

Department of Mathematics and Statistics

**Risk, Return and Market
Condition: A New
Functional-Beta Capital Asset
Pricing Model**

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Declaration

To the best of my knowledge and belief, this thesis contains no material previously published by any other person except where due acknowledgment has been made.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

Signature:

Date: 28 June 2009

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Abstract

In this research, we will focus on investigating the relationship between risk and return. We will propose a new model which leads to a more sensible approach to modelling the relationship between risk and return under different market conditions. It is an extension of the traditional single-index capital asset pricing model (CAPM) which reads as: The return R_i on individual Security i can be decomposed into the specific return $\alpha_i + \varepsilon_i$ (expected specific return α_i and random specific return ε_i) and the systematic return $\beta_i R_m$ owing to the common market return R_m .

In our new model, we suggest a functional-beta single-index CAPM, extending the work of three-beta CAPM (Galagedera and Faff, 2004) that takes into account the condition of market volatility. Differently from the three-beta CAPM, we allow β_i changing functionally with the market volatility σ_m , which is more flexible and adaptable to the changing structure of financial systems. The main contributions of this thesis are summarised as follows:

- A new functional-beta CAPM, taking into account the conditions of market volatility, is proposed under the framework of widely applicable data generating processes of near epoch dependence (NED).
- A semi-parametric estimation procedure based on least squares local lin-

ear modelling technique under NED is suggested with the large sample distributions of the estimators established.

- Simulation study is fully made, illustrating that the suggested estimation procedure for the proposed functional-beta CAPM under near epoch dependence can work well. It provides reasonable estimates of the functional beta in the condition of moderate market volatility.
- By using a set of stocks data sets collected from Australian stock market in the past ten years, empirical evidences of the functional-beta CAPM in Australia are carefully examined under both nonparametric and parametric model structures. Differently from the three- or multi-beta (constant) CAPM in Galagedera and Faff (2005), our new findings show that the functional beta can be reasonably parameterized as threshold (regime-switching) linear functions of market volatility with two or three regimes of market condition. In the condition of extreme market volatility, a threshold functional-beta CAPM is suggested.

The CAPM provides a usable measure of risk that helps investors determine what return they deserve for putting their money at risk. Our new model is no doubt helpful to better understand the relationship between risk and return under different market conditions. It can be potentially applied widely, for example, it may be useful both for market investors and financial risk managers in their investment/management decision-making.

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Chapter 1

Background: Literature and Problems

1.1 Introduction

Modern financial management requires a number of key mathematical concepts. As commented by Smith (1997), this is particularly relevant to financial risk management and risk management products. Because of various information shocks, financial systems are frequently evolving with the time. The reason why we would attempt to study the risk-return relation variation is obvious. For example, Carvalho, Durand and Hock Guan (2002) studied daily returns on a primary sample of the 21 Australian Internet stocks in the Merrill Lynch (Australian) Internet Stock Index at the climax of the boom-bust period, between 21 September 1999 and 20 September 2000. They found evidence that Internet stocks exhibited positive risk adjusted returns in the pre-Crash period but that these returns disappear in the post-Crash period, indicating *the importance of market conditions*.

Classical portfolio theory examines risk/return tradeoffs from a “mean-variance” framework. In this model, the risk of an individual security is measured by the variance (or, equivalently, standard deviation) of its return. Diversification by creating a portfolio of securities makes investors decrease their risk while maintaining their expected return target. In the traditional capital asset pricing model (CAPM), beta is assumed to be constant, where the market portfolio can be considered as: the more securities you choose, the lower risk you have, and the return of portfolio tends to the exact number of the market return.

1.2 Literature review

Capital asset pricing model (CAPM) due to Sharpe(1964) and Lintner(1965) conveys the important information that securities are priced so that their expected return will compensate investors for their expected risk. Symbolically, CAPM is expressed as

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f], \quad (1.1)$$

where R_i is the return on security i , R_f is the return on risk-free asset, R_m is the return on the market portfolio and β_i is the measure of security i 's non-diversifiable risk relative to that of the market portfolio. More generally, (1.1) can be expressed in a non-expected form as

$$R_i = \alpha_i + \beta_i R_m + \varepsilon_i, \quad (1.2)$$

where the return on individual security R_i can be decomposed into the specific return, including expected specific return α_i and random specific return ε_i , and the systematic return, $\beta_i R_m$, owing to the common market return R_m . Clearly, model (1.2) reduces to (1.1) if setting $\alpha_i = (1 - \beta_i)R_f$ and $E(\varepsilon_i) = 0$ in (1.2). In this model, the quantity β_i is particularly important, which is an alternative measure of the risk that the investor has to bear owing to the systematic market movement.

In the traditional CAPM, β_i is assumed to be constant. This assumption has been widely documented to be untrue in the literature. Blume (1971) was among the first to consider the time-varying beta market model, which showed that the estimated beta tended to regress toward the mean; see also Blume (1975). Earlier studies that attempted to apply random coefficient model to beta include, among others, Sunder (1980) and Simonds, LaMotte and McWhorter (1986) who suggested a random-walk coefficient model, and Ohlson and Rosenberg (1982) and Collins, Ledolter and Rayburn (1987) who proposed an ARMA(1,1) model for the beta coefficient. More recent literature has widely recognized that the systematic risk of asset changing over time may be due to both the microeconomic factors in the level of the firm and the macroeconomic factors (see Fabozzi & Francis 1978; Bos & Newbold 1984). Considerable empirical evidences have suggested that beta stability assumption is invalid. The literature is abundant, see, for example, Kim (1993), Bos and Ferson (1992, 1995), Wells (1994), Bos, Ferson, Martikainen and Perttunen (1995), Brooks, Faff and Lee (1992) and Cheng (1997).

The time-varying beta models have also been investigate in Australia. Brooks, Faff and Lee (1992), and Faff, Lee and Fry (1992) were among the first to investigate the time-varying beta models. Faff, Lee and Fry (1992) em-

ployed a locally best invariant test to study the hypothesis of stationary beta, with evident finding of nonstationarity across all of their analysis. The random coefficient model was further suggested by Brooks, Faff and Lee (1994) as the preferred model to best describe the systematic risk of both individual shares and portfolios. However, Pope and Warrington (1996) reported that random coefficient model was appropriate only for a bit more than 10% companies in their studies. Faff, Lee and Fry (1992) investigated the links between betas nonstationarity and the three firm characteristics: riskiness, size and industrial sector, without finding the strong pattern between firm size or industry sector and nonstationarity. Faff and Brooks (1998) modelled industrial betas by different regimes based on market returns and volatility of the risk-free interest rate, their univariate and multivariate tests providing mixed evidence concerning the applicability of a time-varying beta model which incorporates these variables. Groenewold and Fraser (1999) argued that the industrial sectors could be divided into two groups: one of them has volatile and non-stationary betas and the other group has relatively constant and generally stationary beta. Other recent studies include Gangemi, Brooks and Faff (2001), Josev, Brooks and Faff (2001), and others. An interesting study recently made by Yao and Gao (2004) investigated the problem of choosing a best possible time-varying beta for each individual industrial index using the state-space framework, including the random walk models, random coefficient models and mean reverting models, which were examined in detail by using the Kalman filter approach.

When testing the validity of asset pricing models, many studies account for market movements, defined as up and down markets. For example, Kim and Zumwalt (1979) used the average monthly market return, the average risk-free

rate and zero as three threshold levels; when the realized market return is above (below) the threshold level the market is said to be in the up (down) market state. Crombez and Vennet (2000) conducted an extensive investigation into the risk-return relationship in the tails of the market return distribution; they defined up and down markets with two thresholds: zero and the risk-free rate. Further, to define three regimes for market movements, that is substantially upward moving, neutral and substantial bear, different threshold points were used, such as: the average positive (negative) market return, the average positive (negative) market return plus (less) half the standard deviation of positive (negative) market returns, and the average positive (negative) market return plus (less) three-quarters of the standard deviation of positive (negative) market returns. The conditional beta risk-return relation has been found to be stronger if the classification of up and down markets is more pronounced.

Galagedera and Faff (2005) has recently argued as in the finance literature and media that high volatility leads to high returns. High volatility in equity prices in many situations has been related to negative shocks to the real economy. On one hand, the volatility of macro-economic variables may partially explain the equity market price variation. On the other hand, the volatility in equity market prices may also be entrenched more in financial market disturbances. In particular, when the market volatility becomes extreme, it could have an impact on financial markets. Some securities are more susceptible to market volatility than others. Two interesting questions that arise in this setting were posed by Galagedera and Faff (2005): (i) Does the beta risk-return relationship depend on the various market volatility regimes? (ii) Are the betas corresponding to these volatility regimes priced? There have been empirical evidences raising concern about the ability of a single beta to explain

cross-sectional variation of security and portfolio returns. Security or portfolio systematic risk is known to vary considerably over time, as documented in the literature in the above. It is further well known that the volatility of financial time series, particularly in high frequency data, changes over time.

In their pioneering work of three-beta CAPM, Galagedera and Faff (2005) made an assumption that the market conditions can play an important part in explaining a changing beta and could be divided into three states specified as “low”, “neutral” or “high” market volatility. First, they fit a volatility model for daily market returns and obtain the estimates for conditional variance. Then, based on the magnitude of these estimates, Galagedera and Faff classify daily market volatility σ_{Mt} into one of three market volatility regimes, using appropriately defined indicator functions:

$$I_{Lt} = \begin{cases} 1 & \text{if } \sigma_{Mt}^2 < \sigma_L^2 \\ 0 & \text{if } \textit{otherwise} \end{cases} \quad (1.3)$$

$$I_{Nt} = \begin{cases} 1 & \text{if } \sigma_L^2 < \sigma_{Mt}^2 < \sigma_H^2 \\ 0 & \text{if } \textit{otherwise} \end{cases} \quad (1.4)$$

$$I_{Ht} = \begin{cases} 1 & \text{if } \sigma_H^2 < \sigma_{Mt}^2 \\ 0 & \text{if } \textit{otherwise.} \end{cases} \quad (1.5)$$

Here σ_L , σ_N and σ_H are constants: σ_L represents the low market condition, σ_N represents the neutral market condition, σ_H represents high market condition. By investigating empirically on the single factor CAPM $R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$, to estimate the betas in the low, neutral and high volatility markets,

Galagedera and Faff extended the market model given in (1.2), in the form:

$$R_{it} = \alpha_i + \beta_{iL}(I_{Lt}R_{Mt}) + \beta_{iN}(I_{Nt}R_{Mt}) + \beta_{iH}(I_{Ht}R_{Mt}) + \varepsilon_i, \quad (1.6)$$

where $\beta_{iL}, \beta_{iN}, \beta_{iH}$ are three constants defined as the systematic risks corresponding to the low, neutral and high market volatility regimes, respectively. This model is a richer specification than the traditional single factor CAPM. It is a three-state regime-switching model with the percentiles of market volatility used as threshold parameters. The methodology that applies in analyzing the three-beta CAPM model in Galagedera and Faff (2005) consists of beta estimation using time series data, estimation of cross-sectional relationship between returns and betas and accommodating market movement.

1.3 Objective of This Study

In this thesis, following the idea of Galagedera and Faff (2005), we consider new possibility of incorporating market movements into asset pricing models by including the changes in the conditional market volatility. We achieve this by noticing that the model (1.6) can be expressed as

$$R_{it} = \alpha_i + (\beta_{iL}I_{Lt} + \beta_{iN}I_{Nt} + \beta_{iH}I_{Ht})R_{Mt} + \varepsilon_{it} \equiv \alpha_i + \beta_{it}R_{Mt} + \varepsilon_{it}, \quad (1.7)$$

which is a time-varying beta model, with

$$\beta_{it} = \beta_{iL}I_{Lt} + \beta_{iN}I_{Nt} + \beta_{iH}I_{Ht}. \quad (1.8)$$

Note that the volatility of market returns is partitioned into three regimes in (1.3)–(1.5), which are the functions of the size of the conditional market volatility, say, σ_{Mt} . Therefore β_{it} is a simple functional of the market volatility σ_{Mt} , that is

$$\beta_{it} = \begin{cases} \beta_{iL} & \text{if } \sigma_{Mt}^2 < \sigma_L^2, \\ \beta_{iN} & \text{if } \sigma_L^2 \leq \sigma_{Mt}^2 < \sigma_H^2, \\ \beta_{iH} & \text{if } \sigma_{Mt}^2 \geq \sigma_H^2. \end{cases} \quad (1.9)$$

So the three-beta CAPM proposed by Galagedera and Faff (2005) is a simple functional beta model. Although this model is more reasonable than the traditional (constant-beta) CAPM, it still suffers from some obvious shortcomings:

- First of all, why are only three beta's sufficient for characterising the time-varying beta? It lacks an obviously reasonable theoretical underpinning in practice.
- Secondly, how to choose the thresholds of σ_L and σ_H that determine the regimes of market conditions? It lacks a reasonable method on the choice of the thresholds, which are very important in practice, but only chosen subjectively in Galagedera and Faff (2005).
- Thirdly, the empirical performance of the three-beta CAPM that was developed based on the model (1.6) looks quite marginal according to the results reported in Table 2 of Galagedera and Faff (2005), which, as pointed out by themselves (Galagedera and Faff, 2005, Page 84), are inconsistent across the market volatility regimes. This indicates that the model (1.6) proposed by Galagedera and Faff (2005) may not be very reasonable practically.

