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Statistical investigation of geometrical properties of discontinuities Case study: Cavern of Rodbar Lorestan Pumped storage power plant

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ABSTRACT: The geometrical parameters of 639 discontinuities that surveyed in powerhouse cavern of Rodbar Lorestan pumped storage power plant project have been investigated by scanline and areal sampling methods. As regards the processing and correction of bias types, one bedding and three joint sets are existed in the site. The tectonic activities and direction of principal stresses have caused for each of trace length and spacing characteristics, the probability distribution function of joint sets differ to each other as regards their genetic types. The calculated mean trace length by scanline and areal method are very close together for one joint set and for another one the difference is 28%. The actual intensity differences between circular and rectangle sampling windows for joint sets J_1 and J_2 are 7% and 38%, respectively. Meanwhile, the calculation of volumetric intensity by various methods shows the estimation of this characteristic is very difficult in the field.

1 INTRODUCTION

Discontinuities in rock mass have profound effect on deformation, strength, stress-strain relation and failure of rock mass (Ye et al. 2012). So, besides the description of mechanical properties of discontinuities and intact rock the description geometrical properties of discontinuities is very important to investigate of rock mass behavior and to determinate interaction between rock mass and structures. The paper represents geometrical properties of discontinuities in powerhouse cavern of Rodbar Lorestan pumped storage power plant in Iran. They are investigated with using Scanline and areal sampling methods (circular and rectangular). The location of cavern in tensile zone of anticline and the area is tectonized due to the existence of high Zagros zone are one the main characteristics of the study area.

Recognizing of homogenous statistical zone in study area is the first step in modeling of joint geometry in rock mass (Kulatilake et al. 1990). As regards the discontinuities surveyed locations are in different areas, the various statistic tests have been done to control the data homogeneity and to investigate of uniformity of rock mass zones. As regards the 639 surveyed discontinuities and their clustering two joint sets, bedding and another joint set as strike joint set exist in the site. According to uncertainty on surveyed data, the correction of type of errors on dip, dip direction, spacing, frequency and trace length of discontinuities have been done. On the other side, as regards the intrinsic statistical properties of discontinuities geometry, the statistical analysis of the corrected data has been done. Meanwhile, the Fisher constant, intensity (areal and volumetric) and other characteristics are calculated for each joint set.

2 GEOLOGY AND TECTONIC OF THE SITE

According to the structural-sedimentary classification of Nabavi (1976), the zone of Rodbar Lorestan pumped storage power plant is located in sedimentary-structural zone of high Zagros. As regards the defined structural characteristics of this zone, existence of many faults special of trust type, rough topography, crushing and instability in rigid and hard formation and constitution of frequently anticline and syncline are clear. In regional scale, the bedrock of this zone is covered with the sediments belonging to primary until tertiary geological era. On the other hand, as regards the site and laboratory studies and in local scale, the

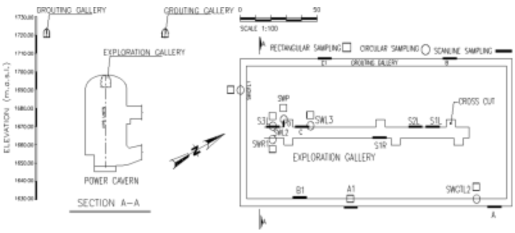

Figure 1. Location of exploratory gallery in cavern roof and cavern peripheral gallery with the location of scanlines and areal sampling.

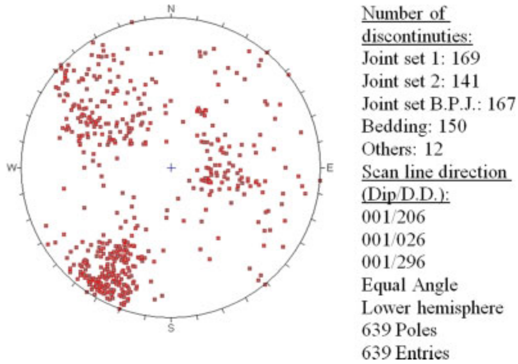

Figure 2. All the surveyed discontinuities poles with number of discontinuities in each set and direction of scanlines.

cavern domain is covered by Dalan formation. The Dalan formation outcrop of with a thickness about 350 to 400 m includes limestone, dark grey dolomitic limestone with medium to high thickness.

