

Quasigeoid comparisons over the French Auvergne test-bed using the Australian Curtin University and Swedish Royal Institute of Technology approaches

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Abstract

Since 2006, several different groups have computed geoid and/or quasigeoid (quasi/geoid) models for the Auvergne test area in central France using various approaches. In this contribution, we compute and compare quasigeoid models for the Auvergne test area using Curtin University of Technology's and the Swedish Royal Institute of Technology's approaches. These approaches differ in many ways, such as their treatment of the input data, choice of type of spherical harmonic model (combined or satellite-only), form and sequence of correction terms applied, and different modified Stokes's kernels (deterministic or stochastic). We have also compared our new results with most of the previously reported studies over Auvergne in order to seek any improvements with respect to time [exceptions are when different subsets of data have been used]. All studies considered here have compared the computed quasi/geoid models with the same 75 GPS-levelling heights over Auvergne. The standard deviation for almost all of the computations (without any fitting) is of the order of 30-40 mm, so there is not yet any clear indication whether any approach is necessarily better than any other nor improving over time. We also recommend more universal standardisation on the presentation of quasi/geoid comparisons with GPS-levelling data so that results from different studies over the same areas can be compared more objectively.

Keywords: Regional quasigeoid computation, Auvergne (France), technique comparison

Introduction

It is now over 170 years since George Gabriel Stokes published his seminal formula for geoid determination from gravity anomalies (Stokes 1849); over 55 years since the English translation of Mikhail Sergeevich Molodensky's book was published on the formula for quasigeoid determination from gravity anomalies (Molodensky et al. 1962); and over 50 years since Martin Hotine's monograph was published on the formula for geoid determination from gravity disturbances (Hotine 1969).

Despite this long-elapsed time, there remains no universal consensus on geoid and/or quasigeoid (quasi/geoid) computation. Arguably, different approaches are necessary in different parts of the world due to, for instance, peculiarities of the data holdings. However, there appears to be some subjectivity in the selection of the computation strategy. As just one example, the third author of this article admits preference for his deterministically modified kernel (Featherstone et al. 1998) for the computation of Australian and New Zealand national gravimetric quasigeoid models (Featherstone et al. 2001, 2011, 2018a; Amos and Featherstone 2009; Claessens et al. 2011). In his defence though, he has compared his kernel with other deterministic modifiers and some simplistic stochastic modifiers (e.g., Featherstone et al. 2004), hence the inclusion of the more sophisticated stochastic modifier embedded in the Swedish Royal Institute of Technology's (KTH) approach in the comparisons presented herein.

In attempts to reach some sort of consensus on quasi/geoid computation, two principal approaches have been endorsed historically by the International Association of Geodesy (IAG): synthetic and empirical.

The creation and use of synthetic gravity fields further comprise two variants. The first is to assume spherical harmonic coefficients of a high-degree Earth Gravitational Model (EGM) are error-free and use them to generate [assumed] self-consistent sets of gravity anomalies and quasi/geoid heights (e.g., Tziavos 1996; Novák et al. 2001; Featherstone 2002). The second synthetic approach is to use forward modelling of gravity anomalies and quasi/geoid heights from digital elevation models (DEMs) (e.g., Haagmans 2000; Kuhn and Featherstone 2003a, 2003b, 2005; Ågren 2004; Baran et al. 2006; Fellner et al. 2012; Vaníček et al. 2013).

Empirical study areas have been proposed in regions with reasonably good coverage and availability of gravity, topographic and GPS-levelling data, most notably Auvergne in central France (Duquenne 2006; Valtý et al. 2012) and Colorado in the USA (e.g., Wang et al. 2020; Claessens and Filmer 2020); Australia has been suggested (Featherstone et al. 2018b) but not yet used by others. In 2006, the French Institut Géographique National (IGN) provided a dataset of ~240,000 land gravity observations and 75 GPS-levelling points over a region surrounding Auvergne in central France along with two DEMs (Duquenne 2006). These two DEMs were later replaced by the SRTM 3" DEM. The Auvergne point gravity observation data are freely available from the Bureau Gravimétrique International (BGI) database.

Since 2006, several published studies have presented quasi/geoid computations for the Auvergne test-bed using several different techniques, which are summarised in Appendix A. We emphasise that the amount of information published on the agreements with the 75 GPS-levelling data is rather inconsistent and we discuss this further in Section 3. In particular, we observe that the reporting of descriptive statistics of the comparison with GPS-levelling data can be inconsistent, which arguably prevents an objective comparison among the different quasi/geoid computation techniques. As such, we present in the Electronic Supplementary Material (ESM) a spreadsheet that others may wish to adopt for a more standardised comparison.

Curtin University of Technology's (CUT) approach to compute the quasigeoid has not been used before for the Auvergne test-bed. In this study, therefore, we compare the CUT and KTH's techniques for quasigeoid modelling so as to add another "data point" to the Auvergne test-bed with a view to determining how well or not the CUT technique performs with respect to some other methods when using the same input and test data. We choose these two approaches only because they are so substantially different to one another.

