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TRANSIENT CFS RESPONSE OVER A TWO-LAYER EARTH

Abstract. A Fourier series approach is used for computation of the time domain CFS response over a two-layer earth. The normalised vertical magnetic fields computed with the help of a digital linear filter are summed at each Fourier component composing the excitation pulse to obtain the CFS time domain response. Alternatively, the theoretical expressions for time domain sounding can be evaluated using numerical integration techniques, namely the Gauss-quadrature method and Simpson's rule. The aim of the present paper is to present a comparative study of these three techniques. An error analysis is also done and the results and performances of various computational approaches analysed. The sets of curves computed for two-layer earth models by these three techniques for varying loop radius, σ_2 / σ_1 ratio and a/h_1 ratio under the influence of a half sinusoidal primary excitation pulse are presented here.

INTRODUCTION

When the primary field is generated by a transient source, the secondary currents and the fields produced by their target decay gradually after the primary field is cut off. The decay of this field is interpreted in terms of the conductivity of the target. Sounding detects the subsurface conductors, since the transient excitation wave-forms contain low to high frequency components. The signals picked up by the receiver gradually decay with time depending on ground conditions. Electromagnetic waves are delayed while passing through the conductor (target). In the case of sinusoidal waves, this delay is measured as a phase shift. The transient signal received is a superposition of the secondary amplitudes of all the frequency components, each with its characteristic delay. The damping characteristics of high frequency components of a transient pulse are related to shallower events. The low frequency components and consequently the signals appearing at late times are governed by conductors at greater depths.

Morrison et al. (1969) have given a quantitative interpretation of transient electromagnetic fields over a layered half-space. Nelson and Morris (1969) have studied the homogenous and layered earth models in an EM input field which consists of a series of half sinusoidal pulses of alternating polarity. Koenigsberger (1939) proposed a new EM induction method, called the central ring induction method, where it is possible to obtain varied depths of penetration by changing the radius of the loop. Yoshizumi et al. (1959) developed the interpretation techniques of this method. Patra (1970) introduced Central Frequency Sounding (CFS) with a large circular loop. He presented 2-layer CFS curves using a contour integration technique. Sanyal (1975) presented three-layer curves using a digital linear filter. Patra (1976) applied the principles of CFS for ground water prospecting. Patra (1978) studied the three-frequency computational method for two-layer CFS data and developed the interpretation scheme. Patra and Shastri

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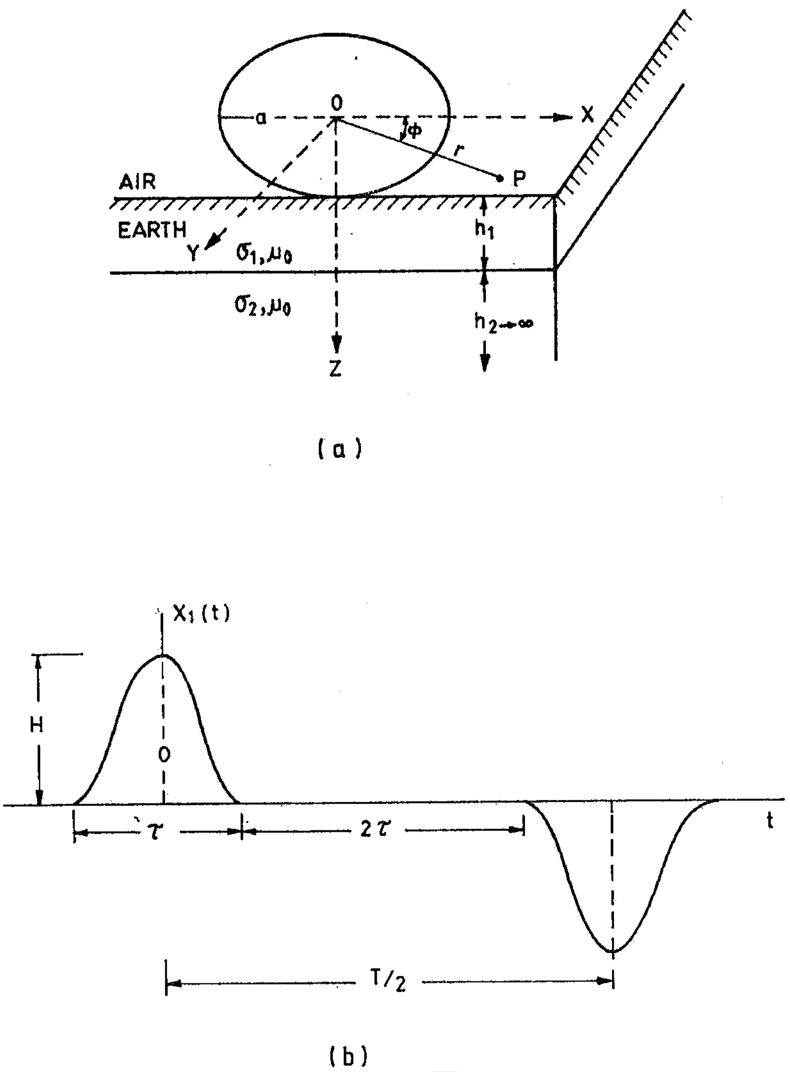

Fig. 1 — a) Transmitter-receiver configuration for two layer earth model; b) half-sinusoidal primary excitation pulse.

