Near-Field Broadband Beamformer Design via Multidimensional Semi-Infinite Linear Programming Techniques

Ka Fai Cedric Yiu, Xiaoqi Yang, Sven Nordholm, and Kok Lay Teo

***Abstract*—Broadband microphone arrays has important applications such as hands-free mobile telephony, voice interface to personal computers and video conference equipment. This problem can be tackled in different ways. In this paper, a general broadband beamformer design problem is considered. The problem is posed as a Chebyshev minimax problem. Using the l_1-norm measure or the real rotation theorem, we show that it can be converted into a semi-infinite linear programming problem. A numerical scheme using a set of adaptive grids is applied. The scheme is proven to be convergent when a certain grid refinement is used. The method can be applied to the design of multidimensional digital finite-impulse response (FIR) filters with arbitrarily specified amplitude and phase.**

I. Introduction

HANDS-FREE audio devices are popular for video conferences, mobile telephony and computer applications. Microphone arrays are usually applied to capture speech under varying acoustic conditions. Most of these applications are near-field applications, which means the aperture of the array is about the same size as the distance from a user to the array. Efficient design methods for near-field broadband beamformer are therefore essential. The traditional way is to consider the minimax design of one-dimensional (1-D) or two-dimensional (2-D) linear phase digital FIR filters via either a linear programming technique [1]–[4] or an exchange algorithm [5], [6]. These techniques are implemented on a single grid and require linear phase.

Recently, more general methods have been considered for the design of near-field multidimensional digital FIR filters for arbitrary amplitudes and phases. In [7] and [8], the near-field-far-field reciprocity relationship is derived and applied to design near-field beamformers via far-field design techniques. Another interesting approach is presented in [9] which makes use of a signal propagation vector representing an ideal point source of acoustic radiation. For situations in which the desired response is known, multidimensional filter design techniques can be applied. In [10], the minimax problem is formulated as a quadratic programming problem and the SQP method is applied. Another method has been explored in [11] where the minimax problem is formulated as a unconstrained nonlinear optimization with the use of a penalty function. This method is modified later in [12] by replacing the penalty function with a root-catching method. However, because of the nonlinearity in these formulations these methods are not very computationally effective.

As pointed out in [10], a quadratic term appears once the linear phase assumption is dropped. However, the quadratic term arises also because the l_2-norm is used as a measure for the magnitude of a complex number. In order to reduce the nonlinearity, the real rotation theorem or alternative norms can be applied instead. In this paper, we use the l_1-norm and the real rotation theorem [13], [14]. With the use of the l_1-norm or the real rotation theorem, the minimax problem can be transformed into an equivalent semi-infinite linear programming problem. Thus, linear programming techniques can again be used. A numerical scheme using a set of adaptive grids is proposed. We then extend the theory described in [15] to show that the adaptive grid scheme gives convergence to the semi-infinite linear programming problem. To be more specific, the theory is extended here to the case with multidimensional arguments and multiple constraints. Finally, the adaptive grid scheme is modified further to allow for the inclusion of near-active constraints only, which greatly reduces the number of linear constraints as the grid is refined. Numerical results are included to illustrate the effectiveness and efficiency of the proposed method.

II. Formulation

Let $G_d(\boldsymbol{r}, f)$ and $G(\boldsymbol{r}, f)$ be the specified desired response and the actual response of the broadband beamformer, respectively, where $\boldsymbol{r}$ is the position vector and f is the frequency. Assume that the array has N elements. Let each microphone signal be sampled at a rate of f_s; and let a L-tap FIR filter be used behind each microphone element. Since we are interested in the frequency response in the nearfield, reflected waves will have less effect and are therefore neglected in the present study. Using a simple spherical model, the transfer function from the source point $\boldsymbol{r}$ to the ith element of the broadband beamformer is given by

$$A_i(\boldsymbol{r}, f) = \frac{1}{\|\boldsymbol{r} - \boldsymbol{r}_i\|} e^{\frac{-j2\pi f \|\boldsymbol{r} - \boldsymbol{r}_i\|}{c}} \tag{1}$$

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where c is the sound speed. The array response vector is therefore given by

$$\boldsymbol{a}(\boldsymbol{r},f) = (A_1(\boldsymbol{r},f),\cdots,A_N(\boldsymbol{r},f))^T \tag{2}$$

while the filter response vector is written as

$$\boldsymbol{d}_0(f) = \left(1, e^{\frac{-j2\pi f}{f_s}},\cdots,e^{\frac{-j2\pi f(L-1)}{f_s}}\right)^T. \tag{3}$$

Denote the filter weights by $\boldsymbol{w}$, the actual response is given by

$$G(\boldsymbol{r},f) = \boldsymbol{w}^T\boldsymbol{d}(\boldsymbol{r},f), \quad \boldsymbol{d}(\boldsymbol{r},f) = \boldsymbol{a}(\boldsymbol{r},f)\otimes\boldsymbol{d}_0(f), \quad \boldsymbol{w}\in\mathbf{R}^n$$

in which $\otimes$ denotes the Kronecker product and $n = N \times L$.

