Western Australian School of Mines

An Implementation of Ant Colony and Genetic Algorithm based Hybrid-Metaheuristic for Cut-off Grade Optimization in Open-Pit Mining Operations

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DECLARATION

It is hereby declared to the best of my understanding and belief that this research thesis contains no material previously published by any other person except where due acknowledgement has been made.

This thesis contains the material which has not been presented or accepted previously for the award of any other degree and diploma in any other University or institution.

Signature: ________________________________

Name: ________________________________

Date: ________________________________

Dedicated to my Father, Muhammad Ehsan Ellahi Qureshi (late),

my mother, Najma Ehsan,

my wife Fatima Asim,

my daughters Saleha and Zainab,

my son Moiez and my siblings

for their prayers, motivations, understanding, care and support

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PUBLICATIONS INCORPORATED INTO THIS THESIS

One of the papers has been published during thesis preparation and its detail is given below:

- Asad, M.W.A., M. Qureshi, and H. Jang. 2016.“A review of cut-off grade policy models for open-pit mining operations.”Resources Policy 49 (2016).

- Qureshi, M A., Asad, M.W.A., (IFORS 2017, July) Paper presented on “A hybrid meta-heuristic algorithm for cut-off grade optimization in open pit mining operations”, Quebec City, Canada.

- Qureshi, M.A., Asad, M.W.A., Jang, H., 2018. Performance evaluation of exact and heuristic algorithms for cut-off grade and production scheduling optimization in open-pit mining operations. (completed working paper to be submitted to *Applied Soft Computing*)

ABSTRACT

An ideal open-pit mining operation comprises of an open-pit, processing mill, and refinery that produces the marketable metal product. An open-pit produces valuable material (ore) and waste. The ore is transported to mill for processing to produce concentrates whereas waste is transported to waste dumps. The concentrate is further refined to produce marketable metal. The material of low-grade ore which has potential value is stored in stockpiles for future processing.

Open-pit mining operation uses economic criterion to distinguish between ore and waste material, which is termed as a cut-off grade. All material within the boundaries of the open-pit comprises of heterogenic nature of the metal content, which may not justify processing, so in economic terms material above cut-off grade is termed as ore (valuable material) and material below is termed as waste (non-valuable material).

It is observed in previous studies, that cut-off grade policy uses economic parameters (metal price, operating cost, and percentage yield and discount factor) in addition to the grade-tonnage distribution of mineralization as input, with the assumption that grade or metal content is uniformly distributed throughout the deposit. However, facts are contrary to the assumption and in reality, the grade is location dependent, where each mining block in an ore body model constitutes a unique grade, and the variation in grade from one mining block to the other is quite imminent. Consequently, the cut-off grade policy over the life of mining operation which is defined using uniformly distributed grade-tonnage curve remains impracticable for short-term plans, and it becomes difficult to synchronize short and long-term plans for defining an optimal cut-off grade policy.

In addition, conventional open-pit production scheduling model takes economic, geologic (ore-body model) and operational parameters as inputs and converts this information

into economic block models, where an economic value based on the breakeven cut-off grade describes ore and waste blocks in the block model, and then this economic value becomes an input to the production scheduling formulation.

The aim of this study is to develop and implement mathematical formulation that overcomes these deficiencies in the existing procedures, and which can effectively find an optimal solution in addition to the yearly production sequence without using economic block values. Therefore, in addition to the economic and operational parameters as inputs, the proposed model takes the realistic ore-body model as a geological input, and an implementation of the model not only derives an optimal cut-off grade policy but also it generates the corresponding optimal sequence of block-by-block and year by year production.

The proposed MILP based mathematical model maximises the net present value (NPV) of the mining operation subject to precedence, production (mine, processing plant and refinery) capacity and grade blending constraints. This bring the proposed model into NP-hard (Atai et al., 2004) category i.e. computationally complex and generating solution to the model that takes realistic block ore-body models with thousands of blocks as an input, becomes impossible. Therefore, to overcome this issue, a hybrid metaheuristic (a combination of genetic and ant colony optimization algorithms) technique is introduced, developed and implemented in this research as an alternative approach for solving the mathematical model, and to define optimal cut-off grade policy to nearest optimum values. The gap analysis is performed in this research to evaluate the performance of both the methods in addition to the graphical representation of the results. The analysis shows that the hybrid-metaheuristic does not generate an exact solution to the problem, but effective in setting up a roadmap for cut-off grade policy which may lead to potential investments and higher returns on investments for future mining projects.

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LIST OF SYMBOLS AND ABBREVIATIONS

$\overline{g}_t$:	*Average grade of blocks mined in time period* t
COG_t:	*Cut-off grade of ore blocks mined in time t*
CO_{LB}:	*Cumulative tonnage of blocks having grade less than or equal to* COG_t *(lower bound)*
CO_{UB}:	*Cumulative tonnage of blocks having grade greater than or equal to* COG_t *(upper bound)*
$Mcap_{t\_ub}$:	*Mining capacity in time period* t *(upper bound)*
$Mcap_{t\_lb}$:	*Mining capacity in time period* t *(lower bound)*
O_i:	*Tonnage of block* i *(every block which has grade equal to or greater than milling head grade.*
$Pcap_t$:	*Processing capacity of a processing plant in time period t*
$Pcap_{t\_ub}$:	*Processing capacity of a processing plant in time period* t *(upper bound)*
$Pcap_{t\_lb}$:	*Processing capacity of a processing plant in time period* t *(lower bound)*
P_t:	*Annual cash flows*
Qc_t :	*Quantity of ore processed in time t*
Qm_t:	*Quantity of material mined in time t*
Qr_t:	*Quantity of ore refined in time* t *where* $Qr_t = Qc_t \times \overline{g_t} \times y$
g_i:	*Grade (metal content) of block i*
g_{lb} :	*Lower bound grade in grade-tonnage distribution curve*
g_{ub}:	*Upper bound grade in grade-tonnage distribution curve*
q_i:	*Tonnage (tons) of block i*
q_{lb}:	*Quantity of material (tons) for a lower bound grade block in grade-tonnage distribution curve*
q_{lb-ub}:	*Sum of the quantity of all blocks (tons) between lower and upper bound grades' blocks*
q_o:	*Quantity of ore*

q_{ub}: *Quantity of material (tons) for an upper bound grade block in grade-tonnage distribution curve*

q_w: *Quantity of waste*

$\overline{v}$: *Overall present value for the future cash flows*

v_c: *Present value if processing bottlenecks the mining operation*

v_m: *Present value if mining bottlenecks the mining operation*

v_r: *Present value if refining bottlenecks the mining operation*

ACO: *Ant colony optimization*

C: *Processing capacity*

COG: *Cut-off grade as an abbreviation*

GA: *Genetic algorithm*

MILP: *Mixed integer linear programming*

LP: *Linear programming*

NPV: *Net present value*

NP: *Non-deterministic polynomial*

PCO: *Particles swarm optimization*

PS: *Production scheduling*

R: *Refining capacity*

TS: *Tabu search*

UPL: *Ultimate pit limit*

FC: *Fixed cost*

M: *Mining capacity*

Q: *Total quantity of material*

T: *Total time period in years*

c: *Processing cost per block (block with grade > 0)*

m: *Mining cost*

***p**:* *Unit price of metal*

r: *Refining cost per unit of marketable mineral*

s: *Selling price per unit of marketable metal*

t:	*Time period in years*
v**:**	*Present value*
y**:**	*Processing recovery or yield*
σ**:**	*Cut-off grade (used in literature review)*
σ_m**:**	*Mine limiting cut-off grade*
σ_c**:**	*Processing plant limiting cut-off grade*
σ_r**:**	*Refining plant limiting cut-off grade*
σ_{mc}**:**	*Mine and processing plant balancing cut-off grade*
σ_{cr}**:**	*Processing plant and refining balancing cut-off grade*
σ_{mr}**:**	*Mine and refining balancing cut-off grade*
v**:**	*Block value for each block*
V**:**	*Block value of each block*
W**:**	*Present value of each cash flow in time t*
V_i**:**	*Block value for each block i*
V_{it}**:**	*Block value for each block i in time period t*
v_i**:**	*Block value for each block i*
v_{max}**:**	*Maximum present value*
2D:	*Two-dimensional*
3D:	*Three-dimensional*
F_n**:**	*Sum of sizes of objects in each set of array in GA fitness function*
Υ**:**	*Overall capacity of blocks mined within constraints each year in GA fitness function*
k**:**	*Constant in GA fitness function*
P_i^e**:**	*The probability of occurrences in ACO probability rule*
e**:**	*Best ant in ACO probability rule*
N_i^e**:**	*Set of feasible solutions in ACO probability rule*

$\boldsymbol{a}, \boldsymbol{b}$**:** *Parameters which determine the relative influence of pheromone trail in ACO probability rule*

$\boldsymbol{\sigma_i}$**:** *The heuristic information at block i in ACO probability rule*

$\boldsymbol{\tau_i}$**:** *Pheromone value at block i in ACO probability rule*

Chapter 1: Introduction

1.1 Introduction:

Open-pit mine planning optimisation for minerals' exploitation is a challenge for scientists and engineers for the last few decades. Different techniques and methods have been introduced and implemented for achieving production optimisation in order to attain maximum profits. The revenue estimated at a certain cut-off point where the value of marketable product successfully justifies the cost of production, generates profit and maximises net present value (NPV). That is the stage where further extraction of mineral transforms profits into operational expenditures.

In an open-pit mining operation (Figure 1.1), minerals of various categories are extracted and sent to appropriate destinations depending on the grade of the ore. The open-pit mine, processing plant, refinery, waste dumps and stockpiles are important components of the mining operation.

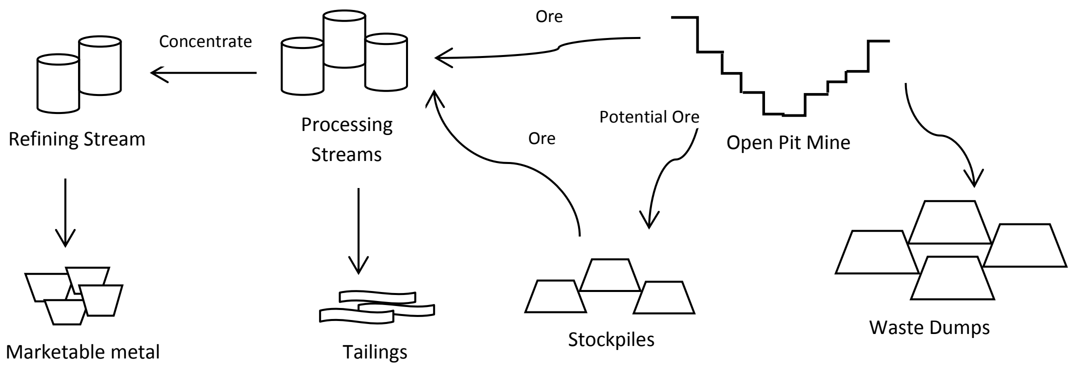


Figure 1.1: Layout plan of open-pit mining operation

The ore and the waste are the two different extremes to be catered for in the mining operations. Ore is sent to a processing plant for producing metal concentrate and then further directed to a refinery to yield marketable mineral (McKee et al., 1995) for attaining returns on investments, whereas waste is hauled to waste dumps. The economics behind this entire operation usually decides the net profit generated from overall mining operations.

Cut-off grade is the significant economic standard (Asad et al., 2016) which specifies the destinations of the ore (to the processing plant), potential ore (to stockpiles) and waste (to waste dumps) in an open-pit mining operation. The material with metal content above cut-off grade is termed as ore, which not only covers the cost of mining, processing, and refining but also generates profit (Asad et al., 2016); whereas the material with grade less than cut-off grade is termed as waste (Dagdelen, 2001). A cut-off grade of the mining operation is a cut-off point, which shows that any further operation (processing waste) transforms profit into operational cost. Cut-off grade policy thus defines a schedule of cut-off grades over the life of mining operation (Asad et al., 2016), along with the quantity of material (ore and waste) mined, quantity of ore processed, and quantity of metal refined (Hirai et al., 1987; Marques and Costa, 2013, Asad et al., 2016). The heterogeneity of the metal content in the deposit limits the process of extraction, processing, and refining within the capacities. The objective of this research is to maximise NPV through optimal cut-off grade strategies in line with the capacity constraints.

Geological block-by-block ore-body model, economic and operational capacities are considered as inputs in general open-pit mine planning operations. A typical ore-body consists of several thousands of blocks; each of which is defined by its location, grade, and associated tonnage. Policy decisions are made in mine planning whether a block is to be mined at a certain specific time, if mined, then whether to send it as an ore to the processing plant or whether it must be stockpiled at this stage. These decisions ultimately define the long-term sequence for production and optimal cut-off grade at the point where NPV is maximised over the life of the mine. Regardless of the block-by-block model of the ore-body, Lane (1964, 1988) show that geological input is transformed into grade-tonnage distribution with respect to the incremental grades along with corresponding quantity of material, within each grade classification increment (Dagdelen, 1992; Asad et al., 2016). The economic parameters such as the price of metal, mining cost, processing cost, refining/selling cost define the economic worth of the material to be mined, whereas operational parameters consist of mining, milling, refining capacities, and metallurgical recovery.

Generally, an optimal cut-off grade strategy to maximise NPV in an open-pit mining operation is subject to mining, processing and marketing constraints (Rendu, 2008; King, 2011). They are usually expressed as annual constraints to the quantity of material excavated, quantity of material sent for processing and the quantity of saleable product in the market. However, there are chances that any one of the constraints bottlenecks the whole operation at any given point of time. In strategic mine planning, two cut-off grade policies are in practice; the breakeven cut-off grade policy and Lane's optimal cut-off grade strategy.

The breakeven model defines the size and extent of the extraction (Dagdelen, 1992), and defines mining cut-off grade and classifies ore and waste blocks within the ore-body model. Many extensions to the breakeven model are made (Dagdelen, 1992; Vickers, 1961; Henning, 1963, Asad et al., 2016) in order to achieve better NPVs and variable annual cut-off grade, but regardless of all of these changes, the breakeven model defines a cut-off grade policy without considering the grade-tonnage distribution and operational capacities over the life of mining operation (Taylor, 1972).

Lane's optimal cut-off grade strategy (Lane, 1964, 1988), on the other hand, is a heuristic approach which takes grade-tonnage distribution of the available mineralization as input, in addition to mining, milling, and processing capacities. Lane's model generates a schedule of dynamic cut-off grades over the life of mining operation especially for the long-term mining. The use of normal distribution of grades and tonnage in Lane's optimal cut-off grade policy entails the development of a mathematical model for cut-off grade optimisation problems (Dagdelen and Kawahata, 2007, 2008) where block-by-block grade and tonnage is considered for defining a cut-off grade policy.

An improvement in Lane's model is proposed in linear (Ganguli et al., 2011) and non-linear (Yasrebi et al., 2015) programming models which are presented in different studies to achieve an optimal value of cut-off grades in open-pit mining operation. Mixed integer linear programming (MILP) formulation is developed (Dagdelen and Kawahata, 2007, 2008) to solve mathematical model considering multiple mines and multiple destinations including processing plants, stockpiles, and waste dumps. However, these models consider grade-tonnage distribution of deposit as an input. MILP models (Dagdelen and Johnson, 1986; Ramazan, 2007; Ramazan and Dimitrakopoulos, 2007; Newman et al., 2010; Ramazan and Dimitrakopoulos, 2012, Topal and Ramazan, 2010; Lamghari and Dimitrakopoulos, 2012) offer production schedule, however, these models depend on the pre-defined economic block values of a mining block, derived from the breakeven cut-off grades (Asad et al., 2016).

Mathematical models are developed both for production scheduling and optimum cut-off grades, but previous studies show that solving the mathematical model using an exact approach for a realistic block model is computationally inefficient, especially in the case of an ore-body model consisting of several thousands of blocks. Generally, the decision variables are defined in these formulations, where variables' size increases exponentially with the increase in the life of mining operation. Therefore, such formulations are termed as NP-hard combinatorial problems (easy to state and difficult to solve) (Atai et al., 2004).

The challenges to the extended solution time lead to near optimum solutions for cut-off grade optimisation over the life of mining operation, which substitutes the development of algorithms based on metaheuristics, and are defined as a set of algorithmic concepts used to improve the heuristic methods to solve complex problems. These concepts are inspired by biological and natural sciences. Metaheuristics are based on evolutionary algorithms like Genetic Algorithm (GA), Ant Colony Optimisation (ACO) (Gilani and Sattarvand, 2016), Tabu Search (TS), Simulated Annealing and Particle Swarm Optimisation (PSO), which is used for solving large-scale MILP formulations for open-pit mine planning problem (Askari-Nasab and Szymanksi, 2006; Denby and Schofield, 1995). The use of metaheuristics is considerably increased for large-scale combinatorial problems for being time efficient and capable of producing high-quality solutions.

1.2. Problem statement:

Existing procedures for the development of cut-off grade policy take economic parameters (price of metal, operating costs, recoveries, and discount rate) and grade-tonnage distribution of the mineralization as inputs. Considering the grade-tonnage curve as an input, one common aspect of the previously discussed studies is the assumption that the metal content or grade is uniformly distributed throughout the deposit. However, in reality, the grade is location dependent, that is, each mining block in an ore-body model constitutes a unique grade. Thus, the variation in grade from one mining block to the next is imminent. Consequently, the optimal cut-off grade policy, i.e. the long-term or life of operation schedule of cut-off grades, resulting from the grade-tonnage curve-based uniformly distributed inputs, remains aloof to the short-term operational plans. Thus, without synchronization of the long and short-term plans, mining engineers find it difficult to implement the optimal cut-off grade policies.

Given this, it is required that a realistic mathematical model for cut-off grade optimisation shall take into account the economic parameters and a three-dimensional ore-body model as an input, such that, the ore-body model retains the grade-tonnage distribution of the deposit on a block-by-block basis, where location (along XYZ-direction), metal content, and the quantity of material of an individual mining block are described exclusively. An ore-body model in practice constitutes thousands of mining blocks, and a MILP-based mathematical model considers these mining blocks as binary (0/1) variables. Thus, a practical instance of the cut-off grade optimisation problem may constitute thousands of binary variables, categorizing it as a computationally complex optimisation problem, and an exact optimal solution to the mathematical model therefore becomes impossible. As such, the solution of this problem requires a consideration of the metaheuristics such as GA or ACO algorithms or their combination, providing a close optimal solution within reasonable time and accuracy (Dagdelen and Johnson, 1987; Dagdelen and Asad, 2002; Johnson et al., 2011; Caccetta and Hill, 2003; Ramazan and Dimitrakopoulos, 2004).

1.3. Objectives

The objectives of the proposed study may be outlined as follows:

1. To develop a MILP based mathematical model for cut-off grade optimisation in open-pit mining operations.

2. To solve the MILP model through an exact optimisation approach using CPLEX optimisation software.

3. To solve the MILP model through hybrid-metaheuristic (combination of GA and ACO algorithms) approach.

4. To evaluate the performance of hybrid-metaheuristic against the exact solution.

1.4. Significance of research

The research is based on a new hybrid-metaheuristic approach which derives the cut-off grade optimisation policy in open-pit mining operations. It helps in solving the mathematical model for a cut-off grade but it also improves the computational efficiency. A hybrid-metaheuristic algorithm is developed while combining two different algorithms namely ACO and GA, where both of these metaheuristics contributed towards problem-solving in many planning, functional and engineering optimisation, especially in the field of open-pit mine production optimisation. The benefit of this research for the mining industry in Australia is to get an alternative cut-off grade optimisation framework. This research plays a significant role in improving metal mine productivity through more accurate results and computer efficient solutions in realistic problems. This study also solves the difficulty of the currently available commercial software applications and achieves more efficient optimisation results through advanced algorithms.

The economy of any country depends on its growth and annual GDP. According to Australian Bureau of Statistics, metal mining has contributed to nearly 41% of the annual production in Western Australia in the last few years. The research in cut-off grade optimisation makes a significant difference, and new investment and employment opportunities in Western Australia could be expected if more efficient production optimisation solutions are developed. This research can be considered as a step forward towards it, and can be expected to contribute effectively towards production optimisation in mining and metallurgical industries, which ultimately benefits the overall economy of Western Australia.

1.5. Research methodology

The following methodology helps achieve the objectives of the proposed study:

- A comprehensive literature review establishes the relevance, innovation and significance of the proposed method for cut-off grade optimisation and production scheduling in an open-pit mining operation.
- The development of mixed integer linear programming (MILP) based mathematical model and a creation of formulation using JAVA programming language and IBM ILOG CPLEX CONCERT technology.
- An implementation of the MILP formulation through exact approach using CPLEX.
- The development of a new algorithm using hybrid-metaheuristics (combination of GA and ACO algorithms) as an alternate approach for solving mathematical model.
- An implementation of hybrid-metaheuristic algorithm using JAVA programming language using realistic block model.
- Comparison of the solutions for MILP model using exact approach and hybrid-metaheuristic approach through gap analysis.

1.6. Structure of the thesis:

Chapter 2 presents a critical overview of the cut-off grade optimisation policies, methods, and variations. It covers a detailed discussion on the relevant work done in previous studies.

Chapter 3 is a detailed analysis on the cut-off grade formulation developed in this research. It describes the step by step methods and techniques used in developing MILP based mathematical model with examples.

Chapter 4 examines the cut-off grade optimisation strategy model developed and implemented using hybrid metaheuristics formulation. It covers the concepts of GA and ACO and their combination.

Chapter 5 is a case study discussion of a hypothetical block model with all geological, operational and economic parameters. The implementation of both MILP formulation and hybrid metaheuristics, with their comparison, is also discussed in this chapter. The realistic block model is also implemented and analysed in this research using hybrid-metaheuristic.

Chapter 6 is the conclusions extracted from the research findings, analysis, and the potential future research.

Chapter 2: Literature review

The material produced from an open-pit mining operation is either ore or waste, which is further transported to different destinations including processing streams for ore, and waste dumps for waste material as shown in Figure 2.1. The ore is transported in raw form to the processing plant and after processing, the concentrate is sent to a refinery for preparing the final saleable product (McKee et al., 1995; Asad et al., 2016). The material movement in the overall mining operation is evaluated in economic terms to finally decide whether to send the material to the processing plant, waste dumps or stockpiles for future processing.

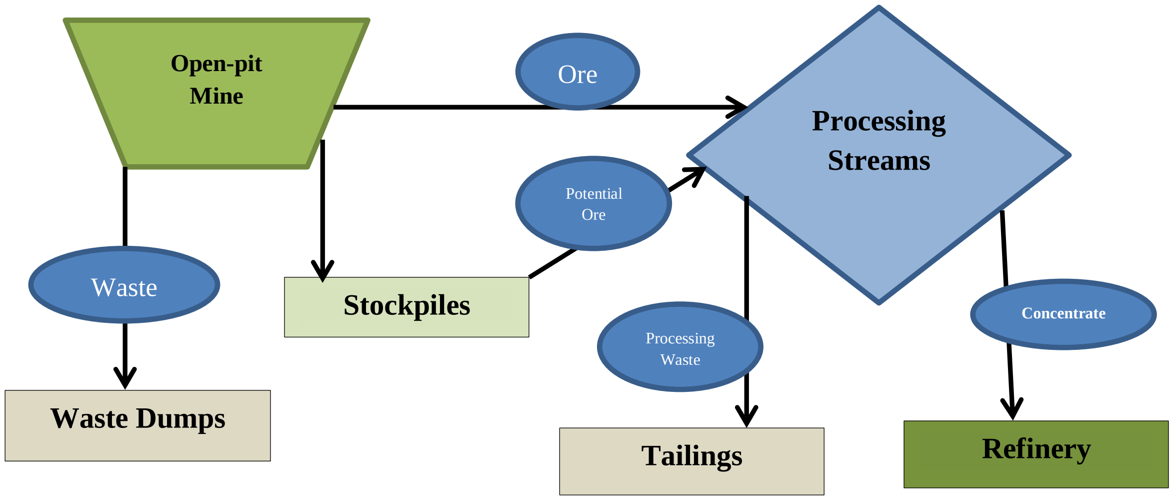


Figure 2.1: Schematic diagram of a model open pit mining system

Cut-off grade is a significant economic standard (Asad et al., 2016), which maximises NPV and defines the final destination of the material (King 1999, 2001; Wooler, 2001; Asad et al., 2016). The metal content present in the ore justifies the cost of mining (including excavation and haulage cost) after processing, and generates the profit. On the other hand, waste increases the cost of mining and if the material present in stockpiles (potential ore) is processed, it supplements the cost with the potential increase in profits. Cut-off grade policy thus delineates annual cut-off grades over the life of mining operation; including the quantity of material mined, the quantity of ore processed and quantity of concentrate refined (Hirai et al., 1987; Marques and Costa, 2013, Asad et al., 2016). Subsequently, this policy defines the net present value (NPV) over the life of mining operation. The strategic mine planning and cut-off grade policies are interrelated and also related to the operational plans of the mining operation (Lane, 1988; Hustrulid et al., 2013; Rendu, 2014; Hall, 2014; Asad et al., 2016).

2.1. Inputs of the cut-off grade model:

The understanding of the inputs to cut-off models is significant for developing cut-off grade polices. Generally, geological, economic, and operational parameters are used as inputs to define cut-off grade strategies (Taylor, 1972; King, 2001, Asad et al., 2016).

The economic parameters are defined as follows:

- Cost of mining $\boldsymbol{m}$ (Price units per tonnes)
- Milling cost $\boldsymbol{c}$ (Price units per tonnes)
- Sale price $\boldsymbol{s}$ (Price units per tonnes; or grams in case of valuable material)
- Fixed cost $\boldsymbol{FC}$ (Price units per period or year)
- Discount rate $\boldsymbol{d}$ (percentage)

As block-by-block configuration of resource mineralization is taken as geological input, both for production scheduling and cut-off grade estimation. The ore-body model generally comprises of thousands to millions of blocks depending on the size with the following specifications;

- Spatial location (X, Y and Z coordinates)
- Grade (metal content in a mining block)

- Quantity of material (tonnes)

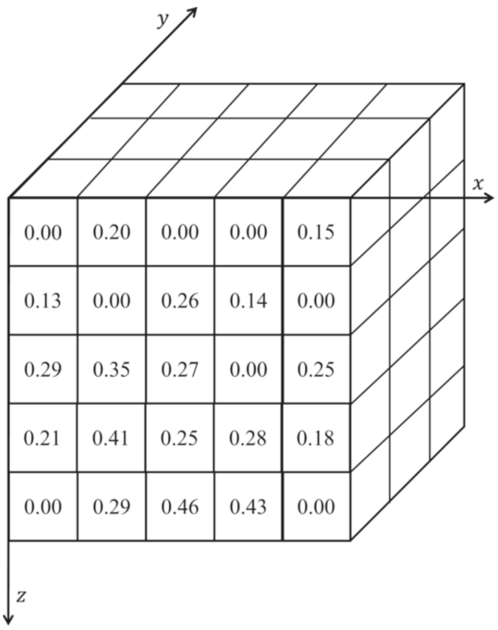


Figure 2.2: Hypothetical ore-body model (grade in % Cu) in 3D (Source: Asad et al., 2016)

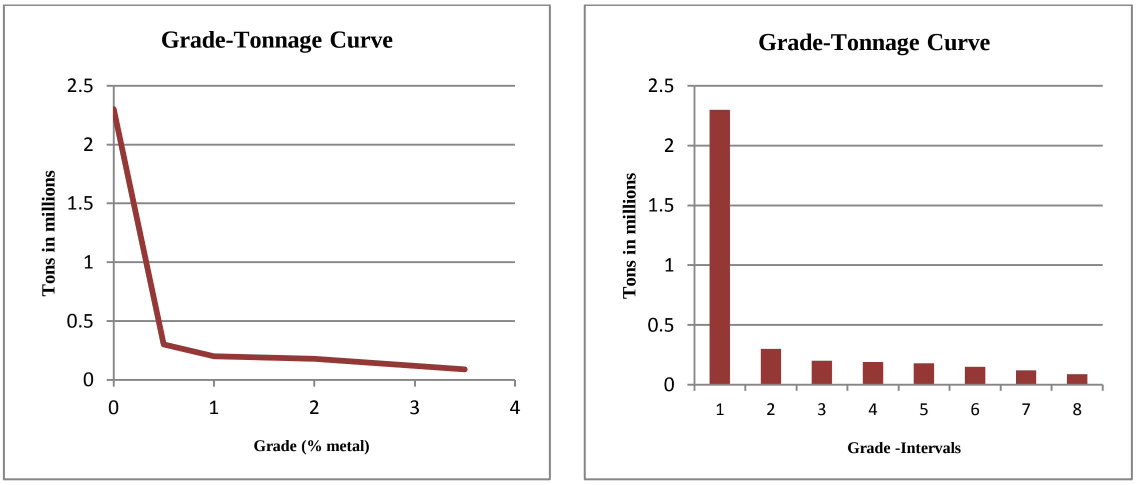
2.2. Grade-tonnage distribution:

The conventional approaches for cut-off grade optimisation take geological ore-body model as primary input, but not in the form of realistic block model instead, the primary input is converted into grade-tonnage distribution curve that constitutes lower and upper bounds of grades and their corresponding material (tonnage) in tonnes (Dagdelen, 1992). The conventional approaches use a grade-tonnage curve especially in developing optimal cut-off grade policies, defining a best schedule of dynamic cut-off grades. Figure 2.3 shows the grade-tonnage distribution of a hypothetical ore-body model (Hustrulid et al., 2013; Asad et al., 2016). Lower and upper bounds are shown as g_{lb} and g_{ub}, whereas the quantity of material within lower and upper bounds of grades is termed as q_{lb-ub}, and together with $\boldsymbol{n}$ number of increments, they can be represented as (Asad et al., 2016):

$[(g_{lb\_1}, g_{ub\_1}), q_{lb\_1-ub\_1}]$, $[(g_{lb\_2}, g_{ub\_2}), q_{lb\_2-ub\_2}]$,.., $[(g_{lb\_n}, g_{ub\_n}), q_{lb\_n-ub\_n}]$ where Q is the total quantity of material.