The main faults in study area are Saravan-Baznavid (S.B.), ChleHatam and Rudbar. Among which, S.B. with a length of 96 km, which is a segment of Zagros main recent fault, is the most important active seismogenic fault (quaternary). S.B has most effect on the values of ground motion parameters in site. Zagros recent fault is a quaternary and it has right-lateral strike slip mechanism. The cavern is located in about 1.3 km from S.B. fault.

3 FIELD INVESTIGATION TO RECORD CHARACTERISTICS OF DISCONTINUITIES

Sampling methods with scanline and areal sampling including circular and window (rectangle and square are used to investigate and estimate the characteristic of geometrical and mechanical discontinuities in powerhouse area domain. A 120 m long exploratory tunnel with 6*6 m horseshoe section is excavated in the cavern roof. A 514 m long grouting gallery with 3 m*4 m (w*h) and horseshoe section is excavated 25 m away from the cavern axis and 20 m above the cavern roof, figure 1. The exploratory gallery cavern roof and the cavern peripheral gallery are surveyed to the international recommendation. The 11 scanlines with a length of 10 to 16 m, each one, are used to survey the discontinuities in these locations.

The total length of scanlines is 118 m. 9 circular sampling windows with radius of 70 cm to 1 m and 12 rectangle sampling windows with maximum dimensions of 1.2*1.6 m are used, as well. The location of scanlines and areal sampling are shown in Figure 1.

As the discontinuities are recorded in two locations, equality of two mean tests and analysis of variance (ANOVA) are done on dip and spacing data at 5% significant level to investigate the data homogeneity. The results showed that data are homogenous.

It is strived to keep areal sampling and scanline close to each other. It has been tried to have some scanlines and areal samplings perpendicular to each other to decrease the orientation bias. In this study only scanline is used for bedding and B.P.J. that are over 16 and 2 m in length, respectively. As regards the J1 and J2 joint sets trace length, rectangle and circular sampling windows are only used to analyze these joint sets. This matter is actually a weak point of 2D surface sampling methods in practical conditions (Weiss, 2008). In such a case, it is recommended to use bigger sampling windows (Zhang et al. 2000). This case is not applicable to Rodbar site, due to the grouting gallery height (2.5 m), exploratory tunnel height (4 m) in the cavern roof and unfavorable condition in the 1 m section of exploratory tunnel wall near to floor.

Length of the all scanline is 20 to 32 times more than the means pacing for all joint sets. This value is 9 at the case study of Arrowhead East Tunnel (Kulatilake et al. 2003). It is tried, to consider the general recommendation in all surveys i.e. to record 130–350 joints in every sampling zone out of which 50% have one visible end (Priest 1993). The 639 discontinuities have been recorded in this study whose poles are shown in Figure 2.

4 GENETIC CLASSIFICATION OF DISCONTINUITY SYSTEM

In genetic classification of discontinuities, the jointing formation is considered. As per principal stresses and dynamic formation, fractures can be classified into three types: dilational, shear and hybrid (dilational-shear) (Mandl 2005). The min. and max. principal stresses vs. dilational and shear fractures are shown in figure 3. The 28 HF and HTPF tests are done in two boreholes to determine the optimum axis of powerhouse cavern and to consider the cavern stability. So, the Direction of the measured maximum horizontal is N130E.

Due to the existence of the right-lateral strike slip fault and occasionally reverse component, young orogeny activities in study area and powerhouse cavern location in tensile zone of anticline (as one of local structures), the direction of site principle stress is perpendicular to prevailing stress direction of Iran. The direction of max. and min. horizontal stresses, cavern direction and all joint sets are shown in figure 3.

