WASM: Minerals, Energy and Chemical Engineering

Production Scheduling of an Open-pit Mining Complex with Waste Dump Constraints

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DECLARATION

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

To the best of my knowledge and belief, this thesis contains no material previously published by another person except where due acknowledgement has been made.

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ABSTRACT

Open-pit mines, especially open-pit mining complexes, are a kind of large-scale, integrated, and complicated operating system, which requires significant initial capital cost and sustaining capital cost. A successful mining operator knows how to maximise benefits from mine development via good strategic mine planning. Open-pit mine production scheduling (OPMPS) involves strategic decision-making that seeks to optimise the mining sequence and the materials flow (i.e. processing streams, stockpiles, waste dumps) within given constraints. The space availability typically is a common constraint during the mining layout study. However, the strategic mine planning for those open-pit mining complex by manually has great difficulty. Meanwhile, the available commercial mining software is unable to be developed by being tailored for those specific open-pit mine scheduling problems.

The research work aims to solve the production scheduling problem for open-pit mining complexes. It establishes a Mixed-Integer Programming (MIP) model that maximises the net present value of future cash flows and satisfies reserve, production capacity, mining block precedence, waste disposal, stockpiling, and pit sequence constraints. The model is validated by using small to medium scale datasets. All formulated constraints have worked correctly based on the validation results.

It is presented using a real data set from a gold mine in Western Australia to test a proposed MIP model. The case study is based on the given mining physicals, which assesses the mine strategy plan of the open-pit mining complex by conjunction with the simultaneous optimisation of extraction sequence and processing stream decisions. On the basis of the same dataset, two scenarios are examined, which indicates that the proposed MIP model can generate a mine schedule to fit the constraint of limited

space. Compared to the original schedule, the chosen schedule has less static income, million dollars. However, the chosen schedule can come through the permitting one and half years earlier, which brings a higher net present value (NPV) through a 10-year period of life-of mine (LOM).

The established MIP model is flexible to adjust the given constraints and decision variables, which can solve more complicated problems of the mining industry.

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LIST OF ABBREVIATIONS

BEV	Block Economic Value
CIL	carbon in leach
CIC	carbon-in-column
DCF	Discounted Cash Flow
LOM	life-of mine
LP	Linear programming
mtpa	million tonnes per annum
MIP	Mixed Integer Programming
MILP	Mixed Integer linear Programs
MINLP	Mixed Integer Nonlinear Programs
NAF	Non-Acid Forming
NPV	net present value
OPMPS	Open-pit mine production scheduling

CHAPTER 1. INTRODUCTION

Surface mining technique, namely open-pit mining or open-cast mining, is a mining method that the rock or minerals are extracted from the earth. As a widely used mineral extraction method, Open-pit mining is chosen when minerals or deposits are found relatively close to the surface. Usually, two or more open-pit mines, processing flows, stocks, mixed options, and products are composed of an open-pit mining complex. Optimising the mining complex scheduling is designed to maximise the net present value (NPV) of cash flow by generating a production plan for the entire mining activity (Goodfellow and Dimitrakopoulos 2016) .

The open-pit mine production scheduling (OPMPS) problem consists of (i) identifying a mineralised zone through exploration with drilling and mapping,

(ii) dividing the field into three-dimensional rectangular blocks, and creating a block model to represent the mineral deposit numerically,

(iii) assigning attributes such as grades that are estimated by sampling analysing drill cores, and

(iv) utilising the attributes to evaluate the economic value of each block, i.e., differences between the expected revenue from selling ore and associated costs such as those related to mining and processing. Given this data, the further work's target is to maximise the mine project’s NPV by determining each block's extraction time in a deposit and confirm the destinations of which the blocks must be sent to the processing streams, stockpiles, or waste dumps.

The quantity and quality of production will be determined, reaching millions of dollars over the life-of-mine (LOM). An optimal sequence for annually extracting the mineralised material is determined by the annual production scheduling of an open-pit mine.

Figure 1.1 describes a typical open-pit mine design process. The process starts with the assumption of initial production capacities and estimates for the related costs and commodity prices. Once the economic parameters are known, the analysis of the ultimate pit limits of the mine is undertaken to determine what portion of the deposit can economically be mined. The ultimate pit shell divides the entire deposit into two subgroups. First, ore reserves is the minable ore within the ultimate pit shell. This is usually done by using the moving cone method or the method of Lerchs and Grossmann (Lerchs 1965).

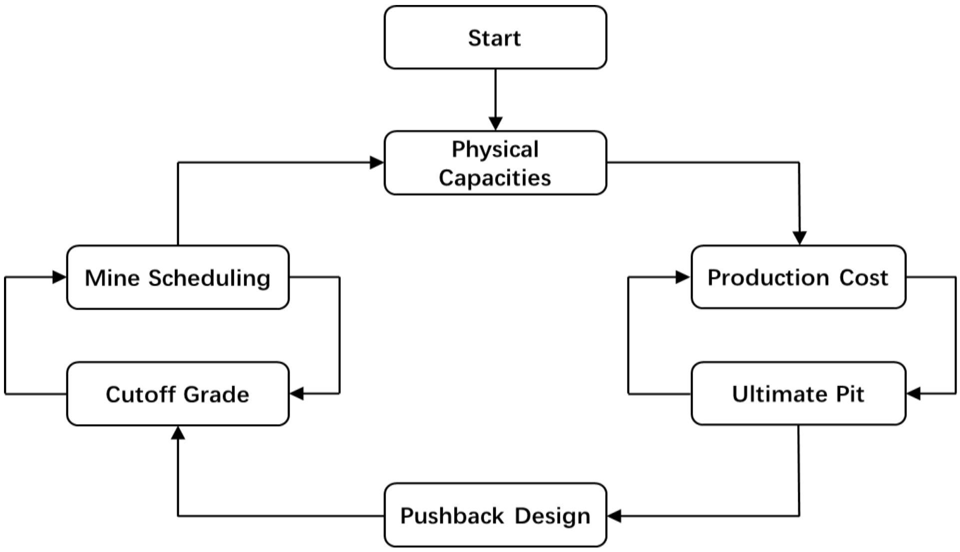

Figure 1.1 Steps of traditional planning by Circular Analysis (Dagdelen, 1985)

Within the ultimate pit limits, pushbacks are further designed to divide deposit into nested pits, going from the smallest pit with the highest value per tonne of ore to the largest pit with the lowest value per tonne of ore. These pushbacks are designed with haul road access and act as a guide during the scheduling of yearly productions from different benches. The cutoff grade strategy is defined as differentiating ore from waste and further determining how the individual blocks should be processed. These steps are repeated circularly as further improvements are made with respect to the adequacy of the production capacities and the estimated costs (Dagdelen 2001).

1.1 Definitions

1.1.1 Block Model

As a simplified representation of the ore body, a block model is treated as a stack of computer-generated cuboids representing volumes of rock in deposit (ore or waste). Block model is created using geostatistics and geological data collected by drilling the prospective ore zone, laboratory samples analysis, and geological mapping.

The block model is essentially a set of specific-sized blocks in the shape of the mineralised ore body, as depicted in Figure 1.2. All blocks have the same size, but each block has different attributes, including the coordinates, dimension of individual block, density, element content, rock type, mineral resource classification, etc.

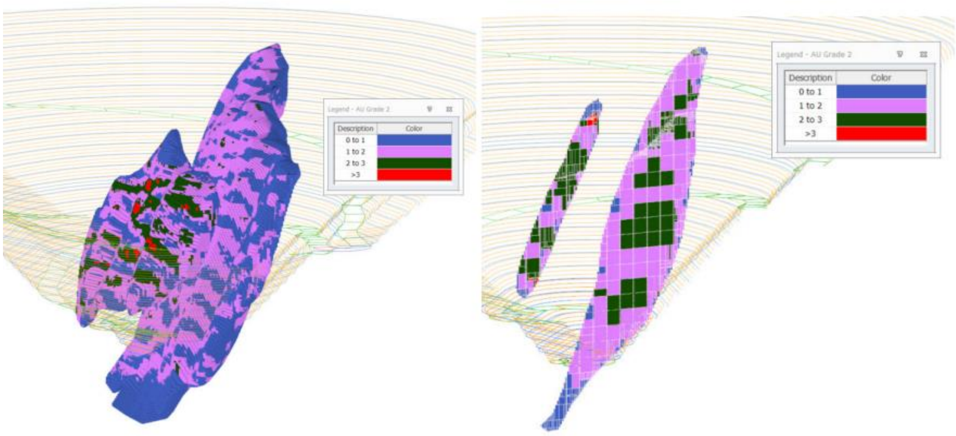

Figure 1.2 A typical block model of ore lode (shell and slice) (Poniewierski and CP 2019)

1.1.2 Block Economic Value (BEV) and Discounted Cash Flow (DCF)

The purpose of open-pit production scheduling is to determine the LOM plan under certain technical and operational constraints to maximise the value of mine. This process involves calculating the net economic value of a single block. According to the estimated economic value of blocks, the optimal solver chooses each block's optimal destination under the given technical constraints to maximise the total pit value. Many researchers have studied the economic value equation of blocks, such as Ataee-pour (Ataee-pour 2005), and Whittle (Whittle and Wooller 1999). The BEV calculation formula is defined as the revenue from selling recovered metal at a specific fixed metal price, minus pit extraction cost, ore processing cost, and other applicable costs. Economic value is then assigned to a single block. The Whittle’s BEV equation is the following (Whittle and Wooller 1999)

$$\mathrm{BEV} = \mathrm{T_oGRP} - \mathrm{T_oC_p} - \mathrm{TC_m} \quad \text{Equation 1}$$

where:

BEV	block economic value, \$,
T_o	ore tonnage of the block,
G	ore grade, unit/tonne,
R	metal recovery rate,
P	unit metal price, \$/unit,
C_p	the unit cost of processing, \$/tonne,
T	rock tonnage of the block,
C_m	the unit cost of mining, \$/tonne.

DCF analysis is used in the process of strategic mine planning (Whittle 2000). It is convenient to consider the pit optimiser as an engine within the planning process that partly answers the first question by finding the pit design that maximises the difference between variable costs and revenues.

In simple cash flow analysis, the question of “whether the project should be proceed or not” is answered in the affirmative if the calculated value is greater than the cost of capital.

DCF analysis adds to this the concept of cash flow discounting. Discounting determines the present value of a future payment or stream of payments, which is performed to remove the capital cost and remove the relevant cost from a particular project. The resultant value of the calculation is the project NPV. This means the project's value is in excess of that is required to pay the cost of capital and compensate for the risk associated with the project. The answer to the second question posed at the beginning of this section is answered if the NPV is positive.

The rules that govern which costs should be included can be stated simply (Whittle 2000):

- All costs which vary according to the amount of waste, ore, or product that is removed, processed, or sold should be included in the pit optimisation model, and any costs which do not so vary should not be included.
- All expenditure that was not included in the pit optimisation model, except for the portion in spending that has already been committed and is irreversible, should be included in the project evaluation calculations.

1.1.3 Ultimate Pit Limit Design

Two principal classical methods are widely used to determine the shape of a surface mine. The first one is the floating cone method (Laurich and Kennedy 1990), which assumes a block as a reference point for expanding the pit upward according to pit slope rules. This upward expansion, which contains all blocks whose removal is necessary for the removal of the reference block's removal, forms a cone whose economic value we can compute. One can then take a second reference block and add to the value of the cone the incremental value associated with the removal of the additional blocks necessary to remove the second reference block; the process then continues. Problems with this method include the following: (1) the final pit design relies on the sequence in which reference blocks are chosen, and (2) many reference blocks might need to be chosen (and the associated value of the cone computed) to achieve a reasonable, although not even necessarily optimal, pit design. Although the floating cone method is used widely in practice, the seminal work of Lerchs and Grossmann (Lerchs 1965), who provide an exact and computationally tractable method for open-pit design. This problem can be cast as an integer program (Hochbaum and Chen 2000), as described below.

$(b, b') \in B$: Set of Precedences between blocks

v_b: value obtained from extracting block b

y_b: 1 if block b is extracted, i.e., if the block is part of the ultimate pit, 0 otherwise (variable).

Equation 2

$$max \sum_b v_b y_b$$

Subject to

$y_b \leq y_{b'} \quad \forall(b, b') \in B$ Equation 3

$0 \leq y_b \leq 1$ Equation 4

1.1.4 Precedence relations

The precedence relationships between blocks geometrically constitute the principal structural constraint geometrically in open-pit mine planning. Subject to the "slope angles", block i cannot be mined before a group of the determined blocks that are "above" block i are removed.

As shown in Figure 1.3, two scenarios of block precedence relationships are depicted. (i): if going to extract block 6, the five blocks above block 6 should be mined out; or, (ii): the nine blocks above block 10 should be mined out prior to extracting block 10. The blocks "above" block 10 include the blocks one level higher and the front, left, right, back, or diagonal with respect to a given block (Espinoza, Goycoolea et al. 2013).

