#### School of Electrical Engineering, Computing and Mathematical Sciences

### Co-Design of Wideband Polarization-Agile Phased-Array Antennas and Low-Noise Amplifiers for the Next Generation Radio Telescopes

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This thesis is presented for the Degree of Doctor of Philosophy of Curtin University of Technology and Eindhoven University of Technology

June 2021 - Revision November 2022

To the best of my knowledge and belief this thesis contains no material previously published by any other person except where due acknowledgement has been made. This thesis contains no material which has been accepted for the award of any other degree or diploma in any university.

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### Abstract

The next generation of radio astronomy telescopes in the Ultra-High Frequency (UHF) or decimetre band require phased-array technologies that can provide a large bandwidth, high polarization purity, and a large Field-of-View (FoV). To achieve the sensitivity required of these instruments, a very low receiver noise temperature is necessary. With a projected size greater than a million elements, further restrictions must be taken into account with respect to the ease of construction, maintenance, and expense of the instrument.

To this end, the thesis will present the Capacitively-Connected Dipole Array (CCDA). As the name suggests, the CCDA is a dense regular array of dipole elements which are capacitively coupled to each other. Because of its connected nature, which effectively elongates the electrical length of each element, a wide-band response is generated. No grating lobes or scan blindness will exist due to the fact the array is dense over the entire frequency range. Finally, because the array consists of simple dipole elements, good polarization purity is to be expected.

As with all dense phased arrays, a good first design step is the consideration of a unit cell in an infinite array since the array response will dominate the singleelement response. In the case of connected arrays, a transmission line equivalent circuit of this unit cell in an infinite array environment exists, which is used as a first order design tool. By expanding on the equivalent circuit model, the necessary conditions for the excitation of detrimental (edge-born) guided wave modes have been derived and subsequently used to show how a simple loading of the array, by means of the resistive part of the input impedance of the Low-Noise Amplifier (LNA), will minimize this finite array effect.

One of the challenges with these large-scale dense connected arrays is the lack of a small-scale intermediate prototyping method that is representative of the larger array response. A novel experimental validation method of these type of large dense arrays is introduced that effectively reduces the measurement effort quadratically. This method consists of placing a linear column array of the CCDA between walls of a parallel-plate waveguide to simulate a larger array response. Using this method, the active reflection coefficient of these elements can be measured accurately, irrespective of the frequency and for the array scanned over the entire H-plane, without the help of phase-shifters. Measurement of the active reflection coefficient using this method shows good agreement with simulation results, justifying the experimental validation method.

Finally, based upon existing frameworks, electromagnetic simulation software, existing circuit models, and the measurement of the prototype a comprehensive description of the performance of the CCDA as a radio-telescope will be given.

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## Abbreviations

ARQZWA	Australian	Radio	Quiet	Zone	WA
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ASKAP Australian Square Kilometre Array (SKA) Pathfinder

**BALUN** BALanced to UNbalanced transformer

CCDA Capacitively-Connected Dipole Array

- ${\bf CD}\,$  Cosmic Dawn
- $\mathbf{DUT}\,$  Device Under Test
- $\mathbf{E}\mathbf{M}$  Electromagnetic

EMBRACE Electronic MultiBeam Radio Astronomy ConcEpt

 $\mathbf{EoR}$  Epoch-of-Reionization

 ${\bf FoV}~{\rm Field}{\rm -of}{\rm -View}$ 

- **FSS** Frequency-Selective Surfaces
- HERA Hydrogen Epoch of Reionization Array
- **IWC** Improved Wheeler Cap
- LFAA Low-Frequency Aperture Array
- LNA Low-Noise Amplifier
- LOFAR LOw-Frequency ARray

LWA Long Wavelength Array

MFAA Mid-Frequency Aperture Array

MRO Murchison Radio-astronomy Observatory

MWA Murchison Widefield Array

**PCB** Printed Circuit Board

**PPWG** Parallel-Plate WaveGuide

RF Radio-Frequency

SARAO South African Radio Astronomy Observatory

 ${\bf SKA}\,$  Square Kilometre Array

SMA Surface-Mounted Assembly

**SMD** Surface-Mounted Device

**TE** Transverse Electric

**TEM** Transverse Electro-Magnetic

**TM** Transverse Magnetic

**UHF** Ultra-High Frequency

 $\mathbf{VNA}\ \mathrm{Vector}\ \mathrm{Network}\ \mathrm{Analyzer}$ 

### Chapter 1

### Introduction and Background

#### 1.1 Radio Astronomy

Radio astronomy is the study of celestial objects and phenomena at Radio-Frequency (RF). The radio-astronomy spectrum ranges from wavelengths of several meters to as small as the millimetre range. The very first observation of a celestial signal at RF was done by Karl Jansky in 1933 [1] when he picked up 'static of an unknown source' during his work at Bell Telephone Laboratories. In 1941 the Milky Way was mapped at RF by Grote Reber by using a 9.6 m parabolic dish antenna constructed in his own backyard. Most current radio-telescopes can be grouped into one of three categories; single-dish reflector telescopes, phased-array telescopes, or interferometric arrays.

The term phased-array refers to an array of electronically scanned antennas. By introducing an electronic time-delay or phase-shift between the signals of the elements of the array the scan direction of the primary beam can be steered to different points in the sky without any mechanical movement of the array antennas. Phased-array transmission was first demonstrated by means of three monopole antennas in an equilateral triangle by Karl Ferdinand Braun [2] back in 1905. The pursuit of increasingly higher resolutions, which could not be achieved by a single radio-telescopes, led to the implementation of radio-interferometry by Martin Ryle, Joseph Lade Pawsey and Ruby Payne-Scott in 1946 [3]. Radiointerferometry combines the signals of several antennas, either single-dish or phased-array, to create a 'virtual telescope' with a very large and incompletelyfilled aperture. The long spacing (baseline) of this sparse array improves the beam resolution, where the rotation of the Earth is used to improve the sampling of the aperture to increase the beam quality.

## 1.2 Sensitivity of a Phased-Array Radio Telescope

By far the most important measure of a (low-frequency phased-array) radiotelescope is the sensitivity of the instrument. The sensitivity is given by the ratio of the effective electromagnetic collecting area of the phased array divided by the system noise temperature. The sensitivity [4] is given by

$$\frac{A_{eff}}{T_{sys}} = \frac{\eta_{rad} \frac{D\lambda_0^2}{4\pi}}{\eta_{rad} T_{sky} + (1 - \eta_{rad}) T_0 + T_{rec}}$$
(1.1)

where  $A_{eff}^{1}$  is the effective area of the phased array and where  $T_{sys}$  is the total system noise temperature. The total system noise temperature consists of three components.  $T_{rec}$  is the total noise temperature added by the receiver signal processing chain, which is dominated by the noise temperature of the Low-Noise Amplifier (LNA) because any noise induced after the LNA is generally considered insignificant as follows from Friis' formula [7].  $T_0$  is the ambient temperature (which is generally defined as 290 K), and  $T_{sky}$  is the average external noise temperature due to the radio sky. The radiation efficiency, given by  $\eta_{rad}$ , determines to which degree the ambient and sky noise temperature contribute to the total system noise temperature. The effective area is a function of the directivity (D) of the phased-array antenna.

<sup>&</sup>lt;sup>1</sup>The IEEE effective area definition assumes an incident plane wave that is polarization matched to the antenna [5, 6].



Figure 1.1: Sky noise temperature as function of frequency when modelled according to Eq. (1.2).

The average sky noise temperature in Kelvin is usually modelled [8] by

$$T_{sky} = \left(\frac{c}{0.2008f}\right)^{2.55} + \left(\frac{f}{f_0}\right)^{1.8} + T_{bg}$$
(1.2)

where f is the frequency of interest, c is the speed of light,  $f_0$  is the reference frequency of 1 GHz, and where  $T_{bg}$  is the background temperature given by 2.7 K. At lower frequencies, i.e.  $f < f_0$ , the model can be approximated by  $\approx 60\lambda^{2.55}$ . Fig. 1.1 shows the sky noise temperature as function of the frequency range of interest for low-frequency radio astronomy, i.e. from 50 MHz up to 1.5 GHz. Commercially available room-temperature LNAs can achieve minimum noise temperatures as low as several dozens of Kelvin, and experimental LNA designs have shown noise figures as low as 0.2 dB [9, 10, 11, 12], which equates to receiver noise temperatures as low as 15 Kelvin. Taking into account the noise mismatch between the antenna and LNA, the system noise temperature of a phased-array



Figure 1.2: Photo of the inverted-vee dipole antennas used by the LOw-Frequency ARray (LOFAR) Low-Band Array in the Netherlands. Photo credit: The Netherlands Institute for Radio Astronomy, ASTRON.

instrument is generally limited by the average sky noise temperate below 250 MHz and limited by the receiver noise temperature above 250 MHz.

### 1.3 The Square Kilometre Array

With the ever increasing capabilities and lower costs of electronic circuitry, astronomers began to dream about the possibilities of the next generation of radiotelescopes in the early 1990's [13, 14]. This eventually progressed into the international consortium now known as the Square Kilometre Array (SKA)<sup>2</sup> [15]. As of 2021, 14 countries have signed up as members of the SKA organization. Two radio-quiet zones <sup>3</sup>, the Australian Radio Quiet Zone WA (ARQZWA) surrounding the MRO [16, 17] in Western Australia and a radio-quiet zone surrounding the

<sup>&</sup>lt;sup>2</sup>https://www.skatelescope.org/

<sup>&</sup>lt;sup>3</sup>Both radio-quiet zones are defined in ITU-R RA.2259-1



Figure 1.3: Photo of the bow-tie (sometimes also referred to as 'spiders') antennas used by the Murchison Widefield Array (MWA) at the Murchison Radio-astronomy Observatory (MRO) in Western Australia. Photo credit: Curtin University and the MWA collaboration.

South African Radio Astronomy Observatory (SARAO) at the Northern Cape in South Africa, are established as the sites of the SKA. SKA will consist of two phases. SKA-1 will consist of SKA-low, a phased array of log-periodic dipole array antennas to cover the frequency range of 50-350 MHz which started construction in Western Australia in early 2021 as well as the SKA-mid, an array of about two hundred dish antennas covering the 350 MHz-14 GHz range to be constructed in South Africa. SKA-1 will have approximately 10% of the collecting area of the full project and started construction this year (2021) and first science with the instrument is expected in 2030. In SKA-2, the telescope will be expanded to its full size and collecting area and will furthermore be expanded with the SKA-mid Mid-Frequency Aperture Array (MFAA), a phased-array radio telescope which will cover the 450 - 1450 MHz frequency band. SKA-2 is not expected to start construction until at least the early 2030's. The next two sections will discuss the



Figure 1.4: Top: Artist's impression of the SKA-Low at the MRO in Western Australia. Bottom: Aperture array verification system prototype station [18, 19] for LFAA/SKAlow consisting of dual-polarized log-periodic antenna elements as constructed in the MRO in Western Australia. Photo credit the Square Kilometre Array Observatory.

two phased-array systems, Low-Frequency Aperture Array (LFAA) for SKA-low and MFAA for SKA-mid, in more detail.

#### 1.3.1 SKA-low: LFAA

Several pathfinders, precursors and design studies have paved the way towards the current SKA-low design. The first of these pathfinders is the LOw-Frequency-ARray LOFAR [20] in the Netherlands. LOFAR consists of two sub-telescopes, the low-band array (LBA) covering the 30-90 MHz frequency range by means of a phased array of inverted-vee dipole antennas as shown in Fig. 1.2 and the highband array covering the 110-250 MHz by means 4x4 phased array of crossed bowtie dipole pairs [21]. The MWA [22] is a SKA precursor constructed in the MRO in Western Australia. The MWA consist of 256 tiles of 16-element phased-array bow-tie elements (sometimes also referred to as 'spiders', see Fig. 1.3). The MWA covers the frequency range of 70 - 300 MHz. A further low-frequency phasedarray SKA-precursor is the Hydrogen Epoch of Reionization Array (HERA) [23] and a further SKA-pathfinder is the Long Wavelength Array (LWA) [24].

Based upon the lessons learned from its precursors and pathfinders, the SKAlow [25] will consist of 512 stations of 256 log-periodic antennas [26] each, placed pseudo-randomly within a station with a diameter of 38 m. An artist impression and the construction of the first aperture array verification prototype [18, 19] station at the MRO can be seen in Fig. 1.4. The SKA-low will be a sparse phased array, i.e. it will have an inter-element spacing of more than half a wavelength. Because the log-periodic elements are electrically short, the noise mismatch, and hence the receiver noise temperature will be significant at the lower end of the SKA-low band. However, because of the high sky noise temperature, and the fact that slightly sparse arrays are actually more directive [27, 28] than dense arrays it does not reduce the sensitivity significantly. Sparse arrays do give rise to unwanted grating lobes, which are randomized over the array aperture by means of the pseudo-random configuration. Furthermore, because of its sparse nature the isolated elements within a sparse array need to be wideband themselves. This is achieved by the log-periodic design [26].

The main science motivator for the SKA-low design, as well as for LOFAR, MWA, HERA and LWA, is the search for the 21 cm emission from the Epoch-of-Reionization (EoR) and Cosmic Dawn (CD) in the early universe [20, 23, 24, 25, 29, 30, 31]. With redshifts of 25 to approximately 6 this occurs in the frequency range from 50 MHz to approximately 200 MHz. Even though the current sparse design solution with the wideband log-periodic array elements is justified in terms

Parameter	Essential requirement		
Frequency of operation	450-1450 MHz		
Maximum zenith scan angle	$\pm 45 \deg$		
Maximum receiver noise temperature	30 K		

Table 1.1: Current relevant technical requirements of the MFAA.

of the achievable sensitivity, considerable concerns are raised with respect to the ability to de-embed the spectral artefacts of the current SKA-low design [32, 33], and its effect on EoR science. As a result, the EoR science community could benefit from a small-scale instrument with a low receiver noise temperature, a high spectral smoothness, high polarization purity and predictable radiation pattern. The increased resolution due to the smaller inter-element spacing would furthermore allow to resolve larger scale sky structures. The work presented in this thesis could potentially address this.

#### 1.3.2 SKA-MID: MFAA

The MFAA envisioned for the SKA is the main motivator for the work undertaken in this thesis. Preliminary design requirements of the MFAA are set out in [34], but it is worth noting that the MFAA project is still very much in infancy and as such the design requirements are not yet set in stone. The preliminary design requirements suggest that the MFAA is expected to at least cover the 450-1450 MHz frequency range. Furthermore, the MFAA is expected to cover a zenith scan angle of up to  $\pm 45^{\circ}$  and the full azimuthal angular range with two orthogonal polarizations. Finally, the active MFAA is expected to have a beamequivalent receiver noise temperature below 30 K over the entire frequency and angular range specified, because it cannot rely on a high sky noise temperature to mask a poor noise match in the upper part of its frequency band. These requirements are summarized in Table 1.1. The SKA-pathfinder EMBRACE [35], part of which can be seen in Fig. 1.5, is a dense phased-array of regular-on-grid connected array of Vivaldi antennas constructed as a prototype for MFAA. It has



Figure 1.5: The SKA pathfinder Electronic MultiBeam Radio Astronomy ConcEpt (EMBRACE) at ASTRON. EMBRACE consist of a dense array of Vivaldi antennas, also constructed in Nancay, France. Photo credit: The Netherlands Institute for Radio Astronomy, ASTRON.

been designed to cover the 0.5-1.5 GHz range. Further work includes the work on the Octagonal Ring Antenna [36] and the Dense Dipole Array [37, 38].

Previous work has shown that connected array can achieve a considerably lower cross-polarization than Vivaldi antennas if common-mode resonances are mitigated by means of a common-mode suppressing feed [38, 39] or careful consideration of the array loading [40, 41].

In this thesis the Capacitively-Connected Dipole Array (CCDA) will be investigated as an alternative technology for the MFAA. More specifically, the first steps towards an active CCDA prototype are taken in an effort to express the CCDA's performance in radio-astronomy relevant parameters. To this end, a fast and reliable measurement of the active reflection coefficient of the CCDA will be vital to understand the effects of mutual coupling and noise coupling and to guide future designs.

### 1.4 Outline of Thesis

This thesis is organized as follows:

- Chapter 2 will introduce the CCDA and use its equivalent circuit to design a dipole element for the MFAA. An extension to the equivalent circuit will be made to explore the finite array effect of detrimental (edge-born) guided waves and how to mitigate their effect on the array performance by means of LNA impedance loading.
- Chapter 3 will introduce the Parallel-Plate WaveGuide (PPWG) as a novel small-scale intermediate prototyping method for the CCDA after which the construction and measurement of the prototype is discussed.
- Chapter 4 will present the characterization of the CCDA in terms of its sensitivity and related parameters.
- Chapter 5 concludes the thesis with a summary and concluding remarks.

### **1.5** Original Contributions of the Thesis

The work presented in this thesis contains the following original contributions.

- A novel small-scale intermediate measurement and validation method by utilizing the periodicity of dense regular-on-grid connected arrays, such as the CCDA. Placing a linear (1-by-N) CCDA between the walls of a PPWG allows the measurement of the active reflection coefficient of each array element as if it were part of a semi-infinite array ( $\infty$ -by-N) irrespective of the frequency and over the entire H-plane without the help of phase-shifters, as shown in Chapter 3.
- The first characterization of the CCDA in radio astronomy relevant terms, such as the sensitivity and receiver noise temperature, as presented in Chapter 4.

- Extensions to the transmission line equivalent circuit of a unit cell of an ∞-by-∞ connected array, as it had been previously derived by Munk [42], to explore the underlying mode solutions of detrimental guided wave modes on these type of planar arrays and what steps can be taken to avoid the excitation altogether, as shown in Chapter 2.
- The work presented in this thesis is explicitly not an attempt to optimize a specific design because the MFAA requirements are not set in stone. Nevertheless, it was concluded that a bandwidth of 1 : 4 to 1 : 4.5, as is the current requirement for the MFAA, can be achieved, especially once an active noise match optimization is actively pursued, as concluded in Chapter 4.
- The work presented in this thesis resulted in three first-author publications that are included at the end of this thesis.

### Chapter 2

# Design of the Capacitively-Connected Dipole Array

## 2.1 Theoretical Background and Equivalent Circuit of the CCDA

As discussed in the introduction of this thesis, the next generation low-frequency phased-array radio-telescopes, such as those part of the Low-Frequency Aperture Array (LFAA) and Mid-Frequency Aperture Array (MFAA) of the Square Kilometre Array (SKA), require low-cost antenna technologies that can provide a large bandwidth, low receiver noise temperature, and a large Field-of-View (FoV) [34]. Furthermore, it has been shown that several science cases require a smooth spectral and scan response [32, 33]. As such, the predictable and smooth response of dense regular arrays are preferred over sparse randomized irregular arrays [43, 44, 45, 46]. The Capacitively-Connected Dipole Array (CCDA) [37, 42, 47, 48, 49] can theoretically provide these characteristics.

The favourable response of the CCDA is explained by the fact that it is



Figure 2.1: Top-view of a three-by-three element cut-out of an infinite-by-infinite singlepolarized capacitively-connected dipole array. The red and blue dotted lines define the parallel ( $\varphi^{//}$ ) and perpendicular ( $\varphi^{\perp}$ ) phase shift walls respectively, creating a unit-cell around the centre element depicted. The inter-element spacing (d) is equal in the  $\hat{x}$ and  $\hat{y}$ -direction for this regular dense array.

an approximation of Wheeler's continuous current sheet [50] concept, a concept that has been utilized by several other radio astronomy instruments [37, 36, 35] including the phased-array feed of the Australian SKA Pathfinder (ASKAP) MkII [51, 52, 53, 54, 55, 56, 57]. Allowing for a continuous current path within the array results in a smooth spectral response. Wheeler furthermore found that the element resistance at zenith scan angle  $\theta$ , was proportional to  $1/\cos\theta$  in the Hplane and  $\cos\theta$  in the E-plane. Fig. 2.1 shows a schematic diagram of the top view of a dual-polarized CCDA, Fig. 2.2 shows a 3-D view of a 4-by-4 dual-polarized



Figure 2.2: 3D view of a 4-by-4 element dual-polarized capacitively-connected dipole array over a finite groundplane realized on printed-circuit boards. Each antenna port is fed down by a pair of microstrip feed lines into a pair of Surface-Mounted Assembly (SMA) connectors on the backside of the groundplane. This example does not include a common-mode suppressing feed. The tip capacitance is realized by small lumpedelement surface mount capacitors and hence not readily visible in this picture.

CCDA. As the name suggests, a dipole in a CCDA is capacitively connected to its neighbouring elements, effectively elongating the electrical length of each dipole which in turn explains its wideband response. If the element spacing is furthermore kept below half a wavelength throughout the frequency of operation, the onset of any grating lobes can be avoided, resulting in a smooth spectral and scan response [42].

The bandwidth of these type of arrays can be extended by placing a dielectric superstrate on top of the dipole elements [42]. Further extension of the bandwidth has been achieved by combining the superstrate with a lossy resistive loop [58] or lossy parasitic strips [59]. Ultra-wideband arrays are achieved by placing a resistive sheet in the substrate and several layers of lossy Frequency-Selective Surfaces (FSS) in the superstrate [60, 61], where bandwidths as high as 46 : 1 have been reported. These additions to the standard connected-array design all introduce loss into the design and are as such unsuitable to be used in a low-noise application such as low-frequency radio astronomy.



Figure 2.3: Equivalent circuit of a unit cell of a **single** polarization of the CCDA over a perfect and infinite ground plane radiating into free space.

#### 2.1.1 Equivalent Circuit

In the case of these large, dense, highly coupled phased arrays, the array response generally dominates the isolated element response. In other words, a unit cell in an infinite array will be a better approximation of the embedded response than an isolated element ever will be. As such, the operating principle and design of the capacitively-connected dipole array of Fig. 2.1 is best demonstrated by its equivalent circuit [42]. Fig. 2.3 shows the basic equivalent circuit of a unit cell of an  $\infty$ -by- $\infty$  capacitively-connected dipole array over a perfect ground plane [42]. The  $\hat{x}$ -directed array in Fig. 2.1 is taken as the starting point for the unit cell derivations that follow. We do note that the equivalent circuit is only valid for a single polarization because it does not account for the cross-polarization in the inter-cardinal planes. As a result, strictly speaking, the equivalent circuit is only valid for the two main scan planes, i.e. scan in the  $\hat{x}\hat{z}$ -plane (H-plane) and scan in the  $\hat{y}\hat{z}$ -plane (E-plane).