Consider a region $\Omega = \cup_{i=1}^m \Omega_i$ in the space-frequency domain, where each Ω_i is a convex set and $\Omega_i \cap \Omega_j = \emptyset$ for $i \neq j$. The minimax design problem can be formulated as

$$\min_{\boldsymbol{w}\in\mathbf{R}^n} \max_{(\boldsymbol{r},f)\in\Omega} \left|\boldsymbol{w}^T\boldsymbol{d}(\boldsymbol{r},f) - G_d(\boldsymbol{r},f)\right|. \tag{4}$$

Expanding the complex functions as

$$\boldsymbol{d}(\boldsymbol{r},f) = \boldsymbol{d}_1(\boldsymbol{r},f) + j\boldsymbol{d}_2(\boldsymbol{r},f),$$
$$G_d(\boldsymbol{r},f) = G_{d_1}(\boldsymbol{r},f) + jG_{d_2}(\boldsymbol{r},f)$$

and denote

$$\boldsymbol{u}(\boldsymbol{r},f) = \left(\boldsymbol{w}^T\boldsymbol{d}_1(\boldsymbol{r},f) - G_{d_1}(\boldsymbol{r},f)\right) \tag{5}$$
$$\boldsymbol{v}(\boldsymbol{r},f) = \left(\boldsymbol{w}^T\boldsymbol{d}_2(\boldsymbol{r},f) - G_{d_2}(\boldsymbol{r},f)\right) \tag{6}$$

the minimax filter design problem can be rewritten as

$$\min_{\boldsymbol{w}\in\mathbf{R}^n} \max_{(\boldsymbol{r},f)\in\Omega} |u(\boldsymbol{r},f) + jv(\boldsymbol{r},f)|. \tag{7}$$

Clearly, if the standard modulus of the complex function is used to measure the complex number, i.e.,

$$\min_{\boldsymbol{w}\in\mathbf{R}^n} \max_{(\boldsymbol{r},f)\in\Omega} \left\{u(\boldsymbol{r},f)^2 + v(\boldsymbol{r},f)^2\right\} \tag{8}$$

a quadratic term arises which increases the nonlinearity of the problem [10]–[12]. Note that this corresponds to the use of a l_2-norm to measure the complex number. Another equivalent way to formulate the problem is to make use of the real rotation theorem and the design problem can be written as

$$\min_{\boldsymbol{w}\in\mathbf{R}^n} \max_{(\boldsymbol{r},f)\in\Omega} \max_{\theta} \{u(\boldsymbol{r},f)\cos\theta + v(\boldsymbol{r},f)\sin\theta\}. \tag{9}$$

If only the angle $\theta = (0, (\pi/2), \pi, (3\pi/2))$ are used, it is just to measure the complex number by the maximum of $|u(\boldsymbol{r},f)|$ and $|v(\boldsymbol{r},f)|$, which is equivalent to the use of a l_∞-norm [16], [17].

Another norm which has a lot of favorable properties is the l_1-norm. Similar to the l_∞-norm, it bounds the l_2-norm with the upper and lower bounds as

$$2^{-\frac{1}{2}}\|\boldsymbol{x}\|_1 \leq \|\boldsymbol{x}\|_2 \leq \|\boldsymbol{x}\|_1 \quad \boldsymbol{x}\in\mathbf{R}^2. \tag{10}$$

Using the l_1-norm, the design problem becomes

$$\min_{\boldsymbol{w}\in\mathbf{R}^n} \max_{(\boldsymbol{r},f)\in\Omega} \{|u(\boldsymbol{r},f)| + |v(\boldsymbol{r},f)|\}. \tag{11}$$

Note that this problem cannot be replicated by any chance of θ in (9).

From the formulation (9) and (11), the real and imaginary parts remain to be linear. Therefore, linear programming techniques are possible to tackle these problems. Define

$$z = \max_{(\boldsymbol{r},f)\in\Omega} \max_{\theta} \{\boldsymbol{u}(\boldsymbol{r},f)\cos\theta + \boldsymbol{v}(\boldsymbol{r},f)\sin\theta\}$$

the design problem (9) is equivalent to the semi-infinite linear programming problem

$$\min_{\boldsymbol{w}\in\mathbf{R}^n, z\in\mathbf{R}} z \tag{12a}$$

subject to

$$\boldsymbol{u}(\boldsymbol{r},f)\cos\theta + \boldsymbol{v}(\boldsymbol{r},f)\sin\theta \leq z \quad \forall\theta, \forall(\boldsymbol{r},f)\in\Omega. \tag{12b}$$

To convert (11) into a semi-infinite linear programming, one way is to introduce the variable

$$z = \max_{(\boldsymbol{r},f)\in\Omega} \{|\boldsymbol{u}(\boldsymbol{r},f)| + |\boldsymbol{v}(\boldsymbol{r},f)|\}$$

and the minimax problem can be converted into

$$\min_{\boldsymbol{w}\in\mathbf{R}^n, z\in\mathbf{R}} z \tag{13a}$$

subject to

$$\boldsymbol{u}(\boldsymbol{r},f) + \boldsymbol{v}(\boldsymbol{r},f) \leq z \quad \forall(\boldsymbol{r},f)\in\Omega \tag{13b}$$
$$\boldsymbol{u}(\boldsymbol{r},f) - \boldsymbol{v}(\boldsymbol{r},f) \leq z \quad \forall(\boldsymbol{r},f)\in\Omega \tag{13c}$$
$$-\boldsymbol{u}(\boldsymbol{r},f) + \boldsymbol{v}(\boldsymbol{r},f) \leq z \quad \forall(\boldsymbol{r},f)\in\Omega \tag{13d}$$
$$-\boldsymbol{u}(\boldsymbol{r},f) - \boldsymbol{v}(\boldsymbol{r},f) \leq z \quad \forall(\boldsymbol{r},f)\in\Omega. \tag{13e}$$

An alternative and more flexible way to convert (11) into a semi-infinite linear programming problem is to control the real part and the imaginary part separately by introducing two new variables as

$$z_1 = \max_{(\boldsymbol{r},f)\in\Omega} |\boldsymbol{u}(\boldsymbol{r},f)|, \quad z_2 = \max_{(\boldsymbol{r},f)\in\Omega} |\boldsymbol{v}(\boldsymbol{r},f)|.$$