2. Comparing and contrasting the CUT and KTH approaches

Both approaches have evolved over time, so we only report on their current status, but with some historical context.

2.1 The CUT approach

The CUT approach has evolved over around 25 years with particular focus on computing Australian models, though it has also been used in New Zealand, Colorado in the USA and the UK (the latter is unpublished). Probably the most Australia-specific aspect is the treatment of the terrestrial gravity data. Usually, refined Bouguer or isostatic gravity anomalies are recommended for gridding as they are smoother and thus more suited to interpolation. In Australia, however, the mean elevation is only ~330 m (max 2228 m) so topographic/isostatic corrections are small and planar simple Bouguer anomalies appear sufficient for interpolation and gridding (Goos et al. 2003; Zhang and Featherstone 2004). There is a side-benefit to this approach because it allows for the so-called reconstruction of Faye anomalies on the topography (Featherstone and Kirby 2000).

In short, point planar simple Bouguer anomalies [including the atmospheric correction from Moritz (1980; 2000)] are computed using a constant topographic bulk density of $2,670 \text{ kg m}^{-3}$. They are then interpolated to the $1'' \times 1''$ resolution of the Australian DEM using the tensioned spline algorithm (*surface* with $T=0.25$) embedded in the Generic Mapping Tools (GMT; Wessel et al. 2013). Molodensky free air anomalies are 'reconstructed' on the topography by applying a reverse planar simple Bouguer correction with the height of each DEM element. Faye gravity anomalies are computed by adding the planar terrain correction from the same DEM as an approximation of the Molodensky G_1 term, recently including error propagation (McCubbine et al. 2017, 2019). These are then block-averaged (GMT routine *blockmean*) to

determine surface-mean Faye gravity anomalies as approximations of Molodensky anomalies for subsequent quasigeoid computation.

The CUT approach has consistently used the highest-available degree of EGM, which is generally a combined model that has merged terrestrial and satellite-only coefficients (e.g., Pavlis et al. 2012, 2013). This is in contrast to the KTH approach that uses a satellite-only EGM so as to avoid correlations in the terrestrial data when used twice (e.g., Vaniček and Sjöberg 1991). The [implicit] rationale for the CUT approach is that, while being fully subject to the undesirable correlation of largely the same terrestrial data being used (most Australian gravity data are in the public domain), the use of a high-degree EGM makes the residual quasigeoid smaller in magnitude and thus less subject to approximation errors in the residual quasigeoid computation. A recent refinement to the treatment of the EGM is to compute ellipsoidal area-mean gravity anomalies on the topography (Featherstone et al. 2018a, Section 2.3.2).

The CUT approach to computing the residual quasigeoid from the residual area-mean gravity anomalies is based on the 1D-FFT (Haagmans et al. 1993) using F77 code that originated from the University of Calgary, Canada, but which has been adapted to include deterministically modified kernels (Featherstone and Sideris 1998; Featherstone 2003). It also now uses Gauss-Legendre quadrature to better-determine area-means for the deterministically modified kernels in the discretised numerical integration (Hirt et al. 2011). The Australian models use the deterministic Featherstone et al. (1998) kernel that is a combination of the Vaniček and Kleusberg (1997) and Meissl (1971) modifiers. This combined modifier aims to reduce the truncation error and improve the rate of convergence to zero of the series expansion of the truncation error. The integer degrees of kernel modification and integration cap radius are chosen empirically through comparisons with GPS-levelling after parameter sweeps versus GPS-levelling data. The ellipsoidal correction is handled by using the geocentric radius to the surface of the GRS80 ellipsoid in Stokes's integral along each parallel of latitude of the computation grid in the 1D-FFT (Claessens 2006, Chapter 6).

2.2 The KTH approach

The stochastically modified kernel used in KTH method comprises a least-squares combination of satellite and terrestrial data (Sjöberg 1981). Since then, the KTH method has been continuously developed and modified (e.g. Sjöberg 1984, 1991, 2003c; Ågren 2004 and references therein). The KTH method follows remove-interpolate-restore-compute strategy for the geoid computations, which contrasts with CUT method that follows an interpolate-remove-compute-restore strategy.

The primary uniqueness of the KTH method lies in the stochastic modification of Stokes's kernel and corrections to the gravity data. Unlike other methods, the direct and indirect effects needed to make the observations accordant with the geodetic boundary value problem are added as separate combined corrections to the approximate geoid estimates obtained using the Stokes integration with un-reduced gridded gravity data.

The KTH method has been used to compute the Swedish national quasigeoid (Ågren et al. 2009b), the Nordic Geodetic Commission 2015 quasigeoid (Ågren et al. 2016). The KTH approach has received much wider application than the CUT approach, with quasi/geoid models for the Baltic countries (Ellmann 2004), Iran (Kiamehr 2006), Tanzania (Ulotu 2009), Greece (Daras et al. 2010), Kazakhstan (Inerbayeva 2010), Sudan (Abdalla and Fairhead 2011), New Zealand (Abdalla and Tenzer 2011), central Turkey (Abbak et al. 2012), Moldova (Danila 2012), Saudi Arabia (Abdalla and Mogren 2015), Uganda (Ssengendo 2015), Poland (Kuczynska-Siehn et al. 2016), peninsular Malaysia (Pa'suya et al. 2019), Estonia (Ellmann et al. 2019) and Jilin province in China (Wu et al. 2020).