Grade % (lower bound)		Grade % (upper bound)	Quantity (Tons in millions)
g_{lb}	-	g_{ub}	q_{lb-ub}
0	-	0.5	2.3
0.5	-	1	0.3
1	-	1.5	0.2
1.5	-	2	0.19
2	-	2.5	0.18
2.5	-	3	0.15
3	-	3.5	0.12
3.5	-	4	0.09

Table 2.1: The grade-tonnage distribution of a hypothetical copper deposit (Lane, 1964; Asad et al., 2016)



Grade (% Metal)

Grade Category Interval (% Metal)

Figure 2.3: The grade-tonnage distribution of a hypothetical copper deposit (Source: Asad et al., 2016)

2.3. Breakeven cut-off grade policy:

Breakeven cut-off grade policy is a fundamental policy for cut-off grades which takes the following inputs and generates profit (P) as follows (Henning, 1963; Taylor, 1972, 1984; Asad et al., 2016):

$$P = (s - r)gy - m - p \qquad (2.1)$$

Using Eq. (2.1) the breakeven cut-off grade is then calculated as follows:

$$\sigma = \frac{m+p}{(s-r)y} \quad (2.2)$$

The unit value of σ is based on the mineral commodity which may be in %, grams per tonne, pounds per tonne, or ounces per tonne (Asad et al., 2016). Equation (2.2) is also significant for determining the size or extent of the mineral extraction i.e. ultimate pit limit (UPL) (Dagdelen, 1992; Asad et al., 2016). Equation 2.2 represents σ as mining cut-off grade, which differentiates between ore and grade. The economic value (V) of each block is calculated using Equation 2.1 and finally an ultimate pit is generated using graph theory based algorithms (Asad et al., 2016). The worth of each block and its actual location i.e. whether it is inside the UPL or not, or whether mining such block helps in covering the cost of removing overlying waste is determined at this stage.

The final contour or the ultimate pit (regardless of the material is a waste or ore) leads to the final destination of all the blocks mined within UPL i.e. whether the material is transported to the processing stream or hauled to the waste dumps. The block is termed to be suitable for processing stream if it contains enough valuable metal content capable to cater the costs of mining, processing, and refining, if not then it shall be destined to waste dumps. Equation 2.2 is then modified to Equation 2.3 as shown below, and it generates processing cut-off grade (Asad et al., 2016).

$$\sigma = \frac{p}{(s-r)y} \quad (2.3)$$

In contrast to the mining cut-off grade, the processing cut-off grade takes a grade-tonnage distribution of the mineralization as input (Asad et al., 2016) within the UPL and defines the cut-off grade policy over the life of mining operation (when all reserves are exhausted). Many researchers worked on the extensions in breakeven model, Vickers (1961) introduced marginal analysis for defining cut-off grade (Asad et al., 2016) policy using graphical representation, with the assumption of maximising profits in breakeven cut-off grades. However, the proposed method delivers a schedule of constant cut-off grades over the life of the operation. Henning (1963) on the other hand, provides a framework for defining cut-off grade while varying the enterprise objectives (Asad et al., 2016). The difference between the total annual profit and associated costs is maximised in Henning's (1963) approach, which gives higher cut-off grades in the initial years, ultimately reducing breakeven value for later years. Dagdelen (1992) depicts a similar strategy where higher breakeven values are achieved in the initial years over the total life of the open-pit mine.

The breakeven model, regardless of many variations plays a significant role in the calculations of the cut-off grade policy using Equation 2.3 (Asad et al., 2016). The gap in the breakeven model is its reliance on economic parameters and it ignores the practicality of grade-tonnage distribution of the mineral deposit, and this policy does not consider operational capacities, and ultimately generates constant cut-off grades over the life of mining operation (Taylor, 1972; Asad et al., 2016).

2.4. Lane's cut-off grade model:

Lane proposes an optimal cut-off grade policy (Lane, 1964, 1988), which considers a grade-tonnage distribution of mineral deposits and maximises NPV subject to operational capacities, including mining, milling and marketing constraints as presented in the following equations. NPV is maximised using discounted cash flows as shown is Equation 2.4.

$$Max\ Z = \sum_{t=1}^{T} \frac{P_t}{(1+d)^t} \quad (2.4)$$

Subject to

$$Qm_t \leq M,\ \forall t \quad (2.5)$$

$$Qc_t \leq C,\quad \forall t \quad (2.6)$$

$$Qr_t \leq R, \quad \forall t \tag{2.7}$$

Where $P_t = (p - r)Qr_t - mQm_t - cQc_t - FC_t$ is the cash flow or profit generated by the mining quantity of material mined Qm_t in time t, the quantity of material processed Qc_t in time t, quantity of material refined Qr_t in time t. The cut-off grade strategy generates a schedule of cut-off grades over the life of mining operation T, in a manner that NPV is maximised and the constraints mentioned in Eq. (2.5)- (2.7) are satisfied (Asad et al., 2016).

Given a grade-tonnage distribution and production capacities, if g_l is taken as the cut-off grade, then quantity of waste (q_w), and quantity of ore (q_o), and the average grade $(\bar{g})$ are considered in terms of cut-off grades are as follows where C is the total processing capacity and y is the metallurgical recovery (% age yield) (Asad et al., 2016).

$$Qc = \begin{cases} C; & if\ q_o > 0 \\ q_o\,; & if\ q_o < 0 \end{cases} \tag{2.8}$$

$$Qm = Qc\left[1 + \frac{q_w}{q_o}\right] \tag{2.9}$$

$$Qr = Qc\,(\bar{g}\,y) \tag{2.10}$$

Here, if Qm (quantity of material) in Equation (2.9) is mined over the time period t, and a cash flow P_t is realized at the end of the time t. However, after mining Qm, $Q - Qm$ quantity of the deposit still exists, and if scheduled to be mined from time period $t + 1$ to T, with possible cash flows P_{t+1} to P_T, and W is the present value of these cash flows in time t, then overall present value $\overline{v}$ for the future cash flows generated from time t to T (Asad et al., 2016) shown in Figure 2.4 (Asad et al., 2016).

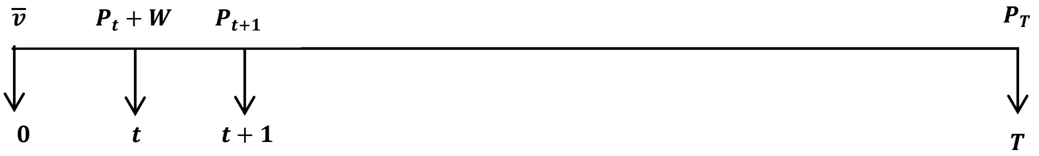


Figure 2.4: Presentation of the cash flows and the present value in Lane's model (Source: Asad et al., 2016)

If, $W = \frac{P_{t+1}}{(1+d)^1} + \frac{P_{t+2}}{(1+d)^2} + \cdots + \frac{P_T}{(1+d)^{T-1}}$

Then $\overline{v} = \frac{P_t + W}{(1+d)^t}$ (2.11)

Thus, increase in present value (v) by mining next Qm quantity of material may be abstracted from Equation 2.11 as follows (Asad et al., 2016):

$$v = \overline{v} - W = P_t - \overline{v}\,dt \quad (2.12)$$

Substituting value of P_t in Equation 2.12 (Asad et al., 2016):

$$v = (s - r)Qr - mQm - pQc - (f + \overline{v}\,d)t \quad (2.13)$$

Where time t is purely dependent upon the either of limiting production capacities, i.e. if mine bottlenecks the operation then $t = \frac{Qm}{M}$, if any of processing plant or refinery delays the operation, the value of t becomes $t = \frac{Qc}{C}$ in case of processing or $t = \frac{Qr}{R}$ in case of refining. The opportunity cost $(f + \overline{v}\,d)$ for either of these three conditions is distributed per unit if material mined, processed or refined respectively, as shown in Equations (2.14- 2.16) (Asad et al., 2016).

$$v_m = (s - r)Qr - (m + \frac{f + \overline{v}\,d}{M})Qm - pQc \quad (2.14)$$

$$v_c = (s - r)Qr - mQm - (p + \frac{f + \overline{v}\,d}{C})Qc \quad (2.15)$$

$$v_r = \left(s - (r + \frac{f+\overline{v}\,d}{R})\right) Qr - mQm - pQc \quad (2.16)$$

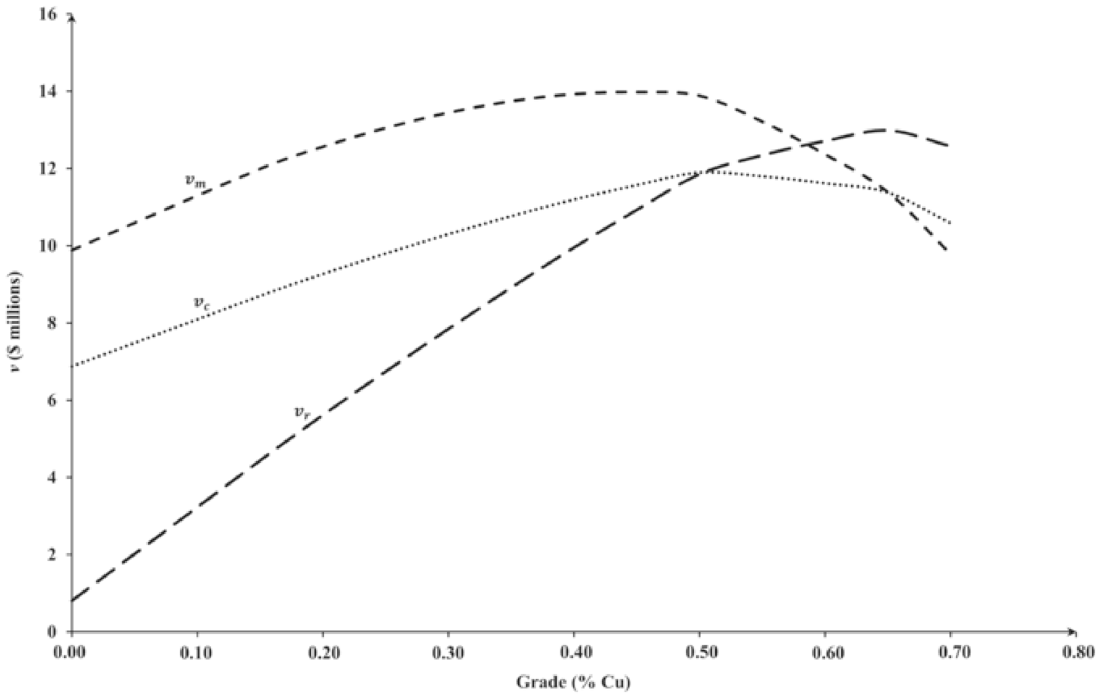


Figure 2.5: A graphical presentation of v_m, v_c and v_r as a function of g_l (Source: Asad et al., 2016)

The optimum value of cut-off grade σ thus calculated as increase in present values v_m, v_c and v_r as a function of g_l. Graphical representation of Lane's model shown in Figure 2.5 depicts that grade at the maximum value of three curves v_m, v_c and v_r represents that either of mining, processing or refining bottlenecks the operation, considering either of values in Equations (2.5 to 2.7) as equality, and generates corresponding mine limiting cut-off grade σ_m, process limiting cut-off grade σ_c and refinery limiting cut-off grade σ_r respectively.

Figure 2.5 also shows that point of intersection of curves v_m and v_c that both mine and processing limit the operation (i.e. Equation 2.5 and 2.6 represents an equality), and the grade at this point is mine and processing plant balancing cut-off grade (σ_{mc}), ensuring maximum throughput in both the stages. Similarly, mine and refinery balancing cut-off grade (σ_{mr}) is a grade where both mine and refinery limit the operation (i.e. Equation 2.5 and 2.7

represents equality) and same in the case if processing and refinery bottlenecks the operation (i.e. Equation 2.6 and 2.7 represents equality), then grade corresponds to the balancing cut-off grade (σ_{cr}) (Asad et al., 2016).

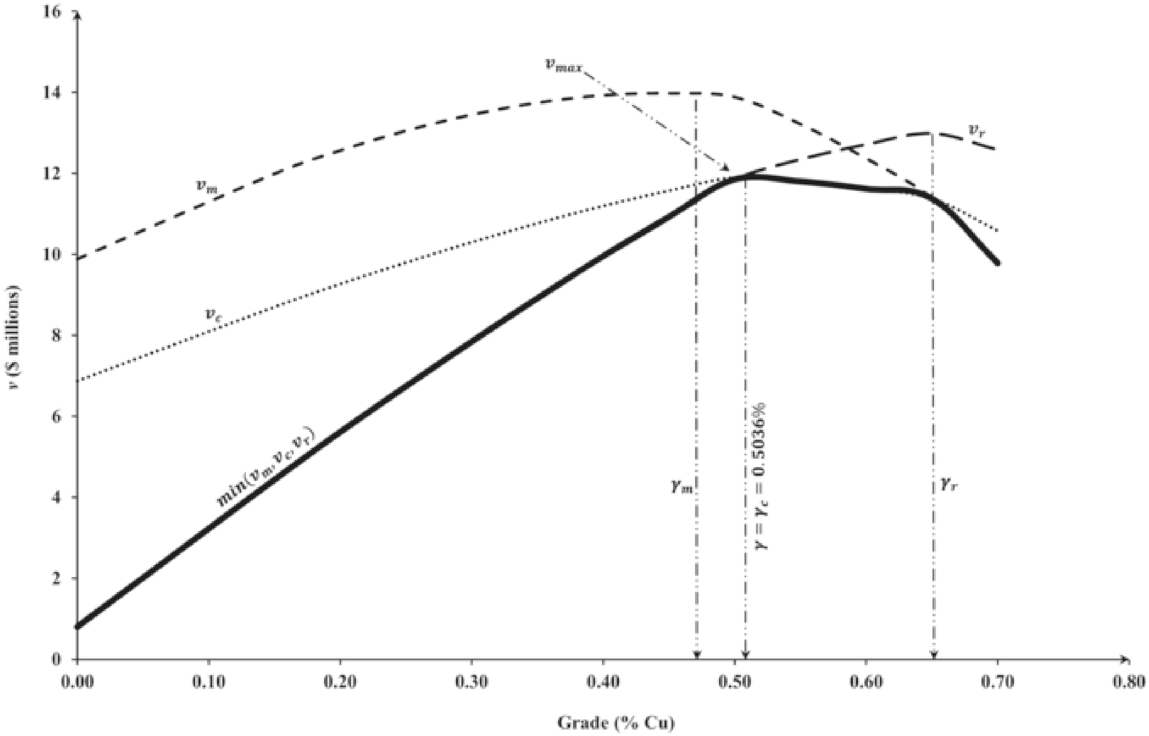


Figure 2.6: Choosing the optimum value in Lane's model (Source: Asad et al., 2016)

Therefore, the optimum cut-off grade (σ) is one of the six limiting and balancing cut-off grades without violating any of the mining, milling and processing constraints (Asad et al., 2016). Figure 2.6 represents Lane's model (Lane, 1988) for estimating the optimal cut-off grade (σ) corresponding to the maximum value of v_{max} among the minimums from the functions v_m, v_c and v_r .

$$v_{max}(\sigma) = max[min\ (v_m, v_c\ , v_r)] \tag{2.17}$$

2.5. Extensions in Lane's model:

Lane's optimal cut-off grade strategy offers a basic methodology for dynamic cut-off grades and it opens an avenue of new innovations in cut-off grade optimisation. The comparison between both the breakeven and Lane's approaches gives an insight about the significance of Lane's approach, as it considers all the aspects of economic, geological and operational parameters for cut-off grades calculations, and this is the reason for its vast acceptance in the mining industry (Asad et al., 2016). Whittle and Vassiliev (1998) implements Lane's model for standard strategic mine planning software.

There are several extensions made to Lane's approach, like Mol and Gillies (1984) suggest advancement into conventional cut-off grade prototypes, to make it more relevant to the iron mining operation, and in this context, marketable material is prioritised to maximise NPV, and material blending is used to attain required grade specifications (Asad et al., 2016).

Dagdelen (1992, 1993) present different algorithms for implementation of Lane's model through a case study, which depict the advantages of this approach. Higher cut-off grades are obtained in the initial years of the mining operation, leading to a faster rate of return on the capital investments. Balancing the cut-off grade using linear interpolation by Dagdelen (1993) is also used for the implementation of Lane's approach (Asad et al., 2016).

Whittle and Vassilev (1998) makes an extension in Lane's cut-off grade calculation model by changing the processing cost, recovery cost, and capacities, based on the stochastic liberation modelling method which provides a recovery prediction system which is contrary to Lane's approach. This takes variable inputs for recovery cost, where Whittle and Wooler (1999) depicts the relation between cut-off grades and the required milling time, which implements stochastic liberation modelling procedure in Whittle and Vassilev (1998). Whittle also presents Opti-Cut®, commercial software for Lane's model for defining optimal strategy for a milling operation, in parallel to mill throughput and mining cut-off grade optimisation (Wooler, 2001; Asad et al., 2016).

Generalised Reduced Gradient (GRG) factor is introduced by Nieto and Bacetin (2006). Bacetin and Nieto (2007) modifies opportunity cost with the addition of optimisation factor, and a solution is extracted considering non-linear characteristics of the model. The optimisation factor helps in converging NPV over different iterative processes, ultimately showing enhancement in the overall NPV of the operation (Asad et al., 2016).

Asad (2007) presents a realistic cut-off grade policy by introducing commodity price escalation as an addition to Lane's optimal model, for the reason that the commodity price is changed annually and operating cost also escalates (depending on fixed escalation rates and economic parameters) over the life of mining operation. Asad (2007) presents calculations based on the hypothetical grade-tonnage distribution, showing impact and importance of price variation and cost escalation in this model, on the overall NPV of the operation. The stockpiling option presented in Asad (2007) is implemented through mathematical modelling in Asad and Topal (2011), which also outlines the strategy to salvage the stockpiled material after the valuable mineral present in the pit is exhausted. Asad and Topal (2011) also compares the cut-off grade policies with and without stockpiling scenarios, and its impact on overall NPV and life of mining operation (Asad et al., 2016).

Osanloo et, al. (2008) extends Lane's model by the incorporation of environmental factors influencing the copper deposits, where the mathematical formulation is developed while catering the operating cost of dumping waste and tailings (acidic and non-acidic generating wastes), which consequently improves NPV relevant to Lane's model, while ensuring environmental impregnability. Narri and Osanloo (2015) also modifies Lane's model on the similar grounds with the incorporation of a reduction in cost due to the findings of environmental impact, in addition to extra revenue generating through waste rock recovery (Asad et al., 2016).

He et al. (2009) employs a hybrid algorithm by combining genetic and neural networks for dynamic optimisation of cut-off grades. Evolutionary algorithms are used in this study to define the solution considering Lane's model as a non-linear approach. Genetic algorithm (GA) generates chromosomes and its combination with neural networks develops a link between revenue factor and chromosomes, where GA performs a search for the global optimal cut-off grade. This method is applied to the iron ore deposit generating cut-off grades, and leads to substantial increase in NPVs (Asad et al., 2016).

Gholamnejad (2008, 2009) incorporates waste dump reclamation cost in Lane's model as a cash flow function, which in return changes the relationships of mining, processing and refining values, with a subsequent shift of optimal point. The case study model elaborates on the waste dump rehabilitation costs, leading to a reduction in the cut-off grade value, and with the decrease in the quantity of waste to be sent to the waste dumps, subsequently sees an increase in NPV while processing low-grade ores (Asad et al., 2016).

King (2009), following the initial studies in King (1999, 2001, 2004) extends Lane's approach by introducing various strategies and their insinuations regarding operating and administrative cost modelling. For example, the study depicts changes to the cut-off grade policy by distributing the cost to two different extremes (cost of mining ore is different to the cost of mining waste). This cost separation is based on the operations which comparatively take less cost for blasting, whereas the cost of hauling ore and waste is always different.

Rendu (2009) introduces a modification in Lane's optimal cut-off grade strategy that exhibits relationships among different policies so that they are in permissible range with different conditions and scenarios, and secondly, it identifies the difference among different cut-off grade policies that maximise NPV or internal rate of return (IRR). In this study, the geological and self-designed variables are considered to define optimal cut-off grade strategy.

Abdollahisharif et al. (2012) introduces an idea of variable production capacities in the optimal Lane's approach (Asad et al., 2016). The modifications made to Lane's model incorporate refinery capacity according to the market demand and incorporate processing and mining capacity as a function of cut-off grade and refinery capacity. Although, this study opposes the accepted mathematical model developed by Lane (1964,1988), but it still shares a crude framework for defining optimum cut-off grades, and reflects a higher NPV as compared to Lane's model and Gholamnejad (2009) (Asad et al., 2016). By using a variable capacity based model, it ensures the processing of low-grade ore and reduces the waste material (Asad et al., 2016).

Khodayari and Jafarnejad (2012) delineate a concept of balancing mine and processing plant in Lane's approach to maximise the quantity of metal (Qr) per year. In this study, an optimum cut-off grade is achieved where maximum metal quantity becomes possible, and it only happens when the optimal value is equal to the mine and processing plant balancing cut-off grade (Asad et al., 2016).

Gama (2013) reforms the profit function in Lane's model for finding the optimal value of cut-off grade. This study discusses the formulation of minimum permissible cut-off grade and maximum stripping ratio (waste to ore ratio). The sensitivity analysis used in the case study verifies optimum cut-off grade, which is not less than the minimum permissible cut-off grade, and secondly the corresponding stripping ratio does not surpass the maximum stripping ratio.

Hustrulid et al. (2013) and Rendu (2014) share a detailed review of both the breakeven and cut-off grade strategy, followed by elaborating the complications in Lane's approach both in open-pit and underground scenarios. These studies also share appreciated case studies with different stockpiling approaches.

Rahimi and Ghasemzadeh (2015) and Rahimi et al. (2015a, 2015b) depict Lane's model as a basis for sharing an innovative approach towards the estimation of cut-off grade policy that takes bio-heap leaching and concentration as processing means in parallel. In addition, it is associated with environmental recoveries and capital costs (Asad et al. 2016). This study depicts a performance evaluation of proposed models using realistic case studies. Rahimi et al. (2015b) covers the development of optimal cut-off grade strategy considering the environmental costs to accommodate low-grade mining operations (Asad et al., 2016). The environmental responsive hydrometallurgical methods are used in this study; in addition to the implementation of mathematical models and projected algorithm for a case study, which prove to be an increase in the NPV in comparison to the basic Lane's approach (Asad et al., 2016).

Lane (1984) followed by Lane (1988) propose an important extension to the novel Lane (1964) model, with the introduction of methods to calculate cut-off grades for multiple economic minerals present in the mineral deposits, whereas, Lane (1964) and the discussions above explains single economic mineral present in the deposit. According to Lane (1984), the procedure of calculating the cut-off grade for a single economic mineral is valid in multiple minerals' mining operations. Such calculation becomes unrealistic if multiple mineral grades are converted to single equivalent grade (Osanloo and Ataei, 2003), specifically if any of the minerals is subject to market demand constraint. Solving this problem, Lane (1984) and (1988) exploit the grid search (GS) technique which is further extended by many researchers.

Dagdelen and Asad (1997) implements the grid search (GS) method for cut-off grade policy using sensitivity analysis with variable production capacities, whereas Cetin and Dowd

(2002) made a comparison for GA based search method for defining cut-off grades with GS method mentioned in Lane's approach.

Osanloo and Ataei (2003) employs equivalent grade factor in Lane's approach and golden search method for deliberating cut-off grade policy, followed by Osanloo and Ataei (2003a) where golden search method (without using equivalent grade factor) is implemented (Asad et al., 2016), whereas, Osanloo and Ataei (2003b) covers the implementation of GA, GS, and equivalent grades methods in the Lane's approach using a case study of lead-zinc ore-body; extended by Osanloo and Ataei (2004) which use Lane's model through algorithmic structure with the combination of GA and GS, resulting in optimal cut-off grades (Asad et al., 2016).

Asad (2005) introduces stockpiles in multi-mineral deposits and solves this through an algorithmic method using GS and implements Lane's model, whereas Cetin and Dowd (2013) improves Lane's model using GS in the multi-mineral deposits. Nieto and Zhang (2013) presents a modification in Lane's approach using equivalent grade distribution and present valued sensitivity analysis that incorporates price variation of secondary mineral.

2.6. Stochastic cut-off grade models:

The context above shows that in both the breakeven and Lane's model, deterministic values of the metal selling price and grade-tonnage distribution are considered (Asad et al., 2016); whereas, realistically it does not remain the same over the life of mining operation and it is subject to variation depending on the demand and supply of a particular metal. Sometimes, the changes in prices are enormous due to the economic recession (Abdel Sabour and Dimitrakopolous, 2011). The production targets are affected with the variable metal content present in the ore which leads to uncertain supply of ore for processing. This study also discusses about the pre-mature shutting down of mining operations due to the poor estimation of the ore and its metal content (Baker and Giacomo, 1988; Vallee, 2000; Asad et al., 2016).

Dimitrakopolous (2011) defines the importance of stochastic models to cater the market and grade uncertainties, as constant inputs of metal prices and grade-tonnage distribution seems to be unviable under such scenarios. Asad and Dimitrakopolous (2013) derives a framework for addressing stochasticity in the grade-tonnage distribution using equal probable realizations and develops a unique cut-off grade policy which addresses low-grade ore bodies; whereas Goodfellow and Dimitrakopolous (2016) develop stochastic models using equally probable realization of metal price and/or grade-tonnage distribution (Asad et al., 2016).

There are several other contributions towards stochastic models for cut-off grade optimisation, Dowd (1976) is the initial contribution that shares a programme model which shows dynamicity and stochasticity, Krautkraemer (1988) shares a hypothetical stochastic model considering anticipated increase or decrease in metal price (Asad et al., 2016). Mardones (1993) develops a cut-off grade strategy using an option valuation approach under market price uncertainty, whereas, Cairns and Shinkuma (2003) presents a model to address the impact of a changing the price on cut-off grades (Asad et al., 2016).

Johnson et al. (2011) develops a mathematical model using partial differential equations that generate dynamic cut-off grades under market uncertainty. Azimi et al. (2012) delineates a comparison based on analysis using real options' evaluations and discounted cash flow. Li and Chang (2012) consider uncertainty in grades and develop a model for calculating the cut-off grades as a multi-stage stochastic programming model. Thompson and Barr (2014) incorporate uncertain selling price and solve the cut-off grade optimisation problem using a numerical approach considering it as a structure of non-linear differential equations (Asad et al., 2016).

2.7. Mathematical programming models:

The heuristic nature of Lane's model that does not follow realistic block model, leads to the development of an approach or mathematical model which takes realistic block model as input, delivers optimal cut-off grade optimisation solution to the mining problem (Dagdelen and Kawahata, 2008; Ganguli et al., 2011; Yasrebi et al., 2015). Mathematical based modelling is used in production scheduling problems, where the solutions are derived through exact approach (methodology to solve MILP based mathematical model), leads toward the development of ultimate pit and production sequences over the life of mining operation. Linear programming (LP) and mixed integer linear programming MILP based models are the most common models for production scheduling.

Dagdelen and Kawahata (2007, 2008) present mathematical formulation using MILP based models for defining optimal cut-off grade for open-pit mining operations (Asad et al., 2016), comprises of multi-sources, multi-destinations (waste dumps, stockpiles, and processing plants). Dagdelen and Kawahata (2007) discusses different scenarios where MILP formulation can be applied successfully, whereas Dagdelen and Kawahata (2008) deploy MILP model on multiple processing streams (run-of-mine leach, crushes one leach, floatation circuit with concentrates fed to autoclave mill, and direct feed to autoclave mill) (Asad et al., 2016), with the development of production schedules, and a schedule of dynamic cut-off grades (Asad et al., 2016). A performance evaluation of cut-off grade policies with stockpiles inclusive leads to further operational intricacies (Asad et al., 2016), due to the increase in the number of variables and their solution time.

Ganguli et al. (2011) uses the theoretical MILP structure bases on cut-off grade policy developed by Dagdelen and Kawahata (2007, 2008), while taking grade-tonnage distribution as input in addition to economic parameters, maximises NPV of the operation, subject to constraints (reserve, mining, milling, blending, precedence) over the life of mining operation. Moosavi et al. (2004) employs an MILP model that develops solution for production sequences and dynamic cut-off grade optimisation problems, which minimizes economic losses subject to constraints (reserve, mining, milling, blending, precedence); whereas the computational intricacies are not discussed in this study. On the other hand, Yasrebi et al. (2015) and Hustrulid (2013) deploy a non-linear model for cut-off grade optimisation, which shows no improvement in NPV when compared with Lane's optimal cut-off grade models (Asad et al., 2016).

