School of Education

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Collaborative Inquiry: A Professional Learning Approach for Middle School Mathematics Teachers

Tsae Wong

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DECLARATION

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Human Ethics - The research presented and reported in this thesis was conducted in accordance with the National Health and Medical Research Council National Statement on Ethical Conduct in Human Research (2007) – updated March 2014. The proposed research study received human research ethics approval from the Curtin University Human Research Ethics Committee (EC00262), Approval Number #SMEC-109-11.

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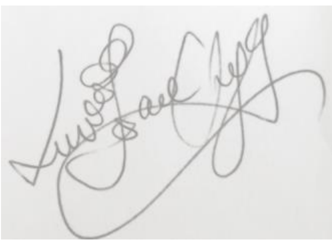

Tsae Wong

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ABSTRACT

This research involved the use of a collaborative inquiry approach as a tool in the professional learning of middle school mathematics teachers to challenge their pedagogical practice. It entailed the building of a professional learning community among these teachers to provoke and share their teaching approaches. This grounded theory study adopted the collaborative inquiry approach via the use of site-based programmed planning time when the teachers were able to discuss, deliberate and share pedagogical practices so as to grow their professional practice. The research was undertaken over a three-year period, and included the use of team planning recordings, student focus groups, teacher survey responses and individual teacher interviews as data to triangulate the teachers' collective professional journey and trajectory of growth and development in their pedagogical understanding and practices.

The findings of this research confirm the importance of such an on-site built-in professional learning model which harnesses the benefits of the site-based context in establishing a professional learning community that enables a group of teachers to consolidate professional learning needs for their collective growth. This approach to professional learning takes into account the teachers' differentiated learning needs and has the added benefits of creating a positive team and school culture that harnesses a critical mass among the teachers to reflect their practice. This mode of professional learning was found to be effective in developing a shared positive disposition and potentially initiated change in teaching practice. The approach adheres to adult learning principles such as focussing on relevant topics to the teachers' work and encouraging dialogues to share personal experience. The impact of the teachers' pedagogical change on the students' engagement was found to be largely dependent on the wider context of the culture and attitude of the school, the teachers' ability to link mathematics to other disciplines, and the mental models and beliefs of the teachers.

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CHAPTER ONE

RESEARCH OVERVIEW

1.1 Introduction

With globalisation and advancing technology, Australia's economic viability is dependent largely on the quality of our human resources. In turn, the quality of our human resources relies heavily on the quality of our educational system. To this end, Australian educational outcomes in relation to future human resources need to be compared at global scale. Hence, the vitally important scrutiny of our educational outcomes against Trends in International Mathematics and Science Study (TIMSS) conducted once every four years (Thomson, Hillman & Wernert, 2012), and through the Programme for International Student Assessment (PISA) (Buckley, Thomson & De Bortoli, 2013) conducted once every three years, are the basic benchmarks of how well the Australian educational system delivers educational outcomes for its youth.

As stated in the TIMSS 2011 report;

> … the goal of TIMSS is to provide comparative information about educational achievement across countries to improve teaching and learning in mathematics and science. It also provides comparative perspectives on trends in achievement in the context of different educational systems, school organisational approaches and instructional practices… ." (Thomson, Hillman & Wernert, 2012, p. 5)

This report provided Australian science and mathematics educators with valuable comparative data to inform school organisational approaches and instructional practices towards better outcomes for our students.

The report focused on the achievement of Australian year eight students (sample size of 7,500 students in 275 schools) in comparison with the international average of Year eight students in 45 countries that participated in this assessment. The students were assessed on two dimensions, namely, content and cognition. The former involved testing mastery of the subject matter of mathematics and science, while the latter involved assessing the thinking processes employed by the students. The four content domains assessed in mathematics were number, algebra, geometry, data and chance. The cognitive domains assessed in each curriculum area included knowing, applying and reasoning (Thomson, Hillman, & Wernert, 2012).

Thomson et al. (2012) state that teachers' job satisfaction and a supportive and ambitious school climate are important to the students' motivation and confidence in achievement. Consequently, it is imperative that the Australian government develops systems that "build accountability and support capacity building for teachers and school management in order to address attitudinal barriers towards teaching and learning" (Thomson et al., 2012, p. xv).

1.2 Background

The senior school students' participation rate in mathematics has dropped significantly over the last decade in Australia (Barrington & Evans, 2014; Forgasz & Leder, 2008). To increase the students' participation in advanced level mathematics, an extension mathematics program was established at my school in 2011 for students in years three to ten. This extension program was designed for students with a proven track record in mathematics achievements. The students were clustered in the same class to challenge one another and engage in the learning of mathematics at a deeper level through interactions with like-minded students, while developing their higher order thinking skills.

Research suggests that the traditional method of teaching mathematics, in which set concepts and skills are taught before exposing students to problems where these concepts and skills are applied, are ineffective and do not develop deep conceptual understanding to enable students to participate meaningfully in senior mathematics in the later years (Yoon, Thompson, & Zawojewski, 2007). Schools are often pre-occupied with putting knowledge into the child's mind rather than perceiving that their main task is to help children to develop useful and appropriate ideas by analysing their own experience. Arguably, this traditional approach of filling the child's mind with knowledge is more prevalent now that the Australian government has legislated the publication of national numeracy and literacy test results known as the National Assessment Plan of Literacy and Numeracy (NAPLAN).

The focus of NAPLAN testing is to collect data about the progress of Australian students in years 3, 5, 7 and 9 in literacy and numeracy (MCEETYA, 2008). The NAPLAN test results have been used as public data by the Australian Federal Government to hold schools accountable for their performances, as well as to determine Federal Government funding levels for schools. Consequently, most schools focus their teaching and learning activities on rehearsing for these high stakes tests, rather than on the development of deeper conceptual understanding of mathematics in the students and the applications and connections of these mathematics concepts to real life situations to enhance the students' problem-solving capabilities. As the NAPLAN tests are benchmark tests for literacy and numeracy, actions by schools to teach to the minimum benchmark levels is likely to deprive the students of the opportunity to develop deeper understanding and appreciation for the relevance of learning mathematics. Such dumbing down of curriculum requirements to address the minimum benchmark needs is neither an ethical nor a responsible approach to educating the students.

Teaching of mathematics is an increasingly important priority for Australia and other developed nations around the world. Mathematics is the foundation for technology and science, which have been identified as two essential areas in the economic development of a country through problem-solving and raising living standards (Atweh, 2007). In recent years, science, technology, engineering and mathematics (STEM) study has emerged as an area of emphasis for educators and schools, in order to better position Australia to develop innovative industries and increase its economic viability in the world. A strong and deep understanding of mathematics is a pre-requisite for our students to engage in nation building and future-proofing innovation and industries.

In a school context, all teachers are required to deliver a differentiated curriculum to cater to the different learning styles and developmental stages of their students. Hence, teachers need to develop their ability to cater for the diverse learning needs of students. However, professional learning activities in schools mostly adopt the one-size-fits-all approach, which often does very little to foster teacher learning and effective change in classroom practices (Goodnough, 2005). Reeves (2009) argues that, to influence change in schools, the school leaders must focus on productive factors such as teacher assignment, professional development, collaboration time and effective use of staff meetings. Reeves explains that the professional development of staff needs to focus on what to teach (content and concepts), how to teach it (pedagogy), how to meet the needs of individual students (differentiation), and how to build internal capacity of the school through these professional learning activities (Reeves, 2009). Hence, Goodnough (2005) proposes that, professional learning activities for teachers at schools need "to offer teachers a range of choices that incorporate many different strategies ranging from traditional workshops to more collaborative, team-based initiatives such as study group or teacher inquiry groups" (Goodnough, 2005, p. 88).

1.3 Statement of the Problem

The TIMSS 2011 report highlights that, Australian students had been performing at a significantly lower level than our Asian counterparts, that is, Korea, Singapore, Chinese Taipei, Hong Kong and Japan, as well as the Russian Federation, even though the 2011 results for Australian students were relatively better than for 2007. The 2011 TIMSS results indicate that the percentage of our students achieving at advanced and high international benchmarks was lower than in the countries identified above. Furthermore, "Australia's scores in mathematics and science have largely stagnated over the past 16 years. Over this same time, a number of other countries have either dramatically improved their results or slowly but surely improved" (Thomson et al., 2012, p. xiv). It was also noted that Australia has a substantial "tail" of underperformance which was deemed unacceptable for us as a highly developed country. Hence, for Australia to remain competitive on the global stage and position itself as one of the top five education systems in the world, the urgency of examining our teaching and learning practice to minimise our underperformance is a priority.

Similar to TIMSS, "PISA assessment focuses on young people's ability to apply their knowledge and skills to real-life problems and situations" (Buckley, Thomson, & De Bortoli, 2013, p. viii). PISA is taken by approximately half-a-million 15-year-old students, out of a possible group of 28 million, from all 34 Organisation for Economic Co-operation and Development (OCED) countries and 31 partner countries. "OECD considers that mathematics, science and technology are so pervasive in modern life that it is important for students to be literate in these areas" (Buckley et al., 2013, p. viii). PISA's aims to assist OECD to develop public policy issues relating to education provision that are of particular interest to this research are: "Are some ways of organising schools and school learning more effective than others?"; "What influence does quality of school resources have on student outcomes?"; and "What educational structures and practices maximize the opportunities of students from disadvantaged backgrounds?" (Buckley et al., 2013, p. viii).

Even though Australia achieved a significantly higher average score in mathematical literacy than the OECD average, it is important to evaluate how our teaching and learning in mathematics can be improved to ensure that Australia is able to challenge the 16 countries/cities — Shanghai, Hong Kong, Taipei, Korea, Macao, Japan, Liechtenstein, Switzerland, the Netherlands, Estonia, Finland, Canada, Poland, Belgium and Germany — which performed better, if Australia is serious about positioning itself as an innovative leading economy. Of greater concern was that 42% of Australian students achieved below the nationally agreed baseline of Level three, even though it was 3% lower than the OECD average; however, the proportion of students scoring below the baseline of Level three was 12% higher than the highest scoring city, Shanghai. Furthermore, Australia's average score in mathematical literacy declined significantly from 2003 to 2012 (Buckley et al., 2013; Hammerman, 1999; Morocco & Solomon, 1999; Senge, 2006; White, Way, Perry, & Southwell, 2006).

Based on Hattie's (2008) synthesis of over 800 meta-analyses relating to student achievement, 19 of the top 30 factors in student performance with the greatest effect size related to teaching and teachers. Hattie (2008) emphasised the importance of the quality of teachers, in terms of what they do and the effect they have on the students. Argent Rogers (2013) quoted Becoats's (2009) comment that "student achievement in mathematics is largely dependent on high quality mathematics teachers who deliver rigorous and engaging lessons during first instruction" (Argent Rogers, 2013, p. 19). Consequently, for Australia or any country to improve on its students' achievement, one of the key aims needs to be to lift the quality of teaching. Accordingly, this study has investigated the effectiveness of collaborative inquiry as a professional learning method for middle school mathematics teachers aimed at developing their pedagogical understanding with the expressed goal of improving students' engagement.

With increased understanding of how to equip our teachers better for their teaching, resulting in increased student engagement, it is anticipated that student achievement will be a by-product in a long run (Bessoondyal, 2005; Utley, 2004). Consequently, if Australian students are positioned for employability and a better economic future, in turn, high achieving citizens will enhance Australia's competitiveness in the global economy.

1.4 Assumptions

The assumptions of this study are:

a. Teachers bring to their teams past experience and knowledge about teaching mathematics which can be shared and modified for better teaching approaches.
b. Teachers have professional curiosity, commitment and desire for professional growth.
c. Teachers are ready to engage in professional dialogues for their own professional growth.
d. On site professional learning opportunities provide effective, relevant and timely professional development in the work context.
e. Teachers with better pedagogical practice improve student learning outcomes.

1.5 The Research Problem

Students' mathematical learning should include the holistic approach of engaging in reading and transforming the physical, economic and social world around them (Atweh, 2007). To this end, there is a need to explore and incorporate a framework for mathematics learning which will develop holistic and deeper meaning making for all students.

For exploring effective pedagogies within middle school mathematics classrooms, I used collaborative inquiry at my research site to enable the teachers to engage in active learning through ongoing professional dialogues about effective pedagogies characteristics (Goodnough, 2005). Collaborative inquiry was selected as a tool to engage the teachers in professional learning and growth because, for the teachers to develop in-depth knowledge and transformative pedagogical practices, it is essential for them to have the opportunity to communicate and network with their colleagues in the workplace (Sun & Liu, 2010). The collaborative inquiry approach to mathematics teachers' professional learning was based on the principle of establishing an onsite professional learning community which focuses on the teachers' learning, professional collaboration and pedagogical change (Garrett, 2010; Lee, 2010; O'Malley, 2010). This approach involved the establishment of a group of mathematics teachers who could work together to investigate strategies for implementing effective pedagogies into their mathematics teaching, with regular collegial feedback and evaluation of their progress in the group (Garrett, 2010; Lee, 2010; O'Malley, 2010). This group would build a consistent, reflective, shared best practice among middle school mathematics teachers, while promoting leadership towards overall school improvement (Garrett, 2010; Lee, 2010; O'Malley, 2010).

Mathematical understanding is constructed by each student as he/she engages in the various critical mental activities, which include actively constructing relationships between and among mathematical ideas by reflecting on problem solutions, extending knowledge by relating new solutions to what has been known previously, and articulating thinking about the mathematics that they have explored (Atweh, 2007; Fennema, Sowder & Carpenter, 1999). By engaging in these activities, each learner assumes ownership of the mathematical knowledge that he/she has constructed. Hence, helping the teachers to approach their pedagogies using these activities will provide the impetus for their professional learning and growth.

To teach for understanding, teachers need to embrace a different notion of what explaining mathematics means. This requires a fundamental shift in teachers' notion of what knowing mathematics entails (Atweh, 2007; Kinach, 2002). The construction of classrooms that promote understanding is dependent on thoughtful and knowledgeable teachers who have participated in professional learning programs that enable the development of their own understanding of mathematics, students' thinking about mathematics, and the interdependence of the two. When this happens, classrooms that promote understanding will proliferate and such understanding and practices will be the desired outcome of teachers who participate in this model of professional learning (Fennema et al., 1999).

The collaborative inquiry approach to professional learning develops a democratic culture of learning and risk taking for the teachers. Given et al. (2009) and T. H. Nelson, Slavit, Perkins and Hathorn (2008) assert that collaborative tolerance and parity among teachers working in groups often provokes changes in teacher practice. Hence, this approach is a professional learning activity that promises powerful, insightful and relevant learning for teachers with more lasting change, compared with the commonly available episodic, surface level experience of workshops, seminars or conferences as professional development learning activities (Feiman-Nemser, 2001). Schifter, Russell and Bastable (1999) argue that regular follow up support for teachers throughout the school year will have a more profound and longer-lasting impact on their pedagogical practice.

The teachers engaging in the collaborative inquiry group are able to engage in planning and professional dialogues to develop their understanding of the approaches to teaching problem solving. It is through these active dialogues that the teachers are able to articulate and clarify their understanding during planning. Managing challenges that emerge both strengthens the community of learners and enhances teachers' abilities to observe, record, analyse, represent and respond to the teaching and learning that occurs in their classrooms, ultimately changing the culture of their learning communities.

In their case study of a transformation of understanding and pedagogical practices that took place for science teachers, Lebak and Tinsley (2010) employed the collaborative inquiry approach to track the professional growth and learning of these teachers. Lebak and Tinsley (2010), following 19 teachers' general research seminar attendance, documented the teachers' journeys as practitioner-researchers by keeping weekly journals of their observations and reflections of these teachers' activities for a 14-week period, as well as examining the teachers' own weekly field notes and video tapes of collaborative dialogue sessions over the research period. In addition, they analysed the classroom teaching tapes of these teachers. They then identified three teachers who had similar professional growth trajectories to theorise changes in the teachers' pedagogical understanding and practices in greater depth.

My study has some similar features to Lebak and Tinsely's (2010) study in the use of a collaborative inquiry method to document and analyse the teachers' understanding and pedagogical practice to ascertain the benefits and limitations of collaborative inquiry as a professional learning tool in schools. However, Lebak and Tinsely's (2010) study focuses on teachers' transformation from a teacher-centric didactic teaching practice to a student-centred engaging teaching practice in science, while my study focused on not just the benefits and challenges, but also on how collaborative inquiry could be implemented as a professional learning tool to challenge and grow middle school mathematics teachers in their practice. Furthermore, I hope to fill the gap and silence in the understanding of collaborative inquiry as a professional learning tool for teachers in schools by developing a collaborative inquiry model that can be easily replicated in schools.

1.6 Research Questions

The objective of this research was to evaluate the effectiveness of collaborative inquiry as a professional learning approach to challenge middle school mathematics teachers' pedagogical practice.

The following research sub-questions were addressed:

a. What benefits and challenges would the implementation of collaborative inquiry as a professional learning approach present for the school and the middle school mathematics teachers?
b. What would be the essential elements in implementing effective collaborative inquiry as a professional learning approach for teachers in the school?
c. How would these essential elements be incorporated in an empirical model to inform the effectiveness of collaborative inquiry as a professional learning approach for middle school mathematics teachers in schools?

This study used a grounded theory approach coupled with a qualitative research methodology in the collection and analysis of data (Creswell, 2013). This qualitative design involved identifying the impacts of the collaborative inquiry approach to regular on-site professional learning for a group of mathematics teachers in terms of their development of understanding of effective pedagogies (Shadish, Luellen & Clark, 2006).

1.7 Significance of Research

The significance of this study is established on several grounds:

- The insights gained through this study are likely to contribute to the growing body of work on collaborative inquiry as a tool to develop teachers' pedagogical practices. The findings regarding the implementation of collaboratie inquiry as a professional learning approach are likely to add to knowledge about the impact of this onsite professional development in mathematics teaching in the educational field.
- The documentation of the professional dialogues and successful trials of various pedagogical practices can be used as a framework for professional development activities in other key learning areas in the college.
- The development of an empirical model to articulate and synthesise collaborative inquiry as an effective tool for teachers' professional learning and development based on grounded theory findings are likely to help school leaders develop principles for the design of professional learning activities and implementation of pedagogical change at schools.
- The experience and documentation of this collaborative inquiry for teacher professional development can be applied across system schools, such as the Lutheran schools in Queensland, with which my research site is affiliated, for the benefit of the other teachers and students in the sytem.
- The research findings can be shared with fellow educational leaders in Australia to enhance further exploration of the implementation of collaborative inquiry as a form of professional development for teachers.

1.8 Vocabulary of the Study

1.8.1 Professional Learning / Professional Development

Professional learning is used interchangeably with professional development in this research. Reese (2010) states that "Professional development involves comprehensive, sustained and systemic learning experiences that are based on identified needs of teachers, and result in improved instructional effectiveness and increased student achievement and performance outcomes" (Reese, 2010, p. 38). Kelchtermans (2004, p. 217), as cited by Wong (2009, p. 22), defines professional development as a “learning process, resulting from the meaningful interaction between teacher and their professional context, both in time and space”. Accordingly, the definition adopted for this study is that professional learning (or professional development) of teachers is the professional growth of teachers in understanding and practice in their role as teachers through the purposeful planning of activities, based on the identified needs of the teachers, as perceived in the context of this research.

1.8.2 NAPLAN (National Assessment Plan for Literacy and Numeracy)

The National Assessment Plan for Literacy and Numeracy (NAPLAN) is run at the direction of the Education Council of Australia. This assessment is administered by Australian schools irrespective of school sectors annually in mid-May to assess the levels of attainment of school children against essential standards in literacy and numeracy (Australian Curriculum Assessment and Reporting Authority, 2013).

NAPLAN is administered annually over a three-day period over less than four hours for Years 3, 5, 7 and 9 students. The NAPLAN results of schools are made available to the public through a government website known as MySchool (https://www.myschool.edu.au/), and comparative data between like schools is made readily available through this website (ACARA, 2016).

1.8.3 Collaborative Inquiry

I adopted a definition of collaborative inquiry as a stance of knowledge negotiation among group members. Employing dialogue grounded in shared experiences and a shared focus, group members question ideas, actions and artefacts; examine varying perspectives and beliefs; and work toward a co-construction of understanding about the focus of their collaborative work (T. H. Nelson, Slavit, Perkins & Hathorn, 2008, p. 1279).

The shared focus, asserted by T. H. Nelson (2009), is grounded in classroom practices, especially in teacher-identified gaps between a negotiated vision of high-quality learning and data about students’ actual accomplishments and understanding.

The collaborative inquiry approach to professional development within the school context involved teams of mathematics teachers in the middle school meeting weekly during their scheduled planning meetings to discuss a range of pedagogical practices.

Because this study was restricted to one site in Australia with a small sample of middle school mathematics teachers, the small sample limits the generalisability of the findings in this study, but still the study provides an in-depth insight into the impact of this approach. However, it was anticipated that the impact of teachers' professional learning on student achievement would be very limited, as student achievement depends on many factors. The assumptions in this study also limited its impact on the evaluation of collaborative inquiry as an effective professional learning tool because the measures of teachers' attitudes towards this professional learning tool before and after their experience in the collaborative inquiry style professional learning activities were not collected for consideration.

1.8.4 Effective Pedagogy

Effective is defined as having an effect according to the Australian Reference Dictionary (Godfrey-Smith, Williams, Hughes, Purchase & Ramson, 1991). Pedagogy is the study of the methods and activities of teaching, according to the Cambridge Dictionary, as quoted by Vander Veldt (2006). Hence, effective pedagogy, when used in this study, refers to the teaching methods and activities which are adequate to accomplish the purpose of engaging students in learning.

The methods and activities used by teachers depend on their epistemological world view (Schraw & Olasfson, 2002, as quoted by Vander Veldt, 2006). These views are framed by the beliefs of the individual teachers, whether realist, contextualist (constructivist) or relativist. The realist teacher adopts the belief "that learning is transferred in a programmatic fashion from teachers to students"; hence, the preferred teaching method is the didactic style of lecturing to impart knowledge (Vander Veldt, 2006, p. 9). The constructivist teacher holds the belief that learners need to construct their knowledge along with others through connecting with the learning environment and by the teachers designing problem-based or inquiry style activities for co-construction of knowledge (Vander Veldt, 2006). Lastly, the relativist teacher holds the belief that the knowledge is relative to the individual students; hence, the curriculum is not standardised but determined by the individual student's specific interests (Vander Veldt, 2006). The learning environment is focused on the individual student's needs to develop self-regulated learners.

Cook (2008), in her study of how to increase middle school students' grades through the implementation of effective teaching practice, concluded that learning is the most effective when the students are actively doing mathematics rather than having mathematics done for them. Consequently, when effective pedagogy (or pedagogies) was being explored in my study, they were dependent on many factors including the teachers' beliefs about pedagogy relating to engaging students in learning, their teaching practices and approaches related to students doing mathematics rather than didactic approach to teaching mathematics.

1.9 Thesis Overview

This thesis reports qualitative research completed over a three-year period of data collection, adopting the grounded theory approach to explore and understand the value of collaborative inquiry as a professional learning tool for teachers in situ, to provide timely and relevant collaborative learning opportunities for collective growth.

Chapter One outlines the background of this research, and then defines the problem in context at the time of the construction of this research. The assumptions and research problem are defined and the research questions delineated. The significance of the research is outlined while the vocabularies pertinent to this research are clearly enunciated.

Chapter Two provides the literature review relevant to the research, even though a literature review is not normally favoured in classical grounded theory research methodology (Oktay, 2012; Teppo, 2015). However, I found that, in order to fully comprehend and prepare to undertake this research, a literature review before, during and after data collection and analysis added richness and value to the research and its findings (Dey, 2004; Lempert, 2007; Stern & Porr, 2011). The chapter reviews the significant documents relevant to this research, such as the Melbourne Declaration and the Australian Professional Standards for Teachers, to provide a backdrop for the research. Further exposition of other literature on effective pedagogies, theories of mathematics teaching, effective professional learning activities for teachers, the concepts of professional learning communities, and collaborative inquiry as a professional learning tool grounds understanding of the subject. During data collection and analysis, as the theory was sampled and saturated, a further literature search reviewed the mental models and beliefs of teachers, adult learning methods, intrinsic and extrinsic motivations for professional learning and change, organisational structure and culture in schools, and facilitation skills in collaborative inquiry.

Chapter Three explains the grounded theory research approach and the appropriateness of this research approach. The research design and plan are outlined, followed by a discussion of how the sample for data collection was determined, including the different sample sizes throughout the three years of this study. Ethical considerations are then discussed, including how I managed my relationship as a researcher, a participant and an employer of the participants and all other associated issues.

Chapters Four, Five and Six document the data analysis and findings of the research. Each chapter is dedicated to reporting the findings that emerged from each stage of data analysis for each phase of data collection. At the end of each chapter, the findings of that phase are summarised.

Chapter Seven synthesises the research findings by discussing the findings in the context of the literature review, drawing conclusions and making recommendations. The chapter explores the extent to which the research questions have been addressed, and lessons learned about the research methodology and about my research skills. The limitations and contributions of this study are discussed, and recommendations for possible future research arising from this study are proposed.

CHAPTER TWO

LITERATURE REVIEW

This chapter reviews theoretical and research literature on collaborative inquiry and professional learning for teachers and pedagogies, especially those pertaining to mathematics teachers and middle schools.

This chapter has six parts covering the educational context in which the study is set, followed by a review of literature on mathematics teaching and learning, professional learning for teachers, collaborative inquiry and factors influencing professional learning. The chapter concludes with an explanation of how the literature review frames the study.

2.1 Educational Context of Study

The Melbourne Declaration on Educational Goals for Young Australians signed by the State, Territory and Commonwealth Education Ministers in 2008, building on the 1989 Hobart Declaration and 1999 Declaration, makes a strong commitment to work with all school sectors and the broader community to achieve educational goals for young Australians. Two of the eight inter-related areas pertaining to this research are: (a) supporting quality teaching and school leadership; and (b) enhancing middle years' development (Minsterial Council on Education Emplyment Training and Youth Affairs, 2008). The Ministers also identified literacy, numeracy and discipline knowledge as cornerstones of schooling for young Australians to be able to think deeply and logically, and obtain and evaluate evidence in a disciplined way as a result of studying fundamental disciplines (MCEETYA, 2008).

The Melbourne Declaration states, "Excellent teachers have the capacity to transform the lives of students and to inspire and nurture their development as learners, individuals and citizens" (MCEETYA, 2008, p. 11). School leaders are charged with the responsibility of creating and sustaining the learning environment and conditions to deliver high-quality teaching and learning in schools (MCEETYA, 2008). The Declaration also identifies the middle years of schooling as an important period of learning during which knowledge of fundamental disciplines is developed, while also being a time of the highest risk of disengagement from learning.

Sun and Liu (2010) define excellent teachers as those who can comprehensively utilise creative thinking ability and creative practice ability to labour creatively, make new products, and make great contributions in the development of the education domain. Sun and Liu's (2010) study confirms that there is a need to differentiate professional learning activities for teachers based on the teachers' learning needs and stages in their career progression. In their study of physics teachers in China, the teachers identified lesson preparation and lesson case discussion as two professional learning activities that were useful for them.

In Sun and Liu's (2010) research, the teachers' self-perception of professional development needs included reflective teaching, self-learning and collegial cooperation. D. Nelson, Strouse, Waechter, and St Maurice (2003) emphasise that reflection-in-action should be a major component of professional development for teachers at all stages of their profession.

It is noted in the TIMSS 2011 report that, around one-third of Year eight mathematics students in Australia were being taught by teachers with no pedagogical training in mathematics (Thomson et al., 2012). This report also registers concern that, even though the teachers might express some confidence in teaching mathematics to the year eight students, "without strong pedagogical and content knowledge of mathematics, the teachers would be more likely to teach to the middle, failing to provide adequate extension for high-achieving students and unable to provide alternative structure for students who are having difficulties" resulting in a decrease in student engagement and future participation in mathematics learning in the later years of education (Thomson et al., 2012, p. xv).

The Australian Institute of Teaching and School Leadership (AITSL) developed the Australian Professional Standards for Teachers (APST) which states that all teachers need to know the students and how they learn while knowing the content and how to teach it. One of the professional practices expected of teachers is to plan for and implement effective teaching and learning and to engage in professional learning and with their colleagues (AITSL, 2014). The emphasis on literacy and numeracy strategies aimed at different levels of competence is a new reality for Australian teachers. It is pointed out that teachers need to master teaching strategies at various degrees of sophistication depending on their stages of development as teachers. Graduate teachers are required to use an appropriate range of strategies, while proficient teachers are required to select and use relevant teaching strategies to further develop their knowledge, skills, problem solving and critical and creative thinking. Highly accomplished teachers are to support their colleagues to select and apply effective teaching strategies to accomplish the goals identified above, while the leading teachers should support their colleagues to review, modify and expand their repertoire.

Clarke (2003) states that it is important to distinguish a teacher's preoccupation with student learning, defined as tasks associated with teaching, from a preoccupation with how students learn, defined as focusing on one's practice. The latter is essential for the ongoing renewal of teacher professional practices. This argument supports the APST outlined by AITSL, of which Standard One states that teachers are expected to know the students and how they learn.

In summary, the TIMSS 2011 report raises alarming statistics about middle years' mathematics teachers' lack of strong pedagogical skills and content knowledge in mathematics. To fulfil the responsibility to create and sustain quality teaching and learning in schools, especially for the important phase of learning in middle years, Australian education ministers from various jurisdictions charged school leaders to support and grow excellent teachers by offering differentiated professional learning activities suitable to their career stages as specified by the professional standards developed by AITSL as a general guide. These differentiated professional learning activities need to incorporate reflective practice, self-learning and collegial cooperation with the goal to engage the students in learning (Clarke, 2003; D. Nelson, Strouse, Waechter & St Maurice, 2003; Sun & Liu, 2010).

2.2 Effective Mathematics Pedagogical Practice

Mathematics teaching is not just an act of delivering a set mathematics curriculum, but also it is a social process involving learning. Teachers must take responsibility for the students' learning irrespective of whether the students appear to lack the motivation or capability to access the curriculum (Nickson, 2000; Seeley, 2009). Mathematics teachers in the twenty-first century assume multifaceted roles, according to Seeley (2009), including being architect, composer, movie director, stockbroker, captain, mayor, bridge builder, recruiter and prospector.

Mathematics teachers construct the learning environment, including a physical layout of the classrooms that is conducive to carrying out the relevant class activities, a safe sanctuary for the sharing of thoughts, feeling and ideas, and an inclusive space to evoke a sense of belonging and welcome (Jeffrey & Beasley, 2012; Seeley, 2009). They design learning activities to engage and stretch the students' intellect as they access mathematics learning as well as orchestrate the pace of learning and discourse of their students' discussions (Horn, 2008). The mathematics teachers help the students to bridge the gaps in their mathematical concepts and link these with the real world, and support connections amongst the students. But most importantly, they never lose hope and sight of their students' potential for mastering mathematical concepts (Horn, 2008; Seeley, 2009).

Schoenfeld and Kilpatrick (2008) assert that the culture and effectiveness of the mathematics classrooms influence the quality and nature of interactions of the teachers and students according to the respective perceptions, values and beliefs that they bring to the classrooms. The teachers are also able to intervene in interactions in a timely and appropriate way within the classrooms to create effective learning opportunities for the students (Nickson, 2000).

Seago (2008), drawing on the work of Darling-Hammond (1990) and Hargreaves (2003), argues that teachers need to be committed to use every opportunity to analyse and interrogate their own practice through professional learning activities, such as: observing and being observed by their colleagues or via virtual learning platforms; collectively diagnosing their school's issues and generating solutions by sharing effective pedagogies; developing curricula; and developing means for evaluating their students' learning progress to improve their learning outcomes. School teachers are now required to not only have knowledge about their students, their subjects, the management of classes and appropriate resources, authentic assessments, and effective pedagogies, but also about the system and structure within which they are teaching (Perks & Prestage, 2008).

Goos (2008) argues that, when students are asked to perform certain mathematical tasks that they are unable to do without help, mathematics teachers should utilise Vygotsky's principle of the Zone of Proximal Development (ZPD) to lead the students into success instead of focusing on their incompetency or weaknesses. For the teachers to be able to assist the students to acquire new knowledge and skills in mathematics, Schoenfeld and Kilpatrick (2008) maintain that teachers must have both breadth and depth in school mathematics knowledge. By breadth, Schoenfeld and Kilpatrick (2008) refer to conceptualisation of the current grade-level content in multiple ways and, by depth, they mean knowledge of where the content comes from and where it might lead.

In Queensland, where this research site was located, the Queensland College of Teachers, a legislative organisation that registers all teachers in Queensland, does not mandate that mathematics teachers possess certain minimum training standards or tertiary education in mathematics. The decisions about who can teach mathematics in the secondary to senior years lie solely with the schools which hire the teachers. This practice, on the one hand, liberates the schools from staffing constraints; but, on the other hand, it creates challenges for schools in the more remote and less desirable areas of the state to appropriately fill their staffing needs in the mathematics learning area, and hence limits the quality of mathematics education for the students.

Gronewold (2009) reports concerns expressed by Harris and Jensz (2006), who state that 75% of teachers of senior mathematics had studied some mathematics to third year at university, while 8% of all secondary mathematics teachers studied no mathematics at university and some 20% of all mathematics teachers studied no mathematics beyond first year (Gronewold, 2009, p. 107)

Teachers' limited exposure in mathematics content knowledge impacts on the effectiveness of reform-oriented teaching practice (Stacey, 2008). Mathematics educators share Metais's (2002) concern, as cited by Forgasz and Leder (2008), that the decline in popularity of mathematics across secondary schools in a number of western countries has led to the pool of specialist mathematics teachers trending downwards. This concern is consistent with the points raised in the TIMSS report outlined in Sections 1.1, 1.2 and 2.1.

Gronewold (2009), Goldsmith and Seago (2013), Obrycki (2009) and Seeley (2009) are in agreement that a good mathematics education sets out to engage all students at all year levels, using age appropriate activities which aim to grow students' conceptual understanding of mathematical knowledge and facts as well as their abilities to tackle non-routine problems systematically. Such an education also assists the students to apply their knowledge to solving real world problems and using associated technologies sensibly; increases the students' fluency and accuracy in performing routine procedural skills; develops their strategic and adaptive mathematical thinking by applying their logical reasoning and conception of the nature of proof; and develops their productive disposition through their understanding and appreciation of major mathematical knowledge and its applications in society.

In order for all students to access a good mathematics education, to uphold and honour The Melbourne Declaration, schools need to support students who have additional learning difficulties to manage the barriers preventing them from accessing mathematics learning (Goos, 2008; Nickson, 2000; Schoenfeld & Kilpatrick, 2008). The usual barriers for these students, according to Seeley (2009), include: the memory challenge of retaining algorithm or procedures; processing issues in auditory processing of oral explanations of mathematics concepts; visual processing difficulties including their inability to process and differentiate mathematics content visually; the motor processing challenge of writing numbers in regular sized spaces or writing them legibly in small spaces; abstract reasoning difficulty including solving problems and learning new concepts; organisational challenges pertaining to time, materials and information; and lastly the inability to stay focused on important information and attend to the information meaningfully.

Ginsbury and Dolan (2011) propose two overarching approaches to teaching which they deem are incompatible models of good pedagogy based on different theories of learning. The first is traditional transmission instruction (TTI) which is based on the theory of learning that students learn by absorbing facts, concepts and understanding from the teachers' or texts' explanations and answering related questions. Procedural knowledge and skills in TTI are mastered through guided and repetitive practice of skills in sequence. The second is constructivist-compatible instruction (CCI) which is based on the theory of learning that understanding is attained through prolonged engagement of the learner in new ideas and explanations built on the learner's prior beliefs. Procedural knowledge and skills in CCI are acquired and called upon via working on concrete problems.

Ravitz, Becker and Wong (2000) suggest that, to understand teachers' choice of pedagogical practices, one must first examine their beliefs about the elements of good instructional practices, because their beliefs impact on their understanding of how students learn. Of course, other factors, such as class sizes, grouping of students (whether heterogeneous or homogeneous), content of resources, site based directives or philosophies of teaching and peer influence would also influence the choice of pedagogical practice (Ravitz et al., 2000). For the purpose of their study on the use of computers as a teaching tool, Ravitz et al. (2000) identify three determinants of teacher pedagogy: (i) teacher background and role orientation (private or collaborative professional practice); (ii) student ability and socio-economic background; and (iii) school professional culture, including aspects of staff development, cohesion and school level leadership.

Manipulatives are very popular materials used by mathematics teachers to help students to visualise abstract concepts; hence, mathematics teachers need to get to know these manipulatives for instruction as well as understanding the didactic objective of these materials (Ravitz et al., 2000). Mathematics teachers also need to develop the diagnostic ability to analyse and evalute the students' learning through the use of manipulatives (Nuhreborgen & Steinbring, 2008). For example, coloured reversible chips, cubes and multi-system blocks are used to facilitate and extend students' learning of mathematical concepts and operations (Nuhreborgen & Steinbring, 2008).

As mentioned in Section 2.1, many mathematics teachers are teaching outside their field of training; consequently, it is important for these teachers to have direct experience of the process of mathematical discovery, investigation and applications; they need to develop skills in learning mathematics independently as well as developing the quality and connectedness of mathematics knowledge and topics to support the students in making the required connections and meanings in their learning of mathematics (Nuhreborgen & Steinbring, 2008).

Goldsmith and Seago (2013) and O'Connor (2009) promote the use of productive mathematical discussions in the classroom as a student-centred pedagogy. However, they warn that the teachers must first establish clear and specific learning goals before selecting an appropriately challenging task, and have clear expectations of creating a respectful culture and guiding the student' reasoning and thinking to meet the pedagogical goals (Smith & Stein, 2011).

Goldsmith and Seago (2013), Gronewold (2009) and Herbel-Eisenmann (2009) develop skill frameworks to assist teachers in analysing the students' responses, comprising attention to thinking and attention to content. The former focuses on interpreting the strength and depth of thinking by distinguishing the descriptive style and interpretative style of responses as well as acknowledging plausible alternative interpretations with appropriate justifications (Goldsmith & Seago, 2013). The latter focuses on representing the mathematical ideas, explanations, arguments and solution approaches (Goldsmith & Seago, 2013). Providing ongoing feedback to students using the framework outlined above would help the teachers to gain insight into the students' mathematical knowledge and skills, and assist the teachers to plan more responsive and effective instruction for all students (Goldsmith & Seago, 2013). Furthermore, the students come to realise that mathematics is about solving and communicating mathematical solutions to problems (Ginsbury & Dolan, 2011).

In summary, every child can learn mathematics and the teachers must never lose hope and sight of each child's potential to master mathematical concepts (Seeley, 2009; Stacey, 2008). Teachers not only need to have strong content knowledge by keeping up to date with mathematics and growing their general mathematics knowledge over time, but also to develop their general pedagogical knowledge (e.g. constructing conducive learning environment, planning learning activities, analysing and responding to data on students and engaging all students in learning) (Nickson, 2000; Seeley, 2009; Perks & Prestage, 2008; Vinner, 2008). To this end, their professional learning activities are important and pivot to the ongoing growth and development of their teaching practice and the engagement of their students.

2.3 Professional Learning for Teachers

Professional development or learning for teachers is defined in this study as the learning experiences of teachers, resulting from meaningful interactions between the teachers and their professional context, which empower them to improve their pedagogical practice, address performance gaps and student assessment in order to facilitate student growth and development (Globe & Horm, 2010; Gronewold, 2009; Obrychi, 2009; Wong 2009). Therefore, the purpose of professional development is to challenge teachers to reflect on their beliefs and practice and to equip them to better serve the children in order to increase their achievement through the use of a performance-centred or problem-centred learning process (Hoy, 2009; Terehoff, 2002; Wong, 2009).

The need for professional learning is universal irrespective of the professional stage of the teachers; teachers need to continually enrich their knowledge and increase their sense of professionalism over the course of their careers (Small, 2011; Wilkins & Shin, 2011). In Queensland, 20 hours of professional learning are mandated each year for all teachers to maintain their annual teacher registration renewals.

To promote effective professional learning activities for teachers, teachers' needs as adult learners and andragogical theories need to be considered. This term andragogy was not widely used until Knowles (1968) contrasted andragogy (for adult learners) to pedagogy (for student learners) and the term became widely associated with Knowles' life work (Hendreke, 2009).

The following factors affect adults' motivation to learn:

1. Adults are more ready to learn if they have a practical use for the knowledge and think that it will benefit them in real life; that is, they like to learn for a specific purpose.
2. Self-esteem and pleasure help to maintain adults' motivation; they tend to become anxious when learning new material, due to age-related changes in their brains; similarly, they are risk averse compared with their students.
3. Adults are more self-reliant and hence resist others dictating what they should learn; that is, they prefer to be in charge of their own learning.
4. Adults tend to be more heterogeneous in readiness to learn than younger students are. (Hashweh, 2003, p. 421)

The concept of andragogy is based on assumptions of the adults needing to know, their self-concept, their motivation and orientation to learn, as well as the role of the adult's learning experiences (Rickey, 2008). Consequently, because effective professional development for teachers must focus on helping them to make personal connections to their work and preferred learning styles to promote active involvement, traditional professional development formats which favour a top-down, non-negotiable and didactic style are ineffective (Rickey, 2008).

Other important principles for adult learning are: adults need follow-up support, such as coaching to help them transfer their new skills into everyday practice, collegial interaction and job embedded, ongoing, personalised, reflective, low risk and non-evaluative support (Cox, 2010; Rickey, 2008; Robert & Pruitt, 2008; Small, 2011; Sun & Liu, 2010; Terehoff, 2002).

Vander Veldt (2006), through a review of numerous K to 12 education reform programs, noted that these reforms did not produce significant and sustained improvements in student achievement, due to a failure to prepare staff, a school culture that did not support sustained change or, most importantly, a failure to connect change and classroom practices through teacher inputs. Teachers need to change their classroom and individual practices in order for the intended reforms and desired changes to take place (Vander Veldt, 2006).

To engender change in the practice of individual classroom teachers, professional development activities must be relevant and effective to meet the learning needs of the teachers (Rickey, 2008). For the individual teachers to make learning practical, relevant, collaborative, voluntary and self-directed, professional development activities need to take adult learning needs or andragogy into account.

Rickey (2008) asserts that, in order for school-based teacher professional development to deliver its desired outcomes, adult learning principles must guide the process. These principles include: (a) setting up an environment for adult learning; (b) involving adult learners in mutual planning; (c) attending to the adult learners' needs and interests; (d) involving adult learners in setting the program's goals and objectives; (e) involving adult learners in designing an effective program; (f) involving adult learners in implementing the program; and (g) involving adult learners in a program's evaluation (Rickey, 2008, p. 51).

Adult learners may find it confronting when their past assumptions are challenged, because they attach their self-worth to their past experience, and may experience a sense of failure, rejection and fear (Rickey, 2008). Consequently, to play it safe, such adult learners may choose to resist change or remain in their known domains instead of stepping over the precipice into the unknown. It is therefore essential to create a trusting context for adult learners to engage in mutual inquiry and growth and to engender transformative learning outcomes for the teachers' professional development. Such transformative learning experiences of the teachers would in turn transform their classroom practices through their mutual accountability for better student outcomes (Roberts & Pruitt, 2009; Taylor & Wallce, 2007).

The model of effective professional development that will sustain and support the change agenda involves teachers in planning, reflecting, and modelling best practices (Terehoff, 2002). Such professional learning opportunities take place in a safe and judgement-free environment which emphasises growth and collective participation, using effective learning strategies while focusing on content knowledge (Roberts & Pruitt, 2009; Taylor & Wallce, 2007).

Hashweh (2003) proposes that, for teachers to continually examine their practice and change, they need a collaborative, trusting and reflective environment. This will support them to have the internal motivation to learn, to be aware and critically examine their own teaching ideas and practices, to have the capacity to construct alternative knowledge, beliefs and practices, and to resolve any conflicts between their own beliefs and the newly acquired perspective. Hashweh (2003) further suggests that any changes in the beliefs and practices of teachers rest with the teachers themselves because, when they face cognitive conflicts between their old and new beliefs, there are three possible outcomes. Progressive teachers will resolve the conflict by adopting the new beliefs and practices. Transitional teachers will continue to live within the unresolved conflicts. Conservative teachers might change some beliefs or practices to preserve their existing beliefs and practices. Sometimes one teacher can experience these three phases of change within themselves at different times and contexts.

Teachers' self-concept can be enhanced if they are given the personal freedom to learn, exercising control over their own professional learning goals, because their motivation for learning and growth will match their other self-directed activities in their personal lives (Vinner, 2008). When school leaders create an environment where teachers have choice and see the relevance of the learning experience, professional development will indeed be the outcome. These self-concepts can be enhanced with appropriate andragogy which values the teachers' past experiences (Terehoff, 2002).

Wong (2009) suggests that, for continual professional development, the teacher needs to make meaning of his/her experience and interact with the social conditions of the context. To develop professionally, the teacher needs to be motivated to learn, understanding their own beliefs within the context of teaching, while having the willingness to change to resolve any conflicts between the old and new beliefs. Wong suggests that professional learning for teachers needs to incorporate three elements: teachers' motivation to learn; how they learn; and their awareness of their beliefs and their changes, as well as the social context within which they are undergoing change (Bequary, 2012; Wong, 2009).

It is difficult to identify the effectiveness of professional development programs for teachers; however, Wong (2009) outlines some common features in several studies which have had positive impacts on teachers' beliefs and practices. These features include (a) providing a similar analogue experience for teachers so that they can duplicate their learning experience with their students; (b) having a strong content focus when teachers report an increase in knowledge and skills which results in a change in classroom practices; (c) involving active learning through the analysis of teaching and learning activities such as reviewing students' work collectively, providing teachers with feedback on their own practice, collaboratively inquiring into students' thinking, and so on; (d) engaging in dialogues amongst themselves to facilitate the process of belief change; and (e) providing long term follow-up support and feedback to teachers after workshops (Wong, 2009). The process of becoming self-reflective can be a strong impetus for the success of teachers' professional learning as adults (Terehoff, 2002). Such self-reflection focuses on the ability to think critically about one's perspective and explore alternatives in practice as the ultimate goal of teacher professional development, leading to change in classroom practice and thus student achievement. In addition, Poekert (2012), Terehoff (2002) and Wong (2009) emphasise that reflection is the ultimate key to one's professional growth as a teacher.

Goos (2008) proposes a framework of three intersecting circles to represent the interconnecting relationships between Vygotsky's zone of proximal development (ZPD) (Vialle, Lysaght, & Verenikina, 2005), Valsiner's (2000) zone of free movement (ZFM) and the zone of promoted action (ZPA), articulating the relationships between the knowledge and beliefs of teachers (represented by ZPD), the professional context of teachers (represented by ZFM), and the sources of assistance for teachers (represented by ZPA). Her representation implies that learning takes place at the intersection of the three zones.

Vygotsky's ZPD concept was introduced in the early twentieth century to explain how a social phenomenon such as a child's interaction with another more capable individual such as parent, teacher or peer is transformed into a psychological phenomenon, an enlightening moment of realisation and understanding (Goos, 2008; Vialle et al., 2005). Valsiner's (2000) ZFM refers to the accessible areas of the learning environment in which to act; while ZPA refers to activities, objects and areas in the environment for which the teacher's actions are promoted. Applying Valsiner's ideas to the classrooms means that the teachers' instructional choices control the students' learning experience (Goos, 2008; Valsiner, 2000).

The professional learning community (PLC) is defined as a set of shared beliefs and norms held by the community of teachers, which influences their interactions and understanding of their role in the community (the school) (Goos, 2008). The professional community has a role to play in changing classroom practice, but its effect is limited by the teachers' mental models which dictate whether they are ready to change and adopt the professional community's proposed pedagogical change (Louis, Anderson & Riedel, 2003). Consequently, Louis, Anderson and Riedel (2003, p. 13) suggest that, "creating significant change in the classrooms must involve sustained engagement with ideas and practices that challenge their taken-for-granted assumptions about themselves, their students, and how best to stimulate learning". That is, teachers' mental models of teaching and learning (Louis et al., 2003).

Garrett (2010), T. H. Nelson (2009) and Roberts and Pruitt (2008) stress that a professional learning community consists of a team of educators who inquire and reflect together by reading, analysing, dialoguing, implementing and evaluating research and strategies for the ultimate gain in students' engagement and achievement. Such a professional learning community has some distinct hallmarks, including: (a) a focus on sustainable teacher learning through inquiry; (b) professional collaboration with a shared vision of student learning; (c) a focus on student learning and attainment; and (d) shared values and norms.

A school-wide culture of success for each child is one in which all the educators at the school work together to identify students who are having trouble and intervene early as a community of professionals (Louis et al., 2003). This is reflected in Dufour's (2007) assertion that a professional learning community is centred on teachers' collaborative work, framed by an inquiry cycle that includes a definition of the inquiry focus based on the gap between the data for student learning and the teachers' learning goals.

Teachers must be provided with an environment that facilitates instructional coaching by the school leaders (T. H. Nelson, 2009; Roberts & Pruitt, 2008). Unless teachers are seeking out and trying new strategies and tools in their classrooms, collaborating with colleagues, creating lessons as a team, analysing student data, and generating interventions as a team, their personal and professional growth will be limited (Habegger & Hodanbosi, 2011). Such a professional learning community is facilitated when school leaders: prioritise allocating time to teachers for collaboration in new instructional strategies and tools; facilitate structured professional learning activities such as professional reading and collegial discussion of student achievement and best practices; enable teachers to try out new strategies and tools; and, overall, generate interventions for a team of trusting professionals (Habegger & Hodanbosi, 2011).

The school communication structure and inclusion of teachers in a team can facilitate ongoing dialogues on professional issues of students' learning and achievement gaps (Roberts & Pruitt, 2008). Other than a common time set aside for teacher collaboration and dialogues, a successful professional learning community needs to focus in their inquiry on the gaps between the shared vision for student learning and the students' actual achievements (T. H. Nelson et al., 2008; Roberts & Pruitt, 2008).

2.4 Collaborative Inquiry as a Professional Learning Tool for Teachers

Collaborative inquiry involves a team of teachers using professional dialogues rich in shared experiences and a shared focus to inquire into ideas, actions and artefacts of student work, examining various perspectives and beliefs so that the team can construct a collective understanding about their work (Bequary, 2012; T. H. Nelson et al., 2008). This professional learning tool is a promising approach to teachers' professional learning in situ, because collaborative inquiry provides the teachers with the platform on which critical reflections can be enhanced by productive dialogues amongst the practitioners over a period of time, catering for challenging and complex changes of mindset and practice (Bequary, 2012; Darling-Hammond & McLaughlin, 2011; T. H. Nelson, 2009; T. H. Nelson et al., 2008).

This sentiment is echoed in the TIMSS 2011 report which notes that students who were taught by teachers who collaborated regularly (such as one to three times per week) with other colleagues, to discuss how to improve their methods of instruction, performed better in the TIMSS testing. These teachers reported that their collaboration involved interactions such as: (a) discussing how to teach a particular topic; (b) collaborating in planning and preparing instructional materials; (c) sharing what they had learned about their teaching experiences; (d) visiting another teacher's classroom to learn more about teaching; and (e) working with other teachers to try out new ideas (T. H. Nelson et al., 2008; Poekert, 2012).

