**School of Economics and Finance**

# On the determinants of the evolution of the volatility surface in the over-the-counter currency option market

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## Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgement has been made. This thesis contains no material which has been accepted for the award of any other degree of diploma in any university.

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Date: ……………………………………………........

## Abstract

This dissertation presents an empirical analysis of the determinants of the evolution of the volatility surface in the over-the-counter currency option market. This topic has been rarely studied in the existing literature. Using principle component analysis, we find that over 95% of variation of the volatility surface can be summarised in four latent factors. By regressing these factors with possible explanatory variables, we find the determinant of the variation of the volatility surface is not exclusively the spot rate return. We show changes in realised volatility, volatility of volatility, market sentiment and interest rate differential all have strong correlation with the volatility surface,and that the size and significance of these explanatory variables differs between currencies and different market conditions (high/low risk aversion).

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# Contents

## List of Tables

## List of Figures

# 1 Introduction

This thesis is concerned with exploring the determinants of the evolution of the shape of the volatility surface over time in the over-the-counter (OTC) European vanilla currency option market. This topic is of paramount importance to both academics and practitioners, as understanding the determinants of the evolution of the volatility surface is crucial for valuation and risk management of both European vanilla options and all other exotic options.

Option volatility is the volatility parameter set in the Black and Scholes (1973) and Merton (1973) (BSM) model to obtain the value of the European vanilla option. When option volatilities are plotted against option maturity and moneyness, the shape we obtain is referred to as the volatility surface. If the BSM model holds, the volatility surface should be a horizontal plane. However, in the market the volatility surface is not horizontal: instead it evidences skew and curvature. To complicate things further, the shape of the volatility surface in the market varies over time.

Of the existing research papers exploring the determinants of the evolution of the volatility surface, those ofMixon (2002) and Chalamandaris and Tsekrekos (2013)are closest to this thesis. Their papers find the determinants of the change of the shape of the volatility surface over time in the S&P 500 index option and the Asian-Pacific currency option, respectively. Both papers apply a factor model to summarise the change of the shape of the volatility surface in several common factors, and then regress the common factor with some explanatory variables.

We differentiate ourselves from these papers by using a new data set that is large in time series and a new set of explanatory variable that has not been extensively examined in these papers. It includes the Global Financial Crisis (GFC) and considers those currencies with the largest trading volumes or the most pronounced volatility surfaces. We adopted a new set of explanatory variables. These variables are the spot rate return, the correlation between spot return and the change in option volatility, spike/crash in the currency rate, the net long position (a proxy for the market sentiment) and the implied interest rate differential. The economic significance of these variables is straight forward. For example, examining the relationship between the spot rate return and the volatility surface can essentially reveal the effectiveness of delta hedging. And examining the relationship between net long position and volatility surface can inform whether option pricing in the market is related to the market sentiment In addition, we hypothesis that the explanatory power of selected explanatory variableson the variation of the volatility surface shifts with changes in market conditions, reflecting the fact that as market conditions changes variables that attract the market concern shift. For example, in one period, the change of interest rate is a particular concern of the market; therefore, the volatility surface tends to be more sensitive to the change of interest rate in this period than others. A multiple structural break test proposed by Bai and Perron (1998) is conducted to test the sample instability of the regression relationship, which was not done by Mixon (2002)orChalamandaris and Tsekrekos (2013).

This approach has led to some major findings. We show that over 95% of the variation in the volatility surface of each of our sample currency pairs can be summarised in four common factors, and that the interpretations of these factors are intuitive. They can be interpreted as the parallel shift, the slope shift of the volatility term structure, the slope shift of the volatility strike structure, and the curvature shift of the volatility term structure. We show that the changes in the volatility surface represented by these four factors are strongly correlated with spot rate return, realised volatility, volatility of volatility, market sentiment and interest rate differential. Furthermore, the size and significance of these variables to changes of the factors varies with currencies, meaning the determinants of the volatility surfaces differ by currency pairs. Finally, we show that the determinant factors of the volatility surface vary over different market conditions: specifically, the determinants of the volatility surface during the period of the GFC are substantially different than at other periods of time. This finding is consistent with our hypotheses that market concerns switch as the market conditions change.

This thesis is organised as follows: Chapter 2 introduces the literature relevant to our topic. Chapter 3 introduces our data and methodology, and we give our justification for the choice of sample period, data frequency and proposed explanatory variables in this empirical investigation. Chapter 4 reports the estimation results and whole sample regression results. Chapter 5 reports the results of the multiple structural break test and a sub-sample regression. A summary follows in Chapter 6.

# 2 Literature Review

## 2.1 Introduction

This chapter reviews the related literatures relevant to our topic. It contains a description of the Black–Scholes–Merton option pricing model, a discussion of the well-known volatility smile, the possible reasons for the volatility smile, and the empirical work relating time-varying option volatilities to state variables.

## 2.2 The Black–Scholes–Merton model

A European vanilla call option gives the owner the right to buy one unit of the underlying asset at strike price $K$, at maturity time $T$. It is a common financial derivative contract that meets the hedging and speculating needs of market practitioners. Black and Scholes (1973) and Merton (1973) derived a closed-form analytical formula (BSM model) for valuing European vanilla options that is widely used in the market:

$$C = e^{-r(T-t)}[\omega S e^{r(T-t)} \Phi(\omega d_1) - \omega K \Phi(\omega d_2)] \quad \text{Eq.2.1}$$

Where

$$d_1 = \frac{\ln\left(\frac{Se^{r(T-t)}}{K}\right) + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}} \quad \text{Eq.2.2}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \quad \text{Eq.2.3}$$

And C is the value of European vanilla call option; $\Phi(x)$ is the standardised cumulative normal distribution function in x; $r$ represents the riskless interest rate; $T-t$ represents the option's annualised time to maturity; $K$ is the option strike price; $S$ is the spot price of the underlying asset; $\sigma$ represents the square root of the instantaneous variance of underlying asset return. If one wants to calculate the value of call (put) option, one can set $\omega = 1$ ($\omega = -1$).

To value the European vanilla call option in the BSM model, the only parameter that is not directly observable is the square root of the instantaneous variance of the underlying asset return. Black and Scholes (1973) and Merton (1973) propose to estimate it from the time series data of the underlying asset price, although, in the market, the estimated volatility parameter from the past time series data of underlying asset is usually not consistent with the option volatility observed in the market.

One of the key contributions of Black and Scholes (1973) and Merton (1973) is that they show that European vanilla options can be replicated by a dynamically rebalanced portfolio that consists of a certain amount of underlying asset and riskless bond; therefore, the option is a redundant security whose price should be equal to the cost of this dynamic replicating strategy. If this is not the case, then there is an arbitrage opportunity, since one can always obtain the identical payoff by investing in the dynamically rebalanced portfolio as by buying an option. In using the concept of hedging and no-arbitrage, BSM can derive a partial differential equation that contains no random factor, and therefore obtain the BSM model. Since the work of Black and Scholes (1973) and Merton (1973), the concept of no-arbitrage has become the main methodology to price options.

## 2.3 The volatility smile

The BSM formula in Black and Scholes (1973) and Merton (1973) is obtained under the assumption that the underlying asset price is modelled as a Geometric Brownian

Motion with constant volatility. In the BSM model, the only unknown parameter is volatility: if the BSM assumptions were true, one should find the same option volatility for all strikes and maturities, as the volatility of the underlying asset is unique. Empirical studies show that option volatility systematically varies by strike and maturity, and that this variation persists. The non-linear pattern when option volatilities are plotted in terms of the moneyness is called the “volatility smile”, and the non-flat surface when option volatilities are plotted in terms of moneyness and maturity is called “the volatility surface”.

Despite the existence of the volatility smile, the BSM model is still widely used in the European vanilla option market for option valuation and risk management purposes(Castagna 2010). For the purpose of risk management using the BSM model, the risk exposure of an option comes not only from the randomness of future stock prices but also from the option volatility: an option is said to be perfectly dynamically hedged only if it is neutral in Delta, Gamma, Vega, Volga and Vanna. Figure 2.1 shows the mathematic representation of these Greek letters.

**Figure 2.1The Greeks**

| Delta | Gamma | Vega | Volga | Vanna |
|---|---|---|---|---|
| $\frac{\partial C}{\partial S}$ | $\frac{\partial Delta}{\partial S}$ | $\frac{\partial C}{\partial \sigma}$ | $\frac{\partial Vega}{\partial \sigma}$ | $\frac{\partial Vega}{\partial S}$ |

Note: this table shows the mathematic representation of the risk exposure of an option examined under the BSM model. Delta represents the change in the value of option price given a unit change in the spot rate. Gamma represents the change in Delta given a unit change in the spot rate. Vega represents the change in the value of option price given a unit change in option volatility. Volga represents the change in Vega given a unit change in option volatility; and Vanna represents the change in Vega given a unit change in the spot rate.

To hedge the first two Greeks (Delta and Gamma) in Table 2.1, one can use either the underlying asset or other options written on the same underlying asset, or both; and to hedge the last three Greeks (Vega, Volga and Vanna), one needs to use other options written on the same underlying asset. Terminology such as Delta, Gamma, and Vega hedging, usually referred to in academic journals or the market place, are references to hedging these Greeks. The five Greeks are the main risk indicators of an option book if one is doing option risk management under the BSM model in the market.

BSM is still widely used in the European vanilla option market, even with the presence of the volatility smile, because there is no better model[1] to replace it; it has become a market convention that the European vanilla option is priced using BSM and the volatility surface. Many efforts have been made to understand the smile effect and develop a better option pricing model.

## 2.4 Theoretical explanation for the volatility smile

Hafner and Wallmeier (2000)surveyed the literature and categorised explanations for the existence of the volatility smile as (1) the misspecification of the underlying asset price dynamics in the BSM model;(2) market frictions; and (3) demand and supply.

### 2.4.1 Model misspecification

Model misspecification causes the volatility smile pattern. This is because in the market, the instantaneous return of underlying asset distribution shows skew and excess kurtosis[2], while as BSM assumes the instantaneous return of underlying asset is normally distributed. Smile curvature is an exogenous correction for excess

[1] A "better model" means a model can consistently do a better job in terms of explaining the market price of option than BSM model over the time

[2] The feature of negative skewed and excess kurtosis of log return distribution of underlying asset price has been documented in, for example, Bakshi, Cao, and Chen (1997), Bates (1996) and Rubinstein (1994)" as a footnote

kurtosis, whereas smile slope is an exogenous correction for skew. More specifically, excess kurtosis implies that out the money (OTM) calls and puts are systematically undervalued in the BSM model; therefore, the symmetric “U” shape of the volatility smile occurs. The negative (positive) skew implies that the upward adjustments for the value of OTM put should be relatively more (less) than the upward adjustments for the value of OTM call; therefore, the smile shape is asymmetric (Bates 1996).

To solve the volatility smile problem or to explain the option prices observed in the market, more complex option pricing models have been developed. The idea behind these advanced models is to impose a richer setting in modelling the dynamics of the underlying asset’s price in a manner that is consistent with the empirical observation of the characteristics of the time series data of the underlying asset price, therebycapturing the volatility smile effect. Some example of these models are the jump diffusion model proposed by Merton (1976), in which the dynamics of the underlying asset’s price includes random jumps; the stochastic volatility models proposed by Heston (1993), Hull and White (1987), Stein and Stein (1991),in which the volatility of the underlying asset’s price is modelled as a random variable; the stochastic jump diffusion model proposed by Bates (1996) in which the dynamics of underlying asset price show both random jumps and stochastic volatility; and the deterministic volatility model proposed by Derman and Kani (1994), Dupire (1994) and Rubinstein (1994) in which volatility is a deterministic function of the level of the underlying asset price and time.

These more complex models, although in most cases able to give a better fit to the option's market prices than BSM model via calibration[3], still present many problemsunderempirical scrutiny. For example, Das and Sundaram (1999) find that the volatility smile in Merton's (1976) jump diffusion model flattens as time to maturity increases, because jumps tend to offset each other in the long run, which is contradictory to what we observed from the market. They also find that the Heston (1993) stochastic volatility model is unable to explain the volatility smile for options with a short time to maturity in the market. Bakshi, Cao, and Chen (1997) tested the pricing and hedging performance of the BSM model against four different versions of the stochastic volatility model: (1) the stochastic volatility model; (2) the model with stochastic interest rate; (3) the model with random jumps; and (4) the model with stochastic interest rate and random jumps. They find that even though the out of sample pricing performance of these models are all superior to the BSM model, their hedging performance is not necessarily better than Delta–Vega hedging under BSM. They also find that the parameters of these stochastic volatility models estimated by calibrating to the market option price are inconsistent with the historical time series of data of the underlying asset price. In fact, they show correlation between spot to ATM volatility, estimated by calibrating these models to market option price, is three times larger than when estimated by calibrating the models to the historical time series data of the underlying asset price. Such undesired features of the empirical performance of these advanced models undermine to some extent the validity of using them to explain the volatility smile effect.

[3] Calibration here means obtaining model parameters based on the best match between model price and market prices.

For the deterministic volatility model derived by Derman and Kani (1994), Dupire (1994) and Rubinstein (1994),Dumas, Fleming, and Whaley (1998) designed an empirical test for their pricing and hedging performance on the S&P 500 index option. They first hypothesized a set of polynomial functional forms for the deterministic volatility, and then estimated the coefficient of the deterministic volatility function by calibrating the deterministic volatility model to the option's market price. They find that even a simple deterministic volatility functional form can fit the market option price exactly, however, the out of sample prediction of option prices and hedging performance of these deterministic volatility models are worse than an ad hoc procedure using BSM with the volatility smile. This suggests that the deterministic volatility model's perfect fit to the market option price is more a result of over-fitting to the option price data than a revelation of the underlying asset's true price dynamics. Dumas et al.'s finding is consistent with that ofHagan et al. (2002), who demonstrate that the dynamic behaviour of smiles and skews predicted by deterministic volatility model is exactly the opposite of the behaviour observed in the market place.

Although more complex models can improve the in sample option pricing substantially, empirical evidence shows they typically suffer two drawbacks, both of which are prima facie evidence of model misspecification. First, the model parameters estimated from calibration to market option price are generally inconsistent with the time series data of the underlying asset price; and second, the model parameter tends to be unstable over time. These models cannot explain the time-varying volatility smile in the market.

Having observed the difficulty of explaining the time-varying volatility smile by introducing more complex settings to describe the dynamics of the underlying asset price, other authors have considered the possibility that market friction and demand and supply play a role in explaining the volatility smile. The literature that explores this possibility is reviewed in the next section.

### 2.4.2 Market friction and demand and supply

The BSM model assumes the market is frictionless and that there is no transaction cost in buying or selling underlying asset or option that does not hold in the market. Longstaff (1995) shows that transaction costs (such as brokerage fees and bid-ask spread) play an important role in explaining the volatility smile, a finding confirmed by Pena, Rubio, and Serna (1999) and Hafner and Wallmeier (2000) in the Spanish and German equity option markets. Both studies find that the width of the bid-ask spread is statistically significant, relating to the time-varying volatility smile. Ederington and Guan (2002) tested whether the true volatility smile in the S&P 500 index option is actually flat, by conducting a trading strategy that longs the option with lowest volatility and shorts the option with highest volatility and then dynamically Delta-Gamma hedging the portfolio. They find their trading strategy yields substantial profit if there is no transaction cost, but the profit disappears when transaction costs are included; they concludethat the occurrence of transaction costs largely explains the presence of the smile shape which will otherwise be arbitraged away.

Bollen and Whaley (2004) studied the S&P 500 index as well as options on equities in the S&P500 and find that net buying pressure on options explains the skew of the volatility smile. Garleanu, Pedersen, and Poteshman (2009) confirmthat the force of the demand and supply imbalance explains the volatility smile; this finding is also consistent with the finding inMcMillan (2011) that the crash of 1987 lessened the supply of put option sellers, while at the same time fund managers showed a higher demand for out-of-the-money puts. Because hedging the risk exposure of written out-of-the-money puts turned out to be expensive, higher prices for out-of-the-money puts were charged; therefore, the volatility smile in the equity option market showed negative skew.

It seems reasonable to conclude that the presence of the volatility smile effect is more or less a mixed result of misspecifications of the underlying asset price dynamics in the BSM model, market friction, and demand and supply.

## 2.5 Empirical work relating the evolution of option volatilities to state variables

To complicate things, option volatility is not only non-constant across moneyness and maturity, but changes dynamically and substantially through time. Research has been devoted to connecting time-varying volatilities to state variables, to discover the determinants of the volatility smile by identifying the contemporaneous or lead-lag relationships between time-varying volatility smiles and state variables. However, most of this research typically focuses on equity or interest rate option markets. For example, Mixon (2002) studied the relationship of changes in the S&P 500 index option volatility surface to economic state variables. These variables are contemporaneous and lagged log returns on the S&P500 index, the contemporaneous return on the Nikkei 225 index, the three-month constant maturity Treasury bill rate, the slope of the yield curve, and the spread of Moody’s AAA index yield over the 30-year constant maturity Treasury bond yield. He finds the contemporaneous return to the S&P 500 is the most important variable to explain the variation of the volatility smile among these economic state variables, capturing roughly 20 percent of the variation of the volatility surface. Han (2008) analysed the effect of market sentiment on the volatility smile on S&P500 index option and finds that the volatility smile tends to be more (less) negatively skewed when the market is bearish (bullish). Deuskar, Gupta, and Subrahmanyam (2007) examined the economic determinants of interest rate option smiles and find that the time-varying shape of the volatility smile is connected to the slope of the yield curve and the future uncertainty in the interest rate markets, which is proxied by the ATM volatility of interest rate option.

The only published paper that examines the determinants of the volatility surface in OTC currency options as far as we know is byChalamandaris and Tsekrekos (2013), who conducted an analysis of 11 Asian-Pacific currencies. However, there are two gaps in their work. First, their designation of the explanatory variables seems to be incomplete, as variables that are statistically significant in other markets are not included in their regression equation. For example, Low (2004) finds that extreme movement in the underlying asset price increases the S&P 100 index option ATM volatility substantially; and Han (2008) finds that market sentiment has a big influence on the option skew on the S&P 500 index option. Both of these variables have not been examined in Chalamandaris and Tsekrekos (2013). Second, they studied the volatility surface only for Asian-Pacific currencies, omitting EUR/USD, which is one of the most highly traded currency options. These possible gaps are the inspiration for our exploration of the determinants of the evolution of the volatility surfaces, with different explanatory variables and a more comprehensive dataset that includes the most highly traded currencies in the market.

Some literature explores the characteristics of the dynamics and predictability of evolution of option volatilities, such as the work byCont and Fonseca (2002), Goncalves and Guidolin (2006), and Chalamandaris and Tsekrekos (2009). These literatures generally reveal that the dynamics of the volatility surface show mean-reverting features and can be modelled using a factor model; however they do not explore its determinant factors. These works inspired us to use a factor model to quantify the change of the volatility surface and to control for the auto-correlation feature of the volatility surface dynamics in our regression setting.

