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Performance of inclined pillars with a major discontinuity

K.V. Jessu *, A.J.S. Spearing

Western Australia School of Mines Kalgoorlie, Curtin University, Australia

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ABSTRACT

Discontinuities are an inherent part of the rock mass and majorly affect the stability of the excavation skin and pillars. The dip of the discontinuities and their properties also have a significant effect on the strength of the pillars. Empirical approaches are commonly used to determine the pillar strength but can overestimate the strength and don't consider the inclination of the pillars and the strength reduction caused by discontinuities. Numerical modeling is a powerful tool and if calibrated can be used to evaluate the strength of the pillars with discontinuities having a range of properties. The effect of a discontinuity on inclined pillars was conducted which has been seldom considered in evaluating the pillar strength. Three-dimensional vertical pillars were simulated, and the pillar strength was calibrated to accepted theoretical results and then the discontinuities were introduced in different pillar inclinations with distinct width to height ratios to gain an insight into the effective pillar strength reduction. Based upon the results, it was found that the discontinuities have a significant effect with the increase in the inclination of the pillars even at a higher width to height ratios.

1. Introduction

Pillars are frequently left in the underground mines to stabilize the relatively large openings. It is necessary to efficiently design the pillars. Under-designed pillars can lead to abrupt failure and can lead to catastrophic pillar failure such as in a room and pillar metal mine, failure of four pillars at the center of the section resulted in a failure of almost 100 pillars [1,2]. Over-designed pillars can lead to an economic loss due to the excess ore left in the mines. The design of the pillars is also dependent on the properties of the ore deposit and surrounding rockmass and the adopted mining method to extract the ore. The flat or shallow dipping ore deposits are generally extracted by employing the room and pillar mining method where the pillars act as the primary roof support in the excavations and often barrier pillars are used as the regional support for the panels. If the ore bodies are steeply dipping, the rib pillars are used as the primary support in the stopes while the sill pillars are used to distinguish the two mining horizons. The crown pillars are the pillars left behind to establish the stability of the mine to the surface. This study deals with the pillars which are used as the primary support.

Depending on the orientation of the ore body, the pillars left in the vertical direction will lead to compression loading and the pillars in the inclined direction will undergo combined compression as well as shear loading as shown in Fig. 1. The loading on the pillars lead to brittle failure in slender pillars and in larger pillars can lead to spalling and shear failure which is the common pillar failure mechanism. Other pillar failure mechanisms include axial splitting [3,4] and in the presence of the discontinuities, sliding along the shear plane [5].

Empirical approaches are mainly relevant on vertical pillars and one of the earliest studies was conducted by Hedley and Grant [6] on the Quartzite pillars in Canada by classifying them into stable, unstable and failed categories and derived a relationship between the pillar strength, and width-to-height ratio. Lunder and Pakalnis [7] introduced the confinement parameter in developing the strength of hard rock pillars and it is the most commonly used empirical approach in the underground mines. The design of the pillars is given as:

$$\sigma_p = K * UCS * (C1 + C2 * \kappa) \quad (1)$$

where σ_p is the ultimate strength of the pillar (MPa), K is the pillar size factor given as 0.44, UCS is the uniaxial compressive strength of the intact rock (MPa), C1 and C2 are the empirical rock mass constants given as 0.68 and 0.52 respectively and κ is the friction term which is calculated as:

$$\kappa = \tan\left[cos^{-1}\left(\frac{1 - C_{pav}}{1 + C_{pav}}\right)\right] \quad (2)$$

$$C_{pav} = Coeff * \left[Log\left(\frac{W}{H} + 0.75\right)\right]^{1.4(W/H)} \quad (3)$$

* Corresponding author.
E-mail address: kashijessu@gmail.com (K.V. Jessu).

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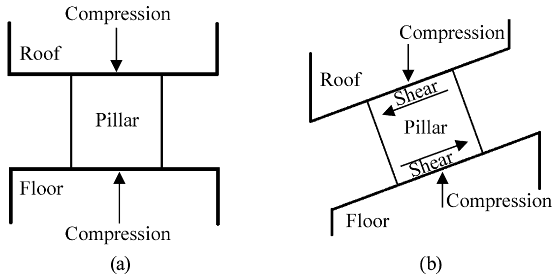

Fig. 1. (a) Vertical pillars undergoing compression (b) inclined pillars undergoing oblique loading.

where C_{pav}, the average pillar confinement and the Coeff is the coefficient of pillar confinement.

Research conducted on the inclined pillars concluded that the pillars dipping at angle undergo oblique loading which is a combination of compressive and shear stress and therefore have lower strength when compared to the vertical pillars [8,9]. However, the empirical designs are only developed based on stable and failed pillars without taking into consideration the failure behavior of the pillars and the effect of discontinuities in the pillars.

