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COMPUTATION OF MODEL CURVES FOR CENTRAL FREQUENCY SOUNDING BY MEANS OF DIGITAL LINEAR FILTER

Summary. Starting with basic equations for CFS response, the computational approach is developed. A linear relationship exists between the magnetic number and the kernel function involved in the integral expression. The normalized vertical magnetic field is determined by subjecting sample values of the kernel function to a digital linear filter. With the help of given filter coefficients, response curves for homogeneous and two-layer Earth models are computed, presented and qualitatively analysed.

Riassunto. Partendo dalle equazioni fondamentali per risposte CFS, viene sviluppato un procedimento per il calcolo di curve teoriche.

Tra il numero magnetico e la funzione di Kernel, contenuta nell'espressione d'integrale, sussite una relazione lineare.

Il valore del campo magnetico verticale viene determinato sottoponendo la funzione di Kernel ad una trasformazione lineare mediante un filtro digitale lineare.

Tramite i coefficienti di tale filtro, vengono calcolate, presentate ed analizzate qualitativamente le curve teoriche per i modelli di terra omogenea ed a due strati.

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1. Introduction

Electromagnetic depth sounding is being increasingly applied for solving geological problems particularly in hard formations. A convenient method of frequency sounding using a comparatively larger loop, known as Central Frequency Sounding (CFS) has been introduced by Patra (1967, 1970) in order to solve shallow problems in hard rocks. CFS measures the existing vertical component of the magnetic field induced at the center of a circular or square loop placed on a layered Earth. Patra (1967) has given an expression for the vertical magnetic field with central frequency sounding. This equation has the form of an infinite integral in which the integrand is the product of two functions, a kernel function, which depends on the parameters of a layer distribution and on the frequency, and a Bessel function. Earlier approaches (Patra, 1967, 1970; Sanyal, 1975) using contour or numerical integration have been found to be time consuming. The present method involves the calculation of the vertical component of the magnetic field by subjecting the kernel function to a linear transformation through a digital linear filter. The method is in line with that Ghosh (1970) used for the computation of model curves in resistivity sounding problems. Verma (1973, 1977) and Patra and Mallick (1980) have discussed the application of the digital linear filter method in the computation of dipole frequency response curves in detail.

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2. Statement of the problem

The measurement of the vertical magnetic field component is of great convenience in the field. The mode of computation of model curves in terms of normalized vertical magnetic field is considered. In earlier approaches of computing CFS electromagnetic response, the evaluation has been made possible only for simple geological models. The contour integration method applied for solving the problem (Patra, 1970) provides only an approximate relation for the computation in the two-layer case. The computation of response curves via numerical integration has so far been restricted for CFS (Sanyal, 1975) only to simple two-layer and three-layer special cases (Patra and Shastri, 1982). Because of some inherent limitations, numerical integration is replaced here by a linear digital filter method leading to a rapid preparation of sets of model curves.

Use is made of the digital linear filter developed by Koefoed et al. (1972) and Verma (1973, 1977) in place of contour and numerical integration approaches. This method subjects the kernel function to a linear transformation through given digital filter coefficients in computing CFS response. A 38-point filter is used in the present case.

3. Theoretical formulations

The vertical component H_z of the magnetic field at the center of the loop is given by the following equation for the case represented in Figs. 1 and 3:

$$H_z = \frac{aI}{2} \int_0^\infty e^{-mz} J_0(mr) J_1(ma)\, m dm \tag{1}$$

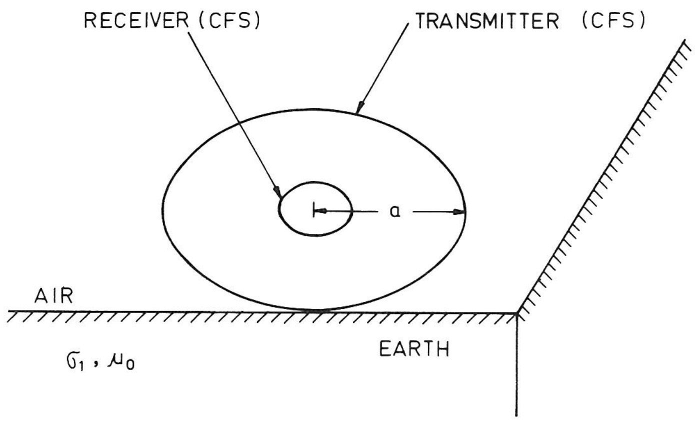

Fig. 1 — CFS system over a homogeneous Earth.