A more complete and fully analytical transmission line equivalent circuit based upon the spectral Green's Functions exists [62, 63]. This improved equivalent circuit rigorously accounts for the inductance of the dipole, for the capacitance of the feeding gap, for the interaction between Transverse Electric (TE) and Transverse Magnetic (TM) waves making it valid for scanning in any generic azimuth plane, for higher order Floquet modes in the input reactance, and for the inter-dipole capacitance. The choice is made to limit the analysis to the simplified equivalent circuit discussed above and in [42] because it suffices to make the high-level design choices required at this stage of the design, and will simplify the extensions to the equivalent circuit to be made in remainder of the thesis. Previous work [64, 65, 39] has shown that the polarization cross-coupling in the inter-cardinal plane of these type of arrays can be substantial, as high as -15 dB. The effect of the cross-coupling in the inter-cardinal planes is one of the subjects beyond the scope of the work presented in this thesis as a consequence of the choice of using the simplified equivalent circuit.

Since the phase-shift walls that define the unit cell boundaries in Fig. 2.3 can be seen as a virtual waveguide that only supports a Transverse Electro-Magnetic (TEM) wave, we can model the unit cell as a transmission line. The phase-shift between the two consecutive phase-shift walls will then define the scan direction of the array, boresight scan is for example achieved by  $(\varphi_n^{\perp} - \varphi_{n+1}^{\perp}) = \pm 180^o, (\varphi_n^{\prime\prime} - \varphi_{n+1}^{\prime\prime}) = 0^o.$ 

The per unit (inductive) reactance of the dipole elements is given by  $X_A$  and the total per unit tip capacitance is denoted by  $C_{tip}$ . Following the transmission line comparison,  $Z_{in}$  is the radiation impedance of the upper halfspace, which in Fig. 2.3 is equal to the free space wave impedance times the scan dependence. In the two principal scan planes, E-plane and H-plane, the radiation impedance is known [50] to be

$$2R_{A0}^{E-plane}(\theta) = \sqrt{\frac{\mu_0}{\varepsilon_0}}\cos\theta = \eta_0\cos\theta \qquad (2.1a)$$

$$2R_{A0}^{H-plane}(\theta) = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{\cos\theta} = \eta_0 \frac{1}{\cos\theta}$$
(2.1b)

with  $\mu_0$ ,  $\varepsilon_0$ , and  $\eta_0$  as the free-space permeability, permittivity and wave impedance respectively. The zenith scan angle is given by  $\theta$ , where  $\theta = 0^\circ$  is defined as zenith or boresight scan. In the transmission line equivalent circuit the groundplane can be seen as a short-circuited termination. As such,  $Z_{gp}(\theta)$  is the wave impedance seen by the array as the result of the reflection of the perfect ground plane and accounts for the total wave impedance of the lower halfspace.  $Z_{gp}(\theta)$  is a function of the height of the dipole element over the ground plane as given by

$$Z_{gp}(\theta) = 2jR_{A0}\tan\left(k_z H_{dip}\right) = 2jR_{A0}\tan\left(\frac{2\pi}{\lambda_0}\cos\theta H_{dip}\right)$$
(2.2)

where  $\lambda_0$  is the wavelength in free-space and  $H_{dip}$  the height of the dipole above the groundplane. Knowing that we have limited ourselves to just the two principle scan planes, the wave-vector component in the  $\hat{z}$ -direction is given by

$$k_z = \sqrt{k_0^2 - k_t^2} = \frac{2\pi}{\lambda_0} \sqrt{1 - \sin^2 \theta} = \frac{2\pi}{\lambda_0} \cos \theta \tag{2.3}$$

where  $k_t$  is the transverse wave component, i.e.  $k_x$  in the E-plane or  $k_y$  in the H-plane. The resulting active scan impedance of a unit cell of the CCDA as function of the zenith scan angle in either of the two principle planes is given by

$$Z_{act}(\theta) = \left(jX_A + \frac{1}{j\omega C_{tip}}\right) + \left(Z_{gp}(\theta) / / Z_{in}(\theta)\right)$$
(2.4)

where // denotes two parallel loads.

With the help of the equivalent circuit we can find the parameters of the singlepolarized CCDA design for the MFAA. With a maximum frequency of 1500 MHz, which corresponds to a minimum wavelength of 20 cm, we set the inter-element spacing (d) to 9 cm, to avoid the onset of grating lobes at the horizon. An inductive wire approximation of a dipole of this length gives the total per unit selfinductance equal to 35 nH [66], depending slightly on the chosen wire thickness. If required, a more accurate estimate of the per unit reactance of the dipoles can be found using the improved equivalent circuit of [62, 63]. We do note that at this stage of the design no choice of low-noise amplifier (LNA) design has been made and as such the design parameters are not optimized to reduce the receiver noise temperature of a specific LNA, nor to maximise the matching efficiency, bandwidth or scanning performance. Future iterations of the design will utilize the vast knowledge in the literature [67, 68, 69] on these topics to achieve the performance requirements of the MFAA. Instead we minimize the reactive part of Eq. (2.4) over the chosen frequency range. The height of the dipole over the groundplane is chosen to be 8.75 cm such that for boresight scan at the center frequency (850 MHz) the total radiation impedance, given by  $(Z_{gp}(\theta))//Z_{in}(\theta))$ , has no reactive part. Finally, the total per unit tip capacitance is chosen to be 1 pF to compensate the inductive reflection of the groundplane ( $Z_{gp}(\theta)$ ) at the lower end of the frequency band. The relevant parameters are summarized in Table 2.1.

Fig. 2.4 shows the calculated active scan impedance of the CCDA using the basic equivalent transmission line circuit for the design parameters discussed at several different scan directions in the E-plane/ $\hat{x}\hat{z}$ -plane and scan in the H-plane/ $\hat{y}\hat{z}$ plane. For presentation clarity a  $Z_0 = 200 \ \Omega$  basis is used for these Smith Charts. Fig. 2.5 shows the same calculated active scan impedance in an impedance plot.

#### 2.2 Realization of the CCDA Dipole Element

Having used the equivalent circuit to find the global parameters of the CCDA for the MFAA we now move on to designing the dipole element on a PCB. The CCDA elements are realized on a 1.6 mm thick FR-4 PCB. The layout of the PCB is shown in Fig. 2.6. The tip-capacitors are realized by two 0402 Surface-Mounted Device (SMD) components soldered on each end of the dipole to one of the three solder pads available. A sufficiently large solder pad for the dipole to be soldered to its neighbouring element is left next to the SMD tip capacitors.



Figure 2.4: Active scan impedance  $Z_{act}$  of the CCDA calculated using the basic equivalent transmission line circuit for the design parameters discussed in Section 2.1.1 for several different scan directions. Smith Charts with a  $\mathbf{Z}_0 = 200 \ \Omega$  basis. Shown are boresight/zenith scan (blue lines), scan in the E-plane/ $\hat{x}\hat{z}$ -plane (red lines), and scan in the H-plane/ $\hat{y}\hat{z}$ -plane (green lines). All for the 200 – 1500 MHz frequency range with frequency ticks at 400 MHz and 1300 MHz and where an increasing frequency moves clockwise.

Although previous work [38] has shown that the tip-capacitance can be realized by overlapping dipole edges instead, avoiding the loss of the SMD components, we have chosen the SMD components to ease the placement of the PCB boards. The differential port of the dipole is fed by a 100  $\Omega$  differential feed line. The


Figure 2.5: Active scan impedance  $Z_{act}$  of the CCDA calculated using the basic equivalent transmission line circuit for the design parameters discussed in Section 2.1.1 for several different scan directions. Shown are boresight/zenith scan (blue lines), scan in the E-plane/ $\hat{x}\hat{z}$ -plane (red lines), and scan in the H-plane/ $\hat{y}\hat{z}$ -plane (green lines).

differential microstrip lines are backed by a groundplane on the backside of the PCB. According to coupled line theory, there will be minimal coupling between the two microstrip lines of the differential feed-line [70]. The differential feed line allows for a full mixed mode de-embedding up to the reference point. The deembedding method, and the reasoning for not using a BALanced to UNbalanced transformer (BALUN) in the design of the CCDA dipole element are discussed in Appendix A. Finally, Fig. 2.6 shows the two reference planes. The measurement plane at the end-launch SMA-connectors (denoted by  $S^{meas}$ ) and the reference plane (denoted by  $S^{DUT}$ ). The reference plane is used as the reference point for the design as it will serve as the input of the future inclusion of the Low-Noise

Quantity	Value	Description	
d	$9~\mathrm{cm}$	Inter-element spacing of the CCDA	
$H_{dip}$	$8.75~\mathrm{cm}$	Height of the dipole over the ground plane.	
$C_{tip}$	1  pF	Total per unit tip capacitance	
$W_{ml}$	$0.3~\mathrm{cm}$	Width of the microstrip feed lines	
$H_{ml}$	$6.7 \mathrm{~cm}$	Height of the microstrip lines	
$D_{ml}$	$1.7 \mathrm{~cm}$	Distance between the microstrip lines	
$W_{fg}$	$0.8~{\rm cm}$	Width of the feed gap	
$W_{dip}$	$0.75~\mathrm{cm}$	Width of the dipole	

Table 2.1: Dimensions of the MFAA design and Printed Circuit Board (PCB)

Amplifier (LNA), once the array is upgraded to an active array. A common-mode suppressing feed-structure such as used in [37, 39] is not included in this design because the common-mode current path between neighbouring elements along the dipole direction will not exist in the specific prototyping method first introduced in this thesis, as will be shown in Chapter 3 and Appendix B.

# 2.3 Analysis of Guided Wave Modes in Planar Connected Arrays for Radio Astronomy

1

#### 2.3.1 Introduction

Previous sections have discussed the CCDA as a potential candidate for a lowfrequency radio-astronomy instrument. Due to its strongly coupled nature the CCDA can support unwanted guided waves modes propagating along the array plane as a result of edge effects in a finite array. In Section 2.3.2 the transverse resonance method [71] is applied to the basic equivalent circuit to find a necessary condition for the excitation of any guided mode in an attempt to avoid the exci-

<sup>&</sup>lt;sup>1</sup>This section is based upon the work in [P3]. The work undertaken in this section predates the previous sections, and as such was done for the LFAA frequency range primarily, but the conclusions will apply equally well to the MFAA frequency range.



Figure 2.6: The CCDA dipole printed on a 1.6 mm FR-4 PCB. The tip capacitance is realized by two 0402 surface mount capacitors placed closely to the edge of the dipole. The overlay shows the reference level  $(S^{DUT})$  and the measurement level  $(S^{meas})$  as well as the definition of all the relevant dimensions, where DUT stands for Device Under Test (DUT). The dashed rectangular line shows the dimension (40 mm by 73.5 mm) of the ground plane of the feed-lines on the back side of the PCB.

tation of any guided mode altogether. For this purpose we will consider a CCDA array designed for the [50 - 350] MHz frequency range using the same high-level design consideration as in Section 2.1.1. Recent work using the improved equivalent circuit [62, 63] to assess edge-born guided wave modes in connected arrays has shown that a sufficiently large source impedance is required to mitigate the detrimental effect of edge-born guided waves [72, 73]. In Section 2.3.3 we investigate a finite-by-finite CCDA and whether the input impedance of a typical



Figure 2.7: Equivalent circuit of a unit cell of a **single** polarization of the CCDA over a perfect and infinite ground plane radiating into free space. Explicitly separating the upwards and downwards travelling waves along the transmission line equivalent circuit by  $Z \uparrow$  and  $Z \downarrow$ .

LNA will provide a sufficiently large source impedance to mitigate the edge-born guided waves.

### 2.3.2 Dispersion Relationship of Guided Wave Modes

The transmission line equivalent circuit of Fig. 2.3 is repeated in Fig 2.7. The transverse resonance method is a well known method to find the propagation constant of a mode solution in (dielectric) waveguides [74]. The transverse resonance method can just as easily be applied to the transmission line equivalent circuit of Fig. 2.7 to find a necessary condition for the excitation of guided wave modes in CCDA's. The transverse resonance method takes advantage of the fact that any mode solution must satisfy the boundary condition at the two phase-shift walls, in our case given by  $\varphi_1$  and  $\varphi_2$ . The condition which constitutes a solution is

Table 2.2: Parameters of the example array for LFAA used in this section

Freq. range	d	$H_{dip}$	$C_{tip}$	Wire radius
50-350 MHz	40 cm	$37.5~\mathrm{cm}$	3.54 pF	$1 \mathrm{cm}$

that after short circuiting the feed port  $(Z_{act} = 0)$ , at any point in the equivalent circuit, the sum of the impedances in opposite directions must equal zero. Applying this to the equivalent circuit as given in Fig. 2.7 we get

$$Z\uparrow + Z\downarrow = 0. \tag{2.5}$$

from which the eigenvalue  $k_z$  is found that satisfies the resonance condition. We once again restrict our analysis to the two principle scan planes, i.e. the  $\hat{x}\hat{z}$ -plane or E-plane and the  $\hat{y}\hat{z}$ -plane or H-plane, due to the limitations of the equivalent circuit used. A guided mode will be purely decaying in the  $\hat{z}$ -direction. This allows us to separate the two-dimensional wave equation to define the eigenvalue as

$$k_z = -j\alpha = -j\sqrt{k_t^2 - k_0^2} = -jk_0\sqrt{s_t^2 - 1}$$
(2.6)

where  $k_t$  is the transverse wave component and where  $s_t$  is the spatial frequency in the transverse direction which, in the invisible space runs between 1 and  $s_{tmax}$ , with

$$s_{tmax} = \frac{\lambda_0}{2d} \tag{2.7}$$

where d is the inter-element spacing of the CCDA. The array can be steered into the invisible space since the inter-element spacing is less than half a wavelength across the entire frequency range. The complex wave impedance in the principle planes is subsequently given as

$$2Z_{A0}^{E-plane} = \frac{k_z}{\omega\epsilon_0} = -j\frac{\alpha}{\omega\epsilon_0}$$
(2.8a)

$$2Z_{A0}^{H-plane} = \frac{\omega\mu_0}{k_z} = j\frac{\omega\mu_0}{\alpha}$$
(2.8b)

both of which are purely imaginary in contrast to the normal operation principle where the radiation resistance is purely real  $(2R_{A0})$ . The equivalent impedance of the reflection of the perfect ground plane in either of the principle planes is given by

$$Z_{gp} = j2Z_{A0}\tan(k_z H_{dip}).$$
 (2.9)

Using the transverse resonance method of Eq. (2.5) below the array plane in Fig. 2.7 any guided mode solution must satisfy

$$Z_{gp} + \frac{Z_{A0}jX_A}{jX_A + Z_{A0}} = 0 (2.10)$$

in which  $X_{A0}$  is the total per unit reactance of the array plane upon short circuiting the feed port, in our case given by the combination of  $X_A$  and  $C_{tip}$ . Solving this equation for the E-plane scan direction gives the dispersion relationship as

$$\tanh(H_{dip}k_0\sqrt{s_t^2-1}) + \frac{X_A}{X_A - \eta_0\sqrt{s_t^2-1}} = 0$$
(2.11)

with  $\eta_0$  being the free-space wave impedance. Similarly, solving the dispersion relationship for the H-plane scan direction gives

$$\tanh\left(H_{dip}k_0\sqrt{s_t^2-1}\right) + \frac{X_A}{X_A + \frac{\eta_0}{\sqrt{s_t^2-1}}} = 0$$
(2.12)

Rewriting the dispersion relationships as

$$X_{A0}^{E-plane}\left(s_{t}\right) = \eta_{0}\sqrt{s_{t}^{2} - 1} \frac{\tanh\left(H_{dip}k_{0}\sqrt{s_{t}^{2} - 1}\right)}{1 + \tanh\left(H_{dip}k_{0}\sqrt{s_{t}^{2} - 1}\right)}$$
(2.13a)

$$X_{A0}^{H-plane}(s_t) = -\frac{\eta_0}{\sqrt{s_t^2 - 1}} \frac{\tanh\left(H_{dip}k_0\sqrt{s_t^2 - 1}\right)}{1 + \tanh\left(H_{dip}k_0\sqrt{s_t^2 - 1}\right)}$$
(2.13b)

gives a necessary condition for the total per unit reactance of the array plane for which any guided mode can exist as function of the (spatial) frequency and the



Figure 2.8: Per unit array surface reactance necessary to support any guided wave mode in the E-plane (solid lines) and the H-plane (dashed lines) following (2.13) for several spot frequencies as function of the spatial frequency. The dimensions of the CCDA under consideration are given in Table. 2.2.

height above a ground plane. Utilizing the equivalent circuit of Fig. 2.3, we design an example CCDA to operate in the LFAA frequency range of 50 to 350 MHz, using a similar approach as in Section 2.2. The parameters of this example array are given in Table 2.2. Having designed the array around 200 MHz, the active impedance of a unit cell of the array pointed at zenith is shown in Fig. 2.9 by the thick black lines. The response is in line with our expectations upon the choice of the design parameters as listed in Table 2.2.

Based upon this design, Fig. 2.8 shows the per unit array surface reactance  $(X_{A0})$  necessary to excite guided wave modes in this LFAA CCDA design as given by Eq. (2.13) plotted as function of the spatial frequency. Shown is the necessary condition for guided wave modes in the E-plane (in the direction of the dipole

connection) and in the H-plane (transverse to the dipole direction) at several spot frequencies within the design band [50 - 350] MHz. The spatial frequency  $(s_t)$  is not a function of the scan direction in the invisible space. Consequently, following the results of Fig. 2.8, a per unit surface reactance outside of the  $-100 \ \Omega$  and  $0 \ \Omega$ range meets the necessary condition for a guided wave mode to exist in at least part of the frequency band. It is concluded it is impossible to design a wideband CCDA to avoid meeting the necessary condition of any guided wave mode. Note here the difference between the per unit surface reactance given by  $X_{A0}$  and the reactance of the active scan impedance given by  $\text{Im}\{Z_{act}\}$ .

# 2.3.3 Mitigation of Edge-Born Guided Waves in CCDA's by Low-Noise Amplifier Impedance Loading

To investigate further, let us consider an 16-by-16 element single-polarized **finite**by-**finite** array of the same fundamental dimensions as given in Table. 2.2. We can simulate the response of this array using FEKO<sup>2</sup>, an Electromagnetic (EM)simulation tool. For computational efficiency the array is modelled as simple wire dipoles with a radius of 1 cm floating above an infinite ground plane. The dipole feeds are modelled using a wire port with a reference impedance of 50  $\Omega$ . The PCB and feed-line structures as given in Fig 2.6 are not included in this simulation. The scattering parameters resulting from this simulation allows the derivation of the mutual coupling matrix  $Z_{array}^{50}$ , from which the active scan impedance  $(Z_{act}^{50})$ of each dipole element within the array is calculated.

Fig. 2.9 shows the real and imaginary part of the active impedance of all 256 elements of the finite-by-finite CCDA as function of the frequency when the array is pointed at zenith ( $\theta_0 = 0^o$ ), i.e. normal to the array plane. Shown in the same graph is the active impedance derived from a unit cell within an infinite array simulation as a comparison. Very strong resonances in the active scan impedance of the finite-by-finite array can be seen between 100 and 200 MHz. In simulation

<sup>&</sup>lt;sup>2</sup>https://www.altair.com/feko/

these resonances can be so strong that it reverses the current direction in some of the array ports, i.e. current flowing into the ports of a transmitting array or current flowing out of the ports of a receiving array. The resonance frequency differs between elements because each element has a different distance to the truncated edge of the array.

Knowing that the per unit surface reactance is positive at the frequencies where the resonances occur the derivations and Fig 2.8 of the previous section suggest that the resonances are likely caused by guided wave modes propagating in the E-plane (in the dipole direction). Finiteness effects in the direction transverse to the dipole direction are considered to be dominated by space wave coupling [75]. We know that we are not seeing common-mode resonances because no feed-lines are included in this simulation Appendix B. From this we can conclude that we are indeed seeing edge-born guided waves [72, 73].

The intensity of edge-born waves travelling along the dipole direction can be mitigated by a sufficiently large source load impedance (approximately 400  $\Omega$ ) as was shown in [72, 73]. Next we will explore whether the input impedance of a commonly used LNA is sufficient to attenuate the resonances seen in the finite array of Fig 2.9. For this we will consider the ATF-54143 by Avago [76], which is used by the Murchison Widefield Array (MWA) radio telescope [22]. This LNA has been characterized [77] for the frequency range of interest here and its input impedance can be modelled by the 4-element equivalent circuit shown in Fig. 2.10. The 4-element equivalent circuit model of the ATF-54143 suggest that the resistive part of it's input impedance does not drop below 150  $\Omega$  for the frequency range under consideration here.