Rock mass consists of fractures and discontinuities that affect the stability of the excavations especially when significant discontinuities traverse the pillars. Discontinuities are discrete plane structures in the rock mass which may or may not lead to slippage along the plane as shown in Fig. 2a. Jaeger and Cook [10] evaluated that the strength of the rock sample reduces significantly in the presence of the discontinuities with respect to its orientation to the stress direction as shown in Fig. 2b. Similar behavior has been observed in the pillars in the presence of the discontinuities such that the strength of the pillar is significantly influenced by the orientation of the discontinuity and the size of the pillar [11].

One of the earliest studies on the influence of the discontinuities has been conducted in the South African Coal fields by estimating the failure of the pillars and classifying the pillars by pillar condition rating system [12]. It was simply based on the visual inspections and scaling of the fractures and limited to the coal pillars in the South African Coal mines database. Iannacchoine [13] conducted studies on determining the dip of the discontinuities effecting the pillars strength and derived that the pillar strength is significantly affected at 60° discontinuity dip angle. Esterhuizen [14,10] conducted numerical modelling studies on vertical pillars with discontinuities and developed a relationship between the strength of the pillars, discontinuity dip direction and size of the pillars in the coal and limestone mines. It was concluded that the smaller pillars are highly affected by the discontinuities and derived strength for every 10° angle of discontinuity.

This paper will present the numerical analysis of inclined pillars with discontinuities. The inclined pillars with varying width to height ratios will be investigated against different orientations of the discontinuities to develop an understanding of the pillar strength reduction.

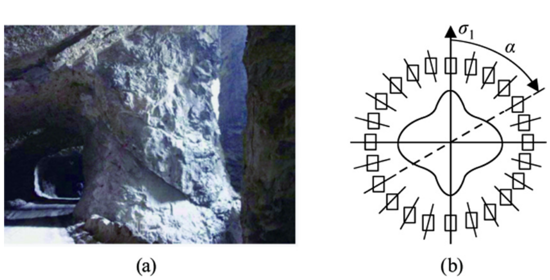

Fig. 2. (a) Example of discontinuity in a pillar [11]. (b) Sample strength along the discontinuity dip angle [10].

2. Model configuration

Numerical modeling is a very powerful tool due to its ability to model complex geometries and incorporate varied material behavior to provide a valuable insight into the failure modes of the pillars if calibrated correctly. FLAC^{3D} [15], a three-dimensional finite difference numerical modeling package was used to simulate the vertical pillars and inclined pillars and to analyze the pillar strength reduction in the presence of the discontinuities. This package was selected as it has the capability to realistically simulate the failure process of the pillars [10]. Vertical pillars and inclined pillars were simulated as shown in Fig. 3.

The x, y, and z coordinate space was used to simulate the pillars in the model. The horizontal plane is represented in the x and y-direction while the vertical plane is presented in z-direction. The inclination of the pillars is towards the x-direction as shown in Fig. 3b. The model consists of main floor, pillar and main roof with height being constant in all the three sections. The height of the pillar was 4 m which was majorly adopted from the Lunder and Pakalnis (1997) database [7]. The pillar width in x and y-direction is varied to achieve different width to height ratios in the model. The extraction ratio of the vertical pillars is kept constant at 75% and for the inclined pillars, the boundaries were established far enough from the pillar when compared to vertical pillars to avoid its influence on the pillar failure behavior. The height of the roof and floor have been kept three times that of the pillar height to avoid boundary effects on the pillars.

Boundary conditions were established by placing fixed supports at the bottom of the floor with which the displacement and velocity are restricted in normal and parallel directions. Roller supports were introduced on the sides to restrict the displacement and velocities in the normal direction. Models were subjected to uniform velocity on the top of the roof to simulate the loading as recommended by Lorig and Cabrera [16] with which vertical pillars undergo compression and the inclined pillars undergo oblique loading.

Stresses were applied to the models to simulate 100 m mining depth with horizontal to vertical stress of about 1:1 and the models were run to equilibrium under elastic conditions. The material properties of the pillar were transformed to bilinear material from the elastic condition after reaching the equilibrium. This was undertaken to reproduce the rock mass behavior before the excavation as elastic; and after the excavation as brittle and plastic. Then the uniform velocity was applied to the model until the pillar had completely failed and reached the residual strength equal to 50% of the peak strength. FISH code, an inbuilt language to implement user-defined functions and variables, was used to develop stress-strain relationship to determine the peak strength of the

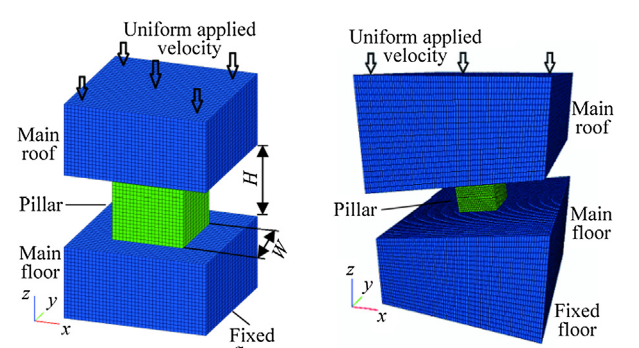

Fig. 3. Numerical models (a) vertical pillar (b) 10° angled pillar.

pillar. The roof and the floor rock mass were modelled using an elastic criterion which is stronger than the pillar to ensure that the failure is induced in the pillar.

