest to the traditional teaching practices of the participants and the professional values of the education system. Loucks-Horsley et al. (2003) suggested that when teachers draw on their teaching experiences to make useful connections between their existing ideas and the new ones, they will be better able to apply the new ideas. On the other hand,

some other elements that are problematic for mathematics teachers are the pedagogical elements of working and valuing difference dimension. There is some debate in the research literature about this dimension; some educators have questioned the empirical evidence and the need for more refinement to the element (Mills et al., 2009). Lingard (2007) who was part of the QRSLs research team argued that the absence of this dimension might reflect critical questions about the observed classroom levels of the elements of the dimension.

Teachers' difficulty in implementing this particular dimension is not unique to these teachers. For example, this dimension does not appear in the NSW model of Quality Teaching which is based on the Authentic Pedagogy and Productive Pedagogies framework. Within this model, the dimension of working and valuing difference is excluded and some of its elements of *narrative*, *cultural knowledge* and *inclusivity* were included under the dimension of *significance* (NSW Department of Education and Training, 2003). Regarding this difference, Mills et al. (2009) suggested that lack of the empirical evidence of some of the elements of working and valuing difference such as *group identity*, *active citizenship* and *narrative* did not warrant the need to omit them but it highlights the need for further enhancement of these elements to improve their focus. They identified that while the elements of *active citizenship* and *group identity* are valid in all curriculum areas, they may not be obvious in the content of the mathematics curriculum. In mathematics education, these concepts are not highly stressed in the mainstream literature.

Challenges in implementing the pedagogical element of active citizenship

While the mathematics participating teachers in this study identified the importance of the pedagogical element of *active citizenship* in preparing learners to be connected to and be more productive in their society, there was no evidence that their classroom practices were directed towards that preparation. The difficulty of implementing the pedagogical element of *active citizenship* may be related to the influence of previous teaching practices that did not reflect democratic practices within the classroom and school contexts. Even though the teachers recognised the importance of students' participation in the democratic practices, it was still difficult for them to transform that recognition into a real practice within their mathematics classrooms and to prepare learning activities that promote the usefulness of public participation in

mathematics classes. Further, the literature in mathematics education does not focus much on this point. Chinnappan (2008) pointed out that while there is a current consideration about preparing learners to be more productive in a globalized world, less has been illustrated about the quality of mathematics knowledge and skills that learners could construct in their mathematics classrooms that would create active citizenship. D'Ambrosio, (2010) suggested that many of the classroom practices do little to provide opportunities for students to learn the knowledge and skills that prepare them to function successfully in their new world as many of the mathematics teachers do not have deep understanding of the connection between mathematics learning and culture. Within the Omani context, the study of Al Kharusi (2011) revealed that one of the challenges that limited civic participation in Omani school practices was the lack of classroom practices that were exposed to issues and concerns in the wider society. In addition, the teachers lacked experiences that created democratic practices that involved engaging teaching approaches. Chinnappan (2008, p. 190) argued that the challenge for mathematics teachers is "how to reorganize school mathematics concepts and conventions in ways that help learners in the new globalized world". As mentioned by Atweh and Brady (2009), mathematics can play a crucial role in preparing students for more productive roles in their society. However, Atweh and Brady cautioned that achieving this role faces challenges related to: a) the traditional dominant view that learning school mathematics is needed for increasing admission of students to higher education and for pursuing careers in mathematics, b) the rapid change of the nature of mathematics in society and c) the complexity faced by teachers to identify and design learning activities that support the usefulness of mathematics knowledge for application in everyday life and for democratic participation.

Challenges in implementing the pedagogical elements of narrative

While most teachers indicated the usefulness of *narrative* element in their mathematics classrooms, their attempts were artificial. Teachers' attempts to apply *narrative* element in their mathematics lessons did not reflect an understanding of the element. For example, in some of the observed lessons some of the processes and content of mathematics lessons were storytelling. Some of the lessons were introduced using a historical story that reflected some indications to Islamic and Arabic values and some of the practices used short stories that added more

enjoyment to the lessons. However, the effective application of these *narrative* styles in the teaching was limited in content and practice. The findings showed that teachers used short stories at the beginning of some lessons as means to increase students' interest in the topic. However, these stories did not relate to the content of the lesson or to the main mathematics concepts and ideas. This limitation points to insufficient experiences of the participating teachers to apply the *narrative* style in their mathematics lessons. Similarly, the literature in mathematics education does not deal much with this element.

The use of *narrative* in lessons is identified in the Productive Pedagogies Classroom Observation Manual as the application of “personal stories, biographies, historical accounts, literary and cultural texts” (Education Queensland, 2010a, p. 22). Applying *narrative* as a pedagogical tool in mathematics classes is aimed at the development of coherent meaning and understanding rather than reproduction of knowledge through constructing and telling stories. Krummheuer (2000) suggested that some of the aspects of narrative are most relevant to mathematics classroom interactions when the classroom activities encourage students to become aware of the social conditions of mathematics learning based on their cultural perspectives. The evolving co-operative and interactive learning processes that result offer opportunities for creative and productive writing. In her articulation of narrative in mathematics classroom activities, Burton (1999) argued that the *narrative* is rare in the mathematics classroom as it is challenging in the teaching and learning of mathematics. She mentioned that nurturing requires telling and communicating stories in a convenient form, maintaining the learners' self-perceptions and cultural beliefs to ensure that the strength and enthusiasm of the classroom learning community is sustained. Mills et al. (2009) indicated that the limitations of applying the *narrative* element in mathematics classrooms are related to: a) the complexity of the relationship between language and narrative approaches, b) the differences in the styles of storytelling and oral traditions within particular societies and c) the limited understanding of the differences between the different narrative approaches.