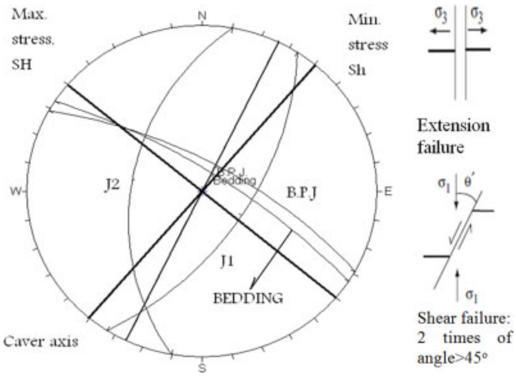

Figure 3. The direction of max. and min. horizontal stresses, cavern direction, all joint sets and direction of stress proportionate shear and dilational fractures.

As seen, the joint sets J1 and J2 are of shear type and B.P.J. joint set is of dilational type. B.P.J. is parallel to the axis of folding. The shear type fractures are characterized by slickenside as seen on J1 and J2 joint sets over site investigations. There are 12 small faults and major joints, but shear fractures do not exist around the faults.

5 GEOMETRICAL DISCONTINUITIES CHARACTERISTICS

5.1 *Orientation of discontinuities*

In most cases, the evaluation of discontinuities and their orientation are the most important geological characteristics in the design of underground spaces and the structures constructed on rock mass. In hard rock mass, discontinuities usually occur in specified orientation (Kulatilake 1986). In general, the fractures formed in the same geologic event have similar characteristics (e.g. preferable orientation). So, clustering the fractures is very important before their statistical processing (Lei et al. 2014). According to clustering, four discontinuity sets (three joint sets plus one bedding) are specified. In view of the geometrical conditions of the third joint set proportionate to the bedding, it is considered a bedding plane joint (B.P.J.). As regards the scanline direction, the orientation data including dip and dip direction are corrected (unbiased data), table 1.

The orientation of all discontinuities is modeled by Fisher distribution, table 2. The Fisher constant is between 0–700, but it is usually between 20 to 300 in rock mechanics and orientation analysis (Kemeny et al. 2003).

According to table 2, all discontinuities except the bedding have Fisher constant less than 20 that is shown high tectonic activities in the study area. The ρ angle is the angle between scanline and fisher mean pole vector. This angle is necessary to convert the linear and areal intensity to volumetric intensity (Wang 2005), table 2.

Table 1. The values of corrected and uncorrected dip and dip direction.

	Dip (°)		Dip Direction (°)	
Type	Uncorrected	Corrected	Uncorrected	Corrected
Bedding	77	76	032	033
J_1	62	60	129	123
J_2	45	46	263	280
B.P.J.	73	70	029	029

Table 2. The values of corrected and uncorrected Fisher constant and ρ angle along scanlines.

	K-Fisher		ρ angle (°)	
Type	Uncorrected	Corrected	Along 001/206 & 001/026	Along 001/296
Bedding	28.6	25.6	15.6	*
J_1	13.5	17.3	84	30
J_2	6.6	9.5	78.6	47
B.P.J.	11	10.4	20.2	*

5.2 *Length of discontinuities*

Length or persistence of discontinuities is one of the most important characteristics of rock mass. It is really difficult to quantify of this characteristic because there is not a direct reliable method in this respect (Brown 1981). Three methods such as scanline, rectangle and circular sampling are used to calculate the mean trace length of discontinuities. As regards the field investigation and site studies in surface space, the bedding trace length is 10 m to 20 m (high persistence) in 80% and longer than 20 m (very high persistence) in 20% case. According to site observation and weighted averaging, the average and standard deviation of bedding trace are considered 16 m and 4 m, respectively. As per the dimension of underground spaces and all 150 surveyed beddings are doubly censored proportionate to Scanline, to evaluate the trace length only trace length of J1, J2 and B.P.J. are considered with above methods.