## 2.6 Conclusion

The BSM model is widely used in practice because the volatility smile makes it "work". Many studies have attempted to solve the smile problem by imposing a richer setting in modelling the dynamics of the underlying asset price; however, these more complex models generally fail to capture the time-varying feature of the volatility smile. On one hand, empirical works reveal that the volatility smile may be a mixed effect of the misspecification of the dynamics of the underlying asset price, market friction, and demand and supply; on the other hand, empirical research has made progress in relating the time-varying volatility smile to state variables. However, little research has been done in the OTC currency option market. Our work aims to fill this gap by exploring the determinants of the time-varying option volatilities in the OTC currency option market.

# 3 Data and methodology

## 3.1 Introduction

This chapter introduces the data and methodology that underpins the empirical analysis. It contains the data description, hypotheses developments, methodology of quantifying the volatility surface, justification for the proposed explanatory variables, the regression model, and the methodology of testing structural breaks.

## 3.2 Data

### 3.2.1 OTC currency option market

In the interbank currency option market, options are traded in terms of option volatilities. The option price is then obtained by using the traded option volatilities as the volatility input in the Garman and Kohlhagen (1983) model (GK model), which is an extension of the BSM model for currency options:

$$C = e^{-rd(T-t)}[\omega S e^{(rd-rf)(T-t)} \Phi(\omega d_1) - \omega K \Phi(\omega d_2)] \qquad \text{Eq.3.11}$$

where

$$d_1 = \frac{\ln\left(\frac{Se^{(rd-rf)(T-t)}}{K}\right) + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}} \qquad \text{Eq.3.12}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \qquad \text{Eq.3.13}$$

The notations are the same as in the BSM model formula given in Chapter 2. The only difference between the GK and BSM models is that the forward price of the underlying asset in the GK model is determined by $rd$ and $rf$ which respectively represent the domestic and foreign riskless interest rate, whereas in the BSM model it is determined by $rd$ only.

Knowing the quoted option volatility along with other model inputs, one can calculate the option's price from the GK model. The benefit of using option volatility as an alternative to option price is that it is effectively a dimensionless variable that facilitates comparison and has established patterns (its smiles and skews are smooth). More specifically, it allows investors to compare option prices across moneyness, maturity, underlying asset price and observation time, aiding them to assess an option's fair value.

The volatility surface is constructed when option volatilities are plotted against maturity and moneyness. Figure 3.1 presents three dimensional graphs, AUDUSD, EURUSD and USDJPY (denoted hereafter as AUD, EUR and JPY). Option moneyness is expressed as the Delta of the GK model. Delta is the first partial derivative of the GK value with respect to the spot rate.

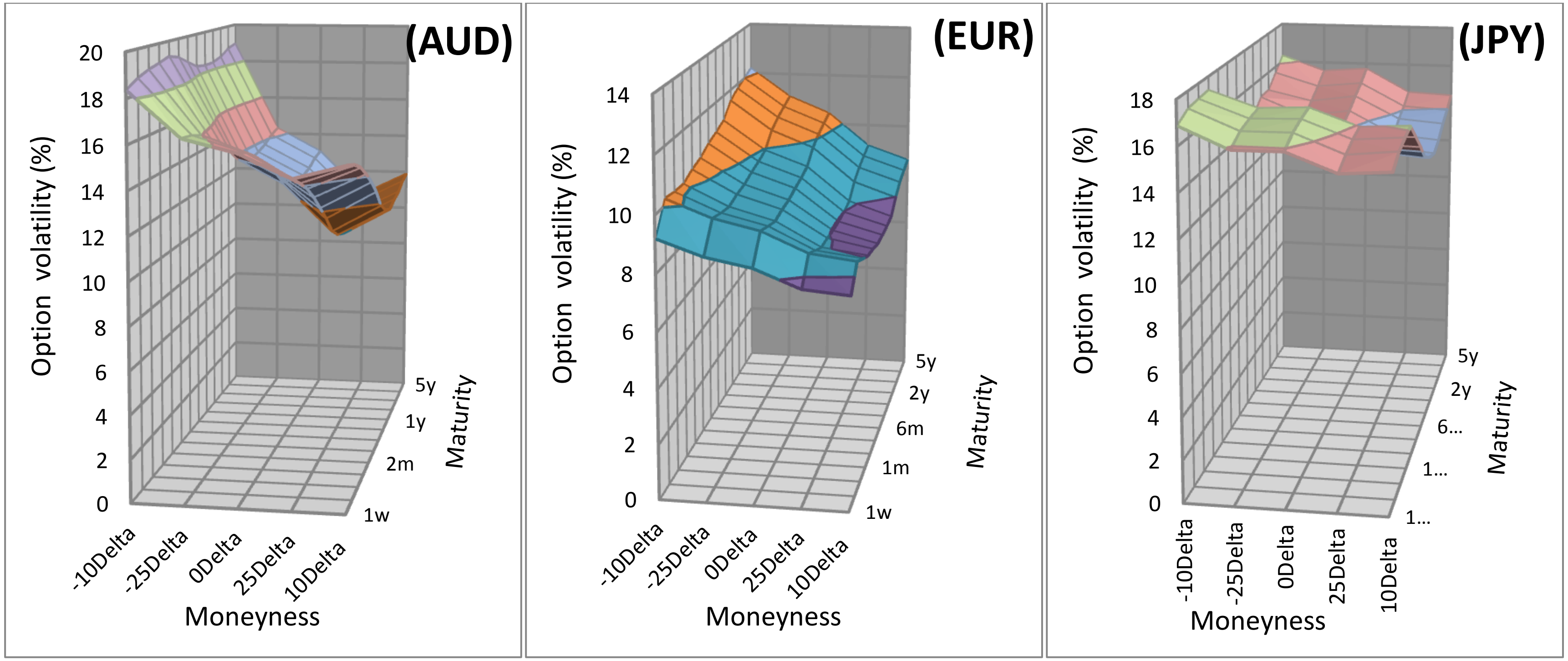
$$Delta = \omega e^{-rf(T-t)}\Phi(\omega \left( \frac{\ln\left(\frac{Se^{(rd-rf)(\mathrm{T}-t)}}{K}\right) + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}} \right)) \qquad \text{Eq.3.14}$$

Therefore, the strike price can be reverse-engineered from Delta and the option volatility. Castagna (2010)justifies the use of GK Delta to define option moneyness instead of strike price as follows:

This way of quoting is smart: it allows us not to worry about small movements of the underlying market during the bargaining process, because the absolute strike level will be defined only after the agreement on the price (in terms of implied volatility), so that the trader is sure to trade an option with given features in terms of exposures both to the underlying pair and to the implied volatility. (p.15)

In particular, in the interbank market traders trade options Delta-neutral since they are reluctant to option Delta risk: that is to say, when a trader longs a call option, a Delta amount of underlying asset will be shorted to give a zero Delta exposure of this trade. Expressing option moneyness by using Delta, one can know directly what amount of underlying asset will be traded along with the option to give a Delta-neutral trading portfolio; this amount does not alter even as a small movement in the spot price during the bargaining process, whereas when expressing option moneyness by using strike during the bargaining process, a small movement in the spot price will alter the amount of the underlying asset needed to trade to achieve a Delta-neutral risk exposure, which may in turn affect the price of this option.

**Figure3.1Illustration of volatility surfacesat 25/06/2013 for AUD, EUR and JPY**




Note: This figure illustrates the volatility surfaces of AUDUSD, EURUSD and USDJPY in the over-the-counter currency option market at 25 June 2013. Source: Bloomberg.

### 3.2.2 Sample currency pair, volatility surface data and sample period

The currency pairs examined in this study are AUD, EUR and JPY. The traded volatility surfaces are sourced from Bloomberg, who provides the traded quotes for the zero Delta straddle, 10 and 25 Delta risk reversals, and 10 and 25 Delta butterflies. Zero Delta straddles are trading portfolios consisting of a long call and a long put option with the same maturity and strike price, which neutralises the portfolio's Delta. Risk reversals are trading portfolios consisting of a long OTM call and a short OTM put option with the same maturity but different strike price, and identical Deltas. Butterflies are long strangle portfolios consisting of a long OTM call and OTM put with the same maturity, different strikes, Delta with equal magnitude but opposite sign, and a short zero straddle that neutralises the Vega position of the portfolio. These commoditised option trading strategies are quoted in terms of option volatility. Delta is calculated from the GK model and used as a market convention to express the option's moneyness. Specifically:

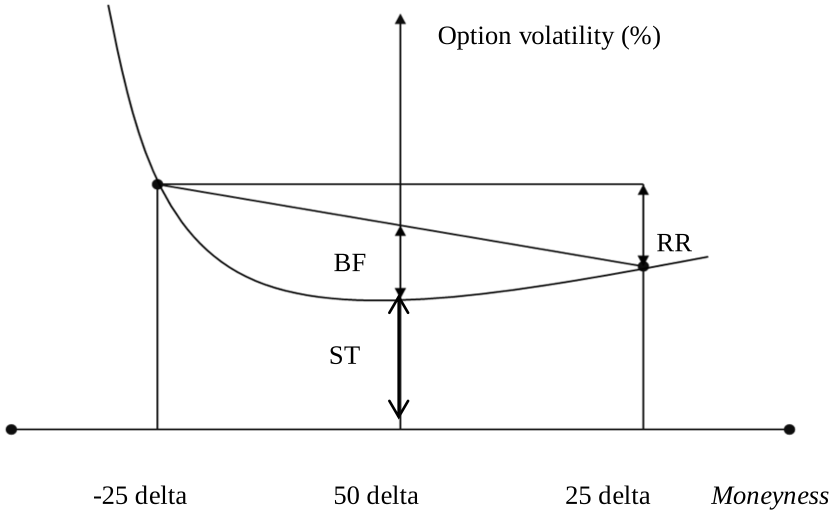
$$Zerodeltastraddle \approx \sigma_{ATM} \qquad \text{Eq.3.15}$$

$$RR_{10(25)delta} = \sigma_{10(25)delta} - \sigma_{-10(-25)delta} \qquad \text{Eq.3.16}$$

$$BF_{10(25)delta} = \frac{\left(\sigma_{10(25)delta} + \sigma_{-10(-25)delta}\right)}{2} - \sigma_{ATM} \qquad \text{Eq.3.17}$$

By solving the simultaneous equation above for each maturity, option volatilities are observed with five different Deltas: OTM puts with Delta= $-0.10$ and Delta = $-0.25$,ATM calls/puts and OTM calls with Delta = 0.10 and Delta= 0.25. In practice, zero Delta straddle volatilities define the level of the volatility smile; and risk reversal and butterfly volatilities define its slope and curvature respectively(Wystup 2003). Thus, collectively, volatility quotes of these three strategies define the volatility smiles in the currency market. Figure 3.2 illustrates this point.

**Figure3.2Illustration of the volatility smile measured by zero Delta straddle, 25 Delta risk reversal and 25 Delta butterfly**




Note: this figure illustrates how the volatility smile is defined by risk reversal (RR), butterfly (BF) and zero Delta straddle (ST) quotes. Source:Wystup (2003)

These trading strategies are so liquid that they are effectively commoditised in the interbank option market, and therefore the potential bias caused by illiquidity observed in other markets is not an issue. The main reasons for the popularity of these commoditised option trading strategies in interbank transactions is that they all have an exclusive risk exposure for GK Greeks, which are often traded between interbank traders for risk control purposes. Table 3.1 illustrates the risk exposure for GK Greeks of zero Delta straddle, 25 Delta risk reversal and 25 Delta butterfly.

**Figure3.3Volatility risk exposures for zero Delta straddle, 25 Delta risk reversal and 25 Delta butterfly**

| | ATM-Straddle | 25 Delta Risk Reversal | 25 Delta Butterfly |
|---|---|---|---|
| Defines | Level | Skew | Curvature |
| Volatility risk | Vega | Vanna | Volga |

Note: This table shows the only volatility risk exposure of option portfolio of zero Delta straddle, 25 Delta risk reversal and 25 Delta butterfly. The definitions of Vega, Vanna and Volga have been given previously.

In Bloomberg there are seventeen different option maturities. We exclude the option volatility data for 4- and 18-month and 7- and 10-year maturity because these data are not available for the whole sample period, leaving us with option volatility data for 1 week, 2 weeks, 3 weeks, 1–3 months, 6 months, 9 months, 1 year, and 2–5 years. Since five option volatilities are observed for each maturity on each observation date, we are able to obtain a matrix of $13 \times 5$ option volatilities per currency pair.

To avoid the noise generated from using daily data frequency, the variation of the volatility surface is studied in weekly frequency. This is consistent with Carr and Wu (2007), who use weekly data to analyse the relationship between ATM volatility and sovereign credit default swap spread for the same reason. The noise we are trying to avoid is the change in the mean volatility quotes, which is not considered substantial. For example, daily mean volatility quotes can fluctuate in a period but never exceed the bid-ask bound of previous trading days. Such a situation can be frequently observed if we use daily data, but not if we use weekly data in our sample period.

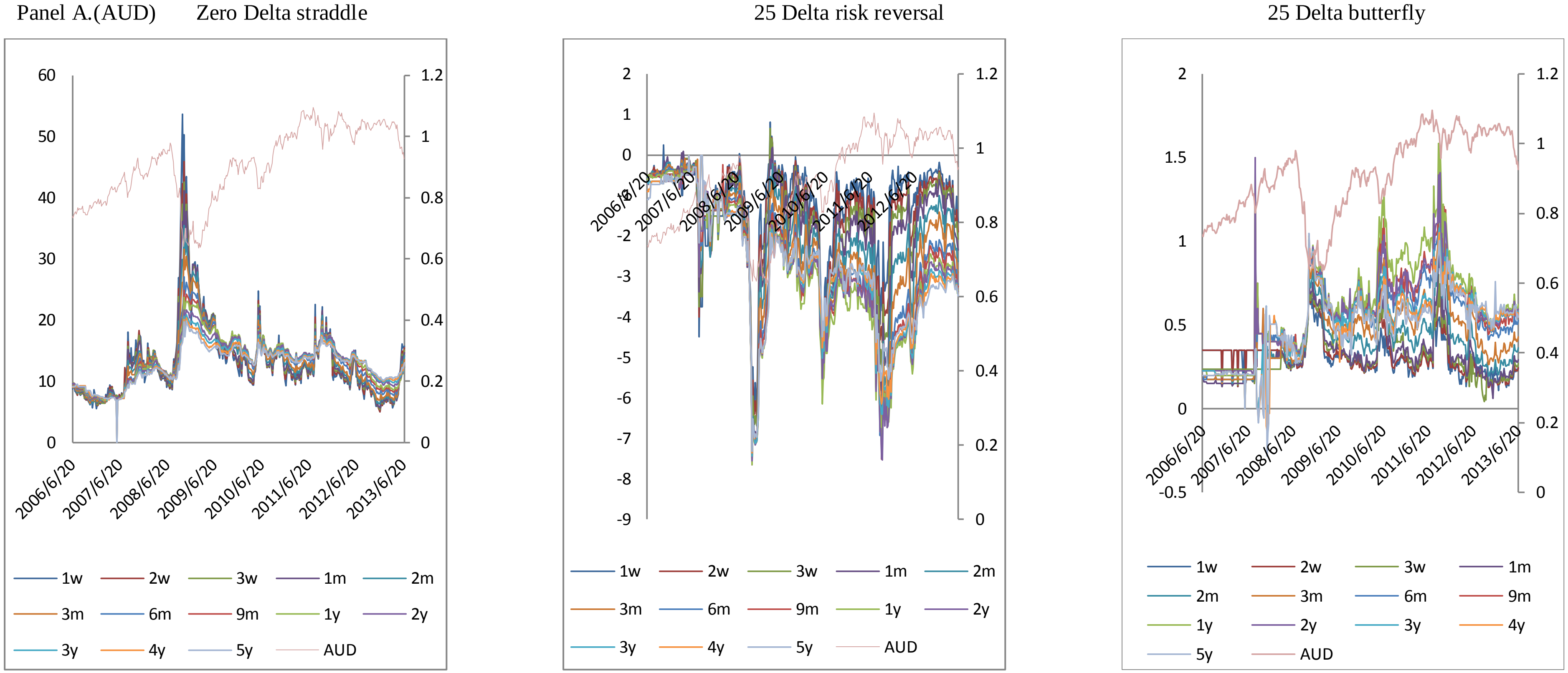
The sample period for this analysis is from 27 June 2006 to 25 June 2013. The sample length is restricted by the data availability of one of our key explanatory variables. However, our sample length of approximately 7 years with 367 weekly observations in total is large in comparison to those in the published literature, and covers a wide range of different behaviours for the option volatilities.

### 3.2.3 Time variation of the component of the volatility smile across maturities

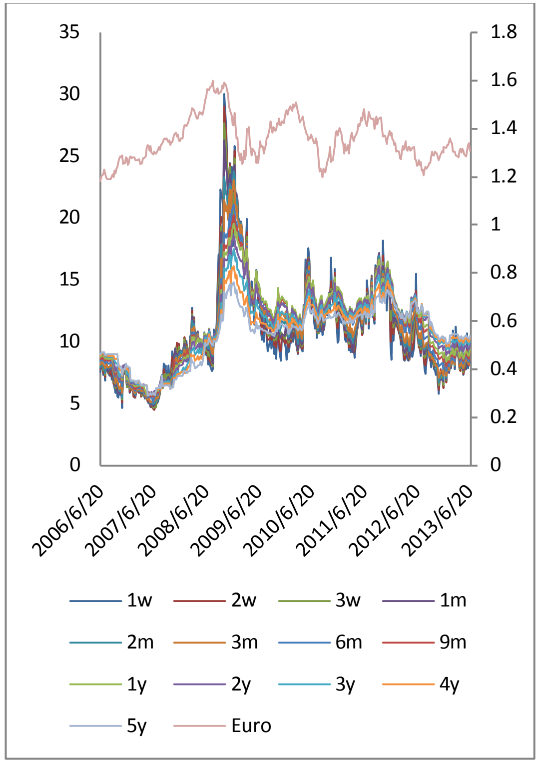
In Figure 3.3 we plot the time series of traded volatilities for zero Delta straddle, 25 Delta risk reversal and 25 Delta butterfly for thirteen different maturities for AUD, EUR and JPY. Since these three traded volatility quotes represent the level, skew and curvature of the volatility smile for each maturity, we are able to compare changes in the shape of the volatility smile between short, medium and long maturities over time. This figure is also informative for the time variation in the volatility surface, which is constructed as a combination of the volatility smile and different maturities. The time variation of the volatility surface exhibits similar features among the three currencies. From early 2007 the level of the volatility surface shows a slow upward trend and then a substantial increase in the period from September to October 2008;a slow trend downward has occurred more recently. Judging from the level of the volatility smile among different maturities for the three currencies, we can see that the term structure of the volatility surface was extremely inverse in the period September to October 2008, owing to the more rapid increase in the level of the short-term volatility smile, and thereafter the term structure of the volatility surface slowly trends from inverse to normal. Notably, the skewness and curvature of the volatility surface also experienced a substantial variation in the period of the global financial crisis (GFC) and remained quite volatile during the rest of our sample period.[4]

[4]You may find 25 Delta butterfly charts have strange looking data at the beginning of the sample for all three currencies. This is because, before 2008, 25 Delta butterfly quotes for these three currencies do not change often in a daily basis.