Challenges in implementing the pedagogical element of cultural knowledge

The Challenges in implementing the pedagogical element of *cultural knowledge* may be related to teachers' initial thinking regarding this element. They thought that

cultural knowledge is applied in non-dominant cultures. Moreover, in mathematics education, especially in Oman, there may be a limited attempt to include the talk about *cultural knowledge* and so it was novel to teachers. The influence of teachers' initial reactions towards applying Productive Pedagogies may explain their limited attempts to implement the *cultural knowledge* element in their teaching practices. Their initial scepticism about the relevance of the element of *cultural knowledge* to the Omani school context in general, and to mathematics teaching in particular, may be one of the reasons that prevented the mathematics teachers from applying it in their mathematics teaching. In the Productive Pedagogies framework, the *cultural knowledge* element requires valuing non-dominant cultures and different social characterisations in the same classroom. Teachers may not have considered this as a relevant feature. However, the idea behind the importance of this element is that some aspects of classroom pedagogy can exclude some students regarding some cultural events. When teachers pay attention to and understand the patterns of differences between their students, their pedagogies become more accessible and equitable. The teachers who participated in this study were under the impression that the study was conducted in the context of a homogeneous culture; the teachers raised questions about the relevance and the appropriateness of the pedagogical element of *cultural knowledge* to the Omani educational context. Henniger (2004) clarified that the differences among students has serious implications for teaching. The interactions in the classroom can be influenced by four main factors: 1) cultural, racial and ethnic diversity, 2) family diversity, 3) diverse student abilities and 4) gender diversity. For example, the difference in: a) the personal and social experiences and problems the students encounter, b) the kinds of family expectations and support that they receive and c) the motivational factors that influence students learning, are all related in one way or another to the differences in cultural and social heritages. Moreover, while the participating teachers did not have students from minority religious or language groups in their classes, not all Omani schools reflected a significant level of homogeneity. In fact, many schools, in particular urban schools, reflected some aspects of diversity. Some schools have students from other countries with different values, beliefs and actions. A segment of the Omani population has its origins in Africa or Persia (Ministry of Information (Oman), 2002). Though Arabic is used for education, official purposes and by the media, there are several regional dialects. Some of the students especially in the capital –Muscat– are from families

with one or both parents who are native speakers of different dialects. The ethnic Baluchi population in Muscat speaks Baluchi, derived from ancient Indo- Iranian language. Many inhabitants in southern coastal towns in Oman speak a Dhofari dialect. Some citizens of the isolated peninsula in the Musandem Governorate speak Kumzari, which is one of the south-western Iranian languages related to Luri. Swahili is also widely spoken in the country due to the historical relations between Oman and Zanzibar in Africa. Moreover, while the Omani culture is steeped in the religion of Islam, Oman has its own form of Islam, known as Ibadhism. There are also both Sunni and Shia Muslims in Oman. Omanis are not only accepting of the beliefs of different Muslim divisions; they are also tolerant towards believers of other faiths, who are allowed to practise their religion in churches and temples (Ministry of Information (Oman), 2002). Thus, as the experience of differences and disadvantages varies in different communities, the implementation of the *cultural knowledge* element may help prepare teachers to successfully educate their students to live as citizens in a multicultural globalized world. The aim of education is to prepare students to live and work in the whole country and in a globalised world, not just in the local context. Cole (1984, p. 153) argued that “preparing students for life in a nation and the world that are characterized by cultural diversity”, assists in promoting an acceptance and understanding of others; education equips them with cultural awareness and prepares them to interact positively with each other in tandem with global changes of the future.

Challenges in implementing the pedagogical element of metalanguage

Teachers’ ability to implement the element of *metalanguage*, the pedagogical element of intellectual quality dimension, in their mathematics classroom practices was limited. The findings showed that the teachers were not sure of the importance of applying the *metalanguage* element in their mathematics lessons. This view may be related to their initial teaching tradition that considered *metalanguage* as being concerned with the development of the language of mathematics itself and thus was outside the usual focus of school mathematics. Also, in this context it is worthwhile to mention that *metalanguage* is not a construct that is dealt with in the literature on mathematics language. However, some of the literature points to the importance of recognizing the role of *metalanguage* in finding new ways of constructing and presenting mathematics knowledge. Schleppegrell (2010) suggested that

mathematics teachers need to be aware of explicating about language features by providing students with opportunities to talk about the mathematical knowledge that they have developed and modelling ways of responding to students' clarifications, descriptions and questions. Rittenhouse (1998) argued that commenting on the mathematical conversation and building up a language for talking about language by story, imagery and questioning are some of the significant vehicles that help students to think about the talk they use and how their talk is able to support their learning.

During their implementation of Productive Pedagogies, mathematics teachers questioned the role of language in mathematics teaching and learning. Mathematics as any school subject is constructed through language but the challenge is that mathematics knowledge incorporates many linguistic features. The linguistic features of mathematics knowledge include the usage of everyday vocabulary that has different meanings to those in everyday usage. They also include technical terms particular to mathematics such as the mathematical verbal words, symbolic language and visual representations. These features form the internal organizations of language as a system. However, these linguistic features of mathematics knowledge provide opportunities for teachers for supporting the development of language in constructing mathematics knowledge (Chronaki & Christiansen, 2005; Pirie, 1998; Schleppegrell, 2010). The literature has documented that one of the key challenges in mathematics teaching is to help students to build on their language that has typically arisen from everyday life and has constructed their knowledge of the world, to enable them to advance their learning (Schleppegrell, 2010). Skovsmose (2009) suggested that the translation from natural language description to mathematical language description may include loss of meaning and lead to misunderstanding. Schleppegrell (2007) identified some of the challenges that mathematics teachers face on transforming from ordinary language to technical meanings of mathematical concepts related to: a) the construction of mathematics knowledge in different language organizations such as verbal, symbolic and visual representations, b) the technical vocabulary and grammatical structuring that bring a variety of meanings of mathematical concepts and relations different from the meanings that have developed in the ordinary language of students and c) the use of conjunctions in mathematics word problems that include implied logical meanings and indirect relational structures that influence students' ability to solve them. Menon (1998) commented that the difficulty for some

students when solving some mathematics word problems is a function of not only mathematical content of the problem but also of the language involved for understanding the problem. For example, some of the main challenges in reading and understanding mathematics symbols are: a) the same symbols may have different meanings and representations, b) specific variables may be used in different contexts and c) symbolic-forms, the implicitly of symbols and the placement of ordering of symbols.

Challenges in implementing the pedagogical element of connectedness to the world beyond the classroom

In this research, teachers attempted to apply the element of *connectedness to the world beyond the classroom* by using some applications of mathematics in real life. The participating teachers attempted to make their lessons interesting to students by offering opportunities for the students to recognise the beauty and the usefulness of some mathematics concepts in nature and in real life, for example, in constructing buildings. From the findings of this study, what appeared to be missing from these attempts of implementing this element was the substantive understanding of the use of mathematics in real life and the teaching practices that encouraged learners to make sense of school mathematics beyond the classroom. Atweh and Bland (2005) suggested that while students believe that studying mathematics is important for their future, they fail to see the relevance of the mathematics content that they study to their immediate life, and in particular in understanding their social world. As mentioned by Gutstein (2009), one of the obstacles facing the pedagogical shift to teaching mathematics as a means of changing the world for the better is the historical common vision of mathematics as objective, neutral and logical. The other challenge is that while there are recent reforms that value the role of mathematics in real life (Gutstein, 2009; Stemhagen, 2009), the ways in which teaching how mathematics can alter students' lives seem hidden and even weak in actual mathematics classroom practices (Stemhagen, 2009). Van Zoot and Enyat (1998) believed that when teachers are convinced of the need to change their teaching practices, the greatest challenge then is the ways by which teachers bring that change to reality in the classroom. Torres-Velasquez and Lobo (2004) emphasized that connecting mathematics to real-life experiences and developing a community of learners is one way to support

students' home culture and support social norms such as beliefs, civilizations and artefacts.