5.2.1 *Scanline method*

For each discontinuity set proportionate to scanline, the semi trace length can be classified into three groups (Kulatilake et al. 2003). As regards the location of scanline and surveyed area in tunnel, the situation of end points of discontinuities are divided into three cases: two end points are visible (first type or 2-2), one end point is visible in up or down of scanline (second type or 3-2 or 2-3) and two end points are censored (third type or 3-3).

Three statistics methods such as Chi-square, Anderson-Darling and Kolmogorov-Smirnov at 5% significance level are used to select best probability distribution function (Pdf). So, the value of Test statistics, P-value and P-P plot are used to rank the Pdfs. The difference in distribution type of trace length is as a result of the difference in joint mechanical formation (Dershowitz et al. 1988).

According to the statistics methods, the trace length of J_1 & J_2 follow Lognormal. These joint sets are shear fracture. As regards the censoring type of B.P.J. end point trace length, the Uniform and Weibull distributions are suitable. These types of distributions are less reported for trace length. As regards this joint set is dilational, the main reason of suitability of Weibull distribution can be the stress regime of the study area.

As per the limited of surveying area, there are censoring and curtailment biases. So, the correction of length bias should be done (Priest et al. 1981). The cut-off length is considered 5 cm to eliminate the curtailment biases. As regards the type of Pdf and curtailment level, the 'ncr' method is used to eliminate the third type discontinuities bias (Priest 1993, Priest et al. 1981). The curtailment levels of J_1, J_2 and B.P.J. are calculated 200, 230 and 400 cm, respectively. So as regards the above explanations, the mean and standard deviation trace length of J_1, J_2 and B.P.J. are calculated 63 & 52.5 cm, 65 & 52.7 cm, 206 & 102 cm, respectively.

5.2.2 *Circular and square window methods*

10 circular sampling windows with diameter larger than the block size and spacing, but very smaller than the dimension of sampling domain are necessary to determine the fracture trace length (Rohrbaugh et al. 2002). 12 rectangle windows and 9 circular windows are used to calculate J_1 & J_2 trace length. The measurements done with window sampling can be affected by orientation, curtailment and censoring biases (Zeeb et al. 2013). As regards the trace length of J_1 & J_2, the rectangle windows and circular windows have max. 1.2*1.6 (m) dimensions and max. radius of 1 m, respectively.

With regard to the low Fisher constant of B.P.J. and its mean trace length calculated by scanline method, the suitable survey circle diameter should be min. 6 m based on numerical analysis. So, according to practical difficulties and there is edge effect due to tunnel dimensions and circle diameter of 6 m, the window method is not used for B.P.J. joint set.

The relations presented by Zhang et al. (Zhang et al. 2010), Mauldon et al. (Mauldon et al. 2001), Andersson and Dverstorp (Ferrero et al. 2011), Pahl (Priest 1993) and Laslett (Priest 1993) are used to calculate trace length with areal sampling. Based on the results of window sampling technique, the mean trace length of J_1 & J_2 joint sets ranges from 25 to 65 cm and from 34 to 110 cm, respectively. The mean trace lengths of J1 & J2 joint sets with areal method (average all windows) are 45 and 67 cm, respectively. As seen, the differences between scanline and areal methods for J_1 & J_2 joint sets are 28% and 3%, respectively.

The standard deviations of J_1 & J_2 joint sets in areal method with the relation presented by Zhang et al. (Zhang et al. 2010) are 13 and 21 cm, respectively. As regards the results of scanline and areal methods, the mean trace lengths of J_1 & J_2 joint sets are calculated 54 and 66 cm, respectively.

Table 3. The mean and standard deviation of trace length and size of joint sets.