(1982) studied the relative performances of central and dipole frequency soundings over a layered earth. They also presented the computational approach of model curves for CFS using digital linear filters (1983a, 1983b). Patra and Shastri (1985, 1988) studied the response characteristics of multi-layer earth using theoretical as well as laboratory simulated models.

CFS can be used to solve the problems associated with a stratified earth. CFS measures the normalised vertical component of the magnetic field induced at the center of the circular loop by an alternating current in the frequency domain. The transmitter-receiver configuration for CFS measurements over a two-layer earth model is shown in Fig. 1a.

The present work uses a Fourier summation approach to compute the time domain response of an earth model consisting of two subsurface layers. Here, the numerical integration techniques of the Gauss-quadrature and Simpson's 1/3 rule, along with the digital linear filter technique, have been utilised to compute the time domain response for a two-layer earth. A widely spaced set of theoretical models have been used to compute the response due to two-layer earth models for various conductivities, thicknesses, and transmitter-receiver separations. The resolution capabilities were also studied for several geometric and parametric variables pertaining to the CFS system.

Another aspect of the study is to compare the results obtained by the two different numerical integration methods against the background of the digital filter method. The computations were carried out on a CYBER 180/840 (CDC).

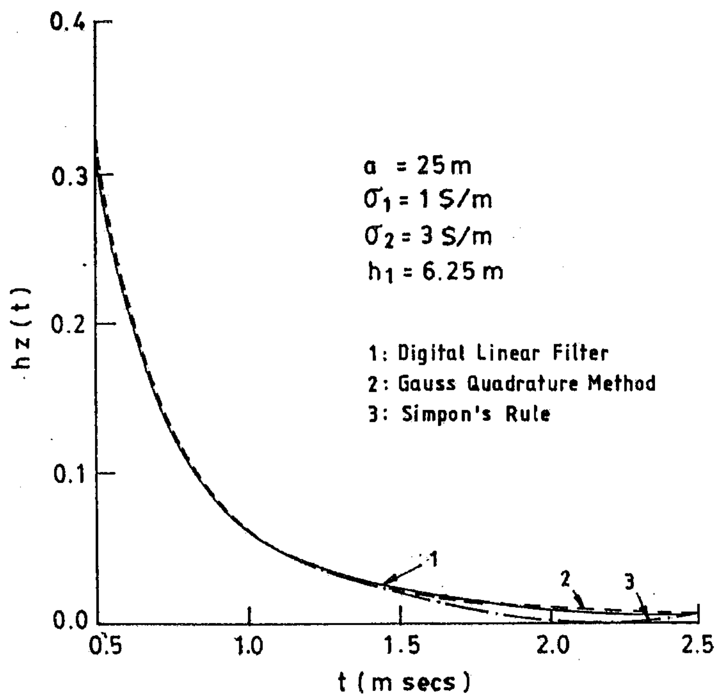

Fig. 2 — Comparison of CFS responses obtained by DLF, the Gauss-quadrature method and Simpson's rule for $\sigma_2 / \sigma_1 = 3$.

METHODOLOGY

Starting from the basic expression for CFS response (Patra and Mallick, 1980, p. 92), the equation for a normalised magnetic field can be given by

$$h_z = \frac{H_z}{H_o} = \frac{a^2}{2} \int_o^{\infty} [1+f(\lambda)]\ \lambda\ J_1\ (\lambda a)\ d\lambda. \tag{1}$$

The integral involved here is solved with the help of digital linear filters, the Gauss-quadrature method and Simpson's rule. Eqn. (1) is simplified as

$$|\ h_z\ | = 1 + a^2 \int_o^{\infty} f_1\ (\lambda)\ \lambda\ J_1\ (\lambda a)\ d\lambda. \tag{2}$$

The kernel function f(λ) is computed from the layer parameters and frequency using the recurrence relation

$$f(\lambda) = f_{o,\ n}\ (\lambda), \tag{3}$$

where the first suffix represents the measurement of field in space above the ground and the second suffix 'n' is the number of layers.

The kernel function for a two-layer earth can be given by

$$f_{o,\ 2} = f(\lambda) = \frac{M_{o,\ 1} + M_{1,\ 2}\ e^{-2h_1 \lambda_1}}{1 + M_{o,\ 1} \cdot M_{1,\ 2}\ e^{-2h_1 \lambda_1}}\ , \tag{4}$$