Obstacles and contextual influences

Some of the previous limitations in implementing the pedagogical elements of Productive Pedagogies were related to the influence of traditional teaching beliefs and practices. In this study, building a professional learning environment within the school community faced some contextual and structural challenges. First, to move teachers from their past experiences of individual preparation of their lessons to collaborative work in preparing common learning and teaching activities and to engage them in reflective learning requires time, effort and support from the teachers themselves and the school administration. The other challenge is that reflective analysis based on providing critical comments on classroom observations was not a common practice among mathematics teachers. These findings are consistent with previous research studies on Productive Pedagogies. Lingard (2007) reported that some of the school contextual structures that contribute to the limitations of applying some of the elements of Productive Pedagogies are large class size, contemporary political and testing policies, teaching work load, time demand of a crowded curriculum and the considerations of teacher professional learning communities.

Time was illustrated as one of school contextual factors that challenged teachers' best efforts to apply Productive Pedagogies. In this research, two aspects of time constraints challenged the participating teachers. First, the participating teachers and the researcher experienced difficulty in finding common school free time so that they could participate in group discussions of lesson analysis and for planning. Even though we arranged for some time to release teachers for participating in this project, the unexpected continuous changes to the school timetable added further demands on teachers' time.

The second aspect of time constraints was the short time-span of the whole project. The whole project took fourteen weeks including the teacher professional development program and the school based implementation. During the study the researcher met the teachers of each school five to six times every two weeks (two classroom observations and three group discussions for each group). However, building new understandings of teaching and learning, reconstructing the teaching

beliefs and expectations and changing practices does not occur in a short time or in one step. It is a progressive process that requires effective guidance, experimenting and support. This finding is consistent with some of the studies on applying Productive Pedagogies. Lingard et al. (2003) found that insufficient school time that was allocated for professional discussions about the most productive ways for quality teaching in some schools was one of the reasons behind the limited implementation of Productive Pedagogies. In their study, Gore et al. (2004) found that time constraints that were mostly linked to everyday pressures of teaching, were widely cited as a factor that limited teachers' ability to explicitly plan for and apply Productive Pedagogies. Guskey (2002) had reiterated that school professional development programs for reflective meetings and discussion of case studies seem to be undesirable in schools because of the limited time and resources. While the most valuable school time of teachers is spent on teaching in classrooms to engender effective learning, finding ways to use the school time for offering opportunities for reflection and group discussions can also contribute to effective learning. As mentioned by Loucks-Horsley et al. (2003), one of the influences that challenged teachers' professional development is finding the ways to effectively use school time for professional development activities.

The findings of this study also showed that the challenges that teachers faced to find sufficient time for reflection and preparation were related to the extensive mathematics curriculum content that teachers had to cover in a limited period of time. This finding is consistent with the results of the study that was conducted by the Oman Ministry of Education (Oman) (MOE) and the World Bank that showed that "time on task is limited by the relatively short school year. While the official target of a 180-day school year is similar in length to that of many countries, in reality the school year is attenuated by the grade 12 public examinations and by special events. These examinations are held twice a year, resulting in cancelled classes. This has an impact even on junior classes, as cycle-one teachers are used as examination overseers. In effect, the number of actual days that students spend in school can be as low as two-thirds of the expected or officially approved number of days" (Ministry of Education (Oman), 2012, p. 232).

This section concludes with some comments on teachers' attempts to implement Productive Pedagogies in their mathematics classrooms. The Productive Pedagogies

framework guides teachers' attempts to improve their teaching practices and provides a common language of reflection and professional dialogue. Recognising and practicing the value of Productive Pedagogies helped mathematics teachers to change the tradition of their teaching practices that is based on transforming mathematics knowledge and to develop teachers' willingness to bring about that change in their classroom practices. The challenges faced in the implementation have demonstrated that mathematics teachers need continual support for better understanding and implementation of Productive Pedagogies. It has also been demonstrated that developing new understanding and ability of implementation of a new knowledge needs time. Secondly, the limited implementation of some of the pedagogical elements point to the need for a greater focus in the research, as well as pre-service training and professional development of serving teachers on these aspects related to mathematics teaching. What perhaps is needed is to increase the knowledge of teachers and to develop curriculum resources that identify connection with the real world. Also textbooks should provide better examples. Maybe teacher resources and activities that illustrate how these elements may be implemented in the classroom are also needed. Finally, the findings emphasize the role of the school community to support teachers' best efforts to apply Productive Pedagogies as a framework to improve their classroom pedagogies and as a language of professional growth.

5.1.4 The appropriateness of implementing Productive Pedagogies framework in the educational system of Sultanate of Oman

The concept of the Productive pedagogies framework is in line with the main objectives of the Omani education system as well as with the current interest of the Ministry of Education towards quality education in general and quality teaching in particular. Education in Oman (Ministry of Education (Oman), 2004b, p. 29) aims to: a) develop the physical, intellectual, psychological and social outcomes of the learners, b) support learners with objective vision, logical inference and scientific and critical thinking approaches, c) "deal with the explosion of knowledge, rapid advancements in technologies and modern inventions and d) support cultural identity of the Omani citizens". The Ministry of Education (Oman) in its recent report *Education in Oman: The Drive for Quality* that was launched in October, 2012 indicated the need to refocus the Basic Education System on quality, in particular

“teaching quality” (Ministry of Education (Oman), 2012, p. 141). It suggests supporting teachers with appropriate in-service professional development, encouraging teacher-peer collaboration to share ideas and develop good practice and making greater use of the expertise of practicing teachers. Productive Pedagogies as a framework of quality teachers and reflection aims to support the intellectual and social outcomes. The dimensions of Productive Pedagogies framework can assist teachers to inform the design of learning experiences to ensure quality intellectual learning, relevant content, supportive learning environment and creating a sense of community and identity (Education Queensland, 2010b). The Productive Pedagogies framework has been used as reference for teachers to analyse, examine and reflect on their teaching practices as well as a language of dialogue with the community of teachers (Hayes et al., 2006).

The participating teachers in this research recognised the value of the Productive Pedagogies framework. That recognition was illustrated in their ability and attempts to: a) introduce the framework into their teaching practices, b) reflect on their teaching, c) open a professional dialogue based on the dimensions of the framework and d) recognise its benefits on improving students’ engagement and learning. It was also illustrated in their desire to continue applying the framework in the future.

