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# 1 Experimental and Numerical Studies of the Shear Resistance

## 2 Capacities of Interlocking Blocks

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### 7 Abstract

8 Interlocking bricks could improve construction efficiency, reduce labour cost, and  
9 provide better mechanical performance for masonry structures. Nevertheless, the shear  
10 properties of mortar-less interlocking bricks have not been systematically investigated  
11 which may impede their wide applications. In this study, the shear performance of a  
12 new type of interlocking brick is investigated in detail. Laboratory shear test is firstly  
13 conducted to study the damage and shear capacity of mortar-less (dry-stacked)  
14 interlocking bricks. Numerical model is then generated with consideration of contact  
15 imperfection and validated with test results. Intensive parametric studies are conducted  
16 to quantify the influences of material strength, axial pre-compression force, friction  
17 coefficients, and contact imperfection at brick interfaces on the shear response of  
18 interlocking prisms. The accuracy of existing methods for predicting the shear  
19 capacities of shear key by design standard and empirical formula are evaluated. Based  
20 on the numerical and laboratory results, an empirical design formula is proposed to  
21 predict the shear capacity of the interlocking brick.

22 **Keywords:** Interlocking blocks; shear key; shear strength; numerical modelling.

### 24 1 Introduction

25 Brick structure is one of the most popular building types all over the world especially  
26 for low-rise buildings. Conventional masonry structure is constructed with mortar and  
27 bricks. Usually, the strength of mortar is lower than the brick material. Hence, damage  
28 tends to occur in the mortar layers when the structure is subjected to in-plane or out-of-  
29 plane shear force, especially when the axial loading level is relatively low [1].  
30 Considering this deficiency, interlocking bricks with shear keys have been introduced  
31 to substitute conventional bricks to improve its mechanical performance. Besides

32 enhanced shear resistance, interlocking bricks also have other attractive features such  
33 as improved construction efficiency with easy alignment [2, 3].

34 Different interlocking keys have been developed and introduced in recent years,  
35 which can provide loading resistance in vertical, horizontal or both directions. The  
36 effect of interlocking connection on brick compressive load bearing capacity has been  
37 investigated primarily through experimental methods [4-10]. Some researchers  
38 reported low compressive load-bearing capacity of interlocking bricks due to the  
39 relatively small contact area because of joint imperfection [11, 12]. Apart from  
40 compressive capacity, the shear mechanisms of brick with different interconnections  
41 have also been studied and reported [13-21], which nevertheless are mostly for  
42 validation of particular products. It shall also be worth noting that most current  
43 structures comprised of interlocking bricks are characterized by small shear keys for  
44 easiness in construction, i.e., easy alignment. The shear tenons do not remarkably  
45 improve the shear resistance of these bricks since the projection area of the keys is  
46 relatively small [22]. Recently some laboratory tests were conducted on interlocking  
47 bricks with large shear keys. Total shear off failure was found on these interlocking  
48 bricks under large axial force; and damages to the tips of the keys were discovered  
49 under low axial compression [23, 24]. Besides, recent studies by Zhang et al. [25, 26]  
50 also observed damages induced by stress concentration at shear keys of segmented  
51 columns comprised of concrete blocks with shear keys under impact and cyclic loading,  
52 which reduced column load bearing capacity against impact and seismic load. Apart  
53 from the above studies on the performances of particular designs by different  
54 researchers, the mechanical properties of interlocking brick still need comprehensive  
55 studies.

56 Various types and shapes of interlocking joints have been developed to improve  
57 the capacity of the interlocking bricks [27-29]. Ahmed and Aziz summarized that  
58 mortar-less joints with multiple keys had better mechanical behaviour than single key  
59 without epoxy, because multiple keys enable stress transfer uniformly between adjacent  
60 bricks and thus lead to better mechanical performance [30]. Although increasing the  
61 number of interlocking keys improves the shear capacity of joints, the equivalent shear  
62 capacity of mortar-less joints with multiple keys is less than that of mortar-less joints  
63 with single key. For example, Alcalde et al. [31] analysed the fracture behaviour of  
64 mortar-less keyed joints subjected to shear load and found that the averaged shear  
65 strength decreased with the increase in the number of keys, because the keys failed

66 sequentially. Nevertheless, this effect became less apparent as axial prestress increased  
67 to 3.0 MPa because a higher normal compressive stress increased the friction resistance  
68 and improved the integrity of the key group. Similar results were observed by Zhou et  
69 al. [32] and Jiang et al. [33]. Moreover, changing the key geometry may also greatly  
70 influence the shear performance of keyed joints [34]. Zhang et al. [35] examined the  
71 direct shear resistances of four different shaped shear keys and concluded that they have  
72 very different shear resistance capacity and shear stiffness because of the difference in  
73 shear flow mechanism of different shaped shear keys.

74 The axial pre-compression level at the interlocking joint is another factor that  
75 influences the shear performance of keyed joint [27, 32, 36-39]. Higher shear strength  
76 and initial stiffness were found on interlocking joints when the axial pre-compression  
77 increases. This is because the increased axial compression could increase both the  
78 contact surface friction and the shear resistance of the interlocking key [40]. Moreover,  
79 with a higher axial pre-compression level, the post-peak ductility of the interlocking  
80 joint was also found to be improved [28]. As expected, material strength also  
81 considerably affects the initial stiffness, the ultimate shear capacity, failure modes and  
82 ductility of interlocking joint [30, 37, 38, 41]. Nevertheless, the increase in the initial  
83 stiffness of the keyed joint is not proportional to the increase of material strength. When  
84 a high strength concrete of over 80 MPa was used for the interlocking joint, limited  
85 improvement was found on the elastic stiffness of the interlocking joint [28].

86 For dry interlocking joints, surface roughness condition could strongly influence  
87 the shear performance of keyed joints [6, 12, 42, 43]. Martínez et al. [8] found that the  
88 uneven surface despite small could change stress distribution at the interface, and  
89 therefore affect contact pressure. Fan et al. [44] studied the contact behaviour of rock  
90 and observed both shear failure and friction failure modes which are influenced by  
91 surface roughness condition. These previous studies indicated that for mortar-less  
92 masonry construction, the surface roughness condition at the joint could significantly  
93 influence the mechanical performance, which are proved by some recent studies on  
94 mortar-less masonry prisms [45-47]. However, the influence of the surface roughness  
95 on the shear capacity has not been properly studied. It is critical for engineering  
96 application to appropriately investigate the effect of contact surface on the failure  
97 modes, as well as stress concentration in the dry-stacking masonry constructions. To  
98 investigate the effect of contact surface roughness, different methods have been applied  
99 to model rough contact surfaces. For example, Bahaaddini [48] employed discrete

100 element method to reproduce the shear behaviour of saw-tooth triangular joints.  
101 Homogenization of the random rough surface into regular rough surface has also been  
102 a popular approach for modelling rough steel surface [49, 50]. The study of influences  
103 and modelling approaches of rough concrete and brick surface is very limited.

104 Different formulae for predicting the ultimate shear capacity of interlocking joints  
105 have been proposed where the difference could be substantial [30]. Some of these  
106 formulae come from theoretical derivation [51, 52], while others are empirical from  
107 laboratory testing and numerical modelling [36, 53, 54]. Most popularly used design  
108 code such as AASHTO [55] assumes the shear force is transferred through the  
109 interlocking joint by both the shear key and surface friction [28]. Some researchers  
110 evaluated the accuracy of AASHTO method in predicting the shear resistance capacity  
111 of different keyed joints. For example, Ahmed and Aziz [56] carried out direct shear  
112 test to study the shear behaviour of mortar-less connections with single and multiple  
113 keys. It was found that the AASHTO design formula could conservatively predict the  
114 shear strength of joints with single key, but overestimate the shear strength of mortar-  
115 less with multiple keys. Similar results were also reported by Zhou et al. [32]. However,  
116 opposite conclusion was reported by Jiang et al. [33] who found AASHTO method  
117 underestimates the shear load of joints with single key made of steel fibre reinforced  
118 concrete (SFRC) but more accurately predicts that of three-keyed dry joints. For  
119 interlocking brick comprising multiple keys, the accuracy of AASHTO and other  
120 prediction methods are not known yet.

121 In this paper, laboratory tests and numerical simulations are performed to  
122 investigate the shear behaviour of interlocking brick prisms. Laboratory shear tests are  
123 firstly conducted on interlocking brick prisms under different axial pre-compressions.  
124 Then, a detailed numerical model considering contact surface roughness is generated  
125 and validated with testing results. Parametric studies are then carried out to investigate  
126 the influence of different design parameters. An empirical formula is proposed to  
127 predict the shear capacity of the mortar-less interlocking brick prism, which can be used  
128 in engineering practice to quickly estimate the shear capacities of interlocking brick  
129 structures with varying material properties, loading conditions, and brick surface  
130 conditions.

131

## 132 2 Laboratory Tests

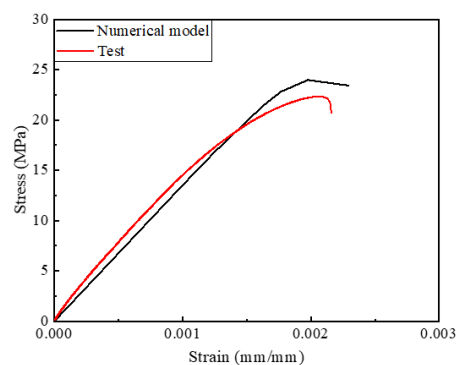
133 Laboratory shear test is carried out to experimentally examine the shear behaviour of  
134 interlocking bricks. Considering the large variation in brick material properties, the  
135 material strength of the studied interlocking bricks is firstly tested through unconfined  
136 uniaxial compressive test. Then, shear test on interlocking brick prisms is setup and  
137 performed to investigate the shear behaviour of interlocking bricks.

### 138 2.1 Material property

139 The interlocking bricks used in the laboratory test are made of cement stabilised  
140 rammed earth material. To determine the brick material properties, uniaxial unconfined  
141 compressive tests are conducted using a SHIMADZU-50 machine in Structural  
142 Laboratory of Curtin University. Brick cores with a height of 100 mm and a diameter  
143 of 50 mm are drilled out of interlocking bricks, and carefully grinded on both the top  
144 and bottom, as shown in Figure 1a. Strain gauges are stuck onto the specimen surfaces  
145 to acquire the axial strain. Following ASTM C140 [57], in the test a loading speed of  
146 0.03 mm per second is adopted to apply the axial compressive load using displacement  
147 control method. The averaged axial stress-strain curve measured in the laboratory tests  
148 are shown in Figure 1b, where the axial stress is calculated by dividing the measured  
149 axial compressive load by the cross-sectional area of the specimen. Strain gauges are  
150 used to measure the axial strain. The numerical prediction of the corresponding stress-  
151 strain curve is also presented in the figure, details of numerical model will be presented  
152 in the subsequent sections.



