Western Australian School of Mines

# Evaluating Flexible Mine Design under Uncertainty

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# DECLARATION

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Signature: …………………………………..

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# PUBLICATIONS INCORPORATED INTO THIS THESIS

Groeneveld, B, Topal, E and Leenders, B, 2009. A new methodology for flexible mine design, in Proceedings of the Orebody Modelling and Strategic Mine Planning Symposium 2009, pp 109-118 (The Australasian Institute of Mining and Metallurgy: Melbourne).

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Groeneveld, B., Topal, E. and Leenders, B., 2017 Examining system configuration in an open pit mine design. (*under review submitted to Computers and Operations Research*)

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# ABSTRACT

The mining industry is characterised by volatility in commodity prices, limited orebody knowledge and large capital investments, that are mostly inflexible once committed. The primary goal of a mining operation is to exploit an ore body in a way that generates maximum benefit for all stakeholders. One of the most critical enablers occurs when the system configuration of design options is developed i.e. mining fleet size, plant capacity and product. Conventional mine planning methods use deterministic values for uncertainties and static system configurations. A method that integrates uncertainties and design options into the optimisation process should generate a system configuration that is more flexible and valuable than conventional methods are likely to achieve. This research provides a methodology for optimising the system configuration with consideration for uncertainty. It utilises Conditional Simulation (CS) to simulate geological uncertainty, Monte Carlo Simulation (MCS) to simulate uncertainty values and Mixed Integer Programming (MIP) to determine the optimal system configurations. Different modelling modes are proposed; a fixed mode which considers a static system configuration under uncertainty, a flexible mode which has a dynamic system configuration that can respond to changes in the uncertainties optimally (however assumes perfect information), a robust mode which can handle multiple simulations in the same model to generate a single system configuration and an operational mode where the system configuration is fixed in early time periods and flexible in later time periods. Value-at-Risk, Frequency of Execution and Frequency of Extraction graphs are utilised to guide the decision making process around design option inclusion. The methodology is applied to a real feasibility study from a mining operation and alternative system configurations are generated based on expected uncertainty at the feasibility date. A back analysis is then undertaken against actual data to compare how these alternative system configurations would have performed. This demonstrates that significant additional value adding is possible by considering flexibility and uncertainty in the design phase of a project.

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# CHAPTER 1. INTRODUCTION

The primary goal of a mining operation is to exploit an ore body in a way that generates maximum benefit for all stakeholders. As the minerals industry is prone to ‘boom’ and ‘bust’ production cycles in which large upfront capital investments are required with uncertain payoffs, a big investment at the wrong time in the economic cycle may be catastrophic. The uncertainty of the future economic state and limited information from which to develop knowledge of the in-situ resource, make correctly interpolating the state of the mining cycle and determining the upside and downside potential difficult, even for the best investors (Vardi, 2013, Evans, 2015). Nonetheless, individual projects can proactively manage their business to minimise the impact of this volatility if aware of its presence (Lamb, 2003). Surprisingly, commonly adopted decision making methodologies fail to take this into consideration in the design phase of a project.

In open pit mining, the surface of the ground is continuously excavated and material that has the potential to be economic is selected as ore, with other uneconomic gangue material being sent to waste (Lerchs and Grossman, 1965). Open pit mines are characterised by high strip ratio (high amount of waste to ore) in contrast to underground mines which are generally low strip ratio (low amount of waste to ore) due to their ability to be more selective. Several factors govern the profitability of an open pit mine:

- Ore grade and tonnage;
- Physical size, shape and structure of the deposit;
- Capital expenditure required;
- Operating cost;
- Pit limits, cut-off grades and stripping-ratios;
- Production rate;
- Bench access;
- Mine design (the bench heights, the road grades, geotechnical and hydrological requirements etc.);
- Location and proximity to communities;
- Regulations and taxes of the host country;
- Environmental conditions; and
- Availability of supplies such as water, chemicals and energy.

Current practice is to generate ‘optimal’ designs by using average values for key variables as a proxy for a distribution that represents the underlying uncertainty (Grobler et al., 2011). The ‘flaw of averages’ is a well-known concept that states: “Plans based on the assumption that average conditions will occur are usually wrong” (Savage, 2000). Additionally, Jensen’s law states that the “average of all possible outcomes associated with uncertain parameters, generally does not equal the value obtained from using the average value of the parameters” (de Neufville and Scholtes, 2011). This holds true for all payoff functions that are non-linear.

Barnes (1986) highlights the inadequacy of using deterministic values (average values) in the mine planning process by highlighting potential contractual violations when uncertainty is considered. Additionally, by only considering deterministic values a design may not be able to handle peaks and troughs in inputs or demand. However, if uncertainty is included in the design phase, it may justify additional cost to provide flexibility, which allows for a quick reaction to change, thereby generating significant real value. Incorporation of uncertainty in the optimisation process is a potential method that allows this flexibility to be included. Some researchers have investigated the impact of changing prices on the mine optimisation decision (Samis et al., 2005, Martinez et al., 2007, Ramazan and Dimitrakopoulos, 2012), however all assume that the maximum system capacity is fixed with no expansion or closure options. Geological uncertainty, in terms of ore grade and tonnage, has been incorporated by a few studies with varying methods of approach (Godoy and Dimitrakopoulos, 2004, Ramazan and Dimitrakopoulos, 2004, Leite and Dimitrakopoulos, 2007, Meagher et al., 2009, Dimitrakopoulos and Abdel Sabour, 2011, Gholamnejad and Moosavi, 2012, Del Castillo and Dimitrakopoulos, 2014, Montiel et al., 2016), most rely on conditional simulation of the orebody.

A decision making method, called real options 'in' projects (ROIP), has been developed to increase the flexibility of an engineering system under uncertainties. It is located midway between financial real options analysis (which does not deal with engineering system flexibility) and traditional engineering approaches (which do not deal with financial flexibility). ROIP benefits by being able to adjust the underlying system in response to the resolution of uncertainties over time. Research into this method has been undertaken by de Neufville and his colleagues with applications in various industries. A frequently used example to explore the concept of ROIP is that of a multi-story car park. Flexibility in this situation is in the design of the footing and columns of the building so that additional levels can be added at a later date. This flexibility comes at a cost, and the designer must determine if this is warranted (de Neufville et al, 2005).

Including flexibility in a mining operation provides the ability to quickly respond to changing conditons; however, flexibility comes at a cost. For example, at a mine that is determining the size of the processing plant to build an initial capacity configuration of 10 Mtpa is proposed. An option exists to increase the size of the foundations and footing to allow for an expansion of the plant to 15 Mtpa at a later date. However, to minimise initial capital investment, this optionality is removed in the final design. Two years after the operation commences, the sale price of the product doubles and costs increase by a marginal amount, while all other variables hold. Thus, under these conditions it is favourable to expand the operation. Unfortunately, the ability to easily expand the operation was removed and the only option is to build a complete new crusher facility. This will require significant additional capital and take longer to construct Consequently, the overall value of the operation has not been maximisied. A decision making tool called Real options ‘in’ projects (ROIP) has been proposed as a method to analyse and justify an appropriate level of flexiblity (de Neufville et al., 2005; de Neufville, 2006; Wang and de Neufville, 2005, 2006; Cardin et al., 2008; Groeneveld et al., 2009).

Making a decision with uncertain information (partial uncertainty) is a common reality, however making a decision without adequately considering known data can lead to failure; even underestimating uncertainty can lead to overstating outcomes. Flexibility in the system design can mitigate this risk, however the underlying uncertainty needs to be included in the optimisation process in order for this to be evaluated. The central hypothesis of this thesis is to propose a methodology and framework that provides the decision maker with a tool to adequately evaluate design options and include the true impact of uncertainty.

## 1.1 PROBLEM DESCRIPTION

A mining investment decision requires several key variables to be analysed and committed during the development of a project. The typical life cycle of a mineral investment decision consists of serveral stages. A simplistic view of this development cycle is; targeted exploration drilling (or other geophysical techniques) of geological anomalies to identify potential mineralisation zones to determine the mineral quantity and quality present. The results form the basis of a geological model interpretation (resource definition) that flows into a mine planning process that assesses the most economic method of extraction and a financial model that predicts the cash flow from the chosen method and design. Based on this a decision to invest or not to invest is made. The mining and processing of the ore is performed to produce a product that can be marketed for sale. Finally, once all economic material from a mine has been exhausted the mine is closed (for simplicity this thesis will refer to this as the mine design). A summary of this process is outlined in Figure 1.


Figure 1: Typical mining life cycle

Throughout the life of an asset, decisions are made based on incomplete data (imperfect information). For example, due to economic reasons, the geological model is developed from a limited set of data points which is unlikely to perfectly replicate the real world. Metallurgical recoveries are generally based on limited test work on a selection of samples obtained from the exploration campaign; so there is likely to be a degree of error and variance in actual returns. Commodity prices are mostly dictated by supply and demand forces that are in a constant state of flux; price movements are likely to occur over the asset life. Production rates can be estimated from similar historical scenarios however are unlikely to be identical to future production. These examples are some of the areas where variations could occur during an asset life cycle. A further (non-exhaustive) list of uncertain parameters is summarised in Figure 2.

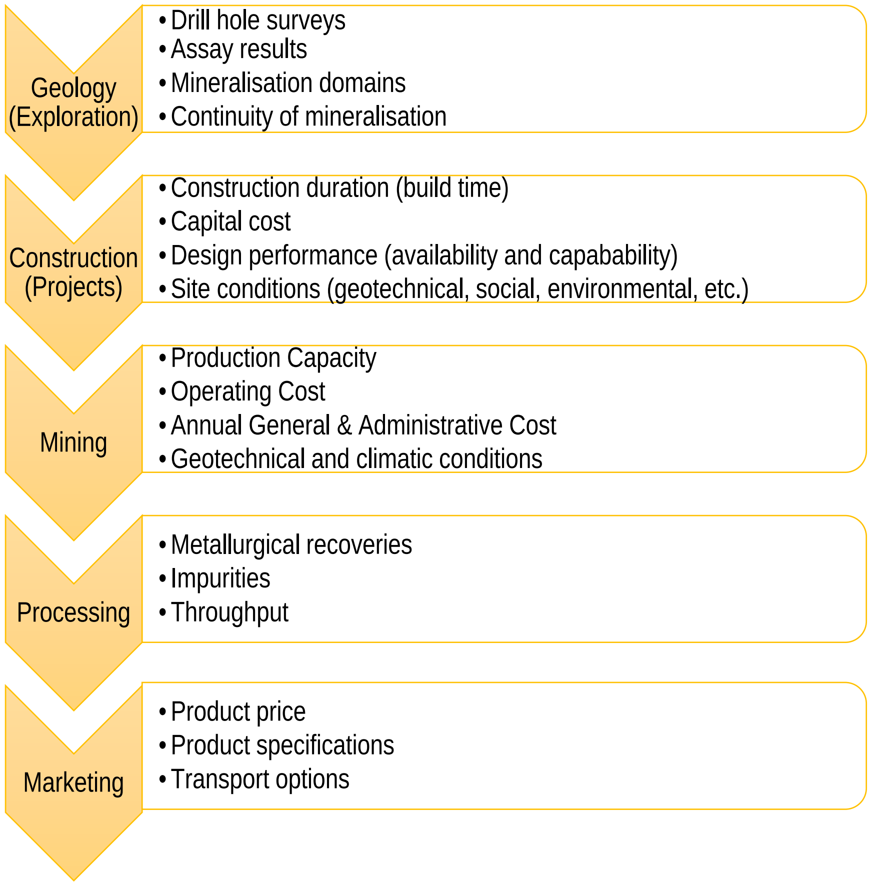



Figure 2: Examples of uncertain variables across the mining value chain

Generally this uncertainty is handled by simplifying the parameters to an average or mean value of expectations (a deterministic scenario). Vallee (2000) reported in the first year of operation, 60% of surveyed mines had an average rate of production less than 70% of the designed capacity. It is hypothesised, that if a wide range of values where tested throughout the mine design process a more accurate and flexible project value would be developed. One potential reason this is not common practice is provided by Stange and Cooper (2008) who state; “Understanding and exploiting value options requires a culture that tolerates, and revels in, ambiguity and uncertainty – something that would make many project teams uncomfortable!”.

A mining system has several design options relating to decisions about mine capacity selection, process plant capacity, location and downstream logistics handling capability. Currently, most techniques require manual selection of the design options and this is computationally intensive to check all valid configurations. Therefore, obtaining an optimal configuration is problematic. For example, a project that had five different mining and processing options would have thirty-two ($2^5$) different possible configurations at the start of the project if each decision was mutual independent. In addition, if the project had two different periods in which the decision could be changed, there would be 1024 ($32^2$) possible design combinations. Likewise, if the decision could change in five periods there would be 33,554,432

($32^5$) possible design combinations. This shows how the complexity increases exponentially as the number of decision points increase in the design environment. Optimisation techniques exist which aim to limit the search requirements for finding the optimal solution. By utilising such techniques, it is hypothesised that if these design options were included in the optimisation, additional design flexibility could be justified. This would generate extra shareholder value, improved efficency and increase sustainability of mining projects.

## 1.2 AIMS AND OBJECTIVES

This research seeks to provide a methodology for decision makers to design for change and incorporate flexibility into the mine design process. It is anticipated that such a framework could increase operational flexibility leading to increased efficiency and profitability of mining projects.

The objectives of this research are to:

I. Introduce a methodology to evaluate mine design decisions within the optimisation process;
II. Incorporate uncertainty into the optimisation process;
III. Develop an optimisation model which can adjust the mine schedule with design changes;
IV. Incorporate in the model the ability to produce fixed, robust and flexible designs;
V. Outline a methodology to determine the value of flexibility; and
VI. Evaluate design for implementation (operational mode) against theoretical maximum value designs (flexible mode) to further understand the impact of decisions.

The fundamental questions this research will seek to answer are:

- How can design option flexibilities be modelled in the mine optimisation process?
- How can uncertain inputs be included in the mine optimisation process?
- What range of value is likely to be achieved by adopting a more flexible operational plan?

## 1.3 SCOPE

This research evaluates open pit planning decisions in the mining value chain from resource definition through to product marketing (pre-downstream processing). The key focus is to determine robust and flexible long-term open pit planning decisions when considering uncertainty in key inputs.

This research will focus on quantifying the flexibility in the mining system, with respect to the following elements:

- Open pit scheduling of blocks, subject to capacity limits, vertical dependencies and ore processing quality requirements;

- Scale and timing of mining and process plant capacity decisions, both expansion and closure;
- Stockpile capability between mining and processing;
- Post-process plant logistics capacity;
- Marketing scenarios, such as what type of product to produced;

Whilst acknowledged as important aspects of the mining problem, specifically excluded from the scope of this research are:

- The ultimate pit limit problem;
- Optimisation of pushback and phase selection;
- Uncertainty estimation methods for both geological and time dependant variables;
- Geotechnical uncertainty and its impact on pit shapes; and
- Detailed scheduling of mining activities.

These issues should be considered in future research. The key value of this research is in understanding how flexibility in the system design can generate value when considering uncertainty in the underlying parameters.

Furthermore, underground mining is acknowledged as an important alternative mining method, but this research has not considered the many and varied extraction techniques available. It is envisaged that future research could incorporate this by modifying the mathematical model without fundamental change. Moreover, the determination of the underlying stochastic processes has not been a focus of this research. It is acknowledged that this is an important part of the decision making process however, the goal of this research is to develop a model which can incorporate this information into mine planning decisions. More detailed modelling of the uncertainties can be undertaken without changing the fundamental methodology.

## 1.4 METHODOLOGY

An optimisation methodology will be developed which has the ability to handle design decisions simultaneously in the optimisation process and allows the model to respond to changes in the uncertainties. Mixed Integer Programming (MIP), a form of Linear Programming (LP), where some variables are constrained to integer values instead of decimals, will be used for optimisation. Monte Carlo Simulation (MCS) will be used for simulation of financial and utilisation uncertainties. Conditional Simulation (CS) will be used to simulate resource uncertainty (the implementation of this process is not a focus of this thesis). LP is selected because it is a mathematical technique that has been commonly applied to planning problems seeking to maximise or minimise an objective while handling multiple constraints. MIP is used because it allows key decision variables to be incorporated in the optimisation process together with mining constraints. Further, MIP also guarantees optimality by rigorously exploring the solution domain and in doing so, bounding the upper limit of the maximisation problem. MCS is used to model realisations of uncertain variables that are then incorporated in the MIP models. Importantly, this allows correlation of variables with a wide range of underlying distributions, notably it does not require geometric Brownian motion to be true. CS is used to produce equally probable realisations of the geological context. Techniques such as data mining, regression analysis and value at risk graphs are used for analysis of the results to help the decision maker understand key value levers.

## 1.5 SIGNIFICANCE AND RELEVANCE

This research will improve the quality of mining investment decisions by making operations more flexible and adaptable. The contributions of this research to the academic pursuit are:

- A new optimisation model that incorporates uncertainties in the mine design process to develop operational decisions that maximise the expected project value is presented. A new MIP formulation is developed with inputs for uncertainties from MCS and CS simulations, inclusion of design options as decision variables (integer or binary variables) combined with decision variables (linear and binary) to handle the mine scheduling of the system. Additionally, multiple modes of the optimisation model are proposed to analyse fixed, flexible, robust and operational system configurations.

- A set of graphical analysis tools that assist decision makers to analyse flexible decisions with respect to uncertainty and provide a basis for choosing between design options. Namely a 'value-at-risk' graph is used to discuss how the results from a flexible mode produce an optimal frontier that the value of fixed, robust and operational designs can be measured against. Additionally, a table of frequency of execution over time is presented to understand how the system changes over time.

- Multiple performance algorithms are outlined that reduce the number of variables in the model (without violating optimality) and lead to a reduction in the solution time for the optimisation model. Early start and late start restrictions are utilised on the execution of the design options, together with the removal of value paths that have a negative marginal cost. Additionally, integer feasibility constraints are included to provide a tighter bounding of the model formulation.

## 1.6 THESIS STRUCTURE

Chapter 1 defines the problem, states the aims, objectives and scope of the study, outlines the methodology that will be used and summarises the key original contributions this project will offer.

Chapter 2 outlines the components of open pit mine design under deterministic conditions with reference to the current literature and commerically available packages. Following this, an outline of the principles applied in the civil engineering industry to include flexibility under uncertainty in engineering system design is provided. Building on the principle of including uncertainty in the evaluation process, a review of the literature which has included uncertainty in the mine design process is outlined. A discussion of the advantages and disadvantages of the various techiniques is provided.

Chapter 3 explains the methodology to be used in this research; specifically, Mixed Integer Programming, Monte Carlo Simulation and Conditional Simulation. Additionally, the components of the model formulation are introduced from a conceptual viewpoint.

Chapter 4 outlines and describes a new mathematical formulation for evaluating flexible mine design under uncertainty. The model can be run in four different modes; fixed, flexible, robust and operational. These modes allow a set of results to be generated and the decision maker can ultimately test decision processes against this data. Additionally, a set of performance improvement algorithms will be outlined that aid in reducing the size of the problem and reduce the solution time for each optimisation model.

Chapter 5 discusses the implementation of these mathematical models and application to a real case study. The case study contrasts how a feasibility investment decision for a project that used traditional deterministic planning could have varied had a flexible approach been considered. A comparison of the performance of the alternative designs under actual project conditions is provided to demonstrate the value of the flexible approach. Further, a discussion of the limitations and challenges of the alternative design approaches is provided. Finally, an overview of the custom software *Mineplex* and user interface is provided to show how this case is setup and the mathematical models generated for passing to a solver.

Chapter 6 concludes this thesis by summarising the key contributions and findings of the research. Ideas and recommendations for future research are advanced.

# CHAPTER 2. LITERATURE REVIEW

This section outlines relevant literature relating to the design of a mining system within a strategic open pit mining context. Strategic mine planning should be distinguished from tactical planning. Strategic planning focuses on longer term value drivers (i.e. installed capacity, sequence and schedule). Tactical planning focuses on maximising efficiency from mature assets with tools like lean manufacturing and six sigma (Montero, 2014). For the purpose of this review it has been assumed that the geological resource has been quantified and an open pit mining method is preferred (whilst other mining methods are not explicitly referenced, the main variation is in the structure of the dependency network), thus geological estimation techniques and underground mine optimisation will not be discussed.

The discussion is divided into three parts; firstly, each element of the traditional deterministic open pit mine design process is discussed with reference to relevant research. Secondly, key research which integrates multiple design elements in the same process is outlined. Finally, a discussion of attempts to include uncertainty into this design process is considered; both from a general perspective and mining specific perspective.

## 2.1 ELEMENTS OF OPEN PIT MINE DESIGN

The universally accepted objective of the mine design process is to develop a system which maximises the Net Present Value (NPV) of the project. NPV is used as a proxy for stakeholder value (Runge, 1998, Brealey et al., 2006). Optimisation can play a critical role in maximising the NPV of an operation and has been shown to increase project value (NPV) by at least 5 to 50% over manual designs (Whittle et al., 2007). The key to most optimisation processes is to have a singular objective which is usually achieved by combining multiple factors into a single overall measure of performance (Hillier and Lieberman, 2010). However, in some circumstances the objective function has multiple objectives that cannot be composited into a singular measure. These could be intangibles such as geopolitical risk, employee relations, environmental and social sensitivities. In these circumstances, a decision maker could use a pareto-optimal curve to compare the trade-off between parameters (Montero, 2014) or multi criteria decision analysis techniques (Musingwini, 2010). This research will assume NPV can be used as a singular measure of stakeholder value.

Maximisation of NPV requires six mine design elements. Generally, this process starts after an exploration program has defined a mineral resource (for which a block model is generated). The first step is to define a likely production capacity based on the estimated resource size. The second step is a cost estimate. The third step is the calculation of the cut-off grade (the cut-off grade defines the grade above which material is economic to process for sale). The fourth step is the calculation of the Ultimate Pit Limit (UPL). The fifth step is the development of pit pushback designs (sequential mining pits). The sixth step is to develop a production schedule for the material inside these pushbacks. The seventh step is to adjust the production capacity, which begins the cycle again. The process would continue until the error between cycles converges. A summary of this circular process is shown in Figure 3.

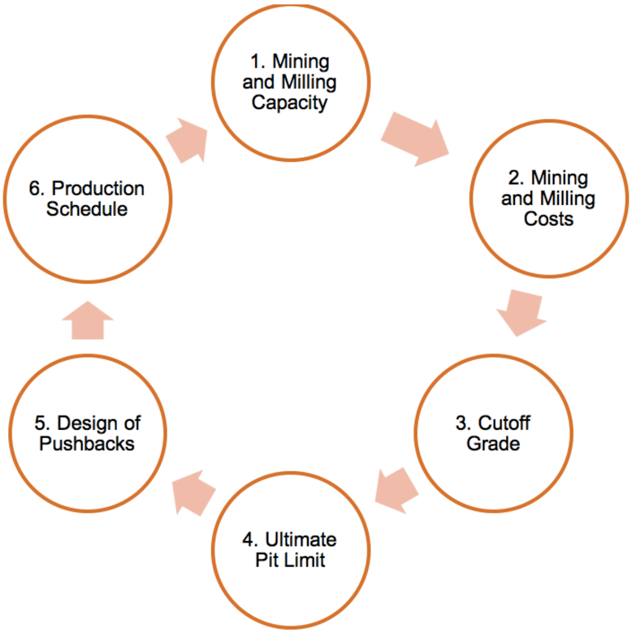



Figure 3: Circular nature of open pit design variables (adapted from Osanloo et al. (2008))

Most of the research to date has been focused on optimising one or two elements of this mine design process and assumed the other elements as fixed. More recent research has attempted to optimise multiple elements of the mine design process. Optimisation of multiple aspects in the same process is likely to generate improved outcomes. The ideal solution is to optimise all aspects of the mine design process in a single step (however, to date this has not been achieved), as shown in Figure 4.

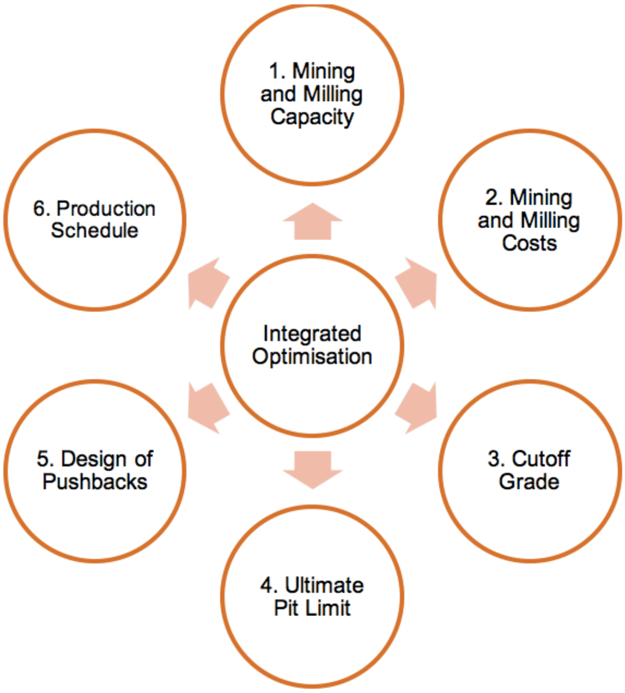



Figure 4: Integrated optimisation of the open pit mine design process

This thesis will explore a process to optimise four of the six elements of the mine design process. This will be outlined in the next chapter.

### 2.1.1 Project Capacity

Project capacity is commonly determined by a manual selection process with iterative changes on the completion of each mine design cycle. Over time the design process will converge on a project capacity solution which maximises NPV. A key risk with a manual iteration process is obtaining a solution that is a local maximum not a global optimum. This concept is graphically represented in Figure 5 where the blue path leads to a local maximum whilst following the green path would lead to a global maximum.

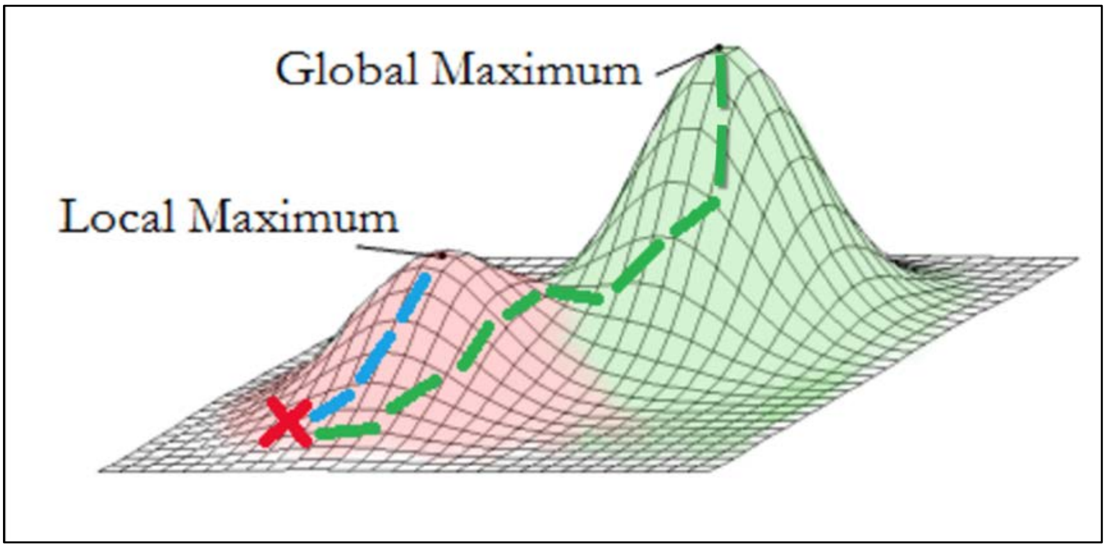



Figure 5: Illustration of a local maximum found by following blue path vs global maximum found by following green path (Sosailaway, 2013)

A few different techniques have been proposed for initial selection of the project capacity (Hotelling, 1931, Smith, 1997, Abdel Sabour, 2002, Cairns and Davis, 2001, Roumpos et al., 2009). However, the most widely accepted and reliable is Taylor's rule (Taylor, 1977) which was empirically derived from a large dataset of mine capacities and reserve tonnages across multiple commodities. The rule estimates the mining capacity based on the reserve tonnage (expressed in short tons) as stated in Equation 1.

$$\text{Mine Capacity (Mtpa)} = 0.147\ (\text{Reserve Tonnage})^{0.75} \quad (1)$$

Once the mine capacity is calculated the estimated mine life can be calculated by dividing the reserve tonnage by the capacity.

A modification to Taylors rule was suggested by Long (2009) by splitting the data by mining method into;

a) Open pit and underground block caving mines as shown in Equation 2.

$$\text{Mine Capacity (short tons per day)} = 0.123\ (\text{Reserve Tonnage})^{0.65} \quad (2)$$

b) Other underground mines as shown in Equation 3.

$$\text{Mine Capacity (short tons per day)} = 0.297\ (\text{Reserve Tonnage})^{0.56} \quad (3)$$

The key assumption in these equations is the reserve tonnage. This can be subjectively deduced by analysing the grade tonnage curve to give a starting point for the planning process.

An alternative method suggested by Whinchup (2010) is to test a range of capacity options based on a rule of thumb. The rule assumes a mine life no less than 5 years or greater than 10 years, which allows the capacity to be determined for an estimated reserve tonnage. For example, the base case treatment rate of a metal resource of ~80Mt could be 16 Mt/year based on a 5-year project life. By applying this rule, a range of capital and processing cost scenarios could be modelled between 10, 12, 14 and 16Mt/yr.

The practice of testing a range of options is common place in order of magnitude studies where some manual iteration through the full mine design process is undertaken. However, time and limited computational capacity make manual iterations tedious resulting in a small number of options being tested. Roman (1973) outline a dynamic programming method to modify the project capacity and production schedule (but not sequence) through working forwards and backwards through the scheduling horizon to exploit the resource, which demonstrates an increase in project NPV. However, due to the heuristic nature of the optimisation this practice is prone to finding local maximums instead of global optimums. A novel approach which integrates the capacity selection into the schedule and destination optimisation problem in an MIP is outlined Groeneveld et al. (2009), Groeneveld et al. (2012) and Groeneveld et al. (2017). This manuscript describes a thesis that builds and develops the novel concepts introduced in those papers.

### 2.1.2 Project Costs

After determining the production capacity for the operation an estimate of the mining and processing costs is developed. Typically, this is broken into capital and operating costs. Estimates are developed from first principles or by benchmarking comparable operations. Commonly both approaches are undertaken to validate cost estimates. The variability and detail expected in cost estimates depends on the level of study (de la Vergne, 2008), typically tolerances are:

- Order of Magnitude studies ± 40-50%;
- Pre-Feasibility studies ± 25-30%;
- Feasibility studies ± 10-15%;

Capital costs are the expenses associated with the construction, planning, development, procurement and acquisition of infrastructure and equipment. These are typically forecast by detailed engineering cost estimates. For mine design purposes they can be broken into project costs and sustaining costs (Hall, 2014). Project costs are related with expansionary activities (additional capability) being one-off costs, whilst sustaining costs are reoccurring costs associated with maintaining a certain level of production. For example, project costs include items like the purchase of new trucks and construction of a new ball mill, whilst sustaining costs include items like the replacement of a truck (due to poor availabilities) and relining of the ball mill (due to wear). Capital cost projections are tainted by long time intervals between planning and expenditure.

Operating costs typically have three components, mining, processing and administration costs. Mining costs include drilling and blasting, load and haul, primary crushing, mine management. Processing costs include secondary/tertiary crushing, leaching, flotation, de-watering, concentration, comminution. Administration costs include site support services and administrative functions. Key cost drivers are typically labour, consumables and energy costs.

Operating costs can be further broken into variable and fixed costs (Hall, 2014). Variable costs are those where the cost should vary in direct proportion to the scale of activity. For example, the amount of fuel required to move material from a pit is directly proportional to the amount of material moved. Fixed costs are those that do not change relative to the level of activity. They are the same for any specified level of activity. Fixed costs are commonly expressed as dollars per year or per month.

Single point estimates of fixed and variable costs do not provide any information on the underlying variability of the cost structure. This thesis will propose incorporating uncertainty of cost parameters in the decision analysis process to improve the understanding of cost structure on design decisions.

### 2.1.3 Economic Cut-off grade

After the cost structure of the operation has been determined it is typical for the cut-off grade of the operation to be determined. The cut-off grade is the grade used to discriminate between ore and waste material (Ganguli et al., 2011). Material above the cut-off grade is sent to the mill for processing, in the same period or stockpiled for later use. Material below the cut-off grade is sent to waste. This is an important aspect of the mine design problem as it determines the amount of material which will become ore; the revenue stream of the operation. Cut-off grade is subject to the cost and scale of the operation.

The cut-off grade concept was first established by Berry (1922). When observing operating mines, it was noted that some superintendents had a tendency to send non-economic material (waste) to the mill when they could still deliver the required average grade. Whilst this practice allowed for an increase in daily tonnages, they were unknowingly destroying the present value of the operation. Berry (1922) further concludes that the present value of an operation may be increased if higher grade material is processed sooner, even if it costs more to access this material. Callaway (1954) defines the break-even cut-off grade as “the grade where revenue equals the production cost attributable to a tonne of material”. Equation 4 shows how the breakeven cut-off grade is calculated.

$$\text{Breakeven cut} - \text{off grade} = \frac{\text{Milling Cost} + \text{Mining Cost } + \text{Sustaining Capital Cost}}{(\text{Price} - \text{Selling Cost})\text{ x Recovery}} \qquad 4$$

Break-even analysis is what we might call a one-dimensional process, it is based on financial parameters only and disregards the impact of time. Asad et al. (2016) provides a detailed review of the historical background of breakeven cut-off grades and discusses the limitations.

Lane (1964) recognised that the cut-off grade of an open pit should not consider the mining cost as the cut-off grade decision is a waste/ore decision not a mine/do not mine decision. The assuption being the pit shape decision was determined by the Ultimate Pit Limit solution; this will be discussed further in the next section.

Lane (1988) further identified that an opportunity cost should be included in the cut-off grade calculation. Opportunity cost is the value forgone by not undertaking an alternative option. Specifically, in the context of cut-off grades, if the cut-off decision impacts when blocks are mined the value of those blocks will change. For example, in a mill constrained operation, if the cut-off grade is decreased, higher grade material will be deferred to a later period and may consequently drop in value (in today's dollars). Thus, this change in value should be captured as a time cost and included in the cut-off calculation (as an opportunity cost). As the mine progresses in its life, less material is left that could be deferred, so the opportunity cost should be reduced. This concept leads to Lane (1988) proposing that a "declining cut-off" grade (dynamic) strategy will always lead to a higher project value than a fixed cut-off policy. This theory led (Lane, 1988) to the construction of a function for the maximization of the NPV. Mathematically, it was shown that the opportunity cost to be included in the cut-off calculation for small time steps can be approximated as shown in Equation 5:

$$\text{Opportunity Cost } (\$/\text{t}) = \text{Discount Rate } \times \text{ Remaining Project Value / Capacity in Time Step} \qquad 5$$

Further, Lane proposes that the optimal cut-off grade for a period, is either, one of the three limiting cut-off grades (mining, milling or selling) determined by the operational constraint or a balancing cut-off grade between any two of the operations components. Limiting cut-off grades vary by how opportunity costs are allocated. Balancing cut-off grades are independent of economics and depend on the grade distribution of material. A consequence of this approach is the requirement for the estimation of future value which can only be achieved by knowing the production schedule. Since the production schedule is dependent on the cut-off grade this creates a circular problem.

However, Lane's approach suffers from many shortcomings (Dagdelen and Kawahata, 2007). First, the algorithm is limited to a single process and single commodity solution which cannot solve for multiple processes or products. Second, it assumes the life of mine extraction sequence is known in advance, however in practice this is a function of cut-off grade. Third, no blending requirements or opportunities are considered. Finally, no allowance for stockpiling of material is possible.

Dagdelen (1993) presents an algorithm which implements Lane's model together with a case study which demonstrates the benefits, in terms of increased NPV, that can be gained by adopting such an approach. Additionally, Dagdelen (1993) proposes a linear interpolation which is more suited to algorithmic optimisation techniques than the graphical approaches of Lane (1988).

Asad (2005) addresses the issue of stockpiling by modifying Lane's model to use a grade distribution concept on material between the break-even and optimum cut-off grade. Asad (2005) outlines a grid search approach and step by step procedure to implement this extension. Cetin and Dowd (2013) extends Lane's model with a grid search algorithm that can handle up to three economic minerals. Nieto and Zhang (2013) use an equivalent factor and apply Lane's model to a rare earth deposit with two minerals. They show that the cut-off grade is sensitive to variability in the by-product mineral.

King (2000) proposes a heuristic algorithm, in the form of an iterative dynamic programming approach, to determine the project value and include stockpile capability. The algorithm starts with a seed solution (which satisfies constraints) to determine a starting project value which can be used in Lane's opportunity cost calculation. Iteratively, the algorithm attempts to converge on a better solution by modifying the cut-off grade policy to improve the project value. Lane's approach assumes a terminal value of zero whilst King's appaorach can include clousre and rehabilitation costs in the optimisation. King (2000) optimises over multiple elements simultaneously using dynamic programming (cut-off grade, dilution and comminution). However, due to the heuristic nature of the algorithm it cannot guarantee a globally optimal solution.

A further detailed review of cut-off grade strategy approaches that can be used to maximise NPV is described in Asad et al. (2016). These approaches all assume that the cut-off grade strategy is an input to the production scheduling problem. However, the preference is for the cut-off grade strategy to be a result of the optimisation process such as in Dagdelen and Kawahata (2007). Dagdelen and Kawahata (2007) formulates the optimisation problem in such a way that material can report to either ore or waste destinations. A flexible model could allow for a variable cut-off grade and adjustable production capacities. This approach will be used in the model proposed in this thesis.

### 2.1.4 The Ultimate Pit Limit Problem

An important step in the open pit design process is defining the ultimate pit limit. A pit limit is a geometric shape that is physically mineable and satisfies geotechnical wall slope requirements. The ultimate pit limit is the point at which it becomes uneconomic (costs exceed revenue) to mine an additional tonne from the pit. The earliest solutions focused on a moving (or floating) cone approach where a cone (dictated by the geotechnical slope constraints) was moved through the block model with the ore flagged as profitable if the cost of removing waste was less than the revenue from the ore (Carlson et al., 1966). However, this approach did not guarantee optimal solutions. This was addressed in later algorithms (Caccetta et al., 1998, Dagdelen, 1985, Gershon, 1987, Caccetta and Giannini, 1988, Lerchs and Grossman, 1965, Matheron, 1973, Matheron and Formery, 1962, Matheron, 1975, Whittle, 1988). Commonly used method, the Lerchs-Grossman (LG) (1965) algorithm, based on graph theory, is used. The Pseudoflow algorithm, based on network flow theory (Hochbaum and Orlin, 2013), has recently been preferred, however.

The LG algorithm determines the optimal extraction shape for a given set of economic and geotechnical parameters. Economic data is deterministic expectations of revenue and costs, generally determined iteratively as part of a feasibility study (Figure 3). The geotechnical splope parameters are specified as overall slope angles based on an engineering study. In the absence of an engineering study a general rule of thumb is to use 45 degrees in average ground conditions. A precedence relationship is established between each block in the model. Figure 6 (a) demonstrates how this relationship is defined in 2D. Block 1 cannot be mined until the block immediately above Block 2 has been mined together with the two immediate neighbouring blocks, Blocks 3 and 4. Figure 6 (b) extends this to the 3D case; here Block 5 cannot be mined until Blocks 6,7,8,9 and 10 have been removed.

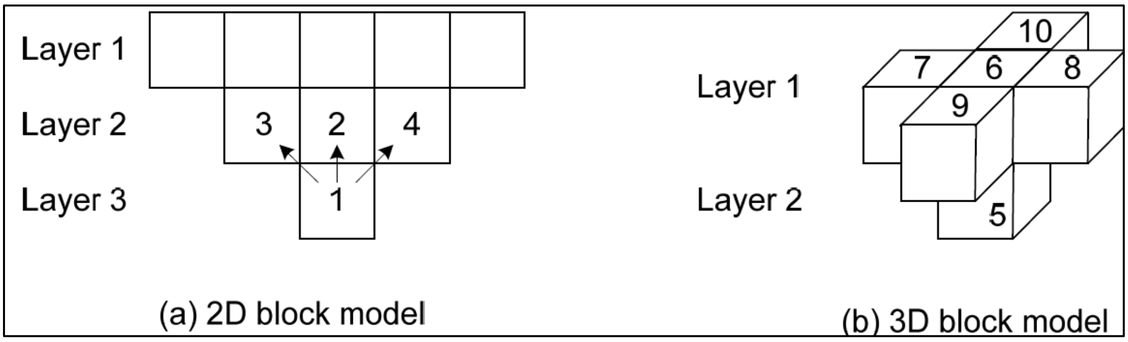



Figure 6: Illustration of block precedence relationships in 2D and 3D (Kozan and Liu, 2011)

The objective of the algorithm is to determine the blocks which satisfy the slope constraints (no capacity constraints are included) and maximises the net profit. The LG algorithm exploits the fact that the block model can be represented as a weighted directed graph in which the vertices represent the economic value of blocks and the arcs represent mining restrictions on other blocks. The graph contains the arc *(x,y)* if the mining of block *x* is dependent on the removal of block *y*. The closure of this graph determines the pit contour by ensuring if block *x* forms part of the pit contour and (*x,y*) is an arc of the graph, then *y* must also be included in the pit contour. This ensures a feasible pit contour is always found and allows the optimum pit contour to be determined by finding closure of the maximum weight for the graph problem (Lerchs and Grossman, 1965, Mart and Markey, 2013).

The LG algorithm was implemented as the Nested Lerchs-Grossmann algorithm (Whittle, 1988) most famously in the Whittle™ software. The Nested algorithm parametrises the pay-off function to produce optimal economic pits for a range of λ parameters which are multiples applied to the expected value of a block. More commonly the λ are referred to as Revenue.

Improving the speed of the LG algorithm has been claimed by techniques that range from dynamic programming to generating a complete set of nested pits in a single run (Caccetta and Giannini, 1988, Seymour, 1994, Zhao and Kim, 1992).

Alternative formulations as a network flow problem have generated the most interest including Push-Relabel (Hochbaum and Chen, 2000, Goldfarb and Chen, 1997), Dual Ultimate Pit (Underwood and Tolwinski, 1998), but most notably Pseudoflow algorithms (Hochbaum, 2008). The Pseudoflow algorithm was simplified and solved faster (Hochbaum and Orlin, 2013) resulting in it being the preferred algorithm. Mathematically the ultimate pit shape is equivalent to the LG algorithm. The performance on a large block model is much quicker but due to some pre-processing can be slower on small block models. A *pseudoflow* on a network satisfies capacity constraints, but may violate flow balance constraints by creating deficits and excesses at each node (Muir, 2007). The Pseudoflow algorithm solves the maximum flow problem with the use of *pseudoflows* instead of masses (as used in the LG algorithm). In the formulation, the mass supported by the root of the strong tree is treated as a pseudoflow and is pushed to the weak root and hence to the dummy root (both source and sink node) (Muir, 2007). The algorithm provides a systematic way to process weak-over-strong vertices by using a distance label concept and proposes two algorithmic variants, the lowest label and highest label variants. Additional information on the implementation of the algorithm can be found in Hochbaum (2008), Hochbaum and Orlin (2013).

In addition, other formulations have been proposed over the years such as the Maxiflow technique (Picard, 1976), parameterization of the final pit contours (Francois-Bongarcon and Marechal, 1976) and others (Muir, 2007, Achireko and Frimpong, 1996, Underwood and Tolwinski, 1996). Amankwah et al. (2014) propose a promising extension of the Maxiflow derivation of Picard (1976) to include multiple time periods subject to spatial and time (temporal) constraints, however can only include a singular overall capacity constraint.

Critically important comments by Francois-Bongarcon and Marechal (1976) recognised that for optimisation purposes small block sizes (with the exception of highly geologically controlled mineralisation such as vein structures), are in fact not statistically dissimilar to larger panel size blocks that more closely represent the data influence zone. This is an important point as it allows for size reduction of the optimisation problem.

A fundamental limitation of all these varied UPL algorithms is the assumption that mining (and by consequence cash flow generation) occurs in the same period, i.e. they do not include the time value of money over the period of extraction. Hence, the defined geometric shape is larger than needed if discounting of cashflows were to occur; as mining in later years would be uneconomic. This is particularly relevant for long life projects (>5 years).

### 2.1.5 Pushback Design

The selection of pushbacks is an aspect of the strategic open pit planning process that is not usually optimised; rather based on practicality, rules of thumb, trial and error or a combination. Generally, the nested pit shells obtained from parametrizing the UPL algorithm are ranked on a pit-by-pit analysis graph and candidate shells for pushback generation selected. A manual mine design is then generated which includes haul roads and access considerations for each shell of material (the delta between consecutive pit shells). The pit-by-pit analysis shows the best and worst present values for each shell, together with the quantity of ore and waste. The best and worst values are determined by scheduling the material according to mining and plant capacity. For the best value scenario, material is mined in order of ascending revenue factor economic pits, for example the material inside a lower revenue factor shell is fully mined before any material in a higher revenue factor shell. For the worst value scenario, each bench within the economic pit is mined consecutively, for example all material on the first bench within the economic pit is mined before the second bench is mined; this produces the worst cash flow scenario. A typical graphical representation of the output is shown in Figure 7.