In this thesis, we will extend the model (1.6) that was suggested by Galagedera and Faff (2005) and propose a general functional-beta model as follows:

$$R_{it} = \alpha_i + \beta_i(\sigma_{Mt})R_{Mt} + \varepsilon_{it}, \quad (1.10)$$

where $\beta_i(\sigma_{Mt})$ is a functional of the market volatility σ_{Mt} . Our main objective is to investigate whether and how securities responses to the market vary depending on changing market volatility. In particular, we investigate whether systematic market risks as measured by betas estimated across different market volatility regimes are useful in explaining asset or portfolio returns.

Generally, our proposed functional-beta model (1.10) can be seen as a kind of varying-coefficient models in the literature of statistics and econometrics, defined by

$$Y_t = \alpha(Z_t) + \beta(Z_t)X_t + \varepsilon_t, \quad (1.11)$$

where Y_t and X_t are the response and regressor variables, respectively, and Z_t is a regime variable that is usually assumed to be observable in the literature of varying-coefficient models. However, note that the market volatility σ_{Mt} in our proposed model (1.10) can not be observed directly; we have to estimate the market volatility based on some reasonable econometric/ statistical models. In general case, we may also assume the intercept α_i depending on σ_{Mt} in the model (1.10), as done in (1.11); however, the value of α_i in model (1.10) is usually very small in practice, and therefore can be seen as a constant for simplicity as in the usual finance literature. For a recent review regarding varying-coefficient statistical models, the reader is referred to Fan and Zhang (2008); see also Cai (2005) and the references therein for the varying-coefficient econometric models under strongly (i.e., α -) mixing data generating processes,

and Gao (2008) for review of applications of models (1.11) in economics and finance.

In this research we focus on the problems on the functional beta in the model (1.10). Detailedly, we are concerned with:

- A new functional-beta CAPM, which is proposed to take into account the conditions of market volatility, under the framework of data generating processes of near epoch dependence (NED).

This framework of data generating processes is more generally verifiable than strongly (i.e., α -) mixing processes and is widely applicable and necessary in the setting of the proposed model (1.10) due to the unobservable nature of market volatility. This is will be discussed in Section 2.2.

- How to estimate the unknown systematic market risk functional $\beta(\cdot)$?

This is a very important and fundamental question in applications. A nonparametric estimation procedure under NED will be suggested in Sections 2.3 and 2.4, with the large sample distributions established in Chapter 3, by applying the least squares local linear modelling technique developed in Lu and Linton (2007).

- How about the finite sample performance of the suggested nonparametric estimation procedure in estimating the functional beta model under near epoch dependence?

Simulation study will be carefully made in Chapter 4, illustrating that in the condition of moderate market volatility, our method provides reasonable estimates of the unknown functional beta.

- What kind of parametric forms that can be taken in the proposed functional-beta CAPM, which is empirically satisfying with real financial data sets?

By using a set of stocks data sets collected from Australian stock market in the past ten years, we are carefully examining the evidences of functional-beta model in Australia, under the structure of nonparametric and parametric models, respectively, in Chapter 5. Differently from the three- or multi-beta CAPM in Galagedera and Faff (2005), our new findings indicate that the functional beta can be reasonably parameterized as threshold (regime-switching) linear functionals of the market volatility, rather than three or more simple constant beta's. The difficult choice of the specific regimes of market condition is suggested in accordance with the nonparametric outcomes of the functional beta.

According to the results from nonparametric estimation, we will select reasonable changing points (thresholds) that are needed in parametric estimation. The problem of how many thresholds we should choose in the functional-beta model will be solved by Akaike's information corrected criterion (AICc). Therefore, we can have a general functional-beta model which can fit the financial real data more adaptively.

We conclude in Chapter 6, with possible future research direction suggested.

1.4 Significance of This Study

By now people get a common idea: the higher the risk the higher is the expected return. This doesn't mean, however, that investing in higher risk valuables will always bring you the best return. It only means that expected returns

will be higher in this kind of investments. How to manage the portfolio total risk in a better way and understand the relationship between risk and return becomes essential.

The traditional CAPM has been widely applied in financial risk management. Similarly, our new model can be utilised extensively. For example, by better understanding the systematic risk and the risk-return relation variation with the market conditions, the managers of companies could make decisions by analyzing the risk-return relationship more rationally. Investors can use this tool to analyze their investment which is potentially more effective and reasonable. Moreover, the empirical research allows to take a close look at the portfolio and the principle it contains in practice. No matter how it is, it is a useful way to better manage financial risk.

Chapter 2

Methodology: Model and Estimation

2.1 Introduction

We first consider the case of single security, that is $i = 1$ in (1.6); for simplicity of notation we rewrite (1.6) as

$$R_t = \alpha + \beta(\sigma_{Mt})R_{Mt} + \varepsilon_t. \quad (2.1)$$

Assume we have the historical time series observations $\{R_t, \sigma_{Mt}, R_{Mt}\}_{t=1}^T$, and we would estimate the unknown functions $\alpha(\cdot)$ and $\beta(\cdot)$ based on these observed data. One method that we will propose is to apply the least squares method with combination of local linear ideas (c.f., Fan and Gijbels, 1996, Lu and Linton, 2007). However, note that the market volatility σ_{Mt} is not observable practically, which needs to be estimated. Before we proceed on estimation, we first of all discuss the data structure that we will assume are easily applicable

in practice.

2.2 Data generating process: Near epoch dependence

Consider to establish a model in a time series context under near epoch dependence. Andrews(1995)'s work discussed nonparametric density and regression estimators based on the local constant paradigm also under near epoch dependence conditions. Lu and Linton (2007) recently established a Central Limit Theorem for the more desirable class of local linear estimators (Fan and Gijbels (1996)) under similar weak dependence conditions, which provides a key tool for developing the theory in this paper.

In time series analysis, the observations $\{R_t, \sigma_t, R_{Mt}\}_{t=1}^T$ are conventionally assumed to be from some stationary stochastic process that satisfies some type of mixing conditions (c.f., Tong(1990), Fan and Yao(2003), Gao(2007)), defined on some probability space $(\Omega, \mathcal{F}, \mathcal{P})$ (throughout the paper all the random variables are defined on this space). Among the widely used mixing conditions, such as ϕ -, ρ -, β - and α - mixings, α - mixing is no doubt the weakest and most popular in the econometric literature. For example, Cai (2005) established the asymptotic theory for the varying-coefficient models (1.11) under α - mixing. The rational behind this assumption is that under some suitable conditions, the stationary solutions of many time series econometric models (linear or nonlinear) are α - mixing; see, e.g., Gorodeskii (1997), Pham and Tran (1985), Masry and Tostheim (1995), Lu (1998), Cline and Pu (1999), Carrasco and Chen (2002), and Saikkonen (2001). For reference, its definition

is stated as follows.

Definition 0. A stationary sequence $X_t, t = 0, \pm 1, \dots$ is said to be α -mixing if

$$\mu(k) = \sup_{A \in \Gamma_{-\infty}^n, B \in \Gamma_{n+k}^\infty} |P(AB) - P(A)P(B)| \rightarrow 0, \quad (2.2)$$

as $k \rightarrow \infty$, where $\Gamma_{-\infty}^n$ and Γ_{n+k}^∞ are two fields generated by $X_t, t \leq n$ and $X_t, t \geq n+k$, respectively. We call $\mu(\cdot)$ the mixing coefficient.

However, from a practical point of view, the α -mixing is hard to verify in practice, especially in the case of compound processes. For the former case, the reader is referred to Andrews (1984). For the latter case, the ARMA process with ARCH/GARCH errors, discussed in Engle (1982) and Weiss (1984) as well as Lin and Li (1997), is well applied in financial econometrics, where the model is composed of two time series models, such as AR and GARCH:

$$R_t = a_0 + a_1 R_{t-1} + \epsilon_t, \quad \epsilon_t = e_t h_t^{1/2}, \quad h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}, \quad (2.3)$$

where a_i 's are the coefficients in the AR model, α_i 's and β_i are the coefficients in the GARCH model, with e_t being i.i.d innovation with mean 0 and variance 1. It is well known that ARCH/GARCH model is α -mixing under some mild conditions (c.f., Lu (1996a, b), Carrasco and Chen (2002)), however, except some special cases, no general results are available to guarantee that R_t defined in (2.3) is α -mixing. For more complex models than (2.3), it

can be imagined that it is harder to verify the α -mixing of data generating processes. Especially, in our functional-beta model (2.1), the market volatility process σ_{Mt} is unobservable, which is difficult to reasonably assume and verify to be α -mixing in general. We will therefore suggest using a generalized version of mixing processes, called stable or near epoch dependent processes, which can easily cover the compounded processes and many nonlinear/ non- α -mixing processes (c.f., Ibragimov (1962), Billingsley (1968) and Mcleish (1975a, 1975b, 1977)) and has been used extensively in econometrics following Bierens (1981), see for example Gallant (1987), Gallant and White (1988), Andrew (1995), Lu (2001) and Lu and Linton (2007).

In general, let Y_t and X_t be both stationary processes, of \mathbb{R}^1 - and \mathbb{R}^d -valued, respectively, defined based on a stationary process $\{\epsilon_t\}$ by

$$Y_t = \Psi_Y(\epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}, \dots), \quad (2.4)$$

$$X_t = (X_{t1}, \dots, X_{td})^\tau = \Psi_X(\epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}, \dots), \quad (2.5)$$

where X^τ denotes the transpose of X (a vector or matrix), $\Psi_Y : \mathbb{R}^\infty \rightarrow \mathbb{R}^1$ and $\Psi_X : \mathbb{R}^\infty \rightarrow \mathbb{R}^d$ are two Borel measurable functions, respectively, and $\{\epsilon_t\}$ may be vector-valued. Let $V > 0$ be a positive real number.

Definition 1. The stationary process $\{(Y_t, X_t)\}$ is said to be near epoch dependent in L_V norm (NED in L_V for simplicity) with respect to a stationary α -mixing process $\{\epsilon_t\}$, if

$$v_V(m) = E \|Y_t - Y_t^{(m)}\|^V + E \|X_t - X_t^{(m)}\|^V \rightarrow 0, \quad (2.6)$$

as $m \rightarrow 0$, where $|\cdot|$ and $\|\cdot\|$ are the absolute value and the Euclidean norm of \mathbb{R}^d , respectively,

$$Y_t^{(m)} = \Psi_{Y,m}(\epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}, \dots, \epsilon_{t-m+1}),$$

$$X_t^{(m)} = ((X_{t1}^{(m)}, \dots, X_{td}^{(m)})^\tau = \Psi_{X,m}(\epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}, \dots, \epsilon_{t-m+1}),$$

$\Psi_{Y,m}$ and $\Psi_{X,m}$ are \mathbb{R}^1 - and \mathbb{R}^d - valued Borel measurable functions with m arguments, respectively. We will call $v_V(m)$ the stability coefficients of order V of the process $\{(Y_t, X_t)\}$.

Clearly, $\{(Y_t^{(m)}, X_t^{(m)})\}$ is an α -mixing process with mixing coefficient

$$\mu_m(k) \leq \begin{cases} \mu_m(k-m) & k \geq m+1 \\ 1 & k \leq m \end{cases} \quad (2.7)$$

The type of the setting where our results are useful is for models with complicated dynamics in both mean and variance for which the usual mixing conditions do not necessarily apply. These sorts of models are common in finance and economics, and near-epoch dependence is sometimes easier to verify in the case of these models.