Thus, in matrix notation, the design problem can be formulated as

$$\min_{\boldsymbol{z}\in\mathbf{R}^{n+2}} \boldsymbol{b}^T\boldsymbol{z} \tag{14a}$$

subject to

$$A(\boldsymbol{r},f)z - \boldsymbol{G} \leq \boldsymbol{0} \quad \forall(\boldsymbol{r},f)\in\Omega \tag{14b}$$

where

$$A(\boldsymbol{r},f) = \begin{pmatrix} \boldsymbol{d}_1(\boldsymbol{r},f)^T & -1 & 0 \\ -\boldsymbol{d}_1(\boldsymbol{r},f)^T & -1 & 0 \\ \boldsymbol{d}_2(\boldsymbol{r},f)^T & 0 & -1 \\ -\boldsymbol{d}_2(\boldsymbol{r},f)^T & 0 & -1 \end{pmatrix},$$

$$\boldsymbol{z} = \begin{pmatrix} \boldsymbol{w} \\ z_1 \\ z_2 \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} 0 \\ \phi_1 \\ \phi_2 \end{pmatrix}, \boldsymbol{G} = \begin{pmatrix} G_{d_1}(\boldsymbol{r},f) \\ -G_{d_1}(\boldsymbol{r},f) \\ G_{d_2}(\boldsymbol{r},f) \\ -G_{d_2}(\boldsymbol{r},f) \end{pmatrix}$$

in which ϕ_1 and ϕ_2 are two different weights for the real and imaginary parts, respectively.

Assume the optimal is achieved at w^* and the acoustic of the room has changed slightly. The transfer function becomes

$\tilde{\boldsymbol{d}}(\boldsymbol{r}, f)$ while the optimal is now achieved at $\tilde{\boldsymbol{w}}^*$. Assume the perturbation is small so that $\tilde{\boldsymbol{d}} = \boldsymbol{d} + \delta\boldsymbol{d}$, $\tilde{\boldsymbol{w}}^* = \boldsymbol{w}^* + \delta\boldsymbol{w}^*$. We therefore have

$$\begin{aligned}\max_{(\boldsymbol{r},f)\in\Omega} &\left|\tilde{\boldsymbol{w}}^{*T}\tilde{\boldsymbol{d}}(\boldsymbol{r}, f) - G_d(\boldsymbol{r}, f)\right| \\ &\leq \max_{(\boldsymbol{r},f)\in\Omega} \left|\boldsymbol{w}^{*T}\boldsymbol{d}(\boldsymbol{r}, f) - G_d(\boldsymbol{r}, f)\right| \\ &\quad + \max_{(\boldsymbol{r},f)\in\Omega} \left|\tilde{\boldsymbol{w}}^{*T}\delta\boldsymbol{d}(\boldsymbol{r}, f) + \delta\tilde{\boldsymbol{w}}^{*T}\boldsymbol{d}(\boldsymbol{r}, f)\right|. \end{aligned} \quad (15)$$

It is advantageous to restrict the size of the weights

$$-1 \leq w_i \leq 1 \quad \forall i \quad (16)$$

so that any perturbations to the design conditions are not magnified by the weights significantly. This will also constrain $\delta\tilde{\boldsymbol{w}}$ within bounds. Furthermore, the magnitudes of the weights should be controlled to avoid the problem of overflow or other filter realization problems.

The complexity of the linear programming technique is rather high, especially for very long filter [18]. Therefore, the method should be applied for off-line design. However, for small perturbations to the acoustic, a real-time implementation is possible. This can be achieved by exploring the simplex algorithm. If the perturbations are small, a careful sensitivity analysis will improve the rate of re-convergence to the new optimal solution significantly. In this way, it is possible to track, e.g., small movements of the speaker, in real-time after the initial solution sought.

III. Algorithm

For a fixed set of $(\boldsymbol{r}, f)$, the design problem (14) is simply a linear programming problem. Similarly, if θ is chosen to be a fixed set, the design problem (12) is also another linear programming problem. Therefore, a discretization solution method with adaptive schemes such as [15], [19] and [20] can be used. Approximating Ω with a uniform grid containing M mesh points in each dimension results in a multi-dimensional grid, denoted by $\hat{\Omega}_M$. In order that the discretization problem obtained is a good approximation of the original problem, the integer M needs to be large. In order to determine a suitable M to solve the problem, a sequence of adaptive meshes can be applied so that the mesh is refined gradually. For convergence, an appropriate refinement scheme is to include all the previous coarse grid points in the finer meshes. As a result, the solution of solving the discrete linear programming problem can be proven to converge to the original semi-infinite linear programming problem as the mesh becomes finer and finer. The proof is given in the Appendix.

Since a simple replacement of Ω by $\hat{\Omega}_M$ leads to a linear programming problem with a large number of constraints as the mesh becomes finer and finer, it is possible to eliminate some unnecessary constraints and to consider only those near-active constraints. Although it is not easy to prove this scheme to be convergent, experiences have shown that this scheme works extremely well in practice. We shall describe and use this scheme in the following.

