Multi-Objective Optimisation for Unidirectional Glass and Carbon Fibre Reinforced Hybrid Epoxy Composites under Flexural Loading

Chensong Dong*

Department of Mechanical Engineering, Curtin University, GPO Box U1987, Perth, Western Australia 6845, Australia

Abstract

A multi-objective optimisation study for unidirectional carbon and glass fibre reinforced epoxy hybrid composites under flexural loading is presented in this paper. The flexural strength of the hybrid composite was conveniently expressed by regression formulas. Two optimisation methods: Weighted Sum Method (WSM) and Non-Dominated Sorting Genetic Algorithm (NSGA-II) were employed for multi-objective optimisation with the cost and weight being the objective functions. When WSM was used, two algorithms: Interior Point Algorithm (IPA) and Genetic Algorithm (GA) were used. The results suggest that positive hybrid effects can be utilised to improve the flexural strength.

Keywords: Polymer-matrix composites; Carbon fibre; Glass fibre; Hybrid; Flexural strength; Cost; Weight; Optimisation

* Corresponding author. Tel: +61 (8) 92669204; fax: +61 (8) 92662681; email: c.dong@curtin.edu.au.

Nomenclature

b	Width of the specimen (mm)
C_c	Cost per unit volume of the hybrid composite (US$/litre)
C_{cc}	Cost per unit volume of the carbon fibre composite (US$/litre)
C_{cg}	Cost per unit volume of the glass fibre composite (US$/litre)
C_{fc}	Cost per unit volume of the carbon fibres (US$/litre)
C_{fg}	Cost per unit volume of the glass fibres (US$/litre)
C_m	Cost per unit volume of the matrix (US$/litre)
D	Maximum deflection (mm)
E_{11}	Longitudinal tensile modulus (GPa)
E_{11c}	Longitudinal tensile modulus of the carbon/epoxy section (GPa)
E_{11g}	Longitudinal tensile modulus of the glass/epoxy section (GPa)
e_h	Hybrid effect
E_F	Flexural modulus (GPa)
G_{12}	Shear modulus (GPa)
h	Depth of the specimen (total thickness of the laminate) (mm)
h_c	Thickness of the carbon/epoxy section (mm)
h_g	Thickness of the glass/epoxy section (mm)
$P_{\max}$	The maximum load encountered before failure (N)
r_h	Hybrid ratio
S	Span of the specimen (distance between to supporting pins) (mm)
S_C	Compressive strength (MPa)

S_{Cc}	Compressive strength of the carbon/epoxy composite (MPa)
S_{Cg}	Compressive strength of the glass/epoxy composite (MPa)
S_F	Flexural strength of the hybrid composite (MPa)
S_{Fc}	Flexural strength of the carbon/epoxy composite (MPa)
S_{Fg}	Flexural strength of the glass/epoxy composite (MPa)
S_{FRoM}	Flexural strength from the rule of mixtures (MPa)
V_{fc}	Fibre volume fraction of the carbon/epoxy section
V_{fg}	Fibre volume fraction of the glass/epoxy section
x, y, z	Coordinates (mm)
ε_F	Flexural strain
ρ_c	Density of the hybrid composite (g/cm^3)
ρ_{cc}	Density of the carbon fibre composite (g/cm^3)
ρ_{cg}	Density of the glass fibre composite (g/cm^3)
ρ_{fc}	Density of the carbon fibres (g/cm^3)
ρ_{fg}	Density of the glass fibres (g/cm^3)
ρ_m	Density of the matrix (g/cm^3)
σ_F	Flexural stress (MPa)

1 Introduction

Hybrid composites reinforced by more than one type of fibres are of great research interest because they provide a convenient way to achieving tailored material properties. Although carbon fibres are well known for high strength, they have low strain-to-failure because of their high stiffness.

Compared to carbon fibres, glass fibres have much higher strain-to-failure due to their lower modulus. From this point, it is possible to increase the strain-to-failure by substitution of carbon fibres for glass fibres.

When considering the mechanical properties of hybrids a general rule of mixtures (RoM) approach may be utilized which quantifies a material property with respect to the volume concentration of its constituents. Many researchers have however noted the existence of hybrid effects in which the material property as predicted by the RoM differs to that observed in reality. A positive or negative hybrid effect is defined as the positive or negative deviation of a certain mechanical property from the RoM behaviour, respectively (Marom et al. 1978).

Dong *et al.* (Dong et al. 2012a, Dong et al. 2012b, Dong and Davies 2013a, Dong and Davies 2013b) studied the flexural properties of unidirectional carbon/glass fibre reinforced hybrid epoxy composites using both experiments and finite element analysis (FEA). It was shown that partial substitution of carbon fibres for glass fibres on the compressive face resulted in improved flexural strength, *i.e.* positive hybrid effect. Dong *et al.* (Dong and Davies 2012, Dong and Davies 2014a, Dong and Davies 2014b) further investigated optimal design of hybrid composites. It was concluded that in order to achieve positive hybrid effects, the fibre volume fraction of the glass/epoxy section needed to be higher than that of the carbon/epoxy section (Dong and Davies 2012, Dong and Davies 2014b).

Traditional design of composites is based on a deterministic approach, and a large factor of safety is needed for incorporating the variability of data. A new alternative approach is probabilistic design (Chamis and Shiao 1992, Shiao and Chamis 1999, Chamis 2004, Chamis and Abumeri 2005, Shaw et al. 2010), which allows the estimation of reliability and inclusion of stochastic variability (Di Sciuva and Lomario 2003).

Variability in the performance of composite materials arises mainly from the variability in constituent properties, fibre distribution, structural geometry, loading conditions and also manufacturing process (Chiachio et al. 2012). Fertig *et al.* (Fertig, III et al. 2014) showed that microstructural variations, especially volume fraction variations, led to significant stress variations in composites. Spurgeon (Spurgeon 2005) showed the variation in fibre volume fraction could be as high as ±1%. Shaw (Shaw et al. 2010) showed a 1.3% standard deviation for fibre volume fraction. Another important source of manufacturing related variation is ply thickness. According to Chamis (Chamis 2004), the coefficient of variation (CoV) could be as high as 5%.

Antonio and Hoffbauer (António and Hoffbauer 2008) studied the uncertainty propagation on structural response of composites using three different approaches: a first-order local method, a Global Sensitivity Analysis supported by a variance-based method and an extension of local variance to estimate the global variance over all domain of inputs. The uncertainty quantification and stochastic modelling approaches in FRP composites were reviewed by Sriramula and Chryssanthopoulos (Sriramula and Chryssanthopoulos 2009).

A multi-objective optimisation study for unidirectional carbon and glass fibre reinforced epoxy hybrid composites under flexural loading is presented in this paper. The flexural strength of the hybrid composite was conveniently calculated by regression formulas. Two optimisation methods: Weighted Sum Method (WSM) and Non-Dominated Sorting Genetic Algorithm (NSGA-II) were employed for multi-objective optimisation with the cost and weight being the objective functions. When WSM was used, two algorithms: Interior Point Algorithm (IPA) (Byrd et al. 1999, Byrd et al. 2000, Waltz et al. 2006) and Genetic Algorithm (GA) (Mitchell 1998) were used.

2 Model Development

2.1 Material Properties

The hybrid composites being investigated in this study were made by embedding both carbon and glass fibres into one common matrix, epoxy. The material structure was inter-ply, *i.e.* both carbon/epoxy and glass/epoxy laminas were present in a hybrid composite. The typical material properties of the fibres and matrix are shown in Table 1. The lamina properties, including the longitudinal modulus E_{11}, the transverse moduli E_{22} and E_{33}, and the shear moduli G_{12}, G_{13} and G_{23}, were derived from the constituent properties using Hashin's model (Chou 1992), and the lamina stiffness matrices were derived.

