

# Event-based leader-follower consensus for multiple Euler-Lagrange systems with parametric uncertainties

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**Abstract**—An adaptive, distributed, event-triggered controller is proposed in this paper to study the problem of leader-follower consensus for a directed network of Euler-Lagrange agents. We show that if each agent uses the proposed controller, the leader-follower consensus objective is globally asymptotically achieved if the directed network contains a directed spanning tree with the leader as the root node. We provide a trigger function to govern the event time; at each event time the controller is updated. In doing so, we also obtain an explicit lower bound on the time interval between events and thus we conclude that the proposed controller does not exhibit Zeno behavior. Simulations are provided which show the effectiveness of the proposed controller. Also shown in the simulations is the piecewise constant nature of the control law; this significantly reduces the number of updates required by each actuator, thereby saving energy resources.

## I. INTRODUCTION

The field of multiagent systems has received extensive attention from the control community in the past two decades. In particular, coordination of a network of interacting agents to achieve a global objective has been seen as a key sub-area within the field. See [1] for a recent survey of multi-agent coordination problems. Leader-follower consensus is a variation of the commonly studied consensus problem where, with all agents having a commonly defined state variable(s), the network of follower agents converge to the state value of the stationary leader. This is achieved by interaction between neighboring agents. By the fact that interaction is only between neighboring agents then each follower agent must use a distributed controller, i.e. agents cannot use global information about the network as a whole [2]. It is well known that the topological constraints of a network are intertwined with the agent dynamics when studying control laws which guarantee achievement of the

consensus objective. Reducing constraints on the topology allows greater flexibility during the design phase.

The Euler-Lagrange equations describe the dynamics of a large class of nonlinear systems (including many mechanical systems such as robotic manipulators, spacecraft and marine vessels) [3], [4]. As a result, there is motivation to study multiagent coordination problems where each agent has Euler-Lagrange dynamics [5]. Leader-follower consensus for directed networks of Euler-Lagrange agents has been studied in [6] using a model-independent controller, and in [7] using an adaptive controller. In both, the topology requires a directed spanning tree.

Recently, event-triggered controllers have been popularised [8], [9], [10], [11], [12]. While each agent has continuous time dynamics, the controller is updated at discrete time instants based on event-scheduling. Because the controller updates occur at specific events, this has the benefit of reducing actuator updates. However, it is important to properly design and analyse the event-scheduling trigger function to exclude Zeno behaviour [13], which can cause the controller to collapse. Numerous results have been published studying consensus based problems using distributed event-triggered control laws. However, the majority study agents with single and double-integrator dynamics [8], [9], [10], [11].

There have been few results published studying event-triggered control for networks Euler-Lagrange agents. A pioneering contribution studied leaderless consensus (but not leader-follower consensus) on an undirected network [14]. The dynamics studied in [6] and [14] are a subclass of Euler-Lagrange dynamics as they do not consider the presence of gravitational forces of each agent. While continuous model-independent algorithms e.g. [6] are easily adapted to be event-triggered, as shown in [14], they cannot guarantee the coordination objective in the presence of gravitational forces. The gravitational term has an effect which is similar to a bounded disturbance. Typical control techniques required to deal with this term include feedback linearization [15], adaptive control [7] and discontinuous control [16]. However, these techniques have not been studied in an event-triggered framework. In fact, discontinuous controllers are unsuited to event-triggered control as the switching controller exhibits Zeno-like behavior.

In this paper, we propose an adaptive, distributed event-triggered controller to achieve leader-follower consensus. This allows for uncertain parameters in each agent, e.g. the mass of a robotic manipulator arm, and includes the gravitational forces omitted in [14]. The proposed controller

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is piecewise constant, which has the benefit of reducing actuator updates and thus conserving energy resources. Furthermore, each agent only requires relative state measurements, and does not require the trigger times of neighboring agents; this reduces the amount of interaction required. We study the agents on a directed network and show leader-follower consensus is globally asymptotically achieved if the graph contains a directed spanning tree. Because directed networks can model unilateral interactions, directed networks offer more flexibility in design of the multiagent system. We design the trigger function such that a strictly positive lower bound between each event can be explicitly given. This excludes the possibility of Zeno behavior.

The paper is structured as follows. Section II provides mathematical notation and background on graph theory and Euler-Lagrange equations. A formal problem definition is also provided. The main results are then given in Section III in two parts; stability of the system and the exclusion of Zeno behavior. Simulations in Section IV show the effectiveness of the proposed controller. Concluding remarks are given in Section V.

## II. PRELIMINARIES

### A. Notations

In this paper,  $\mathbb{R}^n$  denotes the  $n$ -dimensional Euclidean space and  $\mathbb{R}^{m \times n}$  denotes the set of  $m \times n$  real matrices. The transpose of a vector or matrix  $M$  is given by  $M^\top$ . The  $i$ -th smallest eigenvalue of a symmetric matrix  $M$  is denoted by  $\lambda_i(M)$ . Let  $x = [x_1, \dots, x_n]^\top$  where  $x_i \in \mathbb{R}^{n \times n}$  and  $n \geq 1$ . Then  $\text{diag}\{x\}$  denotes a (block) diagonal matrix with the (block) elements of  $x$  on its diagonal, i.e.  $\text{diag}\{x_1, \dots, x_n\}$ . The  $n \times n$  identity matrix is  $I_n$  and  $\mathbf{1}_n$  denotes an  $n$ -tuple column vector of all ones. The  $n \times 1$  column vector of all zeros is denoted by  $\mathbf{0}_n$ . The Euclidean norm of a vector, and the matrix norm induced by the Euclidean norm, is denoted by  $\|\cdot\|$ . The  $L_p$  norm of a function  $f: \mathbb{R}^+ \rightarrow \mathbb{R}$  is defined as  $\|f(t)\|_p = (\int_0^\infty |f(\tau)|^p d\tau)^{\frac{1}{p}}$  for all  $p \in [1, \infty)$ , while  $\|f(t)\|_\infty = \max_{t \geq 0} |f(t)|$  denotes the  $L_\infty$  norm of  $f$ . We say that  $f \in L_p$  if  $\|f(t)\|_p$  exists and is finite and  $f \in L_\infty$  if  $\|f(t)\|_\infty < \infty$ .