Since the field is measured at the center of the loop, the normal field (H_0) for CFS at the center of the loop is given by eq. (1) (setting $r = 0, z = 0$ and thereby $J_0(mr) = 1$) as

$$H_0 = \frac{aI}{2} \int_0^\infty J_1(ma)\, m dm. \tag{2}$$

With the help of the Lipschitz integral,

$$\int_0^{\infty} e^{-bm} J_0(ma)\, dm = \frac{1}{\sqrt{(b^2 + a^2)}} \qquad (3)$$

eq. (2) can be rewritten as

$$H_0 = I/2a \ (\text{since} \int_0^{\infty} J_1(ma)\, m dm = 1/a^2). \qquad (4)$$

This is the basic equation for the normal field in the CFS system used for normalization of field.

The next step is to express and evaluate the magnetic fields for a layered Earth. For this purpose, use of the following basic equation is made (Sanyal, 1975) to obtain the vertical component of the magnetic field of a horizontal circular loop (carrying current $Ie^{j\omega t}$) of radius a placed on the surface of a layered Earth. This can be expressed at any point on the surface at a distance r from the center of the loop as

$$H_z = \frac{aI}{2} \int_0^{\infty} [1 + F(m, h_j, \sigma_j, f)]\, mJ_1(ma)\, J_0(mr)\, dm \qquad (5)$$

where $F(m)$ is the kernel function of layer thickness, conductivity and frequency. Since CFS measures the induced field at the center of the loop, putting $r = 0$, eq. (5) reduces to

$$H_z = \frac{aI}{2} \int_0^{\infty} [1 + F(m)]\, mJ_1(ma)\, dm. \qquad (6)$$

The normalized magnetic field (or magnetic number) can be expressed with the help of eq. (4) as

$$h_z = \frac{H_z}{H_0} = a^2 \int_0^{\infty} [1 + F(m)]\, mJ_1(ma)\, dm. \qquad (7)$$

This is the basic equation for the normalized vertical component of the magnetic field when a circular loop source is placed on the surface of a horizontally stratified Earth. The components of the electromagnetic vector potential in the horizonal plane are zero and the radiation term in Maxwell's equation is neglected.

The kernel function F(m) can be computed from the subsurface layer parameters and the frequency of the primary excitation using a recurrence relation, setting

$$F(m) = F_{o,n}(m). \qquad (8)$$

In eq. (8), the first suffix refers to the space above the ground surface and the second suffix n is the number of subsurface layers. The recurrence relation is expressed as follows (following the notations used by Koefoed et al. 1972, Sanyal, 1975 and Verma, 1977):

$$F_{(j-1),\,n}\,(m) = \frac{M_{(j-1),\,j} + F_{j,n}\,(m)\; e^{-2h_j m_j}}{1 + M_{(j-1),\,j}\; F_{j,n}\,(m)\; e^{-2h_j m_j}} \tag{9}$$

and $F_{n,n}\,(m) = 0$
where

$$m_j = \sqrt{(m^2 + k_j^2)};\; k_j^2 = i2\pi\, \mu_0\, \sigma_j f;\; M_{je} = \frac{m_j - m_e}{m_j + m_e}\,.$$

These are the generalized formulations used to compute the kernel function for any number of subsurface layers.