Let the matrix $\tilde{A}(\boldsymbol{r}, f)$ be partitioned by its rows denoted by $(\tilde{a}_1(\boldsymbol{r}, f)^T, \cdots, \tilde{a}_m(\boldsymbol{r}, f)^T)^T$, where $m = 4$ for the l_1-norm formulation and $m \geq 4$ in the real rotation formulation depending on how many discrete θ's are used. Let

$$\phi_M^i(\boldsymbol{r}, f) = \max_{(\boldsymbol{r},f)\in\hat{\Omega}_M} \left(\tilde{\boldsymbol{a}}_i(\boldsymbol{r}, f)^T\boldsymbol{z} - \boldsymbol{G}_i\right), \quad i = 1, \cdots, m$$

in which $\boldsymbol{z} \in \mathbf{R}^{n+1}$ for the real rotation formulation and $\boldsymbol{z} \in \mathbf{R}^{n+2}$ for the l_1-norm formulation under (14). Define

$$\hat{\Omega}_M^\beta = \Big\{(\boldsymbol{r}, f) \in \hat{\Omega}_M | \tilde{A}(\boldsymbol{r}, f)\boldsymbol{z} - \boldsymbol{G} \geq \left(\phi_M^1(\boldsymbol{r}, f), \cdots, \phi_M^m(\boldsymbol{r}, f)\right)^T - (\beta^1, \cdots, \beta^m)^T\Big\}$$

where $\beta^i \geq \phi_M^i(\boldsymbol{r}, f) + \epsilon$, $i = 1, \cdots, m$. We consider a modified problem

$$\min_{\boldsymbol{z}} \boldsymbol{b}^T\boldsymbol{z} \quad (17a)$$

subject to

$$\tilde{A}(\boldsymbol{r}, f)\boldsymbol{z} - \boldsymbol{G} \leq -\epsilon \quad \forall(\boldsymbol{r}, f) \in \hat{\Omega}_M^\beta. \quad (17b)$$

The number of constraints in the modified problem is much less than that of the original problem. The scheme proposed in [15] is extended here to tackle problem (17). The final algorithm can be summarized as follows.

i) Choose $\boldsymbol{z} = \boldsymbol{z}_0$, ϵ, M and M_l, where M_l is several times larger than M.
ii) Choose a β^i such that $\beta^i \geq \phi_{M_l}^i(\boldsymbol{r}, f) + \epsilon$, $i = 1, \cdots, m$.
iii) Solve the linear programming problem (17) to give $\tilde{\boldsymbol{z}}$.
iv) If ϵ is small enough, stop.

Otherwise, $\epsilon = \epsilon/10$, $M = 2M$, $\boldsymbol{z} = \tilde{\boldsymbol{z}}$, go to ii).

IV. Numercial Results

In the following examples, the weights ϕ_1 and ϕ_2 are set to one for the l_1-norm formulation (14). For the real rotation formulation (12), θ is chosen to have the following eight values

$$\theta = -\pi + (i - 1) * \frac{2\pi}{7} \quad i = 1, \cdots, 8.$$

In the near-active constraints algorithm, β was chosen such that $\beta \geq \max(\beta^1, \cdots, \beta^m)$. MATLAB version 6 is used and the library function LP using the simplex method is applied here.

The desired response function is chosen as a region that would fit into a multimedia or hands-free mobile phone application. This will include the frequency range of human voice, and a range of positions that the microphones intend to receive. To allow for the delay of the speech to reach the microphones, the desired response function in the passband region is chosen as

$$G_d(\boldsymbol{r}, f) = \exp\left(-i2\pi f\left(\frac{\|\boldsymbol{r} - \boldsymbol{r}_c\|}{c} + \frac{L-1}{2}T\right)\right) \quad (18)$$

where $\boldsymbol{r}_c$ is the coordinate for the centre element, $c = 340.9\ m/s$ and the sample increment $T = 125\ \mu s$. It is possible to optimize on the delay, but the choice here will be very close to the best possible choice. In the stopband region, $G_d(\boldsymbol{r}, f) = 0$ in order to filter out the noise. A equispaced linear array with five elements and a seven-tap FIR filter

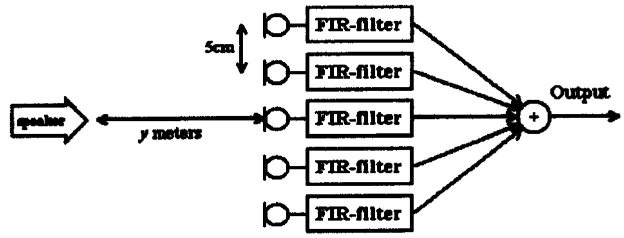

Fig. 1. Configuration of the beamformer.

TABLE I
SUMMARY OF CONVERGENCE FOR THE l_1-NORM METHOD

β	ϵ	M	NC	$\mathcal{I}$	$z_1 + z_2$
2	0.01	20	1600	0.12219	0.25113
0.1	0.001	40	2230	0.	0.30199
0.01	0.0001	40	146	0.	0.30379

behind each element is used. The element spacing is 5 cm to avoid spatial aliasing for the frequency of interest. In the examples, $\boldsymbol{r} = (x, y)$ and the beamformer is specified on an x-axis parallel with, and y meters in front of, the array. This is depicted in Fig. 1. The passband region is defined as

$$\{(x, f) : -0.4\, m \le x \le 0.4\, m,\ 0.5\, kHz \le f \le 1.5\, kHz\}$$

while the stopband regions are defined as

$$\{(x, f) : -0.4\, m \le x \le 0.4\, m,\ 2.5\, kHz \le f \le 4\, kHz\},$$
$$\{(x, f) : 1.5\, m \le |x| \le 2.5\, m,\ 0.5\, kHz \le f \le 1.5\, kHz\},$$
$$\{(x, f) : 1.5\, m \le x \le 2.5\, m,\ 2.5\, kHz \le f \le 4\, kHz\}.$$

