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To cite this article: Y. L. D. Ling et al 2019 IOP Conf. Ser.: Mater. Sci. Eng. 495 012109

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# Cascaded Channel Estimation Technique for Massive MIMO Relay System

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Abstract. This paper concerns with the cascaded channel estimation of massive multipleinput multiple-output (MIMO) relay system. In order to meet the increasing demands for high-speed wireless communication networks, massive MIMO has been recognized as one of the key technologies for the future fifth generation (5G) cellular networks. It is an advanced MIMO technique consists of a very large number of antennas at the base station and serves a smaller number of single-antenna users simultaneously. Basically, the idea of massive MIMO technique is to harvest all the advantages of conventional MIMO system in a much larger scale. To reap the benefits of massive MIMO in practice, an accurate estimation of the channel state information (CSI) is needed. In this paper, the relaying technique has been incorporated with massive MIMO system in order to increase system throughput and improve the coverage in cell-edge users. A relay node is placed in between transmitter and receiver to reduce the path loss and improve the spectral efficiency of massive MIMO system. Cascaded channel estimation technique for massive MIMO relay system is developed in this paper. The mean squared error (MSE) of the cascaded channel estimation for massive MIMO relay system is optimized to obtain accurate CSI.

#### 1. Introduction

The current multiple-input multiple-output (MIMO) technology can significantly improve the capacity and reliability of the wireless communication systems. However, it might unable to cope with the increasing demands for higher data rate in future. Cisco (2017) in white paper shows that the global mobile data traffic in 2016 increased by 63% compared to 2015, which is mainly caused by the rapid cellular technology developments. One of the potential solutions to meet these demands is implementing the massive MIMO technique, which extends the concept of conventional MIMO by scaling up the number of antenna ports at base station to hundreds (Elijah et al., 2016; Khansefid and Minn, 2015; Lu et al., 2014; Wang et al., 2015; Zuo et al., 2015).

Massive MIMO communication system is equipped with a large number of antennas at the base station and served with a much smaller number of single-antenna terminals simultaneously. The concept of massive MIMO technique is to harvest all the advantages of conventional MIMO systems in a larger scale. Massive MIMO is one of the possible fifth generation (5G) wireless system technology. Many companies and industry groups expect that massive MIMO can handle more traffic at higher transmission speed than the conventional MIMO system. As massive MIMO is equipped with up to hundred of antennas at base station for transmitters and receivers, therefore its base station is able to transmit and receive signals from more users at the same time. Nordrum (2016) set up an experiment with 128-antenna array and proved that the large

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number of antennas can increase the capacity and spectrum efficiency of wireless communication networks by a factor of 22 over current existing 4G networks (Nordrum and Clark, 2017).

There are two major duplex manners in wireless communication systems, namely time-division duplex (TDD) and frequency-division duplex (FDD) (Elijah et al., 2016; Lu et al., 2014). For FDD mode, the uplink and downlink channels work on different carrier frequencies at the same time; and hence, the instantaneous CSI is required for both downlink and uplink. As the number of time-frequency resources used for downlink pilots are proportional to the number of antennas at the base station, therefore, massive MIMO system requires hundred times more of time-frequency downlink resources compared to a conventional MIMO system (Nam et al., 2012). Generally, the solution to these limitations is to operate in TDD mode by relying on the reciprocity property. Unlike FDD, same carrier frequencies are allocated for uplink and downlink channels in TDD operation mode. This indicates the reciprocity of both uplink and downlink channels (Lu et al., 2014; Liu et al., 2015). Using this property, the downlink channel can be obtained from the estimated uplink channel without through feedback scheme. Therefore, TDD protocol is considered in this paper.

The application of massive MIMO has been investigated through many research works, yet the researchers mostly assume that the channel estimation is known at the base station with some CSI (Ngo, Larsson, and Marzetta, 2013). The channel estimation is assumed to be done with the uplink pilots by relying on the reciprocity property of TDD operation mode. It is crucial to have an improved channel estimation in order to have better signal transition, as well as recover and retrieve the transmitted signal at the receiver. Therefore, to reap all the benefits of massive MIMO, it is required to have an accurate CSI for the channel estimation.