2.3 Methodology of estimation

2.3.1 Model and observations

By extending the single-factor CAPM, we propose a functional-beta CAPM for Asset i , which reads as:

$$R_{it} = \alpha_i(\sigma_{Mt}) + \beta_i(\sigma_{Mt})R_{Mt} + \varepsilon_{it}. \quad (2.8)$$

In this model, R_{it} is the return of financial asset i at time t , R_{Mt} is the market return at time t , σ_{Mt}^2 is the market volatility at time t , α_i is the conditional expected specific return, ε_{it} is random specific return, and β_i is the coefficient of the contribution due to the market factor, changing with the market volatility. For generality, we allow both α_i and β_i varying with the condition of market volatility σ_{Mt} , where $\beta_i(\cdot)$ is particularly important, which is the systematic risk functional, in capital asset pricing modelling.

Given the historical observations (R_{it}, R_{Mt}) , $t = 1, 2, \dots, T$, we are concerned with how to estimate the unknown functional beta. First of all, we need some way to estimate the unobservable market volatility σ_{Mt}^2 . Using the market returns R_{Mt} , $t = 1, 2, \dots, T$, we can try to estimate σ_{Mt}^2 in various ways. A simple way is to apply the econometric models of ARCH of Engle (1982) or GARCH of Bollersleve (1986), as done in Galagedera and Faff (2005). More involved stochastic volatility models can also be applied (c.f., Gao, 2007, Page 169). Alternatively, we can use realized market volatility as an estimate of σ_{Mt}^2 ; see Allen *et al.* (2008) for a comprehensive review on realized volatility. In the following we assume the market volatility σ_{Mt}^2 has been estimated,

denoted by $\hat{\sigma}_{Mt}^2$, $t = 1, 2, \dots, T$.

2.3.2 Estimation

We now propose our methodology to estimate the unknown functional beta in the model (2.8). As we now are only concerned with the performance of individual asset i below, we drop the subscript i as in (2.1) for notational simplicity. Therefore we write model (2.8) as:

$$R_t = \alpha(\sigma_{Mt}) + \beta(\sigma_{Mt})R_{Mt} + \varepsilon_t. \quad (2.9)$$

Assume we have the historical data (R_t, R_{Mt}) , $t = 1, 2, \dots, T$.

In this research we will estimate the unknown functional $\beta(x)$ at x by least squares local linear modelling technique (c.f. Fan and Gijbels, 1996). Although the Nadaraya-Watson method is central in most nonparametric regression method in the traditional *i.i.d.* series case, it has been well documented (see, for instance, Fan and Gijbels 1996) that this approach suffers from several severe drawbacks, such as poor boundary performances, excessive bias and low efficiency, and that the local polynomial fitting methods developed by Stone (1977) and Cleveland (1979) are generally preferable. Local polynomial fitting, and particularly its special case—local linear fitting—recently have become increasingly popular in the light of recent work by Cleveland and Loader (1996), Fan (1992), Fan and Gijbels (1992, 1995), Hastie and Loader (1993), Ruppert and Wand (1994), and several others. Masry and Fan (1997) have studied the asymptotics of local polynomial fitting for regression under general α -mixing conditions; see also Fan and Yao (2003). Recently, Lu and Linton (2007) extend this approach to the context of our generalized

mixing dependence—NED processes by defining an estimator of regression function based on local linear fitting and establishing its asymptotic properties.

The basic idea of least squares local linear modelling technique with $\beta(\cdot)$ can be described as follows. When σ_{Mt} is close to x , then $\beta(\sigma_{Mt})$ can be approximated by

$$\beta(x) + \beta'(x)(\sigma_{Mt} - x) \equiv \beta_0 + \beta_1(\sigma_{Mt} - x) \quad (2.10)$$

Locally at x , the model can then be approximately expressed as:

$$R_t \approx \alpha + (\beta_0 + \beta_1(\sigma_{Mt} - x))R_{Mt} + \varepsilon_t, \quad (2.11)$$

where though we also assume α depending on σ_{Mt} in model (2.9) and we can also apply local linear idea to $\alpha(\cdot)$, the estimation of $\alpha(\cdot)$ is of less interest in capital asset pricing modelling, therefore in (2.11) only a local constant method is applied to $\alpha(\cdot)$ to reduce the number of unknown local parameters. As explained in Chapter 1, we may assume α is constant as it takes on rather small values in financial practice.

Therefore, replacing σ_{Mt}^2 by $\hat{\sigma}_{Mt}^2$, the least squares local linear estimate of $\alpha(\cdot)$ and $\beta(\cdot)$ in (2.9) can be made by setting $\widehat{\alpha}(x) = \widehat{\alpha}$ and $\widehat{\beta}(x) = \widehat{\beta}_0$, where $(\widehat{\alpha}, \widehat{\beta}_0, \widehat{\beta}_1)$ minimizes:

$$L(\alpha, \beta_0, \beta_1) = \sum_{t=1}^T (R_t - [\alpha + (\beta_0 + \beta_1(\hat{\sigma}_{Mt} - x))R_{Mt}])^2 K\left(\frac{\hat{\sigma}_{Mt} - x}{h}\right), \quad (2.12)$$

where $h = h_T \rightarrow 0$ is bandwidth, $K(x)$ is a kernel function, which may, for

example, take

$$K(x) = \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}. \quad (2.13)$$

Therefore, we have three unknown local parameters α , β_0 , β_1 . Applying partial differentiation, we get the expression of the estimators of the three unknown local parameters at x as follows:

$$\begin{pmatrix} \hat{\alpha} \\ \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix} = A_T^{-1} B_T, \quad (2.14)$$

where

$$A_T = \sum_{t=1}^T \begin{pmatrix} 1 & R_{Mt} & R_{Mt}(\hat{\sigma}_{Mt} - x) \\ R_{Mt} & R_{Mt}^2 & R_{Mt}^2(\hat{\sigma}_{Mt} - x) \\ (\hat{\sigma}_{Mt} - x)R_{Mt} & (\hat{\sigma}_{Mt} - x)R_{Mt}^2 & (\hat{\sigma}_{Mt} - x)^2 R_{Mt}^2 \end{pmatrix} K\left(\frac{\hat{\sigma}_{Mt} - x}{h}\right),$$

$$B_T = \begin{pmatrix} \sum_{t=1}^T R_t K\left(\frac{\hat{\sigma}_{Mt} - x}{h}\right) \\ \sum_{t=1}^T R_{Mt} R_t K\left(\frac{\hat{\sigma}_{Mt} - x}{h}\right) \\ \sum_{t=1}^T (\hat{\sigma}_{Mt} - x) R_{Mt} R_t K\left(\frac{\hat{\sigma}_{Mt} - x}{h}\right) \end{pmatrix}.$$

After that we can plot graphs of $\alpha(\cdot)$ and $\beta(\cdot)$ according to which we will decide the borders of market volatility regimes. Furthermore, we can divide the beta functional into different parts by the thresholds.

2.4 Bandwidth selection

It is obvious that the bandwidth plays an important role in the process of estimation. Therefore, how to choose the bandwidth becomes a serious problem in the whole process. Referring to Cai and Tiwari (2000) and Cai(2002), we choose a simple way to obtain the bandwidth for the suggested estimation procedures. The method is described as follow:

For the given observed values $\{Y_t = R_t\}_{t=1}^T$, the fitted values \widehat{Y}_i can be calculated as

$$\widehat{Y} = H_h Y. \quad (2.15)$$

Here $Y = (Y_1, Y_2, \dots, Y_T)'$ and H_h is called the $T \times T$ smoother or hat matrix associated with the smoothing parameter h , whose i th row is given by

$$H_{h,i} = (1 \quad R_{Mi}) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} (X(\hat{\sigma}_{Mi})' W(\hat{\sigma}_{Mi}) X(\hat{\sigma}_{Mi}) + 0.0001 \cdot I)^{-1} X(\hat{\sigma}_{Mi})' W(\hat{\sigma}_{Mi}),$$

where

$$X(\sigma) = \begin{pmatrix} 1 & R_{M1} & (\hat{\sigma}_{M1} - \sigma) R_{M1} \\ 1 & R_{M2} & (\hat{\sigma}_{M2} - \sigma) R_{M2} \\ 1 & R_{M3} & (\hat{\sigma}_{M3} - \sigma) R_{M3} \\ \vdots & \vdots & \vdots \\ 1 & R_{MT} & (\hat{\sigma}_{MT} - \sigma) R_{MT} \end{pmatrix},$$

$$W(\sigma) = \begin{pmatrix} K\left(\frac{\hat{\sigma}_{M1}-\sigma}{h}\right) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & K\left(\frac{\hat{\sigma}_{MT}-\sigma}{h}\right) \end{pmatrix}.$$

Here $K(\cdot)$ is the kernel function, h is the bandwidth. We apply the following nonparametric version of $AICc(h)$, due to C. M. Hurvich and C. L. Tsai (1989), to select the optimal bandwidth h_{opt} by minimizing

$$AICc(h) = T \log \hat{\sigma}_\varepsilon^2 + T(1 + T_h/T)(1 - (T_h + 2)/T) \quad (2.16)$$

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{T}(Y - \hat{Y})'(Y - \hat{Y}) = \frac{1}{T}Y'(I - H_h)'(I - H_h)Y \quad (2.17)$$

T_h is the trace of the hat matrix H_h . This selection criterion counteracts the over/under-fitting tendency of the generalized cross-validation and the classical AIC.

Chapter 3

Theoretical studies: Large sample property under NED

3.1 Notation and Assumptions

Before we state the main asymptotic distributions, we give the notation and assumptions that will be required below.

3.1.1 Notation and main assumptions

We summarize here the main assumptions we are making on the data generating process (DGP) $(R_t, \sigma_{Mt}, R_{Mt})$ in model (2.9) and the kernel K and bandwidth h used in the estimation method (2.12). These assumptions are basically inherited from Lu and Linton (2007).

Assumptions (A1)-(A4) are related to the nonlinear process itself.

(A1) The DGP $\{(R_t, \sigma_{Mt}, R_{Mt})\}$ in model (2.9) is a strictly stationary NED process (c.f., (2.6)), with order $V = 2 + \delta/2$, with respect to some α -

mixing process $\{\epsilon_t\}$ in Definition 1 in §2.2, where the constant $\delta > 0$ is specified in Assumption (A2) below. For all i and j in \mathbb{Z} , the market volatilities σ_{Mi} and σ_{mj} admit a joint density f_{ij} ; moreover, $f_{ij}(x', x'') \leq C$ for all $i, j \in \mathbb{Z}$, all $x', x'' \in \mathbb{R}^1$, where $C > 0$ is some constant, and f denotes the marginal density of σ_{Mi} .

(A2) The random variables R_{Mt} and ϵ_t in model (2.9) have finite absolute moment of order $(2 + \delta)$, that is, $E[|R_{Mt}^2|^{2+\delta}] < \infty$ and $E[|\epsilon_t^2|^{2+\delta}] < \infty$ for some $\delta > 0$.

(A3) (i) The coefficient functions $\alpha(x)$ and $\beta(x)$ in model (2.9) are twice differentiable with their second derivatives continuous at all x . (ii) The density function of σ_{Mt} , $f(x)$, is continuous at x . (iii) The conditional functions $g_{M,i}(x) = E(R_{Mt}^i | \sigma_{Mt} = x)$ and $g_{M,i}^\epsilon(x) = E(R_{Mt}^i \epsilon_t^2 | \sigma_{Mt} = x)$ are continuous at all x , for $i = 0, 1, 2$.

Assumption (A4) is an assumption of the mixing coefficients in Definition 0.

(A4) For the α -mixing process ϵ_t in Definition 1, the mixing function $\mu(\cdot)$ is such that

$$\lim_{k \rightarrow \infty} k^a \sum_{j=k}^{\infty} \{\mu(j)\}^{\delta/(4+\delta)} = 0 \quad \text{for some constant } a > \delta/(4 + \delta).$$

Assumption (A5) deals with the kernel function $K : \mathbb{R} \rightarrow \mathbb{R}$, to be used in the estimation method (2.12).

(A5)(i) K is a symmetric probability density function, with $|K(u)|$ is uniformly bounded by some constant K^+ .

- (ii) $|K|$ has an integrable second order radial majorant, that is, $Q^K(x) := \sup_{|y| \geq |x|} [|y|^2 K(y)]$ is integrable.
- (iii) K is first order differentiable with its derivative \dot{K} being bounded and Lipschitz continuous of order 1, that is, for some constant $C > 0$,

$$|\dot{K}(u) - \dot{K}(v)| \leq C|u - v| \text{ for any } u, v \in \mathbb{R}.$$

This assumption allows an unbounded support for the kernel function; compare this with Condition 2(i) of Masry and Fan (1997, page 170) who require the kernel function to have a bounded support.