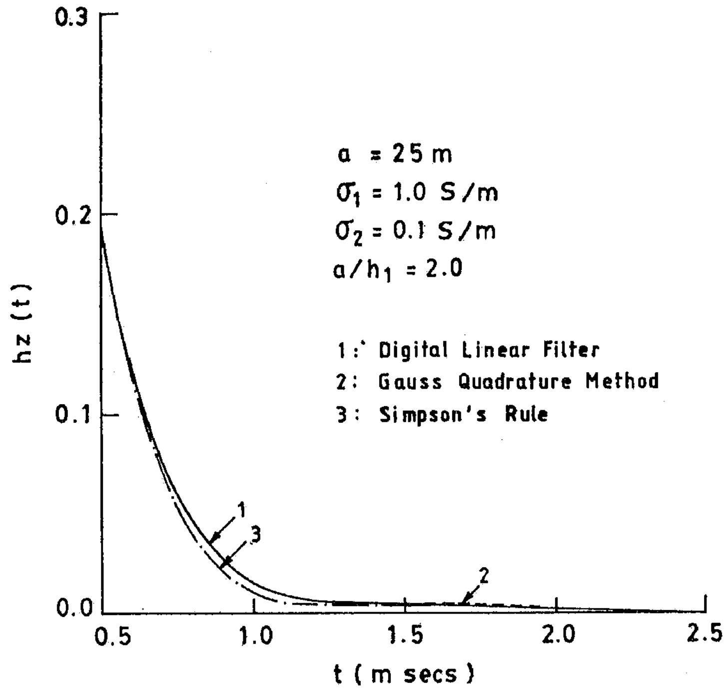

Fig. 3 — Comparison of CFS response obtained by DLF, the Gauss-quadrature method and Simpson's rule for a=25 m.

where

$M_{0,1}=(\lambda_o-\lambda_1)\ /\ (\lambda_o+\lambda_1)$,

$M_{1,2}=(\lambda_1-\lambda_2)\ /\ (\lambda_1+\lambda_2)$,

$\lambda_j^2=(\lambda^2+k_j^2)^{1/2}$,

$k_j^2=i2\pi\mu_0\sigma_j f$, for $j=1, 2$,

λ=variable for integration,

f=frequency,

a=loop radius,

h_1=thickness of top layer,

σ_1=conductivity of top layer,

σ_2=conductivity of bottom layer.

Primary excitation in a transient system can have any shape: step, ramp, sawthooth, square, trapezoidal, Gaussian, half-sinusoidal or pseudo-noise waveforms. In the present study we have considered only one type of excitation: a series of half sinusoidal pulses of alternating polarity (Fig. 1b). The pulse width is 1ms with off-period 2ms. The total period is 6ms, there by giving a repetition of fundamental frequency of about 167 Hz. Such a pulse is represented by

$$\begin{aligned} f(t) &= H_0 \cos pt & -0.5<t<0.5 \text{ ms} \\ &= 0 & 0.5<t<2.5 \text{ ms} \\ &= -H_0 \cos pt & 2.5<t<3.5 \text{ ms} \\ &= 0 & 3.5<t<5.5 \text{ ms}, \end{aligned} \tag{5}$$