### B. Graph Theory

We model a group of  $n$  interacting agents by a weighted directed graph (digraph)  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$  with vertex set  $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$  and edge set  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ . An ordered edge set of  $\mathcal{G}$  is  $e_{ij} = (v_i, v_j)$  and we do not assume  $e_{ij} = e_{ji}$ . The weighted adjacency matrix  $\mathcal{A} = \mathcal{A}(\mathcal{G}) = \{a_{ij}\}$  is the  $n \times n$  matrix given by  $a_{ij} > 0$ , if  $e_{(j,i)} \in \mathcal{E}$  and  $a_{ij} = 0$ , otherwise. In this paper, it is assumed that  $a_{ii} = 0$ . The edge  $e_{ij}$  is incoming with respect to  $v_j$  and outgoing with respect to  $v_i$ . The neighbor set of  $v_i$  is denoted by  $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$ . The  $n \times n$  Laplacian matrix,  $\mathcal{L} = \{l_{ij}\}$ , of the associated directed graph  $\mathcal{G}$  is defined as

$$l_{ij} = \begin{cases} \sum_{k=1, k \neq i}^n a_{ik} & \text{for } j = i \\ -a_{ij} & \text{for } j \neq i \end{cases}$$

A digraph with  $n$  vertexes is called a directed tree if it has  $n - 1$  edges and there exists a root vertex with directed paths to every other vertex. A directed spanning tree of a digraph is a directed tree that contains all vertexes [2]. The following result holds for the Laplacian matrix associated with a directed graph.

**Lemma 1:** (From [2]) Let  $\mathcal{L}$  be the Laplacian matrix associated with a directed graph. Then  $\mathcal{L}$  has a simple zero eigenvalue and all other eigenvalues have positive real parts if and only if  $\mathcal{G}$  has a directed spanning tree.

**Lemma 2:** (From [17]) Suppose a graph  $\mathcal{G}$  contains a directed spanning tree, and there are no edges of  $\mathcal{G}$  which are incoming to the root vertex of the tree. Without loss of generality, number the root vertex as  $v_1$ . Then the Laplacian matrix associated with  $\mathcal{G}$  has the following form:

$$\mathcal{L} = \begin{bmatrix} 0 & \mathbf{0}_{n-1}^\top \\ \mathcal{L}_{21} & \mathcal{L}_{22} \end{bmatrix}$$

and all eigenvalues of  $\mathcal{L}_{22}$  have positive real parts.

### C. Euler-Lagrange Systems

A class of dynamical systems can be described using the Euler-Lagrange equations [3]. The general form for the  $i$ -th agent equation of motion is:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i \quad (1)$$

where  $q_i \in \mathbb{R}^p$  is a vector of the generalized coordinates,  $M_i(q_i) \in \mathbb{R}^{p \times p}$  is the inertial matrix,  $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{p \times p}$  is the Coriolis and centrifugal torque matrix,  $g_i(q_i) \in \mathbb{R}^p$  is the vector of gravitational forces and  $\tau_i \in \mathbb{R}^p$  is the control input vector. For agent  $i$ , we have  $q_i = [q_i^{(1)}, \dots, q_i^{(p)}]^\top$ . We assume each agent is fully actuated. The dynamics in (1) are assumed to satisfy the following properties, details of which are provided in [3].

P(1) The matrix  $M_i(q_i)$  is symmetric positive definite.

P(2) There exist positive constants  $k_{\underline{m}}$  and  $k_{\overline{m}}$  such that  $0 < k_{\underline{m}}I_n \leq M_i(q_i) \leq k_{\overline{m}}I_n$ .

P(3) There exists a scalar constant  $k_C > 0$  such that  $\|C_i(q_i, \dot{q}_i)\| < k_C \|\dot{q}_i\|$ .

P(4) The matrix  $C_i(q_i, \dot{q}_i)$  is related to the inertial matrix  $M_i(q_i, \dot{q}_i)$  by the expression  $x^T (\frac{1}{2}\dot{M}_i(q_i) - C_i(q_i, \dot{q}_i))x = 0$  for any  $q, \dot{q}, x \in \mathbb{R}^p$ .

P(5) There exists a scalar constant  $k_g > 0$  such that  $\|g_i(q_i)\| < k_g$ .

P(6) Linearity in the parameters:  $M_i(q_i)x + C_i(q_i, \dot{q}_i)y + g_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\Theta_i$  for all vectors  $x, y \in \mathbb{R}^p$ , where  $Y_i(q_i, \dot{q}_i, x, y)$  is the regressor and  $\Theta_i$  is a vector for unknown but constant parameters associated with the  $i$ -th agent.

We make the following assumption on the system. This assumption is discussed following the main proof in Remark 2.

**Assumption 1:** Each element of the matrices  $M_i, C_i$  and the vector  $g_i$  is globally Lipschitz and absolutely continuous. From Property P(6), this in turn implies that each element of  $Y_i(q_i, \dot{q}_i, x, y)$  is globally Lipschitz and absolutely continuous. It follows that for bounded values of  $Y_i(q_i, \dot{q}_i, x, y)$ , the time derivative  $\dot{Y}_i$  is bounded. The condition of global Lipschitz and absolute continuity is satisfied for a class of

Euler-Lagrange equations, including robots which only have revolute joints (see Chapter 4 of [3]) and relative translational dynamics of spacecraft [18]. AC motors and voltage-fed induction motors modelled in the Euler-Lagrange framework also satisfy this condition [4].

