Estimation of elasticity tensor from the inversion of traveltimes in spherical shale samples

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Summary

From P-wave traveltime measurements in a spherical shale sample at 40 MPa we find the symmetry axis. We transform the ray velocities from the measurement coordinate system to the symmetry axis coordinate system. Assuming transverse isotropy symmetry, we estimate the elasticity tensor using a very fast simulated annealing algorithm followed by a quasi Newton method.

Introduction

Due to sedimentation pattern of clay minerals, shale formations generally show transverse isotropy (TI) with vertical axis of symmetry. The main motivation of this study is to understand the seismic anisotropy of the overburden shale. Geological formations in the region under study are generally experiencing a horizontal stress field. This stress field may cause azimuthal anisotropy by either tilting the symmetry axis of shale formations and/or causing the directional planes of weakness. To characterize the seismic anisotropy, we have used P-wave traveltimes from a spherical shale core sample from the top of a sand reservoir. Unlike others, e.g Delinger (2005) and Vestrum and Brown (1994), we find the symmetry axis first and then invert for elasticity parameters to reduce the complexity of the inversion. Assuming TI symmetry, we have estimated the elasticity tensor using the Simulating Annealing followed by quasi Newton algorithm.

Sample preparation and measurement

To prepare a spherical sample, a core sample was polished in different directions to obtain a sphere with 50 mm in diameter. The spherical sample has been placed in pressure chamber to measure the ultrasonic velocities in a broad range of confining pressures from ambient pressure to 700 MPa. Traveltimes was measured over the spherical sample at every 15 degrees in azimuthal and polar directions using the transducers at resonant frequency of 2 MHz (Figure 1). This acquisition pattern produced 132 records of P-wave traveltimes.

Estimation of major symmetry axis

To find the symmetry axis of a TI medium, we use the invariance of ray velocity V along the azimuthal direction, expressed by equation (1). To find the axis, we form an objective function S given by equation (2). This requires,
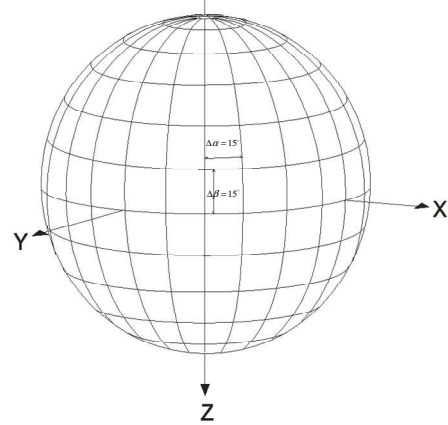

Figure 1: This A schematic of measurements locations on the sphere

transforming the data from the measurement coordinates (α, β) to the symmetry axis coordinates (θ, φ) given by equations (3), (4), and (5) with symmetry axis coordinates (α_0, β_0).

$$\left.\frac{\partial V}{\partial \varphi}\right|_{(\alpha_0,\beta_0)} = \frac{\partial \alpha}{\partial \varphi}\frac{\partial V}{\partial \alpha} + \frac{\partial \beta}{\partial \varphi}\frac{\partial V}{\partial \beta} = 0, \tag{1}$$

$$\left.S\right|_{(\alpha_0,\beta_0)} = \sum_{\alpha=0}^{2\pi}\sum_{\beta=0}^{\pi}\left(\frac{\partial V}{\partial \varphi}\right)^2 \bigg/ \left(\left(\frac{\partial \alpha}{\partial \varphi}\right)^2 + \left(\frac{\partial \beta}{\partial \varphi}\right)^2\right), \tag{2}$$

$$\begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix} = \begin{pmatrix} \sin\beta_0\cos\alpha_0 & \sin\beta_0\sin\alpha_0 & -\cos\beta_0 \\ -\sin\alpha_0 & +\cos\alpha_0 & 0 \\ \cos\beta_0\cos\alpha_0 & \cos\beta_0\sin\alpha_0 & \sin\beta_0 \end{pmatrix}.$$

$$\begin{pmatrix} \cos\beta\cos\alpha \\ \cos\beta\sin\alpha \\ \sin\beta \end{pmatrix}, \tag{3}$$

$$\varphi = \tan^{-1}\left(\frac{(-\sin\alpha_0\cos\alpha + \cos\alpha_0\sin\alpha)/}{(\sin\beta_0\cos\alpha_0\cos\alpha + \sin\beta_0\sin\alpha_0\sin\alpha - \cos\beta_0\tan\beta)}\right), \tag{4}$$

$$\theta = \cos^{-1}\left(\frac{\cos\beta_0\cos\alpha_0\cos\beta\cos\alpha +}{\cos\beta_0\sin\alpha_0\cos\beta\sin\alpha + \sin\beta_0\sin\beta}\right), \tag{5}$$