with

$p=2\pi/T$,

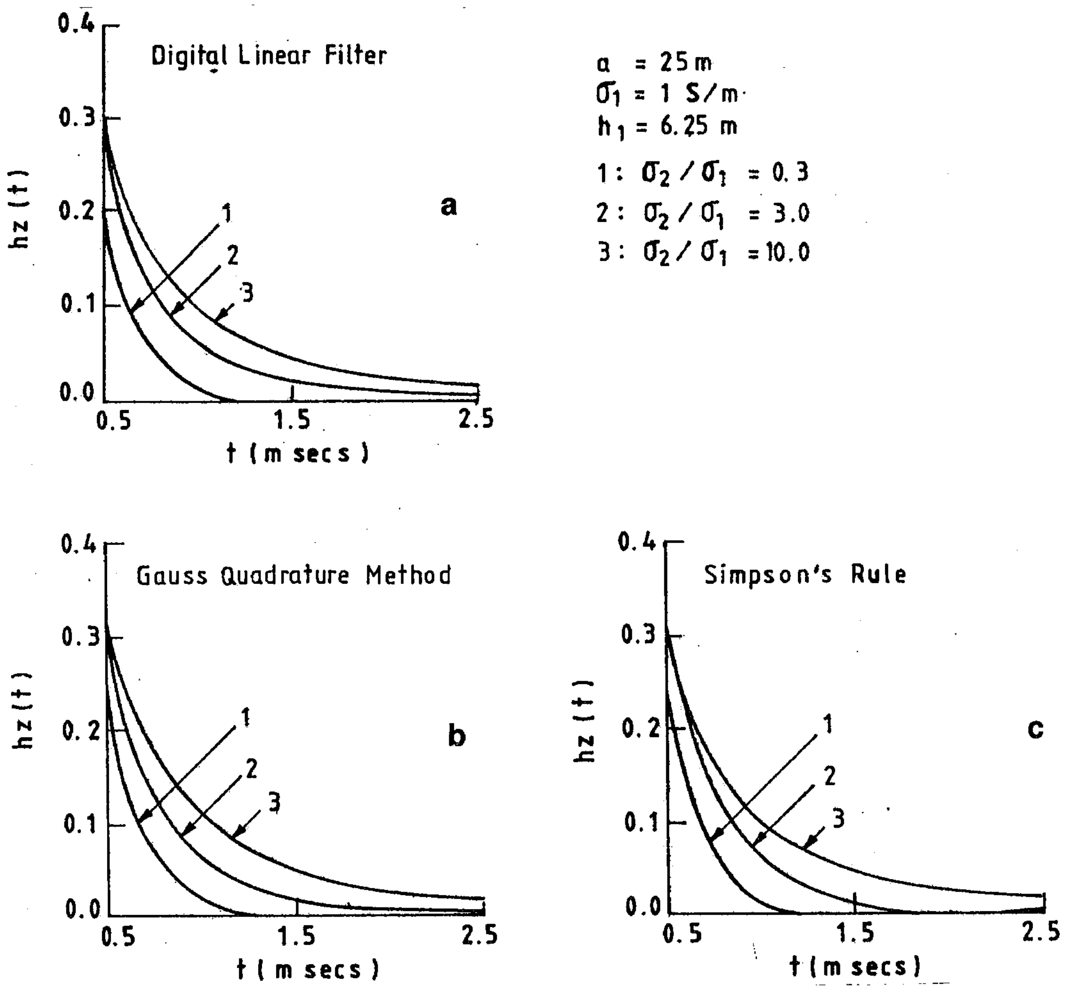

Fig. 4 — CFS response obtained by using: a) Digital Linear Filter, b) Gauss-quadrature method, c) Simpson's rule for different σ_2 / σ_1 ratio.

T = period of the complete sine wave,

H_0 = primary peak value taken as unity.

The pulse train can be expressed in terms of a Fourier series

$$f(t) = \sum_{n=1,\,3,\,5..}^{\infty} F_n \cos (n\omega_0 t), \tag{6}$$

where

$$\omega_0 = 2\pi/T_0;\; T_0 = 6ms,$$

$$F_n = \frac{8\delta \cos n\pi\delta}{\pi\,(1-4n^2\delta^2)} \text{ for } 2n\delta \neq 1 \tag{7}$$

$$= \quad 2\delta \qquad \text{for } 2n\delta = 1,$$

$\delta = \tau/T$ = duration fo the half sinusoid.

The pulse is generated by summing 100 terms. The error at t = 0.5 or –0.5 and at other corners

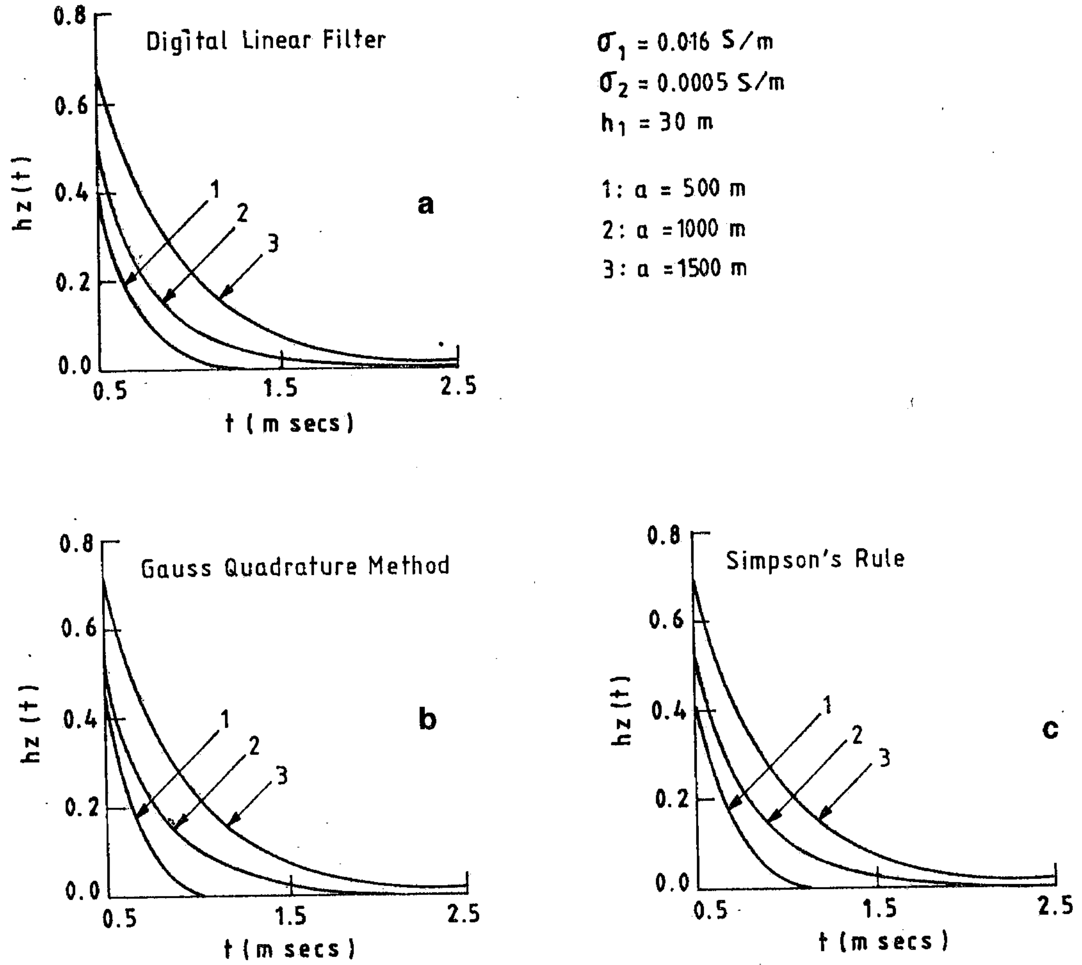

Fig. 5 — CFS response obtained by using: a) Digital Linear Filter, b) Gauss-quadrature method, c) Simpson's rule with varying loop radius.

of the sine pulse due to Gibb's phenomenon is 1%.

