

An experimental Indian gravimetric geoid model using Curtin University's approach

R. Goyal^{1,2,*}, W.E. Featherstone^{2,1}, S.J. Claessens², O. Dikshit¹, N. Balasubramanian¹

1) Department of Civil Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India;

rupeshg@iitk.ac.in, onkar@iitk.ac.in, nagaraj@iitk.ac.in

2) School of Earth and Planetary Sciences, Curtin University of Technology, GPO Box U1987, Perth

WA 6845, Australia; w.featherstone@curtin.edu.au, s.claessens@curtin.edu.au

ORCID: R Goyal ((0000-0002-2178-3265); W.E. Featherstone (0000-0001-9644-4535);

S.J. Claessens (0000-0003-4935-6916); O. Dikshit (0000-0003-3213-8218)

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Three Key Points:

- a) Indian gravimetric geoid and quasigeoid models are developed using the CUT method
- b) The peculiarities of the Indian geodetic data are discussed
- c) Intermediate results of geoid computation methodology are presented for the whole of India

Corresponding author: Ropesh Goyal, rupeshg@iitk.ac.in; ropeshgoyal2809@gmail.com

ABSTRACT

22

23 Over the past decade, numerous advantages of a gravimetric geoid model and its possible suitability
24 for the Indian national vertical datum have been discussed and advocated by the Indian scientific
25 community and national geodetic agencies. However, despite several regional efforts, a state-of-the-
26 art gravimetric geoid model for the whole of India remains elusive due to a multitude of reasons.
27 India encompasses one of the most diverse topographies on the planet, which includes the Gangetic
28 plains, the Himalayas, the Thar desert, and a long peninsular coastline, among other topographic
29 features. In the present study, we have developed the first national geoid and quasigeoid models for
30 India using Curtin University's approach. Terrain corrections were found to reach an extreme of 187
31 mGal, Faye gravity anomalies 617 mGal, and the geoid-quasigeoid separation 4.002 m. We have
32 computed both geoid and quasigeoid models to analyse their representativeness of the Indian normal-
33 orthometric heights from the 119 GNSS-levelling points that are available to us. A geoid model for
34 India has been computed with an overall standard deviation of ± 0.396 m but varying from ± 0.03 m
35 to ± 0.158 m in four test regions with GNSS-levelling data. The greatest challenge in developing a
36 precise gravimetric geoid for the whole of India is data availability and its preparation. More densely
37 surveyed precise gravity data and a larger number of GNSS/levelling data are required to further
38 improve the models and their testing.

39

40 Keywords: Geoid, quasigeoid, India, Curtin University's approach

41

42 1. INTRODUCTION

43 Ideally, Stokes's (1849) integral should be implemented over the entire Earth with continuous gravity
44 anomalies on the geoid and with the condition that there must not be any gravitating masses above it.
45 However, in practice, the availability of gravity observations is limited to a specific area, so the
46 integration domain has to be truncated. Also, the gravity anomalies usually exist discontinuously on
47 or above the Earth's surface so various types of downward continuation and regularisation have been
48 proposed. The gaps between theoretical and practical aspects induce several kinds of errors, which
49 geodesists have tried to reduce, but usually requiring assumptions and approximations.

50 Based on various ideas, philosophies and numerical approaches, what we consider the four
51 most commonly used approaches/techniques are adopted for geoid computation experiments in India.
52 i) geoid/quasigeoid computation methodology developed at the University of Copenhagen, Denmark
53 (Forsberg 1984, 1985; Forsberg and Tscherning 2008) implemented in the public-domain
54 GRAVSOFTE package, ii) the Stokes-Helmert method developed at the University of New Brunswick
55 (UNB), Canada (Vaníček and Kleusberg 1987; Vaníček and Martinec 1994; Vaníček et al. 1999;
56 UNB, 2002; Ellmann and Vaníček 2007), iii) the Least Squares Modification of Stokes formula with
57 Additive Corrections (LSMSAC) method developed at the Royal Institute of Technology (KTH),
58 Sweden (Sjöberg 1984, 1991, 2003; Ågren 2004), and iv) geoid/quasigeoid computation
59 methodology developed at Curtin University of Technology (CUT), Australia (Featherstone 2000,
60 2003; Featherstone et al. 1998, 2001, 2011, 2018). There are of course other approaches, such as
61 radial basis functions (e.g., Li 2018; Liu et al. 2020), but perhaps not yet applied as widely. The
62 application areas of the above four approaches are listed in Goyal et al. (2021b).

63 For India, the first geoid map was developed more than five decades ago. It was based on
64 astrogeodetic observations (Fischer 1961) and with respect to the Everest 1956 ellipsoid (cf. Singh
65 and Srivastava 2018). No more information is available on this geoid, apart from distorted hardcopy
66 contour maps that are difficult to digitise reliably. The levelled height information presently available

67 in India is more than a century old. When these heights were observed, neither the concept of foresight
68 and backsight levelling nor the use of invar staves were considered. Observed gravity values were
69 not available as this was before the development of the low-cost portable relative gravimeter. The
70 Indian vertical datum defined in 1909 was based on constraining the levelling to nine tide-gauges
71 along the Indian coast to zero height (Burrard 1910). We will show later that this approach may have
72 caused a north-south tilt (cf. Fischer 1975; 1977), most probably due to the ocean's time-mean
73 dynamic topography (cf. Featherstone and Filmer 2012).

74 Frequent seismic activity in various parts of the Indian sub-continent and so-caused crustal
75 movement also necessitate the introduction of a new height system, probably to be based on
76 geopotential numbers and Helmert's orthometric heights (or 'rigorous' orthometric heights as
77 formulated by Santos et al. 2006). The Survey of India (SoI) carried out a re-levelling program [2007-
78 2017] with gravity observations at fundamental benchmarks to provide a densified network of
79 Helmert's orthometric heights as a part of the Redefined Indian Vertical Datum 2009 (Singh 2018;
80 G&RB 2018). However, these data are not yet in the public domain, so we are unable to use them to
81 validate our geoid and quasigeoid models. In addition, the national geodetic agencies have proposed
82 to compute a precise national geoid model to serve as the new vertical datum for the country. This
83 can be viewed as following the suit of New Zealand (LINZ 2016), Canada (Véronneau and Huang
84 2016), and the USA (NGS 2017; 2019). Such an approach is being considered in many other countries
85 too.

86 Researchers and government organisations have made some efforts to develop local
87 gravimetric geoid models for regions in India (Singh 2007; Carrion et al. 2009; Srinivas et al. 2012;
88 Mishra and Ghosh 2016, Singh and Srivastava 2018), but only using the GRAVSOFIT package with
89 residual terrain modelling (Forsberg 1985). Despite these efforts, a state-of-the-art national
90 gravimetric geoid model for the whole India remains elusive (Goyal et al. 2017). Therefore, in this
91 study, we present the first-ever nationwide geoid and quasigeoid computation results over India with

92 the available data sets using the CUT method implemented using our own computation package
 93 developed in MATLAB™.