Table 1: Typical properties of fibres and resin

Material	Tensile modulus (GPa)	Tensile strength (MPa)	Density (g/cm^3)	Cost (Gurit.) (US$/litre)
High strength carbon fibres	230	4900	1.8	151.2
S-2 glass fibres	86.9	4890	2.46	103.3
E glass fibres	72	3450	2.58	10.8
High performance epoxy	3.1	69.6	1.09	26.2

2.2 Flexural Strength

With reference to our previous studies (Dong and Davies 2012, Dong et al. 2012a, Dong, et al. 2012b, Dong and Davies 2013a, Dong and Davies 2013b, Dong and Davies 2014a, Dong and Davies 2014b), the stacking configuration for the hybrid composites was achieved by partially substituting carbon/epoxy laminas on the compressive face of a full carbon/epoxy composite laminate for glass/epoxy laminas. A hybrid composite specimen under the three point bending is schematically shown in Figure 1. The deformation and stresses were obtained by FEA. The details can be found in our previous studies (Dong and Davies 2012, Dong and Davies 2014a, Dong and Davies 2014b). Only a brief description is given here for completeness.

Figure 1: A hybrid composite specimen under the three point bending

A commercial FEA software package ANSYS APDL was used for this purpose. FEA models were constructed using PLANE182 element. For the hybrid composite, the FEA model consisted of two sections: carbon/epoxy and glass/epoxy. The material properties, including the longitudinal modulus E_{11} and the shear moduli G_{12}, G_{13} and G_{23}, were derived by Hashin's model (Chou 1992) using the constituent properties and the fibre volume fraction of the lamina. The transverse moduli E_{22} and E_{33} were derived from the stress-strain relationship (Hashin 1983).

Two modelling techniques were employed in this study. For flexural modulus, linear elastic analysis was employed. A bending load, *P*, was applied and the corresponding deflection was obtained, and the flexural modulus was obtained using Eqn. 1.

$$E_F = \frac{\Delta\sigma_F}{\Delta\varepsilon_F} \tag{1}$$

For flexural strength, nonlinear large displacement analysis was employed for determining the flexural strength. The stresses at both surfaces of each section were examined, and the applied load *P* was increased until one of the stresses reached the strength. The flexural strength was calculated from the maximum load, *i.e.*

$$S_F = \frac{3P_{\max}S}{2bh^2} \tag{2}$$

The total thickness of the laminate was kept constant and the thickness of the glass/epoxy section was varied from 0 to *h*. For the purpose of quantitatively characterising the degree of hybridisation and applying the RoM, hybrid ratio is introduced, which is the relative percentage of glass fibres with respect to all fibres, *i.e.*

$$r_h = \frac{h_g V_{fg}}{h_g V_{fg} + h_c V_{fc}} \tag{3}$$

The flexural strength of the hybrid composites according to the RoM is given by

$$S_{FRoM} = S_{Fc}(1 - r_h) + S_{Fg} r_h \tag{4}$$

The hybrid effect is given by

$$e_h = \frac{S_F}{S_{FRoM}} - 1 \tag{5}$$

Because of the hybrid effect, the actual flexural strength is given by

$$S_F = S_{FRoM}(1+e_h) \tag{6}$$

When the fibre volume fractions for both carbon/epoxy and glass/epoxy sections are 50%, the flexural strength and hybrid effect vs. hybrid ratio is shown in Figure 2. It is shown a peak exists for both the flexural strength and the hybrid effect, which corresponds to the critical hybrid ratio, r_{hc} (Dong et al. 2015).

Figure 2: Flexural strength and hybrid effect vs. hybrid ratio

In order to calculate the flexural strength of hybrid composites using Eqns. 3-5, the flexural strengths of the full carbon and full glass composites are needed. Since the flexural strength increases with the fibre volume fraction, the flexural strength of a composite containing only one type of fibres is expressed by the following regression formula.

$$S_F = s_0 + s_1 V_f^{s_2} \tag{7}$$

where s_0, s_1 and s_2 are constants, and can be determined by fitting the FEA data. The constants for the carbon/epoxy, S-2 glass/epoxy and E glass/epoxy composites are shown in Table 2.

Table 2: Constants of regression formulas

Composite	s_0 (MPa)	s_1 (MPa)	s_2
Carbon/epoxy	552	2163	1.6
S-2 glass/epoxy	316	1789	1.65
E glass/epoxy	267	1516	1.56

According to Eqn. 6, the hybrid effect is needed for calculating the flexural strength of hybrid composites. The hybrid effect is expressed by a regression formula by incorporating two boundary values, *i.e.*

$$e_h = \begin{cases} \dfrac{e_{hc}}{r_{hc}} r_h + a r_h (r_h - r_{hc}) & 0 \le r_h \le r_{hc} \\ e_{hc} - \dfrac{e_{hc}}{1 - r_{hc}} (r_h - r_{hc}) + b_1 (r_h - r_{hc})(r_h - 1) & r_{hc} \le r_h \le 1 \\ + b_2 (r_h - r_{hc})(r_h - 1)^2 + b_3 (r_h - r_{hc})(r_h - 1)^3 & \end{cases} \quad (8)$$

where a, b_1, b_2, and b_3 are constants.

It is shown from the FEA results that the critical hybrid ratio decreases with increasing ratio of compressive strengths of the full glass and full carbon composites, and the hybrid effect at the critical hybrid ratio increases with the hybrid ratio. Thus, the critical hybrid ratio and corresponding hybrid effect are expressed by regression models being fitted to the FEA data, *i.e.*

$$\begin{aligned} r_{hc} &= 0.3 \left(\frac{S_{Cg}}{S_{Cc}} \right)^{0.7} \\ e_{hc} &= 0.82 - 0.25 r_{hc}^{-0.63} \end{aligned} \quad (9)$$

The constants a, b_1, b_2, and b_3 in Eqn. 8 were determined using multiple linear regression, and the regression formulas are given by

$$a = 1.1947 \frac{S_{Cc}}{S_{Cg}} + 0.0142 \left(\frac{S_{Cc}}{S_{Cg}} \right)^2 \quad (10)$$

$$b_1 = 1.4994 - 10.68 \frac{S_{Cg}}{S_{Cc}} \left(1 - 1.1073 \frac{E_{11g}}{E_{11c}} \right) \quad (11)$$

$$b_2 = 2.6122 - 35.3102 \frac{S_{Cg}}{S_{Cc}} \left(1 - 1.0849 \frac{E_{11g}}{E_{11c}} \right) \quad (12)$$

$$b_3 = 8.2584 - 38.9616 \frac{S_{Cg}}{S_{Cc}} \left(1 - 0.9364 \frac{E_{11g}}{E_{11c}} \right) \quad (13)$$

The overall approach to calculating the flexural strength is shown in Figure 3. The R^2 of the developed regression model is 96.2%. Thus, a good fit is achieved.