On the other hand, the participating teachers questioned the appropriateness of the pedagogical element of *cultural knowledge* (an element of working and valuing difference dimension) to the Omani educational context as discussed in the previous section. The Productive Pedagogies framework defined the element of *cultural knowledge* as

Cultures are valued when there is explicit valuing of their identity represented in such things as beliefs, languages, practices, and ways of knowing. Valuing all cultural knowledge requires more than one culture being present, and given status, within the curriculum. Cultural groups are distinguished by social characteristics such as gender, ethnicity, race, religion, economic status, or youth. Thus, their valuing means legitimating these cultures for all students, through the inclusion, recognition and transmission of this cultural knowledge. (Education Queensland, 2010a, p. 20)

Based on this definition, the importance of valuing cultural identity is in line with the objectives and philosophy of Omani education that aims to support the elements of Omani cultural identity (Islam and Arabic language), Islam as a faith that stipulates certain values and behaviours and Arabic language as a means of social communication. This definition focuses also on the differences related to the social characteristics of different cultural groups. The participating teachers believed that their classroom context reflected high levels of homogeneity and did not include different group-cultures. Omani culture however, does reflect some aspects of diversity. Though most Omanis are Arabs, the long trading history of Oman has led to an intermingling of Omani of Omani Arabs with other ethnic groups. These ethnic groups still have retained some of their identities; they have their own cultural traditions. They even have their own schools such as Indian schools. Little is known about their educational outcomes, achievement and participation.

A form of structural “gender discrimination and expectations at work in terms of job fields, levels and quality of work” is still one of the factors that have impact in Arabian Gulf Societies (Al-Lamky, 2007, p. 1). Oman is one of the Muslim and Arab countries where the position of women has improved significantly during the last few decades. The growing discernibility and active participation of women in the education sector has spread to other areas of public life including the political arena (Rassekh, 2004). However, the choice of careers to Omani women is influenced by the interactions of cultural, economic and educational forces. While participation of men is in all fields of work, the participation of women is restricted by the social mores in a male dominated society. According to expectations women are able to work mainly in the areas of education and nursing. Moreover, according to Islamic values, the primary role for women is to be a wife and a mother. Thus, some employers believe that this role restricts women’s work productivity (McElwee & Riyami, 2003). Al'Omairi and Amzat (2012, p. 63) stated that “the huge progress that the females have in Oman does not change the old beliefs that society prefers to have males as their Heads of Departments or Deans compared to females. Females hold many high positions in the ministries, but males still dominate the Omani Council. Even though females can look after themselves outside the home, males still believe women's place is the home, and husbands prefer their wives to stay at home to take care of their family. Females contribute to the political development; however, they

do not share the political power as males in Oman”. They suggested that educational policy should re-examine the cultural factors that hinder opening the doors to females to enrol in other specializations and majors in which there are fewer women. The other social characteristics are having their impact on Omani Education outcomes and especially in learners’ participation and achievement. In particular, the size of the gender gap in learning achievement in general and in mathematics in particular, is considerable regarding all national and international assessments in Oman. The Trends in International Mathematics and Science Study (TIMSS) showed that in “In comparison of gender differences in achievement in mathematics in all 48 participating countries in the TIMSS 2007, Oman had the largest gender difference with boys scoring significantly lower than girls in all content areas” (Ministry of Education (Oman), 2012, p. 24). Maintaining the progress of female students as well as opening the door to improve the learning of boys is one of the considerations of the Ministry of Education in the coming years. This research was conducted in schools for girls. The findings showed that applying Productive Pedagogies helped teachers in improving students’ engagement and learning in mathematics classrooms. Productive Pedagogies framework as a framework of quality teaching can be applied for enhancing the quality of students learning by focusing on the pedagogical practices of teachers. Keddie and Mills (2007) drew on the Productive Pedagogies framework to consider some of the key strategies for improving the intellectual and social outcomes of boys. Keddie and Mills suggested that enhancing boys’ educational outcomes can be improved through greater efforts to construct learning environments based on the dimension of Productive Pedagogies that is built around challenging tasks that are socially developed to boys’ interests, providing students with opportunities to experience success in their learning and emphasising on activities that promote their role in public participation. Moreover, participation in international studies such as TIMSS (2007 and 2011, for example) may be useful to provide indications about the quality of educational outcomes in Oman. However, some of the important factors in education take place behind the closed doors of the classroom and there is little evidence available about what happens inside (Ministry of Education (Oman), 2012, p. 138). The idea of applying Productive Pedagogies is on focusing on the pedagogical practice within the classroom and working towards improving its quality. This aim may highlight the necessity to investigate the

classroom practice and examine its effectiveness on providing quality intellectual and social outcomes that are research-based.

5.1.5 Students' perceptions of the change in pedagogy in their mathematics classrooms

Teaching practices in mathematics classrooms can often result in resistance by students in the form of the negative views towards mathematics and its study (Picker & Berry, 2008). Thus, exploring students' perceptions about mathematics and their experience of learning school mathematics can help the teacher to understand their attitudes toward, and their beliefs about the subject. Teachers need to find out what students know, believe, feel and think about learning mathematics. This knowledge informs their decision, planning, classroom activities, tasks and assessments. Picker and Berry (2008) found that teachers' concern about the negative conceptions of mathematics held by their students can play an important role in reforming the classroom pedagogy towards changing students' attitudes towards mathematics. Mills et al. (2009) indicated that student voice around about the classroom practices might provide indications about some issues of pedagogy that support or hinder their learning and provide more understanding of the dimensions of Productive Pedagogies.

