Robust Stabilization of Uncertain Impulsive Switched Systems with Delayed Control ⋆

Honglei Xu [a,b,∗] Kok Lay Teo [a]

[a] *Department of Mathematics and Statistics, Curtin University of Technology, Perth, WA 6845, Australia*

[b] *Department of Mathematics, Guizhou University, Guiyang, 550025, China*

Abstract

In this paper, stability criteria and switching controllers design problems for uncertain impulsive switched systems with input delay are investigated by using the receding horizon method. Some LMI conditions are derived to guarantee asymptotical stability of an impulsive switched system under a certain designed delayed controller. Finally, a numerical example is presented to illustrate the effectiveness of the results obtained.

Key words: Stability analysis, Input delay, Impulsive switched systems, LMI, robust stabilization, Impulsive switchings
PACS:

1 Introduction

Within the past several years, there is an increasing interest in the qualitative theory of impulsive switched systems. The reason is that impulsive switched systems can model nonlinear systems which exhibit not only impulsive dynamical behaviors but also switching phenomena. Nowadays, there are various stability results available in the literature for impulsive switched systems with or without uncertainty. For example, results on the uniformly asymptotical stability of impulsive switched systems with uncertainty are obtained in

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∗ Corresponding author. Tel.: +61 8 92663491; fax: +61 8 92663197.
Email address: H. Xu@curtin.edu.au (Honglei Xu).

[1] by using an LMI approach. Robust stabilization conditions for uncertain impulsive switched systems with definite attenuance are derived in [2], where the corresponding robust H_∞ optimal control law is also presented. In [3], a unified approach is used to the study of stability criteria of impulsive hybrid systems.

In practice, many systems arising in disciplines, such as physics, chemistry, biology and engineering, often involve after effects or time lags. These systems, which are called time delay systems, are often described by functional differential equations with time delays. See, for example, [4]-[6] and the references therein. If a controller contains time delays, it is called a delayed controller. Recently, there are some results focused on the stability analysis of dynamical systems with delayed controllers. For example, a receding horizon method is used in [7] to design a delayed controller to stabilizing a linear system. Stability analysis and control of switched systems with input delay are studied in [8]. However, it appears that no results are available for stability analysis and controller design for impulsive switched systems with delay input.

In this paper, we consider a class of uncertain impulsive switched systems. By using a receding horizon method, some LMI-based sufficient conditions for asymptotic stability of the impulsive switched system are obtained. Furthermore, a design procedure for the construction of a delayed stabilizing controller is given.

The remainder of the paper proceeds as follows. In Section 2, we formulate the problem described by this class of impulsive switched systems with delayed input. In Section 3, we derive sufficient conditions for asymptotic stability of the uncertain impulsive switched system with delayed input. Furthermore, we devise a method for the design of switched delayed controllers. In Section 4, an illustrative example is presented, showing the effectiveness of the results obtained. Section 5 contains some concluding remarks.

2 Problem statement

Consider the following impulsive switched systems with delay input

$$\begin{cases} \dot{x}(t) = (A_{i_k} + \Delta A_{i_k})x(t) + B_{i_k}u(t) + C_{i_k}u(t-h) \\ \qquad -A_{i_k}\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds & t \neq t_k \\ \Delta x(t) = I_k(t,x) = D_k x(t) + D_k\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds & t = t_k \\ x(t) = \varphi(t) & -\tau \leq t \leq 0, \end{cases} \quad (1)$$

where $x(t) \in R^n$, $u(t) \in R^p$, $u(t-h) \in R^q$, with $n, p, q \in \mathcal{N}$, are, respectively, the state and control vectors, while $\mathcal{N}$ denotes the set of all positive natural numbers. A_{i_k}, B_{i_k}, C_{i_k} are constant real matrices of appropriate dimensions. $I_k(t,\cdot) : R^n \to R^n$, $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k) = x(t_k^-) = \lim_{\upsilon \to 0+} x(t_k - \upsilon)$, $x(t_k^+) = \lim_{\upsilon \to 0+} x(t_k + \upsilon)$. $x(t_k) = x(t_k^-)$ means that the solution of the impulsive switched systems (1) is left continuous. h represents a control delay. $t_0 < t_1 < t_2 < ... < t_k < ...(t_k \to \infty$ as $k \to \infty)$. $i_k \in \{1, 2, ...m\}$, with $k, m \in \mathcal{N}$, is a discrete state variable and t_k is an impulsive switching point. $\{t_k, i_k\}$ represents a switching law of the systems (1), *i.e.* at t_k time point, the system switches to the i_k subsystem from the i_{k-1} subsystem. The matrix $\Delta A_{i_k}(\cdot)$ is an unknown real norm-bounded matrix function representing time-varying parameter uncertainty. Assume that admissible uncertainties are of the form