(a) Uniaxial compressive test



(b) Stress-strain curves

153 *Figure 1. Determination of brick material properties*

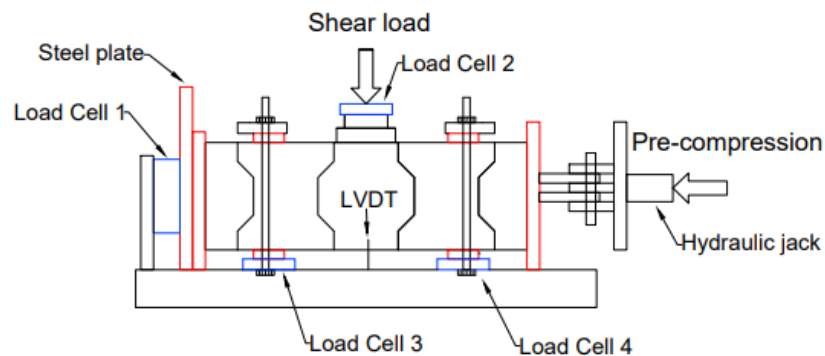
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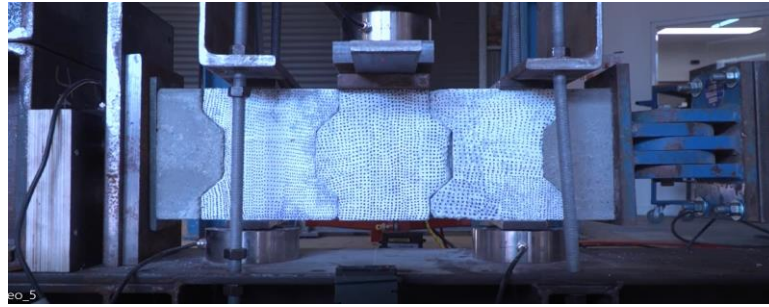
### 155 2.2 Shear test setup

156 Following BS EN 1052-3 [58], shear tests are setup to examine the shear properties  
157 of the interlocking bricks. Figure 2a and b illustrates the test setup, where the specimen

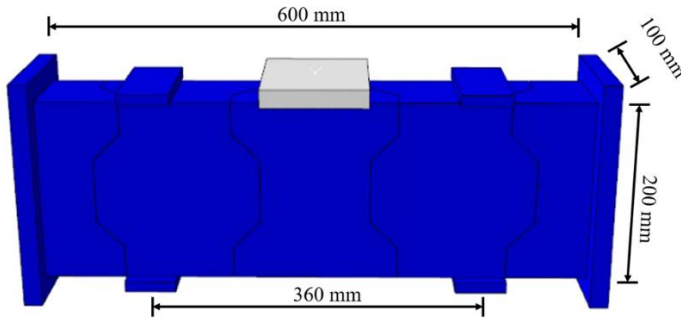
158 dimension is 600mm × 200mm × 200mm (length × height × thickness). The prism  
 159 comprises of three dry-stacking interlocking blocks of 200mm × 100mm × 180mm  
 160 (length × thickness × height) and two pieces of half bricks as end blocks. The interlocking  
 161 blocks have large interlocking keys (35 mm length × 35 mm thickness × 30 mm height),  
 162 which provide shear resistance at the interlocking joints, as shown in Figure 2c. The  
 163 prism is firstly pre-loaded axially in the horizontal direction. Then, the two side blocks  
 164 are fixed using two steel plates to the bottom supporting frames. To minimize the  
 165 flexural bending deformation in the prism, flat bars and wide-angle plates are used to  
 166 fix the two end bricks firmly to the bottom support frame. The rotation of the two side  
 167 bricks is therefore effectively restrained. Displacement controlled loading method is  
 168 applied where the central block is pressed to move downwards at a speed of 1.8 mm  
 169 per minute. Two shear planes are therefore created through this setup. Because of the  
 170 non-symmetric layout of the interlocking keys on the brick, the damage and failure of  
 171 the testing brick prism vary on the frontal surface (Side I) and rear surface (Side II).

172 One LVDT (linear variable differential transformer) is installed to record the  
 173 vertical displacement of the central brick. Another LVDT is used to measure the vertical  
 174 movement of one side brick, so as to monitor the rotational movement of the side brick.  
 175 One loadcell is used to monitor the axial pre-compressive force applied. Another  
 176 loadcell is used on the central brick to measure the shear force applied to the shear  
 177 planes. Two loadcells are installed beneath the two side bricks to ensure the same  
 178 amount of tying forces is applied. Two groups of tests are conducted with 10 kN and  
 179 30 kN axial pre-compression applied to the brick prisms, corresponding to 0.538 MPa  
 180 and 1.614 MPa axial stress, which are the typical vertical stress in masonry wall for a  
 181 single-storey and low-rise masonry building. Three specimens are tested for each group  
 182 in the study.

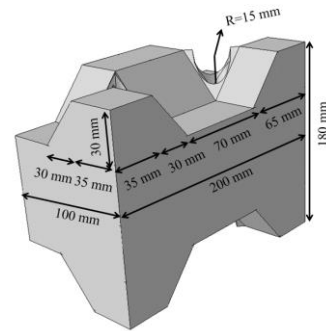




(a) Illustration of test setup



(b) Numerical model



(c) Configuration of interlocking blocks

Figure 2. Experimental and numerical models for the prism shear test

183

### 184 3 Numerical Simulation

185 A three-dimensional finite element model of the interlocking brick prism is developed  
 186 in Abaqus [59] to further investigate the shear behaviour of the interlocking bricks.

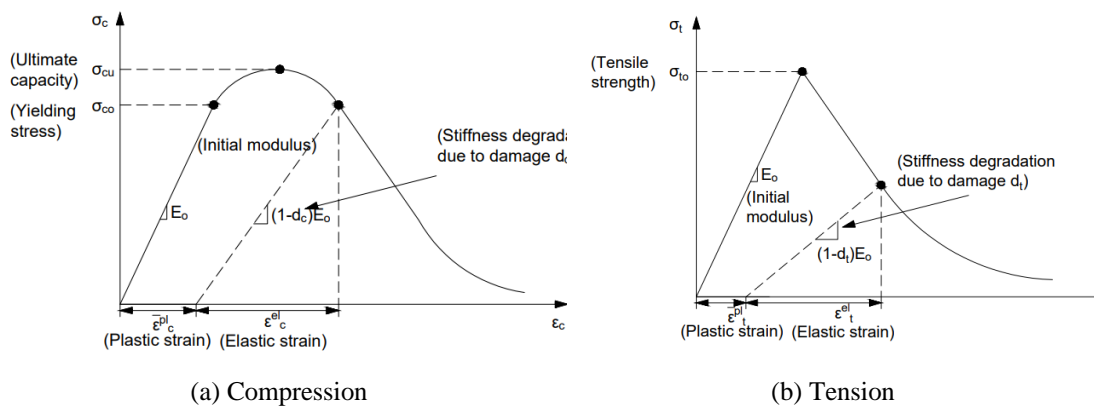
#### 187 3.1 Model details

188 Figure 2b presents the numerical simulation of the interlocking prism, which  
 189 replicates the laboratory test setup. Steel strips of the same dimensions as in the test are  
 190 modelled to fix and load the brick prism, where a friction coefficient of 0.15 is adopted  
 191 between steel and the bricks [8]. Solid element C3D8R in Abaqus is adopted for the  
 192 interlocking brick. In the numerical modelling, three loading steps are implemented,  
 193 i.e., axial pre-compression of the interlocking prism, fixing the two side bricks with  
 194 vertical pre-tying force onto the two supports, and application of vertical load on the  
 195 central brick. The axial pre-compressive force and pre-tying force are applied using  
 196 force control method. For the vertical force, displacement-controlled loading method is  
 197 used as in the laboratory test. Convergence study is implemented by step-reducing the  
 198 mesh sizes. It is found that when the mesh size reduces from 5 mm to 2.5 mm, the  
 199 maximum compressive force in the prism does not change much but it requires a

200 significantly higher computation resource. Therefore, 5 mm mesh size is used in this  
 201 study for the numerical simulation.

### 202 3.2 Material model

203 The material model of concrete damage plasticity (CDP) is employed to simulate  
 204 the nonlinear behaviour and damage of the brick, which is proposed by Lubliner et al.  
 205 [60]. Crushing in compression and cracking in tension can both be modelled. As shown  
 206 in Figure 3, in CDP model, the compressive and tensile stress-strain relationships are  
 207 defined, which are featured by damaged plasticity parameters. The unconfined uniaxial  
 208 compressive strength is acquired via the laboratory material tests depicted in Section  
 209 2.1. The elastic modulus ( $E_0$ ) is taken as the secant modulus determined from the origin  
 210 to the point with a stress level equivalent to 40% of the compressive strength. The  
 211 Poisson's ratio is determined at the same stress level. Table 1 and Table 2 show the  
 212 material properties of the interlocking brick, where  $E_0$  represents the elastic modulus;  $\nu$   
 213 denotes the Poisson's ratio. The tensile strength is taken as  $f_t=0.1 f_c$  following previous  
 214 studies [8, 61]. The unconfined uniaxial compressive behaviour of the brick core is  
 215 modelled to verify the brick material constitutive model. The stress-strain curve from  
 216 the numerical calculation using the CDP model agrees reasonably well with that in the  
 217 laboratory test (see Figure 1b). The compressive and tensile damage parameters ( $d_c$  and  
 218  $d_t$  in Figure 3) can be calculated following literature [62]. An elastic perfect plastic  
 219 material model is adopted for the steel, whose Young's modulus of 210 GPa as well as  
 220 Poisson's ratio of 0.3 are used.



221 *Figure 3. Definitions of concrete damage plasticity (CDP) model [53]*

222 *Table 1. Material properties of interlocking brick*

Elasticity			Plasticity		
Initial Young's modulus,	Poisson's ratio $\nu$	Dilatation angle $\psi$ ( $^\circ$ )	Eccentricity	Biaxial stress	K



E <sub>0</sub> (GPa)		ratio $f_{bo}/f_{co}$			
13.49	0.2	30	0.1	1.16	0.67

223

224

*Table 2. Material constants of concrete damage plasticity model*

Behaviour in compression		Behaviour in Tension	
Yield stress (MPa)	Inelastic strain	Yield stress (MPa)	Cracking strain
15.12	0	1.78	0
15.96	0.00002	0.93	0.0007
16.84	0.00006	0.78	0.0008
17.84	0.00012	0.63	0.0009
10	0.0010	0.48	0.0010
		0.18	0.0012

225

### 226 3.3 Contact algorithm

227 Contact surface strongly influences the behaviour of mortar-less joint of  
 228 interlocking bricks [6]. Three different modelling approaches: perfect contact, random  
 229 rough contact and simplified rough contact, are used to simulate the contact behaviour  
 230 between the mortar-less joints.

#### 231 3.3.1 Perfect contact

232 The perfect contact is the simplest approach used in the engineering field. It  
 233 assumes that contact surfaces between neighbouring bricks are smooth, which leads to  
 234 perfect connection. The surface-to-surface contact is used to simulate the connections  
 235 between the neighbouring interlocking bricks. The tangential behaviour is defined by  
 236 Mohr-Coulomb criterion, the friction coefficient is taken as 0.3 [8, 63]. And the normal  
 237 behaviour is defined by hard contact. The hard contact ensures contact surfaces between  
 238 the adjacent interlocking bricks be in contact without penetration.

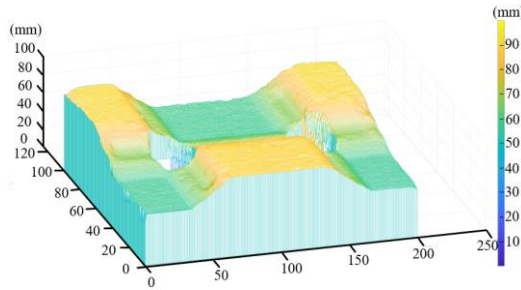
#### 239 3.3.2 Random and simplified rough contact

240 The random rough contact considers brick natural surface condition due to material  
 241 and manufacture tolerance. To examine the true brick surface condition, laboratory test  
 242 is carried out using a laser profile scanner to quantify the surface profile of the bricks.  
 243 As shown in Figure 4a, each brick is cut into halves and placed on a flat testing table,  
 244 and the laser scanner installed on a rigid steel frame scans the top surface profile of the  
 245 interlocking brick. The laser scans the surface for three times, and the averaged value  
 246 of profile is taken as the actual surface roughness. Figure 4b shows one of the typical  
 247 brick surface contours scanned from the test. The above experimentally measured  
 248 contour at the interlocking bricks is then numerically generated with fine mesh as

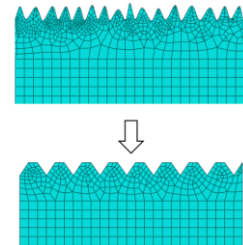
249 illustrated in Figure 4c. To improve computational efficiency, and also to reasonably  
 250 model surface roughness without the need to measure every surface of the interlocking  
 251 bricks, the random surface roughness is simplified by the mean surface roughness value  
 252 and trapezoidal shape roughness profiles (Figure 4c).