Fig. 2.11 shows the active scan impedance of all the elements of the CCDA with the LNA impedance source load, which shows a strong attenuation of the resonances seen in Fig 2.11. To further illustrate the positive effect of resistive loading the array using the LNAs we will calculate the receiver noise temperature



Figure 2.9: Real (solid line) and imaginary (dashed lines) part of the active impedance  $(Z_{act}^{50})$  of all the elements of a CCDA pointed at zenith, in the case of the infinite array (thick black lines) or the 256 elements in a finite array (thin coloured lines). Finite array result with respect to a 50  $\Omega$  source load.

using the ATF-54143 by Avago. The receiver noise temperature is given as

$$T_{rec} = T_{min} + 4T_0 N \frac{|\Gamma_{act} - \Gamma_{opt}|^2}{(1 - |\Gamma_{act}|^2)(1 - |\Gamma_{opt}|^2)}$$
(2.14)

where  $T_{min}$  is the minimum noise temperature of the LNA, N the Lagrange noise ratio given by  $N = R_n \cdot \Re(1/Z_{opt})$ , where  $R_n$  is the noise resistance,  $\Gamma_{opt}$  is the optimum source impedance reflection coefficient of the LNA,  $T_0$  is the ambient temperature (290 K) and  $\Gamma_{ant}$  is the reflection coefficient of the antenna element, which is directly derived from the active impedance. Do note that in this case we do not account for noise coupling between the array elements in the finite array but that Eq. (2.14) is an exact solution for an infinite array [78]. Fig. 2.12 shows the receiver noise temperature for the infinite array, the (averaged) uncoupled



Figure 2.10: 4-element equivalent circuit of the ATF-54143 by Avago.

receiver noise temperature for the finite array with a 50  $\Omega$  source load impedance and the (averaged) uncoupled receiver noise temperature for the finite array with the LNA source load impedance. In Fig. 2.12 the calculated receiver noise temperature of the 50  $\Omega$  source loaded finite array shows significant resonances between 100 - 200 MHz which are directly related to the resonances seen in the active scan impedance. However, the calculated receiver noise temperature of the finite array with the LNA source load impedance does not show these resonances and almost lines up with the infinite array receiver noise temperature.

## 2.3.4 Conclusion

In this section a necessary condition for the excitation of guided wave modes over the array plane of CCDA's has been derived using the transverse resonance method applied to the transmission line equivalent circuit of CCDA's. For wideband CCDA's it was found impossible to design the array in such a way that the necessary condition for guided wave modes is not met at least in part of the frequency band. It is known that the intensity of edge-born guided waves can be



Figure 2.11: Real (solid line) and imaginary (dashed lines) part of the active impedance  $(Z_{act})$  of all the elements within a CCDA pointed at zenith, in the case of the infinite array (thick black lines) or the 256 element in a 16-by-16 single polarized finite array (thin coloured lines). Finite array result with respect to a source load impedance equal to the input impedance of the ATF-54143 by Avago.

mitigated by a sufficiently high source load impedance, and in this section it has been shown that the input impedance of a typical LNA can achieve this. Further analysis will be required to understand the finiteness effects seen in this section and to understand the interplay between noise matching requirements and the requirement of resistive loading for an active low-frequency phased-array CCDA receiver. The conclusions drawn for the LFAA design in this section apply equally well to the MFAA design discussed in previous sections because of the scalability of the CCDA design. Future work will be needed to investigate how large the real part of the LNA impedance needs to be to sufficiently suppress edge-born guided wave modes.



Figure 2.12: Receiver noise temperature of the infinite array element (black), 50  $\Omega$  loaded finite array element (red) and LNA source impedance loaded finite array element (blue) of the CCDA when using the ATF-54143 low-noise amplifier. Finite array receiver noise temperatures are averaged over all 256 elements of the 16-by-16 element single-polarized array.

# Chapter 3

# Measurement and Verification of a Capacitively-Connected Dipole Array with a Parallel-Plate Waveguide

1

# 3.1 Introduction

As with any very large dense phased array, a good first design step is to consider a single unit cell in an infinite array with the help of simulation software or electromagnetic theory. The unit cell is a good first design step since the array response generally dominates the isolated element response in any dense array, as has been done in Chapter 2. However, once the time comes to construct the first practical prototype, one usually resorts to the construction of a passive array of only a few elements to reduce the cost and measurement effort despite the fact that it will result in a poor estimate of the larger array response [79], especially

<sup>&</sup>lt;sup>1</sup>The work in this chapter is based upon [P1] and [P4].

in the case of high mutual coupling arrays such as the Capacitively-Connected Dipole Array (CCDA) [38].

One method to measure the active reflection coefficient of a large array is by placing several elements within a rectangular waveguide<sup>2</sup> [80]. These rectangular waveguides rely on the propagation of a Transverse Electric (TE) and/or a Transverse Magnetic (TM) mode in the waveguide cavity. The downside to this method is that a broadside scan cannot be achieved and that the scan direction of the simulated array becomes a function of frequency. Furthermore, for the CCDA discussed in this thesis, a large number (> 80) of elements would have to be placed inside the rectangular waveguide cavity since dipoles are much shorter than both the cut-off frequency of the rectangular waveguide and the wavelength of operation of the array.

Alternatively, the Improved Wheeler Cap (IWC) method [81, 82, 83] has been used to measure antenna efficiency of 1-D connected arrays with great accuracy. However, for the CCDA we expect the mutual coupling between the elements to be significant, not only in the direction of physical connection but also between the elements in neighbouring rows. Therefore, we expect the 1-D IWC method to provide a rather poor approximation of the larger connected array response.

In this chapter we introduce a novel method to measure the active reflection coefficient of dense arrays based on a Parallel-Plate WaveGuide (PPWG) instead of the usual rectangular waveguides. PPWGs have previously been used to improve the bandwidth of linear arrays [84, 85], and even the bandwidth of a linear CCDA [86, 87]. As we will show, a theoretical 1-by- $\infty$  linear array placed in between the walls of a PPWG will allow us to mimic the response of an  $\infty$ -by- $\infty$ array, effectively reducing the measurement effort quadratically. We can furthermore scan the H-plane and are not frequency limited since the TEM mode within a PPWG has no cut-off frequency. The operating frequency of the array and

 $<sup>^{2}</sup>$ In the literature, these type of structures are called waveguide simulators, since they simulate a larger array response. However, to not confuse the reader with CAD electromagnetic simulation software, we have chosen to refer to these type of structures as just waveguides throughout this thesis.

the PPWG are well below the cut-off frequency of any higher order modes. The PPWG measurement method discussed in this chapter is limited to H-plane scan measurement of a single-polarization of the CCDA only and should not be seen as a replacement for a full-array prototype, much like the rectangular waveguide measurements method of [80].

Section 3.2 will discuss the theoretical background of the CCDA design and the PPWG starting with a recap and summary of the equivalent circuit of a single unit cell in an  $\infty$ -by-  $\infty$  CCDA [42]. Then we will expand on the equivalent circuit to introduce the PPWG [88] measurement structure. The construction of both the prototype CCDA and the PPWG is discussed in Section 3.3. The measurement of the prototype is discussed in Section 3.4.

## 3.2 Theory

In this section we will utilize the basic equivalent circuit of a single unit cell in an  $\infty$ -by- $\infty$  CCDA [42], which was previously used in Chapter 2 to design the CCDA. Together with absorber theory we will get a basic understanding of the underlying electromagnetic theory of the proposed set-up.

#### 3.2.1 Equivalent Circuit in Free Space

As a recap, Fig. 3.1 (a) shows the equivalent circuit of a unit cell of an  $\infty$ -by- $\infty$  $\hat{x}$ -directed CCDA over a perfect ground plane. By restricting the scan direction of the array to the  $\hat{y}\hat{z}$ -plane (H-plane) only, we can replace the phase-shift walls by infinite perfect electrically conducting walls, denoted in Fig. 3.1(a) by  $E_t = 0$ . The (inductive) reactance of the elements is given by  $X_A$ , whereas the tip capacitors are denoted by  $C_{tip}$ . The impedance looking upwards, denoted by  $Z_{in}$ , is in this case equal to the wave impedance in free space given by

$$2R_{A0} = \eta_0 \frac{1}{\cos\theta} \tag{3.1}$$



Figure 3.1: The basic equivalent circuit of a unit cell of a *single* polarization of the infinite dense CCDA over a perfect ground plane, as shown in Fig. 2.1. The array radiating into free space (a), the array radiating into a pyramidal absorber backed by a perfect conductor (b), the array radiating into a layered absorber/ an absorber with a permittivity gradient as function of its height backed by a perfect conductor (c).

where  $\theta$  is the scan direction in the  $\hat{y}\hat{z}$ -plane with  $\theta = 0^{\circ}$  broadside scan, and where  $\eta_0 = 120\pi \approx 377 \ \Omega$ . From the equivalent circuit we can then derive the active scan impedance as

$$Z_{act}(\theta) = \left(jX_A + \frac{1}{j\omega C_{tip}}\right) + \left(Z_{gp}(\theta)/2R_{A0}(\theta)\right)$$
(3.2)

where // denotes parallel loads. The reflection of the infinite perfect ground plane is given as

$$Z_{gp}(\theta) = 2jR_{A0}\tan\left(\frac{2\pi}{\lambda_0}\cos\theta H_{dip}\right)$$
(3.3)

where  $H_{dip}$  is the height of the dipole over the ground plane and  $\lambda_0$  is the wavelength in free space.

## 3.2.2 Parallel-Plate Waveguide Theory

Expanding upon the concept of the equivalent circuit, we can restrict ourselves to only consider a  $\hat{y}\hat{z}$ -plane (H-plane) scan array, which can be achieved by fixing the phase difference between the perpendicular phase-shift walls to  $(\varphi_n^{\perp} - \varphi_{n+1}^{\perp}) =$  180°. This phase-shift can be realized by enforcing a zero transverse electric field at the planes of the phase-shift walls, which in turn can be realized by replacing the perpendicular phase-shift walls of the equivalent circuit with two infinitely extending Perfect Electric Conductor (PEC), which create the same 180° phase shift between the incident and reflected electric field. These two PEC walls are denoted by  $E_t = 0$  in Fig. 3.1 (a), essentially creating an infinitely extending parallel-plate waveguide. As a next step we limit the height ( $\hat{z}$ -direction) of these PEC walls. By doing so, a cavity to free-space boundary at the top of the parallel-plates is created. To reduce the reflections from this cavity to free space boundary the cavity is filled with a (pyramidal) electromagnetic absorber and closed off with another PEC plate on top as is shown in Fig. 3.1 (b).

From here on forward we base our derivation on standard foam absorbers impregnated with a lossy material. We assume the absorber to have a homogeneous and isotropic permittivity throughout which is given by

$$\varepsilon_{abs} = \varepsilon_0 \varepsilon'_r(f) - j \frac{\sigma(f)}{\omega} \tag{3.4}$$

where  $\varepsilon_0$  is the permittivity of free space,  $\varepsilon_r(f)'$  is the (frequency-dependent) relative dielectric constant of the absorber, where  $\sigma$  is the (frequency-dependent) conductivity of the absorber, and where  $\omega(f)$  is the radial frequency. The dielectric loss in the foam material is assumed to be negligible compared to the effect of the conductivity.

Assuming an absorber with a pyramidal cone, we choose to define a filling factor (a) describing the ratio of the surface area in the  $\hat{x}\hat{y}$ -plane filled by the absorber as function of the height. The surface area of the pyramid grows quadratically with the height and is thus given by

$$a(z) = \frac{(z - H_{gap} - H_{dip})^2}{H_{pyr}^2}$$
(3.5)

for the region  $H_{dip} + H_{gap} < z < H_{dip} + H_{gap} + H_{pyr}$ . Using this we can then

homogenize the pyramidal shape of the absorber such that we create an effective equivalent relative permittivity gradient as a function of the height as

$$\varepsilon_{eff}(z) = 1 - a(z) + a(z)\varepsilon_{abs} \tag{3.6}$$

where  $\varepsilon_{abs} = \varepsilon'_{abs} - j\varepsilon''_{abs}$  is the relative permittivity of the absorber material as given by Eq. (3.4). Here we made the assumption that the effective equivalent relative permittivity scales linearly with the filling factor. If we then discretize the pyramidal absorber of varying permittivity into N layers of constant permittivity, as shown in Fig. 3.1 (c), we can find the wave impedance looking upwards as seen by the antenna. First of all, we have to find the scan angle in each layer by using Snell's second law iteratively

$$\theta_1 = \arcsin\left(\frac{1}{\sqrt{\varepsilon_{eff}(z_1)}}\sin(\theta)\right)$$
(3.7a)

$$\theta_n = \arcsin\left(\frac{\sqrt{\varepsilon_{eff}(z_{n-1})}}{\sqrt{\varepsilon_{eff}(z_n)}}\sin(\theta_{n-1})\right)$$
(3.7b)

$$\theta_{abs} = \arcsin\left(\frac{\sqrt{\varepsilon_{eff}(z_N)}}{\sqrt{\varepsilon_{abs}}}\sin(\theta_N)\right)$$
(3.7c)

where  $\theta_n$  and  $\theta_{abs}$  is the angle of the direction of propagation in the  $n^{th}$  homogenized layer and the rectangular part of the absorber respectively. The discretized height is given by

$$z_n = \left(H_{dip} + H_{gap} + H_{pyr}\left(1 - \frac{n}{N}\right)\right). \tag{3.8}$$

To find  $Z_{in}(H_{dip})$  we start from the top of the structure and work backwards towards the antenna level. Starting at  $z = H_{dip} + H_{gap} + H_{pyr}$  we can find the wave impedance looking up as

$$Z_{in}(H_{dip} + H_{gap} + H_{pyr}) = jZ_{abs} \tan\left(\frac{2\pi}{\lambda_n}\sqrt{\varepsilon_{abs}}\cos\theta_{abs}H_{rec}\right)$$
(3.9)

where  $Z_{abs}$  is the wave impedance in the rectangular part of the absorber given by

$$Z_{abs} = \sqrt{\frac{\mu_0}{\varepsilon_{abs}\varepsilon_0}} \frac{1}{\cos\theta_{abs}}.$$
(3.10)

The wave impedance seen looking upwards in each successive layer (moving downwards from layer N towards layer 1) can then be calculated from the wave propagation in each layer and the mismatch between itself and the previous layer. This allows us to write

$$Z_{in}(H_{dip} + H_{gap}) = Z_{in}(H_{dip} + H_{gap} + H_{pyr}) \prod_{n=1}^{N} \frac{1 + \Gamma_{n,n-1} e^{j\beta_n \frac{H_{pyr}}{\cos \theta_n N}}}{1 - \Gamma_{n,n-1} e^{j\beta_n \frac{H_{pyr}}{\cos \theta_n N}}}$$
(3.11)

with  $\beta_n$  the complex wave number in the  $n^{th}$  layer of homogenization given by

$$\beta_n = \frac{2\pi}{\lambda_0} \sqrt{\varepsilon_{eff}(z_n)}.$$
(3.12)

The wave impedance mismatch between two successive layers is given by

$$\Gamma_{n,n-1} = \frac{\sqrt{\varepsilon_{eff}(z_{n-1})} - \sqrt{\varepsilon_{eff}(z_n)}}{\sqrt{\varepsilon_{eff}(z_{n-1})} + \sqrt{\varepsilon_{eff}(z_n)}}.$$
(3.13)

In the equivalent transmission line circuit, which assumes just the excitation of a Transverse Electro-Magnetic (TEM) plane wave, the gap between dipole and pyramidal absorbers  $(H_{gap})$  does not alter the radiation impedance  $(Z_{in})$  seen by the array if the absorbers are truly reflection-less. Increasing the gap between the absorbers and the dipoles is equivalent to adding a matched transmission line in before a reflection-less load which also does not change the impedance at the start of the transmission line, i.e. we assume

$$Z_{in}(H_{dip}) = Z_{in}(H_{dip} + H_{gap}).$$
(3.14)

Following this iterative approach we have calculated  $Z_{in}(H_{dip})$ , which in turn can be used to calculated the active scan impedance of the array, as was done in the free space case via

$$Z_{act} = \left(jX_A + \frac{1}{j\omega C_{tip}}\right) + Z_{gp}/Z_{in}(H_{dip}).$$
(3.15)

It follows directly and intuitively from this stratification of the pyramidal part of the absorber that if the absorber is electrically large and lossy enough it will sufficiently suppress the reflections from the top of the PPWG structure such that the radiation impedance for the upper halfspace  $(Z_{in})$  is equal to the free space impedance given by Eq. (3.1),  $Z_{in}^{abs} = 2R_{A0}$  and thus correctly mimics the infinite array response.

The spacing between the elements in a CCDA is smaller than half a wavelength over the entire frequency range of operation. As such, the width between the PPWG walls, which is equal to the inter-element spacing of the CCDA given by d, will only support the fundamental TEM-mode and not any higher order TE- or TM-modes. This is in contrast with the more commonly used rectangular waveguides [80], which rely on TE- and/or TM-mode propagation. Consequently, where the scan direction and frequency are physically coupled in rectangular waveguides, the PPWG structure can scan the entire H-plane over the entire operating frequency of the array.

#### **3.2.3** Absorber Considerations

As an example, consider a unit cell of an  $\infty$ -by- $\infty$  capacitively connected dipole array designed for the Mid-Frequency Aperture Array (MFAA) frequency range, i.e. 200 – 1500 MHz range, as we have done in Section 2.2. The dipole elements are placed 9 cm apart and are placed 8.75 cm over a perfect ground plane. From the length of the dipole we make the rough estimate that the reactance of these dipoles is purely inductive and equal to 35 nH. The tip capacitance is set to 1 pF. Using the basic equivalent circuit of Fig. 2.3, we show the active scan impedance of a unit cell in an infinite array of these dimensions at broadside in Fig. 3.3 and



Figure 3.2: Normalized error of the radiation impedance seen by a CCDA between radiating into free space and an Electromagnetic (EM) absorber as function of the conductivity of the absorber as calculated per Eq. (3.16). The pyramidal and rectangular part of the absorber are  $H_{rec} = 0.375$  m,  $H_{pyr} = 0.125$  m, respectively. The relative dielectric constant is  $\varepsilon'_r = 2$ . Array is scanning to zenith/ normal incidence to the absorber,  $\theta = 0^{\circ}$ .



Figure 3.3: Real (solid lines) and imaginary (dashed lines) of the calculated active scan impedance of the CCDA at zenith ( $\theta = 0^{o}$ ) radiating into free space (black) and EM absorbers (coloured lines) for three different conductivities of the absorber. The pyramidal and rectangular part of the absorber are  $H_{rec} = 0.375$  m,  $H_{pyr} = 0.125$  m. The relative dielectric constant is  $\varepsilon'_r = 2$ .

Fig. 3.5 (black).

Popular types of EM absorbers have a typical relative permittivity of  $\varepsilon_r^{abs} \approx 2 - j$  [89]. Following this assumption we can use Eq. (3.4) to find that  $\varepsilon'_{abs} = 2$  and that the conductivity  $\sigma = [0.0125 - 0.075]$  S/m range if the absorber is designed for the frequency range of interest, [200 - 1500] MHz. The radiation impedance seen by the CCDA radiating into an absorber of these parameters can be calculated using the stratification method derived in Section 3.2.2. For this we assume that the dielectric constant and conductivity are constant with frequency. We take a total absorber length of 0.5 m which is divided between the pyramidal and rectangular part of the absorber as,  $H_{rec} = 0.375$  m and  $H_{pyr} = 0.125$  m. The difference/error between the resulting radiation impedance and the free space wave impedance is calculated by

$$e = \frac{Z_{in}^{abs}(H_{dip}) - Z_{in}^{fs}}{Z_{in}^{fs}}$$
(3.16)

where  $Z_{in}^{fs} = 2R_{A0}$ .

Fig. 3.2 shows this error as function of frequency for different values of the conductivity within the parameter range defined above. A significant decrease in the error can be seen for increasing conductivity, as expected. We furthermore note that there is no strong frequency dependence because the increasing electrical length of the absorbers is matched by a decrease in the imaginary part of  $\varepsilon_{abs}$  with increasing frequency as can be found from Eq. (3.4).

Fig. 3.3 shows the effect of the different conductivities on the calculated active scan impedance of the CCDA for the array and absorber parameters discussed above. The residual reflections seen in the calculated active scan impedance decrease rapidly with increased values of the conductivity of the absorber.

Fig 3.4 and Fig. 3.5 show the same error in radiation impedance and calculated active scan impedance but now for an absorber that has twice the original length of the rectangular part,  $H_{rec} = 0.75$  m. Increasing the length of the absorber also significantly reduces the residual reflections seen. In the next section we



Figure 3.4: Normalized error of the radiation impedance seen by a CCDA between radiating into free space and an EM absorber as function of the conductivity of the absorber as calculated per Eq. (3.16). The pyramidal and rectangular part of the absorber are  $H_{rec} = 0.75$  m,  $H_{pyr} = 0.125$  m, respectively. The relative dielectric constant is  $\varepsilon'_r = 2$ . Array is scanning to zenith/ normal incidence to the absorber,  $\theta = 0^{\circ}$ .



Figure 3.5: Real (solid lines) and imaginary (dashed lines) of the calculated active scan impedance of the CCDA at zenith ( $\theta = 0^{o}$ ) radiating into free space (black) and EM absorbers (coloured lines) for three different conductivities of the absorber. The pyramidal and rectangular part of the absorber are  $H_{rec} = 0.75$  m,  $H_{pyr} = 0.125$  m. The relative dielectric constant is  $\varepsilon'_r = 2$ .



Figure 3.6: Picture of the realized PPWG for the CCDA while being measured in an anechoic chamber. The wooden encasing is necessary for structural support. The EM absorbers shown in the background are the same ones as used in the construction of this prototype.

will discuss the realization of the CCDA prototype and the PPWG measurement structure.

# 3.3 Description of the Capacitively-Connected Dipole Array and Parallel-plate Waveguide Prototype

In this section we set out to use the theoretical knowledge from the previous section to construct a practical measurement prototype. For this, we will use the CCDA dipole PCB designs as shown in Section 2.2. We once again assume that



Figure 3.7: Picture inside the realized PPWG structure showing the Printed Circuit Board (PCB)s of the linear CCDA soldered to the copper walls before the inclusion of the absorbers on top. The optical mirroring of the copper walls mimics the electromagnetic mirroring integral to the operation of the PPWG structure.

a full-array will be 16-by-16 elements in size, and as such we will use 16 of the CCDA dipoles for our linear prototype array.

The PPWG walls, bottom plate, top plate, and ground plane are all made of copper sheets. This allows for the PCBs to be soldered to the walls. A wooden structure, as can be seen in Fig. 3.6, is added for structural support. Fig. 3.7 shows a picture taken inside the PPWG structure where the PCBs of the linear CCDA are soldered to the copper walls.