Note that this is a union of several convex regions, and more mesh points are allocated to larger regions. A 120 × 120 grid was used to check for the convergence of the l_1-norm formulation. The following function

$$\mathcal{I} = \max_{\boldsymbol{r}, f} \left(\left| \boldsymbol{w}^T \boldsymbol{d}(\boldsymbol{r}, f) - G_d(\boldsymbol{r}, f) \right| \right) - z_1 - z_2 \qquad (19)$$

is used to monitor the convergence. For the real rotation formulation, the following function

$$\mathcal{I} = \max_{\boldsymbol{r}, f} \max_{\theta} \left(\boldsymbol{u}(\boldsymbol{r}, f) \cos\theta + \boldsymbol{v}(\boldsymbol{r}, f) \sin\theta \right) - z \qquad (20)$$

is used instead to monitor the convergence, where sixteen θ values are used. When $y = 1\ m$, the l_1-norm calculation is given in Table I and the amplitude response is shown in Fig. 2. In the table, NC stands for number of constraints. The near-active constraints scheme works quite well in reducing the number of constraints from 6400 to 2230 on a 40 × 40 mesh. The convergence monitoring function $\mathcal{I}$ has indicated that the solution has converged to the solution of the semi-infinite linear programming problem up to the machine precision on the reference grid 120 × 120. The absolute maximum and minimum of the weights is 1 and 0.029 83, respectively. By looking at the Langrange multiplier of the linear programming problem, the (x, f) coordinates correspond to the set of active linear constraints can be sought and is shown in Fig. 3. The real rotation calculation is given in Table II. Note that from the table, the convergence monitoring function does not converge to zero. It is because

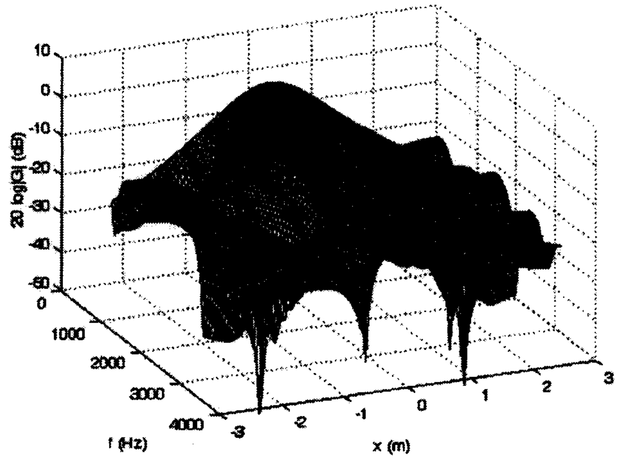

Fig. 2. Optimal design with $N = 5$, $L = 7$ and $y = 1\ m$.

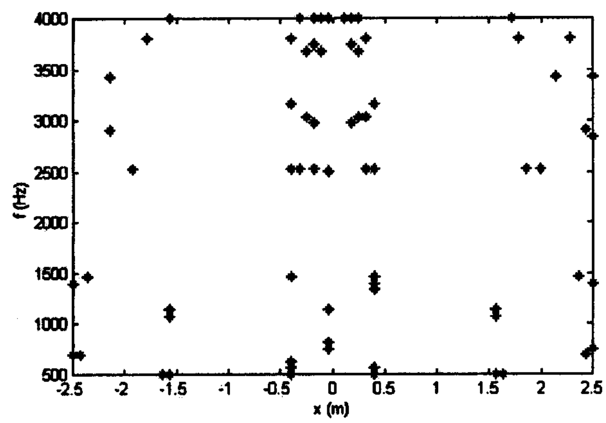

Fig. 3. Set of active points for the first example.

TABLE II
SUMMARY OF CONVERGENCE FOR THE REAL ROTATION METHOD

β	ϵ	M	NC	$\mathcal{I}$	z
2	0.01	20	3200	0.09883	0.13056
0.1	0.001	40	4688	0.05468	0.16020
0.01	0.0001	40	192	0.05378	0.16110

the set of θ is held fixed during the adaptive refinement of the grid. To refine the θ at the same time will significantly increase the number of constraints and hence the size of the linear programming problem. The final response is very similar to the one using l_1-norm is not reproduced here. The absolute maximum and minimum of the weights is 1 and 0.085 52, respectively.

The comparison between different approaches is given in Table III. From the table, both the l_1-norm approach and the real rotation approach perform quite well relative to the least-squares approach in terms of the passband gain. Moreover, the real rotation has the smallest passband ripple among these approaches. However, from comparing Table I and II, the real rotation approach requires at least twice as many constraints as the l_1-norm approach, and is therefore many times more expensive to solve.

TABLE III
COMPARISONS BETWEEN THE l_1-NORM METHOD, THE REAL ROTATION METHOD AND THE LEAST-SQUARES METHOD

$N = 5, L = 7$ & $y = 1m$	passband gain	passband ripple	stopband ripple
l_1-norm	1.01902	0.17508	-14.8244
real rotation	1.0178	0.15724	-13.8018
least-squares	0.93835	0.18308	-12.3667

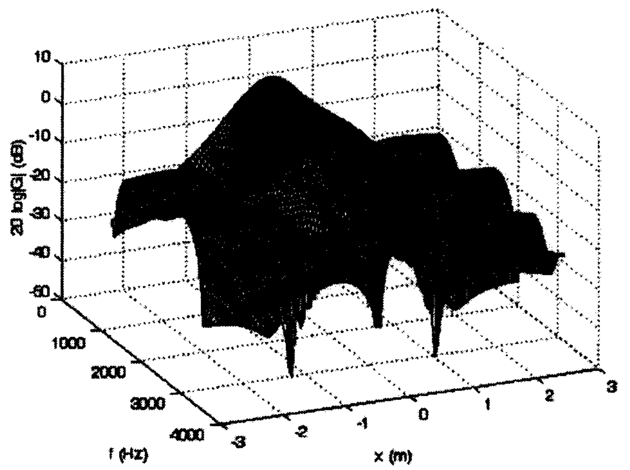

Fig. 4. Off-design performance for weights designed with $y = 1\ m$ and perturbed to $y = 0.5\ m$.