94

95 **2. DATASETS**

96 **2.1 Terrestrial Gravity**

97 Pointwise observed gravity data is confidential in India. Therefore, with this predicament, we
 98 obtained a grid of Indian terrestrial gravity data from GETECH (<https://getech.com/>) that is claimed
 99 to come from the Gravity Map Series of India (GMSI), a joint project of five Indian organisations,
 100 viz., SoI, Geological Survey of India (GSI), Oil and Natural Gas Corporation (ONGC), National
 101 Geophysical Research Institute (NGRI) and Oil India Limited (cf. Tiwari et al. 2014). The GETECH
 102 gravity data comprises a $0.02^\circ \times 0.02^\circ$ grid of simple Bouguer gravity anomalies over India (except a
 103 for few regions in northern India), with an overall estimated precision of ± 1.5 mGal (GETECH,
 104 2006). According to the GETECH manual for Indian gravity data, they used i) the normal gravity
 105 formula from WGS84 (Macomber 1984)

$$106 \quad \gamma_{0_WGS84} = 978032.67714 \left[\frac{1 + 0.00193185438639 \sin^2 \phi}{\sqrt{1 - 0.00669437999013 \sin^2 \phi}} \right] mGal \quad (1)$$

107 ii) a second-order free-air correction given by

$$108 \quad \delta g_{FAC}^{GETECH} = (0.3083293357 + 0.0004397732 \cos^2 \phi) h - 7.2125 \times 10^{-8} h^2 \text{ mGal} \quad (2)$$

109 iii) the following atmospheric correction (Ecker and Mittermayer 1969)

$$110 \quad \delta g_{atm}^{GETECH} = \begin{cases} 0.87 e^{-0.116H^{1.047}} mGal, & H > 0km \\ 0.87 mGal & , H \leq 0km \end{cases} \quad (3)$$

111 and iv) the simple planar Bouguer correction

$$112 \quad \delta g_{BC}^{GETECH} = -0.04191 \rho H \text{ mGal} \approx -0.1119 H \text{ mGal} \quad (4)$$

113 where γ_{0_WGS84} is normal gravity on the WGS84 level ellipsoid, δg_{FAC}^{GETECH} is the free-air correction,
 114 ϕ is the geodetic latitude, h is the ellipsoidal height (in m), H is the elevation (in km for Eqn. 3 and
 115 m for Eqn. 4), δg_{atm}^{GETECH} is the atmospheric correction, δg_{BC}^{GETECH} is the planar Bouguer correction
 116 and ρ is the constant topographical density of 2,670 kg/m³. We re-computed the free-air gravity
 117 anomalies (Δg) from the GETECH data so as to be more compatible with the CUT approach by
 118 using

$$119 \quad \Delta g = \Delta g_{SBA}^{GETECH} + 0.1119H + \gamma_{0_WGS84} - \delta g_{FAC}^{GETECH} - \delta g_{atm}^{GETECH} - \gamma_{0_GRS80} + \delta g_{FAC}^{CUT} + \delta g_{atm}^{CUT} \quad (5)$$

120 where, Δg_{SBA}^{GETECH} are simple Bouguer anomalies from GETECH and

$$121 \quad \delta g_{FAC}^{CUT} = \gamma_{0_GRS80} \left(\frac{2}{a} (1 + f + m - 2f \sin^2 \phi) H - \frac{3}{a^2} H^2 \right) \quad (6)$$

$$122 \quad \gamma_{0_GRS80} = \gamma_a \left(\frac{1 + k \sin^2 \phi}{\sqrt{1 - e^2 \sin^2 \phi}} \right) \quad (7)$$

$$123 \quad \delta g_{atm}^{CUT} = 0.871 - 1.0298 \times 10^{-4} H + 5.3105 \times 10^{-9} H^2 - 2.1642 \times 10^{-13} H^3 + \quad (8)$$

$$9.5246 \times 10^{-18} H^4 - 2.2411 \times 10^{-22} H^5$$

For GRS80, $a = 6378137m$, $e^2 = 0.0066943800229$, $m = 0.0034478600308$, $f = 1/298.257222101$
 125 $\gamma_a = 978032.67715mGal$, $k = 0.001931851353$ (Moritz 2000). The descriptive statistics of the
 126 differences between the free-air anomalies from the GETECH data and re-computed free-air
 127 anomalies are (in mGal): min=-0.001, max=0.188, mean=0.002, STD= \pm 0.007. It should be noted that
 128 we have used H instead of h (ellipsoidal heights) in Eqn. 2 because we believe that there might be
 129 a typographical error in the GETECH manual. The rationale being that with the use of h we would
 130 obtain gravity disturbances and not gravity anomalies (cf. Hackney and Featherstone 2003). A blanket
 131 accuracy estimate of the reconstructed free-air anomalies from the GETECH Bouguer anomalies is

132 ± 2.4 mGal, calculated using the DEM error in the CUT reconstruction technique as per

133
$$\sigma_{FA} = \sqrt{(1.5 \times 10^{-5})^2 + (2\pi G\rho \times 17.3)^2}.$$

134 For the oceanic regions surrounding India, we used free-air gravity anomalies (Version 28.1)
135 from the Scripps Institute of Oceanography (SCRIPPS,
136 https://topex.ucsd.edu/marine_grav/mar_grav.html) which has an overall root mean square error of
137 ± 1.23 mGal (Sandwell et al. 2019). The SCRIPPS data is also accompanied with an error grid that
138 we have shown, for our study area, in Figure 1. The data contains a 1'x1' grid that also covers the
139 land, but we used the SCRIPPS data only for the oceanic region because the land data, in the SCRIPPS
140 dataset, is from EGM2008 to avoid aliasing (Gibbs fringing) at the coasts.

141 **PLACE FIGURE 1 NEAR HERE**

142 We do not have gravity data from the countries neighbouring India and a well distributed
143 sufficient data coverage is not available in the Bureau Gravimetrique International ([https://bgi.obs-](https://bgi.obs-mip.fr/)
144 [mip.fr/](https://bgi.obs-mip.fr/)) archives either (Country: no. of gravity data points - Pakistan: 1270, Bangladesh: 25, Sri
145 Lanka: 48, Myanmar: 71, Afghanistan: 1649, China: 446, Nepal: 617 and Bhutan: 0.). Therefore, we
146 constructed a $0.02^\circ \times 0.02^\circ$ grid of free air anomalies over land using EGM2008 (Pavlis et al. 2012;
147 2013) up to degree and order (d/o) 900 to fill-in the land gravity anomaly data in and around India
148 where the GETECH data is not available, including Nepal, China, Pakistan, Sri Lanka, Bangladesh,
149 Bhutan, Afghanistan and Myanmar. The specific d/o 900 was chosen because EGM2008 uses
150 proprietary data up to d/o 900 (Pavlis et al. 2013).

151 As discussed next, we merged these three datasets to get a complete free-air gravity anomaly
152 grid of $0.02^\circ \times 0.02^\circ$, avoiding aliasing or the contamination of land data (both GETECH and
153 EGM2008 individually) with the marine data or vice-versa.

154 There exist numerous sophisticated space-domain and frequency-domain methods for
155 merging heterogenous gravity anomaly datasets (e.g., Strykowski and Forsberg 1998; Olesen et al.
156 2002; Catalao 2006; McCubbine et al. 2017). However, we chose to work with the comparatively

157 straightforward CUT space-domain method (cf. Featherstone et al. 2011; 2018). This choice is
158 somewhat arbitrary because we are working with the land gravity of unknown quality, and the
159 strategy that we use has already been implemented in the computation of the Australian quasigeoid,
160 which is an island nation and approximately 2.3 times larger than India. Other methods can also be
161 tested, but it is left for the time when sufficient marine and airborne gravity data along with reliable
162 terrestrial gravity data will be available over India.

