

Transition from open-pit to underground – using Mixed Integer Programming considering grade uncertainty

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Abstract

The determination of transition point from open-pit (OP) to underground (UG) is one of the most challenging mining engineering problems. The shallow deposits which vertically extend to considerable depth will have potential to make the transition from OP to UG. In this paper, Mixed Integer Programming (MIP), a well-known operation research technique is introduced to solve the transition problem. Conditional simulation is utilised to generate realisations of the resource model. Thus, the utilisation of the simulated realisations will reduce the geological uncertainty and risk. The aim of this paper is to incorporate MIP model and conditional simulation technique to optimise the transition from OP to UG. As a result, a ‘Transition Envelope’ which represents a zone where the transition might occur is formed.

1 Introduction

Mining projects start with exploration and sampling stage to gather information of deposit and generate a representative geological model. The geological model can be converted into an economic block model. At the stage of seeking an appropriate mining method, the question “What would be the optimal mining method, open-pit (OP), underground (UG) or combination mining?” is often asked (Topal, 2008). The OP mining strategy is usually engaged to mine near surface orebodies and UG is employed to mine deep deposits. However, if the orebody extends to a considerable depth, the deposit will have potential to switch to underground to mine the deeper part of the orebody when OP mining fails to generate sufficient profit over UG mining. This is referred to as the combination mining method and the schematic of this method is shown in Figure 1(a).

In the practice of combination mining method, ‘Transition Problem’ which is to determine the ‘Transition Point’ always exists. ‘Transition Point’ is where the decision has to be made whether expand the pit or make the transition from OP to UG. The general view of the transition problem is illustrated in Figure 1(b).

