

Colorado geoid computation experiment – Overview and summary

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Abstract. The primary objective of the 1-cm geoid experiment in Colorado (USA) is to compare the numerous geoid computation methods used by different groups around the world. This is intended to lay the foundations for tuning computation methods to achieve the sought after 1-cm accuracy, and also evaluate how this accuracy may be robustly assessed. In this experiment, (quasi)geoid models were computed using the same input data provided by the US National Geodetic Survey (NGS), but using different methodologies. The rugged mountainous study area (730 km × 550 km) in Colorado was chosen so as to accentuate any differences between the methodologies, and to take advantage of newly collected GPS/leveling data of the Geoid Slope

Validation Survey 2017 (GSVS17) which is now available to be used as an accurate and independent test dataset. Fourteen groups from thirteen countries submitted a gravimetric geoid and a quasigeoid model in a 1'×1' grid for the study area, as well as geoid heights, height anomalies, and geopotential values at the 223 GSVS17 marks. This paper concentrates on the quasigeoid model comparison and evaluation, while the geopotential value investigations are presented as a separate paper (Sánchez et al. 2021). Three comparisons are performed: the area comparison to show the model precision, the comparison with the GSVS17 data to estimate the relative accuracy of the models, and the differential quasigeoid (slope) comparison with GSVS17 to assess the relative accuracy of the height anomalies at different baseline lengths. The results show that the precision of the 1'×1' models over the complete area is about 2 cm, while the accuracy estimates along the GSVS17 profile range from 1.2 cm to 3.4 cm. Considering that the GSVS17 does not pass the roughest terrain, we estimate that the quasigeoid can be computed with an accuracy of ~2 cm in Colorado. The slope comparisons show that RMS values of the differences vary from 2 to 8 cm in all baseline lengths. Although the 2-cm precision and 2-cm relative accuracy have been estimated in such a rugged region, the experiment has not reached the 1-cm accuracy goal. At this point, the different accuracy estimates are not a proof of the superiority of one methodology over another because the model precision and accuracy of the GSVS17-derived height anomalies are at a similar level. It appears that the differences are not primarily caused by differences in theory, but that they originate mostly from numerical computations and/or data processing techniques. Consequently, recommendations to improve the model precision towards the 1-cm accuracy are also given in this paper.

Keywords Colorado experiment, 1-cm geoid experiment, geoid computation, quasigeoid computation, geoid-quasigeoid separation term, GRAV-D, GSVS17, GPS/leveling.

Acronyms:

AUTH: Aristotle University of Thessaloniki, Greece

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CGS: Canadian Geodetic Survey, Canada

Curtin: Curtin University, Australia

DGFI: Deutsches Geodätisches Forschungsinstitut, Technical University of Munich, Germany

DTU: Technical University of Denmark, Denmark

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NGS: US National Geodetic Survey, NOAA, USA

GSI: Geospatial Information Authority of Japan, Japan

IAPG: Institute for Astronomical and Physical Geodesy, Technical University of Munich, Germany

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NTIS-GEOF: New Technologies for the Information Society, University of West Bohemia, Czech Republic & University of Zagreb, Croatia

Polimi: Politecnico di Milano, Italy

1. Introduction

According to Resolution No. 1, 2015 of the International Association of Geodesy (IAG, Drewes et al. 2016, p. 981), the International Height Reference System (IHRIS, Ihde et al. 2017) is defined in terms of geopotential numbers $C_P = W_0 - W_P$, where W_P is the geopotential at a point of interest P , and $W_0 = 62\,636\,853.4 \text{ m}^2 \text{ s}^{-2}$ (Sánchez et al. 2016) as adopted by the IAG as the reference level for the IHRIS. Since the W_0 value is conventionally fixed, the primary value to be determined is the potential value W_P . As absolute potential values cannot be measured directly, the value W_P is to be determined from observable gravity field data by applying appropriate geoid or quasigeoid determination methods. In the (quasi)geoid computations, the primary quantity to be determined is the disturbing potential $T = W - U$, which is the difference between the actual gravity potential W and the normal gravity potential U generated by the adopted reference ellipsoid. Once T_P is computed, the determination of W_P is straightforward. However, the estimated value of the disturbing potential relies not only on available gravity data but also on gravity field modelling approaches. This includes different methods for handling terrain effects, filtering and combining gravity data, treating long-wavelength errors, formulating mathematical models to continue and transform gravity data and terrain effects, etc. Since there is no standardization in the computation of the disturbing potential and there are many parameter choices when handling gravity and terrain data, potential values estimated by different methods inevitably differ. Thus, different groups can generate quite different (quasi)geoid models from the same input gravity data.

To assess how much the results depend on applied computation methods, fourteen groups from thirteen countries working on regional geoid modelling agreed in August 2017 to determine geoid heights, height anomalies and geopotential values using the same input data and their own modelling strategies. For this purpose, NGS provided GPS/leveling data, terrestrial and airborne gravity data, and a digital terrain model for a test area of about $730 \text{ km} \times 550 \text{ km}$ in Colorado, USA. From 2018 to 2020, about 40 geodesists specialized in (quasi)geoid determination performed iterative computations of local (quasi)geoid models using these data, identifying their discrepancies and homogenizing to a large extent their different processing strategies. This initiative was successful in that the final models presented an agreement of around 2 cm in terms of the standard deviation of their differences. The contributing solutions represent the state-of-the-art in precise high-resolution gravity field modelling. The results of this experiment provide a benchmark for calibration of regional gravity field modelling methods and provide a basis for evaluation and further development of strategies and procedures to increase the achievable accuracy in determination of the regional gravimetric (quasi)geoid.

This paper describes key aspects of this joint international effort known as the Colorado 1-cm geoid experiment. The basic processing requirements agreed for the computation of the geoid and quasigeoid models are given in the Appendix. Section 2 describes the gravity data, the digital elevation model, and the GPS/leveling data provided by NGS for the Colorado experiment as well as the global geopotential models (GGM) used as a reference field in the (quasi)geoid modelling. Section 3 presents the key aspects of the computation methods applied by the different groups. The comparison of estimated quasigeoid models is shown and analyzed in Section 4; the comparison of potential values as the International Height Reference Frame coordinates is discussed in Sánchez et al. (2021). Finally, Section 5 discusses the main findings of the 1-cm geoid experiment, and sets out recommendations for future research.

2. Input data for the Colorado 1-cm geoid experiment

The following datasets were provided for the Colorado experiment:

2.1 Terrestrial and GRAV-D airborne gravity datasets

The terrestrial gravity data covers an area bordered by latitude 35° N to 40° N and longitude 110° W to 102° W, see Fig. 1. This dataset combines the US National Geospatial Intelligence Agency and NGS terrestrial gravity data and contains 59,303 gravity observations. Additionally, airborne gravity data from NGS's GRAV-D project were provided (Damiani et al. 2017). Data from the survey block MS05 was de-biased using the maximum median technique (Wang et al. 2017a), and then resampled at 1 Hz. The gravity observations are given at flight altitude ($\sim 6,000$ m) with positions in the IGS08 reference frame (Rebischung et al. 2012).

Figure 1 shows the spatial distribution of the terrestrial and airborne gravity throughout the region. Sparse distribution of terrestrial gravity data is seen in the northern and southwestern parts of the region along with the most easterly segment of the GSVS17 profile (see $\sim 105.5^{\circ}$ W). Statistics of the terrestrial and airborne gravity data are summarized in Table 1. In general, the airborne data has uniform coverage and consistent quality while the surface data contains outliers and has variable spatial coverage.

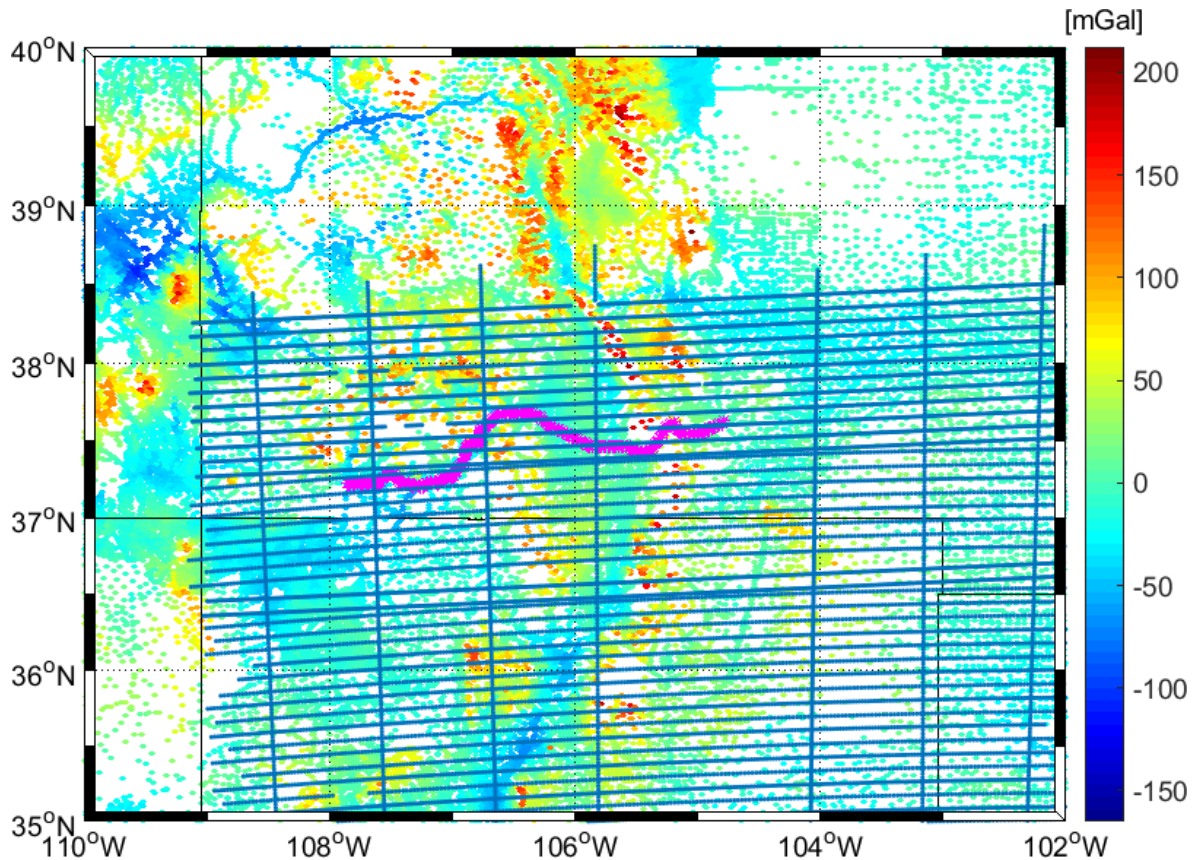


Fig. 1. Distribution of terrestrial and GRAV-D gravity observations. The straight lines (in blue) represent the GRAV-D airborne gravity at flight altitude. The GSVS17 profile (purple curve) is along US Highway 160 from Durango to

Walsenburg. These data were provided as observed gravity values, which the different participants had to convert to gravity anomalies using their own methods. The color ramp represents the free-air gravity anomaly.

Table 1. Statistics of the input gravity data

	Terrestrial (59,303 observations)		Airborne (283,716 observations)	
	Free-air gravity anomaly [mGal]	Orthometric Height [m]	Free-air gravity disturbance [mGal]	Ellipsoidal height [m]
Mean	5.3	2,134	5.7	6,186
STD	38.2	577	29.0	695
RMS	38.5	2,201	29.5	6,225
Min.	-165.7	950	-45.6	5,209
Max.	211.9	4,399	123.5	7,905

2.2 Digital elevation model: SRTM v4. 1

The SRTM v4.1 data (Jarvis et al. 2008) was provided as a 3" resolution model throughout the region (Fig. 2). The topographic heights are fixed for spikes and voids (Ahlgren et al. 2018). Some groups also utilized the topographic potential models dV_ell_Earth2014 (herein called Earth2014) (Rexer et al. 2016) and ERTM2160 (Hirt et al. 2014), which are based on the same SRTM data.