While MILP based cut-off grade optimisation models derive a schedule of cut-off grades over a life of mining operation, several MILP models (Dagdelen and Johnson, 1986; Ramazan, 2007; Ramazan and Dimitrakopoulos, 2007; Newman et al., 2010; Ramazan and Dimitrakopoulos, 2012, Topal and Ramazan, 2010; Lamghari and Dimitrakopoulos, 2012) are available that offer a block-by-block and period-by-period production schedule for open-pit mining operations. The common factor among these models is given an ore-body model, whereas economic parameters are applied to derive economic block models which then become input to the MILP formulation. Breakeven cut-off grade is thus inherently considered as part of this procedure, which dictates the division of mining blocks into ore (block economic value $V > 0$) and waste blocks ($V \leq 0$). MILP based production scheduling formulation then generates optimum production sequence (where NPV is maximised) considering all its significant parameters (geological, economic, operational and slope) (Newman et al., 2010).

The parameters and variables used in MILP formulation for production scheduling are economic value (V_i) of each block i, where I is the total number of blocks (i =*1, 2, …. I*), discount rate (d), binary variable (X_{it}) equal to 1 if block is mined in period t or 0 otherwise, T is the total number of years (life of mining operation i.e. t =*1, 2, ….* T) and q_i is the available quantity of material for a block i. The breakeven cut-off grade is inherently part of the calculation as it helps in defining the block values as $V = (s - r)gy - m - c$ as a function of total tonnage of the block (Newman et al., 2010).

The objective is to maximise the overall net present value (NPV) using the sum of discounted economic block values presented in Equation (2.18) subject to reserve constraint, slope constraint, mining capacity constraint and milling capacity constraint as shown in Equation 2.19 to 2.22.

$$Max\ Z = \sum_{t=1}^{T} \sum_{i=1}^{I} \frac{V_i}{(1+d)^t} X_{it} \tag{2.18}$$

<u>Subject to:</u>

$$\sum_{t=1}^{T} X_{it} \leq 1 \qquad \forall\ i\ , t \tag{2.19}$$

$$X_{it} - \sum_{t=1}^{t}\ X_{jt} \leq 0 \qquad \forall\ t\ , \forall\ j \in N = set\ of\ overlying\ blocks \tag{2.20}$$

$$\sum_{i=1}^{I} q_i X_{it}\ \ \leq Mcap_t \qquad \forall\ t \tag{2.21}$$

$$\sum_{i=1}^{I} q_i X_{it}\ \ \leq Pcap_t \qquad \forall\ i\ \{\ g_i > 0\} \tag{2.22}$$

Despite having said, that linear programming and MILP formulation generates an optimum solution to open-pit mine planning problem for production scheduling and cut-off grade optimisation problems, it leads to complexity for large scale models. Although, previous studies aim to achieve exact solution using MILP formulation both for production scheduling and cut-off grade optimisation, but considering realistic block models (consisting of hundreds and thousands of blocks) as inputs, the solution becomes computationally inefficient and leads to complexity. To overcome this problem, metaheuristics are introduced based on the biological and natural sciences, as an alternative solution to the exact approach, which helps in attaining near optimum solutions in a reasonable time. The following section discusses an overview on the metaheuristics in detail.

2.8. Metaheuristics:

The metaheuristics are defined as follows (Voss, 2001; Osanloo et al., 2004):

> "A meta-heuristic is an iterative master process that guides and modifies the operation of subordinate heuristics to efficiently produce high-quality solutions. It may manipulate a complete single solution or a collection of solutions at each iteration. The subordinate heuristics may be high-level procedures or a single local search, or just a construction method. The family of the metaheuristics includes, but not limited to, Tabu Search, Ant Systems, Greedy Randomized Adaptive Search, Variable Neighborhood Search, Genetic Algorithms, Scatter Search, Neural Networks, Simulated Annealing and their hybrids."

Denby and Schofield (1994) and Denby et al. (1998) used a GA for optimisation problems both for the open-pit and underground mine operations. The following procedure of GA is summarised by Osanloo et al. (2008), as some algorithms support extraction scheduling and cut-off grade optimisation.

1. "*Generation of random pit population;*
2. *Assessment of fitness function, which can be used to assess the suitability of a produced solution. A typical fitness function includes: maximising NPV, minimizing*

early stripping, balancing stripping and balancing ore production of multiple minerals;

3. *Reproduction of pit population using probabilistic techniques;*
4. *Crossover of pits such that between 40 and 60% of the schedules are crossed over;*
5. *Mutation of pits with a probability between 1 and 5% ;*
6. *Normalization of pits to ensure that extraction constraints are not violated;*
7. *Local optimisation of pits to improve the fitness of individual schedules;*
8. *Stopping condition is met when n generations (between 20 and 40) have occurred without any improvement in the best schedule;"*

GA generates better optimisation results in an acceptable time, flexible and effective for UPL and production planning problems (Osanloo, 2008). The results differ in each run of this program, after several generations, due to the stochastic nature of GA (Osanloo, 2008). On the other hand, it also ignores the effect of pit volume on the unit cost of mining. The flow chart of GA is shown in Figure 2.4.

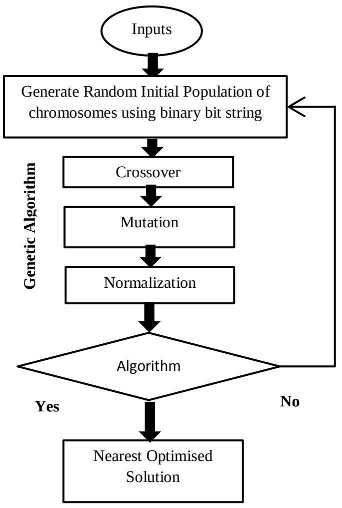


Figure 2.7: Flow chart showing optimised solution using genetic algorithm (Source: Osanloo, 2008)

The cut-off grade of a metalliferous deposit is dynamic in nature and dynamic programming (DP) approach is quite suitable for cut-off grade determination problems (Mishri, 2006). Davey (1979) states that production forecast for an operational property is based on the grade at which the minerals can no longer be processed for profit. DP method discusses where a cut-off grade is calculated as a variable (Dowd, 1976), and can also be extended to a more general case of stochastic programming, which permits future market value in probabilistic terms. Mishri (2006) uses a computer tool in dynamic programming based on mathematical models to solve cut-off grade problems in smaller stages.

Sattarvand and Neimann-Delius (2008) presents three cost components including penalty costs (stockpiling) for production out of required tonnage limit, an average metal content cost for a period exceeding prescribed limits and the cost incurred due to non-uniformity of production. These three cost components are transferred to a single objective problem using a weighing scheme depending on ore-body, sales structure and plant characteristics (Sattarvand and Neimann-Delius, 2008). SA Kumral and Dowd (2005) uses Langragian parameterization for an initial solution proposed by (Dagdelen, 1985). Perturbation and cooling schedule are defined after the initial solution to get the acceptable solutions (Sattarvand and Neimann-Delius, 2008; Kumral and Dowd, 2005).

Artificial ants traveling through the schedule array are used to construct a population of scheduling solution (Sattarvand and Neimann-Delius, 2008), which can also be used for cut-off grade optimisation, where pit depths in each planning period are used in terms of integer variables. A pheromone update procedure runs and either extra reinforcement is given to the best- scheduled blocks or identifying the best ants to deposit pheromones (Sattarvand and Neimann-Delius, 2008; Dorigo and Sttuetzle, 2004). ACO algorithm depicted by Sattarvand (2009) is shown in Figure 2.8.

Initialise (set parameters)
Repeat (loop)
  Each Ant is located on the primary node
  Repeat (loop)
    Each ant employs a state move rule to increase the solution
    Apply the pheromone local update rule
  Until (loop ends)
    All the ants made a complete solution
  Apply a local search method
  Apply the pheromone global update rule
Until (Loop ends)
  the stopping condition is fulfilled

Figure 2.8: Procedure involved in ACO Algorithm (Source: Sattarvand, 2009)

Many other metaheuristics like Tabu Search (TS) (Lamghari and Dimitrakopoulos, 2012; Glover and Laguna, 1997) and Particle Swarm Optimisations (PSO) are being used extensively in the field of open-pit mine optimisation (Sattarvand and Neimann-Delius, 2008). TS uses a neighbourhood search procedure to iteratively move from one solution to another neighbourhood until stopping conditions are achieved where PSO is stochastic and population-based evolutionary algorithm (Sattarvand and Neimann-Delius, 2008; Osanloo et al., 2008), that uses an iterative process of evaluation of fitness while considering the location of the block for the best solution.

Models based on a combination of artificial intelligence (AI) techniques have been articulated (Askari-Nasab, 2006; Askari-Nasab and Awuah-Offei, 2009; Denby et al., 1996; Askari-Nasab et al., 2010; Tolwinsiki and Underwood, 1996). Askari-Nasab et al. (2010) is of the opinion that there is no quality measure to solutions provided through heuristics and AI, comparing against the optimum.

Myburgh et al. (2014) introduce hybrid evolutionary algorithm based engine with one “master” algorithm which manages variation of cut-off grade and extraction sequence while the other two “slaves” or low-level optimisation algorithms which consist of LP algorithm and search technique. First determines an optimal flow of material through multiple processing streams while managing stockpile policy and the second finds the fittest schedule.

Following the same theory, Maptek (2014) introduces "Strategy Engine" shown in Figure 2.9 as follows:

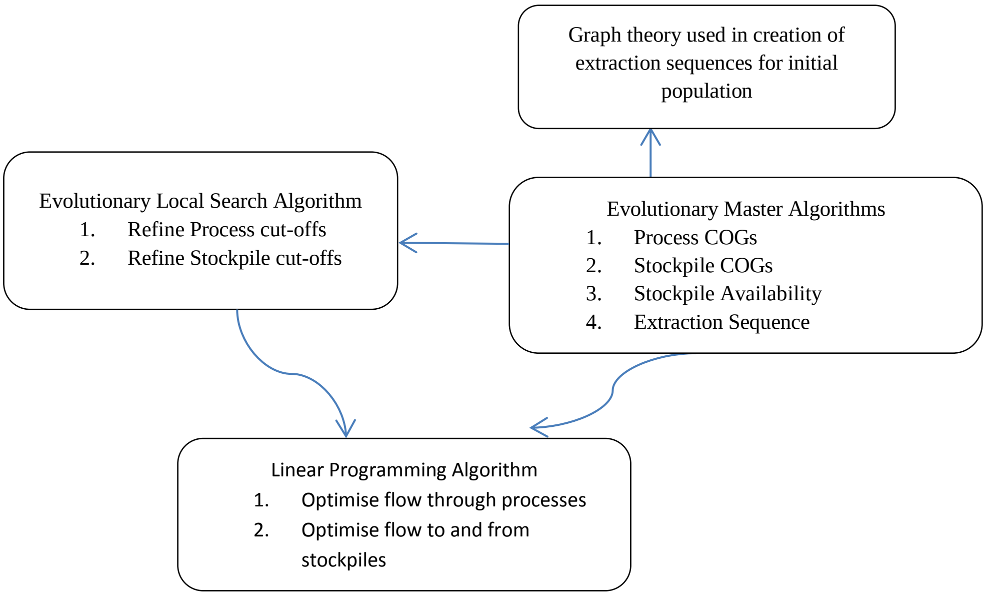


Figure 2.9: Strategy Engine (Source: Maptek 2014)

An extensive and comprehensive literature review regarding cut-off grade optimisation in open-pit mining operations mentioned above shows that there is a need to develop a realistic cut-off grade policy which generates optimum cut-off grades over the operational life of a mine. As discussed earlier, the breakeven cut-off grade model and Lane's optimal cut-off grade policy are restricted to certain conditions, e.g. breakeven model does not consider mining, milling and processing constraints and gives constant mining cut-off grade over the operational life of mine, whereas Lane's optimal policy considers grade-tonnage distribution as input to estimate dynamic processing cut-off grades. The grade-tonnage distribution is unrealistic in view that metal content is not uniformly distributed throughout the deposit. The grade is location dependent, that is, each mining block in the ore-body model constitutes a unique grade. Thus, the variation in grade from one mining block to the other cannot be disregarded in the grade-tonnage curve. Subsequently, the optimal cut-off grade problem, for long-term schedules extracted from the grade-tonnage based normally distributed inputs, remains unresolved. Therefore, this research develops and implements a strategy where short-term and long-term plans are synchronised. The new MILP based mathematical model for defining cut-off grade optimisation policy is defined in the forthcoming sections, which is first solved using exact approach, and later solved using hybrid-metaheuristic. The hybrid-metaheuristic algorithm is introduced, developed and implemented to overcome the complexity of the exact solution in this research and later the results are analysed and evaluated in the form of case studies.

Summary:

This chapter provides a detailed overview of different cut-off grade optimisation models and policies developed and implemented in the previous studies. Two main cut-off grade models, i.e. breakeven and Lane's model with their applied extensions are discussed in detail in the literature review. Later, linear programming and MILP based mathematical models for optimisation are also cited in this chapter. MILP based mathematical model for production scheduling optimisation problem is discussed while mentioning its objective function and constraints considering geological block model, economic parameters and operational capacities as inputs. Keeping in mind the complexities involved in solving mathematical models using exact solution, the literature discusses metaheuristics and hybrid-metaheuristics as an alternative solution to the problem.

Chapter 3: Mixed integer linear programming (MILP) model

The new MILP based mathematical is unique as it maximises NPV using annual cash flows subject to the precedence or slope, production capacity and grade blending constraints. This model takes a three-dimensional ore-body model and economic parameters as inputs and generates a schedule of dynamic cut-off grades as well as block-by-block and period-by-period sequence of production. The general problem for cut-off grade optimisation is mathematically presented as follows:

$$Maximizing\ NPV$$

Subject to

$$Quantity\ of\ Material\ (Q_m) \leq Mining\ Capacity (M)$$

$$Quantity\ of\ Ore\ (Q_c) \leq Processing\ Capacity\ \text{(C)}$$

$$Quantity\ of\ Metal\ Product\ \ (Q_r) \leq Refining\ Capacity\ \ (R)$$

3.1. Inputs to the mathematical model:

An ore-body model is a three dimensional array of fixed size thousands of blocks (Osanloo et al., 2008; Limghari and Dimitrakopolous, 2012; Asad et al. 2016). This ore-body model is considered as a geological resource, where each mining block is defined by its spatial location (XYZ coordinates), metal content (grade) and quantity of material (tonnage). The grade for each block is assigned from the geological data collected from drill cores and by using any of inverse distance weighted interpolation, weighted moving average or kriging techniques (Osanloo et al., 2008).

3.1.1. Economic parameters:

Economic parameters comprise of the market price of metal, mining cost, milling cost, and metallurgical recovery (yield). These parameters normally decide the economic value of each block present in the ore-body model in the conventional production scheduling problem, and depend on the metal content (grade) present in the block. In the new MILP formulation which defines cut-off grade optimisation policy and it does not consider economic block values, and the parameters mentioned above are used to estimate cash flow as part of the formulation. All these parameters are taken as input on yearly basis. As the deterministic block model is considered in this research, therefore, the prices are considered constant over the life of mining operation.

3.2. New optimal cut-off grade model for an open-pit mining operation:

The new cut-off grade formulation is designed and implemented for 2D and 3D block models considering block-by-block realizations as inputs. Table 3.1 discusses parameters and variables used in the MILP formulation.

3.2.1. Objective function:

The objective is to maximise net present value (NPV) using undiscounted cash flow P_t, where P_t is computed as part of MILP formulation (Equation 3.2):

$$Max\ Z = \sum_{t=1}^{T} \frac{P_t}{(1+d)^t} \tag{3.1}$$

3.2.2. Cash flow constraint:

The cash flow P_t in period or year t is determined using Equation (3.2). The values for Qm_t and Qc_t and Qr_t are determined during simulation using mining, processing and milling constraints.

$$P_t - (p - r)Qr_t - mQm_t - cQc_t - FC_t = 0 \qquad \forall\, t \qquad (3.2)$$

$\boldsymbol{i}$	Block index
$\boldsymbol{t}$	Period or year index
$\boldsymbol{T}$	Total life of mining operation, where t =*1, 2… T*
$\boldsymbol{p}$	Price of metal per tonne or ounce or gram of metal (in $/tonne or $/gram)
$\boldsymbol{r}$	Refining cost per tonne or ounce or gram of metal (in $/tonne or $/gram)
$\boldsymbol{m}$	Mining cost per tonne of material (in $/tonne)
$\boldsymbol{c}$	Processing cost per tonne of ore (in $/tonne)
$\boldsymbol{d}$	Discount rate (in %)
$\boldsymbol{y}$	Metallurgical recovery (in %)
$\boldsymbol{g_i}$	Grade (metal content) of block i (in %, ounce or gram of metal)
$\boldsymbol{q_i}$	Quantity of material of block i (in tonnes)
$\boldsymbol{Mcap_t}$	Mining capacity in year or period t (in tonnes)
$\boldsymbol{Pcap_t}$	Processing capacity of a processing plant in period or year t (in tonnes)
$\boldsymbol{Z}$	Net present value (NPV) (in $)
$\boldsymbol{P_t}$	Profit or cash flow in a period or year t (in $)
$\boldsymbol{Qm_t}$	Quantity of material mined in a period or year t (in tonnes)
$\boldsymbol{Qc_t}$	Quantity of ore processed in a period or year t (in tonnes)
$\boldsymbol{Qr_t}$	Quantity of metal refined in a period or year t (in tonnes)
$\boldsymbol{\overline{g}_t}$	Average grade of ore in time period or year t (in %, ounce or gram of metal)
$\boldsymbol{COG_t}$	Cut-off grade of ore blocks mined in a period or year t (in %, ounce or gram of metal)
$\boldsymbol{CO_{LB}}$	Cumulative quantity of material in blocks with grade less than or equal to COG_t (lower bound) (in tonnes)
$\boldsymbol{CO_{UB}}$	Cumulative quantity of material in blocks with grade greater than or equal to COG_t (upper bound) (in tonnes)
$\boldsymbol{X_{it}}$	Binary variable which is equal to 1 if block is mined in a period or year t and 0 otherwise

Table 3.1: Grid of parameters and variables defined for mathematical model

3.2.3. Refining constraint:

Refining constraint ensures that the quantity of metal refined in period t remains within the refining capacity. Qr depends on the quantity of ore processed Qc_t , average grade $\overline{g}_t$ and metallurgical recovery y ($Qr = Qc_t \times \overline{g}_t \times y$) as presented in Equation 3.3.

$$Qr_t - \sum_{i=1}^{I} q_i X_{it} \times \overline{g}_t \times y = 0 \qquad \forall\, i\ \{g_i \geq 0\}, t \qquad (3.3)$$

Where

$$\overline{g}_t = \frac{\sum_{i=1}^{I} g_i q_i X_{it}}{\sum_{i=1}^{I} q_i X_{it}} \qquad \forall\, i\ \{g_i \geq 0\}, t$$

$\overline{g}_t$ is computed as weighted mean of the ore blocks mined in a specific time period. Placing the value of $\overline{g}_t$ in Equation 3.3:

$$Qr_t - \sum_{i=1}^{I} q_i X_{it} \times \frac{\sum_{i=1}^{I} g_i q_i X_{it}}{\sum_{i=1}^{I} q_i X_{it}} \times y = 0 \qquad \forall\, i\ \{g_i \geq 0\}, t$$

$$Qr_t - \sum_{i=1}^{I} g_i q_i X_{it} \times y = 0 \qquad \forall\, i\ \{g_i \geq 0\}, t \qquad (3.4)$$

3.2.4. Mining capacity constraint:

Mining capacity constraint has two parts, first is the quantity of the material mined Qm_t in the time period t must not exceed mining capacity, and second Qm_t must be equal to the cumulative quantity of the material in all the blocks mined in the time period t as shown in Equations 3.5 and 3.6 respectively.

$$Mcap_{t\_lb} \leq Qm_t \leq Mcap_{t\_ub} \qquad for\ \forall\, t \qquad (3.5)$$

$$Qm_t - \sum_{i=1}^{I} q_i X_{it} = 0 \qquad for\ \forall\, i, t \qquad (3.6)$$

3.2.5. Processing capacity constraint:

The processing constraint shows that the quantity of the ore processed Qc_t in the specific time period t must not exceed processing capacity, and secondly Qc_t must be equal to the cumulative quantity of ore ($g_i \geq COG_t$) in the time period t as shown in Equation 3.7 and Equation 3.8 respectively.

$$Pcap_{t\_lb} \leq Qc_t \leq Pcap_{t\_ub} \qquad for\ \forall\ t \qquad (3.7)$$

$$Qc_t - \sum_{i=1}^{I} q_i X_{it} \quad = 0 \qquad for\ \forall\ i\ \{g_i \geq 0\ \}, t \qquad (3.8)$$

3.2.6. Reserve constraint:

The reserve constraint ensures that each block is mined once during the life of the mining operation. It can be mathematically shown as follows:

$$\sum_{t=1}^{T} X_{it} \leq 1 \qquad for\ \forall\ i\ , t \qquad (3.9)$$

3.2.7. Precedence or slope constraint:

The condition for each block i to be mined in any period t, a set N of the overlying blocks which is also termed as set of predecessors in the X, Y and Z location must be mined prior to or in the same period t. This is mathematically shown in Equation 3.10 as a slope constraint.

$$X_{it} - \sum_{t=1}^{t} X_{jt} \leq 0 \qquad for\ \forall\ t,\ \forall\ j \in N \qquad (3.10)$$

3.2.8. Cumulative ore tonnage constraint:

The cumulative ore tonnage constraint helps establish the cut-off grade for a period or year t at different mining sequences, which means a value where neither of mine or processing plant bottlenecks the mining operation.

$$COG_t \times \sum_{i=1}^{I} q_i X_{it} \quad \geq \quad CO_{LB} \qquad for\ \forall\ i\ ,t \tag{3.11}$$

$$COG_t \times \sum_{i=1}^{I} q_i X_{it} \ \leq \quad CO_{UB} \qquad for\ \forall\ i\ ,t \tag{3.12}$$

$$where \quad CO_{LB} = \sum_{i=1}^{I} g_i q_i X_{it} \qquad for\ \forall\ i\,\{0 < g_i \leq\ COG_t)$$

$$where \quad CO_{UB} = \sum_{i=1}^{I} g_i q_i X_{it} \qquad for\ \forall\ i\,\{g_i\ \geq\ COG_t)$$

3.3. Summary of new MILP based cut-off grade formulation:

The summary of the formulation is presented in Equations (3.13 to 3.26).

$$Max\ Z = \sum_{t=1}^{T} \frac{P_t}{(1+d)^t} \tag{3.13}$$

Subject to

$$P_t - (p-r)Qr_t - mQm_t - cQc_t - FC_t = 0 \qquad \forall\ t \tag{3.14}$$

$$Mcap_{t\_lb}\ \leq Qm_t \leq Mcap_{t\_ub} \qquad for\ \forall\ t \tag{3.15}$$

$$Qm_t - \sum_{i=1}^{I} q_i X_{it} \quad = 0 \qquad for\ \ \forall\ i\,,t \tag{3.16}$$

$$Pcap_{t\_lb}\ \leq Qc_t \leq Pcap_{t\_ub} \qquad for\ \forall\ t \tag{3.17}$$

$$Qc_t - \sum_{i=1}^{I} q_i X_{it} \quad = 0 \qquad for\ \forall\ i\ \{g_i \geq COG_t\ \},t \tag{3.18}$$

$$Qr_t - \sum_{i=1}^{I} g_i q_i X_{it}\ \ \times y\ \ = 0 \qquad \forall\ i\,\{g_i \geq COG_t\},t \tag{3.19}$$

$$\text{Where} \quad \overline{g}_t = \frac{\sum_{i=1}^{I} g_i q_i X_{it}}{\sum_{i=1}^{I} q_i X_{it}} \qquad \forall\ i\ \ \{g_i \geq COG_t\ \},t \tag{3.20}$$

$$X_{it} - \sum_{t=1}^{t} X_{jt} \leq 0 \qquad for\ \forall\ t,\ \forall\ j \in N \tag{3.21}$$

$$\sum_{t=1}^{T} X_{it} \leq 1 \qquad for\ \forall\ i\ ,t \qquad (3.22)$$

$$COG_t \times \sum_{i=1}^{I} q_i X_{it} \quad \geq \quad CO_{LB} \qquad for\ \forall\ i\ ,t \qquad (3.23)$$

$$COG_t \times \sum_{i=1}^{I} q_i X_{it} \ \leq \quad CO_{UB} \qquad for\ \forall\ i\ ,t \qquad (3.24)$$

$$where \quad CO_{LB} = \sum_{i=1}^{I} g_i q_i X_{it} \qquad for\ \forall\ i\,\{0 < g_i \leq\ COG_t) \qquad (3.25)$$

$$where \quad CO_{UB} = \sum_{i=1}^{I} g_i q_i X_{it} \qquad for\ \forall\ i\,\{g_i\ \geq\ COG_t) \qquad (3.26)$$

3.3.1 Structure of the new MILP formulation for cut-off grade optimisation:

The MILP based new formulation for cut-off grade optimisation computes the simultaneous estimation of cut-off grades and production sequences; where undiscounted cash flows are estimated as part of MILP formulation to define the objective function. This new cut-off grade formulation is unique in a sense that the objective function which maximises NPV uses cash flow $P_t = (p - r)Qr_t - mQm_t - cQc_t - FC_t$ instead of economic block value (thus ignores breakeven cut-off grades), whilst estimating P_t as a function of quantity of material mined in time t (Qm_t), quantity of ore processed in time t (Qc_t), and quantity of metal refined in time t (Qr_t), in addition to the binary decision variable X_{it} representing a value equals to 1 or 0 depending on whether the respective block i is mined in period or year t or not. Mining, processing and refining capacity constraints are defined in terms of grade g_i for each block other than the slope, and reserve constraints in this formulation. The yearly weighted average grade $\overline{g}_t$ is also computed as part of this formulation as a function of grade g_i for each block. As refining capacity is equal to the product of processing capacity, average ore grade and metallurgical recovery ($Qr_t = Qc_t \times \overline{g}_t \times y$) which is estimated simultaneously depending on the number of blocks mined, whilst running the simulation for the new MILP formulation.

Cumulative ore tonnage constraint is introduced in this formulation to determine cut-off grade COG_t for each period or year t over the life of mine operation. In addition, it indirectly controls the Qc_t and Qr_t constraints. The MILP formulation includes these cumulative tonnage constraints through a random selection of grades available in the ore-body model; however, the minimum value of these randomly selected grades may not be lower than the processing plant required head grade. Cumulative lower bound CO_{LB} and cumulative upper bound tonnage CO_{UB} is defined in this constraint. Cut-off grade for the blocks mined in year t is the *maximum* grade (must be more than or equal to processing plant head grade) value which when multiplied with the sum of the quantities of all ore blocks (with $g_i \leq COG_t$) mined in time period t must be greater than the cumulative quantity of ore blocks at lower bound, and must be less than the cumulative quantity of ore blocks at upper bound. Clearly, the cumulative lower and upper bound tonnage considers the ore blocks having grades lower than the cut-off grade and upper than the cut-off grade respectively.

Figure 3.1 (a) and (b) presents the structure of the precedence constraint using two dimensional (2D) and three dimensional (3D) block models respectively. Layer 1 shows the overlying blocks, whereas layer 2 shows underlying blocks within the ore-body model. The model considers 5 to 1 ratio (as defined in Equation 3.26) for precedence constraint, which infers that for each block to be mined, 5 overlying blocks are mined at XYZ location for a 3D block model as shown in the Figure 3.1 (b). The slope constraints in the formulation consider the same pattern (5:1) and they are mathematically defined in Equation 3.26.