163 In the adopted method, the GETECH-derived free-air anomaly grid is superimposed over the
164 EGM2008 (d/o 900) derived gravity anomalies. The gravity anomalies of the latter dataset at the
165 overlapping grid nodes are replaced by the gravity anomalies from the former dataset. As a result, a
166 $0.02^\circ \times 0.02^\circ$ grid of gravity anomalies on the land is obtained.

167 To concatenate the land and marine gravity anomaly data, $1' \times 1'$ gravity anomalies in the ocean
168 are clipped (or separated) from the complete SCRIPPS dataset, i.e., on both ocean and land. It is then
169 block averaged to the $0.02^\circ \times 0.02^\circ$ grid and is superimposed with the land gravity anomaly grid. The
170 former values were replaced by the latter at overlapping nodes to obtain the $0.02^\circ \times 0.02^\circ$ grid of the
171 merged gravity anomalies. Figure 2 shows the merged free-air gravity anomaly map. To check for
172 any discontinuities at the edges of the merged datasets, we computed and plotted the arctangent
173 (Figure 3a) and logarithmic (Figure 3b) values of the gradients of the merged data. We observe no
174 clear visual indication of any discontinuities at the boundaries of the merged data, but also partially
175 due to the ruggedness of the dataset in our study area that can be obscuring.

176 **PLACE FIGURES 2 AND 3 NEAR HERE**

177

178 **2.2 Digital Elevation Model**

179 The Digital Elevation Model (DEM) is another important input in geoid computation. It is mainly
180 used to compute the topographical effects (e.g., Forsberg 1984). Thus, a precise high-resolution DEM
181 should be used. We would like to mention here that DEM is generally used synonymously with a

182 Digital Surface Model (DSM) (e.g., SRTM, ASTER), but this should be avoided. Quantification of
183 the differences in the topographical effect with the use of DEM and DSM has been investigated by
184 Yang et al. (2019). Since India does not have a national DEM, therefore, after a DEM/DSM analysis
185 (Goyal et al. 2021a), it was decided to work with the best available DEM over India, i.e., the MERIT
186 3"x3" DEM (Yamazaki et al. 2017), for our computations. Though the accuracy of the MERIT DEM
187 varies considerably (± 11.7 m to ± 47.3 m) over different landforms in India, an overall estimate for
188 the whole of India is ± 17.3 m (Goyal et al. 2021a).

189

190 2.3 GNSS-Levelling

191 India has different horizontal and vertical control networks. Therefore, presently there are only a
192 limited number of ground control points where we have the geodetic coordinates (latitude, longitude,
193 ellipsoidal heights) and levelled heights. Moreover, due to several restrictions on the datasets, only a
194 few of these available data points were available to us (Figure 4). The datasets in the Uttar Pradesh
195 west (UP west) and Uttar Pradesh east (UP east) regions were procured from SoI, while the datasets
196 over Hyderabad and Bangalore have been retrieved from Mishra (2018), who also used the SoI
197 dataset. According to Mishra (2018), horizontal and vertical precisions of GNSS data are within ± 12
198 mm to ± 26 mm and ± 31 mm to ± 53 mm, respectively. The vertical precision of the levelling heights
199 is not known to us, but they are from the high precision first level net of India. These heights are from
200 the Indian Vertical Datum 1909 (Burrard, 1910) and are based on the normal-orthometric height
201 system, while those on Indian Vertical Datum 2009 (G&RB 2018) are based on Helmert's
202 orthometric height system. We have not been provided with a clear indication on which heights have
203 been provided to us, and therefore, due to this anonymity of the height system, we consider the
204 levelling heights to be in the normal-orthometric height system (Jekeli 2000; Featherstone and Kuhn
205 2006).

206 **PLACE FIGURE 4 NEAR HERE**

208 3. METHODOLOGY AND RESULTS

209 An overview of the CUT methodology for computing the geoid undulations is shown by a flowchart
210 in Figure 5. The CUT method primarily computes the quasigeoid using the analytical continuation
211 solution (Moritz 1971; 1980) of Molodensky's problem (Molodensky et al. 1962). Moritz (1971)
212 showed that Molodensky's G_1 term can be approximated by the planar terrain correction (TC), which
213 also needs an additional term that is equal to the first-order indirect effect (FOIE). We could not adopt
214 the full CUT method-based reconstruction of Faye anomalies (Featherstone and Kirby 2000) because
215 we already have gridded data, whereas CUT grids point Bouguer anomalies. Instead, we added the
216 block averaged $0.02^\circ \times 0.02^\circ$ grid of TCs (Figure 6a) to the free-air gravity anomaly grid to calculate
217 area-mean Faye anomalies (Figure 6b). The block-averaged $0.02^\circ \times 0.02^\circ$ TC grid was constructed
218 from the $3'' \times 3''$ TC grid computed with the MERIT DEM using the Optimal Separating Radius (OSR)
219 in the spatial-spectral combined method suggested by Goyal et al. (2020). This method of TC
220 guarantees the full convergence of the TC solution, i.e., down to $<1\mu\text{Gal}$.

221 **PLACE FIGURE 5 NEAR HERE**

222 A different approach is used in the CUT method to apply ellipsoidal correction. Unlike other
223 geoid computation strategies considered (UNB or KTH; cf. Huang et al. 2003; Ellmann 2005), the
224 CUT method computes ellipsoidal area-mean free-air gravity anomalies on the topography using a
225 Global Geopotential Model (GGM) (Featherstone et al. 2018). These are subtracted from the mean
226 Faye gravity anomalies to obtain residual gravity anomalies (Figure 6c), which are then Stokes-
227 integrated with the Featherstone-Evans-Olliver (FEO) modified kernel (Featherstone et al. 1998) to
228 obtain the residual height anomalies. The FEO kernel, a deterministic modifier, is the combination of
229 the Meissel (1971) and Vaníček and Kleusberg (1987) modifiers that simultaneously reduces the
230 truncation error and improves the rate of convergence to zero of the series expansion of the truncation
231 error (cf. Featherstone et al. 1998, Featherstone 2003). Additionally, the spherical reference radius in

232 the Stokes integration is set equal to the geocentric ellipsoidal radius of the computation point, and
233 this negates the need for further ellipsoidal corrections to Stokes's integral (Claessens 2006).

234 The residual height anomalies were computed using the following parameter-sweeps of the
235 modification degree (M): 0, 40, 80, 120, 160, 200, 240, 280, 300 and integration cap radius (ψ): 0.2°,
236 0.5°, 0.75°, 1°, 1.5°, 2° (e.g., Figure 6d for M=80, $\psi=1.5^\circ$). The reference height anomalies on the
237 topography are computed using GGMs with a zero-degree term (N_0) from the generalised Bruns's
238 formula (Eqn. 9) (Heiskanen and Moritz 1967) calculated for each latitude parallel, which are added
239 to the residual height anomalies to obtain the required height anomalies. An inconsistent use of Eqn.
240 9 can cause an error of ~ 1 m in the computed geoid undulations/height anomalies. We used normal
241 potential $U_0 (= 62636860.85 \text{ m}^2 \text{ s}^{-2})$ from GRS80 (Moritz 2000) and the geopotential
242 $W_0 (= 62636853.4 \text{ m}^2 \text{ s}^{-2})$ from IHRS (Sanchez et al., 2016).

$$243 \quad N_0 = \frac{GM_G - GM_E}{r\gamma_0} - \frac{W_0 - U_0}{\gamma_0} \quad (9)$$

244 As a small modification to the original CUT method, we added the $FOIE = -\frac{\pi G \rho H^2}{\gamma}$ (Moritz, 1980,
245 Eqn. 48-29; Heiskanen and Moritz 1967, chapter 8) to the computed height anomalies. We note that
246 the negative sign is sometimes omitted (e.g., Sjöberg 2000; Hwang et al. 2020).