The most important part of the numerical modeling is the constitutive model and the input properties to ensure the simulation produces realistic results. Pillars are best simulated by implementing brittle Hoek-Brown criterion [17,18,10]. This criterion basically refers to the brittle nature of the pillars at 0.3–0.5 times of the uniaxial compressive strength which develops brittle cracks in the pillars followed by the shear failure. Therefore, a bilinear strength envelope was introduced in which the strength is equated to one-third of the uniaxial compressive strength and independent of the friction at low confinement subsequently followed by friction hardening at the higher confinement [18].

A constitutive model best representing the bilinear strength envelope can be simulated by bilinear strain hardening/softening ubiquitous joint model which is based on the Mohr Coulomb strength criterion and strain hardening/softening as a function of the deviatoric plastic strain [15]. The input properties for the model are as shown in Tables 1 and 2 [19]. The discontinuities in the pillar (Fig. 4) were introduced in the form of Discrete Fracture Network (DFN) with the properties as shown in Table 2.

The model element size is a critical parameter for the strain softening properties which can be determined by simulating all the models with same element size and calibrating the numerical model to the theoretical results [15]. All the models were run at same element size of 0.5 m * 0.5 m * 0.5 m throughout the model and the cohesion softening was carried out to calibrate the vertical pillar results to that of the Lunder and Pakalnis [7].

2.1. Model calibration

2.1.1. Numerical model validation

The model was simulated with four different width to height (W/H) ratios of 0.5, 1.0, 1.5 and 2.0 to calibrate numerical model results to that of the theoretical results of Lunder and Pakalnis [7] as shown in Fig. 5. The results obtained from the numerical models are similar to that of the theoretical results.

2.1.2. Inclined pillar strength validation

The inclined pillars were simulated next with different inclinations from the horizontal of 0°, 10°, 20°, 30° and 40°. These simulations were carried out with different width to height ratios of 0.5, 1.0, 1.5 and 2.0 and the results are shown in Fig. 6. These results are similar to that of the results presented by Suroineni [9] and Ma [20].

Fig. 6 shows that the pillar strength decreases with the increase in the inclination of the pillars at any width to height ratio. It was also observed that at lower W/H ratio, there is only slight decrease in the strength of the pillars while at higher W/H ratio, there is a drastic reduction in the pillar strength at higher inclinations when compared to the pillar under pure compression. For example, at W/H ratio of 2.0, the strength of the 40° inclined pillar is 43% less than the vertical pillar. It is because the pillars in vertical direction undergo brittle failure in pillars with W/H ratio of 0.5 and the transition from the brittle to shear failure occurs in between the pillars with W/H ratio of 0.5–1.0. While in the inclined pillars, the brittle to shear transition occurs in pillars with higher W/H ratios as the inclination increases. The brittle failure is the dominant failure mechanism with the increase in the inclination of the pillars, and thus cause the reduction in the strength of the inclined pillars [21].

Table 1
Model properties [19].

Rock mass properties	Numerical value
Bulk Modulus	40,000 MPa
Shear Modulus	24,000 MPa
Intact Rock Strength (UCS)	150 MPa
Cohesion (Brittle)	25 MPa
Friction (Brittle)	0°
Cohesion (Mohr-Coulomb)	8.1 MPa
Friction (Mohr-Coulomb)	47.6°
Tensile strength	2.7 MPa
Dilation angle	30°

Table 2
Joint properties [19].

Joint properties	Numerical value
Joint Cohesion	1 MPa
Joint Friction	42°
Joint Tension	0.4 MPa
Joint Dilation	0°

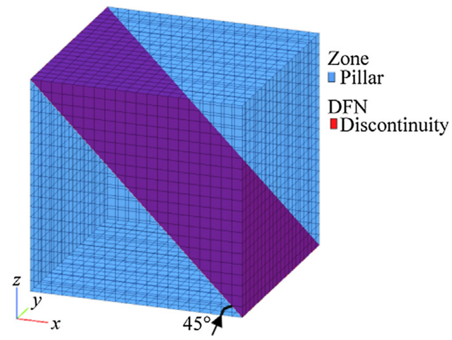

Fig. 4. Discontinuity in the pillar.

