Do we need a Gravimetric Geoid or a Model of the Australian Height Datum to Transform GPS Heights in Australia?

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Abstract
A proposal is made to use a model of the Australian Height Datum (AHD) instead of the classical geoid to provide a more direct transformation of Global Positioning System (GPS) ellipsoidal heights to the AHD. This approach avoids post-survey adjustment of the GPS-AUSGEOID-derived heights in order to align them with existing AHD control. Alternatively, of course, the AHD could be redefined and readjusted such that it is more coincident with the classical geoid, thus allowing the use of a pure gravimetric geoid model in the height transformation. However, the cost and inconvenience associated with implementing a new national vertical datum in the near future render the proposed approach a more practical option in the interim.

1. Introduction

In Australia, Global Positioning System (GPS) ellipsoidal heights can be transformed to heights that approximate Australian Height Datum (AHD) heights by using the new AUSGEOID98 (Johnston and Featherstone, 1998), notwithstanding the geometrical method of occupying local AHD benchmarks with GPS in conjunction with interpolation (eg. Friedlieb *et al*., 1997). However, due to deficiencies in both the gravimetric determination of the long-wavelength component of the geoid and systematic distortions expected to reside in the AHD (eg. Featherstone and Stewart, 1998), the AUSGEOID-based transformation does not routinely yield AHD heights that are fully compatible with the existing vertical control in every survey area.

Any discrepancies between the GPS-AUSGEOID-derived heights and existing AHD heights often have to be removed after the field survey is completed. This can be achieved by occupying existing AHD benchmarks in and around the survey area, which should already be an integral part of the quality assurance process when determining AHD heights from GPS. This allows any discrepancies between the two determinations of AHD height to be modelled and removed (eg. Featherstone *et al*., 1998), thus making the GPS-derived AHD heights compatible with local AHD benchmarks. However, this procedure generates an additional post-survey adjustment of the results, thus increasing survey costs.

In this paper, the arguments are presented in favour of an approach where future Australian gravimetric geoid models are augmented by direct measurements of the separation between the AHD and the WGS84 ellipsoid. An increasing amount of this information is available from GPS surveys conducted at AHD benchmarks. This combination of a gravimetric geoid model with GPS and AHD heights will produce a surface that will allow the *direct* determination of AHD heights from GPS. This surface will be of great benefit to the users of GPS, because it avoids the post-survey adjustment stage in the majority of cases. However, the need to occupy existing AHD benchmarks in and around the survey area will always remain for the purposes of quality assurance.

The approach of using GPS and AHD data to augment a gravimetric geoid model is not new, and is in agreement with GPS height transformation strategies being implemented elsewhere in the world. For example, gravimetric geoid models have been augmented by GPS and levelling measurements in France (Jiang and Duquenne, 1996) and the United States (Milbert, 1995). It is, therefore, proposed that high precision GPS and AHD data are included in future generations of AUSGEOID to yield two solutions:

1. A scientific, gravimetric geoid model for use in geodetic, oceanographic and geophysical sciences (eg. Vanicek and Christou, 1994).

2. A practical model of the separation between the AHD and the WGS84 ellipsoid specifically for the direct transformation of GPS heights to the AHD.

Importantly, the second model could be implemented in Australia until a redefinition and readjustment of the AHD is achieved.

2. Definition of Relevant Heights and Their Associated Reference Surfaces

The Australian geoid has been discussed at some length in this Journal (eg. Gilliland, 1989; Kearsley and Govind, 1991; Steed and Holtznagel, 1994; Featherstone *et al.*, 1997). As is thus well known, the main geodetic application of a geoid model is to transform WGS84 ellipsoidal heights, derived from GPS surveys, to orthometric heights, based on the local vertical datum. This coordinate transformation is necessary because the orthometric height is a physically meaningful quantity in that it is related to the Earth's gravity field. It is measured positively from the geoid to the point of interest along the (curved) plumbline.