Thomson et al. (2012) analyse the work of Hargreaves (2009) and specify that the fourth phase of the post-modern professional paradigm for teachers requires them to engage in systematic and sustained inquiry that helps them to grow as practitioners. This is more evidence of the importance of collaborative inquiry as a tool for teachers' professional learning and growth. They further confirm that there is universal agreement amongst researchers and practitioners that inquiry and reflection in and on teaching practice are essential elements of the teaching profession (Clarke, 2003).

Clarke (2003) defines peer feedback as reciprocal teaching in which paired teachers observe one another as they incorporate new teaching techniques in the classroom to promote professional development, collaboration and self-assessment. This sentiment is echoed by Wilkins and Shin (2011), who assert that collegial training and support ensure that the collaborative inquiry process results in deeper levels of collaboration amongst the teachers and facilitators, resulting in changes in teachers' beliefs and classroom practice.

Professional learning embedded in the work flow of teachers through the formation of collaborative teams is an important factor in change management to bring about school improvement (Cox, 2010; Nisbet, Warren, & Cooper, 2003; Roberts & Pruitt, 2008). In particular, if these collaborative teams focus their deliberations on teacher instruction, such collaboration is linked to positive changes in teacher practices, higher expectations for students, a willingness in teachers to use innovative materials and methods, and improved student achievement (Bequary, 2012; Thibodeau, 2008). This point is supported by Thibodeau (2008), who concludes that, unless teachers are provided with enhanced learning opportunities rather than episodic superficial professional learning opportunities, students will not experience powerful learning. Given et al. (2009) cite Feiman-Nemser (2001) who states that "a valuable approach to teacher learning and professional development lies in creating discourse with other teachers" (Given et al., 2009, p. 37). Through participating in professional learning communities, teachers can exchange their experience and expertise with others, creating a discourse which consists of detailed descriptions of teaching practice supported by evidence, and exploration of alternatives to create new understanding, meaning and purpose.

Perks and Prestage (2008) urge school personnel to organise ways to promote interactive learning among teachers and make the learning visible to one another, in order to facilitate professional dialogues and reflections with clear documentation. Such documentation of professional dialogues, reflections and inquiry affirms the teachers' professionalism and expertise; this practice is stated by the Australian Institute for Teaching and School Leadership to be essential for educators (Given et al., 2009). Risk taking by teachers is to be supported and encouraged in order for the teachers to grow in their practices through cyclical inquiry (AITSL, 2014; Darling-Hammond & McLaughlin, 2011; Given et al., 2009).

Given et al. (2009) reviewed a series of studies on classroom teachers who took the role and responsibility of facilitator of a team of teachers' professional learning, focusing on teachers' reflecting on their practice and becoming thoughtful decision makers. In particular, Nisbet et al. (2003) cited Guseky and Sparks (1991) and Smylie (1988) to argue that, when teachers are given the opportunities to share their best practice, trial new ideas, assess student understanding and share their reflections with their colleagues, they tend to go beyond co-operation into a collaborative approach to professional learning. Nisbet et al. (2003) also cite De Lange (1992) who reasons that, it is critical to the teacher's professional growth to be provided with time to experiment, to gain experience and to build confidence and focus on the topic of inquiry.

Effective professional learning for teachers must include the opportunity to challenge their dominant pedagogical paradigms, inviting them to confront and reflect on the gaps between their own ideals and their own practices and motivating them to put their reflections and learning into action and hence change their pedagogical reality (Nisbet et al., 2003). Nisbet et al. (2003), Schifter et al. (1999) and Wang (1997) support that systematic inquiry by collaborative and self-critical communities of teachers at school will improve teachers' pedagogical understanding and practices, by firstly identifying a problem area for inquiry, and then studying the issue and collecting data on it before making data-informed decisions based on teachers' reflections and evidence.

Watt and Watt (1999) propose some principles for effective professional learning projects for teachers to be incorporated into the design and planning of such activities: a collaborative style in preference to a top-down transmission style; a change process that begins with teachers' prior beliefs and experience of improved student outcomes through the new strategies and acknowledgement that change takes time; regular meetings to provide collegial support and dialogues for sharing and critical reflection, with external facilitators to provide alternate viewpoints; and, lastly but more critically, school management support for such professional learning activities (Nisbet et al., 2003).

Clarke-Peter (1993), as cited by Nisbet et al. (2003) and Stacey (2008), developed a model of professional growth, reproduced here in Figure 2.1.

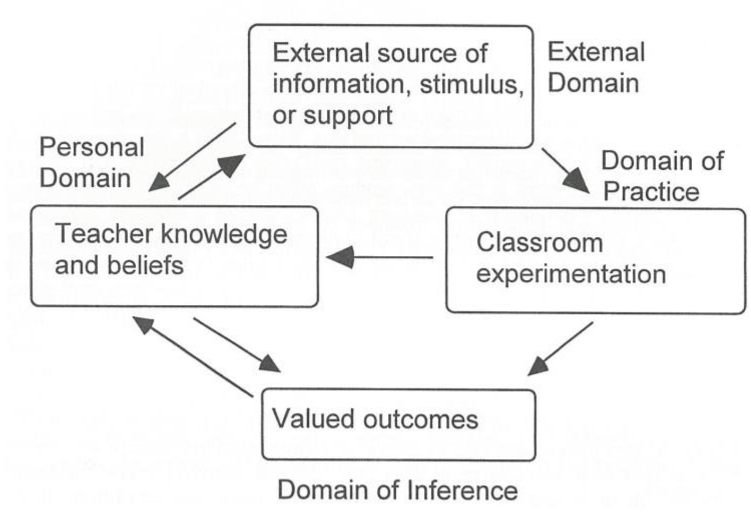


Figure 2.1 The Clarke-Peter model of professional growth. (Nisbet, Warren, & Cooper, 2003, p. 34)

Clarke-Peter's (1993) model recognises the cyclical nature of the professional learning of teachers and teacher change (see Figure 2.1). Their model proposes that, for teachers to change their personal knowledge and beliefs, they need firstly an external source of information, stimulus or support to challenge or reinforce their knowledge and beliefs. However, this new knowledge must first be the focus of classroom experimentation, with student outcomes valued by the teachers, before one can conclude that the change in teachers' knowledge, beliefs and hence practice is the result of teachers' learning (Nisbet et al., 2003).

2.5 Factors Influencing Pedagogical Practice and Professional Learning

The concept of mental models has been developed over the last 40 years, according to Nisbet et al.'s (2003) literature review; it is based on Piaget's observation that people attempt to make sense of the world that they encounter by creating representations of the world as they understand it. In 1943, Craik proposed using the term "mental model" to describe people's internal representations of external processes in terms of words, numbers or symbols (Wilke, 2008). Approximately 40 years later, Johnson-Laird (1983) refined the term, pointing out that human beings construct mental models in their minds and that these mental models are often incomplete and simpler representations of reality. Therefore, a mental model can be defined as a set of personal, internal cognitive constructs or assumptions of reality held about a particular context, phenomenon or system that can be used to anticipate events (Bequary, 2012; Eckert & Bell, 2005; Louis et al., 2003; Manrique & Abchi, 2015; Mason, 2003; Wilke, 2008). In order for teachers to engage in robust inquiry to ascertain their mental model, inquiry skills are essential for managing and conducting face-face interactions, especially when dealing with complex and conflicting issues (Bequary, 2012; Wilke, 2008).

Mental models have been conceptualised by scholars and researchers in the following two ways, according to Wilke (2008):

1) "as an organised and integrated memory representation that can be used to predict some phenomenon;
2) as a transient construction that represents information during cognitive processing." (Wilke, 2008, p. 6)

The first type of mental model reflects beliefs held about a particular system acquired through observation, instruction or inference, while the second type of mental model generates schemata through multiple iterations of transient constructions, according to Seel, Darabi and Nelson (2006), as cited by Wilke (2008). Furthermore, Wilke (2008) cited Seel's (in press) argument that a mental model is not static but in a constant state of flux. This point is confirmed by Wilke (2008) who explains that mental models are dynamic and continue to expand with new information and new experiences.

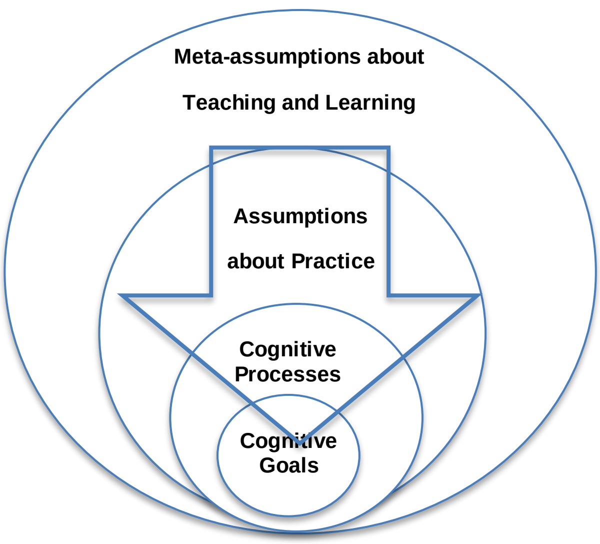
Mental models can impede learning (Bequary, 2012; Mason, 2003). Senge (2006) introduces four mental models about children as learners: children as doers, children as knowers, children as thinkers, and children as knowledgeable. These mental models contrast with Olsen and Bruner's (1996) proposal of two categories of mental models relating to learning: (1) knowledge intake and reproduction; and (2) constructing and transforming knowledge. Figure 2.2 which presents Olsen and Bruner's (1996) proposal graphically to illustrate the teachers' assumptions about teaching and learning is likely to influence their practice which in turn affect the teachers' cognitive goals and processes. Teachers holding these mental models about children's learning would choose a series of actions corresponding to their mental models, as summarised in Table 2.1 below.

Table 2.1: Mental model for learning frameworks (Entwistle & Peterson, 2004)

Mental models of learner minds as classified by Entwistle and Peterson (2004), Olsen and Bruner (1996) and Strauss (2001)	Actions taken by teachers to influence learners with corresponding mental models (Olsen & Bruner, 1996)	Mental models of learning (Olsen & Bruner, 1996)	Mental models of teachers (Entwistle & Peterson, 2004)
"Children as doers"	Modelling or demonstrating	Learning as knowledge intake and reproduction	Espouse model
"Children as knowers"	Telling or showing		
"Children as thinkers"	Collaborating or facilitating	Learning as constructing and transforming knowledge	In-action model
"Children as knowledgeable"	Managing or consulting		

I have combined the two proposed classifications of mental models in Table 2.1 to compare them. The key difference between the two classifications is that Strauss (2001) believes that individuals develop from the first category to the second category, while Entwistle and Peterson (2004) see them as four discrete mental models of learners.

Figure 2.2 Graphical interpretation of components of teacher mental models according to Olsen and Bruner (1996)



Strauss (2001) argues that teachers believe that, through their actions of questioning, describing, summarising, modelling and demonstrating, they are able to influence the mental models of children (as learners) in their classrooms, according to their own mental models of the structure of learner minds and how learning takes place (see Figure 2.2). He further suggests that teachers' mental models have a greater impact on their pedagogy than does the depth of their subject matter knowledge (Strauss, 2001).

Consequently, it is important for the mental models of teachers to be brought to the surface and challenged, so as to lay a strong foundation for teachers' continuous professional learning and growth (Strauss, 2001). Examination of one's mental model requires the development of reflection skills; such skills call for the slowing down of one's thinking processes in order to be more cognisant of how one's mental models are formed and how they affect actions (Senge, 2006). Reflection is a critical skill for teachers to develop and practice, because it is an expected professional practice as stipulated by Australian Institute of Teaching and School Leadership (Senge, 2006)

Teachers' beliefs include the knowledge, concepts, perspectives, images, theories, ideas and so on that teachers hold about a variety of teaching and learning phenomena (AITSL, 2014). Wilke (2008) cites Pajares' (1992) synthesis of the nature of beliefs — that they are formed early in life, reinforce themselves and are often resistant to changes, in spite of reason, time, schooling or experiences. They shape how we view tasks and process information which in turn affects our perceptions, resulting in a conflict with reality at times. Wong (2009) also asserts that change of central beliefs in adulthood is relatively rare, which is confirmed by the works of Rokreach (1968) and Chinn & Brewer (1993). Thompson (1992), cited by Wong (2009), cautions that researching mathematics teachers' beliefs without also taking their knowledge into account, be it knowledge of mathematics content or pedagogy, presents an incomplete picture of the scenario. Ball (1990), cited by Clarke, DePiper, Frank, NIshio, Campbell, Smith and Choi (2014), posits that teachers with very similar mathematical knowledge might teach very differently because they hold different beliefs about mathematics pedagogy. Clarke et al. (2014) comment that the relationship between teachers' beliefs and their instructional practices is dynamic, interactive and cyclical rather than a linear, causal relationship.

It is argued by Hannula, Kaasila, Laine and Pehkonen (2005), as cited by Stacey (2008), that all pre-service teachers' view of mathematics consist of three correlated beliefs; belief in one's own talent, belief in the difficulty of mathematics and one's liking of mathematics.

Mathematics teachers' beliefs about pedagogy will determine their attitudes and consequently their behaviour, actions and practices in the classroom (Clarke et al., 2014; Nickson, 2000; White et al., 2006; Wong, 2009). Drawing on the above findings, teachers' choice of pedagogical practices are likely to be influenced by their mental models/beliefs about how their students learn. This determines whether they choose to use Traditional Transmission Instruction (TTI) or Constructivist-Compatible Instruction (CCI) styles of pedagogical practice. It is also essential to take into consideration Ravitz et al.'s (2000) three determinants of teacher pedagogy outlined in Section 2.3 of this thesis.

White et al.'s (2006) review of frameworks for mathematics teachers' beliefs was divided in the study of Clarke et al. (2014) into two main pedagogical approaches. Clarke et al. (2014) proposed that the behaviourist transmission theory of learning (BTTL) is similar to the TTI method, while their conceptualisation of mathematical learning (CML) is akin to the CCI method.

Ravitz et al. (2000) urge mathematics education researchers to strive to understand the potential influences on teachers' beliefs and awareness while ensuring that any professional development programs designed for mathematics teachers extend beyond increasing the capacity and competencies of the teachers in content, knowledge and pedagogy, to challenging and influencing teachers' beliefs and awareness for improved outcomes.

Belief is defined in this study as a consciously or unconsciously held proposition that predisposes one to a particular kind of action and which can be inferred from what a person says or does (Clarke et al., 2014).

All teachers hold beliefs about the curriculum, nature of student learning, pedagogy and their roles and responsibilities (Wang, 1997). Beliefs held by teachers also impact on their definition of tasks and selection of cognitive tools (Stacey, 2008; Wang, 1997); consequently, their beliefs define and shape their instructional practice. This is particularly important when dealing with the implementation of curricula and pedagogical change.

According to Rokeach (1968), as cited in Wang (1997), belief comprises three components — cognitive, affective and behavioural. These components are linked to a person's knowledge, ability to arouse emotion and decisions about actions. Consequently, by examining a teacher's verbal expressions, tendencies to action, and teaching behaviours, one will be able to infer the beliefs held by the teacher.

In a study of primary school pre-service teachers, Wang (1997) cites the work of Schuck and Grootenboer (2004), who found that prospective primary school teachers generally hold beliefs about mathematics that prevent them from teaching mathematics in ways that that empower children. White et al. (2006) also cite Perry, Vistro-Yu, Howard, Wong and Fong's finding (2002) that the distinct differences between various primary teacher groups in their beliefs about mathematics and its learning gives rise to speculation about the impact of these beliefs upon student achievement

When Hannula, Kaasila, Laine and Pehkonen (2005), as cited by White et al. (2006), explored the structure of 269 Finnish pre-service teachers' views of mathematics and their different belief profiles, they found that the core of the student teachers' views consisted of three correlated beliefs: belief in their own talent; belief about the difficulty of mathematics; and their liking of mathematics.

White et al. (2006), when exploring the debate between educators and researchers about the relationship between teachers' beliefs and practice, compared the positions of Fennema and Franke (1992), Ernest (1988) and Thompson (1984), who found that many researchers argue that teachers' beliefs about the teaching and learning of mathematics do impact their practice; however, Levitt (2001), Shirk (1973), White (2000) and Wilcox-Herzog (2002) found that there were inconsistencies between teachers' beliefs and teaching practice.

It suffices to postulate that there is a link between teachers' beliefs about the teaching and learning of mathematics and their teaching practice.

The evaluation of any professional development program for teachers lies in assessing improvement in the teachers' effectiveness in the classroom and in the student outcomes. Rickey (2008) proposes four elements of andragogical professional development evaluation: reaction, learning, behaviour and results. Teachers' effectiveness in the classroom depends on their reaction to professional learning. They need to have a change of behaviour and practice in the classroom after their past experiences and knowledge are challenged, overcoming their own self-doubts and fear through critical self-reflection to generate an action plan for change.

The commitment to change in teachers is undergirded by their motivation, values and other beliefs (Terehoff, 2002). The motivation for behavioural change is often contingent on the following pre-requisites according to Etchberger and Shaw (1992) and Shaw, Davis, Sidani-Tabbaa and McCarty (1992) (as cited in Wang, 1997): (a) perturbation due to dissatisfaction with the way things are; (b) awareness of a need to change for improvement; (c) commitment to change in the awareness that action is required; (d) ability to envision what the changes involve; and (e) projection into that vision by participating in the change agenda and reflecting through inquiry (Wang, 1997, p. 28). However, change of behaviours or practices will only follow from change in beliefs.

Teachers often feel wary about the effectiveness of innovations proposed by school leaders to create better student outcomes. They assess these proposed changes using three criteria: need and evidence presented; procedural clarity and personal costs; and benefits from engaging in the proposed changes (Fullan, 1991 as cited by Wang, 1997).

School climate affects mathematics achievement and engagement, according to the TIMSS 2011 report (Wang, 1997) revealed that the students who like school and feel a strong sense of belonging also feel safe and have never been bullied and on average, achieve better mathematics and science outcomes (Thomson et al., 2012). In schools where discipline, attendance, prerequisite knowledge, nutrition and sleep are non-issues, students' achievement in mathematics and science on average is higher (Thomson et al., 2012). On the other hand, it is noted in the report that "Australian teachers' reports on their working conditions had no relationship with student achievement in mathematics or science" (Thomson et al., 2012, p. 109).

Thomson et al. (2012) propose that groups of mathematics teachers forming communities of inquiry in schools could provide a structure in which teachers can experiment with and build supportive relationships to challenge and develop one another's mathematics and pedagogical knowledge and beliefs for continuous refinement of their teaching practices. Hammerman (1999) found that: (a) teachers need to recognise the importance of broadening and deepening their own mathematics understanding in order to make good pedagogical decisions grounded in the conception of the discipline; and (b) teachers need to explore new ideas, develop new attitudes and address pedagogical issues explicitly (Hammerman, 1999, p. 189).

Such an approach is only possible when school leaders see the importance of learning in situ while developing a workable structure to facilitate and support these inquiry groups in productive and meaningful professional learning, which are likely to lead to changes in practice and ultimately in improvements in students' engagement and achievement.

School leaders often initiate organisational second-order change for teachers at classroom levels, coupled with an evaluation framework to ascertain if the changes have been implemented by the teachers (Hammerman, 1999). Fullan (1996), as cited by Wang (1997), indicates that organisational restructuring of roles, structures and other mechanisms could enable new cultures to develop. The re-culturing involved a process of developing new values, beliefs and norms in an organization (Wang, 1997).

Wang (1997) warns that getting teachers to change their behaviours without taking into consideration the impact of organisational and cultural norms, as well as the individual teacher's perspectives, could lead to disappointing reform outcomes. School principals and their senior leadership team members such as deputy principals and heads of school, are responsible for the professional development of the teachers and the quality of teaching and learning; they can create a culture of collaboration and teacher autonomy which supports the implementation of collaborative inquiry as a mode of professional learning for teachers (Darling-Hammond & McLaughlin, 2011; Henessya & Deaneya, 2009; Wang, 1997).

If schools wish to have professional development programs which recognise the varying needs of teachers for authentic classroom and cultural change, they have to be willing to change their current structures for professional development, for instance, by allowing teachers to use their time more flexibly for professional development (Fullan & Stiegelbaure, 1991; Krainer & Peter-Koop, 2003). Likewise, school leaders who respect the autonomy of teachers by providing time for reflection and networking are often more successful in transforming practice at schools and assisting their teachers to take ownership of the change (Darling-Hammond-McLaughlin, 2011; Wang, 1997)

Krainer and Peter-Koop (2003) assert that teachers need to create an inquiring culture in their classrooms to facilitate learning; the students and teachers need to engage in exploration of ideas. According to B. S. Nelson (1999), such conditions for learning cannot be achieved unless the teachers themselves experience an inquiry model of learning first, in order for them to be confident and equipped to manage inquiry classrooms effectively. Furthermore, while the teachers are examining their own thinking, they are learning to facilitate students' thinking (Morocco & Solomon, 1999). Consequently, the collaborative inquiry model of professional learning for teachers not only enhances teachers' own professional growth and learning, but also strengthens their ability to deliver an effective inquiry model of learning for their students.

The effectiveness of collaborative inquiry for teachers' professional learning and growth depends on the skills of the facilitator (in this project, the researcher). For teachers to take risks to share their inner-most thoughts, doubts, questions and concerns during collaborative inquiry, a safe and secure environment with mutual respect is needed for intimate professional conversations about teachers' beliefs and practices to be shared. Such an environment can only be created by skilful facilitators creating a context in which teachers grow increasingly comfortable in being guided, and in their beliefs, perceptions and practices being elicited, probed and challenged, so that they share both their successes and failures through reflections (B. S. Nelson, 1999).

Collaborative inquiry, as a tool for teachers' professional learning, therefore uses conversations to assist teachers who are at different levels of understanding and skills to diagnose their own practices (Darling-Hammond & McLaughlin, 2011). This requires the facilitator to possess a high level of facilitation skills, knowledge and flexibility to steer the conversation changes over time (Morocco & Solomon, 1999). This approach is particularly effective if the school leaders help staff members diagnose their needs by teaming teachers according to their learning needs (Morocco & Solomon, 1999). Such teaming, based on the learning needs of the professional development of teachers, provides teachers with opportunities to receive feedback on their ideas, and to share their diverse perspectives on content knowledge, pedagogy and classroom management; collegiality and communication amongst the teachers are strengthened, thus creating a culture of collaboration (Bequary, 2012; Darling-Hammond & McLaughlin, 2011; Terehoff, 2002).

Terehoff (2002) argues that, for collaborative professional learning activities to be effective and successful, the facilitators not only need to be highly skilled and knowledgeable in the specific topic of professional learning, but they also need to be committed and passionate about this style of professional learning for teachers, using a series of activities over a sustained period of time. They need to focus on developing and growing the knowledge, affects and beliefs of the teachers by providing a supportive structure to facilitate sharing, reflection and discussions (Nisbet et al., 2003).

2.6 Literature Review Framing Research Question

The Melbourne declaration has set the stage for this study, in that it states the importance of students' middle years of development, emphasises the importance of students' literacy and numeracy, and recognises the important role of teachers in the engagement and achievement of these students (MCEETYA, 2008). Consequently, the teachers' pedagogical practice and their understanding of their roles in the students' mastery of mathematics are two important factors amongst many others, such as school culture, teachers' content knowledge, …etc., to engage the middle years' students in the learning of mathematics. It is therefore unsurprising that the Australian Institute for Teachers and School Leadership included "the teachers' ability to plan and teach" as one of the standards for teachers in its development of Australian Professional Standards for teachers (AITSL, 2014).

Using collaborative inquiry as a professional learning tool, embedded in the teachers' work schedule in small groups, to challenge their pedagogical practice became the main thrust of this study. The teachers' growth and development, including their mental models and beliefs, as well as their pedagogical practices, have a direct impact on the engagement and achievement of their students. Hence, it is imperative for schools to develop effective professional learning opportunities for teachers to grow and develop. The rise of the professional learning communities model to support teachers' continuous learning and growth has gained much traction in the form of professional learning communities (Dufour, 2007; Nisbet et al., 2003). Other thinkers and researchers have been developing and trialling various professional learning approaches that take into account teachers' adult learning preferences such as active learning, practical, supportive, self-reliant, ongoing follow-up and support of progress, job embedded, reflective and non-evaluative (Hashweh, 2003; Hendreke, 2009; Rickey, 2008; Robert and Pruitt, 2008; Small 2011; Terehoff, 2002; Wilkins & Shin, 2011; Wong, 2009).

These studies largely concur that, as long as the features identified in the preceding paragraphs exist in the professional learning approaches, then teachers are likely to engage with the learning. However, the pace of growth and development in the teachers largely is dependent other contextual factors such as the teacher beliefs and mental models, school culture and support, and the experience and backgrounds of the teachers. A number of studies have adopted a form of collaborative inquiry for professional learning of teachers across systems, districts and schools (Lebak & Tinsley, 2010; Nisbet et al., 2003; T. H. Nelson, 2009; Poekert, 2012; Schnellert, 2011; Schnellert, Kozak & Moore, 2015; Wong, 2009). These studies have taken into consideration the social nature of learning for the teachers to inquire into a question or issue together across a large section of the teaching community in exploring teaching practice in science, physics, writing and drama within a period ranging from 14 weeks to one year. My study was constructed in one school to explore the benefits, limitations and possible wider and more consistent implementation model of collaborative inquiry as a professional learning tool from one setting into another. The middle years' mathematics teachers' professional practice and understanding were the foci of inquiry across the three-year period of my study. This longitudinal study afforded me, the research-participant and school leader, the opportunity to share the journey of the teachers' growth and development.

Teachers' professional growth is impacted by the school climate and structure in which the professional learning opportunities are made available to them. The context in which the teachers learn can affect their intrinsic and extrinsic motivation to grow and change in their practices. Furthermore, the learning outcomes of the students in mathematics are directly impacted by the school climate according to Dufour (2007). To this end, my research explored the benefits and challenges of the collaborative inquiry approach, as perceived by the teacher participants, in order to evaluate its effectiveness for the teachers' professional learning and growth.

2.7 Conclusion

The charge by the Education Ministers of Australian states and territory to school leaders to develop teachers' competency and standards for the benefit of students and, in addition, the TIMSS and PISA data indicating Australian students losing their mathematics capabilities to be competitive on the global stage, provided the impetus for my study.

The conceptualisation of my study centred on identifying the effectiveness of collaborative inquiry as a professional learning tool for teachers. This approach focuses on utilising adult learning mode and job-embedded model of professional learning (as outlined in the earlier sections) for mathematics teachers in the middle years' setting. In the next chapter, the grounded theory research methodology and my research plan are outlined.

CHAPTER THREE

RESEARCH METHODS

3.1 Introduction

Professional learning of teachers supports them to grow in their practice, their professional confidence and pedagogical effectiveness. Most professional learning of teachers in Australia currently still focuses on staff in-service on pupil-free days on site, with learning facilitated by internal staff or external experts, or through sending staff out of the schools to attend conferences, workshops or seminars. Such professional learning practices have a place in teachers' professional growth, and the topics covered in these modes of learning are often varied. In this study, I investigated a means by which teachers can combine their daily practice and reflection on their practice with fellow practitioners to enhance and sustain their professional practice, growth and confidence.

I explored how an onsite professional development approach known as collaborative inquiry can influence mathematics teachers' awareness of effective pedagogies and how their pedagogical change can affect students' achievement in mathematical understanding and engagement. Furthermore, I investigated the benefits and challenges, if any, of implementing collaborative inquiry as a means of effective professional learning for mathematics teachers. In addition, I evaluated how this method of on-site professional learning activity can be incorporated into the schedules of teachers. In this chapter, I outline the research methods adopted for this study.

3.2 Nature and Appropriateness of Grounded Theory Approach

Glaser and Strauss first used grounded theory in 1967. They stated that the aim of grounded theory methodology was to develop middle-range theory at practice settings (Glaser, 2007; Oktay, 2012; Teppo, 2015). In this study, I adopted a constructivist approach to grounded theory research, in that all the reality is constructed, and the constructed perspectives of both the respondents and the researcher are equally valued (Stern & Porr, 2011; Charmaz, 2009).

Grounded theory has been chosen for this research because it is designed to study the interactions between individuals and their practice setting, employing symbolic interaction theory (Oktay, 2012). In my research, I focused on the interactions of the teachers within a collaborative inquiry setting as a team of year level teachers interrogated their own pedagogical practice and its effectiveness. Grounded theory requires that the research sample/population be accessible over a long period of time to enable a multi-stage, abductive process of constantly moving back and forth between data gathering and data analysis. This process explores new emerging ideas, or hypotheses and facilitates more in-depth examination of concepts (Charmaz, 2009; Lempert, 2007). This requirement was adequately satisfied by my research site in that the teachers, other than those teachers who left the school during the first two phases of the research for other employment, were available for a continuous assessment of the process.

This multi-stage process of grounded theory research began with the gathering of data after the selection of the topic. It required that I entered the study with an open mind to minimise researcher bias in generating theory, albeit with some preconceived idea of the problem or question through personal experience and a literature review. A heightened state of theoretical sensitivity is essential to eradicate possible theoretical bias. A literature review is deemed a controversial issue in grounded theory methodology (Glaser, 2007; Teppo, 2015). However, in any research work, reviewing of current theoretical perspectives or models is essential to building on the body of work that is extant in the area of study. In this study, the theory review outlined in Chapter 2 focuses on the current understanding of various concepts relating to the research problem. Ongoing literature review was conducted at each phase when the data analysis showed certain emerging thinking; the latter stages of literature review help to triangulate with my findings (Dey, 2004; Stern & Porr, 2011). The assumptions and understanding of the theoretical concepts and explanations relevant to this study, as stated in Section 1.4, were made prior to the commencement of data collection and analysis. This articulation helped me to stay objective and sensitive to my theoretical understanding of the research problem.

Theoretical sampling is used in grounded theory as the study progresses, allowing the core concepts of the theory to emerge through a range of sampling strategies such as surveys, observations of teaching and dialogues, and focus groups.

Theoretical saturation is reached when the theory and data fit together. At this stage, the theory emerges and is consequently consolidated.

Dey (2004) and Lempert (2007) suggest that, as the research progresses, a literature review will provide additional support to locate a pattern for the emerging ideas and concepts from the data analysis. During the final stages of the research, further literature is reviewed to place the emerging theory in the existing theoretical framework of the area of study; extending the current framework is a process advocated by Teppo (2015).

3.3 Research Design and Plan

Qualitative research is an inquiry into occurrences in the real world context through the use of conversations and observation, to develop greater understanding of our world. Such research adopts an inductive model of inquiry to interpret and explain the phenomena under study (Anderson, 1998). Qualitative research should be shaped by the purpose, context and feasibility of the inquiry rather than with a fixation on the methodology alone (Taylor & Wallace, 2007). This study used a grounded theory method within the qualitative research paradigm.

As a qualitative researcher, I am the principal data collector who uses an emic approach to understanding phenomena and interpreting social reality from my perspective, acknowledging the conceptual and theoretical understanding of the participants' social reality. At the same time, I use an etic perspective by asking what the event or interaction meant to the individual, seeking to understand how participants view their world (Young, 2005).

This research design is divided into three phases as outlined in Table 3.1. In each phase, inductive logic is used to hypothesise the theory based on a literature review and/or data analysis; then deductive logic is applied, using the data collected and analysed to generate and/or refine the theory. After that, the research design is reviewed and refined to further test and refine the theory. Table 3.1 outlines, in each of the three stages, how inductive logic is used to hypothesise the theory using the research questions, and then deductive logic is used when data are analysed and the theory is refined to further test the theory in the following stage.

Table 3.1 Evolution of theory using inductive and deductive logic.

Stages & Evolution of Theory	Inductive Logic Hypothesis of Theory Research Design Based on Theory	Deductive Logic Based on Data Collected to Generate Refined Theory Research Design Reviewed and Refined to Further Test Theory
Stage One	Research questions: a. What benefits and challenges would the implementation of collaborative inquiry as a professional learning approach present for the school and the middle school mathematics teachers? b. What would be the essential elements in implementing effective collaborative inquiry as a professional learning approach for teachers in the school? c. How would these essential elements be incorporated in a theoretical model to inform the effectiveness of collaborative inquiry as an professional learning approach for middle school mathematics teachers in schools?	Research questions: a. What benefits and challenges would the implementation of collaborative inquiry as a professional learning approach present for the school and the middle school mathematics teachers? b. What would be the essential elements in implementing effective collaborative inquiry as a professional learning approach for teachers in the school? c. How would these essential elements be incorporated in a theoretical model to inform the effectiveness of collaborative inquiry as an professional learning approach for middle school mathematics teachers in schools?
Theory evolution	Collaborative inquiry is one effective professional development tool for developing middle school mathematics teachers' understanding and implementation of effective pedagogies. The essential elements are synethsised as optimal environment, authentic assessment and teacher pedagogy.	More focused and better facilitated collaborative inquiry can be an effective professional development tool for middle school mathematics teachers to enhance their teaching practices and hence student achievement. An unexpected emerging element was the mental model and belief of the teachers. Further exploration and attention will be placed on this element in the next stage of the study.
Stage Two	Research Questions: a. What benefits and challenges would the implementation of collaborative inquiry as a professional learning approach present for the school and the middle school mathematics teachers? b. How would collaborative inquiry challenge the teacher mental models and beliefs about effective pedagogical practice? c. What would be the essential elements in implementing effective collaborative inquiry as a professional learning approach for teachers in the school? d. How would these essential elements be incorporated in a theoretical model to inform the effectiveness of collaborative inquiry as an professional learning approach for middle school mathematics teachers in schools?	Research Questions: a. What benefits and challenges would the implementation of collaborative inquiry as a professional learning approach present for the school and the middle school mathematics teachers? b. How would collaborative inquiry challenge the teacher mental models and beliefs about effective pedagogical practice? c. What would be the essential elements in implementing effective collaborative inquiry as a professional learning approach for teachers in the school? d. How would these essential elements be incorporated in a theoretical model to inform the effectiveness of collaborative inquiry as an professional learning approach for middle school mathematics teachers in schools?

Theory evolution	More focused and better facilitated collaborative inquiry can be an effective professional development tool for middle school mathematics teachers to enhance their teaching practices and hence student achievement.	Collaborative inquiry can only be an effective professional development tool for middle school mathematics teachers to enhance their teaching practices, and possibly student outcomes, provided that certain preparations enable staff's mental readiness of for professional growth and learning.
Stage Three	Research Questions: a. What benefits and challenges would the implementation of collaborative inquiry as a professional learning approach present for the school and the middle school mathematics teachers? b. How would collaborative inquiry challenge the teacher mental models and beliefs about effective pedagogical practice? c. What would be the essential elements in implementing effective collaborative inquiry as a professional learning approach for teachers in the school? d. How would these essential elements be incorporated in a theoretical model to inform the effectiveness of collaborative inquiry as an professional learning approach for middle school mathematics teachers in schools?	No further refinement of research questions in this stage as it is the last of the three stages of the process.
Theory evolution	Collaborative inquiry can only be an effective professional development tool for middle school mathematics teachers to enhance their teaching practices, and possibly student outcomes, provided that certain preparations enable staff's mental readiness for professional growth and learning.	Collaborative inquiry is an effective professional development tool for middle school mathematics teachers to enhance their teaching practices provided that the teachers' mental models about teaching and about students' learning are aligned. The focus of collaborative inquiry needs to be on how to create an optimal environment, effective pedagogy and authentic assessment, with the goal of engaging students strongly in learning and developing deep understanding.

This study was designed as a qualitative study using the grounded theory approach to evaluate the effectiveness of collaborative inquiry as a professional development tool for middle school mathematics teachers to improve student achievement. However, as explained in the qualitative grounded theory approach proposed by (Oktay, 2012), grounded theory involves a multistage process to build a theory through the repeated usage of both inductive logic to generate theory and deductive logic to test the theory.

In stage one, this study began by establishing the hypothesis that "collaborative inquiry is an effective professional development tool for middle school mathematics teachers to enhance student achievement." During this stage, I engaged the teachers in collaborative inquiry, using weekly planning meetings to discuss pedagogical theory, critiquing video clips of classroom teaching evidence, and eliciting from the teacher participants any effective pedagogies. This was coupled with conducting student focus group meetings to collect feedback about their preferred learning environment for mathematics. During this stage of research, I found that the teachers were prepared to identify their professional learning needs, open to articulating their pedagogical weaknesses, and ready to critique fellow colleagues, and students were able to discern the subtle change in the teachers' pedagogies due to the collaborative inquiry sessions. The theory developed at the end of the first stage of data collection, and evolved into a more focused and better facilitated collaborative inquiry, can be an effective professional development tool for middle school mathematics teachers to enhance their teaching practices and hence student achievement.

During stage two of the research, a more refined approach with focused discussion topics and better facilitation preparation was adopted. When the data were analysed, it was observed that the teachers' beliefs and mental models were grounded in their upbringings, experience, educational backgrounds, educational philosophies about teaching and learning practices, and student achievement. This stage of the research approach was still informed by my romantic notion that the teachers' professional mindsets enabled them to have professional curiosity, inquisitiveness, commitment and a desire to grow professionally. Furthermore, because the teachers were not able to suggest professional learning needs relating to their teaching of middle school mathematics, the topics chosen by me were sometimes deemed as an academic exercise with little evidence of impact on the teachers' pedagogies. The amended evolving theory after stage two of the research was that collaborative inquiry could be an effective professional development tool for middle school mathematics teachers to enhance their teaching practices and hence student achievement when staff were enabled to develop a mental readiness for professional growth and learning.

Stage three of the research focused on individual interviews with the teachers and discussions to tease out the mindsets of the teachers, while finding areas to align with their mental models. The individual interviews focused on eliciting individual professional insights and reflections in the light of their mental models and mindsets, and evaluating the theoretical model emerging from stage one data. The teachers' feedback from the first set of interviews and stage two data were used to refine stage one's theoretical model.

The research design incorporated various data collection strategies as outlined below.

3.3.1 Field Work

Access to the research site was possible, as it is the school at which I work. The reason for selecting this site was because the school was undergoing curriculum review and seeking to improve teaching and learning at the start of this research, particularly in the numeracy area. Numeracy was central to the priority of curriculum change. The idea of using this site to investigate mathematics teachers' professional learning for better pedagogical practice was met with support and approval from the then Head of College. It was perceived that the findings of this research would greatly benefit the quality of teaching and learning and the approach to professional learning at the college. Written approval for permission to access the site for research purposes was sought from the Head of College (Appendices I and J). Access to the site was approved by the then Head of College. During the life of this research, when the Head of College retired, I was appointed as the next Head of College.

This research site structures its middle school for students entering Year 6, and the final year of middle school is Year 9. Students in Year 6 have a core teacher who normally teaches them all core subjects including English, humanities, mathematics, science and christian studies. Years 7 and 8 core teachers teach the students either mathematics and science or English and humanities, while the Year 9 mathematics teachers either teach the students for mathematics only or teach both mathematics and science to the same class.

Once access approval was received, the mathematics teachers of middle school were invited to a briefing with me for an overview of the value of the research. Amongst the 12 middle school mathematics teachers briefed, seven teachers accepted the invitation to participate in the research. This constituted 58% of the total population of middle school mathematics teachers at that time, which was a high percentage for such research. However, over the three years of data collection during the life of this research, three teachers left the college during the first year, and one teacher left the college in the second year of the research. One teacher joined the research in its second year. In the final year, four of the eight teachers who volunteered to participate in the research remained in the study. These four teachers constituted 50% of the original number of teachers who volunteered for this study and 30% of all the middle school mathematics teachers, making it a viable sample size for this research. All the teachers were provided with the research information and voluntary participation consent forms (Appendices I and K).

The teachers were from three year levels, namely, Years 7, 8 and 9. None of the teachers in Year 6 opted to participate in this research. With the departure of the teacher participants in the research for other employment in the second year of the research, the remaining teachers were teaching Years 7 and 9 in the final year of this research.

Being a senior staff member at the research site, my knowledge of the hierarchical structure of the college and the community of people was strong. In my dual role as a senior staff member/Principal of the college as well as researcher, the challenge was to ensure that my role as researcher was not compromised by my responsibilities, or by the staffing changes

The middle school mathematics teachers met weekly as scheduled in their timetables whenever feasible. The weekly meeting was scheduled as a year level planning meeting. I attended the meetings whenever my schedule permitted, as these meetings were scheduled during school hours and, at the time, my role as senior staff and later Principal required me to focus attention on school business, which sometimes clashed with the scheduled year level planning meetings.

I communicated with the middle school mathematics teachers via email to confirm meeting agendas and staff availability. The scheduled planning meetings were sometimes cancelled due to staff being absent at other meetings, duties, illnesses or external professional learning opportunities. These cancelled meetings were not re-scheduled because of the constraints of the work place agreement which prohibited and restricted my capacity to retain the planned number of sessions.

Student focus groups were organised by emailing the students to attend the focus group discussions. Research information and a parent consent form for student participation were issued to the students (Appendices I and L). The sessions were recorded (audio) on a digital device, and then the recordings were transferred to my computer for transcribing. Field notes were taken to record the contexts, settings and other observations made by me.

Interviews were conducted twice with each of the teachers in the final phase of the research, six months' apart. Each of the teachers was emailed the interview date, time and location. The interview questions were given during the interviews. The interviews took around one hour each. The interviews were recorded using a digital recording device, then the recordings were transferred to my computer for transcribing. Field notes were taken to record the contexts, settings and other observations of mine.

3.3.2 Building Rapport

As a staff member of the research site who knew all the teachers involved in the research for at least two to three years prior to the commencement of this research, I had very good knowledge and rapport with the teachers. However, a possible source of tension was my position as senior staff and later Principal of the research site during the research; hence, I was ultimately the presiding supervisor of all staff. The teachers in this research never raised any relevant concerns with me and I did not hear about their concerns through other sources. However, it can be safely concluded that my full-time employment role at the research site may have influenced teachers' freedom of interactions during this research because of the natural human tendency to avoid conflict with one's supervisors.

3.3.3 Human Issues

Being a staff member and Principal of the institution, I am in constant communication with stakeholders of the institution, which include staff, parents and students. Consequently, I possess additional insights into the research setting that are helpful in shedding light on phenomena observed or under investigation. Furthermore, these insights also clarify value patterns, concepts and beliefs of the participants which otherwise cannot be directly observed.

On the other hand, as the Principal of the institution, these insights may also affect my objectivity as the researcher unless I am cautious and careful to separate the issues or questions arising based on the data collected and analysed. Furthermore, I risk the danger of contaminating the data with reports relating to the teachers' work performance if I fail to exercise extreme care.

3.3.4 Observation

During the classroom observations, because of my dual roles as researcher and Principal, I adopted the participant observer approach by engaging in the regular activities of the classrooms, then periodically withdrawing myself from the setting to check perceptions, record field notes and analyse data.

It was possible that the teachers and students modified their behaviours during my presence to observe teachers' practice because of my dual role. To avoid the Hawthorne effect of interfering with the authenticity of the data collected from classroom observations, I behaved in a way I normally did when I entered the classroom in my role as Principal (Cook, 1962). The teachers and students were used to my regular visits; hence, other than their initial "consciousness" of my presence, after a few minutes, they tended to revert back to their normal behaviour as if I were not present.

3.3.5 Field Notes

Field notes were compiled during classroom observations to facilitate team planning for collaborative inquiry sessions, student focus groups and teacher interviews.

These notes supplemented the digital recordings to assist me to recall non-verbal observations and insights. Furthermore, these descriptive records of research experience, observations, personal reflections and physical descriptions of the setting assisted me to make subsequent decisions about the direction of the research process, which is critical to grounded theory research methodology.

3.3.6 Communications

Several methods were used to communicate with the participants of the research, including face-to-face conversations, group meetings, email messages, paper memos for consent to participate, and outlook calendar appointments to set up individual interviews.

3.3.7 Physical Items

Physical settings of the environments during which data were recorded and collected were noted in the field notes. These included classroom observation settings, team planning and collaborative inquiry session settings, student focus group settings and teacher interview settings, which were all recorded in field notes for future references.

All signed forms and handwritten notes were kept in a locked cabinet in my home office. The only person who had access to these physical items was me with my key.

3.3.8 Data Management

Some quantitative data were collected in the form of a teacher survey in this otherwise qualitative research, as their use was unavoidable. However, they were not used for statistical analysis, but rather the data was analysed to provide another perspective to triangulate the qualitative data analysed. Only the general descriptive properties of the survey responses were used.

All raw data collected were carefully managed through physical storage in locked cabinets at my home office as well as password protected hard drive of my computer. All electronic data were also backed-up on an external password protected cloud server drive.

3.3.9 Data Collection and Analysis

The qualitative data analysis and interpretation were conducted continuously, informed by theory based on the stages of evolution.

I adopted the basic phases of analysis for my data. Firstly, I spent time to immerse myself in the data by listening or watching or reviewing the data collected. After this, I entered the incubation stage during which I spent a time of quiet contemplation on the data to observe significance and patterns. After this stage, illumination emerged with my increased awareness of the meaning presented by the data and new clarity about the data narrative. Through this process of meaning making and the clarity of insights gained, new connections were made, and I prepared my findings by narrating them in writing. Once the findings were identified, I began to tie these findings and research experience together through the creative synthesis phase. It was important to note that, as this was grounded theory research with inductive and deductive processes interfacing, these basic phases were applied repeatedly in a cyclical fashion.

Triangulation of data from the various sources was applied at each phase of data analysis to ensure that the emerging theory was validated at each stage to eliminate bias, error and anomalies.

During each phase of the data analysis and interpretation, I always maintained a heightened state of critical self-reflection and reminded myself to stick to the appropriate modes of representation to honour the ethics of research (Taylor & Wallace, 2007).

3.3.10 Reporting

In this thesis, I aim to report, share and communicate the findings reached after refinement of my thinking, crystallisation of my ideas and internalisation the data. I endeavour to bring my findings to life through the voices and opinions of the participants of this research.

Honouring the principles of grounded theory research, I adopted a reflexive reporting approach to presenting this on site research, in order to capture my close connection with this site as well as my participant-observer role during the data collection stage of this research.

3.3.11 Limitations

There were a number of limitations in my research project, which I kept in mind and sought to manage in this study. Each point is elaborated below:

(a) Objectivity: As I was cognisant of the potential bias as an on-site researcher, I was very careful in my participant observer role throughout the research. During analysis of the data, I also adopted the ethical position of researcher to let the data pattern create the narrative. Consequently, should another researcher carry out this same research design at another site, the findings are likely to be quite similar to mine.

(b) Reliability of informants' information could be another concern: as mentioned earlier in Section 3.3.3, my dual role at the research site could colour the interpretation; however, this limitation was reduced by the triangulation of data.

(c) Quality: The quality of the research data and findings were highly contingent on my research skills and my ability to understand, record, gain insight into and interpret the data collected. This limitation was managed through having ongoing conferences with my research supervisor and regular review of methodologies used by other researchers to ensure that the integrity of research was maintained.

(d) Internal validity: The internal validity of my research was maximised by keeping meticulous records of all sources of information, transcripts and field notes with a clear audit trail, and linking the chains of evidence throughout the phases of the research.

3.3.12 Conclusion

As I used the inductive form of inquiry for this research, I accept that being the main and only data collector meant that I was attached to a set of "baggage" that shaped and informed my opinions, attitudes and ways of looking at phenomena and interpreting the findings of this research.

Being the Principal at the research site provided me with a strong understanding of the research environment and all its political, social, psychological, economic and cultural dynamics. This knowledge was vital in helping me to produce rich, useful and valid findings for this important research.

3.4 Research Instruments

3.4.1 Mathematics Activities

The following mathematics activities were shared and incorporated into the curriculum planning as effective pedagogies for mathematics classrooms at this site:

- Rate of dissolving of Chupa-Chup lolly pops – study of relationships between volume, surface area and dimensions of 3-dimensional objects;

- Bungee Barbie – study of the rate of fall under gravitation acceleration and effect of Hooke's law (elasticity);
- Algebra tiles – modelling of polynomial in algebra using coloured tiles;
- One-metre space – study of relationship of capacity and volume;
- "Psy" Gangnam Style Dance – analyse pattern in dance moves;
- Stackable Chairs – analyse and develop algorithm for height of stacked chairs;
- Fibonacci pattern and Pascal's triangle – analyse and develop patterns;
- Paddle pop sticks – using manipulation to ascertain patterns and relationships between one-dimensional and two-dimensional objects;
- Mapping activity – study of scales and legends;
- Three dimensional objects – study of volume and capacity.

3.4.2 Team Planning Meeting

In order to build professional learning activities into the teachers' work flow, middle school teachers' existing scheduled weekly year level team planning time was used for collaborative inquiry sessions. The team planning was a built-in existing structure in which inquiry was focused on teachers' attention to gaps between their shared vision for student learning and the student achievement . The team planning sessions were recorded in the first two years of the research to collect sufficient data to analyse as well as to analyse the changes in teachers' perceptions and participation in collaborative inquiry over time.

3.4.3 Lesson Observation

Lesson observations were initially planned to serve as resources for collaborative inquiry sessions, during which the teachers watched the recorded lessons and provided feedback to their colleagues to support one another for growth in pedagogical practice. However, after the first year of implementation, it was noted that the teachers were not ready to provide critical evaluation or feedback on another teacher's lessons. I deemed it an ineffective means of encouraging colleagial dialogues; hence, this method was not used in the second year of data collection.

3.4.4 Student Focus Group

Student focus group sessions were conducted to ascertain the effect of teachers' professional learning, that is, collaborative inquiry, and its impact on teachers' observable practice in classrooms in the first year. The student focus group data were used to triangulate the data and findings from collaborative inquiry sessions collected in Year 1 of this research. The focus group survey questions are listed in Appendix E.

It was noted that, during the student focus group sessions, due to the power relationships between students and their teachers, some students were not able to be open and objective in their discussions. A reflection of their inability to articulate their feelings or observations was that they needed a great deal of prompting from the interviewer to solicit responses to address the questions.

In spite of the above limitations, the student focus group feedback provided some insights into the pedagogical practices that they felt would enhance their learning and positively affect their learning outcomes or achievements.

3.4.5 Teacher Survey

Teachers completed a Survey Monkey survey (Appendix D). This survey was designed by me to provide an overview of the teachers' training background, mental models and mindset regarding the effect of professional learning and growth on their effective engagement in collaborative inquiry as a professional learning tool. The survey was anonymous even though, through their profiles and my institutional knowledge of each teacher, it was possible for me to match the profiles to the participants. However, the purpose of the survey was to obtain a representative scan of the beliefs of the teachers about mathematics teaching and learning in order to triangulate the analysed data; consequently the matching of participants and their answers was neither necessary nor completed during the compilation of the survey data. Only 6 out of the 8 teachers completed this survey.

Six teachers involved in this study responded to the teacher survey. The survey was designed to ascertain their collective mental models and mindset as a team of mathematics teachers on this site in relation to professional learning and students' mathematical learning, understanding and engagement.

As this research spanned three years of data collection, one teacher left the institution in the middle of Year 1 of the research, while two teachers left at the end of Year 1; one teacher left in the middle of Year 2 of the research, while a new teacher joined in the middle of Year 2 of the research. [Teacher codes were withheld for the paragraph above to prevent ease of identification of these teachers at the site.]