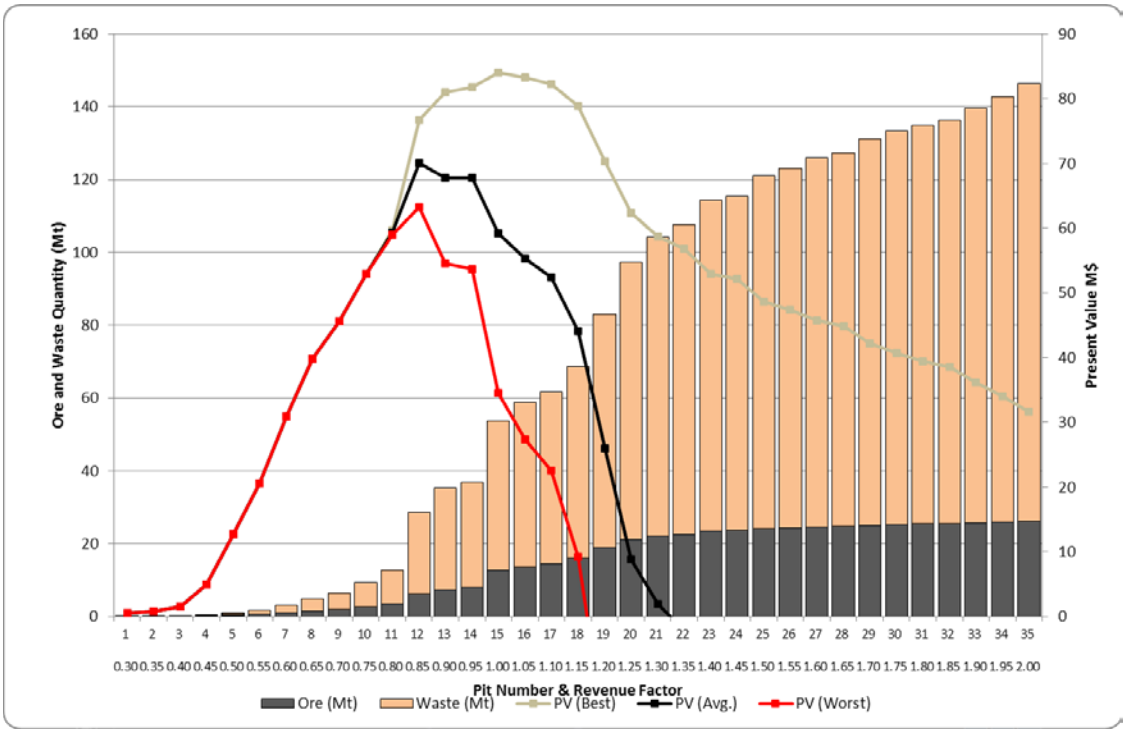



Figure 7: A typical pit-by-pit graphical analysis from Whittle™

From Figure 7, an increase in project value can be seen up to pit shell 15 to 17 when value begins to decline. Logically, one of these shells before pit shell 17 would be selected as the Ultimate Pit Limit. Further, it can be seen that there is a large jump in material movement between shell 11 and 12. This together with the increase in the best value curve, makes shell

11 a good candidate for an interim pushback. Pit shell 14 is then a good trade off between value and total material movement to use as the Utlimate Pit Limt. The large jump between shell 11 and 12 is referred to as the gapping problem and it arises because there is no clear method for choosing the values of λ (Revenue Factor) (Caccetta and Hill, 1999). This is typical of most planning processes currently used.

Various commercial UPL implementations (Milawa NPV and Milawa Balanced are the most common) have a function that allows the user to manually or semi-manually (via a search function) test different pit shells as pushback options by producing an indicative NPV. These algorithms operate by mining the inner most pushback, bench by bench followed by the outer pushbacks bench by bench; possibly with a min/max - lead/lag between stages. However, this approach can generate a solution with a narrow mining width between successive pushbacks or very large pushbacks. This is not practical.

Recent research is attempting to develop an algorithm to solve the pushback design problem, some of the potential techniques are outlined in Meagher et al. (2009) which include densest k Hyper-subgraph theory, approximation algorithms with the LP relaxation of the LP formulation used as a guide to when the solution is close to optimal and Dantzig-Wolf decomposition approaches to the IP formulation. Askari-Nasab and Awuah-Offei (2009) use simulation of pushback designs together with an agent based learning algorithm to converges on the optimal pushback schedule and sequence of material. In effect, this optimises the ultimate pit limit and pushback design problem in the same algorithm. An outcome of this study is to confirm Caccetta and Hill (2003) finding that the optimal final pit limit will be within the UPL determined by the LG algorithm. Gu et al. (2010) propose a model that generates a sequence of optimum pits that is then dynamically evaluated to optimise the number of phases, the geometry, and amount of ore and waste to be mined in each phase. Nanjari and Golosinski (2012) use a dynamic programming with a heuristic algorithm that includes a practical mining width constraint in the optimisation, which provides more practical pushback selections.

Juarez et al. (2014) propose an approximate dynamic programming algorithm in the form of commercially available software called DeepMine™. A heuristic method is used because they regard the problem of pushback design, ultimate pit limit and production scheduling problem as NP-hard. The algorithm creates scenarios for a particular period based on the many ways in which blocks could be selected in a period. For each of these scenarios it develops a number of new possible pathways for subsequent periods, and so forth (Juarez et al., 2014). The consequence of this approach is that a big tree of possible combinations is developed which is too large to return a solution. To overcome this, two strategies are proposed; first, blocks can be extracted in cohesive volumes with a maximum length, maximum sinking rate and minimum width. These are dynamically defined through the progression of the algorithm and second, the results of the LG algorithm are used to guide the solution. This is achieved in two ways by excluding blocks outside the final pit limit and secondly, by using interim pits to guide the solution of the phase shapes (though the authors stress that the way in which this information is included allows for solutions which are much more flexible than just assuming phases will follow pit exact boundaries). Additionally, the total number of phases operational in a period is constrained by equipment availability.

### 2.1.6 Production scheduling

Production scheduling is concerned with the optimal extraction timing in terms of mining block, subject to a number of physical and economic constraints. The output of a production schedule feeds into a financial model which allows cashflows to be determined. These cashflows are discounted by an appropriate discount rate. The discount rate to be used is dependent on the companies funding arrangements plus a risk premium. Summing the discounted cashflows allows the Net Present Value of the project to be determined and scheduling scenarios to be compared.

The most common approach is scheduling material within pushbacks within a UPL (as determined in the previous steps). Generally, deterministic values for physical and economic conditions are used for production scheduling. Additionally, consideration of stockpiling material (delaying production by holding in a pile before sending to the mill) should be handled in this step. Some authors break this out as a separate subject, however this is a time based management decision and it should be managed in the same process.

In early attempts, Lizotte and Elbrond (1982) proposed an open ended dynamic programming approach where the mine and mill capacities were adjusted to determine the best scenario against a range of criteria NPV, Internal Rate of Return (IRR), Present Value Ratio (the quotient of the generated present value cash flow on the present value of the investment) and wealth growth rate (the rate at which the future value of capital outlays equals the future value of generated cash flows).

Other early solutions to the problem where proposed by (Johnson, 1968) with a Linear Programming (LP) formulation which considers the time value of money, different processing types and allows a dynamic cut-off grade. LP techniques have the benefit of being able to provide exact optimised solutions.  Johnson (1968) uses a Dantzig-Wolf decomposition of the problem to solve the master multi-period problem in single periods using sub problems. The LP formulation suffers from partial mining of blocks as the blocks are represented as linear variables. A potential flaw in the model is the unrealistic condition of extracting blocks which are below those not yet mined, as shown in Figure 8.

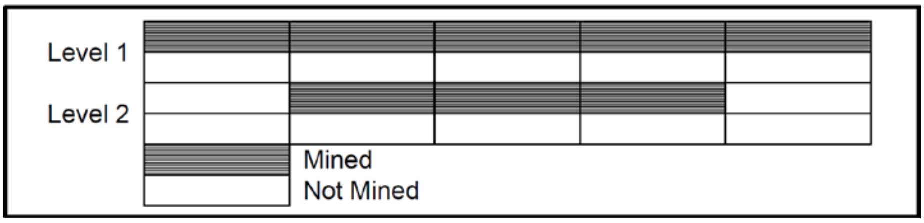



Figure 8: LP formulation allows mining below to occur before full block has been mined

To rectify this problem an additional constraint needs to be included to model the dependency and sequencing constraints between blocks. This requires the introduction of binary variables to the problem and turns the problem into a Mixed Integer Programming (MIP) model. Many papers have been published on implementing MIP to solve mine scheduling problems (Boland et al., 2009, Caccetta and Hill, 2003, Nehring and Topal, 2007, Ramazan, 2007, Ramazan and Dimitrakopoulos, 2004, Topal, 2003). Caccetta and Hill (2003) provide a MIP formulation of the mine scheduling problem commonly referred to as the Open Pit Block Scheduling Problems (OPBSP). The objective of the model is to maximise the discounted profit of mining a block. The decision variables in the model determine the period at which a block is fully mined. The model utilises SMU size block representations, so no allowance for partial mining of blocks is included. The constraints in the model restrict the amount of material that can be mined and milled in a period to within pre-defined upper and lower limits, and prevent mining in a block until all blocks that restrict access have been removed

This provides an optimal solution to the problem of when a block should be mined (mine sequencing problem). When a large number of blocks exist in the model the formulation becomes intractable due to the large number of potential combinations as illustrated by Montiel and Dimitrakopoulos (2015) in Figure 9.

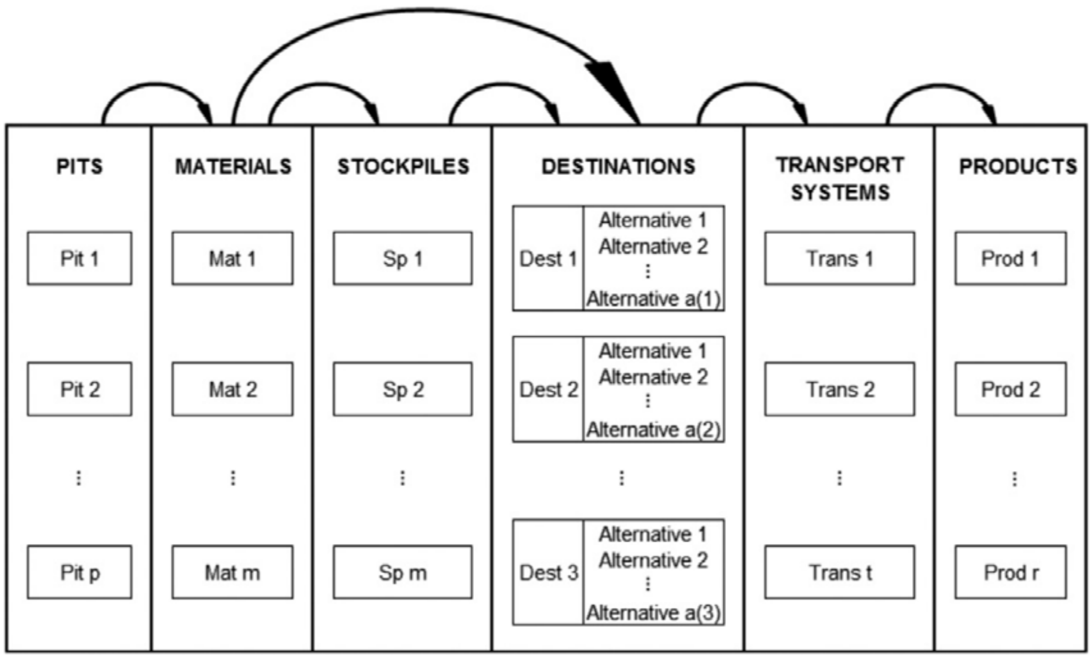



Figure 9: Combinatorial complexity of optimising a mining complex (Montiel and Dimitrakopoulos, 2015)

Following this formulation, others proposed solutions which use techniques such as dynamic programming, genetic algorithms, Tabu search, variable neighbourhood search, neural networks and simulated annealing. The solutions trade-off solution quality (guarantee of optimality) and practicalities (finding a solution in a reasonable time frame).

Alternative integer programming formulation were progressed as outlined in Osanloo et al. (2007), the fundamental difference being that full blocks must be mined at once i.e. no linear variable tracks the amount of the block mined in each time period. Dagdelen (1985) propose a solution to the integer formulation of the problem using Lagrangian relaxation with mining of whole blocks. Lagrangian relaxation is a technique where in a difficult constraint is taken from the model and placed in the objective (in this case the binary variable controlling full mining of blocks). A sub-gradient optimisation process is then proposed to solve the model. Additionally, an extension of the model is proposed to include stockpiling by introducing a variable to reflect the period when a block is mined $t_m$ and when the block is processed $t_p$. Caccetta et al. (1998) note that a 5% gap from optimum is achieved with this formulation.

To improve the ability to solve a large number of blocks with a MIP formulation, various clustering techniques have been proposed. One of the techniques called the “Fundamental Tree Algorithm” is proposed by (Ramazan, 2007). In this method, the number of binary variables is reduced in the model formulation by combining ore and waste blocks together. This is done by utilising a LP formulation that ensures each combination obeys slope constraints and is profitable (revenue exceeds cost). Additionally, a block cannot be a member of more than one Fundamental Tree. Caccetta and Hill (2003) outline a Branch and Cut procedure combined with an algorithmic process to improve the solution speed of the optimisation which broadly speaking combines a best-first search and depth-first search technique, a period by period LP-heuristic and various branching rules to give a spread of quality solutions. Reportedly, this can solve a 210,000 block, 10 time period problem in a reasonable solution time, however full details on the formulation are not provided due to commercial confidentiality. Boland et al. (2009) propose further reductions to the Caccetta and Hill (2003) formulation, with 0-1 knapsack inequalities and aggregation of blocks to form selective units followed by disaggregation. Weintraub et al. (2008) propose a *priori* and a *posteriori* procedure to reduce the model size. Bley et al. (2010) propose a series of clique and cover constraints based on the predetermined production requirement. Chicoisne et al. (2012) outline a formulation based on an IP formulation of the problem and propose a solution which uses an LP decomposition together with a TopoSort local search algorithm to deliver high quality results, however this algorithm can only handle a single capacity constraint per period. Bienstock and Zuckerbeg (2015) propose an algorithm (known as the BZ algorithm) that formulates the problem as a maximum closure problem and they solve it as a minimum s-t cut problem. This approach is reported to be efficient in very large, real world, instances of the problem and can be embedded inside a branch and bound algorithm to solve the original IP problem. This speed up is confirmed in Muñoz et al. (2016). Recently, Vossen et al. (2016) proposed a hierarchical benders decomposition method for the block sequencing problem which also demonstrated a considerable speed up in solution time over traditional techniques, however the solution approach relies on a lower production bound.

Most MIP formulations fix the ore and waste classification when generating the model as a presolve function, however Kumral (2012) proposed a model with a variable cut-off grade policy based on destination of material. Whittle (2010) outline an application of the Prober C algorithim (documented in Whittle (2009)) that performs enterprise optimisation by considering all elements simultaneously to arrive at a value increase of 73.7% over a manual approach. Others have also developed similar implementations (Gholamnejad and Moosavi, 2012, Moosavi et al., 2014, Groeneveld et al., 2010, Groeneveld and Topal, 2011). This approach will be extended in this thesis with the addition of uncertainty in the underlying parameters.

Alternative methods have been proposed to solve the production scheduling problem which have included Heuristic algorithms, Simulated Annealing, Tabu-Search, Particle Swarm and Ant Colony Optimisation (Clement and Vagenas, 1994, Lizotte and Elbrond, 1982, Pendharkar and Rodger, 2000, Osanloo et al., 2007, Sattarvand and Niemann-Delius, 2013, Thomas, 1996, Saavedra-Rosas, 2010, Nanjari and Golosinski, 2012).

The term *heuristic* is of Greek origin, meaning “serving to find out or discover” (Bartz-Beielstein et al., 2010), thus Heuristic methods are solutions to the mathematical problems that seek to increase or improve on the previous solution by trial and error techniques. Some of these methods fall under the class of genetic algorithms which are based on the principles learnt from natural processes, an example is that proposed by Sattarvand and Niemann-Delius (2013) which is based on an ant colony that tackles the scheduling problem with an increase in value of 34% over traditional methods. The main downside of these methods is that they hit local maximum and falsely report these as global maximum, which can only be overcome by running multiple scenarios with varied parameters. The fundamental problem with dynamic programming and meta-heuristic algorithms, is that generally they do not guarantee optimality as they may get stuck on a local maxima in the search process and fail to find the global optimum, similar to that shown in Figure 5.

Additional discussions on open pit mine planning algorithms are provided in Osanloo et al. (2008), Newman et al. (2010), Meagher et al. (2014) and Vossen et al. (2016).

## 2.2 SYSTEM DESIGN UNDER UNCERTAINTY

The ability of a system configuration to respond to change can lead to significant additional value. However, in practice, we “design to specification” when we should “design for variation” (de Neufville and Scholtes, 2011). De Neufville (2004) classified management of uncertainty in three ways; as controlling uncertainty by demand management, protecting passively by building robustness, and lastly by actively creating flexibility that managers can use to react to change. The potential value of alternative system configurations has traditionally been investigated by manually iterating through the design space and comparing options in a financial model.

A financial model will typically include a calculation of NPV, IRR, payback period and capital cost as comparative indicators between scenarios. A limitation of these financial models analysis is the inability of the method to shed light on crucial decisions around optimal timing to invest (Cortazar and Casassus, 1998) and downside risk. For example, a deterministic NPV analysis may indicate a positive NPV for a project, which would lead decision makers to undertake the project. However, if the underlying uncertainties of the project are highly volatile, the project may have a high chance being NPV negative. If decision makers had this insight they could decide to delay the investment in the project. More advanced financial models attempt to value this risk and flexibility with option pricing formulations or Real Options.

Real Options applies concepts from financial option theory to real valuation models Myers (1977). An option in financial option theory is the right but not the obligation to sell or buy a quantity of shares or commodity such as oil or gas, at or before a given date in the future (Galli et al., 1999). Three different techniques can be used to calculate the value a financial option; Black and Scholes (1973) mathematical formula, Binomial Trees (Cox et al., 1979) a simple and transparent process that is mathematically easier to follow and Monte Carlo Simulation (Broadie and Glasserman, 1997) which can handle complex uncertainty distributions. Topal (2008b) contains a comparison of these methods, demonstrating significant additional value is obtained when using Real Option Valuation (ROV).

A simple example of a Real Option is a spare tyre in a car (Wang, 2005). A certain cost was incurred to include the car tyre in the design of the car at a predetermined price. The payoff or benefit of that tyre is unknown, as it is impossible to accurately predict when you will get a flat tyre. The tyre gives the driver the **right**, but **not** the **obligation**, to **change** the tyre of the car at any time.

Another example provided by de Neufville and Wang (2005) is that of the bridge over the Tagus River in Lisbon, Portugal. The original design of the bridge was ‘enhanced’ by including an allowance to build a second deck above the first. This was achieved by increasing the size of the initial footings. These additions to the design resulted in considerable additional cost subsequently the Portuguese government exercised their option to expand the bridge and added a second deck to carry a suburban railroad.

Real Options are a better tool for guiding investment decisions then static NPV analysis. Firstly, rational managers react to changing situations by expanding operations in good times and closing or delaying operations/projects in hard times (Trigeorgis, 1996, Cortazar and Casassus, 1998, Moel and Tufano, 2002). Secondly, suboptimal decisions can be made when using traditional NPV analysis as the optimal timing of a project is not determined (Majd and Pindyck, 1989, Cortazar and Casassus, 1998, Fontes, 2006). Finally, it is hypothesised that low returns are a direct result of errors in NPV analysis which have led to poor capital allocation and inflexible projects (Slade, 2000, Samis et al., 2005).

In the mineral project valuation, several authors have investigated using Real Options with flexibility included by the option to expand, contract, close, delay, abandon or choose (Brennan and Schwartz, 1985, Dixit and Pindyck, 1995, Trigeorgis, 1996, Cortazar and Casassus, 1998, Keenan and Copeland, 1998, Samis, 2003, Tufano and Copeland, 2004, Samis et al., 2005, Mun, 2006, Masunaga, 2007, Evatt et al., 2011). Importantly, Aminul Haque et al. (2016) outline the need to include representative uncertainties in an ROV model to generate a valuation that is not overinflated; in their case commodity price and exchange rate uncertainty. The methodology used by these authors can be classified as Real Options ‘on’ projects. Instead of changing the underlying engineering system design these methods modify the system in the financial model only.

Limited success has been attributed to Real Options ‘on’ Project analysis because of a failure to recognise the significant value that can be generated by modifying key aspects of the project (for example the mine schedule and mine design). Applications of Real Options ‘on’ projects suffer from two fundamental issues. Firstly, a prerequisite assumption when using the Black-Scholes or Binomial Tree evaluation methods is the underlying assumption that uncertainties must follow a Geometric Brownian Motion. Generally, this is not true of an engineering system. For example, the geological variability of an orebody is not controlled by a random walk over time, instead the variability is positionally dependent and related (Grobler, 2015). Secondly, the models have limited ability to capture non-linear reactions of the engineering system to change. For example, in a mining application, in a low price environment, the schedule can be changed so low grade ore is sent to waste instead of through the processing stream.

An alternative is Real Options ‘in’ projects which incorporates flexibility of the engineering system into the decision making process. This methodology is located midway between Real Options ‘on’ projects analysis (which does not deal with system flexibility) and traditional engineering approaches (which does not to deal with financial flexibility) (de Neufville, 2004). Real Options ‘in’ projects explores the multiple uncertainties present in a project in order to determine the optimal design. In contrast, Real Options ‘on’ projects attempts to determine a valuation for the investment (de Neufville and Wang, 2005). Real Options ‘in’ projects benefits by adjusting the underlying system in response to changes in uncertainties over time. A comparison of the differences is provided in Table 1.

Table 1: Comparison of real option 'on' and 'in' projects (de Neufville and Wang, 2005)

| **Real Options ‘on’ projects** | **Real Options ‘in’ projects** |
|---|---|
| Value opportunities | Design flexibility |
| Valuation important | Decision important (‘go’ or ‘no go’) |
| Relatively easy to define | Difficult to define |
| Interdependency/Path-dependency less of an issue | Interdependency/Path-dependency an important issue |

A frequently used example to explore the concept of Real Options ‘in’ projects is that of a multi-story car park (Zhao and Tseng, 2003, de Neufville and Wang, 2004, de Neufville et al., 2005, Cardin, 2007). Flexibility in this situation is in the design of the footing and columns of the building so that additional levels can be added on at a later date. This flexibility comes at a cost, and the designer must determine if this is warranted. Design flexibility does not provide the best design under all circumstances (de Neufville and Scholtes, 2011). For example, if the scenario never eventuated for the carpark to be expanded then the scenario which did not increase the size of the footings and columns would be the preferred solution. Importantly, flexibility in design must improve the overall outcome to be justified.

Significant research using this method has been undertaken by de Neufville and his colleagues, with applications in various industries (de Neufville and Wang, 2004, de Neufville and Wang, 2005, de Neufville et al., 2005, de Neufville and Wang, 2006, de Neufville, 2006, Cardin, 2007, Suh et al., 2007, Cardin et al., 2008, Cardin et al., 2007). Their approaches incorporate a flexible system configuration which can adjust operational strategies in response to change. A flexible system is defined by Mayer and Kazakidis (2007) as; "the ability of a system to sustain performance, preserve a particular cost structure, adapt to internal or external changes in operation conditions, or take advantage of new opportunities that develop during its life cycle by modifying operational parameters."

An important point is that Real Options 'in' projects are interested in system configuration decisions, more so than whether the valuation is correct. With this information the designer can tailor the system in a manner that enhances over all potential economic and physical scenarios, instead of the traditional approach of maximising for a single deterministic scenario.

de Neufville (de Neufville and Wang, 2005, Cardin, 2007) provides two main Real Options 'in' projects methodologies. The first, uses a catalogue of operating plans which defines different system states and operating modes. Cardin (2007) in conjunction with de Neufville, develop this approach by constructing various scenarios dictating how the model should react to changes in parameters. A major strength of this method is that the computational space is considerably reduced, into a few selected cases, however, this does lead to the initial selection of the catalogue of operating plans significantly impacting the optimality.

An alternative method for solving Real Options 'in' projects involves three phases as outlined by de Neufville and Wang (2005); an initial screening model is developed, followed by a simulation model and finally an options analysis. The screening model is a low-fidelity representation of the system options that is easy to implement and quickly solved by linear programming. This allows a large number of possibilities to be tested and focuses attention on the promising designs (de Neufville and Wang, 2004), considerably reducing the design space. Once the interesting designs have been established, they can be incorporated in the simulation model. The simulation model is a high-fidelity model of the system and costly to run. In this stage robustness and reliability of the design is increased, through extensive testing with various technical and economic uncertainties. By utilising a screening model before undertaking a high-fidelity model (which can take a number of days to run) it is argued that a decision maker is provided with a tool to more efficiently evaluate alternative scenarios before proceeding to a detailed design phase (Cardin et al., 2015).

An example of a Real Options ‘in’ a project approach is provided by Cardin et al. (2008) with a case study of the mining project in the ‘Cluster Toki’ region in Chile owned by Codelco. A staged development approach is examined where different operating plans are designed to respond to changing prices. Truck fleet capacity and crusher size were altered in the different operating plans. Three scenarios are considered, which are outlined in Table 2. From this catalogue of operating plans the best plan is selected for each uncertain environment. This is then used to determine the value of the project. A graphical summary of this process is shown in Figure 10.

Table 2: The catalogue of operating plans proposed by (Cardin et al., 2008)

| **Model Assumptions** | **Operating Plans (for Phase II only)** | | |
|---|---|---|---|
| | 1) Positive growth | 2) Zero growth | 3) Negative growth |
| Initial production for Phase II | 12 | 0.6 | 0 |
| Production growth (annual) | 2% | 1% | 0% |
| Initial operating cost ($/metric ton) | $1,200 | $1,000 | $0 |
| Operating cost growth (annual) | 2% | 1% | 0% |
| Fixed cost ($ millions) | $75.00 | $75.00 | $75.00 |
| Fixed cost growth (annual) | 0% | 0% | 0% |
| Discount rate | 10% | 10% | 10% |
| Investment ($ millions) | $3,300 | $3,300 | $0 |

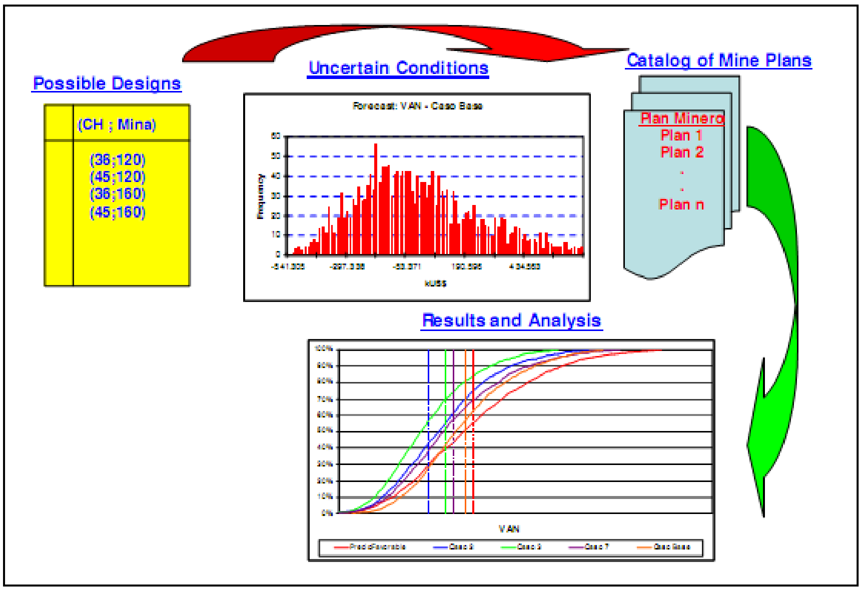



Figure 10: Analysis method proposed for the solving the Real Options 'in' project problem (Cardin et al., 2008)

The methodology used to determine the value is simple conceptually to understand but is enormously more complex in absolute terms. Reasons for this are that the preparation of a mine plan (or operating plan) may involve:

- Representation of the orebody by three-dimensional blocks which may be numerous in quantity (more commonly known as the block model);
- Associating a best estimate of the quantity of ore in each block; and
- Use of a complex optimisation process to calculate the extraction sequence that produces the best cash flow.

This approach varies significantly from the current appraisal where limited possibilities are evaluated. A comparison in approach is provided in Table 3 which shows a better correlation with reality.

Table 3: Contrast between current situation and proposed situation (Cardin et al., 2008)

| | **Reality** | **Current Appraisal Method** | **Proposed Operational Method** |
|---|---|---|---|
| Initial Design (Physical Infrastructure and system) | Many Possibilities | Many Possibilities | Many Possibilities |
| Uncertain Variables (Price, quality of ore, demand for services) | Many Possibilities | Single value for each | Many Possibilities |
| Operating Plan (Mangers adjust best use of existing facilities; development of additional facilities, etc.) | Many Possibilities | Single plan | Some Possibilities – A catalogue of major possible plans |
| Lifetime Performance (Realized Net Present Value, Rate of Return, etc.) | Many Possibilities | Single Cash Flow | Many Possibilities |

The application of this method results in approximately 30 – 50% more accurate project value than current estimates. This value reflects active management of a project beyond a fixed price and demand scenario (Cardin et al., 2008); where management adapt the project design as uncertainties resolve. However, the approach of Cardin et al. (2008) suffer several flaws in a mining context. First, the initial scenario generation process creates a schedule for a single system configuration. This inherently limits the flexibility in the model and likely prevents the optimal schedule and design combination being chosen. Secondly, the model fails to deal with geological and metallurgical variation; a key driver in most mining projects. Finally, the model does not incorporate flexibility at all stages of the mining value chain (the value chain typically stretches from the mine to the port).

Overall, Real Options ‘in’ projects theory provides a strong basis on which to improve the value of a mining project. This is supported by experimental data in Cardin (2011) that concludes project performance can be improved by 36% on average by utilising a Real Options ‘in’ projects design philosophy. Further, Cardin (2011) indicates that this can be enhanced if designers are exposed to flexibility in design concepts prior to the commencing a project design phase. Adopting an approach which considers flexibility is likely to result in improved project outcomes.

## 2.3 MINE PLANNING UNDER UNCERTAINTY

Current mine planning approaches generate optimal designs by using deterministic (average) values for key variables which do not consider the underlying uncertainty distribution (Grobler et al., 2011). Deterministic approaches are not able to deal with grade and geological uncertainties (Gholamnejad and Moosavi, 2012) and more broadly price, cost and utilisation uncertainties. Thus, in reality, some constraints in a deterministic financial model will be violated. The benefits of considering uncertainty were first discussed by Ravenscroft (1993) and Dowd (1994). Over recent years, substantial efforts (Ravenscroft, 1993, Baker et al., 1998, Dimitrakopoulos, 1998, Ahmed et al., 2002, Dimitrakopoulos et al., 2002, Lemelin et al., 2006, Dimitrakopoulos and Abdel Sabour, 2007, Groeneveld and Topal, 2011, Grobler et al., 2011, Montoya et al., 2012, Marcotte and Caron, 2013, Deutsch et al., 2015) have been made to integrate uncertainty into the mine planning process. Whilst accounting for uncertainty involves an increased complexity in the modelling process, these developments have demonstrated ignoring economic or geological uncertainty may lead to unrealistic assessments and inflexible decisions. Additionally, stochastic approaches have shown, an NPV increase in the order of 10–30% (Lamghari and Dimitrakopoulos, 2016). Further, by including stochastic inputs in the optimisation decision, robust and flexible plans can be developed which outperform traditional designs.

### 2.3.1 Sensitivity Analysis approaches

Early attempts to include uncertainty in project decision making, focused on using Sensitivity Analysis (SA). An SA analysis attempts to define the upper and lower limits of possible financial outcomes by ‘flexing’ the inputs to a financial model by a certain percentage and determine the percentage impact on the financial outcome (Mackenzie, 1969). Bai et al. (1997) state that “SA is a reactive approach for limiting risk. It simply measures the sensitivity of a solution to changes in the input data. It provides no direct mechanism for controlling this sensitivity. For this reason, sensitivity analysis may be an inadequate method for dealing with risk in complex decisions.” One of the first attempts focused on evaluating the effectiveness of different operating polices under uncertainty by simulating the exploitation of a mineral deposit Coyle (1973). The key assumption of SA is that the project value behaves in a linear fashion in response to change. In reality the response is generally non-linear. For example, if the gold price drops, most mines will delay production or divert low grade material to waste. This minimises the downside risk. Conversely, if prices increase, previously uneconomic sections of the deposit may be processed and the upside potential increased. This was originally outlined in Brennan and Schwartz (1985) who commented that an obvious deficiency in traditional analysis is the “total neglect for the stochastic nature of output prices <u>and</u> possible managerial responses.”

The next advancement involved the inclusion of uncertainty in the financial valuation model by Monte Carlo simulation of inputs (sometimes multiple factors at once), to understand the impact on the financial valuation. This essentially provided a different mechanism to ‘flex’ inputs to the financial model with the ability to control the distribution of inputs and correlation between variables. However, this type of approach led to the need for more sophisticated models of geological variability that handled the localised uncertainty. Importantly, Krigged models (the standard approach) tend to smooth the spatial variability of grades and underestimate the true variance which can lead to errors in the mine valuation (Bastante et al., 2008). This led to developments which included orebody variability; predominately Conditional Simulation (CS) techniques (discussed in the next chapter).

CS provided the mechanism to include geological variability in the financial calculation which provide a more robust valuation. Ravenscroft (1993) was amongst the first to create multiple mine plans for each simulation of an orebody. A novel approach by Armstrong and Galli (2012) generated a large set of random sequences (using a sequence break point concept) which are then passed through a screening model (with a constant price) and re-evaluated against a set of 100 geostaticially simulated models with 1000 stochastic simulated price paths for a total of 100,000 combinations. However, this approach does not incorporate optimal decision logic around the operating policy of the mine.

### 2.3.2 Heuristic approaches

Heuristic algorithms are customised algorithms that attempt to find good solutions to a problem by exploiting its structure but not necessarily guaranteeing optimality. Various approaches including search methods such as Tabu search, variable neighbourhood descent, network-flow based heuristic and diversified local search have been explored (Lamghari and Dimitrakopoulos, 2016). Lamghari and Dimitrakopoulos (2016) describe a model that has a four-part objective to maximise the discounted profit of mining a block and sending it immediately to the process stream, less the cost of extraction of a blocks in a period, less the cost of delay caused by sending the block to, plus the discounted profit obtained from sending material to the process stream from the stockpile. Metal uncertainty is modelled through a finite set of scenarios, each scenario representing a possible realization of the grade and having an associated probability of occurrence. An initial solution is provided by utilising four different heuristic techniques; tabu search incorporating a diversification strategy (TS), a variable neighbourhood descent heuristic (VND), a very large neighbourhood search based on network flow techniques (NF) and a diversified local search that combines VND and NF. NF and DLS are more efficient. This demonstrated a performance improvement over solving the model as serious of sub-problems for each time step using a branch and cut algorithim.

Montiel and Dimitrakopoulos (2017) outliine a heuristic method using a three stage appaorach, where the first stage solves for an initial solution for the likely outcome, the second stage then calculates the outcome for the simulated scenarios and the third stage pertubates the solution until a stopping criteria is reached to generate the final soultion. Application of the method shows that deviation in processing capacities can be reduced from 9% to 0.2% while increasing expected NPV by 30%. Additionally, the paper demonstrates no additional benefit is obtained by including more than 15 simulations due to the so-called support model. Finally. the authors note the limitation of heuristic approaches and suggest alternative starting solutions should be investigated to search the solution domain further.

Alternative methods have been proposed that attempt to maximise the project value and minimise variation explicitly in the optimisation model and overcome the limitations of heuristic methods to searching the solution domain (Dowd, 1998, Deutsch and Journel, 1998, Dimitrakopoulos, 1998, Dimitrakopoulos et al., 2002). These approaches use meta-heuristic or stochastic integer programming (SIP) techniques.

### 2.3.3 Meta-heuristic approaches

Metaheuristic algorithms are generalised algorithms which use various search techniques in an attempt to find ‘good’ solutions to complex problems. Metaheuristics are not customised to each application instead using generalised search techniques which may be based on natural phenomena. An early approach by Denby and Schofield (1995) outlined a genetic algorithm which maximises value and minimises grade variance for multiple orebody simulations. However, the algorithm draws from a global geological risk distribution which does not consider spatial variability and interdependency.  Later developments were made by Godoy and Dimitrakopoulos (2004) that outlined an approach where Stable Solution Domains (SSDs) are used to generate schedule envelopes which are then used by a simulated annealing algorithm to choose the best combination that maximise system capacity and minimise the uncertainty response penalty. A significant improvement in NPV is recorded, however the implementation is difficult and lengthy. An alternative method by Goodfellow and Dimitrakopoulos (2015) outlines the use of a two-stage stochastic programming approach with simulated annealing, particle swarm optimization and differential evolution heuristics. The objective of the model is to maximise NPV with recourse penalties for deviations. Capacity constraints are handled by predefining lower and upper bounds on each step of the mining value chain. Non-linear relationships in stockpiling formulation can be handled. An application of their technique to a copper-gold mining complex demonstrates an increase in expected NPV of 6.6% over the deterministic equivalent design and 22.6% higher NPV on an industry-standard generated mine plan. Additionally, Goodfellow and Dimitrakopoulos (2015) notes that a production schedule that considers geological uncertainty can be substantially higher than that of a conventional solution as a direct result of managing the impact of risk to production targets. In a similar approach, Rahmanpour and Osanloo (2016) outline a method which uses MIP to optimise a set of mine schedules and possible price paths to generate a schedule. The preferred mine plan is then selected by a multi-criteria ranking of upside potential, downside risk and value at risk (VaR) criteria.

Lamghari et al. (2014) propose a metaheuristic based on variable neighbourhood descent with an efficient solution method which reportedly performs well on large scale problems. The objective is to maximise NPV and minimise the penalty function for production above target. An inequality is included to combine the slope and mining constraints to improve solution speed, but details are not provided. The heuristic process used contains three methods to, Exchange, Shift-After (t+1) and Shift-Before (t-1) a block and all its predecessors to different periods. In conjunction with the heuristic, the linear relaxation is used to provide an estimate of solution quality by measuring the gap variation as the problem is solved. From a case study, a reasonable solution can generally be found within a 2.5% of optimality gap.

Farmer (2016) outline a model to optimise the value chain including capacity decisions using metaheuristic methods that allow non-linear relationships. The algroithm takes 37 hours and 7 minutes to solve but cannot guarantee convergence on a mathamtically optimal solution. The scheudle that integrates capacity optimisation shows a 12% increase in value over the stochastic schedule that does not included capacity decisions. As an addition, the author also outlines a method to take a stochastic schedule and optimise the pushback shape, noting that further work would integrate the schedule and pushback shape selection.

Additional discussions on heuristic and meta-heuristic approaches are contained in Newman et al. (2010) and Osanloo et al. (2008).

### 2.3.4 Stochastic Integer Programming approaches

Stochastic Integer Programming (SIP) method includes elements of uncertainty explicitly in the optimisation and can generate solutions which guarantee optimality. Various authors have proposed SIP techniques (Ramazan and Dimitrakopoulos, 2004, Albor Consuegra and Dimitrakopoulos, 2009, Albor Consuegra and Dimitrakopoulos, 2010, Dimitrakopoulos et al., 2007, Asad and Dimitrakopoulos, 2012a, Asad and Dimitrakopoulos, 2012b, Gholamnejad and Moosavi, 2012, Kumral, 2012, Ramazan and Dimitrakopoulos, 2012, Koushavand et al., 2014, Lamghari et al., 2014, Moosavi et al., 2014, Rahmanpour and Osanloo, 2016). SIP models include a utility function to handle the risk trade-off whilst generating a solution which guarantees optimality. Broadly speaking, two algorithmic approaches have developed, the first, stochastic programming with recourse and the second, robust stochastic optimisation. The two methods differ by objective, in stochastic programming with recourse a decision is made with a sensitivity analysis conducted for some correction, whilst, robust stochastic optimisation incorporates the variability in the decision process and attempts to generate a solution which is insensitive to differences in simulations (Kumral, 2012).

Stochastic programming with recourse approaches were some of the first SIP models proposed. Ramazan and Dimitrakopoulos (2004) outline an approach that maximises NPV for each realisation of the orebody in a two stage optimisation. The first stage generates an optimal schedule for each orebody realisation. A probability is then calculated for each block, zero if it is not mined in any schedule in a given period and a one if it is mined in all schedules in that period. The second part of the model takes all blocks that have probabilities between zero and one into a new optimisation model that calculates the risk-weighted return for each block less a penalty for deviation. As such the objective attempts to maximise the probability of a block being scheduled in a period whilst the deviation penalty seeks to provide a practical schedule. This SIP model strikes a balance between extracting high-value and low-risk material at the beginning of a mine's life and deferring risker material till later periods. Gholamnejad and Moosavi (2012) outline an alternate SIP formulation with a two-part objective that maximises NPV and maximises the probability of a block being ore. Indicator Kriging is used to determine this probability. This results in blocks with a higher degree of confidence being brought forward in the schedule and blocks with low levels of confidence being sent to waste.

A robust stochastic approach is outlined by Moosavi et al. (2014) in which a SIP model produces a mining schedule that considers several equally probable realizations of the orebody model and multiple waste destinations or processing streams. The model optimises the minimization of the economic loss whilst satisfying the production capacity and precedence constraints.

However, SIP solutions are computationally demanding, which has led to more efficient approaches being proposed that use various techniques to return good solutions which may not necessarily be optimal. Ramazan and Dimitrakopoulos (2012) propose a clustering techniques to aggregate blocks, however assumes ore and waste blocks are an a-priori decision before the model is run. The method is applied to a case study of a Nickel deposit that demonstrates considering uncertainty in a mining project may lead to a non-feasible schedule that does not satisfy pre-determined grade or milling constraints when uncertainty is considered.

Models which use a SIP structure but metaheuristic techniques have also been suggested. Koushavand et al. (2014) outline a SIP model with an objective to maximise NPV and minimise cost of uncertainty. The cost of uncertainty is defined as the cost of overproduction and the cost of underproduction. Uniquely, an efficient Fuzzy c-mean clustering technique is proposed to make problem tractable, however does not guarantee optimality. An application to an Alberta sands operation with 50 orebody realisations is made and demonstrates a reduction in the cost of uncertainty when comparing the simulations with an OK model, however the NPV of the project is marginally less, likely due to the base case model overestimating the true value of the project.

### 2.3.5 Ultimate Pit Limit considerations

Optimising the mining schedule for uncertainty, fundamentally can change the solution to the UPL problem. An application of an uncertainty algorithm by Dimitrakopoulos and Abdel Sabour (2011) demonstrated by means of a case study that incorporating uncertainty could lead to an increase the UPL by 15% and increase project value by 10%. However, Meagher et al. (2009)

outlined some theoretical constraints of using a minimum cut algorithm for the optimal ultimate pit limit with commodity price and geological uncertainties. Whilst, Asad and Dimitrakopoulos (2012b) proposed a Lagrangian relaxation of resource constraints that generates a production-phase design and ultimate pit limit under commodity price and geological uncertainty. A heuristic approach is outlined by Deck et al. (2013) using the DeepMine™ algorithim (a metaheuristic algorithm) to evaluate mine design scenarios with price uncertainty whilst changing the UPL outline and system configuration based on fixed trigger conditions. The requirement for set trigger conditions that define when plant expansions occur can be difficult to define as a priori criteria. An application of this method to a case study on Codelco's Radmoric Tomic deposit indicates that an expansion option with flexibility increases expected NPV by $360M over the deterministic equivalent.

A recent network flow approach has been outlined by Chatterjee et al. (2016) which implements a minimum cut algorithm to the production-phase and UPL problem with uncertainty simulated by a smoothing spline and geostatistical sequential simulation of commodity price variation. Whilst including uncertainty in the UPL decision is preferred, computationally this makes the problem intractable to a MIP and may only add limited value compared with the benefits modifying the underlying system configuration.

### 2.3.6 Integrated Capacity Decisions

Most of these approaches outlined so far, rely on the capacity being fixed outside the optimisation process as a priori decision. Consequently, these models require manual iteration of mining and processing capacity options to determine if an optimal solution is being generated. This can lead to solutions which do not guarantee an optimised system configuration.

Some approaches to modifying the system configuration have been outlined to a degree in Montiel and Dimitrakopoulos (2015) and Asad and Dimitrakopoulos (2012a). Montiel and Dimitrakopoulos (2015) outline a model which uses a simulated annealing algorithm (a heuristic method based on the Metropolis algorithm) to iterate over different mining, processing and transport policies to maximise the NPV of the project less any penalties for deviation. Flexibility in the design is included by different operating policies for the processing and transport components. A case study demonstrates an increase in NPV of 5 percent over the base case. Importantly, the base case has large variations when uncertainty is considered, whilst the optimised strategy includes a penalty function to handle deviation in production and metallurgical targets which forces a limit on variation. However, the model does not include uncertainty in financial or utilisation parameters and cannot guarantee an optimal solution. Asad and Dimitrakopoulos (2012a) extend the traditional MIP cut-off grade optimisation to the case of multiple orebody realisations to select the process plant capacity that will maximise the NPV under constant economic conditions. Whilst these methods make some progress at defining flexible system configurations they do not handle capacity selection under uncertainty.

Recent research has attempted to close this gap by focusing on including the system configuration in the optimisation model (Cardin et al., 2008, Groeneveld et al., 2009, Groeneveld et al., 2010, Groeneveld et al., 2012). Further, building on the principles outlined by de Neufville (2006), robust, flexible and operational models have been proposed (Groeneveld et al., 2009, Groeneveld and Topal, 2011, Groeneveld et al., 2012). These models capture the full engineering system flexibility in the optimisation model by being able to adapt the mine plan in response to change. This thesis extends the advances of these papers by refining the mathematical formulation and including resource model uncertainty. Further, an application of these methods to a real case study is used to demonstrate the significant potential value increase that can be obtained.

