School of Science and Engineering

Application of the Real Options in Engineering Design and Decision Making: Focus on Mine Design and Planning at Operational Level

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Doctor of Philosophy
of
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Declaration

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Signature: --

Date: 25/7/18 --

Abstract

Risk and uncertainty have dominated business news headlines in recent times. However, opportunities generated by real or perceived risk and uncertainty are less mentioned. Traditional engineering practices are generally risk-averse, a tendency which diminished opportunities that resulted from positive treatment of risk and uncertainty. Flexibility and strategic adaptability are essential for long-term corporate success, and real options (RO) analysis is applauded as a preferred tool for analysis. Nevertheless, its application in engineering design has been sluggish compared to its financial application.

This thesis is comprised of four journal articles, utilising analytical methodologies including a binomial decision tree that was applied to create a switching option between pits regarding changing ore grades and fluctuating commodity prices, a stochastic process with jump diffusion which was applied to analyse option to delay, to abandon the operations and to stage the investment and predictive data mining which was utilised in the analysis of ROs in managing geological uncertainty for optimal decision making in mining operations. Additionally, a RO identification framework for mine operational decision making has been proposed, a relationship between risk measure (beta) and flexibility (flexibility index) is derived and applied, and a modified smooth pasting condition with the mean value theorem is subsequently applied to estimate the optimal value.

This research argues that engineers and decision makers should view uncertainty as a source of value. Adaptation of this view may shift the debate on how mine planners and engineers approach uncertainty. Thus, the thesis extends the idea that uncertainty cannot be eliminated but rather the opportunities that it presents can be leveraged by having a flexible system. This thesis is one of the few, if not the only research piece to have applied a predictive data mining algorithm for creating managerial flexibility at a mine operational level. The research supports future studies and encourages the use of RO analysis in engineering design and decision making as it enhances understanding of the rationale for flexibility in design. Moreover, the thesis proposes a flexibility domain map for managerial decision making by creating an uncertainty identifications framework and RO application domains within the mining system. This introduction of managerial flexibility on the shop floor is a critical step as ROs have not been used at a mine operational level before.

In addition to furthering the academic debate about the critical role of data utilisation and highlighting the new area of research, particularly the integration of data analytics into mine design and planning process, this thesis has opened up a new research frontier on how the concept of ROs can be integrated with technology, particularly with data analytics. There is currently a disconnect between big data and mining operations which generates huge data that is not adequately utilised to create value.

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List of publications

Ajak, A. D., and Topal, E. (2015). Real option in action: An example of flexible decision making at a mine operational level. *Resources Policy*, 45, 109 - 120. DOI: 10.1016/j.resourpol.2015.04.001

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Ajak, A. D., Lilford, E., and Topal, E. (2017). Application of predictive data mining to create mine plan flexibility in the face of geological uncertainty. *Resources Policy, 55*, 62 - 79. DOI: 10.1016/j.resourpol.2017.10.016

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Statement of contribution of others

Title of Paper	**Real option in action: An example of flexible decision making at a mine operational level**		
Publication Status	☒ Published	☐ Accepted for publication	Publication is refereed: ☒ Yes ☐ No
Publication Details	Ajak, A. D., and Topal, E. (2015). Real option in action: An example of flexible decision making at a mine operational level. *Resource Policy*, 45, 109 - 120. DOI: 10.1016/j.resourpol.2015.04.001		
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Principal Author	Candidate Contribution to Paper 1 - 4	Overall (%)	Signature	Date
Ajak Duany Ajak	Set research question, developed methodology, developed real options identification framework and derived a relation between project beta, flexibility index and volatility, wrote manuscript and acted as corresponding author.	85%		25/7/18

Co-Author Contribution

By signing the statement of Authorship, each author certifies that:

I. the candidate's stated contribution is accurate as stated above;

II. permission is granted for the candidate to include the publication in the thesis; and

III. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Co-Author	Contribution to the Papers	Signature	Date
Erkan Topal	Supervised development of work and reviewed manuscript.		25/7/18
Eric Lilford	Supervised development of work and reviewed manuscript.		20 July 2018

Table of contents

List of Figures

List of Tables

Chapter 1: **Introduction**

The world economy is unsteady and it cannot be precisely predicted. The business environment has become so volatile and uncertainty is greater than ever before. However, risk and uncertainty are not limited to the stock market but have their places in engineering work as well. Traditional engineering practices are generally risk-averse, a tendency which reduces opportunities that result from the positive treatment of risk and uncertainty. Considering that the future cannot be predicted, and the uncertainty is projected to increase as technology is converging the world where information is disseminated instantaneously (Deloitte, 2016), risky businesses such as mining operations cannot continue to rely on traditional capital investment analytical models such as the Discount Cash Flow (DCF) analysis. The most suitable tool for handling uncertainty and for justifying an investment in system flexibility, particularly a mining operation is the real option (RO) tool. The RO theory postulates that uncertainty has value and only those that embrace it can minimise losses or maximise opportunities that come with volatility. Nevertheless, its application in engineering design has been sluggish when compared to its financial application. The main goal in this research is to study the technical application of the RO in mine design and decision making, create an understanding of options selection for flexibility and to identify application domains of the RO in areas such as mine development strategy, mine design and layout strategy, mineral waste management and processing strategy. To make the study manageable and to remain within the research scope, two major research strands are identified. These are the theoretical analysis on the application of the RO in engineering design and its practical application in mine planning and operational decision making. The findings of this research will support future research and encourage the use of RO analysis in engineering design and decision making by enhancing understanding of the rationale for flexibility in mine design. Conducting studies in specific settings would enable scholars to model the impacts of the main variables that affect option values and finally, it will help in the development of normative aspirations of option theory.

1.1 Background

The world is unreliable and worldly events and occurrences cannot be predicted by using traditional analytical models. Thus, it is much better to build a system that can be adapted to change rather than trying to predict the unpredictable and assume the future to be constant as per the present conventional models. Decision makers are fully aware that the world is uncertain but there is still an ongoing struggle around how best operations can be adapted to the new and constantly evolving environment. Adapting to change is critical for survival, whether in the natural world or in the business environment. Traditional decision analysis models which have been relied upon for decades tend to be static and cannot predict the future off the present dynamics.

Good examples of such failures of the deterministic mathematical models were the 2016 UK Brexit referendum and the US presidential elections where the outcomes have been contrary to predictions. According to O'Callaghan et al. (2016), a study of 40 top mining companies indicated that these companies had collectively booked a net loss of US$427 billion in revenue or 37% of their combined market capitalisation in 2016 as these operations were not flexible enough to respond to commodity price fluctuations. Such a loss in capital value is the largest ever recorded in a single reporting period. To put it into context, these losses are equivalent to about 15% of the Australia economy.

Over the last decades the global community has suffered many natural and political disasters including hurricanes in the USA, cyclones and floods in Asia Pacific, landslides in Latin America, earthquakes and tsunamis in New Zealand, Indonesia, Japan and Haiti, the war on terror, the onset of the Global Financial Crisis (GFC) in 2008, market instability and currency crises, hazardous waste and industrial accidents and political turmoil in the Middle East and in North Africa. Any disturbances in economic systems, whether natural or human-made, will usually affect both economic and physical systems in some way. This amounts to some form of risk, whether real or perceived (Risk Intelligence, 2010).

In today's world where systems are linked, the traditional discounted cash flow (DCF) analysis that assumes project outcomes to be constant regardless of future decision of the firm will not hold to these challenges. RO 'in' projects, a methodology to justify an increase in system flexibility under uncertainty (Groeneveld et al., 2010) has been regarded among researchers and economists as a means of better assessing investment proposals under uncertain market conditions which characterise most capital investments (Dimitrakopoulos & Abdelsabour, 2007). In this ever-changing world, engineers play crucial roles in designing structures and systems that can withstand those challenges and fulfil required performance targets as well as being financially viable in the face of uncertainties. They are responsible for the design and construction of private and public projects in all industries including building (commercial and residential), transport systems (roads, bridges, rails and airports), manufacturing, natural resources (mining, oil and gas), energy, IT, telecommunication and many more.

According to Global Construction 2020 (2010), it is estimated that global construction will grow by almost 70% from its current US$7.2 trillion to US$12 trillion by 2020. This growth is associated with an ever expanding global natural resource industry and continued urbanisation in the developing nations. China, the USA, India, Indonesia, Canada, Australia and Russia will account for two thirds of the growth in the global construction industry by 2020 (Global Construction 2020, 2010). Such growth will require innovation and new thinking in engineering practices.

Considering that the future cannot be precisely predicted, and the uncertainty is projected to increase as technology converges the world into one village where information is disseminated instantaneously, risky businesses such as mining operations cannot continue to rely on traditional capital investment analysis models such as DCF analysis. Projects which are not flexible enough are usually the first to fail when the market shakes up because managers of such projects normally have very few options to exercise.

The most suitable tool for handling uncertainty and for justifying investment in system flexibility, particularly in a mining operation, is real option analysis. RO theory postulates that uncertainty has value and only those that embrace it can minimise losses or maximise opportunities that come with associated volatility.

However, identifying where to reflect options is a complicated problem (Milkaelian et al., 2007). Even though the potential benefits of applying RO in engineering design have been recognised by many researchers, it is still confined to the valuation of a defined project with its assumed limitations (Akbari et al., 2009). Research has shown that up to 85% of capital costs are committed during the design and development phase (Sanislo, 2003). However, engineers and project managers do not usually explore the value of flexibility at this crucial stage of the project. There are limited studies on the application of RO in engineering design and decision making at the initial mine design stages. Creating guidelines for implementing options to capture their values should always make the centre piece of RO proposals. Many researchers evaluate RO with the assumption that architectural concepts and functional requirements for variants are pre-determined (Nembhard & Aktan, 2010; Angioletti, 2010) while others analyse projects that have already attained volume and quality of information to support sophisticated economic valuation. Wang (2005) has successfully suggested screening of variables but no systematic way to identify parameters to vary in screening exists.

The key issue that needs to be researched furrther is whether the real option technique is well suited to face challenges involved in mine design without introducing unrealistic simplifications. Groeneveld et al. (2010) demonstrated that incorporating RO into the design creates flexible mine operations with greater net present value (NPV) than the traditional approach.

1.2 Research questions

Seeing that the literature review adds proof that RO increase project value in the face of uncertain economic environments, the overarching questions that require further investigations are:

I. Is the RO technique well suited to face challenges involved in mine design and decision making for creating flexible designs that maximise project value at a mine operational level without introducing unrealistic simplifications?

II. If yes, where within the mining cycle can this flexibility be introduced?

The term 'mine operational level' is very important in this regard because it is the backbone of this research. It is the major distinction between this thesis and any other research that has been done in the past. Past researchers have focused mainly on the application of the RO at the strategic level. Volumes of publications in literature have explored how organisations can apply the RO method to frame an investment strategy and gain a competitive advantage over rivals who have not utilised the RO methodology. Contrary to previous existing studies, this thesis is an investigation on the suitability of utilising the RO methodology at the mine operational level where production decisions are made weekly, monthly, quarterly and annually rather than organisational strategies that are considered for periods spanning several years.

1.3 Aims and objectives

Many engineering practices are directed towards risk elimination as a top priority (Neufville, 2004). Narrowing the focus on system failure leads one to disregard the uncertainties that create opportunities. RO analysis recognises uncertainty as helpful and hosting intrinsic value. However, there is a need to research the applicability and usefulness of RO associated valuations in solving real life engineering design and decision-making problems. As a result, the objectives of the research are as follows:

i. To explore the technical application of RO in mine design and decision making at an operational level in mining operations.

ii. To link the engineering design to financial investment decision making. Engineers are to treat flexibility and uncertainty in complex design as investment decisions.

iii. Creating guidelines for the implementation and developing estimation techniques to capture RO values.

iv. Development of normative aspirations of option theory through the investigation of detailed techniques, definition of practical integrated procedures for RO analysis, connecting the descriptive and normative perspective, integrating the qualitative and quantitative approaches and the determination of application domains. Determinacy where flexibility should be embedded in a design is a serious issue and the option selection where a choice of options occurs has consequences for the valuation of the options.

1.4 Real options in mine design and planning for operational decision making

A complete open pit mining cycle involves drill and blast, loading, hauling and dumping of the materials (Ajak, 2012). All these activities are sequential, starting in the pit and ending at the processing plant or on the waste dump and each of them forms a value centre. In other words, there are always opportunities for introducing flexibility at each stage in order to maximise project value. In this research, all these activities will be grouped into four options:

i. mine development strategy;
ii. dumping strategy;
iii. mine layout; and
iv. processing strategy.

Fig. 1.1 shows a diagram of an open pit mining cycle and highlights where flexibility can be introduced.

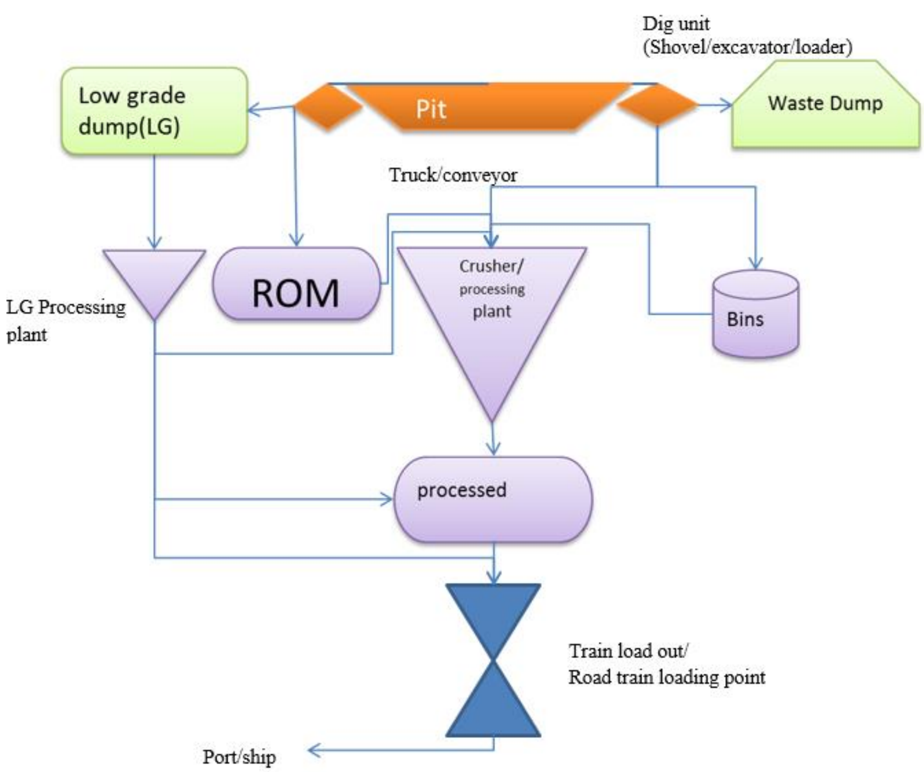


Fig. 1.1, Schematic diagram of mining cycle showing options for flexibility

It should be noted that the Run-of-mine (ROM) material in this thesis refers to ore material that is stockpiled on the pad before crushing and does not include low-grade material. The bins in Figure 1.1 are stockpile compartments designated to hold different material types before reclaiming the material for blending. Anecdotal evidence has shown that in most operations, low–grade material is processed separately to minimise ore contamination and the crushed material is used for blending when the need arises. Therefore, mining operations mine low-grade material and stockpile it in low-grade dumps.

1.5 Major strands and layout of the thesis

This thesis consists of both qualitative and quantitative work used to solve RO. The collective research was a continuous development and flipping between the theoretical analysis of the chosen methodologies and the demonstration of those techniques through case studies, all of which have been published as four journal articles and included in this thesis. To make studies effective and to stay within scope, this research was divided into two major strands as discussed in 1.5.1 and 1.5.2 below.

1.5.1 Theoretical analysis on the application of real options in engineering design

This section presents an opportunity to explore theoretical aspects of RO and collate business objectives and goals that usually dictate design considerations and decision making with possible applications. This has been outlined in the literature review (Chapter 2) and in each of the first sections of this thesis' peer-reviewed and published papers (Chapter 3 – 6).

To complete this section, the following activities were undertaken:

- RO theory was explored before developing a methodology and performing RO analyses on each case study.
- Identification of design objectives and goals in each case study.
- Identification of appropriate assumptions that are relevant to the application of RO in engineering design.

1.5.2 Methodologies and practical application of RO in engineering design

The applicability of RO at the mine's operational level was ascertained through four published papers. RO was applied in various case studies and engineering applications were formulated. Three evaluation approaches were used in this research for RO analysis, these being:

- the binomial decision tree mode (Paper 1 or Chapter 3 in this thesis);
- the stochastic simulation process with jump diffusion (Paper 2 or Chapter 4 herein); and
- the predictive data mining functionality (Paper 3 or Chapter 5 herein).

Finally, the RO identification framework for a mine operational decision making which combined uncertainty, risk and flexibility was developed (Paper 4 or Chapter 6 herein).

1.5.2.1 First analytical methodology - The binomial decision tree model (Chapter 3)

This methodology was the first to be applied to a real case study where it was used to analyse a switching option between pits arising from changing ore grades and fluctuating commodity prices. This demonstration of how RO can be used in designing multiple pits in multi-zone ore deposits was published as Paper 1 (Chapter 3) of this thesis. The resulting values were compared against the traditional analytical methods. Moreover, a structured framework and accompanying analytical steps were followed as shown in Fig. 1.2.

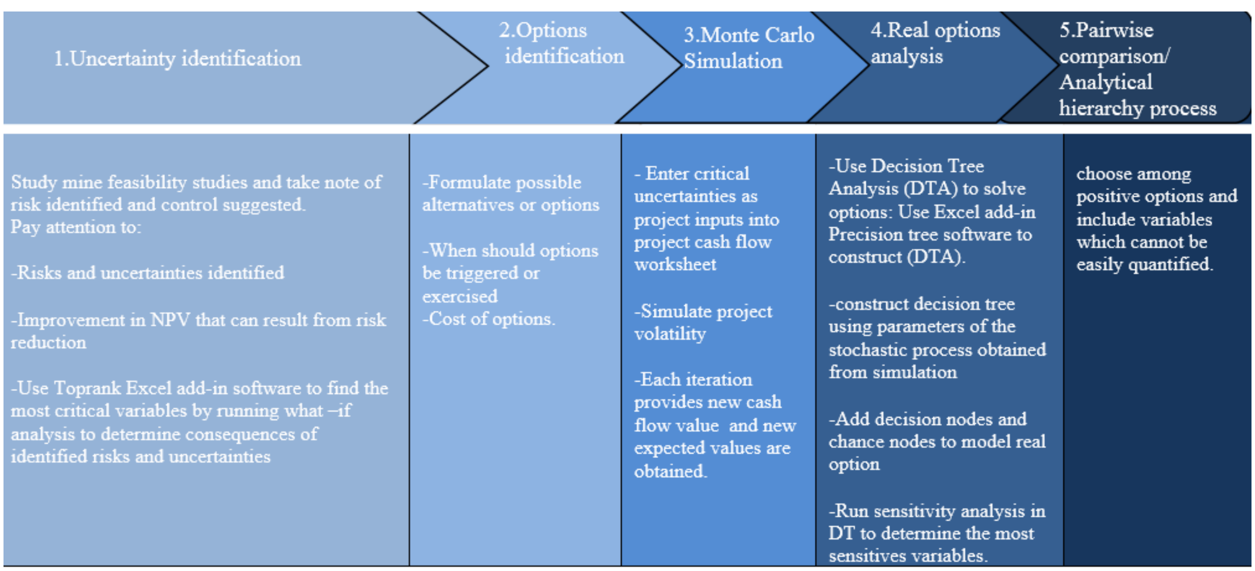


Fig. 1.2, The utilised binomial decision tree model for RO analysis.

1.5.2.2 Second analytical method - stochastic simulation of mean-reversion with jump diffusion (Chapter 4)

The following steps were performed for the stochastic simulation of a physical process required to value managerial flexibility in a mining operation (Paper 2). The paper has considered a stochastic simulation to analyse ROs for a real case iron ore mine which closed in April 2016.

The analysis in Paper 2 (Chapter 4) starts off with developing a deterministic model which closely resembles the real scenario of an iron ore operation. In this deterministic model, the most likely values of the base case input parameters were used, after which a mathematical formula of the NPV was applied which used the values of the input variables and transformed them into the desired output.

Once the deterministic model was identified, risk components to the model were stochastically simulated using mean-reversion with jump diffusion as the risk originates from the stochastic nature of the input variable which was the iron ore price. Therefore, the historical data of the iron ore price was utilised.

After generating a sample of output values from the simulation, options were analysed based on the simulation values, such as the NPV, of the potential investment. Its volatility and other sensitivities were observed from the analysis of the output. In comparing the net present value from the traditional DCF method to “delay”, to “abandon” the operations and to “stage the investment” options, the RO method increased the project value. The increase in additional value depends on the level of volatility. Moreover, the managerial flexibility domain map was proposed in this paper. Thus, this paper confirmed that flexibility in mining operations has the potential to create agility, increase value and mitigate financial losses.

1.5.2.3 Third analytical method - application of predictive data mining to create mine plan flexibility (Chapter 5)

In Paper 3 (Chapter 5), a predictive data mining algorithm, in this case a classification tree, was utilised to solve a mine operation case study where geological uncertainty was identified in a mine plan. The results of the model were used in the creation of various flexibility alternatives or options which managers could exercise to manage the uncertainty associated with clay inclusions in the ore body.

The first step in the data mining process was to develop the data mining model. This ensured that the research questions were well understood and were being appropriately reflected in the mining process. This process involves defining uncertain variables whose values should be mined, identifying how to measure values, what type of data set will be applicable, how the model will be applied, what decision criteria will be used and what type of study will be needed to deploy the model and test the model effectiveness. Moreover, relevant prior knowledge about historical characteristics of the variables were stated as well as the goals of application to the current problem.

The identified data was then prepared. The activities in data preparation include the treatment of outliers and the removal of noise in the data, the development of strategies for handling missing data fields, accounting for time sequence information and data reduction which involves finding useful features, variable reduction and invariant representation of the data.

Rapidminer (2017) and Orange (Demsar et al., 2013) data processing programs were used in arranging, summarising and communicating information in such a way that the meaningful and essential value of the data can be extracted and grasped easily. These precede the use of the chosen mining algorithm. In this phase, the selected parameters were searched for patterns of interest.

After visualisation, transformation, removing redundant patterns during the data processing, analysis of the options, the result of the uncertain variable values obtained through the data mining model and were incorporated to value the flexibility of the project. The outcome of each option was documented and gauged against the chosen decision criteria before any conclusion could be drawn up. Poisson distributions and Monte Carlo simulations were applied to analyse various RO. The paper revealed that operations could minimise losses and create value if the predictive data mining algorithm is applied to create RO.

1.5.2.4 Real option identification framework for a mine's operational decision making: combining uncertainty, risk and flexibility (Chapter 6)

This was the last paper in the series. In Paper 4 (Chapter 6), uncertainty identification framework in a mining operation is proposed. The paper discussed uncertainty as the source of value. It has been highlighted that RO will have no use if uncertainty is not considered. Thus, areas for managerial flexibility and their application domains within the mining cycle were also mapped. To avoid complex mathematical models which have hindered the adoption of RO analysis in mining operations, a relationship between the risk measure (beta) and flexibility (flexibility index) was derived and demonstrated using the data from the analysis performed in Paper 3 (Chapters 4 and 5). This paper proposed that if a project beta is known, then the expected option values and volatility of future cash flows could be precisely estimated using the derived relationship. Moreover, a modified "smooth pasting condition" with the mean value theorem was subsequently applied to estimate the optimal value.

1.6 Significance and contribution of this research

Like any other form of investment, mines are not immune from local and global uncertainties. The irreversibility of a mining investment compounds this problem. Once a firm has committed resources in developing a mine, it has to see it through. As a result, mine managers and engineers require some room to respond rapidly and make informed decisions when conditions change. This thesis has examined the suitability of RO methodology to create managerial flexibility at a mine operational level. To demonstrate the use of RO, four case studies were analysed (Paper 1 – 4 or Chapter 3 – 6 herein) and as a result of those studies, it became apparent that this thesis will have the following significance:

- As a consequence of this research, it is argued that engineers and decision makers should view uncertainty as a source of value. Adaptation of this view may shift the debate on how mine planners and engineers approach uncertainty. Thus, it is furthering the idea that uncertainty cannot be eliminated but, the opportunities it presents can be leveraged by having a flexible system. Therefore, the thesis is enhancing the understanding of the rationale for flexibility in design. It is irrefutable that the future is uncertain and that there is value in having the right but, not the obligation (option) to react to future development. Therefore, the application of RO in engineering design and decision making, particularly in mining operations, has been lagging as there has been no real research on how the method could be implemented. Thus, this research is groundbreaking in applying the concept of RO for flexible decision making at a mine operational level (Paper 1 - 4 or Chapter - 6 herein).

- Its introduction of managerial flexibility at the shop floor is a critical step as many operations operate on a static model that assumes limited changes once the mine plan has been produced and is either being executed or is ready for execution (Paper 1 or Chapter 3 herein). Therefore, this thesis will help researchers and mine designers to understand the importance of building RO into projects and products, giving managers options and capabilities to improve performance. Conducting studies in mine specific areas would help in modelling the impacts of the main variables that affect option values instead of testing the effects of broad variables that provide only indirect evidence of the RO perspective.

- Moreover, the research has led to the development of an uncertainty identification framework (Paper 2 & 4 or Chapter 4 & 6 herein) and RO application domains within the mining system. This is another contribution in the development of normative aspirations of the option theory that identifies better application domains, determines conditions for successful implementation of RO and explores a clear distinction between project elements that fit well with RO logic and those that do not. There is an apparent contradiction between RO value and RO reasoning approaches. The qualitative use of RO analysis should not be underestimated. Empirical research has demonstrated that the quantitative evaluation of the project plays just part of the role in strategic investment and management gives equal if not greater and essential role to strategic considerations emerging from informal processes. At the same time, quantitative evaluation prevents the risk of launching a project whose NPV is negative and justifies the capital investment. Thus, connecting the descriptive and normative perspectives plus integrating the qualitative and quantitative approaches is a valuable contribution to real options analysis.

- Another distinct contribution of this research to both academic and mining professions is the adoption of stochastic simulation with jump diffusion and predictive data mining algorithms to the analysis of RO. This allows managers and analysts to consider all the possible paths which a random variable could take and use such information to create managerial flexibility (Paper 2 or Chapter 4 herein).
- This thesis is one of the few, if not the only one, to have applied a predictive data mining algorithm for creating managerial flexibility at a mine operational and planning level (Paper 3 or Chapter 5 herein). It has also been demonstrated that big data and data mining algorithms present enormous opportunities for a mining operation as this was proven through the application of the predictive machine learning classification tree in the management of geological uncertainty. Thus, the research has opened up a new research frontier on how the concept of real options can be integrated with technology, particularly the integration of Data Analytics into mine design and planning. There is currently a disconnect between big data and mining operations which generate huge data that is not adequately utilised to create value.
- Finally, another significant contribution of this thesis to RO methodology is the derivation of the relationship between project beta, which is the measure of risk, and the flexibility index was derived. This thesis argued that if the project beta is known, then the expected optimal option value and volatility of future cash flows could be precisely estimated. The research has proven that a combination of beta, flexibility index and mean value theorem could be used as a decision criterion for screening various options within the mining project (Paper 4 or Chapter 6 herein).

Chapter 2: **Literature review**

This research is dealing with application of RO in engineering design and decision making. Therefore, terms like option, real option, flexibility and uncertainty will appear a lot throughout this thesis. It is important that they are defined from the onset to clarify and set some boundaries to their meaning and their interpretation. RO users, researchers and academics have put forward various explanations and descriptions of what they think RO, flexibility and uncertainty are. A naive construal of these definitions can assume that they mean the same thing. However, the choice of words used by difference authors does influence the interpretation as well as the application of the RO analysis.

2.1 Definition of key terms

The following definitions may be applied in various parts of this thesis.

Options

- Option is the right to buy but not the obligation to buy (Nembhard & Aktan, 2010).
- Financial option is the right, but not the obligation to buy (or sell) a stock (the underlying asset) at a fixed price (the exercise price) with or at the end of a fixed period maturity (Black & Scholes, 1973; Krychowski & Quenlin, 2010).

Real option

From the literature review, a real option has been defined as follows:

- Real option approach is a language to describe the possibilities the firm has so that the world can be opened up as a map of opportunities (Edelmann & Koivuniemi, 2004).
- It is a methodology to justify an increase in system flexibility under uncertainty (Groeneveld et al., 2010).
- Real option is the quantification of flexibility value (Nembhard & Aktan, 2010).
- Real option is a principle to choose (Malts & Maasen, 2007).
- Real option is an approach that allows people to make optimal decision within their current context (Malts & Maasen, 2007).

- Option in an engineering perspective refers to flexibility, the ability to adjust a design of a system in significant ways that enable the system managers to redirect the enterprise in a way that either avoids down side consequences or exploits upside opportunities (Neufville, 2004).
- Making a decision to develop a pit or to wait and see in mine planning procedure and determination of ultimate pit limit (Akbari et al., 2009).

Flexibility

- Flexibility is the ability to cope with modification made as necessary due to changes in financial, technical or production (Kazakidis and Scoble, 2002).
- According to Dunbar et al. (1998) and cited by Mayer & Kazakidis (2007), flexibility in mining is a measure of the capabilities of a mine production system or a mining company to respond to changes.
- It is the ability to adapt a system's design or operation to fluctuating conditions (Ku 1995), cited by Mayer & Kazakidis (2007).
- Flexibility is the ability to deal with variability and uncertainty, Schirn et al. (1999), cited by Mayer & Kazakidis (2007).

Uncertainty

Uncertainty is defined in the following three contexts:

- Decision making: Situation where there is a gap in knowledge such that the order or nature of things is unknown, the consequences, extent, or magnitude of circumstances, conditions, or events is unpredictable, and credible probabilities to possible outcomes cannot be assigned (Jablonowski et al., 2017; Zagayevskiy & Deutsch, 2014).
- Information theory: Degree to which available choices or the outcomes of possible alternatives are free from constraints (Brown & Innocent, 2013; Bammer & Smithson, 2009; Business Dictionary, 2011).
- Statistical sciences: Situation where neither the probability distribution of a variable nor its mode of occurrence is known (Business Dictionary, 2011; Hawkes & Sierra, 2010; Weiss, 2003).

2.2 Historical context of the option theory: financial options

The option theory was fully brought into light in 1973 by Black and Scholes 'The pricing of options and corporate liabilities' and Merton 'Theory of rational option pricing' (Brealey et al., 2017; Bernstein, 2012; Black, 1989; Black & Scholes, 1973; Merton, 1973). As the name suggested, it was first used in financial markets for trading stocks. Since then, it has received wide acclaim from the financial world. It has undergone several modifications and iterations and many different equations have been derived from it. However, the concept remained the same. This concept can be simplified by equating an option value as a function of stock price and time (Bernstein, 2012; James, 2003; Black, 1989; Black & Scholes, 1973).

Shortly after the introduction of the option theory, Myers (1977) introduced the real option concept when he observed an inverse relationship between corporate borrowing and the proportion of market value accounted for by RO. He postulated that a firm has a collection of tangible and intangible assets at any point in time. Intangible assets are real assets with market values independent of the firm's investment strategy while tangible assets are RO, which are opportunities to purchase real assets on possibly favourable terms.

Johnston et al. (2011) extended the RO concept into employment benefits for young workers. He examined the application of RO to implicit contracts in the context of financial innovation and in regard to an accrual pension benefit plan and cash balance plan. He argued that RO does not contradict contract theory since it presents employees with a choice of switching to cash in order to increase mobility for young workers versus the value of imbedded options of the change of company plan and accrual patterns. In their study on how to deal with external risks after the conclusions of contracts, Mühlbacher et al. (2018) applied financial options to healthcare contracts where financial reimbursement strategies are usually critical when drafting contracts which do carry a high degree of uncertainty due to incomplete information. These authors have linked organisational performances to incentives as the way of preventing losses.

However, the big question is the harmony of the contract theory with the industrial legislations of different countries which undermined the assertion that RO can handle employment benefit issues successfully. Apart from financial and services sectors, financial option theory has also been successfully applied in many other logistics, supply chain, outsourcing and quality control case studies (Nembabhard & Aktan, 2010). However, RO as a theory depends on assumptions for its valuation to be successful and not many of its early critical assumptions were clearly met and could not directly solve engineering problems without modifications (Copeland & Antikarov, 2003).

2.3 Strategic and technical classification of RO

There is a fine line between RO 'in' and 'on' project. This is a classification of RO based on strategic and technical application the RO. Thus, it is pertinent that these two classifications of RO are differentiated early to avoid confusion and to underscore that the scope of research is limited to the technical application of the RO, that is the RO 'in' project.

RO 'on' project	**RO 'in' project**
• RO 'on' projects are aimed at valuing future investment opportunities that are resented by uncertainty (Trigeorgis, 1993, 1996; Dixit & Pindyck, 1995; Wang & Neufville, 2005)	• RO 'in' projects are those imbedded into the system design to create managerial flexibility (Weck et al., 2004; Neufville et al., 2004; Wang & Neufville, 2004, 2005)
• Their application is at strategic and policy level (Trigeorgis, 1996; Wang & Neufville, 2005)	• Their application is at a technical level (Roos et al., 2004; Wang & Neufville, 2004, 2005)
• They are easy to define and compute (Roos et al., 2004; Wang & Neufville, 2004, 2005).	• They are difficult to define due to their technical nature (Roos et al., 2004; Wang & Neufville, 2004, 2005).
• They are important for valuing future strategic decision and policies (Roos et al., 2004; Wang & Neufville, 2004, 2005)	• They are important for future decisions since they are imbedded into the system (Roos et al., 2004; Wang & Neufville, 2004, 2005).
• They do not last for long as they change once the strategy and policy changed (Roos et al., 2004; Wang & Neufville, 2004, 2005)	• Last for a long time since they are part of the technical design (Roos et al., 2004; Wang & Neufville, 2004, 2005)

2.4 RO 'on' project

Mine economics treat proved ore reserves as inventory. They are the end-product of development investment. Natural resources are limited by nature and doomed to decline (Adelman & Watkins, 1995). This unique property has sparked numerous researches in mineral economics. As production declines, mining expense rises per unit of output. This logic is supported by the economy of scale. The margin shrinks between gross value of output and current outlays until production stops when the margin goes to zero, being the "economic limit" (Gupta et al., 2015; Adelman & Watkins, 1995).

Apart from other researchers who touched on the concept of the option theory in natural resources exploitation, Adelman & Watkins (1995) evaluated the theory of the scarcity of the resources and put its hypothesis to test. Using oil reserves in Organisation of the Petroleum Exporting Countries (OPEC) member countries as their case studies, they argued that the amount of resources that can be exploited or extracted are dictated by the cost expended and the price obtained in return. For instance, delaying drilling would increase a petroleum well's present value by the creation of option value but there is no simultaneous creation of the reserves. This assertion puts future prices as the main uncertain variable and gives stakeholders an option of investing or hanging on to their hard-earned cash.

In spite of this ever-increasing level of uncertainty, most corporations make their decisions on DCF based methods such as NPV and internal rate of return (IRR) which are static in nature. Economists have long recognised that future events cast their shadows into the present. Any future value must be discounted down to the present in order to be comparable to any current price (Mahmudul et al., 2016; Chen & Teng, 2015; Guj, 2013; Adelman & Watkins, 1995).

Benaroch & Kauffman (1999) tested the validity of the Black – Scholes option pricing model in the context of the traditional capital budgeting method by using a real-world business case. They examined information technology projects in light of the assumptions of both binomial option pricing and Black – Scholes in a constrained business situation that can also allow the application of the DCF method.

A study done by Samis & Laughton (2007) concluded that valuation influences project uncertainty, structure and value estimation. Lilford & Minnitt (2005) studied various valuation processes and applied each method to a variety of South African gold projects and concluded that any application of the valuation technique depends principally on the project stage such that a single project can produce different values depending on the valuation technique applied.

In their analysis, Lilford & Minnitt (2005) placed option techniques under one of the two major valuation methods which are market and income approaches. The market approach is composed of the Lilford Techno - economic Matrix, the US dollar per unit of commodity (e.g. US$/oz) and Kilburn method (Lilford & Minnitt, 2005). The income approach includes the DCF technique, the tail margin analysis (derived from cash flows) and option (derivative) pricing technique.

Akbari et al. (2009) went ahead and used RO valuation in determining an ultimate pit limit in the face of price uncertainty. Similar to Haque et al. (2016), Samis & Davis (2014) applied RO in the hedging strategy where they explored funding options for the development of a gold mine. Their approaches were aimed at setting a high-level strategy to provide protection from fluctuating commodity prices.

Ayanso & Herath (2010) recommended future research to address the development of complex compound RO with Bayesian learning and the development of comprehensive risk matrices that involve both quantitative and qualitative factors. To optimise project values and to increase robustness of the mining project, Whittle et al. (2007) applied a quantitative risk management framework which tends to reduce risk, optimised various variables and ignores future decisions. Kim (2010) proposed a model to determine the optimum timing of projects by using RO that focus on ownership ratio, synergy effect and payment options. This approach was a RO on system without any technical design involved which is similar to dynamic DCF analysis studied by Herbelot (1994) on coal – fired power plant projects.

2.5 RO 'in' project at strategic level

Production planning and design would be easy if variables such as price follow a known trend. Even though methodologies including linear programming (LP), mix integer programming (MIP) and heuristics can be used to include uncertainty and to optimise value in engineering design and especially in mining industry, they are typically suitable for selecting the best design alternative at an initial stage of the project and particularly for strategic decision making once various scenarios have been analysed (Akbari et al., 2009; Bixby, 2012).

Guj & Harton (2007) argued that techniques such as the modern asset pricing (MAP) model provide a minimum risk adjusted value (floor value) which is better than the DCF but, falls short of the RO method.

Flexibility in design has significant value creation potential and may radically change economic risks in design and embedding RO and therefore may reduce the overall required investment (Kalligeros, 2010; Hassan et al., 2006). This was earlier noted by Zhao & Tseng (2003) as the expected pay-off of the option. Neely & Neufville (2001) summarised the valuation of RO in five phases; setup, analysis, financial perspectives, technological perspective and the sensitivity phase (sensitivity analysis of the key assumptions). Neufville (2002, 2003) developed the value at risk based RO valuation in an effort to propagate systematic thinking and flexibility in engineering designs

Kalligeros & Neufville (2006) used the Monte Carlo simulation methodology in evaluating the flexibility in specific classes of engineering systems for design purposes. Their intention was not a valuation of the system for investors but, as an alternative to engineering design for flexibility prospectiveness. Their method involved step processes to stimulate exogenous uncertainties of each alternative as per organisational objectives. In addition to being tedious and requiring repetitive work to value each and every alternative since there are usually many alternatives and options for every design, it failed on a mix of multiple assets because of the different assumptions.

A follow up study conducted by Kalligeros et al. (2006), introduced an algorithm for qualitative identification of various components that constitute a single platform at multiple levels of system aggregation among variants within a family system. The shortcoming of their method is the assumption that architectural concept and functional requirements for variants are pre-determined. This supposition is not always true in scenarios of any initial design stage. Research conducted by Lin, et al. (2009) concluded that RO in systems design has the following characteristics:

- It presumes that the main variables of the system are partially unknown and indeed unknowable in advance (Lin, et al., 2009).
- Its rationale for flexibility in design is that the future is uncertain and operations can create value in having the right but, not the obligation to react to future developments (Lin, et al., 2009).
- Current engineering design assumes that the unknowable future uses of the system can be and are in fact known (the engineering current practice proceeds on the basis of fixed assumptions), but, the RO method does challenge this assumption.
- It identifies forces that lead to engineering design fix assumptions.
- It treats important drivers of value like demand as social factors outside the engineering purview.
- Complexity in design is eased by simplifying the assumptions.

The greatest issue at present is not to prove that RO increases project value but, to apply it at the operational level and in particular, in the mining industry where projects constantly face an uncertain future.

2.6 RO 'in' project at the operational level: Past attempts to include flexibility at the tactical level

Inadequate information has been the biggest challenge in the planning and design of any natural resource project (Zagayevskiy & Deutsch 2014; Jablonowski et al. 2017). Even though the mining industry has improved value optimisation techniques which enhance value and reduce risk, many algorithms proposed in the past like Arc, cone, network flow analysis, Korbov algorithm and Lerchs & Grossman theory neglect price uncertainty (Akbari et al., 2009).

Dimitrakopoulos & Sabour (2007) proposed a simulation based on real option valuation which can analyse various uncertain variables as well as the volatility in cash flow parameters that is commonly witnessed in mining projects. The limitation of their method was the use of a single option for flexibility and that was abandoned. However, good flexibility should have at least more than two options to see real benefits. Even though the total amount of metal units within a deposit can never be known with certainty, a typical practice in valuing mining investments using the traditional valuation model assumes mine production to be constant both in terms of volume and quality of metal units throughout the mine life under varying future metal prices.

Thompson & Barr (2014) attempted to utilise RO approach in determining the cut-off grade when prices are stochastic, but the method fell short in articulating how this could be executed operationally. Their approach deviated into an optimisation problem that was later tackled by Mohammadi et al. (2017). Lane et al. (2007) used the concept of economic surface, also known as “hill of value”, to represent optimisation outcomes and to discern routes of optimal configuration as well as extending it to incorporate risk associated with variance assumptions and input. This is called dynamic financial analysis (DFA). The drawback of this concept is risk minimisation and it tends to shy away from exploiting opportunities presented by uncertainty when faced with two opposing effects.

Some short-term uncertainties of the mining industry are new taxation regimes, increasing restrictions on new development approvals and accelerating the pace of growth which other parts of a country and economy struggle to match (Ross, 2011). Kazakidis & Scoble (2002) introduced the mine flexibility index as a quotient of option value and passive NPV. Musingwini et al. (2008) stated that a flexibility index of less than 1.0 indicates an inflexible mine. An index of 1.0 indicates a marginal operating flexibility, while an index of greater than 1.0 suggests a flexible operation.

2.7 Why RO is not gaining popularity in engineering design and at the operational level

There are numerous examples where RO has been strategically applied in business strategy and policy analysis. For instance, the RO method was used in the design and management of software engineering in the US Air Force (Olagbemiro, 2008). It was also used in the valuation of a drone development project by Boeing (Datar et al., 2007), in urban development (Morano et al., 2014), in real estate development, in research and development of pharmaceutical products, in manufacturing and in energy resources (Nembabhard & Aktan, 2010). However, there are limited cases where it has been tactically applied at an operational level.

The literature review has shown that the adaption of RO is being hampered by lack of synthesis of the RO process and computation challenges that are explained in the following sections.

2.7.1 Conceptive challenges impeding the adaptation of RO ‘on’ project

Conventionally, any good engineering design must minimise risk. Such a mindset is reactive to risk (Neufville, 2002). According to Samis et al. (2002), industry risk management has been focusing on an aggressive level of company operation and risk management principles applied at the project level or design phases. The combined effect of globalisation, deregulation and reduced technology cycles result in managers facing very volatile environments in their strategic investment decisions, which in turn, limit the range of possible future actions (Fu, 2002).

The lack of a single, well-proven technique is a challenge to valuers and engineers alike (Lilford & Minnitt, 2005; Kazakidis & Scoble, 2002). According to Copeland & Tufano (2004), business leaders frequently use some sort of option approach in evaluating and deciding investment opportunities at the initial stages but many of the users of an option approach who have tried gave up due to technical grounds.

Others argued that Chief executive officers (CEOs) intuitively understand the value of flexibility but there is a disconnect with Chief Financial Officers (CFOs) that predominantly use static DCF analyses (Portfolio Group, 2002). Trigeogis & Smit (2003) attributed a general implementation problem of option pricing methodology to an insufficient set of market quotes of correlated financial instruments or data. According to Benaroch & Kauffman (1999), most people who use the DCF method are ill-equipped to use the option pricing model correctly due to a large number of varied information and assumptions required than are usually used in discount cash flow for NPV concepts. The real options valuation (ROV) and DCF methods differ fundamentally in the way they are discounted. Option pricing takes into account changes in revenue as time passes without parameter adjustment but, considers the asymmetric distribution of the expected revenue and their variability called volatility or the variance of the expected rate of return on the mining project (Haque et al., 2016).

Moreover, there is an overriding question on whether one can analyse non-trading assets using models (Black–Scholes and binomial) formulated to evaluate assets traded in a financial market. Both Black – Scholes and binomial models assume risk neutrality and are not immune to the analyst subjectivity that can lead to overestimation or underestimation of the NPV which can have significant impacts on investment decisions.

2.7.2 Computational challenges facing adaptation of RO 'in' project

The lack of corroborating evidence in existing literature showing how to apply RO valuation in practice, under multiple uncertainties and non-uniform conditions without oversimplifying the reality has also contributed to the low uptake of the RO technique. Wang (2005) studied the application of RO in engineering design, by focusing on option identification and analysis. He used screening and simulation models to identify options and also used stochastic mixed- integer programming to value options. As the author of the thesis, he acknowledged that any use of stochastic mix-integer programming reformulation to value options complicates the analysis and makes the approach less attractive and discourages potential users.

Brandao et al. (2005) have focused on RO analysis techniques. They argue that the use of sophisticated mathematical models to carry out RO analysis takes it onto the path of many other algorithms which turns out to be complicated, lacking intuition and under applied. They used a simple binomial tree and decision tree analysis that is simple and intuitive. It is important to emphasise that the technique applied by Brandao et al. (2005) is pertinent to this research and it will be referred to as the Brandao – Dyer – Hahn approach.

To overcome users fear, Neufville (2002) presented a paper on approaching fundamental engineering issues. He suggested that the RO analysis method can provide a conceptual basis for defining optimal configurations by designing flexible engineering systems that can evolve optimally to meet new challenges and opportunities. However, improper differentiation between risk and uncertainty in the running of operations is impeding the use of RO methodology. It is not uncommon that risk and uncertainty are frequently confused and loosely used (Koleczko, 2012). Brammer & Smithson (2008) attempted to put forward a taxonomy for uncertainty by illustrating what is known and what is unknown, but most importantly, they acknowledge that there is a gap where current practices and perspectives cannot be easily mapped onto existing structures. Thus, there is no definitive guide to managing uncertainty that is currently established. Therefore, providing another, more appropriate and user-friendly guideline is a good justification for the importance of this research.

Even though their approach was aimed at the application of RO at the strategic level, Haque et al. (2014, 2015) made a genuine attempt to promote the use of RO in mine valuation. However, their methodology requires a reasonable understanding of calculus and stochastic processes as they utilised partial differential equations with Matlab simulation to solve RO. The system dynamics approach has also been used for policy development and decision making during feasibility studies (Inthavongsa et al., 2016). However, the Inthavongsa et al. (2016) approach was flawed as the process mimics scenario planning where the option is chosen and executed. Thus, there was no real managerial flexibility designed into the mine operations.

2.8 Real options conventional analytical models

It is the view captured in this thesis that it is important to understand where RO came from, what models are relevant and why those models are utilised to analyse available options when running mining operations. Therefore, this chapter presents summaries of theories that will be used in the case studies as well as being referred to and cited throughout this research to demonstrate how RO can create managerial flexibility at a mine operational level. Options are deduced into mathematical models that are used for calculating the expected value of flexibility.

To cover the option theory adequately in support of the scope of this research, the financial option and option theory models will be explained. The models which will be covered in this chapter include the Black Scholes model, binomial model and lattice, all risk-neutral models, risk measures, DCF and NPV analysis and stochastic processes. It is important to underscore that the theory and summary of the models in this chapter set the ground for the research's chosen methodologies applied in the published papers that will be explored in full detail in Chapter 3 to Chapter 6. A good understanding of the existing theory is critical as most of the models mentioned above will be either referred to directly and called into use in equation derivations when the chosen methods are applied.

Even though the three analytical methodologies chosen for this thesis are the Binomial Tree Analysis, Stochastic Simulation of Mean-Reversion with Jump Diffusion and Predictive Data Mining Decision Tree Classification, the researcher is cognisant that there are many other analytical models that can be used in RO valuations. Techniques including multicriteria dynamic programming (Targiel, 2013), the Datar – Matthew's Method which employs triangular distribution (Datar et al., 2007) and mixed - integer programming as mentioned in Section 2.5 could also be used in option valuations, depending on the project type and its suitability to analysing the associated variables.

As stated by Ajak et al. (2018), the RO methodology is both a qualitative and analytical tool. Therefore, the choice of the RO analytical model does not depend on the quantitative aspect alone but also on a qualitative framework being the real options identification steps. The project structure guides the choice of an analytical model, commensurate with project characteristics and value estimation for the required flexibility (Lilford & Minnitt, 2005; Samis & Laughton, 2007). Therefore, a binomial decision tree was chosen since it is intuitive and several variables can be modelled simultaneously. Additionally, the commodity price, which is one of the main uncertain variables in this thesis, is known to be stochastic, goes through cycles that are annotated with jumps and the returns demonstrate a lognormal distribution. Therefore, the choice of binomial and Mean-Reversion with Jump Diffusion approaches were the most appropriate models. Considering that technological advances have increased data availability, utilising predictive data analytics in RO analysis made the thesis robust and delivers a new perspective to this research topic.

2.8.1 Black and Scholes

When this model was first proposed by Fisher Black and Myron Scholes (Black & Scholes, 1973), it was intended for valuing options on trading assets, especially stocks and bonds. Compared to stochastic and simple geometric Brownian motion (GBM) models, the Black and Scholes model is not applicable to valuing real projects that involve compound American-style RO with multiple uncertainties

(Dimitrakopoulos & Abdelsabour, 2007). The Black and Scholes model is more suitable for theoretical computation of values of European style options. To formulate a mathematical representation of the model, the following assumptions are made:

- The short-term interest rate is known and constant over time.
- Stock price follows the random walk process in continuous time period.
- Stock pays no dividend.
- The options are European style and can only be exercised at a specified date.
- No fees or trading cost when buying or selling a stock or option.
- No penalties for short selling variation.
- Market returns on the underlying asset are normally distributed.

The Black and Scholes formula is shown in Eq. (2.1).

$$c = S_N(d_1) + N(d_2)Ke^{2-rt} \tag{2.1}$$

Both d_1 and d_2 are determined in Eqs. (2.2) and (2.3).

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{s^2}{2}\right)t}{\sqrt[\sigma]{t}} \tag{2.2}$$

$$d_2 = d_1 - \sqrt[\sigma]{t} \tag{2.3}$$

Where;

c = Value of the call option

S = Current stock price

t = Time until option maturity

K = Option striking price

r = Risk-free interest rate

N = Cumulative standard normal distribution

e = Exponential term or constant which is ~ 2.71828

σ = Standard deviation of the stock price

ln = Natural logarithm

As shown by Eqs. (2.2) and (2.3), the Black and Scholes model is divided into two distinctive parts. The first part, *SN(d1)* multiplies the price of the risky asset by the change in the call price associated with a change in the underlying price. This part justifies the expected gains from the option by purchasing the right but not the obligation.

Part two of the equation is *N(d2)Ke^r(T-t).* This part provides the present value of paying the exercise price at the maturity as applies to European options that are exercisable only on expiration day. The value of the option is a difference between these two parts of the equation. The calculation of the Black and Scholes model is based on a mathematical theory such as the geometric Brownian motion theory of stock price behaviour and risk-neutral valuation.

The main strength of the Black-Scholes model is its ability to calculate a vast number of option prices in a short time span. However, the model cannot accurately price options with an American-style exercise as it only calculates the option price at maturity. To approximate an American option, the Black and Scholes model is modified into the Fischer Black Pseudo-American method (Ball & Torous, 1985; Hoadley, 2017).

2.8.2 Standard risk-neutral valuation and probabilities

The most important principle of the risk-neutral valuation which underpins its application is the notion that the value of an option being created is completely independent of the expected rate of return of the underlying asset (Hoadley, 2017). In the application of this method, the fair value of the option on the underlying asset is affected by the volatility of the asset values and the risk-free rate. That is to say, the price of an option is independent of the risk preferences of investors (Goddard, 2015; Mun, 2006; Copeland & Antikarov, 2003). This implies that a risky asset could be valued with the assumption that the return from their underlying assets is the risk-free rate.

The risk-neutral valuation utilises three equations to obtain three parameters of the binomial model assuming that the model behaviour will be similar to those of the risky asset being valued over a short period. The following are the three equations:

- Matching return equation: this formula calculates the value of the expected return of the binomial model over a short period of time Δt, and it matches the calculated value to the expected return in a risk-neutral state.

$$pu + (1 - p)d = e^{r\Delta t} \qquad 2.4$$

Where p is the probability of moving up, u is the upside percentage and d is the down side percentage.

- Matching variance equation: this formula enables the variance of the expected return of the binomial model to match the variance of the expected return in a risk-neutral state.

$$pu^2 + (1 - p)d^2 - \left(e^{r\Delta t}\right)^2 = \sigma^2 \Delta t \qquad 2.5$$

2.8.2.1 Cox-Ross-Rubinstein equation

This equation states that parameter u is inversely proportional to parameter d thus,

$$u = 1/d \tag{2.6}$$

The reorganisation of the above three equations produces the following risk-neutral equations and provides a solution for the three parameters which are p, u and d.

$$p = \frac{e^{r\Delta t} - u}{u - d} \tag{2.7}$$

$$u = e^{\sigma\sqrt{\Delta t}} \tag{2.8}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{2.9}$$

The solution of these parameters is the intention of the first two equations which are to enable the expected value of the binomial model to match the mean and variance of an asset in a risk-neutral state.