$$\Delta A_{i_k}(t) = E_{i_k} F_{i_k}(t) H_{i_k}, \tag{2}$$

where E_{i_k}, H_{i_k} are known real constant matrices, $F_{i_k}(t)$ is an unknown real time-varying matrix satisfying $F_{i_k}^T(t) F_{i_k}(t) < I$, in which I represents the identity matrix of appropriate dimension.

By virtue of the receding horizon method reported in [7], we define, for the impulsive switched systems (1) with delay input,

$$y(t) = x(t) + \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds, \tag{3}$$

where $u(t-h)$ is an arbitrary control, $t \in (t_k, t_{k+1}]$, $k = 1, 2, \cdots$, and $i_k \in \{1, 2, \cdots, m\}$, and $m \in \mathcal{N}$.

Lemma 2.1: The uncertain impulsive switched system (1) is equivalent to

$$\dot{y}(t) = [A_{i_k} + E_{i_k} F_{i_k}(t) H_{i_k}] y(t) + [B_{i_k} + e^{-A_{i_k} h} C_{i_k}] u(t) \tag{4}$$

$$-[A_{i_k} + E_{i_k} F_{i_k}(t) H_{i_k}] \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds$$

$$\Delta y(t) = D_k y(t) \tag{5}$$

$$y(t) = \varphi(t) + \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds \quad -\tau \le t \le 0, \tag{6}$$

where $t \in (t_k, t_{k+1}], k = 1, 2, \cdots, i_k \in \{1, 2, \cdots, m\}$, and $m \in \mathcal{N}$.

Proof. When $t \in (t_k, t_{k+1}]$, define $y(t) = x(t) + \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds$. $y(t)$ can be rewritten as:

$$y(t) = x(t) + \int_{a}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds - \int_{a}^{t-h} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds$$

$$= x(t) + e^{A_{i_k} t} \int_{a}^{t} e^{A_{i_k} -(s+h)} C_{i_k} u(s) ds - e^{A_{i_k} t} \int_{a}^{t-h} e^{A_{i_k} -(s+h)} C_{i_k} u(s) ds,$$

where a is a real number.

Consider the time derivative of $y(t)$, we obtain

$$\dot{y}(t) = \dot{x}(t) + A_{i_k} e^{A_{i_k} t} \int_{a}^{t} e^{-A_{i_k}(s+h)} C_{i_k} u(s) ds + e^{A_{i_k} t} e^{-A_{i_k}(t+h)} C_{i_k} u(t)$$

$$-A_{i_k} e^{A_{i_k} t} \int_{a}^{t-h} e^{-A_{i_k}(s+h)} C_{i_k} u(s) ds - e^{A_{i_k} t} e^{-A_{i_k} t} C_{i_k} u(t-h)$$

$$= \dot{x}(t) + A_{i_k} \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds + e^{-A_{i_k} h} C_{i_k} u(t) - C_{i_k} u(t-h)$$

$$= (A_{i_k} + \Delta A_{i_k}) x(t) + B_{i_k} u(t) + e^{-A_{i_k} h} C_{i_k} u(t)$$

$$= (A_{i_k} + \Delta A_{i_k}) y(t) + (B_{i_k} + e^{-A_{i_k} h} C_{i_k}) u(t)$$

$$-(A_{i_k} + \Delta A_{i_k}) \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds$$

$$= (A_{i_k} + E_{i_k} F_{i_k}(t) H_{i_k}) y(t) + (B_{i_k} + e^{-A_{i_k} h} C_{i_k}) u(t)$$

$$-(A_{i_k} + E_{i_k} F_{i_k}(t) H_{i_k}) \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds$$

Next, when $t = t_k$,

$$\Delta y(t) = y(t_k^+) - y(t_k^-)$$

$$= x(t_k^+) + \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds - (x(t_k^-) + \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds)$$

$$= x(t_k^+) - x(t_k^-) = D_k x(t) + D_k \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s)ds = D_k y(t)$$

For $-\tau \leq t \leq 0$, $y(t) = \varphi(t) + \int_{t-h}^{t} e^{A_{i_k}(t-s-h)} C_{i_k} u(s)ds$ since $x(t) = \varphi(t)$.