#### D. Problem Statement

Denote the leader as agent 0 with  $q_0$  and  $\dot{q}_0$  being the generalized coordinates and generalized velocity of the leader, respectively. The aim is to develop an event-based, distributed algorithm for each Euler-Lagrange follower agent, where the updates are such that  $\tau_i$  is piecewise-constant, to achieve leader-follower consensus to a stationary leader, i.e.  $\dot{q}_0(t) = 0, \forall t \geq 0$ . Leader-follower consensus is said to be achieved if  $\lim_{t \rightarrow \infty} \|q_i(t) - q_0(t)\| = 0, \forall i = 1, \dots, n$  and  $\lim_{t \rightarrow \infty} \|\dot{q}_i(t)\| = 0, \forall i = 1, \dots, n$  are satisfied.

Another aim of this paper is to exclude the possibility of Zeno behavior. We provide a formal definition of Zeno behavior in the sequel. Zeno behavior of an event-based controller means the controller triggers an infinite number of events in a finite time period, which is an undesirable triggering behavior since no practical controller can trigger an infinite number of events in a finite time period.

In this paper, we assume that agent  $i \in 1, \dots, n$  is equipped with sensors which continuously measures the relative generalized coordinates to agent  $i$ 's neighbors. In other words,  $q_i(t) - q_j(t), \forall j \in \mathcal{N}_i$  is available to agent  $i$ . The sensing graph topology for the relative generalized coordinates among the  $n$  followers and the leader is modelled by a fixed, directed graph  $\bar{\mathcal{G}}$  with associated Laplacian matrix  $\bar{\mathcal{L}}$ . That is, if for agent  $i$ , the sensors on agent  $i$  can sense  $q_i(t) - q_j(t), i = 1, \dots, n, j = 0, \dots, n$ , then  $a_{ij} > 0$ , otherwise,  $a_{ij} = 0$ , where  $a_{ij}$  are the elements of the adjacency matrix associated with  $\bar{\mathcal{G}}$ . The scenario where agents collect relative information to execute algorithms can be found in many experimental testbeds, such as ground robots or UAVs equipped with high-speed cameras. It is also assumed that each agent  $i$  can measure its own generalized velocity continuously,  $\dot{q}_i(t)$ .

### III. MAIN RESULTS

Before we present the main results, we introduce variables which allow us to rewrite the multiagent system in a way which facilitates stability analysis. Two lemmas on stability are also provided. To begin, we introduce the following auxiliary variables  $q_{ri}$  and  $s_i$ , which appeared in [7], [19] studying leader-follow problems in directed Euler-Lagrange networks. Define

$$\dot{q}_{ri}(t) = -\alpha \sum_{j=0}^n a_{ij}(q_i(t) - q_j(t)), \quad (2)$$

$$s_i(t) = \dot{q}_i(t) - \dot{q}_{ri}(t) = \dot{q}_i(t) + \alpha \sum_{j=0}^n a_{ij}(q_i(t) - q_j(t)), \quad (3)$$

$i = 1, \dots, n$

where  $\alpha$  is a positive constant,  $a_{ij}$  is the weighted  $(i, j)$  entry of the adjacency matrix  $\bar{\mathcal{A}}$  associated with the directed graph

$\bar{\mathcal{G}}$  that characterises the sensing flows among the  $n$  followers. Let the vertex  $v_0$  represent agent 0 in graph  $\bar{\mathcal{G}}$ , then  $\bar{\mathcal{L}}$  can be written as:

$$\bar{\mathcal{L}} = \begin{bmatrix} 0 & \mathbf{0}_n^T \\ -a_0 & H \end{bmatrix} \quad (4)$$

where  $a_0 = [a_{10}, \dots, a_{n0}]^T$  and  $H = \mathcal{L}_{\mathcal{F}} + \text{diag}\{a_0\}$ . Here,  $\mathcal{L}_{\mathcal{F}}$  is the Laplacian matrix associated with  $\mathcal{G}_{\mathcal{F}}$ , the subgraph of the followers. One can then verify that the compact form of (3) can be written as:

$$\dot{q}(t) = -\alpha(H \otimes I_p)(q(t) - \mathbf{1}_n \otimes q_0) + s(t) \quad (5)$$

The following lemmas will later be used for stability analysis of the networked system.

**Lemma 3:** ([19]) Suppose that, for the system (5), the graph  $\bar{\mathcal{G}}$  contains a directed spanning tree. Then system (5) is input-to-state stable with respect to input  $s(t)$ . If  $s(t) \rightarrow \mathbf{0}_p$  as  $t \rightarrow \infty$ , then  $\dot{q}_i(t) \rightarrow \mathbf{0}_p$  and  $q_i(t) \rightarrow q_0$  as  $t \rightarrow \infty$ .

Note that the proof of the above lemma is part of the proof of Corollary 3.7 in [19].

**Lemma 4:** (From [20]) If a differential equation  $f(t)$  satisfies  $f(t), \dot{f}(t) \in L_\infty$ , and  $f(t) \in L_p$  for some value of  $p \in [1, \infty)$ , then  $f(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

From P(6) and the definition of  $\dot{q}_{ri}$ , we obtain

$$M_i(q_i)\ddot{q}_{ri} + C_i(q_i, \dot{q}_i)\dot{q}_{ri} + g_i(q_i) = Y_i(q_i, \dot{q}_i, \ddot{q}_{ri}, \dot{q}_{ri})\Theta_i, \quad i = 1, \dots, n \quad (6)$$

Note that  $\Theta_i$  is an unknown but constant vector for agent  $i$ . Let  $\hat{\Theta}_i(t)$  be the estimate of  $\Theta_i$  at time  $t$ . We update  $\hat{\Theta}_i(t)$  by the following adaptation law:

$$\dot{\hat{\Theta}}_i(t) = -\Lambda_i Y_i^T(t) s_i(t), \quad i = 1, \dots, n \quad (7)$$

where  $\Lambda_i$  is a symmetric positive-definite matrix.