Poor signal coverage for cell-edge users is one of the major concerns in wireless communication networks. Due to the long transmission path between the transmitter and receiver, base station is hardly to receive the pilot signal from the user terminals. The path loss is higher when the distance is far, which results in weak signal transmission. Relay technology, which acts as a node between transmitter and receiver, can reduce the pathloss between them effectively through cutting a long transmission path into a few short paths (Chen et al., 2016). Hence, the relay system has taken into account in massive MIMO system.

There are two main types of relaying strategies, namely amplify-and-forward (AF) and decodeand-forward (DF) cooperative strategies. AF cooperative strategy amplifies the received signals and then forwards the amplified signals to the receiver; while the DF cooperative strategy decodes and re-encodes the received signals first, before forwards them to the receiver. The results in Zhang et al. (2009) prove that AF relay has larger deployment area than DF scheme and is able to work under more channel conditions. As sophisticated signal processing is not required for AF cooperative strategy, it is easier to implement compared to DF cooperative strategy. In this paper, AF cooperative strategy is considered to be implement in the massive MIMO relay network.

However, the channel estimation for massive MIMO relay system is usually assumed to be available at the base station and only channel statistic is known by user terminals (Chen et al., 2016). Perfect channel state information (CSI) is assumed for both source-relay and relay-destination links by relying on the reciprocity property of TDD protocol (Kudathanthirige and Baduge, 2017; Wang and Jing, 2016). The effectiveness of massive MIMO system is severely affected by the accuracy of the channel estimation (Elijah et al., 2016). Thus, it is crucial to have accurate channel estimation for massive MIMO relay system.

The rest of this paper is organized as follows. The system model of massive MIMO relay system is presented in Section II. The uplink transmission algorithm is developed in Section III, and Section VI optimizes the MSE problem. Last but not least, Section V concludes the paper.

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#### 2. System and channel model

In a single cell, consider K single-antenna user terminals (UTs) transmit signal to the relay node (RN) with N-antennas, and the RN forwards the signal to the base station (BS) with M-antennas after amplified the received signal. The AF half-duplex relay is assumed to be used in the network. The UTs to BS channel is considered to be unavailable because of the severe path loss and shadowing effects. Figure 1 shows the proposed massive MIMO relay network system model consists of L cells. The received noise is modeled as zero mean additive white Gaussian noise (AWGN).

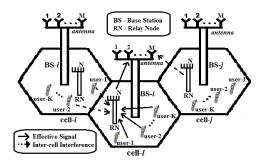


Figure 1. Massive MIMO relay network system model

Same channel model is considered for the channel between UT to RN and the channel between RN to BS, which denoted by  $\mathbf{F}$  and  $\mathbf{G}$  respectively. In *l*th cell, the channels can be expressed as

$$\mathbf{F}_{\mathbf{ll}} = \widehat{\mathbf{F}_{\mathbf{ll}}} \sqrt{\mathbf{D}_{\mathbf{F}_{\mathbf{ll}}}} \quad and \quad \mathbf{G}_{\mathbf{ll}} = \widehat{\mathbf{G}_{\mathbf{ll}}} \sqrt{\mathbf{D}_{\mathbf{G}_{\mathbf{ll}}}} \tag{1}$$

where  $\widehat{F_{ll}}$  and  $\widehat{G_{ll}}$  are the small-scale fast fading that modeled as block Rayleigh fading, and are given by

$$\widehat{\mathbf{F}_{\mathbf{l}\mathbf{l}}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{\mathbf{K}} \otimes \mathbf{I}_{\mathbf{N}})$$
$$\widehat{\mathbf{G}_{\mathbf{l}\mathbf{l}}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{\mathbf{N}} \otimes \mathbf{I}_{\mathbf{M}})$$
(2)

Here  $\otimes$  stands for the matrix Kronecker product. While  $\mathbf{D}_{\mathbf{F}_{\mathbf{II}}}$  and  $\mathbf{D}_{\mathbf{G}_{\mathbf{II}}}$  are the large-scale fading that represent path loss and shadowing effects. The large-scale fading coefficients,  $\beta_k$  and  $\beta_n$ , represent the geometric attenuation and shadow fading of user-k and antenna-n of RN (Ngo, Larsson, and Marzetta, 2014). It changes slowly and assumed to remain constant over a coherence time interval, T.