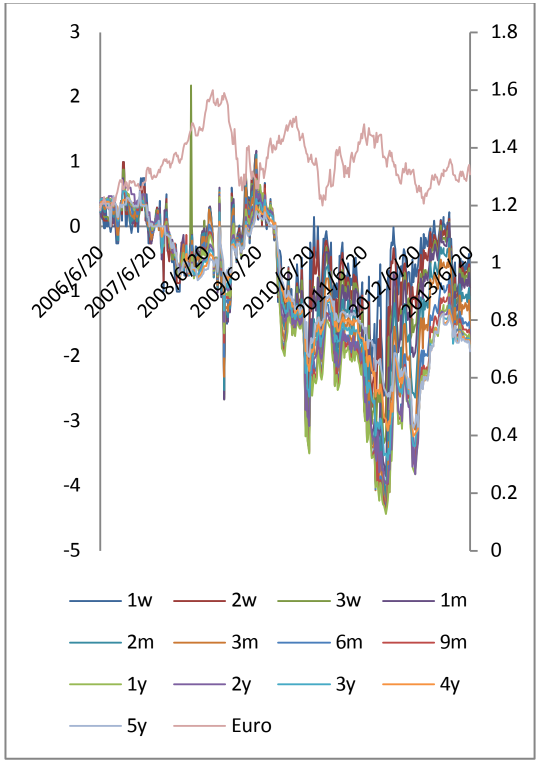
**Figure 3.4Time series data of the traded volatility quotes for zero Delta straddle, 25 Delta risk reversal and 25 Delta butterfly**



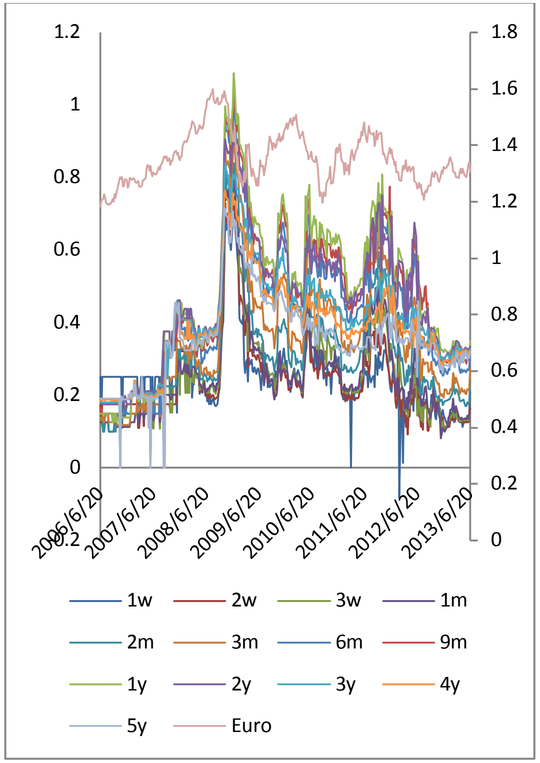
Panel B.(EUR) Zero Delta straddle



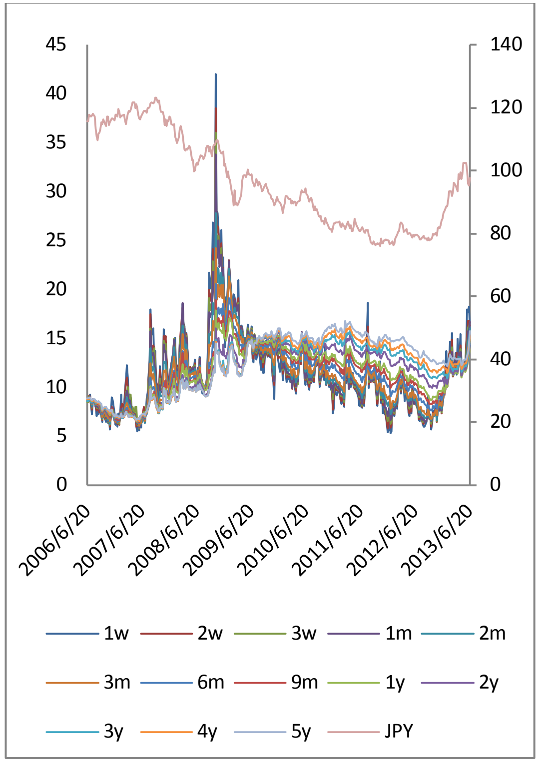

25 Delta risk reversal



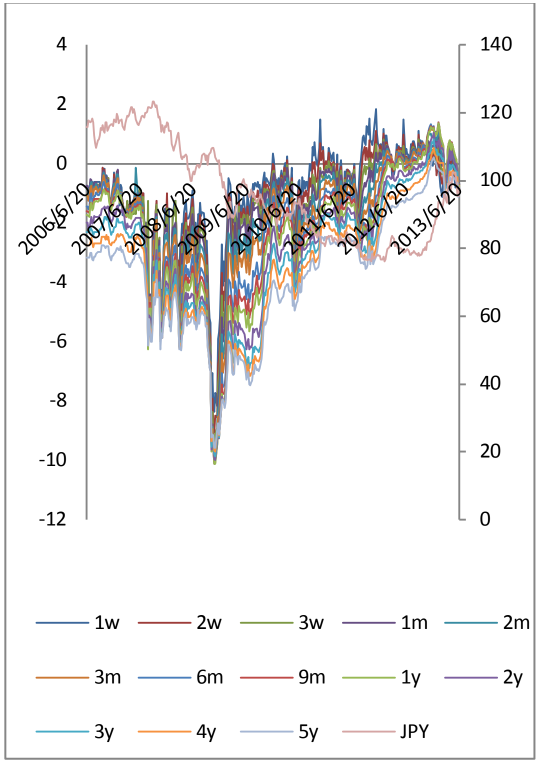

25 Delta butterfly



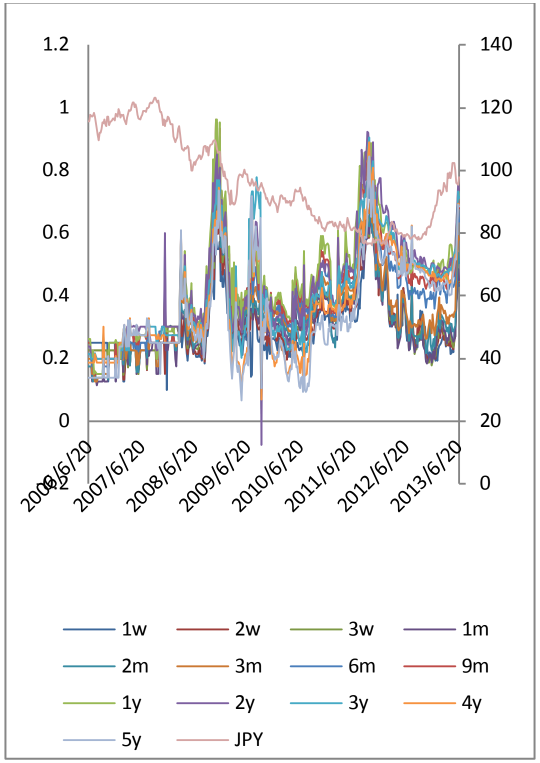
Panel C.(JPY) Zero Delta straddle




25 Delta risk reversal




25 Delta butterfly




Note: This figure shows time series of traded volatility quotes for zero Delta straddle, 25 Delta risk reversal and 25 Delta butterfly of 14 different maturities from 27 June 2006 to 25 June 2013 for AUD, EUR and JPY respectively. The left Y axis represents the volatilities (%); and the right Y axis represents the level of the spot rate of corresponding currency.

### 3.2.4 Summary statistics of option volatilities

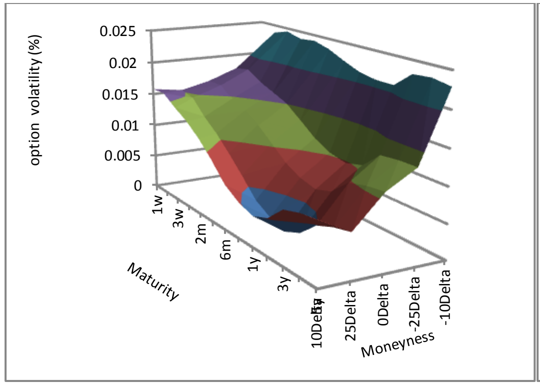
Figure 3.3 displays the mean and standard deviation of weekly change of volatilities for AUD, EUR, and JPY. An apparent feature observed here is that the standard deviation of weekly change of option volatilities dampens with increase in maturity. This finding suggests that long-dated volatilities do not change to the same extent as short-dated volatilities.

Based on the figures, a natural question here is what drives the variation of the volatility surface. Hypotheses developments and methodology of testing these hypotheses are introduced in the following chapter.

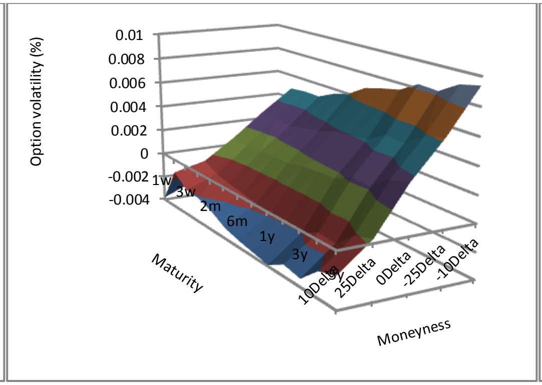
**Figure 3.5Statistics of mean and standard deviation of weekly change of option volatilities**

Panel A. Average weekly change of option volatilities in:

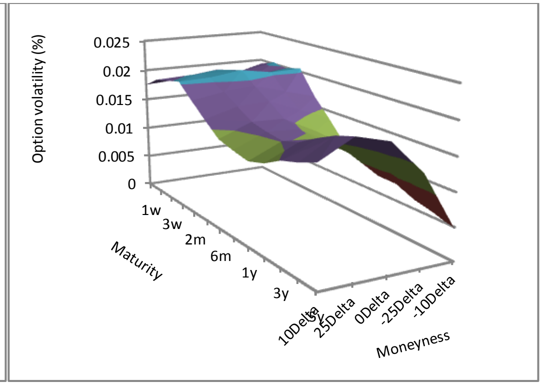
(AUD)




(EUR)

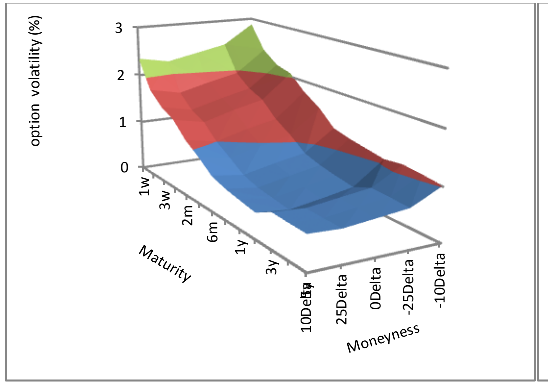



(JPY)

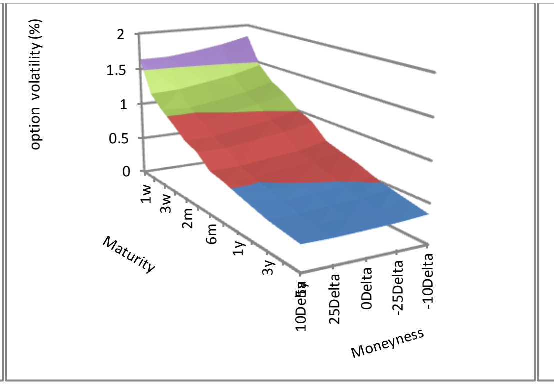


Panel B Standard deviation of weekly change of option volatilities in:

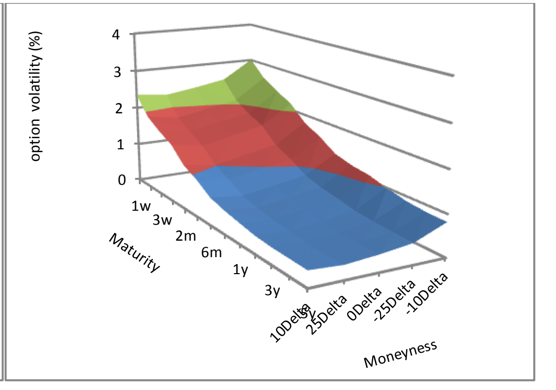
(AUD)




(EUR)




(JPY)




Note: This figure shows the average weekly change and standard deviation of weekly change of option volatilities for 5 different option moneyness and 13 different maturities.

## 3.3 Methodology

### 3.4 Hypotheses developments

*Our first hypothesis* is that the determinant of the variation of the volatility surface is not exclusively the change in the level of the spot rate: other variables have an effect as well. These variables collectively reflect any change in market expectation of the future higher moments of the underlying asset price dynamics, in market sentiment, and in the hedging cost of option books. The proposed set of explanatory variables in this thesis will be discussed in a later section.

*Our second hypothesis* is that under different market conditions there are different exogenous variables governing market concern about the market condition in the future, and therefore of the variation of the volatility surfaces.

Testing the second hypothesis involves employing a structural break test to test the stability of the regression relationship between the volatility surface and explanatory variables; the methodology for doing this will be discussed later. There are two major reasons to test this hypothesis. First, there is no theory to show that the relationship between the explanatory variable and the variation of volatility surface should be constant in the first place. On the contrary, Derman (1999) suggests that the sign of the correlation-coefficient between the spot rate and the level of volatility changes over time as the model that governs the volatility surface switches. Second, since our sample period covers the most recent GFC and Euro-zone sovereign debt crisis (ESDC), by imposing a structural break test we are able to see the sensitivity of the volatility surfaces to proposed explanatory variables changes when market conditions switch from an extremely high degree of risk reversion (during the GFC and ESDC) to a normal degree of risk reversion.

In order to test these two hypotheses, we need a model to quantify the variation of the volatility surface, which will be introduced in the following section.

### 3.4.1 Estimation method

In order to study the determinants of the evolution of the volatility surface, the variation of the volatility surface has to be quantified. We follow Chalamandaris and Tsekrekos (2013) andMixon (2002)in using a factor model to model the change of the volatility surface. The idea behind using a factor model is to reduce the multiple dimensions of option volatility data series with different moneyness and maturity to several common factors which are able to explain a large portion of the total variance of the original multiple dimension series. The model specification is as follows:

$$\Delta\boldsymbol{\sigma_{it}} = \beta_{i1}f_{1t} + \cdots + \beta_{im}f_{mt} + \varepsilon_{it} = \boldsymbol{\beta}'\boldsymbol{f_t} + \boldsymbol{\varepsilon_t} \qquad \text{Eq.3.18}$$

Where, $\Delta\sigma_{it}$ is a $k \times 1$ vector of the volatility change at time period t. $\boldsymbol{\beta}$ is $k \times m$ matrix of factor loadings. $\boldsymbol{f_t}$is $m \times 1$, a vector of common shocks at time period t. $\boldsymbol{\varepsilon}$ is $k \times 1$,a vector of random residuals at time period t.

Unlike the multivariate regression model, both $\boldsymbol{\beta}$and$\boldsymbol{f_t}$have to be estimated. There are many ways to do this. Following Chalamandaris and Tsekrekos (2013), we use the method of asymptotic principal components to estimate consistently the common factors and the coefficient term. The consistency of this estimator has been established by Bai and Ng (2002) and Stock and Watson (2002). The estimates of $\boldsymbol{\beta}$and$\boldsymbol{f_t}$are obtained by solving

$$J(K) = \min_{\boldsymbol{\beta},\boldsymbol{f_t}} \frac{1}{k * \tau} \sum_{t=1}^{\tau} \sum_{i=1}^{k} (\Delta\sigma_{it} - \beta_{im}f_{mt})^{\boldsymbol{2}} \qquad \text{Eq.3.19}$$

Where, $k$is the number of volatilities, $\tau$is the number of the single time period. This is the least square estimation criteria. To ensure the solution does not go to infinity, the optimisation is solved by imposing$\boldsymbol{\beta}'\boldsymbol{\beta} = \mathbf{1}$constraint. It turns out that one of the solutions for *Eq.3.19*is given by $(\widehat{\boldsymbol{\beta}}, \widehat{\boldsymbol{f}_t})$where $\widehat{\boldsymbol{\beta}} \equiv [\widehat{\boldsymbol{\beta}}_{ik}] \equiv [\sqrt{\boldsymbol{k}}\hat{\boldsymbol{e}}_{\mathbf{1}}, \dots, \sqrt{\boldsymbol{k}}\hat{\boldsymbol{e}}_{\boldsymbol{m}}]$ and $\hat{\boldsymbol{e}}_{\boldsymbol{m}}$is the eigenvector of theestimated covariance matrix of the volatility dynamics series with $m$ largest eigenvalue.$\widehat{\boldsymbol{f}_t} \equiv \Delta\boldsymbol{\sigma}_{it}\widehat{\boldsymbol{\beta}}_{ik}/k$.

An alternative is to use the maximum-likelihood method. This assumes$\Delta\sigma_{it}$ are joint normal distribution with a mean of $u$ and the covariance matrix equal to $\boldsymbol{\beta}\boldsymbol{\beta}' + \boldsymbol{\varepsilon}\boldsymbol{\varepsilon}'$**.** It requires an initial guess of the number of the common factor. Since this method requires an extra assumption about the joint distribution of the $\Delta\sigma_{it}$series, it brings additional risk; to avoid this, we employed the asymptotic principal components approach to estimate the factor loading and common factor vector.

There may be concern that the serial correlation property in the volatility series[5]will undermine the consistency of the estimate of the factor loadings and common factors. However, Stock and Watson (2002) demonstrate that such a concern can be eased when one estimates the factor loading and common factor in an asymptotic principal components approach.

The number of factors we employ in the factor model is four. Four factors explain over 95% of our sample. The volatility surface variation and the marginal increase in

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[5] We note that option volatilities show a high degree of serial correlation in our volatility sample and sample period.

explanatory power improves by less than 1.5% when adding an extra factor; hence, using four factors is considered a reasonable choice.

One alternative approach to estimating the volatility surface dynamics is to model the change of the coefficient parameter from the best fitted polynomial equation that quantifies the volatility surface, to represent the change in the shape of the volatility surface over the time. For example, Goncalves and Guidolin (2006)offer the following specification:

$$\ln \sigma_i = \beta_0 + \beta_1 M_i + \beta_2 M_i^2 + \beta_3 \tau_i + \beta_4 (M_i * \tau_i) + \varepsilon_i \qquad \text{Eq.3.20}$$

Where $\varepsilon_i$ is the random error term, $i$=1, 2, 3,. . . ,N and N is the number of options available at each time t. $\ln \sigma_i$ is the natural log of the GK volatility for option i. $M_i$and$\tau_i$are the moneyness and the time to maturity of option i, respectively. To parameterise the volatility surface, term $\beta_1$ is designed to capture the skewness of the volatility smile, while term $\beta_2$ represents the curvature of the volatility smile. $\beta_3$ controls for the slope of the term structure of volatility, and $\beta_4(M_i * \tau_i)$ captures the interaction effect between the skewness of the volatility smile and the time to maturity.

