Bearing-based Formation Control and Network Localization via Global Orientation Estimation

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***Abstract*— Consider a system of single-integrator agents in the *d*-dimensional space whose goal is to achieve a target formation shape specified by some desired bearing vectors. Suppose that the agents do not share a common global reference frame but can sense the relative orientations and bearing information with regard to some neighbor agents. We propose solutions to two problems: global orientation estimation and bearing-only measurement based formation control. Combining the solutions to the two problems, we solve the distributed bearing-based formation control problem. The combined strategy guarantees the desired formation shape is almost globally achieved up to a translation, a rotation and a scaling. Furthermore, we apply the proposed strategy to bearing-based network localization as a dual problem.**

I. INTRODUCTION

Recently, bearing-based formation control and network localization have attracted lots of research interests [1], [2]. Consider a group of multi-rotor drones flying in a formation. Bearing-only algorithms open the possibility of reducing the network's cost by reducing the number of required sensors, which in turn could free up the drones' payloads for other tasks. Additionally, the bearings can be obtained from an onboard camera, which is passive and transmits no signal. As a result, bearing-based algorithms provide an effective solution for security demanding applications.

In the literature, bearing information can be refer to either the bearing vector (unit directional vector to a target) or the subtended angle between two vectors [1]. Since the subtended bearing angle is independent of any coordinate reference frame, many early works studied bearing-only network localization and formation control based on the latter information, see for examples [3]–[5] and the references therein. However, existing results working with subtended bearing angles are restricted to two-dimensional space. On the other hand, with the assumption that all bearing vectors are expressed in a common reference frame, a bearing rigidity theory in $\mathbb{R}^d$ has been proposed in [6]. Using the notions of infinitesimally bearing rigidity, bearing-only formation control and bearing-based network localization have been studied [6], [7]. However, a critical assumption on almost works on bearing vector-based formation control is that all agents/nodes in the system have access to a common reference frame or having onboard compasses [2], [8], [9]. When the agents do not share a common reference frame, an orientation alignment step has been introduced to the formation control law in [10], [11]. However, the orientation alignment algorithm requires some strict conditions on the initial orientations. Additionally, controlling local orientation and linear velocity of the agent simultaneously is not practical for multi-rotor systems. These weaknesses motivate us to propose a distributed bearing-based formation control strategy via global orientation estimation [12]. We combine the orientation estimation step to formations with leader-first follower formation graphs [9], [13]. The orientation estimation step requires exchanging of a few auxiliary variables and can be considered as an input to the bearing-based formation control system, thus reducing the difficulty in the analysis. Under the proposed control strategies, the desired formation shape can be achieved up to a translation, a rotation, and a scale. As a dual problem to formation control, a bearing-based network localization problem is also formulated and similarly analyzed.

The rest of this paper is organized in the following manner. In Section II, we revisit relevant results from algebraic graph theory and bearing rigidity theory. The bearing-based formation control and network localization problems are stated in Section III. In Section IV, we propose the control/estimation strategies and provide the main analytical results. Section V gives simulation results and Section V concludes the paper.

Notations. For a matrix $\boldsymbol{A} \in \mathbb{R}^{d\times d}$, $\boldsymbol{A}^T$ is the transpose of $\boldsymbol{A}$. Let $\mathcal{N}(\boldsymbol{A})$ and $\mathcal{C}(\boldsymbol{A})$ denote $\boldsymbol{A}$'s nullspace and column space, respectively. Then, $\mathcal{C}(\boldsymbol{A}) = \mathcal{N}(\boldsymbol{A}^T)^{\perp}$, where $\mathcal{N}(\boldsymbol{A}^T)^{\perp}$ is the orthogonal space of $\mathcal{N}(\boldsymbol{A}^T)$.

II. BACKGROUND

A weighted graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ consists of a vertex set $\mathcal{V} = \{1, \ldots, n\}$, an edge set $\mathcal{E} = \{e_{ij} = (i,j) |\ i,j \in \mathcal{V}, i \neq j\}$ and a weight set $\mathcal{A} = \{a_{ij} > 0 |\ (i,j) \in \mathcal{E}\}$. If $\mathcal{G}$ is directed, an edge $(i,j) \in \mathcal{E}$ means that only vertex i is connected to vertex j but not vice verse. If $\mathcal{G}$ is undirected, then $(i,j) \in \mathcal{E}$ implies that $(j,i) \in \mathcal{E}$. If j is connected to i, we call j a neighbor of i. Further, we denote $\mathcal{N}_i = \{j \in \mathcal{V} |\ (i,j) \in \mathcal{E}\}$ as the neighbor set of i. A (directed) path is a sequence of distinct vertices $i_1 i_2 \ldots i_l$ such that $(i_k, i_{k+1}) \in \mathcal{E},\ \forall k = 1, \ldots, l-1$. If there is at least one node having directed paths to every other nodes, $\mathcal{G}$ is said to be rooted out-branching. The Laplacian matrix $\boldsymbol{L}(\mathcal{G}) = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with the graph $\mathcal{G}$ is defined by $l_{ij} = -a_{ij}$, for $i \neq j$, and $l_{ii} = \sum_{j=1}^{n} a_{ij}$, where a_{ij}s are the weights of edges of the graph.

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Consider the consensus protocol, written in vector form

$$\dot{\boldsymbol{x}}(t) = -(\boldsymbol{L} \otimes \boldsymbol{I}_d)\boldsymbol{x}(t), \tag{1}$$

where $\boldsymbol{x} = [\boldsymbol{x}_1^T, \ldots, \boldsymbol{x}_n^T]^T, \boldsymbol{x}_i \in \mathbb{R}^d$, $\boldsymbol{I}_d$ is the $d \times d$ identity matrix, and $\otimes$ denotes the Kronecker product. It is well known that if $\mathcal{G}$ is rooted out-branching, the state $\boldsymbol{x}(t)$ converges to a finite point in the equilibrium set $\mathcal{Q} = \{\boldsymbol{x} = [\boldsymbol{x}_1^T, \ldots, \boldsymbol{x}_n^T]^T \in \mathbb{R}^{dn} | \ \boldsymbol{x}_i = \boldsymbol{x}_j, \forall i, j \in \mathcal{V}\}$ of (1) exponentially fast [14].