This completes the proof.

Our objective is to devise a design method for constructing linear switching controllers that can stabilize (6) with admissible uncertainties under an arbitrary switching law.

3 Main results

The following result is well known.

Lemma 3.1[9]. Let E, H and $F(t)$ be real matrices of appropriate dimensions with $F^T(t)F(t) \leq I$. Then, for any scalar $\varepsilon > 0$, it holds that

$$EF(t)H + H^T F^T(t) E^T \leq \frac{1}{\varepsilon} EE^T + \varepsilon H^T H. \tag{7}$$

Assumption 3.1. $\int_{t-h}^{t} y^T(s)\Phi(s)y(s)ds \leq y^T(t)(\int_{t-h}^{t} \Phi(s)ds)y(t)$, where Φ is a symmetric positive definite matrix.

Theorem 3.1. Suppose that Assumption 3.1 holds and that there exist symmetric positive definite matrices P_{i_k}, Q_{i_k} and some positive scalars ε_1, ε_2, ε_3, such that the following conditions are satisfied.

(a)

$$\begin{bmatrix} -(\varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k} + Q_{i_k}) & \varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k} + Q_{i_k} & 0 \\ \varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k} + Q_{i_k} & A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} & P_{i_k} \varphi_{i_k} \\ 0 & \varphi_{i_k}^T P_{i_k} & -I \end{bmatrix} < 0, (8)$$

(b)

$$\begin{bmatrix} P_{i_{k-1}} & (I + D_k)^T P_{i_k} \\ P_{i_k}(I + D_k) & P_{i_k} \end{bmatrix} > 0, \tag{9}$$

where

$$\varphi_{i_k}\varphi_{i_k}^T = -2I + \varepsilon_2^{-1}A_{i_k}A_{i_k}^T + \varepsilon_2 hU_{i_k} + \varepsilon_3^{-1}E_{i_k}E_{i_k}^T + \varepsilon_3 h\hat{U}_{i_k}, \tag{10}$$

while

$$U_{i_k} \geq (B_{i_k} + e^{-A_{i_k}h}C_{i_k})^{-T}C_{i_k}^T(\int_{-h}^{0} e^{-A_{i_k}^T(s+h)}e^{-A_{i_k}(s+h)}ds)C_{i_k} \times(B_{i_k} + e^{-A_{i_k}h}C_{i_k})^{-1} \tag{11}$$

and

$$\hat{U}_{i_k} \geq (B_{i_k} + e^{-A_{i_k}h}C_{i_k})^{-T}C_{i_k}^T(\int_{-h}^{0} e^{-A_{i_k}^T(s+h)}H_{i_k}^T H_{i_k}e^{-A_{i_k}(s+h)}ds)C_{i_k} \times(B_{i_k} + e^{-A_{i_k}h}C_{i_k})^{-1}. \tag{12}$$

Then, the impulsive switched system (1) can be robustly asymptotically stabilized under an arbitrary given switching law by the following switching controller

$$u(t) = -(B_{i_k} + e^{-A_{i_k}h}C_{i_k})^{-1}P_{i_k}y(t). \tag{13}$$

Proof. For $t \in (t_k, t_{k+1}]$, $k = 1, 2, \cdots; i_k \in \{1, 2, \cdots, m\}$; $m \in \mathcal{N}$, define

$$V(t) = y^T(t)P_{i_k}y(t) + \int_{t-h}^{t} y^T(s)Q_{i_k}y(s)ds \tag{14}$$

where $P_{i_k} > 0$, $Q_{i_k} > 0$. We shall show that V is a Lyapunov function.

Taking the differentiation of (14) along the trajectory of system (4)-(6), we obtain

$$\begin{aligned}\dot{V}(t) &= \dot{y}^T(t)P_{i_k}y(t) + y^T(t)P_{i_k}\dot{y}(t) + y^T(t)Q_{i_k}y(t) - y^T(t-h)Q_{i_k}y(t-h)\\ &= [(A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k})y(t) + (B_{i_k} + e^{-A_{i_k}h}C_{i_k})u(t)\\ &-(A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k})\int_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds]^T P_{i_k}y(t)\end{aligned}$$

$$+y(t)^T P_{i_k}[(A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k})y(t) + (B_{i_k} + e^{-A_{i_k}h}C_{i_k})u(t)$$