Once the normalised values of the vertical magnetic field component corresponding to each harmonic have been computed, the Fourier series can be summed over all the harmonics to yield the time domain response:

$$h_z(t) = Re\left[\sum_{n=1,3,5..}^{N} F_n \left| \frac{H_z}{H_o}(\omega)_n \right| e^{i(n\omega_o t + \phi_n(\omega))}\right], \tag{8}$$

where N = number of harmonics.

The summation is over the real part in the time domain. Eqn. (8) can be written as

$$h_z(t) = \sum_{n=1,3,5..}^{\lambda} F_n \left[\left.\frac{H_z}{H_o}\right|_n^r \cos n\omega_o t - \left.\frac{H_z}{H_o}\right|_n^i \sin n\omega_o t \right]. \tag{9}$$

The vertical component of the magnetic field at the center of the loop was calculated using numerical integration techniques, namely, the Gauss-quadrature method and Simpson's 1/3 rule. The results are finally compared with those obtained by using a digital linear filter.

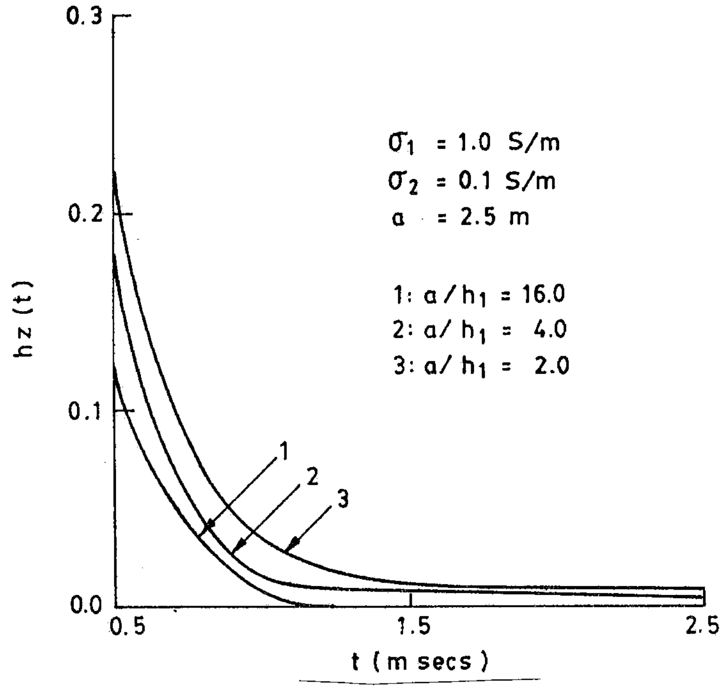

Fig. 6 — CFS response computed by Gauss-quadrature method for different a/h_1 ratio.

ALGORITHMS

The algorithms developed using a digital linear filter, Gauss-quadrature method and Simpson's rule are discussed below:

(i) *Digital Linear Filter*

Step 1: For the purpose of deriving the digital linear filter, eqn. (2) is reproduced using logarithmic scales, and the final expression can be written as (Patra and Shastri, 1983a)

$$|h_z| = 1 - \int_0^\infty e^{2(x-y)} f_1(y) J_1(e^{x-y})\, dy, \tag{10}$$

where

$$x = \ln(a) \quad ; \quad y = \ln(1/\lambda).$$

Step 2: For computing the response function, the following choice is made for the CFS system:

Input function: $-e^{(x-y)} f_1(y, h, \sigma, f)$,

Output function: $e^{(x-y)} J_1(e^{x-y})$.

Step 3: These functions are used for obtaining the normalised maghetic field values from

$$h_z = \text{Convolution sum} = \sum_{k=0,1,2}^{n} C_k \cdot f_1(y_k), \tag{11}$$

where

C_k = filter coefficients along abscissa,

f (y) = Input function,

$y_k = x - y_k = \ln(a) - \eta_o + k\,[(\ln 10/10)]$,

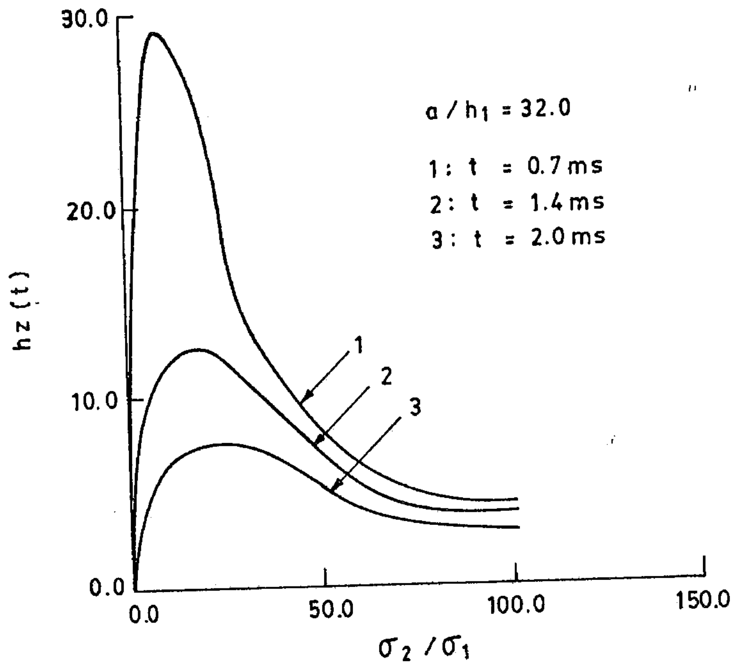

Fig. 7 — CFS response computed by Gauss-quadrature method for different time points with $a/h_1=32$.