# CHAPTER 3. METHODOLOGY

Operations research (OR) techniques are widely used to model real-world problems by mathematical formulation. They help decision makers by:

- Guiding the direction to solving problems;
- Prompting what questions should be asked;
- Focusing efforts on what the key value drivers are;
- Enabling comparisons between scenarios.

It has been determined that a mathematical formulation of the problem is possible. Generally an optimisation model consists of four components. These are (Hillier and Lieberman, 2010):

- An objective function – the function to be maximised or minimised;
- Data – inputs to the model;
- Decision variables – parameters that can be varied to optimise the objective function;
- Constraints – equalities and inequalities that define limits to the values of the decision variables.

Two Operations Research (OR) techniques, simulation and optimisation, was used in this research. Monte Carlo Simulation (MCS) and Conditional Simulation (CS) techniques were chosen to simulate equally probable outcomes based on uncertainty distributions of the input data. Similar approaches were made in the oil and gas industry (Zhang et al., 2007) and in other mining related literatrue .

These simulated values will then be used as inputs to a Mixed Integer Programming (MIP) model. Mixed Integer Programming (MIP) is a sub-category of Linear Programming (LP) where some decision variables are fixed as integer values. Linear Programming (LP) is an optimisation technique commonly used in Operations Research (OR) to determine the optimal solution to a mathematical problem as it can guarantee true optimality. In this research, MIP was used to optimise the selected design options and physicals mining schedule for a set of physical, financial and geological conditions. Integer variables are required to model the design options and mining timing.

The conceptual framework for modelling the problem with these techniques and the analysis tools used for examining the results will be discussed at the end of the chapter. Key terms used in this research are outlined in Appendix A.

## 3.1 SIMULATION

Simulation is commonly used in OR to estimate the performance of complex stochastic systems. It is a versatile technique which can be applied to almost any type of stochastic process (Hillier and Lieberman, 2010). Essentially, the system is modelled as if the random variables were known. The value of these random variables however is drawn from a probability distribution which estimates the underlying stochastic process. With enough simulations the model will closely replicate the probability distribution.

### 3.1.1 Monte Carlo Simulation (MCS)

Several researchers have written about modelling uncertainties in the mining industry and have suggested different types of distributions (Halbe and Smolik, 2003, Pindyck, 2004, Lima and Suslick, 2006, Hall and Nicholls, 2006, Reitz and Westerhoff, 2007, Doran and Ronn, 2008, Shafiee et al., 2009, Yaziz et al., 2013). These models stretch from complex models of supply and demand forces in a dynamic environment (Pindyck, 2004) to time series models using autoregressive conditional heteoskedatic models (Yaziz et al., 2013). Normal, lognormal, triangular, gamma and binomial distributions have also been suggested.

Monte Carlo Simulation (MCS) is as a technique for generating random variables. It was developed for analysis of physical phenomena in the Los Alamos project during World War II by Ulam and Metropolis (1949). The method is based on sampling values from distributions defined by Probability Density Function (PDF), as shown in Figure 11. MCS techniques are well suited to problems that are too laborious or impossible to be solved by closed form analytical equations. The underlying distributions behind a range of uncertainties in mining tend to be in this category.

The process implemented by MCS generally follows the following structure;

1) Define the underlying distribution as a Probability Density Function (PDF);
2) Turn this distribution into a Cumulative Density Function (CDF);
3) Draw a random number between 0 and 1 using methods such as pseudo-random or quasi-random number generation;
4) Determine the value that correlates to this probability value by using the CDF; and
5) Repeat this process for the number of trials or ‘states-of-the-world’ required.

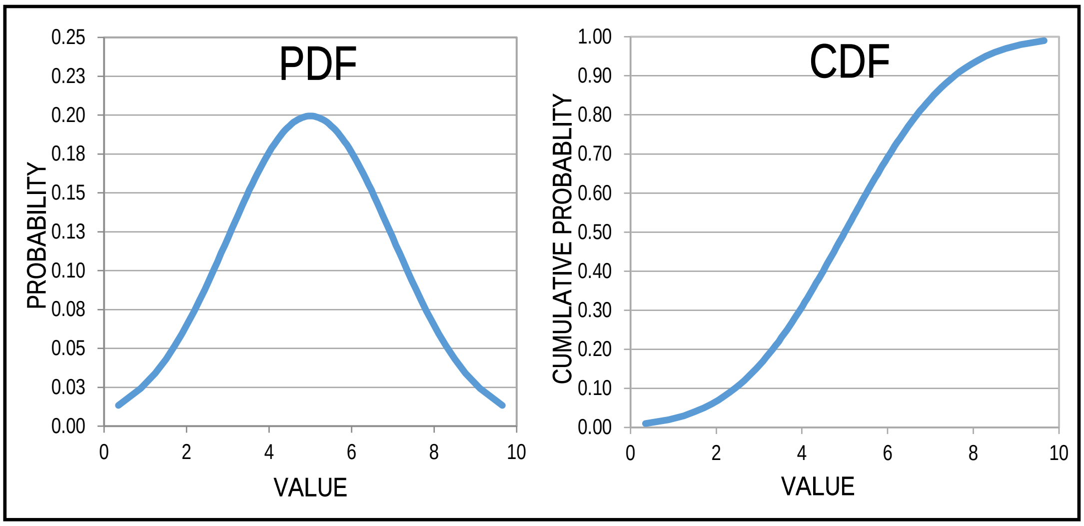



Figure 11: Example of a probability distribution function and corresponding cumulative distribution function for a normal distribution

Using MCS for different distributions is relatively straight forward with commercially available software packages; @Risk™ and Crystal Ball™ are the most well developed packages. Further, most have the ability to define custom functions that allows any type of distribution imaginable to be modelled. This ability to draw numbers in accordance with an underlying distribution gives the user greater flexibility in dictating how various ‘states-of-the-world’ are generated.

A number of alternative sampling methods have been developed in recent years. These include Latin hypercube sampling, polynomial chaos and bootstrapping (Oladyshkin and Nowak, 2012). Most of the improvement in these techniques centres on improving the speed of the sampling process and to a lesser degree the accuracy with which a distribution is reproduced.

For time dependenant uncertainties such as commodity prices and costs they can be classified as either having a constant trend or an average equilibrium level over tiem (Lemelin et al., 2006). For constant trend commodities, Geometric Brownian Motion (GBM) as shown in Equation 6 can be used to simulate the price change from one period to the next, whilst for average equilibrium commodities a Mean Reverting Process as shown in Equation 7 can be used (Shafiee et al., 2009, Aminul Haque et al., 2016).

$$dP = \alpha P\, dt + \sigma P\, dZ \tag{6}$$

$$dP = \eta\,(\ln \bar{P} - \ln P) P\, dt + \sigma P\, dZ \tag{7}$$

Where $P$ is the price of metal, $\alpha$ is the expected trend, $\sigma$ is the standard deviation, $dZ$ is an increment in a standard wiener process, $dt$ in an increment of time and $\eta$ is the speed at which the prices reverts to its long term equilibrium $\bar{P}$ (Azimi, et al. 2013). An important aspect of time series modelling is to simulate paths according to the Markov property (Dixit and Pindyck, 1994), where current and past time information is when forecasting future values.

In addition to the above, heteroskedasticity models with Auto-Regressive functions and Moving Average models have been proposed to model gold price and costs (Shaifee and Topal, 2010, Truck and Liang, 2012, Yaziz, et al. 2013, Kristjanpoller, et al. 2015). An Auto-Regressive Hetoroskedasticity model assumes that the next periods value is dependent on the previous periods error term with consideration for volatility clustering. The process for forecasting with an ARCH1 model is outlined in Engle (1982). A Moving Average Time series model is generally a linear regression of the current observations of the time series against random shocks of prior observations.

Akaike Information Criterion (AIC) is commonly used when comparing possible models as it provides an estimate of the relative strength of a stastical model for an input dataset. Alternatively, an ANOVA and *t* tests can be used however AIC is preferred (Burnham, et. Al. 2011). AIC seeks to recommend a maximum likelihood model whilst peanlising models for the addition of underlying parameters. AIC is calculated as per Equation 8, where K is the number of estimated parameters and $\mathcal{L}$ is the maximum likelihood (Burnham, et. Al. 2011).

$$\mathrm{AIC} = -2\,(\log \mathcal{L}\,) + 2\,K \tag{8}$$

In addition, to the selection of distributions, variables may be correlated with each other over time. This correlation is important to capture in simulation process. Two variables are fully correlated where it has a correlation of one i.e. if price increase by one unit than cost increase by a unti of one. Likewise, two variables are negatively correlated if they have an inverse relationship i.e. time operating is negatively correlated to time not operating. Thus, by correlating variables it is possible to ensure that variables do not drastically diverge and generate extreme outlier events. Correlation is mathematically defined as the Pearson product-moment correlation coefficient shown in Equation 9 (Puth, et al. 2014):

$$\mathrm{Correlation} = \frac{E\left[(X-\mu_x)\left(Y-\mu_y\right)\right]}{\sigma_x \sigma_y} \tag{9}$$

Where $E$ is the expected value, X and Y are random variables defined by there respective time series models, $\mu_x$and $\mu_x$ are average values of the data set and $\sigma_x$ and $\sigma_y$ are standard deviations of X and Y respectively.

Key parameters in a project generally have some degree of positive or negative correlation. For example, generally if the price of a commodity increases it is common that the cost of producing that commodity will likely increase due to an increase in input costs.

### 3.1.2 Conditional Simulation (CS)

Traditionally, Kriging is used to calculate a deterministic geological model, however, Kriging tends to over smooth the estimated values. This results in high-grade values being underestimated and low-grade values being over estimated, similar to that shown in Figure 12. Further, since Kriging provides a single point estimate for a block it does not indicate the possible variability of the orebody (Proulx et al., 2004).

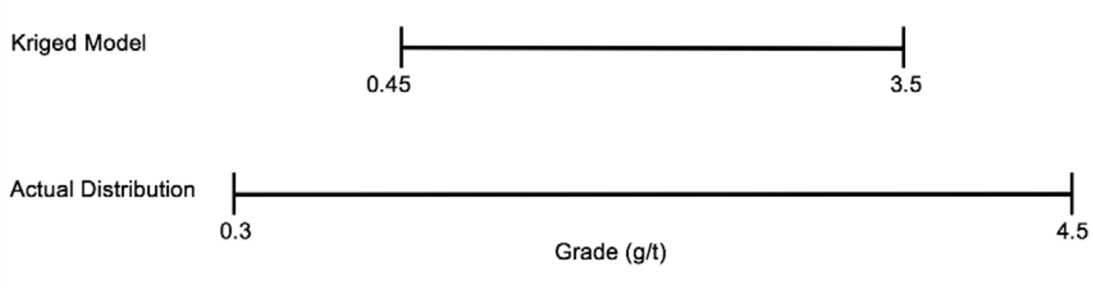



Figure 12: Example of difference between the range of Kriged and Actual model grades for an orebody showing a narrower range due to smoothing for the Kriged model

A key feature of geological estimation is that block values are positionally dependent and related to each other. Conditional simulation (CS) uses a Monte Carlo type simulation to model uncertainty in these spatially distributed attributes. It acknowledges that the ore grade between two known sampling locations cannot be exactly predicted, thus a range of possible values are generated (Ravenscroft, 1993). The simulation process generates equally probable realizations of the in-situ orebody that take into consideration all available information, including the distribution and spatial continuity of the mineralisation (Dimitrakopoulos, 1998). A large collection of conditionally simulated deposits describes the uncertainty of an orebody and tend to overcome the smoothing impact of Kriging models. The most common CS approaches are:

a) Turning Bands simulation;
b) Sequential simulation;
c) Gaussian simulation;
d) Simulating Annealing simulation.

The most common implementation of CS in the early days was contained in the GSLIB library (Deutsch and Journel, 1998). A lack of computing power and code efficiency was a bottleneck to the use of this method (Dimitrakopoulos et al., 2002). However, Dimitrakopoulos (1998) notes that simulation provides more credible models than kriging and whilst they require more time to implement, its ability to demonstrate geological uncertainty adds significant value to any operation.

The most common concept used for conceptually understanding CS is Sequential Gaussian Simulation (SGS). In this method, a conditionally simulated model is generated by sub-dividing a standard mining unit (SMU) or block model cell into a number of grid node points. The values of these grid node points will then be randomly generated according to the process below (Dimitrakopoulos, 1998):

1. Randomly select a grid node yet to be simulated;
2. Estimate the local conditional probability distribution (lcpd) of the grades at the grid node;
3. Randomly draw a grade value from the lcpd;
4. Include the simulated value in the conditioning data set;
5. Repeat points 1 to 4 until all grid nodes have a value;
6. Calculate the properties of all SMU blocks in the model by averaging all grid nodes; and
7. Repeat points 1 to 6 to generate additional equally probable models.

Improvements in the speed of the algorithm have been proposed with techniques such as direct block simulation, simulated annealing, generalize sequential Gaussian simulation and sequential Gaussian simulation (Dimitrakopoulos et al., 2002). These all seek to speed up the relatively slow process of drawing a random sample from an lcpd that is conditioned by every other grid node in the model.

### 3.1.3 Importance of Uncertainty Model Selection

The selection of an appropriate uncertainty distribution is a critical part of model development. It is important to realise that any distribution selection may not be totally representative of reality, thus actual results could be significantly different due to random error or a change in probablity of the underlying events. Additionally, there is also an element of objective selection based on ranking criteria and subjective selection based on specialised knowledge of the parameter.

In this thesis, an appropriate distribution is selected for the non-geological uncertainties using AIC ranking of alternative distributions and geological uncertainties have been modelled with judgment by an experienced geologist. It is acknowledged that there is a degree of subjectivity in the uncertainty distribution selection process which can impact model outcomes. A detailed distribution selection process was not undertaken as the aim of this thesis is to demonstrate a process to handle ‘given’ uncertainty.

## 3.2 LINEAR PROGRAMMING (LP) TECHNIQUES

Linear programming (LP) is a technique that efficiently optimises problems that are linear in nature. Linear variables have a constant rate of change to every other variable in the model. Graphically this is represented in Figure 13 by the solid line (a linear relationship) as opposed to the dashed line (a non-linear relationship). LP only handles linear variables, non-linear variables cannot be included in an LP optimisation. MIP (discussed later) can be used to approximate non-linear relationships by breaking a non-linear function into a piecewise linear function.

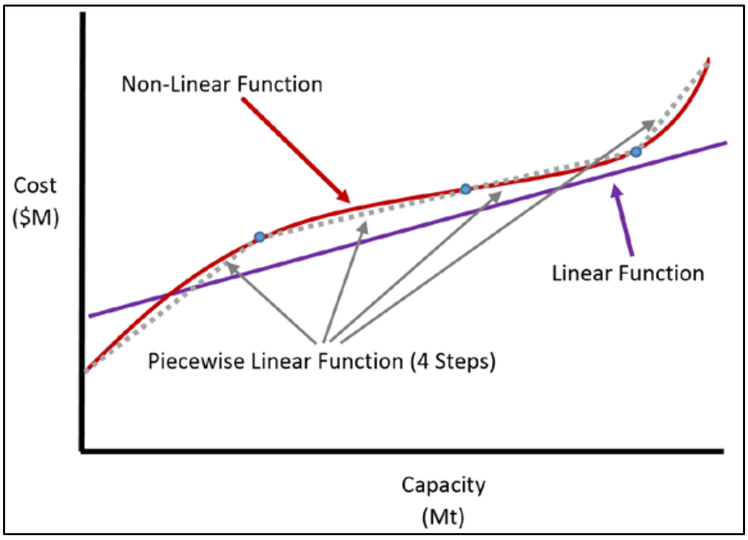



Figure 13: Capital Cost for varying capacities showing linear vs non-linear relationship

An LP model consists of an objective function of the form shown in Equation 10 (Topal, 2008a):

$$Maximise\ (or\ Minimise)\ Z = c_1x_1 + c_2x_2 + \ldots\ldots\ldots\ldots c_nx_n \qquad 10$$

Subject to a set of linear constraints, of the form shown in Equation 11:

$$a_{11}x_1 + a_{12}x_2 + \ldots\ldots a_{1n}x_n \leq b_1 \qquad 11$$

$$a_{21}x_1 + a_{22}x_2 + \ldots\ldots a_{2n}x_n \leq b_2$$

$$\ldots \quad \ldots \quad \ldots \quad \ldots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots\ldots a_{mn}x_n \leq b_m$$

And a set of non-negativity restrictions shown in Equation 12:

$$x_1, x_2, \ldots\ldots x_n \geq 0 \qquad 12$$

In this formulation, 'Z' is the objective function that needs to be maximised or minimised. The decision variables in this scenario are denoted by $x_j$. Constants are denoted by $c_j$ and $a_{ij}$ which are dictated by the nature of the problem. The constant $b_i$ is the right-hand side constraint, which may for example be the resource of $i$.

The optimal solution to an LP model can be solved by using a variety of algorithms; the most popular algorithm is the Simplex process (Dantzig, 1947). In short, the Simplex process selects an extreme point inside the feasible region of an LP problem, known as the Basic Feasible

Solution (BFS), then proceeds to a neighbouring BFS in a manner that increases (decreases in a minimisation problem) the value of the objective function. The inner workings of the most commonly used algorithm 'the Simplex method' is demonstrated further with the use of an online tool which steps the user through each part of the algorithm, available at: http://www.mathstools.com/section/main/simplex_online_calculator# (accessed 3 July 2017).

In practice, several factors impact the solution time of an LP, however in general, it is more dependent on the number of functional constraints in the model than the number of decision variables. A rule of thumb is that the number of iterations is roughly twice the number of functional constraints (Hillier and Lieberman, 2010).

### 3.2.1 Mixed Integer Programming (MIP)

Mixed Integer programming is similar to LP programming with the added restriction that some variables may be full integer values (i.e 1, 6 or 9) or binary variables (0 or 1) but not linear variables (i.e. 1.422, 5.67 or 9.21). A requirement to use MIP may arise in certain situations when some variables need to be linear whilst others represent discrete values whilst some variables are linear and some var discrete values, i.e. packages, people, trucks and decisions. Integer variables are generally used to model discrete items, such as the number of trucks allocated to a loader (you cannot have 0.3 of a truck). Binary variables are generally used to model yes/no or go/no-go decisions, such as to model capacity expansion options to represent when an option is being implemented (unique in this research) or to indicate when benches in a pit are fully mined. Alternatively and a unique approach developed in this research, is to use them to model capacity expansion options to represent when an option is being implemented.

The mathematical model is defined in the same manner as an LP with the additional restriction that some decision variables must be integers as shown in Equation 13:

$$x_1, x_2, \dots \dots . x_n \ are \ integers \qquad 13$$

MIP problems are more computationally difficult than an LP problem due to the restriction of integer decision variables. In MIP models the solution time is more dependent on the number of decision variables than the number of constraints, as more constraints can reduce the feasible solution space in the model (Hillier and Lieberman, 2010). Typically, the solution time is increased exponentially with additional decision variables.

MIP problems are generally solved using a Branch-and-Bound technique which implements a smart search process to reduce the solution time. A simplistic summary of the Branch-and-Bound algorithm follows. The first step is to solve the LP relaxation of the model. The LP relaxation is an LP version of the model where the integer constraint on decision variables is removed. An upper and lower bound on the variables may be introduced for a binary variable; lower bound of 0 and an upper bound of 1. If the decision variables in the solution are integer values then the optimal solution to the model is the LP relaxation solution. If the decision variables are non-integers, the algorithm attempts to enforce the integer restriction by assigning an integer value to either side of the non-integer value, creating a new upper and lower bound for the variable; branches of the decision variable. Two sub LP problems are then solved for the upper and lower bound case of the variable. This process then continues for all decision variables in the model until all branches have been explored and no better objective value is obtained. This process is known as the Branch-and-Bound process. At the start of this process a root node is chosen as the decision variable in which all branching will occur from. As the process goes, upper and lower limits of the objective value are determined and the algorithm seeks to narrow the gap until the upper and lower bound are equal. At this point the solution is optimal.

Various parameters can be set within the Branch-and-Bound algorithm to improve performance. These are generally outlined in the documentation of the solver and vary in mathematical robustness. Tuning these parameters can lead to a reduced solution time. The best way to improve solution time is to reduce the number of integers or restrict the feasible region (where this helps the solver establish tighter bounds). As a general rule, models with tens of thousands of integer variables can be solved within a few hours, models with over one hundred thousand integers may take many days, months or be impossible to solve for an optimal solution. Advances in computer technology will remove this barrier by significantly reducing the amount of time required to solve MIP problems. In particular, the movement of solver engines to cloud computing environments will allow an increase in computing resources allocated to solving a problem, reducing the amount of ‘real time’ required to solve a model. However, to date, very large MIP models cannot be solved by current computational technology.

## 3.3 MODEL FORMULATION CONCEPTS

A mathematical model was used in this research which simultaneously optimises four of the six elements as shown in Figure 14.

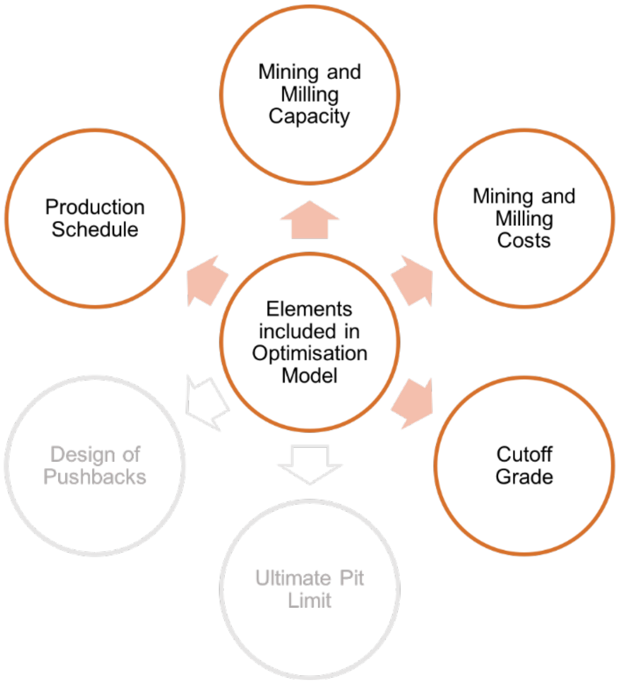



Figure 14: Mine design elements included in the optimisation model

Additionally, to model the mining system, several components needed to be integrated into the mathematical model. This following section explains how concepts of geological resource, design options, decision periods, stockpiling, product generation and key parameter uncertainty are included in the mathematical model.

### 3.3.1 Modelling the Mineable Resource

The mineable resource is represented by a model containing blocks of material with attributes describing the geological, financial and technical properties of each Standard Mining Unit (SMU; see Appendix A). The model can be divided into a set of parcels representing physical scheduling unit constraints that must be completed before the next parcel (a successor) can be commenced. Typically, a parcel will represent a bench of a pushback.

Scheduling dependencies are physical constraints related to the mining sequence. Typically, vertical dependencies are applied on a bench by bench basis within a pushback and horizontal dependencies are applied between benches of successive pushbacks. This concept of physical dependencies is depicted in a typical cross section of an open pit in Figure 15. Pushbacks are successive geometric shape increments (Pushback 1, Pushback 2 and Pushback 3) typically determined from paramertisation of the UPL problelm. Parcels are then generally benches within a pushback (P1_1, P1_2, P2_4, etc.).

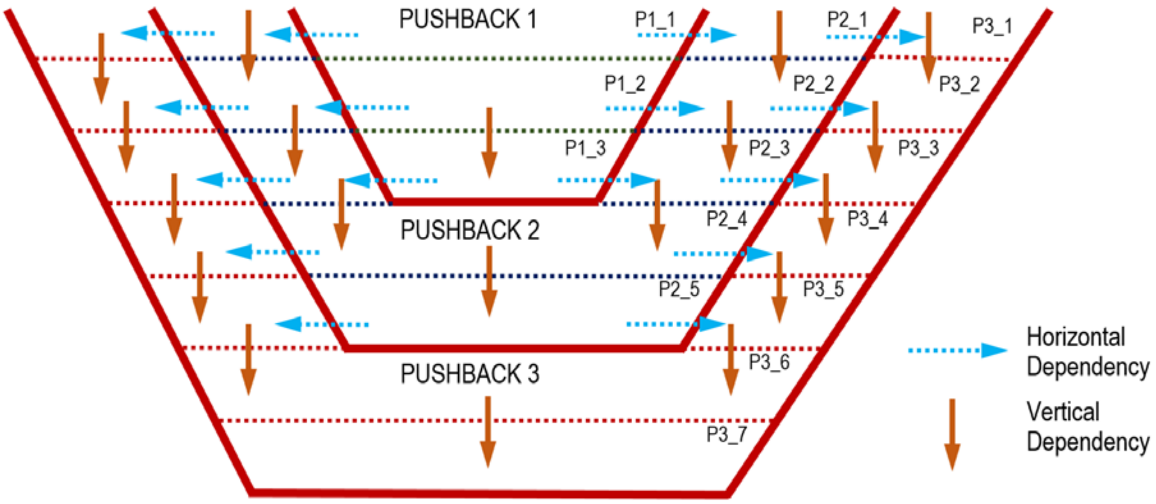



Figure 15: Example of parcel dependencies between pushbacks and parcels

Within a parcel, multiple SMU's which have similar characteristics can be grouped and treated as a single bin of material. Together all bins within a parcel sum to give the total physical and financial attributes of a parcel.

An example of a parcel with aggregated bins is shown in Figure 16. Note, although typical, a parcel does not need to be orthogonal in shape.

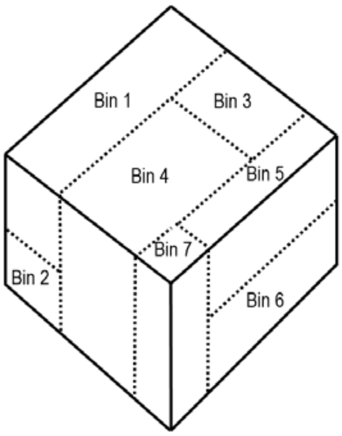



Figure 16: A parcel of material with seven bins

The concept of a bin within a parcel is analyagous to an increment within a grade tonnage distribution. Figure 17 shows the concept of a grade-tonnage curve increment and how the cutoff grade is applied. It can be seen that the cutoff grade location determines which material is sent to the mill or waste dump. It should be noted that a bin material can be scheduled to different destinations depending on the constraints in each period.

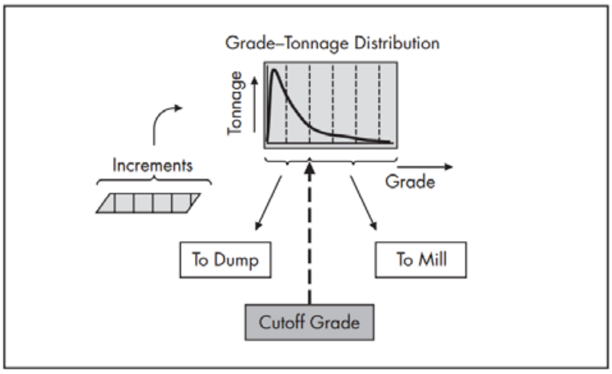



Figure 17: Increments within a grade-tonnage distribution (Ganguli et al., 2011)

Model complexity can be further reduced by aggregating bins of material that have similar physical and financial properties within a defined tolerance. For example, the grade bins in Table 4, can be combined into an aggregated bin without significantly impacting the overall optimality of the model as they group material that is similar together. This reduces the number of variables in the model and has a significant impact on the solution time of a model.

Table 4: Example of grade bin aggregation

| **Grade Bin** | **Tonnes (t)** | **Grade (Au g/t)** | **Oxcode (#)** | **Bin** | **Tonnes (t)** | **Grade (Au g/t)** | **Oxcode (#)** |
|---|---|---|---|---|---|---|---|
| Bin 1 | 2,700 | 0.63 | 1 | Aggregated Bin 1 | 16,300 | 0.60 | 1 |
| Bin 2 | 8,100 | 0.59 | 1 | | | | |
| Bin 3 | 5,500 | 0.60 | 1 | | | | |
| Bin 4 | 5,700 | 0.82 | 3 | Aggregated Bin 2 | 11,000 | 0.84 | 3 |
| Bin 5 | 1,500 | 0.84 | 3 | | | | |
| Bin 6 | 3,800 | 0.87 | 3 | | | | |

The aggregated bins in a parcel are mined in even ratios within a specified tolerance (usually 5%). A diagramtic representation of the schedule showing the dependency between the parcel and the equal mining of bins is shown in Figure 18.

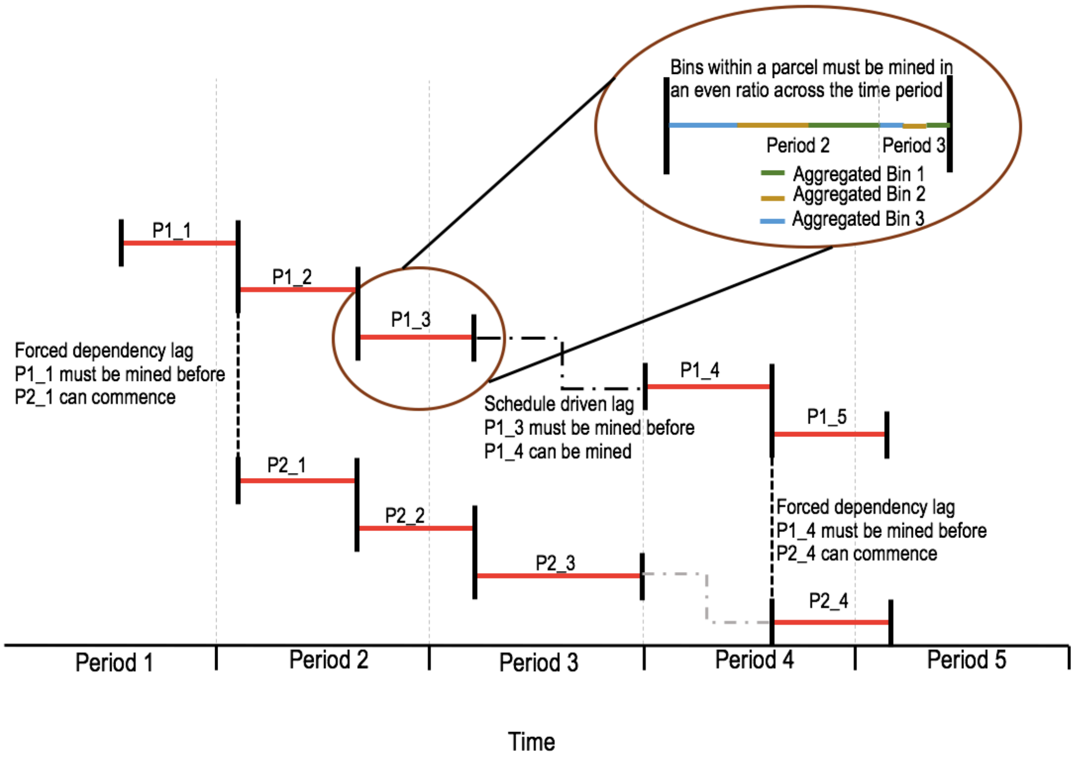



Figure 18: Impact of dependencies on the mining schedule and 'even' mining of bins within a parcel

In Figure 18, the dependencies are shown between parcel P1_2 and P1_1. This is due to P1_1 being located vertically above P1_2. Additionally, P1_3 shows mining of bins in an even ratio over time period 2 and period 3.

### 3.3.2 Design option flexibility

Four types of design options can be dynamically incorporated into the model. These are; mining capacity options, pre-processing stockpiling, processing plants options and capacity constraint options. Each option will be made available in the model and the optimisation process will determine the timing of their execution. Examining this will allow the decision maker to develop a flexible mine plan with the best set of options.

Each option type has different characteristics which need to be explained further. Mine options represent the physical extraction capacity that is required to move material from the pit. They will impose an upper bound on the annual tonnage or an effective flat haul (which considers the difference in flat vs inclined haulage) to move material from the pit. Pre-processing stockpiles are locations that material can be stored after extraction from the ground before processing. Typically these are used for either long term low grade scenarios or fluctuating demand scenarios. They can also be used to model different waste rock storage limits (or locations) if they are a restriction in the system. Processing plant options represent the physical and/or chemical process that ore can be sent to in order to remove gangue material. Processing plants may also contain multiple different circuits which the material may pass through. These circuits may have different beneficiation characteristics with individual capacity limits. Capacity constraint options represent physical constraints which may need to be modelled in the design. These may represent physical systems such as ports, loading facilities, crushers or conveyors. They may be incorporated at any point in the network.

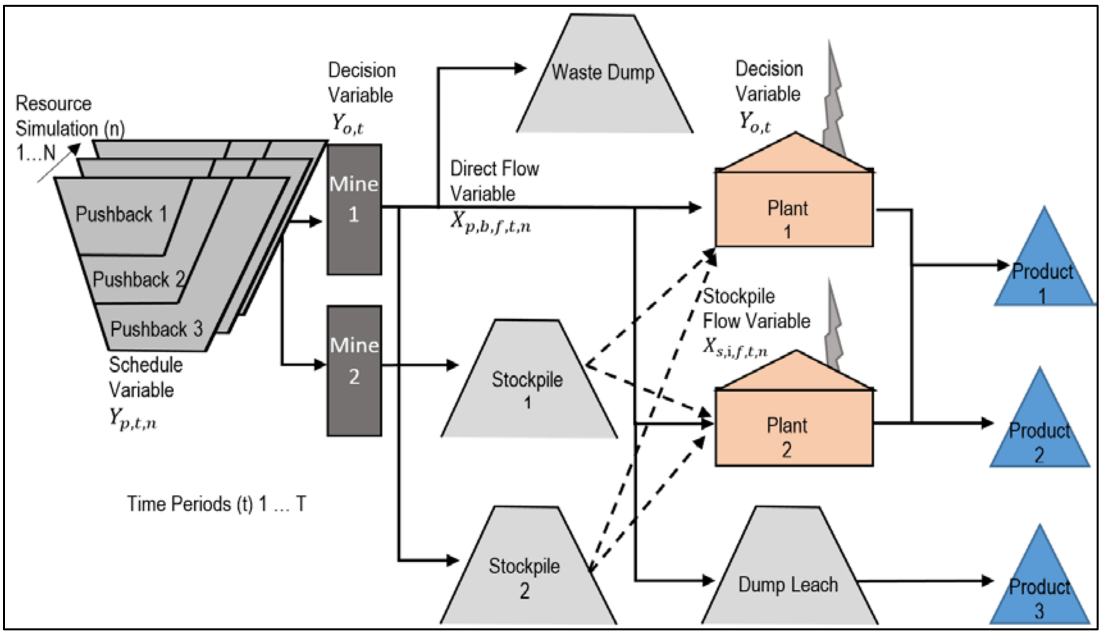



Figure 19: Example of how the flow network between design options is configured with the key decision variables shown

Most optimisation models proposed so far incorporate a minimum mining or processing limit in the optimisation process. This is an artificial constraint which is attempting to restrict the model to certain limits in order to maintain the validity of a certain fixed cost profile which is usually handled as a variable cost. In the proposed model, this can be handled explicitly by incorporating a fixed cost per period that the design option is active. When the model sees benefit in not operating the option the appropriate fixed cost will be removed (after incurring a cost of disposal). This helps more accurately model decisions around when to place an operation into care and maintenance or close it completely.

Each design option can have varying costs modelled, namely fixed, variable, disposal and capital costs. Fixed costs represent ongoing costs for operating the option in each period. Variable costs reflect unit costs for each tonne of material processed through the option and can be tailored based on material characteristics in the block model. Disposal costs are the cost to close down an asset, typically redundancies, clean-up and rehabilitation costs. Sometimes the disposal cost may be negative which would represent a positive payback due to salvage value being higher than the cost of disposal. Capital costs represent the upfront cost of executing a decision, usually purchase of equipment or construction of a plant.

### 3.3.3 Stockpiling Approach

A stockpile exists in the model for three primary purposes; a) it provides a storage location for lower grade material, allowing higher grades to be processed preferentially, b) it provides a source to enable blending to occur from over time and c) it provides a location to store waste material (a stockpile with no flow out path). Calculating the grade *g* on a stockpile is a non-linear problem as the quantity of metal *q* (unknown) needs to be divided by the tonnage of material $x$ (unknown). Since, $g = q/x$ is a non-linear equation this cannot be solved with linear programming. To overcome this issue, linear approximations are used to represent stockpiles. The approach used is to divide the stockpile into a number of grade bins *i* which have an upper and lower grade limit on material which can enter. With these limits, it is possible to determine every bin *b* within a parcel *p* that could report to the stockpile and thus calculate the weighted average grade of all of these bins. This weighted average grade is then used in the optimisation process as the attributes of the material leaving the stockpile.

An additional consideration to be included in the modelling of the stockpiling process is if any degradation of the materials grade on the stockpile will occur over time. For example, in an ore that contains gravity gold,there is potential for some of the gravity gold to wash away over time as rain water percolates through the stockpile. This can be adjusted by applying a *stockpile recovery factor* to the determination of the grade produced to a product from a stockpile bin.

### 3.3.4 Product output generation

The model can handle the generation of multiple products and within each product multiple value streams and grade limits can be setup. A value stream is defined by a revenue payoff function, tonnage recovery and a list of circuits that can produce the value stream. The revenue function determines the possible unit value of each parcel and bin in the resource model. The tonnage recovery sets the factor to be applied to the tonnage of material flowing into a product when determining the capacity limit for the product. Circuits defined for each product determine the possible circuits in a process plant that can generate a certain product. Grade limits are defined by a quantity function and upper and lower limit functions. These determine the factors to be applied to the tonnage of material from a bin within a parcel to determine the units of metal to be limited.

### 3.3.5 Modelling uncertainties in the optimisation model

The model incorporates uncertainty by a finitie number of simulations of various parameters by MCS and resource uncertainty by CS. Each simulation of these values represents a ‘state of the world’ that is equally probable in the future. Various parameters can be incorporated in the model including; price, capital cost, operating cost, equipment utilisation, recovery and time to build. Running a set of simulations is intended to give a representative sample of a future ‘state of the world’. In a robust model, multiple realisations are combined into one model with each simulation contributing to portion of the overall project value. Importantly the schedule can respond to change over time.

### 3.3.6 Time scaling in the optimisation model

The time steps in the model can be differently sized between periods. For example, the decision points could be quarterly intervals for the first two time periods, annual intervals for the next three time periods and biennial for the next time period, as shown in Figure 20. All costs and capacities are scaled for the time interval in the model generation process when generating the optimisation model.

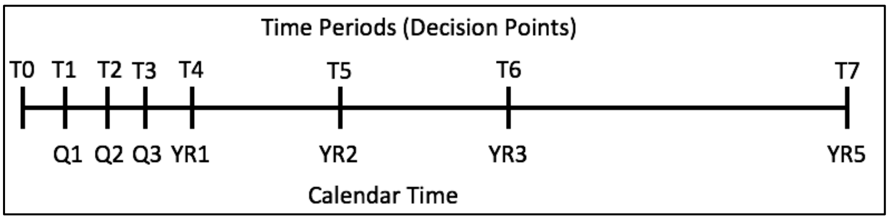



Figure 20: Time scale for key decision steps

## 3.4 TECHNIQUES FOR ANALYSING MODEL OUTPUT

A large amount of data is generated from each run about the configuration of the system and location of mining. Analysing this data to provide relevant information requires the use of certain techniques. To begin with an overview of how the system performs as a whole is best examined with a value-at-risk graph. Insights into how the flexibility in the system is configured can be gained from a frequency of execution table. Details on the location of mining over time can be determined from a frequency of mining table. The following sections provide further explanation of these methods.

### 3.4.1 Value-At-Risk Graph

A value-at-risk graph is a commonly adopted tool for determining the risk profile of multiple scenarios. Commonly it has been used in the financial industry for tasks such as, quantifying the payoff of insurance policies and determining the potential daily trading portfolio losses (Dowd, 1998, Jorion, 2007). References can also be found in the literature where it has been used in an engineering environment such as determining the geotechnical stability risk of slopes (Lai et al., 2009). An application of the method to the NPV outcome of system optimisation for the construction of a parking lot was suggested in de Neufville et al. (2005) (a similar approach was later extended in Cardin et al. (2008) to a mining complex). Further, it was used in Whittle et al. (2007) to quantity the NPV risk of the Marvin copper-gold project.

For the purposes of this research, a value-at-risk graph will display the probability of achieving a Net Present Value (NPV) for a system configuration. The x-axis displays the NPV from the minimum to maximum value, whilst the y-axis shows the probability of achieving at least the NPV of the x-axis. An NPV greater than zero means the system configuration generates a profit whilst an NPV less than zero generate a loss. For example in Figure 21 below, point A indicates 90% of the cases achieve an NPV greater than zero (profit), point B indicates 50% of the cases achieve an NPV exceeding \$67M and point C indicates only 10% of cases achieve an NPV greater than \$140M. Point B is the mode of the results – the point at which 50% of the simulations has a higher value and 50% a lower value. The Expected NPV (ENPV) is the average value of all the simulations. It is important to note that the 50% probability of achieving an NPV value (the mode) may be different to the ENPV (the mean). The dotted line represents a single point estimate of Net Present Value for a deterministic scenario; a scenario where only average values are used as inputs. Further, any scenario with a curve to the right of a given scenario represents one with a higher expected value, likewise a curve to the left represents a lower expected value scenario.

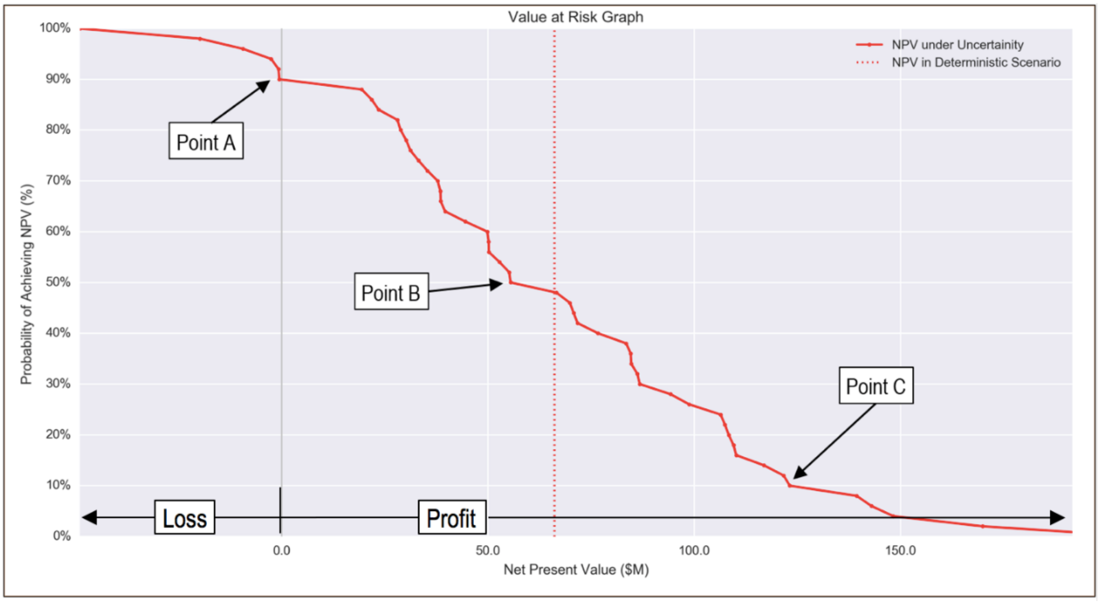



Figure 21: Example Value-at-Risk Graph

### 3.4.2 Frequency of Execution for Options

A frequency of execution table outlines the percentage of simulations in which a particular option is active over time. For example, an execution percentage of 60% would mean that in 60% of the simulated cases the option was active in that period. This means the option has an above average chance of being utilised in that period. Likewise, an execution percentage of 10% indicates that it is unlikely the option will be utilised in that period. This information can help identify which options are more frequently utilised and guide the decision makers focus on decisions which are most likely to occur and avoid allocating resources to options which are unlikely to occur. Equation 14 shows how frequency of execution is calcuated:

$$\text{Frequency of Execution } (\%) = \frac{\text{Number of simulations in which option is being utilised}}{\text{Total number of simulations}} \tag{14}$$

An example of a frequency of execution table is shown in Table 5. From this table it can be seen that Option A is utilised from Period 1 till Period 5 and then the option is disposed. Typically this would represent that a design option has been fixed in the system as there is no flexibility around when it has started or finished. Option B is utilised in 35% of simulations for the first two period, than increases to above 50% of the simulations in period three to five, before 52% of the simulations dispose of the option off in period six, with the remainder disposing of the option in period seven. Option C is not used at all in period one, barely used (<6%) in the next two periods, with a ramp up of usage in the subsequent period to 92% usage in periods six and seven. This suggests that option C is of interest in the later periods of the project when the uncertainties have 'played out'. Alternatively, this can be thought of as an option that should not be executed now but has a 'wait and see' decision process. Finally, option D is not used in periods one or two, with usage in subsequent periods less than a quarter. This indicates that the option adds little value to the system configuration and could be reasonable discarded from further work programs unless unlikely changes in forecast circumstances occur.

Table 5: Example of a Frequency of Execution for Options Table

| Option \ Decision Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Option A | 100 | 100 | 100 | 100 | 100 | | |
| Option B | 35 | 35 | 53 | 65 | 76 | 24 | |
| Option C | | 2 | 6 | 27 | 45 | 92 | 92 |
| Option D | | | 8 | 10 | 12 | 18 | 18 |

Frequency of Execution: 0% 20% 40% 60% 80% 100%

### 3.4.3 Frequency of Extraction for Parcels

A frequency of mining table displays the percentage of time a parcel is mined by a particular period. For example, a value of 50% indicates that in half the simulations the parcel is fully mined by that period. Likewise, a value of 100% indicates that the parcel has been fully mined in this or a previous period. With this information it can be determined by which period a given bench within a pit will be mined by. Additionally, this can be used to help determine when and in which areas of the pit one should focus efforts on to reduce uncertainty. Equation 15 shows the calculation for frequency of extraction:

$$\text{Frequency of Extraction (\%)} = \frac{\text{Number of simulations in which parcel is fully mined}}{\text{Total number of simulations}} \qquad (15)$$

An example of a frequency of mining table is shown in Table 6. The labelling on the y-axis consists of the pushback number (first digit) separated by the bench number (final 4 digits). Big jumps over multiple benches between periods would indicate a high degree of variability around that area of the resource. These areas of variability, which impact on the mining sequence, could be targeted with additional drilling to mitigate the uncertainty. Other areas of the pit that have a higher extraction progression may not actually impact the schedule and financials of the pit.