$$-(A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k}) \int\limits_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds]$$

$$+y^T(t)Q_{i_k}y(t) - y^T(t-h)Q_{i_k}y(t-h)$$

$$= S_1(t) + S_2(t) + S_3(t) \tag{15}$$

where

$$S_1(t) = y^T(t)(A_{i_k}^T P_{i_k} + P_{i_k}A_{i_k})y(t) + y^T(t)Q_{i_k}y(t) - y^T(t-h)Q_{i_k}y(t-h), \tag{16}$$

$$S_2(t) = y^T(t)(H_{i_k}^T F_{i_k}^T(t)E_{i_k}^T + E_{i_k}F_{i_k}(t)H_{i_k})y(t), \tag{17}$$

and

$$S_3(t) = 2u^T(t)(B_{i_k} + e^{-A_{i_k}h}C_{i_k})^T P_{i_k}y(t)$$

$$-2y^T(t)P_{i_k}(A_{i_k} + E_{i_k}F_{i_k}(t)H_{i_k}) \int\limits_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds. \tag{18}$$

By Lemma 3.1, we obtain

$$S_2(t) \le y^T(t)(\varepsilon_1^{-1}E_{i_k}E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k})y(t), \tag{19}$$

and

$$S_3(t) = -2y^T(t)P_{i_k}^2 y(t) - 2y^T(t)P_{i_k}A_{i_k} \int\limits_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds$$

$$-2y^T(t)P_{i_k}E_{i_k}F_{i_k}(t)H_{i_k} \int\limits_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds$$

$$\le -2y^T(t)P_{i_k}^2 y(t) + \varepsilon_2^{-1}y^T(t)P_{i_k}A_{i_k}A_{i_k}^T P_{i_k}y(t)$$

$$+\varepsilon_2(\int\limits_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds)^T \int\limits_{t-h}^{t} e^{A_{i_k}(t-s-h)}C_{i_k}u(s)ds$$

$$+\varepsilon_3^{-1}y^T(t)P_{i_k}E_{i_k}E_{i_k}^T P_{i_k}y(t)$$

$$+\varepsilon_3(\int_{t-h}^{t} H_{i_k} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds)^T \int_{t-h}^{t} H_{i_k} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds. \tag{20}$$

Applying the following inequality to (20),

$$(\int_{t-h}^{t} x(s) ds)^T (\int_{t-h}^{t} x(s) ds) \leq h \int_{t-h}^{t} x^T(s) x(s) ds, \tag{21}$$

we obtain

$$S_3(t) \leq -2y^T(t) P_{i_k}^2 y(t) + \varepsilon_2^{-1} y^T(t) P_{i_k} A_{i_k} A_{i_k}^T P_{i_k} y(t)$$

$$+\varepsilon_2 h \int_{t-h}^{t} (e^{A_{i_k}(t-s-h)} C_{i_k} u(s))^T e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds$$

$$+\varepsilon_3^{-1} y^T(t) P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} y(t)$$

$$+\varepsilon_3 h \int_{t-h}^{t} (H_{i_k} e^{A_{i_k}(t-s-h)} C_{i_k} u(s))^T H_{i_k} e^{A_{i_k}(t-s-h)} C_{i_k} u(s) ds. \tag{22}$$

Substituting the expression of $u(t)$ given by (13) into (22), we obtain

$$S_3(t) \leq -2y^T(t) P_{i_k}^2 y(t) + \varepsilon_2^{-1} y^T(t) P_{i_k} A_{i_k} A_{i_k}^T P_{i_k} y(t)$$

$$+\varepsilon_2 h y^T(t) P_{i_k} (B_{i_k} + e^{-A_{i_k} h} C_{i_k})^{-T} C_{i_k}^T \Phi_{i_k} P_{i_k} y(t)$$

$$+\varepsilon_3^{-1} y^T(t) P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} y(t)$$

$$+\varepsilon_3 h y^T(t) P_{i_k} (B_{i_k} + e^{-A_{i_k} h} C_{i_k})^{-T} C_{i_k}^T \hat{\Phi}_{i_k} P_{i_k} y(t). \tag{23}$$

where

$$\Phi_{i_k} = (\int_{t-h}^{t} e^{A_{i_k}^T (t-s-h)} e^{A_{i_k}(t-s-h)} ds) C_{i_k} (B_{i_k} + e^{-A_{i_k} h} C_{i_k})^{-1} \tag{24}$$

and

$$\hat{\Phi}_{i_k} = (\int_{t-h}^{t} e^{A_{i_k}^T(t-s-h)} H_{i_k}^T H_{i_k} e^{A_{i_k}(t-s-h)} ds) C_{i_k} (B_{i_k} + e^{-A_{i_k}h} C_{i_k})^{-1}. \quad (25)$$

