# A Strengthened Solution to Option Manipulation

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**Abstract**

Thanks to the reduced price and less exposition to sudden crashes or price hikes, the Asian option is among the most favorable hedging instruments that are hard to be manipulated, in both the commodity market and executive compensation plan. Since the creation of the option, the main focus has been more on how to price it accurately while much less on how to explore deeper the benefits that the option offers. In this paper, a new type of path-dependent option, referred to as the average-Asian option, is introduced to reduce further the volatility of the underlying price risk and minimize option manipulation threat. The price is proved to be less than that of the standard option. It is additionally shown by numerical results that, when granted at the money, the proposed option is on average about 49.32% and 5.45% cheaper than the standard and Asian options, respectively. Furthermore, the option is less sensitive than the Asian counterpart, at both the front-end and the back-end price manipulation.

***Keywords:*** Financial derivative, path-dependent option, risk management, price manipulation, executive compensation

## 1 Introduction

Financial derivatives, in particular options, are created to hedge the additional risk arising from, for example, the volatility in the currency exchange rate for a company whose business largely relies on imports and exports. Since the first option was traded on the Chicago Board Options Exchange (CBOE) in 1973, the market has grown dramatically. In 2021, CBOE recorded an annual trading volume of around 3 billion contracts, a jump of nearly 19% from around 2.513 billion contracts in 2020 [CBOE, 2022].

Various perspectives have been consistently looked into to eliminate possible arbitrage opportunities. For instance, sophisticated mathematical models have been developed to price options more appropriately since the seminal Black-Scholes-Merton option pricing model [Black and Scholes, 1973; Merton, 1976], which helped boom further options trading around the world [MacKenzie, 2008].

To minimize the potential option manipulation threat, this paper from a different dimension focuses on how to possibly reduce further the risk associated with the volatility of the asset price by proposing an alternative type of option. Before the so-called average-Asian option is introduced in detail in Section 2, the background and motivation of the study are briefly discussed in Section 1.1 and Section 1.2 first.

### 1.1 Option

In addition to the standard European and American call and put options that are traded on exchanges, a number of exotic options are often traded in large quantities in the over-the-counter (OTC) market, which nowadays is significantly larger than the exchange-traded market [Hull, 2018; Kyprianou et al., 2005]. The exotic options are usually created to meet certain specific needs of particular business or risk management. For example, the binary asset-or-nothing call option pays the asset price if the underlying asset price ends up above the strike price and zero otherwise [Rubinstein and Reiner, 1991].

The payoff of a standard European style option depends on the price of the underlying asset at expiry. There is then a chance that the option may be manipulated, besides that it may be comparatively expensive. In case that there is a potential to manipulate the price of the underlying asset or that a cheaper option is preferred, a popular alternative is the Asian option, the payoff of which is determined by the average underlying price during the life of the option. The averaging feature hence reduces the volatility inherent in the option, making it less exposed to sudden crashes or rallies in the asset price and harder to be manipulated [Wilmott, 2007]. As a result, Asian option is the most popular exotic option chosen by the U.S. non-financial firms for risk management [Bodnar et al., 1998].

Particularly in the commodity market, which is now a mainstream financial investment class [Kyriakou et al., 2016], end users are often exposed to the average price over time. This increases the popular appeal of Asian option. The path-dependent option is especially appropriate to the electricity market, where a contract is written to supply continuous electricity over the life of the option. It is therefore reasonable for the electricity market to refer to the average price over the period of the contract [Fanelli et al., 2016].

Similarly, being exposed to the international business environment, the shipping market faces significant risks resulting from seasonality and other volatility [Tsai et al., 2009]. Shipping derivatives are therefore used to manage freight rate risk. Analogous to the case of the gas and electricity markets, where Asian options are natural hedging instruments for risk management due to the limited possibility of storage leading to continuous purchases for energy consumers, freight rates are non-storable as well in the shipping market. Freight options are hence also Asian-style options where the payoff at settlement depends on the arithmetic average of the spot freight rate [Koekebakker et al., 2007].

In addition, price manipulation by large market participants is harder in the case of an Asian option as compared with a standard option [Chatterjee et al., 2018]. This is critically important for thinly-traded commodities as it is possible to manipulate the price on any given day or near option expiry while not the average price of the underlying asset in general [Linetsky, 2004].

### 1.2 Price Manipulation

In corporate finance, the conflict of interests between the shareholders being the principal who owns the company and the top management being the agent who runs the company on behalf of the principal is referred to as the principal-agent problem. In particular, the shareholders seek to maximize their wealth through the increase in the share price, while the top management as the executive may look for corporate luxury, job security, or increment in its own wealth at the expense of the shareholders. Consequently, stock options are granted as an incentive paid to the executive to align the interests of the two parties, so as to mitigate the principal-agent problem [Berk and DeMarzo, 2020; Brealey et al., 2019]. Meanwhile, despite their popularity, these options could still not adequately align the interests of the two parties, besides that there is additional risk of stock price manipulation by the executive to boost the compensation package [Hall and Murphy, 2003; Tian, 2017]. In a survey of 169 chief financial officers of the U.S. public companies, it is reported that about 20% of the firms misrepresented their economic performance, with the main reason being the desire to influence the stock price [Dichev et al., 2013]. It thus naturally suggests the utilization of the averaging feature in designing the executive stock options (ESOs) to better align the interests of the management and shareholders, as well as to preserve the value of options for both the corporations and employees [Chhabra, 2008]. This indicates that firms should consider granting Asian options instead of standard options as compensation packages [Tian, 2013]. In addition, the payoff structure of an Asian option resembles that of the variable annuity [Bernard et al., 2017], an insurance contract that is typically a long-term investment aimed at generating income for retirement. Insurance companies hence trade Asian options to hedge the embedded option risk. To meet the increasing market demand, the CBOE introduced the Asian FLEX Index Options in 2016.

By incorporating the average stock price into the payoff of an ESO, which is called the Asian executive option in executive compensation, it is shown that the Asian option has an advantage over and is cheaper than the traditional option [Tian, 2013]. This makes the Asian option more appealing as generally a lower up-front premium of an option is more attractive [Wilmott, 2007]. In particular, from the perspective of a risk-averse executive, the Asian option is cheaper to the company while being equally or more desirable to the executive who discounts the value of the stock options due to the aversion to risk. The Asian option is then a more cost-effective form of executive compensation from the perspective of the company and its shareholders, compared with the European counterpart [Tian, 2013].

A new option, whose payoff is based on a power of the stock price at expiry, is further introduced. The so-called power option is even cheaper than the Asian option when priced in the Black-Scholes world. However, the power option requires the expected return of the stock, which is difficult to estimate and violates the risk-neutral valuation assumption of the Black-Scholes world. This makes pricing the option more challenging than the Asian counterpart [Bernard et al., 2016].

### 1.3 Average Option

Since the market crash in 1987, substantial efforts have been made to price the Asian option as the reliable alternative to the vanilla counterpart for financial risk management, particularly in the market where either the volume is low or the volatility is high [Fanelli et al., 2016]. In the meantime, except for the power option in executive compensation, attention still lacks being paid to look into what brings the popularity of the option, that is, the averaging feature, the reduced price, and the safeguard from manipulation threat.

In this paper, a new path-dependent option, referred to as the average-Asian option, is introduced to reduce further the volatility of the asset price risk and minimize the option manipulation threat. To be in line with the Asian and power executive options [Bernard et al., 2016; Tian, 2013], the Black-Scholes environment is considered as well.