In the KTH treatment of the terrestrial gravity data, point free-air gravity anomalies are computed from the observed gravity values on the Earth's surface. These are then reduced point-wise by the long wavelength gravity anomalies from synthesising a satellite-only EGM, the high-frequency part of the topography is removed using Residual Terrain Modelling (RTM; Forsberg 1985) and the atmospheric effect applied to obtain residual point free-air gravity anomalies. These are then interpolated using Least Squares Collocation (LSC), in the *geogrid.f* module of the GRAVSOF package (Tscherning et al. 1992), to the resolution of the desired model to obtain a regular grid of residual gravity anomalies. Since the KTH

method uses un-reduced gravity anomalies, the contributions of the EGM, RTM and atmospheric effect are all computed at the nodes of the grid and restored to the interpolated residual gravity anomalies.

Following Sjöberg (1991, 2003c), approximate values of geoid undulations are computed from the un-reduced gridded gravity anomalies and EGM using the unbiased least squares geoid estimator. This makes use of a stochastic Stokes's modified kernel that simultaneously reduces the errors due to the truncation bias, satellite-only EGM coefficients and the terrestrial gravity data (Sjöberg 1984). Besides the choice of an integration cap radius, the most important step in the computation of approximate geoid in the KTH method is the determination of a-priori estimates of signal and error degree variances. These are necessary for the computation of a *better choice* of modification parameters to be used in the least-squares modification method. Similar to the CUT approach, the integration cap radius is chosen empirically based on parameter sweeps versus GPS-levelling data.

The Tscherning and Rapp (1974) model is generally preferred to compute the gravity signal degree variance. The error degree variance of the EGM gravity is computed from the published error estimates that accompany the EGM coefficients. The error degree variance of terrestrial gravity anomalies are assumed to be a combination of white noise and a reciprocal distance covariance model (Ågren 2004; Ågren and Sjöberg 2009b). The signal and the EGM error degree variances are further rescaled by some empirical factor to best depict the 'reality' of the study area. The stochastically modified Stokes's integral in the geoid estimator is evaluated using the 1D-FFT method (Haagmans et al. 1993), but it has not been modified to include Gauss-Legendre quadrature (cf. Hirt et al. 2011).

Next are the so-called additive corrections from the combined topographic effect (Sjöberg 2000, 2001), atmospheric effect (Sjöberg 1999; Sjöberg and Nahavandchi 2000), ellipsoidal shape of the Earth (Sjöberg 2003b) and downward continuation (Sjöberg 2003a; Ågren 2004), which are added to the approximate geoid to achieve the final geoid.

The KTH method has been designed primarily to compute a gravimetric geoid, but which is then converted to quasigeoid by adding the geoid-quasigeoid separation term (Sjöberg 1995, 2010). However, Sjöberg (2000) and Ågren et al. (2009b) show that if the combined topographic effects are not applied in the computations using the KTH method and if the downward continuation is also adjusted accordingly, the result will be a quasigeoid. This eliminates the need for computing the topographic effects and further correction terms to convert the geoid to quasigeoid. The latter is the approach that was taken in the computations reported in the following section.

3. Results and Discussion

Four separate quasigeoid models of Auvergne were computed at a grid resolution of $0.02^\circ \times 0.02^\circ$ using the CUT and KTH approaches. The computation area encompasses all 75 GPS-levelling points publicly available for validation. The KTH technique was used with the satellite-only DIR_R5 EGM (Bruinsma et al. 2013) up to spherical harmonic degree and order (d/o) 240. The CUT method is used with DIR_R5 to d/o 240 (so as to compare the results between the two methods), EGM2008 to d/o 360 (to compare the results from CUT method with previously published results using some other methods; see Appendix A), and EGM2008 to d/o 2190 (to show the CUT method as it has been used in Australia, New Zealand and the USA). The SRTM 3" x 3" DEM (Farr et al. 2007) is used in all models computed.

In previous studies over Auvergne (Appendix A), the results are presented either with and/or without applying some form of fitting surface. To be consistent with these other studies, we have provided our results with and without surface fitting, which are appended in Table A.1. In addition to simple descriptive statistics (minimum, maximum, mean and standard deviation) that are commonly used in most evaluations of gravimetric quasi/geoid models, we include the mean absolute error (MAE) and skewness, which are given in Table 1. We believe that these additional statistics are informative because the mean and standard deviation alone do not necessarily provide sufficient information to compare two or more methods, as shown later in this section.

The standard deviation alone gives the magnitude of the variation of differences but not the direction, which is better quantified by the skewness. The MAE measures the mean magnitude of differences that is not available in case of arithmetic mean values. Thus, MAE and skewness are necessary along with mean and standard deviation to have an overall estimate of the magnitude and direction of the

differences and their distribution. In Table 1, we also provide the coefficient of determination (R-squared) values for our four quasigeoids after-fitting as a measure of how well the four-parameter regression model explains the total variation of gravimetric quasigeoid with respect to the GPS-levelling points. The closer the R-squared value is to one, better is the regression model.