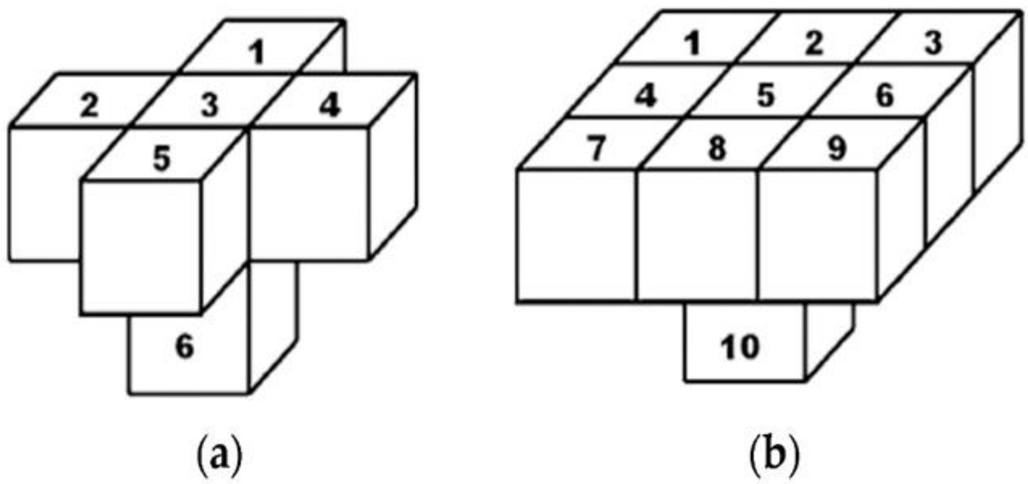

Figure 1.3 two kinds of block precedence relationships. (i): if going to extract block 6, the five blocks above must be extracted; or, (ii): the nine blocks above block 10 must be mined prior to removing block 10.

1.1.5 Strategic mine planning

The mining sequence is determined by strategic mine planning. All blocks within the ultimate pit shell must be mined to maximise the project's NPV with the technical and economic constraints (Newman and Kuchta 2007). The following formulation describes the problem:

Indices and sets:

B: $set\ of\ \ blocks\ b,$

T: $set\ of\ periods\ t,$

$b' \in B_b$: $set\ of\ all\ predecessor\ block\ b'that\ must\ be\ extracted$
$directly\ before\ b,$

Parameters:

C_{bt}: $economic\ value\ obtained\ from\ extracting\ and\ processing$
$\ block\ b\ in\ period\ t,$

m_b: $quantity\ of\ materials\ in\ block\ b,$

M_t^l, M_t^u: $lower\ and\ upper\ limits\ of\ mining\ capacity\ in\ period\ t.$

Decision Variables:

y_{bt}: $1\ if\ block\ b\ is\ to\ be\ extracted;0\ otherwise.$

Objective function:

$$max \sum_{b \in B} \sum_{t \in T} y_{bt} C_{bt}$$ Equation 5

Constraints:

$$\sum_{t \in T} y_{bt} \leq 1 \quad \forall b \in B$$ Equation 6

$$M_t^l \leq \sum_{b \in B} m_b y_{bt} \leq M_t^u \quad \forall t \in T$$ Equation 7

$$y_{bt} \leq \sum_{\tau=1}^{t} y_{b'\tau} \qquad \forall b \in B, \forall b' \in B_b, t \in T$$ Equation 8

$$y_{bt}\ binary \quad \forall\ b \in B, t \in T$$ Equation 9

Constraint (Eq.6) ensure that a block can only be extracted one time. Constraint (Eq.7) limits the number of blocks removed during each period. Constraint (Eq.8) ensures that a precedence constraint is validated.

1.2 Problem statement

A gold deposit features low-grade but large-tonnage, located in Western Australia. The gold project consists of several open-pit mines that form an open-pit mining complex. The open-pit mines were previously mined and are scattered across the leases along the north-south direction. In the past, the mined ore was hauled and treated in a carbon in leach (CIL) plant with the capacity of 3.7 million tonnes per annum (mtpa), about 40 km away from the north of the mine site. It is hard to continue to mine and treat the remanent low-grade gold ore in the CIL plant economically. Considering that heap leaching is a cost-effective technology very suitable for processing low-grade ores (Petersen 2016), the potential for a heap leach project on the mine site is to be evaluated.

There are two main constraints on the mining complex. One is the scattered pits. All mined ores from scattered pits need to be hauled to a heap leach plant for centralised treatment adjacent to Pit 2. Another one is the limited land space. The mining tenement features bell-shaped with narrow side wings where the limited availability of land space is a significant constraint on the layout of waste/ore stockpiles and heap leach facilities. In the initial layout study, the waste dump is located in a large lake district to the south of pits. It significantly increases the haulage cost of waste rock and is detrimental to the heap leach project.

Considering the low content of metal sulfide minerals in the deposit and no underground mining potential, an approach is dumping waste rock to that mined-out pit and mined-out areas of active pits, which could alleviate the tightened land use requirement, but challenge the production scheduling with the synergism problem of mining production and waste rock dump.

Figure 1.4 shows a schematic view of an open-pit mine production scheduling problem with multiple movement destinations of materials. After mining out a designated pit within the mining complex, the mined-out pit can be utilised as an in-pit waste disposal facility and a waste dump on top of the back-filled pit. This option provides a short-haul distance of the waste rock and sufficient storage capacity for waste rock dumping over the project's life of mine.

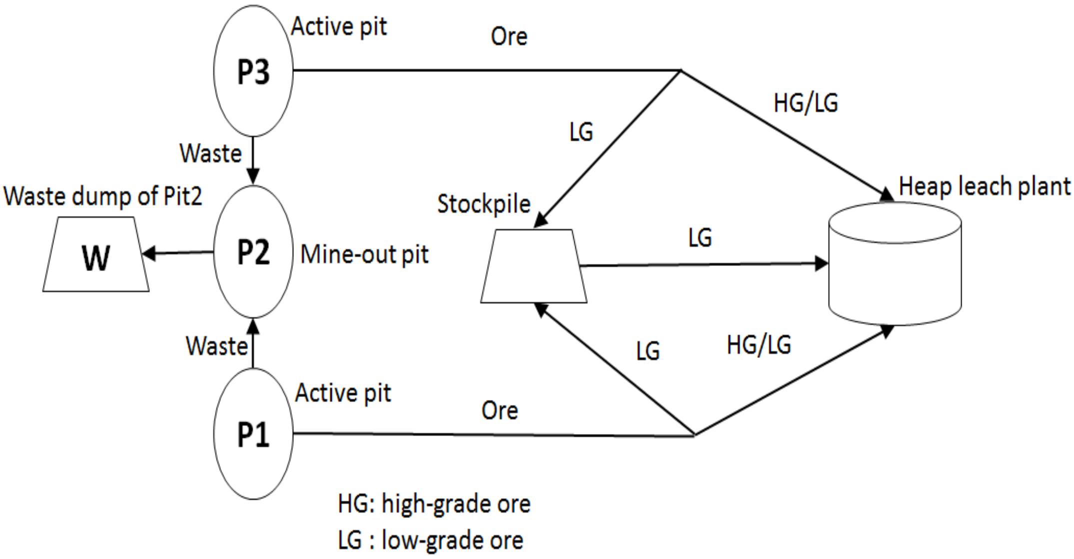

Figure 1.4 In-pit waste disposal problem depiction

1.3 Objectives

This research will develop a Mixed Integer Program (MIP) model to solve production scheduling problems with the constraints related to mining sequence, stockpile, and waste dump, among other constraints. The MIP model will also consider two concerns. Firstly, the mining complex's production scheduling over the LOM will be a large-size problem for MIP formulation, which is unable to be implemented directly. Secondly, the low-grade gold project with considerable capital cost has high investment risks. Fast recovery of cash flow in the early production period is critical to the heap leach project's feasibility assessment. The objectives of the thesis work are the following:

- establish a MIP model with the scheduling problem,
- meet the demands of ore throughput,
- maximise the project’s NPV,
- solve the waste dump constraints

1.4 Research Methodology

(1) Develop the MIP model first.

(2) Create the MIP model in CPLEX.

(3) Validate the proposed model using subsets of data with a small-scale block model.

(4) Implement a model using a real-world dataset.

1.5 Significance

Based on an actual case, the research project gives a new model to solve the mining production schedule for an open-pit mining complex.

The proposed mathematical models will generate a detailed production schedule to achieve different objectives.

The approach uses mathematical programming to solve the particular project's scheduling problem. This demonstrated optimality method is tailored for an individual project, which is more accurate and catering to mine owner's requirements, such as space limitation, gold-producing priority, rather than the scheduling manually by commercial mining software. The process of mathematical modelling is adaptable for adjustments, which will benefit other similar scheduling problems in the mining industry.

1.6 Outline of chapters

The thesis is organised as follows:

Chapter 1 focuses on the introduction of the thesis. The background, the definition of the problem, and research objectives are presented.

Chapter 2 presents a comprehensive literature review on the current MIP technology research. The review identified the current challenge on mining scheduling of multiple open-pit mines and determine the best solution for this study.

Chapter 3 explains the formulations of the developed MIP model. The parameters, variables of the model are defined, and the objective function and constraints are presented. The proposed MIP model is implemented on four sets of datasets of an open-pit mining complex to verify the developed MIP model. The model coding and implementation are operated in IBM ILOG CPLEX Optimization Studio.

Chapter 4 includes implementing the proposed MIP model using the real-world data from an open-pit mining complex. A long-term mine production planning model is applied to the ten-year dataset.

Chapter 5 elaborates the summary, contribution, scope, and limitations of the research are presented that are followed by a conclusion and some suggestions for research in the future.

CHAPTER 2. LITERATURE REVIEW

No commercial software has the function of solving a complicated scheduling problem and generating the optimum production schedule for an open-pit mining complex. Scheduling results are not the optimal solution since humans must choose the mining sequence.

Modeling mathematically is an advanced approach with multiple criteria to find the best scientific solution. Furthermore, since mathematical modeling can be modified to enhance performance, it has been utilized in many fields to study and optimize the systems (Taha 2007).

2.1 Mathematical Modeling

Kallrath (Kallrath 2005)describes a typical process of mathematical modelling. It involves the following steps.

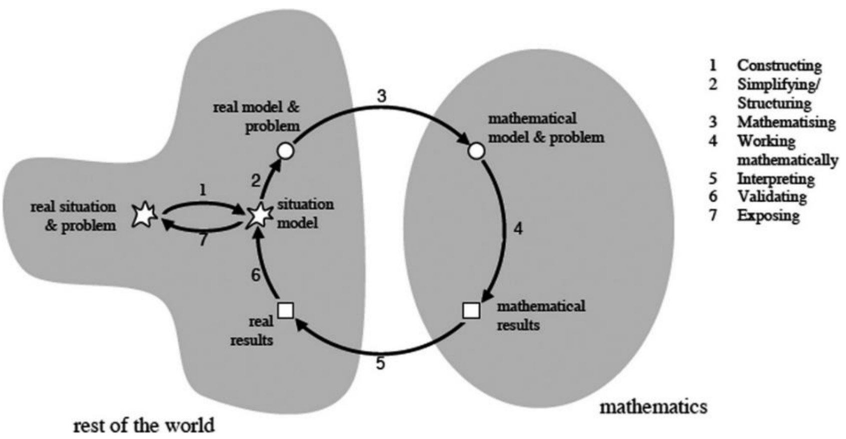

Figure 2.1 Mathematical modelling cycle of Blum & Leiss (Blum and Leiss 2007)

By modelling a real-world issue, a mathematical model is developed.

- Collect inputs for the problem generation;
- Solve the problem and obtain the optimum solution;
- Interpret the solution; and

- Implement the solution to improve the system.

Blum and Leiss (2007) have framed mathematical modelling as a process consisting of subsequent activities. It is usually necessary to iterate the mathematical modelling cycle many times, as shown in Figure 2.1, in order to obtain the optimal representation of problems.

2.2 MIP Model

Linear programming (LP) method is a popular mathematical model for resolving optimization problems. As shown in equations 10 to 12, the generalised LP model is composed of a linear objective function, some linear constraints, and a set of non-negative restrictions (Topal 2003).

Maximise (or minimise) $z = c_1x_1 + c_1x_1 + c_1x_1 \cdots + c_jx_i$

Equation 10

$$\begin{Bmatrix} a_{11}x_1 + a_{12}x_2 & + \cdots & a_{1n}x_i \leq b_1 \\ a_{21}x_1 + a_{22}x_2 & + \cdots & a_{2n}x_i \leq b_2 \\ \vdots \quad\quad \vdots & & \vdots \quad \vdots \\ a_{m1}x_1 + a_{m2}x_2 & + \cdots & a_{mn}x_i \leq b_m \end{Bmatrix}$$

Equation 11

$x_1x_2 \cdots x_i \geq 0$ Equation 12

This objective z (Eq.10) corresponds to the value of interest, which is equal to a function of the decision variables x_i with the corresponding coefficients c. The Z value may stand for the cost or NPV, depending on the formulation. It provides a numerical indicator to compare the solutions. The limiting conditions of the problem are formulated in the constraint sets (Eq.11), and the constant a_{mn} and b_m are derived from the problem. Furthermore, constraint (Eq. 12) restricts the values of x_i.