4. Digital filter coefficients

Eq. (7) can be expressed in a suitable form to compute the integral involved with the help of a digital linear filter as follows:

$$h_z = a^2 \int_0^\infty [1 + F(m)]\, J_1\,(ma)\; mdm$$

$$= a^2 \int_0^\infty mJ_1\,(ma)\; dm + a^2 \int_0^\infty F\,(m)\; mJ_1\,(ma)\; dm. \tag{10}$$

The first term on the right hand side of eq. (10) represents the magnetic field strength in free space. By making use of eq. (3), eq. (10) reduces to:

$$h_z = 1 + a^2 \int_0^\infty F\,(m)\; mJ_1\,(ma)\; dm. \tag{11}$$

In order to derive the digital filter, logarithmic scales are introduced such that

$$x = ln\,(a);\; y = ln\,(1/m). \tag{12}$$

Eq. (11) then becomes

$$h_z = 1 - \int_0^\infty e^{2\,(x-y)}\; F\,(y,\, h_j,\, \sigma_j,\, f)\; J_1\,(e^{x-y})\; dy. \tag{13}$$

It is evident that the relation between the integral as a function of x and the CFS kernel function as a function of y (for given values of frequency and the layer parameters) represents linearity. The integral in eq. (13) is the convolution of two functions. One of these functions is referred to as the filter function and the other as the input function to the filter. The integral itself is termed as the output function. Special care is required in the choice of the filter function, for the reason that not all the choices are equally good for practical application.

The following optimum choice is made for CFS system in computing the response function derived in eq. (13)

$$\begin{aligned} \text{input function} &= -\, e^{x\text{-}y}\, F\,(y,\, h_p,\, \sigma_p,\, f) \\ \text{filter function} &= e^{x\text{-}y}\, J_1\,(e^{x\text{-}y}). \end{aligned} \tag{14}$$

For deducing the digital filter, the input function is approximated as the sum of the sinc functions. The integral in eq. (13) is the output function corresponding to input and filter functions defined in eq. (14). The evaluation of sinc-response of the filter can be made theoretically by substitution of sinc-function for the input function and then by numerical evaluation of this integral for the final output function. This is a time consuming method due to the oscillating character and the slow decay of Bessel functions involved in the integral. An alternate and a more convenient method to determine the sinc-response of the filter is by operating on the spectra of the functions under consideration. The sinc-response of the filter for CFS system is given in Fig. 2.

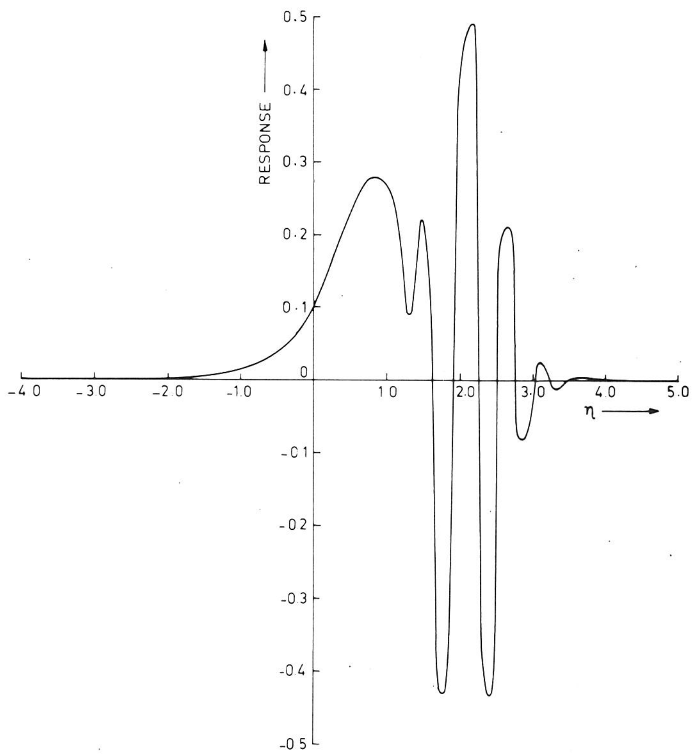

Fig. 2 — Sinc response of filter for CFS system.