(a) 2D block model with predecessors (b) 3D block model with predecessors

Figure 3.1: Block model with predecessors

3.3.2 Case 1: MILP based cut-off grade optimisation using 2D block model (25 blocks):

Case 1 discusses the implementation of the MILP based cut-off grade optimisation model for the one year life of mining operation considering 2D block model, which comprises of 25 blocks as shown in Figure 3.2. The problem in this case study is defined as a new MILP formulation and manually solved through step by step development and implementation of the formulation, while considering 3:1 for 2D model (for each block to be mined three overlying blocks need to be mined) slope constraint. The processing plant head grade is assumed as 0.35 for case 1.For clarity, the block index (i) and the time index (t) the format of the binary variable X_{it} is used as $X_{i..t}$ in the following solved examples.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

0	0	0	0	0
0	0.2	0.4	0.5	0
0.1	0	0.3	0.5	0
0	0.5	0.6	0	0
0	0	0.4	0	0

Figure 3.2: Hypothetical 2D (grid of 25 blocks) block model with a section map

<u>Economic and operational parameters:</u>

Parameters	value	Units
Fixed tonnage per block	150	Tonnes
Mining capacity $\boldsymbol{Mcap_t}$	1500	tonnes/year
Processing capacity $\boldsymbol{Pcap_t}$	700	tonnes/year
Discount rate $\boldsymbol{d}$	15	%
Metal price $\boldsymbol{p}$	1500	$/ tonne
Refining cost $\boldsymbol{r}$	500	$/ tonne
Milling cost $\boldsymbol{c}$	5.3	$/tonne
Mining cost m	1.25	$/tonne
Metallurgical recovery $\boldsymbol{y}$	95	%
Fixed cost $\boldsymbol{FC}$	2500	$/tonne

Table 3.2: Economic and operational parameters for 2D hypothetical model

<u>Objective function:</u>

Using Equation (3.13) and putting value of discount factor

$$Max\ NPV = \frac{P_1}{(1+0.15)^1} \quad (3.27)$$

Putting values of p, r, m and c from Table (3.2) in Equation (3.14)

$$P_1 = (1500 - 500)Qr_1 - (1.25)Qm_1 - (5.3)Qc_1 - 2500 \quad (3.28)$$

Mining capacity constraint:

Applying constraints as mentioned in Equations (3.15) and (3.16):

$$Qm_1 \leq 1500 \quad (3.29)$$

$$Qm_1 - 150(x_{1..1} + x_{2..1} + x_{3..1} + x_{4..1} + \cdots + x_{24..1} + x_{25..1}) = 0 \quad (3.30)$$

Processing capacity constraint:

Applying constraints as mentioned in Equations (3.17) and (3.18):

$$Qc_1 \leq 700 \quad (3.31)$$

$$Qc_1 - 150(x_{7..1} + x_{8..1} + x_{9..1} + x_{11..1} + x_{13..1} + x_{14..1} + x_{17..1} + x_{18..1} + x_{23..1}) = 0 \quad (3.32)$$

Calculating average grade:

Calculating weighted average grade using Equation (3.19):

$$\overline{g}_1 = \frac{150(0.2x_{7..1} + 0.4x_{8..1} + 0.5x_{9..1} + 0.1x_{11..1} + 0.3x_{13..1} + 0.5x_{14..1} + 0.6x_{17..1} + 0.5x_{18..1} + 0.5x_{23..1})}{150(x_{7..1} + x_{8..1} + x_{9..1} + x_{11..1} + x_{13..1} + x_{14..1} + x_{17..1} + x_{18..1} + x_{23..1})}$$

(3.33)

Refining capacity constraint:

Using Equation (3.19):

$$Qr_1 = \{150(x_{7..1} + x_{8..1} + x_{9..1} + x_{11..1} + x_{13..1} + x_{14..1} + x_{17..1} + x_{18..1} + x_{23.1})\} \times \overline{g}_1 \times y \quad (3.34)$$

Precedence or slope constraint:

Applying slope constraints using Equation (3.21)

$$x_{7..1} - x_{1..1} \leq 0 \quad (3.35)$$

$$x_{7..1} + x_{2..1} \leq 0 \quad (3.36)$$

$$x_{7..1} + x_{3..1} \leq 0 \quad (3.37)$$

$$x_{8..1} - x_{2..1} \leq 0 \quad (3.38)$$

$$x_{8..1} + x_{3..1} \leq 0 \quad (3.39)$$

$$x_{8..1} + x_{4..1} \leq 0 \quad (3.40)$$

$$x_{9..1} - x_{3..1} \leq 0 \quad (3.38)$$

$$x_{9..1} + x_{4..1} \leq 0 \quad (3.39)$$

$$x_{9..1} + x_{5..1} \leq 0 \quad (3.40)$$

Reserve constraint:

Reserve constraints are applied using Equation (3.22)

$$\left.\begin{array}{l} x_{1..1} \leq 1 \\ x_{2..1} \leq 1 \\ x_{3..1} \leq 1 \quad \ldots\ldots\ldots\ldots \; x_{9..1} \leq 1 \end{array}\right\} \quad (3.41)$$

Cumulative tonnage constraint:

Given the ore-body model, cumulative tonnage constraints are created using Equations 3.23 to 3.26.

Estimating value of CO_{LB} and CO_{UB} using Equation (3.25) and (3.26):

$$CO_{LB} = (q_7 g_7 x_{7..1} + q_8 g_8 x_{8..1} + q_9 g_9 x_{9..1} + q_{11} g_{11} x_{11..1} + q_{13} g_{13} x_{13..1} + q_{14} g_{14} x_{14..1} + q_{17} g_{17} x_{17..1} + q_{18} g_{18} x_{18..1} + q_{23} g_{23} x_{23..1}) \quad (3.42)$$

$$CO_{UB} = (q_7 g_7 x_{7..1} + q_8 g_8 x_{8..1} + q_9 g_9 x_{9..1} + q_{11} g_{11} x_{11..1} + q_{13} g_{13} x_{13..1} + q_{14} g_{14} x_{14..1} + q_{17} g_{17} x_{17..1} + q_{18} g_{18} x_{18..1} + q_{23} g_{23} x_{23..1}) \quad (3.43)$$

As quantity of block is considered as $q = 150\ tonnes$ for all blocks, Equations (3.42) and (3.43) can be written as follows:

$$CO_{LB} = 150(g_7 x_{7..1} + g_8 x_{8..1} + g_9 x_{9..1} + g_{11} x_{11..1} + g_{13} x_{13..1} + g_{14} x_{14..1} + g_{17} x_{17..1} + g_{18} x_{18..1} + g_{23} x_{23..1}) \quad (3.44)$$

$$CO_{UB} = 150(g_7 x_{7..1} + g_8 x_{8..1} + g_9 x_{9..1} + g_{11} x_{11..1} + g_{13} x_{13..1} + g_{14} x_{14..1} + g_{17} x_{17..1} + g_{18} x_{18..1} + g_{23} x_{23..1}) \quad (3.45)$$

Using Equation 3.23 and 3.24

$$COG_t \times (q_7 x_{7..1} + q_7 x_{8..1} + q_9 x_{9..1} + q_{11} x_{11..1} + q_{13} x_{13..1} + q_{14} x_{14..1} + q_{17} x_{17..1} + q_{18} x_{18..1} + q_{23} x_{23..1}) \geq CO_{LB} \quad (3.46)$$

$$COG_t \times (q_7 x_{7..1} + q_7 x_{8..1} + q_9 x_{9..1} + q_{11} x_{11..1} + q_{13} x_{13..1} + q_{14} x_{14..1} + q_{17} x_{17..1} + q_{18} x_{18..1} + q_{23} x_{23..1}) \leq CO_{UB} \quad (3.47)$$

Solution Process:

Following the constraints it is found from Figure 3.2, only 9 out of 25 blocks are mined in year 1. Therefore, using Equation (3.29 and 3.30) Qm_1 is found as follows:

$$Qm_1 \leq 1500$$

$$Qm_1 - 150(x_{1..1} + x_{2..1} + x_{3..1} + x_{4..1} + \cdots + x_{24..1} + x_{25..1}) = 0$$

$$Qm_1 = 150 \times 9 = 1350 \text{ Tonnes} \quad (3.48)$$

Out of total 9 blocks mined, 4 blocks have $g_1 > 0$, but it is important here to determine COG_t value which is computed from Equation (3.52) during the simulation of the formulation. Therefore, using Equations (3.31), (3.32) and (3.52) only 2 blocks ($x_{8..1}$ and $x_{9..1}$) are considered as ore with $g_1 > COG_t$ and sent for processing, and blocks with $g_1 < COG_t$ are ignored for being considered as waste.

$$Qc_1 \leq 700$$

$$Qc_1 - 150(x_{7..1} + x_{8..1} + x_{9..1} + x_{11..1} + x_{13..1} + x_{14..1} + x_{17..1} + x_{18..1} + x_{23..1}) = 0$$

$$Qc_1 - 150(x_{8..1} + x_{9..1}) = 0$$

$$Qc_1 = 150 \times 2 = 300 \text{ Tonnes} \tag{3.49}$$

Calculation of average grade:

Considering 2 blocks selected for processing and using Equations (3.33) value of $\overline{g}_1$ is calculated as follows:

$$\overline{g}_1 = \frac{150(0.4x_{8..1} + 0.5x_{9..1})}{150(x_8 + x_9)}$$

$$\overline{g}_1 = \frac{0.4+0.5}{2} = 0.45 \tag{3.50}$$

Putting value of Qc_1 from Equation (3.49), value of $\overline{g}_1$ from Equation (3.50), and value of y from Table 3.1 in Equation (3.25)

$$Qr_1 = \{150(x_{8..1} + x_{9..1})\} \times \overline{g}_1 \times y$$

$$Qr_1 = 300 \times 0.45 \times 0.95$$

$$Qr_1 = 128.25 \text{ Tonnes} \tag{3.51}$$

Computing COG_t through cumulative tonnage constraints:

The value of COG_t is selected and checked against the cumulative tonnage constraint. The grade which satisfies all the given conditions of the constraint is termed as COG_t of that year t, else if the condition at any given period t of the constraint are satisfied at the value less than processing plant head grade then plant head grade is considered as COG_t of that particular year.

@ Cumulative tonnage constraint for COG_1 = 0.5

Using Equations (3.44) and (3.45) to find CO_{LB} and CO_{UB} respectively while considering only the ore blocks ($where\ 0 < g_i \leq COG_t$) for CO_{LB} and ore blocks ($where\ g_i \geq COG_t$) for CO_{UB}, which are selected for mining in the first production sequence.

$$CO_{LB} = 150(g_7 x_{7..1} + g_8 x_{8..1} + g_9 x_{9..1} + g_{13} x_{13..1})$$

$$CO_{UB} = 150(g_9 x_{9..1} x_{9..1})$$

$$CO_{LB} = 150(0.2 \times 1 + 0.3 \times 1 + 0.4 \times 1 + 0.5 \times 1) = 210 \text{ tonnes}$$

$$CO_{UB} = 150(0.5 \times 1) = 75 \text{ Tonnes}$$

Considering all ore blocks mined in the first production sequence and putting the value of $COG_t = 0.5$, and $q = 150\ tonnes$, and using Equations (3.46) and (3.47):

$$0.5 \times 150(x_{7..1} + x_{8..1} + x_{9..1} + x_{13..1}) \geq 210$$

$$0.5 \times 150(4) \geq 210$$

$$300 \geq 210$$

Using Equations (3.47)

$$0.5 \times 150(4) \leq 75$$

$0.5 \times 600 \leq 75$

$300 \leq 75$ *(Constraint in Equation 3.24 is violated)*

@ Cumulative tonnage constraint for COG_1 = 0.4

Estimating value of $CO_{LB}$and CO_{UB} using Equation (3.44) to (3.47) and repeating the same process mentioned above while selecting COG_1 = 0.4:

$CO_{LB} = 150(\, 0.2 \times 1 \, + 0.3 \, \times 1 + 0.4 \, \times 1) = 135$ Tonnes

$CO_{UB} = 150(\, 0.4 \, \times 1 + 0.5 \times 1) = \, 135$ Tonnes

$0.4 \, \times 600 \, \geq 135 \; implies \; 240 \geq 135$

$0.4 \, \times 600 \leq 135 \; implies \; 240 \leq 135$ *(Constraint in Equation 3.24 is violated)*

@ Cumulative tonnage constraint for COG_t = 0.3

Estimating Value of CO_{LB} and CO_{UB} using Equation (3.44) to (3.47) and repeating the same process:

$CO_{LB} = 150 \; (\, 0.2 \times 1 \, + 0.3 \, \times 1) = 75$ Tons

$CO_{UB} = 150 \; (\, 0.4 \, \times 1 + 0.5 \times 1 + 0.3 \, \times 1)$

$= \, 180$ Tonnes

Putting the values of COG_t, CO_{LB}, CO_{UB} and $\sum_{i=1}^{I} q_i X_{it}$ in Equations (3.23) and (3.24)

$0.3 \, \times 600 \geq 75 \;\; implies \; 180 \geq 75$ (Condition satisfied)

$0.3 \;\; \times 600 \leq 180 \; implies \; 180 \leq 180$ (Condition satisfied)

It is found that cut-off grade is equal to 0.3, as the constraint is violated for $COG_t > 0.3$. Therefore, through random selection of grades it is found that COG_t = 0.30 satisfies the cumulative tonnage constraint, but for this case minimum processing head grade is assumed as 0.35, so grade below 0.35 will not be considered for processing, which infers COG_t to be equal to the processing plant head grade as mentioned in Equation (3.52)

$$COG_1 = 0.35 \tag{3.52}$$

Putting values of Qm_1, Qc_1 and Qr_1 from Equations (3.42), (3.49) and (3.51) respectively, in Equation (3.28)

$$P_1 = (1000)Qr_1 - (1.25)Qm_1 - (5.3)Qc_1 - 2500$$

$$P_1 = 1000 \times 128.25 - 1.25 \times 1350 - 5.3 \times 300 - 2500$$

$$P_1 = \$122{,}472.50 \tag{3.55}$$

Putting value of P_1 from Eq. (3.55) in Eq. (3.13), $Max\ NPV$ can be computed as follows:

$$Max\ NPV = \frac{122{,}472.50}{(1+0.15)^1} = \$106{,}497.83 \tag{3.56}$$

Compiling all the values for $\boldsymbol{COG_1}$, $\boldsymbol{\overline{g}_1}$, $\boldsymbol{Qm_t}$, $\boldsymbol{Qc_t}$, $\boldsymbol{Qr_t}$, $\boldsymbol{P_1}$ and $\boldsymbol{NPV}$ from Equations (3.52), (3.50), (3.48), (3.49), (3.51), (3.55) and (3.56) respectively are shown in Table 3.3.

Year (t)	COG_t (% Cu)	$\overline{g}_t$ (% Cu)	Qm_t **(tonnes/year)**	Qc_t **(tonnes/year)**	Qr_t **(tonnes/year)**	P_t **($)**	NPV_t **($)**
1	0.35	0.45	1350	300	128	122,472	106,497

Table: 3.3: Results obtained from solving 2D model using new MILP based formulation

3.3.3 Case 2: MILP Cut-off grade optimisation using 2D block model (100 blocks)

Using economic and operational parameters as shown in Table 3.1

Life of mining operation = 3 years

Milling head grade (assumed) = 0.4

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 3.3 (a) Number of blocks in a hypothetical geological Block model

0	0	0	0	0	0	0	0	0	0
0	0	0.3	0.5	0.6	0	0	0	0	0
0	0	0	0.5	0.4	0.6	0	0	0	0
0	0	0	0.3	0.5	0.6	0	0	0	0
0	0	0	0	0.3	0.5	0.6	0	0	0
0	0	0	0	0.3	0.4	0.5	0	0	0
0	0	0	0	0.4	0.3	0.5	0	0	0
0	0	0	0.3	0.5	0.6	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Figure 3.3 (b) Hypothetical geological block model showing blocks with % age grades and waste with 0 grade

Objective function:

Using Equation (3.13) the objective function for maximising NPV whilst taking 3 years life of mining operation is defined as follows:

Using Equation (3.13) and putting value of discount factor from Table 3.2

$$Max\ NPV = \frac{P_1}{(1+0.15)^1} + \frac{P_2}{(1+0.15)^2} + \frac{P_3}{(1+0.15)^3} \qquad (3.57)$$

Putting values of p, r, m and c from Table 3.2 in Equation 3.14

$$P_1 = (1500 - 500)Qr_1 - (1.25)Qm_1 - (5.3)Qc_1 - 2500 \qquad (3.58)$$

$$P_2 = (1500 - 500)Qr_2 - (1.25)Qm_2 - (5.3)Qc_2 - 2500 \qquad (3.59)$$

$$P_3 = (1500 - 500)Qr_3 - (1.25)Qm_3 - (5.3)Qc_3 - 2500 \qquad (3.60)$$

Mining capacity constraint:

Applying mining capacity constraints using Equation (3.15) and (3.16)

$$Qm_1 \leq 1500 \qquad (3.61)$$

$$Qm_1 - 150(x_{1..1} + x_{2..1} + x_{3..1} + x_{4..1} + \cdots + x_{98..1} + x_{99..1} + x_{100..1}) = 0 \qquad (3.62)$$

$$Qm_2 \leq 1500 \qquad (3.63)$$

$$Qm_2 - 150(x_{1..2} + x_{2..2} + x_{3..2} + x_{4..2} + \cdots + x_{98..2} + x_{99..2} + x_{100..2}) = 0 \qquad (3.64)$$

$$Qm_3 \leq 1500 \qquad (3.65)$$

$$Qm_3 - 150(x_{1..3} + x_{2..3} + x_{3..3} + x_{4..3} + \cdots + x_{98..3} + x_{99..3} + x_{100..3}) = 0 \quad (3.66)$$

Processing capacity constraint:

Applying processing capacity constraints using Equation (3.17) and (3.18)

$$Qc_1 \leq 700 \quad (3.67)$$

$$Qc_1 - 150(x_{13..1} + x_{14..1} + x_{15..1} + x_{24..1} + x_{25..1} + x_{26..1} + x_{34..1} + x_{35..1} + x_{36..1} + x_{45..1} + x_{46..1} + x_{47..1} + x_{55..1} + x_{56..1} + x_{57..1} + x_{65..1} + x_{66..1} + x_{67..1} + x_{74..1} + x_{75..1} + x_{76..1}) = 0 \quad (3.68)$$

$$Qc_2 \leq 700 \quad (3.69)$$

$$Qc_2 - 150(x_{13..2} + x_{14..2} + x_{15..2} + x_{24..2} + x_{25..2} + x_{26..2} + x_{34..2} + x_{35..2} + x_{36..2} + x_{45..2} + x_{46..2} + x_{47..2} + x_{55..2} + x_{56..2} + x_{57..2} + x_{65..2} + x_{66..2} + x_{67..2} + x_{74..2} + x_{75..2} + x_{76..2}) = 0 \quad (3.70)$$

$$Qc_3 \leq 700 \quad (3.71)$$

$$Qc_3 - 150(x_{13..3} + x_{14..3} + x_{15..3} + x_{24..3} + x_{25..3} + x_{26..3} + x_{34..3} + x_{35..3} + x_{36..3} + x_{45..3} + x_{46..3} + x_{47..3} + x_{55..3} + x_{56..3} + x_{57..3} + x_{65..3} + x_{66..3} + x_{67..3} + x_{74..3} + x_{75..3} + x_{76..3}) = 0 \quad (3.72)$$

Calculation of yearly average grades:

Weighted average grades $\overline{g_1}, \overline{g_2}$ and $\overline{g_3}$ are calculated using Equation (3.20) as follows:

$$\overline{g}_1 = \frac{\begin{matrix}150(0.3x_{13..1}+0.5x_{14..1}+0.6x_{15..1}+0.4x_{24..1}+0.5x_{25..1}+0.6x_{26..1}+0.3x_{34..1} \\ +0.5x_{35..1}+0.6x_{36..1}+0.3x_{45..1}+0.5x_{46..1}+0.6x_{47..1}+0.3x_{55..1}+ \\ 0.4x_{56..1}+0.5x_{57..1}+0.4x_{65..1}+0.3x_{66..1}+0.5x_{67..1}+0.3x_{74..1}+0.5x_{75..1} \\ 0.4x_{65..1}+0.3x_{66..1}+0.5x_{67..1}+0.3x_{74..1}+0.5x_{75..1}+0.6x_{76..1})\end{matrix}}{\begin{matrix}150(x_{13..1}+x_{14..1}+x_{15..1}+x_{24..1}+x_{25..1}+x_{26..1}+x_{34..1}+x_{35..1}+ \\ x_{36..1}+x_{45..1}+x_{46..1}+x_{47..1}+x_{55..1}+x_{56..1}+x_{57..1}+ \\ x_{65..1}+x_{66..1}+x_{67..1}+x_{74..1}+x_{75..1}+x_{76..1})\end{matrix}} \quad (3.73)$$

$$\overline{g}_2 = \frac{\begin{matrix}150(0.3x_{13..2}+0.5x_{14..2}+0.6x_{15..2}+0.4x_{24..2}+0.5x_{25..2}+0.6x_{26..2}+0.3x_{34..2} \\ +0.5x_{35..2}+0.6x_{36..2}+0.3x_{45..2}+0.5x_{46..2}+0.6x_{47..2}+0.3x_{55..2}+ \\ 0.4x_{56..2}+0.5x_{57..2}+0.4x_{65..2}+0.3x_{66..2}+0.5x_{67..2}+0.3x_{74..2}+0.5x_{75..2} \\ 0.4x_{65..2}+0.3x_{66..2}+0.5x_{67..2}+0.3x_{74..2}+0.5x_{75..2}+0.6x_{76..2})\end{matrix}}{\begin{matrix}150(x_{13..2}+x_{14..2}+x_{15..2}+x_{24..2}+x_{25..2}+x_{26..2}+x_{34..2}+x_{35..2}+ \\ x_{36..2}+x_{45..2}+x_{46..2}+x_{47..2}+x_{55..2}+x_{56..2}+x_{57..2}+ \\ x_{65..2}+x_{66..2}+x_{67..2}+x_{74..2}+x_{75..2}+x_{76..2})\end{matrix}} \quad (3.74)$$

$$\overline{g}_3 = \frac{\begin{gathered}150(0.3x_{13..3}+0.5x_{14..3}+0.6x_{15..3}+0.4x_{24..3}+0.5x_{25..3}+0.6x_{26..3}+0.3x_{34..3}\\ +0.5x_{35..3}+0.6x_{36..3}+0.3x_{45..3}+0.5x_{46..3}+0.6x_{47..3}+0.3x_{55..3}+\\ 0.4x_{56..3}+0.5x_{57..3}+0.4x_{65..3}+0.3x_{66..3}+0.5x_{67..3}+0.3x_{74..3}+0.5x_{75..3}\\ 0.4x_{65..3}+0.3x_{66..3}+0.5x_{67..3}+0.3x_{74..3}+0.5x_{75..3}+0.6x_{76..3})\end{gathered}}{\begin{gathered}150(x_{13..3}+x_{14..3}+x_{15..3}+x_{24..3}+x_{25..3}+x_{26..3}+x_{34..3}+x_{35..3}+\\ x_{36..3}+x_{45..3}+x_{46..3}+x_{47..3}+x_{55..3}+x_{56..3}+x_{57..3}+\\ x_{65..3}+x_{66..3}+x_{67..3}+x_{74..3}+x_{75..3}+x_{76..3})\end{gathered}} \quad (3.75)$$

Refining capacity constraint:

Applying refining capacity constraints Qr_1, $Qr_2$and $Qr_3$are calculated using Equation (3.19)

$$Qr_1 = Qc_1 \times \overline{g}_1 \times y \quad (3.76)$$

$$Qr_2 = Qc_2 \times \overline{g}_2 \times y \quad (3.77)$$

$$Qr_3 = Qc_3 \times \overline{g}_3 \times y \quad (3.78)$$

Precedence and slope constraint:

Applying slope constraints for 3 different years using Equation (3.21)

$$x_{12..1} - x_{1..1} \leq 0$$

$$x_{12..1} - x_{2..1} \leq 0$$

$$x_{12..1} - x_{3..1} \leq 0$$

The above equations' format is generated in LP model file after simulation but to make it simple to write, the above three inequalities can be written as follows:

$$3x_{12..1} - (x_{1..1} + x_{2..1} + x_{3..1}) \leq 0$$

$$3x_{12..2} - (x_{1..1} + x_{2..1} + x_{3..1} + x_{1..2} + x_{2..2} + x_{3..2}) \leq 0$$

$$3x_{12..3} - (x_{1..1} + x_{2..1} + x_{3..1} + x_{1..2} + x_{2..2} + x_{3..2} + x_{1..3} + x_{2..3} + x_{3..3}) \leq 0$$

$$3x_{13..1} - (x_{2..1} + x_{3..1} + x_{4..1}) \leq 0$$

$$3x_{13..2} - (x_{2..1} + x_{3..1} + x_{4..1} + x_{2..2} + x_{3..2} + x_{4..2}) \leq 0$$

$$3x_{13..3} - (x_{2..1} + x_{3..1} + x_{4..1} + x_{2..2} + x_{3..2} + x_{4..2} + x_{2..3} + x_{3..3} + x_{4..3}) \leq 0$$

$$3x_{14..1} - (x_{4..1} + x_{5..1} + x_{5..1}) \leq 0$$

$$3x_{14..2} - (x_{4..1} + x_{5..1} + x_{5..1} + x_{4..2} + x_{5..2} + x_{5..2}) \leq 0$$

$$3x_{14..3} - (x_{4..1} + x_{5..1} + x_{4..2} + x_{5..2} + x_{5..2} + x_{4..3} + x_{5..3}) \leq 0$$

Continue to

$3x_{45..1} - (x_{34..1} + x_{35..1} + x_{35..1}) \leq 0$

$3x_{45..2} - (x_{34..1} + x_{35..1} + x_{35..1} + x_{34..2} + x_{35..2}) \leq 0$

$3x_{45..3} - (x_{34..1} + x_{35..1} + x_{35..1} + x_{34..2} + x_{35..2} + x_{35..2} + x_{34..3} + x_{35..3} + x_{35..3}) \leq 0$

$3x_{46..1} - (x_{35..1} + x_{36..1} + x_{37..1}) \leq 0$

$3x_{46..2} - (x_{35..1} + x_{36..1} + x_{37..1} + x_{35..2} + x_{36..2} + x_{37..2}) \leq 0$

$3x_{46..3} - (x_{35..1} + x_{36..1} + x_{37..1} + x_{35..2} + x_{36..2} + x_{37..2} + x_{35..3} + x_{36..3} + x_{37..3}) \leq 0$

<u>Reserve constraint:</u>

Applying reserve constraints for 3 different years using Equation (3.22)

$x_{1..1} + x_{1..2} + x_{1..3} \leq 1$

$x_{2..1} + x_{2..2} + x_{3..3} \leq 1$

$x_{3..1} + x_{3..2} + x_{3..3} \leq 1$

$x_{4..1} + x_{4..2} + x_{4..3} \leq 1$

Continue to

$x_{99..1} + x_{99..2} + x_{99..3} \leq 1$

$x_{100..1} + x_{100..2} + x_{100..3} \leq 1$

<u>Cumulative tonnage constraint:</u>

Estimating value of CO_{LB} and CO_{UB} for *first* year of mining operation using Equation (3.25) and (3.26):

$$CO_{LB} = (q_{13}g_3x_{13..1} + q_{14}g_{14}x_{14..1} + q_{15}g_{15}x_{15..1} + q_{24}g_{24}x_{24..1} + q_{25}g_{25}x_{25..1} + q_{26}g_{26}x_{26..1} + q_{34}g_{34}x_{34..1} + q_{35}g_{35}x_{35..1} + q_{36}g_{36}x_{36..1} + q_{45}g_{45}x_{45..1} + q_{46}g_{46}x_{46..1} + q_{47}g_{47}x_{47..1} + q_{55}g_{55}x_{55..1} + q_{56}g_{56}x_{56..1} + q_{57}g_{57}x_{57..1} + q_{65}g_{65}x_{65..1} + q_{66}g_{66}x_{66..1} + q_{67}g_{67}x_{67..1} + q_{74}g_{74}x_{74..1} + q_{75}g_{75}x_{75..1} + q_{76}g_{76}x_{76..1}) \quad (3.79)$$

$$CO_{UB} = (q_{13}g_3x_{13..1} + q_{14}g_{14}x_{14..1} + q_{15}g_{15}x_{15..1} + q_{24}g_{24}x_{24..1} + q_{25}g_{25}x_{25..1} + q_{26}g_{26}x_{26..1} + q_{34}g_{34}x_{34..1} + q_{35}g_{35}x_{35..1} + q_{36}g_{36}x_{36..1} + q_{45}g_{45}x_{45..1} + q_{46}g_{46}x_{46..1} + q_{47}g_{47}x_{47..1} + q_{55}g_{55}x_{55..1} + q_{56}g_{56}x_{56..1} + q_{57}g_{57}x_{57..1} + q_{65}g_{65}x_{65..1} + q_{66}g_{66}x_{66..1} + q_{67}g_{67}x_{67..1} + q_{74}g_{74}x_{74..1} + q_{75}g_{75}x_{75..1} + q_{76}g_{76}x_{76..1}) \quad (3.80)$$

Using Equations (3.23) and (3.24)

$$COG_1 \times (q_{13}x_{13..1} + q_{14}x_{14..1} + q_{15}x_{15..1} + \cdots + q_{75}x_{75..1} + q_{76}x_{76..1}) \geq CO_{LB} \quad (3.81)$$

$$COG_1 \times (q_{13}x_{13..1} + q_{14}x_{14..1} + q_{15}x_{15..1} + \cdots + q_{75}x_{75..1} + q_{76}x_{76..1}) \leq CO_{LB} \quad (3.82)$$

Similarly estimating value of CO_{LB} and CO_{UB} for *second* year of mining operation using Equation (3.25) and (3.26):

$$CO_{LB} = (q_{13}g_3x_{13..2} + q_{14}g_{14}x_{14..2} + \cdots + q_{75}g_{75}x_{75..2} + q_{76}g_{76}x_{76..2}) \quad (3.83)$$

$$CO_{UB} = (q_{13}g_3x_{13..2} + q_{14}g_{14}x_{14..2} + \cdots + q_{75}g_{75}x_{75..2} + q_{76}g_{76}x_{76..2}) \quad (3.84)$$

Using Equation (3.23) and (3.24)

$$COG_2 \times (q_{13}x_{13..2} + q_{14}x_{14..2} + q_{15}x_{15..2} + \cdots + q_{75}x_{75..2} + q_{76}x_{76..2}) \geq CO_{LB} \quad (3.85)$$

$$COG_2 \times (q_{13}x_{13..2} + q_{14}x_{14..2} + q_{15}x_{15..2} + \cdots + q_{75}x_{75..2} + q_{76}x_{76..2}) \leq CO_{LB} \quad (3.86)$$

Similarly estimating value of CO_{LB} and CO_{UB} for *third* year of mining operation using Equation (3.25) and (3.26):

$$CO_{LB} = (q_{13}g_3x_{13..3} + q_{14}g_{14}x_{14..3} + \cdots + q_{75}g_{75}x_{75..3} + q_{76}g_{76}x_{76..3}) \quad (3.87)$$

$$CO_{UB} = (q_{13}g_3x_{13..3} + q_{14}g_{14}x_{14..3} + \cdots + q_{75}g_{75}x_{75..3} + q_{76}g_{76}x_{76..3}) \quad (3.88)$$

Using Equation (3.23) and (3.24)

$$COG_3 \times (q_{13}x_{13..3} + q_{14}x_{14..3} + q_{15}x_{15..3} + \cdots + q_{75}x_{75..3} + q_{76}x_{76..3}) \geq CO_{LB} \quad (3.89)$$

$$COG_3 \times (q_{13}x_{13..3} + q_{14}x_{14..3} + q_{15}x_{15..3} + \cdots + q_{75}x_{75..3} + q_{76}x_{76..3}) \leq CO_{LB} \quad (3.90)$$