Moreover, to focus only on the computed quasi/geoid uncertainty, the effect of ellipsoidal and levelling height errors should be removed from the overall error estimate obtained with respect to the GPS-levelling data. The observed ellipsoidal are not correlated with the computed quasi/geoid, but the levelling will have [unknown] correlations if gravity observations have been observed at levelled benchmarks. Therefore, the quasi/geoid uncertainty (σ_N) before any fitting can be obtained using equation 1 (cf. Foroughi et al. 2019).

$$\sigma_N = \sqrt{(\sigma_{overall})^2 - (\sigma_h)^2 - (\sigma_H)^2} \quad 1$$

where σ_h and σ_H are the uncertainties of ellipsoidal and levelling heights, respectively, and $\sigma_{overall}$ is the standard deviation obtained on comparison wrt the ground data (e.g., Table A1).

However, a parametric-fitted quasi/geoid is correlated with the ellipsoidal and levelling heights (see equations for 4-parameter fit in the Electronic Supplementary Material). However, due to the unavailability of the corresponding covariance terms, the quasi/geoid uncertainty of the fitted model can also be computed using equation 1 (cf. Ågren and Sjöberg 2014; Sjöberg and Bagherbandi 2017; Ellmann et al.

2019). It is important to note here that equation 1 is valid iff $\sigma_{overall}$ is greater than $\sigma_{GPS/lev} (= \sqrt{\sigma_h^2 + \sigma_H^2})$.

This condition may not always be met. In this regard, the internally propagated errors from GPS data processing software can be 2-10 times overly optimistic, i.e., too small (Rothacher 2002). Therefore, one solution is to scale up the formally propagated ellipsoidal height errors, as has been done for the Australian data (Featherstone et al. 2019).

For the Auvergne GPS-levelling dataset, Duquenne (2006) had provided an approximate and blanket (not point by point) error estimate of ~20-30 mm for the ellipsoidal heights and 20 mm for the levelled heights. Therefore, the uncertainty of the four quasigeoids (before and after fit) in our study can only be computed if the corresponding $\sigma_{overall}$ is greater than 32 mm ($\sigma_{GPS/lev}$). From Table A.1, it is observed that this condition is true for all the quasigeoid with no fit but not for any quasigeoid after fit. Thus, we computed the uncertainties of the quasigeoid with no fit only using equation (1), and these are provided in Table 1.

We also provide results of relative fit of quasigeoids (Table 2) with respect to the tolerances for differential levelling (cf. Featherstone 2001). Testing for the relative fit of quasi/geoids can also be an analysis tool to investigate quasi/geoid gradients. This parameter is of more interest to land surveyors who use relative GNSS baselines and a quasi/geoid as a replacement for the more time-consuming differential levelling. Moreover, like the parameter-fitting, it also cancels the effect of almost constant zero-degree term (discussed later) irrespective of the choice of reference geopotential (W0) value.

Figures 1 and 2 show scatter plots of the relative difference (magnitude) of the four quasigeoid models before and after parameter fitting, respectively. The curved lines in the figure depict the maximum allowable misclose for first order (lower curve) and third order levelling (upper curve) for all the 2775 baselines computed using equation 2 with c equal to 4 and 12, respectively.

$$r = c\sqrt{d} \quad 2$$

where, r = standard uncertainty, in mm; c = empirically derived factor for a given 'order' of levelling; d = distance between stations, in km. The values adopted for c may vary among countries, and the levelling tolerances for different order levelling in France is unavailable to us, so we have used the values from Australian perspective (ICSM, 2007).

Table 1: Extended analysis of the computed quasigeoids with respect to 75 GPS/levelling data around Auvergne. L is the degree of kernel modification and ψ_0 is the integration cap radius.

	L	Ψ_0		MAE (m)	Skewness	R-squared	Quasigeoid uncertainty (m)
KTH (DIR_R5)	240	1°	No Fit	0.819	-0.312	--	0.016
			4P Fit	0.020	0.712	0.448	NA [#]
CUT (DIR_R5)	240	1°	No Fit	0.871	-0.358	--	0.012
			4P Fit	0.021	0.152	0.362	NA
CUT (EGM08 d/o 360)	360	1°	No Fit	0.982	-0.377	--	0.016
			4P Fit	0.020	0.151	0.445	NA
CUT (EGM08 d/o 2190) *	360	0.1°	No Fit	0.872	-0.413	--	0.018
			4P Fit	0.020	-0.017	0.481	NA

* This solution is almost independent of the modification degree parameter sweeps (analysed for L=20, 40, 60, 80, 120, 140, 180, 240, 360)

[#] Not applicable because $\sigma_{overall}$ (after fit) is smaller than $\sigma_{GPS/lev}$ (cf. Eq. 1 and the discussion after)

Table 2: Relative fit of quasigeoids over (75*74*0.5=) 2775 possible baselines around Auvergne