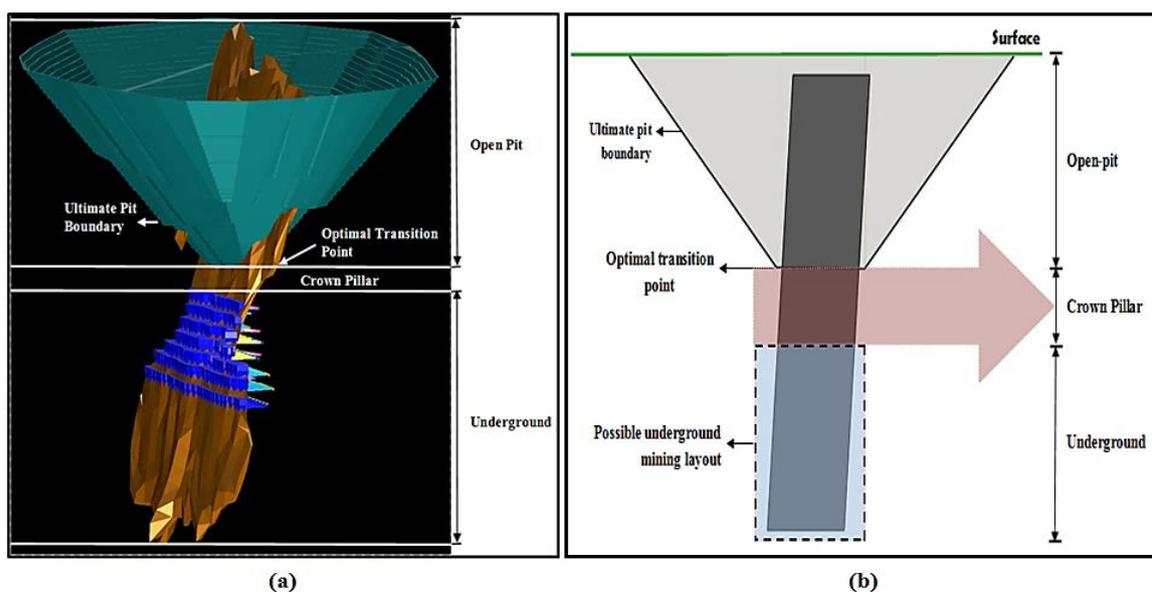


Figure 1. Schematic of (a) combination mining method (b) transition problem

The quantification of uncertainty has the major implications for mine planning and design, as it has a huge financial impact on the mining operation. Mine planning and optimisation have been discussed broadly in the past. Currently, there are several mathematical models and methodologies available to optimise open-pit (OP) and underground (UG) mining operations, such as Lerchs and Grossmann (1964), Johnson (1968), Dimitrakopoulos et al. (2007), Topal and Ramazan (2012), Little and Topal (2011), Brazil et al. (2005), Alford (1995) and others. The key concern of considering both grade uncertainty and mine optimisation is mainly the complexity of the resource model. It is in fact better to be approximately right rather than completely wrong (Dimitrakopoulos et al., 2002). Dealing and managing the risk and uncertainty leads the operations to be performed closer to their potential.

Conventionally, most of the mining projects are completed using a deterministic resource model. The application of a single resource model may lead to over or under-estimate expected value of the project. Therefore, due to the complexity of the mining process, understanding of the uncertainty assessment in term of technical, economic and environmental sources is essential. The technical source is burdensome, particularly in geological characteristics; there is no way that the details of the resource can be fully understood. Therefore, there is a necessity to consider the geological uncertainty in the mine planning and optimization process in order to increase the utility of the resources.

1.1 Background and problem definition

The determination of optimal transition point, which is known as transition problem remains as the most challenging mining engineering issue due to the complexity of the problem. In the literature, many researches have been carried out and discussed by Soderberg and Rausch (1968), Nilsson (1982), Camus (1992), Fuentes (2004), Bakhtavar and Shahriar (2007), Bakhtavar et al. (2012), Opoku and Musingwini (2013) and Newman et al. (2013). However, none of them is able to generate a robust and optimal solution for this problem in three-dimensional space.

Generally, in the mining project with stand-alone resource model, the expected grade and reserves of the deposit can be over or under-estimated. This is known as geological uncertainty and risk. To reduce the geological risk and increase the opportunity of having the real value of the resource, the risk and uncertainty need to be integrated into the mine planning and optimisation process. Thus, the maximum reserve utility can be achieved with minimal operation risk.

Considering grade uncertainty, pit boundary and allocation of UG resources may vary. Thus, transition point may alter under grade uncertainty, as shown in Figure 2. As the transition point is changing according to different resource realisations (considering grade uncertainty), a 'Transition Envelope' can be generated. 'Transition Envelope' is the region where the transition from OP to UG can possibly happened. Figure 2 illustrates how the transition envelope is created to account for the transition problem under the grade uncertainty.

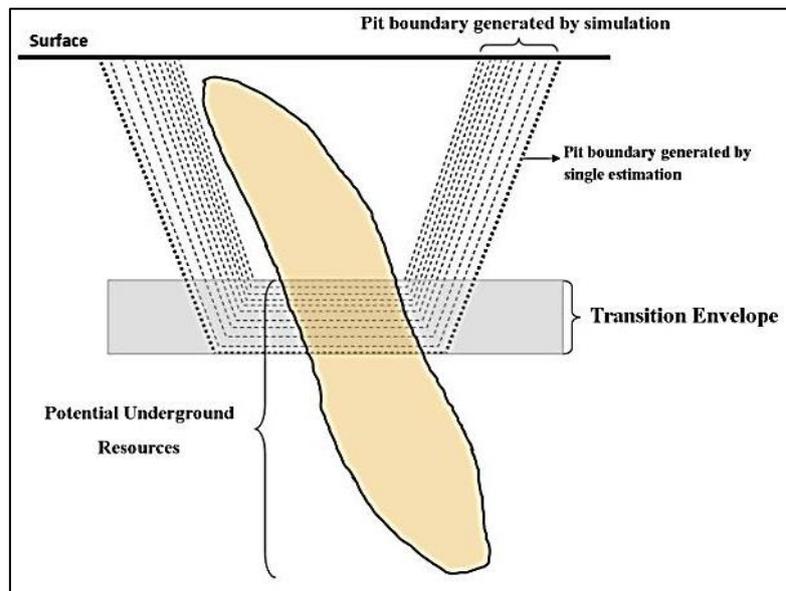


Figure 2. Relation between the transition problem and geological uncertainties

Considering the transition problem, OP and UG reserves have an inversely proportional relation. That is, when the OP reserve increases, the UG reserve will decrease – some resource will be within the pit and others will be left as crown pillar (CP) and vice versa. In order to maximise profit of the mining project, it is imperative that the OP and UG reserves be considered simultaneously, as well as CP. Therefore, the mixed integer programming (MIP) model utilised in this paper will be able to achieve this goal.