It is possible to satisfy the constraints (Eq.11) and (Eq.12) of the LP problem with many different solutions. But only one of those sets of solutions can reach the maximum (or minimum) Z value. Depending on whether maximization or minimisation is desired, this solution set is defined as the optimal solution, which is mathematically proven.

A MIP is a form of LP that restricts some variables to integers and others to continuous values. An integer variable can also be of a binary type. MIP specifies various logical conditions in a binary variable, so the mathematical model can solve the problem more accurately by specifying some logical conditions(Li 2014).

2.3 Solution of MIP Model

After the mathematical model has been constructed, a problem needs to be solved in order to determine the optimum approach. In the past, solving simple LP problems with a graphical method (as shown in Figure 2.2) has been accomplished by following the steps:

Step 1: Formulate the LP problem.

Step 2: Draw the constraint lines on the graph.

Step 3: Determine which constraint line is valid for each situation.

Step 4: Determine the feasible region.

Step 5: Create the objective function on the graph.

Step 6: Determine the optimum point.

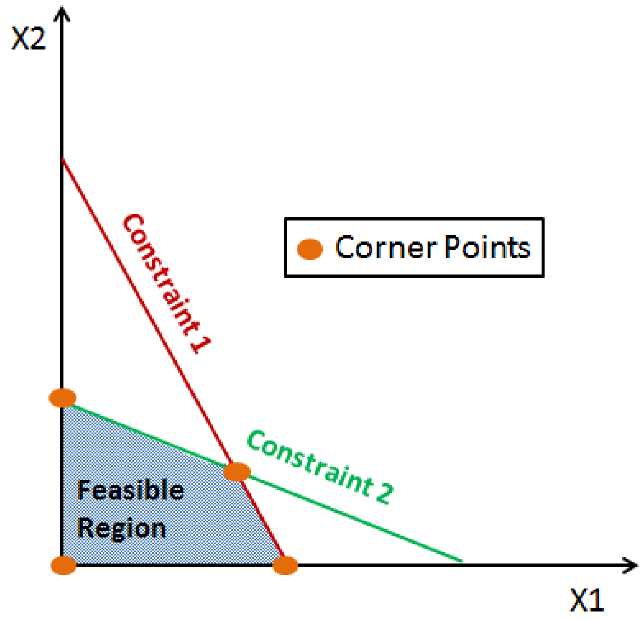

Figure 2.2 Illustration of the graphical method concept

The linear constraints outline the feasible region, where any points (X1, X2) within this region satisfy the condition. The Objective function Z is graphed and the value is calculated. An optimum solution set (X1, X2) can be determined when Z value reaches the maximum or minimum, depending on the optimisation nature of the problem. However, the graphical method becomes impractical when solving problems with many variables and constraints. In the late 1940's, the simplex algorithm was developed by Dantzig for solving more complicated linear programming problems (Fourer, Gay et al. 1990). This method provides a standard approach to solve any linear programming problems. It first converts a problem into standard form. Then the problem is reconstructed to a table form. The derivation of the optimum solution is via a series of row operations on the table. The detailed solving steps are discussed by Taha (2007). With the advancement of computing technology, computerised row operation enables faster and accurate results generation. However, problems involving binary variables cannot be solved by using the simplex method. It requires a branch and bound technique to divide the problem into sub-problems before solving. The branch and bound algorithm first solve the problem using LP relaxation method. The optimal solution is obtained if the solution contains the correct integer value for the integer variable. Otherwise, an integer value will be assigned to either side of the non-integer value to create two new subproblems, which are then solved by standard LP solution procedures. This process continues until all branches for a given 'level' have been evaluated and no better objective value can be calculated. Figure 2-11 illustrates the fundamental of the branch and bound technique.

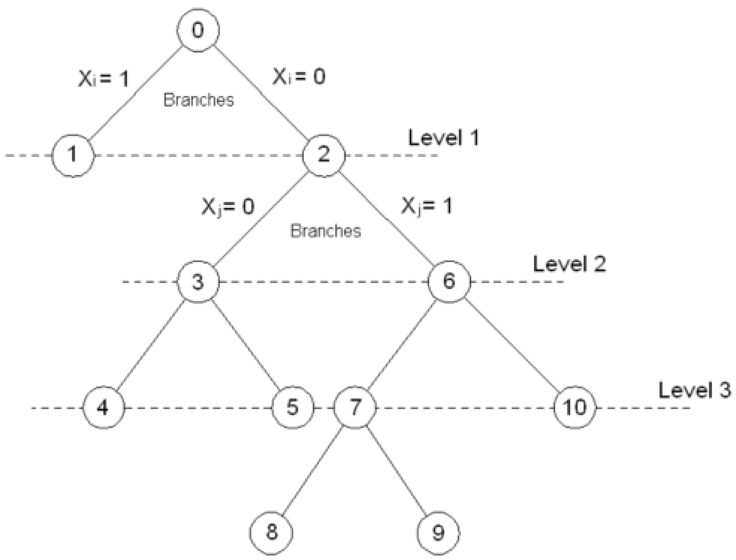

Figure 2.3 Illustration of branch and bond tree (Kianfar 2011)

Modern optimisation engines, such as IBM ILOG CPLEX, have incorporated these algorithms to solve large-scale optimisation problems.

MIP is generally classified into mixed integer linear programming (MILP) and mixed integer nonlinear programming (MINLP), which are recognized in the mathematical sciences community for modelling and finding optimal solutions for large, complex, and highly constrained problems. MILP problems use linear objective functions, which are constrained by linear constraints, to perform minimisation and maximisation of the problem.

MIP has been used in mine planning for cut-off grade optimization, equipment allocation, ore blending, stockpiles, and process stream selection. Most of the studies on mine plans involved selecting blocks to maximize NPV values.

To maximize the project NPV, the scheduling approach prioritizes processing the highest value ore available in the early periods of mining, subject to multiple considerations, such as mill throughput, mining capacity, rock type, ore properties, and waste management.

2.4 MIP models application in the mining industry

MIP technique has been implemented into mining industry for more than 60 years. Many studies are available to provide MIP application and other operations research techniques to optimize various aspects of both open-pit and underground mining operations (Newman and Kuchta 2007, Dimitrakopoulos and Ramazan 2008, Epstein, Goic et al. 2012). Most of the researches on mine scheduling consists in selecting blocks to maximize NPV value.

Mine production scheduling involves the sequence and timing of ore and waste movement during the life of a mine or a complex. It determines the destinations where the block is moving to, such as processing plant, ore stockpile, waste dump, mined-out area of the active pit, and mined-out pit. Various mathematical formulations have been developed to solve the mine scheduling problems.

Hoerger (Hoerger 1999) establish a MIP model to solve the scheduling problem of multiple pits' simultaneous mining and ore delivery to multiple plants. The model group blocks into increments and accounts for multiple stockpiles. The model is successfully implemented at Newmont's Nevada operations, where fifty sources, sixty destinations, and eight stockpiles are present. The given solution will increase the NPV of operations if verified.

Caccetta and Hill (Caccetta and Hill 2003) developed a MIP model with an objective to maximise the Project NPV over the sequenced blocks, which added constraints including extraction sequence, mining, milling, refining capacities, grades of the mill and concentrate, stockpiles, and operational conditions such as pit bottom width and depth limit.

Stone et al. (Stone, Froyland et al. 2018) present the Blasor optimization tool, which addresses using solver ILOG CPLEX to determine the best extraction sequence for multiple pits as MIP.

Wooller (Wooller 2007) introduced Comet software that uses an iterative algorithm to define operational strategies and process routes, such as heap leaching versus concentration, to optimize the plant's yield/recovery rate and cut-off grade.

Zuckerberg (Zuckerberg, Stone et al. 2007) optimized the extraction sequence of bauxite "pods" from the Boddington bauxite mine in south-west Australia. The pod is a distinctive body of medium sized ore located near the surface.

Chanda (Chanda 2007) formulates the delivery of materials from different deposits to metallurgical plants as a network linear programming optimization problem. The model uses a network that includes mines, concentrators, smelters, refineries, and market areas to minimize the costs.

It is suggested that the scale of practical problems makes it difficult to use the integer programming model for mine production. Therefore, it adds heuristics and aggregation techniques to reduce the problem's size. This approach aims to use aggregation techniques to get a suboptimal solution to reduce the number of variables and constraints.

Ramazan (Ramazan 2007) uses a Fundamental Tree algorithm (FT) based on linear programming. This method could lead to aggregate material blocks and reduce integer variables and constraints to form mixed integer programming formulas.

Badiozamani et al. (Badiozamani and Askari-Nasab 2016) used the MIP model to solve the scheduling problems of oil sands mining sequence and tailings pulp management. In this project, two techniques are constructed, and the problem's size was reduced. In addition, this also makes the real cases more valuable and practical.

Ramazan (Ramazan, Dagdelen et al. 2005) regards that simultaneous optimization is a practical and well-developed and suitable way to optimize the mining complex because it can perform global optimization under all constraints.

The application of MIP is regarded as a solution that can solve the production scheduling problem of open-pit mines, especially in large open-pit mines with many blocks, which requires too many variables to deal with the problem of time arrangement problem. Currently, the only common practice is reconstructing the mining blocks before scheduling (Figure 2.2), which creates an aggregate that is a subset of blocks on the same bench and in the same grade group. It is customarily postulated that all blocks' properties should be identical in an aggregate. Thus, the same aggregate blocks will be sent to the same destination at the identical mining productivity (Smith and Wicks 2014, Van-Dunem 2016). In this approach, the optimization problem size obviously reduces, which can significantly save the computing time for solving optimization problems.

S. Ramazan (Ramazan, Dagdelen et al. 2005) proposed a MIP method to solve a multi-element large open-pit's production scheduling problem.

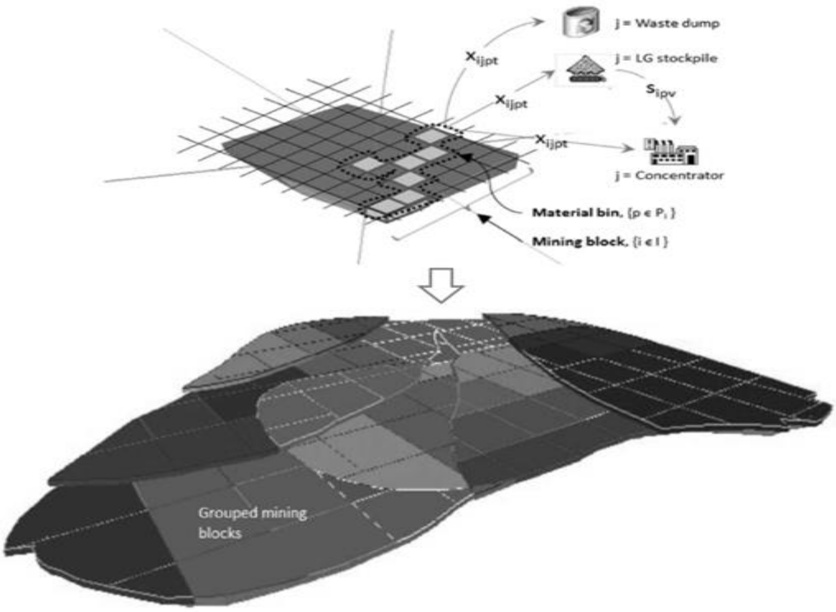

Figure 2.4 A many-to-one relationship between the resource blocks and the grouped blocks used for production planning (Martin L. Smith, 2014)

Most of the previous practices and studies are studying mining scheduling optimization under the in-pit dumping of waste rock. The study of in-pit dumping was also addressed by Zuckerberg (Zuckerberg, Stone et al. 2007) and Adrien (2018). However, there is a number of mine operations that consist of multiple mining operations, but little research has been done to establish a MIP model for solving the mining scheduling problem of multiple mines. The most widely used method for multiple mines with complicated constraints is utilise the commercial software as well as Excel by assigning specific constraints. However, this manual method relies heavily on the people experience and skills, and more likely generates different schedules if conducted by different operators.

2.5 Summary

Extensive studies and practices have demonstrated the significance of MIP technology on solving specific scheduling problems with various constraints. Therefore, MIP technology is necessary to solve open-pit mining scheduling problems under an in-pit dumping waste rock. However, for those multiple open-pit mines, there is currently no MIP model that can be used to determine an optimal mining schedule while considering all the significant constraints. This research requires the development of a new MIP model.

CHAPTER 3. MATHEMATICAL MODEL FORMULATION

This section mainly includes MIP formulation for mine production scheduling of an open-pit mining complex in detail. Firstly, the assumptions, sets, parameters, and decision variables are explained. The proposed operation's NPV maximisation objective function in association with various constraints is presented.