Small-scale fast fading is assumed to be independent of each users and antennas of the receiver. For large-scale fading, it is assumed to be same for different antennas of the receiver but user-independent. Therefore, the antennas of RN have the same large-scale fading.

By referring to (1) and (2), the interference channel between UT in cell-*i* and RN in cell-*j* is denoted as  $\mathbf{F}_{ji} = \widehat{\mathbf{F}}_{ji} \sqrt{\mathbf{D}_{\mathbf{F}_{ji}}}$  for  $j, i \in \{1, ..., L\}$ , while the interference channel between RN in cell-*j* and BS in cell-*l* is denoted as  $\mathbf{G}_{lj} = \widehat{\mathbf{G}}_{lj} \sqrt{\mathbf{D}_{\mathbf{G}_{lj}}}$ .

#### 3. Uplink transmission algorithms

The uplink (UTs to BS) transmission is first estimated as the downlink (BS to UTs) transmission can be estimated from uplink through reciprocity property. Block fading model is considered, such that the channel remains constant over certain coherence interval  $\tau$  and is independent of

each channels. Assume that all UTs are transmitting synchronous pilot sequences of length  $\tau$  at the same time to BS. The total coherence interval,  $T \ge K + N$  time slots are used for uplink data transmission, where the first time slot (UTs to RN) is  $\tau_1 \ge K$  and the second time slot (RN to BS) is  $\tau_2 \ge N$ .

Assume that same transmit power is used for all UTs, the received signal at relay node within one cell is given by

$$\mathbf{Y}_{R,l} = \sqrt{\tau_1 P_{P1}} \mathbf{F}_{ll} \mathbf{\Phi}_{P1} + \sqrt{\tau_1 P_{P1}} \sum_{i=1, i \neq l}^{L} \mathbf{F}_{li} \mathbf{\Phi}_{P1} + \mathbf{N}_{R_l}$$
(3)

where  $\tau_1$  and  $P_{P1}$  are the coherence time for uplink pilot and average pilot transmit power respectively,  $\mathbf{F}_{ll}$  is the channel matrix from UTs to RN,  $\mathbf{\Phi}_{P1}$  is  $K \times \tau_1$  pilot matrix transmitted by each UTs,  $\mathbf{F}_{li}$  is the interference channel matrix from UTs to RN in the *i*th cell, and  $\mathbf{N}_{R_l}$ denotes the received AWGN noise matrix at RN. The noise matrix is assumed to be independent and identical distributed (i.i.d.) additive white Gaussian noise, whose elements are  $\mathcal{CN}(0,1)$ random variables which satisfying  $E[\mathbf{N}_{R_l}\mathbf{N}_{R_l}^H] = \mathbf{I}_{\mathbf{N}R_l}$ .

The relay node amplifies the received signal  $\mathbf{Y}_R$  with an amplification factor  $\alpha$  before forward them to the BS. Therefore, the transmitted signal at relay node can be written as

$$\mathbf{Y}_{BR,l} = \alpha \mathbf{Y}_{R,l} \tag{4}$$

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From Equation (3) and (4), the received signal at the base station is given by

$$\begin{aligned} \mathbf{Y}_{B,l} &= \mathbf{G}_{ll} \mathbf{Y}_{BR,l} + \sum_{j=1, j \neq l}^{L} \mathbf{G}_{lj} \mathbf{Y}_{BR,j} + \mathbf{N}_{B_l} \\ &= \sqrt{\tau_1 P_{P1}} \alpha_{R_l} \mathbf{G}_{ll} \mathbf{F}_{ll} \mathbf{\Phi}_{P1} \\ &+ \sqrt{\tau_1 P_{P1}} \alpha_{R_l} \mathbf{G}_{ll} \sum_{i=1, i \neq l}^{L} \mathbf{F}_{li} \mathbf{\Phi}_{P1} \\ &+ \sqrt{\tau_1 P_{P1}} \alpha_{R_j} \sum_{j=1, j \neq l}^{L} \sum_{i=1}^{L} \mathbf{G}_{lj} \mathbf{F}_{ji} \mathbf{\Phi}_{P1} \\ &+ \sqrt{\tau_1 P_{P1}} \alpha_{R_i} \sum_{j=1}^{L} \mathbf{G}_{lj} \mathbf{N}_{R_l} + \mathbf{N}_{B_l} \\ &= \sqrt{\tau_1 P_{P1}} \alpha_{R_l} \mathbf{G}_{ll} \mathbf{F}_{ll} \mathbf{\Phi}_{P1} + \overline{\mathbf{N}}_{B,l} \end{aligned}$$
(5)