This paper presents a methodology which optimises the transition point and mining strategy selection under the grade uncertainty. The strategy selection and transition problem will be formulated by using MIP and sequential Gaussian simulation (sGs) method will be used to generate equally probable grade realisations to represent the resource model. 20 realizations will be generated and transition point will then be optimised by using the MIP model. The results will be presented and discussed.

2 Stochastic optimisation modelling

2.1 Grade uncertainty modelling

Conditional Simulation is a Monte Carlo simulation approach that is developed to model the uncertainty in the spatially distributed attributes such as grade. The aim of this approach is to produce the equally probable realisations representing grade variability within the deposit. The difference between the conditional simulated deposits will be able to capture the variability in grade (Dimitrakopoulos et al., 2002).

Traditionally, kriging is used to estimate the resources, which uses the standard deviation as the estimation at each grid node. The sGs is introduced in this study with the aim of generating multiple realisations of a pertinent attribute by reproducing the available data. The steps comprised in the sGs process are as follows (Dimitrakopoulos, 2009):

- Step 1: Normalise and standardise the sample data.
- Step 2: Compute and model the variogram of the normalised data.
- Step 3: Choose a random path that goes through each grid node.
- Step 4: Conduct kriging of the normalised value of the selected node using both actual and simulated data to estimate the normal local conditional distribution.
- Step 5: Simulate the value by randomly sampling the estimated normal local conditional distribution which is having the kriging estimate and its variance as mean and variance respectively.
- Step 6: Add the simulated value to the conditioning data set.
- Step 7: Move to the next grid node and go to Step 1; Stop if all nodes are simulated.
- Step 8: “Denormalise” the simulated values and check validity of the results.

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$$

Non – negativity restrictions: $x_1, x_2, \dots, x_n \geq 0$

Where, the z represents the objective function to maximise or minimise the goal and x_j represents the decision variable. The a_{ij} and c_j represent the nature of the problem and the b_j represents the availability of the resource. When the decision variables represent undividable entities, then it needs to be an integer which formed an integer programming (IP). The integers “0” and “1” – binary variables are used to capture the decision of “yes” and “no”; therefore, decision making process formed, 0 and 1 will be indicated to a decision of “non-extracting the block” or “extracting the block” respectively. When the model consists of both integral and continuous variables, the MIP model is then formed.

The MIP model is formulated in 3-D. The objective function of the MIP is to maximise the undiscounted profit from both OP and UG mining. The objective function is formulated as follow:

$$\text{Max } Z = \sum_{i \in B} C_i x_i + \sum_{j \in D} C_j y_j$$

where,

i	<i>Block reference name i for open pit mining</i>
j	<i>Block reference name j for underground mining</i>
C_i	<i>The undiscounted profit to be generated by mining Block i</i>
C_j	<i>The undiscounted profit to be generated by mining Block j</i>
B	<i>Set of all blocks in block model for open pit mining</i>
D	<i>Set of all possible block in block model for underground mining</i>

The constraints of the model are:

- Slope constraint which holds the OP slope restrictions. In the application of 45 degree conventional wall slope, it forces that all of the overlying blocks are required to be removed in order to access an ore block. For example, in three-dimensional case, five overlying blocks will need to be removed in order to mine an ore block underneath.
- UG stope design constraint which prevents the overlapping stope formation. For instance, the blocks consist in a mineable stope profile will not be part of another mineable stope profile.
- ‘Only one mining method for each level’ constraint is constructed to make sure that each level can only be mined through one mining method. For instances, if a block is mined through OP mining, the entire level will mine through OP mining only, vice versa.
- Reserve restriction constraint is used to ensure that each row can be either mined through OP mining, UG mining or leave-in as crown pillar (CP) or waste.
- Crown pillar constraint is set to make sure the number of row to be leave-in as CP, based on geotechnical requirements.
- Non-negativity constraint which satisfied the non-negativity integrality of the variables as required.