3.1 MODEL FORMULATION

Generally, the standard procedures to build a model includes indices, sets, parameters, variables, objective function and constraints.

3.1.1 Assumptions

- Ultimate pit shell is available.
- Key infrastructure layout: the heap leach plant, haul roads, stockpile, and waste dump are known in advance.
- The operating hours are already confirmed in consideration of planned downtime, blasting, weather delays, and availability and utilization rate of the processing plant.
- The selected mining fleet and the equipment of the heap leach plant are kept over the life of mine without replacement or salvage. All sustaining capital costs related to maintenance are considered.
- Capital cost and operating cost of heap leach project are available.

3.1.2 Sets

b set of mining blocks, $b = \{1 \cdots B\}$;

μ_b set of mining blocks overlying block b;

m set of mines, $m = \{1 \cdots M\}$;

p set of processes in mining complex, $p = \{1 \cdots P\}$;

s set of stockpile bins in stockpile, $s = \{1 \cdots S\}$;

w set of waste dumps in mining complex, $w = \{1 \cdots W\}$;

t set of periods or years, $t = \{1 \cdots T\}$;

3.1.3 Parameters

$r =$ metal price in A$ per unit of metal;

$n =$ selling cost of metal in A$ per unit of metal;

$a_m =$ mining cost at mine m in A$ per tonne of material mined;

$c_p =$ processing cost at processing stream p in A$ per tonne of ore processed;

$h =$ stockpile rehandling cost in A$ per tonne of ore rehandled;

$u =$ metallurgical recovery in %;

$i =$ discount rate in %;

$J_t =$ mining capacity in tonnes during period t;

$K_t =$ processing capacity in tonnes of ore to be processed during period t;

$L_t =$ stockpile capacity in tonnes of ore to be stockpiled at the end of period t;

$g_{bm} =$ gold grade of block b in gram per tonne at mine m;

$q_{bm} =$ quantity (tonnes) of material in block b at mine m;

$g_s =$ average grade of material in stockpile bin s;

$V_{bmpt} =$ discounted value of a block b from mine m to process p during period t, $V_{bmpt} = \frac{[(r-n)g_{bm}u-a_m-c_p]q_{bm}}{(1+i)^t}$;

$V_{bmst} =$ discounted value of a block b from mine m to stockpile bin s during period t, $V_{bmst} = \frac{[-a_m]q_{bm}}{(1+i)^t}$;

$V_{bmwt} =$ discounted value of a block b from mine m to waste dump w during period t, $V_{bmwt} = \frac{[-a_m]q_{bm}}{(1+i)^t}$;

$V_{spt} =$ discounted value per tonne of ore from stockpile bin s to process p during period t, $V_{spt} = \frac{[(r-n)g_s u-h-c_p]}{(1+i)^t}$;

3.1.4 Decision Variables

$$X_{bmpt} = \begin{cases} 1, \\ 0, \text{if block b at mine m is mined and sent to process p during period t} \end{cases}$$

$Y_{bmst} = \begin{cases} 1, \\ 0, \text{if block b at mine m is mined and sent to stockpile s during period t} \end{cases}$

$Z_{bmpt} = \begin{cases} 1, \\ 0, \text{if block b at mine m is mined and sent to weste dump s during period t} \end{cases}$

$E_{spt} =$ quantity (tonnes) of ore movement from stockpile s to process p during period t, $E_{spt} \geq 0$

$\underline{E}_{st} =$ quantity (tonnes) of ore in stockpile s at the end of period t, $\underline{E}_{st} \geq 0$;

3.1.5 Objective Function

The objective function for maximizing the NPV is the following.

Max NPV =

$$\sum_{m=1}^{M}\sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{p=1}^{P} \boldsymbol{V_{bmpt}X_{bmpt}} + \sum_{m=1}^{M}\sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{s=1}^{P} \boldsymbol{V_{bmst}Y_{bmst}} + \sum_{m=1}^{M}\sum_{b=1}^{B}\sum_{t=1}^{T}\sum_{w=1}^{W} \boldsymbol{V_{bmwt}Z_{bmwt}} + \sum_{t=1}^{T}\sum_{p=1}^{P}\sum_{s=1}^{S} \boldsymbol{V_{spt}E_{spt}}$$

Equation 13

3.1.6 Constraints

The formulation includes the following constraints.

Reserve constraints

The reserve constraints ensure that all blocks are extracted only once.

$$\sum_{t=1}^{T}\left[\sum_{p=1}^{P} \boldsymbol{X_{bmpt}} + \sum_{s=1}^{S} \boldsymbol{Y_{bmst}} + \sum_{w=1}^{W} \boldsymbol{Z_{bmwt}}\right] \leq \mathbf{1};\ \forall \boldsymbol{mb}$$

Equation 14

Mining block extraction precedence constraints

Prior to extract block b, the immediate predecessor blocks must be extracted.

$$\left[\sum_{p=1}^{P} X_{bmpt} + \sum_{s=1}^{S} Y_{bmst} + \sum_{w=1}^{W} Z_{bmwt}\right] - \left[\sum_{\tau=1}^{t}\left[\sum_{p=1}^{P} X_{\acute{b}mpt} + \sum_{s=1}^{S} Y_{\acute{b}mst} + \sum_{w=1}^{W} Z_{\acute{b}mwt}\right]\right] \leq 0;\ \forall mbt, \acute{b} \in \mu_b$$

Equation 15

Mining capacity constraints

There is extraction capacity upper limit in period t.

$$\sum_{b=1}^{B}\left[\sum_{p=1}^{P} q_{bm}X_{bmpt} + \sum_{s=1}^{S} q_{bm}Y_{bmst} + \sum_{w=1}^{W} q_{bm}Z_{bmwt}\right] \leq J_t;\ \forall mt$$

Equation 16

Processing capacity constraints

The heap leach plant's ore processing capacity in period t is constrained.

$$\sum_{m=1}^{M}\sum_{b=1}^{B} q_{bm}X_{bmpt} + \sum_{s=1}^{S} E_{spt} \leq K_t;\ \forall pt$$

Equation 17

Stockpile constraints

Equation 18 and Equation19 show the balancing equation for stockpiles' inventory through t periods. Inventory of stockpiles at the end of period t is equal to the calculated result from the amount of incoming materials at period t plus the amount of inventory at the end of t-1 minus the amount of outbound at period t. The upper limit of stockpile storage capacity is constrained in period t (Eq.20).

$$\underline{E}_{s(t-1)} + \sum_{m=1}^{M}\sum_{b=1}^{B} X_{bmst} - \sum_{p=1}^{P} E_{spt} - \underline{E}_{st} = 0; \forall s, t \geq 2$$ Equation 18

$$\sum_{m=1}^{M}\sum_{b=1}^{B} q_{bm}X_{bmst} - \sum_{p=1}^{P} E_{spt} - \underline{E}_{st} = 0;\ \forall s, t = 1$$ Equation 19

$$\sum_{s=1}^{S} \underline{E}_{st} \leq L_t;\ \forall t$$ Equation 20

3.2 MIP Model Verification

The proposed MIP model is programmed in the OPL code of IBM ILOG CPLEX Optimization Studio (Version 12.10). The optimality gap is set to 0.01%. To verify the MI

P model, the model is tested with four datasets.

3.2.1 Input Data Set

A mining complex including three open-pit mines is employed for this verification. In datasets, 600, 1,500, 3,000, and 4,500 mining blocks in total are assumed to be mined. All mines of each dataset have the same number of blocks. Each mining block is initially assigned with attributes, such as block coordinates, ore grade, density, etc. Each block is also assigned with a block ID, making each block unique and validating block movement destination in the schedule.

3.2.2 Problem Size and Solution Time

The optimum solution is determined by CPLEX optimizer, which runs on a computer with 256 GB NVM.2 hard drive, 8×2.9 GHz processors, and 48GB of RAM, operating under the Windows 10 environment.

Table 3.1 Problem sizes and solving time

	No. of blocks	Periods(a)	Constraints	Binary variables	CPU time(s)
Dataset-1	600	6	37,038	21,600	9
Dataset-2	1,500	6	93,222	54,000	30
Dataset-3	3,000	6	219,216	108,000	68
Dataset-4	4,500	6	369,198	162,000	8,100

As shown in table 1, as the number of blocks increases from 600 to 4,500, the problems will be amplified and the computing time soar from 9 second to 8,100 seconds.

3.2.3 Implementation Results

With the implementation of four sets of data, the verification results reflect that the optimum solutions satisfy the given constraints. It is ensured that the designated mine is completed first. Secondly, the mining block's extraction is restricted with the mining block precedence under the slope angle constraint. Those constraints, such as the mining capacity, plant capacity, and stockpile storage capacity, have been met. The highest-grade ore is processed in the early stages to the greatest extent.

Table 3.2 Results of the material movement schedule with dataset-1

Mine	Period	NO. of blocks mined	NO. of mined blocks sent to stockpile	NO. of mined blocks sent to mill	NO. of mined blocks sent to waste dump
Pit1	t=1	0	0	0	0
	t=2	1	0	1	0
	t=3	55	10	37	8
	t=4	27	7	16	4
	t=5	29	20	0	9
	t=6	88	14	0	74
Pit2	t=1	100	11	45	44
	t=2	100	35	14	51
	t=3	0	0	0	0
	t=4	0	0	0	0
	t=5	0	0	0	0
	t=6	0	0	0	0
Pit3	t=1	0	0	0	0
	t=2	1	0	1	0
	t=3	32	2	9	21
	t=4	76	11	9	56
	t=5	75	16	0	59
	t=6	16	8	0	8

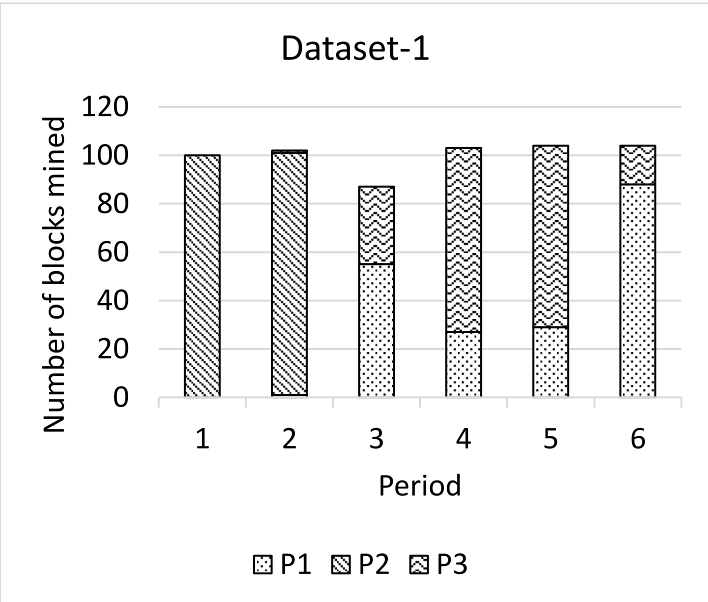

Figure 3.1 Mining sequence of Dataset-1

Table 3.3 Results of the material movement schedule with dataset-2

Mine	Period	NO. of blocks mined	NO. of mined blocks sent to stockpile	NO. of mined blocks sent to mill	NO. of mined blocks sent to waste dump
Pit1	t=1	0	0	0	0
	t=2	0	0	0	0
	t=3	103	0	12	91
	t=4	143	0	19	124
	t=5	125	0	55	70
	t=6	129	1	2	126
Pit2	t=1	250	8	74	168
	t=2	250	18	74	158
	t=3	0	0	0	0
	t=4	0	0	0	0
	t=5	0	0	0	0
	t=6	0	0	0	0
Pit3	t=1	0	0	0	0
	t=2	1	0	1	0
	t=3	135	1	62	72
	t=4	110	9	48	53
	t=5	131	7	15	109
	t=6	123	12	1	110

