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An Analysis of the Australian Mortality

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This thesis is presented for the degree of
Doctor of Philosophy
of
Curtin University

October 2017

Declaration

To the best of my knowledge and belief, this thesis contains no material previously published by any other person except where due acknowledgement has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

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25/10/2017

Abstract

This thesis evaluates and compares the goodness-of-fit of six stochastic mortality models—the Lee-Carter (LC), Renshaw-Haberman (RH), Age-Period-Cohort (APC), Cairns-Blake-Dowd models (CBD), M7, and Plat models—with specific reference to Australian mortality data. The models are fitted to Australian mortality data for both sexes across three age-group stratification (S0, S1 and S2), four look back windows (l=20, 30, 40 and 50 years) and five look forward windows (h=1, 5, 10, 15 and 20 years). For each combination of look back, look forward window and age-groups stratification, years spans were adjusted to forecast mortality rates for the years 2007 to 2011. The six models were evaluated using four different criteria for model selection: the root mean square error (RMSE); Bayesian Information Criterion (BIC); Akaike Information Criterion (AIC) and heat-maps of residual plots (random/non-random), each derived using the StMoMo package in R.

The Results showed that the best look back window is of 20–years and Lee-Carter model is a good choice for forecasting Australian mortality rates for both females and males when all ages 0-100 are considered. However, the mortality rates are better predicted by age wise stratification for ages 60 and higher. The results also indicate that in the long-term there is a decreasing trend in Australian mortality rates with more than two fold decrease in mortality rates between years 2011 and 2061.

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Acknowledgements

I would like to express my special appreciation and thanks to the people who have provided me with the assistance, guidance and inspiration which has made the completion of this thesis possible.

Firstly, I am enormously grateful to my supervisors, Dr Ritu Gupta and Dr Ee Ling Grace Aw for their wholehearted support and insightful comments. This thesis would not have been possible without their invaluable advice, commitment and professionalism. I would also like to thank Professor Dr Yong Hong for his helpful career assistance in general during the hardships that I faced.

Secondly, I also thank my friends (too many to list here but you know who you are!) for providing the support and friendship that I needed. I would like to thank Casey Josman for being supportive throughout my time here and for helping me to correct errors in my R codes. I think of him as a rescuer.

Thirdly, I especially thank my husband, my children, mom, dad, my sisters, and brothers. My hard-working family have sacrificed their lives for me and provided unconditional love and care. I love them so much, and I would not have made it this far without them. I know I always have my family to count on when times are rough. I thank my beloved kids, Bushra, Omar, Dima, and Zahra.

I dedicate this thesis to my fabulous family,
my supportive husband Abdullatif Swan,
and my beloved children
for their constant support and unconditional love.

I love you all dearly.

Chapter 1 Introduction

1.1 Background

The world's population continues to grow, and with it the importance and application of mortality modelling and forecasting. In the course of the 20th century, the population experienced a significant change in mortality as mortality rates decreased dramatically and life expectancy increased. These continue to change, which represents a phenomenon thought to be driven by improving standards of hygiene, education, nutrition, living environment and medical care. In Australia, for example, life expectancy at birth increased from 70.87 years in 1960 to 82.15 years in 2011.

Many authors in the theoretical and empirical literature argued that the ability to accurately project mortality rates, and their impact on the size and structure of populations, is crucial to preparing and providing suitable infrastructure and services for the future. Accurate mortality forecasts are essential to planning for the development of schools and hospitals, the provision of insurance, pensions and welfare, and other important social, economic and health policy decisions (Yeo, Chan and Kogure 2012; Hu 2014; Li and O'Hare 2015). Any ability to improve the fit of mortality forecasting models could have positive implications in these areas. Thus, it is vital that demographers and actuaries can select and apply a mortality model that performs effectively for a particular set of economic, geographic and population–based parameters. In this case, an effective model is one that fits well with historical data and produces apparently plausible forecasts that do not differ significantly from actual outcomes ex–post by Dowd, Cairns, Blake, Coughlan, Epstein, and Khalaf–allah (2010b).

Starting with the 19th century when Gompertz (1825) published his law of mortality, the availability of different models for mortality forecasting has increased dramatically, along with model complexity. Early mortality models are relatively simple, comprising linear or quadratic extrapolation of measures such as life expectancy and age-specific trends. Such models assume that past trends will continue into the future. Although these methods are relatively easy to apply, they often suppose a fixed rate of change and can be a poor fit when trends change or if cohort-specific

events skew data (World Health Organization 1957; Haines 1977; Coale, Demeny and Vaughan 2013; Brass 2015).

Over the years, different models have been developed to describe mortality. Concepts related to mortality, annuities and adverse risk selection were already mentioned in the empirical and theoretical literature of the nineteenth century. In this period, the first mortality tables were created in the United Kingdom, which included a margin to predict changes in mortality, to protect insurers against losses (Olivieri and Pitacco 2009).

In predicting trends for old age mortality, 85 years has been suggested as a biological limit of human life expectancy (Carnes and Olshansky 2007). However, others argue there may be no upper limit (Tuljapurkar and Boe 1998). Thus the quality of simple extrapolations may be enhanced by extending models. For example, the long-term accuracy of linear extrapolations might be improved by anticipating a slowdown in yearly mortality improvements, and mathematically incorporating this into the model (Ediev 2008). However, such additions often involve the subjective use of expert opinion to predict future population dynamics.

In the last 15–20 years, the simple extrapolation methods have rapidly evolved into more complex stochastic methods. These methods rely on statistical techniques and algorithms used to analyse a matrix of disaggregated data, divided by age, sex and/or other variables (Booth and Tickle 2008; Cairns, Blake, Dowd, Coughlane, Epsteine, Ong, and Balevich 2009; Cairns, Blake, Dowd, Coughlane, Epsteine and Khalaf-Allah 2011).

The study of mortality led to the development of graduation models that have been used to soften crude mortality rates and to analyse the behaviour of mortality. With the emergence of new statistical methods, the analysis of mortality was revolutionised. Among these the Poisson models were developed, which estimate the number of deaths in each age given the number of people exposed to risk. Among the stochastic models used to adjust mortality, the most representative are the CMI model (Continuous Mortality Investigation), the Lee-Carter model and its extensions (Renshaw-Haberman, Age-Period-Cohort, CBD, M7 and Plat).

For example, the Lee and Carter, (1992) and its variants and extensions estimate the age pattern of mortality using time series methods based on a matrix decomposition of mortality data, assuming the dynamic nature of mortality trends over time is ruled by a single parameter (Andreozzi, Blacona and Arnesi 2011). Although originally developed for US mortality data, the Lee-Carter model is now the "leading" long–term forecasting model internationally (Kan 2012), and is used to analyse mortality in different countries, from different causes, and over different time periods (Girosi and King 2007). It has become a dominant base model to which extensions or modifications are added, with differing levels of success (Wang 2007).

Even though current studies in the literature prove that is a continuous improvement in mortality rates, with child mortality rapidly decreasing, in particular some specialists consider it unreasonable to expect this rate of improvement to continue (Lewis and McCormick 2012). These stochastic methods require minimal subjective input and are particularly useful in that they include a measure of uncertainty in their extrapolations, allowing the forecast of probability distributions of mortality, rather than a single point forecast for which the degree of uncertainty cannot be quantified.

The research in this thesis is an extension of the work conducted by Kul and Sucu (2015), they compared eight stochastic models that applied on Turkish mortality data for both male and female, aged 5 to 89 over the period 1980 to 2012. The mortality models compared were the Lee-Carter model (M1), the Renshaw-Haberman model (M2), the APC model (M3), the Renshaw-Haberman model (M4), the CBD model (M5), the M6 model, the M7 model and the Plat model (M8). The criteria for model selection were the Bayesian Information Criterion (BlC), the Mean Absolute Percentage Error (MAPE) and unexplained variance. Kul and Sucu found that the Lee-Carter model gave a poor fit; in contrast, Renshaw-Haberman was the best performing model. The data considered deaths and exposed numbers of were grouped into five age bands to fit mortality rates by using Lifemetrics R-code software. Moreover, ARIMA models were used to forecast the general index for both time and cohort period, which was used for life expectancy forecasting from 2013 to 2030. Obviously, they have made a commendable effort in terms of a number of models and tried to evaluate them and show the best. Nevertheless, their analysis did not focus on the age stratification and time factor, which necessarily does not give accurate results. While the current

study compared the six stochastic models—the Lee-Carter model, the Renshaw-Haberman model, the APC model, the CBD model, the M7 model and the Plat model—with particular reference to Australian mortality data; both the modelling period and ages modelling to evaluating these models were taken into consideration. The analysis is performed on data from Australia for both sexes over three different scenarios and for four look back windows and five look forward windows were adjusted to prediction of mortality rates for the years 2007 to 2011. Furthermore, the models are evaluated based on three common statistical model selection criteria and a diagnostic plot in the form of heat maps. Mortality models were fitted and evaluated using the StMoMo package in R. The mortality rates were forecasted for a period of 50 years, from 2012 to 2061 for both sexes. The accuracy of mortality rates forecasting has improved when using age stratification, which used several stochastic mortality models instead of a single model.

1.2 Objective

The main objective of this thesis is to analyse female and male mortality in Australia so as to accurately forecast mortality rates and life expectancy. Thus, six stochastic mortality models are applied to the Australian data between 1961 and 2011 (namely the Lee-Carter model (LC or M1), the Renshaw and Haberman model (RH or M2), the Age-Period-Cohort model (APC or M3), The Cairns-Blake-Dowd models (CBD or M5 and M7) and the PLAT model (M8)). A comparison is then made between the six mortality models using different selection criteria (Root Mean Square Error, Bayesian Information Criterion and Akaike Information Criterion) in order to choose the best-fitted model to be used for mortality and life expectancy forecasting in Australia.

Regarding the investigation and forecasting of mortality rates in Australia, a review of the literature shows that only a few studies have been carried out to model and project mortality. For example, Booth, Maindonald and Smith (2001) apply the Lee-Carter model to the Australian data to show that the assumptions of the model are not always met because of age-time interactions. More recently, Booth and Tickle (2008) use the Lee-Carter model on Australian data for the period 1968–2000 to forecast mortality to 2031. Furthermore, they compare the results obtained with official projections.

Given the lack of recent mortality studies in Australia, this thesis intends to achieve the following objectives:

- To present and explore different models for mortality estimation and forecasting;
- To investigate and estimate six models (the original Lee-Carter model and its extensions) that employ extrapolative class of stochastic mortality models;
- To compare and evaluate the six stochastic models and to use different model selection criteria in order to decide which model is the best to forecast Australian mortality rates and life expectancy;
- To calculate the mortality rates for the Australian population (females and males), for which the methodology necessary for the mortality rates construction will be described and implemented;
- To check if age, period and cohort patterns prove to be important for clarifying mortality patterns;
- To find optimal age/time stratification of data for forecasting mortality rates and life expectancy;
- To use the best-fitted model to project mortality rates and life expectancy for Australian females and males for the next 50 years.

1.3 Significance of research

The research conducted in this thesis offers significant potential contributions to accurate forecasts of mortality levels in Australia. The purpose of this thesis is to explore mortality patterns in Australia and to analyse and predict mortality rates and life expectancy. These contributions are essential in selecting accurate models on which to base future forecasts. The comparison of the six mortality models (starting with the original Lee-Carter model and ending with one of its most recent extensions), with the purpose of selecting the best model for Australian mortality and life-expectancy forecast represent own contributions highlighting the originality and significance of the research.

The thesis is structured over five chapters, starting with the presentation of conceptual and theoretical aspects and continuing to merge with empirical elements that allow testing the applicability of the six mortality models on Australian mortality data.

Chapter 2 — Literature Review — gives a historical development of the mortality models in the research literature. This chapter provides an overview of previous research on mortality and life-expectancy forecasting, focusing on the Lee-Carter model and its extensions. Also, the purpose of this chapter is to introduce the framework for the application of the six stochastic mortality models that represent the main focus of the research conducted in this thesis.

Chapter 3 — Analysis of Australian Female Mortality — compares the results of the six mortality models estimated using Australia female data. This chapter explores mortality patterns in Australia over the period 1961– 2061, analyses and predicts female mortality rates using the best-fitted model (selected using four different selection criteria: the RMSE, the AIC, the BIC and heat-maps of residual plots) and detects and models the eventual presence of cohort effects in mortality patterns. The purpose is to predict mortality rates and compute life expectancy for the next 50 years.

Similarly, Chapter 4 — Analysis of Australian Male Mortality — compares the results of the six mortality models applied to the Australian male data. This is done by using four different selection criteria for model selection (the RMSE, the AIC, the BIC and heat-maps of residual plots). The purpose is to select the best model that can be used to predict mortality rates and life expectancy for the next 50 years.

Finally, Chapter 5 — Summary, Conclusions and Recommendations — concludes and summarises the work and the most important results obtained, presents the main contributions of the thesis focusing on the original elements and results and describes some limitations and directions for future research.

Chapter 2 Literature Review

2.1 Introduction

In recent years, the human population has experienced a vast improvement in the quality of life and health, owing to a variety of scientific, technological, environmental and socio-economic factors (Aro and Pennanen 2011). Life expectancies in developed countries have gone up from 25 to 40 years at the beginning of the 20th century (Maddison 2001) to about 70 years by 1960 (Edwards and Tuljapurkar 2005). Longevity has increased across the industrial world since then at an estimated annual rate of 0.2 years (White 2002).

This improvement in mortality has varied across different countries and demographics (Tuljapurkar and Boe 1998) as well as age groups (Keilman 2008). Behavioural differences, such as smoking has been identified as an important factor influencing mortality rates (Gjonça, Tomassini and Vaupel 1999; Pampel 2002). Improvement in female mortality rates (Oeppen and Vaupel 2002), reduction in infant mortality since World War II (Cheung, Robins, Tu and Caselli 2005) due to immunization, control of infectious disease and better health care for the aged, are some of the drivers of improvement in life expectancies (Booth, Maindonald and Smith 2002).

The unprecedented increase in longevity and lower mortality rates have posed an unprecedented risk on financial institutions (Wang, Huang, Yang and Tsai 2010). To cope with the demands of the growing elderly pensionable population, insurance and pension providers rely on higher capital stores (Plat 2011). Learning from past errors in forecasting longevity, financial institutions continue to develop more complex forecasting models, with the goal of selecting at the model that provides the least uncertainty and best estimate (Haberman and Renshaw 1996). A variety of unrelated, unpredictable events governing the improvement of mortality rates have necessitated the development of stochastic models to quantitatively analyse these improvements with a high degree of confidence (Cairns et al. 2009). Falling interest rates complicate the issue of rising longevity, which together have led to increased

uncertainty and error in financial institutions with respect to insurance and long-term pensions (Sweeting 2008).

Mortality studies in the UK have shown considerable improvements in mortality rates of 65–year-old males after 1960, whereas the same estimations for 25–year-olds showed improvements before 1960 but levelled off in the decades that followed (Dunnell 2007). Mortality rates have decreased significantly compared with the elderly. The pattern of decline or the curve of mortality rates varies from year to year due to prevailing conditions during that period, such as flu epidemics, heat waves, or other such underlying environmental factors (Cairns, Blake and Dowd 2008). This fluctuation of mortality rates leads to considerable uncertainty and volatility in forecasting for future generations.

Traditional risk management approaches used to calculate mortality and life expectancy rates may lead to biased results, introducing errors into calculations. Losses due to miscalculation of mortality rates coupled with volatility in financial markets call for an increased level of precision in predicting mortality rates as well as determining uncertainties associated with such projections (Melnikov and Romaniuk 2006). When calculating mortality and life expectancy rates using direct or indirect methods of age adjustment, a random error may be a significant issue. To solve the problem of error in calculation, researchers quantified the errors and calculated confidence intervals around the measured mortality and life expectancy rates (Anderson and Rosenberg 1998). Furthermore, over the years many statistical models have been developed to address uncertainties associated with changing mortality rates, each with their pros and cons.

2.1.1 Statistical mortality models

Methods for forecasting uncertainties may be static or dynamic, as suggested by Tuljapurkar (1997). Static methods involve making assumptions based on subjective determinations, whereas dynamic methods pertain to stochastic models fitted to historical data (Li 2007: p.2). The latter is more sensitive to parameter changes and hence are more highly valued in actuarial practice (Continuous Mortality Investigation Bureau 2004).

According to Blake (2013), stochastic models may be classified as process-based, explanatory and extrapolative projections. Process-based models are highly subjective based on cause-and-effect relationships. Explanatory models are based on regressive analysis of exogenous causes, such as correlating health and income indicators with mortality rate. These models are again based on assumptions and hypothesis testing (Blake 2013). These models fit each age/period terms and do not capture more complex parameters such as the generational influences or cohorts. Since the mortality rates are influenced by multiple stochastic factors, complex models that fit multiple parameters are needed to optimize estimations (Aro and Pennan 2011). Extrapolatory models use parametric or none-parametric factors tailored for complex datasets spanning multiple ages and periods, designed to produce accurate forecasts which capture complex phenomena. These models are of two types: none-parametric models or the Lee-Carter class of models; and parametric models, or the Cairns-Blake-Dowd class of models (Blake 2013).

Stochastic models are two-dimensional designed to predict mortality rates by age and time. Most stochastic models identify three parameters in mortality data. These parameters are the age effect, time effect and the birth cohort (Kul and Sucu 2015).

2.1.2 The cohort effect

A cohort effect is a difference in health and life patterns because of varying environmental exposures and societal experiences of different cohorts (Willets 2004).

Changing trends in mortality may be associated with several factors. A wide variety of covariates may be included in a dataset ranging from gender, education, ethnicity, occupation, lifestyle indicators such as smoker or non-smoker status and lifestyle indicators such as smoker or non-smoker status. However, computation of several covariates is complex and does not necessarily contribute to the forecast (Cairns et al. 2011).

Mortality rates have been linked to specific generations or year of birth (Richards, Kirkby and Currie 2006). The birth cohort, which refers to people born in the same year, is most commonly added to the age and period effects in most stochastic mortality models. The birth cohort may be seen as an index of barriers and resources that affect health and mortality (Keyes and Li 2010). The birth cohort satisfies the

assumption that people born in the same year or time frame experience similar health effects which in turn affects their mortality rates, as evidenced in the UK in studies conducted by the Continuous Mortality Investigation (2002) and Willets (2004). Investigations showed highly significant improvements in mortality rates in individuals born between 1920 and 1940 in the UK and other European countries, largely attributed to the introduction of healthcare and a drop in the number of smokers (Hunt and Villegas 2015).

The cohort effect has been adopted from various areas of research including epidemiology and social sciences (Willets 2004). The year of birth significantly affects mortality rate by a combination of factors. Richards et al. (2006) discovered that the maximum improvement in mortality rates in England and Wales were experienced by people born in or around 1930. Willets (2004) argued that the rapid decrease in mortality rates for the cohort born between 1925–1944 compared to those born a decade earlier could be attributed to the adverse conditions of war suffered by the previous generation. Cigarette distribution was the highest during the war and declined steadily after 1960; this would be reflected in the improved mortality rates of the cohort born after 1944. The cohort effect at birth continues into old age thus impacting mortality rates for higher age groups. The longevity of the birth cohort born between 1920 and 1940 is highly significant in the present day, as a large number of pensioners drawing from financial reserves over an unpredictably long period of time would impose unprecedented costs on governments, financial institutions, and healthcare organisations (Hunt and Villegas 2015). The presence of a cohort effect in a dataset may be manifested as a marked distortion of mortality curves (Cairns et al. 2009).

2.2 Criteria for comparing stochastic models

There is an attempt to critically examine six such models that empirically explain changing mortality rates concerning actuarial risk management.

The models reviewed in the current thesis, are the Lee-Carter one-factor model, 1992 (M1), the Renshaw-Haberman extension, 2006 (M2), the age-period-cohort simplification, 2006 (M3), the Cairns-Baird-Dowd two-factor model, 2006b (M5), the M7 generalisation of CBD (2007) and Plat's proposed model (M8) which

incorporating the strengths of the existing models while eliminating their weaknesses (Plat 2009).

Typically, statistical models on mortality are assessed on their efficiency to calculate longevity and mortality risks (Plat 2009). Since all mathematical and statistical models are only an approximation of reality (Cairns et al. 2011), they need to be evaluated on the plausibility of their predictions. Models that balance parsimony with flexibility are most preferred (Danesi, Haberman and Millossovich 2015).

A parsimonious or simple, well-specified model is certainly preferred to an unnecessary complex one. Quantitative criteria relate to the consistency of the forecast with historic data and robustness in relation to current data (Cairns et al. 2009). The goodness of fit for a dataset is estimated by the log-likelihood function. Bayes information criterion, (Dowd, Cairns, Blake, Coughlan, Epstein and Khalaf-Allah 2010a: p.2), Akaike information criterion (AIC), and the log-likelihood ratio test (LRT) are some of the model selection criteria suggested (Li, Hardy and Tan 2009).

The Akaike information criterion is:

$$AIC_i = L(\widehat{\emptyset}_i) - n_i \tag{2.1}$$

The Bayes information criterion (BIC) is:

$$BIC_i = L(\widehat{\emptyset}_i) - 1/2\, n_i lnN \tag{2.2}$$

ni = number of parameters estimated in the model.
N= number of observations
$L(\widehat{\emptyset}_i)$ = max. log likelihood.

Both AIC and BIC are measures of fit and parsimony. The model that gives the minimum AIC or BIC is selected (Biffi and Clemente 2014). BIC penalizes complexity more stringently so it is normally used to select for parsimony (Burnham and Anderson 2004).

According to Cairns et al. (2011), the qualitative criteria examined in stochastic mortality models include (a) biological reasonableness, (b) robustness of parameter estimates for extensions in data period and age range, (c) stringency or parsimony, (d) the robustness of forecasting realistic uncertainties, (e) incorporation

of cohort effects, (f) the ease with which the model is interpreted and translated into practical solutions, (g) the capacity to define parameters, generate sample paths and projection intervals so as to allow assessment of uncertainties for future cash flows. (*h*) It is also important for a model to produce, non-trivial correlations between relevant age ranges

These six models were selected based on their suitability for projections on a more aged population, as seen by historical data forecasts (Cairns et al. 2011).

2.3 Basic notations

Stochastic models use either $m_{x,t}$, which is the crude death rate (Dowd et al. 2010a), also referred to as the central mortality rate (Plat 2009), or $q_{x,t}$, which is the initial mortality rate, for year *t* for age *x*.

The central mortality rate is defined as:

$$m_{x,t} = \frac{D_{x,t}}{E_{x,t}} \tag{2.3}$$

$$\frac{D_{x,t}}{E_{x,t}} = \frac{(deaths\ during\ calendar\ year\ t\ at\ age\ x\ last\ birthday)}{average\ population\ during\ calendar\ year\ t\ at\ age\ x\ last\ birthday} \tag{2.4}$$

Simply stated, the central mortality rate is the annual number of deaths divided by the mid-year population (Tuljapurkar, Li, Nan and Boe 2000), the latter representing the exposure to death during calendar year *t* at age *x* last birthday (Kul and Sucu 2015). The data for this estimation is readily available in national statistics offices (Office for National Statistics 2016).

Uncertainties associated with mortality risk may be either unsystematic or systematic. The unsystematic uncertainties arise due to the randomness of the number of deaths which is relatively small for higher populations. On the other hand, systematic uncertainties affect the entire population which is significant in predicting future trends (Cairns et al. 2008). The purpose of mortality model is to account for these uncertainties or risks in future forecasts to ensure appropriate cash flows. This includes short-term risks of catastrophes and epidemic (Cairns, Blake and Dowd 2006b).

The probability that an individual aged *x* will die in the current year *t*, is referred to as the initial mortality rate (Plat 2009) or the true, unobserved mortality rate (Cairns et al. 2008), defined by the equation:

$$q_{(x,t)} = 1 - e^{-m(x,t)} \tag{2.5}$$

Stochastic models assume that the number of deaths follows a Poisson distribution which can be approximated to a normal distribution for large populations (Dowd et al. 2010a). This is important for determining mortality residuals and percent error, implying:

$$D_{(x,t)} \sim Poisson\ (E_{(x,t)} m_{(x,t)}) \tag{2.6}$$

The equation 2.5 can be expressed as:

$$q_{(x,t)} = \frac{m_{(x,t)}}{(1 + 0.5m_{\ (x,t)})} \tag{2.7}$$

2.4 Review of mortality rates

2.4.1 Review of mortality rates by gender:

Variation in mortality rates between the sexes has been a subject of considerable interest for more than two hundred years (Gjonça, Tomassini, Toson and Smallwood 2005). Paleo-demographers, who derive their statistical data from examining prehistoric skeletal remains, contend that these differences were linked to the growth of agriculture in early societies (Bocquet-Appel and Masset 1996). As a rough estimate, this period is presumed to be after the fall of the Roman Empire in the early middle ages (Boldsen and Paine 1995). It is now widely accepted that women live longer than men in both wealthy as well as impoverished countries (Poulain 2012). The reason for this difference has been attributed to a range of biological, cultural, economic and social causes (Robsen 2015).

Among the biological causes one of the main factors believed to influence life-span, is the female-specific hormone, oestrogen (Kirkwood 2010). This hormone is known to protect the brain and central nervous system from degenerative disorders, owing to its ability to remove fat deposits that contribute to plaque formation in the brain tissue. By the same mechanism, it protects women against ischemic heart and

coronary disease (Waldron 1993). Post-menopausal women are more at risk of heart attacks than younger women. However, the incidence of heart attacks in postmenopausal women is lower than that of men in the same age group. Lowered testosterone levels have been associated with higher mortality (Poulain 2012). High testosterone levels exacerbate the risk of cardiovascular failure in men who have a history of cigarette smoking and high cholesterol levels (Webb, McNeill, Hayward, De Zeigler and Collins 1999).

The presence of two X chromosomes in females is another biological factor contributing to their increased longevity. Since men have only one X chromosome, any damage to the genes on the X chromosome is detrimental, whereas in women, it has been postulated that damage to one X chromosome is compensated for by the second X chromosome (Christensen, Kristiansen, Hagen-Larsen, Skytthe, Bathum, Jeune, Andersen-Ranberg, Vaupel and Ørstavik 2000).

Biological factors may also affect behaviour, as high testosterone levels lead to risky and aggressive behaviours (Archer 1994). Cultural factors tend to put men at a higher risk than females (Reddy 1999). Men are more frequently involved in occupations such as mining and heavy engineering. These occupations, while being physically perilous, also tend to impact on health, as they often cause mental stress. Increased stress and insecurity lead to unhealthy behaviours such as cigarette smoking and excessive alcohol consumption. Furthermore, women are known to be more conscious of seeking medical advice for health problems and have a healthier diet. They are, in general, less likely to indulge in dangerous behaviour and insecure lifestyle (Waldron 1993).

In a comparative study on gender-based mortality in three developed western countries over the last 150 years, Wisser and Vaupel (2014) showed that the difference in mortality rates (DMR) between the two sexes has been increasing from 1860, with an excess of male deaths across most age groups. The exception was between the ages 5–15 before 1930, where a distinct female disadvantage is seen. The disadvantage has been attributed to poor housing and higher degree of frailty among young girls, rendering them more susceptible to infectious disease, most notably tuberculosis. It has been argued that the excess in feminine mortality during this period is a consequence of sexual discrimination which deprived girls of education, medical aid,

good nutrition and hygiene (Gjonça et al. 2005). Higher maternal mortality was another factor contributing to the female disadvantage before 1930 (McNay, Humphries, and Klasen 2005).

Following the two world wars, female mortality rates declined over that of their male counterparts. Male mortality rates were highest for ages between 18–25 years. There was also a peak at around the age of 60, owing to a rise in cardiovascular diseases (CVD). This trend continued after the war, the leading causes of deaths in younger males being traffic accidents and CVD continuing to be the main killer in older age groups (Guralnik, Balfour and Volpato 2000). However, since 1980, the difference in mortality rates between the sexes has been narrowing largely due to improved treatments and preventative methods for CVD. This trend was similar in all countries studied. (Wisser and Vaupel 2014).

Gender differences in mortality rates vary by country, with a definite advantage in life-expectancy enjoyed by females in the developed world (Daw 1961). While this is most prominent in developed countries. For instance, in Japan, life expectancy for women is 86 years and 79 years for men, giving the former a 7 years advantage (Yin 2016). However, in India, the life expectancy for women is 64 years, while that of their male counterparts is 63 years (Sulaja 2016).

Following the trend in other developed countries, the current figure for life-expectancy in Australia is 84.4 years on average for females and 80.3 years for males, with a female advantage of 4.1 years. In 1998, the female advantage was 5.4 years, when life expectancy for females was 82 years and 76.6 years for males. As a corollary, the standardised death rate was lower in females at 0.46% as compared to 0.64% for males in 2012 to 2014 (Australian Bureau of Statistics 2016).

As opposed to these figures, the current life expectancy for indigenous Australian females is estimated at 73.7 years and 69.1 years for males (Steering Committee for the Review of Government Service Provision 2014).

In Australia, dangerous life-styles and behaviour are a cause of rampant mortality among teenagers, young adults, and men below the age of forty-five (Wisser and Vaupel 2014). However, despite disturbing patterns of reckless consumption and behaviour, Australia currently has the sixth highest life-span expectancy in the world,

as documented by the Australian Institute of Health and Welfare (World Health Organization 2016).

In the 1889–1890 Australian census, males average were expected to live up to the age of 47.2 years and females on average, until the age of 50.8 years. In the early 1900s, the major cause of death was an infectious disease, which peaked in 1919 with the pandemic of Spanish influenza, but declined drastically thereafter, following the advent of vaccines and antibiotics. Infectious disease was replaced by cardio-and cerebrovascular disorders as the leading cause of fatality among the Australian population. Deaths due to heart disease rose steadily during 1960–70 but has been continuously declining since. With rapid advances in scientific knowledge, coupled with astonishing developments in medicine and technology, the life expectancy for both males and females rose sharply over the next several decades (Australian Institute of Health and welfare 2016).

The highest incidence of deaths for the ages 15–44 is by suicide, drugs and motor-vehicle accidents, as reported by the Australian Bureau of Statistics (ABS 2016). Coronary heart disease is the biggest killer in males aged 45–95 years. In females, the causes of mortality for the same cohort are more varied. Breast cancer is the primary killer of the ages 44–65 years, and lung cancer dominates ages 65–79. Deaths due to lung cancer show a rising trend in women over the last century. This has been attributed to changing lifestyles including increased cigarette smoking. Coronary heart disease is the main cause of fatality after 79 years in females. While mortality due to heart disease is on the decline, that due to Alzheimer's disease and dementia is on the rise (Evershed 2016).

In general, death rates have continuously fallen from 1200 per 100,000 males, and approximately 900 per 100000 females in 1907, to approximately 660 per 100,000 males and 620 per 100,000 females in 2013 (Evershed 2016).

Sixty percent of women survived beyond the age of 85 years in 2012–2014, while only 45% of men in the same cohort survived to the same age. Yet the pensionable age for both genders remains the same. This is currently at 60 years for those born before 1952, and 65 years, 6 months for those born in the following 6 months, increasing by a factor of 6 months for every 2–year cohort, reaching 67 years

by 2023 when those born January 1957 are eligible to access their age pensions (Department of Human Services 2016).

The narrowing gap in life expectancies between men and women may justify their equality in accessing age pensions. However, with changing trends, it would be prudent to examine the changing mortality rates within specific cohorts, to more accurately predict the period of financial dependence of female pensioners, and thereby disburse funds more equitably (Espejo and Montero 2006). Increased lifespans among women would require planning for their health, social as well as economic welfare, based on realistic forecasts (Booth, Hyndman, Tickle and De Jong 2006). Failure to do so could lead to an excessive burden on the country's financial system causing a wide range of repercussions through the economy (Alho, Jensen and Lassila 2008: p11).

In the actual social and demographic context, the analysis, forecasting, modelling and monitoring of the mortality rates are of fundamental importance. For example, Birdsall, Kelley and Sinding (2001) in the book called "*Population matters: demographic change, economic growth, and poverty in the developing world*" argue that demographic forecasting represents a fundamental tool in developing programs and strategies for the economic and social development of a country. The high dimensionality of the data represents a problem in the process of mortality forecasting. In order to deal with this, researchers developed several models and methodologies.

2.4.2 Review of mortality models:

2.4.2.1 The Lee-Carter model (LC or M1)

The stochastic mortality model of Lee and Carter (1992) is the primary stochastic model widely used across several countries including the US and Australia (Booth et al. 2006). The model, which is taken as a point of reference for development of more complex models is basically extrapolative and incorporates demographic considerations of mortality with an autoregressive integrated moving average (ARIMA) time series model analysis and is best suited for long-term forecasting, although variations have been made to fit in short to medium-term projections (Booth et al. 2006). The method is simple and robust for analysis of linear trends of death rates by age-specific calculations (Renshaw and Haberman 2003).

The Lee-Carter model is defined as:

$$ln(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \quad (2.8)$$

where a_x and b_x are age specific constants reflecting age related effects; k_x is the period effect or mortality index reflecting period related effects (Cairns et al. 2009; Booth et al. 2006). The error term $\varepsilon_{x,t}$ is the residual at age *x* and time *t* (Lee and Carter 1992).

The mortality rate is a function of age and calendar year as exemplified by the model the dominant parameter being the mortality index *k* for the calendar year *t*.

The model weighs heavily on past trends; the age-specific constant a_x is the shape of the mortality curve across ages, calculated as a function of average central mortality rates over time (mean $\ln(m_{x,t})$) and b_x is the age-specific relative rate of mortality change attributable to social and historical factors (Lee and Carter 1992). This may be positive or negative depending on the age group. The model is constrained by normalising b_x to sum to unity ($\sum_x b_x = 1$) and k_t sum to zero ($\sum_t k_t = 0$) (Booth et al. 2006).

The period effect k_t is calculated as:

$$k_t = k_{t-1} + d + e_t \quad (2.9)$$

where *d* is the mean annual variation in k_t and e_t denotes uncorrelated error (Booth et al. 2006).

The period effect may be calculated either as a random walk or an ARIMA process (Continuous Mortality Investigation 2007).