(a) Laser scanning  
brick surface roughness



(b) Measured contour of a typical surface  
roughness



(c) The equivalent rough  
surface

Figure 4. Evaluation of brick surface roughness

253

254

## 255 4 Results and Analysis

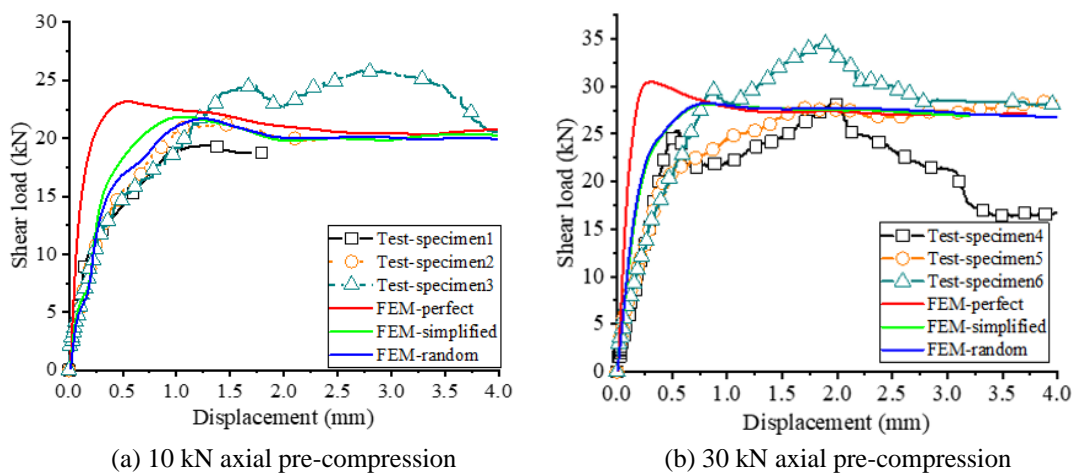
256 Numerical modelling and laboratory testing results are provided in this section. Shear  
 257 load-displacement relationship, failure modes of the interlocking brick, and shear  
 258 capacity are compared to demonstrate the shear behaviour of interlocking bricks.

### 259 4.1 Load-displacement curves

260 Figure 5 presents the shear load versus central brick vertical displacement. In this  
 261 paper, a half of the applied vertical force is taken as the “shear force” experienced by  
 262 the interlocking joint due to the two symmetric shear planes, and this shear force is  
 263 taken as the shear capacity of the interlocking joint. Without losing generality,  
 264 Specimen 2 is taken as an example, when a 10 kN axial pre-compression is applied to  
 265 the brick prism, the shear force increases linearly to about 6.27 kN at about 0.11 mm  
 266 displacement, reflecting an initial stiffness of 56.43 kN/mm, and it corresponds to about  
 267 30% of the maximum shear load of the prism. As the shear load further increases, the  
 268 slope of the curve drops. The shear load increases non-linearly as it approaches the  
 269 maximum shear load of 21.18 kN at a displacement of around 1.46 mm, after which it  
 270 begins to decrease, reflecting the failure of the interlocking prism. Similar trend can be  
 271 found on Specimen 1. It is worth noticing that on Specimen 3 after the initial peak load  
 272 is reached, a 2<sup>nd</sup> peak load is developed. The first peak corresponds to the damage of  
 273 intact prism at the weakest shear key and the sharp drop reveals shearing off on one  
 274 side of the shear key that provides the shear strength  $\tau_1$ . Afterwards, stress at the

275 interlocking connection is redistributed, where block rotation can be observed. The test  
 276 might be influenced by the flexural bending deformation of the prism, which was  
 277 observed in the lab test. The second peak represents the combined contributions from  
 278 the shear strength of the second interlocking key, some friction force in the first  
 279 interlocking key connection due to bending, and membrane effect due to the prism  
 280 deformation and axial pre-compression. Therefore, a higher shear force is recorded for  
 281 the 2<sup>nd</sup> peak load. Similar observation was reported by previous researchers on the  
 282 concrete shear key [64]. Nevertheless, it is worth noting that because of the large depth,  
 283 the prism can be considered as a deep beam, whose flexural deformation is therefore  
 284 not significant. Some variations among the three prisms tested can be observed, which  
 285 are due to the inherent variability of contact surfaces between mortar-less interlocking  
 286 bricks and the non-simultaneous damages of the interlocking bricks at the two shear  
 287 planes.

288 When the prisms are subjected to 30 kN axial pre-compression, similar behaviours  
 289 can be observed, but with a larger initial stiffness. Typically for Specimen 6, the initial  
 290 stiffness is 88.30 kN/mm due to the higher axial pre-compression. A peak shear load of  
 291 about 29.58 kN is achieved at around 0.88 mm displacement. After reaching the  
 292 maximum shear force, the applied force decreases steadily with further increased  
 293 displacement until residual strength is maintained. Larger peak shear resistance is  
 294 observed on the 30 kN pre-compressed prisms as compared to that of the 10 kN pre-  
 295 compressed prisms because the increased axial compression leads to higher inter-  
 296 surface friction [29].



297 *Figure 5. Load-displacement curves from numerical simulation and laboratory test*

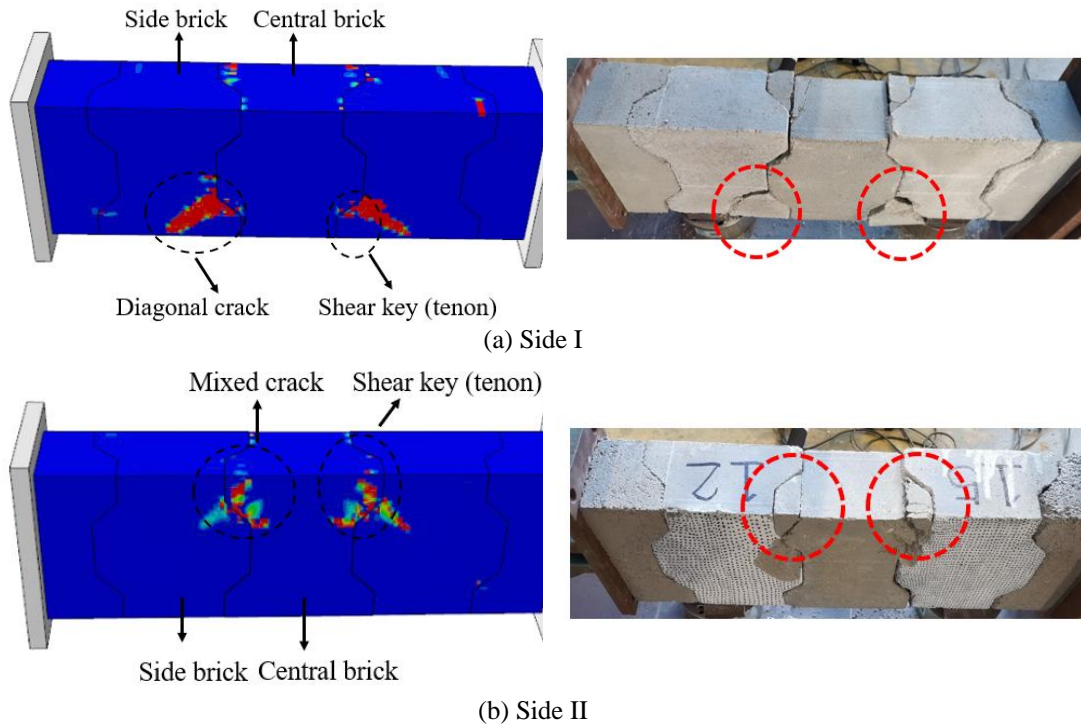
298 The numerically modelled shear force-displacement curves are compared with  
 299 those from the laboratory tests, as shown in Figure 5. It can be observed that the perfect

300 contact models largely overestimate the initial stiffness of the interlocking prisms under  
301 both 10 kN and 30 kN axial pre-compression cases. In comparison, the numerical  
302 models with random and simplified rough surfaces could more closely replicate the  
303 stiffness of the prism. For example, under 10 kN axial pre-compression, the perfect  
304 contact model predicts an initial stiffness of 136.87 kN/mm, while the simplified and  
305 random rough contact models predict 50.86 kN/mm and 38.88 kN/mm, respectively.  
306 Similarly, under 30 kN axial pre-compression, an initial stiffness of 188.31 kN/mm is  
307 predicted by the perfect contact model, which is much higher than those of 102.96  
308 kN/mm and 107.75 kN/mm by the random and simplified contact model. Nevertheless,  
309 these three models predict similar shear loading capacities. For example, under 10 kN  
310 axial pre-compression, the perfect contact model predicts a maximum shear load of  
311 23.17 kN in comparison to 21.86 kN and 21.69 kN for the simplified and random rough  
312 contact models indicating less than 10% difference. Similar trend can be found for the  
313 interlocking bricks under 30 kN axial pre-compression. It is evidenced that modelling  
314 of contact surface is crucial for accurate predictions of interlocking brick shear stiffness  
315 and capacity, under higher axial pre-compression, the difference between the random  
316 and simplified rough surface models is smaller. The simplified and detailed random  
317 rough surface models predict very similar shear capacity because the shear resistance  
318 is primarily provided by the shear key, while the contribution of the surface friction that  
319 is closely related to surface roughness condition is not pronounced. These results  
320 indicate that the random rough surface can be approximately modelled by simplified  
321 trapezoidal rough surface, which give similar predictions of the shear capacity, and  
322 close predictions of the shear stiffness especially when the axial compression force is  
323 relatively large.

#### 324 4.2 Failure mode and crack propagation

325 In the numerical simulation, crack initiation and evolution can be depicted by the  
326 damage contour since continuum element with damage based material model is  
327 employed [65]. Because of the unique shape of the interlocking bricks, there are two  
328 different failure patterns on the frontal and rear sides of the interlocking brick prism as  
329 shown in Figure 6, i.e., Side I and II. On Side I diagonal cracks are developed on the  
330 bottom shear keys of the two side bricks. This is accompanied by a brittle shear failure  
331 at the shear keys because of the principle stress on the plane reaching the failure strength  
332 [52]. Similarly, on Side II cracks initiate on the corner of the tenon of the central brick,

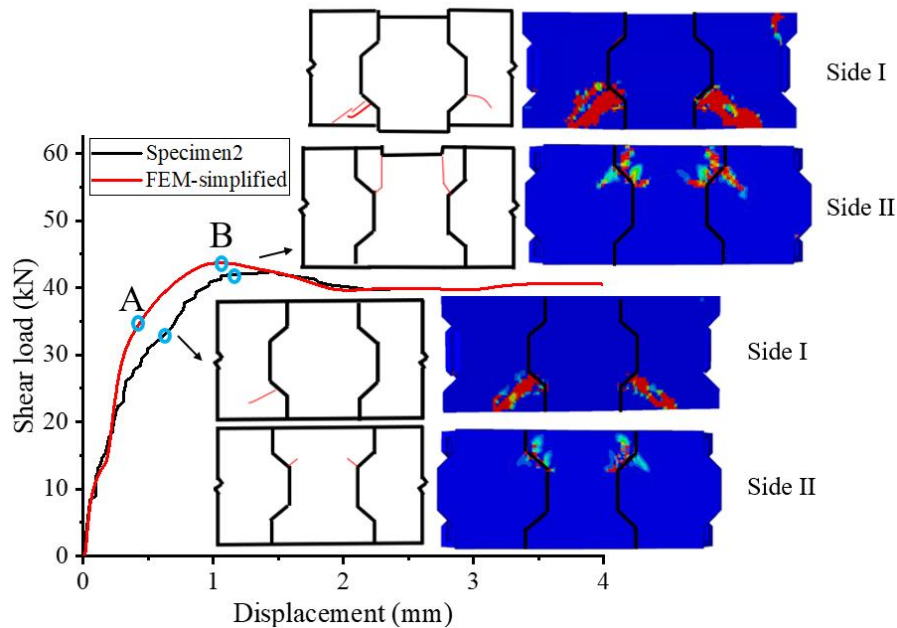
333 and then the cracks extend shortly in the direction perpendicular to the inclined surface  
 334 after which the cracks propagate vertically. Thus, the crack pattern on Side II is a mixed  
 335 crack mode (Figure 6b). The excessive shear stress leads to the eventual damages of  
 336 these tenons. After the failure of these tenons, only surface friction at the interfaces  
 337 resists the shear load which provides the residual shear capacity.