Combining (16), (19) and (23) with (15), it follows that

$$\begin{aligned}
\dot{V}(t) &\leq y^T(t)(A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} + \varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k} - 2P_{i_k}^2) y(t) \\
&+ \varepsilon_2^{-1} y^T(t) P_{i_k} A_{i_k} A_{i_k}^T P_{i_k} y(t) + \varepsilon_2 h y^T(t) P_{i_k} U_{i_k} P_{i_k} y(t) + \varepsilon_3^{-1} y^T(t) P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} y(t) \\
&+ \varepsilon_3 h y^T(t) P_{i_k} \hat{U}_{i_k} P_{i_k} y(t) + y^T(t) Q_{i_k} y(t) - y^T(t-h) Q_{i_k} y(t-h) \\
&= y^T(t)(A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} + \varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k} + Q_{i_k} - 2P_{i_k}^2) y(t) \\
&+ y^T(t)(\varepsilon_2^{-1} P_{i_k} A_{i_k} A_{i_k}^T P_{i_k} + \varepsilon_2 h P_{i_k} U_{i_k} P_{i_k} + \varepsilon_3^{-1} P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} + \varepsilon_3 h P_{i_k} \hat{U}_{i_k} P_{i_k}) y(t) \\
&- y^T(t-h) Q_{i_k} y(t-h), \quad (26)
\end{aligned}$$

where U_{i_k} and $\hat{U}_{i_k}$ are defined in (11) and (12), respectively.

Clearly, $\dot{V}(t) < 0$ is implied by

$$W_{i_k} < 0 \quad (27)$$

where

$$\begin{aligned}
W_{i_k} &= A_{i_k}^T P_{i_k} + P_{i_k} A_{i_k} + \varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k} + Q_{i_k} - 2P_{i_k}^2 + \varepsilon_2^{-1} P_{i_k} A_{i_k} A_{i_k}^T P_{i_k} \\
&+ \varepsilon_2 h P_{i_k} U_{i_k} P_{i_k} + \varepsilon_3^{-1} P_{i_k} E_{i_k} E_{i_k}^T P_{i_k} + \varepsilon_3 h P_{i_k} \hat{U}_{i_k} P_{i_k}. \quad (28)
\end{aligned}$$

Furthermore, $W_{i_k} < 0$ is equivalent to

$$\begin{bmatrix} -I & & \\ & W_{i_k} & \\ & & -I \end{bmatrix} < 0. \quad (29)$$

Define

$$Z_{i_k} = \begin{bmatrix} (\varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k} + Q_{i_k})^{1/2} & 0 & 0 \\ -(\varepsilon_1^{-1} E_{i_k} E_{i_k}^T + \varepsilon_1 H_{i_k}^T H_{i_k} + Q_{i_k})^{1/2} & I & -P_{i_k}\varphi_{i_k} \\ 0 & 0 & I \end{bmatrix}, \quad (30)$$

where

$$\varphi_{i_k}\varphi_{i_k}^T = -2I + \varepsilon_2^{-1}A_{i_k}A_{i_k}^T + \varepsilon_2 hU_{i_k} + \varepsilon_3^{-1}E_{i_k}E_{i_k}^T + \varepsilon_3 h\hat{U}_{i_k}.$$

Then, by left multiplying Z_{i_k} and right multiplying $Z_{i_k}^T$, we obtain condition (a) of the theorem given by (8), which is satisfied by assumption.

Thus, $W_{i_k} < 0$ and hence $\dot{V}(t) < 0$ during the whole continues time parts ($i.e.$, excluding impulsive and switching time points) of the time horizon.

Next, for the impulsive and switching time point t_k, we have

$$\begin{aligned} V(t_k^+) - V(t_k) &= y(t_k^+)^T P_{i_k} y(t_k^+) - y(t_k)^T P_{i_{k-1}} y(t_k) \\ &\leq y(t_k)[(I + D_k)^T P_{i_k}(I + D_k) - P_{i_{k-1}}]y(t_k) < 0. \end{aligned}$$

Clearly, $V(t_k^+) < V(t_k^-)$ is implied by

$$(I + D_k)^T P_{i_k}(I + D_k) - P_{i_{k-1}} < 0. \tag{31}$$

By virtue of Schur complements, the inequality(31) is equivalent to that of condition (b) of the theorem given by (9) , which is satisfied by assumption.