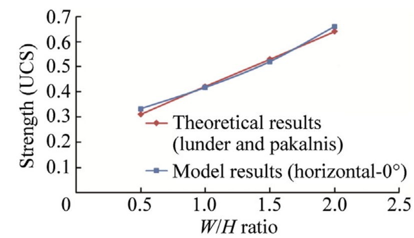

Fig. 5. Validation plot of numerical models to that of the theoretical results.

2.1.3. Validation of discontinuity effects on vertical pillars

To evaluate the strength of the vertical pillars with a major discontinuity, a single large DFN was included in the pillars passing through the center of the pillar extending to the sides of the pillar. The discontinuity dip angle was kept parallel to the pillar face (i.e. Strike = 0°) as shown in Fig. 4. The models were simulated at every 10° inclination increase of the discontinuity from 0° to 90° inclination at W/H ratios of 0.5, 1.0, 1.5 and 2.0. There-

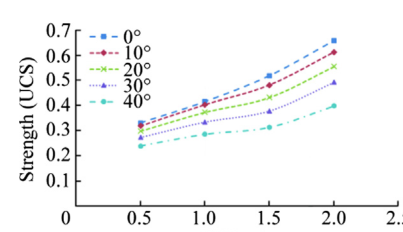

Fig. 6. Strength variation of the inclined pillars at a varied width to height ratios.

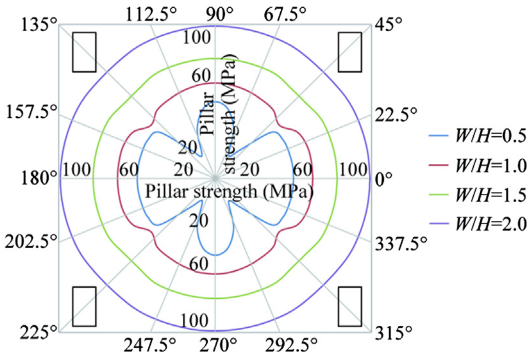

Fig. 7. Effect of discontinuity dip angle on strength of vertical pillars.

fore, in total 36 models were simulated to understand the effect of discontinuity on the strength of the pillars. The results are shown in Fig. 7 in which the graph is shown in the form of polar co-ordinates (r, θ) where r is represented as the pillar strength and the θ is the angle of inclination of the discontinuity. Fig. 7 also shows the results beyond 90° discontinuity inclination which are represented as the mirror image of the results from the first quadrant. For example, the discontinuity dip angle of 45° would bear the same strength as the discontinuity dip angle of 135° as they are both intersecting the pillar at 45°. Therefore, the second, third and fourth quadrants are the mirror images of the first quadrant.

It is observed from Fig. 7 that in pillars with W/H ratio of 0.5, the discontinuity dip angle of 60° results in significant reduction of the pillar strength which resembles the results from Esterhuizen [10]. It can also be observed that at higher W/H ratios, the discontinuity dip angle has a little to no effect on the strength of the pillars. Therefore, it can be noted that the in vertical pillars, the pillars with lower W/H ratio are significantly affected by the discontinuity dip angle parallel to the pillar face while the effect of discontinuity on the pillar strength diminishes with increase in W/H ratio.

3. Results and discussions

3.1. Evaluating the effect of discontinuity on inclined pillar strength

To understand the effect of discontinuity on the strength of the inclined pillars, four different inclination (10°, 20°, 30° and 40°) of the pillars were simulated. The discontinuity dip angles were always measured from the horizontal axis (positive x-axis) of the pillar. As the 45° discontinuity intersecting an inclined pillar is not similar to that of the 135° discontinuity dip angle, therefore, the pillar strength was analyzed at every 10° discontinuity dip angle from 0° to 180°. The discontinuity effect on pillars was measured on four different width to height ratios of 0.5, 1.0, 1.5 and 2.0. Therefore, in total 288 models were simulated to understand inclined pillar strength reduction in the presence of discontinuity.

In inclined pillars, the discontinuity angle can be classified into two categories, where the discontinuity inclined towards the dip of the pillar and the discontinuity inclined against the dip of the pillar. For example, in 20° inclined pillar, the 45° discontinuity dip angle is inclined along the pillar inclination, therefore, this angle can be treated as discontinuity towards the dip of the pillar. While the 135° discontinuity dip angle is inclined opposite to the inclination of the pillar, therefore, this discontinuity can be treated as discontinuity against the dip of the pillar.