338 *Figure 6. Comparison of prism damage modes between numerical modelling and*  
 339 *laboratory test*

340 The crack initiation and propagation processes of the interlocking brick prism  
 341 modelled numerically and recorded in the lab test are plotted in the shear force versus  
 342 displacement curve as shown in Figure 7. As can be observed on Side I diagonal cracks  
 343 initiate on the bottom shear keys of the two side bricks at Stage A. They extend  
 344 diagonally at about 45°, associated with the slight decrease in the stiffness of the  
 345 specimen. With further applied vertical displacement on the central block, cracks  
 346 further develop leading to the further damage of the interlocking brick prism. Unlike  
 347 the numerically modelled cracks occurred simultaneously and symmetrically on both  
 348 sides of the interlocking brick prism, crack in the laboratory tested specimen occurred  
 349 only on one side first because of unavoidable asymmetry of the tested specimens owing  
 350 to imperfectness in preparing the bricks and interlocking specimens. But at the  
 351 maximum shear load, the crack patterns converge between numerical modelling and  
 352 experimental observation. It can also be observed from Figure 7 that on Side II the shear  
 353 key of the central brick cracks under the applied shear load at Stage A, which extend

354 diagonally at about 45° angle. With the further increased shear load, the cracks then  
 355 extend vertically and penetrate through the central brick at the maximum shear load.

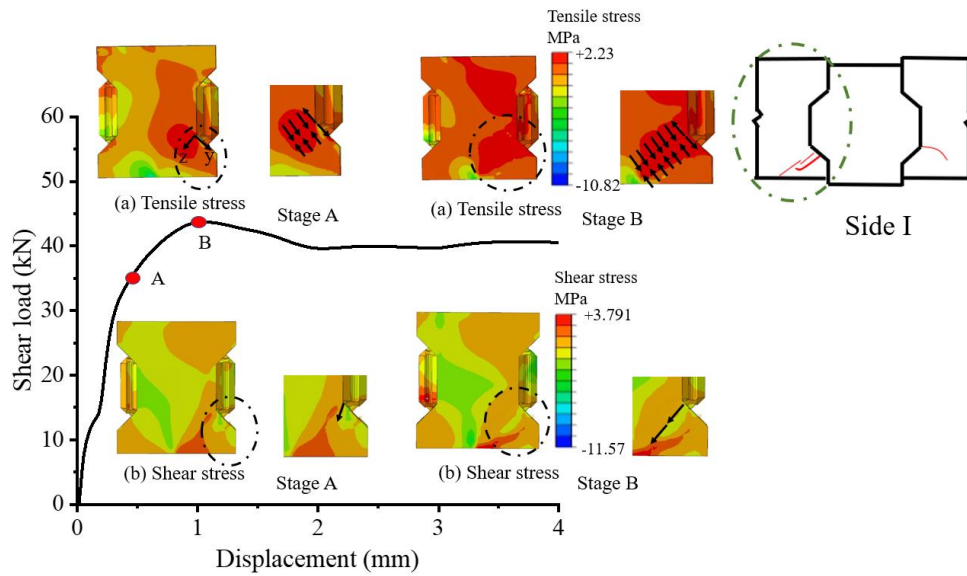


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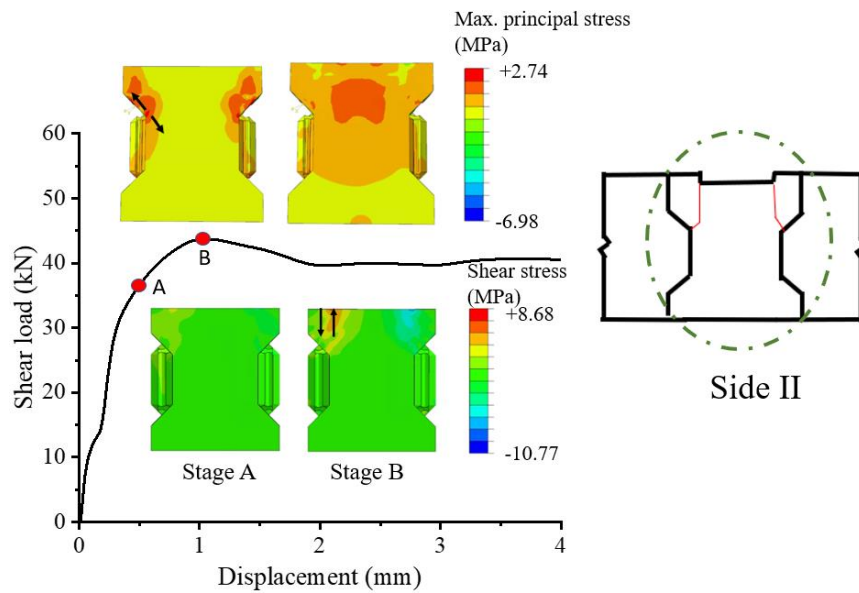
357 *Figure 7. Prism cracks evolution in the numerical simulations and experiments (Specimen*  
 358 *2 under 10 kN axial pre-compression)*

### 359 4.3 Stress distribution and crack evolution

360 To better understand the stress distribution in the interlocking brick prism, the  
 361 tensile stress and shear stress contours generated from the numerical modelling using  
 362 the simplified rough surface model are plotted along with the shear load versus  
 363 displacement curves in Figure 8. On Side I, the applied shear force on the shear plane  
 364 induces a large tensile stress around the shear key of the side brick. At Stage A, diagonal  
 365 crack appears due to excessive tensile stress at the shear key, which extends and  
 366 propagates under the further increased shear force on the interlocking joint. For the  
 367 central block on Side II (Figure 8b), a large tensile stress is generated around the shear  
 368 key because of geometry change induced stress concentration. Tensile cracks (mode I)  
 369 are initiated at Stage A, which extend diagonally at about 45°. As the applied shear load  
 370 gradually increases, the propagation of diagonal cracks ceased because it enters a low  
 371 tensile stress field which therefore would release less strain energy. The formation and  
 372 propagation of the diagonal crack results in the rotation of the shear key and varies the  
 373 boundary condition of the stress zone. As a result, a large shear stress is induced around  
 374 the shear key (as in Figure 8b), which consequentially leads to the further development  
 375 of the crack under the shear stress (mode II crack) until the total failure of the shear key  
 376 on the central block at Stage B.



(a) The diagonal crack evolution in the side brick on Side I



(b) The mixed crack evolution in the central brick on Side II

377

*Figure 8. Crack evolution of prism stress contours*

#### 378 4.4 Shear capacity

379 To examine the shear capacity of the interlocking prism, the wedge crack model  
 380 (WCM) [66] is employed. As illustrated in Figure 9, the vertical force from the shear  
 381 load on the interlocking joint is  $F$  and the horizontal force from axial compression  
 382 acting on the cracking area is  $F'$ , which leads to the crack initiation. The stress intensity  
 383 factor of mode I crack [67] at the crack tip, generated by the wedging forces  $F$  and  $F'$   
 384 ( $K_{Ia}$ ), can be given as follows:

$$K_I = 2(F \sin \theta - F' \cos \theta) \sqrt{\frac{\pi}{(\pi^2 - 4)l}} \quad (1)$$

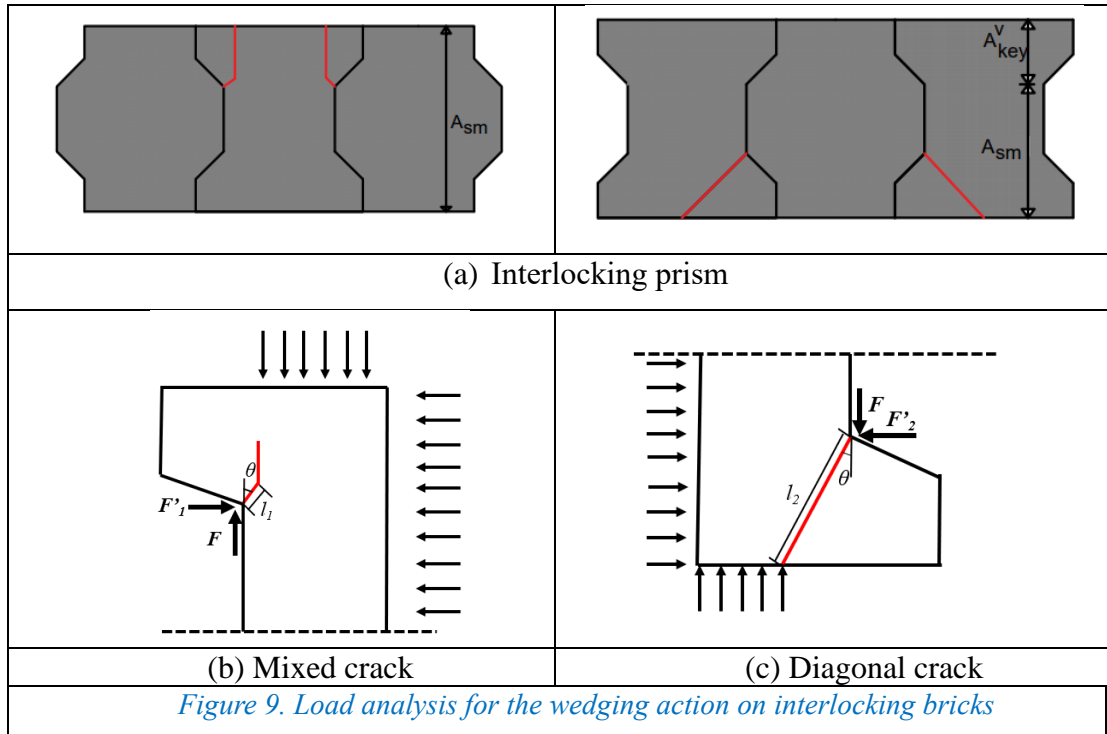
385 where  $l$  stands for the length of the diagonal crack; subscription I represents mode I  
 386 crack. The crack angle  $\theta$  is assumed to be  $45^\circ$  following laboratory observation and  
 387 previous studies [68-70]. For mode I fracture, crack is initiated when the stress intensity  
 388 factor  $K_I$  reaches  $K_{IC}$ , where  $K_{IC} = 0.0443 \text{ MPa}\cdot\text{m}^{1/2}$  denotes the fracture toughness of  
 389 material [71]. Therefore, the maximum shear capacity  $V$  for the interlocking brick at  
 390 the joint can be expressed as:

$$V = F_{pre} \mu + \left( K_{IC} \sqrt{\frac{(\pi^2 - 4)l_1}{\pi}} \times \frac{1}{2} + F'_1 \cos \theta \right) \frac{1}{\sin \theta} \quad (2)$$

$$+ \left( K_{IC} \sqrt{\frac{(\pi^2 - 4)l_2}{\pi}} \times \frac{1}{2} + F'_2 \cos \theta \right) \frac{1}{\sin \theta}$$

391 Where  $F_{pre}$  is the axial pre-compressive force on the interlocking prism, and  $\mu$  is the  
 392 surface friction coefficient, which equals to 0.3. The term  $F_{pre} \mu$  in Eq. (2) accounts for  
 393 the friction resistance force at the interlocking joint. Substituting crack lengths  
 394  $l_1 = 10 \text{ mm}$  for the central brick and  $l_2 = 88 \text{ mm}$  for the side bricks, the maximum shear  
 395 capacity  $V$  for the interlocking brick can be calculated.