	L	Ψ_0		Min (m)	Max (m)	Mean (m)	StD (m)	MAE (m)	Skewness	Mean ppm
KTH (DIR_R5)	240	1°	No Fit	-0.166	0.189	-0.002	0.051	0.040	0.008	0.440
			4P Fit	-0.166	0.170	0.001	0.038	0.029	-0.282	0.354
CUT (DIR_R5)	240	1°	No Fit	-0.152	0.170	0.010	0.048	0.039	-0.114	0.440
			4P Fit	-0.138	0.155	0.002	0.039	0.031	-0.108	0.380
CUT (EGM08 d/o 360)	360	1°	No Fit	-0.150	0.177	0.014	0.049	0.041	-0.122	0.457
			4P Fit	-0.139	0.154	0.002	0.038	0.030	-0.048	0.374
CUT (EGM08 d/o 2190)	360	0.1°	No Fit	-0.159	0.181	0.014	0.050	0.042	-0.121	0.463
			4P Fit	-0.122	0.145	0.002	0.038	0.030	0.011	0.369

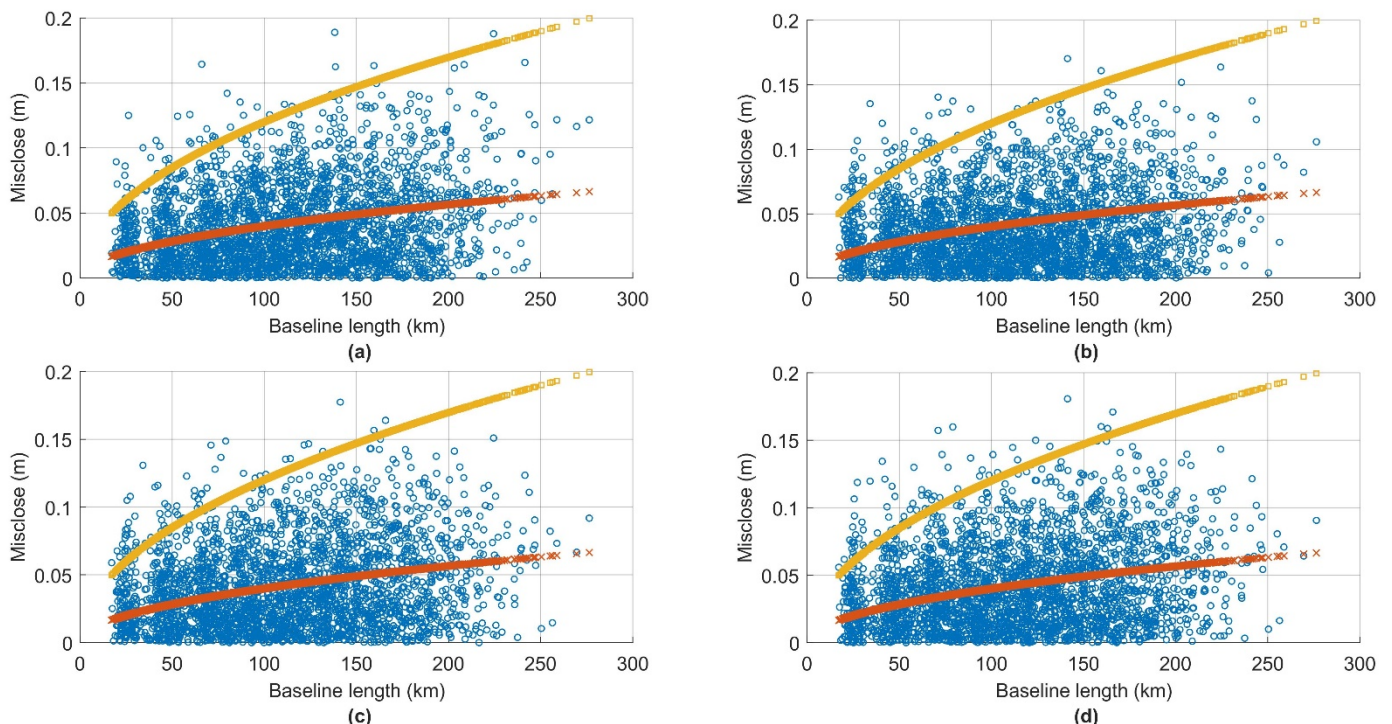


Figure 1: Magnitude of relative differences (circles) for the four quasigeoids before any fitting (a. KTH-DIR_R5, b. CUT-DIR_R5, c. CUT-EGM08_360, d. CUT-EGM08_2190) over 2775 baselines. Crosses and squares represent the maximum permissible error for Australian first order and third order levelling for each baseline, respectively.