2.8.2.2 Cox-Ross-Rubinstein (CRR) with drift

The CRR model with drift is a modified version of the standard CRR model with an arbitrary drift η, applied to parameters u and d to generate a binomial model that can hold its assumptions (Cox et al., 1979). The CRR equations with drift area:

$$u = e^{\eta\Delta t + \sigma\sqrt{\Delta t}} \tag{2.10}$$

$$d = e^{\eta\Delta t - \sigma\sqrt{\Delta t}} \tag{2.11}$$

Any increase in drift term η moves the prices on lattice further away from the current asset value S_0 and if the drift is decreased until its arbitrary value reaches zero, the model with drift term collapses to the original Cox-Ross-Rubinstein binomial model which generates a lattice of prices that is centred around the current asset price S_0 (Goddard, 2015; Mun, 2006).

The Jarrow-Rudd Risk Neutral model which will be discussed in the next section is a specific case of the CRR with drift model. The drift for this model is:

$$\eta = \frac{(\ln(X) - \ln(S_0))}{T} \tag{2.12}$$

Where $ln()$ is the natural logarithm and T is the time to expiry in years.

This equation balances out the tree to attain a symmetric growth as it moves up, moves down and converges in the centre. However, it should be noted that the tree works only for a specific individual strike price and any change in the strike price usually requires a regeneration of the tree.

2.8.2.3 Equal-probability model (Jarrow – Rudd model)

This model was proposed by Jarrow & Rudd (1983), and it is commonly referred to as the Jarrow-Rudd model. The central concept of this model is to provide a mathematical equation that will allow the expected mean of returns and variance of the binomial model (Goddard, 2015) to match those expected in a risk-neutral world over a short period. Since there are three variables in the binomial model (p, u and d), three equations are needed to calculate unique values for them. Two of these equations are the risk-neutral equations, and the third equation proposed by Jarrow and Rudd is:

$$p = 1/2 \tag{2.13}$$

Eq. (2.13) implies that there are two possible outcomes with equal probabilities of the asset price rising or falling. Eqs. (2.14) and (2.15) are used to calculate the parameters for the Jarrow-Rudd binomial model. These parameters are p, u and d which are calculated similarly to those of the standard binomial price tree and uses it for pricing options. The Jarrow-Rudd model is not risk-neutral. The three equations of the Jarrow-Rudd model are:

$$u = e^{\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}} \tag{2.14}$$

$$d = e^{\left(r - \frac{\sigma^2}{2}\right)\Delta t - \sigma\sqrt{\Delta t}} \tag{2.15}$$

To address the main limitations of the Jarrow-Rudd model, which is not risk-neutral, a small modification is applied and this results in what is referred to as the Jarrow-Rudd risk - neutral model. This model is a modified standard Jarrow-Rudd model. Apart from the substitution of the probability equation where the risk-neutral value for p is fixed, the two models are very similar in most aspects. Therefore, the resulting equation is similar to Eq. (2.7). As stated in the previous section, the Jarrow-Rudd risk-neutral model is a special case of the Cox-Ross-Rubinstein with drift model.

2.8.2.4 Tian's model

The advantage of the model proposed by Tian is its ability to match the expected mean and variance of the binomial model to values of the risk-neutral model (Tian, 1993, 1999). The mean and variance are referred to as the first moments of a lognormal distribution. Eqs. (2.16), (2.17) and (2.18) represent Tian's model.

$$pu + (1 - p) = e^{r\Delta t} \tag{2.16}$$

$$pu^2 + (1 - p)d^2 = \left(e^{r\Delta t}\right)^2 e^{\sigma^2 \Delta t} \tag{2.17}$$

$$pu^3 + (1 - p)d^3 = \left(e^{r\Delta t}\right)^3 \left(e^{\sigma^2 \Delta t}\right)^3 \tag{2.18}$$

Similar to a standard binomial model, the calculation of variables p, u and d does not change.

$$u = 0.5e^{r\Delta t}v\left(v + 1 + \sqrt{v^2 + 2v - 3}\right) \tag{2.19}$$

$$d = 0.5e^{r\Delta t}v\left(v + 1 - \sqrt{v^2 + 2v - 3}\right) \tag{2.20}$$

$$v = e^{\sigma^2 \Delta t} \quad 2.21$$

where v is the option value.

2.8.2.5 Leisen-Reimer model

Leisen and Reimer developed this model to make the convergence of the binomial tree smoother (Leisen & Reimer, 1996). As discussed by Goddard (2015), most of the binomial model 'converge to the Black-Scholes solution when the function approaches its limit as the size of the time step Δt is reduced to zero' (Goddard, 2015; Mun, 2006). The Leisen-Reimer binomial tree is generated using the following equations:

$$p = h^{-1}(d_1) \quad 2.22$$

$$p = h^{-1}(d_2) \quad 2.23$$

$$u = e^{r\Delta t}\frac{\bar{p}}{p} \quad 2.24$$

$$d = \frac{e^{r\Delta t} - pu}{1 - p} \quad 2.25$$

where $h - 1()$ is a discrete approximation to the cumulative distribution function for a normal distribution (Goddard, 2015) and can be calculated using the method suggested by Leisen and Reimer which is shown below.

$$h^{-1}(z) = 0.5 + sgn(z)\left[0.25 - 0.25e^{-\left(\frac{z}{n+\frac{1}{3}+\frac{0.1}{n+1}}\right)\left(n+\frac{1}{6}\right)}\right]^{1/2} \quad 2.26$$

Where d_1 and d_2 are the parameters of the Black-Scholes equation and n is the number of time intervals which must be odd within the model that starts from 0 to T inclusively. These two parameters, d_1 and d_2 still take their usual definitions from the Black-Scholes formulation. This equation tends to be complicated and is rarely utilised in RO analysis.

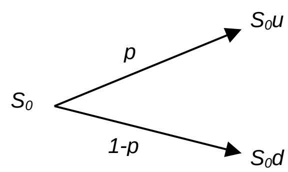
Having explored the risk-neutral valuation techniques an their probabilities in Eqs. (2.4) – (2.26), it is important to highlight that those equations formed the basis of the binomial tree constructions that is commonly referred as the binomial model that is discussed in Section 2.8.3.

2.8.3 Binomial model

The binomial model equations can be traced back to Cox et al. (1979). The application of this model has been discussed in detail in Chapter 3 where it has been applied in the case study. Therefore, this section will state its general principles and central concepts. The binomial model reduces the time to expiration into small steps and value of the risky asset is quantified at each node. The main steps involved in calculating option values in binomial models are:

- Firstly, the expected future values of the risky asset at maturity for a European option or at each period of it is an American option are calculated.
- The payoff of the option is calculated at the maturity for underlying risky values.
- Payoffs are discounted to obtain the present value of the option.

This breakdown generates a tree of the asset value by starting from the present time to maturity of the option. For a single step binomial model, the tree will look as follows:



One step Binomial Model (Cox et al. 1979)

Where;

S_o is the stock price today.

p is the probability of a value rise.

u is the upside factor by which the value rises.

d is the downside factor by which the price falls.

The main computation is the calculation of payoffs at each node corresponding to the time of expiry. This matches to all of the nodes at the right-hand edge of the decision tree.

Put payoff $$V_n = \mathrm{Max}(\mathrm{X} - S_n, 0) \qquad 2.27$$

Call payoff $$V_n = \mathrm{Max}(S_n - \mathrm{X}, 0) \qquad 2.28$$

Where;

n designates a node before expiry.

V_n is the option value.

X is the strike price.

S_n is the price of the underlying asset.

After calculating the payoffs at each step, the binomial model is used to discount the payoffs of the option at the expiry nodes back to today (Goddard, 2015; Mun, 2006; Copeland & Antikarov, 2003).

European Put or Call $$V_n = e^{-r\Delta t}(pV_u + (1-p)Vd) \qquad 2.29$$

American Put $$V_n = \text{Max}(\text{X} - S_n, e^{-r\Delta t}(pV_u + (1-p)Vd) \quad 2.30$$

American Call $$V_n = \text{Max}(S_n - \text{X}, e^{-r\Delta t}(pV_u + (1-p)Vd) \quad 2.31$$

Where;

p is the probability of an upward price movement.

r is the risk-free interest rate.

Δt is the step size between time slices of the model.

Using backward induction, the counter n starts at N (expiry) and decreases to 0 (today).
It is assumed that values of the asset will either move up or down at each interval by a certain amount obtained from the volatility of the asset value. All these steps diverge and converge sequentially, producing a binomial distribution or recombining tree of the asset values.

Option values at each node are obtained by working backwards from the end of the tree to the starting node. It is the value of the starting step that is used to determine the value of the next step and the calculation progress along the path to the initial point. The primary input in the calculation is the upside and downside risk-neutral probabilities of the asset value, the risk-free rate and the time interval of each step. If the value of the risky asset today is S_0 and at time interval Δt it may either move to $S_0 u$ or $S_0 d$. This model assumes that the value of the risky asset underlying the option undergoes a random walk process and a probability p is assigned to the likelihood that the values may rise, and the probability of falling is therefore $1 - p$. This recombining tree is a representation of all the possible paths that the asset values may take during the life of the option and the values at the end of the tree are the intrinsic values of the asset.

The ability of the binomial model to accurately price American options is its main strength over the Black-Scholes model but, its slow speed and the fact that it is cumbersome in handling many nodes is its main drawback. Additionally, lattice techniques are challenging to implement as they are complicated by the rapid growth of lattices as the variables increase. Thus, the technique becomes unmanageable and impractical (Dimitrakopoulos & Abdelsabour, 2007).

2.8.4 DCF analysis and net present value (NPV)

This concept helps in valuing the intrinsic value of a company (or asset). It aim is to work out the value today, based on projections of all the cash that could be made available to investors in the future. DCF analysis adjusts the cash flow to capture the essence of the time value of money. This principle states that any cash in the future is worth less than that cash today Solution Matrix (2014). There is no risk to cash which the investor has currently, while future cash is uncertain. As time passes, the buying power of money reduces due to in inflationary pressures.

2.8.4.1 Discounting the cash flows and NPV

This is where the attractiveness of an investment opportunity is expressed in dollar per period terms (Summa, 2015). The discounted cash flow equation is a derivative of the future value formula and is used in computing the time value of money and compounding returns.

$$DCF = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_n}{(1+r)^n} \tag{2.32}$$

Where;

DPV is the discounted project present value.

DCF is the discounted cash flow.

FV is the value of a cash flow amount in a future period.

CF is the cash flow

r is the discount rate

n is the time period in years

Future value (FV) 'is the value, in non-discounted currency units that actually flows in or out at the future time' (Summa, 2015; Solution Matrix, 2014; Samis & Laughton, 2007; Mun, 2006).

$$FV = DCF.(1+r)^n \tag{2.33}$$

Thus, the present value (PV) which is the value today of the future cash flow (Solution Matrix, 2014; Copeland & Antikarov, 2003; Kritzman, 2003) for a single period is

$$DPV = \frac{CF_n}{(1+r)^n} \tag{2.34}$$

For any project that runs for many years, the total discounted present value is the sum of DPV for all periods.

$$DPV = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t} \tag{2.35}$$

The assumption for these equations is that the interest rate must remain constant for all periods into the future. In scenarios where the cash flow is continuous, an integral formula is used. FV(t) is the rate of cash flow, λ is log (1 + r) and e is the Euler constant. The PV of future cash flow is inversely proportional to the period of more than one period. This means that the difference between the discounted cash flows decreases as periods increase.

NPV is the remainder after the capital, I has been subtracted from the present value (Solution Matrix, 2014; Summa, 2015; Mun, 2006; Copeland & Antikarov, 2003).

$$NPV = -I + \frac{FV_0}{(1+r)^0} + \frac{FV_1}{(1+r)^1} + \frac{FV_2}{(1+r)^2} + \cdots + \frac{FV_n}{(1+r)^n} \qquad 2.36$$

NPV is used as a key decision criterion when deciding on various investment opportunities. The bigger the NPV, the more attractive the investment and the higher the chances that the project will be accepted. However, DCF assumptions are rigid, future decisions are ignored and flexibility is not valued, thus the technique undervalues the project (Purwar et al., 2011).

Therefore, it is important to highlight that this research will mostly be using both the NPV and PV when comparing RO versus the traditional analysis in the assessment of the optimal managerial decision in the face of uncertainty as discussed in Chapter 3 to Chapter 6.

2.8.5 Stochastic processes

It is formally defined as a process that is described by the change of some random variable over time, which may be either discrete or continuous. During this time, events may be happening at various points along the path that may affect the ultimate value of the process (Alao & Oloni, 2015).

2.8.5.1 Brownian Motion for modelling prices

This random stochastic process generates the course of each movement which creates a probability space for all the possible outcomes. The Brownian motion, however, is a deterministic system, which means that the nodes generated can be mathematically calculated.

Let Ω be a set of possible outcomes from an experiment or uncertain event $(\omega_1, \omega_2, \ldots)$, and let's take a random variable X, say the price of the commodity, which is a function from the Ω of the possible outcomes, and let it be a real number $\mathbb{R}$ from the set. An algebraic X needs to be generated from set, and all the subsets of X are referred to as events which are the outcomes of a particular experiment where $X: \Omega \rightarrow \mathbb{R}$.

Therefore, the events of X are those for which a probability can be given that they will occur. Probability is a measure on X, and it is the chance of the event occurring or not.

$$P(\emptyset) = 0, P(\Omega) = 1 \qquad 2.37$$

P (disjoint events) = $\sum P(\text{each event})$

The Brownian and all the other stochastics processes briefly discussed in the next subsections are conceptually probability-based risk assessment tools which quantify risk (Jablonowski et al., 2017).

Therefore, they cannot be independently used to value flexibility without the incorporation of the RO methodology.

2.8.5.2 Wiener process

This process is a form of the Brownian motion stochastic process *Wt* which is characterised by the following three facts:

$W_0 = 0$

W_t is almost certainly continuous (in another words it has a continuous sample path)

W_t has an independent increment with a distribution $Wt\text{-}Ws\sim N(0, t\text{-}s)$.

Thus, the summary equation for the Wiener process is

$$dx = adt + bdW(t) \qquad 2.38$$

Where a and b are constants and the $dx = adt$ can be integrated to $x = x_0 + at$, where x_0 is the initial value and if the time period is T, then the variable is increased by at. bdz accounts for the noise or variability to the path followed by x. The amount of this noise or variability is b times the Wiener process.

2.8.5.3 Geometric Brownian Motion (GBM) and mean-reverting process (MRP)

The models used for generating realisations of the market and economic variables are the GBM and MRP models (Dimitrakopoulos & Abdelsabour, 2007; Mun, 2006). Some researchers in the past used one-factor GBM models to reduce the dimension of the problem to one. This simplification implies that there is a perfect correlation between two completely independent variables such as metal price and the grade of metal. However, such an assumption is erroneous as the grade is an internal variable which is usually project specific and the price is an external variable (Ajak & Topal, 2015; Dimitrakopoulos & Abdelsabour, 2007). Eq. (2.39) shows the GBM model.

$$\frac{\Delta S}{S} = \mu\Delta t + \sigma\varepsilon\sqrt{\Delta t} \qquad 2.39$$

Where;

ΔS is the change in the price of risky asset or stock price.

μ is the expected rate of return.

$\sigma\sqrt{\Delta t}$ is the stochastic component.

ε is the normal distribution.

The MRP model i proposed by Schwartz (1997) is

$$\frac{dS}{S} = k(u\mu - lnS)dt + \sigma dz \qquad 2.40$$

Where k is the reversion speed at which the log of a price reverts to a long-term equilibrium log price μ.

2.8.5.4 Itō's process for computing option value

An Ito process is a generalised form of the Wiener process in which the parameters a and b are functions of the value of the underlying variable x and t. These are both the expected drift and the volatility that can change over time. An Ito process with many dimensions is represented by:

$$x_t = x_0 + \int_0^t a_s\, ds + \int_0^t b_s\, dW_s \qquad 2.41$$

Where W is a m-dimensional standard Brownian motion and a and b are n-dimensional and $a(n * m)$ - dimensional F_t adapted processes, respectively. Eq. (2.38) and the n-dimensional stochastic differential equation forms Eq. (2.42).

$$dX_t = a(X_t + t)dt + b(X_t + t)dW_t \,;\, X_0 = x \qquad 2.42$$

Thus, Eq. (2.42) can be represented as:

$$x_t = x_0 + \int_0^t a(X_s\,, s)\, ds + \int_0^t b(X_s\,, s)\, dW_s \qquad 2.43$$

If the value of a variable x follows the Itō process, the variable x has a drift rate of a with a variance rate of b^2. Itō's lemma shows that a function G of x and t follows the process shown in Eq. (2.44).

$$dG = \left(\frac{\delta G}{\delta x} a + \frac{\delta G}{\delta t} + \frac{1}{2\delta}\frac{\delta^2 G}{x^2} b^2\right) dt + \frac{\delta G}{\delta x} bdz \qquad 2.44$$

Therefore, G also follows an Itō process with the drift rate of**:**

$$\frac{\delta G}{\delta x} a + \frac{\delta G}{\delta t} + \frac{1}{2}\frac{\delta^2 G}{\delta x^2} b^2 \qquad 2.45$$

moreover, the variance of:

$$\left(\frac{\delta G}{\delta x} b\right)^2 b^2 \qquad 2.46$$

A complete study of the Itō's process is beyond the scope of this thesis. Thus, this lemma is not derived as it has been widely described in many calculus books. However, this section will only deal with its application as it will be applied to the analysis of real options later in Chapter 4.

2.8.5.5 Application of Itō's lemma on a forward contract

The formula used in the calculation of a forward contract price is as follows:

$$F_0 = S_0 e^{rT} \qquad 2.47$$

Where F is the forward price as time passes, S is the current stock price, t is the time when the forward contract is being exercised before maturity and $a < T$, r is the rate of return and e is the natural number which is constant. Thus,

$$F_0 = S_0 e^{r(T-t)} \qquad 2.48$$

For instance, if S_0 = \$50, r=0.05 and T= 1 year, then F_0 will be equal to \$52.55

2.8.5.6 Partial derivatives of forward price (F)

Assuming that Eq. (2.39) gives the process for the stock price S. The Itō process can then be used to determine the process for F.

$$\frac{\delta F}{\delta S} = e^{r(T-t)}, \frac{\delta^2 F}{\delta S^2} = 0, \frac{\delta F}{\delta t} = -rSe^{r(T-t)} \qquad 2.49$$

Substitute G function with F, a with uS, and b with σS. The new equation will appear as follows:

$$dF = \left(\frac{\delta F}{\delta x}\mu S + \frac{\delta F}{\delta t} + \frac{1}{2\delta}\frac{\delta^2 F}{x^2}(\sigma S)^2\right)dt + \frac{\delta F}{\delta x}\sigma S dz \qquad 2.50$$

However, it is already shown that:

$$\frac{\delta F}{\delta S} = e^{r(T-t)}, \frac{\delta^2 F}{\delta S^2} = 0, \frac{\delta F}{\delta t} = -rSe^{r(T-t)} \qquad 2.51$$

Therefore substituting this derivative into the equation produces the following:

$$dF = \left(e^{r(T-t)}\mu S - rSe^{r(T-t)} + 0(\sigma S)^2\right)dt + e^{r(T-t)}\sigma S dz \qquad 2.52$$

Since $F_0 = S_0 e^{r(T-t)}$, substituting F for $S_0 e^{r(T-t)}$ produces

$$dF = (u - r)Fdt + \sigma F dz \qquad 2.53$$

Similar to the stock price S, the forward price F follows the GBM. This has an expected growth rate of $u - r$ rather than u. The growth rate of F is the excess return of the risky asset with a risk-free rate.

Assuming variable S follows an Itō process which contains a non-stochastic and a stochastic component, then the following statement is true about any function $G(S,t)$, that is a function of S and t.

$$dG(S,t) = \left(\mu S_t \frac{\delta G}{\delta s} + \frac{\delta G}{\delta t} + \frac{1}{2}\sigma^2 S^2 \frac{\delta^2 G}{\delta S^2} b^2\right) dt + \sigma S_t \frac{\delta G}{\delta S} dW_t \qquad 2.54$$

2.8.5.7 Lognormal distribution

This is a continuous probability distribution of an uncertain variable such as commodity price whose logarithm of the returns is normally distributed (National Institute of Standards and Technology, 2012). A variable modelled as lognormal is a multiplicative product of many independent random variables and each of which is positive with a probability density function shown in Eq. (2.55).

$$f_x(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(lnx-\mu)^2}{2\sigma^{2,}}}, x > 0 \text{ and } X = e^{u+\sigma z} \qquad 2.55$$

In a lognormal distribution X, the parameters denoted by u and σ are, respectively, the mean and standard deviation of the variable's natural logarithm and z is the standard normal variable (z-score). By definition, the variable's logarithm is normally distributed.

This method used a normal distribution of the gains from the risky asset. In another words, the prices of the risky asset are lognormally distributed. Lognormal distributions are much more tailed to the right than the ordinary continuous normal distribution (bell-shaped distribution). Prices of the asset which meet this distribution usually range between zero and infinity but, never go below zero. This can be simply put as saying that stock prices will always stay above zero.

There are a few cases where the prices of the risk asset do not meet the normal distribution criteria and become common when changes are rapid and frequent. Such departures from lognormal distributions are measured by coefficients of skewness and kurtosis of the values.

2.8.5.8 Application of Itō's lemma to lognormal process

The aim here is to produce an equation of $Y_t = F(S_t)$ such that Y_t will not contain any references to S_t. Now the equation $Y_t = log(S_t)$ will be used, but it should be noted that Y_t is not a function of t but S_t.

$$dY_t = \left[0 + \frac{1}{S_t}\mu_t S_t + \frac{1}{2}\left(\frac{1}{S_t^2}\right)\sigma_t^2 S_t^2\right] dt + \frac{1}{S_t}\sigma_t S_t dW_t \qquad 2.56$$

$$dY_t = \left(\mu_t + \frac{\sigma_t^2}{2}\right) dt + \sigma_t dW_t \qquad 2.57$$

$$Y_t = \int_0^t \left(\mu_t + \frac{\sigma_t^2}{2}\right) dt + \int_0^t \sigma_t dW_t \qquad 2.58$$

However, it is already established that $Y_t = \log(S_t)$
Therefore,

$$Y_t = \log(S_0 + \int_0^t \mu_u \, S_u \, d_u + \int_0^t \sigma_u \, S_u \, dw_u) \qquad 2.59$$

and it is also true that,

$$S_t = e^{(Y_t)} \qquad 2.60$$

$$S_t = S_0 \, e^{\left(\int_0^t \left(\mu_u - \frac{\sigma^2}{2}\right) du + \int_0^t \sigma_u \, S_u \, dw_u \right)} \qquad 2.61$$

It should be highlighted that this equation is a Geometric Brownian Motion if the u and σ are constants and the $Y_t = \log(S_t)$ is normally distributed and the S_t is lognormal.

As discussed previously, the application of stochastic processes alone assesses risk but, does not value flexibility. They are commonly utilised in scenario planning where alternatives are assessed, and the decision is made on the scenario with an optimal value. Therefore, there is no room for future decisions as the future is assumed to mimic the simulated path. As a consequence, these processes are backed up with RO which can create the managerial flexibility (Ajak & Topal, 2015; Santos et al., 2014; Guj, 2011).

2.9 Conclusion

The literature review reveals that RO thinking recognises that uncertainty adds value to options being envisaged and that flexibility has value. Current engineering practices in the treatment of risk and uncertainty are based on fixed assumptions which result in undervaluing a project's NPV. This implies that firms that ignore this lose the competitive advantage that stems from applying the RO methodology as a tool in decision making.

Due to a perceived notion that the real option model input variables are difficult to calculate, many mining companies do not use it during a project feasibility study. Therefore, the application of RO in engineering design and decision making is less researched compared to its financial applications. There is no clear path or well-mapped technique in applying and analysing RO. Published literature has indicated that senior executives in corporations that make investment decisions are not convinced by the available research and therefore there is no appreciation of the RO method and therefore have less incentive to apply it. Since the majority of the population is risk-averse, there is no appetite for experimenting with investor monies by applying unconventional methods like RO.

However, real-world problems are askew, not all of them can be solved by using a mathematical model, but, some decision science is needed. Therefore, there is a genuine case to research the application of RO by using simple, visual, intuitive and transparent models (Ajak & Topal, 2015) and analytical processes to eliminate decision maker apprehension and may improve the use of RO in engineering design and decision making.

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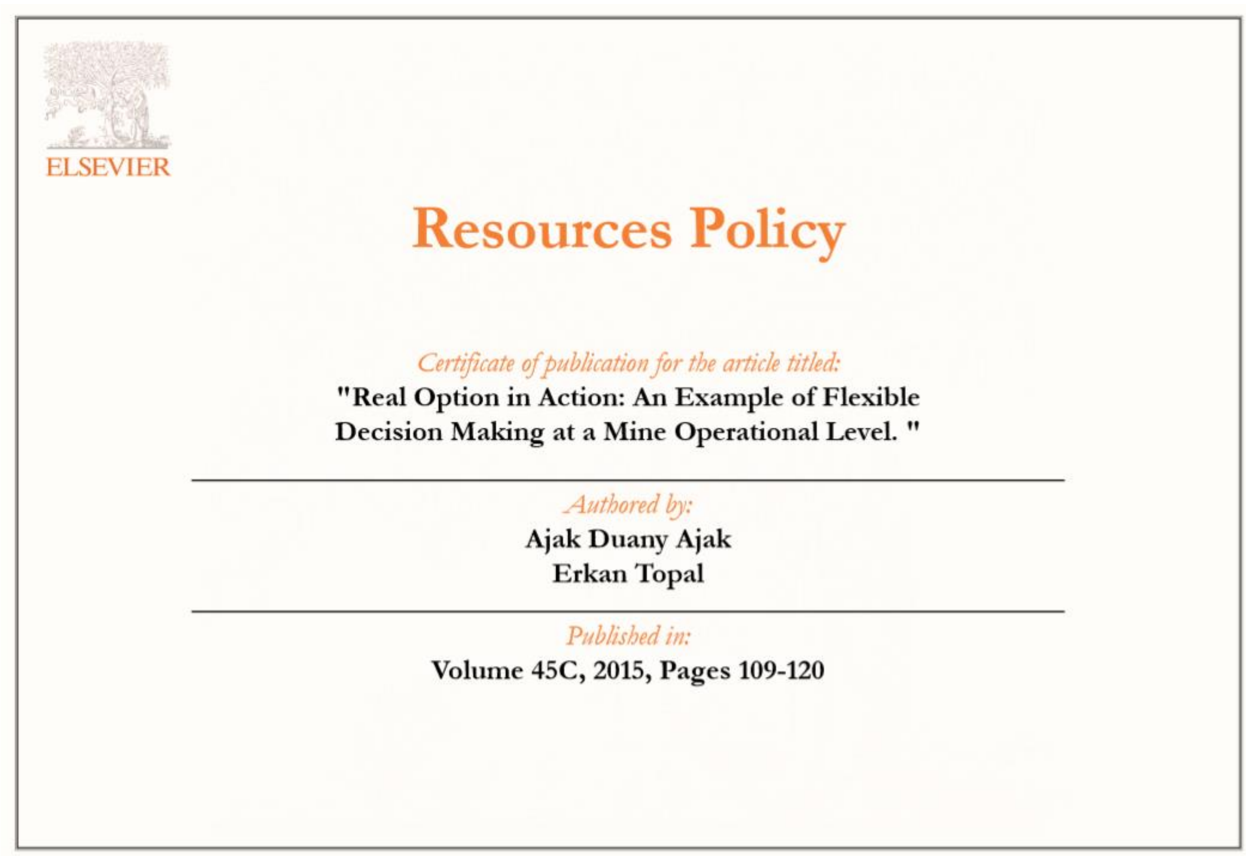
Chapter 3: Real Option in Action: An Example of Flexible Decision Making at a Mine Operational Level.



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Principal Author	**Candidate Contribution to the Paper**	**Overall (%)**	**Signature**	**Date**
Ajak Duany Ajak	Set research question, developed methodology and predictive data mining model, developed case studies and analysed real options, wrote manuscript and acted as corresponding author.	85%	[signature]	25/7/18

Co-Author Contribution

By signing the statement of Authorship, each author certifies that:

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Abstract

Flexibility and operational adaptability are essential for long term corporate success and real option (RO) analysis appears suitable for analysing risky projects. Nevertheless, its application in engineering design has been slow-moving compared to financial uses. Therefore, there is a compelling argument for using visual, intuitive and transparent models, such as the binomial decision tree, which has the potential to eliminate decision maker apprehension and improve RO use in engineering design and decision making. This paper reviews RO applications in mining projects and proposes a new methodology to explore technical applications of RO in mine design and decision making at the mine operational level. The research will investigate the suitability of using RO method at the mine operational level where production decisions are made frequently, rather than organisational strategies that are reviewed after several years. The proposed approach is applied to a case study. This will demonstrate how RO can be used in designing multiple pits in multi-zone ore deposits to create a switching option between pits regarding changing ore grades and fluctuating commodity prices. The main rationale of this option involves deferring waste materials by switching mining activities from a high to low stripping ratio pit. This creates a choice between using new RO thinking and the traditional methodology. The option is analysed using the binomial decision tree. The results summarised in this paper's conclusion reveal that the project's value increased considerably when flexibility was included in the mine design. These increases in project value were between eight to 15 per cent, depending on the number of flexible options incorporated into the design.

3.1 Introduction

The mining industry is one of the riskiest sectors when compared to other industries. A skilled team, courage and an optimistic view are required to attract financial investor support for mining projects. Most natural resource investments are irreversible, implying that if a firm has made a commitment to finance a mine, then it will be difficult to wind back that investment (Martinez & McKibben, 2010). Investors and managers normally face a dilemma regarding whether to continue with the mining investment when the commodity market is worse than expected, or simply forgo the capital already invested and discard the project. Considering the amount of capital investment required to develop a mine, the above choices are not easy for managers to make.

Engineers and project managers involved in the mining industry resort to, and make use of, the unique advantages the mining sector has over other industries. It takes a number of years from the commencement of an investment to the actual production of saleable ore product (SOP). This can range from between three to seven years, providing an opportunity to gather more information and make informed decisions. Most people involved believe that the best way of doing this is to use the real option (RO) approach (Topal et al., 2009). This describes the possibilities a firm has, allowing the world to be opened up as a map of opportunities (Edelmann & Koivuniemi, 2004). The methodology is used to justify an increase in system flexibility under uncertainty (Groeneveld et al., 2010).

Despite an ever-increasing level of uncertainty, most corporations make their financial decisions based on discount cash flow (DCF) methods, such as net present value (NPV) and internal rate of return (IRR), which are static. Economists have long recognised that possible future events can cast shadows on the present. Any future value must be discounted down to the present to be comparable to any current price (Adelman & Watkins, 1995; Topal, 2008). Production planning and design would be easy if variables like price or ore grade followed a known value. Methodologies such as linear programming (LP), mixed integer programming (MIP) and the heuristic method commonly used in engineering design (and especially in the mining industry) can also be used to model flexibility and maximise a project's NPV, based on the assumptions of the DCF analysis (Akbari et al., 2009).

Guj & Harton (2007) have argued that techniques such as the modern asset pricing (MAP) model provide minimum risk-adjusted value (floor value). This is better than the DCF model, but falls short of the RO method. Neufville developed the value at risk base RO valuation in 2003, to propagate systems thinking and flexibility in engineering design; this is known as an RO 'in' system (Neufville, 2003). Flexibility in design has significant value creation and the potential for radically changed economic risks in design-embedded ROs, reducing the overall investment required (Hassan et al., 2006; Kalligeros, 2010). This was noted earlier by Zhao and Tseng (2003), as an expected pay off of the option. Neely & Neufville (2001) summarised valuation of ROs in five phases: setup, analysis, financial perspectives, technological perspectives and sensitivity (sensitivity analysis of the key assumptions). A follow-up study, conducted by Kalligeros et al. (2006), introduced an algorithm for qualitative identification of platform components at multiple levels of system aggregation among variants within a family system. The shortcomings of this method include the assumption that the architectural concept and functional requirements for variants are predetermined. This supposition is not always true in many scenarios of the initial design stage. Neufville et al. (2009) demonstrated that the RO 'in' system design states the following: 1) the major requirements of the system are partially unknown; 2) the future is uncertain; 3) there is value in having the right (but not the obligation) to react to future development; 4) forces that lead to engineering design-fix assumptions are identifiable and 5) the drivers of value like demand are outside engineering previews.

Kim (2010) proposed a model to determine the optimum timing of projects by using ROs that focus on ownership ratios, synergy effects and payment options. This approach is an RO 'on' system, without any technical design involved, and is similar to the dynamic DCF analysis studied by Herbelot (1994) regarding coal-fired power plant projects. The greatest issue at present is not to prove that ROs increase project value, but to apply RO at the 'operational level', particularly in the mining sector, where projects constantly face uncertain futures.

The special characteristics of any mining project are the high levels of uncertainty in ore grade estimation and the volatile fluctuations in commodity prices (Groeneveld & Topal, 2011). Moreover, there are myriad risks and uncertainties associated with individual operations. Some of these uncertainties stem from the industry itself, and the operating environment, as well as the geopolitical factors of the host country. As outlined by Kazakidis & Scoble (2002), the main uncertainties of any mining project can be categorised as either exogenous, endogenous or a combination of both. Those uncertainties that fit into both major categories are shown in Table 3.1.

Table 3.1, Uncertainty categories in the mining industry (Originally presented as a diagram by Kazakidis & Scoble, 2002).

Uncertainty Sources in Mining Projects		
External (Exogenous)		**Internal (Endogenous)**
• Market prices		**Operating**
• Industrial relations		• Grade distribution
• Legislation/regulations		• Ground-related conditions
• Political risks		• Equipment
• Government policies		• Infrastructure
		• Recovery method
		Other
		• Workforce
		• Management/operating team
• Environmental & societal issues		

Regardless of these known uncertainties, the mining sector has continued with a DCF analysis that is rigid and ignores future values of the information. DCF works on fixed assumptions and disregards the notion that situations do change. Groeneveld et al. (2010), and Dimitrakopoulos & Abdelsabour (2007), acknowledge that RO 'in' projects can be used to quantify system flexibility under uncertainty. The mining sector currently uses a 'just in time' production system, which requires that ore is exposed only when prices are high (Archambeault, 2007). The RO paradigm requires that a flexible design can be used to determine parameters such as cut-off grade, production rates and when to mine certain sections of the mine in relation to challenges posed by commodity price fluctuations and uncertainties in Ore Reserves.

Unlike past studies that have focused on the strategic application of the RO approach, this research centres on using RO in design and decision making at the mine operational level. Literature review has indicated very limited studies in this area. Therefore, it is envisioned that this article will highlight and contribute to opening up new research frontiers into RO applications at the operational level. Results from the case study show the clear advantages of using the proposed method in handling project risks at the operation level.

3.2 How do mines become flexible at the operational level?

All mining companies, whether at the strategic or operational level, aim to identify development and production activities that maximise the net present value. The main value creation centre for a mine's commercial viability is its operational level. This objective can be achieved or neglected, depending on the proposed engineering design and how the mine has been planned. Introducing flexibility into a mine's operations is something that cannot be created simply when the mine is in production; rather, it starts at the feasibility study stage. Depending on the Ore Reserve, geological characteristics and the size of capital investment required for building the proposed mine infrastructure, mine planners analyse various options for mining the ore. This follows recognition that capital expenditure can be used for developing either inflexible (conventional mine design) or flexible stage-based mine design to access ore development. Identifying sound operating strategies that take advantage of the ore body's geological structures—such as scaling down a high-cost stage in response to a fall in mineral prices—can determine the mine's survival during tough times. Three common options are used by mine planners to identify these strategies, but they are normally ignored in favour of traditionally created open pit shells using algorithms such as Lerchs-Grossman (L-G):

- Design multiple pits in multi-zone ore deposits to create a switching option between the pits in regard to changing global situations. The main rationale of this option involves deferring waste material by switching mining activities from high to low stripping ratio pits. This creates choice between the RO approach and the traditional method, which is to mine the ultimate pit shell without flexibility in accordance with industry-established practices that maintain the status quo.
- Pre-strip pits that are not planned for mining at the present to expose the ore, creating an expansion option in response to high prices or high demand.
- Design multiple pit entries and develop pit auxiliary infrastructure, even though there are no plans to mine the pit currently but creating flexible options for responding to high commodity prices in the future.

3.3 The binomial decision tree model

RO is completely different from DCF analysis, which is well established in the mining industry. The RO approach is a new paradigm that requires engineers and project managers to view uncertainties as opportunities for value creation; however, quantifying the values that can be derived from the available options can be a daunting task. The mathematical complexity involved in valuing RO has been the main hindrance in its adoption, as well as its application in solving problems related to real world projects. Like any other decision-making tool, RO uses available mathematical and decision analysis techniques. The binomial decision tree model is one technique used in valuing RO. As concluded by Brandao et al. (2005), the binomial decision tree model is the most popular, intuitive and transparent method used so far in RO analysis.

3.3.1 Assumptions of the Model

The best model for tracing the evolution of using key underlying variables to create project flexibility over a discrete time is the Cox, Ross and Rubinstein binomial tree model (Cox et al., 1979). This model was intended as a numerical procedure to solve the Black-Scholes equation.

There are two main assumptions underlying the binomial tree. The first assumption is that of a continuous random walk (Eq. 3.1). This assumption allows the tree to be modelled by a discrete random walk with the following properties $t_0 = 0$, *t0= 0*, $t_0 = \Delta t$. The price of the underlying asset S changes only at discrete times $t_1 = \Delta t, t_2 = 2\Delta t, t_n = n\Delta t = T$, where T is the expiration date of the option and $\Delta t = \frac{T}{n}$ denotes the one time step.

If the price of the underlying asset is $S_{n,i}$ at i and time t_n, then at the next time step it may take only one of two possible values:

$$Sn+1, i+1 = uSn, i>Sn, i \text{ or } Sn+1, i+1 = dSn, i<Sn, i \quad 3.1$$

This is equivalent to assuming there are only two returns possible at each time step $u\text{-}1>0$, $d\ \text{-}1<0$, where u is the up movement and d is the down movement, and these two returns are the same for all time steps. The probability, p of Sn,i moving up to $Sn+1,i+1 = uSn, i$ is known. The same results for the probability q of Sn, i moving down to $dSn+1, i$, since $p+q = 1$.

The second assumption underlying a binomial tree is that of risk-neutrality. This implies that the investor risk preferences are irrelevant to option valuation. This has two implications.

First, the expected return from all traded securities is the risk-free interest rate, rf. This means that the drift term u in the stochastic differential equation for the asset return (Eq. 3.2) is replaced by the risk-free interest rate rf whenever it appears, and Wt is a standard Wiener process:

$$\frac{dS_t}{S_t} = rdt + \sigma dWt \quad 3.2$$

Second, the option value V_n, at $t_n = n\Delta t$ is its expected value E, at $t_{n+1} = (n+1)\Delta t$, discounted by the risk-free interest rate r_f:

$$V_n = E^{[\exp(-r_f \Delta t) V_{n+1}]} \quad 3.3$$

As per this approach, the probabilities p, q, and the returns u, d should reflect the important statistical properties of the continuous random walk, meaning they have to ensure that for $\Delta t \to 0$ the underlying asset S follows the geometric Brownian motion (GBM). In short, the parameters p, q, u, d should give the correct values for the mean and the variance of the underlying asset, which is shown in Eq. (3.4) during a time interval Δt, where $\sigma^2 \Delta t$ is the variance parameter and lnS_n is the conditional distribution:

$$lnS_{tn+1} \approx N(\ln(S_n + \left(b - \frac{\sigma^2}{2}\Delta t, \sigma^2 \Delta t\right) \quad 3.4$$

Consequently, these parameters must solve the following equations:

$$p + q = 1 \quad 3.5$$

$$E = pln(uS_n) + qln(S_n) = \ln(S_n) + (b + \frac{\sigma^2}{2})\Delta t \quad 3.6$$

$$P(ln(uS_n) - E) + q(ln(dS_n) - E)2 = \sigma^2 \Delta t \qquad 3.7$$

Substituting $q = p - 1$ in Eqs. (3.6) and (3.7), there are three unknown parameters and two non-linear equations to solve. To obtain a unique solution, a supplementary restriction for the parameters is needed. Variable b is the option carry cost and is equal to r_f. Cox et al. (1979) chose the restriction $ud = 1$, as it drastically simplifies the tree. At time point tn there are only $i = 1, \ldots, n + 1$ possible nodes and:

$$S_{n,1} = undn - iS_{0,} \qquad 3.8$$

Where S_0 is the asset price in t_0. Solving the Eqs. (3.6) – (3.8) for p, u and d and neglecting the terms smaller than Δt results in:

$$p = \frac{1}{2} + \frac{1}{2}(b - \frac{1}{2}\sigma^2)\frac{\sqrt{\Delta t}}{\sigma}, \quad u = \exp(\sigma\sqrt{\Delta t}), \quad d = \exp(-\sigma\sqrt{\Delta t}), \; u = \frac{1}{d} \qquad 3.9$$

Therefore:

$$\text{for continuously compound growth} \qquad p = \frac{e^{r\Delta t} - u}{u - d}$$

$$\text{for annually compounded growth} \qquad p = \frac{1 + r - d}{u - d} \qquad 3.10$$

The time steps are of equal length, so that the risk-neutral probability p (as calculated by Eq. 3.9) is the same at each node. The option price $Vn,i = V(Sn,i, tn)$, at node i and time tn is the expected pay-off at $tn+1$ discounted at the risk-free interest rate:

$$V_{n,i} = E^{\exp(-r_f \Delta t)[pV_{n+1,},i+1+(1-p)V_n,i+1]} \qquad 3.11$$

Simply, the value of the uncertain variables, the uncertain price value in the next period, $Vt+\Delta t$ is equal to the value of this period Vt multiplied by the continuous growth rate r for an interval Δt. The growth rate r is a random variable that is normally distributed with constant expected growth ($\hat{r}$) and constant standard deviation σ:

$$V_{t+\Delta t} = V_t e^{r\Delta t} \qquad 3.12$$

$$E(V_{\Delta t}) = V_0 e^{r\Delta t} = puV_0 + (1-p)dV_0 \qquad 3.13$$

$$e^{r\Delta t} = pu + (1-p)d \qquad 3.14$$

At the end of the tree, the option price is known. It equals the option value at expiration. For a call option, this is:

$$VN,I = max(0, S_{N},I - K),\ k = 0,\ldots.,n \qquad 3.15$$

Where I is the capital investment and K is the exercise price. The option values at each node, $Vn,i,\ i = 0,\ldots,n$ and $n = N - 1,\ldots,0,$ will then be determined recursively by working backwards through the tree.

Finally, the binomial decision tree model, which is the proposed method for RO analysis in this study, will be applied following a modified four-step process (Fig.3.1) developed by Copeland & Antikarov (2003), and advanced by Brandao et al. (2005). An alternative approach which would also have been used instead of the decision tree analysis is a stochastic simulation process such as Monte Carlo simulation. This method would have produced pretty much similar results but with less intuition. The Black – Scholes option model would also have been used but its application was limited as this method requires options that are European style which can only be exercised at a specified date. The proposed binomial decision tree model is summarised in Fig. 3.1 below.

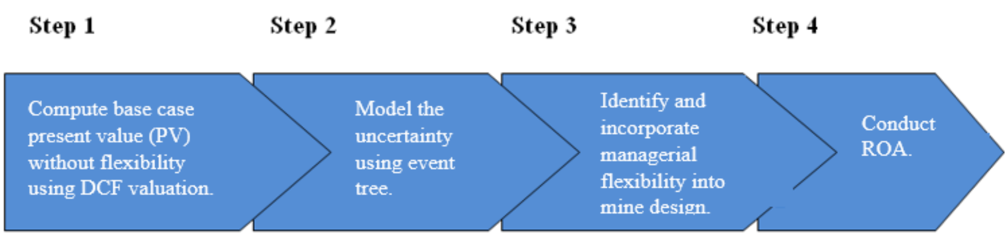


Fig. 3.1, Binomial model application steps (source: Copeland & Antikarov, 2003).

3.4 Implementation of the proposed methodology

Real case mine site data has been used to demonstrate the applicability of the proposed method. The mine is an open pit iron ore operation located in Western Australia (WA). The operations extract 15Mtpa ore and waste materials and process 5.1Mtpa at the mill. The mine production plan is based on high grade (HG) blocks that are contained in the optimised ultimate pit shell using L-G algorithms. Mine exploration has recently developed geological block models of the deposit and categorised the material into 'HG', 'low grade' (LG) and 'waste blocks', as summarised in Table 3. The LG blocks are considered valuable but will not be crushed for processing.

Based on the given information, several scenarios were developed during the bankable feasibility studies (BFS). The most profitable option was determined as developing the mine as one large pit mined bench-by-bench to comply with the proposed design. The processing plant circuit design allows for future modifications. Therefore, the management team is optimistic that LG ore can be concentrated in future.

Management recognises that iron ore prices are currently volatile, and the ore grade will remain uncertain. LG ore will be mined sequentially in the plan and processed at the end of the mine's life once the HG ore has been exhausted; however, these decisions are subject to annual revision, in response to new information. The capital and operating costs were estimated based on currently available mining industry data from individual analysts and research organisations. Some of the sources used to estimate the capital and operating cost include: Price Waterhouse Cooper (Buckeridge et al., 2010), Infomine (InfoMine, 2013), R2Mining (R2Mining, 2013), West Pilbara Iron Ore Project (API, 2010 and Aquila Resources, 2012), Apurimac Iron Ore Project (Strike Resources, 2013), Metals Economics Group (2012), Buckland Project (Iron Ore Holding, 2013) and McNab et al. (2009). According to this information, the current mine design requires a capital cost of about $909.5 million, with a discount rate of 13 per cent for its development (Cambridge Economic Policy Associates Ltd, 2013).

A graphical representation of the current production strategy was obtained by getting a maximum pay-off, which is the difference between the price and cost of producing a tonne of iron ore. This policy states that LG ore can only be processed if the iron ore prices are above $100 per tonne (/t) (Fig. 3.2). The annual operating cost, as indicated in the cost section above, is $56.77/t of SOP. This assumes that only HG ore above the 56 per cent Fe cut-off grade is fed to the crusher. LG ore will be mined and stockpiled with the possibility of processing it at the end of the mine's life, as long as iron ore prices stay above $100/t if the pit is developed based on the initial recommendations, the mine will be closed when the iron ore prices drop below the breakeven point. It must be noted however that other quality parameters such as alumina, silica, phosphorus, manganese and sulphur which affect the cut-off grade decisions are assumed to have been agreed upon between the operations and customers and that their cut-off grades will not change with any change in commodity price.

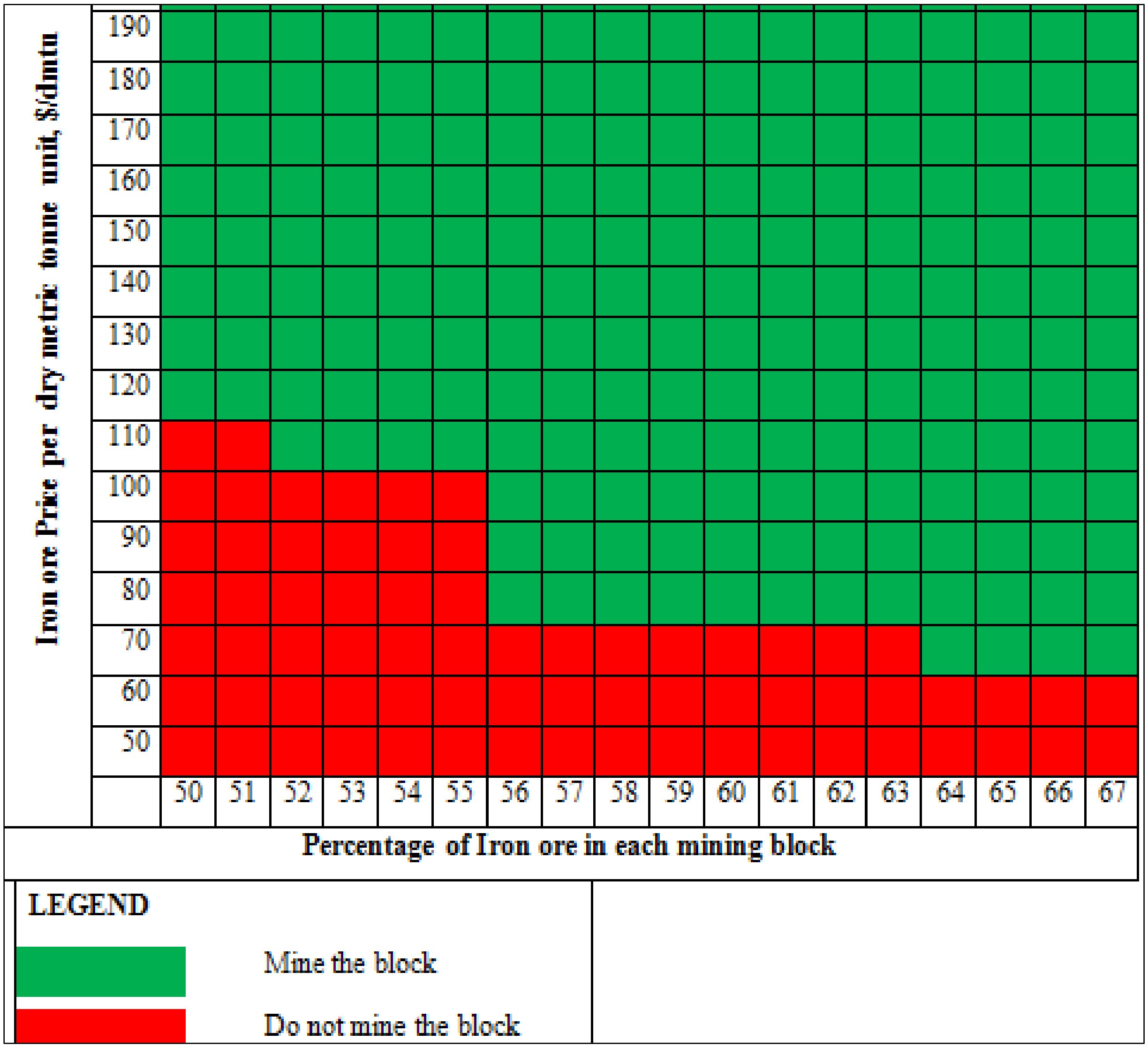


Fig. 3.2, Graphical representation of the real case iron ore mine production strategy.

3.4.1 Project uncertainties

The two major sources of uncertainty relevant to the mine project are the market price for the extracted mineral and the mineral content of blocks in the pit. Anecdotally, there is a strong correlation between the commodity price and cut-off grade. Any commodity price change has a huge influence on the ore cut-off grade. The formation of the mineable deposit and quality of the head grade are dictated by the nature and genesis of the deposit's geological structures in the area. However, a decision to mine a particular ore grade is a function of the price. Management's problem is to set up the operation in such a way that it is resilient and can deal with unfavourable global occurrences.

3.4.1.1 Iron ore price forecast and market outlook used for a real case iron ore mine project

As seen in the section on price forecast, the iron ore price can fluctuate, resulting in either an upside state or a downside state. They both affect the profitability of the mining project. These movements can be sudden, due to exogenous factors; however, the inability to predict commodity prices with any degree of accuracy presents an opportunity to managers and mine planners to seize the options that generate additional value during uncertainty. It is also a fact that during periods when commodity prices are high, mines increase production to sell the most mineral product possible during these lucrative periods. When prices are low, mine operators reduce production, anticipating that prices will rise again in the future. This logic will be used in the analysis of the case study to reflect mine management behaviour.

In the first quarter of 2013, when this iron ore mine project was being considered, the effects of the 2008 global financial crisis (GFC) were still evident in the Australian economy and around the world. The demand for commodities was easing and the general outlook was gloomy in China, Australia's major commodity export market.

Forecasting commodity price movements has been described as something of a 'black art' (FitzGerald, 2013). If a prediction is wrong, then the best laid plans can come undone quickly (Anderson & Greber, 2013). To underscore the level of uncertainty involved in forecasting and predicting iron ore prices, a number of iron ore price predictions (as reported by various news outlets and research organisations) have been outlined (Papadakis, 2013 and Treadgold, 2013). A chart of the actual iron ore spot prices between 2009 and 2013 is also included (Fig. 3.3).

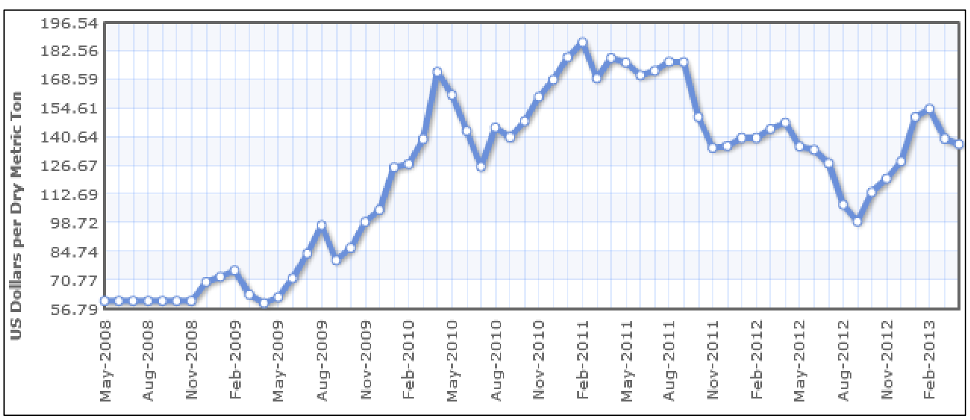


Fig. 3.3, Five years iron ore prices per tonne of mines with 62% Fe (source: indexmundi.com).

Many analysts have predicted that the long-term average price for bench mark 62 per cent iron ore would stay above $120 if the Chinese iron ore producers remained in business (Els, 2012). However, there is recognition that iron ore prices are already 30 per cent lower than their peak in 2010–11 (Macro Business, 2012).