Table 6: Example of a Frequency of Extraction Table

| Parcel \ Decision Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1_2200 | 18 | 24 | 45 | 86 | 90 | 98 | 100 |
| 1_2180 | 18 | 24 | 45 | 86 | 90 | 98 | 100 |
| 1_2160 | 8 | 24 | 45 | 86 | 90 | 98 | 100 |
| 1_2140 | 4 | 20 | 39 | 86 | 90 | 98 | 100 |
| 1_2120 | 0 | 2 | 24 | 86 | 90 | 98 | 100 |
| 1_2100 | 0 | 0 | 0 | 80 | 88 | 98 | 100 |
| 1_2080 | 0 | 0 | 0 | 49 | 86 | 94 | 100 |
| 2_2220 | 6 | 14 | 29 | 80 | 90 | 98 | 100 |
| 2_2200 | 4 | 12 | 22 | 76 | 90 | 98 | 100 |
| 2_2180 | 0 | 8 | 12 | 57 | 88 | 98 | 100 |
| 2_2160 | 0 | 2 | 10 | 53 | 88 | 98 | 100 |
| 2_2140 | 0 | 0 | 10 | 53 | 88 | 96 | 100 |
| 2_2120 | 0 | 0 | 0 | 53 | 88 | 96 | 100 |
| 2_2100 | 0 | 0 | 0 | 49 | 86 | 94 | 100 |
| 2_2080 | 0 | 0 | 0 | 37 | 86 | 94 | 100 |
| 2_2060 | 0 | 0 | 0 | 35 | 86 | 94 | 100 |
| 2_2040 | 0 | 0 | 0 | 35 | 86 | 92 | 100 |
| 2_2020 | 0 | 0 | 0 | 0 | 0 | 0 | 96 |
| 2_2000 | 0 | 0 | 0 | 0 | 0 | 0 | 94 |

Frequency of Extraction: 0% 20% 40% 60% 80% 100%

# CHAPTER 4. MIP MODEL FORMULATION

This section will outline the MIP mathematical formulation used for evaluating design options under uncertainty. The model formulation is developed from the concepts outlined in Section 3.3. The same underlying mathematical model and principles are used for a fixed, flexible, robust and operational modes (refer to Table 7 and Appendix A for a comparison of modes), however they vary by the number of simulations included and whether some decision variables are pre-fixed. In a flexible, fixed or operational model, one simulation (N=1) of uncertainties is included in the optimisation, whilst in a robust model, N simulations of uncertainties (where N >1) are included in the model. Additionally, in a flexible and robust mode, all designs are determined by the optimisation, whilst in a fixed and operational mode some (or all) of the decisions are fixed outside of the optimisation.

## 4.1 MODEL FORMULATION

The terminology used in this model formulation is explained further in Appendix A.

### 4.1.1 Indices

| | |
|---|---|
| $t$ | time period |
| $\acute{t}$ | a period index used when iterating over a range of time periods; usually from the first period to $t$ the current period |
| $p$ | parcel number |
| $b$ | bin number within a parcel $p$ |
| $\acute{p}$ | number of successor parcel of $p$ |
| $\check{p}$ | number of predecessor parcel of $p$ |
| $g$ | grade element number |
| $n$ | simulation number |
| $o$ | design option number |
| $\acute{o}$ | number of option dependent on option $o$ being executed |
| $\hat{o}$ | number of option within a flow path before option $o$ |
| $c$ | number of circuit type in a processing plant |
| $f$ | flow path number |
| $s$ | stockpile number |
| $i$ | number of stockpile grade bin |
| $u$ | product type number |
| $v$ | product value stream number |

### 4.1.2 Sets

$\mathrm{B}_p$ set of all bins within the parcel $p$
$\mathrm{C}_o$ set of all circuits within the process plant option $o$
$\mathrm{F}_s$ set of possible flow paths from the stockpile $s$
$\mathrm{F}_p$ set of possible flow paths from the parcel $p$
$I_s$ set of all stockpile grade bins within the stockpile $s$
$\mathrm{M} \subset \mathrm{O}$ subset of mining options $o$ within all design options $\mathrm{O}$
$\mathrm{O}$ set of all design options $o$
$\mathrm{P}$ set of all parcels $p$
$\mathrm{PIT}_p \subset \mathrm{P}$ subset of parcels $p$ within a Pit or Pushback (a scheduling entity)
$\mathrm{PRE}_o \subset \mathrm{O}$ subset of options $o$ from all design options in $\mathrm{O}_o$ that are located before any stockpiling in the flow network
$\mathrm{POST}_o \subset \mathrm{O}$ subset of options $o$ from all design options in $\mathrm{O}_o$ that are located after any stockpiling in the flow network or paths that do not have any stockpiling
$\mathrm{S}$ set of all stockpiles $s$
$\mathrm{T}$ set of all time periods $t$
$\mathrm{D} \subset \mathrm{S}$ set of stockpiles $s$ that have no exit flow path (waste dumps)
$\mathrm{V}_u$ set of all value streams in a product from $d$

### 4.1.3 Parameters

$\mathrm{BE}_p$ the number of benches in a parcel $p$
$\mathrm{CAP}_{v,u}$ the maximum capacity of value stream $v$ in product $u$
$\mathrm{CAP}_{s,t}$ the stockpile capacity of stockpile $s$ at time period $t$
$\mathrm{GL}_{g,v,u}$ the lower grade limit of element $g$ in value stream $v$ in product $u$
$\mathrm{GU}_{g,v,u}$ the upper grade limit of element $g$ in value stream $v$ in product $u$
$\widehat{\mathrm{N}}$ the total number of simulations
$\mathrm{r}$ the discount rate
$\mathrm{W}_p$ the available tonnage of parcel $p$
$\mathrm{W}_{p,b}$ the available tonnage of parcel $p$ bin $b$
$\mathrm{SR}_t$ the sink rate in period $t$
$\delta$ the percentage tolerance a bin $b$ can be mined in an unequal proportion to its parcel $\mathrm{B}_p$

### 4.1.4 Stochastic Parameters

$CAP_{o,t,n}$ the maximum capacity of option $o$ at period $t$ in simulation $n$
$CAP_{c,o,t,n}$ the maximum capacity of circuit $c$ in option $o$ at period $t$ in simulation $n$
$CO_{o,t,n}$ the fixed cost of operating option $o$ at period $t$ in simulation $n$
$FD_{o,t,n}$ the disposal cost of option $o$ at period $t$ in simulation $n$
$FC_{o,t,n}$ the capital cost of option $o$ at period $t$ in simulation $n$
$V_{o,t,n}$ the unit variable cost of option $o$ at period $t$ in simulation $n$
$V_{s,t,n}$ the unit variable cost of material from stockpile $s$ at time period $t$ in simulation $n$
$G_{g,p,b,f,t,n}$ the metal units of grade element $g$ from parcel $p$ bin $b$ through flow path $f$ with at time period $t$ in simulation $n$
$G_{g,s,i,f,n}$ the calculated average units of grade element $g$ from stockpile $s$ stockpile grade bin $i$ through flow path $f$ in simulation $n$
$R_{v,u,t,n}$ the unit revenue of value stream $v$ in product $u$ at period $t$ (in \$/metal unit) in simulation $n$ less any selling costs
$K_{v,u,p,b,n}$ the tonnage recovery to value stream $v$ in product $i$ from parcel $p$ bin $b$ in simulation $n$
$K_{v,u,s,i,n}$ the calculated average tonnage recovery for all material able to report from stockpile $s$ stockpile grade bin $i$ to value stream $v$ in product $u$ in simulation $n$

### 4.1.5 Variables

$X_{p,b,f,t,n}$ the tonnage mined from parcel $p$, bin $b$ and sent through flow path $f$ at period $t$ in simulation $n$

$X_{s,\mathrm{i},f,t,n}$ the tonnage of material from stockpile $s$ stockpile grade bin $i$ along flow path $f$ at period $t$ in simulation $n$

$$Y_{o,t} = \begin{Bmatrix} 1\ldots n, \text{number of options } o \text{ on at period } t \text{ (if only single execution max 1) ;} \\ \\ 0, \text{option } o \text{ not active in period } t. \end{Bmatrix}$$

$$Y_{p,t,n} = \begin{Bmatrix} 1, \text{if parcel } p \text{ is mined by period } t \text{ in simulation } n; \\ \\ 0, \text{parcel } p \text{ is not mined by period } t \text{ in simulation } n. \end{Bmatrix}$$

$CC_{o,t,n}$ the total capital cost of option $o$ at period $t$ in simulation $n$

$CD_{o,t,n}$ the total disposal cost of option $o$ at period $t$

### 4.1.6 Objective function

The objective function, shown in Equation 16, seeks to maximise the equally weighted before tax net present value (NPV) for all simulations (or a single case for a flexible model). The before tax net present value is calculated by adding the revenue (price function multipled by recovered tonnage) together

$$
\begin{aligned}
\sum_{n=1}^{\mathrm{N}} \frac{1}{\widehat{\mathrm{N}}} \Bigg( \sum_{t=1}^{\mathrm{T}} \frac{1}{(1+\mathrm{r})^{t}} \Bigg[ & \sum_{p,b,f=1|v,u\in f}^{\mathrm{P},\mathrm{B}_p,\mathrm{F}_p} R_{v,u,t,n} X_{p,b,f,t,n} + \sum_{s,i,f=1|v,u\in f}^{\mathrm{S},\mathrm{I}_s,\mathrm{F}_s} R_{v,u,t,n} X_{s,i,f,t,n} \\
& - \sum_{p,b,f,=1|o\in f}^{\mathrm{P},\mathrm{B}_p,\mathrm{F}_p} V_{o,t,n} X_{p,b,f,t,n} - \sum_{s,i,f=1}^{\mathrm{S},\mathrm{I}_s,\mathrm{F}_s} V_{s,t,n} X_{s,i,f,t,n} \\
& - \sum_{o=1}^{\mathrm{O}} CO_{o,t,n} Y_{o,t} - \sum_{o=1}^{\mathrm{O}} CC_{o,t,n} - \sum_{o=1|t\neq 1}^{\mathrm{O}} CD_{o,t,n} \Bigg] \Bigg)
\end{aligned} \qquad 16
$$

The constraints in the model can be divided into five categories: production, design option, stockpiling, product and additional limiting bound constraints.

### 4.1.7 Production constraints

$$\sum_{f,t=1}^{\mathrm{F}_p,\mathrm{T}} X_{p,b,f,t,n} \leq \mathrm{W}_{p,b} \qquad \text{for } \forall\, p,b,n \qquad 17$$

$$\sum_{b,f,\acute{t}=1}^{\mathrm{B}_p,\mathrm{F}_p,t} X_{p,b,f,\acute{t},n} \geq \mathrm{W}_p Y_{p,t,n} \qquad \text{for } \forall\, p,t,n \qquad 18$$

$$\sum_{b,f,\acute{t}=1}^{\mathrm{B}_p,\mathrm{F}_p,t} X_{\acute{p},b,f,\acute{t},n} \leq \mathrm{W}_{\acute{p}} Y_{p,t,n} \qquad \text{for } \forall\, p,t,n \qquad 19$$

$$Y_{p,t-1,n} \leq Y_{p,t,n} \qquad \text{for } \forall\, p,t \neq 1,n \qquad 20$$

$$\mathrm{BE}_p Y_{p,t,n} \leq \mathrm{SR}_t \qquad \text{for } \forall\, p \in PIT_p,t,n \qquad 21$$

$$\frac{1}{\mathrm{W}_{p,b}} \sum_{f=1}^{\mathrm{F}_p} X_{p,b,f,t,n} - \frac{1}{\mathrm{W}_p} \sum_{b,f=1}^{\mathrm{B}_p,\mathrm{F}_p} X_{p,b,f,t,n} \leq \delta \qquad \text{for } \forall\, p,b,t,n \qquad 22$$

$$\frac{1}{\mathrm{W}_p} \sum_{b,f=1}^{\mathrm{B}_p,\mathrm{F}_p} X_{p,b,f,t,n} - \frac{1}{\mathrm{W}_{p,b}} \sum_{f=1}^{\mathrm{F}_p} X_{p,b,f,t,n} \leq \delta \qquad \text{for } \forall\, p,b,t,n \qquad 23$$

The production constraints limit what quantities and grades of material can be produced and when they can be produced. Section 3.3.1 outlines how the mineral resource is modelled. The amount of material extracted from a bin in a pit has an upper bound based on the resource model which is constrained by 17. Constraint 18 and 19 enforce the physical scheduling constraints. Constraint 20 ensures that once a parcel is mined it is flagged as being mined in that period and all later periods. Constraint 21 sets a maximum sink rate constraint that ensures that the turnover of a bench in a pushback does not exceed the nominated value (usually 12 benches per pushback per year). Two further optional constraints (22 and 23) try to minimise grade variability by restricting the model to mine low grade and high grade material within a parcel in an even ratio within a tolerance of $\delta$. It should be noted that whilst this thesis focuses on open pit mining, the nature of these constraints allow for any successor and predecessor relationships to be modelled and could be extended to underground mining with relative ease.

### 4.1.8 Design Option constraints

$$\sum_{p,b,f=1|o\in f}^{\mathrm{P,B}_p,\mathrm{F}_p} X_{p,b,f,t,n} \leq CAP_{o,t,n} Y_{o,t} \qquad \text{for } \forall\ o \in \mathrm{PRE}_o, t, n \qquad 24$$

$$\sum_{p,b,f=1|o\in f}^{\mathrm{P,B}_p,\mathrm{F}_p} X_{p,b,f,t,n} + \sum_{s,i,f=1|o\in \mathrm{f}}^{\mathrm{S,I}_s,\mathrm{F}_s} X_{s,i,f,t,n} \leq CAP_{o,t,n} Y_{o,t} \qquad \text{for } \forall\ o \in \mathrm{POST}_o, t, n \qquad 25$$

$$\sum_{p,b,f=1|c,o\in f}^{\mathrm{P,B}_p,\mathrm{F}_p} X_{p,b,f,t,n} + \sum_{s,i,f=1|c,o\in f}^{\mathrm{S,I}_s,\mathrm{F}_s} X_{s,i,f,t,n} \leq CAP_{c,o,t,n} Y_{o,t} \qquad \text{for } \forall\ \mathrm{c}, o \in \mathrm{POST}_o, t, n \qquad 26$$

$$FC_{o,t,n} Y_{o,t} - FC_{o,t,n} Y_{o,t-1} \leq CC_{o,t,n} \qquad \text{for } \forall\ o, t, n \qquad 27$$

$$FD_{o,t,n} Y_{o,t} - FD_{o,t,n} Y_{o,t-1} \leq CD_{o,t,n} \qquad \text{for } \forall\ o, t, n \qquad 28$$

$$Y_{\acute{o},t} \leq Y_{o,t-1} \qquad \text{for } \forall\ o, t \neq 1 \qquad 29$$

The above constraints handle the design options in the model. Eac h design option is reflected in the model by an integer variable (if it can be executed multiple times) or a binary variable (if it can be executed only once). Multiple execution is used for design options that can be duplicated and have the same underlying characteristics, capacity and cost. An example would be mining fleet capacity which can easily be increased by adding more trucks and excavators. An upper capacity limit for each design option is set through constraint 24 for any design option occurring before a stockpile option in a flow path and constraint 25 for any design option occurring after a stockpile. The difference is the inclusion of material flowing out of a stockpile in constraint 25. Similarly, constraint 26 applies a capacity limit to individual circuits within a process plant. Constraint 27 and 28 apply the capital cost and disposal cost of a design option in period *t.* Option dependency relationships are handled through constraint 29 which ensures a successor (child) design option cannot be built before its predecessor (parent).

### 4.1.9 Stockpiling Constraints

$$\sum_{f,\acute{t}=2|s,i\in f}^{\mathrm{F}_s,t-1} X_{s,i,f,\acute{t},n} \leq \sum_{p,b,f,\acute{t}=1|s,i\in f}^{\mathrm{P},\mathrm{B}_p,\mathrm{F}_p,t} X_{p,b,f,\acute{t},n} \quad \text{for } \forall\ s,i,t \neq 1,n \qquad 30$$

$$\sum_{p,b,f,\acute{t}=1|s,i\in f}^{\mathrm{P},\mathrm{B}_p,\mathrm{F}_p,t} X_{p,b,f,\acute{t},n} - \sum_{f,\acute{t}=2|s,i\in f}^{\mathrm{F}_s,t} X_{s,i,f,\acute{t},n} \leq \mathrm{CAP}_{s,t} \quad \text{for } \forall\ s,i,t,n \qquad 31$$

Stockpiling is handled in the model through the use of variables that track the flow of material into a stockpile bin and out of a stockpile bin. The stockpile grade bin which material can flow into is predetermined in the model generation phase based on the upper and lower grade limits of the . Each grade bin in the resource has exactly one stockpile bin it can enter based on the upper and lower grade limit of the primary element used to enforce the grade limit. Constraint 30 ensures the material leaving the stockpile must not exceed the material entering the stockpile. Constraint 31 limits the total amount of material on a stockpile to the stockpiles' capacity.

An imperative aspect is determining the stockpile bins that should be setup. The number of bins in the stockpile does not adversely impact the model as it creates one additional variable per period. When determining the range of the stockpile bin a general principle is that the bin should be half the difference between the cost of reclaiming the material and the value of the bin. For example if the stockpile cost is \$1.50 and the gold price is \$600/oz at a recovery of 90% then the bin width should be less than 2 x (1.50 / \$17.36g/t) = 0.173g/t. Thus the bin width should be less than 0.17g/t. This is to minimise the chance the optimiser is sending material to a stockpile to increase the grade as it should be penalised for sending material to the stockpile. Note if the operation is process limited this may not necessarily hold true as the optimiser may prioritise lower grades in order to pick up a marginal grade increase by sending material through a stockpile where it gets the calculated average on exit.

### 4.1.10 Product Constraints

$$\sum_{p,b,f=1|v,u\in f}^{\mathrm{P},\mathrm{B}_p,\mathrm{F}_p} K_{v,u,p,b,n} X_{p,b,f,t,n} + \sum_{s,i,f=1|v,u\in f}^{\mathrm{S},\mathrm{I}_s,\mathrm{F}_s} K_{v,u,s,i,n} X_{s,i,f,t,n} \leq \mathrm{CAP}_{v,u} \quad \text{for } \forall\ v,u,t,n \qquad 32$$

$$\begin{aligned} &\sum_{p,b,f=1|v,u\in f}^{\mathrm{P},\mathrm{B}_p,\mathrm{F}_p} \mathrm{GL}_{g,v,u} X_{p,b,f,t,n} + \sum_{s,i,f=1|v,u\in f}^{\mathrm{S},\mathrm{I}_s,\mathrm{F}_s} \mathrm{GL}_{g,v,u} X_{s,i,f,t,n} \\ &\leq \sum_{p,b,f=1}^{\mathrm{P},\mathrm{B}_p,\mathrm{F}_p} G_{g,p,b,f,t,n} X_{p,b,f,t,n} + \sum_{s,i,f=1}^{\mathrm{S},\mathrm{I}_s,\mathrm{F}_s} G_{g,s,i,f,n} X_{s,i,f,t,n} \end{aligned} \quad \text{for } \forall\ g,v,u,t,n \qquad 33$$

$$\sum_{p,b,f=1|v,u\in f}^{\mathrm{P,B}_p,\mathrm{F}_s} \mathrm{GU}_{g,v,u} X_{p,b,f,t,n} + \sum_{s,i,f=1|v,u\in f}^{\mathrm{S,I}_s,\mathrm{F}_s} \mathrm{GU}_{g,v,u} X_{s,i,f,t,n} \geq \sum_{p,b,f=1|v,u\in f}^{\mathrm{P,B}_p,\mathrm{F}_s} G_{g,p,b,f,t,n} X_{p,b,f,t,n} + \sum_{s,i,f=1|v,u\in f}^{\mathrm{S,I}_s,\mathrm{F}_s} G_{g,s,i,f,n} X_{s,i,f,t,n} \quad \text{for } \forall\ g,v,u,t,n \qquad 34$$

The system configuration can produce multiple products allowing different marketing strategies to be analysed. Constraint 32 provides an upper bound to the amount of material that can be produced for any given value stream where a tonnage recovery factor is applied to the material processed. Constraint 33 applies a lower bound grade limit to the material that can be produced into a value stream of a product. Constraint 34 enforces an upper bound grade limit to the material that can be produced into a value stream of a product. In both cases, the grade element is calculated from the bin in the resource for material not flowing through a stockpile and for a stockpile, from the average of the possible grade elements that could report, from the resource, to the stockpile bin.

### 4.1.11 Additional Model Bounds

$$Y_{o,t} - \sum_{\acute{t}=1}^{t} Y_{\hat{o},\acute{t}} \leq 0 \quad \text{for } \forall\ o,t \qquad 35$$

$$\sum_{\acute{t}=1}^{t} Y_{\acute{p},\acute{t},n} \leq \sum_{\acute{t}=1}^{t} Y_{p,\acute{t},n} \quad \text{for } \forall\ p,t,n \qquad 36$$

$$Y_{p,t,n} \leq \sum_{\acute{t}=1,o=1|o\in M}^{t,O} Y_{o,t} \quad \text{for } \forall\ p,o,t,n \qquad 37$$

Constraints 35, 36 and 37 help to further bound the optimal solution space of the model in the optimisation process and do not violate the optimality. This is similar in concept to applying additional cutting planes shown in Figure 22. Essentially these constraints seek to use specific knowledge of the mining problem structure instead of relying solely on generic cutting planes built into the optimisation solver. They are particularly useful in the improving the quality of the solution in the linear relaxation of the problem.

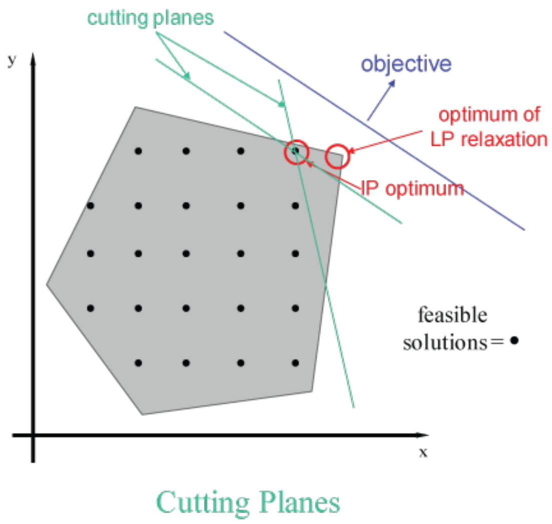



Figure 22: Additional cutting planes to limit the optimal region of a MIP model (Gurobi)

The first cut (Equation 35) requires any option $\hat{o}$ that exists in a flow path before option *o* to be built at or before the option *o*. For example, in a simple setup of one mine and plant producing one product, $\hat{o}$ would be the mining option and *o* would be the plant option, thus the plant could not be built until the mining option had been executed. The second cut (Equation 36) further enforces the parcel sequencing constraint by constraining the dependent parcels *p'* to be less than or equal to the parent parcel *p*. Finally, the third cut (Equation 37) restricts mining of any parcel before a mining option has been built. However, this logic cannot be extended to a processing plant as material could be pre-striped and placed on the stockpile awaiting construction of a processing plant.

### 4.1.12 Non-negativity and Integrality Constraint

Constraints 38 to 43 enforce non-negativity and integrality of the variables, as appropriate.

$$FC_{o,t,n} \geq 0 \qquad \forall o,t,n \qquad 38$$

$$FD_{o,t,n} \geq 0 \qquad \forall o,t,n \qquad 39$$

$$X_{p,b,f,t,n} \geq 0 \qquad \forall p,b,f,t,n \qquad 40$$

$$X_{s,i,f,t,n} \geq 0 \qquad \forall s,i,f,t,n \qquad 41$$

$$Y_{o,t} \text{ binary for single execution or integer for multiple} \qquad \forall o,t \qquad 42$$

$$Y_{p,t,n} \text{ binary} \qquad \forall\, p,t,n \qquad 43$$

## 4.2 DIFFERENT MODEL FORMULATIONS

The above model formulation can be used in three different forms; flexible, robust or operational modes. In a flexible mode the number of simulations is limited to one, such that the model will represent the optimum design for exactly one state of the world. Uncertainty is analysed in this model by running multiple models (~50-100) of a given design option and post optimisation data analysis is used to understand which options are executed most frequently and the risk profile associated with the project. A robust model is a stochastic linear programming where multiple simulations with equal probability of occurrence are used in the same optimisation model with an equal contribution to the objective function. This provides a recommended design that handles changes between these simulations the best. Using 3 simulations from the $25^{th}$, $50^{th}$ and $75^{th}$ percentile outcomes of the flexible model, provides an adequate trade-off between solution time and result quality. The solution time increases exponentially as the number of simulations in the model increase. An operational model is developed after both a flexible and a robust model are analysed. It represents the key insights for the decision maker and creates a practical development pathway for the initial periods of the project whilst allowing flexibility in the later periods to account for uncertainty. Typically, the designer would lock in the first two or three periods and keep the later periods flexible so that the model can react to change. If conditions deteriorate the optimiser can dispose of the option, removing the fixed cost liability. Table 7 provides a comparison of the role of each variable in each mode and explains how it differs.

Table 7: Comparison of model modes

| | **Fixed mode** | **Robust mode** | **Flexible mode** | **Operational mode** |
|---|---|---|---|---|
| Simulation | Single simulation *n* | Multiple in one model 1 to N (or subset) | Single simulation *n* | Single simulation *n* |
| $FC_{o,t,n}$ | Represents the capital cost of simulation $n$ | Represents the weighted average capital cost of each simulation in N | Represents the capital cost of simulation $n$ | Represents the weighted average capital cost of each simulation in N |
| $FD_{o,t,n}$ | Represents the disposal cost of simulation $n$ | Represents the weighted average disposal cost of each simulation in N | Represents the disposal cost of simulation $n$ | Represents the weighted average disposal cost of each simulation in N |
| $X_{p,b,f,t,n}$ and $X_{s,\mathrm{i},f,t,n}$ | Schedule will be unique for each simulation $n$ | Schedule will be unique for each simulation $n$ | Schedule will be unique for each simulation $n$ | Schedule will be fixed for initial time periods (say 3 years) then flexible and unique for each simulation n after the fixed period |
| $Y_{o,t}$ | Fixed design input to model i.e. variables set to 1 or 0 for each period | Optimal design decision for all simulations in N that handles variation the best. | Unique design developed for each simulation $n$ | Fixed design for the initial periods (say 3 years) then flexible for periods afterwards |
| $Y_{p,t,n}$ | Schedule will be unique for each simulation $n$ | Schedule will be unique for each simulation $n$ | Schedule will be unique for each simulation $n$ | Schedule will be fixed for initial time periods (say 3 years) then flexible and unique for each simulation $n$ after that fixed period |
| Advantages | Clear path forward due to fixed system configuration | System configuration is designed to handle change the best | Obtains an optimal system configuration for the simulation | Fixed configuration and clear pathway for initial periods with flexibility maintained in back end of the schedule |
| Disadvantages | System configuration will not react to change | Model is large and slow to solve | Assumes perfect knowledge is available to decision makers | Requires manual input of the initial fixed configuration which requires running of the other modes to determine the best configuration |

## 4.3 STRATEGIES FOR PROBLEM SIZE REDUCTION

The optimisation model contains the basic formulation to get the optimal solution to the problem. Solving for optimality is possible with this formulation, however the computation time can be very long. Investigation of several strategies was undertaken to reduce the number of variables in the mathematical model formulation. These strategies all ensure that optimality of the solution is not violated.

### 4.3.1 Size Reduction Algorithms

Size reduction algorithms focus on removing variables from the model that do not have the potential to contribute to the optimal solution. To implement this the program predetermines all variables required by the optimisation model and then runs the size reduction search to turn off variables. Additional model indicies and variables are defined below:

| | |
|---|---|
| $e$ | sorted index |
| $\mathrm{RW}_{e,t}$ | list of *e* sorted on unit revenue and tonnage for period *t* |
| $ABE_p$ | number of benches required to access parcel *p* |
| $\boldsymbol{CSR_t}$ | cumulative strip ratio in time period *t* |
| $\mathrm{MC}_{t,n}$ | maximum mining capacity in time period *t* for simulation *n* |
| $PV_{p,b,f,t,n}$ | maximum financial value through parcel *p* bin *b* flow path *f* in time period *t* for simulation *n* |
| $PV_{o,t}$ | maximum financial value through option *o* in time period *t* |
| $PV_{o,t,n}$ | maximum financial value through option *o* in time period *t* for simulation *n* |
| $PV_{s,f,t,n}$ | maximum financial value through stockpile *s* flow path *f* in time period *t* |
| $XM_{t,n}$ | mined tonnage in time period *t* simulation *n* |

#### 4.3.1.1 Late start period constraint on execution of option due to remaining time to payback option

When, in period *t*, the maximum value of the remaining resource (assuming nothing has been extracted up to period *t*) is less than the cost of developing and operating design option *o*, then design option cannot be executed in or after period *t*. This can be implemented as an additional constraint on the design option decision variable in period *t*, to not exceed the decision variable value in period *t-1*. A pseudocode outline of the algorithm follows, with a programatic outline in Appendix B:

1. Iterate for each simulation in *N*
2. Iterate for all design options in *O*, for all time periods in *T* and
   a. Generated a two variable list sorted by maximum price with the tonnage for each bin *b* in parcel *p* at that price, as in Equation 44.

$$RW_{e,t} = \left\{\left(max_1^t\left(R_{v,u,t,n}\right), W_{p,b}\right)_{max}, \cdots, \left(max_1^t\left(R_{v,u,t,n}\right), W_{p,b}\right)_{min} \mid p \in P \wedge b \in B\right\} \forall t \qquad 44$$

b. Determine the maximum value $PV_{o,t,n}$ through design option *o* by calculating the maximum revenue possible at the maximum possible capacity less the minium variable cost of the possible flow paths less the capital cost and disposal cost of the option (Equation 46);

$$PV_{o,t,n} = max\left(\mathrm{RW}_{e,t}(R_{v,u,t,n}) \times \mathrm{RW}_{e,t}(W_{p,b}) \text{ where,} \sum \mathrm{RW}_{e,t}(W_{p,b}) = \sum_{t=1}^{t} CAP_{o,t,n}\right) - min_t^T \left(CO_{o,t,n} + V_{o,t,n}\right) - FC_{o,t,n} - FD_{o,t,n} \;\forall\, o,t,n \quad 45$$

3. Update the maximum value for all simulations $PV_{o,t}$ (Equation 48);

$$PV_{o,t} = \max(PV_{o,t}, PV_{o,t,n}) \quad \textbf{46}$$

4. If the maximum financial value $PV_{o,t}$ is less than zero, add constraint 49 such that the decision variable must be less than or equal to the previous period;

$$Y_{o,t} \le Y_{o,t-1} \;\forall\; PV_{o,t} \le 0 \quad 47$$

4.3.1.2 Early start period for a parcel due to mining capacity

The second size reduction algorithm examines each parcel *p* in the resource to determine if it can feasibly be mined by period *t*. If the maximum mining capacity for a period *t* exceeds the extraction capacity required by the sequence to access parcel *p*, then parcel *p* will cannot be mined by period *t*. The pseudocode below outlines the implementation of this size reduction, with a programatic outline in Appendix B:

1. Iterate across all periods *T* for all simulations *N*
2. Set the maximum mining capacity $\mathrm{MC}_{t,n}$ in each period *t* equal to the sum of capacity $\mathrm{CAP}_{o,t,n}$ for all mining design options;

$$\mathrm{MC}_{t,n} = \sum_{o \in M, t=1}^{\mathrm{O},t} \mathrm{CAP}_{o,t,n} \quad 48$$

3. Set the mined tonnage $XM_{t,n}$ to zero;

$$XM_{t,n} = 0 \quad 49$$

4. Iterate through each predecessor $\check{p}$ of parcel *p* in the resource;
   a. Add the parcel tonnage $W_{\check{p}}$ to the mined tonnage $XM_{t,n}$;

$$XM_{t,n} = XM_{t,n} + W_{\check{p}} \quad 50$$

b. If the mined tonnage $XM_{t,n}$ exceeds the mining capacity $\mathrm{MC}_{t,n}$set the mined tonnage variable of parcel $\check{p}$ to zero;

$$X_{\check{p},b,f,t,n} = 0 \qquad \forall\, b, f, XM_{t,n} > \mathrm{MC}_{t,n} \tag{51}$$

c. Step to the next predecessor $\check{p}$ until there are no more predecessors.

5. Iterate through all parcels *P* in the resource;
   a. Set the mined tonnage $XM_{t,n}$ to the parcel tonnage $W_p$
   b. Iterate through each predecessor $\check{p}$ of parcel $p$
      i. Add the tonnage $W_{\check{p}}$ to the mined tonnage $XM_{t,n}$
      ii. If $XM_{t,n}$ exceeds the mining capacity $\mathrm{MC}_{t,n}$ set the decision variable for parcel $\check{p}$ to zero

$$Y_{\check{p},t,n} = 0 \qquad \forall\ XM_{t,n} \geq \mathrm{MC}_{t,n} \tag{52}$$

      iii. Step to the next predecessor $\check{p}$ until there are no more predecessors

An example of this; if the model contains 4 mine options with a total capacity in period 2 of 14Mt (6 Mt in period 1 and 8 Mt in period 2), the parcel considered for reduction is at the 5th bench of the 5th pushback and the shortest path to mining that parcel requires 30 Mt to be removed. Then in period 2 this parcel cannot be mined as there is insufficient capacity. Thus, the corresponding $Y_{p,t,n}$ and $X_{p,b,f,t,n}$ can be removed from the mathematical model. If the mining option can be executed more than once in a period and no upper limit on the number of times in a period it can be executed, then the upper bound on mining capacity cannot be defined and the size reduction cannot be implemented.

#### 4.3.1.3 Early start period for a parcel due to maximum sink rate

The third size reduction can be applied if a maximum sink rate is defined. The maximum sink rate is the maximum vertical progression per period specified in number of parcels per period. If the number of parcels to be extracted inside the same pushback as parcel $p$ exceeds the maximum sink rate in the same period *t* then parcel $p$ decision variables can be removed. A pseudocode outline of the algorithm follows, with a programatic outline in Appendix B:

1. Sort the resource by Pushback (Ascending) and Bench (Descending);
2. Iterate through each parcel *p* within the same Pushback for each time period *t*;
   a. Set the cumulative strip ratio for period *t*

$$CSR_t = \sum_{t=1}^{t} SR_t \tag{53}$$

b. Determine the number of benches $ABE_p$ required to access $Y_p$;

$$ABE_p = BE_p + \sum_{\check{p} \in \mathrm{PIT}_p}^{\check{P}} BE_{\check{p}} \tag{54}$$

c. If the number of benches $ABE_p$ exceeds the cumulative strip ratio $CSR_t$ then set the decision variables for parcel $p$ to zero

$$Y_{p,t,n} = 0 \qquad \forall\, n, ABE_p \geq CSR_t \tag{55}$$

$$X_{p,b,f,t,n} = 0 \qquad \forall\, b, f, n, ABE_p \geq CSR_t \tag{56}$$

For example, if the maximum vertical bench sink rate is 10 benches per year, then in the first year all benches below bench 10 can be removed from the mathematical model. Likewise, in the second year all benches below bench 20 cannot be removed from the model.

4.3.1.4 Removal of flow paths with a negative marginal value

The final size reduction algorithim limits the number of available flow paths. Flow paths that end in a product generation state (not paths to a waste dump or stockpile), must have a marginal value greater than zero for material to flow along the flow path otherwise the material should be sent to the waste dump. Two exceptions could occur where this would not be valid; a) if the capacity of the waste dumps or stockpiles was limited or b) where grade blending may be required to generate a product within the grade limits defined. These exceptions could make material that is un-economic by itself, economic in an overall blend. A pseudocode outline of the algorithm follows, with a programatic outline in Appendix B:

1. Iterate across all time periods *T* for all simulations *N*
   a. Iterate across all flow paths $\mathrm{F}_p$ for all parcels *P* for each bin *b* in $\mathrm{B}_p$
      i. Determine the maximum value $PV_{p,b,f,t,n}$ by summing the unit revenue $R_{v,u,t,n}$ for each value stream $v$ in product $u$ of the flow path $f$ and subtracting the unit variable cost $V_{o,t,n}$ of each option within the flow path $f$ where the option is not a mining option (as mining cost is not a marginal cost item);

$$PV_{p,b,f,t,n} = \sum_{v,u \in f}^{V} R_{v,u,t,n} - \sum_{o \in f, o \notin M_O}^{O} V_{o,t,n} \tag{57}$$

      ii. If $PV_{p,b,f,t,n}$ is less than zero set the decision variable to zero

$$X_{p,b,f,t,n} = 0 \qquad \forall\, PV_{p,b,f,t,n} \leq 0 \tag{58}$$

b. Iterate across all flow paths $\mathrm{F}_p$ for all parcels P for each bin $b$ in $\mathrm{B}_p$ that end at stockpile $s$ which is not a waste dump $s \notin W_s$

   i. Determine the maximum value $PV_{p,b,f,t,n}$ by summing the unit revenue $R_{v,u,t,n}$ for each value stream $v$ in product $u$ that is possible from the stockpile $v \in F_s$ and subtracting the unit variable cost $V_{o,t,n}$ of each option within the flow path $f$ where the option is not a mining option (as mining cost is not a marginal cost item) and subtracting the unit variable cost $V_{s,t,n}$ of removing material from stockpile $s$;

$$PV_{p,b,f,t,n} = \sum_{v \in F_S}^{V} R_{v,u,t,n} - \sum_{o \in f, o \notin M_O}^{O} V_{o,t,n} - V_{s,t,n} \qquad 59$$

   ii. If $PV_{p,b,f,t,n}$ is less than zero set the tonnage recovered from stockpile to zero

$$X_{p,b,f,t,n} = 0 \qquad \forall\, PV_{p,b,f,t,n} \leq 0 \qquad 60$$

a. Iterate across each stockpiles $s$ that is not a waste dump $s \notin D$ for each flow path $f$ in $F_s$

   iii. Determine the maximum value $PV_{s,f,t,n}$ by summing the potential product revenue $P_{v,d,t,n}$ for each value stream $v$ is part of the product $u \in f$ and subtracting the unit variable cost $V_{o,t,n}$ of each option within the flow path $f$ and subtracting the unit variable cost $V_{s,t,n}$ of removing material from stockpile $s$;

$$PV_{s,f,t,n} = \sum_{v \in F_S}^{V} R_{v,u,t,n} - \sum_{o \in f}^{O} V_{o,t,n} - V_{s,t,n} \qquad 61$$

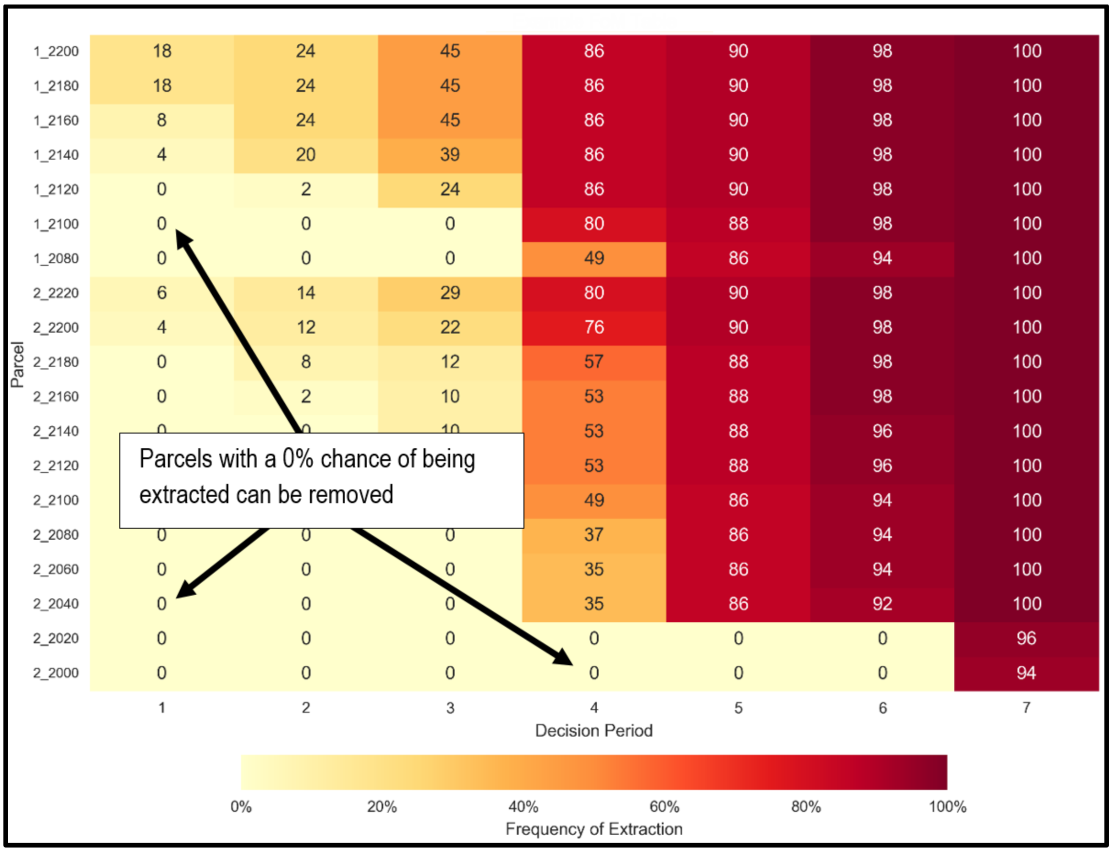
   iv. If $PV_{s,f,t,n}$ is less than zero set the tonnage recovered from stockpile to zero

$$X_{s,i,f,t,n} = 0 \qquad \forall\, i, PV_{s,f,t,n} \leq 0 \qquad 62$$

### 4.3.2 Robust Mode Size Reduction

In the robust mode, the number of variables is large and doubles for each simulation included in the model. Consequently, the solution time for these hybrid models increases exponentially with each simulation included. To counter this impact, an additional performance reduction method is implemented to remove parcel variables not utilised in any scenario when the model is run in the flexible mode. The parcels with a 0% frequency of extraction (usually 0 is not included on the graph, but has been for clarity here) can be identified from the frequency of extraction graphs like the one shown in Table 8.

Table 8: Frequency of Extraction table illustrating parcels with a zero percentage chance of being removed




| Parcel | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1_2200 | 18 | 24 | 45 | 86 | 90 | 98 | 100 |
| 1_2180 | 18 | 24 | 45 | 86 | 90 | 98 | 100 |
| 1_2160 | 8 | 24 | 45 | 86 | 90 | 98 | 100 |
| 1_2140 | 4 | 20 | 39 | 86 | 90 | 98 | 100 |
| 1_2120 | 0 | 2 | 24 | 86 | 90 | 98 | 100 |
| 1_2100 | 0 | 0 | 0 | 80 | 88 | 98 | 100 |
| 1_2080 | 0 | 0 | 0 | 49 | 86 | 94 | 100 |
| 2_2220 | 6 | 14 | 29 | 80 | 90 | 98 | 100 |
| 2_2200 | 4 | 12 | 22 | 76 | 90 | 98 | 100 |
| 2_2180 | 0 | 8 | 12 | 57 | 88 | 98 | 100 |
| 2_2160 | 0 | 2 | 10 | 53 | 88 | 98 | 100 |
| 2_2140 | 0 | 0 | 10 | 53 | 88 | 96 | 100 |
| 2_2120 | [illegible] | [illegible] | [illegible] | 53 | 88 | 96 | 100 |
| 2_2100 | [illegible] | [illegible] | [illegible] | 49 | 86 | 94 | 100 |
| 2_2080 | 0 | 0 | 0 | 37 | 86 | 94 | 100 |
| 2_2060 | 0 | 0 | 0 | 35 | 86 | 94 | 100 |
| 2_2040 | 0 | 0 | 0 | 35 | 86 | 92 | 100 |
| 2_2020 | 0 | 0 | 0 | 0 | 0 | 0 | 96 |
| 2_2000 | 0 | 0 | 0 | 0 | 0 | 0 | 94 |

At these locations, no mining occurs in these time period for these parcels. It can therefore reasonably be assumed that in a robust mode which includes a subset of the same simulations that mining will not occur at these parcels in these periods. This allows the parcel decision variable ($Y_{p,t,n}$) to be removed from the model before optimisation. Further, any tonnage flow variables ($X_{p\prime,b,f,t,n}$ ) from successor parcels can be removed from the model. For example, if parcel 2_2060 has a frequency of execution of 0% in period 1 to 3, then the tonnage variables related to the successor parcels can be removed from the model i.e. $X_{2\_2040,b,f,1,n}$ ,$X_{2\_2040,b,f,2,n}$ and $X_{2\_2040,b,f,3,n}$ (where bench 2_2040 occurs after 2_2060). Additionally, the decision variables for the parcel 2_2060 can be removed from the model $Y_{2\_2060,1,n}$, $Y_{2\_2060,2,n}$ and $Y_{2\_2060,3,n}$. Note, that the frequency of execution graph is based on the binary decision variables when a block is fully mined by period *t* and does not indicate when the mining of a parcel may start. It is feasible for mining to occur in periods 1 through 4 even if the the binary decision variable ($Y_{p,t,n}$) is only set to 1 in period 4. This means, only the binary decision variables for full mining of a parcel can be removed from the model and not the linear variables partial mining of a block. This model reduction strategy has a significant impact on the number of variables in the robust model formulation.