In spite of their departure, as the teachers were involved in the data set collected, their profiles and inputs collected were analysed accordingly. Only six of the other participating teachers completed the survey; two teachers left the school before the survey was administered.

3.4.6 Teacher Interview

Teachers were interviewed to assess their personal opinions and feedback about collaborative inquiry and the emerging theoretical sampling and stage one of the theoretical model. Furthermore, their perceptions of mental models and mindsets, professional learning and growth were discussed and then analysed. See Appendices F and G for the questions asked at the interviews.

3.5 Sample

There were a total of 12 middle school mathematics teachers at the research site in the year when the research began its phase one data collection. After being briefed about this research project and the voluntary nature of the research participation, seven teachers submitted their consent forms to participate in the research. This sample size constitutes 58% of the possible research population who consented to participate in this research.

I then visited each of the middle school classes taught by these seven teachers to explain the purpose of my research and distributed the information sheet and parent consent forms for their parents to invite the students for voluntary participation in the student focus groups of this study. There were a total of 275 middle school students taught by these 7 teachers in the first year. 49 signed student consent forms were returned by the parents. This constituted approximately 18% of the research population who participated in the research.

3.5.1 Selection and Description of Stage One Sample

Collaborative inquiry sessions were conducted over three year levels, namely, Years 7, 8 and 9. The Year 7 team consisted of two teachers, both of whom were participating in this research, converting to 100% participation rate for this year level. The Year 8 team consisted of three teachers; two of the three teachers participated in this research which constituted a 66% participation rate. Of the four teachers in Year 9, three of them participated in this survey, making a 75% participation rate. None of the Year 6 teachers participated in this research. This level of participation was considered very high for the research.

All seven teachers participated in the data collection for this research in Year 1, which included (a) team planning sessions and (b) classroom teaching recorded as a resource for team planning discussions.

18% of approximately 272 students whose parents gave consent for their participation in this research. This sample size was probably due to the students not bringing their consent forms home to show their parents. Furthermore, adolescents, who are still developing their organisational skills, might have misplaced the consent forms. The sample size was unlikely to impact the findings of this study given that the focus group data were intended to be used for triangulation. Two student focus groups were formed from the participating students, each consisting of 8 students, making up 16 of the 49 participants, representing close to 30% of respondents who participated in this research.

3.5.2 Selection and Description of Stage Two Sample

In the second year, collaborative inquiry sessions were conducted over two year levels, Years 7 and 9. The Year 7 team consisted of two teachers, both of whom participated in this research, converting to a 100% participation rate for this year level. One teacher participant in the first year of this study in the Year 8 team left the organisation, and the other Year 8 teacher participant did not teach Year 8 in the second year of this study; consequently there were no Year 8 collaborative inquiry sessions. Of the three participating teachers in Year 9, one left the college. The replacement teacher in Year 9 volunteered to join the research. This level of participation rate was considered very high for the research.

This qualitative research process together with the sample size based on the population available at the site is illustrated in Figure 3.1. The sample size for each phase of the research is clearly displayed showing the challenges of retaining participants in this 3-year longitudinal study.

3.5.3 Selection and Description of Stage Three Sample

In the third year, the four remaining participating teachers from the original eight participating teachers were interviewed. As mentioned in Section 3.5.2, 50% of the teacher participants left their employment at the research site to seek alternative employment in other schools locally and overseas.

All of the remaining teacher participants were interviewed twice a year over a six-month period. This constituted 100% of all the remaining teacher participants.

3.6 Data Collection

The data were collected over a three-year period using a digital recorder for team planning meetings, student focus groups and individual interviews.

3.6.1 Phases and Types of Data Collection

In Phase 1, data were collected throughout the year during the Year 7, Year 8 and Year 9 team planning sessions during which the teachers engaged in the inquiry of their teaching understanding and practices. Two student focus group sessions were undertaken at the end of phase 1.

In Phase 2, data were collected throughout the year during the Year 7 and Year 9 team planning sessions involving slightly different team members due to the timetable changes from phase 1 to phase 2 of the study. Teacher surveys were completed in the middle of phase 2.

In Phase 3, teachers were interviewed individually twice, six-month apart, and data from the interviews were collected.

Each type of data collected was described in greater depth after Figure 3.1.

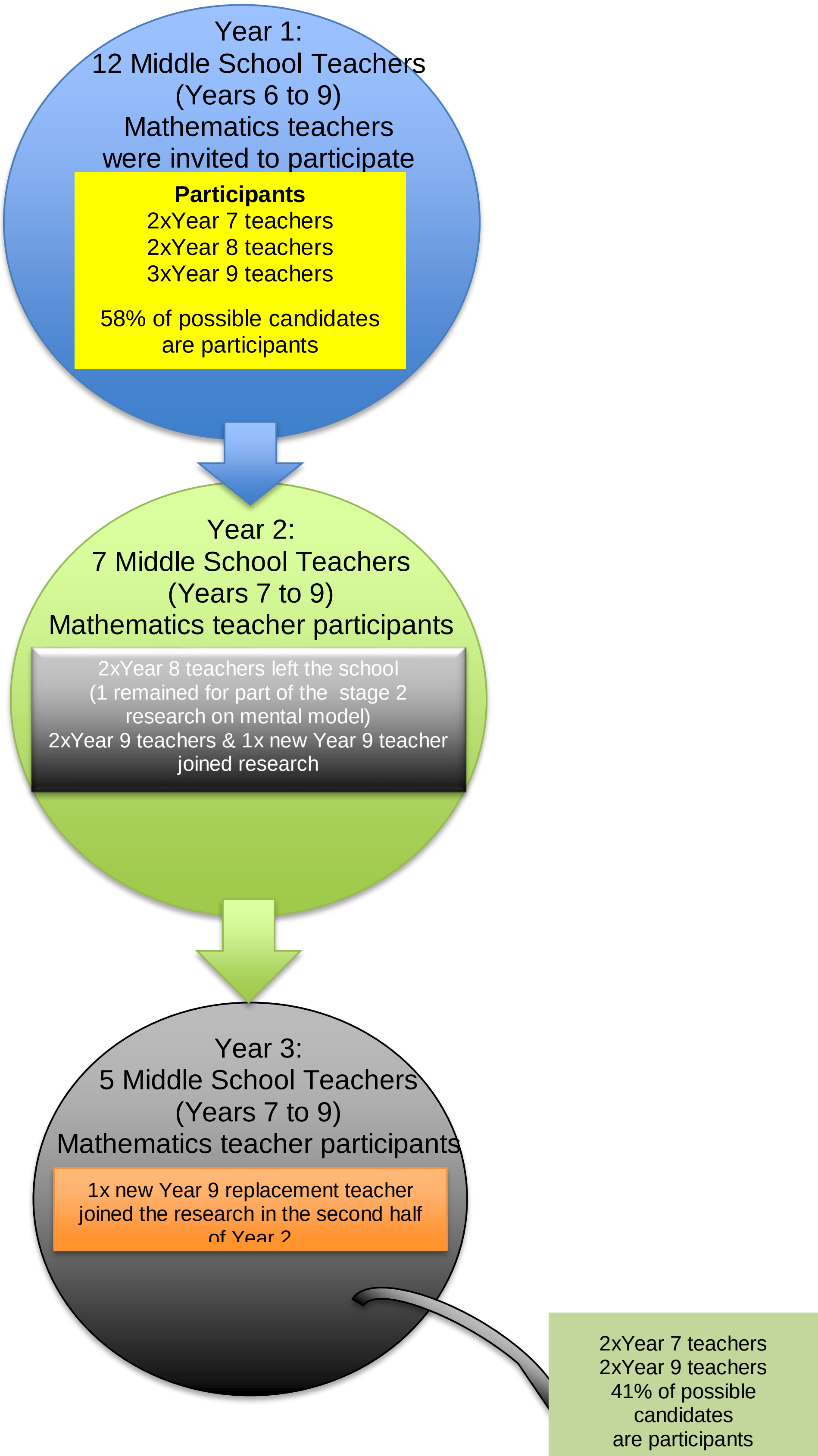


Figure 3.1 Qualitative research sample size

3.6.1.1 Team Planning Sessions – Collaborative Inquiry Sessions

The school year typically ran for 38 weeks, splitting into four terms with each term lasting around nine to 10 weeks. The year level mathematics teachers also met on a weekly basis as scheduled in their timetables, with the exception that the planning times sometimes fell on public holidays or staff in-service days, year level camps or excursion days, or teachers were absent due to external professional learning opportunities or illnesses. In addition, due to teacher workloads during heavy assessment and reporting periods, consensus was often reached to cancel the planning meetings. Another factor that affected the recording of the planning sessions was the clashing of my commitments and the planning meetings, as I was a full time employee at the research site. This deficiency in the data collection plan was compensated for by reviewing the meeting schedule to create a more realistic schedule while still adhering to the intention of collecting enough data for analysis. The reality of the school scheduling meant that such disruptions were unavoidable, but the sample size still was large.

Consequently, the original plan of recording and analysing approximately four planning sessions a term was significantly reduced to approximately two planning sessions per term, resulting in six recorded sessions for Year 7, five recorded sessions for Year 8, and eight recorded sessions for Year 9 in the first year of this research. As there were three year levels involved in this research during its first year, a total of 19 sessions of approximately 30-minute discussion were recorded for analysis.

The number of recorded sessions increased in the second year of the research even though there were no teachers participating in this research who were teaching in Year 8. Therefore, there were two year levels involved in this research during the second year of this research. There were nine recorded sessions for Year Seven and 15 recorded sessions for Year Nine. Hence, there were 24 recorded sessions for analysis, each lasting approximately 30 minutes.

These meetings were scheduled weekly as part of the teacher timetable commitments. However, these meetings took place during preparation time of the teachers, not during additional time allocated for planning. The teachers were scheduled to meet for 50 minutes each week whenever possible during the 38 weeks of each academic year.

The records of the collaborative inquiry group discussions were analysed to track the trajectory of the teachers' change of mindset, understandings about pedagogical change and degree of implementation of the effective pedagogies and, overall, of the effectiveness of collaborative inquiry as a professional learning tool.

I participated as the facilitator and an observer at the fortnightly collaborative inquiry session, keeping observation notes for each participant about their journey of pedagogical understanding through noting their participation in the discussions.

All collaborative inquiry sessions over the 34-week pilot period were recorded for analysis of the teachers' style of questions, depth of understanding and attitude/mindset change.

However, in reality, the opportunities for meetings dropped significantly due to (a) teacher absence because of illness or external professional development or attendance at camps with students; (b) planning days falling on public holidays or staff in-service days when special programs were planned for teachers and (c) periods of heavy assessment setting, marking and reporting each term. In effect, the number of these meetings was significantly reduced to approximately half of the time available.

3.6.1.2 Student Focus Group Sessions

A student focus group conducted for each of the year levels involved in this research was recorded once. The focus group questions and field note templates are found in Appendix E.

Two student focus groups were conducted for Years 7 and 9 students at the end of the first year of the research data collection. The data collected from these two student focus groups were used to triangulate the data collected regarding the preferred mental models, beliefs and pedagogical practices of teachers, in order to engender an optimal learning environment for students and the most effective pedagogical practice from the students' perspective. Student focus groups were only conducted in the first year of this research.

3.6.1.3 Teacher Surveys

A survey (see appendix D) based on Olsen and Bruner's (1996) classification of learner minds and the work of Van Dresar (1996) was sent to the teacher participants via email, to ascertain their mental models and beliefs about teaching and learning of mathematics, professional development and pedagogy. The teacher participants' backgrounds and profiles aimed to provide some insights into their perceptions on the above listed aspects of this study.

The teacher survey aimed to ascertain and triangulate the theory developed from data collected during the collaborative inquiry sessions over the two years. The survey was deemed appropriate in that it was designed to gather how the mental models and beliefs of teachers can impact on their effectiveness as teachers. Their beliefs were formed through their experience as students and training to be teachers. These beliefs, even with very similar mathematical knowledge, may result in these teachers adopting very different pedagogical practices (Clarke et al., 2014).

i. Development of Survey

The development of the survey was accomplished through the literature review of teachers' beliefs about mathematics learning, teaching and achievement as well as the pedagogical practice of these teachers relating to their beliefs, as outlined in Section 2.9 and Section 3.6.1.2.2 (Clarke et al., 2014; Entwistle & Peterson, 2004; Olsen & Bruner, 1996; Strauss, 2001; Van Dresar, 1996).

Furthermore, the teachers' experience, preferences and beliefs about the values and modes of professional learning were also surveyed to triangulate the comments made during the collaborative inquiry sessions, as well as their one-one interviews in phase three of the research.

ii. Survey Questionnaire Procedure

The teachers were emailed the survey which was posted online using Survey Monkey website. I was the only person able to access the survey results. The teachers were also reminded about completing the surveys. All teachers were invited to complete the survey and all of them responded to the survey.

Table 3.2 summarises the educational and teaching backgrounds of teacher participants. This table provides the contextual background of the teachers involved in this study. At the time to the research, only one teacher participant was a developing teacher with under five years' teaching experience, two teachers had experience between five to ten years, four teachers had experience between 11 to 15 years and only one teacher had more than 15 years' experience. Furthermore, only three participants are trained in mathematics teaching. The length of time that the teacher participants working at the study site provides contextual knowledge about the possible site culture in mathematics teaching.

Table 3.2 Training and experience background of teacher participants

Teacher Years at school (de-identified using different coding system)	Professional Training	Educational Training	Years in teaching	Years at the school
A Year 1 - 3	Environmental science degree	Middle schooling education	11 - 15 years	6 - 10 years
B Year 1 - 3	NIL	Primary education	6 - 10 years	6 - 10 years
C Year 1 - 2	Information technology	Mathematics education	15 - 20 years	6 - 10 years
D Year 1	Science	Science education	11 - 15 years	6 - 10 years
E Year 1	Engineering	Science education	6 - 10 years	6 - 10 years
F Year 1 - 3	NIL	Mathematics education degree	0 - 5 years	0 - 5 years
G Year 1 - 2	Exercise science	Health and Physical Education & Mathematics education	11 - 15 years	11 - 15 years
H Year 2 - 3	Exercise Science	Health and Physical Education & Mathematics education	11 - 15 years	0 - 5 years

3.6.1.4 Individual Participating Teacher Interviews

The teachers were interviewed individually twice in this phase. The two interviews were conducted six months apart; the first interview took place in June and the second interview took place in December of the same year. The purpose of the six months' spacing between the two interviews for the same teachers was to provide the teachers with six months to reflect on their practice and observe the value of the collaborative inquiry.

The interview questions are listed in Appendices F and G. The first interview aimed to solicit the teachers' views about professional learning, student learning and achievement. The second interview aimed to clarify the teachers' mental models and beliefs about mathematics teaching, learning and pedagogical practices. The interviews were recorded. The transcripts were sent to the teachers for their approval - whether they were true reflections of the discussions - before coding, categorising and theoretical sampling were completed. The interview data were used to determine the effect of teachers' involvement in a collaborative inquiry approach to the impact of professional learning on the teachers' confidence and competence in planning and teaching through the use of effective pedagogies. The interviews allowed the teachers to provide individual responses, which enabled the teachers to express their opinions without inhibitions.

3.6.1.4.1 Appropriateness of interviewing for the research

The intention of the phase three staff interviews was to hear first-hand the teachers' view about collaborative inquiry and how it can be used to enhance their professional practice and growth for further development. It also sought feedback from the teachers about their views of the model based on stage one collaborative inquiry data.

The staff interviews were used to triangulate the phase one data and clarify and remove any bias that I, as the researcher and a senior staff member of the research site, might have.

3.6.1.4.2 Development of the interview protocol

The participating teachers were sent the invitation to be interviewed, including the purpose of the interview, date, time and venue. Handwritten field notes were taken and interviews recorded on digital recorder.

At the interviews, the introductory scripts were always read to the participating teacher. However, based on the responses of the participating teachers, I asked questions to further prompt and advance in-depth discussion of the topics on hand, while ensuring that the purposes and intents set out in 3.6.1.3 were closely adhered to.

3.6.1.4.3 Interview bias.

The interviewer's bias was managed by using triangulation of data collected at the interviews with the other phases of data collection.

3.6.2 Data Analysis

Grounded theory research methodology calls for continuous and specific information collection and information analysis (Hoy, 2009). Using a grounded theory approach allows information collection and information analysis, continuously directing and redirecting the focus of the study to accurately determine how the educators' perceptions are constructed (Hoy, 2009).

Grounded theory research typically uses coding assisted by memo-writing and diagrams to generate concepts, ideas and insights into the data collected so as to ground the theory.

3.6.2.1 Coding

Coding is used in grounded theory research to analyse the data so that the theory developed is grounded in the data; therefore coding is a core element of the development of a theory (Vollstedt, 2015). The methodology requires that the data are continuously categorised and compared in an analytical fashion with attention to the relationships between phenomena observed through the data analysis process (Vollstedt, 2015).

There are typically three stages of coding: open coding, axial coding and selective coding.

Stage 1: Open coding is a process by which the recorded data are analysed by a two-fold intent of asking questions of the data and constantly comparing the incidents as revealed by the data (Charmaz, 2009; Teppo, 2015). The aim of this coding is to generate a concept from the data so as to generate a phenomenon coding (Charmaz, 2009; Oktay, 2012).

I reviewed all the data from all sources inclusive of field notes, transcriptions of collaborative inquiry sessions, focus groups and interviews, and applied the open coding process by creating categories to describe the phenomena, using the sensitising approach of deciphering the intents and purposes of the participants as they responded in discussions, focus groups and interviews by asking questions.

Stage 2: Axial coding is the state of coding during which one category of the coded data is analysed intensely within the paradigm of the items of this category, such as the conditions and consequences of the incident (Teppo, 2015).

I applied axial coding within each category by examining the context during which the data were presented so as to create an axial line of inquiry to interrogate the data in greater depth.

Stage 3: Selective coding is the stage at which a core category of data is ordered and linked to other categories with expressed relationships to generate and integrate into a theory (Teppo, 2015). Corbin and Strauss (2008) suggest the development of a narrative in order that the researcher can integrate the emerging categories into a flow of ideas as the building blocks of the emerging theory.

During this process, I constantly compared the categories from the axial coding stage to further generate a pattern and narrative of the emerging theory. The data were like pieces in a box of jigsaw puzzle; I needed to start looking at how each piece of data fitted in the categories and how the categories were joined to form the full picture of the theory hidden within the pieces of data.

3.6.2.2 Memo-writing

I adopted Oktay's (2012) recommendation for me to write a "researcher identity memo" originally suggested by Maxwell (2005). These memos helped me to reflect on how my own background as mathematics teacher, and now researcher of mathematics education, shaped my thinking about this research and the data collected. This process also assisted me to remind myself of potential personal bias during the analytic process (Dey, 2004; Lempert, 2007).

During the analytic process, I wrote memos to document the date on which codes were assigned to the associated thoughts, insights and reflections; the coding process provided a rich narrative to assist with the capturing of ideas emerging from the codes and categories, and established the relationships, including the conflicts and contradictions, amongst the concepts which shaped the emerging theory (Stern & Porr, 2011).

These memos created an audit trail of the development of trends and thinking to enable the theoretical sampling to be verified and triangulated. Furthermore, diagrams representing concept mappings to illustrate the theory also assisted with the analysis (Teppo, 2015).

3.6.2.3 Theoretical Sampling and Saturation

Theoretical sampling is achieved by integrating and relating the memos in an organised fashion. However, the sampling strategy is not predetermined, to allow the data coding, categories, concepts and ideas to generate the strategy (Poekert, 2012).

Once the analysis of data did not result in a new category, I knew that the data had reached theoretical saturation; hence, the emergence of the theory grounded by the data (Glaser, 2007; Oktay, 2012).

3.6.2.4 Timeline

This research data collection spanned three years. Data collection started with three phases of data collection, as outlined in Section 3.6.1. The data analysis of phase one data was conducted when phase two data were collected. Phase two data were analysed when phase three data were collected. Hence, the data collection in phase three was influenced slightly by phase one data analysis, as the coding and categorising of data provided me with glimpses of possible theory constructions. However, as the data had not been fully analysed and categories drawn until the latter part of phase three data collection, the influence and “bias” were minimal and deemed inconsequential.

3.7 Ethical Consideration

3.7.1 Researcher-Participant Relationship

The unequal power relationship between the participating teachers and me must be taken into account in this research, because I was a senior executive member of the school leadership team, giving me the position of power; furthermore, the teachers' work schedules were assigned by me. I was also a Year 9 Mathematics teacher in the year preceding the commencement of this research; hence, I had multiple roles in this study. These roles included: a participant whose growth trajectory would be tracked; a member of the collaborative inquiry group who contributed actively to the discussions; a facilitator of the collaborative inquiry group; and an observer of the collaborative group discussions.

To ensure a more equitable relationship with the teachers, informed consent and open communication with the teachers were sent after their classes for the following year were allocated and finalised. The informed consent and communication were managed by the then Associate Dean of Curriculum & Learning who was a critical friend and associate for this research. The teachers were assured that there would be no repercussion for not participating in the research, and that the data collected would be kept strictly confidential and used for the research study only.

3.7.2 Information and Informed Consent

The support and approval of the Head of College, the gatekeeper of the research site, was first sought. All teachers were provided with information about the nature of the research methodology, its purpose, any risks and benefits to the participants, possible outcomes of the resarch, and the exercise of voluntary choice to participate. Specifically, they were made aware that they were free to withdraw from the research at any time without prejudice or negative consequences. Observation logs, interview notes, sound files and digital tapes of the collaborative inquiry meetings and classroom teaching would not be used for any purpose other than this research. This point was made clear to the teachers to allay any fear of being judged by me, a staff member in a position of authority.

Parents and students were informed of this research and informed consent forms were distributed to the parents. Any parents who did not wish their students to participate in this research were not required to complete any part of the data collection for students.

3.7.3 Consideration

During data collection, the teachers experienced minimum disruption to their normal teaching and learning program. Interviews, group meetings and digital records were spread across the three year period, and took approximately one hour of their allocated time release per week for facilitated group discussions, which also included professional reading and reflection. Teachers' lessons were recorded for the sole purpose of this research and, once the footages were shown for the collaborative inquiry sessions for critique and discussions, they were deleted from the device and the recorded data were destroyed.

3.7.4 Anonymity and Confidentiality

All teachers were guaranteed confidentiality and anomymity. All interviews, observation notes and digial recordings were coded for the thesis to ensure confidentiality. Anomymity in the final thesis and any publications that might result from the study were achieved through the use of pseudonyms. Access to data gathered during the research was guaranteed privacy and was only available to my supervisor and myself.

3.7.5 Acknowledgement

My critical friend, Associate Dean of Curriculum and Learning and the teachers involved in the research were given the option as to whether they wished to be acknowledge as having taken part in this research. All the staff who were invovled in this reaserch as critical friend or participants gladly accepted to be listed in my acknowledgement in general terms as a college, but not expressly by names.

3.8 Chapter Summary

This study used the grounded theory approach to analyse the data collected over a three-year period through various data collection methods such as collaborative inquiry sessions, student focus group sessions, teacher survey and individual interviews. The different types of data were used to triangulate the data collected to ensure the validity and reliability of the research while removing the bias of the researcher.

Coding, memo-writing, theoretical sampling and theoretical saturation were the process by which the data were distilled and funnelled to generate a model to represent the narrative existing in the data. Ethical considerations were explored and adherence to protocol was achieved to maintain the anonymity and confidentiality of the participating teachers and students.

CHAPTER FOUR

PHASE ONE DATA ANALYSIS AND FINDINGS

4.1 Introduction

In the previous chapter, the research methods and plan were discussed including the different phases and types of data collected. The data analysis method using the Grounded Theory approach was elaborated. In this chapter, the collected data, which had been transcribed, were analysed using coding, memo writing, theory sampling and theory saturation to develop a model to illustrate the emerging theory for the topic under study.

4.2 Data Analysis

The data were analysed year by year or phase by phase before the subsequent analysed data and findings were used for triangulating, theory sampling and theory saturation. This chapter outlines the analysis of the data collected at team planning sessions (i.e. collaborative inquiry sessions) and how the claims made were supported by the student focus group data and analysis. The findings of phase 1 data analysis are presented under the headings of benefits, limitation and implementation of collaborative inquiry as a professional learning tool for middle school mathematics teachers.

4.2.1 Benefits of Collaborative Inquiry as a Professional Learning Tool

4.2.1.1 Collaborative Inquiry Elicited Pedagogical Strategies

4.2.1.1.1 Sharing the use real life problems

At a session during which the teachers discussed the challenges of helping students understand the relationships between discount and mark-up, profit and loss, using their existing knowledge of percentage, teacher T5 shared how he used a real life experience of making purchases at a local flea market to engage the students in learning these relationships:

> *What I did was told them a story about buying shirts at the Carrara market and selling them on eBay at a marked up [price].* T5

This real life example was familiar to the middle school students in his class because this market was situated in the local area near the school where the students and their parents would have visited sometime in their past experience. It was generally deemed a relevant example to the students because they are familiar with the worldwide online buying and selling website, eBay, and hence the example engaged the students because this real life example was meaningful to them. Teacher T5 shared that once the students' interest and attention were captured by this familiar and relevant real life example, he then brought in the concept of percentage changes to link to the new mathematical relationships to be taught (i.e. discount and mark-up, profit and loss):

Then I did a little percentage on one side of increase and decrease. T5

While discussing with another team of teachers about how the national testing, NAPLAN, seemed to shift teaching of mathematics to test preparation by rote learning rather than students developing deeper understanding in the concepts taught, teacher T2 shared a pedagogical approach learnt at a recent workshop that she attended about the importance of mathematics hands-on activities to engage students in learning, including using familiar and interesting real life examples. Teacher T2 shared how using the well-known dance routines made popular by Korean pop singer Psy helped students to link concrete patterns to the more abstract concepts in algebra:

> *.....you know the Korean Psy dance you know the dance he does, it's a pattern. Make them watch it, make them write down mathematically what he's doing.* T2

Teacher T2 prefaced the pedagogical strategy of linking Psy's dance routine and pattern to algebraic concept by discussing the importance of real life activities to engage the students, in particular, when relating to more abstract mathematics concept such as algebra:

*That model, the Raymar model, is brilliant where you start off where you engage them with real life activities that relate to the unit so I always do this with algebra and patterns. We always go out and draw patterns. They have to do man made ones and find some ones,....*T2

Three teachers were exploring and deciding between two real life problems to be used for an investigative assessment and classroom task for the students. These tasks were chair stacking and Bungy Barbie. It was ascertained by the teachers that they were both suitable for developing their understanding of mathematical modelling involving algebraic expression. Teacher T3 proposed a classroom activity, Bungy Barbie (i.e. the use of the Barbie doll on elastic string to simulate bungy jumping), which was an activity involving real life with which all students understand and are familiar; consequently the task would be meaningful to the students. Furthermore, this task would build their understanding to tackle the chair stacking task:

> *But I think kids should be able to accept that knowledge and build on that knowledge cos I think Bungy Barbie now is very similar to this. They'll get that; they'll have to get that because that's what the question says to do, make that table*. T3

This team of teachers was deliberating and finalising the approach to an assessment which required the students to generate an algorithm and algebraic expression to generalise the strategy to solve a typical school problem of stacking away chairs after a school assembly. This was another real life example that was both familiar and relevant to students in that the stacking and storage of chairs after assembly were very common to schools and occurred weekly. Teacher T3 proposed to make this task more accessible to the students by offering scaffolding such as taking photographs and recording relevant dimensions at each stage while not making the strategy overt to the students and making the assessment open-ended:

> *Take a photo of a stackable chair, you will need to have this in your report, record the dimensions. Then it says here, use a mathematical model. Stack two chairs together and record the height. Continue to add chairs one at a time until you have a stack of five chairs. Measure and record the height of the stack after you have added each chair. In the space below, record your results in a table. So I think that's scaffold without actually having to draw the table and fill the table in because kids need to be able to come up with what their table would be. The only thing we need do is add some graph paper*. T3

The examples are a small selection to represent the extensive evidence available in the data collected that the collaborative inquiry sessions provided teachers with organised forums through which they could share real life problem ideas, which are both familiar and relevant, for class activities.

4.2.1.1.2 Making problems and learning visible

When discussing the reasonableness of solutions to mathematical problems, teacher T4 shared examples of the students' apparent ignorance about grossly erroneous and unreasonable answers, such as:

> *They don't understand the difference between 0.9 and 1.1. They didn't realise that 0.9 was a decrease and 1.1 was an increase. It's just a number*. T4

Teachers T7 shared her experience, strategy and views on the topic which evoked an in-depth discussion of how to develop deep conceptual understanding with students:

> *In NAPLAN there's those questions where you're dividing by a decimal or multiplying by a decimal — what does that actually mean. They have to do it without the calculator. Even if you convert it to a fraction, here's your decimal and here's your fraction. They should be able to multiply by a fraction.* T7

Teach T7 shared a strategy with the team how to use a visual representation to illustrate a problem in order to develop strategies for solution:

> *I'd look at it from the point of view that if you give them a question, how can they visually represent what's happening with whatever – drawings or equipment.* T7

At another collaborative inquiry session, teacher T7 demonstrated a strategy using software, an excel spreadsheet, to help students to visualise and develop better understanding of the concepts of gradient, slope, co-ordinates of mid-point and distance of a line segment between two points on the Cartesian plane:

> *We did all the Excel work before I gave a lesson on slope so they got a good idea. We did midpoint, what else do we have to do, distance. Yes we did distance because that tied in with Pythagoras. That was the first lesson. Then,*

we don't have time to watch, but on the right hand side we had the Excel screen up. I did a direct referencing cell in there, two direct referencing cells where I could change the slope and inset. Get kids to number round the class and put that in and it goes up and down whatever they said. The slope would go up and down like that. T7

Teacher T7's demonstration above provided the teachers at the session with a strategy by which abstract mathematical concepts could be made visible for students who were challenged by developing spatial awareness and understanding in their heads alone.

During the collaborative inquiry sessions, the teachers shared past successful contextual and problem-based approaches that ensured deep conceptual development incorporating higher order thinking. For example, an activity involving Fibonacci numbers and Pascal's triangle was a typical example:

…we did a whole thing on Fibonacci for the extension kids so that ties straight into nature. T1

It does because there's that great little video about nature by number which is brilliant and they watch the (sea) shell and how it comes together. T2

We do Pascal's and we do Fibonacci. We went into Pascal's and then the basic you know we do the machines, the patterning machines. T2

Teachers often explored different ways of deepening students' understanding of challenging concepts during collaborative inquiry sessions because these professional dialogues provided the avenue for teachers to engage in professional dialogues in situ which were both relevant and timely for their teaching duties.

Below are quotes giving another example of shared practice about the teaching of gradient and making it practical and visible by using an iPad application to measure angles of elevation and depression. Teachers T5 and T6 shared how learning can be made visible by taking actual measurements around the school of handrails, roof or other objects:

But we deal with level in Maths A, you probably do it NAME [teacher X], where we calculated, well it's not the gradient but effectively that same thing, *like the handrails and the roof or whatever it happens to be like actually getting them to go out and measure different things.* T6

The board goes so it's equal to delta y and delta x so you find it's equal to tan theta and we have to go to the computer room. What I could have done then is just get any kid to put a level on their iPad, that's what I do for Physics, and just get a level and stick it on there and they can read the angle straight off there and we could do rise on the run like that. T5

It could be like a class activity where we ask how can we measure the gradient of that. Get them to come up with ideas and get them to predict, if I change, I don't know what language I use, if I lift if up, what do you predict the gradient will look like. T6

In the following example, teacher T7 was using coloured tiles to demonstrate distributive law in algebra in order to make the abstract concept visible and accessible to the students. She also demonstrated on the whiteboard in the meeting room how the distributive law can be made easy for the students if set up as shown by her to the teachers on her team:

When you come to doing the distributive law and so forth, I think I've got it on the planner using area and that sort of stuff. So, if it's like x + 3, you'd get them to write, can you just do x(x+3), something like that, and then I'll set up the area so we're going to have x down one side and the x+3 across the top. So you've got your x and you've got to fill in the little spots. Have you used that? So I'm going to get them to use this with no algebra and then use the algebra. T7

These examples above illustrate how the teachers shared strategies that make learning challenging and difficult mathematical concepts visible by using software and video clips to illustrate the more complex concepts using visual stimuli. Furthermore, collaborative inquiry provides the opportunity for teachers to share their pedagogical practice for mutual benefits.

4.2.1.1.3 Scaffolding exercise to develop conceptual understanding

Teachers shared scaffolding practices and ideas to assist students to develop deep conceptual understanding from concrete to abstract. Teachers T1 and T2, when prompted by the facilitator, illustrated this point in their sharing at one of the sessions:

So how do you use those tiles and paddle pop sticks to introduce the relationships, the XY relationship, the linear equation, is that how you do that? F

Yeah, well we set up the pattern and the squares. T1

…and the X and the Y and the data so that they collect it in the table and then they're able to know that the X and the Y is what we have to graph. T1

…and then it leads straight into algebra and it says here, too, from authentic data you've got to be able to interpret and analyse graphs. So that's what we've got to look at, even NAPLAN style questions that are on graphing. Probably just take them from there and make our own little powerpoint. T2

From the quote above, it can be seen that teacher T1 used coloured plastic tiles and paddle pop sticks to construct concrete patterns to help the students to visualise the relationship between the X and Y variables and translate the abstract algebraic expression in concrete representation. Furthermore, the additional step of tabulation of the X and Y variables in columns also helped the students to easily recognise the pattern and relationship between the variables before representing the algebraic relationships in graph.

During another collaborative inquiry session involving teachers T1 and T2, as illustrated in the dialogue below, they were discussing how to provide a scaffold to teach the students how to deconstruct a complex geometrical shape in a problematized situation in a new geometry unit of work:

What's the suggestion, do I just let them make a few patterns first because I was thinking I could do that and try to get them to draw them and write down what they've done and see if they can work out a pattern and then I thought I'd go for the triangle then how we work out the third turn and fourth turn. T2

See I'm just looking at my task for Friday, for Year 8, and I'm going to do a lot of scaffolding to get the kids to know actually what the CCEs [core curriculum elements] are. For the first, if you just put it up there and say, *here's a triangle. How many do eight and ten make, I don't think they would get it.* T1

That's what I mean. I'd work through that one and then it'd be make a little house shape and then work out …. T2

Teacher T1 emphasised, in the dialogue above, the need to provide scaffolding for students to assist them to access the learning of more complex ideas and that just merely telling them without the deconstruction steps would not work. Teacher T2 also offered that students needed to be led from the known (e.g. triangle) to more complex house shape. By house shape, she referred to a drawing of a house which might be a composite shape consisting of squares, triangles and rectangles. Scaffolding was required for students to develop understanding; however, scaffolding for students should not be a "crutch" without which they could not proceed. Teacher T1 shared her intention to use scaffolding to teach the core curriculum elements (CCE) through the concepts of triangles.

In the example below, teacher T1 shared the strategy that she used through a webpage that she created on Learnist to provide the required scaffolding for the students who were being extended beyond the core curriculum, while teacher T2 provided another example of how to assist students to see the link between rectangles and triangles. This example further illustrates that, through collaborative inquiry, teachers are able to share scaffolding ideas to help students to master more complex or advance concepts:

Circles they don't do till Year 8 and I've already got a couple of my extension kids doing it through Learnist (i.e. a website collating all the relevant webpages of the same themed resources) already doing trapezes and rhombuses which isn't till Year 8 as well. And that ties in. Their assignment is designing a house and we give them different areas and they've got to work out the dimensions and then they research how much paint costs and it turns into … T1

There's an activity about triangles where they draw a rectangle and they cut it in half and show that it's both a triangle and a rectangle. T2

Teacher T6, in another team, shared a scaffolding strategy that he used to assist his students to help them to understand the concept behind the Cartesian plane (e.g. left to right) and the convention commonly used to work out the distance between two points. This scaffolding strategy, according to him, steered them away from merely remembering the rules or rote learning that they used while their knowledge was still in its infancy. Furthermore, the students would develop the rule through understanding rather than rote learning by memory without understanding:

> *Yes, we spoke about reading from left to right. We read our book from left to right, that's how we read our graph. If we're going from left to right, the line's going down so that's a vertical step going down. Like we always did the vertical step first, because that's the numerator. We always spoke about that and they developed the Y2 take Y1 over X2 take X1 because they'd done that with midpoint and distance so they were familiar with that, the naming process and coming up with the rule.* T6

The extracts above from the collaborative inquiry sessions illustrate that teachers shared scaffolding strategies to assist students to transit from concrete to abstract learning stages as well as developing deeper conceptual understanding.

When students were challenged with the idea that not every mathematics lesson could be fun, and asked how a teacher could cater for the needs of the students to maintain their engagement, one student responded that "the teacher can cater to different learners by using learning activities and tools to cater to those who favour learning verbally and visually, using a variety of approaches." Another said "Ms X makes us work hard some days and other days are a little relaxed," and another said "Mrs X – some days we did one period of non-stop work; then we played maths game." Another suggested "change of pace – sometimes" suggesting that the variation of pace and activities to cater for various learning styles would enhance students' engagement.

4.2.1.1.4 Memo: Collaborative inquiry elicited pedagogical strategies

It was evident from the collaborative inquiry sessions in Year 1 of the research that, in these sessions, through structured professional dialogues and exchanges, teachers brainstormed and shared teaching strategies with the aims of engaging students in learning and developing deep understanding in the students.

Pedagogical strategies elicited included the use of real life programs to interest and engage students because they are familiar and relevant to them; making problems and learning visible to the students so that they are able to access the learning; and scaffolding activities to develop conceptual understanding of more complex mathematical concepts. These pedagogical strategies elicited from the collaborative inquiry sessions were deemed effective by the teachers who shared them in these collaborative inquiry sessions as pedagogical practices tried and tested by themselves to create a productive learning environment for the students to develop stronger engagement and deeper conceptual understanding. Collaborative inquiry sessions could therefore be seen as structured professional sharing dialogues through which teachers were given the freedom to share their ideas and practice for collective benefit and professional growth.

4.2.1.2 Collaborative Inquiry Aided Authentic Assessment Development

4.2.1.2.1 Developing collaborative insights into authentic assessment of students.

Collaborative insights could be gathered through the collaborative inquiry session; in this case, the inquiry was "what are the best ways to assess students?". Two teachers discussed how to better assess students. I, the facilitator (F), prompted the teachers by questioning the suitability of a particular assessment question designed by the teachers to assess the students' higher order thinking:

> *I just wondered things like, um, Question Ten, right, we used "wh" word such as "who", "what, "why", "where", "when", ...e.t.c.... I wondered whether you want to, I don't know what you expect in an answer because its Standard A, like state the positive. Do you just want them to state or do they actually have to ....."* F

The two teachers discussed how to ensure that they developed common understanding of the requirements of the assessment task before administering it, and making a commitment to moderate through a consistent teacher judgement (CTJ) process, as quoted below:

> *That's why I want us to look at this today because this got taken straight from that task.* (in the context of a CTJ session) T1

…and that's what we do when we mark it. If a child's answer like xxx [name withheld] and I obviously do CTJ and if they answer part of that we would. T2

Collective insight can be generated through teachers' professional discussions to reach common understanding and interpretation of expected student outcomes.

4.2.1.2.2 Using appropriate differentiation to incorporate higher order thinking elements in assessments

In the example below, the teachers discussed the implications of scaffolding in assessment design and how to incorporate scaffolding within a complex assessment task without compromising the integrity of the task. Teacher T2 expressed that some clarity or scaffolding might be needed (e.g. "drop a hint") to make the task more accessible. This suggested that teacher T2 was not sure if more scaffolding would compromise the integrity of the task. However, with the inputs from teacher T1, based on the fact that it was a task to be completed at home with the prospect of parental inputs, the teachers reached the compromise, noting that additional step was taken to ensure that the in-school portion of the assessment was designed to differentiate the students who could demonstrate higher order thinking (i.e. "the one we set in school is going to be a lot more strict" and "therefore it's going to even it out anyway"):

If we're concerned that the kids won't get the ratio thing right, do we drop a hint? T2

Well this is a home one and they're going to get help so I'd rather give a hint in this one so we don't get parents getting all antsy that it's too hard. Give a hint in this one and then at least the one we set in school is going to be a lot more strict therefore it's going to even it out anyway. I don't mean a hint for an A standard, I just mean a clearer wording of what we want to see. T1

But it is complex, unfamiliar. As soon as we drop something in, how much of that complexity and unfamiliarity is there. T2

Such dialogues about complexity and the assessment of higher order thinking were often challenging to the teachers participating in the collaborative inquiry, in that there was a conflict between having high expectations for the students but fearing that they might not be successful at challenging questions.

The example below illustrated that, when the teachers planned at their collaborative inquiry sessions, decisions were always made to assess from simple recall (i.e. "basic knowledge") to higher order thinking skills to provide appropriate differentiation:

> *It's like a real world problem so this one's got you know, you've been picked to be on a crew and you're creating a package for there so you're doing it for a tour using Dreamworld, Seaworld maps and you've got to plot the coordinates for them to go. Write down all the coordinates of how they would enter the park and how they would get to these places by the time they leave the park. Include stops at venues. So they're putting an itinerary together but using coordinates and a Cartesian plane. So it's a real world activity and it's certainly showing some of those higher order thinking but we're going to see their basic knowledge too because if they get those coordinates wrong they are going to end up in the sea or (muted).* T2

4.2.1.2.3 Sharing strategies to address identified students' weakness in learning

During collaborative inquiry, teachers were able to share their diagnosis of students' challenges in mastering certain mathematics concepts by soliciting ideas and suggestions from fellow colleagues to assist the students. This was a powerful approach, as the students were all on the same site with a similar set of external (state and commonwealth legislative requirements) and internal (school) constraints.

Sharing of ideas or challenging one another to explore alternative means by which to teach better was a noble goal for teachers. Teacher T3 shared in the extract below that he used emails as a means to help students who lack confidence to raise questions in class:

> *I'm getting more kids email me about things as their means of communication and from kids who normally wouldn't ask me questions in class..* T3

During another collaborative inquiry session, T7 shared her analysis of the common weaknesses of the students in that year level to assist with the diagnosis of areas for improvement for the students. She went on to remind and challenge the teachers to use mathematical equipment as an effective strategy to teach through investigation:

> *Ratio, measurement, money and algebra. They were the four weakest things.* T7

> *Yes, so it's like an investigation they can work through. Probability is the next one. We're able to do lots of activities with that. There's so much (manipulative equipment) in the storage cupboard. Have you been there, XX (name of teacher withheld)?* T7

Another example of teachers sharing tested and proven strategies to develop deeper understandings of concepts occurred when a non-participating teacher in this research expressed her dismay at her students' struggle to comprehend the concept of 'discount and mark-up' during a collaborative inquiry session. Teacher T4 shared a strategy which was affirmed by teacher T7. Even though the strategy suggested by teacher T4 was a "legitimate strategy for those who are struggling", teacher T4 confirmed that "a lot of them (i.e. students) still want to do discount the slow way". During collaborative inquiry, teachers were working to strategise how to assist the students needing additional assistance:

> *I have to do it with the block of cheese like with my Maths A (students). If the block of cheese is 100% and I take away 10%, how much of my block of cheese is left? They can see that's 90% but now I have my block of cheese plus the extra bit of cheese on it. Now what do I have; oh, it's 120%. So how do I figure out what 100% is, that's when they'll struggle.* S6 [none participating teacher]

> *So isn't that their concept of fractions?* F

> *Yes, whole numbers and part of it; seeing if I take out a portion of it, what's left. A lot of them still want to do discount the slow way; find 10% and add it then take away.* T4

> *That's still a legitimate strategy for those who are struggling but.... You can use some of those resources for the same sort of concept.* T7

4.2.1.2.4 Memo: Collaborative inquiry aided authentic assessment development

The teachers at the same site were confronted with the same set of circumstances, such as student profile, school culture, curriculum emphasis and demands, …etc., and established professional relationships, were able to co-construct and develop authentic assessment tools to support and assess the students' learning and progress.

Collaborative inquiry provided a wonderful platform for the teachers at this site to engage in robust dialogues about students' learning challenges, thereby developing insights into appropriate differentiated activities containing various degrees of high order thinking elements to further enhance the students' learning. The exchanges of these deep discussions were made possible by the collaborative inquiry sessions.

4.2.1.3 Collaborative Inquiry Challenged Dispositions about Pedagogical Approach

During the collaborative inquiry, I was able to probe the teachers about their pedagogical approach, as evident below in a discussion about students engaging in deep learning as opposed to engagement in learning. The challenge as presented by teacher T6 was often the students not seeing the relevance of the learning experience and, hence, lacking in commitment on the students' participation:

> *Back to the topic about deep learning, we know, as teachers, that things spiral. We come back and we build on and we build on. How do we get the kids to learn deeply without them just going through the motions?* F
>
> *They don't see the point of it. They can't see beyond next week or tomorrow or lunch.* T6
>
> *They can't see the relevance. Because a lot of this maths, a lot of kids at this age are going to be, well, when am I going to use it and you can't explain when you're going to use it.* T6

In another example, when I tabled the topic of how to improve student motivation for learning so that they were engaged in their learning of mathematics, it challenged the teachers to think about their pedagogical approach in this regard.

Teacher T5 was able to share a professional learning experience of being challenged by a professional learning presenter to motivate students, and feeling challenged with the new knowledge about his own pedagogical practice:

> *That's obviously the fact that it's life. We all come and work, the pay cheque is probably a good reward but the flip side is we would rather work with people who are passionate about the subject matter and teaching because it's easier to work with that. The same with the kids. If they are enjoying their learning and applying themselves obviously it's a much better group of kids to work with. How do we get that?* F

> *One year a brain man came in and cut a sheep's brain out. He had a whole row of books to sell after the thing and one thing that stuck in my mind forever is that if a kid is motivated, you grow dendrites in your brain and it takes two weeks without pain. If you can get that motivation to a level where that kid is motivated that they want to learn, but that is really difficult and it's more from them than you, but the strategies. I bought his books and I read them but I don't want to go through, but that's really important, I think.* T5

4.2.1.3.1 Memo: Collaborative inquiry challenged dispositions of teachers

As outlined in the earlier memos, the robust dialogues engaged in by the teachers during their engagement in collaborative inquiry brought up ideas, questions and concerns which often challenged the dispositions of the teachers and required them to reflect deeply on their personal beliefs about teaching and their students' achievement.

As the saying goes, "no pain, no gain." The collegial challenge of colleagues who had strong understanding of the site-based constraints and strengths with similar site-based teaching and learning culture, practice and goals could only result in growth and development of the teachers in their practice and beliefs.

4.2.2 Effective Implementation of Collaborative Inquiry

4.2.2.4 Factors Affecting the Effective Implementation of Collaborative Inquiry

4.2.2.4.1 Alignment of teachers' mental models and beliefs about mathematics learning, teaching, schooling and assessment

The teachers were asked by the facilitator what they would do when students wanted to learn only one way of solving the problem and were resistant to learning alternate methods. Teacher T5 shared the following differentiation strategy which was deemed helpful to the team of teachers. He was able to provide opportunities to all ability groups to enjoy success in their engagement of the activity with differentiate approach (e.g. "my bottom group would get, you have to make a list" and "my top group would be given a scaffold where they have to put it into that diagram and they have to make it into relationships between them"):

> *They would struggle to recall and they would struggle to pay attention and participate. I would probably have put my kids into groups, I did this the other day, and gave them a printout sheet with all the recap information of what we've done, because half the time they don't remember, and then each group would get a different way that they have to put that information together. So, my bottom group would get, you have to make a list or you have to make a poster, whereas my top group would be given a scaffold where they have to put it into that diagram and they have to make it into relationships between them. It pushes the level up for the kids who are capable whereas my little ones just have to regurgitate and sort.* T5

His sharing provided the insight that not all students were able to learn multiple strategies; however, instead of teaching multiple strategies to the whole class, group activities can be provided with subtle differentiation to cater for the diverse learning needs of the class.

Beliefs and mindsets of teachers can be impediments to effective engagement in collaborative inquiry, because they tend to look for excuses or reasons for students' lack of progress in learning, rather than looking for solutions to address the learning challenges faced by the students. This belief and mindset is illustrated in the example below. Teacher T5 attributed the lack of academic rigour in his class to the fact that the school's grouping of the highly able academic students in one class resulted in the absence of academic rigour in other classes:

> *You're right, the Physics kids, absolutely no trouble. When you think about, and this is not panning Velocitas, but all the good kids are put in the top end of one class the bottom classes in both 9 and 10 are left and there's not many good kids in; kids have got no role models, or very few; there's a few left. Then they're just living up to the standard of the class, in general. Now, by the same token Velocitas is an excellent idea and wonderful to teach those kids who all seem like that and the big question is what year do we actually attack that and make sure they're all intellectually stimulated - 8 or 6 or…"* T5

This mental model, that the absence of highly able students could lead to low academic rigour, may result in the teachers not being willing to seek creative and alternative approaches to cater for the learning needs of the students in their classes, or failing to set appropriately high expectations for the students to aspire to.

A similar mental model about students and students' capability is expressed by two other teachers. Teacher T4 expressed that the weaker students were not able to reach a certain level of thinking due to the absence of the more able children by stating that "that's really hard in a homogenous, low level class". Teacher T6 expressed that "they're so used to scaffolds …It's almost like learned helplessness":

> *It's that creativity of thought that's really hard in a homogenous, lower level class. Cos you don't want to keep giving them the answers but there's no kid in there that keeps picking them up.* T4
>
> *But they're so used to scaffolds and being led to questions and that. It's almost like a learned helplessness.* T6
>
> *So they're not risk takers.* F
>
> *No.* T6

It is clear that the teacher T4 did not see that all students can be creative in thinking. The absence of the highly able students would give room for the students normally labelled as "average" to showcase their creative thoughts, as they would not be threatened by the presence of the highly able students. It can be their time to shine. This example also illustrates the point that the teachers need to ask appropriate probing or scaffolding questions or use constructivist or Socratic questions to assist the students. While teacher T6 saw scaffolding as handicapping students' ability to learn and grow. This point was raised in Section 4.2.1.1.3 where it was noted that scaffolding should not be a "crutch" for students.

When I (i.e. the facilitator) shared a research article to stimulate the teachers to consider that all students can expect to perform work of high intellectual quality, teacher T4 immediately linked high intellectual quality and student achievement to passing and failing an assessment. This reaction of teacher T4 was an example of how the mental model of teachers could impede the progress of quality collaborative inquiry. In this instance, teacher T4 immediately attributed the students' inability to perform work of high intellectual quality to the school or teachers not putting enough pressure on students to pass their assessments instead of the teachers' expectations and practice:

> *Conversely, these two researchers suggest that when students from all backgrounds are expected to perform work of high intellectual quality overall student achievement and academic performance increases and equity gap diminishes.* F
>
> *That's like the Japanese style that if you fail a test you've got to keep sitting it until you pass, so if we did that, the kids would have to pull their finger out.* T6

The following week, during the weekly collaborative inquiry session, I (facilitator F) asked for examples of higher order thinking strategies adopted in the preceding week in the teachers' classes. Here are the responses:

> *Not really. I've just done the introduction to Pythagoras. It's not really there yet for my class anyway. Haven't really looked at the text book questions yet but I did notice there were a lot of real life examples which is, I guess, similar to this sort of thing. I haven't set them as homework or anything yet, I'm just not there.* T6
>
> *What I did, I don't know if you have come across the IXL website, with all those, there's lots of the real life things where they need to be able to*

visualise it and with a lot of the kids that's a skill they really struggle with. IXL maps. It's got examples and then they can click on say, for example, you're doing Pythagoras, there's basic ones and they can just click on that and work through basic ones and it tells if they're correct when they're going through it and it they are incorrect it works them through the problem. So it's really useful for that and that's sort of like where we're at in terms of being able to interpret a question, being able to draw a diagram, visualise it. T7

It was evident during the discussion that the teachers' mental model or knowledge of higher order thinking constituted using "harder questions" as a tool instead of exploring other higher order thinking tasks or better questioning techniques. It appeared that they considered higher order thinking skills can only be taught after the basics are taught (e.g. "*Not really. I've just done the introduction to Pythagoras"*). This demonstrated that the teachers' own past experience as teachers formed their mental model and beliefs about higher order thinking. However, teacher T7 shared her use of IXL, a mathematics website, to provide differentiated opportunities for the students to access learning at their own levels.

4.2.2.4.2 Optimal school structure, programs and resource

Teachers want authentic opportunities for sharing and being problem solvers rather than just focusing on getting the paper work done. Whatever their mental models and beliefs as teachers, inherently, teachers entered into teaching with the motivation to influence lives and make a difference to children, as expressed by teachers T4 and T6 below:

You want to get through to the kids and when you do, it's yes, I changed that kid's life. T4

I remember in prac, doing Grade 2, and I helped a little girl and she wrote in my book: Thank you for helping me, and I've still got that piece of paper. I saw her and she still remembers. She's in Grade 7 now and she said: Thank you Mr XXX (name withheld). I learnt to write cursive with you. T6

Teacher T4 stated that "I changed that kid's life" and teacher T6 stated that "I've got that piece of paper (from the student)", indicating that teachers are motivated by the noble notion that their work touches and influences lives of students.

In schools, teachers often found the administrative demands on teachers by schools and systems daunting and exasperating, making it difficult to discharge their responsibilities as teachers to educate and make a difference. Each state and territory in Australia imposes legislative and mandatory requirements which are often tied to commonwealth and state funding. These requirements sometimes create conditions which result in competing priorities within schools.

For example, in the previous year, the research site introduced collegial coaching training to equip the teachers with self-evaluation and peer-evaluation strategies through coaching them to provide observation feedback. The teachers viewed the professional learning activities as valuable, but found that they could not fit them into their daily schedule, or that organising peers to observe them and provide feedback was often an afterthought. Teacher T7, who has worked in the school for ten years, expressed the challenge of accessing worthwhile professional learning opportunities when the demands of her role was already overwhelming (e.g. "it's the hardest I've had to ever try and work at this school"):

> *That's exactly where we're all at, though. As much as we admire the idea and think that collegial coaching is extremely valuable, I just can't see how I'm going to fit it in. I'm losing my mind this year. It's the hardest I've had to ever try and work at this school. It's nuts.* T7

When I suggested that such stressors could be better managed to assist the teachers to focus on their pedagogical practices, teacher T5 suggested that teachers' collective wisdom could be harnessed, indicating that teachers did want to generate solutions. The use of adult learning strategies to develop solutions would be beneficial:

> *I think we need a bigger forum and more time so it's done properly. It doesn't need to take a lot of time; by one staff meeting on a Thursday we can approach that problem. People would be happy if there's an agenda and we're going to solve this problem together. I think that's a good idea; a broad forum and it needs to be tightly chaired.* T5

Each school has policies, procedures and programs in place which may restrict the effective implementation of collaborative inquiry; there is pressure from external organisations, such as the International Baccalaureate Organisation (IBO) requirements for Middle Years' Program (MYP) at this research site and state and national requirements such as unit plans for compliance rather than for the learning needs of the children. The discussion below illustrates that such requirements (e.g. "the Areas of Interaction…..need to match up", "we did massive changes at the end of last year", "then if we get MYP look at our program we can say we have covered all the approaches to learning, all the areas of interaction", "the other thing we needed to do was the spelling list") were daunting and demanding tasks for administrative purpose rather than children's learning outcomes:

> *Just for the future, you need to have a look, because the Areas of Interaction and all of that need to match up with what we've got as our yearly summary. We did massive changes at the end of last year so when we're doing these we need to make sure we have that yearly overview that that matches up. With approaches to learning that matches up as well because I've done it in a way that across the year we cover all the approaches to learning. So we need to make sure that this matches what we've done for our yearly planner and the CCEs, I think there were about ten, and we can't teach explicitly all ten so I've chosen in the inter-disciplinary plan, the two that we're targeting for this particular unit; apply in a progression of steps, calculating with and without. So we just need to make sure that matches up. Then if we get MYP look at our program we can say we have covered all the approaches to learning, all the areas of interaction.* T7

> *The other thing we needed to do was the spelling list. You guys have a look and see if you're happy with that before we put it on for the kids.* T7

The school (research site) required that all teachers work on the literacy of the students. The mathematics department required each mathematics teacher in the middle school to create a unit spelling list and assess students and report on their spelling accuracy, without teaching the students a unifying spelling rule or clearly articulating the pedagogical benefits or links to the learning of mathematics. Hence, as far as the mathematics teachers were concerned, spelling was another management, top-down, administrative compliance task, rather than enabling them to explore the pedagogical benefits of spelling and expand their pedagogical tool kit to include literacy skills which are important for the students' mastery of problem-solving skills as illustrated below by teacher T6:

> *Yes, so with the unit I've just done, well they do it [i.e. the spelling list] for homework on the app, and then in class on Wednesday, they actually do it in class. They'll all get 100% on the app, or I hope they would, so then test it in class and that's part of the tracking for (a senior staff- name withheld).* T6

4.2.2.4.3 Facilitator's knowledge and skills

As the collaborative inquiry conducted for this research involved open discussion on the teaching and learning practices of teachers and students' learning, at times the discussions or comments by the teachers required the facilitator to be skilful in steering the professional dialogues back to the topic under inquiry for professional growth, rather than allowing the collaborative inquiry session to become a complaint session:

> *Before we can even get there, they have to be listening and that's a big skill they don't have. We understand from MYP mind, their attention span is extremely short. If you've got to get the concept of the multiplier across, a lot of them have lost it before you get the whole sentence out. They've got boyfriend trouble, girlfriend trouble, zit trouble and peer group pressure trouble, bullying trouble, Facebook trouble and this is all going through their mind like a sea. I'm aware of this looking at them and thinking what's going on here. That's a micro skill but it's in every Year 9 kid, that trouble.* T5

> *It's acknowledging their needs and kids will think we have empathy with them. We are not stone age people who don't understand adolescents. That's what we need to talk about. Sometimes it frustrates me that we have forty minutes or double that but what we get out of the kids is only twenty. That's the reason why we always play catch up.* F

> *We plan fifty minute lessons but you never get that much content into them. Maybe twenty minutes.* T4

> *We need to really focus on quality of learning on their part not our part.* F

In the examples above, teacher T5 blamed the students' lack of listening skills, and hence inability to learn, and his point was supported by teacher T4. As the facilitator of the session, I had to demonstrate acceptance of the issues raised while assisting the teacher to focus on seeking solutions to address the problem. It is often challenging during collaborative inquiry sessions when the teachers adopt a negative mindset when discussing teaching and learning, to bring them back to the moral purpose of their roles in the classrooms.

Another example was a discussion of motivation with the same group of teachers at another session, when teacher T5 raised concern about some research that he had read about motivation suggested that "you can't motivate people, they have to motivate themselves":

> *If kids are motivated towards stuff, it tends to run like a fuel with them but I just read recently that you can't motivate people, they have to motivate themselves. You can facilitate that but you can't talk, like these motivational speakers that you pay a $1000, it lasts for the period of the talk and then people are demotivated again.* T5
>
> *It's not sustainable. When there is no scaffold and structure, that high level of excitement cannot be sustained.* F
>
> *If we could find something that would keep them motivated but with Year 9 Maths and a sea of other stuff going on, it's tricky.* T5

On reflection, I should have challenged the research and requested its source before engaging in this topic. In the instance copied above, the other teachers at the collaborative inquiry session might have concluded that people cannot be motivated and, hence, there was no point in trying to motivate and engage students by exploring productive pedagogies. This approach would leave the responsibility of the motivation for learning solely on the students. Consequently, better facilitation skills would have avoided potential misconception or wrong conclusions being drawn by teachers in attendance.

4.2.2.4.4 Memo: Factors affecting the effective implementation of collaborative inquiry

Even though the advantages of site-based similarity of contextual features are numerous, site based practice such as school structure — personal and curriculum structure and timetable, school programs and availability of resources — can stifle the effectiveness of collaborative inquiry in that the teachers are prevented from entering in-depth discussions and exchanges due to time constraints or unrealistic expectations about the time spent in collaborative inquiry or having too many administrative tasks to complete during the period.

Another factor affecting the effectiveness of the collaborative inquiry sessions is the knowledge and skills of the facilitator; the researcher in this case or in any school setting would be the moderator or team leader of the collaborative inquiry team. The facilitator or moderator or team leader needs strong knowledge of the subject matter, as well as facilitation skills to support the teachers in the collaborative inquiry session and challenge the team members, to ensure the sessions are productive and positive for all involved.

4.2.3 Perceived Influences on Student Engagement

4.2.3.1 Teacher Pedagogy

In order for deep learning to take place, mathematics concepts need to be taught through effective pedagogy, with or without textbooks, with appropriate resources including hands-on tasks or activities, effective questioning and so on. Teachers should fully understand the content knowledge and be able to succinctly make these concepts clear to the students through effective pedagogical practice rather than relying on textbooks alone (e.g. "No, that's not in the book" by teacher T4). The dialogue below indicated that teacher T6 had the advance knowledge to solve the problem under examination; however, it was too advanced a concept for the year level in discussion (e.g. "Can't you use logs for that", "Yes, but this is Year 9") because logarithm is taught in Year 11, not Year 9:

> *…calculate principal, rate and time. I use i=prt. I don't know what you guys use. For compound interest, it rearranges it for all of them but I don't think*

our kids need to be able to do time. I think I put the wrong thing on here; I said interest rate on here, but for time. T7

Can't you use logs for that? T6

Yes, but this is Year 9. T7

That's not in the text and I don't think, I definitely wouldn't have put it in mine. T7

You wouldn't have to do logs with that, but rearranging to find whatever root of …. Have a look at it. T7

Like find the nth root. No, that's not in the book. T4

Teacher T5 commented that his class was not able to "pay attention" due to the nature of the students and their difficulties in focusing, stating that "for me to get 100% attention spread over those times is almost impossible":

I really think in my class for me to get, even after repeating it several times, for me to get 100% attention spread over those times is almost impossible because for a start, it's far too noisy. They're going to be dropping something and fiddling and then half of them are distracted towards that. It's not like Velocitas kids. I had the pleasure of teaching them for a short time last year. T5

It was evident that teacher T5 neither explored different approaches to engaging the students, nor considered the well-researched fact that adolescents' concentration span is short; hence the 50-minute lesson needed to be appropriately planned to cater for the different learning styles, different knowledge entry levels on the subject matter, and so on. He simply blamed the students for not being able to concentrate rather reflecting on his own teaching practice and approach being a potential reason for the students' disengagement.

Teacher pedagogical practices were also commented on by the students during the focus group sessions. When students were asked why "making learning fun" was important for their learning, a student stated that "Miss X makes it easy to (learn) like she'll put songs on her Learnist board to make it easier (for us) to learn it." Another student commented that "she makes it enjoyable" which was agreed with by two other students. Another commented that because "every lesson is fun," the students looked forward to the teacher's lessons and hence were excited about learning.

When the students were asked what their teacher did to help them to improve their results, one student answered that "I wanted to learn; so I learned more and enjoyed learning". All students agreed to this statement.

Based on these observations, enjoyable and interesting lessons were important to the students because they kept the students engaged and they looked forward to learning; hence, learning to them was no longer laborious but enjoyable.

Both groups of students agreed that the attributes of good mathematics teachers were:

- Making learning fun;
- Good depth of knowledge;
- Being highly adaptable;
- Finding out what students like;
- Being strict so that everyone can learn;
- Being nice;
- Not humiliating students in class;
- Change of pace;
- Variation of methods;
- Activities being relevant;
- Using games.

Table 4.1 Attributes of good mathematics teachers according to middle school students

Aspects of Learning	Conducive Learning Environment		Understanding and Engagement	
Categories	Personality traits	Behavioural management strategies	Pedagogical practices	Professional knowledge
Attributes	Highly adaptable Be nice	Be strict so that everyone can learn Not humiliating students in class	Make learning fun Find out what students like Change of pace Variation of methods Activities must be relevant Using games Explain concepts well	Strong content knowledge

These attributes are classified into several categories, summarised in the Table 4.1. Analysing the focus group data in relations to two broad aspects pertinent to learning, I divided them into conducive learning environment and understanding and engagement. I have separated the categories of personality traits, pedagogical practices, behavioural management strategies and professional knowledge of the teachers before I placed the attributes that students identified as helpful for their engagement and learning. Teacher attributes such as being highly adaptable, nice and strict and not humiliating students were considered helpful in creating a conducive learning environment for the students. On the other hand, attributes such as making learning fun, interest in what students like, variation of pace and method for learning, using relevant activities and games, having strong content knowledge and explaining the concepts well were considered essential pedagogical practices for developing deeper understanding and engaging students in learning.

When students were asked if hands-on activities or manipulatives help with their mathematical understanding, one student commented that, when they learned "probabilities, (we) used a pack of card and (it) was helpful to learn about probabilities." Another student commented that "we were able to relate activities to maths" while another mentioned that "hands-on (activities) must be relevant (to the topic)." Two other students identified that assignment tasks relevant to real life problems make learning meaningful and interesting, with comments such as, "house plan assignment integrated hands-on to maths; learning was meaningful," "manipulative was good to make learning visual and learning comes alive," and "algebra, house plan assignment – cool to see how to solve practical problem using mathematics".

4.2.3.2 Linking with Other Disciplines.

The way our brains work suggests that the more cross-fertilisation of disciplines, the better the connections in the brain are wired (O'Connor, 2009). Teacher T3 was clearly aware of this link, and suggested that greater use of students' knowledge and experience outside the subject matter of mathematics, such as Design Technology and Science in the case of volume and capacity, would encourage the students to make meaningful connections of concepts acquired through other disciplines:

> *I want to do a lot of stuff with water to look at volume and capacity. I want to get kids to get those measuring things from Science and drop solids in to measure the capacity of the solid and then work backwards to work out the volume.* T3
>
> [A non-participating teacher mentioned that the students were currently learning density in science.]
>
> *Once XXX [a design technology teacher who did not participate in this project] has done the solids, I'll come and do a demo. I've also asked my kids to bring in three circular objects that are dinner plate size or larger cos I want the kids to measure the circumference, divide it by the diameter, to come up with pi. It's one of those Australian curriculum outcomes in number that I said I want to leave till Term 3 to investigate those types of numbers. So the larger the object, the closer it will be to 3.14.* T3

It works for the little ones but the bigger ones ... That's when I got the callipers out of Science. I think a lot of kids will use the tape measures we've got but if you want to make it more challenging, just don't give them the resources. T3

Similarly, when making conceptual connection, I (facilitator F) shared the example from Physics to encourage the teachers to think about examples outside the discipline of mathematics, so that students were able to connect the mathematical concept of "reasonableness" with their daily experience. Teacher T2 shared how she assisted the students to make such connections on the topic of "money":

But then in maths we talk about reasonableness, that's the same thing. In Physics, they come up with an answer and we say, is that reasonable? F

That's what I say to them all the time. Does that make sense as an answer? If I went to the shop and bought half a litre of milk would it cost $50, no. T2

That's what it says here. When students engage in the construction of knowledge, an element of uncertainty is introduced into the instructional process and makes instruction not always predictable, i.e. the teacher is not even certain what will be produced by the students. F

Teachers T5 also shared his experience of transferring the approach that he used in Physics to Mathematics by using students' ipad application to measure angles of elevation:

The board goes so it's equal to delta y and delta x so you find it's equal to tan theta and we have to go to the computer room. What I could have done then is just get any kid to put a level on their iPad, that's what I do for Physics, and just get a level and stick it on there and they can read the angle straight off there and we could do rise on run like that. T5

4.2.3.3 Teacher Belief and Mental Model

Teachers' mental models about mathematics understanding and engagement impact on pedagogical choice. Teacher T5, quoted below, believed that, by repeatedly completing problems in the textbook, students were able to improve their speed of completion and hence their motivation to tackle more challenging questions even though he acknowledged that repetitive attempts at textbook questions were not consistent with learning theory such as the Bloom's taxonomy (Adams, 2005). Such belief, which was not substantiated by research, was firmly held by teacher T5 which could influence teacher T5's pedagogical practice. Consequently, such belief can affect the students' mathematics understanding and engagement:

> *There's just one thing, I've found that since I've been here, that when they are doing maths, many, many examples out of the book and they start to learn and they get to the hard ones and they start to do it quickly, it is extremely motivating for them. That's nothing to do with any sort of higher, like mathematics books are naturally progressive to higher order thinking, to modern problem solving but I've found that, I don't think we should throw that out, that's really.... we have to motivate kids and the way to motivate them is to give them plenty, plenty of opportunities to work through that. ..... and they start to feel I can do this, then they start to get a love for it and we had one speaker here many years ago who said something that stuck in my brain. To develop dendrites you do not need to do any exercise, you don't need to do practice in maths. All you need is to be motivated and they grow themselves within two weeks. All those pathways connect up to do the problem. It's not like gym where you have to get the lactic acid burning and the muscle comes on, you just sit there and do the problems and it starts, the motivation starts to come. I know it's only a simple bottom of the end, no higher order thinking or Bloom's taxonomy just grunt effort and it's like sport. They start to get a sense I can do it and they go, coach, and he's there or she's there and they start to get the sense 'I can do this' and they start to progress.* T5

> *I think with some kids that's just not going to ever happen with them cos they don't have that..... You can see at tutoring when kids come it helps them they sit there and they have that little bit of motivation as they've come to that and you can see them work, work, work and 'oh, this is working for me' and for some kids we've got to give them other options as well.* T6

Teacher T6, quoted above, shared his view about mathematics mastery by rote learning and repetition alone, ignoring the well-researched and evidence-based practice of scaffolding, differentiation and higher order thinking skills (Nickerson, 2007; Vinner, 2008; Smith & Stein, 2011). When he alluded to the confidence gained in a withdrawal program conducted by a learning enhancement teacher in the context outlined above, he was referring to teacher T5. It was a small group withdrawal program using responses to intervention strategies to improve outcomes.

Teacher T6 reinforced teacher T5's assertion that through mere hard work, students were able to master new concepts, suggesting that they shared the mental model that teachers do not need to differentiate and cater for all learners, when both teachers and students share responsibility for the mastery of new content.

Teacher T4 commented that, when prompted by me (facilitator F), that while all children can master higher order thinking, some struggled and remained on a slightly lower plateau of skill level. Such comments were made during the same collaborative inquiry session, as comments made by teachers T5 and T6 above provided the opportunity for all the teachers present to ponder and examine their own mental model of students' capacity to master higher order thinking:

> *So do we think that some of the children can't do higher order thinking at all?* F

> *Some of them really struggle but I think they are all capable, just a different higher order.* T7

When I prompted "what would be some of the things we would be seeing?" should Year 9 students be provided with the opportunity to operate a higher cognitive level, teacher T7 responded:

> *I guess one that comes to mind is by giving them a question and putting to them that they are going to have to explore. We're starting a new unit, giving them something that will eventually be solved by the end, starting ... that kind of thing. Trying to get them to think that way.* T7

This response came after a period of silence, indicating that the teachers in attendance that day (T4, T5, T6, T7) were all struggling to overcome their mental models of students' ability to master higher order thinking skills.

4.2.3.4 Constraints of External Environments

Most teachers desire the best outcomes for their students and they have the desire to assist the students to develop deep understanding and strong engagement; however, sometimes they felt helpless against the tidal waves of societal norms and practices, as evident by the comment made by teachers T4 “some of the kids are happy to just pass” and T5 “It’s social, generational and the Gold Coast”:

> *Some of the kids are happy to just pass.* T4
>
> *Is this the culture, is this the attitude?* F
>
> *Yeah, it definitely is and I don’t, as a teacher, I don’t know what more to do. It’s social, generational and the Gold Coast.* T5

This expression of exasperation was clearly felt in relation to T5’s concern about students’ mastery of mathematical concepts. If schools were able to address and support the teachers to tackle such external environmental factors, their effectiveness in collaborative inquiry for pedagogical change and hence improving students’ outcomes would be enhanced.

Another source of angst and hence pressure on teachers appears to be time constraints. The introduction of the Australian National Curriculum for Mathematics means that teachers, particularly those in Queensland, feel that the compression and density of the mathematics curriculum requires them to teach many more concepts within a shorter period of time. Teacher T4 clearly articulated her desire to implement hands-on activities to deliver more engaging lessons; however, under time pressure, she admitted that she resorted to the didactic teaching approach:

> *I know I do it from time to time, at least I do some sort of manipulative thing each unit, I’d dearly like to do it more but the term was gone before I knew it, but there are definite opportunities like even with measuring and Pythagoras, there’s stuff we can get the kids out and doing different things and getting them engaged that way.* T4

Another teacher mentioned that, philosophically and theoretically, he agreed with the need for incorporating better pedagogy, hands-on activities and higher order thinking skills in his lessons; however, due to pressure, time constraints and “meaningless administration tasks,” they would not be implemented. The stark contradiction was that the teacher's comment focused on the students' attributes rather than how his own pedagogical practice can be modified to increase students' engagement. His disposition could due largely to his mental model of teaching:

> *I agree with what you're saying almost entirely, but it's the way the time is allocated. We do things, to my way of thinking, that are quite meaningless and don't have anything to do with ... like, you can sack me for this but it it doesn't have anything to do with the kid, it goes to the bottom of the basket. That's it.* T5

When the discussion was turned to how to challenge academically weaker students to engage and develop higher order thinking skills, teacher T5 commented about not being comfortable to challenge the lower level students in his class because students would be disengaged and starting to misbehave. His comment suggested that good pedagogy ought to incorporate an appropriate behavioural management plan, and probably appropriate differentiation strategy, for better student engagement by all students:

> *My problem is not the comfort level of the students but my comfort level. If you chuck a question out like that, XXX (student's name withheld) would be belting someone over the head cos he's bored.* T5

4.2.3.5 Effect of Culture and Attitude on the Learning Attitude of Students

It was noted that students' mathematical understanding and engagement were affected by the school culture and their attitude towards assessments as evidenced in the comment of teacher T7 below:

> *Unfortunately there are a bunch of students going around saying we are the state school of private schools. When we went to XX school and to YY school, our kids were saying oh, look at their uniform, look at their buildings, they're so good, even though we had just beaten them at sport.* T7

Such a self-deprecating attitude created a 'near enough is good enough' attitude in students, while striving for excellence was a mirage and an elusive goal. This in turn affected students' attitude towards the mastery of mathematics. This issue was largely to do with school culture, rather than being an issue which a classroom teacher could tackle and change. It required the management and leadership of the school to create the conditions for a "can do" culture in which the students can achieve excellence.

Misinformation and misconceptions about state level systems could sometimes contribute to students' apathy about their achievement. For example, teacher T6 quoted his frustration with his year 11 students. The concerns that he raised indicate the need to dispel misconceptions about the Year 11 formative assessments in the Queensland senior schooling system, as well as explaining the relationship between formative and summative assessments in the final two years of senior schooling:

> *"What my struggle was with the 11s last year and talking to parents, there's such a mindset amongst the kids that Year 11 doesn't count. Why should we try, we'll leave it till next year."* T6

It was also suggested by teacher T5 that the change could be achieved through educating the parents:

> *"So we need to educate the parents in our school. The shy ones who are shy to come; I can't just solve that but there will be ways to get shy parents involved. You think well what's stopping them getting here. They're shy."* T5

4.2.3.6 Strong Content Knowledge

When the discussion turned to the level of student engagement in mathematics classes, a student mentioned the year when he had a mathematics specialist teacher teaching the class. All students agreed that they liked teachers to have strong content knowledge in order to answer the students' questions confidently and accurately and explain in depth and in detail. Some of the students' comments were: "she started explaining the simple stuff; when she knew you got it, she expanded it"; "we are learning probability and fraction; she took us further into more stuff than year 7 normally learn, higher stuff"; "Mrs X told us the information, then gave us the worksheets, then helped us with the work and explained in real detail which helped"; "we learned in greater depth"; "she had a bar for us. If we reached it, she then extended us by giving us harder work or more work to complete"; "Mrs X knew everything"; "she knew everything"; "they explained well enough for me to understand”; “how well the teachers explain it”; “if teacher explained it well enough, it got interesting; then we learn better”.

Based on the student focus group feedback outlined above, it can be summarised that the students felt confident and engaged when their mathematics teachers possessed strong content knowledge in order to respond to their questions.

4.2.3.7 Memo: Perceived Influences of Student Engagement

Repeatedly during the collaborative inquiry sessions, it was noted that the students’ mathematical understanding and engagement were influenced by many wide ranging factors, such as teacher pedagogy, connections of mathematics learning with other disciplines, teachers’ beliefs and mental models, external environmental constraints, and the culture in which the students learn. These perceptions emerged either from teachers’ direct expressions or were inferred by me in my dual role as teacher participant and researcher.

The degree to which these attributes affected the students’ mathematical understanding and engagement neither was measured in this research nor was it meant to be measured. However, these factors did play a part in the effectiveness and quality of the collaborative inquiry sessions, in that the quality of the teachers’ contributions to the discussion may have been limited by the above factors. They may have felt powerless because of the external environmental constraints or the learning attitudes of students, hence becoming disillusioned and disheartened, and not being able to fully engage in productive and meaningful collegial dialogues.

The students expressed that teachers who were interested in students would find out what they liked and incorporate activities that were enjoyable and engaging for the students. The variation of pace and activities was another feature of classes with the teachers from whom they enjoyed learning, embracing the use of games and hands-on activities to teach concepts and ideas which were both engaging and held their attention. Strong content knowledge was another attribute of teachers with whom the students enjoyed learning.

The students clearly articulated the model that they felt teachers needed: to have strong content knowledge, to know the students well including their likes and dislikes, and to invest time and energy into designing lessons which consisted of well-paced activities including games and hands-on activities which were fun.

4.3 Summary of Year One Findings

When triangulating findings from the collaborative inquiry sessions, students' focus group feedback and other research findings, some trends emerged from the stakeholders in the mathematics classrooms, that is, the teachers and their students.

The benefits of collaborative inquiry used as a professional learning tool were identified. The teachers were able to use the collaborative inquiry sessions to elicit pedagogical strategies from others in their team because they shared the same contexts in the same school. During the teachers' collaborative inquiry, it was noted that, when the topic of discussion focused on teaching strategies, the need to develop strategies for better pedagogical practices was of concern. The teachers shared real life problems for a range of topics, making problems and learning visible, scaffolding exercises for developing conceptual understanding, and how effective pedagogy could assist students to make stronger and deeper conceptual links.

Strategies such as using real life problems, making problems and learning visible and using scaffolding exercises to help students to develop more challenging concepts were affirmed by other studies. Classen (2002) concluded that students are more motivated to learn when they are interested in what they are learning, particularly when they can see its relevance to their lives and outside school. The students in the focus groups felt that the use of games and hands-on activities alone would not necessarily engage them in learning or help them to develop better understanding, unless these games and activities were relevant and appropriate. This position held by the students was confirmed by Attard (2013). In her study of Years 6 to 8 students, she concluded that the notion of fun appeared to be important to middle years' students even though enjoyable lessons did not guarantee mastery of concepts (Attard, 2013).

The collaborative inquiry sessions also brought out the importance of developing and using authentic assessments to develop and assess students' mathematical understanding and engagement. They explored possibilities for authentic assessment procedures to assess higher order thinking skills; they also discussed potential strategies to address students' learning weaknesses, and how differentiation in assessment items and classroom activities can enhance students' mathematical mastery, and hence achievement. The teachers were able to refine their assessments and make them authentic through their collaborative sessions. Through their sharing, they were able to develop greater insights into the authenticity of assessment tasks, gaining ideas about how to differentiate assessment tasks to incorporate higher order thinking challenges and meeting the learning needs of students displaying certain weaknesses in their learning approaches. These collaborative inquiry sessions provided the teachers with on-site and built-in professional learning opportunities from their colleagues. Referring to the teacher profile in Table 3.2, only two of the eight teachers have mathematics education background and only three of the eight teachers have mathematics educational training; hence, these collaborative inquiry sessions provided just-in-time professional learning for the teachers to support one another in their practices and were valuable professional development for these teachers. Such professional development experience empowered the teachers, addressed performance gaps and helped with student assessment with the goal of improving student outcomes, which is consistent with the findings of Hoy (2009), Globe & Horm (2010), Groneword (2009), Obrycki (2009) and Terehoff (2002).

The benefits of collaborative inquiry were numerous, as evident in the data analysis summarised above. However, for collaborative inquiry to be an effective professional learning tool for teachers, several factors affecting the effective implementation of collaborative inquiry must be addressed. One of these factors is the alignment of teacher dispositions about their pedagogical approaches. The pedagogical dispositions of the teachers, as evident in the analysis of data in Section 4.2.1.3, impacted on the threads and quality of collaborative inquiry. For example, when the teachers expressed negative mental models of students' ability to master mathematical concepts or higher order thinking skills, they had the tendency to dismiss the students' learning and, hence, not to seek ways to value-add to the student tool kits or skills. This finding is consistent with those of Dufour (2007), Habegger and Hodanbosi (2011) and Seeley (2009) who retorted that a team of teachers who inquired about an issue arising in their teaching setting (through reading, analysing, dialoguing, implementing and evaluating research) would generate interventions with the goal of student engagement and achievement; they would never lose hope concerning students' ability to achieve the learning goals.

Another factor impacting on the effectiveness of collaborative inquiry in a school setting, which is closely associated to the teachers' mental models and beliefs, was the need expressed for optimal school structure, programs and resources. Optimal school structure would include incorporating and valuing collaborative inquiry as a valid professional learning opportunity for the teachers, as well as programming time for teachers to have guided discovery about their own pedagogy, not just in order to address the organisational pedagogical framework, but also to allow the teachers to examine their professional growth journey in pedagogical practice. Louis, Anderson and Riedel's (2003) assertion that, for professional learning communities in schools to be sustained and practised successfully for long term change in practice school-wide structural implementation is needed which supports my finding. Optimal programs as suggested by the teachers involved teachers as valuable and value-adding members of the team, rather than as receivers of top-down mandated programs from the school leadership team. Based on the teachers' feedback, optimal resource provisions would include cutting down red-tape, emails, staff location and so on, in order to provide the teachers with the time to engage in professional dialogues, both incidental and organised ones.

Implementation of effective collaborative inquiry would also be dependent on the knowledge and skills of the facilitator. As I had the dual role of researcher and facilitator of all the collaborative inquiry and focus group sessions, I needed pedagogical knowledge for effective probing and prompting to engage the teachers in inquiring into their pedagogy. My facilitation skills were also critical to keep the inquiry centred on the topic of effective pedagogy, while being sensitive to the research participants and the research site protocol and priorities. In this instance, I was also the Deputy Principal in phase one and the Principal in both phases two and three of the research. So, I needed to stay objective while facilitating the collaborative inquiry sessions with the teachers and the focus group discussions with the students.

The influences affecting student engagement also emerged from the data. These influences included teacher pedagogy, linkage with other disciplines, teacher mental models and beliefs, constraints of the external environment, culture and attitude of site and teachers' content knowledge. My identification of teacher pedagogical practice and teacher mental models is consistent with the finding that collaborative inquiry elicited pedagogical strategies and challenged disposition of teachers about their pedagogical approaches. Pajares' (1992) study showed that there was a strong link between teachers' educational beliefs and their pedagogical practices, which supported my findings. Similarly, Ravitz, Becker and Wong (2000) concluded that teacher pedagogical practice and teacher mental models and beliefs were closely linked. Hence, the collaborative inquiry approach to meeting the teachers' professional learning needs could challenge the teachers' dispositions about their pedagogical approach and practice.

When the Year 1 findings were synthesising a theoretical model was developed as shown in the diagram Figure 4.1. Table 3.1 illustrates the stage one theory evolution using inductive logic and deductive logic. The theory sampling emerged through the data analysis of teachers' discussions, student focus groups' comments and my field notes that repeatedly reinforced the importance of teacher mental model and belief about teaching and learning in influencing student engagement in learning. Hence, I used this inverted triangle to illustrate the pivotal and foundational role of teacher mental model and belief about teaching and learning as the pointy tip of the inverted triangle in creating a conducive learning environment for students to engage in learning as the ultimate outcomes of learning. The middle section of the triangle shows three inter-connecting circles that identify the three areas that all schools should focus in order to ensure the desired positive outcomes for all students: optimal environment, authentic assessment and teacher pedagogy. These three areas emerged from the theory sampling and saturation process as outlined earlier in this chapter. The desired outcomes for students can be achieved through using collaborative inquiry as a professional learning tool which was illustrated by the ribbon transgressing the three layers of the triangle. I identified "teacher mental model and belief about teaching and learning" as foundational because the data set suggested that they were pivotal to student engagement. I summarised "authentic assessment", "optimal environment" and "teacher pedagogy" as core business of schools to illustrate the three main threads that emerged from the data to suggest that student engagement in learning could be narrowed down to these three main areas.

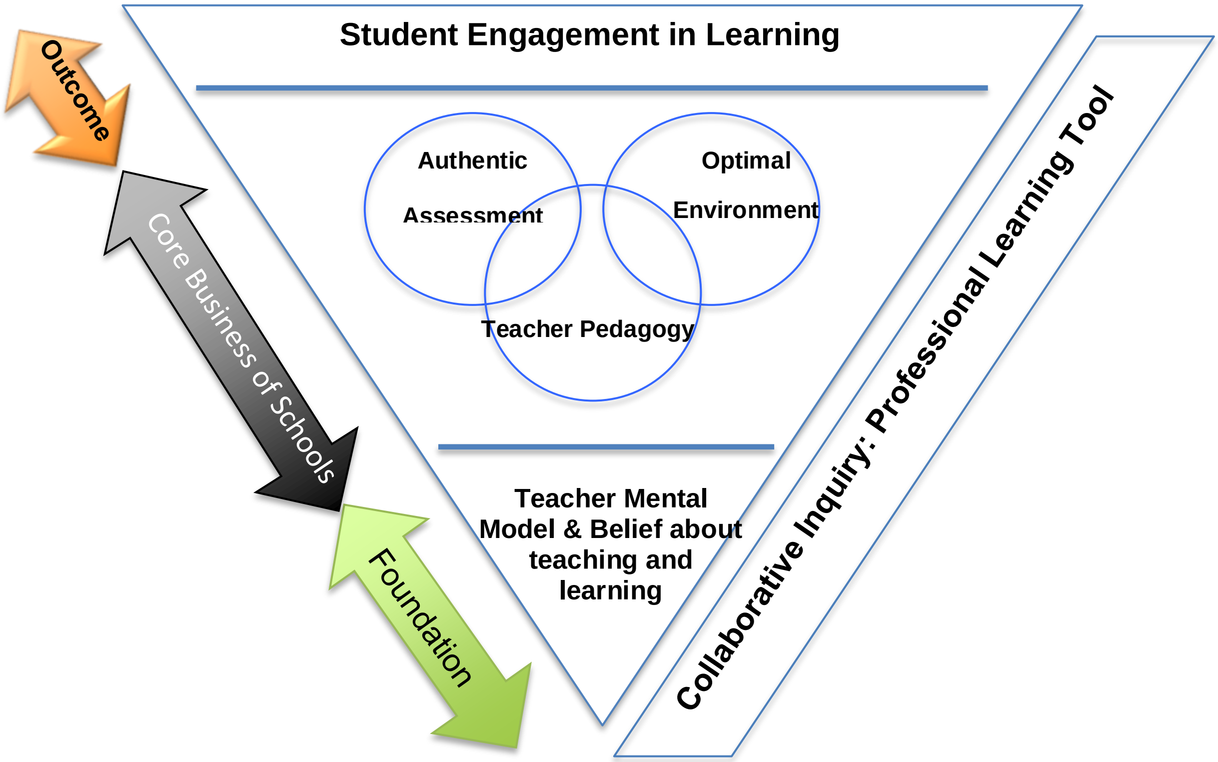


Figure 4.1: Inter-connecting evolutionary links for implementing collaborative inquiry as a professional learning tool for teachers in schools

CHAPTER FIVE

PHASE TWO DATA ANALYSIS AND FINDINGS

5.1 Introduction

In the previous chapter, the emerging theory from the data was conceptualised visually in response to the research question of developing a model which could enable the collaborative inquiry approach of professional development to be replicated across other sites. The previous chapter identified that the benefits of collaborative inquiry as professional learning included that teachers could use it to elicit pedagogical strategies, refine authentic assessment, and challenge their own dispositions about pedagogical approach. Other findings involved how collaborative inquiry can be implemented effectively in schools and some influences on student engagement.

In this chapter, data collected from team planning sessions and teacher survey were analysed and the findings synthesised into two broad groups, namely, teacher centric factors and external factors, which affect the effectiveness of collaborative inquiry as a professional learning tool for teacher professional learning.

5.2 Data Analysis

The data were analysed for this phase and the findings were used for triangulating, theory sampling and theory saturation. It was anticipated that the phase two data would add richness to clarify the emerging theory and model conceptualised from phase one data.

5.2.1 Teacher Profiles and Perceived Professional Learning Preference

Table 5.1 was compiled from the survey data outlining the individual teacher participant's educational background, teaching experience and belief about how children learn. It can be seen from the profiles in Table 5.1 that one of the teachers had less than five years' experience, three had less than ten years' experience, three had less than 15 years' experience, and only one teacher had more than 20 years' experience. It is important to note that the teacher with more than 20 years' experience in teaching, teacher C (not using original code to prevent identification), was not chronologically the oldest teacher participating in this research and, similarly, the teacher with less than five years' experience was not the youngest teacher chronologically. Knowledge of the estimated ages of the teachers was obtained from my observation as the researcher.

Teachers T1 to T8 were randomly re-coded in Table 5.1 to prevent these teachers from being identified. The codes A to H are used only for the data from teacher survey.

Table 5.1 Profiles of the teachers

Characteristics	Teachers							
	A	B	C	D	E	F	G	H
Education background	Science	General primary	Math	Science	Science	Math	Science	Science
Education training	Middle school	General primary	Math	Science	Science	Math	Math	Science
Years of teaching experience	11-15 years	6-10 years	15-20 years	11-15 years	6-10 years	1-5 years	6-10 years	11-15 years
Years teaching middle school	11-15 years	6-10 years	15-20 years	11-15 years	6-10 years	1- 5 years	1-5 years	11-15 years
Years teaching middle school mathematics	11-15 years	6-10 years	15-20 years	11-15 years	1- 5 years	1- 5 years	1-5 years	11-15 years
Children as doers	Well	Well	Very well	NIL	NIL	Very well	Well	Very well
Children as knowers	Very well	Well	Well	NIL	NIL	Well	Well	Well
Children as thinkers	Well	Well	Very well	NIL	NIL	Very well	Very well	Very well
Children as knowledge-able	Well	Well	Well	NIL	NIL	Well	Well	Well

The teacher profile shown on Table 5.1 data was consistent with the TIMSS 2011 report which expressed concern about the lack of mathematics education and training amongst middle school mathematics teachers in Australia. The report stated that one-third of Year 8 students were taught by teachers with no mathematics education or training. One quarter of the teachers in this study had mathematics education and 38% of them had mathematics training similar to Australia wide in terms of the teachers' ability to engage and extend middle years' students in their mathematics learning, which is likely to affect their long term engagement in mathematics learning beyond schools.

Given that a large percentage of the teacher participants was not educated and trained mainly in mathematics, meeting the professional learning needs of the teachers was likely to be helpful in ensuring that they continued to develop their competency and capacity to teach middle years' mathematics. Table 5.2 summarises the teachers' perceived professional learning needs expressed in the teacher survey.

The most sought after professional learning topics listed in Table 5.2, with at least five out of six teachers selecting strongly agree or agree, were: watching teaching videos; sharing with colleagues at team planning meetings or incidental professional dialogues; observing other teachers teach; attending workshops, conference or seminars run by external practitioners; and trialling new ideas or getting feedback about one's teaching. The identification of sharing with colleagues and trialling new ideas or getting feedback about one's teaching was consistent with phase one findings that collaborative inquiry elicited pedagogical strategies and challenged dispositions about pedagogical approaches. "Watching fellow colleagues teaching…" and "listening to fellow colleagues share……" received the strongest support from the teachers as professional learning needs of mathematics teachers which provided support to the proposed collegial inquiry approach to professional learning for middle years' mathematics teachers.

Table 5.2 Degree of agreement of mathematics teachers' perceived professional learning needs

Professional learning needs as mathematics teacher	Strongly agree	Agree	Neither agree or disagree	Disagree	Strongly disagree
Reading a book/article on pedagogical practices helps me with my professional growth and practice		3/6	2/6	1/6	
Watch(ing) videos teaching about pedagogical practices helps me with my professional growth and practice		5/6	1/6		

Watching fellow colleagues teaching in a classroom setting helps me with my professional growth and practice	4/6	2/6			
Listening to fellow colleagues share about an effective pedagogical practice helps me with my professional growth and practice	4/6	2/6			
Attending conferences/seminars by practitioners outside the organisation helps me with my professional growth and practice	1/6	5/6			
Year level planning sessions to share practical and workable pedagogy relevant for the unit help me with my professional growth and practice	1/6	4/6	1/6		
My pedagogical practice can be enhanced by trying new ideas or approaches, then reflecting on their effectiveness.	2/6	3/6	1/6		
My pedagogical practice can be enhanced by listening to other more experienced teachers share ideas	3/6	1/6	2/6		
My pedagogical practice can be enhanced by incorporating mandatory pedagogical ideas into unit plan		1/6	5/6		
My pedagogical practice can be enhanced by getting another staff member to observe me and give me feedback	1/6	5/6			
My pedagogical practice can be enhanced by discussing and sharing teaching ideas during unit planning	1/6	5/6			
My pedagogical practice can be enhanced by reading about/listening to the latest research and approaches and incorporating them into my teaching	1/6	2/6	1/6	2/6	
The most effective professional learning approach differs from teacher to teacher	3/6	3/6			
Teachers need different types of professional learning experience to grow	4/6	2/6			
Professional dialogues amongst teachers are the best professional learning opportunity	1/6	3/6	2/6		
There’s a link between teacher professional development activities and the teacher’s pedagogical practice		4/6	2/6		
There’s a link between teacher professional development activities and the students’ achievement		2/6	3/6	1/6	
There’s a link between teacher pedagogical practice and student achievement	3/6	3/6			

5.2.2 Teacher Centric Factors Influencing Collaborative Inquiry Implementation

In this section, the factors which impacted on the implementation of collaborative inquiry as a professional learning approach at the site were being outlined. These factors below mainly relate to the teachers participating in such professional learning activities.

5.2.2.1 Teachers' Concerns About Student Achievement

Through the teachers' discussions, the teachers for the younger group of students repeatedly expressed concerns about their students' levels of mastery and success, and, from their knowledge of the students, predicted their academic achievement:

> *I'm just really concerned. I teach [names of classes withheld], and looking at what it now is, my kids are heading for failure and it's our very first test of the year.* T2

While discussing the assessment task to be implemented, this teacher T2 expressed concern about the students' general knowledge and their competency to successfully complete that particular task, stating that:

> *My kids won't even know 'votes cast'. They won't even know the terminology to cast a vote.* T2

This same teacher T2 also expressed that her original fear of their unreadiness to tackle the assessment task, due to a lack in foundational mathematics knowledge from the previous year, sadly came true, and she was trying to diagnose the possible reasons for their poor knowledge and background in fractions:

> *Do they have any of their tests for 6? I think I need to sit down with a 6 fraction test, and then a 7 fraction test. I know [teacher name's name withheld] had to put all of that in, and you knew my concerns with it. I emailed [teacher name's name withheld] straight away and said "half of my class is going to fail this," and they did. I wish I was wrong. Is it too much of a challenge? I know that we have to do the maths, but are we trying to put too much language in too early, and they're not understanding it?* T2

When the discussion moved on to whether or not the teachers had too high an expectation of the students by setting too challenging a problem, another teacher T1 asserted the need to maintain the high standard which she was confident they could eventually achieve, stating that:

> *But we can't dumb down either, because we've got capable kids.* T1

However, the concern over students' unsuccessful attempts at assessments could also be attributed to the site being an independent school, and parents having high expectations of their children's academic success; this may be the cause of the same teacher's concern as expressed below at different meetings:

> *I understand that, and that's my concern. Like I said, this is year 7 work. These kids aren't year 7 material.* T2
>
> *But if I just go on my first marks, 50% of my kids have failed.* T2
>
> *One of my kids, in the practice, put a cross in the bubble. If she did that for NAPLAN, she would've failed the whole test because they're electronically marked and you have to shade the whole bubble.* T2

The above statements suggest that the mental model and beliefs of T2 were that all students must have success in all assessments, otherwise the teacher was to be held accountable for the unsuccessful outcomes. This sense of accountability and ownership might not have been widely held by the other participating teachers.

The degree of agreement by teachers on students' mathematics achievement collected from the survey was compiled in Table 5.3. Based on the summary of data in Table 5.3, the teachers seemed quite consistent in their beliefs that student effort, practice and students' beliefs about their mathematics abilities were keys to their mathematics achievement, as well as that success in mathematics is not linked to the students' inherent abilities in mathematics.

As seen in Table 5.3, there was a lesser degree of agreement amongst the teachers that "students need to be rewarded for the right procedures more than getting the right answers," which surprised me because, based on the collaborative inquiry sessions transcribed and analysed, such differences in belief were not evident. However, this could be because those who felt that answers are more important did not air their views during the sessions due to peer pressure.

Table 5.3 Degree of agreement of teachers' beliefs about mathematics achievement

Beliefs about mathematics achievement	Strongly agree	Agree	Neither agree or disagree	Disagree	Strongly disagree
The key to success in maths is effort on the part of the students	1/6	5/6			
Students need to be rewarded for the right procedures more than getting the right answers.	2/6	2/6	1/6	1/6	
Students who do not achieve well in mathematics should try hard, e.g. go to tutorials, do more homework questions.		2/6	4/6		
Mathematics success is linked to students' self-belief in their own abilities.	3/6	2/6	1/6		
To be good in mathematics, students need to have a "mathematical mind."			2/6	1/6	3/6
Practice is the key to mathematics mastery.	2/6	4/6			

Table 5.4 Top three factors affecting student achievement from teacher survey

Top three factors which affect student achievement	Probability
Students not afraid to make mistakes	4/6
Students keep thorough notes and attempt all homework	0/6
Students always start to tackle the simple routine problems first	0/6
Students have strong foundational basis such as number sense and times tables	3/6
Students are happy and engage in real life problem-solving	3/6
Students immerse and enjoy answering "why" questions all the time	0/6
Students transfer their knowledge to unfamiliar situations by linking the old and new concepts	5/6
Students are engaged in critical mental activities such as linking mathematical ideas themselves	0/6
Students construct their understanding by relating ideas and articulating them	3/6
Students accept uncertainty and apparently conflicting ideas in the process of learning	0/6

Another discrepancy in the data in Table 5.3 on the mathematics achievement of students was that, while the teachers held the belief that the students' own efforts and practice were important factors, they did not necessarily hold the belief that the students should be helping themselves by attending tutorials or doing more homework questions. This could relate to the teachers' concern that some students who went through the motions of attending tutorials or doing extra homework did not develop deeper and more complete understanding of the concepts learned, and thus would be deluding themselves that their efforts would yield positive outcomes.

There were apparent contradictions the teachers' responses in the survey. The data in Table 5.3 revealed the teachers' beliefs that students' efforts, practice and belief in their own abilities were strong predictors of mathematical achievement. However, as shown in Table 5.4 and Table 5.5, the teachers rated students' ability to transfer their knowledge in new situations, readiness to take risks, strong foundation in mathematics, engagement in real life problems, and ability to construct their own understanding as important factors as predictors of students' mathematics achievement; none of the teachers selected "attempt all homework" or the factors involving higher order and independent deep thinking as important factors for success, whereas the latter would be required for the students to transfer their knowledge to unfamiliar situations and construct their own understanding.

This survey provides a glimpse of the apparent contradiction in the teachers' articulated beliefs and practice beliefs, which can be ascertained in their responses to their practice and in the collaborative inquiry sessions, where their real beliefs about students and their achievements may have been revealed in context.

Table 5.5 Rank ordering of teachers' identified factors affecting student achievement

Rank ordered factors affecting student achievement	Probability
Ranked 1st	
Students transfer their knowledge to unfamiliar situations by linking the old and new concepts	5/6
Ranked 2nd	
Students not afraid to make mistakes	4/6

Ranked equal 3rd	
Students have strong foundational basis such as number sense and times tables	3/6
Students are happy and engage in real life problem-solving	3/6
Students construct their understanding by relating ideas and articulating them	3/6
Not selected by the participants as top 3 factors at all	
Students keep thorough notes and attempt all homework	0/6
Students always start to tackle the simple routine problems first	0/6
Students immerse and enjoy answering "why" questions all the time	0/6
Students are engaged in critical mental activities such as linking mathematical ideas themselves	0/6
Students accept uncertainty and apparently conflicting ideas in the process of learning.	0/6

5.2.2.2 Teachers Seeing Problems Instead of Solutions

There were many occasions during the collaborative inquiry sessions when the teachers spent an extended time during talking about the students' challenges instead of using the collaborative inquiry sessions to discuss strategies to manage these challenges.

For example, students' ability to manage time during examination setting was discussed, but the teachers did not use the opportunity to devise strategies to be incorporated into the unit plan for future improvement of the course work:

> *My kids were similar. They struggled with time management with the last few. It's only the last three, those are hard.* T6

On another point about a take-home assignment, at the same meeting (number 9), teacher T6 commented that the students focused on the presentation of their written tasks instead of focusing on the quality of the subject matter discussed. However, once again, there was no discussion of future planning to address the students' weakness in completing take-home written tasks:

It's about effort and presentation. I've got one that's nicely typed up and pretty. I've got some who have the diagrams and no effort. T6

On the issue of the students' problem solving approach, two teachers T6 and T7 commented that the students seemed to go to great lengths to avoid using algebra to solve the mathematical worded problems; however, they did not take the opportunity to discuss how to encourage students or create scaffolding for the students to use algebra more intuitively to solve word problems:

They're still very reluctant to use algebra. Even when I throw it in an example, they freak out. T6

They try every other method to avoid it. T7

Similarly, a teacher who taught a different year level also commented in a similar vein about identifying the problems the students encountered (e.g. "They don't know symmetry", "they don't know shapers or area or perimeter", "can't do measurement", "can't convert from millimetres") and neither teacher used the opportunity to turn the students' difficulties in mastery of mathematical concepts into learning opportunity to better equip themselves to brainstorm strategies to address the challenges faced by the students:

I did analysis on that too, and it was all those things. They don't know symmetry, they don't know shapes or area or perimeter. My kids can't do measurement. They can't convert from millimetres. They say to me "do we have to put that little two above it" and I say "you mean squared?". T2

There's no way at the moment, they've only just started and it takes me a lesson to get through it and mark it and maybe do two or three problems that they're getting wrong. Maybe we should go back to… we've got the old spelling mastery books too. These kids can't spell for the life of them. Year 7 neither, [teacher name withheld] must be pulling out his hair with them because we just did a draft for the Bible and every Bible had lower case, every Noah had lower case, because it's a proper noun and I don't even think they even know what I'm talking about. T2

Such opinions could again be attributed to the teachers' mental models and beliefs about teaching and learning or about learners themselves. They might have felt that, by identifying the deficiency in the tool kits held by the students, the teachers did not need to be held accountable, or they could simply put the onus on the students, instead of taking control of the issue.

5.2.2.3 Student Skills and Student Achievement are Closely Linked

Extensive time was devoted by the teachers to discussing the close link between student achievement and examination skills. Such discussions provided insights into the need for the teachers to incorporate teaching management skills in order for the students to master these challenges.

Firstly, time management was identified as a critical skill for student achievement, as evident by the comments made by teacher T1 below:

> *Their time management was horrendous. I said to them, "you have to realise you can't have gotten A's because you didn't do them." I've never had so many kids not finish a test before. I said to them "it's not that it was too long, they just didn't have any time management."* T1

In reality, the teachers needed to go a step further to provide strategies, scaffolding or practice to enhance the students' competency and mastery in time management during timed assessments.

Another scholastic skill of concern that was expressed by teacher T2 was the reading comprehension skill needed for the students to successfully attempt worded problems in mathematics assessments, such as "not understanding the key words":

> *Definitely comprehension because they're not understanding the key words.* T2

> *The mayonnaise one, you've got "14 grams of mayonnaise, 11 grams of fat, what's the percentage?" I said to them, "the amount of percentage of the amount we did," but they turned it into division and subtraction.* T1

The same assessment was made by another teacher, confirming that students need to be taught reading comprehension skills for further improvement:

> *Yes, but I definitely have kids who need it [i.e. comprehension skills in context of the discussion]. I've got –C kids like xxx [name of student withheld].* T1

Reading comprehension skills also required the students to have strong vocabulary. For example, “buying in bulk” and “better deal,” as stated below, would require the students to make inferences about the better deal by evaluating different deals:

We didn’t want them to do that. We wanted them to see that buying in bulk was a better deal. We wanted them to say buying 10 litres was going to be more cost-effective than buying 10 one litre can. T2

The teachers further deliberated the need to set high expectations and use correct terminology such as “full mathematical justification” and “communication and justification.” They also further emphasized the need to teach scholastic skills, such as justification, so that students were able to demonstrate such skills in examination settings:

Maybe we need to make it clear that this has to have full mathematical justification, which is what they didn’t do. T1

Problem being, what if I want to see working? You look at the 11 maths A, B, C, curriculum, they have “communication and justification.” We need to be teaching kids that it’s not just the answer we’re looking for, it’s also how they get there. Can they communicate? I can’t manage to get this one [Penultimate, i.e. the last one in the series of number] to write on the next line. It’s lovely otherwise, because it’s very clear.” T7

The group of teachers teaching Year 9 mathematics also identified specific skills needed for the students to be successful in their academic outcomes.

In the first example below, the teacher T1 prepared a set of basic online mathematical skills tasks for students, such as “the kids (who) are weak”, “kids struggling with fractions”, to consolidate and master basic skills not specifically taught in the current unit, such as “send them back (to the online task) and they can open up any one (task)”. The mastery of basic skills was identified by the teachers in the survey as the important. According to Table 5.5 which ranked “student have strong foundational basis such as a number sense and times tables” as the third highest factor affecting student achievement:

It's more to track where the kids are weak. From what I've seen, it looks quite good in that sense. It's not that game style, but I think for our kids it doesn't need to be. So that would be perfect for you guys who have kids struggling with fractions. You could send them back and they can open up any one. T7

The next example shows that the teacher recommended online tools such as Khan Academy for a student who was absent frequently due to illness (e.g. "he had cancer") and learning disability (e.g. "He is one of our lowest kids", "robbed him of his long term memory, ADHD, anxiety") to better manage his learning progress:

He is one of our lowest kids. He's also got another big thing, because he had cancer and got over it. Now his mum has something. Xxx [student's name withheld] has multiple disorders. He's got cancer, which robbed him of his long-term memory, ADHD, anxiety. T6

Things like Khan Academy. It's an app, and there are little videos he can watch, and activities he can do. T6

Regarding independent thinking skills, a teacher pointed out that, from cohort to cohort, students differ; however, as teachers we, need to focus on our teaching and learning practices and reinforce scaffolded steps, as outlined below. This skill was ranked equal third by the teachers when they ordered the factors affecting student achievement when completing the teacher survey (see Table 5.5):

As a cohort, I think these ones might be a little bit more dependent. They constantly want you to tell them exactly what to do. T7

They constantly need clarifications on whether they are right. I say "I don't care if you're right. Have you followed the steps correctly? If you've followed the steps correctly, you should have the right answer." T6

As in the earlier discussion about the need for reading skills to be reinforced for the mastery of mathematics, the example below identifies other literacy skills required for the students to achieve a higher grade:

That's the literacy coming in. I reckon they're the same in the literacy where they're borderline C to B kids. A lot of the time, there's a few D's here and there. So you've got the same sort of problems. As soon as you get to the literacy side of maths… T6

Another scholastic skill needed was enhancement of visual and spatial ability so that the students could master mathematical concepts such as geometry, trigonometry and co-ordinate geometry, as stated below:

> *With distance, mine was very visual. I put a grid on the board and did it all over the board. I drew two coordinates and said "this is how I'd do it." Then I got someone to draw the next two and got someone else to draw in the triangle.* T6

> *With my lower kids, I said "you could fall back to Pythagoras because every one of you can do Pythagoras. If you can't do it, draw it. Be conscious that it's not going to be the most time-efficient in an exam." Most of them, even my lower ones, said they found formula easier.* T7

The example above also further illustrates the earlier point about the need for the students to manage multiple concepts and steps to tackle more complex problems. It was noted that the top two ranking factors for student achievement in Table 5.5 (i.e. "students transfer their knowledge to unfamiliar situations by linking the old and new concepts" and "students not afraid to make mistakes") were not identified by the teachers during their collaborative inquiry sessions, which suggest a lack of consistency in their perceptions in student achievement.

5.2.2.4 Evidence Used to Ascertain Student Mathematical Understanding and Mastery

Table 5.6 shows the compilation of the teachers' responses to the survey regarding the evidence they used to ascertain students' mastery of mathematical understanding. Collectively, it was noted in Table 5.6 that, the most frequently favoured approaches used by the teachers to ascertain students' mathematics mastery (with five out of six teachers selecting strongly agree or agree) were "they are able to apply the concepts to solve challenging problems", "they are able to explain the new concept to another student", "they are able to articulate new situations to which the new concept can be applied" and "they are able to explain why their solutions are wrong".

Based on my observations of the teachers' classroom practice on my observation field notes, the opportunities for students to engage in discussions in the teacher participants' classes were not evident. Hence, it was surprising to read these survey responses from the teachers. A possible explanation could be that the teachers used these strategies regularly; however, during the few classroom visits these strategies were not suitable for the activities at that time. Another possible interpretation could be that these were the ideal strategies based on the teachers' training; however, the teachers did not employ these strategies in practice.

It was worth noting in Table 5.6 that, "they ask questions to clarify understanding," and "they ask questions to relate new concepts with existing concept," attracted less agreement or more diverse responses.

Table 5.6 Summary of evidence used to ascertain students' mastery of mathematical understanding

Evidence used to ascertain that the students have developed mathematical understanding of the new concept	Strongly agree	Agree	Neither agree or disagree	Dis-agree	Strongly disagree
They are able to solve mathematics problems using the correct formulae		3/6	1/6	2/6	
They are able to apply the concept to solve challenging problems	1/6	4/6			1/6
They are able to explain the new concept to another student	4/6	1/6			1/6
They ask questions to clarify understanding	1/6	2/6	2/6		1/6
They ask questions to relate new concept with existing concept	1/6	3/6	1/6	1/6	
They are able to articulate new situations to which the new concept can be applied	3/6	2/6			1/6
They stick to the algorithms they have mastered to solve all types of problems			4/6	2/6	
They are able to explain why their solutions are wrong	4/6	1/6			1/6

5.2.2.5 Create Authentic Learning Experience

As the discussion ensued over the year at the two collaborative inquiry groups, the teachers repeatedly shared how authentic learning experience provided in the classroom engaged the students strongly in learning while developing in them deep understanding of the subject matter at hand. Even though, in the responses to the teacher survey in Table 5.7 and Table 5.8, "always start a new unit with a real life problem" was not picked as one of the top three most important conditions by any of the teachers. Using the strategy of starting a new unit with a real-life problem could be the teachers' intention; however, such a strategy was not always possible for every new unit of work.

One way of providing authentic learning experience shared was hands-on activities in real life, which appealed to adolescent learners who were still hovering between concrete learners and abstract learners.

Involvement of their senses, such as "going outside to measure things", helped the students to stay involved in a learning experience such as goal-oriented measuring activity:

> *So if you were doing measurement, we'd do things like, instead of them just figuring things out like circumference, we'd go outside and measure things or look at how we measure.* T1

Another way for the teachers to create authentic learning experience for the students during mathematics activity was to involve the students in a novelty such as eating a "chupa-chup" lollypop to measure the rate of melting of the candy in relation to the dimension of the lollypop. Teacher T1 described how such a novel task engaged the students in learning:

> *The thing is that we've got to do it like a test, they can't help each other. So other than timing yourself eating the chupa-chup, you'll take photos and document what you're doing. So open the chuppa-chup and lay it on the wrapper, use dental floss to measure around. That makes sense so far. The first thing that it gets into is "measure the distance on a ruler to get the circumference, that's the distance around in millimetres, and record the data in the table provided." So "consume your chuppa-chup one minute at a time for five minutes, measure the*

> *circumference in millimetres and record the data in the table below." So it's in an x and y table, which is really clear… So they'll go to five minutes. I'll probably have to buy a chuppa-chup today and eat it to see if it gets smaller in five minutes.* T1

Another way to create authentic learning experience was using local objects (e.g. "people and legs" and "cars and wheels") so that the students develop relationships between related objects to create a deep understanding in algebraic equations of dependent and independent variables:

> *Like I said, we just did one activity where we had people and legs. Then I made them do one of their own with cars and wheels.* T2

Teachers T1 and T2 used graphing of the rate of change of heart rates, taking into consideration the factors affecting trends in exercise to help the students to develop deep understanding of how factors such as fatigue affect performance in exercise and hence applied these factors to deduce algebraic relationships:

> *I made them do star jumps; we did star jumps for 10, 20, 30 seconds.* T1

> *Well it was good for mine to extend them because they didn't get a totally linear data, and we discussed "do you think your graph would continue like this?" and they were like "no, we'll get tired." So it was good to get them thinking, but this should be pretty easy.* T1

Using authentic learning experience also gave the teachers the opportunity to ask some relevant, meaningful but challenging questions to help them think critically (e.g. "what could you do to improve it?") and push the students to be more specific (e.g. "do you stick it in your mouth or do you lick it", "by rolling it (in your mouth) does that mean more saliva gets to it") and identifying the rate of reduction, outliers and anomalies in regard to the activities; these questions were more accessible for the Year 7 students to respond to:

> *What could you do to improve it? The rate of reduction.* T2

> *Or give/have a method of doing it, do you stick it in your mouth or do you lick it? Make it accurate.* T1

Do you roll it in your mouth, by rolling it does that mean more saliva gets to it? T2

Yes, we've done that. It says, "with clear working out, calculate the average time for teachers, students, and then all participants." But then for the higher level, it's got "describe two anomalies with the data." T1

So that would be the outlying sets. T2

It would be the outlier and the fact that you're comparing adults and kids when there's definitely a difference… there's more kids, it'll skew your data. What they should see is that the averages would be very different so your finding of everyone… T1

The creation of interesting tasks involving mathematical concepts while incorporating visual and aesthetic features of daily lives, such as stained glass windows, would create greater engagement through these authentic learning activities outlined below:

Yeah, they can do their own shapes and we take out those points. We've got those handouts that we used, the ones with the different squares. T1

They'd go home and flip them. T2

We can use those colouring in things… like drawing on a stain glass window, the kids have to colour in symmetry. T1

So we have two weeks on symmetry and then we go into geometric reasoning, which are all the angle and transverse. T2

Flip, slide, turn, reflective, symmetry, colouring, translations – it's all of them. You gave me these sheets. T1

Using activities involving ballista, bow and arrow, which interest adolescents while making their learning meaningful, further introduces the fun element to learning:

This looks more like a bow and arrow than a ballista. Let's see what they'll do. T2

Ballista and a bow and arrow are essentially the same thing, just on a bigger scale. T1

Similar examples of how the teachers endeavoured to use authentic learning tasks to engage the students were evident in the discussions about teaching higher grade students. This approach was consistent with the condition "authentic and real life assessments", listed on Table 5.7, ranked third most important condition conducive for mathematics learning by the teachers in the survey.

For instance, the use of visual examples was seen as a way of developing deeper understanding while teaching the formula for the midpoint of a segment which can be quite dry and boring. For example, teachers T7 contrasted a horizontal line segment and a diagonal line to help the students develop strategies of finding the co-ordinates of the mid-points of the segments. The students generated the understanding that "its an average of the two numbers (co-ordinates)":

I didn't give them the distance or midpoint formula. They developed it themselves. When I did midpoint with the kids, I started with "okay, this is a horizontal line, what is the midpoint?" They could all visually do that. Then I changed it to a diagonal line and asked what they were going to do. We talked about what we're effectively doing when we're trying to find the midpoint of two. A couple of kids came up with "it's an average of two numbers." *They all know how to find average so we went through that as a class. All of them might not remember the midpoint formula but they all know how to find averages. So in the worst case scenario, they can go back to that.* T7

A teacher shared another activity which was likely to engage students by using the technology available, such as excel spreadsheet and video. Such activity was deemed engaging because it appealed to students who were interested in using and producing multimedia-rich products:

I'd have it as a group thing, and have a group of five. Everyone has a minute and they compile it to make a five-minute video. T6

Potentially, we could have it as a group of 3, 4, or 5, and we evaluate the video as a class. I think that, in the end, it evaluates their Excel skills but it's also collaborative. T8

Real life examples helped students to relate mathematical concepts and enhance their life skills, such as overtime pay, pay rate, heights of students and reaction time, as shared by two teachers T6 and T8 during one collaborative inquiry session:

I think in finance, I'd like to have things like overtime pay, pay rates, etc. T6

I was teaching year 9s. They came up with their own research, like heights of the year 9s, or comparing two numerical things. I think that was too broad for them. The next year they said "you have to use a random sampler" and compare two things in the random sampler and the numerical data. This last year they did reaction time. They tested their reaction times against another variable. T8

In response to the question requiring the teachers to rank three conditions most conducive for mathematics learning, as evident in Tables 5.7 and 5.8, there was a strong agreement that a "supportive learning environment" is the most important, with five out of six teachers (more than 80% of the respondents) selecting it, while approximately 50% of the respondents ranked "good and probing questions" and "students taking ownership of their own learning" as ranking second most important. Even though creating "supportive learning environment" was not identified in phase two collaborative inquiry sessions, it was raised in phase one (see Section 4.2.2.4.2) when "optimal school structures, programs and resources" were identified as important for the implementation of collaborative inquiry as well as for student engagement.

It is noteworthy that "structured learning environment", "teaching algorithms and scaffolding well," and "always start a new unit with a real life problem" were not picked as the top three most important conditions by any of the teachers.

Table 5.7 Summary of rankings of conditions conducive to mathematics learning

Conditions which are conducive for maths learning (each teacher only ranked top 3 conditions).	Probability
Supportive learning environment	5/6
Structured learning environment	0/6
Teach algorithms and scaffolding well	0/6
Strong foundational basis such as number sense and times tables	2/6
Authentic and real life assessments	2/6
Good and probing questions	3/6
Problematised learning environment	1/6
Always start a new unit with a real life problem	0/6
Students are allowed to construct their understanding	2/6
Students taking ownership of their own learning	3/6

Table 5.8 Rank ordered conditions conducive to mathematics learning according to the teachers

Rank ordered conditions which are conducive for mathematicss learning	Probability
Ranked 1st	
Supportive learning environment	5/6
Ranked equal 2nd	
Good and probing questions	3/6
Students taking ownership of their own learning	3/6
Ranked equal 3rd	
Strong foundational basis such as number sense and times tables	2/6
Authentic and real life assessments	2/6
Students are allowed to construct their understanding	2/6
Ranked 4th	
Problematised learning environment	1/6
Not selected by the participants as top 3 factors at all	
Structured learning environment	0/6
Teach algorithms and scaffolding well	0/6
Always start a new unit with a real life problem	0/6

The three conditions, conducive to mathematics learning, listed in the previous paragraph, were not ranked as top three by the teachers could be attributed to the fact that the site was an independent school, having a high expectation that the teachers are able to provide a well-structured learning environment at all times. Hence, the above-mentioned conditions were considered given in the minds of the teachers. Furthermore, because the pedagogical imperative of the school required the teachers to all follow a unit plan which consistently outlined appropriate scaffolding of skills and knowledge, this may also have been considered a given condition. The last condition of starting each unit with a real life problem was a very specific teaching approach, championed by the Japanese lesson study or the constructivist approach to mathematics teaching, which may have been unfamiliar to the teachers at this site (Takahashi, 2014).

5.2.2.6 Mental Models and Beliefs of Teachers Set the Classroom Tone Affecting Student Self-concept

The data supported that the mental models and beliefs of teachers set the tone of the classroom and influenced the self-concept of the students, which in turn affected their academic achievement.

Here teacher T8 espoused his belief system that ability groupings of students suggest that one's ability is finite and stagnant at a certain level. However, he believed that, should the language used change to the amount of mathematics learned, then it creates the scenario of the possibility of growing in one's ability should one decide to put in greater effort. His beliefs were asserted in these statements: "I never talk about ability, I don't like that concept of ability in maths", "it is how much maths they've done", "we can change that with kids", "I'm a big believer of challenging kids, but I also think we should be rewarding effort", "we can't change ability, but we can change effort":

> *Whatever you call it in whatever school – you call it "extension class." What I said the difference sometimes between a maths A and a maths B student... I never talk about ability, I don't like that concept of ability in maths. It's about how much maths they've done. I had a kid in one school who was getting 30% in exams in grade 12. One year later in university, he was getting 90%*

in university exams. It clicked for him. We can change that with kids. It annoys me when a kid looks at me and says "I'm not good at this." T8

I say to them it's because they never had a good maths teacher. One of the reasons I don't like art is because in 8th grade my teacher said, "you get to pass art if you never take it again." Can you imagine saying that to a kid? So I've had that experience. I can't even put a cross on a test because I'm afraid that will say something to the kids. I'm a big believer of challenging kids, but I also think we should be rewarding effort. We can't change the ability, but we can change effort. T8

Another example of the mental model and belief of teachers affecting students' potential to achieve could be the growth mindset that the teachers have about their students. Teacher T8 argued that "they (the students) want adults to give them approval" and that "they want us (the adults) to think that they're good at something" and "we need to have kids in situations where they do experience success":

I still think that you can do this. I'm talking to experienced teachers, when the kids are eight, they still want their parents' approval. When they're 14, they don't necessarily seek their parents' approval. They want adults to give them approval, and that's why we're so important. They want us to think that they're good at something. That's something I grapple with. But on the other hand, we need to have kids in situations where they do experience success. T8

The teacher's mental model and beliefs create an environment conducive for students to take risks and engage in learning. Teacher T8 encouraged students to take risks by trying, saying that "the absolute worst thing that could happen is that you get the answer wrong" and asserted that "it's all about trying":

I say to them, when I ask a question and no hands come up, is, "the absolute worst thing that could happen is that you get the answer wrong. I'm not going to take away your birthday. I'd rather you try. It's all about trying." As you say, it's all about creating the environment. But, another way I thought of, is whether we could utilise the kids who are doing well and finish quickly. T6

The following example shows that, when teachers adopted a mental model or belief that an approach should be adopted to develop deeper understanding, then they would create a learning environment that encouraged the development of skills, such as having the students explain the reasons behind phenomena (e.g. "the reason behind things") or their answers instead of just using mathematical rules blindly:

> *It's just actually going through. I've made a focus this year on the reason behind things. I would say that my kids last year would have had the same knowledge.* T7

One main objective of this research was to investigate on site if collaborative inquiry sessions built into the teacher's work routine would be an effective professional learning tool.

The examples of how the teachers used the sessions to share specific successful strategies with one another, sharing the same context and school culture with the same resource constraints and facilities, show that collaborative inquiry is a powerful means for teachers to grow and develop.

At a subsequent meeting, teacher T8 shared a strategy about another way to use technology to provide students instant feedback during homework time or revision and consolidation time without the teachers' presence:

> *There are kids who rush through it. I, personally, wouldn't do it on the iPad. I used an app called 'Penultimate'. It's just a different way of presenting it. The kids love writing on it. I can see the answers and it's instant feedback. It's energy consumptive for the teacher, but that's our job.* T8

> *Theoretical, I used Khan Academy [for the teaching of compound interest], but I love "show me." What I was wondering about, was that, do you get the class to share it? I believe when kids have to explain something, that's when they learn the most. We talk about the teaching pyramid – you can pass a test, but can you explain the content? Potentially what you could do is to get four different groups and construct a quick "show me."* T8

> *I can see that as a phenomenal use of technology. I tried to do my first "show me" the other night, explaining the factorisation strategy. I tried it four or five times. You have to know exactly what you're saying. That's a presentation*

strategy for kids. Not only are we teaching them maths, we're teaching them presentation. T8

Stage two data analysis of teachers' choices of the types of students' learner minds would be most effective and responsive to mathematics learning and achievement. These classifications: "children as doers", "children as knowers", "children as thinkers" and "children as knowledgeable" were based on the work of Olsen and Bruner (1996). The teachers' responses were tabulated in Table 5.9.

Table 5.9 Teachers' choices of learner minds most effective for mathematics learning and achievement

Categories of learner minds (Olsen & Bruner, 1996)	Children as doers	Children as knowers	Children as thinkers	Children as knowledgeable
Very well	3/6	1/6	4/6	0/6
Well	3/6	5/6	2/6	6/6
Neither bad nor well	0/6	0/6	0/6	0/6
Bad	0/6	0/6	0/6	0/6
Very bad	0/6	0/6	0/6	0/6

On further analysis of the data summarised in Table 5.9, it was noted that the teachers generally agreed that these are important attributes of students who successfully master mathematics. However, the teachers favoured the skills of thinking and doing over knowledge, with only one out of six rating their students' knowledge as very well, while the majority of the participating teachers — five out of six and six out of six — only rated their students' knowledge as well.

It could be inferred, based on Tables 5.1 and 5.9, that the teachers held the belief that, if their students thought and practised regularly, it was more valuable than their existing or acquired knowledge.

The teachers' responses to the teacher survey on their beliefs about mathematics learning, teaching and achievement were tabulated in Table 5.10.

Table 5.10 Teacher beliefs about mathematics learning, teaching and achievement

Beliefs	Strongly agree	Agree	Neither agree or disagree	Disagree	Strongly disagree
Students need to be encouraged to explain and evaluate their solution process.	6/6	0/6	0/6	0/6	0/6
Memorising arithmetic facts (such as times tables) and algorithms is essential to mastering mathematics problem solving.	1/6	2/6	2/6	1/6	0/6
Mathematics curriculum has not changed since I went to secondary school.	0/6	0/6	1/6	2/6	3/6
Mathematics learning is being able to get the right answers quickly.	0/6	0/6	0/6	5/6	1/6
Mathematics learning process may include students experiencing uncertainty, conflict, confusion and surprise.	2/6	4/6	0/6	0/6	0/6
Mathematics learning is independent of students' real life experience.	0/6	0/6	0/6	3/6	3/6

From Table 5.10, it can be noted that the teachers generally agreed that "students need to be encouraged to explain and evaluate their solution process," and "mathematics learning process may include students experiencing uncertainty, conflict, confusion and surprise." There was also agreement that the Mathematics curriculum had changed since teachers' school days as students, that mathematics learning is not about getting the right answers, and that mathematics learning is dependent on the students' real life experience.

It is interesting that there was a greater disparity in opinion about beliefs such as "memorising arithmetic facts such as times tables and algorithms is essential to mastering mathematics problem solving" in that the responses ranged from strongly agree to disagree. This disparity could be due to the different ways in which the respondents interpreted the question, in that some may have responded literally, while others may have felt that there were more essential knowledge and skills to mastering mathematics problem solving than those two identified.

In summary, other than the disparity about what was essential for the mastery of mathematics problem solving, the middle school mathematics teachers at this site shared similar beliefs about the teaching and learning of mathematics.

5.2.2.7 Collaborative Inquiry Facilitates Sharing of Pedagogical Practice

Teacher T7 outlined several strategies at different meetings of using mental routines, games, and online tasks to enhance student engagement (e.g. "see where the kids were at", "the kids loved it") and mastery of mathematics concepts:

> *I would maybe break up a double lesson with a five-minute transition of math game that's doing a mental calculation and that kind of thing.* T7

> *I play games with all of them because they rely on the calculator so much, they need to start thinking.* T7

> *It's a program. It's Dr Barton, and he has hundreds of these. You have to download the program, but it's pretty self-explanatory. So this order of operations, I just search it and there are heaps of different levels. The kids loved it. That's how I started operations, just to see where the kids were at.* T7

> *Maths online. The other one is "I Excel Math.". That's a good one for the kids where they can also practice on the iPad and type the answers. The good thing about it is that if the kids get something wrong, it actually shows them the process.* T7

Another scenario where the collaborative inquiry sessions helped the teachers to grow professionally was the practice of consistent teacher judgement (CTJ). CTJ is a teacher peer moderation process for assessment tasks. This type of sharing is most beneficial for less experienced or newer teachers to the school such as teacher T6, who was in his fifth year of his career as a teacher, in comparison to teacher T7 who had been a teacher for more than ten years at the school. These two teachers were comparing how they assigned grades to the assessment tasks to reach a consistent teacher judgement:

> *I see what you mean. If I have a look at my –B's…* T7

Well that's close and you've given them a B. Those are no attempts. So for mine, instead of getting no attempts I've got three partials. T6

Down here, for the –B, they've got all of those and a half. T7

All and a half for a –B? For all and a half, I've given mine a –B +C. Take them down a peg, xxx [teacher's name withheld]. A partial in question 9. T6

It depends if they've done question 9. You can make a pretty clear distinction there about whether they understood it or not, or if it was just a random guess. If you don't think they actually understand what they're talking about. T7

Further sharing took the form of brainstorming ideas together which not only build the pedagogical skills of the younger teacher T6 but also provided an opportunity for teacher T6 to feel valued in contributing to the team's tool kit. Teachers T6 commented that "the students can't see the flow-on effect in an investigation" to which the more experienced teacher T7 responded "rather than having just match sticks, giving them other patterns and different types":

I think the students can't see the flow-on effect in an investigation. They don't realise what they've done is going to help them with what they do next. It's like, "I've generated a formula here by doing this, I don't know how to apply that in a differently worded question." If it's the same sort of the concepts where… they had to figure out two consecutive numbers and then three consecutive numbers, now what happens when it's 9 consecutive numbers? T6

In the booklet, rather than having just match sticks, giving them other patterns and different types. That might help because they couldn't see the link. T7

Some practical strategies for guiding students in organizational skills, presentation techniques and so on are not outlined in any unit plans or textbooks; these strategies need platforms from which newer career teachers can learn from more experienced ones, as quoted below:

With the assignments, this is the progress check this week. If you're concerned about any of the kids in terms of those first three questions, we can *give them some guidance because they're both C and D questions. It just drops off when it gets to the A's and the B's. So I said to the kids, they're not handing it in and marking it. We're just checking to see if you're on track.* T7

I just told the kids that with their original drawings – like question one there was three they needed to draw. I told them "use blank paper, draw your three-scale drawings." I explained how they can scan it through the photocopier… I want that original diagram in it because I don't want them sharing theirs. T7

On using visual spatial awareness to teach the more abstract aspects of co-ordinate geometry, the formulae to calculate and derive relationships of line segments on the Cartesian plane, the sharing of strategies was useful for teachers' growth and development:

I thought I was missing a few things so I added stuff like "here's the function, what's the perpendicular gradient?" We can do some simple ones, then some complex ones for the higher kids. Then in terms of problem solving, I was looking at the one with the diagonal. Like the Pythagoras one, I would replace it with the parking lot one. "One person walks a diagonal, one person walks the perimeter. How much further is it?" I also added in one to do with the rhombus like "find the property to do with the midpoint of each diagonal. What's special about it?" They should be able to identify that they're perpendicular. T7

"Well you can give them the diagram of the rhombus and say "what's special about this rhombus?" Because you give them the coordinates. T6

At another meeting, a concept to teach Year 9 students algebra was shared and discussed to ensure that the finer points of potential confusion could be avoided. For example, teacher T7 asked "how people explain collecting like terms" and teacher T6 shared how he used technology to show the students how to collect like terms:

I'm always interested to see how people explain collecting like terms. I always discuss A, A squared. I've always found kids struggle with A and A squared – "why can't you add A and A squared?" I talk to the kids about apples. There's *all types of apples but I don't like green apples. A squared might be a green apple. I also talk about it as a concept with a box.* T7

The file is sometimes 70MB or 80MB because it's video and audio. Any laptop becomes an Apple TV so I just plug it in and connect to it. I'll get my stylus out and I'll be able to sit next to the kid and do it so that everyone can see. T6

Another idea shared was to use various online platforms to enhance students' learning experience through just-in-time learning, using technology:

What I've said to the kids is, is you have to sign up via email because when you sign up, you're given the option of using Google or Facebook. It also has to be a school email, and you have to use your first and last name with a capital. They show you the video from where your knowledge is. They assess where you're at and what gaps you have. I think it's a great tool. I see Khan Academy and the use of an iPad app as a tool. It's not all one, it's a combination of everything. I'll let you guys know how I go with that, because I can assign activities to kids, but it can also be self-paced – particularly for students who are ahead. T8

They can do that, but the key thing is that it differentiates where they're at, so if there are holes... Sometimes a kid's learning is a bit like Swiss cheese, there are holes. This will assess them based on that. At the same time, there are different layers you can assign them. Once you've set it up and you actually have coached the kids, you can assign activities. You don't have to assign it to all the kids, you can assign it depending on how much practice they need. T6

Table 5.11 collated the teachers' responses to the frequency of pedagogical practices used recently. According to Table 5.11, the pedagogical strategies which were least used in the immediate past unit of work by the teachers were "setting group work activities for students to develop the concept on their own" and "creating a situation where the students need to ask questions to learn". While on the other hand, "creating a situation where the students need to ask questions to learn" and "explaining to students the theory behind a new concept" were frequently used practice by the teachers.

Table 5.11 Teachers' pedagogical practice frequency and difference in unit of work

Pedagogical practice frequency in recent unit of work	Almost never	Seldom	Some-times	Frequent-ly	All the time
Showing students the fundamental of a new concept			1/6	4/6	1/6
Explaining to students the theory behind a new concept			1/6	3/6	2/6
Modelling how to apply a concept before students try it			2/6	1/6	3/6
Providing scenarios for students to find solutions in groups		1/6	1/6	4/6	
Setting group work activities for students to develop the concept on their own		3/6	2/6		1/6
Helping students learn the fundamentals before giving the challenging tasks			2/6	2/6	2/6
Creating a situation where the students need to ask questions to learn		1/6	3/6	1/6	1/6
Providing scaffolding for students to construct their own understanding			2/6	3/6	1/6

Table 5.12 summarised the most often used teaching strategies by the participant teachers in their classrooms. Data shown in Table 5.12 reinforces the earlier observation that the preferred teaching approach was didactic, in that the teaching activities were mainly teacher led and directed. This summary is also confirmed by the diverse or low preferences of the teachers for student-centred and student-led approaches such as "students working in groups to solve authentic problems" and "encouraging students to figure things out on their own rather than showing them how to solve the problem". Table 5.12 showed that at least 5 out of 6 teachers reported that they used the following strategies "all the time" and "frequently": "showing students how to apply concepts to routine problems", "allowing students to apply concepts to challenging problems on their own" and "asking 'why' questions to check for student understanding" suggested that the teachers attempted to develop the higher order thinking in the students. Teachers also ensured students have strong basic foundation by "ensuring the students rehearse the basic procedures and methods in demonstrated exercises".

Table 5.12 Summary of most often used teaching strategies in the classrooms

Most often used teaching strategies during my mathematics classrooms:	Almost never	Seldom	Sometimes	Frequently	All the time
Showing students how to apply concepts to routine problems			1/6	3/6	2/6
Allowing students to apply concepts to challenging problems on their own		1/6		3/6	2/6
Students working in groups to solve authentic problems		2/6	3/6	1/6	
Asking "why" questions to check for student understanding			2/6	2/6	2/6
Asking clarifying questions to check for understanding			1/6	3/6	2/6
Referring to textbook examples and questions		2/6	3/6	1/6	
Structuring my lessons based on the textbook in order not to confuse the students	3/6		2/6	1/6	
Encouraging students to figure things out on their own rather than showing them how to solve the problem		1/6	3/6	2/6	
Ensuring the students rehearse the basic procedures and methods in demonstrated exercises			1/6	4/6	1/6
Ensuring the students master the algorithms through drill and practice		1/6	2/6	2/6	1/6
Helping students who are stuck straight away, to avoid them developing dislikes or frustrations about maths learning		1/6	2/6	1/6	2/6
Quiet individual work to consolidate their learning and thinking		1/6	3/6	2/6	
Always starting with teaching formal mathematics symbols and algorithms to ensure that they have strong basics and foundation	1/6	1/6	2/6	2/6	

It is interesting to note that the strategy "referring to textbook examples and questions" was not commonly used by the teachers, while during the collaborative inquiry sessions over the two years of the data collection period, the teachers indicated use of textbook examples and questions as their foundational approach. During the collaborative inquiry sessions, units of work were perused and they referred to examples and questions in the textbook, which once again contradicts the teachers' self-reported practice.

The participating teachers' reports of the pedagogical practice most frequently used in the recent unit of work indicated that they tended to use very conventional teaching practices that favoured didactic teaching pedagogy (e.g. "show students the fundamental of a new concept", "explain to students the theory behind a new concept", "model how to apply a concept before students try it", "help students learn the fundamentals before giving the challenging tasks" and "provide scaffold for students to construct their own understanding") (see Table 5.12). This suggests that the common mode of instruction was to focus on show and tell first, and then let students practise, whether individually or in groups, while they assisted.

The above observation can be triangulated with the seldom practised strategy of "setting group work activities for students to develop the concept on their own," or "creating a situation where the students need to ask questions to learn"; it is apparent that this constructivist approach to teaching mathematics was not favoured by this collective group of practitioners.

5.2.2.8 Memo: Teacher centric factors influencing collaborative inquiry implementation

The teachers regularly expressed concerns about their students' success in mathematics. The teachers articulated the problems and challenges faced by the students without offering solutions or strategies to manage these problems and challenges. Even though the teachers identified that students required to experience success in learning, they were focussing on the deficiencies of the students rather turning their attention to address these deficiencies. There appeared to be a "blame" approach by the teachers about student issues.

The mental models and beliefs of the teachers were inferred as setting the classroom tones and affecting the students' self-esteem, and hence their achievements. Authentic learning experience, according to the teachers, strengthened student engagement and understanding.

The teacher profiles indicated that the participants in this research were of differing educational backgrounds, years of experience, opinions and beliefs about the teaching of mathematics and professional learning needs to enhance the student achievement.

It can be deduced that the teachers held the basic beliefs that "children as doers" and "children as thinkers" were significant for student mastery and achievement, while "children as knowers" and "children as knowledgeable" were deemed less significant for student mastery and achievement in mathematics (see Table 5.9).

The survey showed that the teachers also had consistent responses to six given statements on beliefs about the learning of mathematics, while "memorising arithmetic facts (such as times tables) and algorithms are essential to master mathematics problem solving" was the only one that elicited a wide range of responses from the teachers (see Table 5.9).

The teachers' responses to conducive conditions for mathematics learning were consistent with their responses quoted above, in that they selected "structured learning environment,", "teaching algorithms and scaffolding well," and "always start a new unit with a real life problem" as the best strategies

Teachers' beliefs about mathematics achievement were generally consistent with one another except for "students need to be rewarded for the right procedures more than getting the right answers," and "students who do not achieve well in mathematics should try hard, such as go to tutorials, or do more homework questions". The diversity in beliefs for these two attributes was more evident.

The top three factors affecting students' mathematics achievement were voted as "students transfer their knowledge to unfamiliar situations by linking the old and new concepts," "students not afraid to make mistakes," and "students have strong foundational basis such as number sense and times tables"; while "students are happy and engage in real life problem-solving" and "students construct their understanding by relating ideas and articulating them" were ranked equal third (see Tables 3.3 and 3.4). The responses to these factors indicated apparent contradictions in the teachers' articulated beliefs and their practice.

Similarly, the data on most frequently used pedagogies and most often used teaching strategies as reported by the teachers showed inconsistency between teachers' self-reported beliefs and self-reported practice.

Teachers appeared not to favour questioning students to ascertain whether they had developed mathematical understanding of the new concept. The teachers favoured testing their understanding by asking them to explain the concepts or errors in applications to others, or apply them to new situations.

Teachers agreed on the types of effective professional learning activities. They also agreed with the statement that teachers' professional learning activities impact on student achievement, which appeared to contradict their earlier statement choices in this same survey.

5.2.3 External Factors Influencing Collaborative Inquiry Implementation

The external factors that affect the effective implementation of the collaborative inquiry sessions are discussed below.

5.2.3.1 Over Emphasis on Assessment Materials

It was noted in three of the meetings that the whole 50-minute collaborative inquiry sessions were used to discussing the assessment items in great detail. Furthermore, during the other sessions, it was noted that more than 10 minutes or 20% of the collaborative inquiry time, was used to discuss the fine points of assessment items.

Such use of collaborative inquiry time could be due to administrative constraints at the site, where the teachers were required to spend an extensive amount of time finalising their assessments to certain set standards imposed by the head of department of mathematics. Such practice could also be due to organisational challenges, where the teachers may not be teaching close by or located together, so that collaborative inquiry sessions were their only time for extensive deliberation.

5.2.3.2 Expectations and Focus on Teaching Courses Instead of Students

Another factor which would affect the effectiveness of collaborative inquiry sessions for teachers' professional growth was when, instead of using the sessions to discuss strategies for teaching and learning, the teachers focused on administrative tasks or teaching the course, rather than on teaching the students and assessing their progress and understanding. Teachers T2 lamented below that her students lacked some basic mathematics knowledge of "multiple and divide", "can't subtract" and "they don't know how to carry, they can't multiply by 2 digits" to access the Year 7 curriculum. Consequently, she had to resort to "fill the gaps first" before returning to the Year 7 curriculum. Teacher T1 concurred with teacher T2:

> *That's what I mean. I can't do that to them in 5 weeks. I can't possibly dumb them down from –B to a +D in five weeks. So what's going on, at that level? What I've got to do now is, I told [teacher's name withheld], "I can't go on with the year 7 curriculum when these kids can't multiply and divide." So, whether I try to do two lessons of my four a week, trying to fill the gaps and going back. Then I do two lessons a week, which is the content.* T2
>
> *That's what I talked to [teacher's name withheld] about, because I can't fill the gaps and do the curriculum.* T1
>
> *And just picking up things up from that maths mastery, the kids can't subtract, they don't know how to carry, they can't multiply by two digits – the stuff we expect and, no wonder they can't do fractions. So that's how my week works.* T2
>
> *You know, even holding up a 3D object, they didn't even know what a vertice was. So they now know edges, vertices… math mastery goes through all the concepts so I'm seeing a faint light on.* T2

5.2.3.3 Curriculum Compacting Affecting Deep Understanding of Students

In schools, the teachers often feel helpless in their teaching planning because of external pressure outside their control to complete certain concepts and topics within a certain period of time. These expectations are often set by authorities higher than themselves, such as mandatory requirements, curriculum authority, or even the school management.

Such curriculum compacting could hinder students developing deep understanding of the concepts taught, because the teachers are required to complete the units of work within a given time frame in order for assessment and reporting to be completed, as evident in the comment made by teacher T2 below:

I don't have the time to get through the concepts and cover them. I really believe they move through the concepts so fast that the kids don't remember them the next year. They don't get that in-depth learning. We're trying to do fractions, percentage, ratio, index notation, integers, negative and positive, and money, in term one. T2

Another frustration shared by teacher T8 was that the crowded curriculum had taken out of the teachers' hands the power to respond to the students' learning needs instead of just delivering the planned curriculum, whether or not the students were developing any understanding on the subject matter:

The problem being is that we're so driven by the curriculum, but we should explain the question. We should have a collaborative way of saying "can you explain this in your own words?" This [activity that [teacher's name withheld] presented] certainly summarises things really well and shows the kids that there's more than one way of doing it, and there's more than one way of explaining it. It frustrates me when kids say "do I have to do it your way, sir?" As long as they have working and have the answer, and have mathematical reasoning, I'm happy with it. T8

5.2.3.4 Competing Mandatory Requirements Impacting on the Quality of Collaborative Inquiry

Another external factor that often frustrated teachers was the competing attention of mandatory requirements exposed by external agencies such as the national testing regime, which often drew the teachers' attention away from the students' learning needs, due to the need to ensure that the expectations of the legislative organisations or the school management were adequately addressed:

Year 7 don't know the formula for area of a triangle. Because we haven't done measurement yet, we haven't done it, but I thought they would have already done it in grade 6, but they haven't, so that's something I have to do this week. So if it's in NAPLAN, we're in trouble. The other thing they don't know is shapes. They don't know what a prism is. T2

We worked hard enough! I got rid of all the D's, which is delightful. I've got a few –C's but I'm so happy the D's are gone. We're talking about kids who *can't divide by a single digit. I think we're doing okay. But that's the whole cohort; the class would be a bit different.* T2

5.2.3.5 Memo: External Factors Influencing Effectiveness of Collaborative Inquiry

Some of the teachers' discussions concerned factors external to the classrooms, and this focus appears to have impacted on the effectiveness of the collaborative inquiry. The factors included: an over-emphasis on assessments; a focus on teaching courses rather than students; over compaction of the curriculum; and mandatory requirements such as NAPLAN testing and public reporting.

It was also stated by the teachers that the expectations of the mathematics department head in regard to completion of certain administrative tasks reduced the productivity of the collaborative inquiry session as an effective professional learning tool.

More evidence was collected that supported these points. This could be due to the fact the that Year 7 students were in their first year of high school and adjusting to changes in their learning setting, and the year 7 teachers tended to be more caring and gentle with their students. The teachers of Year 9, on the other hand, tended to be concerned with preparation for senior schooling and hence were more detached or focused on the task on hand.

Furthermore, the teachers' mental model that emerged in Year 9 in phase two of data collection matched that emerging from phase one data in that the teachers shifted the focus on students' ability about their ability to learn to how they could change their pedagogical approach to enhance learning. However, the limited change could be due to changes in staffing and culture in the second year of this research project. There seemed to be an increased sharing of strategies for teaching with the aim of collective growth.

5.3 Summary of Year Two Findings

Data in phase two of the research allowed the triangulation of collaborative inquiry data against the teachers' mental model and beliefs survey. As outlined earlier, the objective of the survey was to assist me to develop an understanding of how the teachers' mental models and beliefs about teaching and learning, as well as their professional development, would impact on their professional practice. The triangulation of data from the collaborative inquiry sessions and the survey data provided a glimpse of apparent contradictions between the teachers' articulated beliefs and practice beliefs, as shown in discrepancies between their perceptions of their practice and the collaborative inquiry sessions when their real beliefs about students and their achievements might have been revealed in context. This apparent gap of teachers' espoused mental model and their in-action model was supported by the studies of Entwistle and Peterson (2004) as illustrated in Table 2.1 and the study of Manrique and Abchi (2005). The gap could be verified more effectively in my study if I had observed their practice in the classrooms (i.e. their in-action model) and compare that with their espouse model (i.e. beliefs articulated at collaborative inquiry sessions and teacher survey outcomes).

In the phase two collaborative inquiry sessions (with 42% of the Year 1 teacher participants, namely, teachers T1, T2, T6, T7, and without teachers T3, T4, T5, then the replacement of teacher T7 with T8), a different, more positive and solution-focused tone was observed in Year 2 collaborative inquiry sessions. This change could be attributed to there being fewer negative opinions, and a growth and development of teachers in the collaborative inquiry process, which suggests that collaborative inquiry is an effective professional learning tool for middle year mathematics teachers. Referring to Figure 2.2 to verify this observation, it can be argued that the teachers' meta-assumptions about teaching and learning might have shifted over the one year period due to the professional and reflective dialogues with their colleagues. These dialogues challenged them to unpack their assumptions, adopting various perspectives, resulting in the transformation and evolution of their teachers' belief, mental model and practice (Manrique & Abchi, 2015; Olsen & Bruner, 1996).

Apart from the different opinions about what is essential for the mastery of mathematics problem solving, the data indicate that the middle school mathematics teachers at this site shared similar beliefs about the teaching and learning of mathematics. The teachers held the belief that, if the students are taught and practised regularly, they would develop their existing or acquired knowledge.

The collaborative inquiry sessions also provided the teachers with opportunities to discuss their concerns about their students' achievement or lack thereof, individually or collectively, even when the teachers only saw the problems of the students, but not the solutions. This was facilitated by a collegial setting where concerns and frustrations could be aired and possibly addressed at a later stage. Providing the teachers in the same department and school with the platform to inquire into an issue is one of the benefits of collaborative inquiry which is similar to the phase one finding that such approach to professional learning elicits pedagogical strategies.

For effective implementation of collaborative inquiry as a professional learning tool, some teacher centric and school wide factors need to be taken into consideration in the planning. Accepting and addressing the concern held by the teachers about their student achievement can help them to focus their time during collaborative inquiry on sharing strategies about how "create authentic learning experience" so that the students can engage in "doing rather than having things done for" them (Cook, 2008). As student skills were deemed important by the teachers from both the survey and collaborative inquiry data, strategies to develop these skills can be included as one of the foci of collaborative inquiry implementation. These two foci can be classified as effective pedagogy as outlined in Figure 4.1 as one of the essential elements of the collaborative inquiry model. Were teachers to agree on the inquiry topic, have strong facilitator skills (as identified in Chapter Four), to focus on the assessment materials instead of how to teach in order to better develop the students' deeper understanding, the teachers can be guided back to the agreed topic and focus on inquiring into the pedagogy.

Collaborative inquiry sessions also revealed that the teachers' mental models and beliefs determined the classroom tone, which impacts on the students' self-concept and success. The teachers shared the importance of creating authentic learning experience to strengthen student engagement and understanding. This self-reporting position of the teachers contradicted their self-reporting practice as summarised in Table 5.11 and 5.12 with the seldom practised strategy of "setting group work activities for students to develop the concept on their own" or "creating a situation where the students need to ask questions to learn"; this constructivist approach to learning mathematics was not favoured by the teachers as a collective group of practitioners. This summary of the results reinforces the earlier observation that the preferred teaching approach of the teachers was didactic or traditional transmission instruction according to Ginsbury and Dolan (2011), in that the teaching activities were mainly teacher led and directed instead of the constructivist-compatible instruction approach. Such contradiction can be attributed to the gap between the ideal pedagogy and practice in reality for the teachers (Manrique & Abchi, 2015; Olsen & Bruner, 1996). The gap between perception and practice could be due to one of the three possible determinants of teacher pedagogy as suggested by Ravitz et al. (2000), namely, teacher background and role orientation, student ability and socio-economic background and school professional culture (Jones, Ross, Lynam, Parez & Leitch, 2011). As the research site is a private and high-average fee charging school, it eliminates two of the three determinants (i.e. the socio-economic background of the students and their abilities and the school professional culture) leaving teacher background and role orientation as the main factors. Referring to Table 5.1, even though seven out of the eight teachers have more than five years' teaching experience, only four out of eight have taught middle years' mathematics in excess of ten years and three of whom have had less than five years' experience teaching mathematics to this age group. Hence, a likely reason for the gap between practice and perception is that the teachers have not gained good understanding and experience of effective middle schooling pedagogy.

Another example of the gap between perception and practice was that the strategy of "referring to textbook examples and questions" was not commonly selected by the teachers in the survey but, during the collaborative inquiry sessions over the two years of the data collection period, the teachers indicated that the use of textbook examples and questions was their foundational approach. During the collaborative inquiry sessions, units of work were perused and the teachers referred to examples and questions in the textbook, which again contradicts the teachers' self-reported practice. Jones et al. (2011) and Manrique and Abchi (2015) agree that mental models can be amendable due to the contexts even though they are difficult to elicit, measure or inspect directly. Hence, professional learning approach seeking to change mental model or align mental models of teachers with those of the schools can be challenging; however, being cognisant of the teachers' personal orientations towards new learning and ideas can help (Jones et al., 2011).

In this exploration of whether collaborative inquiry is a valid tool for middle school professional development, it is noteworthy that the teachers revealed through their dialogues with one another their deep concerns about the challenges that they faced in curriculum compacting, trying to cover the courses instead of focusing on the teaching of students, and fulfilling mandatory requirements such as NAPLAN and department needs. Through the dialogues, the teachers shared strategies and approaches through which such frustrations can be managed or resolved, showing collaborative inquiry is a valid forum for sharing their concerns and generate local intervention strategies due to their ownership of the issues and sharing the same constraints and resources at the same site (Dufour, 2007; Garratt, 2010; Habegger & Hodanbosi, 2011; T. H. Nelson, 2009; Robert & Pruitt, 2008).

Some external constraints challenged the effective implementation of collaborative inquiry as a professional learning tool in schools. Constraints other than the one mentioned earlier (e.g. over emphasis on assessment materials) include a focus on teaching courses instead of students, curriculum compaction and competing mandatory requirements. These external factors inadvertently shift the focus of the teachers from using collaborative inquiry to developing their pedagogical practice and creating optimal learning environment to benefit the students in meeting external imposed demands on their planning time.

The theory emerging from the inductive logic at stage two as outlined in Table 3.1 consolidated stage one finding that for collaborative inquiry. Theory evolved at the end of stage two is that “more focused and better facilitated collaborative inquiry can be an effective professional development tool for middle school mathematics teachers to enhance their teaching practices and hence student achievement”. On the other hand, the deductive logic generates the theory that “collaborative inquiry can only be an effective professional development tool for middle school mathematics teachers to enhance their teaching practices, and possible student outcomes, provided that certain preparations enable staff’s mental readiness of the professional growth and learning”. These intermediate sampled theories provide the foundation for stage three data collection and analysis in that the teachers were interviewed individually to provide their views on the initial findings of the study.

CHAPTER SIX

PHASE THREE DATA ANALYSIS AND FINDINGS

6.1 Introduction

The findings of the previous chapter retain the thread of a range of teacher centric and externally imposed factors which are likely to impact on the implementation of collaborative inquiry as a professional learning tool for middle years' mathematics teachers. These factors included the teachers' mental models and beliefs about teacher pedagogy, learning focus, an optimal learning environment, curriculum construction and assessment.

In this chapter, the data analysis and findings focus on the two sets of individual interview data collected to ascertain the teacher perceptions about optimal learning environments, pedagogy, professional learning and assessment. In addition, the feedback from teachers on the theoretical collaborative inquiry model developed using phase one data is analysed and incorporated into the final theoretical model.

6.2 Data Analysis

The data collected for phase three were transcribed and analysed using the process of coding, memo writing, theory sampling and theory saturation process to refine the model developed in Chapter 4 (i.e. Figure 4.1) to illustrate the emerging theory on the topic under research.

6.2.1 Perceptions of Optimal Learning Environment

The teachers shared their opinions and positions on the optimal learning environment, revealing their mental models and beliefs.

6.2.1.1 An Optimal Learning Environment is Critical for Students' Engagement

The teachers expressed that an optimal learning environment in the classrooms is critical to student engagement in learning and their success in mathematics learning.

Teacher T2 suggested that teachers ought to create a safe, warm and inviting learning space (e.g. "feel safe in the space", "you need to be warm and inviting") in which the students can work and learn:

> *School environment is very important – feel safe in the space; you need to be warm and inviting; visually inviting space; comfortable to work; old school not the best today; I think we can do it better to move to 21st century.* T2

Others felt that parents also influenced their children's attitudes to learning, engagement and optimal learning environment (e.g. "Their parents, [influence] whether they are open to learning" and "correlation of parental engagement and children's learning"):

> *Their parents, whether they are open to learning.* T6

> *Depends upon the ability of the students and the parental involvement in the children's education; correlation of parental engagement and children's learning.* T8

The above comments suggest that the optimal learning environment involved the children's personal attitudes, which are influenced by external factors such as parents' engagement, attitudes towards the children's education, physical environment and the sense of wellness of the students.

Teachers T1, T2 and T8 agreed that an optimal learning environment was critical to engaging students in learning. It was seen as foundational by teacher T2 because, "without which (i.e. it), it does not matter how good the teacher's pedagogy is, they are not going to get through their learning". This same position was held by teacher T8 who stated that, "everything happens in school must be optimal environment":

> *Optimal environment – understanding reports; design; go and do it herself; create a video. Authentic assessment we mark.* T1

> *Optimal environment also includes the teacher pedagogy.* T1

> *Of those three, that's [optimal environment] to me is the most vital, without which, it does not matter how good the teacher's pedagogy is, they are not going to get them through their learning.* T2

I like the optimal environment – all encompassing; everything happens in school must be optimal environment; teacher pedagogy is key to that, I would think more than authentic assessment; but may be authentic learning experiences; may be that comes with teaching pedagogy. But then arguably assessment is part of pedagogy as well; I do like the word "authentic" because we have the WHY generation where they want to know why they have to learn or do something before they give their cooperation and efforts. T8

6.2.1.2 Teachers' Responsibility to Create Optimal Classroom Environment

As teachers were often the only adults in the learning environment of the students at school, the teachers interviewed felt that the responsibility of creating an optimal classroom environment lands solely on the teachers.

Teacher T8 identified that the teachers' passion, enthusiasm and knowledge of the subject matter to be taught as well their knowledge of the students (e.g. "teacher's enthusiasm and knowledge of the subject" and "…and passion") were important when asked what were elements essential to the creation of an optimal learning environment:

Teacher's enthusiasm and knowledge of the subject – if they are enthusiastic about their subject matter, and passion; translate the enthusiasm to want the students to want to do the topic and understand the topic; competency – experience in teaching the subject – having taught the subject matter each year, getting greater understanding of the subject matter and the learning needs (how they learn and their challenges) of the students. Develop the strategies that intertwine the concepts. T6

It was noted that, even though the teachers influenced the optimal learning environment, our current educational culture was still one of teacher-centred pedagogy, which was perceived as not good practice because it is not student-centred according to teacher T8.

We are still too teacher-centred in education around the world; there are pockets of good practice but largely teacher-centred. T8

It was suggested that the lack of a student-centred approach in teaching was largely due to time-poor teachers not having the time to design student-centred activities; another factor mentioned was that the teachers needed to be in constant control of their classrooms, and student-centred activities were seen as lowering their control. Teacher T8 commented:

1. *Classroom activities, easier to be teacher-centred*
2. *Nature of people who go into teaching are "attention seeking" – like the control aspect; (is it our concept of teaching?)*
3. *Time factor – needs time to plan worthwhile student – centred activities.* T8

Teachers T1, T2 and T6 considered that the teacher's mental model of teaching and learning was crucial to creating an optimal learning environment. However, they felt that as long as the teachers possessed growth mindsets, their mindsets could be "developed" and "moulded" for the benefit of the students at the site:

> *I think that all three (TP, AA, OE) are important; the hurdle is still teacher's mental model and belief – they will be able to manage.* T1

> *So this is on an individual basis? The teacher's belief and walks into the classroom with? It is the key, if they are not confident, and not engaging [with] what they are teaching; the students will pick up on that; the teachers have to go in with the confidence in the content they are teaching and how they are going to teach it; the tools they are going to use and their passion about what they are teaching; that would come through.* T2

> *Mental model and belief of the teachers can be developed.* T6

6.2.1.3 Catering for Different Needs of Students- Differentiation

Creating an optimal learning environment for all students required the teachers to keep up to date with research through reading or networking with colleagues to determine suitable learning conditions for different students and cater for different learning needs (e.g. "reading, think about research and listen to what others are doing" and "I can do different things to cater to the learning needs of the students") which were stated by teacher T1:

Teacher T1 shared how she allowed some latitude in students' discussions and interests on the subject matter by being open minded and flexible.

> *Reading, think about research and listen to what others are doing; recognise the classes are different; I can do different things to cater to the learning needs of the students; we started with discussions, old school style theory discussions, then practical activities, research or discuss things that pop out during the class discussions. There's a structure, keeping it relevant.* T1

Teacher T2 concurred with the need for the teacher to cater for the diverse learning needs of students in the classrooms through collegial sharing. She asserted that teachers need to be observant (e.g. "intuition of the teachers comes from their observations of the students") at all times to ascertain the students' preferences for learning (e.g." how they learn" and "the teachers need to be aware of the students' preferred learning styles") to enable them to create an optimal learning environment:

> *Teacher is the greatest impact; it's the personality of the teacher to be very diverse to link with the student different learning styles.* T2

> *Students are different and have different learning styles; the intuitiveness of the teachers to be able to discern how they can have positive impact and create an optimal environment for the student learning.* T2

> *Intuition of the teachers comes from their observations of the students; how they learn via which methods, e.g. different learning styles. ... [because] as we get to know them, they give up clues, e.g. how they approach the question, the teachers need to be aware of the students' preferred learning styles. (Their metacognition is not very high or acute); they don't know how to tell you how they prefer to learn.* T2

6.2.1.4 Challenges with Differentiation for Larger Class Sizes

Teacher T2 expressed the practical concern that the number of students in the class impacted on the optimal learning for the students; a larger class size meant a larger variation of student learning needs and hence posed greater challenges to the teachers to differentiate between them:

Number of students in the class affects the optimal learning for the students; more students harder to differentiate their learning needs and achieve success; students in the class from different background have effects on other students' learning in the class. T2

Teacher T6's commented that "you have to know the students in order to develop the pedagogy" illustrated that differentiation can occur when the teachers know the students and hence their learning needs:

You have to know the students in order to develop the pedagogy; see students as puzzles and try to unlock the puzzles. They need to know you care. T6

6.2.1.5 Engagement Means Extending Beyond the Classroom Materials

Teacher T1 shared that, in order for the students to engage in learning and hence, have optimal learning, they must first enjoy the learning experience and want to explore beyond the confinement of the classroom activities:

Engagement is when the students send materials to me to discuss every few days; these are A to C students. They need to enjoy and engage in their learning. T1

Teacher T6 shared that lessons with real world applications often engaged the students when responding to the question of how to create an optimal learning environment to engage students in learning (e.g. "real world applications", "link algebra to something the students already know"):

Real world applications. T6

Link algebra to something the students already know; the idea that it started from and where they will end up. T6

6.2.1.6 External Environment, such as Supportive Management Staff and Additional Resources, Affect One's Performance

When attention turned to what external environmental factors can affect the teachers' ability to create optimal learning environment for students, teachers T6 and T8 pointed out that middle and senior management need to have a growth mindset about their teachers and be supportive of the teacher by creating a work environment that encourages the strategies and initiatives for collaboration and support for collective growth of the staff:

> *I could consider things like needing the support of your line manager within the environment; not just the classroom environment but the great work environment, e.g. your line manager supports your assessment items.* T6

> *If they are given the idea that it's less work, a lot of these teachers will jump on board. I shared everything I did, I shared worksheets, PDF, OneNote. E.g. xxx [teacher's name withheld] used it all the time, she amended it to suit her class.* T6

> *You got to know your students and adapt. Many teachers got stuck because they were under time pressure so that they did not try. So if it is prepared, they heard it at staff meeting, they just have to implement it once a term, then it's something they can develop and it's something that can be developed and the growth mindset and sharing and support.* T6

> *Do the school leaders have the growth mindset of their teachers?* T8

6.2.2 Perceptions on Pedagogy

The second aspect of teachers' beliefs and mental models examined during the individual teacher interviews was pedagogy. Several points were raised about how some internal and external factors impacted upon the teachers' mental models and beliefs about pedagogy.

6.2.2.1 Freedom of Choice of Pedagogical Practice

To create an optimal learning environment, teacher T1 felt that teachers need to be given the freedom and latitude to adopt the most suitable pedagogy for their classrooms; T1, who has taught at other sites, felt that this existed at the site of research:

> *At school, we have the freedom to teach how we wish to teach; it's not prescribed in maths and science.* T1

Teacher T1 shared that, to create an optimal learning environment, she constantly networked with other teachers to trial teaching and learning approaches learned. In addition to her own growth, she felt that it demonstrated and role modelled to students how to take risks and try something new or different for improved outcomes:

> *I like to try something other teachers shared which worked for their classes; we are encouraged to do so and we model to students that we are prepared to take risks and try something.* T1

6.2.2.2 Colleagues Influence Teacher's Motivation

Teacher T1 pointed out that the other colleagues' attitude affects the teacher's enthusiasm and motivation for implementing new pedagogical practices; for example, other teachers' level of interests and engagement in the new practice:

> *Looking for student engagement; I am willing to try; when others do not have the same enthusiasm, I feel frustrated.* T1

6.2.2.3 Teachers' Ongoing Evolution for Growth

Teacher T1 shared that teachers' growth is akin to an evolution for which incremental changes take place over a period of time. Change of pedagogical practice and mastery does not occur overnight but through ongoing reflections and modifications over many years:

> *I reflect on what worked the previous years and tried different ways of doing things to increase engagement.* T1

6.2.2.4 Engagement Generates Learning Interests

Teacher T1 felt that the immediate results of student engagement may not be evident in academic outcomes; at times, increased student engagement translates to a change in students' attitude toward learning, from disinterested to interested:

> *They did not like a subject; but now like the subject even though their results remained unchanged.* T1

6.2.2.5 Engaging Teachers Produce Engaging Students

Teacher T1 shared that teachers were learners who would grow in their profession if they maintained interest and curiosity in their work (e.g. "constantly looking to do new things and change things", "curiosity – teachers who love what they do" and "if teachers are engaged, the students are engaged as well"). They naturally would create an interesting learning environment to arouse the students' curiosity and sustain their interests. In other words, if the teachers were engaged in their profession, their students would be engaged in the learning:

> *Engage everyone in the classrooms. Constantly looking to do new things and change things. Learning by doing is what I focus [on]. Active participation of the learning process – engagement.* T1

> *Curiosity – teachers who love what they do; teachers want the students to enjoy schools; not just doing my job but share my interests;* T1

> *Teachers need to be learners to focus on the learners' needs. Doing stuff with them; keeping checking in; if teachers are engaged, the students are engaged as well.* T1

6.2.3 Perceptions of Professional Learning and Growth

The interviews covered teachers' professional learning and growth and the teachers discussed the types of professional learning platforms preferred by them.

6.2.3.1 Teachers' Professional Development Needs to be in Small Chunks in Small Groups

Teacher T1 shared that teachers would prefer to learn in small chunks instead of a full day workshop or seminar, as it is often challenging for the teachers to apply them immediately. Furthermore, the objectives may be varied during the day, so it is hard for the teachers to engage for such long time in meaningful learning. Teachers having a choice of professional learning topics would increase the level of their engagement in learning:

> *Do not have a whole day thing; need to have choices; smaller groups work better; easy to be lost in the crowd; the PD objective needs to be specific and sometimes too big for staff to engage or apply immediately.* T1

> *Choices are better for people – one-hour long sessions with target audience.* T1

6.2.3.2 Observation - a Good Professional Learning Opportunity for Teachers

Teacher T6 explained that observing other teachers in practice in classrooms would be the most effective professional learning experience, based on his past experience. He felt that real time observation provided first-hand experience of possible strategies in action in the dynamics of a classroom.

> *Release time to see other teachers' classrooms; having the opportunities to observe outstanding teachers will be great PD opportunity for me.* T6

> *You need to see how others manage the lessons and interact with students, …etc. focus on one thing; how to create the opportunities to grow; how to use technology in a real time situation.* T6

> *Observe how students behave in another subject matter; it helps to see how they manage.* T6

> *Teachers want to see how a strategy looks like in a classroom.* T6

When the discussion went on to how open other teachers would share his position about being observed, teacher T6 asserted that all teachers would be open to observing others or to being observed, as long as collegial and non-judgemental relationships exist amongst the teachers to be observed. Any links of observation with performance review or evaluation would be counter-productive according to teacher T6. Teacher T2 shared this same position as teacher T6 about observation being a good learning tool if it is meant for learning, instead of evaluation, by stating that "if the intention of observation is to see all the great things you do in your class, instead of "judgement," then teachers will be welcoming":

> *"Teachers are open for others to observe them and support their learning.'* T6

> *Depends on the relationship between teachers to see if giving feedback may be a little daunting if there's no relationship; need to pre-empt what one is looking for; they would not be stressed about their performance.* T6

> *Comments afterwards must be what you get out of it; not evaluative; you helped me improve; would like to go to the junior school to see how they teach.* T6

If the intention of observation is to see all the great things you do in your class, instead of "judgement," then teachers will be welcoming. It has to come from the perspective of the person watching (to the person being watched) that it's not about you, it's about me. Then people will see that practice very differently. T2

Teachers repeatedly and unanimously expressed the significance of observing other teachers teach in professional learning. However, they stipulated that an observation system should be established on the basis of mutual benefit for growth (e.g. "they would be if they felt safe….it's not judgemental") rather than evaluative judgement. This required a collegial and positive learning culture to be established among the staff in order to facilitate and achieve respectful mutual observation opportunities. This point was strongly advocated by the less experienced teachers amongst the eight participating teachers, T2 and T6, as quoted below:

They would be if they felt safe; felt that it's not judgemental, but the person observing benefits, but going to be critical of the person teaching. T2

You will grow because you will see how in the same year level others work and organise their classes for better ideas. How we teach fractions and share to generate better ideas would be fantastic. T2

But this is how easy that can be for kids or how difficult for them. If you have not seen it in action, then you don't know how hard or easy. When we first did this, the classroom observation was probably the biggest thing to develop our teachers' pedagogy, and your environment, and your assessment. T6

Being able to observe outside subject area and different schools, e.g. see how music and art classes work and the dynamics; but also good to see stuff that I want to do and that's happening in another school, being able to observe that. T6

It is not necessary to go to a structured PD course. My friend in xxx [name of school withheld] is using OneNote and I would like to see how he does it, how he prepares his lessons, how he does that. Go and see how it's implemented. If you cannot see how it looks in his setting, e.g. what technology he has in his class, then how he runs it.. T6

It's important to see teaching in practice; rather than being taught this is how it is done at a workshop. That might just be me. I learn by demonstration and observation. T6

6.2.3.3 New Initiatives Need to be Supported in Small Groups

When the discussion turned to pedagogical practice, teachers T1 and T8 both shared that the internal resources of the school were wonderful. However, for the initiatives to work well, teachers need to be given time and support in smaller groups to yield wonderful outcomes:

Support staff in the applications in classrooms; small group interactions, …etc will be helpful and more targeted. T1

People to watch one another; internal resources to give people time. T8

6.2.3.4 Teacher PD Needs to Cater for Different Learning Needs in Teachers – Differentiation

It was noted by teachers T1, T and T8 that many schools made mistakes in designing staff day professional learning programs by using large group sessions with one "size" fitting all the varying needs with statements such as "need to engage in smaller groups through sharing", "people are different in their preferences in teaching" and "providing differentiated approach to professional development". It was suggested that, as well as the need for an adult learning approach to be adopted for teachers' professional learning on site, differentiation to cater for the varying learning needs of teachers was required for the professional learning exercise to be productive and effective (e.g. "engagement for adult learning" and "hands on physical to graph lines"):

Time factor is real; some of the PD on reading, we do that already in science; engagement for adult learning – some like to speak; some like to reflect – need to engage in smaller groups through sharing. T1

Like to be given something to think about; sometimes we get stuck on things for so long. T1

Going to collegial coaching with ISQ – numeracy coaching academy; made me stop to think that people are different in their preferences in teaching. T1

PD – hands on physical to graph lines; bring ideas together to use them and trial them. T8

Changing from teacher-centred to student-centred implies the teachers need to be given ideas and resources to do it. Start from unit planning, it's hands on, some technology, investigation, ... etc. to show the teachers how to change and what to change. Providing differentiated approach to professional development (PD); personalities as well. T8

6.2.3.5 Teacher Engagement is Necessary for Professional Growth

The school had always been committed to providing learning opportunities to teachers for their professional growth; however, teachers T1 and T8 cautioned that teacher engagement was essential for their growth (e.g. "we need to stay focussed on staff engagement"). Teacher T1 explained that teacher engagement in their profession involved the teachers being learners themselves (e.g. "learners are good teachers"), while teacher T8 suggested that ongoing evaluation of and reflection on one's practice through a structured program would be most helpful, "implemented reflective evaluation….". Teacher T8 commented that accountability could be useful when release time had been offered for teachers to engage in professional development activities (e.g. "paid time to investigate and learning – accountability") and could be effected through the teachers presenting evidence of changes in practice:

We need to stay focused on staff engagement; try to see how we improve our units of work and teaching practices; bigger picture of improvement needs to be looked into; there was not change, then it cannot keep the teachers engaged. T1

Learners are good teachers. Give them choice, they will want to be involved. T1

Video type lessons and upload on portal – would be helpful. T8

Implemented reflective evaluation – through a range of strategies – student evaluation, self-evaluation, ….etc. T8

Paid time to investigate and learn – accountability. T8

6.2.3.6 Right Mindset is Needed for Teachers to Learn

Teachers T2 and T8 commented that, for teachers' professional growth, the teachers need to have the right mindset (e.g. "fixed mindset people are difficult to change" and "change mindset about teaching"). Unless the teachers have open minds and are willing to change, then learning is not possible for them, according to these two teachers:

> *Teacher experience does help to make them more intuitive and develop their skills; personality – some are better at it than others; better personal traits, e.g. too strong minded and dogmatic, cannot see; fixed mindset – I always do it this way why change.* T2
>
> *Fixed mindset people are difficult to change; education is a changing landscape.* T2
>
> *Staff must want to be involved; change mindset about teaching; use lessons in the week to share ideas; come out with ideas – action research, videos, task sheets, activities, ….etc.* T8

6.2.4 Perceptions of Authentic Assessments

Teachers T1, T2 and T6 agreed that, in order for assessments to be authentic and useful to inform the teachers' practice and planning, ongoing evaluation of the authenticity, relevance and effectiveness of the assessments is needed (e.g. "authentic assessment clarifies what the teachers already know"):

They further qualified that all assessments, formative or summative, need to have educational values to justify their place in the assessment regime (e.g. "make sure we have focus and reason for why we assess" and "we need to question – it has educational values"). There were concerns about how externally imposed assessments, such as National Assessment Plan for Literacy and Numeracy (NAPLAN) in Australia and Queensland Senior External Assessments (or examinations), may not be authentic assessments for strong engagement and deep learning of the students (e.g. "we don't teach to NAPLAN, so we know that these are unavoidable assessment but not drive our teaching", "external assessment might drive us to give students more autonomy to try" and "external examination is going to change authentic assessment"):

Assessments – authentic – need to change it; make sure we have focus and reason for why we assess. T1

We need to question – it has educational values; not because we have been doing this all time. T1

Authentic assessment clarifies what the teachers already know; they overall know that they are already there and use the data to justify their final grade. T2

Even have some connections, e.g. NAPLAN is outside our control, we don't teach to NAPLAN, so we know that these are unavoidable assessment but not drive our teaching. T2

Australian curriculum external assessment might drive us to give students more autonomy to try instead of doing too much for them. T6

External examination is going to change authentic assessment in Australian Curriculum. If that stress is taken away, would it drive the lesson. T6

Assessment – through questioning, listening to assess their learning journey and process; plus formal assessment for reporting. T8

6.2.5 Perceptions about Professional Beliefs and Mental Models

6.2.5.1 Teaching Students Without Preconceived Prejudice

Teachers T1, T6 and T8 stated passionately that teachers must not have preconceived ideas and prejudice about students in order for the students to maximise their learning opportunities (e.g. "it's important to not go in with preconceived ideas about the students", "you have better understanding of your subject and the students, then the outcomes would happen" and "not limiting the kids" and "need to be flexible to reach all students").

For me, I don't feel that I go into [the classroom] and treat them differently; it's important to not go in with preconceived ideas about students; challenge higher ability students to think outside the square or outside results; teachers need to have a positive view about the students and their teaching. T1

And it is basically if you are able to show your teacher backpack to help support different times and places; every day you may be teaching algebra, like I have to two year 9 classes, teaching them algebra, I had to teach eight different ways to the two different classes. I have a velocitas and a core class; I still have students getting As in both types of class. I still have kids who get Ds in both; so you still have all the skills available to you and the more you can work on those skills, I believe you have better understanding of your subject and the students, then the outcomes would happen. T6

When you look at all these excellent teachers, they all care in their different ways; they all believe in the children and kids; we are not limiting kids; believe is growth mindset. T8

I believe that everything needs to be fair; different rules for different children; cannot be the same; different rules, tones and expectations; my strategies change with different children; need to be flexible to reach all students; not just engage 20% of the students. T8

6.2.5.2 Teachers Need to Model Risk Taking– Show Growth Mindset

Teachers T1, T2 and T6 all agreed that teachers, as learners themselves, need to model risk takers, with growth mindsets about their own professional practice as well as student capabilities so as to create an optimal learning environment for their students (e.g. "teachers are scared to fail with their attempts", "things do not work, we just move on" and "having no fear …, if the lesson did not work, you get the support of your line manager to keep trying and be innovative"):

Teachers are scared to fail with their attempts; I do that with students and evaluate with them. T1

What drives your positive mental model? Role modelling to students our expectations of the students – I am interested in education; education is changing and evolving; things who do not work, we just move on. T1

Having no fear because of the support of the line manager, if the lesson did not work, you get the support of your line manager to keep trying and be innovative. It encourages the teachers to try new things and be innovative. T6

The ability and willingness to try is dependent on the support you receive from your line manager. T6

6.2.5.3 Passion for One's Vocation – Influencing Lives

More specifically, teachers T1 and T2 pointed out that all teachers, to be effective, must be passionate about their profession and want to make a difference in the lives of the students in their care with statements such as "passionate about them taking on the understanding" and "the care factor from the teachers":

> *I think it's all those things put together; some teachers who influenced me; who are the teachers who remember me; I cannot remember doing homework or its specifics.* T2

> *I am passionate about students engaging in maths and learning about it; not loving it but getting it and having lights come on in them. Passionate about them taking on the understanding.* T1

> *The care factor from the teachers – the relationship with the students; [the optimal learning environment], not just material, e.g. resources.* T2

6.2.5.4 Taking Responsibility for One's Craft and Growth

When discussing teachers' professional growth and development, teachers T1 and T2 shared that teachers needed to take personal responsibility for their own growth and development (e.g. "wanting to be better" and "I get annoyed that I lost the creative juices"). They felt that learning opportunities were available around teachers at their schools, and hence a teacher who was keen to grow would have plenty of resources at their disposal through their colleagues (e.g. "you're surrounded with people who want to do fantastic stuff" and "we have a wealth of knowledge in our school and we do not tap into it enough"):

> *Wanting to be better; you're surrounded with people who want to do fantastic stuff; always gain ideas from other similar mindset practitioners; you become a little ingrained.* T1

> *What happened to the creative things I started during the year; I get annoyed that I lost the creative juices; STEAM has been good for me; not assessed; they learned for the sake of learning – thirst and curiosity.* T1

Certainly I think we have a wealth of knowledge in our school and we do not tap into it enough. We have fantastic teachers here, but teaching is a very lonely job and you are in a room on your own – we don't have the time to go around and look at other ideas; but if we could make really good use of time when we sit together and swap ideas; but we need to have a very open mind to do that; most xxx [name of school withheld] staff are open minded. T2

6.2.6 Feedback on Collaboration Inquiry Implementation

6.2.6.1 Collaborative Inquiry to Fit into Teachers' Work Schedule

When discussing how collaborative inquiry sessions could be more productive and fitted into the work schedule of the teachers better, several suggestions of format and organisation emerged.

For example, a once-a-month session to focus on professional learning and sharing practice instead of mundane administration details was suggested by teacher T2:

Once a month using planning time to focus on learning, certain curriculum priority, CPD – learning time, not planning time. T2

Administrative time is needed for teachers; time is our enemy. T2

6.2.6.2 Possible Collaborative Inquiry Topics

Teacher T6 suggested that using the team planning time to share, as in this study, so that teachers could hear about the practice of other teachers from whom they can learn and grow. He further commented that "we don't need to go out to learn PD. We can do it within our staff's expertise" indicating a recognition of the expertise existed within the staff of the school:

Sharing in planning meetings helps us understand who we can learn what from. T6

We don't need to go out to learn PD. We can do it within with our staff's expertise. T6

Use 4-5 hours in school will be more beneficial than a 6-hour PD away from the college ... [with] only one hour of ideas. T6

Teacher T6 suggested a more structured use of collaborative inquiry sessions by each teacher selecting a priority and focus area of the year (e.g. "technology" "group work in HPE, technology, science; but to apply in maths") so they can report and share their learning with the team in a culture of collective learning:

> *Introduce a way of teaching as trial; one thing to do with technology; one thing to do with group work; then develop a plan and collect the resources*. T6

> *Outcomes – e.g. action research in maths key learning area on two aspects – context is the same; developing our own pedagogy and professional development base; release time to talk with other teachers will be helpful; release time to plan together; group work in health and physical education (HPE), technology, science; but to apply in maths."* T6

Possible topics for collaborative inquiry were also suggested by teacher T8 — pedagogy, assessment and student achievement:

> *Not as much opportunities for us to discuss pedagogy; but we did evaluate assessments and student grades.* T8

> *Planning a meeting takes time; sometimes the planning of these meetings takes time. The time to plan is not really available.* T8

> *Create greater autonomy of teachers to develop units and activities that are productive; how we are teaching them and resources to teach them.* T8

During the discussions about the most effective professional development approach and activities for the teachers to enhance their teaching practices for strong student engagement and deep learning, teachers T2 and T6 suggested the use of technology and behavioural management as a large range of skills that can further equip teachers to enhance their tool kits:

> *Make sure that we move forward with the latest teaching methods; keeping up with the students the way best to teach them; e.g. they are born into the technology which I often feel is my second language; finding the people who can help you with that; and we have that in this environment; find out what our strengths are and then use them; so that people that have better*

understanding of the technology can assist us and do it at a nice and simple level. T2

For example, technology or skill or different ways to do things, you don't have to use it straight away; just one lesson a year or one lesson a term, just make a small adjustment to see what difference it makes; you need the support of your line manager. They are the ones who need to encourage you. If you are in the classrooms and no one sees you and how you do things, you are not going to change. T6

Behavioural management skills but that would come from here [pointing at teacher pedagogy], you can observe from any teachers if it is demonstrated and you can understand what's going on. Some teachers do not know that they are doing it. You walk into a class, they don't even know they are doing it, e.g. proximity, seating plan, teacher signage and cues for them, a little conversation at the end of the lesson with the teacher, you gained some insights. T6

6.2.6.3 Collaborative Inquiry for Action Research

A possible format of collaborative inquiry could also be in the form of action research for more extensive study of a topic relevant to teaching and learning at the site, and focused professional dialogues, as suggested by teacher T8:

Most effective way to change their practice – best value for money is internal action research – external PD – ideas got lost after PD. T8

Maths action research project – want to see in project – contribution of ideas; one-drive – problem solving activities; online place to add and subtract ideas; resources need to come with instructions; catalogue of teaching resources. T8

While evaluating how to make collaborative inquiry sessions more effective for teacher professional development, teacher T6 suggested using year level planning sessions once a month for dedicated sharing and reporting of teaching related strategies such as group work and warm up activities. In this way, other planning sessions can be dedicated to the administration and planning aspects of the teachers' work, thus not putting onerous expectations on teachers to find time to deal with administration and thus reducing their desire to develop their skills and knowledge as teachers. Teacher T8 suggested the strategy of "pairing the fixed and growth mindset teachers together for peer mentoring".:

> *In a planning meeting, [have] a part on organisation so that we know where everyone is up to [in the plan]; if you have a year 9 planning meeting, to know what topics they've done, what resources they've used, then discuss what resources you use —, are you able to show me, do you have a video or student work to share and use as evidence?* T6

> *E.g. Teacher E does amazing things with group work; but I would like to see it.* T6

> *E.g. Teacher T8 does wonderful warm up activities; but I don't get to see it.* T6

> *XXX [name of teacher withheld] made sure he tested the students, to know that they were all at the same level before new materials were introduced.* T6

> *It should not necessarily be weekly but per topic; you need to have at least one catch up per topic, e.g. at least once every two or three weeks, i.e. duration of one topic.* T6

> *Pairing the fixed and growth mindset teachers together for peer mentoring; need to differentiate our PD to staff; need to show evidence; a C+ kid can become A-; core belief – don't know how I came to the belief; need to know where everyone is at; investigation and know our staff better and create mentoring groups.* T8

6.2.6.4 Collaborative Inquiry Provided a Platform for Collective Professional Growth

When the discussion turned to how collaborative inquiry can be used as professional learning tool in schools, teacher T2 explained that the professional dialogues during collaborative inquiry provided teachers with opportunities to discuss and share their practice (e.g. "interesting to see the disparity of how the learning activities work in each class setting" and "it's always good to talk to fellow teachers who are teaching the same area; same year levels"):

Gives us an opportunity to reflect on the student learning experience; interesting to see the disparity of how the learning activities work in each class setting; not about content or subject matter but more about the students and their engagement in the learning experiences provided. T2

It's always good to talk to fellow teachers who are teaching the same area; year levels. T2

Teacher T8 suggested that collaborative inquiry could generate professional practice growth given an appropriate time frame for implementation and assuming that the teachers involved were given specific guidance and support for the pedagogical change agenda:

It's not an overnight transition for teachers to change their pedagogy – at least three years; set targets to increase student centred approach – resources – e.g. Teachers are given one class less, need to use the class loading to work change of practice – teaching and assessments; sharing ideas; multi-faceted – the tool kit, ongoing resources and catalogue to teach a particular topic. Conversations about pedagogy. T8

Changed practice evidence – talk to students; lesson observation; student evaluation; video lessons; professional discussions; teaching ideas being shared – people tried and shared their successes and failings. T8

Teachers T2 and T6 outlined that sharing practices during collaborative inquir provided experienced teachers with fresh perspectives from teachers new to the school or the profession, while the more experienced staff with positions of added responsibilities, such as Head of Learning (Mathematics), could enhance teacher practice:

xxx [a younger teacher on the team, teacher's name withheld] being less experienced in teaching the year level; the younger teachers shared some strategies to share with the classes. A fresh perspective. T2

Talking with fellow teachers about how to teach, and with Head of Learning (Mathematicss) on how one learns and how one teaches are all good professional dialogues. Professional dialogues via planning meeting, *incidental staff room conversations and CTJ conversations are all very useful for my professional reflections. T6*

Professional sharing amongst colleagues of the same school share the same context and can see what it looks like in practice. T6

6.2.6.5 Effectiveness of CI as PD Depends on the Team Members

When discussing the effectiveness of collaborative inquiry as a professional learning tool, it was clear that the teachers, based on their experience, felt that collaborative inquiry's effectiveness for professional growth largely depended on the team members and the levels of their engagement in the collaborative inquiry process:

Depends on the teaching team members – would have different levels of engagement in our professional dialogues. T2

Much dependent on the people involved in the team. Working with newer teachers versus experienced teachers varies. T2

The team dynamics does change and vary from year to year. Would it be a professional learning time. Planning and CTJ – professional learning. T2

Trust and relationship were two key attributes required of all collaborative inquiry team members to ensure the success of collaborative inquiry as professional learning tool:

Create trusting relationships with members of the team so that Heads of Learning going to the classes to observe – welcoming rather than judgemental. T8

Teacher T2 lamented that teaching is a lonely and isolated profession, in that they performed their duties alone in a classroom as the only adults in the rooms, and so did not have opportunities to share practice in their natural work setting unless "interrupted in practice":

Teaching is a very lonely profession; I know what I do in my class, but I don't spend enough time in another teacher's classroom to see what others do. T2

This comment shared by teacher T2 outlines the importance of collaborative and team based approach to teacher professional learning activities so as to connect the isolated teachers sharing the same teaching context, constraints and resources to advance the site based vision for the students as long as the membership of the teachers is carefully planned.

6.2.7 Evaluative Feedback on Proposed CI Model

The teachers were shown Figure 4.1involving inter-connecting evolutionary links for implementing collaborative inquiry as a professional learning tool for teachers. They were asked to provide feedback on the model and suggest the important elements for implementing collaborative inquiry as a learning tool for teachers, so that students can gain deeper understanding and attain strong engagement in learning.

In general, the teachers felt that the model captured the essential elements which would impact on using collaborative inquiry as a professional learning tool at the research site. In spite of the agreement, there were suggestions for modifications to the model to better reflect the site, with relevant and more generic representation of the needs of the profession based on their experience of other sites.

Teachers T1, T6 and T8 all agreed that the mental models of teachers about teaching and learning would definitely impact on their classroom practice and, hence, the way in which they created the classroom environment, their choice of pedagogies and their attitude and approach to assessment:

> *The middle part of the model definitely makes sense to me; teacher mental model/belief – personal teaching philosophy.* T1
>
> *This is what the teachers bring [pointing at the mental model/belief section of the model]; their emotional state, you work your way up and you can see the pedagogy is the skills the teacher has; that's like developed through experience; through the knowledge of your students; stuff like that.* T6
>
> *Relationships; growth mindset; not intersecting from the mathematics*. T8

The suggested modifications included:

(a) Using a cyclical model instead of linear relationships between the elements was suggested by teacher T6; or using a layering of a sphere displaying students in the centre and how other factors are intertwined to achieve the desired outcomes, suggested by teachers T6 and T8; or a cell or atomic structure with the nucleus in the centre, suggested by teacher T6:

A cycle; I can see that if you have a good foundation here [pointing to belief and mental model] – you will branch out to better environment, develop the understanding of the kids; that comes back to your mental model and your belief in yourself so that you can teach better. It keeps building up your confidence. …We kept stumbling here [pointed at assessment on the model] so it worked against the teacher pedagogy and the environment. When you cannot get your assessment item right, you cannot progress in teacher pedagogy and optimal environment, you cannot tell if your assessment is authentic or you cannot tell if your teacher kit is working or your environment is conducive or your little changes in differentiation and stuff is working, because you cannot see it through your assessment outcomes. There is no justification of the results. T6

I like the sphere, basically in the centre, encompassed by everything outside, the supportive environment, the teacher, basically slightly add assessment, but the student needs to understand, they need to develop the deep understanding and show. I see it's more like the nucleus, more so. You have the cell and the things around it still connected but not embedded on it. T6

I like this one like a cell; this is encompassing [pointing to the two linking circles in one larger circle] while the other one is not [pointing to the three interlinking circles]. I like the big circle of optimal environment [OE] outside; look you know me, creating an environment where the children are comfortable. T8

OE – big circle; descriptive word – for pedagogy; prefer assessment – learning experiences; without assessment; should the child be in the middle; start with the student and finish with the students. T8

Ball – peeling away; cut out of earth with layers and centre of the core is student. T8

(b) It was suggested that the interactions between the three interconnecting circles, namely, teacher pedagogy, authentic assessment and optimal environment, ought to be connected in different ways to display the relationships amongst them, not just as interconnecting, but to show the degree of influence:

But you can also be supported by that environment or basically the skills or the teacher tool kit you have; you need both, the support by the environment but also the ability to use that in creating assessment pieces; but I felt that xxx [teacher name withheld] did it well with us where he led us to run our assessment pieces by ourselves with the confidence that there were not issues with what I created. There was no issue with time, difficulty. I always had a nice spread of results; there wasn't all high or all low results. It basically allowed us to create assessment items without stress. T6

First reaction – needs to tease out, the optimal environment needs to be teased out more to (make it explicit). T8

I struggle to like the word of assessment; does it include assessment? Should we use evaluation. T8

Two circles – optimal environment; authentic pedagogy and assessment. T8

(c) Another comment was the lack of an adjective to describe the desirable teacher pedagogy for successful engagement of students and hence deep understanding:

What's authentic? May be the word is engaging pedagogy? Anything fundamental core business? Supportive environment is the optimal environment. T8

(d) Teacher T8 observed that two of the elements in the model represented in Figure 4.1, namely, teacher pedagogy and teachers' mental models and beliefs about teaching and learning, did not have adjectives to illustrate the desirable attributes of these elements:

You gave an adjective OPTIMAL environment; AUTHENTIC assessment; but nothing about pedagogy. T8

(e) Teacher T8 also proposed that using concentric circles or spheres would better illustrate the connection and that the core of all that schools do should be the students. Hence, in the modified model, the outcome of deep understanding and strong engagement in learning for the students as in Figure 4.1 is situated in the centre of the three-dimensional model.:

….should the child be in the middle; start with the student and finish with the students. T8

Ball – peeling away; cut out of a[n] earth with layers and centre of the core is student. T8

(f) Teacher T1 commented that the interconnecting circles of authentic assessment, optimal environment and teacher pedagogy made sense to her, and she felt that the teachers' mental model and beliefs, that is, their personal teaching philosophy, would influence their practice in those three areas:

The middle part of the model definitely makes sense to me; teacher mental model/belief – personal teaching philosophy. T1

Teacher T1 further emphasised that all three areas summarised and presented in the model are important. She confirmed that the main obstacle to students in developing deep understanding and strong engagement in their learning was still the teacher's mental model and belief system about teaching and learning:

I think that all three are important; the hurdle is still teacher mental model and belief – they will be able to manage. T1

Finally, I decided to retain the three elements of effective pedagogy, authentic assessment and optimal environment because they were derived from the research findings supported by literature review, in the modified model in Figure 6.1. However, taking into consideration teacher T1's feedback regarding the pivotal role of the teacher's mental model and belief about teaching and learning, I have combined the four elements into one tetrahedron as shown Figure 6.2 with positive teacher mental model/belief being the base of the tetrahedron. This tetrahedron has four similar triangles and each triangular surface represents an important element that is pivotal for students' strong engagement and developing deeper understanding.

These elements are positive teacher mental model/belief, authentic assessment, optimal environment and effective pedagogy. Positive teacher mental model/belief has been placed at the base of the tetrahedron to illustrate that it forms the foundation for the other three elements because without which, deep understanding and strong engagement of students is unlikely to occur. Figure 6.1 is simply the net of the tetrahedron when the three-dimensional diagram is flatten out into two-dimensional diagram.

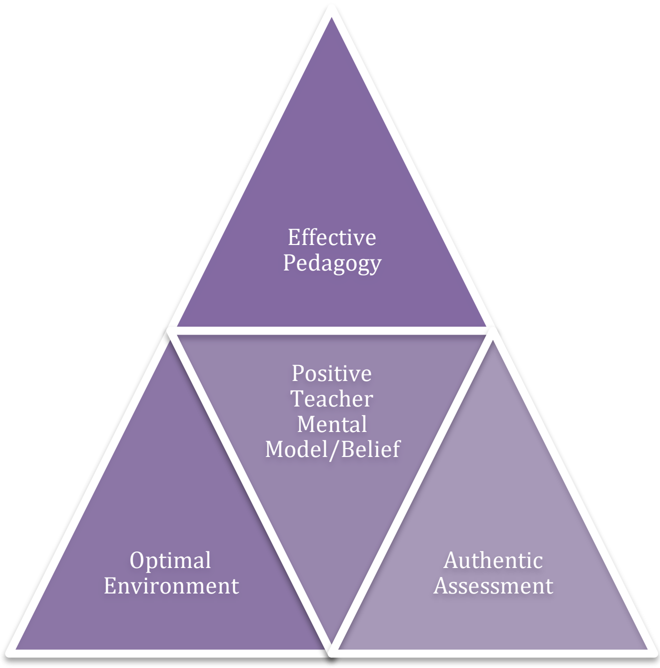


Figure 6.1 Net (two-dimensional representation of a three-dimensional model) of the tetrahedron containing inter-connecting evolutionary elements

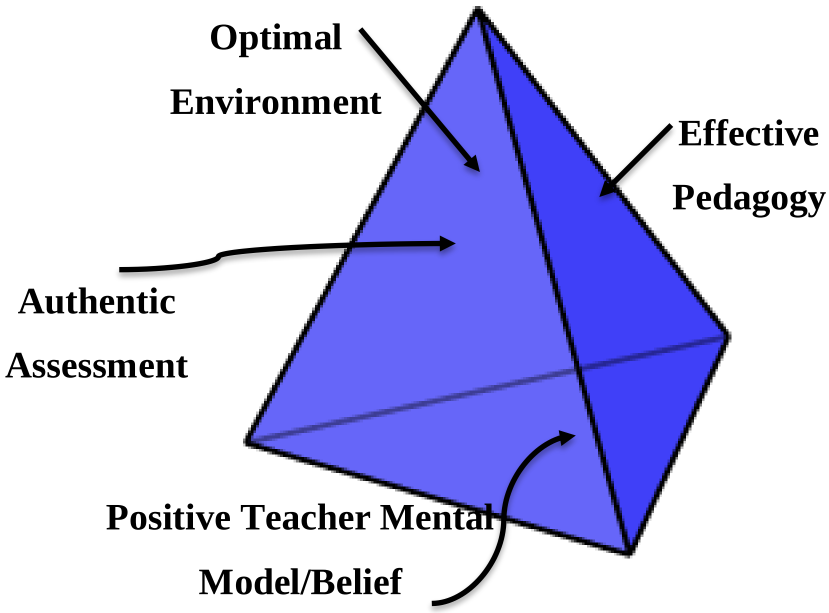


Figure 6.2 Three-dimensional model of the tetrahedron containing equally important inter-connecting evolutionary elements

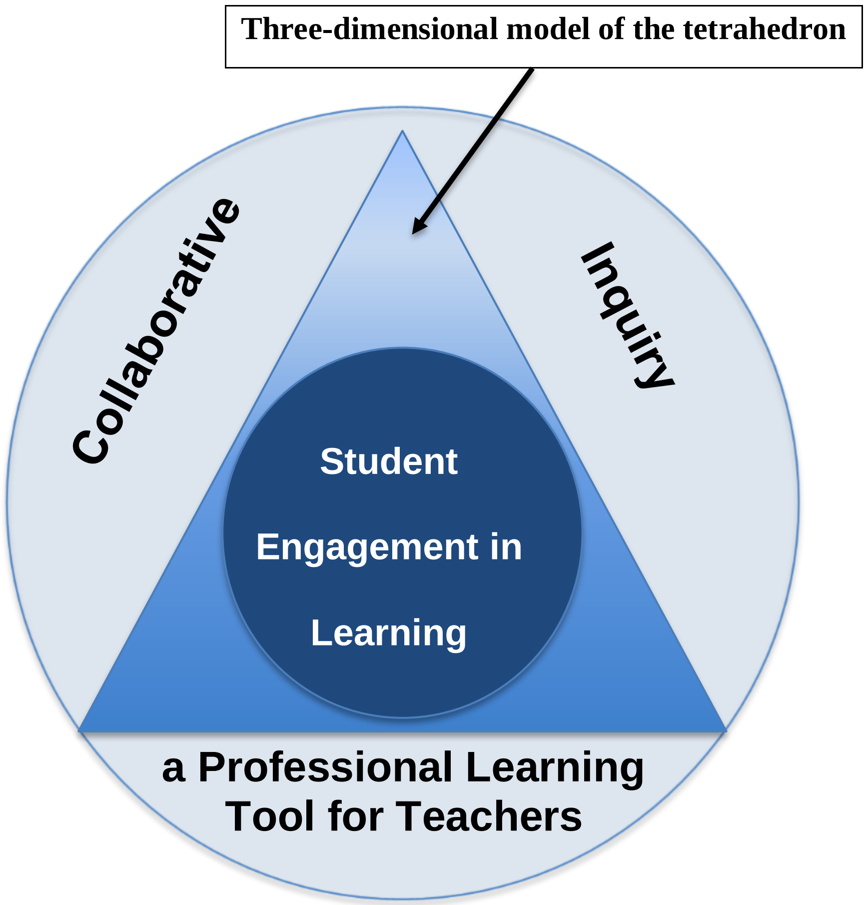


Figure 6.3 Collaborative Inquiry: A professional learning tool for teachers (Inter-connecting evolutionary links)

Figure 6.3 illustrates the full model of the research findings showing the tetrahedron containing a sphere as the core business of schools, which is students' deep understanding and strong engagement in learning. The model completes with a concentric sphere outside the tetrahedron showing that collaborative inquiry as a professional learning tool for teachers is likely to encapsulate the core business of the school in providing learning opportunities for the students.

6. 9 Memo: Teacher Interviews

Teachers' beliefs and mental models of an optimal learning environment can impact on whether classroom learning is optimal, as teachers hold the key to creating such an environment. In turn, the optimal learning environment is a critical ingredient for students' engagement in learning. An optimal learning environment can be created for teachers by choosing and trialling shared best pedagogical practices to cater to the different learning needs of students, in order to differentiate the learning activities and tasks better. The teachers perceived that engagement in learning should be extended beyond the classrooms and that large class sizes presented challenges to effective differentiation.

Teachers' beliefs and mental models about their pedagogy were contingent on colleagues' attitude, which affects teachers' willingness to explore pedagogically. Furthermore, the pedagogical growth of the teachers requires them to undertake ongoing reflections and modifications of their own teaching practice over a period of several years. The teachers perceived that student engagement does not necessarily yield better academic outcomes; however, engagement of teachers and students means learning and growth for both teachers and students.

Teachers' beliefs and mental models about professional learning and growth indicate that they found professional development activities were most effective when delivered in small chunks, catering to their individual learning needs; that is, teachers' professional learning requires a differentiated approach. Effective professional learning activities for teachers include the observation of other teachers in practice, and collaborative inquiry with positive team members. Teachers' professional growth requires them to have the right mindset and to be actively engaged. Any new teaching and learning initiatives need to be supported by a small group of early adopters to ensure success before school-wide implementation.

The teachers stated that teaching is a lonely profession and they often felt isolated in their own classrooms. Consequently, the provision of collaborative inquiry as professional learning opportunities for teachers to engage in productive dialogues with the intention of professional growth and support would be a good approach because it is conducted on site where the contexts are similar. A frequency of once-a-month was suggested for professional dialogues with focus areas such as pedagogy, assessment and student achievement. Another possible approach to collaborative inquiry is for the team to engage in action research on a particular area of interest for professional growth.

The feedback from the teachers on the Year 1 data modelling in Figure 4.1 was overwhelmingly positive, affirming that the model captured the essence of some important elements in creating an environment to engender deep understanding and strong engagement in students' learning. The constructive suggestions for modifying the model included using a cyclical rather than a linear model to indicate the interwoven effects of the elements, and using adjectives to describe the impact of each element clearly.

The teachers reiterated points raised in the first interviews, such as: the teacher's mental model will determine the optimal learning environment; observation of other teachers' practice in classrooms helps with teachers' growth in practice; and teachers need to take responsibility for their own professional growth and pedagogical practices.

New insights into the teachers' views that emerged during the interviews on the creation of an optimal learning environment for students included: teachers need to model risk taking to create an optimal learning environment for their students; they need to have passion for their own vocation and teach their students without preconceived prejudice to be effective; and time is needed to thoroughly plan for optimal learning environment.

Other areas brought to attention during the interviews were: the external environment, such as management support, can improve teachers' willingness to take initiatives and risks by trialling pedagogies; authentic assessments require ongoing evaluation of their effectiveness and relevance to the learning; and possible topics for professional learning of staff include use of technology in the classroom and behavioural management.

On the possible use of collaborative inquiry as a professional development tool for teachers, teachers favoured a frequency of once a month and including dedicated sharing and reporting of teaching strategies, such as group work, warm up activities, growth mindset, and mentoring as possible collaborative inquiry discussion topics.

6.10 Summary of Year Three Findings

The Year 3 research data comprised two sets of teacher interviews with different foci, and the interviews were set six months apart.

Firstly, the data revealed that the teachers were in agreement that their beliefs and mental models about the optimal learning environment were critical for students' engagement, and that they themselves were pivotal in this, because they had the freedom to choose the appropriate pedagogical practices to create such an environment. The teachers concurred that, through mutual sharing during collaborative inquiry sessions, they could enhance and develop their pedagogical tool kits, so that they could create an optimal learning environment to cater for the differing needs of students. In this way, they could ensure that differentiation became a reality in the classrooms, thus engaging students, not just in classrooms, but also beyond the classrooms, through enthusiastic self-directed learning. The finding on the teachers' role in creating optimal learning environment using their pedagogical tool kit to engage students in their learning replicates studies of Attard (2013), Atweh (2007), Fennenma (1999), Kinach (2002) and Fredricks, Blumenfeld amd Paris (2004). These studies asserted that student engagement is malleable which are affected by the individual students and the context in which they learn (Attard, 2013; Fredricks et al., 2004). Furthermore, they stated that teachers and their practice are the core influences of student engagement (Attard, 2013; Fredricks et al., 2004). However, the teachers also expressed concerns about their ability to differentiate effectively in larger classes and when they faced competing demands for their time, which meant that they did not have adequate time to carefully plan for an optimal learning environment.

Secondly, the data confirmed that the teachers' beliefs and mental models about their pedagogy were founded on the culture of the school, which impacted on their desire to grow pedagogically; at the same time, they acknowledged that each of them needed to seek ongoing evolution of their practice in order to grow professionally. Furthermore, they were concerned that external environmental factors such as school management policies, legislative bodies' expectations and so on impacted on their pedagogical practice and growth. As outlined above, even though there was an intrinsic link between teachers' pedagogical practice and the optimal environment. The teachers felt that students' strong engagement did not necessarily translate into positive academic outcomes which Gningue, Peach and Schroder (2013) argued that as long as students are engaged in learning, their motivation will improve and positive academic outcome will be a natural by products. Even though Gningue et al (2013) stated that not many other studies have confirmed the links between student engagement and student outcomes. The link between engagement and outcomes is therefore a gap in the current body of research. Conversely, strong student engagement itself should be considered a positive academic outcome in their attitude towards learning. It was agreed that strong engagement should not be limited to the students; teachers' strong engagement is also required for the collective growth of teachers and students.