Similarly, Bob Kohut of the Bull Daily Newsletter (2013), wrote a comprehensive report on iron predictions that varied widely between \$50/t to \$130/t. The summary of the price prediction used to calculate the value of the iron ore mine project is shown in Table 3.2.

Table 3.2, Price prediction used to forecast the mine cash flows (source: Kohut, 2013).

	Year														
Predictor	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027
Australian Government 2013/2014 Budget Papers	110	110	115												
Australian Miners	125	100	85	85	85										
Credit Suisse and UBS			87	87	80										
Goldman Sachs		126	95												
Deutsche Bank		155													
Morgan Stanley	133					These years prices were filled by calculating rolling averages.									
Bloomberg News polled					96										
BREE					109										
Andy Xie	50														
Predicted Average Prices used in the planing of XYZ Mine	104.5	123	95.5	86	92.5	100.3	99.4	94.7	94.6	96.3	97.0	96.4	95.8	96.0	96.3

3.4.1.2 Ore grade

Product head grade, which determines the quantity of SOP, has always been uncertain and will continue to be uncertain for this iron ore mine. Cut-off grades are raised when prices are low, as the revenue generated from milling the rock is not sufficient to offset the processing and selling costs. If grades turn out to be better than expected (the upside state occurrence), then some LG ore can be fed into the crusher to blend in and increase production. The upside state presents an opportunity to mine both high and low stripping ratio pits to bring down grades to the planned level.

When grades are less than expected (the downside state), the target grades cannot be supplied as required by the market, affecting mine production quantities. This also presents an opportunity to switch to low stripping ratio pits. For the iron ore mine, the sequence in which blocks are mined is predetermined during the short and medium-term mine planning processes, avoiding complications related to slope constraints.

3.5 Traditional versus flexible mine design

The management (of the case study mine) has realised that the existing design is not resilient enough to withstand unfavourable future events. Therefore, they must decide whether to mine with the existing design or use a flexible design that will provide options to switch between low stripping (SP1) and high stripping ratio (SP2).

Mine managers must also decide whether to invest the additional capital required to create flexibility. This includes options to put the two pits into production mode, processing LG by adding a bypass circuit to the processing plant. Creating these options has been costed at approximately $1.13/t SOP ($59.3 million).

Allowing a design switching option for SP2 pit attracts an additional fixed cost of US$62.63/t. This includes an operating cost of US$5.86/t SOP for processing LG, with a concentrator in the circuit and US$4.5/t of total material moved (TMM), based on research by McNab et al. (2009). The SP1 switching option results in a net annual cost saving of US$37.7 million. This implies that when prices are low, the mine operations will have the ability to defer waste by moving approximately 6.6Mtpa of material to achieve 5.1Mtpa of SOP, compared to the base case that recommends a constant TMM at the rate of 15Mtpa, to achieve similar production targets. The SP2 pit alternative increases the project value by five per cent on the upside state as this is an expansion option which allows the mining of simultaneous pits by being in both low and high strip ratio pits at the same time. However, the SP2 pit has a higher volatility than the SP1 pit, due to its high fixed costs.

Table 3.3 below summarises the measured Ore Reserve quantities for both the existing and the flexible design. For the existing design, the mine can be developed as designed, based on a traditional methodology; it will close at any time if the changes in commodity prices or fluctuations in the ore grades make it unviable. Implementing the existing design implies that the operations have to mine two tonnes of waste for every tonne of ore, without any flexibility to defer waste, regardless of price fluctuations. The flexible design has split the pit into zones of high and low stripping ratios (SP1 and SP2). This provides the operations with options to defer waste by mining SP1 when prices are low, or mine SP2 when prices are high. Thus, the flexible design is a loss minimiser and value maximiser.

Table 3.3, Ore reserve for the real case iron ore mine in millions of metric tonnes.

	Existing Design	Flexible Design	
	Large Single Pit	Stage Pit 1 (SP1)	Stage Pit 2(SP2)
High Grade >56% Iron (Fe)	77	32	45
Low Grade (LG) , 50% <LG<56% Fe	51	24	27
Waste ,< 50% Fe	96	17	68
Strip Ratio (Waste:Ore)	1.9:1	0.9:1	2.1:1

3.5.1 Step 1: Computing base case PV without flexibility using DCF valuation

The expected present value (PV) of the project at Time 0 will be determined using traditional DCF analysis, without consideration of any managerial flexibility. The project's free cash flows will be discounted using a risk adjusted discount rate, subsequently determining the project's PV and NPV. An insert of the spread sheet used for DCF analysis is shown in Table 3.4.

Table 3.4, Spread sheet used to estimate project value.

Period in Years	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Price ,$/tonne		105	123	96	86	93	100	99	95	95	96	97	96	96	96	96
Quantity (Mtpa)		5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1	5.1
Operating Cost/tonne		56.8	56.8	56.8	56.8	56.8	56.8	56.8	56.8	56.8	56.8	56.8	56.8	56.8	56.8	56.8
Revenue		536	626	487	439	472	511	507	483	482	491	495	492	489	490	491
EBIT		243.4	336.5	197.5	149.1	182.2	221.7	217.4	193.6	192.8	201.6	205.4	202.2	199.1	200.2	201.7
Capital expenditure	909.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Corporate Taxes (30%)		73.0	100.9	59.3	44.7	54.7	66.5	65.2	58.1	57.8	60.5	61.6	60.6	59.7	60.1	60.5
Free Cash Flow (FCF)	0.0	170.4	235.5	138.3	104.4	127.6	155.2	152.2	135.5	135.0	141.1	143.8	141.5	139.4	140.1	141.2
WACC Discount factor (13%)		0.885	0.783	0.693	0.613	0.543	0.480	0.425	0.376	0.333	0.295	0.261	0.231	0.204	0.181	0.160
PV of FCF	987.5	150.8	184.5	95.8	64.0	69.2	74.6	64.7	51.0	44.9	41.6	37.5	32.6	28.5	25.3	22.6
NPV	78.0															

The approach here has benefited from using a lognormal distribution for the possible values of the underlying asset. Lognormal distribution was the best model that fitted this analysis because the value of the two variables being analysed will never go negative. There are no negative prices for commodity and no negative cut-off grade values. Additionally, this assumption applied a lower mean than a normal distribution, thus eliminating over-optimistic estimations, which are eliminated. Furthermore, up or down percentage price movement is equally likely in this distribution.

Without flexibility, the project would be accepted, as the NPV is positive. The calculated NPV of US$78 million signifies that the project will earn sufficient free cash flow to meet its obligations and also pay the capital gain expected by the investors. This NPV is only achievable if the expected price per tonne of iron ore stays between US$86– US$123, and the mine produces ore grades of 62 per cent Fe for the rest of its life.

3.5.2 Step 2: Model the uncertainty using event trees

In this step, the volatility of the return or the PV of the project will be estimated using the Monte Carlo simulation. Even though the ore grade was identified as an uncertain variable in the preceding sections, it will not be used in the simulation, as it is assumed that the operations would only mine HG ore with within the cut-off grade of 62 per cent Fe (see Step 1). The project’s key uncertainty, the price of the commodity is entered into the ‘@risk excel add in program’, and then the simulation is run.

3.5.2.1 Stochastic inputs

For the simulation, a lognormal distribution will be applied. Lognormal distribution is a continuous probability distribution of a random variable, whose logarithm is normally distributed.

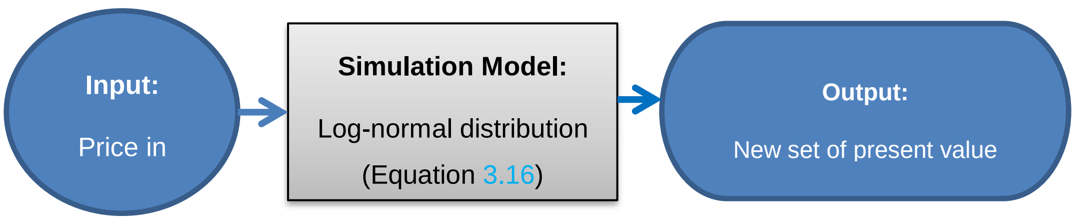
A variable modelled as 'lognormal' is a multiplicative product of many independent random variables, each of which is positive.

The probability density function of a lognormal distribution is:

$$f_x(\mathrm{x}; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{\frac{-(lnx-\mu)^2}{2\sigma^2,}}, where\ x > 0 \text{ and X=e}\mu\text{+}\sigma\text{z} \quad 3.16$$

In a lognormal distribution X, the parameters denoted as μ and σ are, respectively, the mean and standard deviation of the variable's natural logarithm (ln), e is the Euler's constant = 2.718 and z is the a standard normal variable (z-score) and x values must be greater than zero. By definition, the variable's logarithm is normally distributed.

Thus, simulation input is:



The simulation returned a new set of PVs using the formula:

$$V_i = \sum_{i=1}^{n} \frac{c_i}{(1+u)^{i-1}} \quad 3.17$$

Where V_i the project value at period i among n periods, c is the expected free cash flows and u is the weighted average cost of capital (WACC). From the simulation, a random sample of variable z can be determined using the following relationship:

$$z = \ln(\frac{v_i}{v_0}) \quad 3.18$$

Where z is the mean of the distribution of the project returns between Time o and Time i. The estimate of the standard deviation of z, denoted as σ, is obtained from the simulation results, which is the annualised percentage standard deviation of the returns and is estimated from the relationship:

$$\sigma\sqrt{\Delta t} \quad 3.19$$

Where t is the length of the period in years used in the cash flow spread sheet. After 10,000 iterations using the iron ore parameters calculated at the price prediction section above, the volatility of the project PV at time t_0 is 12.7 per cent, but the annualised volatility over the life of the project is 40.3 per cent. The volatility of the project's PV is significantly higher than the volatility of the iron ore price, which is 29.2 per cent.

This proves that the observed volatility of the input variables driving uncertainty is not the same as the volatility of the project. However, it is worth noting that the switching options are special case options, consisting of two operation modes. Therefore, the volatilities of the two pits were estimated separately, with SP2 pit having a volatility of 36 per cent and SP1 pit a volatility of five per cent. The frequency distribution of the annual rate of return (which is shown as project values in Fig. 3.4) was used to generate the project volatility produced as an output of the @risk program.

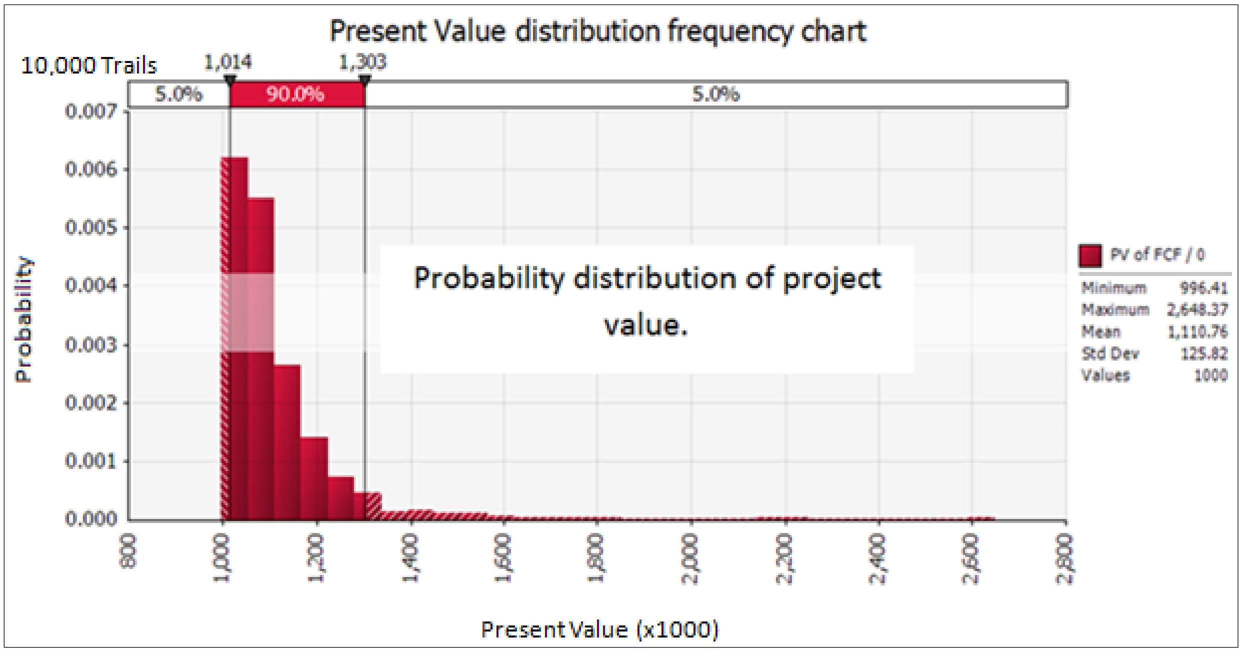


Fig. 3.4, Frequency distribution of the project values.

As the project's volatility has been estimated, the project event tree can now be used to model the project's uncertainty. The question that should be asked by the mine iron ore operations management is, that given a choice between two pits—SP1 and SP2—which one should they choose? The choice of any pit is based on the NPV each pit provides. Such a problem can effectively be solved with a structured decision-making process that requires applying analytical software. A spread sheet with embedded macros that loop through periods was used to construct the project event trees for the two pits. Figures 3.5a and 3.5d show the SP1 and SP2 project value event trees.

3.5.3 Step 3: Identifying and incorporating managerial flexibility to create value

In this step, the project event trees will be transformed through incorporating managerial flexibility into the design of the iron ore mine. The managerial flexibility available to the iron ore mine operations is the ability to switch between SP1 and SP2 pits by paying the exercise prices. As mentioned earlier, SP2 has higher fixed costs than SP1; thus, it is more volatile. These volatilities, as calculated in Step 2, were incorporated into the event tree spread sheet. This new information has altered the risk characteristics of the project; therefore, the cost of capital has changed, producing a new PV of the project, with flexibility (Fig. 3.5a and 3.5d).

As shown in Fig. 3.5b and 3.5c, the management of the iron ore mine operations needs to identify the optimal switching behaviour between the SP1 and SP2 pits for each possible sequence of the global situation. Solving the decision tree starts from the end and works backwards. The mine operations management should ask the following two questions twice at each node:

I. If the mine was operating in SP1 pit at the previous state, would the operations stay in SP1 or switch to SP2, and pay the switching cost.

II. If the mine was operating in SP2 pit at the previous state, would the operations stay in SP2 or switch to SP1, and pay the switching cost.

As switching comes at a cost, the optimal operating pit at a given state depends on the pit that was operating at the previous state, and on the optimal operating pit in the possible following states. As outlined by Copeland & Antikarov (2003), switching will only occur if the switching criteria have been met. An application of these criteria to all nodes in the decision trees resulted in the following optimal decisions:

If mine operations are currently in SP1 pit and the wish is to switch to SP2:

- $SP1_{Node\ i} = Max(PV_{SP1\ node\ i}, PV_{SP2\ node\ i} - switching\ Cost)$
- If $PV_{SP1\ node\ i} > (PV_{SP2\ node\ i} - switching\ Cost)$, then stay in SP1, otherwise switch to SP2

If mine operations are currently in SP2 and the wish is to switch to SP1:

- $SP2_{Node\ i} = Max(PV_{SP2\ node\ i}, PV_{SP1\ node\ i} - switching\ Cost)$
- If $PV_{SP2\ node\ i} > (PV_{SP1\ node\ i} - switching\ Cost)$, then stay in SP2, otherwise switch to SP1.

3.5.4 Step 4: Conducting RO analysis

At this step, the whole project will be valued using a simple algebraic methodology using the values calculated. As shown in Step 3, two ROs have been developed as a result of valuing the flexible iron ore mine design, depending on the starting pit.

At the start of the mine development, iron ore mine operations would have been either in SP1 or SP2 pit. As a result, of the two sets of optimal switching strategies, their corresponding decision trees for each possibility have been developed.

The ability to switch between SP1 and SP2 implies that, if the operations are SP1 pit (Fig. 3.5b & 3.5c quadrants and Fig. 3.6), the management has the option to buy the PV provided by the SP2 pit for an exercise price equal to the switching cost from SP1 to SP2 pit, and vice versa.

It can be deduced that if the two-stage pits are developed and in operation, management will have the ability to mitigate losses in response to low mineral prices or changing ore grades, or alternatively to maximise gains by incurring the cost of exercising these options. If management predicts potential for price increases in future, they can expand, and vice versa if the future appears gloomy.

However, if there is potential for price increases in the future, they can increase production capacity and incur the cost of processing LG ore and moving extra material. Switching to SP1 implies that when prices are low, the mine operations will have the ability to defer waste by moving HG material, which can offer some cost saving, creating value that would have been lost. Thus, the RO expression for the analysis of the project's PVs is:

RO_{SP2}> *ROSP1> Base Case PV*

$1131.5million >1074.5million >$987.5million

The analysis indicates that the option value for the flexible design is higher than the value of mining any pits independently, without switching options. This implies that if this option is available to the mine management, and is used optimally by the mine operations, the flexibility provided by switching between pits would be valuable.

67

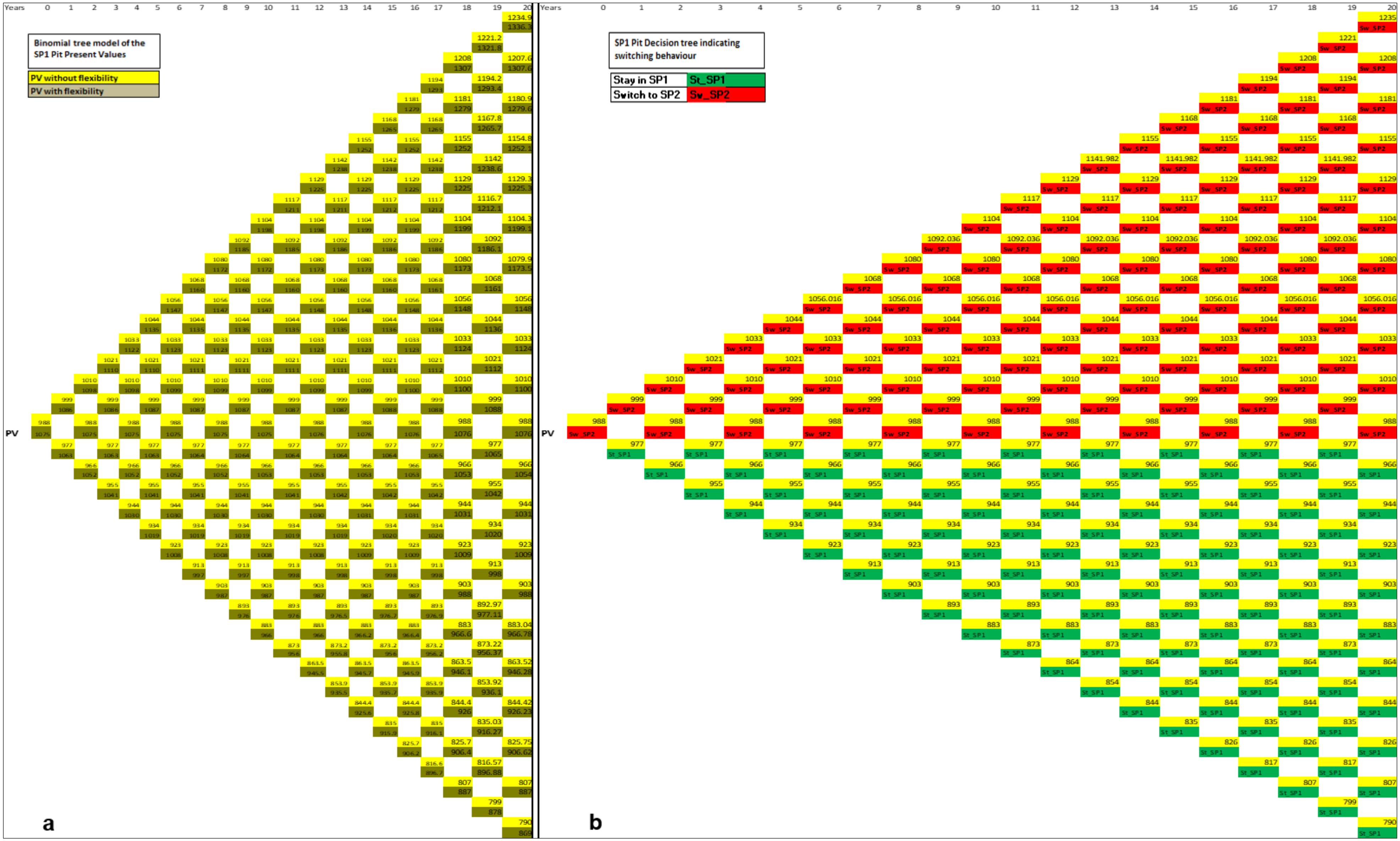


Fig. 3.5, Binomial decision trees for RO analysis in iron ore operations

68

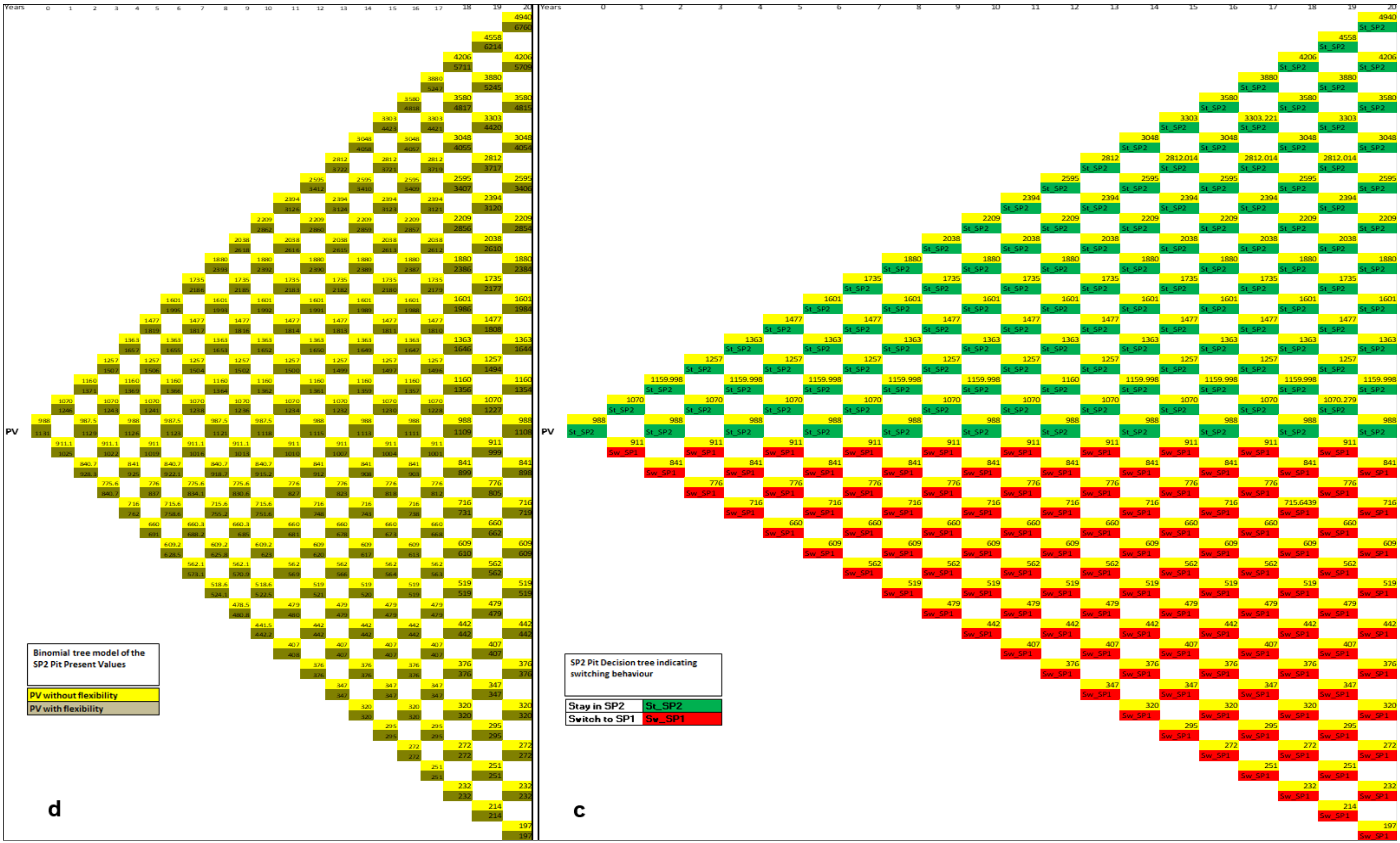


Fig. 3.5, Binomial decision trees for RO analysis in iron ore operation.

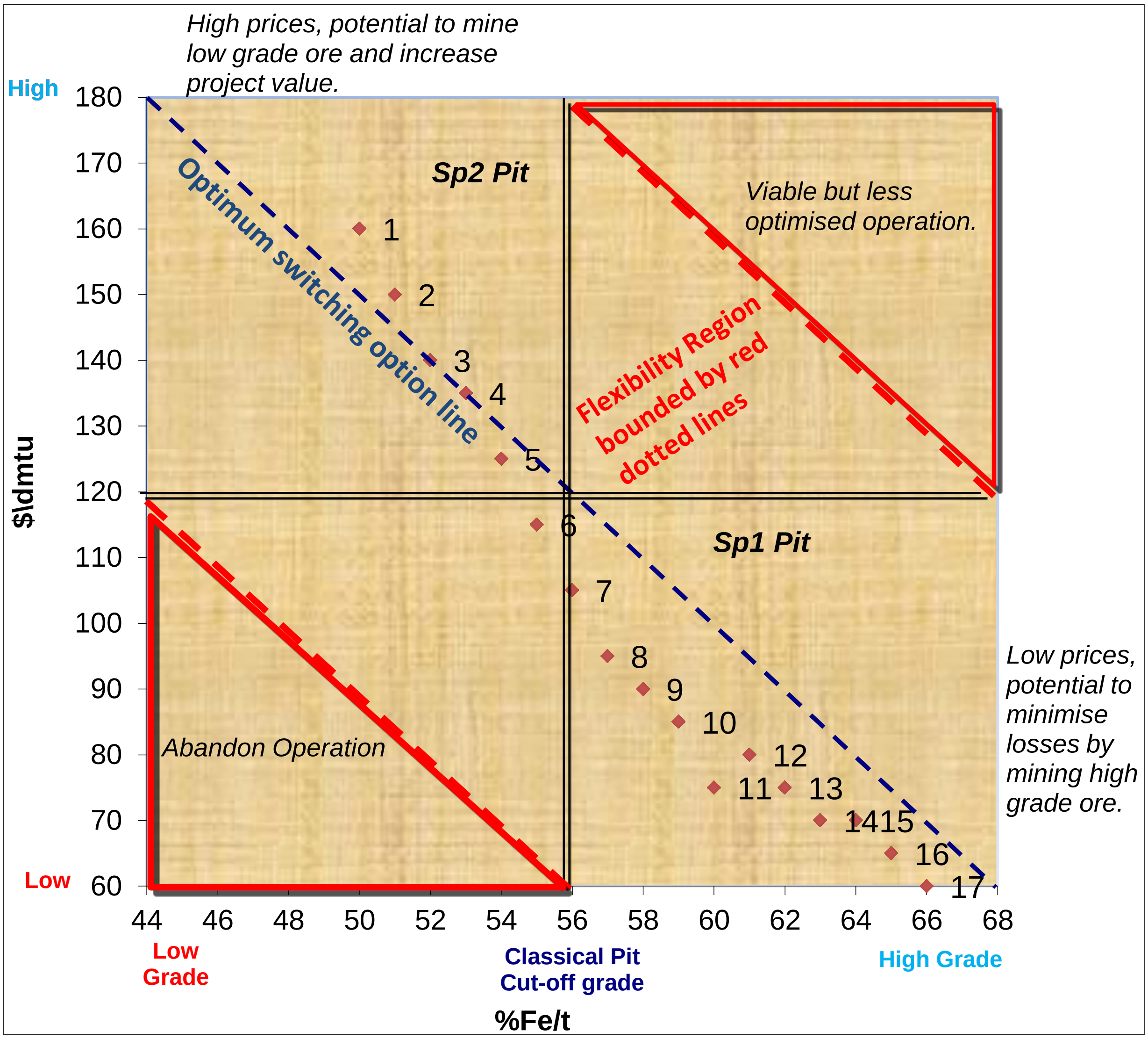


Fig. 3.6, Proposed strategies for flexible operation.

3.6 Result analysis

During this study, the option to switch, and the operating strategy for the real case iron ore mine were identified using RO analysis. Using decision tree analysis has been simultaneously intuitive and visual. It allows project uncertainties to be considered without complicating the analysis. When compared to DCF analysis, RO analysis gives a better estimate of the project value, and it also considers operating environment dynamics. The analysis includes the base case PV, without flexibility plus the option (flexibility) value.

Under high uncertainty, and with the managerial flexibility incorporated into the design, the option value was substantial. As seen in the analysis in the previous sections (Fig. 3.5), the SP2 pit is more uncertain than the SP1 pit. As calculated, the volatility of the SP2 pit was 36 per cent compared to five per cent for the SP1 pit. The SP2 pit volatility was compounded by its multi-zoned ore deposit, more than the SP1 pit, which predominately lies in a HG zone. This means that the mining blocks in SP1 pit were highly likely to be HG blocks whenever they were excavated. Therefore, its fixed costs are lower than those of the SP2 pit. These differences in the fixed costs were reflected in the real case iron ore mine switching behaviour analysed using the decision trees in Fig. 3.5.

As mentioned earlier, the main decision criteria that mine operations management use will be the NPV values. Therefore, the NPV of the flexible design has to be calculated to determine if the flexibility value is adequate, such that it can justify additional investment to create flexible mine designs, by splitting the base case pit into several stage pits and also investing in processing LG ore. During the RO analysis, the expression NPV = PV – Investment was used to obtain various NPV values for each design. Note that all monetary values are in millions of US dollars.

The RO analysis has revealed that switching to the low cost SP1 pit was always the preferable option, whenever there are drops in iron prices. The design with switching options has greater NPV values of $162.7 million for the SP2 pit, and $105.7 million for the SP1 pit, than the base case design, which returns the NPV value of $78 million (Table 3.5).

Table 3.5, Comparison of the NPV for classical pit and flexible design.

Classical Pit Design:	**Flexible Design:**	
Base Case Pit PV = $987.5 *Base Case Pit Investment = $ 909.5* *Base Case Pit NPV= $78*	*SP1 Pit PV = $1074.5* *SP1 Pit Investment = $968.8* *SP1 Pit NPV = $105.7*	*Pit PV =$1131.5* *Pit Investment = $968.8* *Pit PV = $162.7*

Designing the mine with a flexible switching option is the robust choice, regardless of the initial operating pit chosen. However, the mine management can create a larger value if it begins operations in SP2 pit. As showed in the analysis, the obtained option value is an optimal contingent plan for executing the available options. It is also assumed in the analysis that the movement of heavy mining equipment like shovels and excavators during the switch will not be very different from normal relocation of equipment within a single or multiple pits similar to the current mining practices where equipment are walked or floated around and the associated cost is included in the production cost.

If the real case iron ore mine management chooses the traditional DCF analysis and bases its future decisions on the NPVs obtained using this method, substantial value would be put at potential risk; this value is equated to the value of flexibility. A graphical representation for the mine operations value at risk is shown in Fig. 3.7.

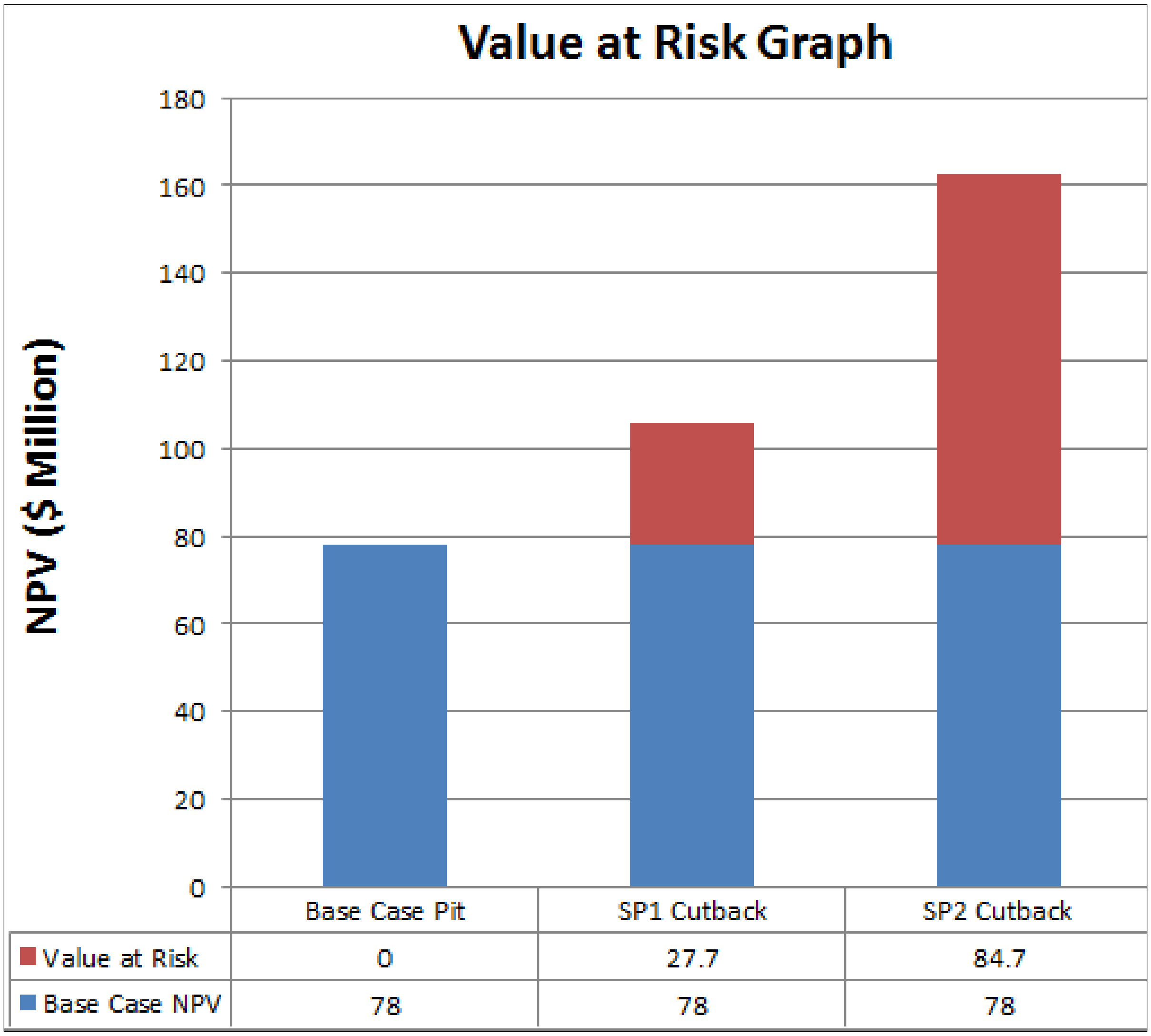


Fig. 3.7, Value at risk for the real case iron ore mine.

In the analysis, the assumption that the commodity prices follow a random walk process was used, allowing use of the GBM model. As the switching options are very different from the rest of the options, each operating pit was modelled independently. This allows the management to choose when to switch and when to maintain the status quo. The switching option model was the most practical RO technique, fitting the desired flexibility for this particular real case mining operation. However, it should be acknowledged that single period or short-term project values could have been valued with DCF analysis. As calculated, the presented RO framework helped to identify options not discoverable from the project description for the base case of the real case iron ore mine operations.

The analysis done in this case study was not intended to shape the firm's strategy, but to help create a design that is flexible under any global situation. Surprisingly, the RO analysis performed could also have been used to influence the firm's overall strategy. Unlike other research, which tests the application of the RO at strategic level, a major benefit of this case study was the ability to test the applicability of RO at the mine operational level. Even though the iron ore mining project was not one of the most complex, it was certainly exemplary in relation to common mining operation challenges.

3.7 Conclusion

Previous research has dwelled mainly on the strategic application of ROs to project development. Little focus has been given to its potential application at the operational level. This article demonstrates that the RO method can be applied to decision making at the mine operational level. It has demonstrated that RO can also be used to create designs that give mine management the flexibility to switch between different pits, depending on varying global situations, without facing the challenge of closing the mine, especially when prices have sunk below the marginal cost. The RO problem was solved using decision tree analysis. This approach provides a definite and flexible way of implementing RO techniques through modelling the decision problem in a spread sheet. Using a spread sheet has advantages, as it is the most common analytical computer application available to almost every person with access to a computer. Spread sheets can also be adapted to solve complex decision-making problems easily, using either macros, or when writing the visual basic for application (VBA) code.

The switching options framework used in the real case study offers a robust approach to the testing applications of ROs in engineering design, and decision making at the operational level. The analysis has shown that the project value increased considerably when flexibility was included in the mine design. These increases in the project value were between eight and 15 per cent, depending on the number of flexible options incorporated into the design. As the analysis is based on scientifically established theories, and on the relationship between uncertainties and ROs, the model as displayed in this paper may be subject to critical discussion with respect to option types and uncertainty categories. Additionally, it is very important to underscore that the switching option can be applicable in some certain mines, which are lateral or with horizontal deposits. If grade increases with depth, the switching option may not be useful in such a situation. However, the basic idea still holds. There are opportunities for other option types and uncertainties, which could easily be integrated into the model.

For the purpose of this research, the method applied to solve RO allows for an intuitive discovery process of options, reflecting the managerial decision-making process. The framework ensures that identifying the options is easy. It uses a four-step approach, which includes: computing the base case PV without flexibility using DCF valuation; modelling uncertainty using decision event trees, then managerial flexibilities are identified and incorporated into the decision tree, thus creating value; finally, the RO analysis is conducted. Similar to any other theory, RO in general, and the binomial option model in particular, depends on the analyst's assumptions and inputs. These assumptions are sometimes a mirror image of individual subjective judgements. Therefore, they may not evaluate all available information correctly, instead focussing on particular variables of interest.

In conclusion, this work has created an opportunity for future research into additional benefits that stem from backfilling or in-pit dumping opportunities which may be created by having multiple pits option. It is expected that there may be some cost savings which can be driven from reduced cycle times as a result of short haulage from the in-pit dumping and the impacts of such savings on the net present value of the operation need to be investigated in much detail.

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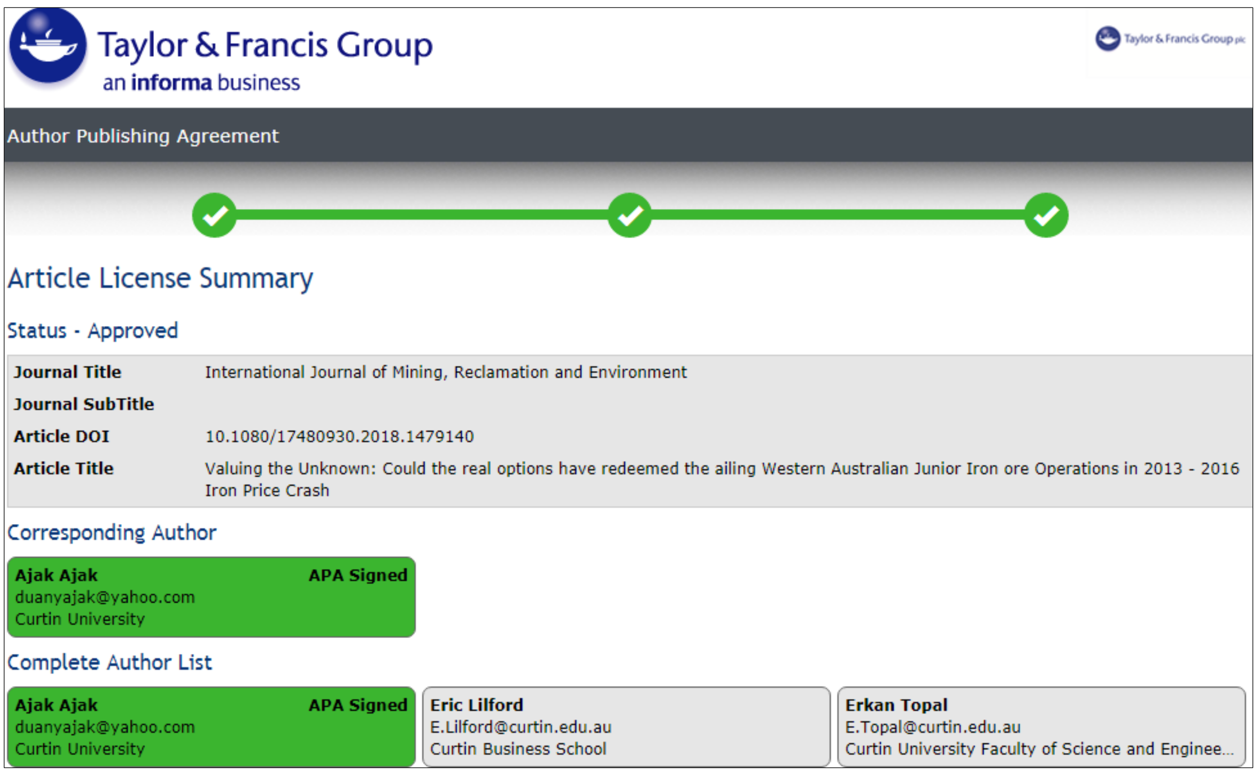
Chapter 4: Valuing the unknown: Could the real options have redeemed the ailing Western Australian junior iron ore operations in 2013 – 2016 iron price crash.



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Principal Author	**Candidate Contribution to the Paper**	**Overall (%)**	**Signature**	**Date**
Ajak Duany Ajak	Set research question, developed methodology and predictive data mining model, developed case studies and analysed real options, wrote manuscript and acted as corresponding author.	85%		25/7/18

Co-Author Contribution

By signing the statement of Authorship, each author certifies that:

I. the candidate's stated contribution is accurate as stated above;
II. permission is granted for the candidate to include the publication in the thesis; and
III. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Co-Author	**Contribution to the Papers**	**Signature**	**Date**
Erkan Topal	Supervised development of work and reviewed manuscript.		25/7/18
Eric Lilford	Supervised development of work and reviewed manuscript.		20 July 2018

Abstract

The inability of existing analytical models to accurately predict future events has, at times, led to the economic failure of mining operations whose financial viabilities were determined based on static assumptions, leaving operational managers with little room to make future decisions. Therefore, the application of a robust decision-making tool, such as real options (RO) can minimise losses and more accurately express uncertainty. This paper has considered a stochastic simulation to analyse ROs for a real case iron ore mine which closed in April 2016. In comparing the net present value from the traditional discounted cash flow (DCF) method to delay, to abandon the operations and to stage the investment options, the ROs method increased the project value by between 56% and 195% depending on the volatility. As a new contribution, a managerial flexibility domain map is proposed in this paper. Thus, flexibility in mining operations creates agility, increases value and mitigates financial losses.

4.1 Introduction

The mining industry is known to be synonymous with risk and market uncertainty. The special characteristics of any mining operation are high levels of uncertainty in ore grade estimation and volatile commodity prices (Groeneveld & Topal, 2011).

Risks and uncertainties surrounding the iron ore market have dominated a large part of media coverage. The observed level of volatility that the world has experienced in recent years is noted, with the future outlook suggesting continued volatility and unpredictability. As published in the Australia Financial Review Weekend (Stevens, 2015), managers and analysts need to imagine the unimaginable. At the start of 2015, no one had imagined that Britain would exit the Eurozone and the possibility that America would have Donald Trump as the next president.

At the time of writing this article, it was reported that more than 22 iron (Fe) ore mining operations and projects with a projected output of 140Mtpa were either suspended or cancelled between July 2014 and January 2015 (Gilroy, 2015). In addition, 130 million high-cost iron ore tonnes could not be produced for the market in 2015 and 125 million tonnes could not be produced for the market in 2014 (FitzGerald, 2016). This collective decline represents a reduction of about 13% per annum of a billion tonne seaborne iron ore supply amount. At the end of 2012, Australia produced approximately 520 Mt of iron ore with the Western Australia Pilbara region producing 97% of overall iron ore production in Australia in that year. This represents approximately 25% of the annual global seaborne iron ore trade (Haque et al., 2015a).

The inability of existing analytical models to accurately forecast future events has led to operational failures whose financial viabilities were determined through static analytical models including the discounted cash flow (DCF) analysis model which assumes constant variables throughout the life of the operation. These DCF models are rigid and operational managers have very little room to evaluate and hence make future decisions. In fact, the majority of the capital investment decisions which were made in the Western Australia iron ore industry during the iron ore boom between 2007 and 2013 were made with the assumption that the long-term average price for the benchmark 62% Fe iron ore would stay above US$120/t if the Chinese iron ore producers were to remain in business (Ajak & Topal, 2015). It has been proven that many of the mining investment decisions are commonly made based on such inaccurate information (Haque et al., 2016b).

When Andy Xie (Goncalves, 2015; Ng, 2015; Ajak & Topal, 2015), predicted iron ore prices to drop to US$50/t in 2012 (Ajak & Topal, 2015), his view was dismissed as extreme and pessimistic. However, his prediction came to pass even though the price drop did not occur at the predicted time. An important learning from this is that predictions will never be accurate or precise and that almost all of the analysts from investment banks and research institutions often get it wrong. The 2015 UK Brexit referendum was another perfect example of how decision makers should never rely solely on statistical analyses when framing operating strategies (Cowling, 2015).

Operational managers need robust methodologies for decision analysis and the real options (RO) methodology, which creates value through flexibility, is well placed to handle uncertainty (Topal et al., 2009). As stated by Jonathan Mun (Mun, 2006), ROs should not be treated as a tool but both as an analytical process and a decision analysis through a process (Mun, 2006), which combine qualitative and quantitative analyses to incorporate decision processes including strategy and risk analysis.

RO thinking on risk and uncertainty can be closely analogised with the view of risk described by Chinese calligraphy and cited by Damodaran (2014), who portrayed risk as a mix of danger and opportunity. According to Willis (2014), the three fundamental things that support any business are the opportunity, the challenge and the volatility of the operating environment. Risks are only taken by investors if they know the odds of the occurrence of the uncertain events, and this point can be bluntly quoted as 'Without risk, there is no enterprise, but no one likes to take risks without knowing the odds', (Willis, 2014, p10).

However, putting a value on the unknown has remained one of the greatest challenges of all mathematical models and particularly to the RO methodology. Lilford and Minnitt (2005) attempted this by studying various valuation processes and applying each method to a variety of South African gold projects. They concluded that any application of a specific valuation methodology depends principally on the project stage such that a single project can produce different values depending on the valuation technique.

The traditional DCF analysis, which is the conventionally accepted method, operates on fixed assumptions and usually fails to value the unknowns. The method therefore eliminates the managerial flexibility to assess future operating conditions and make decisions accordingly.

The main question being asked by this research paper is whether RO analysis can value the unknowns. In addition to determining whether it can, could it have saved the ailing junior iron ore miners in the Australian iron ore industry during the 2012 – 2016 iron ore price crash?

In this study, the stochastic simulation method was used to model the uncertain variables and outputs were applied in analysing the project's ROs. The use of stochastic simulation is justified by the future prices of mineral commodities which tend to be cyclic and fluctuate over time. The process in which prices fluctuate around a constant level or long-term mean is known as a mean – reversion process (Copeland & Antikarov, 2003). Additionally, uncertain events can sometimes occur in a time space in which a time period passes without anything happening instantaneously. The occurrence of such uncertain events can cause a jump or spike in prices and this process is known as a jump diffusion (Ni & Isaksson, 2005). The mean reverting process has been applied to selected commodities in the past but, research has demonstrated that prices for the majority of minerals do actually follow a mean reverting process (Haque et al., 2016a). Therefore, the most appropriate stochastic model to use in this study is the Monte Carlo simulation (MCS) of the mean-reversion with jump diffusion.

4.2 Creating operational flexibility in a mining project

Current investment decisions are based on static methodologies where possibilities are limited to scenario analysis with decision makers ranking the options from the best case to the worst case and choosing one option for implementation (A. T. Kearney Inc., 2014). This leaves operational managers with no flexibility during the running of the operation. The mining industry has adopted such traditional approaches in which investors make decisions to spend capital in developing a mine now in order to meet future commodity demand and benefit from potential gains early on from the fixed investment. Operational managers adhere to such static choices and obviate any room for future decisions. In such an approach, decisions are made now and the future is assumed to mimic present assumptions. Unfortunately, commodity prices have been uncertain and typically never follow the fixed assumptions, unless hedged. Such an approach has been used for decades and this was in fact the case in the Western Australian iron ore sector where investors projected continuously escalating demand and rushed to invest in the development of high-cost iron ore mines.

However, thinking about running a mining operation as a series of decisions that should be made along the way as information emerges gives managers the flexibility to proactively manage risks as well as seize opportunities presented by uncertain events. According to A.T Kearney Inc (2014), the three main ways of increasing the managerial flexibility of the strategy or systems are found in choice, timing and scope. Since RO is both qualitative and quantitative, much of the managerial flexibility value is created through qualitative thinking and processes. Therefore, this research is proposing the following structural map for flexibility domains (Fig. 4.1) to be used in the application of ROs for decision making when running a mining operation.

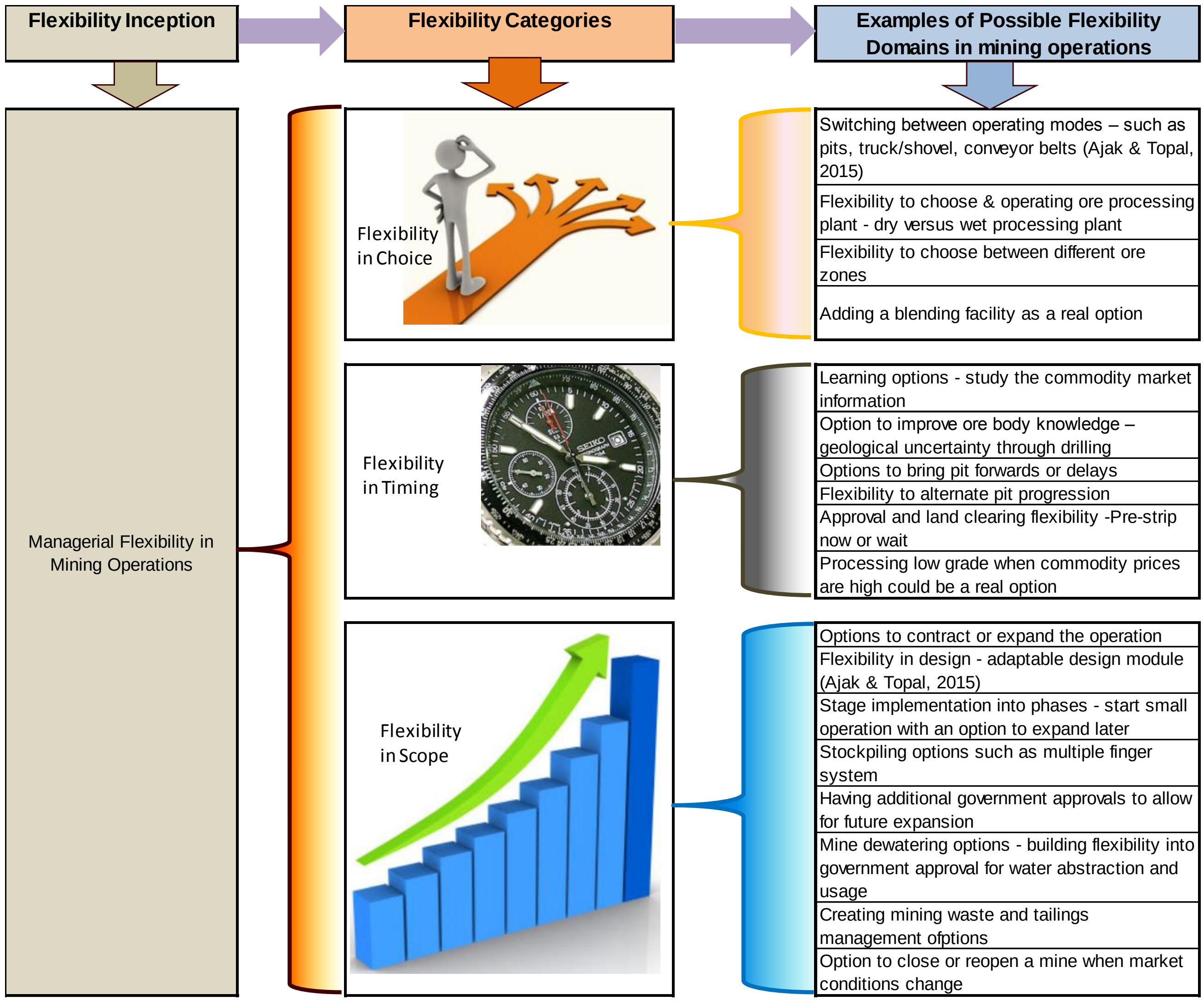

Fig. 4.1, Mapped example of flexibility domains for creating managerial flexibility in mining operations.

Apart from the summary provided in Fig. 4.1, it must be clarified that flexibility in choice is the ability to choose things such as mining method, equipment type, processing plant, haulage system, operating mode and many other choices, while flexibility in scope refers to the ability to either increase or decrease existing capacities or volumes.

4.3 Why real options for managerial flexibility?

An option is typically a security giving the holder the right to buy or sell an asset, subject to certain conditions within a specified period of time (Black & Scholes, 1973). The commonly used traditional approach in capital investment analysis is the DCF method which is used to estimate the attractiveness of an investment opportunity (Summa, 2015). The RO approach is fundamentally different from DCF in the way in which risk is treated.