Pyramidal foam absorbers impregnated with lossy material are used since they were readily available to us at the time of the construction of the prototype. These pyramidal absorbers can be seen in the background of Fig. 3.6. To fill the cavity, 16 of these absorbers are cut to size, one for each dipole. The rectangular

Quantity	Value	Description	
d	9 cm	Inter-element spacing of the CCDA	
$H_{rec}$	$40 \mathrm{~cm}$	Height of the rectangular part of the absorber	
$H_{pyr}$	$25~\mathrm{cm}$	Height of the pyramidal part of the absorber	
$H_{gap}$	$1 \mathrm{cm}$	Average gap between dipole and absorber	
$H_{dip}$	$8.75~\mathrm{cm}$	Height of the dipole over the g.p.	
$C_{tip}$	1  pF	Total per unit tip capacitance	
$H_{tot}$	$75~\mathrm{cm}$	Total height of the copper walls	

Table 3.1: All relevant dimensions of the prototype

part of the absorbers will have a base of 9-by-9 cm with a height of 40 cm, the pyramidal part will have a height of 25 cm.

In the case of the equivalent circuit derived in Section 3.2, as well as in the case of a unit cell simulation, it was found that an increase or decrease in the gap between the absorbers and the array element  $(H_{gap})$  does not alter the radiation impedance for the upper halfspace. However, in the case of the finite 16 element array the TEM plane wave assumption of the transmission line equivalent circuit is no longer strictly true. Nevertheless, to constrain the size of this prototype, the choice was made to place the absorbers right on top of the PCBs. With an average gap of 1 cm between the PCBs and the absorbers, the total structure will be 75 cm tall.

Finally, a choice had to be made how to terminate the cavity in the positive and negative  $\hat{y}$ -directions. Ideally, these cavity to free space boundaries are terminated in the same manner as the cavity to free space boundary at the top of the structure had been terminated. With the proximity of the dipole elements and the absorbers, we expect the open cavities to only significantly affect the two edge elements. To further constrain the size of the prototype the decision had been made to not pursue a completely closed cavity in this first prototype. A schematic side-view cut-out of the structure including the dimensions showing the green PCBs with the absorbers on top is shown in Fig. 3.8.



Figure 3.8: Side-view cut-out of the PPWG structure showing the 16 elements (pictured as green PCB boards) of the  $\hat{x}$ -directed linear CCDA and the electromagnetic absorbers above each element, as well as the element numbering.

# 3.4 Measurement Results

The full 32-port structure was measured with the help of a four-port Vector Network Analyzer (VNA). To fill the entire 32-by-32 scattering matrix 108 distinctive four-port measurements were necessary. For each of these measurements the 28 remaining ports were terminated into readily available 50  $\Omega$  broadband loads, which allowed for easy de-embedding. As a result of this measurement approach, several elements of the scattering matrix, most notably the self-reflections ( $S_{ii}^{meas}$ ), are measured more than once. In these cases the median values at each frequency point are used to fill the scattering matrix.

The measured scattering matrix is then de-embedded [90] up to the DUT reference level, as it is defined in Fig. 2.6. It is assumed no cross-talk exists between the microstrip lines of neighbouring PCBs, which is a reasonable assumption for ground plane confined microstrip lines. The de-embedded measurement results, with the from here on forward suppressed superscript  $^{DUT}$ , are subsequently transformed into a combined differential and common-mode (mixed mode) scattering parameters [91] given by

$$\boldsymbol{S}_{mm} = \begin{pmatrix} \boldsymbol{S}_{dd} & \boldsymbol{S}_{dc} \\ \boldsymbol{S}_{cd} & \boldsymbol{S}_{cc} \end{pmatrix}.$$
 (3.17)

where  $S_{mm}$  is the mixed-mode scattering matrix and where  $S_{dd}$ ,  $S_{dc}$ ,  $S_{cd}$ , and  $S_{cc}$  refer to the differential-to-differential, differential-to-common, common-todifferential and common-to-common mode scattering sub-matrices respectively.

Although not required in this prototype, a full array will require commonmode suppression [47, 38, 40]. As such, even though the measurement gives us the full mixed-mode response, only the purely differential response  $(S_{dd})$  is considered from here on forward, which is the ideal balun response, i.e. the response as if all common-mode currents were completely suppressed.

The active differential reflection coefficient of differential pair number n as function of the zenith scan angle is then calculated using a geometrical weighting method according to

$$\Gamma_{ddact_n}(\theta_0) = \sum_{m=1}^{16} w_{nm}(\theta_0) S_{dd_{nm}}$$
(3.18)

where the weights are given by

$$w_{nm}(\theta_0) = e^{j\frac{2\pi}{\lambda_0}} (m-n)\frac{d}{2}\sin\theta_0$$
(3.19)

and where the subscripts refer to the antenna numbering and not to the single-ended ports at the measurement plane. This allows us to show the active differential reflection coefficient of each dipole as function of the frequency. In Fig. 3.9-3.13 the measured active differential reflection coefficient for 5 different zenith scan angles in the H-plane ( $\theta_0 = [0^o, 15^o, 30^o, 45^o, 60^o], \phi_0 = 90^o$ ) are given by the blue lines. Shown are the active differential reflection coefficient are defined in Fig. 3.8.



Figure 3.9: Active differential reflection coefficient ( $\Gamma_{ddact}$ ) of all of the CCDA elements when the array is scanned to  $\theta_0 = 0^o$ in the H-plane. Simulation of a unit cell in an infinite array (red lines), simulation of a perfect PPWG structure (green lines)), and the measured results of the CCDA prototype placed in the constructed PPWG structure (blue lines)



Figure 3.10: Active differential reflection coefficient ( $\Gamma_{ddact}$ ) of all of the CCDA elements when the array is scanned to  $\theta_0 = 15^o$ in the H-plane. Simulation of a unit cell in an infinite array (red lines), simulation of a perfect PPWG structure (green lines)), and the measured results of the CCDA prototype placed in the constructed PPWG structure (blue lines).







Figure 3.12: Active differential reflection coefficient ( $\Gamma_{ddact}$ ) of all of the CCDA elements when the array is scanned to  $\theta_0 = 45^o$ in the H-plane. Simulation of a unit cell in an infinite array (red lines), simulation of a perfect PPWG structure (green lines)), and the measured results of the CCDA prototype placed in the constructed PPWG structure (blue lines).




To verify the measurement results, and hence the viability of the PPWG measurement concept as well as the CCDA, the measurement results are compared to two different electromagnetic simulation set-ups. The first comparison in Fig. 3.9-3.13 is to a unit cell simulation, as shown by the dash-dotted red lines. A unit cell is simulated by two pairs of periodic boundary conditions in the planes shown by the dashed blue lines in Fig. 2.1. The unit cell then symbolizes an infinitely extending array in both the  $\hat{x}$ - and  $\hat{y}$ -direction. To distinguish between finite array effects and the effects of the non-perfect PPWG structure a second simulation result is shown. This second simulation has the 16  $\hat{x}$ -directed linear array elements but now placed between two  $\hat{y}\hat{z}$ -plane  $E_t = 0$  planes virtually extending to infinity by using a perfectly matched layer (PML) boundary in the far-field. Waves can pass through this PML boundary with minimal reflections and as such this simulation can be seen as a 'perfect PPWG' structure.

The first thing to notice in these results is that the measured active differential reflection coefficient for the edge element shows a significant variation for both scan angles which can be explained by the open boundary at the edge of the PPWG structure, which we expected to see upon making the choice for the open boundary. Furthermore, as expected the edge elements have a significant variation to the unit cell due to finite array effects. However, for all but the edge elements, the measured boresight scan response shows a good match with the simulation. A bigger difference between the measured and simulated results exists for the larger zenith scan angles, an effect that could be explained by the choice for a row of pyramidal absorbers which tend to scatter more irregularly at larger incident angles [92]. Finally, it is worth noting that the edge element at  $\theta_0 = 45^{\circ}$  and  $\theta_0 = 60^{\circ}$  zenith scan still follows the finite array result quite accurately, showing the resonance at the same frequency point.

### 3.5 Conclusion

This Chapter discussed the construction of and verification of a prototype CCDA for the use in low-frequency [200 - 1500 MHz] radio astronomy. It emphasizes the need for a small-scale intermediate prototyping method for large-scale dense connected phased arrays. To this end, a PPWG was constructed. The PPWG structure, allowing a full H-plane scan over the entire frequency of operation, helped to verify the design of the CCDA. The least accurate elements of the CCDA in the constructed PPWG are the edge elements, since the PPWG structure is not terminated in absorbers in the direction orthogonal to the dipole direction because of practical reasons.

## Chapter 4

# Characterization of the Capacitively-Connected Dipole Array as a Radio Astronomy Instrument

In this chapter we will characterize the Capacitively-Connected Dipole Array (CCDA) to be the used as a phased-array radio-astronomy instrument. The majority of the work in this chapter is based around the Mid-Frequency Aperture Array (MFAA) [200 - 1500] MHz frequency range but applies equally well to the Low-Frequency Aperture Array (LFAA) [50-350] MHz frequency range. To characterize the CCDA we will use a cohort of modelled, simulated, and measured data that are applicable and available at the current stage of the design work.

In this chapter we will use the definition for the sensitivity as given in Chapter 1

$$\frac{A_{eff}}{T_{sys}} = \frac{\eta_{rad} \frac{D\lambda_0^2}{4\pi}}{\eta_{rad} T_{sky} + \eta_{rad} T_0 + T_{rec}}.$$
(4.1)

where  $A_{eff}^{1}$  is the effective area of the phased-array,  $T_{sys}$  is the system noise tem-

<sup>&</sup>lt;sup>1</sup>The IEEE effective area definition assumes an incident plane wave that is polarization matched to the antenna [5, 6].

perature, D is the directivity of the phased-array,  $\eta_{rad}$  is the radiation efficiency of the phased-array,  $T_{sky}$  is the average sky noise temperature,  $T_0$  is the ambient temperature, and where  $T_{rec}$  is the receiver noise temperature. The computation and/or characterization of each component of the sensitivity formula will be discussed in the following sections.

#### 4.1 Effective Area

In this section the theoretical background for the computation of the effective area of the CCDA is discussed. Based upon the theory presented and a full-wave simulation of a full-size CCDA the effective area is calculated and presented as function of the scan direction of the array and for several frequency points.

## 4.1.1 Embedded Element Patterns and the Full Array Response

The far-field pattern of an element embedded within a phased array, known as the embedded element pattern [93, 94], can be written as

$$\vec{E}_n(\theta,\phi) = E_{\theta_n}(\theta,\phi)\hat{\theta} + E_{\phi_n}(\theta,\phi)\hat{\phi}$$
(4.2)

where  $\vec{E}_n(\theta, \phi)$  is the electric far-field vector for embedded element n, and where  $E_{\theta_n}(\theta, \phi)$  and  $E_{\phi_n}(\theta, \phi)$  are the electric field components in the  $\hat{\theta}$  and  $\hat{\phi}$  directions, respectively. The zenith angle is given by  $\theta$  and the azimuthal angle is given by  $\phi$ , where  $\theta = 0^o$  is once again the direction along the  $\hat{u}_z$  axis, where  $(\theta = 90^o, \phi = 0^o)$  is along the  $\hat{u}_x$  axis, and where  $(\theta = 90^o, \phi = 90^o)$  is along the  $\hat{u}_y$  axis. The embedded element pattern of each element is computed by exciting said element with a unitary voltage generator with zero phase while all other generator voltages are zero and all other elements are terminated with a known impedance.

Most simulation software only allows the embedded element patterns to be



Figure 4.1: Schematic overview of the problem under consideration in this section showing three elements of the CCDA each terminated in a low-noise amplifier and the beam-former weights  $(w_n)$  and far-field embedded-element patterns  $(\vec{E}_n(\theta, \phi))$ .

computed based upon a real reference impedance, for example  $Z_0 = 50 \ \Omega$ . Fortunately, it is a well-known procedure [95, 96, 97] to re-normalize the embedded element patterns into a different reference impedance, which in our case is the impedance of the low-noise amplifier,  $Z_{LNA}$ . We choose to perform this transformation in the impedance domain instead of the scattering parameters domain since scattering parameters with complex reference impedances are generally illdefined. In our case, the transformation of the embedded element patterns from a 50  $\Omega$  load impedance to the embedded element pattern with a  $Z_{LNA}$  load impedance follows as

$$\boldsymbol{E}_{p}^{Z_{LNA}} = \left(\boldsymbol{Z}_{array} + Z_{LNA}\boldsymbol{I}\right)^{-1} \left(\boldsymbol{Z}_{array} + Z_{0}\boldsymbol{I}\right) \boldsymbol{E}_{p}^{Z_{0}}$$
(4.3)

where the subscript  $_p$  refers to either of the polarizations, where  $Z_{array}$  is the mutual coupling impedances matrix of the array and I defines an identity matrix. From here on forward the superscript describing the loading impedance of the array is suppressed and all electrical fields are assumed to be in reference of the correct loading impedance, i.e.  $Z_{LNA}$ .

Now, the beauty of a phased array is to combine and electronically weight these individual embedded element patterns in such a way that one creates a highly directive far-field pattern. The array far-field pattern upon steering the array to a certain point in the sky, given by zenith and azimuthal angle ( $\theta_0, \phi_0$ ), follows as

$$\vec{\boldsymbol{E}}(\theta,\phi,\theta_0,\phi_0) = \sum_{n=1}^{N} V_{s,n} w_n(\theta_0,\phi_0) \vec{\boldsymbol{E}}_n(\theta,\phi)$$
(4.4)

where  $V_{s,n}$  is the (unitary) generator voltage applied to element n and where  $\boldsymbol{w}_n(\theta_0, \phi_0)$  is the weight applied to element n to achieve constructive interference of the embedded element patterns in the desired scan direction of  $(\theta_0, \phi_0)$ .

In the case of low mutual-coupling arrays, such as sparse arrays, it suffices to use voltage weights based upon the geometry of the array as we have in the previous chapters. The geometrical weights are a function of the position of the feed of each element,  $(x_n, y_n)$  and the scan direction, and given by

$$w_n^{geo}(\theta_0, \phi_0) = \frac{1}{N} e^{-j \left(k_{x0} x_n + k_{y0} y_n\right)}$$
(4.5)

such that  $\sum_{n=1}^{N} |w_n| = 1$ . The wave vector components are given by

$$k_{x0} = \frac{2\pi}{\lambda_0} \sin \theta_0 \cos \phi_0 \tag{4.6a}$$

$$k_{y0} = \frac{2\pi}{\lambda_0} \sin \theta_0 \sin \phi_0 \tag{4.6b}$$

where  $\lambda_0$  is the wavelength in free-space. However, in the case of highly-coupled arrays, knowing that the electric (far)-field of an antenna is excited by the current through the antenna and not the voltage on the antenna ports, this no longer necessarily suffices. More precisely, we know that the current at antenna port  $n(I_n)$  is given by

$$I_n = row_n \left\{ \left( \boldsymbol{Z}_{array} + \boldsymbol{Z}_{LNA} \right)^{-1} \boldsymbol{V} \right\} \neq \frac{V_n}{Z_{array_{n,n}}}$$
(4.7)

where  $\mathbf{Z}_{array}$  is the mutual coupling impedance matrix,  $\mathbf{Z}_{LNA}$  is a diagonal matrix with the complex reference impedance of the Low-Noise Amplifier (LNA). Furthermore, we know that the self-impedance differs between the elements within the array. As a result of this, we have to compensate the weights accordingly. The corrected weights, denoted by the superscript <sup>cur</sup> for current steering, are given by

$$\boldsymbol{w}^{cur}(\theta_0,\phi_0) = \frac{1}{\sum_{n=1}^{N} |w_n^{cur}|} \left( \boldsymbol{Z}_{array} + \boldsymbol{Z}_{LNA} \right) e^{-j \left( k_{x0} \boldsymbol{x} + k_{y0} \boldsymbol{y} \right)}$$
(4.8)

where  $\boldsymbol{x}$  and  $\boldsymbol{y}$  are the position vectors of the entire array.

More elaborate beam-forming methods with weights calculated to maximize directivity, sensitivity or signal-to-noise ratio exist [98, 4, 99] but they generally require a robust knowledge of the phased-array system under consideration and are hence beyond the scope of the work in this thesis.

#### 4.1.2 Directivity and Effective Area

From the computed total electric field, as defined in Eq. (4.4), we can now calculate the directivity and the effective area of the phased array. The IEEE Standard Definition of Terms for Antennas (1993, reaffirmed 2004) defines the directivity of an antenna as: The ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions. The radiation intensity is given by

$$U(\theta, \phi) = \frac{1}{2\eta_0} \left[ |E_{\theta}(\theta, \phi)|^2 + |E_{\phi}(\theta, \phi)|^2 \right]$$
(4.9)

where  $\eta_0$  is the intrinsic impedance of free-space, which in our case is

$$\eta_0 = \frac{\mu_0}{\varepsilon_0} = 120\pi \cong 377 \ \Omega \tag{4.10}$$

where  $\mu_0$  and  $\varepsilon_0$  are the permeability and permittivity of free space respectively. The total radiated power is found by integrating the radiation intensity over the entire solid angle by

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin \theta d\theta d\phi.$$
(4.11)

An isotropic antenna, also known as an omnidirectional antenna, is a theoretical antenna which has the same radiation intensity in all direction. The radiation intensity of an isotropic antenna is

$$U_0 = \frac{P_{rad}}{4\pi}.\tag{4.12}$$

The directivity of an antenna is the ratio of the radiation intensity of the antenna under consideration and the radiation intensity of an isotropic antenna with the same total radiated power, or

$$D(\theta, \phi, \theta_0, \phi_0) = \frac{U(\theta, \phi)}{U_0} = \frac{4\pi U(\theta, \phi)}{P_{rad}}$$
(4.13)

The maximum directivity is then simply defined as

$$D_0(\theta_0, \phi_0) = \max \{ D(\theta, \phi, \theta_0, \phi_0) \}$$
(4.14)

which does not necessarily have to occur at  $(\theta = \theta_0, \phi = \phi_0)$ , but can greatly depend on the choice of the beam-former weights used.

In antenna theory, the effective aperture or effective area is a measure of how effective a receiving antenna (or phased-array antenna for that matter) is at



Figure 4.2: Schematic top-view of the simulation setup for the calculation of the effective area of the CCDA. This schematic shows a 16-by-16 single-polarized CCDA in the  $\hat{x}$ -direction. The simulation includes the Printed Circuit Board (PCB)s as they have been defined in Chapter 2, including the groundplane on the backside of the PCB but not the transmission lines before the reference point defined by  $S^{DUT}$ . The array is placed over an infinite perfect ground plane to reduce the simulation complexity.

receiving electromagnetic radiation. The effective area  $(A_{eff})$  is defined as

$$A_{eff}(\theta, \phi, \theta_0, \phi_0) = \frac{\eta_{rad} D(\theta, \phi, \theta_0, \phi_0) \lambda_0^2}{4\pi}$$
(4.15)

where  $(\theta, \phi, \theta_0, \phi_0)$  is assumed to be known from here on forward.

#### 4.1.3 Simulation Setup and Calculated Effective Area

As in Chapter 2, we once again assume that a full-size CCDA will be at least 16-by-16 elements in size, or 256 element in total, and we furthermore once again



Figure 4.3: Snapshot of the simulation setup in CST Microwave Studio for the effective area calculations showing the discrete feed port at the  $S^{DUT}$  level as defined in Fig. 2.6 and the tip-capacitance realized by a lumped element port.

only consider a single polarization; the  $\hat{x}$ -directed array only. We once again use CST Microwave Studios as our simulation software. A schematic top-view of the simulation setup is shown in Fig. 4.2, which includes the element numbering. Originally, the intention was to simulate the full PCB, as defined in Fig. 2.6, for all 256 array elements as well a finite ground plane. This proved to be too computationally intensive. As such, the Surface-Mounted Assembly (SMA) connectors at the bottom of the PCB as well as the transmission lines from the bottom of the PCB up until the  $S^{DUT}$  level (as it was defined in Fig. 2.6) have not been included in this simulation setup. The ground plane is furthermore modelled as an infinite half-plane of perfect-electric conductor material. The port at the  $S^{DUT}$  level is modelled by a discrete single-ended port, as can be seen in Fig. 4.3. As such, we only consider the purely differential response ( $S_{dd}$ ), which is the ideal balun response. The ground plane on the back of each PCB is included in the simulation setup. Five equidistant simulation frequency points are chosen to be 200, 525, 850, 1175, and 1500 MHz. This setup, consisting of the 256 array elements simulated at five frequency points, took approximately 126 hours on a desktop PC (Windows 10, Intel Core i7-7700K @ 4.20 GHz, 32 GB RAM @ 2133 MHz) to compute.

The embedded element patterns are simulated with respect to a 50  $\Omega$  impedance load. The input impedance at the simulated frequency points of the low-noise amplifier under consideration here, the SAV-541+ by Minicircuits<sup>2</sup>, is given in Table 4.1. The embedded elements are renormalized to these loading impedances following Eq. (4.3). Based upon the inter-element spacing of 9 cm and the mutual coupling matrix of the array the current steering weights of Eq. (4.8) are computed and used to create the total electric far-field, although it should be noted that the effect of using the current steering weights instead of the geometrical weights proved to be minimal.

For now, we will consider an ideal effective area of the array where the radiation efficiency is 100%. This ideal effective area is given by

$$A_{eff}^{ideal} = A_{eff}(\eta_{rad} = 1) = \frac{D\lambda_0^2}{4\pi}$$

$$(4.16)$$

The radiation efficiency  $(\eta_{rad})$  will be considered in the next Section.

The calculated ideal effective area for the five frequency points by using Eq. (4.16) is shown in Fig. 4.4. The top row of results shows the ideal effective area if scanned in the E-plane (i.e. in the direction of the dipoles:  $\hat{x}$ -direction), the middle row shows the calculated ideal effective area if scanned in the H-plane (i.e. in the direction orthogonal to the dipoles:  $\hat{y}$ -direction), and the bottom row of results show the ideal calculated area if the array is scanned in the inter-cardinal plane (i.e.  $\phi_0 = 45^{\circ}$ ). In each of these planes, the array is scanned to five different zenith angles, being  $\theta_0 = [0^{\circ}, 15^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}]$ .