while the  $\overline{\mathbf{N}}_{B,l}$  is calculated as

$$\overline{\mathbf{N}}_{B,l} \triangleq \sqrt{\tau_1 P_{P1}} \alpha_{R_l} \mathbf{G}_{ll} \sum_{i=1, i \neq l}^{L} \mathbf{F}_{li} \mathbf{\Phi}_{P1} + \sqrt{\tau_1 P_{P1}} \alpha_{R_j} \sum_{j=1, j \neq l}^{L} \sum_{i=1}^{L} \mathbf{G}_{lj} \mathbf{F}_{ji} \mathbf{\Phi}_{P1} + \sqrt{\tau_1 P_{P1}} \alpha_{R_i} \sum_{j=1}^{L} \mathbf{G}_{lj} \mathbf{N}_{R_l} + \mathbf{N}_{B_l}$$
(6)

where  $\alpha_{R_l}$  and  $\alpha_{R_j}$  refers to the amplification factor of RN in desired cell and in the *j*th cell, respectively,  $\mathbf{G}_{ll}$  is the channel matrix from UTs to BS,  $\mathbf{G}_{lj}$  is the interference channel matrix from UTs to BS in the *j*th cell,  $\mathbf{N}_{B_l}$  denotes the received AWGN noise matrix at BS, and  $\overline{\mathbf{N}}_{B,l}$  is assumed to be the equivalent noise matrix at BS.

By vectorizing both sides,

$$\mathbf{y}_{B,l} = (\sqrt{\tau_1 P_{P1}} \alpha_{R_l} \mathbf{\Phi}_{P1}^T \otimes \mathbf{I}_M) vec(\mathbf{H}_{ll}) + vec(\overline{\mathbf{N}}_{B,l}) = (\sqrt{\tau_1 P_{P1}} \alpha_{R_l} \mathbf{\Phi}_{P1}^T \otimes \mathbf{I}_M) \mathbf{h}_{ll} + \overline{\mathbf{n}}_{B,l} = \mathbf{S}_{P1} \mathbf{h}_{ll} + \overline{\mathbf{n}}_{B,l}$$
(7)

where  $\mathbf{y}_{B,l} \triangleq vec(\mathbf{Y}_{B,l}), \mathbf{h}_{ll} \triangleq vec(\mathbf{H}_{ll}), \mathbf{\bar{n}}_{B,l} \triangleq vec(\mathbf{\bar{N}}_{B,l}), \text{ and } \mathbf{S}_{P1} \triangleq \sqrt{\tau_1 P_{P1}} \alpha_{R_l} \mathbf{\Phi}_{P1}^T \otimes \mathbf{I}_M.$ Here  $vec(\cdot)$  denotes the vectorization operator that stacks all the column vectors of a matrix on top of each other, and identity of  $vec(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})vec(\mathbf{B})$  is used to obtain Equation (7) from Equation (5).

Linear MMSE estimator is applied to estimate  $\mathbf{h}_{ll}$  due to its simplicity,

$$\widehat{\mathbf{h}}_{ll} = \mathbf{W}^{\mathbf{H}} \mathbf{y}_{B,l} \tag{8}$$

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where  $\widehat{\mathbf{h}}_{ll}$  is an estimation of  $\mathbf{h}_{ll}$  and  $\mathbf{W}^{\mathbf{H}}$  is the weight matrix of the MMSE estimator. As a linear channel estimator is used, the time interval is  $T \ge K + N$ , and the MSE of estimating  $\mathbf{h}_{ll}$  can be calculated as

$$MSE = E[tr((\widehat{\mathbf{h}}_{ll} - \mathbf{h}_{ll})(\widehat{\mathbf{h}}_{ll} - \mathbf{h}_{ll})^{H})]$$
  