5. Frequency-domain sounding

Measurements made by the CFS system with a variety of frequencies form the basis of the Frequency domain CFS.
Frequency plays an important role indirectly with the kernel function F(m) in the input function and obviously in the final output function. A dimensionless parameter called conductivity parameter $B = a\sqrt{(\omega\mu_0 \sigma_1/2)}$ containing the frequency component is introduced in the final solution of the output function. Thus the conductivity parameter is considered as a variable throughout the computation of frenquency-domain responses. This facilitates an easy interpretation in the desired range of frequency. The range of B values is chosen such that the complete frequency domain response due to a stratified Earth model is obtained within the accuracy of measurement. The frequency, loop radius and the top layer conductivity influence the output function and to some extent the filter function particularly in selecting the filter length as illustrated by Koefoed and others (Koefoed et al., 1972). By making use of eq. (14), the integral in eq. (13) is solved by treating the input and filter functions in the frequency-domain. The manner of applying the filter coefficients and their abscissa to the convolution sum along with the filter weights is given as

$$h_z = \text{convolution sum} = \sum_{k=0}^{N} C_k \cdot F(Y_k) \tag{15}$$

where
C_k = the filter coefficients at abscissa values η_k
$F(Y)$ = the input function

$$Y_k = x - \eta_\kappa = ln\,(a) - \eta_0 + k\,(ln\,(10)/10) \tag{16}$$

N = the suffix of the last filter coefficient (= 37, here)
η_0 = the first abscissa value
a = loop radius.

At each and every value of conductivity parameter (i.e., frequency) the convolution summation is performed with the help of filter coefficients (Fig. 2). The number of conductivity parameter values B taken for computation is 24 and the range is between 0.01 and 20.0. The number of filter coefficients is 38. The computations for CFS based on Verma (1977) with sampling interval (ln 10)/10 is found adequate to reconstruct the actual input function between sample points with an absolute error of less than 10^{-5}.

6. Computation of response curves

Response for a homogeneous Earth:

A non-magnetic homogeneous Earth (Fig. 3) is considered for computing the amplitude response. The kernel function in eq. (13) and in the input functions as derived in eq. (14) can be expressed for a homogeneous Earth (letting $j = 1, n = 1$ in eq. (9)) as

$$F_{0,1} = F_m = M_{0,1} = (m_0 - m_1)/(m_0 + m_1) \tag{17}$$

where

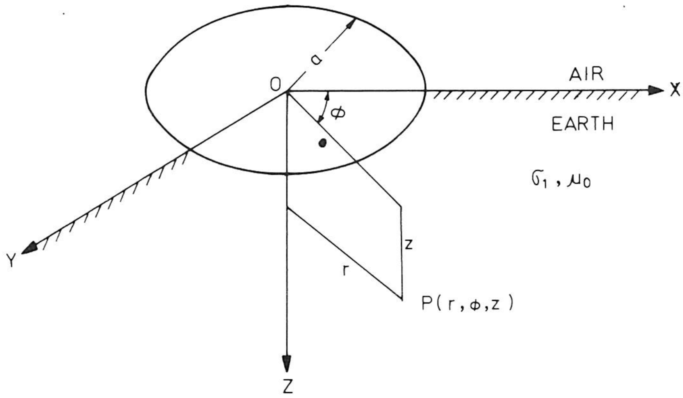

Fig. 3 — Horizontal circular loop over a homogeneous Earth.

$$m_0 = \sqrt{(m^2 + k_0^2)};\ m_1 = \sqrt{(m^2 + k_1^2)}.$$

Since the medium 1 as shown in Fig. 3 is air, $\sigma_0 \rightarrow 0$ and therefore at low frequencies $k_0 \rightarrow 0$ and $m_0 \rightarrow m$. Then the integral equation for a homogeneous Earth is

$$h_z = 1 + a^2 \int_0^\infty [1 + (\frac{m - m_1}{m + m_1})]\, m\, J_1 (ma)\, dm$$

$$= 1 + a^2 \int_0^\infty \frac{2\, m^2}{m + m_1} J_1 (ma)\ dm. \qquad (18)$$

This equation can be rewritten in the digital form with the help of substitutions given in eq. (12). Using the kernel function given in eq. (17) and consequently the expression for the input function given in eq. (14), the convolution summation is performed with the digital notation. In the present study, amplitude response is computed for loop radius (a) = 1000 m and conductivity = 0.001 S/m.