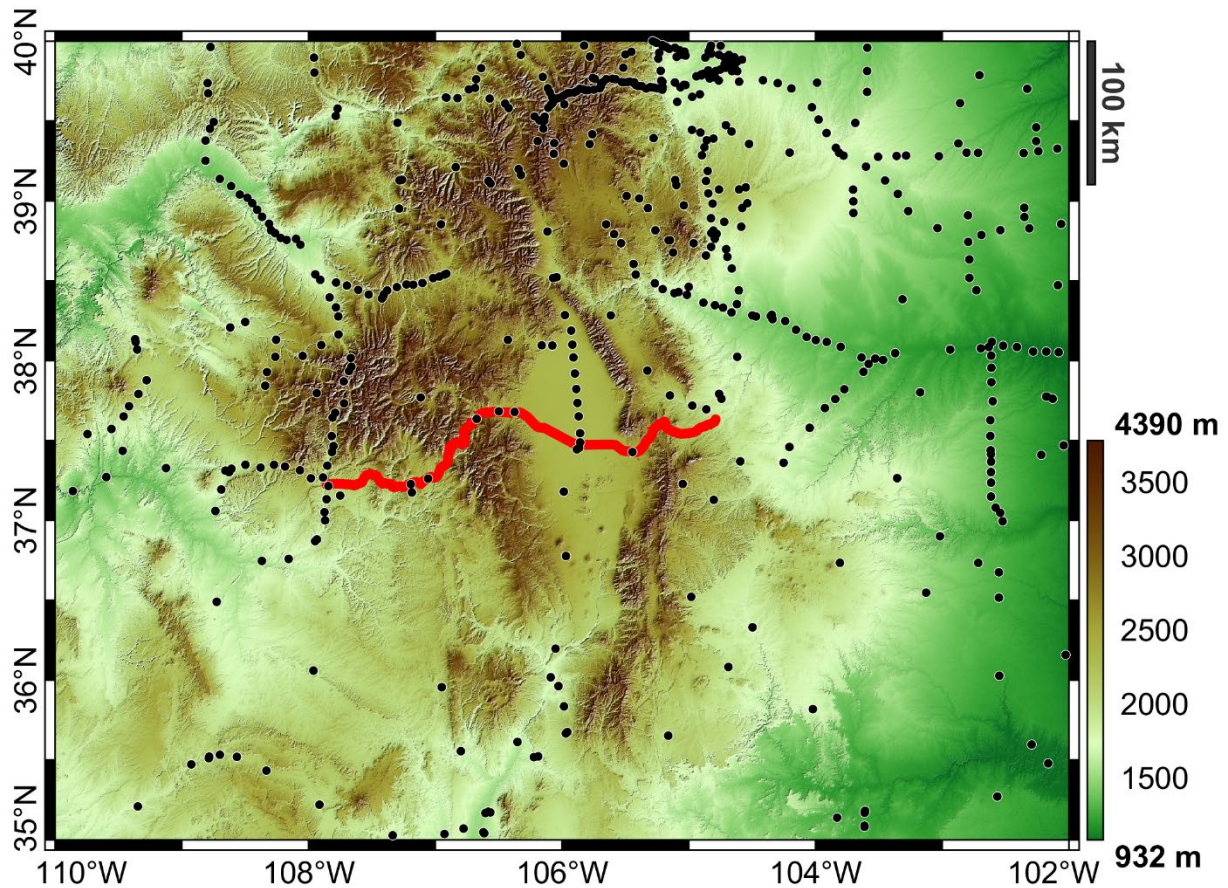


Fig. 2. Topographic heights based on SRTM v4.1 (Jarvis et al. 2008) model in the Colorado area with a spatial resolution of 3 arc-second. The heights have a mean value of 2,017 m and a standard deviation (STD) of 614 m. The minimum height is 932 m and the highest peak is 4,385 m. The red line represents the GSVS17 profile along US Highway 160 from Durango to Walsenburg. The black dots show the distribution of the GPS/leveling data provided to the group.

2.3 GPS/leveling data sets

Two GPS/leveling data sets were used in the experiment. The first set corresponds to NAVD88 benchmarks with GPS observations. This data is quite old with over half of the GPS observations collected in the 1990s, while the leveling lines were observed more sporadically over the last century with most of the data being observed in the 1930s and 1980s. There is no accuracy estimated for the GPS/leveling data-derived height anomalies. However, the comparison results of this experiment indicate a 5 cm accuracy (relative). These data were distributed to the contributing groups with the intention that they are used as initial reference for the quality control in the (quasi)geoid computations. The original ellipsoidal heights were in the NAD83 system, but it were converted to the IGS08 frame for this experiment. Out of 509 benchmarks, 467 benchmarks were from the NGS Integrated Database and the rest from the NGS OPUS-Share Tool (<https://www.ngs.noaa.gov/OPUS/>). The spatial distribution of the benchmarks is sparse due to the mountainous terrain (see Fig. 2).

The second data set is from the GSVS17 survey in the IGS08 frame. GPS, leveling, gravity and astrogeodetic deflections of the vertical were observed at 223 marks spaced at about 1.6 km along Highway US 160 from Durango to Walsenburg in the summer of 2017 (van Westrum et al. 2021). The accuracy of the geoid undulations ($N_{GSVS17} = h_{GPS} - H_{lev}$) inferred from the GPS-based ellipsoidal heights h_{GPS} and the leveling-based orthometric heights H_{lev} is estimated to be around 1.5 cm between the marks (ibid.). The accuracy of the GSVS17 height anomalies ($\zeta_{GSVS17} = h_{GPS} - H^N$), where H^N is the normal height computed from the adjusted geopotential numbers of GSVS17, is estimated as 1.2 cm (see Section 4.3). The GSVS17 dataset was not available to any of the contributing groups (including NGS) and used only as an independent, final comparison.

2.4 Global geopotential models (GGMs)

It was specified that three different GGMs could be used as reference models in the (quasi)geoid computations (Sánchez et al. 2018): xGEOID17REFB (herein called RefB) d/o 2190 (Wang et al. 2017a), XGM2016 d/o 719 (Pail et al. 2017; 2018), and GOCO05S d/o 280 (Mayer-Gürr et al. 2015). It was thus up to the particular geoid modeler to decide whether to use a satellite-only or a combined GGM. All three GGMs are given in the zero-tide system.

3. Key characteristics of the computation methods contributing to the Colorado experiment

The computation methodologies are roughly classified according to the geodetic boundary-value problems (GBVPs). Eleven groups computed quasigeoid models under the scalar-free Molodensky GBVP (e.g., Molodensky et al. 1962; Hofmann-Wellenhof and Moritz 2005; Moritz 1980) and the fixed GBVP (e.g., Koch and Pope 1972; Heck 2011). Three groups computed the geoid based on the scalar-free Stokes GBVP (e.g., Vaníček and Martinec 1994; Huang and Véronneau 2013; Abd-Elmotaal and Kühnreiter 2003, 2020), then converted geoid undulations into height anomalies using the geoid-quasigeoid separation term. Most of the solutions applied the remove-compute-restore (RCR) technique using different parameters for the modification of the Stokes integration kernel or the spectral combination of the different input data (GGM, airborne and terrestrial gravity data, topography gravity signals). Four solutions follow the least squares modification of Stokes's formula with additive corrections (LSMSA) algorithm (Sjöberg 1980, 1981, 1984 and 2003). Three solutions performed the data processing using spherical radial basis functions (SRBF), least-squares collocation (LSC) and residual LSC. The variety of parameter choices for the data processing is so large that it is impractical to provide a detailed description of the different computation strategies in this article. We concentrate therefore on the key characteristics considering three main computation stages: handling of topographic effects on gravity (Table 2), preparation of surface (terrestrial + airborne) gravity data (Table 3), and solution of the GBVP (Table 4). Table 4 also provides the corresponding bibliographical references describing the detailed computation strategies.

Table 2: Methods for estimating the topography gravity effects within the contributing solutions to the Colorado experiment (FFT – fast Fourier transformation, RTM – residual terrain modelling, DEM – digital elevation model)

Solution	Models	Computation
AUTH	Earth2014, ERTM2160, SRTM 3" v4.1	- Gravity anomalies interpolation based on SRTM - GBVP solution using spherical harmonics combination - Terrestrial data: Earth2014 from degree 720 to 2159 and ERTM2160 from degree 2,160 to ~80,000

		- Airborne data: Earth2014 from degree 720 to 2,160
CASM	SRTM 3" v4.1	- RTM for airborne and terrestrial data, rectangular prism integration with a 3" DEM and a 5' mean model
CGS	SRTM 3" v4.1	- Terrain correction and condensed terrain effect on the geoid with a 3"x3" DEM within the inner zone (<50 km) - The direct topographical effects within the near zone (> 50 km and < 3°) with a mean 1' DEM; and far-zone (> 3°) with a spherical harmonic expansion up to d/o 90 - The primary topographical effect within the inner zone (<0.5°) with a mean 30"x30" DEM; near zone (>0.5 and <2°) with a mean 1' DEM; and far-zone (>2°) with a mean 30' global DEM
Curtin	SRTM 3" v4.1	- Terrain correction (for Faye anomalies) based on a binomial expansion up to sixth order using FFT - Topography effects removed in the combination of terrestrial and airborne data based on a rectangular prism integration using a long-wavelength DEM created by spherical harmonic synthesis (maximum degree 300)
DGFI	Earth2014, ERTM2160	- Spherical harmonics combination - Terrestrial data: Earth2014 from degree 720 to 2,159 and ERTM2160 from degree 2,160 to ~80,000 - Airborne data: Earth2014 from degree 720 to 5480
DTU	SRTM 3" v4.1	- RTM for airborne and terrestrial data, rectangular prism integration with a 3" DEM and a 9" mean model. Harmonic correction based on the linear Prey term
Minia	SRTM 3" v4.1	- The windowed topographic–isostatic Airy mass balance principle
GSI	SRTM 3" v4.1, Earth2014	- Spherical terrain correction considering the combined effect of topographical and condensation masses based on three DEMs of different resolution: 3" × 3", 1'×1' and 1° × 1°
IAPG	Earth2014, ERTM2160	- Spherical harmonics combination - Terrestrial data: Earth2014 from degree 720 to 2,159 and ERTM2160 from degree 2,160 to ~80,000 - Airborne data: Earth2014 from degree 720 to 5,400
ITU	SRTM 3" v4.1	- RTM for airborne and terrestrial data, rectangular prism integration with a 3" DEM and a 1' mean model
KTH	SRTM 3" v4.1	- RTM for airborne and terrestrial data, rectangular prism integration with a 3" DEM and a 5' mean model
NGS	SRTM 3" v4.1	- RTM for airborne and terrestrial data, rectangular prism integration with a 3" × 3" DEM and a 1' × 1' mean model
NTIS-GEOF	SRTM 3" v4.1	- RTM for airborne and terrestrial data, rectangular prism integration with a 3" DEM for the inner zone (up to 30 km) and 1' for the outer zone (from 30 to 200 km) - The mean model is a 5' DEM
Polimi	Same as AUTH	Same as AUTH

Table 3. Key aspects of the surface (airborne + terrestrial) gravity data preparation within the contributing solutions to the Colorado experiment