n = suffix of last filter coefficient,

η_o = first value of abscissa,

a = loop radius.

Step 4: The value of h_z is computed for all the harmonics and a Fourier summation of these values using eqn. (9) yields the time domain response.

(ii) *Gauss Quadrature Method of Integration*

Step 1: The integral in eqn (1) is approximated by

$$\int_{o}^{\infty} [1+f(\lambda)]\,\lambda\, J_1(\lambda a)\, d\lambda = \frac{N}{2} \sum_{i=1}^{n} g_i\, f(v_i), \tag{12}$$

where N is taken as large as possible till the integral converges; n = 12 for the 12 - point Gauss - quadrature method, and g_i and v_i are the Gauss coefficients and points obtained from standard tables.

Step 2: The interval [0, N] is divided into subintervals suitably chosen for convergence of the integral. Integration is carried out in each of these subintervals and summed.

Step 3: Steps 1 and 2 are repeated to compute the response at the odd harmonics starting from 167 Hz.

Step 4: A Fourier summation of these responses is done using eqn. (9) to get the time domain response at each time point.

Step 5: Steps 1-4 are repeated for all the time points, i.e., from 0.5 ms to 2.5 ms.

(iii) *Simpson's 1/3 rule Integration*

Step 1: For the numerical evaluation of the integral

$$\int_{o}^{N} [1+f(\lambda)]\,\lambda\, J_1(\lambda a)\, d\lambda \qquad \text{where } N\rightarrow\infty, \tag{13}$$

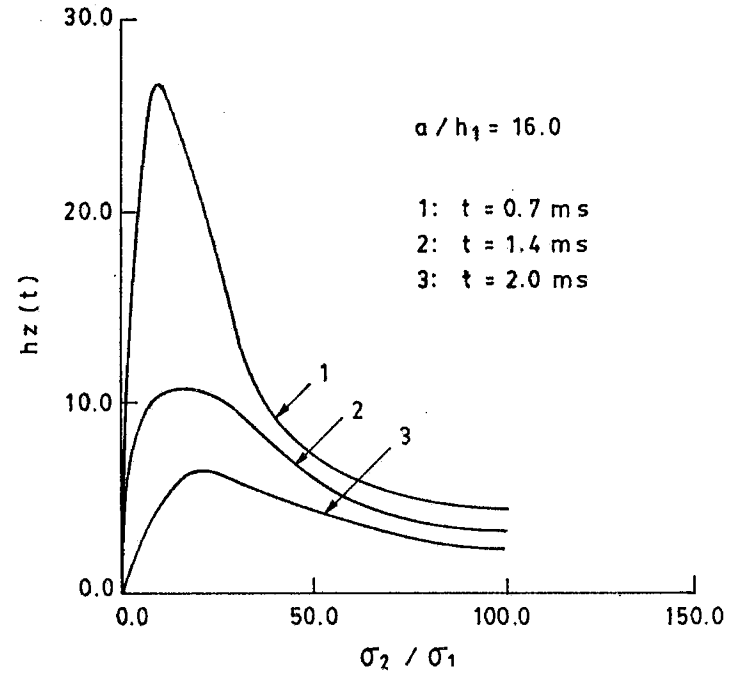

Fig. 8 — CFS response computed by Gauss-quadrature method for different time points with $a/h_1=16$.

we divide the interval [0, N] into a number of subintervals.

Step 2: If n is the total number of subintervals, then the above expression is approximated by

$$S=\frac{h}{3}[f_o+4\ (f_1+f_3+...)\ +2\ (f_2+f_4+...)\ +f_n], \tag{14}$$

where the subscript denotes the interval or iteration number and f_i is the value of the integral in that iteration computed using Simpson's rule.

Step 3: The value of S is found for each of the odd harmonics starting from 167 Hz.

Step 4: A Fourier summation of these values gives the time domain response at each time point.

RESULTS AND DISCUSSIONS

(A) A comparative study of the three tecniques

Model 1: For the two-layer earth model with the configuration a=25m; $\sigma_1=1$ S/m; $\sigma_2=3$ S/m; $h_1=6.25$m, as shown in Fig. 2, the responses are computed using all three techniques and presented in the same figure. Curve 1 shows the response using a digital linear filter, curve 2 represents the response using the Gauss-quadrature method and curve 3 shows the response using Simpson's rule. The responses and the normalised error norm as given by eqn. (15) are presented in Table 1 and 2 respectively.

$$error=\frac{1}{N}\sum_{i=1}^{N}(A_i-B_i), \tag{15}$$

where N=number of sample points, i.e, 5,

A_i, B_i=response at the i-th sample point obtained by two different methods.