For these type of dense arrays the effective area is expected to be equal to

<sup>&</sup>lt;sup>2</sup>https://www.minicircuits.com/pdfs/SAV-541+.pdf

Table 4.1: SAV-541+ low-noise amplifier input impedance

Freq [MHz]	200	525	850	1175	1500
$Z_{LNA} \left[ \Omega \right]$	26.9+j1.9	30.1+j3.4	30.4 + j5.3	30.9+j7.1	31.3+j8.8

the projection of the physical area of the array in the plane of incidence of the incoming radiation. As such, as a comparison, the physical area times  $\cos \theta_0$  is plotted as a reference by the black dashed line. The physical area of the array, which has the same inter-element spacing and number of elements in both directions, is given by

$$A_{phys} = (N_x d_x)(N_y d_y) = (16 \times 0.09)^2 = 2.073 \text{ m}^2$$
(4.17)

where  $N_{x/y}$  and  $d_{x/y}$  is the number of elements and the inter-element spacing in each direction, respectively. The first thing to notice in the effective area results of Fig. 4.4 is that for the three centre frequencies (525, 850, and 1175 MHz) the response is very much in line with expectations, except for when the array is scanned to  $\theta_0 = 60^{\circ}$ , which in the E-plane results in a beam maximum at a slightly lower zenith angle.

At 200 MHz the effective area is very different compared to the expected result. First of all, the scan direction and the direction of the maximum of the beam do not line up at all. Furthermore, the effective area in this case is actually considerably higher than the physical area of the array. A possible explanation for the results seen at 200 MHz is that the array is only approximately one wavelength in size. Because of its connected nature, the full array structure might acts as a secondary radiator. The beam of this secondary radiation is not scanned and hence by definition directed at boresight ( $\theta = 0^{\circ}$ ) which in turn could explain the beam direction pulled towards boresight. Another possible explanation could be the ground plane being very close to the element in terms of wavelength, which makes the beam sharper. In the case of a design for the MFAA, a full MFAA station will be much bigger than 16-by-16 elements, and as such we do not expect



cardinal plane( $\phi_0 = 45^o$ ) respectively. In these planes, the array is scanned to a zenith angle of  $\theta_0 = [0^o, 15^o, 30^o, 45^o, 60^o]$  for the and 1500 MHz respectively. From top-to-bottom for zenith scan in the E-plane ( $\phi_0 = 0^o$ ), H-plane ( $\phi_0 = 90^o$ ), and the interthree scan planes using Eq. (4.16) which assumes a radiation efficiency of  $\eta_{rad} = 100\%$ . From left-to-right for 200, 525, 850, 1175, Figure 4.4: Calculated ideal effective area of the simulated single-polarized 16-by-16 element CCDA for five frequency points and blue, red, green, yellow, and purple lines respectively. As a reference, the projection of the physical area on the scan direction, i.e.  $(A_{phys} \cos \theta)$ , is plotted by the black dashed line.

this effect to be as strong. In the case of a LFAA size array, where 256 elements is likely to be the full-size of an array this effect is more severe and needs to be understood better.

At 1500 MHz, although the scan direction and beam direction do mostly line up, the effective area is less than half that of which is expected from the physical area comparison. It is unclear what the cause of this result is at at this time. Further work will need to be done to determine the cause of the lower than expected effective area and scan direction mismatches at 200 and 1500 MHz. Further work will also need to determine the exact frequency range in which the effective area of the array is along expected results, and at which frequency point the effects at 200 and 1500 MHz become detrimental to the performance of the CCDA.

#### 4.2 System Noise Temperature

Following the definition at the start of this chapter, we have defined the total system noise temperature as

$$T_{sys} = \eta_{rad} T_{sky} (1 - \eta_{rad}) T_0 + T_{rec}$$
(4.18)

The ambient temperature, given by  $T_0$ , is taken to be 290 K throughout this thesis. The derivation and calculation of the radiation efficiency  $(\eta_{rad})$  and receiver noise temperature  $(T_{rec})$  are discussed in the next sections. The sky noise temperature has been discussed in Chapter 1.

#### 4.2.1 Radiation Efficiency

In radio astronomy, the radiation efficiency is defined as the ratio of the radiated power to the power injected into the antenna, i.e.



Figure 4.5: Radiation efficiency of the CCDA dipole element within a unit cell of an  $\infty$ -by- $\infty$  array two different LNA input reference points,  $S^{DUT}$  and  $S^{meas}$  as defined in Fig. 2.6, and for two different PCB substrates, FR4 and Rogers 4003C.

$$\eta_{rad} = \frac{P_{rad}}{P_{inj}} = \frac{P_{rad}}{P_{rad} + P_{loss}} 100\%$$
(4.19)

where the total injected power  $P_{inj}$  is the sum of the power loss and the radiated power. The power loss consists of both the metallic conduction losses in the copper of the antenna and the dielectric losses in the substrate of the PCB and hence given by

$$P_{loss} = P_{metal} + P_{dielectric} \tag{4.20}$$

To calculate the radiation efficiency of our design for the MFAA we will once again use a single-polarization of a unit cell simulation of the board as it is shown in Fig. 2.6. Ideally, we would have used a finite array simulation, similar to the calculation of the effective area in Section 4.1.3, however this once again proved to be too computationally expensive. The copper that makes the transmission lines and the antenna on the PCB is modelled to have conductivity of  $\sigma_{copper} = 5.8e^7$  S/m. The substrate of the FR-4 PCB is modelled to have a relative permittivity of  $\varepsilon_r = 4.3$  and a loss tangent of tan  $\delta = 0.025$ . Fig 4.5 first of all shows the radiation efficiency assuming an LNA is included at the reference level, i.e.  $S^{DUT}$ . This is the definition we have used throughout this thesis. Using this definition and an FR-4 substrate, we see that a radiation efficiency above 90% is achieved between 230 – 1380 MHz.

As a comparison, and to justify the choice of reference level and not the measurement level as the place of the future inclusion of the LNAs we also show the radiation efficiency at the measurement level, i.e.  $S^{meas}$ . As expected, the radiation efficiency is considerably lower due to the extra substrate and metal losses due to the longer transmission lines.

Throughout this thesis we have used FR4 as our PCB substrate despite its high dielectric losses because it is considerably cheaper compared to lower-loss substrates, such as Rogers 4003C. Nevertheless we will investigate effect of using low-loss substrate on the radiation efficiency. The Rogers 4003C substrate is modelled to have relative permittivity of  $\varepsilon_r = 3.55$  and loss tangent of tan  $\delta = 0.0027$ . Fig. 4.5 also shows the radiation efficiency when using a Rogers 4003C substrate for the same two reference planes. As can be seen, the radiation efficiency is higher when using these low-loss substrates, but the effect is minimal when compared to the  $S^{DUT}$  reference level.

#### 4.2.2 Receiver Noise Modelling

To calculate the receiver noise temperature we will base our calculations on the work done on the PPWG simulator of Chapter 3. For this purpose, once again, the SAV-541+ LNA by Minicircuits<sup>3</sup> is being considered. The datasheet supplied noise parameters of this LNA, given at 0.5, 0.7, 0.9, 1.0, and 1.9 GHz, are linearly inter- and/or extra-polated to match the measurement points of the array. In the

<sup>&</sup>lt;sup>3</sup>https://www.minicircuits.com/pdfs/SAV-541+.pdf

case of the unit cell simulation we can follow the well-known standalone receiver noise temperature calculation via

$$T_{rec} = T_{min} + 4T_0 N \frac{|\Gamma_{act} - \Gamma_{opt}|^2}{(1 - |\Gamma_{opt}|^2)(1 - |\Gamma_{act}|^2)}$$
(4.21)

where  $T_{min}$  is the minimum noise temperature of the LNA, where  $T_0$  is the reference ambient temperature of 290 K,  $\Gamma_{act}$  is the active reflection coefficient of the source antenna,  $\Gamma_{opt}$  is the optimum reflection coefficient for minimum noise match and  $N^4$  is the Lange invariant noise parameter [100]. The Lange invariant noise parameter version of Eq. (4.21) correctly accounts for the differential connection of the single-ended LNA [101]. In the case of the measured and simulated finite arrays the noise coupling within the array is accounted for by calculating the beam-equivalent receiver noise temperature [102], which gives a single value array equivalent noise temperature. This calculation is cross-validated with the framework presented in [78]. This calculation is in contrast with the relevant, but incomplete calculation of the receiver noise temperature in Section 2.3.3.

Fig. 4.8 shows the resulting receiver noise temperature at boresight and a  $45^{\circ}$  zenith scan in the H-plane for the three set-ups, the unit cell simulation, the perfect PPWG simulation and the measured prototype of Chapter 3. The extrapolated datasheet values of the minimum noise temperature of the LNA  $(T_{min})$  is shown as a reference. At boresight, a very good match between the calculated noise temperature based upon simulation and measurement can be seen, further proving the validity of the CCDA and Parallel-Plate WaveGuide (PPWG) designs. At boresight, for the measured results, we achieve a calculated receiver noise temperature below the MFAA limit of 30 K for the frequency range of 300 to 1350 MHz, or a bandwidth of 1 : 4.5. The receiver noise temperature result reported here is comparable with contemporary ambient temperature receiver noise results [103, 104, 105, 106].

<sup>&</sup>lt;sup>4</sup>Note that  $N \equiv R_n \Re \{Y_{opt}\}$  is used instead of the customary equivalent noise resistance  $R_n$ , where the admittance  $Y_{opt}$  is related to  $\Gamma_{opt}$ .

At higher zenith scan angles in the H-plane, the noise temperature calculation based upon the measurement and simulations start to differ more, as can be seen in Fig. 4.8 where the noise temperature calculation based upon the measured result actually shows a considerably lower calculated receiver noise temperature. To explain this result we show the active reflection coefficient of the unit cell simulation as well as the active reflection coefficient of element #9 of the constructed prototype in a Smith Chart, for boresight scan and a 45° zenith scan in the H-plane in Fig. 4.6 and Fig. 4.7 respectively. We can see that at the relevant frequencies, the measured active reflection coefficient is more inductive than the simulated active reflection coefficient. As a result, at the center of our frequency band we do not only have a better power match as was seen in Fig. 3.12 but also a better noise match which results in the lower receiver noise temperature seen in Fig. 4.8.

The Smith Charts of Fig. 4.6 and Fig. 4.7 also show that for this particular choice of LNA, and based upon the assumptions about the array response made previously, there is a non-optimal noise match. A better noise match can be achieved by moving the inner loop of the active reflection coefficient contours, which corresponds to the center frequencies of our frequency band, up in the Smith Chart. This suggest making the active differential reflection coefficient more inductive. Referring back to Section 2.1.1 and especially Eq. (2.4), we see several options. This could be achieved by increasing the total per unit inductive reactance  $X_A$  by simply increasing the length of the dipoles. However, if we do not want to change inter-element spacing, a meandering dipole or a dipole overlap [38] will have to be considered. Alternatively, one could reduce the per unit tip capacitance  $C_{tip}$  or increase the height of the dipoles over the groundplane  $(H_{dip})$ . Future work should pursue an active noise match optimization which will include common mode effects, noise coupling and finite array effects.

A receiver noise temperature calculation based upon the measured active reflection coefficient of the CCDA elements and a 30 K receiver noise temperature



Figure 4.6: Extrapolated optimum reflection coefficient of the SAV-541+ LNA by Minicircuits (dotted black line). Active differential reflection coefficient of the unit cell simulation (dash-dot red line) and of element #9 of the measured CCDA prototype (solid blue line), at boresight scan (0°). All for the 300 – 1400 MHz frequency range. Frequency markers at 400 MHz, 700 MHz, 1000 MHz, and 1300 MHz where an increasing frequency moves clockwise. The frequency markers at 1 GHz for both contours have been emphasized for extra clarity.

limit shows that a bandwidth of 1 : 4.5 can easily be obtained at boresight scan with the current design. We expect to further broaden this bandwidth and improve the noise match at higher zenith scan angles once an LNA noise match optimization is actively pursued.



Figure 4.7: Extrapolated optimum reflection coefficient of the SAV-541+ LNA by Minicircuits (dotted black line). Active differential reflection coefficient of the unit cell simulation (dash-dot red line) and of element #9 of the measured CCDA prototype (solid blue line), at a 45° zenith scan in the H-plane. All for the 300 – 1400 MHz frequency range. Frequency ticks at 400 MHz, 700 MHz, 1000 MHz, and 1300 MHz where an increasing frequency moves clockwise. The frequency markers at 1 GHz for both contours have been emphasized for extra clarity.

#### 4.3 Sensitivity of the CCDA

In the previous sections we have derived all the necessary components of Eq. (4.1) to show a first estimation of the sensitivity of the CCDA. First off, we can show the calculated system noise temperature by using Eq. (4.18) in Fig. 4.9. In the [400 - 1350] MHz range the system noise temperature is below 45 K for boresight scan and below 65 K for a  $45^{\circ}$  scan in the H-plane. The calculated sensitivity is shown in Fig. 4.10 for two different zenith scan angles in the H-



Figure 4.8: Receiver noise temperature of a unit cell (dash-dot red line) and the beamequivalent noise temperature of the 16-elements within a linear array of the simulated (dashed green line) and measured (solid blue line) CCDA within a PPWG structure at a boresight/zenith scan angle ( $\theta = 0^{\circ}$ ). Calculated based upon the interpolated datasheet values of the SAV-541+ LNA by Minicircuits assuming a purely differential connection. Minimum noise temperature of the LNA given as reference (black dotted line), where the markers show the datasheet frequency points from where the noise parameters are extrapolated.

plane,  $(\theta_0 = 0^\circ, \phi_0 = 90^\circ)$  and  $(\theta_0 = 45^\circ, \phi_0 = 90^\circ)$ . For this calculation we used the receiver noise temperature calculated based upon the measured 16 linear CCDA elements in the prototype PPWG, as given by the blue lines in Fig. 2.12. The radiation efficiency is based on Section 4.2.2 and the ' $S^{DUT}$  FR4' unit cell simulation result, where any scan dependence on the radiation efficiency is not accounted for.

The effective area effects seen at 200 MHz and 1500 MHz remain unexplained and the frequency points of onset of these effects are unknown. As such, the



Figure 4.9: Calculated system noise temperature  $(T_{sys})$  of the 16-by-16 element singlepolarized CCDA based upon the derivations and assumptions set out in this chapter. System noise temperature shown for two scan directions in the H-plane, the blue line showing  $(\theta_0 = 0^o, \phi_0 = 90^o)$  and the red line showing  $(\theta_0 = 45^o, \phi_0 = 90^o)$ .

sensitivity is calculated based upon the projection of the physical area of the array in the plane of incidence only, i.e.  $A_{phys} \cos \theta_0$ , and the resulting sensitivity is more uncertain below 525 MHz and above 1175 MHz as a result. As a reference, the sensitivity for the five frequency points at which the effective area has been simulated previously are shown by the black ticks in Fig. 4.10.

At boresight in the centre of the frequency band we achieve a sensitivity of  $\approx 0.025 \text{ m}^2/\text{K}$ . With a physical area of 2.073 m<sup>2</sup>, the total system noise temperature is  $T_{sys} \approx 45$  K, a figure which is line with the expectations of an instrument for the MFAA [34]. At a  $\theta_0 = 45^\circ$  zenith scan angle the sensitivity drops by approximately a factor of 2 of which  $\cos 45^\circ = 1/\sqrt{2}$  is explained by the projection



Figure 4.10: Estimation of the sensitivity of 16-by-16 element single-polarized CCDA based upon the derivations and assumptions set out in this chapter. Sensitivity shown for two scan directions in the H-plane, the blue line showing ( $\theta_0 = 0^o, \phi_0 = 90^o$ ) and the red line showing ( $\theta_0 = 45^o, \phi_0 = 90^o$ ). Black ticks show the sensitivity calculated based upon the simulated CCDA effective area at the five frequency points, as given in Fig. 4.4.

effect of the effective area alone.

From this calculation it is immediately apparent that the bandwidth of the current design is limited by both the receiver noise temperature and the radiation efficiency of the design below 400 MHz and above 1350 MHz. However, we expect to further broaden the bandwidth once an LNA noise match optimization design is actively pursued.

## Chapter 5

## Conclusions and Recommendations

This thesis presented work on the next-generation radio astronomy telescopes such as those required for the Mid-Frequency Aperture Array (MFAA) which will be part of the second phase of the Square Kilometre Array (SKA). To this end we explored the capabilities of the Capacitively-Connected Dipole Array (CCDA) [37, 42, 47, 48, 49]. The CCDA is a dense array of dipole elements that are capacitively coupled to each other. This type of highly-connected dense array requires a fundamentally different design approach to sparse arrays. The mutual coupling between the array elements in the CCDA is integral to its broadband response, whereas sparse arrays usually start out by designing a broadband isolated element and only consider the mutual coupling between array elements as a detrimental afterthought. As such, the transmission line based equivalent circuit of a unit cell in a two-dimensional infinite array has been a very useful design tool.

In Chapter 2 we used the transmission line equivalent circuit to get a better understanding of the CCDA and used this understanding to design a CCDA dipole element for the MFAA. Because of its connected nature and high mutual coupling detrimental finite array effects guided wave modes can be excited over the array plane. By applying the transverse resonance method to the transmission line equivalent circuit mode solutions for the excitation of guided wave modes were found. Unable to avoid edge-born guided wave solutions in a broadband design it was then shown that any form of resistive loading of the array, for example by the inclusion of a Low-Noise Amplifier (LNA) in an active array, will sufficiently suppress these unwanted finite array effects.

One of the greatest challenges with these type of large-scale dense connected arrays is to find a small-scale intermediate prototyping method that is representative of the larger array response. The biggest contribution of this thesis has been in finding this novel experimental validation method. In Chapter 3 we discussed how by placing a linear CCDA between the walls of a Parallel-Plate WaveGuide (PPWG) allows us to measure the active reflection coefficient of the linear array as if it were part of a much larger two-dimensional array. Doing so effectively reduced the measurement effort quadratically and furthermore allowed the scanning of the entire H-plane irrespective of the frequency and without the use of phase-shifters.

Finally, in chapter 4, based upon a cohort of measured data, simulated data and modelled data collected we presented the first calculation of the sensitivity the CCDA design for the MFAA. Although several significant uncertainties exist about this calculation, especially in terms of the effective collecting area of the array, it still provides the most comprehensive estimation of the CCDA performance as a radio astronomy instrument to date. We concluded that the required bandwidth for MFAA of roughly 1 : 4 to 1 : 4.5 can be achieved once an active noise match optimization is explicitly pursued.

Several recommendations for future work are suggested, primarily based upon the current characterization of the CCDA as a radio astronomy instrument in Chapter 4:

• An LNA noise match design optimization for the use in the CCDA which results in an active array prototype.

- Exploration of a small-scale prototyping, measurement and validation method which allows for the measurement of the receiver noise temperature of an active CCDA, in contrast to the method presented in Chapter 3 which only allowed the measurement of a passive instrument.
- Further investigation into the effective area of the CCDA and especially the yet unexplained effects seen at the edges of the frequency band.
- Far-field measurement of a prototype CCDA to validate the effective area calculations.
- Exploration of the dual-polarized element, which includes the cross-coupling and polarization purity of the CCDA.
- Further exploration of finite array effects such as the guided wave modes discussed in Chapter 2.

Appendices



Figure A.1: Simplified definition of the de-embedding problem assuming the feed between  $S^{meas}$  and  $S^{DUT}$  utilizes a BALanced to UNbalanced transformer (BALUN).

