School of Civil and Mechanical Engineering

Integrated Frameworks for Effective Multi-criteria Decision Making in Reliability Centred Maintenance of Industrial Machines

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DEDICATED TO

My parents and my beloved sister

The almighty

ABSTRACT

Reliability centred maintenance (RCM) is a corporate maintenance strategy, which provides a structured framework to analyse the functions and potential failures of an asset, focusing mainly on preserving its functions. It emphasizes on the principle of *'maximum availability vs. minimum cost'* of an asset. However, during its implementation, organizations are often confronted with different types of multiple-criteria based decision making situations such as prioritization of failure modes as per their risk levels, identifying their causes and effects (FMECA), diagnosing the faults at the earliest possible opportunity even with missing health indicators, and aiding the system with an appropriate maintenance strategy. The present research work is an attempt to address the aforementioned decision-making issues in a more abstract way and by illustrating the proposed solutions through a case study on a process plant gearbox.

Several integrated multi-criteria decision-making (MCDM) frameworks are proposed to overcome some major shortcomings of the traditional risk priority number (RPN) based failure modes ranking approach. Besides, the impacts of linguistic uncertainties are gradually minimized on the final risk ranking results by proposing the mathematical models of modified fuzzy multi-attributive ideal real comparative analysis (fuzzy MAIRCA), modified fuzzy measurement of alternative and ranking according to compromise solution (fuzzy MARCOS), extended interval type-2 fuzzy decision making trail and evaluation laboratory (IT2F-DEMATEL), IT2F-MAIRCA, IT2F-MARCOS, and modified IT2F-technique for order of preference by similarity to ideal solution (IT2F-TOPSIS). Apart from that, the causes and effects of different failure modes of the considered case study are identified from the triple bottom line (TBL) of sustainability with the aim of easing the implementation of sustainable manufacturing practices. The concept of half quadratic (HQ) minimization is utilized to address the issue of disparate risk ranking results by different MCDM methods through consensus index and trust level values.

Next, a decision support system based on case-based reasoning (CBR) methodology is developed for the fault diagnosis of the gearboxes at the earliest possible opportunity, considering the situation of incomplete information about multiple health indicators. Other than that, the developed system advises the engineers with the best possible maintenance tasks which are required to be performed after fault diagnosis.

Finally, a hybrid artificial intelligence-based framework is proposed for choosing the optimal maintenance strategy after identifying the key performance indicators for sustainability-based

maintenance strategy selection problems. This framework is developed to overcome the drawbacks of the principles of the MCDM methods by exploiting the advantages of both CBR and expert systems (ES). The outcomes of this research have culminated in the publications of four international refereed journal papers, one international conference paper and one book chapter. Another book chapter has also been communicated recently.

Keywords

Maintenance Decision Making, Multi-Criteria Decision Making, Fault Diagnosis, Case-Based Reasoning, Artificial Intelligence, Fuzzy Sets, Expert Systems, Machine Learning.

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List of Abbreviations

Abbreviation	Full Form
4-R	Retrieve, Reuse, Revise, and Retain
AHP	Analytic Hierarchy Process
AI	Artificial Intelligence
AID Solution	Anti-Ideal Solution
ANFIS	Adaptive Neuro-Fuzzy Inference System
ANN	Artificial Neural Network
ANP	Analytic Network Process
AQM	Alternative Queuing Method
BM	Breakdown Maintenance
BN	Bayesian Network
BPNN	Back Propagation Neural Network
BWM	Best-Worst Method
CBM	Condition-Based Maintenance
CBR	Case-Based Reasoning
CI	Consensus Index
CMMS	Computerized Maintenance Management System
CNC	Computer Numerical Control
CNN	Convolution Neural Network
COPRAS	COmplex PRoportional ASsessment
CR	Consistency Ratio
CWT	Continuous Wavelet Transformation
D	Detectability
DAC	Data Acquisition System
DE	Decision Experts
DEMATEL	DEcision MAking Trial and Evaluation Laboratory
D-S Evidence Theory	Dempster-Shafer Evidence Theory
DWT	Discrete Wavelet Transformation
ELECTRE	ELimination Et Choice Translating REality
ERP	Enterprise Resource Planning
ESs	Expert Systems
FIS	Fuzzy Inference System
FL	Fuzzy Logic
FMEA	Failure Modes and Effects Analysis
FMECA	Failure Modes, Effects, and Criticality Analysis
FNN	Fully Connected Neural Network
FNs	Fuzzy Numbers
FSs	Fuzzy Sets
GA	Genetic Algorithm
GRA	Grey Relational Analysis
GRPM	Grey Relational Projection Method
GTM approach	Graph Theory and Matrix Approach
GTrFNs	Generalized Trapezoidal Fuzzy Numbers
HI	Health Indicator
HoR	House of Reliability
HQ Programming	Half Quadratic Programming
HTLTSs	Hesitant 2-Tuple Linguistic Term Sets
ICA	Independent Component Analysis

Abbreviation	Full Form
ID Solution	Ideal Solution
IEC	International Electrotechnical Commission
IFSs	Intuitionistic Fuzzy Sets
ISO	International Organization for Standardization
IT2FN	Interval Type-2 Fuzzy Number
IT2FSs	Interval Type-2 Fuzzy Sets
IT2LFSs	Interval 2-Tuple Linguistic Fuzzy Sets
ITLVs	Interval 2-Tuple Linguistic Variables
IVIFSs	Interval-Valued Intuitionistic Fuzzy Sets
k-NN	k-Nearest Neighbour
KPI	Key Performance Indicator
LMMM Section	Light and Medium Merchant Mill Section
MABAC	Multi-Attributive Border Approximation Area Comparison
MAIRCA	Multi-Attributive Ideal Real Comparative Analysis
MARCOS	Measurement of Alternatives and Ranking according to COmpromise Solution
MCAP	Multi-Criteria Aggregation Procedure
MCDM	Multi-Criteria Decision-Making
MF	Membership Function
MLPNN	Multi-Layer Perceptron Neural Network
MMSM Section	Medium Merchant and Structural Mill Section
MOORA	Multi-Objective Optimization by Ratio Analysis
MSSP	Maintenance Strategy Selection Problem
MULTIMOORA	Multi Objective Optimization by Ratio Analysis plus full Multiplicative Form
NIS	Negative Ideal Solution
O	Likelihood of Occurrence
ORESTE	Organization, Rangement Et Synthese De Donnes Relationnelles
OWG Operator	Ordered Weighted Geometric Operator
PCA	Principle Component Analysis
PdM	Predictive Maintenance
PFSs	Pythagorean Fuzzy Sets
PHM	Prognostics and Health Management
PIS	Positive Ideal Solution
PLTSs	Probabilistic Linguistic Term Sets
PM	Preventive Maintenance
PNN	Probabilistic Neural Network
PROMETHEE	Preference Ranking Organization METHod for Enrichment of Evaluations
PSO	Particle Swarm Optimization
QUALIFLEX	QUALItative FLEXible multiple criteria method
R2F Maintenance	Run-to-Failure Maintenance
RBM	Rule-Based Method
RBR	Rule-Based Reasoning
RCM	Reliability Centred Maintenance
RF Algorithm	Random Forest Algorithm
RoI	Return on Investment
RPN	Risk Priority Number
S	Severity
SAW	Simple Additive Weighting

Abbreviation	Full Form
SDG	Sustainable Development Goal
SVM	Support Vector Machine
T1FSs	Type-1 Fuzzy Sets
T2FSs	Type-2 Fuzzy Sets
TBL	Triple Bottom Line
TBPM	Time-Based Preventive Maintenance
TFNs	Triangular Fuzzy Numbers
TL	Trust Level
TODIM	An acronym in Portuguese for interactive multi-criteria decision making
TOPSIS	Technique for Order Preference by Similarity to Ideal Solution
TPM	Total Productive Maintenance
TrFNs	Trapezoidal Fuzzy Numbers
TrIT2FNs	Trapezoidal Interval Type-2 Fuzzy Numbers
VIKOR	VlseKriterijumska Optimizacija I Kompromisno Resenje or multi-criteria optimization and compromise solution
WSs	Weight Sets

Chapter 1 Introduction

During earlier days, it was perceived that each component of a system has a specific age, and after that the complete overhaul of the system is compulsory to ensure safe and failure free operation. However, the demerits of this thought became clear in the 1960s, during the development of the preventive maintenance (PM) program for the Boeing 747 aircraft. The study investigated the failure characteristics of aircraft components, and the observations were outlined in the *Handbook for the Maintenance Evaluation and Program Development for Boeing 747*, also known as MSG-1 (*Maintenance Steering Group – 1*). Afterwards, MSG – 1 was revised and became MSG – 2. In 1979, the *Air Transport Association* (ATA) again revised the MSG – 2 to finalize the draft of MSG – 3, which was employed for the maintenance of Boeing 757 and Boeing 767 aircraft. In another project, *United Airlines*, funded by *US Department of Defence* prepared a comprehensive report, finding the relationships between maintenance, reliability, and safety. The report was prepared by Nowlan and Heap, and became popularized as *Reliability Centred Maintenance* (RCM) (Nowlan and Heap, 1978). The report found out that the periodic overhauls have impacts on 11% of failures, whereas 89% of the failures have occurred randomly. Therefore, new thinking was sought to deal with these 89% of failures.

It is well known that assets are the centrepiece of any organization. The term *'asset'* can refer to an equipment, machine, tool, vehicles, *etc.* However, in today's rapidly progressing business scenario, industries are deeply concerned about the downtimes of these assets which can be the reasons for the major to minor losses (*e.g.*, loss of production, poor quality of product leading to customer dissatisfactions, *etc.*). Although maintenance engineers put their best efforts to keep the assets running, businesses also try to squeeze as much value out of their assets as possible. Therefore, it is vital for the organizations to implement the asset management practices, which intends to maximize the return on investment (ROI) value of an asset during its entire lifecycle.

Maintenance, being a significant part in the asset lifecycle (refer Figure 1.1) consists of multiple tasks, which are required to be performed efficiently in order to maintain it in the functioning state. Knezvic (Knezevic, 1993) defined maintenance tasks as *"a set of activities which need to be performed, in specified manner, by the user in order to maintain the functionability of the item/system."* While, *maintenance management* focuses on the performance of maintenance activities, coordination of maintenance resources including parts, labours, budgets, *etc.*

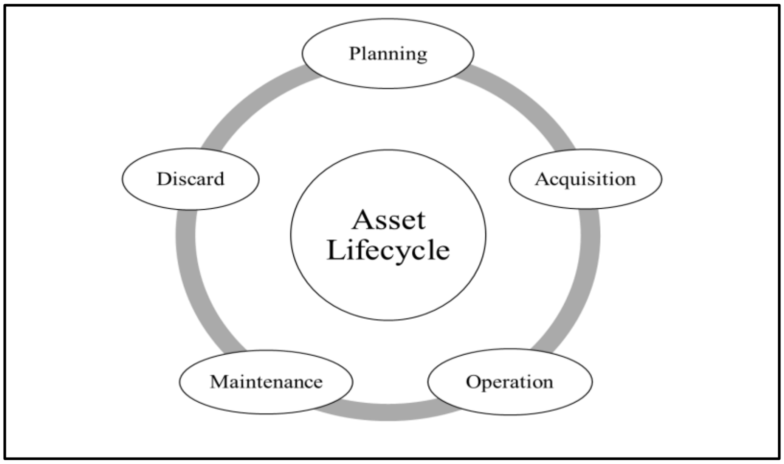

Figure 1.1. Asset lifecycle process

From the historical timeline, the concept of maintenance has advanced from breakdown maintenance (BM)/run-to-failure (R2F) maintenance to PM to condition-based maintenance (CBM). But, now-a-days organizations prefer to perform the maintenance tasks using the just-in-time approach, based on the current health condition of the asset, and without hampering the ongoing operations. This requirement has laid the foundation of CBM philosophy, which can be further classified into two major domains: diagnostics and prognostics. The prior one deals with fault detection, isolation, and identification, whereas the latter one with fault prediction before the occurrence. In other words, fault prediction is a practice to determine whether a fault is impending and to estimate how shortly and how likely the fault will occur. Thus, diagnostics is a posterior event, and prognostics is a prior event analysis, which is more competent than diagnostics to achieve zero down-time performance. However, diagnostics is required when the fault prediction in prognostics fails and a fault ensues.

Considering the financial burdens involved in each maintenance practice, now-a-days, organizations are emphasizing on the concept of *"maximum availability vs. minimum cost"*, which further entails into the implementation of RCM practice. This philosophy considers the maintenance of a system from its functional point of view, instead of the operational perspectives (JA1011_199908, 1999). It emphasizes the cost-effective motive of the organization by identifying and devising operational, maintenance policies and strategies. It is a compatible maintenance strategy for situations of low or limited financial resources, while preserving the critical plant functions (Moubray, 2001). It features the utilization of predictive maintenance (PdM) (*i.e.,* CBM or prognostics and health management (PHM)), in addition to the traditional PM. Basically, RCM considers R2F policy as a viable option for non-critical systems, and preserves the critical equipment by employing the PM or PdM policy. Yet, it is not feasible for an organization to aid every machine with the CBM because of its high implementation cost. Thus, engineers must think about the effectiveness of using the different maintenance philosophies for each machine.

The RCM philosophy starts with defining the system, its surroundings, then its failure modes, their cause(s), effect(s), and criticality ranking by a comprehensive *Failure Mode(s) and Effect(s) and Criticality Analysis* (FMECA); then to detect, isolate, and classify the fault at the earliest possible opportunity; and lastly to support the system with the optimal maintenance strategy.

FMEA[1] is a popular methodology among the risk and reliability engineers to identify the failure mode(s), their cause(s), effect(s), and to rank the failure modes according to their risk levels. This is a proactive approach to mitigate/eliminate the occurrences of failures in a system. The risks of failure modes of a system are calculated by multiplying three risk factors, namely severity (S), occurrence (O), and detection (D), and expressed in terms of risk priority numbers (RPN) (IEC 60812:2018). However, the traditional RPN-based FMEA approach has multiple shortcomings as pointed out by the earlier researchers (Liu *et al.*, 2013, 2019a). Some of the major drawbacks are: not considering the relative importance among the risk factors, linguistic/crisp evaluations of failure modes with respect to the risk factors without considering their inherent uncertainties and vagueness, improper justifications of the multiplicative formula for calculating the RPN values, consideration of only three risk factors without describing the hidden risk implications, *etc*. (Gargama and Chaturvedi, 2011). All of these drawbacks have made researchers to consider the FMECA approach as a complex multiple criteria-based decision-making task, which needs to be focused at the initial stage of the RCM implementation.

Nevertheless, despite the best possible efforts to eliminate the occurrences of failures of the systems, they still occur. So, it is another burden to the engineers to arrest the failures at the earliest possible opportunity, certainly at the onset of the fault. This is usually carried out by observing the deviations of some specified *health indicators* (HIs), *viz.*, different signals emitting from the system (*e.g.*, vibrations, sounds, etc.) and/or images (*e.g.*, thermography), *etc*. Initially, the signal(s) are collected by means of some sophisticated sensors/gadgets, and later they are analysed through some software to diagnose the fault. However, in case of a large and complex system[2], which is challenging to simulate mathematically, the task of collecting the signals from each of the pre-specified point, and further analysing them separately becomes a daunting task to the engineers. Besides, it is believed that the harsh environmental conditions have a considerable impact on the

[1] In this thesis FMECA and FMEA are used interchangeably.
[2] A complex system is a system composed of many components which may interact with each other.

occurrences of faults. In this way, when the considered HIs proliferate in number, it becomes a arduous task for the engineers to map all the HIs from the measurement space to the fault space to diagnose the fault. Then they are compelled to proceed with incomplete or missing data, which is regarded as another multiple criteria-based decision-making problem in RCM.

Finally, the system should be supported with an optimal maintenance strategy for improving the system availability. In the context of sustainable manufacturing practices, it is essential that the associated processes are also sustainable. As the maintenance tasks are considered as an integrated part of the manufacturing/production system, thus, it becomes necessary to identify the pertinent *key performance indicators* (KPIs) for selecting the best maintenance strategy from the *Triple Bottom Line* (TBL) of sustainability (*i.e.,* economic, social, and environmental perspectives). However, when the number of associated criteria and choices/alternatives are increased, and their complex interrelationships are hard to interpret, selection of the best maintenance strategy also becomes a complex decision-making task, which again needs to be appropriately managed.

1.1. Decision-Making Problems in RCM

The foregoing discussions reveal some key decision-making areas in the milieu of implementation of RCM, which need further research. They are summarised below:

- Identification of the potential failure mode(s) of the item(s)/system(s), their cause(s), effect(s), as well as prioritizing them according to their risk levels by performing a detailed FMECA.

- Diagnosing the faults of a system at the earliest possible opportunity to avoid the catastrophic incident.

- Supporting the system with the best (sustainable) maintenance strategy.

In the next sub-sections, the above decision-making areas and their related problems are further detailed for the completeness of the thesis.

1.1.1. Failure Mode(s), Effect(s) and Criticality Analysis

According to IEC 60812:2018 the FMEA is defined as:

> *"…a systematic method of evaluating an item or process to identify the ways in which it might potentially fail, and the effects of the mode of failure upon the performance of the item or process and on the surrounding environment and personnel."*

Whereas, the FMECA is explained as:

> *"…ranking of criticality involves at least the severity of consequences, and often other measures of importance…"*

Before proceeding further, it is required to look through some necessary terms associated with FMECA.

- *Failure mode* is how a failure occurs.
- *Failure effects* are the consequences of a failure, within or beyond the boundary of the system.
- *Failure causes* are the set of circumstances that lead to failure.
- *Failure mechanism* is the process that leads to failure.
- *Likelihood* is the chances of occurring something.
- *Severity* is the relative ranking of potential or actual consequences of a failure/fault.
- *Detection* method implies the means by which a failure mode or incipient failure becomes identified.
- *Criticality* of a failure mode is the importance ranking computed by means of a definite evaluation criterion.

Although FMEA can be performed at the different stages during the life cycle of the system or process, yet maximum benefits are achieved if it is executed at the earlier stages of the life cycle. For example, a preliminary analysis can be carried out during the design or planning phase, followed by a detailed analysis all through the operation stage of the item/process, when more information becomes available. In fact, in the closed loop analysis of FMEA, it allows for the evaluation of the effectiveness of any treatment. According to the domain of application, FMEA can be classified as: system FMEA, process FMEA, design FMEA, service FMEA, software FMEA, manufacturing FMEA, *etc.*

When FMECA is performed appropriately, it provides enhanced outcomes to the organizations, such as improved reliability of the system, reduced environmental effects, procurement costs of the spares, operating costs of the system, enhanced business reputations, etc. Other major reasons for conducting FMEA can be:

- To identify the failure modes which have unwanted effects on system operation.
- To improve the design and development of items or processes in a cost-effective manner by intervening at the earliest possible opportunity.
- To identify the risks as part of the risk management process as given in ISO 31000 (Purdy, 2010).
- To provide a foundation for other associated tasks, such as maintenance analysis, troubleshooting tactics during maintenance, testability analysis, logistics support analysis, mission reliability analysis, availability analysis, etc.
- To develop and support the reliability test programme.
- A basis for implementing RCM (IEC 60300-3-11).
- To efficiently manage the asset management program (ISO 55000:2014 (E)).

Along with the above notable points, it is necessary to prioritize the failure modes according to their risk levels to effectively arrange the maintenance resources at the onset of failures. In IEC 60812:2018, four types of approaches are mentioned for the risk ranking of the failure modes:

a) The criticality matrix,

b) The criticality plot,

c) The RPN-based approach,

d) Alternative risk priority number.

However, among these methods, the criticality matrix, and RPN-based approaches have been widely adopted by the organizations. In RPN-based approach, the RPN values of failure modes are calculated by employing (1.1). The flowchart of traditional RPN-based FMEA approach is presented in Figure 1.2.

$$
\begin{aligned}
&Risk\ Priority\ Number \\
&\qquad = Severity\ (S) \times Likelihood\ of\ Occurrence(O) \\
&\qquad \times Detectability(D)
\end{aligned} \tag{1.1}
$$

The RPN elements (*viz.*, severity, likelihood of occurrence, and detectability) in (1.1) are evaluated either quantitatively (using scale values between 1-10), qualitatively (*i.e.*, low, very low, medium, etc.), or semi-quantitatively by cross-functional experts. Yet, it is observed that instead of providing crisp judgements, experts generally prefer to provide their linguistic judgements while evaluating the failure modes. Even in some cases, the linguistic judgements are converted into crisp numbers by following a scale, and then they are multiplied to obtain the RPN values. However, this approach has some major pitfalls, which are listed below (Gargama and Chaturvedi, 2011; Liu *et al.*, 2019a):

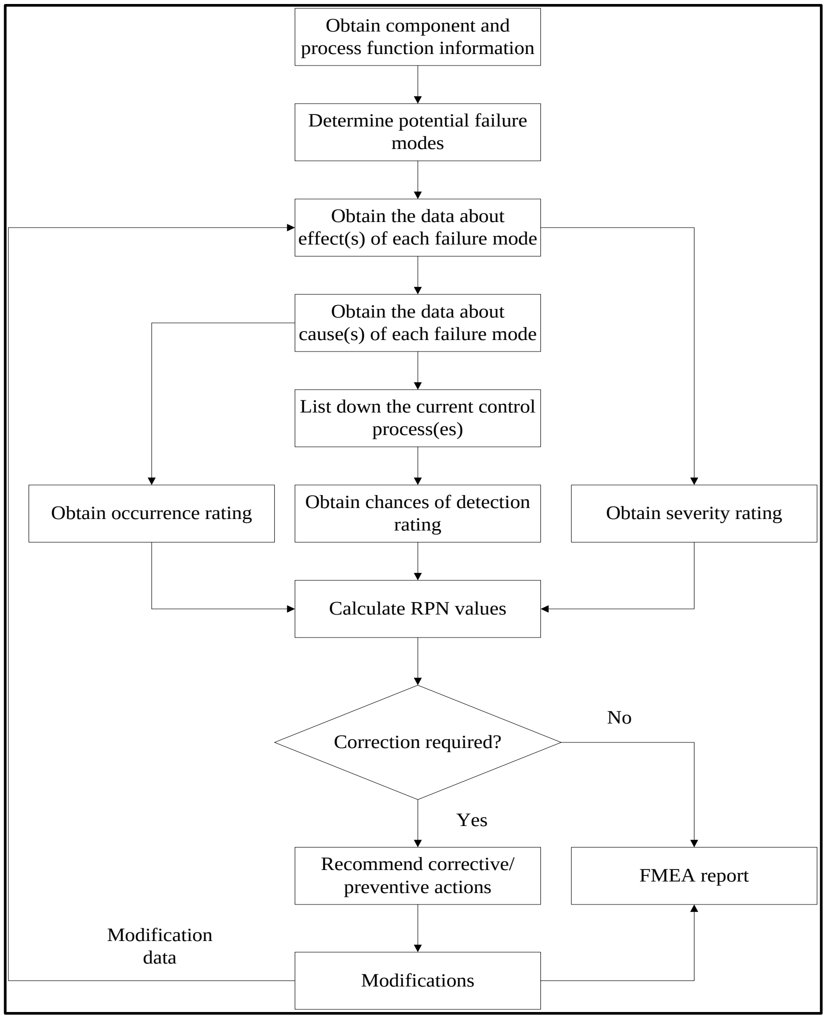

Figure 1.2. Flowchart of traditional RPN-based FMEA approach

- Duplication of RPN values for different combinations of crisp ratings of the risk factors, although their hidden risk implications may be poles apart. For example, let us consider that for *failure mode-1* the ratings are: $S = 2$, $O = 7$, and $D = 8$, then the $RPN = 2 \times 7 \times 8 = 112$. Whereas for *failure mode - 2*, the ratings are: $S = 7$, $O = 2$, and $D = 8$, then the $RPN = 7 \times 2 \times 8 = 112$. Now, it can be noticed that although the failure modes have the same RPN value, their severities differ to a significant extent.

- Higher concentration of RPN values at the lower side of the histogram diagram as shown in Figure 1.3.

- Small variations in the ratings of a risk factors may produce a completely different RPN value. For instance, consider the previous example of failure mode-1. If S becomes 3, instead of 2, then $RPN = 3 \times 7 \times 8 = 168$, which is totally different than the earlier RPN value (viz., $RPN = 112$).

- The weights of the risk factors are not considered during the risk ranking of failure modes.

- In case of linguistic judgements, when they are converted into crisp values, their inherent uncertainties are not considered in both cases.

- The mathematical formula to compute the RPN value as presented in (1.1) is disputed.

- The risk factors are not explicitly defined, (i.e., S may connote different based on the application context).

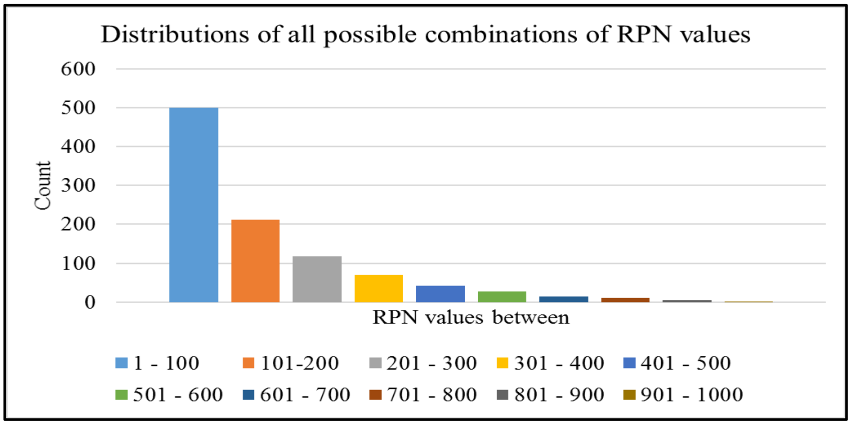

Figure 1.3. Distributions of all possible combinations of RPN values

- Considering only three risk factors may not reveal the hidden risks of the failure modes (viz, in some case studies along with S, economic consequences of failure modes have been explored).

Besides the above, it is already accentuated that the main purpose of RCM is to protect the functionability of the critical systems by using the CBM, however it has a significant initial cost of implementation. Thus, it is always suggested to perform a detailed FMECA of the system and its components, before adopting the CBM. If it is witnessed that the system has a crucial role towards the overall production process, and prone to failures, then CBM is the best option. Otherwise, time/age-based PM can be thought of. Besides, fault diagnosis, being a part of CBM, when employed to a large-scale, complex, and critical to the production facility machine, has several complex decision-making tasks, which are further detailed in the next sub-section.

1.1.2. System Fault Diagnosis in CBM

Reliability has always been a significant facet in the asset management practices. Systems/products having robust design are less prone to failure, but at the same time, the accumulation cost increases. In such instance, the concept of optimal design has been developed. However, no matter how reliable the product design is, the performance of an asset deteriorates over time due to diversified reasons. Fault diagnosis is an important aspect of CBM and is required to be carried out properly to arrest the fault at the earliest possible opportunity. It consists of the following three sequential steps:

a) *Fault detection:* It is a task to indicate whether something is happening wrong in the monitored system.

b) *Fault isolation:* It is a task to pinpoint the faulty component.

c) *Fault classification:* It is the task to categorize the fault of the component.

The above steps can be performed only after sensing the current health state of the asset by means of *data acquisition*, filtering the noises & extracting the useful information by *data processing*, and finally *mapping* the data from the feature space to fault space (Jardine *et al.*, 2006). However, the major decision-making problems arise from the final stage (*viz.*, fault classification: mapping the fault features from the measurement space to fault space) of CBM.

Data acquisition is the process of collecting the useful health information from the targeted system, which is considered as a pivotal step in system fault diagnosis. Acquired data in CBM program can be divided into two broad categories: *event data* and *condition monitoring data*. Event data consists of the information about what happened (*e.g.*, installation, causes, breakdown, and what the causes were) and/or what was done (*e.g.*, minor repair, PM, oil change etc.) to the asset. Whereas, condition monitoring data have the measurements of different HIs, which can be further sub-divided into *value type* (*e.g.*, oil analysis, temperature, pressure, humidity, etc.), *waveform type* (*e.g.*, vibration and acoustic data), and *multidimensional type* (*e.g.*, X-ray image, visual images, etc.) data. Event data are collected manually, and further shifted to the CMMS (computerized maintenance management system) for future utilization. The condition monitoring data are obtained through DAC systems (data acquisition system), portable devices, etc., and can be transferred to the central server (*i.e.*, ERP, or CMMS).

In *data processing*, data are initially cleaned up to discard the noises, and then further analysed. Waveform type data can be investigated in the time-domain, frequency domain, and / or time-frequency domain. Value type data are examined through different statistical tools (*e.g.*, PCA (principle component analysis), ICA (independent component analysis), regression techniques, etc.) (Grimmelius *et al.*, 1995; Yang *et al.*, 2000).

When the system is large and complex, the challenges in fault diagnosis process are further aggravated due to the following reasons:

- *Impacts of operating conditions:* When the system is functioning in hostile environmental conditions, the faults are more likely to occur. However, modelling the impacts of environmental condition on fault occurrences require rigorous mathematical modelling, which is hard to comprehend and often require expert’s involvement.

- *Proper Representations of event type data:* To properly infer the causes of the present fault, engineers often seek the help of prior event data, which are generally recorded in the logbook in terms of natural languages/linguistic terms. Due to lack of awareness of the operators, these data contain some missing and/or redundant information, which are further required to be carefully stored in the database through proper representation. Otherwise, it is obvious that when the data will be recalled in the future, misleading information will be given by the computer program.

- *Considering multiple monitoring techniques:* Experts opine that adopting a single monitoring technique may not always reveal the fault at the earliest possible opportunity. In (Jardine *et al.*, 2006), the authors emphasized that *"case-dependent knowledge and investigation are required to select appropriate signal processing tools among a number of possibilities"* especially for waveform data analysis. Thus, industries would rather prefer to simultaneously consider multiple monitoring techniques. However, choosing the best technique require experts' involvement.

- *Poor accessibility for continuous monitoring:* When the system has a poor and/or inaccessible surrounding environment (*e.g.*, high surrounding temperature), it becomes a difficult task to constantly monitor its health condition. In such circumstances, either organization should install high temperature resistant costly sensors, or they need to monitor it weekly or fortnightly by means of some portable devices to record the pertinent HIs. Later, they are analysed to detect the present health condition of the system.

- *Considerations of large set of HIs:* For the considered type of systems, maintenance engineers need to consider numerous HIs for the accurate fault diagnosis. However, often, it is not possible to accumulate all of the pertinent HIs, and they diagnose the fault with partial information. Furthermore, mapping of incomplete HIs from measurement space to fault space may generate inaccurate results.

As discussed, the next task is to facilitate the system with an optimal maintenance strategy.

1.1.3. Optimal Sustainability-based Maintenance Strategy Selection

The awareness of sustainability emerged from a series of discussions and reports published between the 1970s and 1980s. Initially, it was motivated by some disasters and accidents that happened in chemical plants, as well as resources depletion. The similar idea was pointed out in the 1987 Brundtland Report (Brundtland Commission, 1987):

> *"Major, united changes are occurring in the atmosphere, in soils, in waters, among plants and animals. Nature is bountiful but it is also fragile and finely balanced. There are thresholds that cannot be crossed without endangering the basic integrity of the system. Today we are close to many of those thresholds."*

The same report defined the *sustainable development* as:

> *"…the development that meets the needs of the present without compromising the ability of future generations to meet their own needs."*

In 2005, the *United Nations World Summit* further emphasized on the three significantly interdependent and mutually reinforcing pillars of accomplishing the sustainable development goal (SDG): economic development, social development, and protection of environment (United Nations General Assembly (60th sess.: 2005-2006), 2005). These three pillars are identified as the *TBL of sustainability*.

Manufacturing has had a pivotal role in the global development and growth in terms of wealth generation, and job creations. But it also has significant roles on the environment degradation. For example, in 2006, the *United States* manufacturing sectors have accounted for 36% of the total carbon monoxide emission (Haapala *et al.*, 2013). In the manufacturing industries, sustainability related impacts arise from manufacturing operations and activities, when the systems are exploited to convert the raw materials and energies into marketable products. The definition of sustainable manufacturing as given by *U.S. Department of Commerce* is as follows:

> *"…the creation of manufactured products that use processes that minimize negative environmental impacts, conserve energy and natural resources, are safe for employees, communities, and consumers and are economically sound."*

Thus, when an organization is trying to implement the sustainable manufacturing, its supplementary processes should be sustainable. In essence, the consequences of traditional maintenance practices (*viz.*, R2F, PM, CBM, PdM) are required to be considered from the TBL of sustainability within a sustainable maintenance philosophy.

For large and complex systems, it is essential to prevent the repeated occurrence of failures during the operational phase. Catastrophic failures not only result into substantial cost of repair or replacement, and significant loss of production, but also disrupt the overall safety and environment. To circumvent such adverse impacts, it is important for the organization to assist by using an optimal maintenance strategy. An optimal selection not only alleviates the likelihood of occurrences of failures, but also curtails the maintenance cost, increases the production quantity, as well as the quality of product. However, the optimal maintenance strategy selection problem in the sustainability context can be an arduous task due to the ensuing causes:

- To consider the conventional maintenance philosophies from the sustainability context, initially it is essential to properly identify the pertinent indicators, which is considered as a challenging job for the researchers.

- Thereafter, it is required to collect the data for the identified parameters. However, for some of them, the exact information is easily obtainable, whereas the remaining parameters cannot be quantified, and thus experts' judicious thinking is considered further.

- Selecting of the best experts is completely dependent on the top management. Wrong selection can lead to improper selection of maintenance practices.

- For the non-quantifiable parameters, it is necessary to evaluate them by utilizing proper scale values. However, choosing the appropriate scale is also dependent on experts' judicious thinking.

- From the existing literature, it is observed that *Multi-Criteria Decision Making* (MCDM) methods have been widely adopted to solve the maintenance strategy selection problems (MSSPs). However, these methods fail to solve the problem in case of missing information and/or when the experts are not able to judge a parameter for a particular alternative. Furthermore, it is obvious that each criterion has a different contribution towards the selection of any alternative, thus, they should have different weight values. However, when the numbers of involved criterion are significantly large in number, then the mathematical complexities of the MCDM methods are intensified.

1.2. Objectives

Based on the preceding discussions, it is observed that during the implementation of RCM, organizations are confronted with multiple types of decision-making problems, out of which a comprehensive FMECA with a credible risk rankings of failure modes through linguistic judgements, fault diagnosis of the system with incomplete/missing information and different types of data, and identification of pertinent sustainable criteria and adopting them for choosing the optimal maintenance strategy are of major focus. Besides, these decision-making problems have multiple conflicting criteria, which make them more complex to solve. Thus, these problems are required to be researched further for ease implementation of RCM. Hence, in this research work the following objectives are outlined and addressed further:

- **Objective 1:** To develop mathematical frameworks, which can simultaneously overcome the drawbacks of traditional RPN-based FMEA approach and can efficiently manage the uncertainties involved in experts' linguistic judgements to compute more credible risk ranking results of the failure modes. Further, in the context of sustainability-based manufacturing, it attempts to identify the pertinent risk factors for calculating more realistic risk ranking results.

- **Objective 2:** To develop an automated decision-support system which can assist the organizations in early fault diagnosis of large-scale, and complex systems, along with delivering the best possible solutions, in case of missing and/or incomplete information about HIs. Further, the system should be capable enough to incorporate both the value and event type data and provide the information about the necessary maintenance tasks after detecting the fault.

- **Objective 3:** To identify the pertinent key performance indicators (KPIs) from the TBL of sustainability for different conventional maintenance strategies, and finally developing a decision-making framework for selecting the optimal sustainable maintenance strategy for the machine.

Based on the above objectives, an exhaustive literature survey is conducted and presented later in Chapter 2 to search for the existing solutions, if any, or to discover gaps in the existing solutions to devise strategies to fill those gaps.

1.3. Thesis outline

To manage the problems discussed in previous sections, the rest of the thesis is outlined as follows:

- In Chapter 2, the existing solutions and approaches to overcome the decision-making problems in the published literatures are broadly discussed.

- To eliminate the challenges associated with the traditional FMEA approach, two integrated fuzzy multi-criteria decision making (MCDM)-based mathematical frameworks are proposed, and their potential in risk ranking of failure modes are validated by considering a prior benchmark case study. These constitute the contents of Chapter 3.

- Chapter 4 describes the system (*viz.*, process plant gearbox) on which the proposed frameworks and approaches have been applied and tested. The necessary information before the implementation of RCM is elucidated, which include the surrounding environmental conditions, different failure modes, their causes, effects (especially from TBL of sustainability), faults, relevant HIs, their measuring instruments, *etc.*

- Based on the MCDM-based mathematical risk ranking methods proposed in Chapter 3, Chapter 5 utilizes them for the case study on a process plant gearbox. Unlike other researchers, here the severities of failure modes are considered from economic, social, and environmental point of views in addition to *O*, and *D*. Further, to comprehensively model the uncertainties in experts' subjective assessments, the concept of interval type-2 fuzzy sets (IT2FSs) are explored in conjunction with the developed fuzzy MCDM methods.

- Upon observing the results obtained and observations thereof in Chapter 3, and Chapter 5, Chapter 6 makes an endeavour to further sieve out the uncertainties inherent in subjective assessments by proposing an integrated interval type-2 fuzzy sets (IT2FSs) and half quadratic minimization-based MCDM framework.

- Chapter 7 presents the development of a Case-Based Reasoning (CBR) framework to deal with the problems of the fault diagnosis. The proposed framework is capable enough to deal with incomplete information and resembles the human reasoning process. It also suggests alternatives to the maintenance engineers for the maintenance tasks necessary to be carried out after fault diagnosis.

- Chapter 8 initially identifies the pertinent key performance indicators (KPIs) for the implementation of sustainability-based maintenance practices for different traditional maintenance philosophies. Thereafter, it proposes a hybrid *Artificial Intelligence* (AI)-based framework for the selection of the optimal sustainable maintenance strategy explained through a hypothetical example.

- Chapter 9 concludes the objectives achieved in this research with suggestive and possible directions of future research.

Chapter 2 Literature Review and Research Contributions

The previous chapter brought out some major decision-making problems confronted by the organizations during the implementation of RCM. In this chapter, detailed insights are provided on the adopted methods in the available literature to solve those difficulties. Then, based on the observations, the best methods are selected and/or further extended to achieve the objectives as set out in *Chapter 1 / Section 1.2*.

2.1. Uses of MCDM Methods in FMEA Context

The limitations of the traditional RPN-based risk ranking approach have already been described in *Chapter 1 / Section 1.1.1*. Some of these problems have been endeavoured to be solved in different ways by the previous researchers. They can be initially grouped into following five categories (Liu *et al.,* 2013):

a) MCDM methods,

b) Mathematical programming,

c) AI-based approaches,

d) Integrated approaches, and

e) Other approaches.

MCDM is a well-recognized field of operations research, where alternatives are assessed and ordered from best to worst options, against the judgemental data/values of multiple conflicting criteria, obtained from expert(s)/field. The MCDM methods are closely related to the traditional single-objective optimality concept of operations research, based on a single scalar function maximization with respect to a priori given constraints. However, some researchers claim that MCDM methods belong to the group of vector optimization problem, as two or more scalar-valued objective functions (or criteria) are minimized over a set of feasible solutions. On a similar note, the risk ranking of failure modes with respect to the risk factors in FMEA resembles the main philosophy of the MCDM. Liu *et al.,* (Liu *et al.,* 2013) found out that until 2013, 22.50% of FMEA literature have adopted the MCDM methods to surmount the inadequacies of the traditional RPN-based approach. Besides, utilizations of MCDM methods have also been found in the category of *other approaches* (11.25%). In a more recent work, Liu *et al.,* (Liu *et al.,* 2019a) specifically emphasized on the applications, developments, and/or modifications of different MCDM methods, which have been solely aimed at addressing the single/multiple drawbacks of the traditional FMEA. However, these reviews fail to bring out the advantages and disadvantages of different applied MCDM methods, or omitted the answer to the questions – *"why the particular MCDM method has been chosen?"*, and/or *"what are the necessities to apply a particular MCDM method, where another MCDM method can rank the failure modes according to their risk levels?"*

Apart from the above, it has been already mentioned in *Chapter 1* that due to the unavailability of exact numerical values of the risk factors, experts often desire to provide their subjective perceptions, which certainly contain uncertainties and vagueness. To deal with this problem, the existing literature have often been integrated the MCDM methods with different uncertainty handling tools, such as fuzzy sets, rough sets, Z-numbers, *etc*. However, it is observed that continuous efforts are still now being made by the research fraternity to minimize the impacts of linguistic uncertainties on the final ranking results, especially in a critical area like FMEA.

To provide solutions to the above questions, in the ensuing sub-sections a detailed classification of the recent applications of different MCDM methods (refer Figure 2.1), their advantages as well as disadvantages, and the integrations with different uncertainty handling tools are discussed.

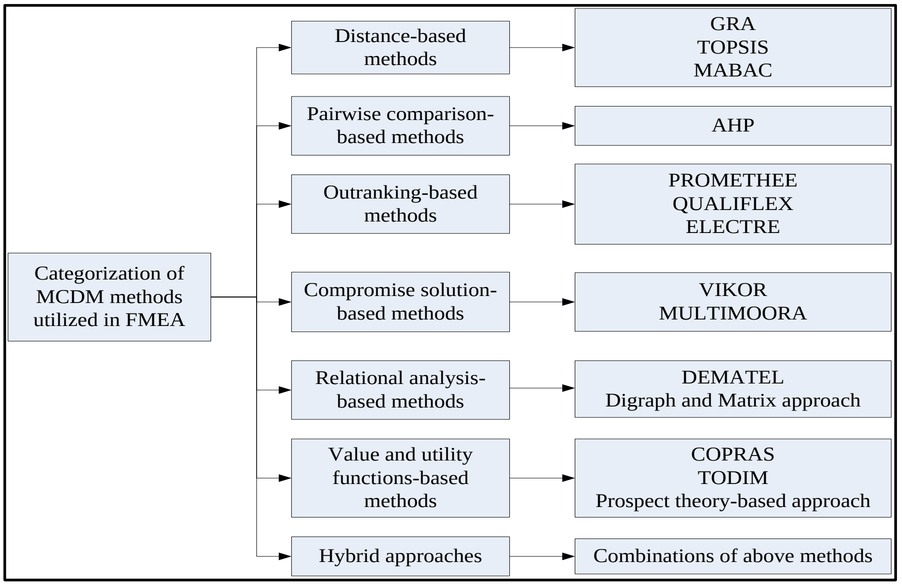

Figure 2.1. Classification of MCDM methods for risk ranking of failure modes in FMEA

2.1.1. Distance-based Methods

Popular candidates in this category are:

- *Grey relational analysis* (GRA),
- *Technique for Order Preference by Similarity to Ideal Solution* (TOPSIS), and
- *Multi-Attributive Border Approximation Area Comparison* (MABAC)

The grey system theory was propounded by J. Deng, along with the development of the concept of a grey set (Deng, 1982). The advantages of this concept are as follows:

- This theory is suitable for the data which are uncertain, have multi-inputs and are discrete in nature.

- It is an efficient method to resolve uncertainty issues under partial information.

GRA is a multi-variate statistical approach of grey theory and has been employed by many previous researchers for ranking of alternatives based on decision criteria. Here, the information about the similar features are considered as a sequence, and by utilizing the degree of grey relation coefficient (that varies between 0 to 1), the correlative degree of two sequences is computed. Finally, the alternatives are ranked based on their corresponding correlation degree. It is a viable option to the decision-makers while dealing with both qualitative and quantitative analyses. The applications of GRA in FMECA case studies are shown in Table 2.1.

Table 2.1. Applications of GRA method FMEA

References	Mathematical tool(s)	Application area(s)	Other information
(Abbasgholizadeh Rahimi *et al.*, 2015)	GRA, and fuzzy logic	Healthcare sector	- GRA was used for risk prioritization, - fuzzy logic dealt the linguistic uncertainties. - considered 19 risk factors, including S, O, and D.
(Liu *et al.*, 2015a)	GRA, and interval 2-tuple linguistic fuzzy sets (IT2LFSs)	C-arm X-ray machine	- GRA was employed for failure modes ranking. - S, O, D were considered as pertinent risk factors. - interval 2-tuple linguistic variables were utilized to handle the linguistic uncertainties.
(Sharma and Sharma, 2015)	Fuzzy inference system (FIS), GRA, and analytic hierarchy process (AHP)	Mechatronic system	- Presented the hybrid applications of FTA and FMEA for accomplishing the reliability needs. - FIS was utilized to model the subjective uncertainties. - GRA was adopted to rank the failure modes. - AHP was used to compute the weights of the risk factors.
(Tsai and Yeh, 2015)	Entropy method, FIS, GRA	Soldering process in surface mount assemblies	- FIS was used to account the subjective uncertainties. - Entropy method computed the weights of the risk factors.

References	Mathematical tool(s)	Application area(s)	Other information
			- GRA was applied for risk prioritization.
(Lo and Liou, 2018)	Intuitionistic grey value mathematics, interval valued best-worst method (IVBWM), GRA	-	- Intuitionistic grey valued numbers were adopted to cope up with linguistic uncertainty. - IVBWM was adopted for computing the weights of the risk factors. - Expected cost was considered as additional risk factor. - Possibility concept was integrated in GRA for risk ranking of failure modes.
(Kumar *et al.*, 2018)	Fuzzy rule-based system, GRA	LPG refueling station	- GRA was adopted in case of unavailability of specified rule for failure mode prioritization.
(Panchal *et al.*, 2018a)	Fuzzy rule-based system, GRA	Heavy commercial vehicle	- GRA was adopted in case of unavailability of specified rule for failure mode prioritization.
(Panchal *et al.*, 2018b)	GRA, fuzzy AHP	Urea fertilizer industry	- fuzzy AHP was employed to compute the weights of the risk factors. - GRA was utilized for risk ranking.
(Li and Chen, 2019)	Grey relational projection method (GRPM), fuzzy belief structure, Dempster-Shafer (D-S) evidence theory	Sheet steel production process	- GRPM was proposed to address the limitations of GRA and used for risk ranking. - Fuzzy belief structure tackled the subjective uncertainties. - D-S evidence theory was employed to aggregate the team members judgements.

TOPSIS is another well-recognized distance based MCDM method which studies a decision-making problem having m alternatives as a geometric system of m points in the n-dimensional space. This method is based on the notion that the finest choice should have the shortest distance from the positive-ideal solution (PIS)/ideal solution and the longest distance from the nadir solution/ negative-ideal solution (NIS)/ anti-ideal solution. Thereafter it elects the best alternative which has the maximum similarity to the PIS (Yoon, 1980; Yoon and Hwang, 1981). When contrasted with GRA, the earlier one only considers either the best/worst point during the generation of reference sequence, while TOPSIS regards both PIS and NIS. The applications of TOPSIS in multiple FMEA case studies are presented in Table 2.2.

Table 2.2. Applications of TOPSIS method in FMEA

References	Mathematical tool(s)	Application area(s)	Other information
(Chang, 2015)	Soft sets, and TOPSIS	Notebook development module company	- Soft set theory was employed to deal with uncertainties, imprecisions, and vagueness in subjective assessments. - TOPSIS was used for risk prioritization.
(Vahdani *et al.*, 2015)	Fuzzy belief structure, and TOPSIS	Sheet steel production process	- Fuzzy belief structure handled the subjective uncertainties. - fuzzy belief TOPSIS was employed for risk ranking of failure modes.
(Selim *et al.*, 2016)	Fuzzy sets, and TOPSIS	Food processing industry	- Fuzzy TOPSIS was adopted for setting the maintenance priority of machines.
(Tooranloo and Ayatollah, 2016)	Intuitionistic fuzzy sets (IFSs), and TOPSIS	Banking sector	- Linguistic uncertainties were dealt by IFSs.

References	Mathematical tool(s)	Application area(s)	Other information
			- Failure modes were prioritized by IFSs-based TOPSIS method.
(Bian *et al.*, 2018)	D-numbers, and TOPSIS	Rotor blades of aircraft turbine	- D-number was employed to overcome certain drawbacks of D-S evidence theory. - D-TOPSIS was adopted for risk ranking.
(Carpitella *et al.*, 2018)	AHP, fuzzy sets, and TOPSIS	Street cleaning vehicle	- AHP was employed to calculate the weights of the risk factors. - Fuzzy TOPSIS was used for failure modes ranking. - Considered risk factors were ooccurrence, time of operation, and modality of execution.
(Tooranloo *et al.*, 2018)	IFSs, and TOPSIS	Oil and gas company	- IFSs were adopted for dealing with subjective uncertainties. - IFS-TOPSIS was used for risk ranking of failure modes.
(Wang *et al.*, 2018a)	D-S evidence theory, and TOPSIS	Gas insulated metal enclosed transmission line	- D-S evidence theory was utilized to deal with cognitive uncertainties. - TOPSIS was used for failure modes ordering purpose.
(Li *et al.*, 2020)	Interval 2-tuple linguistic variables (ITLVs), and TOPSIS	Spindle box system of a CNC machine	- ITLVs was employed to clarify uncertain information and improve analysis accuracy. - ITLV-TOPSIS was adopted for risk prioritization.
(Mangeli *et al.*, 2019)	Support vector machine (SVM), FIS, logarithmic fuzzy preference programming, fuzzy TOPSIS	Occupational accidents in cooper leaching factory	- Predicted the S, and O values using SVM. - FIS was employed to decrease the subjective uncertainties in S and O. - Logarithmic fuzzy preference programming was adopted to calculate the weights of the risk factors. - Revised fuzzy TOPSIS was used to rank the failure modes.
(Başhan *et al.*, 2020)	Single valued neutrosophic sets, and TOPSIS	Ship navigation in maritime industry	- Single valued neutrosophic sets was adopted to deal with linguistic uncertainties. - single valued neutrosophic set-based TOPSIS was used for risk ranking.

Despite the numerous applications, the TOPSIS has the following shortcomings:

- The weights of the distances between the optimal point and the best/worst point are not taken into account (Wang *et al.*, 2018a).
- Multiple research works have reported that TOPSIS suffers from a drastic rank reversal problem in the case of the dynamic decision matrix (Pamučar and Ćirović, 2015).

Based on the limitations of TOPSIS and GRA methods, Pamučar and Ćirović recently devised the concept of another distance based MCDM approach: MABAC (Pamučar and Ćirović, 2015). The ensuing points are noteworthy related to this method:

- The MABAC has added benefits over other popular and established MCDM methods, like – simple additive weighting (SAW), VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), multi-objective optimization by ratio analysis (MOORA), complex proportional assessment (COPRAS), *etc.* in terms of ranking stability and credible rank order computation.

- In the MABAC method, the values of criteria functions are computed for each of the alternatives and the distance of the criteria function from the border approximation area is defined. Based on the distances, the alternatives are ranked and the best one is chosen. Some applications of MABAC method in FMEA context are tabulated in Table *2.3*.

Table 2.3. Applications of MABAC method in FMEA

References	Mathematical tool(s)	Application area(s)	Other information
(Delice and Can, 2017)	MABAC	-	- MABAC method was employed for risk ordering.
(Liu, 2019a)	Interval-valued intuitionistic fuzzy sets (IVIFSs), maximum cross-entropy based linear programming model	Healthcare sector	- IVIF-MABAC was adopted to prioritize the failure modes. - IVIFSs were employed to deal with linguistic uncertainties. - Weights of the risk factors were computed using maximum cross-entropy based linear programming model.

Even with the several benefits, in the MABAC method, the un-biased attitude of the decision-maker for the ranking of the alternatives is not considered.

2.1.2. Pairwise Comparison-based Methods

Pairwise comparison-based methods have a different type of concept than the distance-based MCDM methods. Here, each of the alternative/criterion is pair-wisely compared either subjectively or objectively. Some of the major candidates in this group are:

- AHP,
- *Analytic Network Process* (ANP), and
- *Alternative Queuing Method* (AQM).

The AHP method was formerly devised by Satty (Satty, 1990), and has been adopted by multiple previous researchers to solve many real-world MCDM problems. Its advantages are ease of use, ability to structure the problem methodically, and thereafter computing the criteria weights and alternatives priorities.

For risk ordering purpose in FMEA of a TFT-LCD product, Chang (Chang, 2016) exploited the idea of 2-tuple linguistic fuzzy AHP. The 2-tuple linguistic fuzzy representation model was adopted to minimize the impacts of linguistics uncertainties on the final ranking results. The weights of the risk factors were calculated by AHP. However, the major weakness of AHP is when the counted criteria/alternatives are increased significantly, the number of obligatory pairwise comparisons are also increased by following the formula $n(n+1)/2$, where n denotes the number of criteria.

2.1.3. Outranking-based Methods

Outranking-based methods are established on pairwise comparisons of alternatives against each other (or against a pre-defined norm), on each criterion, ensued by a procedure that aggregates and exploits the information to determine the strength of evidence, supporting one alternative, over another (Govindan and Jepsen, 2016). An outranking relation is a preference model which considers three types of situations: preference, indifference, and incomparability. The details about the mathematics involved in outranking-based methods can be found in (Bouyssou, 2009). Some popular candidates belong to this group are:

- *Preference Ranking Organization METHod for Enrichment of Evaluations* (PROMETHEE),
- *QUALItative FLEXible* multiple criteria method (QUALIFLEX),
- *ELimination Et Choice Translating REality* (ELECTRE).

PROMETHEE has been an extensively studied outranking method to manage intricate MCDM problems (Brans *et al.*, 1986; Brans and Vincke, 1985). It is a more appropriate and flexible approach during the selection of a preference function. Driven by these, its utilizations in FMEA case studies are tabulated in Table 2.4.

Table 2.4. Applications of PROMETHEE method and its variants in FMEA

References	Mathematical tool(s)	Application area(s)	Other information
(Lolli *et al.*, 2015)	PROMETHEE	Plastic bottle manufacturing process	- PROMETHEE was used for sorting the failure modes according to their criticality levels.
(Liu *et al.*, 2017)	Cloud mode theory, and PROMETHEE	Healthcare sector	- Cloud model theory was employed to deal with linguistic uncertainties. - PROMETHEE was used for risk prioritization.
(Zhang *et al.*, 2019)	Z-number, and PROMETHEE-II, distance-based	Geothermal power plant	- Z-number was utilized to cope up with subjective vagueness, and reliability of the linguistic decisions.

References	Mathematical tool(s)	Application area(s)	Other information
	weighting method, Bonferroni mean operator		- Bonferroni mean operator was adopted to aggregate the judgements of the experts. - Distance based weighting method calculated the weights of the risk factors. - PROMETHEE-II was used for risk ordering.
(Zhu *et al.*, 2020a)	Maximizing deviation method, TOPSIS, neutrosophic number, and PROMETHEE	Supercritical water gasification system	- Neutrosophic numbers were adopted to deal with subjective uncertainties. - Maximizing deviation model and TOPSIS methods were combinedly employed to calculate the weights of the risk factors. - PROMETHEE was used for risk ranking.

However, the PROMETHEE method has the following limitations (De Keyser and Peeters, 1996):

- It can only be adopted if the decision maker can express preference between two actions for a given criterion on a rational scale.
- It can only be useful if the decision maker can express the importance s/he attaches to the criteria on a ratio scale.
- The weights of the criteria represent the trade-offs between them.
- It can only be adopted with criteria, where the differences between evaluations are meaningful.
- It is not feasible to consider the discordance when constructing the outrank relations in the PROMETHEE method.

Whereas, the another outranking-based method, QUALIFLEX was propounded by Paelinck (Paelinck, 1978), and the following points are relevant to this method:

- It is a variation of Jacquet-Lagrexe's permutation method.
- It is popular due to its simplicity in mathematical logic, easy computational procedure, and full utilization of information contained in decision analysis.
- Its methodology is based on a metric procedure that evaluates all possible permutations of the considered alternatives, and recognizes the optimal permutation that reveals the greatest comprehensive concordance/discordance index (Liu *et al.*, 2016b).

In the context of FMEA of a C-arm X-ray machine, Liu *et al.,* (Liu *et al.*, 2016b) presented an extended QUALIFLEX approach based on hesitant 2-tuple linguistic terms sets (HTLTSs). The later one was adopted to manage the linguistic uncertainties, and a GRA-based multi-objective optimization model was developed to evaluate the weights of the risk factors. Occurrence, impact on organization, impact on patient, detection, interdependence with other failures, cost due to failures, and corrective action cost, were regarded as the pertinent risk factors. Thereafter. the extended QUALIFLEX approach with an inclusion comparison method was suggested to determine the risk orders of the failure modes.

ELECTRE being the oldest candidate in this category has several variants, and each of them have different application contexts, as tabularized in Table 2.5. The below points are required to be highlighted before discussing on ELECTRE:

- Each variant entail two phases – aggregation and exploitation. In the aggregation phase, within a multi-criteria aggregation procedure (MCAP), the concordance and discordance concepts are employed to make pairwise comparisons of alternatives, which are being characterized by their performances on different criteria.

Table 2.5. Variants of ELECTRE and their conditions for applications

Variants and references	Useful information
ELECTRE – I (Roy, 1968)	- It is applicable to '*choice problematic'* or '*problematic* α*'*, where the objective is to select a smallest set of best alternatives. - Uses the concept of preference threshold which allows to model imperfect knowledge, which may be result of uncertainty, imprecision, and ill-determination of certain data.
ELECTRE – II (Roy and Bertier, 1971)	- Based on true criteria. - Objective is to order the alternatives from the best to worst – '*problematic* γ*'*.
ELECTRE – III (Roy, 1978)	- Objective is to order the alternatives from the best to worst – '*problematic* γ*'*. - Consider the concept of pseudo-criteria. - Consider the criteria weights.
ELECTRE – IV (Roy and Hugonnard, 1982)	- Objective is to order the alternatives from the best to worst – '*problematic* γ*'*. - Does not consider the criteria weights. - Consider the concept of pseudo-criteria
ELECTRE – TRI (Yu, 1992)	- Consider the concept of pseudo-criteria. - Objective is to assign alternatives to a set of pre-defined categories – '*problematic* β*'*.
ELECTRE – IS (Roy *et al.*, 1986)	- It is applicable to '*choice problematic'* or '*problematic* α*'*, where the objective is to select a smallest set of best alternatives.
ELECTRE Iv (v for veto) (Figueira *et al.,* 2016)	- It is applicable to '*choice problematic'* or '*problematic* α*'*, where the objective is to select a smallest set of best alternatives. - Used when veto thresholds are taken into consideration.