Table 1 (a) — Time domain CFS response computed by the three techniques.

Sample point (msec)	Responses as computed by		
	Digital Linear Filter	Gauss Quadrature method	Simpson's rule
0.5	0.3500	0.3414	0.3737
1.0	0.0600	0.0610	0.0611
1.5	0.0250	0.0210	0.0210
2.0	0.0100	0.0110	0.0122
2.5	0.0075	0.0075	0.0070

Table 1 (b) — Error norm between responses obtained by the three techniques.

Normalised error norm between		
DLF & GQ method	DLF & Simpson's rule	GQ method & Simpson's rule
0.00212	0.00470	0.00682

From a close observation of the curves and the tables it is evident that amongst the two numerical integration techniques the Gauss-quadrature method gives a better result than Simpson's rule. The digital linear filter technique was used to generate master curves and it gives accurate results. So assuming it to be the reference method, the results obtained by the numerical integration techniques (Table 1 (a)) have been compared and the error norms calculated (Table 1 (b)). The error norm between the results obtained by the digital linear filter (DLF) and Gauss-quadrature (GQ) method is found to be the least, i.e., 0.00212. The error norm between the DLF and Gauss-quadrature results is 0.00470 and that between Simpson's rule and Gauss-quadrature method is 0.00682.

Model 2: For the two-layer earth model with the configurations a=25m; $\sigma_1=1$ S/m; $\sigma_2=0.1$ S/m; $h_1=12.5$m, as shown in Fig. 3, the time domain CFS responses were calculated by all three techniques as in the previous case. The response values at all the sample points and the normalised error norms have been tabulated in Tables 2(a) and 2(b) respectively.

For this model also, an inspection of the results reveals that the Gauss-quadrature technique for integration yields better results than Simpson's rule. These results were again compared with those obtained using filter theory. The normalised error between responses obtained using the DLF and Gauss-quadrature method is found to be the least, i.e, 0.00276 compared to 0.00334 between DLF and Simpson's rule.

(B) *(i) Effect of conductivity as computed by all the three techniques*

In Fig. 4 the physical configuration and parameters with the respective computed CFS response curves are presented. Fig. 4 shows the response curves obtained by DLF, the Gauss-quadrature method and Simpson's rule.

It is observed from Fig. 4 that the variation in resolution due to a change in layer conductivity is well reflected in the response curves, particularly for values of $\sigma_2 > \sigma_1$. The resolution of the curves is high for early and intermediate time points and completely lost for late time points. In other words, the decay at late times is slow with increase in second layer conductivity.

(ii) Effect of loop radius as computed by all three techniques

Fig. 5 shows the model parameters and the CFS time domain responses computed by the digital filter, the Gauss-quadrature method and Simpson's rule respectively. Responses have been computed for three different loop radii. The nature of variation in response is found to be independent of the loop radius. However, there is an increase in the magnitude with increased loop radius, and a slower decay rate at late times. This results in a very good resolution of

Table 2 (a) — Time domain CFS response computed by the three techniques.

Sample point (msec)	Responses as computed by		
	Digital Linear Filter	Gauss Quadrature method	Simpson's rule
0.5	0.2600	0.2500	0.2800
1.0	0.0150	0.0112	0.0109
1.5	0.0050	0.0050	0.0055
2.0	0.0030	0.0030	0.0030
2.5	0.0010	0.0010	0.0013

Table 2 (b) — Error norm between responses computed by the three techniques.

Normalised error norm between		
DLF & GQ method	DLF & Simpson's rule	GQ method & Simpson's rule
0.00276	0.00334	0.00610

curves at early and intermediate times and very poor resolution at late times. Fig. 5 shows a comparison of the results.

(iii) Effect of top-layer thickness

Fig. 6 shows the model parameters for a two-layer earth, and the time domain CFS response computed by using the Gauss-quadrature method of integration. A comparison of the results shown in Figs. 6, 7, 8 and 9 reflects that the resolution of the response curves due to layer thickness is significant at higher layer conductivity contrasts. The magnitude falls considerably with increase in top layer thickness. The response decays slowly at late times for $\sigma_2 > \sigma_1$ and with decrease in a/h_1, while for $\sigma_2 < \sigma_1$ and at larger value of a/h_1, an early time response saturation is observed.