3 Application and result discussions

3.1 Profile and parameters of the case study

The three-dimension (3-D) hypothetical gold deposit is used in this study to demonstrate how the grade uncertainty of the deposit will affect the optimal transition depth. The hypothetical case study consists of 38,584 blocks with block size of $40 \times 40 \times 20m$. The designed stope size is $2 \times 2 \times 2$ blocks. For safety restriction, at least two levels (40m) need to be retained as CP. Figure 4 illustrates a simulated realisation of the resource model.

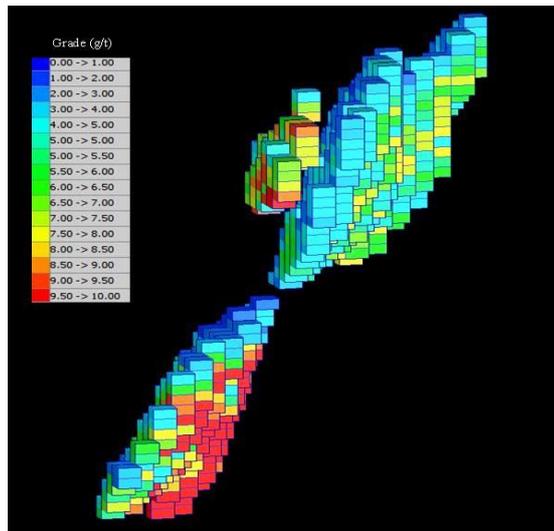


Figure 4. Resource model configuration of a simulated realisation

The MIP models were written using Microsoft Excel VBA (2010). The linear programming problems were solved using ILOG Corp. (2013) on a standard office computer which is Dell OPTIPLEX 9020 with Intel Core i7 3.40 GHz CPU and 8 GB installed. The key statistics are demonstrated in Table 1. It can be seen that each problem with huge number of variables is impossible to be solved manually. With the help of CPLEX solver, the problems are solved with the average solution time of 13.06 minutes.

Components	Model information	
Problem Size	OP Blocks	38,584
	UG Blocks	38,584
	Constraints	187,671
	Binary Variables	77,327
Average Solution Time (minutes)	13.06	

Table 1: Problem size and Solution Time

3.2 Result discussions

As mentioned earlier, 20 realisations have been generated. These block models were then solved with the MIP model to observe the effect of the grade uncertainty on transition point. The steps comprising in the whole process is presented in Figure 5.

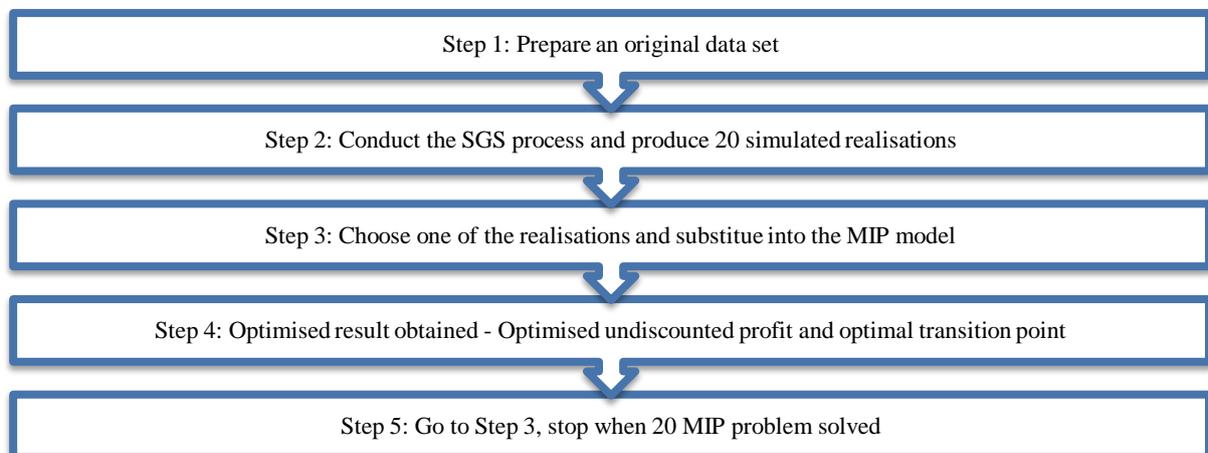


Figure 5. Steps to incorporate the simulation process and the optimisation process