- In methods dealing with problematic α or problematic γ, the alternatives are compared against themselves.

- In problematic β methods, the alternatives under consideration are compared against a set of reference alternatives characterized by norms on different criteria. These pairwise comparisons of the alternatives lead to build one or more outranking relations depending on the specific method in question.

- The second phase comprises of an exploitation procedure specific for the ELECTRE method in question. The procedure is utilized to exploit the outranking relation previously constructed by the MCAP, and it is aimed at constructing and presenting the type of results that are expected for the given problematic.

Driven by the utilities, few researchers have employed this concept in solving FMEA case studies, which are organized in Table 2.6.

Table 2.6. Applications of ELECTRE and its variants in FMEA

References	Mathematical tool(s)	Application area(s)	Other information
(Certa *et al.*, 2017)	ELECTRE-TRI	Dairy industry	- ELECTRE-TRI was employed for risk classifications.
(Liu, 2019b)	IT2LFSs, statistical distance-based approach, ELECTRE	Proton beam radiotherapy	- IT2LVs were adopted to manage the linguistic uncertainties. - Adopted combined weighting methods for computing the weights of the risk factors. - Subjective weights – given by experts, objective weights – statistical distance-based approach. - IT2L-ELECTRE was utilized for risk ranking.
(He *et al.*, 2020)	Probabilistic linguistic term sets (PLTSs), entropy method, ELECTRE - II	Nuclear reheat valve	- PLTSs were utilized to model the llinguistic uncertainties. - PLTS-based entropy method was employed to calculate the weights of the risk factors. - PLTS-ELECTRE – II was used for risk ranking of failure modes.

Despite the various superiorities, the major drawback of normal ranking of ELECTRE is that it involves an additional threshold to be introduced, and the ranking of alternatives depends on the size of this threshold for which there exist no *'correct value'* (Li and Wang, 2007).

2.1.4. Compromise Solution-based Methods

The another major group of MCDM methods have been developed on the concept of the compromise solution, which was established by Yu (Yu, 1973). The compromise solution is a feasible solution, which is closest to the ideal, and a compromise implies an agreement established by mutual concessions. Some key candidates in this group are:

- VIKOR, and
- *Multi Objective Optimization by Ratio Analysis plus full Multiplicative Form* (MULTIMOORA).

The VIKOR method was initially developed for multi-criteria optimization of complex systems (Opricovic, 1998). However before discussing further, the following points should be mentioned:

- The VIKOR method defines the positive and the negative ideal points in the solution space.
- It focuses on ranking and selecting from a finite set of feasible alternatives in the presence of conflicting and non-commensurable (attributes with different units) criteria.
- It evaluates a multi-criterion ranking index based on the *'closeness'* to the *'ideal'* solution. When each alternative is evaluated with respect to each criterion, the compromise ranking can be obtained while comparing the relative closeness measure to the ideal alternative. Thus, the derived compromise solution is a feasible solution, which is the closest to the positive ideal solution and farthest from the negative ideal solution. The term *'compromise'* means an agreement established by mutual concessions made between the alternatives. (Opricovic and Tzeng, 2004).

In FMEA case studies, several researchers have explored its abilities in risk ranking of failure modes. Few of them are presented in Table 2.7.

Table 2.7. Applications of VIKOR method in FMEA

References	Mathematical tool(s)	Application area(s)	Other information
(Liu *et al.*, 2015d)	Fuzzy numbers (FNs), fuzzy AHP, entropy, fuzzy VIKOR	General anesthesia process	- Fuzzy numbers were employed to cope up with linguistic uncertainties. - Combination weighting scheme of risk factors was adopted (subjective weights – fuzzy AHP, objective weights – entropy method).

References	Mathematical tool(s)	Application area(s)	Other information
			- Fuzzy VIKOR was utilized for ranking of failure modes.
(Safari *et al.*, 2016)	FNs, and VIKOR	Enterprise architecture	- Weights of the risk factors were considered as equal. - Fuzzy VIKOR was employed for risk ranking.
(Mohsen and Fereshteh, 2017)	Z-number, entropy method, fuzzy AHP, fuzzy VIKOR	Geothermal power plant	- Z-numbers were adopted to capture the inherent uncertainty in experts' judgements. - Z-evaluations were converted into fuzzy numbers. - Utilized combination weighting approach (subjective weights – fuzzy AHP, objective weights – entropy method). - Failure modes were ranked by the fuzzy VIKOR method.
(Tian *et al.*, 2018)	Proximity entropy weights, similarity entropy weights, fuzzy BWM, fuzzy VIKOR	Grinding wheel system	- Team members weights were computed by proximity entropy weight, and similarity entropy weight. - Risk factors weights were evaluated by fuzzy BWM. - Risk ranking of failure modes were done by fuzzy VIKOR method.
(Wang *et al.*, 2018b)	House of reliability (HoR), rough set theory, and VIKOR	Transmission system of vertical machining center	- HoR was adopted to recognize the dependencies among the failure modes, the link between failure modes and the considered risk factors. - Rough VIKOR was employed to rank the failure modes.
(Garg *et al.*, 2020)	Granularized Z-number and VIKOR	-	- Granularized Z numbers handled the linguistic uncertainties. - Granularized Z-VIKOR was used to rank the failure modes.
(Rathore *et al.*, 2020)	Fuzzy numbers, and VIKOR	Food grain supply chain	- Fuzzy numbers were integrated with VIKOR to deal with subjective vagueness, and to rank the failure modes.

However, one of the major problems with the VIKOR method is the selection of the value of weight of the strategy of *'the majority of the attributes'* (ν) (or, *'maximum group utility'*). Its value lies between 0 and 1. Usually, in most earlier works, $\nu = 0.5$ has been preferred. The compromise can be selected with *voting by majority* ($\nu > 0.5$), *with consensus* ($\nu = 0.5$), or *with veto* ($\nu < 0.5$). When the $\nu = 1$, it represents a decision-making process that can adopt the strategy of maximizing the group utility. While $\nu = 0$ represents a process that can adopt a *minimum individual regret strategy*, which is found among maximum individual regrets/gaps of lower-level criteria for each alternative. The value of ν affects the ranking of the alternatives and is usually determined explicitly by the decision experts.

Another method MOORA was initially propounded by Baruers and Zavadskas (Brauers and Zavadskas, 2006), combining Ratio System and Reference Point Approach. Later, (Brauers and Zavadskas, 2010) extended and improved MOORA to MULTIMOORA by adding *Full Multiplicative Form* and adopting *Dominance Theory* to attain a final integrated ranking based on the results of these triple subordinate methods. *Ratio System* and *Full Multiplicative Form* belong to *Value Measurements methods*, while *Reference Point Approach* originates from *Goal or Reference Level models* (Hafezalkotob *et al.*, 2019). The brief details about the triplet candidates are presented in Table 2.8.

Table 2.8. Descriptions of the approaches used to develop MULTIMOORA method

Ratio system approach	- It is applicable when the 'independent criteria' exists in a problem. - It is not capable to consider 'dependent criteria'. - It uses arithmetic weighted aggregation operator which is also a 'full compensatory' model. - The best alternative based on ratio system has the maximum utility and the ranking of this method is obtained in descending order. - It can provide the opportunity to compensate the poor performance of an alternative on one criterion by the performances on another criterion.
Reference point approach	- It utilizes Tchebycheff Min-Max Metric, a 'compensatory' method useful for the cases where the optimal choice for decision-makers is the alternative that does not have a very bad performance on none of the criteria. - This approach, as a non-compensatory model, first finds the alternatives ratings with the worst performance with respect to each criterion, and finally selects the overall best value (i.e., the minimum value) from these worst ratings. - The best alternative in reference point approach has the minimum utility and the ranking is produced in ascending order. - Reference Point Approach sometimes cannot differ on two or more alternatives and leads to same ranking.
Full multiplicative form	- It uses the geometric weighted aggregation operator, is an incompletely compensatory model (viz., small normalized values of an alternative could not be completely compensated by the same degree of large values. Thus, this issue leads to the perception that an alternative with moderate performance may be superior to an alternative with moderate performances with respect to different criteria. - The best alternative based on this approach has the maximum utility and the ranking of this technique is generate in descending order. - In utility formulae of Full Multiplicative Form, multiplying normalized ratings with weights leads to the same result as the situation in which no weights are considered. Thus, weights should be considered as exponent in utility equation.

Considering its capabilities, few research articles have applied it in solving FMEA case-studies, which are tabulated in Table 2.9.

Table 2.9. Applications of MOORA and its variants in FMEA

References	Mathematical tool(s)	Application area(s)	Other information
(Zhao *et al.*, 2017)	IVIFSs, entropy method, MULTIMOORA	Steel production process	- IVIFSs were used to tackle the linguistic uncertainties. - Weights of the risk factors were computed by IVIF-continuous weighted entropy method. - Failure modes were ranked by IVIF-MULTIMOORA method.
(Ghoushchi *et al.*, 2019)	Z-number, fuzzy BWM, MOORA	Automotive spare parts	- Z-numbers were adopted to handle the subjective uncertainties. - fuzzy BWM was employed to calculate the weights of the risk factors. - Z-MOORA was utilized for risk ranking of failure modes.
(Wang *et al.*, 2019b)	Interval type-2 fuzzy sets (IT2FSs), MULTIMOORA	Steel company	- IT2FSs were employed to manage the subjective uncertainties.

References	Mathematical tool(s)	Application area(s)	Other information
			- The weights of the risk factors were computed by TOPSIS, in terms of crisp numbers. - IT2F-MULTIMOORA was developed for risk ranking of failure modes.
(Chen *et al.*, 2020)	Fuzzy numbers, ordered weighted geometric averaging operator, Choquet integral, MULTIMOORA	CNC grinding machine	- Order weighted geometric averaging operator was adopted to experts' evaluations. - Adopted trapezoidal fuzzy numbers (TrFNs) ranking method based on the preference relation was coupled with Choquet integral to model the interactions among the risk factors and to calculate their weight values. - Extended MULTIMOORA was employed to rank the failure modes.
(Li *et al.*, 2020)	Pythagorean fuzzy sets (PFSs), maximization deviation model, MULTIMOORA	Water diversion project	- Weights of the risk factors were directly assigned by the experts (objectively). - Extended interval valued PF-MULTIMOORA was developed for risk prioritization of failure modes. - The IVPFPWA (interval valued Pythagorean fuzzy priority weight average) operator and IVPFPGA (interval valued Pythagorean fuzzy priority geometric average) operator were introduced into ratio system and the full multiplicative model to avoid information loss. - Euclidian distance was calculated between the evaluation information and the reference point in the reference point method.

The major drawback of MULTIMOORA method lies in the aggregation part of the ranking results produced by *Ratio System Approach*, *Reference Point Approach*, and *Full Multiplicative Form*. The original MULTIMOORA method uses *Dominance Theory* approach for this aggregation, which has some major weaknesses as listed below:

- Obtaining ranks of alternatives is hard as the theory is not yet automated.
- The theory only uses ordinal values by neglecting the relative importance of alternatives.
- Circular reasoning happens in some cases which leads to identical ranks that is not desirable.

To surmount these flaws, some other aggregation methods, like Dominance-directed graph, rank position method, technique of precise order preference, Borda and improved Borda rules, *Organization, Rangement Et Synthese De Donnes Relationnelles* (ORESTE) method, etc. have been applied, but each of them has some advantages and drawbacks. Selection of proper aggregation method is explicitly reliant on the experts' choice and improper selection may lead to a misleading result.

2.1.5. Relational Analysis-based Methods

In this group, the major candidates are:

- *DEcision MAking Trial and Evaluation Laboratory* (DEMATEL), and
- Digraph and Matrix approach.

DEMATEL was originally developed by the Battelle Memorial Institute, Geneva Research *Centre* (Gabus and Fontela, 1973). The original DEMATEL method was aimed at the fragmented and antagonistic phenomenon of world societies and searched for integrated solutions. DEMATEL is counted as an effective technique for the identification of the cause-effect chain components of a complex system. It deals with evaluating interdependent relationships among factors and finding the critical ones through a visual structural diagram (or influential relation map) – causal diagram. The DEMATEL can not only be employed to determine the ranking of alternatives, but also to find out critical evaluation criteria, and measure their weights. When contrasted with the AHP, both can compute the weights of the risk factors, and rank the alternatives, but the later one presumes that the criteria are independent and neglects their interactions and dependencies. Meanwhile, it is worth mentioning that ANP, an advanced form of AHP, can deal with the dependencies, along with the feedback between the criteria, but the assumption of equal weight for each cluster to obtain the weighted super-matrix in the ANP is not reasonable in practical situations (Si *et al.*, 2018).

In the FMEA domain, the applications of DEMATEL method are mainly adopted to either find out the relationship among the failure modes, and/or for their risk ranking. The recent applications of this method are shown in Table 2.10.

Table 2.10. Applications of DEMATEL method in FMEA

References	Mathematical tool(s)	Application area(s)	Other information
(Chang *et al.*, 2018)	Ordered weighted geometric operator, and DEMATEL	Extreme low-k dielectric integration	- The ordered weights of the risk factors were calculated by ordered weighted geometric (OWG) operator. - DEMATEL was used for risk ranking.
(Liu *et al.*, 2015c)	Fuzzy number, and DEMATEL	TFT-LCD product	- Fuzzy DEMATEL approach was utilized for the risk ranking of the failure modes.
(X. Wang *et al.*, 2016)	Entropy method, and DEMATEL	CNC machining center	- Considered risk factors in their case study were failure mode ratio, failure effect probability, and failure rate. - The weights of the risk factors were computed by combination weighting method

References	Mathematical tool(s)	Application area(s)	Other information
			(entropy for objective judgements, and experts' opinions for subjective evaluation). - DEMATEL was employed to derive the causal dependencies among the failure modes and to prioritize them.

Digraph and matrix approaches is another facet of relational analysis approach. The foundation of this approach belongs to the *graph theory and matrix algebra*. The matrix approach is a viable option during the analysis of a graph/digraph model and to compute the system function, & index to meet the objectives. The digraph and matrix approach can model the criteria interactions and generate hierarchical models. Further, representing the graph/digraph by a matrix offers ease in computer processing. Their applications in FMEA problems are tabulated in Table 2.11.

Table 2.11. Applications of Digraph and Matrix approaches in FMEA

References	Mathematical tool(s)	Application area(s)	Other information
(Liu *et al.*, 2016a)	Fuzzy numbers, digraph and matrix approach	Steam valve system in a power generation plant	- Triangular fuzzy numbers (TFNs) were adopted to deal with linguistic uncertainties, as they are appropriate for quantifying the vague information in most of the decision-making problems for their intuitiveness and computational-efficient representation. - Fuzzy digraph and matrix approach was utilized to rank the failure modes.
(Baykasoğlu and Gölcük, 2020)	Fuzzy numbers, graph-theoretical matrix approach, fuzzy preference programming, and fuzzy cognitive mapping	ERP system	- Fuzzy preference programming was adopted to derive the ratings of the risk factors. - Causal dependencies among the risk factors were modelled by fuzzy cognitive mapping. - Fuzzy graph-theoretical matrix (GTM) approach was utilized for ranking of failure modes.

The major disadvantages of this diagraph and matrix approach are listed below:

- If the numbers of risk factors increase, then there is no provision of hierarchical representation and to calculate their relation.
- When the numbers of risk factors increase, the visual representations among their interrelationships become so complex and hard to interpret.
- When the numbers of risk factors increase, then a lot of comparisons are required to be made to represent their relationships in terms of matrix form.

2.1.6. Value and Utility Functions-based Methods

The major candidates in this group are as below:

- *COmplex PRoportional ASsessment* (COPRAS),
- *TODIM* (an acronym in Portuguese for *interactive multi-criteria decision making*),
- Prospect theory-based applications

COPRAS was developed by Zavadskas *et al.*, (Zavadskas *et al.*, 1994), whose functions are similar to *Simple Additive Weighting* (SAW) method. SAW is one of the most straightforward and accepted MCDM method. However, SAW can employ just maximizing attributes, and the minimizing criteria must be converted into the maximizing factor before the utilization, which is not a trivial task, as it may cause the contradictory outcomes (Mousavi-Nasab and Sotoudeh-Anvari, 2017). Fortunately, this inadequacy is eliminated in COPRAS, and its applications for risk prioritization in FMEA case-studies are presented in Table 2.12.

Table 2.12. Applications of COPRAS method in FMEA

References	Mathematical tool(s)	Application area(s)	Other information
(Wang *et al.*, 2016)	IVIFSs, ANP, COPRAS	Healthcare facility	- IVIFSs managed the subjective uncertainties. - IVIF-ANP derived the weights of the risk factors. - IVIF-COPRAS was adopted for risk ranking of failure modes.
(Wang *et al.*, 2017)	Fuzzy soft set theory, COPRAS	Main engine crankcase explosions	- Fuzzy soft set theory was adopted to manage the linguistic uncertainties. - soft- COPRAS was developed to rank the failure modes.
(Nie *et al.*, 2018)	Multi-granular linguistic distribution, BWM, maximizing deviation method, COPRAS	Supercritical water gasification system	- BWM and maximizing deviation methods were adopted to compute the weights of the risk factors. - COPRAS method was utilized to rank the failure modes.

Karande and Chakraborty (Karande and Chakraborty, 2012) emphasized the following drawbacks of the COPRAS method:

- It has a hard, and complex computation procedure,
- The final ranking results are often changed by the normalization process,
- It is affected largely by the criteria weights.

TODIM method was initially propounded by Gomes and Lima (Gomes and Lima, 1992). It pair-wisely compares the alternatives, calculates the dominance of one alternative over another for each criterion by using the value function introduced by (Kahneman and Tversky, 2013). This value function, which has an S-shaped growth curve, allows to model the behaviour of the decision maker with respect to gains and losses. Finally, the overall performance of each alternative is calculated by employing an additive function. Its applications in FMEA case studies are tabularized in Table 2.13. In spite of the several applications, the traditional TODIM method is reported to be vulnerable when the weights of the gains/losses function are considered that may lead to inconsistent risk ranking results (Lee and Shih, 2016).

Table 2.13. Applications of TODIM in FMEA case studies

References	Mathematical tool(s)	Application area(s)	Other information
(Huang *et al.*, 2017)	Linguistic distribution assessment, entropy method, and TODIM	Grinding wheel of a CNC machine	- Linguistic distribution assessments were adopted to manage the linguistic uncertainties. - Combination weighting method was employed (objective – entropy method, subjective- experts' judgements). - TODIM method was utilized for risk ranking.
(Huang *et al.*, 2019)	PLTSs, TOPSIS, PLTS-TODIM	Enterprise architecture, and information system	- PLTSs tackled the subjective uncertainties. - TOPSIS based objective weighting method was adopted to compute the weights of the risk factors. - PLTS-TODIM was employed to rank the failure modes.
(Wang *et al.*, 2019a)	Generalized TrFNs (GTrFNs), Choquet integral, entropy method, TODIM	Steam valve system in a power generation plant	- GTrFNs were adopted to tackle the linguistic uncertainties. - Choquet integral with entropy method was adopted to measure the importance degree of the criteria, and their interactions. - GrTFN-TODIM was employed to rank the failure modes.
(Wang *et al.*, 2019c)	TrFNs, Fuzzy measure, Shapley index, TODIM	Main engine crankcase explosion failures	- TrFNs managed the linguistic uncertainties. - Fuzzy measure and Shapley index were adopted to model the relationships among the risk factors, and to compute their weights. - Extended TODIM was adopted to rank the failure modes.
(Sagnak *et al.*, 2020)	FNs, AHP, and TODIM	Hot-dip galvanizing process	- Fuzzy AHP was employed to calculate the criteria weights. - Fuzzy TODIM was utilized to rank the failure modes.

The concept of Prospect theory was developed by Kahneman and Tversky (Kahneman and Tversky, 2013). It is a depictive theory to predict individual authentic decision-making behaviours towards risks and uncertainties. Their applications in solving FMEA problems are summarized in Table 2.14.

Table 2.14. Utilizations of Prospect theory in solving FMEA problems

References	Mathematical tool(s)	Application area(s)	Other information
(Liu *et al.*, 2018)	Hesitant linguistic term sets (HLTSs), entropy method, prospect theory	Blood transfusion process	- HLTSs were employed to manage the subjective judgements' uncertainties. - Entropy method was adopted to calculate the weights of the risk factors.

References	Mathematical tool(s)	Application area(s)	Other information
			- HLTS-based Prospect theory was used for risk ranking of failure modes.
(Wang *et al.*, 2018)	Fuzzy numbers, relative preference relation method, Prospect theory, Choquet integral, and entropy method.	Aircraft landing system	- Fuzzy numbers were adopted to deal with linguistic uncertainties. - Relative preference relation was used for ranking of fuzzy numbers. - Prospect theory was adopted to determine the risk evaluation of the risk factors. - Interacting relationships among the risk factors were calculated by fuzzy measures, and Choquet integral. - Entropy weighting method was adopted to obtain the overall RPN values associated with each failure modes.

The major limitation of *Prospect theory*-based approach is that in case of a dynamic environment it fails to update the reference point and is mostly dependent on the past decisions.

2.1.7. Hybrid Approaches

Some researchers have also been taken the endeavours to combine the previously discussed methods to compute risk ranking results in FMEA. Some of them are presented in Table 2.15.

Table 2.15. FMEA research works combining different MCDM methods

References	Mathematical tool(s)	Application area(s)	Other information
(Liu *et al.*, 2015b)	Modified VIKOR, DEMATEL, AHP	Diesel engine turbocharger	- Modified VIKOR was employed to compute the effects of the failure modes on together. - DEMATEL was adopted to construct the influential relation map among the failure modes and causes of failures. - AHP was utilized to calculate the weights of the risk factors. - DEMATEL was used to calculate the risk ranking.
(Hu *et al.*, 2019)	GRA, TOPSIS, maximizing deviation method	Healthcare sector	- Maximizing deviation method calculated the weights of the risk factors, - GRA-based TOPSIS was adopted to compute the risk ranking of failure modes.
(Liu *et al.*, 2019b)	Fuzzy sets (FSs), AHP, fuzzy graph theory and matrix (fuzzy GTM) approach, DEMATEL	Rotary switch	- FSs were adopted to manage the linguistic uncertainties, - fuzzy AHP computed the weights of the risk factors, - fuzzy GTM was adopted to calculate the risk effect indexes of the failure modes, - DEMATEL was utilized to derive the dependencies among the failure modes and to rank them.
(Lo *et al.*, 2020)	DEMATEL, SAW, VIKOR, GRA, COPRAS	CNC rotary machine	- Influential relations among the failure modes were evaluate by DEMATEL method, - A TOPSIS based method comprising of SAW, VIKOR, GRA, and COPRAS was utilized to rank the failure modes.
(Akram *et al.*, 2021)	Pythagorean fuzzy hybrid (PFH) TOPSIS, and PFH-ELECTRE - I	Color super-twisted nematic	- PFH-TOPSIS was adopted to compute the distances of the failure modes from the Pythagorean fuzzy PIS and NIS. - PFH-ELECTRE- I was used to calculate the fuzzy concordance and discordance matrices.

2.2. Fault Diagnosis by AI-based approaches

The difficulties generally confronted with the fault diagnosis of large and complex machines have already been presented in *Chapter 1 / Section 1.1.2*, which can be managed by adopting any of the following approaches (Jardine *et al.*, 2006):

a) Statistical methods,

b) AI-based methods, and

c) Other methods.

In this context, it is required to mention that when discussing about large and complex machines, gearboxes are considered as essential systems in most of the mechanical industries, especially in manufacturing and process plants. This is because of their ability to transmit the torque and speed from one position to another with minimal losses of power, which is quite different when compared with the belt drives. A large and complex gearbox consists of multiple gears (may be of different types, *e.g.*, spur gears, helical gears, *etc.*), bearings (*e.g.*, roller bearings, sliding bearings, *etc.*), and shafts of different dimensions. Due to their high costs, gearboxes are not readily kept in stock for replacement in event of catastrophic failure. Therefore, gearbox failure can potentially lay off the plant for months, due to the long lead time for repair and/or replacement.

AI-based methods have been found to be more efficient while dealing with fault diagnosis of such type of system, as implementing these methods does not require to mathematically simulate the system, where the system has multiple components. Apart from this, recently, rapid progress has been noticed in employing the AI-based methods in solving fault diagnosis problems because of their capability of managing exhaustive data coming from different sources, and are of different types (*viz.*, value type, waveform type, and/or event type). Additionally, they are considered as viable choices to the engineers, whenever the current problem does not have a pre-defined model, as fault diagnosis of gearboxes. These methods generally provide the solutions based on some prior case histories, which are typically available in the central data-server of the organizations and are kept unexploited. Based on this, in Table 2.16, some applications of AI-based methods for fault diagnosis of gearbox, and their associated components are discussed.

Table 2.16. Applications of AI-based approaches for fault diagnosis of gearboxes and/or their associated components

References	Adopted AI-based method(s)	Considered HIs	Considered fault(s)	Other information
(Samanta, 2004)	Artificial Neural Network (ANN),	Vibration signals	Pitting failure of gear	- Numbers of nodes in the hidden layer of ANN, radial basis function kernel

References	Adopted AI-based method(s)	Considered HIs	Considered fault(s)	Other information
	Support Vector Machine (SVM), and Genetic Algorithm (GA)			parameter for SVM, along with the selection of input features were optimizing using GA, - Classification accuracy of SVM was better than ANN, without employing GA, - Classification accuracies of SVM, and ANN are comparable when employed along with GA.
(Rafiee *et al.*, 2007)	Wavelet transformation, multi-layer perceptron neural network (MLPNN)	Vibration signals	Worn teeth, and broken teeth of gear	- Proposed an intelligent condition monitoring approach for motor-cycle gearbox by calculating the standard deviations of the wavelet packet coefficients from the acquired signals, - Later, these signals were fed to MLPNN to examine the condition of the gearbox.
(Ebersbach and Peng, 2008)	*'IF-THEN'* rule-based expert systems (ESs)	Vibration signals, wear debris, and oil analysis	Fault of bearings (both roller and journal bearings), coupling, fans, spur gears, belt, and general faults	- ES was developed based on the knowledge adopted through tri-axial frequency spectra, demodulated spectra, time domain features, and technical handbooks, - Wear debris and oil analyses are time consuming and are not considered as viable option from application point of view. - The developed ES was totally dependent on developed rules.
(Wu and Chan, 2009)	ANN	Acoustic signals	Broken teeth of gear	- Feature vectors were collected from the continuous wavelet transformation (CWT) of the acoustic signals. - ANN was adopted for condition monitoring and fault diagnosis of a three-stage gearbox.
(Wu *et al.*, 2009)	Adaptive neuro-fuzzy inference system (ANFIS)	Vibration signals	Wear teeth (single local defect, multiple local defect)	- The elucidated approach combined DWT and ANFIS for fault identification and classification purposes, - The proposed approach utilized the qualitative approximation ability of FL and ANFIS.
(Saravanan *et al.*, 2009)	Decision tree	Vibration signals	Broken teeth, crack at the root of the teeth, gear with face wear	- A rule set was formed from the extracted features and fed to fuzzy classifier for discriminating fault condition of the gearbox.
(Jayaswal *et al.*, 2010)	ES, ANN, and FL	Vibration signals	Antifriction bearing faults – inner race, outer race, and ball defect	- A hybrid ES was developed using ANN, FL, FNs and WT to detect and diagnose the faults of rotating components.
(Hajnayeb et al., 2011)	MLPNN	Vibration signals	Crest removed gear, and complete removal of teeth of a gear	- UTA, and GA algorithms was applied to discard the unimportant features related to gear faults. - GA provided better results than UTA while eliminating the nonimportant features than UTA.
(Hashemi and Saeed Safizadeh, 2013)	FL	Vibration signals	Corrosion of teeth, and removal of teeth	- The gear faults were detected by a rule-based FL model. - Authors mentioned that for the early fault detection a lot of vibration signals are required to captured to increase the system efficiency.

References	Adopted AI-based method(s)	Considered HIs	Considered fault(s)	Other information
				- Selections of membership functions for healthy state, semi-faulty state, and faulty state were completely defined by the experts.
(Shao *et al.*, 2014)	PCA, and kernel PCA	Vibration signals	Tooth root crack, pitch circle crack, tooth wear faults of gears	- Utilized vibration signals to extract the features from vibration signals and used them in PCA and kernel PCA (KPCA) for fault detection and classification of gearbox.
(Tyagi and Panigrahi, 2017)	Back propagation NN, and GA	Vibration signals	Gear with missing tooth, chipped tooth, and surface wear	- An ANN classifier trained by a hybrid GA-BP method for gear fault diagnosis was presented. - Both time domain and frequency domain features were used, - DWT was employed for pre-processing of signals.
(Dou and Zhou, 2016)	K-nearest neighbor (KNN) algorithm, probabilistic NN (PNN), particle swarm optimization (PSO) optimized SVM, and a Rule-based method (RBM)	Vibration signals	Inner race defect, outer race defect, and rolling elements defects of rolling bearings	- Seven numbers of time domain features, and five numbers of dimensionless frequency domain features were adopted to classify the faults from the healthy state. - PSO-based SVM can more accurately classify the faults, - RBM can trails the PSO-based SVM in terms of classification accuracy. - RBM is most user friendly.
(Jing *et al.*, 2017)	Convolution neural network (CNN)	Vibration signals	Case- 1: Gear: chipped teeth, and broken teeth; Bearing: Inner race defect, ball defect; Shaft: Bent shaft, imbalance. Case 2: Six types of defects of gears – chipped tooth, pitting tooth, chaffing tooth, wear root crack tooth, root crack tooth, and worn tooth.	- The helical gear set was considered for the study, - Both time-domain (eight nos.), frequency domain (32 nos.), and wavelet domain features (5 nos.) were used for fault diagnosis, - 50% of data were used to train the CNN, and 50% for testing. - CNN outperforms fully connected NN (FNN), SVM, and random forest (RF) algorithm.
(Praveenkumar *et al.*, 2018)	Decision tree	Vibration signals	Gear: tooth breakage, Bearing: Outer race crack	- Both time domain and frequency domain features were extracted from the vibration data, - Time domain features when normalized outperforms the frequency domain data, - Normalized time domain data were used to train the decision tree, - An online condition monitoring window was developed for ease fault diagnosis.

However, the major problems observed during the exploitations of these methods are:

- These methods are ineffective, when the exhaustive prior faulty data are unavailable with the organization.

- Despite the availability of the exhaustive database, when exceptional cases occur frequently.

Due to these limitations, it became necessary to explore other potential methodologies, which can provide a similarity-based solution to the fault diagnosis problems.

Case-based reasoning (CBR) is an amalgamated domain of both AI and human cognitive process (Aamodt and Plaza, 1994). It is a powerful methodology, which mimics the human reasoning process, and has a computational model that is rather innate and easy to comprehend. This methodology has been explored successfully in various fields, such as medical applications (Fan *et al.*, 2011), energy management (Faia *et al.*, 2017), product configuration (Tseng *et al.*, 2005), machine tool selection (Chakraborty and Boral, 2017), material selection (Amen and Vomacka, 2001), etc. It has also been adopted by prior researchers to cope up with different difficulties in fault diagnosis process, which are presented in Table 2.17.

Table 2.17. Utilizations of CBR methodology for fault diagnosis of different systems

References	Targeted system	Other information
(Varma, 1999)	Off-board locomotive	- Considered different types of faults and their repair histories. - Didn’t consider the faults having partial information.
(Tsai, 2009)	Injection moulding machine	- Fault tree analysis (FTA) was combined with CBR in the proposed fault diagnosis approach. - Considered limited types of faults due to the increase in complexity of the model while employing FTA. - Computational complexity was increased due to combining CBR with FTA.
(Wong, 2011)	Car fault diagnosis	- Integrated rule-based reasoning (RBR) with CBR. - Deducing the rules from incomplete and/or partial information may lead to wrong conclusions.
(Olsson and Funk, 2012)	Industrial robots, welding, and cutting machines	- They only used acoustic signals for fault diagnosis, - Didn’t consider the case of incomplete or missing information.
(Dendani-Hadiby and Khadir, 2013)	Steam turbine	- The proposed model integrated the domain knowledge in an ontological form and focused on similarity-based retrieval step, - The proposed model relied on extensive experts’ opinions and failed to bring out the potentiality of CBR to deal with incomplete information.
(Zhao, 2013)	Numerically controlled (NC) machine	- The developed work used the object-oriented method to represent the fault of NC machine, - Didn’t consider the incomplete information.
(Deng *et al.*, 2014)	Aircraft gear landing system	- Combined both RBR and CBR for fault diagnosis, - CBR was utilized as a complimentary part.
(Xu *et al.*, 2018)	Fault diagnosis of loaders	- Due to the deficiency of pertinent cases, ontology based RBR approach was applied through building *Semantic Web Rule Language*.

		- Extensive rule-base was developed for providing solutions.

2.3. Uses of MCDM Methods in Maintenance Strategy Selection Problems

The problems confronted with the selection of the pertinent parameters in best (sustainable) maintenance strategy selection have already been described in *Chapter 1 / Section 1.1.3*. It has also been pointed out that MCDM methods have been widely adopted by the prior researchers to manage these problems, and is evident from the review work of Shafiee (Shafiee, 2015). For the sake of the benefits to the readers, the available literature on the above-mentioned problem after 2015 are presented in tabular format in Table 2.18.

Table 2.18. Some recent researches on maintenance strategy selection problem

References	MCDM method(s)	Factors chosen	Considered maintenance strategies	Area of application
(Kirubakaran and Ilangkumaran, 2015)	ANP, GRA-TOPSIS	Safety, cost, added value, feasibility	CM, PdM, TBPM, CBM	Pumps of paper mill
(Joshua *et al.*, 2016)	ANP	Cost (repair cost, annual maintenance cost, spares inventory), safety (personal safety, facility, machine security), strategic workers and union's acceptance, dispatch plan and training for employees) and time requirement (production shift, spare parts availability and workforce).	Scheduled maintenance, CM, PdM, Reactive maintenance.	Casting plant
(Lazakis and Ölçer, 2016)	Fuzzy AHP and TOPSIS	Maintenance cost, maintenance type efficiency, system reliability, management commitment, crew training, company investment, spare parts inventories, minimization of operation loss	CM, PM, and PdM	Diesel engine generator of a vessel
(Kirubakaran and Ilangkumaran, 2016)	Fuzzy AHP, GRA-TOPSIS	Safety, cost, added value, feasibility	CM, PdM, TBPM, CBM	Pumps of paper mill
(Özcan *et al.*, 2017)	AHP, TOPSIS, and goal programming	Warehouse backup, maintenance pre-conditions, failure period, possible consequences, availability of measuring instrument, static, dynamic or electrical property of the equipment, troubleshooting time, detectability of failure, additional work requirement	CM, PdM, PM, Revision maintenance strategy	Machinery of hydroelectric power plant
(Shafiee *et al.*, 2019)	ANP	Maintenance implementation costs and failure intensities	Failure based, risk based, TBPM, CBM	Wind turbine
(Seiti *et al.*, 2017)	Rough AHP	Added values, safety, cost, reliability and feasibility, time, efficiency and damage	CM, TBM, CBM, BM, TPM.	Rolling mill company

References	MCDM method(s)	Factors chosen	Considered maintenance strategies	Area of application
(Hemmati *et al.*, 2018)	Fuzzy ANP	Cost, risk, added value	CM, TBPM, CBM and shutdown maintenance.	Sulphuric acid production plant.
(Emovon *et al.*, 2018)	Delphi-AHP and Delphi-AHP-PROMETHEE	Cost (spare parts inventories, maintenance cost, crew training cost, equipment damage cost), safety (personnel, equipment, and environment), added value (minimization of operation loss, equipment reliability) and applicability (system failure characteristics, available monetary resources and equipment risk level).	CM, PM, and CBM	Ship machinery system
(Borjalilu and Ghambari, 2018)	Fuzzy ANP	Organization, safety, administration, staff and technical requirements.	TBPM, CM, CBM, RCM, PdM.	5-MW powerhouse.
(Seiti *et al.*, 2018)	Fuzzy axiomatic design	Added value, safety, cost, feasibility, damage, efficiency, reliability, time.	CBM, CM, BM, TPM, and TBPM.	Rolling mill

In the context of sustainability based MSSP, very few research works are present in the literature. Nezami and Yildirin (Nezami and Yildirim, 2013) first reduced the number of contributory factors by using the *factor analysis method* and applied fuzzy VIKOR approach for selecting the optimal maintenance strategy. The approach was validated by taking an example of a manufacturing plant. In (Wang *et al.*, 2015), the authors established an evaluation index by considering six aspects, *viz.*, input cost, risk of failure, fault duration, maintenance cost, social factors, and environmental factors. The relative importance among the factors were calculated by AHP whereas the final decision was made by employing the VIKOR approach. Ighravwe & Ayoola Oke (Ighravwe and Ayoola Oke, 2017) solved a sustainable maintenance strategy selection problem by using the fuzzy axiomatic design principle and fuzzy TOPSIS approach. The entropy method was utilized to calculate the relative importance between the sustainable factors. A cement producing plant in Africa was taken as a case study to show the efficacy of their approach.

2.4. Observations

Based on the extensive surveys presented in previous sections, the observations are outlined in the following sub-sections:

2.4.1. Observations from FMEA Literature

From the *Section 2.1*, it can be observed that to overcome the inadequacies of the RPN-based risk ranking approach, prior researchers have extensively employed the MCDM methods. The main aim of these implementations is to eliminate the practice of multiplicative formula for RPN value computations along with overcoming the other shortcomings. Other observations are as below:

1. *Utilization of different linguistic uncertainty handling tools:* To manage the uncertainties and vagueness involved in experts' subjective assessments, until now, multiple approaches have been integrated with the MCDM methods. A year-wise applications in this context is depicted in Figure 2.2, and the subsequent points are noted:

    - **From 2015-2020, FSs (more specifically FNs), have been mostly adopted by the previous researchers**, which was originally proposed by Zadeh (Zadeh, 1965). Additionally, despite the availabilities of various types of FNs (*e.g.,* TFNs, TrFNs, *etc.*), most of the earlier researchers have adopted the TFNs to rank the potential failure modes, due to their simple calculation steps, and ability to model the uncertainties through an abstract way.

    - The theory of FSs has been proliferated to different facets, such as IFSs (Atanassov, 2012), HLTSs (Liu *et al.,* 2018), PFSs (Akram *et al.,* 2020), Z-number (Zadeh, 2011), *etc.* However, most of these extended versions have higher mathematical complexities, when contrasted with traditional FSs.

    - Despite the several advantages of FSs, it has a crisp membership function (MF) value. Further, most of the variants of FSs have either crisp and/or interval number-based MF values, which surely be another obstacle in modelling the uncertainties. To address this problem, Zadeh further extended the concept of FSs (or type-1 FSs) to type-2 fuzzy sets (T2FSs) (Zadeh, 1975), where the MFs are themselves fuzzy in nature. Besides, it can consider the intra and inter personnel uncertainties of human perceptions and is more useful in case of presence of more fuzziness in a judgement. However, T2FSs involve substantial computational and mathematical complexities. To overcome it, researchers have further improved the concept of T2FSs to IT2FSs (Mendel, 2009) by defining an interval-valued MF. However, it can be witnessed from Figure 2.2 that the utilizations of IT2FSs in combination with MCDM methods is very limited: only Wang *et al.* (Wang *et al.,* 2019b) integrated the concept of IT2FSs and MULTIMOORA for risk prioritization. The major limitation of their work is that they directly computed the weights of the risk factors in terms of crisp numbers, which certainly instigated information distortions at the earlier stages of decision-making. Overall, they have not properly exploited the potentials of the

IT2FSs, **which motivates this research work to carry out further investigations by employing them in FMEA case studies.**

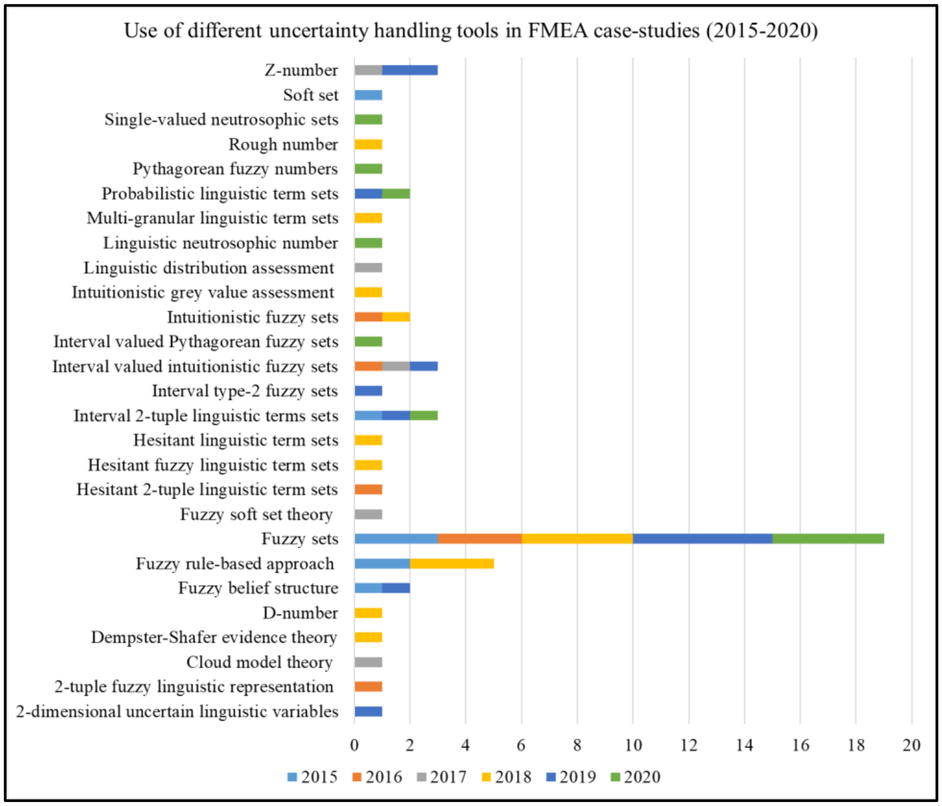

Figure 2.2. Use of different uncertainty handling tools in FMEA case-studies

- The other group consists of the implementations and/or developments of rough numbers (Wang *et al.*, 2018b), soft sets (Chang, 2015), Dempster-Shafer evidence theory (Wang *et al.*, 2018a), D-numbers (Bian *et al.*, 2018), etc, and their different properties. These theories also aimed to abate the linguistic uncertainties, but when compared to FS theory, these are still developing.

2. *Considerations of different risk factors:* Most of prior FMEA case studies have opted for three conventional risk factors: S, O, and D. Yet, for more rational risk prioritization, few researchers have decoupled these risk factors and/or added additional risk factors. An overview of them are presented in Table 2.19, which reveals that **in the context of sustainable manufacturing pratice, and when the research community is immensely focusing on the philosphy of Industry 4.0, none of the earlier research has judged the risk factors from sustainability point of view, *viz.*, from *TBL* of sustainability.**

Table 2.19. Research considering additional risk factors

References	Considered risk factors
(Abbasgholizadeh Rahimi *et al.*, 2015)	Severity (impact, core process, typicality, affected range, customer participation, service encounter, interdependency, bottleneck possibility, hardness of isolation, resource distribution), occurrence (frequency, repeatability, failure visibility, single point failure), and detection (chance of un-detection, method of systematic detection, customer/employee detection, hardness of proactive inspection)
(Liu *et al.*, 2016b)	Occurrence, impact on organization, impact on patient, detection, interdependence with other failures, cost due to failure, and corrective action cost
(Selim *et al.*, 2016)	Severity, occurrence, detection, current technology, substitutability, capacity utilization, and contribution to profit
(Wang *et al.*, 2016)	Failure mode ratio, failure effect probability, failure rate
(Wang *et al.*, 2018b)	Severity (personal security, environmental security, strength, stiffness, precision, stability, maintenance costs, direct losses, indirect losses, disassembly difficulty, maintenance time), probability, non-detectability
(Certa *et al.*, 2017)	Severity, occurrence, and detection
(Carpitella *et al.*, 2018)	Occurrence, time of operation, and modality of execution
(Lo and Liou, 2018)	Severity, occurrence, detection, and expected cost

3. *Methods for calculating the weights of the risk factors:* Different methods have been adopted to calculate the weights of the risk factors as presented in Table 2.20. However, the following points are worth mentioning:

- In most of the works, subjective judgements have been delivered by the experts.
- Thereafter, the **mostly employed method is either AHP and/or fuzzy AHP.**
- In the objective weighting category, most of the research has considered the directly given values of the weights, ensuing by the Entropy method.
- Few of them have adopted the combination and incomplete weighting methods.

Table 2.20. Use of different methods to calculate the weights of the risk factors

Classification	Method(s)	References	Count
Subjective weighting methods	Expert judgement	(Abbasgholizadeh Rahimi *et al.*, 2015; Başhan *et al.*, 2020; Bian *et al.*, 2018; Chen *et al.*, 2020; Li *et al.*, 2020; Liu *et al.*, 2015a, 2015c; Rathore *et al.*, 2020; Safari *et al.*, 2016; Selim *et al.*, 2016; Tooranloo *et al.*, 2018; Tooranloo and Ayatollah, 2016; Wang *et al.*, 2019b; Wang *et al.*, 2018a)	14
	AHP	(Carpitella *et al.*, 2018; Chang, 2016; Liu *et al.*, 2019b; Sharma and Sharma, 2015)	4
	Grey interval based BWM	(Lo and Liou, 2018)	1
	Fuzzy AHP	(Panchal *et al.*, 2018a, 2018b; Sagnak *et al.*, 2020)	3
	Fuzzy AHP and Logarithmic fuzzy preference programming	(Mangeli *et al.*, 2019)	1
	Z-TOPSIS	(Zhang *et al.*, 2019)	1
	Fuzzy BWM	(Ghoushchi *et al.*, 2019; Tian *et al.*, 2018)	2
	Fuzzy digraph	(Liu *et al.*, 2016a)	1
	Fuzzy preference programming	(Baykasoğlu and Gölcük, 2020)	1

Classification	Method(s)	References	Count
	ANP	(Wang *et al.*, 2016)	1
	Fuzzy measure and Shapley index	(Wang *et al.*, 2019c)	1
Objective weighting methods	Entropy	(He *et al.*, 2020; Liu *et al.*, 2018; Tsai and Yeh, 2015; Wang *et al.*, 2018; Zhao *et al.*, 2017)	5
	Directly given	(Certa *et al.*, 2017; Chang, 2015; Kumar *et al.*, 2018; Li and Chen, 2019; Liu *et al.*, 2017, 2015b; Lolli *et al.*, 2015; Vahdani *et al.*, 2015; Wang *et al.*, 2018b)	9
	Randomly generated	(Delice and Can, 2017)	1
	Maximum deviation model	(Hu *et al.*, 2019b; Li *et al.*, 2020; Zhu *et al.*, 2020b)	3
	Ordered weighted geometric operator	(Chang *et al.*, 2018)	1
	Choquet integral	(Wang *et al.*, 2017)	1
	TOPSIS based method	(Huang *et al.*, 2019)	1
	DEMATEL	(Lo *et al.*, 2020)	1
Combination weighting	Experts' linguistic judgements and statistical distance	(Liu, 2019b)	1
	Fuzzy AHP and entropy	(Liu *et al.*, 2015d; Mohsen and Fereshteh, 2017)	2
	Experts' judgements and Entropy method	(Huang *et al.*, 2017; Wang *et al.*, 2016)	2
	BWM method and maximizing deviation method	(Nie *et al.*, 2018)	1
	Choquet integral and Entropy method	(Wang *et al.*, 2019a)	1
	Experts' judgements and normal distribution based ordered ratings	(Akram *et al.*, 2020)	1
Incomplete weighting	Maximum cross-entropy based linear programming model	(Liu, 2019a)	1
	GRA based multi-objective optimization model	(Liu *et al.*, 2016b)	1

4. *Utilizations of MCDM methods for risk prioritizations:* Until now, numerous MCDM methods have been explored and their potentials have been examined in the risk ranking of failure modes. A graphical analysis of them is presented in Figure 2.3. It can be observed that **TOPSIS and GRA are widely adopted methods (belongs to distance based MCDM method)**, followed by **VIKOR, MULTIMOORA (belongs to compromise-solution based MCDM method)**, TODIM, hybrid methods, and so on.

5. *Application areas:* From Figure 2.4, it can be observed that most of the previous FMEA case studies pertain to the mechanical sector (16 papers). However, as mentioned earlier, **gearboxes being an important asset in most of the manufacturing and processing plants, have not been considered for detailed FMECA, especially from sustainability context.**

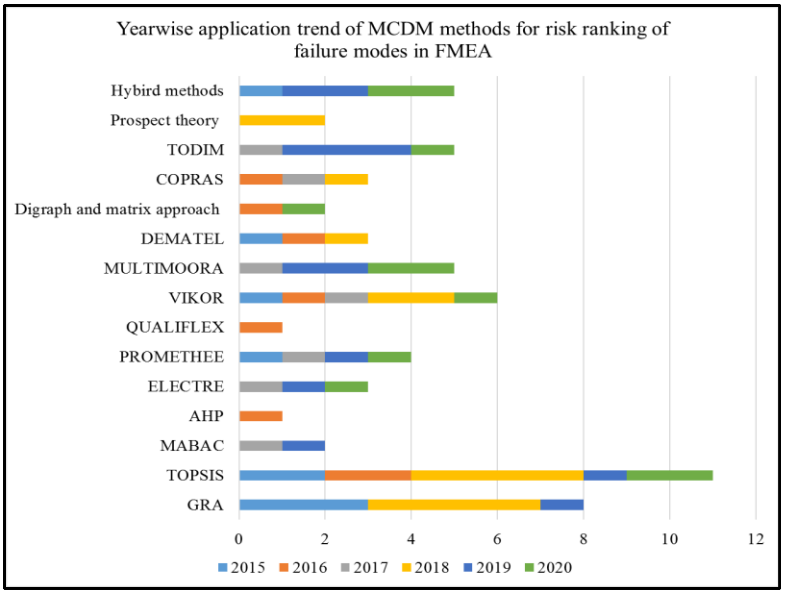

Figure 2.3. Use of MCDM methods for failure modes ranking

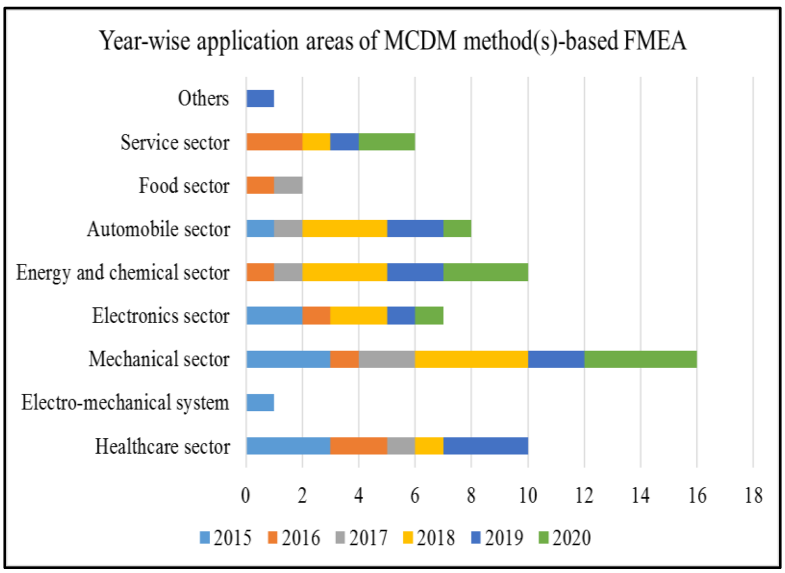

Figure 2.4. Areas of applications for FMECA case studies

2.4.2. Observations from Literature Related to AI-based Fault Diagnosis Approaches

Based on the survey presented in *Section 2.2*, the following points are observed:

1. *Effects of environmental conditions on fault initiation:* Most of the previous research works have been performed in the laboratory, without considering to the effects of environmental parameters (*viz.*, high humidity, high dust level, high temperature, etc.) on fault initiation.

2. *Considerations of multiple monitoring approaches:* Most of the prior researches have adopted single type of monitoring technique (*i.e.,* vibration signal). However, the fault propagation graph confirms that at different stages of the fault, distinct types of signals become prominent. Thus, engineers always desire to rely upon multiple types of monitoring techniques.

3. *Dealing with bottom-up concept:* Earlier researches have implemented the *'top-down'* approach of fault diagnosis (*i.e.,* fault is induced in a part of a system, then the resultant effects are collected). Whereas, in industries it is impractical to follow this policy. Instead, engineers are frequently confronted with the *'bottom-up'* approach (*i.e.,* based on the detected parameters and/or signals, fault diagnosis is performed).

4. *Experts' involvement to interpret the vibration signals:* Vibration signal analysis techniques are case-specific, and often require experts' involvements for the suitable interpretation. For example, by performing *Fast Fourier Transform* (FFT) of vibration signals, hardly any information can be extracted in case of any gear fault, because several sidebands usually appear around the gear mesh frequency (GMF). In such circumstances, experts generally prefer to adopt another analysis technique, such as Wavelet analysis or Cepstrum analysis.

5. *Capturing signals from pre-defined locations:* For a large and complex system (*e.g.,* gearbox), if the vibration signal is captured from a single point, it may not reveal the exact location and type of fault. This problem is further intensified when the system is functioning in a condition of lots of surrounding noises. So, engineers must collect the vibration signals from the pre-defined locations. Thereafter, these captured signals are either analysed by some sophisticated software or by some high-valued gadgets incorporated with different analysis techniques. This is also a tedious and time-consuming task.

6. *Dealing with large numbers of HIs:* Mapping of large numbers of HIs from the measurement space to fault space require experts' intervention. However, in industry, experts are not available all the time.

7. *Considerations of multiple types of data:* Earlier research work either considered the waveform data, or event data, or value type data separately. However, engineers often desire the combination of these (*i.e.,* after identification of type and location of fault, what will be the necessary tasks to be carried out?).

8. *Dealing with incomplete and/or missing information:* Earlier CBR-based fault diagnosis researches have not considered the situation of dealing with the incomplete information, which frequently happens during the fault diagnosis of large-scale and complex systems.

2.4.3. Observations from Maintenance Strategy Selection Problems Literature

From the discussions carried out in *Section 2.3*, it can be remarked that the MSSP has been thoroughly studied by earlier researchers. Very few researchers have taken the endeavour and studied the sustainable MSSP. However, none of them have broadly identified the pertinent KPIs from the perspective of sustainability's TBL.

Apart from that, earlier contributors have solved the MSSP through employing MCDM methods, which basically deals with a set of data, and each of the time new alternatives/criteria are added to the list of consideration, either experts need to re-evaluate the weights of the criteria or they need to re-evaluate the new alternative with respect to criteria. This makes the process a tedious one. Furthermore, in case of qualitative KPIs, each time the experts' judgement may not be consistent or available, and thus lead to improper selection of the maintenance strategy which subsequently become a financial burden for the organization.

2.5. Selection of Method(s)/Tool(s) for Utilization and/or Development

From the observations presented in the previous sections, the following method(s)/tool(s) are chosen for further implementations and/or developments.

2.5.1. Method(s)/Tool(s) Adopted and/or Developed in FMECA

- *Tools to deal with linguistic uncertainties:* The capabilities of FSs and IT2FSs to manage the uncertainties in linguistic opinions have already been presented in *Section 2.4.1*. Additionally, it has been also witnessed that most of the FMEA literatures have adopted the FSs in their FMEA case studies. Thus, initially in this thesis work, FSs (more specifically, TFNs) have been chosen as the potential candidate to deal with linguistic uncertainties. However, despite the profound abilities of FSs, as mentioned earlier, it has been criticized in prior literatures. To further refine the subjective vagueness, the applications of IT2FSs are thereafter examined in this thesis. The motivation behind using the IT2FSs is as follows:

a) Ranking of Failure modes according to their criticality levels is a sensitive application area from the organization's point of view. Thus, it is always desired to minimize the linguistic uncertainties to the utmost level, such that failure modes can be properly ordered.

b) Only a single research work has employed the IT2FSs in combination with MCDM method for risk ranking. However, in that work, the information distortions happened at the preliminary stages, which necessitated the further investigations.

- *MCDM methods to calculate the weights of the risk factors:* Initially, in this thesis work the concept of AHP method is adopted to compute the weights of the risk factors. The reasons behind this selection are:

a) It has been widely adopted in prior literatures to compute the risk factors' weights by subjective weighting approach.

b) It can consider the consistency ratio of the experts' evaluations and can consider the judgements of multiple experts in a group decision making scenario.

However, the proposed scale in the original AHP method is crisp in nature (*i.e.*, linguistic judgements are converted into crisp numbers*)*, and are not capable to consider human's vague thoughts (Kutlu and Ekmekçioğlu, 2012). To improve that, different versions of fuzzy AHP have been developed by the earlier researchers (Buckley, 1985; Chang, 1996; Van Laarhoven and Pedrycz, 1983). Although, Chang's extent analysis-based fuzzy AHP method has been extensively applied to solve different FMEA problems (Kutlu and Ekmekçioğlu, 2012), it has received a lot of criticism. Wang *et al.*, (Wang *et al.*, 2008) specifically mentioned that *"extent analysis method cannot estimate the true weights from a fuzzy comparison matrix and has led to quite a number of misapplications in the literature".* Whereas, Buckley's fuzzy AHP has not received any criticism till date and to the best knowledge, its application in FMEA literature is very limited. Thus, it is further utilized in this thesis to compute the weights of the risk factors, considering the subjective judgements.