Solution to problem:

Using Equations (3.61) to (3.66) values of Qm_1, Qm_2 and Qm_3 are calculated as follows:

$Qm_1 \leq 1500$

$Qm_1 - 150\ (x_{2..1} + x_{3..1} + x_{4..1} + x_{5..1} + x_{6..1} + x_{13..1} + x_{14..1} + x_{15..1} + x_{24..1}) = 0$

$$Qm_1 = 150 \times 9 = 1350 \text{ Tonnes} \quad (3.91)$$

$Qm_2 \leq 1500$

$Qm_2 - 150(x_{7..2} + x_{8..2} + x_{16..2} + x_{17..2} + x_{25..2} + x_{26..2} + x_{35..2}) = 0$

$$Qm_2 = 150 \times 7 = 1050 \text{ Tonnes} \quad (3.92)$$

$Qm_3 \leq 1500$

$Qm_3 - 150\ (x_{1..3} + x_{9..3} + x_{12..3} + x_{18..3} + x_{23..3} + x_{27..3} + x_{34..3} + x_{36..3} + x_{45..3}) = 0$

$$Qm_3 = 150 \times 9 = 1350 \text{ Tonnes} \quad (3.93)$$

Considering values COG_1, COG_2 and COG_3 from Equation (3.103) to (3.105) respectively, ore blocks ($g_i \geq COG_t$) are selected for processing. Using Equations (3.67) to (3.72), the values of Qc_1, Qc_2 and Qc_3 of are calculated as follows:

$Qc_1 \leq 700$

$Qc_1 - 150\ (x_{14..1} + x_{15..1} + x_{24..1}) = 0$

$$Qc_1 = 150 \times 3 = 450 \text{ Tonnes} \quad (3.94)$$

$Qc_2 \leq 700$

$Qc_2 - 150\ (x_{25..2} + x_{26..2} + x_{35..2}) = 0$

$$0 \leq Qc_2 = 150 \times 3 = 450 \text{ Tonnes} \quad (3.95)$$

$Qc_3 \leq 700$

$Qc_3 - 150\ (x_{36..3}) = 0$

$$Qc_3 = 150 \times 3 = 450 \text{ Tonnes} \quad (3.96)$$

Using Equations (3.73) to (3.75), the values of $\overline{g_1}$, $\overline{g_2}$ and $\overline{g_3}$ are calculated as follows:

$$\overline{g}_1 = \frac{150\ (0.5x_{14..1}+0.6x_{15..1}+0.5x_{24..1})}{150\ (x_{13..1}+x_{14..1}+x_{15..1}+x_{24..1})}$$

$$\overline{g}_1 = \frac{0.5+0.6+0.5}{3} = 0.53 \quad (3.97)$$

$$\overline{g}_2 = \frac{150\ (0.4x_{25..2}+0.6x_{26..2}+0.5x_{35..2})}{150\ (x_{25..2}+x_{26..2}+x_{35..2})}$$

$$\overline{g}_2 = \frac{0.4+0.6+0.5}{3} = 0.50 \quad (3.98)$$

$$\overline{g}_3 = \frac{150\ (0.6x_{36..3})}{150\ (x_{36..3})}$$

$$\overline{g}_3 = \frac{0.6}{1} = 0.60 \quad (3.99)$$

Putting values of Qc_1, Qc_2 and Qc_3 from Equations (3.94) to (3.96) and values of $\overline{g}_1$, $\overline{g}_2$ and $\overline{g}_3$ from Equations (3.97) to (3.99) in Equations (3.75) to (3.77) to find values Qr_1, Qr_2 and Qr_3 as follows:

$$Qr_1 = 450 \times 0.53 \times 0.95$$

$$Qr_1 = 226.58 \text{ Tons} \quad (3.100)$$

$$Qr_2 = 450 \times 0.50 \times 0.95$$

$$Qr_2 = 213.75 \text{ Tons} \quad (3.101)$$

$$Qr_3 = 150 \times 0.60 \times 0.95$$

$$Qr_3 = 85.50 \text{ Tons} \quad (3.102)$$

Cut-off grade for year 1:

Using Equations (3.79) to (3.82)

@ Cumulative tonnage constraint for COG_1 = 0.6

$CO_{LB} = 150\,(0.3 + 0.5 + 0.5 + 0.6) = 2485$ Tons

$CO_{UB} = 150(\,0.6) = 90$ Tons

$0.6 \times 600 \geq 285$ *implies* $360 \geq 285$

$0.6 \times 600 \leq 90$ *implies* $360 \leq 90$ *(Constraint in Equation 3.24 is violated)*

@ Cumulative tonnage constraint for COG_1 = 0.5

$CO_{LB} = 150\,(0.3 + 0.5 + 0.5) = 195$ Tons

$CO_{UB} = 150(\,0.5 + 0.5 + 0.6\,) = 240$ Tons

$0.5 \times 600 \geq 195$ *implies* $300 \geq 195$

$0.5 \times 600 \leq 240$ *implies* $300 \leq 240$ *(Constraint in Equation 3.24 is violated)*

@ Cumulative tonnage constraint for COG_1 = 0.40

$CO_{LB} = 150(0.3) = 45$ Tons

$CO_{UB} = 150(0.5 + 0.6 + 0.5) = 240$

$0.40 \times 600 \geq 45$ *implies* $240 \geq 45$ (Condition satisfied)

$0.40 \times 600 \leq 240$ *implies* $240 \leq 240$ (Condition satisfied)

$$COG_1 = 0.40 \tag{3.103}$$

<u>Cut-off grade for year 2:</u>

Using Equations (3.83) to (3.86)

$CO_{LB} = 150(0.3 + 0.3) = 90$ Tons

$CO_{UB} = 150(0.4 + 0.6 + 0.3 + 0.5 + 0.5 + 0.3) = 390$ Tonnes

@ Cumulative tonnage constraint for COG_2 = 0.50

$CO_{LB} = 150(0.3 + 0.3 + 0.4 + 0.5 + 0.5) = 300$ Tonnes

$CO_{UB} = 150(0.6 + 0.5 + 0.5) = 240$ Tonnes

$0.5 \times 900 \geq 300$ *implies* $450 \geq 300$

$0.5 \times 900 \leq 240$ *implies* $450 \leq 240$ *(Constraint in Equation 3.24 is violated)*

@ Cumulative tonnage constraint for COG_2 = 0.4

$CO_{LB} = 150(0.3 + 0.3 + 0.4) = 150$ Tonnes

$CO_{UB} = 150(0.4 + 0.6\ + 0.5\ + 0.5 + 0.3)$ = 345 Tonnes

$0.4 \times 900 \geq 150\ implies\ 360 \geq 150$

$0.4 \times 900 \leq 345\ implies\ 360 \leq 345$ *(Close but constraint in Equation 3.24 is still violated)*

It is possible in this scenario that the cut-off grade below 0.4 satisfies the constraint, but as the minimum processing plant head grade is assumed to be 0.4 so the grade does not consider below 0.4 is not considered for processing. In that case, the processing plant grade is considered as cut-off grade, and it remains 0.4 for the similar cases.

COG_2 = 0.40	(3.104)

Cut-off grade for year 3:

Figure 3.4 shows that only one ore block with grade 0.6 is mined in year 3 in addition to other waste blocks. Using Equations (3.87) to (3.90)

@ Cumulative tonnage constraint for COG_3 = 0.6

$CO_{LB} = 150(0.6)\ = 30$ tonnes

$CO_{UB} = 150(0.6\,)$ = 30 tonnes

$0.6\ \times 150 = 30 \geq 30$

$0.6\ \times 150 = 30 \leq 30$

COG_3 = 0.60	(3.105)

Putting the values of Qm_1, Qc_1 and Qr_1 in Equation (3.50)

$P_1 =\ (1000)Qr_1 - (1.25)Qm_1 - (5.3)Qc_1 - 2500$

$P_1 =\ 1000 \times 226.58 - 1.25 \times 1350 - 5.3 \times 450 - 2500$

$P_1 = \$220{,}007.50$	(3.106)

Putting the values of Qm_1, Qc_1 and Qr_1 in Equation (3.51)

$P_2 =\ (1000)Qr_2 - (1.25)Qm_2 - (5.3)Qc_2 - 2500$

$P_2 =\ 1000 \times 213.75 - 1.25 \times 1050 - 5.3 \times 450 - 2500$

$$P_2 = \$207{,}552.50 \quad (3.107)$$

Putting values of Qm_3, Qc_3 and Qr_3 in Equation (3.52)

$$P_3 = (1000)Qr_1 - (1.25)Qm_1 - (5.3)Qc_1 - 2500$$

$$P_3 = 1000 \times 85.5 - 1.25 \times 1350 - 5.3 \times 150 - 2500$$

$$P_3 = \$80{,}517.50 \quad (3.108)$$

Putting values of P_1, P_2 and P_3 from Equations (3.106) to (3.108) in Equation (3.57)

$$Max\ NPV = \frac{\$263382.50}{(1+0.15)^1} + \frac{\$207552.50}{(1+0.15)^2} + \frac{\$164427.5}{(1+0.15)^3} = \$\ 1467402.80$$

$$Max\ NPV = \$1467402.80 \quad (3.109)$$

Compiling all the values from Equations (3.91 to 3.109), the results are shown in Table 3.4.

Year (t)	COG_t (% Cu)	$\bar{g}_t$ (% Cu)	Qm_t **(tonnes/year)**	Qc_t **(tonnes/year)**	Qr_t **(tonnes/year)**	P_t **($ in millions)**	NPV_t **($ in millions)**
1	0.40	0.53	1350	600	270	0.22	0.37
2	0.40	0.50	1050	450	213	0.21	0.23
3	0.60	0.60	1350	150	85	0.08	0.07

Table 3.4: Results obtained from solving 2D model for 3 years using new MILP based formulation

Figure 3.4 shows the production scheduling for three years life of mining operation.

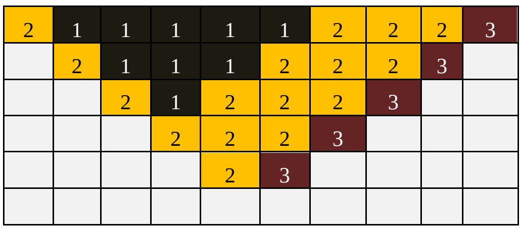


Figure 3.4 Production scheduling for 3 years of mining operation

The MILP formulation is developed as software application and solved using exact approach and implemented for the hypothetical model. It is found that the number of variables increases exponentially leads increase in time of simulation; thus the solution becomes computationally inefficient and termed as NP-hard; therefore, hybrid-metaheuristic is introduced as an alternate approach to solve this MILP based mathematical formulation. Although, it is not possible to get an optimal solution using hybrid-metaheuristic, but this approach generates a near optimal solution and can generate a roadmap for investment and returns on investment in a very reasonable time. The next section discusses the development of hybrid-metaheuristic followed by its implementation and analysis as a software application.

Chapter 4: Theory and development of hybrid-metaheuristic

4.1. Introduction:

Computational complexity for solving a mathematical model using exact approach leads to the development of a heuristic approach to achieve a near-optimal solution in a reasonable time. The theory and development of hybrid-metaheuristics which employs a combination of two evolutionary algorithms (metaheuristics) known as genetic algorithm (GA) and ant colony optimisation (ACO) algorithms. These are introduced in this research as an alternative approach for solving mathematical model for cut-off grade optimisation. Both algorithms are used independently or in combination for solving NP-hard problems efficiently and provide an alternative solution to exact approach. Hybrid-metaheuristic uses GA and ACO in combination to find the best and fittest solution, whilst reducing any complexities in solving the mathematical model.

4.2. Development of hybrid-metaheuristic for open-pit mining problem:

4.2.1. Objective function:

Considering the objective function defined and constraints in the MILP formulation, hybrid-algorithm accounts for the geological (block model), economic and operational parameters, generates dynamic cut-off grade policy and yearly production sequence.

The hybrid-metaheuristic is used as an alternative solution technique for MILP model developed for the cut-off grade optimization. The solution construction of GA and ACO are independently discussed in the subsequent section followed by the hybrid-algorithm which is developed by combining of GA and ACO.

4.2.2. Solution construction using GA:

Step 1: Initial population:

Given the ore-body model and the operational capacities, the initial population is the random selection of feasible blocks that satisfies the capacity constraints. The random selection of blocks at different Z location (top to bottom approach) builds the initial solutions. These initial solutions are termed as parent chromosomes in GA terminology. As the objective is to maximise NPV, the criterion to select the best set of blocks with high grades and their predecessors through multiple iterations are the ones which have the potential to generate maximum profits, while catering for the cost of mining, and processing. The multiple chromosomes are generated in this process.

Step 2: Generate chromosmes and fitness function:

The best multiple chromosomes, are the initial selection of mineable blocks after multiple generations. These chromosomes define a fitness function and generate production sequences based on the maximum NPV, depending on the selection of best set of selected blocks. The limit of mining ore blocks and the overall mining constraints including both high-grade blocks and their predecessors generating maximum profit decides the best of the chromosomes. Several generations in GA finalize the fittest among the best as shown in Figure 4.1. The fitness function (Falkenaur and Delchambre, 1992) of individuals is mathematically presented in Equation (4.1):

$$f = \sum_{i=1}^{N} (F_n / \Upsilon)^k / N \quad (4.1)$$

Where f represents fitness function, N is number of blocks, F_n is the sum of sizes of objects in each set of array, Υ is overall capacity of blocks mined within constraints each year, k is constant, and thus gives the best fitness function.

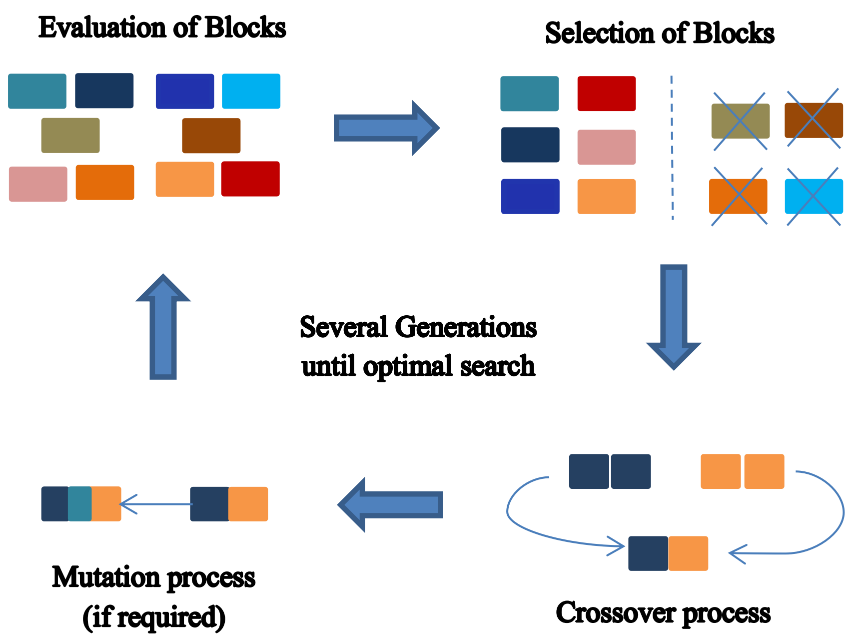


Figure 4.1: Schematic diagram showing procedures of Genetic Algorithm

Step 3: Crossover and mutation:

Crossover and mutation are two different processes developed in GA which work simultaneously and generate best of the offsprings through a crossover of chromosomes (initial population) already generated after the selection of the fittest, and also complete the selection process of the fittest individuals. Crossover generates the best children, which means sequences are updated from the parental chromosomes, if parents consist of blocks already mined, they must not be included in the next generation of new individuals (blocks and their production schedules); whereas the mutation process fills a gap of block with another potential mineable block or a predecessor which is needed to complete the sequence, whilst satisfying mining and processing constraints. This ultimately defines the optimal strategy of cut-off grades and yearly production sequences.

GA uses discounted cash flow to maximise NPV through different generations, where a set of ore blocks is selected with possible higher grades and their predecessors, subject to all the given constraints. The final selection of the initial solution and production sequence which maximises NPV is based on the possible future production, which can save the cost of mining for later production.

4.2.3. Solution contruction using ACO:

Step 1: Initial population using ants and pheromones:

ACO algorithm follows the colonization behaviour of ants and their pheromones and generates the initial selection of blocks (ore and predecessors) following their behaviour. In this research, each ant is assumed as a single block and ants together with their pheromones (shortest distance to targetted food) are considered as production sequences. ACO algorithm selects the best among the pheromones (populations) as an initial solution and then develops further schedules on the basis of initial population. ACO develops dynamic cut-off grades simultaneously while maximising NPV considering the lowest grade as cut-off grade. This is equal to or more than the required processing plant head grade in the schedule, and eventually after several iterations (selection of best ants and pheromones) achieves the objective of the proposed mathematical model.

Step 2: Defining fitness function:

ACO model defines ants and pheromones giving the best of the production schedule over the life of the mining operation. The colonization of the ants defined in this algorithm builds super-sequences of the strings of the blocks. The choice of characters (predecessors) by ants in the strings of blocks depends on the pheromone trails which are actually the conditions of the constraints. The ant is defined as the one which utilizes a probabilistic rule (Equation 4.2) and selects the possible blocks to the lowest depth of the pit at different levels starting from the top level. The pheromone trail is then built for each block. Equation (4.2) shows the probability of choosing ore-blocks at the different levels starting from the top level at a given period within the constraints.

$$P_i^e = \frac{[\tau_i]^a[\sigma_i]^b}{\sum_n^{N_i^e}[\tau_n]^a[\sigma_n]^b} \tag{4.2}$$

Where P_i^e represents the probability of occurrences of best ant e selecting block i , τ_i is pheromone value at block i, σ_i is the heuristic information which makes that block value as best, a and b are the factors to determine the relative impact of pheromone trail and heuristic evidence respectively, N_i^e is set of feasible solutions.

Step 4: Pheromones' evaporation and defining production sequences:

The first feasible solution defines the pheromones comprising the best high-grade blocks and their predecessors that is termed as the first production sequence. This sequence is not considered in the next iteration, and the process of selecting one fittest schedule (where NPV is maximised subject to all constraints) and ignoring the other possible best solutions is termed as pheromones' evaporation. This process is iterative and continues until each set of ants and pheromones (production sequences) for each year are selected. Each production sequence and the preceding sequences are not considered in defining the future sequences, and ACO finds the most feasible solution for each year till the stopping conditions are achieved.

Step 5: Finding the cut-off grade and average grade:

The block grade defines the destination of each block, whether they are sent to the processing plant, dumped as waste or sent to stockpile. The processing head grade is considered as the lowest grade for each block to be sent to the mill for processing. Schedules comprise of a set of blocks with high grades, set of overlying blocks (predecessor blocks necessary to mine to complete the sequence). The minimum grade of the blocks mined, which is equal to or more than the processing head grade is considered as cut-off grade, as this is the grade where NPV is maximised and the objective function is achieved (i.e. NPV is maximised). The average grade of the blocks mined in each schedule is also computed in this step.

Step 6: Stopping condition:

The stopping conditions are accomplished when optimum or nearest optimum value is achieved. As the best among the population is selected, the optimum population gives the optimum solution, and at this stage all the corresponding values ($\bar{g}_t, Qm_t, Qc_t$ and Qr_t) are evaluated.

4.2.4. Flow diagram of ACO solution construction:

Figure 4.2 shows a flow chart for the ant colony optimisation (ACO) algorithm:

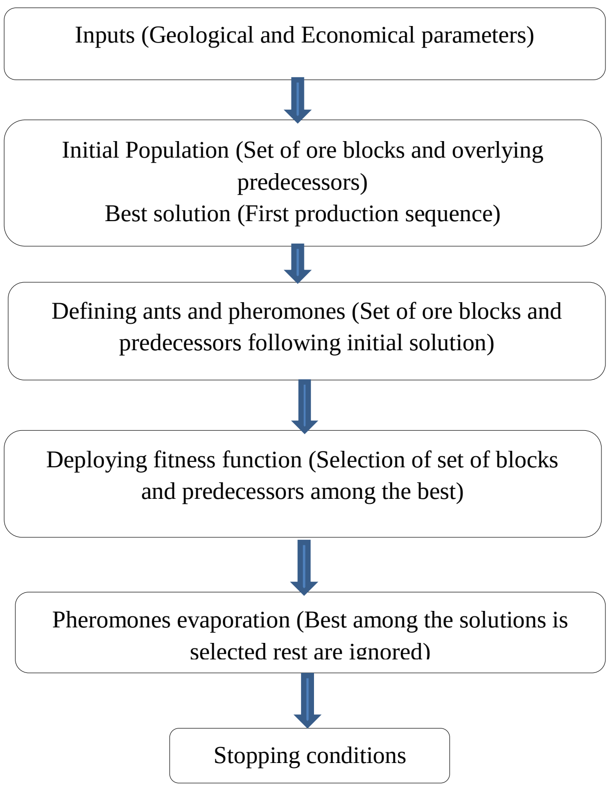


Figure 4.2 Flow diagram for ACO algorithm

4.3. Hybrid-metaheuristic:

Hybrid-metaheuristic is an important contribution to this research, as it combines GA and ACO and provides an alternative methodology for solving the mathematical formulation discussed earlier, while other heuritic algorithms such as Partical Swarm optimization (PSO), Tabu Search (TS) and dynamic programming are available to solving combinatorial hard problems (Dorigo & Stützle, 2004). The limitation of these algorithms for solving production scheduling problems, and the structure of the proposed cut-off grade model become the reason for the selection of the GA and ACO based hybrid-metaheuristic.

The GA is used as a search algorithm for finding the best offsprings (set of ore blocks or production sequence) within the mining, milling and refining constraints as shown in Figure 4.3. The search in this algorithm is based on the best ore grade, but it is difficult to find the best set of the ore blocks, which satisfy the given constraints, and that if mined maximises NPV.

The hybrid-metaheuristic algorithm performs two heuristics at the same time. Firstly, GA searches the best offspring (blocks with possible high grades), and after running a procedure of mutation, crossover and normalization as discussed above, it defines the fittest blocks and production sequences. The searching of best offsprings is performed in the descending order of Z values across X and Y location in the given block model. These are the fittest set of blocks obtained after running GA combined with their predecessors develop an initial solution that maximises NPV subject to all constraints. This fittest set of blocks are saved in an array for further processing and validation using ACO algorithm.

The fittest set of blocks which need to be mined with their predecessors are taken as ants (blocks with possible high grades) and pheromones are processed using ACO method for the fittest production schedule in the first year providing the initial solution. The process of pheromones' evaporation is also a significant part of this algorithm, where after selecting the fittest solution, remaining solutions are ignored and considered as evaporated pheromones in addition to the blocks which are already mined in the earlier sequences, and they are not considered in the preceding selection process of ants and pheromones (production schedules). The set of ore blocks and their predecessors which are satisfying all the constraints are processed through repeated iterations using GA and ACO simultaneously until the final solution is achieved and at this stage the best among the fittest is selected, and the stopping conditions are applied (Figure 4.3). The number of iterations are run to validate this algorithm while setting different time periods.

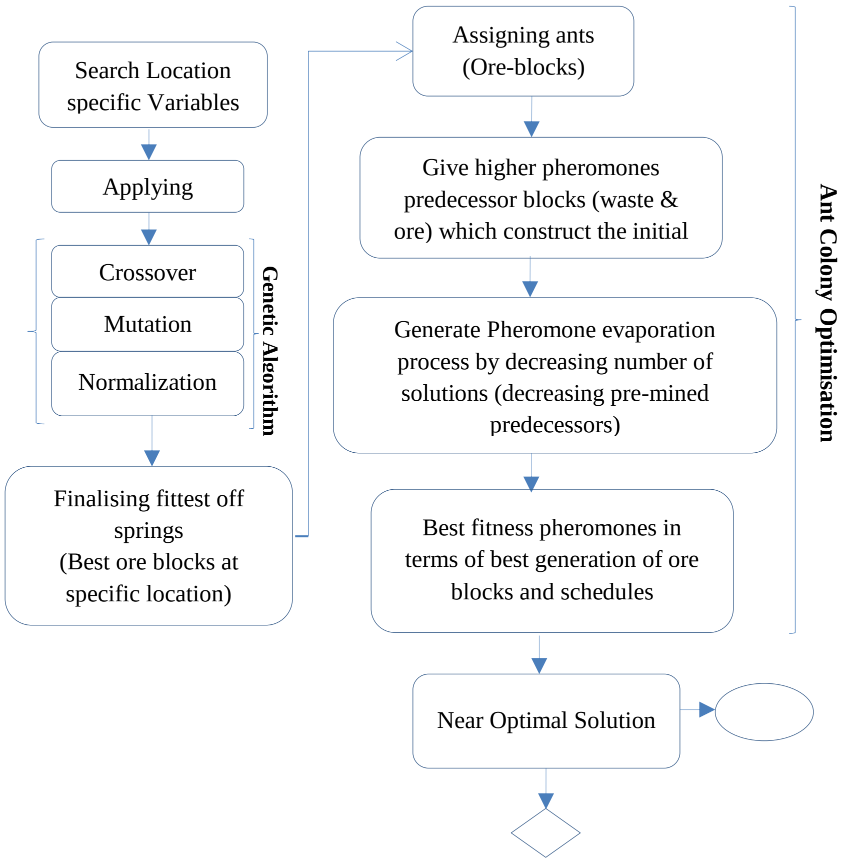


Figure 4.3: Flow chart for running hybrid meta-heuristic algorithm

Although this technique gives a near-optimal solution, the results are comparable with the optimum values. The following case study describes the implementation of a mathematical model using hybrid-metaheuristics solution.

4.3.1. Hybrid-algorithm pseudocode:

<u>Genetic Algrithm Sequence</u>

Initialise (set parameters)

Repeat (loop)

Initialize generation

Search best of array of ore blocks at each Z value within the constraints giving maximum NPV

Crossover the arrays to select fittest among the best solution

Mutate array values to redefine best of array while filling any gaps or repalcements

Finalse the selection of array which maximises NPV

Repeat generation until loop ends (search for better solution)

Fittest set of blocks with grades and their predecessors

Stopping conditions for GA

<u>Ant Colony Optimisation sequence</u>

Initialise (set parameters)

Selection of each Ant (block with grade) is located on the primary node

Repeat (loop)

Assign ants to the best of the array of blocks including their predecessors

Each ant employs a state move rule to increase the solution

Make selection of ants among the selected arrays

Validating the GA search Array

Assign ants to the best of the array of blocks including their predecessors

Find the pheromones ants and set f ore blocks need to be mined, relevant to the selected ants (ore blocks)

Apply the pheromone local update rule

Until (loop ends)

Evaporation of the pheromones once used (evaporating non optimal solutions and the array of blocks, already mined in the previous sequence

Update the ants and pheromones giving the maximium NPV (best solution)

Automate iterations till the best solution is achieved

Until (Loop ends)

Stopping condition is fulfilled

Stoppping condition apply

New loop

Automate the near optimal solution

The hybrid algorithm takes minimum ore-grade as input, which is equal to or higher than the processing plant head grade, and that is the grade where the yearly schedule is finalized. This grade discriminates ore and waste and it is termed as a cut-off grade. The grade higher or equal to the dynamic cut-off grade that is generated each year is termed as ore and is sent to mill for processing and the grade which is lower than cut-off grade is either stockpiled or sent to the waste dump. At this level, maximum possible NPV is achieved. The average grade of the ore for each schedule is also determined. The iterations are continued for the life of mining operation until the best production schedules are destined and optimum cut-off grade is obtained.

The practical implementation of new MILP formulation and hybrid-metaheuristic considering cut-off grade optimisation problems are discussed in the form of different case studies in the next section followed by the solutions and analysis.

Chapter 5: Implementations of cut-off grade models

The new MILP based mathematical model is implemented and solved, primarily by exact solution and secondly by hybrid-metaheuristic as an alternate approach. The models are defined as case studies and their solutions are implemented, analysed and discussed in the following sections. In the first instance, the hypothetical block model is taken as input, which comprises of location, grades, tonnage, and recovery in addition to the given economic and operational parameters. The simulation results attained using both the approaches for a certain period or years are analysed and compared through gap analysis.

The hypothetical model is further solved using conventional production scheduling formulation that generates yearly production sequences. Later, hybrid-metaheuristic is used to solve mathematical model considering realistic block model as input for obtaining a near optimal solution (Appendix 2).

5.1. Hypothetical block model – Case study: 1

5.1.1. Input parameters

<u>Geological inputs:</u>

A geological 3D block model with 501 blocks is considered in the case. Table 5.1 shows the geological parameters for a hypothetical block model. The block model is saved in a text file format (Appendix 1) in the directory of software application.

Parameters	**Value**	**Units**
Number of blocks	501	
Slope angle	45	degrees
Bench height	10	meters
Number of benches	4	
Processing head grade	0.40	% Cu

Table 5.1: Geological parameters for hypothetical block model

Economic inputs:

The economical parameters used for the hypothetical block model are given in Table 5.2.

Parameters	Value	Units
Discount rate $\boldsymbol{d}$	15	percent
Time period or years	4	years
Metal price $\boldsymbol{p}$	5300	$/tonne
Refining cost $\boldsymbol{r}$	1220	$/tonne
Milling cost $\boldsymbol{c}$	9.3	$/tonne
Mining cost $\boldsymbol{m}$	1.57	$/tonne
Fixed cost $\boldsymbol{FC}$	800,000	$/tonne

Table 5.2: Economic parameters for hypothetical block model

Operational inputs:

The mining and processing capacities' constraints has a direct impact on the overall mining cost and NPV estimation. As discussed in the previous chapters, that if mining and processing capacities are considered inaccurately, it could lead to over budgetting, which consequently converts profits into losses. Therefore, the capacity constraints considered for case study 1 are given in Table 5.3.