The main issue with the traditional valuation approaches such as DCF is not that they are incorrect but rather are incomplete when applied to model uncertainty (Mun, 2006). DCF views risk as an unnecessary aspect that should be eliminated or mitigated at the initial stages of the project, while the RO methodology is used to create system flexibility under uncertainty (Groeneveld et al., 2010; Ajak & Topal, 2015). DCF is a discounting technique based on the principle of time preference (Centre for Social Impact Bonds, 2013). The DCF concept is measured by a real interest rate of return which indicates an individual time preference, therefore, it is an adjusted cash flow based on a discount factor, Dt.

$$D_t = \frac{1}{(1+r)^t} \qquad 4.1$$

Where r is the discount rate, t is the time period in years and D_t is the discount factor.

The DCF equation is a derivative of the future value formula above that is used in calculating the time value of money and compounding returns. DCF analysis incorporates the weighted average cost of capital (WACC) to discount future free cash flows.

$$DCF = \sum \frac{FCF_t}{(1+r)^t} \text{ or } \sum \frac{FCF_t}{D_t} \qquad 4.2$$

Where FCF is the free cash flow.

The main assumption in these equations is that the interest rate must remain constant for all periods into the future. This means that the difference between the discounted cash flow values decreases year-on-year as the time period increases. The net present value (NPV), which is the key decision criteria when deciding on various investment opportunities, is the resulting value after expended capital (I) has been subtracted from the present value, being the summation of all the periodic discounted cash flows. The relationship between DCF and NPV is shown in Eq. (4.3).

$$NPV = \sum \frac{FCF_t}{(1+r)^t} - I \qquad 4.3$$

Therefore, in general and mindful of the relationship between upfront capital expenditure and the free cash flows, the greater the NPV, the more attractive the investment and the greater the chances that it will be chosen from among other capital investment projects. The discount rate *r*, can be substituted by the WACC. The shortcomings of the traditional DCF analysis were stated by Mun (2006) and are pictorially summarised in Fig. 4.2.

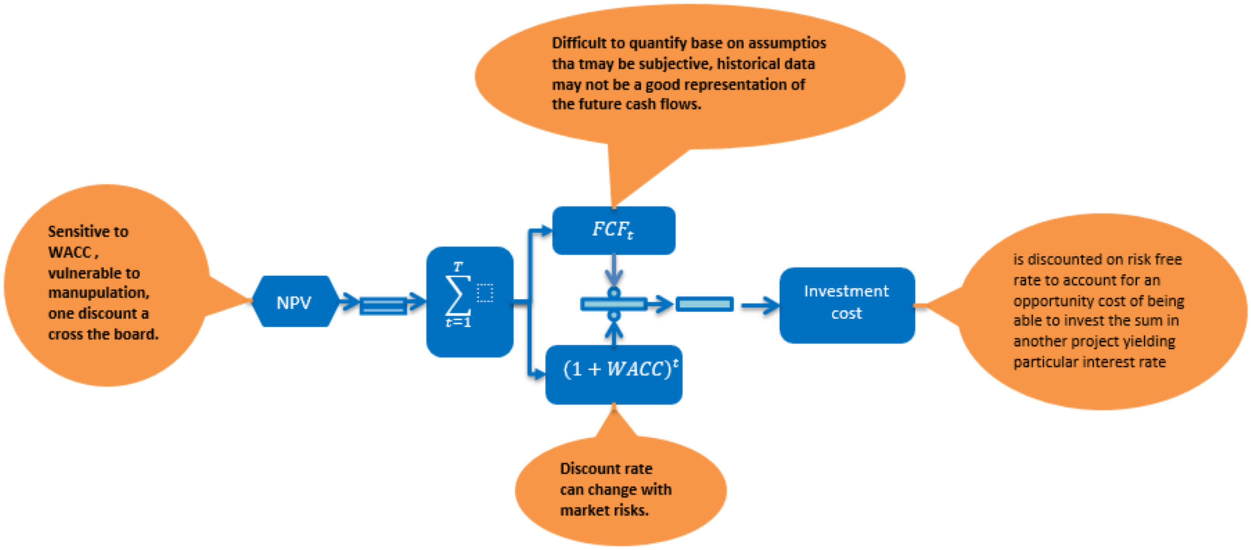

Fig. 4.2, Schematic diagram of the DCF analysis limitations as well as showing the relationship between DCF and NPV.

4.4 Stochastic process: mean-reversion simulation with jump diffusion

A process is an event that evolves over time with the intention of achieving a goal. Normally, the time period for a process is from 0 to T. During this time, events may be happening at various points along the path that may have an effect on the eventual value of the process. A stochastic process is therefore formally defined as a process that can be described by the change of some random variable over time, which may be either discrete or continuous. A stochastic process with the expression $\{Wt: 0 \leq t \leq \infty\}$ is a standard Brownian motion if $W0 = 0$. It has a continuous sample path and independent and normally distributed increments. When an independent increment has a distribution $Wt\text{-}Ws \sim N(0,t\text{-}s)$, it is then referred to as a Wiener process which is also normally distributed, $N(\mu, \sigma^2)$ with an expected value of μ and sample variance of σ. In a situation where two or more independent variables need to be correlated, a modified Wiener process applies, known as Geometric Brownian Motion (GBM) which is represented as:

$$\frac{\Delta p}{p} = \mu\Delta t + \sigma\varepsilon\sqrt{\Delta t} \qquad 4.4$$

Where ΔP is the change in the commodity price, μ is the expected rate of return, σ is the volatility of the price, $\sigma\sqrt{\Delta t}$ is the stochastic component and ε is the standard normal distribution. A generalised form of the Wiener process in which the parameters a and b are functions of the value of the underlying variable *x* and *t* is

$$x_t = x_0 + \int_0^t \alpha(X_s, s)\,\delta s + \int_0^t \sigma(X_s, s)\,\delta W_s \qquad 4.5$$

Note that it is beyond the scope of this research to derive this lemma as it has been widely described in many calculus books and the literature includes various journal articles published on this topic. However, this section will only deal with its application as it will be applied to the analysis of the RO.

4.4.1 Stochastic simulation

Two main stochastic simulation processes that are used to solve ROs are the GBM and mean-reversion process (MRP) (Henley Business School, 2004). In most cases, such as modelling the commodity prices, MRP is modified by adding a price jump component that is also a random process. Each of these methods can be applied in both risk-neutral and real simulation models. As stated in the article published by Henley Business School (2004), real simulation is better suited for solving ROs compared to risk-neutrals that are intended for financial derivatives as this model utilises risk-neutral probabilities. The mathematical representations of these two simulation models are explained in the sections below. The commodity price is assumed to follow the GBM stochastic process given by Eq. (4.4).

If the knowledge of the capital asset pricing model (CAPM) is applied as a start to treat the drift as if it was a risk premium, the equation of total investment return, $\mu = \alpha + \delta$ can then be applied. Where μ is the risk-adjusted discount rate and δ is the convenience yield or the average return between two successive periods and the standard normal distribution, is denoted as ε. However, the real simulation of GBM utilises the drift term αt, resulting in future price Pt (Pontificia Universidade Catolica, 2008).

$$\mathrm{P_t} = P_0 \exp\{(\alpha - 0.5\sigma^2)\Delta t + \sigma \mathrm{N}(0,1)\sqrt{\Delta \mathrm{t}}\,\} \qquad 4.6$$

4.4.1.1 Estimating reversion speed from historical iron ore prices

The MRP model proposed by Schwartz (1997) is defined by:

$$\frac{\delta x}{x} = \eta(\bar{X} - lnX)\delta t + \sigma\delta z \qquad 4.7$$

Where η is the reversion speed at which the log of a price reverts to a long-term equilibrium log price $\bar{X}$. Following the above logic, a single period mean-reversion equation for a commodity price such as iron ore can be expressed as follows:

$$\mathrm{P}_{\mathrm{t+1}} = P_t - \eta(P_t - E(P)) + \varepsilon_t \qquad 4.8$$

$$= \eta\mathrm{E}(\mathrm{P}) + (1-\eta)P_t + \varepsilon_t$$

$$= \alpha + \beta P_t + \varepsilon_t$$

If the methodology proposed by Copeland and Antikarov is applied (Copeland & Antikarov, 2003, p254), the beta term, β which is the slope of the equation can be related to the Pearson coefficient or r^2 between values of the random variables between the adjacent periods.

$$r^2 = \frac{COV(P_t, P_{t+1})}{\sigma_t \sigma_{t+1}} \qquad 4.9$$

Similarly, β which is the slope is:

$$\beta = \frac{COV(P_t, P_{t+1})}{VAR(P_t)} \qquad 4.10$$

and assuming the time series of the random variable to be stationary, the following direct relationship can be derived:

$$(P_t) = \sigma^2(P_{t+1}), \text{ thus } \sigma_t = \sigma_{t+1} \qquad 4.11$$

This leads to

$$\sigma_t \sigma_{t+1} = \sigma_t^2 = VAR(P_t) \qquad 4.12$$

The above equation shows that β is equal to r^2 and since β is equal to $(1-\eta)$, then the speed of the reversion η is

$$\eta = 1 - \mathrm{r}^2 \qquad 4.13$$

Applying this to the case study, Fig. 4.3 showed the time series of the historical monthly iron ore prices between August 1985 and June 2016.

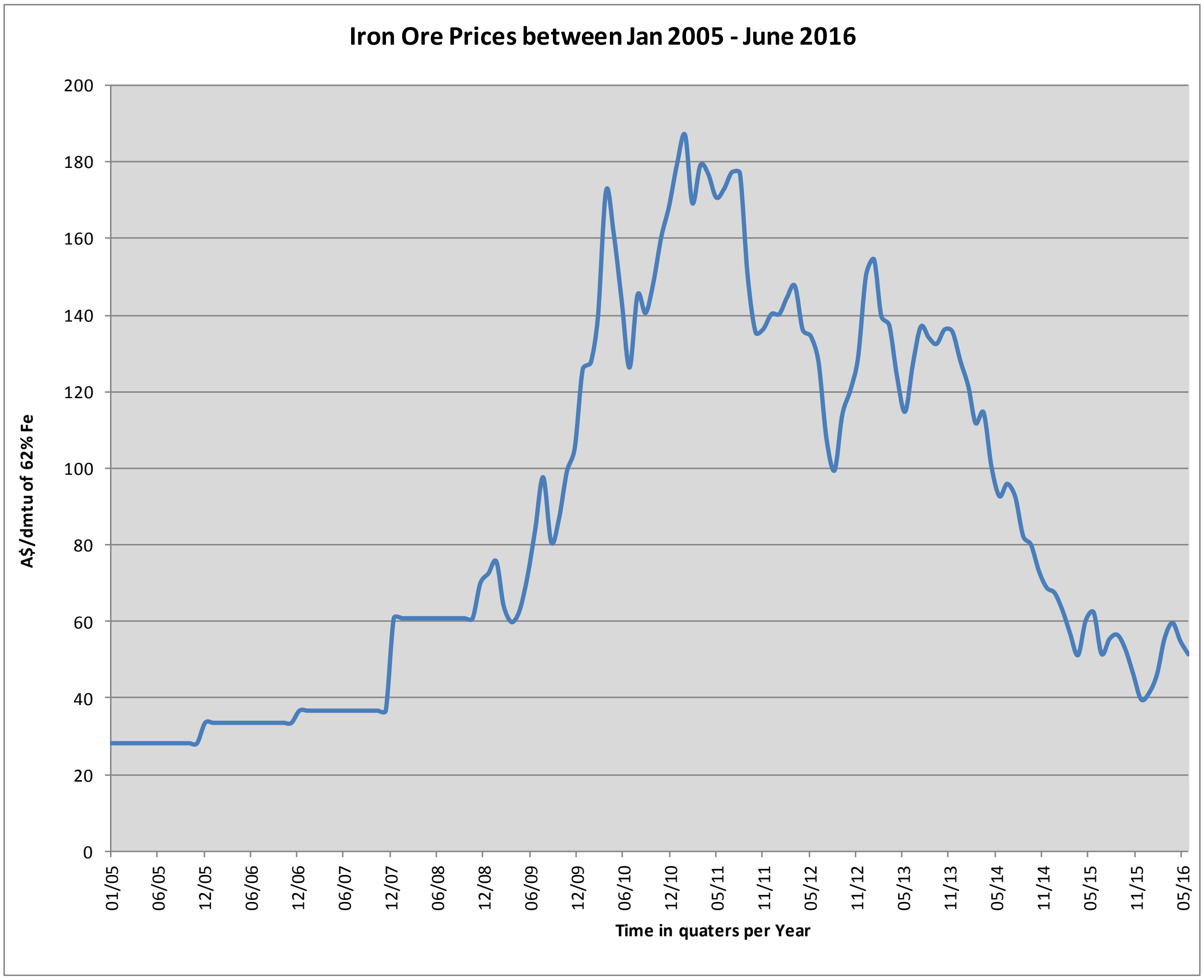

Fig 4.3, Actual iron ore prices goodness fit trend line. (Actuals sourced from IndexMundi, 2016).

4.4.1.2 Mean reversion with jump diffusion

In order to model iron ore prices, mean – reversion with jump diffusion was chosen as the iron ore prices have shown a stochastic behaviour with a long-run steady state inflationary price level since 1885 (Deverell, 2012). For the last forty years, iron ore prices have had at least 3 random spikes and have then reverted to equilibrium over time (Coble -Neal, 2011). Therefore, applying any other model which does not fit the identified characteristic of the iron ore price may produce unreliable results. The following section will explain the mean-reversion process.

If the uncertain variable in the single period mean reversion Eq. (4.6) is denoted as x, it is propagated that x can revert to its equilibrium level $\bar{x}$ in any of the future periods and the equation can be re-written as the stochastic process for variable x (Owen, 2013).

$$dx = \eta(\bar{x} - x)dt + \sigma dt \qquad 4.14$$

The Ito integration of the equation produces Eq. (4.15) which is made up of the mean, variance and normal distribution of the process:

$$\underbrace{X(T) = \mathrm{x}(0)\mathrm{e}^{-\mathrm{Ht}} + \left(1 - \mathrm{e}^{-\eta\mathrm{T}}\right)\bar{X}}_{\text{Mean of X at Time T}} \qquad \underbrace{+\sigma\mathrm{e}^{-\eta\mathrm{T}} \int_0^T \mathrm{e}^{\eta\mathrm{t}} dz(t)}_{\text{Variance of X at Time T}} \tag{4.15}$$

The integration of variance can be written as $(1 - e^{-2\eta T}) * \frac{\sigma^2}{2\eta}$ where the mean is the weighted average of the initial value and long-run level $\bar{X}$ which is the function of time and reversion speed. As explained by Oksendal (1996), the variance converges to $\frac{\sigma^2}{2\eta}$ as the time simultaneously increases and approaches infinity. The time it takes the stochastic variable x, moving at mean-reversion speed η to reach half of the distance toward steady state $\bar{x}$ is called its half-life (H), whose relationship is given by $H = \frac{\ln(2)}{\eta}$. Simulation of the future values can only be performed once the discrete-time equation of the process is obtained (Henley Business School, 2004; Pontificia Universidade Catolica, 2008).

$$X_t = X_{t-1}e^{-\eta\Delta t} + \bar{X}\left(1 - e^{-\eta\Delta t}\right) + \sigma\sqrt{\frac{1 - \exp[-2\eta\Delta t]}{2\eta}}\,\mathrm{N}(0,1) \tag{4.16}$$

As explained in Henley Business School (2004) and Owen (2013), variable X(t) is normally distributed and the stochastic process, such as the price of iron ore P, must have a positive value only. The association between the long-run level $\bar{X}$ of variable X(t) and the steady price $\bar{P}$ of the commodity, such as iron ore, is expressed as follows:

$$\bar{X} = \ln(\bar{P}) \tag{4.17}$$

Consequently, the long-run equilibrium level for the commodity is:

$$\bar{P} = \exp(\bar{X}) \tag{4.18}$$

Once the process is simulated, the expected price at time T is shown in Eq. (4.19):

$$E[P(T)] = \exp\{\mathrm{x}(0)\mathrm{e}^{-\eta\mathrm{T}} + \left(1 - \mathrm{e}^{-\eta\mathrm{T}}\right)\bar{X}\} \tag{4.19}$$

However due to the failure of the direct calculation resulting from inflation of the distribution by the unnecessary addition of variance, the added half of the variance is subtracted as shown in Eq. (4.20) (Henley Business School, 2004; Pontificia Universidade Catolica, 2008).

$$P(t)] = \exp\{\mathrm{x(t)} - 0.5Var[\mathrm{x(t)}]\} \qquad 4.20$$

Now the sample path for price P can be simulated by sampling the normal distribution of X(t) in Eq. (4.16), then the variance Var[X(t)] and Eq. (4.20) are applied to estimate the value of P(t). A combination of Eqs. (4.16), (4.19), and (4.20) results in Eq. (4.21), which produces a direct real simulation of the stochastic process for P(t).

$$P(t) = \exp\left\{\ln[[\mathrm{p(t-1)}]\exp[-\Delta\eta\mathrm{t}]] + [\ln(\overline{\mathrm{P}})(1 - \exp[-\eta\Delta\mathrm{t}])] - \left[(1 - \exp[-2\eta\Delta\mathrm{t}])\frac{\sigma^2}{4\eta}\right] + \sigma\sqrt{\frac{1 - \exp[-\eta\Delta\mathrm{t}]}{2\eta}}\mathrm{N}(0,1\right\} \qquad 4.21$$

Where the first and second terms of the equation are drifts which are the weighted average of the initial price and long-run average, the third term of the equation is what is referred to as the convex adjustment (Henley Business School, 2004) and the last term is the normal distribution which samples the simulation. It must be noted that the volatility is only used in the third and fourth terms of the equation.

4.4.1.3 Addition of jump diffusion to continuous MRP simulation

Jump diffusions (JD) are estimated using the Lévy process, which is a family of any continuous-time stochastic processes that have stationary independent increments (Ni & Isaksson, 2005). Therefore, both the Poisson and Wiener are stochastic processes as they share the characteristic of being stationary independent increments (Ni & Isaksson, 2005). Poisson denotes any occurrence of the uncertain event in a time space in which a time period passes without anything happening instantaneously, a jump occurs whose distribution of the expected increment is a positive value while that of the Wiener process is zero. Therefore, the Poisson process is used to model unexpected jumps. Following Matsuda (2004), the JD can be summarised as an exponential of the Levy's process as shown below:

$$S_t = S_0 e^{L_t} \qquad 4.22$$

Where S0 and St are values of the uncertain variable at time T0 and Tt, respectively and the process for this variable is St ; 0 ≤ t ≤ T and the Levy's model to this process is represented by Lt ; 0≤t≤T. According to Merton and cited by Matsuda (2004), the Levy process is a form of Brownian motion which has two components to its equation. The first part is a drift which is a continuous diffusion process while the second portion is the Poisson process that is discontinuous in nature and represents the summation of all the jumps that have occurred over the time period:

$$\ln(\frac{S_t}{S_o})(L_t = \left(\alpha - \frac{\sigma^2}{2} - \lambda k\right)t + \sigma B_t + \sum_{\mathrm{i=1}}^{\mathrm{N_i}} \mathrm{Y_i} \qquad 4.23$$

Where Bt ; 0 ≤ t ≤ T is the standard Brownian motion and $\left(\alpha - \frac{\sigma^2}{2} - \lambda k\right)t + \sigma B_t$ is a Brownian motion with a drift component and $\partial q = \sum_{\mathrm{i=1}}^{\mathrm{N_i}} \mathrm{Y_i}$ is the compound Poison jump process. Where:

$$\partial q = \begin{cases} 0\ with\ probability\ 1 - \lambda\partial t \\ \emptyset\ with\ probability\ \lambda\partial t \end{cases}$$

Comparing JD to the Black – Scholes option model { $c = S_N(d_1) + N(d_2)Ke^{2-rt}$}, Matsuda underscored that it is the addition of the jump that differentiates the two models. The jump process has two random portions which are the Poisson process dN_t with lambda (λ) which is the average number of jumps in a time period that caused the uncertain variable, the commodity price in this case to spike up or down. Additionally, the size of the jump once it has occurred is also uncertain and hence it is random in nature. Using Merton's assumption that the log of the price is normally distributed, normal (μ, δ^2), the normal distribution formula in Eq. (4.27) can be applied to represent this randomness. The continuous MRP is usually referred to as Schwartz's Model or Dias Marlim (Henley Business School, 2004). The only difference between this model and the discrete-time model shown in Eq. (4.14) is the simplification of the relationship between $\bar{X}$ and $\bar{P}$. Where price P is used directly to simulate the future price as expressed:

$$\bar{X} = ln\bar{P} - \frac{\sigma^2}{2\eta} \qquad 4.24$$

Therefore, the future value of price P at time t or real simulation can be obtained using Eq. (4.24), and as time approaches infinity, the values converge as

$$F = E[P(\infty)] = Expected\ long - run\ future\ price = exp\left(\bar{X} - \frac{\sigma^2}{4\eta}\right) = \boldsymbol{exp}\left(ln\bar{P} - \frac{\sigma^2}{2\eta}\right)$$

$$= \bar{P}exp\left(-\frac{\sigma^2}{4\eta}\right) \qquad 4.25$$

Recall that the price of the commodity at time t in Eq. (4.20), in a real process simulation, with the long run average price $\bar{X}$ ($\ln(\bar{P})$) subtracted from the equation, resulting in the following:

$$X_t = X_{t-1}e^{-\eta\Delta t} + \ln(\bar{P})\left(1 - e^{-\eta\Delta t}\right) + \sigma\sqrt{\frac{1 - \exp[-2\eta\Delta t]}{2\eta}}\ \mathrm{N}(0,1) + jumps \qquad 4.26$$

To utilise the above stochastic formula in a spreadsheet, the Box-Muller transformation was applied. This is a form of Gaussian distribution which takes random variables from one distribution as inputs and outputs random variables in a new distribution function, thus changing the uniformly distributed random variables (Design, 2016; Thistleton, et.al, 2006). The square of the random variable R^2 has a x^2distribution and U = exp($-\frac{1}{2}R^2$) uniformly distributed and its inverse transformation is (Thistleton.et.al., 2006):

$$R = (\sqrt{(-2lnx_1)\ cos(2\pi x_2)} \quad 4.27$$

4.5 The real case study

Let's take the case of a medium sized iron ore mining project that had received mining approval to develop a 10Mtpa magnetite operation in Western Australia's Pilbara region with a mine life of 15 years. The iron ore company had five years to bring the operation into production by July 2012, and the management planned to ship the first ore to China by July 2013.

Apart from other operational and market risks, iron ore price volatility has the highest impact on the project value and therefore this study will concentrate on iron ore price uncertainty (Ajak & Topal, 2015; Haque et al., 2015b, 2016a). The project viability decisions were made in regard to the expected prices as modelled by the World Bank (Kolesnikov, 2015; Knoema, 2014). Even though there were other parameters such as discount rate and exchange rate, these factors were not detrimental to iron ore operations as the value of the AUD moved with iron ore prices (Haque et al., 2015, 2016). In fact, these factors had a positive effect on the operational bottom line because a low (weak) AUD was a good thing for Australian companies that were exporting raw materials.

With strict timelines to deliver and meet the project milestones, operations did indeed deliver the first shipment to China at the end of July 2013. This milestone was celebrated by the company and its stock price increased commensurately. Table 4.1 contains an average cost summary of iron operations in Australia.

Table 4.1, Australian iron ore mines cost analysis between 2009 & 2014 (Researcher's own analysis).

Item	Capital	Data Source
Average Capital Cost per Anuual tonne (US$)	152.0	Researcher's own analysis
Operating Cost per tonne (US$)	51.2	
Company Tax	30%	Ajak & Topal, 2015
Royalties (included in total cost)	7.5%	
Risk free rate	5%	Kleymenova, et al. 2009
Market Risk Premium	6%	
Gearing	17%	
Equity	83%	
Gearing/Equity	20%	
Cost of debt	12%	
Capital	100%	
ROE	16%	
WACC Iron Ore Mine	15.43%	
Long Foreign exchange rate AUD/USD	0.77	Researcher's own analysis of Ozforex data from 1990 to 2015 (ozforex, 2015).

****Note:* *the raw data used in the analysis of the iron ore cost is confidential and only available on request.*

4.5.1 Step 1: Static DCF Analysis

The viability of the project was assessed using a DCF analysis by applying Eq. (4.2). This conventional method showed a positive NPV, as summarised in Table 4.2.

Table 4.2, Traditional DCF analysis model utilising World Bank iron ore price predictions (Kolesnikov, 2015; Knoema, 2014).

Extraction rate Mtpa	10.00
Cost per tonne (US$)	51.2
Risk free interest	5.02%
WACC	15.43%
Current Iron ore price- EOY 2012 (US$)	128.5

Period in years	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027
Price; US$/dmtu		128.0	94.0	98.0	99.0	100.0	101.0	101.0	102.0	102.0	103.0	103.0	104.0	105.0	102.0	102.3
Cost, US$M		511.7	511.7	511.7	511.7	511.7	511.7	511.7	511.7	511.7	511.7	511.7	511.7	511.7	511.7	511.7
Revenue; US$M		1,280.0	940.0	980.0	990.0	1,000.0	1,010.0	1,010.0	1,020.0	1,020.0	1,030.0	1,030.0	1,040.0	1,050.0	1,020.0	1,023.0
EBIT, US$		768.3	428.3	468.3	478.3	488.3	498.3	498.3	508.3	508.3	518.3	518.3	528.3	538.3	508.3	511.3
Corporate Tax, US$		230.5	128.5	140.5	143.5	146.5	149.5	149.5	152.5	152.5	155.5	155.5	158.5	161.5	152.5	153.4
Free Cash Flows, US$		537.8	299.8	327.8	334.8	341.8	348.8	348.8	355.8	355.8	362.8	362.8	369.8	376.8	355.8	357.9
Discount Factor		0.9	0.8	0.7	0.6	0.5	0.4	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.1	0.1
DCF, US$	(1,520)	465.9	225.0	213.2	188.6	166.8	147.5	127.8	112.9	97.8	86.4	74.9	66.1	58.4	47.7	41.6
NPV, US$	601															

Based on the conventional investment logic, the project had a positive NPV and it was then accepted. The investment was subsequently made and mine development progressed. However, the iron ore price dropped below US$40/tonne between April 2015 and quarter one (Q1) of 2016 (IndexMundi, 2016), and being a high-cost magnetite mine, the operation incurred huge losses and ultimately collapsed in 2016. The analysis showed that the operation reached its breakeven point and was already unprofitable when the prices plunged below US$40/tonne at the end of 2015 (IndexMundi, 2015). In line with the expectation as supported by the historical actual prices, the project lost value just two years into its operational life by having a negative NPV of (US$750 million) immediately before it collapsed (Table 4.3).

Table 4.3, Operations returned losses and negative NPV when prices plunged below US$40/tonne in 2015 (IndexMundi, 2016).

Period in years	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027
Price; US$/dmtu		135.4	96.8	55.2	51.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Cost, US$M		511.7	511.7	511.7	511.7											
Revenue; US$M		1353.6	968.4	552.1	514.6											
EBIT, US$		841.9	456.7	40.4	2.9											
Corporate Tax, US$		252.6	137.0	12.1	0.9											
Free Cash Flows, US$		589.3	319.7	28.3	2.0											
Discount Factor		0.9	0.8	0.7	0.6											
DCF, US$	(1,520)	510.6	239.9	18.4	1.1											
NPV	(750)															

If the management had asked what could happen to iron ore prices and if the analysis model considered the impact of uncertainty, the decision would probably have been different prior to investing in developing this mine. However, the traditional DCF analysis assumed the future to be static and ignored the possibility of making alternative (deferred) investment decisions in the future. Therefore, the management of the real case mine did not have any flexibility. They had no choice but to close the mine as the initial investment had become a sunk cost with little prospects for providing a return on the capital development outlay. Looking at the current state of the world, would ROs have made a difference if they were built into the real case mine operations' original analysis? The answer to this question will be revealed in the following sections.

4.5.2 Step 2: Stochastic simulation process

The MCS method generates random numbers and uses probability to find the best solution (Hoffman, 1998; James, 1980). It uses a parameter of a hypothetical population and a random sequence of numbers to construct a sample of the population, from which statistical estimates of the parameter can be obtained (James, 1980 and citing Halton, 1970). It has the advantage of simulating multiple uncertainties described by complex stochastic models that can be handled without the need of transforming a real complex problem into a simple, but highly unrealistic problem. Metal units' variability over time and cash flows' non-uniformity can be incorporated into the valuation process (Dimitrakopoulos & Abdelsabour, 2007). Uncertainty is propagated by examining how random variation, lack of knowledge or error affects the sensitivity, performance, or reliability of the system that is being modelled (Vertex42, 2015). Fig. 4.4 presents a schematic representation of the MCS method.

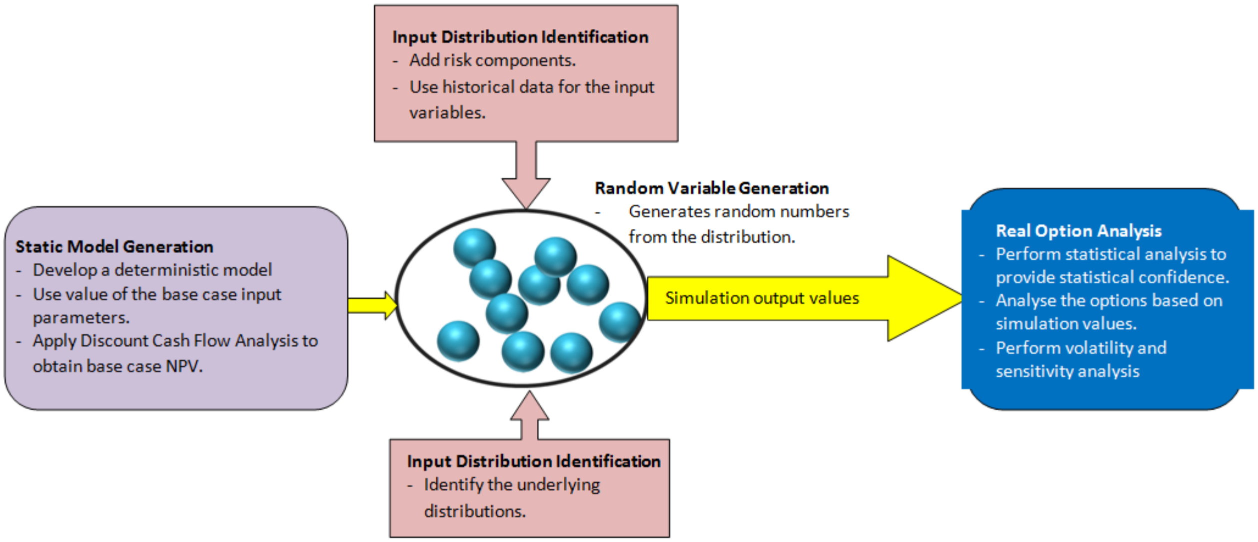

Fig. 4.4, The proposed stochastic simulation process for real options analysis.

4.5.2.1 Stochastic simulation of iron ore prices using mean – reversion model

The main uncertainty impacting the iron ore industry is the fluctuating iron ore price as China's economy consolidates its previous prolific growth. Using Eq. (4.26) and applying a spreadsheet-based linear congruential generator (LCG), $x_i = a_0 + a_1 x_{i-1}$ mod M, where mod M is an integer modulus, 5,000 iterations of the possible future price paths for iron ore were simulated (Fig. 4.5).

4.5.3 Project present value from stochastic simulation of the iron ore prices

Following the price simulation, future project values were set as the simulation outputs or target variables which were also stochastically simulated.

Fig. 4.6 is the histogram of project PVs. Plotting the frequency of PV in the 5000 iterations helps in visualising the most likely obtainable PV if a commitment was made to invest in this project. Moreover, the histogram assisted in the determination of skewness which measured the symmetry and kurtosis which assisted in checking whether the simulated values were heavy-tailed. Fig. 4.7 is the spread of the simulated PVs which would be used to determine the confidence levels, especially when two standard deviations were considered. It would provide the expected range for resultant PVs to be estimated with 95% confidence level.

As explained in the preceding paragraph, results of the MCS indicated that there is 95% chance that the project present values will range from (US$367) to US$1,993 million (Fig. 4.7). Additionally, Fig. 4.6 and

4.7 indicated that the project PVs have positive skewness and leptokurtosis. These two measures imply that the project will experience numerous small negative cash flow outcomes, but extreme bad scenarios are not as likely. Therefore, as the commodity price fluctuates, there will be frequent small losses and few extreme gains as the iron ore prices are not expected to have high jumps. When the mean of the simulated prices is used in the DCF model, the project NPV improves from negative -$US750 to -$US328 million as shown in Table 4.4 and 4.5.

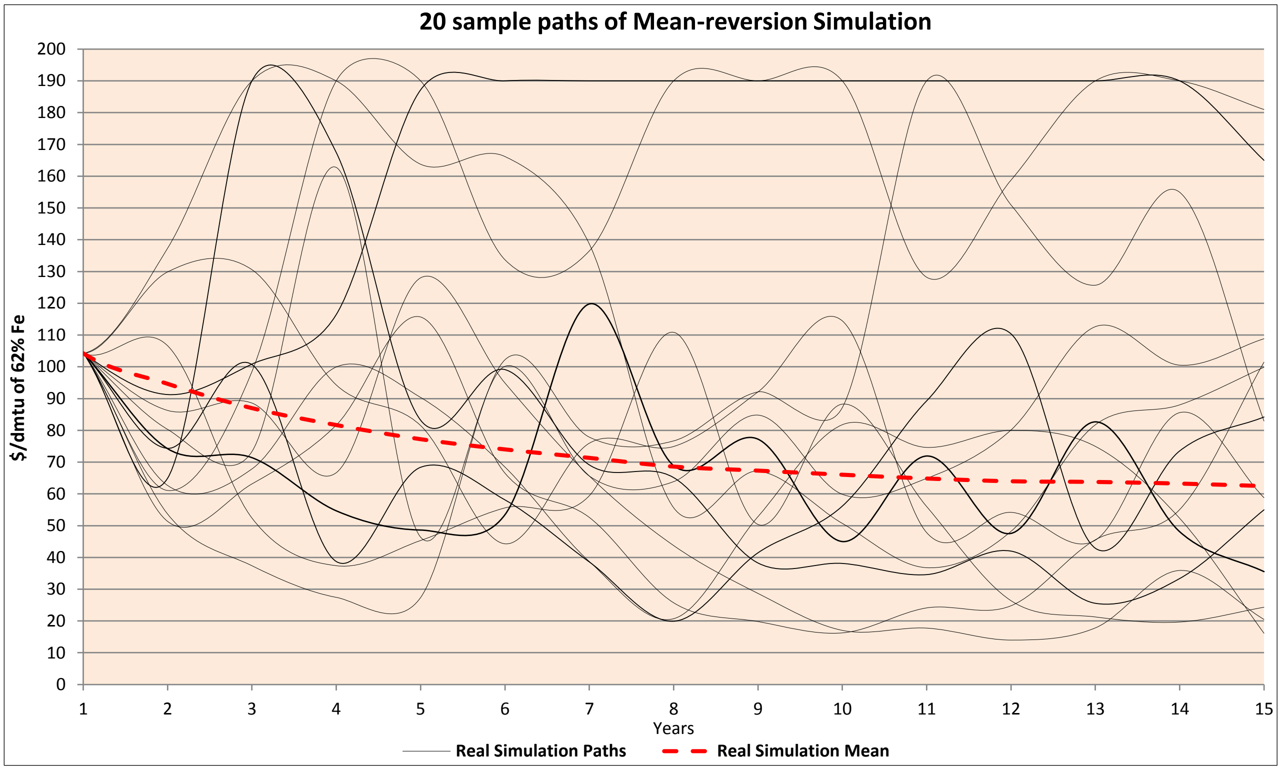

Fig. 4.5, 20 sample paths of iron ore price mean-reversion model from 5,000 iterations.

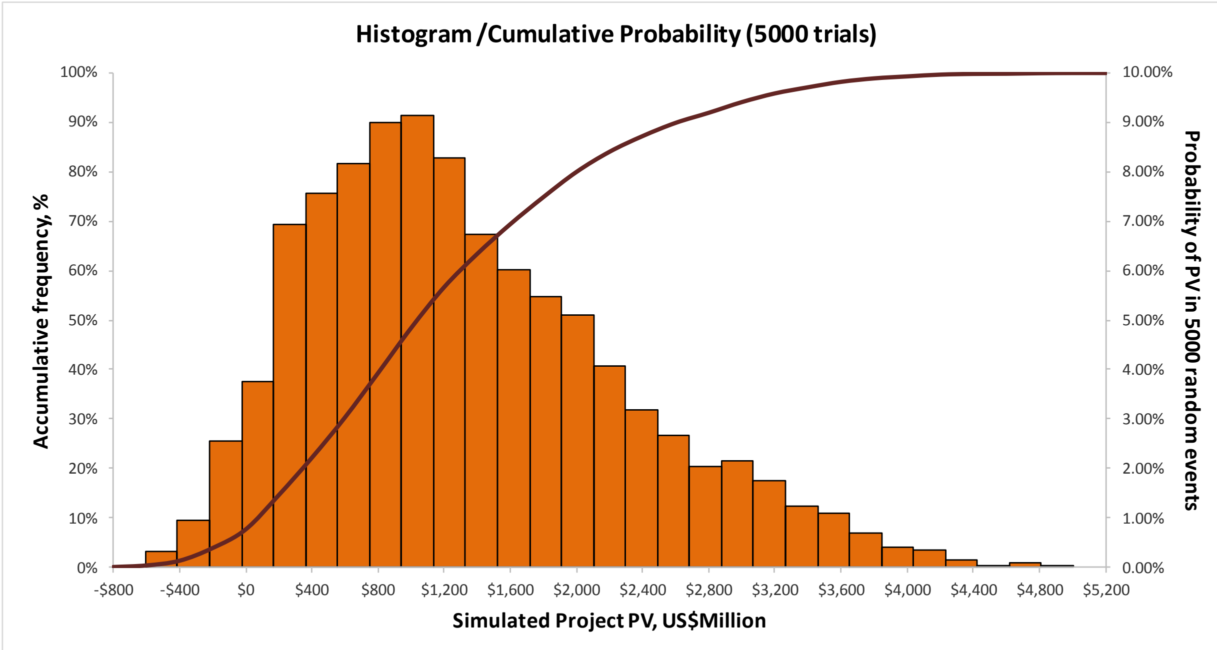

Fig. 4.6, Histogram of project PVs for 5000 iterations.

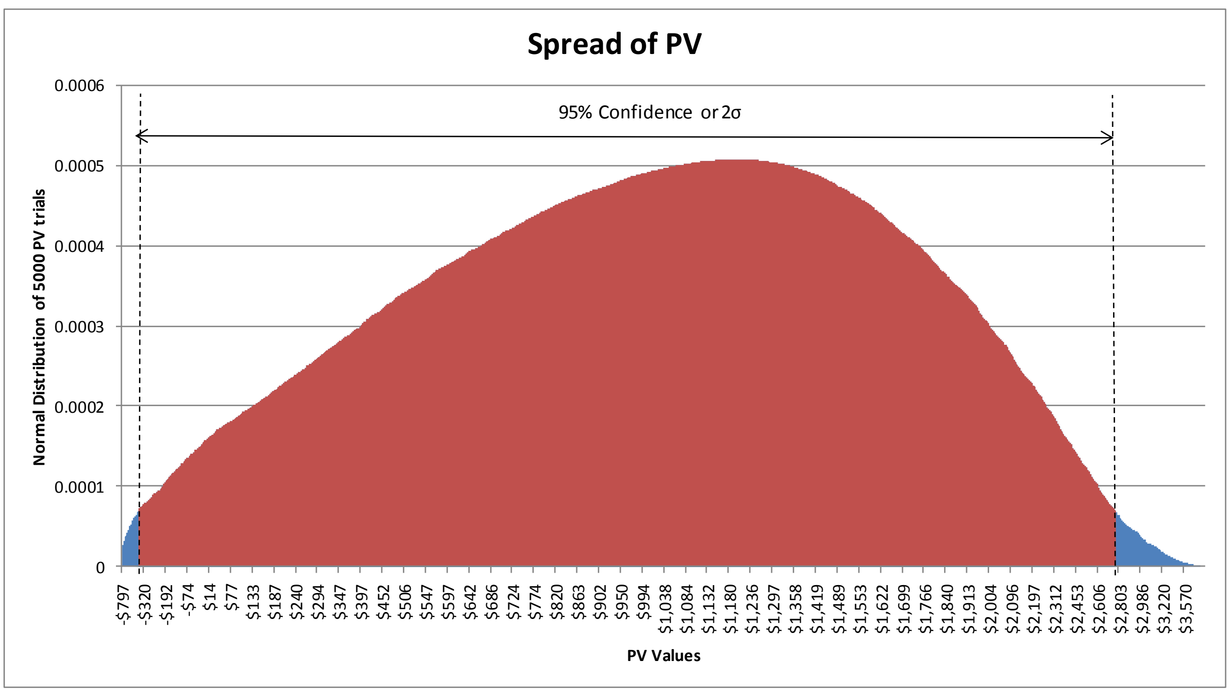

Fig. 4.7, Spread of project PV for 5000 iterations (US$'million).

Table 4.4, Summary statistics of the stochastic simulation of the project PVs.

Average Simulated PV	$ 1,204.9
Max PV	$ 4,876.2
Min PV	-$ 797.4
PV Standard deviation	49%
Probability of profitability; PV >= $1,520	33%
Probability of Loss; PV < $1,520	67%
Skewness	0.70
Kurtosis	0.13

Table 4.5, Stochastic simulation of real case study using mean – reversion with jump (US$'mill).

Period in years	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027
Price; US$/dmtu	129	104	94	87	80	77	73	71	69	67	66	64	64	63	63	62
Cost, US$		512	512	512	512	512	512	512	512	512	512	512	512	512	512	512
Revenue;US$		1,043	945	870	801	768	733	711	686	673	656	642	636	632	625	618
EBIT; US$		531	433	358	290	257	222	199	175	161	144	131	124	121	114	106
Corporate Tax, US$		159	130	107	87	77	67	60	52	48	43	39	37	36	34	32
Free Cash Flows, US$		372	303	251	203	180	155	140	122	113	101	91	87	84	80	75
Discount Factor		1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
DCF; US$	(1,520)	322	228	163	114	88	66	51	39	31	24	19	16	13	11	9
PV; US$	(328)															

4.5.4 Step 3: Creating flexible option in mine planning and development

ROs were analysed based on the NPVs calculated from the simulated values of the potential investment. The analysis of the various ROs has shown that the real case operation would have created value from the market uncertainty if the management had considered flexibility and treated it as an investment decision.

4.5.4.1 Option to abandon

If the management had considered an option to abandon the operations once the prices start to drop and exercised this option early rather than holding out until the company collapsed, the company would have reflected a better value. This is with the assumption that the company would have already started talking to prospective buyers who may have been ready to pay a fair, yet depreciated value in the year of exercising this option.

For this real case study, this option should have been exercised at the end of 2015. The salvage value was obtained as the median of estimated values using the diminishing value method (Australian Taxation Office, 2013).

$$Value, \$ = base\ value\ \times \frac{days\ held}{365} \times 200\% \quad 4.28$$

The Black and Scholes (BS) model was used to value the project with an abandon option

$$c = S_N(d_1) + N(d_2)Ke^{2-rt} \quad 4.29$$

with both d_1 and d_2 determined below:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{s^2}{2}\right)t}{\sqrt[\sigma]{t}} \quad 4.30$$

$$d_2 = d_1 - \sqrt[\sigma]{t} \quad 4.31$$

Where c = value of the call option, S = current stock price, t = time until option maturity, K = option strike price, r = risk-free interest rate, N = cumulative standard normal distribution, e = exponential term or constant which is ~ 2.71828, σ = standard deviation of the stock price and natural logarithm (ln). The NPV for the abandon option was calculated by discounting the present value of the project without flexibility by the yield rate or cost of delay and then taking away the capital investment that has been discounted by the risk-free rate plus the future expected salvage value that has also been discounted by the risk-free rate. Thus, the abandon option decision criteria is to abandon the operations if Max (0, X -V).

Table 4.6, NPV of the project with an option to abandon (US$'mill).

Summary of Abandon Option	
Present Value of project if fund is committed	$ 1,205
Annualised Standard Deviation of PV	49%
PV Variance	24%
Mine life	15.00
Investment	$ 1,520
Median Salvage value if abandoned	$ 407
Risk free rate	5%
Cost per year for abandonment, 1/expiry time	7%
Black - Schole Parameters	
d1 =	1.384
N(d1) =	0.917
d2 =	-0.494
N(D2) =	0.310
Abandon Option NPV, $M	$ 243

As shown in Table 4.6, the NPV of the project with an abandon option is US$243 million which suggests the project being a worthwhile investment. Therefore, this project would have mitigated losses which the operations incurred when the prices dropped in 2015 if the option to abandon was built into the mine plan at its inception.

4.5.4.2 Option to delay investment

A decision to invest now or to wait until the market improves is a RO whose exercise price is the capital development cost, while the underlying asset value, being an acquisition value, is the PV of net cash flows. However, each year beyond which the project is delayed, the investment foregoes cash flows that would have been earned. This cost is equivalent to $\frac{1}{T}$, where T is the time to option expiry (Damodaran, 2015).

The Black-Scholes model was also used to value the delay option. The NPV for the delay option was calculated by discounting the present value of the project without flexibility by the yield rate or the cost of delay and then taking away the upfront capital investment that has been discounted by the risk-free rate.

Table 4.7, NPV of the project with an option to delay investment (US$'mill).

Delay Option	
Present Value of project if fund is committed	$ 1,205
Annualised Standard Deviation of PV	49%
PV Variance	24%
Mine life	15
Investment required for mine development	$ 1,520
Risk free rate	5%
Cost per year of delay,1/expiry time	7%
Black Schole Parameters	
d1=	0.682
N(d1)=	0.753
d2=	-1.196
N(D2)=	0.116
Delay Option NPV,$M	$ 250

As shown in Table 4.7, the NPV of the project with delays is US$250 million which is US$7 million higher than the option to start and abandon the operation. Therefore, if the management of the real case operation had considered the option to delay the investment, the operations will have mitigated the losses which occurred when prices dropped in 2015.

4.5.4.3 Staged investment option

The management of the real case operation also held an option to start a small operation for 5Mtpa that could be run by contractors using mobile crushing facilities and mining the open pit using a conventional truck and shovel system. This option provides management with the flexibility to learn more about the market and the operations can be either expanded if prices increase or discontinued whenever it becomes unprofitable. Secondly, the benefit of this option is that there is no need to spend capital in buying mining equipment and in building the processing plant as these will be provided by the contractor. However, an initial cost of US$194 million must be paid for the development of mine services such as accommodation camp, bore field, power, pipelines for camp and concentrate processing. The cost for these services was estimated based on a similar sized mine project, being the Jupiter Mt Ida mine (Jupiter Mines Ltd, 2011) and a 10% contingency was applied to this cost (Brockman Resources, 2010). Additionally, the contractor cost for running the operations was estimated to be 25% more than the estimated owner-operated cost where 20% is the contractor profit margin and 5% is the contingency cost (Khairo and Davies, 2009). This option was valued using the stochastic model which simulates all the possible price outcomes that trigger the decision to either expand the operation when prices jump to 30% above the cost, to maintain (shrink) the current small size operation or to shut the mine when the discounted cash flows are less than zero (Fig. 4.7). The decision criteria for the option is to expand operations if prices increase if [(% of expansion * Vt) – X] > Vt , otherwise maintain (shrink) the existing operating profile.

Table 4.8, NPV of the stochastic simulation of the option to staged investment option (US$'mill).

Staged Investment and Expansion Option			
Extraction rate in Mtpa	5.00	Expansion Option; Mtpa	5.0
owner's operator cost; US$	51.2	Expand if price is 30% above cost, US$/t	83.2
Contractor Contigency cost	5%	Expansion Cost, US$/t	64.0
Contractor Profit margin	20%	Cost of shuting; US$	
Mine services development cost, US$/annual t	38.7		
Contractor cost for mobile crushing, US$/t	**64.0**		

Period in years	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027
Price; US$/dmtu	128.5	104.8	92.6	49.9	41.4	45.2	51.9	83.9	71.0	45.5	26.7	32.9	20.7	34.9	33.5	49.8
Free Cash Flows; US$		143.1	200.8	(49.1)	(78.8)	(65.5)	(42.1)	139.7	24.6	(64.5)	(130.5)	(108.7)	(151.3)	(101.8)	(106.6)	(49.5)
Discount Factor		0.9	0.8	0.7	0.6	0.5	0.4	0.4	0.3	0.3	0.2	0.2	0.2	0.2	0.1	0.1
DCF; US$	(193.5)	123.9	150.7	-	-	-	-	51.2	7.8	-	-	-	-	-	-	-
Flexible Operation			*Expand*	*Shut*	*Shut*	*Shut*	*Shut*	*Expand*	*shrink*	*Shut*	*Shut*	*Shut*	*Shut*	*Shut*	*Shut*	*Shut*
NPV; US$	140.1															
Mean NPV for 5000 Iterations; US$	**715.0**															

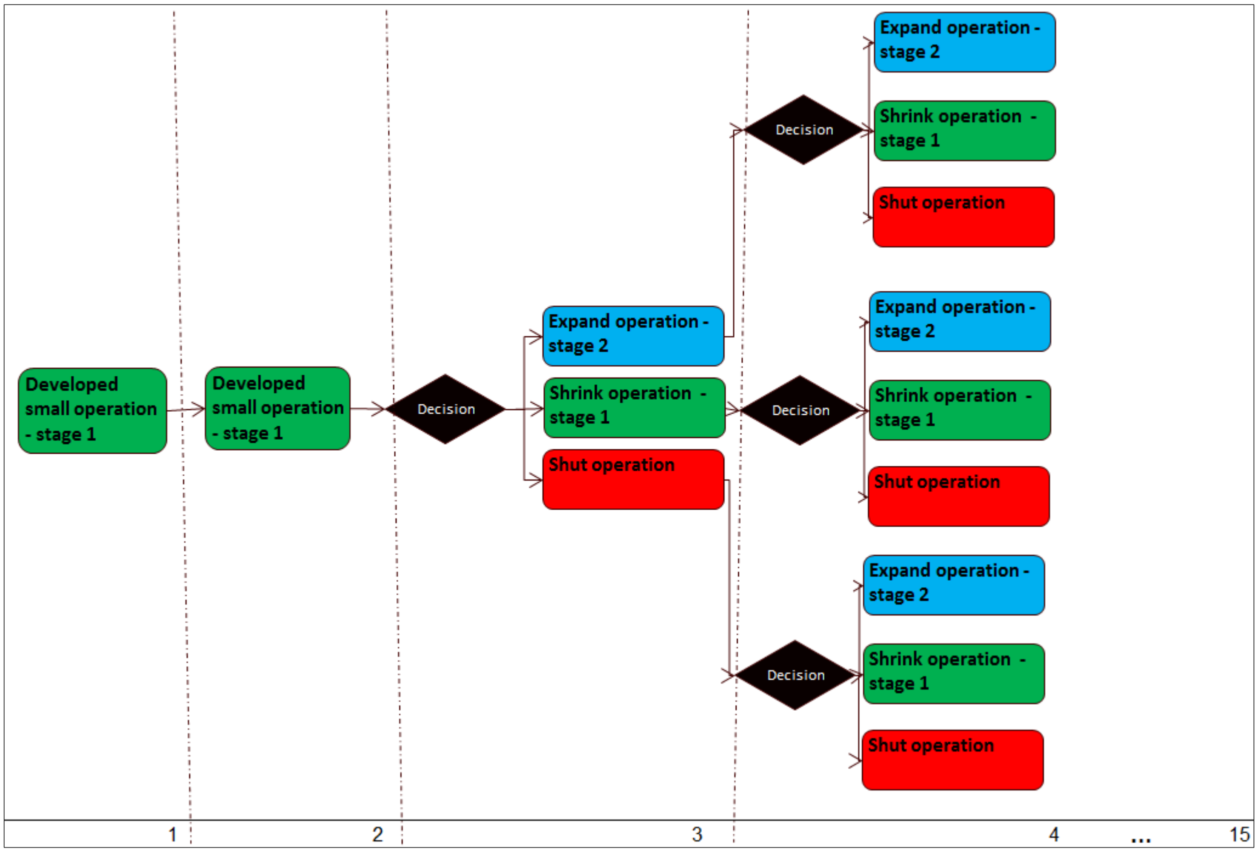

Fig. 4.8, Schematic diagram of management decision for staged investment option.

From the analysis summarised in Table 4.8 and Fig. 4.8, staging the investment by starting a small operation now and expanding it in the future when prices improve has the largest NPV of US$715 million among other options. This implies that the operations would benefit from the present supported prices and would minimise large capital outlays by contracting out the operations. The real gains are embedded in the ability to expand when prices increase and scale back or discontinue operations when cash flows are less than zero. Even though such option commonly yields better value than others, staged investment option depends on the circumstances and setting of an individual project. Therefore, its outcome is a case-specific and not a generalised result.

4.5.5 Step 4: Real option analysis and result discussion

In comparing the NPVs of flexibility operations against the actually achieved project NPV immediately before it collapsed in April 2016, it is now clear from the analysis that it was a costly mistake to base a significant investment on DCF analysis alone without considering what could happen to iron ore prices in future.

The three options to abandon the project, to delay the investment and to stage the investment by starting a small operation and to expand in the future if the prices increase have all returned positive NPVs of US$243 million, US$250 million and US$715 million, respectively compared to the actual project NPV of negative $750 million before it collapsed in April 2016. Additionally, it is not surprising that the use of a stochastic simulation model is more reliable than the traditional DCF analysis as it has reliably predicted the future prices and returned a negative NPV of US$328 million. Thus, the application of the stochastic simulation without considering options would have stopped the management from committing capital to an unviable investment in the first place.