# CHAPTER 5. IMPLEMENTATION OF THE MATHEMATICAL MODEL

This chapter applies the mathematcial model to a previous an investment decision and examines the alternative decision pathway. The case study is based on a feasibility study completed for a gold deposit in a developing country. The exact feasibility study outcomes are not presented as they are confidential in nature, however a financial model was developed which is comparable in outcome. The investment decision for the feasibility study was presented and then analysed through the prism of uncertainty. Alternative flexible system configurations will be considered with the analysis revealing the value of including design flexibility. A comparison of the implemented feasibility design and the proposed operational system to the actual project performance will be presented. This also indicates the benefit of including flexibility in the system. A short description of the custom program developed for the implementation of this model is discussed towards the end of this chapter.

## 5.1 INITIAL PROJECT FEASIBILITY STUDY

### 5.1.1 Project Description

The project is a greenfield open pit gold mine in a developing country with a moderate level of political instability. Exploration drilling was undertaken to determine the size and quality of the resource. Approximately 200 Reverse Circulation (RC) holes, 100 RC holes with diamond tails and 6 diamond core holes had an aggregate of 43,000 metres of drilling. From this drilling a geological resource model was developed and used for mine planning and design of the project. Metallurgical testing of the orebody indicated a gravity circuit combined with a Carbon-in-Leach (CIL) circuit would allow for gold recoveries in the mid 90%'s, whilst heap leaching would provide recoveries in the mid 50's. It was determined that the product to be produced on site would be Gold Dore which would be exported to a refinery for further processing. The primary drivers behind this were the scale, location and process characteristics of the operation. No other products were considered economically viable.

### 5.1.2 In-situ Resource Model

The geology of the resource is dominated by a sub volcanic porphyry of granitic composition. Within this, two main ore bearing rock types were identified in the feasibility study drilling. The dominant mineralized zone is contained in a relatively undefo rmed, granodiorite unit known as the "felsic unit". A second smaller but higher grade brittle-ductile "mafic shear" zone contains a well-defined planar dipping lode. Geological interpretations of the drilling results were generated in conjunction with 1m and 3m composited drillhole data to generate an in-situ resource model of the deposit. A three dimensional block model was constructed with 20m E x 25m N x 5m RL parent cell size and 2.5m E x 6.25m N x 1.25m RL sub-celling dimensions. A total of 1,577,361 blocks were modelled. The grade distribution of these two distinct lithological units were modelled using different techniques. Ordinary Kriging (OK) was used for modelling of the mafic shear. Due to the highly skewed nature of the felsic unit variogram a non-linear method called Multiple Indicator Kriging (MIK) was used to estimate the probability a block was above a certain cutoff grade. A post-processing process was used to convert the MIK simulations to whole block E-type grade estimates for a 5m E x 6.25m N x 5m RL selective mining unit size. An Au cut-off grade of 0.70 g/t was used for the mineralised envelope definition. It was noted that selectivity issues within the orebody could be encountered in regions if a mineralised cut-off grade above 0.9 g/t was used. The total resource reported in the feasibilty study is shown in Table 9.

Table 9: In-situ Au mineralisation resource classification above 0.70 g/t cut-off grade

| | Tonnes (Mt) | Grade (g/t) | Ounces (Oz) |
|---|---|---|---|
| Indicated | 11.79 | 2.00 | 759,000 |
| Inferred | 0.18 | 1.98 | 11,000 |
| Total | 11.97 | 2.00 | 770,000 |

### 5.1.3 Mine Planning Process

The original feasibility study followed a deterministic mine planning process, with the ultimate pit limit being determined from a Whittle™ optimisation using average values for geotechnical and financial inputs. From this limit, a series of interim pushbacks .were designed and then mine blocks were scheduled using a priority rules based algorithm in Minesched™. A summary of the financial inputs used for the optimisation process are outlined in Table 10, whilst geotechnical inputs are outlined in Table 11. All monetary values are in United States (US) dollars.

Table 10: Financial inputs for pit optimisation process

| Variable | Units | Value |
|---|---|---|
| Gold Price | $/oz | 600 |
| Royalty | % | 2% |
| TCRCs | $/oz | 4.0 |
| Total Selling Cost | $/oz | 12.0 |
| Average Mining Cost* | $/t | 1.58 |
| Mine Supervision | $/pa | 1.20 |
| Grade Control | $/pa | 1.10 |
| Mine Rehab | $/pa | 0.50 |
| De-watering | $/pa | 0.30 |
| Mining Rate | Mtpa | 10.5 |
| Mine Overhead Cost | $/pa | 0.21 |
| Process Rate | Mtpa | 2.20 |
| Process Recovery | % | 94% |
| Processing Cost - Oxide | $/t | 4.50 |
| Processing Cost - Fresh | $/t | 6.30 |
| G&A | $/pa | 2.20 |
| G&A Unit Process Cost | $/t | 1.00 |
| Crusher Feed | $/t | 0.32 |
| Discount Rate | % | 10 |

* A bench by bench mining cost was developed to account for the increase in haulage cost with pit depth

Table 11: Geotechnical assumptions used for pit optimisation

| Rock Type | Bearing (°) | Face Angle (°) |
|---|---|---|
| Oxide | 30 | 37 |
| | 130 | 45 |
| | 160 | 37 |
| | 200 | 41 |
| | 220 | 43 |
| | 270 | 45 |
| Transitional Material | All bearings | 50 |
| Fresh | All bearings | 50 |

An isometric view of the pushback designs is shown in Figure 23 and a plan view in Figure 24. Pushbacks 1 to 4 were used as mining inventory in the feasibility study. Pushback 5 was deemed to contain a large amount of material in the “Inferred” category and was consequently left out of the mine plan. Pushback 6, the Ultimate Pit Limit, was generated with a Revenue Factor (parameter that the revenue of a block is multiplied by) of 1.6, thus was deemed uneconomic at the planned gold price. Pushback 5 and 6 will be included in the flexible system design with geological uncertainty represented by the conditionally simulated models. This allows the model to value the upside potential of the resource.

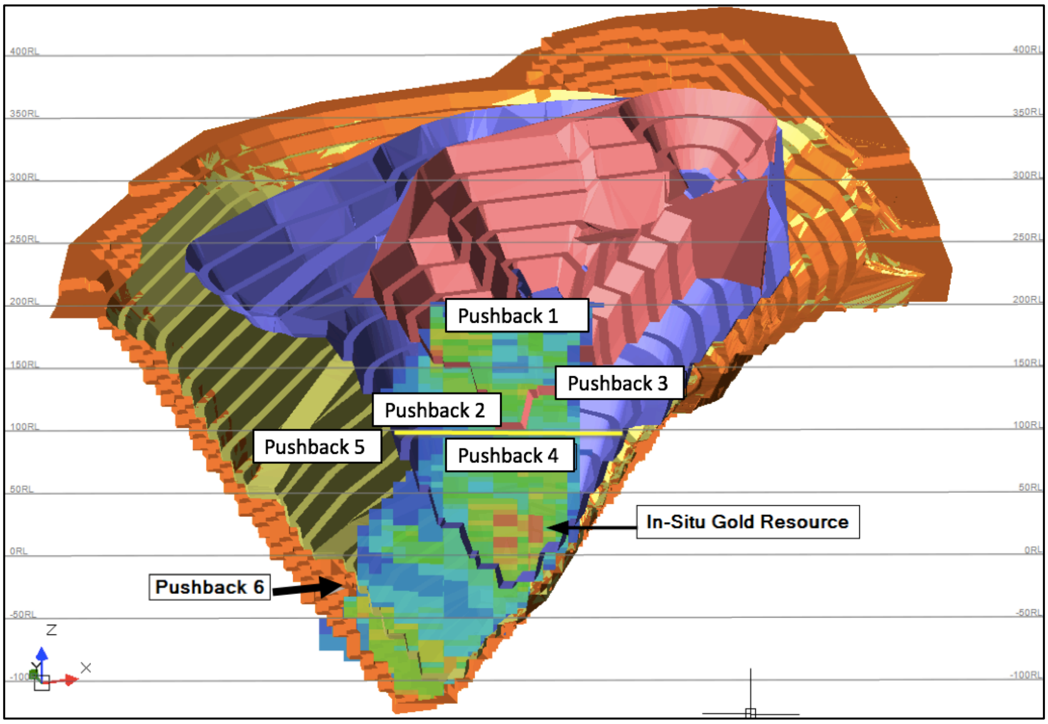



Figure 23: Cross-section of pushback designs with a section through the gold resource

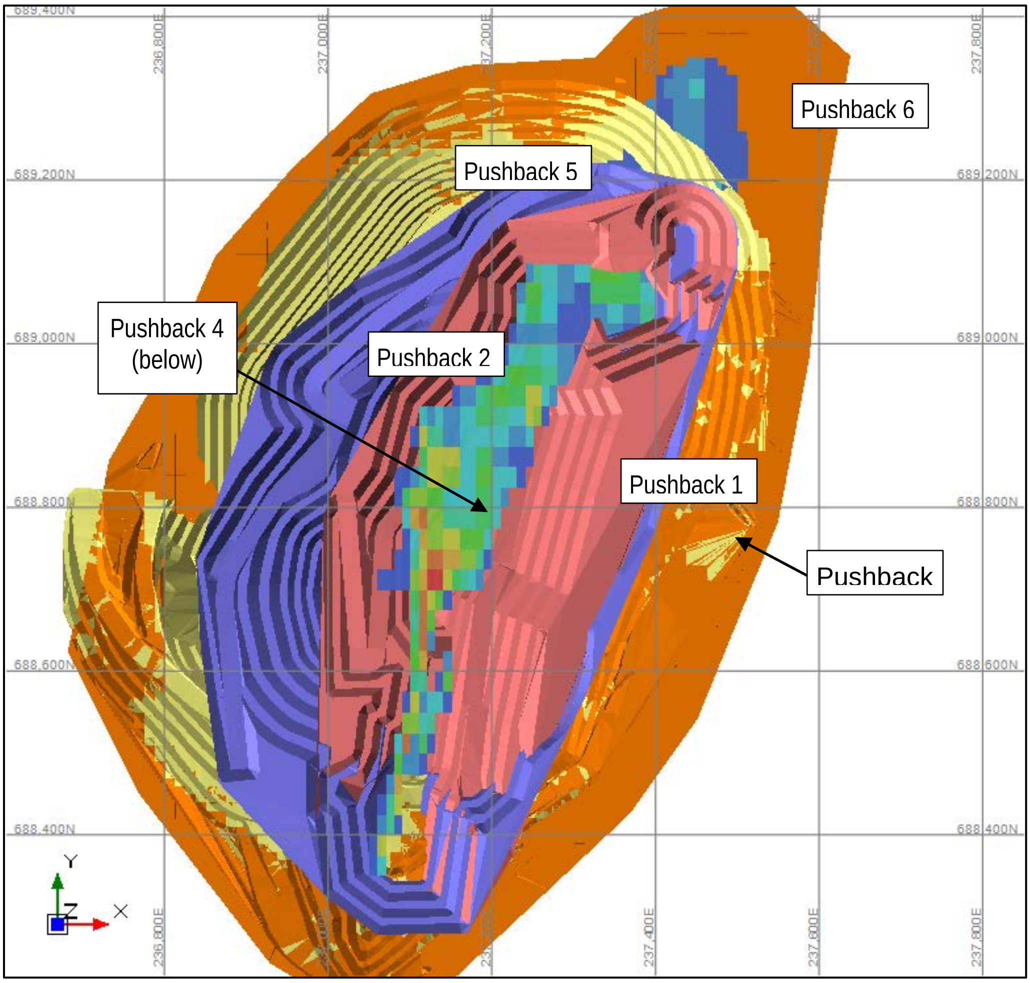



Figure 24: Plan view of pushbacks with a section through the gold resource

From these designs a mine schedule was developed for the life of the operation. The life of the operation was determined to be a maximum of 11 years. For decision making purposes it was determined that 7 decision points were required to adequately model the operation. It is possible to scale the decision period with different time steps as shown in Figure 25.

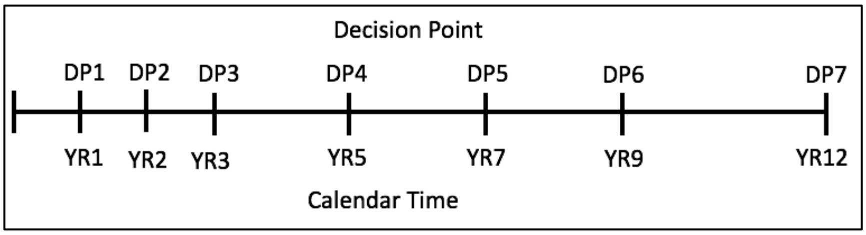



Figure 25: Scaling of decision periods compared to calendar time

A set of sequencing constraints are required for modelling the practical constraints of open pit mining of the resource model, similar to those outlined in Figure 15. The constraints will represent the vertical mining dependency for bench progression; mining of a block above must occur before mining of a block below. Additionally, a set of constraints to enforce mining of successive pushbacks is defined. The constraints are developed from the pushback designs used in the feasibility study, summarised in Table 12.

Table 12: Predecessor to successor dependency relationships

| Predecessor | Successor |
|---|---|
| Inter Pushback Dependencies | |
| Pushback 1 Bench 1….n | Pushback 2 Bench 1….n |
| Pushback 2 Bench 1….n | Pushback 3 Bench 1….n |
| Pushback 3 Bench 1….n | Pushback 4 Bench 1….n |
| Pushback 4 Bench 1….n | Pushback 5 Bench 1….n |
| Pushback 5 Bench 1….n | Pushback 6 Bench 1….n |
| Intra Pushback Dependencies | |
| Pushback 1 Bench 1….n-1 | Pushback 1 Bench 2….n |
| Pushback 2 Bench 1….n-1 | Pushback 2 Bench 2….n |
| Pushback 3 Bench 1….n-1 | Pushback 3 Bench 2….n |
| Pushback 4 Bench 1….n-1 | Pushback 4 Bench 2….n |
| Pushback 5 Bench 1….n-1 | Pushback 5 Bench 2….n |
| Pushback 6 Bench 1….n-1 | Pushback 6 Bench 2….n |

For reasons which are commercial in confidence the financial analysis that underpins the feasibility study has been excluded from this thesis. However, a mine schedule which forms a reasonable approximation was forged by running the fixed system mode through the mathematical model. From this mine schedule, a financial analysis of the project was undertaken at a gold price of $600/oz and a sensitivity completed at ±$100/oz (similar to the method used to approximate uncertainty in the feasibility study). The financial outcomes are shown in Table 13 and Figure 26.

Table 13: Financial Sensitivity of Feasibility Study Outcome

| | | Gold Price | | |
|---|---|---|---|---|
| **Inputs** | **Units** | **$500/oz** | **$600/oz** | **$700/oz** |
| Cash Surplus (Pre capital) | $M | 177.5 | 251.6 | 325.8 |
| Net Cash | $M | 96.7 | 170.8 | 245.0 |
| NPV @ 10% discount rate | $M | 55.46 | 112.9 | 170.3 |
| IRR | % | 32.3 | 53.1 | 73.0 |
| Payback Period | months | 32 | 18 | 13 |

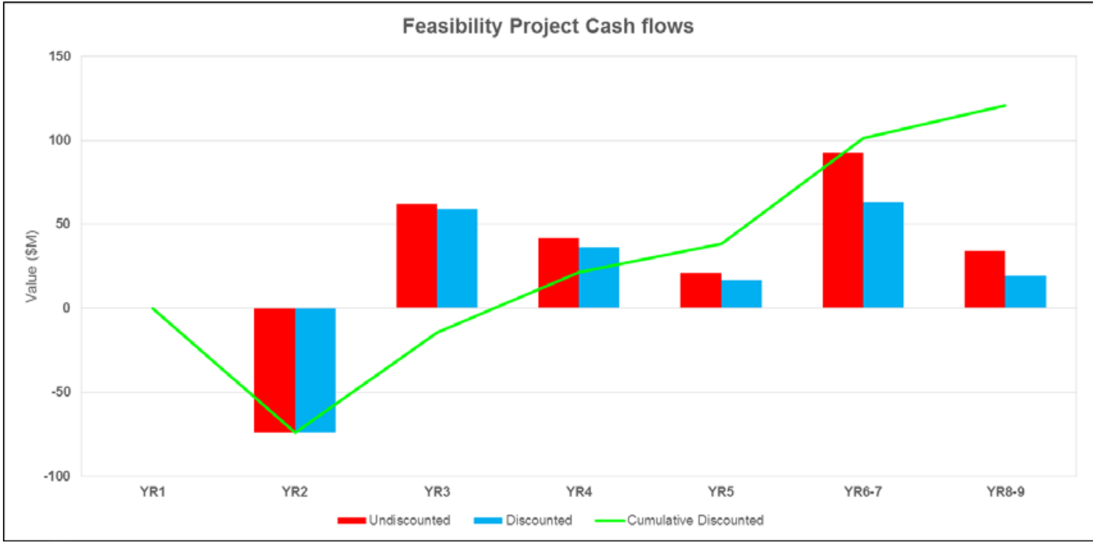



Figure 26: Feasibility project cash flows on deterministic values

The feasibility study recommended a positive investment decision in the project with the following key recommendations:

- Commencement of site works in Year 1 Q3 (Quarter 3);
- Open pit mining commencement in Year 1 Q4 (with a fleet capacity of 18Mtpa);
- Gold production to commence from a plant capable of processing 2Mtpa of fresh rock in Year 2 Q1;
- A fixed Au cut-off grade of 0.60 g/t for fresh and 0.50 g/t for oxide rock;
- Upside potential in a nearby deposit was identified but not studied.

Finally, the resulting ore inventory for this study is shown in Table 14. Note that the difference to Table 9 is due to a variable cut-off grade used in the schedule.

Table 14: Scheduled project ore inventory.

| | **Tonnes (Mt)** | **Grade (g/t)** | **Gold Ounces (Oz)** |
|---|---|---|---|
| High Grade | 9.7 | 2.27 | 705,500 |
| Low Grade | 3.4 | 0.82 | 89,500 |
| Total Ore | 13.1 | 1.89 | 795,000 |

## 5.2 CONSIDERING UNCERTAINTY IN THE PROJECT

This section examines the impact of uncertainty (not taking average values) in the key parameters of the system. A key step when including uncertainty in an analysis selecting appropriate distributinos for uncertainties as this can impact the outcome. The parameters used for simulation of the uncertainty distributions are outlined. These scenarios will then be processed through the mathematical model in the fixed system mode with the system configuration generated from the feasibility process. From the resulting outcomes of the mathematical model an analysis of the impact to project value will be generated using a VARG.

### 5.2.1 Simulation of Uncertainties using Monte Carlo simulation

The case study handles uncertainty in the gold price, mining cost, processing cost, mine utilisation, process utilisation and administration cost. It is important to note that the distrbutions used for simulation of input parameters will directly impact the ouctome of the model. Monte Carlo simulation was chosen as an appropriate method to generate values for these parameters. Simulation of the uncertainties was undertaken in quarterly time steps using the @Risk software according to the parameters in Table 15. These parameters were derived with the use of distribution fitting in @Risk on the actual variability in parameters over time. @Risk provides a recommended distribution based on the highest ranking Akaike Infromation Criterion (AIC). Mining, processing and administration cost have been modelled as index values from the first period. This is to allow modelling of variability with varying cost bases as capacity is scaled up and down (i.e. large plant capacity options have a lower cost per tonne).

Table 15: Uncertainty parameters for Monte Carlo Simulation

| Uncertainty | Distribution | Parameters | 95% probability interval | Correlation |
|---|---|---|---|---|
| Recovery (%) | Triangular | Minimum: 86.4%<br>Most Likely: 94.3%<br>Maximum: 97.4% | 87.8% - 96.4% | Independent |
| Gold Price ($/oz) | First-order Auto-Regressive Conditional Heteroskedastic Time Series (ARCH1) | Mean: 7.08<br>Volatility: 796.49<br>Error Coefficient: 0.793<br>Starting Price: 600 | $322/oz - $1539/oz | See Table 16 |
| Mining Cost Index (#) | First-order Moving Average Time Series (MA1) | Mean: 1.068<br>Volatility: 0.292<br>Moving Average Coefficient: 0.486<br>Initial Error Term: -0.236<br>Start Value: 1 | 0.48-1.18 (first period)<br>1.34-2.79 (last period) | See Table 16 |
| Processing Cost Index (#) | First-order Moving Average Time Series (MA1) | Mean: 1.00<br>Volatility: 0.1742<br>Moving Average Coefficient: 0.23042<br>Initial Error Term: -0.0991<br>Start Value: 1 | 0.71-1.35 | See Table 16 |
| Administration Cost Index (#) | First-order Moving Average Time Series (MA1) | Mean: 1.00<br>Volatility: 0.2878<br>Moving Average Coefficient: 0.1533<br>Initial Error Term: 0<br>Start Value: 1 | 0.51-1.48 | See Table 16 |
| Process Plant Utilisation (%) | Triangular | Minimum: 7.35%<br>Most Likely: 92.5%<br>Maximum: 100% | 21.4% - 95.8% | 0.762 to Mining Utilisation |
| Mining Fleet Utilisation (%) | Triangular | Minimum: 2.25%<br>Most Likely: 95.4%<br>Maximum: 100% | 17.7% - 98.5% | 0.762 to Process Plant Utilisation |

Additionally, to control the possible divergence of the gold price and cost variables (i.e a significant increase in the gold price whilst the cost was decreasing) they were correlated together in the Monte Carlo Suimulation with the coefficients shown in Table 16.

Table 16: Correlation of gold price and different costs between periods

| | Administration Cost Index | Mining Cost Index | Processing Cost Index | Gold Price |
|---|---|---|---|---|
| Administration Cost Index | 1.00 | | | |
| Mining Cost Index | 0.48 | 1.00 | | |
| Processing Cost Index | 0.71 | 0.57 | 1.00 | |
| Gold Price | 0.76 | 0.57 | 0.77 | 1.00 |

Geological uncertainty is simulated by conditionally simulated models for 100 realisations as shown in section 5.2.2.

A sample set of one hundred simulations were generated as representations of possible 'states-of-the-world' to be used as an input to the mathematical model to represent uncertainty in the case study. A summary of the ranges of simulated values for each variable is shown in Figure 27 through to Figure 33 with a comparison to the actual value in red. For a given time step (i.e an annual period consisting of four quarters) the average of value of all quarters was used for the time step in the optimisation model.

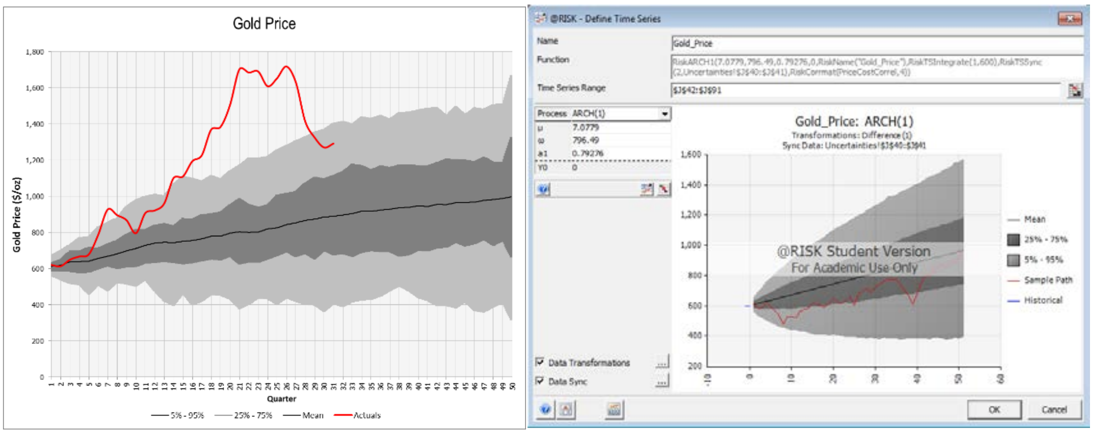



Figure 27: Range of simulated gold prices with comparison to actuals (red line) and distribution in @Risk

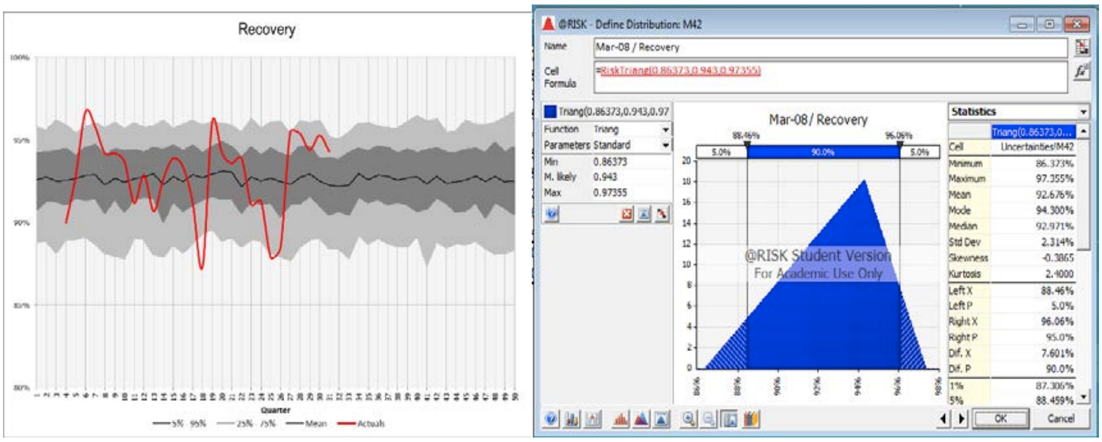



Figure 28: Range of simulated recoveries over time and comparison to actuals (red line)

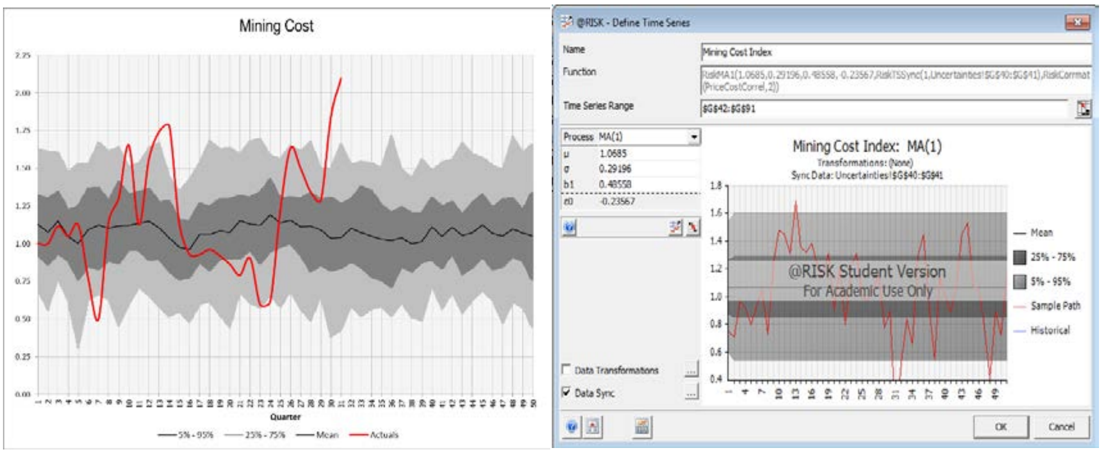



Figure 29: Range of simulated mining cost indices over time and comparison to actuals (red line)

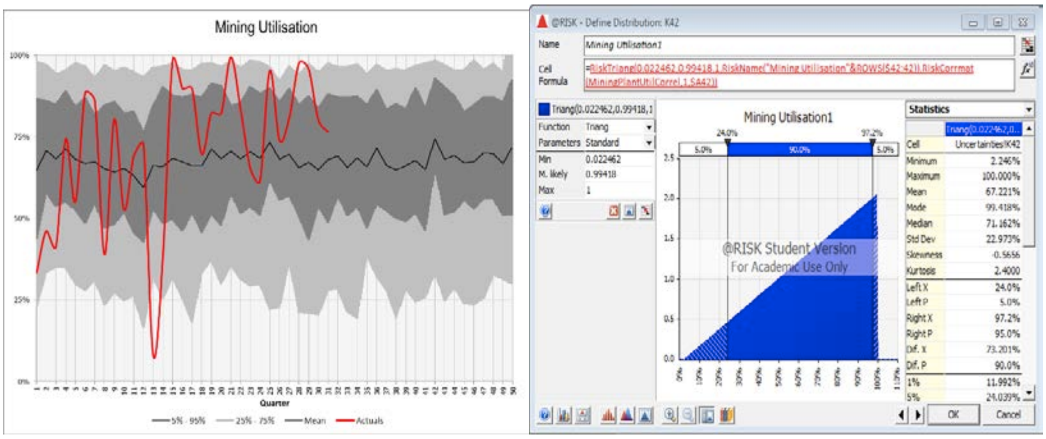



Figure 30: Range of simulated mining utilisations over time and comparison to actuals (red line)

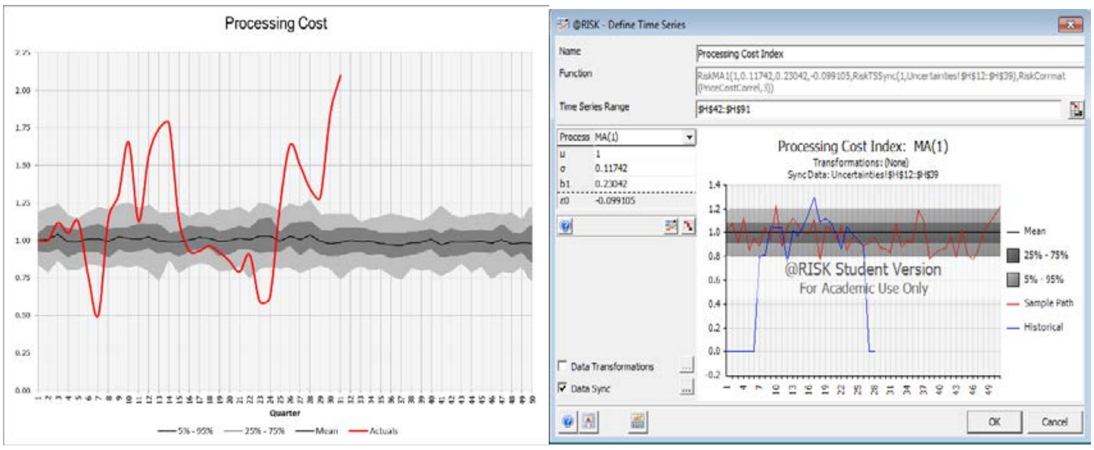



Figure 31: Range of simulated processing costs over time and comparison to actuals (red line)

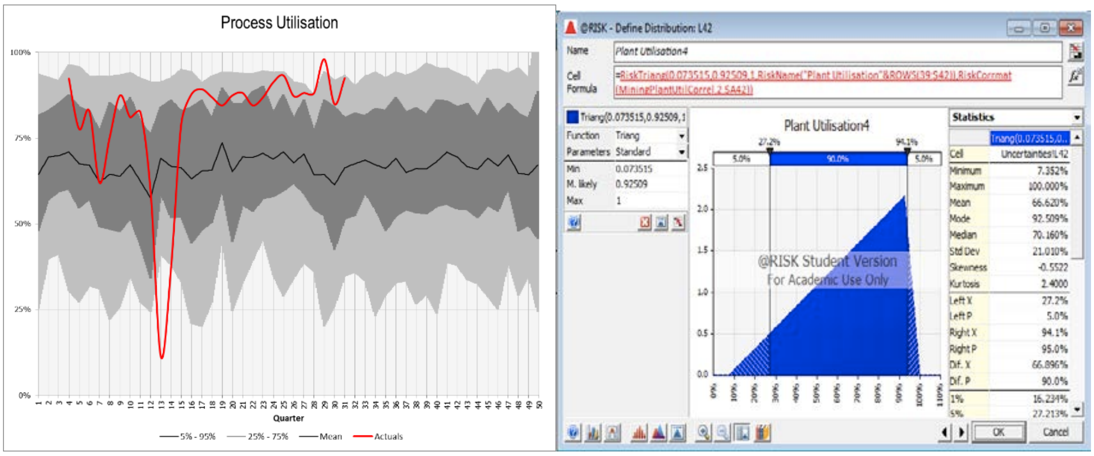



Figure 32: Range of simulated processing utilisations and comparison to actuals (red line)

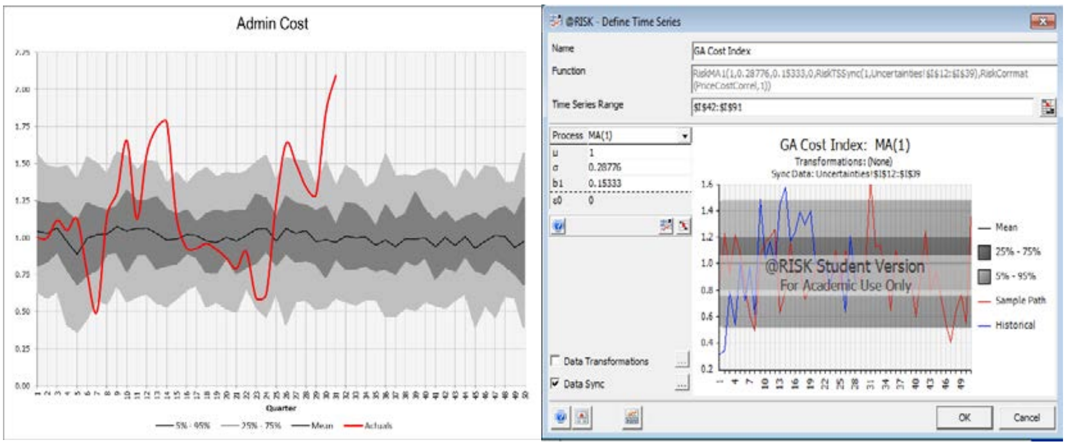



Figure 33: Range of administration cost indices and comparison to actuals (red line)

### 5.2.2 Simulation of geological uncertainty using Conditional Simulation

Geological uncertainty is generally a key parameter to a mining project . This case study chose to represent geological uncertainty by a conditional simulation (CS) model of the felsic unit to test the strength of the MIK estimation. The mafic shear unit, a high grade structure, was considered too have limited variability and did not warrant simulation. The CS model for the felsic unit was merged with the Kriged model for the mafic shear structure. Each simulation of the geological model will represent one possible realisation of the orebody. A cross section of the orebody model used in the feasibility study is shown next to three realisations from the CS model in Figure 36.

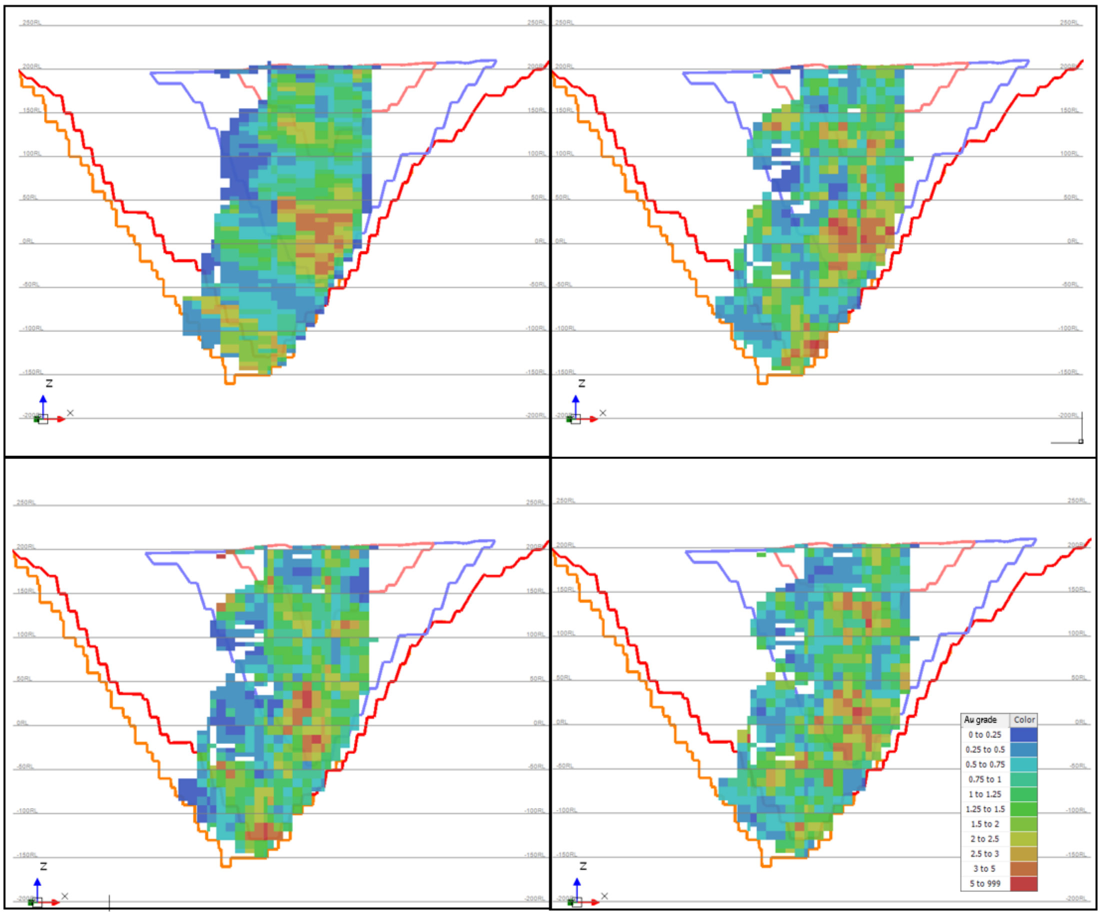



Figure 34: Cross-section comparison of Au distribution between the Kriged feasibility resource model (top left) and conditionally simulated models

A comparison of the tonnage, grade and contained ounces for each CS model within the economic pit limit used in the feasibility study (Pushback 4) is shown in Figure 35.

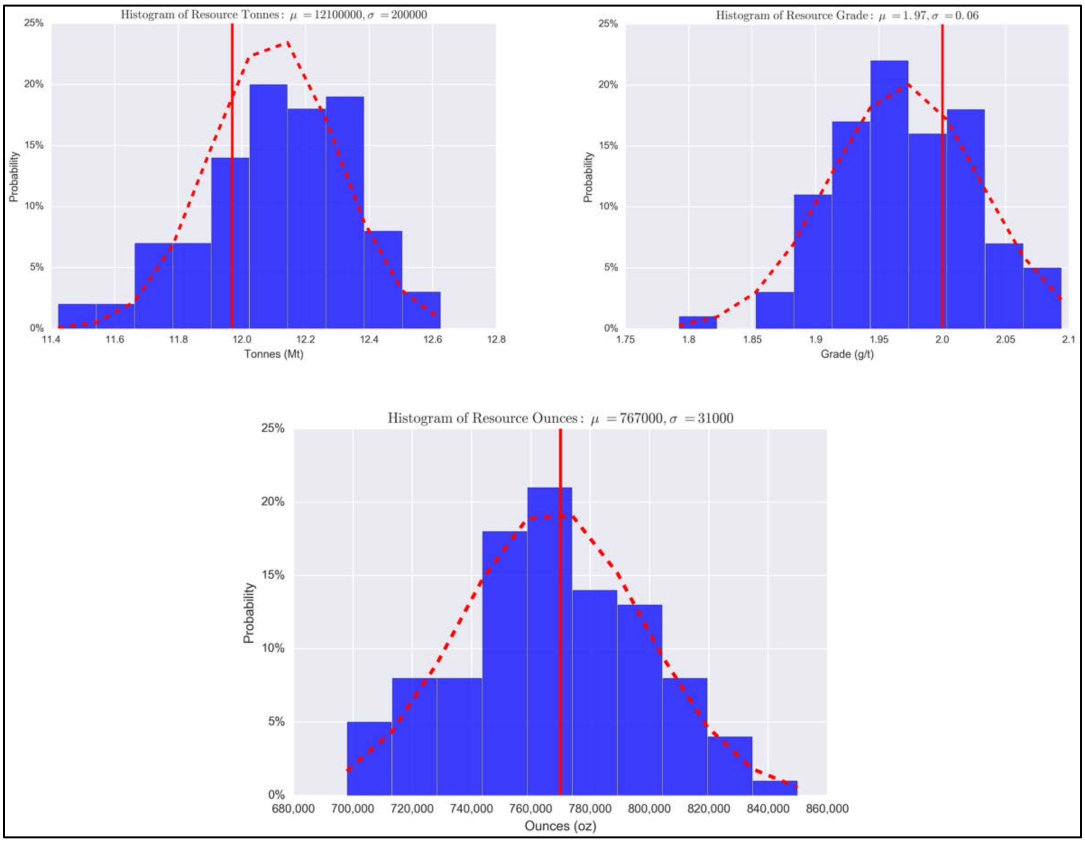



Figure 35: Comparison of the range of recovered reserve tonnes, grade and ounces within Pushback 1 to 4 for the conditionally simulated models (blue bars) vs the feasibility model (vertical red line)

### 5.2.3 Product configuration

The products built into the model are a Carbon in Leach (CIL) gold dore and Dump Leach (DL) gold dore. They contain a lower grade limit of 0.3 g/t which reflects the lowest grade that is representative in the in-situ geological model. Grades below this are considered outside the detection limits of the model generation. Importantly, this lower grade limit identifies gold mineralisation and should not be considered an economic cutoff grade as the model will typically select a grade higher than this as the distinction point between ore and waste.

## 5.3 MODEL SIZE AND PERFORMANCE

A purpose built software package and user interface called *Mineplex* has been developed in Visual Basic .NET to output the mathematical models and interperate results (discussed later). The output from *Mineplex* is a Mixed Integer Programming formulation in the industry standard LP File Format. This file is then passed to *Gurobi™* which generates the solution to the model formulation. The solution file is then passed back to *Mineplex* to generate a set of result files which are analysed with a set of Python scripts which make use of open source data visualisation libraries. A summary of the number of model variables for each model mode is shown in Table 17. It should be noted that the number of variables in an operational model after variable reduction is significantly reduced and the number of variables in a robust model is significantly more then any other model mode.

Table 17: Summary of variables for each model mode pre and post reduction strategies

| Model Mode | Linear Variables Pre-Reduction | Linear Variables Post-Reduction | Binary Variables Pre-Reduction | Binary Variables Post-Reduction |
|---|---|---|---|---|
| Fixed Mode - Feasibility Design | 321,688 | 137,505 | 1,018 | 465 |
| Flexible Mode | 321,696 | 137,548 | 1,018 | 465 |
| Operational Mode | 318,702 | 68,774 | 507 | 231 |
| Robust Mode | 812,333 | 281,891 | 2,326 | 779 |
| Flexible Mode PB1-4 Only | 159,429 | 71,839 | 543 | 262 |
| Operational Mode PB1-4 Only | 318,702 | 68,774 | 913 | 231 |
| Robust Mode PB1-4 Only | 468,316 | 217,620 | 1,405 | 677 |

The solution time of the models by *Gurobi™* varies by system configuration as shown in Table 18. Models with few design decisions are quicker to solve to optimal whilst models with more design decisions or more simulations take longer. The fixed system mode is the quickest whilst the robust model mode is the slowest. Note that the models with a very low minimum time (where the average is much higher) are generally due to the linear relaxation of the model returning an upper bound of zero.

Table 18: Summary of solution times for different model mode's

| | Duration (seconds) | | | |
|---|---|---|---|---|
| **Model Mode** | Average | Maximum | Minimum | Standard Deviation |
| Fixed Mode - Feasibility Design | 17 | 28 | 4 | 6 |
| Flexible Mode | 7,215 | 24,171 | 27 | 5,267 |
| Operational Mode | 1,492 | 32,568 | 168 | 4,372 |
| Robust Mode | 86,128 | NA | NA | NA |
| Flexible Mode - PB1-4 Only | 468 | 845 | 6 | 169 |
| Operational Mode - PB1-4 Only | 55 | 152 | 4 | 19 |
| Robust Mode - PB 1-4 Only | 48,542 | NA | NA | NA |

Further, an investigation into the impact of the variable reduction strategy on the robust mode model was done with two sample model. The model solution was identical for both models however the model with variable reduction strategy implemented contained significantly fewer variables, as shown in Table 19.

Table 19: Example of the variable reduction in a robust mode model

| Type of Variable | State | Test Model 1 | Test Model 2 |
|---|---|---|---|
| Binary Variables | Before | 4,253 | 4,250 |
| | After | 1,986 | 1,986 |
| | Reduction | 2,267 (-53%) | 2,264 (-53%) |
| Linear Variables | Before | 1,143,861 | 1,146,181 |
| | After | 642,194 | 645,137 |
| | Reduction | 501,667 (-44%) | 501,044 (-44%) |

It is noted that the scale of computing resources available today reduces the computational barrier to performing these optimisation tasks. This research used a combination of cloud computing and high performance computing environments to facilitate solving multiple models simultaneously. The resources used include the following:

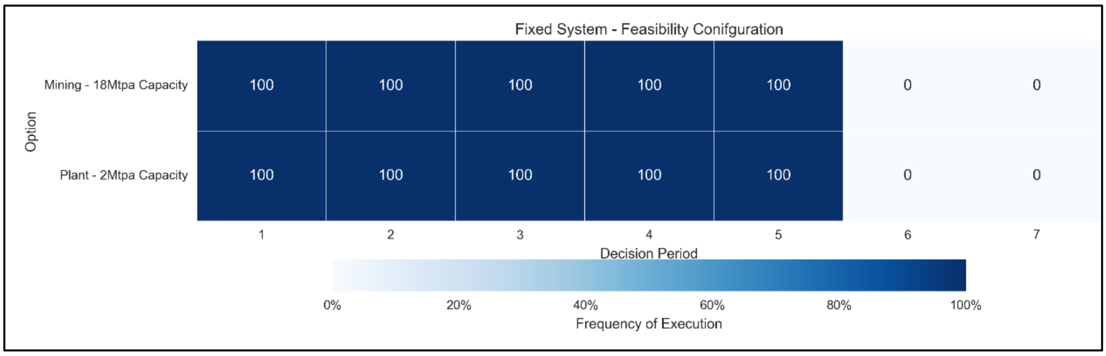
- Zeus cluster at the Pawsey Supercomputer Centre (Curtin University) – 16 Core nodes with Intel Xeon E5-2670 @2.6Ghz processors

- Australian Government funded research cloud (NeCTAR). Resources are requested on demand and the OpenStack system allocates request to one of the cloud computing facilities located in each State capital city and Canberra (NeCTAR, 2016). A typical instance (m1.xxlarge) consists of an AMD Opteron 6276 16 core @ 2.3GhZ processor with 64Gb of RAM.