The control algorithm is now proposed. Let the triggering time sequence of agent  $i$  be  $t_0^i, t_1^i, \dots, t_k^i, \dots$  with  $t_0^i := 0$ . The event-triggered controller for follower agent  $i$  is designed as:

$$\tau_i(t) = -K_i s_i(t_k^i) + Y_i(t_k^i) \hat{\Theta}_i(t_k^i), \quad i = 1, \dots, n, \quad t \in [t_k^i, t_{k+1}^i) \quad (8)$$

where  $K_i > 0$  is a symmetric positive definite matrix. It is observed that the control torque remains constant in the time interval  $[t_k^i, t_{k+1}^i)$ , i.e.  $\tau_i(t)$  is a piecewise-constant function in time. From the definitions of  $q_{ri}$  and  $s_i$ , calculations show that the system in (1) can be written as

$$M_i(q_i)\dot{s}_i(t) + C_i(q_i, \dot{q}_i)s_i(t) = -K_i s_i(t_k^i) + Y_i(t_k^i) \hat{\Theta}_i(t_k^i) - Y_i(t) \Theta_i \quad (9)$$

Before the trigger function is presented, we define two types of measurement errors:

$$e_i(t) = s_i(t_k^i) - s_i(t);$$

$$\varepsilon_i(t) = Y_i(t_k^i) \hat{\Theta}_i(t_k^i) - Y_i(t) \hat{\Theta}_i(t); \quad (10)$$

The trigger function is proposed as follows:

$$f_i(\varepsilon_i(t), e_i(t)) = \|\varepsilon_i(t)\| + \lambda_{\max}(K_i) \|e_i(t)\| - \frac{\gamma_i}{2} \lambda_{\min}(K_i) \|s_i(t)\| - \mu_i(t) \quad (11)$$

where  $0 < \gamma_i < 1$ ,  $\mu_i(t) = \sigma_i \sqrt{\lambda_{\min}(K_i)} \exp(-\kappa_i t)$  with  $\sigma_i > 0$  and  $0 < \kappa_i < 1$ . The  $k$ -th event for agent  $i$  is triggered as soon as the trigger condition  $f_i(\varepsilon_i(t), e_i(t)) = 0$  is fulfilled at  $t = t_k^i$ . For  $t \in [t_k^i, t_{k+1}^i)$ , the control input is  $\tau_i(t) = \tau_i(t_k^i)$ ; the control input is updated when the next event is triggered. Furthermore, every time an event is triggered, and in accordance with their definitions, the measurement errors  $\varepsilon_i(t)$  and  $e_i(t)$  are reset to be equal to zero. Thus  $f_i(\varepsilon_i(t), e_i(t)) \leq 0$  for all  $t \geq 0$ .

We define Zeno behavior as follows. Let a finite time interval be  $t_Z = [a, b]$  where  $0 \leq a < b < \infty$ . If, for some finite  $k \geq 0$ , the sequence of event triggers  $\{t_k^i, \dots, t_\infty^i\} \in [a, b]$  then the system exhibits Zeno behavior. Thus, one can prove that Zeno behavior does not occur for  $t \in [0, b]$  by showing that for all  $k \geq 0$  there holds  $t_{k+1}^i - t_k^i \geq \xi$  where  $\xi > 0$  is a strictly positive constant.

**Remark 1:** Henceforth, we refer to the function  $\mu_i(t)$  as an offset, and is designed to ensure the controller avoids Zeno behavior throughout the evolution of system (9).

We now present our main result.

**Theorem 1:** Consider the multi-agent system (1) with control law (8). If  $\mathcal{G}$  contains a directed spanning tree with the leader as the root vertex (and thus with no incoming edges), then leader-follower consensus ( $\|q_i - q_0\| \rightarrow 0$  and  $\|\dot{q}_i\| \rightarrow 0$ ,  $i = 1, \dots, n$ ) is globally asymptotically achieved as  $t \rightarrow \infty$  and no agent will exhibit Zeno behavior.

*Proof:* We divide our proof into two parts. In the first part, we focus on the stability analysis of the system (9). Notice that (9) is non-autonomous in the sense that it is not self-contained ( $M_i, C_i$  depend on  $q_i$  and  $q_i, \dot{q}_i$  respectively). However, study of a Lyapunov-like function shows leader-follower consensus is achieved. In the second part, analysis is provided to show the exclusion of Zeno behavior.

1) *Stability analysis:* In this part, we make use of abuse of notation by omitting the argument of time  $t$  for time-dependent functions when appropriate, e.g.  $q_i$  denotes  $q_i(t)$ .