The paper is organized as follows. In Section 2, the average-Asian option is introduced and properties of the price as well as the impact of price manipulation are examined. In Section 3, the average-Asian option is priced by using two numerical procedures, followed by a discussion on the option price w.r.t. the underlying parameters. Section 4 concludes the study.

# 2 Average-Asian Option

In Section 2.1, the motivation for considering the new option is first discussed and the average-Asian option is then defined. In Section 2.2, properties with respect to the expected value as well as payoff are looked into, from both the analytical and numerical perspectives. Impact of price manipulation is further examined based on both the front-end and the back-end gaming sensitivity in Section 2.3.

## 2.1 Specification

Let $T \in (0, \infty)$ and $(\Omega, \mathcal{F}, \mathbf{P})$ be a complete probability space with a filtration $\mathbb{F} = \{\mathcal{F}_t\}_{t\in[0,T]}$ of $\sigma$-algebras satisfying the usual conditions. Suppose $S = \{S_t\}_{t\in[0,T]}$ is an $\mathbb{F}$-adapted stochastic process.

The payoff of a standard European-style option depends on the price of the underlying asset at expiry $t = T$. The payoff of a call is $\max(S_T - K, 0)$ and that of a put is $\max(K - S_T, 0)$, where $S_T$ is the price of the underlying asset at expiry and $K$ is the strike price. For an Asian option, the payoff depends on the average price of the underlying asset during the life of the option. The payoff of an Asian call option is $\max(\bar{S}_0^T - K, 0)$ and that of a put option is $\max(K - \bar{S}_0^T, 0)$, where $\bar{S}_0^T$ is the average price of the underlying asset from $t = 0$ to $t = T$, inclusive. That is,

$$\bar{S}_0^T = \frac{1}{T}\int_0^T S_t dt. \tag{1}$$

More generally, for a payoff function being a weighted average of the price at expiry and the average price from time $t = 0$ to $t = T$, that is,

$$f(\alpha S_T + \beta \bar{S}_0^T), \tag{2}$$

where $\alpha$ and $\beta$ are non-negative constants such that $\alpha + \beta = 1$, by the fundamental theorem of arbitrage-free asset pricing, the option price is

$$\mathrm{E}\big(e^{-rT} f\big(\alpha S_T + \beta \bar{S}_0^T\big)\big),$$

where $r$ is the risk-free interest rate [Jourdain and Sbai, 2007]. Apparently, it is a European option in the case that $\beta = 0$ and an Asian option in the case that $\alpha = 0$.

Assume that the underlying price $S = \{S_t\}_{t\in[0,T]}$ follows a geometric Brownian motion. That is,

$$dS_t = \mu S_t dt + \sigma S_t dW_t, t \in [0, T], \tag{3}$$

where $\mu$ is the expected rate of return of the asset price, $\sigma$ is the volatility of the return, $dS_t$ is the change in the stock price in relation to the current price $S_t$ over a short time interval $dt$, and $dW_t$ is the change in the standard Wiener process $W = \{W_t\}_{t\in[0,T]}$. By Monte Carlo simulation with $140,000$ sample paths, the prices for the option whose payoff is given by (2) are shown in Table 1, with $S_0 = 100, K = 100, T = 1$ and three different combinations of the risk-free interest rate $r$ and the volatility $\sigma$. As can be expected, the cheapest option is obtained when $\alpha = 0$ and $\beta = 1$, that is, when no weight is assigned to $S_T$. The price keeps increasing when more weight is allocated to $S_T$. The payoff hence brings no grounded advantage in terms of price reduction, compared with that of the Asian option. An alternative not in the equivalent form of the first-order sum of $S_T$ and $\bar{S}_0^T$ may then be further explored.

Table 1: Prices of Weighted Average Options with Different Values of $\alpha$ and $\beta$

| $\alpha$ | $\beta$ | $r = 0.05, \sigma = 0.20$ | $r = 0.10, \sigma = 0.30$ | $r = 0.15, \sigma = 0.45$ |
|---|---|---|---|---|
| 0.0 | 1.0 | 5.75 | 9.03 | 13.09 |
| 0.1 | 0.9 | 6.14 | 9.67 | 14.08 |
| 0.2 | 0.8 | 6.56 | 10.38 | 15.09 |
| 0.3 | 0.7 | 6.99 | 11.10 | 16.14 |
| 0.4 | 0.6 | 7.43 | 11.83 | 17.28 |
| 0.5 | 0.5 | 7.91 | 12.65 | 18.36 |
| 0.6 | 0.4 | 8.39 | 13.43 | 19.55 |
| 0.7 | 0.3 | 8.90 | 14.22 | 20.75 |
| 0.8 | 0.2 | 9.39 | 15.04 | 21.95 |
| 0.9 | 0.1 | 9.96 | 15.86 | 23.19 |
| 1.0 | 0.0 | 10.46 | 16.73 | 24.44 |

In general, the greater the volatility, the greater the option value is expected. The variance of $S_T$ is greater than that of $\bar{S}_0^T$ [Kemna and Vorst, 1990]. To reduce further the variance, in the payoff function where the weighted average of $S_T$ and $\bar{S}_0^T$ is taken, it may be considered replacing the mean of $S_T$ and $K$ for $S_T$, that is, $\frac{S_T+K}{2} - K$ instead of $S_T - K$. This then motivates to introduce the average-Asian option, defined in (4) and (5) for the call and put, respectively.

**Definition 1** *An option is called an average-Asian option if the payoff equally depends on*

- *the average price of the underlying asset during the life of the option, and*
- *the mean of the underlying price at expiry and the strike price.*

The payoff of an average-Asian call option is

$$\frac{1}{2}\max\left(\bar{S}_0^T - K + \frac{S_T + K}{2} - K, 0\right) = \frac{1}{4}\max\left(2\bar{S}_0^T + S_T - 3K, 0\right) \tag{4}$$

and that of an average-Asian put option is

$$\frac{1}{2}\max\left(K - \bar{S}_0^T + K - \frac{S_T + K}{2}, 0\right) = \frac{1}{4}\max\left(3K - (2\bar{S}_0^T + S_T), 0\right), \tag{5}$$

where $S_T$ is the underlying price at expiry $t = T$, $\bar{S}_0^T$ is the average price of the underlying asset from time $t = 0$ to $t = T$ as given in (1), and $K$ is the strike price.

**Remark 1** *The payoff given in* (4) *can be rewritten as* $\gamma \max\left(\alpha\bar{S}_0^T + \beta S_T - K, 0\right)$ *and that given in* (5) *can be rewritten as* $\gamma \max\left(K - (\alpha\bar{S}_0^T + \beta S_T), 0\right)$, *with* $\alpha = \frac{2}{3}$, $\beta = \frac{1}{3}$, *and* $\gamma = \frac{3}{4}$. *The option specified in Definition 1 can hence be generalized to a class of options with payoff* $\gamma \max\left(\alpha\bar{S}_0^T + \beta S_T - K, 0\right)$ *and* $\gamma \max\left(K - (\alpha\bar{S}_0^T + \beta S_T), 0\right)$ *for call and put, respectively, where* $\alpha + \beta = 1$ *and* $\alpha, \beta, \gamma \in [0, 1]$. *In other words, it provides an intuitively-interpretable or meaningful example for the generalized case.*

## 2.2 Price

Assume that the underlying price follows a geometric Brownian motion defined by (3). For the scenarios that $S_0 = 100$, $r = 0.1$, $\sigma = 0.3, 0.4, 0.5$, $T = 1$, and $\Delta t = \frac{T}{m}$ with $m = 100$, the respective expected means and variances of $S_T$, $\bar{S}_0^T$, and $\tilde{S}_0^T = \frac{1}{4}(2\bar{S}_0^T + S_T)$ [1] are shown in Table 2 for both the arithmetic and geometric averages, where the simulation is performed for $n = 200,000$ times.