In this real case study, ROs analysis appears to have provided insight with improved results compared to DCF analysis where constant assumptions had initially returned a positive NPV that led to capital being invested but, eventually ended up in a collapsed operation. The RO models accounted for uncertainty.

Therefore, if ROs and the stochastic model were applied during the bankable feasibility study, it would have been highly likely that the management would have either waited by delaying the investment or staged the operations by starting a small mine with the view to expanding it if the market improved. Fig. 4.9 is the tornado diagram of all the NPV outcomes that compare flexible options to stochastic simulation and traditional DCF analysis.

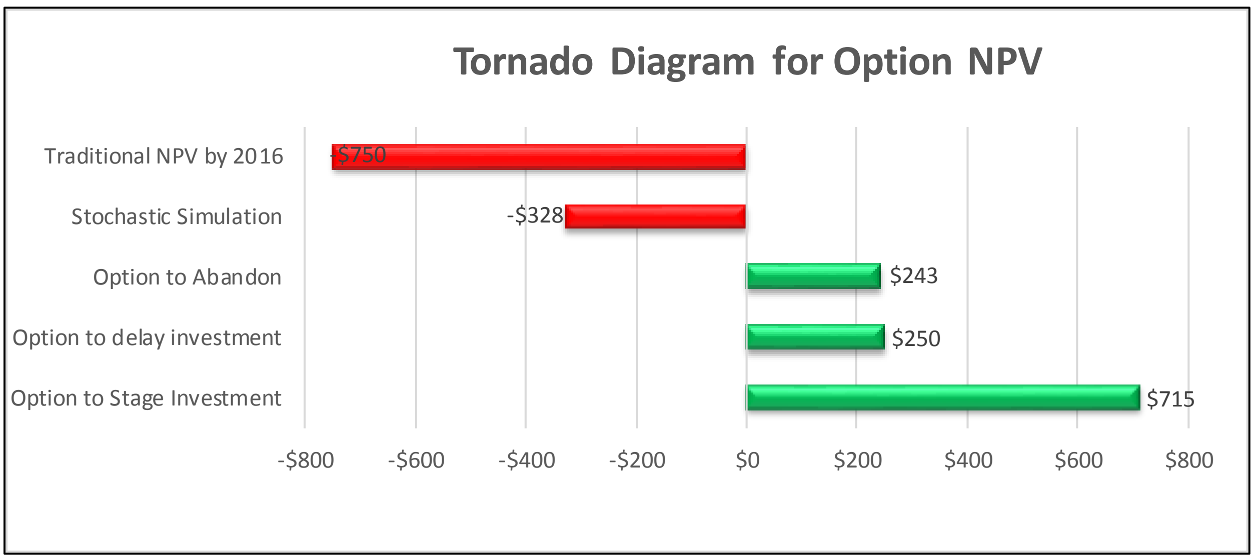

Fig. 4.9, Comparison of project NPV for various flexible options to traditional DCF analysis (US$'million).

Option value is expressed as a difference between the benefit multiplied by the standard distribution of the probable outcome and cost which is also multiplied by the standard distribution of the probable outcome as propagated in the Black-Scholes model (Eq. 4.29).

$$Option\ value = BenefitsN(d_1) - CostN(d_2), \quad 4.32$$

Thus, this can also be expressed in terms of NPV

$$Option\ value = NPV\ of\ flexible\ design\ (NPV_f) - Traditional\ NPV \quad 4.33$$

In applying the above concepts, the flexibility values for the option NPV calculated in Step 3 are summarised in Table 9.

Table 4.9, Flexibility values for various mine options.

Analysis Model	NPV, $M	NPV_f - *NPV*	Option Value, $M
Traditional DCF Analysis	-$750	(-$750) - (-$750)	$0
Stochastic Simulation	-$328	(-$328) - (-$750)	$422
Option to Abandon	$243	$243 - (-$750)	$993
Option to Delay	$250	$250 - (-$750)	$1,000
Staged Investment	$715	$715 - (-$750)	$1,465

Note: *Table 4.9 showed estimations of individual option values but not a summation of the combined options.*

As seen from the analysis (Fig. 4.9), the use of ROs has indicated that this project is economically attractive and therefore it is worthwhile acquiring the right (the option) to hold and potentially own it. If the Johnathan Mun approach of using minima and maxima criteria in an investment decision-making process is applied (Mun, 2006), it is clear from Table 4.8 that the worst outcome would be when the project collapsed, in April 2016, when reflecting a NPV of negative US$750 million. Therefore, applying DCF analysis alone and ignoring the value of future information and commensurate flexibility only shows negative impacts on the project bottom-line. On the other hand, applying a stochastic simulation methodology reduced the uncertainty and, importantly, the potential loss can be completely avoided by deferring the investment decision to a future point in time. Naturally, the future opportunity would not exist if the right to hold the investment, being the option, is not paid for upfront.

The flexibility values of the stochastic model, the abandon, the delay and the staged investment options are US$422 million, US$993 million, US$1,000 million and US$1,465 million, respectively (Table 4.8). These figures reflect the value of learning the future information or 'the known unknown' which is the uncertainty. It is referred to as 'the known unknown' because the known part is the iron ore price which is uncertain, but what is unknown is the level of volatility which is stochastic in nature and can take any value (Fig. 4.9).

In conclusion, the real case study has demonstrated that building flexibility into a mining operation makes it more agile, increases its value and mitigates losses. Therefore, if the operators within the junior iron ore sector had considered using ROs in investment decision making, their respective capital development programmes may have been deferred. This analysis has proven the suitability of RO analysis when making decisions around developing and running iron ore operations.

4.6 Conclusion

Investments in mining operations are irreversible once the initial capital development outlay has been expended and becomes a sunk cost. This implies that companies hold an opportunity cost of investing now rather than waiting. Therefore, a robust analysis methodology such as ROs, that can factor in uncertainty and future decisions, is crucial. This real case study is a good representation of existing iron ore mines in Australia and particularly in the Pilbara region. The research has demonstrated that junior iron ore miners may have avoided losses which resulted from price fluctuations had the operations utilised ROs to model managerial flexibility instead of using the traditional DCF analysis whose assumptions are typically constant throughout the life of a project.

Valuing the unknown involves embracing uncertainty as an opportunity for creating value and accepting that investment decisions are not once-off events but can be made as new information emerges. As shown in the analysis of the real case study, projects which have managerial flexibility built into them reflect a larger NPV than that of a traditional DCF method. The option values increase proportionally with an increase in the level of flexibility. Thus, the analysis has highlighted the importance of asking the relevant questions by managers. Instead of basing investment decisions on the outcomes of a traditional DCF analysis, the RO analysis demonstrates what could happen to iron ore prices and what that would mean for the project.

However, considering any other model, the reliability of the results is dependent on the accuracy of the input assumptions. This study has utilised data from reliable sources that are widely accessible and has applied the stochastic simulation processes to forecast future project values. It was clear from the analysis that the application of the stochastic simulation method produced more reliable estimates than the traditional DCF NPV method. Additionally, the Black – Scholes option model was used in valuing the ‘delay’ and ‘abandon’ options. This method was chosen over other models such as the binomial and risk-neutral models as it presents a lower bound value and does not typically result in overestimation.

In comparing the NPV of the traditional method to the ‘delay’, the ‘abandon’ and ‘staged investment’ options, the RO method returned higher NPVs for all three scenarios. The case study has also demonstrated that ignoring the extent of the unknown and its implications is a significant management error.

The notable distinction between this research and the available literature is its initiative in the creation of the Managerial Flexibility Domain Map. This simple structured map may be of great help to operational mine managers who can qualitatively apply the RO methodology prior to using complex quantitative models. Since RO is both qualitative and quantitative, much of the managerial flexibility value is created through qualitative thinking and processes. Therefore, this research proposes the use of this structured guide in identifying which area within the mining continuum where flexibility can be created, and then relying on the use of ROs for decision making.

This contribution to the field of ROs may encourage future research into the application of ROs in valuing uncertainty and it may eventually lead to the methodology being conventionally accepted in mine planning and development as its suitability and application have been demonstrated in this research. Even though this paper has demonstrated that there is value in unknown information, the way of identifying which information add value is not yet explored and it required further research. The science of data analytics is abetting the concept of creating value from data. Thus, there are opportunities for real option researchers to study how data analytics can help in identifying the pieces of information that add value to the mining project.

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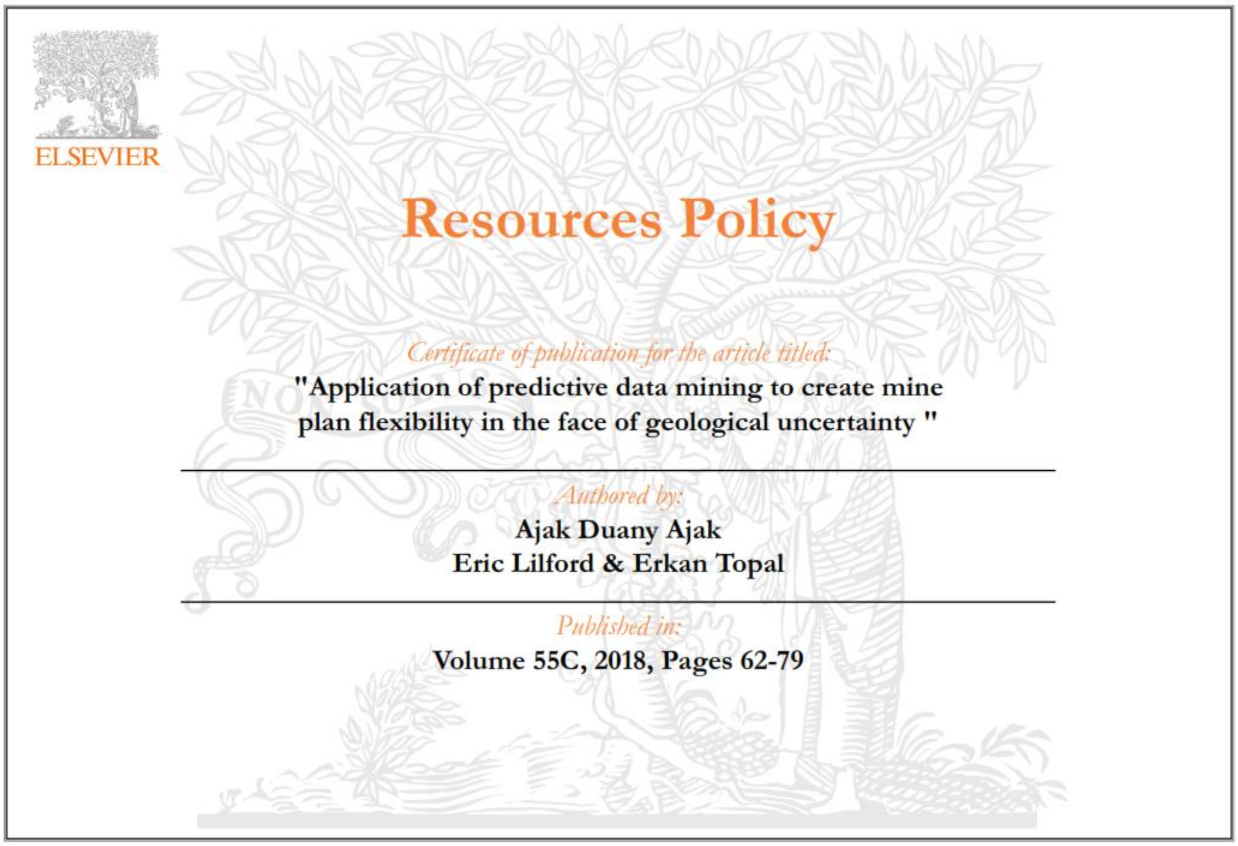
Chapter 5: Application of predictive data mining to create mine plan flexibility in the face of geological uncertainty



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Principal Author	Candidate Contribution to the Paper	Overall (%)	Signature	Date
Ajak Duany Ajak	Set research question, developed methodology and predictive data mining model, developed case studies and analysed real options, wrote manuscript and acted as corresponding author.	85%		25/7/18

Co-Author Contribution

By signing the statement of Authorship, each author certifies that:

I. the candidate's stated contribution is accurate as stated above;
II. permission is granted for the candidate to include the publication in the thesis; and
III. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Co-Author	Contribution to the Papers	Signature	Date
Erkan Topal	Supervised development of work and reviewed manuscript.		25/7/18
Eric Lilford	Supervised development of work and reviewed manuscript.		20 July 2018

Abstract

Geological uncertainty represents an inherent threat to all mining projects. Mining operations utilise resource block models as primary sources of data in planning and decision making. However, such operational decisions are not free from risk and uncertainty. For the majority of iron ore mines, as an example, uncertainties such as clay pods, variability in grades and tonnages can have significant impacts on project viability. However, a paradigm shift on how uncertainty is treated and a willingness to invest in areas that create operational flexibility can mitigate potential losses. Data analytics is touted as one of the major disruptions in the 21st century and operations that properly utilise data can create real opportunities in the face of an uncertain future. Since organisations have abundant definite geological data, a combination of data mining and real options can provide a competitive advantage. In the present study, predictive data mining algorithms were applied to a real case mine operation to predict the probability of encountering problematic ore in a mining schedule. The data mining model outputs were used to generate possible real options that the operations could exercise to deal with clay uncertainty. The most suitable data mining algorithm chosen for this task was the classification tree, which predicted the occurrence of clay with 78.6% precision. Poisson distribution and Monte Carlo simulations were applied to analyse various real options. The research revealed that operations could minimise unscheduled losses in the processing plant and could increase a project's present value by between 12% and 21% if the predictive data mining algorithm was applied to create real options.

5.1 Introduction

Understanding the ore body and operational uncertainty are crucial for every mining project as these parameters affect the ultimate delivery of the planned targets. Geological uncertainty is an inherent threat that all mining operations have to manage. However, managing risks posed by the lack of adequate ore body knowledge is a major challenge to mine planning engineers, geologists and operational managers who are under constant pressure to produce mine plans and deliver the required tonnes of ore at specified grades their executives and to the market. Even though mine geologists and planning engineers have a reasonable level of information obtained from the resource model regarding ore boundaries, tonnages, geochemical grades, lithological units and geometallurgical characteristics such as ore strength, respond to crushing, grinding and floatation processes (La Rosa et al., 2014) that are necessary in order to undertake proper planning and to warrant mining of the deposit (Cornah, 2013; U.S. Bureau of Mines and The U.S. Geological Survey, 1980), there is still geological information that is either 'known unknowns' or 'unknown unknowns' (Brown & Innocent, 2012; Brammer & Smithson, 2008). Known unknown is the availability of the information needed to work out the unknown variable while unknown unknown is existence of a variable that is not yet explored and no realisation something is missing (Brown & Innocent, 2012; Brammer & Smithson, 2008). Geological processes that lead to ore formation tend to be very complex and not every parameter can be easily estimated (Spalla et al., 2010). Therefore, geological models contain uncertainties which require quantification (Wellmann and Regenauer-Lieb, 2012). It has been proven that most mining investment decisions are commonly made based on either incomplete or inaccurate information (Haque et al., 2016).

For most iron ore deposits, uncertainties such as the presence of fibre material and clay pods are seldom modelled into ore block models (OBMs). These are the known unknowns to the majority of the iron ore operations whose production can be interrupted by the unexpected appearance of the unwanted material. Clay pods shock the processing circuit if unknowingly fed into the crusher and can cause a significant amount of downtime and reduction in productivity in the plant (AJM, 2011). Clay material accounts for approximately 15% of the feed for fine iron ore deposits and it is generally sent to the tailings dam (Clout, 2013). However, for balanced iron ore operations, the clay proportion in the plant feed can range between 6% and 11% but constitutes between 23% and 46% of product in the wet process. Thus, its potential overall impact on individual operations should never be ignored during the mine planning process.

In operations where the ore body has a high clay content which is modelled into the ore body, a desand process has usually been implemented. This is a two-stage process that is a combination of the traditional wet processing plant and a desand circuit. Desand plants are very expensive to build and operate at a high cost. Apart from the Fortescue Metals Group Limited (FMG) Cloudbreak Mine, these plants are sparingly used in the Pilbara region in Australia as they require a constant supply of clean water that must be supplied in large volumes to operate. In addition, they are not commonly built as there is usually very limited data on the prevalence of the clay material in the ore deposit to justify the inclusion of a desand plant, which is capital intensive. Therefore, most operations in the Pilbara use wet plants that integrate either one- or two-stage hydrocyclone circuits due to their ability to upgrade products (Clout, 2013). However, these circuits are not intended to process clay material. For mining operations that operate one or two production pits or those that utilise a single processing plant, any downtime caused by clay material feed can cumulatively translate into significant financial losses, which are published at the end of each reporting period.

Big data is touted as one of the major disruptions in the 21st century (Deloitte, 2016) and has recently been the focus of many big firms. Data are expected to grow as technology improves and business units within operations become more connected. Companies that use and treat data as an important asset can create value through predictive analytics. Underground patterns can be visualised easily and relationships to the available information can be revealed, as well as options created and utilised to avoid or mitigate losses. However, mining operations are not making full use of this abundant data. Regardless of the reluctance to adopt data mining, it remains a superior method for obtaining knowledge of what is hidden inside the messier real-world data than the standard statistical techniques that are commonly applied (Berson et. al., 1999).

Considering there is already plenty of data available for improving decision making, especially for mining operations, a literature search returned limited cases of the application of predictive data mining in solving clay material geological uncertainty in mining operations, particularly in the creation of managerial flexibility. It is important to highlight that there is a disconnect between technical personnel such as mine planning engineers and data managers who focus on analysing activities that are geared to improve productivity and efficiencies rather than creating flexibility that should be considered at the early stages of mine planning and design. Therefore, the problem that the present study is aiming to address is how to use data mining methodologies for creating real options, particularly when dealing with internal uncertainty that results in operational risks.

5.2 Methodology

In the present study, the predictive data mining algorithms such as decision tree classification or ID3 machine learning model will be utilised for predicting the occurrence of the problematic ore in mine schedules. The outputs will then be applied for the purpose of creating and analysing RO in a real-world case study. Orange (Demsar et al., 2013) and RapidMiner (2017) software programs that combine both statistical and machine learning capabilities will be applied to create both the training and machine learning models for predicting clay pod occurrences.

Once the model is confirmed, it will then be applied to an already generated mine plan that has utilised OBM data to predict any problematic ore via qualitative classification of the risks in each block that has been scheduled for mining during each period. The results of the analysis will then be utilised to create various real options that provide managerial flexibility in the running of their respective mining operations.

5.2.1 Predictive data mining with id3 decision tree classification

Predictive analytics is the process of applying various mathematical formulae to discover the best decision for a given situation and to eliminate guess work about the future (Mishra et. al., 2010). There is a large number of algorithms that can be used to perform predictive data mining (PDM). This data mining activity aims to quantify the probability of intercepting a problematic ore or clay pods during the planning of the real case mine study. This will assist in the creation of real options for managing geological uncertainty as the predictive model will be generated and applied to create the necessary managerial flexibility to reduce plant downtimes and unnecessary maintenance. A decision tree classification, which is also known as the ID3 algorithm, will be utilised to demonstrate how data mining can be applied to create real options analysis. In the next sections, the ID3 algorithm will be explained through examples. Since the field of data mining is a specialised area of study, it is beyond the scope of this research to explore the presented theories in detail. Instead, the research will focus on the chosen techniques among other classes of data mining summarised in Fig. 5.1. This is because the use of geological data including structure, lithology and mineralogy, ore types and associated elements, geophysical and geochemical data of an ore deposit is considered to be the fundamental method in descriptive models which were suggested and consequently developed by Cox & Singer (1986). These models have a major disadvantage as follows:

- The geological uncertainty is not observed (visual assessment) with these methods while the geological uncertainty is an obvious and salient feature which has to be always predicted for mining scheduling (Yasrebi, 2013).

Data mining has proved their superiority to the classical statistical and conventional geological methods as follows:

- In classical statistics, frequency distribution of a desired attribute in an intended area must adhere to a normal distribution. To do this, different populations based on mean and standard deviation should be carried out with normalised data. This is not met in data.

- In addition, local neighbourhood statistics can provide less statistical information which is less biased than that of global statistics, such as mean and SD because the utilised data generally satisfy non-normal distributions and contain outliers (Agterberg et al., 1993; Zhang et al., 2007, Yasrebi, 2014).

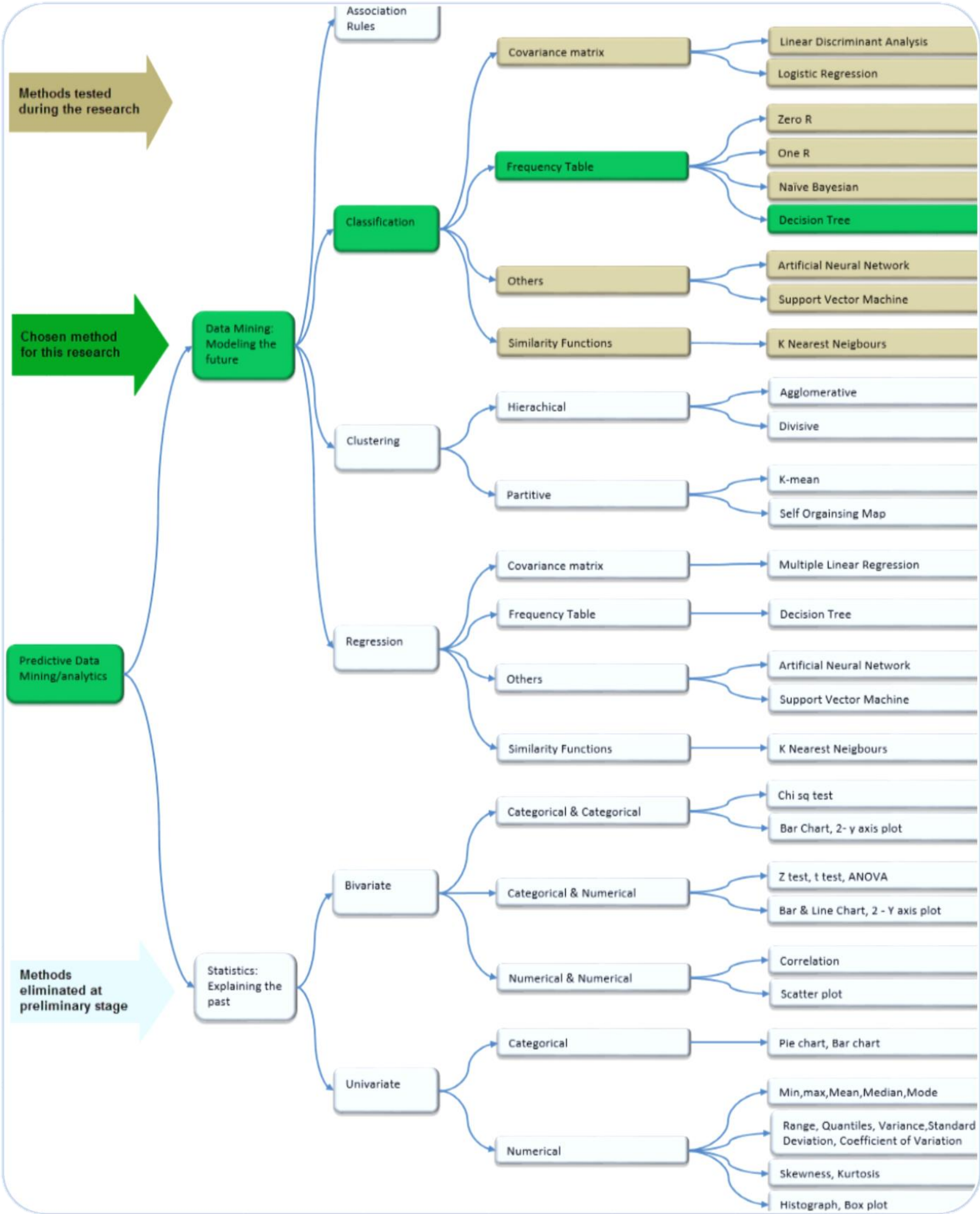


Fig. 5.1,Schematic diagram summarising various data mining models (Sayad, 2017)

5.2.2 Modelling clay uncertainty in a mining operation using decision tree classification

Decision trees are a class of regression models that have been restructured in the form of a tree that is more visual than conventional statistical regression models. Thus, it is a form of machine learning that analyses the past data and uses it to predict future events of similar characteristics. The input data, which is also referred to as the predictor, is broken down into smaller units and the model continues to break down the data until the target variable is predicted. The theory behind decision tree classification is based on the ID3 algorithm. This algorithm deploys what is referred to as "greedy search" where all the possible branches are explored in the probability space without backtracking. This algorithm uses entropy and information gain (also referred to as gain ratio) in the RapidMiner (2017) and Orange (Demsar et al., 2013) software programs (Sayad, 2017). These will be utilised in the present study.

The decision tree was chosen as the best model for this research due to the following reasons (Peng et al., 2007):

- It has the ability to generalise unobserved instances that have features that are correlated with target variables.
- It is efficient and intuitive in its computation.
- The resultant tree provides a conceptual representation that is transparent and easy to understand.

5.2.2.1 Building a decision tree using entropy and information gain ratio

The decision tree is built from the top to the bottom as shown in Fig. 5.1. The dataset is subdivided into a small group of similar values or with homogenous properties and then their entropy is calculated. If the dataset is completely homogenous, it is classed as having an entropy of zero whereas an entropy of one applies if the data is equally divided.

Therefore, Eq. (5.1) is the underlying equation for the entropy of a single predictor:

$$E(S) = -\sum_{1}^{n} p_i \, log_2 p_i \tag{5.1}$$

and Eq. (5.2) is for more than one predictor:

$$E(T,X) = \sum_{C \in X} P(c)E(c) \tag{5.2}$$

Where, E(S) is the entropy of the attribute and P is the probability of the attribute occurring.

Note that Log_2 can be replaced with Log_N where N is the number of variable classes to normalise the entropy. Therefore, the entropy of the target variable, which is the ore processing risk, will be calculated in the next sections.

To clearly explain the mathematical model of the decision tree classification which will be used to predict clay uncertainty in the present case study, 35 samples were randomly selected from the training data which was composed of 832 mining blocks. The main variables were $\mathrm{Iron}_{\mathrm{fe}}$, $\mathrm{Silica}_{\mathrm{SiO_2}}$, $\mathrm{Alumina}_{\mathrm{Al_2O_3}}$, $\mathrm{Magnesium}_{\mathrm{mgo}}$, $\mathrm{Calcium}_{\mathrm{cao}}$, and $\mathrm{Phosphorous}_{\mathrm{p}}$.The influence of these variables on clay pods were tested as geologists have long noticed associations between these variables, particularly between SiO_2, Al_2O_3 and clay. As shown in Table 5.1, the target variable is the overall processing risk and the variables to the left of the table are the predictors. To illustrate the concept of entropies and information gain, the numerical values of predictors shown in Table 5.1 are converted into categorical data based on the risk rating of each block depending on the ease of processing the ore. As mentioned previously, the key features of each block, which are associated with process risk, are lump (%), fine alumina and water reactive clay values if intercepted. These characteristics have been classified as shown in Table 5.2 in accordance with the ore rating that is commonly used in most grade control processes in iron ore mines.

Riskex's risk score calculator (Fig. 5.2) was then used to obtain the rating for sample blocks. Thus, the numerical values of this data were encoded into categorical data as shown in Table 5.3 as this will be utilised in the calculation logic to be explained in three steps (shown in the following subsections).

Table 5.1, Grade control sample data for illustrating the ID3 algorithm

Regular variables/Identifiers				Predictors categorical data							Target
Block ID	Pit_Name	Reduced_Level	Block_Name	Lump%	Fe%	Si%	Al%	Fines Al%	Loi%	Water Reactive Clay %	Overal Processing Risk
1	CASEPIT3	640	164	34.9	64.92	0.96	1.25	1.56	4.47	0.0	Moderate
2	CASEPIT3	640	164	34.83	64.67	1.09	1.3	1.59	4.59	0.0	Moderate
:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:
33	CASEPIT2	630	30	35.56	63.97	1.54	1.54	2.01	4.96	1.3	Substantial
34	CASEPIT3	630	124	35.98	63.78	1.54	1.79	2.09	4.9	0.3	Substantial
:	:	:	:	:	:	:	:	:	:	:	:
832	CASEPIT1	630	16	30.2	7.45	55.38	15.98	18.81	12.37	0.0	High

****Note that the training data had 832 mining blocks and what is showed here in this paper is a small sample for illustration purposes.*

Table 5.2, Ore block processing risk rating

Ore Block Feature	Probability	Exposure	Consequence	Riskex Risk Score Calculator Rating
Lump % <32.99	Quite possible	Frequent	Important	65.5
Lump % <35.99	Unusual but possible	Frequent	Important	23.7
Lump % <42.99	Conceivable	Frequent	Noticeable	2.1
Lump % >43	Unusual but possible	Frequent	Important	23.7
Al & Fines Al % <3	Conceivable	Frequent	Noticeable	2.1
Al & Fines Al % >3	Unusual but possible	Frequent	Important	23.7
Water Reactive Clay % > 0.5	Quite possible	Frequent	Important	60.7
Fe, Si & Loi %	Conceivable	Frequent	Noticeable	2.1

Processing Risk	Riskex Risk Score Calculator Rating
Low	2.1
Moderate	2
Substantial	65.2
High	338.1
Very High	775

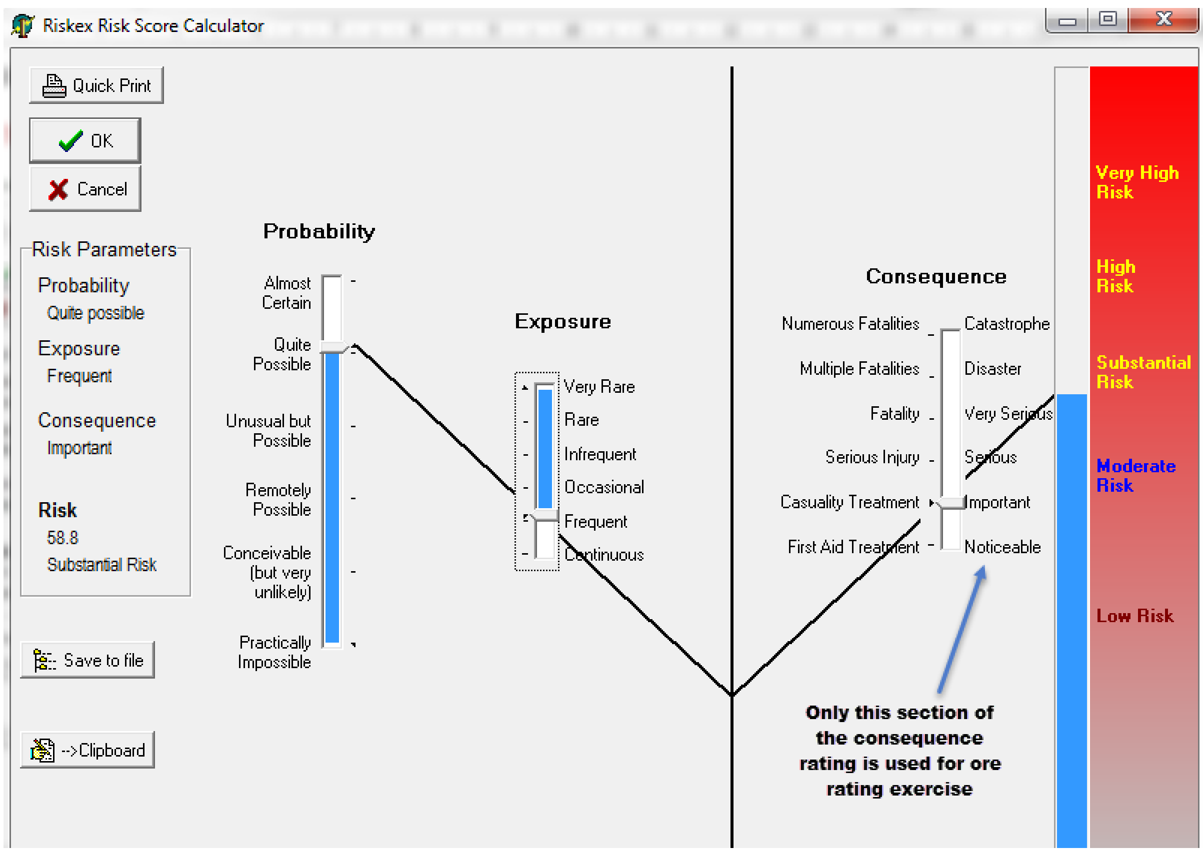


Fig. 5.2, Riskex's risk score calculator (Riskex, 2017).

Table 5.3, Conversion of numerical grade control data to categorical for explanation of entropy and information gain concepts

Regular variables/Identifiers				Predictors categorical data							Target	Real Option Decision
Block ID	**Pit_Name**	**Reduced _Level**	**Block_ Name**	**Lump**	**Fe**	**Si**	**Al**	**Fines Al**	**Loi**	**Water Reactive Clay**	**Overal Processing Risk**	**Decision Criteria**
1	CASEPIT3	640	164	Moderate	Low	Low	Low	Low	Low	Low	Moderate	Create Option
2	CASEPIT3	640	164	Moderate	Low	Low	Low	Low	Low	Low	Moderate	Create Option
3	CASEPIT2	630	142	Moderate	Low	Low	Low	Low	Low	Substantial	Substantial	Create Option
4	CASEPIT1	620	100	Moderate	Low	Low	Low	Low	Low	Low	Moderate	Create Option
5	CASEPIT1	620	101	Moderate	Low	Low	Low	Low	Low	Low	Moderate	Create Option
6	CASEPIT2	630	30	Moderate	Low	Low	Low	Low	Low	Substantial	Substantial	Create Option
7	CASEPIT2	620	101	Low	Low	Low	Low	Low	Low	Low	Low	No Action
8	CASEPIT1	620	140	Substantial	Low	Low	Moderate	Moderate	Low	Low	Very High	Create Option
9	CASEPIT1	630	127	Substantial	Low	Low	Moderate	Moderate	Low	Substantial	Very High	Create Option
10	CASEPIT2	640	114	Substantial	Low	Low	Moderate	Moderate	Low	Substantial	Very High	Create Option
11	CASEPIT2	620	101	Substantial	Low	Low	Moderate	Moderate	Low	Substantial	Very High	Create Option
12	CASEPIT3	630	116	Substantial	Low	Low	Moderate	Moderate	Low	Substantial	Very High	Create Option
13	CASEPIT1	620	25	Substantial	Low	Low	Moderate	Moderate	Low	Substantial	Very High	Create Option
14	CASEPIT3	650	133	Low	Low	Low	Low	Low	Low	Low	Low	No Action
15	CASEPIT1	620	103	Low	Low	Low	Low	Low	Low	Low	Low	No Action
16	CASEPIT2	640	106	Low	Low	Low	Low	Low	Low	Low	Low	No Action
17	CASEPIT1	620	103	Low	Low	Low	Low	Low	Low	Low	Low	No Action
18	CASEPIT1	620	103	Low	Low	Low	Low	Low	Low	Low	Low	No Action
19	CASEPIT1	630	157	Low	Low	Low	Low	Low	Low	Low	Low	No Action
20	CASEPIT2	620	101	Low	Low	Low	Low	Low	Low	Low	Low	No Action
21	CASEPIT3	650	133	Low	Low	Low	Low	Low	Low	Low	Low	No Action
22	CASEPIT1	630	125	Low	Low	Low	Low	Low	Low	Low	Low	No Action
23	CASEPIT2	640	106	Low	Low	Low	Low	Low	Low	Low	Low	No Action
24	CASEPIT3	630	124	Low	Low	Low	Low	Low	Low	Low	Low	No Action
25	CASEPIT2	640	106	Low	Low	Low	Low	Low	Low	Low	Low	No Action
26	CASEPIT2	640	106	Low	Low	Low	Low	Low	Low	Low	Low	No Action
27	CASEPIT3	630	124	Low	Low	Low	Low	Low	Low	Low	Low	No Action
28	CASEPIT3	650	133	Low	Low	Low	Low	Low	Low	Low	Low	No Action
29	CASEPIT2	640	106	Low	Low	Low	Low	Low	Low	Low	Low	No Action
30	CASEPIT3	650	133	Low	Low	Low	Low	Low	Low	Low	Low	No Action
31	CASEPIT2	630	145	Substantial	Low	Low	Low	Low	Low	Substantial	High	Create Option
32	CASEPIT1	630	107	Moderate	Low	Low	Moderate	Moderate	Low	Substantial	High	Create Option
33	CASEPIT1	620	124	Substantial	Low	Low	Low	Low	Low	Substantial	High	Create Option
34	CASEPIT2	630	145	Substantial	Low	Low	Low	Low	Low	Substantial	High	Create Option
35	CASEPIT2	630	140	Substantial	Low	Low	Low	Low	Low	Substantial	High	Create Option

****Note that the training data had 832 mining blocks and what showed here a sample of 35 blocks to be used in explanation and illustration of the entropy and gain ratio concepts.*

5.2.2.2 Entropy of ore processing risk as per Equation 2.

Eq. (5.2) will be applied in this section to calculate the entropy of ore processing risks based on Table 5.4.

Table 5.4, Overall Ore Processing Risk Frequency table.

Overall Ore Processing Risk				
Very High	High	Substantial	Moderate	Low
6	5	2	4	18

Entropy (Processing Risk) = Entropy (6, 5, 2, 4, 18)

$$= -\{ (0.171 \log_5 0.171) + (0.143 \log_5 0.143) + (0.057 \log_5 0.057) + (0.114 \log_5 0.114) + (0.514 \log_5 0.514) \}$$

$$= \mathbf{0.829}$$

When a fitting is based on the notion of decreasing entropy once the sample data have been split, it is referred to as the information gain ratio. The algorithm loops through the data until the attribute with the highest gain ratio is found and this is then used to construct the decision tree. The information gain is the most popular among other classification tree algorithms and this is the default classification in both the RapidMiner (2017) and Orange (Demsar et al., 2013) software programs, which will be used in this research. The equation for the entropies is shown below.

$$Gain\ (T, X) = Entropy\ (T) - Entropy\ (T, X)$$

5.2.2.3 Entropy and gain ratio of predictors

To explain how the entropies and gain ratio were achieved, the entropy and gain ratio of lump (%) are shown as an example below:

$$\mathbf{E}(Processing\ Risk, Lump\ (\%)) = \mathbf{P}(Very\ High)*\mathbf{E}(0,0,0,0,0) + \mathbf{P}(High)*\mathbf{E}(0,0,0,0,0) + \mathbf{P}(Substantial)*\mathbf{E}(6,4,0,0,0) + \mathbf{P}(Moderate)*\mathbf{E}(0,1,2,4,0) + \mathbf{P}(Low)*\mathbf{E}(0,0,0,0,18)$$

$$= 0 + 0 + \frac{10}{35}(0.418) + \frac{7}{35}(0.594) + \frac{10}{35}(0)$$

$$= 0.238$$

$$Gain\ (Processing\ Risk, Lump(\ \%)) = 0.8269 - 0.238$$

$$= 0.522$$

Table **5.5** shows the frequencies and summary of the calculation for ore processing risk using the other predictors.

Table 5.5, Predictor frequencies calculated entropies and gain ratios.

		Overall Ore Processing Risk					
		Very High	High	Substantial	Moderate	Low	
Lump %	Very High	0	0	0	0	0	0
	High	0	0	0	0	0	0
	Substantial	6	4	0	0	0	10
	Moderate	0	1	2	4	0	7
	Low	0	0	0	0	18	18
	Gain	*0.590*	*Entropy (Processing Risk, Lump %)*			*0.238*	35

		Overall Ore Processing Risk					
		Very High	High	Substantial	Moderate	Low	
Fines Al %	Very High	0	0	0	0	0	0
	High	0	0	0	0	0	0
	Substantial	0	0	0	0	0	0
	Moderate	6	1	0	0	0	7
	Low	0	4	2	4	18	28
	Gain	*0.267*	*Entropy (Processing Risk, Fines Al %)*			*0.562*	35

		Overall Ore Processing Risk					
		Very High	High	Substantial	Moderate	Low	
Water Reactive Clay	Very High	0	0	0	0	0	0
	High	0	0	0	0	0	0
	Substantial	5	4	0	2	1	12
	Moderate	0	0	0	0	0	0
	Low	1	0	0	4	18	23
	Gain	*0.307*	*Entropy (Processing Risk, Water Reactive Clay)*			*0.522*	35

		Overall Ore Processing Risk					
		Very High	High	Substantial	Moderate	Low	
Al %	Very High	0	0	0	0	0	0
	High	0	0	0	0	0	0
	Substantial	0	0	0	0	0	0
	Moderate	6	1	0	0	0	7
	Low	0	4	2	4	18	28
	Gain	*0.267*	*Entropy (Processing Risk, Al %)*			*0.562*	35

		Overall Ore Processing Risk					
		Very High	High	Substantial	Moderate	Low	
Fe %	Very High	0	0	0	0	0	0
	High	0	0	0	0	0	0
	Substantial	0	0	0	0	0	0
	Moderate	0	0	0	0	0	0
	Low	6	5	2	4	18	35
	Gain	*0.000*	*Entropy (Processing Risk, Fe %)*			*0.829*	35

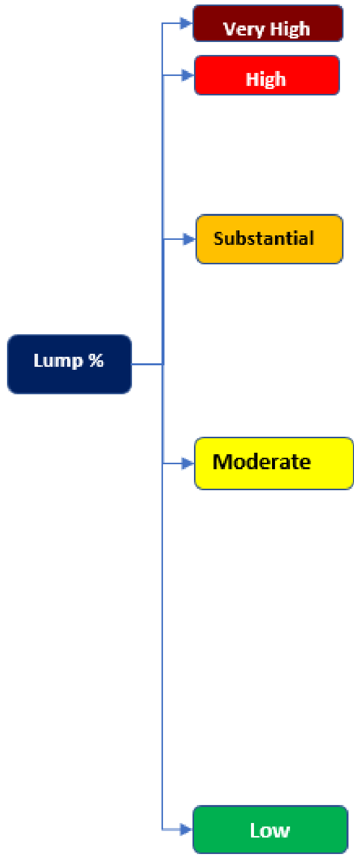
		Overall Ore Processing Risk					
		Very High	High	Substantial	Moderate	Low	
Loi %	Very High	0	0	0	0	0	0
	High	0	0	0	0	0	0
	Substantial	0	0	0	0	0	0
	Moderate	0	0	0	0	0	0
	Low	6	5	2	4	18	35
	Gain	*0.000*	*Entropy (Processing Risk, Loi %)*			*0.829*	35

		Overall Ore Processing Risk					
		Very High	High	Substantial	Moderate	Low	
Si %	Very High	0	0	0	0	0	0
	High	0	0	0	0	0	0
	Substantial	0	0	0	0	0	0
	Moderate	0	0	0	0	0	0
	Low	6	5	2	4	18	35
	Gain	*0.000*	*Entropy (Processing Risk, Si%)*			*0.829*	35

Gain (ProcessingRisk, Lump) = 0.590
Gain (ProcessingRisk, Clay) = 0.307
Gain (ProcessingRisk, Fines Al) = 0.267
Gain (ProcessingRisk, Al) = 0.267
Fe, Si & Loi have a gain of zero or closer to zero

5.2.2.4 Identifying decision mode

As shown in subsection b), the element with the highest gain ratio of 0.522 is iron ore lump (%) (Table 5.5). Thus, lump (%) will be used as the decision node as summarised in Fig. ***5.3*** and Fig. 5.4. It should be noted that any attribute with a gain of zero will not be a priority during the algorithm search. This process is iterative and is best performed by a machine learning tool such as RapidMiner (2017) or Orange (Demsar et al., 2013) software programs. The decision rule in this analysis that will be used to create operational flexibility is that 'If an overall processing risk is Very High, High, Substantial or Moderate then the option must be created, otherwise no action is required'.



Lump	Fe	Si	Al	Fines Al	Loi	Water Reactive Clay	Overal Processing Risk	Decision Criteria
Substantial	Low	Low	Moderate	Moderate	Low	Low	Very High	Create Option
Substantial	Low	Low	Moderate	Moderate	Low	Substantial	Very High	Create Option
Substantial	Low	Low	Moderate	Moderate	Low	Substantial	Very High	Create Option
Substantial	Low	Low	Moderate	Moderate	Low	Substantial	Very High	Create Option
Substantial	Low	Low	Moderate	Moderate	Low	Substantial	Very High	Create Option
Substantial	Low	Low	Moderate	Moderate	Low	Substantial	Very High	Create Option
Substantial	Low	Low	Low	Low	Low	Substantial	High	Create Option
Substantial	Low	Low	Low	Low	Low	Substantial	High	Create Option
Substantial	Low	Low	Low	Low	Low	Substantial	High	Create Option
Substantial	Low	Low	Low	Low	Low	Substantial	High	Create Option
Moderate	Low	Low	Low	Low	Low	Low	Moderate	Create Option
Moderate	Low	Low	Low	Low	Low	Low	Moderate	Create Option
Moderate	Low	Low	Low	Low	Low	Substantial	Substantial	Create Option
Moderate	Low	Low	Low	Low	Low	Low	Moderate	Create Option
Moderate	Low	Low	Low	Low	Low	Low	Moderate	Create Option
Moderate	Low	Low	Low	Low	Low	Substantial	Substantial	Create Option
Moderate	Low	Low	Moderate	Moderate	Low	Substantial	High	Create Option
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action
Low	Low	Low	Low	Low	Low	Low	Low	No Action

Fig. 5.3, Illustration of decision rule and decision node identification.

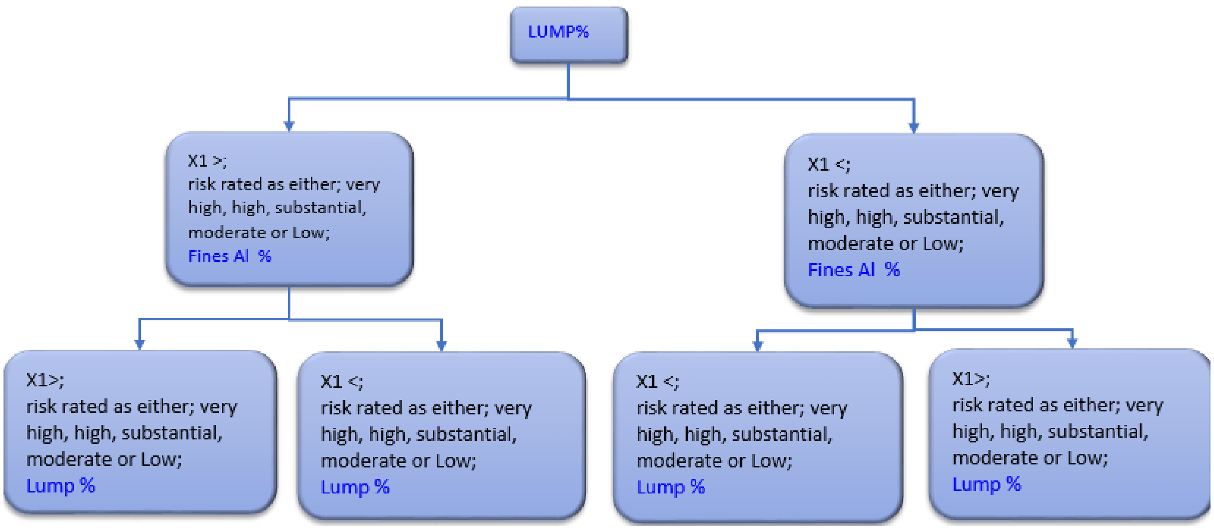


Fig. 5.4, Predictive decision tree classification model for clay material uncertainty in a mine plan based on ID3 algorithm.

5.3 Predictive data mining problem framing

The main problem to be solved by data mining is how to predict clay material in the ore before it is fed into the processing plant. Even though there is enough drilling and assayed data, the issue in the industry is that most of the mining professionals do not perform in-depth data mining. Apart from performing simplistic reconciliations and descriptive statistics for mine conformance purposes, mining operations do not undertake serious data analytics exercises and data is only accumulated without exploiting its potential benefits.

Even though data mining has gained acceptance in market research and the financial sectors, it has seldom been properly applied to solve any mine planning and operational problems, and nor has it been utilised in geological ore boundaries modelling. The commonly used deterministic models do not correctly quantify the size and occurrence of uncertain variables as they are based on probabilistic statistics and spatial data as mentioned previously, or simply on expert opinions. The real concern is that important decisions that can easily destroy the economic value of the mining operation in question are based on managerial intuitions that are backed up with standard analysis that have been proven to fail and therefore it is the aim of data mining to remove the guess-work when making investment decisions (Berson et.al., 1999). Performing proper data analytics can assist in making informed decisions. Therefore, this data mining task aims to achieve the following:

- Find a coefficient, predictor or the best model that will be able to indicate the likelihood of encountering reactive clay material and isolate such material prior to being fed into the processing plant.
- Utilise the result of the data mining model to create real options that management can use to avoid or mitigate losses and capitalise on opportunities presented by geological uncertainty.

To successfully implement PDM to solve the problematic ore uncertainty for the real case mining operation, the following questions were asked throughout this research and during the PDM application:

- How do iron ore geochemical elements, particularly Fe, SiO_2, Al_2O_3 and Phosphorus relate to reactive clay and what pattern can be revealed by analysing these elements?
- What associations do these geochemical parameters have on clay and the uncertain parameters when they are used in predicting its occurrence?
- Which element has the highest association for predicting clay pods?
- How can actual data be utilised to predict potential clay areas in mine plans?
- How can the predicted uncertain values be utilised to identify real options in a mine plan and create managerial flexibility in mine operations?

The aim of this case study is not to pinpoint single values of the uncertain variables. Rather the aim is to qualitatively predict the likelihood of intercepting problematic ore in mine plans and to estimate the probability of such occurrence. Any machine learning algorithm that can qualitatively rate the planned ore with more than 50% chance or with a good precision, and strong correlation between the predictors and target variable will be deemed a success. This would then have achieved the main objective of this research, which is to change the direction of uncertain variables in the ore being rated from low to high risk.

5.3.1 Data mining process

Implementation of any data mining project requires the use of structured procedures and rules, which are commonly known as Cross Industry Standard Process for Data Mining (CRISP–DM). Such logic will guide the analysis in this research but in simplified steps that are suitable to the research scope. The sequence that will be followed is showed in Fig. 5.5 below.

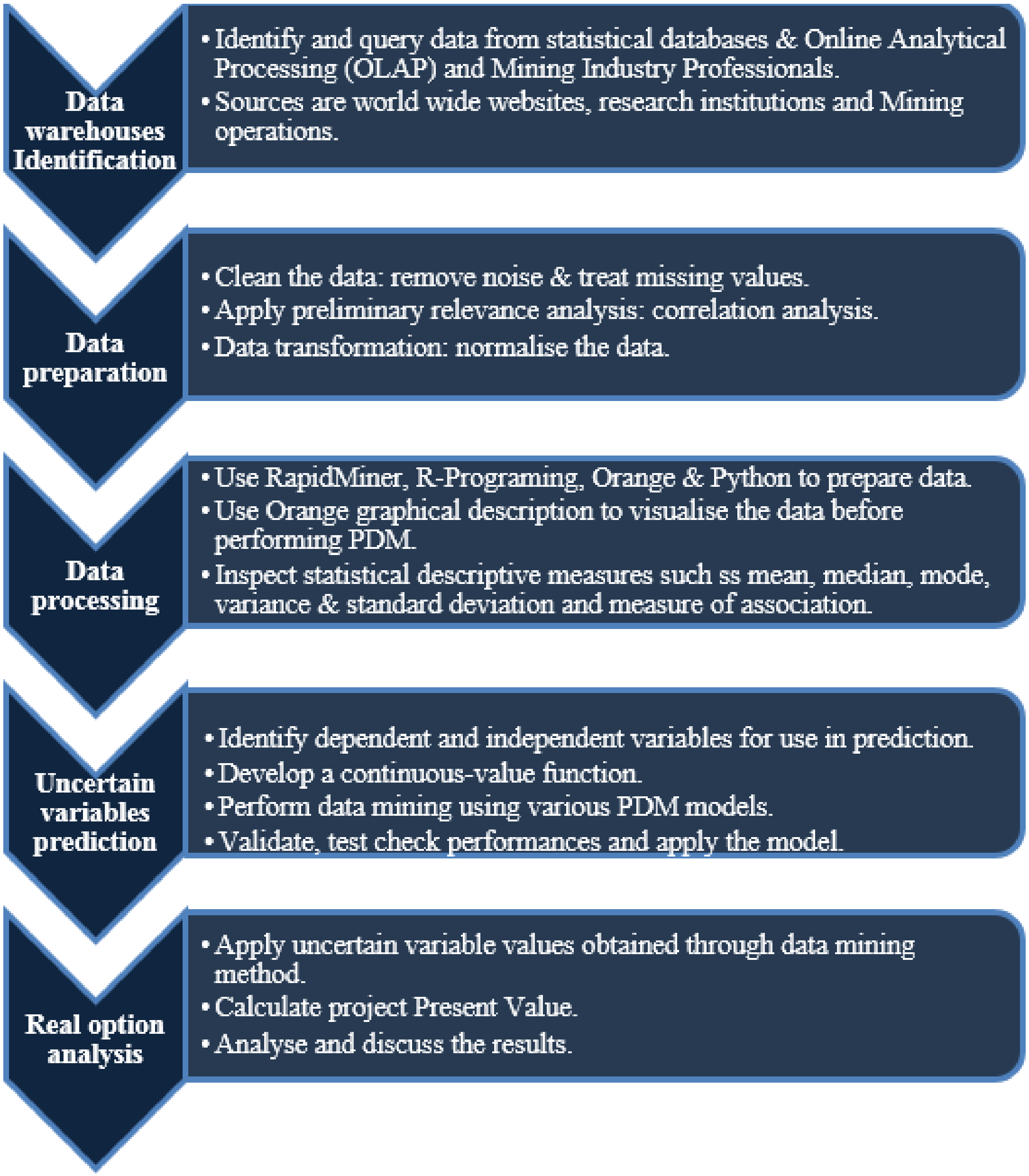


Fig. 5.5, The proposed predictive data mining functionality for real options analysis

5.3.1.1 Understanding data from real case mining operations

This research will utilise actual industry data from an operating mine in Western Australia. The actual grade control assay data will be used for training and testing the model and the mine plan data will be used to apply the model for real options analysis.