In order to study the dynamics of the volatility surface, at each time t such a parametric equation is fitted to the volatility surface and the estimated coefficients are recorded. Since each of the coefficients capture an aspect of the volatility surface, the analysis of the dynamics of the volatility surface is then reduced to the dynamics of the estimated coefficients.

We do not consider this approach to estimate the variation of the volatility surface because although this parametric model of quantifying the volatility smile is convenient for interpretation, as are all parametric models, it effectiveness depends on its goodness of fit to the volatility data. Therefore, an effort has to be made to get the best parametric equation possible. For example,Pena, Rubio, and Serna (1999)compared the goodness of fit of a set of parametric equations to the volatility smile data before conducting regression analysis. Such an extra step causes inconvenience in operation; when moving on to analyse many countries' volatility surfaces, this inconvenience is magnified, as each volatility surface may need a different form of parametric equation to describe it.

### 3.4.2 The explanatory variables

The statistical factor model simply summarises the variation in the volatility surface through the estimated factor loadings, and cannot make any prediction or theoretical explanation of the future evolution of the volatility surface. This is because the latent factors are endogenously obtained from the linear combination of the option volatilities; the latent factors will not be known until the option volatilities are known. If we want to understand the driving force behind the volatility surface for predictive purposes, we need to build a model to explain the dynamics of the latent factors in the first place. For example, Chalamandaris and Tsekrekos (2009) model the latent factors using vector auto regression to test the predictability of volatility surface dynamics. Cont and Fonseca (2002) propose using mean reverting Ornstein–Uhlenbeck processes to model the latent factor dynamics of option volatilities. Since understanding the relationship between state variables and the dynamics of volatility surface is the focus of this thesis, the latent factor is modelled by a related state variable, using a standard linear multivariate regression model. The following paragraphs introduce the proposed related explanatory variables.

### *a.* Spot rate (SPOT)

There are two basic reasons that motivate us to include the weekly spot return as an explanatory variable. First, even though the relationship between spot return and option volatility has been studied extensively in the equity market by Bollerslev and Zhou (2006),Derman (1999),Hibbert, Daigler, and Dupoyet (2008) and others, there are few analyses of the relationship between spot return and option volatility in the currency market. Most of the existing literature either focuses on single option volatility(usually money volatility), as do Kim and Kim (2003), or on option volatilities with single maturity, as do Deuskar, Gupta, and Subrahmanyam (2007).We fill this gap by studying the relationship between the spot rate return and variation of volatility surface in the currency option market. Secondly, exploring the correlation between the spot return and the variation of volatility surface has an important hedging implication, as the effectiveness of a Delta hedge of an option largely depends on the assumption that option volatility is uncorrelated with spot return. By studying the spot return and volatility surface relationship, we should be able to determine the effectiveness of the Delta hedge for options across maturity and moneyness. We do not use forward rate return because we want to see the separate impacts of the spot rate and interest rate differentials on the variation of volatility surface.

We calculate the weekly spot return as:

$$\Delta SPOT_t = \frac{Spotrate_t - Spotrate_{t-1}}{Spotrate_{t-1}} \qquad \text{Eq.3.21}$$

### *b.* Realised volatility (RV)

The relationship between realised volatility and option volatility can be better understood with a close look at the instantaneous trading profit and loss (P&L) for a trader who longs an option and dynamically Delta hedges its position(Carr and Madan 2001). It is given by:

$$P\&L = \frac{1}{2}\Gamma S^2(\left(\frac{\Delta S}{S}\right)^2 - \sigma_B^2 \Delta t) \qquad \text{Eq.3.22}$$

Where$\sigma_B^2$ is the option volatility at which the option was purchased and $(\Delta S/S)^2$ is the spot volatility realised during the small time interval$\Delta t$. $\Gamma = N'(d_1)/(S\sigma_B^2\sqrt{\tau})$ represents GK Gamma of the option. "B" denotes BSM.

One can see from the equation above that in order to yield an instantaneous positive payoff from a long option position combined with a dynamic Delta hedging strategy, a trader wants the realised volatility to be larger than the option volatility. A trader who shorts an option and dynamically Delta hedges wants the realised volatility to be smaller than the option volatility. In an equilibrium condition, the realised volatility should be equal to the implied volatility to give a zero expected P&L; hence, it is considered that volatility should be closely and positively related with realised volatility. It is noted that the instantaneous P&L of dynamic Delta hedging is also dependent on the path of the spot rate since $\Gamma$ is dependent on the level of the spot rate. Empirical work such as that of Chalamandaris and Tsekrekos (2013)demonstrates the close relationship between option volatility and realised volatility. The estimated realised volatility is calculated as:

$$RV_t = \sqrt{\frac{\sum_{k=1}^{n}(R_{t-k+1} - \hat{\mu})^2}{n-1}} \qquad \text{Eq.3.23}$$

Where $\mathrm{R}_{\mathrm{t-k+1}}$ is the daily return of currency rate on $\mathrm{t-k+1}$ day. $\hat{\mu}$ is the estimated mean return during the past $n$ trading days. $n$ is set as 30 to give more weight on the recent realised volatility.

### *c.* Volatility of volatility (VOV)

Volatility of volatility measures the stability of the volatility surface itself. This variable is calculated as the standard deviation of one-month straddle quotes over 30 trading days. The estimated volatility of volatility is calculated as:

$$VOV_t = \sqrt{\frac{\sum_{k=1}^{n}(V_{t-k+1} - \hat{V})^2}{n-1}} \qquad \text{Eq.3.24}$$

Where $\mathrm{V}_{\mathrm{t-k+1}}$ is the daily percentage change of one-month straddle quotes on the $\mathrm{t-k+1}$ day. $\hat{\mathrm{V}}$ is the estimated mean daily percentage change of volatility during the past $n$ trading days. $n$ is set as 30 to give more weight to recent information.

The reason for considering this variable is that the hedging cost of an option book is closely related to the stability of the volatility surface. Specifically, in a period with stable/unstable volatility surface, fewer/more hedging activities have to be undertaken by market markers to hedge an additional option to achieve a neutral position or a certain risk limit; therefore, the price of an option should increase/decrease in this period, and this should be reflected in the currency option market as a change in the volatility surface.

### *d.* Correlation between spot rate and ATM volatility (COR)

Spot-ATM volatility correlation is calculated as:

$$COR_t = \frac{\left(\frac{\sum_{k=1}^{n}(R_{t-k+1}-\widehat{R})(V_{t-k+1}-\widehat{V})}{n}\right)}{\sigma_R \sigma_V} \qquad \text{Eq.3.25}$$

where, $R_{t-k+1}$ is the daily return of spot rate; $V_{t-k+1}$ is the daily percentage change of the one-month zero Delta straddle quotes. $\widehat{R}$ is the mean daily percentage return ofin the past $n$ trading days. $\widehat{V}$ is the mean daily percentage change of one-month zero Delta straddle quotes in the past $n$ trading days. $\sigma_R$ is the standard deviation of the daily percentage return of spot rates in the past $n$ trading days. $\sigma_V$ is the standard deviation of the daily percentage change of one-month zero Delta straddle quotes in the past $n$ trading days. To be consistent with the calculation of realised volatility and volatility of volatility, n is set as 30.

The reason for including this variable is that from the traders' point of view there is an advantage to long risk reversal (buy OTM call and sell OTM put) when they believe that there is a positive correlation between ATM volatility and spot return. This advantage comes from the positive Vanna position of this trading portfolio. Vanna is the change of Vega of an option portfolio when there is a unit change in the underlying asset price. When there is a positive correlation between spot rate and option volatility, the future value of an option portfolio with positive Vanna will always dominate the future payoff of a portfolio with negative Vanna. To illustrate the point, if an investor has a positive Vanna position where volatility and spot rate positively correlate with each other, an increase in option volatility will increase the Vega position as a contemporaneous increase in the spot rate. A decrease in volatility with a decreased Vega position is due to a contemporaneous decrease in the spot rate. A positive correlation between volatility and spot rate makes a long positive Vanna strategy a win-big and lose-small situation; consequently, the price of a positive

Vanna strategy should be closely related to the correlation between volatility and spot rate. Since long 25 Delta risk reversal is an option portfolio with positive Vanna, we can expect the correlation between volatility and spot rate to be positively related to the price of a 25 risk reversal trading portfolio; hence the skew of the volatility surface. In addition, correlation between spot rate and option volatility is also informative about the third moment of the underlying asset distribution. Specifically, with the positive (negative) spot rate and option volatility, we expect to see a positive (negative) skewed distribution of underlying asset return (Heston 1993), and consequently a change in correlation between spot rate and option volatility, related to the variation of volatility surface.

### *e.* Positive and negative extreme returns (CRASH and SPIKE)

We define the occurrence of extreme return as any daily returns that exceed two standard deviation of weekday daily return in our sample period. This is consistent with Vilkov and Yan (2013), who use two standard deviation of currency daily return to represent a market spike or crash. The assumption here is straightforward: that extreme movement causes good/bad market sentiment, which results in change to the volatility surface. In the S&P 500 option market, Low (2004) finds extreme return increase option volatility in a large magnitude.

### *f.* Net long position (NL)

This variable is the weekly change of the net long position in the currency futures contracts of large speculators.[6] NL reveals the average expectation of the future direction of the currency rate, and serves as a proxy for market sentiment[7]. A comprehensive empirical study of the market sentiment effect on S&P500 volatility skew can be found in Han (2008), who argues that the skew moves in the same direction as market sentiment. We include this variable to test whether market sentiment and the belief of the future direction of the currency rate have the same impact on the shape of the volatility smile in the currency option market. The data for this variable is available from the official website of the US Commodity Futures Trading Commission, recorded every Tuesday and released every Friday. To be consistent with the recorded date of net long position, our weekday data for all other variables starts on every Tuesday.

### *g.* Implied interest rate differential (INTDF)

Implied interest rate differential is calculated using the 12-month forward rate. This variable assesses the possible connection between changes in the currency interest rate differential and variation of volatility surface dynamics. It is calculated as

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6 The Commodity Futures Trading Commission defines larger traders in the three categories of market dealer, asset manager and leveraged funds; following this, asset managers and leveraged funds are deemed large speculators in this thesis.

7 "Investor sentiment, defined broadly, is a belief about future cash flows and investment risks that is not justified by the facts at hand" quoted from Malcolm Baker and Jeffrey Wurgler (2007). Han used three proxies to measure the market sentiment: 1. Investors intelligence's weekly surveys of approximately 150 investment newsletter writers" 2. Net position of large speculators; the data is from CFTC. 3. Sharpe's (2002) valuation errors of S&P 500 index. I used the net position of large speculators as a market sentiment proxy.

$$INTDF = \frac{F}{S} \quad \text{Eq.3.26}$$

Where, F is the 12 month forward rate and S is the spot rate.

### 3.4.3 The regression model

To study the determinants of the evolution of the volatility surface, for each currency pair the variation of the volatility surface is summarised in four latent factors. Each factor is modelled as follows:

$$F_{it}^{j} = C + \beta_{i1}^{j}\Delta SPOT_{t}^{j} + \beta_{i2}^{j}\Delta RV_{\mathrm{t}}^{j} + \beta_{i3}^{j}\Delta VOV_{t}^{j} + \beta_{i4}^{j}\Delta COR_{t}^{j} + \beta_{i5}^{j}CRASH_{t}^{j} + \beta_{i6}^{j}SPIKE_{t}^{j} + \beta_{i7}^{j}\Delta NL_{t}^{j} + \beta_{i8}^{j}\Delta INTDF_{t}^{j} + e_{t}^{j} \quad \text{Eq. 3.27}$$

$$e_{t}^{j} = \varepsilon_{t}^{j} + b\varepsilon_{t-1}^{j} \quad \text{Eq. 3.28}$$

Where $F_{it}^{j}$ represents the $i$th latent factor of $j$ currency pair. Following Mixon (2002), the error term is modelled as a first order moving average process. $\varepsilon_{t}^{j}$ represents the residual of modelling the error term. The description of these explanatory variables is illustrated in Table 3.1. It is noted that $\Delta cor_{t}^{j}$ and$\Delta nl_{t}^{j}$ are weekly absolute changes, whereas other variables are all percentage changes. The reason for setting it this way is that correlation and net long position can be negative values; therefore, if a percentage change were to be used, the result would be misleading. For example, a positive return should be recorded when a value moves from negative to positive, but when using percentage change to measure this change of value, the return is recorded as negative.

**Figure3.6The explanatory variables**

| Explanatory variables | Description |
|---|---|
| $\Delta SPOT_t^j$ | Weekly return of currency j |
| $\Delta VOV_t^j$ | Weekly percentage change of volatility of volatility of spot rate for currency j |
| $\Delta COR_t^j$ | Weekly absolute change of spot and one month at the money correlation for currency j |
| $CRASH_t^j$ | Dummy variable receiving “1” if there has been an extreme negative daily return occurred in the last week days, otherwise“0” |
| $SPIKE_t^j$ | Dummy variable receiving “1” if there has been an extreme positive daily return occurred in the last week days, otherwise “0” |
| $\Delta NL_t^j$ | Weekly absolute change of net long position of larger speculators for currency j |
| $\Delta INTDF_t^j$ | Weekly percentage change of the interest rate differential between two currencies for currency pair j. |

### 3.4.4 Methodology of detecting structural breaks

To test our second hypothesis that under different market conditions there are different exogenous variables governing market concern about the future market condition, and therefore the variation of volatility surfaces, we employ a structural break test to examine the stability of the regression relationship between the variation of the volatility surface and possible explanatory variables. To detect the number of breaks and break dates in the regression relationship during our sample period, we employ the method of sequential estimation introduced by Bai and Perron (1998). This amount to first testing the null hypothesis of no structural break against the alternative hypothesis of a single break: if the null hypothesis is rejected, then a new null hypothesis of a single break versus an alternative hypothesis of two break points is tested. This procedure repeats until the test fails to reject the null hypothesis.

Under this method, to test the null hypothesis of no additional structural break, the break date must first be estimated by numerically finding a break date that minimises the sum of squared regression error (SSE). The significance of this break date is then judged by whether the amount of reduction in the SSE in the new model exceeds some critical value. This critical value at some different significant level is provided in Bai and Perron (2003). If the proposed break date is too near the beginning or end of the sample, the estimates and tests can be misleading; therefore, the candidate break dates are usually bounded within the region $[T_1, T_2]$,which are generally set as the top and bottom 15% (the trimming percentage) of the observation dates.

Since, in our regression setting we have two dummy variables that respectively capture spike and crash, we find that a low trimming percentage causes the issue of a singular matrix in the numerical procedure of finding break dates. We set the trimming percentage at a level that prevents this issue. In the case of AUD and JPY, the trimming percentage is set at 20%. For EUR, the trimming percentage is set at 25%. Since we have a relative small sample with 367 observations in terms of performing structural break test and large trimming percentages, we accept a maximum of two breaking dates in the numerical procedure to avoid misleading estimates, which may be raised from issues of perfect sub-sample fit because of tiny sub-sample sizes. To be consistent with the settings in the previous regression, we use the Newey–West estimator to correct for the presence of heteroskedasticity and serial correlation in the residual series. In addition, in finding the break dates, we allow the residual distribution to differ across the sub-samples.

# 4 Empirical results

## 4.1 Introduction

The previous chapter introduced the data and methodology for exploring the determinants of the evolution of the volatility surface. This chapter presents the results of this empirical analysis. It contains a description of the estimation results of volatilities from the factor model, and the full sample regression result analysis, structural break analysis, and sub-sample regression result analysis.

## 4.2 Estimation result

### 4.2.1 Principle component analysis

Table 4.1 demonstrates the proportion of variance explained by the four latent factors for AUD, EUR and JPY. This proportion is calculated from the eigenvalue of the corresponding eigenvector of the option volatility correlation matrix, divided by the total eigenvalue. For each currency during the sample period, factor 1 and factor 2 together explain approximately 90% of the variance of the volatilities; factor 1 alone explains around 80% of the variance. Factors3 and 4 together account for approximately 7% of the variance. The four factors collectively explain over 95% of the sample the volatility surface variation.

**Table 4-1The proportion of variance explained by the four factors for AUD, EUR and JPY**

| | AUD | | EUR | | JPY | |
|---|---|---|---|---|---|---|
| | Eigenvalue | Proportion | Eigenvalue | Proportion | Eigenvalue | Proportion |
| Factor1 | 48.21 | 74% | 52.56 | 81% | 52.24 | 80% |
| Factor2 | 8.46 | 13% | 5.54 | 9% | 5.79 | 9% |
| Factor3 | 3.48 | 5% | 2.38 | 4% | 3.12 | 5% |
| Factor4 | 2.05 | 3% | 1.66 | 3% | 1.37 | 2% |
| Total variance explained | | 96% | | 96% | | 96% |

Note: This table shows the proportion of variance of the variation of the volatility surface explained by the first four factors for AUD, EUR and JPY respectively. These factors are estimated by the asymptotic principle component approach, as shown inStock and Watson (2002).

### 4.2.2 Factor loading

Factor loadings represent the sensitivity of option volatilities to the corresponding latent factor. In the study of volatility in currency options, factor loadings are plotted against two dimensions, since volatilities can be described by moneyness and maturity. Figures 4.1, 4.2 and4.3 display the three-dimensional graph for the factor loadings of volatility dynamics in the AUD, EUR and JPY currency options. These factor loadings have not been rotated since the patterns of the original factor loadings across moneyness and maturity have already given a clear indication of the character of all four factors.

For the three currencies, factor 1 may be interpreted as a shock that affects volatilities for all maturity and moneyness, in the same sign and by approximately same size. This factor can therefore be referred to as parallel shift, meaning that any increase in this factor leads to a uniform increase in the level of the volatility surface across both maturity and moneyness.

For the three currencies, factor 2 may be interpreted as a shock that affects both the long maturity and short maturity volatilities, with opposite signs. In particular, the magnitude of the sensitivity of volatilities to these factors shows strong linear dependence on maturity for EUR and JPY, and less for AUD. We interpret this factor as the slope shift of the term structure of volatility.

Factor 3 (in the case of EUR and JPY) and Factor 4 (in the case of AUD) may be interpreted as the slope shift of the strike structure, meaning a positive shock in this factor increases the OTM call and decreases the OTM put volatilities. Inspection shows that long maturity volatilities are more sensitive to this factor than short maturity volatilities. For JPY, 25 Delta volatilities tend to be more sensitive to this factor than 10 Delta volatilities.

Factor 3 (in the case of AUD) and Factor 4 (in the case of EUR and JPY) may be interpreted as the curvature of the term structure of volatilities, meaning that any increase in this factor increases both long maturity and short maturity volatilities, but decreases medium maturity volatilities.