On the other hand, the GPS-derived ellipsoidal height is a purely geometrical quantity since it is unrelated to the Earth's gravity field. It is measured positively from the surface of the WGS84 ellipsoid to the point of interest along the (straight) ellipsoidal normal. Similarly, the geoid height or N value, which is an explicit function of the Earth's gravity field, is measured positively from the surface of the WGS84 ellipsoid to the geoid along the ellipsoidal normal. The latter two definitions also hold for reference ellipsoids other than WGS84.

The transformation between ellipsoidal and orthometric heights is documented as being extremely straightforward. However, by algebraically subtracting the geoid height (N) from the ellipsoidal height (h) does not exactly yield the orthometric height (H). One reason for this is the curvature of the plumbline with respect to the ellipsoidal normal. The offset is manifested as a combination of the deflection of the vertical, which is the angular separation of ellipsoidal normal and plumbline at the geoid, and the curvature of the plumbline itself.

Now assume that in Australia, the deflection of the vertical reaches a maximum of about 30 arc seconds and the curvature of the plumbline is of the order of, say, 10 arc seconds in mountainous regions. These values allow an estimate of the maximum error affecting this vertical coordinate transformation to be made. Using the extreme case of Mount Kosciusko (the tallest mountain in Australia with an AHD height of 2228m), the error of the simple algebraic approximation (ie. $H = h - N$) is less than a millimetre. Given, that the most precisely determined GPS ellipsoidal heights in Australia are of the order of 4cm to 8cm (eg. Morgan *et al.*, 1996 and Stewart, 1998), a full equality is justified in this transformation equation.

3. What Type of Height System is Used in Australia?

The Australian Height Datum (Roelse *et al.*, 1971) has also been discussed in this Journal (eg. Kearsley *et al.*, 1988; Morgan, 1992; Stewart, 1998). However, it is important to revisit the conceptual model of AHD heights in relation to the commonly accepted height systems described above, in order to provide a basis for the arguments that follow in this paper.

Roelse *et al.* (1971) show that the miscloses in the spirit levelling loops used to establish the AHD were reduced when a particular approximation of the orthometric correction was applied. The orthometric correction is a term that accounts for the non-parallelism (in a geometrical sense) of the equipotential surfaces of the Earth’s gravity field. As the plumblines and equipotential surfaces are orthogonal at every point, this can be conceptualised as a correction to spirit-levelling measurements that accounts for the curvature of the plumbline.

In practice, only an approximation of the true orthometric correction can be made because the value of the Earth's gravity acceleration is needed at all points along the plumbline; a quantity that can not be practically observed for a national levelling network. Instead, surface gravity measurements and a hypothesis concerning the mass density distribution in the topography are used. A common assumption is that a constant topographic density (usually 2,670 kgm^{-3}) is applied via the Poincaré-Prey gravity gradient in the orthometric correction formula (eg. Torge, 1991). When this approximated form of the orthometric correction is applied to levelling observations, it yields the so-called “Helmert orthometric height”.

In Australia, however, the orthometric correction was applied slightly differently. Instead of using observed values of gravity, normal gravity - generated by the Geodetic Reference System 1967 mean Earth ellipsoid model - was used in the orthometric correction formula (Roelse *et al.*, 1971). Therefore, the height system that is utilised by the AHD is more accurately termed a “normal-orthometric height” (Holloway, 1988). However, this type of correction does not always accurately model the Earth’s true gravity field because of lateral density variations within the Earth. The study by Mitchell (1973) shows that the difference between using observed and normal gravity is negligible in comparison with other effects on the AHD. However, Friedlieb (1995) shows that the Helmert orthometric correction, based on observed gravity data, reaches 30mm over 30km near Perth, Western Australia. In addition, Featherstone and Kirby (1998) show that the difference between normal and orthometric heights reaches 150mm over Australia.