In this research, the general outcomes that were identified from students' perceptions of the change in pedagogy in their mathematics classrooms resulted in their concern about the issues of "engagement" and "connecting". The initial reactions of students at the beginning of semester two indicated negative attitudes toward studying mathematics. The findings showed that two of the main initial negative attitudes that were commonly reflected in students' minds were "mathematics is not an enjoyable subject" and "it is not important". Maybe one of the assumptions is that the apparent lack of engagement of the students is related to some attitudinal and affective variables that are related to their previous academic success in mathematics. For example, factors such as students' confidence in learning mathematics, their motivation and their own experiences and expectations of success in mathematics may affect their engagement. Students' comments pointed to lack of their engagement in learning mathematics. As a result of having been involved in learning mathematics subject at school for many years, the students who were interviewed in

this study had formed a view about mathematics and identified some of the factors that shaped their attitudes. Firstly, students viewed mathematics in terms of getting the right answer, as involving memorising formulas and as an opportunity to be successful in higher education. Secondly, students found mathematics lessons often overlapped and they could not identify the differences between them. Thirdly, the students faced difficulty in solving mathematics problems in the final examinations. While these negative conceptions related directly to mathematics as subject, they point to a possible failure that is reflected in many concerns about the teaching and learning practices that those students experienced in their previous learning. The research was conducted in the schools of Basic Education Cycle two (grades 5-10) which is the middle between basic education Cycle one (grades 1-4) and Post Basic Education years (grades 11, 12). Students' negative attitudes towards learning mathematics may be reflected in their lack of interest in mathematics and negative attitudes towards learning mathematics in the early school-grades. Engaging middle-years students in their mathematics classes and motivating them "have attracted serious attention in recent years" (Singh et al., 2002, p. 324). Studies show that learners become more disengaged as they progress from elementary to middle to high school-grade levels (Brewster & Fager, 2000; Klem & Connell, 2004). Morony and Stocks (2005) found that disengagement of some students from learning in mathematics was noted as a significant issue in the middle years of schooling and could be reinforced by the teaching practices that students experienced that lacked connections of the mathematics that students learned with everyday contexts. Singh et al. (2002) maintained that the attitudinal variables such as self-concept, confidence in learning mathematics, mathematics interest and motivation have their impact on academic engagement and achievement in mathematics. Many students in middle school years are able to develop expectations, to deal with abstractions, to develop the propensity to be reflective and to be able to construct their ideas and express them. Most students look for something that interests them, motivates their desire to learn mathematics and allows them to connect and interact with the world (Morony & Stocks, 2005).

Common among students' perceptions was also the fear of mathematics examinations and the difficulty to transfer their mathematical knowledge to other situations that they encounter in their learning of mathematics. While this research

did not focus on finding out how students performed in assessment tasks and how teachers' practices were integrated with the assessment tasks, students' perceptions reflected negative attitudes towards mathematics examinations. The teaching and learning experiences in mathematics lessons may affect success in final examinations. That was consistent with teachers' views about their previous teaching practices that relied on completing exercises provided in mathematics textbooks to prepare students for the final examinations. Students' fear of examinations was also reflected in their high reliance on tests to evaluate their performance in mathematics. It also reflects students need to understand more about the different approaches that are used to evaluate them under the Basic Education System. Under this system, there is an emphasis on continuous assessment by using a wide range of assessment approaches such as quizzes, homework, individual and group-projects and final examinations (Ministry of Education (Oman), 2008). Sometimes teachers tend to play down the importance of the classroom assessment tasks and place more emphasis on preparing students for the final examinations, adding more anxiety on students. Comprehensive assessment of students involves the use of various evaluation instruments including continuous tests, informal measures, student self-reports, parents' comments and progress observing data. Students need to know their performance expectations in clear, detailed and reasonable information. Productive Pedagogies emphasized the importance of involving students with overall frequent and detailed statements regarding their performance in classroom learning tasks, activities and assignments (Education Queensland, 2010a).

The other reasons for students' disengagement may be related to the attitudes that are formed by social conditions; those related to teachers' and parents' expectations and to the influence of mathematics achievement in future career opportunities. Teacher's expectations, especially their expectations towards students as learners of mathematics may be one of the factors that affect students' conceptions and attitudes towards leaning mathematics. In this research, teachers indicated that in their previous years, in their initial expectations of low-achieving students, they tended to engage those students less often than high-achieving students in activities that were of intellectual quality. Moreover, students' comments pointed to the teacher's role and her major responsibility for their learning; they also blamed the teacher for their failure to understand the subject. Parents' expectations also impact students'

motivation for learning school mathematics. The findings showed that students need mathematics because it is required in future careers and to have opportunities for entering universities or colleges. The findings also showed that some of students' attitudes towards their school learning were considered to be the bridge to specific higher educational institutes that parents wanted regardless of their desires. In the Omani context, the continued pressure from parents on females to be restricted to specific careers such as teaching and nursing directs their expectations, engagement and learning. Until recently, some scientific disciplines such as electronic engineering, architectural and space-science disciplines that rely heavily on mathematics were not popular options for females. These views influence the learner's motivation, engagement and expectations. While some of these views change rapidly, still they influenced some of the students. Rassekh (2004) showed that the numbers of registered female students for higher education institutions are in arts and social sciences specializations while male students outnumber women in engineering, commerce, economics and agriculture. The Ministry of Education (Oman) in its recent report *Education in Oman: The Drive for Quality* indicated that "although girls are performing better than boys in the education system, they are underrepresented in the workforce" (Ministry of Education (Oman), 2012, p. 230).

The results from the students' focus group interviews that were conducted at the end of semester two also showed that the students from the participating mathematics classrooms identified some of the teaching practices that motivated them, like giving them opportunities to collaborate, discuss and present and enabled them to have many roles in classroom discussions and activities. While the evidence of improvement in students engagement to and learning of mathematics needs more time to be investigated, the findings showed that the change in teachers' practices by applying Productive Pedagogies have made some changes in students' negative conceptions about mathematics. Students' change of attitudes can be related to specific teacher attempts to implement particular pedagogical elements. For example, students pointed directly to the idea of connectedness and the classroom activities that encouraged higher order thinking. The students also identified some classroom discussions about real-life problems that they discussed in mathematics classrooms and have relevance to the Omani context. They also pointed to the classroom discussions that provided opportunities for freely expressing their views, ideas and

feelings. Assisting teachers to change their beliefs and expectations, and more importantly, their practices may contribute to changing students' disengagement in learning mathematics. Guskey (2002) suggested that the new teaching models or frameworks that are combined with their evidence of improving students' learning are more desirable to be implemented by teachers. The findings also showed that providing attention and practical guidance to teachers by using the Productive Pedagogies framework to enhance the quality of their teaching practices resulted in some improvement in students' engagement in learning mathematics as well as drew the attention of teachers to focusing on developing students' thinking skills that supported students' involvement in mathematics. The Productive Pedagogies framework targets teaching practices that provide students with meaningful experiences that help them to build deep understanding of the knowledge, to transform their meanings and applications, make connections and show enthusiasm and engagement and take responsibility for their learning (Education Queensland, 2010a).

In conclusion, students' initial perceptions reflected negative attitudes towards learning mathematics. Students' disengagement from learning mathematics may be associated in one way or another to: a) attitudinal and motivational factors, b) the types of teaching and learning experiences that they experienced in their previous school-grade levels, c) the expectations from teachers and parents towards students as learners of mathematics and d) the opportunities for future careers. Students' perceptions of the change in pedagogy in their mathematics classrooms at the end of semester two pointed to some improvement; on students' attitudes towards learning mathematics, their participation in mathematics classrooms and their engagement in mathematics lessons. Even students' perceptions gave some indication of change in teachers' practices in their mathematics classrooms. These general findings open directions for future more involved research.