A framework in a d-dimensional space is denoted by $\mathcal{G}_b(\boldsymbol{p})$. Here, $\mathcal{G}_b = (\mathcal{V}_b, \mathcal{E}_b)$ is a directed graph of $|\mathcal{V}_b| = n$ vertices and $|\mathcal{E}_b| = m$ edges, and $\boldsymbol{p} = [\boldsymbol{p}_1^T, \ldots, \boldsymbol{p}_n^T] \in \mathbb{R}^{dn}$ is a realization of $\mathcal{G}_b$ in the global reference frame ${}^g\Sigma$. Denote $\mathcal{N}_i = \{j \in \mathcal{V}_b | \ (i,j) \in \mathcal{E}_b\}$ as the neighbor set of vertex i. For each edge $(i,j) \in \mathcal{E}_b$, we define a corresponding displacement vector $\boldsymbol{e}_{ij} = \boldsymbol{p}_j - \boldsymbol{p}_i$. For an arbitrary labeling of the edges in $\mathcal{E}_b$, let $\boldsymbol{H}_b \in \mathbb{R}^{m \times n}$ denote the corresponding incidence matrix. The stacked displacement vector[1] is defined as $\boldsymbol{e} = [\boldsymbol{e}_1^T, \ldots, \boldsymbol{e}_m^T]^T = (\boldsymbol{H}_b \otimes \boldsymbol{I}_d)\boldsymbol{p} = \bar{\boldsymbol{H}}_b \boldsymbol{p} \in \mathbb{R}^{dm}$.

For $\boldsymbol{p}_i \neq \boldsymbol{p}_j$, the bearing vector $\boldsymbol{b}_{ij}$ is the unit vector from $\boldsymbol{p}_i$ to $\boldsymbol{p}_j$, i.e., $\boldsymbol{b}_{ij} = \boldsymbol{p}_j - \boldsymbol{p}_i / \|\boldsymbol{p}_j - \boldsymbol{p}_i\| = \boldsymbol{e}_{ij}/\|\boldsymbol{e}_{ij}\|$. The orthogonal projection matrix corresponding to $\boldsymbol{b}_{ij}$ is $\boldsymbol{P}_{\boldsymbol{b}_{ij}} = \boldsymbol{I}_d - \boldsymbol{b}_{ij}\boldsymbol{b}_{ij}^T$. Note that $\boldsymbol{P}_{\boldsymbol{b}_{ij}}$ is symmetric, idempotent, positive semidefinite ($\boldsymbol{P}_{\boldsymbol{b}_{ij}} = \boldsymbol{P}_{\boldsymbol{b}_{ij}}^T = \boldsymbol{P}_{\boldsymbol{b}_{ij}}^2 \geq 0$), and $\mathcal{N}(\boldsymbol{P}_{\boldsymbol{b}_{ij}}) = span\{\boldsymbol{b}_{ij}\}$.

With the same labeling of edges in $\mathcal{E}_b$, we denote the stacked bearing vector $\boldsymbol{b} = [\boldsymbol{b}_1^T, \ldots, \boldsymbol{b}_m^T]^T \in \mathbb{R}^{dm}$. A directed graph is called a leader-first follower (LFF) graph if it is built up from a bearing-based Henneberg construction [13]. In a LFF graph, there is a leader (vertex 1) having no neighbor, a first follower (vertex 2) with one directed edge $(2,1)$, and the other are followers (vertices $i = 3, \ldots, n$) with two directed edges (i,j) with $1 \leq j < i$ [13]. Figure 2c shows an example of the LFF graph.

III. Problem formulation

Consider a group of n agents with single integrator dynamics in $\mathbb{R}^d$:

$$\dot{\boldsymbol{p}}_i^i = \boldsymbol{u}_i^i, \ i = 1, \ldots, n, \tag{2}$$

where $\boldsymbol{p}_i^i \in \mathbb{R}^d$ is the position and $\boldsymbol{u}_i^i \in \mathbb{R}^d$ is the control input of agent i written in its local reference frame ${}^i\Sigma$. Suppose that the local reference frame ${}^i\Sigma$ is rotated from the global reference frame ${}^g\Sigma$ by a rotation matrix $\boldsymbol{R}_i \in \mathrm{SO}(d)$ as depicted in Fig. 1. It holds that $\boldsymbol{R}_i^T = \boldsymbol{R}_i^{-1}$, $\boldsymbol{R}_i^T\boldsymbol{R}_i = \boldsymbol{I}_d$, and $\det(\boldsymbol{R}_i) = +1$.

We assume further that each agent i ($i = 1, \ldots, n$) can sense the relative orientations and exchange information with its neighbor $j \in \mathcal{N}_i$. Specifically, if j is a neighbor of i, then i senses the relative orientation of j's local reference frame with regard to the i's local reference frame $\boldsymbol{R}_{ij}$, which can be written as:

$$\boldsymbol{R}_{ij} = \boldsymbol{R}_i^{-1}\boldsymbol{R}_j = \boldsymbol{R}_i^T\boldsymbol{R}_j. \tag{3}$$

[1]For an edge $e_k = (i,j) \in \mathcal{E}_b$, we use $\boldsymbol{e}_{ij}$ and $\boldsymbol{e}_k$ interchangeable in the paper to denote the corresponding displacement vector.

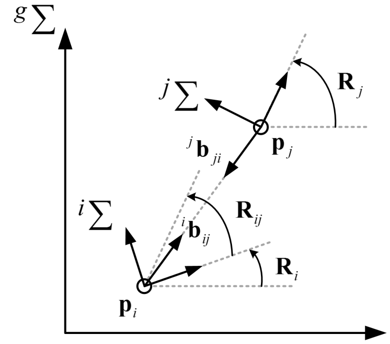


Fig. 1: Example: the orientation and bearing sensing topologies between agents i and j in 2D. Here, i senses both relative orientation $\boldsymbol{R}_{ij}$ and $\boldsymbol{b}_{ij}^i$ while j senses only $\boldsymbol{b}_{ji}^j$.

The relative orientation information might be obtained by analyzing images of the neighbor agent using vision based techniques [2], [11]. From these relative orientation information, each agent i tries to estimate its own orientation $\boldsymbol{R}_i$ with respect to the global reference frame. This estimation is characterized by the matrix $\hat{\boldsymbol{R}}_i \in SO(d)$. The directed graph $\mathcal{G}_o = (\mathcal{V}_o, \mathcal{E}_o, \mathcal{A}_o)$ characterizes the (relative) orientation sensing between agents in the system. Note that $\mathcal{V}_o = \{1, \ldots, n\}$, $\mathcal{E}_0 \subseteq \mathcal{V}_o \times \mathcal{V}_o$ and $\mathcal{A}_o$ is a set of positive scalar weights a_{ij} corresponding to each edge $(i,j) \in \mathcal{E}_o$. Let $\boldsymbol{L}_o \in \mathbb{R}^{n \times n}$ be the graph Laplacian of $\mathcal{G}_o$. We make the following assumption on the orientation sensing graph:

Assumption 1: The orientation sensing graph $\mathcal{G}_o$ is rooted out-branching.

The first problem studied in this paper is estimating global orientation.