The Lee-Carter model forecasts mortality rates by age, using extrapolated values of k_t and fixed values of age related measures a_x and b_x . When $k_t$varies linearly with time, age-specific variation in mortality takes place at a constant exponential rate. When $k_t$tends to negative infinity the age-specific rates tend to zero. The model is robust in that it excludes the occurrence of negative mortality rates (Lee and Carter 1992).

The model permits the construction of period-specific life tables and age-specific mortality tables, using the single parameter *k* as the index of mortality. Further, the model uses variations in a single parameter to forecast mortality trends, fulfilling the criterion of parsimony. Uncertainties in measurements of mortality rates are computed by:

$$ln(m_{x,t+s}) = (\hat{a}_x + \alpha_x) + (\hat{k}_{t+s} + u_{t+s})(\hat{b}_x + \beta_x) + \varepsilon_{x,t+s,} \quad (2.10)$$

where α_x and β_x are errors in calculating a_x and b_x respectively and u_{t+s} is the error in forecasting *k* for a period of s from starting point *t*.

The goodness of fit is seen to decrease in ages where the mortality rates are low and underestimates death rates for lower ages (Lee and Miller 2001: pp 537–549).

The assumptions in the model are transparent and calculations are plausible. It is possible to estimate parameters in relatively accessible computer software such as Excel and R (Gustafsson 2011).

Incorporating data for the period 1950–1994 for ages ranging from 0–105, Tuljapurkar et al. (2000) found that mortality rates in each of the G7 countries (Canada, France, Germany, Italy, Japan, UK, US) declined exponentially at a constant rate, the value of k_t following a linear trend. They also demonstrated that the decline in mortality for each age group is dependent on the age factor b_x and is a product of k_t and b_x .

It is therefore feasible to forecast life-expectancy at birth in the G7 countries using the long-term linear decline of mortality index k_t (Tuljapurkar et al. 2000). Adjusting the value of k_t to account for historical deaths in each year, and incorporating it into a stochastic model of decline:

$$\hat{k}_{(t+1)} = \hat{k}_{(t)} - z + \epsilon_t \quad (2.6)$$

where *z* is the rate of decline of mortality and ϵ_t denotes background stochastic disturbances (Tuljapurkar et al. 2000).

Using the adjusted value of k_t, decline in mortality rate *z* for different countries was calculated for the period 1995 to 2050, and ranged from 0.3 to 0.5 per year in North America and Europe to a high of 0.8 per year in Japan (Tuljapurkar et al. 2000). This rapid fall in mortality has been attributed to immunization and better health care, but is expected to slow down or stop with novel causes of death that balance out the progressive factors promoting longevity (Tuljapurkar et al. 2000). One of these factors suggested by Lee and Carter (1992) is the emergence of infectious diseases such as AIDS. However, with successful antiretroviral therapies, the rates are likely to continue declining.

Plat (2009) and Cairns et al. (2009) have pointed out many weaknesses in the Lee-Carter model. This being a single factor model has a trivial correlation structure in that the mortality improvements across all ages appears perfectly correlated. It is potentially inflexible with respect to age (Renshaw and Haberman 2003). As the improvement rate (b_x) decreases with increasing age, life-expectancies at higher ages are likely to be underestimated (Plat 2009).

Furthermore, since it does not take into consideration cohort effect, it may not be consistent with historical trends in countries with a significant cohort effect, as evidenced by Turkish studies (Kul and Sucu 2015).

Another observation is that the model suffers from being too precise at higher ages, suggesting that uncertainties are underestimated (Cairns et al. 2011). The model is not suitable for outliers, such as increased deaths resulting from the flu of 1918 or increased suicide rates or HIV deaths in more recent times (Renshaw and Haberman 2003). The Lee-Carter model is preferred when assessing linear trends of historical data and when a subjective judgement of the data is not needed (Di Cesar and Murphy 2009).

2.4.2.2 The Renshaw-Haberman model (RH or M2)

The inflexibility of the Lee-Carter model was exemplified in mortality forecasting using data from England and Wales for the period 1950–1998 (Renshaw and Haberman 2003). Importantly, raw data clearly showed the spike in male mortality in the age group 20–39 from the last quarter of the 20th century but this was not captured by the Lee-Carter model.

To provide for such anomalous but significant data, Renshaw and Haberman (2003) the Lee-Carter model modified to include the cohort or birth-time effect. This was the first model developed incorporating the cohort effect. This is a multifactor model defined as:

$$ln(m_{x,t}) = a_x + b_x^{(1)} k_t + b_x^{(0)} \gamma_{t-x} + \varepsilon_{x,t} \quad (2.7)$$

As with the Lee-Carter model, a_x , $b_x^{(1)}$ and $b_x^{(0)}$ are age parameters and k_t is the period effect or mortality index at time *t*. The random cohort effect γ_{t-x} is a function of the year of birth, *t–x*, derived as:

$$\gamma_{t-x} = \gamma_c = \gamma_{c-1} + \mu_\gamma + \alpha_\gamma \left(\gamma_{c-1} - \gamma_{c-2} - \mu_\gamma \right) + \emptyset Z_\gamma(c) \quad (2.8)$$

where μ_γ and α_γ are drifts, $\emptyset$ denotes an unknown parameter class, and $Z_\gamma(c)$ is a normal innovation independent of the period effect (Kul and Sucu 2015).

Estimation of the cohort effect γ_{t-x} is facilitated by an iterative scheme based on the Newton-Raphson algorithm (Cairns et al. 2011). The underlying assumption of the model is that the cohort effect γ_{t-x} is independent of the period effect k_t.

The model is constrained by normalizing values to sum k_t and γ_{t-x} to zero and $b_x^{(1)}$ and $b_x^{(0)}$ to unity.

When using the model for forecasting at a specific time, intervals are chosen to check for cohort effects for specific time periods. Estimation of parameters can be performed via available packages in R (Hunt and Villegas (2015); Spedicato, Kang, Yalamanchi and Yadav (2016)). Fixed age effects are computed using SVD for the least square solution (Gustaffsson 2011).

The RH model had some improvements over the LC model, the most apparent of which was that the standardised residuals were independent of the year of birth for certain periods, unlike the LC model where a high dependency was seen for certain periods; in other words, the error in the LC model was period related and this is probably suggestive of a cohort effect (Cairns et al. 2008).

The RH model provides a good fit for historical data from countries with significant cohort effect observed in the past (Plat 2009). In a recent Turkish study on mortality rates (Kul and Sucu 2015), the Renshaw-Haberman model gave a better fit to data for both males and females as compared to the Lee-Carter model, using the Bayes information criterion (BIC) for comparison. In another study involving mortality data for males aged 64–89 for the period 1961 and 2007 from England, Wales and the US, the Renshaw-Haberman model had the highest BIC ranking in terms of goodness of fit (Dowd et al. 2010a). Similar ranking was seen for Turkish data (Kul and Sucu 2015).

However, Hunt and Villegas (2015) argue that the main drawback of the model is its lack of robustness to changes in data. In some situations, the identifiability is inherent in the model but not serious. For example, in mixture models the lack of

identifiability relates to re-labelling the components, but this still leads to the same parameter estimates within the re-labelled components and in general there are no issues with convergence. These computational flows are related to the identifiability problems associated with the parameter estimation. This model is also less parsimonious than the Lee-Carter model, with an extra parameter included in the calculation. Parametric convergence in the iterative scheme for maximum likelihood estimations has been found to be very slow suggesting inherent computational flaws in the model (Cairns et al. 2009).

The parametrisation $ln(m_{x,t}) = a_x + b_x^{(1)}k_t + b_x^{(0)}\gamma_{t-x} + \varepsilon_{x,t}$ in the Renshaw and Haberman leads to the same values of $ln(m_{x,t})$ as LC model. Although Cairns et al. (2009) imposed several constraints to fix the identifiability problem, their results still suggest that "*parameter values in the iterative scheme converge very slowly to their maximum likelihood estimates*".

The model suffers from a trivial correlation structure, as it gives perfect correlation between all ages (Cairns et al. 2009).

In addition to iterative complexity, the model may overestimate the cohort effect and run the risk of higher levels of uncertainty in forecasting (Gustafsson 2011). In a comparative study of stochastic models in forecasting mortality for male populations in England and Wales, Cairns et al. (2011) demonstrated the higher levels of uncertainty in the Renshaw-Haberman projections for mortality rates at age 65–75. The biological reasonableness or plausibility of the model was also questionable as fan chart results and ARIMA models for uncertainty suggested lower uncertainties at age 85 than at 65. Apart from being counter-intuitive, these observations contradict the results obtained from the other models tested and historical evidence of higher volatility in death rates at higher ages (Cairns et al. 2011). Comparable results were obtained when the Renshaw-Haberman model was fitted to mortality data from the US, indicating consistent flaws in this model.

2.4.2.3 The Age-Period-Cohort model (APC or M3)

The Age-Period-Cohort model originally described by Currie (2006) is a simplification of the Renshaw-Haberman model and is represented as:

$$ln(m_{x,t}) = a_x + k_t + \gamma_{t-x} + \varepsilon_{x,t} \quad (2.9)$$

This is a simplification of the RH model where age effects $b_x^{(0)}$ and $b_x^{(1)}$ are normalized to unity, but here are set to 1. The model is constrained by summing the period effect k_t and the cohort effect γ_{t-x} to *zero*.

The APC model has been used to study population trends in demography, epidemiology and social-sciences and was first used as far back as 1885 by Farr (1885). In this model, the effects of age, period (calendar year) and birth cohort (year of birth) additively combine to give the logarithmic mortality rate. The model can be evaluated using tests for maximum likelihood.

The model was fitted with the same data, from England and Wales for male mortality at ages 60–89 between 1961 and 2004 (Cairns et al. 2011), as for the previous models. Values of age, period and cohort parameter effects were determined using a Newton-Raphson iterative scheme. ARIMA (0, 2, 1) models were used for time-series evaluation of the cohort effect, and model selection was made based on the BIC statistic for parsimony and goodness of fit (Cairns et al. 2011).

In terms of goodness of fit, the APC model ranked lower than the RH model. It also suffered from the problem of a trivial correlation structure. However, forecasts for the England and Wales data, illustrated by fan chart outputs, showed higher uncertainties at higher ages. This is consistent with historical findings, suggesting plausibility and biological reasonableness of the model. Robustness of the forecast was tested by incorporating varying cohort effects into the model. The APC model showed good robustness with respect to the data studied, as the cohort variations in the forecast were relatively moderate. Similar findings were obtained from the US data.

Based on BIC statistics, the model ranks second for data from the Netherlands (Plat 2009) and is an improvement over the Renshaw-Haberman model in terms of parsimony and ease of implementation (Cairns et al. 2009).

Expert opinion and analysis indicate a number of weaknesses in the APC model. The first is that there are no unique set of parameters, as they are all dependent (age and period) (Wilmoth 1990) since:

$$\text{Cohort} + \text{age} = \text{period} \tag{2.10}$$

Second, it is criticised as being too simplistic in its approximation of reality. This is evidenced by the assessment that the model assumes an ageindependent period effect when in reality, the rate of improvement of mortality rates over different time periods does, in fact, vary with age. Moreover, there has been a perceptible shift in the increase in the rate of mortality improvement with age. This is likely to impact each generation or cohort. Since cohort effects-such as prenatal maternal nutrition-accumulates over time, the impact is more likely to be manifested in the elderly, and therefore affect mortality rates at higher ages (Willets 2004). This impact is not necessarily captured by the model.

Moreover, the inclusion of the cohort effects necessitates prior knowledge. (Wilmoth 1990). For instance, back-tracking the cohort effect of years up to 1945 from present-day mortality rates for a population past middle-age would verify existence of a cohort that impacts health at higher ages in 2016; however, extrapolating cohort effects which may or may not impact future years, (such as AIDS, substance abuse or terrorism in the 1980–2000 cohort) is more hypothetical and is likely to give volatile projections at higher ages (Plat 2009). The need for the cohort effect was also questioned by Cairns et al. (2011) who suggested the replacement of a linear cohort effect in the APC model, by linear adjustments to age and period effects.

The most robust feature of the model is its non-trivial correlation structure, making it a reasonable model insolvency calculation. It is also well fitted to historical data and is applicable to a full age range (Plat 2009). The APC model is preferred when a clear cohort effect is present in the data such as lung cancer or influenza, which is of value in health care forecasting (Di Cesar and Murphy 2009).

2.4.2.4 The Cairns-Blake-Dowd models (M5 and M7)

Unlike the Lee-Carter class of models which use the central mortality rate ($m_{x,t}$) for estimations, the Cairns-Blake-Dowd model or CBD employs the initial mortality rate $q_{x,t}$(Cairns et al. 2008).

The M5 model (Cairns et al. 2008) is a stochastic version of the Perks (1932) model and a modification of the CBD two-factor model (Cairns et al. 2006a) and is defined as:

$$logit\left(q_{x,t}\right) = k_t^{(1)} + k_t^{(2)}\left(x - \bar{x}\right) + \varepsilon_{x,t} \tag{2.11}$$

$$logit\left(q_{x,t}\right) = ln\left(\frac{q_{x,t}}{1 - q_{x,t}}\right) \tag{2.12}$$

where $\bar{x}$ is the average age in the data range; $k_t^{(1)}$ and $k_t^{(2)}$ denote a bivariate random walk with drift (Plat 2009) that is:

$$k_t = k_{t-1} + \mu + CZ_t \tag{2.13}$$

where μ and C are constants and Z is a two-dimensional standard normal variable, all independent of each other (Cairns et al., 2008).

The *logit* transformation of $q_{x,t}$ linearizes its relation to age. The parameters $k_t^{(1)}$ and $k_t^{(2)}$ are easy to interpret and may be considered as candidates for mortality indices (Chan, Li and Li, 2014).

The model is unconstrained and does not take into account the cohort effect. When compared with other models for England and Wales male mortality data between 1981 and 2004, the M5 model showed the highest percent error on standardised residuals and ranked most poorly for goodness of fit (Dowd et al. 2010a).

The model is relatively parsimonious and focuses more on advanced ages of 60–89 (Cairns et al. 2008). Based on fan charts, the model shows the highest degree of volatility at higher ages (>85), which is consistent with historical data. The model satisfies the criteria of plausibility (Cairns et al. 2011).

In terms of robustness, Chan et al. (2014) have reported the M5 model to possess the property of new-data-invariance. This means that the addition of a new year's data to existing mortality figures does not affect mortality indices. This is a significant advantage since it makes it possible to track historical values. Hence the CBD model M5 is most suited to index development.

The M7 model of Cairns et al. (2008) is a generalisation of the M5 model with the addition of the cohort effect and a quadratic term for age:

$$logit\left(q_{x,t}\right) = k_t^{(1)} + k_t^{(2)}(x - \bar{x}) + k_t^{(3)}((x - \bar{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}^{(4)} + \varepsilon_{x,t} \qquad (2.14)$$

where $\hat{\sigma}_x^2$ is the mean of $(x - \bar{x})^2$. The additional age-period effect $k_t^{(3)}(x - \bar{x})^2$ is the quadratic age term.

As with other models that include the cohort effect, M7 also suffers from the parameter identification problem. Hence three constraints are applied by summing the three period effects (k_t) and the cohort effect (γ_{t-x}) to *zero*. The quadratic function appears to smoothen out unevenness due to cohort effects (Cairns et al. 2009).

Using BIC ranking, the M7 model ranks the highest of all the models studied for robustness for English and Welsh data from 1961–2004 or from 1981–2004. Updating the existing dataset by adding or removing data does not change parameter estimates (Cairns et al. 2009). The model ranks second (after M8) for goodness of fit and parsimony. Standardized residuals satisfy the assumptions of randomness.

Using the M7 model on US and English and Welsh data the following insights were gleaned.

Wong-Fupuy and Haberman (2004) showed that changes to the period effects k_t over time are approximately linear as revealed from M7 forecasts on England - Wales as well as US data. The model gave plausible forecasts showing the continuing decline of mortality rates over time; the rate of decline reduces in advanced years. The logit mortality rates and quadratic functions help estimate uncertainties at higher ages (>89). Cohort effects are clearly revealed, and were seen to be more prominent in England and Wales than in the US.

The M5 and M7 model have a non-trivial correlation structure, but are limited to higher age cohorts. For the full range of ages, the predictions are biologically unreasonable, and the models fit poorly for full age ranges (Plat 2009).

2.4.2.5 Plat model (M8)

Since each of the models belonging to either the Lee-Carter class or the CBD-Perks class of models have their merits and demerits, Plat (2009) formulated a model to eliminate the defects with the hope of arriving at the most robust, plausible model best fitted to all age ranges. The model attempts to use the salient features of both classes

of models. It uses the central mortality rate as in the case of the Lee-Carter class of models, but incorporates the parameters incorporated in CBD-Perks class of models.

The definition of the model is:

$$ln(m_{x,t}) = a_x + k_t^{(1)} + k_t^{(2)}(x - \bar{x}) + k_t^{(3)}(x - \bar{x})^+ + \gamma_{t-x}^{(4)} + \varepsilon_{x,t} \quad (2.20)$$

where the a_x is the shape of the mortality curve across ages, calculated as a function of average central mortality rates over time, as in the Lee-Carter model. $(\,)^+$ the positive part of the expression.

$\gamma_{t-x}^{(4)}$ is the cohort effect and $k_t^{(1)}, k_t^{(2)},$ and $k_t^{(3)}$ are the three period effects, as in the M7 model. The first period effect $k_t^{(1)}$ captures long-term effects of all ages, the second $k_t^{(3)}$ is more age specific and the third $k_t^{(2)}$ relates to ages <50. The model is constrained by the age effect (a_x) summed to one and the four stochastic parameters (period effects and cohort effect) summed to *zero*. Owing to these constraints issues relating to parameter identifiability are resolved.

The inclusion of $k_t^{(2)},$ and $k_t^{(3)}$ makes the model favourable as a non-trivial correlation structure.

This model is flexible accommodating all ages. It can be adjusted for specific age groups; for instance, by removing the component $k_t^{(3)}(x - \bar{x})^+$ it can be better fitted for higher ages (Plat 2009).

On a BIC ranking for mortality data for males aged 20–90 over four decades (1961–2004) the model ranked the highest for US and Netherlands data, and ranked second after the RH model for English and Welsh data (Plat 2009).

Analysis by Cairns et al. (2009) showed that with certain datasets, the M8 model gave implausible results for the US data as it showed increased mortality rates rather than decreasing trends. Cairns et al. (2011) suggested that this may be due to the inadequacy of the model to fit age-period effects and that it suffers from overfitting the cohort effect. The biological reasonableness of the model diminishes at higher ages (Cairns et al. 2011).

The Plat model has been criticised as being over-parameterized and is not as parsimonious as some of the earlier models. However, since it seems to have removed

some of the disadvantages of the earlier models and is robust across all ages, this has been suggested as the ideal candidate for pricing of longevity associated annuities.

2.5 Conclusion

Mortality trends are of infinite importance in a large array of social, economic and financial forecasting. This includes economic planning for social security and health, retirement income funds, hedge funds as well as actuarial systems in pricing and reserving of annuities (Renshaw and Haberman 2006).

The objective of all the complex iterations using the various models stated above is financial forecasting. This includes the calculation of annuity or pensions and to adjust longevity risks to pensions and similar annuities. Mortality trends need to be extrapolated to the valuation of pension and insurance funds as well as assets and liabilities for solvency risk assessments. This will be mandatory under the new Solvency Capital Requirement or S2 as elaborated by Plat (2011).

Mortality rates derived from the best-fitted models are used to estimate the age-specific survivor index. For example, the survivor index $S_{(t,65)}$ denotes the proportion of individuals aged 65 at the start of the calendar year t_0 who will remain alive at the start of year $t_0 + t$.

Assuming a constant interest rate, the value of an annuity P payable annually for the next 25 years to an individual aged 65 at the start of year t_0 is calculated by the formula:

$$P = \sum_{t=1}^{25} v^t \, S_{(t,65)} \tag{2.15}$$

where v is the discount factor.

Cairns et al. (2011) concluded in their comparative study on six stochastic models for forecasting mortality from male population in England and Wales, that the differences in annuity payable are moderate despite variations in mortality rates using different forecasting models.

Wang et al. (2010) studied mortality data from the US using both the Lee-Carter and CBD model to extrapolate changes in annuity pricing with changes in

mortality rate. Their results and extrapolations showed that changed to annuity prices following sudden changes in mortality rates can be significantly large. Hence forecasters and actuaries need to have contingency measures to hedge longevity risks.

Quantitative evaluation of different models evaluated by the Bayes information criterion (BIC) for data on mortality rates of males aged 20–89 during the period 1961–2005 in England and Wales gave the ranking of different models for the goodness of fit and parsimony. The Renshaw-Haberman model ranked the highest for this dataset, whereas the Plat modification was best suited to data for US males aged 20–84 during the same period and for Netherlands data from 1951 for males aged 20–90 (Plat 2009). The Turkish data for males aged 0–90 for the period 1980 to 2012, was best fitted by the Renshaw-Haberman model (Kul and Sucu 2015).

In a separate Italian study, the Renshaw-Haberman model ranked highly for ages below 40, while the CBD model was best fitted for the higher age groups of 60–89 for the period 1952–2003 (Biffi and Clemente 2014).

Cohort effects were evident for data from England and Wales as well as the US data, but are less pronounced after the age of 60 for the US data (Cairns et al. 2009).

RH, CBD and Plat work well for short time-spans. The LC model is good for long-term forecasting, while CBD has the worst BIC ranking for long-term forecasting. Projections with ARIMA processes to forecast time-series parameters by calendar years were compared for different models to obtain forecasted annuity values. The Plat model gave the best fit for short-term data (2004– 2008) for ages 60–70 but tends to overestimate survival probabilities in data for higher ages, resulting in lower forecast values for annuities. This is also true of the CBD whereas annuities projected by the RH model was higher for the Italian data (Biffi and Clemente 2014).

Biological reasonableness or plausibility is another crucial factor to be considered while selecting the best model for forecasting mortality rates. The RH model ranked the highest for the England and Wales data in terms of goodness of fit and parsimony, but gave unrealistic forecasts. This may not be true for other datasets.

The robustness of the model to variation in data is yet another feature to be considered. The ease of implementation, and fitting into available computational programmes is of practical significance. The RH programme lacked the ease of

implementation but was improved by the APC model. These comparisons are seen in Table 2.1 and Table 2.2 below.

Table 2.1 BIC ranking of six stochastic mortality models fitted for data from different countries (from Plat, 2009; and Kul and Sucu, 2015)

Model	Model	U. S	England& Wales	Netherlands	Turkey
LC (1992)	M1	4	4	4	6
RH(2006)	M2	2	1	3	1
Currie (2006)	M3	3	3	2	4
Cairns et al. (2006b)	M5	6	6	6	3
Cairns et al. (2009)	M7	5	5	5	5
Plat (2009)	M9	1	2	1	2

Table 2.2 A comparison of six stochastic mortality models based on criteria satisfaction (from Plat, 2009)

Criterion	Model					
	LC	RH	APC	M5	M7	M8
Positive mortality rates	+	+	+	+	+	+
Consistency historical data	+/–	+	+	+	+	+
Long-term biological reasonableness	+	+	+	+	+	+
Robustness	+	–	+	+	+	+
Forecasts biological reasonable	+/–	+	+	+	+	+
Ease of implementation	+	+	+	+	+	+
Parsimony	+	+/–	+/–	+	+/–	+/–
Possibility generating sample paths	+	+	+	+	+	+
Allowance for parameter uncertainty	+	+	+	+	+	+
Incorporation cohort effects	–	+	+	–	+	+
Non-trivial correlation structure	–	+/–	+/–	+	+	+
Applicable for full age range	+/–	+/–	+/–	–	–	+

* +: criterion completely satisfied; +/–: criterion partly satisfied; –: criterion not satisfied.

The main strengths of the mortality forecasting methods used in this thesis are their robustness and the fact that they avoid error in calculation compared to traditional methods of mortality and life-expectancy rates calculations. The use of the Lee-Carter model and its extensions allow for the errors in mortality rates and life-expectancy to be examined. Thus, it is possible to assess the degree of confidence associated with the conclusions formulated based on of mortality and life-expectancy

estimates. Furthermore, using appropriate model selection criteria such as RMSE, AIC or BIC, it makes it possible to evaluate the reliability of the estimated rates.

As seen in these studies, there is no single model that best suits all age groups or demographics. No single model is consistently better than the others under all conditions. Each model has its merits and demerits; selection of the optimum model would depend on the aim of the forecast.

From comparative studies on six different models, the Plat model seems to be best suited for data from a few different countries (Table 2.1) and satisfied almost all the required qualitative criteria. However, it too has its flaws under some conditions. Hence, it would be prudent to use at least two different models in order to reduce uncertainties and avoid unrealistic forecasts. With the variety of models to choose from and continuously changing mortality rates alongside the emergence of hitherto unforeseen cohorts, it would be best to assess existing and emerging mortality models for each dataset and select the one that best suits the dataset in quantitative and qualitative terms to produce accurate and realistic forecasts to plan future annuities.

The purpose of this research is to find models that explain trends in different age cohorts of male and female population mortality rates in Australia.

Chapter 3 Analysis of Australian Female Mortality

3.1 Introduction

The development of mortality rate analysis has progressed with the availability of much more data and with the advancement of computational tools (Hill, Thomas, Abou Zahr, Walker, Say, Inoue, Suzuki and Maternal Mortality Working Group 2007). The modelling and forecasting of long-term trends in mortality is essential for the correct pricing of life insurance policies and in the projection of population levels. The accurate projection of life-expectancies is important at the national level as it directly impacts social security costs (Richards and Currie 2009).

Section 2.4.1 described that the size of the gap between the male and female mortality rates in Australia has varied and increased over the last 150 years. In addition, mortality and life-expectancy for both sexes has dramatically improved. This chapter is dedicated comprehensively to examine the mortality of just the Australian female. Figure 3.1 shows an increase in worldwide average longevity of worldwide for females from 54 years in 1960 to 73.6 years in 2014. In Australia, female lifespan has increased, on average, from 74 years in 1960 to 84.3 in 2012 and remained unchanged in 2013 and 2014.

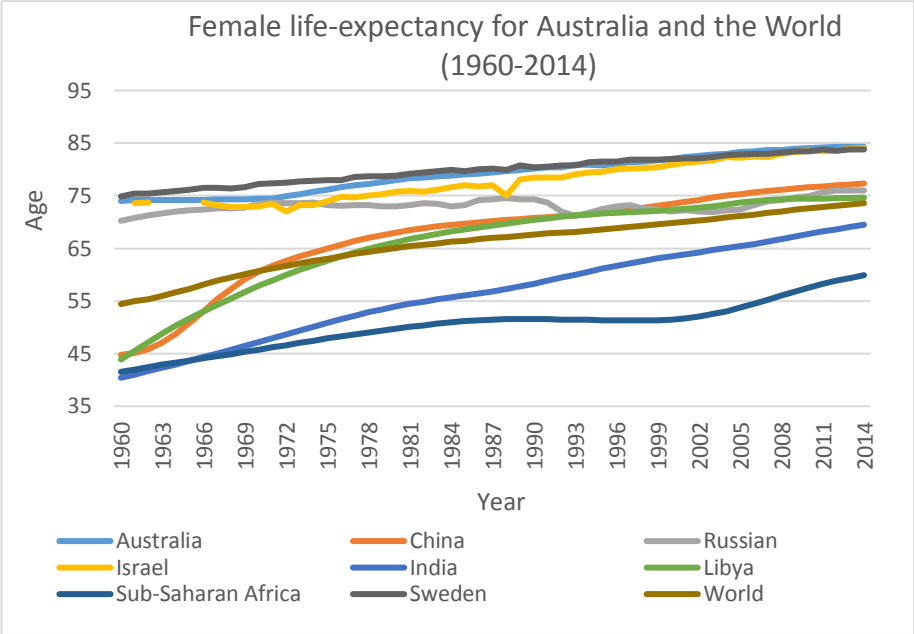

Figure 3.1 Female life-expectancy for Australia and the World - 1960 –2014.
Data source: World Development Indicators 2016.

In section 2.4.2, we discussed six stochastic mortality models which are currently amongst the most popular models of mortality forecasting. These models are the Lee-Carter Model (LC or M1), Renshaw-Haberman model (RH or M2), the Age-Period-Cohort model (APC or M3), the Cairns-Blake-Dowd model (CBD or M5), M7 and the Plat model or M8. All of these models will be examined for modelling of Australian Female data. The choice to use Australian demographic data is based on its availability, local context and on the scarcity of actuarial research on this demographic.

3.2 Objectives

The main objectives of this chapter are to compare the fit of the six stochastic mortality models discussed in section 2.4.2 and to identify the model that is best suited to model Australian female mortality rates and life-expectancy. The comparison is based on selection criteria encompassing, goodness-of-fit and parsimony of the model.

Using historical data from past decades (‘look back’ windows) (Dowd et al. 2010b), each of these models will be tested for their robustness in making accurate future predictions (referred to as ‘look forward’ windows) and age brackets based on a varied length of historical data. This will allow us to understand the data requirements for making accurate predictions. It will allow us to ask questions such as: Do we need 20

years or 30 years of data (look back window) to make 10 years forward (look forward window) prediction? Do we need separate models for different age groups?

After closely assessing each model for efficiency, the best model will then be used for projecting death rates and life-expectancy for the Australian female population.

3.3 Selection criteria for mortality models

As mentioned in section 2.2, the following statistical criteria enable the evaluation of the goodness-of-fit, flexibility and parsimony of the model with respect to the dataset in question (Perna and Sibillo 2012: pp. 231–234):

- C1: the Akaike information criterion (AIC)
- C2: the Bayesian information criterion (BIC)
- C3: Colour maps of residual plots, and
- C4: Root-mean-square-error (RMSE).

It is important to note that, while predicting mortality trends and life-expectancies, these predictions are only approximations of reality. Heat-maps of the residual plots are a diagnostic and it allows evaluation of models. However, the residual plots are subjective and based on judgement, unlike the other three criteria. The robustness of these approximations will depend on the property of the model to retain and process all the information contained in the dataset, with minimum loss of information (Burnham and Anderson 2004). This loss of information is assessed by the AIC (Fabozzi, Focardi, Rachev and Arshanapalli 2014: p.399) and BIC statistics (Ando 2008). The root mean square error is an absolute measure of the difference between values predicted by the model and observed values. Hence, the RMSE determines prediction errors and is a recommended determinant of the goodness-of-fit of the model (Hyndman and Koehler 2006).

The model with the lowest AIC, BIC and RMSE would be the most suitable. Validity of the model is tested by the randomness of residual plots as described by Cairns et al. (2009).

3.4 Data synthesis and analysis

Data was sourced from the Human Mortality Database (2016). This dataset consists of male and female yearly mortality rates in Australia for the period 1921 to 2010. The data included the number of deaths, births, exposures as well as mortality rates and life-expectancy at birth. Data for age groups beyond 100 years were not considered.

The death rate for the age group x in the year t was modelled as a Poisson distribution.

$$d_{(t,x)} = E_{(t,x)} * m_{(t,x)} \tag{3.1}$$

where the number of deaths [$d_{(t,x)}$] is the product of the exposure [$E_{(t,x)}$] defined as the average population during the calendar year t at age x at the last birthday, and the mortality rate for the specific age cohort for a specific year [$m_{(t,x)}$] (Plat 2009).

Mortality models were fitted and evaluated using the StMoMo package in R, which is available at (http://CRAN.R–project.org/package=StMoMo). Version 0.3.0 has been used for this study (Villegas, Kaishev and Millossovich 2015).

3.5 Methodology

The fitting and robustness of mortality models are investigated for two main components. The first part was to determine an appropriate number of years of data required for modelling look back window. The second was to identify if modelling for different strata of age-groups adds any accuracy to the prediction of mortality rates.

3.5.1 Study 1: Sensitivity of the look back modelling period

The sensitivity of past time periods used in modelling would provide insight into the number of years of past data that is required to make accurate predictions. This period is called the 'look back window' (l). The 'look back window' defines the number of years of past data used for forecast. For instance, a 'look back window' of 20 years would project mortality outcomes set for a specific year, based on data from the past twenty years. The length of the projection would constitute the 'look forward window' (h). For example, 5–years look forward window (h=5) and 20–years look back window (l), for forecasting of mortality for years 2011(t) will be based on the data for years 1986–2006 [$t–h–l$, $t–h$].

For a 2–year forecast (h=2) for the year 2011, the modelling period would be 1989–2009 for l=20. Similarly, for a 3–year forecast (h=3) for 2011 the mode time would be 1988–2008 for look back window of 20 years. For a look back window 20–year for the year 2011, with a projection of h=20 years, the data modelled would extend from 1971– 1991. The look back window of 20, 30, 40 and 50 years will be referred as 'Data 20', 'Data 30', 'Data 40' and 'Data 50' respectively.

For different values of l (l=20,30,40, 50) and h (h=1,5, 10, 15, 20), stochastic models (LC, RH, APC, CBD, M7 and Plat) that are being fitted encompassing data for years [$t–h– l$, $t–h$] and making h years forward predictions. The t varies from years 2011 to 2007. All these scenarios tested are summarised in Table 3.1 for l =20.

Table 3.1 Lists the scenarios that will be tested six stochastic mortality models for look back window 20–years.

Model time [$t–h–l, t–h$]	Year of prediction (t)
Look forward window: $h = 1$	
1990–2010	2011
1989–2009	2010
1988–2008	2009
1987–2007	2008
1986–2006	2007
Look forward window: $h = 5$	
1986–2006	2011
1985–2005	2010
1984–2004	2009
1983–2003	2008
1982–2002	2007
Look forward window: $h = 10$	
1981–2001	2011
1980–2000	2010
1979–1999	2009
1978–1998	2008
1977–1997	2007
Look forward window: $h = 15$	
1976–1996	2011
1975–1995	2010
1974–1994	2009
1973–1993	2008
1972–1992	2007
Look forward window: $h = 20$	
1971–1991	2011
1970–1990	2010
1969–1989	2009
1968–1988	2008
1967–1987	2007

Similar scenarios of data are constructed for l =30 year, 40–year and 50–year data and tested for h (h=1, 5, 10, 15, 20) and t=2007, 2008…2011.