Throughout, for convenient reference, we are listing here some conditions on the asymptotic behavior, as $T \rightarrow \infty$, of the bandwidth $h = h_T$ that will be used for generality in the sequel, where Assumption (B1) below is standard, while Assumptions (B2) through (B4) below look complex: some simple and verifiable conditions on the stability and mixing coefficients to ensure they hold can be given as in the main theorem of Lu and Linton (2007, Theorem 3.1 and Corollary 3.1).

(B1) The bandwidth h tends to zero in such a way that $Th \rightarrow \infty$ as $T \rightarrow \infty$.

(B2) There is a positive integer $m = m_T \rightarrow \infty$ such that the stability coefficients, defined in (2.6) with $V = 2$ and $V = 2 + \delta/2$, satisfy

$$T^{2+4/\delta} h^{-(3+2/\delta)} v_2(m) \rightarrow 0, \quad \text{and} \quad h^{-4(2+2/\delta)} v_2(m) = O(1),$$

and

$$h^{-(4+\delta/2+4/\delta)} v_{2+\delta/2}(m) = O(1).$$

(B3) There exist two sequences of positive integer vectors, $p = p_T \in \mathbb{Z}$ and $q = q_T = 2m_T \in \mathbb{Z}$, with $m = m_T \rightarrow \infty$ such that $p = p_T = o((Th)^{1/2})$, $q/p \rightarrow 0$ and $T/p \rightarrow \infty$, and $Tp^{-1}\mu(m) \rightarrow 0$.

(B4) h tends to zero in such a manner that $qh = O(1)$ such that

$$h^{-\delta/(4+\delta)} \sum_{t=q}^{\infty} \{\mu_m(t)\}^{\delta/(4+\delta)} \rightarrow 0 \quad \text{as } T \rightarrow \infty. \quad (3.1)$$

Remark. Assumption (B1) is standard on the bandwidth, the same as that in the *i.i.d.* case; Assumption (B2) is concerned with the conditions on the stability coefficients related to the bandwidth; and Assumptions (B3) and (B4) are on the mixing coefficients which are associated with the bandwidth, among which (B3) together with (B1) is similar to the conditions specified for the strongly mixing processes in Condition 3 of Masry and Fan (1997, page 172). Assumptions B2-B4 are phrased as restrictions on the decay rates of the stability and mixing coefficients for a given bandwidth, although one could rewrite these conditions as restrictions on the bandwidth (and hence the implied rate of convergence of the estimator) for a given decay rate thereby allowing greater dependence at the cost of slower convergence. Although Assumptions (B2) through (B4) look somewhat complex, some milder and more specific conditions can be derived from them with the bandwidth set as a power function of the number of observations, as is generally the case in practice. For the details, see Theorem 3.2.1 together with corollary and the remark there in Section 3.2.

Finally, in the least squares local fitting (2.12), we assume the market volatility σ_{Mt} has been estimated by $\hat{\sigma}_{Mt}$, as explained in Section 2.3.1, with the property

$$(B5) \max_{1 \leq t \leq T} |\hat{\sigma}_{Mt} - \sigma_{Mt}| = o((Th)^{-1/2}).$$

This assumption (B5) is easily satisfied when the market volatility is estimated by some parametric volatility models, which is in fact of root- T convergence rates.

3.2 Theorems

Let $\hat{\vartheta} = (\widehat{\alpha}(x), \widehat{\beta}(x))'$, which is defined in (2.12), and $\vartheta = (\alpha(x), \beta(x))'$, defined in model (2.9). Then $\hat{\vartheta}$ is a consistent estimator of ϑ with asymptotic distribution as follows.

Theorem 3.2.1 Assume the assumptions (A1)-(A5) and (B1-B5) hold. Then

$$(Th)^{1/2} \left(\hat{\vartheta} - \vartheta - \frac{1}{2} h^2 C_1^*(x) \right) \rightsquigarrow N(\mathbf{0}_2, A^*(x)^{-1} C_2^*(x) A^*(x)^{-1} / f(x)),$$

where $\mathbf{0}_2 = (0, 0)'$, $C_1^*(x) = (\ddot{\alpha}(x)g_{M,0}(x) + \ddot{\beta}(x)g_{M,1}(x), \ddot{\alpha}(x)g_{M,1}(x) + \ddot{\beta}(x)g_{M,2}(x))' \kappa_{21}$,

$$A^*(x) = \begin{pmatrix} 1 & g_{M,1}(x) \\ g_{M,1}(x) & g_{M,2}(x) \end{pmatrix},$$

$$C_2^*(x) = \begin{pmatrix} g_{M,0}^\varepsilon(x) & g_{M,1}^\varepsilon(x) \\ g_{M,1}^\varepsilon(x) & g_{M,2}^\varepsilon(x) \end{pmatrix} \kappa_{02},$$

and $\kappa_{ij} = \int u^i K^j(u) du$, $g_{M,i}(x) = E(R_{Mt}^i | \sigma_{Mt} = x)$ and $g_{M,i}^\varepsilon(x) = E(R_{Mt}^i \varepsilon_t^2 | \sigma_{Mt} = x)$.

The following corollary specifies more verifiable conditions on the mixing and stable coefficients, as derived similarly in Theorem 3.1 of Lu and Linton (2007, Page 47).

Corollary 3.2.2 *Let Assumptions (A1), (A2), (A3), (A5) and (B5) hold, with $v_{2+\delta/2}(x) = O(x^{-\mu})$ and $\alpha(x) = O(x^{-\lambda})$ for some $\mu \geq \max\{4(\kappa_1 - 1), \kappa_3/(1 + \delta/4)\}\kappa_2$ and some $\lambda > (a+1)(1+4/\delta)$ with $a > \delta/(4+\delta)$, such that $T^{2+4/\delta}h^{\mu/\kappa_2-\kappa_1} \rightarrow 0$, $Th^{[1+2/\{(a(1+4/\delta)\}]} / \log T \rightarrow \infty$, and $Th^{[2\lambda/\{(a(1+4/\delta)\}-1]} \log T \rightarrow 0$ as $T \rightarrow \infty$, where $\kappa_1 = 3 + 2/\delta$ and $\kappa_2 = a(1 + 4/\delta)(1 + \delta/4)/d$, $\kappa_3 = 4 + \delta/2 + 4/\delta$. Then the conclusion of Theorem 3.2.1 holds.*

Further, we can derive the following corollary which gives the conditions under which the usually used optimal bandwidth, $h = O(T^{-1/5})$, is achievable, as done in Corollary 3.1 of Lu and Linton (2007, Page 48).

Corollary 3.2.3 *Let Assumptions (A1), (A2), (A3), (A5) and (B5) hold, with $v_{2+\delta/2}(x) = O(x^{-\mu})$ and $\alpha(x) = O(x^{-\lambda})$ for some $\mu \geq \max\{4(\kappa_1 - 1), \kappa_3/(1 + \delta/4), (\kappa_4 + \kappa_1)\}\kappa_2$ and some $\lambda > \max\{(a+1), 3a\}(1 + 4/\delta)$ with $a > \delta/(4 + \delta)$, and $h = O(T^{-1/5})$, where κ_1, κ_2 and κ_3 are specified in Corollary 3.2.2, and $\kappa_4 = 10(1 + 2/\delta)$. Then the conclusion of Theorem 3.2.1 holds.*

3.3 Sketch of Proof

As done in Lu and Linton (2007), a fundamental technique which is required to study (2.14) is the following approximation to an NED process $\{(Y_t, X_t)\}$ by an α -mixing process $\{(Y_t^{(m)}, X_t^{(m)})\}$ defined in Definition 1, that is

$$Y_t = Y_t^{(m)} + (Y_t - Y_t^{(m)}) := Y_t^{(m)} + \delta_{Y,t}^{(m)}, \quad (3.2)$$

$$X_t = X_t^{(m)} + (X_t - X_t^{(m)}) := X_t^{(m)} + \delta_{X,t}^{(m)}, \quad (3.3)$$

where

$$E [\delta_{Y,t}^{(m)}]^2 = O(v_2(m)), \text{ and } E [\delta_{X,t}^{(m)}]^2 = O(v_2(m)), \text{ as } m \rightarrow \infty, \quad (3.4)$$

and the mixing coefficients of $\{(Y_t^{(m)}, X_t^{(m)})\}$ satisfy

$$\alpha_m(k) \leq 1 \text{ for } k = 0, 1, \dots, m, \quad \text{and } \alpha_m(k) = \alpha(k-m) \text{ for } k \geq m+1, \quad (3.5)$$

with $\alpha(\cdot)$ defined in Definition 0. For details the reader is referred to Lu and Linton (2007).

Proof of Theorem 3.2.1: Set $\hat{\vartheta} = (\hat{\alpha}, \hat{\beta}_0)'$, $\vartheta = (\alpha(x), \beta(x))'$, $\Gamma = \text{diag}\{1, 1, 0\}$ a diagonal matrix, and $\hat{\theta} = (\hat{\alpha}, \hat{\beta}_0, \hat{\beta}_1)'$ and $\theta = (\alpha, \beta_0, \beta_1)' = (\alpha(x), \beta(x), \dot{\beta}(x))'$. Then it follows from (2.14) that

$$\hat{\vartheta} - \vartheta = \Gamma(\hat{\theta} - \theta) = \Gamma A_T^{-1}[B_T - A_T\theta] = \Gamma A_T^{-1}C_T, \quad (3.6)$$

where $C_T = B_T - A_T\theta$. For notational simplicity, we will denote $\eta_{mt} = (1, R_{Mt}, R_{Mt}(\sigma_{Mt} - x))'$. Then applying Assumptions (B5) and (A5)(iii), we easily have

$$A_T = \sum_{t=1}^T \eta_{mt} \eta_{mt}' K((\sigma_{Mt} - x)/h) [1 + o_P((Th)^{-1/2})],$$

$$B_T = \sum_{t=1}^T \eta_{mt} R_t K((\sigma_{Mt} - x)/h) [1 + o_P((Th)^{-1/2})],$$

and

$$C_T = \sum_{t=1}^T \eta_{mt} [R_t - \eta_{mt}'\theta] K((\sigma_{Mt} - x)/h) [1 + o_P((Th)^{-1/2})],$$

where $o_P(1)$ is uniform in t .

First of all, let $\gamma_T = \text{diag}\{1, 1, h\}$ be a diagonal matrix, $g_{M,i}(x) = E(R_{Mt}^i | \sigma_{Mt} = x)$, $\kappa_{ij} = \int u^i K^j(u) du$ and $f(x)$ be the probability density function of σ_{Mt} . Note that $\kappa_{11} = \kappa_{31} = \kappa_{12} = \kappa_{32} = 0$ for $K(\cdot)$ is symmetric. It follows from Lemma 3.1 of Lu and Linton (2007) that

$$(Th)^{-1} \gamma_T^{-1} A_T \gamma_T^{-1} \xrightarrow{P} A(x) f(x),$$

where

$$A(x) = \begin{pmatrix} 1 & g_{M,1}(x) & 0 \\ g_{M,1}(x) & g_{M,2}(x) & 0 \\ 0 & 0 & g_{M,2}(x) \kappa_{21} \end{pmatrix}.$$

Applying (2.9) with Taylor's expansion, we have

$$\begin{aligned} R_t - \eta'_{mt} \theta &= \dot{\alpha}(x)(\sigma_{Mt} - x) + \frac{1}{2} \ddot{\alpha}(x + \zeta_1(\sigma_{Mt} - x))(\sigma_{Mt} - x)^2 \\ &\quad + \frac{1}{2} \ddot{\beta}(x + \zeta_2(\sigma_{Mt} - x))(\sigma_{Mt} - x)^2 R_{Mt} + \varepsilon_t, \end{aligned}$$

where $|\zeta_i| < 1$ for $i = 1, 2$, by which we can decompose C_T into two parts

$$C_T = C_{T1} + C_{T2}, \quad (3.7)$$

where

$$\begin{aligned} C_{T1} &= \sum_{t=1}^T \eta_{mt} [\dot{\alpha}(x)(\sigma_{Mt} - x) + \frac{1}{2} \ddot{\alpha}(x + \zeta_1(\sigma_{Mt} - x))(\sigma_{Mt} - x)^2] K((\sigma_{Mt} - x)/h) \\ &\quad + \sum_{t=1}^T \eta_{mt} \frac{1}{2} \ddot{\beta}(x + \zeta_2(\sigma_{Mt} - x))(\sigma_{Mt} - x)^2 R_{Mt} K((\sigma_{Mt} - x)/h), \end{aligned} \quad (3.8)$$

and

$$C_{T2} = \sum_{t=1}^T \eta_{mt} \varepsilon_t K((\sigma_{Mt} - x)/h). \quad (3.9)$$