Solution	Preprocessing airborne and terrestrial data	Combination airborne and terrestrial data
AUTH	<i>Airborne data:</i> - Molodensky-type free-air anomalies - No filtering	- Residual anomalies (Molodensky-type anomalies with GGM (XGM2016, d/o = 720) and topography effects removed)

<ul style="list-style-type: none"> - Downward continuation using LSC with planar logarithmic covariance function using residual anomalies (with GGM (XGM2016, d/o = 720) and topography effects removed) - Covariance function parameters determined from terrestrial data <p>Terrestrial data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies 	<ul style="list-style-type: none"> - Unweighted combination of downward continued airborne and terrestrial surface residual anomalies - Gridding: Spline interpolation (tension factor 0.10) to 1' × 1'. 1' × 1' gridded GGM and topography effects restored
<p>CASM</p> <p>Airborne data:</p> <ul style="list-style-type: none"> - Gravity disturbances - Filtering with Gaussian filter, length 120s - Downward continuation implicit in the spectral combination with terrestrial data <p>Terrestrial data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies 	<ul style="list-style-type: none"> - Spectral combination with weights defined according with white and colored noise. Terrestrial data: correlation distance 0.5°, weights for white and colored noise: 3.0 mGal and 1.0 mGal, respectively. Airborne gravity: Correlation distance 0.5°, weights for white and colored noise: 1.5 mGal, 0.5 mGal, respectively. - Gridding: GEOGRID weighted means interpolation to 1' × 1' based on residual gravity disturbances (airborne data) and residual gravity anomalies (terrestrial data). In both cases, GGM (RefB, d/o = 2,190) and topography effects removed before interpolation and 1' × 1' gridded GGM and topography effects restored after interpolation
<p>CGS</p> <p>Airborne data:</p> <ul style="list-style-type: none"> - Observable: gravity disturbances - Filtering: a weighted 3-point smoothing - Downward continuation: inverse Poisson <p>Terrestrial data:</p> <ul style="list-style-type: none"> - Free-air anomalies 	<ul style="list-style-type: none"> - Spectral combination of the GGM RefB (up to degree 150 from where it goes into transition with the surface gravity data up to degree 210) and terrestrial gravity data (up to degree 10,800 with a smooth transition at the high-degree end. - Airborne gravity correction for the medium-band (between degrees 210 and 790 with smooth transition backwards at each end), followed by a Gaussian smoothing to the airborne geoid residuals - Mean topographical contribution up to degree 10,800 by the primary indirect effect with SRTM v4.1 - Gridding: LSC at the mean flight height and Earth's surface
<p>Curtin</p> <p>Airborne data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies - Filtering with Gaussian filter, length 80s - Downward continuation implicit in the LSC used for the combination with terrestrial data <p>Terrestrial data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies 	<ul style="list-style-type: none"> - Residual anomalies (Molodensky-type anomalies with GGM (DIR_R6, d/o = 300) and topography effects removed) - 3D LSC with planar logarithmic covariance function - Gridding: LSC interpolation to 1' × 1' included in the airborne and terrestrial data combination - 1' × 1' gridded GGM and topography effects restored.

DGFI	<i>Airborne data:</i>	<ul style="list-style-type: none"> - Gravity disturbances - Applying a cubic polynomial (CuP) function with smoothing features as a low-pass filter to remove the high-frequency noise in the airborne data, afterwards down sampling to 1/8 Hz - Downward continuation implicit in the SRBF approach 	<ul style="list-style-type: none"> - Relative weighting using the method of variance component estimation and regularization parameter determined using the L-curve method - No gridding required
	<i>Terrestrial data:</i>	<ul style="list-style-type: none"> - Gravity disturbances 	
DTU	<i>Airborne data:</i>	<ul style="list-style-type: none"> - Molodensky-type free-air anomalies - Filtering with Gaussian filter, length 30s - Downward continuation implicit in the LSC used for the combination with terrestrial data 	<ul style="list-style-type: none"> - Residual anomalies (Molodensky-type anomalies with GGM (XGM2016, d/o = 300) and topography effects removed) - 3D LSC with planar logarithmic covariance function, a-priori error airborne data 2 mGal, a-priori error terrestrial data 1 mGal - Gridding: LSC interpolation to 1' × 1' included in the airborne and terrestrial data combination - 1' × 1' gridded GGM and topography effects restored
	<i>Terrestrial data:</i>	<ul style="list-style-type: none"> - Molodensky-type free-air anomalies 	
Minia	<i>Airborne data:</i>	<ul style="list-style-type: none"> - No airborne data used 	<ul style="list-style-type: none"> - Residual anomalies with GGM (RefB, d/o = 2,190) - Gridding: Kriging interpolation
	<i>Terrestrial data:</i>	<ul style="list-style-type: none"> - Molodensky-type free-air anomalies 	
GSI	<i>Airborne data:</i>	<ul style="list-style-type: none"> - Molodensky-type free-air anomalies - Down sampling to 500 m - Downward continuation implicit in the LSC used for the combination with terrestrial data 	<ul style="list-style-type: none"> - Residual anomalies (Molodensky-type anomalies with GGM (XGM2016, d/o = 719) and topography effects removed) - 3D LSC with planar logarithmic covariance function, a-priori error airborne data 2.16 mGal, a-priori error terrestrial data 1.64 mGal - Gridding: LSC interpolation to 1' × 1' included in the airborne and terrestrial data combination. 1' × 1' gridded GGM and topography effects restored
	<i>Terrestrial data:</i>	<ul style="list-style-type: none"> - Molodensky-type free-air anomalies 	
IAPG	<i>Airborne data:</i>	<ul style="list-style-type: none"> - Gravity disturbances - Filtering with a Gaussian low-pass filter (with a half-width at half-maximum of 3 km), down sampling to 1/32 Hz - Downward continuation implicit in the residual LSC approach 	<ul style="list-style-type: none"> - Relative weighting based on a-priori stochastic modelling - No gridding required
	<i>Terrestrial data:</i>	<ul style="list-style-type: none"> - Gravity disturbances 	

ITU	<p>Airborne data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies - Filtering with Gaussian filter, length 120s, down sampling to 1 km - Downward continuation implicit in the LSC used for the combination with terrestrial data <p>Terrestrial data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies 	<ul style="list-style-type: none"> - Residual anomalies (Molodensky-type anomalies with GGM (XGM2016, d/o = 719) and topography effects removed) - 3D LSC with planar logarithmic covariance function, a-priori error airborne data 2 mGal, a-priori error terrestrial data 1 mGal - Gridding: LSC interpolation to 1' × 1' included in the airborne and terrestrial data combination - 1' × 1' gridded GGM and topography effects restored
KTH	<p>Airborne data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies - Down sampling to 3' (~5.5 km) - Downward continuation implicit in the LSC used for the combination with terrestrial data <p>Terrestrial data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies 	<ul style="list-style-type: none"> - Residual anomalies (Molodensky-type anomalies with GGM (Ref_B, n = 2,190) and topography effects removed) - 3D LSC with spherical covariance function (Tscherning and Rapp, 1974), uncorrelated white noise assumed for both airborne and terrestrial data with standard deviations 2 mGal and 1 mGal, respectively - Gridding: LSC interpolation to 0.5' × 0.5' included in the airborne and terrestrial data combination - 0.5' × 0.5' gridded GGM and topography effects restored
NGS	<p>Airborne data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies - Gridded to 5' × 5' and converted to spherical harmonic series, degree 2190 - Downward continuation implicit in the spectral combination with terrestrial data <p>Terrestrial data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies 	<ul style="list-style-type: none"> - Spectral combination with weights defined according to white and colored noise. - Terrestrial data: correlation distance 0.5°, weights for white and colored noise: 3.5 mGal and 3.0 mGal, respectively - Airborne gravity: Correlation distance 0.25°, weights for white and colored noise: 1.5 mGal, 1.0 mGal, respectively. - Gridding: LSC interpolation to 1' × 1' using refined Bouguer anomalies - 1' × 1' gridded Bouguer plate and topography effects restored.
NTIS-GEOF	<p>Airborne data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies - Filtering with Gaussian filter, length 120s - Downward continuation implicit in the LSC used for the combination with terrestrial data <p>Terrestrial data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies 	<ul style="list-style-type: none"> - Residual anomalies (Molodensky-type anomalies with GGM (XGM2016, d/o = 500) and topography effects removed) - 3D LSC with planar logarithmic covariance function, a-priori error airborne data 3.4 mGal, a-priori error terrestrial data 2.6 mGal - Gridding: LSC interpolation to 1' × 1' included in the airborne and terrestrial data combination - 1' × 1' gridded GGM and topography effects restored
Polimi	<p>Airborne data:</p> <ul style="list-style-type: none"> - Molodensky-type free-air anomalies - No filtering 	<ul style="list-style-type: none"> - Residual anomalies (Molodensky-type anomalies with GGM (XGM2016, d/o = 720) and topography effects removed)

- Downward continuation using LSC with planar logarithmic covariance function (Forsberg 1987) using residual anomalies (with GGM (XGM2016, d/o = 719) and topography effects removed). Covariance function parameters determined from terrestrial data
- Unweighted combination of downward continued airborne and terrestrial surface residual anomalies
- Gridding: Spline interpolation (tension factor 0.10) to $1' \times 1'$
- $1' \times 1'$ gridded GGM and topography effects restored

Terrestrial data:

- Molodensky-type free-air anomalies

Table 4. Key aspects of the geoid and quasigeoid computation within the contributing solutions to the Colorado experiment

Solution	GVBP type and input data	Solution	Results
AUTh	<ul style="list-style-type: none"> - Molodensky scalar-free - GGM: XGM2016, d/o = 719 - Surface data: $1' \times 1'$ Molodensky-type free air anomalies 	<ul style="list-style-type: none"> - RCR, modified Stokes integral (1D FFT) - Wong-Gore kernel modification (best-fitting with GPS/leveling), no cap, transition degrees 450 – 719 	<ul style="list-style-type: none"> - Height anomaly - Converted to geoid using refined Bouguer anomaly - Details: Grigoriadis et al. (2021)
CASM	<ul style="list-style-type: none"> - Molodensky scalar-free with terrestrial data - Fixed GBVP with airborne data - GGM: RefB, d/o = 2,190 - Surface data: $1' \times 1'$ terrestrial Molodensky-type free air anomalies and $1' \times 1'$ residual airborne gravity disturbances 	<ul style="list-style-type: none"> - Spectral combination approach - Computation of height anomalies from terrestrial and airborne data independently - Spectral combination of the height anomalies of the GGM (up to degree 114 from where it goes into transition with the surface gravity data up to degree 217) and terrestrial and airborne gravity (between degrees 217 and 1,000 with smooth transition at each end) with SRTM model (up to degree 10,800), cap size 1° 	<ul style="list-style-type: none"> - Height anomaly - Converted to geoid using refined Bouguer anomalies and the gradient of the gravitational potential - Details: Jiang et al. (2020)
CGS	<ul style="list-style-type: none"> - Stokes-Helmert scalar-free - GGM: RefB, d/o = 2190 - Surface data: $1' \times 1'$ airborne and terrestrial Helmert gravity anomalies on the geoid 	<ul style="list-style-type: none"> - Modified degree-banded Stokes kernels - Modification degree $L = 210$ - Cap size 6° - FFT for numerical integration 	<ul style="list-style-type: none"> - Geoid undulation - Converted to height anomaly using simple Bouguer anomalies and refined Bouguer anomalies (two results) at the GSVS17 marks - Details: Huang and Véronneau (2013)
Curtin	<ul style="list-style-type: none"> - Molodensky scalar-free. - GGM: RefB, d/o = 2,190 - Surface data: $1' \times 1'$ Faye anomalies (Molodensky- 	<ul style="list-style-type: none"> - RCR, modified Stokes integral (1D FFT) - Wong-Gore kernel modification (best-fitting with 	<ul style="list-style-type: none"> - Height anomaly - Converted to geoid using simple Bouguer anomaly - Details: Claessens and Filmer (2020)