## Appendix A

# Mixed-mode Considerations on De-embedding and the Use of BALUN Supported Elements

Assume an 2N-port network, given here by Device Under Test (DUT), which is fed by N identical RF transformers. These RF transformers transform the two-port S-parameters (plus and min) at the DUT level to one-port (singleended) S-parameters at the measurement level with the help of a BALUN. At the measurement level we have the relationship between the measured ingoing ( $\boldsymbol{a} = [a_1, a_2, \ldots, a_N]$ ) and outgoing ( $\boldsymbol{b} = [b_1, b_2, \ldots, b_N]$ ) power wave amplitudes as

$$\boldsymbol{b} = \boldsymbol{S}_{meas} \boldsymbol{a} \tag{A.1}$$

with  $S_{meas}$  the measured scattering matrix. Assuming we have fully characterized the response of the RF-transformers, we know the relationship between the ingoing and outgoing power waves at the single-ended port (<sub>s</sub>), the minus port (<sub>m</sub>) and the plus port (<sub>p</sub>) we can then write the following relationship

$$\begin{pmatrix} \boldsymbol{b} \\ \boldsymbol{a}_{p} \\ \boldsymbol{a}_{m} \end{pmatrix} = \begin{pmatrix} \boldsymbol{S}_{ss}^{RF} & \boldsymbol{S}_{sp}^{RF} & \boldsymbol{S}_{sm}^{RF} \\ \boldsymbol{S}_{ps}^{RF} & \boldsymbol{S}_{pp}^{RF} & \boldsymbol{S}_{pm}^{RF} \\ \boldsymbol{S}_{ms}^{RF} & \boldsymbol{S}_{mp}^{RF} & \boldsymbol{S}_{mm}^{RF} \end{pmatrix} \begin{pmatrix} \boldsymbol{a} \\ \boldsymbol{b}_{p} \\ \boldsymbol{b}_{m} \end{pmatrix}$$
(A.2)

where I is a N-by-N identity matrix and  $S_{ss}^{RF}$  an element of the scattering matrix of the three-port RF-transformer. Combining Eq. (A.1) and the first row of Eq. (A.2) we can find a as

$$\boldsymbol{a} = \left(\boldsymbol{S}_{meas} - \boldsymbol{S}_{ss}^{RF}\right)^{-1} \left(\boldsymbol{S}_{sp}^{RF} \boldsymbol{b}_{p} + \boldsymbol{S}_{sm}^{RF} \boldsymbol{b}_{m}\right)$$
(A.3)

Filling this in into the second and third row of Eq. (A.2) allows us to write

$$\boldsymbol{a}_{p} = \left(\boldsymbol{S}_{ps}^{RF} \left(\boldsymbol{S}_{meas} - \boldsymbol{S}_{ss}^{RF}\right)^{-1} \boldsymbol{S}_{sp}^{RF} + \boldsymbol{S}_{pp}^{RF}\right) \boldsymbol{b}_{p} +$$
(A.4)

$$\left(\boldsymbol{S}_{ps}^{RF}\left(\boldsymbol{S}_{meas}-\boldsymbol{S}_{ss}^{RF}\right)^{-1}\boldsymbol{S}_{sm}^{RF}+\boldsymbol{S}_{pm}^{RF}\right)\boldsymbol{b}_{m}=$$
(A.5)

$$\hat{\boldsymbol{S}}_{pp}^{DUT}\boldsymbol{b}_{p}+\hat{\boldsymbol{S}}_{pm}^{DUT}\boldsymbol{b}_{m} \tag{A.6}$$

$$\boldsymbol{a}_{m} = \left(\boldsymbol{S}_{ms}^{RF} \left(\boldsymbol{S}_{meas} - \boldsymbol{S}_{ss}^{RF}\right)^{-1} \boldsymbol{S}_{sp}^{RF} + \boldsymbol{S}_{mp}^{RF}\right) \boldsymbol{b}_{p} +$$
(A.7)

$$\left(\boldsymbol{S}_{ms}^{RF}\left(\boldsymbol{S}_{meas}-\boldsymbol{S}_{ss}^{RF}\right)^{-1}\boldsymbol{S}_{sm}^{RF}+\boldsymbol{S}_{mm}^{RF}\right)\boldsymbol{b}_{m}=$$
(A.8)

$$\hat{\boldsymbol{S}}_{mp}^{DUT}\boldsymbol{b}_{p}+\hat{\boldsymbol{S}}_{mm}^{DUT}\boldsymbol{b}_{m} \tag{A.9}$$

Finally, the scattering parameters of the DUT  $(\boldsymbol{S}^{DUT})$  can be found via

$$\begin{pmatrix} \boldsymbol{b}_{p} \\ \boldsymbol{b}_{m} \end{pmatrix} = \begin{pmatrix} \hat{\boldsymbol{S}}_{pp}^{DUT} & \hat{\boldsymbol{S}}_{pm}^{DUT} \\ \hat{\boldsymbol{S}}_{mp}^{DUT} & \hat{\boldsymbol{S}}_{mm}^{DUT} \end{pmatrix}^{-1} \begin{pmatrix} \boldsymbol{a}_{p} \\ \boldsymbol{a}_{m} \end{pmatrix} = \boldsymbol{S}^{DUT} \begin{pmatrix} \boldsymbol{a}_{p} \\ \boldsymbol{a}_{m} \end{pmatrix}$$
(A.10)

from which one may **incorrectly** conclude that it is possible to fully characterize  $2N S^{DUT}$  system with just N-port measurements  $S^{meas}$  if the BALUN utilizing feed lines, given by  $S^{RF}$  are known. However, if we rewrite the above equations in matrix form we find that we attempted to find the 4N unknowns of  $S^{DUT}$  with only N known values,  $S^{meas}$ . One may argue that  $S^{RF}$  is known as well, but having fully characterized the transformer in isolation does not provide further information on the  $S^{meas}$ . As such, the above gives **one** solution to the problem, but not a unique solution and not necessarily the solution we are looking for.

#### **Pure Differential Assumption**

Now assume that we are only interested in the differential mode, i.e. assume a perfect RF transformer. We can derive the mixed mode scattering parameters of the RF-transformer as

$$\boldsymbol{S_{mixedmode}^{RF}} = \begin{pmatrix} S_{ss}^{RF} & S_{sd}^{RF} & S_{sc}^{RF} \\ S_{ds}^{RF} & S_{dd}^{RF} & S_{dc}^{RF} \\ S_{cs}^{RF} & S_{cd}^{RF} & S_{cc}^{RF} \end{pmatrix} = \boldsymbol{M}\boldsymbol{S}^{RF}\boldsymbol{M}^{-1}$$
(A.11)

with M given as

$$\boldsymbol{M} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$
(A.12)

when the first port of  $\mathbf{S}^{RF}$  is the single ended measurement port and port 2 and port 3 make up the differential port. Assuming the RF-transformer is ideal and completely blocks any common mode transmission, we can write

$$\boldsymbol{S}_{mixedmode}^{RF} = \begin{pmatrix} S_{ss}^{RF} & S_{sd}^{RF} & 0\\ S_{ds}^{RF} & S_{dd}^{RF} & 0\\ 0 & 0 & 1 \end{pmatrix}$$
(A.13)

This in turn allows us to write the **differential** response of the DUT as

$$\boldsymbol{S}_{dd}^{DUT} = \left(\boldsymbol{S}_{ds} \left(\boldsymbol{S}_{meas} - \boldsymbol{S}_{ss}\right)^{-1} \boldsymbol{S}_{sd} + \boldsymbol{S}_{dd}\right)^{-1}$$
(A.14)

However, as was also noted in [38], the Capacitively-Connected Dipole Array (CCDA) under consideration is prone to unwanted common mode transmission. A full mixed-mode de-embedding is hence preferred.

#### Full Mixed-mode De-embedding

In this section we show the de-embedding method used in this thesis, such that the full mixed-mode 2*N*-port scattering parameters of  $S^{DUT}$  can be de-embedded from the full mixed-mode 2*N*-port scattering parameters of  $S^{meas}$  as they were defined in Fig. A.2. This problem is presented simplified in Fig. A.2. For this we will use the method in [90] in which the following sub-matrices are defined as

$$\Gamma_{00} = \begin{pmatrix} S_{33}^{RF} & S_{34}^{RF} \\ S_{43}^{RF} & S_{44}^{RF} \end{pmatrix}$$
(A.15a)


Figure A.2: Simplified definition of the de-embedding problem assuming just two straightforward feed lines between  $S^{DUT}$  and  $S^{meas}$ , as is the problem under consideration in this thesis as given by Fig. 2.6.

$$\Gamma_{01} = \begin{pmatrix} S_{13}^{RF} & S_{14}^{RF} \\ S_{23}^{RF} & S_{24}^{RF} \end{pmatrix}$$
(A.15b)

$$\Gamma_{10} = \begin{pmatrix} S_{31}^{RF} & S_{32}^{RF} \\ S_{41}^{RF} & S_{42}^{RF} \end{pmatrix}$$
(A.15c)

$$\Gamma_{11} = \begin{pmatrix} S_{11}^{RF} & S_{12}^{RF} \\ S_{21}^{RF} & S_{22}^{RF} \end{pmatrix}$$
(A.15d)

The relationship between the DUT and measurement level is then given by these equations:

$$\boldsymbol{S}^{meas} = \boldsymbol{\Gamma}_{00} + \boldsymbol{\Gamma}_{01} \left[ \boldsymbol{I} - \boldsymbol{S}^{DUT} \boldsymbol{\Gamma}_{11} \right]^{-1} \boldsymbol{S}^{DUT} \boldsymbol{\Gamma}_{10}$$
(A.16)

and the inverse relationship is give by

$$\boldsymbol{S}^{DUT} = \left(\boldsymbol{\Gamma}_{01}^{-1}\boldsymbol{\Gamma}_{00} - \boldsymbol{\Gamma}_{01}^{-1}\boldsymbol{S}^{meas}\right) \left(\boldsymbol{\Gamma}_{11}\boldsymbol{\Gamma}_{01}^{-1}\boldsymbol{\Gamma}_{00} - \boldsymbol{\Gamma}_{10}\right)^{-1}$$
(A.17)

where I is an identity matrix of size 2N-by-2N and where the sub-matrices have been multiplied with the identity matrix of size 2N-by-2N.



Figure B.1: Three elements of the Capacitively-Connected Dipole Array (CCDA) fed by a differential feed line. The current is separated to show differential- (red arrows) and common-mode (blue) resonance operation. An example of an enforced boundary condition is given by the two  $E_t = 0$  planes.

## Appendix B

# Common-mode Current and Resonances in the CCDA

The short inter-element spacing and capacitive connection between the elements in a CCDA could result in common-mode resonances. Fig. B.1 shows three elements of a single polarization of the CCDA along the direction of physical connection. The direction of the current under differential operation of the array is shown by the red arrows. A secondary unwanted common-mode current path exists between the positive feed line of element n and the negative feed line of element n - 1, as is depicted in Fig. B.1 by the blue arrows. A common-mode resonance occurs when the current path length is exactly 360°, where the path length is a function of the inter-element spacing, the length of the feed-lines, the frequency, and the scanning direction. The current path dictates that neighbouring elements have an opposite common-mode current direction. Previous work has shown that either the inclusion of a common-mode suppressing feed [38, 39] or a careful consideration of the array loading [40] is required to reduce the unwanted common-mode currents and utilize the entire potential bandwidth of a connected array.

The common-mode resonances are not a function of the number of elements in the CCDA and can exists both in finite and theoretical infinite arrays. However, this changes when a (perfect) boundary condition is imposed. Perfect boundary conditions are imposed when an infinite array is modelled by a unit cell with phase-shift boundaries in simulation software, perfect phase-shift boundary conditions are assumed in the basic unit cell equivalent circuit [42] discussed in Chapter 2, and in Chapter 3 we have approximated a perfect boundary condition by placing a linear CCDA between the walls of a Parallel-Plate WaveGuide (PPWG). An example is given in Fig. B.1 where two perfect  $\hat{y}\hat{z}$ -plane electric boundaries are imposed, denoted by  $E_t = 0$ . Upon first sight, both the differential- (red arrows) and common-mode (blue arrows) currents are correctly following the  $E_t = 0$ boundary condition. However, the effective current on the positive and negative feed-lines for an element placed between the boundary condition planes will be

$$i_{eff}^{\pm} = \sum_{n=1}^{N} i_n^{\pm} = \begin{cases} = 0 & \text{common-mode} \\ \neq 0 & \text{differential-mode} \end{cases}$$
(B.1)

as follows directly from the current direction in Fig. B.1. A common-mode current path does not exist upon imposing the boundary condition, and as such we do

not see common-mode resonances in the prototype linear CCDA of Chapter 3 once placed between the PPWG.

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## List of Publications

### **Journal Publications**

[P1] R.A.C. Baelemans, A.T. Sutinjo, D. Prinsloo, A.B. Smolders, D.B. Davidson, "Fast and Accurate Prototyping of Capacitively-Connected Dipole Arrays With a Parallel-Plate Waveguide", IEEE Open Journal of Antennas and Propagation, in preparation

[P2] R. A. C. Baelemans, A. T. Sutinjo, P. J. Hall, A. B. Smolders, M. J. Arts and E. de Lera Acedo, "Closed-Form Jones Matrix of Dual-Polarized Inverted-Vee Dipole Antennas Over Lossy Ground," in IEEE Transactions on Antennas and Propagation, vol. 65, no. 1, pp. 26-35, Jan. 2017, doi: 10.1109/TAP.2016.2630665.

## **Conference** Publications

[P3] R. Baelemans, A. Sutinjo, B. Smolders, D. Davidson, U. Johannsen and R. Wayth, "Analysis of Surface Waves Modes in Planar Connected Arrays for Radio Astronomy," 2018 International Conference on Electromagnetics in Advanced Applications (ICEAA), Cartagena des Indias, 2018, pp. 446-449, doi: 10.1109/ICEAA.2018.8520459.

[P4] R. Baelemans, D. Prinsloo, B. Smolders, A. Sutinjo, D. B. Davidson and R. Wayth, "Parallel Plate Waveguide Simulator of a Dense Connected Dipole Array," 2020 14th European Conference on Antennas and Propagation (EuCAP), 2020, pp. 1-4, doi: 10.23919/EuCAP48036.2020.9135267. [P5] R. Baelemans, A. Sutinjo, P. Hall and B. Smolders, "Analysis of the Polarization Properties of Dual Polarized Inverted Vee Dipole Antennas over a Ground Plane," 2016 IEEE International Symposium on Antennas and Propagation (AP-SURSI), 2016, pp. 1883-1884, doi: 10.1109/APS.2016.7696648.

[P6] D. B. Davidson et al., "Recent Progress on the Design of Aperture Arrays for Radio Astronomy," 2018 IEEE Radio and Antenna Days of the Indian Ocean (RADIO), 2018, pp. 1-2, doi: 10.23919/RADIO.2018.8572328.

### Design and Verification of a Capacitively-Connected Dipole Array With a Parallel-Plate Waveguide

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Abstract—This work presents the design and experimental verification of a capacitively-connected dipole array designed for low-frequency radio-astronomy such as the Mid-Frequency Aperture Array of the Square Kilometre Array. Integral to the verification of the design was the construction of a parallel-plate waveguide. The parallel-plate waveguide allows the measurement of the active reflection coefficient of a linear array as if it was part of a finite-by-infinite array, effectively reducing the measurement effort quadratically. The active reflection coefficient can be measured irrespective of the frequency and for the array scanned over the entire H-plane without phase-shifters. Measurement of the active reflection coefficient shows good agreement with simulation results, justifying the measurement approach. Based upon a boresight receiver noise temperature calculation limit of 30 K we can confidently claim a 1 : 4.5 bandwidth for the current capacitively-connected dipole array design.

Index Terms—radio astronomy, phased arrays, antenna measurement, wide-band, connected arrays

#### I. INTRODUCTION

THE next generation low-frequency phased-array radiotelescopes, such as those part of the Mid-Frequency Aperture Array (MFAA) of the Square Kilometre Array (SKA) [1], require low-cost antenna technologies which can provide a large bandwidth, low receiver noise temperature, and a large Field-of-View [2]. Furthermore, it has been shown that several science cases require a smooth spectral and scan response [3], [4]. As such, the predictable and smooth response of dense regular arrays are preferred over sparse randomized irregular arrays. The capacitively-connected dipole array (CCDA) [5]-[8] can provide these requirements.

The favourable response of the CCDA is explained by the fact that it is an approximation of Wheeler's continuous current sheet [9] concept. Allowing for a continuous current path within the array results in a smooth spectral response. Wheeler furthermore found that the element resistance at zenith scan angle  $\theta$ , was proportional to  $1/\cos\theta$  in the H-plane and  $\cos\theta$  in the E-plane. Fig. 1 shows an artist's impression of the top view of a dual-polarized CCDA. As the name suggests, a

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The bandwidth of these type of arrays can be extended by placing a dielectric superstrate on top of the dipole elements [5]. Further extension of the bandwidth has been achieved by combining the superstrate with a lossy resistive loop [10] or lossy parasitic strips [11]. Ultrawideband arrays are achieved by placing a resistive sheet in the substrate and several layers of lossy frequency selective surfaces (FSS) in the superstrate [12], [13], where bandwidths as high as 46 : 1 have been reported. These additions to the standard connected array design all introduce loss into the design and are as such unsuitable to be used in a low-noise application such as lowfrequency radio-astronomy.

As with any very large dense phased array, a good first design step is to consider a single unit cell in an infinite array with the help of simulation software or electromagnetic theory. The unit cell is a good first design step since the array response generally dominates the isolated element response in any dense array. However, once the time comes to construct the first practical prototype, one usually resorts to the construction of a passive array of only a few elements to reduce the cost and measurement effort despite the fact that it will result in a poor estimate of the larger array response [14], especially in the case of high mutual coupling arrays such as the CCDA [15].

One method to measure the active reflection coefficient of a large array is a rectangular waveguide<sup>1</sup> [16]. These rectangular waveguides rely on the propagation of TE and/or TM modes in the cavity. The downside to this method is that a broadside scan cannot be achieved and that the scan direction of the simulated array becomes a function of frequency. Furthermore, for the CCDA discussed in this paper, a large number of elements would have to be placed inside the rectangular waveguide cavity since the length of the elements is much shorter than both the cut-off frequency of the rectangular waveguide and the wavelength of operation of the array.

Alternatively, the (improved) Wheeler cap (IWC) method [17]-[19] has been used to measure antenna

<sup>1</sup>Historically, these type of structures are called waveguide simulators, since they simulate a larger array response. However, to not confuse the reader with CAD electromagnetic simulation software, we have chosen to refer to these type of structures as just waveguides throughout this paper.



Fig. 1. Top view of an infinite dual-polarized capacitively-connected dipole array, where  $^2$  depicts a dual-polarized feed port. Note that the orthogonal arrays are **not** electrically connected. The blue dashed line confines a dual-polarized unit cell of the infinite array. The inter-element spacing (*d*) is equal for the  $\hat{x}$ - and  $\hat{y}$ -directed polarization for this regular dense array.

efficiency of 1-D connected arrays with great accuracy. However, for the CCDA we expect the mutual coupling between the elements to be significant, not only in the direction of physical connection but also between the elements in neighbouring rows. Therefore, we expect the 1-D IWC method to provide a rather poor approximation of the larger connected array response.

In this paper we introduce a new experimental validation method for dense arrays based on a parallel-plate waveguide (PPWG) instead of the usual rectangular waveguides. PPWGs have previously been used to improve the bandwidth of linear arrays [20], [21], and even the bandwidth of a linear CCDA [22], [23]. In this paper the PPWG is used as an experimental verification tool of a 2D array and as such our goal is to mimic the 2D array response as accurately as possible. As we will show, a theoretical 1-by- $\infty$  linear array placed in between the walls of a PPWG will allow us to simulate an  $\infty\mbox{-by-}\infty$  array, effectively reducing the measurement effort quadratically. We can furthermore scan the H-plane and are not frequency limited since the TEM mode within a parallel-plate waveguide has no cut-off frequency. The operating frequency of the array and the PPWG are well below the cut-off frequency of any higher order modes.

The Mid-Frequency Aperture Array (MFAA) is the main motivator for the work undertaken in this paper. The CCDA is designed to meet the current requirements of the MFAA [2]. The MFAA is expected to at least cover the 450-1450 MHz frequency range and as such we design the CCDA to cover a wider frequency range of 200-1500 MHz. Furthermore, the MFAA is expected to cover a zenith scan angle of up to  $\pm 45^{\circ}$  and the full azimuthal angle with two orthogonal polarizations. Finally, the active MFAA is expected to have a



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Fig. 2. 3-D view of a 4-by-4 element dual-polarized capacitively-connected dipole array over a finite groundplane realized on printed-circuit boards. Each antenna port is fed down by a pair of microstrip feed lines into a pair of SMA connectors on the backside of the groundplane. The tip capacitance is realized by small lumped-element surface mount capacitors and hence indistinguishable in this picture.

### TABLE I CURRENT RELEVANT TECHNICAL REQUIREMENTS OF THE MFAA.

Parameter	Essential requirement
Frequency of operation	450-1450 MHz
Maximum zenith scan angle	$\pm 45 \text{ deg}$
Maximum receiver noise temperature	30 K

beam-equivalent noise temperature below 30 K within these boundaries. These requirements are summarized in Table I. We assume a full CCDA array tile to be 16-by-16 elements in size, or 256 elements per polarization. This choice guarantees that the full array is a wavelength in size, even at the lower end of the frequency band, minimizing finite array effects [24].

Section II will discuss the theoretical background of the CCDA design and the PPWG starting with a review and summary of the equivalent circuit of a single unit cell in an  $\infty$ -by-  $\infty$  CCDA [5] and showing how this equivalent circuit can be used to design a CCDA. Then we will expand on the equivalent circuit to introduce the PPWG [25] measurement structure. The construction of both the prototype CCDA and the PPWG is discussed in Section III. The measurement of the prototype is discussed in Section IV. Finally, Section V will present some initial receiver noise temperature modelling to justify the design for the use in low-frequency radio-astronomy.

#### II. THEORETICAL BACKGROUND

#### A. Equivalent circuit

Fig. 3 (a) shows the equivalent circuit of a *single* polarization of a unit cell of an  $\infty$ -by- $\infty$  CCDA over a perfect groundplane [5]. The  $\hat{x}$ -directed array in Fig. 1 is taken as the starting point for the unit cell derivations that follow. We do note that the equivalent circuit, and hence the soon to be derived PPWG structure, are only valid for a single polarization because it does not account for the cross-polarization in the inter-cardinal planes. As a result, strictly speaking, the equivalent circuit is only valid for the two main scan planes, i.e. scan in the  $\hat{y}\hat{z}$ -plane (H-plane) and scan in the  $\hat{x}\hat{z}$ -plane



Fig. 3. (a) Equivalent circuit of a unit cell of a **single** polarization of the CCDA over a perfect and infinite ground plane radiating into free space, where the scan-direction is limited to  $i\hat{p}^2$ -plane (H-plane) scan only allowing us to replace the phase shift walls with infinitely extending perfect electric conductors, denoted by  $E_{\rm e} = 0$ . (b) Model of the proposed parallel-plate awarguide structure where the array radiates into a pyramidal absorber. (c) Conceptual drawing where the pyramidal part of the absorber is replaced by N homogenized layers.

(E-plane). However, we expect the coupling between the two orthogonal polarizations to be minimal.

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Since the unit cell boundaries can be seen as a virtual waveguide that only supports a TEM wave, we can model the unit cell as a transmission line. The per unit (inductive) reactance of the dipole elements is given by 
$$X_A$$
 and the total per unit tip capacitance is denoted by  $C_{tip}$ . Following the transmission line comparison,  $Z_{in}$  is the radiation impedance of the upper halfspace, which in Fig. 3 (a) is equal to the free space wave impedance. In the two principal scan planes, E-plane and H-plane, the free-space wave impedances is known [9] to be

$$Z_{fs}^{E-plane}(\theta) = \sqrt{\frac{\mu_0}{\varepsilon_0}}\cos\theta \tag{1a}$$

$$Z_{fs}^{H-plane}(\theta) = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{\cos\theta}$$
(1b)

with  $\mu_0$  and  $\varepsilon_0$  as the free-space permeability and permittivity respectively. The zenith scan angle is given by  $\theta$ , where  $\theta =$  $0^o$  is defined as zenith or boresight scan. In the transmission line equivalent circuit the groundplane can be seen as a short termination. As such,  $Z_{gp}(\theta)$  is the wave impedance seen by the array as the result of the reflection of the perfect ground plane and accounts for the total wave impedance of the lower halfspace.  $Z_{gp}(\theta)$  is a function of the height of the dipole element over the ground plane as given by

$$Z_{gp}(\theta) = j Z_{fs} \tan\left(\frac{2\pi}{\lambda_0} \frac{H_{dip}}{\cos\theta}\right)$$

where  $\lambda_0$  is the wavelength in free-space and  $H_{dip}$  the height of the dipole above the groundplane. The resulting active scan

and angle in either of the two principle planes is given by 
$$(1 - 1)^{1/2}$$

$$Z_{act}(\theta) = \left(jX_A + \frac{1}{j\omega C_{tip}}\right) + \left(Z_{gp}(\theta)//Z_{in}(\theta)\right) \quad (3)$$

where // denotes two parallel loads.