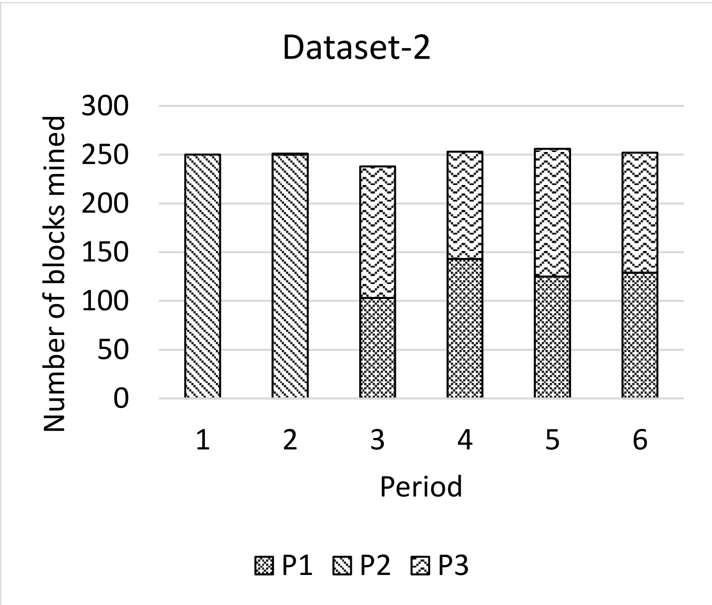

Figure 3.2 Mining sequence of Dataset-2

Table 3.4 Results of the material movement schedule with dataset-3

Mine	Period	NO. of blocks mined	NO. of mined blocks sent to stockpile	NO. of mined blocks sent to mill	NO. of mined blocks sent to waste dump
Pit1	t=1	0	0	0	0
	t=2	0	0	0	0
	t=3	82	0	30	52
	t=4	273	0	21	252
	t=5	281	0	17	264
	t=6	364	0	78	286
Pit2	t=1	513	12	142	359
	t=2	487	1	139	347
	t=3	0	0	0	0
	t=4	0	0	0	0
	t=5	0	0	0	0
	t=6	0	0	0	0
Pit3	t=1	0	0	0	0
	t=2	0	0	0	0
	t=3	289	1	116	172
	t=4	269	0	123	146
	t=5	255	1	127	127
	t=6	187	0	59	128

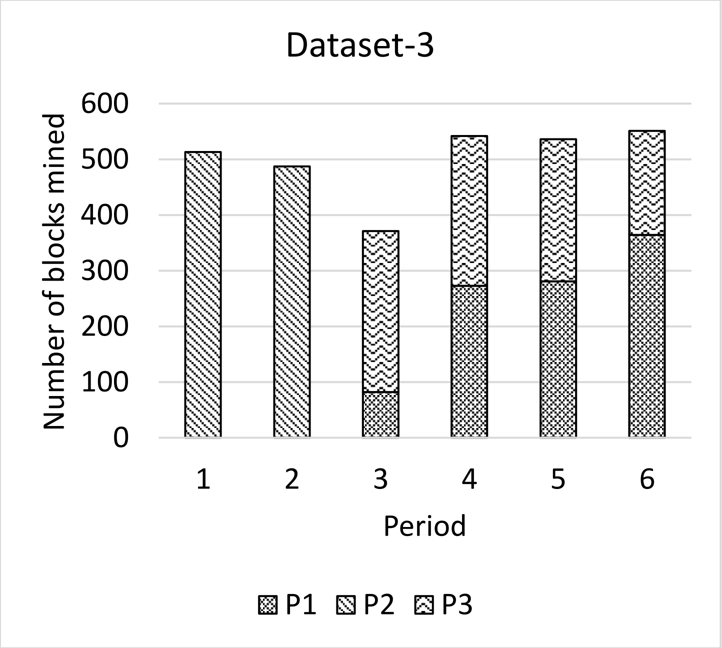

Figure 3.3 Mining sequence of Dataset-3

Table 3.5 Results of the material movement schedule with dataset-4

Mine	Period	NO. of blocks mined	NO. of mined blocks sent to stockpile	NO. of mined blocks sent to mill	NO. of mined blocks sent to waste dump
Pit1	t=1	0	0	0	0
	t=2	0	0	0	0
	t=3	458	0	76	382
	t=4	471	0	44	427
	t=5	493	2	59	432
	t=6	78	5	66	7
Pit2	t=1	735	11	171	553
	t=2	765	11	165	589
	t=3	0	0	0	0
	t=4	0	0	0	0
	t=5	0	0	0	0
	t=6	0	0	0	0
Pit3	t=1	0	0	0	0
	t=2	0	0	0	0
	t=3	179	0	102	77
	t=4	447	4	121	322
	t=5	396	2	106	288
	t=6	478	3	22	453

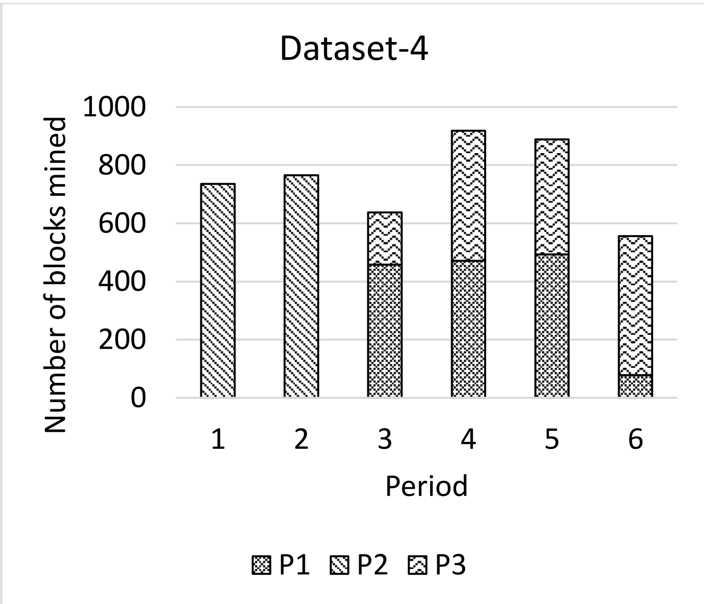

Figure 3.4 Mining sequence of Dataset-4

In summary, the mining scheduling model is developed to solve the waste rock dumping problems and maximize the project's NPV. All mining blocks and pits are extracted by following the given constraints. The proposed MIP model generates the reliable and valid solution, which is a useful function for creating the production plan of the specific mining complex.

CHAPTER 4. MIP MODEL IMPLEMENTATION

This section demonstrates the real-world implementation of the developed MIP models. It includes an introduction to the mining project, MIP problem solving, results in generation and analysis and mining sequence scenarios.

4.1 Background

The project, located 10 km south-west of Kalgoorlie, Western Australia, is divided into two parts, the north and south. Some infrastructures such as highway, railway, and portable water pipeline are sitting in the middle. This study case is focusing on the southern part of the project.
The open-pit mining complex includes several independent pits which were previously mined and are scattered across the leases along the north-south direction. The estimated resource within the leases is 2.7 Moz of gold in total including Pit 1 (26.7mt @ 0.84g/t Au for 614 Koz), Pit 2 (16.2mt @ 0.95g/t Au for 493 Koz), and Pit 3 (26.8mt @ 0.96g/t Au for 826koz). In the past, the mined ore was hauled and treated in a carbon in leach (CIL) plant with the capacity of 3.7 million tonnes per annum (mtpa), approx. 40 km away from the mine site. It is hard to continue to mine and treat the remanent low-grade resource in the CIL plant economically. Considering that heap leach technology has a cost advantage in treating low-grade ores, the potential to construct a heap leach plant on the mine site has been evaluated technically and economically, and a leach plant with a throughput of 6.5 million tonne per year will be built.

There are two main constraints on the open-pit mining complex. One is the scattered pits. All mined ores from scattered pits need to be hauled to a heap leach plant for centralized treatment adjacent to Pit 1, and minimizing the haulage distance of waste rock is a challenge to the mine design. Another one is the limited land space. The mining tenement features bell-shaped with narrow side wings where the limited availability of land space is a significant constraint on the layout of waste/ore stockpiles and heap leach facilities. In the initial layout study, the waste dump is in a large lake district to the south of pits, which is hard to be approved by the government. It significantly increases the haulage cost of waste rock and detrimental to the heap leach project.

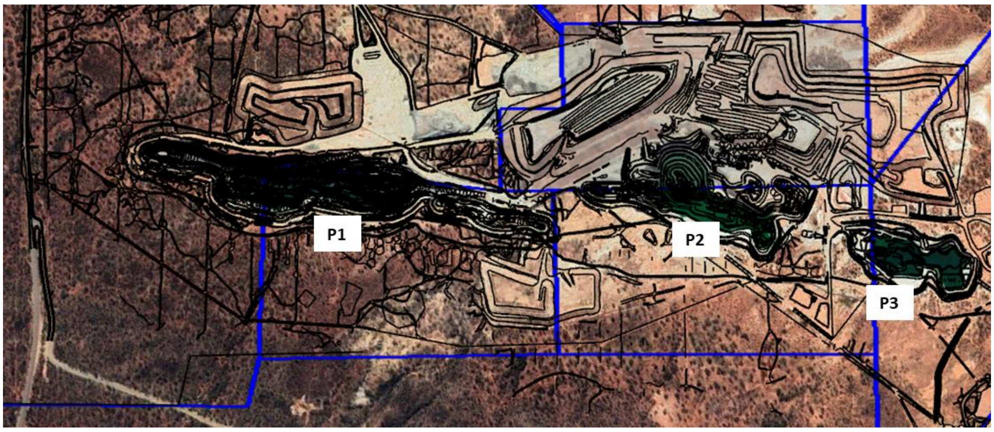

Figure 4.1 Mine site topography as mined

Considering the Non-Acid Forming (NAF) deposits of this project and no underground potential, an proposed approach is dumping waste rock to those mined-out pits, minimizing the waste haulage cost and alleviating the tightly land use requirement but challenge the production scheduling with the synergism problem of mining production and waste rock dump.

4.2 Geology

The prospect of this project is considered to be primarily composed of an epiclastic sedimentary sequence and a suite of felsic porphyritic intrusions that occur along a Fault. The regional metamorphic grade is lower to mid-greenschist facies based on petrographic studies. Historically, the epiclastic sediments are considered to be conformable with the overlying conglomerates of the Kurrawang Syncline.

Porphyritic intrusions can be distinguished into two groups based on alteration; hematite altered, and sericite altered. The sericitic porphyry is compositionally uniform, incorporating blocky white feldspar phenocrysts in a glassy, sericitic pale green altered, groundmass. Differentiation of the haematitic porphyry is based on the altered groundmass, which is dark red. The feldspars, similar to the sericitic porphyry, are blocky and white and the groundmass remains glassy. The relationship between the groups has not been established. Numerous porphyritic conglomerates or breccia aprons are present throughout the tenement package, comprised of rounded clasts of porphyry, ranging in size from centimetre scale to 0.5m in diameter. These flows are believed to have formed by the over steeping of dome structures and are observed to have an agglomeratic texture with a very fine grained glassy matrix. Occasionally within the matrix are euhedral white feldspar phenocrysts, as well as clasts of fine grained sandstones and siltstones. The silt-mudstones are commonly laminated, compositionally uniform with a well developed regional foliation. Inter-bedded silt and mud layers within the sandstones occur commonly throughout the sedimentary sequence. The dominant unit in this project area is a sequence of sandstones (arenites) of varying grain size separated by siltstones. Typically the sequence is graded, from fine grained to very coarse grained with well developed bedding and cross-bedding. Soft sediment structures are observed throughout, involving mainly dewatering flame structures and impacted pebbles. The sandstone is generally dominated by rounded quartz with minor amounts of feldspar and rock fragments. Pebble lags occur occasionally within the series, although these intervals are discontinuous. The thick to massive bedded sandstone is characterized by a lack of well developed sedimentary structures, a coarse to very coarse grain size, and an immature and sub-arkosic composition.

4.3 Mine Design

The project concept involves using the existing Hitachi EX3600 excavator and Cat 789 trucks to mine waste pre-strip, then joined by Hitachi EX2600 excavator and Cat 777 truck fleet to mine ore and waste.

The case study will be based on the given mining physicals. The mine design reflects experience at the previously mined pits. A geotechnical assessment was conducted for pit design purposes, but ongoing geotechnical work will be required during operations. The proposed pits will be up to 260 m deep at the Southern end, 4.5 km long, and 450 m wide.

The overall slope varies from 42 degrees in the south end to 53 degrees in the north. Batter angles vary from 40 degrees in oxide to 70 degrees in fresh rock. Ore will generally be mined in 10 m benches. Figure 4.1 shows the ultimate pit shells and the blocks with the final pits.

As the pit will be deepened to approximately 180m depth, sumps and pit dewatering equipment will be required to dewater the pit. From the pit sumps water will be nominally discharged to Pit 2. The pit dewatering plumbing system will also deliver clear water to the Stormwater collection ponds at the Heap Leach pad site where it can be used as make up water. As shown in Figure 4.3 and Figure 4.4, additional or expanded waste dump designs are required to accommodate approximately 149 million tonnes of excavated waste during the ten year operation. Backfilling of Pit 2 allows the existing waste dumps to be expanded.