5.2 Limitations of the Study

One of the limitations of the present study is that this study is a case study of small sample size. This study employed qualitative naturalistic classroom observation methods with a small size of female mathematics teachers as well as unstructured individual interviews, students' focus groups and teachers' group discussions. Using

these methods help the research issues to be examined in detail and in depth but the findings of the research cannot be generalized to a larger population of school mathematics teachers. The findings are limited to the context of the schools involved, the classes and grade levels of mathematics content in which this research was conducted.

The other limitation of this study is that quantitative findings of coded classroom observations using the *Productive Pedagogies Classroom Observation Manual* has limitations in providing a true indication of the development of teachers' implementation of Productive Pedagogies during the observation time. Some of the practices that took place in the mathematics classrooms are not captured adequately. That is because the data from the coded observations are heavily dependent on the individual skills of the teachers and more easily influenced by their limited experience in using the *Classroom Observation Manual Instrument*. Teachers need more training in the use of the manual for coding their observations.

Moreover, the appropriateness of the language of the Arabic translated version of the *Productive Pedagogies Classroom Observation Manual* may also be one of the issue in this study. Translating from one language to another language may have an impact on some of the special meanings of the materials included in this manual. In addition, there was no attempt to examine the inter-rater reliability of this scale. Despite these limitations, the quantitative data is aimed to provide some indication of the development of teachers' ability of implementation during the observation time and this data are supported by more in-depth qualitative methods such as consistent classroom observations and individual teachers' interviews as well as teacher-group interviews.

While this study is conducted in approximately one full academic school semester, still this allocated time cannot capture significant changes in students' engagement and learning. As the change needs time, the researcher relied on the findings from classroom observations and focus group interviews as well as teachers conceptions to identify indications of positive change in students' motivation and engagement in learning mathematics. This research does not explicitly investigate students' engagement and achievement in mathematics by direct classroom observations. It relied on students' reports of their engagement.

5.3 Contributions and Implications of the Research

This section draws attention to possible research implications related to the theoretical contributions and implications for future research of this study. It also discusses some of the practical implications of this research. The following subsections will deal with each of these implications of the research in turn.

5.3.1 Research implications

Theoretical contributions

One of the theoretical contributions of the research is that this study is the first study conducted in an Omani Educational context with the specific aim to promote the quality teaching of practicing mathematics teachers. While the concept of Productive Pedagogies has acquired a high interest in research in Australia (Hayes et al. 2006), there are few attempts to introduce the framework into teaching practices within the context of Arab countries. Alsarif's study (2011) was the first attempt to use the Productive Pedagogies framework in Arab countries within a teachers' pre-service unit in mathematics education by examining their understanding and ability of implementation during their field experience. While the design of this study was not aimed to make generalisations, it is my hope that some of the benefits that teachers obtained, the obstacles that they faced, the mathematics lessons in which the teachers attempted to apply Productive Pedagogies may be used as resources for future professional discussions on educational reform in the country, and specifically in mathematics education. The theoretical framework of Productive Pedagogies and experience of applying the framework by the participating teachers could be a useful tool for school professional development efforts to promote the quality teaching within schools.

Moreover, the objectives of this research to enhance quality teaching are in line with the current targets of the Basic Educational System in Oman that aims to enhance the quality of school teaching and learning to improve students' outcomes. This research gave an indication of the possibility of developing new knowledge and understanding of possible ways for enhancing quality teaching from a research base.

While Productive Pedagogies as a framework of quality teaching and reflection has been examined by few research studies in the field of mathematics education and

school teaching, examining the students' voice about the teaching practices that they experienced in mathematics was one of the contributions of this study. Notwithstanding the limitations discussed above, students' perceptions about their teachers' practices may inform their teachers' decisions in planning and practices towards reaching high quality intellectual and social outcomes of their students.

Raising some theoretical questions about the meaning of the pedagogical element of *cultural knowledge* is also one of the theoretical contributions of this study. Further theorizing may be needed regarding the possible ways to identify the concept of difference in different cultural contexts and how Productive Pedagogies can support that. More research may be needed to investigate the role of the dimension of working and valuing difference in the homogenous contexts and how to support teachers to take advantage of the idea of this dimension to ensure that no individual or group of students are excluded in these contexts.

Suggestions for future research

The culture of research in Oman is just evolving, so doing research like this may increase the chances of Omani academics to participate in international dialogue about the quality of teaching and the possible ways and tools that support teachers to enhance their teaching practices towards higher intellectual and social students' outcomes. One in particular is the cultural expectations; to repeat this research in mixed classes in Basic Education Cycle One or in mixed cultural groups classes like in schools of the capital, Muscat. Looking in depth on the *cultural knowledge* in these contexts might be useful. Moreover, the findings showed that the participating female teachers are interested in applying Productive Pedagogies in their classroom practices. This provides a springboard for future researchers to conduct further research in introducing Productive Pedagogies by male teachers and in other school subject areas.

The experience of applying Productive Pedagogies by participants presents a variety of opportunities for future research. In particular, it will hopefully stimulate interest on researching the support available coupled with teachers' needs that is essential for enhancing their teaching practices. While investigating the kinds of support and their effects was not the main objective of the research, the supportiveness of teachers' pedagogies by the researcher or by the group discussions of reflection and

preparation found in this study are necessary to be examined in depth. We know from this study that some structural features of a school can support or limit creating an effective learning community for teachers. Maybe further research is also important on examining the actual school professional practices and conversations that are aimed to support effective pedagogical practices to enhance students' outcomes.

Moreover, it is important to recognise the need for building effective teacher professional learning opportunities through schools. The research can be extended to investigate the role of school principals and school mathematics supervisors to recognise the centrality of teachers' ongoing learning and the importance to offer opportunities for reflections and conversation for teachers and to support the demands of Productive Pedagogies among school teachers in the Omani context.

This study is concentrated on investigating the development of teachers' understanding of Productive Pedagogies and their ability in implementing the framework. In terms of the depth of this study, it does not concentrate on students' learning. However, improving students' learning is the enduring concern of schooling. So, there is a room here for research to concentrate on that issue. Equally important is researching assessment practices because of the coherent link between classroom pedagogies, assessment practices and students' performance and their significant relation to the specific goals of schooling and to the educational outcomes.