Capacities	Value	Units
Mining capacity $\boldsymbol{M}$	53550	tonnes/year
Milling capacity $\boldsymbol{P}$	15300	tonnes/year

Table 5.3: Operational parameters of hypothetical block model

Software program for MILP formulation:

The software application is developed using *JAVA* programing platform and *CPLEX* concert technology in case of MILP formulation; whereas only *JAVA* programing platform is used to implement hybrid metaheuristics. The geological inputs are entered in the software program using "File Reader" input function in *JAVA***,** whereas input and operational parameters are taken using a graphical user interface (GUI) as shown in Figure 5.1. The program developed for MILP formulation has the ability to generate linear programming (*.lp)* model file (Appendix 1) which can be independently solved using *CPLEX* executable (*cplex.exe*) file, generates solution (*.sol*) file (Appendix 1). The optimised results are obtained in the same program or can be processed while importing results in the spreadsheet.

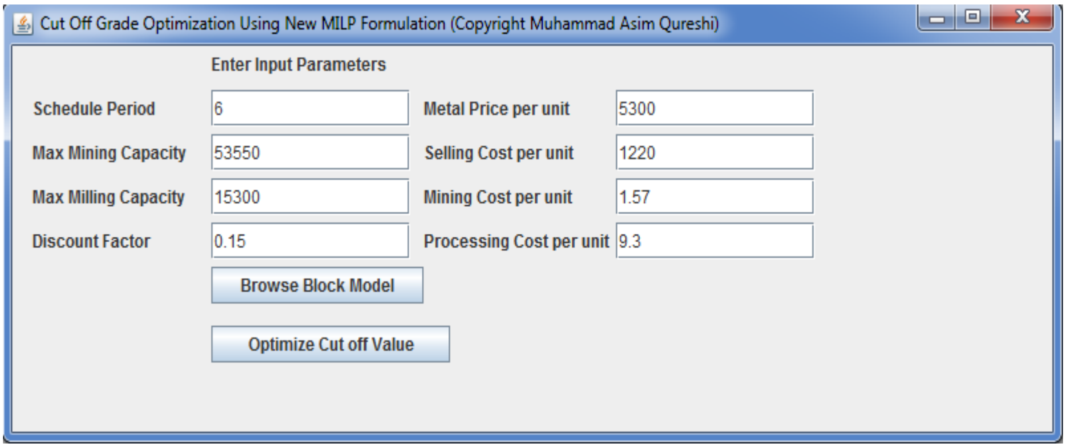


Figure 5.1: GUI for taking economic and operational parameters as inputs

<u>System specifications and limitations:</u>

The program is run using following system specifications:

Processor: Intel (R) Core (TM) i7-4770 CPU @ 3.40GHZ

RAM: 8GB

Harddisk: 600GB

System Type: 64-bit Windows 8.01 Operating systems

5.1.2. Implementation of MILP based mathematical model using CPLEX concert Technology

The algorithm for MILP formulation is implemented in *JAVA* programing platform with the integration of *CPLEX* concert technology to generate an exact solution.

<u>Optimsed results using MILP formulation</u>

The algorithm for MILP formulation is implemented in *JAVA* programming language while importing *CPLEX* concert technology to generate an exact solution. The detail on the working of the software program is mentioned in Appendix 1.

Year (t)	COG_t (% Cu)	$\overline{g}_t$ (% Cu)	Qm_t (tonnes/year)	Qc_t (tonnes/year)	Qr_t (tonnes/year)	P_t (millions)	NPV_t (millions)
1	0.45	0.54	52020.00	9180.00	4496	$10.18	$45.39
2	0.42	0.87	52020.00	4590.00	3583	$6.49	$42.03
3	0.40	0.77	50490.00	12240.00	8444	$26.26	$41.84
4	0.40	0.74	52020.00	12240.00	8169	$25.13	$21.85
Total	__	__	*206550.00*	*38250.00*	*24692*	*$68.06*	__

Table 5.4 Optimisation results for cut-off grade and production scheduling optimisation using new-MILP formulation

Table 5.4 shows the results after solving MILP problem using exact approach, which shows dynamic cut-off grades over 4 years life of mining operation, where dynamic values of COG_t are generated for each period. The $\overline{g}_t$ shows that the grade of blocks selected for mining is above the processing head grade over the life of mining operation, where Qc_t does not achieve the target processing capacity (15300 tonnes). Given the structure of the ore-body and the distribution of ore blocks only limited quantity of ore is sent to the processing plant, which ultimately yields less marketable material Qr_t. However, it is expected that a higher or an unlimited mining capacity would help meet the demand of ore at the processing plant. Figure 5.2 presents 3D graphical view of the pit which is generated after simulating new MILP formulation for 4 years.

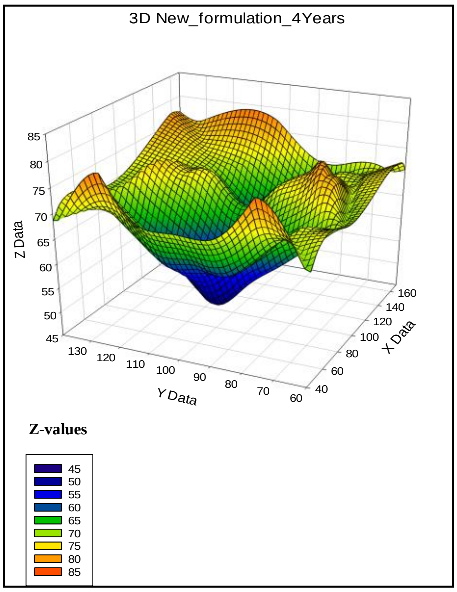


Figure 5.2: 3D-View of production scheduling for 4 years simulation using new MILP formulation

Figure 5.3 shows the graphical representation of the optimised results for cut-off grades, average grades and NPVs, obtained while solving the mathematical model for 4 years life of mining operation using exact approach for hypothetical ore-body model.

The plan and section maps of the production sequences developed in X, Y and Z plane using new MILP formulation, are given in *Appendix 3*.

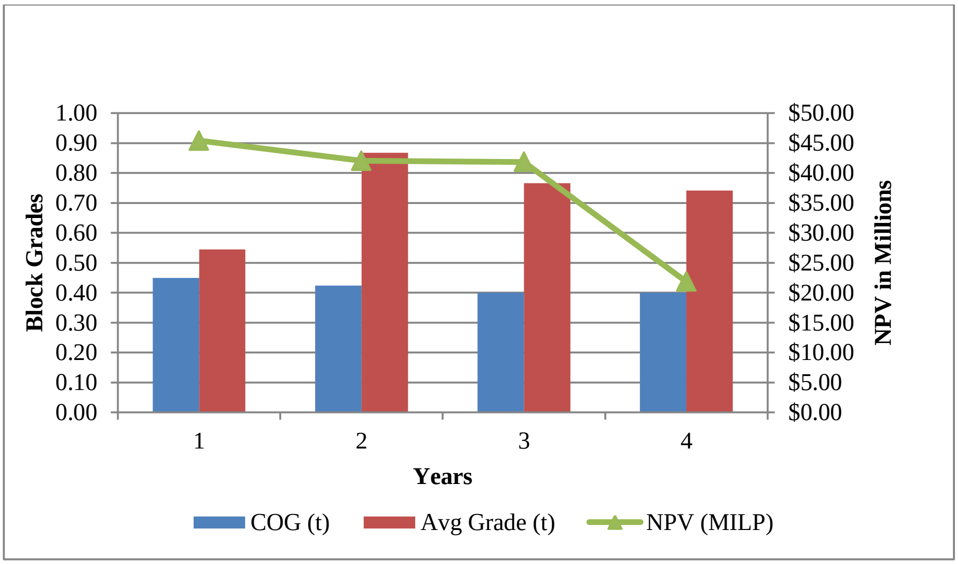


Figure 5.3: Graphical results of average grades, cut-off grades and NPV generated using new MILP for 4 years using a hypothetical block model

In addition, with 2004 binary variables in the MILP model for life of operation equal to 4 years, the corresponding solution time is 4 hours 35 minutes. This solution time may increase exponentially as the number of binary variables increase at relatively higher life of operation, however, this is sufficient for validation or performance evaluation of the hybrid-metaheuristic (Lamghari and Dimitrakopoulos, 2012).

5.1.3. Implementation of mathematical model using hybrid-metaheuristic

Table 5.5 shows the solution of mathematical model using hybrid-metaheuristic, which generates the near optimal values for cut-off grades. Although, better dynamic cut-off grades are obtained over 4 years life of mining operation using hybrid-metaheuristic, but still there is gap in NPV in comparison to the exact approach. The reason inferred from the results that using heuristic approach the value of Qc_t and Qr_t is somehow consistent but still does not achieve the target processing capacity which shows that half of the processing capacity is not utilized. As mentioned earlier, a higher or unlimited mining capacity will help meet the demand for the processing plant.

Year (*t*)	COG_t (% Cu)	$\bar{g}_t$ (% Cu)	Qm_t **(tonnes/year)**	Qc_t **(tonnes/year)**	Qr_t **(tonnes/year)**	P_t **($ in millions)**	NPV_t **($ in millions)**	**Gap%**
1	0.4	0.45	41310	10710	4588	13.56	$41	0.10
2	0.57	0.76	53550	7650	5537	17.44	$33.8	0.25
3	0.69	0.80	44370	7650	5844	18.70	$21	0.96
4	0.45	0.67	29070	4590	2900	6.74	$5.9	2.73
4	_	_	*168300*	*30600*	*18870*	*56.45*	_	

Table 5.5: Optimisation results for cut-off grade and production scheduling optimisation for 4 years using hybrid-metaheuristics

The comparison between the material Qc_t sent for processing using exact solution and the hybrid solution is evaluated against the processing capacity as mentioned in Figure 5.4.

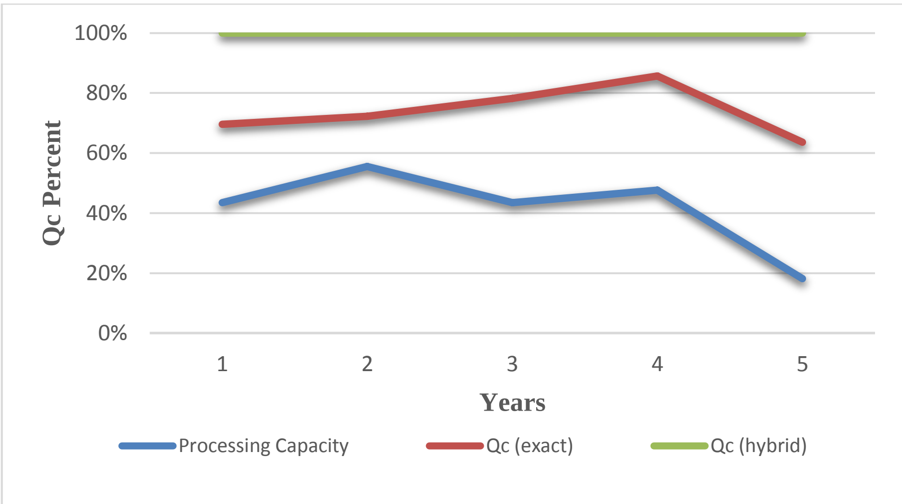


Figure 5.4: Comparison in % age for Qc_t (exact) ad Qc_t (hybrid) and their difference with processing capacity over the life of mining operation

Figure 5.5 presents 3D graphical view of the pit generated after simulating new MILP formulation for 4 years. The period-by-period production sequences are also developed using hybrid-metaheuristic. The time recorded at the end of the simulation is 4-5 minutes which is significantly less as compare to the exact solution. The plan and section maps of the production sequences developed in X, Y and Z plane using hybrid-metaheuristic, are given in *Appendix 4.*

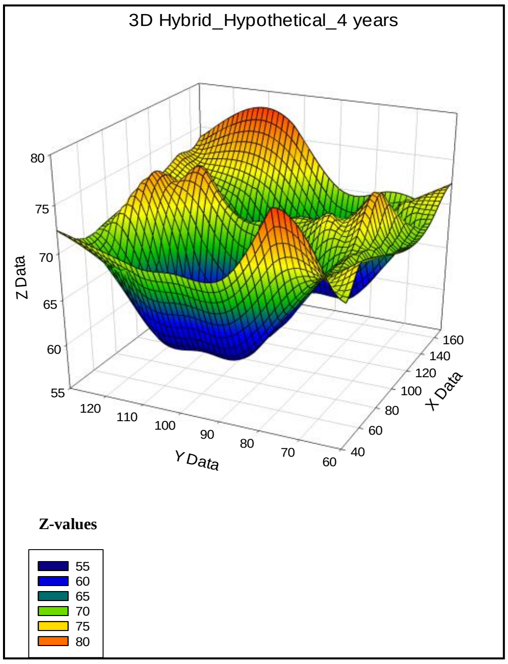


Figure 5.5: 3D-View of production scheduling for 4 years using hybrid-metaheuristics

Figure 5.6 shows the graphical representation of the optimised results for cut-off grades, average gardes and NPVs, for 4 years life of mining operation using hybrid-metaheuristic for hypothetical block model.

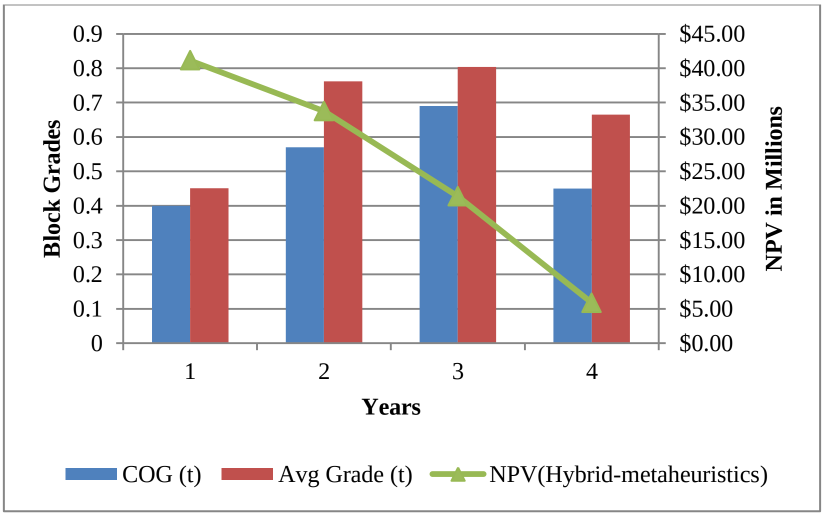


Figure 5.6: Graphical results of average grades, cut-off grades and NPV generated using hybrid-metaheuristic for 4 years for hypothetical block model

Figure 5.7 shows the graphical representation of gap analysis between exact and the hybrid-metaheuristic approaches. It is deduced from the gap analysis that near optimum values are achieved in a significantly less time in the case of hybrid-metaheuristic, showing small initial variance in NPV as compare to exact approach which increases in later years. It can be inferred from Figure 5.7, the gap between NPVs is very nominal for both the solutions (hybrid and exact solution), and at some stage, curves of NPVs are parallel, which shows that the NPVs obtained for hybrid-metaheuristic solution are comparable with the exact solution, and provides a roadmap in estimating the exact optimized values of cut-off grades. The increase in the gap shows that most of the material considered as waste while using hybrid-metaheuristic resulting in lower NPVs as compared to the exact approach.

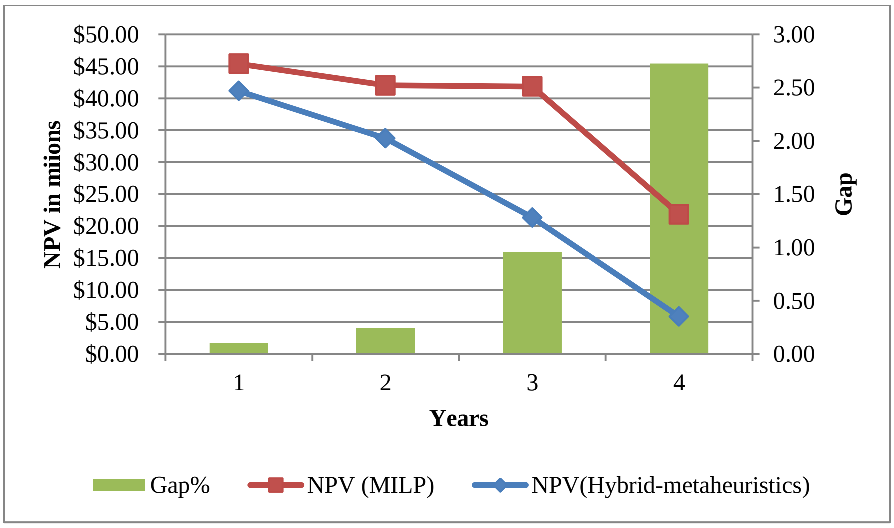


Figure 5.7: % Gap analysis between new MILP formulation and hybrid-metaheuristic

As discussed earlier, solving mathematical problem for larger datasets, is neither time efficient nor computational efficient especially in the case if MILP formulation solved using exact approach. Therefore, computers or super-computers with high capacity memory and processing speeds are recommended to find the solution.

In contrary to the exact solution, hybrid-metaheuristic finds near optimal solutions for cut-off grades and production sequences for long and short term plans of the mining operation in a very reasonable time.

Year (t)	COG_t (% Cu)	$\overline{g}_t$ (% Cu)	Qm_t (tonnes/year)	Qc_t (tonnes/year)	Qr_t (tonnes/year)	P_t (millions)	NPV_t (millions)
1	0.40	0.43	41310	7650	3145	$12.70	$95.83
2	0.57	0.76	44370	7650	5538	$22.46	$93.50
3	0.69	0.85	44370	7650	6195	$25.14	$89.67
4	0.45	0.79	52020	4590	3424	$13.85	$77.98
5	0.64	0.65	48960	4590	2824	$11.41	$75.83
6	0.66	0.89	45900	7650	6492	$26.35	$75.79
7	0.59	0.79	42840	7650	5758	$23.35	$60.81
8	0.73	0.83	42840	6120	4852	$19.68	$46.58
9	0.61	1.03	45900	7650	7514	$30.52	$33.89
10	0.50	0.55	52020	4590	2413	$9.72	$8.45
Total	_	_	*460530*	*65790*	*48160*	*$195.16*	_

Table 5.6: Optimisation results for cut-off grade and production scheduling optimisation using hybrid metaheuristics for 10 years

Thus, considering same hypothetical model, the simulation is run for 10 years life of mining operation using hybrid-metaheurictics, leads to the results shown in Table 5.6, where Figure 5.8 shows the 3D graphical presentation of the simulation results. The time recorded at the end of simulation after different runs is 7-8 minutes.

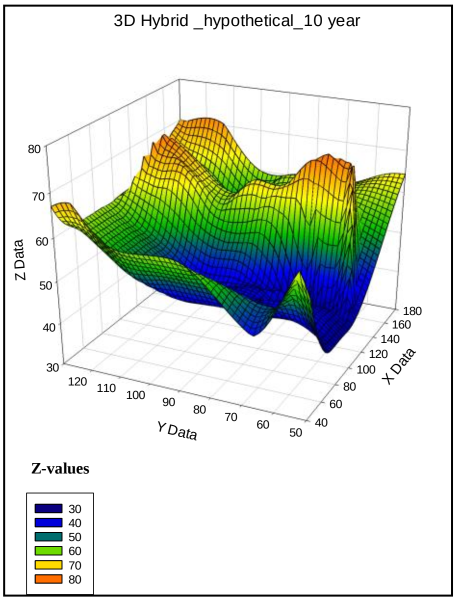


Figure 5.8: 3D-View for near optimised results for 10 years using hybrid-metaheuristics for hypothetical model

Figure 5.9 shows graphical results for average grades, dynamic cut-off grades and NPVs generated for each year, for 10 years life of mining operation.

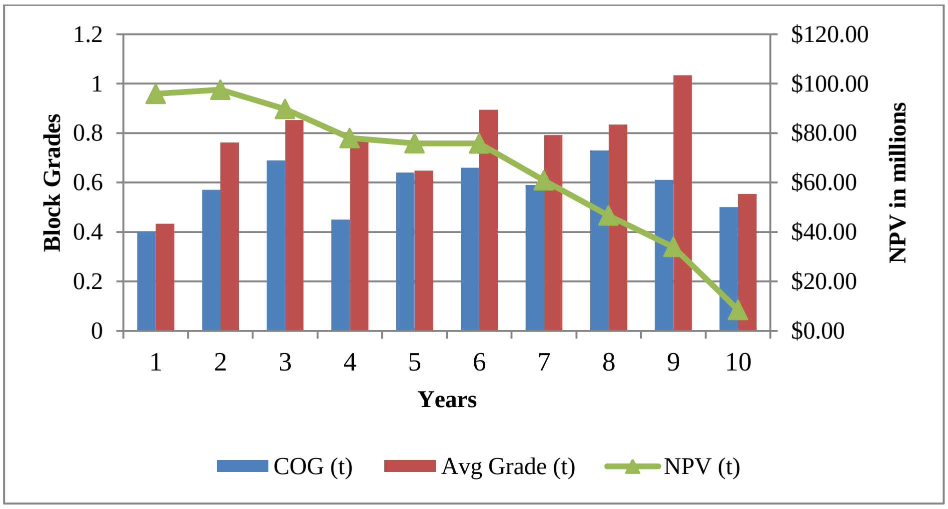


Figure 5.9: Graphical results of average grades, cut-off grades and NPV generated using hybrid_metaheuristic for 10 years for hypothetical block model

5.1.4. Implementation of MILP based conventional production scheduling model using breakeven cut-off grade policy:

The hypothetical block model with 501 blocks is also used to develop production sequence using MILP based production scheduling (Newman et al., 2010). The block-by-block economic block values are used in MILP based formulation for production scheduling, where it considers breakeven cut-off grade strategy. The gap analysis shows that production scheduling generates higher NPV in the initial years, whereas still the quantity of material processed Qc_t does not justify processing. The time recorded after the end of simulation is 4 hrs 23 minutes. However, this insignificant negative gap is owing to the quantity of ore processed at the processing plant during the life of mining operation.

The results in Table 5.7 show that better NPVs are achieved in the case of conventional production scheduling as compared to the new MILP model for cut-off grade optimisation, whereas the time of simulation is comparatively less than the time of simulation of new MILP formulation.

Year (t)	$\overline{g}_t$ (% Cu)	Qm_t (tonnes/year)	Qc_t (tonnes/year)	Qr_t (tonnes/year)	P_t (Millions)	NPV_t (Millions)	Gap%
1	0.85	48960	4590	4360.5	$16.87	$130.77	-$0.40
2	0.76	39780	13770	13081.5	$52.38	$121.13	-$0.27
3	0.73	41310	12240	11628	$46.46	$86.69	$0.06
4	0.69	38990	14510	13784.5	$55.24	$48.03	$0.44
Total		*169040*	45110	42854.5	$170.96		

Table 5.7: Results obtained after simulation of conventional production scheduling formulation

Figure 5.10 shows graphical results for average grades, dynamic cut-off grades and NPVs generated for each year, for 10 years life of mining operation using conventional production scheduling.

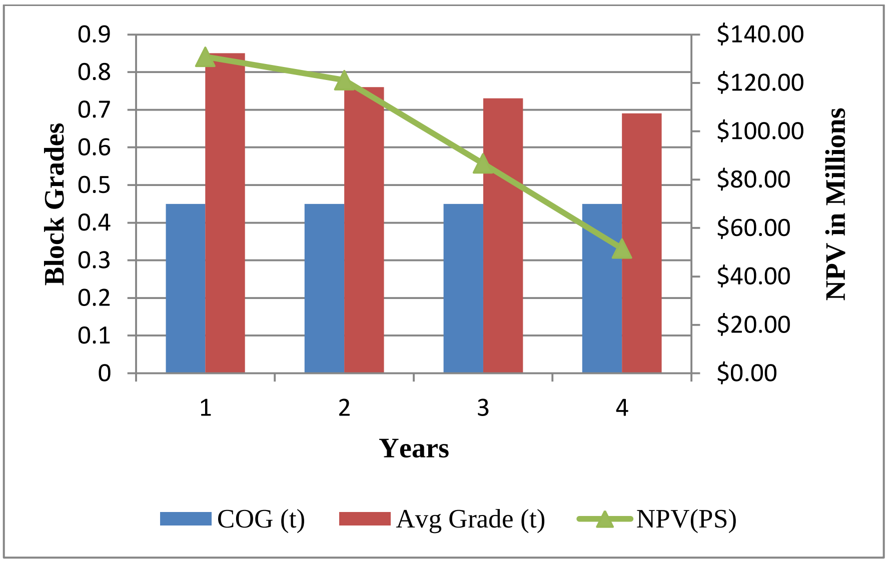


Figure 5.10: Graphical results of average grades, cut-off grades and NPV generated using conventional production scheduling for 4 years for hypothetical block model

The gap analysis of production sequences using breakeven cut-off grade and new MILP cut-off grade formulation is shown in Table 5.7. It can be derived from the results that even though economic block values help in generating more reliable and efficient production sequences with higher NPVs value, but cut-off grade remains constant over the life of mining operation for its reliance on breakeven cut-off grade strategy. However, this higher NPV is owing to the satisfaction of targetted ore production during the life of mining operation.

5.2. Realistic block model (copper deposit) – Case study 2

Case study 2 shows the solution of MILP formulation using realistic ore-body model using hybrid-metaheuristic.

5.2.1. Input parameters

<u>Geological inputs:</u>

Characteristics	Value	Units
Number of Blocks	142296	
Slope Angle	45	degrees
Bench Height	20	meters
Number of Benches	32	
Processing head grade	0.40	% Cu

Table 5.8: Geological parameters for realistic block model

A geological 3D block model comprising of 142296 blocks (Appendix 2) is considered in a realistic case study (2) using copper deposit (ore-body).The charateristics of the block model is given in Table 5.7.

<u>Economic inputs:</u>

Table 5.9 shows the economic parameters for the realistic block model (Appendix 2)

Parameters	Value	Units
Discount Rate d	15	percent
Simulation time	10	years
Metal Price p	3747	$/tonne
Refining cost r	881	$/tonne
Milling Cost c	9	$/tonne
Mining Cost m	1	$/tonne
Fixed Cost FC	25,000,000	$

Table 5.9: Economic parameters for realistic block model

<u>Operational inputs:</u>

Table 5.10 shows the operational capacities' constraints:

Capacities	Value	Units
Mining Capacity ***M***	27,000,000	tonnes/year
Milling Capacity ***P***	7,500,000	tonnes/year

Table 5.10: Operational parameters for realistic block model

5.2.2. Implementation of mathematical model using hybrid-metaheuristic:

New MILP formulation for the realistic case study comprising of 142296 blocks for cut-off grade optimisation and long term production scheduling generates hundred thousands of variables to solve realistic problem, which is computationally not possible. Given the validity of hybrid-metaheuristic approach as shown through the gap analysis in Figure 5.7 for a small hypothetical problem, the mathematical model for realistic case study is solved using hybrid-metaheuristic.

Firstly, a subset of block model is developed using new MILP formulation for cut-off grade optimisation and used as a benchmark for production sequences developed using hybrid-metaheuristic (Figure 5.11).

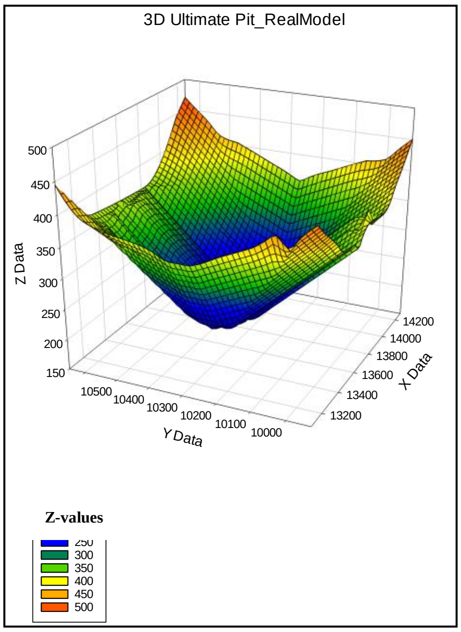


Figure 5.11: 3D-View for near optimised subset for 10 years using Hybrid metaheuristics for realistic block model

The simulation using hybrid- metaheuristic generates near optimal solution for 10 years life of mining operation for realistic case study and is shown in Table 5.11.

Year (t)	COG_t (% Cu)	$\overline{g}_t$ (% Cu)	Qm_t **(tonnes/year)**	Qc_t **(tonnes/year)**	Qr_t **(tonnes/year)**	P_t **(millions)**	NPV_t **(millions)**
1	0.517	0.731	9450000	7171200	4980040	$13,949	$43,787.46
2	0.518	0.71	8132400	4914000	3314493	$9,197	$36,406.58
3	0.531	0.719	9331200	4860000	3319623	$9,211	$32,670.57
4	0.531	0.709	10486800	4644000	3127966	$8,662	$28,360.16
5	0.529	0.709	10411200	4255200	2866090	$7,916	$23,952.18
6	0.525	0.705	10087200	3823200	2560588	$7,044	$19,629.01
7	0.521	0.698	9601200	3175200	2105475	$5,746	$15,529.36
8	0.522	0.698	9288000	3013200	1998053	$5,440	$12,112.76
9	0.524	0.709	8715600	2916000	1964072	$5,344	$8,489.68
10	0.527	0.724	8780400	2721600	1871916	$5,082	$4,419.13
Total	_	_	*94284000*	*41493600*	*28108316*	*77591*	_

Table 5.11: Optimisation results for cut-off grade and 10 year production scheduling using hybrid metaheuristics for a realistic block model

Table 5.11 shows the solution of MILP formulation for a realistic copper deposit simulating for 10 years life of mining operation, which provides a near optimal solution, and defines a new cut-off grade with simultaneous generation of year-by-year production sequence. Secondly, the NPV estimated (Table 5.11) is depending on the cash flows P_t which are derived from Qm_t, Qc_t and Qr_t; where Qc_t and Qr_t do not achieve the target processing capacity of the ore and also varies with the size of the block model; where dynamic COG_t values are obtained and $\overline{g}_t$ values show that the grades of the ore sent for processing are above milling head. The production target can only be achieved by relaxing mining capacity but it directly affects the cost of production. On the other hand, in case the formulation is solved using exact solution, an exponential increase in variable size (10×142296) makes it more complex for realistic copper deposit, and it requires an extraordinary time and computational capacity for simulating the results.