3.5.2 Study 2: Sensitivity of ages in modelling

As the mortality changes with age, the second component of the study was to determine the impact of age stratification in achieving prediction accuracy. The following scenarios for age stratification are considered. All ages are described in years.

- Scenario 0: [0–100] {S0}
- Scenario 1: [0–40] [40–60] [60–100] {S1A, S1B, S1C}
- Scenario 2: [0–60] [60–80] [80–100] {S2A, S2B, S2C}

The best model was identified based on lowest values of the AIC and BIC criteria and RMSE, as well as randomly distributed residuals, derived using R. The R Code excerpt and the R-Code steps for fitting of mortality models for the Australian data are described in the flowcharts and Appendix A.1 and A.2.

The workflow followed in this study is presented in Figure 3.2 below.

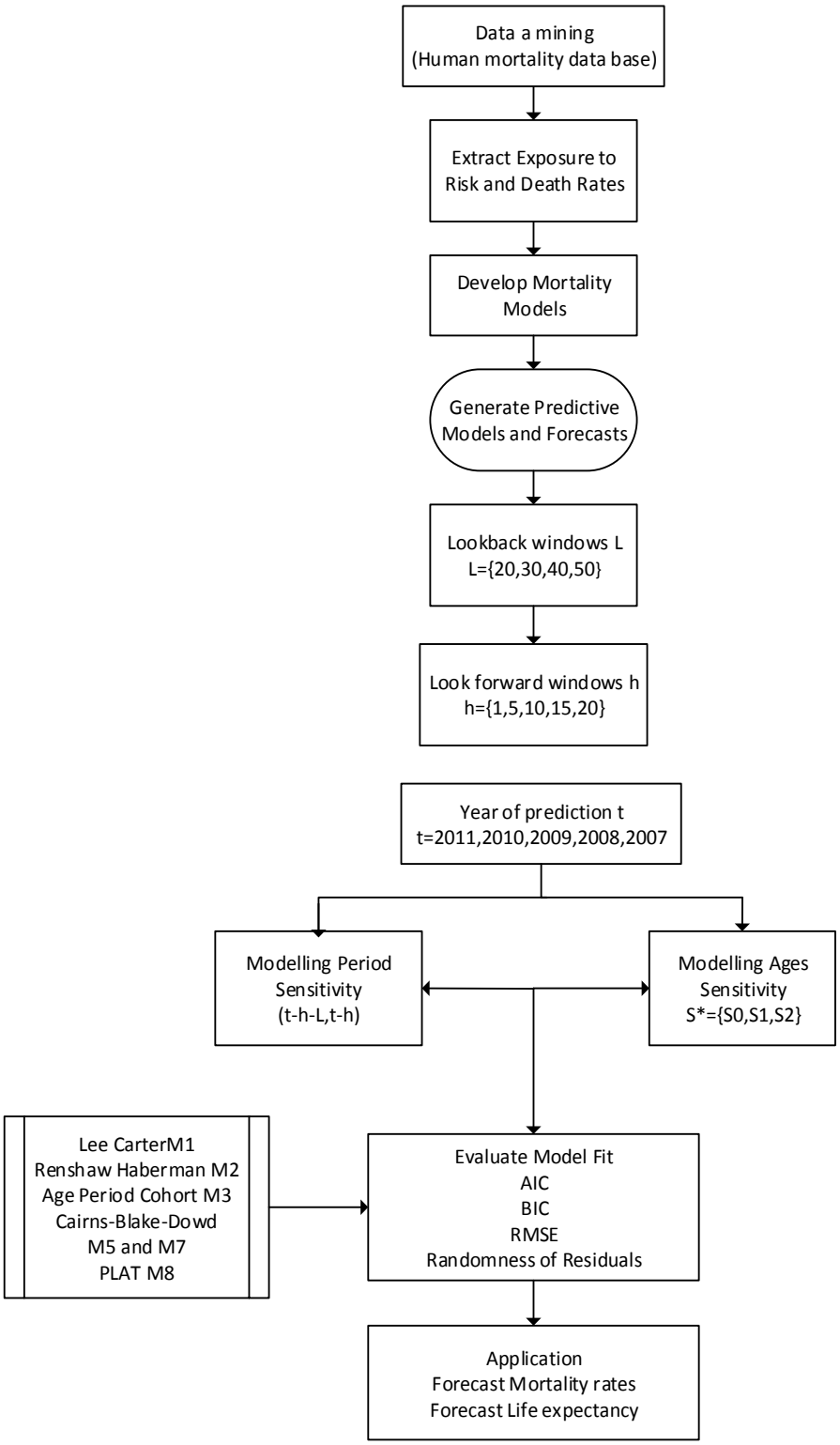

Figure 3.2 The methodology for the present study for construction of mortality models for Australian data.

Table 3.2 below illustrates an example of primary results of the four statistical criteria using LC model with data 20 when h=1 for all model time [$t–h–l$, $t–h$].

Table 3.2 The results of the goodness of fit criteria of data 20 and *h*=1 using LC model, Australian female.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	18143.17	5848.036	3797.521	8853.514	9508.833	4390.699	4647.904
BIC	19393.95	6328.604	4046.952	9334.082	10235.74	4640.131	4897.336
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.002809	0.000044	0.000112	0.004398	0.000085	0.0004962	0.00586
RMSES0	_____	0.000044	0.000133	0.004408	0.000086	0.0004990	0.00614
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	18286.57	5906.980	3805.728	8938.976	9567.028	4402.172	4686.508
BIC	19537.35	6387.547	4055.159	9419.543	10293.94	4651.604	4935.939
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.002856	0.000039	0.000157	0.004491	0.000092	0.000564	0.005682
RMSES0	_____	0.000044	0.000136	0.004482	0.000088	0.000577	0.006244
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	18243.45	5916.788	3781.983	8915.724	9545.261	4388.719	4682.373
BIC	19494.23	6397.356	4031.415	9396.291	10272.17	4638.151	4931.805
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.005795	0.000055	0.000215	0.009124	0.000142	0.000653	0.012162
RMSES0	_____	0.000069	0.000204	0.009094	0.000130	0.000706	0.012687
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	18282.10	5950.097	3781.322	8918.401	9570.246	4398.710	4674.845
BIC	19532.88	6430.665	4030.753	9398.968	10297.16	4648.142	4924.277

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.004243	0.000045	0.000154	0.00667	0.000095	0.00055	0.00994
RMSES0	_____	0.000046	0.000143	0.00666	0.000092	0.00055	0.00929
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	18356.92	5977.464	3779.008	8949.530	9591.525	4426.559	4676.186
BIC	19607.71	6458.031	4028.440	9430.097	10318.44	4675.990	4925.617
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.004929	0.000051	0.000224	0.007897	0.000137	0.000732	0.01118
RMSES0	_____	0.000058	0.000245	0.007735	0.000151	0.000663	0.01079

l: look back window 20–years

t: year of prediction

[*t–h– l*, *t–h*]: modelling time

Scenario 0 (S0): [0–100]

Scenario 1 (S1): S1A=[0–40], S1B=[40–60], S1C=[60–100]

Scenario 2 (S2): S2A=[0–60], S2B=[60–80], S2C=[80–100]

For each of the scenarios (S0, S1 and S2), look forward window (h=1, 5, 10, 15, 20) prediction for year t (t=2007, 2008, 2009, 2010, 2011), each of the six models (LC, RH, APC, CBD, M7, and PLAT) are fitted. The goodness-of-fit for each case is evaluated using the criteria C1 to C4 as above. The RMSES0 is calculated for S0 model for S1 and S2 stratification ages. Selection of these results are presented in Appendix B, Table B.1–Table B.5.

The fitting of Australian mortality data for some scenarios using R, package StMoMo was challenging. There was converge issues, largely due to the default choice of ARIMA (0, 0, 1). In some cases, choice of ARIMA (1, 1, 0) model or other form resolved the issues. In other cases, especially with M7 and RH models, reasonable fits could not be obtained. These cases are then not included in subsequent comparisons.

3.6 Results

The following sections will present the comparisons of the average RMSE values for the six models for different look back windows, look forward windows and different age stratification. Each section corresponds to each age stratification, namely S0, S1 and S2.

3.6.1 Comparison of average RMSE for different look back and look forward windows for Scenario S0

The following sections will present the comparisons of the average RMSE values for the six models for different look back windows, look forward windows for Scenario S0.

First, we consider Data20, which is 20 years look back window. The average root mean square error (RMSE) is computed for each of the six models (Lee-Carter, RH, APC, CBD, M7 and Plat) and five look forward windows (h=1, h=5, h=10, h=15 and h=20). The results are presented in Table 3.3

Table 3.3 Average RMSE, Look back Window =20 years, Ages: 0–100, Australian female.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.004126**	**0.005014**	**0.007865**	**0.012181**	**0.016696**
RH	0.021682	0.056287	0.049121	0.047695	0.08731
APC	**0.004288**	0.056287	0.049121	0.047695	0.08731
CBD	0.016772	**0.015972**	**0.016417**	**0.013202**	**0.021916**
M7	**0.009326**	**0.027856**	0.048935	0.06883	0.080737
Plat	**0.004191**	**0.006529**	**0.011968**	0.018304	**0.021822**
P-value	0.0000	0.0002	0.0000	0.0000	0.0000

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported in the last row are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. Red highlight is the case with smallest RMSE.

As evident from the Table 3.3, the average root mean square error increases with an increase in the value of *h*. The smallest average error for all models is observed for the 1–year look forward window, while the highest average RMSE is observed by the 20–year look forward window. As expected we are making more error for higher look forward window.

A single factor analysis of variance (ANOVA) comparing average RMSE across models for *h*=1, indicate that average RMSE for all models is not the same. Similar results are observed for each value of *h*.

For each look forward window LC model results in lowest RMSE values. Next comparable model is Plat Model for all look forward windows. The average RMSE for other models was at least 2 folds higher.

For *h*=1, performance of LC, M7, APC and Plat are comparable, while for *h*=5, performance of LC, M7, Plat and CBD are comparable. Meanwhile, for *h*=10 and *h*=20, performance of LC, Plat and CBD are comparable, whilst for *h*=15, performance of LC and CBD are comparable. These results are illustrated in Table 3.3

Secondly Data30, which is 30 years look back window was investigated. The Table 3.4 below presents the results of average RMSE of the six stochastic models and five distinct look forward windows fitted under non-stratified data, ages 0–100.

Table 3.4 Average RMSE, Look back Window =30 years, Ages: 0–100, Australian female

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.004078**	**0.005239**	**0.007918**	**0.01096**	**0.011148**
RH	0.013844	0.057087	0.079713	0.075225	0.057769
APC	**0.005973**	**0.012563**	0.019403	0.022913	0.02388
CBD	0.016655	0.01675	0.015798	**0.01179**	**0.009459**
M7	0.012727	0.035073	0.058505	0.074904	0.084198
Plat	**0.004751**	**0.009417**	0.016088	**0.017513**	**0.016618**
P-value	**0.0001**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported in the last row are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. Red highlight is the case with smallest RMSE.

As evident from the Table 3.4, the RMSE increases with an increase in the value of '*h'*. The smallest average error for all models is given by the 1–year look forward window, while the highest average RMSE is given by the 20–year look forward window.

A one-way ANOVA for RMSE comparing average RMSE across models for each look forward window, indicate that average RMSE for models is not the same for each look forward window.

As seen in Table 3.4, the groups with lowest and similar average RMSE are presented in bold. For all look forward windows LC model results in lowest RMSE values. The next comparable models is Plat Model for look forward windows h= 1, 5, 15 and 20.

For *h*=1 and *h*=5, the performance of LC, Plat and APC are comparable, meanwhile, for *h*=15 and *h*=20, the performance of LC, Plat and CBD are comparable. While *h*=10, only LC model is the best performing model.

Thirdly, we examine Data 40 and the average root mean square error (RMSE) is computed for each of the six stochastic models and five look forward windows (*h*=1, *h*=5, *h*=10, *h*=15 and *h*=20) under the non–stratified data ages: 0–100. The results are presented in Table 3.5

Table 3.5 Average RMSE, Look back window=40 years, ages: 0–100, Australian female.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.004143**	**0.005039**	**0.006844**	**0.010417**	**0.012902**
RH	0.011745	0.030677	0.011771	0.040301	0.047769
APC	0.008209	0.014756	0.019718	0.023762	**0.025839**
CBD	0.016699	0.016253	**0.006844**	**0.010195**	**0.010045**
M7	0.016722	0.042032	0.065176	0.079735	0.086733
Plat	**0.00572**	**0.010451**	0.014815	**0.01625**	**0.015783**
P-value	0.0000	0.0000	0.0000	0.0000	0.0000

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported in the last row are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. Red highlight is the case with smallest RMSE.

Table 3.5 above shows increasing error and decreasing goodness-of-fit with increasing values of '*h*'. The minimum RMSE is 0.0041 given by the 1–year look forward window using LC model and the maximum RMSE is 0.0867 given by the 20–year look forward window using M7 model. As expected, the error is larger for higher look forward window.

A one-way ANOVA for RMSE comparing average RMSE across models for each look forward window, indicate that average RMSE for models is not the same for each look forward window

As seen in Table 3.5, the groups with lowest and similar average RMSE are presented in bold. For each look forward window LC model results in lowest RMSE values. The next comparable models are Plat Model (*h*= 1, 5, 15 and 20) and CBD (*h*= 10,15and 20).

For *h*=1 and *h*=5, performance of LC and Plat are comparable, while for *h*=10, performance of LC and CBD are comparable. Meanwhile, for *h*=15, performance of LC, Plat and CBD are comparable, whilst for *h*=20, performance of LC, Plat, APC and CBD are comparable.

Finally, Table 3.6 below show the comparative RMSE for different look forward windows for the six mortality model, fitted using non-stratified data for the ages 0–100 years using look back window 50-years.

Table 3.6 Average RMSE, Look back window=50 years, ages: 0–100, Australian female.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.00415**	**0.005198**	**0.007609**	**0.01096**	**0.013148**
RH	**0.005655**	0.013601	0.020135	0.075225	2.948101
APC	0.010102	0.016232	0.021168	0.022913	**0.024911**
CBD	0.016508	0.015663	0.014583	**0.01179**	**0.008242**
M7	0.019765	0.048111	0.070402	0.074904	0.088191
Plat	**0.006761**	0.01104	0.014778	**0.017513**	**0.011751**
P-value	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported in the last row are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. Red highlight is the case with smallest.

As evident from the Table 3.6, the average RMSE increases with an increase in the value of *h*. The smallest average error for all models is given by the 1-year look forward window, while the highest average RMSE is given by the 20-year look forward window.

A single factor analysis of variance (ANOVA) comparing average RMSE across models, shows that for each look forward windows, the average RMSE criterion varied significantly between all models (*p-value*<0.05).

The groups with lowest and similar average RMSE are presented in bold in Table .6. For all look forward window LC model results in lowest RMSE values. Plat model is the second best model for look forward windows *h*= 1, 15 and 20.

For *h*=5and *h*=10, only LC is the best performing model, while for *h*=1, performance of LC, Plat and CBD are comparable. Meanwhile, for *h*=15, performance of LC, Plat and CBD are comparable, whilst for *h*=20, performance of LC, Plat, APC and CBD are comparable.

Now to understand the sensitivity of length of data required for forward prediction, the best performing model is selected for each *h*, and average RMSE is compared for each decade of data acquisition. The results are presented in Figure 3.3.

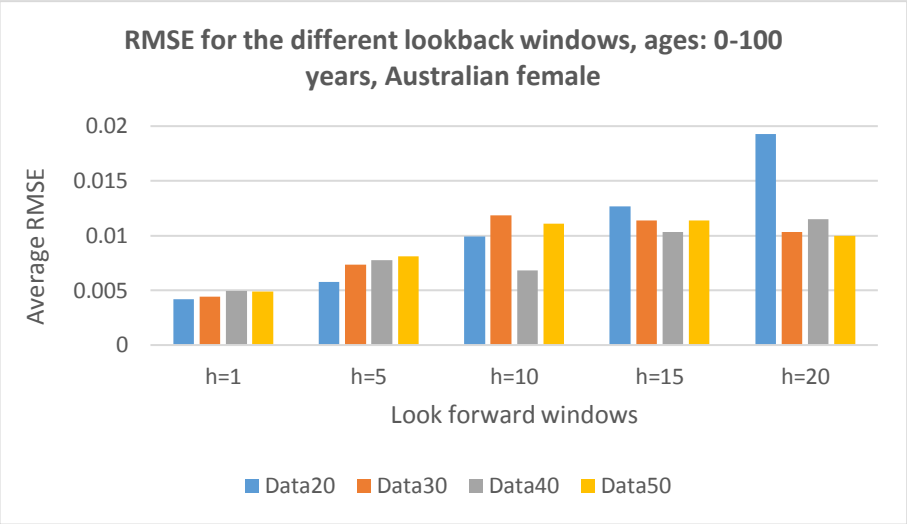

Figure 3.3 RMSE for the different look back and look forward windows, ages: 0–100 years, Australian female.

For each value of *h*, one way ANOVA comparing average RMSE across 4 decades of look back window resulted in *P-valu*e greater than 0.05, indicating equality of mean. There is not much improvement in RMSE as additional 10 years of modelling data is used. Keeping data acquisition simple, 20 years of data can be safely used for modelling.

The final summary of the best performing modelling method and look back window is presented in Table 3.7.

Table 3.7 Summary of the best of look back and look forward windows using mortality models, S0.

Forward prediction	Modelling method	Look back window
***h*=1**	LC and Plat	20 years or more
***h*=5**	LC and Plat	20 years or more
***h*=10**	LC	20 years or more
***h*=15**	LC and CBD	20 years or more
***h*=20**	LC, Plat and CBD	20 years or more

In the following sections, we only focus on Data20, which is 20 years look back window.

In the next section we present the comparisons of average RMSE values for the six models under the second scenario S1 for different forward-looking windows.

3.6.2 Comparison of average RMSE for different look forward windows for stratified data (Scenario S1)

In this section, we consider Data 20 only. The average RMSE is computed for each of the six models (Lee-Carter, RH, APC, CBD, M7 and Plat) and five look forward windows (h=1, h=5, h=10, h=15 and h=20) for ages 0–40 (Scenario S1A). The results are summarized in Table 3.8 below.

Table 3.8 Average RMSE, Look back window=20 years, ages: 0–40, Australian female.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.000047**	**0.000069**	**0.000097**	**0.000137**	**0.000087**
RH	**0.000054**	**0.000093**	**0.000076**	**0.001422**	**0.000411**
APC	**0.000056**	**0.000076**	**0.000067**	**0.000074**	**0.000104**
CBD	0.000548	0.000548	**0.000550**	**0.000548**	**0.000548**
M7	0.000816	0.001995	0.022943	0.448562	0.118579
Plat	**0.000053**	**0.000057**	**0.000088**	**0.000103**	**0.000075**
P-value	0.0000	0.0000	0.0000	0.0000	0.0000
Mean	0.0001	0.0001	0.0001	0.0001	0.0001
t-Statistics	1.4914	0.7804	6.5527	–1.7699	0.1836
P(T<=t)	0.1051	0.2394	0.0014	0.0757	0.4316

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. The Means, t-Statistics and P (T<=t) in the last three rows are from paired t-test for comparing RMSE from best performing model under S1 and S0 for corresponding ages. Red highlight is the case with smallest RMSE.

The model with the best fit for this dataset for short-term forecasting (h =1) was the Lee-Carter model with an RMSE of 0.000047. The model best fitted for this cohort for long-term forecasting (h=20) was the Plat model with an average RMSE of 0.000075 (See Table 3.8). One way ANOVA show that average RMSE is not the same across all models for each look forward windows (p-*value*<0.05).

A student's t-test to compare the average RMSE and average RMSES0 values shows that comparable results are obtained for age strata S1A if we model age group 0–100 years or 0–40 for look forward window h=1,5, 15 and 20. On the other hand, for look forward windows h= 10 smaller RMSE are obtained if we model age groups 0–40 years instead of 0–100 years (P<0.01) (See table 3.8).

Second, we consider age-group 40–60, using the same look back window20 years. The average root mean square error (RMSE) is computed for each of the six models (Lee-Carter, RH, APC, CBD, M7 and Plat) and five look forward windows (h=1, h=5, h=10, h=15 and h=20). The results are presented in Table 3.9

Table 3.9 Average RMSE, Look back window=20 years, ages: 40–60, Australian female.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.000172**	**0.000219**	**0.000239**	0.000331	**0.000334**
RH	**0.000161**	**0.000213**	0.000847	0.000322	**0.000811**
APC	**0.000170**	**0.000176**	**0.000176**	**0.000278**	**0.000322**
CBD	**0.000166**	**0.000208**	**0.000199**	**0.000306**	**0.000323**
M7	**0.000175**	**0.000215**	**0.000243**	0.000387	**0.000491**
Plat	**0.000173**	**0.000215**	**0.000202**	**0.000244**	**0.000298**
P-value	0.8665	0.2167	0.0398	0.0001	0.3676
Mean	0.0003	0.0003	0.0004	0.0003	0.0004
t-Statistics	4.1376	7.4931	6.4872	0.8362	0.6589
P(T<=t)	0.0072	0.0008	0.0015	0.2250	0.2730

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-alues reported are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. The Means, t-Statistics and P (T<=t) in the last three rows are from paired t-test for comparing RMSE from best performing model under S1 and S0 for corresponding ages. Red highlight is the case with smallest RMSE.

For both the long-term (h=20) and short-term (h=1,5) forecasting performance of all modelling methods are comparable. For h=10 and 15, APC, CBD and Plat are comparable and best-performing models.

A student's t-test to compare the average RMSE and average RMSES0 values shows that comparable results are obtained, for age strata S1B if we model age group 0–100 years or 40–60 for look forward window h=15 and 20. On the other hand, for look forward windows h= 1, 5 and 10 smaller RMSE are obtained if we model age groups 40–60 years instead of 0–100 years (See Table 3.9).

Finally, Table 3.10 below shows the comparative RMSE for different look forward windows for the six mortality model, fitted using scenario S1 data for the ages 60–100 years.

Table 3.10 Average RMSE, Look back window=20 years, ages: 60–100, Australian female.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.006516**	**0.008007**	**0.012551**	**0.019668**	**0.026682**
RH	0.036263	0.072645	0.101996	0.128354	0.133283
APC	**0.006679**	**0.012995**	**0.021889**	0.034421	**0.042235**
CBD	**0.012667**	**0.012225**	**0.014294**	**0.012965**	**0.021186**
M7	**0.006847**	**0.016358**	0.029952	0.051601	0.063263
Plat	**0.006351**	**0.005959**	**0.007153**	**0.017438**	**0.024888**
P-value	0.0000	0.0004	0.0000	0.0000	0.0000
Mean	0.0066	0.0102	0.0188	0.0287	0.0343
t-Statistics	0.3344	4.5675	21.3177	7.5591	2.9284
P(T<=t)	0.3774	0.0051	0.0000	0.0008	0.0214

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. The Means, t-Statistics and P (T<=t) in the last three rows are from paired t-test for comparing RMSE from best performing model under S1 and S0 for corresponding ages. Red highlight is the case with smallest RMSE.

A single factor analysis of variance (ANOVA) comparing average RMSE across models for h=1, indicate that average RMSE for all models is not the same. Similar results are observed for each value of h. (p-$value$<0.05).

A student's t-test to compare the average RMSE and average RMSES0 values shows that comparable results are obtained, for age strata S1C if we model age group 0–100 years or 60–100 for look forward window h=1. On the other hand, for look forward windows of 5 or more year, smaller RMSE are obtained if we model age groups 60–100 years instead of 0–100 years (P<0.01) (See Table 3.10).

Overall, for stratified scenario S1 for group 60–100 years, the Lee-Carter and Plat models had the lowest average RMSE for all values of different look forward windows 'h'. As seen in Table 3.10, the groups with lowest and similar average RMSE are presented in bold.

The following is a comparison of the average RMSE values for the six models for all age cohort groups of the third scenario for different look forward windows.

3.6.3 Comparison of average RMSE for different look forward windows for stratified data (Scenario S2)

Further stratification of data in scenario (S2) for stratified age data (Scenario S2: A=0–60, B=60–80, C=80–100) as seen below in Tables 3.11–3.13 that compare the goodness-of-fit in terms of RMSE between different look forward windows for the six different models using Data for 20–years.

Table 3.11 Average RMSE, Look back window=20 years, ages: 0–60, Australian female.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.000110**	**0.000140**	**0.000161**	**0.000226**	**0.000227**
RH	**0.000170**	**0.001478**	**0.000125**	**0.000187**	**0.000774**
APC	0.000107	**0.000117**	**0.000113**	**0.000152**	**0.000194**
CBD	0.000538	0.000559	**0.000574**	**0.000605**	**0.000602**
M7	0.000656	0.000912	0.001224	0.001848	0.007118
Plat	**0.000105**	**0.000103**	**0.000124**	**0.000139**	**0.000173**
P-value	0.0000	0.0000	0.0000	0.0000	0.0000
Mean	0.0001	0.0001	0.0001	0.0001	0.0001
t-Statistics	–0.5487	1.4692	–2.9186	0.6301	–2.2056
P(T<=t)	0.3062	0.1079	0.0217	0.2814	0.0460

Table 3.12 Average RMSE, Look back window=20 years, ages: 60–80, Australian female.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.000599**	**0.000680**	**0.001098**	**0.001578**	**0.001381**
RH	0.002200	0.012529	0.001585	0.012797	0.015169
APC	**0.000624**	**0.000744**	**0.000746**	**0.000891**	**0.001473**
CBD	0.000910	0.000884	**0.001012**	**0.001481**	**0.001454**
M7	**0.000575**	0.000802	**0.000951**	**0.001550**	**0.002032**
Plat	**0.000581**	**0.000673**	**0.000949**	**0.001540**	**0.001801**
P-value	0.0336	0.0000	0.0030	0.0000	0.0000
Mean	0.0005	0.0005	0.0008	0.0011	0.0016
t-Statistics	–1.0817	–2.0233	–1.7828	–3.5489	–2.8948
P(T<=t)	0.1701	0.0565	0.0746	0.0119	0.0222

Table 3.13 Average RMSE, Look back window=20 years, ages: 80–100, Australian female.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.008964**	**0.010661**	**0.017297**	**0.028374**	**0.038343**
RH	0.038253	0.113141	0.195040	0.195454	0.168864
APC	**0.008782**	**0.015506**	0.026796	0.046440	**0.051861**
CBD	0.023231	0.021833	0.027173	**0.019453**	**0.030748**
M7	**0.009522**	0.017743	0.021693	**0.024532**	**0.044230**
Plat	**0.008486**	**0.007197**	**0.009120**	**0.021993**	**0.029137**
P-value	0.0003	0.0000	0.0000	0.0000	0.0000
Mean	0.0092	0.0143	0.0262	0.0401	0.0478
t-Statistics	1.4658	4.3374	7.6438	10.6797	2.0363
P(T<=t)	0.1083	0.0061	0.0008	0.0002	0.0557

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. The Means, t-Statistics and P (T<=t) in the last three rows are from paired t-test for comparing RMSE from best performing model under S1 and S0 for corresponding ages. Red highlight is the case with smallest RMSE.

One way ANOVA for the goodness-of-fit for different values of *h* revealed that there was no significant difference in the goodness-of-fit for all models fitted with stratified data for scenario S2 (*p-value*<0.05) (See Tables 3.10–3.13).

A student's t-test to compare the average RMSE and average RMSES0 values shows that for *h*=15, modelling for age groups 60–80 or 80–100 gives better prediction (*P*<0.01) (See Table 3.12–3.13). On the other hand, for age groups 0–60 years models fitted for ages 0–100 are comparable to model fitted for stratified ages (*P*>0.05) (See Table 3.11). There is not much again in stratification at other look forward windows.

In summary, based on the results of the RMSE for look forward windows, for stratified scenario S2 for group 80–100 years, it may be inferred that the Lee-Carter model and Plat model models had the lowest average RMSE for all values of different look forward windows '*h*'. The groups with lowest and similar average RMSE are presented in bold (See Table 3.13).

The next section will compare the goodness-of-fit for each of the six stochastic models separately based on all the selection criteria for each of age cohort scenarios.

3.6.4 Comparison of average AIC and BIC for different scenarios

The purpose of this section is to illustrate that the best performing models on RMSE are also good performing models on AIC and BIC criterion.

The comparisons between the average AIC and BIC values for 1-–year look forward window using look back window 20-years were illustrated in Figure 3.4– Figure 3.10 below. The comparison shows that the best fitted models for scenario S0 (0–100 years, non-stratified data) are (LC, Plat and APC) also have lower AIC and BIC values.

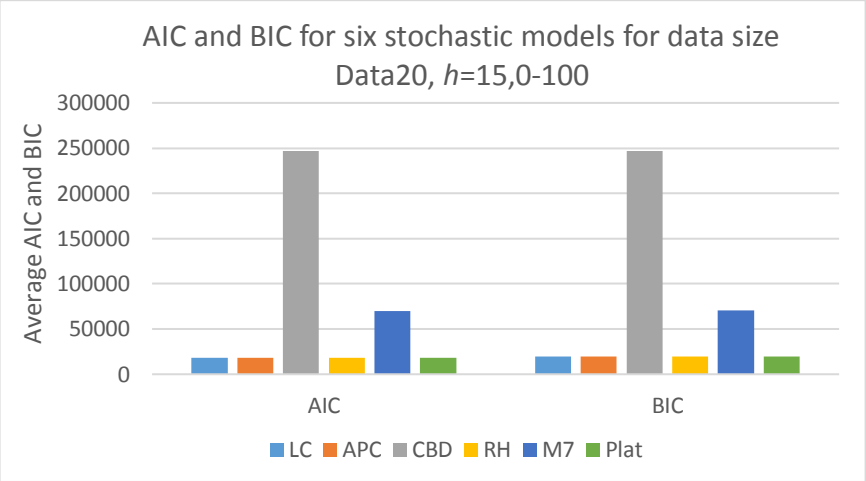

Figure 3.4 A comparative assessment of the six mortality models with respect to S0 for ages 0–100, h=15, Data 20, Australian female.

Figure 3.5 below shows that once again for scenario S1A (0–40 years) the best fitted model according to RMSE (APC, Plat, LC and RH) are also the models with a lower range of AIC and BIC values.

For the same scenario, stratified data 40–60 years, the Renshaw-Haberman and the APC models give the best results according to the AIC and RMSE criteria, while according to the BIC criterion the Lee-Carter models and APC are the best fitted model for this dataset (Figure 3.6 below). However, AIC and BIC values for best performing models for RMSE (Plat, APC and CBD) are comparable to the best scenario.

Figure 3.7 below, presents AIC and BIC values for S1C (60–100 years). Once again, the best performing models for RMSE (Plat and LC) have lower range of AIC and BIC values.

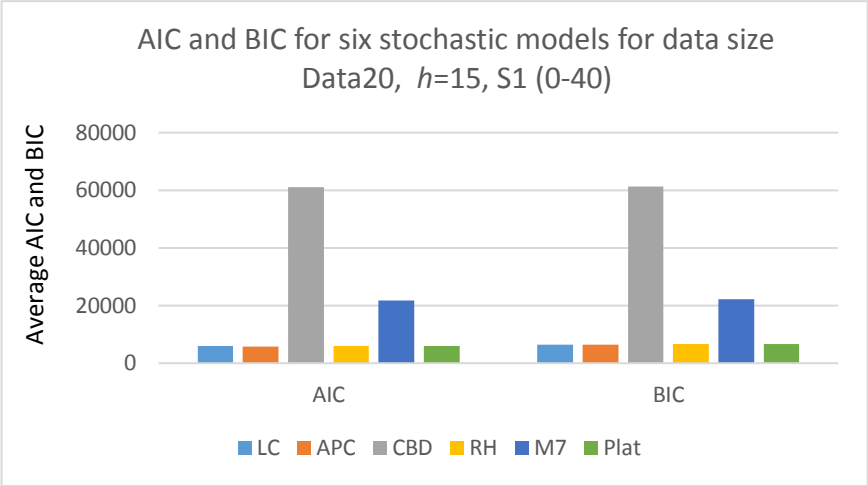

Figure 3.5 **A comparative assessment of the six mortality models with respect to S1 for ages 0–40, h=15, Data 20, Australian female.**

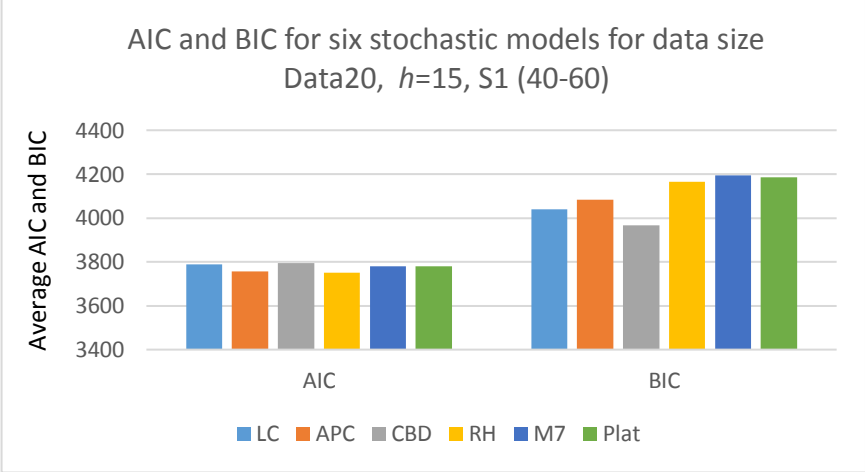

Figure 3.6 **A comparative assessment of the six mortality models with respect to S1 for ages 40–60, h=15, Data 20, Australian female.**

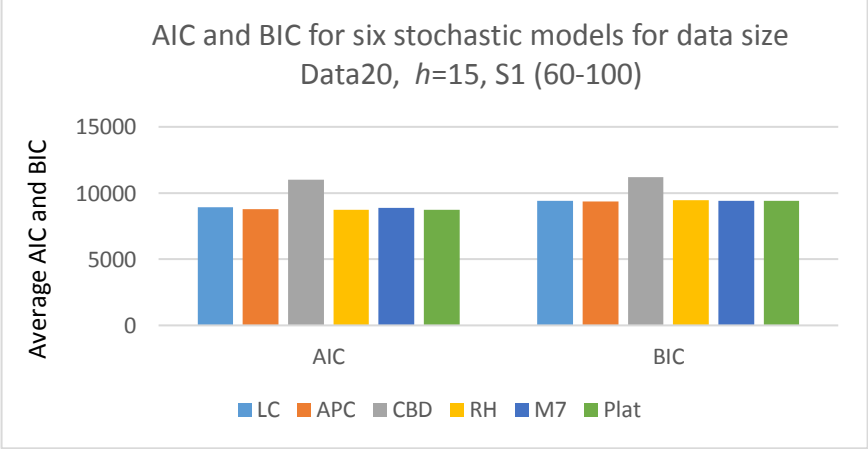

Figure 3.7 **A comparative assessment of the six mortality models with respect to S1 for ages 60–100, h=15, Data 20, Australian female.**

Finally, for scenario S2A (0–60 years), data 20, 15–year look forward window, the results show that according to the AIC criterion the best fitted models are the Plat and the APC models. The BIC and RMSE criteria give the LC, Plat and RH models as the best models to estimate mortality and life-expectancy rates for this dataset. On the other hand, the LC and the APC models are the best fitted models for scenario S2B (60–80 years) according to the AIC and the BIC criterion, while according to RMSE the Plat and APC models are the best. Finally, for scenario S2C (80–100 years) the AIC criterion gives the RH and Plat models as the best fitted models, the BIC criterion gives the Lee-Carter and the APC models, while the minimum RMSE values are given by the Plat and the CBD models (Figure 3.8 – Figure 3.10 below).