5.3.2 Practical implications

One of the practical implications of this research is to use the Productive Pedagogies framework in pre-service teacher preparation as a standard language to talk about in teaching. Moreover, the framework can be a part of a professional development program in the plan that is prepared annually by Omani school mathematics supervisors for training in-service mathematics teachers who are under their supervision. The Directorate General of Human Resources Development department in the Ministry of Education conducts several professional development programs annually at the beginning of each year for the new service teachers to prepare them for teaching in schools. In particular, the stages of professional growth (preparation, developing and consolidation) discussed in this research may provide guidance for

the design of school teacher professional experiences by introducing the Productive Pedagogies framework for in-service teachers. Thus, Productive Pedagogies can be used as a professional program for these teachers in order to be a standard language to prepare for, practice in and reflect on teaching.

In introducing Productive Pedagogies to mathematics teachers in this study during a teacher professional development program, the findings suggest that support is needed to allow for effective use of Productive Pedagogies. Despite some of the contextual obstacles that limited the best efforts of teachers to apply Productive Pedagogies, the confidence of applying Productive Pedagogies by the participants and their recognition of its value highlighted the need for an integrated in-school professional development program to help teachers to implement the teaching practices within the classroom context and to open opportunities for them to build substantive conversations about their teaching practices based on the dimensions of Productive Pedagogies. Schools should offer sufficient time for teacher professional development. The research pointed to the support that is needed to constitute schools as reflective communities of practice. In particular, schools need to base their professional development programs on teachers' needs. There is a need to refocus on the pedagogical practices within classrooms by encouraging reflection and peer collaboration. In particular, there is a need to offer opportunities to provide teachers with regular analysis and feedback on their teaching practices. There is a need to find more practical ways to help teachers to translate their new knowledge into mathematics classroom practice, keeping in mind the obstacles related to their experience of implementation. The findings of this research emphasized the potential value of applying Productive Pedagogies framework in providing a language of analysis and substantive professional conversations for teachers to reflect on their teaching practices. Lingard et al. (2003) suggested that providing sufficient school time that is allocated for professional discussions supports teachers' efforts towards achieving the most productive ways of quality teaching. Loucks-Horsley et al. (2003) believed that building effective professional development programs for teachers requires considering the issues of: a) finding times for continuous learning and reflection, b) building a collaborative professional learning culture, and c) providing policies, resources and structures that make professional development dominant and sustainable.

In addition, this study is related to the teaching practices of mathematics in this context. The overall limited understanding of some of the pedagogical elements by participants and their potential applications across teaching mathematics support the need for its integration as a part of school professional developments programs. For example, the general lack of focus on the pedagogies that promote *active citizenship*, *metalanguage* and *cultural knowledge* and the complexity of incorporating these elements in mathematics teaching highlight the potential for teacher education and school professional development to promote greater understanding of and support for the role of these pedagogical elements in teaching and learning mathematics.

5.4 Conclusion

The comprehensive framework provided by Productive Pedagogies challenges the pedagogical core of Omani mathematics teachers by bringing the quality of classroom practices into focus. Drawing on the research findings, mathematics teachers identified the value of Productive Pedagogies framework to guide their preparation of their lessons and to change their teaching practices towards improving classroom practices. Underpinning these positive effects, the practicing teachers developed a desire to include the framework to be a part of their future teaching practices and as a teacher professional development program within the school community.

The challenges faced by mathematics teachers in their experience of implementing Productive Pedagogies involved various factors of conceptual, pedagogical and contextual limitations indicating that teachers' understanding of the pedagogical elements is still developing. New skills are required in providing lessons that encourage learning mathematics for all students to develop them intellectually and to be of relevance to students' lives in a supportive classroom environment. The fact that these challenges influenced mathematics teachers' best efforts to apply Productive Pedagogies in their mathematics classrooms suggests the need to support these challenges.

The findings also supported the importance and the responsibility of the school community to provide teachers with a school culture of professional dialogue and development. More opportunities of support from schools to be constituted as reflective communities can enhance the implementation of Productive Pedagogies

and can promote professional dialogue around its value. The development of teachers' understanding and ability of implementation involves reconstructing of teachers' beliefs and experiences, requires time for reflection and designing effective teaching and learning experiences and needs support with strategies for professional growth. Thus, the pedagogy within mathematics classroom practice needs to become the focus in professional conversations in Omani schools.

Investigating the appropriateness of implementing the Productive Pedagogies framework in the educational system of Oman showed that the concept of the framework is in line with the main objectives of the Omani Basic Educational System as well as with its current interest towards enhancing quality teaching. However, the appropriateness of the pedagogical element of *cultural knowledge* has to be considered and to be examined in more depth in the context of other mixed cultural groups in Oman.

Students' perceptions about studying school mathematics subjects reflect ambivalent views about the desired outcomes of classroom practices that should be valued. Putting forward the importance of improving students' engagement in the learning of mathematics informs the aspects of the pedagogical practices that are necessary to meet these demands. The Productive Pedagogies framework emphasizes the value of the intellectual and social outcomes of students and its importance to be reflected in classroom practices.

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Memorandum

To	Khoula Alhosni, SMEC
From	Pauline Howat, Administrator, Human Research Ethics Science and Mathematics Education Centre
Subject	Protocol Approval SMEC-96-11
Date	4 November 2011
Copy	Bill Atweh, SMEC

Office of Research and Development
 Human Research Ethics Committee
 Telephone 9266 2784
 Facsimile 9266 3793
 Email hrec@curtin.edu.au

Thank you for your "Form C Application for Approval of Research with Low Risk (Ethical Requirements)" for the project titled "*Using Productive Pedagogies as a framework for promoting the quality teaching of Omani mathematics teachers*". On behalf of the Human Research Ethics Committee, I am authorised to inform you that the project is approved.

Approval of this project is for a period of twelve months **2nd November 2011 to 1st November 2012**.

The approval number for your project is **SMEC-96-11**. *Please quote this number in any future correspondence.* If at any time during the twelve months changes/amendments occur, or if a serious or unexpected adverse event occurs, please advise me immediately.



PAULINE HOWAT
 Administrator
 Human Research Ethics
 Science and Mathematics Education Centre

Please Note: The following standard statement must be included in the information sheet to participants:
This study has been approved by the Curtin University Human Research Ethics Committee (Approval Number SMEC-96-11). If needed, verification of approval can be obtained either by writing to the Curtin University Human Research Ethics Committee, c/- Office of Research and Development, Curtin University of Technology, GPO Box U1987, Perth, 6845 or by telephoning 9266 2784 or hrec@curtin.edu.au

Curtin University of Technology

School of Science and Engineering/ SMEC

Teachers' Information Sheet

My name is Khoula Alhosni. I'm currently completing a piece for my research for my Doctoral degree of mathematics education at Curtin University of Technology

Research title:

Using Productive Pedagogies as a framework for promoting the quality teaching of Omani mathematics teachers.