4. Is the AHD Coincident with the Geoid?

For many reasons, the AHD defines a non-equipotential vertical datum surface that is not coincident with the geoid, according to its classical definition as a single equipotential surface of the Earth's gravity field. The subject of vertical datums in relation to the geoid is treated in more detail by, for example, Mather *et al.* (1976), Featherstone (1995), Vanicek (1991), and Kearsley *et al.* (1993), and will not be duplicated here. However, the points that are pertinent to the proposal made in this paper are summarised in what follows.

Firstly, the 30 tide gauges around Australia that were held fixed to zero during the adjustment of the AHD (Roelse *et al.*, 1971) do not necessarily give a measurement of the geoid according to its classical definition. In essence, the geoid can be defined in two subtly different, but quite distinct, ways (cf. Vanicek, 1991):

1. The 'classical geodesist' uses the geoid as a unique equipotential surface of the Earth's gravity field (W_0 = constant) that is defined from the principles of potential theory and broadly agrees with mean sea level in the open oceans on a global scale.

2. The practising geodesist uses the geoid in terms of a definition based on mean sea level measurements at tide gauges, which is realised as a vertical datum through levelling (and gravity measurements), usually on a regional scale.

A problem that now faces Australian geodesy and surveying is that these different definitions and realisations of the geoid are now becoming apparent (eg. Featherstone and Stewart, 1998; Forsberg and Featherstone, 1998).

4.1 The gravimetric geoid

The gravimetric geoid, as used by 'classical geodesists', is a specific equipotential surface that can be computed from gravity measurements via Stokes's integral. This definition of the geoid is realised through Stokes's formulation and solution of the boundary value problem of physical geodesy. This classical definition is of fundamental importance in the geodetic, oceanographic and geophysical sciences (eg. Vanicek and Christou, 1994). It also has a vital role to play in the unification of the global vertical datum (eg. Rapp and Balasubramania, 1992). This is because the gravimetric geoid is not necessarily identical to the vertical datum used in a particular country (eg. Mather *et al.*, 1976; Rapp, 1994; Featherstone, 1995).

Improvements in the computational and theoretical aspects of gravimetric geoid determination have meant a great deal of progress has been achieved in this field over the last decade. However, the determination of the gravimetric geoid is imperfect because it remains subject to a number of error sources, three of which are summarised below:

1. The theories and equations used in the gravimetric determination of the geoid contain several approximations and assumptions, which preclude the most accurate geoid solution.

2. The global geopotential coefficients used a basis for a regional geoid computation contain errors that are of long wavelength in nature and cannot be completely corrected by local gravity data (eg. Vanicek and Featherstone, 1998).

3. The terrestrial gravity and height data used to compute gravity anomalies are also in error, which propagates into all components of the gravimetric geoid. For instance, the Australian gravity measurements were collected primarily for resource exploration purposes and their heights determined using inaccurate barometric methods, which renders the gravity anomalies inappropriate for the demands of physical geodesy.

Therefore, the computation of the gravimetric geoid is subject to a number of algorithmic and data errors, which are still attracting the attention of physical geodesists. It is estimated that the realistic accuracy of the Australian geoid is probably in the vicinity of 10-30cm, notwithstanding the scale error introduced by the zero-degree term (Kirby and Featherstone, 1997). Gravimetric geoid results in other countries are often quoted with an accuracy of the order of a few centimetres through comparisons with GPS and levelling data. However, this does not necessarily imply that the Australian geoid is inaccurate. Instead, these accuracy estimates are generally over-optimistic because the systematic differences, due principally to geoid errors and vertical datum differences, have been modelled through the use of a plane (eg. Sideris and She, 1995) or surface (Milbert, 1995).

Nevertheless, the new generation of Australian gravimetric geoid, released by the Australian Surveying and Land Information Group (AUSLIG) as AUSGEOID98 (Johnston and Featherstone, 1998), shows inconsistencies of up to one metre with heights derived from GPS networks (eg Stewart *et al.*, 1997; Morgan *et al.*, 1996) that are co-located with spirit-levelled benchmarks on the AHD. Importantly, these inconsistencies remain even when a plane is used to attempt to absorb the long-wavelength differences. This implies that there are more significant medium and short wavelength differences among these height data. Given that the accuracy of the ellipsoidal heights from national GPS networks is between 4cm and 8cm (Stewart, 1998), these discrepancies are more likely lie in the gravimetric geoid model, the AHD, or both.