Table 2: Expected Means and Variances of $S_T$, $\bar{S}_0^T$, and $\tilde{S}_0^T$

| | | Arithmetic | | | Geometric | | |
|---|---|---|---|---|---|---|---|
| | | $S_T$ | $\bar{S}_0^T$ | $\tilde{S}_0^T$ | $S_T$ | $\bar{S}_0^T$ | $\tilde{S}_0^T$ |
| Mean | $\sigma = 0.3$<br>$\sigma = 0.4$<br>$\sigma = 0.5$ | 110.37 | 105.12 | 80.15 | 110.48 | 103.75 | 79.49 |
| Variance | $\sigma = 0.3$ | 1148.31 | 345.71 | 294.98 | 1150.18 | 330.88 | 287.55 |
| | $\sigma = 0.4$ | 2111.82 | 626.50 | 537.71 | 2113.34 | 585.45 | 516.56 |
| | $\sigma = 0.5$ | 3453.69 | 997.68 | 865.25 | 3458.86 | 915.22 | 822.99 |

As illustrated in Table 2, the average-Asian option reduces the price volatility effectively.

### 2.2.1 Analytical Properties of Expected Value

Rewrite the asset price $S_T$ at $t = T$ and the average price $\bar{S}_0^T$ from time $t = 0$ to $t = T$ as

$$S_T = \sum_{i=0}^{m} \frac{1}{m+1} S_T \quad \text{and} \quad \bar{S}_0^T = \sum_{i=0}^{m} \frac{1}{m+1} S_{t_i},$$

where $t_i = t_0 + \frac{t_m - t_0}{m} i$, $i = 0, \ldots, m$, with $t_0 = 0$ and $t_m = T$. The values of the standard, Asian, and average-Asian call options are then respectively given by

$$c_{\text{Standard}} = e^{-rT} \mathrm{E}\big[\max\big(\sum_{i=0}^{m} \frac{1}{m+1} S_T - K, 0\big)\big],$$

$$c_{\text{Asian}} = e^{-rT} \mathrm{E}\big[\max\big(\sum_{i=0}^{m} \frac{1}{m+1} S_{t_i} - K, 0\big)\big],$$

and

$$\begin{aligned} c_{\text{A.Asian}} &= e^{-rT} \mathrm{E}\big[\frac{1}{4} \max\big(2\sum_{i=0}^{m} \frac{1}{m+1} S_{t_i} + \sum_{i=0}^{m} \frac{1}{m+1} S_T - 3K, 0\big)\big] \\ &= e^{-rT} \mathrm{E}\big[\frac{1}{2} \max\big(\sum_{i=0}^{m} \frac{1}{m+1} S_{t_i} - K + \sum_{i=0}^{m} \frac{\frac{S_T + K}{2} - K}{m+1}, 0\big)\big]. \end{aligned}$$

**Proposition 1** *The expected value of an average-Asian call option is less than or equals to that of the corresponding standard call option.*

[1] A more equivalent form to $S_T$ and $\bar{S}_0^T$ would be, for instance, $\frac{1}{4}(2\bar{S}_0^T + S_T + S_0)$. Since $S_0$ is fixed, the additional term $\frac{1}{4}S_0$ does not affect the variance, which is the main focus of Table 2. $\tilde{S}_0^T = \frac{1}{4}(2\bar{S}_0^T + S_T)$ is hence directly looked at here.

*Proof.* The following relationship holds [Kemna and Vorst, 1990],

$$\mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_{t_i}-K,0\big)\big]\leq \mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_T-K,0\big)\big].$$

In addition,

$$\begin{aligned}\mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{\frac{S_T+K}{2}-K}{m+1},0\big)\big] &= \mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{\frac{S_T+K}{m+1}}{2}-K,0\big)\big]\\ &= \frac{1}{2}\mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_T-K,0\big)\big]\\ &\leq \mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_T-K,0\big)\big].\end{aligned}$$

Then, since either

$$\mathrm{E}\big[\frac{1}{2}\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_{t_i}-K+\sum_{i=0}^{m}\frac{\frac{S_T+K}{2}-K}{m+1},0\big)\big]\leq \mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_{t_i}-K,0\big)\big]\leq \mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_T-K,0\big)\big]$$

or

$$\mathrm{E}\big[\frac{1}{2}\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_{t_i}-K+\sum_{i=0}^{m}\frac{\frac{S_T+K}{2}-K}{m+1},0\big)\big]\leq \mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{\frac{S_T+K}{2}-K}{m+1},0\big)\big]\leq \mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_T-K,0\big)\big],$$

it hence follows that

$$c_{\mathrm{A.Asian}}=e^{-rT}\mathrm{E}\big[\frac{1}{2}\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_{t_i}-K+\sum_{i=0}^{m}\frac{\frac{S_T+K}{2}-K}{m+1},0\big)\big]\leq e^{-rT}\mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_T-K,0\big)\big]=c_{\mathrm{Standard}}.$$

□

**Remark 2** *For the relationship between the expected value of an average-Asian call option and that of the corresponding Asian option, it shows that taking the average price over the period reduces the price volatility by about 42% [Kemna and Vorst, 1990]. In addition, as indicated later in Table 10, it could be hypothesized that, when $S_0 \geq K$,*

$$\mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{\frac{S_T+K}{2}-K}{m+1},0\big)\big]\leq \mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_{t_i}-K,0\big)\big].$$

*Then, a conjecture for the case that $S_0 \geq K$ could be*

$$c_{\mathrm{A.Asian}}=\mathrm{E}\big[\frac{1}{4}\max\big(2\sum_{i=0}^{m}\frac{1}{m+1}S_{t_i}+\sum_{i=0}^{m}\frac{1}{m+1}S_T-3K,0\big)\big]\leq \mathrm{E}\big[\max\big(\sum_{i=0}^{m}\frac{1}{m+1}S_{t_i}-K,0\big)\big]=c_{\mathrm{Asian}}.$$

**Remark 3** *For the relationship between the expected value of an average-Asian put option and that of* *the Asian counterpart, as indicated in Table 10, a corresponding conjecture could be*

$$p_{\text{A.Asian}} = \mathrm{E}\big[\frac{1}{4}\max\big(3K-(2\sum_{i=0}^{m}\frac{1}{m+1}S_{t_i}+\sum_{i=0}^{m}\frac{1}{m+1}S_T),0\big)\big] \leq \mathrm{E}\big[\max\big(K-\sum_{i=0}^{m}\frac{1}{m+1}S_{t_i},0\big)\big] = p_{\text{Asian}} \leq p_{\text{Standard}}.$$

### 2.2.2 Numerical Properties of Payoff

A counterpart to the payoff of an Asian call option $\max(\bar{S}_0^T - K, 0)$, that of a power call option is defined as $\psi \max(S_T^{\varphi} - \frac{K}{\psi}, 0)$, where $\varphi = \frac{1}{\sqrt{3}}$ and $\psi = S_0^{1-\varphi}\exp\{(\frac{1}{2}-\varphi)(\mu - q - \frac{\sigma^2}{2})T\}$. Here $\mu$ is the expected return of the asset price, $\sigma$ is the volatility of the return, and $q$ is the dividend yield. To evaluate the payoffs of different options, the same example used for the study of the Asian and power options is adopted, for the sake of consistence. The five-year at-the-money (ATM) and in-the-money (ITM) options issued on both July 1, 2003 and July 1, 2008 are based on the stock price of Legg Mason [Bernard et al., 2016].