Conversely, when the number of risk factors are increased in numbers (*i.e.*, above 11 in numbers), it is difficult to compute the criteria weights by employing AHP. Further, decision-makers always attempt to classify the cause and effects groups of criteria (*viz.*, causal dependency), along with their weights. To deal with these, two such viable options are Choquet integral and ANP methods. However, for the earlier one, identification of fuzzy measures is an arduous task for the decision-makers. For example, suppose that there are n number of criteria, and then the Choquet integral requires $(2^n - 2)$ coefficients to be defined. Whereas, for the latter one, it requires too many comparisons, and in some situations the pairwise comparison questions might be difficult to intercept. Further, in ANP the network structure of the decision problem is to be known a priori.

As previously mentioned, DEMATEL is a popular relational analysis MCDM method, and has been mainly adopted in the FMEA literature to derive the relationships among the failure modes, and to rank them according to their risk levels. But in this context, the reasons for choosing DEMATEL as a viable option are presented below:

a) It can simultaneously depict the causal dependencies among the risk factors and calculate their weights,

b) Until now, none of the FMEA case studies have reported the depiction of the causal dependencies among the risk factors, by utilizing the DEMATEL method.

- *MCDM methods to prioritize the failure modes:* From Figure 2.3, it can be seen that whenever a new MCDM method is developed by the researchers, its abilities in solving diversified decision-making problems are examined. Obviously, the developed method should have some advantages over the earlier ones and must be capable of computing more credible and robust ranking results.

MAIRCA *(Multi-Attributive Ideal Real Comparative Analysis)* is a newly developed distance-based MCDM method, originally proposed by Pamucar *et al.*, (Pamučar *et al.*, 2014). It has proved its superiority over other well-established MCDM methods, like TOPSIS, ELECTRE, *etc.* because of its linear normalization method, characterized by a simple mathematical apparatus, and solution stability. In this method, the decision maker is unbiased from the very initial stage towards the ordering of alternatives. Later, based on the minimum distance between the matrix of theoretical ponder and actual ponder the best alternative (or the most critical failure mode) is chosen and other failure modes are ranked.

MARCOS *(Measurement of Alternatives and Ranking according to COmpromise Solution)* is a compromise solution-based MCDM method recently developed by Stević *et al.,* (Stević *et al.,* 2020). It is based on defining the relationship between alternatives and reference values (ideal and anti-ideal alternatives). Based on the defined relationship, the utility functions of alternatives are determined, and compromise ranking is made in relation to ideal and anti-ideal solutions. The utility function represents the position of an alternative about an ideal and anti-ideal solution. The best alternative is the one that is closest to the ideal and at the same time furthest from the anti-ideal reference point. Authors have proven that it has a better ranking stability than other popular MCDM methods (*i.e.,* TOPSIS, ELECTRE, etc.).

The above two methods, along with the well-established TOPSIS are opted in this thesis work at different chapters for the risk ranking purpose. The specific reasons of utilizing them are as follows:

a) Distance-based and Compromise-ranking based MCDM methods have been mostly adopted by the former researchers to rank the failure modes. Similarly, MAIRCA and TOPSIS belongs to the first group, and MARCOS belongs to the latter group.

b) Until now, none of the earlier research works have examined the capabilities of MAIRCA and MARCOS methods for failure modes' risk ranking in FMECA.

2.5.2. Method(s)/Tool(s) Adopted for Fault Diagnosis

The observations from the literature review (refer *Section 2.4.2*) reveals that multiple AI-based methods have been exploited during fault classification (*e.g.,* ANN and/or their variants, SVM, ESs, FL, etc.). However, each of them has their own benefits and limitations as presented in Table 2.21.

Table 2.21. Comparisons of different AI-based approaches for fault diagnosis of gearbox

Method by	Advantages	Drawbacks	Computational time	Complexity of the method	Decision maker's involvement
ANN	- Model the structure of reasoning of human brain. - Ability to learn and detect the relationships between several inputs and outputs.	- Requires extensive training for accurate result. - Perform like a black box. - Selection of number of hidden layers requires expert knowledge. - Weightages between parameters are given by experts.	Moderate	Moderate (increases with number of parameters)	High
ESs	- Decision making problems become simplified. - Decision are taken by several 'IF-THEN' rules.	- Constructing several 'IF-THEN' rules are tedious task and it is necessary to extract knowledge from cases.	Very low	Moderate	High

Method by	Advantages	Drawbacks	Computational time	Complexity of the method	Decision maker's involvement
		- Unable to handle with novel cases.			
FL	- Efficiently deal with imprecise data. -Relationship between the parameters are taken into consideration for decision making.	- Creating a rule-base for final decision making requires expert's involvement. - Unable to deal with incomplete data.	Moderate	High	Moderate
BN	- Can handle complex and non-linear problems. - Can handle missing data and allows combination of data with domain knowledge (Uusitalo, 2007). - Can avoid over-fitting of data. -Can provide good decisions with small sample size.	-Decisions are taken by prior distributions. When the prior distributions are violated then improper outcomes are provided. - To calculate probability of any branch of the network, all branches must be calculated previously. - It is useful, when prior knowledge is reliable. -Ability to deal with continuous data is limited.	High	High	High
SVM	- Efficiently handle data with unknown distributions.(Auria and Moro, 2008), - Can provide an efficient out of sample generalization, - Can provide a unique solution, when optimality problem is convex.	- Lack of transparency of result. - Choice of kernel is considered as a major problem. - Not able to handle efficiently with discrete data. -high algorithmic complexity and extensive memory requirements.	High	Very High	High
CBR system	- Can perform efficiently in unstructured domain. - Can able to provide decisions with incomplete information, and able to replace human experts. - No requirement to derive knowledge by inductive rules. Inferences are provided directly from cases. -Superior ability of adaptation and learning over time.	- Accuracy depends on the size of case-base. - Case-base maintenance is a major issue.	Moderate (depends on the volume of case-base)	Very low	Low

On the contrary, The CBR methodology is well-suited in the following instances:

- Historical cases are available and easily accessible. References to historical cases are beneficial while dealing with recurrent problems.

- If a domain does not have a basic model or has a model that is impossible to comprehend and to model mathematically, the CBR approach becomes flexible in that area. Prior experiences are ample to develop a CBR model without a profound understanding of the problem state (Chakraborty and Boral, 2017).

- When exceptional cases are frequently encountered for solving a problem. Incremental learning and adaption qualities are two major significant features those incorporated in the process model of CBR (Kolodner and Riesbeck, 2014).

Further, from the prior applications of CBR in fault diagnosis problems, it is perceived that the benefits of the methodology have not been properly explored, which motivates to further employ it in a methodical way in this thesis work.

2.5.3. Method(s)/Tool(s) Adopted for Optimal Sustainable Maintenance Strategy Selection

The recent developments of computerized programmes and different philosophies of data-driven AI-based approaches can sort out this problem in a well-structured manner. In an organization, it is a common practice to store all the maintenance related information in the centralized database, which can be a valuable input to the computerized maintenance management system (CMMS). So, it is always preferable to consider these data for any decision-making purpose. However, in case, when the exact data are not available, then AI-based approaches outperform the MCDM methods to provide an approximate reasoning.

Besides, the potential of CBR approach, and ES have already been explained in Table 2.21. It can be clearly observed that ES performs well in the case of presence of exact matches, whereas CBR system performs well when there is no exact match in the database. Thus, both of their advantages can be further explored in the context of sustainable maintenance strategy selection problem by overcoming the drawbacks of MCDM methods.

2.6. Research Contributions

Based on the preceding discussions the contributions of this research are summarised as:

1. To develop and devise integrated MCDM-based frameworks to overcome the drawbacks of the traditional RPN-based FMEA approaches, along with managing the uncertainties involved in subjective judgements. This is further sub-categorized as follows:

   a) Initially, two integrated fuzzy MCDM decision-making frameworks are proposed for the risk ranking of failure modes in a benchmark FMEA case-study. In the first framework, Buckley's fuzzy AHP is integrated with the developed fuzzy MAIRCA, and in second framework, fuzzy AHP is integrated with the proposed modified fuzzy MARCOS method for the risk ranking of failure modes.

   Alike the previous literature, where the weights of the risk factors have been calculated in terms of crisp numbers and can cause early information distortion, Buckley's fuzzy AHP is here adopted to determine the fuzzy weights of the risk factors, and the developed fuzzy MAIRCA and modified fuzzy MARCOS are employed for risk ranking of failure modes individually.

b) Then, a real time FMEA case-study of large-scale process plant gearbox is performed. The failure modes are ranked by proposing two mathematical frameworks consist of IT2F-DEMATEL – fuzzy MAIRCA and IT2F-DEMATEL – fuzzy MARCOS methods.

In the context of sustainable development, the severities of failure modes are considered from the TBL of sustainability by identifying the pertinent KPIs. Then the causal dependencies and weights of the risk factors are determined by IT2F-DEMATEL method. Finally, the risk prioritization of failure modes is carried out by fuzzy MAIRCA and modified fuzzy MARCOS, individually. Validations of the risk ranking results are carried out and sensitivity analyses of the integrated approaches are performed.

c) Proposing an integrated IT2FSs and multiplicative half quadratic programming based MCDM framework for computing the aggregated risk ranking of failure modes in FMEA.

Here, initially the IT2F-DEMATEL method is further extended to obtain the IT2FNs-based weights of the risk factors. Then the mathematical models of IT2F-MAIRCA, IT2F-MARCOS, and modified IT2F-TOPSIS are developed. After that, half quadratic minimization-based approach is adopted to compute final aggregated risk rankings of the failure modes, along with the consensus index and trust level. The case-study of process plant gearbox is utilized to validate the suitability of the integrated framework.

2. To develop a CBR-based system and exploit its ability in fault diagnosis of the considered gearbox. The system is capable to address the problems as mentioned in the *Section 2.4.2*.

Further, the developed system can provide the approximate location and type of fault while dealing with incomplete data. After fault diagnosis, it can provide information about the most appropriate maintenance tasks to be carried out to minimize the downtime of the production facility.

3. Initially to identify the pertinent KPIs for each available maintenance strategy. Then, using those KPIs, a hybrid AI-based framework by integrating the concepts of CBR and ESs is developed. This hybrid framework will aid the organization in selecting the optimal sustainable maintenance strategy while dealing with incomplete information.

As mentioned previously, the next Chapter develops the mathematical models for the risk ranking of failure modes of a popular FMEA example.

Chapter 3 Integrated Fuzzy MCDM Frameworks for Risk Prioritization in FMEA

Based on the discussions carried out in the previous two chapters, the following contributions are made here[3]:

1. Proposing the mathematical model of modified fuzzy MAIRCA, and thereafter developing an integrated fuzzy MCDM framework by combining Buckley's fuzzy AHP and modified fuzzy MAIRCA.

2. Suggesting the mathematical model of modified fuzzy MARCOS, and then developing an integrated fuzzy MCDM framework by combining Buckley's fuzzy AHP and modified fuzzy MARCOS.

3. Validating the capabilities of the proposed frameworks for the risk ranking of failure modes by considering a popular FMEA case study as given in (Kutlu and Ekmekçioğlu, 2012).

4. Comparisons of performances between the two developed frameworks in terms of ranking stability and robustness. Thereafter, performance of each developed framework is compared with the original approach (*viz.*, fuzzy AHP-fuzzy TOPSIS), as developed by Kutlu and Ekmekçioğlu (Kutlu and Ekmekçioğlu, 2012) .

3.1. Preliminaries

The necessary terms, definitions, and arithmetic operations required to develop the mathematical models are presented below for the sake of completeness.

Fuzzy Set: A FS $\tilde{A}$ can be defined mathematically by a membership function $\mu_{\tilde{A}}(x)$, which assigns each element x in the universe of discourse X to a real number in the interval $[0,1]$.

[3] The contributions of this chapter can be found in the following two published papers:

a) Boral, S., Howard, I., Chaturvedi, S.K., McKee, K., Naikan, V.N.A., 2020. An integrated approach for fuzzy failure modes and effects analysis using fuzzy AHP and fuzzy MAIRCA. *Engineering Failure Analysis* 108, 104195.
b) Boral, S., Chaturvedi, S.K., Howard, I., McKee, K., Naikan, V.N.A. 2020. An Integrated Approach for Fuzzy Failure Mode and Effect Analysis Using Fuzzy AHP and Fuzzy MARCOS. *In Proceedings of IEEE International Conference on Industrial Engineering and Engineering Management (IEEM)*, Singapore. pp. 395-400.

Fuzzy Number: A FN is a fuzzy subset in the universe of discourse X, which is both convex and normal. A fuzzy set $\tilde{A}$ in the universe of discourse X is convex if and only if for all x_1, x_2 in X, $\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\,(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$, where $\lambda \in [0,1]$. The similar fuzzy set $\tilde{A}$ is called a normal fuzzy set if $\exists x_i \in X, \mu_{\tilde{A}}(x_i) = 1$.

Triangular Fuzzy Number: A TFN is represented by a triplet $\tilde{A} = [a_l, a_m, a_u]$, and is depicted in Figure 3.1. Membership function of a TFN is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_l}{a_m - a_l} & , a_l \leq x \leq a_m \\ \dfrac{x - a_u}{a_m - a_l} & , a_m \leq x \leq a_u \\ 0 & , otherwise \end{cases} \tag{3.1}$$

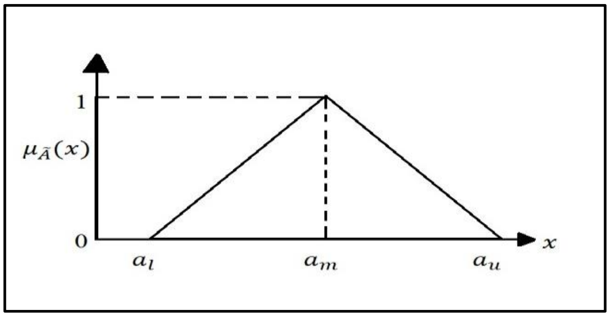

Figure 3.1 Graphical representation of a TFN

The TFNs can be applied in a situation, where the decision maker is not sure about the type of MF associated with a FN.

Arithmetic Operations: Most commonly used arithmetic operations on two FNs $\tilde{A} = [a_l, a_m, a_u]$, and $\tilde{B} = [b_l, b_m, b_u]$, where $a_l \leq a_m \leq a_u$ and $b_l \leq b_m \leq b_u$ are provided in (3.2)-(3.6):

Addition:

$$\tilde{A} \oplus \tilde{B} = [a_l, a_m, a_u] \oplus [b_l, b_m, b_u] = [a_l + b_l, a_m + b_m, a_u + b_u] \tag{3.2}$$

Subtraction:

$$\tilde{A} \ominus \tilde{B} = [a_l, a_m, a_u] \ominus [b_l, b_m, b_u] = [a_l - b_u, a_m - b_m, a_u - b_l] \tag{3.3}$$

Multiplication:

$$\tilde{A} \otimes \tilde{B} = [a_l, a_m, a_u] \otimes [b_l, b_m, b_u] = [a_l \times b_l, a_m \times b_m, a_u \times b_u], \text{ if } a_l \geq 0 \text{ and } b_l \geq 0. \tag{3.4}$$

Division: $$\tilde{A} \oslash \tilde{B} = [a_l, a_m, a_u] \oslash [b_l, b_m, b_u] = \left[\frac{a_l}{b_u}, \frac{a_m}{b_m}, \frac{a_u}{b_l}\right], \text{ if } a_l \geq 0 \text{ and } b_l > 0 \quad (3.5)$$

Multiplication by a scalar, k: $$k \otimes \tilde{A} = \begin{cases} (ka_l, ka_m, ka_u)\ , if\ k > 0 \\ (ka_u, ka_m, ka_l), if\ k < 0 \end{cases} \quad (3.6)$$

A TFN can be considered as a special case of a TrFN, and is defined by a quadruplet $\tilde{A} = (a_l, a_{m1}, a_{m2}, a_u)$. Arithmetic operations of two TrFNs are almost similar to that of TFNs and interested readers may refer any textbook on FST.

Defuzzification: The process of converting a fuzzy number to its crisp value is known as defuzzification. A crisp number is needed for several purposes, such as comparison, ranking, etc. There are several popular methods available to accomplish this purpose, and among them the graded mean average (equations (3.7) and(3.8) is the most popular one due to its simple calculation step and robust mathematical foundation.

$$A = \frac{a_l+4a_m+a_u}{6} \text{ (for TFNs)} \quad (3.7)$$

$$A = \frac{a_l+4a_m+a_u}{6} \text{ (for TrFNs)} \quad (3.8)$$

3.2. Criteria Weights Calculation by Fuzzy AHP

The steps for computing the criteria weights in terms of TFNs are as follows:

Step 1: Construct the pairwise comparison matrices for n number of criteria/sub-criteria by converting the linguistic judgements to the corresponding TFNs, utilizing Table 3.1. If K cross-functional experts participate in the decision-making process, then each element $\tilde{a}_{ij}^K$ of the pairwise comparison matrix $\tilde{A}^K$ is a TFN. The mathematical representation of this step is given in (3.9).

$$\tilde{A}^K = \begin{bmatrix} 1 & \tilde{a}_{12}^K & \cdots & \tilde{a}_{1n}^K \\ \tilde{a}_{21}^K & 1 & & \tilde{a}_{2n}^K \\ \vdots & & \ddots & \vdots \\ \tilde{a}_{n1}^K & \tilde{a}_{n2}^K & \cdots & 1 \end{bmatrix} \quad (3.9)$$

where, $\tilde{a}_{ij}^K = (a_{l_{ij}}^K, a_{m_{ij}}^K, a_{u_{ij}}^K)$, *and* $K = 1,2, \ldots, k$.

Table 3.1. Fuzzy evaluation scores for the weight vectors

Linguistic terms	Triangular fuzzy numbers
Absolutely strong (AS)	(2,5/2,3)
Very strong (VS)	(3/2,2,5/2)
Fairly strong (FS)	(1,3/2,2)
Slightly strong (SS)	(1,1,3/2)
Equal (E)	(1,1,1)
Slightly weak (SW)	(2/3,1,1)
Fairly weak (FW)	(1/2,2/3,1)
Very weak (VW)	(2/5,1/2,2/3)
Absolutely weak (AW)	(1/3,2/5,1/2)

Step 2: Compute the aggregated fuzzy pairwise comparison matrix by employing (3.10).

$$\tilde{A} = \begin{bmatrix} 1 & \frac{\tilde{a}_{12}^1 \oplus \dots \oplus \tilde{a}_{12}^k}{K} & \cdots & \frac{\tilde{a}_{1n}^1 \oplus \dots \oplus \tilde{a}_{1n}^k}{K} \\ \frac{\tilde{a}_{21}^1 \oplus \dots \oplus \tilde{a}_{21}^k}{K} & 1 & \cdots & \frac{\tilde{a}_{2n}^1 \oplus \dots \oplus \tilde{a}_{2n}^k}{k} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\tilde{a}_{n1}^1 \oplus \dots \oplus \tilde{a}_{n1}^k}{K} & \frac{\tilde{a}_{n2}^1 \oplus \dots \oplus \tilde{a}_{n2}^k}{K} & \cdots & 1 \end{bmatrix} \tag{3.10}$$

$$Or,$$

$$\tilde{A} = \begin{bmatrix} 1 & (a_{l_{12}}, a_{m_{12}}, a_{u_{12}}) & \cdots & (a_{l_{1n}}, a_{m_{1n}}, a_{u_{1n}}) \\ (a_{l_{21}}, a_{m_{21}}, a_{u_{21}}) & 1 & \cdots & (a_{l_{2n}}, a_{m_{2n}}, a_{u_{2n}}) \\ \vdots & \vdots & \ddots & \vdots \\ (a_{l_{n1}}, a_{m_{n1}}, a_{u_{n1}}) & (a_{l_{n2}}, a_{m_{n2}}, a_{u_{n2}}) & \cdots & 1 \end{bmatrix}$$

Step 3: Check the consistency[4] of the fuzzy aggregated pairwise comparison matrix as obtained in Step 2. To check the consistency of fuzzy pairwise comparison matrix, elements of the pairwise comparison matrix are de-fuzzified by employing (3.7).

Step 4: Compute the fuzzy geometric mean for each row of the matrix as shown in (3.11). The fuzzy geometric means of the first parameters of the triangular fuzzy numbers in each row are calculated as follows:

$$\begin{aligned} a_{l_1} &= [1 \times a_{l_{12}} \times \dots \times a_{l_{1n}}]^{\frac{1}{n}} \\ a_{l_2} &= [a_{l_{21}} \times 1 \times \dots \times a_{l_{2n}}]^{\frac{1}{n}} \\ a_{l_n} &= [a_{l_{n1}} \times a_{l_{n2}} \times \dots \times 1]^{\frac{1}{n}} \end{aligned} \tag{3.11}$$

[4] If $\tilde{A} = [\tilde{a}_{ij}]$ is a fuzzy positive reciprocal matrix, and $A = [a_{ij}]$ is the defuzzified positive reciprocal matrix, then $\tilde{A}$ is said to be consistent iff A is consistent (Buckley, 1985). The general procedures for measuring the consistency ratio in AHP is given in (Gugaliya *et al.,* 2019). If, in case, the result is not consistent, experts need to re-evaluate the pairwise comparisons.

Similarly, compute the geometric means of second and third parameters of the TFNs in each row.

Step 5: Assuming that the sums of the geometric mean values in the row are a_{l_s},a_{m_s}, and a_{u_s}, respectively, then fuzzy criteria weights are calculated as (3.12).

$$\widetilde{W} = \begin{bmatrix} \widetilde{w}_1 \\ \widetilde{w}_2 \\ \vdots \\ \widetilde{w}_n \end{bmatrix} = \begin{bmatrix} \left(\frac{a_{l_1}}{a_{u_s}}, \frac{a_{m_1}}{a_{m_s}}, \frac{a_{u_1}}{a_{l_s}}\right) \\ \left(\frac{a_{l_2}}{a_{u_s}}, \frac{a_{m_2}}{a_{m_s}}, \frac{a_{u_2}}{a_{l_s}}\right) \\ \left(\frac{a_{l_n}}{a_{u_s}}, \frac{a_{m_n}}{a_{m_s}}, \frac{a_{u_n}}{a_{l_s}}\right) \end{bmatrix} \tag{3.12}$$

3.3. Ranking of Alternatives by Modified Fuzzy MAIRCA Method

Ranking results of alternatives are computed by employing the ensuing steps:

Step 1: Construct the initial linguistic decision matrix (D_L) based on the linguistic evaluation of alternatives with respect to the considered criteria. Let K numbers of experts be involved to judge m alternatives with respect to n numbers of criteria. The decision matrix is shown in (3.13):

$$D_L = \begin{pmatrix} L_{11}^1, \dots L_{11}^k & L_{12}^1, \dots L_{12}^k & \dots & L_{1n}^1, \dots L_{1n}^k \\ L_{21}^1, \dots L_{21}^k & L_{22}^1, \dots L_{22}^k & \dots & L_{2n}^1, \dots L_{2n}^k \\ \vdots & \ddots & & \vdots \\ \vdots & \ddots & & \vdots \\ L_{m1}^1, \dots L_{m1}^k & L_{m2}^1, \dots L_{m2}^k & \dots & L_{mn}^1, \dots L_{mn}^k \end{pmatrix} \tag{3.13}$$

Here, L_{mn}^k implies that the $m-th$ alternative is linguistically evaluated with respect to the $n-th$ criterion by the $k-th$ expert, and $K = 1,2, \dots, k$.

Step 2: Following the scale for converting the linguistic judgements into corresponding TFNs (*e.g.*, Table *3.2*), replace each of the linguistic decision as in (3.14):

$$\widetilde{D}^{(K)} = \begin{pmatrix} \tilde{A}_{11}^{(K)} & \tilde{A}_{12}^{(K)} & \dots & \tilde{A}_{1n}^{(K)} \\ \tilde{A}_{21}^{(K)} & \tilde{A}_{22}^{(K)} & & \tilde{A}_{2n}^{(K)} \\ \vdots & & \ddots & \vdots \\ \tilde{A}_{m1}^{(K)} & \tilde{A}_{m2}^{(K)} & \dots & \tilde{A}_{mn}^{(K)} \end{pmatrix} \tag{3.14}$$

Table 3.2 Fuzzy evaluation scores to rate the alternatives

Linguistic terms	Fuzzy score
Very poor (VP)	(0,0,1)
Poor (P)	(0,1,3)
Medium poor (MP)	(1,3,5)
Fair (F)	(3,5,7)
Medium good (MG)	(5,7,9)
Good	(7,9,10)
Very good (VG)	(9,10,10)

Step 3: Using (3.2) and (3.6), construct the fuzzy aggregated decision matrix which is represented by (3.15).

$$\widetilde{D} = \begin{pmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \cdots & \tilde{A}_{1n} \\ \tilde{A}_{21} & \tilde{A}_{22} & & \tilde{A}_{2n} \\ \vdots & & \ddots & \vdots \\ \tilde{A}_{m1} & \tilde{A}_{m2} & \cdots & \tilde{A}_{mn} \end{pmatrix} \tag{3.15}$$

where, $\tilde{A}_{11} = \frac{\tilde{A}_{11}^{(1)} + \tilde{A}_{11}^{(2)} + \cdots + \tilde{A}_{11}^{(k)}}{K}$, and $\tilde{A}_{ij} = \left(a_{l_{ij}}, a_{m_{ij}}, a_{u_{ij}}\right)$.

Step 4: Since any alternative P_{A_i} can be chosen with equal probability, the preferences for each of them can be represented by (3.16). This step implies that the decision maker is un-biased towards the selection of an alternative.

$$P_{A_i} = \frac{1}{m};\ \sum_{i=1}^{m} P_{A_i} = 1 \tag{3.16}$$

Step 5: Compute and determine the elements of the fuzzy theoretical evaluation matrix ($\tilde{T}_{P_A}$) by multiplying preferences according to alternatives P_{A_i}, and fuzzy criteria weights as obtained by fuzzy AHP. This step is mathematically represented in (3.17).

$$\tilde{T}_{P_A} = \begin{pmatrix} \frac{1}{m}\widetilde{w}_1 & \frac{1}{m}\widetilde{w}_2 & \cdots & \frac{1}{m}\widetilde{w}_n \\ \frac{1}{m}\widetilde{w}_1 & \frac{1}{m}\widetilde{w}_2 & & \frac{1}{m}\widetilde{w}_n \\ \vdots & & \ddots & \vdots \\ \frac{1}{m}\widetilde{w}_1 & \frac{1}{m}\widetilde{w}_2 & \cdots & \frac{1}{m}\widetilde{w}_n \end{pmatrix} = \begin{pmatrix} \tilde{t}_{p11} & \tilde{t}_{p12} & \cdots & \tilde{t}_{p1n} \\ \tilde{t}_{p21} & \tilde{t}_{p22} & & \tilde{t}_{p2n} \\ \vdots & & \ddots & \vdots \\ \tilde{t}_{pm1} & \tilde{t}_{pm2} & \cdots & \tilde{t}_{pmn} \end{pmatrix} \tag{3.17}$$

where, $\tilde{t}_{pij} = \left(t_{l_{pij}}, t_{m_{pij}}, t_{u_{pij}}\right), 1 \le i \le m, 1 \le j \le n.$

Step 6: Normalize[5] the fuzzy aggregated decision matrix, obtained in Step 3 to compute the fuzzy normalized decision matrix $\tilde{N} = [\tilde{n}_{ij}]_{m\times n}$. Where, $\tilde{n}_{ij} = \left(n_{l_{ij}}, n_{m_{ij}}, n_{u_{ij}}\right)$ is computed by (3.18)[6].

$$n_{l_{ij}} = \frac{a_{l_{ij}}}{\sqrt{\sum_{i=1}^{m}\left[\left(a_{l_{ij}}\right)^2 + \left(a_{m_{ij}}\right)^2 + \left(a_{u_{ij}}\right)^2\right]}}$$
$$n_{m_{ij}} = \frac{a_{m_{ij}}}{\sqrt{\sum_{i=1}^{m}\left[\left(a_{l_{ij}}\right)^2 + \left(a_{m_{ij}}\right)^2 + \left(a_{u_{ij}}\right)^2\right]}} \qquad (3.18)$$
$$n_{u_{ij}} = \frac{a_{u_{ij}}}{\sqrt{\sum_{i=1}^{m}\left[\left(a_{l_{ij}}\right)^2 + \left(a_{m_{ij}}\right)^2 + \left(a_{u_{ij}}\right)^2\right]}}$$

Step 7: Calculate the fuzzy elements of the actual ponder matrix $(\tilde{T}_{r_A})$. This step is executed by multiplying the elements of the normalized decision matrix (refer (3.18) to the elements of the matrix of actual ponder (refer (3.17) by employing (3.19).

$$\tilde{T}_{r_A} = \begin{pmatrix} \tilde{t}_{r11} & \tilde{t}_{r12} & \cdots & \tilde{t}_{rn1} \\ \tilde{t}_{r21} & \tilde{t}_{r22} & & \tilde{t}_{rn2} \\ \vdots & & \ddots & \vdots \\ \tilde{t}_{rm1} & \tilde{t}_{rm2} & \cdots & \tilde{t}_{rmn} \end{pmatrix}$$
$$= \begin{pmatrix} \tilde{n}_{11}\times\tilde{t}_{p11} & \tilde{n}_{12}\times\tilde{t}_{p12} & \cdots & \tilde{n}_{n1}\times\tilde{t}_{pn1} \\ \tilde{n}_{21}\times\tilde{t}_{p21} & \tilde{n}_{22}\times\tilde{t}_{p22} & & \tilde{n}_{n2}\times\tilde{t}_{pn2} \\ \vdots & & \ddots & \vdots \\ \tilde{n}_{m1}\times\tilde{t}_{pm1} & \tilde{n}_{m2}\times\tilde{t}_{pm2} & \cdots & \tilde{n}_{nm}\times\tilde{t}_{pmn} \end{pmatrix} \qquad (3.19)$$

where, $\tilde{t}_{rij} = \left(t_{l_{rij}}, t_{m_{rij}}, t_{u_{rij}}\right), 1 \le i \le m, 1 \le j \le n.$

Step 8: Compute the gap between the theoretical and actual evaluation of each alternative with respect to each criterion, by computing the total gap matrix $\tilde{G}$ (refer (3.20)[7]). Although, in other works (Chatterjee *et al.*, 2018; Gigović *et al.*, 2016; Pamučar *et al.*, 2017, 2014, 2019; Pamucar *et al.*, 2018), the authors suggested to perform the simple fuzzy subtraction operation between $\tilde{T}_{P_A}$ and $\tilde{T}_{r_A}$, however in some cases it has been examined that in the output, the upper bound of the TFN

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[5] The normalization procedure of the decision matrix is carried out to increase its comparable capability.
[6] A normalization technique, in fuzzy MAIRCA is used to reduce the complexity involved in hard computation as well as to improve the accuracy of numeration. Another benefit of using this procedure is that the decision-maker need not be concerned about the nature of the criteria (*i.e.*, benefit or cost criteria). Generally, these types of situations are frequently encountered when the decision-maker is dealing with a large set of criteria.
[7] The distance measurement formulae between two fuzzy numbers as given in (Kutlu and Ekmekçioğlu, 2012) is used here. The reason for using this may be supplemented as it is a well-established technique in MCDM approaches (*e.g.*, Fuzzy TOPSIS).

tends to very high value, which further leads to a very high de-fuzzified crisp number, and attaining the wrong ordering of alternatives. In this work, the chosen procedure not only reduces the computational complexity, but also helps in computing the values without the requirement of de-fuzzification process.

$$g_{ij} = \sqrt{\frac{1}{3}\left[\left(t_{l_{pij}} - t_{l_{rij}}\right)^2 + \left(t_{m_{pij}} - t_{m_{rij}}\right)^2 + \left(t_{u_{pij}} - t_{u_{rij}}\right)^2\right]} \tag{3.21}$$

Step 9: Sum up the gap values for each alternative to obtain the final criteria function value by utilizing (3.22). These values are further arranged in ascending order to find the final ranking results of the alternatives.

$$Q_i = \sum_{j=1}^{n} g_{ij}\,, where\ i = 1,2, \dots, m \tag{3.22}$$

3.4. Ranking of Alternatives by Modified Fuzzy MARCOS Method

In this section the mathematical steps involved in developing the modified fuzzy MARCOS method are presented.

Steps 1-3: Same as described for fuzzy MAIRCA in *Section 3.3*.

Step 4: Generate the extended fuzzy initial decision matrix from the fuzzy aggregated decision matrix. In this step, the fuzzy ideal (ID) and fuzzy anti-ideal (AID) solutions are determined[8]. The matrix is represented as in (3.23).

$$\widetilde{D} = \begin{pmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \dots & \tilde{A}_{1n} \\ \tilde{A}_{21} & \tilde{A}_{22} & & \tilde{A}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_{m1} & \tilde{A}_{m2} & \dots & \tilde{A}_{mn} \\ \tilde{A}_{i1}^{(ID)} & \tilde{A}_{i2}^{(ID)} & \dots & \tilde{A}_{in}^{(ID)} \\ \tilde{A}_{i1}^{(AID)} & \tilde{A}_{i2}^{(AID)} & \dots & \tilde{A}_{in}^{(AID)} \end{pmatrix} \tag{3.23}$$

where, $\tilde{A}_{11} = \left(a_{l_{ij}}, a_{m_{ij}}, a_{u_{ij}}\right)$,

[8] This is similar to the determination of the best and worst alternative respectively using the distance minimization based ranking method (Asady and Zendehnam, 2007)

$$\tilde{A}_{ij}^{(ID)} = \left(a_{l_{ij}}^{(ID)}, a_{m_{ij}}^{(ID)}, a_{u_{ij}}^{(ID)}\right) = max\{rank\ value(\tilde{A}_{ij})\}, \forall i\ and\ if\ j \in C_B,$$

$$\tilde{A}_{ij}^{(ID)} = \left(a_{l_{ij}}^{(ID)}, a_{m_{ij}}^{(ID)}, a_{u_{ij}}^{(ID)}\right) = min\{rank\ value(\tilde{A}_{ij})\}, \forall i\ and\ if\ j \in C_C,$$

$$\tilde{A}_{ij}^{(AID)} = \left(a_{l_{ij}}^{(AID)}, a_{m_{ij}}^{(AID)}, a_{u_{ij}}^{(AID)}\right) = min\{rank\ value(\tilde{A}_{ij})\}, \forall i\ and\ if\ j \in C_B,$$

$$\tilde{A}_{ij}^{(ID)} = \left(a_{l_{ij}}^{(AID)}, a_{m_{ij}}^{(AID)}, a_{u_{ij}}^{(AID)}\right) = max\{rank\ value(\tilde{A}_{ij})\}, \forall i\ and\ if\ j \in C_C,$$

C_B represents beneficial criterion, and C_C represents cost criterion.

Step 5: Normalize the extended fuzzy initial decision matrix by employing (3.24) and (3.25).

$$\tilde{n}_{ij} = \left(n_{l_{ij}}, n_{m_{ij}}, n_{u_{ij}}\right) = \left(\frac{a_{l_{ij}}}{a_{u_{ij}}^{(ID)}}, \frac{a_{m_{ij}}}{a_{m_{ij}}^{(ID)}}, \frac{a_{u_{ij}}}{a_{l_{ij}}^{(ID)}}\right)\ if\ j \in C_B, \tag{3.24}$$

$$\tilde{n}_{ij} = \left(n_{l_{ij}}, n_{m_{ij}}, n_{u_{ij}}\right) = \left(\frac{a_{l_{ij}}^{(AID)}}{a_{u_{ij}}}, \frac{a_{m_{ij}}^{(AID)}}{a_{m_{ij}}}, \frac{a_{u_{ij}}^{(AID)}}{a_{l_{ij}}}\right)\ if\ j \in C_C, \tag{3.25}$$

where, $1 \le i \le n$, and $1 \le j \le m$.

Step 6: Compute the elements of the weighted normalized decision matrix $\left(\tilde{V} = \left(\tilde{v}_{ij}\right)_{m \times n}\right)$ (3.26). The fuzzy criteria weights obtained from (3.12) are utilized here.

$$\tilde{v}_{ij} = \widetilde{w}_j \otimes \tilde{n}_{ij} \tag{3.26}$$

Step 7: Calculate the sum of row elements of fuzzy weighted normalized decision matrix by using (3.27). Similarly, $\tilde{S}^{(ID)}$ and $\tilde{S}^{(AID)}$ are also calculated.

$$\tilde{S}_i = \sum_{j=1}^{n} \tilde{v}_{ij}\ \text{for all}\ 1 \le i \le m \tag{3.27}$$

Step 8: De-fuzzify[9] the elements $\tilde{S}_i$, $\tilde{S}^{(ID)}$, and $\tilde{S}^{(AID)}$ using (3.7). The de-fuzzified values of them are represented as S_i, $S^{(ID)}$, and $S^{(AID)}$, respectively. Then, compute the utility degree of the alternatives in relation to the ID and AID by (3.28) and (3.29):

[9] It is observed that when the alternatives are evaluated according to the TFN values given in Table *3.2*, and the steps presented in (Stanković et al., 2020), the upper bound of the TFNs tend to infinity.

$$UD_i^{(ID)} = \frac{S_i}{S^{(ID)}} \tag{3.28}$$

$$UD_i^{(AID)} = \frac{S_i}{S^{(AID)}} \tag{3.29}$$

Step 9: Compute the utility function values of the alternatives in relation to the ID $\left(UF_i^{(ID)}\right)$ and AID $\left(UF_i^{(AID)}\right)$ solutions by adopting (3.30) and (3.31).

$$UF_i^{(ID)} = \frac{UD_i^{(AID)}}{UD_i^{(ID)} + UD_i^{(AID)}} \tag{3.30}$$

$$UF_i^{(AID)} = \frac{UD_i^{(ID)}}{UD_i^{(ID)} + UD_i^{(AID)}} \tag{3.31}$$

Step 10: Determine the utility function values of the alternatives by employing (3.32).

$$UF_i = \frac{\left(UD_i^{(ID)} + UD_i^{(AID)}\right)}{1 + \left(\frac{UD_i^{(ID)}}{UD_i^{(AID)}}\right) + \left(\frac{UD_i^{(AID)}}{UD_i^{(ID)}}\right)} \tag{3.32}$$

Step 11: Rank the alternatives based on the final values of the utility function. It is preferred that the best alternative has the maximum utility and the worst alternative has the least utility.

3.5. Proposed Integrated Fuzzy MCDM Frameworks for Risk Prioritization

Based on the mathematical steps presented for fuzzy AHP, modified fuzzy MAIRCA, and modified fuzzy MARCOS in the earlier sections, here two integrated MCDM frameworks are developed, and are highlighted below.

3.5.1. Framework-I: Integrated MCDM Framework Using Fuzzy AHP-Modified Fuzzy MAIRCA

The integrated framework for the risk prioritization of failure modes by using fuzzy AHP and fuzzy MAIRCA methods is depicted in Figure 3.2. The procedural steps for calculating the weights of the risk factors, and the ranking of failure modes are already discussed *Section 3.2*, and *Section 3.3*, respectively.

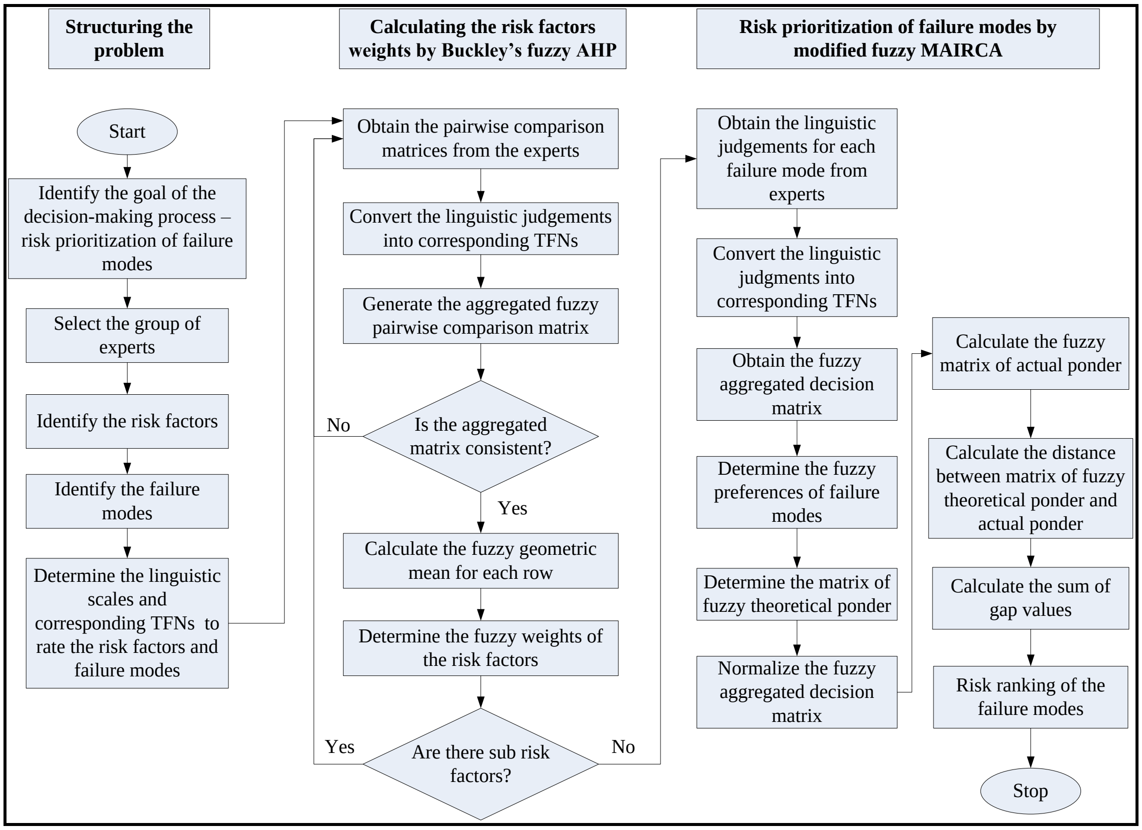


Figure 3.2 Proposed framework by integrating fuzzy AHP and modified fuzzy MAIRCA

3.5.2. Framework-II: Integrated MCDM Framework Using Fuzzy AHP-Modified Fuzzy MARCOS

Like the previous section, here also the weights of the risk factors are computed by the fuzzy AHP method (refer *Section 3.2*), and the failure modes are ranked by the modified fuzzy MARCOS method (refer *Section 3.4*). The framework, along with its steps are further portrayed in Figure 3.3.

3.6. FMEA Case Study of Automotive Industry

To examine the potential of the developed frameworks, as presented in *Section 3.6*, a popular FMEA case study of automotive industry is reconsidered here from the work of Kutlu and Ekmekçioğlu (Kutlu and Ekmekçioğlu, 2012). The potential failure modes were:

- non-conforming material (FM1),
- wrong die (FM2),
- wrong program (FM3),
- excessive cycle time (FM4),
- wrong process (FM5),
- damaged goods (FM6),
- wrong part (FM7), and
- incorrect forms (FM8).

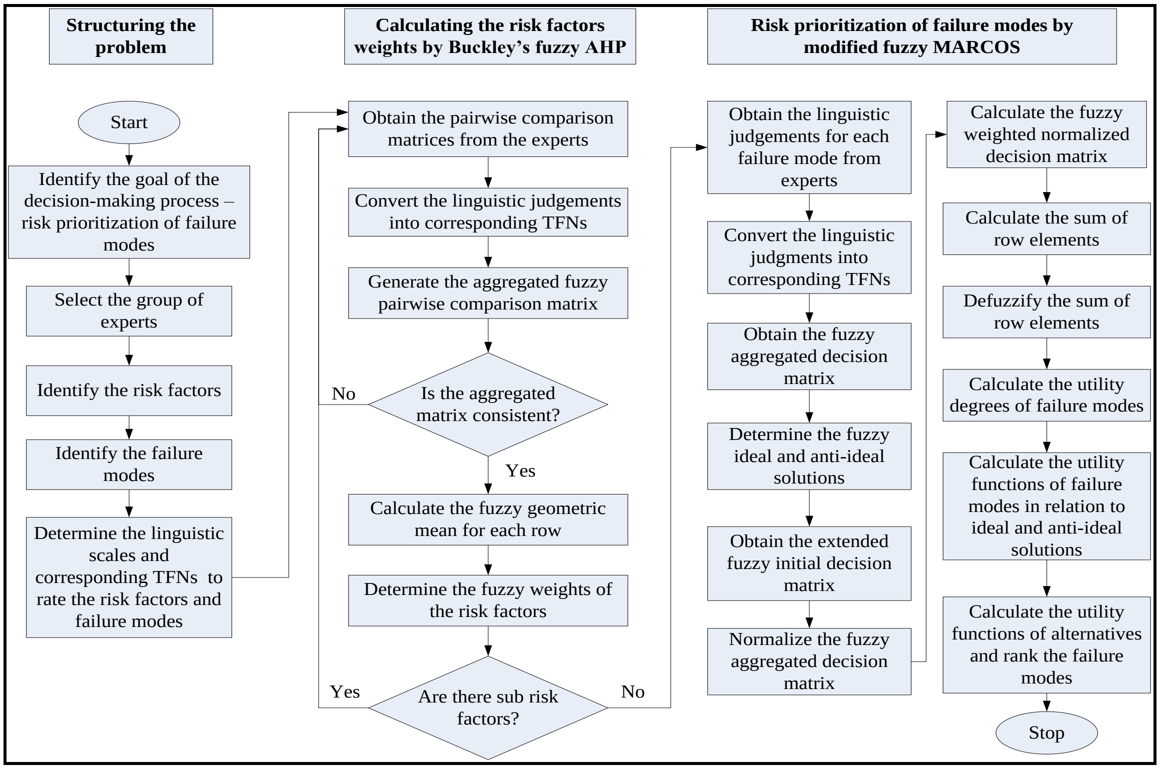


Figure 3.3 Proposed framework by integrating fuzzy AHP and modified fuzzy MARCOS

Three participating experts (DM1, DM2, and DM3) provided their linguistic judgements in the decision-making process. The pairwise comparisons data among the risk factors through the linguistic terms are presented in Table 3.3. Linguistic evaluations of the failure modes in relation to the risk factors are replicated in Table 3.4.

Table 3.3 Linguistic evaluations for obtaining criteria weights

Risk factors	Severity			Occurrence			Detection		
	DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3
Severity	E	E	E	FS	FS	VS	SS	SS	SS
Occurrence	-	-	-	E	E	E	SS	FW	E
Detection	-	-	-	-	-	-	E	E	E

Table 3.4. Linguistic evaluations of failure modes with respect to the risk factors

Failure Modes	Severity			Occurrence			Detection		
	DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3
FM1	F	F	MP	F	MG	MG	G	MG	G
FM2	P	MP	MP	VG	G	VG	MP	MP	P
FM3	MP	P	MP	VG	G	G	VP	MP	P
FM4	MP	F	MP	F	MG	MG	G	MG	G
FM5	F	F	MP	MG	MG	G	G	VG	G
FM6	MG	MG	F	MG	G	MG	MP	MP	F
FM7	P	MP	VP	VG	VG	VG	VP	MP	P

Failure Modes	Severity			Occurrence			Detection		
	DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3
FM8	VP	VP	P	VP	VP	VP	VP	VP	VP

3.6.1. Risk Ranking of Failure Modes by Framework - I

Employing the mathematical steps as presented in *Section 3.2,* & *Section 3.3*, the fuzzy weights of the risk factors are computed (refer Table 3.5) and the failure modes are ranked (refer Table 3.6).

Table 3.5 Fuzzy weights of the risk factors

Criteria	Fuzzy weights
Severity	(0.293,0.388,0.565)
Occurrence	(0.203,0.267,0.386)
Detection	(0.234,0.345,0.420)

The computed consistency ratio (CR) of the aggregated pairwise comparison matrix in fuzzy AHP method is 0.050 (which is <0.10). In Table 3.6 the computed ranking results are compared with the original work.

Table 3.6 Risk ranking results of failure modes by framework-I

Failure modes	Severity	Occurrence	Detection	Value of criteria functions	Ranking by framework-I	Ranking obtained by(Kutlu and Ekmekçioğlu, 2012)
FM1	0.040	0.030	0.029	0.0986	2	2
FM2	0.045	0.027	0.038	0.1103	5	5
FM3	0.045	0.028	0.040	0.1121	6	7
FM4	0.041	0.030	0.029	0.1004	4	4
FM5	0.040	0.029	0.028	0.0963	1	1
FM6	0.034	0.029	0.036	0.0986	3	3
FM7	0.048	0.027	0.040	0.1147	7	6
FM8	0.051	0.036	0.042	0.1291	8	8

It can be observed from Table 3.6 that the proposed *Framework-I* ranks the failure modes in the following order: $FM5 > FM1 > FM6 > FM4 > FM2 > FM3 > FM7 > FM8$.

3.6.2. Risk Ranking of Failure Modes by Framework-II

Here, the risk ranking of the failure modes for the considered case study are computed (refer Table 3.7) by employing the methods proposed in *Section 3.2*, *Section 3.4*.

Table 3.7 Risk ranking results of failure modes by framework-II

Failure modes	$\boldsymbol{S_i}$	$\boldsymbol{UD_i^{(ID)}}$	$\boldsymbol{UD_i^{(AID)}}$	$\boldsymbol{UF_i^{(ID)}}$	$\boldsymbol{UF_i^{(AID)}}$	$\boldsymbol{UF_i}$	Ranking by framework-II
FM1	0.685	16.069	0.758	0.0450	0.955	21.2028	2

Failure modes	S_i	$UD_i^{(ID)}$	$UD_i^{(AID)}$	$UF_i^{(ID)}$	$UF_i^{(AID)}$	UF_i	Ranking by framework-II
FM2	0.477	11.185	0.528	0.0450	0.955	21.2028	5
FM3	0.435	10.204	0.481	0.0450	0.955	21.2028	6
FM4	0.652	15.314	0.722	0.0450	0.955	21.2028	4
FM5	0.750	17.599	0.830	0.0450	0.955	21.2028	1
FM6	0.662	15.546	0.733	0.0450	0.955	21.2028	3
FM7	0.403	9.460	0.446	0.0450	0.955	21.2028	7
FM8	0.043	1.000	0.047	0.0450	0.955	21.2028	8
$S^{(ID)}$	0.043						
$S^{(AID)}$	0.903						

From Table 3.7, it can be observed that the proposed Framework-II generates the same results as in proposed Framework-I, and are quite similar to the original work Thus, it can be said that the developed methods produce credible ranking results.

3.7. Validation of the Ranking Results

This section presents twofold validations to examine the sensitivities of the suggested frameworks.

3.7.1. Validation 1: Comparisons with Other Fuzzy MCDM Methods

Here, initially the same case study is solved by other popular fuzzy MCDM methods: fuzzy VIKOR (Opricovic, 2011), fuzzy COPRAS (Zarbakhshnia *et al.*, 2018), fuzzy MOORA (Akkaya *et al.*, 2015), fuzzy MABAC (Bozanic *et al.*, 2018), fuzzy TOPSIS (Kutlu and Ekmekçioğlu, 2012), original fuzzy MAIRCA (Pamučar *et al.*, 2014), fuzzy MARCOS (Stanković *et al.*, 2020), and the outputs are compared with the ranking results obtained by the proposed Framework-I and Framework-II (refer Table 3.8) to examine that feasibility of the outcomes.

Table 3.8 Risk ranking of failure modes generated by other fuzzy MCDM methods

Failure modes	Fuzzy VIKOR	Fuzzy COPRAS	Fuzzy MOORA	Fuzzy MABAC	Fuzzy TOPSIS	Fuzzy MAIRCA	Fuzzy MARCOS	Framework-I	Framework-II
FM1	2	2	2	2	2	2	Do not able to rank the failure modes. Algorithm got stuck in Step 8 as pointed out in section 3.4	2	2
FM2	5	5	5	5	5	5		5	5
FM3	6	6	6	6	7	6		6	6
FM4	3	4	4	4	4	4		4	4
FM5	1	1	1	1	1	1		1	1
FM6	4	3	3	3	3	3		3	3
FM7	7	7	7	7	6	7		7	7
FM8	8	8	8	8	8	8		8	8

The following observations are made from the results presented in Table 3.8:

- The approach given in (Kutlu and Ekmekçioğlu, 2012) computes slightly different risk ranking results when compared with the proposed frameworks. This is probably due to adopting the unlike criteria weight calculation method. In the original work *Chang's extent analysis method* (Chang, 1996) was employed, but in the proposed frameworks Buckley's fuzzy AHP method is exploited (Buckley, 1985). The reasons for not using extent analysis method have already been discussed in *Chapter 2 / Section 2.5.1.*

- All fuzzy MCDM methods rank FM5 as the most critical failure mode, followed by FM1.

- Fuzzy VIKOR method ranks FM4 as the third critical failure mode, and FM6 as the fourth critical failure mode. This is probably due to the chosen value of $v = 0.5$, a weight for the strategy of maximum group utility. Otherwise, all other fuzzy MCDM methods rank FM6 as the third critical failure mode and FM4 as the fourth critical failure mode. Fuzzy COPRAS and Fuzzy MOORA produce the same risk ranking results when compared with the proposed approaches.

- Although the Fuzzy COPRAS method does not need a distinct transformation of the values of the beneficial/cost attribute in the normalized matrix and the total ranking index of each alternative is computed using proportional evaluation, the fuzzy COPRAS method has a more complicated procedure of combination of the values of the alternatives.

- In some circumstances, Fuzzy COPRAS method shows some degree of contradiction. For example, if the value of the dominant attribute for the non-beneficial criterion is the smallest and the highest weight of the criteria relates to that criterion, then the aggregation of weighted values is found in the denominator of the aggregated function. This can lead to erroneous decisions (Stević *et al.*, 2020).

- In the MOORA method, the ratio system and reference point approaches are combined to obtain the best alternative. Ratio system employs arithmetic weighted aggregation operator, and it is useful in applications where the attributes are independent to each other. However, it has the defect when the dependent attributes are considered for the decision-making process. The reference point approach uses the Min-Max metric which is useful for the cases where the optimal choice for decision-makers is the alternative that does not show bad performance on any of the attributes (Brauers and Zavadskas, 2006).

- For Fuzzy MOORA, it is assumed that the criteria are independent, which is not feasible in this considered FMEA example, and such assumption can lead to a wrong ranking result.

Although the fuzzy version of the original MAIRCA method and the proposed fuzzy MAIRCA method generate the same risk ranking results, the ensuing salient features highlight the superiority of the proposed Framework-I:

- It incorporates a normalization technique which helps in reducing the hard computation complexity and increases calculation accuracy. When the number of criteria increases, it becomes quite difficult for the decision maker to identify the benefit and cost criteria. However, in the adopted normalization technique, there is no such requirement of that identification.

- Secondly, the step of de-fuzzification after obtaining the total gap matrix is eliminated. To do that, the fuzzy Euclidian distance between elements of the matrix of theoretical and actual ponder are calculated, which is no doubt more realistic than the simple fuzzy subtraction operation.

When comparisons are made between the fuzzy MARCOS method presented in (Stanković *et al.*, 2020)*, and the proposed modified fuzzy MARCOS method, it is observed that the original work was not able to rank the failure modes, as it got stuck in step 8, as pointed out in Section 3.4.* While the proposed fuzzy MARCOS method properly ranks the failure modes, which also shows a good uniformity with the other fuzzy MCDM methods. Although, a drastic rank variation between the original work and the proposed approaches are not noticed, it can be asserted that the applications of these hybrid approaches are new in the FMEA domain, mathematically easier, easy to interpret, and require less computational steps.

3.7.2. Validation 2: Effects of Changing of Risk Factors Weights

In practical scenarios, it is often required to change the risk factors' weights according to the application need and the MCDM method should be robust enough to prevent the drastic rank reversals. Therefore, here the weights of the risk factors are interchanged to visualize their subsequent impacts on the risk ranking results (refer Table 3.9). Additionally, the Spearman's rank correlation coefficients of the generated ranking results after changing the criteria weights are computed and contrasted with the original work of Kutlu and Ekmekçioğlu (Kutlu and Ekmekçioğlu, 2012) and the fuzzy version of MAIRCA (Pamučar *et al.*, 2014).