Now we consider

$$\begin{aligned} & (Th)^{-1} \gamma_T^{-1} C_{T1} \\ &= (Th)^{-1} \sum_{t=1}^T \gamma_T^{-1} \eta_{mt} [\dot{\alpha}(x)(\sigma_{Mt} - x) + \frac{1}{2} \ddot{\alpha}(x + \zeta_1(\sigma_{Mt} - x))(\sigma_{Mt} - x)^2] K((\sigma_{Mt} - x)/h) \\ &+ (Th)^{-1} \sum_{t=1}^T \gamma_T^{-1} \eta_{mt} \frac{1}{2} \ddot{\beta}(x + \zeta(\sigma_{Mt} - x))(\sigma_{Mt} - x)^2 R_{Mt} K((\sigma_{Mt} - x)/h) \\ &= \dot{\alpha}(x)(Th)^{-1} \sum_{t=1}^T \gamma_T^{-1} \eta_{mt} (\sigma_{Mt} - x) K((\sigma_{Mt} - x)/h) \\ &+ \frac{1}{2} \ddot{\alpha}(x)(1 + o_P(1))(Th)^{-1} \sum_{t=1}^T \gamma_T^{-1} \eta_{mt} (\sigma_{Mt} - x)^2 K((\sigma_{Mt} - x)/h) \\ &+ \frac{1}{2} \ddot{\beta}(x)(1 + o_P(1))(Th)^{-1} \sum_{t=1}^T \gamma_T^{-1} \eta_{mt} (\sigma_{Mt} - x)^2 R_{Mt} K((\sigma_{Mt} - x)/h) \\ &= [h\dot{\alpha}(x)C_{10}(x) + \frac{1}{2}h^2\ddot{\alpha}(x)C_{11}(x) + \frac{1}{2}h^2\ddot{\beta}(x)C_{12}(x)](1 + o_P(1))f(x), \quad (3.10) \end{aligned}$$

where $C_{10}(x) = (0, 0, g_{M,1}(x)\kappa_{21})'$, $C_{11}(x) = (g_{M,0}(x)\kappa_{21}, g_{M,1}(x)\kappa_{21}, 0)'$, $C_{12}(x) = (g_{M,1}(x)\kappa_{21}, g_{M,2}(x)\kappa_{21}, 0)'$, and the final equality is derived by applying Lemma 3.1 of Lu and Linton (2007, page 45).

The remaining is to establish the asymptotic normality of C_{T2} . By applying Lemma 3.4 of Lu and Linton (2007)

$$\begin{aligned} & (Th)^{-1/2} \gamma_T^{-1} C_{T2} \\ &= (Th)^{-1/2} \sum_{t=1}^T \gamma_T^{-1} \eta_{mt} \varepsilon_t K((\sigma_{Mt} - x)/h) \\ &\rightsquigarrow N(\mathbf{0}, C_2(x)f(x)), \quad (3.11) \end{aligned}$$

where \rightsquigarrow stands for the convergence in distribution, $\mathbf{0} = (0, 0, 0)'$ and

$$C_2(x) = \begin{pmatrix} g_{M,0}^\varepsilon(x)\kappa_{02} & g_{M,1}^\varepsilon(x)\kappa_{02} & 0 \\ g_{M,1}^\varepsilon(x)\kappa_{02} & g_{M,2}^\varepsilon(x)\kappa_{02} & 0 \\ 0 & 0 & g_{M,2}^\varepsilon(x)\kappa_{22} \end{pmatrix},$$

with $g_{M,i}^\varepsilon(x) = E(R_{Mt}^i \varepsilon_t^2 | \sigma_{Mt} = x)$.

Then combining (3.6)–(3.11) leads to

$$\begin{aligned} (Th)^{1/2} \left(\hat{\vartheta} - \vartheta - \frac{1}{2} h^2 \Gamma A(x)^{-1} [\ddot{\alpha}(x) C_{11}(x) + \ddot{\beta}(x) C_{12}(x)] \right) \\ \rightsquigarrow N(\mathbf{0}, \Gamma A(x)^{-1} C_2(x) A(x)^{-1} \Gamma / f(x)). \end{aligned}$$

Note that

$$\Gamma A(x)^{-1} [\ddot{\alpha}(x) C_{11}(x) + \ddot{\beta}(x) C_{12}(x)] = A^*(x)^{-1} C_1^*(x)$$

$$\Gamma A(x)^{-1} C_2(x) A(x)^{-1} \Gamma / f(x) = (A^*(x))^{-1} C_2^*(x) (A^*(x))^{-1} / f(x)$$

where $C_1^*(x) = \ddot{\alpha}(x) C_{11}(x) + \ddot{\beta}(x) C_{12}(x) = (\ddot{\alpha}(x) g_{M0}(x) + \ddot{\beta}(x) g_{M1}(x), \ddot{\alpha}(x) g_{M1}(x) + \ddot{\beta}(x) g_{M2}(x))' \kappa_{21}$. The proof is completed.

Chapter 4

Simulation studies: Finite sample performance

In this section, we report the results of a small Monte Carlo study of the method given in this paper, the purpose of which is to illustrate that local linear estimate of functional beta with a bandwidth gained by a method of AICc can work well in finite samples.

4.1 GARCH Model for market volatility

We will model the market volatility by an GARCH model as follows.

As is known, GARCH model is commonly used in many researches based on CAPM model, which can be expressed for market return process:

$$\begin{cases} R_{Mt} = a_0 + a_1 R_{M,t-1} + \epsilon_t \\ \epsilon_t = e_t \sigma_{Mt}^{1/2} \\ \sigma_{Mt}^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{M,t-1}^2 \end{cases} \quad (4.1)$$

In the conditional mean model, in (4.1), the returns, R_{Mt} , consist of a simple constant, plus an uncorrelated, white noise disturbance, ϵ_t . This model is often sufficient to describe the conditional mean in a financial return series. Most financial return series do not require the comprehensiveness that an ARMAX model provides.

In the conditional variance model, in(4.1), the variance forecast, σ_{Mt}^2 , consists of a constant plus a weighted average of last period's forecast, ϵ_{t-1}^2 , and last period's squared disturbance, σ_{Mt}^2 and e_t being an i.i.d. random sequence with $Ee_t = 0$ and $Ee_t^2 = 1$. Once more, according to GARCH(1,1) model, $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0$.

Since $\alpha_1 + \beta_1 < 1$ in (4.1) with some suitably regular conditions (cf. Carrasco and Chen, 2002), the ϵ_t in GARCH(1,1) model is strongly α - mixing with a geometrically decaying mixing coefficient.

4.2 Simulation study

In this simulated example, we will simulate samples of size 100, 300 and 500, respectively, from the following model, to examine the estimation of the beta functional.

According to expression (2.8), our model reads:

$$R_{it} = \alpha_i(\sigma_{Mt}) + \beta_i(\sigma_{Mt})R_{Mt} + \epsilon_{it}, \quad (4.2)$$

where

$$\alpha_i(x) = 1.6x, \quad (4.3)$$

$$\beta_i(x) = 2x + \exp(-16(x - 0.5)^2) - 1, \quad (4.4)$$

and

$$R_{Mt} = a_0 + a_1 R_{M,t-1} + \epsilon_t \quad (4.5)$$

$$\begin{cases} \epsilon_t = e_t \sigma_{Mt} \\ \sigma_{Mt}^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{M,t-1}^2 \end{cases} \quad (4.6)$$

We took the parameters $a_0 = 0.001682$, $a_1 = 0.020602$, $\alpha_0 = 0.137526$, $\alpha_1 = 0.094518$, $\beta_1 = 0.726777$, which are the parameter estimates of the model (4.1) obtained from the real data of the FT100 Index by using the maximum likelihood method procedure in the GARCH module of S-plus, which is referred to (Lu and Linton 2007). As showed in Lu and Linton (2007, Page 50), R_{Mt} and σ_{Mt} are NED of order $2 + \delta$ w.r.t. a strongly (α -) mixing process, if $E|\epsilon_t|^{2+\delta} < \infty$, with stable coefficients

$$v_{2+\delta}(k) = E \left| R_{Mt} - R_{Mt}^{(k)} \right|^{2+\delta} = O \left(|a_1|^{(2+\delta)k} \right),$$

decaying at a geometric rate, where $R_{Mt}^{(k)} = a_0/(1 - a_1) + \epsilon_t + \sum_{j=1}^k a_1^j \epsilon_{t-j}$, and $w_k = \sum_{j=k+1}^{\infty} a_1^j = O(a_1^k)$. Here the conditions to ensure $E|\epsilon_t^2|^{2+\delta} < \infty$ can be found in Carrasco and Chen (2002), and therefore $E|R_{Mt}^2|^{2+\delta} < \infty$ can be guaranteed. Hence it follows from (4.2) that $(R_{it}, R_{Mt}, \sigma_{Mt})$ is NED of order $2 + \delta$.

Corresponding to (2.8) in chapter 2, we have:

$$R_{it} = 1.6\sigma_{Mt} + (2\sigma_{Mt} + e^{(-16(\sigma_{Mt}-0.5)^2)} - 1)R_{Mt} + \varepsilon_{it}$$

The boxplots of the local estimators of $\beta(\cdot)$, at 50 equally partitioned points, based on 100 replications with each sample size equal to 100, 300 and 500, respectively, are depicted in Figures 4.1, 4.5, and 4.9. The boxplots of the local

estimators of $\alpha(\cdot)$ are also plotted in Figures 4.3, 4.7, and 4.11 for different sample sizes.

Overall, the simulation results in the example adapt very well to our asymptotic theory: with the sample size increasing, the locally estimated curves with an AICc choice of bandwidth become more stable and fit better to actual curve lines both for the $\beta(\cdot)$ and $\alpha(\cdot)$ functions. Clearly, for beta functional, the dashed line which represents the median value of the estimated results of the 100 simulations tends to the solid line standing for the true value in our assumed model with the sample size increasing in Figures 4.2, 4.6, 4.10, respectively. Also, as Figures 4.4, 4.8, 4.12 show, the difference between the median value of 100 simulation outcomes and the true value of alpha function is decreasing as the sample size increases. In addition, from Figures 4.1-4.12, it is quite obvious that the curves of $\beta(\cdot)$ and $\alpha(\cdot)$ at the extreme points of market volatility are less well estimated.

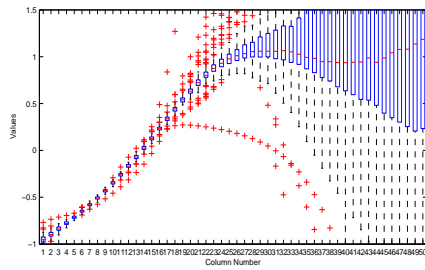


Figure 4.1: sample size=100 boxplot of $\beta_i(x)$

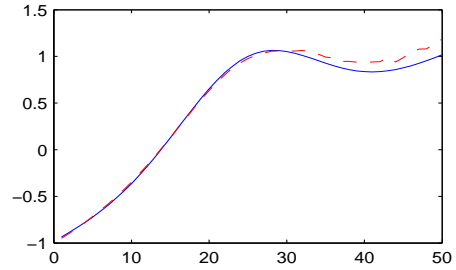


Figure 4.2: sample size=100 The dash line represents median line of $\beta_i(x)$, the other is the true value of $\beta_i(x)$.

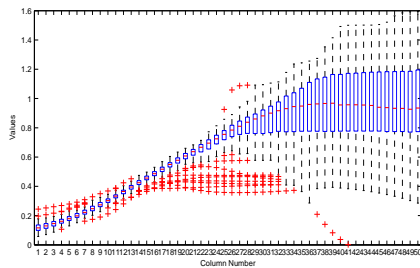


Figure 4.3: sample size=100 boxplot of $\alpha_i(x)$

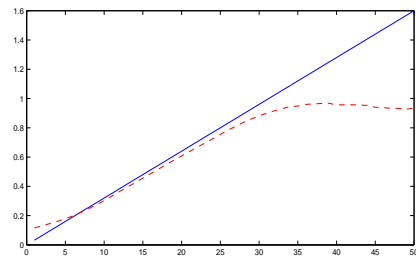


Figure 4.4: sample size=100 The dash line represents median line of $\alpha_i(x)$, the other is the true value of $\alpha_i(x)$.