Forsberg and Featherstone (1998) give a notable example of these height discrepancies, where gravimetric geoids of Scandinavia and Australia were computed using identical numerical techniques. Subsequent comparisons with GPS and levelling data show agreements of the order of a few centimetres in Scandinavia and agreements of the order of a few tens of centimetres in Australia. Assuming that the GPS and geoid data are of commensurate precision in each region, the poorer fits among the three height sources in Australia could be attributed to the AHD. However, this observation alone is inconclusive because the Scandinavian gravity data are probably more accurate than the Australian gravity data.

Featherstone and Stewart (1998) give a similar source of evidence that distortions exist in the AHD. The Western Australian GPS network, called STATEFIX (Stewart *et al*., 1997), was used in conjunction with the EGM96 global geopotential model (Lemoine *et al*., 1997) to indicate that the height discrepancies with the AHD are systematic. Moreover, there is a remarkable correlation between these discrepancies and the differences between the minimally (one tide gauge) and fully constrained (30 tide gauges) levelling network adjustments shown in Roelse *et al*. (1971). These two sources of evidence, though each implicitly dependent upon the accuracy of each gravimetric geoid, reinforce the argument that the largest source of distortion in the AHD is due to fixing the 30 tide gauges to zero.

4.2 Tide gauges and levelling

The surface defined through tide gauge measurements of mean sea level and spirit-levelling, converted to normal orthometric heights, is both conceptually and practically different to a gravimetric geoid. This is the case in Australia, and evidence is continuing to accumulate to prove that the AHD is not coincident with the classical geoid (eg. Mitchell, 1973; Mather *et al*., 1976; Featherstone, 1995; Rapp, 1994). Two principal factors cause this:

1. Tide gauge measurements of mean sea-level do not define the classical geoid. This is because of a phenomenon called sea surface topography (cf. land topography), which can reach two metres. Sea surface topography is the departure of the mean sea surface from the classical geoid defined as a unique equipotential surface of the Earth's gravity field. It is extremely difficult to quantify close to the coast. In the case of the AHD, the tide gauges were often located near estuaries that are subject to local oceanographic effects, mixing of fresh and salt water (eg. Eckman, 1994), and the prevention of out-flow by coastal reefs. As such, they are not ideal for determining the geoid. Also, the tide gauge records used to define a vertical datum should also be averaged over the full lunar cycle of 18.67 years in order to remove long-period tidal effects.

   Therefore, by holding 30 tide gauges *fixed* to zero in the 1971 adjustment distorts the AHD to coincide with coastal mean sea-level rather than a single equipotential surface. It is preferable to model the sea surface topography and *constrain* the tide gauge heights to zero (Vanicek, 1991). The effect of fixing the tide gauges to zero is partly evident through the one-metre discrepancy between free and fixed adjustments of the AHD in Roelse *et al*. (1971). However, this difference must also be qualified by the presence of a systematic error in the levelling conducted in northern Queensland (eg. National Mapping Council, 1986; Morgan, 1992).

2. In Australia, the tide gauge measurements of mean sea level were effectively assigned a zero AHD height and is carried to the interior of the continent by spirit levelling. As orthometric corrections based on observed gravity data were not used, the levelling measurements do not coincide with the (most probably non-geoidal) equipotential surface defined at each tide gauge. Moreover the spirit levelling measurements are mostly of third-order tolerance (Morgan, 1992), and are thus subject to their own error budget. Systematic errors are also present in any levelling measurements through factors such as equipment calibration and atmospheric refraction. Such errors are more problematic because they are not necessarily detected by closed surveys. The combination of all these errors are spatially variant over the Australian continent and may cause errors of more than a metre in magnitude. However, it should be stated that despite its documented inadequacies, the AHD has served Australia well as a vertical datum. From the practical point of view, the detection and correction of gross errors in the AHD is probably more pressing than the redefinition and readjustment of the AHD at a continental scale.