=  $tr[(\mathbf{W}^{\mathbf{H}}\mathbf{S}_{P1} - \mathbf{I}_{Q})\mathbf{R}_{h_{ll}}(\mathbf{W}^{\mathbf{H}}\mathbf{S}_{P1} - \mathbf{I}_{Q})^{H}$   
+  $\mathbf{W}^{\mathbf{H}}\mathbf{R}_{\overline{n}_{B_{l}}}\mathbf{W}]$  (9)

where  $\mathbf{I}_Q$  is identity vector with dimension of  $M \times K$ ,  $\mathbf{R}_{h_{ll}}$  is the channel covariance matrix, and  $\mathbf{R}_{\overline{n}_{B,l}}$  is the noise covariance matrix. The *m*th column of  $\mathbf{H}_{ll}$  is given as  $[\mathbf{H}_{ll}]_m = \beta_{G_{ll},m}^{1/2} \mathbf{A}_{G_{ll}} \mathbf{H}_{G_{ll,w}} \mathbf{B}_{G_{ll}}^H \mathbf{A}_{F_{ll}} \mathbf{H}_{F_{ll,w}} \mathbf{K}_{F_{ll}}^H \beta_{F_{ll,m}}^{1/2}, m = 1, ..., N$ , where  $\beta_{G_{ll,m}}$  and  $\beta_{F_{ll,m}}$  are the *m*th diagnoal element of  $\mathbf{D}_{G_{ll}}$  and  $\mathbf{D}_{F_{ll}}$ , respectively. By using Lemma 1, the covariance matrix of  $[\mathbf{H}_{ll}]_m$  can be calculated as

$$E[[\mathbf{H}_{ll}]_{m}[\mathbf{H}_{ll}]_{m}^{H}]$$

$$= \beta_{G_{ll}}\beta_{F_{ll}}tr(\mathbf{A}_{F_{ll}}\mathbf{A}_{F_{ll}}^{H}\mathbf{I}_{N})\mathbf{I}_{M}$$

$$= \beta_{G_{ll}}\beta_{F_{ll}}N\mathbf{I}_{M}$$
(10)

From (10), the channel covariance matrix  $\mathbf{R}_{h_{ll}}$  can be obtained as

$$\mathbf{R}_{h_{ll}} = \mathbf{D}_{F_{ll}} \otimes \beta_{G_{ll}} N \mathbf{I}_M \tag{11}$$

Using (6) and Lemma 1, the covariance matrix of  $E[\overline{\mathbf{N}}_{B,l}\overline{\mathbf{N}}_{B,l}^{H}]$  can be written as

$$E[\overline{\mathbf{N}}_{B,l}\overline{\mathbf{N}}_{B,l}^{H}]$$

$$= \tau_{1}P_{P1}\alpha_{R_{l}}^{2}\sum_{i=1,i\neq l}^{L}E(\mathbf{G}_{ll}\mathbf{F}_{li}\boldsymbol{\Phi}_{P1}\mathbf{\Phi}_{P1}^{H}\mathbf{F}_{li}^{H}\mathbf{G}_{ll}^{H})$$

$$+ \tau_{1}P_{P1}\alpha_{R_{j}}^{2}\sum_{j=1,j\neq l}^{L}\sum_{i=1}^{L}E(\mathbf{G}_{lj}\mathbf{F}_{ji}\boldsymbol{\Phi}_{P1}\mathbf{\Phi}_{P1}^{H}\mathbf{F}_{ji}^{H}\mathbf{G}_{lj}^{H})$$

$$+ \alpha_{R_{i}}^{2}\sum_{j=1}^{L}E(\mathbf{G}_{lj}\mathbf{N}_{R_{l}}\mathbf{N}_{R_{l}}^{H}\mathbf{G}_{lj}^{H}) + E(\mathbf{N}_{B_{l}}\mathbf{N}_{B_{l}}^{H})$$

$$= \tau_{1}P_{P1}\alpha_{R_{l}}^{2}\sum_{i=1,i\neq l}^{L}tr(\boldsymbol{\Phi}_{P1}\boldsymbol{\Phi}_{P1}^{H}\mathbf{D}_{F_{li}}\otimes\mathbf{D}_{G_{ll}})\mathbf{I}_{M}$$

$$+ \tau_{1}P_{P1}\alpha_{R_{j}}^{2}\sum_{j=1,j\neq l}^{L}\sum_{i=1}^{L}tr(\boldsymbol{\Phi}_{P1}\boldsymbol{\Phi}_{P1}^{H}\mathbf{D}_{F_{ji}}\otimes\mathbf{D}_{G_{lj}})\mathbf{I}_{M}$$

$$+ \alpha_{R_{i}}^{2}\sum_{j=1}^{L}\beta_{G_{lj}}N\mathbf{I}_{M} + \mathbf{I}_{M}$$
(12)