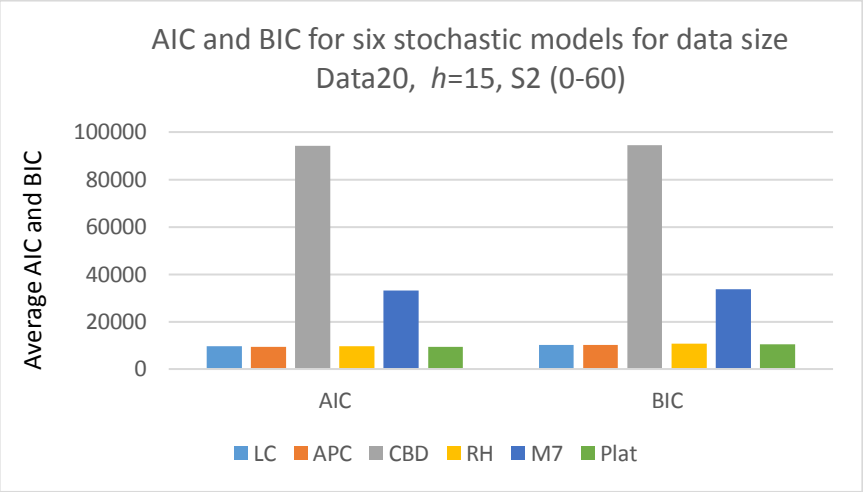

Figure 3.8 A comparative assessment of the six mortality models with respect to S2 for ages 0–60, h=15, Data 20, Australian female.

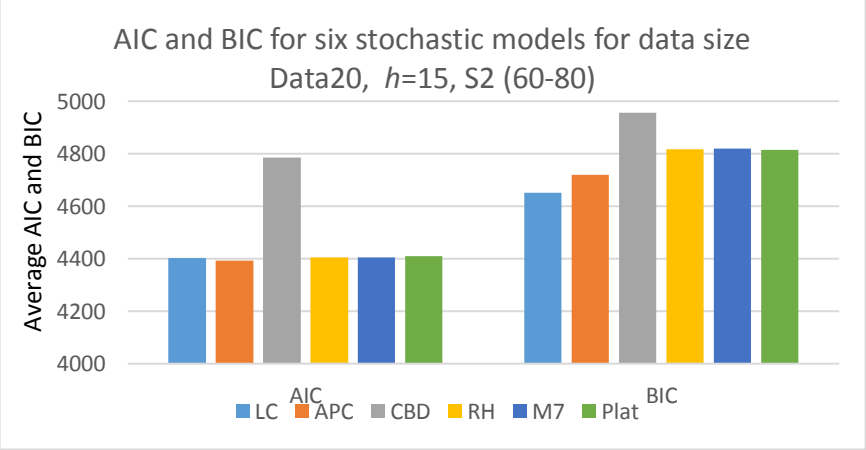

Figure 3.9 A comparative assessment of the six mortality models with respect to S2 for ages 60–80, h=15, Data 20, Australian female.

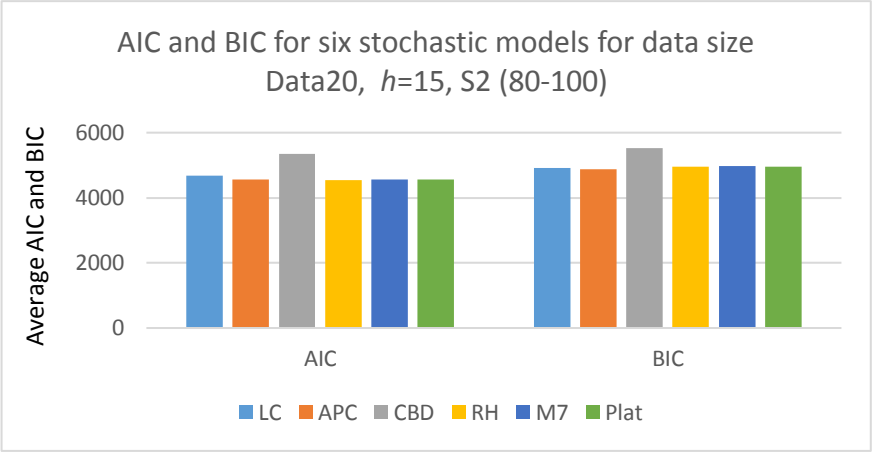

Figure 3.10 A comparative assessment of the six mortality models with respect to S2 for ages 80–100, h=15, Data 20, Australian female.

In summary, the best performing models for RMSE (LC and Plat) have lower ranges of AIC and BIC values.

3.6.5 Randomness of residuals

The fourth criteria to be satisfied were the randomness of residuals. The residuals for the models were randomly distributed, as computed using the StMoMo R package. (Figure 3.11 and Figure 3.12).

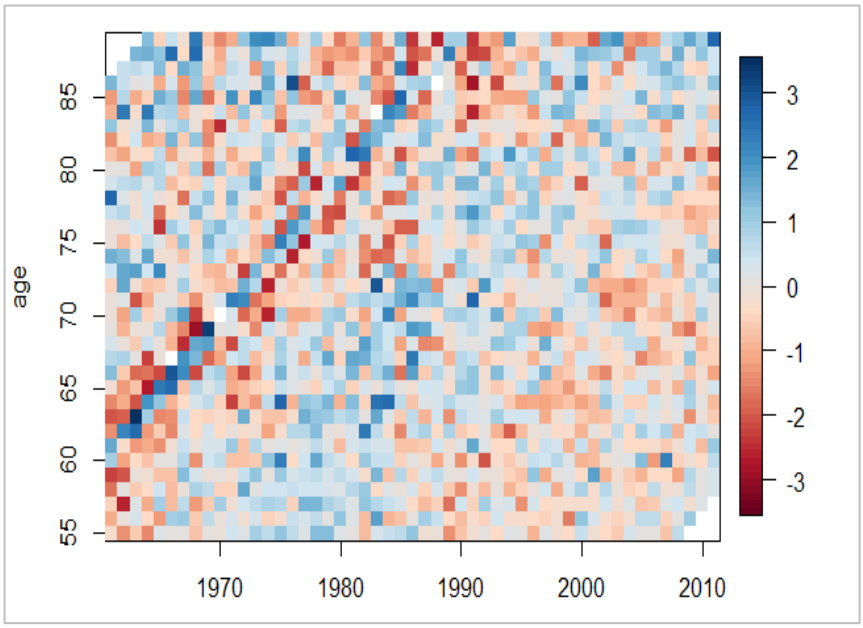

Figure 3.11 Residual plot for the Lee-Carter model, Australian female.

The distribution of residuals for the Lee-Carter model is random and indicates deterioration in mortality (red cells). The decline in mortality is seen across all ages represented in the model.

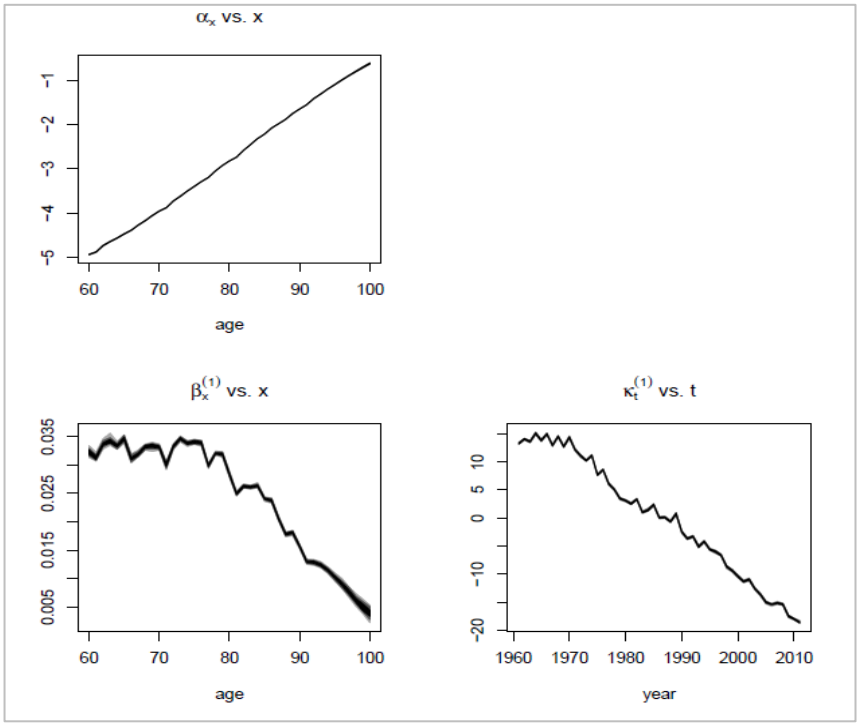

Figure 3.12 Parameter estimation for Lee-Carter model, Australian female.

Parameter estimations for the age-coefficients (α_x) and (β_x) and the period effect or mortality index (k_t) follow exponential and linear trends respectively, which are typical for the Lee-Carter model. No cohort effect is expected or apparent. The central mortality rate predicted by the model varies logarithmically with α_x, β_x and k_t as $\ln m_{(x,t)} = \alpha_{(x)} + \beta_{(x)} k_{(t)} + \varepsilon_{(x,t)}$ (Cairns et al. 2011).

3.6.6 Summary of results

In summary, the following assumptions can be made. Based on the four criteria for model selection, the Lee-Carter model appears to be a good model across all scenarios. In addition, the model with the lowest error (RMSE) across all look back and look forward windows, age-based stratification and data sizes, also had the lowest BIC and a low AIC. Hence, on this basis, the best model for estimating mortality and life-expectancy for this dataset would be the Lee-Carter model. The CBD and M7

models had the highest values for the RMSE as well as BIC and AIC, suggesting a poor selection for this dataset.

Regarding the size of the data (look back windows), there was no significant difference in the goodness-of-fit for the Australian female mortality data, between the different data sizes (Data 20 to Data 50) for the six stochastic models studied. However, using age stratification data for ages 60 to 100 or 80 to 100 led to improved results in the model fit.

3.7 Calculating mortality

Mortality $m_{(x,t)}$ and the number of deaths between the ages x and $x_{(t+1)}$ were determined using the StMoMo package for all the models used. The Lee-Carter model, which ranked the highest, based on the selection criteria of goodness-of-fit, BIC, AIC and random distribution of residuals, was compared with the CBD model which fitted most poorly to the data and ranked the lowest with respect to the selection criteria.

The mortality rate $m_{(x,t)}$ and the number of deaths for age x at time t $D_{(x,t)}$ were computed using the StMoMo package for the R statistical programme and obtained from the original mortality tables (www.mortality.org). Central mortality rates $m_{(x,t)}$ are calculated using the equations (2.3) and (2.4) in Chapter 2.

The life-expectancy for a specific age was calculated based on the method described by Arias (2014), Strauss, Shavelle and Brook, www.lifeexpectancy.org (2016); Anderson (1999) and Slud (2001).

The residual life-expectancy, at age x, ($e_{x,}$ or the average survival time at age x) is calculated as:

$$e_x = sum\,(T_x / l_x) \tag{3.2}$$

where l_x is the survivor function for age x, and T_x is the total number of persons alive after the age of x years.

T_x is obtained from the mortality tables, while l_x was calculated using the formula:

$$l_x = d_x / m_x \tag{3.3}$$

where d_x is the number of deaths between the ages x and $x+1$, and m_x is the mortality rate computed by the model.

The estimated mortality rates and number of deaths from the Lee-Carter model and CBD model are tabulated in Table 3.14. Calculated life-expectancy using mortality rates from both models and population data from life-tables (www.mortality.org) is presented in Table 3.15.

The forecasted mortality rates for *h1*, *h5*, *h10* and *h*20 for the age group 60–100 are presented in Tables 3.16 and 3.17 with and Figures 3.13–3.15 below.

Mortality rates from life-tables (www.mortality.org) for a 65–year–old Australian woman in 2011 was 0.00613. The mortality rate for the same cohort derived from the Lee-Carter model is 0.00602, and from the CBD model, is 0.00491 (see Table 3.14).

Table 3.14 Mortality rate $[m_{(x,t)}]$ and number of deaths $[D_{(x,t)}]$ computed by the CBD and Lee-Carter Models and original Australian female data for year=2011, ages=60—100.

	Mortality rate $[m_{(x,t)}]$			Number of deaths $[D_{(x,t)}]$		
Age	**CBD**	**LC**	**original**	**CBD**	**LC**	**original**
60	0.002513	0.003966	0.00416	238.5375	376.4541	394
61	0.002874	0.00429	0.00466	271.6138	405.511	439
62	0.003286	0.004729	0.00471	309.1104	444.9459	442
63	0.003756	0.005124	0.00483	351.742	479.8065	452
64	0.004294	0.005632	0.00525	400.1742	524.8221	488
65	**0.004909**	**0.006023**	**0.00613**	455.05	558.354	567
66	0.005611	0.006988	0.00643	516.9533	643.8092	590
67	0.006413	0.007735	0.00746	587.039	708.0515	680
68	0.007328	0.008326	0.00802	665.8698	756.5207	726
69	0.008374	0.00932	0.00927	754.768	840.1002	832
70	0.009566	0.010327	0.01034	854.3225	922.2054	919
71	0.010927	0.011834	0.01188	965.8188	1045.955	1044
72	0.012479	0.012887	0.01342	1089.969	1125.616	1164
73	0.014249	0.014	0.01428	1227.918	1206.531	1222
74	0.016265	0.01592	0.01701	1381.772	1352.522	1433
75	0.018561	0.017594	0.01727	1550.233	1469.538	1430
76	0.021174	0.019626	0.01956	1738.201	1611.122	1590
77	0.024145	0.023115	0.02261	1943.776	1860.854	1800
78	0.027523	0.025654	0.02546	2166.111	2019.072	1978
79	0.031357	0.028957	0.02785	2405.85	2221.723	2107
80	0.035706	0.034109	0.03407	2664.274	2545.139	2500

	Mortality rate [$m_{(x,t)}$]			Number of deaths [$D_{(x,t)}$]		
Age	**CBD**	**LC**	**original**	**CBD**	**LC**	**original**
81	0.040633	0.039281	0.03734	2930.36	2832.831	2644
82	0.046207	0.044496	0.04518	3210.159	3091.287	3069
83	0.052504	0.050529	0.05251	3486.521	3355.353	3398
84	0.059605	0.057116	0.05746	3755.523	3598.666	3519
85	0.067599	0.065475	0.0677	4021.28	3894.917	3895
86	0.076577	0.07457	0.07965	4257.059	4145.515	4258
87	0.086637	0.086089	0.09063	4447.443	4419.335	4451
88	0.097878	0.098516	0.10455	4588.8	4618.707	4658
89	0.110401	0.110403	0.12488	4661.741	4661.829	4963
90	0.124306	0.125646	0.14225	4631.955	4681.867	4949
91	0.139687	0.142639	0.15577	4513.858	4609.222	4670
92	0.156631	0.160524	0.17263	4329.842	4437.438	4393
93	0.175212	0.176416	0.194	4073.862	4101.842	4112
94	0.195486	0.195922	0.2247	3741.416	3749.752	3866
95	0.217488	0.215715	0.24836	3321.694	3294.62	3374
96	0.241223	0.237768	0.27733	2870.315	2829.198	2898
97	0.266666	0.260787	0.30829	2400.261	2347.346	2404
98	0.293753	0.284737	0.34108	1937.598	1878.127	1922
99	0.322383	0.309331	0.37547	1506.818	1445.814	1478
100	0.35241	0.334132	0.41115	1126.655	1068.22	1090

Table 3.15 Calculation of life-expectancy of Australian female from modelled mortality rates for year=2011, ages=60-–100.

	Number of survivors [l_x]				Life-expectancy [$e_{(x,t)}$]		
Age	**CBD**	**LC**	**(Original)**	**T_x (original)**	**CBD**	**LC**	**(Original)**
60	95028.46	94890.55	94915	2514660	26.462177	26.500638	26.49381
61	94643.39	94509.49	94521	2419942	25.569056	25.605281	25.602162
62	94211.89	94076.05	94082	2325640	24.685207	24.72085	24.719287
63	93730.26	93602.19	93640	2231780	23.810667	23.843245	23.833618
64	93239.83	93115.18	93188	2138366	22.934041	22.964742	22.946796
65	92732.95	92629.65	92700	2045422	22.057122	22.081721	22.064962
66	92183.05	92056.19	92133	1953006	21.186173	21.215368	21.197682
67	91545.96	91424.95	91542	1861168	20.330422	20.357332	20.331301
68	90876.13	90785.48	90862	1769966	19.476688	19.496135	19.479716
69	90107.23	90021.9	90136	1679467	18.638537	18.656205	18.632589
70	89281.68	89213.79	89305	1589746	17.80596	17.819509	17.80131

	Number of survivors [l_x]				Life-expectancy [$e_{(x,t)}$]		
Age	**CBD**	**LC**	**(Original)**	**T_x (original)**	**CBD**	**LC**	**(Original)**
71	88339.18	88259.04	88386	1500901	16.990207	17.005634	16.981207
72	87296.03	87260.38	87342	1413036	16.186715	16.193328	16.178196
73	86114.08	86135.47	86178	1326276	15.401384	15.397559	15.38996
74	84796.23	84825.48	84956	1240709	14.631653	14.626608	14.604136
75	83405.77	83486.46	83523	1156470	13.865588	13.852186	13.846126
76	81784.8	81911.88	82093	1073662	13.127892	13.107525	13.078606
77	80149.22	80232.15	80503	992364	12.381455	12.368658	12.327044
78	78336.89	78483.93	78703	912762	11.651752	11.629922	11.59755
79	76297.15	76481.28	76725	835048	10.944681	10.918332	10.883649
80	74060.73	74179.86	74617	759377	10.253437	10.23697	10.176997
81	71686.64	71784.17	72118	686009	9.5695515	9.55655	9.5123132
82	68907.84	69026.71	69474	615213	8.9280551	8.9126799	8.8552984
83	65987.48	66118.65	66405	547274	8.2936037	8.2771507	8.2414577
84	62649.48	62806.33	63007	482568	7.7026661	7.6834288	7.6589585
85	58985.72	59112.08	59488	421321	7.1427627	7.1274937	7.0824536
86	55230.94	55342.49	55592	363781	6.5865436	6.5732682	6.5437653
87	51144.56	51172.67	51334	310318	6.0674688	6.0641359	6.0450773
88	46745.2	46715.29	46883	261210	5.5879534	5.5915308	5.5715291
89	42221.26	42221.17	42225	216656	5.1314434	5.1314541	5.1309888
90	37593.05	37543.13	37262	176912	4.7059768	4.7122333	4.7477859
91	32748.14	32652.78	32313	142124	4.3399103	4.3525853	4.3983536
92	27983.16	27875.56	27644	112145	4.0075892	4.023058	4.0567573
93	23570.14	23542.16	23251	86698	3.6782984	3.68267	3.7287859
94	19509.58	19501.25	19139	65503	3.357478	3.3589132	3.4224881
95	15817.31	15844.38	15273	48297	3.0534276	3.0482102	3.1622471
96	12402.69	12443.8	11899	34712	2.7987488	2.789501	2.9172199
97	9498.739	9551.654	9001	24262	2.5542338	2.5400837	2.6954783
98	7063.402	7122.873	6596	16463	2.3307466	2.3112864	2.4959066
99	5089.182	5150.186	4674	10828	2.1276502	2.1024482	2.3166453
100	3547.345	3605.78	3197	6893	1.9431436	1.911653	2.1560838

Table 3.16 Mortality forecasts for forward look windows *h*=1, *h*=5 and *h*=10.

	2012		2016		2021	
Age	**LC**	**CBD**	**LC**	**CBD**	**LC**	**CBD**
60	0.003886	0.002442	0.003581	0.002176	0.003232	0.001884
61	0.004206	0.002794	0.003887	0.002495	0.003522	0.002167

	2012		2016		2021	
Age	**LC**	**CBD**	**LC**	**CBD**	**LC**	**CBD**
62	0.00463	0.003196	0.004253	0.002862	0.003824	0.002492
63	0.005014	0.003656	0.004599	0.003282	0.004128	0.002867
64	0.005515	0.004182	0.00507	0.003763	0.004563	0.003297
65	0.005893	0.004784	0.005399	0.004315	0.004839	0.003792
66	0.006852	0.005471	0.006335	0.004947	0.005743	0.004361
67	0.007581	0.006257	0.006995	0.005671	0.006326	0.005014
68	0.008154	0.007155	0.007499	0.0065	0.006753	0.005765
69	0.009127	0.00818	0.008393	0.00745	0.007558	0.006628
70	0.010114	0.009351	0.009304	0.008538	0.008382	0.007619
71	0.011615	0.010688	0.010777	0.009783	0.009813	0.008757
72	0.012621	0.012214	0.01161	0.011207	0.010457	0.010063
73	0.013701	0.013954	0.012563	0.012836	0.011271	0.011561
74	0.015587	0.015939	0.014322	0.014698	0.012882	0.01328
75	0.017224	0.0182	0.015818	0.016825	0.014219	0.01525
76	0.019216	0.020775	0.017659	0.019255	0.015886	0.017507
77	0.022691	0.023706	0.021068	0.022028	0.019198	0.020092
78	0.02515	0.02704	0.023229	0.025189	0.021028	0.023049
79	0.028393	0.030827	0.026243	0.028791	0.023776	0.026429
80	0.03352	0.035125	0.031261	0.03289	0.028643	0.03029
81	0.038687	0.039998	0.036398	0.037551	0.033719	0.034695
82	0.043792	0.045515	0.041083	0.042843	0.037921	0.039714
83	0.04974	0.051752	0.0467	0.048843	0.043148	0.045425
84	0.056221	0.05879	0.052771	0.055634	0.048739	0.051913
85	0.064547	0.066719	0.060957	0.063307	0.056732	0.05927
86	0.073536	0.075632	0.069527	0.071957	0.064801	0.067595
87	0.085065	0.085625	0.081079	0.081686	0.076336	0.076994
88	0.097512	0.096801	0.093586	0.092599	0.088879	0.087578
89	0.109277	0.109261	0.104872	0.104804	0.099586	0.099459
90	0.124552	0.123107	0.12026	0.118408	0.115075	0.112753
91	0.141637	0.138434	0.137687	0.133513	0.132882	0.127572
92	0.159427	0.155331	0.155101	0.150218	0.149829	0.144022
93	0.175273	0.173875	0.170763	0.168606	0.165254	0.1622
94	0.194781	0.194124	0.190268	0.188745	0.184739	0.182183
95	0.214603	0.216114	0.210194	0.21068	0.204776	0.204029
96	0.236724	0.239853	0.232583	0.234427	0.227478	0.227765
97	0.259834	0.265318	0.256045	0.25997	0.251359	0.253384
98	0.283908	0.292446	0.280604	0.287251	0.276508	0.280835
99	0.30865	0.321136	0.305934	0.316172	0.302559	0.310026

	2012		2016		2021	
Age	LC	CBD	LC	CBD	LC	CBD
100	0.333612	0.351242	0.331536	0.346589	0.328951	0.340814

Table 3.17 Mortality forecasts of Australian female for forward look windows *h*=15, *h*=20 and *h*=50.

	2026		2031		2061	
Age	LC	CBD	LC	CBD	LC	CBD
60	0.002918	0.001631	0.002634	0.001412	0.001424	0.000594
61	0.003191	0.001881	0.002891	0.001633	0.001598	0.000699
62	0.003438	0.002171	0.003091	0.00189	0.001631	0.000824
63	0.003705	0.002504	0.003325	0.002187	0.001736	0.000971
64	0.004107	0.002889	0.003696	0.002531	0.001963	0.001144
65	0.004337	0.003332	0.003887	0.002928	0.002012	0.001347
66	0.005206	0.003844	0.004719	0.003388	0.002615	0.001587
67	0.00572	0.004434	0.005172	0.00392	0.002824	0.00187
68	0.006082	0.005113	0.005476	0.004534	0.002916	0.002203
69	0.006805	0.005896	0.006127	0.005245	0.003259	0.002595
70	0.007551	0.006799	0.006801	0.006066	0.003628	0.003056
71	0.008935	0.007838	0.008135	0.007015	0.004627	0.003599
72	0.009418	0.009035	0.008482	0.008111	0.004516	0.004238
73	0.010111	0.010412	0.009069	0.009376	0.004713	0.00499
74	0.011585	0.011997	0.010418	0.010837	0.005496	0.005875
75	0.012779	0.01382	0.011483	0.012522	0.006032	0.006915
76	0.014289	0.015915	0.01285	0.014466	0.006779	0.008138
77	0.017491	0.018323	0.015933	0.016707	0.009081	0.009576
78	0.019031	0.021086	0.017221	0.019287	0.009423	0.011264
79	0.021537	0.024256	0.019504	0.022258	0.010721	0.013246
80	0.026238	0.027889	0.024031	0.025674	0.014133	0.015572
81	0.031231	0.032048	0.028921	0.029598	0.018179	0.018298
82	0.034993	0.036804	0.032284	0.034101	0.019828	0.021491
83	0.039855	0.042236	0.036803	0.039261	0.022718	0.025227
84	0.045001	0.048428	0.041536	0.045166	0.025553	0.029593
85	0.052784	0.055475	0.049096	0.05191	0.03163	0.034688
86	0.060375	0.06348	0.056234	0.059599	0.036505	0.040622
87	0.071849	0.072551	0.067606	0.068345	0.046681	0.047523
88	0.084387	0.082804	0.080101	0.078268	0.058322	0.055527
89	0.094539	0.094358	0.089723	0.089492	0.065215	0.064788
90	0.110085	0.107335	0.105286	0.102148	0.080208	0.07547

Age	2026		2031		2061	
	LC	CBD	LC	CBD	LC	CBD
91	0.128218	0.121858	0.123696	0.116365	0.099356	0.087748
92	0.144706	0.138041	0.139729	0.132269	0.11281	0.101804
93	0.15989	0.155992	0.154667	0.149978	0.126208	0.11782
94	0.179335	0.175801	0.174056	0.169595	0.144933	0.135975
95	0.199464	0.197536	0.194255	0.1912	0.165167	0.156431
96	0.222453	0.221238	0.217507	0.214846	0.18951	0.179326
97	0.246731	0.246909	0.24216	0.240546	0.215961	0.204758
98	0.272449	0.274508	0.268428	0.26827	0.245101	0.232774
99	0.299204	0.303947	0.295871	0.297935	0.276333	0.263354
100	0.326376	0.335085	0.323811	0.329405	0.308648	0.296399

As seen in Table 3.16 - Table 3.17, the mortality rates over different values of *h* (*h*1–*h*50) were not significantly different between the Lee-Carter model and CBD model as determined by the student's t-test ($P>0.05$).

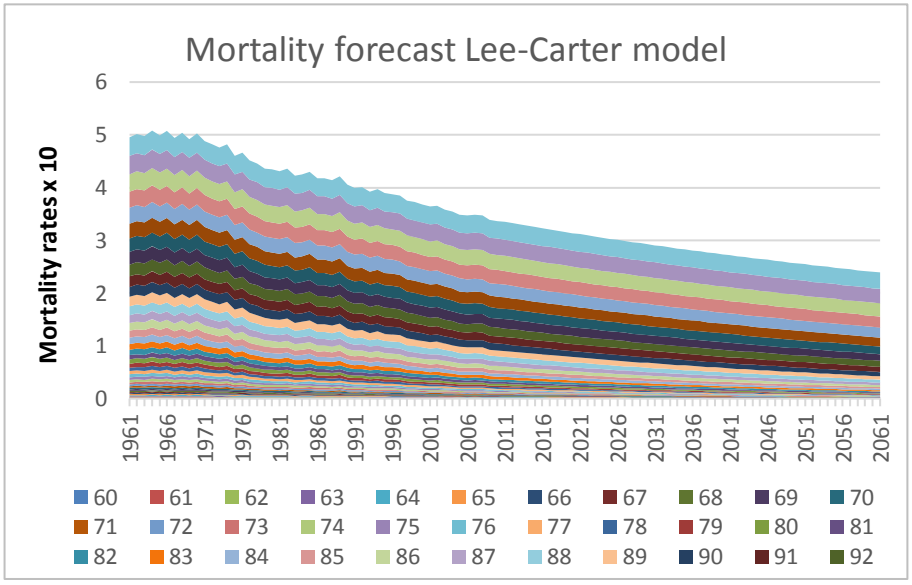

Figure 3.13 Mortality forecast of Australian female - Lee-Carter model.

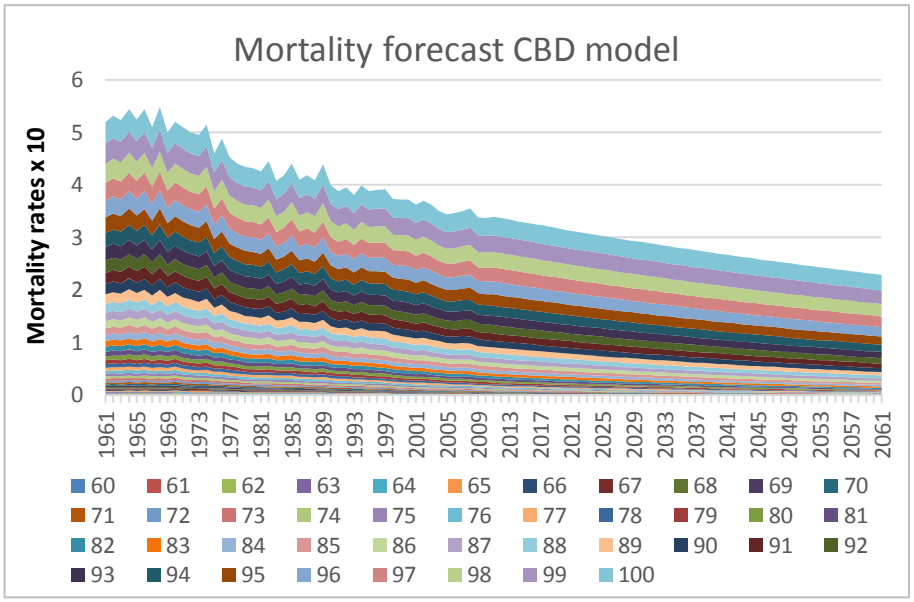

Figure 3.14 Mortality forecast of Australian female-CBD model.

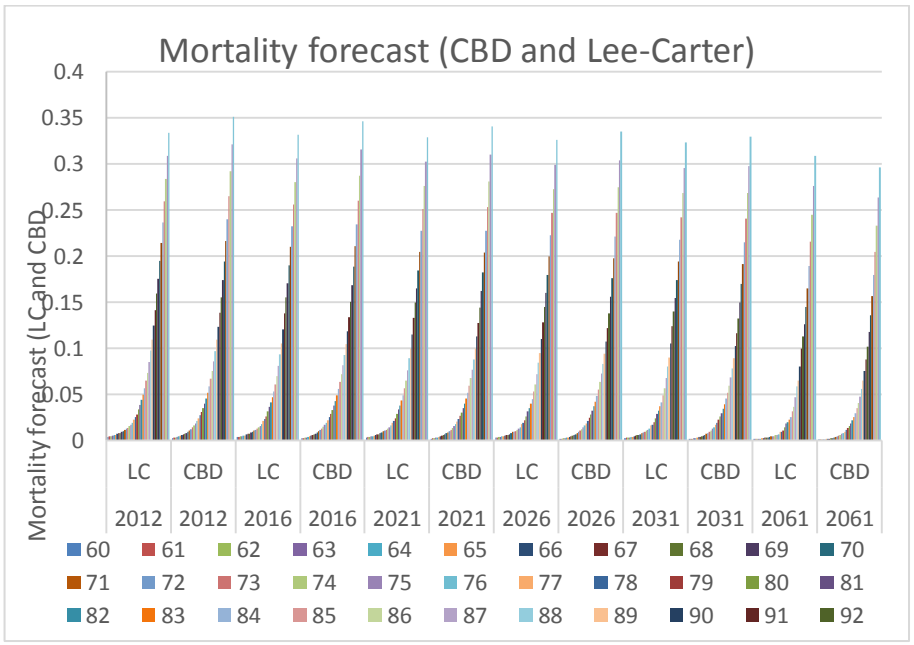

Figure 3.15 Comparative mortality forecasts by the Lee-Carter and CBD models using Australian female data.

The graphs from Figure 3.13 to Figure 3.15 presented the mortality rates predicted by the Lee-Carter and CBD models, as observed, the mortality rates of Australian female population aged 60–100 decline over 50 years in the period from 1961 to 2011. This age cohort was fitted into the models to predict female mortality with a forecasting period of 1 year (2012), 5 years (2016), 10 years (2021) 15 years, (2026), 20 years (2031) and fifty years (2061). Based on current population figures, the residual life-expectancy for a 65-year-old Australian woman in 2011 is 22.06 years,

with an expected life–span of 87.06 years. Using mortality rates predicted by the Lee-Carter model, the residual life-expectancy for an Australia woman aged 65 in 2011, is 22.081 while the CBD model gives a residual life-expectancy of 22.057 years, for the same individual.