Two-layer Earth:

Considering the case represented in Fig. 4, the amplitude of the normalized magnetic field is computed over a two-layer Earth. The corresponding kernel function derived is as follows:

Letting $j = 1$ and $n = 2$, eq. (9) takes the form

$$F_{0.2} = F(m) = \frac{M_{0,1} + M_{1,2}\ e^{-2h_1 m_1}}{1 + M_{0,1}\ M_{1,2}\ e^{-2h_1 m_1}} \qquad (19)$$

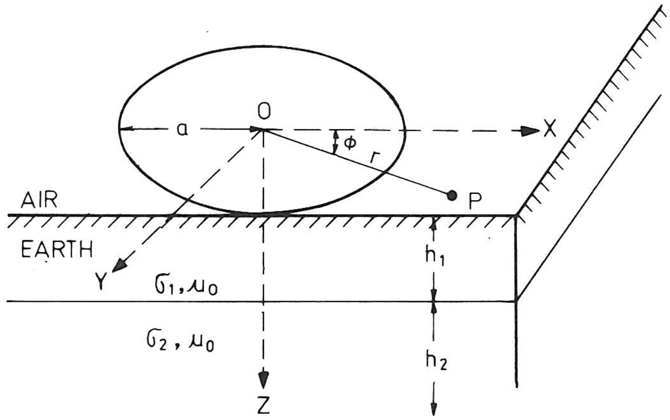

Fig. 4 — Horizontal circular loop over a two-layer Earth.

where

$M_{0,1} = (m_0 - m_1) / (m_0 + m_1)$
$M_{1,2} = (m_1 - m_2) / (m_1 + m_2)$
$m_0 = \sqrt{(m^2 + k_0^2)}; m_1 = \sqrt{(m^2 + k_1^2)}; m_2 = \sqrt{(m^2 + k_2^2)}$
$\sigma_0 \rightarrow 0$ as $k_0 \rightarrow 0$ and $m_0 \rightarrow m$ in air with
$k_j^2 = i\, 2\pi\, \mu_0\, \sigma_j f$ where $j = 1,2$.

The kernel function defined in eq. (19) is substituted in eq. (14) to obtain the input function. Once the input function for a two layer Earth is found, digital summation is performed with the usual procedure to obtain the final output function. To compute frequency-domain responses for a two-layer Earth, the following layer parameters are considered:

Set I - Loop radius (a) = 50,500,1000 and 1500 m.
σ_2 / σ_1 = 0.030 and h_1 = 30 m.
Set II - Loop radius (a) = 25 m
σ_2 / σ_1 = 0.01, 0.1, 0.3, 3.0, 10.0, 30.0, 100.0, ∞
a/h_1 = 32,16,4,2.0.

Frequency-domain responses are computed for the above models to study the resolution trends and its behaviour with different conductivity ratios, thickness ratios and loop radii.

7. Results

A gradual fall of amplitude over a homogeneous Earth (Fig. 5) is observed with the increase of conductivity parameter until a saturation is achieved. Similar nature of a homogeneous Earth response (Fig. 5) and two-layer response curve (for a = 50 m, Fig. 6) implies that the effect of a second layer is not felt even for a thickness of 30 m and for a loop radius a = 50 m, while for larger values of loop radius, a sharp fall of amplitude indicates the effect of the existing second layer (Fig. 6).