Problem 1: Consider an n agent system whose orientation sensing graph satisfies Assumption 1. For a common orientation which is identified by $\boldsymbol{R}^* \in SO(d)$, design an estimation law such that $\boldsymbol{R}_i\hat{\boldsymbol{R}}_i \to \boldsymbol{R}^*, \forall i \in 1, \ldots, n$ asymptotically, based on measurements of relative orientation (3).

We can now set up the bearing-based formation control problem. Suppose that each agent can sense the bearing vector toward each of its neighbor agents. The bearing sensing graph is given by a graph $\mathcal{G}_b = (\mathcal{V}_b, \mathcal{E}_b)$, with $\mathcal{V}_b = \{1, \ldots, n\}$ and $\mathcal{E}_b \subseteq \mathcal{V}_b \times \mathcal{V}_b$. If two agents i and j are neighbors in $\mathcal{G}_b$ and their positions are not collocated, agent i can sense the bearing vector

$$\boldsymbol{b}_{ij}^i = \frac{\boldsymbol{p}_j^i - \boldsymbol{p}_i^i}{\|\boldsymbol{p}_j^i - \boldsymbol{p}_i^i\|} = \boldsymbol{R}_i^T \frac{\boldsymbol{p}_j - \boldsymbol{p}_i}{\|\boldsymbol{p}_j - \boldsymbol{p}_i\|} = \boldsymbol{R}_i^T\boldsymbol{b}_{ij}, \tag{4}$$

in ${}^i\Sigma$, and agent j can sense $\boldsymbol{b}_{ji}^j = \boldsymbol{R}_j^T\boldsymbol{b}_{ji}$ in ${}^j\Sigma$. In equation (4), $\boldsymbol{p}_i, \boldsymbol{p}_j \in \mathbb{R}^d$ are the position of two agents i and j in ${}^g\Sigma$, respectively. It is observed that the *local* bearing vectors obtained by i and j are generally different, i.e., $\boldsymbol{b}_{ij} = -\boldsymbol{b}_{ji}$ while $\boldsymbol{b}_{ij}^i \neq -\boldsymbol{b}_{ij}^j$. The target formation shape is given by a set of desired bearing vectors $\mathcal{B} = \{\boldsymbol{b}_{ij}^*\}_{(i,j)\in\mathcal{E}_b}$ expressed in global reference frame ${}^g\Sigma$. Each agent i is given all $\boldsymbol{b}_{ij}^*$ for all $j \in \mathcal{N}_i$ and can sense all bearing vectors $\boldsymbol{b}_{ij}^i$. We make the following assumptions on the set of desired bearing vectors and the target formation.

Assumption 2: The set $\mathcal{B}$ is feasible, i.e., there exists a configuration $\boldsymbol{p}^* \in \mathbb{R}^{dn}$ such that $(\boldsymbol{p}_j^* - \boldsymbol{p}_i^*)/\|\boldsymbol{p}_j^* - \boldsymbol{p}_i^*\| = \boldsymbol{b}_{ij}^*$, for all $\boldsymbol{b}_{ij}^* \in \mathcal{B}$.

Assumption 3: $\mathcal{G}_b$ is a leader-first follower graph built-up from a bearing-based Henneberg construction [13].

The bearing-based formation control problem can be formally stated as follows:

Problem 2: Consider the system of n agents with dynamics (2). Under Assumptions 1–3 and for a common reference frame ${}^c\Sigma$ identified by $\boldsymbol{R}^* \in \text{SO}(d)$, design a control strategy for each agent using only local bearings and relative orientation information such that the system almost globally asymptotically achieves a target formation shape up to a translation, a scale and a rotation identified by $\boldsymbol{R}^*$.

In other words, the objective of Problem 2 is achieving a formation with $(\boldsymbol{p}_j - \boldsymbol{p}_i)/\|\boldsymbol{p}_j - \boldsymbol{p}_i\| \to \boldsymbol{R}^* \boldsymbol{b}_{ij}^*, \forall (i,j) \in \mathcal{E}_b$, as $t \to \infty$.

We next formulate the bearing-based localization problem. Consider a network of n stationary nodes in $\mathbb{R}^d$, whose locations and sensing graphs satisfy Assumptions 1 and 3. The nodes in the network do not have a common reference frame. Through the interaction graph $\mathcal{G}_b$, we assume that each node i estimates its position $\hat{\boldsymbol{p}}_i$ and communicates its estimation with its neighbors. Thus, each node i senses or receives information about $\boldsymbol{R}_{ij}$, $\boldsymbol{b}_{ij}^i$, $\hat{\boldsymbol{p}}_j$ for all $j \in \mathcal{N}_i$. Let $\hat{\boldsymbol{p}} = [\hat{\boldsymbol{p}}_1^T, \ldots, \hat{\boldsymbol{p}}_n^T]^T$, we can now formulate the bearing-based localization problem as follows:

Problem 3: Consider the network of n nodes satisfying Assumptions 1–3. For a common reference frame ${}^c\Sigma$ identified by $\boldsymbol{R}^* \in \text{SO}(d)$, design a distributed estimation law such that the agents can localize the network in ${}^c\Sigma$ up to a translation, a scale, and a rotation identified by $\boldsymbol{R}^*$, or i.e., estimating $\hat{\boldsymbol{p}} \in \mathbb{R}^{dn}$ such that $(\hat{\boldsymbol{p}}_j - \hat{\boldsymbol{p}}_i)/\|\hat{\boldsymbol{p}}_j - \hat{\boldsymbol{p}}_i\| \to (\boldsymbol{R}^*)^T \boldsymbol{b}_{ij}, \forall (i,j) \in \mathcal{E}_b$.

IV. Main results

A. Orientation estimation

Consider Problem 1. We define the matrix $\boldsymbol{Q}_i \in SO(d)$ as

$$\boldsymbol{Q}_i := \boldsymbol{R}_i^T \boldsymbol{X}, \ \forall i \in \mathcal{V}_o, \tag{5}$$

where $\boldsymbol{X} \in SO(d)$ and $\boldsymbol{X}$ is a constant matrix. According to (3), it follows that

$$\boldsymbol{Q}_i = \boldsymbol{R}_{ij} \boldsymbol{Q}_j, \ \forall (i,j) \in \mathcal{E}_o. \tag{6}$$

Finding the steady-state solution of $\hat{\boldsymbol{R}}_i$ in Problem 1 is equivalent to agent i finding a $\boldsymbol{Q}_i$ satisfying the equality (6). Then, Problem 1 can be restated as follows:

Problem 4: Under Assumption 1, design an algorithm for $\boldsymbol{Q}_i : [0, \infty) \to \mathbb{R}^{d \times d}$ such that

- $\boldsymbol{Q}_i(t) \in SO(d), \forall t \in [0, \infty)$,
- $\boldsymbol{Q}_i(t) \to \boldsymbol{R}_{ij} \boldsymbol{Q}_j(t)$, as $t \to \infty, \forall (i,j) \in \mathcal{E}_o$.