4.3 Using GPS and the geoid

As the error characteristics of a gravimetric geoid and AHD are different, there is a distinct difference between the surface that defines the AHD and the surface defined by a gravimetric geoid model. The consequences for the GPS user are then self-evident. If a gravimetric geoid model is used to transform GPS ellipsoidal heights, the resulting heights do not necessarily coincide with the AHD as realised through third-order levelling and fixed tide gauge measurements. Examples of this difference over relatively small areas (~60km square) are given in Featherstone *et al*. (1998), where local AHD benchmarks must be occupied by GPS and used to adjust the GPS-AUSGEOID93 (Steed and Holtznagel 1994) heights (post-survey) to make them compatible with the AHD.

One point worthy of note is the way in which local GPS surveys are currently conducted in Australia. The geodetic stations used as three-dimensional control for the GPS network are often derived from Australian Geodetic Datum (AGD) horizontal coordinates by a transformation to the Geocentric Datum of Australia 1994 or GDA94, which is compatible with WGS84, using AUSLIG's transformation parameters (see http://www.anzlic.org.au/icsm/gdatm/gdatm.htm).

The ellipsoidal heights of these GPS-control stations are often derived from AHD and AUSGEOID (ie. $h = H + N$). In the latter coordinate transformation, any systematic discrepancy between the AHD and AUSGEOID is accounted for to a large extent before the GPS survey begins. That is, any common errors or biases will cancel over short GPS baselines. However, as the lateral extent of the survey area increases, the discrepancy no longer cancels, and further discrepancies often result. It is these discrepancies that have to be modelled and minimised by a further post-survey adjustment of the GPS-derived heights (eg. Featherstone *et al.*, 1998).

This problem may become more pronounced if the ellipsoidal heights at some of the GPS-control stations have been established using only GPS, such as a station in the Australian National Network (ANN), whereas others have been transformed from the AHD. Because of the inconsistency between the observed (pure) and transformed ellipsoidal heights, miscloses of several decimetres can occur, depending upon the distortion between the AHD and AUSGEOID and the lateral extent of the survey. As a practical solution to this problem, the following can be used:

- the transformed ellipsoidal heights could be excluded from the final network adjustment;
- the pure ellipsoidal heights could be excluded from the final network adjustment; or
- only transformed ellipsoidal heights be used consistently, even at the stations where pure ellipsoidal heights are available.

As long as a consistent choice is made, the common errors will cancel. Nevertheless, whichever approach is taken, the problem of the discrepancies between the AHD and gravimetric geoid model will remain.

In short, the AHD is not coincident with a gravimetric geoid model so that discrepancies often occur between GPS-derived and spirit-levelled AHD heights. These discrepancies raise the following important question. Does the surveyor really need the gravimetric geoid to transform GPS ellipsoidal heights to AHD heights?

5. A Practical Solution to the Problem

There are two approaches that can be taken to avoid the need to adjust the results of GPS-AUSGEOID height surveys. One is to redefine and readjust the AHD including the improved measurements made since 1971, models of sea-surface topography at tide gauges and orthometric corrections based on observed gravity data (eg. Featherstone and Stewart, 1998) and to include GPS heights and gravimetric geoid models (eg. Vanicek, 1991; Kearsley *et al.*, 1993; Stewart, 1998). These approaches would align the new realisation of the AHD more closely with the classical geoid. This would then allow a gravimetric geoid model to be used directly in the vertical coordinate transformation, and thus avoid post-survey adjustment of the GPS-AUSGEOID derived AHD heights in many cases.