247 **PLACE FIGURE 6 NEAR HERE**

248 The geoid undulations are calculated by adding the quasigeoid-geoid separation term (Figure
249 6e; Flury and Rummel 2009) to the height anomalies. The more rigorous quasigeoid-geoid separation
250 term from Flury and Rummel (2009) differs quite considerably from the approximate formula given
251 in Heiskanen and Moritz (1967, p 328). The difference in the quasigeoid-geoid separation term from
252 the two methods is shown in Figure 6f. Acknowledging that the number and distribution of the
253 GNSS/levelling data points are not sufficient for reliable fitting (Kotsakis and Sideris 1999;

254 Fotopoulos 2003), we have not presented hybrid geoid and hybrid quasigeoid models for this
255 experiment over India.

256 The geoid should be validated with orthometric (Helmert or rigorous (Santos et al. 2006))
257 heights and the quasigeoid validated with normal heights. A more rigorous validation approach would
258 be to convert the normal-orthometric heights to Helmert’s orthometric height and normal heights for
259 validating geoid and quasigeoid, respectively. Examples of this are Foroughi et al. (2017) and Janák
260 et al. (2017) over Auvergne, France, where normal heights were converted to rigorous heights for
261 validation of their developed geoid models. However, Indian levelled heights are based on the
262 normal-orthometric height system for which there is no specific choice of reference surface, i.e.,
263 either geoid or quasigeoid. Therefore, we are only able to “validate” the developed geoid and
264 quasigeoid models with the Indian normal-orthometric heights on an uncertain vertical datum
265 (Section 2.3).

266 Absolute and relative testing (Featherstone 2001) of both height anomalies and geoid
267 undulations are done in this study. The absolute testing is realised through point-wise subtraction of
268 gravimetric geoid undulations obtained using Stokesian integration (N) and the geometrical geoid
269 undulation ($h - H$) obtained using GNSS/levelling data (Eqn. 10).

270
$$\varepsilon_i^{abs} = N_i - (h_i - H_i) \quad \forall i = 1, 2, 3, \dots, n \quad (10)$$

271 where n is the total number of discrete GNSS/levelling data points. It is important to acknowledge
272 that absolute accuracy is only an assumption. This is principally because the levelled heights that
273 refer to the local vertical datum are not necessarily coincident with the geoid. This has been discussed
274 in detail by Featherstone (2001). The descriptive statistics of ε_i^{abs} are in Table 1.

275 **PLACE TABLE 1 NEAR HERE**

276 The relative testing of geoid and quasigeoid (Eqn. 11) is an analysis tool to investigate their
277 gradients. This type of analysis is of more interest to land surveyors who use relative GNSS baselines
278 and a geoid/quasigeoid gradients as a replacement for the more time-consuming differential levelling.

$$279 \quad \varepsilon_{ij}^{rel} = \Delta N_{ij} - (\Delta h_{ij} - \Delta H_{ij}) \quad \forall i, j = 1, 2, 3, \dots, n; i \neq j \quad (11)$$

280 The descriptive statistics of ε_{ij}^{rel}), and the ratio of mean differences to the mean baseline length in
281 parts per million (average ppm in mm/km) for the geoid and quasigeoid are in Table 2.

282 **PLACE TABLE 2 NEAR HERE**

283 The variation of standard deviation in the Indian geoid and quasigeoid models, on testing with
284 GNSS/levelling data, for different combinations of modification degree and integration cap are shown
285 in Figures 7a and 7b, respectively. Table 1 depicts the region-wise (UP west, UP east, Hyderabad,
286 Bangalore, and all together) descriptive statistics for the geoid and quasigeoid for the combination of
287 $M=80$ and $\psi=1.5^\circ$. Though the standard deviation for the whole of India is smaller with the
288 combination of $M=40$ and $\psi=1.5^\circ$ compared to $M=80$ and $\psi=1.5^\circ$ (cf. Figure 7), standard deviations
289 for the four individual regions are less than or equal to the combination of $M=80$ and $\psi=1.5^\circ$
290 compared to $M=40$ and $\psi=1.5^\circ$. Therefore, $M=80$ and $\psi=1.5^\circ$ was chosen to present our results. The
291 results of the relative testing are shown in Figures 8a and 8b, and Table 2. The computed Indian
292 gravimetric geoid (IndGG-CUT2021) and corresponding contours (at a 2 m contour interval) are
293 shown in Figures 9a and 9b, respectively.

294 **PLACE FIGURES 7, 8 AND 9 NEAR HERE**

295

296 **4. DISCUSSION, RECOMMENDATIONS AND CONCLUSIONS**

297 Though the number (119) and the distribution (Figure 4) of the GNSS/levelling data points are
298 insufficient to draw concrete conclusions about the quality of the computed gravimetric geoid and
299 quasigeoid models, the following are some major observations from our experimental results:

- 300 1. Since the study area comprises the most complex topography varying from the Himalayas to the
301 Gangetic plains and a long peninsular coastline, Figure 6 possibly depicts the extreme (maximum
302 and minimum) values of planar TC, Faye gravity anomaly, and quasigeoid-geoid separation on
303 the planet.
- 304 2. From the viewpoint of the “cm-level accurate” geoid, Figure 6f suggests that a more rigorous
305 method (e.g., Flury and Rummel 2009) should be preferred for calculating the quasigeoid-geoid
306 separation over a simple approximate formula (e.g., Heiskanen and Moritz 1967). There exist
307 other formulas for the quasigeoid-geoid separation term (e.g., Sjöberg 2010; Foroughi and Tenzer
308 2017), but they are not tested here.
- 309 3. Figures 7 suggest that the FEO kernel (Featherstone et al. 1998) is not numerically unstable for
310 higher modification degrees, as shown in Featherstone (2003), Li and Wang (2011), Featherstone
311 et al. (2018) and Claessens and Filmer (2020). However, this observation can also result from our
312 choice of parameter sweeps and limited datasets for validation, thus requiring further
313 investigation.
- 314 4. Generally, standard deviations versus GNSS/levelling are large for lower modification degrees
315 and larger integration radii (Featherstone et al. 2018; Claessens and Filmer 2020). However,
316 Figure 7 shows an opposite trend in India, with smaller standard deviations for lower modification
317 degrees and larger integration radii. This is primarily attributable to the north-south tilt in the
318 India height datum (cf. Table 1). However, the smaller number of GNSS/levelling data and their
319 poor distribution are also likely to contribute to this observation.
- 320 5. Figure 7 shows that the Indian levelling heights are marginally better referred to the quasigeoid
321 (std = ± 0.389 m) than the geoid (std = ± 0.396 m). However, Table 1 shows that the geoid has an
322 equal or better precision estimate than the quasigeoid (in terms of standard deviation) in each of
323 the four regions individually. The difference in the standard deviations of the quasigeoid and
324 geoid comparison for the whole of India seems to be a consequence, mostly, of the smaller mean

325 of the quasigeoid (0.690 m) than the geoid (0.751 m) comparison over Bangalore. Also, with the
326 given precision estimate of the data points, there can yet be no preferred choice between geoid or
327 quasigeoid for the Indian vertical datum. Hence, a larger set of data points are needed for any
328 possible claim of reference surface for India. Though the overall standard deviation of the
329 computed geoid/quasigeoid (Table 1) is $\sim\pm 0.40$ m, it varies from $\sim\pm 0.03$ m to $\sim\pm 0.16$ m if only
330 evaluated individually in the four small test regions.