Figure 3: Overall approach to calculating flexural strength

2.3 Density

The density of the carbon/glass fibre reinforced hybrid composite is given by (Kalantari et al. 2016a, Kalantari et al. 2016b)

$$\rho_c = \rho_m + \frac{V_{fc}V_{fg}}{r_h V_{fc} + (1-r_h)V_{fg}}\left(\rho_{fc} - \rho_m\right) + \frac{r_h V_{fc}V_{fg}}{r_h V_{fc} + (1-r_h)V_{fg}}\left(\rho_{fg} - \rho_{fc}\right) \tag{14}$$

2.4 Cost

The material cost per unit volume of the carbon/glass fibre reinforced hybrid composite is given by

$$C_c = \frac{C_{fc}(1-r_h)V_{fc}V_{fg} + C_{fg} r_h V_{fc}V_{fg} + C_m\left[r_h V_{fc}(1-V_{fg}) + (1-r_h)V_{fg}(1-V_{fc})\right]}{r_h V_{fc} + (1-r_h)V_{fg}} \tag{15}$$

3 Optimisation

3.1 Weighted Sum Method

The regression formulas can be convenient used for multi-objective optimisation with the cost and weight as the objective functions. Since there are two objective functions, the simplest way is using the WSM. In order to make the cost and density comparable, they need to be normalised. When E glass fibres are used, because the cost of E glass fibres is lower than that of the epoxy, the glass fibre composite of 70% fibre volume fraction has the lowest cost and the carbon fibre composite of 70% fibre volume fraction has the highest cost. The normalised cost is given by

$$\bar{C}_c = \frac{C_c - C_{cg}\left(V_f = 0.7\right)}{C_{cc}\left(V_f = 0.7\right) - C_{cg}\left(V_f = 0.7\right)} \tag{16}$$

When S-2 glass fibre are used, the glass fibre composite of 30% fibre volume fraction has the lowest cost. The normalised cost is given by

$$\bar{C}_c = \frac{C_c - C_{cg}\left(V_f = 0.3\right)}{C_{cc}\left(V_f = 0.7\right) - C_{cg}\left(V_f = 0.3\right)} \tag{17}$$

Likewise, the normalised density is given by

$$\bar{\rho}_c = \frac{\rho_c - \rho_{cc}\left(V_f = 0.3\right)}{\rho_{cg}\left(V_f = 0.7\right) - \rho_{cc}\left(V_f = 0.3\right)} \tag{18}$$

After normalisation, the normalised costs and densities are both in the range between 0 and 1. The optimisation problem is formulated as

$$\begin{aligned} &\min\ w\bar{C}_c + (1-w)\bar{\rho}_c \\ &s.t.\ S_F \geq S_{F0} \end{aligned} \tag{19}$$

The design variables are the fibre volume fractions of the carbon/epoxy and the glass/epoxy laminas and the hybrid ratio. Two optimisation algorithms, IPA and GA were used. The constraint flexural strength was varied from 500 MPa to 2000 MPa.

3.2 Non-Dominated Sorting Genetic Algorithm

NSGA-II (Deb et al. 2002) is an *a posteriori* method to find the Pareto optimal solution of the MO and MORO problems, *i.e.* the objective preference is not considered in the early stages of the solution, instead, the set of trade-offs between the solution are obtained and then a preference based strategy used to determine the final single optimum solution. A posteriori methods can be classified into mathematical programing-based and evolutionary algorithms. The evolutionary algorithms produce the Pareto optimal set in a single run and thus are generally faster with NSGA-II being selected in the present case. NSGA-II starts with an initial population and tries to improve the results in a number of generations based on non-domination and crowding distance sorting.

The optimisation problem is formulated as

$$\begin{aligned} &\min \ C_c, \rho_c \\ &s.t. \ S_F \geq S_{F0} \end{aligned} \quad (20)$$

The constraint flexural strength was also varied from 500 MPa to 2000 MPa, and the optimisation was implemented using MATLAB.

4 Results and Discussion

4.1 Weighted Sum Method

4.1.1 Carbon/E Glass Hybrid Composites

According to Eqn. 19, when weight is more important than cost, the weighting factor *w* should be less than 0.5. As an example, for the carbon/E glass hybrid composites, when *w* = 0.3, the optimisation results by IPA and GA are shown in Table 3 and Table 4, respectively. It is shown from Table 3 that most optimal designs from IPA are the hybrid composites of hybrid ratio around 0.2. Both the fibre volume fractions of the carbon/epoxy and glass/epoxy sections increase with the required flexural strength. It is shown from Table 4 that the optimal designs from GA are close to the full carbon fibre composites of fibre volume fraction 30% when the required flexural strength is less than 850 MPa. As the required flexural strength further increases, the optimal designs become various hybrid composites and full carbon fibre composites. The maximum achievable flexural strength is slightly above 1850 MPa. Thus, if the required flexural strength is greater than 1850 MPa, no feasible solutions can be found.

Table 3: Optimisation results for carbon/E glass hybrid composites by Interior Point Algorithm with weighting factor 0.3

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.34	0.37	0.2081	500	986	59	1.389
0.34	0.37	0.208	550	986	59	1.389
0.34	0.37	0.2079	600	986	59.01	1.389
0.34	0.37	0.2078	650	986	59.01	1.389
0.34	0.37	0.2077	700	985	59.02	1.389
0.34	0.37	0.2076	750	985	59.01	1.389
0.34	0.37	0.2076	800	985	59	1.389
0.34	0.37	0.2078	850	985	58.97	1.389
0.34	0.37	0.208	900	984	58.93	1.388
0.4	0.5	0.2303	950	1171	64.83	1.461
0.4	0.51	0.2339	1000	1182	64.86	1.465
0.41	0.53	0.2367	1050	1220	66.02	1.479
0.42	0.55	0.2413	1100	1246	66.53	1.488
0.42	0.58	0.2526	1150	1277	66.35	1.497
0.44	0.62	0.2566	1200	1341	68.17	1.520
0.3	0.7	0.3281	1250	1250	55.32	1.447
0.49	0.61	0.237	1300	1417	73.42	1.551
0.52	0.64	0.1788	1350	1432	80.09	1.548
0.54	0.65	0.1845	1400	1473	81.18	1.564
0.56	0.65	0.1746	1450	1520	84.2	1.579
0.6	0.65	0.122	1500	1561	91.92	1.581
0.54	0.67	0.2402	1550	1559	77.92	1.599
0.6	0.65	0.2166	1600	1638	83.89	1.626
0.65	0.64	0.2013	1650	1719	88.97	1.653
0.66	0.66	0.2029	1700	1755	89.9	1.663
0.67	0.67	0.2041	1750	1790	90.88	1.674
0.68	0.68	0.2035	1800	1825	92.13	1.684
0.7	0.69	0.2038	1850	1858	93.16	1.694

Table 4: Optimisation results for carbon/E glass hybrid composites by Genetic Algorithm with weighting factor 0.3

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.3	-	0	500	871	63.94	1.305
0.31	0.3	0.0539	550	877	62.58	1.323
0.3	-	0	600	869	63.83	1.304
0.3	-	0	650	867	63.67	1.303
0.3	0.64	0.01	700	881	63.97	1.310
0.3	-	0	750	867	63.67	1.303
0.3	-	0	800	867	63.67	1.303
0.3	0.3	0.0138	850	867	63.31	1.308
0.3	0.56	0.07	900	909	61.9	1.327
0.32	0.67	0.0692	950	953	64.21	1.342
0.36	0.5	0.075	1000	1002	68.27	1.373
0.41	0.39	0.1489	1050	1076	68.39	1.425
0.31	0.64	0.2373	1100	1102	58.33	1.404
0.37	0.68	0.2963	1150	1309	61.74	1.491
0.48	0.33	0.0167	1200	1213	85.17	1.438
0.45	0.7	0.2054	1250	1347	72.76	1.512
0.38	0.69	0.2796	1300	1300	63.04	1.489
0.54	0.67	0.0748	1350	1401	89.19	1.513
0.45	0.7	0.3125	1400	1420	66.93	1.569
0.64	0.38	0.0735	1450	1478	95.86	1.556
0.62	0.57	0.0573	1500	1549	98.54	1.557
0.62	-	0	1550	1561	103.78	1.531
0.64	-	0	1600	1604	105.83	1.542
0.61	0.7	0.1716	1650	1650	89.43	1.619
0.69	0.42	0.0156	1700	1701	109.62	1.581
0.7	0.7	0.1974	1750	1863	94.18	1.694
0.68	0.7	0.2031	1800	1839	92.5	1.687
0.69	0.69	0.2007	1850	1851	93.35	1.691

When weight and cost are equally important, the weighting factor *w* should be 0.5, and the optimisation results by IPA and GA are shown in Table 5 and Table 6, respectively. It is shown from Table 5 that the optimal designs from IPA are close to the full glass fibre composites for lower required flexural strengths (< 600 MPa). As the required flexural strength increases, the optimal design approaches the hybrid composites of hybrid ratio around 0.2. The fibre volume fractions increase with the required flexural strength. It is shown from Table 6 that the optimal designs from GA are similar to those from IPA.