Because energy propagates with ray velocity, we use the following equation to relate ray and phase velocities,

$$V^2 = v^2 + \left(\partial v / \partial \theta\right)^2 \; , \tag{11}$$

where $\partial v / \partial \theta$ can be expressed as a function of directional cosine n_3,

$$\partial v / \partial \theta = -\sqrt{1 - n_3^2} \; \partial v / \partial n_3 \; , \tag{12}$$

$\partial v / \partial n_3$ is the derivative of phase velocity (8) with respect to directional cosine n_3. To express n_3 as a function of ray angle and ray velocity, we follow the same approach given in Ursin and Hokstad (2003),

$$n_3^2 = \left(\cos \alpha + \sin \alpha \frac{d \ln V}{d \alpha} \right) \Bigg/ \left(1 + \left(\frac{d \ln V}{d \alpha} \right)^2 \right), \tag{13}$$

where α is the ray angle. $d \ln V / d \alpha$ can be simply found numerically using a four-term finite difference operator. Noise in measurements may cause instability in numerical differentiation. Hence, we fit a high order polynomial to the measured ray velocities and constrain it by $\partial V / \partial \alpha \big|_{\alpha=0} = 0$, $\partial V / \partial \alpha \big|_{\alpha=180} = 0$. Figure 5 shows measured ray velocities in symmetry axis coordinate system in azimuth $45°$. Figure 6 shows the numerically and analytically (polynomial fitting) calculated phase angles as a function of ray angle. The imposed constrain results in more stable phase angles.
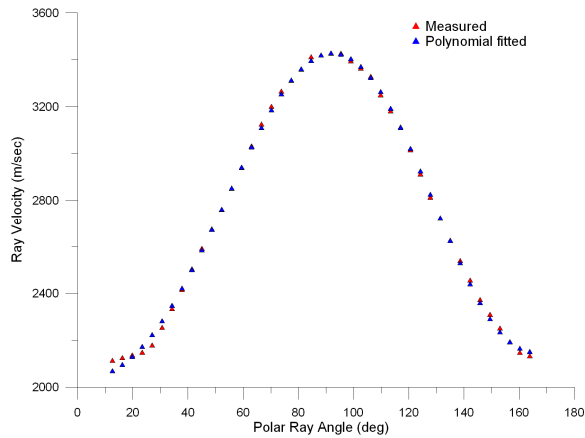

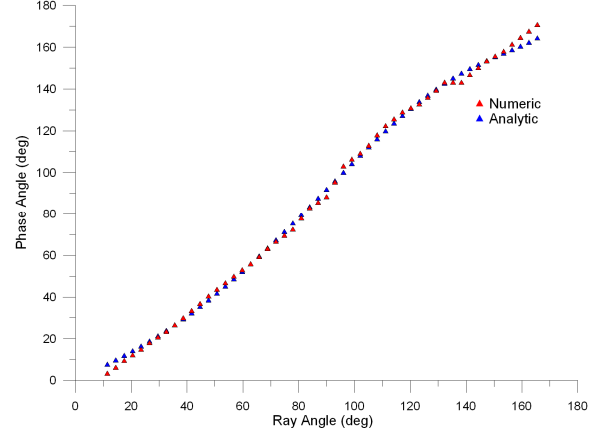Figure 5: Measured and polynomial fitted ray velocities.

Figure 6: Numeric and analytically computed phase angles

Inverse modeling and results

A quadratic objective function, without imposing a specific model space structure given by Tarantola (2005), can be used to minimize the residual error as:

$$f = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} \left(V_{ij}^{obs} - V_{ij}^{syn} \right) \left[C_D^{ii} \right]^{-1} \left(V_{ij}^{obs} - V_{ij}^{syn} \right), \tag{14}$$

where i and j correspond to azimuth and polar angles respectively. N and M are number of interpolated ray velocities V in symmetry axis coordinate over the entire azimuth and polar angles. C_D is the covariance matrix of the data. We assume there is no correlation between the data, hence, the off-diagonal elements of covariance matrix are zero. The diagonal elements or the variances could be the errors in picking the traveltimes or here, error in measuring the ray velocities.