331 6. Table 2 indicates that the largest misclosures in Figure 8 are probably due to the tilt in the Indian
332 height datum and the relative closeness of data points in Hyderabad and Bangalore, which also
333 explains the larger ppm values found in those regions. Spikes in Figures 8a and 8b at distances of
334 approximately [0-50] km, [450-550] km, [900-1200] km, and [1200-1900] km are due to the
335 errors and differences (north-south tilt) in the baselines for [Bangalore and Hyderabad,
336 individually], [Bangalore to Hyderabad], [UP west, UP east to Hyderabad], and [UP west, UP
337 east to Bangalore], respectively.

338 7. On comparison of validations of the Indian gravimetric geoid with the CUT method and the GGM
339 (Table 3), it is observed that though the overall mean values are improved for all regions except
340 Bangalore, an improvement in the standard deviation beyond ± 0.01 m is observed only for UP
341 east. However, the standard deviation of gravimetric geoid in UP west is degraded by ± 0.03 m
342 as compared to the EIGEN-6C4. A degradation in the standard deviation of the gravimetric geoid
343 is also observed in Featherstone and Sideris (1998). This was, and similarly is, attributed to errors
344 in either one or more of terrestrial gravity data, GGMs and the GNSS/levelling data.

345 There is little to no improvement with the inclusion of the terrestrial gravity data with the CUT
346 method because it makes use of the highest available degree-order GGMs. Also, the GETECH
347 data is possibly already included in the high degree-order GGM (e.g., EGM2008, Pavlis et al.,
348 2012; 2013)

349 8. The Faye gravity anomaly (Figure 6b), geoid (Figure 9a), and contour map (Figure 9b) somewhat
350 depict the separation line of the Indian and the Eurasian plate. Thus, the results presented in this
351 study could be important for geophysical studies. The contour pattern around the location of 24°N
352 and 82°E seems intriguing for some gravimetric studies in that region. It should also be noted that
353 the area comprises one of the largest coalfields of India with the thickest and different varieties
354 of coal seams.

355 As a final remark, first experimental geoid and quasigeoid models for India have been computed with
356 a standard deviation of ± 0.396 m and ± 0.389 m, respectively, with respect to a small number of test
357 regions. However, for the four regions individually, the standard deviation varies from ± 0.030 m to
358 ± 0.158 m for the geoid and ± 0.032 m to ± 0.158 m for the quasigeoid. Though all the results presented
359 herein are the first from India, the geoid/quasigeoid must be improved with dense, precise gravity
360 data. Moreover, a larger number of GNSS/levelling data points must become available for more
361 rigorous validation of the gravimetric geoid/quasigeoid. For the re-computation of the Indian
362 geoid/quasigeoid with the CUT method and additional gravity data, the TC and the quasigeoid-geoid
363 separation term need not be computed again unless a high-resolution and more precise DEM is
364 available. Further, due to the complexities of the Indian topography and geomorphic characteristics,
365 other geoid/quasigeoid computation strategies should also be tested over India.

366

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Figure Legends

576 Figure 1: Error map of the SCRIPPSv28.1 marine gravity-anomaly data (units in mGal).

577 Figure 2: Merged gravity anomaly data from GETECH, EGM2008 (d/o 900) and SCRIPPS (units
578 in mGal)

579 Figure 3: Arctangent (a) and logarithmic (b) plot of gradients of merged gravity anomaly data to
580 attempt to identify discontinuities at the edges of the merged grids.

581 Figure 4: Spatial coverage of the available 119 GNSS/levelling data points

582 Figure 5: Flowchart of the CUT methodology of geoid/quasigeoid computation as applied in India
583 for these experiments.

584 Figure 6: a) Block averaged planar TC, b) Faye anomaly, c) residual Faye anomaly, d) residual
585 quasigeoid ($M=80$ and $\psi=1.5^\circ$), e) quasigeoid-geoid separation term (Flury and Rummel 2009), f)
586 difference in quasigeoid-geoid separation term (Flury and Rummel 2009 minus Heiskanen and
587 Moritz 1967). [*all on a $0.02^\circ \times 0.02^\circ$ grid*] (units of a, b, c in mGal and d, e, f in m)

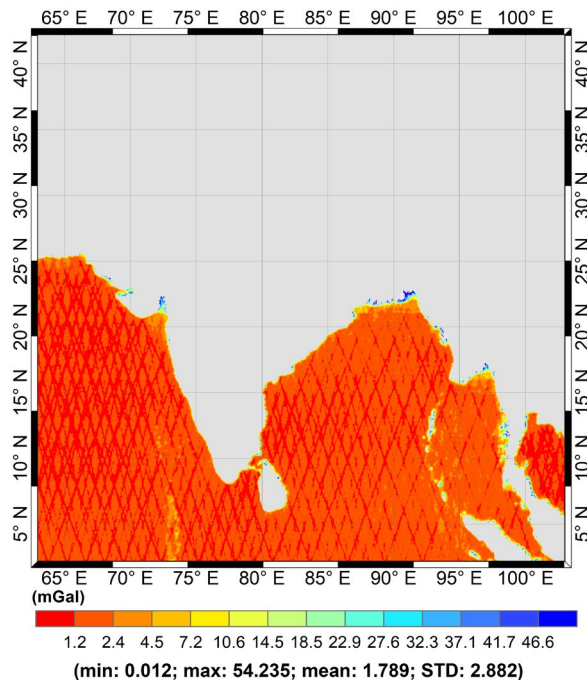
588 Figure 7: Standard deviation of a) geoid and b) quasigeoid of India for different combinations of
589 modification degree and integration cap (units in m).

590 Figure 8: Magnitude of relative differences (blue circles) for the a) geoid, and b) quasigeoid. Orange
591 and yellow circles represent the maximum permissible in-field misclose for Indian high-precision (k
592 = 3) and double tertiary ($k = 12$) levelling for each baseline, respectively (units in m).

593 Figure 9: a) Indian gravimetric geoid computed using the CUT method (units in m), and b)
594 corresponding 2 m geoid contours.