Figure 5.12 presents a 3D view of the pit shape for a realistic block model for 10 years production scheduling. The COG over 10 years for a realistic model is based on the heuristic approach which picks the blocks with maximum grades first and then eliminates the blocks (pheromone update) subject to the mining and processing capacity constraints and maximizing NPV, therefore the variations are not so eminent but still dynamic cut-of grades are generated each year.

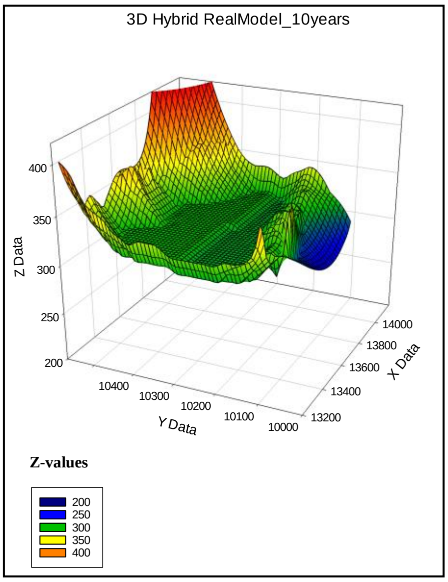


Figure 5.12: 3D-View for near optimised for 10 years production scheduling using hybrid metaheuristics for realistic block model

Figure 5.13 shows graphical results for average grades, dynamic cut-off grades and NPVs generated for each year, for 10 years life of mining operation using realistic copper deposit.

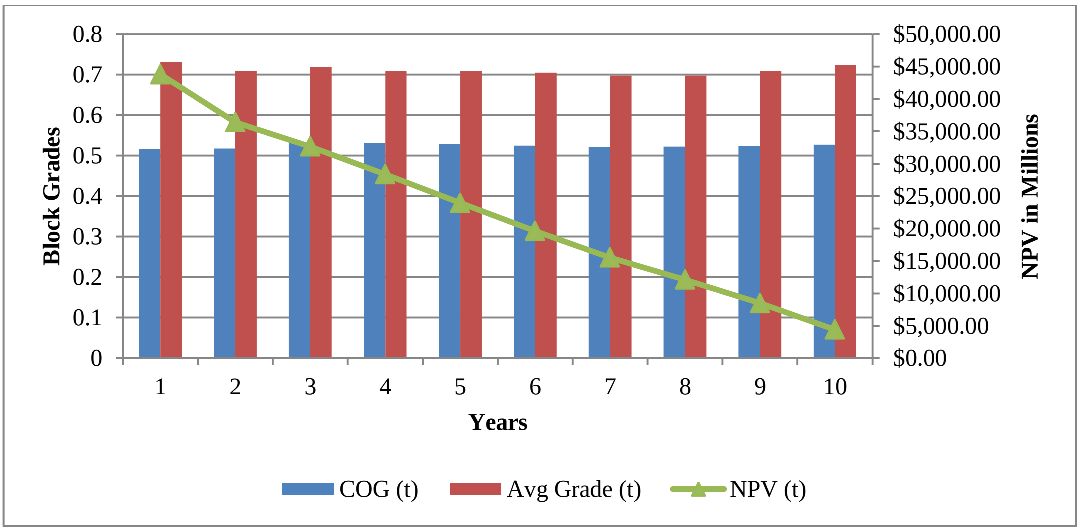


Figure 5.13: Graphical results of average grades, cut-off grades and NPV generated using hybrid-metaheuristic for 10 years for industrial ore-body block model

It is inferred from the above discussion that new MILP based mathematical model defines cut-off grade optimisation policy uses realistic block model and disregards the fact of using grade-tonnage distribution for generating optimal cut-off grade policy. This model is solved using two different techniques, exact approach (using CPLEX concert technology) and hybrid-metaheuristic (combination of GA and ACO), and it generates dynamic cut-off grades with yearly production sequences. The exact solution although generates optimal solution but proves to be NP-hard and computational inefficient, for the large dataset (block model with hundred thousand of blocks). On the other hand, hybrid metaheuristic solves the mathematical model in a reasonable time and generates a near optimal solution. The gap analysis between these two methods shows that hybrid-metaheuristic generates a near optimal solution and derives a road map for future investments and returns on investments.

It is also deduced from the above discussion that new cut-off grade policy generates optimum results without depending on breakeven cut-off grade strategy. On the other hand, the conventional production scheduling formulation which also uses realistic block model, relies on breakeven cut-off grades strategy and economic block values to generate production schedules.

Chapter 6: Conclusions and Recommendations

6.1. Conclusions:

This thesis shares a mixed integer linear programming (MILP) based mathematical model and its implementation for the development of optimal cut-off grade policy and a sequnce of production for open pit mining operations. The structure of the model helps overcome a number of shortcomings in the previous studies. Thus, this effort offers the following major contributions:

1. The model accounts for economic and operational parameters apart from a block-by-block ore-body model (mining block location, grade, and available quantity of material) as a geological input, and then as opposed to converting the ore-body model into a uniform grade-tonnage distcribution or an economic block model (mining block economic values) as practised in the existing mathematical models, it develops a schedule of cut-off grades and a sequence of production *simultaneously*.
2. The model overcomes a compromise on considering breakeven cut-off grade as a basis for delineating ore and waste blocks within the economic block models, which in turn becomes input to the traditional production scheduling methods.
3. It also overcomes a limitation on considering a uniformly distributed grade-tonnage curve of the ore-body, rather, it considers the realistic location-dependent block-by-block information to develop a cut-off policy that *aligns* or synchronises the life-of-operation long-term plans with operational short-term plans, and consequently resolves an important grade control issue.
4. The CPLEX Concert Technology coupled with Java based simple interface develops the MILP formulation, which is then solved to generate exact solution of

the problem in a small-scale hypothetical instance (@ number of variables = 2004). However, the exponential increase in the number of binary variables in the formulation, owing to the the increase in the number of mining blocks in the ore-body model, leads to excessive solution times. Therefore, this implementation confirms the computational complexity of the MILP model.

5. A hybrid-metaheuristic that combines the application of GA and ACO algorithms is then developed to implement the MILP model in a simple Java based interface. This implementation helps address the computational complexity of the problem.
6. A performance evaluation of the metaheuristic in a small-scale hypothetical instance validates the computational efficiency (saving in solution time = approximately 4 hours and 30 minutes) of this new hybrid algorithm through an acceptable (0.10%) gap between the exact (optimal @ NPV = $45.39M) and heuristic (near-optimal @ NPV = $41.13M) solutions.
7. Consequently, an implementation of the MILP model through hybrid algorithm in a practical instance of a copper mining operation generates optimal solution to this large-scale problem within acceptable time (@ solution time = 7-8 minutes).

6.2. Recommendations:

The MILP based mathematical model does not account for creation and management of stockpiles. In addition, it considers deterministic inputs in the context of economic and geological parameters. Similarly, the implementation of the model through hybrid-metaheuristic is restricted to GA and ACO algorithms. Therefore, the future research may incorporate the option to stockpile potential ore, consider the inherent uncertainty in economic and geological inputs, and given the characteristics individual algorithms, other heuristics such as Tabu Search, Particle Swarm Optimisation, Nearest Neighbourhood algorithm may be explored. Moreover, a study into the structure of the proposed MILP model may help address the computational complexity and improve the implementation of the exact approach.

References:

1. Abdel Sabour, S.A., Dimitrakopoulos, R., 2011. Incorporating geological and market uncertainties and operational flexibility into open-pit mine design. J. Min.Sci. 47 (2), 191–201.

2. Abdollahisharif, J., Bakhtavar, E., Anemangely, M., 2012. Optimal cut-off grade determination based on variable capacities in open-pit mining. J. South. Afr. Inst. Min. Metall. 112 (12), 1065–1069

3. Asad, M., M. Qureshi, and H. Jang. 2016."A review of cut-off grade policy models for open-pit mining operations."Resources Policy 49. In press.

4. Asad, M.W.A., 2005. Cutoff grade optimisation algorithm with stockpiling option for open-pit mining operations of two economic minerals. Int. J. Surf. Min. Reclam.Environ. 19 (3), 176–187.

5. Asad, M.W.A., 2007. Optimum cut-off grade policy for open-pit mining operations through net present value algorithm considering metal price and cost escalation Eng. Comput.: Int. J. Comput.-Aided Eng. Softw. 24 (7), 723–736.

6. Asad, M.W.A., Dimitrakopoulos, R., 2013. A heuristic approach to stochastic cut-off grade optimisation for open-pit mining complexes with multiple processing streams. Resour. Policy 38, 591–597.

7. Asad, M.W.A., Topal, E., 2011. Net present value maximisation model for optimum cut-off grade policy of open-pit mining operations. J. South. Afr. Inst. Min. Metall. 111 (11), 741–750.

8. Askari-Nasab, H. and Szymanksi, J., (2006), "Intelligent Agent Framework for Long-Term Mine Planning", Proceeding of Fifteenth International Symposium of Mine Planning and Equipment Selection (MPES), Torino, Italy, Sep. 20-22. , pp. 1076-1082.

9. Ataei, M., Osanloo, M., 2003. Determination of optimum cut-off grades of multiple metal deposits by using the Golden Section search method. J. South. Afr. Inst. Min. Metall. 103 (8), 493–499.

10. Ataei, M., Osanloo, M., 2003. Methods for calculation of optimal cut-off grades in complex ore deposits. J. Min. Sci. 39 (5), 499–507.

11. Ataei, M., Osanloo, M., 2004. Using a combination of genetic algorithm and the grid search method to determine optimum cut-off grades of multiple metal deposits. Int.J. Surf. Min. Reclam. Environ. 18 (1), 60–78.

12. Azimi, Y., Osanloo, M., 2011. Determination of open-pit mining cut-off grade strategy using combination of nonlinear programming and genetic algorithm. Arch. Min. Sci. 56 (2), 189–212.

13. Azimi, Y., Osanloo, M., Esfahanipour, A., 2012. Selection of the open-pit mining cutoff grade strategy under-price uncertainty using a risk based multi-criteria ranking system. Arch. Min. Sci. 57 (3), 741–768.

14. Azimi, Y., Osanloo, M., Esfahanipour, A., 2013. An uncertainty based multi-criteria ranking system for open-pit mining cut-off grade strategy selection. Resour. Policy 38, 212–223.

15. Baker, C.K., Giacomo, S.M., 1998. Resource and reserves: their uses and abuses by the equity markets. In: Ore Reserves and Finance: A Joint Seminar between Australasian Institute of Mining and Metallurgy and ASX, Sydney.

16. Bascetin, A., Nieto, A., 2007. Determination of optimal cut-off grade policy to optimise NPV using a new approach with optimisation factor. J. South. Afr. Inst. Min. Metall. 107 (2), 87–94.

17. Caccetta, L and Hill S P, 2003. An application of branch and cut to open pit mine scheduling, Journal of Global Optimization, 27:349-365.

18. Cairns, R.D., Shinkuma, T., 2003. The choice of the cut-off grade in mining. Resour. Policy 29, 75–81.

19. Cetin E. Dowd P.A. 2013. Multi-mineral cut-off grade optimisation by grid search J. South. Afr. Inst. Min. Metall. 113 (8), 659–665

20. Cetin, E., Dowd, P.A., 1998. The use of genetic algorithms for multiple cut-off grade optimisation. In: Proceedings of the 30th International Symposium on the Application of Computers and Operations Research in the Minerals Industries (APCOM2002), pp. 769–780.

21. Clement, S. and Vagenas, N.: Use of Genetic Algorithms in a Mining Problem. Int. J. Surface Min., Reclamation Environ. 8 (1994), pp. 131–136.

22. Dagdelen K. & Francois-Bongarcon D. (1982) “Towards the complete double parameterization of recovered reserves in open-pit mining”, Proceedings of 17th APCOM symp., Colorado School of Mines, pp. 288 – 296

23. Dagdelen K. (2001) “Open-pit optimisation—strategies for improving economics of mining projects through mine planning”, 17th International Mining Congress and Exhibition of Turkey- IMCET, Ankara, pp. 117-121

24. Dagdelen, K. (1992), “Cutoff grade optimisation”, Proceedings of the 23rd International Symposium on Application of Computers & Operations Research in Minerals Industry, pp. 157-65.

25. Dagdelen, K. (1993), “An NPV optimisation algorithm for open-pit mine design”, Proceedings of the 24th International Symposium on Application of Computers & Operations Research in Minerals Industry, pp. 257-63.

26. Dagdelen, K., 1992. Cut-off grade optimisation. In: Proceedings of the 23rd International Symposium on the Application of Computers and Operations Research in the Minerals Industries (APCOM1992), pp. 157–165.

27. Dagdelen, K., 1993. An NPV optimisation algorithm for open-pit mine design, In: Proceedings of the 24th International Symposium on the Application of Computers and Operations Research in the Minerals Industries (APCOM1993), pp. 257–263.

28. Dagdelen, K., Asad, M.W.A., 1997. Multi mineral cut-off grade optimisation with option to stockpile. In: Proceedings of Society of Mining, Metallurgy, and Exploration, Inc. annual meeting, preprint # 97 186, pp. 1–12.

29. Dagdelen, K., Kawahata, K., 2007. Cut-off grade optimisation for large scale multimine, multi process mining operations. In: Proceedings of the International Symposium on Mine Planning and Equipment Selection. Curran Associates, Bangkok, pp. 226–233.

30. Dagdelen, K., Kawahata, K., 2008. Value creation through strategic mine planning and cut-off grade optimisation. Min. Eng. 60 (1), 39–45.

31. Denby, B. and Schofield, D.: The Use of Genetic Algorithms in Underground Mine Scheduling. In: Proceedings of the 25th International Symposium Application of Computers

and Mathematics in the Mineral Industries, Brisbane, Queensland, Australia, 1995, pp. 389–394.

32. Denby, B., Schofield, D. and Surme, T.: Genetic Algorithms for Flexible Scheduling of Open-pit Operations. In: Proceedings of the 27th International Symposium Application of Computers and Mathematics in the Mineral Industries, 1998.

33. Dimitrakopoulos R, Farrelly C, Godoy MC (2002) Moving forward from traditional optimisation: grade uncertainty and risk effects in open-pit mine design. Trans IMM A Mining Industry 111(1):82–88. doi:10.1179/mnt.2002.111.1.82

34. Dimitrakopoulos, R., 2011. Stochastic optimisation for strategic mine planning: a decade of developments. J. Min. Sci. 47 (2), 138–150.

35. Dorigo M. & Gambardella L. M. (1997a) "Ant colonies for the travelling salesman problem" BioSystems, 43(2), pp. 73–81

36. Dorigo M. & Gambardella L. M. (1997b) "Ant Colony System: A cooperative learning approach to the traveling salesman problem" IEEE Transactions on Evolutionary Computation, 1(1), pp. 53–66

37. Dorigo M. & Stütz..T. (2004) "Ant Colony Optimisation" A Bradford Book, ISBN 0-262-04219-3, pp. 67-11

38. Dowd, P., 1976. Application of dynamic and stochastic programming to optimise cutoff grades and production rates. Trans. Inst. Min. Metall., Sect. A 85, 22–31. Gama, C.D., 2013. Easy profit maximisation method for open-pit mining. J. Rock. Mech. Geotech. Eng. 5, 350–353.

39. Ganguli, R., Dagdelen, K., Grygiel, E., 2011. Mine scheduling and cut-off grade optimisation

40. Gholamnejad, J., 2008. Determination of the optimum cut-off grade considering environmental cost. J. Int. Environ. Appl. Sci. 3 (3), 186–194.

41. Gholamnejad, J., 2009. Incorporation of rehabilitation cost into the optimum cut-off grade determination. J. South. Afr. Inst. Min. Metall. 108 (2), 89–94.

42. Gilani SO, Sattarvand J (2016) Integrating geological uncertainty in long-term open-pit mine production planning by ant colony optimisation. Comp Geosci 87:31–40. doi:10.1016/j.cageo.2015.11.008

43. Goodfellow, R.C., Dimitrakopoulos, R., 2016. Global optimisation of open-pit mining complexes with uncertainty. Appl. Soft Comput. 40, 292–304.

44. Hall, B., 2014. Cut-off grades and optimising the strategic mine plan. Australas. Inst. Min. Metall. 20,

45. He, Y., Zhu, K., Gao, S., Liu, T., Li, Y., 2009. Theory and method of genetic-neural optimizing cut-off grade and grade of crude ore. Expert. Syst. Appl. 36, 7617–7623.

46. Henning, U., 1963. Calculation of cut-off grade. Can. Min. J. 84 (3), 54–57.

47. Hirai, H., Katamura, K., Mamaclay, F.P., Fujimura, T., 1987. Development and Mine Operation at Rio Tuba Nickel Mine. Int. Journal. Mineral. Process. 19, 99–114.

48. Hustrulid, W. and Kuchta, M.: Open-Pit Mine Planning and Design. A.A. Balkema, Rotterdam, Brookfiled, 1, 1995, pp. 512–544.

49. Hustrulid, W., Kuchta, M., Martin, R., 2013. Open-pit Mine Planning and Design 3rd Edition CRC Press/Balkema.

50. Johnson, P.V., Evatt, G.W., Duck, P.W., Howell, S.D., 2011. The determination of a dynamic cut-off grade for the mining industry. Electr. Eng. Appl. Comput., Lect. Notes Electr. Eng. 90, 391–403.

51. Khodayari, A., Jafarnejad, A., 2012. Cut-off grade optimisation for maximising the output rate.

52. King, B., 1999. Cash flow grades - scheduling rocks with different throughput characteristics. In: Proceedings of the Strategic Mine Planning Conference, Perth, Australia, pp. 1–8.

53. King, B., 2001. Transparency in cut-off grade optimisation ‘clear-cut’. In: Proceedings of the Strategic Mine Planning Conference, Perth, Australia, pp. 1–10. 12 Resources Policy xxx (2016) xxx-xxx

54. King, B., 2004. Integrated strategy optimisation for complex operations. In: Proceedings of the Orebody Modelling and Strategic Mine Planning Conference, Perth, Australia, pp. 391–398.

55. King, B., 2009. Optimal mining principles. In: Proceedings of the Orebody Modelling and Strategic Mine Planning Conference, Perth, Australia, pp. 1–6.

56. Krautkraemer, J.A., 1988. The cut-off grade and the theory of extraction. Canadian J.Econ. 21 (1), 146–160.

57. Lamghari, A., and R. Dimitrakopoulos. 2012. "A Diversified Tabu Search Approach for the Open-Pit Mine Production Scheduling Problem with Metal Uncertainty." European Journal of Operational Research 222 (3): 642–652. doi:10.1016/j.ejor.2012.05.029.

58. Lane, K.F., 1964. Choosing the optimum cut-off grade. Q. Colo.Sch. Mines 59, 811–829.

59. Lane, K.F., 1984. Cut-off grades for two minerals. In: Proceedings of the 18th International Symposium on the Application of Computers and Operations Research in the Minerals Industries (APCOM1984), London, pp. 485–492.

60. Lane, K.F., 1988. The Economic Definition of Ore: Cut-off Grades in Theory and Practice. Mining Journal Books, London, UK.

61. Lane, K.F.: Choosing the Optimum Cut-Off Grade. Quart Colorado School Mines 59 (4) (1964), pp. 811–829.

62. Lane, K.F.: The Economic Definition of Ore – Cut-Off Grades in Theory and Practice. Min. Journal Books Limited, London (1988), 145.

63. Li, Sh, Chang, Y., 2012. An optimum algorithm for cut-off grade calculation using multistage stochastic programming. J. Theor. Appl. Inf. Technol. 45 (1), 117–122.

64. Lamghari, A., & Dimitrakopoulos,R.(2012). A diversified tabu search approach for the open-pit mine production scheduling problem with metal uncertainty. *European Journal of Operational Research, 222*,642–652.

65. Lamghari, A., Dimitrakopoulos, R., & Ferland, J.A.(2014). A variable neighbourhood descent algorithm for the open-pit mine production scheduling problem with metal uncertainty. *Journal of the Operational Research Society, 65*, 1305–1314.

66. Mardones, J.L., 1993. Option valuation of real assets: application to a copper mine with operating flexibility. Resour. Policy 19, 51–65.

67. Marques, D.M., Costa, J.F.C.L., 2013. An algorithm to simulate ore grade variability in blending and homogenization piles. Int. J. Mineral. Process. 120, 48–55.

68. McKee, D.J., Chitombo, G.P., Morrell, S., 1995. The relationship between fragmentation in mining and comminution circuit throughput. Miner. Eng. 8 (11), 1265–1274.

69. Mol, O., Gillies, A.D.S., 1984. Cut-off grade determination for mines producing direct shipping iron ore. Proceedings of the Australasian Institute of Mining and Metallurgy, 289, November/December 1984, pp. 283–287.

70. Moosavi, E., Gholamnejad, J., Ataee-pour, M., Khorram, E., 2014. Optimal extraction sequence modelling for open-pit operation considering dynamic cut-off grade. Mineral. esour. Manag. 30 (2), 173–186

71. Narri, S., Osanloo, M., 2015. Optimum cut-off grade's calculation in open-pit mines with regard to reducing the undesirable environmental impacts. Int. J. Min.,Reclam. Environ. 29 (3), 226–242.

72. Newman, A.M., Rubio, E., Caro, R., Weintraub, A., Eurek, K., 2010. A review of operations research in mine planning. Interfaces 40 (3), 222–245.

73. Nieto, A., Bascetin, A., 2006. Mining cut-off grade strategy to optimise NPV based on multiyear GRG iterative factor. Trans. Inst. Min. Metall.: Min. Technol. 115 (2), 59–64.

74. Nieto, A., Zhang, K.Y., 2013. Cut-off grade economic strategy for by-product mineral commodity operation: rare earth case study. Trans. Inst. Min. Metall.: Min. Technol.122 (3), 166–171.

75. Osanloo, M., Ataei, M., 2003. Using equivalent grade factors to find the optimum cutoff grades of multiple metal deposits. Miner. Eng. 16, 771–776

76. Osanloo, M., Rashidinejad, F., Rezai, B., 2008. Incorporating environmental issues into optimum cut-off grades modelling at porphyry copper deposits. Resour. Policy 33, 222–229.

77. Rahimi, E., Ghasemzadeh, H., 2015. A new algorithm to determine optimum cut-off grades considering technical, economic, environmental and social aspects. Resour. Policy 46, 51–63.

78. Rahimi, E., Oraee, K., Shafahi Tonkaboni, Z.A., Ghasemzadeh, H., 2015. Considering environmental costs of copper production in cut-off grades optimisation. Arab. Geosci. 8 (9), 7109–7123.

79. Rahimi, E., Oraee, K., Shafahi, Z., Ghasemzadeh, H., 2015. Determining the optimum cut-off grades in sulphide copper deposits. Arch. Min. Sci. 60 (1), 313–328.

80. Ramazan, S., 2007. The new fundamental tree algorithm for production scheduling of open-pit mines. European Journal of Operational Research 177, 1153–1166.

81. Ramazan, S., Dimitrakopoulos, R., 2007. Stochastic optimisation of long term production scheduling for open-pit mines with a new integer programming formulation. The Australasian Institute of Mining and Metallurgy, Spectrum Series 14, 385–392.

82. Ramazan, S., Dimitrakopoulos, R., 2012. Production scheduling with uncertain supply: a new solution to the open-pit mining problem. Optimisation and Engineering. http://dx.doi.org/10.1007/s11081-012-9186-2.

83. Rao, S.S.: Engineering Optimisation (Theory and Practice). Wiley–Interscience Publication, John Wiley & Sons Inc., New York, 3rd edition, 1996, p. 903.

84. Rardin, R.L.: Optimisation in Operations Research. Prentice-Hall International, Inc., 1998, p. 919.

85. Rendu, J.M., 2009. Cut-Off Grade Estimation – Old Principles Revisited – Application to Optimisation of Net Present Value and Internal Rate of Return. Orebody Modelling and Strategic Mine Planning Conference, Perth, pp. 165-169.

86. Rendu, J.M., 2014. An Introduction to Cut-off Grade Estimation. Society of Mining, Metallurgy, and Exploration (ISBN: 978-0-87335-393-9), 2nd Edition.

87. Taylor, H.K., 1972. General background theory of cut-off grades. Trans. Inst. Min. Metall.: Sect. A 160–179.

88. Taylor, H.K.: Cutoff Grades – Some Further Reflections. Inst. Min. Metall. Trans. (October, 1985), pp. A204–216.

89. Thompson, M., Barr, D., 2014. Cut-off grade: a real options analysis. Resour. Policy 42, 83–92.

90. Topal, E., Ramazan, S., 2010. A new MIP model for mine equipment scheduling by minimizing maintenance cost. European Journal of Operational Research 207, 1065–1071.

91. Vallee, M., 2000. Mineral resource + engineering, economic and legal feasibility = ore reserve. CIM Bull. 93, 53–61.

92. Vickers, E.L., 1961. Marginal analysis – its applications in determining cut-off grade. Min. Eng. 13, 579–582.

93. Whittle D. Wooller R. 1999. Maximising the economic performance of your comminution circuit through cut-off optimisation Int. J. Surf. Min. Reclam. Environ. 13 (4),147–153

94. Whittle, D., Vassiliev, P., 1998. Synthesis of stochastic recovery prediction and cut-off optimisation. In: Proceedings of the Mine to Mill Conference, Australasian Institute of Mining and Metallurgy, Brisbane, Australia, pp. 53–56.

95. Wooler, R., 2001. Cut-off grades beyond the mine – optimising mill throughput. Mineral. Resour. Ore Reserve Estim. – AusIMM Guide Good Pract., Australas. Inst.Min. Metall. 459–468.

96. Yasrebi, A.B., Wetherelt, A., Foster, P., Kennedy, G., Ahangaran, D.K., Afzal, P.,Asadi, A., 2015. Determination of optimised cut-off grade utilising non-linear programming. Arab. J. Geosci. 8 (10), 8963–8967.

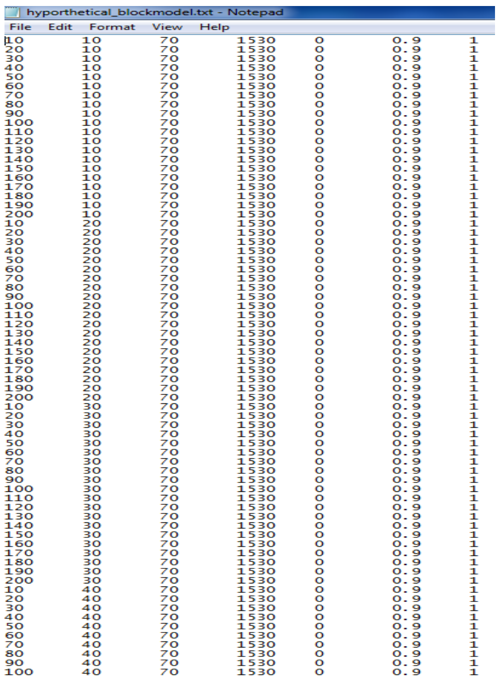
Appendix 1: User manual for software application (solving MILP model based cut-off grade optimisation policy using CPLEX concert technology)

The methods to take input parameters and the design of graphical user interface (GUI) for both MILP and metaheuristic program is same, for the reason that same inputs are considered for both the programs; whereas the functionality and solution methodology developed in both the programs are different. Java programming language is used for developing code for MILP as well as metaheuristic using ECLIPSE editor.