It can be observed from Fig. 8a that in 10° inclined pillars with W/H ratio of 0.5, the 70° and 130° discontinuity dip angles significantly affect the strength of the pillar. In 20° inclined pillar with W/H ratio of 0.5 (Fig. 8b), pillar strength is majorly affected at

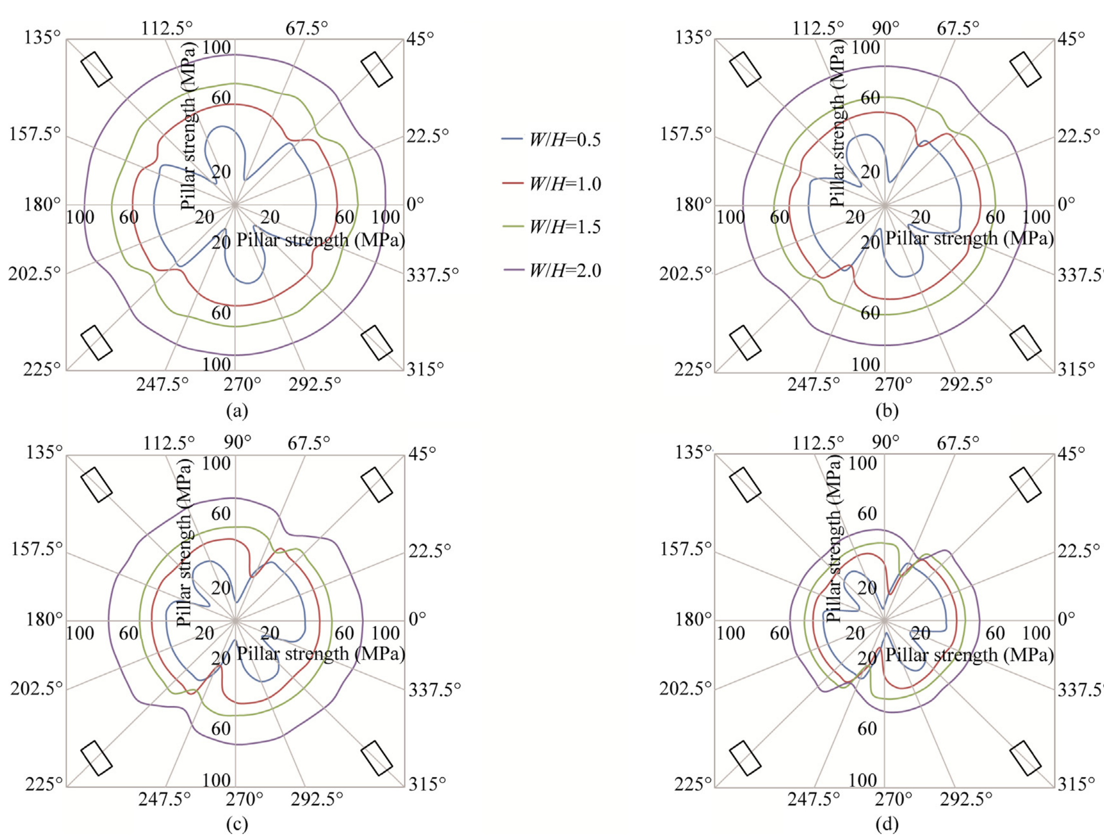

Fig. 8. Discontinuity effects on the pillars of inclination (a) 10° (b) 20° (c) 30° (d) 40°.

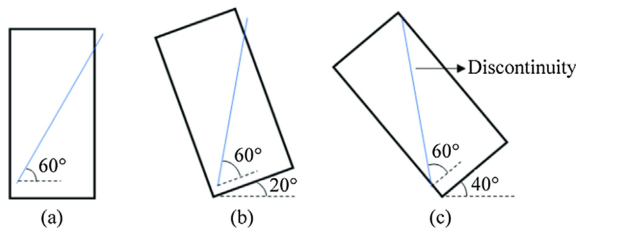

Fig. 9. Discontinuity dip angle of 60° in Pillars with Inclinations (a) 0° (b) 20° (c) 40°.

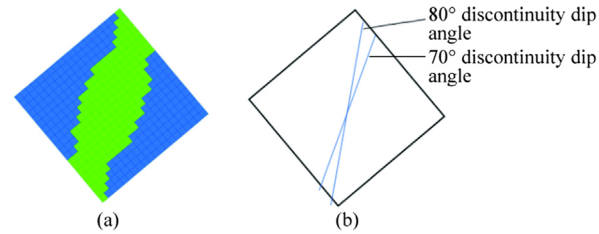

Fig. 11. (a) Failure mechanism of the 40° inclined pillar without discontinuity [21]. (b) The discontinuity angles that can lie in the failure region. Note: Blue shows elastic elements and Green shows yielded elements.

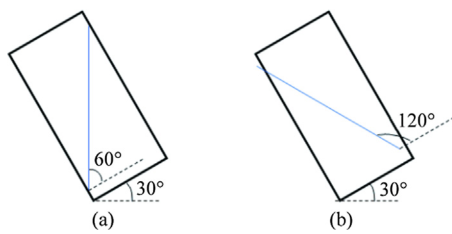

Fig. 10. Discontinuity dipping (a) towards the pillar inclination (b) against the pillar inclination.