The CBD model was compared with the Lee-Carter model to determine whether the difference in selection criteria would influence the actual mortality rate estimation.

In all cases, the difference between CBD model, LC and the life-expectancy from the life-tables at www.mortality.org were not significantly different as determined by the *t*-test ($P>0.05$) indicating that the statistical goodness-of-fit did not significantly impact on the forecasting of life-expectancy, for this dataset, for ages 60–100 years.

Hence, the choice of the model for forecasting mortality and life-expectancy is perhaps based on qualitative rather than quantitative criteria, as was earlier suggested (Biffi and Clemente 2014; Plat 2009). The Lee-Carter model also gave a similar estimate of life-expectancy as the CBD model, which has previously been recommended for older age groups (Biffi and Clemente 2014).

3.8 Discussion

Stochastic modelling of mortality data has been a preferred choice for forecasting survival of the human population over the last hundred years (Dellaportas, Smith and Stavropoulos 2001). The main advantage of using stochastic models is the facility to compare data and predict changes over time and space (Pollard 1989). On this principle, the current research has addressed the issue of female mortality and life-expectancy forecasting in Australia. Because the number of studies on the topic of mortality forecasting in Australia is limited, the current study fills in the gaps in the literature. This research can be considered a great contribution to the existing literature, so that the projections obtained, without leaving aside the limitations of the methodologies used, are a complement to the set of official values.

Some of the desirable attributes of a stochastic model are parsimony, robustness of forecasting, predicting positive mortality rates and flexibility (Congdon 1993). Additionally, the ease with which all age groups can be incorporated in the

model is also seen as an advantage to demographers and actuaries (Heligman and Pollard 1980). In recent years, many user-friendly stochastic models have been developed to accommodate large datasets in different parts of the world, so as to give a more realistic estimate of lifespans over a wide range of demographics, while taking into account the uncertainties in the prediction (Aro and Pennanen 2011). The first of these was the model proposed by Lee and Carter (1992), which is an extension of the general linear model. While the Lee-Carter model remains the most commonly used model (Li, Hardy and Tan 2009), the use of multiple models has been shown to improve uncertainties surrounding mortality (Oeppen and Vaupel 2002).

The Renshaw-Haberman model (Renshaw and Haberman 2003) was an extension of the Lee-Carter model with an added age cohort effect. The Cairns, Blake and Dowd model (2006b) was an extension of the Renshaw-Haberman wherein parametric uncertainty was incorporated. This enables demographers and actuaries to consider natural disasters and other extraneous factors contributing to trends in mortality (Dowd, Blake and Cairns 2016).

The popularity of the Lee-Carter model and its extensions in the theoretical and empirical literature, as well as the extensive investigation of their limitations, represent reasons for applying this methodology in the current research. Thus, the analysis performed in this chapter involved applying the Lee-Carter model and five of its extensions (RH, APC, CBD, M7 and Plat) on Australian female mortality data between 1961 and 2011. Based on a comparison between the six models, the Lee-Carter model has proved to be the most appropriate to forecast the female mortality rate and life-expectancy in Australia for the next 50 years. Results showed that the Lee-Carter model gave a comparatively good fit to the Australian data.

In the empirical literature, many authors use performance measures such as the Mean Absolute Performance Error (MAPE) (Lee, Baek, Kim and Oh 2016), and the Mean Squared Error (MSE) and its derivative or the Root Mean Squared Error (RMSE) (Adhikari and Agarwal 2013) or both MAPE and RMSE (Husin, Zainol and Ramli, 2015) to evaluate the forecasting accuracy of a model. Other model selection methods commonly used in the literature are the AIC and the BIC criteria (Li et al. 2009). Based on the common use of the RMSE in the existing empirical literature and taking into consideration its relative simplicity of use, this is one of the criteria used to

determine the goodness-of-fit for the Australian female dataset in the current study. The other two criteria used are the AIC and the BIC.

The Lee-Carter forecasts of mortality rates are comparable to the official mortality rates, while the forecasts of life-expectancy are a little lower than official numbers, but the difference is not significant. On the other hand, experience has shown that the Lee-Carter method tends to under-predict life-expectancy (Lee and Miller 2001: pp 537–549). Thus, the observed values of life-expectancy could even be higher than the Lee-Carter forecasts.

Booth et al. (2006) applied the Lee-Carter model and its variants to forecast gender-specific mortality rates in ten different countries and concluded that deviation between models in predicting mortality rates did not extend to differences in calculating life-expectancy. In their study, they found that, while the more complex variants of the model were less prone to error, the strongest attributes of the original Lee-Carter model are its simplicity and robustness in forecasting linear trends. In their study, they did not study the effect of the length of the forecasting period on the forecast accuracy.

In contrast with the analysis performed by Booth et al. (2006), the current study takes into consideration the effect of the fitting period (*h*). This was seen to have a significant effect on the RMSE and hence the predictive accuracy of the model. Short-term forecasts (e.g. *h1*) were more accurate than long-term predictions (e.g. *h*20). Moreover, based on the results of the RMSE for look forward windows, the comparison revealed that the Lee-Carter model is better suited for non–stratified data. In addition to the lowest error (RMSE) across almost all look forward windows, age-based stratification and data sizes, the Lee-Carter model also had the lowest BIC and a low AIC.

For the data set considered in the current study (different age cohorts, look forward windows and data size), the comparison techniques used revealed there are some notable differences amongst the six different models. However, none of the models analysed performs well in all tests and no model clearly dominates the others. This conclusion is consistent with the conclusion formulated by Dowd et al. (2010a) after assessing the goodness-of-fit of several mortality models applied to the English and Wales data. For the Australian female data set considered we find that the Lee-

Carter model gives the lowest RMSE for all look forward windows and is better suited for non-stratified data. However, this model does not clearly stand out from the rest of the models analysed. The Plat model gave a comparatively good fit to the Australian female mortality data, while the Cairns-Blake-Dowd model gave the poorest fit to this data.

While long-term forecasts are important for planning economic, social-welfare and health objectives, short-term forecasts have enhanced application in identifying errors in long-term predictions (Booth et al. 2006). Using a variant of the Lee-Carter model, Lee and Miller (2001: pp 537–549) discovered that using a long-term data to make a short-term forecast resulted in a jump-off error and a bias of 0.6 years. This is because the random age effect in the model (β_x) varies over time and the assumption that it is a constant leads to fallacious results, especially when fitting the model with extended historical data encompassing several decades.

The comparison of the six stochastic models used in the current study revealed that the best-fitted models for the dataset were the Lee-Carter Model and the Plat model, based on the four selection criteria that quantify the goodness-of-fit and the parsimony of the models. On the same basis, the CBD model gave the poorest fit on all counts. However, when the data was fitted into both models to calculate mortality rates, the two models showed comparable results despite their differences. Similarly, life-expectancy was comparable between the Lee-Carter and CBD forecasts. For example, the mortality rate predicted by the Lee-Carter model for an 80-year-old Australian woman in 2011 is 0.034, while the CBD model predicted a mortality rate of 0.035 for the same individual. Also, using mortality rates predicted by the Lee-Carter model, the residual life-expectancy for an Australia woman aged 80 in 2011, is 10.23 while the CBD model gives a residual life-expectancy of 10.25 years, for the same individual.

This comparability between the forecasts given by the Lee-Carter and CBD models is similar to the results obtained when the models were applied to the English and Wales male data. The CBD model gave the highest error and ranked poorly for the goodness-of-fit (Dowd et al. 2010a), but its forecasts were comparable to the ones given by the Lee-Carter model. The explanation behind this could be that the CBD

model is known to perform well for older cohorts, and produces realistically volatile results for older age groups, resulting in robust forecasts (Chan, Li, and Li 2014).

In the current study, mortality rates were forecasted for women in Australia for a period of 1, 5, 10, 15, 20 and 50 years, using the CBD and Lee-Carter models. A cohort of 60–100 years was selected. Mortality rates obtained by both models showed a continued trend in declining mortality. The Lee-Carter model was more conservative in its prediction than the CBD model, estimating a steady fall in mortality rates from 0.004 in 2011 to a predicted 0.003 in 2021. With the CBD model, mortality rates declined from 0.0025 in 2011 to 0.00188 in 2021. Further forecasting indicated a further decline to 0.0014 for the Lee-Carter model and 0.0059 for the CBD model in 2061. Based on this forecast it is likely that the number of women over 60 years of age could increase by 25% by 2021.

Clearly, the forecasting results obtained in the current study indicate that in the long-term there is a decreasing trend in mortality rates and an increasing trend in life-expectancy for the Australian female population. These forecasts are consistent with the results obtained by other authors (for example, the studies of Li et al. (2009); Booth (2004); Cairns et al. (2011); Hollmann, Mulder and Kallan (1999); Debón, Montes and Puig (2008)).

As demonstrated by of Booth et al. (2006), the Lee-Carter model and its extensions have a tendency to underestimate errors for the younger ages (0–40) and overestimate them for higher age-groups (60–100). However, the life-expectancy calculated by the Lee-Carter model, which had a low RMSE (0.002) did not significantly differ from the life-expectancy calculated by the CBD model, in which the error was notably overestimated with RMSE of 0.02. Hence, the current study agrees with Booth's hypothesis that the level of error in the model does not translate into an erroneous estimation of life-expectancy (Booth et al. 2006).

The results obtained after applying the Lee-Carter and CBD models suggest that female mortality in Australia has been decreasing in the last decades and, looking at the mortality rates forecasts for the next 50 years, it can be concluded that this population factor will continue to decrease. One of the implications of this research for practice is that the implementation of a model such as the Lee-Carter model for the demographic information of a country can suggest population dynamics that allow

revising the formulations that base the models for pension and insurance funds. Under the assumption that insurance and pension companies are regionally concentrated, it would be interesting for future research purposes to conduct a study that differentiates the mortality rates by regions. This may suggest a differential risk per region and therefore higher or lower pension costs for different participants in the sector.

Even though some previous studies in the literature have proved that in general the more complex variants of the Lee-Carter are less prone to error, the best-fitted model for the Australian female data has proved to be the basic Lee-Carter model. Thus, the main implication for research is the suggestion that none of the models analysed performs well in all tests and no model clearly dominates the others. Furthermore, this research disclosed further research paths concerning mortality rate analysis in Australia, which may help in fostering research on this topic.

The contributions and implications of this research are logical to consider that mortality and life-expectancy in Australia will continue to improve gradually over time, due to a variety of factors, rather than experiencing a substantial abrupt improvement. Among these factors might be great medical advances, greater access to health, reductions in tobacco consumption, etc.

3.9 Conclusion

It has been estimated that, in the last 60 years, female life-expectancy in Australia has been increasing at a steady rate of 3 months every year (Oeppen and Vaupel 2002). The increasing trends in life-expectancies in general have led some demographers to assume that human lifespans could become unlimited (Tuljapurkar et al. 2000). Then again, others have argued that the declining mortality trends may level off, owing to factors such as obesity (Olshansky, Carnes and Désesquelles 2001), lifestyle deterioration (Loladze 2002) and environmental factors (Olshansky, Passaro, Hershow, Layden, Carnes, Brody, Hayflick, Butler, Allison and Ludwig 2005).

From the present study, based on the results of the RMSE for various look forward windows, it may be inferred that the Lee-Carter model and Plat model are more suited to fit a non–stratified model for ages 0 to 100 (See Tables 3.3–3.6). Results across all look back windows are comparable, hence for computational convenience the data

20 can be used to model mortality for ages 60 to 100 or 80 to 100, and this is an improvement over the model for all ages (See Tables 3.10 and 3.13).

The current study concurs with arguments in favour of long-term declining trends in mortality, as the degree of uncertainty with which these predictions are made, is not well defined. Although the selection of mortality models is based on statistical criteria, additional considerations such as flexibility and robustness of forecasting are features that need to be considered in choosing the best stochastic model for predicting human mortality.

Most studies in mortality modelling are calibrated for male populations (Dowd et al. 2010a). The current study is a focus on the female population, and continuing trends in higher survival rates of women would necessitate a review of the financial and healthcare systems. These departments would, in turn, need to remodel themselves to cope with higher numbers of single or widowed older women, solely dependent on state welfare and an over-extended social security system.

The following chapter dedicated to comprehensively examine the mortality of just the Australian male.

Chapter 4 Analysis of Australian Male Mortality

4.1 Introduction

Life-expectancy in Australia has changed significantly over the last 60 years for both males and females (Section 2.4.1). Over this period, male life-expectancy at birth has increased by more than ten years.

In this chapter we comprehensively examine the mortality of Australian male. Figure 4.1 below illustrates the evolution of male life-expectancy in Australia, Russia, Sweden, China, Israel, India, Libya, Sub-Saharan Africa, and the world between 1960 and 2014. There is an increase in worldwide male longevity from 52.4 years in 1960 to 71.4 years in 2014. In Australia, male life-expectancy increased from 70.8 years in 1960 to 85.2 in 2014. Similarly, there has been an increasing trend in male life-expectancy in countries mentioned above, over the last 54 years.

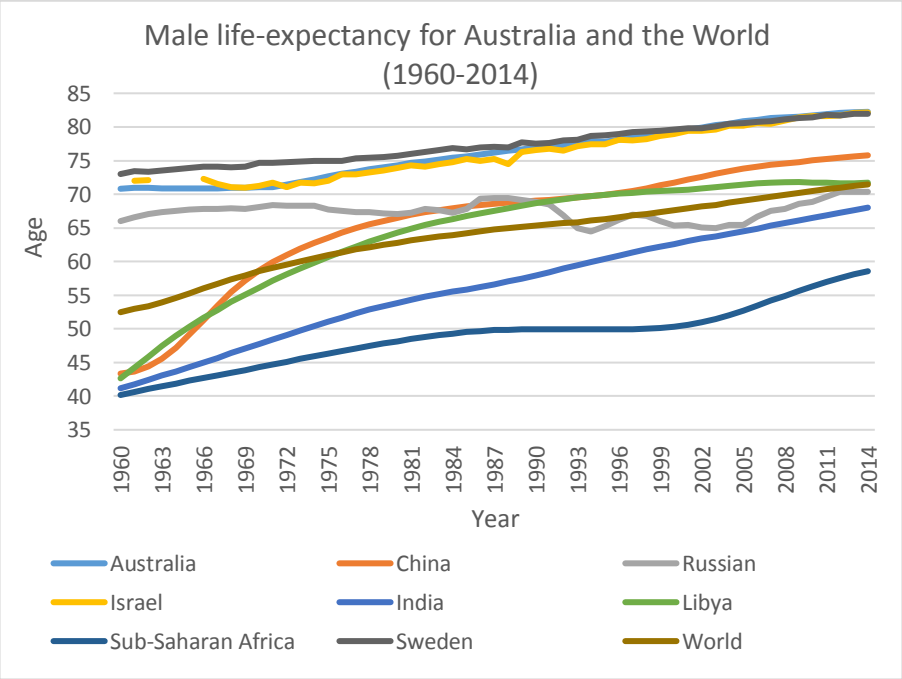

Figure 4.1 Male life-expectancy for Australia and the World - 1960 –2014.
Data source: World Development Indicators 2016.

4.2 Objectives and Research Methodology

The main objectives of this chapter are to compare the fit of the six stochastic mortality models discussed in section 2.4.2 and to identify the model that is best suited to model Australian male mortality rates and life-expectancy. The comparison is based on selection criteria encompassing goodness-of-fit and parsimony of the model.

The research methodology is the same as section 3.5 and is not repeated here. The modelling results for h=1, Data 20 for LC model (similar to Table 3.2) are presented in Table 4.1. A selection of the results is presented in Appendix B, Table B.6–Table B.10.

Table 4.1 The results of the goodness of fit criteria of data 20 and h=1 using LC model, Australian male.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	19462.72	6604.14	4094.06	8797.85	10839.06	4691.57	4293.72
BIC	20713.50	7084.71	4343.49	9278.42	11565.97	4941.00	4543.16
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.006699	0.000065	0.000196	0.010453	0.000179	0.000834	0.014366
RMSES0	____	0.000076	0.000321	0.010511	0.000198	0.000824	0.014664
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	19483.45	6658.46	4095.69	8804.60	10846.40	4700.25	4286.95
BIC	20734.23	7139.03	4345.12	9285.17	11573.31	4949.68	4536.38
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.005622	0.000106	0.000157	0.008657	0.000147	0.001092	0.010877
RMSES0	_____	0.000119	0.000247	0.008821	0.000174	0.001048	0.012290
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	19563.33	6743.80	4106.07	8793.98	10913.30	4706.38	4275.71
BIC	20814.11	7224.37	4355.50	9274.55	11640.21	4955.81	4525.14
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.008509	0.000087	0.000155	0.013326	0.000151	0.000693	0.018447
RMSES0	____	0.000085	0.000232	0.013354	0.000153	0.000751	0.018645

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	19656.72	6835.20	4084.38	8802.76	10955.66	4715.76	4270.26
BIC	20907.50	7315.76	4333.82	9283.33	11682.57	4965.19	4519.69
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.009254	0.000084	0.000282	0.014648	0.000168	0.000914	0.021227
RMSES0	_____	0.000089	0.000273	0.014522	0.000176	0.000907	0.020285
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	19677.04	6919.72	4066.80	8772.86	10968.01	4687.86	4256.10
BIC	20927.82	7400.28	4316.23	9253.43	11694.92	4937.29	4505.53
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.005184	0.000079	0.000322	0.008319	0.000237	0.001390	0.012537
RMSES0	_____	0.000084	0.000475	0.008130	0.000287	0.001321	0.011285

l: look back window 20–years

t: year of prediction

[*t–h– l*, *t–h*]: modelling time

Scenario 0 (S0): [0–100]

Scenario 1 (S1): S1A=[0–40], S1B=[40–60], S1C=[60–100]

Scenario 2 (S2): S2A=[0–60], S2B=[60–80], S2C=[80–100]

As mentioned in section 3.5, the fitting of male mortality data for some scenarios using R, package StMoMo was challenging. There was converge issues, largely due to the default choice of ARIMA (0, 0, 1). In some cases choice of ARIMA (1, 1, 0) model or other form resolved the issues. In other cases, especially with M7 and RH models, reasonable fits could not be obtained. These cases are then not included in subsequent comparisons.

4.3 Results

The following sections present the comparisons of the average RMSE values for the six mortality models for different look back windows, look forward windows and age stratification. Each section corresponds to each age stratification, namely S0, S1 and S2.

4.3.1 Comparison of average RMSE for different look back and look forward windows for Scenario S0

The following sections will present the comparisons of the average RMSE for the six models for different look back, look forward windows under non-stratified data (Scenario S0, ages: 0:100) using Australian males data.

First, we consider Data20, which is 20 years look back window. RMSE is computed for each of the six models (Lee-Carter, RH, APC, CBD, M7 and Plat) and five look forward windows (h=1, h=5, h=10, h=15 and h=20). The results are presented in Table 4.2.

Table 4.2 Average RMSE, Look back Window =20 years, Ages: 0–100, Australian male.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.007053**	**0.008151**	**0.016352**	**0.014889**	**0.018894**
RH	0.022115	0.100593	0.047853	0.036723	0.088449
APC	**0.007470**	0.012341	**0.015663**	**0.021075**	**0.028446**
CBD	0.027415	0.025969	**0.023423**	**0.015227**	**0.017157**
M7	0.024462	0.044227	0.067026	0.089023	0.102262
Plat	**0.006912**	**0.009109**	**0.014839**	**0.017061**	**0.022196**
P-value	0.0000	0.0002	0.0000	0.0000	0.0000

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported in the last row are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. Red highlight is the case with smallest RMSE.

As evident from the Table 4.2, Australian male's results are comparable to Australian female's results for the average root mean square error. The smallest average error for all models is given by the 1-year look forward window, while the highest average RMSE is given by the 20-year look forward window. The average root mean square error increases with an increase in the value of *h*.

A single factor analysis of variance (ANOVA) comparing average RMSE across models for *h*=1, indicate that average RMSE for models is not the same. Similar results are observed for value of *h*=5, 10, 15, 20.

LC and Plat models obtained the lowest RMSE values for different look forward windows. For *h*=1 and *h*=5, performance of LC and Plat are comparable, while for *h*=15 and *h*=20 performance of LC, Plat and CBD are comparable. Meanwhile, for *h*=10, performance of LC, Plat and APC are comparable.

Secondly Data30, which is 30 years look back window was investigated. Table 4.3 below is the results of average RMSE of the six stochastic models and five different look forward windows fitted under non-stratified data, ages 0–100.

Table 4.3 Average RMSE, Look back Window =30 years, Ages: 0–100, Australian male

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.007700**	**0.009098**	**0.013127**	**0.013841**	**0.017470**
RH	0.040990	0.059602	0.057581	0.099908	0.108796
APC	**0.009197**	**0.015932**	**0.021814**	0.023412	0.024006
CBD	0.027204	**0.025182**	**0.023241**	**0.012879**	**0.011172**
M7	0.028952	0.051942	0.078188	0.095815	0.105402
Plat	**0.007285**	**0.011408**	**0.019275**	**0.016604**	**0.016213**
P-value	0.0000	0.0000	0.0000	0.0000	0.0000

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported in the last row are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. Red highlight is the case with smallest RMSE.

As expected, the errors are higher for a higher look forward window. The smallest average error for all models is given by the 1-year look forward window while the highest average RMSE is given by the 20-year look forward window.

A one-way ANOVA for RMSE comparing average RMSE across models for each look forward window, indicate that average RMSE for models is not the same for each look forward window.

For different look forward windows LC model results in lowest RMSE values. The next comparable model is Plat Model for all look forward windows.

For h=5 and h=10, the performance of LC, Plat, CBD and APC are comparable, meanwhile, for h=15 and h=20, the performance of LC, Plat and APC are comparable. For h=1, performance for LC and Plat model are comparable.

Thirdly, we examine results for Data40. The RMSE is computed for each of the six stochastic models and five look forward windows (h=1, h=5, h=10, h=15 and h=20) under the non-stratified data ages: 0–100. The results are presented in Table 4.4

Table 4.4 Average RMSE, Look back window=40 years, ages: 0–100, Australian male.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.007782**	**0.009292**	**0.012441**	**0.013077**	**0.016728**
RH	0.017953	0.038867	**0.023918**	0.093303	**0.097766**
APC	**0.012939**	**0.019791**	**0.022754**	0.023863	**0.024411**
CBD	0.027151	**0.024134**	**0.018610**	**0.010296**	**0.013295**
M7	0.032966	0.059969	0.085493	0.100702	0.108059
Plat	**0.008856**	**0.013347**	**0.018047**	**0.015097**	**0.016097**
P-value	0.0000	0.0001	0.0000	0.0000	0.0054

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported in the last row are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. Red highlight is the case with smallest RMSE.

Table 4.4 above shows increasing error and decreasing goodness-of-fit with increasing values of 'h'. The minimum RMSE is 0.00778 given by the 1-year look forward window using LC model and the maximum RMSE is 0.1081 given by the 20-year look forward window using M7 model. As expected, the error is higher for a higher look forward window.

A one-way ANOVA for RMSE comparing average RMSE across models for each look forward window indicates that the average RMSE for the models is not the same for each look forward window.

For *h*=1, performance of LC, Plat and APC are comparable, while for *h*=10 and *h*=20, the performance of all the models are comparable except M7. Meanwhile, for *h*=15, the performance of LC, Plat and CBD are comparable, whilst for *h*=5 the performance of LC, Plat, APC and CBD are comparable.

Finally, Table 4.5 below shows the comparative RMSE for different look forward windows for the six mortality model, fitted using non–stratified data for the ages 0–100 years using look back window 50-years.

Table 4.5 Average RMSE, Look back window=50 years, ages: 0–100, Australian male.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.007800**	**0.008882**	**0.012038**	**0.013841**	**0.016641**
RH	0.013652	0.042924	**0.010439**	0.099908	11.743798
APC	0.016384	0.022114	0.024087	0.023412	**0.022876**
CBD	0.026768	**0.022547**	0.016738	**0.012879**	**0.017814**
M7	0.036162	0.066781	0.091420	0.095815	0.111108
Plat	**0.010978**	**0.014783**	0.018467	**0.016604**	**0.014390**
P-value	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported in the last row are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. Red highlight is the case with smallest.

As is evident from the Table 4.5, the average RMSE increases with an increase in the value of *h*. The smallest average error for all models is given by the 1–year look forward window, while the highest average RMSE is given by the 20–year look forward window.

A single factor analysis of variance (ANOVA) comparing average RMSE across models, shows that for each look forward windows, the average RMSE criterion varied significantly between all models (*p-value*<0.05).

For *h*=5 and *h*=15, performances of LC, Plat and CBD models are comparable, while for *h*=1, performances of LC and Plat are comparable. Meanwhile, for *h*=10, performances of LC and RH are comparable, whilst for *h*=20, performances of the six models are comparable.

To understand the sensitivity of length of data required for forwarding prediction, the best performing model is selected for each *h*, and the average RMSE is compared for each decade of data acquisition. The results are presented in Figure 4.2

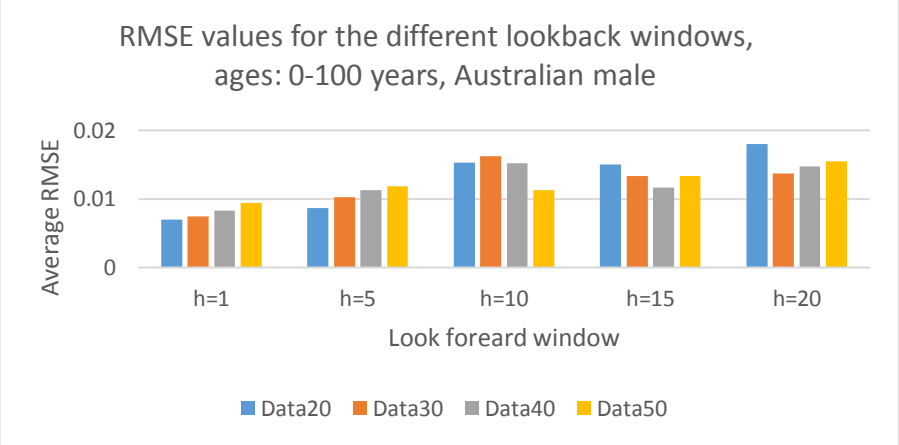

Figure 4.2 RMSE for the different look back and look forward windows, ages: 0–100 years, Australian male.

For each value of *h*, one-way ANOVA comparing average RMSE across 4 decades of look back window resulted in *P-value* greater than 0.05, indicating equality of mean. There is not much improvement in RMSE as additional 10 years of modelling data is used. Consequently, keeping data acquisition simple, 20 years of data can be safely used for modelling. This is consistent with the modelling of female mortality data.

The final summary of the best performing modelling method and look back window is presented in Table 4.6.

Table 4.6 Summary of the best of look back and look forward windows using mortality models, S0.

Forward prediction	Modelling method	Look back window
***h*=1**	LC and Plat	20 years or more
***h*=5**	LC and Plat	20 years or more
***h*=10**	LC	20 years or more
***h*=15**	LC, Plat and CBD	20 years or more
***h*=20**	LC, Plat and CBD	20 years or more

In the following sections we only focus on Data20 for parsimony and investigate if we can improve on the results here by fitting models stratified over the age.

4.3.2 Comparison of average RMSE for different look forward windows for stratified data (Scenario S1)

This section will present the comparisons of the average RMSE for the six models for different look forward windows with stratified data under second scenario (Scenario S1, ages: 0:40, 40:60, 60:100) using look back window 20-years only.

To begin with, the second scenario S1 with age-group 0–40. A summary of the average RMSE for the different age cohorts is illustrated in Table 4.7 below.

Table 4.7 Average RMSE, Look back window=20 years, ages: 0–40, Australian male.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.000084**	**0.000192**	**0.000293**	**0.000309**	**0.000358**
RH	**0.000084**	0.000556	0.005832	**0.000153**	**0.000362**
APC	**0.000080**	**0.000071**	**0.000112**	**0.000145**	**0.000210**
CBD	0.000703	0.000701	0.000729	0.000734	0.000731
M7	0.001120	0.003415	0.053159	0.976010	15.588462
Plat	**0.000077**	**0.000101**	**0.000238**	**0.000239**	**0.000192**
P-value	0.0000	0.0000	0.0000	0.0000	0.0000
Mean	0.0001	0.0001	0.0001	0.0001	0.0002
t-Statistics	–0.5200	–3.4524	–6.1107	–3.0209	–0.5051
P(T<=t)	0.3153	0.0130	0.0018	0.0196	0.3200

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. The Means, t-Statistics and P (T<=t) in the last three rows are from paired t-test for comparing RMSE from best performing model under S1 and S0 for corresponding ages. Red highlight is the case with smallest RMSE.

The model with the best fit for this dataset for short-term and long-term forecasting (h =1 and h=20) was the Plat model follow by APC Aand LC models (See Table 4.7).

One way ANOVA for RMSE comparing models shows that there is a significant variation in the goodness-of-fit by average RMSE within the five different look forward windows (*p-value*<0.05).

A student's t-test to compare the average RMSE and average RMSES0 values shows that comparable results are obtained for age strata S1A if we model age group 0–100 years or 0–40 for look forward window h=1 and 20. On the other hand, for look

forward windows *h*=5, 10, and 15 smaller RMSE are obtained if we model age groups 0–40 years instead of 0–100 years (*P*<0.01) (See Table 4.7)

Second, we consider age-group 40–60, using data 20. RMSE is computed for each of the six models (Lee-Carter, RH, APC, CBD, M7 and Plat) and five look forward windows (*h*=1, *h*=5, *h*=10, *h*=15 and *h*=20). The results are presented in Table 4.8

Table 4.8 Average RMSE, Look back window=20 years, ages: 40–60, Australian male.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.000223**	0.000418	0.000588	**0.000656**	**0.000673**
RH	**0.000427**	**0.000217**	**0.000347**	**0.001418**	**0.003401**
APC	**0.000197**	**0.000217**	**0.000246**	**0.000298**	**0.000522**
CBD	**0.000220**	0.000443	0.000588	**0.000597**	**0.000573**
M7	**0.000208**	0.000439	0.000546	**0.000475**	**0.000624**
Plat	**0.000207**	0.000396	0.000493	**0.000340**	**0.000576**
P-value	0.5594	0.0000	0.0000	0.1374	0.0001
Mean	0.0002	0.0002	0.0004	0.0005	0.0009
t-Statistics	–2.1123	0.2553	13.7764	6.6593	2.6636
P(T<=t)	0.0511	0.4055	0.0001	0.0013	0.0281

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. The Means, t-Statistics and P (T<=t) in the last three rows are from paired t-test for comparing RMSE from best performing model under S1 and S0 for corresponding ages. Red highlight is the case with smallest RMSE.

For *h*=1 and 15 forecasting performance of all modelling methods are comparable. For *h*=5, 10 and 20 the APC and RH comparable and best performing models.

A student's t-test to compare the average RMSE and average RMSES0 values shows that comparable results are obtained for age strata S1B if we model age group 0–100 years or 40–60 years for look forward windows *h*=1 and 5. On the other hand, for look forward windows of 10 or more years, smaller RMSE are obtained if we model age groups 40–60 years instead of 0–100 years *(P<0.01)* (See Table 4.8).

Finally, Table 4.9 below show the comparative RMSE for different look forward windows for the six mortality model, fitted using scenario S1 data for the ages 60–100 years.

Table 4.9 Average RMSE, Look back window=20 years, ages: 60–100, Australian male.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.011080**	**0.012699**	**0.025896**	**0.023120**	**0.029113**
RH	0.020474	0.142091	0.095944	0.086075	0.063505
APC	**0.011443**	**0.018429**	**0.025205**	**0.033550**	**0.043604**
CBD	0.018196	0.022285	**0.030235**	**0.029740**	**0.034221**
M7	**0.014901**	**0.014491**	**0.012285**	**0.018481**	**0.027721**
Plat	**0.010871**	**0.011178**	**0.011292**	**0.016169**	**0.020636**
P-value	0.2054	0.0000	0.0000	0.0008	0.0026
Mean	0.0108	0.0108	0.0233	0.0268	0.0348
t-Statistics	–0.0332	–0.4311	5.4475	2.6829	1.5863
P(T<=t)	0.4875	0.3443	0.0028	0.0275	0.0939

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. The Means, t-Statistics and P (T<=t) in the last three rows are from paired t-test for comparing RMSE from best performing model under S1 and S0 for corresponding ages. Red highlight is the case with smallest RMSE.

Table 4.9, clearly shows that the Plat, Lee-Carter and M7 models give the lowest average RMSE values and that the RH model gives the highest average RMSE values for all look forward windows. The modeling method with the best fit for this dataset for all look forward windows is the Plat model.

A one-way ANOVA for average RMSE across models shows that for look forward windows of 5 year or more, the average RMSE criterion varied significantly between all models (*p-value*<0.05).

A student's t-test to compare the average RMSE and average RMSES0 values shows that comparable results are obtained, for age strata S1C if we model age group 0–100 years or 60–100 years for *h*=1, 5, 15 and 20. On the other hand, for look forward windows *h*=10 smaller RMSE are obtained if we model age groups 60–100 years instead of 0–100 years (*P<0.01)* (See Table 4.9)

The following is a comparison of the average RMSE values for the six models for all age cohort groups of the third scenario for different look forward windows.

4.3.3 Comparison of average RMSE for different look forward windows for stratified data (Scenario S2)

Further stratification of data in scenario (S2) for stratified age data is (Scenario S2: A=0–60, B=60–80, C=80–100). Tables 4.10–4.12 are the summary of the goodness-of-fit in terms of RMSE between different look forward windows for the six different models using Data 20-years under the third scenario S2.