	Trace Length (cm)			Diameter (cm)		
Type	Mean	S.D.	C.O.V	Mean	S.D.	C.O.V.
Bedding	1600	400	0.25	2075	277.6	0.13
J_1	54	52.5	0.97	38.2	34.2	0.90
J_2	66	52.7	0.80	55.2	39.8	0.72
B.P.J.	206	102	0.50	222.8	93.94	0.42

5.3 *Size (diameter) of discontinuities*

Persistence (size) defined as areal extension or discontinuity size along a plane can be quantified by trace length (Brown 1981). Since there isn't a reliable direct method to measure it, it is really difficult to quantify it. As regards the insufficient knowledge of discontinuities shape, the joint trace length and its Pdf are considered the same along the dip and strike (Zhang et al. 2000, Robertson 1970, Song 2006). Of course, there is not general agreement in this case.

In this study, the discontinuities are assumed to be circle in shape. The mean and standard deviation of trace length and diameter for four discontinuity sets are calculated, Table 3.

5.4 *Discontinuities spacing*

Spacing is distance between adjacent discontinuities. The value of spacing is applicable to describe continuum medium and input parameters for rock mass classification, to estimate of RQD, to describe of rock block geometries (Mahtab 1995). In the study area, three types of spacing including: total spacing, spacing of each joint set along scanline (set spacing) and along discontinuity normal vector are investigated. For spacing data, three statistics and three methods explained in part 5-2-1 are used to evaluate the best Pdf and ranking. The amount of total spacing doesn't obey with any distribution.

The bias is corrected according to suggestion of Perist to calculate the normal spacing (Priest, 1993). The prevalent Pdf for spacing are Lognormal and exponential (Kulatilake 2003, Priest 1981), but Weibull and Gamma function are usable (Torres 2008, Stavropoulou 2014), as well. The suitable Pdf for J_1 & J_2 joint sets that are shear type is Weibull and for bedding and B.P.J. that is dilational type is Lognormal. Accordingly, the mechanism of discontinuity formation, type and direction of applied stress on discontinuity surface are main reasons for the difference of joint set spacing Pdf types. This matter shows that the local stress factor in site has effect on Pdf.

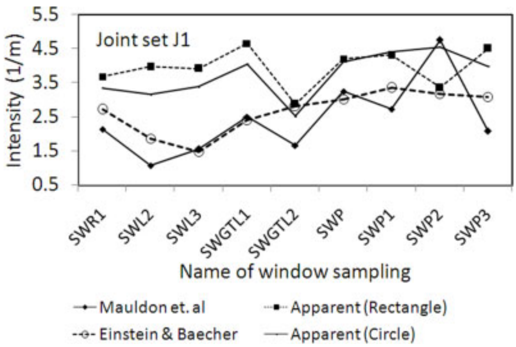

Figure 4. The actual and apparent areal intensity of J1 joint set.

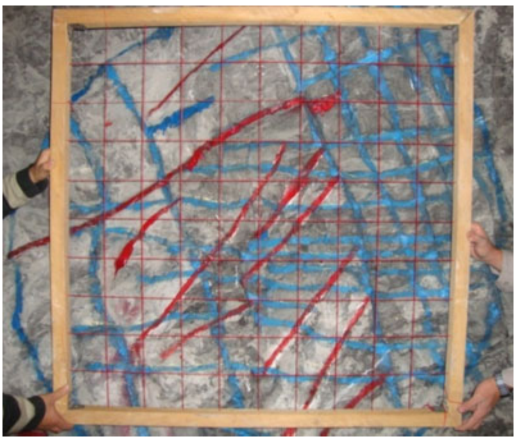

Figure 5. The window sampling with 10*10 cm grids in SWL2 location.

5.5 *Intensity of discontinuity*

Discontinuity frequency (intensity) is one of the fundamental measures of the degree of fracturing in a rock mass (Priest 1993). The intensity has three types including: linear (P_{10}), areal (P_{21}) and volumetric (P_{32}). The dimension all of them is L^{-1}.