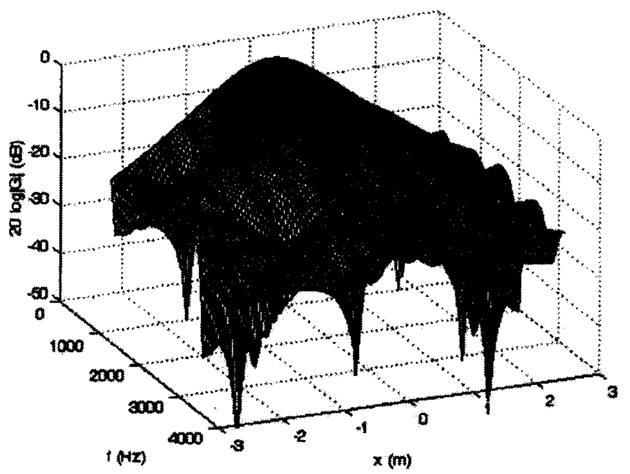

Fig. 5. Off-design performance for weights designed with $y = 1\ m$ and perturbed to $y = 1.5\ m$.

In order to study the off-design performance of the design for source moving, the filter is first designed with $y = 1\ m$ and the source is subsequently displaced to calculate the final amplitude response. Here, the source location is perturbed to $y = 0.5\ m$ and $y = 1.5\ m$ and the results are depicted in Fig. 4 and Fig. 5, respectively. The amplitude responses show that the designed weights are not sensitive to small perturbations to the source location.

Finally, the source is placed very near to the beamformer ($y = 0.2\ m$) and the number of taps are increased to $L = 21$. The

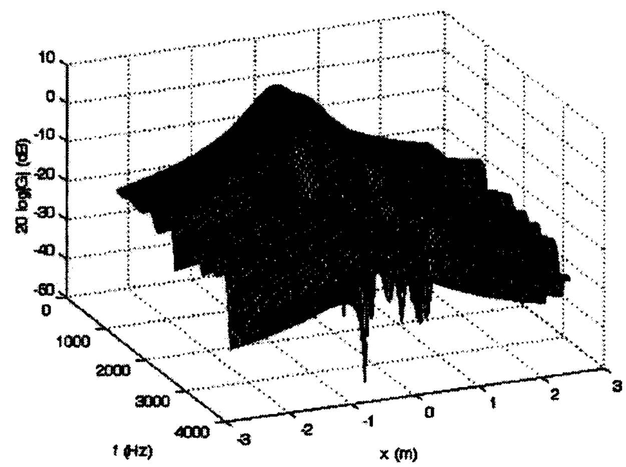

Fig. 6. Optimal design with $N = 5$, $L = 21$ and $y = 0.2\ m$.

TABLE IV
COMPARISONS BETWEEN THE l_1-NORM METHOD AND THE LEAST-SQUARES METHOD

$N = 5, L = 21$ & $y = 0.2m$	passband gain	passband ripple	stopband ripple
l_1-norm	1.01993	0.28823	-9.8975
least-squares	0.95602	0.41464	-11.4242

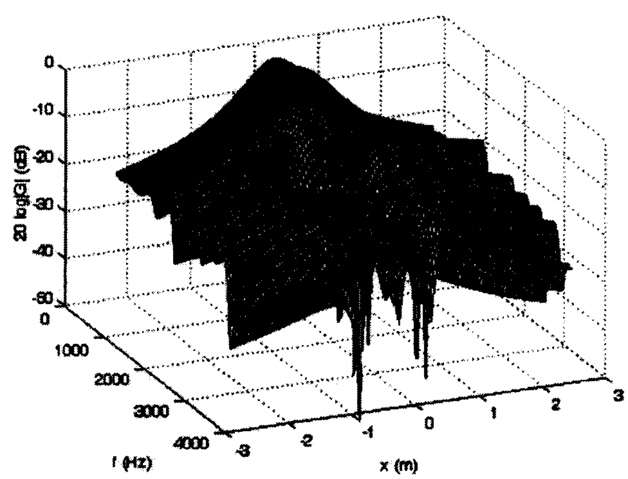

Fig. 7. Off-design performance for weights designed with $y = 0.2\ m$ and perturbed to $y = 0.3\ m$.

transition region is also shorter so that the stopband regions now become

$$\{(x,f) : -0.4\ m \leq x \leq 0.4\ m,\ 1.8\ kHz \leq f \leq 4\ kHz\},$$
$$\{(x,f) : 1.5\ m \leq |x| \leq 2.5\ m,\ 0.5\ kHz \leq f \leq 1.5\ kHz\},$$
$$\{(x,f) : 1.5\ m \leq x \leq 2.5\ m,\ 1.8\ kHz \leq f \leq 4\ kHz\}.$$

The l_1-norm approach is applied and the amplitude response is shown in Fig. 6. The absolute maximum and minimum of the weights is 1 and 0.006 51, respectively. The minimax design is compared with the least-squares design and the comparison is summarized in Table IV. The off-design performance of the design is studied by varying the source location back and fore 0.1 m. The amplitude responses are shown in Fig. 7 and 8. In this

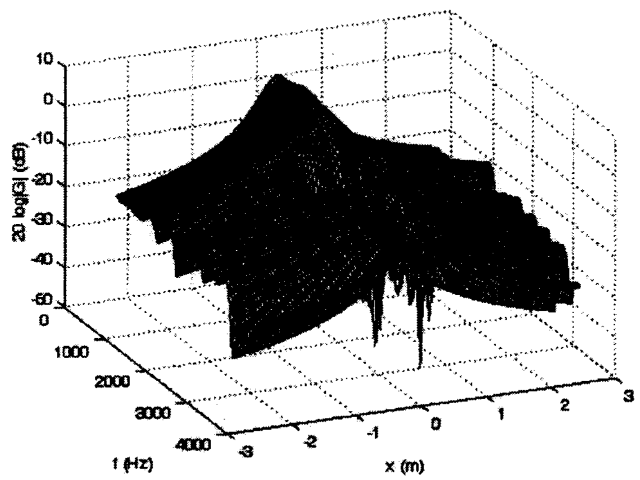

Fig. 8. Off-design performance for weights designed with $y = 0.2\ m$ and perturbed to $y = 0.1\ m$.

example, the design performs rather well under small perturbations to the source location. Moreover, since the beamformer is design over a region of passband, super-directivity would not be incurred despite of the closeness of the element spacing and the source position.