Over 95% of the variation in volatility surfaces takes the forms of a shift in level, a slope shift of the volatility strike structure, a slope shift of the volatility term structure, and a curvature shift of the volatility term structure. For all three currencies, the shift in level and the slope shift of the volatility term structure are the dominant form of variation; hence it is considered that the volatility surface dynamics for these three currencies are acting in a consistent way[8], they are acting in a consistent way in

[8] Visual inspection of Figure 4 indicates that the factor loading for Factor 2 appears different for the EUR/USD than the other currencies. In the absence of theoretical guidance, we cannot comment on this further.

a sense that the dominant forms of variation of all the three volatility surfaces are the level shift of volatility surface and slope shift of volatility strike structure.

The next section presents the summary statistics and regression result of factors on the related variables proposed in the previous chapter.

**Figure 4.1Estimated factor loading in the case of AUD**

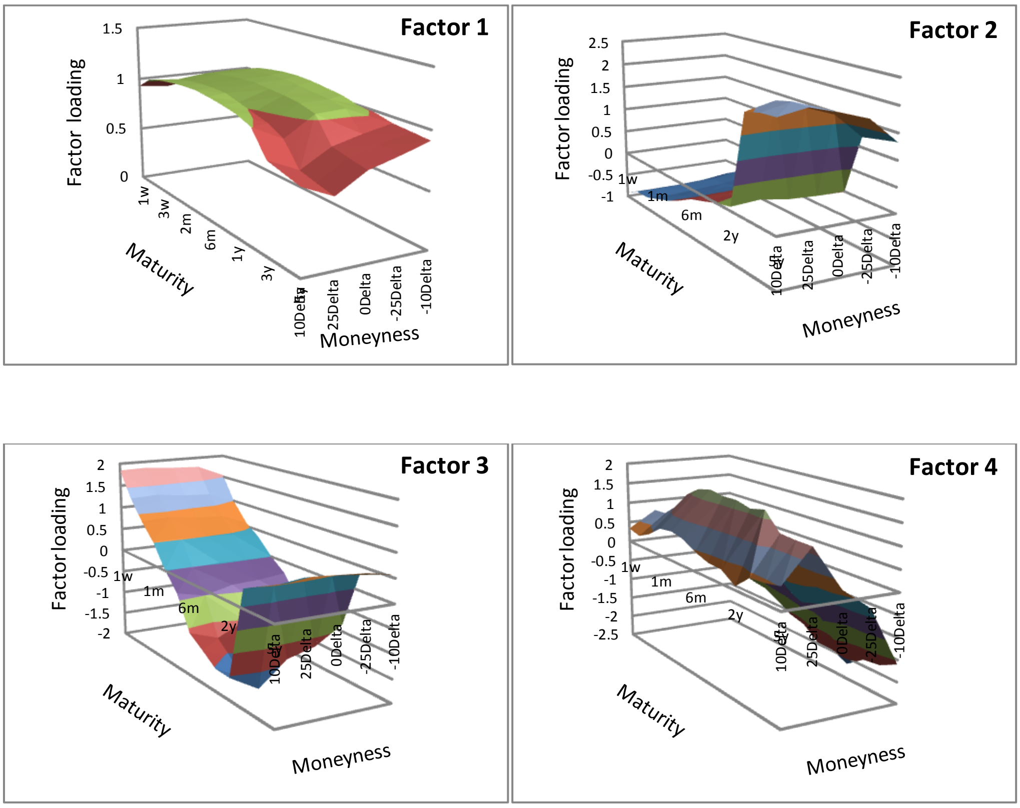
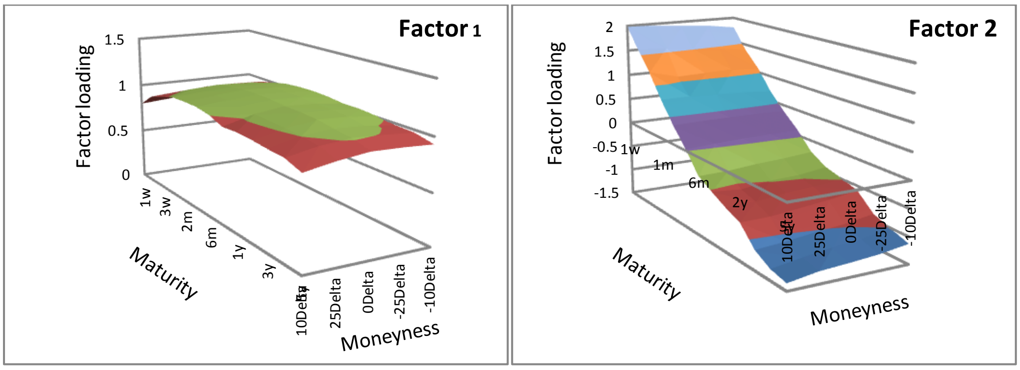



**Figure4.2Estimated factor loading in the case of EUR**



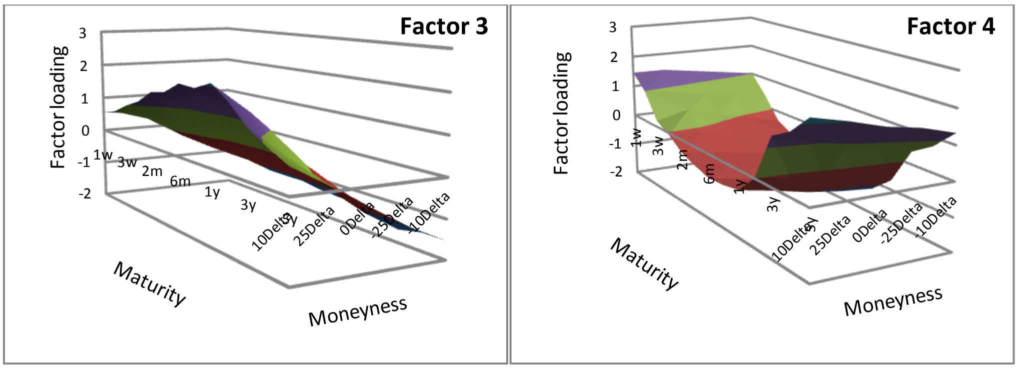
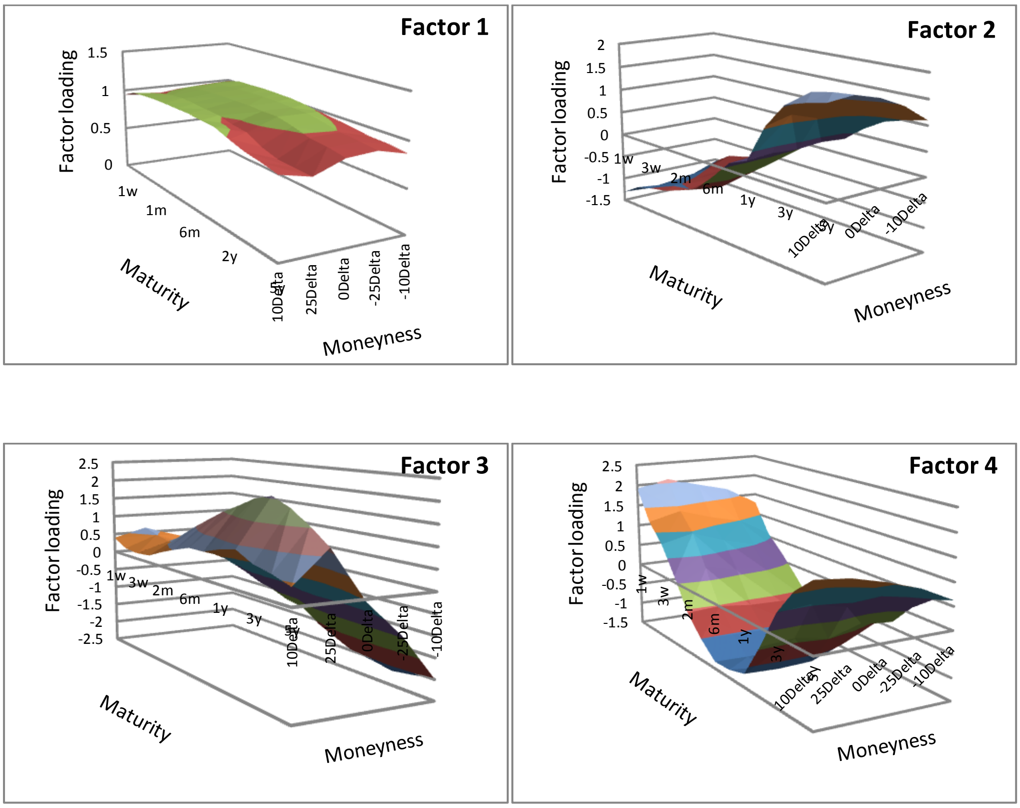



**Figure4.3Estimated factor loading in the case of JPY**




Note: Figures 4, 4.2 and 4.3 show the estimated factor loading of the first four factors in the case of AUD, EUR and JPY. These four factors come from the factor model that quantifies any variation of the volatility surface for AUD, EUR and JPY. The factor loadings are estimated using an asymptotic principle component approach, as shown in Stock and Watson (2002).Our sample period for the volatility surface data is from 27 June 2006 to 25 June 2013.

## 4.3 Regression result

### 4.3.1 Summary statistics and introduction to the regression result

Table 4.2 shows the summary statistics of the dependent and independent variables. Jarque–Bera statistics show these variables are all non-normally distributed, with significant excess kurtosis and skewness. The augmented Dicky Fuller test shows these variables are stationary, which allows one to conduct standard multivariate regression. Among the factors for all three currencies, standard deviation is the largest for factor1 and smallest for factors 3 or 4, which is consistent with factor estimation in an asymptomatic principle component setting, where the first few factors should explain most of the variation of the volatility surface.

In Tables 4.3, 4.4 and 4.5, Panel A reports the regression result for each of the four factors on the related explanatory variables for AUD, EUR and JPY. Since these factors represent four sources of variation in the volatility surface, we shall refer to them in the tables as the parallel shift, the slope shift of the volatility term structure, the slope shift of the volatility strike structure, and the curvature shift of the volatility term structure. Following Mixon (2002), the first-order moving average term (MA (1)) is included to correct for possible auto-correlation in the regression residual. To compare the explanatory power of individual explanatory variables to the variance of each of the four forms of the volatility surface variation, Panel B presents the marginal effect of one standard deviation of positive change in explanatory variables to the four factors.

Since Tables 4.3, 4.4 and 4.5 are not straightforward illustrations of how the volatility surface changes given a change in individual explanatory variables, to aid understanding and interpretation, Figure 4.4 describes the expected changes in the volatility surfaces conditional on one standard deviation positive change in the significant explanatory variables. We shall start our analysis with Tables 4.3, 4.4 and 4.5 to search for major determinants of the volatility surface, then move to Figure 4.4 to identify how a change in each individual major determinant variable shapes the volatility surface.

**Table4-2 Summary statistics of the dependent and independent variables**

| | Mean | Median | Maximum | Minimum | Std. Dev. | Skew | Kurtosis | Jarque-Bera | ADF |
|---|---|---|---|---|---|---|---|---|---|
| **Panel A Summary statistics (AUD)** | | | | | | | | | |
| Factor1 | 0.0130 | -0.0352 | 5.4111 | -4.6961 | 1.0433 | 0.9395 | 9.6103 | 722.169* | -21.05* |
| Factor2 | -0.0002 | 0.0064 | 3.0043 | -2.9154 | 0.4367 | -0.0432 | 16.7278 | 2881.847* | -25.13* |
| Factor3 | 0.0035 | -0.0010 | 3.0316 | -2.6924 | 0.4984 | -0.1461 | 12.6455 | 1423.988* | -23.553* |
| Factor4 | -0.0046 | 0.0007 | 1.2988 | -1.7091 | 0.2340 | -0.8521 | 16.1712 | 2697.219* | -18.371* |
| ΔSPOT | 0.0008 | 0.0020 | 0.0898 | -0.1092 | 0.0196 | -0.5056 | 7.1208 | 275.3013* | -18.375* |
| ΔRV | 0.0067 | 0.0006 | 0.5894 | -0.2819 | 0.1165 | 1.3673 | 7.5344 | 428.761* | -9.890* |
| ΔVOV | 0.0148 | -0.0052 | 1.8842 | -0.5746 | 0.1939 | 4.8247 | 43.7570 | 26825.3* | -8.904* |
| ΔCOR | -0.0004 | 0.0049 | 0.4867 | -0.4815 | 0.1223 | -0.0799 | 5.2527 | 77.988* | -19.028* |
| ΔNL | -0.0231 | 0.0923 | 6.1181 | -5.7354 | 1.2083 | -0.1748 | 6.1342 | 152.085* | -15.971* |
| ΔINTDF | 0.0000 | -0.0001 | 0.0062 | -0.0044 | 0.0014 | 0.6956 | 5.6049 | 133.362* | -15.023* |
| **Panel B Summary statistics (EUR)** | | | | | | | | | |
| Factor1 | 0.0028 | -0.0015 | 5.0725 | -3.0421 | 0.6948 | 0.9474 | 12.2316 | 1354.403* | -22.72* |
| Factor2 | 0.0004 | -0.0075 | 2.6132 | -1.9709 | 0.4485 | 0.2266 | 8.3312 | 436.566* | -17.3074* |
| Factor3 | -0.0034 | -0.0046 | 0.7444 | -0.5786 | 0.1348 | 0.4228 | 8.8487 | 532.572* | -19.371* |
| Factor4 | -0.0001 | 0.0050 | 0.6149 | -0.8485 | 0.1510 | -0.4511 | 6.9238 | 247.204* | -13.564* |
| ΔSPOT | 0.0002 | 0.0012 | 0.0832 | -0.0408 | 0.0152 | 0.1803 | 4.8633 | 54.931* | -18.520* |
| ΔRV | 0.0023 | -0.0024 | 0.2854 | -0.2339 | 0.0762 | 0.2862 | 3.8197 | 15.244* | -8.377* |
| ΔVOV | 0.0100 | -0.0020 | 0.8236 | -0.3661 | 0.1456 | 1.6171 | 10.0816 | 924.277* | -9.887* |
| ΔCOR | -0.0013 | -0.0037 | 0.4900 | -0.5652 | 0.1264 | 0.1975 | 5.0970 | 69.437* | -13.475* |
| ΔNL | -0.0110 | 0.0553 | 4.0028 | -4.1563 | 1.3886 | 0.1148 | 3.3974 | 3.202* | -19.353* |
| ΔINTDF | 0.0001 | 0.0001 | 0.0052 | -0.0094 | 0.0013 | -1.6954 | 16.5957 | 2994.194* | -12.084* |
| **Panel C Summary statistics (JPY)** | | | | | | | | | |
| Factor1 | 0.0160 | -0.0492 | 10.3495 | -5.9276 | 1.0500 | 2.4774 | 31.3893 | 12699.72* | -26.340* |
| Factor2 | -0.0011 | 0.0189 | 3.9432 | -5.2306 | 0.5705 | -1.3493 | 27.9383 | 9621.547* | -28.193* |
| Factor3 | 0.0023 | 0.0100 | 1.0172 | -1.5385 | 0.2502 | -1.3768 | 13.2426 | 1720.208* | -10.327* |
| Factor4 | 0.0020 | -0.0007 | 3.0957 | -2.2936 | 0.3567 | 0.9100 | 23.1570 | 6263.724* | -19.242* |
| ΔSPOT | -0.0005 | -0.0006 | 0.0580 | -0.0467 | 0.0143 | 0.0754 | 4.2299 | 23.477* | -19.253* |
| ΔRV | 0.0103 | -0.0020 | 0.9215 | -0.5502 | 0.1371 | 1.1372 | 10.5720 | 955.843* | -12.087* |
| ΔVOV | 0.0188 | -0.0051 | 1.6027 | -0.6480 | 0.2164 | 2.9895 | 18.6187 | 4276.954* | -9.296* |
| ΔCOR | 0.0004 | 0.0008 | 0.4965 | -0.9998 | 0.1406 | -0.8461 | 10.4824 | 899.908* | -12.532* |
| ΔNL | -0.0198 | 0.0104 | 6.7878 | -8.6920 | 1.6568 | 0.0613 | 6.4797 | 185.388* | -18.263* |
| ΔINTDF | 0.0001 | 0.0001 | 0.0071 | -0.0068 | 0.0012 | 0.4989 | 11.7645 | 1189.877* | -17.700* |

Note: Factors 1, 2, 3, and 4 in panels A, B, and C are factors that summarise changes in the volatility surface of AUD, EUR and JPY. ΔSPOT is the weekly return of the spot rate. ΔRV is the weekly percentage change of the realised volatility. ΔVOV is the weekly percentage change of the volatility of one-month zero Delta straddle quotes. ΔCOR is the absolute change in the correlation between the spot rate and the one-month zero Delta straddle quotes. ΔNL is the absolute weekly change in the net long position in AUD currency by large speculators, divided by 10,000 for comparison purposes. ΔINTDF is the weekly percentage change in interest rate differential, calculated using the one-year forward rate divided by spot rate. It can be understood as the weekly percentage change in the ratio of domestic to foreign interest rate. The Jarque–Bera and ADF test values are also reported (an intercept has been included in the test equation). * denotes rejection of the null hypothesis at the 1% level. The null hypothesis for the Jarque–Bera and ADF tests is that the series is normally distributed and has a unit root.

### 4.3.2 Regression results of factors on the related explanatory variables

Table 4.3 shows that for AUD, spot return and realised volatility are the only variables that show significance. Panel B shows that the magnitude of the marginal effect of the spot return is about twice as large as realised volatility, meaning the volatility surface in AUD is particularly sensitive to spot rate changes. It is worth noting that there is no significant evidence to show that jumps in spot return are related to the variation of volatility surface, which contradicts Low (2004) who argues that option market risk perception is closely related to extreme events.