5.5.1 *Areal intensity*

The areal intensity is the fracture trace length per unit area of sampling (Rohrbaugh et al. 2002). The relation presented by Mauldon et al. (Mauldon et al. 2001) and Einstein & Baecher (Zhang et al. 2000) are used to calculate the P_{21} as actual intensity by circular window. The apparent intensities by circular and rectangular windows are calculated for joint set J_1 and J_2 as well, figure 4. The values of mean actual intensity by two above methods and apparent intensity by circular and rectangular windows of all sampling windows for joint set J_1 are 2.4, 2.6, 3.9 and 3.7 (1/m), respectively and for joint set J_2 are 3.5, 2.15, 5.3 and 4.7 (1/m), respectively.

As seen, for above joint sets the value of apparent intensity by circular and rectangular methods is close together (the max. difference is 11%) and they are bigger than the actual intensity. According to the variation of areal actual intensity for J_1 and J_2 joint sets, it is better to increase the dimension of sampling window.

Table 4. The volumetric intensity (1/m) of all joint sets by Kulatilake et al. method and the average volumetric intensity of all windows sampling for J_1 and J_2 joint sets by Zhang & Einstein method.

Type	Kulatilake et al.	Zhang & Einstein
Bedding	1.81	*
J_1	4.08	2.21
J_2	4.65	3.92
B.P.J.	2.02	*

5.5.2 *Volumetric intensity*

The P_{32} is the fracture area to unit volume of rock. The calculation of P_{32} of discontinuities is very difficult. The P_{32} is calculated by Kulatilake et al. (Kulatilake et al. 1993) and by Zhang & Einstein (Ferrero et al. 2011) methods. In Zhang & Einstein method, the numbers of discontinuities in unit volume are necessary. So, a window with 1*1 dimension and grids with 10*10 cm dimensions is made, figure 5.

The values of volumetric intensity of all joint sets by Kulatilake et al. method and average results of all sampling windows of J_1 and J_2 joint sets by Zhang & Einstein method are shown in table 4. As seen, the calculated differences of P_{32} between two methods for J_1 and J_2 joint sets are 46% and 16%, respectively. This matter shows the estimation of this characteristic is very difficult in site.

6 CONCLUSION

As regards the most significance and key role of discontinuities in mechanical and hydraulic behavior of rock mass, the investigation of geometrical and mechanical characteristics of discontinuities is necessary. In this paper, the geometrical characteristics of discontinuities of Rodbar Lorestan pumped storage power plant cavern are investigated as a case study. According to 639 surveyed discontinuities in cavern domain, three joint sets plus bedding are specified. The important characteristics of the study area are young orogeny activity, existence of active faults, tectonic uplift, many trusts and location of the cavern in tensile zone of anticline. According to the stress regime and site tectonic, two joint sets are dilational type and another on is shear type. As regards the mechanism formation of discontinuities and their genetic types, the probability distribution functions (Pdf) of trace length and spacing of shear and dilational joint sets are different together. This matter shows that the structural mechanism and stress condition of site study have profound effect on Pdf type of discontinuities geometrical parameters.

The tectonic conditions caused the Fisher constant of the three joint sets decrease. The Fisher constant of bedding that formed in compressive mechanism is 53% to 73% larger than the Fisher constant of three joint sets. For joint sets J_1 and J_2, the calculated trace length differences between scanline and areal method are 3% and 28%. So, it is recommend for the trace length smaller than 1 m, the radius of circle and the dimension of rectangle windows should be minimum 1 m and 1.8 m*1.8 m, respectively.

The values of apparent intensity by circular and rectangle windows are close together, but they are larger than actual intensity. The calculation of volumetric intensity is very difficult in the site. According to the results, the differences between two methods of volumetric intensity calculation are 46% and 16% for joint sets J_1 and J_2, respectively. Of course, as regards the type of this parameter 15% difference is acceptable. The value of intensity are affected by trace length. With regards to the calculated values of intensity by circular windows, which have fluctuations for joint sets J_1 and J_2 and the value of Fisher constant is low, it is suggested to increase the radius of window sampling to 1.5 m.

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