Suppose that each agent i generates d auxiliary variables $\boldsymbol{z}_{i,k} \in \mathbb{R}^d, \ k = 1, \ldots, d$. Let $\boldsymbol{Q}_i$ be represented as $\boldsymbol{Q}_i = [\boldsymbol{q}_{i,1}, \boldsymbol{q}_{i,2}, \ldots, \boldsymbol{q}_{i,d}]$ where $\boldsymbol{q}_{i,k} \in \mathbb{R}^d, \ k = 1, \ldots, d$, is a column vector of $\boldsymbol{Q}_i$. The orthonormal column vectors of $\boldsymbol{Q}_i$ can be constructed based on the following Gram-Schmidt process with any independent vectors $\boldsymbol{z}_{i,k}, \ k = 1, \ldots, d$, as follows:

$$\begin{aligned} \boldsymbol{v}_{i,1} &:= \boldsymbol{z}_{i,1}, & \boldsymbol{q}_{i,1} &:= \tfrac{\boldsymbol{v}_{i,1}}{\|\boldsymbol{v}_{i,1}\|}, \\ \boldsymbol{v}_{i,2} &:= \boldsymbol{z}_{i,2} - \langle \boldsymbol{z}_{i,2}, \boldsymbol{q}_{i,1} \rangle \boldsymbol{q}_{i,1}, & \boldsymbol{q}_{i,2} &:= \tfrac{\boldsymbol{v}_{i,2}}{\|\boldsymbol{v}_{i,2}\|}, \\ &\vdots & &\vdots \\ \boldsymbol{v}_{i,d} &:= \boldsymbol{z}_{i,d} - \textstyle\sum_{k=1}^{d-1} \langle \boldsymbol{z}_{i,d}, \boldsymbol{q}_{i,k} \rangle \boldsymbol{q}_{i,k}, & \boldsymbol{q}_{i,d} &:= \sigma \tfrac{\boldsymbol{v}_{i,d}}{\|\boldsymbol{v}_{i,d}\|}, \end{aligned} \tag{7}$$

where $< \cdot, \cdot >$ denotes the inner product, and

$$\sigma := \text{sign}\left(\det \left(\left[\boldsymbol{q}_{i,1}, \ldots, \boldsymbol{q}_{i,d-1}, \frac{\boldsymbol{v}_{i,d}}{\|\boldsymbol{v}_{i,d}\|} \right] \right) \right). \tag{8}$$

Note that σ is chosen to ensure that $\det(\boldsymbol{Q}_i) = +1$ and subsequently $\boldsymbol{Q}_i \in SO(d)$. As a result of this process, the first goal of Problem 4 is guaranteed to be satisfied.

Consider now the second goal of Problem 4. We propose an estimation law for the auxiliary variables as follows:

$$\dot{\boldsymbol{z}}_{i,k}(t) = \sum\nolimits_{j \in \mathcal{N}_i} a_{ij} (\boldsymbol{R}_{ij} \boldsymbol{z}_{j,k}(t) - \boldsymbol{z}_{i,k}(t)), \tag{9}$$

where $\boldsymbol{R}_{ij} = \boldsymbol{R}_i^T \boldsymbol{R}_j$ is available from relative orientation measurements. Note that each agent i is simultaneously running d estimation laws for the d auxiliary variables $\boldsymbol{z}_{i,1}, \ldots, \boldsymbol{z}_{i,d}$. By letting $\boldsymbol{z}_k(t) = [\boldsymbol{z}_{1,k}^T, \ldots, \boldsymbol{z}_{n,k}^T]^T \in \mathbb{R}^{dn}$, the estimation law (9) can be rewritten in the matrix form as

$$\dot{\boldsymbol{z}}_k(t) = \boldsymbol{N} \boldsymbol{z}_k(t), \tag{10}$$

where $\boldsymbol{N} = -\boldsymbol{D}(\boldsymbol{L}_o \otimes \boldsymbol{I}_d)\boldsymbol{D}^{-1}$, $\boldsymbol{D} = \text{blkdiag}(\boldsymbol{R}_1^T, \ldots, \boldsymbol{R}_n^T)$ and $\boldsymbol{D}^{-1} = \boldsymbol{D}^T = \text{blkdiag}(\boldsymbol{R}_1, \ldots, \boldsymbol{R}_n)$ due to properties of the rotation matrix.

By introducing a coordinate transformation $\boldsymbol{z}_{i,k} = \boldsymbol{R}_i \boldsymbol{y}_{i,k}$, and let $\boldsymbol{y}_k(t) = [\boldsymbol{y}_{1,k}^T, \ldots, \boldsymbol{y}_{n,k}^T]^T \in \mathbb{R}^{dn}$, we have $\boldsymbol{z} = \boldsymbol{D}^{-1} \boldsymbol{y}$. Thus, (10) can be rewritten as follows:

$$\dot{\boldsymbol{y}}_k(t) = -(\boldsymbol{L}_o \otimes \boldsymbol{I}_d) \boldsymbol{y}_k(t), \ \forall k = 1, \ldots, n. \tag{11}$$

Let $\mathcal{Q}_{\boldsymbol{y}_k} := \{\boldsymbol{y}_k \in \mathbb{R}^{dn} | \ \boldsymbol{y}_{1,k} = \boldsymbol{y}_{2,k} = \ldots = \boldsymbol{y}_{n,k}\}$. For the estimation of orientation, we have to avoid the convergence of variables $\boldsymbol{y}_k$ to zero. Thus, the desired equilibrium set is defined by $\mathcal{S}_{\boldsymbol{y}_k} := \mathcal{Q}_{\boldsymbol{y}} \backslash \{\boldsymbol{0}\}$. We have the following theorems whose proof can be found in [12].

Theorem 1: Consider the system (11). Under Assumption 1, there exists a finite point $\boldsymbol{y}_k^\infty \in \mathcal{S}_{\boldsymbol{y}_k}$ $(k = 1, \ldots, d)$ such that $\boldsymbol{y}_k$ exponentially converges if and only if the initial value $\boldsymbol{y}_k(0)$ $(k = 1, \ldots, d)$ is not in $\mathcal{C}(\boldsymbol{L}_o \otimes \boldsymbol{I}_d)$.

Theorem 1 implies $\lim_{t \to \infty} \boldsymbol{z}_k = \boldsymbol{D} \boldsymbol{y}_k^\infty, \forall k = 1, \ldots, d$. Since $\boldsymbol{Q}_i \in SO(d)$ is computed from $\boldsymbol{z}_{i,k}, \forall k = 1, \ldots, d$, there exists a $\boldsymbol{Q}_i^\infty \in SO(d)$ to which $\boldsymbol{Q}_i$ converges.