	type free-air + terrain correction)	GPS/leveling), cap size 0.3° , modification degree 360	
DGFI	<ul style="list-style-type: none"> - Fixed GBVP - GGM: XGM2016, d/o = 719 - Surface data: Point-wise gravity disturbances 	<ul style="list-style-type: none"> - RCR, SRBF - The Shannon kernel used for the terrestrial data; and the CuP kernel used for the airborne data - The unknown coefficients associated to the kernels determined by the parameter estimation procedure, instead of numerical integration 	<ul style="list-style-type: none"> - Height anomaly - Converted to geoid using simple Bouguer anomalies - Details: Liu et al. (2020)
DTU	<ul style="list-style-type: none"> - Molodensky scalar-free - GGM: XGM2016, d/o = 360 - Surface data: $1' \times 1'$ Molodensky-type free air anomalies 	<ul style="list-style-type: none"> - RCR, modified Stokes integral (spherical FFT) - Wong-Gore kernel modification (best-fitting with GPS/leveling data), no cap, transition degrees 180 - 190, 100% zero padding 	<ul style="list-style-type: none"> - Height anomaly - Converted to geoid using simple Bouguer anomaly - Details: Forsberg and Tscherning (1981), Forsberg (1987), Forsberg and Featherstone (1998)
Minia	<ul style="list-style-type: none"> - Stokes scalar-free - GGM: RefB d/o = 2190 - Surface data: $1' \times 1'$ residual (isostatic) anomalies on the geoid 	<ul style="list-style-type: none"> - RCR, modified Stokes integral - Meissl (1971) modified kernel, cap size 0.5° 	<ul style="list-style-type: none"> - Geoid undulation - No conversion to height anomaly - Details: Abd-Elmotaal and Kührtreiber (2003, 2020)
GSI	<ul style="list-style-type: none"> - Stokes scale-free - GGM: XGM2016, d/o = 719 - Surface data: $1' \times 1'$ residual Helmert gravity anomalies on the geoid 	<ul style="list-style-type: none"> - RCR, modified Stokes integral - Meissel-Molodensky modified kernel (best-fitting with GPS/leveling), cap size 0.2°, modification degree 719 	<ul style="list-style-type: none"> - Geoid undulation - Converted to height anomaly using simple Bouguer anomalies - Details: Matsuo and Forsberg (2021)
IAPG	<ul style="list-style-type: none"> - Fixed GBVP - GGM: XGM2018, d/o = 719 (very close to XGM2016 but with improved covariance information) - Surface data: Point-wise gravity disturbances 	<ul style="list-style-type: none"> - RCR, residual LSC - Statistical least-squares optimization - Covariance matrices for all input quantities (GGM, topographic reduction, observation accuracy) to result in a realistic modelling of the gravity field and the error budget 	<ul style="list-style-type: none"> - Height anomaly - Converted to geoid using simple Bouguer anomalies - Details: Willberg et al. (2019, 2020)
ITU	<ul style="list-style-type: none"> - Molodensky scalar-free - GGM: XGM2016, d/o = 719 - Surface data: $1' \times 1'$ Molodensky-type free air anomalies 	<ul style="list-style-type: none"> - LSMSA - Kernel modification based on unbiased solution of least squares modification parameters, standard deviation of surface gravity anomalies 3 mGal, cap size 0.5°, kernel modification degree equal to 	<ul style="list-style-type: none"> - Height anomaly - Converted to geoid using simple Bouguer anomalies - Details: Işık et al. (2021)

		maximum GGM coefficient (degree 719)	
KTH	<ul style="list-style-type: none"> - Molodensky scalar-free- - GGM: GOCO05S, d/o = 240 - Surface data: 0.5' × 0.5' Molodensky-type free air anomalies 	<ul style="list-style-type: none"> - LSMSA - Least-squares modification with satellite only GGM Signal degree variances according to Tscherning and Rapp (1974) - Error degree variances as a combination of white noise ($\sigma = 1$ mGal) and colored noise ($\sigma = 0.5$ mGal with correlation-length 0.5 degrees), cap size 2° 	<ul style="list-style-type: none"> - Height anomaly - Converted to geoid using simple Bouguer anomalies - Details: Ågren et al. (2009)
NGS	<ul style="list-style-type: none"> - Molodensky scalar-free- - GGM: RefB, d/o = 2190 - Surface data: 1' × 1' Molodensky-type free air anomalies 	<ul style="list-style-type: none"> - RCR, modified Stokes integral - Wong-Gore kernel modification (best-fitting with GPS/leveling data), cap size 1°, modification degree 980 	<ul style="list-style-type: none"> - Height anomaly - Converted to geoid using simple Bouguer anomalies - Details: Wang et al. (2020)
NTIS- GEOF	<ul style="list-style-type: none"> - Molodensky scalar-free- - GGM: XGM2016, d/o = 500 - Surface data: 1' × 1' Molodensky-type free air anomalies 	<ul style="list-style-type: none"> - LSMSA - Kernel modification based on a biased solution of least squares modification parameters, cap size 0.5° 	<ul style="list-style-type: none"> - Geoid undulation - Converted to height anomaly using refined Bouguer anomalies- - Details: Varga et al. (2021)
Polimi	<ul style="list-style-type: none"> - Molodensky scalar-free- - GGM: XGM2016, d/o = 719 - Surface data: 1' × 1' Molodensky-type free-air anomalies 	<ul style="list-style-type: none"> - RCR, LSC - Signal variances according to the spherical covariance function of Tscherning and Rapp (1974) 	<ul style="list-style-type: none"> - Height anomaly - Converted to geoid using refined Bouguer anomalies. - Details: Grigoriadis et al. (2021)

4. Model comparison and validation

All the groups delivered a geoid model on a 1'×1' grid and geoid heights on the 223 GSVS17 marks; they also provided the geopotential values on the marks, for the comparison of these values, see Sánchez et al. (2021). All the groups (except Minia) also provided a quasigeoid model on a 1'×1' grid and height anomalies on the GSVS17 marks. To simplify the analysis and to focus on the main results, we present in this paper the comparison between the quasigeoid models only; the comparison between the geoid models produces very similar results and conclusions. As mentioned, 11 contributing solutions delivered a quasigeoid model as their primary result, while three groups computed the geoid directly (cf. Table 4). These three geoid models were converted into quasigeoid models by applying exactly the same geoid-quasigeoid conversion term (see section 4.1). In this way, we avoid possible sources of discrepancy caused by the different conversion terms used in the *converted* models (cf. Table 4) and expect that the discrepancies between the results are mainly caused by the different data processing and GBVP solution strategies.

4.1 Geoid-quasigeoid separation term

The simple geoid-(quasi)geoid separation (GQS) uses the Bouguer anomaly Δg_B (Heiskanen and Moritz 1967, section 8-13) to approximate the difference between the mean *real* gravity and mean normal gravity:

$$N - \zeta = \frac{\Delta g_B(P)}{\bar{\gamma}} H, \quad (1)$$

where $\bar{\gamma}$ is the mean normal gravity (GRS80) along the normal plumb line, H is the orthometric height, N and ζ are the geoid undulation and the height anomaly, respectively.

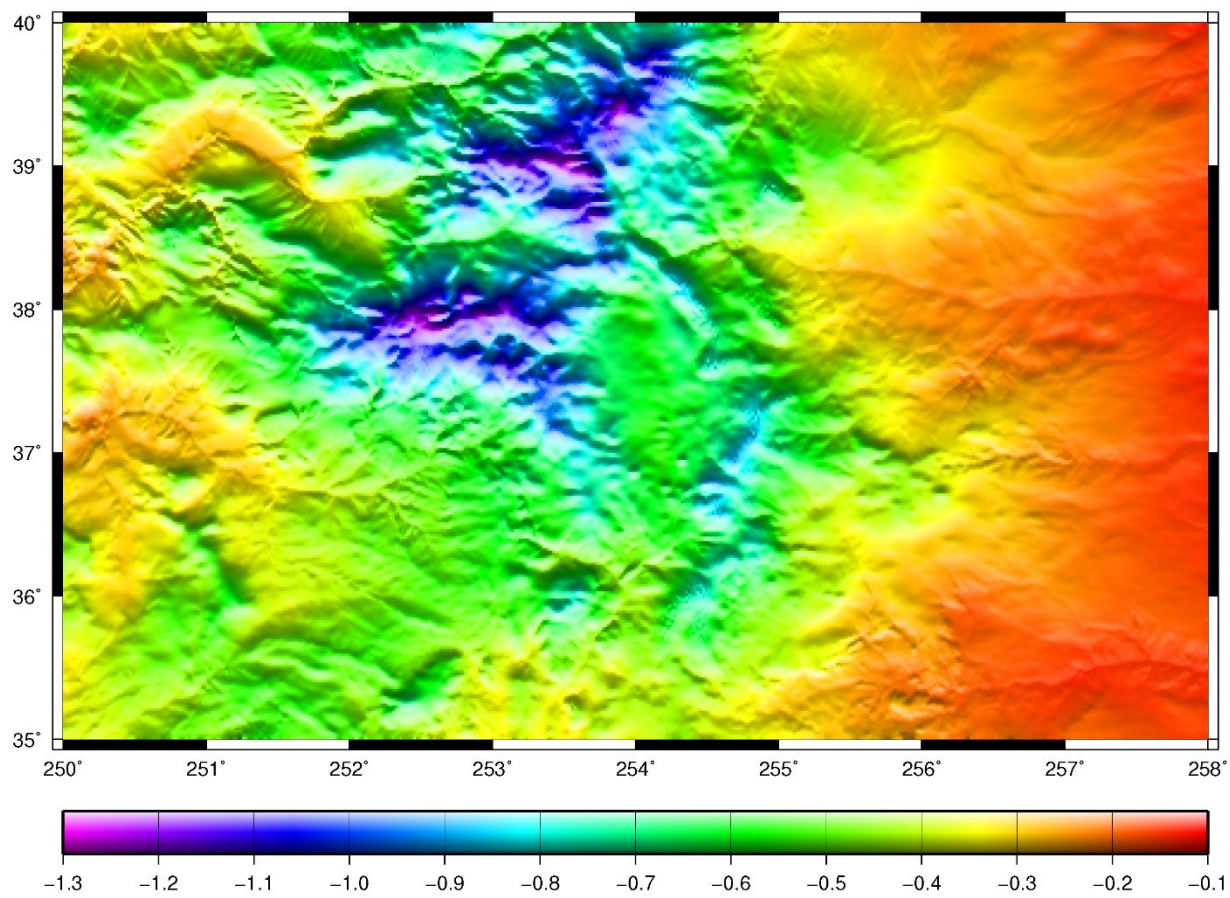
More accurate formulations were developed by, e.g., Flury and Rummel (2009) and Sjöberg (2010). In this paper, the separation GQS is computed in terms of the Bouguer disturbance δg_B using Wang et al. (2021):

$$N - \zeta = \frac{\delta g_B(P)}{\bar{\gamma}} H + \frac{V_t(Q) - V_t(P)}{\bar{\gamma}} - \frac{H^2}{2\bar{\gamma}} \frac{\partial \delta g_B}{\partial H}, \quad (2)$$

where the second and third terms in Eq. (2) are known as the potential and gravity gradient corrections, respectively. V_t is the gravitational potential of the topographic masses. P is located on the Earth's surface and Q is the projection (along the plumb line) of P on the geoid. To distinguish the simple GQS in Eq. (1) from the separation defined in Eq. (2), the latter is called the complete separation term hereafter.