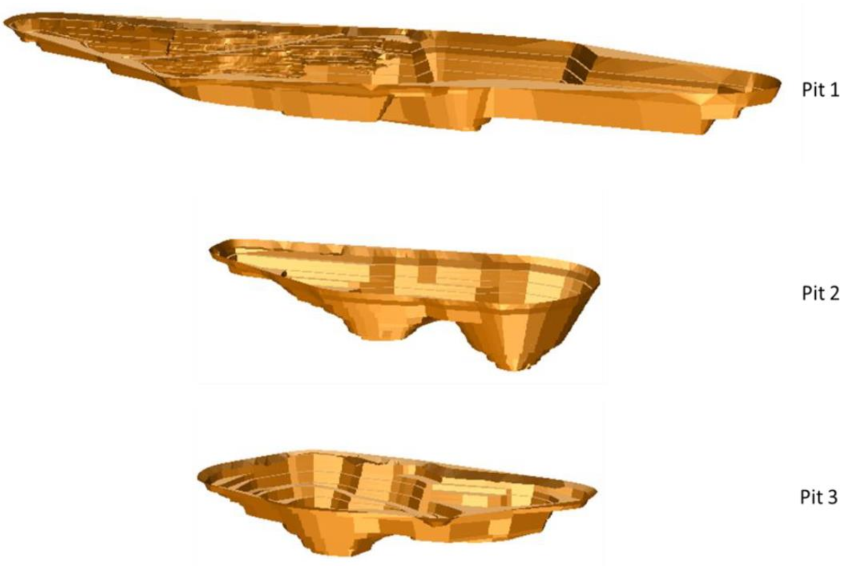

Figure 4.2 Ultimate pit design

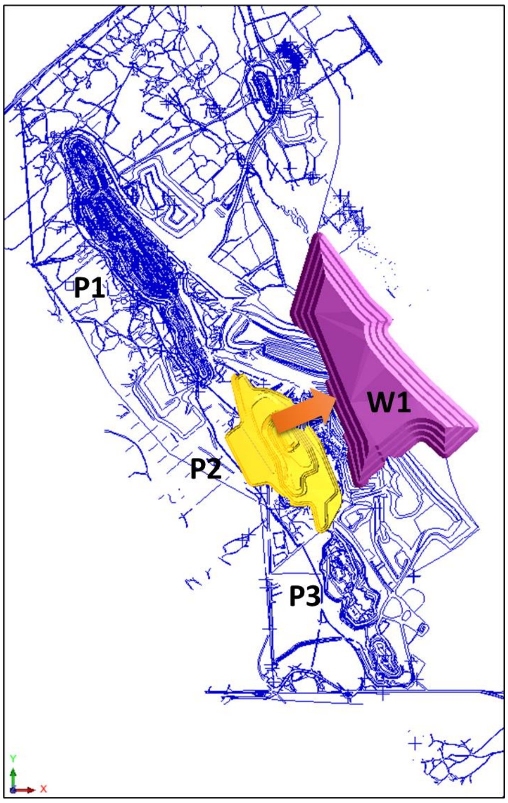

Figure 4.3 Waste rock dumping strategy: waste dump W1 for P2 extracting

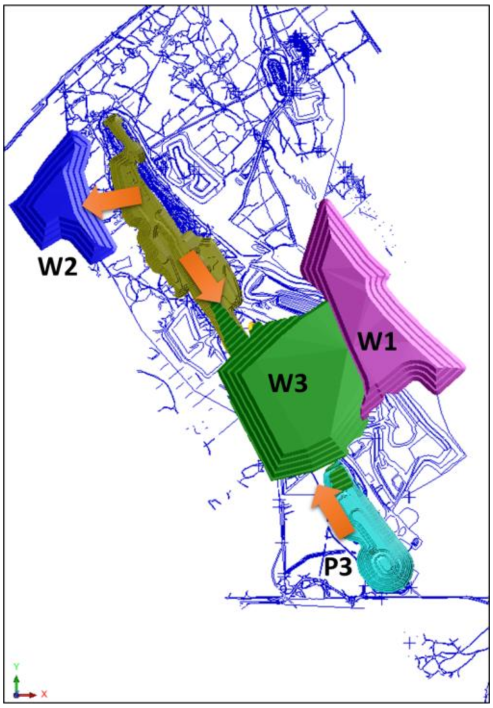

Figure 4.4 Waste rock dumping strategy: waste dump w3 designed for P1 and P3 extracting, and waste dump W2 for the excavation of P1's north end.

The total mining physicals are categorized in two classes of materials based on the following cut-off grade parameters:

- Ore: Au Grade ≥ 0.30 g/t.
- Waste: Au Grade < 0.30 g/t.

4.4 Heap Leach Plant

The heap plant circuit with a throughput of 6.5 mtpa is a two-stage crushing circuit followed by a High-Pressure Grinding Roll (HPGR) to feed an agglomeration drum. The agglomerated ore is transported to the stacking system. Following stacking, the material is irrigated with a dilute sodium cyanide barren leach solution, and the resulting gold-bearing solution is collected into the pregnant solution pond. The leach solution is processed by carbon-in-column (CIC) adsorption, elution, and electro-winning to produce gold doré.

4.5 Input Data

The information provided by the company for the realization of this thesis is presented as follows:

blocks contained on ultimate pits	5,810
block dimension x axis	40 m
block dimension y axis	40 m
block dimension z axis	10 m
density	as attribute
gold grade	as attribute
interest rate	6%
gold price	2,200 A$/oz
processing recovery	74%
average mining cost of ore	3.4 A$/t
average mining cost of waste	3.2 A$/t
rehandling of stockpiled ore cost	0.6 A$/t
processing cost	10 A$/t
selling cost	52.5 A$/oz

4.6 Current Practice of Mining Scheduling

A manual schedule was conducted based the given dataset. Geovia Whittle mine planning software is used to schedule the open-pit mining production of the mines. Regarding with the mining physicals generated by Whittle, it has been partially adjusted in order to meet the production

4.7 Results and Analysis

4.7.1 Overview

Table 4.1 shows the MIP model implementation results of the project. The results indicate that a reasonable NPV is estimated comparing to the original project study based on the existing method in mining software. The computing time is acceptable to the actual use.

Table 4.1 Summary of results

No. of blocks	Periods(a)	Constraints	Binary variables	CPU time(s)	NPV
5,810	10	1,451,990	360,000	18,737	A$ 211.1 million

4.7.2 Results

As depicted in Figure 4.5, the pre-production stripping is done in year one with low stripping volume because all open-pit mines are previous-mined. The stripping ratio from year one to year three is continuously increased. Between year 4 and year 8, there is a relatively stable mining production with a strip ratio of 2.6 tonnes of waste per tonne ore.

Figure 4.6 shows the mine’s current strategy. Mine extraction of Pit 1 and Pit 3 were launched after mine Pit 2 closed in year 3. Since from year 4, the waste rock will be dumped to mine-out Pit 2. The ore mined is relatively sufficient to meet the requirements of mill load, which facilitates the balanced production with the stockpile.

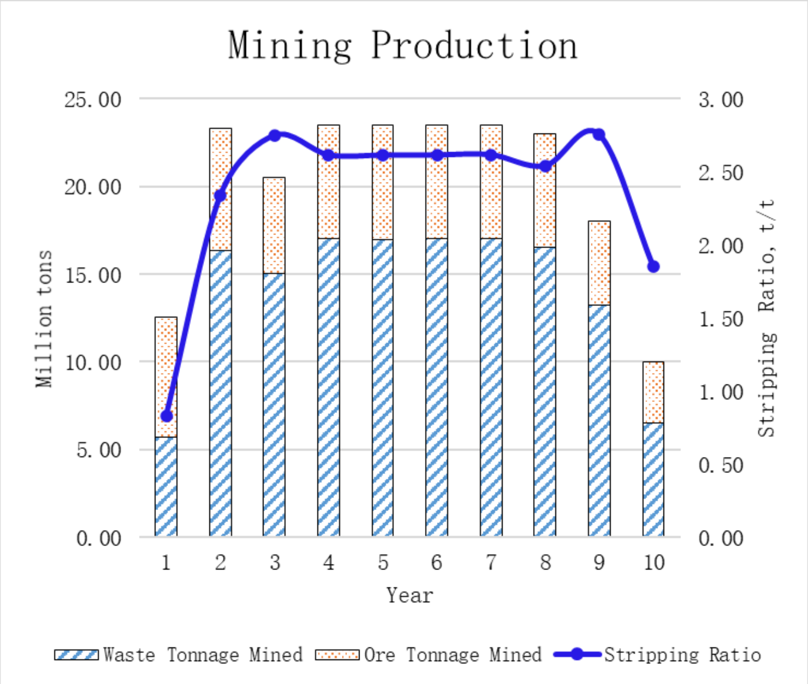

Figure 4.5 Mining production per period

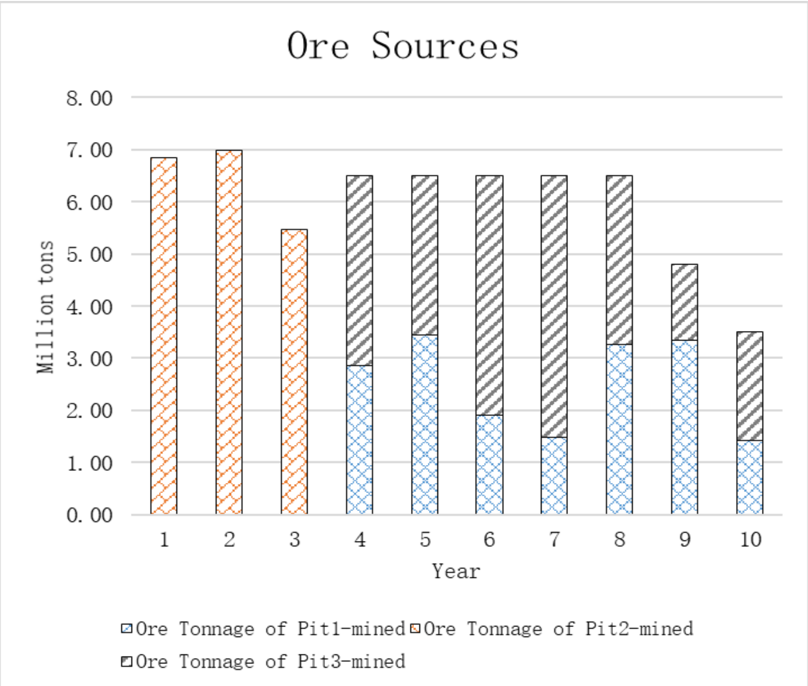

Figure 4.6 Ore sources from pit excavation per period

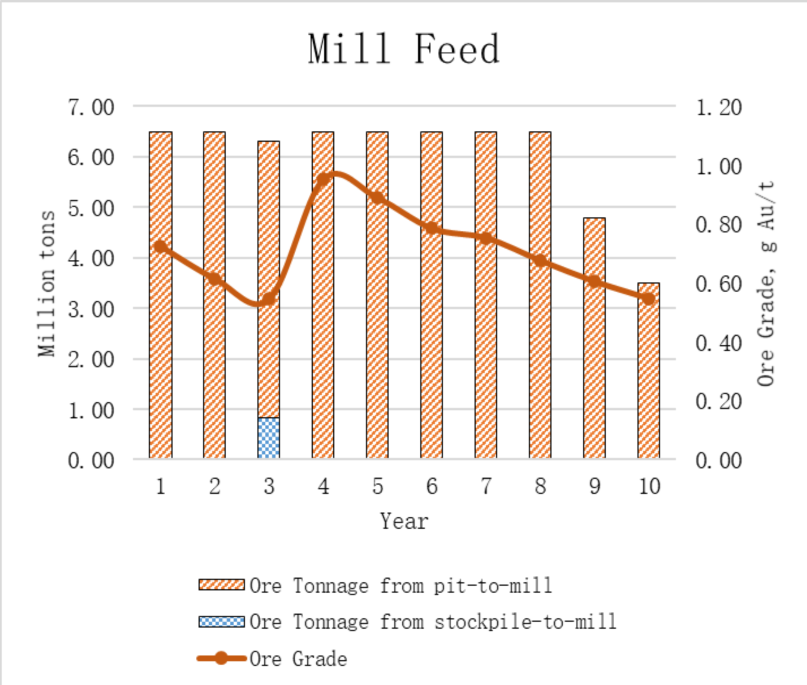

Figure 4.7 Mill production per period

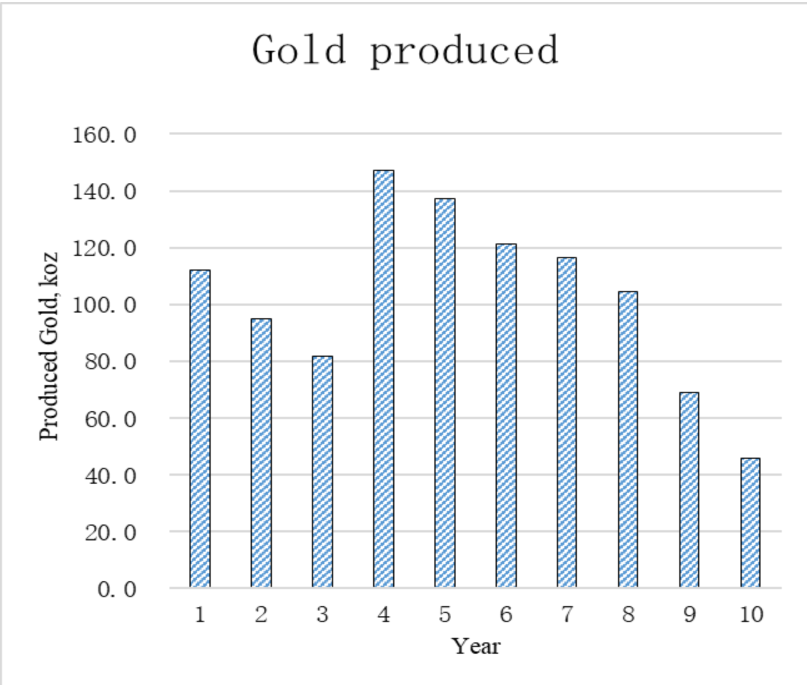

Figure 4.8 Gold produced per period

In order to cope with the smooth transition in production from Pit1 mining operations to the mining operations of Pit 2 and Pit 3, sufficient ore storage in stockpile is required. As shown in Figure 4.6, it is going to extract and recover as many ounces as possible at early stage by processing high-grade ore and stockpiling medium and low-grade ore for later. Considering the mine strategy, the ore grade is decreased from year 1 to year 3, and then significantly increased from year 4 due to the mining commencement of Pit 2 and Pit 3. In year 3, the feed is composed of stockpile inventory and the ore of Pit 2. The production plan delivers a relatively stable and consistent ounce profile until the tenth year.