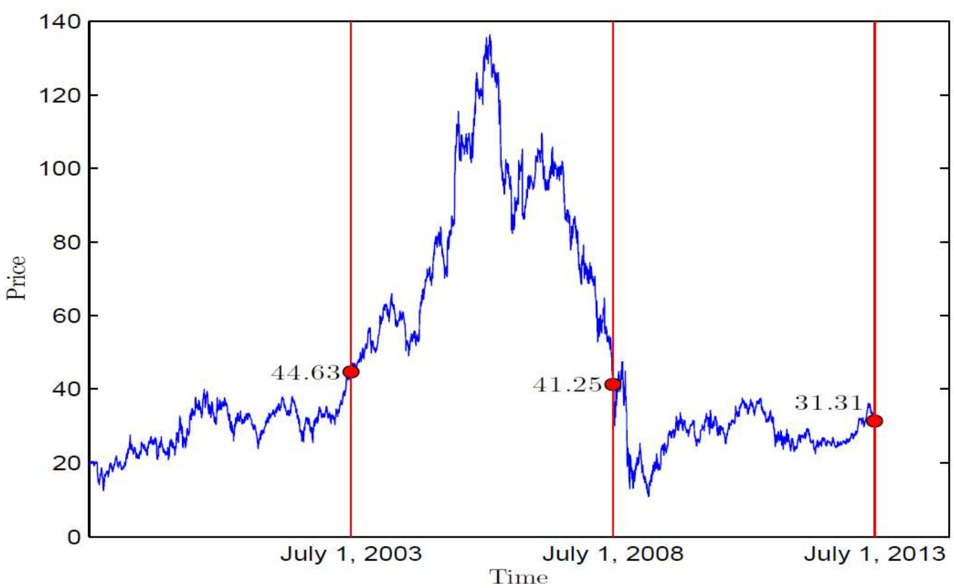


Figure 1: Closing Price of Legg Mason for the Time Period of July 1, 1998 to July 1, 2013

As illustrated in Figure 1, on July 1, 2003, the stock price was \$44.63 and it fell down to \$41.25 on July 1, 2008, while during the five-year time period, the price actually rose dramatically, yielding a high average price. The price further dropped to \$31.31 on July 1, 2013. Meanwhile, the price went down even further during this second five-year time period, resulting in a low average price. For each option, the price at expiry $S_T$, the average stock price $\bar{S}_0^T$ during the life of the option, the strike price $K$, and the option payoff are given in Table 3, where in the case of power options, $\mu - q = 0.262$ and $\sigma = 0.444$ for the option issued in 2003, and $\mu - q = 0.053$ and $\sigma = 0.322$ for the option issued in 2008.

As shown in Table 3, the payoffs of power options are not consistent with those of the Asian options, which are popular hedging instruments and well understood by market participants, besides that the power option is complex and difficult to price [Bernard et al., 2016]. There are scenarios where the Asian option is in the money but the corresponding power option is out of money, and vice versa. This would

Table 3: Payoffs of Standard, Asian, Power, and Average-Asian Call Options on Price of Legg Mason

| Issue Date | $S_T$ | $\bar{S}_0^T$ | Moneyness | $K$ | Payoff | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | Standard | Asian | Power | A.Asian |
| July 1, 2003 | 41.25 | 77.94 | ATM | 44.63 | 00 | 33.31 | 00 | 15.81 |
| | | | ITM | 30 | 11.25 | 47.94 | 10.04 | 26.78 |
| July 1, 2008 | 31.31 | 27.54 | ATM | 41.25 | 00 | 00 | 00 | 00 |
| | | | ITM | 30 | 1.31 | 00 | 5.16 | 00 |

be a matter of concern for a holder of the option. Meanwhile, the average-Asian option is consistent with the Asian option and is cheaper than the latter particularly in the case when the average underlying price rises dramatically. Hence, the average-Asian option may be considered as an alternative hedging instrument where the asset price could rise or fall unexpectedly during the life of the option or where a cheaper option is preferred.

Table 4 compares the payoffs of the standard, Asian, and average-Asian options in more detail, given the initial asset price being 100. Apparently, not only the payoff of the average-Asian option is less than that of the Asian option but also it is more stable in all cases with different parameter value combinations. The standard deviation of the payoff of the average-Asian option is consistently less than that of the Asian option. The dependence on both the price at expiry and the average price makes the average-Asian option less sensitive to sudden price jumps either near the option expiry or during the life of the option.

The price of crude oil is one of the leading indicators for forecasting economic trends [Kyriakou et al., 2016]. A typical example where an Asian option could potentially be undesirably expensive is the crude oil price for the time period of January 2017 to January 2019, as shown in Figure 2. Although the terminal price in January 2019 was even lower than the initial price in January 2017, the high average price during the two-year time period makes an Asian option an expensive choice. In such cases, dependence on both the average price and that at expiry would clearly have advantage over the average price alone.

## 2.3 Impact of Price Manipulation

To analyze the sensitivity of option price to price manipulation in the executive compensation context, the potential gain from asset price manipulation is measured at both the front-end, when the option is contracted, and the back-end, when the option expires [Tian, 2017].

In case of a call option, the front-end gaming involves a downward manipulation of the stock price to gain from better terms, e.g., a lower exercise price. After the option is contracted, gain in the option payoff can only come from a higher asset price, which is called the back-end gaming. In case of a put option, the front-end gaming and the back-end gaming involve upward manipulation and downward manipulation of the stock price, respectively.

### 2.3.1 Front-End Gaming Sensitivity

The front-end gaming sensitivity (FEGS) is defined as

$$\frac{c[S_0(1+\delta), K] - c[S_0, K]}{\delta c[S_0, K]},$$

Table 4: Payoffs of Standard, Asian, and Average-Asian Call Options

| $S_T$ | $\bar{S}_0^T$ | $K=100$ | | | $K=90$ | | |
|---|---|---|---|---|---|---|---|
| | | Standard | Asian | A.Asian | Standard | Asian | A.Asian |
| 120 | 140 | 20 | 40 | 25 | 30 | 50 | 32.5 |
| | 130 | | 30 | 20 | | 40 | 27.5 |
| | 120 | | 20 | 15 | | 30 | 22.5 |
| | 110 | | 10 | 10 | | 20 | 17.5 |
| | 100 | | 00 | 05 | | 10 | 12.5 |
| | 90 | | 00 | 00 | | 00 | 7.50 |
| | 80 | | 00 | 00 | | 00 | 2.50 |
| | 70 | | 00 | 00 | | 00 | 00 |
| Standard Deviation | | | 15.81 | 9.80 | | 19.59 | 11.76 |
| 110 | 130 | 10 | 30 | 17.5 | 20 | 40 | 25 |
| | 120 | | 20 | 12.5 | | 30 | 20 |
| | 110 | | 10 | 7.5 | | 20 | 15 |
| | 100 | | 00 | 2.5 | | 10 | 10 |
| | 90 | | 00 | 00 | | 00 | 05 |
| | 80 | | 00 | 00 | | 00 | 00 |
| Standard Deviation | | | 12.65 | 7.19 | | 16.33 | 9.35 |
| 100 | 120 | 00 | 20 | 10 | 10 | 30 | 17.5 |
| | 110 | | 10 | 05 | | 20 | 12.5 |
| | 100 | | 00 | 00 | | 10 | 7.5 |
| | 90 | | 00 | 00 | | 00 | 2.5 |
| | 80 | | 00 | 00 | | 00 | 00 |
| Standard Deviation | | | 8.94 | 4.47 | | 13.04 | 7.16 |
| 90 | 110 | 00 | 10 | 2.5 | 00 | 20 | 10 |
| | 100 | | 00 | 00 | | 10 | 05 |
| | 90 | | 00 | 00 | | 00 | 00 |
| Standard Deviation | | | 5.77 | 1.44 | | 10.00 | 5.00 |

where $\delta$ is the percentage manipulation in the asset price $S_0$. It is positive for upward manipulation and negative for downward manipulation.