396



397



398 In the meanwhile, following design code EN-1052-3 [58], the equivalent shear  
399 strength for the interlocking bricks can be calculated for simplified engineering  
400 application by assuming two shear planes created between the central block and the two  
401 side blocks as shown in Figure 6. The equivalent shear strength is calculated using the  
402 shear load on the interlocking joint (one shear plane) dividing the cross-sectional area  
403 of the joint (200mm height  $\times$  100mm depth minus the area of two holes). Table 3  
404 summarizes the peak shear load on the interlocking joint ( $V_1$ ), the equivalent shear  
405 strength ( $\tau_1$ ), the associated displacement ( $\delta_1$ ) and the initial shear stiffness. Among the  
406 three specimens of each group of tests, a coefficient of variation (CoV) about 10% is  
407 found which indicates that the test results vary in an acceptable small range. When  
408 subjected to 10 kN axial compression, an averaged equivalent shear strength of 1.17  
409 MPa is measured, which is 27% lower than that under 30 kN axial pre-compression  
410 (1.48 MPa). This is expected as axial pre-compression could influence both the friction  
411 resistance and the shear resistance of the shear key. Therefore, axial pre-compression  
412 level should be considered when evaluating the shear strength of interlocking bricks.

413 *Table 3. The results obtained from the shear tests*

Prisms	$V_1$ /kN	$\tau_1$ /MPa	$\delta_1$ /mm	Initial shear stiffness/(kN/mm)
Preload-10-1	19.42	1.04	1.31	57.12
Preload-10-2	21.18	1.14	1.46	56.42
Preload-10-3	24.51	1.32	1.74	53.00
Average	21.70	1.17	1.50	55.51
Preload-30-1	25.31	1.36	0.54	107.40
Preload-30-2	27.80	1.50	1.75	164.00
Preload-30-3	29.58	1.59	0.88	163.51
Average	27.56	1.48	1.06	144.97

414

#### 415 4.5 Comparison with design formula

416 Comparison is made between the above tested shear capacity for the interlocking  
417 brick and existing empirical formula and design method to evaluate the accuracy and  
418 suitability of these existing methods. In engineering practice, AASHTO design code (in  
419 Eq. (3)) [55] and the semi-empirical formula proposed by Rombach and Specker [32,  
420 72, 73] (in Eq. (4)) are the commonly used methods. As shown, both methods separate  
421 the shear capacity  $V$  of a keyed joint into two parts: a) resistance from the interlocking  
422 key; and b) interface friction.

$$V = A_{key}^V (0.006792 f_{cm})^{0.5} \times (12 + 2.466 \sigma_n) + \mu A_{sm} \sigma_n \quad (3)$$

$$V = 0.14 f_{cm} A_{key}^V + 0.65 (A_{key}^V + A_{sm}) \sigma_n \quad (4)$$

423 where  $A_{key}^V$  is the projection area of shear keys on the failure plane ( $\text{mm}^2$ ); and  $A_{sm}$  is  
 424 the contacting area between flat contact surfaces in the failure plane ( $\text{mm}^2$ ), as shown  
 425 in Figure 9a;  $f_{cm}$  is the characteristic compressive strength of material (MPa);  $\sigma_n$  is the  
 426 average compressive stress across the key base area (MPa); and  $\mu$  is the friction  
 427 coefficient between the contacting surfaces, which AASHTO recommends as 0.6.

428 *Table 4. Comparison of shear capacities between laboratory testing results, numerical*  
 429 *modeling and existing empirical and design formulae*

Pre- load kN	Maximum shear force kN	Numerical simulation kN	error %	AASHTO kN	error %	Rombach & Specker kN	Error %	Theoretical prediction kN	error %
10	21.70	21.86	0.74	21.19	-2.37	15.29	-29.55	22.04	1.57
30	27.56	28.16	2.18	34.16	23.96	28.29	2.64	32.09	16.44

430 Note: AASHTO, Rombach & Specker and Theoretical prediction denote the results from Eq. (3); Eq. (4); and Eq.  
 431 (2), respectively.

432  
 433 Table 4 compares the shear capacity of the interlocking bricks under different axial  
 434 pre-compressions and those estimated by AASHTO, Rombach and Specker's formula  
 435 and the theoretical derivation presented above. Rombach and Specker carried out  
 436 parametric study using numerical modelling and provided an empirical formula for  
 437 estimation of the shear capacity of interlocking joint [73, 74]. Comparing the prediction  
 438 results using their formula with the current testing results, it can be found that Rombach  
 439 and Specker's formula substantially underestimates the shear capacity by about 30%  
 440 when the prism is subjected to 10 kN axial pre-compression. This is because in their  
 441 study, very small concrete shear keys were considered and direct key shear off failure  
 442 was the primary failure mode, which differs to the failure mode of the interlocking brick  
 443 prisms in this study. As the axial pre-compressive force applied to the prism increases,  
 444 the prediction error using Rombach and Specker's formula reduces, which only slightly  
 445 overestimates the 30kN pre-compression cases by 2.64%. This is because the  
 446 contribution percentage of friction resistance in the overall shear capacity increases and  
 447 that of shear key reduces with the increase in axial pre-compression. As a result, the  
 448 relative error reduces. The AASHTO formula predicts different shear capacities of the  
 449 interlocking brick prism, which slightly underestimates the shear capacity of the  
 450 interlocking prism by -2.37% when 10 kN axial pre-compression is applied, but it  
 451 overestimates the prism shear capacity by 23.96% when it is subjected to 30 kN axial  
 452 pre-compression. This prediction error by the AASHTO formula could be attributed to  
 453 the following two reasons: firstly, the AASHTO formula is empirically derived based

454 on a large amount of testing data on concrete joints with small shear keys, which being  
455 similar to the Rombach and Specker's method is not necessarily suitable for prediction  
456 of the shear capacity of large shear key. Secondly, AASHTO specifies a large friction  
457 coefficient of 0.6, which could overestimate the friction resistance at the joint between  
458 interlocking bricks. Therefore, under low axial pre-compression, AASHTO method  
459 underestimates the shear resistance of the shear key but overestimates the friction  
460 coefficient, whose effects cancel each other and ends up a closer match with the lab  
461 testing results. But when the axial pre-compression level is high, the contribution of  
462 friction becomes more pronounced. The AASHTO method gives a much higher  
463 prediction on the shear capacity of interlocking bricks, which is very similar to the  
464 observation given by Zhou et al. on the precast concrete joint, who reported consistently  
465 higher shear capacity was predicted using AASHTO method than that using the  
466 Rombach and Specker's formula. Therefore, the existing methods may not accurately  
467 predict the shear capacity of interlocking bricks. The theoretical derivation based on  
468 fracture mechanics theory overestimates the shear capacities of the interlocking brick  
469 prism by +1.57% and +16.44% when subjected to 10 kN and 30 kN pre-compression,  
470 respectively. This is possibly because the rough surface of the interlocking bricks is not  
471 considered, and thus the friction resistance estimation is not accurate.

472

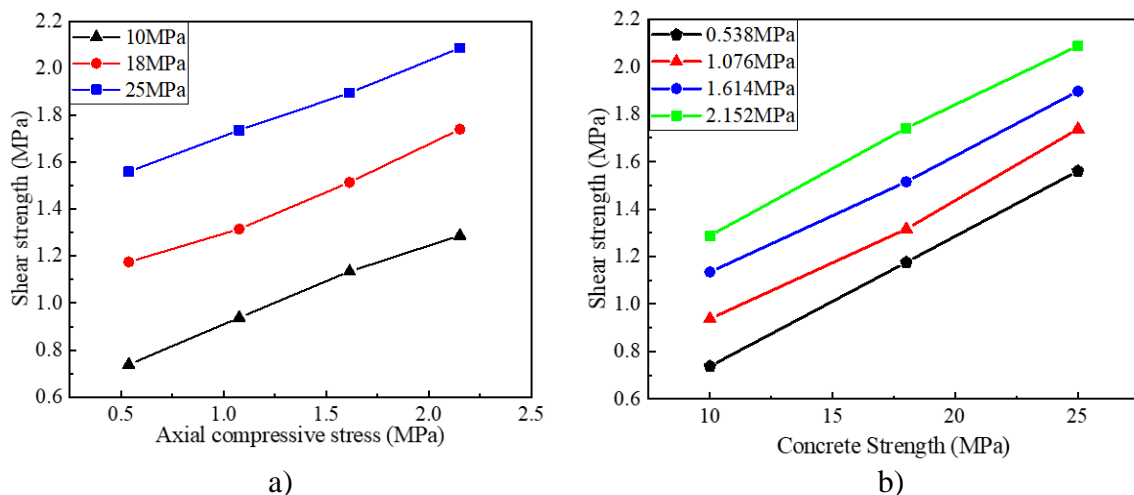
## 473 **5 Parametric study**

474 To evaluate the influence of several design parameters on the shear capacity of  
475 interlocking bricks, and to derive an empirical formula for prediction of the shear  
476 capacity of interlocking bricks for engineering applications, parametric studies are  
477 carried out by varying the axial pre-compression level, surface friction coefficient,  
478 contact surface roughness, and concrete strength.

### 479 **5.1 Effects of axial pre-compression and concrete strength**

480 To quantify the influences of axial pre-compression and brick material  
481 compressive strength on the maximum shear load bearing capacity of the interlocking  
482 brick prism, a number of numerical simulations are conducted. The dimension of the  
483 brick is 200 mm × 100mm × 180 mm (length × thickness × height) as default. The mean  
484 brick surface roughness is assumed to be 0.3 mm, and the coefficient of friction is 0.3,  
485 which are based on the default brick configuration. Four different axial pre-  
486 compression levels are modelled, i.e. 0.538 MPa, 1.073 MPa, 1.614 MPa and 2.152

487 MPa, which correspond approximately to the axial stress levels in a ground floor wall  
 488 for single- or low rise multiple-storey buildings [75]. Three different material strengths  
 489 with  $f_c=10$  MPa, 18 MPa, and 25 MPa are considered which are commonly used for  
 490 concrete masonry units. Figure 10a shows the equivalent shear strength versus axial  
 491 pre-compressive stress. As shown, the equivalent shear strength increases with the axial  
 492 pre-compressive stress. For example, for the interlocking prism with material strength  
 493 of 25 MPa, the equivalent shear strength is 1.56 MPa when it is subjected to a 0.538  
 494 MPa axial pre-compressive stress, and it increases to 2.09 MPa when axial pre-  
 495 compressive stress is 2.15 MPa. The relationship between concrete strength and the  
 496 equivalent shear strength is shown in Figure 10b. As expected, brick material strength  
 497 also strongly influences the brick shear strength. For instance, as the material  
 498 compressive strength increases from 10 MPa to 25 MPa, the equivalent shear strength  
 499 of the interlocking brick prisms (under 2.152 MPa axial pre-compressive stress)  
 500 increases by +62.06%.



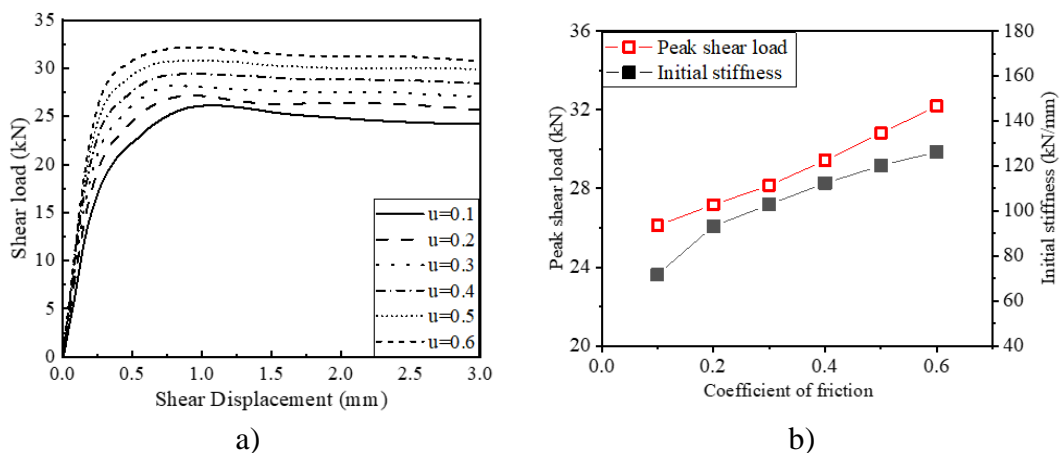
501 *Figure 10. Relationships between a) prism equivalent shear strength with axial compressive*  
 502 *stress; b) prism equivalent shear strength with concrete strength*

503

## 504 5.2 Effect of surface roughness and friction coefficient

505 Both the shear resistance of the interlocking shear key and the interface friction  
 506 contribute to the shear resistance at interlocking joint. Interface friction as a macro-  
 507 level effect and surface roughness as a micro-level effect could both influence the  
 508 friction induced shear resistance of interlocking bricks. To quantify the influence of  
 509 interface friction coefficient on the prism shear capacity, a sensitivity analysis is carried  
 510 out, where a mean brick surface roughness is assumed to be 0.3 mm, the axial pre-  
 511 compression is 30 kN, the material strength is 18 MPa, and the dimension of the

512 interlocking brick is as default. Friction coefficient  $\mu$  is varied from 0.1 to 0.6 with a  
 513 0.1 increment. Figure 11 shows the modelling results. It can be observed that with the  
 514 increase of friction coefficient from 0.1 to 0.6, the initial stiffness increases from  
 515 71.74kN/mm to 126.29kN/mm by 76%. This is because a large shear force is needed to  
 516 initiate the inter-block slip when the friction coefficient increases. The peak shear  
 517 resistances of the interlocking brick prisms also increase as the friction coefficient  
 518 increases. With friction coefficient increases from 0.1 to 0.6, the peak shear load  
 519 increases from 26.12kN to 32.19kN by 23.24%. This is expected because friction  
 520 resistance contributes to the shear capacity of the interlocking brick.