However, such an approach, whilst theoretically rigorous, leads to several practical problems. One particularly strong argument against a redefinition and readjustment of the AHD is that it already forms a single, national height datum across mainland Australia, upon which a great deal of infrastructure depends. This is unlike the AGD, which is used as its 1966 and 1984 realisations in different States and Territories, thus partly justifying the move to the GDA94. In Tasmania, however, the AHD(Tas) is not directly connected to the mainland AHD (National Mapping Council, 1986) and differences of approximately 10-40cm have been quoted (eg. Rizos *et al.*, 1991; Rapp, 1994). This is because the AHD(Tas) forms a separate vertical datum. Therefore, a bias of similar magnitude can be expected between long-baseline GPS-AUSGEOID-derived heights and AHD heights across the Bass Strait. However, as described earlier, this difference will not be observed for mainland- and Tasmania-only surveys because of the way GPS surveys are often conducted.

If the AHD is to be redefined and readjusted, it should occur in the near future. This would allow its implementation to coincide with that of the GDA94, thus reducing some of the costs associated with a later readjustment and redefinition of the AHD. However, the demand (both nationally and internationally) for a unified vertical datum is far less pressing than for the horizontal datum, where the differences are considerably more noticeable (1-2m versus 100-900m). Moreover, the unification of the global vertical datum is at a far less advanced stage than the unification of global horizontal datums (eg. Rapp and Balasubramania, 1992).

If the AHD is not redefined, the problem of the inconsistencies between Australian gravimetric geoid models and the AHD will remain. It is inevitable that with the wider acceptance and increasing use of GPS, its users will demand a more direct means of transforming ellipsoidal heights to the AHD without the inconvenience and expense of post-survey adjustments. Nevertheless, the need to occupy existing AHD benchmarks with GPS in and around each survey area will always remain for the purposes of quality assurance.

The alternative to redefining and readjusting the AHD, which is more easily implemented and less costly to the user, is to adjust future Australian gravimetric geoid models such that they are coincident with the AHD. This can be achieved by including the increasing number of GPS and AHD measurements into a combined solution. This approach is not new (eg. Kearsley *et al*., 1993) and is now being used in other countries, such as France (Jiang and Duquenne, 1996) and the United States (Milbert, 1995). This data combination acts to 'warp' the gravimetric geoid model such that it is more closely aligned with the vertical datum in the country in which it will be used. The benefit of this to the GPS user in Australia is clear: it will allow the *direct* determination of AHD heights from a GPS survey in a majority of cases.

Conceptually, this combined method is an extension of the geometrical modelling technique (eg Friedlieb *et al*., 1997), where GPS measurements at benchmarks allow a surface to be used to model the separation between the local vertical datum and the WGS84 ellipsoid. However, the geometrical method is only effective when the distances between vertical control stations is small (<10km, say) so that the separation between these surfaces may be modelled with sufficient accuracy. At present, the national coverage of GPS ellipsoidal heights at AHD benchmarks does not currently satisfy this requirement.

The proposed approach of using a combination of GPS and AHD heights with a modern gravimetric geoid avoids this scenario. The reasoning is as follows. The gravimetric geoid solution provides the detailed structure of the geoid down to the resolution of the gravity and terrain data used in its computation, but is usually deficient in the long wavelengths (eg. Sideris and She, 1995). Conversely, GPS has a relatively homogeneous precision and is thus more reliable over longer distances (eg. Morgan *et al*., 1996; Stewart, 1998). As argued earlier, the AHD is strongly suspected to contain regional distortions. The combination of these data sources will provide a surface that relies upon GPS to define the long-wavelength vertical control, whereas the medium- and short-wavelength undulations are provided by the gravimetric geoid model, especially in regions where no GPS data are currently available.

Bearing in mind that the objective is to directly determine AHD heights from GPS, it becomes logical to make the gravimetric estimate of the geoid coincident with the AHD at GPS control points. This approach produces a practically useful geodetic product that simultaneously absorbs the errors in the gravimetric geoid and AHD, whilst allowing the GPS surveyor to achieve a direct transformation of AHD heights from GPS. Essentially, the proposal is to adjust the value of N such that at every point where ellipsoidal and AHD heights are available, the equation $h = H - N$ is satisfied exactly. This condition also makes heights slightly easier to administer, since a geodetic database can store either the observed ellipsoidal or observed AHD height because the N value is mutually consistent.