Purposes of the research and your role

I would like to introduce Productive Pedagogies framework to you with a group of three mathematics teachers, one of them will be your peer in your school who teach the same grade, in order to enhance the quality of your teaching and students' learning.

While you agree to participate, I would like you:

- To attend to a professional development program to introduce this framework which will be 15th – 18th January for 5 hours per day including break times.
- To observe and code a minimum of two mathematics classroom's practice of your peer every two weeks during the semester 2/2012.
- To contribute to a minimum of one group discussion with your peer and the researcher after each classroom observation.
- To conduct to three individual interviews during the semester to discuss with you your' experience in applying Productive Pedagogies. The interview process will take approximately 40 minutes.

Further Comments

- A series of workshops will be conducted during the professional development program such as seminars, group discussions and video-taped lessons observations.
- Drawing on your understanding of Productive Pedagogies framework, a classroom observation scale will be used for coding.
- The researcher will also observe and code a minimum of two classroom's practice every two weeks during the whole semester. The lessons to be observed will be nominated by you.

- The group discussions will be conducted to analyse and discuss what we observed, discuss any obstacles that will limit your ability to implement Productive Pedagogies, and plan for learning activities and assessment tasks. Two group discussions will be conducted every two weeks in semester2.
- I would like to investigate your students' perceptions on implementing Productive Pedagogies in their mathematics classrooms. Hence, at most a group of five students will be interviewed from your classroom during the semester.
- I'm interested in giving you the opportunity to present a paper, about your experience of implementing Productive Pedagogies, in the main local educational conference that is hold annually at the end of each year.

Consent to participate

Your involvement in the research is entirely voluntary. You have the right to withdraw at any stage without it affecting your rights or my responsibilities. When you have signed the consent form I will assume that you have agreed to participate and allow me to use your data in this research.

Confidentiality

The information participants provide will be kept separate from your personal details, and only me, my supervisor and my thesis Committee will have access to this. The interview transcript will not have your name or any other identifying information on it and in adherence to university policy, the interview tapes and transcribed information will be kept in a locked cabinet for at least five years, before a decision is made as to whether it should be destroyed.

Further Information

This research has been reviewed and given approval by Curtin University of Technology Human Research Ethics Committee (Approval Number.....). If you would like further information about the study, please feel free to contact me by my email address kh10dream@yahoo.com. Alternatively, you can contact my supervisor, Bill Atweh, on his email address B.Atweh@curtin.edu.au

Thank you very much for your involvement in this research

Your participation is greatly appreciated

Curtin University of Technology

School of Science and Engineering/ SMEC

Principal's Information Sheet

My name is Khoula Alhosni. I'm currently completing a piece for my research for my Doctoral degree of mathematics education at Curtin University of Technology.

Research title:

Using Productive Pedagogies as a framework for promoting the quality teaching of Omani mathematics teachers.

I would like to introduce Productive Pedagogies framework to a group of two mathematics teachers from your school who teach the same grade, in order to enhance the quality of their teaching and their students' learning. **This group of teachers will have:**

- To attend to a professional development program to introduce this framework which will be 15th – 18th January for 5 hours per day including break times.
- To observe and code a minimum of two mathematics classroom's practice every two weeks during the semester 2/2012. Each teacher will observe her group' peer.
- To contribute to a minimum of one group discussion with the researcher after each classroom observation.
- To conduct to three individual interviews during the semester to discuss their experience in applying Productive Pedagogies. The interview process will take approximately 40 minutes.

While you agree that your school and the teachers will participate:

- I would like to have access to enter the school. Every two weeks I'll visit your school three alternative days in the first week and two alternative days in the second week during the whole semester2/2012.
- I would like to offer a quiet room for group discussions and interviews.
- I would like to use one of the technological labs in the school for the professional development Program during the period (15th – 18th January 2011).

Further comments:

- The researcher will also observe and code a minimum of two classroom's practice every two weeks for each participant during the whole semester.
- At most a group of five students will be interviewed from each case-classroom during the semester.

- The participants will present a paper, about their experience of implementing Productive Pedagogies, in the main local educational conference that is held annually at the end of each year.

Consent to participate

Your school's involvement in the research is entirely voluntary. Your school has the right to withdraw at any stage without it affecting its rights or my responsibilities. When you have signed the consent form I will assume that you have agreed to participate and allow me to use the data obtained in this research.

Confidentiality

The information participants and school provide will be kept separate from their personal details, and only me, my supervisor and my thesis Committee will have access to this. The interview transcripts for the teachers and students will not have their names and school's name or any other identifying information on it and in adherence to university policy, the interview tapes and transcribed information will be kept in a locked cabinet for at least five years, before a decision is made as to whether it should be destroyed.

Further Information

This research has been reviewed and given approval by Curtin University of Technology Human Research Ethics Committee (Approval Number.....). If you would like further information about the study, please feel free to contact me by my email address kh10dream@yahoo.com. Alternatively, you can contact my supervisor, Bill Atweh, on his email address B.Atweh@curtin.edu.au.

**Thank you very much for your involvement in this research
Your participation is greatly appreciated**

PRINCIPALS CONSENT FORM

- I understand the purpose and procedures of the study.
 - I have been provided with the participation information sheet.
 - I understand that the procedure itself may not benefit me.
 - I understand that my involvement is voluntary and I can withdraw at any time without problem.
 - I understand that no personal identifying information like my name and address will be based in any published materials.
 - I understand that all information will be securely stored for at least 5 years before a decision is made as to whether it should be destroyed.
 - I have been opportunity to ask questions about the research.
 - I agree to participate in the study outlined to me.
-

Name:

School:

Signature:

Date:

TEACHER'S CONSENT FORM

- I understand the purpose and procedures of the study.
 - I have been provided with the participation information sheet.
 - I understand that the procedure itself may not benefit me.
 - I understand that my involvement is voluntary and I can withdraw at any time without problem.
 - I understand that no personal identifying information like my name and address will be based in any published materials.
 - I understand that all information will be securely stored for at least 5 years before a decision is made as to whether it should be destroyed.
 - I have been opportunity to ask questions about the research.
 - I agree to participate in the study outlined to me.
-

Name:

Signature:

Date:

STUDENT'S CONSENT FORM

- The student will be observed under the normal observations in mathematics classroom without any interruptions.
 - The students will be interviewed within the class and in the school day without any disturbances to student' study.
 - If the student does not want to be observed and interviewed, the observation will still be conducted; however it will omit any data from that student.
-

- **I agree to be observed.**
- **I agree to be interviewed**

Student's Name:

Class:

Student's Signature:

Parent's Signature

Date: