

Deterministic conversion of uncertain manpower planning optimization problem

Bo Li, Yuanguo Zhu, Yufei Sun, Grace Aw and Kok Lay Teo

Abstract—Manpower planning is a very important component of human resource management. However, there are many indeterminate factors that should be taken into consideration in manpower planning. For example, the decision of employees to quit the job is determined by their preference, which is beyond the control of human resource department. It can be realistically modeled as a random variable when the historical data of quitting rate is large enough. Otherwise, it can only be regarded as an uncertain variable when the historical data is inadequate. In this paper, we discuss a manpower planning optimization problem for a manufacturing company with hierarchical system, where the quitting rate of employees is modeled as an uncertain variable. First, we formulate a mathematical model for this uncertain manpower planning optimization problem, where the influence on the production outputs by employees is taken into consideration. Second, we present a deterministic conversion method to transform this uncertain manpower planning optimization problem into an equivalent deterministic discrete-time optimization problem. It is further converted into an equivalent linear programming model with an equality constraint and an inequality constraint. Finally, we use the real data from Singapore, Denmark and China to carry out a numerical simulation and make a comparison with the results obtained based on stochastic model to show the advantages of our method.

Index Terms—Manpower planning, quitting rate, uncertainty theory, uncertain variable, linear programming.

I. INTRODUCTION

WITH the rapid development of economy and increasing globalization, manpower planning has become a critical issue for human resource department in today's competitive world, especially for the international corporations and large organizations. It requires to develop an optimal management strategy to match the requirement of the staffs and the available positions for achieving specific goals.

However, there are many indeterminate factors that should be taken into consideration in manpower planning, such as labor demand, working life and economic environment. At present, many different stochastic manpower planning models

have been established and discussed in the literature. For instance, Chattopadhyay and Gupta [2] developed a stochastic manpower planning model under the set up, where the survival rates and the number of workers at different ages are treated as random variables. Yan et al. [18] discussed two long-term stochastic demand planning models for air cargo terminal manpower supply planning in long-term operations, where the labor demand is described as a random variable. Young and Vassiliou [23] considered a non-linear stochastic model of hierarchically structured management staffs in commercial and industrial organizations, where the promotion of employees is modeled as a random variable.

However, for most of the related literature, the effect of the optimal planning strategy on the production outputs has not been taken into full consideration. In 2016, Sun et al. [16] constructed a new model of a manpower planning optimization problem for a manufacturing company with different types of employees, where the types of workers have direct influence on the production outputs, and the quitting rate of employees was assumed as a random variable with known expected value and variance. Then, the manpower planning optimization problem was formulated as a stochastic discrete time optimization model.

When we need to describe a subjective imprecise quantity, the concept of belief degree will be used. In order to deal with personal belief degrees, an uncertainty theory was introduced by Liu [10] in 2007 and refined by Liu [14] in 2010 based on normality, duality, subadditivity and product axioms. After that, uncertainty theory has been investigated by many researchers. Sheng and Kar [15] provided a new formula using inverse uncertainty distribution to describe the moment of uncertain variable. Yao [20] proposed a formula to calculate the variance of an uncertain variable via the inverse uncertainty distribution. Yao and Li [22] presented a kind of uncertain process, called uncertain alternating renewal process and developed an uncertain renewal theory. Zhu [24] introduced and dealt with an uncertain optimal control problem with application to a portfolio selection model. Up to now, uncertainty theory has become a branch of axiomatic mathematics and contains many topics, such as uncertain programming (Gao [8], Liu [13]), uncertain process (Chen [3], Liu [11]), uncertain finance (Chen and Gao [4], Chen et al. [6]) and uncertain differential equation (Chen and Liu [5], Yao et al. [21]).

As mentioned above, the quitting rate of employees for a manufacturing company is usually uncertain in nature, which should be taken into consideration during the decision making process in human resource management. Hence, the manpower

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planning optimization problem under uncertain environment is as important as in stochastic environment. In this paper, we consider a manpower planning optimization problem for a manufacturing company with hierarchical system under uncertain environment, where the quitting rate is modeled as an uncertain variable. We shall compare our results with those obtained using the stochastic model proposed in Sun et al. [16].

The rest of this paper is organized as follows. Some basic concepts in uncertain theory are reviewed in Section II. In Section III, the manpower planning optimization problem is formulated as an uncertain discrete time optimization model. In Section IV, we transform it into a deterministic discrete time optimization model and propose a method to solve it. Based on the real data from Singapore, Denmark and China, a numerical simulation is carried out in Section V. The last section gives a conclusion.

II. PRELIMINARY

For the formulation and discussion of the manpower planning optimization problem under uncertain environment, many basic concepts, such as uncertain variable, uncertainty distribution and uncertain expected value, in uncertainty theory will be used in Section III. Thus, they will be introduced in this section.

Let Γ be a nonempty set, and \mathcal{L} a σ -algebra over Γ . Each element $\Lambda \in \mathcal{L}$ is called an event. A set function \mathcal{M} defined on the σ -algebra over \mathcal{L} is called an uncertain measure if it satisfies the following axioms: (**normality axiom**) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ ; (**duality axiom**) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ ; (**subadditivity axiom**) $\mathcal{M}\{\bigcup_{i=1}^{\infty} \Lambda_i\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}$ for every countable sequence of events $\Lambda_1, \Lambda_2, \dots$.

The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space. A product uncertain measure \mathcal{M} is defined by Liu [12] to produce an uncertain measure of compound event: (**product axiom**) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \dots$. Then the product uncertain measure \mathcal{M} is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\}, \quad (1)$$

where Λ_k are arbitrarily chosen events from \mathcal{L}_k for $k = 1, 2, \dots$, respectively.

For modeling the quantities under uncertain environment, a concept of uncertain variable is defined by Liu [10] as a function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that for any Borel set of real numbers, the set $\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$ is an event. In order to describe uncertain variable in practice, the uncertainty distribution Φ of an uncertain variable ξ is defined by $\Phi(x) = \mathcal{M}\{\xi \leq x\}$, for any real number x .

An uncertain variable ξ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad x \in \mathfrak{R} \quad (2)$$

denoted by $\xi \sim \mathcal{N}(e, \sigma)$, where e and σ are real numbers with $\sigma > 0$.

To describe the average value of an uncertain variable ξ , the concept of expected value is defined in Liu [10] as

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq x\} dx - \int_{-\infty}^0 \mathcal{M}\{\xi \leq x\} dx \quad (3)$$

provided that at least one of the two integrals is finite. The variance of ξ is defined as

$$Var[\xi] = E[(\xi - E[\xi])^2]. \quad (4)$$

Let ξ and η be independent uncertain variables with finite expected values. Then, for any real numbers a and b ,

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta]. \quad (5)$$

Remark 1: Uncertainty theory (Liu [10], Liu [14]) is a branch of mathematics for modeling belief degrees, while probability theory (Kolmogorov [9]) is for modeling frequencies. The main different is that the product uncertain measure is the minimum of uncertain measures of uncertain events, i.e.,

$$\mathcal{M}\{\Lambda_1 \times \Lambda_2\} = \mathcal{M}\{\Lambda_1\} \wedge \mathcal{M}\{\Lambda_2\},$$

for uncertain events Λ_1 and Λ_2 . The product probability measure is the product of probability measures of random events, i.e.,

$$\Pr\{\Delta_1 \times \Delta_2\} = \Pr\{\Delta_1\} \times \Pr\{\Delta_2\},$$

for random events Δ_1 and Δ_2 . It implies that uncertain variables and random variables obey different operational laws.

III. PROBLEM FORMULATION

Manpower planning consists of putting right number of employees, right kind of employees at the right place, right time, doing the right tasks for which they are suited for the achievement of different goals. The hierarchical system can be interpreted as a pyramid with lowest rank of job category at the base, for example, junior production worker. Moving up to higher levels, there are higher ranks of job categories, such as senior production worker, supervisor, senior supervisor, and manager. For each level, there will be personnel change through recruitment, promotion, dismissal, as well as employees quitting. The first three activities are regarded as decision variables because they can be decided by human resource department. But, the quitting rate is determined by employees' preference, which is beyond the control of human resource department.

In 2016, Sun et al. [16] constructed a stochastic discrete time optimization model for a manpower planning problem in stochastic environment, where the quitting rate is considered as a random variable. The objective is to minimize the expected human resource cost while maximizing the total expected production outputs. However, the quitting rate can be realistically regarded as a random variable only when the historical data of quitting rate is large enough. This is a fundamental premise. When the historical data is inadequate, it will be more appropriate to formulate it as an uncertain variable. In the following, we consider a manpower planning optimization problem for a manufacturing company with hierarchical system under uncertain environment.

Let M be the number of staff levels and T be the number of time periods considered. The initial total number of staff in the company is N . To formulate the mathematical optimization model, we introduce the following notations:

- $x_i(t)$: the number of staff in level i at the end of period t ,
- $R_i(t)$: the recruitment number of staff being recruited to level i at the end of period t ,
- $P_i(t)$: the promotion number of staff being promoted to level i at the end of period t ,
- $S_i(t)$: the dismissal number of staff in level i at the end of period t ,
- $Q_i(t)$: the quitting number of staff in level i at the end of period t .

Here, i represents the i th rank of job category.

For $i = 1, \dots, M$, $R_i(T) = 0$ (there is no recruitment at the end of the planning horizon), $P_i(0) = S_i(0) = 0$ (at the beginning of the time horizon the company only recruits, i.e., no promotion nor dismissal of staff) and $P_{M+1}(t) = 0$ (no staff can be promoted into level $M + 1$ at the end of period t), for $t = 0, \dots, T$. Here, we model the quitting rate $Q_i(t)$ as a normal uncertain variable with expected value μ_i and variance σ_i^2 . It is assumed that the quitting rates $Q_{i_1}(t_1)$ and $Q_{i_2}(t_2)$ are independent for any different periods t_1 and t_2 ($t_1, t_2 = 1, \dots, T$) or levels i_1 and i_2 ($i_1, i_2 = 1, \dots, M$).

Because the quitting number $Q_i(t)$ ($t = 1, \dots, T, i = 1, \dots, M$) is an uncertain variable, the number of staff in each hierarchy and period can be described by the following system of uncertain difference equations:

$$\begin{aligned} x_i(t+1) &= x_i(t) + R_i(t) + P_i(t) - P_{i+1}(t+1) \\ &\quad - S_i(t+1) - Q_i(t+1), \end{aligned} \quad (6)$$

$$t = 0, \dots, T-1, i = 1, \dots, M.$$

To maintain the company's normal operation, there should be enough of workers in each level. This situation is modeled by the following chance constraints:

$$\mathcal{M}\{x_i(t) \geq p_i\} \geq q_i, \quad t = 1, \dots, T, i = 1, \dots, M, \quad (7)$$

where p_i is the pre-set minimum number of staff in level i and q_i is the given confidence level in level i with $0 < q_i < 1$.

The objective of decision maker is to find an optimal strategy of recruitment, promotion and dismissal for minimizing the expected human resource cost while maximizing the total expected production outputs at the end of the planning horizon. Hence, $R_i(t)$, $P_i(t)$, and $S_i(t)$ are decision variables, which are crisp in nature. Then the expected human resource cost and the expected total production outputs can be formulated as

$$g_1 = E \left[\sum_{t=0}^{T-1} \sum_{i=1}^M a_i x_i(t) \right] + \sum_{t=0}^{T-1} \sum_{i=1}^M \{ b_i^{\text{ex}} R_i(t) + b_i^{\text{in}} P_i(t+1) + c_i S_i(t+1) \}, \quad (8)$$

$$g_2 = E \left[\sum_{t=0}^{T-1} \sum_{i=1}^M n_i x_i(t) \right], \quad (9)$$

respectively, where a_i , b_i^{ex} , b_i^{in} and c_i are the salary cost, recruitment and training cost of external workers, training cost

of internal workers and dismissal cost per worker per period in level i , respectively, n_i is the number of units that the workers in level i can produce in each time period.

Obviously, this manpower planning optimization problem is a bi-objective optimization model. A common method for solving the bi-objective optimization model is to convert it into a single objective optimization model by introducing a weighting parameter, see, for example, Cai et al. [1], Deng et al. [7] and Sun et al. [16]. Likewise, we minimize the following function by introducing the weighting parameters ν and $1 - \nu$:

$$\begin{aligned} G_0 &= \nu g_1 - (1 - \nu) g_2 \\ &= \nu E \left[\sum_{t=0}^{T-1} \sum_{i=1}^M a_i x_i(t) \right] + \nu \sum_{t=0}^{T-1} \sum_{i=1}^M \{ b_i^{\text{ex}} R_i(t) \\ &\quad + b_i^{\text{in}} P_i(t+1) + c_i S_i(t+1) \} \\ &\quad - (1 - \nu) E \left[\sum_{t=0}^{T-1} \sum_{i=1}^M n_i x_i(t) \right]. \end{aligned} \quad (10)$$

Here, we use $-(1 - \nu)$ because the original objective is to maximize the expected total production outputs function g_2 . When $\nu = 1$, the decision maker pays all its attention to minimize the expected human resource cost. Conversely, $\nu = 0$ means that the decision maker just wants to maximize the expected production outputs. In this paper, we assume that $0 < \nu < 1$.

Then, we formulate the manpower planning problem under uncertain environment as the following uncertain discrete time optimization model, named as Problem **P**₀:

$$\begin{aligned} \min \quad G_0 &= \nu E \left[\sum_{t=0}^{T-1} \sum_{i=1}^M a_i x_i(t) \right] + \nu \sum_{t=0}^{T-1} \sum_{i=1}^M \{ b_i^{\text{ex}} R_i(t) \\ &\quad + b_i^{\text{in}} P_i(t+1) + c_i S_i(t+1) \} \\ &\quad - (1 - \nu) E \left[\sum_{t=0}^{T-1} \sum_{i=1}^M n_i x_i(t) \right], \end{aligned} \quad (11)$$

$$\begin{aligned} \text{s.t.} \quad x_i(t+1) &= x_i(t) + R_i(t) + P_i(t) - P_{i+1}(t+1) \\ &\quad - S_i(t+1) - Q_i(t+1), \\ &\quad t = 0, \dots, T-1, i = 1, \dots, M, \\ x_i(0) &= x_i^0, \quad i = 1, \dots, M, \\ \mathcal{M}\{x_i(t) \geq p_i\} &\geq q_i, \quad t = 1, \dots, T, i = 1, \dots, M, \\ R_i(t), P_i(t), S_i(t) &\geq 0, \quad t = 0, \dots, T, i = 1, \dots, M, \end{aligned} \quad (12)$$

where $0 < \nu < 1$.

Remark 2: Although the objective function in Sun et al. [16] contains the term of quitting cost, it has no influence on the transformation and optimization processes. Here, we assume that there is no withdrawal benefit for workers quitting the job themselves. We note that the summation operation at discrete time t means the human resource cost and production outputs during period $t + 1$. Hence, the summation operation over t from 0 to $T - 1$ includes all the human resource cost and production outputs.

Because $R_i(t)$, $P_i(t)$, and $S_i(t)$ are decision variables, they are crisp rather than uncertain in nature. Then it follows from equations (5), (6) and Theorem 8 in Yang [19] that

$$E \left[\sum_{t=0}^{T-1} \sum_{i=1}^M a_i x_i(t) \right] = \Delta - \sum_{t=0}^{T-1} \sum_{i=1}^M \sum_{j=0}^t a_i E[Q_i(j+1)], \quad (13)$$

where the function Δ represents all crisp items. Thus, we can swap the order of the expected value operator E and summation operator. The objective function G_0 can be rewritten as

$$G_0 = \nu \sum_{t=0}^{T-1} \sum_{i=1}^M \{ a_i E[x_i(t)] + b_i^{\text{ex}} R_i(t) + b_i^{\text{in}} P_i(t+1) + c_i S_i(t+1) \} - (1-\nu) \sum_{t=0}^{T-1} \sum_{i=1}^M n_i E[x_i(t)]. \quad (14)$$

IV. DETERMINISTIC TRANSFORMATION

In this section, we first transform Problem \mathbf{P}_0 into an equivalent deterministic problem and then present a method for solving it.

Let $z_i(t)$ and $w_i(t)$ be new state variables defined by

$$z_i(t) = E[x_i(t)], w_i(t) = \text{Var}[x_i(t)], \quad t = 0, \dots, T, \\ i = 1, \dots, M. \quad (15)$$

It follows from equations (5) and (6) that for $t = 0, \dots, T-1$ and $i = 1, \dots, M$,

$$\begin{aligned} z_i(t+1) &= E[x_i(t+1)] \\ &= E[x_i(t) + R_i(t) + P_i(t) - P_{i+1}(t+1) \\ &\quad - S_i(t+1) - Q_i(t+1)] \\ &= z_i(t) + R_i(t) + P_i(t) - P_{i+1}(t+1) \\ &\quad - S_i(t+1) - \mu_i. \end{aligned} \quad (16)$$

Because the variables $x_i(t)$ and $Q_i(t+1)$ are essentially two independent normal uncertain variables, for $t = 0, \dots, T-1$ and $i = 1, \dots, M$, it follows from Theorem 5 in Yao [20] that

$$\sqrt{w_i(t+1)} = \sqrt{\text{Var}[x_i(t+1)]} = \sqrt{w_i(t)} + \sigma_i. \quad (17)$$

We can find, in equation (16), the decision variables contain those in the past time t as well as the present time $t+1$. For achieving the unity of time t , we define

$$v_i(t) = P_i(t+1), t = 0, \dots, T-1, i = 1, \dots, M, \quad (18)$$

$$v_{M+i}(t) = S_i(t+1), t = 0, \dots, T-1, i = 1, \dots, M. \quad (19)$$

Then we transform $P_i(t)$ and $S_i(t)$ from decision variables into state variables with initial condition $P_i(0) = S_i(0) = 0$ and introduce a new decision variable $v_i(t)$, $t = 0, \dots, T-1$, $i = 1, \dots, 2M$. Hence, the decision variables are represented by $R_i(t)$, $v_i(t)$, $v_{M+i}(t)$ with $R_i(t)$, $v_i(t)$, $v_{M+i}(t) \geq 0$, $t = 0, \dots, T-1$, $i = 1, \dots, M$.

The objective function (14) becomes

$$G_0 = \sum_{t=0}^{T-1} \sum_{i=1}^M [(a_i \nu - (1-\nu)n_i) z_i(t) + b_i^{\text{ex}} R_i(t) + b_i^{\text{in}} v_i(t) + c_i v_{M+i}(t)]. \quad (20)$$

Theorem 1: The chance constraints (7) are equivalent to the following constraints.

$$z_i(t) - p_i + \Phi^{-1}(1 - q_i) \sqrt{w_i(t)} \geq 0, \quad t = 1, \dots, T, \\ i = 1, \dots, M, \quad (21)$$

where Φ^{-1} is the inverse uncertainty distribution of normal uncertain variable with expected value 0 and variance 1.

Proof Because $x_i(t)$ is a normal uncertain variable with expected value $z_i(t)$ and variance $w_i(t)$, we know that

$$\frac{x_i(t) - z_i(t)}{\sqrt{w_i(t)}}, \quad t = 1, \dots, T, i = 1, \dots, M,$$

is a normal uncertain variable with expected value 0 and variance 1. Thus, the chance constraints (7) are equivalent to

$$\mathcal{M} \left\{ \frac{x_i(t) - z_i(t)}{\sqrt{w_i(t)}} \geq \frac{p_i - z_i(t)}{\sqrt{w_i(t)}} \right\} \geq q_i,$$

i.e.,

$$\Phi^{-1}(1 - q_i) \geq \frac{p_i(t) - z_i(t)}{\sqrt{w_i(t)}},$$

where Φ^{-1} is the inverse uncertainty distribution of normal uncertain variable with expected value 0 and variance 1. Thus, constraints (7) are equivalent to the constraints (21). The theorem is proved.

Because state variable $w_i(t)$ can be solved directly from equation (17) with initial value 0, it can be obtained that

$$w_i(t) = t^2 \sigma_i^2, \quad t = 0, \dots, T, i = 1, \dots, M. \quad (22)$$

Combining the state variables but excluding $w_i(t)$, for $t = 0, \dots, T-1$, the new dynamical system can be written as

$$\begin{pmatrix} z_1(t+1) \\ \vdots \\ z_M(t+1) \\ P_1(t+1) \\ \vdots \\ P_M(t+1) \\ S_1(t+1) \\ \vdots \\ S_M(t+1) \end{pmatrix} = \begin{pmatrix} z_1(t) + R_1(t) + P_1(t) - v_2(t) \\ -v_{M+1}(t) - \mu_1 \\ \vdots \\ z_M(t) + R_M(t) + P_M(t) - 0 \\ -v_{2M}(t) - \mu_M \\ v_1(t) \\ \vdots \\ v_M(t) \\ v_{M+1}(t) \\ \vdots \\ v_{2M}(t) \end{pmatrix}, \quad (23)$$

with initial condition $[x_1^0, \dots, x_M^0, 0, \dots, 0]^\tau \in \mathbb{R}^{3M}$.

Based on the above operators, Problem \mathbf{P}_0 is equivalent to the following deterministic discrete time optimization model (24), named as Problem \mathbf{P}_1 , where $0 < \nu < 1$.

A. Solving Problem \mathbf{P}_1

In this section, we convert Problem \mathbf{P}_1 into a standard linear programming problem such that it can be solved more easily.

The new state and decision variables can be rewritten in vector forms:

$$\mathbf{y}(t) = [z_1(t), \dots, z_M(t), P_1(t), \dots, P_M(t), \\ S_1(t), \dots, S_M(t)]^\tau, \quad (27)$$

$$\mathbf{h}(t) = [R_1(t), \dots, R_M(t), v_1(t), \dots, v_{2M}(t)]^\tau. \quad (28)$$

$$\min G_0 = \sum_{t=0}^{T-1} \sum_{i=1}^M [(a_i \nu - (1 - \nu)n_i)z_i(t) + b_i^{\text{ex}} R_i(t) + b_i^{\text{in}} v_i(t) + c_i v_{M+i}(t)] \quad (24)$$

$$\text{s.t.} \quad \begin{pmatrix} z_1(t+1) \\ \vdots \\ z_M(t+1) \\ P_1(t+1) \\ \vdots \\ P_M(t+1) \\ S_1(t+1) \\ \vdots \\ S_{2M}(t+1) \end{pmatrix} = \begin{pmatrix} z_1(t) + R_1(t) + P_1(t) - v_2(t) - v_{M+1}(t) - \mu_1 \\ \vdots \\ z_M(t) + R_M(t) + P_M(t) - 0 - v_{2M}(t) - \mu_M \\ v_1(t) \\ \vdots \\ v_M(t) \\ v_{M+1}(t) \\ \vdots \\ v_{2M}(t) \end{pmatrix}, t = 0, \dots, T-1,$$

$$[z_1(0), \dots, z_M(0), P_1(0), \dots, P_M(0), S_1(0), \dots, S_M(0)]^\tau = [x_1^0, \dots, x_M^0, 0, \dots, 0]^\tau,$$

$$z_i(t) - p_i + \Phi^{-1}(1 - q_i)\sqrt{w_i(t)} \geq 0, t = 1, \dots, T, i = 1, \dots, M, \quad (25)$$

$$R_i(t), v_i(t), v_{M+i}(t) \geq 0, t = 0, \dots, T-1, i = 1, \dots, M. \quad (26)$$

It follows from equations (27) and (28) that equation (24) can be rewritten as:

$$G_0 = \sum_{t=0}^{T-1} (\delta \mathbf{y}(t) + \eta \mathbf{h}(t)), \quad (29)$$

where δ and η are both $3M$ dimension row vectors given by

$$\delta = [(a_1 \nu - (1 - \nu)n_1), \dots, (a_M \nu - (1 - \nu)n_M), 0, \dots, 0], \quad (30)$$

$$\eta = [b_1^{\text{ex}}, \dots, b_M^{\text{ex}}, b_1^{\text{in}}, \dots, b_M^{\text{in}}, c_1, \dots, c_M]. \quad (31)$$

Let \mathbf{I} be an $M \times M$ identity matrix, $\mathbf{0}$ be an $M \times M$ zero matrix, $\mathbf{\Lambda}$ be an $M \times M$ matrix defined by

$$\mathbf{\Lambda} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix},$$

and

$$\alpha = \begin{pmatrix} \mathbf{I} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}, \beta = \begin{pmatrix} \mathbf{I} & -\mathbf{\Lambda} & -\mathbf{I} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{pmatrix}.$$

Theorem 2: For $t = 1, \dots, T$, the state variable $\mathbf{y}(t)$ can be expressed in terms of decision variable $\mathbf{h}(t)$ as given below:

$$\mathbf{y}(t) = \alpha^t \mathbf{y}^0 + \sum_{k=0}^{t-1} (\alpha^{(t-1)-k} \beta \mathbf{h}(k) + \alpha^k \zeta), \quad (32)$$

where $\mathbf{y}(0) = \mathbf{y}^0 = [x_1^0, \dots, x_M^0, 0, \dots, 0]^\tau \in \mathbb{R}^{3M}$ and ζ is a $3M$ dimensional column vector given by

$$\zeta = [-\mu_1, \dots, -\mu_M, 0, \dots, 0]^\tau. \quad (33)$$

Proof Here, we use the induction method. It is clear that

$$\mathbf{y}(1) = \alpha \mathbf{y}^0 + \beta \mathbf{h}(0) + \zeta.$$

Suppose that equation (32) is satisfied when $t = n$. Then, at $t = n + 1$, we have

$$\begin{aligned} \mathbf{y}(n+1) &= \alpha \mathbf{y}(n) + \beta \mathbf{h}(n) + \zeta \\ &= \alpha \left[\alpha^n \mathbf{y}^0 + \sum_{k=0}^{n-1} (\alpha^{(n-1)-k} \beta \mathbf{h}(k) + \alpha^k \zeta) \right] \\ &\quad + \beta \mathbf{h}(n) + \zeta \\ &= \alpha^{n+1} \mathbf{y}^0 + \sum_{k=0}^n (\alpha^{n-k} \beta \mathbf{h}(k) + \alpha^k \zeta). \end{aligned}$$

The theorem is proved.

Also, the state constraints (25) can be written in a matrix form given by

$$\kappa \mathbf{y}(t) \geq \lambda(t), \quad t = 1, \dots, T, \quad (34)$$

where κ is an $M \times 3M$ matrix, and $\lambda(t)$ is an M dimensional column vector defined by

$$\kappa = (\mathbf{I} \quad \mathbf{0} \quad \mathbf{0}) \quad (35)$$

and

$$\lambda(t) = \begin{pmatrix} p_1 - \Phi^{-1}(1 - q_1)\sqrt{w_1(t)} \\ \vdots \\ p_M - \Phi^{-1}(1 - q_M)\sqrt{w_M(t)} \end{pmatrix}. \quad (36)$$

For translating the inequality constraints (34) into equality constraints, we add an auxiliary decision variable to the left hand side of (34), yielding

$$\kappa \mathbf{y}(t) - \theta(t) = \lambda(t), \quad t = 1, \dots, T, \quad (37)$$

where $\theta(t) = [\theta_1(t), \dots, \theta_M(t)]^\tau \in \mathbb{R}^M$ with $\theta_i(t) \geq 0$, $i = 1, \dots, M$.

Define

$$\omega = [\mathbf{h}(0)^\tau, \dots, \mathbf{h}(T-1)^\tau, \theta(1)^\tau, \dots, \theta(T)^\tau]^\tau \in \mathbb{R}^{4MT}. \quad (38)$$

Obviously, we have

$$\omega \geq \mathbf{0}, \quad (39)$$

where $\mathbf{0}$ is a $4MT$ dimensional column vector.

According to equations (29), (32) and (38), the objective function G_0 can be expressed as a linear function:

$$G_0 = \mathbf{c}^T \omega, \quad (40)$$

where \mathbf{c} is a $4MT$ dimensional column vector defined by

$$\mathbf{c} = \left[\sum_{t=0}^{T-2} \delta \alpha^t \beta + \eta, \sum_{t=0}^{T-3} \delta \alpha^t \beta + \eta, \dots, \delta \beta + \delta \alpha \beta + \eta, \delta \beta + \eta, \eta, \mathbf{0} \right]^T. \quad (41)$$

Substituting equation (32) into equation (37), the constraints (37) can be expressed as

$$\mathbf{A} \omega = \mathbf{b}, \quad (42)$$

where \mathbf{A} and \mathbf{b} are given below as an $MT \times 4MT$ matrix

$$\begin{pmatrix} \kappa \beta & \bar{\mathbf{0}} & \dots & \bar{\mathbf{0}} & -\mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \\ \kappa \alpha \beta & \kappa \beta & \dots & \bar{\mathbf{0}} & \mathbf{0} & -\mathbf{I} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \kappa \alpha^{T-1} \beta & \kappa \alpha^{T-2} \beta & \dots & \kappa \beta & \mathbf{0} & \mathbf{0} & \dots & -\mathbf{I} \end{pmatrix} \quad (43)$$

and an MT column vector

$$\begin{pmatrix} \lambda(1) - \kappa \alpha y^0 - \kappa \zeta \\ \lambda(2) - \kappa \alpha^2 y^0 - \kappa \zeta - \kappa \alpha \zeta \\ \vdots \\ \lambda(T) - \kappa \alpha^T y^0 - \sum_{t=0}^{T-1} \kappa \alpha^t \zeta \end{pmatrix}, \quad (44)$$

respectively. Here, $\bar{\mathbf{0}}$ is an $M \times 3M$ zero matrix.

Based on equations (39), (40) and (42), Problem \mathbf{P}_1 can be formally stated as a standard linear programming problem as given in the following:

$$\begin{aligned} \min \quad & \mathbf{c}^T \omega \\ \text{s.t.} \quad & \mathbf{A} \omega = \mathbf{b}, \\ & \omega \geq \mathbf{0}. \end{aligned} \quad (45)$$

Then, we use the interior-point linear programming solver SDPT3 (MATLAB platform) proposed in Toh et al. [17] to solve the converted standard linear programming problem (45). After obtaining the value of ω^* , the procedures for obtaining the solution of the original Problem \mathbf{P}_0 are summarized as follows:

Step 1: Calculate the values of $h(0), \dots, h(T-1)$ by equation (38);

Step 2: Calculate the values of decision variables of the original Problem \mathbf{P}_0 according to equations (18), (19) and (28).

V. NUMERICAL SIMULATION

In this section, we use the same data and parameters, shown in Tables I and II, as in Sun et al. [16] to carry out a numerical simulation and make a comparison. All values in Table II are in US dollars. Also, we choose initial workforce $N = 1000$, planning horizon $T = 12$ months, confidence level $q_i = 0.8$

($i = 1, 2, \dots, 6$) and weighting parameter $\nu = 0.3$. Because the quitting cost per worker is chosen as 0 in Sun et al. [16], the optimization problem can be modeled as defined in Section III.

Here, we use the interior-point linear programming solver SDPT3 (MATLAB platform) proposed in Toh et al. [17] to solve the converted standard linear programming problem. From the calculation procedure presented in Section IV, the optimal manpower strategies of recruitment, promotion and dismissal for three different countries can be calculated. The obtained solutions are given in Tables III, V and VII. Our mathematical model for the uncertain manpower planning optimization problem provides results that realistically reflect the conditions of each of the countries considered. We choose three countries (Singapore, Denmark and China) that are distinct in nature and especially in salaries. One is a developed country, Denmark, which has relatively stable domestic labor with relatively high wages. At the other extreme is China, a developing country with relatively low wages and plenty of available manpower. The third is Singapore. While a developed country, Singapore utilizes a huge number of cheap unskilled foreign labor to keep manpower costs down at the lower levels. The results of our model demonstrate that the inclusion of the effect of production outputs produces results that are realistic and takes into consideration the differences of each country.

Singapore, with relatively cheap and abundant low-skilled foreign labor, results in an optimal workforce that encourages recruitment at the lowest two levels as shown in Table III, and also at the more expensive and skilled managerial levels. It is cheaper to recruit than promote internal skilled staff at the managerial levels. Denmark, having the most stable workforce with relatively higher wages than the other two countries, achieves workforce stability in a much shorter time than Singapore and China. From Table V, their human resource strategy stabilizes after nine months while there are no signs of workforce stability in Singapore and China. With regards to recruitment and promotion, the strategy suitable for Denmark is to recruit at all levels rather than promote internal staff as the promotion cost is higher than the recruitment cost for the country. China has a huge number of cheap domestic workers. Hence, it is hardly surprising that their optimal workforce recommends recruiting a huge, even excessive, number of workers at all levels as in Table VII. Their model does not recommend recruitment or promotion after the initial recruitment.

The optimal manpower structures in each level for three different countries can be found in Tables IV, VI and VIII, which are demonstrated in Figures 1, 2 and 3, respectively. From Tables IV, VI and VIII or Figures 1, 2 and 3, there are more staff at the end of each period for China than Singapore and Denmark. This is reasonable because China has abundant and cheap domestic workforce. The optimal manpower cost and units of output are \$21,339,468.64 and 957,150 for Singapore, \$59,635,567.72 and 952,830 for Denmark, \$5,778,962.99 and 1,421,890 for China.

Modeling the quitting rate to better reflect reality results in more employees in the optimal manpower structure. While more employees result in higher manpower cost across all countries, it also leads to higher number of produced outputs.

TABLE IX
THE UNIT COST OF TWO DIFFERENT MODELS

Country	Sun et al. [16] model	Our model
Singapore	22.3476	22.2948
Denmark	62.4880	62.5878
China	4.1055	4.0643

Comparing our results with Sun et al. [16], the percentage increase in manpower cost is 2.2302%, 2.1731% and 3.4213% for Singapore, Denmark and China, respectively. The corresponding increase in production output as a result of the manpower increase is 2.4728%, 2.0103% and 4.4715%. Table IX illustrates the unit output cost based on our optimal manpower structure compared to the optimization model in Sun et al. [16].

Our model, with more stable retention of employees, sees a lower unit cost for Singapore and China, but a higher unit output cost for Denmark. This, indeed, reflects more accurately the real situation, as the manpower cost in Singapore is approximately four times that of China, while in Denmark, the manpower cost is approximately ten times that of China. Thus, unless the quality of the product produced in Denmark is substantially higher than that of Singapore and China, the manufacturing industry is unable to compete with that of Singapore and China.

Our model confirms that for labour intensive companies producing low value products, it is more profitable to base their operations in low labour cost countries like China or countries where the labour cost is relatively lower such as Singapore for which a large number of low cost foreign workers is used. In conclusion, for high labour cost countries like Denmark, only high end products will be compatible and survive.

VI. CONCLUSION

In this paper, we discussed a manpower planning optimization problem for a hierarchical system under uncertain environment because the quitting rate of employees is usually uncertain in nature. The manpower planning optimization problem was formulated as an uncertain discrete time optimization model for minimising the expected human resource cost while maximizing the expected production outputs. Our model considered the actual personnel change through recruitment, promotion, dismissal, as well as employees quitting and took into account the influence of the production outputs of workers. Then, we transformed it into an equivalent deterministic problem and presented a solution method for solving it. In order to show the effectiveness and rationality of our model, we remodeled the practical example considered in Sun et al. [16] by our method. The obtained results showed that our model reflects more accurately the real situation than the stochastic model.

ACKNOWLEDGMENT

This work is supported by the National Natural Science Foundation of China (No.61673011).

REFERENCES

- [1] X. Cai, K. L. Teo, X. Yang, and X. Zhou, "Portfolio optimization under a minimax rule," *Manag. Sci.*, vol. 46, no. 7, pp. 957-972, 2000.
- [2] A. Chattopadhyay and A. Gupta, "A stochastic manpower planning model under varying class sizes," *Ann. Oper. Res.*, vol. 155, no. 1, pp. 41-49, 2007.
- [3] X. Chen, "Variation analysis of uncertain stationary independent increment processes," *Eur. J. Oper. Res.*, vol. 222, no. 2, pp. 312-316, 2012.
- [4] X. Chen and J. Gao, "Uncertain term structure model of interest rate," *Soft Comput.*, vol. 17, no. 4, pp. 597-604, 2013.
- [5] X. Chen and B. Liu, "Existence and uniqueness theorem for uncertain differential equations," *Fuzzy Optim. Dec. Making*, vol. 2, no. 9, pp. 69-81, 2010.
- [6] X. Chen, Y. Liu, and D. Ralescu, "Uncertain stock model with periodic dividends," *Fuzzy Optim. Dec. Making*, vol. 12, no. 1, pp. 111-123, 2013.
- [7] X. Deng, Z. Li, and S. Wang, "A minimax portfolio selection strategy with equilibrium," *Eur. J. Oper. Res.*, vol. 166, no. 1, pp. 278-292, 2005.
- [8] Y. Gao, "Uncertain models for single facility location problems on networks," *Appl. Math. Model.*, vol. 36, no. 6, pp. 2592-2599, 2012.
- [9] A. Kolmogorov, *Grundbegriffe der Wahrscheinlichkeitsrechnung*, Berlin, Germany: Julius-Springer, 1933.
- [10] B. Liu, *Uncertainty Theory*, 2nd ed., Berlin, Germany: Springer-Verlag, 2007.
- [11] B. Liu, "Fuzzy process, hybrid process and uncertain process," *J. Uncertain Syst.*, vol. 2, no. 1, pp. 3-16, 2008.
- [12] B. Liu, "Some research problems in uncertainty theory," *J. Uncertain Syst.*, vol. 3, no. 1, pp. 3-10, 2009.
- [13] B. Liu, *Theory and Practice of Uncertain Programming*, 2nd ed., Berlin, Germany: Springer-Verlag, 2009.
- [14] B. Liu, *Uncertainty Theory: A Branch of Mathematics for Modeling Human Uncertainty*, Berlin, Germany: Springer-Verlag, 2010.
- [15] Y. Sheng and S. Kar, "Some results of moments of uncertain variable through inverse uncertainty distribution," *Fuzzy Optim. Dec. Making*, vol. 14, no. 1, pp. 57-76, 2015.
- [16] Y. Sun, G. Aw, C. Yu, K. L. Teo, and X. Chen, "Multi-period stochastic manpower and productivity optimization in a hierarchical system," *Pac. J. Optim.*, vol. 12, no. 3, pp. 521-539, 2016.
- [17] K. Toh, M. Todd, and R. Tütüncü, "On the implementation and usage of SDPT3-a Matlab software package for semidefinite-quadratic-linear programming, version 4.0," *Int. Ser. Oper. Res. Manag. Sci.*, vol. 166, pp. 715-754, 2012.
- [18] S. Yan, C. Chen, and M. Chen, "Stochastic models for air cargo terminal manpower supply planning in long term operations," *Appl. Stoch. Model. Bus.*, vol. 24, no. 3, pp. 261-275, 2008.
- [19] X. Yang, "On comonotonic functions of uncertain variables," *Fuzzy Optim. Dec. Making*, vol. 12, no. 1, pp. 89-98, 2013.
- [20] K. Yao, "A formula to calculate the variance of uncertain variable," *Soft Comput.*, vol. 19, no. 10, pp. 2947-2953, 2015.
- [21] K. Yao, J. Gao, and Y. Gao, "Some stability theorems of uncertain differential equation," *Fuzzy Optim. Dec. Making*, vol. 12, no. 1, pp. 3-13, 2013.
- [22] K. Yao and X. Li, "Uncertain alternating renewal process and its application," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 6, pp. 1154-1160, 2012.
- [23] A. Young and P. Vassiliou, "A non-linear model on the promotion of staff," *J. Roy. Stat. Soc. A*, vol. 173, no. 4, pp. 584-595, 1974.
- [24] Y. Zhu, "Uncertain optimal control with application to a portfolio selection model," *Cybernet Syst.*, vol. 41, no. 7, pp. 535-547, 2010.



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TABLE I
INITIAL HIERARCHY STRUCTURE, MINIMUM STAFF NUMBER, PRODUCTIVITY, MEAN AND VARIANCE OF QUITTING RATE

	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
x_i^0	350	250	150	120	80	50
p_i	250	170	100	75	50	25
n_i	100	120	150	40	60	90
μ_i	30	20	10	5	3	1
σ_i^2	9	6.25	4	0.5625	0.25	0.01

TABLE II
VARIOUS MANPOWER COST OF EACH HIERARCHY LEVEL

	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
Singapore						
a_i	800.00	960.00	1728.00	3110.40	5598.72	11197.44
b_i^{ex}	2000.00	2400.00	4320.00	6842.88	12317.18	19035.65
b_i^{in}	2000.00	1056.00	1900.80	3421.44	6158.59	12317.18
c_i	800.00	960.00	1728.00	3110.40	5598.72	11197.44
Denmark						
a_i	3206.67	4329.00	5844.15	8181.81	11454.53	16036.35
b_i^{ex}	8016.67	10822.50	14610.38	17999.98	25199.97	27261.79
b_i^{in}	8016.67	4761.90	6428.57	8999.99	12599.99	17639.98
c_i	3206.67	4329.00	5844.15	8181.81	11454.53	16036.35
China						
a_i	206.00	247.20	321.36	514.18	1079.77	3239.31
b_i^{ex}	515.00	618.00	803.40	1131.19	2375.49	5506.82
b_i^{in}	515.00	271.92	353.50	565.59	1187.75	3563.24
c_i	206.00	247.20	321.36	514.18	1079.77	3239.31

TABLE III
OPTIMAL HR STRATEGY OF SINGAPORE

Recruitment	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
t=0	0	0	0	0	0	0
t=1	0	0	0	0	0	0
t=2	0	0	0	0	0	0
t=3	29	15	0	0	0	0
t=4	32	33	0	0	0	0
t=5	32	33	0	0	0	0
t=6	32	33	0	0	0	0
t=7	32	33	0	0	0	0
t=8	32	33	0	5	0	0
t=9	32	33	0	5	3	0
t=10	32	33	0	5	3	1
t=11	32	21	0	5	3	1
t=12	0	0	0	0	0	0
Promotion	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
t=0	0	0	0	0	0	0
t=1	0	0	0	0	0	0
t=2	0	0	0	0	0	0
t=3	0	0	0	0	0	0
t=4	0	0	7	0	0	0
t=5	0	0	11	0	0	0
t=6	0	0	11	0	0	0
t=7	0	0	11	0	0	0
t=8	0	0	11	0	0	0
t=9	0	0	11	0	0	0
t=10	0	0	11	0	0	0
t=11	0	0	11	0	0	0
t=12	0	0	0	0	0	0
Dismissal	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
t=0	0	0	0	0	0	0
t=1	0	0	0	0	0	14
t=2	0	0	0	0	0	0
t=3	0	0	0	0	0	0
t=4	0	0	0	0	0	0
t=5	0	0	0	0	0	0
t=6	0	0	0	0	0	0
t=7	0	0	0	0	0	0
t=8	0	0	0	0	0	0
t=9	0	0	0	0	0	0
t=10	0	0	0	0	0	0
t=11	0	0	0	0	0	0
t=12	0	0	0	0	0	0

TABLE IV
OPTIMAL MANPOWER STRUCTURE OF SINGAPORE

No. of workers	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6	Total
t=0	350	250	150	120	80	50	1000
t=1	320	230	140	115	77	35	917
t=2	290	210	130	110	74	34	848
t=3	260	190	120	105	71	33	779
t=4	259	178	110	100	68	32	747
t=5	261	180	107	95	65	31	739
t=6	263	182	108	90	62	30	735
t=7	265	184	109	85	59	29	731
t=8	267	186	110	80	56	28	727
t=9	269	188	111	80	53	27	728
t=10	271	190	112	80	53	26	732
t=11	273	192	113	80	53	26	737
t=12	275	193	114	80	53	26	741

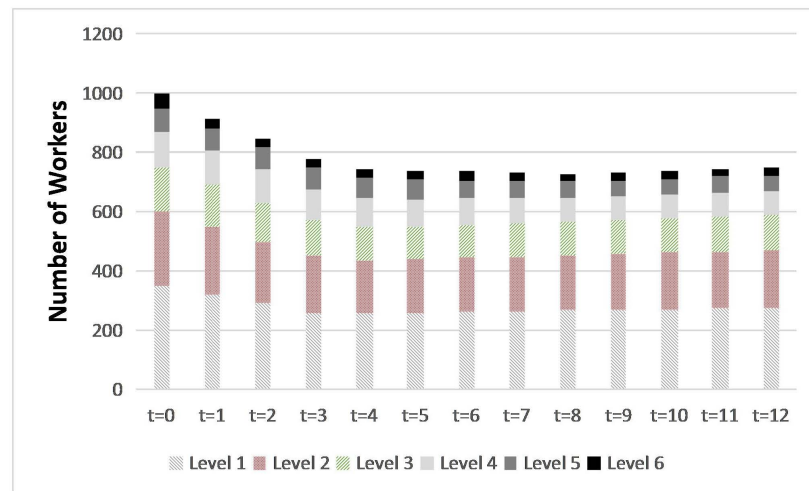


Fig. 1. Optimal Manpower Structure of Singapore

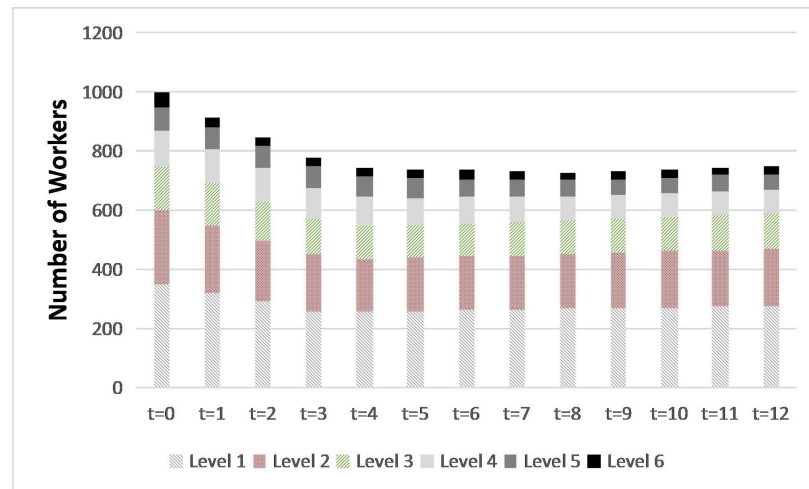


Fig. 2. Optimal Manpower Structure of Denmark

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TABLE V
OPTIMAL HR STRATEGY OF DENMARK

Recruitment	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
t=0	0	0	0	0	0	0
t=1	0	0	0	0	0	0
t=2	0	0	0	0	0	0
t=3	29	7	0	0	0	0
t=4	32	21	7	0	0	0
t=5	32	21	11	0	0	0
t=6	32	21	11	0	0	0
t=7	32	21	11	0	0	0
t=8	32	21	11	5	0	0
t=9	32	21	11	5	3	0
t=10	32	21	11	5	3	1
t=11	32	21	11	5	3	1
t=12	0	0	0	0	0	0
Promotion	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
t=0	0	0	0	0	0	0
t=1	0	0	0	0	0	0
t=2	0	0	0	0	0	0
t=3	0	0	0	0	0	0
t=4	0	0	0	0	0	0
t=5	0	0	0	0	0	0
t=6	0	0	0	0	0	0
t=7	0	0	0	0	0	0
t=8	0	0	0	0	0	0
t=9	0	0	0	0	0	0
t=10	0	0	0	0	0	0
t=11	0	0	0	0	0	0
t=12	0	0	0	0	0	0
Dismissal	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
t=0	0	0	0	0	0	0
t=1	0	0	0	0	0	14
t=2	0	0	0	0	0	0
t=3	0	0	0	0	0	0
t=4	0	0	0	0	0	0
t=5	0	0	0	0	0	0
t=6	0	0	0	0	0	0
t=7	0	0	0	0	0	0
t=8	0	0	0	0	0	0
t=9	0	0	0	0	0	0
t=10	0	0	0	0	0	0
t=11	0	0	0	0	0	0
t=12	0	0	0	0	0	0

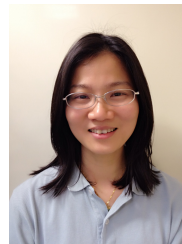
TABLE VI
OPTIMAL MANPOWER STRUCTURE OF DENMARK

No. of workers	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6	Total
t=0	350	250	150	120	80	50	1000
t=1	320	230	140	115	77	35	917
t=2	290	210	130	110	74	34	848
t=3	260	190	120	105	71	33	779
t=4	259	177	110	100	68	32	746
t=5	261	178	107	95	65	31	737
t=6	263	179	108	90	62	30	732
t=7	265	180	109	85	59	29	727
t=8	267	181	110	80	56	28	722
t=9	269	182	111	80	53	27	722
t=10	271	183	112	80	53	26	725
t=11	273	184	113	80	53	26	729
t=12	275	185	114	80	53	26	733



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TABLE VII
OPTIMAL HR STRATEGY OF CHINA

Recruitment	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
t=0	287	182	88	0	0	0
t=1	0	0	0	0	0	0
t=2	0	0	0	0	0	0
t=3	0	0	0	0	0	0
t=4	0	0	0	0	0	0
t=5	0	0	0	0	0	0
t=6	0	0	0	0	0	0
t=7	0	0	0	0	0	0
t=8	0	0	0	8	0	0
t=9	0	0	0	8	0	0
t=10	0	0	0	8	0	1
t=11	0	0	0	5	0	1
t=12	0	0	0	0	0	0
Promotion	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
t=0	0	0	0	0	0	0
t=1	0	0	0	0	0	0
t=2	0	0	0	0	0	0
t=3	0	0	0	0	0	0
t=4	0	0	0	0	0	0
t=5	0	0	0	0	0	0
t=6	0	0	0	0	0	0
t=7	0	0	0	0	0	0
t=8	0	0	0	0	0	0
t=9	0	0	0	0	3	0
t=10	0	0	0	0	3	0
t=11	0	0	0	0	3	0
t=12	0	0	0	0	0	0
Dismissal	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6
t=0	0	0	0	0	0	0
t=1	0	0	0	0	0	14
t=2	0	0	0	0	0	0
t=3	0	0	0	0	0	0
t=4	0	0	0	0	0	0
t=5	0	0	0	0	0	0
t=6	0	0	0	0	0	0
t=7	0	0	0	0	0	0
t=8	0	0	0	0	0	0
t=9	0	0	0	0	0	0
t=10	0	0	0	0	0	0
t=11	0	0	0	0	0	0
t=12	0	0	0	0	0	0

TABLE VIII
OPTIMAL MANPOWER STRUCTURE OF CHINA

No. of workers	Level 1	Level 2	Level 3	Level 4	Level 5	Level 6	Total
t=0	350	250	150	120	80	50	1000
t=1	607	412	228	115	77	35	1474
t=2	577	392	218	110	74	34	1405
t=3	547	372	208	105	71	33	1336
t=4	517	352	198	100	68	32	1267
t=5	487	332	188	95	65	31	1198
t=6	457	312	178	90	62	30	1129
t=7	427	292	168	85	59	29	1060
t=8	397	272	158	80	56	28	991
t=9	367	252	148	80	53	27	927
t=10	337	232	138	80	53	26	866
t=11	307	212	128	80	53	26	806
t=12	277	192	118	80	53	26	746



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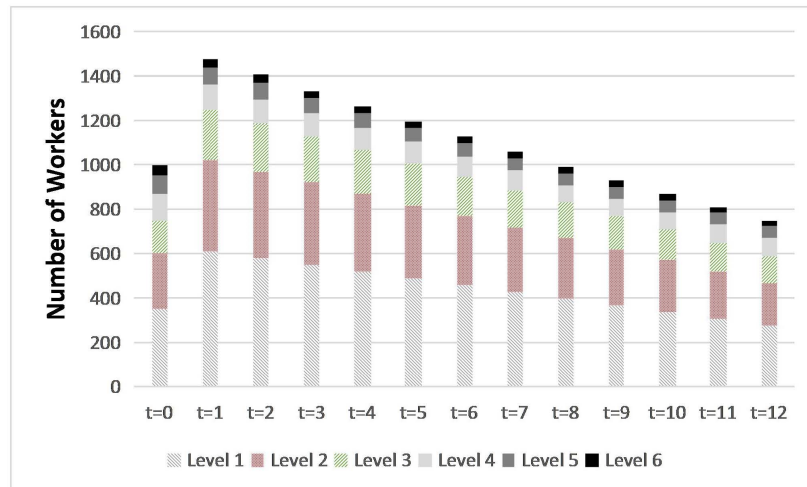


Fig. 3. Optimal Manpower Structure of China