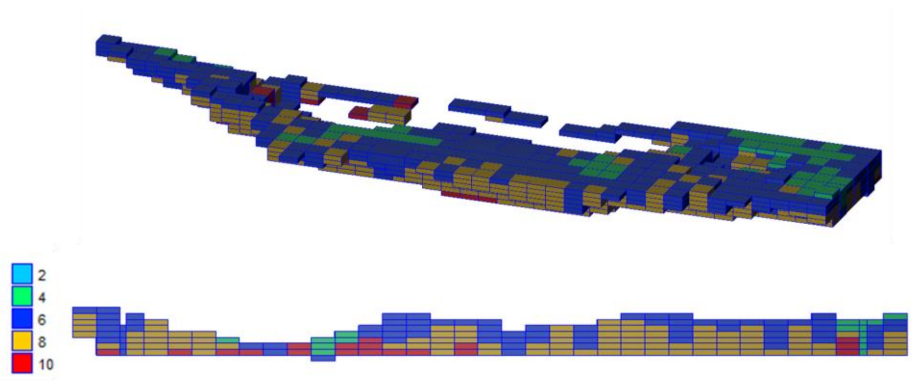

Figure 4.9 Pit 1 extraction sequence cross-section

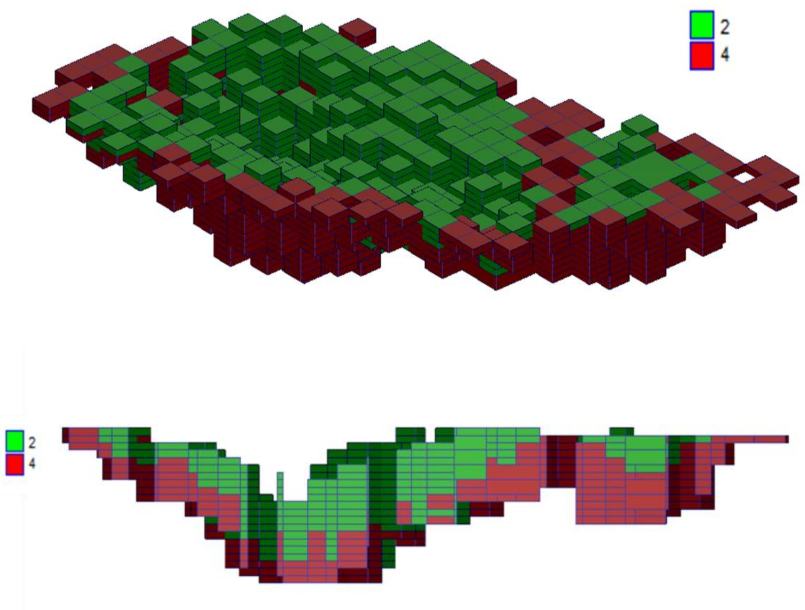

Figure 4.10 Pit 2 extraction sequence cross-section

Figure 4.9, Figure 4.10 and Figure 4.11 present the cross-sections of extraction sequences for each pit. This highlights the ability of the simultaneous stochastic optimizer to capitalize on extra production while managing the impact of risk throughout the life of the operation.

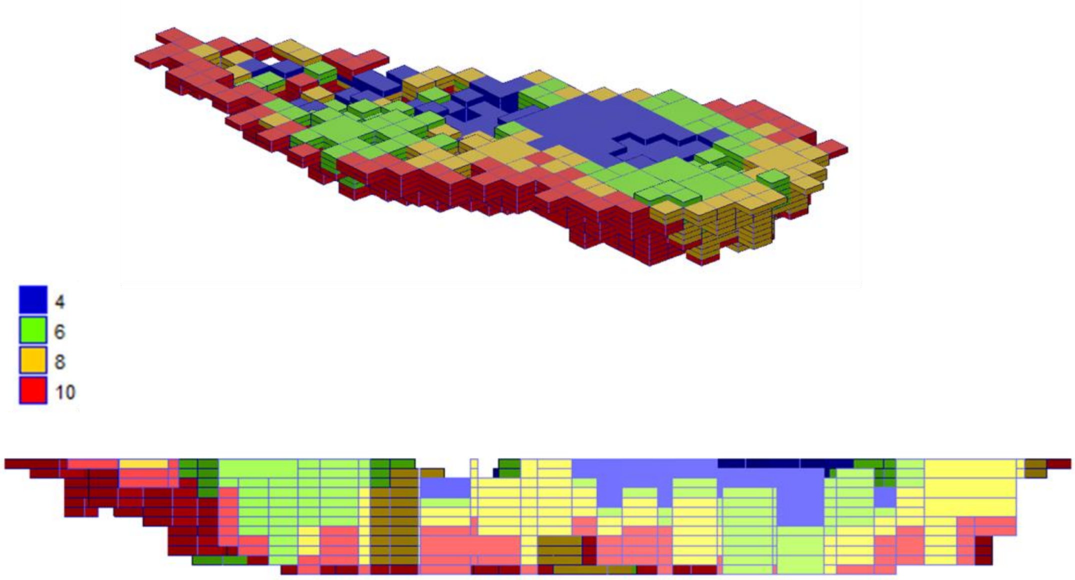

Figure 4.11 Pit 3 extraction sequence cross-section

4.7.3 Waste Rock Dumping

Due to the constraints of waste rock dumping, the early completion of Pit 2 mining is critical for the extraction of Pit 1 and Pit 3. It can be seen from Figure 4.11 that the P 2 waste rock is directly transported to the planned dump site and completed in year 3, after which the waste rock of Pit 1 and Pit 3 is discharged to Pit 2. Combining the previously described mining production plan, the MIP model developed by the constraints of waste rock dumping, an operational production plan can still be formulated, which solves the problem of gold ore resource development in this area.

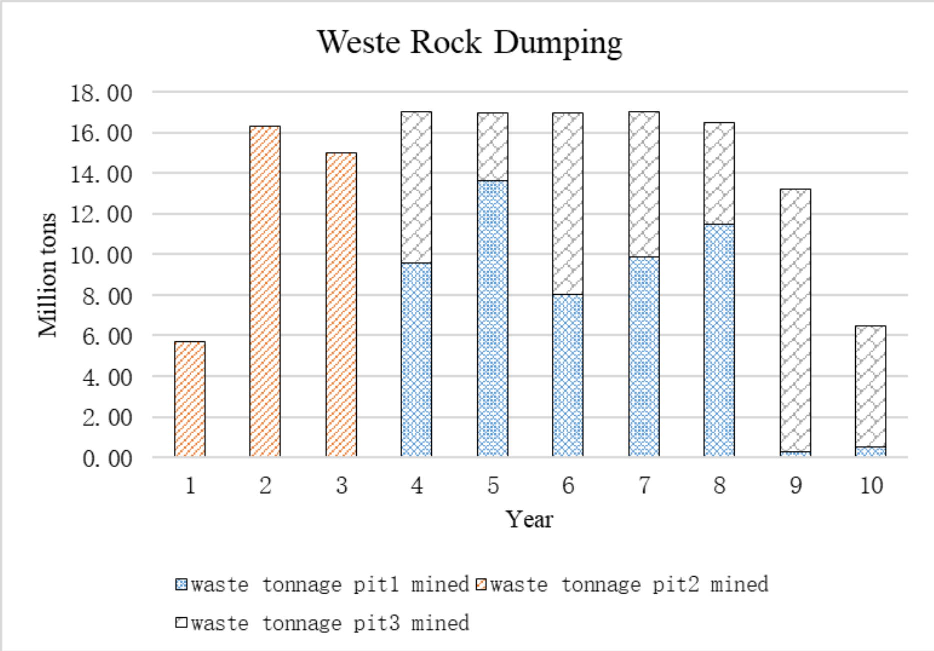

Figure 4.12 Waste dumping schedule

4.7.4 Discount Rate Sensitivity Analysis

In order to study the sensitivity of the project's NPV, discount rate, which is a significant input parameter, has been selected as the factor on NPV sensitivity analysis in this project financial model. Table 4.2 provides NPV values for a series of discount rates between 6% and 10%. Rising interest rates result in a reduction in NPV because the cash flow gets discounted later with higher interest rates.

Table 4.2 Discount Rate Sensitivity Analysis

NPV, million Australia dollar	Discount Rate, %	Difference, %
229.4	5	8.6
211.1	6	/
194.1	7	-8.1
178.3	8	-15.6
163.6	9	-22.5
149.9	10	-29.0

It can be observed from Figure 4.13 that the pre-tax NPV is 417.0 million Australian dollars with a discount rate of 5%, and it drops gradually to 272.6 million Australian dollars with the discount rate increase to 10%.

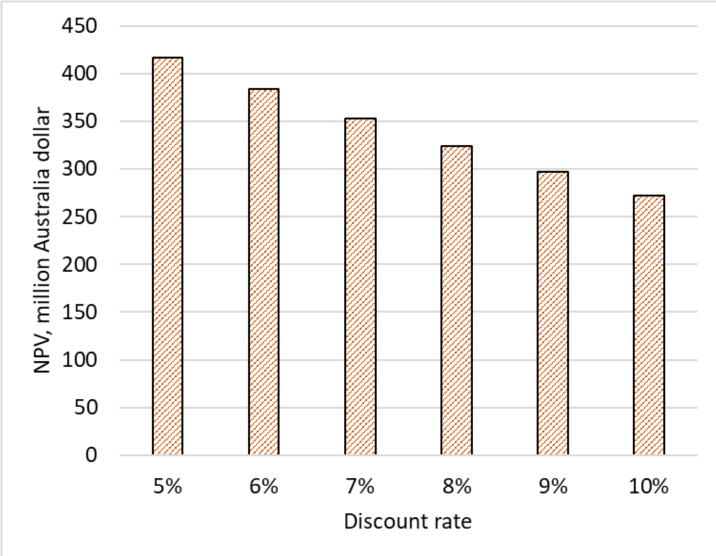

Figure 4.13 Sensitivity analysis of discount rate

4.8 Mine Schedule without Pit Extraction Constraint

In order to deal with those components and decisions that have a significant impact on the value of the project over the long term, a new scenario without pit extraction constraint has also been studied. It assumes that land space of the project is sufficient for waste dumping over the LOM.

Without pit extraction constraints, working faces are available, thus creating a more stable ore supply to the mill. As shown in Figure 4.14 and Figure 4.15, three open-pit mines are operating over the life of mine. Only in year 9 and year 10, the tonnage of mined materials begins to decrease. Mill throughput is very stable from year 1 to year 9, and the feed grade has a downward trend, indicating a good cash flow in the early stage in Figure 4.16 and Figure 4.17. It is observed that the stockpile has more rehandling ore in year 9 due to many ore storages in the pit after blasting, which saves an extensive rehandling cost of ore. The pre-tax NPV of this scenario is 363.8 million AUD, which increases 62% compared to the base scenario. This also provides an option if there is potential to solve the layout issue of the waste dump.