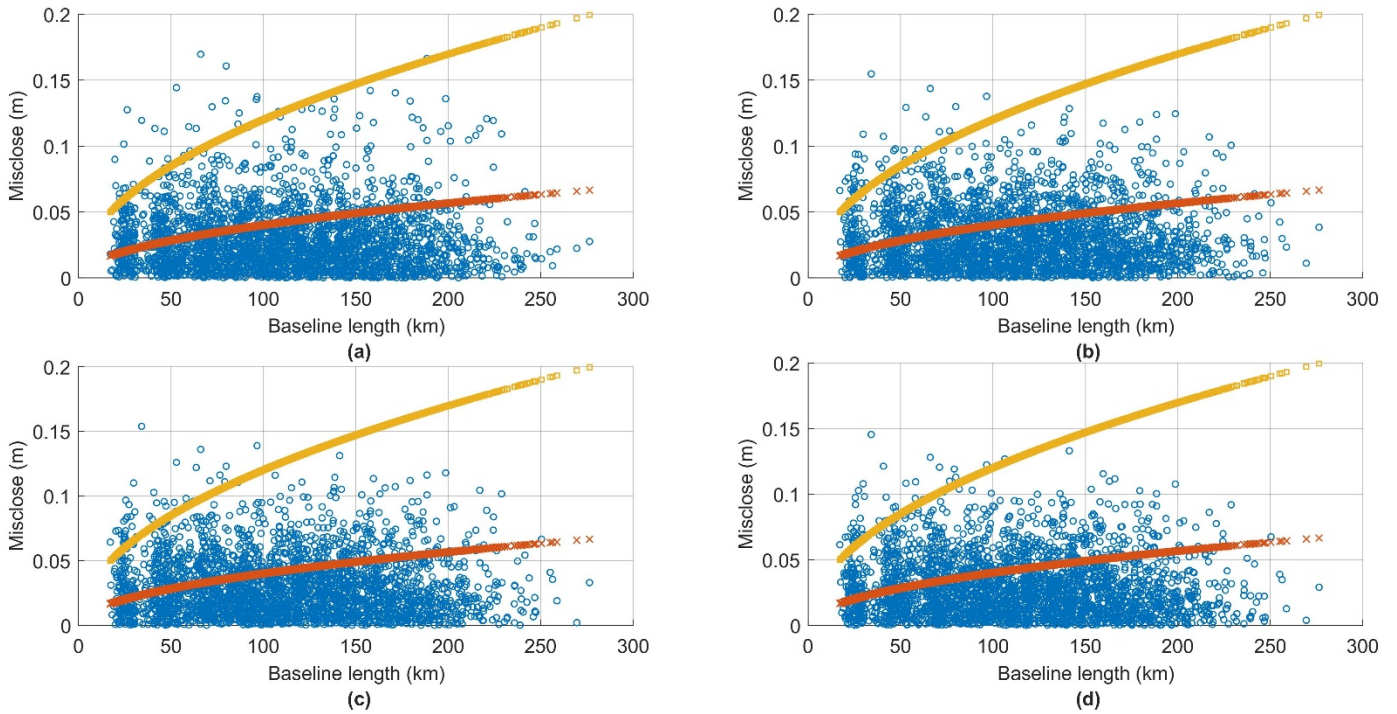


Figure 2: Magnitude of relative error (circles) for the four quasigeoids after four-parameter fitting (a. KTH-DIR_R5, b. CUT-DIR_R5, c. CUT-EGM08_360, d. CUT-EGM08_2190) over 2775 baselines. Crosses and squares represent the maximum permissible error for Australian first order and third order levelling for each baseline, respectively.

Following are our key observations from Tables 1, 2 and A.1, coupled with further discussion:

- a) For the solutions without any fitting of the computed quasi/geoid and the GPS-levelling data, the mean differences of approximately -133 mm and -184 mm by the UNB group and Duquenne (2006), respectively, are attributed to a vertical datum shift for France (Rülke et al. 2012). However, for the computations using both the methods here (KTH and CUT) and the KTH method in Yildiz et al. (2012), the mean difference between quasigeoid and GNSS/levelling is, on average, -863 mm. This is almost 730 mm greater in magnitude as compared to the other reported studies. This large difference is due to the inconsistency in the application of the zero-degree term (cf. Smith 1998) by different groups. While this will cancel when the quasi/geoid is used over baselines (cf. Featherstone 2001), it will not if used with single point positioning techniques (such as PPP). A practical solution if one does not choose to use a parametric fit, a one parameter fit can be used (i.e., a constant terms) to simultaneously absorb the zero-degree term and any constant offset in the local vertical datum.
- b) There are considerable differences among the results computed using the KTH method by Ågren et al. (2009a), Yildiz et al. (2012) and our current study. These differences can be attributed to three reasons: 1) the use of different EGMs (and degrees of modification); 2) different spatial resolutions of the computed quasigeoid (i.e. interpolation error); and 3) inconsistent or lack of reporting on the zero-degree term. Ågren et al. (2009a) use EIGEN_GL04C (d/o360) to compute a quasigeoid at a resolution of 1'x1' with no zero-degree term applied. Yildiz et al. (2012) use EGM2008 (d/o360) and computed a quasigeoid at resolution of 0.02°x0.025° with a zero-degree term applied. However, the results are presented after removing the mean value, so we are unable to distinguish what proportion is due to their value of the zero degree term and any constant offset in the French vertical datum. In this study, we used DIR_R5 (d/o 240) with a resolution of 0.02°x0.02°. Our zero-degree term was applied using the W_0 value used in the International Height Reference System

(Sánchez et al. 2016). We also used the GRS80 ellipsoid and scaled the even degree harmonics as per, e.g., Smith (1988).