Consider the following Lyapunov-like function

$$V = \frac{1}{2} \sum_{i=1}^N s_i^T M_i(q_i) s_i + \frac{1}{2} \sum_{i=1}^N \tilde{\Theta}_i^T \Lambda_i^{-1} \tilde{\Theta}_i \quad (12)$$

where

$$\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i \quad (13)$$

The derivative of  $V$  along the solution of (9) is

$$\begin{aligned} \dot{V} &= \frac{1}{2} \sum_{i=1}^N s_i^T \dot{M}_i(q_i) s_i + \sum_{i=1}^N s_i^T M_i(q_i) \dot{s}_i + \sum_{i=1}^N \tilde{\Theta}_i^T \Lambda_i^{-1} \dot{\tilde{\Theta}}_i \\ &= \sum_{i=1}^N s_i^T \left( \frac{1}{2} \dot{M}_i(q_i) - C_i(q_i, \dot{q}_i) \right) s_i - \sum_{i=1}^N s_i^T K_i s_i(t_k^i) \\ &\quad + \sum_{i=1}^N s_i^T Y_i(t_k^i) \hat{\Theta}_i(t_k^i) - \sum_{i=1}^N s_i^T Y_i \Theta_i + \sum_{i=1}^N \tilde{\Theta}_i^T Y_i^T s_i \end{aligned}$$

From P(4) we have  $\frac{1}{2} \dot{M}_i(q_i) - C_i(q_i)$  is skew-symmetric and

with  $\Theta_i = \tilde{\Theta}_i + \hat{\Theta}_i$ , we obtain

$$\begin{aligned} \dot{V} &= - \sum_{i=1}^N s_i^T K_i s_i(t_k^i) + \sum_{i=1}^N s_i^T Y_i(t_k^i) \hat{\Theta}_i(t_k^i) \\ &\quad - \sum_{i=1}^N s_i^T Y_i(\tilde{\Theta}_i + \hat{\Theta}_i) + \sum_{i=1}^N \tilde{\Theta}_i^T Y_i^T s_i \end{aligned}$$

By recalling the definition of  $e_i$  and  $\varepsilon_i$  in (10), we have

$$\dot{V} = - \sum_{i=1}^N s_i^T K_i s_i - \sum_{i=1}^N s_i^T K_i e_i + \sum_{i=1}^N s_i^T \varepsilon_i$$

Since  $K_i$  is a symmetric positive definite matrix, the upper bound of  $\dot{V}$  is expressed as

$$\begin{aligned} \dot{V} &\leq - \sum_{i=1}^N \lambda_{\min}(K_i) \|s_i\|^2 + \sum_{i=1}^N \lambda_{\max}(K_i) \|s_i\| \|e_i\| \\ &\quad + \sum_{i=1}^N \|s_i\| \|\varepsilon_i\| \end{aligned}$$

Note that the trigger condition  $f_i(\varepsilon_i(t), e_i(t)) = 0$  guarantees that  $\|e_i\| + \lambda_{\max}(K_i) \|e_i\| \leq \frac{\gamma_i}{2} \lambda_{\min}(K_i) \|s_i\| + \mu_i(t)$  holds throughout the evolution of system (9). By further introducing the definition of  $\mu_i(t)$  in (11), we obtain

$$\begin{aligned} \dot{V} &\leq - \sum_{i=1}^N \lambda_{\min}(K_i) \|s_i\|^2 + \sum_{i=1}^N \frac{\gamma_i}{2} \lambda_{\min}(K_i) \|s_i\|^2 \\ &\quad + \sum_{i=1}^N \sqrt{\lambda_{\min}(K_i)} \|s_i\| \sigma_i \exp(-\kappa_i t) \end{aligned}$$

Because there holds  $|xy| \leq \frac{\gamma_i}{2} x^2 + \frac{1}{2\gamma_i} y^2, \forall x, y \in \mathbb{R}$ , for  $0 < \gamma_i < 1$ , analysis of the right hand side of the above inequality implies that  $\dot{V}$  can be further upper bounded as

$$\dot{V} \leq \sum_{i=1}^N (\gamma_i - 1) \lambda_{\min}(K_i) \|s_i\|^2 + \sum_{i=1}^N \frac{\sigma_i^2}{2\gamma_i} \exp(-2\kappa_i t) \quad (14)$$

Integrating both sides of (14) for any  $t > 0$  yields:

$$\begin{aligned} V + \sum_{i=1}^N (1 - \gamma_i) \lambda_{\min}(K_i) \int_0^t \|s_i(\tau)\|^2 d\tau \\ \leq V(0) + \sum_{i=1}^N \frac{\sigma_i^2}{4\gamma_i \kappa_i} \end{aligned} \quad (15)$$

which implies that  $V$  is bounded. Since  $V$  is bounded, according to (12), both  $s_i$  and  $\tilde{\Theta}_i(t)$ , for all  $i \in \{1, \dots, n\}$ , are bounded. Now we return to (6) and obtain that

$$\|Y_i \Theta_i\| \leq \|M_i\| \|\ddot{q}_{ri}\| + \|C_i\| \|\dot{q}_{ri}\| + \|g_i\|$$

By recalling that the linear system (5) is input-to-state stable and the fact that  $s$  is bounded, we conclude that  $q_i$  and  $\dot{q}_i$  are both bounded. Because  $q_i$  and  $\dot{q}_i$  are bounded then, from their definitions, so are  $\dot{q}_{ri}$  and  $\ddot{q}_{ri}$ . Then from P(2), P(3) and P(5), the assumed properties of Euler-lagrange equations, we have that  $\|Y_i\|$  is upper bounded by a positive value. From the above conclusions, it is straightforward to see that the

right hand side of (9),  $M_i$ ,  $C_i$  and  $s_i$  are all bounded. We thus obtain that  $\dot{s}_i$  is bounded. From this, it is obvious that  $s_i, \dot{s}_i \in L_\infty$ . Turning to (15), it follows that

$$\sum_{i=1}^N (1 - \gamma_i) \lambda_{\min}(K_i) \int_0^t \|s_i(\tau)\|^2 d\tau \leq V(0) + \sum_{i=1}^N \frac{\sigma_i^2}{4\gamma_i \kappa_i} \quad (16)$$

which indicates that  $\int_0^t \|s_i(\tau)\|^2 d\tau$  is bounded and thus  $s_i \in L_2$ . By applying Lemma 4, we have that  $s_i \rightarrow \mathbf{0}_p$  as  $t \rightarrow \infty$ . Then by applying Lemma 3, we conclude that  $q_i - q_0 \rightarrow \mathbf{0}_p$  and  $\dot{q}_i \rightarrow \mathbf{0}_p$  as  $t \rightarrow \infty$ . The leader-follower objective is globally asymptotically achieved.