5.3.1.2 Data preparation and processing

Data processing involves checking for outliers and missing values as well as visually exploring the data (Fig. 5.6). In the Orange (Demsar et al., 2013) software program, variables were assigned to roles that will each plan machine learning. The special variable in this analysis was the processing risk, which was set as the 'Label' because it is the target.

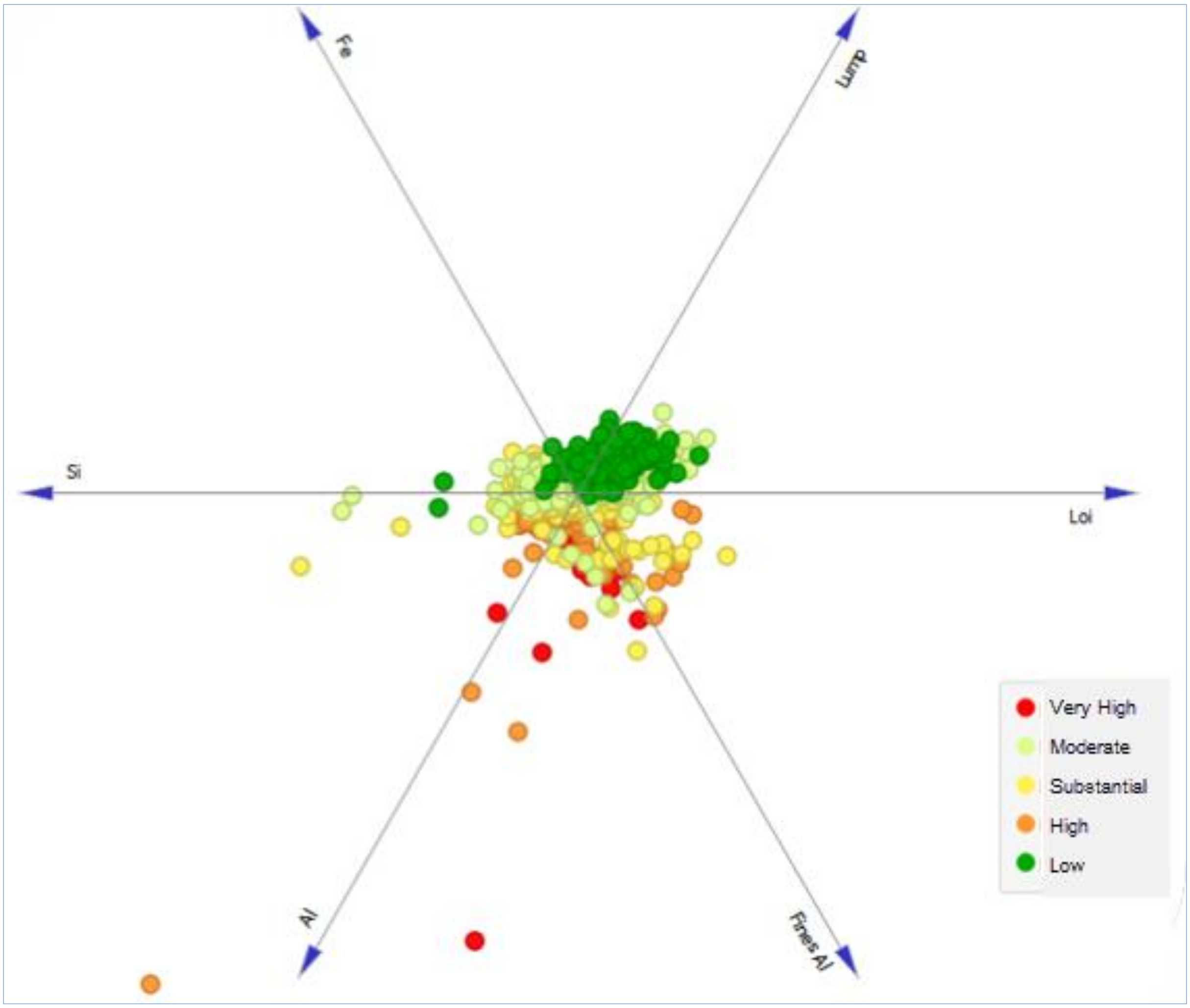
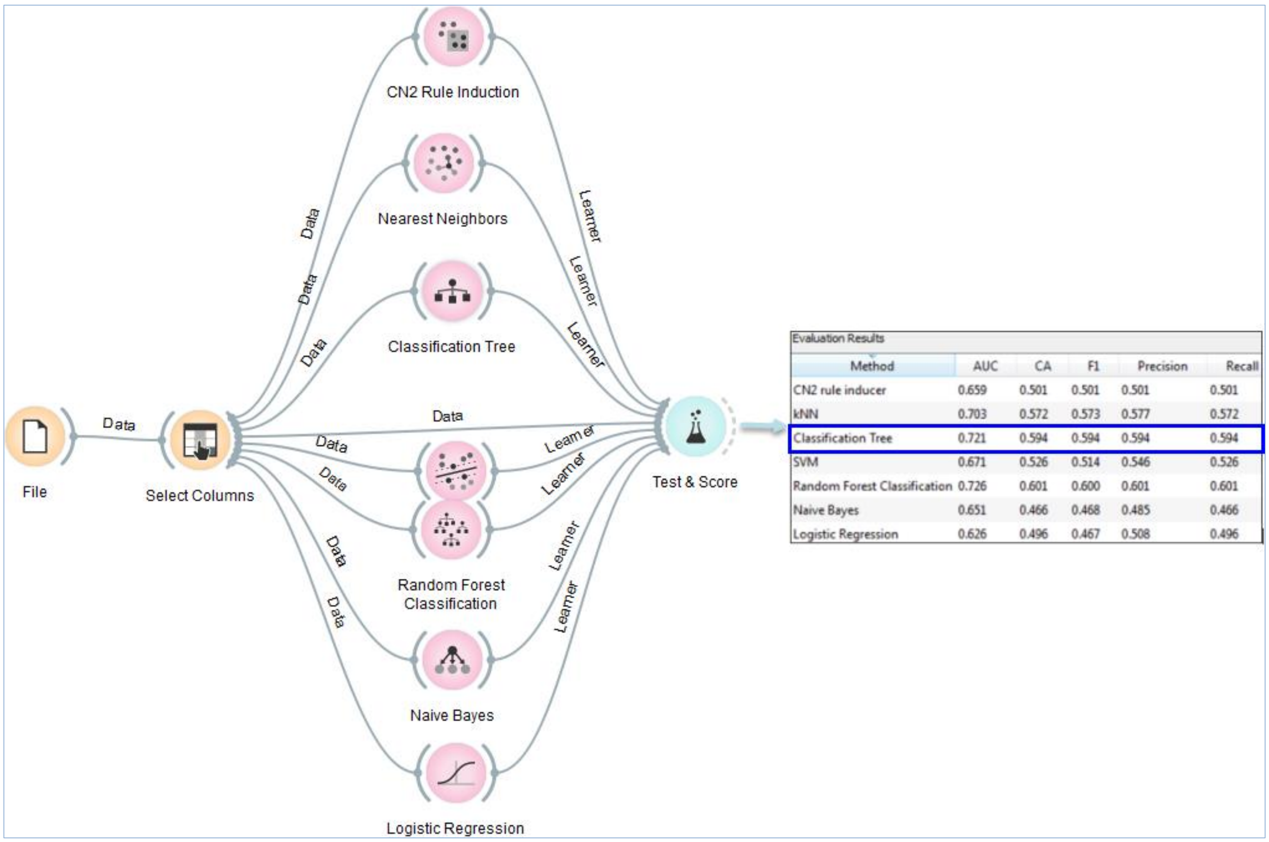


Fig. 5.6, Visualising outliers using Orange linear projection (Demsar et al., 2013).

This visual inspection indicated that alumina and lump (%) will play an important role when predicting processing risk as they are the best predictors of clay occurrence. Secondly, there were extreme cases of a few blocks with very high alumina percentages and those blocks were already ranked as high risk during block logging and grade control processes.

5.3.1.3 Data mining model selection

Most of the applicable algorithms were tested in the Orange (Demsar et al., 2013) software program as shown in Fig. 5.7. It was apparent from the test results that the decision tree and random forest classifications produced precise results with better accuracy. To choose between the two, further evaluation was performed in RapidMiner, with the results showing that the decision tree produced better results. Therefore, this was chosen for implementation.



Method	AUC	CA	F1	Precision	Recall
CN2 rule inducer	0.659	0.501	0.501	0.501	0.501
kNN	0.703	0.572	0.573	0.577	0.572
Classification Tree	0.721	0.594	0.594	0.594	0.594
SVM	0.671	0.526	0.514	0.546	0.526
Random Forest Classification	0.726	0.601	0.600	0.601	0.601
Naive Bayes	0.651	0.466	0.468	0.485	0.466
Logistic Regression	0.626	0.496	0.467	0.508	0.496

Fig. 5.7, Model selection based on classification accuracy.

5.3.1.4 Implementing the data mining model

The decision tree classification model was implemented in the RapidMiner (2017) software program. As described previously, the training data was loaded and the processing risk was set as the target variable. Cross validation, a nested activity where an algorithm can be changed, was applied and tested at the same time. Finally, the datamining model was applied to real mine plan data and the output data that contains predictions that are required for real options analysis was exported into a spreadsheet. Additionally, RapidMiner produced a visual tree classification of the data. The resultant tree contained 256 nodes, which could only be shown in a circular format due to its large size (Fig. 5.8). The algorithm confirmed the expectation that lump and fine alumina percentages where the priority attributes. The dataset was first split based on lump and fine alumina percentages and it then cascaded down to other elements. For clarity, a truncated version of the tree is shown in Fig. 5.9.

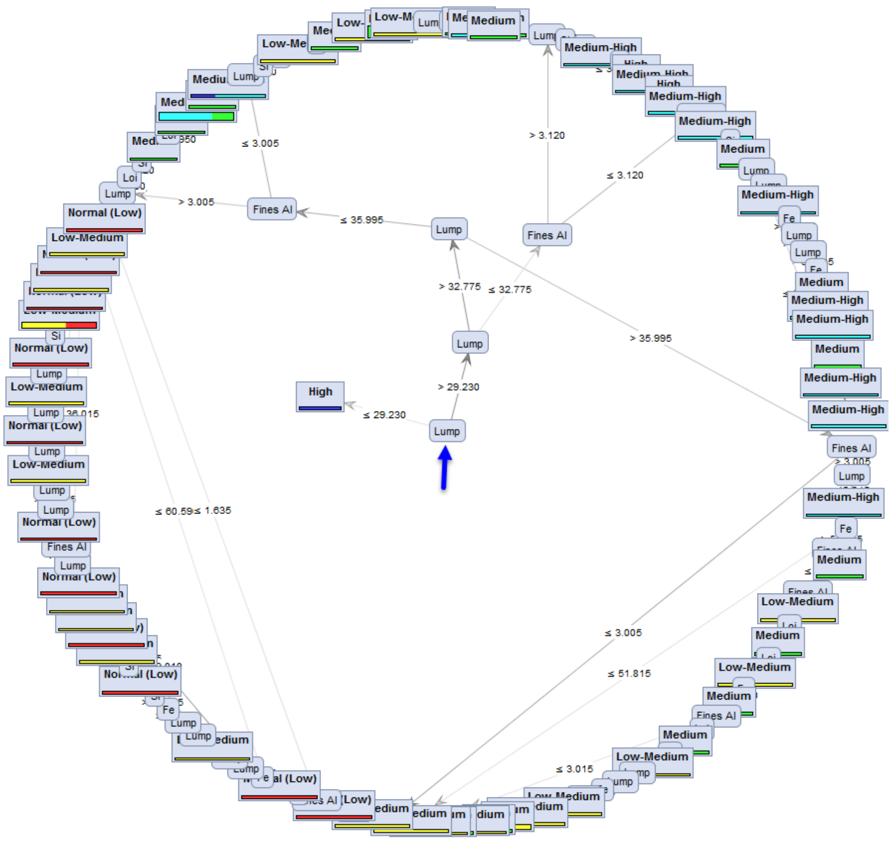


Fig 5.8, Circular view of decision tree classification of clay material.

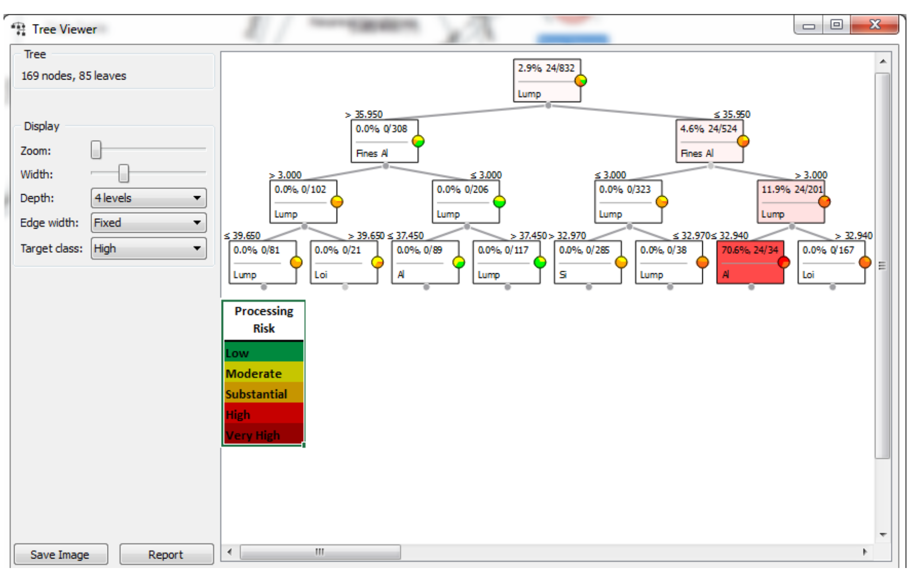
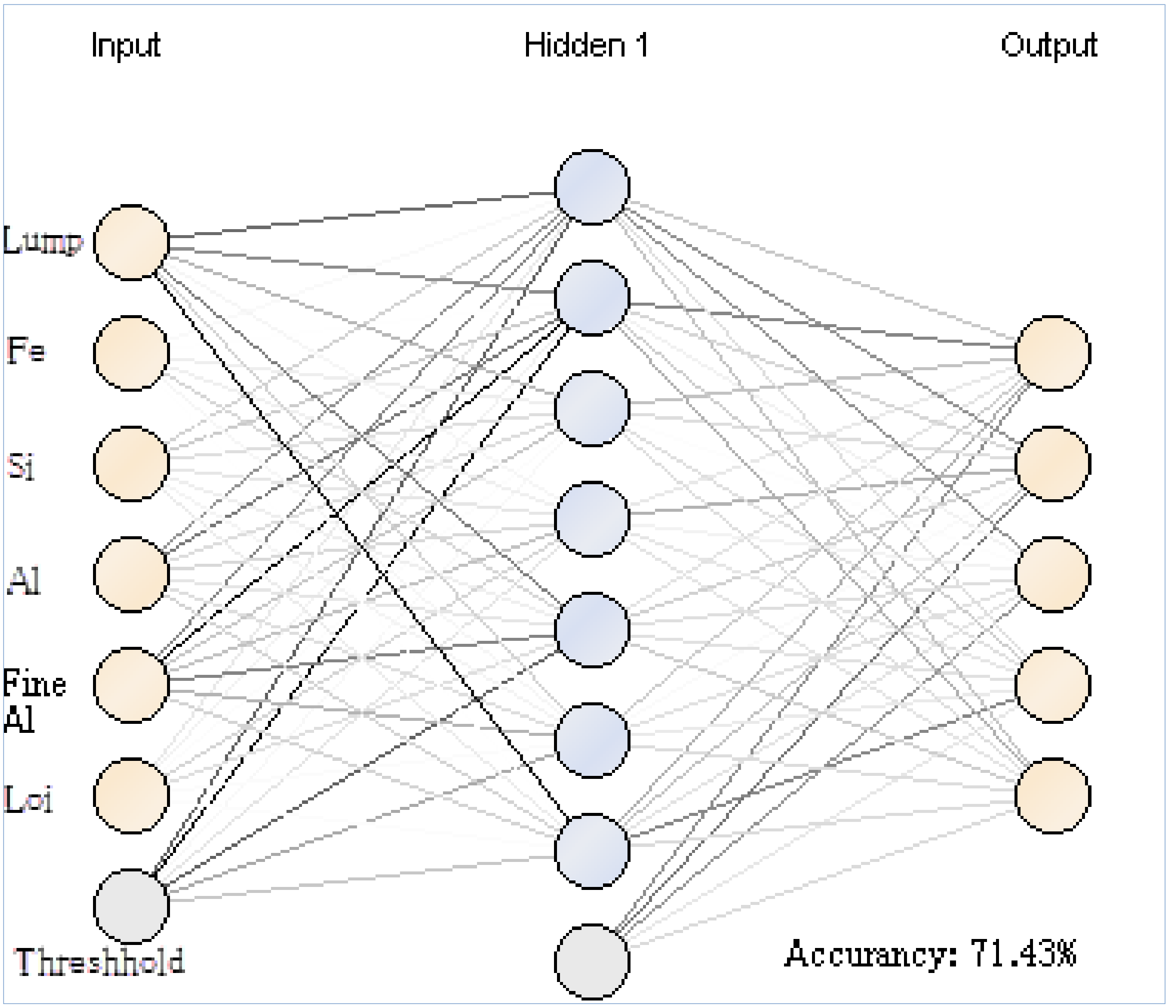


Fig. 5.9, Tree viewer decision tree classification of clay material.

5.3.1.5 Evaluating the data mining model

As stated in the preceding section, cross-validation of the model was performed to measure how well the chosen algorithm was performing. During the cross-validation exercise, a neural network algorithm was also evaluated but was found to be less accurate (71%; Fig. 5.10) than the decision tree classification that had an accuracy of 78.6% (Fig. 5.11).



Accuracy → 71.43%; Correlation→ 0.803; Squared correlation→ 0.644

Fig. 5.10, Neural network analysis of the problematic ore.

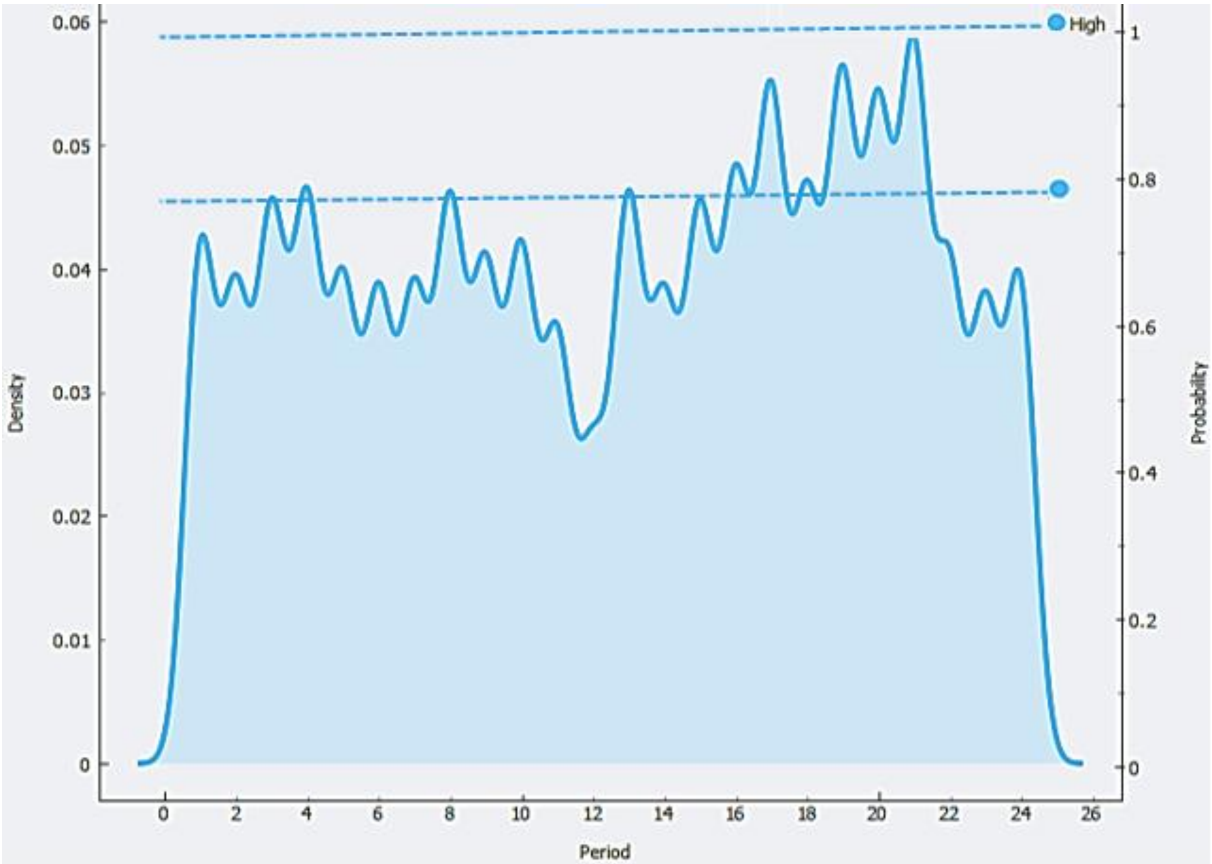


Fig 5.11, Probability density for problematic ore occurrence.

The accuracy of the classification tree model that has been utilised in this research is 78.6%. The results are acceptable as the aim of this research was not to exactly predict the future but to have the indicative direction of what could happen to crusher feed if there is clay material in planned blocks contrary to the resource model prediction. Therefore, the objective of this research has been met by these results as the essence of this data mining was to help in providing a real option for creating flexibility that can give managers the ability to make future decisions. Consequently, it could be statistically inferred in this case that there is a 78.6% probability that the planned crusher feed or tonnes contains problematic ore or clay material, which could result in processing plant downtime. The summary of the model performance is shown by the confusion matrix (Table 5.6). This matrix indicated that 78.6% of the attributes were classified correctly. Moreover, the model classified blocks with a precision of 65.38%. This implied that if a block was identified for instance as medium risk, the model would predict that block as medium risk with 65.38% precision. However, the model has a root mean square error of 0.454 but with strong correlation of 87% which is a good performance as per the purpose of this research.

Table 5.6, Confusion matrix for problematic ore prediction.

Performance Vector					
Accuracy: 78.57%					
Confusion Matrix:					
True:	High	High-Medium	Medium	Low-Medium	Normal (Low)
High:	0	1	0	0	0
Medium-High:	0	7	1	0	0
Medium:	0	0	16	6	0
Low-Medium:	0	0	0	8	0
Normal (Low):	0	0	0	1	2
Weighted Mean Precision: **65.38%,** Weights: **1, 1, 1, 1, 1**					
Root Mean Squared Error: **0.454 +/- 0.000**					
Correlation: **0.870**					
Squared Correlation: **0.758**					

5.4 Real options analysis

Before embarking on real options analysis, it is essential that the aim of the analysis is clearly articulated. One of the most important assumptions in the analysis of this real case study is that the operation is already an established mine that has been running for a few years and it has already paid off the initial starting capital cost. Therefore, the emphasis in this analysis is to demonstrate tangible impacts of embedding managerial flexibility in mine planning and running of the mining operation. In that view, comparing present values (PVs) of the traditional and real options analysis for decision making is the most appropriate measure. Thus, this research will compute the traditional PVs of the operation and compare the outcomes with various real options. Such calculations will be based on various technical and economic assumptions outlined in the sections below.

5.4.1 Technical parameters and economic assumptions

The assumptions listed below were applied in the analysis of this real case study. These assumptions were based on publicly available information and on primary data sourced from the industry at the time of writing:

- Processing plant effective utilisation. The average effective utilisation of the processing plant across the mining industry was estimated at 75% (Connelly, 2013; Board of Governors, 2017). This implies that all the time-usage metrics such as availability, use of availability and operating efficiency must range between 90% and 93% to obtain such effective utilisation.
- Unscheduled loss in effective utilisation due to clay. Based on the analysis performed on the actual processing plant's unscheduled loss data from a real operating mine in Western Australia, it was revealed that the presence of clay in crusher feed could reduce plant utilisation to between 70% and 73.8%. In this research, the mine analysed was a balanced iron ore operation where clay is expected to be between 6% and 11% (Clout, 2013). The reduction in effective utilisation due to clay was estimated to be 3.3 days per year or 1.21% (that is approximately 5% of availability, 5% of use of availability and 4.9 % of operating efficiency). For small operations, such a reduction can make a huge difference to the operation's bottom-line. It must also be noted that this estimated reduction is very optimistic as the loss could be higher for a mine whose ore body is known to contain clay.
- Probability of problematic ore occurring. The probability of intercepting clay material in planned ore blocks was estimated to be 78.6%. This was the output of the classification tree algorithm from the RapidMiner (2017) program (Table 5.6). This variable will be used in the Poisson Distribution (PD) model in the next section before the Monte Carlo simulations.
- Ore recovery. For most iron ore operations, globally and in Western Australia, ore recovery is estimated to range between 80% and 90% depending on the percentage split between fine and coarse material (CPCB, 2007). For simplicity, an average value of 85% was assumed as the recovery rate for the real case mine operations.
- Operating cost. The operating cost of the real case study was estimated to be between US$47.7 and US$43.7 per tonne of saleable ore product in 2015 and 2016, respectively (SNL.com, 2017).
- Risk - free interest rate. The risk-free interest rate was estimated to be 5% (Kleymenova et al., 2009), which is in line with the long-term risk-free interest rate figure used for most mining projects in Australia.
- Iron ore price. The actual realised iron ore prices for 2015 and 2016 where obtained from the IndexMundi.com online database (IndexMundi, 2017).

5.4.1.1 Step 1: Poisson model

To utilise the data mining probability value (Table 5.6) in creating operational flexibility in this case study, the PD was used in interpreting the result and hence incorporated into the spreadsheet model for the real options (RO) analysis. Importantly, the Poisson function is a consequence of the arrival process. This stochastic process works on the premise that events occur one at a time in successive periods at a certain time interval (*0*,*t*) such that the probability of event *X* occurring is function *f(x, t)*. However, the following conditions must exist for this principle to hold:

That the probability of a single event occurring in a very short time interval is $(t, t + \Delta t)$ and summarised in Eq. (5.3).

$$f(x, t + \Delta t) = f(x, t)f(0, \Delta t) + f(x - 1, t)f(1, \Delta t) \quad 5.3$$
$$= f(x, t)f(1 - \alpha\Delta t) + f(x - 1, t)\alpha\Delta t$$

Thus, producing Eq. (5.4).

$$f(1, \Delta t) = \alpha\Delta t \quad 5.4$$

This implies that events are mutually exclusive and two events will never occur at the same time under this assumption. The probability of more than one event occurring in one period is negligible.

The probability of an event occurring in the current period is independent of any occurrences in previous periods. This means that the events are disjoined and the probabilities of the events are obtained through the multiplication rule.

The derivative of the Poisson function f(x, t) with respect to t is Eq. (5.5).

$$\frac{df(x, t)}{dt} = \lim(\Delta t \rightarrow 0)\frac{f(x, t + \Delta t) - f(x, t)}{\Delta t} \quad 5.5$$

Following the differential product rule produces Eq. (5.6).

$$\frac{df(x, t)}{dt} = \frac{\alpha x(\alpha t)^{x-1}e^{-\alpha t}}{x!} - \frac{\alpha(\alpha t)^{x}e^{-\alpha t}}{x!} \quad 5.6$$
$$= \alpha\left\{\frac{(\alpha t)^{x-1}e^{-\alpha t}}{(x - 1)!} - \frac{(\alpha t)^{x}e^{-\alpha t}}{x!}\right\}$$
$$= \alpha\{f(x - 1, t) - f(x, t)\}$$

It should be noted that PD is from a family of binomial distributions whose probability space n increases indefinitely but the expected successful outcome μ = np remains constant for all the trials.

For easy understanding, it is proper for one to look at the binomial calculus shown in Eq. (5.7).

$$b\ (x; n, p)\ =\ \frac{n!}{(n-x)!\,x!}p^{x}(1-p)^{n-x}\ =\ \frac{u^{x}}{x!}.\frac{n!}{(n-x)!\,n^{x}}(1-\frac{u}{n})^{n}(1-\frac{u}{n})^{-x} \quad 5.7$$

On applying the limits of the components derive Eq. (5.8)

$$Lim\ (n \to \infty)\left(1-\frac{u}{n}\right)^{n} = e^{2-u} \quad 5.8$$

and Eq. (5.9).

$$Lim\ (n \to \infty)\left(1-\frac{u}{n}\right)^{-x} = 1 \quad 5.9$$

It becomes clear that the resulting binomial function in Eq. (5.10) is the Poisson's function:

$$Limit\ (n\ \to \infty)\ b\ (x; n, p)\ =\ \frac{u^{x}e^{-u}}{x!} \quad 5.10$$

Therefore, in this real case study on the occurrence of clay material being fed into the processing plant, the issue has been reduced to two possible mutually exclusive events, which are that the truck arriving at the crusher loaded with ore will either contain a clay material in that load and that this material will cause problems in the processing plant, or the load will turn out to be normal without issues. In applying the probability of encountering problematic ore as predicted by the machine learning, the anticipated average performance of the plant can be estimated as a success rate u =np based on the probability of the ore being either problematic or normal.

In applying the Poisson's logic as it is the most suitable one in this particular situation, realisable production targets for the traditional design were estimated and the operation's PV profile for the two-year periods of this mine was computed as shown in Table 5.7.

From Table 5.8, it can be stated that the planned production targets for the successive years would not be achieved due to clay uncertainty and there is 78.6% chance that the planned crusher feed will contain clay material. If the problematic material is contained in the delivered load, it is definite that the plant throughput rate could be reduced. When the standard DCF analysis was applied, the realistic and achievable PV of the case study was estimated at US$113.8 million at a risk-free rate of 5% as this operation is assumed to have already gone through the capital pay-off period and the only obligations are its operating costs. This would have been the most likely outcome of this operation assuming that management did nothing other than their mandatory jobs. As mentioned earlier, the traditional DCF method applied in this step is static in nature as it ignores the possibility of future decisions as well as not valuing the uncertainty posed by clay.

Table 5.7, Analysis of a traditional non-flexible operations using the Poisson function.

Processing Plant Parameters

Probability of occuring	0.7857	Plant Effective Utilisation if the Uncertain Problematic material Occurs	73.8%
Probability of Not occuring	0.2143	Plant Effective Hours in 30 day calendar month (hrs)	540
Plant Effective Utilisation in good material (Connelly, 2013; Board of Governors, 2017)	75.0%	Risk free rate(Kleymenova, et al. 2009)	5%
Unscheduled loss time due to problematic material (Researcher's ownanalysis)	1.21%	Ore Recovery 80% > X < 90 % (CPCB,2007)	85%

Non Flexible Operation PV Using Deterministic Model of Poisson Function

Months (Jan 2015 - Dec 2015)	1	2	3	4	5	6	7	8	9	10	11	12
Nominal Plant feed as planned - Mine plan data (***tonnes***)	599,736	610,590	610,641	632,399	610,641	609,133	567,023	654,258	566,822	533,257	467,933	415,640
Planned Plant Thoughput Rate (***t/h*** r)	1,111	1,131	1,131	1,171	1,131	1,128	1,050	1,212	1,050	988	867	770
Expected Plant Feed if Problematic Ore occurs less function u = np (***tonne*** s)	592,134	602,850	602,900	624,383	602,900	601,412	559,836	645,964	559,637	526,497	462,002	410,371
Price, ***$/t*** of Concentrate in 2015 (IndexMundi,2017)	$69.1	$63.8	$58.2	$52.1	$60.4	$61.8	$52.2	$57.3	$57.0	$53.7	$47.2	$40.9
Revenue ***($M)***	$34.8	$32.7	$29.8	$27.7	$31.0	$31.6	$24.8	$31.5	$27.1	$24.0	$18.5	$14.3
Cost ***$M*** (***$47.7/t*** of Concentrate in 2015; SNL.com,2017),	$24.0	$24.4	$24.4	$25.3	$24.4	$24.4	$22.7	$26.2	$22.7	$21.3	$18.7	$16.6
Average Maintenance Cost, ***$1.73/t feed*** (material, grinding media & wear liners) if problematic ore occur, (McNab et.al, 2009)	$1.024	$1.043	$1.043	$1.080	$1.043	$1.040	$0.969	$1.118	$0.968	$0.911	$0.799	$0.710
DCF Per Period ($M)	**9.3**	**6.9**	**4.1**	**1.2**	**5.2**	**5.9**	**1.1**	**4.0**	**3.3**	**1.7**	**-1.0**	**-2.9**

Months (Jan 2016 - Dec 2016)	13	14	15	16	17	18	19	20	21	22	23	24
Nominal Plant feed as planned - Mine plan data (***tonnes***)	654,258	545,215	654,258	564,683	654,258	544,963	654,258	588,832	620,339	609,272	562,174	654,258
Planned Plant Thoughput Rate (***t/h*** r)	1,212	1,010	1,212	1,046	1,212	1,009	1,212	1,090	1,149	1,128	1,041	1,212
Expected Plant Feed less Poisson Function ***u = np*** (***tonne*** s)	645,964	538,304	645,964	557,525	645,964	538,055	645,965	581,368	612,476	601,549	555,048	645,964
Price, ***$/t*** of Concentrate in 2015/2016 (IndexMundi,2017)	$42.2	$46.5	$56.5	$61.0	$55.9	$52.3	$57.3	$60.9	$57.7	$59.0	$74.1	$79.4
Revenue *($M)*	$23.2	$21.3	$31.0	$28.9	$30.7	$23.9	$31.5	$30.1	$30.0	$30.1	$35.0	$43.6
Cost ***$M*** (***$43.7/t*** of Concentrate in 2016; SNL.com,2017),	$24.0	$20.0	$24.0	$20.7	$24.0	$20.0	$24.0	$21.6	$22.8	$22.3	$20.6	$24.0
Average Maintenance Cost, ***$1.73/t feed*** (material, grinding media & wear liners) if problematic ore occur, (McNab et.al, 2009)	$1.118	$0.931	$1.118	$0.965	$1.118	$0.931	$1.118	$1.006	$1.060	$1.041	$0.960	$1.118
DCF Per Period ($M)	**-1.8**	**0.3**	**5.7**	**6.9**	**5.3**	**2.9**	**6.1**	**7.1**	**5.9**	**6.4**	**12.8**	**17.6**

Present Value of Existing Operation without Real Option ($M)	**$113.8**

5.4.1.2 Step 2: Monte Carlo simulation (MCS) of crusher feed

Having constructed a DCF model in Step 1, MCS was used to simulate the possible crusher feed targets. Even though it is known that clay will be present in some loads that would be dumped into the processing plant each month, it was not possible to estimate with precision the number of tonnes that will contain clay. However, the data mining model analysis does indicate that there is a 78.6% chance that the feed will contain clay and, if that happens, then there will be an unscheduled loss of 1.2% in effective time for every period the plant is running. This means that the plant in this case study can operate with an effective time of between 73.8% and 75%, which resulted in the minimum and maximum scheduled feed. Having now established the simulation boundaries, the crusher feed can be treated as a random process of the form of $crusher\ feed = a + (b - a) * Random()$, where a and b are the minimum and maximum feed rates respectively. This model was iterated 1,000 times and the resultant average PV was calculated to be $111.5

million with a standard deviation of $372,000. Fig. 5.12 shows details of the monthly PV profile and measures of the normal distribution model.

As shown in Fig. 5.12, it could be stated with 95% confidence that the PV of this operation is likely to range between $110.8 million and $112.3 million. It is also possible that there may be a few occasions where the PV approaches $113 million, which may be closer to the PD (special Binomial Model) model whose value has been estimated in Step 1. When comparing the PV estimated stochastically to the traditional DCF in Step 1 above, it would be logical to conclude that the traditional DCF analysis appears to have overstated the PV for this operation. Such assertions make sense because the traditional DCF method utilises single point average values in computation with little regard to fluctuations and, therefore, the PV can either be overestimated or underestimated.

P(PV)	@95%	@5%
Random values of the Project PV (X$1000)	$112,262	$110,805
Mean (X$1000)	$111,533	
Standard Deviation (X$1000)	$372	
Z Score	1.96	

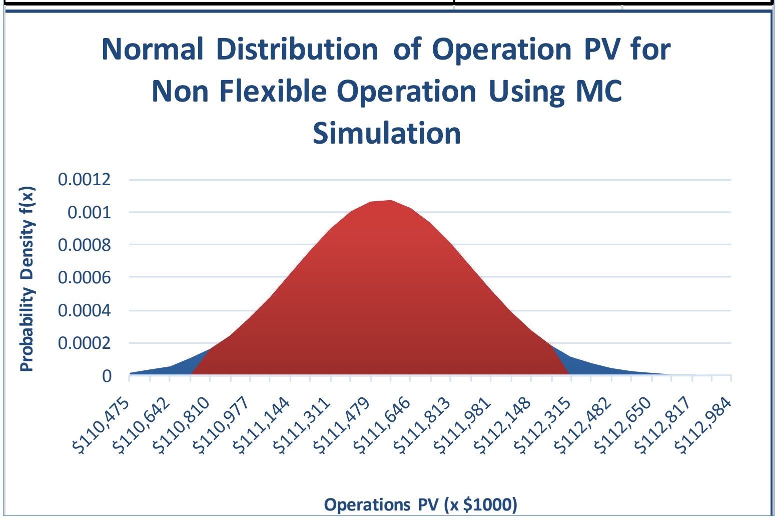


Fig. 5.12, Normal distribution of PV for non-flexible operation using MCS.

5.4.1.3 Step 3: Integrating flexibility into mining operations

A. Material rehandling option

Since the PDM has clearly established that the PV of the real case mining operation will be reduced owing to the possible reduction in planned crusher feed as a result of the clay material uncertainty, management may decide to create a bigger stockpile system (Fig. 5.13). Rehandling of material does come at a cost and the operational managers do not easily approve the establishment of large stockpiles. Anecdotal evidence from the mining industry reveals that mining operations do stockpile material in stockpiles that are large enough to feed a crusher for about one to two weeks only, but most mine stockpiles only have enough material for one week. This level of stockpiling is enough for small scale blending and the levels are constantly maintained to be used as a buffer in case of a major unfavourable event occurring. However, management has an option to create a bigger stockpile finger system by paying the rehandling cost of $4.5/t (Ajak & Topal, 2015). A specialised stockpile can be used to store the problematic ore and the stored quantities can be substituted with rehandle from the stockpiles that had been previously built. Any stockpiled problematic ore would usually be trickle fed into the crusher as long as the quantities are kept below the minimum clay feed proportion of 6% for a balanced iron ore operation (Clout, 2013). Thus, there will be no loss in ore and reduction in crusher feed can be avoided. The economic impact of the rehandling option on the operation's PV was assessed and the PV would increase to $138.2 million if this option was built into the mine plan. Moreover, this option produced a very compacted PV value distribution with a standard deviation of $59,000 (Fig 5.14).

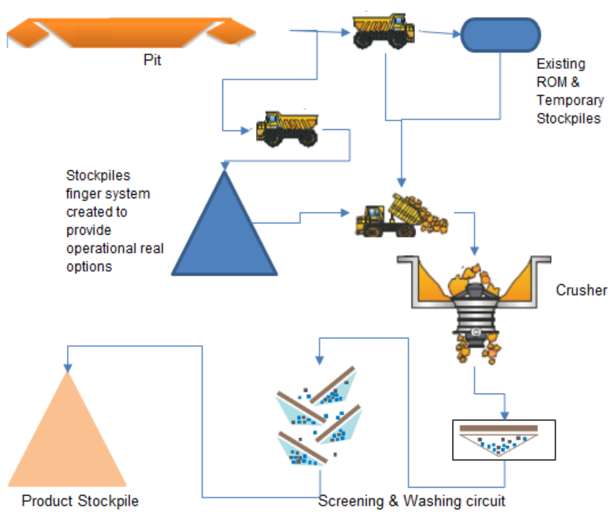


Fig. 5.13, Schematic diagram of rehandling option using MCS.

P(PV)	@95%	@5%
Random values of the Project PV (X$1000)	$138,259	$138,027
Mean (X$1000)	$138,143	
Standard Deviation (X$1000)	$59	
Z Score	1.96	

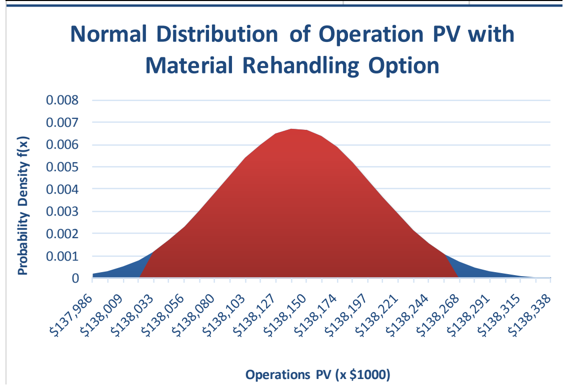


Fig. 5.14, Normal distribution of PV of rehandling option using MCS.

B. Additional mining area option

Uncertainty arises when the location of clay cannot be precisely pin-pointed. However, the operations can develop an additional dig face and incur the development cost to create the required flexibility that operational managers need to make future decisions (Fig. 5.15). However, developing additional dig face attracts development costs in terms of the area clean up, drilling and blasting. For the purpose of this case study, the main cost item that will be incurred to create the required flexibility is the drilling and blasting cost as the other costs such as bench clean up and grading is assumed to be absorbed into the normal daily mine development schedule. According to research carried out by Afum & Temeng (2015), drilling and blasting is usually approximately 15% of the operational cost. When this assumption was utilised together with MCS of what could happen to crusher feed considering that the data mining model has predicted a 78.6% chance of intercepting problematic ore, it was estimated that the project PV could increase to $127.4 million (Fig. 5.16). It could be estimated with 95% confidence that the PV could not drop below $127.2 million and there are also very few events that could increase the PV beyond $127.5 million. Furthermore, the values were tied together with a standard deviation of $73,000.

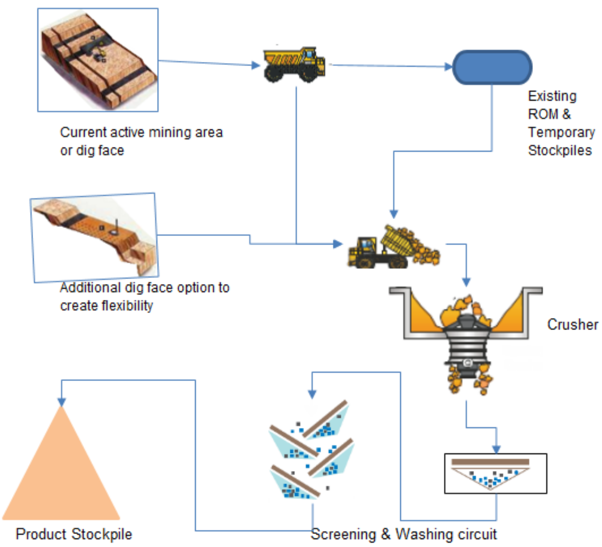


Fig. 5.15, Schematic diagram of PV of an additional mining area using MCS.

P(PV)	@95%	@5%
Random values of the Project PV (X$1000)	$127,480	$127,195
Mean (X$1000)	$127,337	
Standard Deviation (X$1000)	$73	
Z Score	1.96	

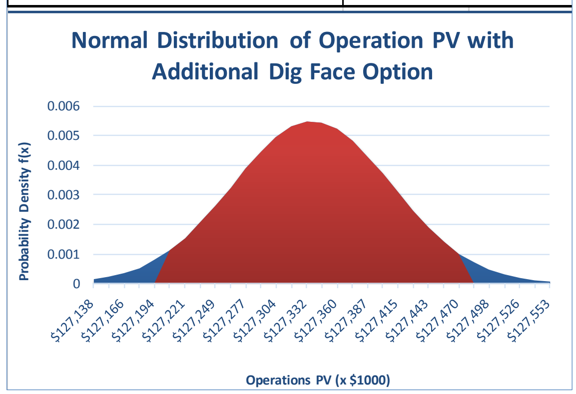


Fig. 5.16, Schematic diagram and normal distribution of PV of an additional mining area using MCS.

C. Building a bypass circuit option

Since it was established earlier in this research that feeding clay material unknowingly into the processing plant does bog the processing circuit, management could consider investing in a bypass circuit (Fig. 5.17). The bypass circuit would only be used when the primary or existing circuit has shut down due to problematic ore issues. The rationale of using these circuits alternatively or intermittently is to minimise any increase in milling cost as the intention of this option is to avoid losses associated with plant reliability issues due to clay material. Since processing plants are built and operated in two modules, then each module carries an equal proportion of the total plant capacity. Therefore, the bypass circuit must be at least a module that can handle 50% of the product and the average capital cost was estimated at $6.8 million based on $1.13 per life of mine tonne of concentrate (Ajak & Topal, 2015). The financial analysis of this option indicated that incorporating a bypass circuit into the mine operations will increase the PV to $131.8 million (Table 5.8).

Table 5.8, Analysis of a traditional non-flexible operations using the Poisson function

Months (Jan 2015 - Dec 2015)	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Nominal Plant feed as planned - Mine plan data (***tonnes***)	599,736	610,590	610,641	632,399	610,641	609,133	567,023	654,258	566,822	533,257	467,933	415,640
Planned Plant Thoughput Rate (***t/h*** r)	1,111	1,131	1,131	1,171	1,131	1,128	1,050	1,212	1,050	988	867	770
Reduction in Plant Feed if Problematic Ore occurs (***tonnes***)	9,676	9,851	9,852	10,203	9,852	9,827	9,148	10,555	9,145	8,603	7,549	6,706
Price, ***$/t*** of Concentrate in 2015/2016 (IndexMundi,2017)	$69.1	$63.8	$58.2	$52.1	$60.4	$61.8	$52.2	$57.3	$57.0	$53.7	$47.2	$40.9
Revenue (*$M*)	$35.2	$33.1	$30.2	$28.0	$31.4	$32.0	$25.1	$31.9	$27.5	$24.3	$18.8	$14.4
Cost ***$M*** (***$47.7/t*** of Concentrate in 2016; SNL.com,2017),	$24.3	$24.8	$24.8	$25.6	$24.8	$24.7	$23.0	$26.5	$23.0	$21.6	$19.0	$16.9
DCF Per Period ($M)	**10.4**	**8.0**	**5.2**	**2.3**	**6.3**	**7.0**	**2.1**	**5.1**	**4.3**	**2.6**	**-0.2**	**-2.3**

Months (Jan 2016 - Dec 2016)	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Nominal Plant feed as planned - Mine plan data (tonnes)	654,258	545,215	654,258	564,683	654,258	544,963	654,258	588,832	620,339	609,272	562,174	654,258
Planned Plant Thoughput Rate (t/hr)	1,212	1,010	1,212	1,046	1,212	1,009	1,212	1,090	1,149	1,128	1,041	1,212
Reduction in Plant Feed if Problematic Ore occurs (tonnes)	10,555	8,796	10,555	9,110	10,555	8,792	10,555	9,500	10,008	9,830	9,070	10,555
Price, $/t of Concentrate in 2015/2016 (IndexMundi,2017)	$42.2	$46.5	$56.5	$61.0	$55.9	$52.3	$57.3	$60.9	$57.7	$59.0	$74.1	$79.4
Revenue, ***$M***	$23.5	$21.5	$31.4	$29.3	$31.1	$24.2	$31.9	$30.5	$30.4	$30.5	$35.4	$44.2
Cost (***$43.7/t*** of Concentrate in 2016; SNL.com,2017), ***$M***	$24.3	$20.3	$24.3	$21.0	$24.3	$20.2	$24.3	$21.9	$23.0	$22.6	$20.9	$24.3
DCF Per Period ($M)	**-0.8**	**1.2**	**6.8**	**7.9**	**6.5**	**3.8**	**7.2**	**8.2**	**7.0**	**7.5**	**13.9**	**18.9**

Strike price for building a bypass circuit to create switching option ($1.13/t concentrate: Ajak & Topal,2015), ***$M***	$6.8
Present Value of flexible operation with bypass circuit for switching option ($M)	**$131.8**

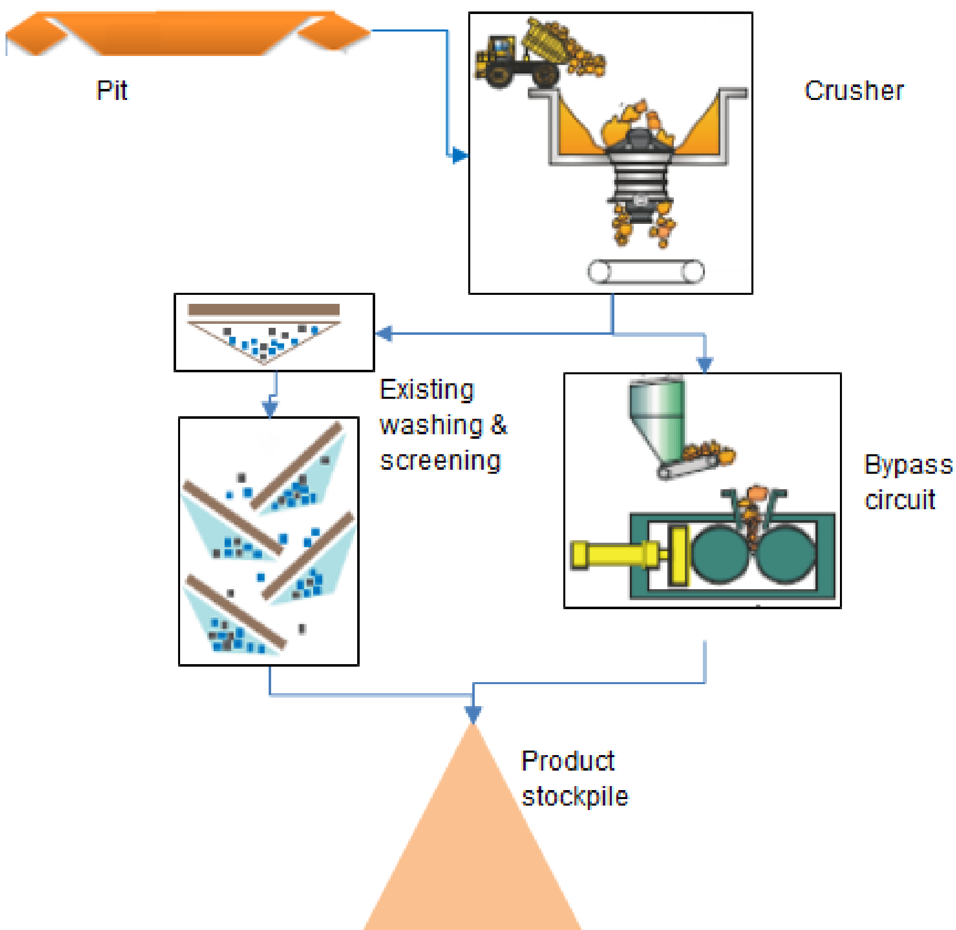


Fig. 5.17, Schematic diagram and bypass circuit option.

5.5 Analysis of results

A detailed discussions of all options are provided in the following sections.

5.5.1 Do nothing option – utilise traditional mine planning strategy

This method is deterministic in nature. It is the current practice in the industry where multiple scenarios are run in general mine planning software and the operations would then choose the scenario that management thinks has the best chance achieving desired outcomes. There is always no flexibility built into the system because the mine plan is usually executed as planned with the assumption that everything will remain as scheduled. Thus, there is no consideration for uncertainty. Therefore, the management had this choice of doing nothing and let the project run as planned. The ultimate outcome is a lower PV of $113.8 million as shown in Table 5.7.

5.5.2 Utilisation of stochastic model – Monte Carlo simulation (MCS)

This approach includes treating the physical outcomes of the mine plan as a random process due to geological and geochemical properties of the ore body. Rather than treating the mine plan as static, managers should allow room for fluctuations and lower expectations. The main advantage of this model is that it does not suffer from an overestimation of the expected values. Compared to a traditional mine planning model, managers would always be in a better position to properly develop the mine as resources can be properly distributed. It also encourages a proactive management of uncertain events compared to a traditional method, which is reactive in nature. It should be noted that a stochastic approach alone is not a real option. However, it does reveal opportunities for real options integration into the system. Based on a MCS, the average project PV is estimated at $111.5 million (Fig. 5.12). Thus, the traditional DCF model had overestimated the project PV by $2.3 million.

5.5.3 Material rehandling option

Management can decide to build a specialised stockpile where any material that is intercepted and contains clay can be stockpiled. However, rehandling of material is commonly minimised by mine managers because it is costly to rehandle the same tonne twice. In the environment where cost and efficiency pressures are at the forefront for every operation, rehandling has always been targeted for cost cutting regardless of the known benefits such as acting as a buffer and to the creation of blending flexibility. However, the analysis of this real case study demonstrated that the rehandling option increased the project PV by 21% (Fig. 5.20) or a total PV figure of $138.1 million. This option appeared to be the most valuable for creating managerial flexibility in the face of clay uncertainty. Apart from increased variable costs, the rehandling option is easy to put into place and does not require any additional capital outlays.