Table 4.10 Average RMSE, Look back window=20 years, ages: 0–60, Australian male.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.000176**	**0.000318**	**0.000417**	**0.000461**	**0.000521**
RH	0.000506	15770	225644861	0.017645	0.003554
APC	**0.000135**	**0.000165**	**0.000243**	**0.000354**	**0.000549**
CBD	0.000711	0.000774	0.000869	0.000866	0.000761
M7	0.000896	0.001212	0.001640	0.002207	0.005382
Plat	**0.000126**	**0.000138**	**0.000262**	**0.000394**	**0.000599**
P-value	0.0051	0.0185	0.4154	0.4629	0.0001
Mean	0.0001	0.0001	0.0003	0.0004	0.0007
t-Statistics	0.2647	0.1293	0.0348	1.8581	2.1248
P(T<=t)	0.4022	0.4517	0.4869	0.0683	0.0504

Table 4.11 Average RMSE, Look back window=20 years, ages: 60–80, Australian male.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.000985**	**0.001005**	**0.002327**	**0.004492**	0.006251
RH	0.001030	0.008218	0.002669	0.005529	**0.003891**
APC	0.001010	0.001555	**0.001407**	**0.001589**	**0.003149**
CBD	0.001217	**0.000998**	**0.002347**	**0.004302**	0.006313
M7	**0.000960**	**0.000998**	0.002821	**0.004513**	0.006533
Plat	**0.000978**	**0.001026**	**0.002483**	**0.004503**	0.006918
P-value	0.3881	0.0238	0.4405	0.0004	0.0013
Mean	0.0009	0.0014	0.0011	0.0011	0.0022
t-Statistics	–2.1483	–3.8087	–2.7619	–6.2329	–1.9425
P(T<=t)	0.0491	0.0095	0.0254	0.0017	0.0620

Table 4.12 Average RMSE, Look back window=20 years, ages: 80–100, Australian male.

	Look Forward Window				
Model	***h*=1**	***h*=5**	***h*=10**	***h*=15**	***h*=20**
LC	**0.015491**	**0.017684**	0.034903	**0.028538**	**0.036265**
RH	0.046805	0.212568	0.241935	0.161524	0.150989
APC	**0.015246**	**0.020968**	**0.031371**	0.044432	0.049402
CBD	0.028128	0.038309	0.041721	**0.031483**	0.040538
M7	**0.013835**	**0.017871**	**0.023784**	**0.031847**	0.045663
Plat	**0.015143**	**0.016434**	**0.017029**	**0.033037**	**0.036372**
P-value	0.0002	0.0000	0.0000	0.0003	0.0012
Mean	0.0151	0.0199	0.0325	0.0374	0.0485
t-Statistics	–0.0248	1.2459	5.4467	0.4116	1.6064
P(T<=t)	0.4907	0.1404	0.0028	0.3509	0.0917

Each cell in the table presents average RMSE for prediction years 2007–2011.
P-values reported are from one-way ANOVA model followed Tukey's HSD pairwise comparisons where appropriate. Groups with lowest and similar average RMSE are presented in **bold**. The Means, t-Statistics and P (T<=t) in the last three rows are from paired t-test for comparing RMSE from best performing model under S1 and S0 for corresponding ages. Red highlight is the case with smallest RMSE.

Table 4.10–Table 4.12 above show clearly for both short-term (*h*=1, 5) and long-term (*h*=15, 20) forecasting performance of LC and Plat modelling methods are comparable. For *h*=10, APC, M7 and Plat are comparable and best performing models. The model with the poorest fit for this age cohort was the RH model for almost all look forward windows.

A student's t-test to compare the average RMSE and average RMSES0 values shows that comparable results should be obtained if we model age group 0–100 years than 0–60 years for all *h* value**s.** On the other hand, for look forward windows *h*=5 and 15 smaller average RMSE are obtained if we model age groups 60–80 years instead of 0–100 years; likewise, for *h*=10, age groups 80–100 years (*P<0.01)* (See Tables 4.10–4.12).

4.3.4 Comparison of average AIC and BIC for different scenarios

The purpose of this section of this section is to illustrate that the best performing models on RMSE are also good performing models using the AIC and BIC criterion.

The comparisons between the average AIC and BIC values for 15-year look forward window using look back window 20-years are illustrated in Figure 4.3–4.9

below. The comparison shows that the best fitted models for scenario S0 (0–100 years, non–stratified data) are (Plat, LC, and APC) also have lower AIC and BIC values.

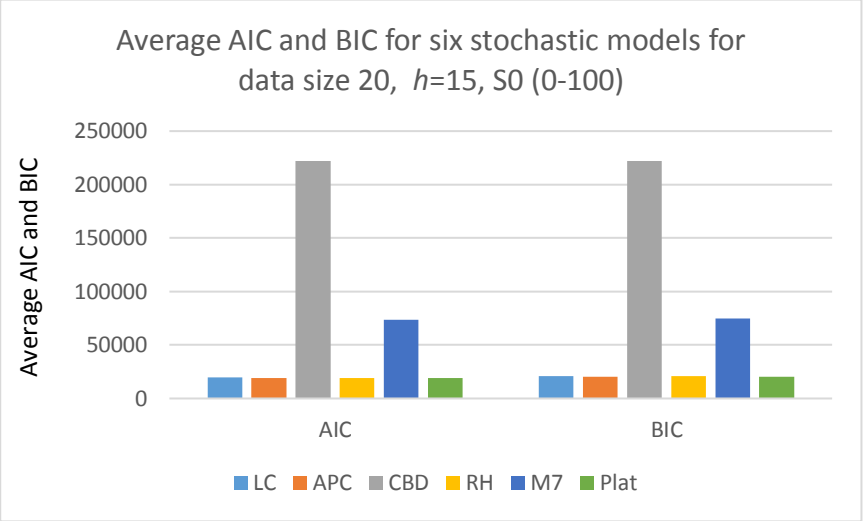

Figure 4.3 A comparative assessment of the six mortality models with respect to S0 for ages 0–100, h=15, Data 20.

Figure 4.4 below shows that once again for scenario S1A (0–40 years) the best fitted model according to RMSE (Plat, APC, RH and LC) are also the models with lower AIC and BIC values.

For the same scenario, stratified data 40–60 years, the Renshaw-Haberman and the APC models give the best results according to the AIC and RMSE criteria, while according to the BIC criterion the Lee-Carter models and APC are the best fitted model for this dataset (Figure 4.5 below). However, AIC and BIC values for best performing models for RMSE (all modelling methods) are comparable to the best scenario.

Figure 4.6 below, presents AIC and BIC values for S1C (60–100 years). Once again, the best performing models for RMSE (Plat, LC and APC) have lower AIC and BIC values.

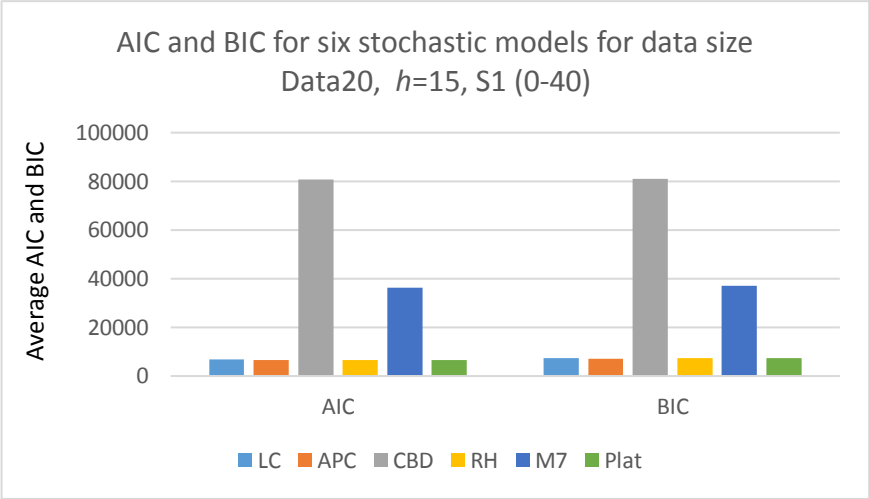

Figure 4.4 **A comparative assessment of the six mortality models with respect to S1 for ages 0–40, h=15, Data 20.**

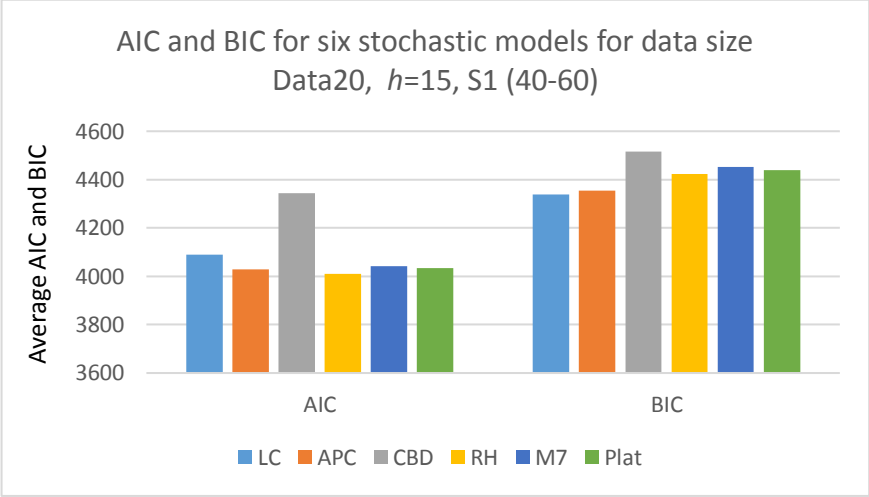

Figure 4.5 **A comparative assessment of the six mortality models with respect to S1 for ages 40–60, h=15, Data 20.**

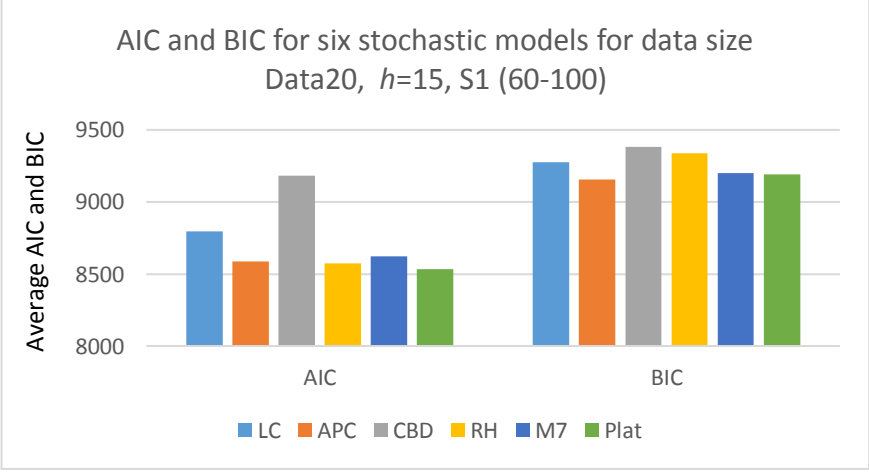

Figure 4.6 **A comparative assessment of the six mortality models with respect to S1 for ages 60–100, h=15, Data 20.**

For scenario S2A (0–60 years), data 20, 15–year look forward window, the results show that according to the AIC criterion the best fitted models are the Plat and the RH models. The BIC and RMSE criteria give the Plat and the APC models as the best models to estimate mortality and life-expectancy rates for this dataset. The M7, CBD and the APC models are the best fitted models for scenario S2B (60–80 years) according to the AIC and the BIC criterion, while according to RMSE the M7 and Plat models are the best. Finally, for scenario S2C (80–100 years) the AIC criterion gives the APC and M7 models as the best fitted models, the BIC criterion gives the Lee-Carter and the APC models, while the minimum RMSE values are given by the Plat and the M7 models (Figure 4.7–4.9 below).

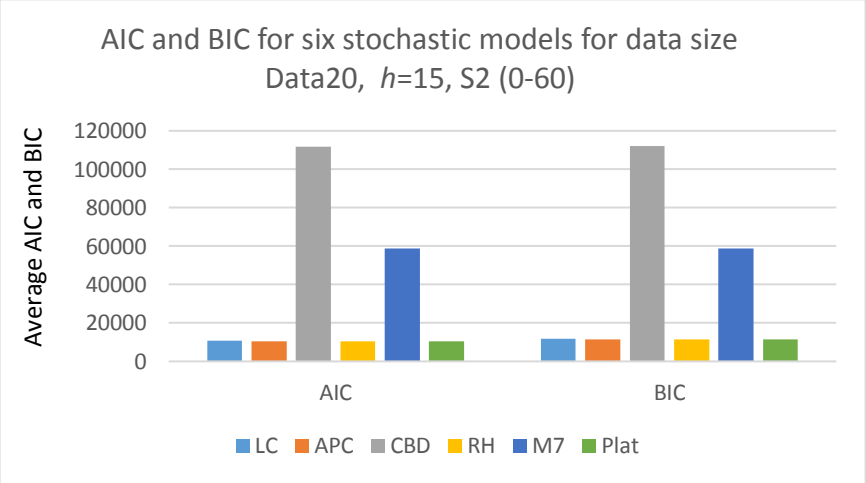

Figure 4.7 A comparative assessment of the six mortality models with respect to S2 for ages 0–60, h=15, Data 20.

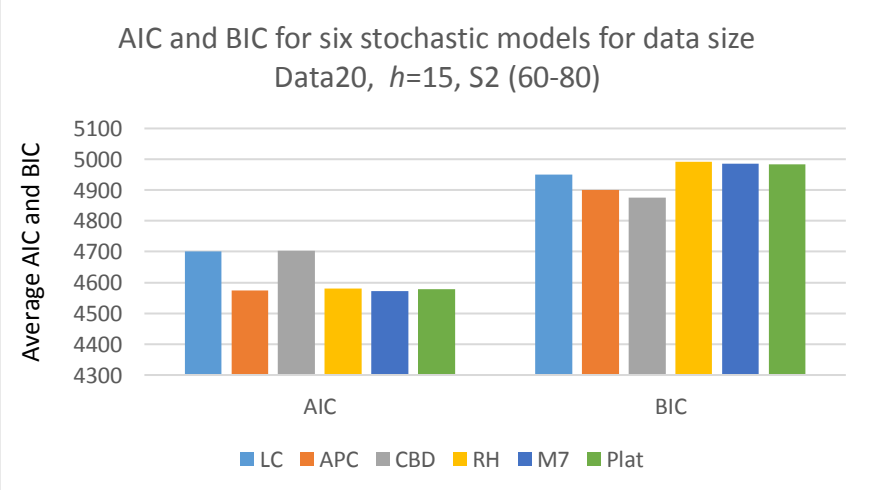

Figure 4.8 A comparative assessment of the six mortality models with respect to S2 for ages 60–80, h=15, Data 2.

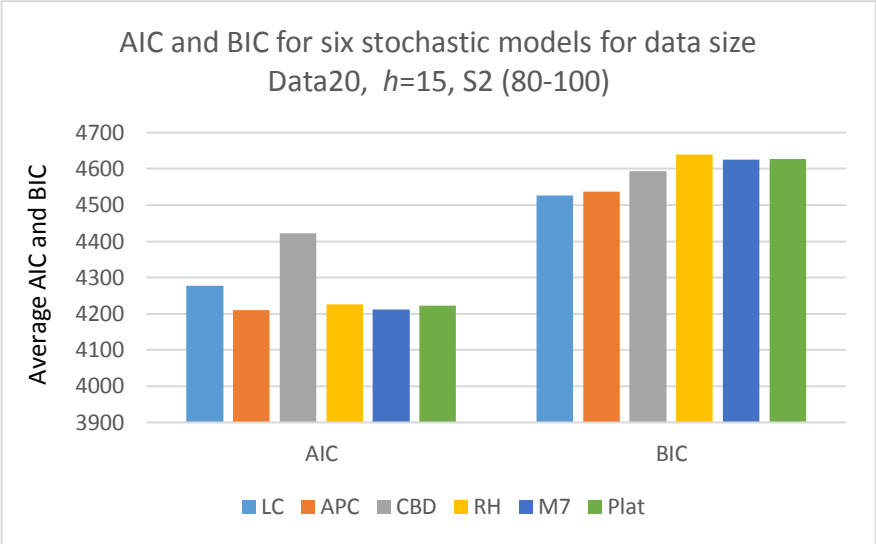

Figure 4.9 A comparative assessment of the six mortality models with respect to S2 for ages 80–100, h=15, Data 20.

Overall, the best performing models for RMSE (LC and Plat) have lower AIC and BIC values. This is consistent with the modelling of female mortality data.

4.3.5 Randomness of residuals

The fourth criterion to be satisfied for model selection is the randomness of residuals. The scatter plot of the residuals for the six models shows they are disordered and do not show any pattern, which means they are randomly distributed.

Figure 4.10 shows that the distribution of residuals for the Lee-Carter model is random and indicates deterioration in mortality (red cells). The decline in mortality is seen across all ages represented in the model.

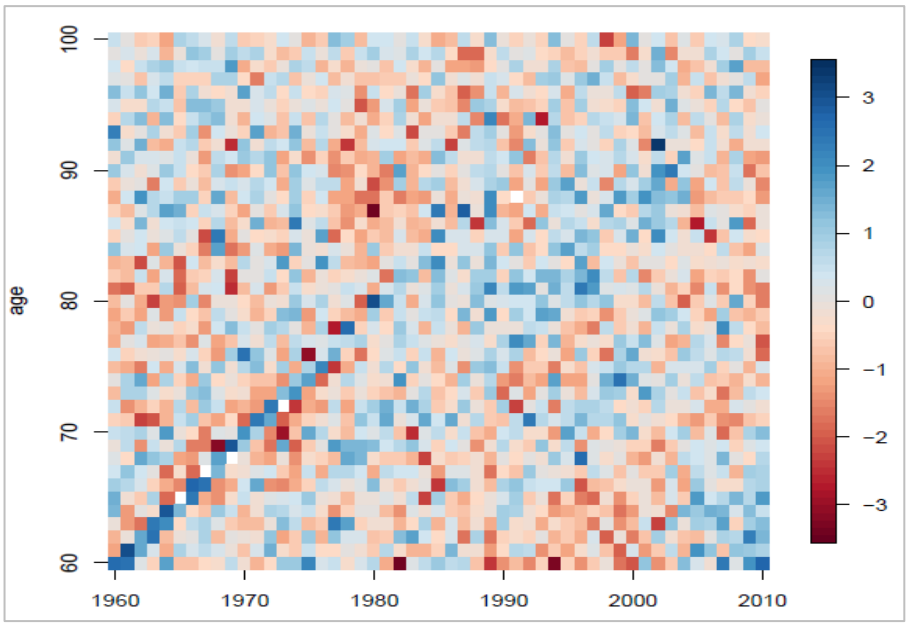

Figure 4.10 Residual plot for the Lee-Carter model.

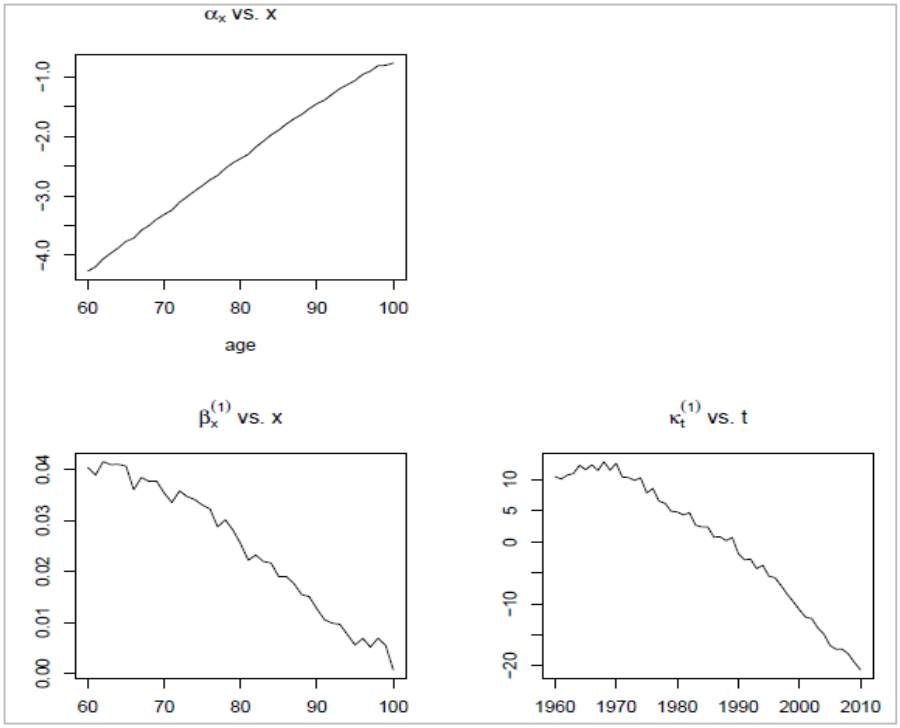

Figure 4.11 Parameter estimation for Lee-Carter model.

Figure 4.11 above shows the plots of the estimated age-coefficients α_x and β_x reflecting age-related effects, and the mortality index k_t, reflecting time-related effects. Typical for the Lee-Carter model, these coefficients follow exponential and linear trends, with:

- coefficient α_x independent of time;
- coefficient β_x reflecting how rapidly or slowly mortality at each age changes when the mortality rate changes and
- Coefficient k_t showing the general level of mortality rate.
- No issues with residuals was noted for best performing cases of RMSE.

4.3.6 Summary of results

Based on the analysis performed in the previous sections of this chapter, the following assumptions can be made about the model selection.

According to the four criteria for model selection, the Lee-Carter model is the best-fitted model to the dataset (Data 20, age cohort 60–100). In addition to the lowest error (RMSE) across most of the look forward windows, low values across age-based stratification also had the lowest BIC values and low AIC values. Results showed there was no significant difference in the goodness-of-fit for the Australian male mortality data, between the different data sizes (Data 20 to Data 50) for the six stochastic models studied. Furthermore, a significant difference was detected in all the quantitative criteria across the look back windows (or the length of the forecast, *h*=1, *h*=5, *h*=10, *h*=15).

In conclusion, the Lee-Carter and Plat models are the best model for estimating male mortality rates and life-expectancy. The CBD and M7 models had the highest values for the RMSE as well as BIC and AIC, suggesting a poor selection for this dataset.

4.4 Calculating mortality

The estimated mortality rates and number of deaths from the Lee-Carter model and CBD model are tabulated in Table 4.13. Calculated life-expectancy using mortality rates from both models and population data from life-tables (www.mortality.org) is presented in Table 4.14.

Table 4.13 **Mortality rate $[m_{(x,t)}]$ and number of deaths $[D_{(x,t)}]$ computed by the CBD and Lee-Carter Models and original Australian male data for year 2011, ages 60–100.**

	Mortality rate $[m_{(x,t)}]$			Number of deaths $[D_{(x,t)}]$		
Age	**CBD**	**LC**	**original**	**CBD**	**LC**	**original**
60	0.005302	0.00608	0.00678	658.7804	755	619
61	0.005972	0.00674	0.00762	728.1093	822	691
62	0.006726	0.007253	0.00793	808.7256	872	714
63	0.007576	0.008104	0.00882	919.252	983	787
64	0.008533	0.008891	0.00984	1016.752	1059	870
65	0.00961	0.009938	0.01031	1051.369	1087	902
66	0.010824	0.011489	0.01171	1091.423	1158	1014
67	0.012192	0.012492	0.01293	1155.029	1184	1105
68	0.013732	0.013686	0.01366	1213.294	1209	1152
69	0.015466	0.015182	0.01503	1295.743	1272	1250
70	0.01742	0.017236	0.01644	1385.673	1371	1346
71	0.01962	0.019241	0.01879	1479.44	1451	1511
72	0.022098	0.021123	0.02172	1580.31	1511	1712
73	0.02489	0.023582	0.02306	1678.661	1590	1777
74	0.028034	0.026437	0.02716	1782.382	1681	2042
75	0.031575	0.029464	0.02909	1866.944	1742	2126
76	0.035563	0.033197	0.03372	1944.829	1815	2388
77	0.040055	0.038407	0.03881	2050.864	1966	2651
78	0.045115	0.041834	0.04109	2181.727	2023	2697
79	0.050814	0.048111	0.04808	2343.998	2219	3018
80	0.057232	0.054856	0.05327	2535.562	2430	3179
81	0.064462	0.063066	0.06018	2689.656	2631	3393
82	0.072604	0.069903	0.06828	2781.616	2678	3611
83	0.081775	0.079087	0.07754	2839.255	2746	3813
84	0.092105	0.088398	0.0871	2861.583	2746	3945
85	0.103739	0.101103	0.0979	2838.844	2767	4043
86	0.116843	0.111997	0.11403	2776.782	2662	4237
87	0.131602	0.125655	0.11935	2647.618	2528	3946
88	0.148225	0.14093	0.13756	2494.539	2372	4002
89	0.166949	0.159452	0.161	2311.432	2208	4037
90	0.188037	0.182115	0.17951	2065.738	2001	3798
91	0.211789	0.202353	0.2063	1698.029	1622	3601
92	0.238541	0.225155	0.21326	1319.9	1246	3017
93	0.268672	0.248302	0.2464	1106.772	1023	2773
94	0.30261	0.274512	0.28028	938.2485	851	2425

	Mortality rate [$m_{(x,t)}$]			Number of deaths [$D_{(x,t)}$]		
Age	**CBD**	**LC**	**original**	**CBD**	**LC**	**original**
95	0.340834	0.307899	0.30273	739.8596	668	1956
96	0.383887	0.326042	0.33251	576.111	489	1564
97	0.432378	0.378175	0.36369	425.3694	372	1207
98	0.486994	0.383811	0.39606	300.4901	237	897
99	0.548509	0.388653	0.42937	196.6516	139	642
100	0.617795	0.467283	0.46333	123.1389	93	442

Table 4.14 Calculation of life-expectancy of Australian male from modelled mortality rates for year 2011, ages 60–100.

	Number of survivors [l_x]				Life-expectancy [$e_{(x,t)}$]		
Age	**CBD**	**LC**	**(Original)**	**T_x (original)**	**CBD**	**LC**	**(Original)**
60	90972.22	90876	91631	2132822	23.444762	23.4697	23.28
61	90283.891	90190	91012	2041501	22.612018	22.635519	22.43
62	89512.274	89449	90321	1950834	21.794039	21.809479	21.6
63	88687.748	88624	89607	1860870	20.982267	20.997437	20.77
64	87804.248	87762	88821	1771656	20.177338	20.187165	19.95
65	86899.631	86864	87951	1683270	19.370278	19.378278	19.14
66	85956.577	85890	87048	1595771	18.564851	18.579348	18.33
67	84879.971	84851	86035	1509229	17.780744	17.786711	17.54
68	83716.706	83721	84930	1423747	17.006725	17.005907	16.76
69	82481.257	82505	83777	1339393	16.238756	16.234071	15.99
70	81142.327	81157	82528	1256241	15.481944	15.479154	15.22
71	79702.56	79731	81182	1174386	14.734608	14.729323	14.47
72	78089.69	78159	79670	1093960	14.00902	13.996515	13.73
73	76280.339	76369	77959	1015146	13.308095	13.29273	13.02
74	74399.618	74501	76182	938076	12.608613	12.59143	12.31
75	72273.056	72398	74140	862915	11.93965	11.919068	11.64
76	70069.171	70199	72014	789838	11.272261	11.251483	10.97
77	67575.136	67660	69626	719018	10.640275	10.627006	10.33
78	64794.273	64953	66976	650717	10.042817	10.018281	9.72
79	61935.002	62060	64279	585089	9.446823	9.4278435	9.1
80	58725.438	58831	61261	522319	8.8942547	8.8783371	8.53
81	55392.344	55451	58082	462648	8.3522012	8.3434301	7.97
82	51907.384	52011	54689	406262	7.8266707	7.8110962	7.43
83	48238.745	48332	51078	353379	7.3256259	7.31148	6.92
84	44403.417	44519	47265	304207	6.8509818	6.8332573	6.44
85	40481.156	40553	43320	258915	6.3959389	6.3845626	5.98

	Number of survivors [l_x]				Life-expectancy [$e_{(x,t)}$]		
Age	**CBD**	**LC**	**(Original)**	**T_x (original)**	**CBD**	**LC**	**(Original)**
86	36500.218	36615	39277	217616	5.9620465	5.9432947	5.54
87	32392.382	32512	35040	180457	5.5709703	5.5504703	5.15
88	28599.461	28722	31094	147390	5.1535936	5.1315652	4.74
89	24780.568	24884	27092	118298	4.7738212	4.7539111	4.37
90	20989.262	21054	23055	93225	4.4415568	4.4278324	4.04
91	17558.971	17635	19257	72069	4.1043977	4.0867903	3.74
92	14336.1	14410	15656	54612	3.8094042	3.7898237	3.49
93	11532.228	11616	12639	40465	3.5088623	3.483514	3.2
94	8927.7515	9015	9866	29212	3.2720445	3.2404236	2.96
95	6701.1404	6773	7441	20558	3.067836	3.035451	2.76
96	4907.889	4995	5484	14096	2.8721106	2.8221923	2.57
97	3495.6306	3549	3921	9393	2.6870688	2.6466945	2.4
98	2413.5099	2477	2714	6076	2.5174954	2.4527921	2.24
99	1620.3484	1678	1817	3810	2.3513462	2.2710203	2.1
100	1051.8611	1082	1175	2314	2.1999102	2.1389066	1.97

Table 4.15 Mortality forecasts for forward look windows *h*=1, *h*=5 and *h*=10.

	2012		2016		2021	
Age	**LC**	**CBD**	**LC**	**CBD**	**LC**	**CBD**
60	0.005932	0.005124	0.005374	0.004565	0.00475	0.003951
61	0.006581	0.005771	0.005979	0.005155	0.005304	0.004476
62	0.007069	0.006499	0.00638	0.005821	0.005612	0.005071
63	0.007902	0.007319	0.007145	0.006572	0.006299	0.005744
64	0.008671	0.008241	0.007842	0.00742	0.006917	0.006506
65	0.009693	0.009278	0.00877	0.008376	0.007738	0.007369
66	0.011236	0.010445	0.010277	0.009453	0.009192	0.008344
67	0.012199	0.011756	0.011092	0.010669	0.009849	0.009448
68	0.013369	0.01323	0.012174	0.012038	0.010829	0.010697
69	0.01483	0.014886	0.013504	0.013581	0.012012	0.012108
70	0.016859	0.016745	0.015432	0.015319	0.013818	0.013703
71	0.018842	0.018832	0.017329	0.017275	0.015607	0.015505
72	0.02066	0.021174	0.018909	0.019475	0.016928	0.017539
73	0.023076	0.0238	0.021159	0.02195	0.018985	0.019835
74	0.025883	0.026742	0.023782	0.024732	0.021394	0.022425
75	0.028864	0.030037	0.026582	0.027856	0.023982	0.025344
76	0.03254	0.033725	0.030038	0.031361	0.027179	0.028632
77	0.037729	0.037847	0.035132	0.035292	0.032136	0.032332

	2012		2016		2021	
Age	**LC**	**CBD**	**LC**	**CBD**	**LC**	**CBD**
78	0.041054	0.042451	0.038078	0.039696	0.034659	0.036493
79	0.047279	0.047587	0.044093	0.044623	0.040412	0.041166
80	0.054001	0.05331	0.050713	0.050131	0.046883	0.046409
81	0.062213	0.059678	0.058913	0.056278	0.055034	0.052283
82	0.068913	0.066754	0.065091	0.063128	0.060611	0.058855
83	0.078026	0.074601	0.073923	0.07075	0.069097	0.066195
84	0.08723	0.083289	0.082712	0.079215	0.077393	0.074378
85	0.099923	0.092887	0.095341	0.088595	0.089907	0.083483
86	0.110689	0.103466	0.105607	0.098967	0.099581	0.093589
87	0.124287	0.115097	0.118962	0.110406	0.112625	0.104779
88	0.139569	0.12785	0.134253	0.122987	0.127891	0.117133
89	0.15803	0.141789	0.152464	0.136781	0.145782	0.130732
90	0.180758	0.156974	0.175432	0.151854	0.168994	0.145649
91	0.201095	0.173456	0.196138	0.168265	0.190114	0.161951
92	0.223848	0.191277	0.218697	0.18606	0.212425	0.179694
93	0.246884	0.210462	0.241292	0.205272	0.234481	0.198919
94	0.273167	0.231022	0.267852	0.225918	0.261353	0.219651
95	0.306824	0.252947	0.302562	0.247992	0.297317	0.241891
96	0.324435	0.276205	0.318085	0.271467	0.310322	0.265616
97	0.377464	0.300741	0.374635	0.296288	0.371127	0.290775
98	0.38211	0.326474	0.375377	0.322374	0.367129	0.317288
99	0.386938	0.353295	0.380153	0.349616	0.371838	0.345043
100	0.467674	0.381074	0.469243	0.377877	0.471211	0.373895

Table 4.16 Mortality forecasts of Australian male for forward look windows *h*=15, *h*=20 and *h*=50.