**Table 4-3 Regression result of factors on the explanatory variables in AUD**

| In the case of AUD | | | | |
|---|---|---|---|---|
| Panel A Regression result | The parallel shift | The slope shift of volatility term structure | The slope shift of volatility strike structure | The curvature shift of volatility term structure |
| Intercept | 0.0080 | -0.0189 | 0.0307 | -0.0041 |
| | [0.218] | [-1.336] | [1.785] | [-0.511] |
| ΔSPOT | -20.5407 | 4.8329 | -5.9672 | 4.6734 |
| | [-3.659]** | [2.996]** | [-2.342]* | [4.605]** |
| ΔRV | 1.9866 | -0.7936 | 1.2280 | -0.0739 |
| | [2.771]** | [-2.222]* | [2.544]* | [-0.553] |
| ΔVOV | 0.0524 | 0.0731 | -0.0694 | -0.1229 |
| | [0.131] | [0.667] | [-0.438] | [-1.207] |
| ΔCOR | 0.1224 | -0.0226 | 0.0825 | 0.1239 |
| | [0.242] | [-0.110] | [0.302] | [0.916] |
| SPIKE | -0.3313 | 0.1205 | -0.2191 | 0.11 |
| | [-1.279] | [1.079] | [-1.509] | [1.515] |
| CRASH | 0.3663 | 0.0974 | -0.1173 | -0.1176 |
| | [1.289] | [0.928] | [-0.785] | [-1.574] |
| ΔNL | -0.0249 | 0.0294 | 0.0097 | -0.0071 |
| | [-0.386] | [1.030] | [0.302] | [-0.517] |
| ΔINTDF | 31.8051 | -10.5764 | 9.7767 | -7.3206 |
| | [0.795] | [-0.531] | [0.360] | [-0.541] |
| MA(1) | -0.1724 | -0.3017 | -0.2743 | -0.0109 |
| | [-2.096]** | [-3.811]** | [-3.053]** | [-0.080] |
| Observations: | 367 | 367 | 367 | 367 |
| R-squared: | 0.3950 | 0.2430 | 0.2270 | 0.326 |
| F-statistic: | 25.9400 | 12.7100 | 11.6200 | 19.167 |
| Probability (F-stat): | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Panel B standardised marginal effect of significant variables | | | | |
| ΔSPOT | -0.4186 | 0.0985 | -0.1216 | 0.0952 |
| ΔRV | 0.2447 | -0.0977 | 0.1513 | |
| ΔVOV | | | | |
| ΔCOR | | | | |
| ΔNL | | | | |
| ΔINTDF | | | | |

Note: ΔSPOT is the weekly return of the AUD. ΔRV is the weekly percentage change of the realised volatility. ΔVOV is the weekly percentage change of the volatility of one-month zero Delta straddle quotes. ΔCOR is the absolute change in the correlation between the spot rate and the one-month zero Delta straddle quotes. SPIKE is the AUD experienced extremely positive daily return in the past five trading days. CRASH is the AUD experienced extremely negative daily return in the past five trading days. The extreme negative daily return is defined as two standard deviations of daily spot rate return in the sample period. ΔNL is the absolute weekly change in the net long position in the AUD currency by large speculators, which is divided by 10,000 for comparison purposes. ΔINTDF is the weekly percentage change in interest rate differential, calculated by using the one- year forward rate divided by spot rate. It can be understood as the weekly percentage change in the ratio of domestic interest rate to foreign interest rate. The dependent variables are the four factors that summarise the dynamics of the volatility surface. MA(1) refers to the moving average correction of order 1. The standardised marginal effect of significant variable is calculated as the coefficient times one standard deviation of positive change in that variable. The Newey–West estimator is used to construct the variance-covariance matrix of the residual term. Sample period is from 27th June 2006 to 25th June 2013. One and two asterisks denote rejection of the null hypothesis of a zero coefficient at the 1% and 5% level, respectively.

In EUR, we find volatility of volatility and interest rate differentials play a major role in explaining the parallel shift and the slope shift of the volatility term structure. It is worth noting that spot return is not significantly related to the parallel shift in the volatility surface; however, it is significantly related to the slope shift of the volatility strike structure. This implies that the spot return affects the OTM call volatility and OTM put volatility in different signs. Although some significance shows up on spot-ATM volatility correlation and net long position, the small standardised marginal effect shows these cannot be considered major determinants. As with AUD, we find no jumps affecting the volatility surface in EUR.

**Table 4-4 Regression result of factors on the explanatory variables in EUR**

| In the case of EUR | | | | |
|---|---|---|---|---|
| Panel A Regression result | The parallel shift | The slope shift of volatility term structure | The slope shift of volatility strike structure | The curvature shift of volatility term structure |
| Intercept | -0.0412 | -0.0176 | -0.0005 | 0.001 |
| | [-1.885] | [-1.266] | [-0.068] | [0.193] |
| ΔSPOT | -3.0428 | -0.6398 | 5.0445 | 1.049 |
| | [-0.604] | [-0.315] | [6.626]** | [1.321] |
| ΔRV | 0.4383 | 0.2701 | 0.0957 | 0.0481 |
| | [0.886] | [1.023] | [1.065] | [0.472] |
| ΔVOV | 1.1770 | 0.4540 | -0.0970 | -0.0554 |
| | [4.768]** | [3.061]** | [-1.920] | [-1.235] |
| ΔCOR | 0.3525 | 0.0679 | 0.1076 | -0.0791 |
| | [1.318] | [0.408] | [2.385]* | [-1.261] |
| SPIKE | 0.0572 | -0.0256 | -0.0184 | -0.0069 |
| | [0.406] | [-0.229] | [-1.003] | [-0.258] |
| CRASH | 0.2171 | 0.1223 | -0.0153 | -0.0005 |
| | [1.794] | [1.878] | [-0.760] | [-0.026] |
| ΔNL | 0.0008 | 0.0079 | -0.0118 | 0.0087 |
| | [0.022] | [0.463] | [-2.002]* | [1.636] |
| ΔINTDF | -117.4362 | -36.7124 | 16.0713 | -0.0996 |
| | [-3.432]** | [-1.317] | [1.864] | [-0.008] |
| MA(1) | -0.2833 | -0.3671 | 0.0783 | -0.3139 |
| | [-3.953]** | [-5.217]** | [0.909] | [-5.429]** |
| Observations: | 367 | 367 | 367 | 367 |
| R-squared: | 0.2560 | 0.1850 | 0.4020 | 0.098 |
| F-statistic: | 13.5750 | 8.9800 | 26.5600 | 4.308 |
| Probability (F-stat): | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Panel B standardised marginal effect of significant variables | | | | |
| ΔSPOT | | | 0.0777 | |
| ΔRV | | | | |
| ΔVOV | 0.1831 | 0.0706 | | |
| ΔCOR | | | 0.0135 | |
| ΔNL | | | -0.0163 | |
| ΔINTDF | -0.1567 | | | |

Note: ΔSPOT is the weekly return of the EUR. ΔRV is the weekly percentage change of the realised volatility. ΔVOV is the weekly percentage change of the volatility of one-month zero Delta straddle quotes. ΔCOR is the absolute change in the correlation between the spot rate and the one-month zero Delta straddle quotes. SPIKE is the EUR experienced extremely positive daily return in the past five trading days. CRASH is the EUR experienced extremely negative daily return in the past five trading days. The extreme negative daily return is defined as two standard deviations of the daily spot rate return in the sample period. ΔNL is the absolute weekly change in the net long position in the EUR currency by large speculators, which is divided by 10,000 for comparison purposes. ΔINTDF is the weekly percentage change in interest rate differential, calculated by using a one-year forward rate divided by spot rate. It can be understood as a weekly percentage change in the ratio of domestic interest rate to foreign interest rate. The dependent variables are the four factors that summarise the dynamics of the volatility surface. The standardised marginal effect of each significant variable is calculated as the coefficient times one standard deviation of positive change in that variable. The Newey–West estimator is used to construct the variance–covariance matrix of the residual term. MA(1) refers to the moving average correction of order 1. The sample period is from 27 June 2006 to 25 June 2013. One and two asterisks denote rejection of the null hypothesis of a zero coefficient at the 1% and 5% level, respectively.

As shown in Table 4.5, unlike AUD and EUR, for JPY negative jumps and net long position (to measure market sentiment) significantly contribute to the variation of volatility surface. Crash possess the largest standardised marginal effect (about three times larger than other significant variables) to the parallel shift and to the slope shift of volatility strike structure, which indicates that large and rapid appreciations in the value of JPY relative to USD induce large changes in the volatility surface. Spot return, realised volatility and volatility of volatility also show significant contributions to the variation of volatility surface used, and share similar standardised marginal effects as net long position.

Before we leave this sub-section, we should note that a highly significant auto-correlation residual is also observed in the regression result. This suggests that there may be a potential risk of omitted variables and model misspecification in our regression setting. [9] This auto-correlation feature seems to be embedded in the variation of the volatility surface, since the same feature is observed in other studies of the variation of volatility surfaces, such as those by Chalamandaris and Tsekrekos (2013), Goncalves and Guidolin (2006), and Mixon (2002)[10]. In the next section we shall have a close look at how each individual major determinant variable identified in this sub-section affects the volatility surface.

[9]We considered including demand and supply and liquidity variable in our analysis to rich our model. However, since we are analysing the over-the-counter option data, the data for market frictions such as bid and ask spread and demand and supply is very difficult to obtain. The bid and ask spread data for OTC options that Bloomberg provide is not the real inter-bank data rather it is calculated from a certain model, so using it may come to an unreliable result.

[10]One robustness test we tried, but did not report in the dissertation, was to add polynomial terms for all explanatory variables as addition of the existing specification to explore the possible nonlinear relationship between explanatory variable and volatility surfaces. This did not change the inferences made in the dissertation

**Table 4-5 Regression result of factors on the explanatory variables in JPY**

| In the case of JPY | | | | |
|---|---|---|---|---|
| Panel A Regression result | The parallel shift | The slope shift of volatility term structure | The slope shift of volatility strike structure | The curvature shift of volatility term structure |
| Intercept | -0.0465 | 0.0086 | 0.0273 | 0.0059 |
| | [-1.498] | [0.580] | [2.595]** | [0.669] |
| ΔSPOT | 9.0179 | -2.3099 | -0.6573 | 0.3897 |
| | [2.818]** | [-1.353] | [-0.737] | [0.367] |
| ΔRV | 1.2325 | -0.8805 | -0.0571 | 0.3756 |
| | [3.062]** | [-4.024]** | [-0.548] | [2.755]** |
| ΔVOV | 0.6135 | -0.1965 | -0.2198 | 0.0794 |
| | [2.426]* | [-1.358] | [-2.783]** | [0.782] |
| ΔCOR | -0.3461 | 0.1937 | 0.1411 | -0.1246 |
| | [-1.195] | [1.299] | [1.610] | [-1.201] |
| SPIKE | -0.1441 | 0.0941 | 0.0704 | 0.0063 |
| | [-0.757] | [1.078] | [1.072] | [0.097] |
| CRASH | 0.5912 | -0.0912 | -0.2766 | -0.0722 |
| | [2.680]** | [-0.821] | [-4.666]** | [-0.736] |
| ΔNL | 0.0880 | -0.0347 | -0.0278 | 0.0267 |
| | [3.828]** | [-2.865]** | [-4.168]** | [3.949]** |
| ΔINTDF | -18.4087 | -1.3492 | 0.1159 | -8.1259 |
| | [-0.365] | [-0.049] | [0.008] | [-0.513] |
| MA(1) | -0.3984 | -0.5000 | 0.0432 | -0.5146 |
| | [-3.470]** | [-4.775]** | [0.658] | [-5.521]** |
| Observations: | 367 | 367 | 367 | 367 |
| R-squared: | 0.3060 | 0.2910 | 0.2790 | 0.247 |
| F-statistic: | 17.4740 | 16.2740 | 15.3510 | 12.999 |
| Probability(F-stat): | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Panel B standardised marginal effect of significant variables | | | | |
| ΔSPOT | 0.1249 | | | |
| ΔRV | 0.1817 | -0.1298 | | 0.0554 |
| ΔVOV | 0.1443 | | -0.0517 | 0.0187 |
| ΔCOR | | | | |
| ΔNL | 0.1441 | -0.0568 | -0.0455 | 0.0437 |
| ΔINTDF | | | | |

Note: ΔSPOT is the weekly return of the JPY. ΔRV is the weekly percentage change of the realised volatility. ΔVOV is the weekly percentage change of the volatility of one-month zero Delta straddle quotes. ΔCOR is the absolute change in the correlation between the spot rate and the one-month zero Delta straddle quotes. SPIKE is the JPY experienced extremely positive daily return in the past five trading days. CRASH is the JPY experienced extremely negative daily return in the past five trading days. The extreme negative daily return is defined as two standard deviations of the daily spot rate return in the sample period. ΔNL is the absolute weekly change in the net long position in the JPY currency by large speculators, which is divided by 10,000 for comparison purposes. ΔINTDF is the weekly percentage change in interest rate differential, calculated by using the one-year forward rate divided by spot rate. It can be understood as a weekly percentage change in the ratio of domestic interest rate to foreign interest rate. MA(1) refers to the moving average correction of order 1. The dependent variables are the four factors that summarise the dynamics of the volatility surface. The standardised marginal effect of the significant variable is calculated as the coefficient times one standard deviation of positive change in that variable. The Newey–West estimator is used to construct the variance–covariance matrix of the residual term. Sample period is from 27 June 2006 to 25 June 2013. One and two asterisks denote rejection of the null hypothesis of a zero coefficient at the 1% and 5% level, respectively.

### 4.3.3 Visualising expected change in the volatility surface given a change in its major determinant

Although the previous section identified the significant relationship between the factors and the explanatory variables, the results are not straightforward enough to illustrate how these significant explanatory variables affect the shape of the volatility surface. Figure 4.5 demonstrates the expected change in the shape of the volatility surfaces conditioned on one standard deviation positive change[11] in its major determinants among the three currencies. These determinants are identified from the previous sub-section.

Several observations can be drawn from the table. A positive change in the spot rate shifts the volatility surface downwards in AUD. It has the largest effect centred on OTM put volatilities and option volatilities for a short maturity option, and no significant effect on option volatilities for which the option has maturity over three years. In EUR, unlike in AUD, we find that a positive change in spot rate increases the OTM call volatilities but decreases the OTM put volatilities. It has the largest effect for option volatilities far out of the money. This confirms the previous observation that the spot rate in EUR significantly contributes to the change of the slope of volatility strike structure, showing that the volatility surface tends to be less negatively skewed after a positive change in the spot rate. In JPY, we show that a positive change in the spot rate raises the volatility surface in a parallel manner, which is quite different to the case of AUD and EUR. However, combined with the

[11] Since change in volatilities is modelled as a linear function of these explanatory variables, the marginal effect of negative change in these variables is a mirror image of positive change around horizontal axis.

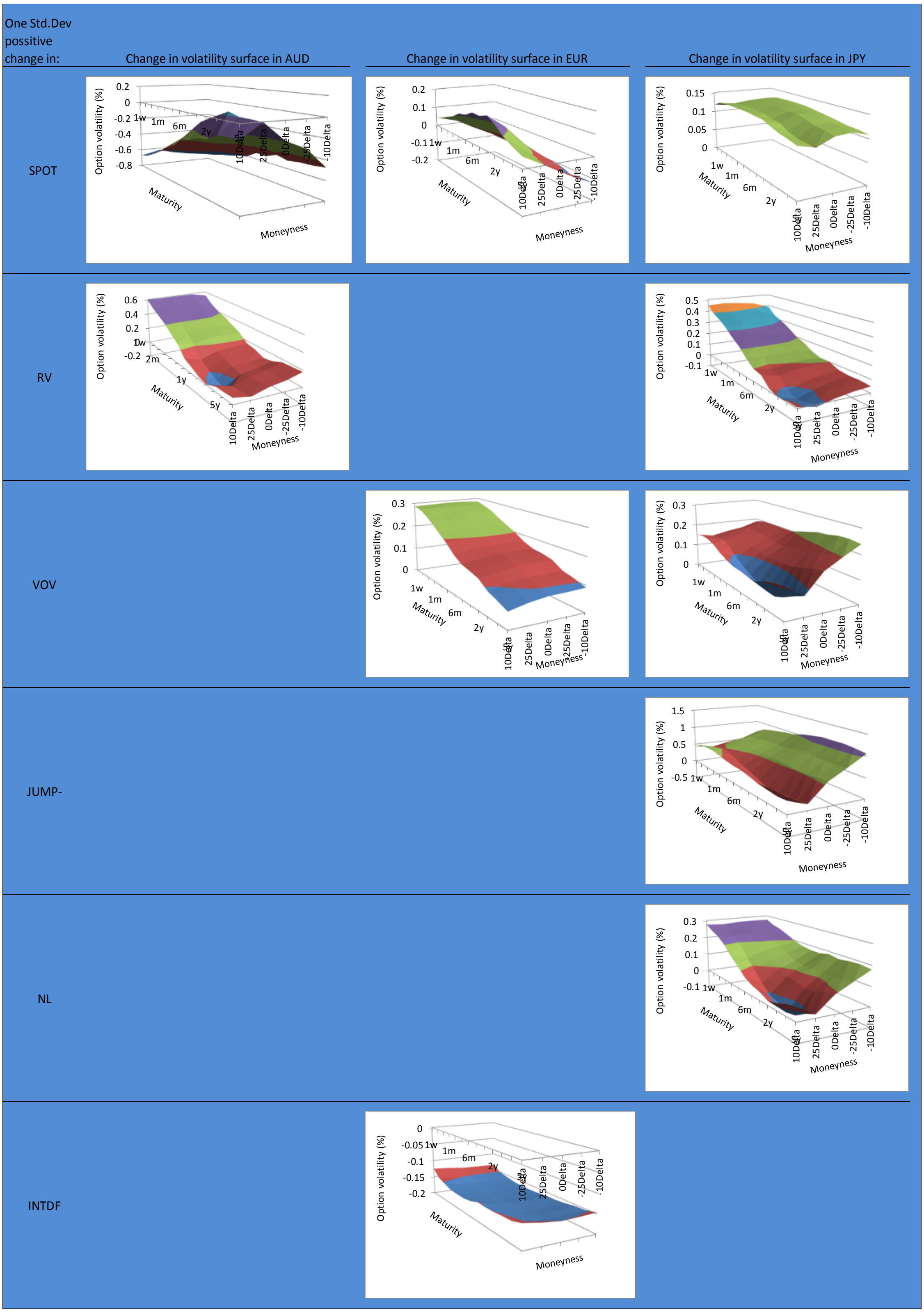
negative relationship between the spot rate and the level of the volatility surface in AUD, this suggests that any appreciation in the value of USD relative to AUD and JPY increases the level of the volatility surface in AUD and JPY. Moreover, a positive change in realised volatility raises the volatility surface while the size of its marginal effect dampens linearly with an increase in maturity in AUD; it has no significant effect on option volatilities for which the option has maturity over than 3 year. Such realised volatility effect on the volatility surface in JPY is quite similar to that of AUD, although that we do not find statistical evidence that realised volatility has an effect on the volatility surface in EUR. The reason for this could be that the effect of realised volatility has been diluted by the volatility of volatility, as we do observe a higher degree of correlation between realised volatility and volatility of volatility.

The change in volatility of volatility does not statistically significantly contribute to the variation of option volatilities in AUD. However, it is positively related to the level and slope of term structure of the volatility surface in EUR. Notably, the magnitude of its marginal effect is largest for the short-term maturities compared with other variables, and dampens linearly across maturity. This implies that for EUR the volatility surface is particularly sensitive to the stability of the volatility surface itself, and changes to its stability results in a more inverted volatility term structure. In JPY, a positive change in the volatility of volatility also raises the level of the volatility surface, although unlike EUR the magnitude of the marginal effect shows uneven distribution across moneyness, being larger for OTM put volatility than for OTM call volatilities.

Market sentiment and crash increase the volatilities only in JPY. Their marginal effects are centred on OTM put volatilities rather than OTM call volatilities and their effect on option volatilities do not seem to differ significantly across maturities. In particular, their strong positive effect on the OTM put volatility for a positive change in market sentiment suggests that when the market is bullish JPY, we will see a more negatively skewed volatility surface. This finding in JPY is consistent with that of Han (2008), who finds that when the market is bullish (bearish) in the S&P index, the volatility smile for S&P 500 options tends to have a positive (negative) skew.