3.2.1 Economic comparison of the simulated realizations

The results generated by the MIP model using 20 realisations are presented in Figure 6. As can be seen from Figure 6, the project undiscounted profit varies drastically, there are about 85% of the outcomes targeted more than \$216 million undiscounted profit and 30% of the outcomes achieved more than \$219 million of undiscounted profit.

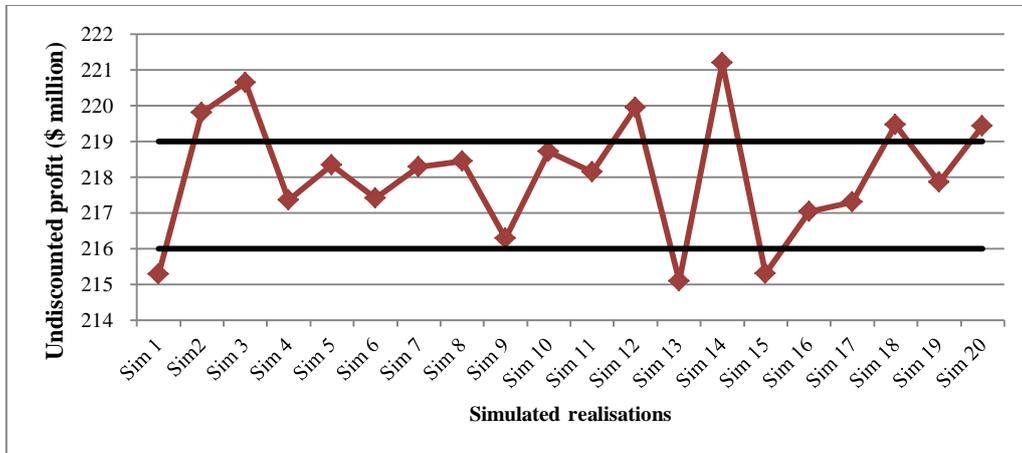


Figure 6. Undiscounted profit for each simulation

The results generated and presented in Figure 6 have been transformed into normal distribution and illustrated in Figure 7. In Figure 7, it shows that there is 90% probability that the project will be returning an undiscounted profit more than approximately \$215 million. The mean undiscounted profit from the conditional simulation is approximately \$218.4 million and there is a probability of 10% that the project will be able to achieve equal or more than an undiscounted profit of \$220 million. From the results from both Figure 6 and Figure 7, it can be observed that if Sim 1 is considered as the base case, it can lead to underestimate the value of the project. In addition, if Sim 14 is considered as the base case, then it will lead to over-estimated of the project's value.