	2026		2031		2061	
Age	**LC**	**CBD**	**LC**	**CBD**	**LC**	**CBD**
60	0.003251	0.003419	0.002731	0.002958	0.003157	0.001241
61	0.00367	0.003886	0.003099	0.003373	0.003567	0.001442
62	0.003785	0.004417	0.003158	0.003847	0.003671	0.001677
63	0.004278	0.00502	0.003582	0.004386	0.004152	0.00195
64	0.004704	0.005704	0.003941	0.005001	0.004565	0.002267
65	0.00527	0.006482	0.004417	0.005701	0.005115	0.002636
66	0.006526	0.007365	0.005576	0.006499	0.006355	0.003064
67	0.006837	0.008366	0.005781	0.007407	0.006646	0.003561
68	0.007558	0.009503	0.006407	0.008442	0.00735	0.004139

	2026		2031		2061	
Age	**LC**	**CBD**	**LC**	**CBD**	**LC**	**CBD**
69	0.008383	0.010793	0.007106	0.00962	0.008152	0.004811
70	0.009841	0.012256	0.00842	0.01096	0.009585	0.005591
71	0.011316	0.013914	0.009763	0.012484	0.011037	0.006496
72	0.012049	0.015792	0.010307	0.014217	0.011735	0.007547
73	0.013608	0.01792	0.011678	0.016187	0.01326	0.008766
74	0.015458	0.020329	0.013314	0.018425	0.015073	0.01018
75	0.017482	0.023054	0.01512	0.020966	0.017058	0.011819
76	0.019991	0.026134	0.01736	0.023848	0.01952	0.013719
77	0.02444	0.029613	0.021552	0.027116	0.023926	0.01592
78	0.025962	0.033539	0.022735	0.030817	0.025386	0.018467
79	0.030917	0.037966	0.027339	0.035006	0.030281	0.021412
80	0.036836	0.042951	0.032974	0.03974	0.036153	0.024815
81	0.044645	0.048558	0.040555	0.045085	0.043926	0.028744
82	0.048689	0.054854	0.044029	0.05111	0.047868	0.033273
83	0.056157	0.061914	0.051055	0.057892	0.05526	0.038487
84	0.0631	0.069815	0.057452	0.065511	0.062108	0.044481
85	0.07508	0.07864	0.069115	0.074055	0.074036	0.051359
86	0.083142	0.088474	0.076529	0.083613	0.081985	0.059234
87	0.095197	0.099406	0.088122	0.094279	0.093962	0.068229
88	0.110178	0.111523	0.102885	0.106148	0.108909	0.078477
89	0.127037	0.124912	0.119254	0.119315	0.125686	0.090115
90	0.150663	0.139656	0.142922	0.13387	0.149325	0.103286
91	0.172746	0.15583	0.165309	0.149898	0.171466	0.118131
92	0.194265	0.173499	0.186452	0.167475	0.192921	0.13479
93	0.21474	0.192715	0.206238	0.18666	0.213278	0.153389
94	0.242364	0.21351	0.234111	0.207495	0.240949	0.174038
95	0.28177	0.235892	0.274905	0.229997	0.280598	0.196822
96	0.287645	0.259846	0.277791	0.254158	0.285955	0.221786
97	0.360559	0.285323	0.355806	0.279934	0.359751	0.248936
98	0.342912	0.312245	0.33233	0.307247	0.3411	0.278221
99	0.347422	0.340497	0.33675	0.335981	0.345594	0.309531
100	0.477306	0.369931	0.480133	0.365984	0.477783	0.342692

Tables 4.15–4.16 contain forecasted mortality rates for Australian males over different values of look forward windows (h=1, h=5, h=10, h=15, h=20 and h=50).

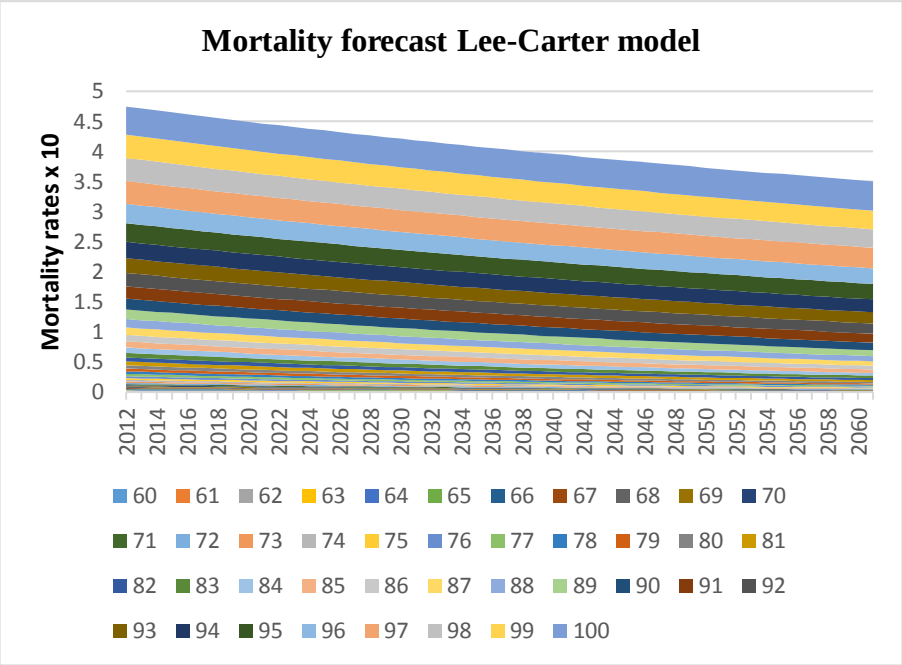

Figure 4.12 Mortality forecast - Lee-Carter model.

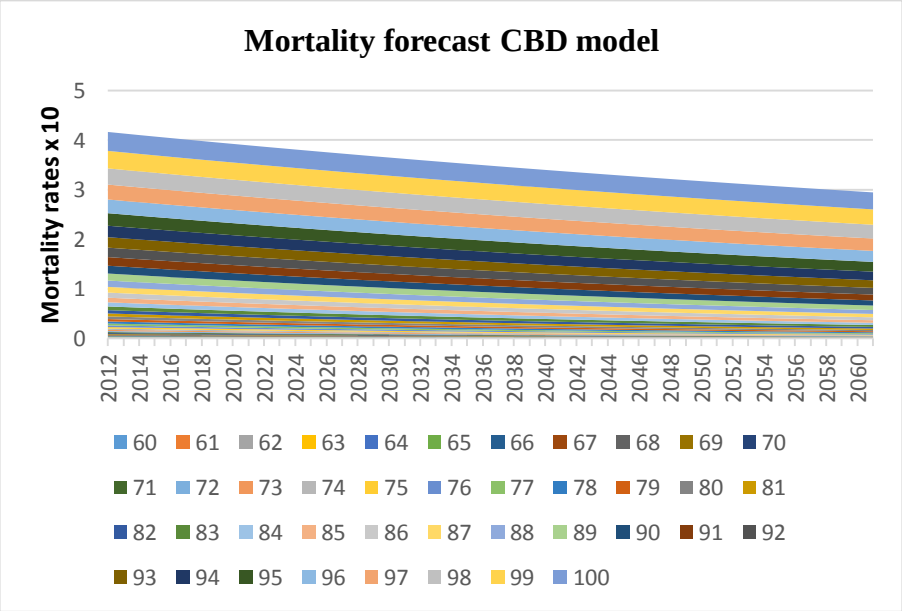

Figure 4.13 Mortality forecast - CBD model.

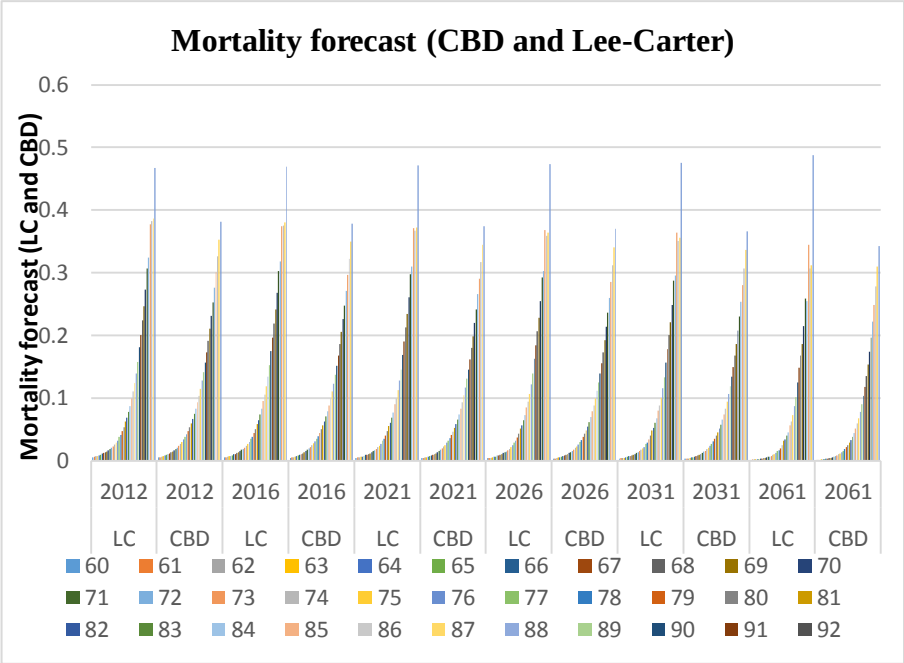

Figure 4.14 Comparative mortality forecasts by the Lee-Carter and CBD models.

The above graphs (Figures 4.12 to 4.14) are graphical representation of the mortality rates predicted by the Lee-Carter and CBD models, illustrating a predicted decline in mortality rates over a 50 - year period. The 50–year data from 1961–2011 for the Australian male population aged 60–100 was fitted into the models to predict male mortality for this cohort with a forecasting period of 1 year (2012), 5 years (2016), 10 years (2021) 15 years, (2026), 20 years (2031) and fifty years (2061). Based on current population figures, the residual life-expectancy for 65-year-old Australian men in 2011 was 19.14 years. Using mortality rates predicted by the Lee-Carter and the CBD models, the residual life-expectancy for an Australia man aged 65 in 2011, was 19.37 years.

As seen in the previous chapter analysing female mortality and life-expectancy, the Lee-Carter model had a similar estimate as the CBD model. The t-test ($P > 0.05$) indicated that the statistical goodness-of-fit did not significantly impact on the forecasting of life-expectancy, for this dataset, for ages 60–100 years.

4.5 Discussion

As mentioned in section 3.8, the development of stochastic mortality models has led to many changes in the analysis of population evolution, supporting new hypotheses, formulations and tools to generate conclusions about topics related to mortality

projections. The present study evaluates the goodness of fit of six stochastic mortality models using Australian male mortality data for a period of 50 years, between 1961–2011. Starting with the Lee-Carter model, and continuing with five of its extensions, the current research has addressed the issue of male mortality and life-expectancy forecasting in Australia. Due to the limited number of studies on the topic of male mortality forecasting in Australia, the current study can be considered a great contribution to the existing literature. The results obtained, in combination with others, contribute to the study of the Australian demographic component and its implications for the development of the country.

The objective of understanding the dynamics of mortality, based on the demographic perspective used to be based only on linear studies using age and known information, which generated over-estimates and inaccurate information. Such results did not allow for a very thorough analysis in terms of insurance and pensions to be performed. Studies of this type began with deterministic models such as that of Gompertz in 1825 (Gompertz 1825) which presents a satisfactory estimate of mortality. However, the main disadvantage of the model is that for ages greater than 80 years, the mortality indicator is overestimated (Koissi and Shapiro 2008).

In 1980, Heligman and Pollard (1980) in their article called "*The age pattern of mortality*" continued developing the subject of dynamic mortality models. Starting from interpolations, Helligman and Pollard took initial steps toward the objective of mortality projection in order to structure the actuarial studies that sustain the insurance and pension business. More recently, the attention of researchers has focused mainly on the stochastic model presented by Lee and Carter. Since first presented by the two authors in 1992 in their article called "*Modeling and forecasting US mortality*", this model has been used for demographic and actuarial applications around the world.

The popularity of the Lee-Carter model and its extensions, as well as their simplicity of use and understanding, represent the main reasons for choosing these methodologies. Thus, the analysis performed in this chapter involved applying the Lee-Carter model and five of its extensions (RH, APC, CBD, M7 and Plat) on Australian male mortality data with the purpose of choosing the best-fitted model to predict male mortality in Australia. Based on a comparison between the six models, the Lee-Carter model has proved to be the most appropriate to forecast male mortality rate and life-

expectancy in Australia for a window of 50 years. Results showed that the Lee-Carter model gave a comparatively good fit to the Australian data for both females and males.

The approach of the Lee-Carter model is based on the projection of the historical tendency presented by the variable, *Kt*. Likewise, its probabilistic composition allows, through time series, to generate analysis on the future behaviour life such as forecasts, and confidence intervals. Despite its limitations, the approach defined by Lee and Carter in 1992 is widely used in the demographic environment and manages to be present in a large percentage of the studies on the subject. The most important differentiation presented by Lee and Carter in their article was the incorporation of the information in two dimensions, which is mortality through periods of time. Specifically, the model assumes that the mortality dynamics responds to a parameter, the mortality rate, generated by the regression. Starting from this index, the behaviour of mortality using a classic model of time series such as Box-Jenkins can be forecasted. Moreover, the model suggests several conclusions about life-expectancy and mortality tables.

The Lee-Carter model, designed to predict mortality and to analyse its dynamics, has the purpose of analysing the interaction between variables such as mortality rate, life-expectancy, and mortality index. However, beyond this intention, the model is limited by the existing patterns in this area, which means it does not include information regarding accidents, advances in medicine, wars or other events that may mark an inflexion point (Lee 2000).

The Lee-Carter forecasts of mortality rates are comparable to the official mortality rates, while the forecasts of life-expectancy are a little higher than official numbers but the difference is not significant. On the other hand, experience has shown that the Lee-Carter method has proved to under predict life-expectancy (Lee and Miller 2001: pp 537–549). Therefore, the observed values of life-expectancy could even be higher than the Lee-Carter forecasts.

Similar to the Australian female dataset, for the male mortality data the comparison techniques used revealed there are some notable differences amongst the six different models. However, none of the models analysed performs well in all tests and no model clearly dominates the others. This conclusion is consistent with the conclusion formulated by Dowd et al. (2010a) after assessing the goodness of fit of

several mortality models applied to the English and Wales male data. For the Australian male dataset considered we find that the Lee-Carter model gives the lowest RMSE for all look forward windows and is better suited for non–stratified data. However, this model does not clearly stand out from the rest of the models analysed. The Plat model gave a comparatively good fit to the Australian male mortality data, while the Cairns-Blake-Dowd model gave the poorest fit to this data.

However, when the data was fitted into the Lee-Carter and CBD models to calculate male mortality rates, the two models showed comparable results. Similarly, life-expectancy was comparable between the Lee-Carter and CBD forecasts. For example, the mortality rate predicted by the Lee-Carter model for an 80–year–old Australian man in 2011 is 0.054, while the CBD model predicted a mortality rate of 0.057 for the same individual. Also, using mortality rates predicted by the Lee-Carter model, the residual life-expectancy for an Australia woman aged 80 in 2011, is 9.00 while the CBD model gives a residual life-expectancy of 9.02 years, for the same individual.

Tabeau, van den Berg Jeths, and Heathcote (2001) described in their paper the models of graduation and prediction recently developed and concluded that there is a need to integrate techniques from different disciplines with the aim of obtaining satisfactory predictions. The Lee-Carter model and its extensions have been used to predict mortality rates in many countries, starting with the United States (Lee and Carter 1992), Canada (Lee and Nault 1993), Chile (Lee and Rofman 1994), Australia (Booth, Maindonald and Smith 2001; Booth and Tickle 2003; Booth 2004; and Erbas, Ullah, Hyndman, Scollo and Abramson 2012), England and Welsh (Renshaw and Haberman 2003; Griffiths and Brock 2003) and Spain (Debón et al. 2008; Debón, Martínez-Ruiz and Montes 2010; and Debón, Montes and Martínez-Ruiz 2011).

Mortality rates were forecasted for men in Australia for period of 1, 5, 10 and 15, 20 and 50 years, using the CBD and Lee-Carter models. A cohort of 60–100 years was selected, based on statistical selection criteria. Mortality rates obtained by both models showed a continuing trend in declining mortality. The Lee-Carter and CBD models were conservative in their prediction, estimating a steady fall in mortality rates from 0.006 in 2011 to a predicted 0.004 in 2021 (Lee-Carter) while with the CBD model, mortality rates declined from 0.005 in 2011 to 0.003 in 2021. Further forecasting indicated the Lee-Carter model is more conservative in its prediction than

the CBD model, predicting a decline to 0.003. On the other hand, the CBD model predicted a further decline in male mortality rates of 0.001 in 2061.

The forecasting results obtained in the current study indicate that in the long-term there is a decreasing trend in mortality rates and an increasing trend in life-expectancy for the Australian male population. The increasing trend in life-expectancy projections and the decreasing trend in mortality rates are consistent with most of the results in the literature when applying the Lee-Carter model to different datasets from different countries.

These methods, however, have several weaknesses. First of all, it is likely to obtain biased results by directly projecting life-expectancy, since it is simply a synthetic and non-linear measure of death rates by age. Even if these mortality rates continued to fall at constant exponential rates according to age, life-expectancy would increase at a decreasing rate due to the reduction in entropy.

Booth et al. (2006) proved that the Lee-Carter model and its extensions tend to underestimate errors for the younger ages (0–40) and overestimate errors for higher age-groups (60–100). However, the life-expectancy calculated by the Lee-Carter model, which had a low RMSE did not significantly differ from the life-expectancy calculated by the CBD model, in which the error was notably overestimated. Hence, the current study agrees with Booth's hypothesis that over-estimation or underestimation error does not translate into an erroneous estimation of life-expectancy.

The results obtained after applying the Lee-Carter and CBD models suggest that male mortality in Australia has been decreasing in the last decades and, looking at the mortality rates forecasts for the next 50 years, it can be concluded that this population factor will continue to decrease. One of the implications of this research for practice is that the implementation of a model such as the Lee-Carter model for the demographic information of a country can suggest population dynamics that allow revising the formulations that base the models for pension and insurance funds. The main implication for research is the suggestion that none of the models analysed performs well in all tests and no model clearly dominates the others. Also, this research disclosed further research paths concerning mortality rate analysis in Australia, which may help in fostering research on this topic.

The contributions and implications of this research are logical to consider that both female and male mortality and life-expectancy in Australia will continue to improve gradually over time, due to a variety of factors, rather than experiencing a substantial abrupt improvement. Among these factors might be great medical advances, greater access to health, reductions in tobacco consumption, etc.

4.6 Conclusions

Male mortality and life-expectancy rates in Australia have significantly improved in the last 60 years with particularly high rates of mortality improvement for the cohort of Australian males born in 1925–35. However, historical rates of mortality improvement have shown significant variations by age, time and cohort (Berry, Tsui and Jones 2010).

Although there is a growing range of models that can be used for forecasting mortality and life-expectancy rates, none of these models has proved to be ideal. Research in this area shows that a combination of extrapolation and explanation is required to ensure that forecasts are reasonable and take into account all the information that is available and relevant (Cairns et al. 2011).

Dowd et al. (2010a and b) use goodness-of-fit tests and out-of-sample back-testing to show that under some criteria, some models are better than others but none of the models is superior under all the criteria considered.

From The current study, based on the results of the RMSE for various look forward windows, it may be inferred that the Lee-Carter model and Plat model are more suited fit to non-stratified model for ages 0 to 100 (See Tables 4.1–4.4). Results across all look back windows are comparable, hence for computational convenience data 20 can be used for modelling within above settings, for ages 60 to 100 or 80 to 100, providing an improvement in model fit compared to model fitted for all ages (See Tables 4.8 and 4.11).

The present study brings empirical evidence and arguments in favour of long-term declining trends in male mortality by selecting the best mortality model using statistical criteria. However, the degree of uncertainty with which the predictions are made is that it incorporates, and flexibility and robustness of forecasting are features that should also be considered in choosing the best stochastic model for predicting human mortality.

The results obtained in this chapter allowed the projection of male life-expectancy and mortality tables in Australia for the period 2012–2061. According to the forecasts, a life-expectancy of 28.72 years is estimated by 2061 for age 60, that is to say, an increase of 5.5 years with respect to the life-expectancy at birth observed in 2011. From a social, economic and individual point of view, living longer is, in itself, a fact of great importance. But the utility of the presented results must also be measured in terms of their possible applications.

In this sense, the projection of life-expectancy and mortality tables has direct implications on the calculation of premiums and annuities in the insurance industry. If dynamic or cohort tables are considered for the death insurance quote, the premiums would decrease their level by increasing the ability to be placed in the market, as a result of considering possible future reductions in the risk of death in the quotation.

It is important to consider the case of life insurance, in which the projected life-expectancy gains during the rest of the passive stage will relativize the periodic purchasing power of the capital with which the rent is purchased (the retirement). Furthermore, at the same time, the premium would be higher because of the projected improvements in survival, which would result in a commercial and technical problem (in terms of maintaining in real terms the income from the current static tables).

In social security, the use of these values affects the projected estimates of pensions for death and retirement pensions. In the first case, it concerns not only the expected amount of discharges per year but also the expected amount to be paid, since the surviving beneficiary will have an expected future mortality reflected in tables such as those presented in the present study.

With respect to the retirement and pension scheme (Antolin 2007), the expected increase in lifespan can be used to plan progressive changes with the objective of maintaining the financing of years gained in longevity.

Finally, from a methodological point of view, one of the main merits of the Lee-Carter model is that incorporates the analysis of mortality level and structure, allowing an easy stochastic projection of mortality.

Chapter 5 Summary, Conclusions and Recommendations

5.1 Summary of the work

The main purpose of this research is to analyse female and male mortality in Australia and to forecast mortality rates and life-expectancy. This is done by choosing the most appropriate model after comparing six stochastic models used in the empirical literature for mortality and life-expectancy forecasting (Lee-Carter, Renshaw-Haberman, Age-Period-Cohort, CBD, M7 and Plat).

The analysis presented in Chapter 3 and Chapter 4 of this thesis illustrates the results obtained after applying the Lee-Carter model and its extensions, which are Renshaw-Haberman-RH, Cairns-Blake-Dowd-CBD, Age-Period-Cohort-APC, M7 and Plat models, to forecast the mortality rate and life-expectancy of the Australian population. Mortality data from the Human Mortality Database was applied to each of the models, separately for men and women. A comparison was made between the six mortality models using different criteria (root mean square error, Bayesian Information Criterion and Akaike Information Criterion) in order to choose the best model to be used for mortality and life-expectancy forecasting.

The comparison of models with measures of goodness of fit showed that the original Lee-Carter provides better prediction results of mortality rates and life-expectancy than the other models. The comparison shows that, despite the greater complexity of the extended models, this model better describes Australian data. Furthermore, the general index of mortality and its future projections were calculated between the years 2012 and 2061, using the best-fitted model for the Australian data (the Lee-Carter model). The results were then compared with the forecasted values given by the CBD model. Furthermore, one mortality indicator was calculated: life-expectancy. The improvement in mortality rates and life-expectancy reflects the increase in the standard of living in the Australian population during the last decades.

The analysis started by comparing the root mean square values, for five different looks back windows (*h=1, h=5, h=10, h=15 and h=20*) for each of the three scenarios considered (*S0, S1 and S2*). For scenario S0 (non-stratified data, 0–100 years), the results were similar for both male's and female's data, providing evidence

that the differences in the goodness of fit, as measured by the RMSE values between the five look back windows were not significant (p-value < 0.05 for all the six models). The Lee-Carter model gave the lowest root mean square values for almost all look-back windows considered (*h=1, h=5, h=10, h=15 and h=20)*, followed by the Plat model, for both male and female mortality data.

For scenario S1 (stratified data, age cohort 0–40 years), the models with the best fit for short-term and long-term forecasting (h =1 and h=20) were the APC and Plat models for both females and males data. For scenario S1 (stratified data, age cohort 40–60 years), the models with the best fit for short-term and long-term forecasting (h =1 and h=20) for female data were the CBD, APC and LC models, while for male data, the APC and Plat models proved to be the best-fitted. For the same scenario S1 but age cohort 60–100 years, the best-fitted models were the APC, Plat and Lee-Carter, for both male and female data.

For scenario S2 (stratified data, age cohort 0–60 years), the models with the best fit for short-term and long-term forecasting (*h =1 and h=20*) were the RH, Lee-Carter and Plat models for female data. The Plat and the APC models proved to be the best models for the same scenario. Results showed that for age cohort 60–80 years, for female data, the best-fitted models for short-term and long-term forecasting (*h =1 and h=20*) were the Lee-Carter, Plat and RH models. On the other hand, the analysis conducted for male data revealed that the APC model was the best fitted from all six models. Finally, for scenario S2, stratified data, age cohort 80–100 years, the models with the best fit for short-term and long-term forecasting (*h =1 and h=20*) were the APC, M7 and Plat models.

Furthermore, a comparative analysis of the root mean square values between the six different mortality models for the three scenarios with the different look back windows was presented for both female and male mortality data. The results for female data supported the hypothesis that the variance in RMSE for the different data sizes (data=20, 30, 40 and 50) is not statistically significant, except for scenario S0 where the CBD and the M7 models were significantly different in the goodness of fit compared to the other four models. The results for male data supported the hypothesis that the variance in RMSE for the different data sizes (data=20, 30, 40 and 50) is

statistically significant, showing that the means of observations grouped by different data sizes are different.

Comparing the root mean square values between the six models estimated for both male and female data and all look back windows considered (20, 30, 40 and 50), the Lee-Carter model proved to be the best-fitted model for scenario S0. Furthermore, age cohort 60–100 years (S1) was selected to determine the differences between RMSE for stratified data using varying periods of forecasting (Data=20 – Data=50), as this comprised the pensionable population. In this case, the analysis performed for female mortality showed the Lee-Carter model was best fitted, while for male mortality, the APC model was superior to the other five models considered.

Finally, for scenario S2, female data, the APC was best fitted for ages 0–60 and 60–80, while the Lee-Carter model was best fitted for the older ages of 80–100 years. On the other hand, for the same scenario, age cohort 0–60 years, the Plat model was the best-fitted model, while for ages 60–80 and 80–100 the Lee-Carter and CBD models gave the best fit.

Next, the Bayesian and Akaike Information Criteria (BIC and AIC) were compared with the purpose of choosing the best model between the six mortality models estimated in the previous chapters of this thesis. Thus, a comparison was made first of all between the BIC scores obtained for the six different models (Lee-Carter, RH, APC, CBD, M7 and Plat) for a data mined between the 20–year period 1990–2010, for female and male data. This comparison showed that the BIC values decreased with increasing ages in the case of stratified data and the models with the lowest BIC values were the Lee-Carter model and the APC model for both female and male data. Furthermore, a comparison between the AIC scores obtained for the six different models was made. In this case, results showed the models with the lowest AIC values were RH model and the Plat for female data. The values for the Lee-Carter model and the APC model were not statistically different from the RH or Plat models as indicated by the student's *t*-test (*p-value > 0.05*). The same comparison analysis between the AIC values, obtained for the six mortality models estimated for Australian male mortality data showed that the Plat, APC and RH models were the best-fitted models.

Other criteria to be satisfied when choosing the best mortality model were the randomness of residuals. For both datasets (female and male mortality), the residuals for the Lee-Carter models were randomly distributed.

In summary, based on the four criteria for model selection analysed in the sections of previous chapters, the best model for estimating mortality and life-expectancy for female and male datasets is the Lee-Carter model. As a comparison, the CBD models had the highest values of root mean square error as well as BIC and AIC, suggesting a poor selection for both datasets. Because there was no significant difference in the goodness-of-fit for the Australian female and male mortality data, between the different data sizes (*data=20, data=30, data=40 or data=50*), a dataset of 50 years was chosen to calculate mortality and life-expectancy. Thus, the Lee Carter and the CBD models were used to calculate mortality rates, the number of deaths, the number of survivors, and life-expectancy in 2011 for age cohort 60–100 years. Furthermore, mortality rates and life-expectancy were forecasted for a period of 50 years, between 2012 and 2061 using the Lee Carter and the CBD models.

Comparing the mortality rates over different values of *h* (*h=1, h=5, h=10, h=15, h=20 and h=50*), results showed they were not statistically different between the Lee-Carter model and the CBD model. Similarly, the difference between the CBD and the Lee-Carter life-expectancy forecasting results were not significantly different, indicating that the statistical goodness-of-fit did not have any significant impact on the forecasting of life-expectancy, for age cohort 60 – 100 years.

Based on current population figures, the residual life-expectancy for a 65–year–old Australian woman in 2011 was 22.06 years. Using mortality rates predicted by the Lee-Carter model, the residual life-expectancy for an Australian woman aged 65 in 2011 was 22.081 while the residual life-expectancy for 65–year–old Australian men in 2011 was 19.14 years. Using mortality rates predicted by the Lee-Carter model, the residual life-expectancy for an Australia men aged 65 in 2011, was 19.37 years.

One important aspect regarding data availability is that it has been easy to access the Human Mortality Databases with information on the number of deaths, number of survivors, life-expectancy, death rates, population size, etc. Thus, it can be said that at country level there is a good source of information about what is happening in Australia's demographics. Starting with 1921, the Human Mortality Database provides detailed information about mortality and population data.

5.2 Contributions

The purpose of this thesis is to examine in detail, the mortality patterns in Australia, over the period 1961– 2061, to analyse and predict mortality rates and life-expectancy, as well as to detect and model the eventual presence of cohort effects in mortality patterns. These contributions are important in selecting accurate models on which to base future forecasts.

The comparison of the six mortality models (starting with the original Lee-Carter model and ending with one of its most recent extension), with the purpose of selecting the best model for Australian mortality and life-expectancy forecast represent contributions highlighting the originality of the thesis.

Among the most important results and conclusions learned from this analysis, we mention the following:

- The results of this analysis contribute to the existing empirical literature by considering not only the Lee-Carter model but also a selection of extensions from the Lee Carter model to estimate and forecast Australian mortality rates and life-expectancy.
- The main contribution of this research is the comparison of six different stochastic mortality forecasting methods estimated for the Australian mortality data, based on three different criteria (RMSE, BIC and AIC values). This comparison is a useful method contributing to the identification of the most appropriate model to be used for mortality and life-expectancy forecasting.
- The separation of age, period and cohort patterns can contribute in important ways to a better understanding of the mortality rate decline in Australia over the last decades.

5.3 Limitations and directions for future research

As a result of the estimation of the Lee-Carter model and its extensions for the population of Australia, a prognosis of the future mortality rates and life-expectancy for the next 50 years has been made. While the current study concurs with arguments in favour of long-term declining trends in mortality, the degree of uncertainty with which these predictions are made is not well defined. Although the selection of

mortality models is based on statistical criteria, additional considerations such as flexibility and robustness of forecasting are features that need to be considered in choosing the best stochastic model for predicting human mortality.

A challenging problem for the study of human longevity is the available demographic data for the elderly. Because the numbers of observations for the elderly are seldom sufficient, false decisions may be caused by their graduated mortality rates that are likely to be very different from the true values.

The analysis and the results presented generated a set of questions which can reinforce even more the interest in the field of stochastic mortality forecasting and can guide future research efforts. Thus, based on the set of limitations discovered and described, a set of directions for future research can be learnt and taken into consideration for future analysis.

One of the main contributions of this research refers to direct implications for the projections of life-expectancy and mortality tables on the calculation of premiums and annuities in the insurance industry. Under the assumption that insurance and pension companies are regionally concentrated, it would be interesting for future research purposes to conduct a study that differentiates the mortality rates by regions. This may suggest a differential risk per region and therefore higher or lower pension costs for different participants in the sector. However, data availability may limit the analysis conducted and its results.

After modelling the mortality curve using the best statistical method chosen from six mortality models based on the original Lee-Carter approach, it would be interesting to implement a more complex model based on a stochastic differential equation (SDE). For example, Milevsky and Promislow (2001) have studied mortality patterns using stochastic differential equations associated with an additive noise based on Brownian motion. Also, Giacometti, Ortobelli and Bertocchi (2011) make a generalisation of SDE models, considering the case with an additive noise but with a diffusion coefficient dependent on time and not only as a constant function. The authors conclude that the use of stochastic mortality equations to model the mortality rates and life-expectancy reflects more accurately the behaviour of mortality and opens a field in the study of this subject.

Some authors in their models tried to adjust the mortality rate by assuming perfect correlations between generations. However, common intuition suggests that between close generations these correlations are high but not perfect. To explain this relationship, Giacometti, Bertocchi, Rachev and Fabozzi (2012) consider short-dependency models such as ARMA (*p, q*), and in particular, suggest the use of an AR (1). The approach that could be explored in a future study is to describe mortality with a model that takes into account long-range dependence. For this it would be appropriate to use a generalisation of the Milevsky-Promislow model, integrating this dependence. The research would try to fit a stochastic differential equation with additive noise based on a fractional Brownian motion (*fBm*), instead of the standard Brownian motion.

According to the Milevsky-Promislow model, the mortality $m_x(t)$ is given by the following equation:

$$m(t) = m_0 exp(\alpha_0 t + \alpha_1 Y_t) \tag{5.1}$$

where: $h_0, \alpha_0, \alpha_1 > 0$.

The stochastic process Y_t (fractional Ornstein-Uhlenbeck process) satisfies the stochastic differential equation:

$$dY_t = -\lambda Y_t d_t + \gamma dB_t^H \tag{5.2}$$

$$Y_0 = 0 \tag{5.3}$$

where B_t^H is a fractional Brownian motion with a Hurst parameter $1/2 \leq H < 1$ and parameters $\lambda, \gamma > 0$.

Using real historical data from the Australian population, the main objective would be to estimate the parameters $H, \gamma, \lambda, \alpha_0, \alpha_1$.

Another direction for future research would be to use other models that overcome the limits of the Lee-Carter model and its extensions by trying to understand how risk factors (like smoking, hypertension, etc.) will affect the different causes of death. In order to do this, multivariate models such as generalised linear models (GLM) could be used, which also attempt to measure changes in mortality by introducing multiple variables that are interrelated.

APPENDICES

Appendix A R Code Excerpt

This appendix shows an R Code excerpt illustrating R commands and input-output that were used.