Input Text files:

- **Block Model File (blockmodel.txt):**

  The block model file shown in Figure 1 is taken as input text file using hard coding in the program. So whatever file is used as an input for geological block model must be saved with the name blockmodel.txt comprising of X,Y, Z tonnage, grades, recovery and MCAF columns separated by tab as shown in Figure 1.



hyporthetical_blockmodel.txt - Notepad

File Edit Format View Help

```
10	10	70	1530	0	0.9	1
20	10	70	1530	0	0.9	1
30	10	70	1530	0	0.9	1
40	10	70	1530	0	0.9	1
50	10	70	1530	0	0.9	1
60	10	70	1530	0	0.9	1
70	10	70	1530	0	0.9	1
80	10	70	1530	0	0.9	1
90	10	70	1530	0	0.9	1
100	10	70	1530	0	0.9	1
110	10	70	1530	0	0.9	1
120	10	70	1530	0	0.9	1
130	10	70	1530	0	0.9	1
140	10	70	1530	0	0.9	1
150	10	70	1530	0	0.9	1
160	10	70	1530	0	0.9	1
170	10	70	1530	0	0.9	1
180	10	70	1530	0	0.9	1
190	10	70	1530	0	0.9	1
200	10	70	1530	0	0.9	1
10	20	70	1530	0	0.9	1
20	20	70	1530	0	0.9	1
30	20	70	1530	0	0.9	1
40	20	70	1530	0	0.9	1
50	20	70	1530	0	0.9	1
60	20	70	1530	0	0.9	1
70	20	70	1530	0	0.9	1
80	20	70	1530	0	0.9	1
90	20	70	1530	0	0.9	1
100	20	70	1530	0	0.9	1
110	20	70	1530	0	0.9	1
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200	20	70	1530	0	0.9	1
10	30	70	1530	0	0.9	1
20	30	70	1530	0	0.9	1
30	30	70	1530	0	0.9	1
40	30	70	1530	0	0.9	1
50	30	70	1530	0	0.9	1
60	30	70	1530	0	0.9	1
70	30	70	1530	0	0.9	1
80	30	70	1530	0	0.9	1
90	30	70	1530	0	0.9	1
100	30	70	1530	0	0.9	1
110	30	70	1530	0	0.9	1
120	30	70	1530	0	0.9	1
130	30	70	1530	0	0.9	1
140	30	70	1530	0	0.9	1
150	30	70	1530	0	0.9	1
160	30	70	1530	0	0.9	1
170	30	70	1530	0	0.9	1
180	30	70	1530	0	0.9	1
190	30	70	1530	0	0.9	1
200	30	70	1530	0	0.9	1
10	40	70	1530	0	0.9	1
20	40	70	1530	0	0.9	1
30	40	70	1530	0	0.9	1
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50	40	70	1530	0	0.9	1
60	40	70	1530	0	0.9	1
70	40	70	1530	0	0.9	1
80	40	70	1530	0	0.9	1
90	40	70	1530	0	0.9	1
100	40	70	1530	0	0.9	1
```
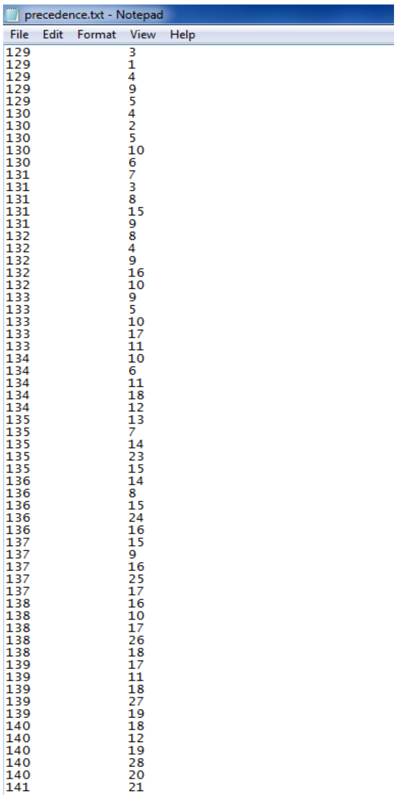

Figure 1: Input text files (Geological block model-blockmodel.txt)

- **Precedence file (precedence.txt):**

The generation of precedence file is based on a stand-alone program developed using C++ and saved as exe file (*precedence.exe*) as shown in Figure 2. This file is saved in the separate folder and processed separately to generate *precedence.txt* file. This file generates the predecessors for each block with the ratio 1:5, which means that for each block if mined, the 5 predecessors (overlying blocks) in three dimensions must be mined as well, and this is one of the important constraints of the MILP model.


precedence.txt - Notepad

File Edit Format View Help

129	3
129	1
129	4
129	9
129	5
130	4
130	2
130	5
130	10
130	6
131	7
131	3
131	8
131	15
131	9
132	8
132	4
132	9
132	16
132	10
133	9
133	5
133	10
133	17
133	11
134	10
134	6
134	11
134	18
134	12
135	13
135	7
135	14
135	23
135	15
136	14
136	8
136	15
136	24
136	16
137	15
137	9
137	16
137	25
137	17
138	16
138	10
138	17
138	26
138	18
139	17
139	11
139	18
139	27
139	19
140	18
140	12
140	19
140	28
140	20
141	21

Figure 2: Precedence file (generated using precedence.exe program)

- **Operational Parameters:**

  Insert operational parameters (mining and processing capacities) are taken as inputs using GUI.

- **Economic Parameters File (economic_parameter.txt):**

  There are two ways, how economic parameters are taken as inputs in the software.

  - Browsing input text files (Figure 3) with economic parameters are saved in the sequence of metal price; selling cost, milling cost and mining cost each separated by tab, and then uploads the text file in the same directory where other text files are saved.

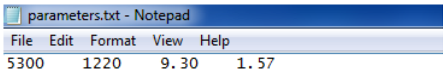


Figure 3: Economic parameters file

- Or, insert economic parameters using GUI as mentioned earlier.

How MILP formulation works as Software:

After taking inputs according to the methods given above, formulation is developed using CONCERT Technology in JAVA. ILO integer variables are used to defining binary variables, whereas and ILO expressions are used to define decision variables and constraints of MILP formulation. The following features are explained in detail.

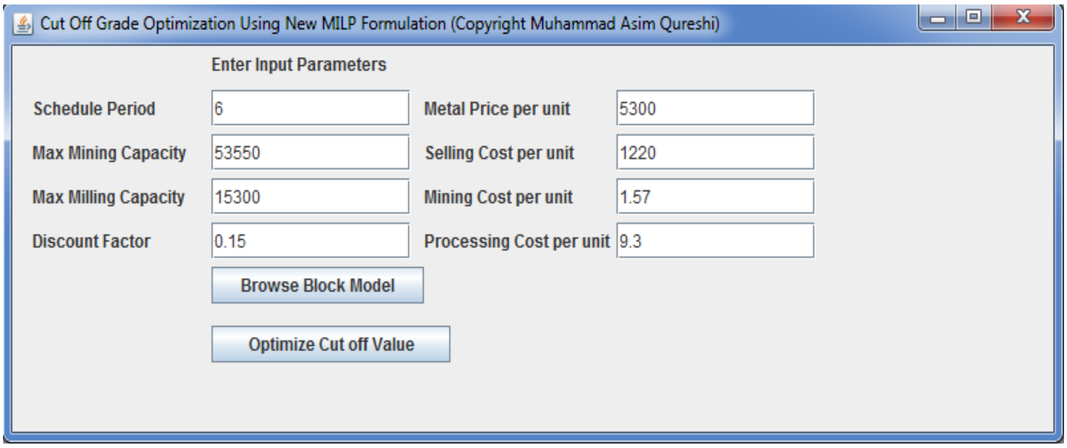


Figure 4: Graphical user interface (GUI) for taking economic and operational inputs for MILP formulation

- *Blockmodel.txt* file is read using file input/output function and all columns separated by tab command saved in separate arrays for location (X, Y and Z coordinates), grades, tonnage and recovery.

- *Precedence.txt* file is read using file input/output function and saved in separate arrays for block number and their predecessors.
- *Eco-parameter.txt* is either read using file input/output function or taking direct input using GUI and saved in separate variables for metal price, refining cost, processing cost and mining cost.
- Variables $xval, yval, zval, grad, cog, avg, p, c, r, m$ are used to assign values for location (X, Y and Z coordinates), grade, cut-off grade, average grade, metal price, processing cost, refining cost and mining cost respectively.
- Variable x is used to define a binary variable and assigning it two values with the condition that if block is mined x = 1 or if not then x = 0.
- Objective function is defined in terms of discounted cash flow P_t .
- Discounted P_t is defined in terms of following cash flow statement

$$P_t = (p - r)Qr_t - cQc_t - mQm_t$$

- The economic value of each single block in the block model is not considered in this formulation so need to define decision variables including quantity of material mined Qm_t, quantity of material processed Qc_t , quantity of material refined Qr_t and fixed annual cost FC_t are defined, and their ranges are also specified in the program.
- Objective function is defined on the basis of discounted cash flow

$$Maximize\ Obj = \sum_{t=0}^{T} \frac{P_t}{(1+d)^t}$$

- The CPLEX expressions are introduced to define the following constraints:
    - Cash flow constraint
    - Quantity of material mined and mining capacity constraint
    - Quantity of material processed and processing capacity constraint
    - Slope constraint
    - Reserve constraint
    - Cut-off grade constraints

Running MILP simulation and Outputs:

Initially the simulation is run to generate the actual year by year production schedules and cut-off grade policy, the MILP simulation is run to generate the following two outputs.

Output 1:

An independent model file (*filename.lp*) is generated as an output of the program. This file can be run separately using CPLEX exe file using CPLEX commands as shown in Figure 5.

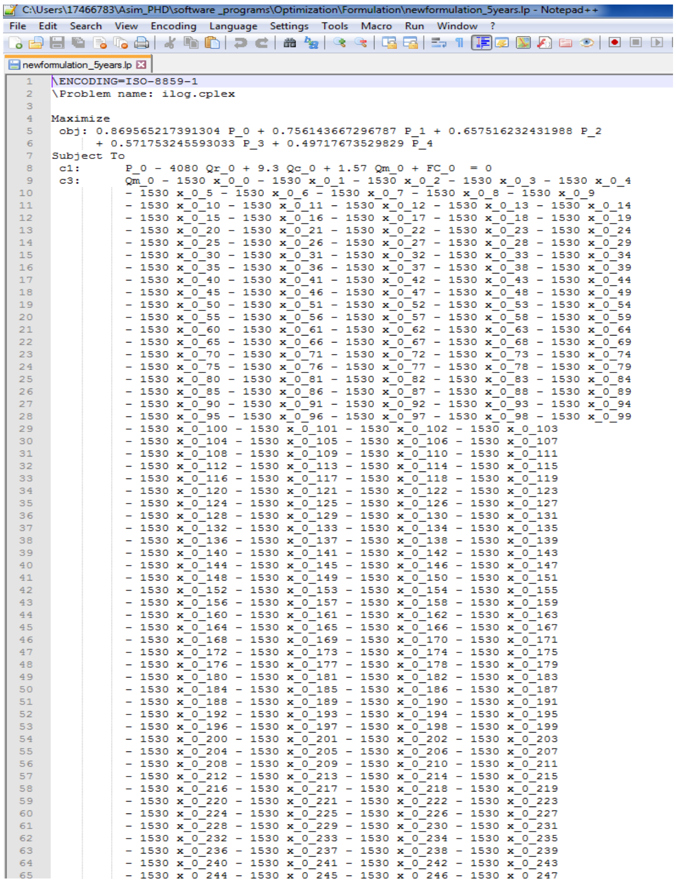


Figure 5: Graphical user interface (GUI) for taking economic and operational inputs for MILP formulation

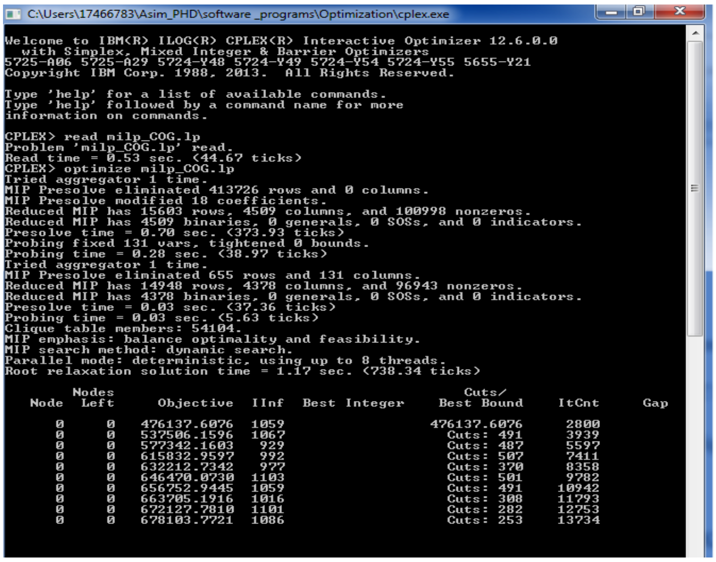


Figure 6: CPLEX exe file (reading and running *lp* model file)

The solution file *(filename.sol)* is also generated which shows the production scheduling (number of blocks mined as 1 and 0 for blocks not mined) and shown in Figure 7.

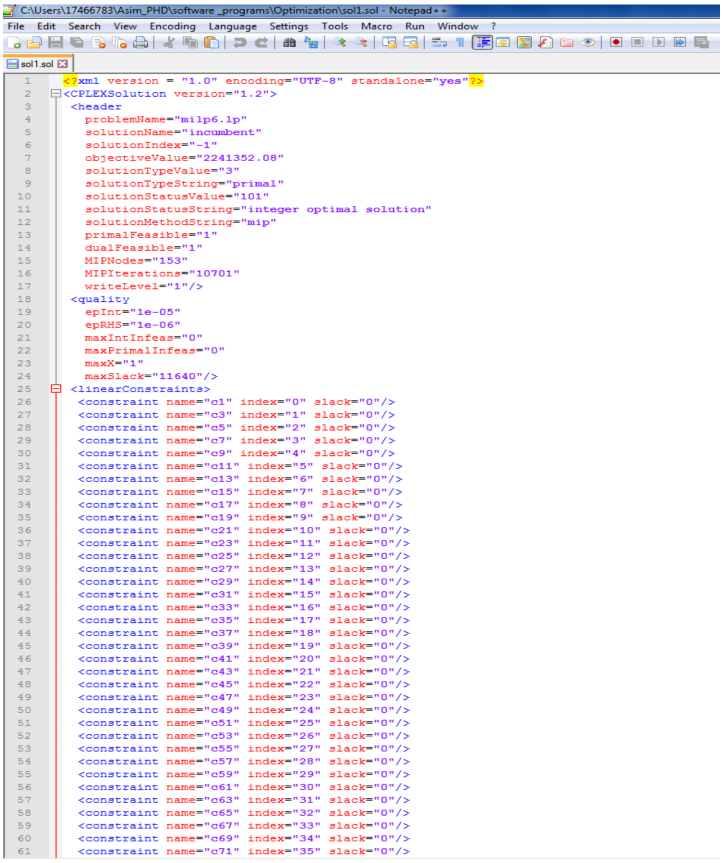
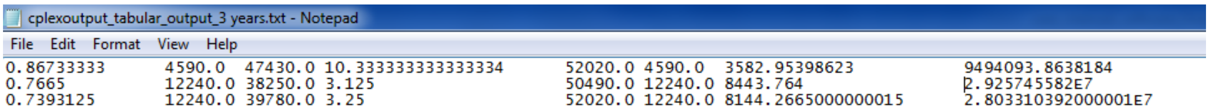


Figure 7: Solution file generated after simulation of lp model file ( )

Output 2:

The CPLEX concert technology is used in the same MILP program which generates simulation results as been compiled separately in CPLEX exe file. The output of the software program is recorded in a text file format, showing simulation results in terms of columns separated by tab as shown in Figure 8.



cplexoutput_tabular_output_3 years.txt - Notepad

File Edit Format View Help

```
0.86733333	4590.0	47430.0	10.333333333333334	52020.0	4590.0	3582.95398623	9494093.8638184
0.7665	12240.0	38250.0	3.125	50490.0	12240.0	8443.764	2.925745582E7
0.7393125	12240.0	39780.0	3.25	52020.0	12240.0	8144.2665000000015	2.803310392000001E7
```

Figure 8: Tabular output of the simulated results using concert technology in MILP formulation

The results of a text file are then imported to Excel spreadsheet, where the production schedules and data is compiled and analysed as shown in Figure 9. The spreadsheet results can be used in any mine planning software to develop 3D schedules.

Appendix 2: User manual for software application (cut-off grade optimisation policy using hybrid metaheuristic)

How hybrid-metaheuristic works as Software:

The mathematical model is also solved using hybrid-metaheuristic. The method of taking inputs using GUI and the input text files is same as mentioned in above in MILP formulation. Figure 1 shows the GUI for taking input parameters for hybrid-metaheuristic. The simulation generates near optimum results and gap analysis shows the difference.

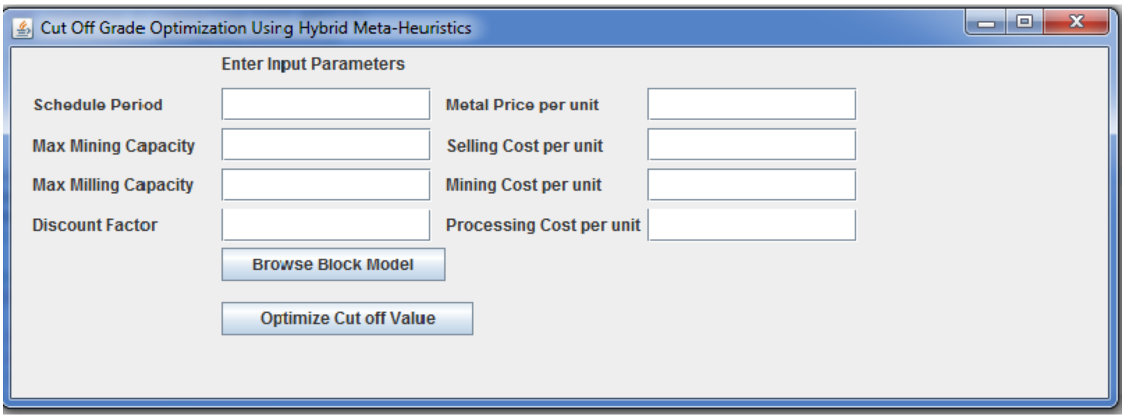


Figure 1: Graphical user interface (GUI) for taking economic and operational inputs for MILP formulation

Running Simulation for hybrid-metaheuristic and Outputs:

Step1: Search using Genetic Algorithm (GA)

Initial search of blocks with highest possible grades which can generate maximum profit at lowest operational cost are searched using GA after several generations. The GA continues unless the best among the best possible solution is selected.

- Condition and constraints:
  - Search blocks in vertical (top to bottom) and along horizontal.
  - Block grade must be greater than minimum required head grade
  - Blocks generating maximum profit with predecessors generating minimum cost
  - Blocks selected for processing must be less than or equal to processing capacity
  - All blocks selected for mining with their predecessors must be less than or equal to the mining capacity

- Search all the possibilities of ore blocks with potential grade horizontally

- Crossover:
  - Using crossover technique checks the best solution of ore block by rearranging them with the condition of maximum profit with the conditions and constraints.
- Mutation:
  - Using mutation process any missing ore block or predecessor is checked for the best among the fittest solutions, abiding by the given constraints.

Step 2: Applying Ant Colony Optimisation (ACO)

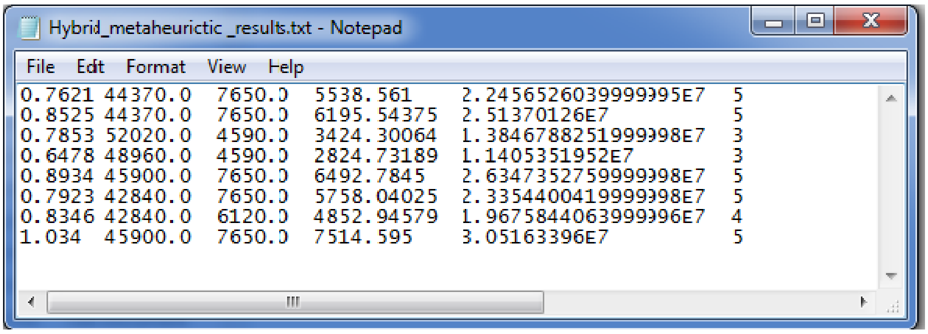
The selection of possible set of mineable blocks in Step 1 is again validated by applying ACO algorithm.

- ANTS:
  - The ants are defined as possible potential set of blocks (grade > 0) as part of ACO algorithm.
- Pheromones
  - Pheromones are defined as ants (potential blocks with grade > 0) and their predecessors (generating best of solutions) fulfilling the given constraints mentioned.
- Pheromone evaporation
  - Only best among the fittest solutions (pheromones) are selected and rest of the pheromones are evaporated (means not selected). The selection of the set of mineable blocks using GA also validated in the selection of ACO and best among them is selected.

Program compilation and Outputs:

The interface shown in the Figure 1 is similar to the one designed for MILP model. The program is designed in such a way that pheromones are generated giving schedule for each year. The schedules (ore blocks and predecessors) mined in the previous schedules are not considered in the next genetic search and considered as evaporated pheromones to generate upcoming schedules in ant colony algorithm. The output of the simulation is recorded in a text file (*hybrid_metaheuristic_results.txt*) shown in Figure 11, and the path of the output files is made as default, which means they are saved in the same folder where other input files are saved. The option to change the path for saving the file is also given.

Figure 11 shows the output text file after running hybrid metaheuristic program. This program generates results after several iterations of genetic algorithm and ant colony optimisation algorithm. The stopping condition achieves when optimum value or near optimum value is achieved. So, in this way the mathematical model is solved using the objective function which maximises NPV subject to all slope, reserve and operational capacity constraints. In addition, it generates dynamic cut-off grades and production sequences over the life of mining operations. The results are finally exported to spreadsheet for analysis and production scheduling.


Hybrid_metaheurictic _results.txt - Notepad

File Edit Format View Help

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0.7853	52020.0	4590.0	3424.30064	1.3846788251999998E7	3
0.6478	48960.0	4590.0	2824.73189	1.1405351952E7	3
0.8934	45900.0	7650.0	6492.7845	2.6347352759999998E7	5
0.7923	42840.0	7650.0	5758.04025	2.3354400419999998E7	5
0.8346	42840.0	6120.0	4852.94579	1.9675844063999996E7	4
1.034	45900.0	7650.0	7514.595	3.05163396E7	5

Figure 11: Output text file generated by running hybrid metaheuristic program

Appendix 3: Production scheduling using new MILP formulation (Generated using MS Excel)

1. Plan view of production schedules developed at different Z location for 4 years life of mining operation where numbers inside the colour boxes are showing the years of production sequence:

X

170																
160																
150																
140							3									
130					4	3	2	2								
120				4	3	2	2	2	2			4	1			
110			4	3	2	2	2	2	2	2	4	1	1	1		
100			3	3	3	2	2	2	2	2	1	1	1	1	1	
90			4	3	3	3	2	2	2	4	1	1	1	1	1	1
80				4	3	3	3	2	4			1	1	1	1	1
70					4	3	3	4					1		1	
60							4									
50																
40																
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170

Y

Figure 1: Plan map of production sequences at Z = 70

X

170																
160																
150																
140																
130						4										
120					4	3	2	2								
110				4	3	2	2	2	2			4	1			
100				3	3	3	2	2	2	4	4	1	1	1		
90				4	3	3	3	2	4			1	1	1	1	
80					4	3	3	4					1		1	
70							4									
60																
50																
40																
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170

Y

Figure 2: Plan map of production sequences at Z = 60

X

170																
160																
150																
140																
130																
120							4									
110					4		2	2								
100					3	3	3	2	4			4	1			
90					4	3	3						1			
80							4									
70																
60																
50																
40																
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170

Y

Figure 3: Plan map of production sequences at Z = 50

X

170																
160																
150																
140																
130																
120																
110							4									
100						3	3	4								
90							4									
80																
70																
60																
50																
40																
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170

Y

Figure 4: Plan map of production sequences at Z = 40

150																
140																
130																
120																
110																
100							4									
90																
80																
70																
60																
50																
40																
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170

X (rows), Y (columns)

Figure 5: Plan map of production sequences at Z = 30

2. Section view of production schedule developed at different Y locations for 4 years life of mining operation:

70				4	3	2	2	2	2	2	2	4	1	1	1		
60					4	3	2	2	2	2			4	1			
50						4	3	2	2								
40								4									
30																	
20																	
10																	
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170

Z (rows), X (columns)

Figure 6: Section map of production sequences at Y = 110

70				3	3	3	2	2	2	2	2	1	1	1	1	1	
60					3	3	3	3	2	2	4	4	4	1	1		
50						3	3	3	2	4			4	1			
40							3	3	4								
30								3									
20																	
10																	
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170

Z (rows), X (columns)

Figure 7: Section map of production sequences at Y = 100

Z																	
70				4	3	3	3	2	2	2	4	1	1	1	1	1	1
60					4	3	3	3	2	4			1	1	1	1	
50						4	3	3	4					1			
40								4									
30																	
20																	
10																	
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170

X

Figure 8: Section map of production sequences at Y = 90

Z																	
70					4	3	3	3	2	4			1	1	1	1	1
60						4	3	3	4					1	1	1	
50								4									
40																	
30																	
20																	
10																	
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170

X

Figure 9: Section map of production sequences at Y = 80

3. Section view of production schedule developed at different X locations for 4 years life of mining operation:

Z													
70							1	1	1	1	1	1	
60								1	1	1	1		
50									1	1			
40													
30													
20													
10													
	10	20	30	40	50	60	70	80	90	100	110	120	130

Y

Figure 10: Section map of production sequences at X = 140

Z													
70									1	1	4		
60										4			
50													
40													
30													
20													
10													
	10	20	30	40	50	60	70	80	90	100	110	120	130

Y

Figure 11: Section map of production sequences at X = 130

Z													
70							4	2	2	2	2	2	2
60								4	2	2	2	2	
50									4	2	2		
40										4			
30													
20													
10													
	10	20	30	40	50	60	70	80	90	100	110	120	130

Y

Figure 12: Section map of production sequences at X = 90

Z														
70						4	3	3	2	2	2	2	2	3
60							4	3	3	2	2	2	4	
50								4	3	3	2	4		
40									4	3	4			
30										3				
20														
10														
	10	20	30	40	50	60	70	80	90	100	110	120	130	140

Y

Figure 13: Section map of production sequences at X = 80

Z														
70							3	3	3	2	2	2	3	
60								3	3	3	2	3		
50									3	3	3			
40										3				
30														
20														
10														
	10	20	30	40	50	60	70	80	90	100	110	120	130	140

Y

Figure 13: Section map of production sequences at X = 70

Appendix 4: Production scheduling using hybrid-metaheuristic

1. Plan view of production schedules developed at different Z location for 4 years life of mining operation using hybrid-metaheuristic where numbers inside the colour boxes are showing the years of production sequence:

170																	
160																	
150																	
140					4	4	5	5									
130					4	4	5	5	5								
120		4	4	4	4	4	4	5	5			3	4	4	4		
110	4	4	4	3	4	4	5	5	3	5	3	3	3	4	4		
100		4	4	2	3	3	3	3	2	3	3	3	2	1	1	1	1
90			2	2	2	2	2	2	2	2	3	1	1	1	1	1	1
80				2	2	2	2	2	2		1	1	1	1	1	1	1
70					2	2	2	2				1	1	1	1		
60													1				
50																	
40																	
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180

X (rows), Y (columns)

Figure 1: Plan map of production sequences at Z = 70

170																	
160																	
150																	
140																	
130					10	10											
120					4	4	5	5						9	10		
110			10	4	3	3	3	3		9		3	4	9	9	10	
100			8	3	2	2	2	2	3	8	3	3	4	4	10	8	10
90			7	2	2	2	2	2	2	6	8	2	1	1	1	8	8
80			5	5	2	2	2	2	6	6	6	1	1	1	1	7	
70				5	5	5	6	6	6	6			1	6	6	7	7
60					5	6	6	6	6					6	7	7	
50																	
40																	
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180

X (rows), Y (columns)

Figure 2: Plan map of production sequences at Z = 60

X

170																	
160																	
150																	
140																	
130																	
120					10	10											
110				10	4	4	5	5	9	9					10		
100				8	3	3	3	3	8	8		3	4	8	8	10	
90				7	2	2	2	2	6	8			2	6	7	8	
80				5	5	5	6	6	6	6			1	6	6	7	
70					5	6	6	6	6					6	7		
60																	
50																	
40																	
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180

Y

Figure 3: Plan map of production sequences at Z = 50

X

170																	
160																	
150																	
140																	
130																	
120																	
110					10	10											
100					8	9	9	9	9						10		
90					7	7	7	7	8					8	8		
80					5	6	6	6	6					6	7		
70																	
60																	
50																	
40																	
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180

Y

Figure 4: Plan map of production sequences at Z = 40

2. Section view of production schedule developed at different Y locations for 4 years life of mining operation using hybrid-metaheuristic:

Z

70					4	4	4	5	5				
60						10	10						
50													
40													
30													
20													
10													
	10	20	30	40	50	60	70	80	90	100	110	120	130

X

Figure 5: Section map of production sequences at Y = 130

Z

70					4	4	4	4	5			10	3
60						9	9	9					
50							10						
40													
30													
20													
10													
	10	20	30	40	50	60	70	80	90	100	110	120	130

X

Figure 6: Section map of production sequences at Y = 120

Z

70		9	4	4	4	4	4	5	9	3	3	3	3	5
60			9	4	4	4	5	9			4	4	5	
50				9	4	5	9					5		
40					9									
30														
20														
10														
	10	20	30	40	50	60	70	80	90	100	110	120	130	140

X

Figure 7: Section map of production sequences at Y = 110

Z

70				3	3	3	3	3	3	2	3	3	3	2	1	1
60					8	2	2	2	2	2		3	3	3	3	3
50						8	3	3	3	8			3	3		
40							8	3	8							
30								9								
20																
10																
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160

X

Figure 7: Section map of production sequences at Y = 100

3. Section view of production schedule developed at different X locations for 4 years life of mining operation using hybrid-metaheuristic:

Z

70			6	6	7	7	6	6	6	6				
60				7	7	7	8	10	7					
50						8		7						
40														
30														
20														
10														
	40	50	60	70	80	90	100	110	120	130	140	150	160	170

Y

Figure 8: Section map of production sequences at X = 170

Z

70				7	5	1	1	7	9					
60					6	5	7	8						
50					6	6	8							
40						8								
30														
20														
10														
	40	50	60	70	80	90	100	110	120	130	140	150	160	170

Y

Figure 9: Section map of production sequences at X = 160