Using the identity of vectorization and (12), the noise covariance matrix can be obtained as

$$\mathbf{R}_{\overline{\mathbf{n}}B,l} = E[\overline{\mathbf{n}}_{B,l}\overline{\mathbf{n}}_{B,l}^{H}]$$

$$= \mathbf{I}_{T} \otimes [\tau_{1}P_{P1}\alpha_{R_{l}}^{2}\sum_{i=1,i\neq l}^{L} tr(\mathbf{\Phi}_{P1}\mathbf{\Phi}_{P1}^{H}\mathbf{D}_{F_{li}}\otimes\mathbf{D}_{G_{ll}})\mathbf{I}_{M}$$

$$+ \tau_{1}P_{P1}\alpha_{R_{j}}^{2}\sum_{j=1,j\neq l}^{L}\sum_{i=1}^{L} tr(\mathbf{\Phi}_{P1}\mathbf{\Phi}_{P1}^{H}\mathbf{D}_{F_{ji}}\otimes\mathbf{D}_{G_{lj}})\mathbf{I}_{M}$$

$$+ \alpha_{R_{i}}^{2}\sum_{j=1}^{L}\beta_{G_{lj}}N\mathbf{I}_{M} + \mathbf{I}_{M}]$$
(13)

Matrix W that minimizing MSE is given by

$$\mathbf{W} = (\mathbf{S}_{P1}\mathbf{R}_{h_{ll}}\mathbf{S}_{P1}^{H} + \mathbf{R}_{\overline{n}_{B,l}})^{-1}\mathbf{S}_{P1}\mathbf{R}_{h_{ll}}$$
(14)

where  $(\cdot)^{-1}$  stands for matrix inversion. The desired signal is assumed to be uncorrelated with noise and interference, the optimal matrix W can be obtained as

$$\mathbf{W} = (\mathbf{S}_{P1}\mathbf{R}_{h_{ll}}\mathbf{S}_{P1}^{H})^{-1}\mathbf{S}_{P1}\mathbf{R}_{h_{ll}}$$
$$= (\mathbf{S}_{P1}^{H})^{-1}\mathbf{R}_{h_{ll}}^{-1}\mathbf{S}_{P1}^{-1}\mathbf{S}_{P1}\mathbf{R}_{h_{ll}}$$
$$= (\mathbf{S}_{P1}^{H})^{-1}$$
(15)

By substituting (15) into (9), MSE of estimating  $\mathbf{g}_{BU}$  can be rewritten as

$$MSE = tr[(\mathbf{S}_{P1}^{-1}\mathbf{S}_{P1} - \mathbf{I}_Q)\mathbf{R}_{h_{ll}}(\mathbf{S}_{P1}^{-1}\mathbf{S}_{P1} - \mathbf{I}_Q)^H + \mathbf{S}_{P1}^{-1}\mathbf{R}_{\overline{n}_{B,l}}(\mathbf{S}_{P1}^H)^{-1}] = tr[\mathbf{R}_{h_{ll}} + \mathbf{S}_{P1}^{-1}\mathbf{R}_{\overline{n}_{B,l}}(\mathbf{S}_{P1}^H)^{-1}] = tr[(\mathbf{R}_{h_{ll}}^{-1} + \mathbf{S}_{P1}^H\mathbf{R}_{\overline{n}_{B,l}}^{-1}\mathbf{S}_{P1})^{-1}]$$
(16)