Table 5: Optimisation results for carbon/E glass hybrid composites by Interior Point Algorithm with weighting factor 0.5

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.3	0.3	0.999	500	500	21.59	1.537
0.3	0.3	0.9542	550	550	23.48	1.526
0.3	0.3	0.8827	600	600	26.49	1.510
0.3	0.3	0.4253	650	650	45.76	1.403
0.3	0.3	0.3537	700	700	48.77	1.386
0.3	0.3	0.3028	750	750	50.92	1.374
0.3	0.3	0.2607	800	800	52.69	1.364
0.3	0.3	0.2236	850	850	54.25	1.355
0.3	0.38	0.2232	900	952	55.72	1.369
0.4	0.57	0.2534	950	1241	64.86	1.483
0.41	0.59	0.256	1000	1274	65.8	1.495
0.42	0.6	0.2572	1050	1294	66.38	1.502
0.42	0.6	0.2575	1100	1298	66.52	1.504
0.42	0.59	0.2543	1150	1295	66.78	1.503
0.43	0.61	0.2549	1200	1330	68	1.516
0.44	0.63	0.2603	1250	1365	68.56	1.528
0.44	0.67	0.2737	1300	1397	67.99	1.536
0.38	0.7	0.3083	1350	1352	62.02	1.507
0.47	0.66	0.2608	1400	1435	70.97	1.553
0.5	0.66	0.2509	1450	1485	73.96	1.572
0.54	0.65	0.2356	1500	1538	77.85	1.592
0.59	0.64	0.2227	1550	1590	82.11	1.617
0.64	0.63	0.2027	1600	1683	87.51	1.642
0.65	0.65	0.2049	1650	1717	88.33	1.652
0.66	0.66	0.2038	1700	1754	89.73	1.663
0.67	0.67	0.2033	1750	1789	91	1.674
0.68	0.68	0.2038	1800	1824	92.06	1.684
0.7	0.69	0.204	1850	1858	93.13	1.694

Table 6: Optimisation results for carbon/E glass hybrid composites by Genetic Algorithm with weighting factor 0.5

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	**Glass/epoxy**	**ratio**	**Objective**	**Actual**	**(US$/litre)**	**(g/cm^3)**
0.31	0.31	0.9961	500	511	21.63	1.545
0.3	0.31	0.9316	550	578	24.4	1.529
0.31	0.31	0.912	600	600	25.22	1.533
0.34	0.31	0.7922	650	650	30.57	1.514
0.3	0.3	0.3493	700	704	48.96	1.385
0.3	0.3	0.2915	750	763	51.4	1.371
0.33	0.3	0.2447	800	833	55.52	1.382
0.3	0.36	0.1445	850	900	58.38	1.343
0.31	0.3	0.1952	900	901	56	1.354
0.31	0.39	0.2447	950	953	55.8	1.384
0.35	0.44	0.2759	1000	1003	58.4	1.436
0.33	0.57	0.1965	1050	1057	60.89	1.398
0.3	0.61	0.259	1100	1101	56.79	1.405
0.39	0.55	0.2773	1150	1179	62.5	1.481
0.45	0.57	0.1987	1200	1277	71.82	1.497
0.43	0.67	0.2027	1250	1298	71.15	1.494
0.38	0.69	0.2796	1300	1300	63.04	1.489
0.45	0.7	0.2078	1350	1351	72.7	1.513
0.44	0.7	0.2662	1400	1414	69.1	1.540
0.6	0.5	0.1892	1450	1456	83.33	1.588
0.57	0.64	0.2066	1500	1571	82.61	1.602
0.61	0.57	0.2004	1550	1553	84.72	1.614
0.7	0.57	0.2041	1600	1620	90.28	1.668
0.61	0.68	0.2375	1650	1659	83.64	1.651
0.68	0.67	0.2339	1700	1722	88.75	1.696
0.68	0.62	0.1909	1750	1755	92.15	1.667
0.67	0.7	0.2178	1800	1808	90.3	1.687
0.7	0.69	0.2047	1850	1857	93.11	1.695

When cost is more important than weight, the weighting factor *w* should be greater than 0.5. When *w* = 0.7, the optimisation results by IPA and GA are shown in Table 7 and Table 8, respectively. It is shown from Table 7 that the optimal designs from IPA are close to the full glass fibre composites for low to medium required flexural strengths (< 1200 MPa). As the required flexural strength increases, the optimal design approaches the hybrid composites of hybrid ratio around 0.2. The fibre volume fractions increase with the required flexural strength. It is shown from Table 8 that the optimal designs from GA are close to the full glass fibre composites when the required flexural strength is below 1400 MPa. As the required flexural strength increases, the optimal design approaches the hybrid composites of hybrid ratio around 0.2.

Table 7: Optimisation results for carbon/E glass hybrid composites by Interior Point Algorithm with weighting factor 0.7

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.43	0.3	0.999	500	500	21.59	1.537
0.3	0.3	0.9542	550	550	23.48	1.526
0.3	0.31	0.9119	600	600	25.23	1.532
0.3	0.35	0.9191	650	650	24.77	1.586
0.3	0.39	0.9237	700	700	24.39	1.635
0.3	0.42	0.9265	750	750	24.09	1.680
0.3	0.46	0.9281	800	800	23.84	1.723
0.3	0.49	0.9295	850	850	23.59	1.764
0.32	0.52	0.932	900	900	23.26	1.806
0.33	0.55	0.934	950	950	22.96	1.845
0.34	0.58	0.9355	1000	1000	22.69	1.884
0.36	0.6	0.9366	1050	1050	22.44	1.920
0.37	0.63	0.9374	1100	1100	22.21	1.956
0.38	0.65	0.9379	1150	1150	22.01	1.990
0.44	0.63	0.26	1200	1364	68.6	1.528
0.5	0.59	0.2277	1250	1418	74.67	1.553
0.54	0.64	0.2313	1300	1529	78.1	1.590
0.52	0.62	0.2304	1350	1469	76.14	1.570
0.46	0.67	0.2649	1400	1434	70.39	1.551
0.51	0.64	0.2396	1450	1487	75.53	1.575
0.58	0.68	0.2296	1500	1638	82.04	1.625
0.57	0.65	0.2242	1550	1589	81.14	1.610
0.6	0.65	0.2177	1600	1635	83.64	1.625
0.61	0.67	0.2189	1650	1683	85.06	1.640
0.66	0.67	0.2071	1700	1758	89.36	1.664
0.67	0.67	0.2045	1750	1795	90.99	1.675
0.69	0.7	0.2062	1800	1859	92.8	1.694
0.7	0.69	0.2039	1850	1858	93.16	1.694

Table 8: Optimisation results for carbon/E glass hybrid composites by Genetic Algorithm with weighting factor 0.7