Table 3.9 Set of criteria weights for sensitivity analysis

W_{SOD}	Weight set - 1	Severity	(0.293, 0.388, 0.565)	CR = 0.05<0.10
		Occurrence	(0.203, 0.267, 0.386)	
		Detection	(0.234, 0.345, 0.420)	
W_{SDO}	Weight set - 2	Severity	(0.293, 0.388, 0.565)	CR = 0.05<0.10
		Occurrence	(0.234, 0.345, 0.420)	
		Detection	(0.203, 0.267, 0.386)	
W_{ODS}	Weight set - 3	Severity	(0.203, 0.267, 0.386)	CR = 0.05<0.10
		Occurrence	(0.234, 0.345, 0.420)	
		Detection	(0.293, 0.388, 0.565)	
W_{OSD}	Weight set - 4	Severity	(0.203, 0.267, 0.386)	CR = 0.05<0.10
		Occurrence	(0.293, 0.388, 0.565)	
		Detection	(0.234, 0.345, 0.420)	
W_{DSO}	Weight set - 5	Severity	(0.234, 0.345, 0.420)	CR = 0.05<0.10
		Occurrence	(0.293, 0.388, 0.565)	
		Detection	(0.203, 0.267, 0.386)	
W_{DOS}	Weight set - 6	Severity	(0.234, 0.345, 0.420)	CR = 0.05<0.10
		Occurrence	(0.203, 0.267, 0.386)	
		Detection	(0.293, 0.388, 0.565)	

The computed ranking results of the failure modes by adopting the different criteria weights are presented in Table 3.10. It is noteworthy that the changes produce the same results for both proposed frameworks (*viz.*, Framework-I, and Framework-II). The average of Spearman's rank correlation coefficient for different ranking results generated by the proposed frameworks is 96.3%. whereas, the average spearman's rank correlation coefficient of the results presented in (Kutlu and Ekmekçioğlu, 2012) was 94.2%. *Thus, it can be said that the proposed approaches are superior to the integrated fuzzy AHP-fuzzy TOPSIS approach.*

Table 3.10 Variations in ranking results by changing the risk factors weights (both Framework-I and Framework-II)

Failure modes	W_{SOD}	W_{SDO}	W_{ODS}	W_{OSD}	W_{DSO}	W_{DOS}
FM1	2	3	2	2	3	2
FM2	5	5	5	5	5	5
FM3	6	6	6	6	6	6
FM4	4	4	3	3	4	3
FM5	1	1	1	1	1	1
FM6	3	2	4	4	2	4
FM7	7	7	7	7	7	7
FM8	8	8	8	8	8	8

Further, it can be observed from Table 3.10 that when the risk factors' weights are altered, there are little variations in preferences ranking. Hence, the proposed methods are sensitive to the changes, but not much. FM5 is consistently ranked as the most critical failure mode. There are changes in ranking positions of FM1, FM4 and FM6. However, any changes in ranking positions of FM2, FM3, FM7 and FM8 are not witnessed. In fact, stability of the ranking results are better than the method in (Kutlu and Ekmekçioğlu, 2012). *Hence, it can be said that the ranking result obtained by our proposed approach is confirmed, credible and robust.*

3.8. Chapter Summary

In this chapter, at first two fuzzy MCDM methods have been developed in the group decision making environment: one is modified fuzzy MAIRCA and another one is modified fuzzy MARCOS. Then to address the shortcomings of the traditional RPN-based FMEA approach, each of these two methods has been combined with Buckley's fuzzy AHP method to develop two integrated frameworks: fuzzy AHP- modified fuzzy MAIRCA, and fuzzy AHP-modified fuzzy MARCOS. To validate the potentiality of the proposed integrated frameworks a benchmark example of FMEA has been considered which was earlier solved by the fuzzy AHP-fuzzy TOPSIS method. In that approach, fuzzy AHP (based on *extent analysis method*) was considered to calculate the relative importance of the risk factors. However, the extent analysis based fuzzy AHP has received several criticisms as discussed in Chapter 2 */Section 2.5.1*. In this work, using the same pairwise comparison matrix and linguistic scale, Buckley's geometric mean based fuzzy AHP has been considered to calculate the weights of the risk factors. Next, to reduce the hard-computational complexity of the fuzzy extension of the traditional MAIRCA method, a modified fuzzy MAIRCA method has been proposed. Further, in the next work, after observing the inability of the fuzzy MARCOS method developed in (Stanković *et al.*, 2020) for risk ranking of failure modes in the considered FMEA problem, a modified fuzzy MARCOS method has been developed. When the obtained results have been compared with the fuzzy AHP-fuzzy TOPSIS approach, it has been observed that both developed integrated approaches produce more stable ranking results than the original one. Apart from that, when the number of risk factors and failure modes are increased, proposed approaches are capable of handling that situation and are still able to provide credible ranking results.

Chapter 4 System Description

4.1. Schemes for Selecting the System

As discussed in *0 / Section 1.1* that during the RCM implementation, the first task is to select the appropriate system. To do that, the next questions to confront are to choose *which system* and on *what basis*? One possible solution is to choose all the systems inside the plant, but this contradicts with the main philosophy of RCM (*viz.,* cost effectiveness). It is observed from the practical experience that many of the system rarely fail in their whole operating cycle, while others fail randomly, and some have predefined failure patterns, which can easily fit a statistical distribution. These random failures have serious consequences on the plant operation and incur a significant maintenance cost. Thus, the following schemes may be adopted in this thesis work while selecting the system:

- Due to the random failures, the system has undergone many CM tasks in recent times,
- The system with repeated PM tasks or associated costs in recent years,
- Combination of above two points,
- The system has high cost of CM tasks in recent years,
- Catastrophic failure of the system can lead to stoppage of the total production process and production flow,
- Catastrophic failure of the system can affect the safety of the operator,
- Hazardous wastes are generated after failure or during maintenance tasks of the system.

Apart from the above points, it is also mandatory to discuss with the experts and shop floor engineers regarding the feasibility of the study while implementing the RCM program. Next, the considered systems in this thesis work are illustrated.

4.2. The System for the Study – Gearboxes of a Steel Rolling Mill

After consulting with the experts and utilizing the previously mentioned schemes, the gearboxes installed in the light and medium merchant mill (LMMM) section of a leading steel processing plant are considered for further study in this thesis.

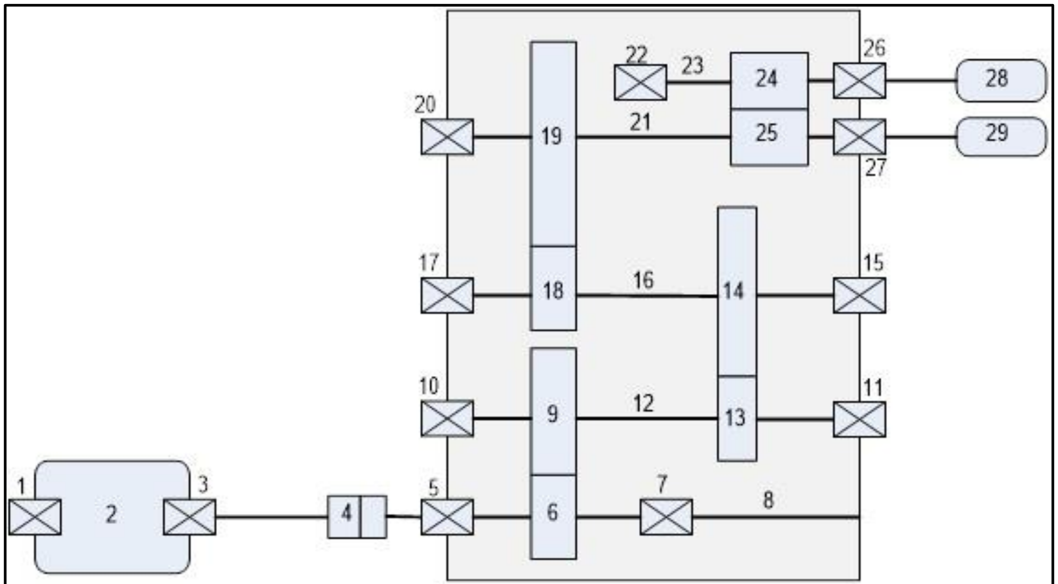
A typical integrated steel plant has several sections, such as coke-oven plant and coal chemical plant, sinter plant, blast furnace, steel melt shop, continuous casting department, *etc.* The cast blooms from the continuous casting department are heated and rolled in the two high speed and fully automated rolling mills namely LMMM and medium merchant and structural mill (MMSM). The billets produced in LMMM are further rolled in the bar mill or wire rod mill. The finished products include wire rods and long products like reinforcement bars, rounds, squares, flats, angles, channels, billets, etc.

In the LMMM section, a total of seven gearboxes are operating, each having a set of attached rollers for the compression of hot steel billets, coming from the continuous casting department. This section is very much critical in terms of providing final shape, size and surface quality to the hot steel billets. Each of the gearbox is driven by an DC induction motor. Generally, two types of gearboxes are installed there, one for vertical compression and another for horizontal compression of the hot steel billets. The block diagrams of the gearboxes are shown in Figure 4.1 and Figure 4.2.

From the figures, it is clearly observed that these gearboxes are quite large, and complex. The gearbox shown in Figure 4.1 comprised of a single gearbox, whereas, in Figure 4.2 it is highlighted that there are two gearboxes – bottom gearbox, and top gearbox. Additionally, all the schemes described in Section 4.1 has been observed to be relevant for these gearboxes. Thus, after consulting the experts and engineers, it was decided to implement the RCM for these gearboxes.

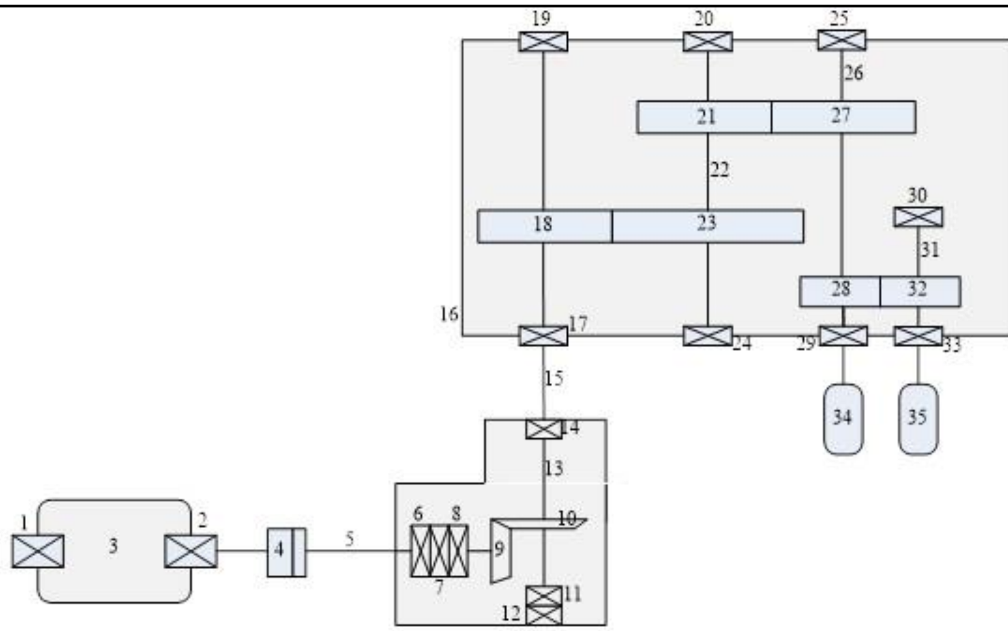
4.3. System Boundary and Surrounding Environment

During the visit to the LMMM section of the steel plant, it was observed that the gearboxes were covered by metal covers. So, in this study, all the components inside the gearbox cover will be focused to address the objectives. Apart from that, other noticeable things about the surrounding environment are as follows:



1) Bearing 1., 2) DC induction motor, 3) Bearing 2., 4) Flexible coupling, 5) Bearing 3., 6) Gear 1., 7) Bearing 4., 8) Input shaft, 9) Gear 2, 10) Bearing 5., 11) Bearing 6., 12) 1st intermediate shaft, 13) Gear 3, 14) Gear 4, 15) Bearing 8., 16) 2nd intermediate shaft, 17) Bearing 7., 18) Gear 5, 19) Gear 6, 20) Bearing 9., 21) 1st Output shaft, 22) Bearing 11., 23) 2nd output shaft, 24) Gear 7 (top), 25) Gear 7 (bottom), 26) Bearing 12., 27) Bearing 10., 28) Top roller, 29) Bottom roller.

Figure 4.1. Gearbox for horizontal compression of the steel billets



1) Bearing 1, 2) Bearing 2, 3) DC induction motor, 4) Flexible coupling, 5) Input shaft for bottom gearbox, 6) Bearing 3, 7) Bearing 4, 8) Bearing 5, 9) Gear 1, 10) Gear 2, 11) Bearing 7, 12) Bearing 6, 13) Output shaft for bottom gearbox, 14) Bearing 8, 15) Input shaft for top gearbox, 16) Top gearbox , 17) Bearing 9, 18) Gear 3, 19) Bearing 10, 20) Bearing 12, 21) Gear 5, 22) Top gearbox 1st intermediate shaft, 23) Gear 4, 24) Bearing 11, 25) Bearing 14, 26) Top gearbox 1st output shaft, 27) Gear 6, 28) Gear 7, 29) Bearing 13, 30) Bearing 16, 31) Top gearbox 2nd output shaft, 32) Gear 8, 33) Bearing 15, 34) Roller 1, 35) Roller 2.

Figure 4.2. Gearbox for vertical compression of the steel billets

- These gearboxes are being operated in a hostile operating condition (refer Table 4.1), where the surrounding dust level, humidity and temperature are high. The dust and humidity once entrapped in the gear oil can initiate the faults which can further lead to failures.

Table 4.1 Specifications of environmental conditions

Environmental Parameter	Normal	Marginal	Hostile
Relative humidity (%)	<80.0	80.0-90.0	>90.0
Ambient temperature (°C)	<35.0	35.0-45.0	>45.0
Dust level (R-scale)	Very very low (1) – Low (3)	Moderate (4)-High (5)	Very high(6)- Very very high(7)
Duty cycle	Constant	Varying	Shock
Surrounding vibration	Low	Moderate	Severe
Ease of maintenance	Good	Restricted	Very complicated

- The surrounding vibration level is very high. This is because several gearboxes are being operated in the surrounding area, and each of them has its own vibration level.

- As the hot cast blooms are being passed through the rollers attached to the gearboxes, it is almost impossible for the operators to stand by the side of the gearboxes for a long time.

4.4. Different Failure Modes, Causes and Effects of the Components of the Gearboxes

To identify the most pertinent failure modes of the considered gearboxes, a team of cross-functional experts are formed consisting of a deputy manager (DE1), an assistant manager another (DE2), and an operator (DE3). They have different level of expertise based on their level of knowledge and the tenure of service. To carry out the FMEA, the failure modes are initially identified, along with their causes, and effects from the *TBL* of sustainability (*viz.*, economic, social, and environmental effects) (refer Table 4.2).

Table 4.2 Different failure modes of the components of the gearbox, their causes and effects

Components	Notations	Failure Modes	Failure causes	Failure effects	
Gears	FM1	Wear of Teeth	- Due to excessive load on tooth profile. This load is larger than the endurance limit of the material, - improper mounting of gears, - poor lubricating condition, - improper heat treatment of gear material,	Economical	- Delay in timely delivery of the final product, - Production of out-of-design final product, - For excessive wear, other costs are incurred, like procurement cost, ordering cost, lost production, etc.
				Social	- Excessive wear leads to increased noise and vibration, increase of smear by lubricant, which can harm the operators physically,

Components	Notations	Failure Modes	Failure causes	Failure effects	
			-impurities in the lubricating oil.		- Lost production time is compensated with excess labour hours to meet the output target, - Worker's mind-set is changed due to repetitive failure and interruption.
				Environmental	- Produce harmful and toxic gases due to burning of lubricants, - Proper disposal of burnt lubricating oil, waste material and their recycling are a major problem.
	FM2	Broken Teeth	- Unexpected heavy load on gears. - fatigue breakage from cyclic loading, - excessive wear of teeth and thinning of teeth, etc.	Economical	Excessive lead time, out-of-design final product preparation, and other procurement and installation related costs.
				Social	- Can obstruct the smooth movement of red-hot cast bloom and subsequently those blooms can fall out of the pathway, which will harm the operator, - excess working hours for the operators, and change of their mind-set, etc.
				Environmental	- Same as FM1.
	FM3	Pitting of Gear	- Improper consideration of hardness, texture and load. Actual born load generally exceeds the endurance limit, - excessive hardening or crispiness of gear surface, - use of improper lubricating oil, etc.	Economical	-Same as FM1 and FM2
				Social	-Same as FM1 and FM2
				Environmental	-Same as FM1 and FM2
	FM4	Axial Shift of Gear	- Improper mounting due to lack of knowledge of the operator, - sudden excessive load, -improper design of teeth profile, etc.	Economical	-It can incur huge economic losses, in terms of lost production, higher lead time, damage of final product, etc.
				Social	- Can harm the operator physically and fatal accident may occur.
				Environmental	- Same as FM1, - at the initial stage, more energies are required to move the other meshing gears, etc.
	FM5	Scoring of gears	- poor quality of lubricating oil, with improper viscosity, - poor matching of material, -improper cooling of lubricating oils, larger loads, etc.	Economical	- Same as FM1
				Social	- Scoring of gear can lead to other types of failures, like wear, breakage of teeth or axial shifts and can cause similar types of damages as mentioned above.

<table>
<tr><th>Components</th><th>Notations</th><th>Failure Modes</th><th>Failure causes</th><th colspan="2">Failure effects</th></tr>
<tr><td></td><td></td><td></td><td></td><td>Environmental</td><td>- Same as FM1 and FM4.</td></tr>
<tr><td rowspan="12">Bearings</td><td rowspan="3">FM6</td><td rowspan="3">Brinelling</td><td rowspan="3">- shock or excessive loads due to improper mounting,
- excessive static or impact load during operation,
- improper installation and handling, etc.</td><td>Economical</td><td>- Can damage other components of the bearing which will lead to total replacement,
- excessive vibration can damage the final dimension of the cast blooms, etc.</td></tr>
<tr><td>Social</td><td>- At the later stage, due to excessive vibration, can lead to fatal accident,
- replacement of bearing will cause increased lead-time, affects the worker's mind-set to a great extent, etc.</td></tr>
<tr><td>Environmental</td><td>- Can damage the lubrication oil, which in turns damage the other parts, and can produce some toxic gases at the burnt condition,
- draw excessive energy for operation, etc.</td></tr>
<tr><td rowspan="3">FM7</td><td rowspan="3">Cage defect</td><td rowspan="3">- Excessive vibration caused due to damage of other components,
- contamination and insufficiency of lubricating oil,
- fluctuation in the rotating speed due to shocks coming from cast blooms,
- improper alignment of balls, etc.</td><td>Economical</td><td>- Same as FM6</td></tr>
<tr><td>Social</td><td>- Same as FM6</td></tr>
<tr><td>Environmental</td><td>- Same as FM6</td></tr>
<tr><td rowspan="3">FM8</td><td rowspan="3">Crack on raceways</td><td rowspan="3">- Excessive interference,
- excessive load and shock load,
- flaking progression,
- generation of heat due to creep,
- poor taper angle of tapered shaft, etc.</td><td>Economical</td><td>- Same as FM6</td></tr>
<tr><td>Social</td><td>- Same as FM6</td></tr>
<tr><td>Environmental</td><td>- Same as FM6</td></tr>
<tr><td rowspan="3">FM9</td><td rowspan="3">Crack of rollers</td><td rowspan="3">- Almost similar causes like FM8</td><td>Economical</td><td>- Same as FM6</td></tr>
<tr><td>Social</td><td>- Same as FM6</td></tr>
<tr><td>Environmental</td><td>- Same as FM6</td></tr>
<tr><td>Shafts</td><td>FM10</td><td>Bent shaft</td><td>- Mostly due to improper installation and setup activities,
- heavy shock loads during operation,</td><td>Economical</td><td>- Can lead to replacement of the shaft, which will take severe delay in production, huge monetary losses to the organization,
- affect the final dimension of the output product, etc.</td></tr>
</table>

Components	Notations	Failure Modes	Failure causes	Failure effects	
			- thermal expansion or contraction caused due to other reasons, etc.	Social	- can lead to fatal accidents of the operators, - affects the working mentality of the operator, - overtime work to meet the output target, etc.
				Environmental	- Generates a lot of waste material which in turn creates a proper disposal problem, - creates excessive heat and burns the lubricating oil. Burnt lubricating oil generates toxic gases, etc.
	FM11	Crack of shaft	- metallurgical abnormalities, - cyclic fatigue, - excessive torque, - increased stress due to misalignment, etc.	Economical	- Same as FM10
				Social	- Same as FM10
				Environmental	- Same as FM10
	FM12	Fracture of shaft	- Heavy loads, - cyclical stress, - poor design, etc.	Economical	- Same as FM10
				Social	- Same as FM10
				Environmental	- Same as FM10

4.5. Major Faults, Symptoms, Health Indicators, and Measuring Instruments

Some frequently occurring faults, and their symptoms are presented in Table 4.3. In Table 4.4 the useful HIs to identify those faults and the measuring instruments are presented. All this information is collected either from the experts, or from the previous data stored in the central database of the organization. Based on this information, the remaining part of the work is developed.

Table 4.3 Major faults along with their notable symptoms

Type of faults		Symptoms
Component	Mode	
Gear	Teeth wear (abrasive)	Abnormal sound and vibration or both.
	Breakage of teeth	Abnormal vibration, abnormal sound, or both, with rise in temperature of bearing housing.
	Pitting	Abnormal vibration or sound.
	Improper meshing	Abnormal sound and vibration with rise in temperature of bearing housing.
	Axial shift of gear	Abnormal sound, abnormal vibration, or combination of them.
Bearing	Fatigue failure (flaking, pitting and surface erosion)	Abnormal sound, vibration, or both.
	Improper mounting and installation	Abnormal vibration, leading to abnormal sound and temperature rise of bearing housing.
	Lubrication failure	Abnormal temperature rise at first, then abnormal vibration and sound.
	Cage defect	Abnormal vibration, sound and temperature or combination of them.

	Race defect (both inner and outer)	Abnormal vibration level with abnormal temperature of housing.
Shaft	Bent shaft	Abnormal sound, vibration and temperature or combination of them.
	Mild and severe crack	Abnormal vibration and sound, later rise in temperature of bearing housing.
	Fretting corrosion	Abnormal vibration or sound.

Table 4.4 HIs, their relevance with different types of faults and respective measuring instruments

Considered health indicators	Relevance with different types of faults	Measuring instrument
RMS value of vibration in vertical direction taken from bearing housing (mm/sec)	High value of this indicator generally represent shock originating due to improper meshing of gear, tooth breakage, different types of bearing defects and looseness of components.	Tri-axial accelerometer.
RMS value of vibration in horizontal direction taken from bearing housing (mm/sec)	High value of this indicator represents unbalance cause due to bent shaft, improper fitting of gears or bearings etc.	
RMS value of vibration in axial direction taken from bearing housing (mm/sec)	High value of this indicator represents misalignment of several components, like gears, bearings etc.	
RMS value of foundation vibration (mm/sec)	This indicator is greatly affected by looseness of several parts of the gearbox. However, vibration at foundation do originate from all moving components in the gearbox.	Uniaxial Accelerometer.
Oil flow rate (litre/min)	Significant reduction of this indicator from its threshold range implies improper meshing of gears, severe bearing defects, or broken teeth etc. Sometimes, faults may present in the lubricating system.	Flow meter.
Oil temperature (˚C)	Major increase of this indicator implies improper bearing function, pitting and breakage of gear teeth etc.	Manually.
Temperature of bearing housing (˚C)	For such gearboxes, operating in a hostile environment, if an operator touches the casing nearer to bearing housing and feels uncomfortable, then it is assumed that it is in unhealthy state. Increased value of this indicator generally indicates slipping of races of bearings, improper lubrication etc.	Infrared thermometer / Laser gun.

4.6. Chapter Summary

In this chapter, the systems considered to address the major decision-making problems as discussed in *Chapter 1* are described. According to RCM philosophy, at first the block diagrams of the considered gearboxes have been presented. Thereafter, a detailed description about the system boundary and surrounding environment are presented. Next, major failure modes, their causes, and effects have been highlighted from the TBL of sustainability. After that, frequently occurring faults, their symptoms, measuring instruments are listed out. These data are further utilized in the next chapters for analyses purposes.

Chapter 5 Integrated MCDM Frameworks for Risk Ranking of Failure Modes of a Gearbox using Linguistic Data

5.1. Introduction

The drawbacks of the traditional RPN-based approach have also been elicited in *Chapter 1 / Section 1.1.1*. There, it has been discussed that to implement the sustainability-based manufacturing practices in the industry, its associated processes should also be made sustainable. For the past two decades or so, manufacturing industries have been heavily pushed to consider sustainability aspects as a major point of concern to save the planet. The Governments have started formulating and enforcing the statutory regulations (*e.g.*, *Clean Air Act* (1970) (Rogers, 1970), *Resource Conservation and Recovery Act* (1976) (Andersen, 1978), and *Toxic Substance Control Act* (1976) (McRae *et al.*, 1978)). These acts are mainly aimed at reducing the environmental impacts of hazardous waste produced by systems/machinery during their operational, maintenance phases or after their failure. Apart from these regulations, there are several other standards like ISO 45001 (related to health and safety), ISO 37001 (anti-bribery management systems), ISO 14064 (greenhouse gases), and TS 14067 (carbon footprint of products) that have also broadened the concept of sustainable development to include economic, social and environmental aspects (Silvestre and Ţîrcă, 2019).

During the RCM study of the process plant gearboxes, it was observed that in its maintenance phase & operating phases, and/or in case of catastrophic failures, different types of hazardous waste (*e.g.*, burnt oil, grease, hazardous gases) are produced, which have significant impacts on the working surroundings. Besides, when the gearboxes are operated in degraded conditions, excess energy is consumed, and sometimes toxic substances are also produced, thereby, adversely affecting the environment. Additionally, the degraded state causes a significant economic and ecological loss by producing poor quality products and a substantial amount of scrap. Socially, this state also intensifies the chances of occurrence of fatal accidents affecting the workers' moral, besides causing delays and frustration in completing other assigned and related tasks. Detailed descriptions of the different potential failure modes of the gearbox, their cause(s), and effect(s) from TBL of sustainability have been described in *Chapter 4 / Table 4.2*.

System FMEA being an integrated part of design/manufacturing process should be considered from a holistic and sustainability point of view. Therefore, initially, it is essential to decouple the severities of failure modes from the *Triple Bottom Line (TBL)* of sustainability. By considering the severities with respect to TBL of sustainability, many risk factors/sub risk factors are emanated and required to be considered simultaneously to accurately estimate the associated risks of failure modes. However, for most of these risk factors, exact numerical data are not obtainable within the organization, and thus they are linguistically evaluated by the experts. Besides, computation of weights of risk factors needs the elimination of subjective uncertainties and/or manage them in a much abstract way, such that they have minimal impacts on the outcome (*viz.*, final ranking results). Additionally, when the involved criteria in a decision-making process are increased in number, the following problems are further developed:

- Organizations often wish to categorize the risk factors into cause and effect groups.

- If the previously described fuzzy AHP based weight calculation method is adopted (refer *Chapter 3 / Section 3.2*), then a lot of pairwise comparisons are required to be performed. For example, if n represents the number of involved criteria in a decision-making process, then in AHP total $\frac{n(n+1)}{2}$ pairwise comparisons are required to be made, which become an arduous task to the decision experts.

Based on the above discussions, this chapter makes the following contributions[10]:

a) Identifying the pertinent risk factors of the failure modes from the *TBL* of sustainability by considering a case study of process plant gearbox (refer *Chapter 4*).

b) Applying IT2F-DEMATEL method to assess the causal dependencies among the risk factors, by considering the subjective assessments of the experts. This method is also utilized to compute the weights of the identified risk factors.

c) Employing the proposed modified fuzzy MAIRCA (refer *Section 3.3*) and modified fuzzy MARCOS (refer *Section 3.4*) methods, to obtain the risk ranking of failure modes.

d) Performing the detailed comparative analyses between the modified fuzzy MAIRCA and the modified fuzzy MARCOS methods in terms of their ranking stability and robustness.

[10] The publication from this work can be found in the following paper:

a) Boral, S., Howard, I., Chaturvedi, S.K., McKee, K., Naikan, V.N.A., 2020. A novel hybrid multi-criteria group decision making approach for failure mode and effect analysis: An essential requirement for sustainable manufacturing. *Sustainable Production and Consumption* 21, 14-32.
b) Boral. S., Chaturvedi, S.K., Liu. Y., Howard, I. (2021). Integrated fuzzy MCDM frameworks in risk prioritization of failure modes. *In Advances in Performability Engineering*. (Communicated).

5.2. Interval Type-2 Fuzzy Sets: Definitions and Arithmetic Operations

5.2.1. Type-2 Fuzzy Set

Definition 1 (Baykasoğlu and Gölcük, 2017)*:* A Type-2 Fuzzy Set (T2FS) $\tilde{\tilde{A}}$ in the universe of discourse X is represented by ((5.1).

$$\tilde{\tilde{A}} = \left\{((x,u), \mu_{\tilde{\tilde{A}}}(x,u) | \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{\tilde{A}}}(x,u) \leq 1 \right\} \tag{5.1}$$

where, $\mu_{\tilde{\tilde{A}}}$ is a type-2 membership function, and J_x denotes an interval in$[0,1]$.

A type-2 fuzzy set $\tilde{\tilde{A}}$ can also be represented as in ((5.2).

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u)/(x,u) \tag{5.2}$$

where $J_x \subseteq [0,1]$ and $\iint$ denotes union over all admissible $x \; and \; u$.

Definition 2 (Baykasoğlu and Gölcük, 2017)*:* Let $\tilde{\tilde{A}}$ be a T2FS in the universe of discourse X, represented by the type-2 membership function $\mu_{\tilde{\tilde{A}}}$. If all $\mu_{\tilde{\tilde{A}}}(x,u) = 1$, then $\tilde{\tilde{A}}$ is called an IT2FS. An IT2FS $\tilde{\tilde{A}}$ can be interpreted as a special case of a type-2 fuzzy set, represented as follows:

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1/(x,u) \tag{5.3}$$

where $J_x \subseteq [0,1]$. In other words, if all the secondary grades are equal to 1, then T2FS is known as IT2FS.

Definition 3 (Baykasoğlu and Gölcük, 2017)*:* The upper and lower MF of an IT2FS are T1FSs. Figure 5.1 graphically represents a trapezoidal IT2FS $\tilde{\tilde{A}}$, and its mathematical representation is given by (5.4). In other words, when the upper MF and lower MF of an IT2FS are of linear type, it is considered as the trapezoidal IT2FS.

$$\tilde{\tilde{A}} = (\tilde{A}_i^U, \tilde{A}_i^L)$$
$$= \left((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U;\ H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U)), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L;\ H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L)\ \right) \tag{5.4}$$ [11]

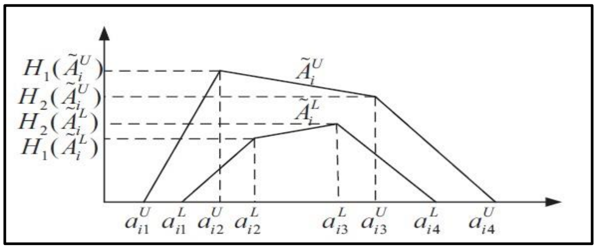


Figure 5.1 A trapezoidal interval type-2 fuzzy set

5.2.2. Basic Arithmetic Operations

Let there be two TrIT2FSs, *i.e.*,

$\tilde{\tilde{A}}_1 = \left((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U;\ H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L;\ H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)) \right)$, and

$\tilde{\tilde{A}}_2 = \left((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U;\ H_1(\tilde{A}_2^U), H_2(\tilde{A}_2^U)), (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L;\ H_1(\tilde{A}_2^L), H_2(\tilde{A}_2^L)) \right)$.

Addition (Baykasoğlu and Gölcük, 2017)**:**

$$\begin{aligned}\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 = ((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U \\ + a_{24}^U; \min\left(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)\right), \min(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U))), (a_{11}^L \\ + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L \\ + a_{24}^L;\ \min\left(H_1(\tilde{A}_1^L); H_1(\tilde{A}_2^L)\right), \min(H_2(\tilde{A}_1^L); H_2(\tilde{A}_2^L))))\end{aligned} \tag{5.5}$$

Subtraction (Baykasoğlu and Gölcük, 2017)**:**

$$\begin{aligned}\tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 = ((a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U \\ - a_{21}^U; \min\left(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)\right), \min(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U))), (a_{11}^L \\ - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L \\ - a_{21}^L;\ \min\left(H_1(\tilde{A}_1^L); H_1(\tilde{A}_2^L)\right), \min(H_2(\tilde{A}_1^L); H_2(\tilde{A}_2^L))))\end{aligned} \tag{5.6}$$

[11] $\tilde{A}_i^U$ and $\tilde{A}_i^L$ are type-1 fuzzy sets, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U$, $a_{i1}^L, a_{i2}^U, a_{i3}^U$ and a_{i4}^L are the reference points of IT2FS $\tilde{\tilde{A}}_i$. $H_j(\tilde{A}_i^U)$ denotes the membership value of the element $a_{j(j+1)}^U$ in the upper trapezoidal membership function $(\tilde{A}_i^U)$, where $1 \le j \le 2$. $H_j(\tilde{A}_i^L)$ denotes the membership value of the element $a_{j(j+1)}^L$ in the lower trapezoidal membership function $(\tilde{A}_i^L)$, where $1 \le j \le 2$. $H_1(\tilde{A}_i^U) \in [0,1], H_2(\tilde{A}_i^U) \in [0,1], H_1(\tilde{A}_i^L) \in [0,1], H_2(\tilde{A}_i^L) \in [0,1]$ and $1 \le i \le n$.

Multiplication (Baykasoğlu and Gölcük, 2017)**:**

$$\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 \cong ((a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U; \min\left(H_1(\tilde{A}_1^U); H_1(\tilde{A}_2^U)\right), \min(H_2(\tilde{A}_1^U); H_2(\tilde{A}_2^U))), (a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L; \min\left(H_1(\tilde{A}_1^L); H_1(\tilde{A}_2^L)\right), \min(H_2(\tilde{A}_1^L); H_2(\tilde{A}_2^L)))) \tag{5.7}$$

Scaling (Baykasoğlu and Gölcük, 2017)**:** Multiplication and division of a TrIT2FN with a crisp value k can be calculated as:

$$\tilde{\tilde{A}}_1 \times k = \left(\left(a_{11}^U \times k, a_{12}^U \times k, a_{13}^U \times k, a_{14}^U \times k; H_1(\tilde{A}_1^U), H_2(\tilde{A}_1^U)\right), \left(a_{11}^L \times k, a_{12}^L \times k, a_{13}^L \times k, a_{14}^L \times k; H_1(\tilde{A}_1^L), H_2(\tilde{A}_1^L)\right)\right) \tag{5.8}$$

$$\frac{\tilde{\tilde{A}}_1}{k} = \left(\left(a_{11}^U \times \frac{1}{k}, a_{12}^U \times \frac{1}{k}, a_{13}^U \times \frac{1}{k}, a_{14}^U \times \frac{1}{k}; H_1(\tilde{A}_1^U)\right), H_2(\tilde{A}_1^U)\right), \left(a_{11}^L \times \frac{1}{k}, a_{12}^L \times \frac{1}{k}, a_{13}^L \times \frac{1}{k}, a_{14}^L \times \frac{1}{k}; H_1(\tilde{A}_1^L)\right), H_2(\tilde{A}_1^L)\right) \tag{5.9}$$

Expected value of an IT2FN (Baykasoğlu and Gölcük, 2017)**:**

$$E\left(\tilde{\tilde{A}}\right) = \frac{1}{2}\left(\frac{1}{4}\sum_{i=1}^{4}(a_i^L + a_i^U)\right) \times \frac{1}{4}\left(\sum_{i=1}^{2}(H_i(A^L) + H_i(A^U))\right) \tag{5.10}$$

where, $\tilde{\tilde{A}} = \left(\left(a_1^U, a_2^U, a_3^U, a_4^U; H_1(\tilde{A}^U), H_2(\tilde{A}^U)\right), \left(a_1^L, a_2^L, a_3^L, a_4^L; H_1(\tilde{A}^L), H_2(\tilde{A}^L)\right)\right)$.

5.3. Interval Type-2 Fuzzy DEMATEL

The IT2F-DEMATEL comprises of the ensuing steps (Baykasoğlu and Gölcük, 2017):

Step 1: Let k number of cross-functional experts provides their subjective judgement to fill-out the influence matrices, which are further converted to respective IT2FNs by using any scale. Table 5.1 provides a typical translation from linguistic to TrIT2FN. The generated IT2FNs-based influence matrices are denoted as $\tilde{\tilde{Y}}^{(1)}, \tilde{\tilde{Y}}^{(2)}, \tilde{\tilde{Y}}^{(3)}, \dots, \tilde{\tilde{Y}}^{(k)}$.

Table 5.1. Linguistic judgements and their corresponding IT2FNs to rate the risk factors

Linguistic variable	Corresponding TrIT2FNs
Very-very low (VVL)	((0,0.1,0.1,0.2;1,1), (0.05,0.1,0.1,0.15;0.9,0.9))
Very low (VL)	((0.1,0.2,0.2,0.35;1,1), (0.15,0.2,0.2,0.3;0.9,0.9))
Low (L)	((0.2,0.35,0.35,0.5;1,1), (0.25,0.35,0.35,0.45;0.9,0.9))

Linguistic variable	Corresponding TrIT2FNs
Medium (M)	((0.35,0.5,0.5,0.65;1,1), (0.4,0.5,0.5,0.6;0.9,0.9))
High (H)	((0.5,0.65,0.65,0.8;1,1), (0.55,0.65,0.65,0.75;0.9,0.9))
Very high (VH)	((0.65,0.8,0.8,0.9;1,1), (0.7,0.8,0.8,0.85;0.9,0.9))
Very-very high (VVH)	((0.8,0.9,0.9,1;1,1), (0.85,0.9,0.9,0.95;0.9,0.9))

Step 2: Compute the average IT2F influence matrix by (5.11).

$$\tilde{\tilde{Y}} = \frac{\tilde{\tilde{Y}}^{(1)} \oplus \tilde{\tilde{Y}}^{(2)} \oplus \tilde{\tilde{Y}}^{(3)} \oplus \ldots \oplus \tilde{\tilde{Y}}^{(k)}}{k} \tag{5.11}$$

where, $\tilde{\tilde{Y}}$ is known as initial direct relation matrix and represented by (5.12),

$$\tilde{\tilde{Y}} = \begin{bmatrix} 0 & \tilde{\tilde{y}}_{12} & \cdots & \tilde{\tilde{y}}_{1m} \\ \tilde{\tilde{y}}_{21} & & \ddots & \vdots \\ \vdots & & & \\ \tilde{\tilde{y}}_{m1} & \tilde{\tilde{y}}_{m2} & \cdots & 0 \end{bmatrix} \tag{5.12}$$

where, $\tilde{\tilde{y}}_{ij} = \left(\left(a_{ij}, b_{ij}, c_{ij}, d_{ij}; H_1\left(\tilde{y}_{ij}^U\right), H_2\left(\tilde{y}_{ij}^U\right)\right), \left(e_{ij}, f_{ij}, g_{ij}, h_{ij}; H_1\left(\tilde{y}_{ij}^L\right), H_2\left(\tilde{y}_{ij}^L\right)\right)\right)$.

Step 3: Calculate the normalized direct relation matrix by reorganizing the IT2F-initial direct relation matrix as shown in (5.13)[12]. Thus, a total of eight $m \times m$ matrices are constructed.

$$Y_{a'} = \begin{bmatrix} 0 & a'_{12} & \cdots & a'_{1m} \\ a'_{21} & & \ddots & a'_{2m} \\ \vdots & & & \vdots \\ a'_{m1} & a'_{m2} & \cdots & 0 \end{bmatrix}, Y_{b'} = \begin{bmatrix} 0 & b'_{12} & \cdots & b'_{1m} \\ b'_{21} & & \ddots & b'_{2m} \\ \vdots & & & \vdots \\ b'_{m1} & b'_{m2} & \cdots & 0 \end{bmatrix}, \ldots,$$
$$Y_{h'} = \begin{bmatrix} 0 & h'_{12} & \cdots & h'_{1m} \\ h'_{21} & & \ddots & h'_{2m} \\ \vdots & & & \vdots \\ h'_{m1} & h'_{m2} & \cdots & 0 \end{bmatrix} \tag{5.13}$$

As $Y_{d'}$ contains the greatest element, it is further utilized to calculate the normalization coefficient. The normalized direct relation matrix is represented by (5.14):

$$\tilde{\tilde{N}} = \begin{bmatrix} \tilde{\tilde{n}}_{11} & \tilde{\tilde{n}}_{12} & \cdots & \tilde{\tilde{n}}_{1m} \\ \tilde{\tilde{n}}_{21} & & \ddots & \tilde{\tilde{n}}_{2m} \\ \vdots & & & \vdots \\ \tilde{\tilde{n}}_{m1} & \tilde{\tilde{n}}_{m2} & \cdots & \tilde{\tilde{n}}_{mm} \end{bmatrix} \tag{5.14}$$

[12] It is observed that heights of IT2FNs do not affect the results and hence they are omitted from the subsequent calculations.

Elements of the normalized direct-relation matrix are calculated as follows:

$$\tilde{\tilde{n}}_{ij} = \frac{\tilde{\tilde{y}}_{ij}}{v} = \left(\left(\frac{Y_{a'_{ij}}}{v}, \frac{Y_{b'_{ij}}}{v}, \frac{Y_{c'_{ij}}}{v}, \frac{Y_{d'_{ij}}}{v};\ H_1(\tilde{y}_{ij}^U), H_2(\tilde{y}_{ij}^U)\right) \times \left(\frac{Y_{e'_{ij}}}{v}, \frac{Y_{f'_{ij}}}{v}, \frac{Y_{g'_{ij}}}{v}, \frac{Y_{h'_{ij}}}{v};\ H_1(\tilde{y}_{ij}^L), H_2(\tilde{y}_{ij}^L)\right)\right) \tag{5.15}$$

The normalization coefficient v is calculated as follows:

$$v = max\left(\underset{1 \le i \le m}{max} \sum_{j=1}^{m} Y_{d'_{ij}}, \underset{1 \le j \le m}{max} \sum_{i=1}^{m} Y_{d'_{ij}}\right) \tag{5.16}$$

Step 4: Compute the total relation matrix like in *Step 3*. The IT2F-normalized direct-relation matrix can be represented by eight crisp matrices as below:

$$N_{a''} = \begin{bmatrix} 0 & a''_{12} & \cdots & a''_{1m} \\ a''_{21} & & \ddots & a''_{2m} \\ \vdots & & & \vdots \\ a''_{m1} & a''_{m2} & \cdots & 0 \end{bmatrix}, N_{b''} = \begin{bmatrix} 0 & b''_{12} & \cdots & b''_{1m} \\ b''_{21} & & \ddots & b''_{2m} \\ \vdots & & & \vdots \\ b''_{m1} & b''_{m2} & \cdots & 0 \end{bmatrix}, \ldots, \quad N_{h''} = \begin{bmatrix} 0 & h''_{12} & \cdots & h''_{1m} \\ h''_{21} & & \ddots & h''_{2m} \\ \vdots & & & \vdots \\ h''_{m1} & h''_{m2} & \cdots & 0 \end{bmatrix} \tag{5.17}$$

The total relation matrix $\tilde{\tilde{T}}$ is denoted as follows:

$$\tilde{\tilde{T}} = \begin{bmatrix} \tilde{\tilde{t}}_{11} & \tilde{\tilde{t}}_{12} & \cdots & \tilde{\tilde{t}}_{1m} \\ \tilde{\tilde{t}}_{21} & & \ddots & \tilde{\tilde{t}}_{2m} \\ \vdots & & & \vdots \\ \tilde{\tilde{t}}_{m1} & \tilde{\tilde{t}}_{m2} & \cdots & \tilde{\tilde{t}}_{mm} \end{bmatrix} \tag{5.18}$$

where, $\tilde{\tilde{t}}_{ij} = \left(\left(a'''_{ij}, b'''_{ij}, c'''_{ij}, d'''_{ij}; H_1(\tilde{t}_{ij}^U), H_2(\tilde{t}_{ij}^U)\right), \left(e'''_{ij}, f'''_{ij}, g'''_{ij}, h'''_{ij}; H_1(\tilde{t}_{ij}^L), H_2(\tilde{t}_{ij}^L)\right)\right)$.

The elements of the matrix in (5.18) are computed as below:

$$\begin{gathered} [a'''_{ij}] = N_{a''} \times (I - N_{a''})^{-1} \\ [b'''_{ij}] = N_{b''} \times (I - N_{b''})^{-1} \\ \vdots \\ [h'''_{ij}] = N_{h''} \times (I - N_{h''})^{-1} \end{gathered} \tag{5.19}$$

Step 5: Utilize the elements of the total-relation matrix $\tilde{\tilde{t}}_{ij}$ to accomplish the structural correlational analysis by using (5.20) and (5.21), respectively.

$$\tilde{\tilde{R}}_j = \sum_{i=1}^{m} \tilde{\tilde{t}}_{ij} \ \ where\ j = 1,2,3, \dots , m \tag{5.20}$$

$$\tilde{\tilde{D}}_i = \sum_{i=1}^{m} \tilde{\tilde{t}}_{ij} \ \ where\ i = 1,2,3, \dots , m \tag{5.21}$$

To portray the causal dependencies among the risk factors, expected values of the ordered pairs $\left(\tilde{\tilde{D}}_i \oplus \tilde{\tilde{R}}_i\right) and \ \left(\tilde{\tilde{D}}_i \ominus \tilde{\tilde{R}}_i\right)$ are calculated.

Step 6: Compute the importance of each criterion by employing (5.22):

$$w_i = \sqrt{\left(E\left(\tilde{\tilde{D}}_i \oplus \tilde{\tilde{R}}_i\right)\right)^2 + \left(E\left(\tilde{\tilde{D}}_i \ominus \tilde{\tilde{R}}_i\right)\right)^2} \tag{5.22}$$

$E\left(\tilde{\tilde{D}}_i \oplus \tilde{\tilde{R}}_i\right)$ = Expected prominence and $E\left(\tilde{\tilde{D}}_i \ominus \tilde{\tilde{R}}_i\right)$= expected relation. Finally, the normalized importance degree of each criterion is computed as in (5.23):

$$nw_i = \frac{w_i}{\sum_{i=1}^{m} w_i} \tag{5.23}$$

Local weights of the sub-risk factors are computed in a similar way as presented above. Then to calculate their global weights, the local weight of the main risk factor is multiplied with the local weight of the sub-risk factor.

5.4. Proposed Frameworks

5.4.1. Framework-I: Integrating IT2F-DEMATEL and Modified Fuzzy MAIRCA

The following four steps are involved in proposed the integrated MCDM framework:

i) Structuring of the problem,

ii) Modelling interactions among the risk factors by IT2F-DEMATEL (refer *Section 5.3*),

iii) Computing the weights of the risk factors by IT2F-DEMATEL (refer *Section 5.3),*

iv) Risk ranking of failure modes by modified fuzzy MAIRCA (refer *Chapter 3 / Section 3.3*).

The workflow diagram of the proposed integrated approach is depicted in Figure 5.2. The framework is further utilized for risk ranking of failure modes of the considered gearbox.

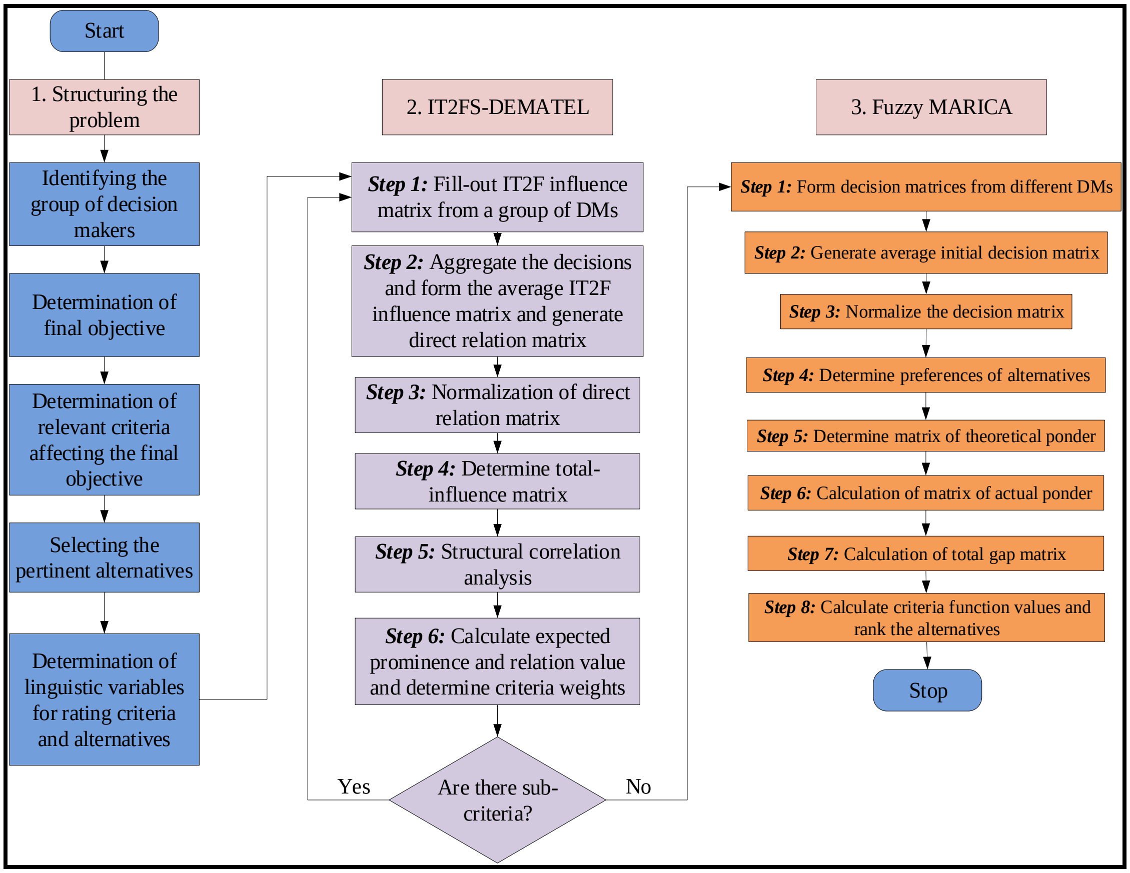


Figure 5.2 Workflow diagram of the proposed framework - I

5.4.2. Framework-II: Integrating IT2F-DEMATEL and Modified Fuzzy MARCOS

Here, the framework for the risk ranking of the failure modes is developed and proposed by combining IT2F-DEMATEL (refer *Section 5.3*) and modified fuzzy MARCOS (refer *Chapter 3 / Section 3.4*) methods. All the steps are like the earlier section, except replacing *Step iv)* to obtain the risk ranking by employing modified fuzzy MARCOS method rather than modified fuzzy MAIRCA. The workflow diagram of the proposed approach is depicted in Figure 5.3.

5.5. Case Study: FMEA of Process Plant Gearbox

The detailed descriptions of the gearboxes, their potential failure modes, cause(s), and effect(s) identified by a team of three cross functional experts (viz., DE1, DE2, and DE3), from economic, social, and environmental point of view have been discussed in *Table 4.2 of Chapter 4*. Now it is required to rank the failure modes as per their criticality levels by employing the frameworks developed in *Section 5.4*. In this chapter, it is assumed that the experts have equal expertise and thus are assigned the same weights, however, this is not a limitation of the proposed methods.

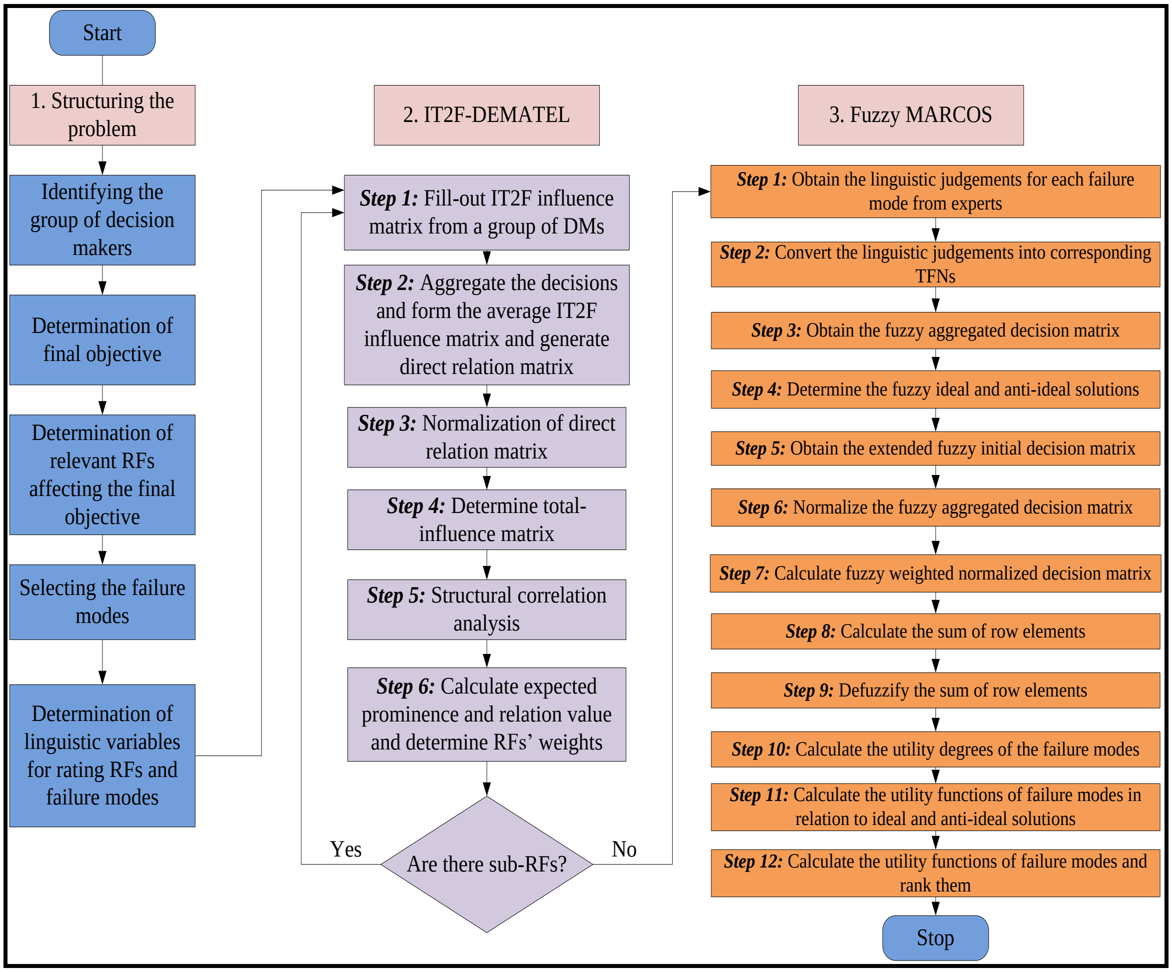


Figure 5.3 Workflow diagram of the proposed framework - II

5.5.1. Structuring the problem

The hierarchical structure of the identified risk factors is shown in Figure 5.4. The experts fill out the influence matrices by their linguistic judgements for the considered risk factors, and are shown in Table 5.2- Table 5.6. The linguistic judgements of the failure modes with respect to the risk factors are presented in Table 5.7. The scales adopted to convert the linguistic judgements to the corresponding of IT2FNs are shown in Table 5.1 (for IT2F-DEMATEL), and

Table *3.2* (for modified fuzzy MAIRCA and modified fuzzy MARCOS), respectively.