Suppose a company announces that it will award its executives a call option granted at the money, i.e., an exercise price equal to the current stock price $S_0$. To receive a lower strike price, the executives manipulate the asset price downward to $S_1 = (1+\delta)S_0$ on or near the option grant date. The option price then reduces to $c_1 = c(S_1, K)$.

The FEGS measure is shown in Table 5 for the standard, Asian, and Average-Asian call options where the strike price is 90, 100, and 110, $S_0 = 100$, $r = 10\%$, $\sigma = 40\%$, and $T = 3$, for different values of $\delta$. The FEGS measure is negative as the asset price is manipulated downward.

An FEGS measure of $-1$ indicates that a 1% downward manipulation in the asset price results in a 1% gain in the option value. Interestingly, the Asian option is the most vulnerable to the front-end manipulation and the standard option is the least vulnerable, whereas the average-Asian option is less vulnerable to price manipulation threats than the Asian option.

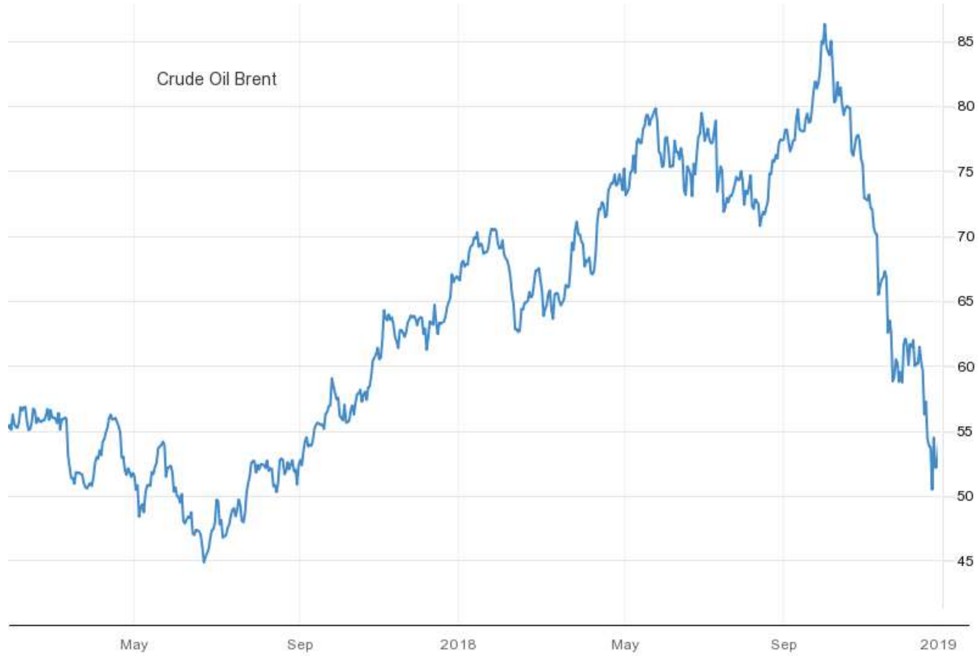


Figure 2: Brent Crude Oil Price for the Time Period of January 2017 to January 2019

Table 5: Front-End Gaming Sensitivity Measure (%) Given $S_0 = 100$, $r = 0.1$, $\sigma = 0.4$, and $T = 3$

| $\delta$ | $K = 90$ | | | $K = 100$ | | | $K = 110$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | Standard | Asian | A.Asian | Standard | Asian | A.Asian | Standard | Asian | A.Asian |
| -0.01 | -1.9592 | -3.0365 | -2.6736 | -2.0615 | -3.3716 | -2.4277 | -2.1601 | -3.7054 | -2.7401 |
| -0.05 | -2.1034 | -3.3716 | -2.7732 | -2.2184 | -3.7701 | -2.9788 | -2.3295 | -4.1716 | -3.3145 |
| -0.10 | -2.3133 | -3.8914 | -3.1283 | -2.4475 | -4.3953 | -3.3780 | -2.5778 | -4.9107 | -3.6624 |
| -0.25 | -3.2464 | -6.6963 | -4.8741 | -3.4765 | -7.8832 | -5.4150 | -3.7034 | -9.1694 | -5.9327 |

### 2.3.2 Back-End Gaming Sensitivity

Once an option contract is written, gain in the option payoff can only be attained from manipulation in the asset price, usually at the time of the option expiry in the case of a standard option and during the life of the option in the case of an Asian option. The option payoff sensitivity, or the back-end gaming sensitivity (BEGS) is defined as

$$\frac{\max[S_T(1+\delta) - K, 0] - \max[S_T - K, 0]}{\delta c_0 e^{rT}}$$

for a standard call option,

$$\frac{\max[\bar{S}_0^T(1+\delta)^{\frac{\Delta t}{T}} - K, 0] - \max[\bar{S}_0^T - K, 0]}{\delta c_0 e^{rT}}$$

for an Asian call option, and

$$\frac{\max[2\bar{S}_0^T(1+\delta)^{\frac{\Delta t}{T}}+S_T-3K,0]-\max[2\bar{S}_0^T+S_T-3K,0]}{\delta c_0 e^{rT}}$$

for an average-Asian call option. Here $T$ is the life in years of the option, $S_T$ is the asset price at expiry, $\bar{S}_0^T$ is the average price from time $t=0$ to $t=T$, $K$ is the strike price, $r$ is the risk-free interest rate, $\delta$ is the percentage change in the asset price due to manipulation, $\Delta t$ is the manipulation period, and $c_0$ is the option price at the time of option contract. Accordingly, $c_0e^{rT}$ is the option price at $t=T$.