- c) From Table A.1, we observe that for any method (KTH, UNB or GRAVSOFT), since 2006, there is no clear trend of improvement in the results without a corrector surface. Of all the studies (in Table A.1, with no surface fitting) the smallest standard deviation of 29 mm is obtained using the Radial Basis Function (RBF) method (Lin et al. 2019), while the Finite Element Method (FEM) method (Janák et al. 2014) provided the largest standard deviation of 97 mm. The KTH method has provided the smaller standard deviations of 24 mm (Yildiz et al. 2012), 25 mm (Abbak & Ustun 2015) and 26 mm (this study) after four-parameter, seven-parameter and four-parameter surface fitting, respectively. Utilising the high degree-order EGM2008 (d/o 2190), the CUT method (this study) also provided a standard deviation of 26 mm after four-parameter fitting.
- d) Different geoid modellers have had different views on whether more than a one-parameter model should be used during the GPS-levelling evaluation or not. One argument for this is that different permanent tide systems are used for the GPS ellipsoidal heights, levelled heights and terrestrial gravity data (cf. Poutanen et al. 1996; Ekman 1989). It is not mentioned in Duquenne (2006) that the corresponding data sets have been transformed to a common permanent tide system, which means that they most likely are in their default tide systems (e.g., non-tidal for RGF93-ETRS89, unknown for NGF-IGN69 and mean for IGSN71). This will result in a systematic tilt effect in the north-south direction with the magnitude of a few centimetres (Ekman 1989), which will be absorbed by a four-parameter surface. Based on this and the comments in part (a), we recommend that both one- and four-parameter fits are used in future Auvergne evaluations.
- e) Based on the standard deviation (σ) from Table A.1 with DIR_R5 EGM, the CUT method ($\sigma=34$ mm) appears marginally “better” than the KTH method ($\sigma=36$ mm) without any surface fitting, whereas the KTH method ($\sigma=26$ mm) is marginally “better” than the CUT method ($\sigma=27$ mm) after four-parameter surface fitting. We use the term marginal because of the blanket error budget used for the GPS-levelling and we are not at the ability to compute millimetre-precise quasigeoid models. However, for the same EGM, Table 1 shows that after surface fitting, the KTH method provides results which are significantly (~ 4.5 times) more positively skewed compared to the CUT method. Larger positive skewness represents asymmetrical distribution of differences with more values being clustered on the left tail of the distribution and therefore, a larger positive difference. The same pattern of results is also observed for the relative fit of the quasigeoids computed using the CUT and KTH methods (Table 2). We believe this is why the skewness is an additional and useful metric of quasi/geoids versus GPS-levelling.
- f) Figures 1 (a,b) and 2(a,b) show that the KTH method (with DIR_R5 EGM d/o 240) provides a larger number of baselines beyond 150 km that have misclosures greater than 150 mm and 100 mm, respectively, as compared to the CUT method. Moreover, the CUT method with EGM2008 (d/o 2190) after a four-parameter fit (Figure 2d) results in a misclosure of less than 100 mm for all baselines greater than 200 km. Hence, with the available data, the CUT method (as used for Australia and New Zealand) can be regarded as a “better” method for larger baselines compared to the KTH method, but we acknowledge that this may be because the French gravity data have been used in the construction of EGM2008 (Pavlis et al. 2012, 2013).

4. Conclusions and recommendations

In this study, quasigeoid models of Auvergne were computed using the CUT and KTH techniques and compared. The results were also compared with respect to previously published studies on quasi/geoid determination over Auvergne. The mean differences of ~ 730 mm among different techniques (e.g., CUT, KTH, UNB, GRAVSOFT/LSC) are mainly due to different treatments of the zero-degree term, but offsets in the French vertical datum cannot be eliminated as a candidate. Small (sub-centimetre) differences among

standard deviations can be due to, one some or all of, the choice of different EGMs, modification degrees, cap radii, DEMs, terrain corrections, quasi/geoid resolution and the gridding of the point anomaly data. However, all these terms are inseparable, so we are unable to point to any particular candidates. We, through our analysis, suggest that the practice of commenting on the pre-eminence of one method over other based on only standard deviation is not completely justified.

It is therefore recommended to establish some commonly adopted guidelines to define a statistical table for reporting the results of the quasi/geoid computations. A tentative list of statistical parameters can be adapted from Tables 1, 2 and A1. These will be important to 1) have an improved understanding of the “accuracy” of the method in use, and 2) more objectively compare the results with other computation approaches over the same region. This recommendation perhaps may be further taken up by either Sub-Commission 2.2 or 2.4 of the International Association of Geodesy.