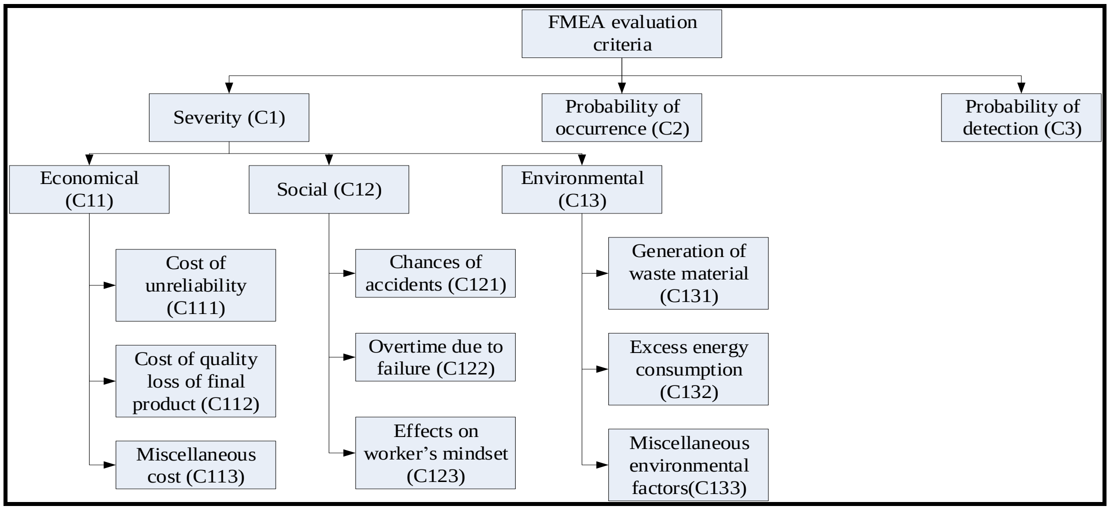


Figure 5.4 Hierarchy of risk factors identified from TBL of sustainability

Table 5.2 Dependency degrees among severity, occurrence and detection

Risk factors	Domain experts	Severity	Occurrence	Detection
Severity	DE1	-	M	VH
	DE2	-	VL	VVH
	DE3	-	L	VH
Occurrence	DE1	L	-	-
	DE2	M	-	-
	DE3	VL	-	-
Detection	DE1	H	-	-
	DE2	VH	-	-
	DE3	H	-	-

Table 5.3. Dependency degrees among economic, social, and environmental severity

Risk factors	Domain experts	Economic severity	Social severity	Environmental severity
Economic severity	DE1	-	H	M
	DE2	-	M	H
	DE3	-	H	M
Social severity	DE1	M	-	VH
	DE2	VH	-	-
	DE3	H	-	-
Environmental severity	DE1	H	M	-
	DE2	M	H	-
	DE3	H	M	-

Table 5.4. Dependency degrees among cost of unreliability, cost of quality loss, and miscellaneous cost factors

Risk factors	Domain experts	Cost of unreliability	Cost of quality loss	Miscellaneous cost factors
Cost of unreliability	DE1	-	VH	M
	DE2	-	H	H
	DE3	-	M	VH
Cost of quality loss	DE1	L	-	-
	DE2	H	-	-
	DE3	M	-	-
Miscellaneous cost factors	DE1	H	-	-
	DE2	VH	-	-
	DE3	H	-	-

Table 5.5. Dependency degrees among chances of accident, overtime due to failure, and effects on workers' mind-set

Risk factors	Domain experts	Chances of accident	Overtime due to failure	Effects on workers' mind-set
Chances of accident	DE1	-	-	VH
	DE2	-	-	H
	DE3	-	-	M
Overtime due to failure	DE1	VL	-	H
	DE2	H	-	M
	DE3	L	-	M
Effects on workers' mindset	DE1	M	H	-
	DE2	H	M	-
	DE3	VL	L	-

Table 5.6. Dependency degrees among generation of waste material, excess energy consumption, and miscellaneous environmental factors

Risk factors	Domain experts	Generation of waste material	Excess energy consumption	Miscellaneous environmental factors
Generation of waste material	DE1	-	VH	M
	DE2	-	H	H
	DE3	-	VH	M
Excess energy consumption	DE1	M	-	L
	DE2	L	-	M
	DE3	H	-	H
Miscellaneous environmental factors	DE1	M	-	-
	DE2	H	-	-
	DE3	L	-	-

Table 5.7 Linguistic evaluations of the failure modes with respect to the risk factors

Failure modes	Domain experts	C111	C112	C113	C121	C122	C123	C131	C132	C133	C2	C3
FM1	DE1	MG	MP	P	P	F	G	F	F	F	P	MG
	DE2	G	P	P	VP	MP	MG	F	MP	P	MP	F
	DE3	MG	MP	VP	P	MP	G	F	P	P	F	MG
FM2	DE1	G	P	P	P	G	G	F	MP	P	F	VG
	DE2	VG	MP	MP	P	VG	MG	MG	F	P	MP	G
	DE3	G	P	P	MP	G	MG	F	F	MP	MP	G
FM3	DE1	F	VP	F	P	MG	G	MP	MP	P	F	G
	DE2	MP	P	MP	P	MG	G	F	P	P	F	MG
	DE3	MG	MP	P	MP	G	G	P	MP	MP	MP	G
FM4	DE1	VG	F	P	F	MP	MG	P	G	P	P	MP
	DE2	G	MG	F	MP	MP	F	VP	G	F	MP	F
	DE3	VG	F	MP	F	P	F	P	F	P	F	MP
FM5	DE1	P	P	P	P	G	MG	P	MP	P	F	F
	DE2	MP	P	P	P	G	G	P	P	MP	MP	F
	DE3	P	MP	MP	MP	MG	G	F	P	MP	MP	MG
FM6	DE1	MG	MP	P	P	F	G	F	MP	P	F	MP
	DE2	F	F	P	P	MG	MG	F	P	F	MG	MP
	DE3	F	MP	P	MP	MG	F	P	P	MP	MG	F

Failure modes	Domain experts	C111	C112	C113	C121	C122	C123	C131	C132	C133	C2	C3
FM7	DE1	G	F	G	F	P	P	VG	G	F	P	VG
	DE2	VG	MG	G	P	F	P	VG	F	F	MP	VG
	DE3	G	F	VG	MP	F	MP	G	MG	P	P	G
FM8	DE1	VG	MG	G	P	P	MP	MP	G	MP	G	VG
	DE2	VG	F	F	P	MP	F	MP	MG	P	G	VG
	DE3	G	F	G	MP	P	MP	P	G	P	VG	VG
FM9	DE1	G	MG	F	P	P	MP	VG	G	F	F	G
	DE2	VG	G	F	P	P	P	VG	G	P	MP	MG
	DE3	G	F	G	MP	F	P	G	G	F	F	VG
FM10	DE1	G	G	F	MG	P	P	F	G	F	MP	F
	DE2	MG	MG	MP	MG	P	P	F	VG	P	F	F
	DE3	G	G	MP	F	MP	VP	P	G	F	MP	MG
FM11	DE1	MG	G	G	F	MP	F	F	G	MP	MP	F
	DE2	G	G	VG	MG	P	F	MP	G	F	P	F
	DE3	VG	F	G	F	MP	F	F	G	F	P	MG
FM12	DE1	VG	VG	P	VG	P	F	VG	VG	F	P	VG
	DE2	VG	G	P	G	P	MG	G	G	G	VP	VG
	DE3	G	G	MP	VG	MP	F	G	VG	G	P	G

5.5.2. IT2F-DEMATEL: Computing Causal Dependencies and Weights of the Risk Factors

Employing the steps presented in *Section 5.3*, the expected prominence and expected relation values of the risk factors are computed and are shown Table 5.8. Based on these values the causal dependency diagrams of the risk factors can be portrayed.

Table 5.8 Expected prominence and expected relation values of the RFs

Risk factors	Expected prominence value $\left[E\left(\tilde{\tilde{D}}_i \oplus \tilde{\tilde{R}}_i\right)\right]$	Expected relation value $\left[E\left(\tilde{\tilde{D}}_i \ominus \tilde{\tilde{R}}_i\right)\right]$	Nature of the risk factor
Severity (C1)	3.231	0.131	Cause
Occurrence (C2)	1.303	0.032	Cause
Detection (C3)	2.716	-0.163	Effect
Economic severity (C11)	5.388	-0.161	Effect
Societal severity (C12)	4.814	-0.475	Effect
Environmental severity (C13)	4.654	0.636	Cause
Cost of unreliability (C111)	3.074	0.086	Cause
Cost of quality loss (C112)	1.831	-0.206	Effect
Misc. cost factors (C113)	2.139	0.119	Cause
Chances of accident (C121)	2.418	-0.311	Effect

Risk factors	Expected prominence value $\left[E\left(\tilde{\tilde{D}}_i \oplus \tilde{\tilde{R}}_i\right)\right]$	Expected relation value $\left[E\left(\tilde{\tilde{D}}_i \ominus \tilde{\tilde{R}}_i\right)\right]$	Nature of the risk factor
Overtime due to failure (C122)	2.390	0.579	Cause
Effects on workers' mindset (C123)	3.173	-0.268	Effect
Generation of waste material (C131)	3.330	0.337	Cause
Excess energy consumption (C132)	2.674	0.356	Cause
Misc. environmental factors (C133)	2.470	-0.693	Effect

The causal relationships among severity (C1), occurrence (C2) and detection (C3) are depicted in Figure 5.5. Note that the cause group is known as the influencing factors and the effect group is known as the influenced factors. From Figure 5.5, it is recommended that the decision makers should concentrate on the cause group elements, which will successively control the effect group elements. The following remarks/suggestions could be drawn from the above analysis and for this case study.

- Utmost priority to be given to control the severity (C1) and occurrence (C2) factors. Again, severity (C1) is decoupled into multiple sustainable indices: economical (C11), societal (C12) and environmental (C13), which should be further taken up for the improvement of the system.

- The chances of occurrence (C2) of a failure mode could be decreased if suitable maintenance measures are taken up at this stage or address them in future designs. Thus, it can be recommended to explore the possibility of equipping the gearbox with technologically advanced and cost-effective modern fault detection instruments aiming at automated fault diagnosis.

- Further, it is observed from Table 5.8 that severity (C1) has the highest prominence (3.231), and relation value (0.1309). It suggests that severity (C1) is the most influencing factor in comparison to the other two factors.

- A high relation value implies that it is not affected by other factors and sub-factors. In this situation, it is always better to pay attention to other factors for overall system improvement. The only member in the effect group is detection (C3), whose $(D - R)$ value is quite low (-0.1632). This implies that chances of improvement with this factor are the highest.

Similarly, the causal dependencies among other risk factors under severity (C1) are computed and shown in Figure 5.6 - Figure 5.9. Like previous analysis, C1 is decoupled and as it also belongs to the cause group (*or* influencing factor), it is required to analyse it further. From Figure 5.6, it is observed that:

- The economical aspect (C11) has the highest prominence value (5.388), but a lower relation value (-0.1607). Furthermore, it belongs to the effect group along with the social aspects (C12), which is obviously influenced by the environmental aspects (C13), whose prominence value (4.654) is lower that the economic aspects but has the highest relation value (0.6359). In such a scenario, the utmost importance should be given to environmental aspects (C13), as it is the strongest influencing one.

As environmental aspects belong to the cause group, it is further analysed for better inferences. From Figure 5.7, it can be remarked that,

- The generation of waste material (C131) has the highest prominence value (3.330) and lower relation value (0.3367). It also belongs to the cause group along with excess energy consumption (C132), which has less prominence value (2.674) and highest relation value (0.3557). These imply that generation of waste material (C131) is the most worrying sub-factor rather than excess energy consumption (C132), when the severity is considered from environmental point of view.

- Other miscellaneous environmental factors (C133) include generation of toxic gases, fumes, dusts, *etc.*

Figure 5.8 (*viz.*, societal point of view) reflects that:

- The effects on workers' mind-set due to failure (C123) have the largest prominence value (3.173) with largest local priority (0.394), although it belongs to the effect group. It is due to that fact that each worker is normally allocated with a set of tasks during their working hours. However, when a gearbox encounters a catastrophic failure, their mind-set and efficiency change.

- Overtime due to failure (C122) belongs to the cause group, as it influences the chances of accidents due to improper vigilance and system failure. Chances of accidents has the least relation value (-0.3112) and implies that it can be further improved by preventing failure of the system.

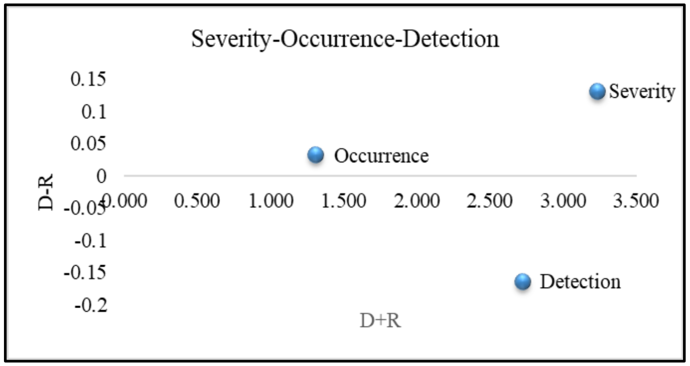


Figure 5.5 Causal dependency diagram of severity-occurrence-detection

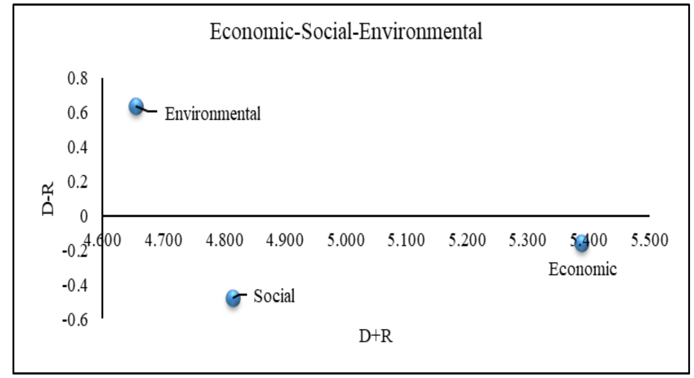


Figure 5.6 Causal dependency diagram of economic-social-environmental severity

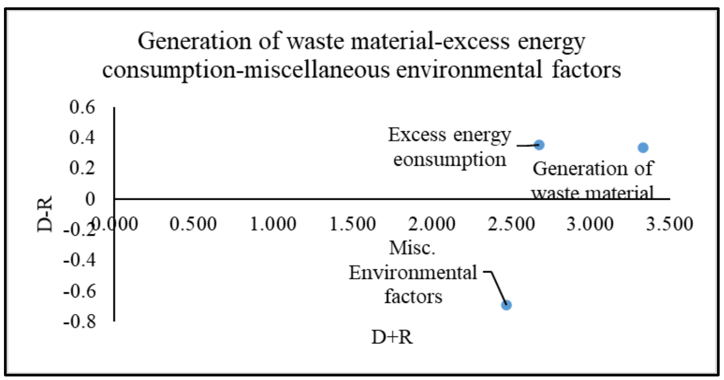


Figure 5.7. Causal dependency diagram of generation of waste material-excess energy consumption-miscellaneous environmental factors

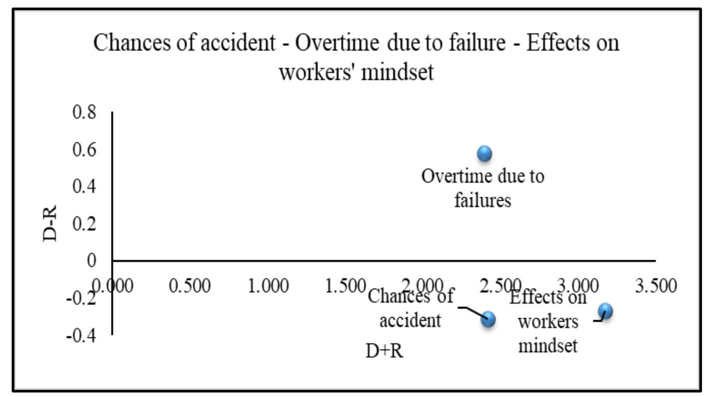


Figure 5.8. Causal dependency diagram of chances of accident - overtime due to failure - effects on workers' mind-set

From an economical point of view:

- Cost of unreliability (C111) and miscellaneous cost (C113) belongs to the cause group and have local priorities of 43.6% and 30.3 %, respectively, as shown in Figure 5.9. Cost of unreliability (C111) is increased with number of failures, which can be controlled if the detection level is high enough and complemented by proactive maintenance efforts.

- Miscellaneous cost includes several factors such as inventory costs, procurement costs, etc. whose data are usually difficult to get, hence they are depicted linguistically.

- The only factor that belongs to the effect group is cost of quality loss of final product (C112), whose $(D - R)$ value is least (-0.2056) and can only be improved if the other two indices, C11 and C113 are improved.

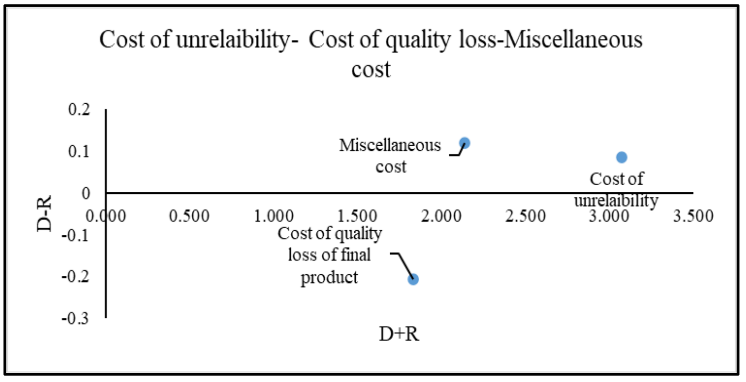


Figure 5.9. Causal dependency diagram of cost of unreliability-cost of quality loss of final product-miscellaneous cost factors

After providing insights about the causal dependencies among the risk factors, their weights are shown in Table 5.9 which are calculated by using (5.22), and (5.23).

Table 5.9. Crisp weights of the risk factors computed by IT2F-DEMATEL

Factors	Crisp Weight values	Sub factors	Crisp sub-factors weight values	Sub-sub factors	Crisp sub-sub factors weight values	Crisp weights
Severity	0.446	Economical	0.361	Cost of unreliability	0.436	0.070
				Cost of quality loss	0.261	0.041
				Miscellaneous cost	0.303	0.049
		Social	0.324	Chances of accidents	0.302	0.044
				Overtime due to failure	0.304	0.044

Factors	Crisp Weight values	Sub factors	Crisp sub-factors weight values	Sub-sub factors	Crisp sub-sub factors weight values	Crisp weights
				Effects on workers mind-set	0.394	0.057
		Environmental	0.315	Generation of waste material	0.389	0.055
				Excess energy consumption	0.313	0.044
				Miscellaneous environmental factors	0.298	0.042
Occurrence	0.180	----	----	----	----	0.180
Detection	0.375	----	----	----	----	0.375

The next section utilizes these weights for the risk ranking of failure modes according to the proposed modified fuzzy MAIRCA and modified fuzzy MARCOS methods, and to examining their potential further in terms of ranking stability.

5.5.3. Risk Ranking of the Failure Modes by Framework-I

Following the mathematical steps of modified fuzzy MAIRCA, (refer *Section 3.3*) and utilizing the weight values of the risk factors as computed by IT2F-DEMATEL, the de-fuzzified gap values and the ranking orders of the failure modes are obtained and are presented in Table 5.10.

Table 5.10 Risk ranking results of failure modes by using Framework-I

Failure Modes	Notations	Criteria function values	Ranking by Framework-I
Wear of Teeth	FM1	0.0732	9
Broken Teeth	FM2	0.0700	5
Pitting of Gear	FM3	0.0711	6
Axial Shift of Gear	FM4	0.0744	12
Scoring of gears	FM5	0.0738	11
Brinelling	FM6	0.0733	10
Cage defect	FM7	0.0696	4
Crack on raceways	FM8	0.0669	1
Crack of rollers	FM9	0.0693	3
Bent shaft	FM10	0.0722	8
Crack of shaft	FM11	0.0719	7
Fracture of shaft	FM12	0.0688	2

Thus, the *Framework-I* identifies crack on raceways (FM8) as the most critical failure mode, followed by fracture of shaft, crack on rollers, cage defects, and so on. Next, these ranking results are again cross-examined by the modified fuzzy MARCOS approach.

5.5.4. Risk Ranking of Failure Modes by Framework-II

Like the previous section, here, again the identified failure modes are ranked according to their risk levels, but by using the Framework-II. The weights of the risk factors are obtained from the outputs of IT2F-DEMATEL method. The computed risk ranking results are presented in Table 5.11, from which it can be discerned that *Framework-II* also identified the crack on raceways (FM8) as the most critical one, followed by fracture of shaft (FM12). However, the *Framework-II* identified cage defect (FM7) as the third critical failure mode and crack on rollers (FM9) as the fourth critical failure mode, which are just opposite to Framework-I.

Table 5.11. Ranking of failure modes by the proposed Framework-II

Failure Modes	$\tilde{S}_i$	S_i	$UD_i^{(AID)}$	$UD_i^{(ID)}$	$UF_i^{(AID)}$	$UF_i^{(ID)}$	UF_i	Ranking by Framework-II
FM1	(0.299, 0.476, 0.668)	0.4787	2.259	0.507	0.183228	0.817	0.4868	9
FM2	(0.474, 0.646, 0.780)	0.6400	3.020	0.678	0.183228	0.817	0.6508	5
FM3	(0.387, 0.573, 0.740)	0.5695	2.688	0.603	0.183228	0.817	0.5791	6
FM4	(0.230, 0.407, 0.592)	0.4083	1.927	0.432	0.183228	0.817	0.4152	12
FM5	(0.248, 0.428, 0.622)	0.4302	2.030	0.455	0.183228	0.817	0.4375	10
FM6	(0.241, 0.428, 0.627)	0.4297	2.028	0.455	0.183228	0.817	0.4370	11
FM7	(0.519, 0.667, 0.779)	0.6607	3.118	0.699	0.183228	0.817	0.6719	3
FM8	(0.622, 0.764, 0.849)	0.7547	3.562	0.799	0.183228	0.817	0.7674	1
FM9	(0.494, 0.665, 0.803)	0.6594	3.112	0.698	0.183228	0.817	0.6706	4
FM10	(0.321, 0.506, 0.692)	0.5063	2.390	0.536	0.183228	0.817	0.5149	8
FM11	(0.344, 0.528, 0.708)	0.5272	2.488	0.558	0.183228	0.817	0.5361	7
FM12	(0.568, 0.700, 0.791)	0.6934	3.273	0.734	0.183228	0.817	0.7052	2
$\tilde{S}^{(AID)}$	(0.071, 0.203, 0.390)	0.2119						
$\tilde{S}^{(ID)}$	(0.822, 0.961, 1.00)	0.9445						

Thus, this necessitated to further analyse the results by comparing them with other popular fuzzy MCDM methods and through sensitivity analysis.

5.6. Validations and Discussions on the Ranking Results

This section deals with the following two aspects:

i) Comparing the ranking result with other existing and popular fuzzy MCDM methods available in FMEA literature.

ii) Performing a sensitivity analysis by changing the values of criteria weights to different levels and observing the subsequent effects on the ranking results.

5.6.1. Comparisons with Other Fuzzy MCDM Methods

In the FMEA domain, to reach at a consensus about the ranking positions of the failure modes, it is often favoured to relate the results with other established methods or validated through the historical evidences. Liu *et al.*, (Liu *et al.*, 2019a) observed that the most frequently used fuzzy MCDM methods in FMEA context are fuzzy extensions of TOPSIS, VIKOR, COPRAS, MOORA, and MABAC. Thus, the ranking results obtained in this work are compared with them, along with the fuzzy extension of the MAIRCA method proposed by (Pamučar *et al.*, 2014), fuzzy MARCOS method developed by Stanković *et al.* (Stanković *et al.*, 2020). The results are shown in Table 5.12 and the following points are noted:

Table 5.12. Comparisons of results with other fuzzy MCDM methods

<table>
<tr><th>Failure modes</th><th>Fuzzy TOPSIS</th><th>Fuzzy VIKOR</th><th>Fuzzy MABAC</th><th>Fuzzy COPRAS</th><th>Fuzzy MOORA</th><th>Fuzzy MAIRCA</th><th>Framework-I</th><th>Fuzzy MARCOS</th><th>Framework-II</th></tr>
<tr><td>FM1</td><td>7</td><td>7</td><td>9</td><td>7</td><td>7</td><td>7</td><td>9</td><td rowspan="12">Do not able to provide outputs.</td><td>9</td></tr>
<tr><td>FM2</td><td>4</td><td>3</td><td>5</td><td>3</td><td>3</td><td>4</td><td>5</td><td>5</td></tr>
<tr><td>FM3</td><td>6</td><td>5</td><td>6</td><td>6</td><td>6</td><td>6</td><td>6</td><td>6</td></tr>
<tr><td>FM4</td><td>12</td><td>12</td><td>12</td><td>12</td><td>12</td><td>12</td><td>12</td><td>12</td></tr>
<tr><td>FM5</td><td>10</td><td>10</td><td>10</td><td>8</td><td>8</td><td>10</td><td>11</td><td>10</td></tr>
<tr><td>FM6</td><td>11</td><td>11</td><td>11</td><td>9</td><td>9</td><td>11</td><td>10</td><td>11</td></tr>
<tr><td>FM7</td><td>3</td><td>4</td><td>3</td><td>4</td><td>4</td><td>3</td><td>4</td><td>3</td></tr>
<tr><td>FM8</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>FM9</td><td>5</td><td>2</td><td>4</td><td>5</td><td>5</td><td>5</td><td>3</td><td>4</td></tr>
<tr><td>FM10</td><td>9</td><td>9</td><td>8</td><td>10</td><td>10</td><td>9</td><td>8</td><td>8</td></tr>
<tr><td>FM11</td><td>8</td><td>8</td><td>7</td><td>11</td><td>11</td><td>8</td><td>7</td><td>7</td></tr>
<tr><td>FM12</td><td>2</td><td>6</td><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td></tr>
</table>

- Every method identified the cracks on raceways (FM8) as the most critical failure mode.

- The fracture of the shaft (FM12) is identified as the second critical failure mode by all other fuzzy MCDM methods, except fuzzy VIKOR (Opricovic, 2011), where it is ranked as the sixth critical failure mode. *This is probably due to choosing the value of ν (0.5 in this case)*. A detailed description about this concept has been delivered in *Sub-Section 2.1.4*.

Thus, it can be said that the results obtained by the proposed integrated approaches are almost similar with the other fuzzy MCDM methods and produces credible results. However, further validations are still needed to compare the performances of the proposed approaches, which are presented in the next section.

5.6.2. Sensitivity Analyses by Varying the Risk Factors' Weights

In this section, the weights of the risk factors are varied to different levels, and their effects on the final ranking results are examined. As FMEA is an iterative and ongoing task, the weights of the risk factors may often vary. A good MCDM method should compute the ranking results in such a way that changing the risk factors' weights should not significantly alter the ranking order of failure modes. The superiority of any approach is determined through calculating the inter-Spearman's rank correlation coefficient, which is nothing but the Spearman's rank correlation coefficient between the ranking results generated by changing a particular criterion's weight to 20%, 40%, 60%, and 80% of its originally computed crisp weight.

While varying the risk factors' weights, a total of 44 scenarios are generated, which are further divided into four equal parts (as there are total 11 numbers of criteria included in this work). For each scenario, the relative importance of each criterion is increased to different levels (*e.g.*, 20%, 40%, 60%, and 80%), while the other criteria are decreased by the same level. Additionally, $\sum_{i=1}^{n} w_i = 1$ condition is satisfied for each instance. Further, the overall Spearman's rank correlation coefficients are calculated for the ranking results generated in the 44 scenarios.

- For the sub-risk factor, cost of unreliability (C111), the changes in risk ranking results are portrayed in Figure 5.10 (for framework-I), Figure 5.11 (for framework-II), respectively. *The inter-Spearman's rank correlation coefficient for the first integrated approach is 96.7%, while for the second integrated approach it is 97.4%. Thus, it can be observed that for changing the cost of unreliability (C111), the first integrated approach generates more robust result than the latter one.*

- For cost of quality loss of the final product (C112), the changes in the ranking results are displayed in Figure 5.12 (framework-I), and Figure 5.13 (framework-II), respectively. The earlier one generates an inter-Spearman's rank correlation coefficient of 95.7%, while the latter one produces the value of 96.5%. *Thus, in case of C112, the second approach demonstrates more stability than the first one.*

- Similar for miscellaneous cost factors (C113), the changes are shown in Figure 5.14 (framework-I) Figure 5.15 (framework-II), respectively. The first one has less ranking stability (inter-Spearman's rank correlation coefficient 94.9%), than the second one (inter-Spearman's rank correlation value 96.8%). *Hence, for C113, fuzzy MAIRCA shows less stability than the fuzzy MARCOS.*

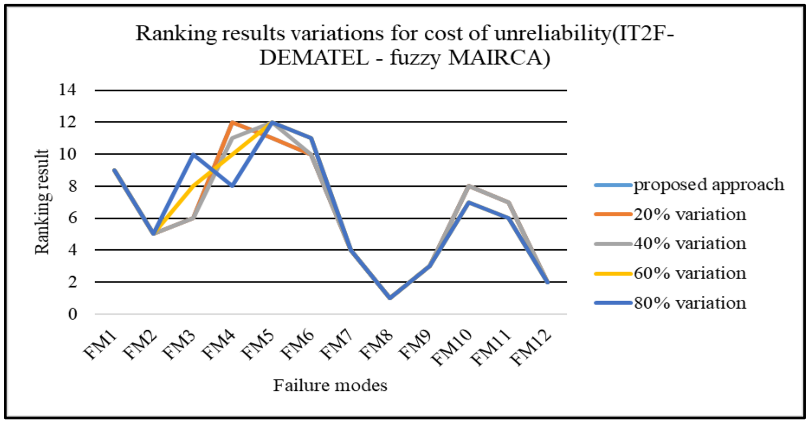


Figure 5.10. Variations in risk ranking results by changing the weights of C111 in framework - I

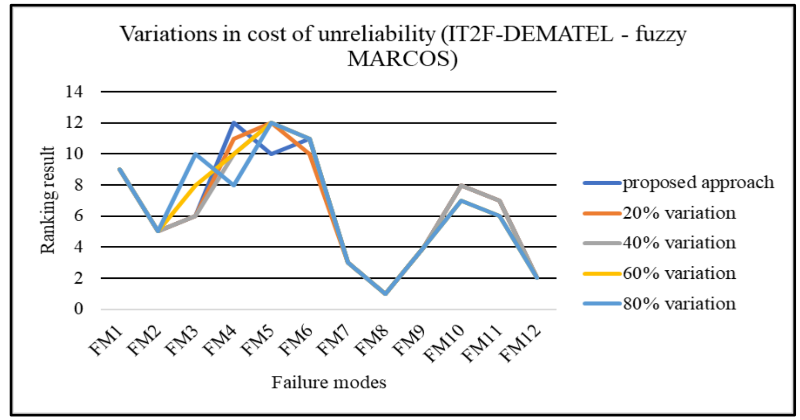


Figure 5.11. Variations in risk ranking results by changing the weights of C111 in framework-II

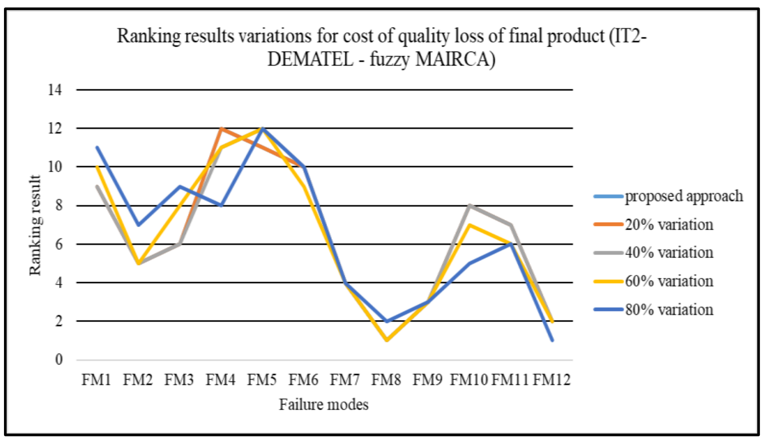


Figure 5.12. Variations in risk ranking results by changing the weights of C112 in framework-I

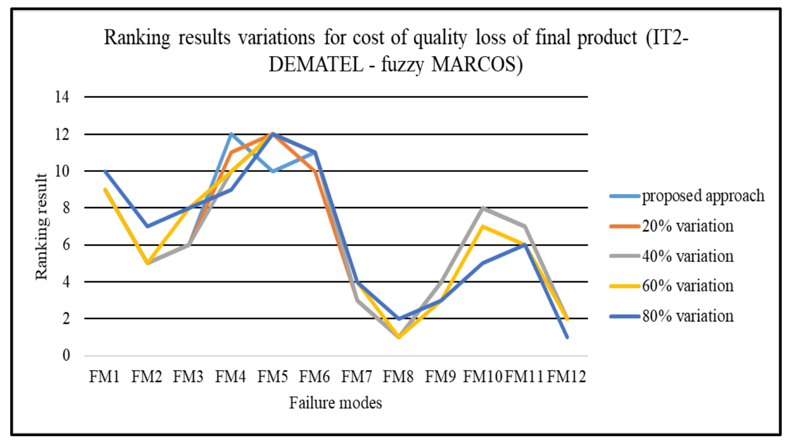


Figure 5.13. Variations in risk ranking results by changing the weights of C112 in framework-II

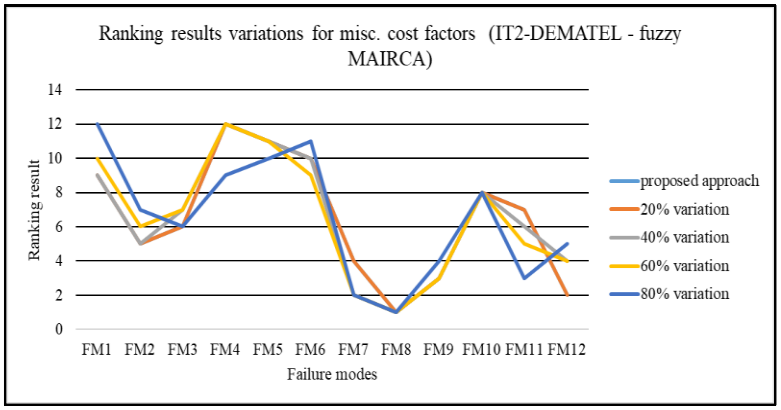


Figure 5.14. Variations in risk ranking results by changing the weights of C113 in framework-I

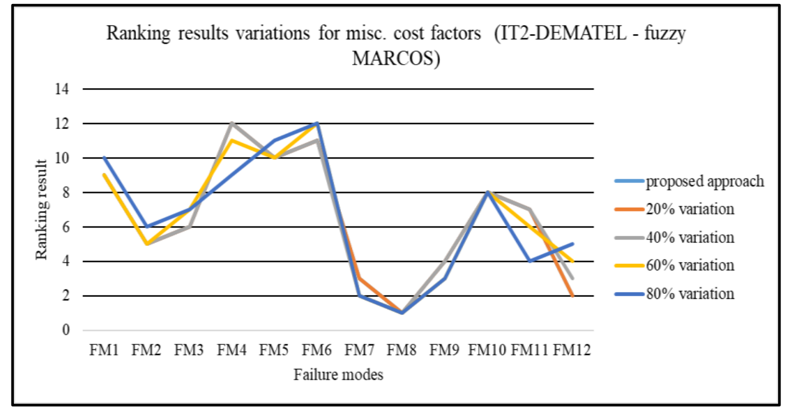


Figure 5.15. Variations in risk ranking results by changing the weights of C113 in framework-II

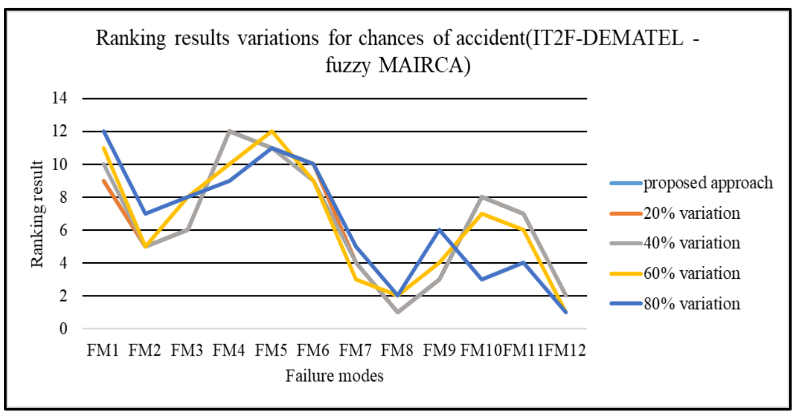


Figure 5.16. Variations in risk ranking results by changing the weights of C121 in framework-I

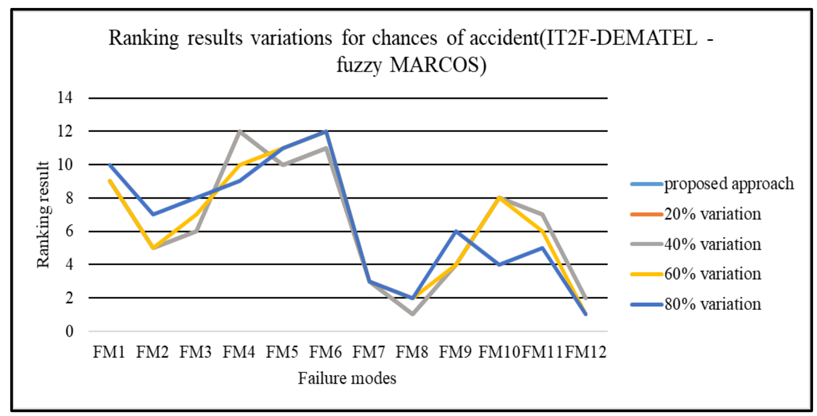


Figure 5.17. Variations in risk ranking results by changing the weights of C121 in framework-II

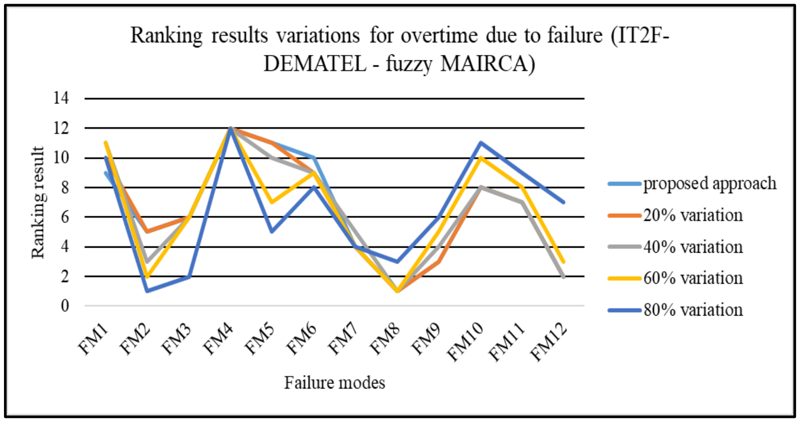


Figure 5.18. Variations in risk ranking results by changing the weights of C122 in framework-I

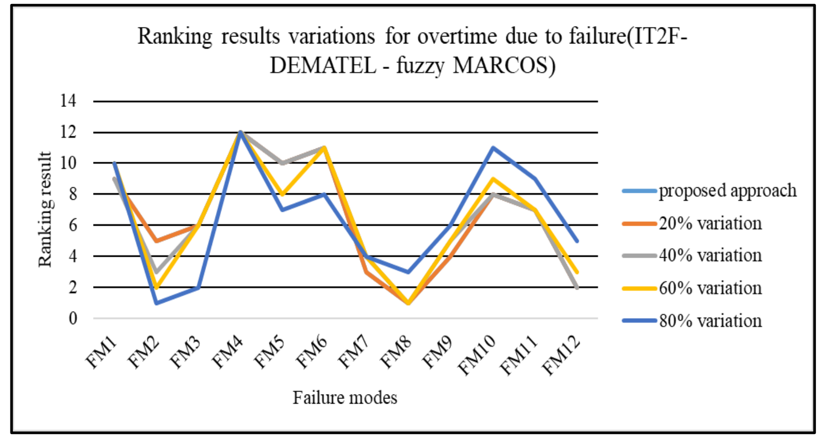


Figure 5.19. Variations in risk ranking results by changing the weights of C122 in framework-II

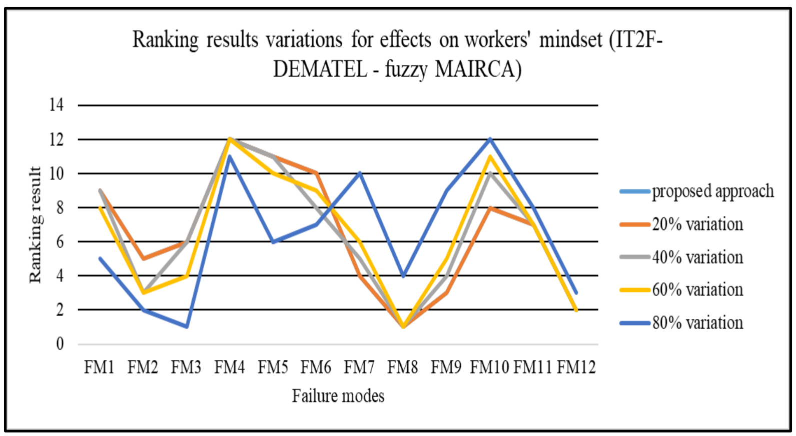


Figure 5.20. Variations in ranking results by changing the weights of C123 in framework-I

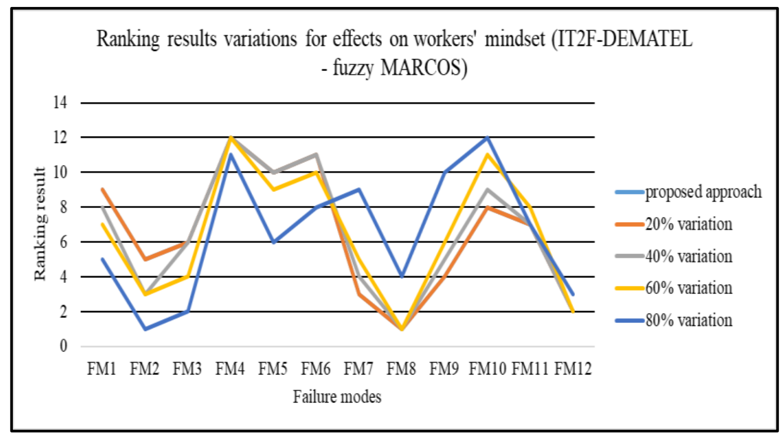


Figure 5.21. Variations in risk ranking results by changing the weights of C123 in framework-II

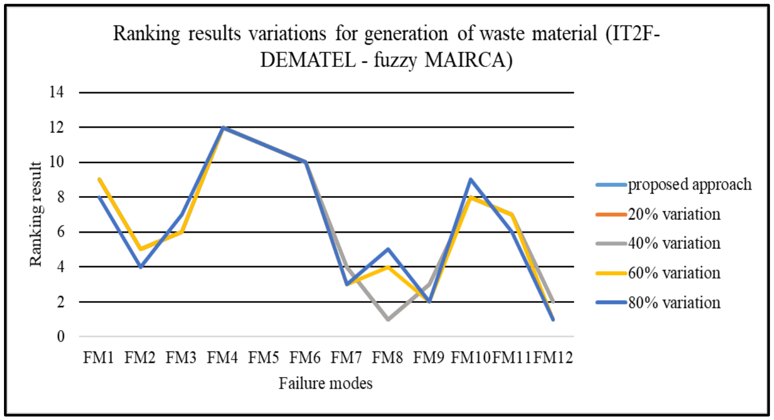


Figure 5.22. Variations in risk ranking results by changing the weights of C131 in framework-I

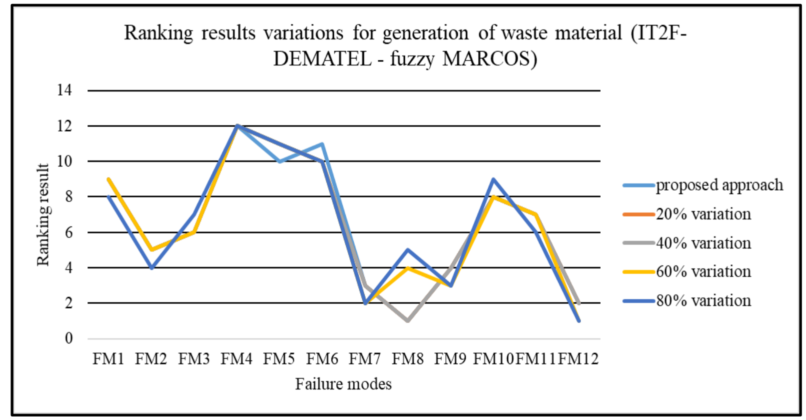


Figure 5.23. Variations in risk ranking results by changing the weights of C131 in framework-II

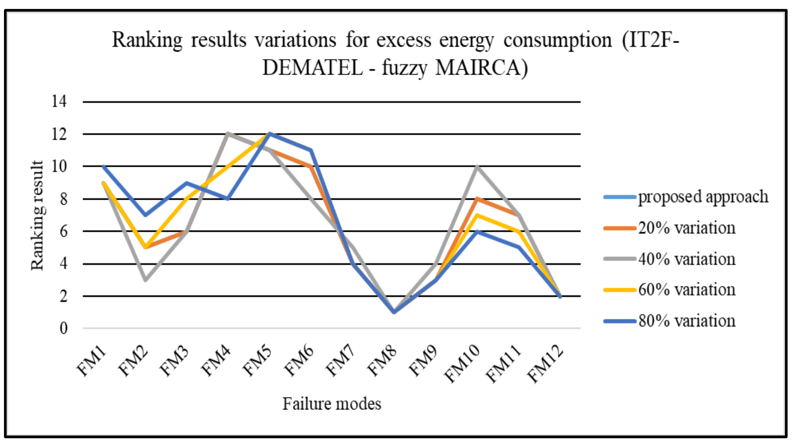


Figure 5.24. Variations in risk ranking results by changing the weights of C132 in framework-I

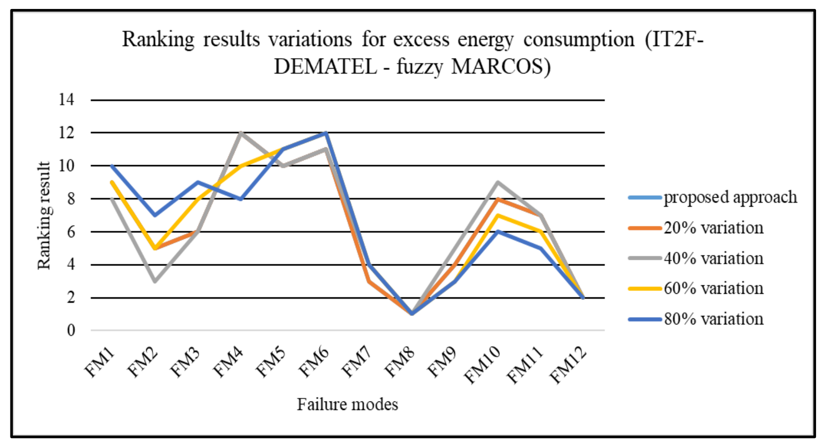


Figure 5.25. Variations in risk ranking results by changing the weights of C132 in framework-II

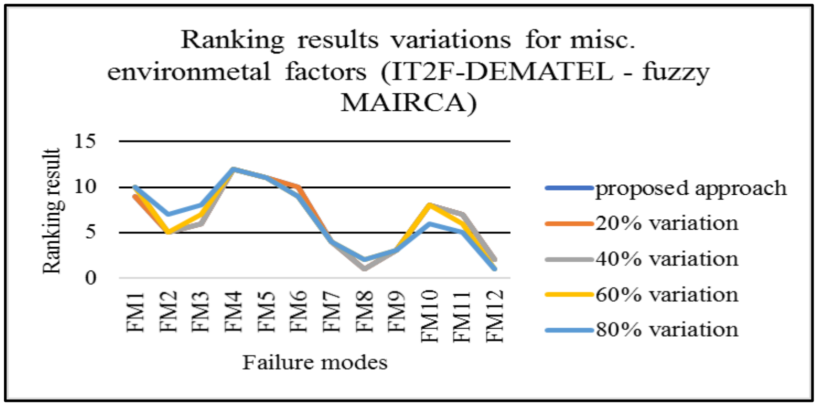


Figure 5.26. Variations in risk ranking results by changing the weights of C133 in framework-I

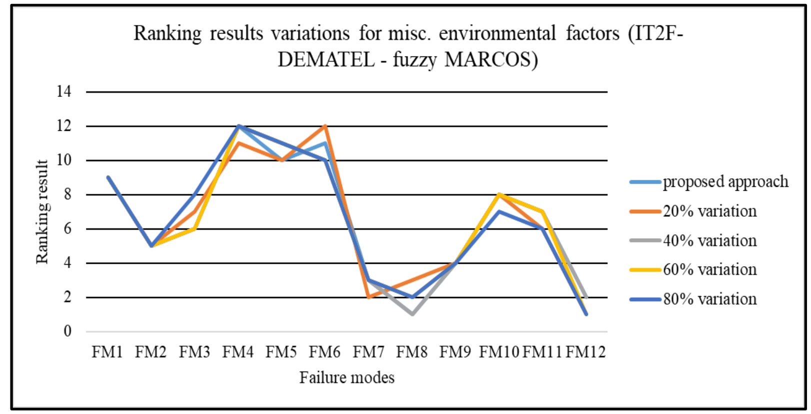


Figure 5.27. Variations in risk ranking results by changing the weights of C133 in framework-II

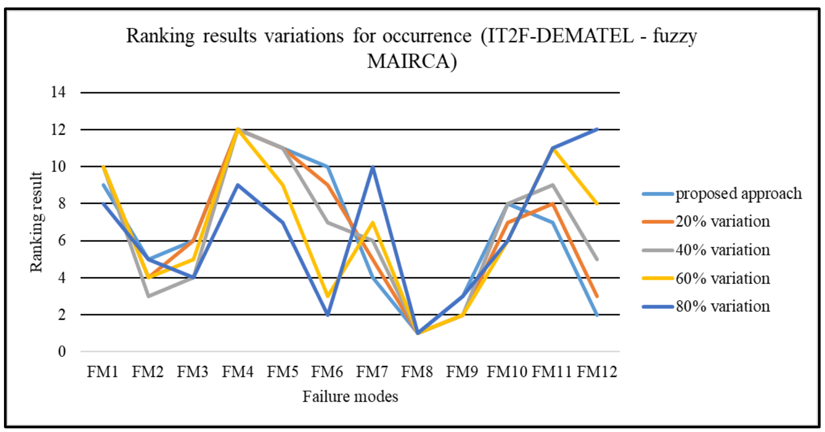


Figure 5.28. Variations in risk ranking results by changing the weights of C2 in framework-I

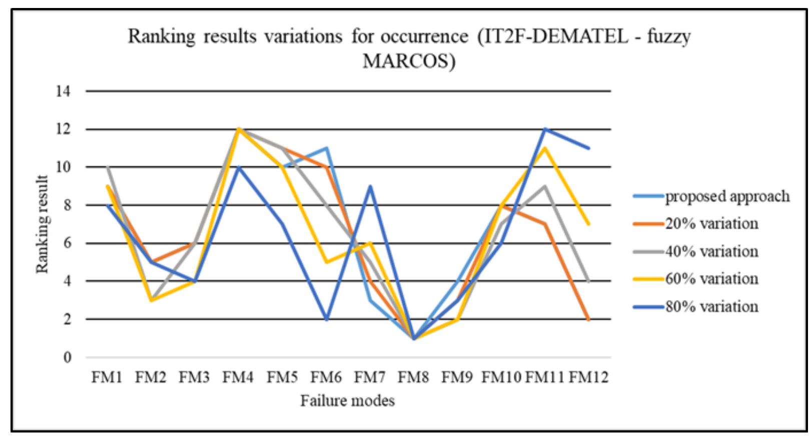


Figure 5.29. Variations in risk ranking results by changing the weights of C2 in framework-II

- In case of chances of accident (C121), the similar variations are presented in Figure 5.16 (framework -I), and Figure 5.17 (framework -II). *In this case, fuzzy MARCOS (inter-Spearman's rank correlation coefficient 95.1%) outperforms fuzzy MAIRCA (inter-Spearman's rank correlation coefficient 92.5%) in terms of ranking stability.*

- Similarly, for overtime due to failure (C122), the consequences in ranks variations are displayed in Figure 5.18 (framework -I), and Figure 5.19 (framework -II), respectively. *In this case also the latter one (92.2.%) outperforms the first one (inter-Spearman's rank correlation coefficient 88.7%), in terms of stability.*

- In case of effects on workers' mindset (C123), the changes observed in framework-I and framework-II are presented in Figure 5.20, and Figure 5.21, with an inter-Spearman's rank correlation coefficients of 84.1%, and 85.3%, respectively. *Here also the later one shows higher ranking stability.*

- For excess energy consumption (C132), the changes are exhibited in Figure 5.24 (framework-I), and Figure 5.25 (framework-II), with inter-Spearman's rank correlation coefficients of 96.6%, and 96.4%, respectively. *Surprisingly, in this case, the earlier one shows better ranking stability.*

- For miscellaneous environmental factors (C133), the criterion weight value changing effects are described through Figure 5.26 (framework-I), and Figure 5.27 (framework-II), with inter-Spearman's rank correlation coefficients of 97.9%, and 98.3%, respectively. *In this case, the later one is showing higher ranking stability.*

- In case of occurrence (C2), the effects are presented in Figure 5.28 (framework-I), and Figure 5.29 (framework-II), with inter-Spearman's rank correlation coefficients of 82.0%, and 80.6.%, respectively. Here also, *the proposed integrated method-1 is noticed to be superior than the second one in terms of ranking stability.*

- For the criterion – detection (C3), the effects are showed in Figure 5.30 (framework-I), and Figure 5.31 (framework-II), with inter-Spearman's rank correlation coefficient values 98.9%, and 99.2%, respectively. *In this case, the latter one outperforms the earlier one by means of ranking stability*.

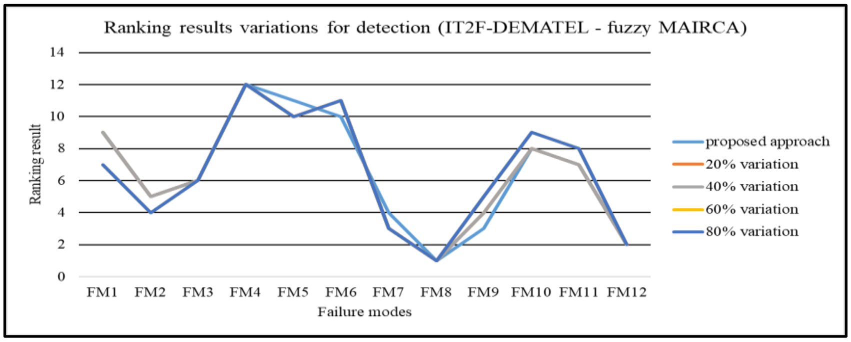


Figure 5.30. Variations in risk ranking results by changing the weights of C3 in framework-I

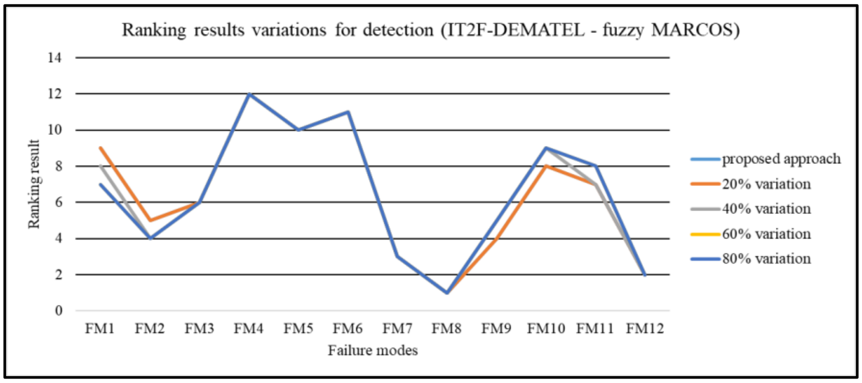


Figure 5.31. Variations in risk ranking results by changing the weights of C3 in framework-II

5.6.3. Concluding Remarks on the Results of Sensitivity Analysis

From the sensitivity analyses the following critical points are noted:

- The statement observed in Chapter 3, where both the modified fuzzy MAIRCA and modified fuzzy MARCOS have presented the same ranking stability, is found to be misleading.

- In terms of ranking stability, the *Framework-II* is found to be superior than *Framework-I* on most of the occasions. The reasons for this superiority could be the consideration of ideal and anti-ideal solutions from the very beginning of the decision-making process, which is not present in the mathematical framework of modified fuzzy MAIRCA method.

- Further, it is observed that the average of inter-Spearman's Rank correlation coefficient for *Framework-II* is 94.1% and for *Framework-I* is 93.2%.

- Again, considering all 44 scenarios, the fuzzy MARCOS (rank correlation coefficient 84.65%) is observed to be superior than the fuzzy MAIRCA (rank correlation coefficient 81.95%).

- The results of this chapter indicate that when the number of criteria and alternatives are increased, fuzzy MARCOS provides more ranking stability than the fuzzy MAIRCA method.

Another comparative analysis is performed from the consequences produced by changing the weights of the risk factors. Here, the number of times a method is able to retain its original ranking position is examined, and the results are presented in Table 5.13.

Table 5.13. Comparisons of rank retention frequency

Ranking	IT2F-DEMATEL - Fuzzy extension of MAIRCA (Pamučar et al., 2014)	IT2F-DEMATEL – fuzzy TOPSIS	Proposed framework-I	Proposed framework-II
1st critical FM	FM8: 35 times. FM12: 7 times Others: 2 times	FM8: 33 times. FM12: 8 times Others: 2 times	FM8: 35 times. FM12: 7 times Others: 3 times	FM8: 34 times. FM12: 8 times Others: 2 times
2nd critical FM	FM12: 28 times, FM8: 5 times	FM12: 25 times, FM8: 6 times	FM12: 27 times, FM8: 5 times FM9: 5 times	FM12: 28 times, FM7: 6 times FM8: 5 times
3rd critical FM	FM7: 24 times FM9: 11 times	FM7: 22 times FM9: 11 times	FM9: 26 times FM7: 7 times	FM7: 20 times FM9: 14 times
4th critical FM	FM9: 21 times FM7: 9 times, Others: 14 times	FM9: 20 times FM7: 11 times, Others: 13 times	FM7: 25 times, FM9: 6 times, FM2: 5 times	FM9: 18 times, FM7: 13 times, FM2: 4 times

The ensuing observations are noted from Table 5.13:

- By employing the modified fuzzy MARCOS approach, FM7 is ranked as 3rd critical failure mode for 20 times, and FM9 is ranked as the 4th critical failure mode for 14 times. Whereas, FM9 is ranked as 4th critical for 18 times, FM7 as 4th critical for 13 times, and FM2 as 4th critical for 2 times.

- In modified fuzzy MAIRCA based approach, FM9 is ranked as the 3rd critical failure mode for 26 times, and FM7 is ranked as the 4th critical failure mode for 7 times. Similarly, for the 4th critical failure mode, FM7 retains the position for 25 times, FM9 for 6 times, and FM2 for 5 times.

Thus, it is obvious that although modified fuzzy MARCOS has greater inter-Spearman's rank correlation than modified fuzzy MAIRCA, for some critical failure modes, the earlier one has the lower rank retention capability than the latter one. However, as both the integrated approaches have the rank correlation values greater than 80%, the solutions provided by them can be acceptable. On the contrary, as the modified fuzzy MARCOS based approach has greater rank correlation coefficient value, it is suggested that the results obtained by that approach are more credible than the modified fuzzy MAIRCA based approach.

5.7. Chapter Summary

In this chapter, at first the severity of a failure mode has been decoupled from the TBL of sustainability, and then two integrated MCDM frameworks have been developed and applied for risk ranking of failure modes in a FMEA problem. With the increasing number of indices available for describing the severity, it becomes difficult for organizations to manage their exact values. In such circumstances, subjectively assessed experts' opinions are often utilized, but they themselves bear a vagueness that can lead to erroneous selection of critical failure mode(s). To overcome such eventualities, this study has used the concept of IT2FSs and integrated it with DEMATEL method for depicting the causal dependencies among the risk factors, as well as to calculate their weight values. Further, previously developed modified fuzzy MAIRCA and modified fuzzy MARCOS methods are separately utilized to rank the failure modes. The proposed frameworks have been implemented for the considered case study of the process plant gearbox.