On the subject of professional learning and growth, the teachers asserted that they needed to learn in small chunks in small groups with similar learning needs; in other words, teachers' professional learning activities need to be differentiated. Hasweh (2003) and Wong (2009) acknowledge that, like students, teachers need differentiation in the professional learning, but the degree of differentiation required for teachers is smaller due to their self-reliance, self-esteem and motivation to learning. The findings also suggest that, to create an impetus for professional growth, the teachers needed to take responsibility for their own professional growth and demonstrate greater teacher engagement with their professional practice through a growth mindset. The teachers agreed that observing other teachers in practice was a good professional learning opportunity for them to see teaching in action. My findings that teachers taking responsibility for their own professional growth and observing other teachers as a useful professional learning activity are consistent with the study undertaken by Seagon (2008) who made many suggestions for possible professional learning avenues, many of which were site-based learning opportunities.

The teachers stressed that teaching is a lonely profession. To this end, collaborative inquiry would be a good platform for collective professional growth because it offers collegial support in the same professional setting, thereby creating a sense of teamwork and camaraderie. There were some excellent suggestions about the various formats and organisational approaches that could create successful and effective approaches to professional learning through collaborative inquiry. These suggestions could be incorporated into the next stage of this study to expand the theoretical model with more details and examples.

The teachers agreed that they need to view their profession as a calling or vocation, not just a job. Only through their passion about their roles, they felt, would they be able to avoid teaching with preconceived ideas and be willing to model risk taking, thus creating an optimal learning environment for their students' engagement. Such a vocational mental model is essential for the teachers because, according to Pajares (1992), there is a strong relationship between teachers' educational beliefs and their pedagogical practice.

Based on the teachers' feedback, the theoretical model developed in phase one of this study (see Figure 4.1) was remodelled and a final theoretical model emerged as in Figure 6.3. The evolution of the model is mainly in the deliberation of the terms adopted in the model and a better diagrammatic representation of the inter-connecting relationships of these elements. Figure 6.3 illustrated in a three-dimensional representation how student engagement in learning is the core purpose of collaborative inquiry by teachers. The teachers identified that, in order for student engagement to remain the goal in the inquiry, the inquiry needs to focus on examining four elements, namely, optimal learning environment, effective pedagogy, authentic assessment and teacher mental model and belief. These four elements were combined into a tetrahedron to illustrate the equal importance of each element. These four elements emerged as overarching themes for teacher professional learning and growth.

CHAPTER SEVEN

SUMMATIVE DISCUSSIONS AND CONCLUSIONS

7.1 Introduction

The last three chapters contain analyses, summaries and discussions of the findings relevant to each phase of the research. The analyses and discussions of the findings focussed on the core themes emerging from the data and the findings were then discussed in the light of the literature review.

This chapter begins with combining the discussions of the last three chapters to synthesise the findings from the three phases into one section containing overarching summative discussions before conclusions for this study are drawn towards the end of the chapter.

7.2 Review of the Study

I finalise my research model in Figures 6.3 to illustrate the theory grounded by the data and literature review and draw conclusions to the research questions that I refined throughout the research process. I also outline the recommendations for refining my research methodology as well as suggestions for future directions for this research topic for the benefit of the wider audience.

To recap, the objective of this research was to evaluate the effectiveness of collaborative inquiry as a professional learning approach to challenge middle school mathematics teachers' pedagogical practice.

The following research sub-questions were addressed:

a. What benefits and challenges would the implementation of collaborative inquiry as a professional learning approach present for the school and the middle school mathematics teachers?
b. What would be the essential elements in implementing effective collaborative inquiry as a professional learning approach for teachers in the school?

c. How would these essential elements be incorporated into a theoretical model to inform the effectiveness of collaborative inquiry as an professional learning approach for middle school mathematics teachers in schools?

7.3 Summative Discussion of Findings

The sections below discuss the findings related to the benefits and challenges of collaborative inquiry, essential elements of effective collaborative inquiry and how these essential elements incorporated in the theoretical model for collaborative inquiry inform the effectiveness of collaborative inquiry drawn from the last three chapters.

7.3.1 Benefits and Challenges

The benefits and challenges of implementing collaborative inquiry in schools to challenge middle school mathematics teachers' pedagogical practice are numerous.

7.3.1.1 Contextual Advantage

It is proposed that collaborative inquiry is set within a similar context, for example, in the same school, same department or same year level, as occured in this research. The contextual advantage is that the teachers' inquiry for their own professional learning and growth arises out of similar challenges within the same context, such as the students, school culture and structure, school goals and priorities for teaching and learning. This finding resonates with the positions presented by Cox (2010), Small (2011) and Wilkins and Shin (2011). They argue that the best professional learning for teachers is job-embedded and ongoing, since the teachers are immersed in the same context at the same school with the same local and legislative constraints with regards to organizational flexibility, time allocation and mandatory requirements pertaining to funding (Cox, 2010; Small, 2011; Wilkins & Shin, 2011). In other words, such site based professional learning also provides a similar backdrop to local and legislative commitments and requirements, such as school culture, staff composition and so on. The teachers share the same school goals and priorities as well as student characteristics, making the professional learning opportunity an authentic and high impact one.

7.3.1.2 Collective Growth Enhances School Culture

As outlined in the literature review in Chapter Two, unless professional development activities adhere to adult learning principles (andragogy), then the professional development activities are likely to be futile (Rickey, 2008). Collaborative inquiry therefore meets the requirement of adult learning principles, namely, involving the adults in setting up the learning environment and planning for their needs and interests while giving them control over the set goals and objectives of their learning (Henschke, 2009; Rickey, 2008; Terehoff, 2002). Some of these principles include focusing on personal connections to their work, preferred learning styles and promoting active involvement in planning, designing, implementing and evaluating the professional learning activities (Rickey, 2008; Terehoff, 2002).

In other words, collaborative inquiry provides the natural setting in the school for the teachers to plan, design, implement and evaluate their professional learning activities via the collaborative inquiry group to meet the group's identified learning needs, not only giving the control of teacher professional learning back to the teachers, but also adhering to their preferred learning approaches. As mentioned in the previous section, when the teachers share the same contextual advantages and constraints, the professional learning gained by the teachers is likely to have greater impact on the site at which the teachers work. Furthermore, the collective implementation of professional learning will be further enhanced by ongoing moral and professional support of colleagues in the same school to generate collective growth; hence, there will be continual improvement of the teaching and learning at the same site in addition to the collective professional growth of the teachers. This finding concurs with those points raised by Kruse, Louis and Bryk (1994) and T. H. Nelson et al. (2008), who conclude that the proximity of the teachers within the team provides a natural setting for their collective inquiry into their practice and growth.

Collaborative inquiry also reinforces and bolsters the team and school culture through the purposeful and structured professional dialogues that are enabled. This approach to professional learning also builds mutual trust and a team approach to learning, and generally allows general greater consistency of practice at one site, building a stronger learning community (Louis et al., 2003; T. H. Nelson et al., 2008; Roberts & Pruitt, 2008; Terehoff, 2002).

Now that the argument of how teachers' collective growth enhances the school culture is established, the school leadership team members should see the importance of promoting and facilitating high quality professional development programs for the teachers. According to Argent Rogers (2013), whose assertion is affirmed in this study, unless the school management commits to collaborative inquiry as an effective professional learning tool by providing resources and constructing a school structure that enables the collaborative inquiry groups to operate within the existing school structure and organisation, then the collaborative inquiry approach to professional learning of teachers will not be as effective. The school management also needs to have a long-term commitment to this approach of professional development to engender long term sustainable improvement in practice.

7.3.1.3 Professional Learning Community

Within the context of a school culture, structure and goals, the professional learning needs of the teachers can be consolidated to a few foci. With the identified foci, the school management can introduce experts as in-house consultants to facilitate the collaborative inquiry sessions, with one or more groups sharing the same instructional goal. Such a collaborative inquiry approach will also provide the consultants and the teachers with more focused professional exchanges and dialogues for developing useful implementation strategies for the site. This finding is consistent with the claim of Roberts and Pruitt (2008) who state the view that teachers are more ready to learn if the professional learning opportunities are deemed to meet their needs of immediate and practical use.

Collaborative inquiry focuses on the needs of a small group of teachers sharing similar contextual requirements; accordingly, with the replication of these collaborative inquiry groups throughout the school, the particular school would establish a network of collaborative inquiry groups which form a professional learning community sharing the same organisational goals and priorities. A professional learning community within a school context provides the background for a relevant and productive action research approach to teachers' professional learning and inquiry. Collaborative inquiry provides a structure and framework through which this action research involving trialling and implementing new ideas to enhance pedagogy within the school context will advance the school goals for continual improvement. In addition, it provides wider educational community data and findings to generate best practice in situ. This finding of the positive impact of the network of collaborative inquiry groups in a school setting as an effective shared learning environment adds to the work of Dufour (2007), Garrett (2010), T. H. Nelson et al. (2008), Roberts and Pruitt (2008) and Louis et al. (2003). These scholars argue the benefits of a professional learning community for schools to empower the growth of their teachers and school culture.

School culture has been defined as "the set of norms, values and beliefs, rituals and ceremonies, symbols and stories that make up the 'persona' of the school" (Roberts & Pruitt, 2008, p. 8). School culture is the way in which staff at the school do things unique to them based on a set of commonly held beliefs; for collaborative inquiry to be effectively and successfully adopted as a professional learning tool for a school, the school management needs to create a healthy school culture characterised by shared values and planned opportunities for continuous learning, collaboration and empowerment of teachers (T. H. Nelson et al., 2008; Roberts & Pruitt, 2008; Terehoff, 2002). Immersion in a positive and supportive school culture will go a long way to addressing the isolation experienced by the teachers in their respective classrooms during their daily work, by helping them to see their isolated work as part of the whole learning community of the school (Brahier, 2005); this sense of isolation was evident in my study when the teachers expressed that "teaching is a lonely profession" .

Waring and Evans (2015) state that "collaboration with colleagues impacts teachers' professionalism and professional development" (Waring & Evans, 2015, p. 3). My study confirms that, with collaborative inquiry and through organised sessions for collegial support and professional dialogues, teachers can be part of a professional learning community to further develop their understanding of and stance in relation to pedagogy. The collaborative inquiry approach to professional learning provides the opportunity for the teachers to identify with others through similarities in workplace constructs and constraints, as well as to co-create solutions and strategies with their colleagues while "forging collegiality, collaboration and openness to change" for the ultimate benefits of the students (Waring & Evans, 2015, p. 3). Teachers' professional learning activities need to incorporate differentiation strategies to meet the diverse needs of teachers at one school site, which is in agreement with Hashweh (2003)'s position on this matter. As suggested above, the collaborative inquiry groups can be established by grouping the learning needs of teachers and also ensuring a prolonged period of working in the same group to allow the professional learning to be structured in small bite sizes, which will ensure long term and deep learning takes place for enduring change in practice. This approach is supported by Schnellert (2011) who promotes that teachers professional development should be designed "with", not "for", the teachers.

Over a period of three to six months to a year, the improvement in teachers' practice and students' engagement and achievement will be more measurable, providing hard evidence for the school management to evaluate the effectiveness of this collaborative inquiry professional learning mode as well as the topics under investigation (Hasweh, 2003; T. H. Nelson et al., 2008; T. H. Nelson, 2009). For example, during this research, the teachers shared interests in topics such as effective pedagogies, behavioural management and technology as high impact topics to support their classroom practice. In this study, the effectiveness of collaborative inquiry was evaluated based on the researcher's observations over the three-year research period and the teachers' own perceptions. Therefore, it opens up an opportunity in further research to measure the extent of the benefits outlined in this study to further consolidate the effectiveness of collaborative inquiry's effectiveness in schools.

7.3.1.4 Critical Mass for Change

Through the collaborative inquiry sessions, the teachers can dialogue and develop conditions for an optimal learning environment for the students. As the group meets at a predetermined frequency to deliberate and discuss how to create an optimal learning environment for students, the collaborative inquiry group of teachers becomes the change agent for the teaching and learning practice at the site, which is consistent with the findings of Nisbet et al. (2003). A sufficient number of collaborative inquiry groups focusing on critical issues at the site would become the critical mass for sustained, deep and long term change, because the change is initiated by the teachers, the practitioners, who know firsthand the issues and challenges at the coalface. The critical mass needed for change at a school will happen if the teachers are given autonomy in their inquiry into a change paradigm for transformative practice, as advocated by Krainer and Peter-Koop (2003).

Other than the juxtaposition of belief with practice, another factor that can affect the teachers' motivation to change their pedagogical practice is the individual's mindset. Teachers with growth mindsets will endeavour to find ways to refine and improve their practices for life-long learning, such as seeking out all learning opportunities for professional growth, while those teachers with fixed mindsets will find excuses to maintain the status quo (Dweck, 2006). Moreover, teachers who enjoy their jobs are often excited about the growth and development of their students (Brahier, 2005). It is the teachers' individual responsibility to take an active role in their own learning and growth in addition to the opportunities provided by the schools in which they work; Brahier (2005, p. 352) asserts that "a teacher does not get 'made' in four years' tertiary education; a teacher grows, develops and becomes a professional over a period of many years". Furthermore, teachers must continually examine their practice.

7.3.1.5 Facilitation Skills of Facilitator

My experience in this study confirmed that, for collaborative inquiry to be used as an effective professional learning tool for teachers, the facilitation and scaffolding skills of the researcher or facilitator are very important to ensure that the collaborative inquiry implementation is productive and effective. With skilful facilitation and appropriate and thorough documentation, the teachers can reflect on the topic at hand and share their practice with colleagues, and teachers can identify themselves as experts in both their classrooms and within their communities of practice. Recognising the importance of facilitation skills is a possible gap in the implementation of collaboration inquiry in that the studies being reviewed were silent on this issue except for a brief reference by Schnellert & Butler (2014). Hence, collaborative inquiry facilitation is an area requiring further exploration and study to provide the right implementation conditions.

7.3.2 Essential Elements for Effective Collaborative Inquiry

7.3.2.1 Effective Pedagogy

My finding indicated that effective pedagogy is one of the important influences on student engagement in mathematics is consistent with the assertions made by Perks and Prestage (2008) and Seago (2008) that school teachers need to undertake ongoing interrogation of their practices so as to develop effective pedagogies with the sole purpose of improving student outcomes. In addition, the linkage the subject matter to be taught with other disciplines, teacher beliefs and mental models, the constraints of environmental factors, and the culture and attitude of students are the other important factors influencing students' engagement and understanding of mathematics. These findings are supported by the work of Nickson (2000) on the impact of the beliefs of teachers on student outcomes. Stacey (2008) also emphasises that mathematics teachers need to possess general pedagogical knowledge as well as pedagogical content knowledge for the students to experience a good mathematics education.

Effective pedagogy was identified by the student focus groups as vital to their engagement in the learning of mathematics. The students identified that teachers who made learning fun and took time to find out what their students liked were the ones who engaged them in learning both during class time and after school. However, the students added the qualification that only the use of relevant and appropriate games and hands-on activities were helpful for their learning. The teachers considered that the incorporation of real life problems helped students to maintain their interest and engagement in learning, which in turn led to deeper understanding and, hence, better achievement in mathematics. This finding is supported by Ravitz et al. (2000) who propose that teachers need to balance the use of the two models of good pedagogy, namely, traditional transmission instruction focusing on the communication of content knowledge by the teachers, and constructivist-compatible instruction involving engaging the students consistently in learning activities. Nuhreborgen and Steinbring's (2008) argument, that informed usage of manipulatives for teaching by mathematics teachers is likely to greatly assist them in the diagnosis of students' learning, is consistent with the findings in this study.

Teachers are often the sole adults in the classrooms, unless team teaching arrangements are in place or teacher assistants are assigned; thus they are charged with the responsibility to plan their classes and must be given the freedom to choose the pedagogies for their classes (Johnson, 2010).

Waring and Evans (2015) argue that:

> …a teacher's understanding of and stance in relation to pedagogy is fundamental to informing how and where they position themselves as professional and (re)frame their vision of education and its future, including the types of learning, learners and society that they want to promote. (Waring & Evans, 2015, p. 26)

In some settings, schools may adopt a certain pedagogical framework; however, unless the classes are visited daily, the teachers are able to circumvent the management's directives and continue with their preferred pedagogical approaches while paying lip service to the management. Consequently, the only way for the schools to have consistent pedagogical approaches is to engage the teachers in regular dialogues through collaborative inquiry, using an adult learning model of sharing and collaborating to engender change of practice to improve student learning and achievement (Mullen, 2009). Smyth (1991) advocates that, to develop a sustainable and critical form of pedagogy, the teachers need to be able to describe what they do and the meaning behind their practice, interrogate their current practices and then reconstruct how they might do things differently for the benefit of their students' strong engagement. Hence, inquiry into effective pedagogy needs to be a core element in teacher professional learning.

Beetham and Sharpe (2007), as cited by Waring and Evans (2015), consider pedagogy as "guidance to learn: learning in the context of teaching and teaching that has learning as its goal" (Waring & Evans, 2015, p. 26). Penney and Waring (2000), cited by Waring and Evans (2015), propose a holistic conceptualisation of pedagogy which puts the "active critical learner", that is, the student, in the centre of its model, which is driven by the forces of "power relationship", "political agenda", "social processes" and "culture of practice" (Waring & Evans, 2015, p. 28). This model outlines the conditions under which active critical learning is further facilitated: (a) design of learning environment; (b) careful selection and application of styles; (c) supporting learner autonomy, including the choices in learning and student voice; and (d) exploring student and teacher beliefs/modelling and support (Waring & Evans, 2015, p. 28). Penny and Waring's (2005) findings echo the findings of my study that, unless student learning and achievement are central to all professional learning of the teachers, the learning opportunities do not result in school improvement. However, the reality of schools, the challenge of political and funding constraints and externally imposed demands on schools, such as NAPLAN testing and the publication of these results, put pressure on schools to focus on test results rather the global learning outcomes of the students. "Just in time" data such as NAPLAN are used to evaluate and inform the teachers' practices resulting in them operating under the constant stress about how these data are used in league tables to evaluate the school's performance.

The teacher participants considered that linking mathematics learning with other disciplines is an effective pedagogical strategy for engaging the students in their learning and engagement, and hence enabling deeper understanding and better outcomes. Linkage of mathematics learning with other disciplines naturally provides the platform for the teachers to create context-rich and relevant learning experiences for the students by associating mathematical concepts with other disciplines, thus assisting with meaning-making in the learning of mathematics. This finding shares the position presented by Obrycki (2009), Seeley (2009) and Stacey (2008), who found that a good mathematics education involves helping students to apply their knowledge in mathematics to solving real world problems.

Ongoing professional dialogues amongst teachers through collaborative inquiry is likely to facilitate a mature, in situ professional learning and growth culture for teachers to refine their pedagogical practices through sharing and learning from one another and sharing the same set of constraints and resources.

A grounded theory approach to collaborative inquiry in this study confirmed that, unless the teachers practise effective pedagogy by putting the students in the centre of the teaching and learning with the intent of improving outcomes and practice ongoing reflection and evaluation, then the students' outcomes will suffer.

7.3.2.2 Positive Teacher Mental Models and Beliefs

Teachers' mental models and beliefs determine how they set up their classrooms, view their students' capability and capacity to learn and succeed, and choose the pedagogical practice and approach. Irrespective of the school culture and external context, if the teacher's supervisor does not monitor his/her practice and growth, the teacher is master of his/her own domain once the classrooms doors are closed. Fennema and Franke (1992), as cited by Brahier (2005), describe "a model for classroom teaching that shows how the teacher's beliefs and knowledge (of content, pedagogy, and learning psychology) interact during a class" (Brahier, 2005, p. 348). Furthermore, Schoenfeld (1998), as cited by Brahier (2005), states that "the actions of a teacher and the decisions made while that teacher is leading a lesson are the results of three interacting factors, the teacher's goals, beliefs, and knowledge" (Brahier, 2005, p. 350).

The teacher's lesson goals can be predetermined by the school, department or curriculum specifications; however, the teacher's beliefs and knowledge are uniquely held by the teacher. As previously discussed, the teacher's pedagogical practice may be influenced by the school culture or guidelines to some extent. However, the privacy of an isolated classroom can be a fertile ground for the teacher's lack of content knowledge and poor pedagogical practice to be hidden for a long time.

Roberts and Pruitt (2008) state that "school culture has been defined as the set of norms, values and beliefs, rituals and ceremonies, symbols and stories that make up the 'persona' of the school" (Roberts & Pruitt, 2008, p. 8). They pointed out that we can see culture's imprint on attitudes towards professional learning and development as well as on the willingness to change (Roberts & Pruitt, 2008). This study shows that the teacher participants demonstrated the sub-culture of the site and their attitudes towards learning mathematics in middle school, even though they displayed quite clear organisational cultures and attitudes based on my observation as researcher over the period of this study. Specifically, in the first year of the study, some teacher participants were not as aligned in their cultural values and hence their attitudes towards the students' understanding and engagement were more inclined to the blame approach. However, in the second and third years, with the departure of teacher participants due to resignations, the culture and tone of the collaborative inquiry session and interview data showed a significant change to more positive and engaging dialogues centred on strategies to support students' learning and engagement.

Strauss (2001) suggests that some of the ways in which teachers believe that they can act at a distance are questioning, describing, summarising, modelling and demonstrating; even though they are not able to enter a student's mind directly, they believe that their instruction can influence his or her mind. Furthermore, the teachers' choices to instruct or influence others is related to their mental models of the structure of a learner's mind and how learning takes place in it (Strauss, 2001). This is affirmed by my study's findings regarding the extent to which the teachers' pedagogical change was subjected to their mental models and beliefs about students' learning and thinking (Loughran, 2005; Strauss, 2001).

It is evident from Table 3.2 that, at this research site, the teachers regarded the skills of students as thinkers and doers as indicators of deep mathematical understanding and hence mastery. The teacher perceptions on their learners suggest that the students favour modelling by their teachers as well as collaborating according to Olsen and Bruner (1996). These two skills, thinking (i.e. "as thinkers") and doing (i.e. "as doers"), identified by the teachers appear to be held by teachers with opposing mental models based on Entwistle and Peterson (2004). They believe that teachers who consider the children as doers hold the espouse model of teaching (see Table 2.1) while teachers who consider the children as thinkers favours in-action model of teaching (Entwistle & Peterson, 2004). Such contrasting positions about students' learning skills is are likely to be held by equal number of teachers holding the opposing perceptions about children.

In general, the teacher participants at this site shared similar beliefs about the teaching and learning mathematics, with very minor disparities. This suggests that the culture and values of the site, that is, the inherent beliefs, did influence the dispositions of the teachers over time to create a homogenous set of beliefs about teaching and learning. This finding is supported by Darling-Hammond and McLaughlin (2011) and Louis et al. (2003) who conclude that over time a school wide culture emerges due to the sustained engagement of ideas and practices at a site. Another possible explanation is that the School Principal recruits teachers with a certain teaching and learning dispositions.

Furthermore, the teachers also held the belief that students' mathematical achievements were dependent largely on their efforts, practice and self-belief. However, none of the teachers selected "attempting all homework" as a critical factor affecting student achievement in mathematics. This contradiction suggests a possible gap between teachers' articulated beliefs and their practice; this gap can be challenged through collaborative inquiry so that the teachers can resolve such apparent conflicts between perceptions and strategies (Darling-Hammond & McLaughlin, 2011; Globe & Horm, 2010; Hashweh, 2003; Manrique & Abchi, 2015). This can be attributed to the ideals that teachers may have held but which were not supported by their actions. Another possible explanation of this contradiction is that the teachers disown their responsibility for the students' learning because, by so doing, they do not need to motivate or assist the students in accessing the curriculum (Nickson, 2000; Seeley, 2009).

Another belief firmly held by the teachers at this site is that teachers need to continue to evolve in their practice in order for them to learn and grow as educators. This may be ascribed to the principles of Dweck's (2006) theory on growth mindset, which is widely advocated and articulated at this site for the students and the teachers. Professional learning opportunities at this research site are widely available through workshops and seminars during staff in-service days, as well as external professional learning opportunities through conferences, workshops and seminars off site.

There are mixed reports about how teachers' mental models and beliefs affect their classroom practice. However, most agree that the successful connection of beliefs to practice is contingent on the contextual factors outlined above (Jones et al., 2011; Provest, 2013). My study's findings show that, unless the teachers have positive beliefs about mathematics teaching and learning for students and achievement, then teachers in isolated classrooms will continue to practise based on their long held beliefs rather than modifying their practice to support the school's strategic vision and desired pedagogical practice. Provest (2013) cites the work of Wood et al. (1991) that "the belief and practice relationship seems to be dialectical in nature" in terms of "both beliefs influencing practice and practice influencing belief" (Provest, 2013, p. 214). Consequently, some teachers will continue to grow and develop in their beliefs when opportunities are presented for them to reflect and challenge their practice for change, which reinforces the importance of in situ professional learning opportunities in the form of collaborative inquiry by the teachers, to enable closer alignment of beliefs and practice in accordance with the school's general program to benefit the students (Bessoondyal, 2005; Bishop, 2006; Mowlaie & Rahimi, 2010; Utley, 2004). Wong (2009) also affirms that collaborative inquiry contributes to close the gap of belief-to-practice alignment; his study synthesises many research finding that "dialogue amongst teachers was found to facilitate a process of belief change" (Wong, 2009, p. 25).

During the collaborative inquiry sessions, when students' achievement was discussed, it is clear that the teachers held the belief that any preconceived ideas about students and their potential for achievement can be attributed to a fixed mindset approach and hence were not considered favourable at this site, which promotes a growth mindset.

Teachers in this study articulated clearly that teachers must have passion and treat their profession as a calling and vocation to be effective. This belief can be attributed to the site-based conviction that all teachers are called to their roles by God and that their teaching roles are divine and sacrosanct, as it is a faith based school. Setting the site bias aside, Cox (2010), when advocating his proposal of Critical Friend Group — that it is important for educators to prioritise time to meet with fellow colleagues at their schools — stated that teachers need to be "passionate about providing positive, motivating learning experiences to all students" and "rejecting status quo" (Cox 2010, p. 33). Lai (2004), as cited by Sun and Liu (2010), explains that:

> …excellent teachers mean those teachers who could comprehensively utilize creative thinking ability and creative practice ability to labour creatively, explore new supposes, make new products, and make great contributions in the development of [the] education domain, and can be confirmed by the experts in the same vocation. (Sun & Liu, 2010, p. 201)

The teachers in this study clearly accepted the nobility of teaching as a high calling and a vocation.

The teacher's mental model and beliefs, hence, will impact their commitment to creating an optimal learning environment for deep understanding and strong engagement of the students. They will also determine how the teacher will use assessment and the quality of assessment, which may result in the authenticity of the assessment being compromised.

7.3.2.3 Authentic Assessment

The teachers argued that improved scaffolding in assessments can better assess higher order thinking skills. In addition, they considered that students' mathematical mastery and achievement can be enhanced by the use of data derived from the assessments to discuss potential strategies to address students' learning weaknesses, and by differentiation in assessment items and classroom activities. A pedagogical approach strongly advocated by Goldsmith and Seago (2013), Gronewold (2009) and Herbel-Eisenmann (2009) is real time feedback on students' performance on well designed and appropriately challenging tasks; its effectiveness as a diagnostic tool for mathematics teachers to assess the mathematical thinking of their students is supported by this research finding.

As a team of teachers sharing the same understandings about the students at the same site with the same expectations from legislative requirements, the teachers in this study shared collaborative insights about teaching and learning challenges unique to the site. It can be concluded that it would be helpful for the teaching community to share ways of developing assessments, assessment practice and teaching approaches that would address the site-based needs of students. The teachers can work together to develop strategies to address students' weaknesses by using appropriate differentiation strategies that incorporate higher order thinking elements to assess the students. Similarly, the teachers through their collaborative insights, teachers can link teaching practice to examination skills required to assist the students to achieve.

Brahier (2005) asserts that teachers should plan how to accurately assess students' understanding and clearly identify the ways by which assessment is done during lesson planning. Hence, via collaborative inquiry sessions, the teachers are able to analyse the students' learning by developing assessments that not only accurately evaluate the students' mastery, but are authentic and will continue to collect evidence of the students' level of mastery of the concepts taught.

The teachers shared the need to continually evaluate the assessment instruments and the process of implementing the assessments, to ensure that authentic assessments are developed. Authentic assessments should focus on assessing the learning rather than collecting data for reporting to the parents and students. This was affirmed by Morocco and Solomon (1999), who argue that assessment tools should be used to reveal students' understanding and thinking, and that collaborative assessment of students' work by a team of teachers helps to measure the level of the students' mastery and develop a deeper understanding of how they learn and think. Wong (2009) echoes the same sentiment in that teachers need to engage in the analysis of their teaching and learning by reviewing students' work and using the students' progress as feedback on their practice. Such teachers' inquiry into students' thinking through assessments are likely to assist them to develop effective interventions to support students' learning (B. S. Nelson, 1999). The teachers play a critical role in selecting and organising the tasks, guiding the process through careful questioning techniques, and assessing student progress along the way. Changing assessment practices, for many, involves realising that assessment has other purposes than assigning grades, and that traditional methods do not provide the total picture of student achievement and attitudes.

It is evident from analysis of the data collected in this study that the teachers engaged in ongoing dialogues to seek team members' inputs and feedback in order to refine assessment tools. The dialogues also elicited ideas for classroom activities that would engender strong student engagement and deep understanding of the concepts being taught. Numerous research findings reveal that authentic assessment is critical to the learning process, in that multiple techniques of collecting assessment data provide the teacher with a broad and diverse method for determining each student's level of development. In particular, formative assessment can be used throughout the teaching and learning life cycle to determine whether the progress of an individual is of acceptable rate, even though it only provides a snapshot of that individual's progress, while summative assessment can be used as a final evaluation of an individual's performance (Brahier, 2005).

The teachers were also able to discuss and deliberate on the types and frequency of assessments to be administered for student learning. As the teachers incorporate assessments into their lesson planning, the assessments will be deemed authentic if they exist for the sole purpose of assessing the students' progress in learning rather than merely assessing their learning. Any assessment is merely a snapshot of student performance on a particular topic on a certain day (Brahier, 2005). The former way of using assessment focuses on the mastery of learning and demands that the teachers find ways to continue to engage the students in learning towards mastery, while the latter focuses on reporting what the students have learned and is not concerned with what the students have not mastered unless an appropriate feedback protocol is instituted.

The teachers in this study expressed concerns about students failing their assessments and that the students might not be motivated by their learning goals. Students also need to respond to the challenges of dealing with parental disappointment or their own disappointment, or teacher embarrassment. Consequently, it is essential to constantly evaluate the purpose of assessments as a tool for learning rather than an end point in the evaluation of success.

7.3.2.4 Optimal Environment

The findings also indicate that collaborative inquiry challenges teachers' pedagogical perceptions, and that alignment of teachers' mental models and beliefs is necessary to create an optimal learning environment for students. Cox (2010) and Small (2011) both argue that the best professional learning for teachers is ongoing, personalised and job-embedded, which is supported by the findings in this study in which the teachers articulated that collaborative inquiry needed to be scheduled into teachers' work schedule to be effective. Coupled with an optimal school structure, programs and resources, this professional learning model can add value to the teachers' efforts to create an optimal learning environment. Most teachers favour self-directed programs rather than top-down programs mandated by the school leadership team. Bessoondyal (2005), Mowlaie and Rahimi (2010), Nickson (2000), Stacey (2008) and Utley (2004) assert that teachers' beliefs about pedagogy determine their attitude and choices of classroom activities. This point is supported by this study's finding that the creation of an optimal learning environment is contingent on the mental models of the teachers. Morocco and Solomon (1999) and B. S. Nelson (1999) confirm that teachers need to have an inquiring approach in their own professional learning for them to be able to implement a constructivist-compatible instructional environment. It is also noted that the implementation of effective collaborative inquiry will be dependent on the knowledge and skills of the facilitator. With these conditions, the goal to enhance student engagement in mathematical thinking through immersion in the new concepts and ideas can be attained.

An optimal learning environment in a school is founded on the school culture because it defines the way things are done in a school. Furthermore, the culture of a school binds the stakeholders together and in turn influences how people in the school behave, what they think and how they feel. In order to create an optimal learning environment in the classrooms, the school culture needs to be student focused, highly collaborative and collegial, with its members sharing the belief that everyone is a learner. The teachers who work in such a culture often engage in action research and problem-solving, examining their practice with the ultimate aim of improving student performance (Darling-Hammond & McLaughlin, 2011; Roberts & Pruitt, 2008).

Teachers play an important role in shaping the culture of a school, and vice versa. School administrators should support their teachers to develop the kind of environmental identity that strengthens a school culture grounded in environmental education (Darling-Hammond & McLaughlin, 2011; Mason, 2003). In this study, it can be seen that, over the three years of data collection, the tone of the collaborative inquiry sessions changed significantly as a more collaborative culture was emerging from the teachers; a change of Principal also engendered some positive school reform and empowerment of teachers to change their practice. This finding is supported by Poekert's (2012) observation that "teachers who participate in collaborative professional development and receive individualised feedback on their teaching engage in reform efforts to improve school culture and teaching practice" (Poekert, 2012, p. 110).

In order for the classroom learning environment to be optimal, Waring and Evans (2015) propose an assessment model which includes "activating students as owners of their own learning" as one of the five effective strategies (Waring & Evans, 2015, p. 136). Waring and Evans (2015) state that "encouragement of student responsibility in the feedback process" is one of many key principles for effective feedback to students (Waring & Evans, 2015, p. 152). This point was clearly identified by the teacher participants in this study; they maintained that the students need to take ownership for their learning as active members of the learning environment to attain optimal conditions. The student population in a school is never homogeneous, even in selective schools or independent schools. Consequently, the school culture will set the scene for the tone of the classrooms; and when teachers collaborate and have a consistent approach to the type of desirable learning environment, then they can create a high expectation that the students will take ownership for their learning as one important part of the puzzle. This includes assisting the students to align their learning goals with evaluation and assessment. OECD (2013), as cited by Waring and Evans (2015), identified that one of the priorities of assessment policy, arising from an evaluation of assessment and evaluation practice in 28 OECD countries, is to "integrate student assessment and school evaluation into a single framework"; this confirms this research's finding that aligning students' learning goals with evaluation and assessment is important (Waring & Evans, 2015, p. 136).

Student engagement in students' learning and academic outcomes was not investigated in this study. The teacher participants in my study suggested that students' level of engagement could be measured by the extent to which the students continued learning on their own beyond the confines of the classrooms and school hours. Highly engaged students tend to continue investigation, research and learning beyond the classroom instructions because their teachers inspire them to continue inquiring as lifelong learners rather than merely learning because they have to. Even though past studies have not made a conclusive link between student engagement and student outcomes, this study revealed that student engagement is the goal of the teachers; this is supported by Gningue et al. (2013) who conclude that the effectiveness of teaching can be measured by students' engagement. Their study suggests that, with student engagement in learning, student outcome is a likely by-product of that learning (Gningue et al., 2013).

My data revealed that the teacher participants repeatedly expressed concerns about how the external environmental constraints limited their capacity and opportunities to create an optimal learning environment for the students to enhance their learning and engagement. External factors, such as shifting demographics, new technology, varying societal demands, government legislation changes and compliance demands, put constraints and pressure on schools which sometimes limit the schools' capacity to sustain an optimal learning environment when they have limited access to relevant resources (Roberts & Pruitt, 2008).

Another possible constraint in creating an optimal learning environment is the parents. Parents may not be educators, but they base their judgement of the optimal learning environment on their own past experience as students in schools, without taking into consideration the changes in technology, new knowledge from brain science, and shifting demands in the economic forces and the workforce. The teachers in this study found that the parents expected learning to be done in a certain way and were often critical or opinionated about the ways in which the teachers taught. Consequently, a unified approach to creating an optimal learning environment by the teachers through collaborative inquiry, supported by clear and concise communication from the school management, would assist in addressing some of the external constraints.

In this study, I have developed the theory that, unless the learning environment, including external contexts such as international and national priorities, school culture, school leadership and management priorities, is conducive for learning, then the students' engagement and deep understanding in mathematics will suffer.

Using grounded theory for collaborative inquiry, this study has suggested that, for deep understanding and strong engagement of students, an optimal environment is an essential ingredient.

7.3.3 Emerging Model for Effective Implementation

As illustrated by Figure 6.3 in Chapter Six, a theoretical model for effective implementation of collaborative inquiry as a professional learning tool emerged from the findings. The three-dimensional model places student engagement in learning firmly in the centre to help the teachers and school management to focus on the sole goal of teacher professional learning. I originally used student outcomes as the sole purpose; however, the data collected and analysed were not sufficient to inform student outcomes, but student engagement as perceived by the teachers could be substantiated by the evidence. The lack of conclusive studies to link student engagement and student outcomes or achievement also solidifies my choice of the goal as student engagement in learning (Gningue et al., 2013).

The core outlined above is encased by a tetrahedron making up four surfaces representing the four essential elements that emerged in this study as four equally important areas of professional learning required by teachers in order to engage their students in learning (see Figure 6.2). These elements are effective pedagogy, positive teacher mental model and belief, optimal environment and authentic assessment. The importance of these four elements was discussed in the previous chapters and previous section. The final outer sphere is the collaborative inquiry approach as a professional learning approach for teachers in schools.

This model illustrated in Figure 6.3 differs from the one proposed by Schnellert and Butler (2014) in that their focus of collaborative inquiry in the actions of the iterative inquiry cycles in the practice environment (e.g. schools) supported by professional learning resources. My study focussed on the content (e.g. elements) on which the teachers ought to inquire during their on-site professional learning to make their profession learning effective and relevant to their work as teachers.

One similarity between my collaborative inquiry model and that proposes by Schnellert and Butler (2014) is that both models identify that teachers bring their experience, assumptions and beliefs with them to the professional learning activities.

7.4 Limitations of Study

This study took place at one site over a three-year period from a possible sample population of 12 middle school mathematics teachers, and the final sample size of four teachers limits the applicability of the findings to other sites. However, this grounded theory study is supported by a literature review and observations made during the data collection and analysis to clarify my thinking and questions. Hence, this qualitative research, by using a grounded theory approach as research methodology, revealed concepts and theories which can be widely applicable to all schools, with localised amendments and adjustments during implementation.

Furthermore, the teachers' change in pedagogical practice over the three years was not measured, which would be another angle by which to measure the effectiveness of the proposed method of professional learning for teachers.

7.5 Contributions of the Study

The insights gained in this study contribute specifically to the body of work on the professional learning of teachers using a collaborative inquiry approach and are likely to add weight to the importance of this model of teacher professional learning. More generally, it adds value to the body of work on how the professional learning and growth of teachers link with school culture, structure and pedagogical approach, as well as clarifying the school leadership's role and responsibility for creating a conducive learning environment for the teachers in order that the students' learning and learning outcomes are improved.

Another contribution from my dual role as a researcher and Principal of the research site was an exploration of a cost effective and meaningful way by which teachers' professional learning can be incorporated into their work routine; the proposed model involves team planning time as collaborative inquiry learning time, to add value to the time when the teachers meet, with effective facilitation to improve their professional performance. I hope to share this approach to professional learning with fellow Principals, so that schools can put into effect the collaborative inquiry approach to professional learning of teachers as a valid, effective, collaborative and supportive learning environment for the teachers while benefiting the schools for their continual improvement.

7.6 Future Directions

Future research which can be undertaken as an extension to this study includes:

- Extending the study to a larger scale of collaborative inquiry for the whole school rather than at several year levels in one department;
- Examining the effectiveness of collaborative inquiry by measuring the growth trajectory of the teachers' practice or student engagement over a one-year period;

- Measuring the impact of teachers engaging in collaborative inquiry into a specific areas of teaching (e.g. teaching of higher order thinking, by evaluating students' engagement and learning outcomes);
- Evaluating the impact of facilitation skills on the effectiveness of collaborative inquiry.

7.7 Conclusions

This research has been a most rewarding and humbling experience for me. I found that all the teacher and student participants were open and engaged in the research process, seeking to add value to the research. Without their invaluable inputs, this study would not have been possible.

In conclusion, this three-year longitudinal study set out to evaluate the effectiveness of collaborative inquiry as a professional learning approach to challenge middle school mathematics teachers' pedagogical practice. Using the structured teacher planning time built into the teacher work schedule, a teacher survey, student focus group interviews and teacher interviews were used to gain insights about collaborative inquiry. The key findings outlined below respond to the statement of problem identified, research objectives and research questions outlined in Chapter One.

My first key finding is that the benefits of collaborative inquiry as a professional learning tool for teachers at a school site outweigh its challenges, which can be managed through careful planning. Consequently, schools should endeavour to implement such a professional learning approach for all the teachers. The school leadership's role of supporting the teachers' professional learning and growth is to provide a conducive and relevant learning platform and structure to make learning relevant while meeting adult learning principles. A built-in approach for teachers' learning is deemed more relevant when it is structured into the teachers' work routine than when professional learning involves external presenters at conferences, workshops and seminars, as these presenters do not have the same understanding of the contextual constraints and preferences.

The bolt-on professional learning approach has a place in the whole suite of professional learning opportunities and plans for teachers; however, the retention and implementation of learning, and hence its effectiveness for change and improvement, is likely to be lower than a built-in collaborative inquiry approach to learning supported by the teacher participants. The school leadership needs to evaluate the way in which professional learning is conducted for the staff to engender a highly collaborative and collegial tone for growth and development amongst its teachers. Professional learning for the teachers occurs in a mix of different forms such as short-burst learning through conferences, seminars and workshops on topics of interests, or long-term sustainable internal collaborative inquiry groups to drive the school's culture and values on topics relevant to the school in stages. The collaborative inquiry groups and sessions can be organised according to teachers' professional learning needs and goals based on the broader school's strategic goals and directions. Another way to use external experts for a specific area of staff learning needs is to bring them into the school as consultants so that the learning is targeted and contextualised for the staff of the school. In the event that the cost of bringing the external consultants into the school is prohibitive, the school leadership can pool together funding from several local or system schools having the same learning needs to fund the external experts for staff in-service days. Should this approach not be possible, then a more structured approach can be to send staff to these external learning opportunities to return to the schools to share and implement desired changes for school improvement.

The school culture sets the tone for the school's expectations of all its stakeholders, including staff, students and parents. The school culture needs to be clear, positive and consistent, so that all aspects of the school will complement one another to enhance the culture of the school. A positive and clearly articulated school culture supported by policies and practices will provide the foundation for the teachers to set the tone of their classrooms, including the expectations of their students. The school culture also determines the mental model and beliefs of the school through the teachers' pedagogical practice and professional learning needs and approaches. These beliefs will then determine how the school leadership structures its classes, curriculum and professional learning activities. Every decision at schools needs to focus on the student engagement and their learning outcomes. When adopting collaborative inquiry as a professional learning tool for teachers, the goals of the learning must ultimately be improving student outcomes. Without such a sharp focus, the investment of time and funding in professional learning is likely to be futile.

My second finding was that, for collaborative inquiry to be effective at a school, the teacher professional learning must incorporate one or more of the following four elements depending on the context and setting of the school within its internal and external environment: positive teacher mental model and belief, effective pedagogy, optimal environment and authentic assessment. All teachers are pre-disposed by their background to hold a certain mental model; they can either have growth or fixed mindsets. Collaborative inquiry will be an effective professional learning tool for teachers if the teachers' mental models and beliefs are aligned with the school's culture and values as well as its preferred pedagogical practice. Such alignment will form a strong foundation for collaborative inquiry, as it removes the potential challenge of needing to negotiate and manage the differences before finding common ground on which to build teachers' knowledge and skills for the benefit of the students. To create an optimal environment, the teachers need to have a tool kit of effective pedagogy. The pedagogical practice of teachers dictates how they run their classrooms and how the tone of the classrooms is set. Pedagogical practice grows over time when the teachers' tool kits grow, as we develop greater understanding of how student brains learn and grow, how to effectively communicate certain complex and challenging concepts, the innovative usage of available technology to engage students, and the development of students' deep understanding and consolidated learning. The control of the pace and type of learning activities to engage the students, including the incorporation of effective differentiation strategies focusing on the students and their learning needs, lies with the teachers. In order for the teachers to develop relevant and meaningful learning activities, they need to have the right mindset and beliefs about teaching and learning and students' capacity to learn and achieve, to have strong subject content knowledge so that appropriate activities are selected to enhance learning, and to take risks with their choices of activities to model to the students about risk taking. No amount of relevant and engaging activities can create an optimal learning environment if the teacher does not create a supportive environment that encourages risk taking and the safety to make mistakes.

Ongoing professional dialogues that are structured and facilitated (i.e. via collaborative inquiry) are likely to ensure that the ongoing evolution of the optimal environment can be modified and moderated to respond to emerging needs of the school and changing educational landscapes and demands. Through adopting a collaborative inquiry approach to the professional learning of teachers by involving the teachers in regular dialogues and professional exchanges with expert facilitation, a teacher-empowered model of the optimal learning environment can be shaped and reinforced at the school site, with greater investment of energy and belief from the teachers and a greater consistency of practice amongst them.

To ensure that optimal learning environments exist in schools, the teachers need to be committed to developing interesting and engaging lessons while teaching the students to take ownership of their learning.

The last finding is the development of a theoretical model which was shown as Figure 6.3. The model was developed based on the findings of the grounded theory study supported by a progressive literature review to verify my thinking and progress of theory sampling and saturation. However, the model is not tested in context to refine it to apply this emerging theory in practice. It is grounded in the theory emerging from this research, and articulates that, for collaborative inquiry to be an effective professional learning tool for teachers at school, the students must be at the core of the inquiry represented by the sphere in the centre of the model. The theoretical model (Figure 6.3) does not fully explored the various ways in which collaborative inquiry could be implemented in the specific school context and the logistics involved in making this tool come to life. Hence, a model of collaborative inquiry akin to the one developed in this study can be developed and shared with school leaders to support AITSL's (2014) advocacy of teacher professional standards.

The topics of the teachers' collaborative inquiry are then grouped into four equally important and pivotal elements — optimal environment, effective pedagogy, authentic assessment and positive teacher belief and mental model — forming a tetrahedron encasing the spherical core representing the student outcomes as show in Figure 6.2. The outermost sphere (see Figure 6.3) represents the collaborative inquiry approach to teachers' professional learning in schools, rounding out the skills and knowledge of the teachers through their inquiry into those identified four elements with the expressed and single focus of enhancing students' strong engagement in learning and developing their deep understanding of mathematics.

Inquiry is the very specific skill of asking questions and reflecting on the evidence presented in order to investigate ways by which problems can be resolved and solutions developed. Collaboration is also a very specific skill, requiring all those involved to share a common mission and goals, and to possess excellent listening and communication skills with well-developed team norms. Hence, for collaborative inquiry to be used as a preferred professional learning tool for teachers in schools, the development of the above mentioned skills will be essential to ensure the effectiveness of such approach to professional learning.

Other than building the collaborative inquiry skills of teachers in order for each inquiry team to be functioning within the agreed norms, a facilitator who is trained in specific facilitation skills is required for each inquiry team. Ground rules that are clearly outlined and team goals and purpose, including terms of reference, are obligatory to avoid misunderstanding or lack of progress. Furthermore, the terms of reference need to outline the milestones and periodic reporting mechanisms so that progress is made.

For collaborative inquiry to work effectively in schools, it is essential to equip leaders to facilitate and lead these collaborative inquiry groups to ensure that they are productive in meeting both the school's and individual teachers' needs.

In the development of this collaborative inquiry model, there were several resource and organisational restrictions incorporated:

i. The focus and agenda of the collaborative inquiry model must be clearly articulated by the school leadership each year after consultation with staff;
ii. A time allocation for collaborative inquiry is needed to ensure that such meetings do not encroach on the teachers' work load;
iii. Competing demands must be streamlined and prioritised by the school leadership to provide a conducive environment for the teachers to engage in productive dialogues and learning.

Collaborative inquiry provides the teachers with an effective professional learning approach that meets their learning needs in context and is embedded in their job schedule. To teach, the teachers need to first engage the students in learning. To engage students, teachers need to create an optimal environment conducive for learning which requires effective pedagogy to make learning relevant, practical and useful. Effective pedagogy contains both generic and school relevant pedagogical strategies consistent with the school's mission, vision and values. Students' learning must be continually assessed authentically to ascertain their progress in the mastery of the learning. However, teachers bring with them mental models and beliefs which require alignment with the team's or school's values and approaches so that consistency of practice can be attained. Within the general direction of effective pedagogy endorsed by the schools, teachers still have the latitude to make their own professional choices based on their respective class contexts, taking into account their training and teaching experience. Collaborative inquiry is well placed to challenge these mental models and beliefs while offering the teachers regular forums to close the gaps between their perceptions and practice. All of these elements about teaching and learning are captured in the model in Figure 6.3 entitled "Collaborative Inquiry: A professional learning tool for teachers (Inter-connecting evolutionary links".

7.9 Recommendations

Schools should be action research centres where continual improvement of teachers' practice can be affected by managing pedagogical change through the use of collaborative inquiry as a professional learning tool for teachers.

REFERENCES

ACARA. (2013). National Assessment Program. Retrieved from http://www.nap.edu.au/naplan.naplan.html

ACARA. (2016). MySchool. Retrieved from https://www.myschool.edu.au/

Adams, N. E. (2015). Bloom's taxonomy of cognitive learning objectives. *Journal of the Medical Library Association*, *103*(3), 152-153. Retrieved from https://search-proquest-com.dbgw.lis.curtin.edu.au/docview/1705546386?accountid=10382

AITSL. (2014). Australian Institute for Teaching and School Leadership. Retrieved from http://www.teacherstandrards.aitsl.edu.au/OrganisationStandards/Organisation

Anderson, G. (1998). *Fundamentals of educational research* (2nd ed.). New York: Falmer.

Argent Rogers, M. (2013). *A study of the relationship between professional development evaluation and middle school mathematics achievement.* (Ed. D thesis). Retrieved from http://search.proquest.com.dbgw.lis.curtin.edu.au/docview/1462058433?accountid=10382

Attard, C. (2013). “If I had to pick any subject, it wouldn’t be maths”: Foundations for engagement with mathematics during the middle years. *Mathematics Educational Research Journal*, *25*, 569-587.

Atweh, B. (2007). *Pedagogy for socially response-able mathematics education.* Paper presented at the Annual Conference of the Australian Association of Research in Education, held in Fremantle, Perth: http://www.aare.edu.au/publications-database.php/5299/pedagogy-for-socially-response-able-mathematics-and-science-education.

Barrington, F., & Evans, M. (2014). Participation in Year 12 mathematics 2004 – 2013. Retrieved from http://amsi.org.au/publications/participation-year-12-mathematics-2004-2013/

Bessoondyal, H. (2005). *Gender and other factors impacting on mathematics achievement at the secondary level in Mauritius.* (Ed. D thesis). Curtin University, Perth

Bequary, C. B. (2012). *Key experiences, changes in mental models, and teachers' development of proficiency in differentiating instruction.* (Ed. D thesis). University of Connecticut, Storrs.