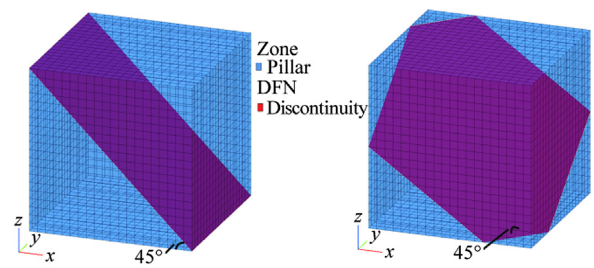

Fig. 12. Discontinuity with 45° dip and (a) 0° dip direction (b) 45° dip direction.

80° and 140° discontinuity dip angle. Similarly, in 30° inclined pillars with W/H ratio of 0.5 (Fig. 8c), the discontinuities effecting the pillars strength are 90° and 150° while in 40° inclined pillars (Fig. 8d), they are 100° and 160°. This can be attributed to the largest discontinuity angle for sliding in the W/H ratio of 0.5 at any pillar inclination is 60° as shown in the Fig. 9. Therefore, it can be concluded that when the discontinuity dip angle is measured relative to pillar inclination, the discontinuity dip angle of 60° and 120° relative to pillar inclination significantly effects the pillar strength irrespective of any inclination.

In Fig. 8a, it can be observed that the pillar of W/H ratio 0.5 with a 70° discontinuity dip angle has similar strength (33% of the actual pillar strength) to that of the 130° discontinuity dip angle. While in the 30° inclined pillar with W/H ratio of 0.5 (Fig. 8c), 90° discontinuity dip angle (Fig. 10a) reduces the pillar strength to 30% while the 150° discontinuity dip angle (Fig. 10b) reduces the pillar strength to 45%. This is due to the oblique loading on the pillars aiding the slippage for the discontinuity inclining towards the pillar dip. While for the discontinuity inclining against the dip, the oblique loading aided the closure of the discontinuity which resulted in lower strength reduction. Similar results are seen in pillars with higher W/H ratios and higher inclination pillars. Therefore, it can be concluded that in inclined pillars at any W/H ratio, the discontinuity inclining towards the pillar dip show a significant reduction in strength than the discontinuity inclining against the pillar dip.

In vertical pillars, at higher W/H ratios, the discontinuity has little effect on the strength of the pillars. It was observed that in highly inclined pillars even at high W/H ratios, the discontinuity reduces the strength of the pillars. For example, in 40° inclined pillar (Fig. 8d) with W/H ratio of 1.0, with discontinuity of 90°, the strength reduces to 40% while with W/H ratio of 1.5, the discontinuity dip angle of 80° lead to strength reduction to 60%. Therefore, it can be concluded that discontinuities play a major role in defining the strength of the inclined pillars when dipping along the pillars even at higher W/H ratios.

In 40° inclined pillars (Fig. 8d), it is interesting to note that the pillar with W/H ratio of 0.5 has higher strength than the pillar with W/H ratio of 1.0 when intersected by a discontinuity of angle 80°. When discontinuity dip angle is 70°, the pillar strength is higher for pillars with W/H ratio of 0.5 than the pillars with W/H ratio of 1.0 and 1.5. Pillars with W/H ratio of 1.5 and 2.0 had equal pillar strength when intersected with discontinuity dip angle of 60°. This is due to the failure mechanism of the 40° inclined where the brittle initiation occurs at the two corners of the pillars which coincides with the discontinuity dip angle and thus reducing the strength of the pillars with higher W/H ratio far less than the strength of lower W/H ratios. For example, the failure mechanism of the 40° inclined pillar with W/H ratio of 1.0 is shown in the Fig. 11 [21]. The discontinuity dip angle of 70° and 80° lie in that failure region, therefore, at those angles, there is a sharp reduction in strength of the pillar.

Similarly, in 30° inclined pillars (Fig. 8c), the pillar with W/H ratio of 1.0 intersected with discontinuity dip angle of 60° has higher strength than the pillar with W/H ratio of 1.5. Therefore, the critical discontinuity angles lie in between the 60° and 90° for all the inclined pillars, where the smaller pillars can have higher strength than the larger pillars. This can lead to higher economic yield in the mines if properly executed.

3.2. Influence of discontinuity dip direction on inclined pillars

To understand the influence of discontinuity dip direction on the inclined pillar strength, models were simulated as vertical pillars and 40° inclined pillars at W/H ratio of 0.5 and 1.0. Three different dip directions were simulated which are 0°, 45°, and 90°. An example of discontinuity dip direction of 0° and 45° is shown in Fig. 12. In vertical pillars, the discontinuity dip direction at 0° and 90° represent the similar discontinuity because these are parallel to the pillar face. In inclined pillars, the discontinuity dip direction of 0° represents the discontinuity parallel to the pillar face and towards the pillar inclination, dip direction of 90° also indicates the discontinuity perpendicular to pillar inclination and dip direction of 45° refer to an intermediate of the two mentioned above. Strike and Dip Direction would be used interchangeably from this point.