Amplitude responses for a two-layer Earth model, presented in Figs. 7-10 show the effect of variation in the layer conductivity of the separation of curves. This change is reflected on the response curves for larger condcutivity contrast. Separation is poor for small conductivity contrast between top and bottom layers. Fall of amplitude is slow and gradual with the increase of second layer conductivity and it decays sharply with the decrease of bottom layer conductivity. Amplitude response versus B with a/h_1 as curve parameter presented in Figs. 11 and 12, shows a significant resolution of layer conductivity for values down to $a/h_1 = 2.0$, meaning thereby that the layers are detected with ease at large conductivity contrast between top and bottom layers.

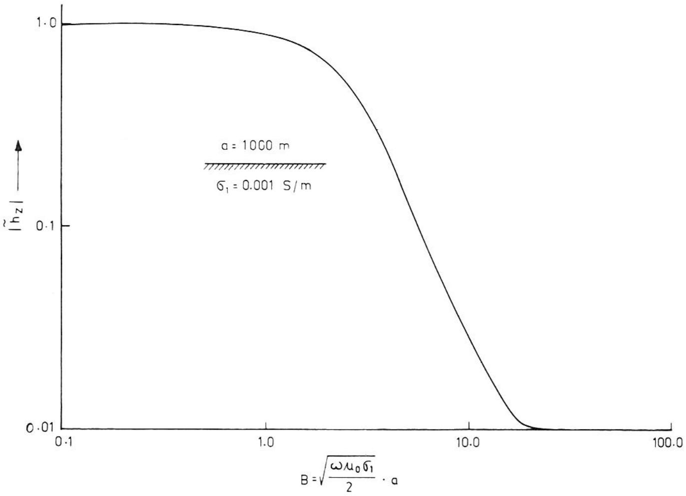

Fig. 5 — Amplitude response curve for a homogeneous Earth.

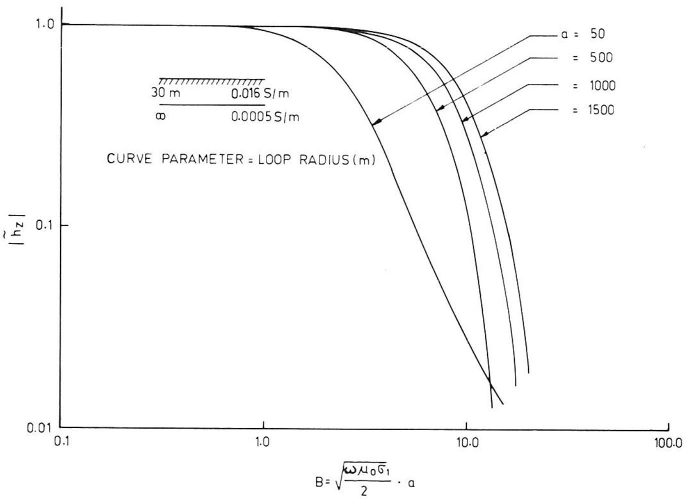

Fig. 6 — Amplitude response curve for a two-layer Earth.

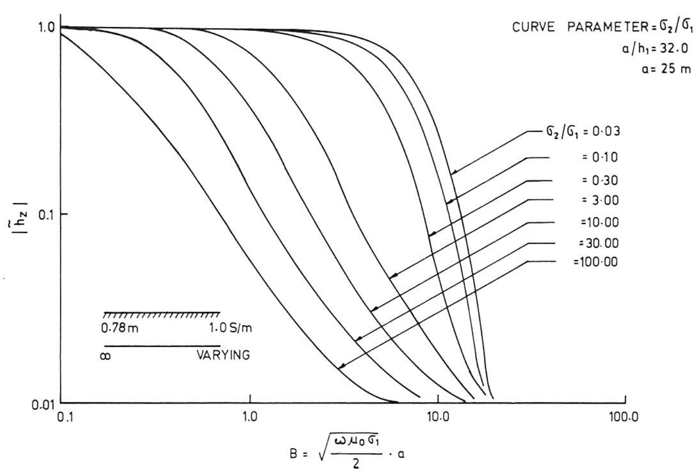

Fig. 7 — Amplitude response curve for a two-layer Earth.