During the sensitivity analyses, it has been observed that although the fuzzy MARCOS provides better rank correlation than the fuzzy MAIRCA, the earlier one has poor rank retention capability. This contradictory situation initiates the necessitates for further investigations, possibly by modelling the linguistic uncertainties in more abstract way and computing the weights of the risk factors in terms of IT2FNs. Furthermore, another generated problem is that different fuzzy MCDM methods produce unlike ranking results, which may perplex the decision makers. Thus, focus must be given to compute an aggregated ranking results with a certain percentage of reliability. These investigations are carried out in the next chapter.

Chapter 6 An Integrated IT2FSs-based MCDM Framework for Calculating the Ensemble Risk Ranking Results in FMECA

6.1. Introduction

The outcomes of the *Chapter 5* revealed the following points:

- IT2F-DEMATEL has been utilized to compute the crisp weights of the risk factors, which have further been adopted in modified fuzzy MAIRCA and fuzzy MARCOS methods for the risk ranking of the failure modes. However, these crisp values can be the cause of information loss at the very beginning stage of the decision-making process, which further impacts on the ranking results. Thus, it is better to calculate these weights in terms of IT2FNs.

- Although modified fuzzy MARCOS has the higher inter-Spearman's rank correlation coefficient value than the modified fuzzy MAIRCA, the rank retention capability of this approach is poorer than the latter one. Thus, it is felt to further improve the linguistic uncertainty modelling approach.

- Different MCDM methods compute dissimilar risk ranking results of failure modes, as each of them follows different mathematical treatment. However, these dissimilar results generally confuse a researcher and/or the decision-makers, if two or more methods are applied for a given problem.

Based on the above points, this chapter extends the work presented in the *Chapter 5* and an attempt is made to lessen the dilemma or confusion of the researchers. More specifically, the contributions of this chapter are[13]:

a) Modelling the linguistic uncertainties more accurately by using the concept of IT2FSs throughout this chapter. The motivations for employing IT2FSs have been described in *Chapter 2 / Section 2.5.1*.

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[13] The contribution of this work can be found in the below published paper:

a) Boral, S., Chaturvedi, S.K., Howard, I., Naikan, V.N.A., McKee, K., 2021. An integrated interval type-2 fuzzy sets and multiplicative half quadratic programming-based MCDM framework for calculating aggregated risk ranking results of failure modes in FMECA. *Process Safety and Environmental Protection* 150, 194-222.

b) The mathematical model of extended IT2F-DEMATEL method is proposed to compute the weights of the risk factors in terms of IT2FNs.

c) For the risk prioritization of failure modes, the concepts of IT2F-MAIRCA, and IT2F-MARCOS are proposed. Further, for the same purpose, a modified IT2F-TOPSIS method is developed.

d) After noticing that each of the IT2F-based MCDM method (*viz.*, IT2F-MAIRCA, IT2F-MARCOS, and IT2F-TOPSIS) produces dissimilar ranking results for the same case study (*viz.*, FMECA of the process plant gearbox), multiplicative HQ programming approach is adopted to compute the ensemble risk ranking results of the failure modes. The results are supplemented with a consensus index (CI) and trust level (TL) to aid the decision makers in making a confident decision.

e) By means of sensitivity analyses, the stability and rank retention capabilities of each developed method is compared and highlighted.

6.2. Proposed Framework

The essential definitions and arithmetic operations of T2FS and/or IT2FSs can be referred to *Chapter 5 / Section 5.2*. This section details the proposed framework and its associated steps for the risk ranking of failure modes. The workflow diagram of the proposed framework is depicted in Figure 6.1. The framework is divided into five blocks as described in the below sub-sections:

6.2.1. Block I: Organizing the Problem

Here, the goal of the study is set out, which is obviously the risk ranking of the failure modes in FMEA. Additionally, participating cross-functional experts are chosen, who further identify the relevant alternatives (*viz.*, failure modes), and the criteria (*viz.*, risk factors). The linguistic terms along with their corresponding TrIT2FNs values are identified and/or selected, to compute the criteria weights (*e.g.*, refer Table 5.1), and finally rank the alternatives (*e.g.*, refer Table 6.1).

Table 6.1. Linguistic variables for rating of alternatives and their corresponding TrIT2FNs

Linguistic terms	Trapezoidal interval type-2 fuzzy number
Very poor (VP)	((0,0,0,0.1;1,1),(0.050,0,0,0.050,0.9,0.9))
Poor (P)	((0,0.1,0.1,0.3;1,1),(0.05,0.1,0.1,0.25;0.9,0.9))
Medium poor (MP)	((0.1,0.3,0.3,0.5;1,1),(0.15,0.3,0.3,0.45;0.9,0.9))

Fair (F)	((0.3,0.5,0.5,0.7;1,1),(0.35,0.5,0.5,0.65;0.9,0.9))
Medium good (MG)	((0.5,0.7,0.7,0.9;1,1),(0.55,0.7,0.7,0.85;0.9,0.9))
Good (G)	((0.7,0.9,0.9,1;1,1),(0.75,0.9,0.9,0.95;0.9,0.9))
Very good (VG)	((0.9,1,1,1;1,1),(0.95,1,1,0.95;0.9,0.9))

6.2.2. Block II: Extended Interval Type-2 Fuzzy DEMATEL Method

The initial steps to calculate the IT2FNs-based weights of the risk factors are the same as mentioned in *Chapter 5 / Section 5.3*, (*i.e.*, *Step 1-5*). The additional steps are shown below:

Step 6: Represent the elements of the matrices $\tilde{\tilde{D}}_i \oplus \tilde{\tilde{R}}_i$ and $\tilde{\tilde{D}}_i \ominus \tilde{\tilde{R}}_i$ by (6.1).

$$\tilde{\tilde{D}}_i \oplus \tilde{\tilde{R}}_i = [\tilde{\tilde{u}}_{i1}]_{m\times 1}, \text{ and } \tilde{\tilde{D}}_i \ominus \tilde{\tilde{R}}_i = \left[\tilde{\tilde{v}}_{i1}\right]_{m\times 1} \tag{6.1}$$

where,

$$\tilde{\tilde{u}}_{i1} = \left(\tilde{u}_{i1}^U, \tilde{u}_{i1}^L\right) = \begin{pmatrix} \left(a_{i1}^{D+R}, b_{i1}^{D+R}, c_{i1}^{D+R}, d_{i1}^{D+R}; H_1\left(\tilde{u}_{i1}^U\right), H_2\left(\tilde{u}_{i1}^U\right)\right), \\ \left(e_{i1}^{D+R}, f_{i1}^{D+R}, g_{i1}^{D+R}, h_{i1}^{D+R}; H_1\left(\tilde{u}_{i1}^L\right), H_2\left(\tilde{u}_{i1}^L\right)\right) \end{pmatrix}, \text{ and}$$

$$\tilde{\tilde{v}}_{i1} = \left(\tilde{v}_{i1}^U, \tilde{v}_{i1}^L\right) = \begin{pmatrix} \left(a_{i1}^{D-R}, b_{i1}^{D-R}, c_{i1}^{D-R}, d_{i1}^{D-R}; H_1\left(\tilde{v}_{i1}^U\right), H_2\left(\tilde{v}_{i1}^U\right)\right), \\ \left(e_{i1}^{D-R}, f_{i1}^{D-R}, g_{i1}^{D-R}, h_{i1}^{D-R}; H_1\left(\tilde{v}_{i1}^L\right), H_2\left(\tilde{v}_{i1}^L\right)\right) \end{pmatrix}$$ [14]

Generate the combination matrix ($\tilde{\tilde{O}}$) as in (6.2).

$$\tilde{\tilde{O}} = [\tilde{\tilde{o}}_{i1}]_{m\times 1} = \begin{bmatrix} \sqrt{(\tilde{\tilde{u}}_{11}\otimes\tilde{\tilde{u}}_{11})\oplus(\tilde{\tilde{v}}_{11}\otimes\tilde{\tilde{v}}_{11})} \\ \sqrt{(\tilde{\tilde{u}}_{21}\otimes\tilde{\tilde{u}}_{21})\oplus(\tilde{\tilde{v}}_{21}\otimes\tilde{\tilde{v}}_{21})} \\ \vdots \\ \sqrt{(\tilde{\tilde{u}}_{m1}\otimes\tilde{\tilde{u}}_{m1})\oplus(\tilde{\tilde{v}}_{m1}\otimes\tilde{\tilde{v}}_{m1})} \end{bmatrix} \tag{6.2}$$

where, $\tilde{\tilde{o}}_{i1} = \left(\tilde{o}_{i1}^U, \tilde{o}_{i1}^L\right) = \begin{pmatrix} \left(a_{i1}^{o}, b_{i1}^{o}, c_{i1}^{o}, d_{i1}^{o}; H_1\left(\tilde{o}_{i1}^U\right), H_2\left(\tilde{o}_{i1}^U\right)\right), \\ \left(e_{i1}^{o}, f_{i1}^{o}, g_{i1}^{o}, h_{i1}^{o}; H_1\left(\tilde{o}_{i1}^L\right), H_2\left(\tilde{o}_{i1}^L\right)\right) \end{pmatrix}$.

[14] The superscripts D + R and D − R used above are for notation purposes only.

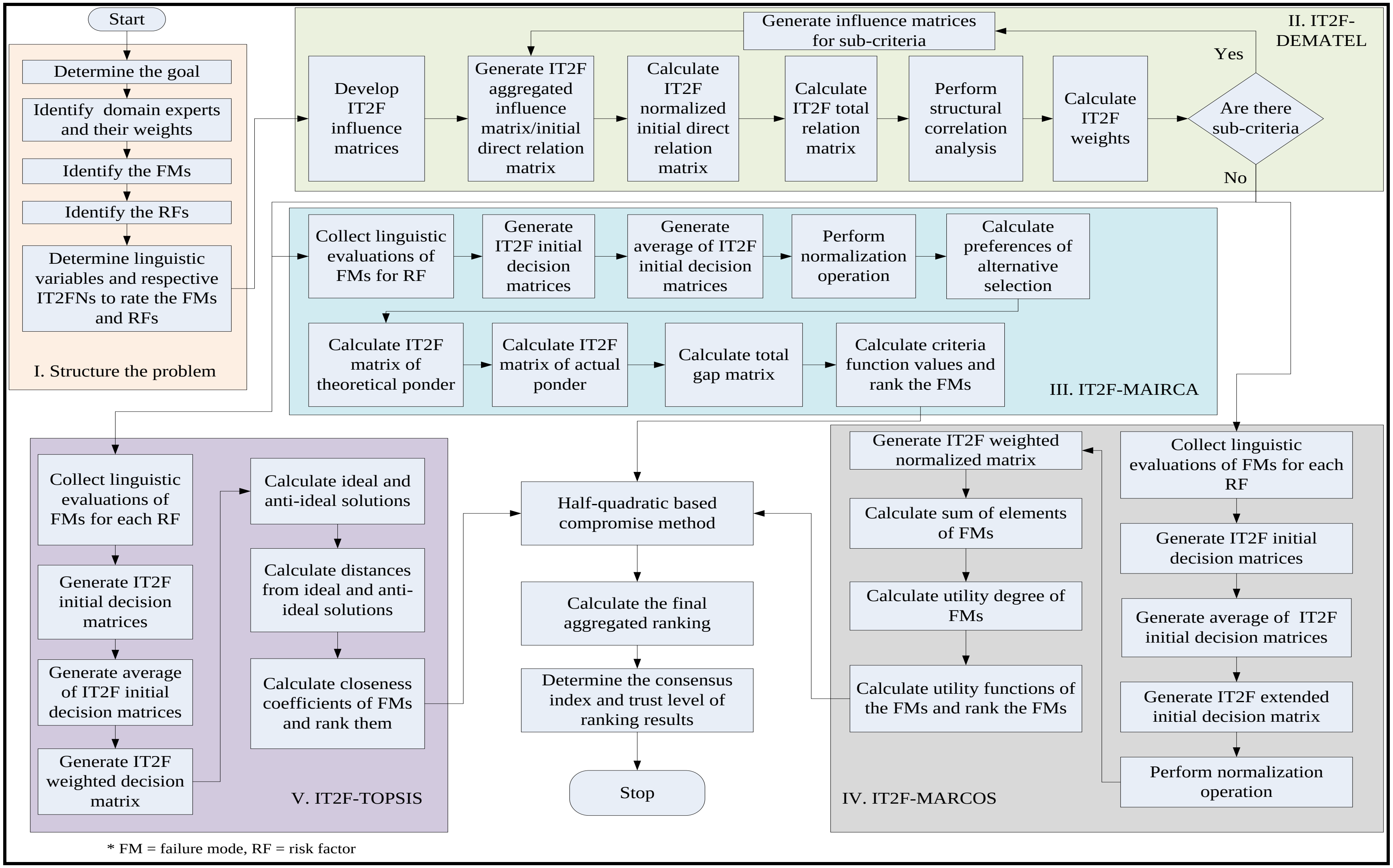

Figure 6.1. Workflow diagram of the proposed integrated framework

Then, perform the normalization operation of $\tilde{\tilde{O}}$ to compute the IT2FNs-based weights of the risk factors as given in (6.3)[15].

$$\tilde{\tilde{W}} = \left[\tilde{\tilde{w}}_{i1}\right]_{m\times 1} \tag{6.3}$$

where,

$$\tilde{\tilde{w}}_{i1} = \begin{pmatrix} \left(\frac{a_{i1}^{o}}{\sum_{i=1}^{m} d_{i1}^{o}}, \frac{b_{i1}^{o}}{\sum_{i=1}^{m} c_{i1}^{o}}, \frac{c_{i1}^{o}}{\sum_{i=1}^{m} b_{i1}^{o}}, \frac{d_{i1}^{o}}{\sum_{i=1}^{m} a_{i1}^{o}}; H_1\left(\tilde{o}_{i1}^{U}\right), H_2\left(\tilde{o}_{i1}^{U}\right)\right), \\ \left(\frac{e_{i1}^{o}}{\sum_{i=1}^{m} h_{i1}^{o}}, \frac{f_{i1}^{o}}{\sum_{i=1}^{m} g_{i1}^{o}}, \frac{g_{i1}^{o}}{\sum_{i=1}^{m} f_{i1}^{o}}, \frac{h_{i1}^{o}}{\sum_{i=1}^{m} e_{i1}^{o}}; H_1\left(\tilde{o}_{i1}^{L}\right), H_2\left(\tilde{o}_{i1}^{L}\right)\right) \end{pmatrix}.$$

In case of clustered representation of the risk factors, at first the local weights to them and their sub-risk factors are calculated. The local weights of the sub-risk factors are multiplied thereafter with the local weight of the main factor to determine the global weights of the sub-risk factors.

6.2.3. Block III: Risk Ranking of Failure Modes by the Proposed IT2F-MAIRCA Method

This section presents the mathematical steps associated with the development of the IT2F-MAIRCA method.

Assume that a set of n failure modes $[X = \{X_1, X_2, \dots, X_n\}]$ are to be prioritized with respect to m numbers of risk factors $[C = \{C_1, C_2, \dots, C_m\}]$. If C_B denotes the set of benefit criteria, C_c denotes the set of cost criteria, then $C_B \cap C_C = \emptyset$, and $C_B \cup C_C = C$, then the major steps in IT2F-MAIRCA are as below:

Step 1: Linguistically assessed failure modes for each of the risk factor are converted into respective IT2FNs by using any standardized or customized scale (*e.g.*, for instance Table 6.1). For l experts (where $l = 1,2,\dots,k$), a total of k initial decision matrices are generated, denoted by $\tilde{\tilde{Z}}^{(1)}, \tilde{\tilde{Z}}^{(2)}, \dots, \tilde{\tilde{Z}}^{(k)}$, respectively. Also, each expert is given a weight value λ_l based on their level of expertise and job tenure.

Step 2: Calculate the IT2F-average initial decision matrix $\left(\tilde{\tilde{Z}}\right)$as in (6.4).

[15] Here also the heights of the IT2FNs are remain unchanged.

$$\tilde{\tilde{Z}} = \left(\lambda_1 \times \tilde{\tilde{Z}}^{(1)}\right) + \left(\lambda_2 \times \tilde{\tilde{Z}}^{(2)}\right) + \cdots + \left(\lambda_k \times \tilde{\tilde{Z}}^{(k)}\right) \quad (6.4)$$

The matrix $\tilde{\tilde{Z}}$ would be like ((6.5), where, $\tilde{\tilde{z}}_{ij}$ represents the IT2F rating of $i-th$ alternative, with respect to $j-th$ criteria, and $1 \leq i \leq n, 1 \leq j \leq m$. To maintain the similarity with the earlier notations, let us assume that:

$$\tilde{\tilde{z}}_{ij} = \left(\tilde{z}_{ij}^U, \tilde{z}_{ij}^L\right) = \left(\left(a_{ij}, b_{ij}, c_{ij}, d_{ij}; H_1\left(\tilde{z}_{ij}^U\right), H_2\left(\tilde{z}_{ij}^U\right)\right), \left(e_{ij}, f_{ij}, g_{ij}, h_{ij}; H_1\left(\tilde{z}_{ij}^L\right), H_2\left(\tilde{z}_{ij}^L\right)\right)\right).$$

$$\tilde{\tilde{Z}} = \left[\tilde{\tilde{z}}_{ij}\right]_{n\times m} \quad (6.5)$$

Step 3: Normalize the IT2F-average initial decision matrix. For each risk factor, calculate$\left(range_j = \left[\max_{1\leq i\leq n} d_{ij} - \min_{1\leq i\leq n} a_{ij}\right]\right)$, and $1 \leq j \leq m$. If the criterion belongs to C_B, then perform the normalization operation as in (6.6), and for C_C follow (6.7). The normalized IT2F matrix is represented as $\tilde{\tilde{N}} = \left[\tilde{\tilde{n}}_{ij}\right]_{n\times m}$.

$$\tilde{\tilde{n}}_{ij} = \left(\tilde{n}_{ij}^U, \tilde{n}_{ij}^L\right) = \left(\begin{pmatrix} \frac{a_{ij} - \min_{1\leq i\leq n} a_{ij}}{range_j}, \frac{b_{ij} - \min_{1\leq i\leq n} a_{ij}}{range_j}, \frac{c_{ij} - \min_{1\leq i\leq n} a_{ij}}{range_j}, \frac{d_{ij} - \min_{1\leq i\leq n} a_{ij}}{range_j}; \\ H_1\left(\tilde{n}_{ij}^U\right), H_2\left(\tilde{n}_{ij}^U\right) \end{pmatrix}, \begin{pmatrix} \frac{e_{ij} - \min_{1\leq i\leq n} a_{ij}}{range_j}, \frac{f_{ij} - \min_{1\leq i\leq n} a_{ij}}{range_j}, \frac{g_{ij} - \min_{1\leq i\leq n} a_{ij}}{range_j}, \frac{h_{ij} - \min_{1\leq i\leq n} a_{ij}}{range_j}; \\ H_1\left(\tilde{n}_{ij}^L\right), H_2\left(\tilde{n}_{ij}^L\right) \end{pmatrix}\right) \quad (6.6)$$

$$\tilde{\tilde{n}}_{ij} = \left(\tilde{n}_{ij}^U, \tilde{n}_{ij}^L\right) = \left(\begin{pmatrix} \frac{\max_{1\leq i\leq n} d_{ij} - a_{ij}}{range_j}, \frac{\max_{1\leq i\leq n} d_{ij} - b_{ij}}{range_j}, \frac{\max_{1\leq i\leq n} d_{ij} - c_{ij}}{range_j}, \frac{\max_{1\leq i\leq n} d_{ij} - d_{ij}}{range_j}; \\ H_1\left(\tilde{n}_{ij}^U\right), H_2\left(\tilde{n}_{ij}^U\right) \end{pmatrix}, \begin{pmatrix} \frac{\max_{1\leq i\leq n} d_{ij} - e_{ij}}{range_j}, \frac{\max_{1\leq i\leq n} d_{ij} - f_{ij}}{range_j}, \frac{\max_{1\leq i\leq n} d_{ij} - g_{ij}}{range_j}, \frac{\max_{1\leq i\leq n} d_{ij} - h_{ij}}{range_j}; \\ H_1\left(\tilde{n}_{ij}^L\right), H_2\left(\tilde{n}_{ij}^L\right) \end{pmatrix}\right) \quad (6.7)$$

Step 4: Calculate the preferences of alternative selection P_{X_i} employing (6.8)[16].

$$P_{X_i} = \frac{1}{n}; \sum_{i=1}^{n} P_{X_i} = 1 \tag{6.8}$$

Step 5: Derive the matrix of theoretical ponder $\left(\tilde{\tilde{T}}_p\right)$ as in (6.9)17

$$\tilde{\tilde{T}}_p = \begin{bmatrix} \tilde{\tilde{t}}_{p_{11}} & \tilde{\tilde{t}}_{p_{12}} & \cdots & \tilde{\tilde{t}}_{p_{1m}} \\ \tilde{\tilde{t}}_{p_{21}} & \tilde{\tilde{t}}_{p_{21}} & \cdots & \tilde{\tilde{t}}_{p_{2m}} \\ \tilde{\tilde{t}}_{p_{31}} & \tilde{\tilde{t}}_{p_{32}} & \cdots & \tilde{\tilde{t}}_{p_{3m}} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{t}}_{p_{n1}} & \tilde{\tilde{t}}_{p_{n2}} & \cdots & \tilde{\tilde{t}}_{p_{nm}} \end{bmatrix} = \begin{bmatrix} P_{X_1} \times \tilde{\tilde{w}}_{11} & P_{X_1} \times \tilde{\tilde{w}}_{21} & \cdots & P_{X_1} \times \tilde{\tilde{w}}_{m1} \\ P_{X_2} \times \tilde{\tilde{w}}_{11} & P_{X_2} \times \tilde{\tilde{w}}_{21} & \cdots & P_{X_2} \times \tilde{\tilde{w}}_{m1} \\ P_{X_3} \times \tilde{\tilde{w}}_{11} & P_{X_3} \times \tilde{\tilde{w}}_{21} & \cdots & P_{X_3} \times \tilde{\tilde{w}}_{m1} \\ \vdots & \vdots & \vdots & \vdots \\ P_{X_n} \times \tilde{\tilde{w}}_{11} & P_{X_n} \times \tilde{\tilde{w}}_{21} & \cdots & P_{X_n} \times \tilde{\tilde{w}}_{m1} \end{bmatrix} \tag{6.9}$$

where,

$$\tilde{\tilde{t}}_{p_{ij}} = \left(\tilde{t}^U_{p_{ij}}, \tilde{t}^L_{p_{ij}}\right) = \begin{pmatrix} \left(a_{t_{p_{ij}}}, b_{t_{p_{ij}}}, c_{t_{p_{ij}}}, d_{t_{p_{ij}}}; H_1\left(\tilde{t}^U_{p_{ij}}\right), H_2\left(\tilde{t}^U_{p_{ij}}\right)\right), \\ \left(e_{t_{p_{ij}}}, f_{t_{p_{ij}}}, g_{t_{p_{ij}}}, h_{t_{p_{ij}}}; H_1\left(\tilde{t}^L_{p_{ij}}\right), H_2\left(\tilde{t}^L_{p_{ij}}\right)\right) \end{pmatrix}$$

Step 6: Calculate the matrix of actual ponder $\left(\tilde{\tilde{T}}_r\right)$as in (6.10).

$$\tilde{\tilde{T}}_r = \begin{bmatrix} \tilde{\tilde{t}}_{r_{11}} & \tilde{\tilde{t}}_{r_{12}} & \cdots & \tilde{\tilde{t}}_{r_{1m}} \\ \tilde{\tilde{t}}_{r_{21}} & \tilde{\tilde{t}}_{r_{21}} & \cdots & \tilde{\tilde{t}}_{r_{2m}} \\ \tilde{\tilde{t}}_{r_{31}} & \tilde{\tilde{t}}_{r_{32}} & \cdots & \tilde{\tilde{t}}_{r_{3m}} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{t}}_{r_{n1}} & \tilde{\tilde{t}}_{r_{n2}} & \cdots & \tilde{\tilde{t}}_{r_{nm}} \end{bmatrix} = \begin{bmatrix} \tilde{\tilde{t}}_{p_{11}} \otimes \tilde{\tilde{n}}_{11} & \tilde{\tilde{t}}_{p_{12}} \otimes \tilde{\tilde{n}}_{12} & \cdots & \tilde{\tilde{t}}_{p_{1m}} \otimes \tilde{\tilde{n}}_{1m} \\ \tilde{\tilde{t}}_{p_{21}} \otimes \tilde{\tilde{n}}_{21} & \tilde{\tilde{t}}_{p_{22}} \otimes \tilde{\tilde{n}}_{22} & \cdots & \tilde{\tilde{t}}_{p_{2m}} \otimes \tilde{\tilde{n}}_{2m} \\ \tilde{\tilde{t}}_{p_{31}} \otimes \tilde{\tilde{n}}_{31} & \tilde{\tilde{t}}_{p_{32}} \otimes \tilde{\tilde{n}}_{32} & \cdots & \tilde{\tilde{t}}_{p_{3m}} \otimes \tilde{\tilde{n}}_{3m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{t}}_{p_{n1}} \otimes \tilde{\tilde{n}}_{n1} & \tilde{\tilde{t}}_{p_{n2}} \otimes \tilde{\tilde{n}}_{n2} & \cdots & \tilde{\tilde{t}}_{p_{nm}} \otimes \tilde{\tilde{n}}_{nm} \end{bmatrix} \tag{6.10}$$

where,

$$\tilde{\tilde{t}}_{r_{ij}} = \left(\tilde{t}^U_{r_{ij}}, \tilde{t}^L_{r_{ij}}\right) = \begin{pmatrix} \left(a_{t_{r_{ij}}}, b_{t_{r_{ij}}}, c_{t_{r_{ij}}}, d_{t_{r_{ij}}}; H_1\left(\tilde{t}^U_{r_{ij}}\right), H_2\left(\tilde{t}^U_{r_{ij}}\right)\right), \\ \left(e_{t_{r_{ij}}}, f_{t_{r_{ij}}}, g_{t_{r_{ij}}}, h_{t_{r_{ij}}}; H_1\left(\tilde{t}^L_{r_{ij}}\right), H_2\left(\tilde{t}^L_{r_{ij}}\right)\right) \end{pmatrix}$$

[16] It basically enumerates that the decision-maker is unbiased towards selection of a failure mode and each of the failure mode has the equal probability of selection as the most critical one.
[17] The IT2FNs-based criteria weights are adopted from the output of IT2F-DEMATEL method.

Step 7: Calculate the total gap matrix (G) by de-fuzzifying, and later on subtracting the $\tilde{\tilde{T}}_p$[18] and $\tilde{\tilde{T}}_r$[19] matrices (refer (6.11)). The ranking-based defuzzification method is reproduced in (6.12)[20] (Kahraman *et al.*, 2014).

$$t_{r_{ij}} = \frac{\left[\frac{\left(d_{t_{r_{ij}}} - a_{t_{r_{ij}}}\right) + \left(H_1\left(\tilde{t}^U_{r_{ij}}\right) \times b_{t_{r_{ij}}} - a_{t_{r_{ij}}}\right) + \left(H_2\left(\tilde{t}^U_{r_{ij}}\right) \times c_{t_{r_{ij}}} - a_{t_{r_{ij}}}\right)}{4} + a_{t_{r_{ij}}}\right] + \left[\frac{\left(h_{t_{r_{ij}}} - e_{t_{r_{ij}}}\right) + \left(H_1\left(\tilde{t}^L_{r_{ij}}\right) \times f_{t_{r_{ij}}} - e_{t_{r_{ij}}}\right) + \left(H_2\left(\tilde{t}^L_{r_{ij}}\right) \times f_{t_{r_{ij}}} - e_{t_{r_{ij}}}\right)}{4} + e_{t_{r_{ij}}}\right]}{2} \quad (6.12)$$

$$G = \left[g_{ij}\right]_{n \times m} = \left[T_p - T_r\right] \quad (6.13)$$

Step 8: Evaluating the criteria function values $(Q_i)$for each failure mode as in (6.14).

$$Q_i = \sum_{j=1}^{m} g_{ij}\,; i = 1,2,\dots,n \quad (6.14)$$

The failure modes are prioritized according the ascending order values of criteria functions[21].

6.2.4. Block IV: Risk Ranking of Failure Modes by the Proposed IT2F-MARCOS Method

The steps associated in the IT2F-MARCOS method are described below:

Step 1: Calculate the extended IT2F initial matrix by using the IT2F average initial decision matrix (refer (6.5)), as shown in (6.15). The ideal (ID) and anti-deal (AID) solutions are computed by using (6.12), and are shown in (6.16) - (6.17)[22].

---

[18] $T_p = \left[t_{p_{ij}}\right]_{n \times m}$

[19] $T_r = \left[t_{r_{ij}}\right]_{n \times m}$

[20] The defuzzification method is explained by utilizing the notations of $\tilde{\tilde{t}}_{r_{ij}}$.

[21] It is always desirable that the gap between the theoretical ponder and actual ponder tends to zero, because the failure mode having the smallest difference is identified as the most critical one.

[22] The anti-ideal solution implies worst alternative/least critical failure mode, and ideal solution denotes the best alternative/most critical failure mode.

$$\tilde{\tilde{Z}} = \left[\tilde{\tilde{z}}_{ij}\right]_{n\times m} = \begin{array}{c} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \\ X_{AID} \\ X_{ID} \end{array} \begin{bmatrix} \tilde{\tilde{z}}_{11} & \tilde{\tilde{z}}_{12} & \cdots & \tilde{\tilde{z}}_{1m} \\ \tilde{\tilde{z}}_{21} & \tilde{\tilde{z}}_{22} & \cdots & \tilde{\tilde{z}}_{2m} \\ \tilde{\tilde{z}}_{31} & \tilde{\tilde{z}}_{32} & \cdots & \tilde{\tilde{z}}_{3m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{z}}_{n1} & \tilde{\tilde{z}}_{n2} & \cdots & \tilde{\tilde{z}}_{nm} \\ \tilde{\tilde{z}}_1^{AID} & \tilde{\tilde{z}}_2^{AID} & \cdots & \tilde{\tilde{z}}_m^{AID} \\ \tilde{\tilde{z}}_1^{ID} & \tilde{\tilde{z}}_2^{ID} & \cdots & \tilde{\tilde{z}}_m^{ID} \end{bmatrix} \tag{6.15}$$

$$\tilde{\tilde{z}}_j^{AID} = \min_{1\le i\le n}\left[\text{rank value}(z_{ij})\right], if\ j \in C_B;\ \tilde{\tilde{z}}_j^{AI} = \max_{1\le i\le n}\left[\text{rank value}(z_{ij})\right], if\ j \in C_C \tag{6.16}$$

$$\tilde{\tilde{z}}_j^{ID} = \max_{1\le i\le n}\left[\text{rank value}(z_{ij})\right], if\ j \in C_B;\ \tilde{\tilde{z}}_j^{I} = \min_{1\le i\le n}\left[\text{rank value}(z_{ij})\right], if\ j \in C_C \tag{6.17}$$

where,

$$\tilde{\tilde{z}}_{ij} = \left(\tilde{z}_{ij}^U, \tilde{z}_{ij}^L\right) = \left(\left(a_{ij}, b_{ij}, c_{ij}, d_{ij}; H_1\left(\tilde{z}_{ij}^U\right), H_2\left(\tilde{z}_{ij}^U\right)\right), \left(e_{ij}, f_{ij}, g_{ij}, h_{ij}; H_1\left(\tilde{z}_{ij}^L\right), H_2\left(\tilde{z}_{ij}^L\right)\right)\right),$$

$$\tilde{\tilde{z}}_j^{AID} = \left(\tilde{z}_j^{AID_U}, \tilde{z}_j^{AID_L}\right) =$$

$$\left(\left(a_j^{AID}, b_j^{AID}, c_j^{AID}, d_j^{AID}; H_1\left(\tilde{z}_j^{AID_U}\right), H_2\left(\tilde{z}_j^{AID_U}\right)\right), \left(e_j^{AID}, f_j^{AID}, g_j^{AID}, h_j^{AID}; H_1\left(\tilde{z}_j^{AID_L}\right), H_2\left(\tilde{z}_j^{AID_L}\right)\right)\right)$$

, and

$$\tilde{\tilde{z}}_j^{ID} = \left(\tilde{z}_j^{ID_U}, \tilde{z}_j^{ID_L}\right) =$$

$$\left(\left(a_j^{ID}, b_j^{ID}, c_j^{ID}, d_j^{ID}; H_1\left(\tilde{z}_j^{ID_U}\right), H_2\left(\tilde{z}_j^{ID_U}\right)\right), \left(e_j^{ID}, f_j^{ID}, g_j^{ID}, h_j^{ID}; H_1\left(\tilde{z}_j^{ID_L}\right), H_2\left(\tilde{z}_j^{ID_L}\right)\right)\right).$$

Step 2: Normalize the IT2F-extended initial decision matrix. Elements of the IT2F normalized matrix $\tilde{\tilde{N}} = \left[\tilde{\tilde{n}}_{ij}\right]_{n\times m}$ are obtained by employing (6.18)-(6.19).

$$\tilde{\tilde{n}}_{ij} = \begin{pmatrix} \left(\dfrac{a_{ij}}{d_j^{ID}}, \dfrac{b_{ij}}{c_j^{ID}}, \dfrac{c_{ij}}{b_j^{ID}}, \dfrac{d_{ij}}{a_j^{ID}}; min\left(H_1\left(\tilde{z}_{ij}^U\right), H_1\left(\tilde{z}_j^{ID_U}\right)\right); min\left(H_2\left(\tilde{z}_{ij}^U\right), H_2\left(\tilde{z}_j^{ID_U}\right)\right)\right), \\ \left(\dfrac{e_{ij}}{h_j^{ID}}, \dfrac{f_{ij}}{g_j^{ID}}, \dfrac{g_{ij}}{f_j^{ID}}, \dfrac{h_{ij}}{e_j^{ID}}; min\left(H_1\left(\tilde{z}_{ij}^L\right), H_1\left(\tilde{z}_j^{ID_L}\right)\right); min\left(H_2\left(\tilde{z}_{ij}^L\right), H_2\left(\tilde{z}_j^{ID_L}\right)\right)\right) \end{pmatrix} \tag{6.18}$$

$$if\ j \in C_B$$

$$\tilde{\tilde{n}}_{ij} = \left(\begin{array}{c} \left(\frac{a_j^{ID}}{d_{ij}}, \frac{b_j^{ID}}{c_{ij}}, \frac{c_j^{ID}}{b_{ij}}, \frac{d_j^{ID}}{a_{ij}}; min\left(H_1\left(\tilde{z}_{ij}^U\right), H_1\left(\tilde{z}_j^{ID_U}\right)\right), min\left(H_2\left(\tilde{z}_{ij}^U\right), H_2\left(\tilde{z}_j^{ID_U}\right)\right)\right), \\ \left(\frac{e_j^{ID}}{h_{ij}}, \frac{f_j^{ID}}{g_{ij}}, \frac{g_j^{ID}}{f_{ij}}, \frac{h_j^{ID}}{e_{ij}}; min\left(H_1\left(\tilde{z}_{ij}^L\right), H_1\left(\tilde{z}_j^{ID_L}\right)\right), min\left(H_2\left(\tilde{z}_{ij}^L\right), H_2\left(\tilde{z}_j^{ID_L}\right)\right)\right) \end{array} \right) \quad (6.19)$$

$$if\ j \in C_C$$

Step 3: Calculate the IT2F-weighted normalized decision matrix as in (6.20)[23].

$$\tilde{\tilde{R}} = \left[\tilde{\tilde{r}}_{ij}\right]_{n\times m} = \begin{array}{c} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \\ X_{AID} \\ X_{ID} \end{array} \begin{bmatrix} \tilde{\tilde{r}}_{11} & \tilde{\tilde{r}}_{12} & \dots & \tilde{\tilde{r}}_{1m} \\ \tilde{\tilde{r}}_{21} & \tilde{\tilde{r}}_{22} & \dots & \tilde{\tilde{r}}_{2m} \\ \tilde{\tilde{r}}_{31} & \tilde{\tilde{r}}_{32} & \dots & \tilde{\tilde{r}}_{3m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{r}}_{n1} & \tilde{\tilde{r}}_{n2} & \dots & \tilde{\tilde{r}}_{nm} \\ \tilde{\tilde{r}}_1^{AID} & \tilde{\tilde{r}}_2^{AID} & \dots & \tilde{\tilde{r}}_m^{AID} \\ \tilde{\tilde{r}}_1^{ID} & \tilde{\tilde{r}}_2^{ID} & \dots & \tilde{\tilde{r}}_m^{ID} \end{bmatrix}$$

$$= \begin{bmatrix} \tilde{\tilde{n}}_{11}\otimes\tilde{\tilde{w}}_{11} & \tilde{\tilde{n}}_{12}\otimes\tilde{\tilde{w}}_{21} & \dots & \tilde{\tilde{n}}_{1m}\otimes\tilde{\tilde{w}}_{m1} \\ \tilde{\tilde{n}}_{21}\otimes\tilde{\tilde{w}}_{11} & \tilde{\tilde{n}}_{22}\otimes\tilde{\tilde{w}}_{21} & \dots & \tilde{\tilde{n}}_{2m}\otimes\tilde{\tilde{w}}_{m1} \\ \tilde{\tilde{n}}_{31}\otimes\tilde{\tilde{w}}_{11} & \tilde{\tilde{n}}_{32}\otimes\tilde{\tilde{w}}_{21} & \dots & \tilde{\tilde{n}}_{3m}\otimes\tilde{\tilde{w}}_{m1} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{n}}_{n1}\otimes\tilde{\tilde{w}}_{11} & \tilde{\tilde{n}}_{n2}\otimes\tilde{\tilde{w}}_{21} & \dots & \tilde{\tilde{n}}_{nm}\otimes\tilde{\tilde{w}}_{m1} \\ \tilde{\tilde{n}}_1^{AID}\otimes\tilde{\tilde{w}}_{11} & \tilde{\tilde{n}}_2^{AID}\otimes\tilde{\tilde{w}}_{21} & \dots & \tilde{\tilde{n}}_m^{AID}\otimes\tilde{\tilde{w}}_{m1} \\ \tilde{\tilde{n}}_1^{ID}\otimes\tilde{\tilde{w}}_{11} & \tilde{\tilde{n}}_2^{ID}\otimes\tilde{\tilde{w}}_{21} & \dots & \tilde{\tilde{n}}_m^{ID}\otimes\tilde{\tilde{w}}_{m1} \end{bmatrix} \quad (6.20)$$

Step 4: Calculate the sum of elements for each alternative as in (6.21).

$$\tilde{\tilde{S}} = \left[\tilde{\tilde{s}}_i\right]_{n\times 1} = \begin{array}{c} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \\ X_{AID} \\ X_{ID} \end{array} \begin{bmatrix} \tilde{\tilde{s}}_1 \\ \tilde{\tilde{s}}_2 \\ \tilde{\tilde{s}}_3 \\ \vdots \\ \tilde{\tilde{s}}_n \\ \tilde{\tilde{s}}^{AID} \\ \tilde{\tilde{s}}^{ID} \end{bmatrix} = \begin{bmatrix} \tilde{\tilde{r}}_{11} \oplus \tilde{\tilde{r}}_{12} \oplus \dots \oplus \tilde{\tilde{r}}_{1m} \\ \tilde{\tilde{r}}_{21} \oplus \tilde{\tilde{r}}_{22} \oplus \dots \oplus \tilde{\tilde{r}}_{2m} \\ \tilde{\tilde{r}}_{31} \oplus \tilde{\tilde{r}}_{32} \oplus \dots \oplus \tilde{\tilde{r}}_{3m} \\ \vdots \\ \tilde{\tilde{r}}_{n1} \oplus \tilde{\tilde{r}}_{n2} \oplus \dots \oplus \tilde{\tilde{r}}_{nm} \\ \tilde{\tilde{r}}_1^{AID} \oplus \tilde{\tilde{r}}_2^{AID} \oplus \dots \oplus \tilde{\tilde{r}}_m^{AID} \\ \tilde{\tilde{r}}_1^{ID} \oplus \tilde{\tilde{r}}_2^{ID} \oplus \dots \oplus \tilde{\tilde{r}}_m^{ID} \end{bmatrix} \quad (6.21)$$

Step 5: Employ the defuzzification method (refer (6.12)) to the matrix shown in (6.21) to de-fuzzify each element. The utility degree of alternatives in relation to the ID and AID solution is evaluated as in (6.22)-(6.23).

[23] The IT2FNs-based weights of the risk factors as computed in IT2F-DEMATEL method are utilized here.

$$K_i^{+} = \frac{s_i}{s^{ID}} \tag{6.22}$$

$$K_i^{-} = \frac{s_i}{s^{AID}} \tag{6.23}$$

Step 6: Determine the utility function in relation to the ideal solution and anti-ideal solution by adopting (6.24) - (6.25). Then calculate the utility function of the failure modes as in (6.26), which basically represents the compromise of the failure modes with respect to ID and AID solutions.

$$f(K_i^{-}) = \frac{K_i^{+}}{K_i^{+} + K_i^{-}} \tag{6.24}$$

$$f(K_i^{+}) = \frac{K_i^{-}}{K_i^{+} + K_i^{-}} \tag{6.25}$$

$$f(K_i) = \frac{K_i^{+} + K_i^{-}}{1 + \frac{K_i^{+}}{K_i^{-}} + \frac{K_i^{-}}{K_i^{+}}} \tag{6.26}$$

Step 7: Rank the failure modes based on the descending order values of the utility function. The failure mode having the highest utility function value is identified as the most critical one.

6.2.5. Block V: Risk Ranking of Failure Modes by Modified IT2F-TOPSIS

Here, the modified IT2F-TOPSIS method is proposed, which is a modified version of the work presented in (Chen and Lee, 2010).

Step 1: Utilize the same IT2F-average initial decision matrices (refer (6.5)), to calculate the IT2F-weighted decision matrix, as in (6.27).

$$\tilde{\tilde{K}} = \left[\tilde{\tilde{k}}_{ij}\right]_{n\times m} = \begin{array}{c} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{array}\begin{bmatrix} \tilde{\tilde{k}}_{11} & \tilde{\tilde{k}}_{12} & \dots & \tilde{\tilde{k}}_{1m} \\ \tilde{\tilde{k}}_{21} & \tilde{\tilde{k}}_{22} & \dots & \tilde{\tilde{k}}_{2m} \\ \tilde{\tilde{k}}_{31} & \tilde{\tilde{k}}_{32} & \dots & \tilde{\tilde{k}}_{3m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{k}}_{n1} & \tilde{\tilde{k}}_{n2} & \dots & \tilde{\tilde{k}}_{nm} \end{bmatrix} = \begin{bmatrix} \tilde{\tilde{z}}_{11}\otimes\tilde{\tilde{w}}_{11} & \tilde{\tilde{z}}_{12}\otimes\tilde{\tilde{w}}_{21} & \dots & \tilde{\tilde{z}}_{1m}\otimes\tilde{\tilde{w}}_{m1} \\ \tilde{\tilde{z}}_{21}\otimes\tilde{\tilde{w}}_{11} & \tilde{\tilde{z}}_{22}\otimes\tilde{\tilde{w}}_{21} & \dots & \tilde{\tilde{z}}_{2m}\otimes\tilde{\tilde{w}}_{m1} \\ \tilde{\tilde{z}}_{31}\otimes\tilde{\tilde{w}}_{11} & \tilde{\tilde{z}}_{32}\otimes\tilde{\tilde{w}}_{21} & \dots & \tilde{\tilde{z}}_{3m}\otimes\tilde{\tilde{w}}_{m1} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{\tilde{z}}_{n1}\otimes\tilde{\tilde{w}}_{11} & \tilde{\tilde{z}}_{n2}\otimes\tilde{\tilde{w}}_{21} & \dots & \tilde{\tilde{z}}_{nm}\otimes\tilde{\tilde{w}}_{m1} \end{bmatrix} \tag{6.27}$$

Step 2: Calculate the ranking values of each element of the matrix $\tilde{\tilde{K}}$ using the ranking based defuzzification method, as presented in (6.12).

Step 3: Determining the ID and AID solutions, as in (6.28) – (6.29).

$$k_j^{ID} = \begin{cases} \max\limits_{1\le j\le m} \left\{rank\left(\tilde{\tilde{k}}_{ij}\right)\right\}, if\ j \in C_B \\ \min\limits_{1\le j\le m} \left\{rank\left(\tilde{\tilde{k}}_{ij}\right)\right\}, if\ j \in C_C \end{cases} \tag{6.28}$$

$$k_j^{AID} = \begin{cases} \min\limits_{1\le j\le m} \left\{rank\left(\tilde{\tilde{k}}_{ij}\right)\right\}, if\ j \in C_C \\ \max\limits_{1\le j\le m} \left\{rank\left(\tilde{\tilde{k}}_{ij}\right)\right\}, if\ j \in C_B \end{cases} \tag{6.29}$$

Step 4: Calculate the distance between each failure mode and ID solutions $d^{ID}(x_i)$ as well as AID solution $d^{AID}(x_i)$ by employing (6.30)-(6.31).

$$d^{ID}(x_i) = \sqrt{\sum_{j=1}^{m} \left(rank\left(\tilde{\tilde{k}}_{ij}\right) - k_j^{ID}\right)^2}\text{; for } 1 \le i \le n \tag{6.30}$$

$$d^{AID}(x_i) = \sqrt{\sum_{j=1}^{m} \left(rank\left(\tilde{\tilde{k}}_{ij}\right) - k_j^{AID}\right)^2}\text{; for } 1 \le i \le n \tag{6.31}$$

Step 5: Compute the degree of closeness of each failure mode with respect to ID and AID solution by adopting (6.32).

$$CC(x_i) = \frac{d^-(x_i)}{d^+(x_i) + d^-(x_i)} \tag{6.32}$$

Step 6: Sort the values of $CC(x_i)$ in descending order. The most critical failure mode has the highest value of $CC(x_i)$.

6.2.6. Final Ensemble Risk Ranking of Failure Modes

In the least square fitting regression technique, the Euclidian norm is exploited as the loss function. Although it is simple and produces a closed form solution, it is extremely sensitive to outliers and displays poor performance in noisy environments. Further, the outliers from the tails of the distribution are heavily weighted, and bad data points can make the regression result insignificant.

Recently, in (Mohammadi and Rezaei, 2020) the authors adopted the concept of HQ programming approach in MCDM theory, and proposed a compromise ensemble method to compute the ensemble ranking results of the alternatives. HQ functions belong to the family of robust estimators, more specifically M-estimators, which are not convex, but their optimum can be obtained by HQ minimization with an assured convergence. There are several approaches of M-estimator available in the literature, like Huber, Cauchy, Welsch, Tukey, etc. (Zhang, 1996). Literature also indicates that the influence of large errors linearly decreases with respect to their size in case of Huber and Cauchy functions. However, the Welsch function is free from this error and has been used extensively in diversified domains, like image processing, signal processing, *etc.* for noise reduction purpose. Apart from that, it provides the CI and TL that helps the decision makers to make a rational decision.

Assume that there are H MCDM methods (where $H = 1,2,3, \ldots, h$), and each of them are utilized to compute the ranking positions of n failure modes with respect to m risk factors for the same FMECA case study. The ranking result produced by the $h-th$ MCDM method is represented by R^h; and the final ranking is denoted by R^θ. The optimization problem to compute the final ranking results is given by (6.33).

$$\min_{R^\theta} \frac{1}{2} \sum_{h=1}^{H} g\left(||R^h - R^\theta||_2\right) \tag{6.33}$$

where, $g(.)$ denotes the HQ function of the M-estimator.

The HQ function and the minimizer function of the Welsch M-estimator are displayed in (6.34)-(6.35).

$$g(l_i) = 1 - exp\left(-\frac{l_i^2}{\sigma^2}\right) \tag{6.34}$$

$$\delta(l_i) = exp\left(-\frac{l_i^2}{\sigma^2}\right) \tag{6.35}$$

where, l_i denotes the $i-th$ element of a vector l, and $1 \leq i \leq n$.

Although (6.33) is not convex but can be solved by the HQ programming approach. Adopting the HQ multiplicative form, (6.33) can be written as (6.36).

$$\min_{R^\theta, \alpha} J\left(R^\theta, \beta\right) = \sum_{h=1}^{H} \beta_h\left(||R^h - R^\theta||_2^2\right) + \psi(\beta_h) \tag{6.36}$$

Where $\beta \in R^H$ is the HQ auxiliary variable, and $\psi(.)$ is the complex conjugate of $g(.)$. The following steps are to be iterated until the convergence is reached.

$$\beta_h = \delta\left(||R^h - R^\theta||_2\right), h = 1,2, \dots, H \tag{6.37}$$

$$R^\theta = \arg\min_{R^\theta} \sum_{h=1}^{H} \beta_h ||R^h - R^\theta||_2^2 \tag{6.38}$$

The solution of (6.37) can be obtained by using the minimizer function shown in (6.36) and the optimum for (6.38) is obtained by setting the derivative to zero. Thus, the final rankings of the alternatives are obtained, and the weights assigned to each MCDM method are shown in (6.39)-(6.40).

$$R^\theta = \sum_{h=1}^{H} w_h R^h \tag{6.39}$$

$$w_h = \frac{\beta_h}{\sum_{j=1}^{H} \beta_j} \tag{6.40}$$

where j denotes the respective column of MCDM ranking, and $w_h > 0$.

6.2.6.1. Degree of Similarity: The Consensus Index

Next, it is required to find the degree to which the MCDM methods agree upon the final ranking result[24]. The CI of a given final ranking R^θ, with respect to ranking R^h is calculated as in (6.41).

$$CI\left(R^\theta\right) = \frac{1}{nH} \sum_{i=1}^{n} \sum_{h=1}^{H} q_{ih}\,, where\ q_{ih} = \frac{\mathcal{N}_\sigma\left(R_i^\theta - R_i^h\right)}{\mathcal{N}_\sigma(0)} \tag{6.41}$$

where, $\mathcal{N}_\sigma(.)$ is the pdf of the Gaussian distribution having a mean of zero and standard deviation of σ, and $\mathcal{N}_\sigma(0)$ is used for normalizing the similarity computation. Thus, $CI\left(R^\theta\right) \in [0,1]$. If there is a complete agreement between the different ranking, then $CI\left(R^\theta\right) = 1$.

6.2.6.2. Reliability of Final Ranking Result: The Trust Level

The TL indicates the reliability of the final ranking result. If ranking produced by any MCDM method deviates significantly from other MCDM methods, it takes the lower weights and has a less

[24] It is the similarity between the individual ranking results with the final ranking result.

impact on the final ranking result. Hence, it has a lower impact on the TL. It can be computed by (6.42)[25].

$$TL(R^{\theta}) = \frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{H} w_h q_{ih} \tag{6.42}$$

6.3. Case Study: FMEA of a Gearbox

The case study of the gearbox dealt in *Chapter 4* is re-produced here to demonstrate the proposed integrated framework.

6.3.1. Block I: Structuring the Problem

- *Goal:* Risk ranking of failure modes of the gearbox.
- *Selecting the experts:* Three cross-functional experts (DE1: deputy manager, DE2: assistant manager, and DE3: operator) participated in the FMEA. The assigned weights of them are $\lambda_1 = 0.4$, $\lambda_2 = 0.35$, and $\lambda_3 = 0.25$.
- *Determining the failure modes, their cause(s), and effect(s):* Refer Table 4.2
- *Determining the pertinent risk factors:* Refer Figure 5.4.
- *Determining and/or choosing the linguistic variables and their respective TrIT2FNs*: Refer Table 5.1 and Table 6.1, respectively.

6.3.2. Block II: Results by IT2F-DEMATEL method

The linguistic evaluations made by domain experts to depict the causal dependencies among the risk factors and to calculate their IT2FNs-based weights have already been presented in Table 5.2 – Table 5.6. Utilizing the steps presented in *Section 5.3*, the expected prominence and expected relation values for the risk factors are computed (refer Table 5.8), and their causal dependencies have been depicted (refer Figure 5.5 - Figure 5.9). Using the step presented *Section 6.2.2*, the weights of the risk factors are computed as given in Table 6.2.

[25] If the weights of the MCDM methods are identical then the TL is equivalent to CI.

Table 6.2. IT2FNs-based weights of the risk factors

Risk factors	IT2F local weights	IT2F global weights
C1	((0.1902,0.4469,0.4469,0.9771;1,1), (0.2389,0.4469,0.4469,0.8098;0.9,0.9))	-
C2	-	((0.0851,0.1770,0.1770,0.4724;1,1), (0.0942,0.1770,0.1770,0.3738;0.9,0.9))
C3	-	((0.1717,0.3762,0.3762,0.7874;1,1), (0.2108,0.3762,0.3762,0.6550;0.9,0.9))
C11	((0.1519,0.3567,0.3567,0.8590;1,1), (0.1619,0.3567,0.3567,0.8043,0.9,0.9))	-
C12	((0.1477,0.3265,0.3265,0.7695;1,1), (0.1557,0.3265,0.3265,0.7212;0.9,0.9))	-
C13	((0.1148,0.3167,03167,0.7849;1,1), (0.1251,0.3167,0.3167,0.7333;0.9,0.9))	-
C111	((0.1902,0.4360,0.4360,0.9546;1,1), (0.2337,0.4360,0.4360,0.7973;0.9,0.9))	((0.0055,0.0695,0.0695,0.8012;1,1), (0.0090,0.0695,0.0695,0.5193;0.9,0.9))
C112	((0.1347,0.2637,0.2637,0.5883;1,1), (0.1538,0.2637,0.2637,0.4848;0.9,0.9))	((0.0039,0.0420,0.0420,0.4938;1,1), (0.0059,0.0420,0.0420,0.3158;0.9,0.9))
C113	((0.1262,0.3003,0.3003,0.6741;1,1), (0.1559,0.3003,0.3003,0.5583;0.9,0.9))	((0.0036,0.0479,0.0479,0.5658;1,1), (0.0060,0.0479,0.0479,0.3636;0.9,0.9))
C121	((0.1482,0.3024,0.3024,0.6976;1,1), (0.1644,0.3024,0.3024,0.5989;0.9,0.9))	((0.0042,0.0441,0.0441,0.5245;1,1), (0.0061,0.0441,0.0441,0.3497;0.9,0.9))
C122	((0.0974,0.2998,0.2998,0.7456;1,1), (0.1242,0.2998,0.2998,0.6369;0.9,0.9))	((0.0027,0.0437,0.0437,0.5606;1,1), (0.0046,0.0437,0.0437,0.3720;0.9,0.9))
C123	((0.1793,0.3978,0.3978,0.9099;1,1), (0.2063,0.3978,0.3978,0.7849;0.9,0.9))	((0.0050,0.0580,0.0580,0.6842;1,1), (0.0077,0.0580,0.0580,0.4584;0.9,0.9))
C131	((0.1511,0.3925,0.3925,0.9216;1,1), (0.1830,0.3925,0.3925,0.7893;0.9,0.9))	((0.0033,0.0556,0.0556,0.7068;1,1), (0.0055,0.0556,0.0556,0.4687;0.9,0.9))
C132	((0.1148,0.3106,0.3106,0.7414;1,1), (0.1418,0.3106,0.3106,0.6307;0.9,0.9))	((0.0025,0.0440,0.0440,0.5686;1,1), (0.0042,0.0440,0.0440,0.3745;0.9,0.9))
C133	((0.1632,0.2970,0.2970,0.6672;1,1), (0.1776,0.2970,0.2970,0.5705;0.9,0.9))	((0.0036,0.0420,0.0420,0.5117;1,1), (0.0053,0.0420,0.0420,0.3388;0.9,0.9))

Next, using these weights, failure modes are prioritized.

6.3.3. Block III: Risk Ranking of Failure Modes by IT2F-MAIRCA method

Using the same linguistic judgements (refer Table 5.7), and adopting the steps presented in *Section 6.2.3*, the total gap matrix (G) is obtained, as shown in Table 6.3. The criteria function values (Q_i), and the corresponding ranking results of the failure modes are shown in Table 6.4.

Table 6.3. Total gap matrix obtained from IT2F-MAIRCA method

FMs	C111	C112	C113	C121	C122	C123	C131	C132	C133	C2	C3
FM1	0.0019	0.0065	0.0093	0.0089	0.0055	0.0011	0.0051	0.0063	0.0058	0.0116	0.0124
FM2	0.0005	0.0072	0.0080	0.0078	0.0003	0.0024	0.0041	0.0055	0.0072	0.0110	0.0024
FM3	0.0064	0.0081	0.0061	0.0077	0.0014	0.0005	0.0077	0.0075	0.0072	0.0085	0.0056
FM4	0.0004	0.0030	0.0066	0.0046	0.0071	0.0039	0.0118	0.0014	0.0061	0.0116	0.0228
FM5	0.0118	0.0073	0.0082	0.0077	0.0007	0.0012	0.0094	0.0081	0.0063	0.0097	0.0156
FM6	0.0047	0.0050	0.0088	0.0077	0.0027	0.0022	0.0065	0.0081	0.0055	0.0055	0.0236

FMs	C111	C112	C113	C121	C122	C123	C131	C132	C133	C2	C3
FM7	0.0005	0.0030	0.0004	0.0059	0.0060	0.0101	0.0003	0.0021	0.0041	0.0133	0.0014
FM8	0.0004	0.0029	0.0017	0.0077	0.0080	0.0069	0.0087	0.0009	0.0068	0.0011	0.0007
FM9	0.0005	0.0017	0.0032	0.0077	0.0077	0.0096	0.0003	0.0004	0.0046	0.0088	0.0049
FM10	0.0013	0.0008	0.0055	0.0022	0.0083	0.0114	0.0065	0.0003	0.0046	0.0099	0.0156
FM11	0.0013	0.0012	0.0003	0.0031	0.0073	0.0050	0.0061	0.0004	0.0039	0.0131	0.0156
FM12	0.0004	0.0003	0.0082	0.0003	0.0083	0.0040	0.0004	0.0003	0.0005	0.0153	0.0014

Table 6.4. Criteria function values and ranking result obtained by IT2F-MAIRCA method

FMs	Criteria function values	Ranking result
FM1	0.07454	9
FM2	0.05633	5
FM3	0.06672	8
FM4	0.07918	10
FM5	0.08612	12
FM6	0.08033	11
FM7	0.04705	3
FM8	0.04586	2
FM9	0.04944	4
FM10	0.06651	7
FM11	0.05746	6
FM12	0.03935	1

The ranking results generated by IT2F-MAIRCA are as follows: $FM12 > FM8 > FM7 > FM9 > FM2 > FM11 > FM10 > FM3 > FM1 > FM4 > FM6 > FM5$.