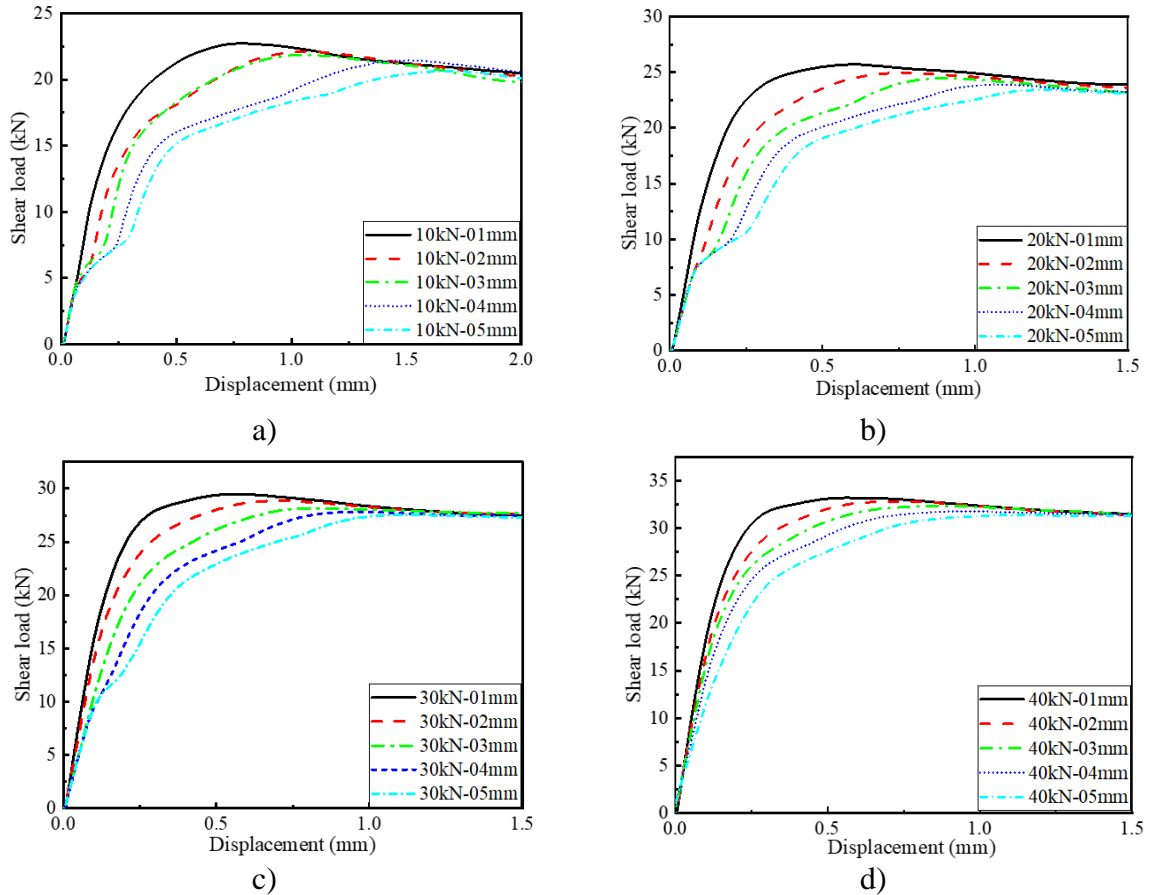


521 *Figure 11. Effect of surface friction coefficient on a) shear load-displacement curves and*  
 522 *b) peak shear load and initial stiffness*

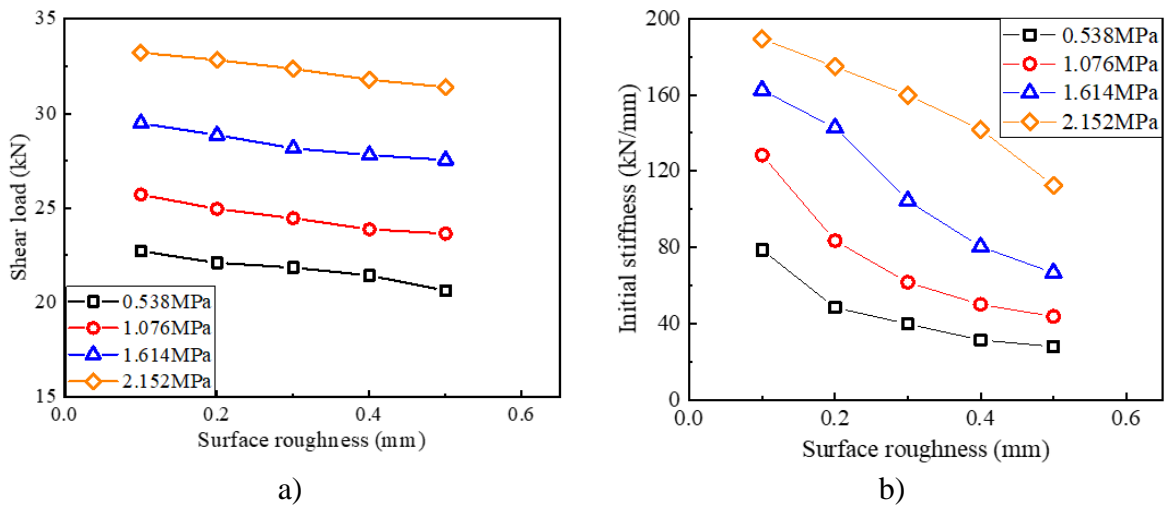
523

524 To quantify the influence of brick surface roughness on the shear resistance of the  
 525 interlocking prism, surface roughness with height ranging between 0.1 and 0.5mm at  
 526 an interval of 0.1mm is numerically modelled on the interlocking bricks. The  
 527 unconfined uniaxial compressive strength of the brick material is 18MPa; the axial pre-  
 528 compression varies from 10 kN to 40 kN, and the friction coefficient is 0.3. The shear  
 529 force versus displacement relationships of specimens with different surface roughness  
 530 conditons as shown in Figure 12. For the interlocking brick prisms under 10 kN and 20  
 531 kN axial pre-compression, non-linear behaviour can be observed in the rising sections  
 532 of the curves when the surface roughness is above 0.2 mm. This is because of the local  
 533 compaction of the rough surfaces under axial compression, which is not obvious when  
 534 the surface roughness is 0.1 mm. Under a higher axial pre-compression, this non-linear  
 535 behaviour becomes unrecognizable. It can also be observed that with increased surface  
 536 roughness, the displacement at the peak shear load increases. This is because a larger  
 537 displacement is needed for the asperities in the rough surface to achieve the maximum

538 shear resistance. Similar influence of surface roughness can be found on the initial  
 539 stiffness. As summarized in Figure 13, under 20 kN axial pre-compression, the initial  
 540 stiffness is 128.42 kN/mm for the interlocking brick prism with 0.1 mm surface  
 541 roughness, which decreases to 83.55 kN/mm and 43.77 kN/mm when the surface  
 542 roughness increases to 0.2 mm and 0.5 mm.



543 *Figure 12. Shear load versus displacement curves for interlocking brick with different*  
 544 *surface roughness a) under 10kN axial pre-compression; b) 20 kN pre-compression; c) 30 kN*  
 545 *pre-compression; d) 40 kN pre-compression*



546 *Figure 13. Effect of the surface roughness on a) peak shear load; and b) initial stiffness*

547

## 548 **6 Empirical Formula**

549 The above results demonstrate existing analysis and design formulae may not provide  
550 accurate predictions of the shear resistance of the interlocking brick prism. This could  
551 be attributed to the different shear key failure mechanism, inappropriate surface friction  
552 coefficient used in the calculation, and lack of consideration of contact surface  
553 roughness [40]. Based on the laboratory test results and numerical parametric study  
554 results, a material failure based empirical prediction formula is proposed herein.

### 555 6.1 Material failure model

556 The following equation with reference to AASHTO is employed to define the  
557 shear resistance capacity of the interlocking brick prism as:

$$V_j = A_{key}^V f'_c (C_1 + C_2 \sigma_n) + \mu A_{sm} \sigma_n \quad (5)$$

558 where  $A_{key}^V f'_c (C_1 + C_2 \sigma_n)$  defines the contribution from the shear keys, and  $\mu A_{sm} \sigma_n$  is the  
559 contribution from the friction resistance.

560 With large shear keys in interlocking bricks, the damage and failure of shear keys  
561 differ from those of small shear keys as defined in AASHTO. It is therefore necessary  
562 to properly re-examine the stress state and define the failure.

563 The failure envelope is employed herein which is based on the modification  
564 suggested by Hofbeck et al.[76]. The detailed derivation is presented in Appendix A.  
565  $C_1$  is the coefficient of shear strength, which takes into account the strength provided  
566 by interlocking keys ignoring the axial pre-compression,  $C_1$ , can be written as

$$C_1 = \frac{0.2125 \cos \theta}{\sqrt{\left(\frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H}\right)^2 + 1} - \frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H} \sin \theta} \quad (6)$$

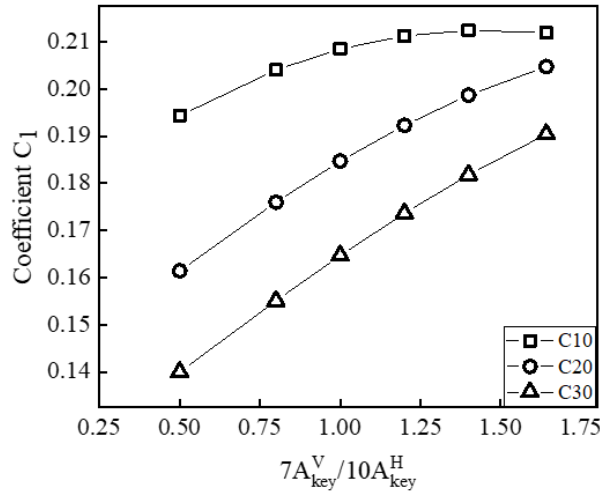
567 where  $\theta$  is the inclined angle of the line  $L_2$  relative to stress axis  $\sigma$ , which is tangent to  
568 the Mohr's circle at failure under uniaxial tension (see Figure A-2);  $A_{key}^H$  is the  
569 horizontal projection area of the interlocking key along the direction of pre-compressive  
570 force,  $A_{key}^V$  is the vertical projection area of the shear key along the direction of applied  
571 vertical force.