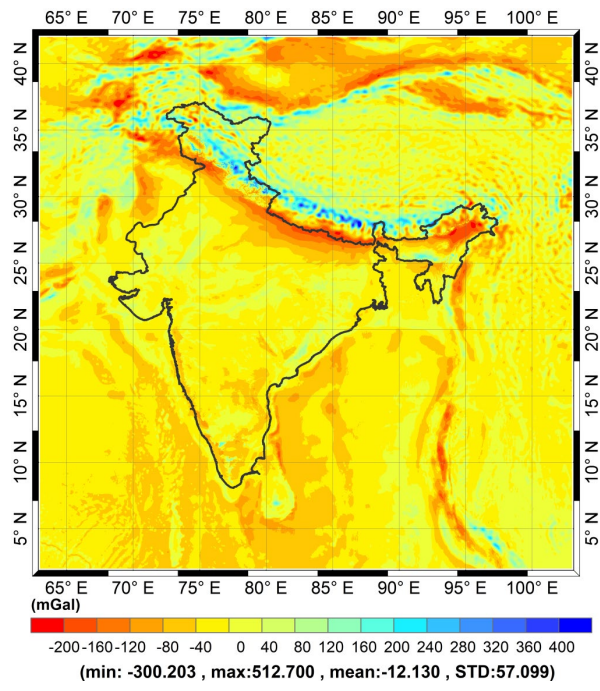
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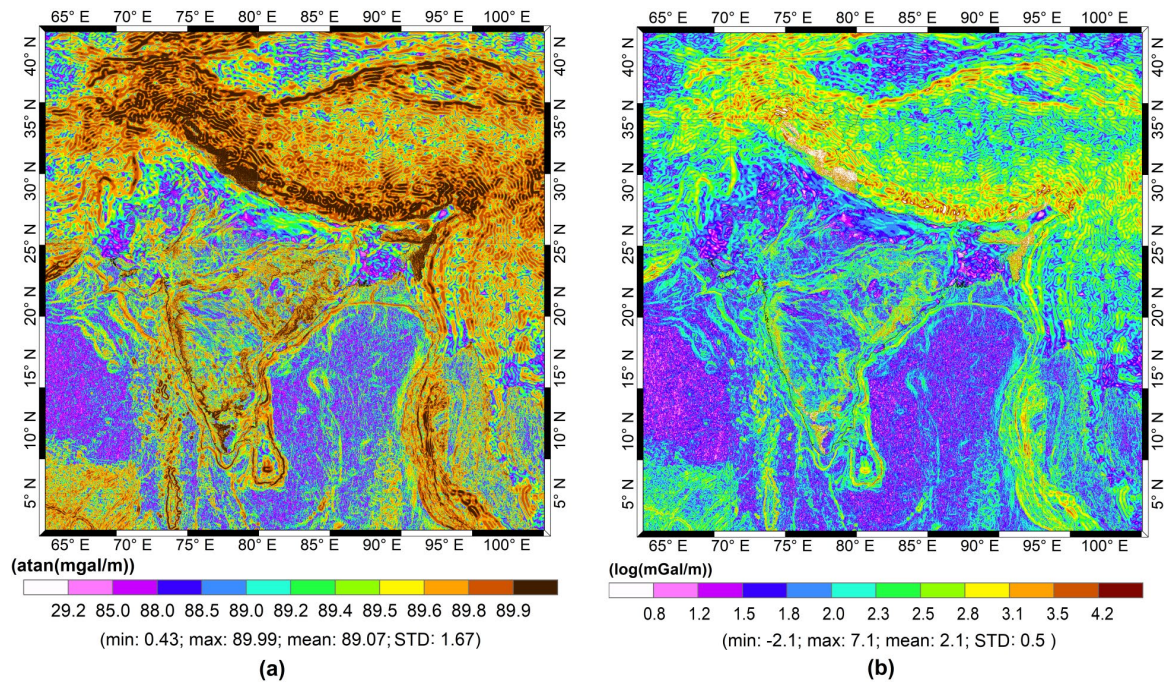
599 Figure 1: Error map of the SCRIPPSv28.1 marine gravity-anomaly data (units in mGal).

600



602 Figure 2: Merged gravity anomaly data from GETECH, EGM2008 (d/o 900) and SCRIPPS (units
 603 in mGal)

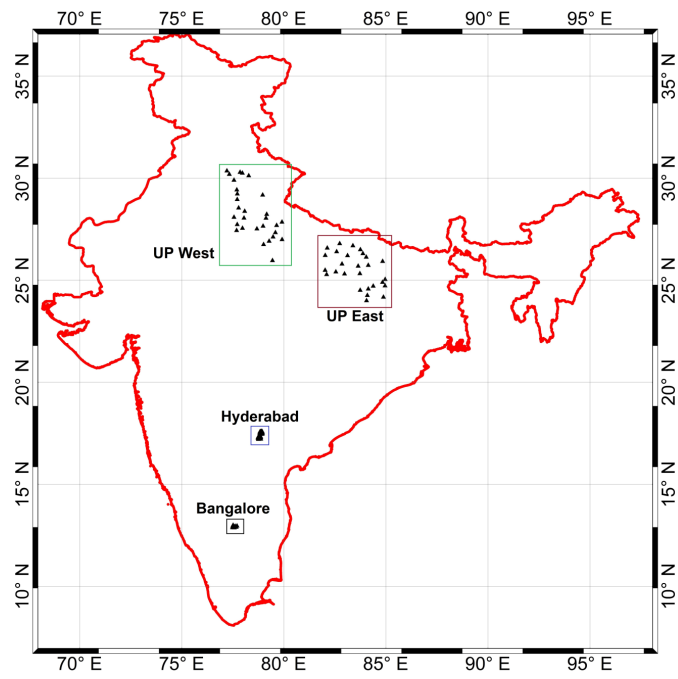
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606 Figure 3: Arctangent (a) and logarithmic (b) plot of gradients of merged gravity anomaly data to
 607 attempt to identify discontinuities at the edges of the merged grids.