A suggested method of implementation is as follows. A gravimetric geoid model of Australia is computed and stored for geodetic, oceanographic and geophysical purposes. In the United States, this has been referred to as the scientific solution. This solution should not be widely distributed because it defines a different datum to the AHD. In fact, there is the possibility that an accurate gravimetric geoid, in that it coincides with the classical geoid, may produce worse comparisons with GPS and AHD data simply because of distortions in the AHD.

The scientific geoid solution is then augmented by geometrical heights from National and State/Territory GPS networks, where ellipsoidal heights have been observed at as many AHD benchmarks as possible. This augmentation provides a direct measure of the separation between the AHD and the WGS84 ellipsoid. Clearly, this changes the definition of the problem at hand, where a model of the AHD is desired instead of a model of the classical geoid. It is beneficial in that it accounts for the datum difference between the AHD and classical geoid, and also absorbs the long-wavelength errors in the gravimetric geoid and the distortions in the AHD. The resulting combined model will undoubtedly be practically more useful product for the GPS surveyor.

However, such an approach can be seen as unscientific because it ignores the underlying reasons for the discrepancies. Nevertheless, it is indisputable that the approach does produce a practically useful geodetic product. As the GPS and AHD observations are currently sparse (typically at a 100km spacing), the medium- and high-frequency gravimetric geoid undulations are preserved such that they provide a model of the majority of the undulations in the AHD between GPS control stations. As a complement, the GPS observations control the medium- and long-wavelength differences between the gravimetric geoid and AHD.

Many modern 'geoids' around the world now model the datum differences and long-wavelength gravimetric geoid errors with planes or surfaces. Notable examples are the use of a plane (eg. Sideris and She, 1995) or a 'corrective surface' determined using least squares techniques (eg. Milbert, 1995; Jiang and Duquenne, 1996). In Australia, however, the use of a plane makes no appreciable difference to the fit among AUSGEOID98 (Johnston and Featherstone, 1998), GPS and AHD heights. Therefore, some form of surface fitting technique should be employed in Australia to account for what are now strongly suspected to be distortions in the AHD. The most likely candidates at this stage for the data combination are least squares collocation (Moritz, 1980) or the tensioned spline algorithm of Smith and Wessel (1990), which are relatively easy to implement and particularly suited to potential field data.

It should be noted that after a gravimetric geoid has been adjusted in this way, the fit among the 'corrected' model, GPS and AHD data should be perfect at the points used to create the model. This is only to be interpreted as an indication of the fit of the combined solution to the datum difference defined by GPS and AHD data, rather than an indication of the accuracy of the gravimetric geoid in relation to the classical geoid.

6. Conclusions and Recommendations

The above summary of direct and anecdotal evidence confirms that the AHD is not coincident with the classical geoid, nor necessarily is the geoid as determined by gravimetric methods. Therefore, if AHD heights are required from GPS surveys, it is preferable to use a technique that accounts for the discrepancies between the vertical datum and gravimetric geoid model. The proposed approach essentially yields a model of the separation between the AHD and the WGS84 ellipsoid, which can be used to determine AHD heights *directly* from GPS surveys. This will provide an interim and practically useful geodetic product for the GPS user community in Australia until any readjustment and redefinition of the AHD is achieved.

In order to meet this aim, it is recommended that:

- Two solutions of AUSGEOID are produced; one a pure gravimetric geoid model held as a scientific solution for exclusive use in geodesy, oceanography and geophysics, and one as a practical product for the GPS user community to transform GPS heights to the current AHD in a more direct and efficient way.
- In order to allow for the best possible combined solution, all States and Territories should make their GPS and AHD data available; GPS surveys should be conducted on existing AHD benchmarks where feasible; and spirit levelling should be conducted to existing high precision GPS points, or *vice versa*. The larger the number of GPS and AHD data that are included in any future combined solution, the more effective it will be in the transformation of GPS heights in these areas.

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