With the help of the equivalent circuit we can find the parameters of the CCDA. With a maximum frequency of 1500 MHz, which corresponds to a minimum wavelength of 20 cm, we set the inter-element spacing (d) to 9 cm, to avoid the onset of grating lobes at the horizon. An inductive wire approximation of a dipole of this length gives the total per unit self-inductance equal to 50 nH [26], depending slightly on the chosen wire thickness. We do note that at this stage of the design no choice of low-noise amplifier (LNA) design has been made and as such the design parameters are not optimized to reduce the receiver noise temperature of a specific LNA. Instead we minimize the reactive part of Eq. 3 over the chosen frequency range. The height of the dipole over the groundplane is chosen to be  $8.75\ \mathrm{cm}$  such that for boresight scan at the center frequency (850 MHz) the total radiation impedance, given by  $(Z_{gp}(\theta)//Z_{in}(\theta))$ , has no reactive part. Finally, the total per unit tip capacitance is chosen to be 1 pF to compensate the inductive reflection of the groundplane  $(Z_{gp}(\theta))$  at the lower end of the frequency band.

#### B. PPWG Equivalent Circuit

Expanding upon the concept of the equivalent circuit, if we restrict ourselves to only consider a  $\hat{y}\hat{z}$ -plane (H-plane) scan array, the  $\hat{y}$ -directed phase-shift walls of the equivalent circuit can be replaced with two infinitely extending perfect

(2)

#### TABLE II All relevant dimensions

Quantity	Value	Description
Quantity	value	Description
d	9 cm	Inter-element spacing of the CCDA
$H_{rec}$	40 cm	Height of the rectangular part of the absorber
Habs	25 cm	Height of the pyramidal part of the absorber
Hgap	1 cm	Average gap between dipole and absorber
H <sub>dip</sub>	8.75 cm	Height of the dipole over the g.p.
C <sub>tip</sub>	1 pF	Total per unit tip capacitance
H <sub>tot</sub>	75 cm	Total height of the copper walls
$W_{ml}$	0.3 cm	Width of the microstrip feed lines
$H_{ml}$	6.7 cm	Height of the microstrip lines
$D_{ml}$	1.7 cm	Distance between the microstrip lines
$W_{fg}$	0.8 cm	Width of the feed gap
Wdip	0.75 cm	Width of the dipole

electric conductors (PEC). These two PEC walls are denoted by  $E_t = 0$  in Fig. 3 (a), essentially creating an infinitely extending parallel-plate waveguide. As a next step we limit the height ( $\hat{z}$ -direction) of these PEC walls. By doing so, a cavity to free-space boundary at the top of the parallel-plates is created. To reduce the reflections from this cavity to free space boundary the cavity is filled with a (pyramidal) electromagnetic absorber and closed off with another PEC plate on top as is shown in Fig. 3 (b).

As was shown in [25], this addition to the equivalent circuit can be analysed by considering the pyramidal part of the absorber as N infinitesimal small homogenized layers with increasing electromagnetic properties, as is conceptually depicted in Fig. 3 (c). This enables the derivation of the radiation impedance of the upper halfspace  $Z_{in}$  as seen by the array for the direct path of an incident electric field incident at zenith angle  $\theta$  as the result of N consecutive reflections. Previous work [25] based upon this homogenization of the pyramidal part of the absorber has shown that if the absorber is electrically large and lossy enough it will sufficiently suppress the reflections from the top of the PPWG structure such that the radiation impedance for the upper halfspace  $(Z_{in})$  is equal to the free space impedance given by Eq. 1, and thus correctly mimics the infinite array response.

In the equivalent transmission line circuit, which assumes just the excitation of a TEM plane wave, the gap between dipole and pyramidal absorbers  $(H_{gap})$  does not alter the radiation impedance  $(Z_{in})$  seen by the array if the absorbers are truly reflectionless. Increasing the gap between the absorbers and the dipoles is equivalent to adding a matched transmission line in before a reflectionless load which also does not change the impedance at the start of the transmission line.

The spacing between the elements in a CCDA is smaller than half a wavelength over the entire frequency range of operation. As such, the width between the PPWG walls, which is equal to the inter-element spacing of the CCDA given by d, will only support the fundamental TEM-mode and not any higher order TE- or TM-modes. This is in contrast with the more commonly used rectangular waveguides [16], which rely on TE- and/or TM-mode propagation. Consequently, where the scan direction and frequency are physically coupled in rectangular waveguides, the PPWG structure can scan the



Fig. 4. The CCDA dipole printed on a 1.6 mm FR-4 PCB. The tip capacitance is realized by two 0402 surface mount capacitors placed closely to the edge of the dipole. The overlay shows the reference level ( $S^{DCD}$ ) and the measurement level ( $S^{meas}$ ) as well as the definition of all the relevant dimensions. The dashed rectangular line shows the dimension (40 mm by 7.3.5 mm) of the groundplane of the feed-lines on the back side of the PCB.

entire H-plane over the entire operating frequency of the array. In the next section we will discuss the realization of the CCDA and the PPWG.

#### III. DESCRIPTION OF PROTOTYPE

#### A. Dipoles

The sixteen dipoles of the linear CCDA are realized on a 1.6 mm thick FR-4 printed-circuit board (PCB). The layout of the PCB is shown in Fig. 4. The tip-capacitors are realized by two  $0402\ {\rm SMD}$  components soldered on each end of the dipole to one out of the three solder pads available. A big enough solder pad for the dipoles to be soldered to the copper walls of the PPWG is left next to the SMD tip capacitors. The differential port of the dipole is fed by a 100  $\Omega$  differential feed line. The differential microstrip lines are backed by a groundplane on the other side of the PCB. According to coupled line theory, there will be minimal coupling between the two microstrip lines of the differential feed-line [27]. The differential feed line allows for a full mixed mode deembedding up to the reference point. Finally, Fig. 4 shows the two reference planes. The measurement plane at the end-launch SMA-connectors (denoted by  $S^{meas}$ ) and the reference plane (denoted by  $S^{DUT}$ ). The reference plane is used as the reference point for the design as it will serve as the input of the future inclusion of the low-noise amplifiers (LNA), once the array is upgraded to an active array.



Fig. 5. Picture of the realized PPWG for the CCDA while being measured in an anechoic chamber. The wooden encasing is necessary for structural support. The absorbers shown in the background are the same ones as used in the construction of this prototype.



Fig. 6. Picture inside the realized PPWG structure showing the PCBs of the linear CCDA soldered to the copper walls before the inclusion of the absorbers on top. The optical mirroring of the copper walls mimics the electromagnetic mirroring integral to the operation of the PPWG structure.

#### B. Parallel-plate waveguide structure

The PPWG walls, bottom plate, top plate, and ground plane are all made of copper sheets. This allows for the dipoles to be soldered to the walls. A wooden structure, as can be seen in Fig. 5, is added for structural support. Fig. 6 shows a picture taken inside the PPWG structure where the PCBs of the linear CCDA array are soldered to the copper walls.

Unfortunately, commercially available electromagnetic absorbers rarely provide the electromagnetic properties of the material. Instead, their performance is defined in terms of a reflection coefficient and/or absorption coefficient. As a result, despite the modelling work done in [25], the size and the properties of the used absorbers are not optimized. Instead, we have chosen to use standard pyramidal carbon-infused absorbers as commonly used in anechoic chambers which were readily available to us at the time of the construction of the prototype. These pyramidal absorbers can be seen in the background of Fig. 5. To fill the cavity, 16 of these absorbers



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Fig. 7. Side-view cut-out of the PPWG structure showing the 16 elements (pictured as green PCB boards) of the  $\hat{x}$ -directed linear CCDA and the electromagnetic absorbers above each element, as well as the element numbering.

are cut to size, one for each dipole. The rectangular part of the absorbers will have a base of 9-by-9 cm with a height of 40 cm, the pyramidal part will have a height of 25 cm.

Not knowing the exact electromagnetic properties of the absorber material we assume the permittivity of the absorber to be  $\varepsilon_r=2,\,\sigma=8.5e^8$  S/m [28]. This allows us to do a unit cell simulation with an absorber of these dimensions from which we found that a sufficient suppression of the reflections is achieved. CST STUDIO SUITE  $^2$  is used as our simulation software.

In the case of the equivalent circuit derived in Section II, as well as in the case of a unit cell simulation, it was found that an increase or decrease in the gap between the absorbers and the array element ( $H_{gap}$ ) does not alter the radiation impedance for the upper halfspace. However, in the case of the finite sixteen element array the TEM plane wave assumption is no longer strictly true. Nevertheless, to constrain the size of this prototype, the choice was made to place the absorbers right on top of the PCBs. With an average gap of 1 cm between the PCBs and the absorbers, the total structure will be 75 cm tall.

Finally, a choice had to be made how to terminate the cavity in the positive and negative  $\hat{y}$ -directions. Ideally, these cavity to free space boundaries are terminated in the same manner as the cavity to free space boundary at the top of the structure had been terminated. With the proximity of the dipole elements and the absorbers, we expect the open cavities to only significantly affect the two edge elements. To further constrain the size of the prototype the decision had been made to not pursue a completely closed cavity in this first prototype. A schematic side-view cut-out of the structure including the dimensions showing the green PCBs with the absorbers on top is shown in Fig. 7.

#### IV. MEASUREMENT RESULTS

The full 32-port structure was measured with the help of a four-port Vector Network Analyzer (VNA). To fill the entire 32-by-32 scattering matrix 108 distinctive four-port measurements were necessary. For each of these measurements the 28 remaining ports were terminated into a 50  $\Omega$ 

<sup>2</sup>https://www.3ds.com/products-services/simulia/products/cst-studio-suite/



Fig. 8. From left-to-right, active differential reflection coefficient ( $\Gamma_{ddact}$ ) of the CCDA of a centre element #9, the second-to-last element #15 and edge element #16. Simulation of a unit cell in an infinite array (red dash-dot lines), simulation of a perfect PPWG structure (green dashed lines), and the measured results of the CCDA prototype while placed in the constructed PPWG structure (blue solid lines). Array scanned to boresight/zenith ( $\theta = 0^{\circ}$ ) and 45° in the H-plane for the top and bottom row respectively.

broadband load. As a result of this measurement approach, several elements of the scattering matrix, most notably the self-reflections ( $S_{ii}^{meas}$ ), are measured more than once. In these cases the median values at each frequency point are used to fill the scattering matrix.

The measured scattering matrix is then de-embedded [29] up to the DUT reference level, as it is defined in Fig. 4. It is assumed no cross-talk exists between the microstrip lines of neighbouring PCBs. The de-embedded measurement results, with the from here on forward suppressed superscript  $^{DUT}$ , are subsequently transformed into a combined differential and common-mode (mixed mode) scattering parameters [30] given by

$$\boldsymbol{S}_{mm} = \begin{pmatrix} \boldsymbol{S}_{dd} & \boldsymbol{S}_{dc} \\ \boldsymbol{S}_{cd} & \boldsymbol{S}_{cc} \end{pmatrix}.$$
 (4)

where  $S_{mm}$  is the mixed-mode scattering matrix and where  $S_{dd}, S_{dc}, S_{cd}, and S_{cc}$  refer to the differential-to-differential, differential-to-common, common-to-differential and common-to-common mode scattering sub-matrices respectively.

Even though the measurement gives us the full mixedmode response, only the purely differential response ( $S_{dd}$ ) is considered from here on forward, which is the ideal balun response. The active differential reflection coefficient of differential pair number n as function of the zenith scan angle is then calculated according to

$$\Gamma_{ddact_n}(\theta) = \sum_{m=1}^{16} w_{nm}(\theta) S_{dd_{nm}}$$
(5)

where the weights are given by

$$w_{nm}(\theta) = e^{j\frac{2\pi}{\lambda_0}} (m-n)\frac{d}{2}\sin\theta$$
(6)

6

and where the subscripts refer to the antenna numbering and not to the single-ended ports at the measurement plane. This allows us to show the active differential reflection coefficient of each dipole as function of the frequency. In Fig. 8 the measured active differential reflection coefficient for boresight and for a  $45^{\circ}$  zenith scan in the H-plane is shown in blue. Shown are the active differential reflection coefficient of centre element #9, the second to last element #15 and edge element #16.

To verify the measurement results, and hence the viability of the PPWG measurement concept as well as the CCDA, the measurement results are compared to two different electromagnetic simulation set-ups. The first comparison in Fig. 8 is to a unit cell simulation, as shown by the dash-dotted red lines. A unit cell simulated by two pairs of periodic boundary conditions in the planes shown by the dashed blue lines in Fig. 1. The unit cell then symbolizes an infinitely extending array in both the  $\hat{x}$ - and  $\hat{y}$ -direction. Secondly, to distinguish between finite array effects and the effects of the non-perfect PPWG structure a second simulation result is shown. This second simulation also has the 16  $\hat{x}$ -directed linear array elements but now placed between two 'virtual infinite'  $\hat{y}\hat{z}$ plane  $E_t = 0$  planes instead of the finite copper plates of the constructed structure. As such, this simulation can be seen as



Fig. 9. Receiver noise temperature of a unit cell (dash-dot red line) and the beam-equivalent noise temperature of the 16-elements within a linear array of the simulated (dashed green line) and measured (solid blue line) CCDA within a PPWG structure at a boresight/zenith scan angle ( $\theta = 0^\circ$ ). Calculated based upon the SAV-541+ LNA by Minicriruits assuming a purely differential connection. Minimum noise temperature of the LNA given as reference (black dotted line), where the marker show the datasheet frequency points from where the noise parameters are extrapolated.

the 'perfect PPWG' structure as no reflections are present as if perfect absorbers were used.

The first thing we notice in Fig. 8 is that the measured active differential reflection coefficient for the edge element shows a significant variation for both scan angles which can be explained by the open boundary at the edge of the PPWG structure, which we expected to see upon making the choice for the open boundary. Furthermore, as expected the edge elements have a significant variation to the unit cell due to finite array effects. However, for all but the edge elements, the measured boresight scan response shows a good match with the simulation. A bigger difference between the measured and simulated results exists for the  $45^{\circ}$  scan angle, but this is still a very reasonable result. The measured result differs more at the lower frequency ranges, we expect this can be explained by the fact that the absorbers are electrically smaller at these frequencies and hence cause a larger distortion. Finally, it is worth noting that the edge element at  $45^{\circ}$  zenith scan still follows the finite array result quite accurately, showing the resonance at the same frequency point.

#### V. RECEIVER NOISE MODELLING

Finally, to validate the CCDA design for the use in the MFAA low-frequency radio-astronomy application, a calculation of the receiver noise temperature is pursued. For this purpose the SAV-541+ low-noise amplifier (LNA) by Minicircuits<sup>3</sup> is being considered. The datasheet supplied noise parameters of this LNA, given at 0.5, 0.7, 0.9, 1.0, and 1.9 GHz, are linearly inter- and/or extra-polated to match the measurement points of the array. In the case of the unit cell

3https://www.minicircuits.com/pdfs/SAV-541+.pdf

simulation we can follow the well-known standalone receiver noise temperature calculation via

$$T_{rec} = T_{min} + 4T_0 N \frac{|\Gamma_{act} - \Gamma_{opt}|^2}{(1 - |\Gamma_{opt}|^2) (1 - |\Gamma_{act}|^2)}$$
(7)

where  $T_{min}$  is the minimum noise temperature of the LNA, where  $T_0$  is the reference temperature of 290 K,  $\Gamma_{act}$  is the active reflection coefficient of the source antenna,  $\Gamma_{opt}$  is the optimum reflection coefficient for minimum noise match and  $N^4$  is the Lange invariant noise parameter [31]. In the case of the measured and simulated finite arrays the noise coupling within the array is accounted for by calculating the beam-equivalent receiver noise temperature [32], which gives a single value array equivalent noise temperature. This calculation is cross-validated with the framework presented in [33].

Fig. 9 shows the resulting receiver noise temperature at boresight and a  $45^{\circ}$  zenith scan in the H-plane for the three set-ups, the unit cell simulation, the perfect PPWG simulation and the measured prototype. The extrapolated datasheet values of the minimum noise temperature of the LNA ( $T_{min}$ ) is shown as a reference. At boresight, a very good match between the calculated noise temperature based upon simulation and measurement can be seen, further proving the validity of the CCDA and PPWG designs. At boresight, for the measured results, we achieve a calculated receiver noise temperature below the MFAA limit of 30 K for the frequency range of 300 to 1350 MHz, or a bandwidth of 1 : 4.5.

At higher zenith scan angles in the H-plane, the noise temperature calculation based upon the measurement and simulations start to differ more, as can be seen in Fig. 9 where the noise temperature calculation based upon the measured result actually shows a considerably lower calculated receiver noise temperature. To explain this result we show the active reflection coefficient of the unit cell simulation as well as the active reflection coefficient of element #9 of the constructed prototype in a Smith Chart, for boresight scan and a 45° zenith scan in the H-plane in Fig. 10 and Fig. 11 respectively. We can see that at the relevant frequencies, the measured active reflection coefficient is more inductive than the simulated active reflection coefficient. As a result, at the center of our frequency band we do not only have a better power match as was seen in Fig. 8 but also a better noise match which results in the lower receiver noise temperature seen in Fig. 9.

The Smith Charts of Fig. 10 and Fig. 11 also show that for this particular choice of LNA, and based upon the assumptions about the array response made previously, there is a nonoptimal noise match. A better noise match can be achieved by moving the inner loop of the active reflection coefficient contours, which corresponds to the center frequencies of our frequency band, up in the Smith Chart. This suggest making the active differential reflection coefficient more inductive. Referring back to Section II A, and especially Eq. 3, we see several options. This could be achieved by increasing the total per unit inductive reactance  $X_A$  by simply increasing

<sup>4</sup>Note that  $N \equiv R_n \Re \{Y_{opt}\}$  is used instead of the customary equivalent noise resistance  $R_n$ , where the admittance  $Y_{opt}$  is related to  $\Gamma_{opt}$ .



Fig. 10. Extrapolated optimum reflection coefficient of the SAV-541+ LNA by Minicircuits (dotted black line). Active differential reflection coefficient of the unit cell simulation (dash-dot red line) and of element #9 of the measured The function for the second s extra clarity.

the length of the dipoles. However, if we do not want to change inter-element spacing, a meandering dipole or a dipole overlap [15] will have to be considered. Alternatively, one could reduce the per unit tip capacitance  $C_{tip}$  or increase the height of the dipoles over the groundplane  $(H_{dip})$ . Future work pursue an active noise match optimization which will include common mode effects, noise coupling and finite array effects.

#### VI. CONCLUSION

This paper discussed the design and verification of a CCDA for the use in low-frequency radio-astronomy. It emphasizes the need for a small-scale intermediate prototyping method for large-scale dense connected phased arrays. To this end, a PPWG was constructed. The PPWG structure, allowing a full H-plane scan over the entire frequency of operation, helped to verify the design of the CCDA. The least accurate elements of the CCDA in the constructed PPWG are the edge elements, since the PPWG structure is not terminated in absorbers in the direction orthogonal to the dipole direction because of practical reasons. A receiver noise temperature calculation based upon the measured active reflection coefficient of the CCDA elements and a 30 K receiver noise temperature limit shows that a bandwidth of  $1:4.5\ \mathrm{can}\ \mathrm{easily}\ \mathrm{be}\ \mathrm{obtained}\ \mathrm{at}$ boresight scan with the current design. We expect to further broaden this bandwidth and improve the noise match at higher zenith scan angles once a LNA noise match optimization is actively pursued.

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+i1.0

+j0.5

+j0.2

0.0

-j0.2

Unit cell

•••• Γ<sub>opt</sub>

GИż

Measurement

+i5.0

-j5.0

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## Analysis of Surface Waves Modes in Planar Connected Arrays for Radio Astronomy

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Abstract-A planar array of closely spaced capacitively coupled antennas over a ground plane can be designed to have a broadband and smooth response, making it an interesting candidate for a low-frequency radio-astronomy instrument. How ever, surface wave modes propagating over the array plane can seriously degrade the response. We will show the derivation of a necessary condition for which surface wave modes will propagate along a planar array of closely spaced capacitively connected antennas. The derivation will be based upon the transverse resonance method [2] applied to the equivalent circuit of a unit cell of an infinite capacitively connected array. From the necessary condition a simple design consideration can be derived which would prevent the excitation of unwanted surface waves on the structure. Finally, we will show how adding resistive loading in the form of the low-noise amplifiers can dampen the unwanted surface wave effects.

Index Terms—planar arrays, radio astronomy, surface waves, connected arrays

#### I. INTRODUCTION

Recently, an increased interest has been shown in the spectral smoothness of low-frequency phased-array radio astronomy receivers [1], [2]. This, in turn, sparked interest in low-frequency radio astronomy receiver solutions that have an intrinsic good spectral smoothness. The most promising concept to realize this is given by Wheeler's current sheet [3]. A practical approximation of the current sheet is given by the connected array [4], [5] as given in the works of Ben Munk [6]. As the name suggests, the connected array consist of a large array of capacitively connected elements arranged into a regular grid placed over some sort of backing plane, an illustration can be seen in Fig. 1. However, due to its strongly coupled nature, the connected array can support unwanted surface wave modes, which we will discuss in this paper.