V. Conclusions

In this paper, a general broadband beamformer minimax design problem has been considered. The l_1-norm measure and the real rotation theorem have been applied to reduce the minimax problem to a semi-infinite linear programming problem. A numerical algorithm using a set of adaptive grids has been proposed. The convergence of this kind of discretization methods for a general semi-infinite problem with multiple constraints and multidimensional arguments has been proven. The scheme is then extended to include only the near-active constraints in the linear programming problem as the mesh is refined. Since the linear programming is used instead of the quadratic programming or penalty functions, the nonlinearity of the problem has been reduced and the method can make use of newly developed linear programming solvers. It would certainly be of interest to look at random perturbations to the filter responses as an extension to the present study.

Appendix

VI. Discretization Methods for SIP

In this section, the method reported in [15] will be extended for semi-infinite problems with multiple constraints and multidimensional arguments to prove that the adaptive grid scheme is a convergent method for the semi-infinite problems.

Consider the semi-infinite program (SIP)

$$\begin{aligned} &\min \quad f(x) \\ &\text{s.t.} \quad x \in X, \quad g_p(w,x) \le 0, \quad \forall w \in \Omega,\ p = 1, \cdots, P \end{aligned}$$

where X is a closed subset of $\mathbf{R}^n$, $f : \mathbf{R}^n \to \mathbf{R}$ is a locally Lipschitz function, $g_p : \mathbf{R}^k \times \mathbf{R}^n \to \mathbf{R}(p = 1, \cdots, P)$ is a locally Lipschitz function and Ω is a compact set in $\mathbf{R}^k$.

Assume that $\|\cdot\|$ is a norm in $\mathbf{R}^k$. Let the feasible set be

$$\mathcal{F} = \{x \in X : g_p(w,x) \le 0, \quad \forall w \in \Omega,\ p = 1, \cdots, P\}$$

and the interior of $\mathcal{F}$ be defined by

$$int(\mathcal{F}) = \{x \in \mathcal{F} : g_p(w,x) < 0, \quad \forall w \in \Omega,\ p = 1, \cdots, P\}$$

and x^* an optimal solution of (SIP) that is assumed to exist.

Let $\epsilon > 0$ and the following condition hold:

$$|g_p(w_1,x) - g_p(w_2,x)| \le K_p\|w_1 - w_2\|,\ p = 1, \cdots, P. \quad (21)$$

For such ϵ and K_p,$p = 1, \cdots, P$, assume that a grid of Ω, e.g., $\Omega_M (M \ge 1)$ has been obtained such that the following condition is satisfied:

$$\sup_{x \in \Omega} \inf_{\bar{w} \in \Omega \cap \Omega_N} \|w - \bar{w}\| \le \min_{1 \le p \le P} \frac{\epsilon}{K_p}. \quad (22)$$

Then, consider the following discretized problem ($\text{SIP}_{\epsilon,\Omega_M}$):

$$\min \quad f(x) \text{ s.t. } g_p(w,x) \le -\epsilon, \quad \forall w \in \Omega \cap \Omega_M,\ p = 1, \cdots, P.$$

Let

$$\mathcal{F}_{\epsilon,\Omega_M} = \{x \in X : g_p(w,x) \le -\epsilon, \quad \forall w \in \Omega \cap \Omega_M,\ p = 1, \cdots, P\}.$$

Lemma 1: Let $\epsilon > 0$ be given and K_p satisfy (21). If there is a grid Ω_M such that (22) holds, then any feasible solution for ($\text{SIP}_{\epsilon,\Omega_M}$) is feasible for (SIP).

Proof: Let x be feasible for ($\text{SIP}_{\epsilon,\Omega_M}$). Then

$$g_p(\bar{w},x) \le -\epsilon, \quad \forall \bar{w} \in \Omega \cap \Omega_M,\ p = 1 \cdots, P.$$

For $w \in \Omega$, from (22), there exists a $\bar{w} \in \Omega \cap \Omega_M$ such that

$$\|w - \bar{w}\| \le \frac{\epsilon}{K_p},\ p = 1, \cdots, P.$$

By (21), we have

$$g_p(w,x) - g_p(\bar{w},x) \le K_p\|w - \bar{w}\|,\ p = 1, \cdots, P.$$

Thus

$$\begin{aligned} g_p(w,x) &\le K_p\|w - \bar{w}\| + g_p(\bar{w},x) \\ &\le K_p\|w - \bar{w}\| - \epsilon \\ &\le K_p \frac{\epsilon}{K_p} - \epsilon \\ &\le 0. \end{aligned}$$

Therefore, $x \in \mathcal{F}$, and hence the conclusion holds.

Remark 1: Let, for example, Ω be contained in a box $[a_1, b_1] \times [a_2, b_2] \times \cdots [a_k, b_k]$ and the norm is chosen as the l_∞ norm, i.e., $\|w\|_\infty = \max_{1 \le i \le k} |w_i|$.

Assume that we partition the interval $[a_i, b_i]$ into M segments. Let $\bar{w} = (w^1, w^2, \cdots, w^k) \in \mathbf{R}^k$ be one grid point in Ω_M. Its ith component ($i = 1, \cdots, k$) can be given as

$$w^i = a_i + \frac{j(b_i - a_i)}{M}$$

where $j \in \{0, 1, \cdots, M\}$. If

$$M \geq \frac{K_p}{\epsilon} \max_{1 \leq i \leq k} |b_i - a_i|, \; p = 1, \cdots, P$$

then the relation (22) is satisfied.