The results show that in vertical pillars with W/H ratio of 0.5 (Fig. 13a), the strike of the discontinuity has little effect on the strength of the pillars. While in inclined pillars (Fig. 13b), the strike

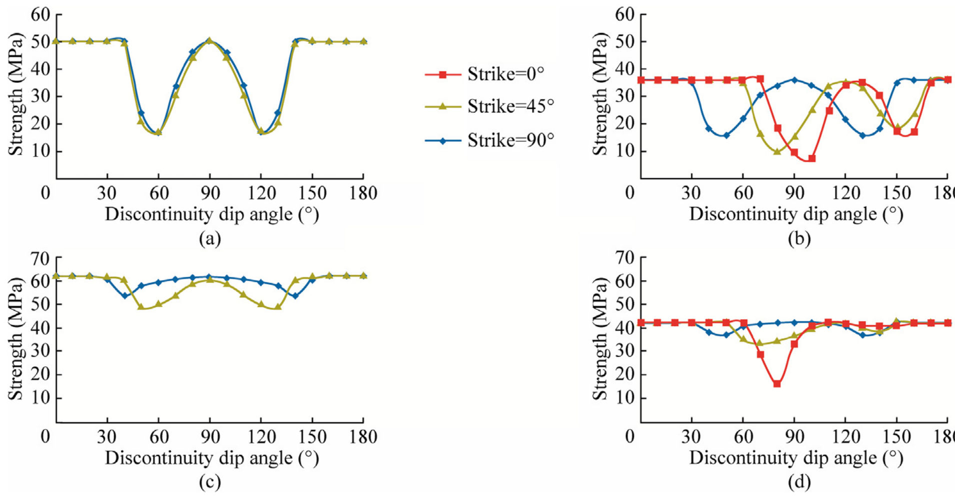

Fig. 13. Influence of discontinuity dip direction on (a) vertical pillars (W/H = 0.5) (b) 40° inclined pillars (W/H = 0.5) (c) vertical pillars (W/H = 1.0) (d) 40° inclined pillars (W/H = 1.0).

of the discontinuity has a significant influence on the pillar performance. When the discontinuity is orientated towards the pillar inclination (Strike = 0°), the discontinuity angle of 100° and 160° significantly effects the pillar strength while when the strike is 45°, the discontinuity angle which is affecting the pillar strength the most is 80° and 150° and when the discontinuity orientation is perpendicular to pillar inclination, 50° and 130° dip angles reduce the strength considerably. It can be observed that when the discontinuity orientation is perpendicular to pillar inclination (Strike = 90°), the plot looks like that of the vertical pillars. The maximum strength reduction was observed when the orientation was towards the pillar inclination and discontinuity angle was 100°.

In vertical pillars with W/H ratio of 1.0 (Fig. 13c), the strike direction of 45° has a higher influence on the pillar strength at 50° discontinuity dip. In 40° inclined pillars with W/H ratio of 1.0 (Fig. 13d), it can be observed that the discontinuity orientated toward the pillar inclination (Strike = 0°), has the largest influence on the pillar strength at 80°. As the strike direction is oriented away from the pillar inclination such as strike of 45° and 90°, the pillar strength was reduced by 20% and 10% respectively. This shows that the strike needs to be considered along the discontinuity dip to evaluate the strength of the pillars in vertical as well as inclined pillars.

4. Conclusions

The effect of discontinuity on inclined pillars was simulated and the following conclusions can be made:

- In pillars with W/H ratio of 0.5, it can be concluded that the 60° discontinuity dip angle (Towards pillar inclination) with respect to the pillar inclination has a significant drop in pillar strength irrespective to the pillar dip. Therefore, it can also be projected that for pillars with W/H ratio less than 0.5, the largest angle for sliding in the vertical pillars will be the same for the inclined pillars.
- Higher safety factors should be employed when the slender pillars of 0.5 (vertical or inclined) are encountered with discontinuities within the range of an angle higher than the friction angle and the largest angle bisecting the pillar. It is best proposed to avoid the slender pillars of W/H ratio less than 0.5 due to its high sensitivity towards the discontinuity dip angle.
- In inclined pillars with higher W/H ratios, the largest discontinuity (i.e. discontinuity from one corner to another corner) in the pillar dip direction has a significant influence on the strength of the pillar.
- The discontinuity dip towards the pillar inclination coincides with the failure mechanism of the inclined pillars thus making these discontinuities much vulnerable than the discontinuities dipping against the pillar inclination.
- In highly inclined pillars with higher W/H ratio, the discontinuities lead to very low strength due to the combination of the failure mechanism and the slippage in the discontinuity. Therefore, design of the inclined pillars should avoid the discontinuities that coincide with the failure mechanism of the inclined pillars. For example, 50° discontinuity angle with respect to pillar inclination for W/H ratio of 1.0 in vertical as well as inclined pillars.
- Alternatively, the results from highly inclined pillars show that the strength of pillar with W/H ratio of 1.5 and discontinuity dip angle of 70° has strength lower than the pillar with W/H ratio of 0.5 with the same discontinuity dip angle. As the discontinuity dip angle of 70° is largely affecting the higher W/H ratios, the excavations can be slightly reduced to accommodate the pillars with W/H ratio of 0.5. This will potentially reduce the problem of pillar failure and increase the productivity.
- The strike direction of the discontinuity is very important aspect in the inclined pillars. Pillar strength reduction due to discontinuity dip angle depends on the strike of the discontinuity.