5.5.4 Developing additional mining area (dig face) option

This option is based on the industry known fact that mining operations are more likely to meet production targets if they have more dig faces that are available for mining. Whenever there is an issue or unfavourable event that affects the active mining area, mining equipment can be floated to the alternative area. However, mine planners and managers tend to focus primarily on operating cost minimisation. Thus, such an operating strategy appeared to have a limited view of the value that can be created by building flexibility into the mine. The conventional mine planning techniques heavily rely on static analysis of the future production targets, which are based on averages. Such planning assumes that the choices made by managers today will continue to hold true or will be implemented as planned.

Since decisions are based on optimal design and sequence, there is usually no real justification for incurring additional costs to create flexibility by developing additional pit areas that may not be mined for the foreseeable future. However, the strength of the real option methodology is its ability to justify additional spending (Groeneveld et. al., 2010; Ajak & Topal, 2015). In this real case study, the operation can introduce flexibility by developing an additional dig face that can be mined if the clay material in the current operating face is intercepted. This will introduce a switching option where management can decide to change the mining location provided that the development cost of drilling and blasting the area has been paid to create this option. The analysis of this option indicates that the project PV could be increased from $111.5 million to $127.3 million if this option is designed into the system.

5.5.5 Building bypass circuit option

Having more than two modules in a processing plant makes operations more flexible and agile. Building a bypass circuit is an option that the management of the real case iron ore project could consider. As explained in sub-section C in section 5.4.1.3 above, this option would increase the project PV to $131.8 million.

5.5.6 Optimal managerial decision

When project PVs were ranked after all the flexible alternatives were evaluated, it was clear that rehandling and building a bypass circuit options increase the PV by $26.6 million and $20.3 million, respectively.

From a practical point of view, management could easily implement the rehandling option as it required no additional capital cost. However, the rehandling cost has to be paid in this option. Thus, the optimal decision is to build a larger stockpile and pay rehandling costs. Distinctive differences in real option values were made clearer via a pairwise comparison in a value radar (Fig. 5.18). A value radar is made up of dynamic rings whose radii increase by $20 million, with the inner ring being $120 million. Each quadrant represents a comparison of option alternatives to the traditional base value. As shown in Fig. 5.18, the option with the largest PV of US$138.14 million was the rehandling option as it was the closest to the outer ring, and thus it has the largest option value. Structuring alternatives visually and graphically helps decision makers understand the impact of various alternatives. Since this option has both the largest PV and option value, it is an obvious choice.

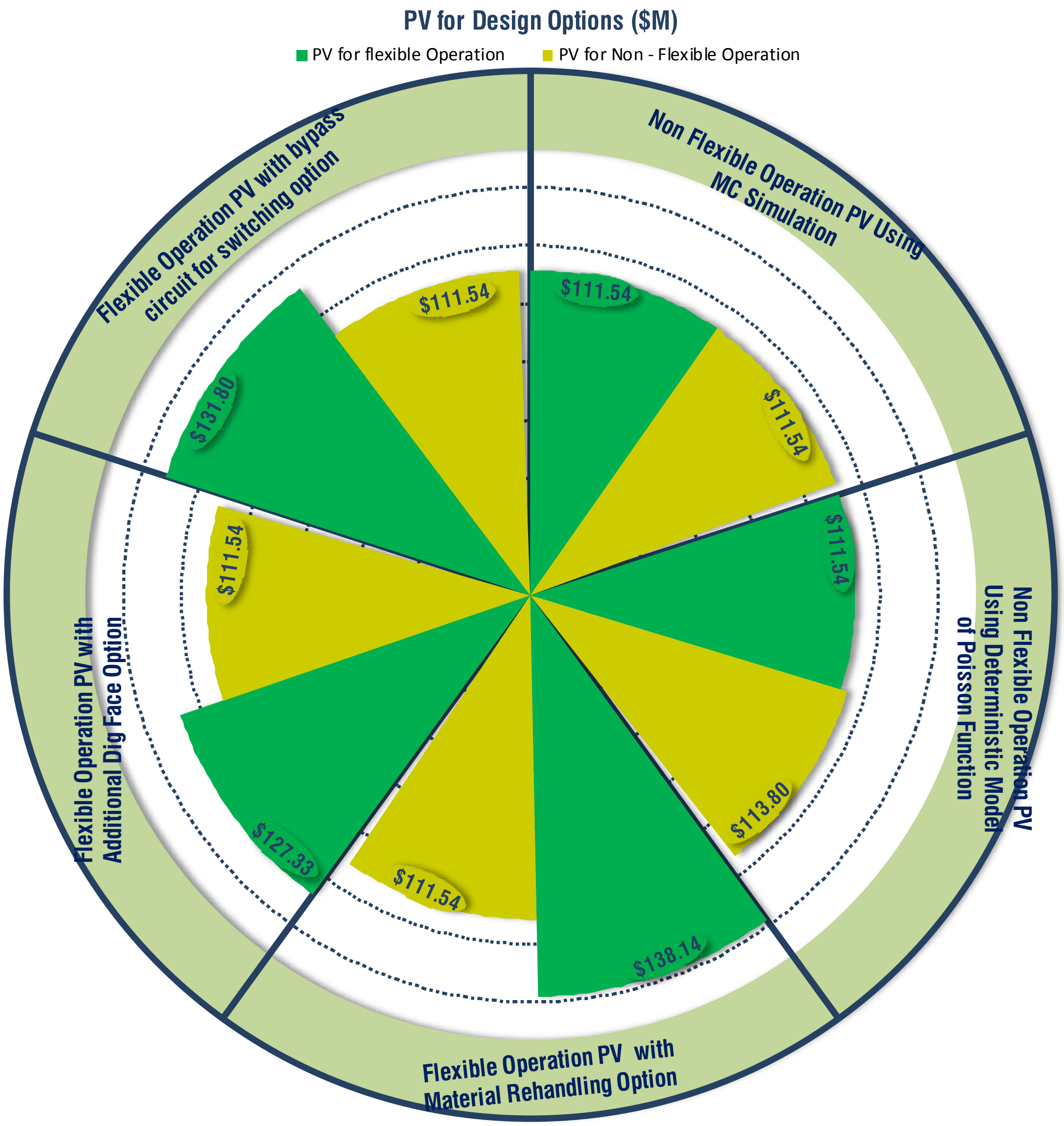


Fig. 5.18, Pairwise comparison of real option alternatives with traditional DCF model.

When the real impact of different options was assessed, the analysis showed that real options increase the real case project PV by between 12% and 21% (Fig 5.20) depending on the option chosen as per the mine operating conditions and environment. This is a vindication that uncertainty has value if well assessed and embraced rather than avoided. Therefore, any option is better than either doing nothing or solely depending on the traditional model.

The ability to generate several options is critical in real options analysis as it allows managers to compare all the available alternatives and weigh each opportunity in terms of cost and value being added to the project PV. The traditional model fully relies on a calculated single average value, which makes it risky as each single point value cannot be sustained if an unfavourable event occurs. The MCS of a non-flexible operation showed a more spread PV than the flexible operations where various real option alternatives have been assessed. In line with expectations, any investment in creating flexible operations minimises losses, resulting in a tighter distribution as shown in Fig. 5.19 where the values increased from left to right of the graph as various options were evaluated. Exploring different alternatives guarantees robustness in the real options analysis and ensures that the option being chosen is the best for creating value and can be practically applied when required.

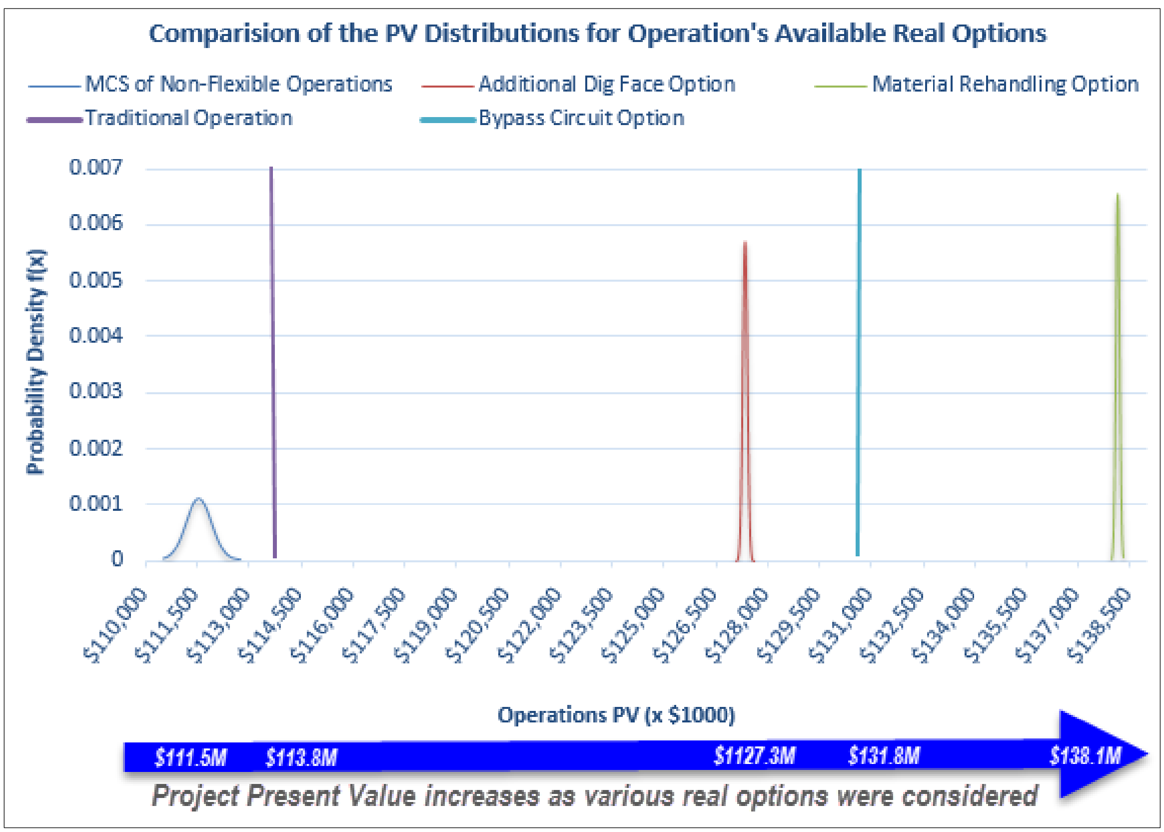


Fig. 5.19, Comparison of the project present values for various real options.

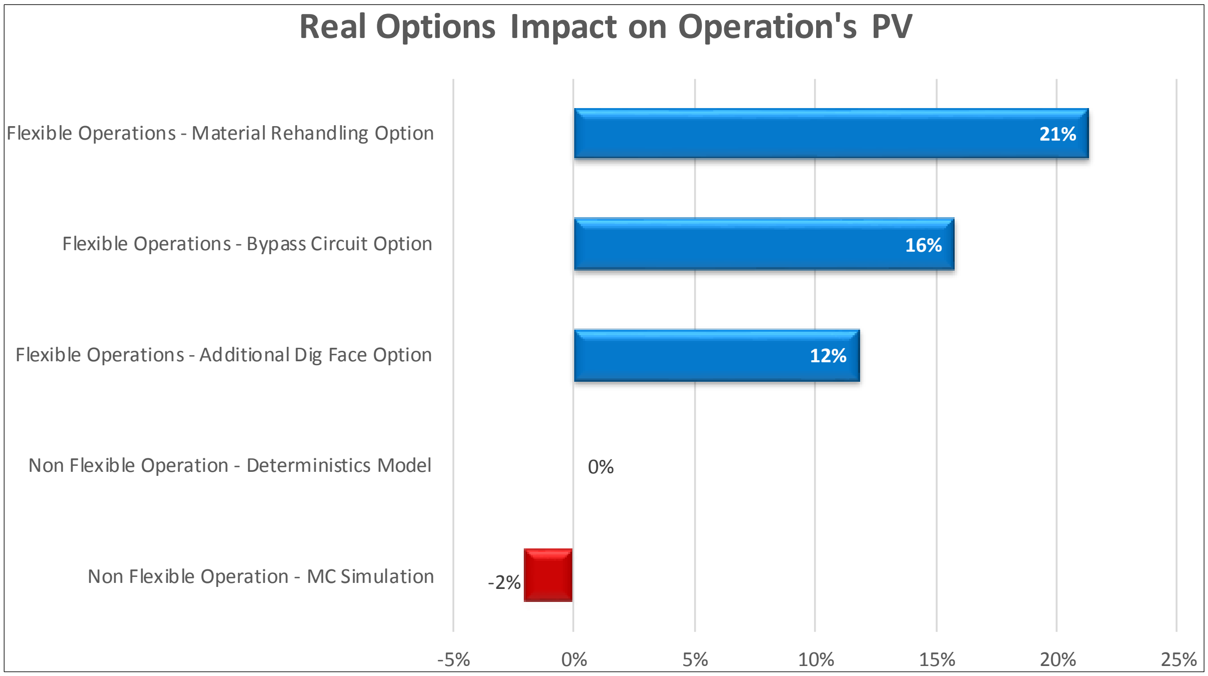


Fig. 5.20, Impact of various real option alternatives on the project present value

In summary, it is clear from the available five options studied in the preceding sections, the analysis revealed that the optimal decision for the management of the real case mine was to build a larger stockpile system and increase rehandling by paying the associated cost to create this option. This option would increase the project value by 21% to $138.2 million from $111.5 million (Figs. 5.18, 5.19 and 5.20).

5.6 Conclusions

The literature search revealed that predictive data mining algorithms such as decision tree classification have never been applied before to model the uncertainty of clay pods or for creating RO at a mine operational level. As stated in the introduction section, clay is an elusive variable and it is not always flagged in the mining block model. Thus, it remains a geological uncertainty. Moreover, clay is difficult to process due to its fine particles. This research has applied a predictive data mining algorithm to reveal a hidden pattern in the existing actual grade control data that are normally collected by mine geologists from blast sampling and cone logging processes.

It has been demonstrated that machine learning can assist operations to create flexibility, which could have a significant effect on PV. Therefore, the importance of data mining and the value that can be created if it is properly and skilfully utilised have been comprehensively highlighted in the research.

In line with the research objective, the predictive decision tree classification model for managing clay material uncertainty in a mine plan based on an ID3 algorithm was developed and explained using a sample of scheduled ore blocks that were randomly selected from actual assay data obtained from a real case mine operation. The model was then implemented using RapidMiner (2017) and Orange (Demsar et al., 2013) software programs, which are R programming and Python based datamining tools. Processing, model selection, evaluation and performance testing tasks were performed to ensure robustness of the model. Among all the algorithms tested in Orange, the decision tree classification returned the highest classification accuracy of 72% and probability of 78.6%. This prediction accuracy was accepted as sufficient for the analysis of the real case mine operation as the initial intention of this research was not to exactly pinpoint the prediction to a single point value but to demonstrate the possible trend no matter how small it might be. The model was implemented to analyse a mine plan. Based on a developed data mining model, several RO were created to make the operation more flexible by allowing for future managerial decisions to respond if clay material was intercepted. This would thus avoid production delays and minimise unscheduled plant shutdowns as a result of clay clogging the processing circuit.

When comparing the RO analysis to the traditional DCF analysis model, options such as establishing additional dig face, building a bypass citcuit and increasing material rehandling increase the project PV by 12%, 16% and 21%, respectively. However, the optimal decision was to increase the material rehandling since it added the greatest value with minimal cost. This is clear proof that managers can reduce losses and increase PV if uncertainty is embraced as a source of value via deployment of data mining strategies. Moreover, this research has shown that a combined application of predictive data mining and real options analysis can provide a robust analysis for justifying and promoting capital investment in managerial flexibility. The real options approach can help decision makers break away from what this research termed 'Scorecard Fever'. This is a situation where managers focus on achieving short-term key performance indicators (KPIs) to maximise their incentives but destroy long-term flexibility (Hope & Player, 2012). Anecdotal evidence has also shown that managers do not want to spend money on activities that do not have a direct short-term impact on their KPIs and balanced score cards. Such attitude is compounded by traditional analytical techniques that ignore the value of future information. Therefore, it has been proven in this research that the management of the real case operations would have increased the project PV if the predictive data mining and analysis were considered when dealing with geological uncertainty presented by clay material in the resource model.

Moreover, this research is expected to contribute to the broader knowledge and application of real options in mine planning and operations in the following ways:

- It will encourage the application of data mining algorithms for uncertainty quantification that can lead to overall use of a real option methodology in running mine operations, particularly in the planning and design stages.
- It will simplify the identification of important variables that have real impacts on the uncertain variables.
- It will ensure the use of data for robust decision making and eliminate the flaws associated with managerial intuition and expert opinions.
- It places data analysis ahead of decision making rather than the widely practiced norms where managers decide base on intuition and then use statistical analysis to justify the imminent decisions that have secretly been made.
- Geological uncertainty is real in mining operations and it is unlikely that it will disappear. Therefore, there is an opportunity to adopt the data mining and real options analysis put forward in this research as the basis of future research on how data mining can be simplified and implemented in mining operations.

Since this research has demonstrated that a combination of predictive data mining and real options analysis increases the PV in the face of geological uncertainty, the process of how to integrate data mining and real options methodologies into present mine planning practice needs to be researched as there are no established approaches on how and to what extent this could be achieved.

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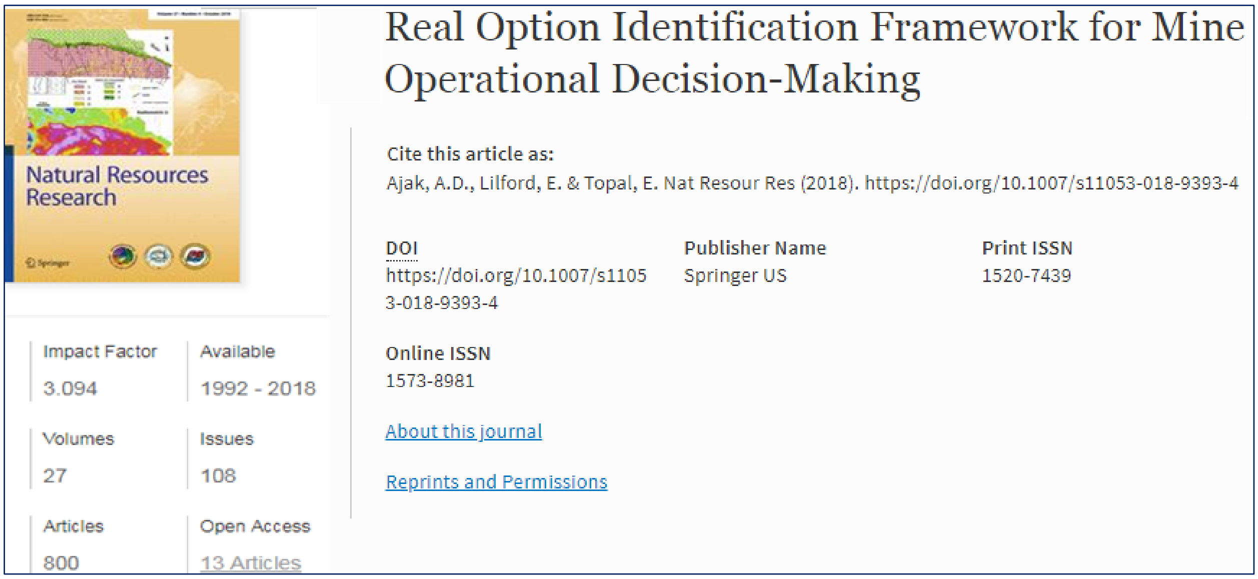
Chapter 6: A real option identification framework for mine operational decision making



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Principal Author	**Candidate Contribution to the Paper**	**Overall (%)**	**Signature**	**Date**
Ajak Duany Ajak	Set research question, developed methodology and predictive data mining model, developed case studies and analysed real options, wrote manuscript and acted as corresponding author.	85%		25/7/18

Co-Author Contribution

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III. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Co-Author	**Contribution to the Papers**	**Signature**	**Date**
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Abstract

Identification of opportunities for applying real options (RO) in mining operations is a major challenge to decision makers. In order to make optimal decisions in uncertain times, managers require a full understanding of the relationships between risk, uncertainty and flexibility. RO analysis, which captures the value of any managerial flexibility that may exist in a project, provides a proactive management of uncertainty. Thus, it enhances optimal decision making. However, it is important that a structured framework is created to identify project uncertainties and areas available to cultivate flexibility. In this paper, uncertainty identification framework in a mining operation is proposed, and areas for managerial flexibility and their application domains within the mining cycle are mapped as well. To avoid complex mathematical models, which hinder the adoption of RO analysis in mining operations, a relationship between risk measure (beta) and flexibility (flexibility index) is derived and applied. This implies that if a project beta is known, then the expected option values and volatility of future cash flows can be precisely estimated. Once the option value is calculated using the derived equation, a modified smooth pasting condition with the mean value theorem is subsequently applied to estimate the optimal value. This combination of beta, flexibility index and mean value theorem can be used as a decision criterion for screening various options within a mining project.

6.1 Introduction

The mining industry has become extremely volatile due to the tendency for commodity prices to fluctuate markedly within short timeframes (Latimer 2016). Mining operations risk renders them less viable if they are not flexible enough to respond to such changes. Maintaining investor and customer confidence during uncertain times is directly connected to the financial viability of an operation. Wade (2015) once stated that planning and executing long-term strategy has become a hindrance to operational or business success, as predicting a future state in a rapidly changing operating environment is becoming increasingly difficult. The increased volatility in the mining sector has rendered traditional evaluation approaches less reliable. Traditional methodologies, such as discounted cash flow analysis (DCF), which have static views, result in systematic undervaluation and can introduce error to development decisions (Purwar et al. 2011).

Traditionally, mine operating strategies are based on analysis of historical data that helps to understand the current state of affairs. Then, a desired future state can be determined and plans to achieve certain operational objectives or production targets can be made. However, managers have the tendency to reduce flexibility due to pro-cyclical investment policies that tend to balance budgets and smooth revenues (Sick and Gamba 2015). This phenomenon has been referred to as 'scorecard fever' by Ajak et al. (2017).

As the world becomes very volatile, pressure is high on operational managers to continuously deliver and improve on the production targets of existing mining operations. The special characteristics of mining projects are their high levels of operating environment uncertainty and volatile commodity prices (Groeneveld and Topal 2011). The main point of differentiation among mining operations is the creation of flexibility. Any mining system that has allowed the use of the real options (RO) methodology tends to be agile and can minimize losses during tough times, especially in periods when commodity prices or ore grades are uncertain (Ajak and Topal 2015; Ajak et al. 2017). Operational flexibility gives managers the ability to make decisions as the operating environment changes and to reliably deliver production targets with minimal losses.

Mine operations are complete systems that have several segments within the production cycles. There are usually management opportunities within this system where managerial flexibility can be created. There is no question that flexibility adds value (Ajak and Topal 2015) but since managerial decisions aim to achieve set goals and targets, they need real values that can be shown to investors. In essence, the RO value is the premium that an operational manager should expect when spending capital to create flexibility. A mining operation with positive net present value (NPV) boosts the confidence of shareholders—who are technically the residual owners of the mine—thus pushing up the firm's share price, which also fluctuates with commodity prices. Therefore, every investor puts money into a mine and expects a return depending on the market risk premium, which is the difference between the expected rate of return on investment and the risk-free rate of return over the same period (Law 2014). The coefficient between the expected return and the risk premium is known as beta. RO analysis requires the use of discounted cash flows which are calculated using a discount rate. Therefore, the market risk premium and beta are required in the estimation of the discount rate.

Business environments are always complex, and managers will continue to focus on achieving short-term key performance indicators (KPIs) to maximise their incentives at the expense of flexibility (Sick and Gamba 2005; Hope and Player 2012; Ajak et al. 2017). The real questions are: What makes it difficult to adopt the RO view? What can be done to encourage the use of RO? Sick and Gamba (2005) answered the first of these important questions. Apart from pro-cyclical investment strategies, other factors include the activity-based compensation system, which encourages managers to exercise RO early (Sick and Gamba 2005), and earnings smoothing by managers, which, due to 'structural uncertainty', is a description of the unclassified sources of uncertainty that are connected neither to known parameters nor to the methodology being utilised (Bojke et al. 2009) or what is also known as the information gap (Knight 1921; Dosi and Egidi 1991; Langlois 1984; Kyläheiko 1998), makes decision makers tend towards optimism when the odds of success are sufficient (Kahneman 2011) while overlooking uncertain factors that could paralyse a project.

Additionally, the RO analysis, which has proven suitable for valuing flexibility (Groeneveld and Topal 2011; Ajak et al. 2017), is gaining recognition in academia but is not yet universally accepted in the mining industry compared to discount cash flow analysis (DCF), whose application is well rooted. Hindrances to the adoption of RO include the lack of an existing frame of reference. According to Kahneman (2011), the frame of reference is anything that is known or existing knowledge from past experience that can be used as the basis of dealing with the present situation. Additionally, the difficulty in calculating option values due to complex mathematical models that require large numbers of varied information and assumptions to value dynamics systems like mining operations has made RO less attractive to engineers and analysts in mining industry (Benaroch and Kauffman 1999; Haque et al. 2017, 2016). The only way operational managers can properly manage uncertainty is by having the ability to differentiate between uncertainty and risk. It is not uncommon for these two terms to be confused and loosely used (Koleczko 2012). Koleczko (2012) equated project managers' success in delivering objectives with their ability to manage risk and face uncertainty. However, such effective decision making can only occur if risk and uncertainty can be properly identified and isolated.

As stated by Copeland and Antinakor (2003), any innovation or introduction of new concepts, like RO, must be simple and based on an existing system in order for it to be accepted. In this research, the authors also argue that those factors which hinder the application of RO can be overcome if managers are provided with a structured framework for differentiating between risk and uncertainty and identifying areas of managerial flexibility. Such a framework must be backed up with a simple tool that provides managers with acceptable option values that set limits on optimal option values, that is intuitive, that is transparent and simple to apply, and is based on universal conventional concepts.

Based on previous research conducted by Ajak et al. (2017), we postulate that there is an existing association among the quotient of the option value and the NPV of rigid operations, which is referred to as the flexibility index, the project beta and volatility of project cash flows. Such an assertion has strong similarities with the work of Sick and Gamba (2005), who stated that ROs are unlike financial options which neither create nor destroy value. Rather, ROs are on real assets that cannot be replicated. Thus, ROs create value, and have an existing relationship with risk, which is measured by beta and volatility. However, ROs also have an operating leverage, as their costs and benefits are not perfectly correlated, because the underlying value of uncertain assets works against the option value.

Therefore, this research attempted to derive a relationship between flexibility index, project beta and the volatility of cash flows or prices. It is envisaged that the derived relationship can then be used to estimate option value. Once the option value is calculated, modified smooth pasting conditions with the value mean theorem can then be utilized to estimate the minimal and optimal option values. It is important to underscore that the estimation of minimum and maximum option value will provide managers with decision criteria which can be used to decide when to exercise the existing option. To test the validity of the proposed approach, the research utilized existing analytical results of the previous work by Ajak et al. (2017). As a result, this research proves that the commodity beta and price volatility can be used to easily and quickly estimate the value of managerial flexibility before embarking on complex modelling tasks that consume resources and time.

6.2 Difference between risk and uncertainty

The RO methodology is made up of four major distinct parts, which are the (1) computation of the base case values, (2) model uncertainty variables, (3) identifying and incorporating managerial flexibility and (4) conducting RO analysis (Copeland and Antikarov 2003). Since RO can only be beneficial when uncertainty is under consideration, it is critical that decision makers must have a clear understanding of differences between risk and uncertainty as they require different treatments. If a variable is identified as a risk, it could then be modelled using probabilistic techniques (Jablonowski et al. 2017) while uncertainty requires the use of RO. It should be highlighted that RO is both a decision process as well as an analytical technique. The inquiries and line of questioning shift the view on uncertainty, leading to the creation of alternative future actions to be taken and, eventually, help managers to justify investment in creating such flexibility within the system design.

Therefore, it is important that the differences between risk and uncertainty are briefly outlined as any confusion of the two terms has led some analysts to treat uncertainty with a flawed thinking that project uncertainty could be computed through probability and confidence intervals (Zagayevskiy and Deutsch 2014). Therefore, providing a summary of differences between risk and uncertainty (Table 6.1) is contextually helpful in understanding when to apply RO, where can it be applied and how to apply it.

Table 6.1. Differences between risk and uncertainty.

Risk	Uncertainty
Information is available about the extent and likelihood of occurrence of unfavourable events (Pass et al. 2005).	Incomplete information leads to uncertainties in derivative computations (Zagayevskiy and Deutsch 2014; Jablonowski et al. 2017).
Statistical analysis is performed on historical data to predict outcomes. The level of risk depends on the activity and its surrounding environment or factors.	Real options analysis is conducted to estimated flexible project value when uncertainty in input variables are considered and how the output variables responded in the model (Zagayevskiy and Deutsch 2014; Ajak and Topal 2015)
Risk scenarios are quantified using probabilistic model (Jablonowski et al. 2017).	The real option is used to value flexibility required to mitigate losses from uncertainty (Ajak et al. 2017; Ajak and Topal 2015).
Risk can be managed by applying Risk Management principles which include utilisation of the hierarchy of control such as to eliminate, to mitigated or reduce the impact (International Standards Organisation 2009).	Uncertainty cannot be eliminated, but RO analysis can be conducted in the project (Olivia 2011). Examples of uncertainty in the mineral industry are commodity price, exchange rate, geologic uncertainties such as rock and fluid properties, grade variability (Ajak and Topal 2015; Ajak et al. 2017; Haque et al. 2017; Jablonowski et al. 2017).
Risk management is more concerned with understanding the project's risk profile and focuses on its probability, impacts and mitigation mechanisms.	Uncertainty management is concerned with understanding the behaviours of key variables which are the real source of uncertainty and necessary for real options analysis.
Risk can be quantified and measured on input and output variables.	The ways that factors interact with one another cannot be easily quantified as some other sources of uncertainty are neither related to parameters nor analytical methodology applied (Bojke et al. 2009; Olivia 2011).

6.3 Exploring uncertainty and flexibility

As stated by Ajak et al. (2018), decision-makers should imagine the unimaginable. The world is unreliable, and it cannot be precisely predicted. The business environment has become so volatile, and uncertainty is higher than ever (Ajak et al. 2018). Thus, it is much better to build a system that can be adapted to change than trying to predict the unpredictable and assumed the future to be constant as per the present conventional models. The traditional valuation models such as DCF do not trigger the possibilities of the future decision. However, RO asks what could happen in the future since the model revolves around uncertainty and emergence of new information in the future. In the next subsections, uncertainty and flexibility, which are very central to the application of RO, are explained in detail.

6.3.1 Uncertainty

Brammer and Smithson (2008) put forward a taxonomy for uncertainty by distinguishing the known from the unknown (Table 6.2) and acknowledged the difficulty of integrating uncertainty into existing structures. Therefore, having a framework for uncertainty identification can enhance the operational ability to identify existing options.

Table 6.2. Uncertainty taxonomy as presented by Brammer and Smithson (2008).

		Known	*Unknown*
Primary level	*Known*	Known knowns	Unknown knowns (tacit knowledge)
	Unknown	Known unknowns (conscious ignorance)	Unknown unknowns (meta-ignorance)

Known knowns involve situations where information is known with certainty and strategies can be put in place to deal with the problem.

Known unknowns (conscious ignorance) is a situation where organisations focus effort and resources on finding out what has been recognised as being unknown (e.g, a drop in commodity prices is known but the amount cannot be reasonably approximated).

Unknown knowns (tacit knowledge) involve situations where there is uncertainty identified via experience or intuition, but no one can pinpoint the details (e.g, when and how an event will occur). In operations and industry, unknown knowns are common in the fields of technological innovation and data analytics. It is unknown what future innovation will look like, but it is known that technology presents uncertainty.

Unknown unknowns (meta-ignorance) are unexpected, such as black swan events.

6.3.2 Can uncertainty be reduced?

'Managing unknowns is just as important as making maximum use of what is known when responding to real world problems' (Brammer and Smithson 2008). Conventional wisdom is that uncertainty is reduced. However, this notion is being debated across various disciplines and sectors such as in economics, where there is an argument that certain knowledge is possible given enough time and effort (Brammer and Smithson 2008; Kasperson 2008). In this research, there is partial agreement with the economic notion of possible knowledge. It is postulated that there is value in unknown future information by creating RO in paying costs (effort) and to have the necessary needed flexibility ahead of time such that it can be exercised when needed.

Since uncertainty cannot be eliminated, there will always be uncertainty during the life of a mining project, but knowledge of the uncertain variable can be gained through further learning or inquisition. However, it is possible for one to argue that decision makers, particularly in mining operations, deploy a process which this research calls the 'transferential approach'. The transferential approach is a situation where decision makers can either change the sequence of activities to avoid dealing with an uncertainty now or delay knowledge acquisition. This approach disintegrates a complex state into small management events. For instance, complex geological information is simplified by starting mine development based on the assay data of an interim pit while the resource block model is continually updated as the information emerges from the exposed dig faces (Ajak et al. 2017). This implies that problems change as processes are fine-tuned and researched. In essence, information is utilized as it emerges and the knowledge of the situation is gained gradually.

6.3.3 Flexibility

Flexibility has its roots in manufacturing, where the concept was referred to as the flexible manufacturing system (FMS). This concept was brought in at a time when manufacturing was expanding, new markets were emerging and volatility was equally increasing. Thus, companies looked for a system that could allow them to increase productivity and lower operating costs (Singh and Skibniewski 1991). This required reductions in rework, elimination of processes that did not add value, avoidance of unnecessary delays and increased equipment utilisation.

Processes involved in mining production share similarities with those of manufacturing. The mining industry saw a huge expansion in the 21st century, especially from 2004 to 2015 when capital investment in mines was very high, new mines commenced, and production rates increased (Commonwealth of Australia 2015). This boom was driven by the expansion of the Chinese economy, which grew at a rate of more than 7% per year during that period (World Bank 2016). As a consequence, mining companies were caught off-guard; those whose production systems were rigid could not rapidly respond to the changing demands for bulk commodities like iron ore and coal. By the time that the mining companies responded to this demand by investing in the development of new mines and starting to produce, the coal and iron ore markets had already flattened-out, economic growth in China had started to slow down and, as a result of these two factors, iron ore and coal prices dropped sharply. These caused companies to pursue aggressive cost-cutting, and many projects were either suspended or collapsed.

Flexibility is defined as being more concerned with the ability of a system to sustain performance, preserve a particular cost structure, adapt to internal or external changes in operating conditions, or take advantage of new opportunities that develop during a mine's life cycle by modifying operational parameters. The ability to respond has to be rapid and cost-effective in order for its impact to be felt. This illustrates the point that current production processes and practices in the mining industry are well rooted in traditional methods that are very rigid and not easily adaptable to changing operational environments.

Even though the mining industry has adopted manufacturing processes that promote efficiency, such as lean methodology, with the hope that these processes will provide competitive advantages, many operations do succumb to volatility caused by external factors (Kazakidis and Scoble 2003). This is the main reason why operations collapse. Although they strive to be efficient, they fail to be agile, which comes from flexibility. Moreover, creating flexibility that leads to agility comes at a cost; however, it is where operational managers should focus.

The current trending topic in the mining industry is productivity improvement, which is tied to eliminating wastage and increasing efficiency. Management tends to narrow this down in terms of cutting costs. The main idea of this research is not to minimize the importance of managing costs. It must be emphasiszed that managing operational costs is very important and always will be, but it should not be the only thing that managers consider and focus on. The moment that cost-cutting takes the central stage, operations reduce their possibilities for creating RO in uncertain conditions and miss opportunities to mitigate losses and increase or create project value.

However, the 'key to improving productivity is being flexible and being able to adapt to changing conditions' (Honeywell Analytic 2015). Decision makers are very much aware that the world is uncertain, but there is a debate on how operations can best adapt to new environments. Adapting to change is critical for operational success in a tough business environment. The necessary ingredients required to succeed, which this research propagates as being applicable to creating RO in mining operations, are listed below:

- The ability to renew, adapt, change quickly and succeed in a rapidly changing, ambiguous, turbulent environment is referred to as agility (McKinsey & Company 2015). For organizations to succeed, they require speed and flexibility to respond and adapt rapidly (EY 2018). Thus, flexibility is the real source of business agility.

- Agility and capabilities that come from RO: Capabilities are essential for competing effectively in a given position, and agility is essential for making shifts in that position in response to a changing environment. Therefore, any option under consideration must meet both capability and agility tests if it is to add value to a mining operation. This means that mine planners and managers may need to do more than simply produce mine plans and aspire to achieve targets in these uncertain times.
- Hyperawareness that RO must provide: A company's ability to identify and monitor changes in its business environment and digital disruption that is marked by high market turbulence and shifting industry boundaries (McKinsey & Company 2015; Wade 2015; Deloitte 2016). This can be achieved through the use of data-mining methods. Organizations can deploy predictive data mining techniques where elements of mining operations, like maintenance and geological changes, can be predicted and options put in place to respond to help in making an informed decision (Ajak et al. 2017). For instance, if prices changes, organisations can use predictive data mining technique as a way of reducing cost by being proactive in controlling unscheduled breakdowns in mining equipment and processing plant. Such unplanned losses can be predicted in time, and RO can be created to mitigate losses.
- Informed decision making: A company's ability to make the most appropriate decision in a given situation (Wade 2015).
- Fast execution: A company's ability to execute its plans quickly and effectively (Wade 2015).

Therefore, mining operations must view RO not a simply a strategy but also an instrument for gaining the ability to be agile and are a key driver of future success.

6.4 Uncertainty identification framework, and incorporating flexibility into the design and planning of mining operations

In order to identify uncertain and risky variables, and to incorporate flexibility into a base case mine design, mine planners, analysts or project managers must ask certain essential questions. Some of the key questions this research used to analyze case studies of RO at the mining operation level are listed below. These have been translated into the uncertainty identification framework tree shown in Figures 6.1, 6.2, 6.3, 6.4 and 6.5.

i. What are the main uncertain parameters or variables driving value or influencing outcomes?
ii. What could possibly happen to these variables?
iii. Where can design or processes be focused?
iv. What is the value of the flexibility created?
v. How much does flexibility cost?

6.4.1 Uncertainty identification framework

The main reason for developing this framework was that the application of RO is still not definite. It is not structured, and areas of its application in mining cycles are not clearly demarcated. This framework is based on the Cardin et al. (2016) four-step process concept but, is very different in that the framework is tailored towards the application of RO at a mining operational level. The framework will be composed of a standard base case design, uncertainty recognition, concept generation and design space exploration where flexibility areas are identified.

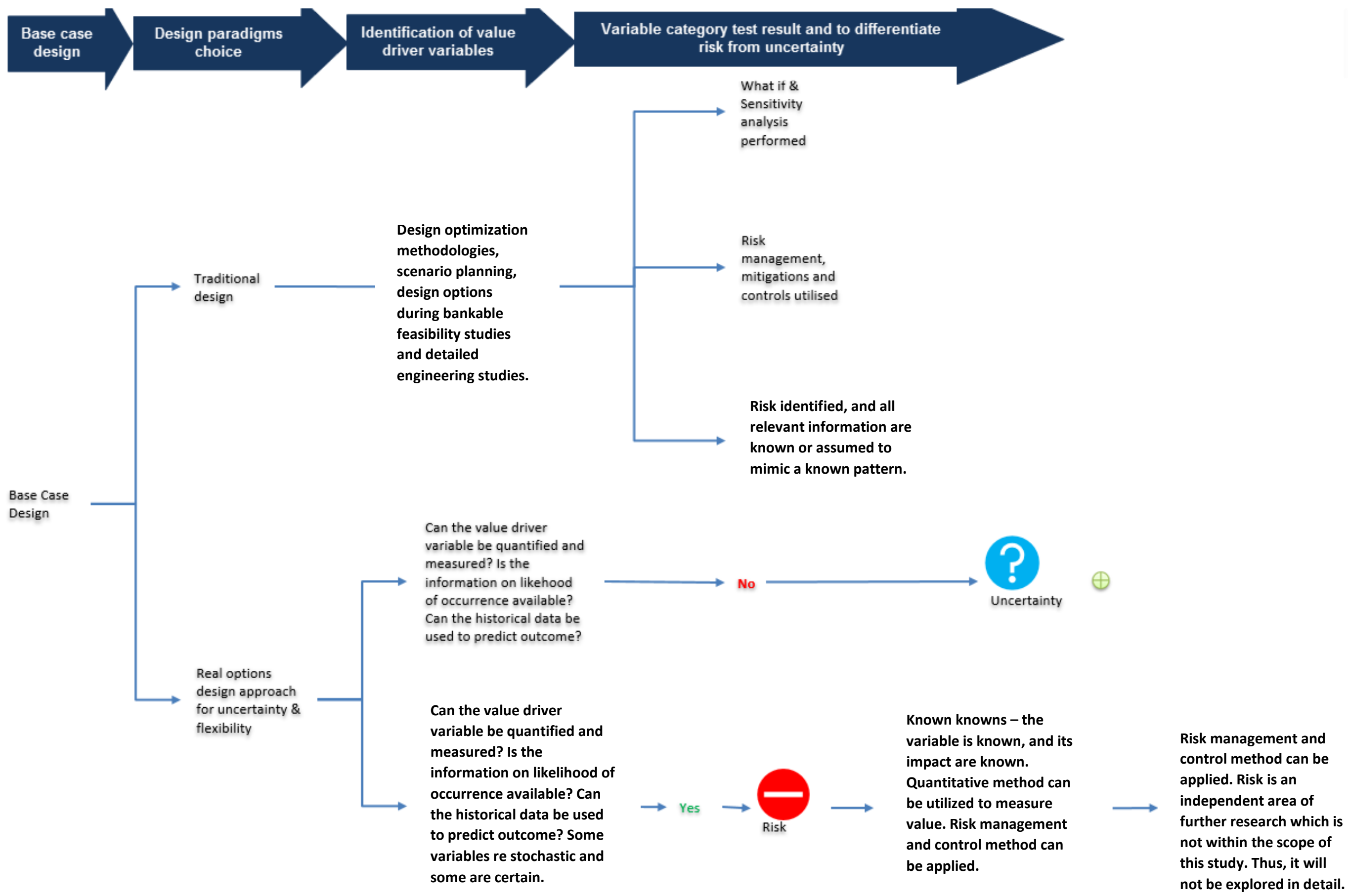

Fig. 6.1, Standard base case design utilised as the starting point before applying uncertainty framework.

** ⊕ indicates that there are more collapsed branches associated with the last visible parent branch.

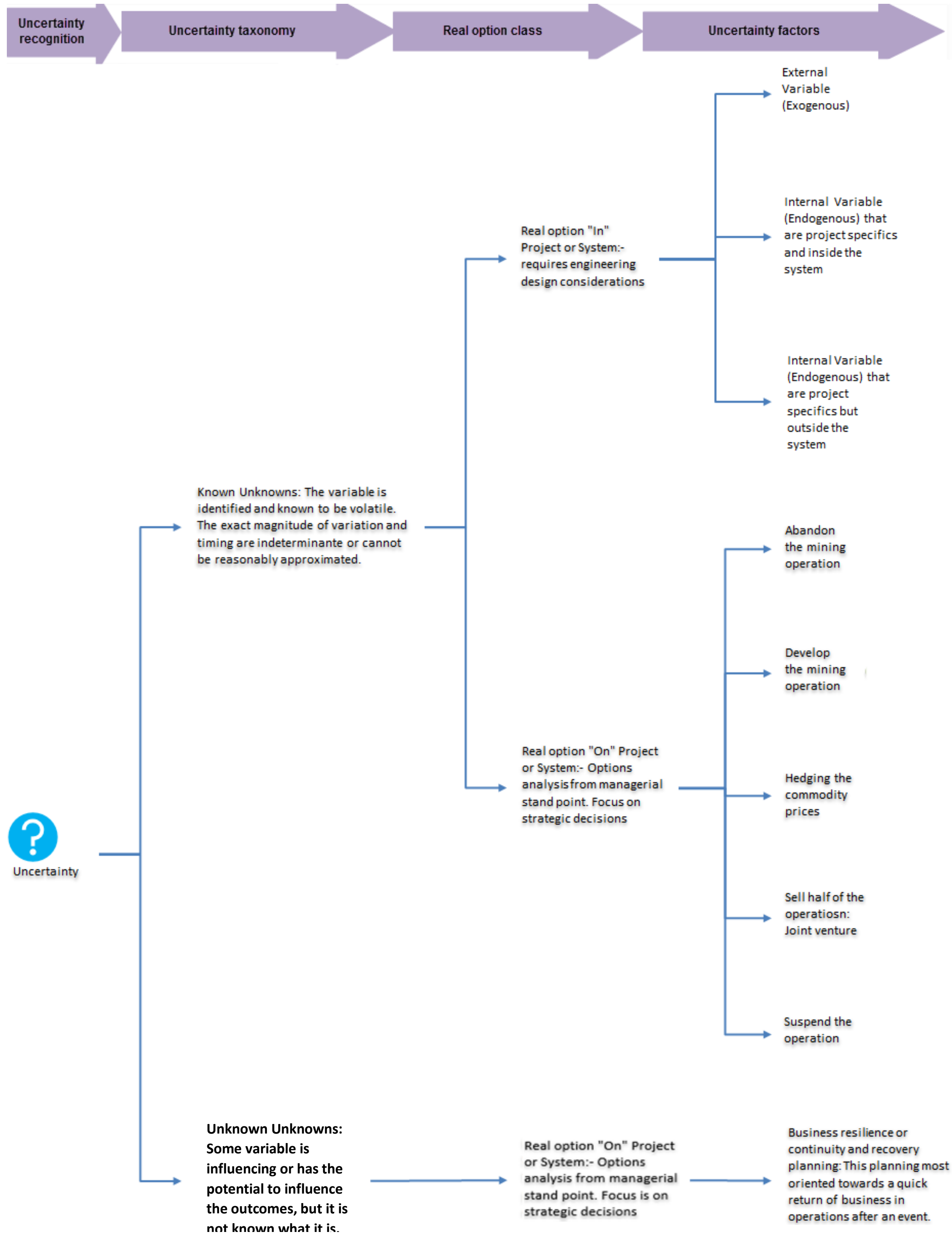

Fig. 6.2, Uncertainty recognition, taxonomy, and real option classes.

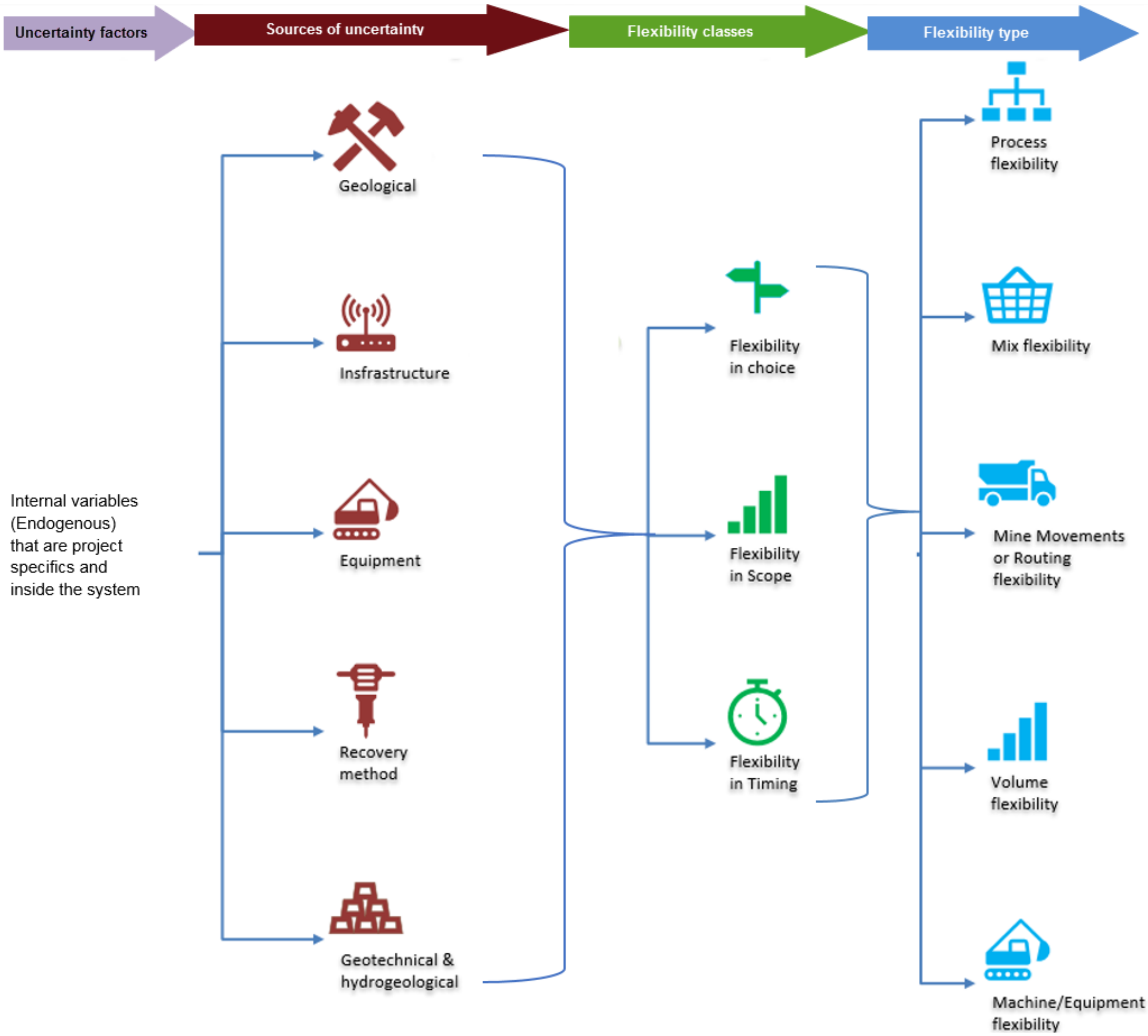

Fig. 6.3, Identifying internal sources of uncertainty within the design of mining operations, as well as available flexibility classes and types.

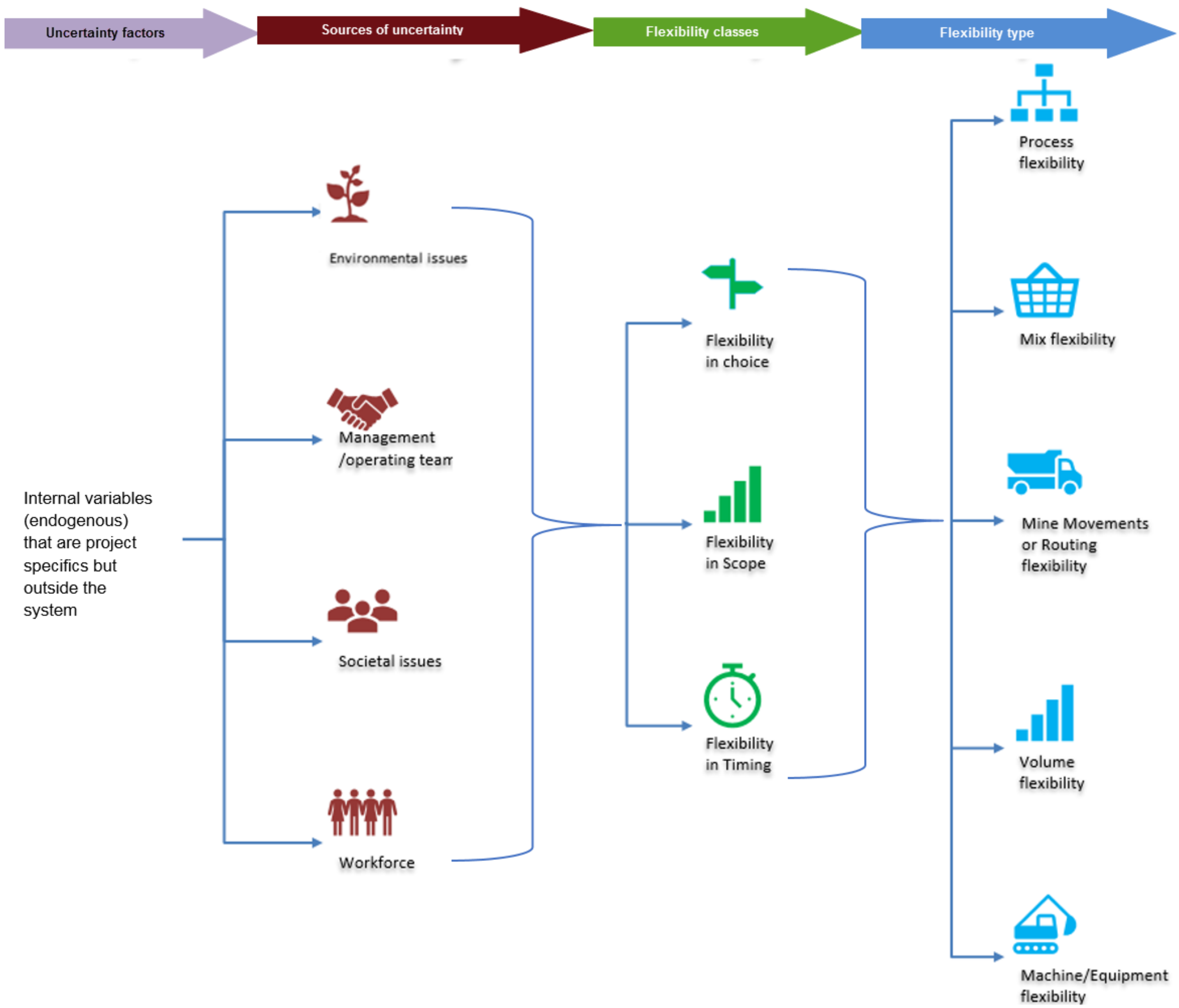

Fig. 6.4, Identifying internal sources of uncertainty which are outside the design of mining operations, as well as available flexibility classes and types.