Finally, interest rate differential tends to shift the volatility surface downward in a parallel manner only in EUR, which suggests an increase in interest rate in USD relative to EUR systematically decreases volatilities in the EUR currency options.

**Figure 4.4Visualising the expected change in volatility surfaces with one standard deviation change in significant explanatory variables**




Note: this figure shows the expected change in the volatility surface given one standard deviation positive change in the significant explanatory variables, assuming the volatility surface is flat at the beginning. Readers may refer to Section 3.4.2for a full explanation of these variables.

Of the full sample regression result analysis, based on the factor analysis, we found that over 95% of the variation in the volatility surfaces can be described as the parallel shift, the slope shift of the volatility term structure, the slope shift of the volatility strike structure and the curvature shift of volatility term structure. Based on the regression result of these four factors on the related variables, in general we can conclude that:

(1). Variables affect the volatility surface, but not uniformly in size or significance, across currencies. Therefore, explaining the evolution of the volatility surface requires the recognition of the currency-specific characteristics. This finding echoes found in Chalamandaris and Tsekrekos (2013) who also showed that some country's volatility surface tends to be more correlated with the momentum of exchange rate while others tend to be more with real effective exchange rate.

(2). Spot rate return is not a domain variable that affects the volatility surface. In fact, we have statistical evidence to show that the realised volatility (in the case of AUD and JPY), stability of the volatility surface itself (in the case of EUR and JPY), and market sentiment (in the case of JPY) show dominant effects in moving the volatility surface.

(3). The second major variation, the slope shift of the volatility term structure, can be attributed to the fact that long-term volatility is less sensitive to changes in market conditions (realised volatility and stability of the volatility surface in particular) than relatively short-term volatility.

(4). In general, the OTM put option volatility tends to be more sensitive to the state variable than the OTM call option volatility; and short maturity option volatility is more sensitive to the state variable than long maturity option volatility.

## 4.4 Structural break analysis and sub-sample regression

### 4.4.1 Introduction

The previous section analysed the relationships between the variation of volatility surfaces and a set of proposed explanatory variables, by implementing a full sample multivariate regression. This chapter presents the results of the stability tests of the estimated regression coefficients, conducted by structural-break tests and subsample regression analysis.

### 4.4.2 Break dates analysis

Table 4.5 shows the results of a structural break test on each of the regressions on the factors which quantify volatility surface variation. We can see that the structural break in the relationship between the variation of the volatility surface and the proposed explanatory variable happens in all three sample currencies. Among the currencies, we find that the first structural break dates occurs roughly around November 2008, at the peak of the GFC; the second structural change occurs roughly one to two years after the GFC, during the recovery phrase from GFC. By looking closely, we see that unlike AUD and JPY, the first break in EUR in terms of modelling the parallel shift of the volatility surface happens on11September 2009, which coincides with the beginning of the Greek sovereign debt crisis in late 2009. This pattern of detected structural breaks clearly supports our expectation that the sensitivity of the volatility surface to the proposed explanatory variables changes with changes in market conditions. The next section presents the results of a sub-sample regression which incorporates the effect of the identified structural break.

**Table 4-6 Results of structural break test**

| Structural break test in modelling: | Break dates | Break Test | F-statistic | Scaled F-statistic | Critical Value** | Trimming percentage |
|---|---|---|---|---|---|---|
| Panel A in the case of AUD | | | | | | |
| Parallel shift | 5/17/2010 | 0 vs. 1 * | 3.4235 | 30.8116 | 24.91 | 20% |
| | 11/24/2008 | 1 vs. 2 * | 6.7374 | 60.6367 | 26.92 | |
| Slope of volatility term structure | | 0 vs. 1 | 1.0919 | 9.8277 | 24.91 | 20% |
| Slope of volatility strike structure | | 0 vs. 1 | 2.5890 | 23.3011 | 24.91 | 20% |
| Curvature of volatility term structure | | 0 vs. 1 | 1.4922 | 13.4303 | 24.91 | 20% |
| Panel B in the case of EUR | | | | | | |
| Parallel shift | 11/09/2009 | 0 vs. 1 * | 4.1280 | 37.1526 | 24.18 | 25% |
| | | 1 vs. 2 | 1.6822 | 15.1398 | 26.28 | |
| Slope of volatility term structure | 11/10/2008 | 0 vs. 1 * | 3.4117 | 30.7059 | 24.18 | 25% |
| | | 1 vs. 2 | 2.3707 | 21.3368 | 26.28 | |
| Slope of volatility strike structure | 11/30/2009 | 0 vs. 1 * | 3.2177 | 28.9601 | 24.18 | 25% |
| | | 1 vs. 2 | 1.2785 | 11.5067 | 26.28 | |
| Curvature of volatility term structure | 11/17/2008 | 0 vs. 1 * | 3.3262 | 29.9358 | 24.18 | 25% |
| | | 1 vs. 2 | 1.7705 | 15.9349 | 26.28 | |
| Panel C in the case of JPY | | | | | | |
| Parallel shift | 10/13/2008 | 0 vs. 1 * | 4.5375 | 40.8379 | 24.91 | 20% |
| | 3/21/2011 | 1 vs. 2 * | 4.0327 | 36.2951 | 26.92 | |
| Slope of volatility term structure | | 0 vs. 1 | 2.0400 | 18.3608 | 24.91 | 20% |
| Slope of volatility strike structure | 11/24/2008 | 0 vs. 1 * | 3.2674 | 29.4073 | 24.91 | 20% |
| | 7/19/2010 | 1 vs. 2 * | 3.3338 | 30.0045 | 26.92 | |
| Curvature of volatility term structure | | 0 vs. 1 | 2.1322 | 19.1901 | 24.91 | 20% |

Note: The break dates testing method is the sequential estimation approach introduced by Bai and Perron (1998). The F-statistics are scaled by the number of time-varying regressors. The critical value of the scaled F-statistics is at five percent significance level. The trimming percentage restricts the regime where the break date occurs.

### 4.4.3 Sub-sample regression analysis

Table 4.7 shows the regression results incorporated with the identified structural break in the case of AUD. The most important finding here is that, contrary to the full sample regression, the sub-sample regression shows that the level of the volatility surface is sensitive to the market sentiment of AUD in certain periods. Specifically, volatility tended to go up (down) when the market was bullish (bearish) AUD during the period 16 June 2006 to 17 November 2008; however, volatility tended to go down (up) when the market was bullish (bearish) AUD from 24 November 2008 to 19 April 2010. This may be because, during the GFC period, investors were facing a highly complex investing environment caused by the US sub-prime debt crisis; any increased bearish (bullish) sentiment to USD (AUD) induced a fear of further market crashes in the world's largest capital market, USA. This bad sentiment was transmitted to the currency option market as an increase in option volatility. However, during the period of recovery from the GFC, increased bearish (bullish) sentiment to USD (AUD) no longer represented a fear of further capital market crashes in the US; rather, it confirmed the market expectation that the Australian economy would show stable growth and the US government would launch a quantitative easing policy; the negative sentiment to the USD is therefore signalling the effectiveness of government policy and a reflection of Australia's economic fundamental stability; and consequently, option volatility decreases. Note that during the recovery period, the market sentiment works as a dominant determinant of the parallel shift of the volatility surface. This seems to suggest that the variation of volatility in the AUD currency option in this sensitive period was largely determined by whether the markets believed in the efficacy of US government monetary policy and Australia's economic stability.

## Table 4-7 Regression result incorporated with the identified structural break in the case of AUD

| In the case of AUD | | | | | | |
|---|---|---|---|---|---|---|
| Panel A Regression result | | | | | | |
| | Parallel shift | | | Slope shift of volatility term structure | Slope shift of volatility strike structure | Curvature shift of volatility term structure |
| | 27 June 2006–17 November 2008 | 24 November 2008–19 April 2010 | 26 April 2010–25 June 2013 | 27 June 2006–25 June 2013 | 27 June 2006–25 June 2013 | 27 June 2006–25 June 2013 |
| Intercept | 0.0674 | -0.1213 | -0.0302 | -0.0189 | 0.0307 | -0.0041 |
| | [1.226] | [-1.162] | [-0.672] | [-1.336] | [1.785] | [-0.511] |
| ΔSPOT | -27.9799 | 9.0086 | -29.2844 | 4.8329 | -5.9672 | 4.6734 |
| | [-5.135]** | [1.915] | [-4.325]** | [2.996]** | [-2.342]* | [4.605]** |
| ΔRV | 1.8614 | 2.9680 | 1.3157 | -0.7936 | 1.2280 | -0.0739 |
| | [1.536] | [1.906] | [2.238]* | [-2.222]* | [2.544]* | [-0.553] |
| ΔVOV | 0.0427 | -0.8448 | -0.3197 | 0.0731 | -0.0694 | -0.1229 |
| | [0.115] | [-0.677] | [-0.827] | [0.667] | [-0.438] | [-1.207] |
| ΔCOR | -0.3339 | 0.4960 | -0.0679 | -0.0226 | 0.0825 | 0.1239 |
| | [-0.556] | [0.610] | [-0.079] | [-0.110] | [0.302] | [0.916] |
| SPIKE | -0.3313 | -0.5469 | -0.5408 | 0.1205 | -0.2191 | 0.11 |
| | [-0.692] | [-1.792] | [-1.112] | [1.079] | [-1.509] | [1.515] |
| CRASH | 0.5221 | -0.1206 | 1.3433 | 0.0974 | -0.1173 | -0.1176 |
| | [1.401] | [-0.365] | [2.510]* | [0.928] | [-0.785] | [-1.574] |
| ΔNL | 0.1729 | -0.4571 | -0.0489 | 0.0294 | 0.0097 | -0.0071 |
| | [2.945]** | [-2.661]** | [-0.878] | [1.030] | [0.302] | [-0.517] |
| ΔINTDF | 22.3971 | -30.0732 | 70.7000 | -10.5764 | 9.7767 | -7.3206 |
| | [0.409] | [-0.377] | [0.933] | [-0.531] | [0.360] | [-0.541] |
| MA(1) | -0.2116 | 0.1460 | -0.1220 | -0.3017 | -0.2743 | -0.0109 |
| | [-1.072] | [0.888] | [-1.293] | [-3.811]** | [-3.053]** | [-0.080]** |
| Observations: | 127 | 74 | 166 | 367 | 367 | 367 |
| R-squared: | 0.535 | 0.163 | 0.571 | 0.243 | 0.227 | 0.326 |
| F-statistic: | 16.869 | 2.580 | 25.409 | 12.710 | 11.620 | 19.167 |
| Prob(F-stat): | 0.000 | 0.013 | 0.000 | 0.000 | 0.000 | 0.000 |
| Panel B standardised marginal effect of significant variables | | | | | | |
| ΔSPOT | -0.5703 | | -0.5968 | 0.0985 | -0.1216 | 0.0952 |
| ΔRV | | | 0.1621 | -0.0977 | 0.1513 | |
| ΔVOV | | | | | | |
| ΔCOR | | | | | | |
| ΔNL | 0.2049 | -0.5418 | | | | |
| ΔINTDF | | | | | | |

Note: ΔSPOT is the weekly return of the AUD. ΔRV is the weekly percentage change of the realised volatility. ΔVOV is the weekly percentage change of the volatility of one-month zero Delta straddle quotes. ΔCOR is the absolute change in the correlation between the spot rate and the one-month zero Delta straddle quotes. SPIKE is the AUD experienced extremely positive daily return in the past five trading days. CRASH is the AUD experienced extremely negative daily return in the past five trading days. The extreme negative daily return is defined as two standard deviations of the daily spot rate return in the sample period. ΔNL is the absolute weekly change in the net long position in the AUD currency by large speculators, divided by 10,000 for comparison purposes. ΔINTDF is the weekly percentage change in interest rate differential calculated by using the one-year forward rate divided by spot rate. It can be understood as the weekly percentage change in the ratio of domestic interest rate to foreign interest rate. The dependent variables are the four factors that summarise the dynamics of the volatility surface. MA(1) refers to the moving average correction of order 1. The standardised marginal effect of the significant variable is calculated as the coefficient times one standard deviation of positive change in that variable.

Table 4.8 shows the regression result incorporated with the identified structural breaks in the case of EUR. We can see that compared with AUD, the sensitivity of the volatility surface variation is more centred on changes in interest differential, volatility of volatility and spot rate. Specifically, during the period of the GFC, changes in the volatility of volatility and realised volatility are the major determinants of the volatility surface variation, whereas the spot rate and interest rate differential explain most variation of the volatility surface post-GFC, from 16 November 2009 to 24 June 2013.

The market sentiment affected the slope of the volatility strike structure during the period of GFC, judging from the sign of its coefficient, which implies that when markets become bullish (bearish) EUR (USD), the slope of volatility strike tends to be more negatively skewed, suggesting an increased downward jump risk in EUR. However, given its tiny marginal effect, it is not considered that market sentiment is a major determinant in this period.

## Table 4-8 Regression result incorporated with the identified structural breaks in the case of EUR

Panel A Regression result

| | Parallel shift | | Slope shift of volatility term structure | | Slope shift of volatility strike structure | | Curvature shift of volatility term structure | |
|---|---|---|---|---|---|---|---|---|
| | 03 July 2006–09 November 2009 | 16 November 2009–24 June 2013 | 03 July 2006–10 November 2008 | 17 November 2008–24 June 2013 | 03 July 2006–30 November 2009 | 07 December 2009–24 June 2013 | 03 July 2006–17 November 2008 | 24 November 2008–24 June 2013 |
| Intercept | -0.0314 [-0.930] | -0.0638 [-1.795] | 0.0190 [0.983] | -0.0283 [-1.749] | -0.0180 [-2.328]* | 0.0134 [1.173] | 0.0022 [0.259] | 0.0029 [0.471] |
| ΔSPOT | 5.5798 [0.815] | -14.1055 [-4.080]** | 0.0534 [0.012] | -0.6367 [-0.296] | 6.3914 [9.626]** | 2.3619 [2.398]* | 1.798 [1.408] | -0.1815 [-0.188] |
| ΔRV | 1.5680 [2.350]* | -0.3758 [-0.670] | 0.5732 [1.198] | 0.0257 [0.088] | 0.0501 [0.430] | 0.1177 [1.056] | -0.0335 [-0.239] | 0.0397 [0. |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | 356] |
| ΔVOV | 1.2549 | 0.5316 | 0.2215 | 0.5613 | -0.0428 | -0.1722 | 0.0367 | -0.1026 |
| | [3.350]** | [2.088]* | [1.403] | [2.556]* | [-0.756] | [-2.109]* | [0.453] | [-1.615] |
| ΔCOR | 0.1457 | 0.4587 | -0.0062 | 0.1056 | 0.1106 | 0.0571 | -0.0483 | -0.1221 |
| | [0.460] | [1.242] | [-0.028] | [0.497] | [2.272]* | [0.857] | [-0.706] | [-1.276] |
| SPIKE | 0.0409 | 0.0294 | 0.5379 | -0.0827 | 0.0018 | -0.0347 | 0.0093 | -0.0105 |
| | [0.151] | [0.287] | [3.459]** | [-0.920] | [0.073] | [-1.189] | [0.084] | [-0.408] |
| CRASH | 0.2316 | 0.2047 | -0.0494 | 0.1462 | 0.0411 | -0.0502 | -0.0364 | 0.0079 |
| | [0.985] | [2.122]* | [-0.346] | [2.637]** | [1.642] | [-2.659]** | [-0.562] | [0.465] |
| ΔNL | -0.0026 | 0.0435 | -0.0005 | 0.0124 | -0.0187 | -0.0033 | 0.0093 | 0.0102 |
| | [-0.055] | [1.302] | [-0.020] | [0.614] | [-3.426]** | [-0.364] | [1.128] | [1.417] |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| ΔINTDF | -85.3789 | -223.9070 | -82.4734 | 7.6304 | 11.2280 | 42.5107 | -23.0367 | 27.3061 |
| | [-1.792] | [-5.771]** | [-2.193]* | [0.279] | [0.921] | [3.591]** | [-1.725] | [2.437]* |
| MA(1) | -0.2945 | -0.1695 | -0.4586 | -0.5398 | 0.0797 | 0.0808 | -0.102 | -0.481 |
| | [-2.781]** | [-2.029]* | [-4.063]** | [-7.676]** | [0.761] | [0.771] | [-0.859] | [-8.066]** |
| Observations: | 176 | 189 | 124 | 241 | 179 | 186 | 125 | 240 |
| R-squared: | 0.221 | 0.413 | 0.377 | 0.189 | 0.572 | 0.319 | 0.089 | 0.114 |
| F-statistic: | 6.539 | 14.000 | 9.287 | 7.238 | 27.526 | 10.658 | 2.351 | 4.418 |
| Prob(F-stat): | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.018 | 0 |
| Panel B standardised marginal effect of significant variables | | | | | | | | |
| ΔSPOT | | -0.2174 | | | 0.0985 | 0.0364 | | |
| ΔRV | 0.1230 | | | | | | | |
| ΔVOV | 0.1952 | 0.0827 | | 0.0873 | | -0.0268 | | |
| ΔCOR | | | | | 0.0138 | | | |
| ΔNL | | | | | -0.0258 | | | |
| ΔINTDF | | -0.2987 | -0.1100 | | | 0.0567 | | 0.0364 |

Note: ΔSPOT is the weekly return of the EUR. ΔRV is the weekly percentage change of the realised volatility. ΔVOV is the weekly percentage change of the volatility of one-month zero Delta straddle quotes. ΔCOR is the absolute change in the correlation between the spot rate and the one-month zero Delta straddle quotes. SPIKE is the EUR experienced extremely positive daily return in the past five trading days. CRASH is the EUR experienced extremely negative daily return in the past five trading days. The extreme negative daily return is defined as two standard deviations of daily spot rate return in the sample period. ΔNL is the absolute weekly change in the net long position in the EUR currency by large speculator, divided by 10,000 for comparison purposes. ΔINTDF is the weekly percentage change in interest rate differential, calculated by the one-year forward rate divided by spot rate. It can be understood as weekly percentage change in the ratio of domestic interest rate to foreign interest rate. The dependent variables are the four factors that summarise the dynamics of the volatility surface. MA(1) refers to the moving average correction of order 1. The standardised marginal effect of significant variable is calculated as the coefficient times the one standard deviation of positive change in that variable.

Table 4.9 shows the regression result incorporated with the identified structural break in the case of JPY. The major finding here is that, during the period of GFC, the major determinants of variation of the volatility surface are changes in the volatility of volatility, negative jumps, net long position and interest rate differential, whereas post-GFC we show the major determinants of variation of the volatility surface switch to change in spot rate, realised volatility, and interest rate differential. This finding is somewhat similar to the case of EUR, in which the major determinants of the volatility surface differ substantially during and post-GFC. Spot-volatility correlation and net long position shows some significance post-GFC; however, given their small marginal effect, they are not considered major determinants even though their net long position has a dominant role during the period of the GFC.