References

[1] Dismuke SR, Forsyth WW, Stewart SBV. The evolution of mining strategy following the collapse of the window area at the Magmont mine, Missouri. In: Proceedings of the CIM annual general meeting, District 6, metal mining. p. 3–8.
[2] Zipf RK. Toward pillar design to prevent collapse in room-and-pillar mines. 108th annual exhibit and meeting, society for mining, metallurgy and exploration, Denver, 2001.
[3] Brady BHG, Brown ET. Rock mechanics for underground mining. UK: Chapman & Hall; 2007.
[4] Fakhimi A, Hemami B. Axial splitting of rocks under uniaxial compression. Int J Rock Mech Min Sci 2015;79:124–34.
[5] Fakhimi A, Hosseini O, Theodore R. Physical and numerical study of strain burst of mine pillars. Comput Geotech 2016;74:36–44.
[6] Hedley DGF, Grant F. Stope and pillar design for the Elliot Lake uranium mines. Can Inst Min Metall Bull 1972;65:37–44.

[7] Lunder PJ, Pakalnis R. Determining the strength of hard rock mine pillars. Bull Can Inst Min Metall 1997;90:51–5.
[8] Martin CD, Maybee WG. The strength of hard rock pillars. Int J Rock Mech Min Sci 2000;37:1239–46.
[9] Suorineni FT, Mgumbwa JJ, Kaiser PK, Thibodeau D. Mining of orebodies under shear loading Part 2 – failure modes and mechanisms. Min Technol (Trans Inst Min Metall A) 2014;123(4):240–9.
[10] Jaeger JC, Cook NGW. Fundamentals of rock mechanics. 3rd ed. New York: Chapman and Hall; 1994.
[11] Esterhuizen GS. An evaluation of the strength of slender pillars. In: Transactions of Society for Mining, Metallurgy, and Exploration, Inc.. Littleton, CO: Society for Mining, Metallurgy, and Exploration, Inc.; 2006. p. 69–76.
[12] Madden BJ, Canbulat I, Jack BW, Prohaska GD. A reassessment of coal pillar design procedures, safety in mines research advisory committee. COL 1995;021:1–148.
[13] Iannacchione AT. Analysis of pillar design practices and techniques for U.S. limestone mines. Trans Inst Min Metall (Sect A: Min Indust) 1999;108 (September-December):A152–60.
[14] Esterhuizen GS. Effect of structural discontinuities on coal pillar strength as a basis for improving safety in the design of coal pillar systems. In: Safety in mines research advisory committee, COL 005a. p. 1–104.
[15] Itasca Consulting Group. Fast Lagrangian analysis of continua in 3Dimensions. Minneapolis, Minnesota, USA; 2018.
[16] Lorig LJ, Cabrera A. Pillar strength estimates for foliated and inclined pillars in schistose material. In: Zhu H, editor. Proceedings of 3rd international FLAC/DEM symposium, Hangzhou, China, October 2013. Paper: 01-01. Minneapolis: Itasca International Inc.; 2013.
[17] Martin CD, Kaiser PK, McCreath DR. Hoek-Brown parameters for predicting the depth of brittle failure around tunnels. Can Geotech J 1999;36(1):136–51.
[18] Kaiser PK, Diederichs MS, Martin CD, Sharp J, Steiner W. Underground works in hard rock tunnelling and mining. In: Proceedings of GeoEng2000, Melbourne, ICMS. p. 841–926.
[19] Dolinar DR, Esterhuizen GS. Evaluation of the effects of length on strength of slender pillars in limestone mines using numerical modelling. In: Proceedings of the 26th International conference on ground control in mining. Morgantown, WV: West Virginia University; 2007. p. 304–13.
[20] Ma T, Wang L, Suroineni FT, Tang C. Numerical analysis on failure modes and mechanisms of mine pillars under shear loading. Hindawi Publ Corp Shock Vibr 2016;2016:1–14.
[21] Jessu KV, Spearing AJS. Effect of dip on hard rock pillars. South Afr Inst Min Metal 2018;118:765–76.