2) *Absence of Zeno behavior*: Let  $\xi = t_{k+1}^i - t_k^i$  denote the lower bound of the inter-event interval. In this part of the proof, we show that  $\xi$  is strictly positive and thus no Zeno behavior can occur. For any  $k \geq 0$ , and any  $i \in \{1, \dots, n\}$ , consider the time interval  $[t_k^i, t_{k+1}^i]$ . From the definition of  $e_i(t)$  in (10) and the fact that  $s_i(t_k^i)$  is a constant, we observe that the derivative of  $\|e_i(t)\|$  with respect to time satisfies

$$\frac{d}{dt} \|e_i(t)\| \leq \|\dot{s}_i(t)\| \quad (17)$$

Note that the boundedness of  $\dot{s}_i(t)$  has been proved in *Part 1*. By letting a positive constant  $B_e$  represent the upper bound of  $\|\dot{s}_i(t)\|$ , we obtain

$$\frac{d}{dt} \|e_i(t)\| \leq B_e$$

Analysis similar to (17) shows that the derivative of  $\|\varepsilon_i(t)\|$  satisfies

$$\begin{aligned} \frac{d}{dt} \|\varepsilon_i(t)\| &\leq \left\| \frac{d}{dt} (Y_i(t) \hat{\Theta}_i(t)) \right\| \\ &= \left\| \dot{Y}_i(t) \hat{\Theta}_i(t) + Y_i(t) \dot{\hat{\Theta}}_i(t) \right\| \end{aligned}$$

From Assumption (1),  $\dot{Y}_i$  is bounded. *Part 1* of our stability analysis concluded that both  $Y_i(t)$  and  $\hat{\Theta}_i(t)$  are bounded. Lastly, from the definition of  $\hat{\Theta}_i(t)$  in (7) and the fact that  $Y_i(t)$  and  $s_i(t)$  are bounded, it is easy to obtain that  $\dot{\hat{\Theta}}_i(t)$  is bounded. According to the above discussions, we have

$$\frac{d}{dt} \|\varepsilon_i(t)\| \leq B_\varepsilon$$

where  $B_\varepsilon$  is a positive constant and indicates the upper bound of the time derivative of  $\|\varepsilon_i(t)\|$ . Let  $B = \max\{B_\varepsilon, \lambda_{\max}(K_i) B_e\}$ . It follows that

$$\|\varepsilon_i(t)\| + \lambda_{\max}(K_i) \|e_i(t)\| \leq 2 \int_{t_k^i}^t B dt = 2B(t - t_k^i) \quad (18)$$

for  $t \in [t_k^i, t_{k+1}^i)$  and for any  $k$ . Recall the trigger function (11) and that both  $\varepsilon_i(t)$  and  $e_i(t)$  are reset to zero at  $t_k^i$ . It follows that the next event time  $t_{k+1}^i$  is determined by the changing rates of  $\varepsilon_i(t)$  and  $e_i(t)$ , and the threshold  $s_i(t) + \mu_i(t)$ . It is obvious that

$$\|\varepsilon_i(t)\| + \lambda_{\max}(K_i) \|e_i(t)\| = \frac{\gamma_i}{2} \lambda_{\min}(K_i) \|s_i(t)\| + \mu_i(t)$$

holds at the next trigger time  $t_{k+1}^i$ . In *Part 1* we conclude that  $s_i(t) \rightarrow \mathbf{0}_p$  as  $t \rightarrow \infty$  but notice that in the evolution of the system,  $s_i(t) = \mathbf{0}_p$  may also hold at  $t_{k+1}^i$ . However, this does not imply leader-follower consensus is reached as  $\dot{s}_i(t)$  can be nonzero at  $t_{k+1}^i$ . Consequently, at  $t_{k+1}^i$ , the triggering of the event can only occur according to the following two cases:

- Case 1: If  $\|s_i(t_{k+1}^i)\| \neq 0$ , the equality  $\|\varepsilon_i(t_{k+1}^i)\| + \lambda_{\max}(K_i) \|e_i(t_{k+1}^i)\| = \frac{\gamma_i}{2} \lambda_{\min}(K_i) \|s_i(t_{k+1}^i)\| + \mu_i(t_{k+1}^i)$  is satisfied.
- Case 2: If  $\|s_i(t_{k+1}^i)\| = 0$ , the equality  $\|\varepsilon_i(t_{k+1}^i)\| + \lambda_{\max}(K_i) \|e_i(t_{k+1}^i)\| = \mu_i(t_{k+1}^i)$  is satisfied.