Table 6: Back-End Gaming Sensitivity Measure (%) Given $S_0=100$, $r=0.1$, $\sigma=0.4$, and $T=3$

| $\Delta t$ | $\delta$ | $K=90$ | | | $K=100$ | | | $K=110$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Standard | Asian | A.Asian | Standard | Asian | A.Asian | Standard | Asian | A.Asian |
| | 0.1 | 1.4559 | 0.3320 | 0.2869 | 1.5390 | 0.3654 | 0.3097 | 1.6154 | 0.3960 | 0.3210 |
| 0.5 | 0.2 | 1.4823 | 0.3222 | 0.2779 | 1.5737 | 0.3560 | 0.3010 | 1.6601 | 0.3856 | 0.3207 |
| | 0.3 | 1.5032 | 0.3130 | 0.2695 | 1.5991 | 0.3458 | 0.2918 | 1.6990 | 0.3762 | 0.3122 |
| | 0.1 | 1.4588 | 0.6754 | 0.5825 | 1.5370 | 0.7436 | 0.6288 | 1.6151 | 0.8075 | 0.6709 |
| 1.0 | 0.2 | 1.4833 | 0.6634 | 0.5708 | 1.5736 | 0.7357 | 0.6194 | 1.6606 | 0.7999 | 0.6619 |
| | 0.3 | 1.5023 | 0.6518 | 0.5592 | 1.5996 | 0.7251 | 0.6084 | 1.6981 | 0.7947 | 0.6540 |

As shown in Table 6, the standard option is the most vulnerable to the back-end gaming. If the option is granted at the money, i.e., $K=100$, the BEGS measure is 1.5390, 1.5737, and 1.5991 when the percentage change in the asset price due to manipulation is 10, 20, and 30 percent, respectively, and the manipulation is maintained for 6 months. Here a BEGS measure of 1.5390 means that, an increase of 1% in the asset price can provide a gain of 1.5390% in the expected payoff of the option. When $\delta$ is 10, 20, and 30 percent, respectively, and $\Delta t$ is 6 months, the BEGS measure for the Asian option is 0.3654, 0.3560, and 0.3458, which is 76.25%, 77.38%, and 78.38% less sensitive than the corresponding standard option. Meanwhile, for the same values of $\delta$ and $\Delta t$, the BEGS measure for the average-Asian option is 0.3097, 0.3010, and 0.2918, respectively. Hence, the average-Asian option is 79.87%, 80.87%, and 81.75% less sensitive than the corresponding standard option, and 15.24%, 15.46%, and 15.60% less sensitive than the corresponding Asian option. This indicates that the average-Asian option is the least sensitive to asset price manipulation at the back-end gaming. The average-Asian option is hence generally less sensitive than the Asian option not only against the front-end gaming but also against the back-end gaming.

# 3 Option Pricing

In this section, a number of methods are presented to price the standard, Asian, and average-Asian options, followed by an illustration on the differences between them in Table 9. For completeness, the main steps of the methods used in Table 9 are briefly discussed.

A variable changing with time is usually descried by using a differential equation and a variable changing with time randomly is often modeled with a stochastic differential equation (SDE), for example, the Black-Scholes model under the assumption that the price follows a geometric Brownian motion SDE as specified in (3).

Solving (3) yields the dynamics of the asset price

$$S_T = S_0 \exp\big[(\mu - \frac{\sigma^2}{2})T + \sigma\sqrt{T}\varepsilon\big], \tag{6}$$

where $\varepsilon$ follows a standard normal distribution $\mathcal{N}(0,1)$.

### 3.1 Standard Option

For $S = \{S_t\}_{t\in[0,T]}$ given by (3), the price of a stock option $f = f(S,t)$, a function of the underlying stock price $S$ and time $t$, satisfies (7) [Itô, 1951],

$$df = \big(\frac{\partial f}{\partial t} + \frac{\partial f}{\partial S}\mu S + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\big)dt + \frac{\partial f}{\partial S}\sigma S dW. \tag{7}$$

A riskless portfolio composed of the option and stock can then be created to eliminate the Brownian motion from (7) [Black and Scholes, 1973], which results in the Black-Scholes partial differential equation

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf. \tag{8}$$

In risk-neutral valuation, the expected rate of return $\mu$ from an asset is the risk-free interest rate $r$. Hence, $\mu$ is replaced by $r$ in (8). The solution to (8) with terminal and boundary conditions of the European-style call option is the well-known Black-Scholes-Merton option pricing formula for European-style call option in the risk-neutral world, i.e.,

$$c = S_0 N(d_1) - Ke^{-rT}N(d_2),$$

where $N(d_1)$ and $N(d_2)$ are the cumulative probability distribution functions for a standard normal distribution with

$$d_1 = \frac{\ln\frac{S_0}{K} + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}.$$

By put-call parity, that of the corresponding European put option is

$$p = Ke^{-rT}N(-d_2) - S_0 N(-d_1).$$

### 3.2 Asian Option

For an Asian option, the payoff depends on the average price of the underlying asset during the life of the option. The payoff of an Asian call option is $\max(\bar{S}_0^T - K, 0)$ and that of an Asian put option is $\max(K - \bar{S}_0^T, 0)$, where $\bar{S}_0^T$ is the average price of the underlying asset from $t = 0$ to $t = T$, inclusively, and $K$ is the strike price. Two popularly referred types are the geometric and arithmetic average options.

### 3.2.1 Geometric Asian Option

The continuous arithmetic average is given by

$$A_0^T = \frac{1}{T}\int_0^T S_t dt,$$

and the continuous geometric average is defined as

$$G_0^T = \exp\big(\frac{1}{T}\int_0^T \ln S_t dt\big).$$

The Black-Scholes model relies on the assumption that the underlying price follows a lognormal distribution. The geometric average $G_0^T$ of a lognormally-distributed random variable is also lognormally distributed. The expectation and variance of $G_0^T$ can then be derived to price the European style average call and put options [Kemna and Vorst, 1990]. Specifically, for the continuous case,

$$\log G_0^T = \mathcal{N}\Big(\frac{1}{2}\big(r - \frac{1}{2}\sigma^2\big)T + \log S_0, \frac{1}{3}\sigma^2 T\Big),$$

where $\mathcal{N}(a,b)$ represents a normal distribution with mean $a$ and variance $b$. Substituting the expectation and variance of $G_0^T$

$$\mu_G = \frac{1}{2}\big(r - \frac{1}{2}\sigma^2\big)T + \log S_0 \quad \text{and} \quad \sigma_G^2 = \frac{1}{3}\sigma^2 T$$

into the generalized version of Black's model [Black, 1976]

$$c = e^{-rT}\Big(e^{\mu_G + \frac{1}{2}\sigma_G^2} N\big(\frac{\mu_G + \sigma_G^2 - \log K}{\sigma_G}\big) - KN\big(\frac{\mu_G - \log K}{\sigma_G}\big)\Big)$$

yields the formulas for geometric Asian call and put options

$$c = e^{-rT}\big[F_0 N(d_1) - KN(d_2)\big] \quad \text{and} \quad p = e^{-rT}\big[KN(-d_2) - F_0 N(-d_1)\big],$$

where for $a = \frac{1}{2}(r - \frac{\sigma^2}{6})$, $F_0 = S_0 e^{aT}$, and for $\tilde{\sigma} = \frac{\sigma}{\sqrt{3}}$, $N(d_1)$ and $N(d_2)$ are the cumulative probability distribution functions of a standard normal distribution, with

$$d_1 = \frac{\ln(\frac{S_0 e^{aT}}{K}) + \frac{1}{2}\tilde{\sigma}^2 T}{\tilde{\sigma}\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \tilde{\sigma}\sqrt{T}.$$

### 3.2.2 Arithmetic Asian Option

The arithmetic average price of the underlying asset, $\bar{S}_0^T$ is defined as

$$\bar{S}_0^T = \frac{1}{m+1}\sum_{i=0}^{m} S_{t_i},$$

where $t_i = t_0 + \frac{t_m - t_0}{m} i$, $i = 0, \ldots, m$, with $t_0 = 0$ and $t_m = T$.

The Black-Scholes model and Black's model rely on the assumption that the underlying price follows a lognormal distribution, while the arithmetic average of a lognormally-distributed random variable is not lognormally distributed in general.