Theorem 2: Let $\boldsymbol{Q}_i \in SO(d)$ for the i-th agent be derived from procedures (7) and (9). There exists a common matrix $\boldsymbol{R}^* \in SO(d)$ such that $\boldsymbol{Q}_i$ converges to $\boldsymbol{R}_i^T \boldsymbol{R}^*$ as $t \to \infty$ for all $i \in \{1, \ldots, n\}$.

Proof: By using the coordinate transformation, we define $\boldsymbol{y}_{i,k} \in \mathbb{R}^d$ such that $\boldsymbol{z}_{i,k} = \boldsymbol{R}_i \boldsymbol{y}_{i,k}$ for all $i = 1, \ldots, n$, and $k = 1, \dots, d$. Procedure (7) can be rewritten (for $\sigma = 1$) as follows:

$$\begin{aligned} \boldsymbol{v}_{i,1} &= \boldsymbol{R}_i^T \boldsymbol{y}_{i,1} := \boldsymbol{R}_i^T \boldsymbol{r}_{i,1}, \\ \boldsymbol{v}_{i,2} &= \boldsymbol{R}_i^T \boldsymbol{y}_{i,2} - \left\langle \boldsymbol{R}_i^T \boldsymbol{y}_{i,2}, \frac{\boldsymbol{v}_{i,1}}{\|\boldsymbol{v}_{i,1}\|} \right\rangle \frac{\boldsymbol{v}_{i,1}}{\|\boldsymbol{v}_{i,1}\|} := \boldsymbol{R}_i^T \boldsymbol{r}_{i,1}, \\ &\vdots \qquad (12) \\ \boldsymbol{v}_{i,k} &= \boldsymbol{R}_i^T \boldsymbol{y}_{i,d} - \sum_{k=1}^{d-1} \left\langle \boldsymbol{R}_i^T \boldsymbol{y}_{i,d}, \frac{\boldsymbol{v}_{i,k}}{\|\boldsymbol{v}_{i,k}\|} \right\rangle \frac{\boldsymbol{v}_{i,k}}{\|\boldsymbol{v}_{i,k}\|} := \boldsymbol{R}_i^T \boldsymbol{r}_{i,d} \end{aligned}$$

Replacing $\boldsymbol{v}_{i,k}$ by $\boldsymbol{y}_{i,k}$ from the above procedure gives:

$$\begin{aligned} \boldsymbol{r}_{i,1} &= \boldsymbol{y}_{i,1}, \\ \boldsymbol{r}_{i,2} &= \boldsymbol{y}_{i,1} - \left\langle \boldsymbol{y}_{i,2}, \frac{\boldsymbol{r}_{i,1}}{\|\boldsymbol{r}_{i,1}\|} \right\rangle \frac{\boldsymbol{r}_{i,1}}{\|\boldsymbol{r}_{i,1}\|}, \\ &\vdots \qquad (13) \\ \boldsymbol{r}_{i,d} &= \boldsymbol{y}_{i,d} - \sum_{k=1}^{d-1} \left\langle \boldsymbol{y}_{i,d}, \frac{\boldsymbol{r}_{i,k}}{\|\boldsymbol{r}_{i,k}\|} \right\rangle \frac{\boldsymbol{r}_{i,k}}{\|\boldsymbol{r}_{i,k}\|}. \end{aligned}$$

Based on Theorem 1, for each $k = 1, \dots, d$ there exists a finite point $\boldsymbol{y}_k^\infty \in \mathbb{R}^n$ as $t \to \infty$ for all $i \in \mathcal{V}_o$. Thus, there exists a finite vector $\boldsymbol{r}_k^\infty \in \mathbb{R}^d$ such that $\{\boldsymbol{r}_{i,1}, \boldsymbol{r}_{i,2}, \dots, \boldsymbol{r}_{i,d}\} \to \{\boldsymbol{r}_1^\infty, \boldsymbol{r}_2^\infty, \dots, \boldsymbol{r}_d^\infty\}$ as $t \to \infty$ for all $i \in \mathcal{V}_o$. Comparing (7) and (13), it is easy to see that $\boldsymbol{q}_{i,k} = \boldsymbol{R}_i^T \frac{\boldsymbol{r}_{i,k}}{\|\boldsymbol{r}_{i,k}\|}$. Thus, in steady state, we have

$$\lim_{t\to\infty} \boldsymbol{Q}_i(t) = \boldsymbol{R}_i^T \left[\frac{\boldsymbol{r}_1^\infty}{\|\boldsymbol{r}_1^\infty\|}, \dots, \frac{\boldsymbol{r}_n^\infty}{\|\boldsymbol{r}_d^\infty\|} \right] = \boldsymbol{R}_i^T \boldsymbol{R}^*, \quad (14)$$

for all $i = 1, \dots, n$. ■

According to Theorem 1, the value of the estimated solution depends on the initial values of auxiliary variables $\boldsymbol{z}_{i,k},\ i = 1, \dots, n,\ k = 1, \dots, d$. Since the set of initial auxiliary variables resulting in nonexistence of solution is a set of Lesbegue measure zero in $\mathbb{R}^{dn}$, we say the estimation law (9) *almost globally asymptotically* solves the Problems 1 and 4. We refer the readers to [12] for a detailed discussion on this issue.

Corollary 1: Suppose that the graph $\mathcal{G}_o$ is rooted-out branching and the root node (say node 1) has information about the global coordinate frame ${}^g\Sigma$, and thus does not update its orientation estimation, i.e., $\dot{\boldsymbol{z}}_{1,k} = \boldsymbol{0}$ while other nodes follows the orientation estimation (9), then the estimated orientation $\boldsymbol{Q}_i$ converges to $\boldsymbol{R}_i^T \boldsymbol{R}_1$ as $t \to \infty$, for all $i = 1, \dots, n$.