Therefore, $V(t)$ defined by (14) decreases along the whole trajectory of system (4)-(6) and is a Lyapunov function. Thus, the impulsive switched system (1) is robustly asymptotically stable under the switching controller (13).

This completes the proof.

As a consequence, the following results are valid for system (1) with no switching.

Corollary 3.1. Suppose that Assumption 3.1 holds and that there exist symmetric positive definite matrices P, Q and some positive scalars ε_1,ε_2,ε_3, such that the following LMIs are satisfied.

(a)

$$\begin{bmatrix} -(\varepsilon_1^{-1}EE^T + \varepsilon_1 H^T H + Q) & \varepsilon_1^{-1}EE^T + \varepsilon_1 H^T H + Q & 0 \\ \varepsilon_1^{-1}EE^T + \varepsilon_1 H^T H + Q & A^T P + PA & P\varphi \\ 0 & \varphi^T P & -I \end{bmatrix} < 0, \tag{32}$$

(b)

$$\begin{bmatrix} P & (I+D_k)^T P \\ P(I+D_k) & P \end{bmatrix} > 0, \tag{33}$$

where

$$\varphi\varphi^T = -2I + \varepsilon_2^{-1} AA^T + \varepsilon_2 hU + \varepsilon_3^{-1} EE^T + \varepsilon_3 h\hat{U}, \tag{34}$$

while

$$U \geq (B + e^{-Ah}C)^{-T} C^T (\int_{-h}^{0} e^{-A^T(s+h)} e^{-A(s+h)} ds) C (B + e^{-Ah}C)^{-1} \tag{35}$$

and

$$\hat{U} \geq (B + e^{-Ah}C)^{-T} C^T (\int_{-h}^{0} e^{-A^T(s+h)} H^T H e^{-A(s+h)} ds) C (B + e^{-Ah}C)^{-1}. \tag{36}$$

Then, system (1) without switchings can be robustly asymptotically stabilized by the following controller

$$u(t) = -(B + e^{-Ah}C)^{-1} P y(t). \tag{37}$$

4 A numerical example

In this section, an illustrative example will be presented to show the effectiveness of the results obtained. Consider the impulsive switched systems with the following specifications

$$A_1 = \begin{bmatrix} -0.24 & -0.8 \\ -0.6 & -2.2 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.5 & 0.4 \\ 0.2 & 0.4 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.7 & 0.7 \\ 0.7 & 0.7 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.8 & 0.9 \\ 1.2 & 1.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2.2 & -0.6 \\ -0.6 & -2 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.3 & 0.1 \\ 0.6 & 0.4 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0.7 & 0.7 \\ 0.7 & 0.7 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.3 & 1.1 \\ 0.8 & 0.5 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.8 & 0.6 \\ 0.3 & 0.5 \end{bmatrix}, \quad D_1 = D_2 = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}, \quad h = 0.3.$$

Choose $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$. Then, by solving LMIs (8)-(9), we obtain the following symmetric positive define matrices,

$$P_1 = \begin{bmatrix} 1.6423 & -1.3092 \\ -1.3092 & 2.0908 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.8959 & -1.3228 \\ -1.3228 & 1.8825 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 0.1514 & -0.0106 \\ -0.0106 & 0.3096 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.5443 & 0.1320 \\ 0.1320 & 0.2655 \end{bmatrix}.$$

By Theorem 3.1, the following switching controller

$$u_1(t) = \begin{bmatrix} -2.2635 & 2.4620 \\ 2.2194 & -2.6337 \end{bmatrix} y(t), \quad u_2(t) = \begin{bmatrix} -2.5453 & 2.5298 \\ 2.7588 & -2.9888 \end{bmatrix} y(t).$$

is obtained. It asymptotically stabilizes the impulsive switched system according to Theorem 3.1.

5 Conclusion

This paper studied a class of uncertain impulsive switched systems with delayed input. Based on the receding horizon method, these systems can be transformed into switched systems without time delay. Some LMIs conditions are derived ensuring asymptotical stability of the impulsive switched systems under delayed controllers obtained. A numerical example is solved, from which we see that results obtained are effective.

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