The computation of the Bouguer disturbance and the topographic potential at the points P and Q requires global integrations. Because of the canceling effect between $V_t(P)$ and $V_t(Q)$, and taking into consideration the high frequencies contained in the corrections, the integration radius was limited to a one-degree cap. The complete GQS was computed for the experimental geoid 2020 (Wang et al., 2021) and the Colorado area is a part of it. Figure 3a and 3b show the GQS term computed using Eq. (1) and the separation difference computed using Eqs (1) and (2), respectively. The differences computed using Eq. (1) and Eq. (2) reach 18 cm in the extreme case, and 1.9 cm in terms of root mean square (RMS) values (see Fig. 3b).

The improvement offered by Eq. (2) is implied by the fact that the CGS model (under the Helmert-Stokes scheme) gets closer to NGS model (under the Molodensky theory) when the complete separation term is used. The STD of the differences between these two models are reduced from 3.1 cm to 2.2 cm. For this reason, the complete separation is applied to the three geoid models (CGS, Minia and GSI) in the following analysis.



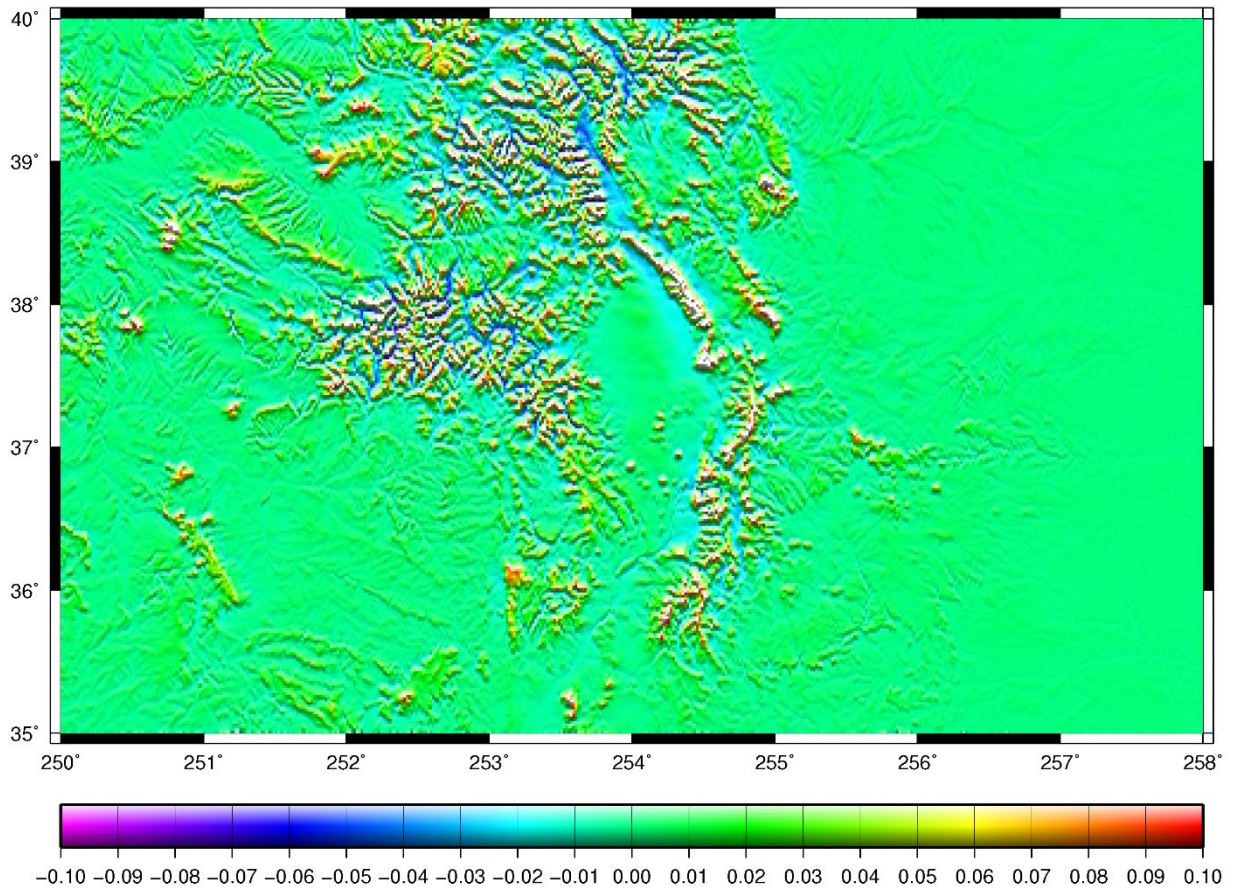


Fig. 3b. Differences between the complete and simple GQS term (Eq. (2) – Eq. (1)), unit in meter. Mean = -0.005, Min = -0.094, Max = 0.277, STD = 0.021, RMS = 0.021.

4.2 Quasigeoid area comparisons (grids of 1'×1')

The models provided by the contributing groups are given on a 1'×1' grid over the study area between latitudes 36° N to 39° N and longitudes 109° W to 103° W. This study area has been reduced by a one-degree border along each side with respect to the area covered by the gravity data (Fig. 1) to reduce edge effects in the quasigeoid models. As the grids are submitted in different formats (point values and lines of scan) and in different cell registrations (center and corner), the first step was to transform the grids to a common format, and then to interpolate the quasigeoid models onto 1'×1' grids with a common registration (hereafter called standardized grids). To show the precision of the models, the 14 quasigeoid models were compared with the group mean. In the end, the height anomalies were interpolated on the GSVS17 marks from the 14 standardized (quasi)geoid grids and compared, without using the values provided by the research groups.

Figure 4 illustrates the mean height anomalies obtained from the 14 standardized grids. An 85 cm bias had to be added to the Minia model as its zero-degree geoid term is related to the first GPS/leveling data (see Section 2) and it reflects the bias of the NAVD88 datum with respect to the global vertical datum introduced by the IHRF W_0 value (see more details in Section 4.3).

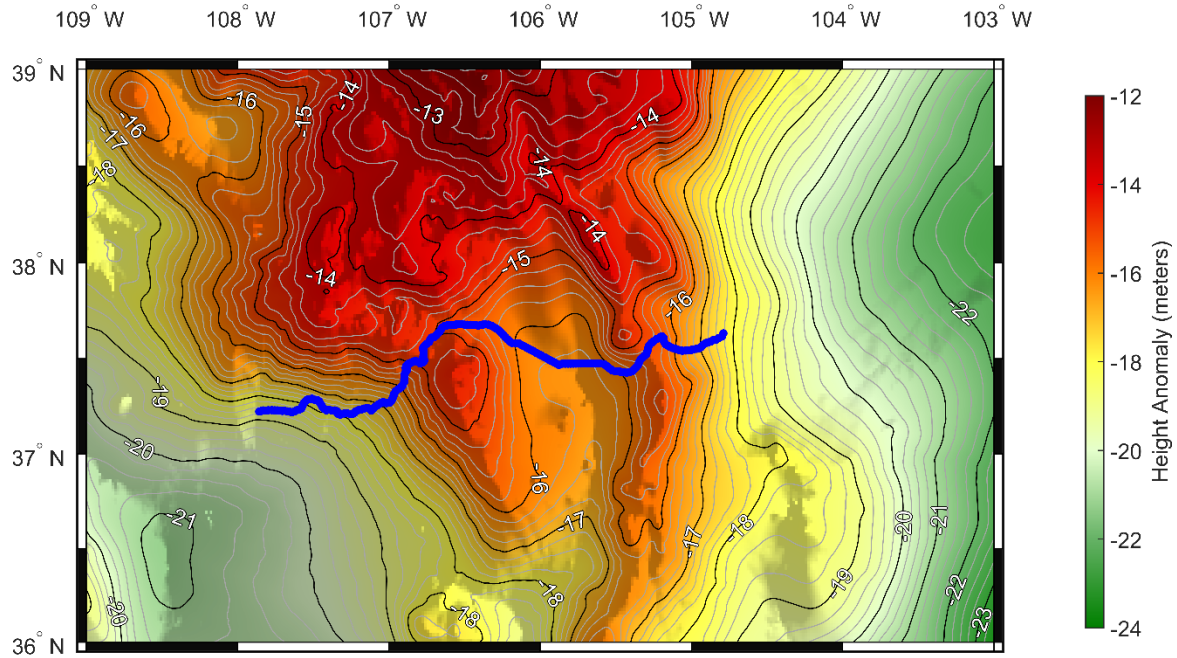


Fig. 4 The mean quasigeoid model at the 1'x1' grid computed from the 14 quasigeoid models. The blue line represents the GSVS17 profile along the highway US 160 from Durango to Walsenburg.

To highlight the agreement (or disagreement) among the models, the statistics of the model differences to the mean of all quasigeoid models caused by the different computation methods, different reference models and different data handling among the groups are listed in Table 5.

Table 5. Statistics of the differences among the individual quasigeoid models and the mean of all quasigeoid models at the 1'x1' grid; 85 cm was added to Minia model. Units in cm.

	Mean	STD	RMS	Min	Max	Range
AUTh	0.4	1.8	1.9	-11.2	6.3	17.5
CASM	0.5	1.9	1.9	-12.0	17.1	29.1
CGS	-0.4	2.0	2.0	-10.8	14.7	25.5
Curtin	0.3	2.3	2.3	-17.8	10.1	27.9
DGFI	-0.6	1.5	1.6	-9.1	7.0	16.1
DTU	1.1	2.8	3.1	-18.1	18.2	36.3
Minia	0.5	5.1	5.1	-12.9	17.7	30.6
GSI	-0.7	2.9	3.0	-29.4	17.9	47.3
IAPG	-1.1	1.8	2.1	-13.8	6.5	20.3
ITU	0.5	5.3	5.3	-45.2	22.9	68.1
KTH	-0.3	1.7	1.8	-11.6	8.8	20.4
NGS	0.0	1.9	1.9	-7.9	13.9	21.8
NTIS-GEOF	-1.6	2.7	3.2	-19.6	13.2	32.8
Polimi	1.2	1.8	2.2	-6.5	14.8	21.3

The RMS values of the individual solutions vary between 1.6 and 5.3 cm and have a median of 2.0 cm. From this comparison, we suggest that ~ 2 cm can be considered a reasonable approximation for the precision of the 14 quasigeoid models. This is a significant result in this rugged mountainous area after two guided iterations, and a good accuracy indication of current quasigeoid computations. However, the precision does not meet the 1-cm target, demonstrating the need for further research, including the capability to be able to accurately validate sub-cm quasigeoid models. Table 5 also indicates large variations in the differences among the ranges, with extreme cases of more than 40 cm. Indeed, the median value of the difference among ranges is 26.7 cm, which is also quite large.

To visualize the spatial distribution of the discrepancies between the models, the standard deviation values of the height anomaly differences (14 samples) in each $1' \times 1'$ cell were computed and are shown in Fig. 5. Large differences are strongly correlated with the roughness and height of the topography, and in areas where terrestrial gravity data is missing or sparse (see Fig. 1).