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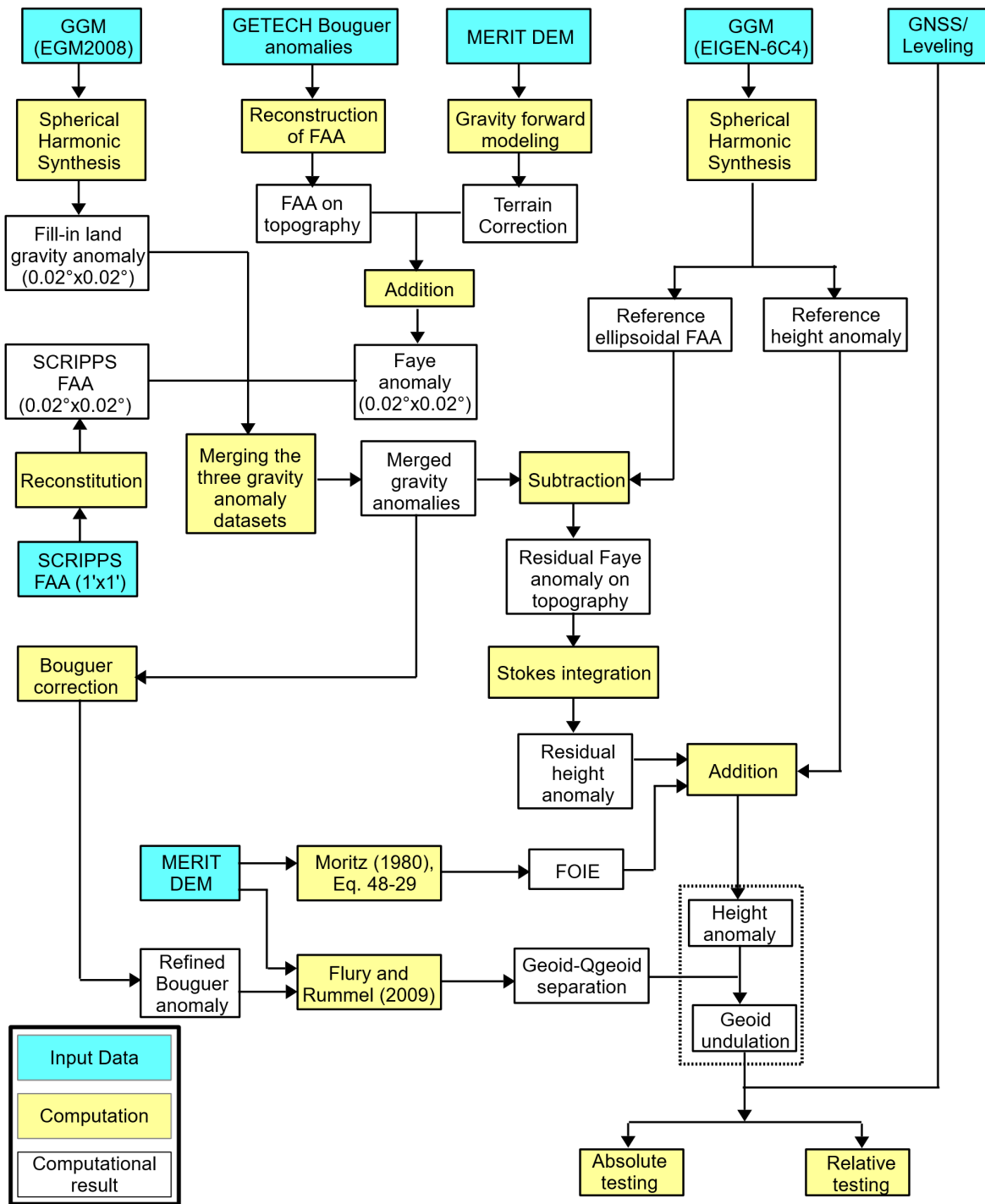
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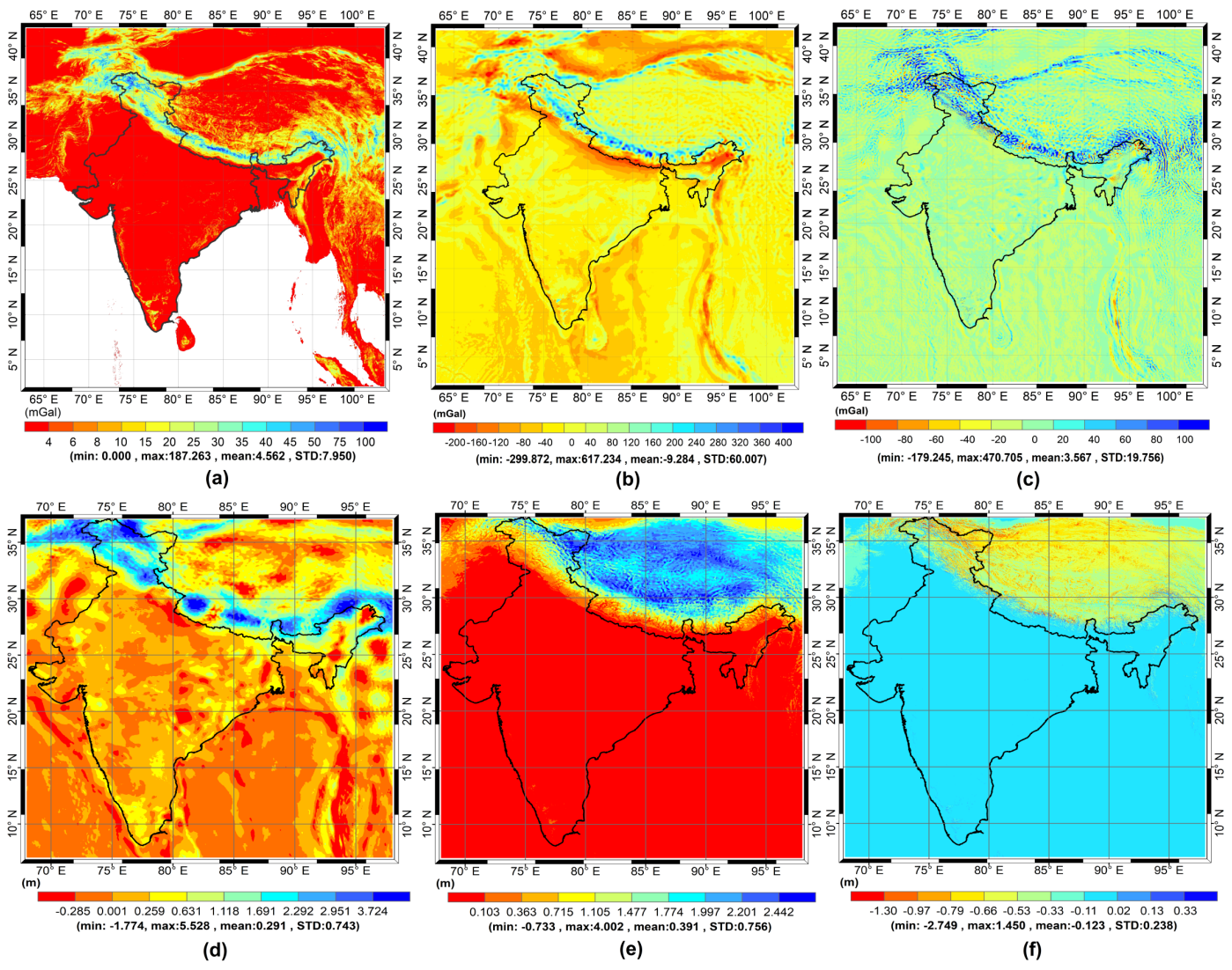
Figure 4: Spatial coverage of the available 119 GNSS/levelling data points

612



614 Figure 5: Flowchart of the CUT methodology of geoid/quasigeoid computation as applied in India
 615 for these experiments.

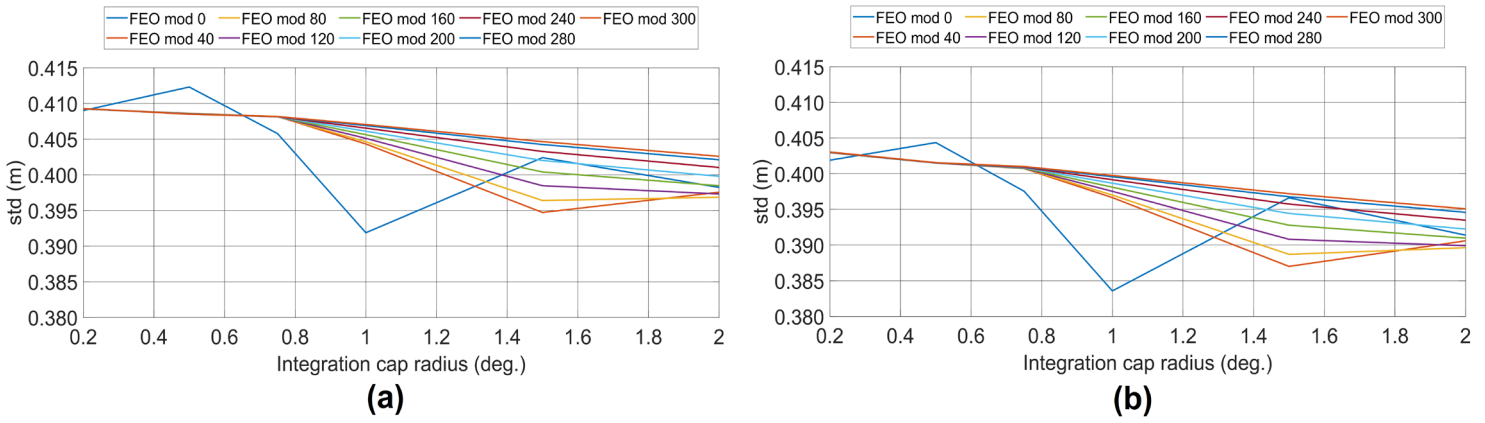
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618 Figure 6: a) Block averaged planar TC, b) Faye anomaly, c) residual Faye anomaly, d) residual
 619 quasigeoid ($M=80$ and $\psi=1.5^\circ$), e) quasigeoid-geoid separation term (Flury and Rummel 2009), f)
 620 difference in quasigeoid-geoid separation term (Flury and Rummel 2009 minus Heiskanen and
 621 Moritz 1967). [all on a $0.02^\circ \times 0.02^\circ$ grid] (units of a, b, c in mGal and d, e, f in m)