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.3	0.31	0.9981	500	513	21.49	1.551
0.32	0.33	0.979	550	559	22.11	1.571
0.42	0.32	0.9263	600	602	24.55	1.550
0.33	0.35	0.9093	650	651	25.26	1.578
0.67	0.33	0.8669	700	701	27.34	1.583
0.37	0.47	0.9926	750	753	19.39	1.792
0.36	0.47	0.9589	800	800	21.69	1.769
0.31	0.52	0.9774	850	856	19.87	1.850
0.42	0.62	0.8241	900	1165	31.51	1.865
0.32	0.55	0.9313	950	953	23.16	1.844
0.39	0.69	0.9559	1000	1203	20.04	2.062
0.44	0.59	0.9338	1050	1050	22.66	1.920
0.42	0.61	0.8992	1100	1110	25.46	1.908
0.7	0.7	0.9468	1150	1268	20.65	2.102
0.5	0.67	0.8454	1200	1281	30.18	1.957
0.45	0.7	0.9242	1250	1281	23	2.050
0.43	0.7	0.8775	1300	1308	27.35	1.994
0.65	0.69	0.8637	1350	1356	28.75	2.037
0.64	0.64	0.3318	1400	1414	76.09	1.707
0.57	0.61	0.2146	1450	1551	81.31	1.600
0.64	0.64	0.2337	1500	1618	84.85	1.658
0.67	0.56	0.2028	1550	1592	88.51	1.651
0.6	0.67	0.2043	1600	1642	85.36	1.624
0.64	0.64	0.2166	1650	1662	86.41	1.650
0.61	0.7	0.2329	1700	1706	84.56	1.654
0.7	0.7	0.25	1750	1751	88.82	1.721
0.7	0.7	0.2031	1800	1872	93.68	1.698
0.7	0.7	0.2056	1850	1860	93.06	1.696

Overall, it is seen that many optimal designs for high required flexural strengths are the hybrid composites of hybrid ratio around 0.2. This implies that positive hybrid effects (Dong, Duong et al. 2012, Dong, Ranaweera-Jayawardena et al. 2012, Dong and Davies 2013, Dong and Davies 2013) can be utilised to improve the flexural strength.

The hybrid ratios of the optimal designs when IPA and GA are used are shown in Figure 4. It is seen that larger variations are present when GA is used. When the weighting factor increases, which indicates cost becomes more important, the hybrid ratio increases to include more E glass fibres.

Figure 4: Hybrid ratios of optimal designs for carbon/E glass hybrid composites when IPA (left) and GA (right) are used

4.1.2 Carbon/S-2 Glass Hybrid Composites

Similarly, for the carbon/S-2 glass hybrid composites, when w = 0.3, the optimisation results by IPA and GA are shown in Table 9 and Table 10, respectively. It is shown from Table 9 that all optimal designs from IPA are the hybrid composites of hybrid ratio between 0.2 and 0.3. The fibre volume fractions increase with the required flexural strength. It is shown from Table 10 that the optimal designs from GA are close to the full carbon fibre composites of fibre volume fraction 30% when the required flexural strength is below 850 MPa. As the required flexural strength further increases, the optimal design approaches the hybrid composites of hybrid ratio between 0.2 and 0.3. The maximum achievable flexural strength is slightly above 1950 MPa. Thus, if the required flexural strength is greater than 1950 MPa, no feasible solutions can be found.

Table 9: Optimisation results for carbon/S-2 glass hybrid composites by Interior Point Algorithm with weighting factor 0.3

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.36	0.41	0.2362	500	1091	67.92	1.408
0.36	0.41	0.2362	550	1091	67.91	1.408
0.36	0.41	0.2362	600	1090	67.89	1.408
0.35	0.41	0.2362	650	1090	67.87	1.408
0.35	0.41	0.2362	700	1090	67.85	1.407
0.35	0.41	0.2362	750	1089	67.82	1.407
0.35	0.41	0.2363	800	1088	67.79	1.407
0.35	0.41	0.2362	850	1088	67.75	1.407
0.35	0.41	0.2362	900	1087	67.72	1.406
0.35	0.41	0.2361	950	1086	67.69	1.406
0.35	0.41	0.236	1000	1086	67.68	1.406
0.38	0.47	0.2479	1050	1186	71.72	1.442
0.39	0.48	0.2514	1100	1202	72.17	1.447
0.4	0.51	0.2579	1150	1253	74.03	1.464
0.42	0.54	0.2628	1200	1308	76.1	1.482
0.42	0.56	0.2678	1250	1336	76.88	1.491
0.42	0.58	0.2719	1300	1354	77.26	1.496
0.31	0.67	0.3657	1350	1361	67.25	1.454
0.43	0.65	0.295	1400	1456	79.28	1.523
0.41	0.68	0.3151	1450	1476	77.95	1.522
0.54	0.59	0.2384	1500	1537	88.24	1.564
0.6	0.6	0.2221	1550	1663	94.85	1.604
0.58	0.61	0.2322	1600	1637	92.83	1.595
0.64	0.6	0.2139	1650	1747	98.96	1.630
0.6	0.66	0.2439	1700	1732	95.35	1.622
0.67	0.65	0.2189	1750	1847	102.24	1.658
0.65	0.68	0.2321	1800	1854	101.13	1.658
0.68	0.68	0.2279	1850	1908	103.57	1.674
0.69	0.68	0.2233	1900	1940	105.22	1.683
0.7	0.69	0.225	1950	1961	105.77	1.688

Table 10: Optimisation results for carbon/S-2 glass hybrid composites by Genetic Algorithm with weighting factor 0.3

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.32	0.3	0.0113	500	896	65.66	1.318
0.3	0.34	0	550	867	63.67	1.303
0.3	0.5	0	600	867	63.67	1.303
0.3	0.3	0.0047	650	875	64.17	1.307
0.3	0.41	0	700	867	63.67	1.303
0.3	0.56	0	750	867	63.67	1.303
0.3	0.49	0	800	867	63.67	1.303
0.3	0.42	0	850	867	63.67	1.303
0.33	0.31	0.0326	900	918	66.83	1.331
0.35	0.35	0.031	950	960	69.64	1.347
0.39	0.32	0.1929	1000	1054	69.39	1.403
0.43	0.43	0.0391	1050	1123	79.51	1.409
0.45	0.66	0.1907	1100	1350	81.69	1.490
0.36	0.57	0.2058	1150	1150	70.78	1.418
0.31	0.61	0.2918	1200	1205	66.38	1.417
0.3	0.68	0.301	1250	1257	66.54	1.422
0.32	0.67	0.338	1300	1339	68.65	1.454
0.38	0.62	0.3096	1350	1353	73.37	1.482
0.45	0.67	0.2256	1400	1400	81.59	1.507
0.45	0.66	0.286	1450	1491	81.35	1.535
0.57	0.51	0.1999	1500	1509	90.54	1.559
0.54	0.69	0.2075	1550	1584	91.5	1.571
0.59	0.59	0.2245	1600	1634	93.45	1.595
0.54	0.69	0.2764	1650	1680	90.75	1.606
0.65	0.57	0.1942	1700	1719	99.65	1.622
0.66	0.64	0.1975	1750	1797	102.1	1.642
0.67	0.67	0.1779	1800	1808	103.89	1.642
0.68	0.65	0.1957	1850	1858	104.59	1.659
0.7	0.7	0.2233	1900	1968	106.17	1.690
0.7	0.7	0.2265	1950	1974	106.09	1.692

When the weighting factor *w* should be 0.5, and the optimisation results by IPA and GA are shown in Table 11 and Table 12, respectively. It is shown from Table 11 that the optimal designs from IPA are the hybrid composites of hybrid ratio between 0.2 and 0.3. It is shown from Table 12 that the optimal designs from GA are close to the full carbon fibre composites of fibre volume fraction 30% when the required flexural strength is below 750 MPa. As the required flexural strength further increases, various hybrid composites and full carbon fibre composites are found to be the optimal designs.