A.1 R Code Excerpt

```
Mortality<-function(age.range,year.range,wd,h,yr){
  #age.range and year.range are in the format a:b e.g. age.range=60:80
  #wd must be working directory in the form "C://Users//..."
  #h is the forward time window (integer) e.g. h=1
  #yr is the forecast year (integer) e.g. yr=2011


  #File name
  filename<-paste("A-",min(age.range),"-",max(age.range),"Y-",min(year.range),"-
",max(year.range),"h-",h,"FC-",yr,sep="")

  #Libraries
  library(demography) #loads demography package
  library(StMoMo) # loads StMoMo package

  #Preamble (Model Difinitions) -> log implies log-Poisson model
  LC <- lc(link = "log")
  CBD <- cbd(link = "log")
  RH <- rh(link = "log", cohortAgeFun = "1")
  APC <- apc(link = "log")
  M7 <- m7(link="log")

  f2 <- function(x, ages){mean(ages) - x}
  constPlat <- function(ax, bx, kt, b0x, gc, wxt, ages){
    nYears <- dim(wxt)[2]
    x <- ages
    t <- 1:nYears
    c <- (1 - tail(ages, 1)):(nYears - ages[1])
    xbar <- mean(x)
    #nsum g(c)=0, nsum cg(c)=0, nsum c^2g(c)=0
    phiReg <- lm(gc ~ 1 + c + I(c^2), na.action = na.omit)
    phi <- coef(phiReg)
    gc <- gc - phi[1] - phi[2] * c - phi[3] * c^2
    kt[2, ] <- kt[2, ] + 2 * phi[3] * t
    kt[1, ] <- kt[1, ] + phi[2] * t + phi[3] * (t^2 - 2 * xbar * t)
    ax <- ax + phi[1] - phi[2] * x + phi[3] * x^2
    #nsum kt[i, ] = 0
    ci <- rowMeans(kt, na.rm = TRUE)
    ax <- ax + ci[1] + ci[2] * (xbar - x)
    kt[1, ] <- kt[1, ] - ci[1]
    kt[2, ] <- kt[2, ] - ci[2]
    list(ax = ax, bx = bx, kt = kt, b0x = b0x, gc = gc)
  }
  PLAT <- StMoMo(link = "log", staticAgeFun = TRUE,periodAgeFun = c("1", f2),
cohortAgeFun = "1",constFun = constPlat)
```

```
  #Read in data and create appropriate subsets

  mx <- read.table("C:\\Users\\15245634\\Desktop\\AUSdata\\Mx_1x1.txt", skip = 2,
header = TRUE, na.strings = ".")
  pop <- read.table("C:\\Users\\15245634\\Desktop\\AUSdata\\Exposures_1x1.txt", skip
= 2, header = TRUE, na.strings = ".")

  obj <- list(type = "mortality", label = "AUS", lambda = 0)
  obj$year <- sort(unique(mx[, 1]))
  n <- length(obj$year)
  m <- length(unique(mx[, 2]))
  obj$age <- mx[1:m, 2]
  mnames <- names(mx)[-c(1, 2)]
  n.mort <- length(mnames)
  obj$rate <- obj$pop <- list()
  for (i in 1:n.mort) {
    obj$rate[[i]] <- matrix(mx[, i + 2], nrow = m, ncol = n)
    obj$rate[[i]][obj$rate[[i]] < 0] <- NA
    obj$pop[[i]] <- matrix(pop[, i + 2], nrow = m, ncol = n)
    obj$pop[[i]][obj$pop[[i]] < 0] <- NA
    dimnames(obj$rate[[i]]) <- dimnames(obj$pop[[i]]) <- list(obj$age,
                                                              obj$year)
  }
  names(obj$pop) = (names(obj$rate) <- tolower(mnames))
  obj$age <- as.numeric(as.character(obj$age))
  if (is.na(obj$age[m]))
    obj$age[m] <- 2 * obj$age[m - 1] - obj$age[m - 2]

  AUSdata<- obj
  class(AUSdata) = "demogdata"

  #Fit models
  #LC
  fitAusMaleLC<-fit(LC,
Dxt=Dxt_Aus_Male,Ext=Ext_Aus_Male,ages=AUSdata$age,years=AUSdata$year,ages.fit=age.ra
nge, years.fit=year.range)
  #CBD
  fitAusMaleCBD<-fit(CBD,
Dxt=Dxt_Aus_Male,Ext=Ext_Aus_Male,ages=AUSdata$age,years=AUSdata$year,ages.fit=age.ra
nge, years.fit=year.range)
  #APC
  fitAusMaleAPC<-fit(APC,
Dxt=Dxt_Aus_Male,Ext=Ext_Aus_Male,ages=AUSdata$age,years=AUSdata$year,ages.fit=age.ra
nge, years.fit=year.range)
  #M7
  fitAusMaleM7<-fit(M7,
Dxt=Dxt_Aus_Male,Ext=Ext_Aus_Male,ages=AUSdata$age,years=AUSdata$year,ages.fit=age.ra
nge, years.fit=year.range)
  #PLAT
  fitAusMalePLAT<-fit(PLAT,
Dxt=Dxt_Aus_Male,Ext=Ext_Aus_Male,ages=AUSdata$age,years=AUSdata$year,ages.fit=age.ra
nge, years.fit=year.range)
  #RH - NOTE: start.ax = fitausmaleLC$ax, start.bx = fitausmaleLC$bx, start.kt =
fitausmaleLC$kt, are set to fitted objects from LC model
  fitAusMaleRH<-fit(RH,
Dxt=Dxt Aus Male,Ext=Ext Aus Male,ages=AUSdata$age,years=AUSdata$year,ages.fit=age.ra
nge, years.fit=year.range, start.ax = fitAusMaleLC$ax,start.bx = fitAusMaleLC$bx,
start.kt = fitAusMaleLC$kt,maxIter=100000)

  Models<-
list(fitAusMaleLC,fitAusMaleCBD,fitAusMaleAPC,fitAusMaleM7,fitAusMalePLAT,fitAusMaleR
H)
  names(Models)<-
c("fitAusMaleLC","fitAusMaleCBD","fitAusMaleAPC","fitAusMaleM7","fitAusMalePLAT","fit
AusMaleRH")
```

```
  #Model Comparisons
  AICModelComparisons<-
data.frame(c(LC=as.numeric(AIC(fitAusMaleLC)),CBD=as.numeric(AIC(fitAusMaleCBD)),
APC=as.numeric(AIC(fitAusMaleAPC)),M7=as.numeric(AIC(fitAusMaleM7)),PLAT=as.numeric(A
IC(fitAusMalePLAT)),RH=as.numeric(AIC(fitAusMaleRH))))
  colnames(AICModelComparisons)<-"AIC"
  AICModelComparisons<-
AICModelComparisons[order(AICModelComparisons$AIC),,drop=FALSE]

  BICModelComparisons<-
data.frame(c(LC=as.numeric(BIC(fitAusMaleLC)),CBD=as.numeric(BIC(fitAusMaleCBD)),
APC=as.numeric(BIC(fitAusMaleAPC)),M7=as.numeric(BIC(fitAusMaleM7)),PLAT=as.numeric(B
IC(fitAusMalePLAT)),RH=as.numeric(BIC(fitAusMaleRH))))
  colnames(BICModelComparisons)<-"BIC"
  BICModelComparisons<-
BICModelComparisons[order(BICModelComparisons$BIC),,drop=FALSE]

  #Comparative Forecasts
  temp<-AUSdata$rate
  actual<-temp$male[,paste(yr)]
  fcLC<-forecast(fitAusMaleLC,h=h)
  fcRates<-fcLC$rates
  fcCBD<-forecast(fitAusMaleCBD,h=h)
  fcAPC<-forecast(fitAusMaleAPC,h=h)
  fcM7<-forecast(fitAusMaleM7,h=h,gc.order=c(0,0,1),method="ML")
  fcPLAT<-forecast(fitAusMalePLAT,h=h)
  fcRH<-forecast(fitAusMaleRH,h=h)
  RatesLC<-fcLC$rates
  RatesCBD<-fcCBD$rates
  RatesAPC<-fcAPC$rates
  RatesM7<-fcM7$rates
  RatesPLAT<-fcPLAT$rates
  RatesRH<-fcRH$rates

  if (h==1){
    RMSEModel<-data.frame(c(LC=sqrt(mean((RatesLC-
actual[names(RatesLC)])^2)),CBD=sqrt(mean((RatesCBD-actual[names(RatesCBD)])^2)),
APC=sqrt(mean((RatesAPC-actual[names(RatesAPC)])^2)),M7=sqrt(mean((RatesM7-
actual[names(RatesM7)])^2)),PLAT=sqrt(mean((RatesPLAT-
actual[names(RatesPLAT)])^2)),RH=sqrt(mean((RatesRH-actual[names(RatesRH)])^2))))
  } else {
    RMSEModel<-data.frame(c(LC=sqrt(mean((RatesLC[,paste(yr)]-
actual[names(RatesLC[,paste(yr)])])^2)),CBD=sqrt(mean((RatesCBD[,paste(yr)]-
actual[names(RatesCBD[,paste(yr)])])^2)), APC=sqrt(mean((RatesAPC[,paste(yr)]-
actual[names(RatesAPC[,paste(yr)])])^2)),M7=sqrt(mean((RatesM7[,paste(yr)]-
actual[names(RatesM7[,paste(yr)])])^2)),PLAT=sqrt(mean((RatesPLAT[,paste(yr)]-
actual[names(RatesPLAT[,paste(yr)])])^2)),RH=sqrt(mean((RatesRH[,paste(yr)]-
actual[names(RatesRH[,paste(yr)])])^2))))
  }
  colnames(RMSEModel)<-"RMSE"
```

```
#Residuals
  LCres<-residuals(fitAusMaleLC)
  CBDres<-residuals(fitAusMaleCBD)
  APCres<-residuals(fitAusMaleAPC)
  M7res<-residuals(fitAusMaleM7)
  PLATres<-residuals(fitAusMalePLAT)
  RHres<-residuals(fitAusMaleRH)

  if (h==1){
    MeanDiff<-mean(actual[names(fcRates)]-fcRates)*100
    MeanAbsDiff<-mean(abs(actual[names(fcRates)]-fcRates))*100
  } else{
    MeanDiff<-mean(actual[names(fcRates[,paste(yr)])]-fcRates)*100
    MeanAbsDiff<-mean(abs(actual[names(fcRates[,paste(yr)])]-fcRates))*100
  }
  if (h==1){

assign(paste0("data",filename,sep=""),data.frame(actual[names(fcRates)],fcRates),envi
r = .GlobalEnv)
  } else {

assign(paste0("data",filename,sep=""),data.frame(actual[names(fcRates[,paste(yr)])],f
cRates),envir = .GlobalEnv)
  }

  #T-Test
  if (h==1){
ttLC<-t.test(x=actual[names(RatesLC)],y=RatesLC,paired=TRUE)
ttCBD<-t.test(x=actual[names(RatesCBD)],y=RatesCBD,paired=TRUE)
ttAPC<-t.test(x=actual[names(RatesAPC)],y=RatesAPC,paired=TRUE)
ttM7<-t.test(x=actual[names(RatesM7)],y=RatesM7,paired=TRUE)
ttPLAT<-t.test(x=actual[names(RatesPLAT)],y=RatesPLAT,paired=TRUE)
ttRH<-t.test(x=actual[names(RatesRH)],y=RatesRH,paired=TRUE)

  } else {
ttLC<-t.test(x=actual[names(RatesLC[,paste(yr)])],y=RatesLC[,paste(yr)],paired=TRUE)
ttCBD<-
t.test(x=actual[names(RatesCBD[,paste(yr)])],y=RatesCBD[,paste(yr)],paired=TRUE)
ttAPC<-
t.test(x=actual[names(RatesAPC[,paste(yr)])],y=RatesAPC[,paste(yr)],paired=TRUE)
ttM7<-t.test(x=actual[names(RatesM7[,paste(yr)])],y=RatesM7[,paste(yr)],paired=TRUE)
ttPLAT<-
t.test(x=actual[names(RatesPLAT[,paste(yr)])],y=RatesPLAT[,paste(yr)],paired=TRUE)
ttRH<-t.test(x=actual[names(RatesRH[,paste(yr)])],y=RatesRH[,paste(yr)],paired=TRUE)
```

```
#Generate TXT for Results
if (file.exists(paste(wd,"//","Results",sep=""))){
  setwd(paste(wd,"//","Results",sep=""))
} else{
  dir.create(paste(wd,"//","Results",sep=""))
  setwd(paste(wd,"//","Results",sep=""))
}

sink(file=paste(filename,".txt",sep=""))

print("Model Parameters")
print(Models)

print("Model Comparisons")
print(AICModelComparisons)
print(BICModelComparisons)
print(RMSEModel)
print(paste("Average Number of Deaths Predicted in Error for the Year ", yr))
print(RMSEModel*10000)

print("Residuals")
print("LC")
print(LCres)
print("CBD")
print(CBDres)
print("APC")
print(APCres)
print("M7")
print(M7res)
print("PLAT")
print(PLATres)
print("RH")
print(RHres)

print("Actual Values")
print(actual)

print("Forecast Values")
print(fcRates)

print("Mean Difference for LC (in %)")
print(MeanDiff)
print("Mean Absolute Difference for LC (in %)")
print(MeanAbsDiff)

print("T-Test Results")
print(ttLC)
print(ttCBD)
print(ttM7)
print(ttAPC)
print(ttPLAT)
print(ttRH)


sink()
```

```
  #Generate PDF for Plots
  if (file.exists(paste(wd,"//","Plots",sep=""))){
    setwd(paste(wd,"//","Plots",sep=""))
  } else{
    dir.create(paste(wd,"//","Plots",sep=""))
    setwd(paste(wd,"//","Plots",sep=""))
  }

  pdf(file=paste(filename,".pdf",sep=""))

  plot(LCres, type = "colourmap", reslim = c(-3.5, 3.5), main="LC")
  plot(fitAusMaleLC)
  plot(CBDres, type = "colourmap", reslim = c(-3.5, 3.5),main="CBD")
  plot(fitAusMaleCBD)
  plot(APCres, type = "colourmap", reslim = c(-3.5, 3.5),main="APC")
  plot(fitAusMaleAPC)
  plot(M7res, type = "colourmap", reslim = c(-3.5, 3.5),main="M7")
  plot(fitAusMaleM7)
  plot(PLATres,type = "colourmap", reslim = c(-3.5, 3.5),main="PLAT")
  plot(fitAusMalePLAT)
  plot(RHres, type = "colourmap", reslim = c(-3.5, 3.5),main="RH")
  plot(fitAusMaleRH)
  if (h==1){
    plot(actual[names(fcRates)],fcRates,xlab="Actual Rates",ylab=paste("Predicted
Rates For The Year",yr))
  } else {
    for (q in 1:h){
      plot(actual[names(fcRates[,paste(yr)])],fcRates[,q],xlab="Actual
Rates",ylab=paste("Predicted Rates For The Year",colnames(fcRates)[q]))
    }
  }

  graphics.off()

}
```

A.2 Example of Results

```
Mortality(age.range = 0:100,year.range = 1990:2010,wd ="C:\\Users\\15245634\\Desktop",h = 1,yr = 2011)
```

Appendix B Results of the goodness-of-fit criteria of data 20 and *h*=1, using different mortality models, for Australian data

This appendix shows tables of modelling results using RH, APC, CBD, M7 and Plat models for 1–year look forward window and 20–years look back window, for both female and male Australian data.

Table B.1 The results of the goodness of fit criteria of data 20 and *h*=1 using RH model, Australian females.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	17936.62	5849.17	3749.38	8692.95	9448.69	4389.64	4546.71
BIC	19866.56	6615.23	4162.37	9459.01	10588.04	4802.64	4959.70
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.004228	0.000049	0.000113	0.004827	0.000078	0.000804	0.006525
RMSES0	____	0.000053	0.000120	0.006636	0.000083	0.000594	0.009256
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	18004.45	5892.82	3753.79	3753.79	9485.77	4396.84	4557.60
BIC	19934.39	6658.87	4166.79	4166.79	10625.11	4809.83	4970.59
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.024000	0.000041	0.000151	0.000151	0.000106	0.001695	0.019164
RMSES0	_____	0.000052	0.000176	0.000176	0.000111	0.002632	0.052591
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	17987.77	5899.53	3754.06	8718.42	9485.77	4398.46	4541.19
BIC	19917.71	6665.58	4167.06	9484.48	10625.11	4811.45	4954.18
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.020406	0.00005	0.000193	0.040772	0.000106	0.000738	0.046702
RMSES0	_____	0.000057	0.000267	0.037668	0.000111	0.002218	0.044714

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	18030.71	5920.51	3753.17	8723.87	9511.68	4403.67	4544.74
BIC	19960.65	6686.57	4166.16	9489.92	10651.02	4816.66	4957.73
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.026401	0.000057	0.000130	0.045301	0.000096	0.001920	0.059578
RMSES0	____	0.000045	0.000184	0.041437	0.000114	0.002788	0.057847
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	18040.35	5929.98	3743.17	8735.86	9514.24	4435.76	4537.32
BIC	19970.29	6696.04	4156.16	9501.91	10653.59	4848.75	4950.31
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.033376	0.000067	0.000219	0.054585	0.000450	0.005844	0.059297
RMSES0	____	0.000067	0.000326	0.052384	0.000199	0.003424	0.073135

Table B.2 The results of the goodness of fit criteria of data 20 and *h*=1 using APC model, Australian females.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	17981.15	5825.547	3762.567	8743.599	9411.628	4393.167	4553.714
BIC	19339.46	6396.519	4089.690	9314.571	10236.49	4720.290	4880.838
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.002858	0.000047	0.000127	0.004456	0.000083	0.000460	0.006015
RMSES0	_____	0.000044	0.000117	0.004485	0.000077	0.000399	0.006255
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	18088.49	5864.828	3758.057	8791.698	9455.997	4382.847	4556.801
BIC	19446.81	6435.799	4085.181	9362.669	10280.86	4709.971	4883.924
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.001838	0.000037	0.000165	0.008015	0.000086	0.000684	0.003639
RMSES0	_____	0.000038	0.000117	0.002884	0.000075	0.000567	0.004004
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	18109.02	5868.507	3759.679	8807.803	9464.675	4386.902	4548.052
BIC	19467.33	6439.478	4086.802	9378.775	10289.54	4714.025	4875.175
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.004680	0.000056	0.000168	0.007913	0.000109	0.000501	0.013258
RMSES0	_____	0.000065	0.000139	0.007344	0.000094	0.000738	0.010235
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	18118.59	5891.327	3755.508	8823.177	9479.767	4385.603	4555.923
BIC	19476.90	6462.298	4082.631	9394.149	10304.63	4712.727	4883.046

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.005353	0.000057	0.000156	0.010084	0.010084	0.000625	0.009002
RMSES0	_____	0.010235	0.000171	0.008400	0.000111	0.000480	0.011728
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	18127.93	5904.274	3742.996	8836.469	9481.285	4416.838	4545.521
BIC	19486.24	6475.245	4070.120	9407.440	10306.15	4743.962	4872.645
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.006712	0.000081	0.000237	0.010084	0.000156	0.000850	0.011996
RMSES0	_____	0.000077	0.000227	0.010534	0.000147	0.000556	0.014710

Table B.3 The results of the goodness of fit criteria of data 20 and *h*=1 using CBD model, Australian females.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	192062.4	58152.75	3797.256	11433.95	88610.16	4824.963	5427.545
BIC	192300.1	58352.59	3968.996	11633.79	88826.69	4996.703	5599.285
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.015989	0.000517	0.000123	0.013313	0.000502	0.000685	0.024297
RMSES0	_____	0.000559	0.000288	0.025088	0.000487	0.003923	0.034853

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	194601.50	59746.70	3804.07	11245.59	91724.09	4817.84	5390.34
BIC	194839.20	59946.54	3975.81	11445.43	91940.62	4989.57	5562.08
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.017031	0.000521	0.000106	0.011512	0.000488	0.000965	0.022024
RMSES0	_____	0.000556	0.000346	0.026724	0.000499	0.003849	0.037150
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	195811.50	61020.94	3793.82	11003.49	94251.75	4782.73	5334.68
BIC	196049.20	61220.78	3965.56	11203.33	94468.28	4954.47	5506.42
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.011523	0.000530	0.000179	0.019978	0.000529	0.000910	0.035620
RMSES0	____	0.000573	0.000352	0.018076	0.000513	0.004382	0.024904
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	197756.00	62616.54	3799.72	10822.22	97203.36	4772.02	5297.84
BIC	197993.70	62816.38	3971.46	11022.06	97419.89	4943.76	5469.58
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.019744	0.000570	0.000135	0.009286	0.000543	0.000964	0.018966
RMSES0	_____	0.000605	0.000381	0.030982	0.000544	0.003803	0.043135

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	199045.10	63882.37	3861.60	10618.45	99787.56	4729.02	5278.77
BIC	199282.80	64082.21	3949.85	10818.29	100004.09	4900.76	5450.51
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.019573	0.000599	0.000285	0.009249	0.000629	0.001028	0.015249
RMSES0	_____	0.000641	0.000305	0.030713	0.000554	0.003233	0.042800

Table B.4 The results of the goodness of fit criteria of data 20 and *h*=1 using M7 model, Australian females.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	52432.19	20734.90	3782.18	8887.00	31646.89	4402.45	4567.13
BIC	53456.59	21310.63	4195.17	9462.73	9462.73	4815.44	4815.44
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.007580	.000751	0.000127	0.006034	0.000614	0.000469	0.006043
RMSES0	_____	0.000574	0.000414	0.011879	0.000528	0.001718	0.016514
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	53490.20	21162.59	3777.35	8857.20	32353.40	4396.43	4569.20
BIC	54514.60	21738.32	4190.34	9432.93	33080.31	4809.43	4982.20

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.008849	0.000767	0.000143	0.004464	0.000604	0.000633	0.005738
RMSES0	_____	0.000577	0.000357	0.013875	0.000516	0.001764	0.019308
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	54115.37	21570.79	3781.25	8833.07	32968.11	4397.27	4564.27
BIC	55139.76	22146.52	4194.25	9408.80	33695.02	4810.26	4977.26
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.004625	0.000795	0.000175	0.011388	0.000635	0.000579	0.013086
RMSES0	_____	0.000732	0.000490	0.007218	0.000665	0.002250	0.009850
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	55156.43	22232.08	3781.10	8840.90	33807.69	4402.41	4569.41
BIC	56180.82	22807.81	4194.10	9416.63	34534.60	4815.40	4982.40
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.012459	0.000857	0.000173	0.005621	0.000670	0.000490	0.009650
RMSES0	_____	0.000670	0.000373	0.019542	0.000590	0.001836	0.027250
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	55908.93	22735.80	3777.75	8865.01	34301.44	4430.97	4563.52
BIC	56933.32	23311.53	4190.75	9440.74	35028.35	4843.97	4976.51
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.013115	0.000909	0.000259	0.006730	0.000756	0.000705	0.013095
RMSES0	____	0.000765	0.000485	0.020569	0.000687	0.001419	0.028706

Table B.5 The results of the goodness of fit criteria of data 20 and *h*=1 using Plat model, Australian females.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	17904.01	5846.51	3785.58	8736.93	9404.40	4407.98	4558.79
BIC	19369.85	6507.88	4190.39	9398.31	10327.22	4812.79	4963.61
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.003227	0.000052	0.000135	0.005335	0.000082	0.000455	0.005328
RMSES0	_____	6.238040	0.000118	0.005064	0.000083	0.000401	0.007065
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	17963.88	5884.24	3781.30	8719.36	9447.54	4400.70	4554.91
BIC	19429.73	6545.61	4186.12	9380.73	10370.36	4805.52	4959.73
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.001770	0.000037	0.000149	0.003183	0.000092	0.000642	0.003056
RMSES0	_____	0.000053	0.000119	0.002776	0.000082	0.000568	0.003852
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	17972.36	5884.56	3783.76	8720.11	9450.91	4403.63	4544.78
BIC	19438.21	6545.93	4188.58	9381.49	10373.72	4808.45	4949.59
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.007267	0.000057	0.000177	0.013115	0.000101	0.000554	0.016213
RMSES0	_____	0.000056	0.000159	0.011405	0.000102	0.000666	0.015923
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	18032.01	5909.40	3780.21	8733.77	9467.65	4405.70	4554.21
BIC	19497.86	6570.77	4185.02	9395.14	10390.47	4810.51	4959.03

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.003588	0.000052	0.000161	0.003879	0.000105	0.000539	0.006180
RMSES0	_____	0.000052	0.000154	0.005630	0.000100	0.000465	0.007853
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	18046.20	5924.49	3772.84	8748.62	9470.87	4436.21	4546.30
BIC	19512.05	6585.87	4177.66	9410.00	10393.69	4841.03	4951.12
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.005103	0.000066	0.000243	0.006244	0.000149	0.000715	0.011655
RMSES0	_____	0.000071	0.000242	0.008008	0.000153	0.000579	0.011176

Table B.6 The results of the goodness of fit criteria of data 20 and *h*=1 using RH model, Australian males.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	18817.79	6455.28	4035.57	8587.89	10300.99	4562.08	4255.80
BIC	20747.73	7221.34	4448.57	9353.95	11440.33	4975.07	4668.79
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.007638	0.000102	0.001353	0.008409	0.000119	0.001445	0.037331
RMSES0	_____	0.000145	0.000140	0.011987	0.000145	0.001656	0.016676

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	18916.58	6497.87	4010.65	8605.63	10356.08	4588.30	4248.24
BIC	20846.52	7263.93	4423.64	9371.69	11495.42	5001.29	4661.23
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.013408	0.000080	0.000164	0.009337	0.000109	0.000496	0.044572
RMSES0	_____	0.000076	0.000251	0.021043	0.000160	0.002585	0.029312
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	18997.14	6567.32	4012.22	8589.62	10433.69	4587.61	4230.16
BIC	20927.08	7333.37	4425.22	9355.67	11573.03	5000.60	4643.16
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.020366	0.000070	0.000173	0.013620	0.000118	0.000661	0.049246
RMSES0	_____	0.000096	0.000359	0.031964	0.000222	0.003777	0.044538
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	19002.09	6620.62	4005.17	8570.28	10459.55	4585.75	4213.69
BIC	20932.02	7386.68	4418.16	9336.33	11598.89	4998.75	4626.68
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.028500	0.000064	0.000189	0.024538	0.000124	0.000973	0.021189
RMSES0	____	0.000126	0.000300	0.044731	0.000202	0.004544	0.062392

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	18969.18	6698.63	3989.84	8513.74	10581.38	4574.26	4185.83
BIC	20899.12	7464.68	4402.83	9279.79	11720.72	4987.26	4598.83
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.040664	0.000105	0.000257	0.046463	0.002058	0.001574	0.081684
RMSES0	_____	0.000149	0.000550	0.063823	0.000345	0.006342	0.089005

Table B.7 The results of the goodness of fit criteria of data 20 and *h*=1 using APC model, Australian males.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	18898.55	4023.74	8563.82	10477.29	9411.63	4393.17	4553.71
BIC	20256.86	7067.92	4350.87	9134.79	11302.16	4884.00	4566.27
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.005387	0.000085	0.000153	0.008063	0.000124	0.001372	0.011279
RMSES0	_____	0.000099	0.000143	0.008454	0.000117	0.001266	0.011751
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	18088.49	5864.83	3758.06	8791.70	9456.00	4382.85	4556.80
BIC	20320.08	7079.69	4356.97	9176.29	11323.21	4911.66	4551.61

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.005905	0.000081	0.000185	0.009348	0.000111	0.000486	0.013014
RMSES0	_____	0.000089	0.000157	0.009267	0.000117	0.000474	0.012940
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	18109.02	5868.51	3759.68	8807.80	9464.68	4386.90	4548.05
BIC	20401.55	7146.48	4362.16	9184.73	11395.09	4914.33	4540.57
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.007362	0.000074	0.000161	0.011824	0.000118	0.000508	0.018632
RMSES0	_____	0.000060	0.000171	0.011554	0.000110	0.000512	0.016136
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	18118.59	5891.33	3755.51	8823.18	9479.77	4385.60	4555.92
BIC	20418.60	7197.82	4358.09	9162.57	11453.59	4905.87	4527.95
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.009908	0.000066	0.000206	0.015084	0.000110	0.001106	0.019102
RMSES0	_____	0.000069	0.000183	0.015550	0.000122	0.001036	0.021719
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	18127.93	5904.27	3743.00	8836.47	9481.29	4416.84	4545.52
BIC	20429.76	7275.01	4345.67	9128.37	11502.28	4890.31	4502.97
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.008786	0.000096	0.000281	0.012899	0.000212	0.001575	0.014202
RMSES0	_____	0.000097	0.000262	0.013789	0.000173	0.001387	0.019226

Table B.8 The results of the goodness of fit criteria of data 20 and *h*=1 using CBD model, Australian males.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	215922.80	76983.69	4341.81	9285.16	104015.50	4722.56	4451.83
BIC	216160.50	77183.53	4513.55	9485.00	104232.10	4894.30	4623.57
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.027430	0.000643	0.000177	0.017860	0.000671	0.001497	0.028538
RMSES0	_____	0.000701	0.000623	0.043044	0.000681	0.002948	0.060072
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	219134.30	78796.22	3804.07	9195.51	107594.80	4729.35	4418.55
BIC	219372.00	78996.06	4511.02	9395.35	107811.30	4901.09	4590.29
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.025853	0.000727	0.000164	0.017164	0.000709	0.000854	0.029268
RMSES0	_____	0.000783	0.000838	0.040566	0.000808	0.003574	0.056570
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	221870.50	80592.70	4355.92	9164.83	111275.80	4716.48	4410.00
BIC	222108.30	80792.54	4527.66	9364.67	111492.30	4888.22	4581.74
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.022872	0.000721	0.000199	0.025850	0.000712	0.000805	0.037417
RMSES0	_____	0.000792	0.000837	0.035886	0.000813	0.003494	0.050021
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	224958.50	82770.11	4340.76	9171.39	115855.80	4703.02	4423.73
BIC	225196.20	82969.95	4512.50	9371.23	116072.30	4874.76	4595.47

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.032008	0.000715	0.000208	0.014598	0.000676	0.001319	0.021659
RMSES0	_____	0.000784	0.001021	0.050227	0.000879	0.003250	0.070112
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	228238.50	84896.70	4338.25	9096.97	120273.10	4644.84	4406.58
BIC	228476.20	85096.5	4509.99	9296.81	120489.60	4816.58	4578.32
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.028910	0.000711	0.000354	0.015506	0.000788	0.001610	0.023758
RMSES0	_____	0.000788	0.000770	0.045365	0.000788	0.002773	0.063328

Table B.9 The results of the goodness of fit criteria of data 20 and *h*=1 using M7 model, Australian males.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	69882.51	34315.32	4038.72	8654.02	55099.85	4558.37	4248.68
BIC	70906.91	34891.05	4451.71	9229.75	55826.77	4971.37	4661.67
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.021075	0.001055	0.000177	0.014934	0.000864	0.001228	0.011906
RMSES0	_____	0.001074	0.000355	0.033060	0.000905	0.001352	0.046175

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	71915.68	35207.16	4038.90	8640.63	56925.41	4571.85	4226.99
BIC	72940.07	35782.88	4451.89	9216.36	57652.32	4984.85	4639.98
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.021259	0.001114	0.000174	0.015255	0.000878	0.000656	0.009786
RMSES0	_____	0.000928	0.000254	0.033353	0.000775	0.002308	0.046552
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	73767.76	36334.44	4049.59	8631.70	58692.73	4572.45	4217.73
BIC	74792.15	36910.17	4462.58	9207.43	59419.64	4985.45	4630.72
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.019039	0.001121	0.000194	0.020100	0.000903	0.001000	0.019663
RMSES0	_____	0.000960	0.000345	0.029866	0.000812	0.002353	0.041673
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	75990.23	37553.05	4038.95	8617.86	60620.77	4581.44	4194.07
BIC	77014.63	38128.7	4451.95	9193.59	61347.69	4994.44	4607.07
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.032009	0.001108	0.000195	0.011572	0.000854	0.000676	0.017911
RMSES0	_____	0.000832	0.000270	0.050232	0.000700	0.001917	0.070162

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	77854.02	38686.20	4037.51	8578.03	62150.76	4574.10	4175.68
BIC	78878.42	39261.93	4450.50	9153.76	62877.67	4987.09	4588.67
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.028928	0.001200	0.000301	0.012645	0.000983	0.001240	0.009910
RMSES0	_____	0.001103	0.000528	0.045389	0.000956	0.001545	0.063402

Table B.10 The results of the goodness of fit criteria of data 20 and *h*=1 using Plat model, Australian males.

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
[t–h– l, t–h] = 1990–2010; t = 2011							
AIC	18712.10	6488.83	4038.76	8551.94	10362.55	4567.99	4254.46
BIC	20177.95	7150.20	4443.57	9213.31	11285.37	4972.81	4659.28
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.005120	0.000084	0.000161	0.008122	0.000103	0.001249	0.010836
RMSES0	_____	0.000080	0.000149	0.008035	0.000109	0.001196	0.011169
[t–h– l, t–h] = 1989–2009; t = 2010							
AIC	18760.43	6500.81	4033.14	8546.14	10386.15	4580.17	4237.33
BIC	20226.28	7162.19	4437.95	9207.51	11308.96	4984.99	4642.14

	Age Full	Age Stratified					
		Scenario 1			Scenario 2		
LC	**S0**	**S1A**	**S1B**	**S1C**	**S2A**	**S2B**	**S2C**
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.006273	0.000102	0.000178	0.010110	0.000120	0.000586	0.013515
RMSES0	_____	0.000081	0.000153	0.009844	0.000112	0.000484	0.013747
[t–h– l, t–h] = 1988–2008; t = 2009							
AIC	18839.38	6569.94	4042.92	8544.32	10457.30	4583.69	4223.85
BIC	20305.23	7231.31	4447.74	9205.70	11380.11	4988.51	4628.67
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.008884	0.000079	0.000181	0.016361	0.000110	0.000759	0.020900
RMSES0	_____	0.000084	0.000167	0.013944	0.000117	0.000579	0.019475
[t–h– l, t–h] = 1987–2007; t = 2008							
AIC	18887.55	6604.38	4032.06	8524.24	10512.30	4587.18	4211.85
BIC	20353.39	7265.76	4436.87	9185.61	11435.12	4992.00	4616.67
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.008142	0.000053	0.000215	0.011605	0.000128	0.000903	0.018633
RMSES0	_____	0.000065	0.000171	0.012778	0.000113	0.000969	0.017844
[t–h– l, t–h] = 1986–2006; t = 2007							
AIC	18906.30	6671.87	4024.13	8490.74	10570.63	4573.44	4188.69
BIC	20372.15	7333.24	4428.95	9152.11	11493.44	4978.25	4593.51
Res	Random	Random	Random	Random	Random	Random	Random
RMSE	0.006142	0.000068	0.000299	0.008155	0.000167	0.001395	0.011829
RMSES0	_____	0.000062	0.000302	0.009637	0.000184	0.001260	0.013415

l: look back window 20–years

t: year of prediction
[*t–h– l*, *t–h*]: modelling time
Scenario 0 (S0): [0–100]
Scenario 1 (S1): S1A=[0–40], S1B=[40–60], S1C=[60–100]
Scenario 2 (S2): S2A=[0–60], S2B=[60–80], S2C=[80–100]

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