We can see that during the period of GFC, when negative jump occurs—that is to say, a large and fast depreciation (appreciation) in the value of USD (JPY)—the volatility surface rises by an amount five times bigger than one standard deviation change in the other significant variables; notably, the market seems to be sensitive only to negative jumps, not positive jumps; and the effect of negative jumps on change of volatility surface vanishes after the GFC. This phenomenon may be because during the GFC when the markets fear a crash in the US capital market, the major global market concern is the US performance. With a large and fast depreciation (appreciation) in the value of USD (JPY), the markets' belief of a further crash in the US capital market strengthened; this introduced bad sentiment and increased volatility in currency options. However, as the GFC passed, the market interpreted negative jumps as a normal reflection of the US government's QE policy, and became less sensitive to them.

We show the sign of coefficient for interest differentials on the variation of volatility surface differs during and post-GFC, which is also why the effect of interest differentials does not show on the full sample regression. Specifically, during the GFC, increase (decrease) in the interest rate in JPY (USD) relative to USD (JPY) increases volatilities, whereas post-GFC this relationship reversed. This finding clearly suggests that under GFC, increase (decrease) in the interest rate in JPY (USD) relative to USD (JPY) is bad news for the market and volatility increases, while the post-GFC market interprets the decrease (increase) in the interest rate in JPY (USD) relative to USD (JPY) as good news, and volatility decreases. This explanation may also be applied to the finding of a relationship between interest rate differential and volatility surface in EUR during our sample period. This finding again supports our hypotheses that the same event can be interpreted differently under different market conditions, and therefore has different effects on the volatility surface under different market conditions.

**Table 4-9 Regression results incorporated with identified structural breaks in the case of JPY**

| Panel A Regression result | Parallel shift | | | Slope shift of volatility term structure | Slope shift of volatility strike structure | | | Curvature shift of volatility term structure |
|---|---|---|---|---|---|---|---|---|
| | 19 June 2006–13 October 2008 | 20 Oct 2008–21 March 2011 | 28 March 2011–24 June 2013 | 19 June 2006–24 June 2013 | 19 June 2006–24 Nov 2008 | 01 Dec 2008–19 July 2010 | 26 July 2010-24 June 2013 | 19 June 2006–24 June 2013 |
| Intercept | -0.0926 | -0.0451 | -0.0278 | 0.0086 | 0.0320 | 0.0328 | 0.0125 | 0.0059 |
| | [-1.702] | [-2.999]** | [-0.723] | [0.580] | [1.743] | [1.208] | [1.320] | [0.669] |
| ΔSPOT | -0.3058 | 4.2529 | 8.4655 | -2.3099 | -0.2197 | -1.4696 | 2.2065 | 0.3897 |
| | [-0.068] | [1.576] | [2.203]* | [-1.353] | [-0.126] | [-1.245] | [2.842]** | [0.367] |
| ΔRV | 0.7548 | 1.0099 | 1.1580 | -0.8805 | -0.1087 | -0.1918 | 0.102 | 0.3756 |
| | [1.052] | [1.224] | [1.989]* | [-4.024]** | [-0.346] | [-1.164] | [1.553] | [2.755]** |
| ΔVOV | 0.6912 | 0.7367 | -0.2294 | -0.1965 | -0.2567 | -0.3180 | -0.0591 | 0.0794 |
| | [2.064]* | [2.609]* | [-0.511] | [-1.358] | [-2.077]* | [-2.381]* | [-1.733] | [0.782] |
| ΔCOR | -0.5053 | 0.8916 | -0.8208 | 0.1937 | 0.1851 | -0.0673 | 0.2463 | -0.1246 |
| | [-0.902] | [1.478] | [-2.358]* | [1.299] | [0.959] | [-0.467] | [3.604]** | [-1.201] |
| SPIKE | 0.1174 | -0.1557 | 0.1540 | 0.0941 | -0.0780 | 0.2513 | -0.0665 | 0.0063 |
| | [0.445] | [-0.764] | [0.734] | [1.078] | [-0.826] | [3.870]** | [-1.636] | [0.097] |
| CRASH | 0.8066 | 0.1143 | 0.4746 | -0.0912 | -0.3309 | -0.1888 | -0.0436 | -0.0722 |
| | [3.212]** | [0.503] | [1.774] | [-0.821] | [-4.499]** | [-3.009]** | [-1.256] | [-0.736] |
| ΔNL | 0.0896 | 0.1291 | -0.0155 | -0.0347 | -0.0200 | -0.0417 | -0.0181 | 0.0267 |
| | [2.622]** | [2.319]* | [-0.380] | [-2.865]** | [-1.969] | [-2.302]* | [-2.479]* | [3.949]** |
| ΔINTDF | 122.6724 | -179.1793 | -220.3766 | -1.3492 | -20.3792 | 38.0432 | 2.5504 | -8.1259 |
| | [2.910]** | [-3.102]** | [-2.494]* | [-0.049] | [-1.043] | [2.078]* | [0.213] | [-0.513] |
| MA(1) | -0.1660 | -0.45 | -0.2393 | -0.5000 | -0.1093 | 0.2753 | 0.0522 | -0.5146 |
| | [-1.243] | [-3.777]** | [-2.482]** | [-4.775]** | [-0.9324] | [2.1334]* | [0.5433] | [-5.521]** |
| Observations: | 122 | 190 | 118 | 367 | 128 | 86 | 153 | 367 |
| R-squared: | 0.434 | 0.403 | 0.294 | 0.291 | 0.381 | 0.371 | 0.212 | 0.247 |
| F-statistic: | 11.341 | 15.289 | 5.003 | 16.274 | 8.067 | 6.574 | 6.221 | 12.999 |
| Prob(F-stat): | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Panel B Standardised marginal effect of significant variables | | | | | | | | |
| ΔSPOT | | | 0.1172 | | | | 0.0306 | |
| ΔRV | | | 0.1707 | -0.1298 | | | | 0.0554 |
| ΔVOV | 0.1626 | 0.1733 | | | -0.0604 | -0.0748 | | |
| ΔCOR | | | -0.1157 | | | | 0.0347 | |
| ΔNL | 0.1467 | 0.2113 | | -0.0568 | | -0.0683 | -0.0341 | 0.0437 |
| ΔINTDF | 0.1654 | -0.2415 | -0.2971 | | | 0.0513 | | |

Note: ΔSPOT is the weekly return of the JPY spot rate. ΔRV is the weekly percentage change of the realised volatility. ΔVOV is the weekly percentage change of the volatility of one-month zero Delta straddle quotes. ΔCOR is the absolute change in the correlation between the spot rate and the one-month zero Delta straddle quotes. SPIKE is the JPY experienced extremely positive daily return in the past five trading days. CRASH is the JPY experienced extremely negative daily return in the past five trading days. The extreme negative daily return is defined as two standard deviations of the daily spot rate return in the sample period. ΔNL is the absolute weekly change in the net long position in the JPY currency by large speculators, divided by 10,000 for comparison purposes. ΔINTDF is the weekly percentage change in interest rate differential, calculated by using the one-year forward rate divided by spot rate. It can be understood as a weekly percentage change in the ratio of domestic interest rate to foreign interest rate. The dependent variables are the four factors that summarise the dynamics of the volatility surface. MA(1) refers to the moving average correction of order 1. The standardised marginal effect of the significant variable is calculated as the coefficient times one standard deviation change in that variable.

This chapter shows how using a structural break test identifies the structural breaks in regression relationships. After imposing a sub-sample regression, in general we find that:

(1) The determinants of the volatility surface vary under different market conditions. In our case the market condition means the risk aversion degree in the market.

(2) Market sentiment, volatility of volatility, interest rate differential and large and fast depreciation in USD play dominant roles in affecting the volatility surface when the market is in a high degree of risk reversion. In our case, this is the period of the GFC. This finding is equivalent to saying that the Vega risk is especially centred in the high degree of risk reversion market conditions.

(3) In periods when the market experiences normal risk reversion, the spot rate, realised volatility and interest rate differential (in the case of JPY and EUR) are the major determinants of volatility surface variation.

(4) The interest rate differential has a significant effect on the volatility surface in the case of JPY and EUR in both high risk aversion and low risk aversion conditions; however, the sign of the coefficients differs across market conditions. These findings suggest that whether a change in interest rates causes an increase or decrease in the volatility of the underlying currency rate depends on the market conditions.

# 5 Conclusion

It is of considerable interest for both academics and practitioners to understand the volatility smile effect and its time-varying feature. Instead of developing more complex models to try to capture both, it might be better to conduct empirical research on possible determinants of the time variance of option volatilities. This topic has rarely been studied in the currency option market. Although this thesis is not the first investigation into this topic, we differentiate our research from the existing published work by testing the explanatory power of a different set of variables on the variation of volatility surface, using a more comprehensive dataset that includes the most highly traded currencies in the market and studying the stability of regression relationships between variation of volatility surface and its determinant variables.

This research looks at the relationship between the time-varying volatility surface and possible exogenous variables for AUD, EUR and JPY in the OTC currency option market. Our sample covers the most recent data, from 27 June 2006 to 25 June 2013. The principal components analysis indicates that four latent factors are sufficient to explain major variations in the time-varying volatility surfaces among all three currencies. For all currencies, factor 1 and factor 2 represent a parallel shift of the volatility surface, and slope shift of the volatility term structure. A change in factor 1 shifts the entire option volatilities in the same sign and approximately equal magnitudes. A change in factor 2 changes the slope of the option volatility term structure. Factor 3 (in EUR and JPY) and factor 4 (in AUD) represent the slope shift of the volatility strike structure. A positive change in this generally increases the difference between OTM call option volatilities and OTM put option volatilities.

Factor 4 (in EUR and JPY) and factor 3 (in AUD) explain the curvature shift of the volatility term structure. It changes the difference between the medium term option volatilities and long and short expiration option volatilities.

The relation of the implied volatility surface to possible explanatory variables is also examined by conducting a multivariate regression analysis on the estimated factors with the possible explanatory variables. These proposed variables are the weekly currency spot return, the percentage weekly change in the realised volatility of currency spot return, the percentage weekly change in the volatility of option volatility, the absolute weekly change in the correlation between the spot return and option volatility, the weekly change in the market sentiment, and the weekly change in the implied interest rate differential. After conducting the multivariate regression in the full sample period, we find that a strong explanatory power for the variation of the volatility surface is not only evidenced by the spot rate return, but by realised volatility, volatility of volatility, interest rate differential and market sentiment as well. The size and significance of these variables affect variation of the volatility surface across currencies.

Since we expected that under different market conditions different exogenous variables govern the variation of the volatility surfaces, we conducted a structural break test by using the sequential estimation approach introduced by Bai and Perron (1998) on the regression relationship. We find the break dates are coincident with changes in market conditions. More specifically, we find the first identified structural break date for all three currencies coincided with the period when the GFC was at its peak; and for the EUR the second structural break dates coincided with the beginning of the Greek sovereign debt crisis in late 2009. This finding supports our hypotheses that the relationship between the variation of volatility surface and the exogenous variable is not constant over the time and is strongly affected by market conditions.

After imposing a sub-period regression analysis, we find that during the period of GFC market sentiment, volatility of volatility, interest rate differential and large and fast depreciation in USD were dominant in affecting the volatility surface, whereas post-GFC, the spot rate, realised volatility and interest rate differential (in the case of JPY and EUR) were the major determinants of volatility surface variation. In particular, the interest rate differential had a significant effect on the volatility surface in the case of JPY and EUR, in both high-risk and low-risk aversion conditions; however, the sign of coefficients differed across market conditions. These findings suggest that whether the direction of change in interest rate causes an increase or decrease in the volatility of the underlying currency rate depends on the market conditions.

The empirical evidence that shows the spot rate return is not the only source contributing to the variation of volatility surface implies a multifactor model for pricing currency options that takes into account the change of interest rate differential and change of higher moment of underlying asset return. The empirical evidence that shows that market sentiment has a significant effect on the variation of the volatility surface in JPY and AUD, suggests that the assumption of the complete market typically made for building option models using risk-neutral valuation techniques fails in the real market, as risk aversion does show an influence on the price of options.

A possible future research direction naturally following from this research is to address the question whether the simultaneous reaction of the option volatilities to the proposed explanatory variables that are not directly related to the characteristics of the underlying asset distribution is due to market over-reaction; this variable in our regression settings is market sentiment. This type of research falls into the category of testing the rationality of the simultaneous relationship between the exogenous variable and the variation of option volatilities in the OTC currency option market.

## References


Bai, Jushan, and Serena Ng. 2002. “Determining the Number of Factors in Approximate Factor Models.”*Econometrica* 70 (1): 191-221.

Bai, Jushan, and Pierre Perron. 1998. “Estimating and Testing Linear Models with Multiple Structural Changes.”*Econometrica* 66 (1): 47-78.

———. 2003. “Critical Values for Multiple Structural Change Tests.”*Econometrics Journal* 6 (1): 72-78.

Bakshi, Gurdip, Charles Cao, and Zhiwu Chen. 1997. “Empirical Performance of Alternative Option Pricing Models.”*Journal of Finance* 52 (5): 2003-2049.

Bates, David S. 1996. “Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options.”*The Review of Financial Studies* 9 (1): 69-107.

Black, Fischer, and Myron Scholes. 1973. “The Pricing of Options and Corporate Liabilities.”*Journal of Political Economy*: 637-654.

Bollen, Nicolas B, and Robert E Whaley. 2004. “Does Net Buying Pressure Affect the Shape of Implied Volatility Functions?”*Journal of Finance* 59 (2): 711-753.

Bollerslev, Time, and Hao Zhou. 2006. “Volatility Puzzles: A Simple Framework for Gauging Return-Volatility Regressions.”*Journal of Econometrics* 131 (1): 123-150.

Carr, Peter, and Dilip Madan. 2001. “Towards a Theory of Volatility Trading.”*Option pricing, Interest rates and Risk management, Handbooks in mathematical finance*: 458-476.

Carr, Peter. and Liuren Wu. 2007 "Theory and Evidence on the Dynamic Interactions between Sovereign Credit Default Swaps and Currency Options", Journal of Banking &Finance 31: 2383-2403.

Castagna, Antonio. 2010. *Fx Options and Smile Risk*: John Wiley & Sons.

Chalamandaris, Georgios, and Andrianos E Tsekrekos. 2009. “Predictable Dynamics in Implied The volatility surfaces from Otc Currency Options.”*The Jorunal of Business* 79 (3): 1591-1635.

Chalamandaris, Georgios, and Andrianos ETsekrekos. 2013. “Explanatory Factors and Causality in the Dynamics of volatility surfaces Implied from OTC Asian-Pacific Currency Options.”*Computational Economics* 41 (3): 327-358.

Cont, Rama, and Jose da Fonseca. 2002. “Dynamics of Implied volatility surface.”*Quantitative Finance* 2: 45-60.

Das, Sanjiv R, and Rangarajan K Sundaram. 1999. “Of Smiles and Smirks: A Term-Structure Perspective.”*Journal of Financial and Quantitative Analysis* 34 (2):211-239

Derman, Emanuel. 1999. “Volatility Regimes.”*Risk* 12 (4): 55-59.

Derman, Emanuel, and Iraj Kani. 1994. “Riding on a Smile.”*Risk* 7 (2): 32-39.

Deuskar, Prachi, Anurag Gupta, and Marti G.Subrahmanyam. 2007. “The Economic Determinants of Interest Rate Option Smiles."*Journal of Banking & Finance* 32 (5): 714-728.

Dumas, Bernard, Jeff Fleming, and Robert E Whaley. 1998. “Implied Volatility Functions:Empirical Tests.”*Journal of Finance* 53 (6): 2059-2106.

Dupire, Bruno. 1994. “Pricing with a Smile." *Risk* 7: 32-39.

Ederington, Louis H, and Wei Guan. 2002. “Why Are Those Options Smiling."*Journal of Derivatives* 10 (2): 9-34.

Garman, Mark B, and Steven W Kohlhagen. 1983. “Foregin Currency Option Values.”*Journal of International Money and Finance* 2 (3): 231-237.

Garleanu, Nicolae, Lasse Heje Pedersen, and Allen M. Poteshman. 2009. "Demand-based option pricing." *Review of Financial Studies* 22(10): 4259-4299.

Goncalves, Silvia , and Massimo Guidolin. 2006. “Predictable Dynamics in the S&P 500 Index Option Implied volatility surface.”*Journal of Business* 79 (3): 1591-1635.

Hafner, Reinhold, and Martin Wallmeier. 2001. “The Dynamics of DAX Implied Volatilities." International Quaterly Journal of Finance 1(1): 1-27

Hagan, Patrick, Kummar Deep, Lesniewski Andrew, and Woodeward Diana. 2002. “Managing Smile Risk.”*Wilmott* 18 (11): 84-108.

Han, Bing. 2008. “Investor Sentiment and Option Prices.”*The Review of Financial Studies* 21 (1): 387-414.

Heston. 1993. “A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options.”*Review of Financial Studies* 6 (2): 327-343.

Hibbert, Ann Marie, Robert T Daigler, and Brice Dupoyet. 2008. “A Behavioral Explanation for the Negative Asymmetric Return-Volatility Relation.”*Journal of Banking & Finance* 32 (10): 2254-2266.

Hull, John, and Alan White. 1987. “The Pricing of Options on Assets with Stochastic Volatilities.”*Journal of Finance* 42 (2): 281-300.

Kim, Minho, and Minho Kim. 2003. “Implied Volatility Dynamics in the Foreign Exchange Markets.”*Journal of International Money and Finance* 22: 511-528.

Longstaff, Francis A. 1995. “Option Pricing and the Martingale Restriction.”*Review of Financial Studies* 8 (4): 1091-1124.

Low, Cheekiat. 2004. “The Fear and Exuberance from Implied Volatility of S&P 100 Index Options.”*Journal of Business* 77 (3): 527-546.

McMillan, Lawrence G. 2011. *McMillan on Options:*John Wiley & Sons.

Merton, Robert C. 1973. “Theory of Rational Option Pricing.”*The Bell Journal of Economics and Management Science* 4 (1): 141-183.

———. 1976. “Option Pricing when the Underlying Stock Returns are Discontinuous.”*Journal of Financial Economics* 4: 125-144.

Mixon, Scott. 2002. “Factors Explaining Movements in the Implied volatility surface.”*Journal of Future Markets* 22 (10): 915-93.

Pena, Ignacio, Gonzalo Rubio, and Gregorio Serna. 1999. “Why Do We Smile? On the Determinants of the Implied Volatility Function.”*Journal of Banking & Finance* 23 (8): 1151-1179.

Rubinstein, Mark. 1994. “Implied Binomial Trees.”*Journal of Finance* 49 (3): 771-818.

Stein, Elias M, and Jeremy C Stein. 1991. “Stock Price Distributions with Stochastic Volatility: An Analytic Approach.”*The Review of Financial Studies* 4 (4): 727-752.

Stock, James H, and Mark W Watson. 2002. “Forecasting Using Principal Components from a Large Number of Predictors.”*Journal of the American Statistical Association* 97 (460): 1167-1176.

Vilkov, Grigory, and Xiao Yan. 2013. “Option-Implied Information and Predictability of Extreme Returns.” Working paper,Geothe University Frankfurt.

Wystup, Uwe. 2003. “The Market Price of One-Touch Options in Foregin Exchange Markets.”*Derivatives Week* 12 (13): 8-9.