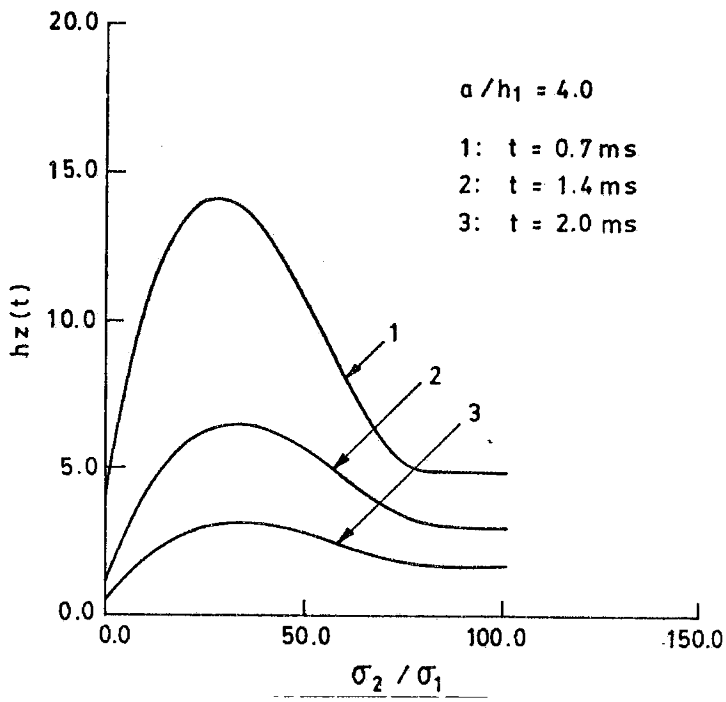

Fig. 9 — CFS response computed by Gauss-quadrature method for different time points with $a/h_1 = 4$.

The plots presented in Figs. 7-9 help in the interpretation of layered earth parameters. The variation in transient response due to layer conductivity contrast in the chosen range of a / h_1 is substantial, but the resolution becomes poor with decrease in a / h_1 at late times.

From a study of the responses computed for the models shown in Figs. 2, 3, 4 and 5 and Tables 1-2, it is evident that responses obtained using the Gauss-quadrature method and a digital linear filter are very close. Since digital filters have been used for the computation of the master curves and give accurate results, we assume this to be the best method, and compare any other technique with it.

For a routine analysis, the Gauss-quadrature method can be prescribed because of the less computational time and storage it requires compared to DLF. Although Simpson's rule gives fairly close results and requires less computation time, a compromise is made between accuracy and cost-effectiveness. The Gauss-quadrature method is extended for any future computation of time domain CFS response.

CONCLUSIONS

Previous studies indicate that CFS is applicable to geo-engineering and shallow hydrogeological problems. The multi-frequency response characteristics of CFS have established the utility of frequency domain sounding over a multi-layer earth. The present study shows the utility of the Gauss-quadrature algorithm in computing the time domain CFS response over a two-layer earth. This algorithm has been extended for computation of the response over a multi-layer earth, and the results will be published subsequently.

The variation in layer conductivity is well reflected in the time domain response curves for two-layer earth models, particularly at late times and at high conductivity contrasts between the first and second layers. The resolution of layer thickness in two-layer models is not so significant, although it is improved with the increase in second layer thickness. A larger loop radius resolves the layer parameters considerably better. These results are useful in resolving two-layer earth structures with conductive overburden when conventional multifrequency methods fail due to skin effect.

REFERENCES

Koenigsberger J.G.; 1939: *Elektrische Vertical-Sounding von der Erdoberflache aus mit der Zentral - Induktions Methode,* Beitrage Zur Angewandten Geophysik, **7**, 112-161.

Morrison H.P., Phillips R.J. and O'Brien D.P.; 1969: *Quantitative interpretation of transient electromagnetic fields over a layered half space,* Geophy. Prospect., **17**, 82-101.

Nelson P.H. and Morris D.B.; 1969: *Theoretical response of time domain EM system,* Geophysics, **34**, 729-738.

Patra H.P.; 1970: *Central frequency sounding in shallow engineering and hydrogeological problems,* Geophy. Prospect., **18**, 236-254.

Patra H.P.; 1976: *Electromagnetic depth sounding for groundwater with particular reference to CFS: Principles, interpretation and applications,* Geo-expl., **14**, 254-258.

Patra H.P.; 1978: *A three frequency computational method for two layer CFS data,* Boll. Geof. Teor. Appl., **21**, 35-45.

Patra H.P. and Mallick K.; 1980: *Geosounding Principles II, Time varying Geoelectric soundings,* Elsevier, Amsterdam, p. 92.

Patra H.P. and Shastri N.L.; 1982: *Relative performances of Central and Dipole frequency soundings over a layered earth,* PAGEOPH, **120**, 527-537.

Patra H.P. and Shastri N.L.; 1983a: *Computation of model curves for central frequency sounding by means of digital linear filter,* Boll. Geof. Teor. Appl., **25**, 119-130.

Patra H.P. and Shastri N.L.; 1983b: *Theoretical central frequency sounding response curves over a generalised three - layer earth,* PAGEOPH, **121**, 317-325.

Patra H.P. and Shastri N.L.; 1985: *Response characteristics of CFS over a multilayer earth,* Boll. Geof. Teor. Appl., **27**, 41-46.

Patra H.P. and Shastri N.L.; 1988: *Multifrequency sounding results of laboratory simulated homogenous and two-layer earth models,* IEEE Trans. on Geoscience and Remote Sensing, **26**, 749-752.

Sanyal N.; 1975: *Some studies on electromagnetic depth sounding for shallow groundwater exploration problems,* Ph.D thesis, IIT Kharagpur.

Yoshizumi E., Taniguchi K. and Kiyono T.; 1959: *Vertical electrical sounding by central ring induction method,* Mem. Fac. Eng. Kyoto Univ., **21**, 154-169.