## 5.4 VARG FOR FEASIBILITY SYSTEM CONFIGURATION

The system configuration generated as part of the feasibility study is shown in Table 20. The fixed mode of the mathematical model was used to generate a mine schedule and valuation for this system under average conditions.

Table 20: System configuration of fixed system from feasibility study




In this mode, the system configuration is fixed and the schedule is flexible having the ability to change between simulations. This approach reflects the flexibility management have to vary the mining schedule but not the system configuration in response to change in uncertainties over time.

Figure 36 shows the VARG for the fixed feasibility system configuration under uncertainty for two scenarios; the first with (red line) geological uncertainty and the second without geological uncertainty (blue line). The 'with' case utilises the conditionally simulated resource models, whilst the 'without' scenarios uses the deterministic resource model used in the original feasibility study.

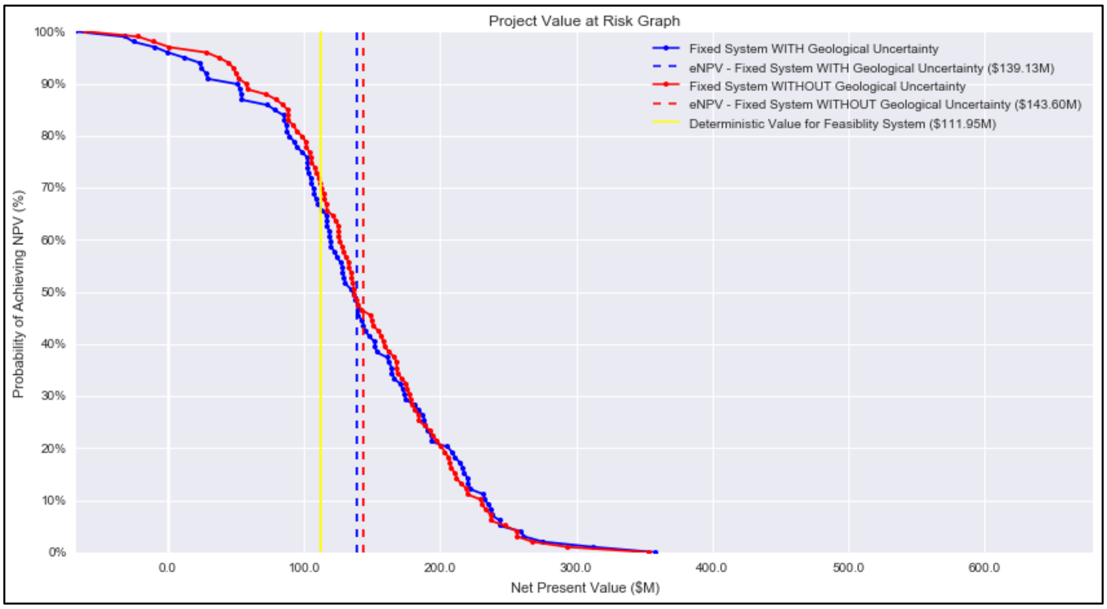



Figure 36: Fixed System Configuration Value-at-Risk Graph

A key observation from this graph is that by considering uncertainty it is possible to determine ~5% of the simulations for the project will lose money. A decision maker cannot determine this information from traditional deterministic modelling. Further, the range of possible project values is high at ~$350M. Additionally, it can be seen that on average the model which includes geological uncertainty (conditionally simulated models), generates a lower value than the deterministic model (feasibility model) as the red line is further to the left than the blue line. This means for this project it is likely that a decision maker would overestimate the project value if only utilising a deterministic model.

Further insight into the variation between the scenarios can be seen by comparing the frequency of extraction for each parcel over time. For the three scenarios – deterministic, fixed system with geological uncertainty and fixed system without geological uncertainty - frequency of extraction graphs have been generated as shown in Table 21, Table 22 and Table 23.

Table 21: Frequency of Extraction for Feasibility Fixed System with Deterministic values

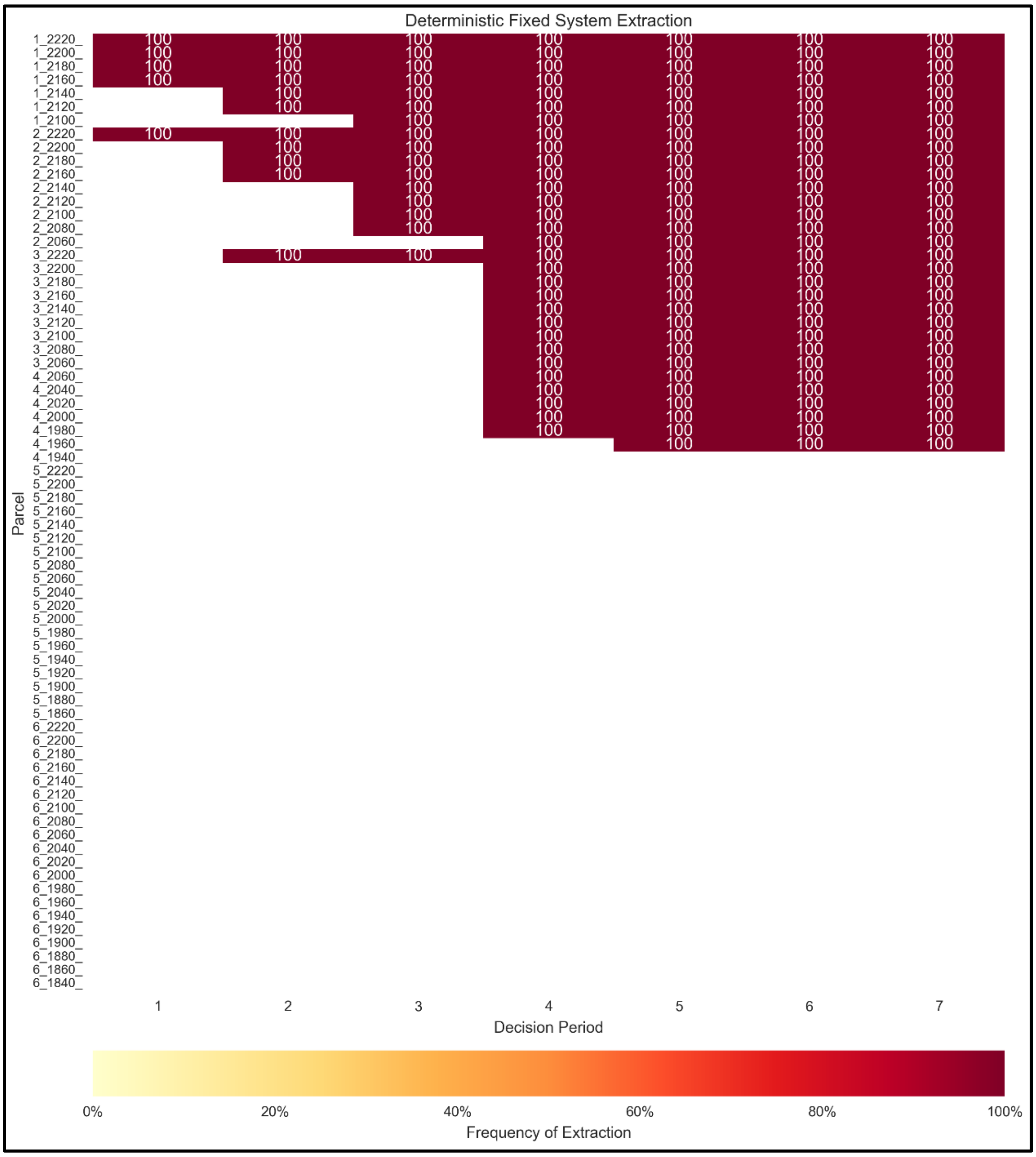



Table 22: Frequency of Extraction for Fixed System without Geological Uncertainty

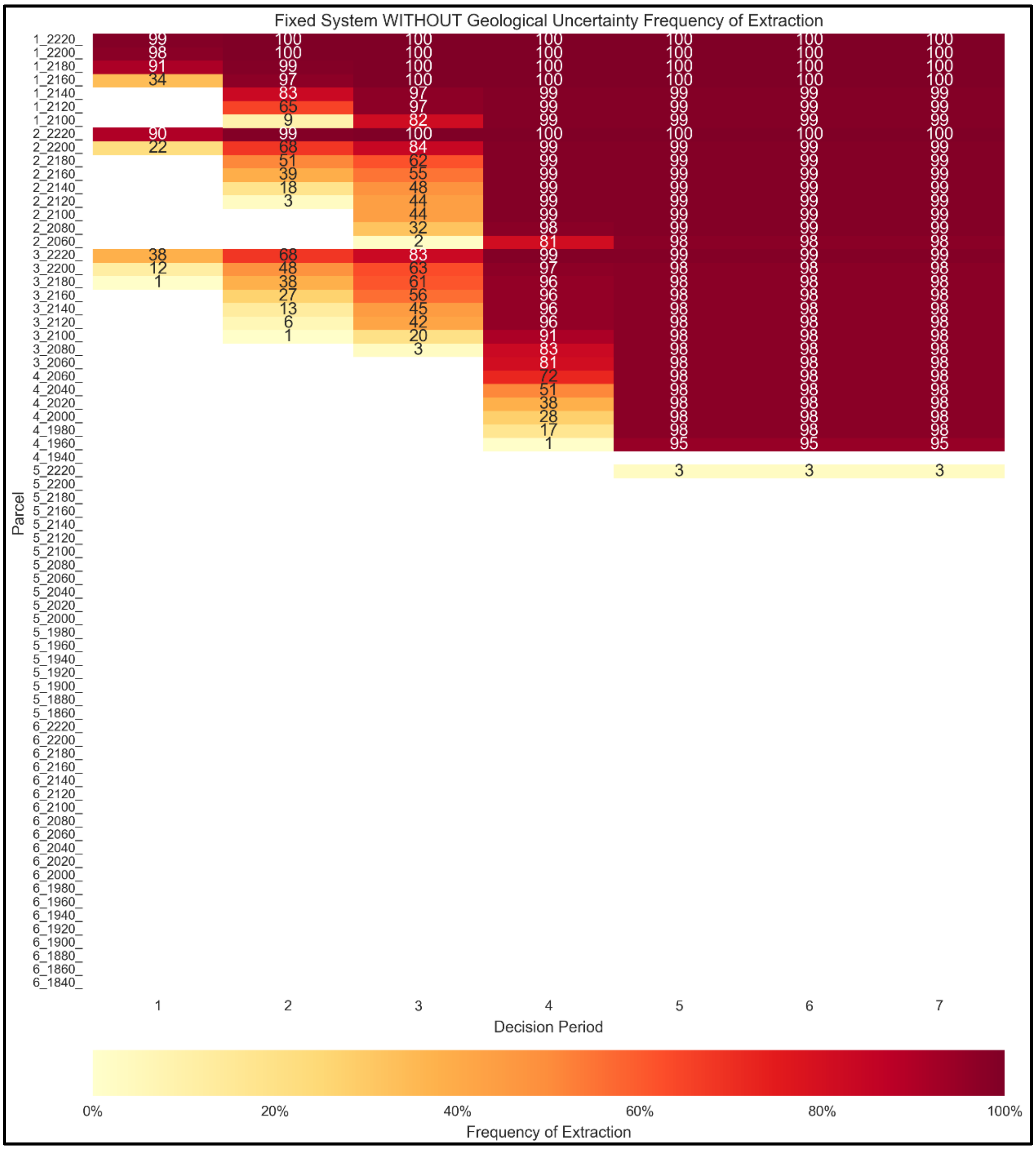
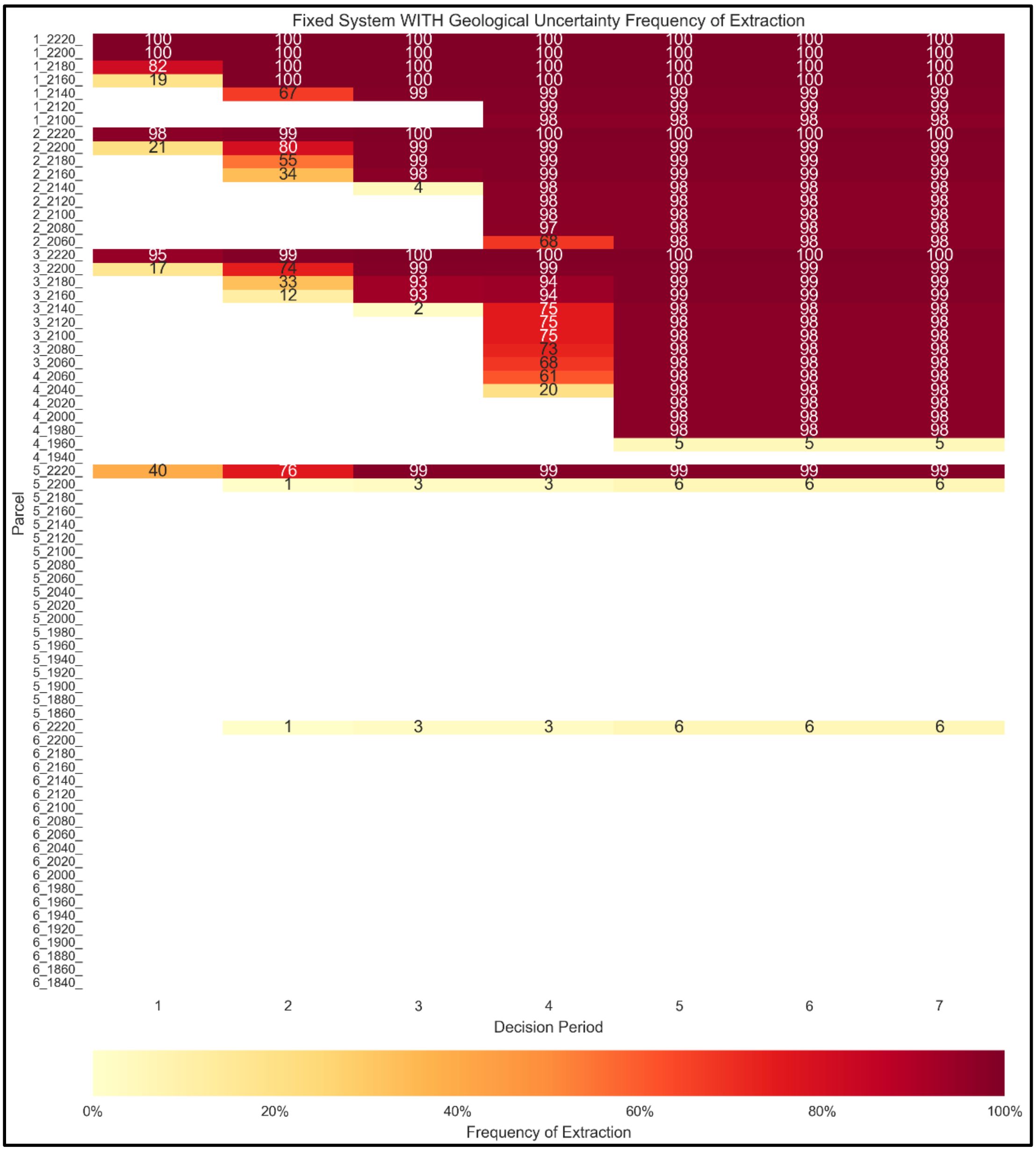



Table 23: Frequency of Extraction for Fixed System with Geological Uncertainty




From these figures it is evident that there is variability in the schedule when geological uncertainty is included in the model. This variability is due to the schedule responding to the change. It should be noted that in Table 22 and Table 23 mining occurs on the first bench(es) of pushback 5 and 6. This is due to the optimisation model targeting high grade material from the mafic shear zone which pays for stripping on the whole bench. There is potential to improve the pit design further and remove this additional waste.

## 5.5 CONSIDERING SYSTEM FLEXIBILITY IN THE PROJECT

This section examines how including flexibility coupled with uncertainty in the system design can lead to an improved project outcome. The potential design options are outlined followed by a graphical layout of the system design network. These options were then processed through the flexible mode of the mathematical model to determine the optimal system configuration in each scenario. In effect this will generate an optimal frontier of value to target in later design selection decisions.

### 5.5.1 Alternative Design Options

The potential alternative design options to be included in the optimisation, centre on three main themes; (a) different mining extraction rates (capacity), (b) different CIL process plant capacities and (c) alternative process routes (CIL vs dump leach). Additional capacity constraints and multiple products were not considered relevant in this case study. The attributes of the options included in the model are summarised in Table 24.

Table 24: Options included in the case study with capacity and financial assumptions

| **Option Name** | **Type** | **Capacity* (Mtpa)** | **Capital ($M)** | **Disposal Cost ($M)** | **Fixed Cost ($Mpa)** | **Variable Cost ($/t) (Fresh/ Oxide)** |
|---|---|---|---|---|---|---|
| Mining | Mine | 18.00 | 17.80 | 3.50 | 1.20 | Per Block Mining Cost |
| Mining Additional Units | Mine | 8.00 | 8.50 | 0.50 | 0.36 | Per Block Mining Cost |
| Dump Leach | Plant | 2.00 | 5.00 | 1.50 | 2.50 | 4.00 |
| Plant 2 Mtpa | Plant | 2.00 | 63.00 | 3.00 | 1.00 | 6.62/4.82 |
| Plant 3 Mtpa | Plant | 3.00 | 90.00 | 4.00 | 1.10 | 5.82/4.12 |
| Plant 5 Mtpa | Plant | 5.00 | 140.00 | 7.00 | 1.40 | 5.52/3.82 |
| Plant 2 Mtpa with 3Mtpa Expansion Option Ready | Plant | 2.00 | 68.50 | 3.00 | 1.00 | 6.62/4.82 |
| Plant Expansion of 3Mtpa | Plant | 3.00 | 77.00 | 4.00 | 0.40 | 4.79/3.15 |
| HG Stockpile | Stockpile | 1.00 | | | | $1.00/t removed |
| LG Stockpile | Stockpile | 10.00 | | | | $1.75/t removed |
| Waste Dump | Stockpile | unlimited | | | | |

* Capacity stated for 100% fresh rock feed. An adjustment for oxide throughput is made by the ratio 1.375 oxide tonne:1 fresh tonne.

Overall, as the capacity of an option increases the relative fixed cost component and variable cost component decrease, due to the ‘economies of scale’ that are gained by running a larger operation. Further, the ready plant expansion option provides the ability to model an option type where a 2 Mtpa plant is constructed initially with the consideration for a 3 Mtpa plant expansion in a later period. The idea being that the plant expansion is allowed for when the initial plant capacity is built by developing the plant on a bigger footprint that allows for an easy tie-in of additional capacity to the existing plant at a later date. Initially the cost will be higher for the 2 Mtpa plant by $5.5M, but a benefit will be gained due to a lower cost of plant expansion.

### 5.5.2 System Network Design

The network configuration of the system design options is outlined in Figure 37. This defines how material flows, in and out through these options. Each step in the flow network is represented by a decision variable which models whether the option is utilised or not. The optimisation process will decide which options should be executed and in what time period to maximise value.

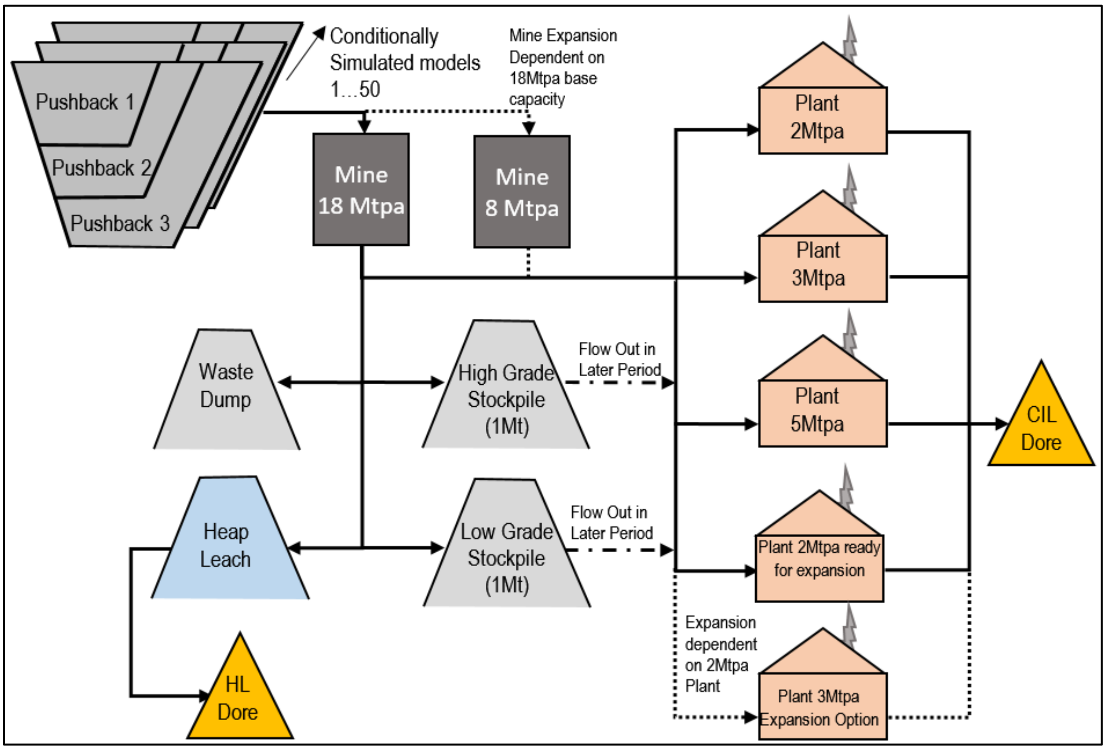



Figure 37: Alternative design option layout for case study analysis

### 5.5.3 Flexible Design Mode of Mathematical Model

A flexible mode mathematical model was generated based on the network of design options outlined in 5.5.2 with 100 simulations of the uncertainties. Each of these models were solved to optimality with the distribution of project values shown in Figure 38.

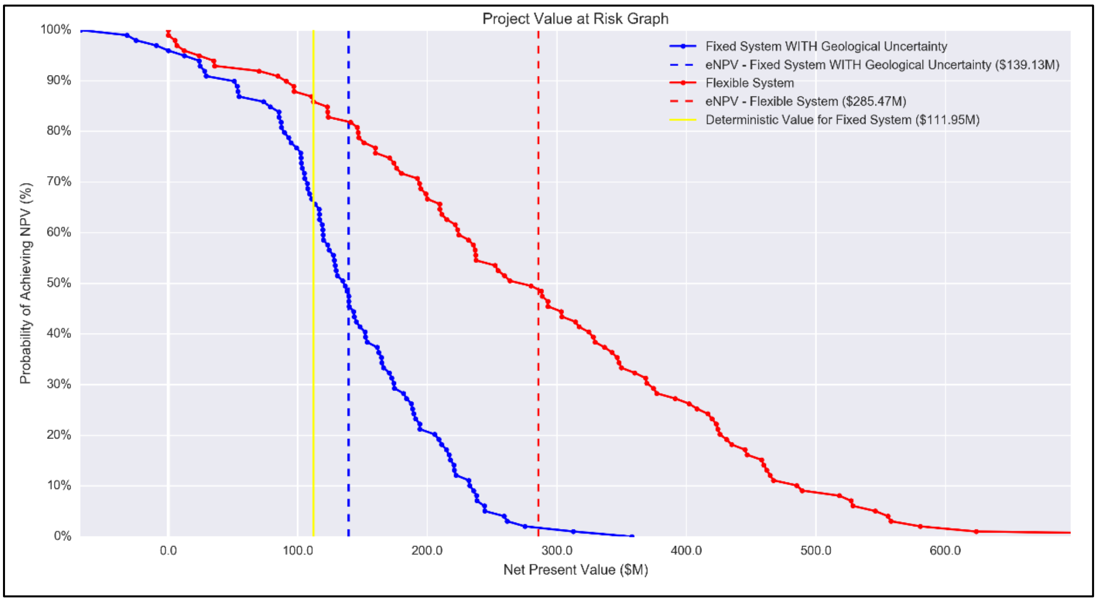
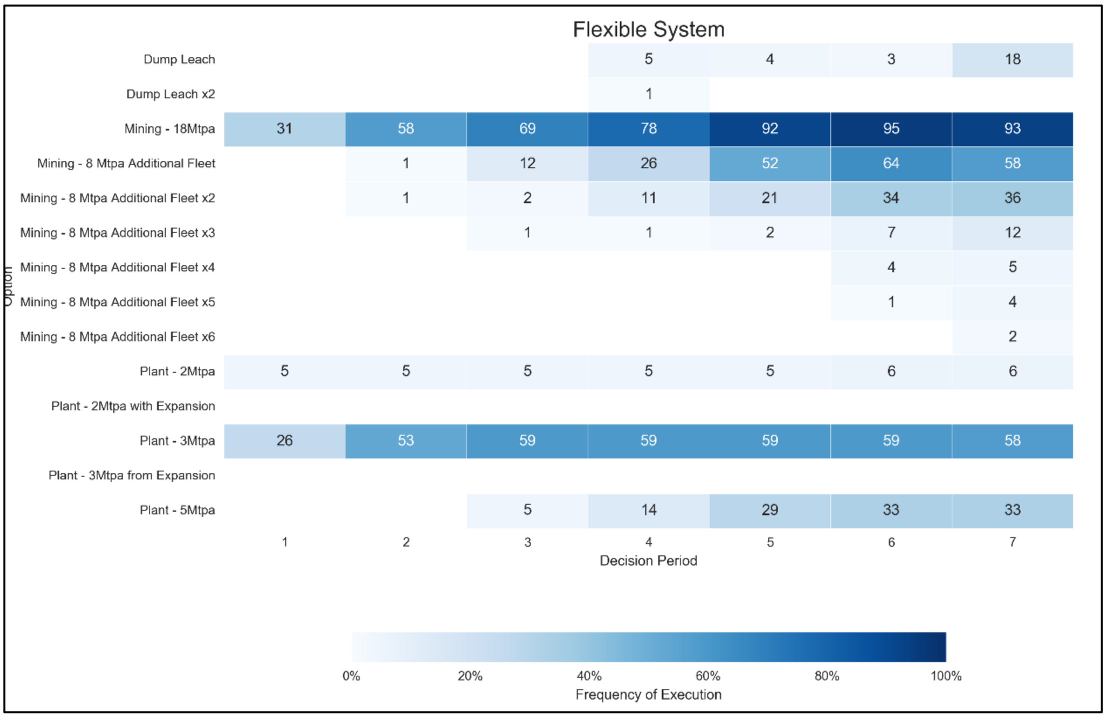



Figure 38: VARG comparing flexible system with a fixed system

The flexible system line (red line) effectively forms an 'optimal frontier' for the project as it determines the best configuration for each simulation. In effect, this would only be possible if perfect information was available i.e. knowledge of the exact future state when making a decision. In reality this never occurs. A decision maker can test alternative system configurations to reduce the value gap between the fixed system and the flexible system.

From the flexible mode it is possible to generate a frequency of execution map that will show which options are utilised most often across scenarios, as shown in Table 25.

Table 25: Frequency of execution of design options in flexible mode



Flexible System

| Option | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Dump Leach | | | | 5 | 4 | 3 | 18 |
| Dump Leach x2 | | | | 1 | | | |
| Mining - 18Mtpa | 31 | 58 | 69 | 78 | 92 | 95 | 93 |
| Mining - 8 Mtpa Additional Fleet | | 1 | 12 | 26 | 52 | 64 | 58 |
| Mining - 8 Mtpa Additional Fleet x2 | | 1 | 2 | 11 | 21 | 34 | 36 |
| Mining - 8 Mtpa Additional Fleet x3 | | | 1 | 1 | 2 | 7 | 12 |
| Mining - 8 Mtpa Additional Fleet x4 | | | | | | 4 | 5 |
| Mining - 8 Mtpa Additional Fleet x5 | | | | | | 1 | 4 |
| Mining - 8 Mtpa Additional Fleet x6 | | | | | | | 2 |
| Plant - 2Mtpa | 5 | 5 | 5 | 5 | 5 | 6 | 6 |
| Plant - 2Mtpa with Expansion | | | | | | | |
| Plant - 3Mtpa | 26 | 53 | 59 | 59 | 59 | 59 | 58 |
| Plant - 3Mtpa from Expansion | | | | | | | |
| Plant - 5Mtpa | | | 5 | 14 | 29 | 33 | 33 |

Decision Period

From this graphical summary of the options executed in each scenario it is possible to determine the most preferred options. In this situation it can be seen that the Mining Fleet of 18 Mtpa is executed in nearly all scenarios ~95% by the end of the project; the maximum is obtained in period 6 with some scenarios disposing of the mining fleet in period 7. Further, from a process plant perspective, in over half the scenarios the 3 Mtpa plant is the preferred option, followed by the 5 Mtpa plant in a third of the scenarios, with the rest of these scenarios using a 2Mtpa plant or Dump Leach process or a combination of these.

The problem with the flexible system is that determining what decision should be made now is unclear as no design option has a 100% execution in period 1. To overcome this issue it is proposed that a robust and operational configuration be investigated.

Prior to investigation of the robust and operational systems, it is worth considering if the original feasibility system design was actually an optimal selection from the design possibilities (system design network). To determine this, a flexible mode model with average deterministic values instead of uncertainty distributions. The system configuration from this optimisation model was then compared to the feasibility study. The optimised configuration is shown in Table 26 with the optimised mining schedule in Table 27.

Table 26: System configuration for deterministic parameters

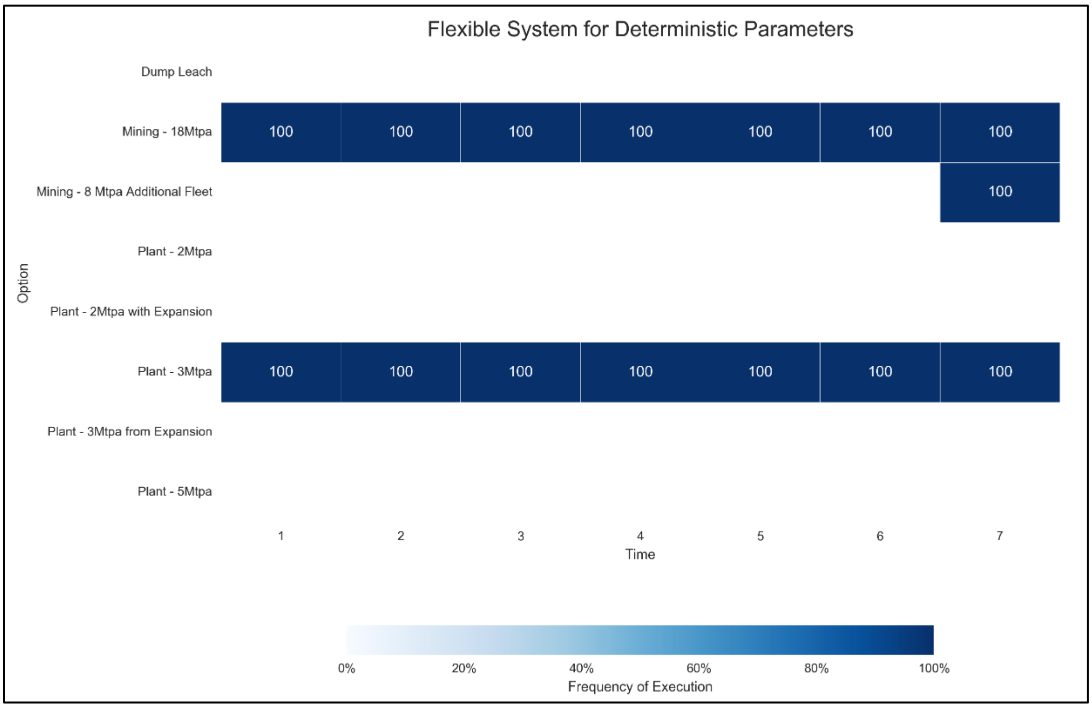
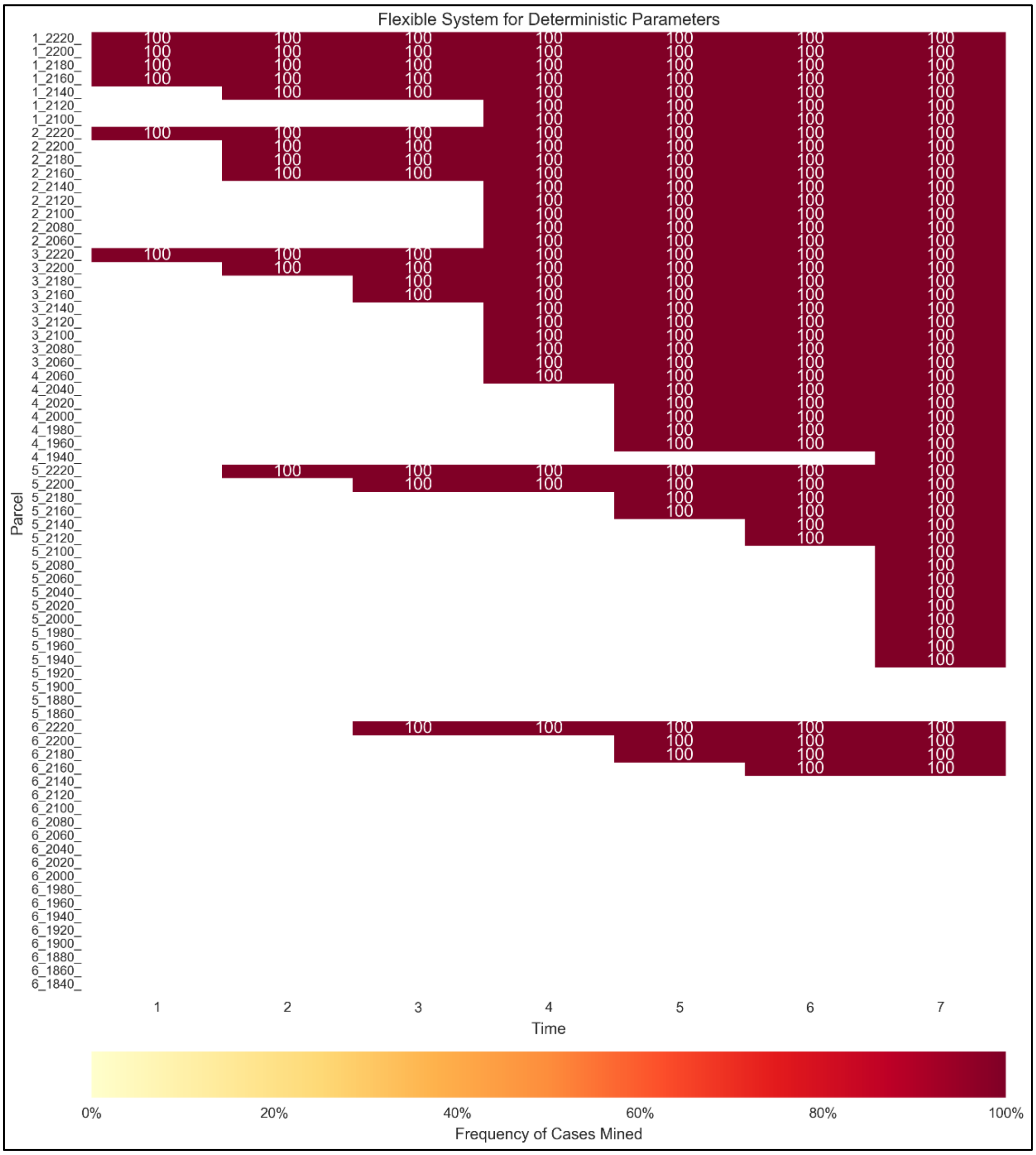



Table 27: Frequency of extraction for flexible system for deterministic parameters showing mine schedule




This system configuration suggests that a larger plant (3 Mtpa) capacity should have been installed initial. It suggests that the operation should have run through all time periods (1-7) and mining capacity should have been expanded in period 7. These key changes allow for the generation of an additional $127.8M in NPV ($266.9M NPV compared with $139.1M NPV from the feasibility study).

This suggested system configuration can be run in the fixed system mode with the simulations of uncertainty included. A VARG can then be generated to compare this system configuration with the other possibilities as shown in Figure 39 with the black line. This shows that whilst this design has a better NPV than the feasibility design it also carries more risk due to a wider distribution of potential project values.

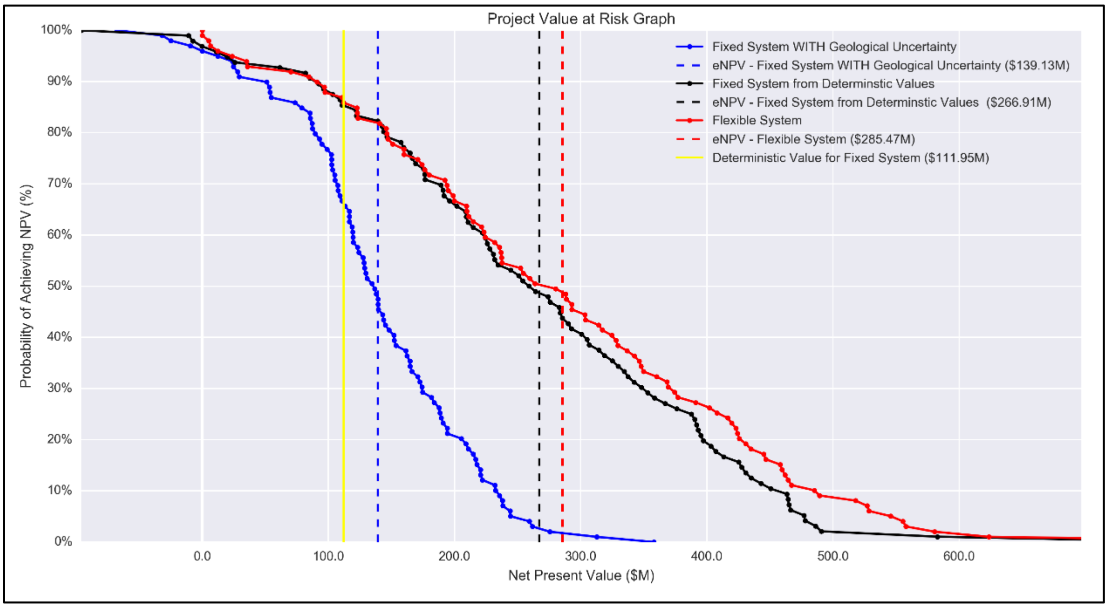



Figure 39: VARG comparing feasibility system with optimised design and flexible system (NB: the slight crossover of the black line on the red are due to differences in stockpiling grade bin ranges between models)

Due to the large deviation between the fixed system from the feasibility study and the proposed fixed system by optimising on the deterministic values, the revised optimised design (Table 26) will be referenced as the base configuration going forward.

## 5.6 DEVELOPING AN OPERATIONAL DESIGN

A robust system approach is proposed as a means to generate a system configuration that can handle variability better. It achieves this by considering different simulations inside the same optimisation process. It is hypothesised that by considering the variability in the design phase the system will be better placed to handle uncertainty and thus increase shareholder value. To generate a robust system the first step is to select which simulation runs will be used as the subset to represent the variability. A representative sample of the possible project variability should be taken. To achieve this, three simulations that correspond to the 25$^{th}$, 50$^{th}$ and 75$^{th}$ percentile of the VARG for the project were selected, as shown in Figure 40.

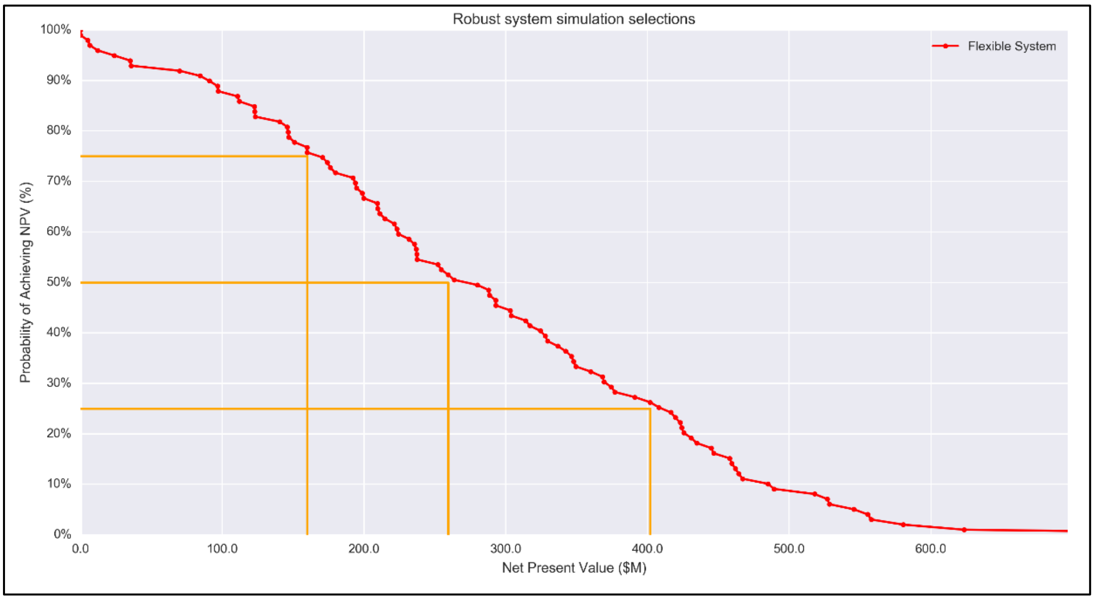
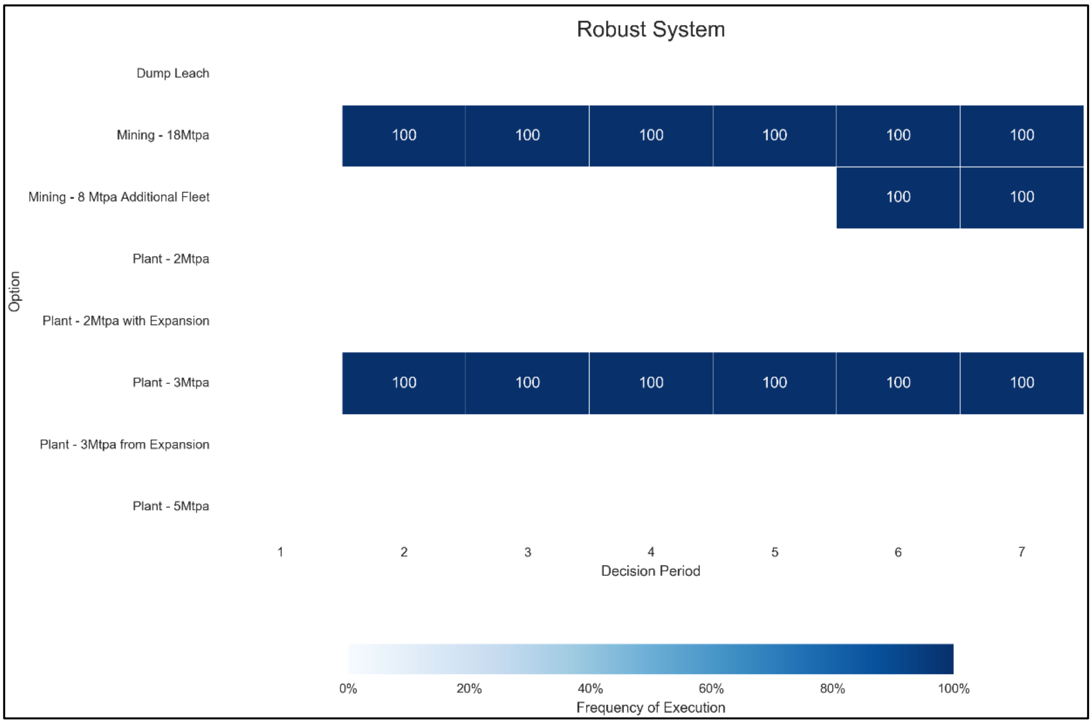



Figure 40: The simulations selected from the flexible system VARG (25th, 50th and 75th percentile scenarios)

The selected simulations are then combined into an optimisation model which solved for a singular system configuration. The robust configuration generated by the optimisation is displayed in Table 28.

Table 28: Robust system configuration




From a project value perspective, this robust design can be analysed in a similar fashion to the feasibility system configuration. That is fixing the system configuration and running a fixed mode model. This allows the impact of uncertainty to be determined on the distribution of project values as shown in Figure 41. Additionally, the variation in the mining schedule is shown in Table 29.