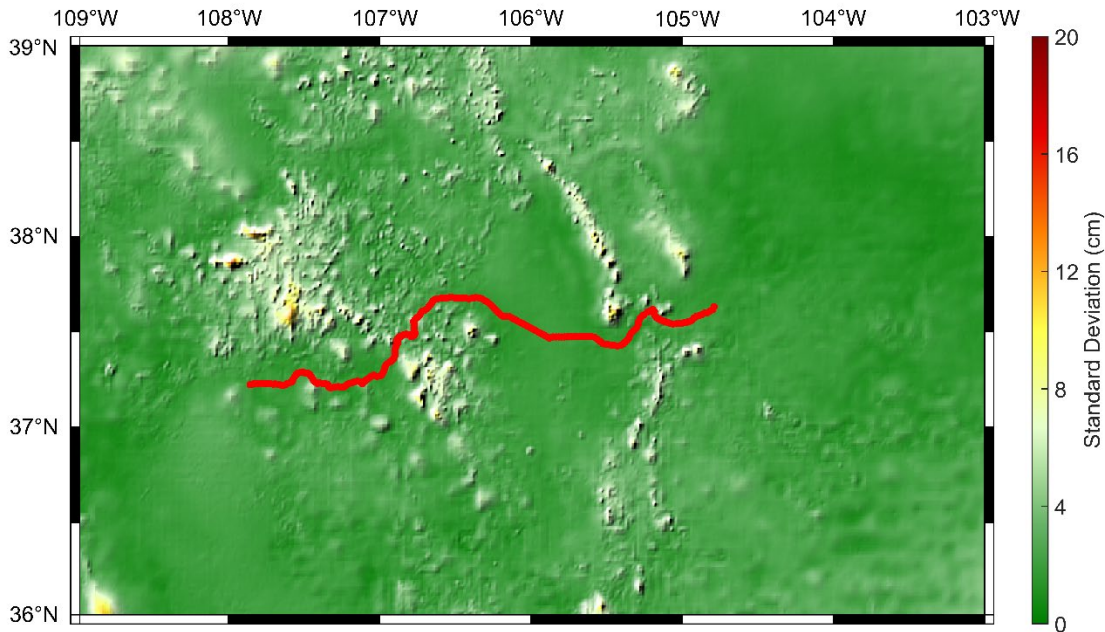


Fig. 5 Standard deviation (STD) values of the height anomalies from all models at a $1' \times 1'$ grid, unit in cm. The STD values have the area mean of 2.7 cm and range from 0.6 to 13.1 cm. The red line represents the GSVS17 profile along the highway US 160 from Durango to Walsenburg. It did not pass the roughest terrain in the region.

4.3 GSVS17 GPS/leveling comparisons

The GSVS17 geopotential numbers are adjusted by holding the mark (ID 223) in Walsenburg fixed with the height of 1908.624 meters (NAVD88) multiplied by the observed gravity value at this point to get a reference potential value. The normal heights H^N are computed using the adjusted GSVS17 geopotential numbers and GRS80 normal gravity values. Afterwards, the GSVS17 height anomalies ζ_{GSVS17} are obtained by removing the normal height H^N from the ellipsoidal height h_{GPS} on the 223 marks ($\zeta_{GSVS17} = h_{GPS} - H^N$). Note that the GSVS17 data was not available to

any of the contributing groups, so that the GSVS17 data could not be used to tune quasigeoid models, but were used solely for an independent comparison.

Figure 6 illustrates the differences between the height anomalies estimated by the contributing groups and the GSVS17-derived height anomalies.

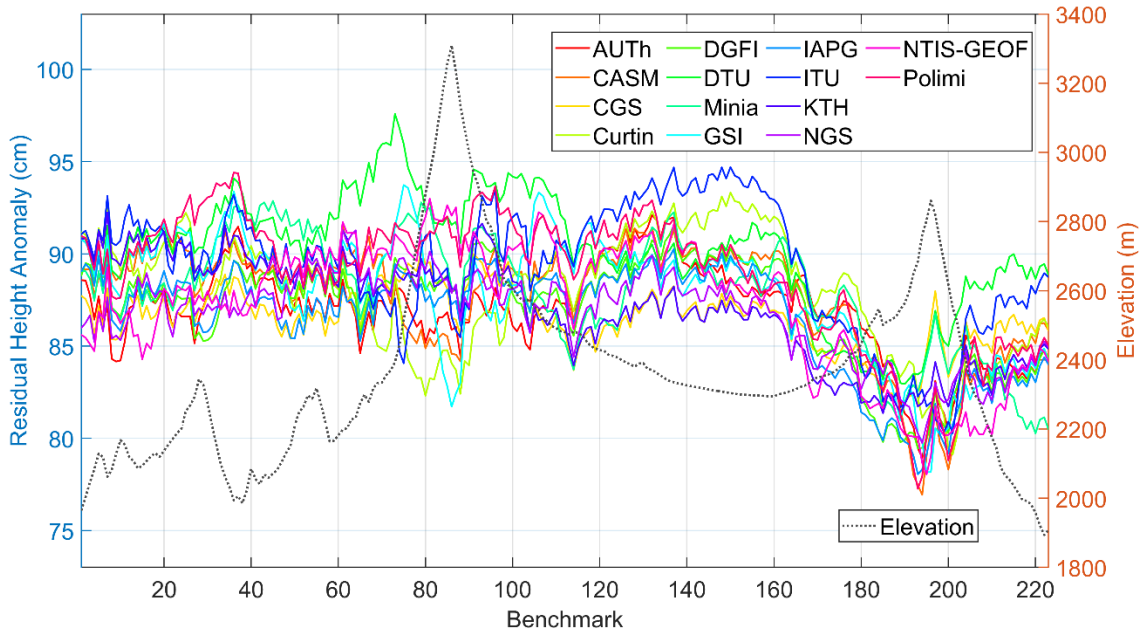


Fig. 6. Differences among the individual residual height anomalies from the quasigeoid solutions and the GSVS17-derived height anomalies ($\zeta_{model} - \zeta_{GSVS17}$). 85 cm was added to the Minia solution.

Figure 6 shows the 88 cm bias (cf. Table 6) between the models and the GSVS17 height anomalies. This bias can be explained primarily as the W_0 value adopted for this experiment differs from that defined by the NAVD88 datum, which is tied to the tide gauge at Rimouski (Zilkoski et al. 1992), and secondly, by errors in the NAVD88 data (GPS and leveling data). Excluding the bias, the profiles in Fig. 6 show no apparent tilt from mark 1 to mark 160 (about 250 km distance) and the differences are limited to a narrow range. A noticeable minimum after mark 160 is seen. All quasigeoid models behave similarly. At this point, we do not know the causes of the minimum. More studies are needed to determine whether it is caused by errors in the GSVS17 data or in the gravity data, or in both. The statistics of the height anomaly differences between the quasigeoid models and GSVS17 are given in Table 6.

Table 6. Statistics of the differences between the height anomalies provided by the individual solutions and the height anomalies inferred from the leveling-based geopotential numbers and GPS positioning along the GSVS17 profile ($\zeta_{model} - \zeta_{GSVS17}$), units in cm.

	Mean	STD	RMS	Min	Max	Range
AUTh	86.9	2.5	87.0	78.9	92.1	13.2
CASM	87.6	3.1	87.7	76.9	91.5	14.6

<i>CGS</i>	<i>86.5</i>	<i>1.7</i>	<i>86.5</i>	<i>81.5</i>	<i>90.1</i>	<i>8.6</i>
Curtin	88.0	3.2	88.0	79.1	93.3	14.2
DGFI	87.0	2.9	87.1	79.4	91.5	12.1
DTU	90.2	3.0	90.2	82.9	97.6	14.6
<i>Minia</i>	<i>7.0</i>	<i>3.1</i>	<i>7.6</i>	<i>0.7</i>	<i>11.9</i>	<i>12.6</i>
<i>GSI</i>	<i>87.8</i>	<i>3.2</i>	<i>87.9</i>	<i>78.2</i>	<i>93.7</i>	<i>15.6</i>
IAPG	86.5	2.9	86.6	78.1	90.3	12.2
ITU	89.4	3.4	89.4	80.3	94.7	14.4
KTH	87.0	2.8	87.0	81.4	92.2	10.9
NGS	86.6	2.3	86.7	79.7	91.3	11.6
NTIS-GEOF	87.2	3.5	87.3	78.0	93.0	15.0
Polimi	88.9	3.6	88.9	77.3	94.4	17.1
Mean*	87.9	2.6	88.0	80.4	91.3	10.9

*Mean value excludes the Minia model. Values of three converted models (CGS, Minia and GSI) are in italics.

After removing the mean values, the quasigeoid models agree with the GSVS17 height anomalies in the range from 1.7 to 3.6 cm in terms of the STD at the 223 marks. The median of the STD values is 3.1 cm. The CGS model agrees most closely with the GSVS17 data with a STD value of 1.7 cm. If the errors in the GSVS17 GPS/leveling data and in the quasigeoid models are uncorrelated (most likely), then the STD of 1.7 cm cautiously suggests that the accuracy of the GSVS17 height anomalies is not worse than this value. Of course, it is always possible that some errors in the gravimetric quasigeoid and the GPS/leveling data are the same and canceled out. Because this is an independent test using the GSVS17 data, we assume in the following analysis that the errors in both the GSVS17-derived height anomalies and the gravimetric height anomalies from the quasigeoid models are also independent.

Based on the preliminary analysis of the GSVS17 data, especially the height anomalies computed from the deflections of the vertical (Ahlgren, personal communication, 2021), we estimate the accuracies of the GSVS17-derived height anomalies may be 1.2 cm. Consequently, accuracies of the gravimetric quasigeoid models are estimated between 1.2 and 3.4 cm. The mean quasigeoid model is also evaluated using the GSVS17-based height anomalies, and agrees better than most of the individual models. Its accuracy is estimated as 2.3 cm.

To demonstrate more clearly the agreement between the gravimetric models and the GSVS17 data, the height anomaly differences of the CGS and the mean models are plotted in Fig. 7.

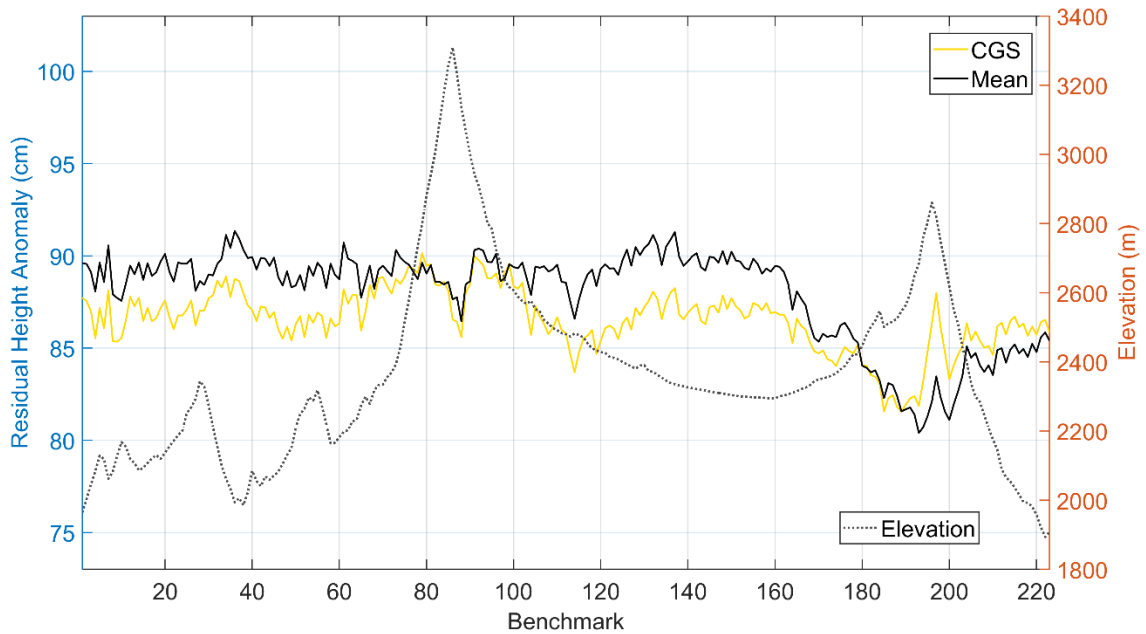


Fig. 7. Differences between the CGS, the mean of all quasigeoid models and the GSVS17-derived heights anomalies ($\zeta_{model} - \zeta_{GSVS17}$).