To value the arithmetic average Asian options by using Black's model, heuristically, it is assumed that $\bar{S}_0^T$ is lognormally distributed with respect to the first and second moments $M_1$ and $M_2$ [Turnbull and Wakeman, 1991],

$$M_1 = \frac{e^{rT}-1}{rT}S_0 \quad \text{and} \quad M_2 = \frac{2e^{(2r+\sigma^2)T}S_0^2}{(r+\sigma^2)(2r+\sigma^2)T^2} + \frac{2S_0^2}{rT^2}\left(\frac{1}{2r+\sigma^2} - \frac{e^{rT}}{r+\sigma^2}\right).$$

Then

$$c = e^{-rT}\big[F_0N(d_1) - KN(d_2)\big] \quad \text{and} \quad p = e^{-rT}\big[KN(-d_2) - F_0N(-d_1)\big],$$

where $F_0 = M_1$ and for $\sigma^2 = \frac{1}{T}\ln\frac{M_2}{M_1^2}$, $N(d_1)$ and $N(d_2)$ are the cumulative probability distribution functions of a standard normal distribution,

$$d_1 = \frac{\ln\frac{F_0}{K} + \frac{\sigma^2}{2}T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}.$$

## 3.3 Average-Asian Option

The average-Asian option is contingent on both the price at expiry and the average price during the life of the option. In the absence of a closed-form solution, numerical methods such as the binomial option pricing model and Monte Carlo simulation are usually the appropriate ones for option pricing.

### 3.3.1 Binomial Option Pricing Model

For the binomial option pricing model [Cox et al., 1979; Mintz, 1997], consider an $m$-step binomial tree and sample from the $2^m$ possible paths. Suppose that the probability of an up movement is $p$ and that of a down movement is $1-p$. The procedure is as follows.

1. At each node, a uniformly distributed random number between 0 and 1 is generated. It takes an up movement if the number is greater than or equal to $1-p$ and a down movement otherwise.

2. Once the path from the initial node $t=0$ to the end of the tree $t=T$ is complete, the price at expiry $S_T$ as well as the average price $\bar{S}_0^T$ are obtained.

3. The payoff of the average-Asian option is then calculated. This completes one trial.

4. Additional trials are generated by following the same procedure.

5. The mean of the payoffs is discounted at the risk-free interest rate for an estimate of the value of the average-Asian option.

To illustrate the method, as shown in Table 7, consider a three-month arithmetic average-Asian option with $S_0 = 100$, $K = 100$, $r = 10\%$, $\sigma = 40\%$, $T = 0.25$, and divide the life of the option into seven intervals, i.e., $\Delta t = \frac{T}{m} = \frac{0.25}{7} = 0.0357$. At each interval, the underlying price moves up or down by a factor $u$ or $d$, with $u \geq 1$ and $0 < d \leq 1$. By the binomial option pricing model, with $u = e^{\sigma\sqrt{\Delta t}} = e^{0.4\times\sqrt{0.0357}} = 1.0785$, $d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.4\times\sqrt{0.0357}} = 0.9272$, and $a = e^{r\Delta t} = e^{0.1\times 0.0357} = 1.0036$, the risk-neutral probability of an upward movement is $p = \frac{a-d}{u-d} = \frac{1.0036-0.9272}{1.0785-0.9272} = 0.50475$. The upward and downward movements are then $S_u = S_0u$ and $S_d = S_0d$, respectively. The prices at the other notes are calculated similarly.

Table 7: Binomial Tree for Stock Price Movement with Seven Time Steps

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | 169.749 |
| | | | | | | 157.39 | |
| | | | | | 145.931 | | 145.931 |
| | | | | 135.306 | | 135.306 | |
| | | | 125.455 | | 125.455 | | 125.455 |
| | | 116.321 | | 116.321 | | 116.321 | |
| | 107.852 | | 107.852 | | 107.852 | | 107.852 |
| 100 | | 100 | | 100 | | 100 | |
| | 92.7194 | | 92.7194 | | 92.7194 | | 92.7194 |
| | | 85.9688 | | 85.9688 | | 85.9688 | |
| | | | 79.7097 | | 79.7097 | | 79.7097 |
| | | | | 73.9063 | | 73.9063 | |
| | | | | | 68.5255 | | 68.5255 |
| | | | | | | 63.5364 | |
| | | | | | | | 58.9105 |

For more details, Table 8 outlines results of twenty trial paths. Based on the sample paths shown in Table 8, the value of an Asian call option is $c_{\text{Asian}} = 5.70e^{-0.1\times 0.25} = 5.56$ and that of an average-Asian call option is $c_{\text{A.Asian}} = 4.37e^{-0.1\times 0.25} = 4.26$.

Table 8: Trial Paths for Pricing Asian and Average-Asian Options by Binomial Option Pricing Model Given $S_0 = 100$, $K = 100$, $r = 0.1$, $\sigma = 0.4$, and $T = 0.25$

| Trial | Path | $S_T$ | $\bar{S}_0^T$ | Asian Payoff | A.Asian Payoff |
|---|---|---|---|---|---|
| 1 | UUDUUDD | 107.85 | 112.25 | 12.25 | 8.09 |
| 2 | UUUDUUD | 125.46 | 119.02 | 19.02 | 15.87 |
| 3 | UDDUUUU | 125.46 | 106.28 | 6.28 | 9.50 |
| 4 | DUUUUUD | 125.46 | 112.89 | 12.89 | 12.81 |
| 5 | UDUDDDD | 79.71 | 96.76 | 0 | 0 |
| 6 | UUDUDDD | 92.72 | 106.11 | 6.11 | 1.24 |
| 7 | UUDDDUU | 107.85 | 104.07 | 4.07 | 4.00 |
| 8 | UDUUDDD | 92.72 | 104.07 | 4.07 | 0.22 |
| 9 | UDUDUDU | 107.85 | 103.93 | 3.93 | 3.93 |
| 10 | UUUUDDD | 107.85 | 116.82 | 16.82 | 10.37 |
| 11 | UDUDUDD | 92.72 | 102.03 | 2.03 | 0 |
| 12 | DUDDUUU | 107.85 | 96.50 | 0 | 0.21 |
| 13 | UDUUDUD | 107.85 | 108.01 | 8.01 | 5.97 |
| 14 | DUUUDUD | 107.85 | 106.11 | 6.11 | 5.02 |
| 15 | DDDUDUD | 79.71 | 86.22 | 0 | 0 |
| 16 | UDUUUDD | 107.85 | 110.21 | 10.21 | 7.07 |
| 17 | DUDUUUD | 107.85 | 102.18 | 2.18 | 3.05 |
| 18 | DDUDUUU | 107.85 | 94.74 | 0 | 0 |
| 19 | DDDUDUD | 79.71 | 86.22 | 0 | 0 |
| 20 | DDDUUUD | 92.72 | 91.23 | 0 | 0 |
| Mean | | | | 5.70 | 4.37 |

In practice, more time steps, usually at least 30, as well as more simulation trials are necessary to obtain an accurate approximation. In Table 9, which shows the prices of the standard, Asian, and average-Asian arithmetic and geometric call options using different methods, 100 equal time intervals and 140,000 simulations are conducted for the binomial option pricing model.

### 3.3.2 Monte Carlo Simulation

The binomial option pricing model assumes that the price movement follows a binomial distribution, which approaches the lognormal distribution implied by the Black-Scholes-Merton option pricing model for a large number of trials. The two resulting prices are then expected to coincide [Cox et al., 1979].