6.3.4. Block IV: Risk Ranking of Failure Modes by IT2F-MARCOS method

Utilizing the steps given in *Section 6.2.4* the sum of elements matrix $\left(\tilde{\tilde{S}}\right)$, and their de-fuzzified values (s_i) are given in Table 6.5.

Table 6.5. Sum of elements matrix and their de-fuzzified values

FMs	Sum of elements $(\tilde{\tilde{s}}_i)$	De-fuzzified values (s_i)
FM1	((0.092,0.490,0.490,5.251;1,1),(0.140,0.490,0.490,3.127;0.9,0.9))	1.3093
FM2	((0.161,0.670,0.670,5.954;1,1),(0.230,0.670,0.670,3.598;0.9,0.9))	1.5612
FM3	((0.138,0.592,0.592,5.386;1,1),(0.197,0.592,0.592,3.247;0.9,0.9))	1.4019
FM4	((0.050,0.423,0.423,5.315;1,1),(0.087,0.423,0.423,3.120;0.9,0.9))	1.2722
FM5	((0.081,0.438,0.438,4.709;1,1),(0.124,0.438,0.438,2.792;0.9,0.9))	1.1713
FM6	((0.070,0.444,0.444,5.226;1,1),(0.107,0.444,0.444,3.096;0.9,0.9))	1.2731
FM7	((0.164,0.689,0.689,6.327;1,1),(0.236,0.689,0.689,3.796;0.9,0.9))	1.6426

FMs	Sum of elements ($\tilde{\tilde{s}}_i$)	De-fuzzified values (s_i)
FM8	((0.230,0.788,0.788,6.042;1,1),(0.312,0.788,0.788,3.669;0.9,0.9))	1.6561
FM9	((0.150,0.678,0.678,6.245;1,1),(0.216,0.678,0.678,3.761;0.9,0.9))	1.6186
FM10	((0.087,0.521,0.521,5.713;1,1),(0.134,0.521,0.521,3.397;0.9,0.9))	1.4136
FM11	((0.079,0.544,0.544,6.401;1,1),(0.127,0.544,0.544,3.805;0.9,0.9))	1.5598
FM12	((0.166,0.726,0.726,6.843;1,1),(0.241,0.726,0.726,4.111;0.9,0.9))	1.7649
FM^{ID}	((0.245,1,1,8.503;1,1),(0.338,1,1,5.194;0.9,0.9))	2.2601
FM^{AID}	((0.026,0.2,0.2,2.757;1,1),(0.053,0.2,0.2,1.522;0.9,0.9))	0.6400

The calculated utility degree of alternatives in relation to ideal and anti-ideal solution (K_i^+, K_i^-), utility function in relation to ideal and anti-ideal solution $[f(K_i^+), f(K_i^-)]$, utility function of alternatives $[f(K_i)]$, and ranking results of failure modes are highlighted in Table 6.6.

Table 6.6. Ranking results obtained by using IT2F-MARCOS method

FMs	K_i^+	K_i^-	$f(K_i^+)$,	$f(K_i^-)$	$f(K_i)$	Ranking result
FM1	0.5793	2.0458	0.7793	0.2207	0.5452	9
FM2	0.6908	2.4394	0.7793	0.2207	0.6501	5
FM3	0.6203	2.1905	0.7793	0.2207	0.5838	8
FM4	0.5629	1.9879	0.7793	0.2207	0.5298	11
FM5	0.5183	1.8303	0.7793	0.2207	0.4878	12
FM6	0.5633	1.9893	0.7793	0.2207	0.5302	10
FM7	0.7268	2.5667	0.7793	0.2207	0.6841	3
FM8	0.7328	2.5877	0.7793	0.2207	0.6897	2
FM9	0.7162	2.5291	0.7793	0.2207	0.6740	4
FM10	0.6255	2.2088	0.7793	0.2207	0.5887	7
FM11	0.6901	2.4372	0.7793	0.2207	0.6496	6
FM12	0.7809	2.7577	0.7793	0.2207	0.7350	1

The ranking results generated by the IT2F-MARCOS method are as follows: $FM12 > FM8 > FM7 > FM9 > FM2 > FM11 > FM10 > FM3 > FM1 > FM6 > FM4 > FM5$. When a comparison is made between the outputs of IT2F-MAIRCA, and IT2F-MARCOS, it is observed that ranking positions of $FM4$ and $FM6$ are changed in the current approach. This necessitates further examinations, probably with a well-adopted MCDM method - modified IT2F-TOPSIS.

6.3.5. Block V: Risk Ranking of Failure Modes by IT2F-TOPSIS Method

Employing the steps elucidated in *Section 6.2.5*, the ranking values of each failure mode, along with the ideal and anti-ideal solution values for each risk factor is computed as shown in Table 6.7.

Table 6.7. Ranking values of failure modes in IT2F-TOPSIS method

FMs	C111	C112	C113	C121	C122	C123	C131	C132	C133	C2	C3
FM1	0.1776	0.0463	0.0285	0.0243	0.0734	0.1589	0.1135	0.0672	0.0521	0.0731	0.2800
FM2	0.1941	0.0389	0.0447	0.0393	0.1344	0.1539	0.1257	0.0797	0.0381	0.0956	0.3828
FM3	0.1254	0.0276	0.0670	0.0393	0.1216	0.1659	0.0821	0.0532	0.0381	0.1104	0.3496
FM4	0.1952	0.0878	0.0614	0.0760	0.0554	0.1252	0.0326	0.1239	0.0496	0.0731	0.1742
FM5	0.0636	0.0365	0.0419	0.0393	0.1296	0.1579	0.0612	0.0463	0.0470	0.0956	0.2475
FM6	0.1454	0.0633	0.0349	0.0393	0.1066	0.1453	0.0961	0.0463	0.0559	0.1465	0.1661
FM7	0.1941	0.0878	0.1359	0.0603	0.0679	0.0513	0.1707	0.1155	0.0699	0.0525	0.3924
FM8	0.1956	0.0890	0.1197	0.0393	0.0444	0.0891	0.0699	0.1293	0.0420	0.1986	0.3992
FM9	0.1941	0.1026	0.1019	0.0393	0.0486	0.0565	0.1707	0.1350	0.0649	0.1062	0.3565
FM10	0.1846	0.1131	0.0740	0.1049	0.0416	0.0350	0.0961	0.1358	0.0649	0.0935	0.2475
FM11	0.1844	0.1084	0.1361	0.0944	0.0527	0.1114	0.1013	0.1350	0.0725	0.0545	0.2475
FM12	0.1956	0.1190	0.0419	0.1284	0.0416	0.1235	0.1697	0.1364	0.1068	0.0280	0.3924
k_j^{ID}	0.1956	0.1190	0.1361	0.1284	0.1344	0.1659	0.1707	0.1364	0.1068	0.1986	0.3992
k_j^{AID}	0.0636	0.0276	0.0285	0.0243	0.0416	0.0350	0.0326	0.0463	0.0381	0.0280	0.1661

The distance between each failure mode and ideal as well as anti-ideal solutions, closeness coefficients and risk ranking of failure modes are presented in Table 6.8.

Table 6.8. Distance of each failure mode from ID and AID solutions, their closeness coefficients and ranking results

FMs	$d^{ID}(x_i)$	$d^{AID}(x_i)$	$CC(x_i)$	Ranking results
FM1	0.2699	0.2278	0.4578	9
FM2	0.2091	0.3189	0.6040	5
FM3	0.2364	0.2684	0.5317	6
FM4	0.3257	0.2032	0.3843	11
FM5	0.3138	0.1882	0.3748	12
FM6	0.3136	0.2074	0.3980	10
FM7	0.2153	0.3334	0.6075	4
FM8	0.1938	0.3527	0.6454	1
FM9	0.2022	0.3143	0.6086	3
FM10	0.2679	0.2327	0.4649	8
FM11	0.2467	0.2543	0.5076	7
FM12	0.2201	0.3570	0.6187	2

In this method, the ranking positions of the failure modes are as follow: $FM8 > FM12 > FM9 > FM7 > FM2 > FM3 > FM11 > FM10 > FM1 > FM6 > FM4 > FM5$. The IT2F-TOPSIS identifies FM8 as the most critical one, instead of FM12, which is identified as the most critical one by earlier two proposed methods.

At this stage, it is necessary to compute the ensemble risk ranking results of failure modes, as different methods produce different ranking results.

6.3.6. Ensemble Risk Ranking Results of the Failure Modes

Adopting the steps presented in *Section 6.2.6*, ensemble risk ranking results, along with *CI* and *TL* are computed, and are shown in Table 6.9.

Table 6.9. Aggregated ranking results of failure modes along with consensus index and trust level

FMs	Risk ranking by IT2F-MAIRCA	Risk ranking by IT2F-MARCOS	Risk ranking by IT2F-TOPSIS	Ensemble risk ranking by HQ programming	Consensus index	Trust level
FM1	9	9	9	9	0.819	0.980
FM2	5	5	5	5		
FM3	8	8	6	8		
FM4	10	11	11	11		
FM5	12	12	12	12		
FM6	11	10	10	10		
FM7	3	3	4	3		
FM8	2	2	1	2		
FM9	4	4	3	4		
FM10	7	7	8	7		
FM11	6	6	7	6		
FM12	1	1	2	1		

6.4. Discussions and Sensitivity Analyses

In the previous chapter, the FMEA case study was solved by using two integrated MCDM approaches: IT2F-DEMATEL-modified fuzzy MAIRCA, and IT2F-DEMATEL-modified fuzzy MARCOS. The current proposed work has fourfold differences, in contrast to the methods proposed in previous chapter, as below:

a) Although the provision of participation of multiple experts have been considered in the earlier proposed methods, but their expertise levels (*i.e.*, weights) have been ignored, which has been taken into account in the method proposed in this Chapter, *i.e.*, participations of multiple experts are considered along with their weight values.

b) Although IT2F-DEMATEL method has been adopted to compute the weight values of the risk factors, but those values are crisp in nature. Whereas, in this chapter, the modified IT2F-DEMATEL method is proposed to calculate the IT2F-weights of the risk factors.

c) Thirdly, to model the linguistic uncertainties in more abstract way, this chapter proposes the concepts of IT2F-MAIRCA, IT2F-MARCOS, and modified IT2F-TOPSIS, instead of fuzzy MAIRCA, fuzzy MARCOS, and fuzzy TOPSIS.

d) Finally, after observing that each combination of IT2FSs-based MCDM method produces different risk ranking results, the compromise ensemble method based on HQ theory is adopted to compute the final aggregated risk ranking results of the failure modes. These aggregated ranking results are supplemented with a *CI* and *TL*.

6.4.1. Sensitivity Analysis - I

To validate the robustness of the proposed integrated framework, in this sensitivity analysis different weight sets (WSs) are generated as given in Table 6.10. Here, WSs are generated by interchanging their values at the same level, while other global weights are automatically generated at subsequent levels. For example, when the weight value of severity at level 1 is changed, then global weights of risk factors are generated automatically, considering the new IT2F local weight of severity. Following this way, a total of 30 number of weight sets are generated. However, it is noteworthy that WS-1, 7, 13, 19, and 25 generate the same global weights as in Table 6.2.

Table 6.10. Generated weight sets

Weight sets	Risk factors	IT2F weights
WS-1	Severity	((0.1902,0.4469,0.4469,0.9771;1,1),(0.2389,0.4469,0.4469,0.8098;0.9,0.9))
	Occurrence	((0.0851,0.1770,0.1770,0.4724;1,1),(0.0942,0.1770,0.1770,0.3738;0.9,0.9))
	Detection	((0.1717,0.3762,0.3762,0.7874;1,1),(0.2108,0.3762,0.3762,0.6550;0.9,0.9))
⋮	⋮	⋮
	⋮	⋮
	⋮	⋮
WS-6	Severity	((0.1717,0.3762,0.3762,0.7874;1,1),(0.2108,0.3762,0.3762,0.6550;0.9,0.9))
	Occurrence	((0.0851,0.1770,0.1770,0.4724;1,1),(0.0942,0.1770,0.1770,0.3738;0.9,0.9))
	Detection	((0.1902,0.4469,0.4469,0.9771;1,1),(0.2389,0.4469,0.4469,0.8098;0.9,0.9))
WS-7	Economic severity	((0.1519,0.3567,0.3567,0.8590;1,1),(0.1619,0.3567,0.3567,0.8043;0.9,0.9))
	Social severity	((0.1477,0.3265,0.3265,0.7695;1,1),(0.1557,0.3265,0.3265,0.7212;0.9,0.9))
	Environmental severity	((0.1148,0.3167,03167,0.7849;1,1),(0.1251,0.3167,0.3167,0.7333;0.9,0.9))
⋮	⋮	⋮
	⋮	⋮
	⋮	⋮
WS-12	Economic severity	((0.1148,0.3167,03167,0.7849;1,1),(0.1251,0.3167,0.3167,0.7333;0.9,0.9))
	Social severity	((0.1477,0.3265,0.3265,0.7695;1,1),(0.1557,0.3265,0.3265,0.7212;0.9,0.9))
	Environmental severity	((0.1519,0.3567,0.3567,0.8590;1,1),(0.1619,0.3567,0.3567,0.8043,0.9,0.9))
WS-13	Cost of unreliability	((0.1902,0.4360,0.4360,0.9546;1,1), (0.2337,0.4360,0.4360,0.7973;0.9,0.9))
	Cost of quality loss	((0.1347,0.2637,0.2637,0.5883;1,1), (0.1538,0.2637,0.2637,0.4848;0.9,0.9))
	Miscellaneous cost factors	((0.1262,0.3003,0.3003,0.6741;1,1), (0.1559,0.3003,0.3003,0.5583;0.9,0.9))
⋮	⋮	⋮
	⋮	⋮
	⋮	⋮
WS-18	Cost of unreliability	((0.1262,0.3003,0.3003,0.6741;1,1), (0.1559,0.3003,0.3003,0.5583;0.9,0.9))

Weight sets	Risk factors	IT2F weights
	Cost of quality loss	((0.1347,0.2637,0.2637,0.5883;1,1), (0.1538,0.2637,0.2637,0.4848;0.9,0.9))
	Miscellaneous cost factors	((0.1902,0.4360,0.4360,0.9546;1,1), (0.2337,0.4360,0.4360,0.7973;0.9,0.9))
WS-19	Chances of accident	((0.1482,0.3024,0.3024,0.6976;1,1),(0.1644,0.3024,0.3024,0.5989;0.9,0.9))
	Overtime due to failure	((0.0974,0.2998,0.2998,0.7456;1,1),(0.1242,0.2998,0.2998,0.6369;0.9,0.9))
	Effects on workers' mind-set	((0.1793,0.3978,0.3978,0.9099;1,1),(0.2063,0.3978,0.3978,0.7849;0.9,0.9))
⋮	⋮	⋮
	⋮	⋮
	⋮	⋮
WS-24	Chances of accident	((0.1793,0.3978,0.3978,0.9099;1,1),(0.2063,0.3978,0.3978,0.7849;0.9,0.9))
	Overtime due to failure	((0.0974,0.2998,0.2998,0.7456;1,1),(0.1242,0.2998,0.2998,0.6369;0.9,0.9))
	Effects on workers' mind-set	((0.1482,0.3024,0.3024,0.6976;1,1),(0.1644,0.3024,0.3024,0.5989;0.9,0.9))
WS-25	Generation of waste material	((0.1511,0.3925,0.3925,0.9216;1,1),(0.1830,0.3925,0.3925,0.7893;0.9,0.9))
	Excess energy consumption	((0.1148,0.3106,0.3106,0.7414;1,1),(0.1418,0.3106,0.3106,0.6307;0.9,0.9))
	Miscellaneous env. factors	((0.1632,0.2970,0.2970,0.6672;1,1),(0.1776,0.2970,0.2970,0.5705;0.9,0.9))
⋮	⋮	⋮
	⋮	⋮
	⋮	⋮
WS-30	Generation of waste material	((0.1632,0.2970,0.2970,0.6672;1,1),(0.1776,0.2970,0.2970,0.5705;0.9,0.9))
	Excess energy consumption	((0.1148,0.3106,0.3106,0.7414;1,1),(0.1418,0.3106,0.3106,0.6307;0.9,0.9))
	Miscellaneous env. factors	((0.1511,0.3925,0.3925,0.9216;1,1),(0.1830,0.3925,0.3925,0.7893;0.9,0.9))

Based on the generated weight sets, the failure modes are again ranked by IT2F-MAIRCA, IT2F-MARCOS, and modified IT2F-TOPSIS methods. The variations in risk ranking results are depicted in Figure 6.2 - Figure 6.4. The variations in the compromise ensemble risk ranking results are depicted in Figure 6.5. The results are further summarized in Table 6.11 and Figure 6.6.

From Table 6.11 and Figure 6.6, the following observations can be made:

- With both IT2F-MAIRCA and IT2F-MARCOS, FM12 retains the most critical failure mode position 26 times. While in the IT2F-TOPSIS method, FM8 retains this position for all scenarios. In case of aggregated risk ranking, the results are obtained same as the IT2F-MAIRCA, and IT2F-MARCOS.

- While selecting the second critical failure mode, IT2F-MAIRCA shows greater rank stability than IT2F-MARCOS, and IT2F-TOPSIS. However, the obtained aggregated risk ranking results are like IT2F-MAIRCA.

- Apart from these two failure modes, in all other cases, IT2F-MARCOS shows greater rank stability than IT2F-MAIRCA, and IT2F-TOPSIS methods. However, the aggregated rankings are like that produced by the IT2F-MAIRCA method.

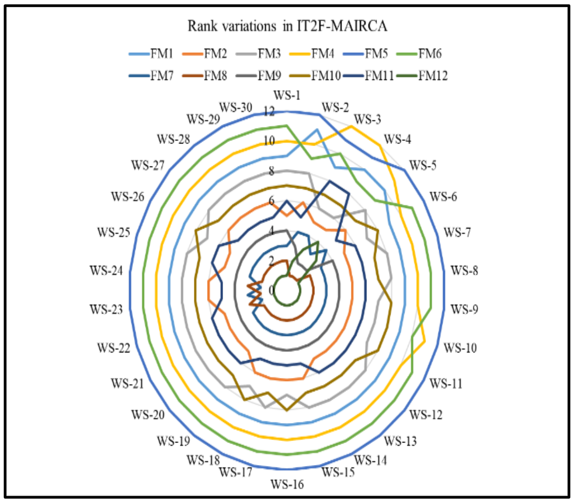

Figure 6.2. Variations in risk ranking results by IT2F-MAIRCA

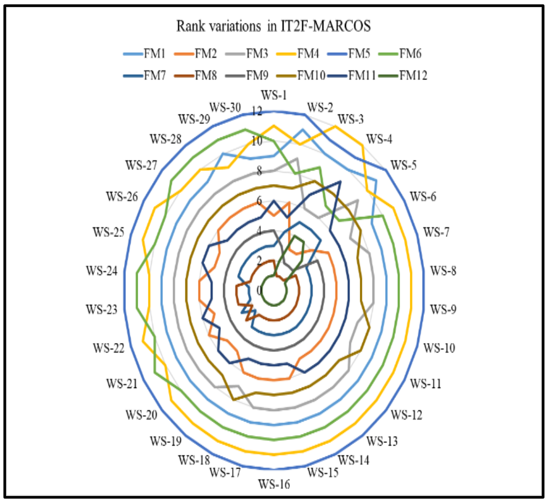

Figure 6.3. Variations in risk ranking results by IT2F-MARCOS

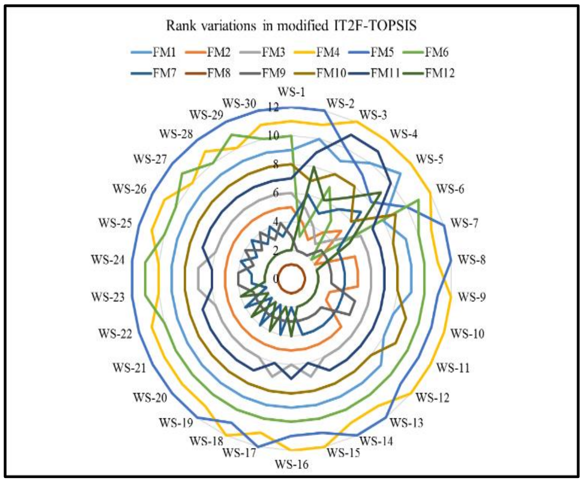

Figure 6.4. Variations in risk ranking results by IT2F-TOPSIS

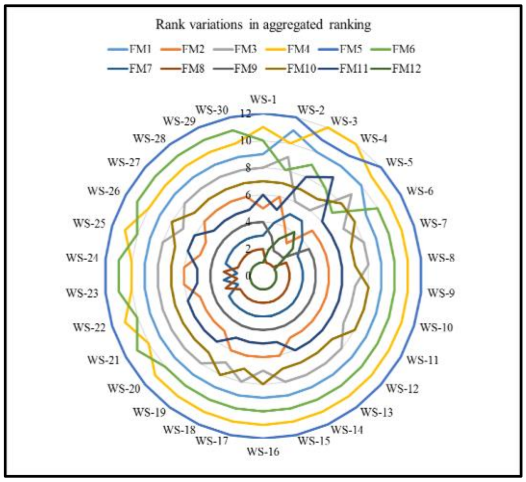

Figure 6.5. Variations in risk ranking results by HQ theory-based compromise ensemble method

Table 6.11. Number of times failure modes retain their rank

Position	IT2F-MAIRCA	IT2F-MARCOS	IT2F-TOPSIS	Aggregated ranking
1st critical	FM12: 26 times, FM4: 4 times.	FM12: 26 times, FM4: 4 times.	FM8: 30 times.	FM12: 26 times, FM4: 4 times.
2nd critical	FM8: 24 times, FM9: 3 times, FM7: 2 times, FM12: 1 time.	FM8: 22 times, FM7: 4 times, FM9: 3 times, FM12: 1 time.	FM12: 21 times, FM7: 4 times, FM9: 3 times, FM2: 1 time, FM6: 1 time.	FM8: 24 times, FM9: 3 times, FM7: 2 times, FM12: 1 time.
3rd critical	FM7: 25 times, FM8: 2 times,	FM7: 22 times, FM8: 4 times,	FM9: 20 times, FM7: 5 times,	FM7: 24 times, FM8: 2 times,

Position	IT2F-MAIRCA	IT2F-MARCOS	IT2F-TOPSIS	Aggregated ranking
	FM12: 2 times, FM9: 1 time.	FM2: 2 times, FM9: 1 time, FM12: 1 time.	FM2: 3 times, FM3: 1 time, FM6: 1 time.	FM12: 2 times, FM2: 1 time, FM9: 1 time.
4th critical	FM9: 26 times, FM7: 3 times, FM12: 1 time.	FM9: 26 times, FM12: 2 times, FM2: 1 time, FM7: 1 time.	FM7: 17 times, FM9: 5 times, FM12: 4 times, FM2: 2 times, FM3: 2 times.	FM9: 26 times, FM7: 2 times, FM2: 1 time, FM12: 1 time.
5th critical	FM2: 18 times, FM11: 12 times.	FM2: 15 times, FM11: 12 times, FM7: 3 times.	FM2: 24 times, FM9: 2 times, FM3: 1 time, FM6: 1 time, FM7: 1 time, FM12: 1 time.	FM2: 17 times, FM11: 11 times, FM7: 2 times.
6th critical	FM11: 16 times, FM2: 12 times, FM3: 2 times.	FM11: 16 times, FM2: 12 times, FM3: 2 times.	FM3: 22 times, FM11: 4 times, FM7: 2 times, FM10: 1 time, FM12: 1 time.	FM11: 17 times, FM2: 11 times, FM3: 2 times.
7th critical	FM10: 23 times, FM3: 7 times.	FM10: 23 times, FM3: 4 times, FM6: 2 times, FM11: 1 time.	FM11: 22 times, FM3: 4 times, FM6: 1 time, FM7: 1 time, FM10: 1 time, FM12: 1 time.	FM10: 22 times, FM3: 7 times, FM6: 1 time.
8th critical	FM3: 21 times, FM10: 7 times, FM11: 2 times.	FM3: 22 times, FM10: 7 times, FM6: 1 time.	FM10: 25 times, FM1: 3 times, FM5: 1 time, FM12: 1 time.	FM3: 19 times, FM10: 8 times, FM6: 2 times, FM11: 1 time.
9th critical	FM1: 27 times, FM6: 3 times.	FM1: 25 times, FM3: 2 times, FM4: 1 time, FM6: 1 time, FM11: 1 time.	FM1: 24 times, FM10: 3 times, FM5: 1 time, FM11: 1 time, FM12: 1 time.	FM1: 26 times, FM3: 2 times, FM6: 1 time, FM11: 1 time.
10th critical	FM4: 26 times, FM1: 2 times, FM6: 2 times.	FM6: 19 times, FM4: 8 times, FM1: 3 times.	FM6: 21 times, FM4: 4 times, FM1: 2 times, FM5: 2 times, FM11: 1 time.	FM6: 18 times, FM4: 9 times, FM1: 3 times.
11th critical	FM6: 25 times, FM4: 2 times, FM5: 2times, FM1: 1 time.	FM4: 19 times, FM6: 7 times, FM1: 2 times, FM5: 2 times.	FM4: 15 times, FM5: 7 times, FM6: 5 times, FM11: 2 times, FM1: 1 time.	FM4: 19 times, FM6: 8 times, FM5: 2 times, FM1: 1 time.
12th critical	FM5: 28 times, FM4: 2 times.	FM5: 28 times, FM4: 2 times.	FM5: 19 times, FM4: 11 times.	FM5: 28 times, FM4: 2 times.

- Figure 6.6 depicts the values of rank correlation coefficients obtained for different WSs. For WS-1 to WS-6, IT2F-MAIRCA shows greater rank stability than IT2F-MARCOS, IT2F-TOPSIS, and aggregated ranking results. Basically, when the weights of the risk factors are changed at level-1, drastic rank reversals are expected. This is because the changes at level-1 will affect the weights of all the sub-risk factors. When the rank correlation coefficients are calculated for all the WSs (*viz.*, WS-1 to WS-30), the stability of the ranking results shows the following order: IT2F-MAIRCA>aggregated ranking approach> IT2F-MARCOS> IT2F-TOPSIS. However, as the aim of this study is to propose aggregated risk ranking results of failure modes, and as the correlation coefficient of each method is above 80%, the stability of the final results is credible and can be accepted by the decision makers.

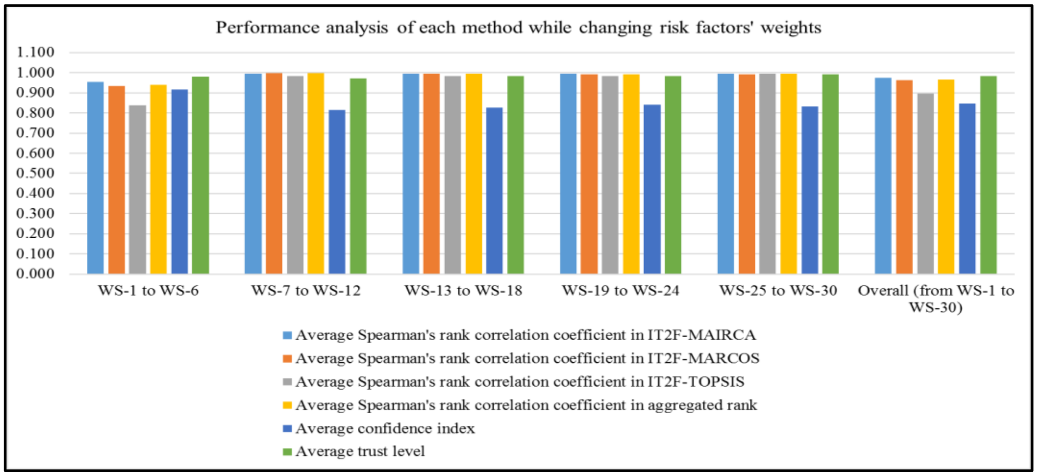

Figure 6.6. Performance analysis of each IT2F-MCDM method and aggregated risk ranking

6.4.2. Sensitivity Analysis - II

This analysis has two motivations: (a) understanding the robustness of the ranking results in uncertain conditions, (b) the analysis of performances of the IT2F-MCDM methods in the conditions of a dynamic IT2F-intial decision matrix of decision-making.

This Section discusses the effects of eliminating the failure modes from the decision matrix. For each of the IT2F-MCDM method (*viz.*, IT2F-MAIRCA, IT2F-MARCOS, and modified IT2F-TOPSIS), the alternatives are deliberately deleted according to their criticality level (both from most critical failure mode to least critical failure mode, and *vice versa*), and the effects on the final ranking results are observed.

In Figure 6.7, the rank reversals in IT2F-MARICA method are depicted, when the failure modes are eliminated from most critical to least critical order. Here, the ranks of FM12, FM8, FM7, FM9, FM11 and FM16, are steadily decreased. While for other failure modes, irregular changes are observed in the ranking results.

Similarly, it can be observed from Figure 6.8 that when the failure modes are removed from the decision matrix (least critical to most critical order), only FM10 and FM3 show abnormal variations in their ranking positions.

Figure 6.9 portrays the rank alterations resulting from the IT2F-MARCOS method, when the failure modes are deleted from the decision matrix from most critical to least critical order. It is observed that only FM9, FM10 and FM3 follow a steady decrease in their ranking positions. However, for the same situation, when IT2F-MAIRCA is contrasted with IT2F-MARCOS, the earlier one shows better robustness.

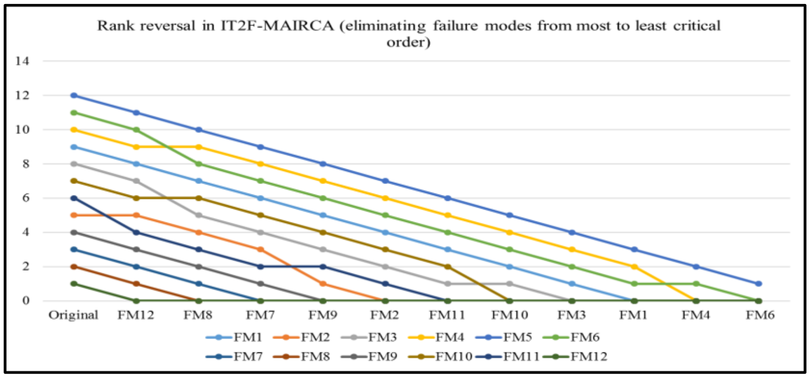

Figure 6.7. Effects on risk ranking in IT2F-MAIRCA (deleting failure modes from decision matrix – most critical to least critical order)

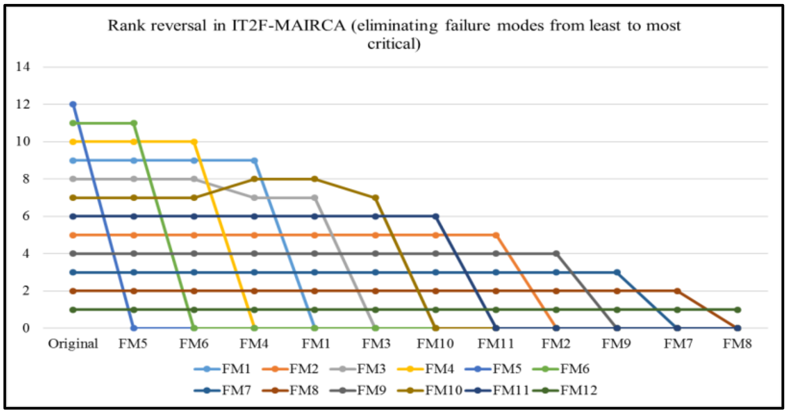

Figure 6.8. Effects on risk ranking in IT2F-MAIRCA (deleting failure modes from decision matrix – least critical to most critical order)

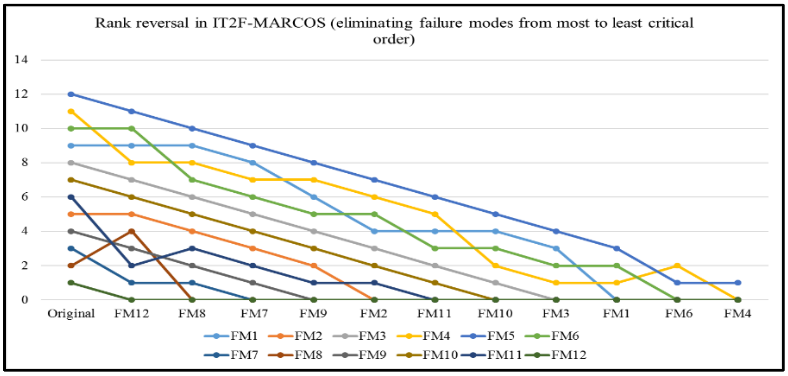

Figure 6.9. Effects on risk ranking in IT2F-MARCOS (deleting failure modes from decision matrix – most critical to least critical order)

Likewise, when the failure modes are eliminated from the decision matrix in IT2F-MARCOS (least critical to most critical sequence), abnormal variations are observed in the ranking positions of FM7 and FM8, as shown in Figure 6.10. Comparing Figure 6.8 with Figure 6.10, it seems like both IT2F-MAIRCA and IT2F-MARCOS have the same robustness. However, in IT2F-MARICA, variations are observed for FM10 (7th critical), and FM3 (8th critical); whereas, in IT2F-MARCOS, the variations are observed for FM8 (2nd critical), and FM7 (3rd critical), which is not desirable. This is because the 2nd and 3rd critical failure modes possess higher risks when compared with 7th and 8th critical failure modes.

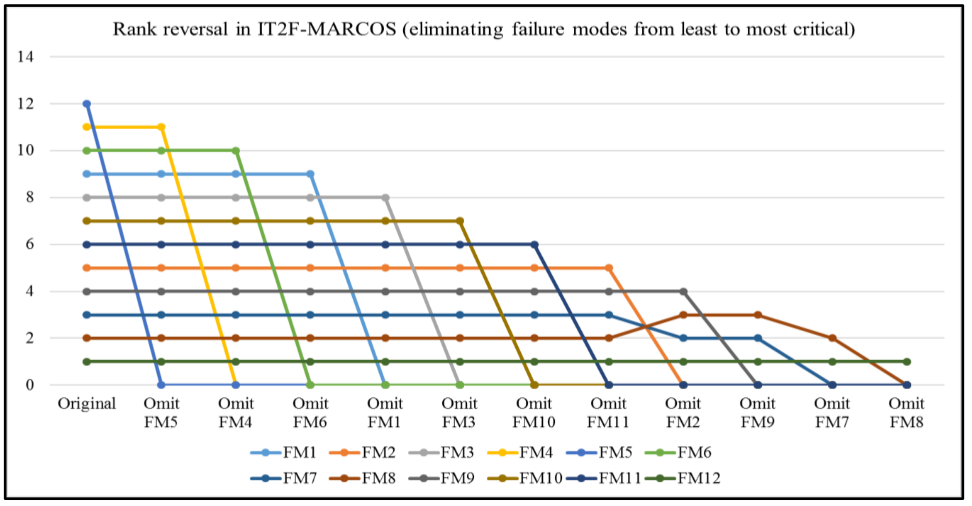

Figure 6.10. Effects on risk ranking in IT2F-MARCOS (deleting failure modes from decision matrix – least critical to most critical order)

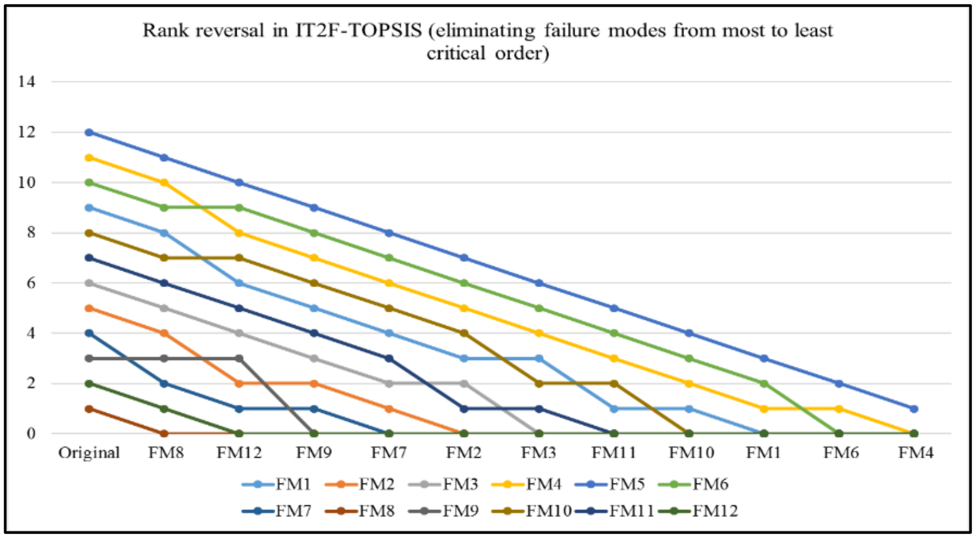

Figure 6.11. Effects on risk ranking in IT2F-TOPSIS (deleting failure modes from decision matrix – most critical to least critical order)

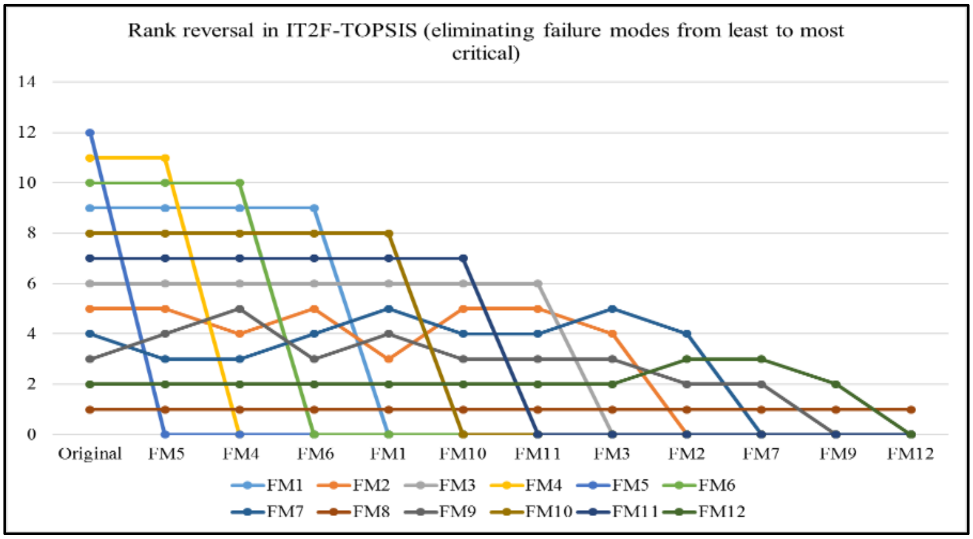

Figure 6.12. Effects on risk ranking in IT2F-TOPSIS (deleting failure modes from decision matrix – least critical to most critical order)

Figure 6.11 displays the rank variations in the IT2F-TOPSIS method, while eliminating failure modes from the decision matrix in the most critical to least critical sequence. FM8, FM12, and FM4 portray steady decrease in their ranking positions. When Figure 6.9 and Figure 6.11 are compared with each other, it can be observed that in the case of IT2F-MARCOS, some failure modes change their ranking position sharply (*e.g.*, FM8, FM14), whereas in later situations, this does not occur. Thus, it can be said that although IT2F-MARCOS has greater average rank correlation than IT2F-TOPSIS, here it shows poor rank stability.

However, as shown in Figure 6.12, when the failure modes are eliminated from the decision matrix in least critical to most critical order, FM2, FM7, FM9 and FM12 show irregular changes in the ranking position. Thus, in such a scenario, IT2F-MARCOS shows greater robustness than IT2F-TOPSIS.

6.5. Chapter Summary

Considering the potential of IT2FSs in precisely modelling the linguistic uncertainty in a decision-making problem, an integrated IT2FSs and HQ minimization-based decision-making framework has been proposed in this chapter. At first, an extended IT2F-DEMATEL method has been developed to depict the causal dependencies among the RFs as well as to calculate their weights. Then, the mathematical models of IT2F-MAIRCA, IT2F-MARCOS, and modified IT2F-TOPSIS have been proposed and adopted for the risk ranking of failure modes. After observing that each of the proposed method produces unlike ranking results, HQ minimization-based approach has been used to generate the ensemble ranking results of the failure modes along with CI and TL. Finally, sensitivity analyses of each of the developed ranking method are carried out to examine their ranking stability and robustness.

Chapter 7 A Case-Based Reasoning System for Fault Diagnosis of Gearboxes with Incomplete Information

7.1. Introduction

The challenges confronted with the fault diagnosis process of large and complex machines have been presented in *Chapter 1, Section 1.1.2*. To address these challenges, *Chapter 2, Section 2.2* has elaborated the utilizations of different AI-based methods, with the selection of the *Case-Based Reasoning* (CBR) as a viable solution. Here, the proposed approach is illustrated by taking the process plant gearbox as a case study. The gearbox details and its environmental and operating conditions, commonly observed faults, their symptoms, relevant HIs, and their correlations with the faults, and the measuring instruments have been described in *Chapter 4*. This research work carried out and presented in this Chapter complement & augments the research presented in earlier Chapters, with the following significant contributions[26]:

a) Proposition and development of a CBR-based decision-support system to automate the fault diagnosis process with minimal human interventions.

b) Considering the case of incomplete/missing information and simultaneously using the value type, event type as well as the features obtained from waveform type data during the fault diagnosis process.

c) Assisting the maintenance engineers by informing them about the necessary maintenance tasks that are needed to be carried out after diagnosing the fault.

The proposed approach explores the power of CBR which has been mentioned vaguely in earlier published research but not explored, and thus is being highlighted below:

- With the partial number of attributes, it has the potential to solve the current problem, by retrieving the past case with greatest similarity, which makes it superior to several variants of ANN. Further, in case of addition of new case in the case-base, there is no need to train the system again, when contrasted with ANN.

[26] The contributions of this chapter can be further referred to the below mentioned paper:

a) Boral, S., Chaturvedi, S.K., Naikan, V.N.A., 2019. A case-based reasoning system for fault detection and isolation: A case study on complex gearboxes. *Journal of Quality in Maintenance Engineering* 25(2), 213-235.

- It is not necessary to exactly match the numerical or textual attributes, which differentiate its capability with the principle of ES.

For the sake of completeness, the ensuing section presents a brief overview of CBR methodology.

7.2. Working Principle of CBR Methodology

The fundamental idea of the CBR methodology is to amalgamate the concepts of both AI and human cognitive process. Since its development, it has been considered as a powerful and intelligent decision-making tool, which can proficiently handle imprecise, uncertain and ill-structured decision making problems (Kolodner, 1992).

CBR consists of three basic terms, namely - *case*, *based* and *reasoning*. *Case* is a contextualized experience of some previously solved problems, which are stored in case-base. *Based* implies that the reasoning is directly specified from the prior cases, and no initiatives are taken to extract knowledge from the cases. Whereas, *reasoning* is the procedure to provide solutions to the problem in hand, by exploiting the information stored in the prior cases. Undoubtedly, it has a *memory model*, which is adopted to represent, index and organize the cases in the case-base, and a *process model* which is employed to provide reasoning to the current problem. The existing literature suggests two types of process models, namely *4-R model* (**R**etrieve, **R**euse, **R**evise and **R**etain) (Aamodt and Plaza, 1994) and *Leake's model* (Leake, 1996). However, the 4-R model is much more popular and has found a niche in the literature and is illustrated in Figure 7.1.

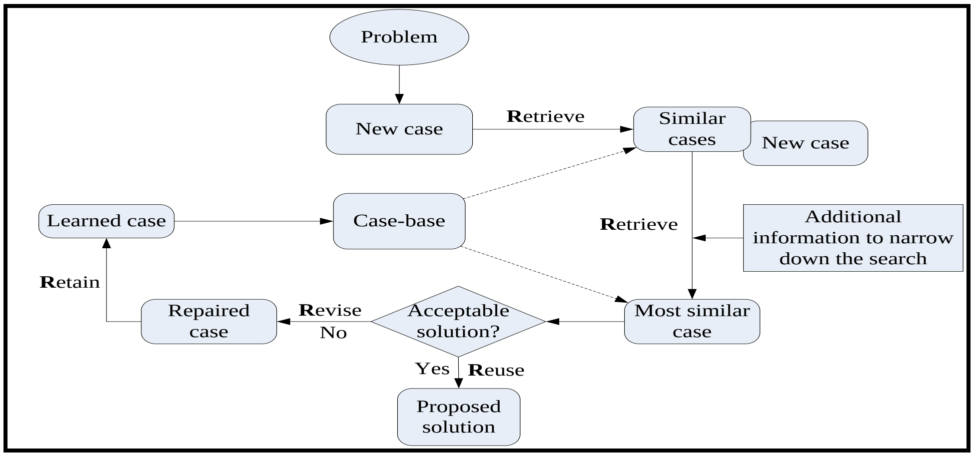

Figure 7.1. '4-R' cycle or CBR cycle

7.2.1. Retrieval of similar cases

Based on the richness of features, CBR primarily retrieves a set of similar cases where attention is directed towards the case(s) with highest similarity for solving the present problem. For instance, let a case-base have N cases, $CB = \{C_1, C_2, C_3, \dots, C_N\} | (i = 1,2,3, \dots, N)$; where $N =$ total number of cases in the case-base. Each case has some associated features and solutions, *i.e.*,$\{F_j (j = 1, 2, 3, \dots, n)\}$ and $\{S_k \, (k = 1,2,3, \dots m)\}$. An $i - th\ case$ in the case-base can be considered as a $(n + m)$ dimensional vector, and is represented as $C_i = \{x_{i1}, x_{i2}, x_{i3}, \dots, x_{in}, \theta_{i1}, \theta_{i2}, \theta_{i3}, \dots, \theta_{im}\}$. Where, x_{ij} are the values of features and θ_{ik} are the solutions associated with $i - th\ case$.

Suppose, for each feature F_j, a weight w_j has been assigned, such that $\sum_{j=1}^{n} w_j = 1$ and $w_j \in (0,1)$, to indicate the importance of $j - th$ feature, then for any pair of (stored and input) case, $C_S$and C_I, respectively, the weighted distance metric is calculated by (7.1).

$$d_{IS}^{w} = d^{w}(C_S, C_I) = \sum_{j=1}^{n} w_j \frac{abs\,(C_{Ij} - C_{Sj})}{(max_{Sj} - min_{Sj})} \tag{7.1}$$

If all features are assigned with equal weightages then d_{IS}^{1} is calculated, where d_{IS}^{1} implies that all features are equally weighted. Ultimately similarity measure in percentage is calculated as in (7.2).

$$Sim(C_S, C_I) = (1 - d_{IS}^{w}) \times 100 \tag{7.2}$$

For symbolic attributes distance is measured as in ((7.3).

$$d_{symb}(a, b) = \begin{cases} 0 \text{ [if a } = \text{ b]} \\ 1 \text{ [if a } \neq \text{ b]} \end{cases} \tag{7.3}$$

To classify an input case, the *k*-NN algorithm is adopted to search the *k*- nearest cases related to the problem in hand, and delineated to the CBR system, using some distance measuring techniques, such as Hamming distance as shown in (7.1) (Pal and Shiu, 2004).

7.2.2. Reuse, Revision and Retention of Case

After retrieving the most similar case from the case-base, it can be directly used to provide solution or phase of revision and retain occur. If the proposed solution is not suitable to the current problem, then it is revised with the advice of experts. After revision of the case, it is stored in the case-base for solving future problems.

7.3. Proposed CBR System for Fault Diagnosis of Gearbox

A systematic flow chart of the proposed decision-making framework is depicted in Figure 7.2. However, it is important and reiterated that the selection of HIs with relevant and respective symptoms, identification of operating and environmental parameters impacting the occurrence of faults and to structure them with prior fault cases (refer Figure 7.3) entails judicious thinking, judgement and experts' interference(s) before building an efficient and effective CBR system. The ensuing discussions elaborate on the proposal. After observing any abnormal symptom(s) (may be either noticeable by the operator, or by instruments/gadgets) from the system, the CBR system starts functioning.

In the proposed CBR system, case retrieval process, and consequent inferences can be carried out in two phases for operational simplicity and proper understanding of an end-user. Following this, the CBR system is developed whose stepwise operational procedures are presented hereunder:

a) In the primary selection window (*refer* Figure 7.4 later), the end-user can choose the observed symptom(s) originating from the machine. Then, the proposed system primarily retrieves a list of probable causes of faults from the case-base by employing (7.3). After that, the system guides the end-user to the final selection window. In the present context, it is noteworthy that the traditional indexing method is adopted for its simplicity of operation and ease of computation. The applied case organization technique is commonly referred to as *flat memory structure* in the literature of CBR.

b) In the final selection window (*refer* Figure 7.5 later), the end-user can choose the available HIs from the lists. Here, an option to provide a range of the selected HIs is also incorporated, considering the principle of time-varying degradation of the observed system, and possibilities of sensors and human observational errors. This proposed system takes these ranges intelligently, because it is obvious that for some HIs, maximum values are more pertinent to diagnose the fault whereas minimum values are considered for oil flow rate, oil pressure, etc.

c) The observed values of different sensors are provided as input parameters to the CBR system by the end-user. After that the system retrieves the best possible cause of fault from its case-base by using Equations (7.1) - (7.3), respectively.

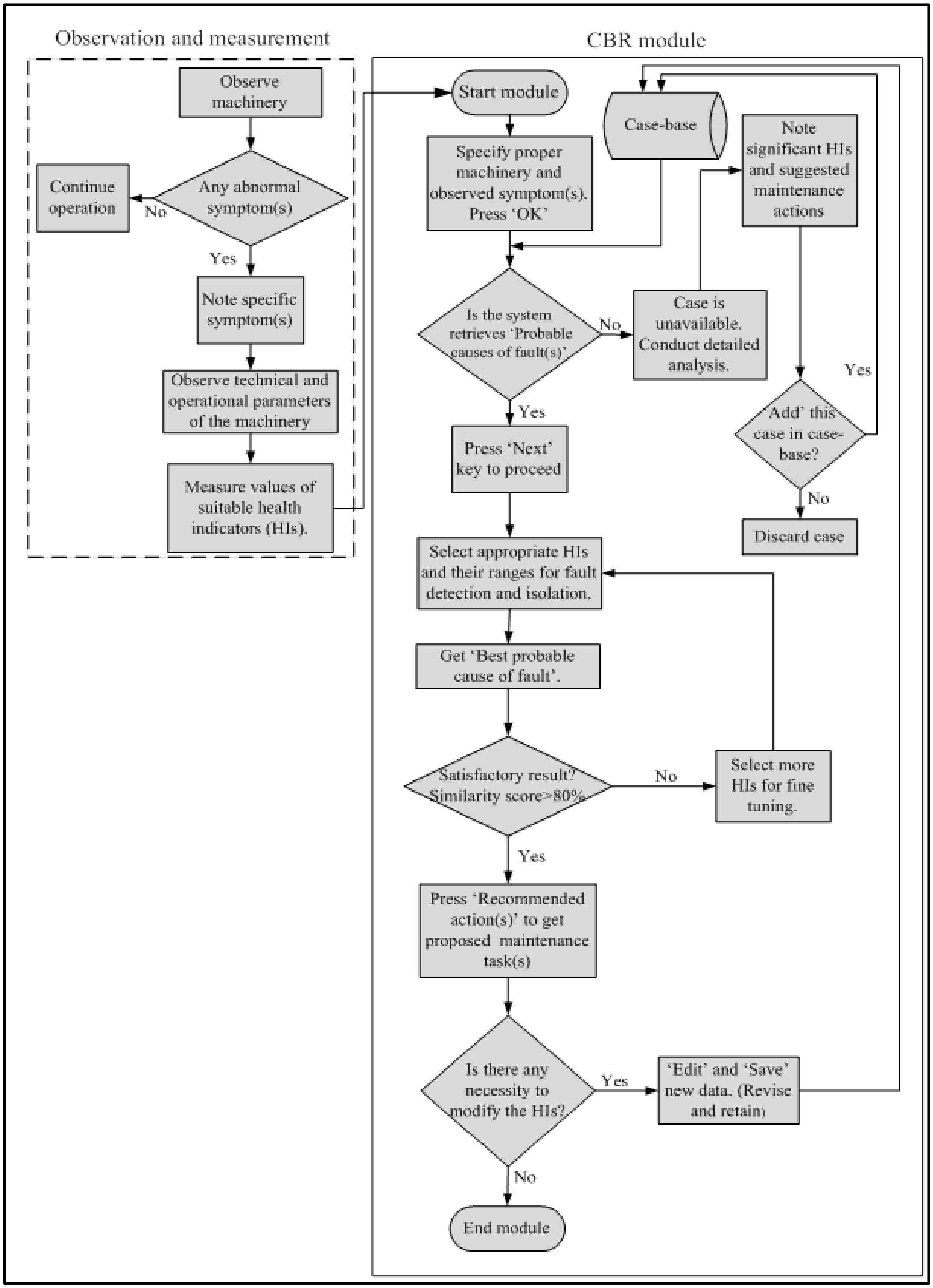

Figure 7.2. Proposed CBR-based fault diagnosis approach

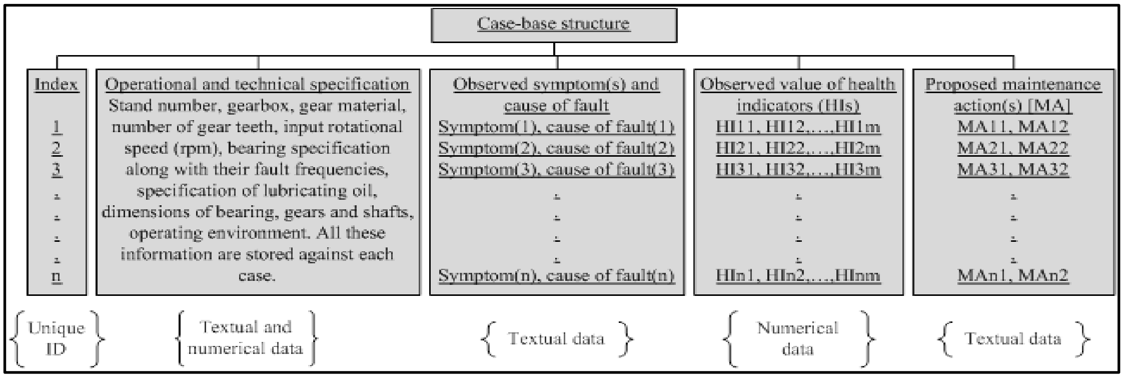

Figure 7.3. Structure of cases in the case-base

It is worth to highlight that *k*-NN is adopted in this work for classification purposes only. It is a single part of the CBR system, while other dimensions of CBR make it a promising tool while solving data intensive fault diagnosis problems.

Though all possible combinations are included in the developed CBR system, however, for some worst cases wherein in absence of previous experiences or solution with considerable similarity score, an option is also incorporated to incrementally augment those cases along with their features in the case-base in consultation with the experts for future use.

7.4. Fault Diagnosis and Suggesting the Maintenance Tasks for the Gearbox

In order to demonstrate the efficacies of the proposed framework in fault diagnosis, the case study of gearboxes is adopted, along with developing a *Graphical User Interface* (GUI) in Microsoft Visual Basic 6.0, running on a desktop with Intel® Dual Core processor, CPU G4400 @3.30 GHz, 4.00 GB RAM, and Windows® 10 as OS.

7.4.1. Steps Involved in Building the CBR System for Fault Diagnosis

Referring to the procedural flowchart as shown in Figure 7.2, the ensuing steps are required to be carried out to develop the CBR system.

Step 1: Collect the technical and operational details of system and its components. Numbers of teeth in each gear, their rotational speeds, specifications about fault frequencies of bearings, materials of components, dimensions of shafts etc. can also be included in the case-base for future use.

Step 2: Consult with the experts and process engineers, and collect the information about frequently observed faults, their symptoms (*refer* Table 4.3). For this case study, among the several HIs, the most pertinent six HIs are identified in consultation with the experts and are described below:

- *RMS value of vibration level (mm/sec) at vertical, horizontal and axial directions in time-domain:* It has been suggested by experts and technical handbooks (Taylor, 2005) that vibration can be measured in units of displacement (peak to peak movement in mm.), velocity (zero to peak in mm/sec), and acceleration (zero to peak in multiple of 'g'). Acceleration emphasizes on high frequencies, displacement on low frequencies and velocity gives equal emphasize on all frequencies. For measuring vibration by means of portable accelerometer (*range: 2Hz-20 KHz*) from the bearing housings of gearboxes, it was advised by the experts to consider the RMS of vibration in *velocity unit in time domain*. Significance of measuring vibration along each direction has been presented earlier in Table 4.4.

- *RMS value of foundation vibration (mm/sec):* As a gearbox is mounted on a foundation, this parameter is considered as a major indicator in fault diagnosis. Any anomaly found in the vibration level taken from the foundation simply implies the presence of abnormalities in the gearbox.

- *Oil flow rate (lt/min):* It is a significant indicator of presence of fault in any gearbox. Lubricating oil are used for smooth functioning of the components by preventing direct metal to metal contact as well as to dissipate the generated heat. Generally, two types of lubrication systems are used in the gearbox. Either all components are immersed to a certain level in the lubricating oil or flashed from the top of the gearbox, and directed towards different prime locations such as the contact region of gear and pinion, shaft and bearings etc. If any fault is occurred in any of its component, at first temperature of the lubricating oil is increased. However due to immense localized heat, some amount of lubricating oil is evaporated causing flow rate to decrease from the threshold level.