572 Considering brick material characteristic compressive strength  $f_{cu,k}$  varying from  
573 10 MPa to 30 MPa which are common range for concrete masonry units, and various  
574 interlocking brick geometry  $A_{key}^V/A_{key}^H$ , the coefficient factor  $C_1$  can be calculated and  
575 shown in Figure 14. A conservative  $C_1=0.14$  is determined with the current brick

576 material strength and shear key geometry. As derived in the Appendix, when the axial  
 577 pre-compression exists, the shear resistance by the shear key comprises coefficient  $C_2$   
 578 which can be expressed as

$$C_2 = \frac{-B + \sqrt{B^2 - 4AC}}{2A\sigma_x f'_c} - \frac{0.2125 \cos \theta}{\sigma_x \left( \sqrt{\left( \frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H} \right)^2 + 1} - \frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H} \sin \theta \right)} \quad (7)$$

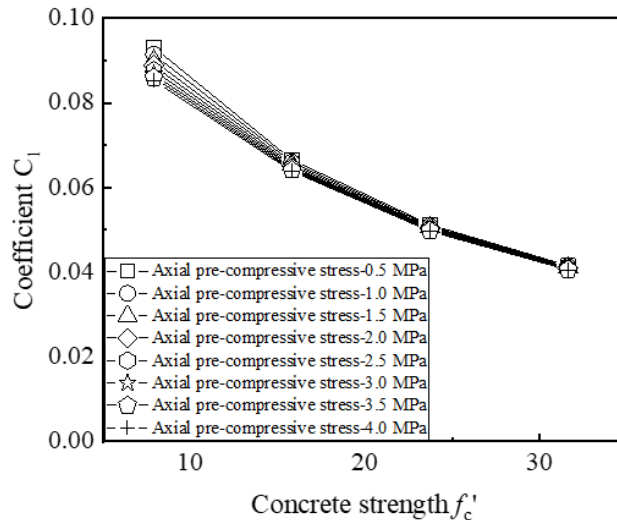
579 where  $\sigma_x$  is the normal stress due to axial pre-compression, and  $f'_c$  is the concrete  
 580 compressive strength;  $A$ ,  $B$  and  $C$  represent the geometry coefficients (see Appendix A  
 581 for the details).



582

583 *Figure 14. Coefficient  $C_1$  with respect to different geometries of interlocking key*

584



585

586 *Figure 15. Coefficient  $C_2$  with varying material strength*

587 The variation of  $C_2$  with material strength as well as axial pre-compression is  
 588 shown in Figure 15. It is found that the coefficient  $C_2$  changes insignificantly with axial



589 pre-compression stress  $\sigma_n$  ( $\sigma_x = \sigma_n$  in Eq. 5). However, the coefficient  $C_2$  decreases, as  
 590 the material compressive strength increases. The relationship between the coefficient  
 591  $C_2$  and material compressive strength  $f'_c$  are linearly fitted and shown in Eq. (8).

$$C_2 = -0.002f'_c + 0.10076 \quad (8)$$

592 Substituting Eq. (8) and  $C_1 = 0.14$  into Eq. (5), the shear capacity of interlocking  
 593 brick is expressed using the following equation.

$$V_j = A_{key}^V f'_c (0.14 + (-0.002f'_c + 0.10076)\sigma_n) + uA_{sm}\sigma_n \quad (9)$$

594 It is worth noting that this equation is applicable to the interlocking brick in this  
 595 study, whose geometry was optimized with proved best mechanical performance [77].

596

## 597 6.2 Modified design formula

598 To consider the influence of the brick surface roughness, modification is made by  
 599 introducing correction factors,  $f(h_{imp})$  and  $g(h_{imp})$  in the analytical solution of Eq. (9)  
 600 based on the results from the numerical simulations and laboratory tests, which account  
 601 for the influence of surface roughness on the shear resistance for the shear key and the  
 602 rest flat regions. The shear capacity of an interlocking brick prism,  $V_{j,imp}$ , is given as  
 603 follows:

$$V_{j,imp} = f(h_{imp}) \cdot A_{key}^V f'_c (0.14 + (-0.002f'_c + 0.10076)\sigma_n) + \mu \cdot g(h_{imp}) A_{sm}\sigma_n \quad (10)$$

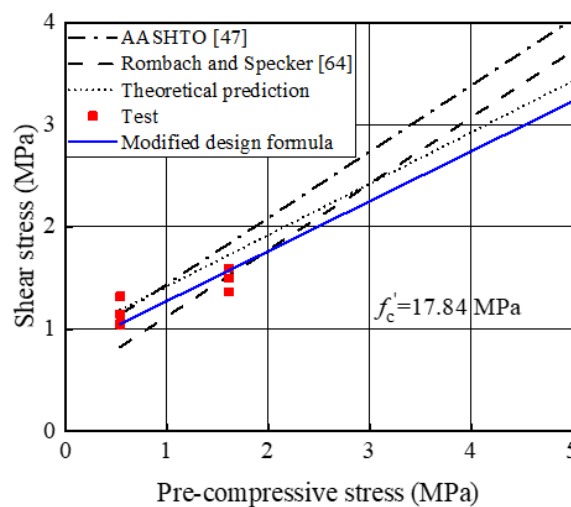
604 Regression analysis on the simulations and laboratory testing is carried out to obtain  
 605 the above modification coefficients in the proposed formula. The coefficient of  
 606 determination ( $R^2$ ) is found to be 95.44% for Eq. (11), which shows the predicted results  
 607 are in good agreement with the values from the test and numerical modelling. The  
 608 predicted prism shear strength is positively related to the material compressive strength,  
 609 and inversely proportional to the roughness amplitude.

$$V_{j,imp} = (-0.3033h_{imp} + 1.7519)A_{key}^V f'_c (0.14 + (-0.002f'_c + 0.10076)\sigma_n) + \mu(-0.0884h_{imp} + 0.5353)A_{sm}\sigma_n \quad (11)$$

610 where  $h_{imp}$  is the surface roughness varying from 0.1 mm to 0.5 mm;  $f'_c$  denotes  
 611 material compressive strength varying from 10 MPa to 30 MPa;  $\sigma_n$  stands for the  
 612 normal stress from axial pre-compressive stress ranging from 0.54 MPa to 2.15 MPa;  
 613 and  $\mu$  is the friction coefficient ranging from 0.1 to 0.6.

614 The predicted shear strength using the above proposed formula, existing design  
 615 methods, and laboratory testing data are compared in Figure 16. The AASHTO design  
 616 specification and the theoretical prediction is unconservative in predicting the shear  
 617 strength of the interlocking key. Rombach and Specker's formula underestimates the

618 shear capacity when the pre-compression is low and overestimates it when the pre-  
 619 compression is high. Using the modified design formula, the shear strength of the  
 620 specimen with 10 kN pre-compression is estimated to be 1.048 MPa, which agrees well  
 621 with the 1.17 MPa shear strength obtained from the laboratory tests, yielding only about  
 622 10% difference. When the pre-compression 30 kN, the shear strength is estimated to be  
 623 1.57 MPa, while the tested strength is 1.48 MPa, indicating a discrepancy of only 6%.  
 624 Therefore, it can be concluded that the proposed formula can better predict the shear  
 625 strength of the interlocking brick under different conditions as compared to the  
 626 AASHTO, Rombach and Specher's method and existing theoretical prediction.



627  
 628 *Figure 16. Comparison between different design models*  
 629

## 630 7 Conclusions

631 In this study, numerical modeling and laboratory tests are conducted to investigate the  
 632 shear behavior of interlocking brick. The failure modes of mortar-less interlocking  
 633 brick prisms are investigated. Three-dimension (3D) numerical models of the  
 634 interlocking brick prism are developed using three different contact modelling  
 635 approaches, which are validated against the laboratory testing results. Parametric study  
 636 is conducted to evaluate the influences of friction coefficient, axial pre-compression,  
 637 brick material strength and interface roughness because of brick surface unevenness on  
 638 the shear capacity. Combining the testing results and numerical simulation, a modified  
 639 analytical formula is proposed for prediction of the shear strength capacity of the  
 640 interlocking brick prism. The following conclusions have been drawn:

- 641 • Laboratory test and numerical modeling show the shear strength of the  
 642 interlocking prism is dependent on the pre-compression level.

- 643       • Numerical simulations with three different contact modelling approaches  
644       demonstrate that modelling the brick surface roughness is important for the  
645       reliable prediction of interlocking brick shear behavior. The simplified rough  
646       contact model is found to be able to give a good prediction of prism initial  
647       stiffness, and shear capacities, whereas the model with perfect contact leads to  
648       large prediction error.
- 649       • Existing design and analysis method may not accurately predict the shear  
650       strength of the interlocking brick with large keys because of the different shear  
651       failure mechanism, negligence of interface roughness, and inappropriate friction  
652       coefficient.
- 653       • Parametric study evaluates the influences of the coefficient of friction, axial pre-  
654       compression, interface roughness, and material compressive strength on the  
655       interlocking prism shear strength.
- 656       • A modified analysis and design formula with consideration of brick surface  
657       condition is proposed for prediction of the shear capacity of interlocking brick  
658       prism.

659

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860  
861

862 **Appendix – Interlocking shear resistance model**

863 The shear resistant mechanism of the interlocking brick prism comprises of two parts,  
 864 i.e., shear keys and interface friction, which can be expressed as

$$V_j = A_{key}^V f'_c (C_1 + C_2 \sigma_n) + u A_{sm} \sigma_n \quad (A-1)$$

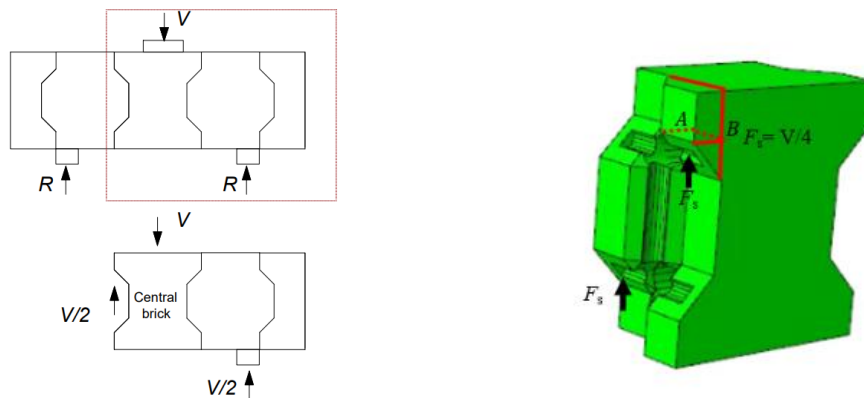
865 The contribution of shear key is influenced by material strength  $f'_c$ , normal stress  
 866 from axial pre-compression  $\sigma_n$ , and shear key geometry. To determine coefficient  $C_1$   
 867 and  $C_2$ , the influence of material shear strength on shear key resistance is analysed first.  
 868 Figure A-1(a) illustrates the free body diagram of the interlocking brick prism.  $V$  is the  
 869 applied vertical force on the brick prims. The force on each interlocking brick joint is  
 870  $V/2$  due to symmetry. Since there are two shear keys on each joint bearing this force,  
 871 the shear force  $F_s$  on each shear key equals to  $V/4$ . Take a typical element on the shear  
 872 key for stress analysis which experiences axial compressive stress  $\sigma_x$  from axial pre-  
 873 compression, shear stress  $\tau$  and normal stress  $\sigma_y$ , which can be calculated with the  
 874 applied forces on the prism as

$$\tau = \frac{F_s}{A_{key}^V} = \frac{V}{4A_{key}^V} \quad (A-2)$$

$$\sigma_x = \frac{N}{A_{brick}} \quad (A-3)$$

$$\sigma_y = \frac{7F_s}{10A_{key}^H} = \frac{7V}{40A_{key}^H} \quad (A-4)$$

875 where  $V$  is the applied vertical force on the brick prims,  $N$  is the axial pre-compression  
 876 force,  $A_{key}^H$  is the horizontal projection area of the interlocking key along the direction  
 877 of pre-compressive force, and  $A_{key}^V$  is the vertical projection area of the interlocking  
 878 key along the direction of the applied vertical shear force,  $A_{brick}$  is the cross-sectional  
 879 area of the interlocking brick perpendicular to the axial pre-compression direction, as  
 880 shown in Figure A-1(c). Detailed derivation of  $\sigma_y$  is provided in Figure A-1.