In order that the relation (22) holds, the grid needs to be chosen to be fine enough, i.e., the number of points in Ω_M may be very large. Hence the number of constraints in $(\text{SIP}_{\epsilon,\Omega_M})$ is very large. So it would be inefficient to compute the subproblem $(\text{SIP}_{\epsilon,\Omega_M})$. An adaptive scheme is introduced in [15] and the references cited therein. For a given $y \in \mathbf{R}^n$, define

$$\Omega_M^\epsilon(y) = \{w \in \Omega \cap \Omega_M : g_p(w, y) > -2\epsilon, \; p = 1, \cdots, P\}.$$

We also assume that $g_p, p = 1, \cdots, P$ is Lipschitz in $x : \exists L_p > 0$ such that

$$|g_p(w, x) - g_p(w, z)| \leq L_p \|x - z\|, \quad \forall w \in \Omega, x, z \in \mathcal{F}, \forall p. \tag{23}$$

Each $t_i \in \Omega_M^\epsilon(y)$ is called an ϵ-active point (with respect to y). In general, the points in $\Omega_M^\epsilon(y)$ would be significantly smaller than that in Ω_M.Consider the following problem:

$$\left(\text{SIP}^y_{\epsilon,\Omega_M}\right) : \quad \min \quad f(x)$$
$$\text{s.t.} \quad x \in \mathcal{F}^y_{\epsilon,M} \cap B\left(y, \frac{\epsilon}{L}\right)$$

where $L = \min_{1 \leq p \leq P} L_p$ and

$$\mathcal{F}^y_{\epsilon,M} = \{x \in X : g_p(w, x) \leq -\epsilon, \; \forall w \in \Omega \cap \Omega_M^\epsilon(y), \forall p\},$$
$$B\left(y, \frac{\epsilon}{L}\right) = \left\{x \in \mathbf{R}^n : \|x - y\| \leq \frac{\epsilon}{L}\right\}.$$

Following the proof of Lemma 4.1, it is not difficult to show the next Lemma.

Lemma 2: Assume that (21), (22) and (23) hold. Then

$$\mathcal{F}^y_{\epsilon,M} \cap B\left(y, \frac{\epsilon}{L}\right) \subseteq \mathcal{F}. \tag{24}$$

The following is a conceptual convergence which generalizes the result in [15] to semi-infinite problems with multiple constraints and multidimensional arguments.

Theorem 1: Suppose that f is continuous, and (21), (22), (23) and the following nonempty interior condition are satisfied:

$$\exists \, \bar{x} \in \mathcal{F} \text{ such that } \alpha\bar{x} + (1-\alpha)x^* \in int(\mathcal{F}), \alpha \in (0, 1). \tag{25}$$

Then, for each $\epsilon > 0$, there exists $y(\epsilon)$ such that the sequence formed by the solutions $x^{*y(\epsilon)}_{\epsilon,\Omega_M}$ of $(\text{SIP}^{y(\epsilon)}_{\epsilon,\Omega_M})$ satisfies

$$\lim_{\epsilon \to 0^+} f\left(x^{*y(\epsilon)}_{\epsilon,\Omega_M}\right) = f(x^*).$$

Proof: It follows from (25) that there exists an $\bar{x} \in int(\mathcal{F})$ such that

$$x_\alpha = \alpha\bar{x} + (1-\alpha)x^* = x^* + \alpha(\bar{x} - x^*) \in int(\mathcal{F}), \quad \alpha \in (0, 1).$$

By the continuity of f, for any $\delta_1 > 0$, there exists an $\alpha_1 \in (0, 1]$ such that

$$f(x_\alpha) \leq f(x^*) + \delta_1, \quad \alpha \in (0, \alpha_1). \tag{26}$$

Choose $\alpha_2 = (\alpha_1/2)$. Then it is clear that $x_{\alpha_2} \in int(\mathcal{F})$. Thus there exists a $\delta_2 > 0$ such that $g(x_{\alpha_2}, w) < -\delta_2$ for all $w \in \Omega$. If we choose $\epsilon = \delta_2$ and Ω_M according to (22), then it follows from Lemma 1 that $x_{\alpha_2} \in \mathcal{F}_{\epsilon,\Omega_M}$. Choose $y(\epsilon)$ such that $\|y(\epsilon) - x_{\alpha_2}\| \leq \epsilon/L$. So, we have

$$x_{\alpha_2} \in \mathcal{F}_{\epsilon,M(\epsilon)} \cap B\left(y(\epsilon), \frac{\epsilon}{L}\right) \subseteq \mathcal{F}^{y(\epsilon)}_{\epsilon,M(\epsilon)} \cap B\left(y(\epsilon), \frac{\epsilon}{L}\right).$$

Thus

$$f\left(x^{*y(\epsilon)}_{\epsilon,M(\epsilon)}\right) \leq f\left(x_{\alpha_2}\right). \tag{27}$$

By Lemma 2

$$f(x^*) \leq f\left(x^{*y(\epsilon)}_{\epsilon,M(\epsilon)}\right). \tag{28}$$

By [(26)–(28)], we have

$$f(x^*) \leq f\left(x^{*y(\epsilon)}_{\epsilon,M(\epsilon)}\right) \leq f(x^*) + \delta_1.$$

Let $\delta_1 \to 0$, the proof is complete.

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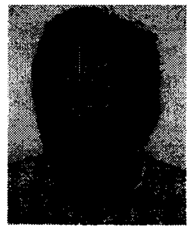

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