We emphasize again that  $\varepsilon_i(t)$  and  $e_i(t)$  are reset to zero at  $t_k^i$ . Compare the above two cases, and note that  $\|s_i(t_{k+1}^i)\| > 0$  for any  $\|s_i(t_{k+1}^i)\| \neq 0$ . It is straightforward to conclude that it takes longer for the quantity  $\|\varepsilon_i(t)\| + \lambda_{\max}(K_i) \|e_i(t)\|$  to increase to be equal to the quantity  $\frac{\gamma_i}{2} \lambda_{\min}(K_i) \|s_i(t_{k+1}^i)\| + \mu_i(t_{k+1}^i)$  (i.e. Case 1) than to increase to be equal to the quantity  $\mu_i(t_{k+1}^i)$  (i.e. Case 2) and thus triggering an event and resetting  $\varepsilon_i(t)$  and  $e_i(t)$ . This implies that  $\xi_{Case 2} < \xi_{Case 1}$  and proving that there exists a strictly positive  $\xi_{Case 2}$  leads to the conclusion that no Zeno behavior occurs. According to (18),

$$2B\xi_{Case 2} \geq \mu_i(t) = \sigma_i \sqrt{\lambda_{\min}(K_i)} \exp(-\kappa_i(t_k^i + \xi_{Case 2}))$$

This implies that the inter-event time  $\xi_{Case 2}$  is lower bounded by the solution of the following equation

$$2B\xi_{Case 2} = \sigma_i \sqrt{\lambda_{\min}(K_i)} \exp(-\kappa_i(t_k^i + \xi_{Case 2}))$$

The solution is time-dependent and strictly positive at any finite time because  $B$  is upper bounded. Zeno behavior is thus excluded for all agents. ■

**Remark 2:** The Lipschitz condition in Assumption 1 is required to ensure that  $\|\dot{Y}_i\|$  is bounded for bounded values of  $\ddot{q}_i, \dot{q}_i, \ddot{q}_i$  and  $q_i$ . This in turn ensures that  $\frac{d}{dt} \|\varepsilon_i(t)\|$  is upper bounded, which is required to prove that no Zeno behavior occurs. Specifically, the piecewise-constant nature of both the controller (8), and the adaptive parameter estimation (7) suggests that in order to avoid Zeno behavior,  $\|\dot{Y}_i\|$  be bounded for bounded values of the state trajectory. In future work, we will consider relaxation of Assumption 1 to include more general forms of the Euler-Lagrange equations.

## IV. SIMULATIONS

In this section, we provide a simulation to demonstrate our event-triggered control algorithm for application to industrial robotic manipulators. We assume that all two-link manipulators share the same dynamic models and parameters.

The Euler-Lagrange equation for the  $i$ -th two-link manipulator is given as [21]:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_i^{(1)} \\ \ddot{q}_i^{(2)} \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_i^{(1)} \\ \dot{q}_i^{(2)} \end{bmatrix} + \begin{bmatrix} g_i^{(1)} \\ g_i^{(2)} \end{bmatrix} = \begin{bmatrix} \tau_i^{(1)} \\ \tau_i^{(2)} \end{bmatrix}$$

TABLE I: Agents' initial states used in simulation

	$q_i^{(1)}(0)$	$q_i^{(2)}(0)$	$\dot{q}_i^{(1)}(0)$	$\dot{q}_i^{(2)}(0)$
Agent 0	$\pi/6$	$\pi/3$	0.0	0.0
Agent 1	$\pi/5$	$\pi/6$	0.8	0.2
Agent 2	$\pi/6$	$\pi/4$	-0.2	0.3
Agent 3	$\pi/9$	$\pi/6$	0.6	-0.4
Agent 4	$\pi/8$	$\pi/4$	0.5	0.1
Agent 5	$\pi/9$	$\pi/6$	0.1	0.0

The elements in  $M$ ,  $C$  matrices and  $g$  vector are given below:

$$M_{11} = (m_1 + m_2)d_1^2 + m_2d_2^2 + 2m_2d_1d_2 \cos(q_i^{(2)})$$

$$M_{12} = M_{21} = m_2(d_2^2 + d_1d_2 \cos(q_i^{(2)}))$$

$$M_{22} = m_2d_2^2$$

$$C_{11} = -m_2d_1d_2 \sin(q_i^{(2)})\dot{q}_i^{(2)}$$

$$C_{12} = -m_2d_1d_2 \sin(q_i^{(2)})\dot{q}_i^{(2)} - m_2d_1d_2 \sin(q_i^{(2)})\dot{q}_i^{(1)}$$

$$C_{21} = m_2d_1d_2 \sin(q_i^{(2)})\dot{q}_i^{(1)}$$

$$C_{22} = 0$$

$$g_i^{(1)} = (m_1 + m_2)gd_1 \sin(q_i^{(1)}) + m_2gd_2 \sin(q_i^{(1)} + q_i^{(2)})$$

$$g_i^{(2)} = m_2gd_2 \sin(q_i^{(1)} + q_i^{(2)})$$

where  $g$  is the acceleration due to gravity,  $d_1$  and  $d_2$  are lengths of the 1<sup>st</sup> and 2<sup>nd</sup> links of the manipulator, respectively;  $m_1$  and  $m_2$  are mass of the 1<sup>st</sup> and 2<sup>nd</sup> of the manipulator. The physical parameters of each manipulator are selected as  $g = 9.8 \text{ m/s}^2$ ,  $d_1 = 1.5 \text{ m}$ ,  $d_2 = 1 \text{ m}$ ,  $m_1 = 1 \text{ kg}$ ,  $m_2 = 2 \text{ kg}$ . On the next page, the uncertain parameter vector  $\Theta_i$  for each manipulator is given in (21) and the regression matrix is given in (22). Note that  $\Theta_i$  is unknown for manipulator  $i$ . It can be verified that all the elements in (22) satisfy Assumption 1.

The directed sensing graph  $\mathcal{G}$  associated with the five follower manipulators and the leader manipulator has the following weighted Laplacian

$$\bar{\mathcal{L}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2.55 & -1.55 & 0 & 0 & 0 \\ -1 & 0 & 3.1 & -2.1 & 0 & 0 \\ 0 & 0 & -2.1 & 2.1 & 0 & 0 \\ 0 & -1.35 & -1.75 & -2.35 & 5.45 & 0 \\ 0 & 0 & 0 & -2.9 & -1.25 & 4.15 \end{bmatrix}$$

and it contains a directed spanning tree rooted at  $v_0$ . There are no incoming edges to  $v_0$ . The control gain matrix (in (8)) for all follower manipulators is chosen as  $K_i = I_2$ . The parameter  $\gamma_i$  required in the trigger function (11) is selected as 0.6 for manipulator  $i = 1, \dots, 5$ . Lastly,  $\mu_i(t) = 5 \exp(-0.6t)$  is used in the trigger functions for all manipulators in the follower network. The initial states of each manipulator are shown in Table I.