To minimize the objective function (14), we implemented two different inversion algorithms. A very fast simulated annealing (VFSA) algorithm (Stofa and Sen, 1995) was used initially to get a solution close to global minimum from a prior model randomly drawn from a wide range uniform distribution for three elasticity coefficients C_{11}, C_{13}, and C_{33}. Without the shear wave velocities it is not possible to uniquely estimate C_{13} and C_{44}. Hence, we kept C_{44} constant during the minimization. A prior value for C_{44} was estimated from the converted shear waves from a VSP survey. Following the VFSA we implemented a quasi Newton algorithm with BFGS method (Press et al., 2002,

Nocedal and Wright, 1999) with a prior model set at the solution which was found from VFSA, to reach the exact solution quickly. Using an iterative scheme we update the prior model vector m according to:

$$m_{i+1} = m_i - \alpha_i C_M \nabla^T f C_D^{-1} e_i, \tag{15}$$

where, C_M is the covariance matrix for model parameters, e is the residual vector and it is the difference between measured and computed ray velocities. α is the step length and could be computed either by inexact line search criteria such as Wolfe conditions (Nocedal and Wright, 1999) or exact methods such as Brent algorithm (Press et al., 2002). ∇f is the derivative of the objective function with respect to model parameters (16) and T is transpose operator. In practice, we assume that the data, as well as model parameters are independent, hence, the correlation between two different elements is zero. So, the covariance matrices of model C_M and data C_D are diagonal and contain the variances. Since we assume that C_D is proportional to identity matrix, we look at the product of gradient $\nabla^T f$ and data residual vector $e = V_{obs} - V_{syn}$.

$$\frac{\partial f}{\partial m_k} = \nabla_f^T e = -\sum_{i=1}^{N} \sum_{j=1}^{M} \frac{\partial V_{ij}}{\partial m_k} e_{ij}, \tag{16}$$

where the indices i, j vary over the number of azimuth and polar angles, and k over model parameters.

Following are the normalized elasticity tensor elements and Thomsen anisotropy parameters for the sample at 40 MPa:

$$C_{11} = 13.43 \, \text{GPa}, \qquad C_{13} = 6.68 \, \text{GPa}, \qquad C_{33} = 9.48 \, \text{GPa},$$

$$\delta = 0.20, \text{ and } \varepsilon = 0.21, \text{ where } C_{44} = 2.25 \, \text{GPa}.$$

Figure 7 shows the measured ray velocities in the symmetry axis coordinate system and Figure 8 shows the residuals in the same coordinate system. As can be seen from these figures, TI symmetry may not be a good approximation, hence a lower symmetry class, such as tetragonal or orthorhombic, may need to be considered. However, this would require shear wave traveltimes as well.

Conclusions

We have estimated the elasticity tensor for a spherical shale sample using the measured ultrasonic P-wave traveltimes in different azimuth and polar angles. Because of horizontal stress field, shale formation may not be approximated by transverse isotropy symmetry well and lower symmetries may need to be considered. We have used a global optimization approach using Simulated Annealing which is followed by local optimization method using the quasi Newton algorithm.
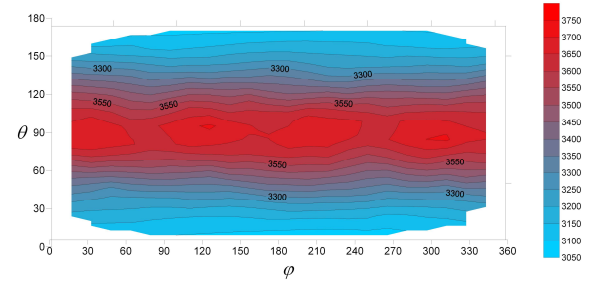

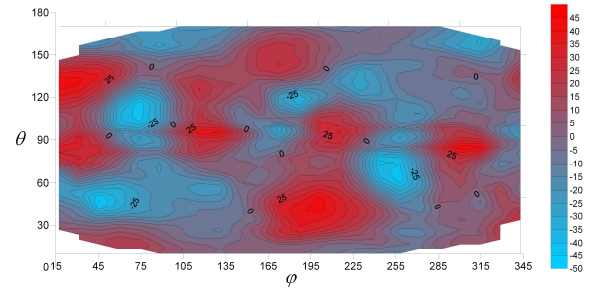Figure 7: Measured ray velocities in symmetry axis coordinate system.

Figure 8: Residual ray velocities in symmetry axis coordinate system coordinate