Figure 7 shows that the mean model has similar smaller features as the CGS model before the mark 160, and the mean model is 1.4 cm higher than the CGS model. The offset is consistent with the 1.1 cm standard deviation of the mean residuals shown in Table 6, which characterize the uncertainty due to methodology. Both sets of height anomalies decrease after mark 160, and the CGS model decreases less: the differences of the mean model have a 10-cm slope relative to the GSVS17 data from mark 160 to 193, whereas the slope of the CGS model is 5 cm. The smaller slope of the CGS model after the mark 160 is reflected in the smaller STD of the differences (1.7 cm) than the STD values of the mean model (2.6 cm). Another notable feature is that both models agree well at the first peak of height 3,310 meters (mark 86), where the elevation raises from 2,200 to 3,300 meters in about 60 km distance.

4.4 GSVS17 differential quasigeoid (slope) comparisons

Similar to the GSVS comparisons (Smith et al. 2013; Wang et al. 2017b; van Westrum et al. 2021), and to assess the relative accuracy of the quasigeoid models, a differential quasigeoid comparison is performed for all the contributing solutions. In the same way as described in Smith et al. (2013), the 24,531 pairs were grouped into 12 bins, containing approximately 2,000 pairs each. Then the RMS of height anomaly differences were computed for the 12 bins. The results are illustrated in Fig. 8. The five models in best agreement are highlighted. The RMS values range from 2 to 3 cm for these five models at baseline lengths from 2 to 100 km, and then, they disagree up to 6 cm at longer baseline lengths. The CGS model presents RMS values between 2 and 3 cm in every baseline length.

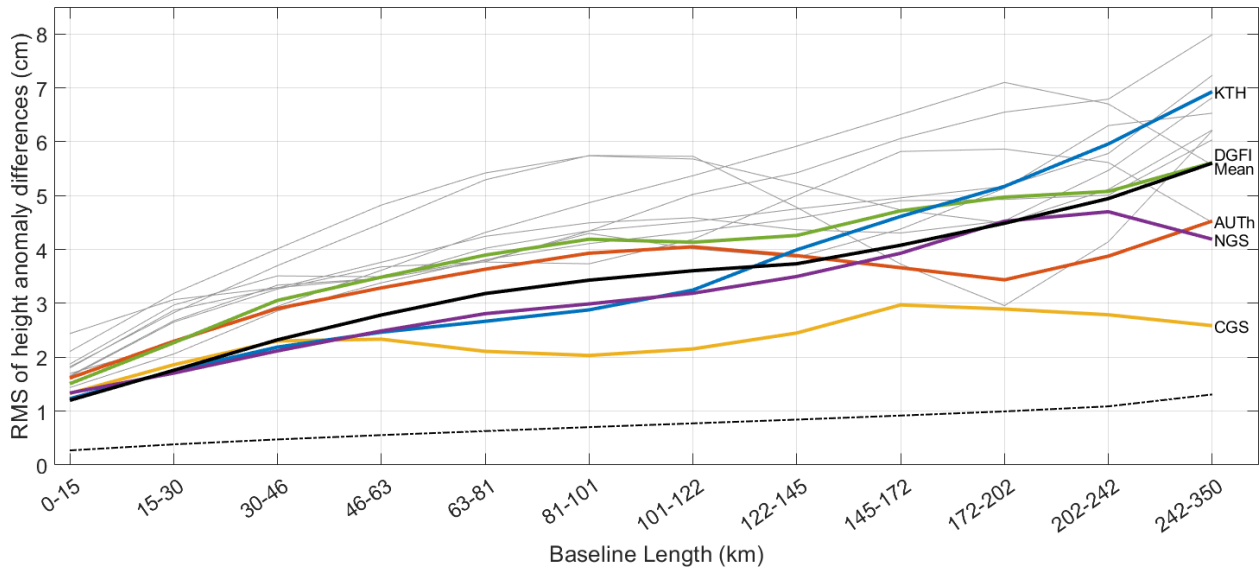


Fig. 8. Quasigeoid slope comparisons. Colored lines highlight the five best fitting models. Black line represents the mean value based on the 14 individual solutions. The formal error of the levelled height is shown as the dotted line. Highlighted models are the following: Mean=black, AUTH=red, CGS=yellow, DGFI=green, KTH=blue, NGS=purple.

The RMS values of the differences for the remaining models vary from 2 to 8 cm for different baseline lengths. The RMS values increase with respect to the baseline length, which resembles the trend of propagated leveling error. The dotted line is the formal error of the GSVS17 levelled heights based on the empirical formula $0.7\sqrt{d}$ mm (Zilkoski et al. 1992), where d is the distance between two marks in km. The formal error reaches its maximum at 1.6 cm at the distance of 350 km and cannot explain the larger RMS values in Fig. 8. To identify the error sources, the RMS differences were computed only for the portion of the line between marks 1 and 160 and plotted in Fig. 9.

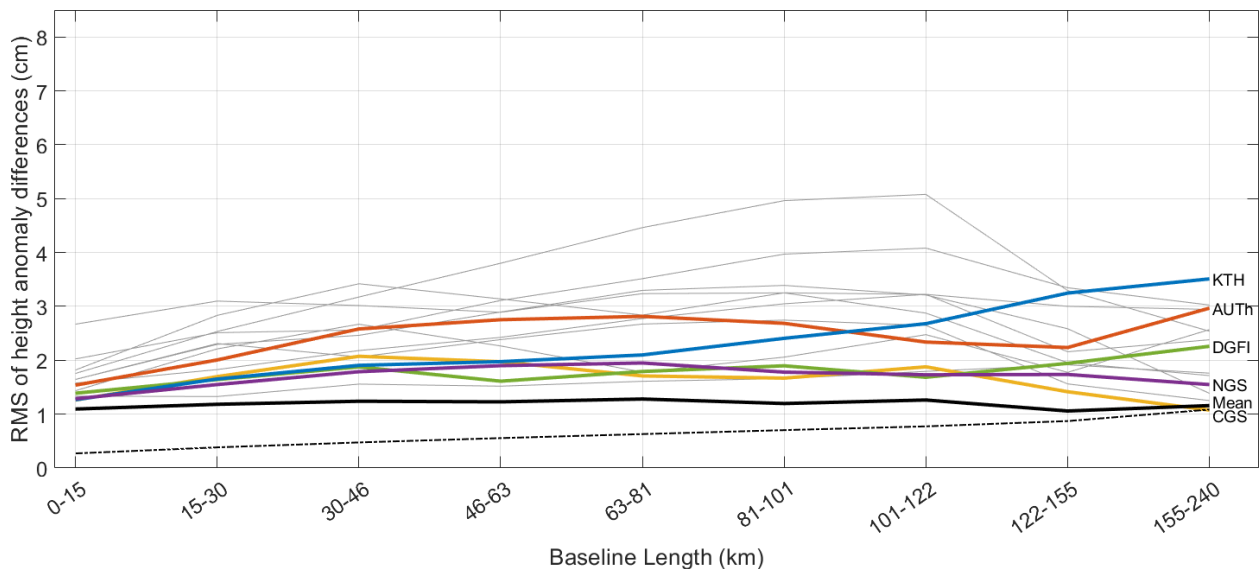


Fig. 9. Quasigeoid slope comparisons between marks 1 and 160 (baseline length 250 km). The formal error of the levelled height is shown as the dotted line. Highlighted models are the following: Mean=black, AUTH=red, CGS=yellow, DGFI=green, KTH=blue, NGS=purple.

Figure 9 shows the RMS differences of quasigeoid slopes from mark 1 to 160. In comparison with Fig. 8, the systematic trend is missing for most of the models. The large differences after mark 160 (see Fig. 6) are the major contributor to the large RMS differences and the systematic trend in Fig. 8. It is also interesting to see that the mean model agrees with the GSVS17 data the best at every baseline length, due to its averaging effect to the random errors in all the individual models. Since the RMS differences contain the errors of the quasigeoid models and the GPS/leveling, the formal leveling error should not be larger than that. The formal error of the levelled height in Fig. 9 indicates that the formal error is properly assigned. Another observation from Fig. 9 is that the mean model reaches 1-cm (relative) accuracy in this best scenario (mark 1 to 160), but the precision of the individual models still needs to be improved further.

The RMS values obtained in the quasigeoid slope comparison of the individual models with the GSVS17 data at different baseline lengths are listed in Table 7. The differential comparisons of the geoid slope provide very similar results as the height anomaly slope comparisons and so are not included here.

Table 7. RMS values obtained in the quasigeoid slope comparisons at different baseline lengths, number of pairs is the count of height anomaly differences at a particular baseline length. Unit in cm.

ID	Baseline length in km // number of pairs											
	0-15 // 2038	15-30 // 2034	30-46 // 2067	46-63 // 2070	63-81 // 2068	81-101 // 2153	101-122 // 2063	122-145 // 1984	145-172 // 2057	172-202 // 1975	202-242 // 2012	242-350 // 2010
AUTH	1.6	2.3	2.9	3.3	3.6	3.9	4.0	3.9	3.7	3.4	3.9	4.5
CASM	1.8	2.9	3.3	3.8	4.2	4.5	4.6	4.4	4.3	4.5	5.5	6.8
CGS	1.3	1.9	2.3	2.3	2.1	2.0	2.2	2.5	3.0	2.9	2.8	2.6
Curtin	1.8	2.8	3.7	4.5	5.3	5.7	5.7	4.8	3.7	3.0	4.1	6.2
DGFI	1.5	2.3	3.1	3.5	3.9	4.2	4.1	4.3	4.7	5.0	5.1	5.6
DTU	1.6	2.7	3.3	3.7	3.8	3.7	4.2	5.0	5.8	5.9	5.6	4.5
Minia	1.9	3.0	3.5	3.5	3.8	4.3	4.0	3.9	4.4	5.1	6.3	6.5
GSI	2.4	3.1	3.3	3.5	4.0	4.3	4.5	4.8	5.0	5.2	5.8	7.2
IAPG	1.4	2.1	2.9	3.4	3.8	4.1	4.3	4.6	4.9	4.9	5.0	6.0
ITU	2.1	3.2	4.0	4.8	5.4	5.7	5.7	5.2	4.7	4.5	5.1	6.2
KTH	1.2	1.8	2.2	2.5	2.7	2.9	3.2	4.0	4.6	5.2	6.0	6.9
NGS	1.3	1.7	2.1	2.5	2.8	3.0	3.2	3.5	3.9	4.5	4.7	4.2
NTIS-GEOF	1.7	2.2	2.9	3.6	4.3	4.9	5.4	5.9	6.5	7.1	6.7	5.6
Polimi	1.7	2.7	3.3	3.5	3.8	4.3	5.0	5.4	6.1	6.5	6.8	8.0
Mean	1.2	1.8	2.3	2.8	3.2	3.4	3.6	3.7	4.1	4.5	4.9	5.6

5 Discussion and conclusions

Fourteen groups from thirteen countries participated in the Colorado 1-cm geoid experiment. The member groups computed geoid heights, height anomalies and geopotential values at the 223 GSVS17 marks. In addition, the groups submitted 14 geoid models and 13 quasigeoid models at 1'×1' grids covering the Colorado area. This paper evaluates the results in terms of height anomalies by comparing the quasigeoid models in three different ways: a) comparison of the 1'×1' grids to assess the model precision, b) comparison with the GSVS17 data to estimate the model accuracy (relative), and c) a slope comparison to evaluate the agreement between the GSVS17 height anomalies and the gravimetric quasigeoid models at different baseline lengths.