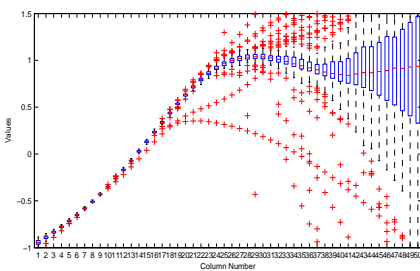


Figure 4.5: sample size=300 boxplot of $\beta_i(x)$

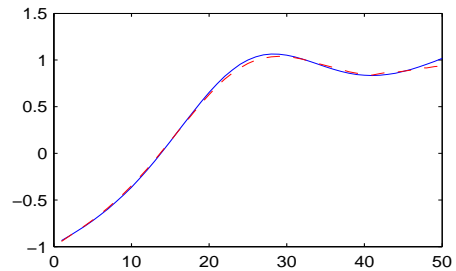


Figure 4.6: sample size=300 The dash line represents median line of $\beta_i(x)$, the other is the true value of $\beta_i(x)$.

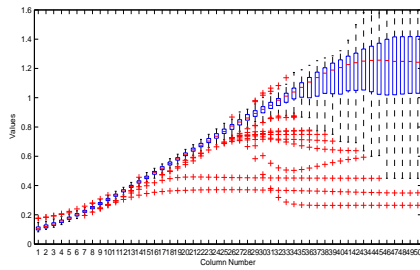


Figure 4.7: sample size=300 boxplot of $\alpha_i(x)$

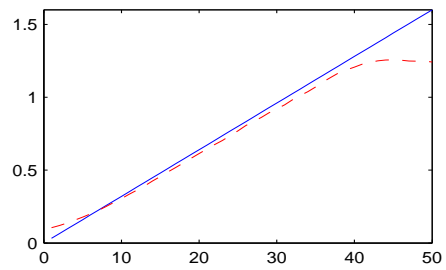


Figure 4.8: sample size=300 The dash line represents median line of $\alpha_i(x)$, the other is the true value of $\alpha_i(x)$.

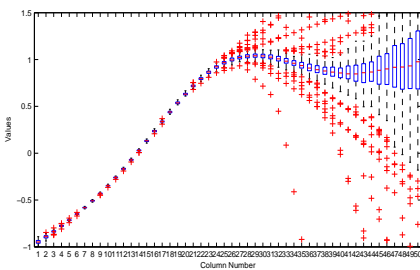


Figure 4.9: sample size=500 boxplot of $\beta_i(x)$

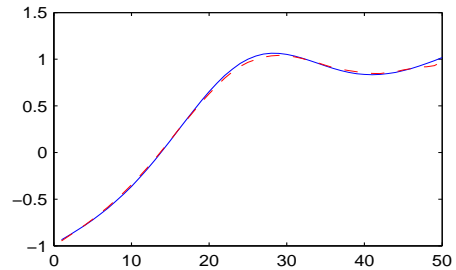


Figure 4.10: sample size=500 The dash line represents median line of $\beta_i(x)$, the other is the true value of $\beta_i(x)$.

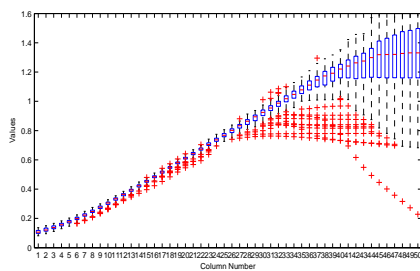


Figure 4.11: sample size=500 boxplot of $\alpha_i(x)$

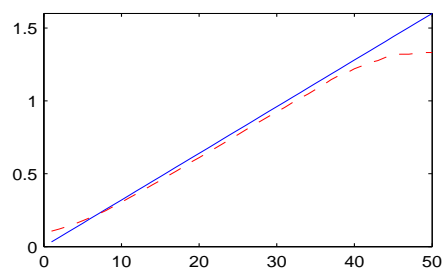


Figure 4.12: sample size=500 The dash line represents median line of $\alpha_i(x)$, the other is the true value of $\alpha_i(x)$.

Chapter 5

Empirical studies: Australian evidence

5.1 Data

We will use a set of stocks data sets collected from Australian stock market and the exchange rate data in the past ten years. The reason why we use the Australian data is because we believe an Australian dataset is ideal for this task. The Australian evidence regarding the CAPM is well studied by Ball, Brown and Officer (1976); Faff (1991); Wood (1991); Faff (1992); Brailsford and Faff (1997); and Faff and Lau (1997) as well as Yao and Gao (2004). Ball, Brown and Officer (1976) may be the first published test of the CAPM using Australian data. They employed the basic univariate testing methodology in vogue at the time and found evidence supporting the model. On the one hand it can be seen that a relatively few, very large companies dominate the Australian market. For example, around forty per cent of market capitalization and trading value is produced by just 10 stocks, whereas a similar number of

the largest US stocks constitute only about 15 per cent of the total US market. Moreover, there are typically prolonged periods in which many smaller Australian companies do not trade. On the other hand, despite the above argument, the Australian equity market belongs to the dominant group of developed markets. For instance, as at the end of 1996 it ranked tenth among all markets in terms of market capitalization at about \$US312,000 million. Interestingly, this is not greatly dis-similar from the size of the Canadian market which ranked sixth. (Faff, Brooks, Fan 2004)

According to ASX Indices (including All Ordinaries Index, ASX 200 GICS Sectors Index), we take sample size 986, from August 2nd 2004 to August 8th 2008, for an illustration. The sectors indexes include ASX 200 GICS Energy, ASX 200 GICS Materials, ASX 200 GICS Health Care, ASX 200 GICS Financials, ASX 200 GICS Finance-x-property trusts and ASX 200 GICS Telecomm. Moreover, we also take two groups of individual stock data which are ANZ bank group limited and Common Wealth bank of Australia as survey sample of individual stock analysis.

At first we review the market return of Australia Index from August 2nd 2004 to August 8th 2008. The daily return data as R_t (for individual sector index or for individual security), can be expressed as:

$$R_t = (\log P_t - \log P_{t-1}) \times 100, \quad (5.1)$$

where P_t represents the closing price of individual sector index in day t . The daily market return data, R_{Mt} , can be expressed as:

$$R_{Mt} = (\log P_{Mt} - \log P_{M,t-1}) \times 100, \quad (5.2)$$

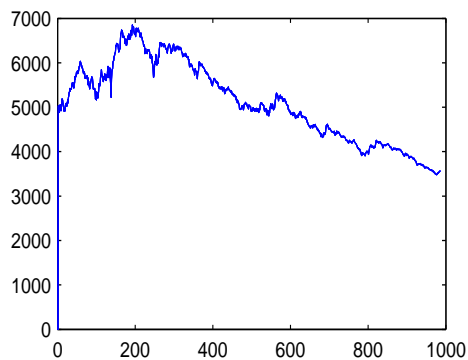


Figure 5.1: All Ordinaries Index in Australia from August 2nd 2004 to August 8th 2008. Sample size =986.

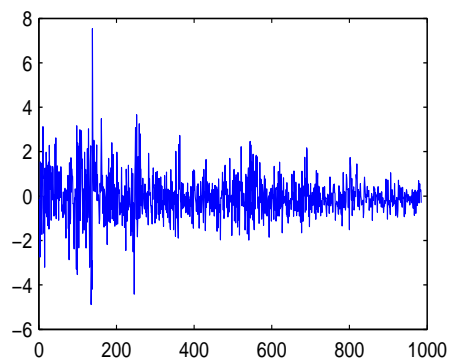


Figure 5.2: the market returns of All Ordinaries Index in Australia from August 2nd 2004 to August 8th 2008. Sample size =985.

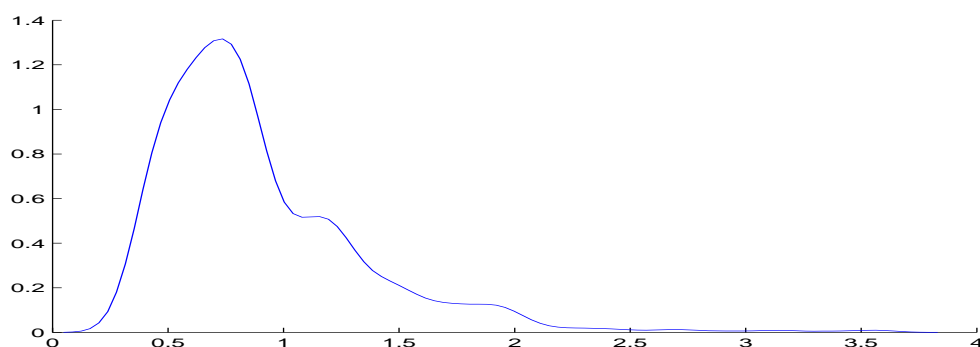


Figure 5.3: The density of market volatility of the returns on All Ordinaries Index

and P_{Mt} represents the closing price of all ordinaries index in day t , both of which are plotted in Figures 5.2 and 5.1, respectively.

Further, we produce the market volatility by GARCH(1,1) model according to All Ordinaries Index. In the GARCH(1,1) model (4.1), we use MATLAB to calculate the parameters with: $a_0 = -0.10591$, $a_1 = -0.06262$, $\alpha_0 = 0.006257$, $\alpha_1 = 0.11541$, $\beta_1 = 0.88163$. The kernel density estimator of the estimated market volatility is plotted in Figure 5.3.

Both the market return and the estimated market volatility are summarized

in the following Table 5.1:

Table 5.1: Some statistics data for the Australia index

	mean	std	skewness	kurtosis	median	min	max
Market return	-0.0354	0.9833	0.5324	8.6204	-0.0766	-4.8826	7.5389
Market volatility	0.896	0.4449	1.8904	8.6092	0.7876	0.3133	3.5655

5.2 Nonparametric evidence

Referred to Section 2.3, we use the real data to calculate the ideal bandwidth for each Sector index. The values of AICc against 25 points of the bandwidth h for eight groups of data (with bandwidth ranging from 0.1 to 0.7 with partition interval of length 0.025) can be found in Figure 5.4. Hence by minimizing the value of AICc, we can have the chosen bandwidths as follows:

Table 5.2: Bandwidth Selection

	a	b	c	d	e	f	g	h
Bandwidth	0.4	0.3	0.475	0.325	0.225	0.15	0.325	0.15

According to Section 2.3, the results of nonparametric estimation of beta functional can be plotted in graphs. For each of eight groups of data (mentioned in Section 5.1), we can have a curve of beta function plotted in the solid line in Figures 5.5-5.12, respectively. As all the beta functions are positive, it means that the market return has positive effects on all individual return. Moreover, the time changing of the beta factor is obvious; it also shows that the individual returns are influenced by the market returns under conditions of market volatility at different levels.

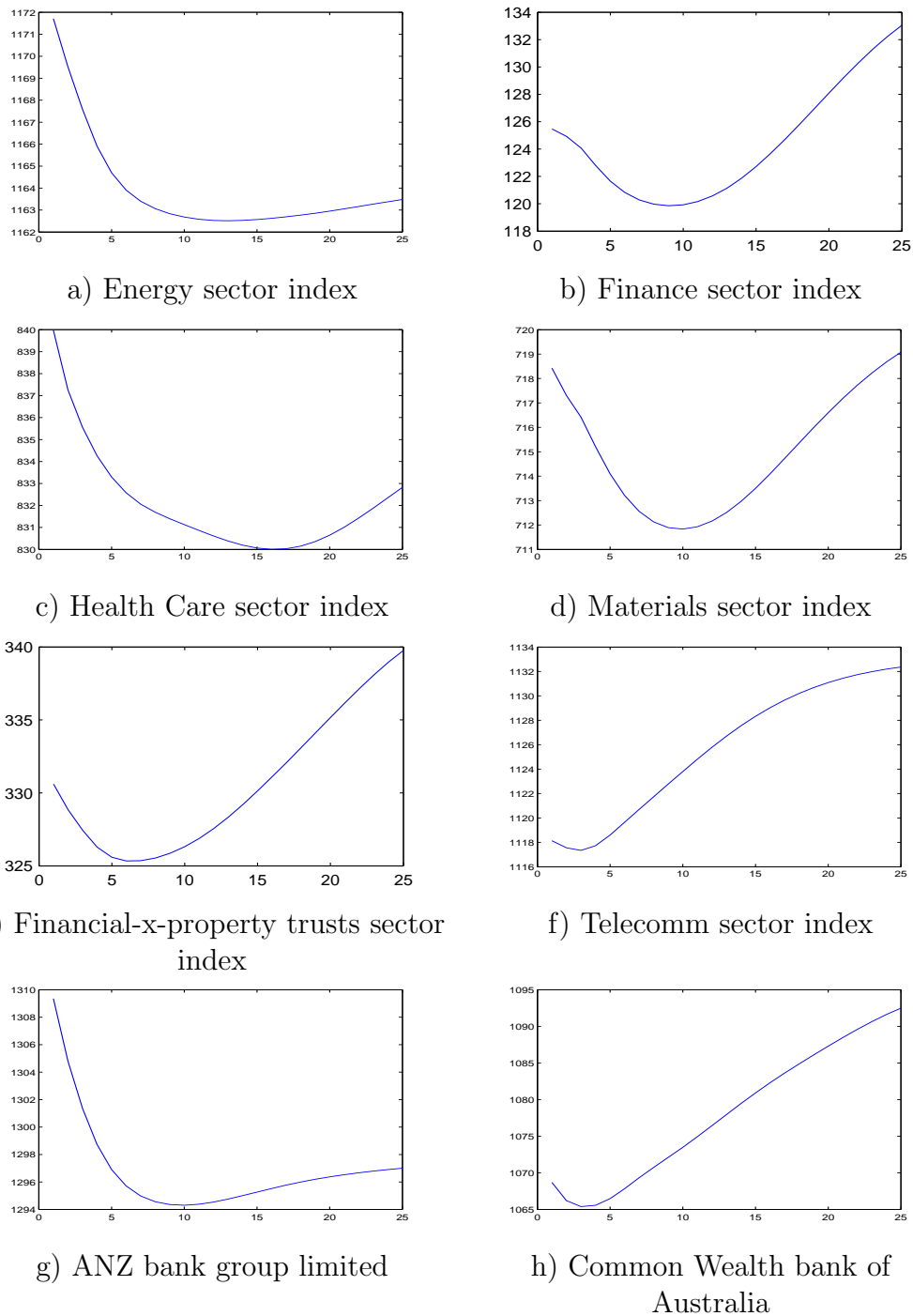


Figure 5.4: The AICc against the bandwidth h for different sector indexes.