Table 11: Optimisation results for carbon/S-2 glass hybrid composites by Interior Point Algorithm with weighting factor 0.5

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.33	0.4	0.2434	500	1052	65.23	1.390
0.33	0.4	0.2434	550	1052	65.23	1.390
0.33	0.4	0.2435	600	1052	65.23	1.390
0.33	0.4	0.2435	650	1052	65.22	1.390
0.33	0.4	0.2435	700	1052	65.22	1.390
0.33	0.4	0.2435	750	1052	65.21	1.390
0.33	0.4	0.2435	800	1052	65.2	1.390
0.33	0.4	0.2435	850	1051	65.19	1.390
0.33	0.4	0.2434	900	1051	65.18	1.390
0.33	0.4	0.2434	950	1051	65.18	1.390
0.33	0.4	0.2433	1000	1051	65.18	1.390
0.38	0.49	0.2584	1050	1203	71.6	1.445
0.39	0.52	0.2639	1100	1235	72.63	1.456
0.39	0.53	0.2669	1150	1253	73.15	1.461
0.39	0.54	0.2729	1200	1273	73.5	1.466
0.4	0.58	0.2808	1250	1327	75.2	1.484
0.42	0.61	0.2853	1300	1384	77.24	1.502
0.43	0.63	0.2904	1350	1429	78.6	1.516
0.42	0.64	0.2984	1400	1428	77.75	1.513
0.44	0.66	0.2965	1450	1490	80.5	1.534
0.48	0.67	0.2861	1500	1563	84.55	1.561
0.53	0.66	0.2681	1550	1619	88.59	1.584
0.52	0.67	0.271	1600	1627	88.62	1.586
0.55	0.69	0.2677	1650	1683	91.13	1.603
0.59	0.68	0.2542	1700	1752	95.08	1.626
0.63	0.68	0.2416	1750	1810	98.53	1.645
0.64	0.67	0.235	1800	1826	99.8	1.650
0.66	0.69	0.2335	1850	1892	102.4	1.669
0.69	0.69	0.2257	1900	1953	105.43	1.686
0.7	0.69	0.2259	1950	1958	105.58	1.687

Table 12: Optimisation results for carbon/S-2 glass hybrid composites by Genetic Algorithm with weighting factor 0.5

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.3	0.33	0.0001	500	869	63.81	1.304
0.3	0.55	0	550	867	63.67	1.303
0.3	0.31	0.0025	600	868	63.67	1.304
0.31	0.3	0.0144	650	881	64.5	1.312
0.3	0.3	0.0041	700	867	63.61	1.304
0.35	0.31	0.2891	750	876	63.82	1.395
0.3	0.3	0.0026	800	873	64.1	1.306
0.31	0.3	0.032	850	878	64.01	1.314
0.3	0.61	0.0396	900	900	63.93	1.316
0.33	0.34	0.2428	950	965	63.52	1.377
0.45	0.38	0.2158	1000	1155	75.77	1.459
0.32	0.5	0.3004	1050	1131	65.67	1.414
0.37	0.57	0.1659	1100	1138	72.71	1.416
0.35	0.51	0.2495	1150	1152	69.02	1.422
0.39	0.64	0.2212	1200	1259	74.68	1.453
0.44	0.69	0.2229	1250	1395	81.04	1.501
0.3	0.67	0.3375	1300	1306	66.7	1.437
0.51	0.51	0.2248	1350	1409	84.57	1.529
0.45	0.7	0.2301	1400	1424	81.91	1.511
0.41	0.66	0.304	1450	1450	77.86	1.516
0.69	0.45	0.163	1500	1641	100.64	1.609
0.57	0.57	0.1976	1550	1557	92.34	1.571
0.54	0.7	0.2581	1600	1672	91.27	1.599
0.57	0.68	0.2253	1650	1656	93.46	1.596
0.67	0.54	0.1901	1700	1715	100.27	1.624
0.67	0.69	0.1231	1750	1759	105.76	1.619
0.7	0.7	0.227	1800	1964	105.78	1.690
0.66	0.69	0.2247	1850	1864	102.05	1.660
0.7	0.7	0.222	1900	1967	106.24	1.690
0.7	0.7	0.2278	1950	1971	106.05	1.692

When $w = 0.7$, the optimisation results by IPA and GA are shown in Table 13 and Table 14, respectively. It is shown from Table 13 that the optimal designs from IPA are close to the full glass fibre composites for low required flexural strengths (< 700 MPa). As the required flexural strength increases, the optimal design approaches the hybrid composites of hybrid ratio between 0.2 and 0.3. The fibre volume fractions increase with the required flexural strength. The optimal designs from GA are similar to those from IPA, as shown in Table 14.

Table 13: Optimisation results for carbon/S-2 glass hybrid composites by Interior Point Algorithm with weighting factor 0.7

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.45	0.3	1	500	561	49.3	1.501
0.45	0.3	1	550	561	49.3	1.501
0.3	0.3	0.9686	600	600	49.75	1.495
0.3	0.3	0.9092	650	650	50.61	1.483
0.3	0.3	0.4558	700	700	57.12	1.393
0.3	0.3	0.3759	750	750	58.27	1.377
0.3	0.3	0.3209	800	800	59.06	1.367
0.3	0.3	0.2754	850	850	59.71	1.358
0.3	0.3	0.2349	900	900	60.3	1.350
0.3	0.35	0.2336	950	973	61.61	1.359
0.3	0.41	0.256	1000	1023	62.53	1.373
0.37	0.56	0.2868	1050	1277	72.32	1.463
0.37	0.53	0.275	1100	1241	71.81	1.454
0.38	0.55	0.2802	1150	1265	72.43	1.461
0.39	0.57	0.284	1200	1299	73.61	1.472
0.39	0.59	0.2916	1250	1320	73.79	1.477
0.39	0.61	0.3012	1300	1357	74.39	1.486
0.4	0.64	0.3081	1350	1415	76.14	1.504
0.42	0.67	0.3118	1400	1473	78.18	1.523
0.43	0.7	0.3144	1450	1531	80.28	1.542
0.44	0.7	0.3129	1500	1546	81.03	1.547
0.45	0.7	0.3111	1550	1551	81.43	1.549
0.48	0.7	0.2966	1600	1601	84.96	1.571
0.56	0.68	0.2617	1650	1687	91.89	1.606
0.58	0.68	0.2572	1700	1723	93.71	1.617
0.6	0.68	0.2502	1750	1771	96.22	1.632
0.65	0.66	0.2276	1800	1827	100.59	1.651
0.67	0.67	0.2264	1850	1885	102.88	1.667
0.68	0.68	0.2261	1900	1923	104.3	1.678
0.7	0.69	0.2261	1950	1958	105.56	1.687

Table 14: Optimisation results for carbon/S-2 glass hybrid composites by Genetic Algorithm with weighting factor 0.7

Fibre volume fraction		Hybrid	Flexural strength		Cost	Density
Carbon/epoxy	Glass/epoxy	ratio	Objective	Actual	(US$/litre)	(g/cm^3)
0.32	0.3	0.9978	500	565	49.36	1.501
0.3	0.3	0.9781	550	589	49.62	1.497
0.3	0.3	0.9501	600	624	50.33	1.496
0.31	0.31	0.9282	650	657	51.34	1.503
0.3	0.31	0.4868	700	703	57.19	1.405
0.3	0.31	0.3636	750	771	58.72	1.377
0.31	0.35	0.422	800	800	60.19	1.411
0.3	0.3	0.2764	850	852	59.76	1.358
0.3	0.33	0.2696	900	904	60.92	1.365
0.32	0.32	0.2255	950	963	63.28	1.369
0.33	0.41	0.1881	1000	1014	65.99	1.376
0.31	0.49	0.23	1050	1050	64.28	1.378
0.43	0.63	0.4796	1100	1304	78.4	1.615
0.32	0.61	0.2451	1150	1151	67.04	1.405
0.39	0.57	0.2223	1200	1217	73.65	1.446
0.45	0.45	0.2141	1250	1253	77.74	1.473
0.43	0.57	0.2317	1300	1312	78.06	1.483
0.45	0.65	0.2137	1350	1369	81.19	1.498
0.43	0.68	0.2435	1400	1407	80.2	1.505
0.56	0.48	0.1991	1450	1455	88.37	1.543
0.42	0.69	0.3151	1500	1507	79.02	1.531
0.57	0.56	0.227	1550	1562	91.23	1.580
0.57	0.64	0.2487	1600	1654	92.59	1.603
0.56	0.68	0.2426	1650	1664	92.43	1.598
0.6	0.66	0.2278	1700	1721	96.27	1.619
0.61	0.68	0.2419	1750	1774	97.03	1.633
0.69	0.67	0.2294	1800	1883	103.98	1.678
0.66	0.68	0.2425	1850	1856	101.86	1.671
0.67	0.7	0.2356	1900	1912	103.01	1.675
0.7	0.69	0.2254	1950	1954	105.49	1.686

The hybrid ratios of the optimal designs when IPA and GA are used are shown in Figure 5. Similar to that of the carbon/E glass hybrid composites, GA yields larger variations. When the weighting factor increases, which indicates cost becomes more important, the hybrid ratio increases to include more S-2 glass fibres.