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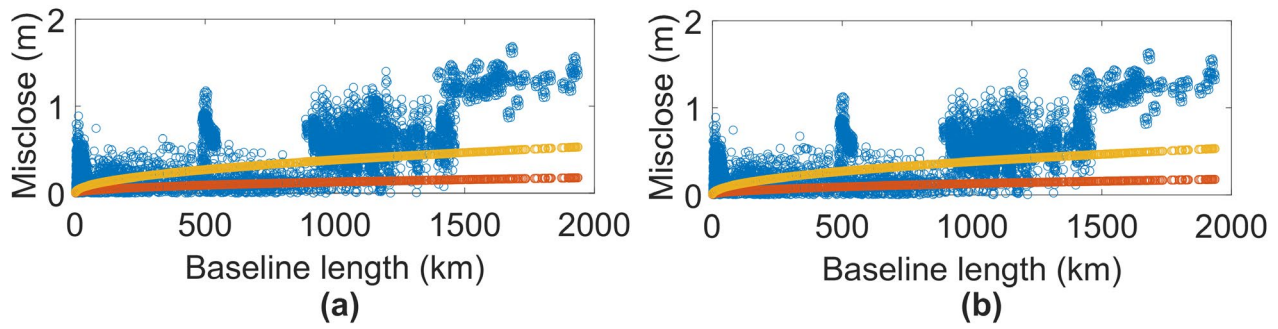
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628 Figure 7: Standard deviation of a) geoid and b) quasigeoid of India for different combinations of
629 modification degree and integration cap (units in m).

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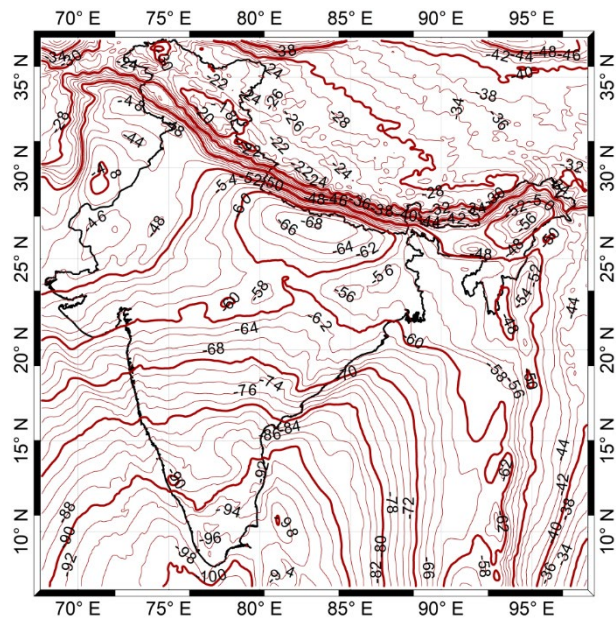
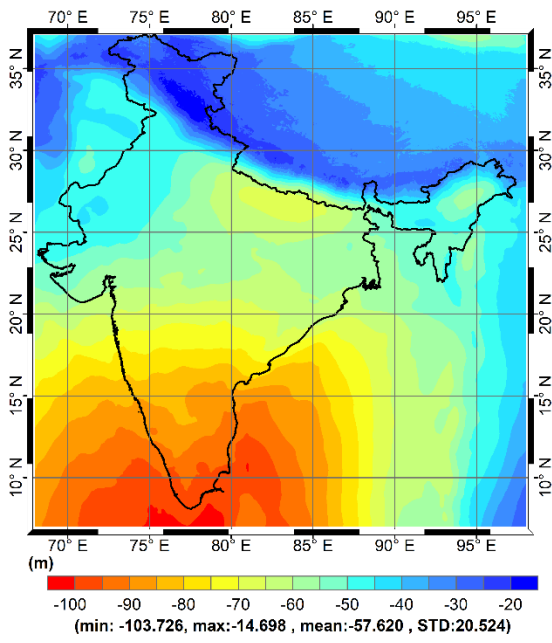
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633 Figure 8: Magnitude of relative differences (blue circles) for the a. geoid, and b. quasigeoid. Orange
634 and yellow circles represent the maximum permissible in-field misclose for Indian high-precision (k
635 $= 3$) and double tertiary ($k = 12$) levelling for each baseline, respectively (units in m).

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639 Figure 9: a) Indian gravimetric geoid computed using the CUT method (units in m), and b)
 640 corresponding 2 m geoid contours.

641

642

Tables643 Table 1: Statistics for the region-wise geoid/quasigeoid (for $M=80$ and $\psi=1.5^\circ$) absolute testing (units in m)

Region	Geoid				Quasigeoid			
(no. of points)	min	max	mean	STD	Min	Max	Mean	STD
India (119)	-0.897	0.788	-0.171	± 0.396	-0.906	0.726	-0.185	± 0.389
UP west (29)	-0.897	-0.154	-0.532	± 0.138	-0.906	-0.164	-0.548	± 0.142
UP east (27)	-0.712	-0.338	-0.521	± 0.114	-0.711	-0.340	-0.523	± 0.114
Hyderabad (56)	-0.385	0.501	0.070	± 0.158	-0.400	0.488	0.057	± 0.158
Bangalore (7)	0.709	0.788	0.751	± 0.030	0.645	0.726	0.690	± 0.032

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647 Table 2: Statistics for the region-wise geoid/quasigeoid (for $M=80$ and $\psi=1.5^\circ$) relative testing

Region	Geoid					Quasigeoid				
	min (m)	max (m)	mean (m)	STD (m)	Average ppm	min (m)	max (m)	mean (m)	STD (m)	Average ppm
India (713.46 km)	-0.620	1.684	0.373	± 0.418	3.371	-0.625	1.632	0.368	± 0.408	3.362
UP west (197.28 km)	-0.605	0.743	0.040	± 0.191	1.111	-0.602	0.742	0.057	± 0.193	1.118
UP east (169.33 km)	-0.374	0.367	0.015	± 0.161	1.048	-0.372	0.367	0.018	± 0.161	1.052
Hyderabad (18.67 km)	-0.620	0.886	-0.031	± 0.221	13.032	-0.625	0.888	-0.032	± 0.221	13.025
Bangalore (14.08 km)	-0.074	0.079	-0.005	± 0.044	3.113	-0.077	0.081	-0.008	± 0.046	3.281

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649

650 Table 3: Comparison of EIGEN-6C4 and IndGG-CUT2021 validated with GNSS/levelling data (Units in m).

		min	max	mean	STD
India	EIGEN-6C4	-1.203	0.463	-0.428	±0.410
	IndGG-CUT2021	-0.897	0.788	-0.171	±0.396
UP west	EIGEN-6C4	-1.203	-0.643	-0.870	±0.105
	IndGG-CUT2021	-0.897	-0.154	-0.532	±0.138
UP east	EIGEN-6C4	-1.034	-0.361	-0.742	±0.144
	IndGG-CUT2021	-0.712	-0.338	-0.521	±0.114
Hyderabad	EIGEN-6C4	-0.612	0.258	-0.154	±0.157
	IndGG-CUT2021	-0.385	0.501	0.070	±0.158
Bangalore	EIGEN-6C4	0.379	0.463	0.422	±0.029
	IndGG-CUT2021	0.709	0.788	0.751	±0.030

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