Fig. 1 shows the rendezvous of the generalized coordinates  $q_i^{(1)}, q_i^{(2)}, i = 1, \dots, 5$  to agent 0 (leader). In Fig. 2, we can see that generalised velocities of all the follower manipulators tend to zero. The piecewise-constant control torques are

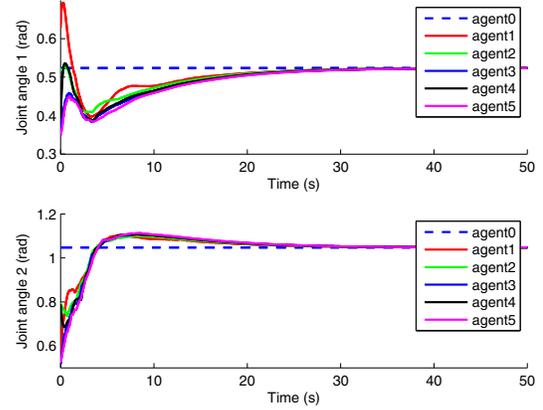


Fig. 1: Plot of generalized coordinates

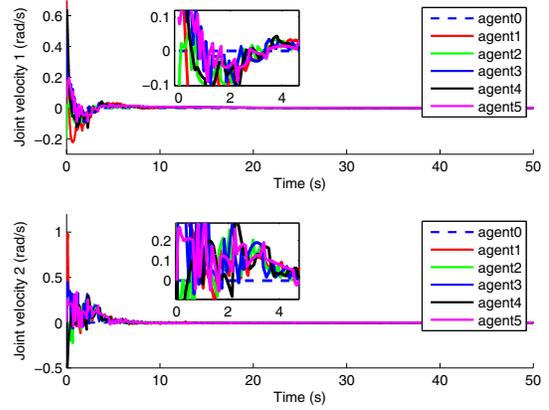


Fig. 2: Plot of generalized velocities

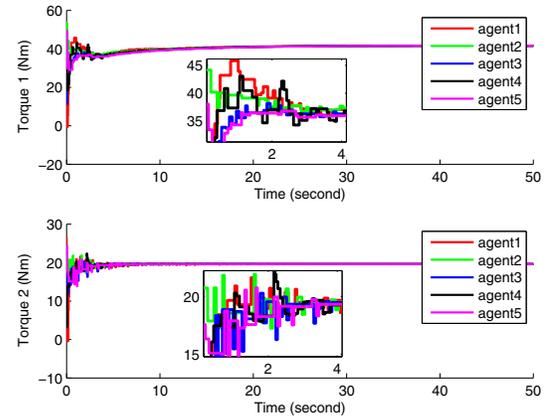


Fig. 3: Plot of control torque

$$\Theta_i = [(m_1 + m_2)d_1^2 + m_2d_2^2 \quad m_2d_1d_2 \quad m_2d_2^2 \quad (m_1 + m_2)gd_1 \quad m_2gd_2]^T \quad (21)$$

$$Y_i = \begin{bmatrix} x_1 & 0 \\ 2x_1 \cos(q_i^{(2)}) + x_2 \cos(q_i^{(2)}) - y_1 \sin(q_i^{(2)})\dot{q}_i^{(2)} - y_2 \sin(q_i^{(2)})\dot{q}_i^{(2)} - y_2 \sin(q_i^{(2)})\dot{q}_i^{(1)} & x_1 \cos(q_i^{(2)}) + y_1 \sin(q_i^{(2)})\dot{q}_i^{(1)} \\ x_2 & x_1 + x_2 \\ \sin(q_i^{(1)}) & 0 \\ \sin(q_i^{(1)} + q_i^{(2)}) & \sin(q_i^{(1)} + q_i^{(2)}) \end{bmatrix}^T \quad (22)$$

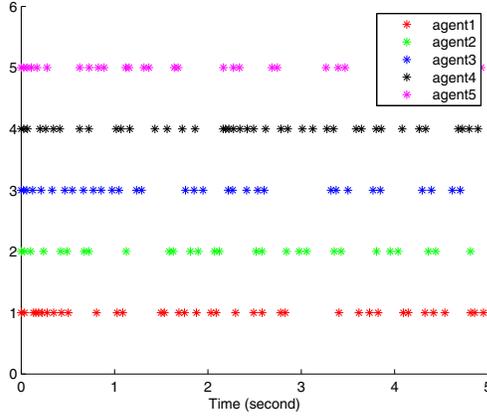


Fig. 4: Plot of event instants

shown in Fig. 3. To ensure figure clarity, Fig. 4 shows event instants for each follower from  $t = 0$  to  $t = 5s$  only.

## V. CONCLUSIONS

In this paper, we proposed a distributed event-triggered control based on the commonly used adaptive algorithms within the framework of networked Euler-Lagrange systems. By showing that the trajectories of the networked system remain bounded, we are able to conclude that the coordination objective is globally asymptotically achieved. A trigger function is designed to ensure stability and exclude the possibility of Zeno behavior. We verify this by deriving an explicit lower bound on the time interval between events. Simulations are provided. Future work will involve several directions. A major focus is to relax the condition of continuous measurement of relative generalized coordinates. Other future work include modelling real-world network effects as time delay in the network or quantization of measured variables.

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