Figure 5: Hybrid ratios of optimal designs for carbon/S-2 glass hybrid composites when IPA (left) and GA (right) are used

4.1.3 Comparison of Carbon/E Glass and Carbon/S-2 Glass Hybrid Composites

When the weight factor is 0.7, the hybrid ratios of the optimal designs for the carbon/E glass and carbon/S-2 glass hybrid composites when IPA and GA are used are shown in Figure 6. Because of the significant lower cost of E glass fibres compared to S-2 glass fibres, more glass fibres are found in the optimal design when low to medium flexural strengths are required.

Figure 6: Hybrid ratios of optimal designs for *w* = 0.7 when IPA (left) and GA (right) are used

4.2 Non-Dominated Sorting Genetic Algorithm

The Pareto fronts from NSGA-II for the carbon/E glass and carbon/S-2 glass hybrid composites are shown in Figure 7.

Figure 7: Pareto fronts from NSGA-II for carbon/E glass (left) and carbon/S-2 glass (right) hybrid composites

It is shown from Figure 7 that form the low to medium flexural strength (<= 1200 MPa) carbon/E glass hybrid composites, the Pareto optimal fronts show a wide range of densities. As the required flexural strength increases, the range of densities decreases significantly. This is because more carbon fibres are needed to achieve the required flexural strength. For the carbon/S-2 glass hybrid composites, the cost is significantly higher compared to that of the carbon/E glass hybrid composites, because of the significantly higher cost of S-2 glass fibres. The density is significantly lower than that of the carbon/E glass hybrid composites. This suggests more carbon fibres are included in the optimal designs.

The Pareto optimal designs for the carbon/E glass hybrid composites subjected to the required flexural strength 1000 MPa and 1500 MPa are shown in Table 15 and Table 16, respectively. It is clearly shown that the hybrid ratios significantly decrease as the flexural strength increase from 1000 MPa to 1500 MPa.

Table 15: Pareto optimal designs for carbon/E glass hybrid composites subjected to required flexural strength 1000 MPa

Fibre volume fraction		Hybrid ratio	Actual flexural strength	Cost (US$/litre)	Density (g/cm^3)
Carbon/epoxy	Glass/epoxy				
0.33	0.68	0.0912	1000	65.32	1.3626
0.3	0.6	0.1769	1013	59.5	1.3721
0.33	0.61	0.1135	1002	64.2	1.3684
0.68	0.7	1	1136	15.41	2.133
0.35	0.6	0.9336	1044	22.71	1.9112
0.31	0.58	0.5178	1015	47.39	1.5417
0.34	0.58	0.7506	1048	35.9	1.7324
0.31	0.58	0.8583	1029	28.51	1.803
0.31	0.59	0.4615	1048	50.03	1.5139
0.31	0.59	0.3491	1124	54.42	1.4551
0.33	0.6	0.6557	1049	41.6	1.6619
0.49	0.66	0.9945	1082	16.5	2.0743
0.45	0.65	0.9543	1134	20.49	2.0123
0.32	0.59	0.7082	1036	38.31	1.689
0.33	0.61	0.807	1098	32.34	1.7949
0.33	0.61	0.2984	1193	58	1.451
0.48	0.64	0.9721	1091	18.89	2.0224
0.32	0.59	0.8985	1035	25.56	1.8538

Table 16: Pareto optimal designs for carbon/E glass hybrid composites subjected to required flexural strength 1500 MPa

Fibre volume fraction		Hybrid ratio	Actual flexural strength	Cost (US$/litre)	Density (g/cm^3)
Carbon/epoxy	Glass/epoxy				
0.6	0.57	0.0003	1501	100.87	1.5145
0.57	0.68	0.1316	1502	87.94	1.5611
0.53	0.7	0.313	1500	73	1.641
0.59	0.62	0.0896	1500	92.79	1.5521
0.53	0.7	0.313	1500	73	1.641
0.54	0.69	0.1774	1504	82.76	1.5695
0.58	0.68	0.1006	1508	91.52	1.5546
0.59	0.63	0.0701	1501	94.7	1.545
0.56	0.68	0.1521	1501	85.64	1.5654
0.53	0.68	0.2023	1501	79.92	1.5727
0.53	0.68	0.2583	1538	75.83	1.6
0.6	0.6	0.005	1500	100.46	1.5166
0.59	0.66	0.0472	1502	96.73	1.5351
0.54	0.69	0.1868	1505	81.84	1.5722
0.56	0.68	0.1571	1509	85.41	1.5691
0.6	0.62	0.0315	1501	98.29	1.5292
0.58	0.68	0.1112	1509	90.52	1.5586
0.6	0.57	0.0003	1501	100.87	1.5145

The Pareto optimal designs for the carbon/S-2 glass hybrid composites subjected to the required flexural strength 1000 MPa and 1500 MPa are shown in Table 17 and Table 18, respectively. Compared to the carbon/E glass hybrid composites, the hybrid ratios of the Pareto optimal designs are significantly lower. Most hybrid ratios are around 0.2. This is in agreement with the results from WSM.

Table 17: Pareto optimal designs for carbon/S-2 glass hybrid composites subjected to required flexural strength 1000 MPa

Fibre volume fraction		Hybrid ratio	Actual flexural strength	Cost (US$/litre)	Density (g/cm^3)
Carbon/epoxy	Glass/epoxy				
0.3	0.6	0.1674	1000	64.53	1.3656
0.3	0.45	0.2574	1000	63.13	1.3888
0.3	0.5	0.2159	1002	63.78	1.3781
0.3	0.57	0.1804	1000	64.36	1.3692
0.3	0.52	0.2023	1002	64.06	1.3751
0.3	0.54	0.1939	1001	64.17	1.3728
0.3	0.48	0.2257	1001	63.55	1.38
0.3	0.6	0.1674	1000	64.53	1.3656
0.3	0.53	0.1986	1001	64.09	1.374
0.3	0.59	0.1758	1003	64.47	1.3683
0.3	0.45	0.2432	1004	63.2	1.3839
0.3	0.51	0.2057	1000	63.94	1.3755
0.3	0.49	0.221	1003	63.7	1.3794
0.3	0.46	0.2336	1001	63.38	1.3817
0.3	0.5	0.2119	1000	63.82	1.3769
0.3	0.46	0.2372	1002	63.29	1.3824
0.3	0.55	0.1889	1000	64.24	1.3715
0.3	0.57	0.1806	1000	64.36	1.3692

Table 18: Pareto optimal designs for carbon/S-2 glass hybrid composites subjected to required flexural strength 1500 MPa