Proof: Let $\boldsymbol{z}_{i,k} = \boldsymbol{R}_i \boldsymbol{y}_{i,k}$. The dynamics (9) can be rewritten in terms of $\boldsymbol{y}_{i,k}$ as follows:

$$\dot{\boldsymbol{y}}_k(t) = -(\boldsymbol{L}_o \otimes \boldsymbol{I}_d)\boldsymbol{y}_k(t),\ \forall k = 1, \dots, n. \quad (15)$$

Since $\dot{\boldsymbol{z}}_{1,k} = \boldsymbol{0}$, the matrix $\boldsymbol{L}_o$ has the following form

$$\boldsymbol{L}_o = \begin{bmatrix} 0 & 0 \ \cdots \ 0 \\ -a_{21} & \\ \vdots & \boldsymbol{L}_r \\ -a_{n1} & \end{bmatrix},$$

where $\boldsymbol{L}_r$ is a reduced Laplacian matrix. Due to the property of triangular block matrices, the eigenvalues of $\boldsymbol{L}_r$ are the same as the eigenvalues of $\boldsymbol{L}_o$, except for a single zero eigenvalue. Since $\mathcal{G}_o$ is rooted out-branching, $\boldsymbol{L}_o$ has a single zero eigenvalue. Thus, $\boldsymbol{L}_r$ is a Hurwitz matrix. Let us denote $\tilde{\boldsymbol{y}}_{i,k} = \boldsymbol{y}_{i,k} - \boldsymbol{y}_{1,k}$. Note that $\boldsymbol{y}_{1,k}$ is invariant under (15). We further define $\tilde{\boldsymbol{y}}_k = (\tilde{\boldsymbol{y}}_{2,k}, \dots, \tilde{\boldsymbol{y}}_{n,k})$. The time derivatives of $\tilde{\boldsymbol{y}}_k$ are given as

$$\dot{\tilde{\boldsymbol{y}}}_k(t) = -(\boldsymbol{L}_r \otimes \boldsymbol{I}_d)\tilde{\boldsymbol{y}}_k(t),\ \forall k = 1, \dots, n. \quad (16)$$

Obviously, $\tilde{\boldsymbol{y}}_k(t) \to \boldsymbol{0}$ exponentially. From the definition of $\boldsymbol{r}_{i,k}$ as in the proof of Theorem 2, $\boldsymbol{R}^*$ is determined by the value of $\boldsymbol{y}_{i,k},\ \forall k \in \{1, \dots, d\}$. By initializing the auxiliary variables for the root note (node 1) as $\boldsymbol{z}_{1,k} = [\boldsymbol{R}_1]_{*k},\ k = 1, \dots, d$ (i.e. the k-th column vector of $\boldsymbol{R}_1$), under the orientation estimation law (15), we have

$$\lim_{t\to\infty} \boldsymbol{Q}_i(t) = \boldsymbol{R}_i^T \left[\frac{\boldsymbol{r}_1^\infty}{\|\boldsymbol{r}_1^\infty\|}, \dots, \frac{\boldsymbol{r}_n^\infty}{\|\boldsymbol{r}_d^\infty\|} \right] = \boldsymbol{R}_i^T \boldsymbol{R}_1, \quad (17)$$

for all $i = 2, \dots, n$. Thus, if node 1's orientation is aligned with the global reference frame (i.e. $\boldsymbol{R}_1 = \boldsymbol{I}_d$), we have $\boldsymbol{Q}_i \to \boldsymbol{R}_i^T$ for all $i = 2, \dots, n$. This concludes the proof. ■

B. Bearing-based formation control

To solve Problem 2, we propose the following bearing-only distributed control law for each agent i ($i = 1, \dots, n$):

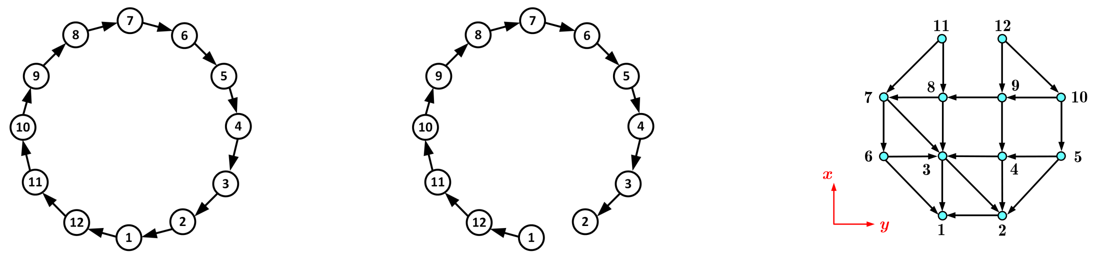
$$\boldsymbol{u}_i^i = -\sum\nolimits_{i\in\mathcal{N}_i} \boldsymbol{P}_{\boldsymbol{b}_{ij}^i} \boldsymbol{Q}_i \boldsymbol{b}_{ij}^*, \quad (18)$$

where $\boldsymbol{Q}_i$ is the estimated orientation matrix obtained at time t based on (9), $\boldsymbol{P}_{\boldsymbol{b}_{ij}^i} = \boldsymbol{I}_d - \boldsymbol{b}_{ij}^i (\boldsymbol{b}_{ij}^i)^T$ is the projection matrix obtained from bearing measurements in ${}^i\Sigma$, and $\mathcal{N}_i$ denotes the set of neighbor of agent i in the bearing sensing graph $\mathcal{G}_b$.

Note that $\boldsymbol{P}_{\boldsymbol{b}_{ij}^i} = \boldsymbol{I}_d - \boldsymbol{b}_{ij}^i (\boldsymbol{b}_i^i)^T = \boldsymbol{I}_d - \boldsymbol{R}_i^{-1} \boldsymbol{b}_{ij} \boldsymbol{b}_{ij}^T \boldsymbol{R}_i = \boldsymbol{R}_i^{-1}(\boldsymbol{I}_d - \boldsymbol{b}_{ij}\boldsymbol{b}_{ij}^T)\boldsymbol{R}_i = \boldsymbol{R}_i^T \boldsymbol{P}_{\boldsymbol{b}_{ij}} \boldsymbol{R}_i$. Further, since $\boldsymbol{Q}_i$ is a rotation matrix, we can write $\boldsymbol{Q}_i = \boldsymbol{R}_i^T \boldsymbol{R}_{\Delta i} \boldsymbol{R}^*$, where $\boldsymbol{R}^*$ is dependent on initial orientations of all agents, $\boldsymbol{R}_{\Delta i} \in SO(d)$ is the rotation error during the estimation procedure at time $t \geq 0$ which is unknown to agent i. Since the dynamics of each agent i written in the global reference frame is given by $\dot{\boldsymbol{p}}_i = \boldsymbol{R}_i \boldsymbol{u}_i^i$, from (18), we can write

$$\begin{aligned} \dot{\boldsymbol{p}}_i &= -\boldsymbol{R}_i \sum_{j\in\mathcal{N}_i} \boldsymbol{P}_{\boldsymbol{b}_{ij}^i} \boldsymbol{Q}_i \boldsymbol{b}_{ij}^* \\ &= -\boldsymbol{R}_i \sum_{j\in\mathcal{N}_i} \boldsymbol{R}_i^T \boldsymbol{P}_{\boldsymbol{b}_{ij}} \boldsymbol{R}_i \boldsymbol{R}_i^T \boldsymbol{R}_{\Delta i} \boldsymbol{R}^* \boldsymbol{b}_{ij}^* \\ &= -\sum_{j\in\mathcal{N}_i} \boldsymbol{P}_{\boldsymbol{b}_{ij}} \boldsymbol{R}_{\Delta i} \boldsymbol{R}^* \boldsymbol{b}_{ij}^* \\ &= -\sum_{j\in\mathcal{N}_i} \boldsymbol{P}_{\boldsymbol{b}_{ij}} \boldsymbol{R}^* \boldsymbol{b}_{ij}^* + \sum_{j\in\mathcal{N}_i} \boldsymbol{P}_{\boldsymbol{b}_{ij}} (\boldsymbol{I}_d - \boldsymbol{R}_{\Delta i}) \boldsymbol{R}^* \boldsymbol{b}_{ij}^* \\ &= -\sum_{j\in\mathcal{N}_i} \boldsymbol{P}_{\boldsymbol{b}_{ij}} \boldsymbol{b}_{ij}^\infty + \boldsymbol{h}_i(t), \end{aligned} \quad (19)$$