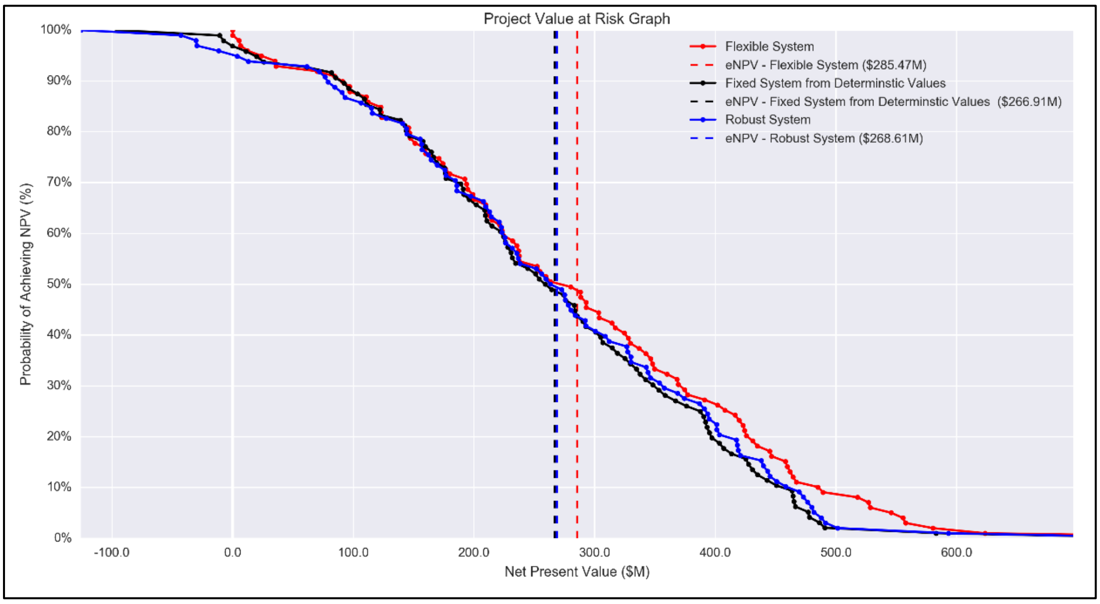
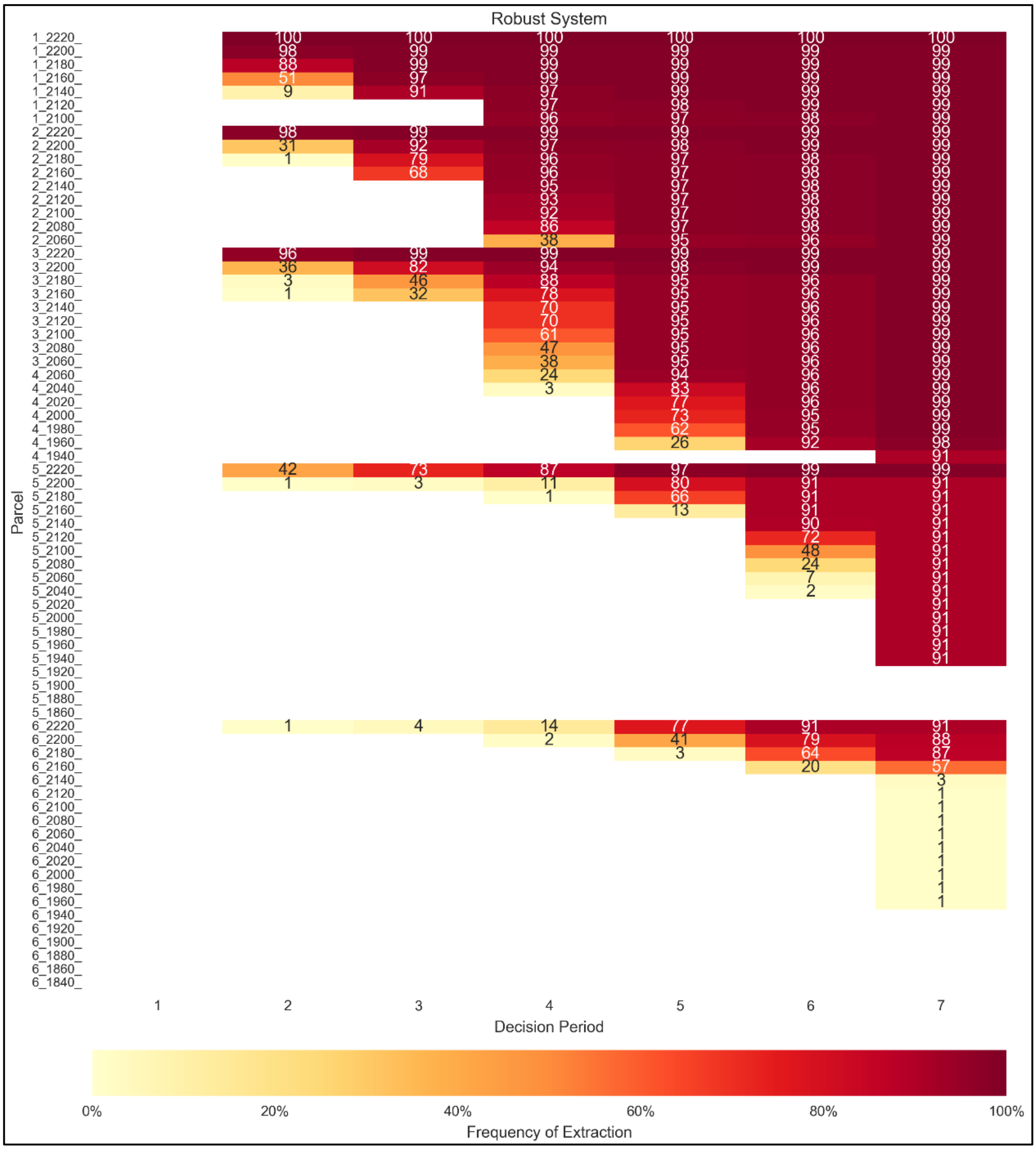



Figure 41: VARG comparing robust system with flexible and fixed systems

Table 29: Frequency of extraction for robust mode showing the variation in mine schedule over time




It can be seen in Figure 41 that there is still a gap between the ‘optimal frontier’ (flexible mode) configuration and the robust system configuration. It is proposed that this gap can be narrowed by developing an operational design which commits to a decision pathway for the first couple of periods with flexibility allowed in later periods. Based on the results of the previous system configuration modes the system configuration in Table 30 is proposed. This fixes the system configuration for the first 3 periods and allows flexibility in the later periods.

Table 30: Operational system configuration over time (N = Not Executed, E = Executed (Fixed), F = Flexible (to be optimised))

| | Time period | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Dump Leach Stockpile | N | N | N | F | F | F | F |
| Mining - 18Mtpa Capacity | N | E | E | F | F | F | F |
| Mining - Additional 8 Mtpa | N | N | N | F | F | F | F |
| Plant - 2Mtpa Capacity | N | N | N | F | F | F | F |
| Plant - 2Mtpa Capacity with Exp. | N | N | N | F | F | F | F |
| Plant - 3Mtpa Capacity | N | E | E | F | F | F | F |
| Plant - 5Mtpa Capacity | N | N | N | F | F | F | F |
| Plant - Expansion 3Mtpa | N | N | N | F | F | F | F |

This system configuration mode is a hybrid between a flexible and fixed mode. It can be evaluated under uncertainty to generate a frequency of execution and VARG, shown in Table 31 and Figure 42 respectively.

Table 31: Frequency of Execution for operational system configuration

Operational System

| Option \ Decision Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Dump Leach | | | | 5 | 2 | 6 | 30 |
| Dump Leach x2 | | | | 1 | | | |
| Mining - 18Mtpa | | 100 | 100 | 99 | 99 | 94 | 93 |
| Mining - 8 Mtpa Additional Fleet | | | | 25 | 51 | 76 | 56 |
| Mining - 8 Mtpa Additional Fleet x2 | | | | 4 | 9 | 20 | 19 |
| Mining - 8 Mtpa Additional Fleet x3 | | | | | | 5 | 5 |
| Mining - 8 Mtpa Additional Fleet x4 | | | | | | 1 | 3 |
| Mining - 8 Mtpa Additional Fleet x5 | | | | | | 1 | 2 |
| Mining - 8 Mtpa Additional Fleet x6 | | | | | | | 1 |
| Plant - 2Mtpa | | | | 2 | 2 | 5 | 5 |
| Plant - 2Mtpa with Expansion | | | | | | | |
| Plant - 3Mtpa | | 100 | 100 | 99 | 99 | 98 | 95 |
| Plant - 3Mtpa from Expansion | | | | | | | |
| Plant - 5Mtpa | | | | | | 2 | 2 |

Frequency of Execution: 0% 20% 40% 60% 80% 100%

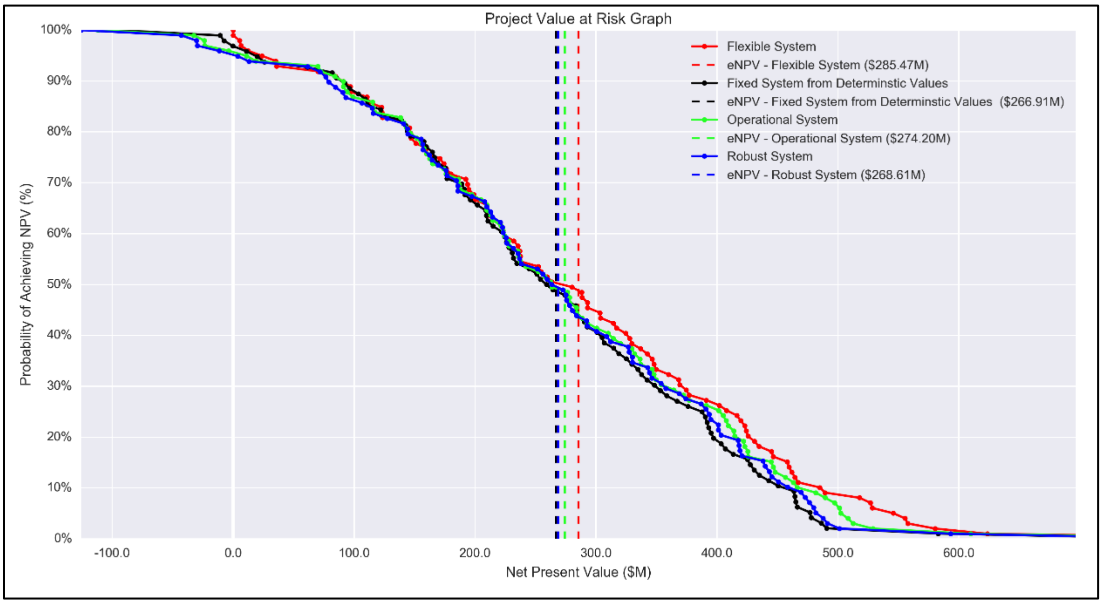



Figure 42: VARG showing operational system value compared to fixed, flexible and robust modes

Based on these results it can be seen that there is significant value potential by including flexibility in the system configuration in later periods. Further, when this flexibility is included in the decision a different initial system configuration is usually recommended.

## 5.7 COMPARISON OF THE MODEL OUTCOMES

A comparison of the models generated centres on investigating the expected value and expected variation (where applicable) in the project value. A summary of this is shown in Table 32.

Table 32: Comparison of system configuration expected values

| | **Expected Project Value ($M NPV)** | **Standard Deviation ($M)** | **Confidence Interval* ($M)** | **Cases which lose money (%)** |
|---|---|---|---|---|
| Flexible System | 285.5 | 164.5 | 5.8 - 569.7 | 0.0 |
| Fixed System from Deterministic Values | 266.9 | 148.8 | -4.6 - 488.9 | 4.0 |
| Feasibility System with Geological Uncertainty | 139.1 | 74.5 | -17.5 - 268.9 | 4.0 |
| Feasibility System without Geological Uncertainty | 143.6 | 68.3 | -4.7 - 262.6 | 3.0 |
| Robust System | 268.6 | 156.1 | -30.0 - 496.8 | 5.0 |
| Deterministic Value for Feasibility System | 111.9 | - | - | 0.0 |
| Operational System | 274.2 | 160.2 | -23.7 - 521.1 | 5.0 |

* 95$^{th}$ Percentile Confidence Interval from outcomes

From this comparison in Table 32 it can be seen that there is an improvement in expected value between the robust mode and the operational mode of $5.59M (2.2%) with a greater upside potential and lower downside potential between the confidence intervals. This is the value of keeping flexibility in the system configuration. By committing to a configuration pathway (in order to progress the project) there is a reduction in the expected value of $11.27M (the fully flexible mode). A reduction in value will always occur because the flexible mode configuration assumes perfect information. In reality, information is costly so the decision maker needs to balance the cost of obtaining more information with the expected value of the project.

## 5.8 COMPARISON TO THE ACTUAL OUTCOME

The project that forms part of this case study has been executed over the past 10 years and mining has recently been placed on care and maintenance whilst the economics of the later pushback stages (5 and 6) are assessed. Since the construction period no significant changes to the feasibility system configuration have been made in terms of mining or process plant. A comparison of the actual cash flow to the forecasted cash flow from the project is shown in

Figure 43.

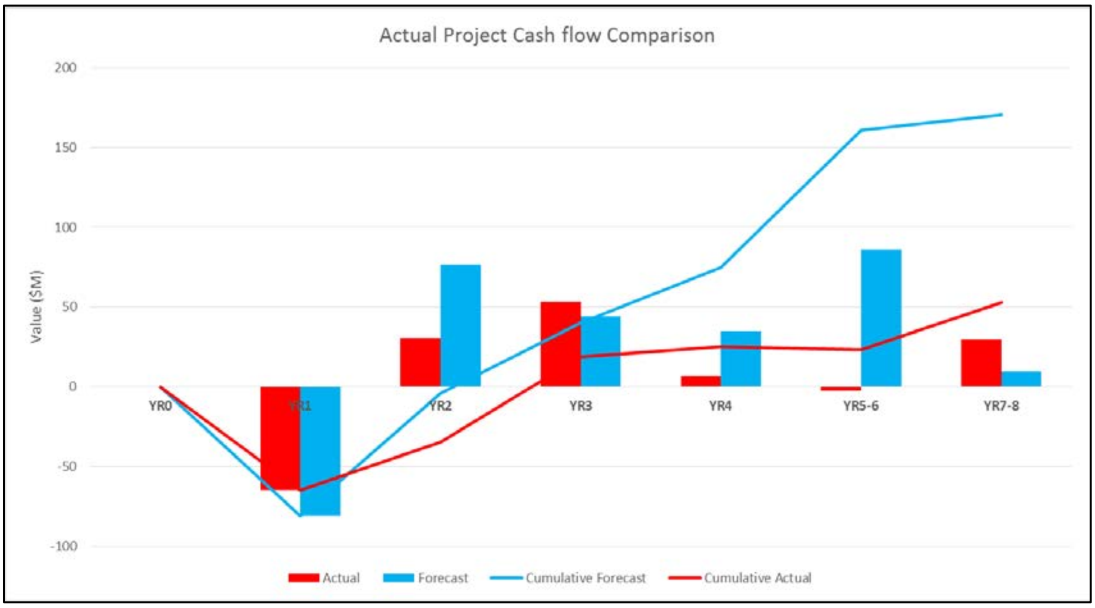



Figure 43: Forecast project cash flow compared to actual cash flow

Based on this; it can be seen that the project had a marginal return and when compared with the forecasted value was much lower. This was mainly due to much higher costs than originally forecasted and a disturbance to operations due to regional civil unrest.

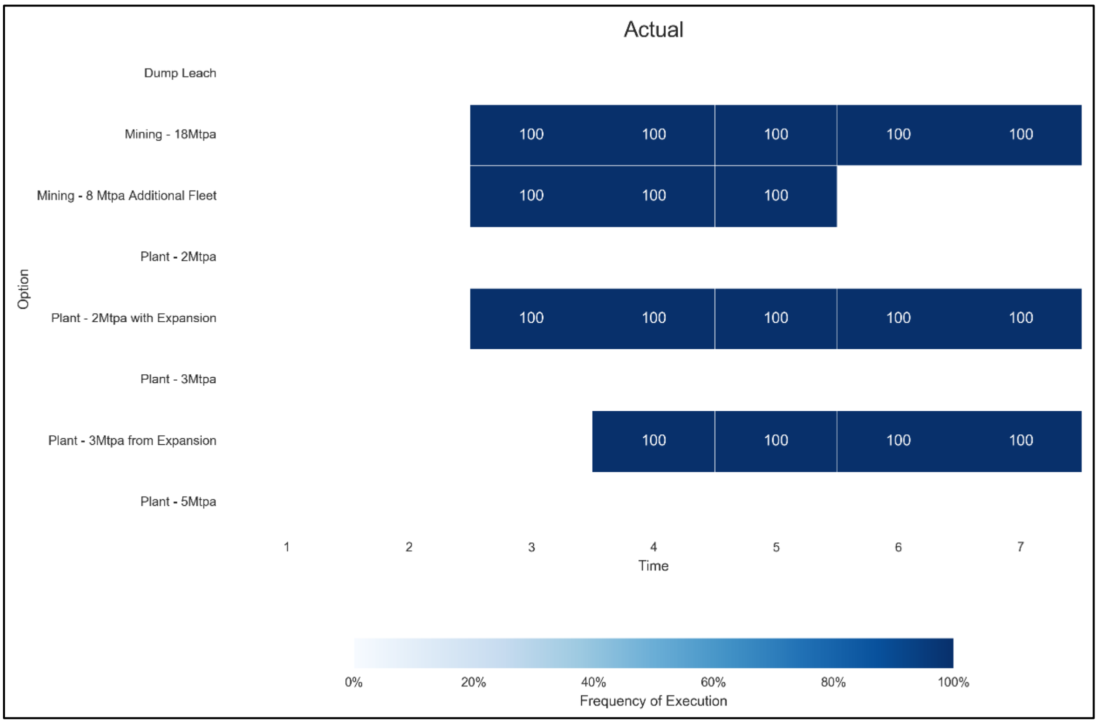
A process of back optimisation was used to assess the actual outcome of theoretical system configurations. This process used the actual uncertainty outcomes (for prices, costs, utilisations, grade and recovery) as the input to an optimisation model. The model was run in the different configuration modes for the system configuration determined earlier in the Chapter. The results are shown in Table 33. It is important to note, the much higher actual project value possible from back optimising, is due to the system maximising production in year 4 when the gold price peaked at $1,600/oz. In reality, the actual cash flow in year 4 was only marginally positive which resulted in a significant missed management opportunity.

Table 33: Expected value for the system configurations when optimised for actual outcomes compared with the planned and actual project value

| Name | Expected Discounted Project Value ($M) |
|---|---|
| Feasibility System | 222.30 |
| Fixed System | 261.23 |
| Robust System | 283.70 |
| Operational System | 287.15 |
| Planned Project | 112.90 |
| Actual Project | 52.76 |

Additionally, with the benefit of hindsight it is possible to determine the optimal system configuration for the project using the actual outcomes. The expected value of the project under these conditions would have been $305M NPV and the system configuration is shown in Table 34. This suggests that a 2 Mtpa plant should have been constructed after a delay of two years with a 3 Mtpa expansion the following year to take the total capacity to 5 Mtpa in Year 4.

Table 34: Optimal system configuration for actual outcomes of uncertainties



## 5.9 EXTENSION OF THE CASE STUDY TO EVALUATE THE IMPACT OF REDUCED RESOURCE

A further extension of the modelling process was considered with Pushback 5 and 6 resources removed from the resource model. In the feasibility study, the ore in these pushbacks was flagged as being inferred with a low level of confidence due to the sparse drillhole spacing. Removing Pushback 5 and 6 from the resource model allows the impact of a reduction in the resource potential to be investigated and the impact the selection of plant and fleet capacities to be determined with Pushback 1-4 only.

Figure 44 shows with dash-dot lines for the system configurations if Pushback 5 and 6 are removed from the resource model. Frequency of option execution graphs for the flexible, robust and operational system configurations are shown in Table 35, Table 36 and Table 37.

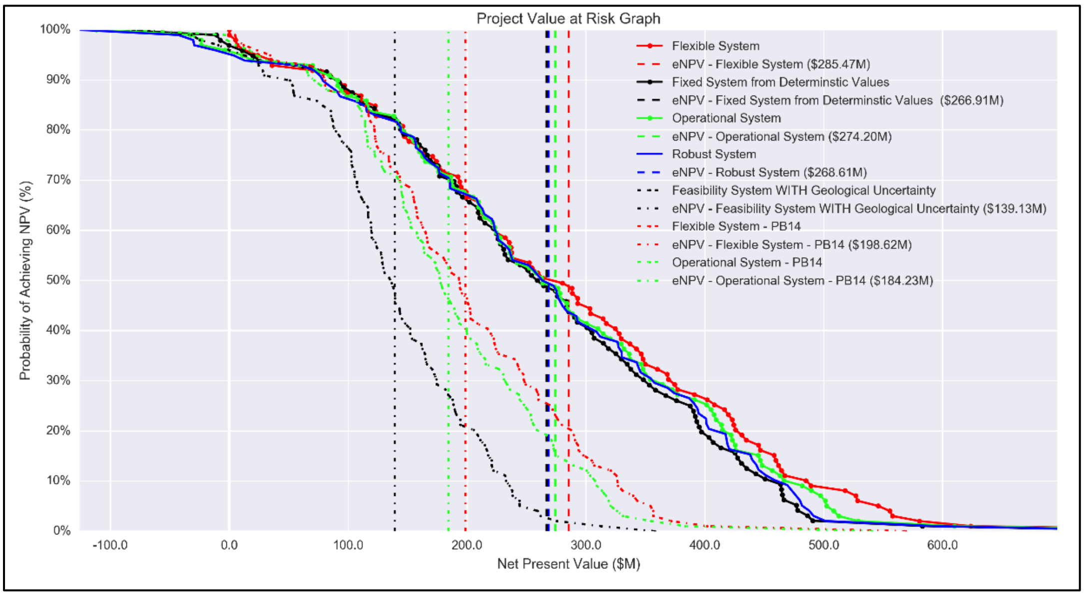



Figure 44: VARG comparison for system configuration with Pushback 5 and 6 removed from the resource model

Table 35: Frequency of execution for flexible system configuration

Flexible System - PB1-4 Only

| Option \ Decision Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Dump Leach | | | | 5 | 1 | 1 | 6 |
| Dump Leach x2 | | | | 2 | | | |
| Mining - 18Mtpa | 32 | 52 | 67 | 80 | 89 | 65 | 30 |
| Mining - 8 Mtpa Additional Fleet | | | | | | 1 | |
| Plant - 2Mtpa | 25 | 32 | 35 | 35 | 34 | 33 | 32 |
| Plant - 2Mtpa with Expansion | | | | | | | |
| Plant - 3Mtpa | 7 | 20 | 32 | 45 | 57 | 58 | 34 |
| Plant - 3Mtpa from Expansion | | | | | | | |
| Plant - 5Mtpa | | | | | | 2 | 2 |

Frequency of Execution: 0% 20% 40% 60% 80% 100%

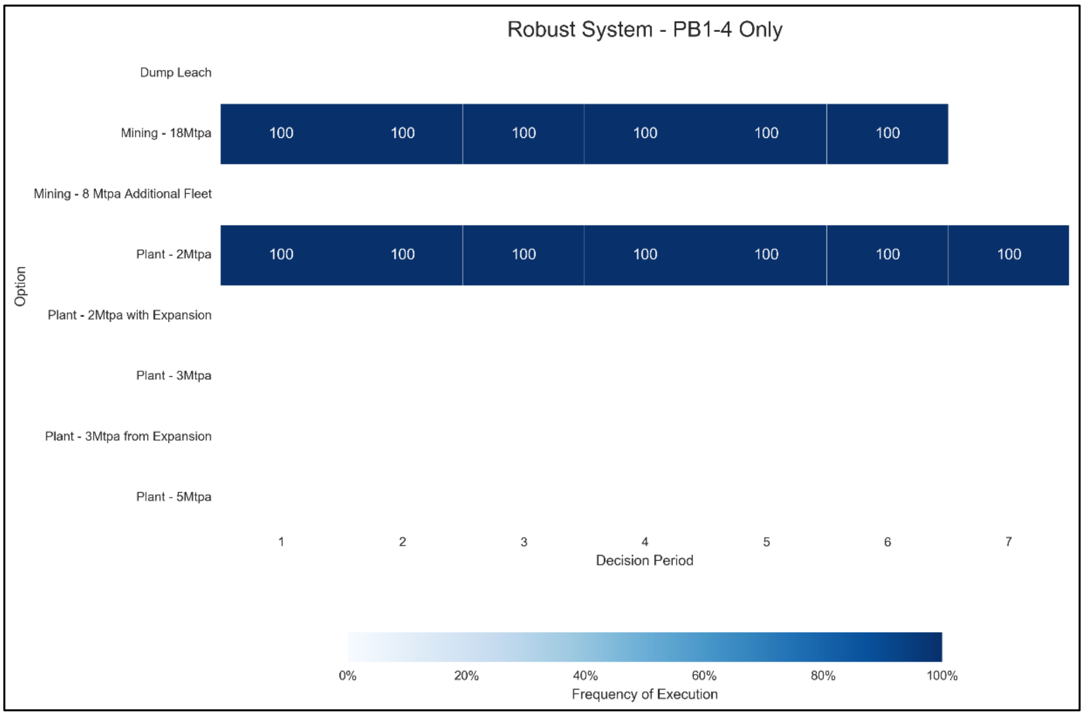
Table 36: Frequency of execution for operation system configuration

Operational System - PB1-4 Only

| Option \ Decision Period | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Dump Leach | | | | 2 | 1 | | 1 |
| Dump Leach x2 | | | | 1 | | | |
| Mining - 18Mtpa | 100 | 100 | 100 | 99 | 97 | 44 | 16 |
| Mining - 8 Mtpa Additional Fleet | | | | | | | |
| Plant - 2Mtpa | | | | | | | |
| Plant - 2Mtpa with Expansion | | | | | | | |
| Plant - 3Mtpa | 100 | 100 | 100 | 99 | 98 | 94 | 40 |
| Plant - 3Mtpa from Expansion | | | | | | | |
| Plant - 5Mtpa | | | | | | | |

Frequency of Execution: 0% 20% 40% 60% 80% 100%

Table 37: Frequency of execution for robust system configuration




It can be seen that removing Pushback 5 and 6 from the model has a significant impact on the project value. This is likely the reason why a smaller plant capacity was chosen in the feasibility study. Had the uncertainty around these later pushbacks been included in the optimisation process the system configuration may have been different. Had uncertainty been included in a quantitative framework as opposed to a qualitative estimate of the sparse drillhole spacing, a more robust system configuration may have resulted. It may have justified additional resource definition drilling to ‘prove up’ the additional resource before commencement of the project.

This could have provided confidence in selection of a different process plant configuration. A different system configuration could have allowed significant additional shareholder value to be realised, potentially in the order of $100M NPV.

## 5.10 APPLICATION USER INTERFACE

The case study utilised a software program developed as part of this research, code named *Mineplex* and developed in Microsoft Visual Basic .NET, that implements the mathematical model proposed in Chapter 4. The program manipulates input data into a structured framework that facilitates an efficient model development process. Models are output as LP files for input into a Mixed Integer Programming solver, of which there are many alternatives. Gurobi (preferred) and CPLEX (commercial solvers, with academic licensing options) were used in this research as they are approximately ten times quicker than the open source solvers (Mittelmann, 2016). The resulting output is sent back to the *Mineplex* environment through a .sol file and post processing routines are run to generate aggregated data. These results are then passed through a routine developed in Python to generate the relevant graphs. The basic process for running an example case is outlined below.

### 5.10.1 Block model setup

The initial resource model import screens are shown in Figure 45 and Figure 46. These allow the selection of the comma separated text file to be made which then populates the field list. Fields can then be dragged and dropped from the ‘Fields’ column to the parcel ID field (a unique block ID value), primary grade (main element field which will be used for stockpiling), tonnage (parcel tonnage) and other fields, or selection of the appropriate fields from the dropdown boxes. Once the setup is complete, the model is imported and the resulting imported data structure can be seen in the resource view, shown in Figure 47.

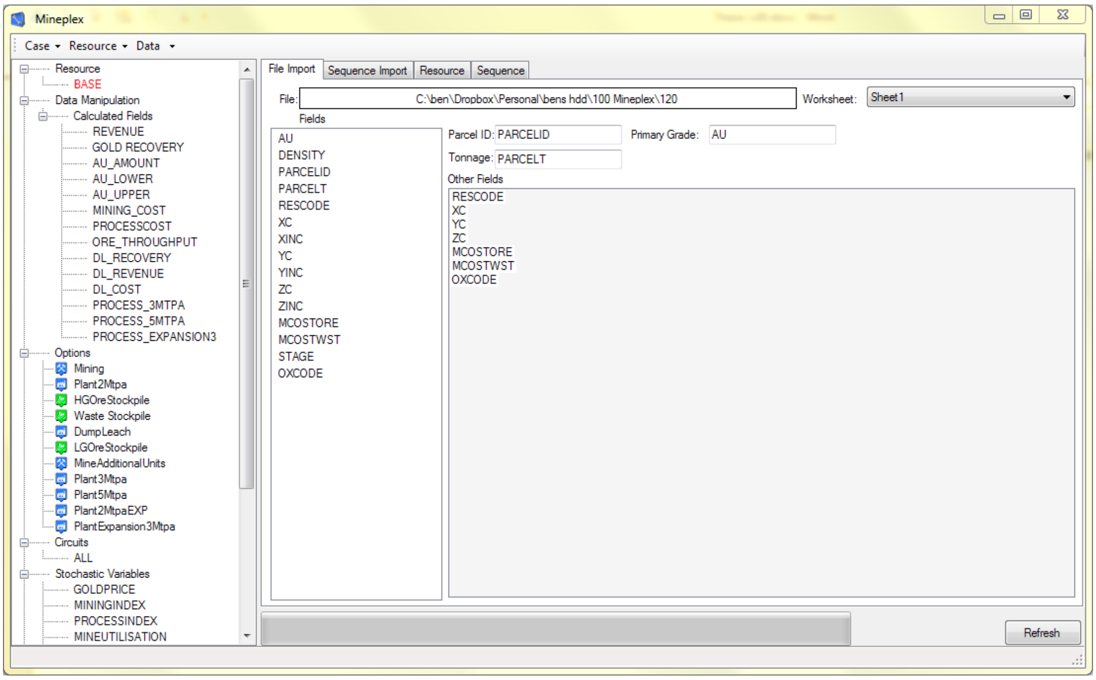



Figure 45: Import resource model

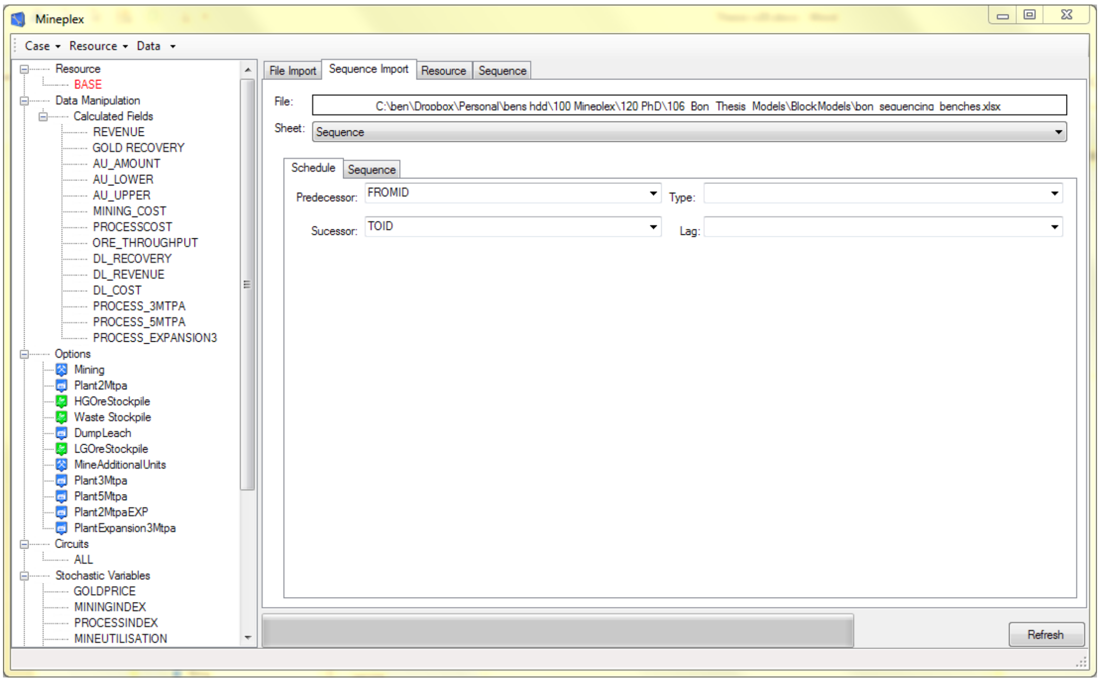



Figure 46: Import sequence dependencies

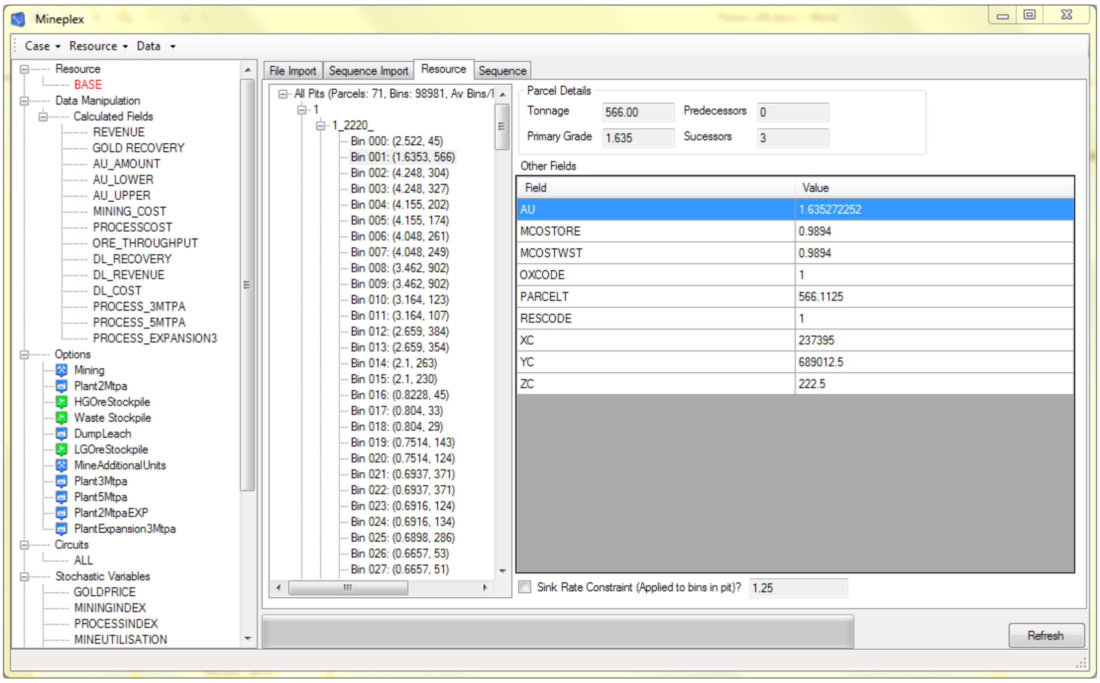



Figure 47: Imported resource model view

## 5.10.2 Block model bin aggregation

Next, the model needs to be aggregated to reduce the number of bins, hence linear variables in the model. This is achieved by selecting Resource -> Aggregate from the menu bar. The size of the bin width can then be set as shown in Figure 48. This determines the accuracy of the model being solved. It is a trade-off between a small bin width that more accurately represents the resource and the solution time of a model. If a small block model is being used than step is not required.

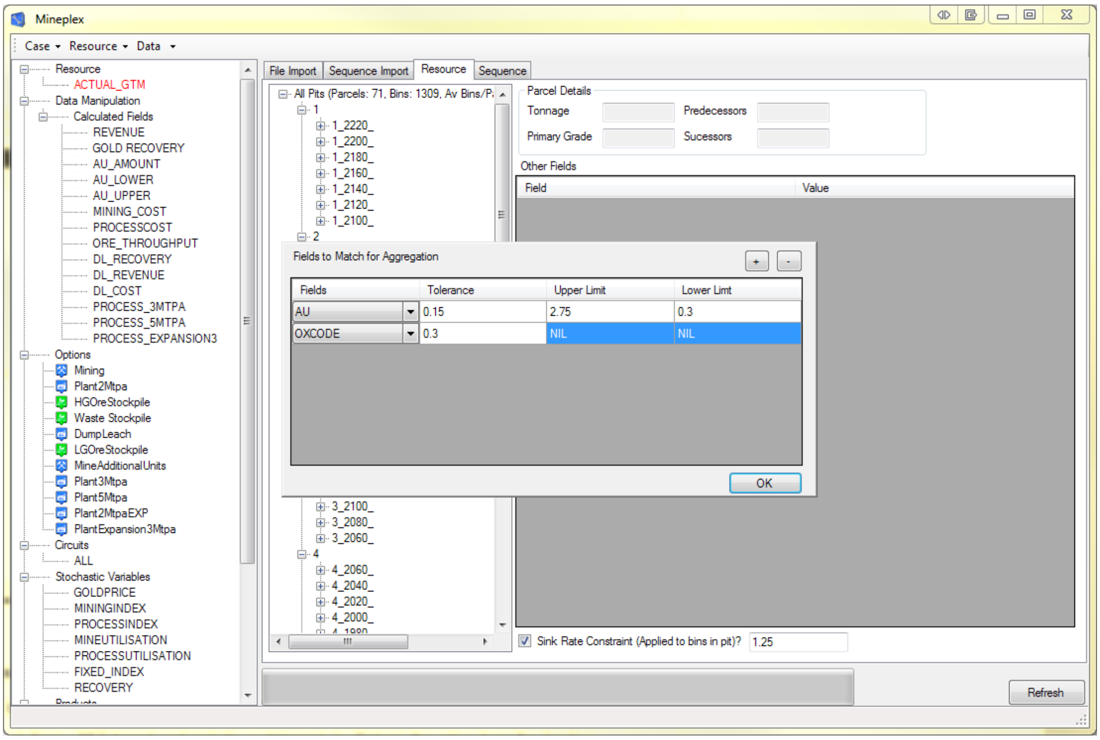



Figure 48: Define the model aggregation tolerances and limits

Calculated fields can be setup which will run the code dynamically on model generation. The calculation code needs to be in a Visual Basic .NET format and include a statement to ‘Return’ the value from the function. These fields provide flexibility by allowing fields from the resource model to be referenced as well as stochastic variables (defined later) from within the code. For example, Figure 49 shows the setup of the ‘Gold Recovery’ field where the [OXCODE] and [AU] values from the resource model are used, together with the ‘Recovery’ variable (referenced as {RECOVERY, time} – ‘time’ indicates that recovery value changes over time and the relevant period value should be return).

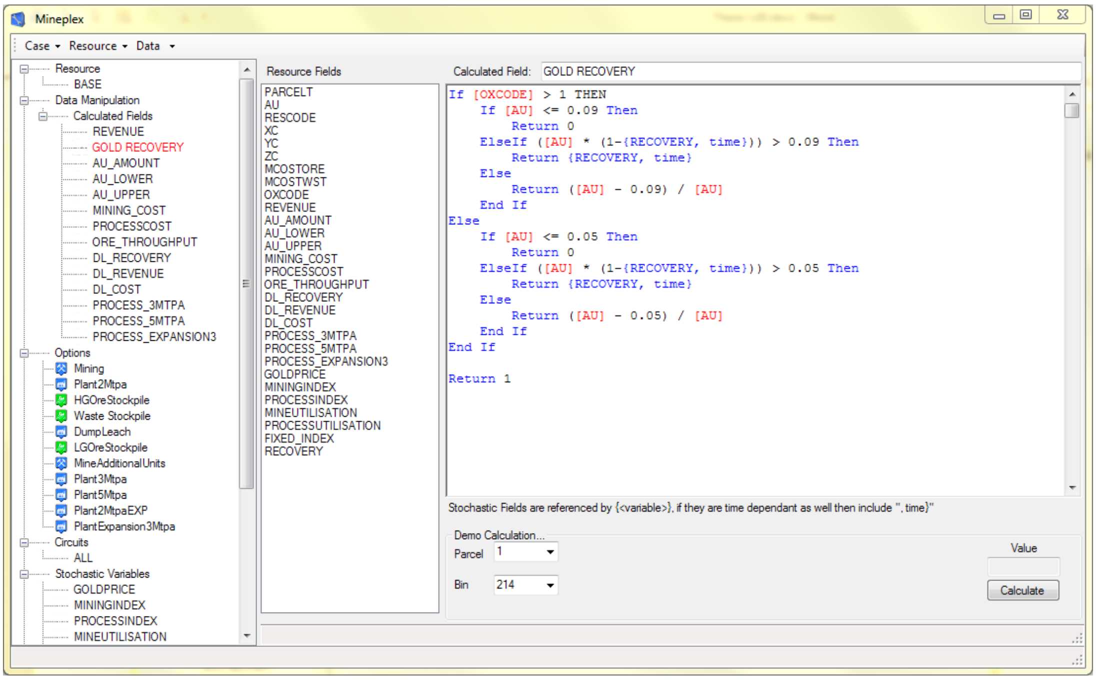


Figure 49: Calculation of recovery field setup

### 5.10.3 Design option configuration

A new design option can be added by right clicking on the Options node. Once a new option is created the relevant option parameters will be configured, in particular; the type of option, the flow in and the flow out to other options or products, financial, capacity and stochastic parameters for the option. Input field vary slightly for each type of option; mining (shown in Figure 50), processing (shown in Figure 51) and stockpile (shown in Figure 52). After each design option is setup the system network design can be viewed by selecting the Options node, as shown in Figure 53.