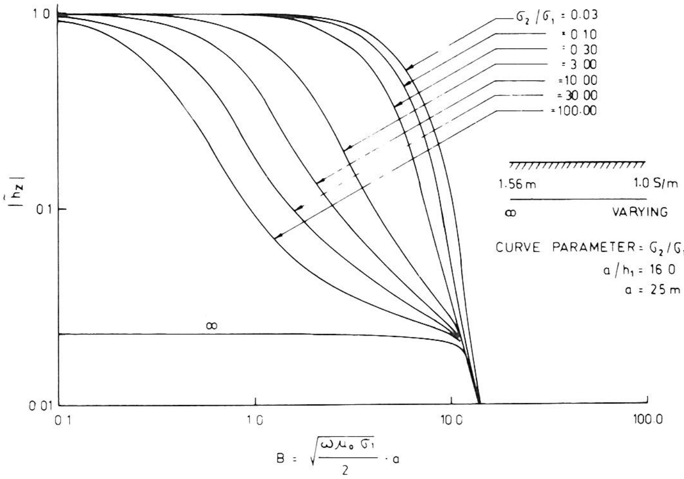

Fig. 8 — Amplitude response curve for a two-layer Earth.

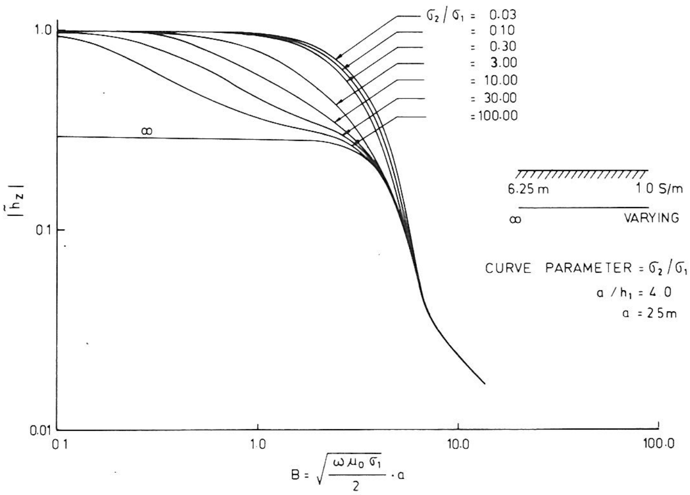

Fig. 9 — Amplitude response curve for a two-layer Earth.

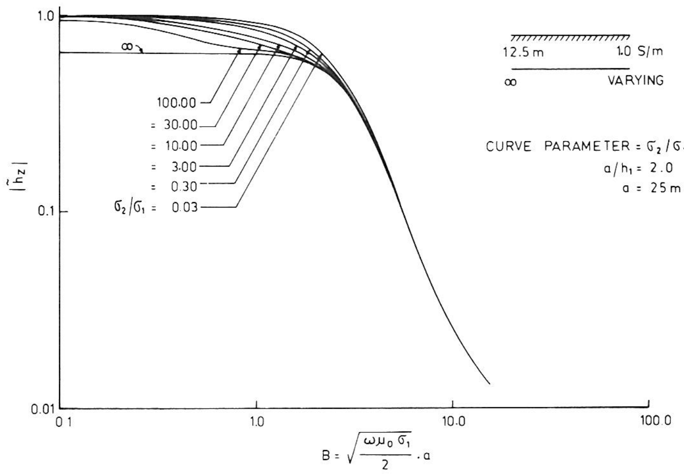

Fig. 10 — Amplitude response curve for a two-layer Earth.