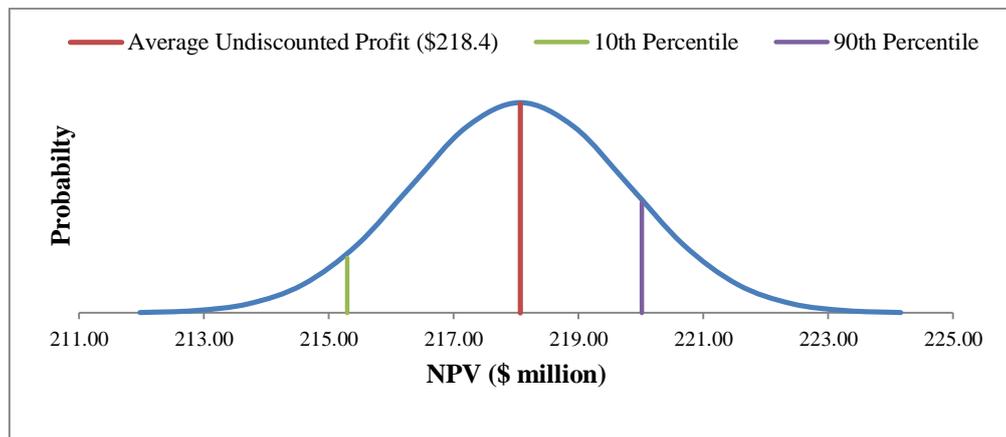


Figure 7. Distribution of undiscounted profit from Conditional Simulated Realizations

3.2.2 Profile of optimal mining strategy and transition envelope

The result generated from the MIP model also gives guidance to the optimal mining strategy selection. Figure 8 illustrates the optimal mining strategies for each proportion of the simulated deposits. There is no consistency of the CP proportion. As mentioned in section 3.1, minimum two rows need to be retained as CP. Additionally, a block or row can be treated in three ways which are: (1) mine out (2) leave as a waste (3) leave as CP. Therefore, if there are more than two rows retained as CP, it can be defined as the combination of CP and waste.

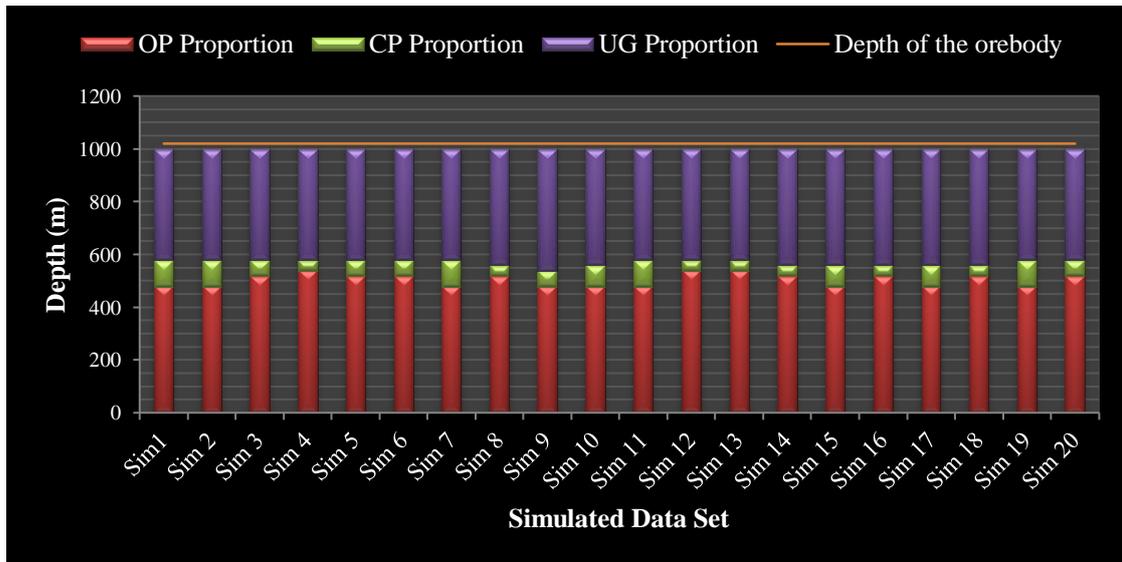


Figure 8. Chart represents the proportion for each mining strategy for each simulated realisations

From the results presented in Figure 8, the optimal transition point created by each simulated realisation is plotted in Figure 9. In Figure 9, it is clearly seen that there are three optimal transition depths obtained by the 20 simulated realisations which are 480m, 520m and 540m. According to the section 1.1 (Figure 2), ‘transition envelope’ created when there is a range of optimal transition depth created by the simulated realizations. Therefore, the simulated realizations used in this paper have created the optimal transition envelope which is between 480m and 540m as shown in Figure 9.