- *Rise in oil temperature (°C):* Any fault in the gearbox will produce some amount of heat, which in turn affects the overall rise in temperature of the lubricating oil. This temperature can be easily detected from the outlet point.

- *Rise in temperature of bearing housing (°C):* Shafts are usually mounted over bearings and when some thrust load is impinged on the gears, it is directly transmitted to the bearings.

If they are unable to withstand these loads, then some faults will be developed. If the causes are not eliminated, then a sudden rise in the temperature of bearing housing will be noticed.

- Significant *reduction in the gearbox oil pressure (in bar)* also indicates the presence of faults. Generally, at the top of each gearbox, an oil pressure measuring gauge is mounted to measure it properly.

Step 3: Along with these HIs, environmental temperature, dust level and humidity are also considered. Values of these parameters are continuously monitored and when a fault is observed, average values of these parameters can be taken from last few days' history (expert's suggested it for two-days for the current CBR system), as the average value of them is enough to highlight their impact on the occurrence of the fault.

Step 4: Collection of the past recommended maintenance actions and preventive actions carried out from ERP section of the organization.

After collecting all these details, cases are developed, and then indexed and organized in a separate schema file using MS Access (in RDBMS form) (refer Figure 7.3). Then, exploiting those cases, a user friendly and easy to operate GUI is developed to automate the decision-making process.

7.4.2. A Sample Case: Abnormal Sound and Vibration from 1st Rolling Stand

This case is taken from an occurrence history of a similar fault to confirm the capability of the CBR system (refer Figure 4.1).

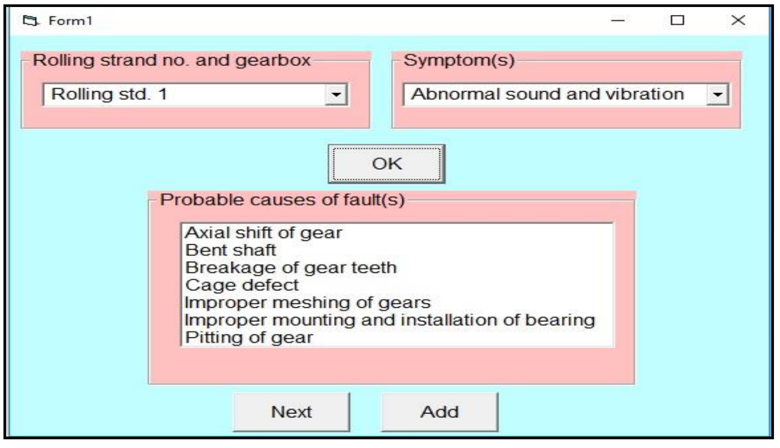

Figure 7.4. Primary input selection window of the developed CBR system

In this case study, abnormal sound and vibration are noticed by the process engineers during their regular visit at the rolling mill. When the specified symptoms along with rolling stand number is proffered to the CBR system, the CBR system provided a set of probable causes of fault as shown in Figure 7.4. When the *'Next'* functional key is pressed, the user is guided to the final selection window as shown in Figure 7.5. Here, the end-user can select the most pertinent HIs from the list-boxes to properly identify and isolate the fault.

For this example, twenty HIs are chosen (refer Figure 7.5) from the entire set of HIs considered during the development of the CBR. After providing the ranges of selected HIs in those activated text boxes shown in green colour, pressing the *'Best possible cause of fault'* key renders *'Breakage of teeth G2'* as the best retrieved past case, which is almost identical to the current problem. Along with the available HIs provided by the end-user, other unavailable HIs are also shown to make the decision properly. After isolating the fault, it is necessary to get the recommended actions at the present scenario, which is also provided when *'Recommended actions'* key is pressed. It is also necessary to mention that if the retrieved best case is not acceptable with the current problem, *'Edit'* and *'Save'* options are also provided to modify the present case (*Revise and Retain of CBR*), and to store it in the case-base for further use.

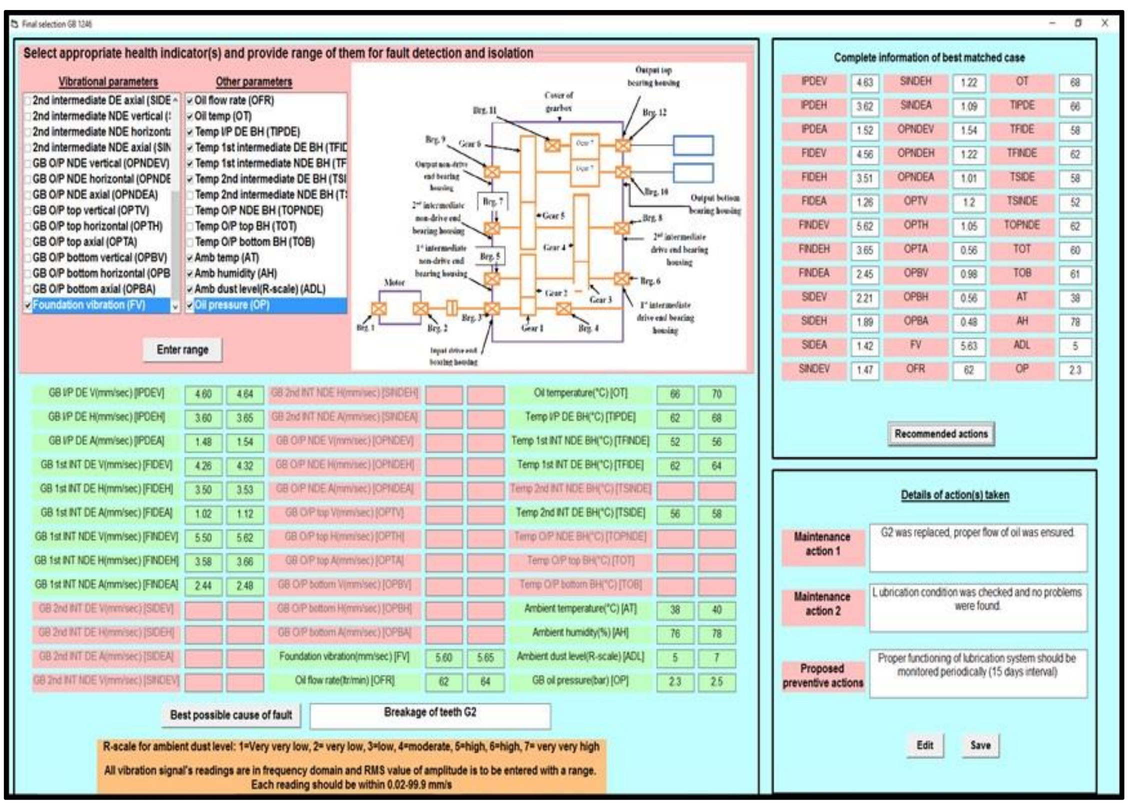

Figure 7.5. Output window for fault diagnosis and recommending the maintenance tasks

7.4.3. Discussions on Output

Decision making related to the fault diagnosis of complex gearboxes are not well understood and experts' interventions are often required to discern such problems. The presented case-study highlights the following noticeable points:

- Whenever there are vague information and scarcity of expert knowledge, previous cases with a well-documented and automated form may be utilized to solve the present problems.
- Although the system may not provide the exact matching of cases due to missing information, it can retrieve the most similar cases from the history (89.2%, refer Table 7.1).
- The values provided to the CBR system need not to belong in the exact ranges of the retrieved case. It can provide approximate solution to the current problem, that's why it is also known as *case-based approximate reasoning method* (Aamodt and Plaza, 1994).
- Additionally, it can provide results with the least amount of available knowledge with the inexperienced process engineer without knowing all the details of the monitored system.
- In the present study, the teeth of the second gear (G2) mounted on the first intermediate shaft were found to have broken resulting in a large rise in the RMS value of vibration level.
- Second gear is meshed with the first gear causing a rise in the vibration level when being monitored from the nearest bearing housing.
- Vibration parameters of other locations have somehow changed from the threshold values, monitored during healthy condition as given in Table 7.2-Table *7.3*.
- Bearing housing temperature near to G2 has also changed *e.g.*, breakage of teeth gives a shock loads to other meshing components such as bearings and shaft.

Table 7.1. Input case and best retrieved case

Name of HIs		Input range		Retrieved value
Input drive end shaft vibration [mm/sec]	Vertical	4.60	4.64	4.63
	Horizontal	3.60	3.65	3.62
	Axial	1.48	1.54	1.52
First intermediate shaft drive end vibration [mm/sec]	Vertical	4.26	4.32	4.56
	Horizontal	3.50	3.53	3.51

	Axial	1.02	1.12	1.26
First intermediate shaft non-drive end vibration [mm/sec]	Vertical	5.50	5.62	5.62
	Horizontal	3.58	3.66	3.65
	Axial	2.44	2.48	2.45
Second intermediate shaft drive end vibration [mm/sec]	Vertical	NK	NK	2.21
	Horizontal	NK	NK	1.89
	Axial	NK	NK	1.42
Second intermediate shaft non-drive end vibration [mm/sec]	Vertical	NK	NK	1.47
	Horizontal	NK	NK	1.22
	Axial	NK	NK	1.09
Output shaft non-drive end vibration [mm/sec]	Vertical	NK	NK	1.54
	Horizontal	NK	NK	1.22
	Axial	NK	NK	1.01
Output shaft drive end (top) vibration [mm/sec]	Vertical	NK	NK	1.2
	Horizontal	NK	NK	1.05
	Axial	NK	NK	0.56
Output shaft drive end (bottom) vibration [mm/sec]	Vertical	NK	NK	0.98
	Horizontal	NK	NK	0.56
	Axial	NK	NK	0.48
Foundation vibration [mm/sec]		5.60	5.65	5.63
Oil flow rate [liter/min]		62	64	62
Oil temperature [°C]		66	70	68
Temperature of bearing housing, input shaft drive end[°C]		62	68	66
Temperature of bearing housing, first intermediate shaft drive end[°C]		62	64	58
Temperature of bearing housing, first intermediate shaft non-drive end[°C]		52	56	62
Temperature of bearing housing, second intermediate shaft drive end[°C]		56	58	58
Temperature of bearing housing, second intermediate shaft non-drive end[°C]		NK	NK	52
Temperature of bearing housing, output shaft non-drive end[°C]		NK	NK	62
Temperature of bearing housing, output shaft drive end (top)[°C]		NK	NK	60
Temperature of bearing housing, output shaft drive end (bottom)[°C]		NK	NK	61
Ambient temperature[°C]		38	40	38
Ambient humidity [%]		76	78	78
Ambient dust level [R-scale]		5	7	5
Oil pressure in gearbox [bar]		2.3	2.5	2.3
Best possible cause of fault = breakage of teeth G2 [Similarity = 89.2% ≥ 80%(acceptable)]				

NK- Missing or unknown Values

Clearly, it is hard to extract an exact relationship between these HIs without knowing the details of several analysis techniques and technical details of the gearbox. However, as the aim of this study is to aid the novice process engineers for making an instant decision regarding the fault diagnosis, CBR is a more suitable methodology other than other available AI methods. Also, similarity of retrieved case is 89.2% with the previous case, which indicate that the solution can be taken into consideration. If the similarity score is decreased to a very lower level, a fine tuning of the HIs and a greater number of HIs are required to be provided for better accuracy. The computational time of the developed system is almost less than one second, which makes it to be appropriate for providing solution to current decision-making problem.

Table 7.2. Average and standard deviation of vibrations from different locations in healthy state

Location	Direction of measurement	Average and standard deviation values of amplitude (RMS value) in time domain (2Hz-20KHz) [mm/sec]	
		Average of RMS values	Standard deviation
Input shaft drive-end bearing housing	Vertical	0.398	0.114
	Horizontal	0.343	0.290
	Axial	0.318	0.083
First intermediate shaft drive end	Vertical	0.374	0.236
	Horizontal	0.276	0.145
	Axial	0.380	0.144
First intermediate shaft non-drive end	Vertical	0.393	0.180
	Horizontal	0.514	0.286
	Axial	0.368	0.103
Second intermediate shaft drive end	Vertical	0.321	0.187
	Horizontal	0.362	0.180
	Axial	0364	0.165
Second intermediate shaft non-drive end	Vertical	0.531	0.420
	Horizontal	0.570	0.249
	Axial	0.415	0.104
Output shaft non-drive end	Vertical	0.378	0.366
	Horizontal	0.456	0.285
	Axial	0.327	0.115
Output shaft (top) drive-end	Vertical	0.365	0.155
	Horizontal	0.425	0.221
	Axial	0.447	0.127
Output shaft (bottom) drive-end	Vertical	0.310	0.092
	Horizontal	0.291	0.165
	Axial	0.430	0.421
Foundation vibration	-	4.889	1.080

Table 7.3. Average and standard deviation values of other HIs in healthy condition

HIs	Average value	Standard deviation
Oil flow rate [liter/min]	60.667	1.341
Oil temperature [°C]	68.265	2.561
Input shaft drive end bearing housing temperature [°C]	52.612	3.142
First intermediate shaft drive end bearing housing temperature [°C]	56.321	2.121
First intermediate shaft non-drive end bearing housing temperature [°C]	53.362	2.625
Second intermediate shaft drive end bearing housing temperature [°C]	55.965	1.982
Second intermediate shaft non-drive end bearing housing temperature[°C]	54.263	2.125
Output shaft non-drive end bearing housing temperature [°C]	56.326	1.569
Out shaft drive end (top) bearing housing temperature [°C]	59.632	1.236
Out shaft drive end (bottom) bearing housing temperature [°C]	58.965	1.856
Oil pressure in the gearbox [bar]	2.2	0.385

7.5. Chapter Summary

In this chapter, a generalized framework for fault diagnosis of large-scale gearbox using the data fusion technique in combination with CBR methodology has been proposed. The proposed CBR system can aid engineers to easily detect and isolate the faults without in-depth knowledge of various vibration signal processing techniques and system structure. The complications of analysing complex and noisy vibration signals have been eliminated by measuring it simply in the time-domain through portable accelerometers. As noted, the time-domain analysis is still commonly performed by analysts in industries to detect several faults of gearboxes such as imbalance or tooth cracks (Rafiee *et al.*, 2007). The proposed approach has not relied just on single vibration signals but has considered other HIs along with environmental parameters for carrying out such complex decision-making task. In the proposed CBR system, hamming distance has been utilized for measuring the similarity along with the principle of *k*-NN. The system has presumed an equal weightage to each of the HIs as detailed inter-relationship among them might not be predicted at the earlier stage. After detecting and isolating the faults, the possible recommended actions such as corrective and preventive measures, have also been suggested by the system to the engineers for the ease of decision making. From the reliability point of view, the solution provided by this CBR system is totally dependent on the volume of the case-base. As the volume of case-base is increased, more accurate solutions could be rendered to the current problem(s).

Chapter 8 A Hybrid AI-Based Conceptual Decision-Making Framework for Sustainable Maintenance Strategy Selection

8.1. Introduction

Over the decades, the concept of manufacturing has been evolved from mass production to the lean manufacturing, to green manufacturing, up to the recent concept of sustainable manufacturing. Sustainable manufacturing is one of the key contributors in the sustainable development since it allows creating products through the rational use of resources and with new cleaner technologies focused to preserve the environment and ensure peoples' safety and health. Further, sustainable manufacturing involves the need to move from the linear economy to the circular economy based on reducing waste through recycle, reuse, remanufacturing, and recovering the material. In other words, the goal of integrating the sustainability concept in the traditional maintenance process is to eliminate and/or mitigate the breakdown, energy waste, and reducing the internal and external costs.

As emphasized in *Chapter 1/ Section 1.1.3*, to progress towards economic, environmental, and social developments of any organization, it is necessary that all the business processes should be sustainable, ensuring the availability and reliability of the systems' components, guaranteed safety of employees and community, and minimizing the environmental impact.

Maintenance, being a key component of the manufacturing/production process, plays a pivotal role for the uninterrupted and/or trouble-free operations. Thus, to successfully implement the sustainable manufacturing philosophy, it is required that the associated maintenance practices should also be sustainable.

Recently, Tornese *et al.* (Tornese *et al.,* 2014) proposed a framework for selecting the most favourable environmental performance measurement methodology, where maintenance was noted to be a significant contributing factor. Authors also emphasized that it should be sustainable with the development of new maintenance services in line with the circular economy and sustainable manufacturing. In a similar note, Pires *et al.* (Pires *et al.*, 2015) argued that new researches must discuss the impact of industrial maintenance on organizational sustainability and *vice-versa*. More recently, similar ideas have also been propagated in (Ejsmont *et al.*, 2020; Holgado *et al.*, 2020; Jasiulewicz-Kaczmarek *et al.,* 2020).

Based on the above and observation from the literature survey as outlined in *Chapter 2/ Section 2.3*, it is obvious that although the earlier researchers have identifies multiple types of KPIs for different maintenance practices, recent trends suggest to consider them from the *TBL* of sustainability without compromising on technical criteria. Besides, the difficulties experienced by the decision makers during the selection of optimal maintenance practice by employing different MCDM methods have also been highlighted in *Chapter 2/ Section 2.4.3*. Considering these, the novelties of the work presented in this chapter could be [27]:

a) To identify the pertinent *KPIs* of different maintenance philosophies from the TBL of sustainability, with a special highlight to the case study of process plant gearboxes (refer *Chapter 4* for more details about the gearboxes).

b) Integrating the ES (Expert System) in the model of CBR (Case Based Reasoning) in a new approach for the selection of optimal maintenance strategy. This integration helps in exploiting the benefits of both CBR and ES. However, the proposal is illustrated with a hypothetical example due to unavailability of data at the time of finishing this research.

8.2. Preliminary Ideas

A brief overview about CBR has already been presented in the previous chapter, here only the brief descriptions on sustainability with the pertinent notions of ESs are presented.

8.2.1. Sustainability

Sustainability is a complex issue, and an elusive one. It is very significant since it has to do with the chances of humankind surviving on this planet. At the rate at which the individuals/groups/nations are exploiting the scarce resources it seems that unless measures are taken now, and if there is still time, the upcoming civilization, at least as it is understandable now, is uncertain to say the least. It follows that such a complex subject has no simple and straightforward treatment, especially when one must understand that sustainability is not a goal but an endless

[27] The contributions of this chapter can be found in the below published book-chapter:

a) Boral, S., Chaturvedi, S. K., Naikan, V. N. A., & Howard, I. M. (2019). A Hybrid AI-Based Conceptual Decision-Making Model for Sustainable Maintenance Strategy Selection. *In Advanced Multi-Criteria Decision Making for Addressing Complex Sustainability Issues (Ed.)*, Chapter 4, pp. 63-93. DOI: 10.4018/978-1-5225-8579-4, IGI Global.

process. It leads to a better life for the present generation and survival for generations in years to come by enhancing their ability to cope with the world that they will inherit from the present.

The terms *sustainability* and *sustainable developments* are often used interchangeably and since 1980, it has gradually turned into an emerging area of research in R&D groups of government and industrial sectors with the theme - *"sustainable development is a kind of development that fulfils the needs of the present generation without compromising the ability of future generations to meet their needs."* This theme consists of three prime terms, *viz.*, development, present and future.

After looking at the word '*development*', one instinctively thinks about economic development, however, from the sustainable perspective, it means the advancement in every area, *viz.*, from industrial innovations to economic development knitted with ethics and societal progress, without damaging the environment of the Earth we inhabited. '*Present*' refers to acting in a structured way with a view of achieving growth not only in the economical context, but also in social and environmental contexts. The term '*future*' does not refer to the immediate future, but to the long-term future which will be inhabited by future generations.

8.2.2. Expert System

An ES differs from the conventional programs in several ways, for instance:

- It is knowledge intensive, highly interactive, and divides the experts' knowledge into separate rules.
- An ES consists of a knowledge base, a working memory, an inference engine, system analysis and graphical software and a user interface, as shown in Figure 8.1.
- The knowledge contained in the knowledge base can be either prior knowledge or posterior knowledge. This knowledge can be represented as rules, semantic nets, frames, scripts, object-oriented structures, conceptual graphs and so on.
- An inference engine examines the knowledge base and reasons the answer (how and why) to the end-user, which is also known as pruning. This task is carried out by following any of the methods, *viz.*, production rules, structured objects and predicate logic. Production rules consists of a rule set, a rule interpreter that specifies when and how to apply the rules and a working memory which holds the data, goals and intermediate results. Structured

objects use vector representation of essential and accidental properties; whereas, predicate logic uses propositional and predicate calculi.

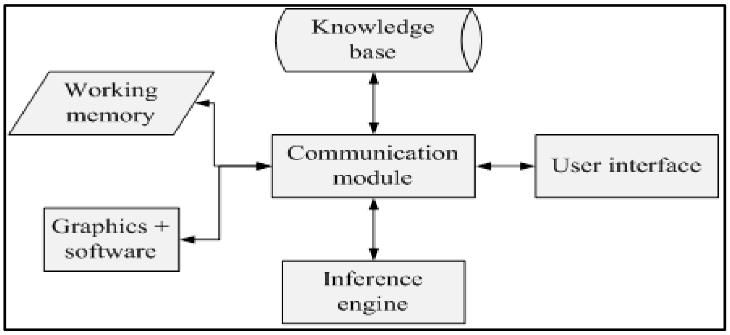

Figure 8.1. Architecture of an expert system

The process of building an ES/knowledge-based system is known as *knowledge engineering*, wherein knowledge engineers embed the knowledge of human experts in it. This embedding process is carried out by decoding the linguistic terms given by the experts into suitable programming codes, and in the absence of such experts, the ES provides decisions to a problem domain with which it has been constructed. Generally, ES provides solutions, which are derived from its knowledge base and contains declarative facts, as well as procedural (or heuristic rules) knowledge about the problem domain by using a reasoning process embedded in its inference engine, the '*thinking part*' of the system. It uses any of these three methods, *viz.*, backward chaining (top-down reasoning), forward chaining (bottom-up reasoning) or abduction as the basis of inference. At first, it looks for the 'most likely' hypothesis and then searches for the evidence for the hypothesis. If after receiving all the relevant information from the end-user, the initial hypothesis cannot be supported then it looks for the 'next most likely hypothesis' and so on (Lucas and Van Der Gaag, 1991).

Usually, to develop a Graphical User Interface-based ES in an industry, the following broad steps could be adopted:

- Define the deliverable or the outputs which are expected from an ES.
- Several interviews are conducted with the active participation of knowledge engineers.
- Store the Acquired knowledge in the knowledgebase.
- Several rules are derived by exploiting these knowledges.

- Store the problems and their associated solutions in the inference engine.

- Develop a GUI by using any graphics enabled software or alike (e.g., Microsoft Visual Basic 6.0) to exploit the knowledge base and to interact with end-user.

ES is a useful tool in a decision-making process when the following bottlenecks are observed:

- When there is a scarcity of experts' knowledge in an organization for a decision-making purpose.
- When enough prior cases are available, but they are unexploited and simply stored in the database.

In other words, the concept of ES is useful when there are enough cases available. In most of the instances, an ES deduces solutions from these stored cases and from an exhaustive 'IF-THEN' rule-base. Despite its several advantages, the major problems associated with ES are:

- Knowledge acquisition barrier, linguistic barrier, cognitive barrier, representation barrier.

- It works on the principle of direct matching.

- A rule-base model becomes unviable, when the number of rules is increased. This not only creates the problem of assessment of rules by experts, but also for many of the combinations, it becomes difficult to assess the consequent part of certain rule(s) even by the best expert available. This necessitates incorporation of an appropriate rule-reduction technique.

8.3. KPIs Considerations in Sustainable Maintenance

KPIs involved in a sustainable maintenance strategy selection problem can be initially divided into qualitative and quantitative components, which in turn have their own sub-factors, *viz.*, economic, technical, social and environmental. Table 8.1 provides a summary of the factors that may be considered for a sustainable maintenance strategy selection problem.

Table 8.1. Sustainability-based criteria, sub-criteria, and their desired nature

	Factors	Sub-factors	Desired nature
Qualitative factors	Economic	Quality of output product after maintenance	↑
		Ease of maintenance	↑
	Technical	Technical feasibility	↑
		Technical complexity	↓

		Flexibility of maintenance program	↑
	Social	Worker's safety	↑
		Acceptance by workers	↑
		Compliance with government regulations	↑
	Environment	Compliance with environmental standards	↑
		Toxicity of generated wastes	↓
Quantitative factors	Economic	Hardware cost	↓
		Spare parts cost	↓
		Software cost	↓
		Manpower cost	↓
		Training cost	↓
		Cost of production loss	↓
		Return on investment	↑
		Cost of cleaning the waste	↓
	Technical	Mean time between failures	↑
		Mean time to repair	↓
		Availability of spare machinery	↑
		Risk level of system/machinery	↓
	Social	Level of performance of employees	↑
	Environment	Amount of toxic substance emission	↓
		Amount of waste material generated	↓

[↑ = higher the better, ↓ = lower the better]

In fact, many factors, qualitative or quantitative falls under the gamut of maintainability engineering and are briefly outlined below:

8.3.1. Qualitative KPIs

These type of KPIs are either assessed linguistically or by ordinal values[28] by the domain experts. The identified candidates of KPIs in this category are grouped and presented below:

8.3.1.1. Economic criteria

- *Quality of output product after maintenance:* Maintenance of a complex system/machine is carried out by repairing, replacing or carrying even minor maintenance tasks such as cleaning or adjustment. The repair actions may be *perfect* (*viz.,* as good as new/renewed), *minimal* (*viz.,* as bad as old), and/or in-between these two extremities (Doyen and Gaudoin, 2004). Generally, after performing any type of maintenance, it is usually expected that the output of the machine should be improved from the previous state (viz., just before the maintenance). This can be measured in terms of improvement of rejection rate through some identified and indicative parameter of performance.

[28] Usually employing any well-established scale (e.g., Liker scale), where very high = 9, high =7, medium = 5, low =3, very low =1.

- *Ease of maintenance:* It is the way of maintaining a system/machine and mitigate faults and/or failures. It can be measured by considering different factors: design of the machine, condition of maintenance platform, time on the maintenance platform, ergonomics & human factors and workforce needed on the maintenance platform.

8.3.1.2. Technical criteria

- *Technical feasibility:* It is necessitated sometimes that the system cannot be considered under a specific maintenance strategy due to either cost or technical infeasibility. This infeasibility might be due to space impediment, climatic conditions and/or maintenance platform. In such a situation, experts usually suggest the next feasible strategy for the considered machine.

- *Technical complexity:* Each maintenance strategy has its own technical complexity in terms of prior knowledge requirements, employing the software, etc. It further affects the judicious and optimal utilization of the needed resources. Such resources are maintenance supply support, maintenance test and support equipment, maintenance personnel, maintenance facility, maintenance technical data and maintenance computer resources (Knezevic, 1997, 1993).

- *Flexibility of maintenance:* It measures the readiness of response of a maintenance strategy to unwarranted incidents of a critical system/machine. It is desirable that the chosen optimal strategy must be flexible enough (*viz.*, adaptive, responsive and agile) in terms of response time and needed efforts (Garg and Deshmukh, 2009). Further, the elapsed time are influenced by the personnel factors (e.g., motivation, skill, physical, attitude), conditional factors influencing the operating environment and consequence of failure on the component, and environmental factors (*viz.*, humidity, noise, vibrations, time of the day, etc.). It can also be measured in terms of maintenance capacity, maintenance facility, vertical integration, managerial flexibility, etc. There are also different techniques for flexible maintenance, such as distribution integration, risk pooling, multifunctional staffs, maintenance outsourcing, etc.

8.3.1.3. Social criteria

- *Workers' safety:* It is major concern while selecting the optimal maintenance strategy for a system/machine. For instance, a pressurized boiler cannot be facilitated with CM or R2F, as each failure of such critical system can lead to severe fatalities of the operator. Hence, CBM or PdM should be adopted for failure free operation.

- *Acceptance by workers:* Workers are the primary drivers of the observation of any abnormalities in the system/machinery. The chosen maintenance strategy should therefore be accepted by the workers or the operators operating the system/machinery as well. Sometimes, it might be possible that a complicated maintenance strategy is not well-accepted by the workers. In this situation, management must train workers to help them understand, through training programs/simulators/brainstorming events from time to time, its advantages.

- *Compliance with government regulations:* Apart from the above, the organization must follow the statutory regulations set out by government regulatory bodies to avoid unwarranted litigations.

8.3.1.4. Environmental criteria

- *Compliance with environmental standards:* The sustainability needs that an organization's liability and responsibility must have to protect the environment through recycling or safe disposal of the worn-out item. There are standards, e.g., ISO 14001, that specify the requirement of an environmental management system for small- and large-scale organizations. For each of the considered strategies, this standard should be followed by the organization.

- *Toxicity of generated waste:* During the maintenance activities multiple types of hazardous/non-hazardous wastes are generated. Their level of toxicities must be considered while opting a maintenance strategy.

8.3.2. Quantitative KPIs

Some identified KPIs under this category are:

8.3.2.1. Economic Criteria

- *Hardware cost:* Usually, it includes cost of electrical/electronic/computer hardware components, *e.g.*, sensors used to detect the condition of the system to determine the overall health.

- *Spare parts cost:* Cost of spares, tools, special supplies and related inventories needed to support the maintenance process used during the maintenance procedure.

- *Software cost:* This includes the cost of different computerized systems or diagnostic software employed to detect the faults and/or determine the health conditions of the system/machine.

- *Manpower cost:* Costs of labour/workers, technicians/engineers to bring the system back to an operative condition.

- *Training cost:* Cost for training the operators to make them acquainted with the tools and their handling techniques (*e.g.*, signal processing in CBM) used in monitoring and maintenance procedures.

- *Cost of production loss:* Due to failure of a critical equipment/machinery in a system or production line, the total system/production line may be shut down, causing significant revenue/production loss.

- *Return on investment:* It is the ratio of net profit gained from that critical system/machinery and cost of investment made on them.

- *Cost of cleaning the waste:* In each maintenance practice, the careful extraction, storage and recycling of waste entails cost, which might be substantial. Now-a-days, in most of the world-class organizations, to comply with the industrial regulations, toxic wastes are cleaned by robots or automated machines.

8.3.2.2. Technical Criteria

- *Mean time between failures:* This is the arithmetic mean time between successive failures of a repairable system. Usually, failures of a repairable system are measured in global time, if the failure times are recorded as time since the initial start-up of the system. Whereas, the same are measured in terms of local time if the failure times are recorded as time since previous failure. For a chosen maintenance strategy, the mean time between failures should tend to increase rather than get worse (Rigdon and Basu, 2000) for the optimal strategy.

- *Mean time to repair:* It measures the average time required to bring back a failed system/machinery to the working condition. In other words, repair time of a system/machinery is the quantification of time that it is out of production due to some occurred faults and/or failures. For a good maintenance practice, this criterion should be understood as the lower the better.

- *Availability of spare machine:* Availability is considered as an important metric for repairable system performance and combines both reliability and maintainability. It is defined as the probability that the system is available for use when demanded. In the worst situations of maintenance (*e.g.,* breakdown maintenance), it is required to replace the system/machinery with the redundant one, which should be in good working condition or available on demand.

- *The risk level of the system/machine:* ISO 9000:2015 defines risk as the *"effects of uncertainty on an expected result."* For a system/machine, risk of a failure can be considered from aspects, such as financial, technical, operational, environmental, health, safety, and impact on business and social objectives. While selecting a maintenance strategy for a system/machine, the level of risk on the total production process must be considered, which may be extended to the system and/or component level.

8.3.2.3. Social Criteria

- *Performance levels of employees:* Operators and/or employees are primarily responsible to carry out maintenance tasks. Their level of performance would be a major concern for selecting the optimal maintenance strategy for the system/machine. Multiple scales are available to measure their performance, such as Global Vigor and Affect (GVA) scale, NASA TLX scale and the Subjective Workload Assessment Technique (SWAT) scale.

8.3.2.4. Environmental Criteria

- *Amount of toxic substance emissions:* Toxic substances (*i.e.,* lubricating oils and their fumes, different harmful gases, like carbon monoxide, sulphur di-oxides, etc.) are produced when the system/machine operates in poor health condition or at faulty condition. For instance, this toxicity can be produced during the task of preventively changing the lubricating oils from gearboxes, or through the generation of harmful gases due to oil-evaporation during a bearing failure of a gearbox.

- *Amount of waste materials generated in a maintenance strategy:* In each maintenance practice, various waste materials are generated. For example, during PM of a system/machine, multiple components are replaced with a new one to mitigate the associated risks. Whereas, in CBM, components can run just prior to failure, and after than generally discarded/scraped/repaired, which generates wastes.

Now, employing the above KPIs, two examples are provided initially, to move forward to the development of the decision-making framework. In these examples, the various factors in the groups are deliberately kept generic to assist in the illustration and to make the process easier to comprehend.

8.4. Illustration of the Proposed Framework

For the considered case study of gearbox, as presented in *Chapter 4*, management sought to adopt the sustainable maintenance practice. To do that, initially, experts selected six sustainable factors, say, *A*, *B*, *C*, *D*, *E* and *F*. As per the requirement, a higher value of factors, *A*, *B* and *F*, whereas a lower value of *C*, *D* and *E* is an indication of rendering benefits. *A*, *B*, and *C* are assumed to be quantitative factors. For instance, *A* is the *mean time between failures* of the gearbox, *B* is *availability of a spare gearbox*, and *C* is the *cost of production loss* due to the shutdown of the gearbox; whereas, *D*, *E* and *F* are supposed to be qualitative factors. Here, *D* represents *technical complexity*, *E* is *toxicity of generated waste*, and *F* is *ease of maintenance*. It is noteworthy that the qualitative factors were initially assessed subjectively (linguistically), and then were translated back to problem specific designed, customized and agreed scale values[29]. It is assumed that all these data are previously stored in the central database of the organization according to the structure shown in Table 8.2. Here, the *Case ID* represents the *'primary key'* associated with each case.

[29] Absolutely high = 9, very very high = 8, very high = 7, high = 6, medium = 5, low = 4, very low = 3, very very low = 2, absolutely low = 1.

Table 8.2. Stored cases in database

	Antecedent factors							Consequent output
Case ID	Machine ID	A (Hrs.)	B (dimensionless)	C (Dollars)	D (scale)	E (scale)	F (scale)	Maintenance strategy
1	A03	130-140	0.80-0.82	100-112	2-3	5-6	2-3	TBPM
2	A05	97-102	0.90-0.92	45-50	8-9	2-3	7-8	CBM
3	A04	170-175	0.75-0.78	198-206	3-4	7-8	5-6	CM
4	A02	150-154	0.84-0.86	112-118	4-5	5-6	2-3	Age based PM
:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:
N-1	A01	110-115	0.94-0.96	78-88	7-8	3-4	8-9	CBM
N	A07	98-102	0.81-0.83	96-104	1-2	8-9	4-5	CM

8.4.1. Input Case - 1

Now suppose, the gearbox is to be supported with this sustainable maintenance strategy, and the decision-maker has requirements of the following values for each selected factor as shown in Table 8.3.

Table 8.3. Data for input case - 1

Case-input						
Factors	A	B	C	D	E	F
Input values	132	0.81	112	3	5	2

This example can be solved from the inputs by using a hypothetical rule-based *ES-1*:

a. The ES-1 builds an initial rule in the following manner:

$$"IF\ A\ is\ 132\ AND\ B\ is\ 0.81\ AND\ C\ is\ 112\ AND\ D\ is\ 3\ AND\ E\ is\ 5\ AND\ F\ is\ 2"$$

b. The built-up rule is then forwarded to the knowledge base and then the knowledge base finds the most similar rule in it, which is stored in the following manner:

$$"IF\ A\ is\ BETWEEN\ (130\ and\ 140)\ AND\ B\ is\ BETWEEN\ (0.80\ and\ 0.82)$$

$$AND\ C\ is\ BETWEEN\ (110\ and\ 112) AND\ D\ is\ BETWEEN\ (2\ and\ 3)\ AND$$

$$E\ is\ BETWEEN\ (5\ and\ 6) AND\ F\ is\ BETWEEN\ \ (2\ and\ 3)\ "$$

c. Thereafter, the knowledge base triggers the inference engine for getting the attached solution with the above rule. As shown in Table 8.2, the solution is attached in the following manner:

$$"IF\ A\ is\ BETWEEN\ (130\ and\ 140)\ AND\ B\ is\ BETWEEN\ (0.80\ and\ 0.82)$$

$$AND\ C\ is\ BETWEEN\ (110\ and\ 112) AND\ D\ is\ BETWEEN\ (2\ and\ 3)\ AND$$

$$E\ is\ BETWEEN\ (5\ and\ 6) AND\ F\ is\ BETWEEN\ \ (2\ and\ 3)\ "\ THEN\ Solution\ is\ "TBPM".$$

Clearly, the ES-1 advices to opt for TBPM.

8.4.2. Input Case- 2

Now suppose the decision maker sought the solution for the values given in Table 8.4. It can be observed by comparing the values in Table 8.2 and Table 8.4 that none of the stored cases are matching with the given input, *e.g.,* for factor A, the input value is 145, but there is no value range in the stored cases between which 145 can fall. Similarly, for criterion B, 0.94 matches with Case-ID (N-1), C matches with none, D matches with Case-ID 1 & 3, etc. The *ES-1* module will then fail to provide any feasible solution. Note that these types of situations are usually encountered in the real world. To solve this problem, an *AI based hybrid decision-making model* is described in the next section.

Table 8.4. Data for input case -2

Case-input-2						
Factors	A	B	C	D	E	F
Factor weight	0.18	0.22	0.1	0.2	0.16	0.14
Input values	145	0.94	62	3	4	2

8.5. Proposed AI-Based Hybrid Decision-Making Model

A generic flowchart of the suggested model is proposed and shown in Figure 8.2. The model consists of mainly two modules, *viz., ES-1* plus a *CBR module*. Within CBR, there is another *ES-2* whose purpose is to carry out the refinement on the outcome at the *retrieval phase* of the CBR module. The steps are:

Step 1: Referring to the previously mentioned example, when a new problem case arrives, the end-user initially provides inputs like number of gears in a gearbox, their diameters, module of gears, etc. to seek for the machine which has almost the same technical specifications with the new one and is already available within this model as global input data. After that the end-user selects the relevant criteria to be used in the decision-making process.

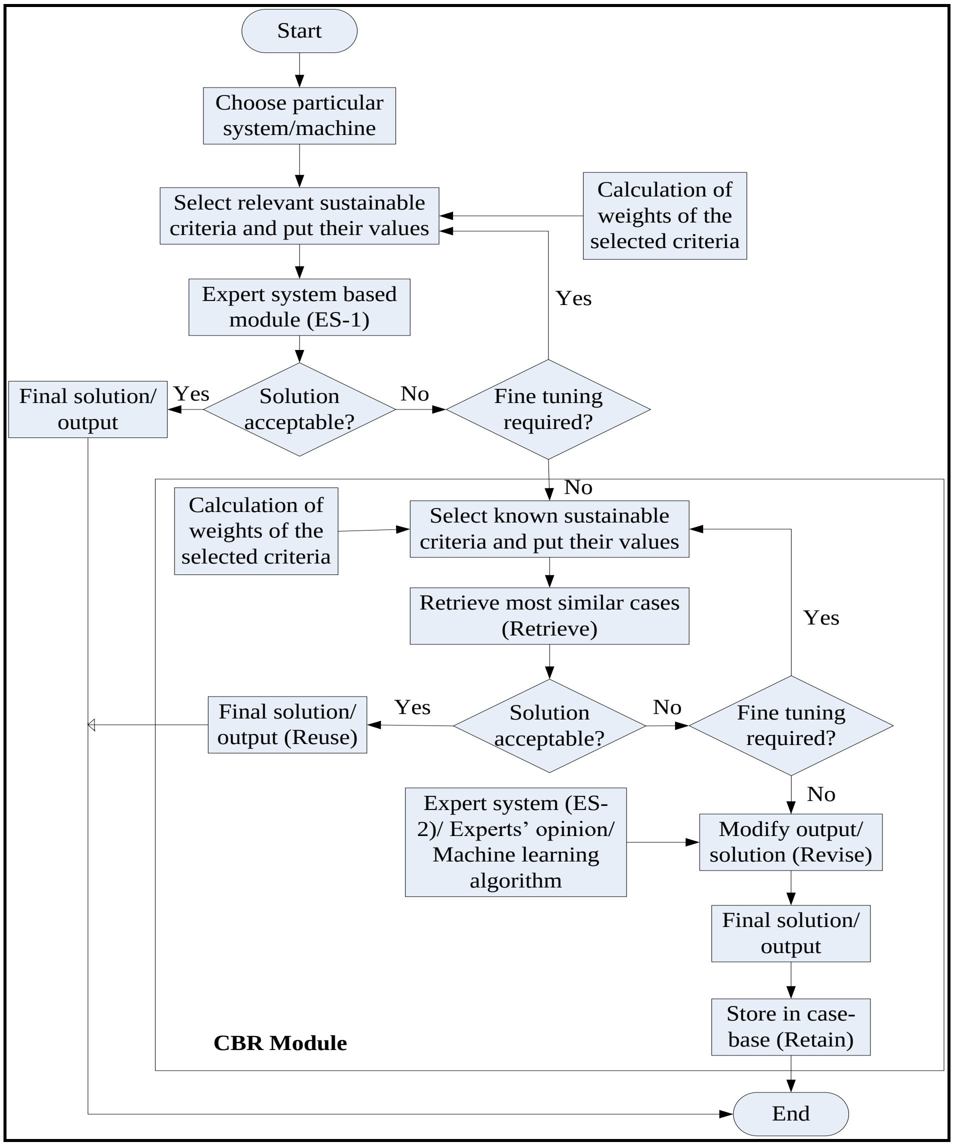

Figure 8.2. Workflow diagram of the hybrid model

Step 2: Since several factors/criteria are given as input to the system, it is required to compute their weight values, as they have varied impact on the outcome. These can be accomplished by any popular approach highlighted in *Chapter 2, Section 2.1.1*, or can be provided by the experts directly.

Step 3: The model now triggers the *ES-1* module that searches its knowledge base for the closest similarity with the input. It is worth mentioning that ES-1 makes the final decision based on a *backward reasoning* (top-down approach) approach. Thereafter, the best matched rule is forwarded to the inference engine for finding its associated solution. In the worst situation, when the ES-1 is unable to provide a solution to the end-user as confronted in *case-input-2*, it may prompt for fine-tuning of the factors, or it will enter into the CBR module for providing an approximate solution to the complex problem.

Step 4: The CBR module of the model searches for the best matched case from the case-base (*flat memory*, *serial search process*), without extracting any knowledge from them. A set of prior similar cases are presented to the end-user along with their similarity score values with the input. However, it is noteworthy that the *CBR module* may not deliver exactly matching solutions with the stored cases, rather it may provide solutions relevant to other installed machines in that organization having similar technical specifications.

Step 5: If the solution provided by the previous step has attained the desired similarity score value, then it can be accepted by the end-user. However, there are situations when the end-user disagrees on the solution and it becomes necessary to fine-tune the criteria for a better similarity match. This fine-tuning process may be carried out by selecting additional criteria and providing their values. Additionally, based on the output, there may be a requirement to adjust the values of input parameters to arrive at the optimal decision. If after fine-tuning the system is still not able to provide a level of similarity (*e.g.,* a threshold similarity score) then it is required to modify the output (*viz., Revision* step of CBR).

Step 6: The revision can be carried out either by means of ES-2, experts' opinions and/or by machine learning algorithms. However, in this work, it is carried out by building an *ES-2*. Mainly, in *ES-2* knowledge from knowledge engineers are incorporated to carry out the revision process of a retrieved case. Based on the output, provided by retrieve phase of a CBR system, different 'IF-THEN' rules are formed to carry out the revision task and to arrive at the optimal decision.

Step 7: After this revision phase, the final decision is suggested to the end-user, and is stored in the case-base for its future use (*viz.,* Retain phase of CBR).

Now, the previously presented case-inputs are again solved by the developed model.

8.6. Illustrative Cases of the Proposed Hybrid Model

8.6.1. Input Case - 1

The proposed and developed model solves the problem by exploiting the *ES-1* module only. Hence, there is no necessity to enter the CBR module, and the final outcome is the same as previous.

8.6.2. Input Case – 2

When the input query is provided to the model, the model works as follows:

a. *Step 1: ES-1* of the model will fail to offer a solution as already explained earlier and therefore, the input is passed to the CBR module. It is assumed here that criteria have weight values of: $A = 0.18$, $B = 0.22, C = 0.1, D = 0.2, E = 0.16$ and $F = 0.14$.

b. *Step 2:* the CBR system calculates the hamming distances (*refer Chapter 7, Section 7.2.1*) between each of the stored cases and the current case, for each of the considered factor (refer Table 8.5). Thereafter, it calculates the weighted similarity score for all the prior cases (refer Table 8.6)[30].

Table 8.5. Hamming distances between input and stored cases.

Case ID	A	B	C	D	E	F
1	0.064	0.571	0.236	0.125	0.143	0.143
2	0.551	0.095	0.106	0.625	0.286	0.857
3	0.385	0.762	0.845	0.000	0.429	0.571
4	0.115	0.381	0.311	0.125	0.143	0.143
⋮	⋮	⋮	⋮	⋮	⋮	⋮
N-1	0.385	0.095	0.099	0.500	0.143	1.000
N	0.551	0.524	0.211	0.250	0.571	0.429

Table 8.6. Similarity scores after considering criteria weights

Case ID	A	B	C	D	E	F	Weighted sum	Similarity score = (1-weighted sum)
1	0.012	0.126	0.024	0.025	0.023	0.020	0.229	0.771
2	0.099	0.021	0.011	0.125	0.046	0.120	0.421	0.579
3	0.069	0.168	0.084	0.000	0.069	0.080	0.470	0.530
4	0.021	0.084	0.031	0.025	0.023	0.020	0.203	0.797
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N-1	0.069	0.021	0.010	0.100	0.023	0.140	0.363	0.637
N	0.099	0.115	0.021	0.050	0.091	0.060	0.437	0.563

In this example, let the threshold value of similarity score be 75%. The best similarity score provided by the CBR module belongs to *case-ID 4*. The solution attached to this score outputted

[30] In this context, it is to be noted that for calculating the weighted distance, higher/lower values are taken from the prior cases for 'higher the better/lower the better' factors.

by the model is *Age based PM* (similarity score = 79.7%). If the similarity score was less than 75%, then there exists a need for case-revision that can be carried out either by experts' opinion, or by ES-2.

8.7. Chapter Summary

In this chapter, a scalable hybrid AI-based conceptual decision-making model has been proposed for a sustainable maintenance strategy selection. The model utilized the benefits of ES and CBR techniques. Several influencing and indicative criteria (economic, social, technical and environmental) have also been provided. The model is relevant in any industrial decision-making problem wherein the end-user/decision makers are forced to arrive at an optimal decision with the available information/data- structured or unstructured, complete or incomplete. Moreover, keeping in mind the recent trend of moving towards sustainable-based approaches, the proposed hybrid maintenance strategy selection model would serve a handy tool for decision-maker to narrow the gap and would help choosing a better, perhaps closer to an optimal one, sustainable maintenance strategy in a timely manner with much rapidity and ease.

Chapter 9 Conclusions and Future Scope of Study

9.1. Conclusions

RCM is a well-known cost-constrained maintenance philosophy, which is always preferable by organizations. In this research, an attempt has been made to address some of the problems in making maintenance decisions following the RCM philosophy. However, during the implementation process, several decision-making problems arise as highlighted below:

- There may exist numerous types of machine installed in an organization. The question may arise: What will be the scheme for selecting a set of machines for implementing the RCM philosophy? Others are,

- What are the different types failure modes, their causes, and effects associated with the RCM aided machine? Further, which failure modes are critical? Which risk factors are to be considered during the risk rankings of failure modes?

These questions can be answered by performing a comprehensive FMEA. However, FMEA is a task, where cross functional experts participate during the process. Then, the next questions are who are those experts, and what are their expertise level? Again, these experts always prefer to evaluate the failure modes with respect to the risk factors linguistically, and each linguistic evaluation contains some uncertainty, which surely affects the decision outcome, that is risk ranking of the failure modes. How to address these uncertainties? Apart from that, the RPN-based traditional FMEA approach has been criticized for many other drawbacks. How can the organization overcome those drawbacks?

Despite taking all possible precautionary measures to prevent the failures of the machine, development of faults is inevitable. The only cost-effective way to detect the fault at the earliest possible opportunity of the large-scale, complex, and critical machine is to adopt the condition monitoring techniques based CBM approach. However,

- when the machine is large and complex, mathematical modelling becomes difficult, if not impossible, with added assumptions.

- Most often, an equipment internal details are covered by an enclosure, upon observing fault symptom(s), engineers become perplexed about the exact location of the fault, type of fault, etc.

- Moreover, the impacts of surrounding environment on the occurrence of fault is most often unknown and require rigorous mathematical modelling. In these circumstances, engineers generally collect information about multiple HIs from different location of the machine (i.e., bearing housings, machine foundation, etc.). This information may be vibration signals, oils, temperature at different part of the machine. However, to know about the location, and type of the fault, each of these data are required to be analyzed by sophisticated information processing techniques, such as vibration signal analysis, oil analysis, etc. which are case-specific and often require experts' intervention.

- Furthermore, for a large-scale machine, it is not possible to collect the HIs from each point, and thus engineers often proceed with incomplete data to analyze the fault. In this scenario, to map the fault information from the measurement space to fault space is considered as a daunting and a pivotal decision-making task in fault diagnosis of such machines.

- The next decision-making problem associated with such machines, is to choose which maintenance strategy is viable in some optimal sense to mitigate the occurrence of failures and to aid the sustainable manufacturing practices? It is thus required to carefully identify those pertaining sustainable parameters from the TBL of sustainability that could aid the sustainable manufacturing practice. Furthermore, using these parameters, and their ordinal and/or cardinal values, selecting the optimal maintenance strategy from a set of alternatives is another critical decision-making task.

A point-wise listing of the novelties and contributions of the work contained in this thesis can be summarized as below:

1. Different MCDM problems during the RCM implementation has been addressed by considering a case study of process plant gearboxes installed in the rolling mill of a steel plant.

2. MCDM-based frameworks have been developed for risk ranking of failure modes in a FMEA problem.

- To deal with the imprecisions and vagueness associated with linguistic judgements while calculating the relative importance among the risk factors, and risk ranking of failure modes, two integrated fuzzy MCDM approaches have been proposed. In the first approach, Buckley's fuzzy AHP method has been integrated with the developed fuzzy MARICA method. Fuzzy AHP has been used to calculate the relative importance of the risk factors, and fuzzy MAIRCA for risk ranking of failure modes. While in the second approach, Buckley's fuzzy AHP has been integrated with the developed modified fuzzy MARCOS method. In this work, only the potential of these integrated approaches in risk ranking of failure modes in a FMEA problem have been verified. To do that, the benchmark example given in (Kutlu and Ekmekçioğlu, 2012) has been used. **It has been observed that the ranking stability of the proposed approaches is greater than the original work. Further, for this example, it has been observed that the ranking stability of both the developed approaches are same**. A detailed discussion on this work can be found in *Chapter 3*.

- The factor 'severity' of a failure mode in RPN methodology have been considered from the TBL of sustainability perspective, thus, 11 risk factors have been generated. Two integrated MCDM approaches have been proposed for the risk ranking of the failure modes by combining the concepts of IT2F-DEMATEL, fuzzy MAIRCA, and modified fuzzy MARCOS. The proposed methods are well illustrated by a case study on a process plant gearbox. Based on the linguistic judgements of risk factors for the chosen failure modes, IT2F-DEMATEL has been used to depict the causal dependencies among the failure modes, and to calculate their relative importance in terms of crisp values. Then fuzzy MAIRCA and modified fuzzy MARCOS have been separately used for risk ranking of failure modes. **It has been observed that when the number of risk factors has been increased in number, then fuzzy MARCOS has the greater ranking stability than fuzzy MAIRCA**. A detailed discussion on this work can be found in *Chapter 5*.

- To further improve the associated imprecision and vagueness in linguistic judgements, and to deal with the phenomenon of production of different risk ranking results by different MCDM methods, a hybrid IT2Fs and half quadratic minimization based MCDM framework has been proposed and developed. The same case-study of gearbox FMEA has been reconsidered here, and the relative importance of the risk factors were calculated in terms of IT2FNs by a modified IT2F-DEMATEL method. This IT2F-DEAMTEL has been modified to prevent the early information distortion while calculating the risk factors' weights in terms of crisp numbers. Then for the risk ranking of failure modes, IT2F-MAIRCA, IT2F-MARCOS, and modified IT2F-TOPSIS have been proposed. After observing that each of these developed IT2F-based MCDM approaches produce different ranking results, **the concept of half-quadratic minimization has been used to generate an aggregated ranking result, along with the consensus index and trust level. Further, it has been observed that in IT2F-domain the ranking stability of IT2F-MARICA is greater than IT2F-MARCOS, and modified IT2F-TOPSIS.** A detailed discussion on this work can be found in *Chapter 6*.

3. A CBR-based framework has been developed for the fault diagnosis of the identified case of gearboxes. This CBR-based framework has used the information of different HIs stored in the central database. Different types of HIs have been considered for accurate fault diagnosis, based on the generated symptom(s). Both event type and value type data have been considered. **This proposed system has the potential to diagnose the fault with incomplete information, which is considered as an added advantage over other AI-based approaches (*i.e.*, ANN, SVM, ESs, etc.) present in the literature. Further, after diagnosing the fault, this system has also aided the maintenance engineers with the suggested maintenance actions. It has been observed that the developed CBR-system was able to diagnose the fault with 89.2% similarity.** A detailed discussion on this work can be found in *Chapter 7*.

4. A hybrid AI-based framework, integrating the concept of ESs and CBR has been proposed and developed for optimal sustainable maintenance strategy selection of the gearboxes. Before doing that, the criteria for this selection process has been selected from TBL of sustainability. The desired nature of these criteria has also been highlighted in *Chapter 8*. Due to the unavailability of real time data, this framework has **considered two hypothetical examples, and it has been observed that the developed framework was able to approximately select the optimal sustainability-based maintenance strategy for the problem scenarios**.

9.2. Future Scope of Research

The research work carried out in and presented in this thesis can be further extended which are likely to be carried out in the near future. The possible future scopes are:

- During addressing the drawbacks of traditional RPN-based FMEA approach the following improvements can be done:
    - To extend the developed frameworks into other uncertainty handling tools like spherical fuzzy sets, Pythagorean fuzzy sets, Fermatean fuzzy sets, etc. may be explored.
    - In the developed frameworks, participations of only three experts have been considered. However, if a lot more experts participate, which may be 40 or 50 in number, AI-based approaches can be integrated for clustering purposes. Further, while considering these large group of experts, a better consensus reaching model can be developed.
    - A more developed FMEA approach can consider simultaneously the relative importance among the failure modes as well as the risk factors.
    - Further investigations are needed to decouple the risk factors from the TBL of sustainability. It is worth mentioning that in Industry 4.0, sustainable manufacturing is a key concept to achieve sustainable development goal (SDG) by 2030.
    - The outputs of FMEA can be further used in other analysis, like reliability, availability analysis and maintenance modelling.
- From the CBR-based fault diagnosis approach the following future research areas are possible:
    - Although the developed CBR system has dealt with an exhaustive case-base, now-a-days, in the era of big data, industries are compelled to deal with a stream and variety of data. Data from other fault analysis techniques (*e.g.*, image processing in thermography, music analysis, etc.) can be incorporated.

- Thus, this framework can be extended into the big-data environment.

- The revision and retention capability of the CBR system has not been considered during the development of the framework, which can be incorporated by future researchers, exploring other tools.

- When the number of cases increases in the case-base, it has been observed that in many cases, the best matched case can be overlooked due to poor indexing structure. Thus, other indexing structure, apart from the considered 'flat-memory' can be explored.

- This system can be extended for other similar types of decision-making problems, such as supplier selection, personnel selection, etc.

* From the optimal sustainable maintenance strategy selection work, the following future research work can be a possibility:

    - In this thesis, only a few of the parameters from TBL of sustainability have been identified for the optimal sustainable maintenance strategy selection. However, the research is still needed to incorporate other parameters to be incorporated in the framework.

    - CBR can be combined with other AI-based approaches, like ANN, SVM for better classification accuracy.

Last but not the least, all these decision-making frameworks can be integrated with the CMMS for ease in strategy making.

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The references have been ordered alphabetically as per the first author's surname and then the second author's. For the same authors within the alphabetical list, arrangement is chronological.

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1. **Boral, S.**, Howard, I., Chaturvedi, S. K., McKee, K., Naikan, V. N. A., 2020. An integrated approach for fuzzy failure modes and effects analysis using fuzzy AHP and fuzzy MAIRCA. *Engineering Failure Analysis* 108, 104195. DOI: *https://doi.org/10.1016/j.engfailanal.2019.104195*.

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4. **Boral, S.,** Chaturvedi, S. K., Howard, I., K., McKee, Naikan, V. N. A., An integrated interval type-2 fuzzy sets and multiplicative half quadratic programming based MCDM framework for risk ranking of failure modes in FMECA, *Process Safety and Environmental Protection* 150, 194-222. DOI: *https://doi.org/10.1016/j.psep.2021.04.006*

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2. **Boral, S.**, Chaturvedi, S.K., Liu, Y., Howard, I., 2021. Integrated fuzzy MCDM frameworks in risk prioritization of failure modes. *In Advances in Performability Engineering*. (**Communicated**).

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1. **Boral, S.,** Chaturvedi, S. K., Howard, I., K., McKee, & Naikan, V. N. A., 2020. An Integrated Approach for Fuzzy Failure Mode and Effect Analysis Using Fuzzy AHP and Fuzzy MARCOS. *In 2020 IEEE International Conference on Industrial Engineering and Engineering Management* (14-17 December 2020). pp. 395-400. DOI: 10.1109/IEEM45057.2020.9309790.

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