By the Black-Scholes model, assume that the asset price follows a geometric Brownian motion specified by (3), the solution to which yields the dynamics of the asset price given in (6). The average-Asian option can then be priced using Monte Carlo simulation by following the procedure below.

1. Divide the option life $T$ by the number of time intervals $m$, i.e., $dt = \frac{T}{m}$.
2. Simulate the stock price given by the solution to the geometric Brownian motion SDE (6), using the time step $dt$ instead of $T$ and the risk-free interest rate $r$ instead of $\mu$.
3. Repeat step 2 for $m$ times by updating continuously $S_0$ with $S_T$ for each time and calculate the cumulative price at each step.
4. After $m$ times, the final price is $S_T$ and the final average price is $\bar{S}_0^T$, which are used in the payoff of the average-Asian option. This completes one trial.
5. Repeat steps 1 to 4 to perform $n$ trials.
6. Discounting the average payoff from the $n$ trials with the risk-free interest rate yields the average-Asian option price.

Values of the standard, Asian, and average-Asian call options using Monte Carlo simulation, along with other methods, are reported in Table 9, where 140,000 simulations are performed and for each simulation, 100 time steps are taken, with $S_0 = 100$, $K = 100$, $r = 10\%$, $\sigma = 40\%$, and $T = 1$.

Table 9: Prices of Standard, Asian, and Average-Asian Options by Using Different Methods Given $S_0 = 100$, $K = 100$, $r = 0.1$, $\sigma = 0.4$, and $T = 1$

| Pricing Method | Standard | Asian | | A.Asian | |
|---|---|---|---|---|---|
| | | Arithmetic | Geometric | Arithmetic | Geometric |
| Black-Scholes-Merton Model | 20.32 | - | - | - | - |
| Binomial Option Pricing Model | 20.27 | 11.12 | 10.25 | 10.38 | 9.94 |
| Monte Carlo Simulation | 20.33 | 11.11 | 10.26 | 10.38 | 9.95 |
| Kemna and Vorst [1990] | - | - | 10.27 | - | - |
| Turnbull and Wakeman [1991] | - | 11.23 | - | - | - |

A '-' means that a price cannot be obtained by using that particular method.

As illustrated in Table 9, the two prices, obtained by using the binomial option pricing model and Monte Carlo simulation method, indeed converge.

Table 10 presents prices of the standard, Asian, and Average-Asian call and put options with two sets of parameter values and different strike prices.

Table 10: Prices of Standard, Asian, and Average-Asian Options with Different Strike Prices

| $S_0$ | $r$ | $\sigma$ | $T$ | $K$ | | Arithmetic | | | Geometric | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | Standard | Asian | A.Asian | Standard | Asian | A.Asian |
| 100 | 0.15 | 0.45 | 1 | 90 | Call | 29.5142 | 18.5279 | 16.3836 | - | 17.2313 | 15.7362 |
| | | | | 95 | | 26.8550 | 15.6389 | 14.2697 | - | 14.3920 | 13.6410 |
| | | | | 100 | | 24.4169 | 13.0875 | 12.3747 | - | 11.9187 | 11.7770 |
| | | | | 105 | | 22.1986 | 10.8866 | 10.7018 | - | 9.7933 | 10.1411 |
| | | | | 110 | | 20.0891 | 8.9695 | 9.1930 | - | 7.9594 | 8.6825 |
| | | | | 90 | Put | 6.9426 | 3.1156 | 3.0346 | - | 3.4871 | 3.2365 |
| | | | | 95 | | 8.6260 | 4.5421 | 4.1641 | - | 4.9772 | 4.3883 |
| | | | | 100 | | 10.5116 | 6.3152 | 5.5122 | - | 6.7892 | 5.7360 |
| | | | | 105 | | 12.5417 | 8.3809 | 7.0347 | - | 8.9445 | 7.3064 |
| | | | | 110 | | 14.8014 | 10.7885 | 8.7806 | - | 11.4261 | 9.0774 |
| 50 | 0.1 | 0.4 | 1 | 46 | Call | 12.1803 | 7.7255 | 6.7821 | - | 7.2453 | 6.5500 |
| | | | | 48 | | 11.1109 | 6.5649 | 5.9336 | - | 6.1158 | 5.7156 |
| | | | | 50 | | 10.1553 | 5.5520 | 5.1856 | - | 5.1188 | 4.9659 |
| | | | | 52 | | 9.2438 | 4.6454 | 4.4994 | - | 4.2441 | 4.2989 |
| | | | | 54 | | 8.4276 | 3.8787 | 3.9043 | - | 3.4960 | 3.7054 |
| | | | | 46 | Put | 3.8133 | 1.7693 | 1.7123 | - | 1.9463 | 1.8077 |
| | | | | 48 | | 4.5779 | 2.4320 | 2.2339 | - | 2.6300 | 2.3346 |
| | | | | 50 | | 5.4004 | 3.2107 | 2.8262 | - | 3.4382 | 2.9411 |
| | | | | 52 | | 6.3047 | 4.1205 | 3.5022 | - | 4.3741 | 3.6269 |
| | | | | 54 | | 7.2873 | 5.1503 | 4.2550 | - | 5.4265 | 4.3845 |

A '-' indicates that a price does not exist.

As shown in Table 10, for different values of the strike price, an important factor in determining the option value, the average-Asian call option is consistently cheaper than the corresponding Asian call option when the option is in the money and slightly cheaper when the option is at the money. For example, for the first set of parameters with $S_0 = 100$, $r = 15\%$, $\sigma = 45\%$, and $T = 1$, when an option is granted deep in the money, i.e., $K = \$90$, the standard option costs \$29.5142, the Asian option costs \$18.5279, whereas the average-Asian option costs \$16.3836. The price of the latter is hence 55.511% and 88.426% of those of the standard and Asian counterparts, respectively. Similarly, when the option is granted at the money, i.e., $K = \$100$, the price of the average-Asian option is 50.68% of that of the standard option and 94.55% of that of the Asian option accordingly.

Table 10 suggests that the resulting price of the average-Asian option, compared with those of the standard and Asian counterparts, is relatively more stable in the real-life world, where the strike price is usually unlikely to be too far out of the money. The average-Asian option is hence an alternative worth considering to reduce the volatility inherent in the option price.

# 4 Conclusion

In this paper, a new path-dependent option, referred to as the average-Asian option, is introduced to reduce the price of the option and to minimize the adverse effect of asset price jumps and the potential market manipulation threat.

The price of the average-Asian call option is proved to be less than that of the standard option. Based on option pricing in the Black-Scholes world, numerical results show that the proposed average-Asian option is cheaper than the Asian counterpart in almost all the practical scenarios. It also reduces more effectively the potential market manipulation threat and adverse effect of sudden price jumps.

In the executive compensation context, the average-Asian option is more cost effective than the Asian counterpart when the option is granted both in the money and at the money. Besides, the average-Asian option is also less sensitive to managerial manipulation at both the front-end and back-end gaming.

As discussed in Section 1.2, an option with a lower up-front premium is generally more desirable. The average-Asian option may then have non-negligible practical importance. The dependence of the option payoff on both the average price of the underlying during the life of the option and the price at expiry makes the average-Asian option a potentially valuable hedging instrument. In addition, as the Asian option is commonly adopted in the financial markets and well understood by market participants as well as academics, the proposed average-Asian option would be naturally straightforward to be perceived and traded, instead of being considered as an abstract, technical, and complicated derivative.


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