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Yildiz et al. (2012)	RCR	quasigeoid	GEOCOL	EGM 2008 (360)	NA	1°	WG	RTM	SRTM 3"x3"	NA	NA	1P	-0.209	-0.075	-0.133	0.030
												4P	-0.067	0.058	0.000	0.029
	KTH	quasigeoid	Geolab	EGM 2008 (360)	NA	1°	LSM	Combined topographic effect	SRTM 3"x3"	no	no	1P	-1.118	-0.961	-1.040	0.035
												4P	-0.051	0.095	0.000	0.024
Janák et al. (2014)	FEM	quasigeoid	ANSYS	GOCE SGG/TIM-R1 (224)	NA	NA	NA	NA	SRTM 1"x1"	NA	NA	none	0.609	1.054	0.863	0.097
Abbak (2014)	KTH	geoid	NA	GGM05S (180)	NA	NA	LSM	Combined topographic effect	SRTM 3"x3"	yes	yes	4P	-0.068	0.073	- 0.0004	0.028 (RMS)
									ASTER 1"x1"			4P	-0.065	0.072	- 0.0003	0.028 (RMS)
Abbak & Ustun (2015)	KTH	geoid	LSMSSOFT	ITG-GRACE2010S (120)	NA	1°	LSM	Combined topographic effect	SRTM 3"x3"	yes	yes	1P	NA	NA	NA	0.099
												7P	-0.064	0.047	0.0005	0.025
Foroughi et al. (2017a)	UNB	geoid	NA (may be SHGeo)	DIR_R5 (140)	NA	0.75°	VK	DTE, PITE, SITE	NA (may be SRTM 3"x3")	yes	NA (may be yes)	none	range	0.163	--	--
				DIR_R5 (160)	NA	0.75°	VK					none	--	--	--	0.033
Foroughi et al. (2017b)	UNB	geoid	NA (may be SHGeo)	DIR_R5 (160)	NA	0.75°	VK	DTE, PITE, SITE	SRTM 3"x3"/ density model	yes	yes	none	---	---	0.133	0.033
	KTH	quasigeoid	NA (may be Geolab)	NA	NA	NA	LSM	Combined topographic effect	SRTM 3"x3"	yes	yes	none	0.021	0.213	0.125	0.034
Janák et al. (2017)	UNB	geoid	NA (may be SHGeo)	DIR_R5 (160)	NA	0.75°	VK	DTE, PITE, SITE	SRTM 3"x3"; ACE2; JGP95	yes	yes	none	0.028	0.207	0.124	0.034

									SRTM 3"x3"; ACE2; JGP95; density model			none	0.024	0.222	0.133	0.033
Foroughi et al. (2019)	UNB	geoid	NA (may be SHGeo)	DIR_R5 (140)	NA	1°	VK	DTE, PITE, SITE	ACE2; density model	yes	yes	none	-0.030	-0.190	-0.130	0.033
Goli et al. (2019)	UNB	geoid	NA (may be SHGeo)	GOCO05S (280)	NA	1.5°	VK	DTE, PITE, SITE	SRTM 3"x3"; SRTM 30"x30"	NA	NA	none	0.043	0.206	0.124	0.034
			NA				Modified VK					none	0.046	0.205	0.124	0.033
Lin et al. (2019)	RBF (Free positioned point mass)/RCR	quasigeoid	GRAVSOF	EGM 2008 (360)	NA	NA	NA	RTM	SRTM 3"x3"	NA	NA	none	-0.187	-0.045	-0.133	0.029
	RBF (Fixed positioned point mass)/RCR												-0.197	-0.050	-0.134	0.030
This Study	KTH	quasigeoid	Geolab	DIR_R5 (240)	yes	1°	LSM	Not required	SRTM 3"x3"	yes	yes	none	-0.922	-0.734	-0.819	0.036
												4P	-0.061	0.108	0.000	0.026
	CUT	quasigeoid	FFT1Dmod	DIR_R5 (240)	yes	1°	FEO	Terrain correction	SRTM 3"x3"	yes	yes	none	-0.958	-0.788	-0.871	0.034
												4P	-0.067	0.086	0.000	0.027
	CUT	quasigeoid	FFT1Dmod	EGM 2008 (360)	yes	1°	FEO	Terrain correction	SRTM 3"x3"	yes	yes	none	-0.988	-0.811	-0.892	0.036
												4P	-0.070	0.083	0.000	0.027
	CUT	quasigeoid	FFT1Dmod	EGM 2008 (2190)	yes	0.1°	FEO	Terrain correction	SRTM 3"x3"	yes	yes	none	-0.974	-0.794	-0.873	0.036
												4P	-0.075	0.069	0.000	0.026

Approach:

RCR= Remove-Compute-Restore

LSC= Least Squares Collocation

KTH=Royal Institute of Technology's method

UNB= University of New Brunswick's method

RBF= Radial Basis Function

FEM= Finite Element Method

CUT= Curtin University of Technology's method

Kernel modification:

WG= Wong-Gore (Wong and Gore 1969)

VK= Vaníček-Kleusberg (Vaníček and Kleusberg 1987)

Modified VK= Modified Vaníček-Kleusberg (Novák 2003)

LSM= Least-squares modification (Sjöberg 1984, 1991)

FEO=Featherstone-Evans-Olliver (Featherstone et al. 1998)