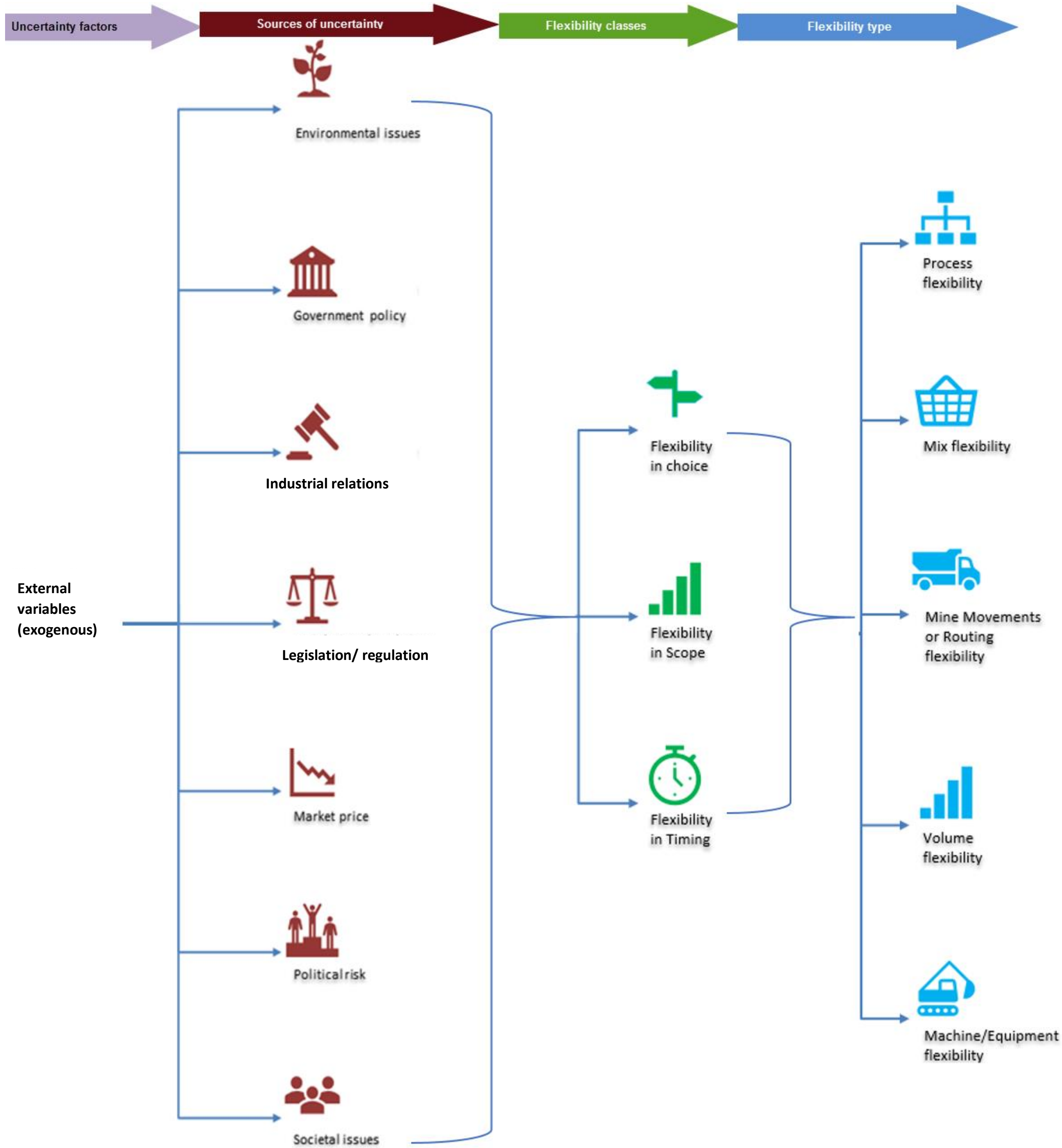

Fig. 6.5, Identifying external sources of uncertainty that are outside the control of mine management, as well as available flexibility classes and types.

6.4.2 Application domain of the highlighted flexibility types in a mining operation

Table 6.3 shows examples of areas or domains where flexibility can be incorporated into a mine design or operation. It must be noted that the domain list is not exhaustive, nor should it be, as RO open up a design system to a sea of opportunities. Therefore, this research only provides a framework that can be used as a guide, and maps out important classes and types of flexibility.

Table 6.3, Examples of real option domains in a mining operation.

Flexibility Type	Domain Examples
Process flexibility	• Different pits that can be planned to be mined in alternate or consecutive periods. • Changes in pit or bench sequencing and design? • Which machine to use for digging and in which pit?
Mix flexibility	• Blending material types in term of grades, hardness, sizes or locations. • Stockpiling capacity can create RO when operations are in dire need of material for crusher feeding or blending of different materials.
Routing flexibility	• Which pit or bench material is to go to which stockpile? • Which dumping location is to be used? • Which machine should be trammed and when? This is relevant to reducing effective flat haul and cycle times. • Which route to schedule and when?
Volume flexibility	• When to increase material going into the crusher? • Which pit to campaign? • When to reduce or increase the saleable ore product? • When to increase ore to reduce the total material movement in and out of mine? • When to increase or decrease truck capacity (lighter trays, more volume), dig unit capacity (same dig unit, different bucket size)? • When to increase or decrease total material movement, pre-stripping? • When to commence land clearing or rehabilitation of disturbed areas?
Machine or equipment flexibility	• When to shut the dig unit or park up trucks? • When to commission more trucks? • Which machine to use and in which pit? • Which machines should be moved around in the case of unplanned breakdowns? • Are there any options for machine automation?

6.5 Nexus between beta and the flexibility index: a tool for estimating operational option value

6.5.1 Beta and its relevance to RO?

Opportunities for creating real value for shareholders through the implementation of RO methodology have always been missed, and they will continue to be missed if operational managers and the market as a whole do not start to view engineering design and decision-making analysis as investment option rather than being treated as solely technical processes. Even though there have been discussions on the need to re-innovate and change how mines are planned and operated, the industry response to the changing business conditions has not been rapid. The re-innovation must involve both processes and attitude toward uncertainty. The main issue in mining is the laxity in adopting new ways as the industry is risk averse and treats uncertainty as a bad thing. Mine managers and engineers tend to utilize tools that have been tested in other industries and provide certainty. Therefore, it is upon the academia and researchers to prove the case that RO is the better tool compared to DCF. Secondly, the means of valuing flexibility has to be linked to the existing acceptable methods in order to mount a convincing case on why managers must invest in creating flexibility which is the real source of operational agility.

The most important characteristic of beta that make it relevant in the estimation of RO value as demonstrated in this research is its ability to measure the risk of additional investment on a diversified portfolio. Since beta is a standardized value that is closer to 1.0 since it measures the relative risk of an asset, it appears to have some association with flexibility index, which measures the worthiness of various options and to justify investing the additional capital needed to create such options (Kazakidis and Scoble 2003).

Therefore, beta is a measure of a stock's volatility in relation to the whole market (Goddard 2005). The whole market is taken as the basis of comparison and is treated as diversified with a beta value of 1.0, while the individual asset an investor wants to buy is ranked according to its variance. A security whose beta is greater than 1.0 is riskier than the market as a whole but can potentially provide greater rewards, while a security with a beta of less than 1.0 is less risky than the market as a whole, but its returns will probably be lower. Resource stocks that are normally dependent on commodity prices tend to be much more volatile compared to retail stocks and, thus, they have a higher beta. Thus, beta is a quantifiable measure of risk (Goddard 2005). As such, companies have developed their own beta bottom-line and these can be obtained from investment websites such as Yahoo Finance, MSN Money and many other financial databases. Beta can also be calculated using regression analysis by applying the following linear relationship:

$$y = a + \beta x \tag{6.1}$$

where y is the dependent variable such as the stock or portfolio, a is the bond yield, β is the beta value and x is the independent variable, such as a stock market index like the S&P 500 or ASX 200.

It is important to emphasise that beta is also a measure of the sensitivity of a stock relative to the market. Therefore, it can be used as a predictive tool. For instance, if a market goes up by 5%, a stock with a beta of 1.5, such as a mining stock, will be expected to go up by 7.5%.

6.5.2 Utilisation of project beta in RO analysis

The RO methodology utilized the concept of NPV, which is calculated from discounted cash flows. However, the discount rate that is utilized in DCF analysis is derived from capital asset pricing models (CAPM), which measure investment risk relative to the whole market. However, an important input in CAPM calculation is the project beta. Most analytical models utilize in financial markets measure risk as a variance of realized returns and expected returns (Damodaran 1998). Beta measures only the risk associated with an additional investment in a diversified portfolio. Based on such propagation, risk is categorized as being either firm-specific on investment or applying to the entire market.

Even though beta can be calculated using methodologies such as linear regression, relative volatility approach and pure play method, it was deemed unnecessary to show such calculation in this research as those techniques have been widely covered in most financial textbooks and plenty of journal articles have been published on those methods. Therefore, this paper explains project beta using DCF technique where discount rate is estimated from CAPM.

Considering that mining projects are always unique, with each having its own risk, the true representation of a project's cash flow is the project itself (Copeland and Antikarov 2003). This concept is known as the market asset disclaimer (MAD) assumption, where the true market value of the project is the present value of the project itself, without flexibility (Copeland and Antikarov 2003). The MAD assumption has gained acceptance in RO analyses of engineering projects, particularly in valuing mining operation flexibility, as demonstrated by Ajak and Topal (2015) and Ajak et al. (2017). However, the single period present value (*PV*) is the quotient of the project free cash flow (*FCF*) which is discounted by the risk-adjusted rate calculated as:

$$PV = \frac{FCF}{1 + Risk - adjusted\ discount\ rate} \qquad 6.2$$

However, from the CAPM, the risk-adjusted discount rate is the expected rate of return, which is expressed as:

$$\overline{r_a} = r_f + \beta_a(\overline{r_m} - r_f) \qquad 6.3$$

where r_f is the risk-free rate, β_a is the beta of security (expressed as relative movement of a specific security relative to the return the investor wants), $\overline{r_m}$ is the expected market return of the highly diversified portfolio, and $\overline{r_a}$ is the expected return of the risky asset.

Equation 6.3 shows that $(\overline{r_m} - r_f)$ is the risk premium r_p, which investors put on the underlying asset. An analysis of the historical risk premiums from 1883 to 2010 showed that the average r_p in Australia was 6.1% (Damodaran 1998; Damodaran Online 2017) and 6.5% for government bonds and bills (Brailsford et al. 2012). For most mining projects, particularly iron ore, r_p is taken as 6% with a beta of 1.55 and r_f of 5% (Kleymenova et al. 2009). Since the r_p and r_f assumptions have been very consistent in the major analyses and case studies that have been performed (Damodaran 1998; Kleymenova et al. 2009; Brailsford et al. 2012; Ajak and Topal 2015; Ajak et al. 2017), the only variable that should be derived in the equation is the project $\overline{r_m}$. Equation 6.3 can be rearranged to derive $\overline{r_m}$

$$\overline{r_m} = \frac{(r_a - r_f)}{\beta_a} + r_f \qquad 6.4$$

If Eqs. 6.3 and 6.4 were utilized to estimate the required discount rate for the analysis of RO in an iron ore project, the discount rate of the case study is 5% + 1.55*6% = 14.3%. Therefore, the beta of a single period can be calculated as a covariance of the commodity spot price and the FCF of the project. Since *PV* is a summation of the DCFs of future periods, the associated covariance is calculated over successive periods and the sample size reduced as the RO approaches maturity or the project reaches its end.

6.6 What is the flexibility index?

The flexibility index (γ) is the ratio of the NPV with flexibility to the NPV without flexibility. Alternatively, it can be expressed as a percentage of the option value to the NPV of the rigid design (Kazakidis and Scoble 2003):

$$Flexibility\ index, F(\%) = \frac{Option\ Value}{NPV\ Passive} \times 100, \text{Option value} > 0 \qquad 6.5$$

If an option is not properly analyzed and the capital is committed to create it, such an investment will become irreversible regardless of the management decision to exercise the option or not. As the option values are not usually obvious, the blurredness in expected option values has been the limiting factor in the application of real option methodologies. Thus, having some kind of a measure like the flexibility index can ease investment decision making to create RO in mine design.

Based on Equation 6.5, if γ is less than 1.0, it implies that the mine operations are very rigid with too many constraints and there are possibly many bottlenecks within the system. As a result, the operations development is impeded as the mine is entirely lacking flexibility and the planned objectives may not be achieved for a set period. If γ is equal to 1.0, it means that the operation is neither rigid nor flexible, but it remains vulnerable to unforeseen events that can lead to losses and may even cause the operation to fail. If γ is greater than 1.0, then the operation is flexible and it can withstand any unforeseen events. Therefore, γ is a robust measurement that promotes proactive management of uncertainty.

6.6.1 Relationship between beta and flexibility index

To illustrate this concept, summarized data from the options analysis of Ajak et al. (2017; 2018) is used. These authors analyzed two case studies by applying a stochastic simulation process to obtain all the possible price paths using mean-reversion with jump diffusion before applying the RO methodology to value managerial flexibility when running a mining operation. The first case study (Ajak et al. 2018) was an analysis of an iron operation in the Western Australian Pilbara region. These operations closed in April 2016 when iron ore prices crashed to less than $US40 per tonne. Ajak et al. (2018) assessed various flexibility options that were available to this mining operation from the onset, which included abandon, delay and staged investment options. From the analysis of these options under uncertain iron ore prices, Ajak et al. (2018) demonstrated that RO increased the project's value by between 56% and 195% depending on the degree of volatility. Table 6.4 shows the present values of various options and this will be used to show an existing relationship between project beta (β), the flexibility index (γ), and project volatility (σ).

Table 6.4, Summary of a project's NPV with the real option case analysis conducted by Ajak et al. (2018)

Analysis model	**NPV, $M**	NPV_f - ***NPV***	**Option value, $M**
Traditional DCF analysis	($750)	($750)– ($750)	$0
Stochastic simulation	($328)	(-$328) – (-$750)	$422
Option to abandon	$243	$243 – (-$750)	$993
Option to delay	$250	$250– (-$750)	$1,000
Staged investment	$715	$715– (-$750)	$1,465

In order to reveal the underlying relationship between the β, the γ of various options, and σ of the project whose RO results are shown in Table 6.4 (Ajak et al. 2018), the industry beta for iron ore was estimated to be 1.55 (Kleymenova et al. 2009), as shown in Table 6.5. The analysis of historical iron ore prices between 1985 and 2015 (IndexMundi 2015) showed that the standard deviation of past prices was 42.4%. Thus, the percentage of the price up-side movement (μ) is ≈ 1.53. Additionally, the project presented values of the stochastic model of the real case study to have a volatility of 49.9% (Table 6.5) and $\mu \approx 1.62$, while the flexibility index of the stochastic model was 1.56 which is similar to $\mu = e^{\sigma\sqrt{\Delta t}}$. Where Δt is the time step and e is the Euler's constant.

Table 6.5, Summary of similarities between project beta and the stochastic model (Ajak et al. 2018).

Standard deviation	**Risk Neutral movement**		**Flexibility index**	**Industry Iron ore beta**
	Project volatility (σ)	Price up-side movement (μ)	(γ)	(β)
Historical Price Stochastic Simulated PV	42% 49%	1.528 1.624	1.56	1.55

The stochastic simulation model was applied to the base case in the analysis to assess the underlying risk, which could not be revealed by the traditional DCF method. It should be noted that stochastic simulation does not replace RO, but it enhances the process. Therefore, this technique was applied before various options were considered. It is apparent from the stochastic simulation model that the beta for iron ore is almost equal to the minimum flexibility index and is also equivalent to the μ value in the risk-neutral equation.

When the differences between the beta and flexibility indices of the various options were computed (Table 6.6), it was revealed that their difference in the stochastic simulation model without options was almost zero, but the absolute values of the iron ore beta and flexibility of the model were almost similar. However, when various options were considered, it appeared that the flexibility index of the project with flexibility was actually a sum of the beta and the volatility of the project. As expected, as the volatility increased, the option value also increased. Thus, indicating a high level of uncertainty (Fig. 6.6). This relationship does intuitively make sense because option values are directly tied to volatility.

Table 6.6, Differences between beta and flexibility indices of various options.

Real Options	Flexibility index	Delta of Flexibility Index vs Beta
Traditional NPV by 2016	1.00	-35.5%
Stochastic simulation	1.56	0.8%
Option to abandon	2.32	49.9%
Option to delay investment	2.33	50.5%
Option to stage investment	2.95	90.5%

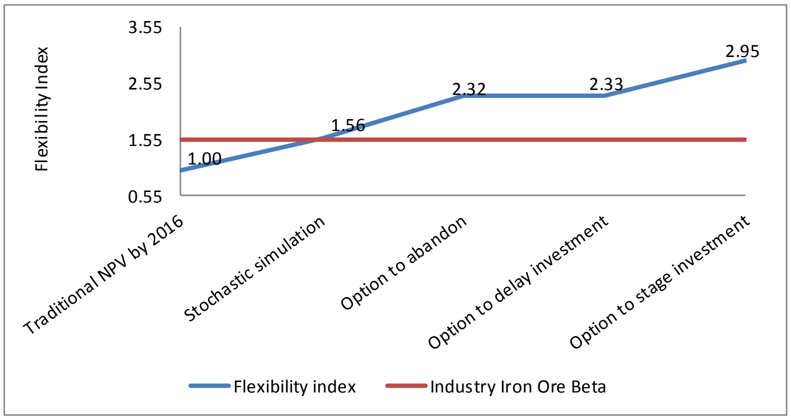

Fig. 6.6, Flexibility indices of the project with various options versus project betas.

Since the beta value for any commodity is widely available from various industry databases, the minimum option value of the flexible design can easily be estimated using the derived the relationship between beta β, volatility of the project value σ, which is estimated as the standard deviation of the natural logarithms of returns, flexibility index γ, and the risk-neutral relationship for upside μ. Thus, this observed relationship can be expressed as:

$$\beta \approx \gamma \approx \mu = e^{\sigma\sqrt{\Delta t}} \quad 6.6$$

This implies that

$$\sigma \approx \ln(\beta) \quad 6.7$$

$$\gamma = (e^{\ln(\beta)\sqrt{\Delta t}})+\sigma \quad 6.8$$

Following the same logic explained above, it is possible to draw some relationship between these two measures (Eqs. 6.7 and 6.8). Even though the size of uncertainty cannot be easily quantified, it can be observed directly in a response variable (Zagayevskiy and Deutsch 2014). In line with the proposition of Zagayevskiy and Deutsch (2014) where the associated uncertain values can be computed through probability and confidence intervals if the coefficients of the model are found to fit the data reliably, namely Eqs. 6.7 and 6.8, to construct a regression model as shown in Figure 6.7. Hence, an increase in beta or volatility will, in turn, increase option values and result in a higher flexibility index. As shown in Figure 6.7, the commodity beta and flexibility index are strongly associated:

$$\gamma = 3.0726\ln(x) + 0.1018 + \sigma \quad 6.9$$

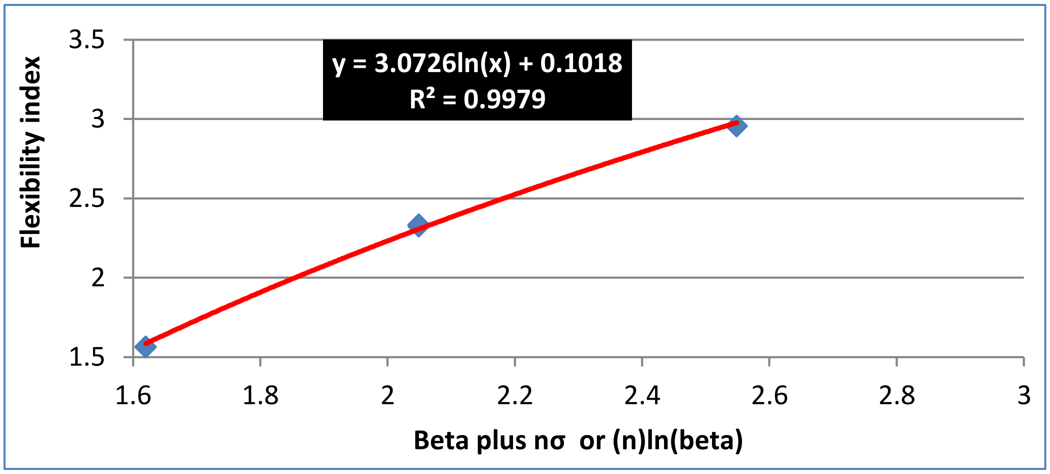

Fig. 6.7, Relationship between the flexibility index and project beta.

6.6.2 Determining the optimal option value as calculated using beta and flexibility linear function

The greatest dilemma facing the implementation of RO, especially in translating analytics to investment, is lack of clarity in determining which option value is the most acceptable out of all other options. A leading study on the concepts of optimal RO values is that of Sick and Gamba (2005). These authors outlined the following four main issues or that hinder the adoption of RO.

6.6.2.1 The smooth pasting condition

The main premise of the smooth pasting condition is that there is an optimality condition that describes an optimal trigger point of the real option. To describe this concept, as put forward by Sick and Gamba (2005), a schematic plot of option values versus underlying asset value (thick purple line in Figure 6.8) was utilized. When various options were added onto the plot, it was apparent that an optimal option value exists between points *a* and *b* on Figure 6.8.

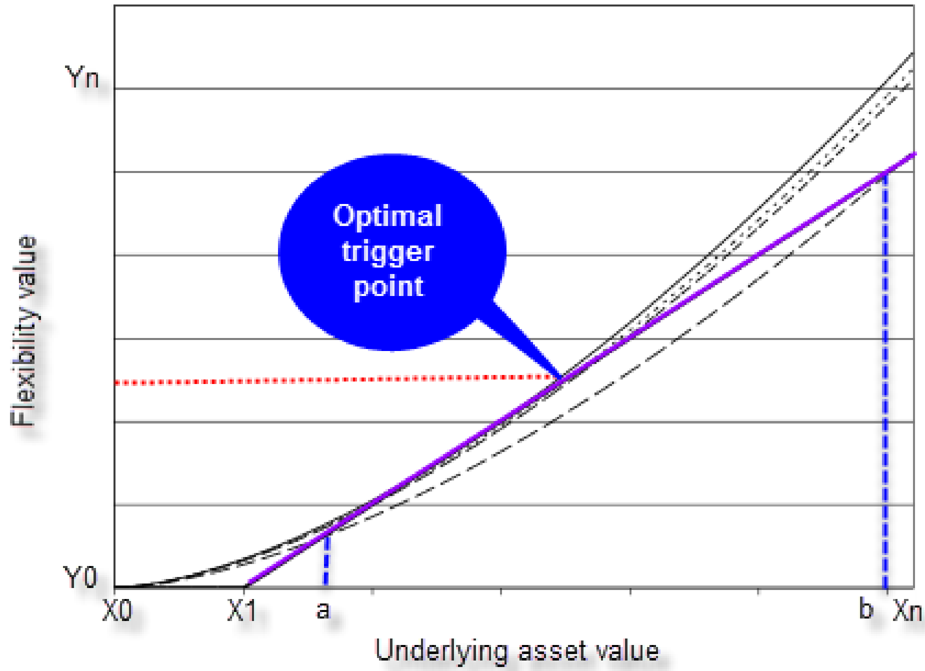

Fig. 6.8, Smooth pasting condition illustration (adopted from Sick & Gamba, 2005).

In Figure 6.8, if an option is exercised early between points *X1* and *a*, then the value of that option is lost. Therefore, finding a *trigger value* of the underlying asset, which is the *payoff* from the option, is the better way of making an optimal decision. The *trigger* or *payoff value* is defined as:

$$\text{Max}\ (0, \text{PV} - \text{k}), \tag{6.10}$$

where *k* is the exercise cost. Therefore, the trigger can be expressed as $a < PV < b$

However, Sick and Gamba (2005) postulated that the option value must not be less than the payoff when exercised. This leads to the application of a partial differential, thus:

$$Max(0, PV - k) = \frac{\partial W}{\partial PV} \tag{6.11}$$

where *k* is the exercise price and *W* is the option value.

However, the main drawback of the smooth pasting condition is its tendency to encourage poor discipline, as it allows managers to apply rules of thumb (Sick and Gamba 2005), as many never perform the complicated calculation. To overcome the subjective characteristics of smooth pasting and to simplify the calculation, the mean value theorem (Stewart 2003, p. 291) was applied to find the optimal trigger point.

A quick summary of this theorem (Stewart 2003, p. 291) is that f is a function that satisfies the following hypothesis:

- f is continuous on a closed interval (a, b)

- f is differentiable on the open interval (a, b)
- there is a number c in (a, b), such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \tag{6.12}$$

where $f'(c)$ is the slope of the tangent line of point $(c, f'(c))$. If this concept is applied to Figure 6.8 above, it is very clear that $f'(c)$ is the flexibility index γ of the project.

However, how can a decision maker work out (a, b) or the project values at points $(a, f(a))$ and $(b, f(ab))$? To explain this, the flexibility and beta values derived were applied to case study data published by Ajak et al. (2017). In their research, they applied predictive data mining to solve geological uncertainty presented by clay material discovered during mining operations, and they went ahead and created various flexibility options for the mining operation and assessed the impacts of these options on project value over two years period. Table 6.7 presents the results of the RO analysis performed by Ajak et al. (2017). Details of the case study and in-depth analysis are available in Ajak et al. (2017).

Table 6.7, Summary of the iron ore project value with various flexibility alternatives (Ajak et al. 2017).

Period	Price	Traditional design non-flexible operations				MC simulation of non flexible operation					Operation with material rehandling option					Operation with additional dig face option					Operation with bypass circuit for switching option				
Months	Historical price, $/t (IndexMundi ,2017)	Monthly plant feed (t)	FCF, US$M	DCF, US$M	PV @ start of period, US$M	Monthly plant feed (t)	FCF, US$M	DCF, US$M	PV, US$M	Delta of traditional DCF vs MCS, US$M	Monthly plant feed (t)	FCF, US$M	DCF, US$M	PV, US$M	Flexibility value, US$M	Monthly plant feed (t)	FCF, US$M	DCF, US$M	PV, US$M	Flexibility value, US$M	Monthly plant feed (t)	FCF, US$M	DCF, US$M	PV, US$M	Flexibility value, US$M
Jan-15	$69.1	592,134	$9.7	$9.3	$113.8	592,345	$9.7	$9.3	$111.2	-$2.7	591,672	$10.9	$10.3	$138.1	$24.3	594,482	$10.3	$9.8	$127.3	$13.5	599,736	$10.6	$10.1	$132.1	$18.3
Feb-15	$63.8	602,850	$7.2	$6.9	$104.5	601,023	$7.2	$6.8	$101.9	-$2.7	610,495	$8.4	$8.0	$127.8	$23.2	601,661	$7.8	$7.4	$117.5	$13.0	610,590	$8.1	$7.7	$122.0	$17.5
Mar-15	$58.2	602,900	$4.3	$4.1	$97.7	602,355	$4.3	$4.1	$95.0	-$2.6	603,045	$5.4	$5.1	$119.8	$22.1	604,082	$4.9	$4.7	$110.1	$12.5	610,641	$5.1	$4.9	$114.4	$16.7
Apr-15	$52.1	624,383	$1.3	$1.2	$93.5	623,255	$1.3	$1.2	$90.9	-$2.6	629,860	$2.4	$2.2	$114.7	$21.1	624,431	$1.9	$1.8	$105.4	$11.9	632,399	$2.1	$2.0	$109.5	$15.9
May-15	$60.4	602,900	$5.5	$5.2	$92.3	600,934	$5.5	$5.2	$89.7	-$2.6	604,870	$6.6	$6.3	$112.4	$20.1	609,443	$6.1	$5.8	$103.7	$11.3	610,641	$6.3	$6.0	$107.5	$15.1
Jun-15	$61.8	601,412	$6.2	$5.9	$87.1	608,742	$6.3	$6.0	$84.5	-$2.6	608,491	$7.3	$7.0	$106.1	$19.0	607,137	$6.8	$6.5	$97.9	$10.7	609,133	$7.0	$6.7	$101.5	$14.4
Jul-15	$52.2	559,836	$1.2	$1.1	$81.2	560,063	$1.2	$1.1	$78.6	-$2.7	560,533	$2.1	$2.0	$99.2	$17.9	560,942	$1.7	$1.6	$91.4	$10.2	567,023	$1.9	$1.8	$94.8	$13.6
Aug-15	$57.3	645,964	$4.2	$4.0	$80.1	644,118	$4.2	$4.0	$77.4	-$2.7	646,143	$5.3	$5.1	$97.2	$17.0	650,981	$4.8	$4.6	$89.8	$9.6	654,258	$5.0	$4.8	$93.0	$12.9
Sep-15	$57.0	559,637	$3.4	$3.3	$76.2	560,581	$3.5	$3.3	$73.5	-$2.7	561,774	$4.4	$4.2	$92.1	$15.9	562,594	$4.0	$3.8	$85.2	$9.0	566,822	$4.2	$4.0	$88.2	$12.0
Oct-15	$53.7	526,497	$1.8	$1.7	$72.9	528,031	$1.8	$1.7	$70.2	-$2.7	527,229	$2.7	$2.5	$87.9	$15.0	530,120	$2.3	$2.2	$81.4	$8.5	533,257	$2.4	$2.3	$84.2	$11.3
Nov-15	$47.2	462,002	-$1.0	-$1.0	$71.2	463,184	-$1.0	-$1.0	$68.5	-$2.7	464,362	-$0.2	-$0.2	$85.3	$14.1	460,969	-$0.5	-$0.5	$79.2	$8.0	467,933	-$0.4	-$0.4	$81.9	$10.7
Dec-15	$40.9	410,371	-$3.1	-$2.9	$72.2	413,362	-$3.1	-$3.0	$69.5	-$2.7	409,621	-$2.4	-$2.3	$85.5	$13.4	413,350	-$2.7	-$2.5	$79.7	$7.5	415,640	-$2.6	-$2.5	$82.3	$10.1
Jan-16	$42.2	645,964	-$1.9	-$1.8	$75.1	649,463	-$2.2	-$2.1	$72.4	-$2.7	648,043	-$0.9	-$0.8	$87.8	$12.7	650,052	-$1.2	-$1.2	$82.2	$7.1	654,258	-$1.1	-$1.1	$84.8	$9.7
Feb-16	$46.5	538,304	$0.3	$0.3	$76.9	536,640	$0.3	$0.3	$74.5	-$2.5	542,436	$1.3	$1.2	$88.7	$11.7	542,154	$0.9	$0.9	$83.4	$6.5	545,215	$1.0	$1.0	$85.9	$8.9
Mar-16	$56.5	645,964	$5.9	$5.7	$76.6	651,651	$5.6	$5.3	$74.2	-$2.4	648,515	$7.1	$6.8	$87.4	$10.8	647,968	$6.6	$6.3	$82.5	$5.9	654,258	$6.8	$6.5	$84.9	$8.3
Apr-16	$61.0	557,525	$7.2	$6.9	$71.0	558,920	$7.1	$6.7	$68.8	-$2.1	564,292	$8.3	$7.9	$80.7	$9.7	556,210	$7.8	$7.4	$76.2	$5.3	564,683	$8.0	$7.6	$78.4	$7.4
May-16	$55.9	645,964	$5.6	$5.3	$64.1	650,863	$5.3	$5.0	$62.1	-$2.0	647,620	$6.7	$6.4	$72.8	$8.7	652,510	$6.3	$6.0	$68.8	$4.7	654,258	$6.5	$6.2	$70.7	$6.6
Jun-16	$52.3	538,055	$3.0	$2.9	$58.8	542,118	$2.8	$2.6	$57.1	-$1.7	540,098	$4.0	$3.8	$66.3	$7.6	538,010	$3.6	$3.4	$62.9	$4.1	544,963	$3.7	$3.6	$64.6	$5.8
Jul-16	$57.3	645,965	$6.4	$6.1	$55.9	650,850	$6.1	$5.8	$54.4	-$1.5	644,873	$7.5	$7.2	$62.5	$6.6	649,720	$7.0	$6.7	$59.4	$3.5	654,258	$7.3	$6.9	$61.0	$5.1
Aug-16	$60.9	581,368	$7.5	$7.1	$49.9	587,352	$7.1	$6.8	$48.7	-$1.2	582,813	$8.6	$8.2	$55.4	$5.5	585,775	$8.1	$7.7	$52.8	$2.9	588,832	$8.3	$7.9	$54.1	$4.3
Sep-16	$57.7	612,476	$6.2	$5.9	$42.7	619,593	$5.8	$5.5	$41.9	-$0.9	614,524	$7.3	$7.0	$47.2	$4.5	614,827	$6.8	$6.5	$45.0	$2.3	620,339	$7.1	$6.7	$46.2	$3.5
Oct-16	$59.0	601,549	$6.8	$6.4	$36.8	607,157	$6.4	$6.1	$36.3	-$0.5	604,733	$7.9	$7.5	$40.2	$3.4	602,783	$7.4	$7.0	$38.5	$1.7	609,272	$7.6	$7.2	$39.5	$2.6
Nov-16	$74.1	555,048	$13.4	$12.8	$30.4	555,079	$13.3	$12.6	$30.2	-$0.2	553,517	$14.5	$13.8	$32.7	$2.3	557,223	$13.9	$13.3	$31.5	$1.1	562,174	$14.3	$13.6	$32.2	$1.8
Dec-16	$79.4	645,964	$18.5	$17.6	$17.6	643,887	$18.4	$17.6	$17.6	-$0.1	652,556	$19.9	$18.9	$18.9	$1.3	650,460	$19.1	$18.2	$18.2	$0.6	654,258	$19.6	$18.6	$18.6	$1.0

From Table 6.7 we can make the same conclusions as Ajak et al. (2017). It is clear that the optimal decision was to increase material rehandling to maintain the crusher feed whenever the operations encountered clay material (Ajak et al. 2017). Therefore, this option will be used to demonstrate the proposed concept.

As shown here, the option value can be calculated using $\mu = e^{\sigma\sqrt{\Delta t}}$ and $\gamma = (e^{\ln(\beta)\sqrt{\Delta t}}) + \sigma$. Table 6.8 can be expressed in terms of the project beta, the flexibility index and project volatility. The main uncertain variable that was utilized to calculate the project volatility was the commodity price. Additionally, the boundaries of the acceptable option value can be obtained as shown below:

$$f'(a) = e^{\sigma\sqrt{\Delta t}} \text{ and } f'(b) = (e^{\ln(\beta)\sqrt{\Delta t}})+\sigma \qquad 6.13$$

These are the minimum and maximum flexibility indices for the real option, as shown in Table 6.8. This logic is based on the fact that an investor will not accept investment in a project that will return a negative cash flow. The investor always expects an upside swing; therefore, setting $a = \mu = e^{\sigma\sqrt{\Delta t}}$ provides the minimum flexibility index, which can be used to calculate the minimum flexibility value, while the maximum value is the function of $f'(b) = (e^{\ln(\beta)\sqrt{\Delta t}})+\sigma$ as shown above.

What would happen if an option is changed? In a traditional methodology, different options result in different option values but the project beta remained constant. However, in the model proposed in this paper, beta would dynamically respond to any change. When an option is changed, flexibility index would also change because the NPV has changed. As a consequence of this change of option, the volatility of project value is also changed as the associated expected cash flows would have changed. Therefore, Equation 6.7 can be applied to calculate a new beta value. Finally, any change in option will trigger a change in all calculated values. Thus, the derived relationship remains valid even when variables change.

T**able 6.8**, Derived values of flexibility indices and trigger option values which can be used to decide when to exercise the existing option.

Period	% returns of iron ore spot price from Jan 15 - Dec 16	Standard deviation of the natural log of return				MC simulation of non flexible operation	Operation with material rehandling option	Operation with with additional dig face option	Operation with bypass circuit for switching option	Acceptable flexibility indices and option values			
Months	Market return	ln(return)	(ln(return) - average)^2	Periodic σ	Annualised σ per period	Flex index	Flex index	Flex index	Flex index	Minimum acceptable flex index; $\mu = e^{\sigma\sqrt{\Delta t}}$	Optimum flexibility index $\gamma = (e^{\ln(\beta)\sqrt{\Delta t}})+\sigma$	Optimal PV of flexible operations	Optimal PV of flexible operations
Jan-15	0.0%	0.00	0	0.0%	0	0.00	0.00	0.00	0.00	0.00	0	0	0
Feb-15	-7.6%	-0.0794	0.0073	8.5%	30%	0.9767	1.2136	1.1188	1.1612	1.103001047	1.64	$186.09	$72.29
Mar-15	-8.8%	-0.0922	0.0096	9.8%	34%	0.9746	1.2223	1.1242	1.1675	1.123004784	1.65	$172.27	$67.74
Apr-15	-10.4%	-0.1102	0.0135	11.6%	40%	0.9730	1.2267	1.1275	1.1710	1.152928987	1.67	$162.72	$65.05
May-15	16.0%	0.1481	0.0203	14.2%	49%	0.9719	1.2257	1.1271	1.1701	1.017391076	1.69	$158.31	$64.76
Jun-15	2.3%	0.0231	0.0003	1.7%	6%	0.9715	1.2173	1.1225	1.1640	1.19227821	1.57	$144.73	$52.38
Jul-15	-15.6%	-0.1700	0.0309	17.6%	61%	0.9700	1.2183	1.1231	1.1648	1.092526909	1.73	$150.37	$63.24
Aug-15	9.9%	0.0943	0.0078	8.8%	31%	0.9670	1.2209	1.1250	1.1669	1.012018519	1.64	$133.10	$51.87
Sep-15	-0.6%	-0.0061	0.0001	1.2%	4%	0.9665	1.2124	1.1201	1.1606	1.068072515	1.56	$125.16	$45.03
Oct-15	-5.8%	-0.0600	0.0043	6.6%	23%	0.9649	1.2091	1.1182	1.1580	1.143746173	1.62	$123.06	$46.90
Nov-15	-12.1%	-0.1285	0.0180	13.4%	47%	0.9633	1.2055	1.1163	1.1553	1.161095797	1.68	$122.75	$49.87
Dec-15	-13.4%	-0.1435	0.0223	14.9%	52%	0.9623	1.1980	1.1119	1.1497	1.026295917	1.70	$121.00	$49.80
Jan-16	3.2%	0.0318	0.0007	2.6%	9%	0.9629	1.1851	1.1044	1.1402	1.095262283	1.58	$113.72	$41.56
Feb-16	10.2%	0.0968	0.0083	9.1%	32%	0.9646	1.1696	1.0949	1.1286	1.209114126	1.64	$123.24	$48.14
Mar-16	21.6%	0.1957	0.0361	19.0%	66%	0.9681	1.1521	1.0838	1.1157	1.072090452	1.74	$133.88	$56.93
Apr-16	7.8%	0.0754	0.0048	7.0%	24%	0.9681	1.1412	1.0769	1.1077	1.097263791	1.62	$124.09	$47.47
May-16	-8.3%	-0.0870	0.0086	9.3%	32%	0.9700	1.1365	1.0741	1.1043	1.074062006	1.64	$116.59	$45.62
Jun-16	-6.4%	-0.0656	0.0051	7.1%	25%	0.9691	1.1353	1.0738	1.1036	1.08859849	1.62	$103.92	$39.83
Jul-16	9.5%	0.0907	0.0072	8.5%	29%	0.9711	1.1285	1.0692	1.0986	1.0559514	1.63	$96.10	$37.32
Aug-16	6.2%	0.0603	0.0030	5.4%	19%	0.9738	1.1188	1.0633	1.0914	1.061836443	1.60	$89.70	$33.79
Sep-16	-5.3%	-0.0542	0.0036	6.0%	21%	0.9763	1.1108	1.0581	1.0853	1.016436398	1.61	$80.26	$30.41
Oct-16	2.2%	0.0221	0.0003	1.6%	6%	0.9799	1.1051	1.0543	1.0810	1.25037364	1.57	$66.93	$24.20
Nov-16	25.8%	0.2293	0.0499	22.3%	77%	0.9866	1.0928	1.0466	1.0719	1.065130984	1.77	$65.29	$28.48
Dec-16	7.1%	0.0689	0.0040	6.3%	22%	0.9939	1.0773	1.0372	1.0605	1.111015116	1.61	$49.01	$18.63
	1.15%	0.006	0.0111	10.5%	36.5%	0.931	1.122	1.049	1.082	1.054	1.66		

6.7 CONCLUSION

In this research, a distinction between risk and uncertainty was drawn. As a consequence, this helped in the creation of an uncertainty identification framework. It is plausible to assert that operational managers can properly manage uncertainty if they have the means of differentiating risky variables from uncertain ones. This has been made possible, or is at least aided by this research, as various uncertainty sources, factors and flexibility opportunities for a mining operation have been clearly shown. Additionally, the research has also provided some examples of real option domains as an indication of the capability of the proposed framework. Thus, it fulfilled the initial research objectives which identified the qualitative aspect of RO to be very important for adoption by the mining industry.

Moreover, the greatest distinction between this research and the available literature is its derivation of a linear function between project beta, which is the measure of risk and the flexibility index. This implies that if the project beta is known, then the expected optimal option value and volatility of future cash flows can be precisely estimated. Moreover, the relationship between beta and the flexibility index can be used as a decision criterion to screen various options, as well as to help the managers know when to exercise options by using the derived expected option value as a benchmark and trigger.

Thus, this important contribution to the field of RO will encourage future research into the application of RO in valuing uncertainty. It may eventually lead to the methodology being conventionally accepted in mine planning and development, as its suitability has been demonstrated in the available literature. In the course of mapping uncertainty, flexibility classes and types, and RO application domains, the research has opened up some areas of future research, as follows:

- The concept of agility vs operational excellence. In this research, it was stated that flexibility is a key requirement for operational agility. However, the mining industry is more focused on operational excellence, which is normally measured by productivity or doing more with less. Mining operations do embark on aggressive cost reductions in order to increase productivity as a means of achieving agility. However, there is a limit to cost-cutting; thus, productivity without flexibility may not be a solution. This argument needs to be tested through further research.
- Reduction of uncertainty. There is an argument that if effort is put into gaining further knowledge in terms of time and money, it is possible that uncertainty may be reduced. However, the practicality and effectiveness of such an argument needs to be tested through further research.
- Transferable nature of uncertainty. There is also a view that delaying or swapping areas of operations does not eliminate uncertainty, but rather transfers or postpones it. For instance, if a mining operation delays capital investment in dewatering, or if the pit is redesigned by high

grading some sections of the mine, such operations may only have delayed the problem, as the future areas of the pit may become uneconomical. However, if the impact of such decisions which may appear like RO is not measured, then management cannot precisely state how things will look in the future. Thus, this is a new research frontier in the field of uncertainty and warrants further research.

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Chapter 7: Conclusions, challenges, and future research

In line with the main aim of this research, it has been proven that the traditional valuation methods are unreliable and the suitability of RO as the best tool that can be used in mine engineering design and also for operational decision making has been demonstrated in Paper 1 – 4 (Chapter 3 – 6). It must be put clearly that the future value of the project is trapped in its system but unlocking the value that is hidden requires better decision tools like RO that must be backed up by proper predictive data analytics. Operations can enhance value through the creation of flexible design which can be incorporated in any segment of the mining cycle. However, unlocking the value of the unknown future information does not come free but must be bought by paying what is referred to as the strike price as well as the option premium for creating the original option without any obligation to exercise it. To demonstrate the application of the RO at the tactical level, the study has utilised three analytical methodologies. Binomial decision tree analysis was applied to create a switching option between mining pits regarding changing ore grades (Paper 1), Stochastic simulation for the mean-reversion with jump diffusion was used to create operational options to delay, to abandon the operations and to stage the investment options (Paper 2), and a predictive data mining model was applied in the analysis of RO in managing geological uncertainty and for optimum decision making in running of mining operations (Paper 3). In paper 4, an uncertainty identification framework in a mining operation has been proposed and areas for managerial flexibility and their application domains within the mining cycle were also mapped. Additionally, a relationship between risk measure (beta) and flexibility (flexibility index) was derived and applied to calculate optimal option values.

7.1 Conclusion

The research objectives as initially outlined have been sufficiently achieved as demonstrated by the following results:

- **General outcomes**
  - ✓ It has also been proven that flexibility is the source of agility as this was demonstrated through case studies. Due to the common usage of the traditional valuation techniques including the DCF methodologies, their static view has been accepted as a regular proposition in the mining industry

and are even applied unconsciously by a large number of their users who rarely question the underlying assumptions. Their acceptance is enshrined in engineering, analytical and decision-making processes and the outputs of these traditional tools such as DCF are taken as the absolute truth.

- ✓ Most analysts who use DCF believe that the method does work, and it provides a fair value. The problem with such skewed beliefs is that it creates enormous confidence with insufficient clarity on the flaws of the underlying assumptions. It is a serious problem because the industry generally resists change and ignores suitable tools such as RO. Traditional models assume the future to be static, thus eliminating the tremendous possibilities of allowing for what could happen due to uncertainty. DCF fixed the top and bottom boundaries of the project values through utilisation of sensitivity analyses. However, RO set the minimal value and has no limit as it considers the future to be a sea of possibilities that could be exploited.
- ✓ This thesis has highlighted some of the operating myths and notions that are commonly associated with a project's financial success. There is an established argument that project success is solely dictated by analysing and managing risk. However, the study of RO and its application in mine planning and operations management has revealed that the project success depends on its flexibility to respond to change caused by either external or internal factors (Paper 1).
- ✓ It is vital to underscore that the success of every project does not wholly depend on managing risk alone in its operating environment but primarily they are dependent on the responsive mechanisms designed into them. It is the main reason some projects collapse while others which are operating within the same environment flourish and achieve great financial success. It is not being said that the operating environment does not play a role. It does, but it affects all operations (Paper 2). The operating environment will continue to be very volatile, and this will be exacerbated by technological disruptions.
- ✓ The study of RO has revealed that comparing the project financial performance against its initial investment which is usually the base case is the fairest comparison because it shows whether the project has achieved the original investment results intended. In paper 4, the thesis has derived a relationship between the flexibility index and project beta. Thus, the best measure of the real project value is the value created by flexibility because it implies that losses have been mitigated and uncertainty has been embraced as a source of opportunity to increase value.
- ✓ Finally, this thesis argued that the future can never be predicted, and probabilistic models cannot be relied upon without any other support tool. The main issue is that probability uses past data to

predict the future. However, the past information is merely a history which has no direct translation into the future. Statistics looks at the past and describes it either through graphical representation or using statistical measures. However, what is hidden in the vast information is the pattern of things that may not precisely predict the future but predict the associated similarities or features that may repeat them themselves, and it is the reason why data mining must be embraced when running mining operations. The past is like dead wood, but the only way it can be made important is by discovering what was in it (Paper 3).

- **Paper 1**

  - ✓ Paper 1 has demonstrated that RO can also be used to create designs that give mine management the flexibility to switch between different pits, depending on varying global situations, without closing the mine.
  - ✓ The switching options framework used in the real case study offers a robust approach for testing the applications of RO in engineering design, and decision making at the operational level.
  - ✓ The analysis has shown that the project value increased considerably when flexibility was included in the mine design.
  - ✓ This paper underscored that switching options could be applicable in certain mines, which host lateral or horizontal deposits.
  - ✓ A binomial decision tree was implemented in a four-step approach, which includes: computing the base case PV without flexibility using DCF valuation; modelling uncertainty using decision event trees, then managerial flexibilities are identified and incorporated into the decision tree thus creating value; finally, the RO analysis was conducted.

- **Paper 2**

  - ✓ As articulated in the objectives, the stochastic simulation of mean-reversion with jump diffusion was applied to analyse ROs for a real case iron ore mine.
  - ✓ It is propagated that valuing the unknown involves embracing uncertainty as an opportunity for creating value and accepting that investment decisions are not one-off events but can be made as new information emerges.
  - ✓ The option values increase proportionally with an increase in the level of flexibility.
  - ✓ Managerial flexibility is also created through qualitative thinking and processes by asking what could happen to the uncertain variable and what would that mean for the project.

- ✓ The notable contribution of this paper is its initiative in creating a Managerial Flexibility Domain Map which can qualitatively be applied to the RO methodology before using complex quantitative models.

- **Paper 3**

- ✓ In line with the research objectives, the predictive decision tree classification model for managing geological uncertainty in a mine plan based on an ID3 algorithm was developed and explained using a sample of scheduled ore blocks that were randomly selected from actual assay data obtained from a real case mine operation.
- ✓ It has been demonstrated that machine learning can assist operations to create flexibility, which can have a significant effect on PV.
- ✓ The predictive data mining algorithms such as decision tree classification have never been applied before to model the uncertainty of clay pods or for creating RO at a mine operational level.
- ✓ This research has applied a predictive data mining algorithm to reveal a hidden pattern in the existing actual grade control data that is usually collected by mine geologists from blast sampling and core logging processes.
- ✓ Therefore, the importance of data mining and the value that can be created if it is appropriate and skilfully utilised have been comprehensively highlighted in the research.
- ✓ The RO approach can help decision makers to break away from what this research termed the 'Scorecard Fever'. This is a situation where managers focus on achieving short-term key performance indicators (KPIs) to maximise their incentives but destroy long-term flexibility.

- **Paper 4**

- ✓ In fulfilment of the research objectives, the qualitative aspect of RO has been identified to be very important for its adoption by the mining industry. A distinction between risk and uncertainty was drawn and the uncertainty identification framework was created as various uncertainty sources, factors and flexibility opportunities for a mining operation were highlighted.
- ✓ As a significant contribution to RO methodology, a relationship between project beta, which is the measure of risk, and the flexibility index was derived.
- ✓ It has been argued that if the project beta is known, then the expected optimal option value and volatility of future cash flows could be precisely estimated.

- ✓ The relationship between the beta and the flexibility index can be used as a decision criterion to screen various options, as well as to help the managers to know when to exercise options by using the derived expected option values as benchmarks and triggers.
- ✓ Thus, it has been demonstrated that a combination of beta, flexibility index and mean value theorem could be used as a decision criterion for screening various options within the mining project.

7.2 Challenges

During the study of this thesis, the following challenges were encountered:

- Predictive data analytics was only applied to grade control data. This was adequate for this thesis but not sufficient for making a generalised conclusion. Even though this research demonstrated that data analytics could be used to create value, getting complete data for the whole mine's value chain was a huge challenge. Organisations have multiple data storage warehouses, and each business unit guards its data tightly. Apart from mining operations, it was not possible to obtain data from other sections of the mine such as maintenance, processing plant, railing, shipping and marketing. Data access remains a limitation in this study as the predictive data mining was only applied to mining operations and was not completely integrated into the mine value chain.
- The study was looking at the application of RO from the mine engineering and planning perspective. Thus, the thesis incorporated a perspective of how engineers make decisions in light of their options. However, the organisational mechanisms that are necessary to capture the full benefits of the RO approach, including organisational structure, incentivising for management, and allocation of the decision were not explored.
- This thesis has predominantly utilised publicly available information in the analysis of the case studies as obtaining the day to day financial data was a challenge. Due to confidential reasons, organisations have strict policy and stringent approval procedures that need to be followed before any access to financial data is granted. Getting such approval requires a sign-off from high ranking staff such as the managing director or the chief financial officer and the chances of obtaining approval in time were remote. Therefore, securing access to internal data was a challenge in this thesis, and it is not likely to change in the foreseeable future.

7.3 Future research

As outlined in the next subsections, this thesis has identified a number of future research areas that require further investigation.

- Impacts of RO on mining activities such as in-pit dumping and changing haulage network: In conclusion, the work outlined in Paper 1 has created an opportunity for future research into additional benefits that stem from in-pit dumping opportunities which may be created by having multiple pit options. There may be some cost savings which can be derived from reduced cycle times as a result of short haulage from in-pit dumping, and the impacts of such savings on the net present value of the operation need to be investigated in detail.
- Identification of crucial information: The way of identifying which information adds value is not yet explored and it requires further research. The science of data analytics is assisting the concept of creating value from data. Thus, there are opportunities for real option researchers to study how data analytics can help in identifying the pieces of information that add value to the mining project.
- The concept of agility versus operational excellence: In this research, it was stated that flexibility is a key requirement for operational agility. However, the mining industry is more focused on operational excellence, which is normally measured in terms of productivity or doing more with less. Mining operations embark on aggressive cost reductions to increase productivity as a means of achieving agility. However, there is a limit to cost-cutting, thus productivity without flexibility may not be a solution. This argument needs to be tested through further research.
- Reduction of uncertainty: There is an argument that if effort is put into gaining further knowledge by spending time and money in research and developopment, it is possible that uncertainty may be reduced. However, the application and effectiveness of such an argument needs to be tested through further research.
- Transferable nature of uncertainty: There is also a view that delaying or swapping areas of operations does not eliminate uncertainty, but rather transfers or postpones it. For instance, if a mining operation delays capital investment in dewatering, or if the pit is redesigned by high grading some sections of the mine, such operations may only have delayed the problem, as the future areas of the pit may become uneconomical. However, if the impact of such decisions which may appear like RO is not measured, then management cannot precisely state how things will look like in the future. Thus, this is a new research frontier in the field of uncertainty and warrants further research.

- Integrating data analytics and RO into a mine plan: Since this research has demonstrated that a combination of predictive data mining and RO analysis increases the present value in the face of geological uncertainty, the process of how to integrate data mining and RO methodology into present mine planning practices needs to be researched as there are no established approaches on how this could be done and to what extent could it be done.

Appendices

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