where $\boldsymbol{b}_{ij}^\infty = \boldsymbol{R}^* \boldsymbol{b}_{ij}^*$, and $\boldsymbol{h}_i(t) = \sum_{j\in\mathcal{N}_i} \boldsymbol{P}_{\boldsymbol{b}_{ij}} (\boldsymbol{I}_d - \boldsymbol{R}_{\Delta i}) \boldsymbol{R}^* \boldsymbol{b}_{ij}^*$. Let $\boldsymbol{p} = [\boldsymbol{p}_1^T, \dots, \boldsymbol{p}_n^T]^T \in \mathbb{R}^{dn}$, $\boldsymbol{h}(t) = [\boldsymbol{h}_1^T, \dots, \boldsymbol{h}_n^T]^T \in \mathbb{R}^{dn}$, and $\boldsymbol{b}^\infty = [(\boldsymbol{b}_1^\infty)^T, \dots, (\boldsymbol{b}_m^\infty)^T]^T \in$



(a) (b) (c)

Fig. 2: Simulation setup: (a) & (b) The orientation sensing graphs ($\mathcal{G}_o$) used in simulations 1 and 2, respectively; (c) The bearing sensing and controlling graph $\mathcal{G}_b$ (a LFF graph) and a desired formation $\boldsymbol{p}^*$ in two-dimensional space.

$\mathbb{R}^{dn}$. From equation (19), we can write the overall dynamics as follows:

$$\dot{\boldsymbol{p}} = \boldsymbol{f}(\boldsymbol{p}) + \boldsymbol{h}(t). \tag{20}$$

Thus, $\boldsymbol{h}(t)$ can be considered as an input to the unforced system:

$$\dot{\boldsymbol{p}} = \boldsymbol{f}(\boldsymbol{p}) = -\bar{\boldsymbol{H}}_b^T \mathrm{diag}\left(\boldsymbol{P}_{\boldsymbol{b}_k}\right) \boldsymbol{b}^{\infty}, \tag{21}$$

Moreover, there holds

$$\|\boldsymbol{h}_i(t)\| \leq \sum\nolimits_{j\in\mathcal{N}_i} \|\boldsymbol{P}_{\boldsymbol{b}_i}\| \|\boldsymbol{I}_d - \boldsymbol{R}_{\Delta i}\| \|\boldsymbol{R}^*\| \|\boldsymbol{b}_{ij}^*\|.$$

Since each component in the left hand side is bounded, $\|\boldsymbol{h}_i(t)\|$ is also bounded. Based on Theorem 2, $\boldsymbol{R}_{\Delta i} \to \boldsymbol{I}_d$ exponentially as $t \to \infty$. Thus $\|\boldsymbol{h}_i(t)\| \to 0$ exponentially.

The system (21) has two equilibrium points: the equilibrium $\boldsymbol{p} = \boldsymbol{p}_d^{\infty}$ corresponding to $\boldsymbol{b}_{ij} = \boldsymbol{b}_{ij}^{\infty},\ \forall (i,j) \in \mathcal{E}_b$ is almost globally exponentially stable; while the equilibrium $\boldsymbol{p} = \boldsymbol{p}_u^{\infty}$ corresponding to $\boldsymbol{b}_{ij} = -\boldsymbol{b}_{ij}^{\infty},\ \forall (i,j) \in \mathcal{E}_b$ is exponentially unstable [13]. The main result of this paper is stated in the following theorem whose proof is omitted due to space restriction.

Theorem 3: Under Assumptions 1–3, the equilibrium $\boldsymbol{p} = \boldsymbol{p}_d^{\infty}$ corresponding to $\boldsymbol{b}_{ij} = \boldsymbol{b}_{ij}^{\infty},\ \forall (i,j) \in \mathcal{E}_b$ of the system (20) is almost globally asymptotically stable.

It is worth remarking that if the hypothesis of Corollary 1 is satisfied, any trajectory of the system (20) almost globally asymptotically converges to the desired equilibrium $\boldsymbol{p} = \boldsymbol{p}_d^{\infty}$ where $\boldsymbol{b}_{ij} = \boldsymbol{b}_{ij}^*,\ \forall (i,j) \in \mathcal{E}_b$.

C. Bearing-based network localization

Consider Problem 3. Since network localization and formation control are dual problems, the following position estimation law is proposed for each agent:

$$\dot{\hat{\boldsymbol{p}}}_i = -\sum\nolimits_{j\in\mathcal{N}_i} \boldsymbol{P}_{\hat{\boldsymbol{b}}_{ij}} \boldsymbol{Q}_i^{-1} \boldsymbol{b}_{ij}^i. \tag{22}$$

Here, $\boldsymbol{P}_{\hat{\boldsymbol{b}}_{ij}}$ is calculated from $\hat{\boldsymbol{b}}_{ij} = (\hat{\boldsymbol{p}}_j - \hat{\boldsymbol{p}}_i)/\|\hat{\boldsymbol{p}}_j - \hat{\boldsymbol{p}}_i\|$, and $\hat{\boldsymbol{p}}_j$ is received from agent j. Thus, this estimation law uses only local sensing information and communicated information.