Bishop, A. (2006). Values and beliefs: Introduction. In F. K. Leung, K. D. Graf & F. J. Lopez-Real (Eds.), *Mathematics education in different cultural traditions: A comparative study of East Asia and the West* (pp. 427 - 433). New York: Springer.

Brahier, D. J. (2005). *Teaching secondary and middle school mathematics* (2nd ed.). Boston: Pearson.

Buckley, S., De Bortoli, S. & Thomson, L., (2013). *PISA 2012: How Australia measures up*. Melbourne: Retrieved from http://www.acer.edu.au/documents/PISA-2012-Report.pdf

Butler, D. L., & Schnellert, L. (2012). Collaborative inquiry in teacher professional development. *Teaching and Teacher Education*, *28*(8), 1206-1220. http://dx.doi.org/http://dx.doi.org/10.1016/j.tate.2012.07.009

Butler, D. L., MacNeil, K. & Schnellert, L., (2015). Collaborative inquiry and distributed agency in educational change: A case study of a multi-level community of inquiry. *Journal of Educational Change, 16*(1), 1-26.

Chinn, C. A., & Brewer, W. F. (1993). The role of anomalous data in knowledge acquisition: A theoretical framework and implications for science instruction. *Review of Educational Research*, *63*, 1 - 49.

Clarke, A. E. (2003). Teacher inquiry: A defining feature of professional practice. In A. D. Clarke (Ed.), *Teacher inquiry: Living the research in everyday practice*, 201 - 211. London: Routledge Falmer.

Clarke, A. E., DePiper, J. N., Frank, T. J., NIshio, M., Campbell, P. F., Smith, T. M., & Choi, Y. (2014). Teacher characteristics associated with mathematics teachers' beliefs and awareness of their students' mathematical dispositions. *Journal for Research in Mathematical Education*, *45*(2), 246 - 284.

Classen, M. (2002). Apply math to real life. *School Librarian in Canada, 22*(1), 30.

Cook, D. (1962). The Hawthorne effect in education research. *Phi Delta Kappan, 44*(3), 116 - 122. Retrieved from http://www.jstor.org/stable/20342865

Cook, L. L. (2008). *Increasing middle grades mathematics achievement through effective teaching practices.* (Ed. D thesis). Walden University, Minneapolis.

Corbin, J. M., & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Thousand Oaks, CA: Sage Publications, Inc.

Cox, E. (2010). Criticial friends groups: Learning experience for teachers. *School Library Monthly, XXVII*(1), 32 - 34.

Creswell, J. W. (2013). *Qualitative inquiry & research design: Choosing among five approaches* (3rd ed.). Thousand Oaks, CA: SAGE Publications.

Darling-Hammond, L., & McLaughlin, M. W. (2011). Policies that support professional development in an era of reform. *Phi Delta Kappan, 92*(6), 81-92. http://dx.doi.org/doi:10.1177/003172171109200622

Dufour, R. (2007). Professional learning communities: A bandwagon, an idea worth considering, or our best hope for high levels of learning. *Middle School Journal, 39*(1), 4 - 8.

Dweck, C. (2006). *Mindset: The new psychology of success*. New York: Random House.

Eckert, E., & Bell, A. (2005). Invisible force: Farmers' mental models and how they influence learning and actions. *Journal of Extension, 43*(3), 1-6

Entwistle, J. N., & Peterson, E. R. (2004). Conceptions of learning and knowledge in higher education: Relationships with study behaviour and influences of learning environments. *International Journal of Education Research, 41*, 407 - 428.

Feiman-Nemser, S. (2001). Helping novices learn to teach. *Journal of Teacher Education, 52*(1), 17-30.

Fennema, E., Carpenter, T. P. & Sowder, J., (1999). Creating classrooms that promote understanding. In E. Fennema & T. A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp. 311 - 334). Mahwah, NJ: Lawrence Erlbaum Associates

Forgasz, H. J., & Leder, G. C. (2008). Beliefs about mathematics and mathematics teaching. In P. W. Sullivan & T. Wood (Eds.), *The international handbook of mathematics teacher education: Knowledge and beliefs in mathematics* *teaching and teaching development* (Vol. 1, pp. 173 - 192). Rotterdam: Sense Publishers.

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59-109.

Fullan, M. (2010). *Motion leadership: The skinny on becoming change savvy*. Thousand Oaks, CA: Corwin.

Fullan, M., & Stiegelbaure, S. (1991). *The new meaning of education change*. New York: Teachers College Prss.

Garrett, K. (2010). Professional learning communities: Allow a transformational cutlure to take root. *The Education Digest, 76*(2), 4-9.

Ginsbury, H., & Dolan, A. (2011). Assessment. In F. Fennell (Ed.), *Acheiving fluency: Special education and mathematics* (pp. 85 - 103). Reston, VA: National Council of Teachers of Mathematics.

Given, H., Kuh, L., Leekeenan, D., Mardell, B., Redditt, S., & Twombly, S. (2009). Changing school culture: Using documentation to support collaborative inquiry. *Theory Into Practice, 49*(1), 36-46.

Globe, C. B., & Horm, D. M. (2010). Taking charge of your personal and professional development. *YC Young Children, 65*(6), 86 - 91.

Gningue, S. M., Peach, R., & Schroder, B. (2013). Developing effective mathematics teaching: Assessing content and pedagogical knowledge, student-centered teaching, and student engagement. *The Mathematics Enthusiast, 10*(3), 621-645.

Godfrey-Smith, A., Hughes, J., Purchase, S., Ramson, W. S. and Williams, J. H., (1991). *Oxford: The Australian reference Dictionary*. Oxford University Press Australia: Oxford.

Golafshani, N. (2005). *Secondary teachers professsed beliefs about mathematics, mathematics teaching and mathematics learning: Iranian perspective.* Ph.D thesis University of Toronto, Toronto.

Goldsmith, L. T., & Seago, N. M. (2013). *Examining mathematics practice through classroom artifacts*. Boston: Pearson.

Goodnough, K. (2005). Fostering teaching learning through collaborative inquiry. *The Clearing House, 79*(2), 88 - 92.

Goos, M. (2008). Sociocultural perspectives on learning to teach mathematics. In B. Jaworski & T. Wood (Eds.), *The mathematics teacher educator as a developing professional* (pp. 75 - 91). Rotterdam: Sense Publishers.

Gronewold, P. A. (2009). Math is about thinking: From increased participation to conceptual talk. In B. Herbel-Eisenmann & M. Girillo (Eds.), *Promoting purposeful discourse* (pp. 45 - 56). Reston, VA: National Council of Teachers of Mathematics.

Habegger, S., & Hodanbosi, P. (2011). Embedded instructional coaching: WHAT works. *Principal Leadership, 11*(6), 36-37,40-41.

Hammerman, J. K. (1999). Teacher inquiry groups: Collaborative explorations of changing practice. In M. Z. Solomon (Ed.), *The diagnostic teacher: Constructing new approaches to professional development* (pp. 187 - 200). New York: Teachers College Press.

Hashweh, M. Z. (2003). Teacher accommodative change. *Teaching and Teacher Education, 19*(4), 421-434. http://dx.doi.org/10.1016/S0742-051X(03)00026-X

Hattie, J. (2008). *Visible thinking: A systematic synthesis of over 800 meta-analyses relating to achievement*. Hoboken, NJ: Taylor and Francis.

Hennessya, S., & Deaneya, R. (2009). The impact of collaborative video analysis by practitioners and researchers upon pedagogical thinking and practice: A follow-up study. *Teachers and Teaching, 15*(5), 617 - 638. http://dx.doi.org/10.1080/13540600903139621

Henschke, J. A. (2009). Beginnings of the history and philosophy of andragogy 1833-2000. In V. Wang (Ed.), *Integrating adult learning and technology for effective education: Strategic approaches* (pp. 1 - 30). Hershey, PA: Global.

Herbel-Eisenmann, B. (2009). Some essential ideas about classroom discourse. In B. Herbel-Eisenmann & M. Girillo (Eds.), *Promoting purposeful discourse* (pp. 29 - 42). Reston, VA: National Cournoil of Teachers of Mathematics.

Horn, I. S. (2008). Turnaround students in high school mathematics: Constructing identities of competence through mathematics worlds. *Mathematical Thinking and Learning, 10*(3), 201 - 239.

Hoy, C. W. (2009). *Education perceptions: The relationshiop between student achievement, professional development and instructional practices in* *suburban secondary matheamtics program.* (Ed. D thesis). Saint Joseph's University, Philadelphia.

Jeffrey, L. & Beasley, K. (2010). Create the perfect play space: Learning environments for young children. Retrieved from http://www.ecrh.edu.au/docs/default-source/resources/ipsp/Create-the-perfect-play-space-learning-environments-for-young-children.pdf?sfvrsn=12

Joergensen, P. (2001). *Proposed professional development program for Middle School Mathematics teachers based on the perceived needs of teachers, Principals, and administrators in Philadelphia.* (Ph.D thesis). Temple University, Philadelphia.

Jones, N., Leitch, A., Lynam, T., Ross, H., &Perez, P. (2011). Mental models: An interdisciplinary synthesis of theory and methods. *Ecology and Society, 16*(1), 46-68.

Johnson, W. (2010). *Teacher collaboration in professional development: Building the social capital of elementary mathematics teachers.* (Ed. D thesis). University of Massachusetts, Lowell.

Kinach, B. M. (2002). A cognitive strategy for developing pedagogical content knowledge in the secondary mathematics methods course: Toward a model of effective practice. *Teaching and Teacher Education, 18*(1), 51 - 71.

Krainer, K., & Peter-Koop, A. (2003). The role of the principal in mathematics teacher development. In A. Peter-Koop, V. Santos-Wagner, C. Breen & A. Begg (Eds.), *Collaboration in teacher education: Examples from the context of mathematics education* (pp. 169 - 190). Dordrecht: Kluwer Academic Publishers.

Kruse, S., Bryk, A., & Louis, K. S. (1994). Building a professional community. Retrieved from https://www.learner.org/workshops/principals/about/index.html

Lebak, K., & Tinsley, R. (2010). Can inquiry and reflection be contagious? Science teachers, students, and action research. *Journal of Science Teacher Education, 21*(8), 953 - 979. Retrieved from http://dx.doi.org/10.1007/s10972-010-9216-x

Lee, M. (2010). 7 principles of highly collaborative professional development. *Science and Children, 47*(9), 28 - 31.

Lempert, L. B. (2007). Asking questions of the data: Memo writing in the grounded theory tradition. In A. Bryant & K. Charmaz (Eds.), *The SAFE handbook of grounded theory* (pp. 245 - 264). London: The SAGE Publications Ltd.

Loughran, J. (2005). *Developing a pedagogy of teacher education: Understanding teaching & learning about teaching*. Hoboken: Taylor and Francis.

Louis, S. K., Anderson, A. R., & Riedel, E. (2003). Implementing arts for academic achievement: The impact of mental models, professional community and interdisciplinary teaming. Retrieved from https://www.researchgate.net/publication/242095191_Implementing_Arts_for_Academic_Achievement_The_Impact_of_Mental_Models_Professional_Community_and_Interdisciplinary_Teaming

Manrique, M. S., & Abchi, V. S. (2015). Teachers´ practices and mental models: Transformation through reflection on action. *Australian Journal of Teacher Education*, *40*(6), 13 - 32.

Mason, H. M. (2003). *Environment identify development: Exploring the formative experiences and mental models of teachers engaged in environmental education.* (Master's thesis). Retrieved from http://search.proquest.com.dbgw.lis.curtin.edu.au/docview/1490996534?accountid=10382

Maxwell, J. A. (2005). *Qualitative research design: An interactive approach* (2nd ed.). Thousand Oaks, CA: Sage Publications.

MCEETYA. (2008). Melbourne declaration on educational goals for young australians. Retrieved from http://www.curriculum.edu.au/verve/_resources/National_Declaration_on_the_Educational_Goals_for_Young_Australians.pdf

Morocco, C. C., & Solomon, M. Z. (1999). Revitalising professional development. In M. Z. Solomon (Ed.), *The diagnostic teacher: Constructing new approaches to professional development* (pp. 247 - 267). New York: Teachers College Press.

Mowlaie, B., & Rahimi, A. (2010). The effect of teachers? attitude about communicative language teaching on their practice: Do they practice what they preach? *Procedia - Social and Behavioral Sciences, 9*, 1524-1528. http://dx.doi.org/10.1016/j.sbspro.2010.12.359

Mullen, C. A. (2009). Introducing collaborative communities with edge and vitality. In C. A. Mullen (Ed.), *The handbook of leadership and professional learning communities* (pp. 1 - 19). New York: Palgrave Macmillan.

Nelson, B. S. (1999). Reconstructing teaching: Interactions among changing beliefs, subject-matter knowledge, instructional repertoire, and professional culture in the process of transforming one's teaching. In M. Z. Solomon (Ed.), *The diagnostic teacher: Constructing new approaches to professional development* (pp. 1 - 21). New York: Teachers College Press.

Nelson, D., St Maurice, H., Strouse, K., &Waechter, C. (2003). How we have grown: Reflections on professional development. In A. D. Clarke (Ed.), *Teacher inquiry: Living the research in everyday practice* (pp. 115 - 126). London: RoutledgeFalmer.

Nelson, T. H. (2009). Teachers' collaborative inquiry and professional growth: Should we be optimistic? *Science Education, 93*(3), 548 - 580.

Nelson, T. H., Hathorn, T., Perkins, M., & Slavit, D. (2008). A culture of collaborative inquiry: Learning to develop and support professional learning communities. *Teacher College Record, 110*(6), 1269 - 1303.

Nickerson, S. D. (2007). Terms of practising mathmeatics teachers: Developing teacher professionals. In K. Krainer & T. Wood (Eds.), *Participants in mathematics teacher education: Individuals, teams, communities and networks* (pp. 89 - 110). Rotterdam: Sense Publishers.

Nickson, M. (2000). *Teaching and learning mathematics: A teacher's guide to recent research and its application*. London: Cassell Education.

Nisbet, S., Cooper, T., &Warren, E. (2003). Collaboration in professional development. In A. Peter-Koop, V. Santos-Wagner, C. Breen & A. Begg (Eds.), *Collaboration in teacher edition: Examples from the context of mathematics education* (pp. 23 - 40). Dordrecht: Kluwer Academic Publishers.

Nuhreborgen, M., & Steinbring, H. (2008). Manipulatives as tools in matheamtics teacher education. In D. W. Tirosh (Ed.), *The international handbook of mathematics teacher education: Tools and processes in mathematics teacher education* (Vol. 2, pp. 155 - 181). Rotterdam: Sense Publishers.

O'Connor, C. (2009). Reflecting on and adjusting one's own talk. In B. Herbel-Eisenmann & M. Cirillo (Eds.), *Promoting purposeful discourse: Teacher* *research in mathematics classrooms* (pp. 179 - 184). Reston, VA: National Council of Teachers of Mathematics.

O'Malley, G. S. (2010). Designing induction as professional learning community. *The Educational Forum, 74*(4), 318 - 327.

Obrycki, J. (2009). Listening to my students' thoughts on mathematics. In B. Herbel-Eisenmann & M. Cirillo (Eds.), *Promoting purposeful discourse: Teacher research in mathematics classrooms* (pp. 167 - 202). Reston, VA: National Council of Teachers of Mathematics.

Oktay, J. S. (2012). *Grounded theory*. New York: Oxford Press.

Olsen, D. R., & Bruner, J. S. (1996). Folk psychology and folk pedagogy. In D. R. Olsen (Ed.), *Handbook of education and human development* (pp. 9 - 22). London: Blackwell.

Pajares, M. F. (1992). Teachers' belief and educational research: Cleaning up the messy. *Review of Educational Research, 62*(3), 307-332.

Penney, D., & Waring, M. (2000). The absent agenda: Pedagogy and physical education. *Journal of Sport Pedagogy, 6*(1), 4 - 37.

Perks, P., & Prestage, S. (2008). Tools for learning about teaching and learning. In B. Jaworski & T. Wood (Eds.), *The international handbook of mathematics education: The mathematics teacher educator as a developing professional* (Vol. 4, pp. 263 - 280). Rotterdam: Sense Publishers.

Poekert, P. E. (2012). Examining the impact of collaborative professional development on teacher practice. *Teacher Education Quarterly, 39(4),* 97 - 118.

Provest, L. E. (2013). *The multifaceted nature of mathematics knowledge for teaching: Understanding the use of teachers' specialised content knowledge and hte role of teachers' beliefs from a practice-based perspective and the role of teachers' beliefs from a practice-based perspective.* (Ph. D thesis). ProQuest Dissertations Publishing. Retrieved from http://search.proquest.com.dbgw.lis.curtin.edu.au/docview/1462612419?accountid=10382

Ravitz, J. L., Becker, H. J., & Wong, Y. T. (2000). *Constructivist-compatible beliefs and practices among United States teachers*. Retrieved from http://www.crito.uci.edu/tlc/html/findings.html

Reese, S. (2010). Bring effective professional development to educators. Available from http://www.aceteonline.org/ Proquest education Retrieved August 18, 2011

Reeves, D. B. (2009). *Leading change in your school: How to conquer myths, build commitment, and great results*. Alexandria, VA: Association for Supervision and Curriculum Development.

Rickey, D. (2008). *Leading adults through change: An action research study of the use of adult and transformational learning theory to guide professional development for teachers.* (Ph.D thesis). Retrieved from http://search.proquest.com.dbgw.lis.curtin.edu.au/docview/89240043?accountid=10382

Roberts, S. M., & Pruitt, E. (2008). *Schools as professional learning communities: Collaborative activities and strategies for professional development* (2nd ed.). Thousand Oaks, CA: Corwin Press Inc.

Rokeach, M. (1968). *Beliefs, attitudes, and values: A theory of organization and change.* San Francisco: Jossey-Bass.

Schifter, D. R., Bastable, V., &Russell, S. J. (1999). Teaching to the big ideas. In M. Z. Solomon (Ed.), *The diagnostic teacher: Constructing new approaches to professional development* (pp. 22 - 47). New York: Teachers College Press.

Schnellert, L., Kozak, D., & Moore, S. (2015). Professional development that positions teachers as inquirers and possibilizers. *LEARNing landscapes, 9*(1), 217 - 236.

Schoenfeld, A., & Kilpatrick, J. (2008). Towards a theory of proficiency in teaching mathematics. In D. Tirosh & T. L. Wood (Eds.), *Tools and processes in mathematics teacher education* (pp. 321 - 354). Rotterdam: Sense Publishers.

Seago, N. (2008). Mathematics teaching profession. In K. Krainer & T. L. Wood (Eds.), *Participants in mathematics teacher education: Individuals, teams, communities and networks* (pp. 331 - 352). Rotterdam: Sense Publishers.

Seeley, C. L. (2009). *Faster isn't smarter: Messages about math, teaching, and learning in the 21st century: A resource for teachers, leaders, policy makers, and families*. Sausalito, CA: Maths Solutions.

Senge, P. M. (2006). *The fifth discipline: The art and practice of the learning organization*. London: Random House.

Shadish, W. R., Luellen, J. K., & Clark, M. H. (2006). Propensity scores and quasi-experiments: A testimony to the practical side of Lee Sechrest. In R. R. Bootzin & P. E. McKnight (Eds.), *Strengthening research methodology: Psychological measurement and evaluation* (pp. 144 - 165). Washington, DC: American Psychological Association.

Small, D. (2011). Patience and partnership. *Principal Leadership, 12*(1), 26-30.

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions* Reston, VA: National Council of Teachers of Mathematics.

Smyth, J. (1991). *Teachers as collaborative learners: Challenging dominant forms of supervision*. Buckingham: Open University Press.

Stacey, K. (2008). Mathematics for secondary teaching: Four components of discipline knowledge for a changing teacher workforce. In P. Sullivan & T. L. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp.87 - 113). Rotterdam: Sense Publishers.

Stein, M. K., & Porr, C. J. (2011). *Essentials of accessible grounded theory*. Walnut Creek: Left Coast Press Inc.

Strauss, S. (2001). Folk psychology, folk pedagogy, and their relations to subject-matter knowledge. In B. Torff (Ed.), *Understanding and teaching the intuitive mind, student and teacher learning* (pp. 217 - 242). Mahwah, NJ: Lawrence Erlbaum Associates.

Sun, H., & Liu, T. (2010). The survey and analysis of excellent senior high school physics teachers' professional growth actuality. *International Education Studies*, *3*(3), 201-204.

Takahashi, A. (2014). Supporting the effective implementation of a new mathematics curriculum: A case study of school-based lesson study at a Japanese public elementary school. In Y. LI & G. Lappan (Eds.), *Mathematics curriculum in school education* (pp. 417 - 441). Dordrecht Springer.

Taylor, P., & Wallace, J. (2007). *Contemporary qualitative research: Exemplars for science and mathematics educators*. Dordrecht: Springer.

Teppo, A. R. (2015). Grounded theory method. In A. Bilner-Ahsbah, C. Knipping & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education: Examples of methodology and methods* (pp. 2 - 21).New York: Springer.

Terehoff, I. (2002). Elements of adult learning in teacher professional development. *National Association of Secondary School Principals Bulletin, 86*(632), 65-77.

Thibodeau, G. M. (2008). A content literacy collaborative study group: High school teachers take charge of their professional learning. *Journal of Adolescent & Adult Literacy, 52*(1), 54-64.

Thomson, S., Hillman, K., & Wernert, N. (2012). *Monitoring Australian year 8 students achievement internationally: TIMSS 2011*. Camberwell: ACER.

Utley, J. (2004). *Impact of a non -traditional geometry course on prospective elementary teachers' attitudes and teaching efficacy.* (Ph. D thesis). Oklahoma State University, Stillwater.

Valsiner, J. (2000). *Culture and human development: An introduction*. London: Sage.

Van Dresar, V. (1996). *Relationships of a mathematics content course for elementary/middle school teachers with preservice teachers' attitudes/beliefs about mathematics and the teaching of mathematics.* (Ph. D thesis). Retrieved from http://search.proquest.com.dbgw.lis.curtin.edu.au/docview/304272652/

Vander Veldt, M. (2006). *Exploring the relationship between teachers' beliefs in mathematics and their instructional practice.* (Ph. D thesis). Retrieved from http://search.proquest.com.dbgw.lis.curtin.edu.au/docview/304962919/

Vialle, W., Lysaght, P., & Verenikina, I. (2005). *Psychology for educators*. Melbourne: Thomson Learning.

Vinner, S. (2008). Some missing dimensions in matheamtics teacher education. In D. Tirosh & T. L. Wood (Eds.), *Tools and processes in mathematics teacher education* (Vol. 2, pp. 305 - 320). Rotterdam: Sense Publishers.

Vollstedt, M. (2015). To see the wood for the trees: The development of theory from empirical interview data using grounded theory. In A. Bilner-Ahsbah, C. Knipping & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education examples of methodology and methods* (pp. 23 - 48). Normal, IL: Springer.

Wang, L.-C. (1997). *Taiwanese elementary teachers' mathematics beliefs and the relationship to instructional practices in the context of constructivist* *curriculum change.* (Ph. D thesis). Retrieved from http://search.proquest.com/docview/304391007/

Waring, M., & Evans, C. (2015). *Understanding pedagogy: Developing a critical approach to teaching and learning*. Abington, OX: Routledge.

Watt, M. L., & Watt, D. L. (1999). Doing research, taking action, and changing practice with collaborative support. In M. Z. Solomon (Ed.), *The diagonostic teacher: Constructing new approaches to professional development* (pp. 48 - 77). New York: Teachers College Press.

White, A. L., Way, J., Perry, B., & Southwell, B. (2006). Mathematical attitudes, beliefs and achievement in primary pre-service mathematics teacher education. *Mathematics Teacher Education and Development, 7*(2005/2006), 33-52.

Wilke, R. (2008). *Developmental changes in preservice teachers' mental models of learning and instruction.* (Ph. D thesis). Retrieved from https://auth.library.curtin.edu.au/gw?url=http://search.proquest.com/docview/250773844/

Wilkins, E. A., & Shin, E. K. (2011). Peer feedback: Who, what, when, why, and how: Using data-driven practice, such as peer feedback, teachers can improve instruction and student learning. *Kappa Delta Pi Record, 46*, 112 - 117.

Wong, Y. T. (2009). Believing in(to) the profession: An investigation of the change in beliefs about drama education as a result of the advanced post graduate in drama and drama education. Retrieved from http://www.academia.edu/

Yoon, C., Thompson, M., & Zawojewski, J. (2007). John Dewey revisited — Making mathematics practical versus making practice mathematical. In R. Lesh, E. Hamilton & J. J. Kaput (Eds.), *Foundations for future in mathematics education* (pp. 268 - 279). Mahwah, NJ: Lawrence Erlhaum Associates, Inc.

Young, J. (2005). On insiders (emic) and outsiders (etic): Views of self, and othering. *Systemic Practice and Action Research, 18*(2), 151-162.

APPENDICES

Appendix A: Collaborative Inquiry: Team Meeting Field Note Template

Researcher:

Place:

Purpose: To record team meeting discussion to demonstrate Intellectual Quality (topic for this term) for Collaborative Inquiry sessions; to observe how pedagogical discussions affect teachers' practice in the light of the higher order thinking (IQ).

Knowledge as problematic: are students critiquing and second-guessing texts, ideas and knowledge?

Higher order thinking: are students using HOT operations within a critical framework?

Depth of knowledge: does the lesson cover operational fields in any depth, details or level of specificity?

Depth of students' understanding: does the work and response of the students provide evidence of depth of understanding of concepts or ideas?

Date/Time: ____day, _______, 2013, _______a/pm

Room set up: _______________

Participants:

Notes and observations:

Appendix B: Collaborative Inquiry: Classroom Observation Field Note Template

Researcher: Tsae Wong

Place:

Purpose: To record clips to demonstrate Intellectual Quality (topic for this term) for Collaborative Inquiry sessions; to observe teacher practices and student reactions in the light of the higher order thinking (IQ).

Knowledge as problematic: are students critiquing and second-guessing texts, ideas and knowledge?

Higher order thinking: are students using HOT operations within a critical framework?

Depth of knowledge: does the lesson cover operational fields in any depth, details or level of specificity?

Depth of students' understanding: does the work and response of the students provide evidence of depth of understanding of concepts or ideas?

Date/Time: Monday, May 20, 2013, 2.30 to 3.10 pm

Room set up: M3 – computer room

Participants: ELLL and Year 9 V Maths class

Topic: Function and excel spreadsheet

Time	Activities observed	

Appendix C: Collaborative Inquiry: Teacher Profile

Date:

Name:

Education background:

Teaching experience:

What main adjustments you made to transit to this school?

What do you think are the key strategies to improve teaching and learning practices?

How do you describe your teacher practice?

What do you see are the suitable professional support for teacher professional growth?

How do you teach higher order thinking skills in your class?

APPENDIX D Collaborative Inquiry: Teacher Survey

Teacher Professional Learning and Growth

Thank you for agreeing to assist me with my research. I hope you can assist me further by completing this survey which will take around 15 - 30 minutes.

I would appreciate it if you could complete the survey by 31 October 2014 so that I can start analysing the data.

1. Education background and training:

   Choose from:

   General Primary Education; Middle School Mathematics Education; Others (please specify)

   My education background is predominantly in

   Choose from:

   General Primary Education; Middle School Mathematics Education; Others (please specify)

2. My teaching experience is:

   Choose from:

   Less than 1 year; 1 – 5 years; 6 – 10 years; 11 – 15 years; 15 – 20 years; more than 20 years

   The total number of years experience I have teaching middle school is:

   Choose from:

   Less than 1 year; 1 – 5 years; 6 – 10 years; 11 – 15 years; 15 – 20 years; more than 20 years

The total number of years I have teaching middle school mathematics is:

Choose from:

Less than 1 year; 1 – 5 years; 6 – 10 years; 11 – 15 years; 15 – 20 years; more than 20 years

3. Beliefs

The following statements are about your belief about mathematics learning, teaching and achievement. There is no right or wrong answer to these items. Please read each statement carefully, then select the response that best indicates how you think about the statement using the scale provided.

Mathematics learning:

Olsen and Bruner (1996) classified the learner minds. Based on your experience teaching Middle School mathematics, rate how successful each of these minds would be most effective in/responsive to mathematics learning and hence achievement.

Very badly/badly/neither badly or well/well/very well

Children as doer

Children as knowers

Children as thinkers

Children as knowledgeable

4. The following statements are about your belief about mathematics learning, teaching and achievement. There is no right or wrong answer to these items. Please read each statement carefully, then select the response that best indicates how you think about the statement using the scale provided.

   Strong disagree/disagree/neither agree or disagree/agree/strongly agree

   Students need to be encouraged to explain and evaluate their solution process.

   Memorising arithmetic facts (such as times tables) and algorithms are essential to master mathematics problem solving.

   Mathematics curriculum has not changed since I went to secondary school.

   Mathematics learning is being able to get the right answers quickly.

   Mathematics learning process may include students experiencing of uncertainty, conflict, confusion and surprise.

   Mathematics learning is independent of students' real life experience.

5. Select the top 3 conditions from the list below which are conducive

   Supportive learning environment

   Structured learning environment

   Teach algorithm and scaffold well

   Strong foundational basis such as number sense and times tables

   Authentic and real life assessments

   Good and probing questions

   Problematised learning environment

Always start a new unit with a real life problem

Students are allowed to construct their understanding

Students taking ownership of their own learning

6. Maths achievement

Please rate the level of your agreement on each statement below.

Strong disagree/disagree/neither agree or disagree/agree/strongly agree

The key to success in mathematics is effort on the part of the students.

Students need to be rewarded for the right procedures more than getting the right answers.

Students who do not achieve well in mathematics should try hard, e.g. go to tutorials, do more homework questions.

Mathematics success is linked to students' self-belief of their own abilities.

To be good in mathematics, students need to have a "mathematical minds".

Practice is the key to mathematics mastery.

7. List your top 3 factors which you think affect student achievement from the list below

Students not afraid to make mistakes

Students keep thorough notes and attempt all homework

Students always start to tackle the simple routine problems first

Students have strong foundational basis such as number sense and times tables

Students are happy and engage in real life problem-solving.

Students immerse and enjoy answering “why” questions all the time.

Students transfer their knowledge to unfamiliar situations by linking the old and new concepts.

Students are engaged in critical mental activities such as linking mathematical ideas themselves

Students construct their understanding by relating ideas and articulating them.

Students accept uncertainty and apparent conflicting ideas in the process of learning.

8. Pedagogical practice

In each of the teaching practice, identify the frequency most appropriate describes your use of each strategy in the recently completed unit of work.

Almost never/Seldom/Sometimes/Frequently/All the time

Showing students the fundamental of a new concept.

Explaining to students the theory behind a new concept.

Modelling how to apply a concept before students try them.

Providing scenarios for students to find solution in groups.

Setting group work activities for students to develop the concept on their own.

Help students learn the fundamentals before giving the challenging tasks.

Creating a situation where the students need to ask questions to learn.

Providing scaffold for students to construct their own understanding.

In my mathematics classrooms, I use the following teaching strategies most often:

Almost never/Seldom/Sometimes/Frequently/All the time

Showing students how to apply concepts into routine problems.

Allowing students to apply concepts into challenging problems on their own.

Students working in groups to solve authentic problems.

Asking “why” questions to check for student understanding.

Asking clarifying questions to check for understanding.

Referring to textbook examples and questions.

Structuring my lessons based on the textbook in order not to confuse the students.

Encourage students to figure things out on their own rather than showing them how to solve the problem.

Ensure the students rehearse the basic procedures and methods in demonstrated.

Ensure the students master the algorithms through drill and practice.

Help students who are stuck quickly to avoid developing dislikes or frustrations about maths learning.

Quiet individual work to consolidate their learning and thinking.

Always start with teaching formal mathematics symbols and algorithms to ensure that they have strong basics and foundation.

9. I know the students have developed mathematical understanding on the new concept when

   Strong agree/agree/neither agree or disagree/disagree/strongly disagree

   There are able to solve mathematics problems using the correct formulae.

   They are able to apply the concept to solve challenging problems.

   They are able to explain the new concept to another student.

   They ask questions to clarify understanding.

   They ask questions to relate new concept with existing concept.

   They are able to articulate new situations by which the new concept can be applied.

   They stick to the algorithms they mastered to solve all types of problems.

   They are able to explain why their wrong solutions [ed. solutions are wrong].

10. Professional learning needs

    Strong agree/agree/neither agree or disagree/disagree/strongly disagree

    Reading a book / article on pedagogical practices helps me with my professional growth and practice.

    Watch videos teaching about pedagogical practices helps me with my professional growth and practice.

Watching fellow colleagues teaching in a classroom seating helps me with my professional growth and practice.

Listening to follow colleagues share about an effective pedagogical practice helps me with my professional growth and practice.

Listening to fellow colleagues share about an effective pedagogical practice helps me with my professional growth and practice.

Attending conferences/seminars by practitioners outside the organisation helps me with my professional growth and practice.

Year level planning sessions to share practical and workable pedagogy relevant for the unit helps me with my professional growth and practice.

My pedagogical practice can be enhanced by trying new ideas or approaches; then reflect on their effectiveness.

My pedagogical practice can be enhanced by listening to other more experience teachers share ideas.

My pedagogical practice can be enhanced by incorporating mandatory pedagogical ideas into unit plan.

My pedagogical practice can be enhanced by getting another staff member to observe me and give me feedback.

My pedagogical practice can be enhanced by discussing and sharing teaching ideas during unit planning.

My pedagogical practice can be enhanced by reading about/listening to the latest research and approaches and incorporate them into my teaching.

The most effective professional learning approach differs from teacher to teacher.

Teachers need different types of professional learning experience to grow.

Professional dialogues amongst teachers are the best professional learning opportunity.

There's a link between teacher professional development activities and the teacher's pedagogical practice.

There's a link between teacher professional develop activities and the student achievement.

There's a link between teacher pedagogical practice and student achievement.

Appendix E: Collaborative Inquiry: Student Focus Group Interview

Questions and Field Note Template

Year level:

Date:

1. Introduction – overview of researcher's learning journey and explain about masters and PhD.
2. Warm up question: How has 2013 been for you in your mathematics class?
3. Questions:
   a. Focusing on mathematics, can you tell me, if your teacher teaches mathematics differently from previous years?
      i. interviewer prompted "did she make it easy or more challenging?"
      ii. Interviewer prompted "did you like the harder stuff, or you just went my brain is hurting?"
      iii. Interviewer prompted "did you find the explanation any difference (from previous years)? Or do you do more hands-on stuff than last year?"
      iv. Interviewer prompted "did she make things harder and more challenging?"
   b. If you compare last year with this year, you identified that the teachers taught you differently from last year, did it help you to improve your results?
   c. Does enjoying the learning, help you do better work? How does the teacher makes?
   d. What did you teacher do this year which you liked?

e. Would doing things in maths help?

f. Do you have teachers do not use the right pedagogies?

g. How does the school ensure that teachers cater?

h. Are the teachers important in your achievement?

i. Would the teachers’ knowledge be important for your learning?

j. Teachers who care about your learning, tended to make you try harder?

k. Interviewer summary of discussion: Students wanted teachers who are:

l. What impressed you most this year in maths? Why?

Appendix F: Collaborative Inquiry: Staff Individual Interview 1 Questions

Script: Thank you for agreeing to assist me with this research through this one-one interview. I will be asking you a few questions concerning your ideas about teacher beliefs, pedagogy, teacher professional learning activities, student engagement and student achievement. This interview will be recorded for research analysis purpose. A transcription of this recording will be sent to you for your approval before analysis. Please relax and respond to the question to your best ability.

Teacher's name:

Date:

Time:

Venue:

Teacher's belief and mental model on optimal learning environment

1. What influences, if any, do you think our college environment and/or community have on our students' learning, engagement and achievement in the middle years' mathematics? Please explain your answer.

Teacher's belief and mental model on his/her pedagogy

2. What influences, if any, do you think our college environment and/or community have your teaching of the middle years' mathematics? Please explain your answer.

Teacher's belief and mental model on professional learning and growth

3. What role and value do your think the year level mathematics planning meetings have to improve teacher’s belief, pedagogical practice, student engagement and ultimately achievement?

4. Do you think mathematics planning meetings through collaborative inquiry can serve effectively as a professional learning platform for teachers if it is appropriately facilitated?

5. Do you see collaborative inquiry through the use year level mathematics planning time as allocated weekly teaching load as a valuable professional learning opportunity for the teachers in the year level teaching team to investigate certain aspects of teaching mathematics? Please explain your answer.

6. In your personal experience, what were some useful professional learning activities you would recommend for effective mathematics teaching to middle year students? Elaborate why it was useful for you.

7. If given the opportunity to be teacher researcher to conduct collaborative inquiry with your fellow teachers, what aspects about mathematics teaching would you be most interested? Please explain your answer.

8. Based on your past experience, what were some professional learning activities directly affected your teaching practice? What were these activities? How did they impact on your teaching practice? Why did you think these activities affect your practice?

Thank you for your time in answering these questions. I would interview you again in Term 4. I appreciate your participation in this project.

Interview ending time:

Appendix G: Collaborative Inquiry: Staff Individual Interview 2 Questions

Script: Thank you for agreeing to assist me with this research through this one-one interview. I will be asking you a few questions concerning your ideas about teacher beliefs, pedagogy, teacher professional learning activities, student engagement and student achievement. This interview will be recorded for research analysis purpose. A transcription of this recording will be sent to you for your approval before analysis. Please relax and respond to the question to your best ability.

Teacher's name:

Date:

Time:

Venue:

Teacher's belief and mental model on professional learning and growth

1. The teacher survey data conducted last year indicated that the following methods were good professional growth means for teachers:

(i) professional dialogues with colleagues through planning meetings, other share ideas

(ii) external conferences/seminars presented by practitioners

(iii) watch others teach or demonstrate

(iv) professional practice and reflection

Which of these methods is most useful for you? Please explain your answer.

Which of these professional learning activities have you accessed in the last 2 years?

Teacher's practice on creating an optimal learning environment for students

2. Teachers who undertook the survey last year ranked a supportive learning environment as most conducive for mathematics learning. Could you discuss how you create such learning environment in your classes?

Teacher's belief and mental model on planning and pedagogy

3. For a recent unit of middle year mathematics you taught, if you had unlimited time and resources to re-teach this unit, what would you do different?

Teacher's belief and mental model on assessment

4. The teacher survey outcome suggested that the best evidence used to ascertain students' mathematical understanding of new concept are:

(i) they are able to explain the new concept to another student;

(ii) why they have the wrong solutions;

(iii) able to apply concept to solve challenging problems;

(iv) able to articulate new situations by which the new concept can be applied. Based on this small sample findings, do you think our assessment items in middle years use such evidence? If so, what were the assessment items. If not, what change do we need to make to our assessment tasks?

Teacher's pedagogical practices for greater student engagement and achievement

5. It was noted in the teacher survey that "students' ability to transfer their knowledge to unfamiliar situations by linking the old and new concepts" as well as "students not afraid to make mistakes" as the top 2 factors which affect student

achievement. Could you share what strategies you adopt or apply to achieve learning environment that encourage these two behaviours in students?

6. The following three factors were not identified as top 3 factors in affecting student achievement at all.

(i) students immerse and enjoy answering “why” questions all the time;

(ii) students are engaged in critical mental activities such as linking mathematical ideas themselves;

(iii) students accept uncertainty and apparent conflicting ideas in the process of learning.

Could you comment on your view of the values of these three factors in assisting students to achieve very high or high achievement in mathematics?

Teacher’s belief and mental model on professional learning and growth in relation to student engagement and student achievement

7. Given the discussions above about student engagement and achievement, what do you think the teachers can do, within their realm of control, to maximise the student engagement and achievement in mathematics? Please explain your answers.

Thank you for your time in answering these questions. I appreciate your participation in this project.

Interview ending time:

Appendix H: Collaborative Inquiry: Research Ethics Approvals

Curtin University

Memorandum

To	Tsae Wong, SMEC
From	Pauline Howat, Administrator, Human Research Ethics Science and Mathematics Education Centre
Subject	Protocol Approval **SMEC-109-11**
Date	21 February 2012
Copy	Bill Atweh, SMEC

Office of Research and Development

Human Research Ethics Committee

Telephone 9266 2784
Facsimile 9266 3793
Email hrec@curtin.edu.au

Thank you for your "Form C Application for Approval of Research with Low Risk (Ethical Requirements)" for the project titled *"Collaborative inquiry as a professional learning approach for middle school mathematics teachers using productive pedagogies"*. On behalf of the Human Research Ethics Committee, I am authorised to inform you that the project is approved.

Approval of this project is for a period of twelve months **3rd January 2012** to **2nd January 2013**.

The approval number for your project is **SMEC-109-11**. *Please quote this number in any future correspondence*. If at any time during the twelve months changes/amendments occur, or if a serious or unexpected adverse event occurs, please advise me immediately.

Pauline

PAULINE HOWAT
Administrator
Human Research Ethics
Science and Mathematics Education Centre

Curtin University

Memorandum

To	Tsae Wong, SMEC
From	Pauline Howat, Administrator, Human Research Ethics, Science and Mathematics Education Centre
Subject	**PROTOCOL APPROVAL – EXTENSION**
Date	5 February 2013
Copy	Rekha Koul, SMEC

Office of Research and Development

Human Research Ethics Committee

TELEPHONE 9266 2784
FACSIMILE 9266 3793
EMAIL hrec@curtin.edu.au

Thank you for keeping us informed of the progress of your research. The Human Research Ethics Committee acknowledges receipt of your Form B progress report for the project *"Collaborative inquiry as a professional learning approach for middle school mathematics teachers using productive pedagogies."*

Approval for this project is extended for the year to 2nd January 2014.

Your approval number remains SMEC-109-11. Please quote this number in any further correspondence regarding this project.

Please note: An application for renewal may be made with a Form B three years running, after which a new application form (Form A), providing comprehensive details, must be submitted.

Thank you.

Pauline

PAULINE HOWAT
Administrator
Human Research Ethics
Science and Mathematics Education Centre

Appendix I: Collaborative Inquiry: Research Information Sheet for Participants

Curtin University of Technology

Doctorate in Education

Research Study Information Sheet for Participants

Title of Project:

Collaborative Inquiry as a Professional Learning Approach for Middle School Mathematics Teachers Using Productive Pedagogies

Principal Investigator:

Tsae Wong

Aims of Project:

The aim of the project is to study how effective is the collaborative inquiry approach for middle school mathematics teachers using productive pedagogies. The following research sub-questions will be addressed:

a. What are the values and problems in using collaborative inquiry as a way of professional learning?

b. How does collaborative inquiry help teachers to use productive pedagogies in their daily practices?

c. Will the change in teachers' teaching affect students' achievement in mathematical understanding and learning?

Participation Activities:

Student participation over one semester (2013) for this study of how middle school mathematics teachers use the collaborative inquiry model to learn about how to implement productive pedagogies in to the middle school mathematics teaching and learning practices occurs in two forms:

a. Classroom activities will be video-typed and analysed by the participating teachers once a term for the sole purpose of this study, i.e. to be viewed and analysed by the participating teachers for their professional learning and by the researcher for the purpose of this research.

b. Five volunteered students, with parental signed consent, from the mathematics classes of these teachers will attend 3 30-min focus group sessions to ascertain how the teacher's pedagogical change affects the students' achievement in mathematical understanding and engagement.

Only data of participants who give written consent will be used for this project.

All data for this project will be accessed by the investigator and the thesis committee members only. All data will be kept in the home and office locked cabinets of the investigator.

The data collected will be used for this project only. The data will be coded for research purpose of this project and all identities of the participants will be not be identifiable in any published material.

All participation is completely voluntary and all participants are at liberty to withdraw at any time during this project without prejudice or negative consequences to the participants. Non-participation will not affect the individual's right and access to the professional learning opportunity.

The benefits to all participants are:

1. Better understanding and practice of using collaborative inquiry method of professional learning
2. Better understanding and practice of productive pedagogies
3. Build collegial relationship with fellow mathematics teaching colleagues

Contact Information of the Project:

Investigator

Name: Tsae Wong

Telephone numbers: (07) 555 68 285

Email address: tsae.wong@postgrad.curtin.edu.au

Supervisor

Name: Dr Rekha Koul

Telephone number: (08) 9266 4074

Email address: R.koul@curtin.edu.au

Human Research Ethics Committee (Secretary)

Should participants wish to make a complaint on ethical grounds, please make contact with the following office.

Telephone: (08) 9266 2784 or

Email: hrec@curtin.edu.au or

In writing to:

C/- Office of Research and Development

Curtin University of Technology

GPO Box U1987, Perth WA 6845)

This project has been approved by the Curtin University Human Research Ethics Committee, approval number: SMEC-109-11.

Please sign and return the attached consent form if you support your child's voluntary participation in this project.

Appendix J: Collaborative Inquiry: Gate keeper consent for access of research site

Collaborative Inquiry as a professional learning approach for middle school mathematics teachers using productive pedagogies

INFORMATION SHEET (Head of College)

Why is the research being conducted?

The purpose of the study is to investigate how collaborative inquiry can be used as a professional learning tool for teachers during middle school mathematics team planning time.

This research will be viewed as a site base research pilot project in view of wider application of the collaborative inquiry to all teachers in the coming years as a professional development tool to meet the varying needs of teachers.

The expected benefits of the research

This study will seek to measure the effectiveness of teacher collaborative inquiry as a tool to provide onsite, relevant and pertinent professional learning approach to meet the emerging needs of teacher development and the strategic priorities of the college.

Your participation is voluntary

Participants will be sought from all middle school mathematics teachers, parents and students. Informed consent will be sought and withdrawal will be accepted at any point in the study.

What will staff, parents and students be asked to do?

Middle school mathematics teachers will be required to engage in collaborative inquiry during their team planning time by identifying professional learning needs to promote intellectual rigour, reading and discussing research articles relevant to the identified needs, and applying lessons learned to lessons. All meeting discussions will be taped and transcribed for analysis by the researcher for research purpose only. Lessons will be taped for discussions and analysis during the meeting for research purpose only.

Parents will be asked to provide written consent for their children to be interviewed.

Students will be interviewed in groups so that the change in the teacher professional journey can be triangulated from the students' perspective.

Your confidentiality

Your confidentiality is highly valued. The school will not be identified anywhere in the research material. We ask that you respect the privacy of the participants who may be interviewed. The information provided will be kept completely confidential. The meeting discussions and classroom observations will be recorded and transcribed by the researcher and recordings will be destroyed upon completion of the non-identified transcription to maintain the confidentiality. Names or any identifier that could be interpreted by a third party will not appear anywhere on the recording or transcript. Participation in this research is voluntary and participants can withdraw from the study at any time. The school administration will not be informed of the identity of participating staff. In order to maintain confidentiality:

no demographic details will be taken

all recordings will be deleted once transcribed

participants may view their transcript before inclusion in the study

Risks staff/school

Although there may be some personal comments during the discussions, it is unlikely that participants will experience any discomfort or harm from taking part in the study. The personal experiences of the teachers will be vital and the researcher's opinion is that each individual's professional journey will be relevant to this study. Participants will be offered to view the results prior to publication. The school will not be identified anywhere in the research material.

Questions / further information

If you have any questions regarding this research, please contact Dr Rekha Koul at R.koul@curtin.edu.auor (08) 9266 4074.

The ethical conduct of this research

Curtin University of Technology conducts research in accordance with the National Statement on Ethical Conduct in Human Research (2007). If potential participants have any concerns or complaints about the ethical conduct of the research project they should contact the Administrator, Human Research Ethics, Science and Mathematics Education Centre on (08) 9266 2784 or hrec@curtin.edu.au.

Feedback to you

If you would like to review the results of this study once completed, please do not hesitate to contact Dr Rekha Koul at R.koul@curtin.edu.au.

Expressing consent

By completing the attached, you will be deemed to have consented to your site being used as research site. Please detach this sheet and retain it for your later reference.

Privacy Statement

The conduct of this research involves the collection, access and / or use of your identified personal information. The information collected is confidential and will not be disclosed to third parties without your consent, except to meet government, legal or other regulatory authority requirements. A de-identified copy of this data may be used for other research purposes. However, your anonymity will at all times be safeguarded.

Consent Form

Research Team

Names: Dr Rekha Koul and Tsae Wong

Science and Mathematics Education Centre

Curtin University of Technology, Perth, Western Australia

Contact Phone: 08 9266 7073.

Contact Email: R.Koul@curtin.edu.au or t.wong@student.curtin.edu.au

By signing below, I confirm that I have read and understood the information package and in particular have noted that:

I understand that staff and student involvement in this research will involve meeting discussions and focus group meetings that are conducted by a researcher. The meetings and focus should last for about 50 minutes. The discussions will be conducted once per week this year for a maximum of 17 sessions for staff and once a term for students;

I understand teacher and student participants have the right to withdraw at any time without penalty or comment.

I understand that the recording will be transcribed and destroyed after analysis. Only the research team will have access to the recording;

I have had any questions answered to my satisfaction;

I understand the risks involved;

I understand that there will be direct benefit to teacher and student participants from their participation in this research. There will be benefit for their school;

I understand that participation in this research project is completely voluntary. Teacher and student participants have the right to withdraw at any time without penalty or comment by those conducting the research. They can choose not to answer certain questions in the research collection or undertake certain activities.

I understand that if I have any additional questions I can contact the research team;

I understand that I can contact the Administrator, Human Research Ethics, Science and Mathematics Education Centre on (08) 9266 2784 or hrec@curtin.edu.au. if I have any concerns about the ethical conduct of the project; and

I agree to participate in the project.

Head of College, Trinity Lutheran College, Ashmore, Queensland

Name : ____

Signed: ________________________________ Date:________

Appendix K: Collaborative Inquiry: Adult Participant Consent Form

Curtin University of Technology

Doctorate in Education

Research Study Participant Consent Form

Title of Project:

Collaborative Inquiry as a Professional Learning Approach for Middle School Mathematics Teachers Using Productive Pedagogies

Principal Investigator:

Tsae Wong

Statements of Confirmation:

I have been informed of and understand the purpose of this study.

I have been given an opportunity to ask questions.

I understand I can withdraw at any time without prejudice.

Any information which might potentially identify me will not be used in published material.

I agree to participate in the study as outlined to me.

Name of Participant: ________________________________

Signature of Participant: ____________________________ Date: ___/ ____ / _____

Appendix L: Collaborative Inquiry: Student Participant Consent Form

Curtin University of Technology

Doctorate in Education

Research Study Participant Consent Form

- Student Participant -

Title of Project:

Collaborative Inquiry as a Professional Learning Approach for Middle School Mathematics Teachers Using Productive Pedagogies

Principal Investigator:

Tsae Wong

Statements of Confirmation:

I have been informed of and understand the purpose of this study.

I have been given an opportunity to ask questions.

I understand I/my child can withdraw from the study at any time without prejudice.

Any information which might potentially identify me/my child will not be used in published material.

I agree to /I given consent for my child to participate in the study as outlined to me.

Name of Student Participant: ______________________________________

Signature of Student Participant: ___________________________________

Date: ___/ ____ / _____

Name of Parent of Student Participant: _______________________________

Signature of Parent of Student Participant: ____________________________

Date: ___/ ____ / _____

Please return the signed copy of the consent form to your Mathematics or MASC teacher by the end of this week, 3 May 2013.