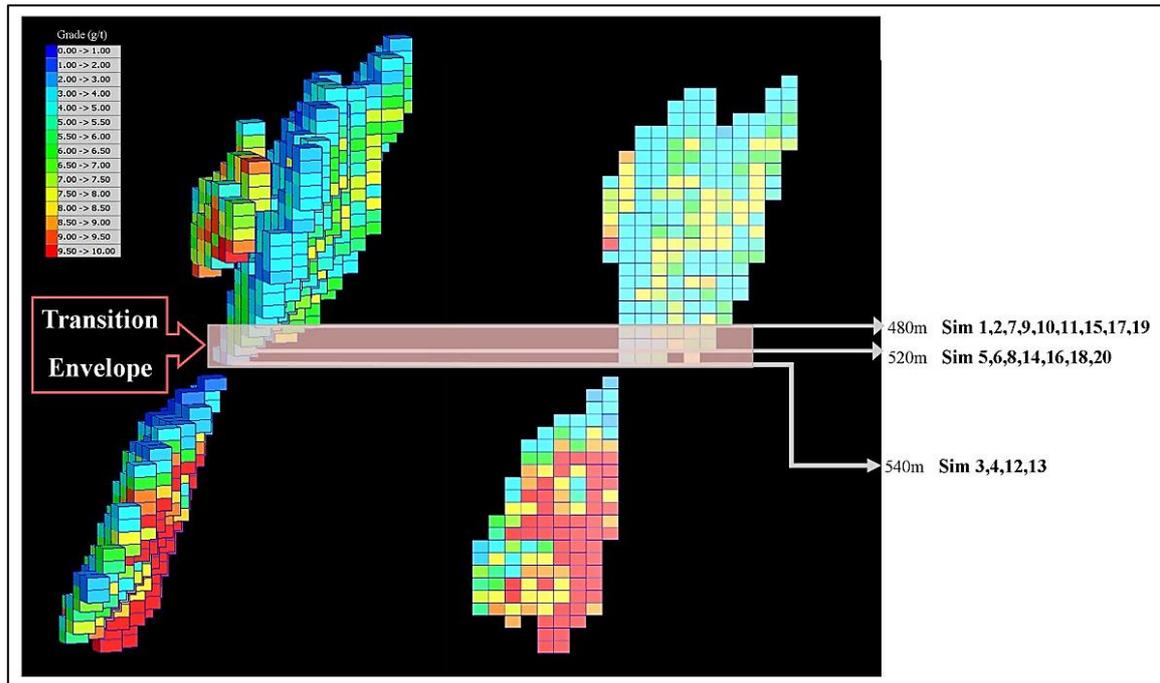


Figure 9. Optimal transition points for the simulated realisations

4 Conclusions and recommendations

The optimisation of the transition from OP to UG mining has been discussed broadly over the past decades. Despite in fact that some researches have been complete in the past, the optimisation of the transition problem still remains unsolved. Thus, MIP model is proposed to solve the transition problem optimally in 3-D basis. The MIP model used in this paper is able to determine: (1) optimal transition point and depth (2) maximised project’s value and (3) appropriate mining strategy.

Traditionally, in the process of mining a deposit, a single resource model has been used for the study and planning purposes. However, in this case, the over or under-estimated the project's value will occur as the geological uncertainty and risk are not taken into consideration. Therefore, in order to reduce the risk and uncertainty, the paper proposes to incorporate the conditional simulation – sGs technique with the mine planning and optimisation process of the transition problem. As a result, a '*transition envelope*' which indicates the possible transition zone for combination mining method was formed.

The 3-D case study consisting 38,584 blocks was used to demonstrate the process of mine planning and optimisation incorporating geological uncertainty. In the grade uncertainty modelling stage, 20 equally probable simulated realisations were generated by using the SGS algorithms. These realisations are able to demonstrate the variability of the resource model. Each of the 20 realisations was considered as an independent input of the MIP model; therefore, 20 MIP problems were generated and solved. The results obtained from the MIP model considering the simulated realisations that: (1) there is approximately 85% probability that the project will return more than \$216 million undiscounted profit (2) the optimal transition envelope was formed between level 24-- 480m and level 27-- 540m.