Substituting $\boldsymbol{Q}_i = \boldsymbol{R}_i^T \boldsymbol{R}_{\Delta i} \boldsymbol{R}^*$ and $\boldsymbol{b}_{ij}^i = \boldsymbol{R}_i^T \boldsymbol{b}_{ij}$ into (22), after some derivations, we have

$$\dot{\hat{\boldsymbol{p}}}_i = -\sum\nolimits_{j\in\mathcal{N}_i} \boldsymbol{P}_{\hat{\boldsymbol{b}}_{ij}} \boldsymbol{b}_{ij}^{\infty} + \boldsymbol{h}_i. \tag{23}$$

Note that in equation (23), $\boldsymbol{b}_{ij}^{\infty} = (\boldsymbol{R}^*)^T \boldsymbol{b}_{ij}$, and $\boldsymbol{h}_i = \sum_{j\in\mathcal{N}_i} \boldsymbol{P}_{\hat{\boldsymbol{b}}_{ij}} (\boldsymbol{R}^*)^T (\boldsymbol{I}_d - \boldsymbol{R}_{\Delta i}^T) \boldsymbol{b}_{ij}$. Let $\boldsymbol{h} = [\boldsymbol{h}_1^T, \ldots, \boldsymbol{h}_n^T]$ and $\boldsymbol{b}^{\infty} = [(\boldsymbol{b}_1^{\infty})^T, \ldots, (\boldsymbol{b}_m^{\infty})^T]^T$. Then, the overall dynamics of n nodes can be written in the following compact form:

$$\dot{\hat{\boldsymbol{p}}} = \boldsymbol{f}(\hat{\boldsymbol{p}}) + \boldsymbol{h}. \tag{24}$$

It can be observed that equation (24) has the same form with equation (20). Thus, we immediately have the following result:

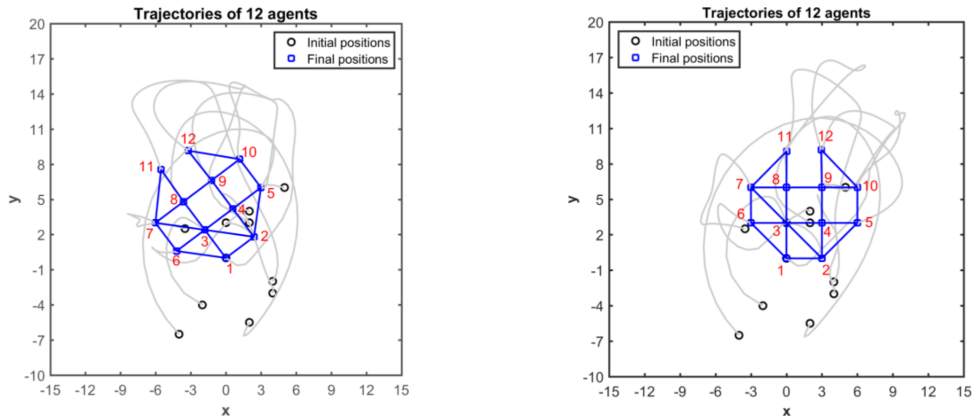
Theorem 4: Consider a network of n nodes satisfying Assumptions 1–3. Under the orientation estimation law (9) and the position estimation law (22), if $\hat{\boldsymbol{p}}_i(0) \neq \hat{\boldsymbol{p}}_j(0), \forall i \neq j,\ i,j = 1, \ldots, n$, then $(\hat{\boldsymbol{p}}_j - \hat{\boldsymbol{p}}_i)/\|\hat{\boldsymbol{p}}_j - \hat{\boldsymbol{p}}_i\| \to (\boldsymbol{R}^*)^T \boldsymbol{b}_{ij}$ almost globally asymptotically, $\forall (i,j) \in \mathcal{E}_b$.

V. Simulations

Consider a system consisting of six agents in a two-dimensional space. The initial positions and orientations of the agents are given in Table I. Note that the orientation matrix $\boldsymbol{R}_i$ in 2D can be fully defined by an angle $\theta_i \in (-\pi, \pi]$. The desired bearing vectors are chosen according to the realization depicted in Fig. 2c.

In simulation 1, the orientation sensing topology between agents in this simulation is a directed cycle graph and the bearing sensing and controlling graph is a leader-first follower graph as depicted in Figs. 2a and 2c. Initial conditions are shown in Table I. Simulation results are given in Fig. 3a. The initial/final positions of the agents are colored black/blue respectively. The trajectories of 12 agent are depicted by gray lines. It can be observed from Fig. 3 that the agents asymptotically form a desired formation shape in the space. The end formation differs from the configuration given in Fig. 2c by a translation, a rotation and a scaling. The position of agent 1 fixed the formation's translation. The distance between agents 1 and 2 fixed the formation scale. The common offset of all agents' estimated orientations determine the formation's orientation and is given by the matrix $\boldsymbol{R}^* = \begin{bmatrix} 0.7992 & -0.6010 \\ 0.6010 & 0.7992 \end{bmatrix}$, which corresponds to the angle $\theta^* = 0.6448$ [rad].

In simulation 2, we use the same initial conditions but replace the orientation sensing graph to the one in Fig. 2a. This time, vertex 1 is the only root of $\mathcal{G}_o$. Simulation results



(a) Simulation 1 (b) Simulation 2

Fig. 3: Simulation of the 12-agent system in Fig. 2 under the proposed formation control strategy.

TABLE I: Simulation parameters

i	1	2	3	4	5	6	7	8	9	10	11	12
θ_i [rad]	0	$\frac{\pi}{6}$	$\frac{\pi}{8}$	$-\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{9}$	$\frac{\pi}{10}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$-\frac{\pi}{2}$	$-\frac{\pi}{5}$	$\frac{3\pi}{8}$
$\boldsymbol{p}_i(0)$	$\begin{bmatrix}0\\0\end{bmatrix}$	$\begin{bmatrix}0\\3\end{bmatrix}$	$\begin{bmatrix}-2\\-4\end{bmatrix}$	$\begin{bmatrix}4\\-3\end{bmatrix}$	$\begin{bmatrix}-4\\-6.5\end{bmatrix}$	$\begin{bmatrix}2\\4\end{bmatrix}$	$\begin{bmatrix}-3.5\\2.5\end{bmatrix}$	$\begin{bmatrix}2\\-5.5\end{bmatrix}$	$\begin{bmatrix}5\\6\end{bmatrix}$	$\begin{bmatrix}-1\\6\end{bmatrix}$	$\begin{bmatrix}4\\-2\end{bmatrix}$	$\begin{bmatrix}2\\3\end{bmatrix}$

are given in Fig. 3b. All agents can estimate their precise orientations and the desired formation is achieved, i.e., $\boldsymbol{R}_i \to \boldsymbol{R}^* = \boldsymbol{I}_2, \forall i = 1, \ldots, n$, and $\boldsymbol{b}_{ij} = \boldsymbol{b}^*_{ij}, \forall (i,j) \in \mathcal{E}_b$. Thus, the simulation results are consistent with our analysis.

VI. Conclusion

This paper proposed a distributed strategy to solve the bearing-based formation control and network localization via global orientation estimation. The proposed strategy does not require any common reference frames and any further local measurements other than the relative orientations and the local bearing vectors.

For future work, it is not practical to assume that the orientation estimation law can be conducted in continuous time due to communication delays. An event-triggered control setup for the orientation estimator may give a more realistic solution. In addition, although the analysis is valid for d-dimensional space, it is not easy to obtain the relative orientation $\boldsymbol{R}_{ij}$ when $d > 2$. Finally, an implementation of the proposed control strategy is also left for future study.

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