5.3 Parametric evidence

In this part we will focus on further investigation according to the previous work of nonparametric outcomes in Section 5.2, which provide us with some possible ways of parametrization of the beta functional. In a recent pioneering work of three-beta CAPM, Galagedera and Faff (2005) who made an assumption that the market conditions can play an important part in explaining a changing beta and could be divided into three states specified as -"low", "neutral" and "high".

Here we consider four types of parametric models to examine which one appears more flexible and better fitted to the real data. (i) The first one is the traditional CAPM with a constant beta as coefficient. (ii) The second one is similar to the first one but it has a linear functional beta. (iii) In the third one, we divide the market volatility into two regimes and the beta functional can be parameterized as a two stepwise linear function. (iv) In the fourth one, we divide the market volatility into three regimes and the beta functional can be parameterized as a three stepwise function. Specifically,

$$\begin{aligned}
 \text{(i)} \quad R_{it} &= \alpha_i + \beta_i R_{Mt} + \varepsilon_{it} & \beta_i &= c \\
 \text{(ii)} \quad R_{it} &= \alpha_i + \beta_{it} R_{Mt} + \varepsilon_{it} & \beta_{it} &= \beta_{i0} + \beta_{i1} \sigma_{Mt} \\
 \text{(iii)} \quad R_{it} &= \alpha_i + \beta_{it,L}(I_{Lt} R_{Mt}) + \beta_{it,H}(I_{Ht} R_{Mt}) + \varepsilon_{it} \\
 & \left\{ \begin{array}{l} \beta_{it,L} = \beta_{i0} + \beta_{i1} \sigma_{Mt} \quad \sigma_{Mt} < \sigma_L \\ \beta_{it,H} = \beta_{i2} + \beta_{i3} \sigma_{Mt} \quad \sigma_L < \sigma_{Mt} \end{array} \right. \\
 \text{(iv)} \quad R_{it} &= \alpha_i + \beta_{it,L}(I_{Lt} R_{Mt}) + \beta_{it,N}(I_{Nt} R_{Mt}) + \beta_{it,H}(I_{Ht} R_{Mt}) + \varepsilon_{it}
 \end{aligned}$$

$$\begin{cases} \beta_{it,L} = \beta_{i0} + \beta_{i1}\sigma_{Mt} & \sigma_{Mt} < \sigma_L \\ \beta_{it,N} = \beta_{i2} + \beta_{i3}\sigma_{Mt} & \sigma_L < \sigma_{Mt} < \sigma_H \\ \beta_{it,H} = \beta_{i4} + \beta_{i5}\sigma_{Mt} & \sigma_H < \sigma_{Mt} \end{cases}$$

where all the $\alpha_i, \beta_{i0}, \beta_{i1}, \beta_{i2}, \beta_{i3}, \beta_{i4}, \beta_{i5}$ are constants to be estimated by linear regression method.

With reference to the results of non-parametric estimates in Figures 5.5-5.12, the market volatility changing regime points σ_L and σ_H are listed below in Table 5.3. Quite amazingly, these regime points are quite similar for different sector indexes.

Table 5.3: Value of σ_L and σ_H

	a	b	c	d	e	f	g	h
σ_L	1.8	1.8	1.7	1.8	1.78	1.25	1.85	1.93
σ_H	2.9	2.8	2.8	3.1	2.6	2.5	2.98	2.68

One important problem in practice is the model selection, that is, which model is the best suitable for a real data set among Models (i)–(iv). In order to verify which type of CAPM best suits each group of data respectively, a criterion of Akaike's information corrected criterion, $AICc$, is applied in this part by minimizing the value of $AICc(m)$. Note that all 4 models (i)–(iv) can be expressed in a linear model in the form $R_i = (R_{i1}, \dots, R_{iT})' = X\mathbf{b} + (\varepsilon_{i1}, \dots, \varepsilon_{iT})'$ by suitably defining a $T \times m$ matrix X and a $m \times 1$ vector \mathbf{b} of unknown parameters. Then we can define

$$\begin{aligned} AICc(m) &= T \log \hat{\sigma}^2 + T \frac{1 + m/T}{1 - (m+2)/T} \\ \widehat{R}_i &= HR_i \quad H = X(X'X)^{-1}X' \end{aligned} \quad (5.3)$$

$$\hat{\sigma}^2 = \frac{1}{n}(R_{it} - \widehat{R}_{it})'(R_{it} - \widehat{R}_{it}) = \frac{1}{n}R_i'(I - H)(I - H)R_i, \quad (5.4)$$

where m is the number of parameters in each of the four CAPMs (See Table 5.4). The results in Table 5.4 show that all the data sets select either Model (iii) or Model (iv), which means the beta functional could be divided into two or three regimes. Models (i) and (ii) may be too simple to describe the relationship between market return and individual return.

Table 5.4: $AICc(m)$ and The Type of CAPM Chosen

Model	(i): m=2	(ii): m=3	(iii): m=5	(iv): m=7	chosen CAPM
Energy	1167.4258	1169.1786	1160.6101	1163.0514	(iii)
Finance	151.5946	151.8064	126.1782	129.5255	(iii)
HealthCare	860.1192	846.1207	830.3054	828.052	(iv)
Materials	741.8942	725.0961	716.733	720.3224	(iii)
F-x-P Trusts	351.267	352.6266	335.7377	337.4171	(iii)
Telecomm	1140.1429	1133.2938	1126.3665	1122.3202	(iv)
ANZ bank	1304.3262	1306.2986	1303.5346	1305.626	(iii)
Common	1074.3087	1073.4483	1091.7984	1070.8326	(iv)

The two-stepwise beta functional in Model (iii) estimated by using the common changing point σ_L for each data set is plotted in dashed line in the right panel of Figures 5.5-5.12, respectively, and the three-stepwise beta functional in Model (iv) in dashed line in the left panel of Figures 5.5-5.12, respectively. Obviously, due to the sparseness of highly extreme market volatility σ_{Mt} (c.f., Figure 5.1), the results of nonparametric estimation are poor and unreliable at extreme market volatility, while the parametric results of two-stepwise or three-stepwise beta functionals provide reasonable outcomes in Figures 5.5-5.12. Under moderate market volatility, both nonparametric and parametric outcomes are pretty consistent.

In the Galagedera and Faff (2005)' work, the functional beta is assumed as three constants over three regimes, which is a special case of Model (iv) with $\beta_{i1} = \beta_{i3} = \beta_{i5} = 0$. To examine their work, here we test the significance of β_{i1} , β_{i3} , β_{i5} by applying T-statistics:

$$H_0 : \beta_{i1} = \beta_{i3} = \beta_{i5} = 0 \quad (5.5)$$

Applying linear regression method we get $\hat{\mathbf{b}}$, and residuals \hat{r} :

$$\hat{\mathbf{b}} = (X'X)^{-1}X'R_i, \text{ where } \mathbf{b} = (\alpha_i, \beta_{i0}, \beta_{i1}, \beta_{i2}, \beta_{i3}, \beta_{i4}, \beta_{i5})', \quad (5.6)$$

and let δ stand for the standard deviation of $\hat{r} = R_i - X\hat{\mathbf{b}}$. Then the T-statistics value of each estimated parameter, $T_j = \frac{\hat{b}_j}{\sqrt{(XX^T)^{-1}_{jj}\delta^2}}$, \hat{b}_j represents the j th element of $\hat{\mathbf{b}}$. In a standard normal distribution, only 5% of the values fall outside the range plus-or-minus 2. Hence, as a rough rule of thumb, a t-statistic larger than 2 in absolute value would be significant at the significance level of 5%. The outcomes of the T-statistics with p-values for Model (iv) and Model (iii) are listed in Tables 5.5–5.12, respectively, which indicate that the Galagedera and Faff (2005)' three-beta model is basically rejected.

Table 5.5: T-statistics value, p-value for each parameter and the linear estimate value of $\hat{\beta}$ for Energy Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}
T statistic (3 step)	-1.2059	11.5787	-2.4497	-0.3729	2.5969	-0.6727	0.9653
Estimate value	-0.0426	1.4396	-0.2692	-0.189	0.5783	-3.1511	1.3262
p-value (3 step)	0.2278	0	0.0143	0.7092	0.0094	0.5012	0.3344
T statistic (2 step)	-1.2432	11.5622	-2.4409	0.3315	2.6166		
Estimate value	-0.0439	1.4387	-0.2685	0.1295	0.4277		
p-value (2 step)	0.2138	0	0.0147	0.7402	0.0089		

Table 5.6: T-statistics value, p-value for each parameter and the linear estimate value of $\hat{\beta}$ for Finance Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}
T statistic (3 step)	0.429	8.9431	5.1589	5.8264	-2.179	0.8613	-0.5422
Estimate value	0.009	0.6729	0.3479	1.6381	-0.2728	2.3877	-0.4408
p-value (3 step)	0.6679	0	0	0	0.0293	0.3891	0.5877
T statistic (2 step)	0.4577	8.9454	5.1521	6.89	-2.2583		
Estimate value	0.0096	0.6733	0.3475	1.5058	-0.2095		
p-value (2 step)	0.6472	0	0	0	0.0239		

Table 5.7: T-statistics value, p-value for each parameter and the linear estimate value of $\hat{\beta}$ for Health care Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}
T statistic (3 step)	-0.5522	11.7196	-5.1893	-0.4069	2.2513	1.6625	-1.5448
Estimate value	-0.0165	1.3798	-0.5759	-0.1363	0.3496	6.5701	-1.7904
p-value (3 step)	0.5808	0	0	0.6841	0.0244	0.0964	0.1224
T statistic (2 step)	-0.6096	11.6717	-5.163	1.0723	1.1938		
Estimate value	-0.0182	1.3785	-0.5748	0.2865	0.1413		
p-value (2 step)	0.5421	0	0	0.2836	0.2325		

Figure 5.5: a) For Energy sector index:

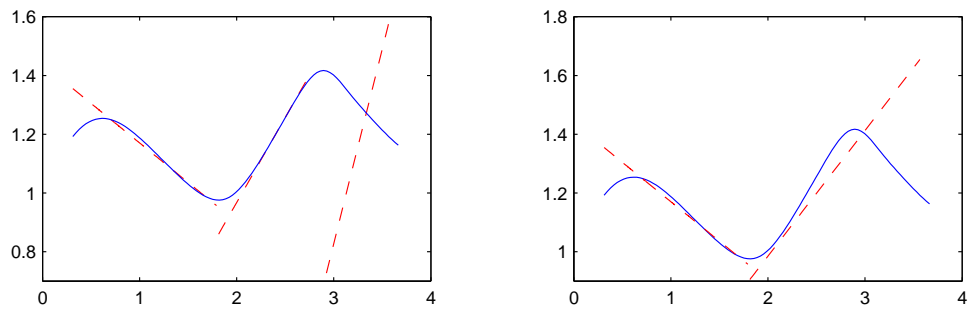


Figure 5.6: b) For Finance sector index:

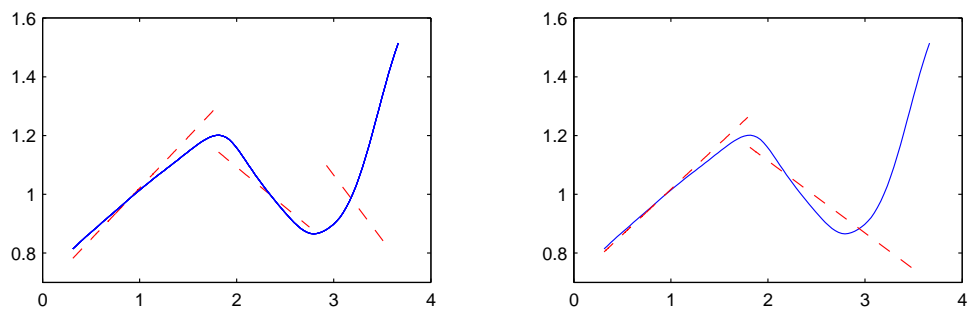


Figure 5.7: c) For Health care sector index:

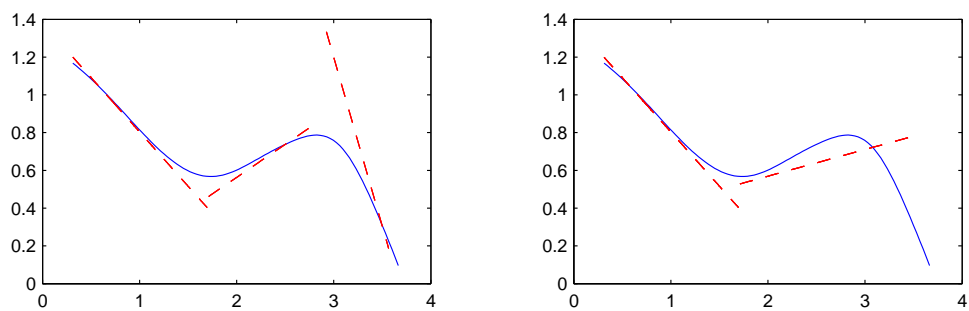


Figure 5.8: d) For Materials sector index:

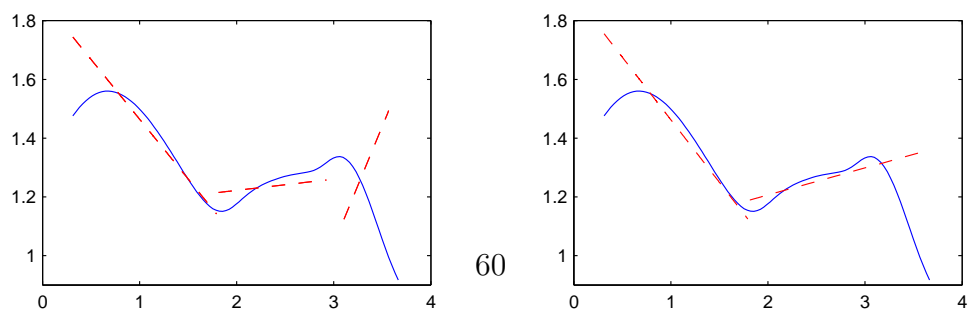


Figure 5.9: e) For Financial-x-Properties Trusts sector index:

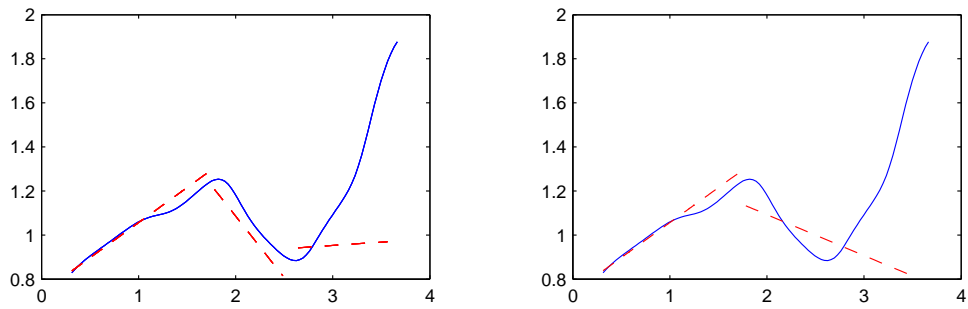


Figure 5.10: f) For Telecomm sector index:

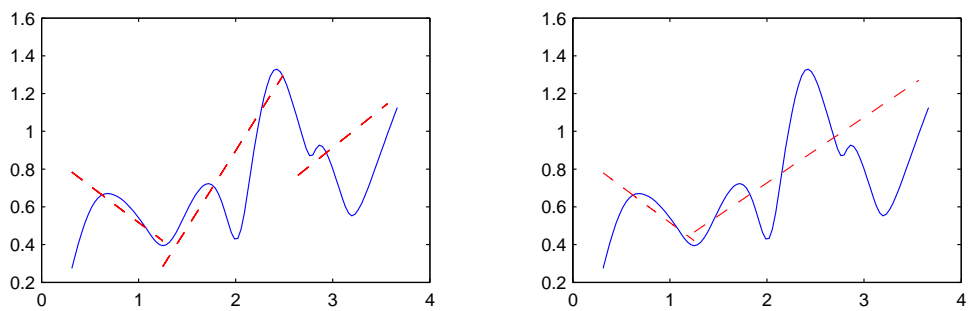


Figure 5.11: g) For ANZ bank group limited:

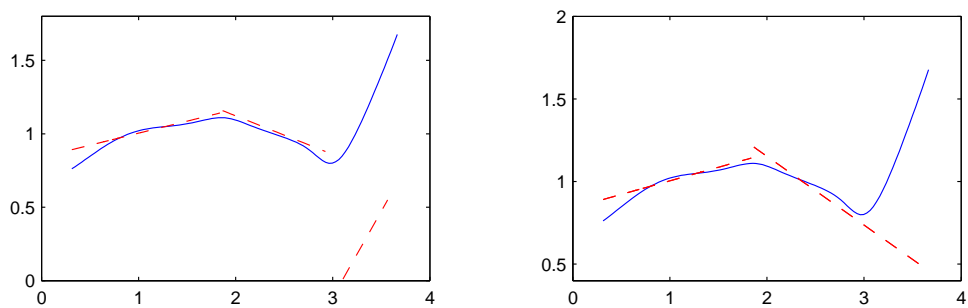


Figure 5.12: h) For Common Wealth bank of Australia:

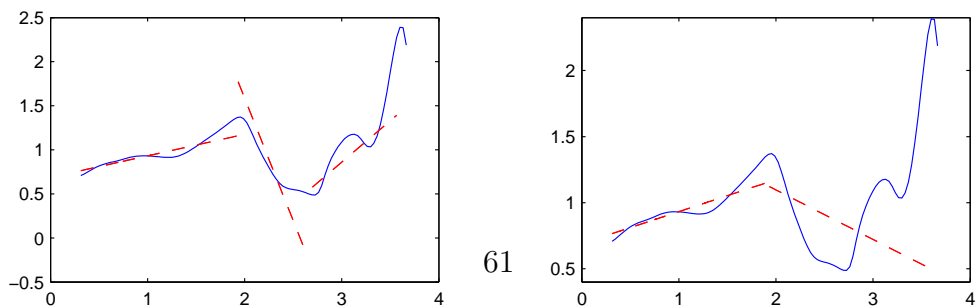


Table 5.8: T-statistics value, p-value for each parameter and the linear estimate value of $\hat{\beta}$ for Materials Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}
T statistic (3 step)	-0.221	19.0194	-4.8549	2.7496	0.2854	-0.3508	0.7135
Estimate value	-0.0062	1.8888	-0.4262	1.1126	0.0507	-1.3254	0.7902
p-value (3 step)	0.8251	0	0	0.006	0.7753	0.7257	0.4756
T statistic (2 step)	-0.22	19.0152	-4.854	3.2799	0.7051		
Estimate value	-0.0062	1.8888	-0.4262	1.0225	0.092		
p-value (2 step)	0.8258	0	0	0.001	0.4808		

Table 5.9: T-statistics value, p-value for each parameter and the linear estimate value of $\hat{\beta}$ for Financial-x-trusts Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}
T statistic (3 step)	0.1413	8.8193	4.2536	3.8508	-1.9635	1.1098	0.1096
Estimate value	0.0033	0.7375	0.3187	2.1994	-0.5568	0.8636	0.0297
p-value (3 step)	0.8876	0	0	0.0001	0.0496	0.2671	0.9128
T statistic (2 step)	0.2208	8.8239	4.2347	6.0216	-1.7991		
Estimate value	0.0051	0.7387	0.3177	1.4638	-0.1857		
p-value (2 step)	0.8253	0	0	0	0.072		

Table 5.10: T-statistics value, p-value for each parameter and the linear estimate value of $\hat{\beta}$ for Telecomm Sector Index

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}
T statistic (3 step)	1.1926	4.9352	-1.9434	-2.1654	4.1877	-0.2755	1.017
Estimate value	0.0415	0.9055	-0.3884	-0.7364	0.8169	-0.3193	0.4112
p-value (3 step)	0.233	0	0.052	0.0304	0	0.7829	0.3091
T statistic (2 step)	1.0371	4.8863	-1.91	0.1592	3.5012		
Estimate value	0.0361	0.9002	-0.3833	0.0316	0.3476		
p-value (2 step)	0.2997	0	0.0561	0.8735	0.0005		

Table 5.11: T-statistics value, p-value for each parameter and the linear estimate value of $\hat{\beta}$ for ANZ bank group limited

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}
T statistic (3 step)	0.7384	6.6172	1.5053	2.4294	-0.9158	-0.7127	0.7853
Estimate value	0.028	0.8407	0.1637	1.6479	-0.2627	-3.6239	1.1705
p-value (3 step)	0.4603	0	0.1322	0.0151	0.3598	0.476	0.4323
T statistic (2 step)	0.7024	6.6044	1.5098	3.9594	-2.0641		
Estimate value	0.0266	0.8399	0.1644	1.9847	-0.416		
p-value (2 step)	0.4824	0	0.1311	0.0001	0.039		

Table 5.12: T-statistics value, p-value for each parameter and the linear estimate value of $\hat{\beta}$ for Common Wealth bank of Australia

	α_i	β_{i0}	β_{i1}	β_{i2}	β_{i3}	β_{i4}	β_{i5}
T statistic (3 step)	-0.4701	6.3163	2.6896	5.0352	-4.2774	-1.734	2.3851
Estimate value	-0.0158	0.6872	0.2446	7.138	-2.7761	-1.9581	0.9395
p-value (3 step)	0.6383	0	0.0072	0	0	0.0829	0.0171
T statistic (2 step)	-0.307	6.2653	2.6284	3.4314	-1.7933		
Estimate value	-0.0105	0.6903	0.242	1.8461	-0.3747		
p-value (2 step)	0.7588	0	0.0086	0.0006	0.0729		

Chapter 6

Conclusions

In our new model, we have suggested a functional-beta single-index CAPM, extending the work of three-beta CAPM (Galagedera and Faff, 2004) that takes into account the condition of market volatility. Differently from the three-beta CAPM, we allow systematic risk β_i changing functionally with the market volatility σ_m , which is more flexible and adaptable to the changing structure of financial systems. The main contributions of this thesis are summarised as follows:

- A new functional-beta CAPM, taking into account the conditions of market volatility, has been proposed under the framework of widely applicable data generating processes of near epoch dependence (NED).
- A semi-parametric estimation procedure based on least squares local linear modelling technique under NED has been suggested with the large sample distributions of the estimators established.
- Simulation study is fully made, illustrating that the suggested estimation procedure for the proposed functional-beta CAPM under near epoch

dependence can work well. It provides reasonable estimates of the functional beta in the condition of moderate market volatility.

- By using a set of stocks data sets collected from Australian stock market in the past ten years, empirical evidences of the functional-beta CAPM in Australia have been carefully examined under both nonparametric and parametric model structures. Differently from the three- or multi-beta (constant) CAPM in Galagedera and Faff (2005), our new findings have convincingly showed that the functional beta can be reasonably parameterized as threshold (regime-switching) linear functionals of market volatility with two or three regimes of market condition. In the condition of extreme market volatility, a parametric threshold functional-beta CAPM is found useful.

The CAPM provides a usable measure of risk that helps investors determine what return they deserve for putting their money at risk. Our new model is no doubt helpful to better understand the relationship between risk and return under different market conditions. It can be potentially applied widely, for example, it may be useful both for market investors and financial risk managers in their investment/management decision-making.

As done in Galagedera and Faff (2005), it is interesting to investigate how the functional beta systematic risk is priced in the real financial assets, which is left for future work.

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