The transmission power consumed at the user terminals is

$$\tau_1 P_{P1} tr(\boldsymbol{\Phi}_{P1} \boldsymbol{\Phi}_{P1}^H) \tag{17}$$

Transmission power consumed at the relay node is

$$tr[E(\mathbf{Y}_{BR,l}\mathbf{Y}_{BR,l}^{H})] = \tau_{1}P_{P1}\alpha_{R_{l}}^{2}tr(\mathbf{D}_{F_{ll}}^{1/2}\mathbf{\Phi}_{P1}\mathbf{\Phi}_{P1}^{H}\mathbf{D}_{F_{ll}}^{1/2})tr(\mathbf{I}_{N}) + \tau_{1}P_{P1}\alpha_{R_{l}}^{2}\sum_{i=1,i\neq l}^{L}tr(\mathbf{D}_{F_{li}}^{1/2}\mathbf{\Phi}_{P1}\mathbf{\Phi}_{P1}^{H}\mathbf{D}_{F_{li}}^{1/2})tr(\mathbf{I}_{N}) + \alpha_{R_{l}}^{2}tr(\mathbf{I}_{N}) = \tau_{1}P_{P1}\alpha_{R_{l}}^{2}Ntr(\mathbf{\Phi}_{P1}\mathbf{\Phi}_{P1}^{H}\mathbf{D}_{F_{ll}}) + \sum_{i=1,i\neq l}^{L}\mathbf{\Phi}_{P1}\mathbf{\Phi}_{P1}^{H}\mathbf{D}_{F_{li}}) + \alpha_{R_{l}}^{2}N$$
(18)

where N is the number of antennas at the relay node.

#### 4. Optimization Problem

The optimal training matrices and the optimal  $\alpha_R$  can be obtained by solving the optimization problem from (16)-(18) as following

$$\min_{\alpha_R, \Phi_{P1}} tr[(\mathbf{R}_{h_{ll}}^{-1} + \mathbf{S}_{P1}^H \mathbf{R}_{\overline{n}_{B,l}}^{-1} \mathbf{S}_{P1})^{-1}]$$
(19)

subject to 
$$\tau_1 P_{P1} tr(\mathbf{\Phi}_{P1} \mathbf{\Phi}_{P1}^H) \le K P_K, \tag{20}$$
$$\tau_1 P_{P1} \alpha_P^2 N tr(\mathbf{\Phi}_{P1} \mathbf{\Phi}_{P1}^H) \mathbf{D}_E$$

$$+\sum_{i=1,i\neq l}^{L} \boldsymbol{\Phi}_{P1} \boldsymbol{\Phi}_{P1}^{H} \mathbf{D}_{F_{li}}) + \alpha_{R_l}^2 N \leq P_R$$

$$(21)$$

where  $P_K$  and  $P_R$  are the transmission power available at each user and the relay node, respectively.

The optimization problem can be solved by using Karush-Kuhn-Tucker (KKT) method by using the gradient and complementary slackness conditions. By using bi-section search, the non-negative  $\Phi_{P1}$  can be found with fixed  $\alpha_{R_l}$ ,  $\tau_1$  and  $P_{P1}$  for each user terminals.

On the other hand, the optimal  $\alpha_{R_l}$  value is able to obtain by applying golden section search (GSS) method explained in Table I, where  $\phi > 0$  is the reduction factor and  $\varepsilon$  is a positive constant value close to 0. The golden ratio of optimal  $\phi = 1.618$  is analysed in Antoniou and Lu (2007). The GSS is a technique to find the extremum (maximum or minimum) of a unimodal function, which can perform up to the desired accuracy without input the iterations number. On the other hand, it may require more iterations number than the Fibonacci search.

 Table 1. Method to apply GSS

1. Set a feasible region [a,b] on  $\alpha$ .

2. Define 
$$x = (\phi - 1)a + (2 - \phi)b$$
 and  
 $y = (2 - \phi)a + (\phi - 1)b.$ 

3. Solve (19) - (21) for  $\alpha = x$ , and compute the MSE value

in (19). Then, repeat for  $\alpha = y$ . 4. If  $f_{MSE}(x) < f_{MSE}(y)$ , set b = y. Else, set a = x. 5. If  $|b - a| \le \varepsilon$ , then end. Else, go to Step 2.

### 5. Conclusion and future work

The cascaded channel estimation for massive MIMO relay system is investigated. The uplink transmission algorithms for channel estimation are developed to reduce the MSE in the massive MIMO relay system. The optimization problem for MSE is obtained to obtain accurate CSI. In order to improve the MSE value, the pilot contamination issue has to be mitigated as it has severely degrades the system throughput. Therefore, additional techniques are required to mitigate the impact of pilot contamination effectively in massive MIMO relay system for better MSE in the overall system.

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