- a) The grid comparison suggests that the precision of the models is ~2.0 cm. This is an important achievement considering the large topographic gradients existing in Colorado. However, this precision does not reach the target value of 1 cm. Further steps are needed to identify the causes of the differences and to improve the precision of the computation methods, or methodologic differences below 1 cm.
- b) Most models agree with the GSVS17 height anomalies to ~2 cm before mark 160, after which the models behave similarly with each other, but differ from the leveling by 5 to 10 cm. The CGS model behaves similarly, but agrees with GSVS17 better than the rest of the models after that mark. This is reflected by the smallest STD of the height anomaly differences (1.7 cm). The STD of other models range from 2.1 cm to 3.6 cm. The median of the STD is 3.1 cm. Under the no-correlation assumption between the errors in gravimetric quasigeoid models and the GSVS17 GPS/leveling data, and using 1.2 cm accuracy of the GSVS17 heights anomalies, the CGS quasigeoid model is estimated to have 1.2 cm accuracy on those GSVS17 marks. Because the GSVS17 traverse does not pass the roughest terrain in the region, a 2 cm accuracy for the quasigeoid model that best agrees with GSVS17 could be a reasonable assessment for this experiment. The GSVS17 comparisons indicate that the 1-cm geoid may be possible even in rugged mountains if accurate and well distributed gravity data, as well as detailed DEMs are available, as Fig. 7 and Fig. 9 suggested for the segment before mark 160.
- c) The differential (slope) quasigeoid comparisons show the RMS differences increase with respect to the baseline length generally (Fig. 8). The RMS values range from 2 to 3 cm for the five highlighted models at baseline lengths from 2 to 100 km, and then they disagree up to 8 cm at longer baseline lengths. The larger RMS differences and the systematic trend in Fig. 8 are caused mostly by the large differences after mark 160.

The Colorado experiment has indicated that quasigeoid models can be computed to 2 cm precision, and that the accuracy may reach 2 cm in the region. A caveat here is the assumption that the GSVS17 used as the test data is independent and has an accuracy of 1.2 cm. The small difference (STD 1.7 cm) between the CGS model and GSVS17-derived height anomalies tends to support this accuracy assessment, noting that providing test data with sub-cm accuracy to verify that 1-cm geoid accuracy has been achieved may be as challenging as achieving the 1 cm geoid itself.

At this point, even if the model under the Helmert-Stokes scheme agrees with the GSVS17 data the best, it is not sufficient to claim its superiority over other methods. A consensus is that the theoretical approaches should be equivalent and up to the same accuracy, so that the differences occur in the numerical computations and the data processing techniques. Differences in data editing, interpolation, data combination and gridding all contribute to differences, as do differences in use of the reference gravity models (kernel modification/truncation), and different ways to combine terrestrial and airborne gravity data. These issues need to be investigated further for their effects on the differences.

It needs to be pointed out that the models computed for the experiment are under the constant density assumption. This assumption affects the geoid computation directly, but it would not affect the computation of the quasigeoid if the gravity data was given everywhere. In practice, the topography has been used to aid the interpolation of gravity data into a regular grid (e.g., use of the Bouguer gravity anomaly or RTM gravity), and thus the constant density assumption is not completely exempt from the quasigeoid computation. Whether its effect is negligible needs to be assessed in future studies to ensure the 1-cm geoid accuracy.

The Colorado experiment is an effort by the international geodetic community coordinated by IAG to examine (quasi)geoid disagreements caused by computation methods and software used by different groups. In addition, the experiment tries to assess the accuracy of the computed (quasi)geoid models and geopotential values (Sánchez et al. 2021) by using the newly collected GSVS17 data in Colorado. Because of the relative sense of leveling data, the accuracy assessment in this paper is considered as relative.

It is challenging to explain fully the differences among the (quasi)geoid models for this experiment due to specific data processing techniques, computational procedures, and combination schemes. To achieve 1-cm geoid computations among all the methods, a series of recommendations has been set out that will be used to guide further collaborative experiments:

- Examine topographical reductions, data interpolation, and downward continuation steps. For example, RTM-based free-air anomalies vs TC-based Bouguer anomalies for interpolation
- Tune the spectral and LSC combinations of global model, airborne and terrestrial gravity data. For instance, adjust weights and error variances optimally
- Select a common reference model, a proper radius of numerical integration, and optimal kernel modification parameters
- Study and understand biases among the models, especially between LSC solutions and others among other causes
- Compare various (quasi)geoid models computed by different international teams
- Improve agreement with the released GSVS17 GPS/leveling and deflections of the vertical data, which are the best independent validation datasets available
- Further evaluation of GSVS17 and other high accuracy test sites with sub-cm accuracies to ensure the 1-cm geoid can be verified when it is achieved

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experiment in Colorado (Wang and Forsberg, 2019); the study group 0.15: Regional geoid/quasi-geoid modelling - Theoretical framework for the sub-centimeter accuracy of the IAG Inter-Commission Committee on Theory – ICCT (Huang and Wang 2019), and the working group 0.1.2: Strategy for the realization of the IHRS of the Focus Area Unified Height System of the Global Geodetic Observing System - GGOS (Sánchez, 2019; Sánchez and Barzaghi 2020). The authors are especially indebted to the National Geodetic Survey for providing the gravity, topographic and GPS/leveling data sets for this experiment. The authors thank three anonymous reviewers, the Associate Editor and Editor for their time, constructive suggestions and comments, which helped improving the initial version of this manuscript.

Author contributions: YMW, LS, JÅ, JH and RF conceptualized and coordinated the Colorado experiment, outlined the standards for the data processing, participated in the computation of the solutions NGS, DGFI, KTH, CGS and DTU and prepared the original draft. YMW, XL, KA and JK computed the solution NGS and performed the comparison/evaluation of the delivered solutions. VNG, GSV and DAN computed the solution AUTH. TJ computed the solution CASM. MVé and JH computed the solution CGS. SC and MF computed the solution Curtin. QL, MS and LS computed the solution DGFI. RF computed the solution DTU. HAAE computed the solution Minia. KM computed the solution GSI. MW, PZ and RP computed the solution IAPG. BE, MSI and SE computed the solution ITU. JÅ computed the solution KTH. MV, TB, MP and PN computed the solution NTIS-GEOF. RB, DC and ÖK computed the solution Polimi. All authors contributed to review and editing.

Data availability statement: The data that support the findings of this study are publically available here:

<https://geodesy.noaa.gov/GEOID/research/co-cm-experiment/>

<https://geodesy.noaa.gov/GEOID/GSVS17/index.shtml>

The airborne gravity data are available at NGS: https://www.ngs.noaa.gov/GRAV-D/data_ms05.shtml

The standardized 1' x 1' geoid, quasigeoid grids, input and validation data sets are available at the International Service for the Geoid (Reguzzoni et al. 2021)

https://www.isgeoid.polimi.it/Projects/colorado_experiment.html

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Appendix Fundamental parameters, zero-degree harmonic treatment and computation requirements

Since a main objective of the Colorado experiment is to compare gravity functionals (i.e., geoid, quasigeoid, potential values) determined using different methodologies, it was necessary to outline a set of basic (minimum) requirements to standardize as much as possible the data processing to avoid gross discrepancies as those caused for instance using different reference ellipsoids or permanent-tide systems. In this way, in addition to the use of the same input data (described in Section 3), we agreed to follow the conventions summarized in Table A1.

Table A1. Constants and conventions agreed within the Colorado experiment (cf. Sánchez et al. 2018)

Constant/Convention	Comment
General Constants	
G : $6.674\ 28 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	Universal gravitational constant
GM_{GGM} : $3.986\ 004\ 415 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$	Geocentric gravitational constant, including the mass of the Earth's atmosphere (Ries et al. 1992)
W_0 : $62\ 636\ 853.4 \text{ m}^2 \text{ s}^{-2}$	Conventional reference potential value of the geoid (IHR reference value adopted by the IAG Resolution No. 1 (2015), Drewes et al. 2016, p. 981)
ρ : 2670 kg m^{-3}	Average density of topographic masses (for geoid computation)
Reference ellipsoid	
Geodetic Reference System 1980 (GRS80)	Standard for computation of gravity anomalies, gravity disturbances, disturbing potential, geodetic coordinates, geoid heights, height anomalies, etc. All parameters are specified in Moritz (2000).
Permanent tide	
Tide-free (non-tide) system	According to the IAG Resolution No. 16 (1983) (Tscherning, 1984), the zero-tide system is to be used for the Earth's gravity field and crust modelling. According to the IAG Resolution No. 1 (2015), the mean system should be used for the final potential values. However, we decide to use the tide-free system in the Colorado experiment because the gravity data, geometric coordinates and GPS/leveling validation data are provided by NGS in the tide-free system. If everything is consistent, this should not influence the comparison results.
Conversion of coefficient C_{20}	
$\underline{C}_{20}^{TF} = \underline{C}_{20}^{ZT} + 3.11080 \cdot 10^{-8} \times 0.3/\sqrt{5}$	\underline{C}_{20}^{TF} and \underline{C}_{20}^{ZT} are the second-degree coefficients of the GGM in the tide-free and zero-tide system, respectively. For the Colorado computation, C_{20} has to be expressed in the tide-free system.
First-degree terms	
Coefficients:	
$C_1=C_{11}=S_{11}=0$	The disturbing potential T is given by (cf. Eq. 2-170, Heiskanen and Moritz, 1967):
Disturbing potential:	
$T_1(\theta, \lambda) = 0$	$T(\theta, \lambda) = T_0 + T_1(\theta, \lambda) + \sum_{n=2}^{\infty} T_n(\theta, \lambda)$

The first-degree terms are assumed to be zero to align the Earth's center of masses with the origin of the International Terrestrial Reference System (ITRS); i.e., the geometric coordinate system.

Zero-degree term

Disturbing potential:

$$T_0 = \frac{(GM_{GGM} - GM_{GRS80})}{r_P}$$

The zero-degree term T_0 must include the difference between Earth's and reference ellipsoid's GM constant (cf. Eq. 2-172, Heiskanen and Moritz, 1967). r_P is the geocentric radial distance of the computation point P .

Height anomaly*:

$$\zeta_0 = \frac{(GM_{GGM} - GM_{GRS80})}{r_P \gamma_Q} - \frac{(W_0 - U_0)}{\gamma_Q}$$

For the height anomaly (ζ) and the geoid undulation (N), in addition to the difference between the two GM values, the difference between the IHRS reference potential W_0 value and the potential U_0 on the reference ellipsoid must be considered. It is recommended to compute the (quasi)geoid starting with $d/o = 2$ and then add the corresponding zero-degree term ζ_0 or N_0 .

Geoid undulation*:

$$N_0 = \frac{(GM_{GGM} - GM_{GRS80})}{r_{P_0} \gamma_{Q_0}} - \frac{(W_0 - U_0)}{\gamma_Q}$$

** P is the computation point on the Earth's surface, Q is the projection of P along the normal plumb line on the telluroid, P_0 is the projection of P along the real plumb line on the geoid, and Q_0 is the projection of P_0 on the ellipsoid.*