881

(a) Free body diagram

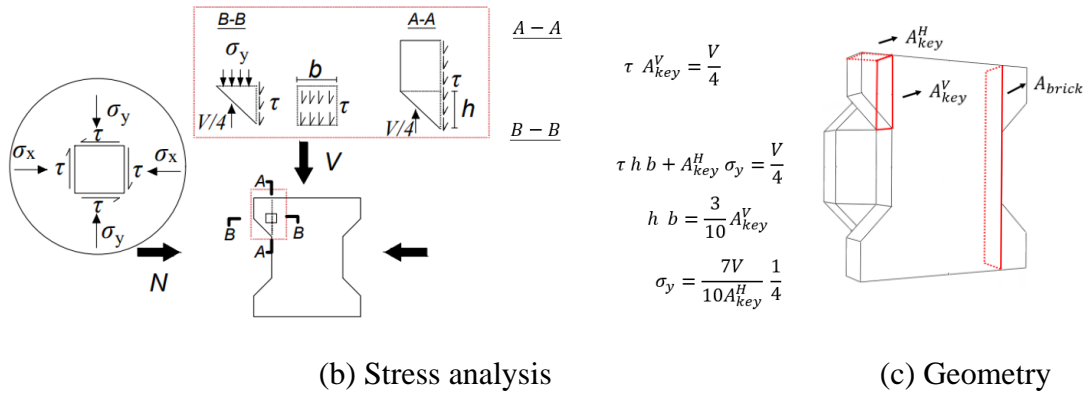


Figure A-1. Free body diagram and stress state

882

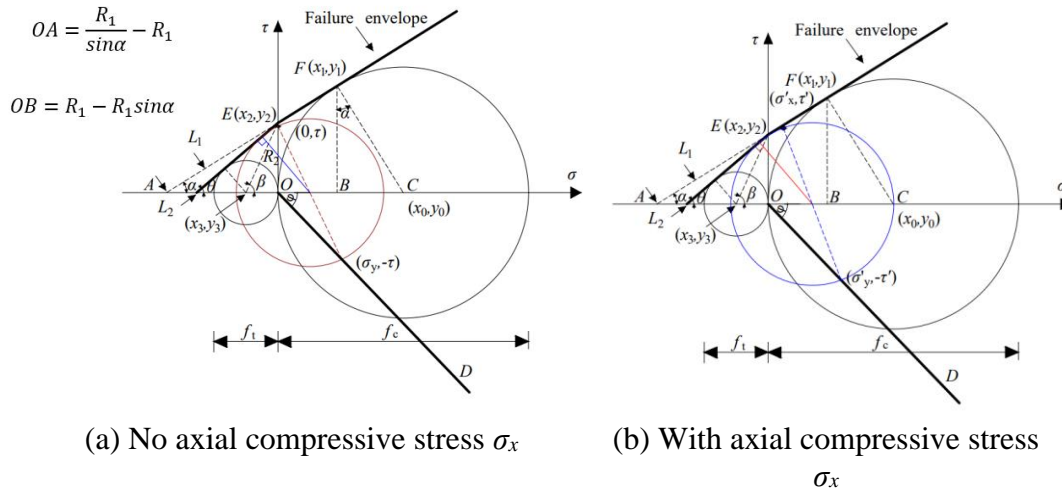


Figure A-2. Stress state and failure envelope

883

884 To define the failure of the brick, modified Mohr-Coulomb criteria is used. As  
 885 shown in Figure A-2a, line  $L_1$  defines the original concrete failure surface, which is  
 886 inclined at  $\alpha=37^\circ$  to the normal stress axis  $\sigma$  and tangent to the Mohr's circle for  
 887 uniaxial compression. The tangent point of  $L_1$  with the Mohr's circle for uniaxial  
 888 compression is  $(x_1, y_1)$ . The coordinate of  $(x_1, y_1)$  can be written as

$$x_1 = x_0 - R_1 \sin \alpha = f_c/2 - f_c/2 \cdot \sin \alpha = f_c/2 \cdot (1 - \sin \alpha) \quad (\text{A-5a})$$

$$y_1 = y_0 + R_1 \cos \alpha = 0 + f_c/2 \cdot \cos \alpha = f_c/2 \cdot \cos \alpha \quad (\text{A-5b})$$

889 where  $\alpha=37^\circ$ ,  $R_1$  is radius of the Mohr's circle for uniaxial compression that equals to  
 890  $f_c/2$ .

891 The point of intersection of line  $L_1$  with the  $\tau$  axis is  $(x_2, y_2)$ , in which  $x_2 = 0$ .  
 892 Considering triangle similarity rule between  $AOE$  and  $ABF$ ,  $y_2$  can be expressed as

$$y_2 = \frac{OA}{AB} \cdot y_1 = \frac{1}{1 + \frac{OB}{OA}} \cdot y_1 = \frac{1}{1 + \frac{R_1 - R_1 \sin \alpha}{\frac{R_1}{\sin \alpha} - R_1}} \cdot R_1 \cdot \cos \alpha = f_c/2 \cdot \frac{\cos \alpha}{1 + \sin \alpha} \quad (\text{A-6})$$

893 Substituting  $\alpha = 37^\circ$  in Eq. A-6,  $y_2 = 0.25f_c$ .

894 Line  $L_2$  is the modified concrete failure surface in the tensile region, which is drawn  
 895 from the point of intersection with  $\tau$  axis, and is tangent to the Mohr's circle for  
 896 uniaxial tensile failure. The tangent angle is relative to stress axis is  $\theta$ . The centre  
 897 coordinate of the uniaxial tensile strength circle is  $(x_3, y_3) = (-f_t/2, 0)$ . The angle of the  
 898 line connecting point  $(x_2, y_2)$  and  $(x_3, y_3)$  relative to  $\sigma$  axis is  $\beta$ , which can be calculated  
 899 by

$$\tan \beta = \frac{|y_2|}{|x_3|} = \frac{0.25f_c}{f_t/2} = 0.5 \frac{f_c}{f_t} \quad (\text{A-7a})$$

$$\beta = \tan^{-1}(0.5 \frac{f_c}{f_t}) \quad (\text{A-7b})$$

900 Typically for concrete like material, the uniaxial compression strength  $f_c$  is taken as  
 901  $0.85 f_c'$  and the tensile strength  $f_t$  is taken as  $0.604\sqrt{f_c'}$  MPa [78]. Therefore,

$$\beta = \tan^{-1}(0.7\sqrt{f_c'}) \quad (\text{A-8})$$

902 The angle for the modified failure envelop can be calculated by

$$\theta = 2\beta - 90^\circ \quad (\text{A-9})$$

903 To determine  $C_l$  in Eq. A-1, take a stress state of non-confinement  $\sigma_x = 0$ , when the  
 904 stress reaches the failure state under gradually increased shear load, line  $L_2$  is tangent  
 905 to the Mohr's circle, and runs across line OD at point  $(\sigma_y, -\tau)$ . The point of intersection  
 906 of the Mohr's circle with the  $\tau$  axis is  $(0, \tau)$ . The distance from centre of the Mohr's  
 907 circle, i.e., point  $(\sigma_y/2, 0)$ , to Line  $L_2$  is

$$R_2 = \left( \frac{y_2}{\tan \theta} + \frac{\sigma_y}{2} \right) \sin \theta \quad (\text{A-10a})$$

908 where  $R_2$  is radius of the Mohr's circle.

909 Substituting Eqs. A-2, A-4 and A-6 into A-10 together with  $\alpha = 37^\circ$ , it yields

$$R_2 = 0.25 \cdot f_c \cdot \cos \theta + \left( \frac{7A_{key}^V}{10A_{key}^H} \right) \cdot \frac{\tau}{2} \cdot \sin \theta \quad (\text{A-10b})$$

910 Since the radius of the stress circle  $R_2$  can also be written as

$$R_2 = \sqrt{(\sigma_y - \frac{\sigma_y}{2})^2 + (-\tau - 0)^2} \quad (\text{A-11a})$$

911 Substituting Eq. A-2 and A-4 in,

$$R_2 = \left( \sqrt{\left( \frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H} \right)^2 + 1} \right) \tau \quad (\text{A-11b})$$

912 With Eqs. A-10b and A-11b, the shear stress  $\tau$  is expressed using the following  
 913 equation.

$$\begin{aligned}\tau &= \frac{0.25 \cdot f_c \cdot \cos\theta}{\sqrt{\left(\frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H}\right)^2 + 1 - \frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H} \sin\theta}} \\ &= \frac{0.2125f_c' \cos\theta}{\sqrt{\left(\frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H}\right)^2 + 1 - \frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H} \sin\theta}}\end{aligned}\quad (A-12)$$

914 Using Eq. A-12, the coefficient  $C_1$  in Eq. A-1 can be expressed with variables  $A_{key}^V$ ,  
 915  $A_{key}^H$  related to shear key geometry and concrete failure angle  $\theta$ .

$$C_1 = \frac{0.2125 \cos\theta}{\sqrt{\left(\frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H}\right)^2 + 1 - \frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H} \sin\theta}} \quad (A-13)$$

916

917 To determine the coefficient  $C_2$ , when the interlocking brick is subjected to axial  
 918 pre-compressive stress,  $\sigma_x$  is introduced and the corresponding Mohr's circle enlarges,  
 919 which is nevertheless still tangent to the strength envelope line  $L_2$ . As shown in Figure  
 920 A-2b, point  $(\sigma'_x, \tau')$  and  $(\sigma'_y, -\tau')$ , respectively. The centre coordinates of the Mohr's  
 921 circle is  $\left(\frac{\sigma'_x + \sigma'_y}{2}, 0\right)$ . The radius of the circle can be calculated as

$$R_3 = \sqrt{\tau'^2 + \left(\frac{\sigma'_x}{2} - \frac{\sigma'_y}{2}\right)^2} \quad (A-14)$$

922 Similar to Eq. A-10, line  $L_2$  is tangent to the Mohr's circle. So the radius can also  
 923 be calculated as

$$R_3 = \left(\frac{y_2}{\tan\theta} + \frac{\sigma'_x + \sigma'_y}{2}\right) \sin\theta = 0.25 \cdot f_c \cdot \cos\theta + \left(\frac{\sigma'_x}{2} + \frac{\sigma'_y}{2}\right) \sin\theta \quad (A-15)$$

924 With Eq. A-14 and A-15, the quadratic equation of shear stress  $\tau$  is expressed using  
 925 the following equation.

$$A\tau^2 + B\tau + C = 0 \quad (A-16)$$

926 where the coefficient A, B and C can be written as

$$A = 1 + \left(\frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H}\right)^2 \cdot \cos^2\theta \quad (A-17a)$$

$$B = -[\sigma_x(1 + \sin^2\theta) + (0.5f_c \cdot \sin\theta \cos\theta)] \cdot \left(\frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H}\right) \quad (A-17b)$$

$$C = \left(\frac{\sigma_x^2}{4}\right) \cos^2\theta - 0.25\sigma_x \cdot f_c \cdot \sin\theta \cos\theta - 0.0625f_c^2 \cdot \cos^2\theta \quad (A-17c)$$

927 The root of Eq. A-16 is expressed as follows:

$$\tau = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad (A-18)$$

928 The shear stress under pre-compression can be written as

$$\tau = \frac{-B + \sqrt{B^2 - 4AC}}{2A} - \frac{0.2125f_c' \cos\theta}{\sqrt{\left(\frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H}\right)^2 + 1 - \frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H} \sin\theta}} \quad (A-19)$$

929 Referring to Eq. (A-1), the coefficient  $C_2$  is expressed using the following equation.

$$C_2 = \frac{-B + \sqrt{B^2 - 4AC}}{2A\sigma_x f_c'} - \frac{0.2125 \cos\theta}{\sigma_x \left( \sqrt{\left(\frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H}\right)^2 + 1 - \frac{1}{2} \times \frac{7A_{key}^V}{10A_{key}^H} \sin\theta} \right)} \quad (A-20)$$

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