Fibre volume fraction		Hybrid ratio	Actual flexural strength	Cost (US$/litre)	Density (g/cm^3)
Carbon/epoxy	Glass/epoxy				
0.6	0.52	0.0016	1500	100.83	1.515
0.57	0.66	0.1321	1510	95.58	1.5654
0.59	0.65	0.0651	1506	98.88	1.5438
0.55	0.67	0.1776	1500	92.15	1.5706
0.56	0.66	0.1607	1502	93.48	1.5693
0.57	0.65	0.1454	1501	94.46	1.5663
0.52	0.66	0.2421	1512	88.42	1.5833
0.54	0.67	0.1978	1501	90.85	1.5738
0.58	0.65	0.1043	1501	96.73	1.5552
0.53	0.67	0.2147	1503	89.76	1.5763
0.6	0.64	0.0192	1502	100.4	1.5237
0.52	0.66	0.2421	1512	88.42	1.5833
0.6	0.52	0.0016	1500	100.83	1.515
0.52	0.66	0.2277	1502	88.9	1.5779
0.56	0.65	0.163	1502	93.35	1.5698
0.54	0.67	0.197	1502	90.92	1.5738
0.59	0.65	0.0955	1504	97.34	1.5535
0.6	0.64	0.0328	1503	100.03	1.5301

5 Conclusions

A multi-objective optimisation study for unidirectional carbon and glass fibre reinforced epoxy hybrid composites under flexural loading is presented in this paper. The flexural strength of the hybrid composite was conveniently expressed by regression formulas. Two optimisation methods: WSM and NSGA-II were employed for multi-objective optimisation with the cost and weight being the objective functions. When WSM was used, two algorithms: IPA and GA were used. It is seen from the results many optimal designs for high required flexural strengths are the hybrid composites of hybrid ratio around 0.2. This implies that positive hybrid effects can be utilised to improve the flexural strength. Although carbon and glass fibres are used in this study, the methodology is suitable for other fibre types provided they have different strains-to-failure.

References

António, C.C. and Hoffbauer, L.N., (2008) From Local to Global Importance Measures of Uncertainty Propagation in Composite Structures, Composite Structures, 85(3), pp. 213-225.

Byrd, R.H., Gilbert, J.C. and Nocedal, J., (2000) A Trust Region Method Based on Interior Point Techniques for Nonlinear Programming, Mathematical Programming, 89(1), pp. 149-185.

Byrd, R.H., Hribar, M.E. and Nocedal, J., (1999) An Interior Point Algorithm for Large-Scale Nonlinear Programming, SIAM Journal on Optimization, 9(4), pp. 877-900.

Chamis, C.C., (2004) Probabilistic Simulation of Multi-Scale Composite Behavior, Theoretical and Applied Fracture Mechanics, 41(1–3), pp. 51-61.

Chamis, C.C. and Abumeri, G.H., (2005) Probabilistic Dynamic Buckling of Composite Shell Structures, Composites Part A: Applied Science and Manufacturing, 36(10), pp. 1368-1380.

Chamis, C.C. and Shiao, M.C., (1992) IPACS – Integrated Probabilistic Assessment of Composite Structures: Code Development and Application, Third NASA Advanced Composite Technology Conference.

Chiachio, M., Chiachio, J. and Rus, G., (2012) Reliability in Composites – A Selective Review and Survey of Current Development, Composites Part B: Engineering 43(3), pp. 902-913.

Chou, T.-W., (1992) Microstructural Design of Fiber Composites, Cambridge, U.K., Cambridge University Press.

Deb, K., Pratap, A., Agarwal, S. and Meyarivan, T., (2002) A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation, 6(2), pp. 182-197.

Di Sciuva, M. and Lomario, D., (2003) A Comparison between Monte Carlo and Forms in Calculating the Reliability of a Composite Structure, Composite Structures, 59(1), pp. 155-162.

Dong, C. and Davies, I.J., (2012) Optimal Design for the Flexural Behaviour of Glass and Carbon Fibre Reinforced Polymer Hybrid Composites, Materials & Design, 37, pp. 450-457.

Dong, C. and Davies, I.J., (2013) Flexural Properties of E Glass and TR50S Carbon Fiber Reinforced Epoxy Hybrid Composites, Journal of Materials Engineering and Performance, 22(1), pp. 41-49.

Dong, C. and Davies, I.J., (2013) Flexural Properties of Glass and Carbon Fiber Reinforced Epoxy Hybrid Composites, Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials Design and Applications, 227(4), pp. 308-317.
Dong, C. and Davies, I.J., (2014) Flexural and Tensile Moduli of Unidirectional Hybrid Epoxy Composites Reinforced by S-2 Glass and T700S Carbon Fibres, Materials & Design, 54, pp. 893-899.
Dong, C. and Davies, I.J., (2014) Flexural and Tensile Strengths of Unidirectional Hybrid Epoxy Composites Reinforced by S-2 Glass And T700S Carbon Fibres, Materials & Design, 54, pp. 955-966.
Dong, C., Duong J. and Davies, I.J., (2012) Flexural Properties of S-2 Glass and TR30S Carbon Fiber-Reinforced Epoxy Hybrid Composites, Polymer Composites, 33(5), pp. 773-781.
Dong, C., Kalantari, M. and Davies, I.J., (2015) Robustness for Unidirectional Carbon/Glass Fibre Reinforced Hybrid Epoxy Composites Under Flexural Loading, Composite Structures, 128, pp. 354-362.
Dong, C., Ranaweera-Jayawardena, H.A. and Davies, I.J., (2012) Flexural Properties of Hybrid Composites Reinforced by S-2 Glass and T700S Carbon Fibres, Composites Part B: Engineering, 43(2), pp. 573-581.
Fertig, III, R.S., Jensen, E.M. and Malusare, K.A., (2014) Effect of Fiber Volume Fraction Variation Across Multiple Length Scales on Composite Stress Variation: The Possibility of Stochastic Multiscale Analysis. 55th AIAA/ASME/ASCE/AHS/SC Structures, Structural Dynamics, and Materials Conference, National Harbor, MD, USA.
Gurit, Guide to Composites, Retrieved June 1, 2017, from www.gurit.com.
Hashin, Z., (1983) Analysis of Composite Materials—A Survey, Journal of Applied Mechanics, 50(3), pp. 481-505.
Kalantari, M., Dong C. and Davies, I.J., (2016) Multi-Objective Analysis for Optimal and Robust Design of Unidirectional Glass/Carbon Fibre Reinforced Hybrid Epoxy Composites Under Flexural Loading, Composites Part B: Engineering, 84, pp. 130-139.
Kalantari, M., Dong C. and Davies, I.J., (2016) Multi-Objective Robust Optimisation of Unidirectional Carbon/Glass Fibre Reinforced Hybrid Composites Under Flexural Loading, Composite Structures, 138, pp. 264-275.
Marom, G., Fischer, S., Tuler, F. R. and Wagner, H. D., (1978) Hybrid Effects in Composites: Conditions for Positive or Negative Effects versus Rule-of-Mixtures Behaviour, Journal of Materials Science, 13(7), pp. 1419-1426.
Mitchell, M. (1998) An introduction to Genetic Algorithms Cambridge, MA, USA, MIT Press.
Shaw, A., Sriramula, S., Gosling, P.D. and Chryssanthopoulos, M.K., (2010) A Critical Reliability Evaluation of Fibre Reinforced Composite Materials Based on Probabilistic Micro and Macro-Mechanical Analysis, Composites Part B: Engineering, 41(6), pp. 446-453.
Shiao, M.C. and Chamis, C.C., (1999) Probabilistic Evaluation of Fuselage-Type Composite Structures, Probabilistic Engineering Mechanics, 14(1–2), pp. 179-187.
Spurgeon, W.A., (2005) Thickness and Reinforcement Fiber Content Control in Composites by Vacuum-Assisted Resin Transfer Molding Fabrication Processes, DTIC Document.
Sriramula, S. and Chryssanthopoulos, M.K., (2009) Quantification of Uncertainty Modelling in Stochastic Analysis of FRP Composites, Composites Part A: Applied Science and Manufacturing, 40(11), pp. 1673-1684.
Waltz, R.A., Morales, J.L., Nocedal, J. and Orban, D., (2006) An Interior Algorithm for Nonlinear Optimization that Combines Line Search and Trust Region Steps, Mathematical Programming, 107(3), pp. 391-408.