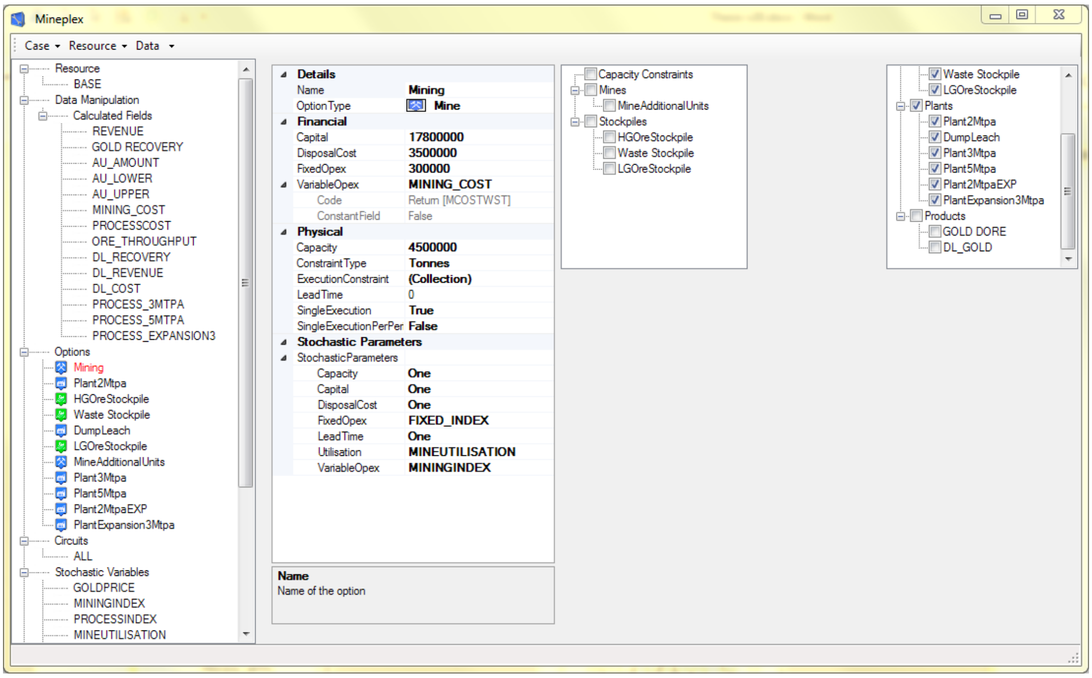


Figure 50: Setup for mining options

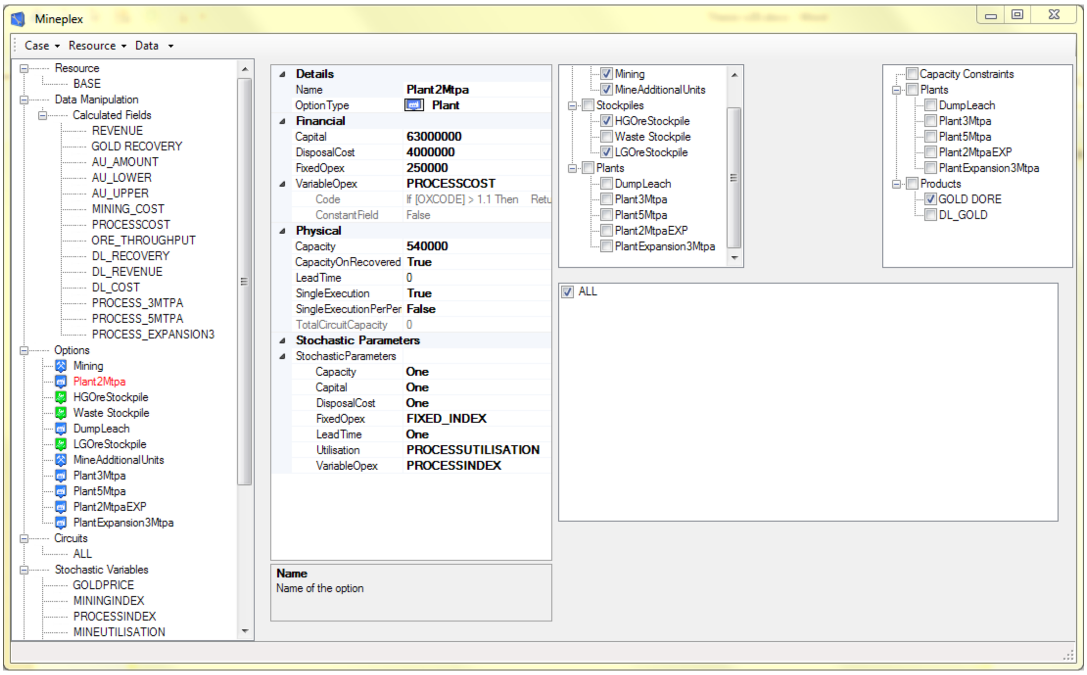


Figure 51: Setup for plant options

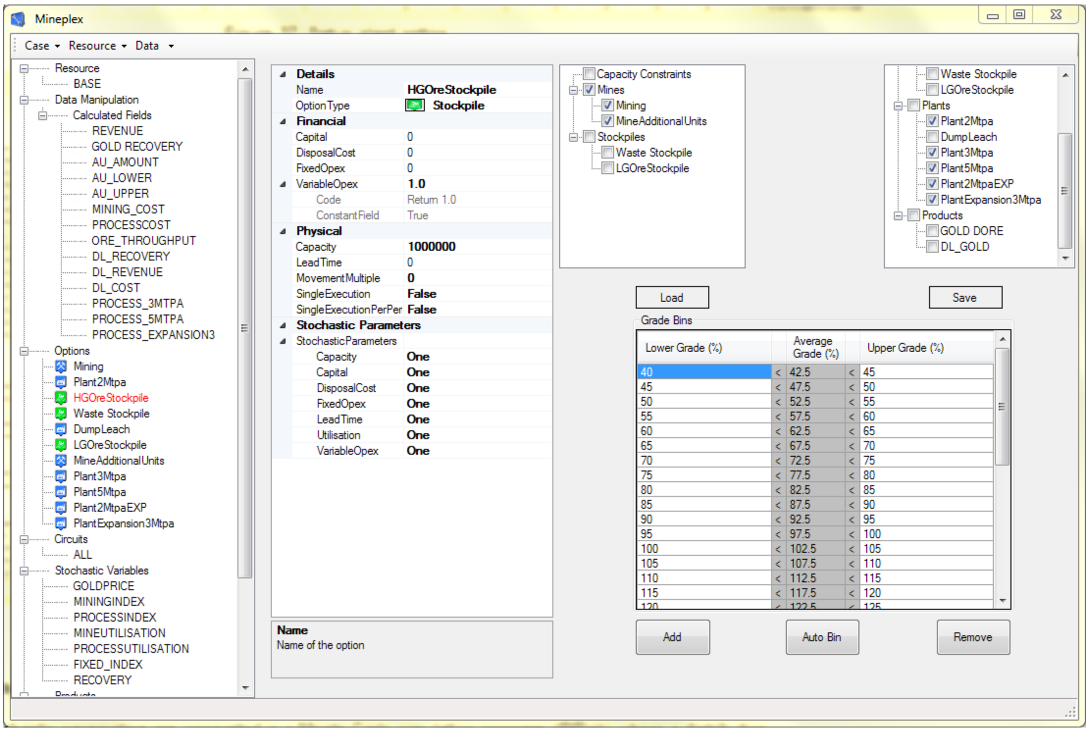



Figure 52: Setup for stockpile options

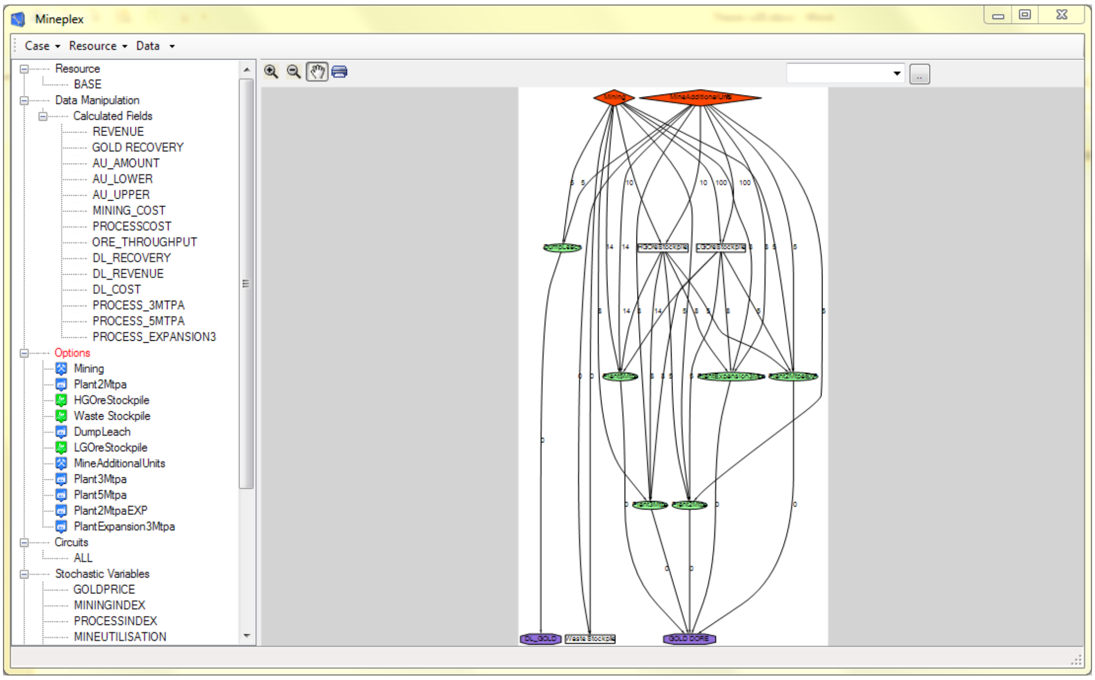



Figure 53: System network design showing configuration of design options

## 5.10.4 Stochastic parameters generation

Next, the stochastic parameters are generated in a Monte Carlo simulation program (@Risk) where a distribution of potential outcomes is setup with a correlation between variables. These variables are then imported into the program via a Microsoft Excel file through the interface shown in Figure 54. These values will then be used in the model generation phase.

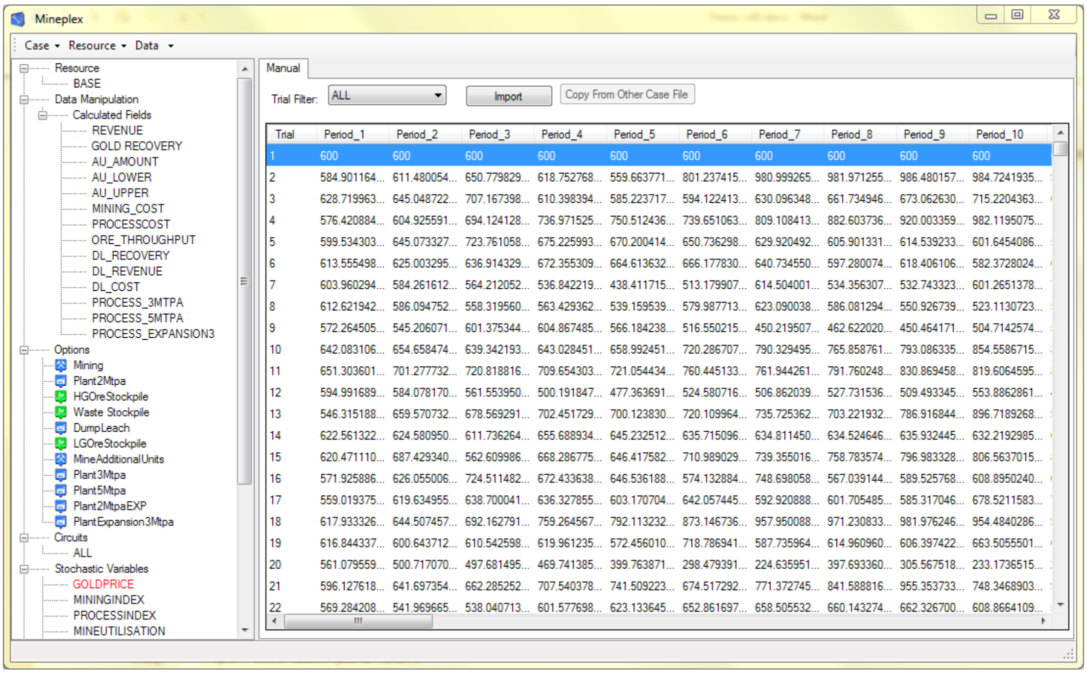



Figure 54: Stochastic value import screen

### 5.10.5 Product Type configuration

The possible products that can be generated by the system are configured as shown in Figure 55. A product is defined by a maximum capacity, value streams which can enter the product and grade limits of the saleable product. Any calculated field can be chosen as the value function which will multiple the tonnage with the corresponding function to determine the revenue from the value stream. The recovery field defines a multiple to the insitu tonnage which adjusts the post process tonnage when considering capacity constraints (in the case study it was used to adjust for differences in oxide and fresh rock throughput capacities). Grade limits can be defined by selecting calculated fields for the lower, upper and quantity functions. Finally, once the product is setup these can be selected in the design option configuration setup as ‘Flow Out’ destinations.

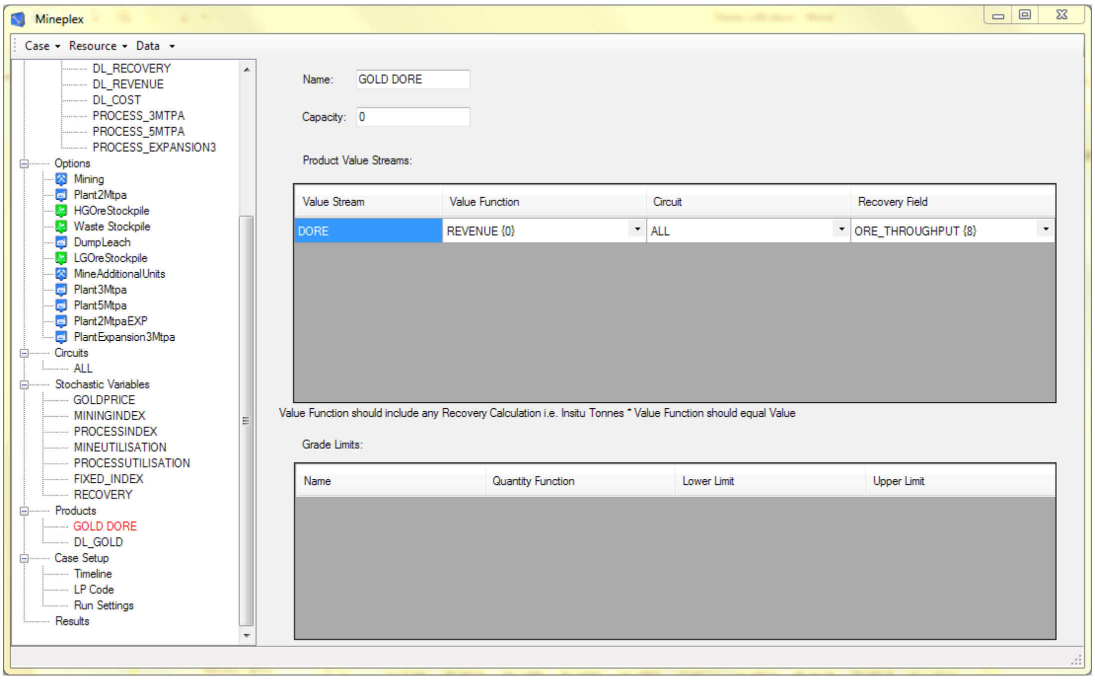



Figure 55: Product setup screen with value and recovery functions

### 5.10.6 Scheduling Period Configuration

The timeline used in the optimisation can be scaled to relevant decision points. The decision points represent meaningful periods at which decisions should be made (scheduling periods). In the example case study, the configuration shown in Figure 56 was adopted. This setup had decision points at annual periods for the first 3 years (3 steps of 4 quarter intervals), biennial periods for the next 6 years (3 steps of 8 quarter intervals) and triennial periods for the next 3 years (1 step of 12 quarter intervals).

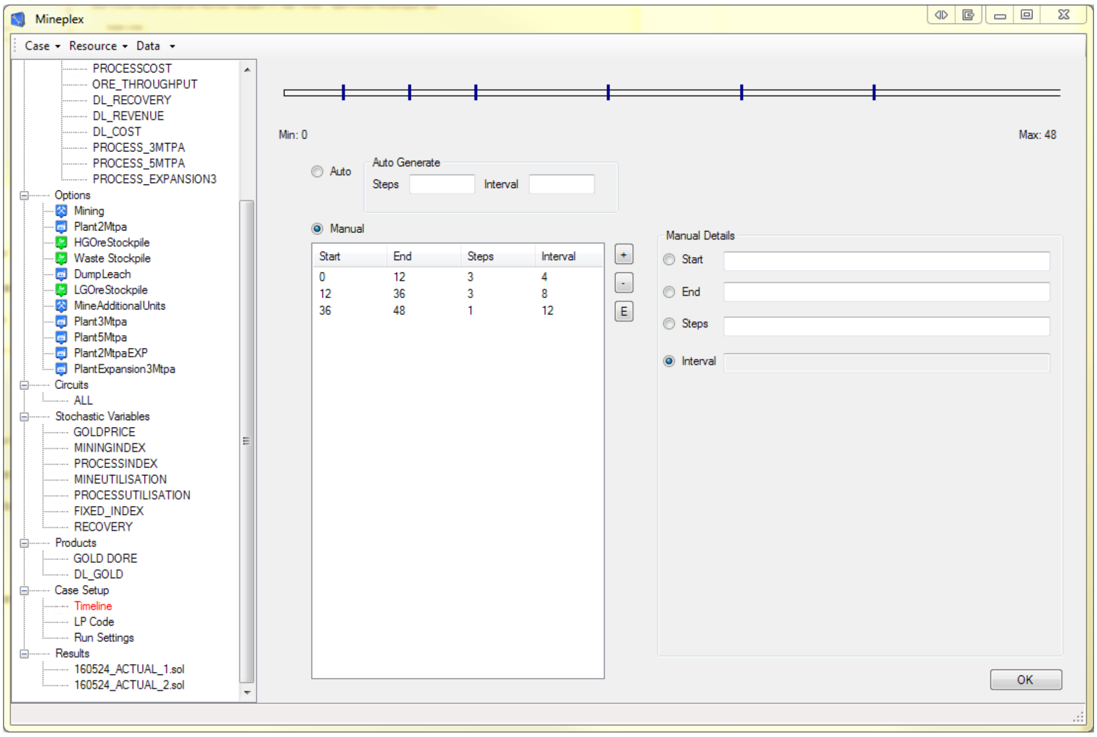


Figure 56: Project timeline showing key decision steps

### 5.10.7 Scenario configuration

Several run parameters can be defined which change the way the model is generated. The input of these parameters is done on the Run Settings tab shown in Figure 57.

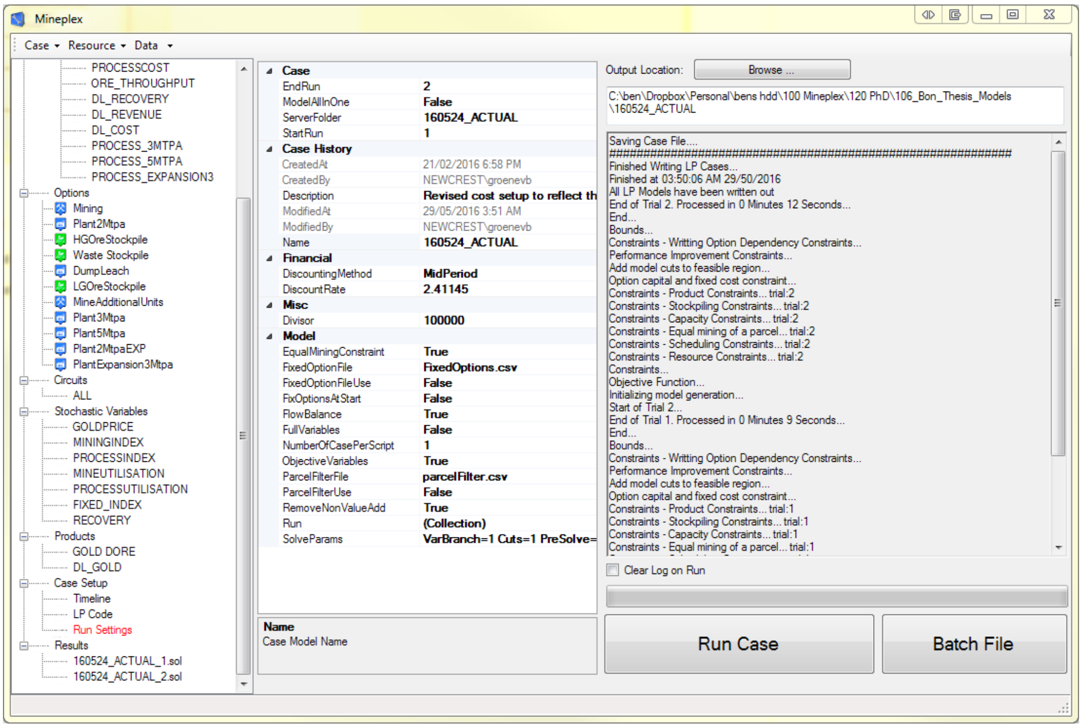



Figure 57: Run parameter settings

The definition for each of these parameters is outlined in Table 38.

Table 38: Description of run parameters

| Category | Parameter | Definition |
|---|---|---|
| Case Details | StartRun | First simulation model generation is to run from. |
| | EndRun | Last simulation model generation is run too. |
| | ModelAllInOne | Set as True to combine all Runs (defined by StartRun to EndRun) into one model i.e. Robust or Set as False to write out individual models for each Run. |
| | ServerFolder | Directory models are located in on the Server. |
| Case History | CreatedAt | The system time the model is created. |
| | CreatedBy | Username of the user who created the model. |
| | Description | Description of what the case is trying to achieve. |
| | ModifiedAt | The system time the model is modified. |
| | ModifiedBy | Username of the user which modified the model. |
| | Name | Name of the model files to be generated. |
| Financial | DiscountingMethod | Define when the discounting method should be applied; start of the period, middle of the period or end of the period. |
| | DiscountRate | Discount rate per time period to be used in financial model (in the case study this is the quarterly equivalent discount rate). |
| | Divisor | Value to scale financial parameters (by dividing with this value). |
| Model | EqualMiningConstraint | Include the equal mining of parcels constraint in the model. |
| | FixedOptionFile | Specify a Fixed Option File to determine if an option is executed in the model. The file format should contain each option on a new row with the value across decision periods defined by the columns. If the value is defined as “Any” it will be left flexible in the model. |
| | FixedOptionFileUse | Set as True to use the FixedOptionFile or Set as False to use the predefined design in the Mineplex code (first two options executed for periods 1 to 5). |
| | FixOptionsAtStart | Fix all mining and processing options based on the FixedOptionFile. |
| | FlowBalance | Set as True to use the flow balance equations to shorten the equal mining constraint equations or Set as False to not introduce additional variables and have long equations. |
| | FullVariables | Set as True to write out full model variables i.e. YL0T1 or XP1_2000_B1T1 or False to write out numbered variables i.e. Y1 or X3. |
| | NumberOfBatchRuns | Define how many batch run files should be created. |
| | ObjectiveVariables | Set as True to include variables to capture the key components of the objective function i.e. REV1 or COST1. Set as False to write contributing variables directly to the objective function i.e. $1250 * XP1_2000_B1L0L2T3. |
| | ParcelFilterFile | File name of the parcel filter file which contains the Parcel ID and time period of parcel variables that should be excluded from the scheduling process. |
| | ParcelFilterUse | Set as True to filter out when parcels can be mined in the model based on the ParcelFilterFile. |

| Category | Parameter | Definition |
|---|---|---|
| | | Set as False to include possibility to mine all parcels at any period. |
| | RemoveNonValueAdd | Set as True to run the performance improvement algorithms to reduce the model size. Set as False to include all variables in the model. |
| | Run | Select the "…" button to setup specific runs to be added instead of using all Runs between the StartRun and EndRun. This is useful when defining the Robust simulations to use. |
| | SolveParams | String of parameters to be set when calling the solver at the start of the optimisation process. |

### 5.10.8 Model generation and optimisation

Models are generated by selecting the "Run Case" button in Figure 57. Compressed LP files will be generated which can then be uploaded to an Ubuntu Linux machine for solving in Gurobi. A batch script file is generated with the models to simplify running multiple models.

### 5.10.9 Model solution analysis

When a model is solved to optimality, Gurobi will generate a solution file which is a column separated text file containing the objective value and the value of each model variable. This file is then uploaded into Mineplex by right clicking on the Results tab and selecting the solution files. In turn, this will generate a summary csv file containing the objective value and execution state of design options and parcels.

With the summary file, several Python scripts can be run to generate VARG graphs, frequency of option execution tables and frequency of parcel extraction maps. These Python scripts makes use of Matplotlib and Seaborn graphing libraries (Hunter et al., 2014, Waskom, 2015) to generate the VARG and Frequency of Execution graphs.

# CHAPTER 6. CONCLUSIONS

This research has demonstrated the value potential of including flexibility in the optimisation process by introducing design options and uncertainty in geological, price, cost and utilisation estimation. The current literature has focused on solving the problems related to determining the Ultimate Pit Limit and scheduling of material within this geometric shape given deterministic conditions. Recent research has attempted to include the impact of uncertainty by including price uncertainty and/or geological uncertainty in the schedule optimisation. However, these approaches do not address the ability to change the system configuration in response to the change in these uncertainties. The methodology outlined addresses these limitations.

The methodology uses a combination of Monte-Carlo Simulation and Conditional Simulation to generate simulations of the uncertainties. This is coupled with a MIP model which incorporates design option flexibility in the optimisation process which allows an optimal system configuration to be determined. Whilst the model cannot generate a single optimal system configuration for all simulations, different modes are proposed which allow the decision maker to step through a logical process to determine which design options contribute to an optimal system configuration. From this process an initial fixed system configuration can be determined for the early decision periods with flexibility maintained into the future. Additionally, the value lost by fixing the initial configuration can be quantified (flexible vs operational). The benefit of fixing the configuration in the early periods is that critical decisions around project implementation (capacity, cost and construction) can be made with flexibility in the later periods maintained (also quantified in the same process). This allows the uncertainties to unfold over time and management to react to this change as they have flexibility already included in the system configuration.

Value-at-Risk, Frequency of Execution and Frequency of Extractions analysis tools have been proposed to assist the decision making process. Examination of the Frequency of Execution graph enables the decision maker to determine which design options are most frequently, and by extension, most valuable to the system configuration. A similar process with the Frequency of Extraction graph highlights when mining is likely to occur in different areas of the deposit, if at all. Finally, the Value-at-Risk graph provides a graphical comparison of the variability of each system configuration and allows alternative configurations to be compared.

Following on from this process, differing hypothetical scenarios can be tested and evaluated, as model outcomes are subject to representative uncertainty distribution selection. For example, the uncertainty distribution can be modified, the model rerun and expected value impact evaluated. Then required effort to gain additional information can be compared – i.e. does an upside modelling case on the geological uncertainty (narrower variation) change the expected NPV by more than 10%? If so, then does additional drilling to determine whether this hypothesis is true cost more in dollars and time than a 10% increase in NPV? Value of

Information theory is an avenue that could be explored to capture these scenarios in a systematic framework.

Performance improvement algorithms and constraints have been introduced and utilised which have a significant impact on reducing the solution time. However, a limitation of the mathematical model is still the solution time required to solve the model (particularly in a robust mode). Additional algorithms (greedy or pruning) and mathematical solution techniques (heuristics) could be formulated to assist this further.

The case study shows, by including uncertainty in the analysis of the deterministic feasibility system configuration, 4% of the cases are likely to achieve a value less than zero for an Expected NPV (ENPV) of $143.6M. Further, the original financial analysis undertaken on the project highlighted an ENPV of $112.9M with a sensitivity analysis on gold price identifying a drop in value to $55.46M for a drop a $100/oz drop in gold price. The decision maker was not informed of the loss making potential of the project.

Improvement on the system configuration was possible by including the design options in the optimisation process using a flexible mode for a single deterministic scenario. This recommended a system configuration that constructed a larger process plant with a capacity of 3Mtpa and a mine expansion in the later periods which generated an increase in ENPV of $123.3M to $266.9M (85%). Next, by including uncertainty into the optimisation process by using the robust mode, a fixed system configuration which generated an increase in ENPV of $3.1M to $268.6M was possible. This recommended delaying the commencement of the operation by 1 period. By including flexibility into the system configuration in the later periods and only fixing the configuration in the early periods it was possible to generate an operational strategy which generated an additional increase in ENPV of $5.6M to $274.2M. This was achieved by implementing the suggested configuration from the robust mode for the first three years with full flexibility to modify the system configuration in the later years. Next, a fully flexible system configuration was able to generate an increase in ENPV of $11.3M to $285.5M. However, this serves as an upper bound on the potential value of the system configuration and is unlikely to be achieved. This is because it assumes perfect information is available to management in order for optimal decisions to be made in every period. Finally, a back analysis of these alternative system configuration by using actual data from the project confirms these observations.

## 6.1 RECOMMENDATIONS FOR FUTURE RESEARCH

Several possibilities for future avenues of investigation were determined throughout this research project. These include the following:

- Extension of the concept of including design options in the system configuration to an underground mine planning environment. It may be possible to restructure the resource model such that it is linked to the different decision possibilities in a way that allows the decision space to be explored efficiently.

- Use of a design structure matrix to remove combinations of options that the decision maker can reasonably exclude as possible viable options (Bartolomei et al., 2007). For example, a mining capacity of only 8 Mtpa with a plant capacity of 40 Mtpa is unlikely to ever be economic.
- Formulation of a greedy algorithm that expands on the early start for a parcel due to mining capacity by including a precedence graph for parcels that relates to mining of other parcel. For example, if mining occurs on the 10$^{th}$ bench of Pushback 1 in period 1 and this requires all mining capacity available to reach this parcel in this period, than any mining in Pushback 2 can be excluded by linking the parcel decision variables.
- Development of an algorithm to provide a lower bound starting basis for the optimisation model. Use of a greedy or network flow algorithm may allow this to be done efficiently as a starting solution. This may help tighten the starting basis of the optimisation model.
- Solving of the model as a two-part model and exploiting the structure of the problem. Specifically the design network and mine schedule components of the model may be exploited through techniques such as Variable Disaggregation (Ahmed et al., 2002) or Benders Decomposition (West-Hansen and Sarin, 2005).
- Integrating pushback selection into the optimisation model. Potential exists to include the pseudoflow algorithm (or variant) or fundamental trees into the optimisation process either directly or indirectly through a guided search and callback from the model solver.
- Reduction in model detail without losing significant value from the optimisation by forcing full parcels to be mined in a period instead of allowing mining activity to exist in a parcel across periods. If a small number of decision periods are used in the model mining across periods has a limited benefit that is likely within the error of the modelling process.
- Refinement of the stockpiling approach in the model to a First-In-Last-Out (FILO) approach by developing flow path variables from the period the block is mined to the period the block is processed for each stockpile and including a binary variable to determine when a stockpile is emptied.
- Development of further algorithms to reduce the solution time by exploiting the mathematical relationships efficiently. This is import as the solving a model by scaling with additional computing resources does not have a linear payoff since there is an overhead in terms of memory management and ‘cross chatter’ between Central Processing Units (CPUs). Furthermore, it is likely that we will see a physical limitation

on the improvement in CPU speed, effectively an end of the benefits of Moore's law (Gordon, 2016). We need smarter algorithms and not just a reliance on brute force to crunch the solution to a problem.

- Use of a genetic algorithm to explore the system configuration matrix with the scheduling sub problem solved with MIP. In theory, the algorithm would learn which design options are beneficial over time and which ones have no value thus converging on a solution quicker than branch-and-bound.

- Additional testing on the number of simulations that are processed through the flexible mode to give a representative distribution of the results. Further, can the question of the number of simulations required for a representative sample be answered?

- Training of minerals industry decision makers - mine planners, geologists, financial analysts and executives to name a few - in uncertainty estimation (Read, 1994) and the impact of incorporating uncertainty into non-linear processes to estimate the true risk of an investment.

Finally, this research has demonstrated significant value creation is possible if uncertainty and flexibility are considered in the design phase of a project. It is hoped a new field of research into operational flexibility from both a strategic and tactical level will be embraced from by the mining industry.

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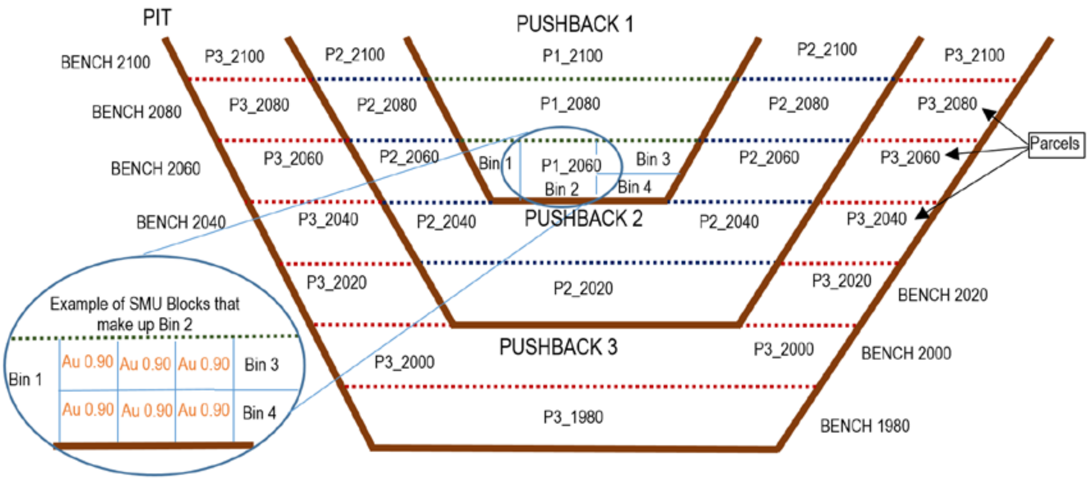
# APPENDIX A – KEY TERMINOLOGY DEFINITIONS

Geological Definitions

The following key terms are used when referring to the geological model:

| | |
|---|---|
| In-situ resource | The block model of mineralisation where tonnes, grade and physical attributes have been determined before mining dilution has been applied. |

Mining Definitions




The following key terms are used when referring to the in-situ resource model when adjusting for mining factors:

| | |
|---|---|
| Resource | A block model containing a set of selective mining units (SMU) which have dilution included. Dilutions occurs when mixing ore and waste material is mixed and results in an increase in mineralised SMU tonnage and reduction in grade. In addition, the SMU's also contain metallurgical recoveries. |
| Reserve | That part of the resource that can be extracted economically at base economic and geotechnical parameters. The optimisation process will determine how much of the Pit can be mined economically in each scenario. |
| Ultimate Pit | The total extraction boundary shape in which the resource could be practically excavated. It is a geometrically feasible shape that could be possibly mined under future economic conditions. Typically the shape is generated by using an Ultimate Pit Limit (UPL) algorithm (Pseudoflow or LG) with a Revenue Factor greater than 1. |
| Economic Pit | The extraction boundary shape that contains the reserve for a specific economic scenario. It may also contain economic interim pushbacks. |
| Pushback | Pushbacks are nested stages (or shells) within a Pit that are a feasible mining shape. They are generated by detailed design or from nested shells by parameterisation of a UPL algorithm. Typically multiple pushbacks will exists within a Pit, with the inner most pushback being the highest value. |
| Bench | A mining bench is a portion of material between vertical elevations, typically a blasting horizon within a pit of 10-20 metres in height. Bench numbering is common between pushbacks. The bench on the inner most pushback is mined before the same bench on the next outer pushback. |
| Parcel | A parcel represents a physical scheduling unit that must be completed before the next parcel (a successor) can be commenced. A parcel can consist of multiple bins of material. Typically a parcel will contain all bins within a bench inside a pushback. |
| Bin | A bin of material that has unique physical and financial attributes determined from the sum of all SMU blocks within the bin. |
| SMU Block | A mineable block of material with dimensions governed by size of equipment, blasting pattern, geology (orientation, variability, dip and structural features) and drillhole spacing. This SMU block will contain physical and financial attributes that define the properties of the block. |
| Mineralised Waste Cutoff Grade | The cutoff grade at which the costs of mining (ex-pit), processing and sustaining capital of one tonne of material equals the net revenue generated from one tonne of material, at <u>future economic prices</u>. Generally used to define mineralised waste material. |
| Economic Cutoff Grade | The cutoff grade at which the costs of mining (ex-pit), processing and sustatining capital for one tonne of material equals the net revenue generated from one tonne of material at <u>current economic prices</u>. Generally this defines the ore and waste classification unless mineralised waste is being stockpiled. |
| Breakeven Cutoff Grade | The cutoff grade at which the cost of mining, processing, administration and sustaining capital for one tonne of material equals the revenue generated from one tonne of material at <u>current economic prices</u>. |

Definition of model components and modes

Four possible system modes can be used to determine the system configuration.

Figure 58 explains graphically how different system configurations may evolve over time depending on the system mode.

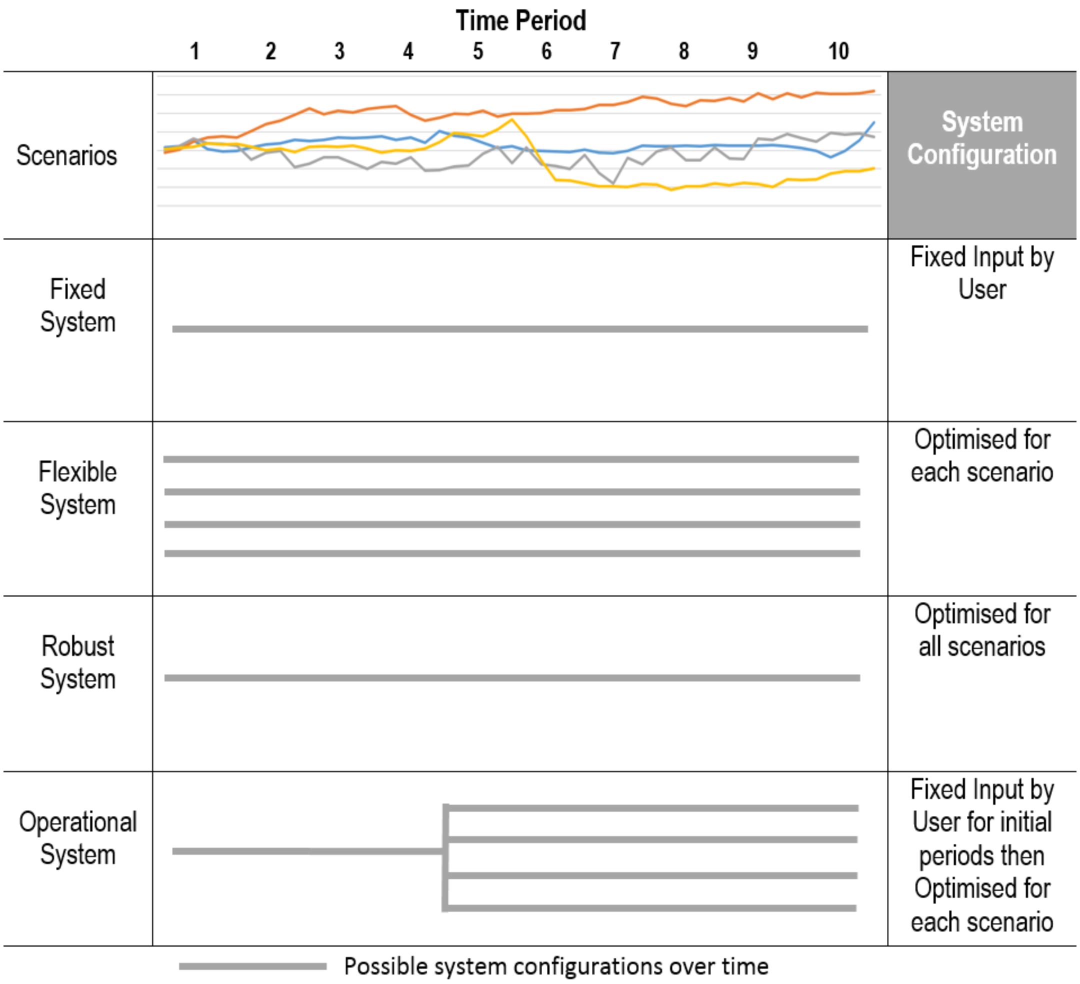



Figure 58: Graphical representation of possible system configurations in each system mode

The following is a list of key terms used when referring to the model design:

| | |
|---|---|
| Design Option | A possible mining, processing, stockpiling or product options. |
| System Design | A network of potential design options that could form part of the mine development. |
| Scenario | A selected set of Design Options from the System Design that form a valid possible solution. |
| Fixed System | A system mode where the configuration is fixed as an input to the optimisation model. This is similar to the current typically adopted approach of fixing mine design on input to the optimisation. |
| Flexible System | A system mode where the configuration is flexible in each scenario. The optimisation model determines the best system configuration for each scenario, thus multiple unique system configurations are possible. |
| Robust System | A system mode where the best system configuration is determined for all scenarios at once in the same optimisation model. This produces one system configuration for all scenarios. |
| Operational System | A system mode where the best system configuration is determined for the initial periods (number of periods defined as an input), then the best system configuration determined for each scenario in the later periods. In theory, this is a hybrid between a flexible and robust system mode and leave flexibility in the back end of the projects life whilst determining a system configuration to adopt and commit to in the early periods. |
| Simulation | A set of economic, physical and geological conditions for a possible future state. |

# APPENDIX B – PSEUDO CODE OF PERFORMANCE IMPROVEMENT

Late start period constraint on execution of option due to remaining time to payback option

When, in period *t*, the value of the potential remaining resource is less than the cost of developing and operating an option *o* then the option cannot be executed in or after period *t*. Since the option execution variables represent whether an option is 'on' or 'off' then this must be implemented as a constraint with the variable remaining in the model.

$MPR_{p,n}$ the maximum value from parcel *p* in simulation *n*

Initialise a sorted list to $RW_{e,t}$ to store the value vs tonnage data sorted by maximum to minimum value

FOR each simulation *n* in N
FOR each parcel *p* in P
FOR each value stream v in V$_d$
FOR each time period *t* in T
IF *max_rev* is less than the revenue ($R_{v,t,n}$ x $W_p$) from parcel *p* through value stream *v* THEN
SET *max_rev* = $R_{v,t,n}$ x $W_p$
END IF
END FOR
END FOR
ADD to the value matrix $RW_{e,t}$ the *max_rev* and parcel tonnage $W_p$
ENDFOR
ENDFOR

SORT the value matrix $RW_{e,t}$ by unit sale price $R_{v,t,n}$in descending order for each simulation *n*

FOR each option o in O
FOR time *t* iterate backwards from T to 2
FOR simulation *n* iterate from 1 to N
SET $\mathrm{MC}_{o,t,n}$ to the sum of option capacity ($\sum_{t=1}^{t} \mathrm{C}_{o,t,n}$) for all possible option builds up to period *t*
FOR flow path *f* in set of all flow paths F that contains option *o*
IF the operating cost ($CO_{o,t,n} + V_{o,t,n}$) is greater then $PV_{o,t,n}$ THEN
SET $MV_{o,t,n}$ = -($CO_{o,t,n} + V_{o,t,n}$)
END IF
END FOR

DO UNTIL *step_ton* is greater than $\sum_{t=1}^{t} \boldsymbol{CAP}_{o,t,n}$OR resource is exceeded
IF next revenue $R_{v,u,t,n}$ – ($MV_{o,t,n}$multiplied by next $W_{p,b}$) THEN
ADD next $R_{v,u,t,n}$ – ($MV_{o,t,n}$multiplied by next $W_{p,b}$) to $PV_{o,t,n}$
END IF
ADD next tonnage $W_{p,b}$ to *step_ton*
LOOP

'Determine minimum fixed cost of capital and fixed operating cost if execution option in this period
SET $PV_{o,t,n}$ to ($PV_{o,t,n}$ – $MV_{o,t,n}$ )
SUBTRACT cost of disposing of option $FD_{o,t,n}$in period T from $PV_{o,t,n}$

```
                    SUBTRACT discounted capital cost FC_{o,t,n} in period t from PV_{o,t,n}

                    IF PV_{o,t,n} > PV_{o,t} THEN
                        SET PV_{o,t} to PV_{o,t,n}
                    ENDIF
                    ENDFOR

                    IF PV_{o,t} is less than the zero THEN
                        ADD constraint Y_{o,t} ≤ Y_{o,t-1}
                    ENDIF
        END FOR
END FOR
```

## Early start for a parcel due to mining capacity

The first size reduction algorithm examines each parcel *p* in the resource to determine if it can feasibly be mined by a given period *t*. If the maximum mining capacity for a period *t* exceeds the extraction required by the sequence to access parcel *p* then parcel *p* will not be mined in period *t*. The pseudocode below outlines the implementation of this size reduction:

```
FOR each simulation n in N
        FOR each time period t in T
            SET MC_{t,n} to the sum of option capacity (Σ_{o∈M,t=1}^{O,t} C_{o,t,n}) for all possible mining
            option builds up to period t
                FOR each parcel p within the set of all parcels P
                    FOR each predecessor p̌ in parcel p relationships
                        DO WHILE p̌ has predecessor parcels
                            ADD parcels p̌ tonnage W_p, to XM_{t,n}
                            IF XM_{t,n} is greater than MC_{t,n} THEN
                                SET X_{p,b,f,t,n} to 0
                            ENDIF
                            STEP to next predecessor of p̌
                        END WHILE
                    END FOR

                    'Determine if mining of a full parcel p cannot be completed in period t
                    - allows Y_{p,t,n} to be removed
                    SET X_{t,n} to parcel p tonnage W_p
                    FOR each predecessor p̌ in parcel p relationships
                        DO WHILE p̌ has predecessor parcels
                            ADD parcels p̌ tonnage W_p, to X_{t,n}
                            IF X_{t,n} is greater than MC_{t,n} THEN
                                SET Y_{p̌,t,n} to 0
                            ENDIF
                            STEP to next predecessor of p̌
                        END WHILE
                    END FOR
                ENDFOR
        ENDFOR
ENDFOR
```

## Early start for a parcel due to maximum sink rate

The second size reduction can be applied if a maximum sink rate is defined. The maximum sink rate is the maximum vertical progression per period specified in number of parcels per period. If the number of parcels to be extracted inside the same pushback as parcel *p* exceeds the maximum sink rate in the same period *t* than parcel *p* decision variables can be removed.

```
FOR each simulation n in N
    FOR each time period t in T
        FOR each parcel p in P within the same Pit (or pushback) in order from of
        highest bench to lowest:
            SET CSR_t to the sum of strip ratios up to period t ∑_{t=1}^{t} SR_t
            SET ABE_p to the benches in parcel p BE_p and the sum of the benches in
            each parcel p on shortest path to the surface ∑_{p̌∈PIT}^{P̌} BE_p̌
            IF ABE_p is greater then CSR_t across all periods to t THEN
                SET Y_{p,t,n} to 0
                SET X_{p,b,f,t,n}  to 0 for all bins b and flow paths f inside parcel p
            END IF
        END FOR
    END FOR
END FOR
```

## Removal flow paths with a marginal value less than zero

Flow paths that end in a product generation state (not paths to a waste dump or stockpile), must have a marginal value greater than zero through the flow path otherwise they should be sent to the waste dump. By default, in the maximisation of the model material will not flow on these paths so they can be removed from the model. Two exceptions could occur where this would not be valid; a) if the capacity of the waste dumps or stockpiles was limited or b) where grade blending may be required to generate a product within the grade limits defined. This may make material that if processed by itself is un-economic actually economic in an overall blend.

```
FOR each simulation n in N
   FOR each time period t in T
     FOR each bin b within B_p from all parcels p within P
         FOR each flow path f from bin b where the flow path ends at a product
            FOR each value stream v in the products value streams
               ADD the unit revenue R_{v,u,t,n} from value stream to the PV_{p,b,f,t,n}
            END FOR

            FOR each option o in flow path f where the option is not a mining option
            o ∉ M
               SUBTRACT the variable unit cost of option V_{o,t,n} from the PV_{p,b,f,t,n}
            END FOR

            IF PV_{p,b,f,t,n} is less than zero THEN
               SET X_{p,b,f,t,n}  to 0
            END IF
         END FOR
     END FOR

     'Determine if material placed on a stockpile could be economic if not remove X_{p,b,f,t,n}
```

FOR each flow path *f* from bin *b* where the flow path ends at a stockpile *s* where *s* is not a waste dump $s \notin \mathrm{D}$
FOR each value stream *v* in the product *d* from stockpile *s* that is possible $v \in F_s$;
ADD product revenue $R_{v,u,t,n}$ to the $PV_{p,b,f,t,n}$
END FOR

FOR each option *o* in flow path *f*
SUBTRACT unit variable cost of option $V_{o,t,n}$ from $PV_{p,b,f,t,n}$
END FOR

SUBTRACT unit variable cost of removing material from stockpile $V_{s,t,n}$ from $PV_{p,b,f,t,n}$

IF $PV_{p,b,f,t,n}$ is less than zero THEN
SET $X_{p,b,f,t,n}$ to 0
ENDIF
END FOR

FOR each flow path *f* in $F_s$
FOR each stockpile *s* that is not a waste dump $s \notin \mathrm{D}$
FOR each potential value stream *v* from stockpile *s*
ADD product revenue $R_{v,u,t,n}$ to $PV_{s,f,t,n}$
END FOR

FOR each option *o* in flow path *f*
SUBTRACT unit variable cost of option $V_{o,t,n}$ from $PV_{s,f,t,n}$
END FOR

SUBTRACT unit variable cost of removing material from stockpile $V_{s,t,n}$ from $PV_{s,f,t,n}$

IF $PV_{s,f,t,n}$ is less than zero THEN
SET $X_{s,\mathrm{i},f,t,n}$ to 0 for all *i*
ENDIF
END FOR
END FOR
END FOR
END FOR