The process of incorporating MIP approach with the geological uncertainty for transition problem is still an immature methodology. Further improvement is required to solve computationally intensive problems using proposed method. Thus, further research will be undertaken in the future to improve the performance of the current method.

5 References

- Alford, C. (1995). Optimization in Underground Mine Design. Proceedings of 25th International APCOM Symposium, Australia: Australasian Institute of Mining and Metallurgy.
- Bakhtavar, E. and K. Shahriar (2007). Optimal Ultimate Pit Depth Considering an Underground Alternative. Proceedings of Fourth AACHEN International Mining Symposium-High Performance Mine Production, Germany.
- Bakhtavar, E., K. Shahriar, and A. Mirhassani (2012). Optimization of the Transition from Open-Pit to Underground Operation in Combined Mining Using (0-1) Integer Programming. Journal of the Southern African Institute of Mining and Metallurgy 112 (12), 1059-1064.
- Brazil, M., A.T. Doreen, J.F. Weng, J.H. Rubinstein, and D.H. Lee (2005). Cost Optimisation for Underground Mining Networks. Optimization and engineering 6 (2), 241-256.
- Camus, J.P. (1992). Open Pit Optimization Considering an Underground Alternative. Proceedings of 23th International APCOM Symposium, Tucson. Society of Mining, Metallurgy, Exploration. pp435-441.
- Dimitrakopoulos, R. (2009). Ore Reserve Risk and Mine Planning Optimization. In Stochastic Models and Optimization with Applications, Course Notes MIME 525 & MIME 513. Canada: McGill COSMO - Stochastic Mine Planning Laboratory, Department of Mining and Materials Engineering.
- Dimitrakopoulos, R., L. Martinez, and S Ramazan (2007) . A Maximum Upside/Minimum Downside Approach to the Traditional Optimization of Open Pit Mine Design. Journal of Mining Science 43 (1), 73-82.
- Dimitrakopoulos, R., C.T. Farrelly, and M. Godoy (2002). Moving Forward from Traditional Optimization: Grade Uncertainty and Risk Effects in Open-Pit Design. Mining Technology 111 (1), 82-88.
- Fuentes, S.S. (2004). Going to an Underground Mining Method. Proceedings of MassMin Conference, Santiago, Chile.
- ILOG Corp. 2013. Cplex.
- Johnson, T.B. 1968. Optimum Open Pit Mine Production Scheduling. Berkeley.

- Lerchs, H., and F.I. Grossmann. (1964). Optimum Design of Open-Pit Mines. Operation Research: Institute Operations Research Management Sciences.
- Little, J. and E. Topal (2011). Strategies to Assist in Obtaining an Optimal Solution for an Underground Mine Planning Problem Using Mixed Integer Programming. International Journal of Mining and Mineral Engineering 3 (2), 152-172.
- Microsoft Excel VBA. 2010.
- Newman, A.M., C. A. Yano, and E. Rubio (2013). Mining above and Below Ground: Timing the Transition. IIE Transactions 45 (8), 865-882.
- Nilsson, D. (1982). Open-Pit or Underground Mining. In W.A Hustrulid, editor, Underground Mining Methods Handbook. The American Institute of Mining, Metallurgical, and Petroleum engineers, Inc., New York, pp70-87.
- Opoku, S. and C. Musingwini. (2013). Stochastic Modelling of the Open Pit to Underground Transition Interface for Gold Mines. International Journal of Mining, Reclamation and Environment 27 (6), 407-424.
- Soderberg, A. and D.O. Rausch. (1968). Pit Planning and Layout. In E.P Pfeider, editor, Surface Mining. The American Institute of Mining, Metallurgical, and Petroleum Engineers, Inc., New York, pp142-143.
- Topal, E. (2008). Early Start and Late Start Algorithms to Improve the Solution Time for Long-Term Underground Mine Production Scheduling. Journal of the South African Institute of Mining & Metallurgy 108 (2), 99-107.
- Topal, E. and S. Ramazan (2012). Strategic Mine Planning Model Using Network Flow Model and Real Case Application. International Journal of Mining, Reclamation and Environment 26 (1), 29-37.