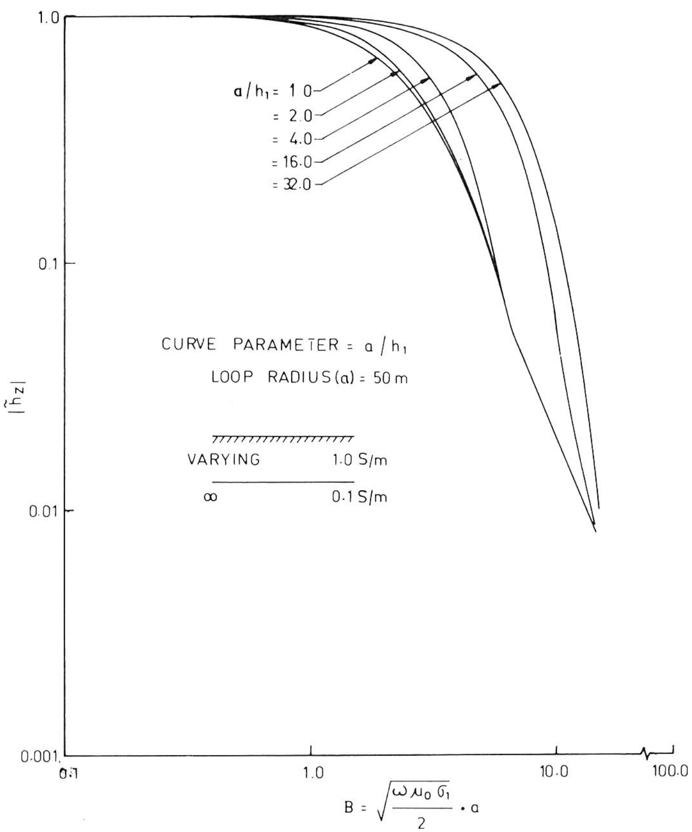

Fig. 11 — Amplitude response curve for a two-layer Earth.

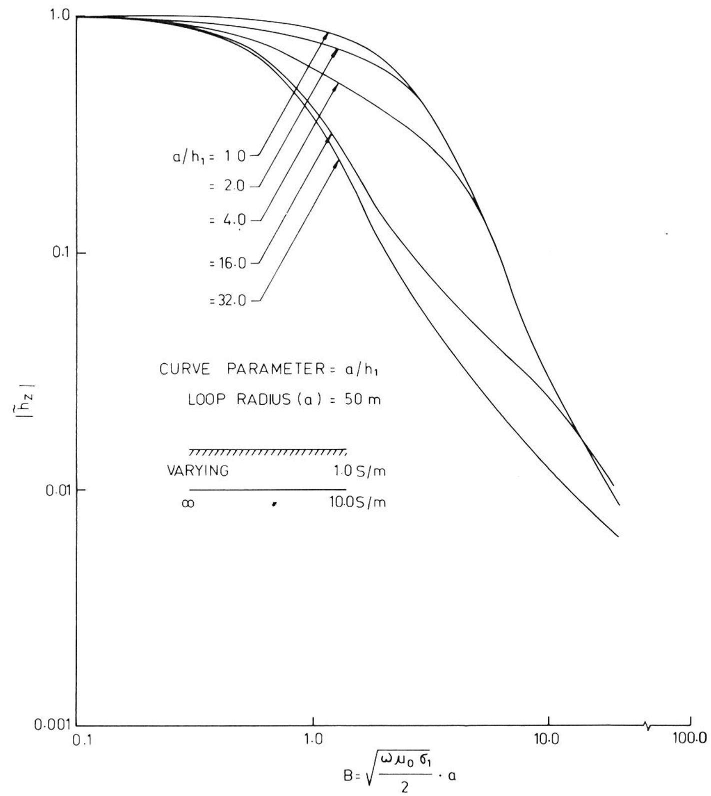

Fig. 12 — Amplitude response curve for a two-layer Earth.

8. Conclusion

The linear digital filter method adopted here in the computation of CFS model curves is accurate and fast compared to numerical integration approach. The examination of model curves computed for the cases of a homogeneous and a two-layer Earth indicates the possibility of a fair resolution of layer parameters. From an analysis of sets of curves, the depth attainable by the method is inferred to be approximately equal to half the loop radius.

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