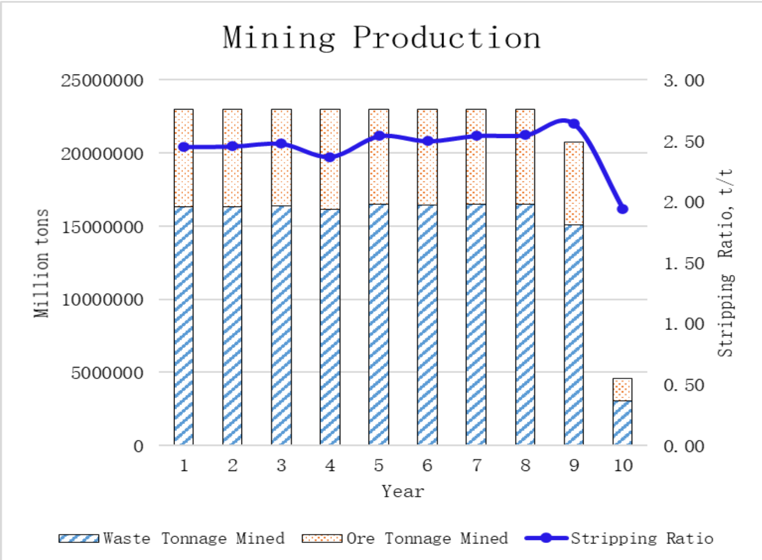

Figure 4.14 Mining production per period without pit extraction constraint

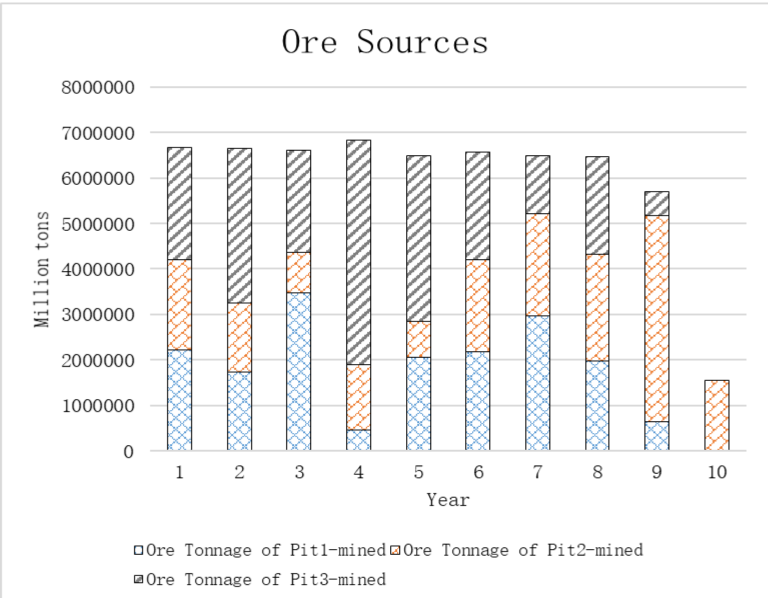

Figure 4.15 Ore sources from pit excavation per period without pit extraction constraint

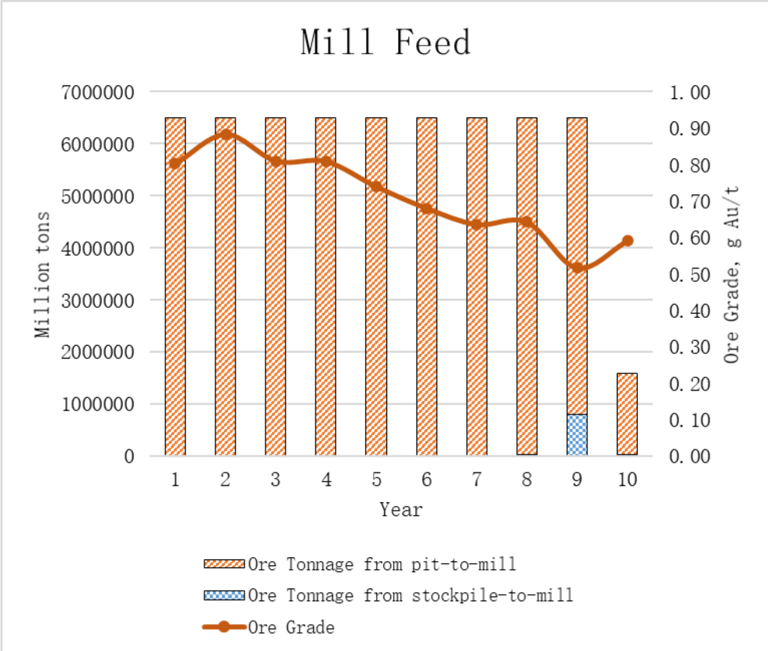

Figure 4.16 Mill production per period without pit extraction constraint

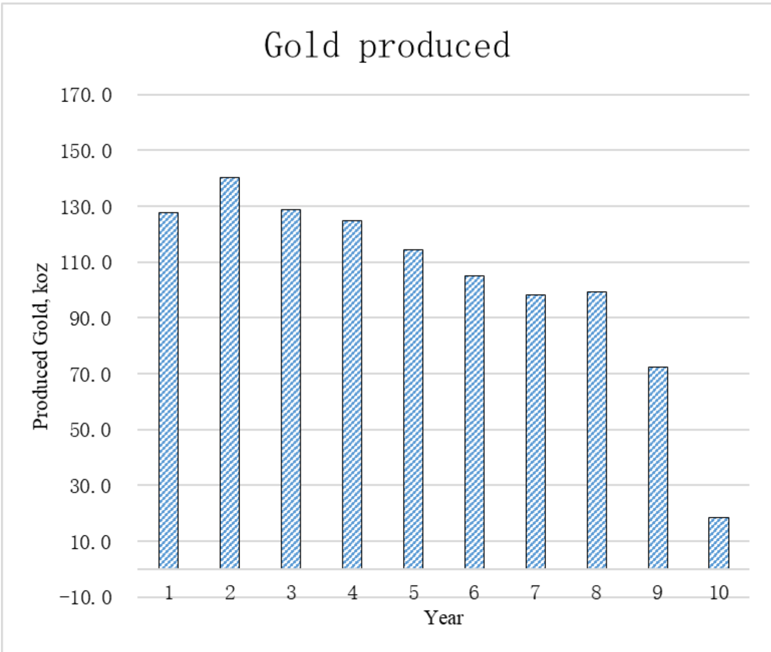

Figure 4.17 Gold produced per period without pit extraction constraint

CHAPTER 5. CONCLUSIONS

5.1 Summary

The research work aims to solve the production scheduling problem for open-pit mining complexes. It develops a MIP model that maximises the net present value of future cash flows and satisfies reserve, production capacity, mining block precedence, waste disposal, stockpiling, and pit sequence constraints.

- The model is validated by using small to medium scale datasets. The validation results have proved that all formulated constraints are working correctly.
- The proposed MIP model is applied to a real data set from a gold mine located in Western Australia. The case study is based on the given mining physicals, which assesses the mine strategy plan of the open-pit mining complex by conjunction with the simultaneous optimisation of extraction sequence and processing stream decisions.
- Two scenarios are studied using the same data set, which indicates that the proposed MIP model can provide a practical mine schedule to satisfy all given constraints. Compared to the original schedule, the chosen schedule has less static income, million dollars. However, the chosen schedule can come through the permitting one and half years earlier, which brings a higher NPV through a 10-year period of LOM.
- The established MIP model is flexible to adjust the given constraints and decision variables, which can solve more complicated problems of the mining industry.

5.2 Future works

The MIP model developed in this thesis has provided a contribution for strategic mine planning with limited space constrains. Besides, there is still a need on mine scheduling. The next recommendations could add value to the knowledge about this topic.

1. Rerun the developed MIP model with optimized ultimate pit limits.
2. Development of a MIP model that represents a mining complex with multiple mines, waste dumps and processing streams.
3. Solution of the MIP model through available algorithms (exact methods) considering a subset of the orebody model.
4. Development and implementation of new solution algorithms for the proposed MIP model.
5. Performance evaluation of the new solution algorithms against available methods.
6. An evaluation of the value created through simultaneous optimisation of open-pit production and waste dump scheduling. This will require repeating steps 1-3 for traditional methods that generate open-pit production schedules and waste dump schedules sequentially.

APPENDIX

OPL Code

```
//Sets
int B = ...;
range b = 1..B;
int M = ...;
range m = 1..M;
int P = ...;
range p = 1..P;
int S = ...;
range s = 1..S;
int W = ...;
range w = 1..W;

int T = ...;
range t = 1..T;

//Parameters
float r=...;
float n=...;
float a[m]=...;
float c[p]=...;
float h=...;
float u=...;
float i=...;
float Jmax[t]=...;
float Jmin[t]=...;
float Kmax[t]=...;
float Kmin[t]=...;
float L[t]=...;
float g[b][m]=...;
float q[b][m]=...;
float gg[s]=...;



float VMTP[bb in b][mm in m][tt in t][pp in p]=((r-n)*g[bb][mm]*u-a[mm]-
c[pp])*q[bb][mm]/((1+i)^tt);
```

```
float VMTS[bb in b][mm in m][tt in t]=(-a[mm])*q[bb][mm]/((1+i)^tt);

float VMTW[bb in b][mm in m][tt in t]=(-a[mm])*q[bb][mm]/((1+i)^tt);

float VSTP[tt in t][pp in p][ss in s]=((r-n)*gg[ss]*u-h-c[pp])/((1+i)^tt);

(Epstein, Goic et al.) ub[m][b]=...;

int t2=...;

//Variables

dvar boolean X[b][m][t][p];
dvar boolean Y[b][m][t][s];
dvar boolean Z[b][m][t][w];

dvar float+ E[t];
dvar float+ F[t];

//Objective Function

dexpr float object = sum(mm in m, bb in b, tt in t, pp in
p)VMTP[bb][mm][tt][pp]*X[bb][mm][tt][pp]
+sum(mm in m, bb in b, tt in t, ss in s)VMTS[bb][mm][tt]*Y[bb][mm][tt][ss]
+sum(mm in m, bb in b, tt in t, ww in w)VMTW[bb][mm][tt]*Z[bb][mm][tt][ww]
+sum(tt in t, pp in p, ss in s)(VSTP[tt][pp][ss])*E[tt];

maximize sum(mm in m, bb in b, tt in t, pp in
p)VMTP[bb][mm][tt][pp]*X[bb][mm][tt][pp]
+sum(mm in m, bb in b, tt in t, ss in s)VMTS[bb][mm][tt]*Y[bb][mm][tt][ss]
+sum(mm in m, bb in b, tt in t, ww in w)VMTW[bb][mm][tt]*Z[bb][mm][tt][ww]
+sum(tt in t, pp in p, ss in s)(VSTP[tt][pp][ss])/((1+i)^tt)*E[tt];

subject to {
ct1: forall(mm in m, bb in b, pp in p, ss in s, ww in w, tt in t)
{sum(tt in 1..t2)(X[bb][2][tt][pp]+Y[bb][2][tt][ss] +Z[bb][2][tt][ww])==1;
sum(tt in t2..T)(X[bb][1][tt][pp]+Y[bb][1][tt][ss] +Z[bb][1][tt][ww])==1;
```

```
sum(tt in t2..T)(X[bb][3][tt][pp]+Y[bb][3][tt][ss] +Z[bb][3][tt][ww])==1;
}

ct2:forall(mm in m, bb in b, bbb in ub[mm][bb], tt in 1..T, pp in p, ss in s, ww in w)
sum(ttt in 1..tt)(X[bb][mm][ttt][pp]+Y[bb][mm][ttt][ss]+Z[bb][mm][ttt][ww])-
sum(ttt in 1..tt)(X[bbb][mm][ttt][pp]+Y[bbb][mm][ttt][ss]+Z[bbb][mm][ttt][ww])
<=0;

ct3:forall(pp in p, ss in s, ww in w, bb in b)
{sum(tt in t2+1..T)(X[bb][2][tt][pp]+Y[bb][2][tt][ss]+Z[bb][2][tt][ww])==0&&
sum(tt in 1..t2-1)(X[bb][1][tt][pp]+Y[bb][1][tt][ss]+Z[bb][1][tt][ww])==0&&
sum(tt in 1..t2-1)(X[bb][3][tt][pp]+Y[bb][3][tt][ss]+Z[bb][3][tt][ww])==0;
};

ct4: forall(tt in t, pp in p, ss in s, ww in w)
Jmin[tt]<=sum(bb in b,mm in
m)(q[bb][mm]*X[bb][mm][tt][pp]+q[bb][mm]*Y[bb][mm][tt][ss]+q[bb][mm]*Z[b
b][mm][tt][ww]) <=Jmax[tt];

ct5: forall(tt in t, pp in p)
Kmin[tt]<=sum(mm in m, bb in
b)q[bb][mm]*X[bb][mm][tt][pp]+E[tt]<=Kmax[tt];

ct6:forall(mm in m, bb in b, ss in s)
{sum(mm in m, bb in b)q[bb][mm]*Y[bb][mm][1][ss]-E[1]-F[1] ==0;
};

ct7: forall(tt in 2..T, mm in m, bb in b, ss in s)
{F[tt-1] + sum(mm in m, bb in b)q[bb][mm]*Y[bb][mm][tt][ss]-E[tt]-F[tt] ==0;
};

ct8:forall(mm in m, bb in b, ss in s)
{sum(mm in m, bb in b)q[bb][mm]*Y[bb][mm][1][ss]-E[1]-F[1] ==0;
};

ct9:forall(tt in t)
{
F[tt] <=L[tt];
```

```
E[tt] <=Kmax[tt];
};

}

 execute {
   writeln(object)

  }
```

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