#### II. THEORY

A. Equivalent circuit

The operation principle and design of the capacitively connected dipole array of Fig. 1 is best demonstrated by its equivalent circuit [6]. Assuming a singly-polarized infinite array placed over an infinite PEC ground plane, the equivalent circuit of a unit cell within the array is given by Fig. 2. From

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the equivalent circuit, the driving impedance can be derived

$$Z_d = \left(jX_A + \frac{1}{j\omega C_{tip}}\right) / (2R_{A0}) / Z_{gp} \tag{1}$$

where // describes parallel connected loads, where  $\omega$  is the angular frequency, where  $R_{A0}$  is the radiation resistance of free space,  $C_{tip}$  is the value of the (lumped element) tip capacitor and  $X_A$  is the imaginary part of the antenna impedance which in the case of a wire dipole array is the wire inductance. Lastly, the reflection of the perfect reflector ground plane appears as

$$Z_{ap} = 2jR_{A0}\tan\left(k_zH\right) \tag{2}$$

where  $k_z$  is the transverse wave number and where H is the height of the dipole over the ground plane. Utilizing the equivalent circuit of Fig. 2, we design an example connected array to operate in the frequency range of 50 to 350 MHz. The parameters of this array are given in Table. I. The inter-element spacing is deliberately kept to less than half a wavelength across the frequency range to avoid the onset of grating lobes. Having designed the array around 200 MHz, we show the driving impedance of the infinite connected array pointed at zenith in Fig. 6. The smooth and broadband response shown justifies the design parameters listed in Table. I.

#### B. Dispersion relationship

Using the equivalent circuit, we can now apply the transverse resonance method [7] to find surface wave mode solutions. The transverse resonance method is a well known method to find the propagation constant of a mode solution in (dielectric) wave guides [8]. Upon inspection of the twodimensional equivalent circuit of Fig. 2 one may realize that the transverse resonance method can as easily be applied to this configuration. The transverse resonance method takes advantage of the fact that any mode solution must satisfy the boundary condition at the two phase-shift walls, in our case given by  $\varphi_1$  and  $\varphi_2$ . The condition which constitutes a solution is that after short circuiting the feed port ( $Z_d = 0$ ), at any point in the equivalent circuit the sum of the impedances in opposite directions must be zero. Applying this to the equivalent circuit as given in Fig. 2 we get

$$Z \uparrow +Z \downarrow = 0.$$
 (3)

from which the eigenvalue  $k_z$  is found that satisfies the resonance condition. If we restrict our analysis to the  $\hat{x}\hat{z}$ -plane,


Fig. 1. Top view of an infinite dual-polarized capacitively connected dipole array, where  $^2$  depicts a dual-polarized feed port. Note that the orthogonal dipoles are **not** physically connected. The blue dashed line confines a dual-polarized unit cell of the infinite array.



Fig. 2. A two dimensional equivalent circuit of the infinite *singly*-polarized capacitively connected dipole array over an infinite PEC ground plane.

and we furthermore know that a surface wave guided by the structure will be purely decaying in the  $\hat{z}$ -direction, we can separate the two-dimensional wave equation as

$$k_z = -j\alpha = -j\sqrt{k_x^2 - k_0^2} = -jk_0\sqrt{s_x^2 - 1}$$
(4)

where  $s_{x}$  is the spatial frequency which, in the invisible space runs between  $1\,$  and  $s_{xmax},$  with

$$s_{xmax} = \frac{\lambda}{2D_x} \tag{5}$$

where  $D_x$  is the inter-element spacing. We can steer the array into the invisible space since the inter-element spacing is less than half a wavelength across the entire frequency range. The

Freq. range	$D_{x/y}$	H	$C_{tip}$	Wire radius	
50-350 MHz	40 cm	37.5 cm	3.54 pF	1 cm	
		TABLE I			

PARAMETERS OF THE EXAMPLE ARRAY USED THROUGHOUT THIS PAPER

wave impedance normal to the array plane for a TM an TE surface wave mode  $\left[9\right]$  is given as

$$Z^{TM} = \frac{k_z}{\omega\epsilon_0} = -j\frac{\alpha}{\omega\epsilon_0} \tag{6a}$$

$$Z^{TE} = \frac{\omega\mu_0}{k_z} = j\frac{\omega\mu_0}{\alpha}$$
(6b)

both of which are purely imaginary as expected and in contrast to the normal operation principle where the radiation resistance is purely real  $(R_{A0})$ . The equivalent impedance of the reflection of the perfect ground plane is given by

$$Z_{gp}^{TM/TE} = jZ^{TM/TE} \tan(k_z H). \tag{7}$$

Using the transverse resonance method of eq. 3 below the array plane in Fig. 2 surface wave mode solutions must satisfy

$$Z_{gp}^{TM/TE} + \frac{Z^{TM/TE}jX}{jX + Z^{TM/TE}} = 0$$
 (8)

in which X is the reactance of the array plane upon short circuiting the feed port, in our case given by the combination of  $X_A$  and  $C_{tip}$ . Solving this equation for TM modes gives the dispersion relationship as

$$anh(Hk_0\sqrt{s_x^2-1}) + \frac{X}{X-Z_0\sqrt{s_x^2-1}} = 0$$
(9)

with  $Z_0$  being the free space wave impedance. Similarly, the dispersion relationship for TE surface wave modes is

$$\tanh\left(Hk_0\sqrt{s_x^2-1}\right) + \frac{X}{X + \frac{Z_0}{\sqrt{s_x^2-1}}} = 0 \qquad (10)$$

Rewriting the dispersion relationships as

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2

$$\chi_{0}^{TM}(s_{x}) = Z_{0}\sqrt{s_{x}^{2} - 1} \frac{\tanh\left(Hk_{0}\sqrt{s_{x}^{2} - 1}\right)}{1 + \tanh\left(Hk_{0}\sqrt{s_{x}^{2} - 1}\right)} \quad (11a)$$

$$X_0^{TE}(s_x) = -\frac{Z_0}{\sqrt{s_x^2 - 1}} \frac{\tanh\left(Hk_0\sqrt{s_x^2 - 1}\right)}{1 + \tanh\left(Hk_0\sqrt{s_x^2 - 1}\right)} \quad (11b)$$

gives us the array surface impedance for which a surface wave mode can exist as function of the (spatial) frequency and the height above a ground plane. Fig. 3 shows the array surface reactance solution as given by (11) plotted as function of the spatial frequency for the array under consideration in this paper. To completely avoid the onset of any surface wave, we would have to design the array such that the surface reactance would be between  $-100\ \Omega$  and  $0\ \Omega$  for the entire frequency range. Approximating the wire inductance and the tip capacitor as a first order series LC-circuit we would need a wire inductance of 6.6 nH and a tip capacitor of 31.1 pF to satisfy this condition. It is practically impossible to design a



Fig. 3. Array surface impedance necessary to support a TM (solid lines) or TE (dashed lines) surface wave mode along the array surface following (11). The dimensions of the array are given in Table. I.



Fig. 4. 4-element equivalent circuit of the ATF-54143 by Avago.

wire dipole with a length of 40 cm to have an inductance that low. Luckily, as will be shown in the next section, adding a (complex) load to the array ports reduces the surface waves.

# III. FINITE ARRAY EXAMPLE

To investigate further, let us consider an 18-by-18 element dual-polarized finite array of the same fundamental dimensions as given in Table. I. Within this array we will actively feed the inner 16-by-16 elements. We can simulate the response of this array using FEKO, an EM-simulation tool. From the simulation we can derive the mutual coupling matrix Z, from which we in turn derive the driving impedance. Fig. 5 shows the real and imaginary part of the driving impedance of all 256 elements of a single polarization as function of the frequency when the array is pointed at zenith, i.e. normal to the array plane. Shown in the same graph is the expected driving impedance derived from an infinite array simulation. Between 100 and 200 MHz, the driving impedance of certain elements peaks. This can be explained by surface waves travelling across the finite array plane creating a standing wave pattern. The standing wave pattern can be so strong that it reverses the current direction in some of the array ports, i.e. current flowing into the ports of a transmitting array or current flowing out of the ports of a receiving array. From Fig. 11 we deduce that we are in fact seeing TE surface waves modes since we know that the surface reactance is negative at these frequencies. The fact



Fig. 5. Real (solid line) and imaginary (dashed lines) part of the driving impedance  $(Z_d)$  of the capacitively connected dipole array at zenith, in the case of the infinite array (thick black lines) or the 256 elements in a finite array (thin coloured lines). Finite array result **without** any LNA loading.

that the infinite array response seems unaffected by surface wave modes does not mean they are not excited in an infinite array. They are excited, but the left and right-travelling planar surface waves over the infinite array plane will never reach an edge and thus will never create the standing wave pattern that so badly degrades the response.

However, we are considering active instead of passive arrays that have the added benefit that they are either fed by a voltage generator in the case of a transmitting array or is delivering the incident energy to an amplifier in the case of a receiving array. Both will add a significant resistive component connected to each port, which will ideally attenuate any potential surface wave. To illustrate this, we will once again consider the finite 256-element connected array. Once again, we will calculate the driving impedance of each element and compare it to the driving impedance of the infinite array but this time we will load each element with the impedance of a low-noise amplifier (LNA). For this we will use the ATF-54143 by Avago [10], which is used by the Murchison Widefield Array (MWA) radio telescope [11]. This LNA has been characterized [12] for the frequency range of interest here, and its input impedance can be modelled by the 4element equivalent circuit shown in Fig. 4. The current of each port upon loading the array is then given by

$$\boldsymbol{I} = \left(\boldsymbol{Z} + \boldsymbol{Z}^{LNA}\right)^{-1} \boldsymbol{V} \tag{12}$$

where in this case Z is the 256-by-256 impedance matrix, where  $Z^{LNA}$  is a diagonal matrix with the input impedance of a single isolated LNA, see Fig. 2, and where V is the voltage vector. The driving impedance of element n in turn is

Ζ

$$I_{d_n} = \frac{V_n}{I_n} - Z^{LNA} \tag{13}$$

Fig. 6 shows the driving impedance of the connected array including the LNA impedance loading, which strongly atten-



Fig. 6. Real (solid line) and imaginary (dashed lines) part of the driving Fig. or near (some me) and magning (dashed mes) pair of the driving impedance  $(Z_d)$  of the capacitively connected dipole array at zenith, in the case of the infinite array (thick black lines) or the 256 element in a finite array (thin coloured lines). Finite array result with the LNA loading.

uated the surface wave effects. Note that the unloaded driving impedance calculation of Fig. 5 is done using eq. 13 and eq. 12 but without the LNA terms. To further illustrate the positive effect of resistive loading the array using the LNA's we will calculate the receiver noise temperature using the ATF-54143 by Avago. The receiver noise temperature is given as

$$T_{rec} = T_{min} + 4T_0 N \frac{|\Gamma_{ant} - \Gamma_{opt}|^2}{(1 - |\Gamma_{ant}|^2)(1 - |\Gamma_{opt}|^2)}$$
(14)

where  $T_{min}$  is the minimum noise temperature of the LNA, N is the Langrange noise ratio given by  $N = R_n \cdot \Re(1/Z_{opt})$ , where  $R_n$  is the noise resistance,  $\Gamma_{opt}$  is the optimum impedance reflection coefficient of the LNA,  $T_0$  is the ambient temperature (290 K) and  $\Gamma_{ant}$  is the reflection coefficient of the antenna element, which is directly derived from the driving impedance. Fig. 7 shows the (averaged) receiver noise temperature for the infinite array, the unloaded finite array and the loaded finite array using (14).

### IV. CONCLUSION

In this paper we have derived a necessary condition for which surface wave modes will propagate over the array plane of capacitively connected arrays using the transverse resonance method. Knowing that, at least for the example array shown in this paper, we can not avoid surface wave modes if we want to operate the array over a large frequency band we then showed how resistive loading the elements will drastically improve the array response. Further analysis will be required to understand the interplay between noise matching requirements and the requirement of resistive loading for an active low-frequency phased array capacitively connected dipole array receiver.

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Fig. 7. Receiver noise temperature of the infinite array element (black), unloaded finite array element (red) and loaded finite array element (blue) of the connected array when using the ATF-54143 low-noise amplifier. Finite array receiver noise temperatures are averaged over all 256 elements.

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# Parallel Plate Waveguide Simulator of a Dense Connected Dipole Array

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Abstract—In this paper we propose the use of a parallelplate waveguide simulator as a useful design verification step of very large phased-array systems. We base the derivation of the theoretical concept upon the wideband capacitively connecteddipole array. It is shown to be key to correctly terminate the cavity to the free-space boundary with the use of electromagnetic absorbers to minimize reflections.

Index Terms-radio astronomy, phased arrays, antenna measurement

# I. INTRODUCTION

The international collaboration known as the Square Kilometre Array (SKA) [1] ushers in the next generation of radiotelescopes. The SKA Mid frequency aperture array (MFAA) project investigates receiver solutions for the frequency band of 400 MHz upwards. Examples of MFAA precursors include EMBRACE [2] which uses Vivaldi antennas and the Orthogonal Ring Array (ORA) [3]. The wideband and spectrally smooth response of the capacitively connected-dipole array (CCDA) [4],[5] suggests that it makes an interesting alternative technology for future low-frequency phased-array radio-telescopes. Fig. 1 shows an artist impression of the top view of the dual-polarized CCDA. As the name suggests, the dipoles in a CCDA are capacitively connected to its neighbouring elements, effectively elongating the electrical length of each dipole which explains its wideband response. If the element spacing is furthermore kept below half a wavelength throughout its frequency band, the onset of any grating lobes can be avoided, resulting in a smooth spectral and scan response.

As with any very large dense phased array a good first design step is to consider a single unit cell in an infinite array with the help of simulation software or electromagnetic theory. The unit cell is a good first design step since the array effects are generally dominating the isolated element response in any dense array. However, once the time comes to construct the first practical prototype one usually resorts to constructing an array of only a few elements to reduce the cost and measurement effort despite the fact that it will result in a poor estimate of the larger array response.

One method to accurately measure the active element response of a large array is to use a rectangular waveguide simulator [6]. These rectangular waveguide simulators rely



Fig. 1: Top view of an infinite dual-polarized capacitively-connected dipole array, where <sup>2</sup> depicts a dual-polarized feed port. Note that the orthogonal dipoles are **not** physically connected. The blue dashed line confines a dualpolarized unit cell of the infinite array.  $D_x$  and  $D_y$  indicate the interelement spacing for the  $\hat{x}$ - and  $\hat{y}$ -directed arrays.

on the propagation of TE and/or TM modes in the cavity. As a result, broadside scan cannot be achieved and the scan direction of the simulated array will become a function of frequency. Furthermore, for the CCDA discussed in this paper, a considerable number of elements would have to be placed inside the waveguide since the elements are much shorter than the cut-off frequency of a rectangular waveguide.

Alternatively, the improved Wheeler cap (IWC) method [7], [8] has been used to measure antenna efficiency of 1-D connected arrays with great accuracy. However, for the CCDA we expect the mutual coupling between the elements to be significant not only in the direction of physical connection but also between the elements in neighbouring rows. Therefore the 1-D IWC method will only be a poor approximation.

In this paper we propose a waveguide simulator based on a parallel-plate waveguide (PPWG) since it supports a TEM



Fig. 2: The equivalent circuit of a unit cell of a *single* polarization of the infinite dense capacitively connected-dipole array over a perfect ground plane, as shown in Fig. 1. The array radiating into free space (a), the array radiating into a pyramidal absorber backed by a perfect conductor (b), the array radiating into a layered absorber/ an absorber with a permittivity gradient as function of its height backed by a perfect conductor (c).

plane wave. As we will show, the proposed 1-by- $\infty$  array will allow us to simulate an  $\infty$ -by- $\infty$  array. We can furthermore scan the H-plane and are not limited by frequency since the TEM mode within a parallel-plate waveguide has no cut-off frequency. The operating frequency of the array and the PPWG simulator are well below the cut-off frequency of any higher order modes. where  $\theta$  is the scan direction in the  $\hat{y}\hat{z}\text{-plane}$  with  $\theta=0^o$  broadside scan, and where  $\sqrt{\frac{\mu_0}{\varepsilon_0}}=120\pi.$  From the equivalent circuit we can then derive the active scan impedance as

$$Z_{act} = \left(jX_A + \frac{1}{j\omega C}\right) + Z_{gp}//Z_{fs} \tag{2}$$

where  $//\ denotes$  parallel loads. The reflection of the infinite perfect ground plane is given as

$$Z_{gp} = j Z_{fs} \tan\left(\frac{2\pi}{\lambda} H_{dip}\right) \tag{3}$$

where  $H_{dip}$  is the height of the dipole over the ground plane and  $\lambda$  is the wavelength in free space.

# B. Parallel plate waveguide and absorbers

As a next step, we limit the height of the PEC walls. To avoid reflections from the cavity to free space boundary the cavity is filled by an electromagnetic absorber. Assuming an absorber with a pyramidal cone, we choose to define a filling factor (f) describing the ratio of the surface area in the  $\hat{x}\hat{y}$ plane filled by the absorber as function of the height. This surface area of the pyramid grows quadratically with the height thus giving us the filling factor as

$$f(z) = \frac{(z - H_{gap} - H_{dip})^2}{H_{pyr}^2}$$
(4)

for the region  $H_{dip} + H_{gap} < z < H_{dip} + H_{gap} + H_{pyr}$ . Using this we can then homogenize the pyramidal shape of the absorber such that we create a relative permittivity gradient as a function of the height as

$$\varepsilon_{eff}(z) = 1 - f(z) + f(z)\varepsilon_{abs} \tag{5}$$

# II. THEORY

In this section we will utilize the equivalent circuit of a single unit cell in an  $\infty$ -by- $\infty$  capacitively connected dipole array [4]. Together with absorber theory we will get a basic understanding of the underlying electromagnetic theory of the proposed set-up.

# A. Equivalent circuit in free space

Fig. 2 (a) shows the equivalent circuit of a unit cell of an  $\infty$ -by- $\infty$  capacitively connected dipole array over a perfect ground plane. By restricting the scan directions of the array to the  $\hat{y}\hat{z}$ -plane (H-plane) only, we can replace the phase-shift walls by infinite perfect electrically conducting walls, denoted in Fig. 2(a) by  $E_t = 0$ . The (inductive) reactance of the elements is given by  $X_A$ , whereas the tip capacitors are denoted by C. The impedance looking upwards, denoted by  $Z_{in}$ , is in this case equal to the wave impedance in free space given by

$$Z_{fs} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{\cos\theta} \tag{1}$$

where  $\varepsilon_{abs} = \varepsilon'_{abs} - j\varepsilon''_{abs}$  is the relative permittivity of the absorber material. Here we made the assumption that the effective homogenized permittivity scales linearly with the filling factor. If we then discretize the pyramidal absorber of varying permittivity into N layers of constant permittivity, as shown in Fig. 2 (c), we can find the wave impedance looking upwards as seen by the antenna. First of all, we have to find the scan angle in each layer by using Snell's second law iteratively

$$\theta_1 = \arcsin\left(\frac{1}{\sqrt{\varepsilon_{eff}(z_1)}}\sin(\theta)\right)$$
(6a)

$$\theta_n = \arcsin\left(\frac{\sqrt{\varepsilon_{eff}(z_{n-1})}}{\sqrt{\varepsilon_{eff}(z_n)}}\sin(\theta_{n-1})\right)$$
(6b)

$$\theta_{abs} = \arcsin\left(\frac{\sqrt{\varepsilon_{eff}(z_N)}}{\sqrt{\varepsilon_{abs}}}\sin(\theta_N)\right) \tag{6c}$$

where  $\theta_n$  and  $\theta_{abs}$  is the angle of the direction of propagation in the  $n^{th}$  homogenized layer and the square absorber respectively. The discretized height is given by

$$z_n = \left(H_{dip} + H_{gap} + H_{pyr}\left(1 - \frac{n}{N}\right)\right). \tag{7}$$

To find  $Z_{in}(H_{dip})$  we start from the top of the structure and work backwards towards the antenna level. Starting at  $z=H_{dip}+H_{gap}+H_{pyr}$  we can find the wave impedance looking up as

$$Z_{in}(H_{dip} + H_{gap} + H_{pyr}) = jZ_{abs} \tan\left(\frac{2\pi}{\lambda}\sqrt{\varepsilon_{abs}}\frac{H_{sqr}}{\cos\theta_{abs}}\right)$$
(8)

where  $Z_{abs}$  is the wave impedance in the square absorber given by

$$Z_{abs} = \sqrt{\frac{\mu_0}{\varepsilon_{abs}\varepsilon_0} \frac{1}{\cos\theta_{abs}}}$$

The wave impedance seen looking upwards in each successive layer (moving downwards from layer N towards layer 1) can then be calculated from the wave propagation in each layer and the mismatch between itself and the previous layer. This allows us to write

$$Z_{in}(H_{dip} + H_{gap}) =$$

$$Z_{in}(H_{dip} + H_{gap} + H_{pyr}) \prod_{n=1}^{N} \frac{1 + \Gamma_{n,n-1} e^{j\beta_n} \frac{H_{pyr}}{cos \theta_n N}}{1 - \Gamma_{n,n-1} e^{j\beta_n} \frac{H_{pyr}}{cos \theta_n N}}$$
(10)

with  $\beta_n$  the complex wave number in the  $n^{th}$  layer of homogenization given by

$$\beta_n = \frac{2\pi}{\lambda} \sqrt{\varepsilon_{eff}(z_n)} \tag{11}$$

The mismatch between two successive layers is given by

$$\Gamma_{n,n-1} = \frac{\sqrt{\varepsilon_{eff}(z_{n-1})} - \sqrt{\varepsilon_{eff}(z_n)}}{\sqrt{\varepsilon_{eff}(z_{n-1})} + \sqrt{\varepsilon_{eff}(z_n)}}$$
(12)



Fig. 3: Real (solid lines) and imaginary (dashed lines) of the active scan impedance of the CCDA PPWG simulator calculated using the equivalent circuit as function of frequency. Array radiating into free space (blue), array radiating into a 25 cm spramidal absorber backed by cavity completely filled with the absorber material of 25 cm and terminated by a perfect electric conductor (red), and the array radiating into a 12.5 cm pyramidal absorber backed by cavity completely filled with the absorber material of 37.5 cm and terminated by a perfect electric conductor (green). All for broadside ( $\theta = 0^{\circ}$ ) scan.

Following this iterative approach allows us to derive  $Z_{in}(H_{dip})$ , which in turn can be used to calculated the active scan impedance of the array, as was done in the free space case via

$$Z_{act} = \left(jX_A + \frac{1}{j\omega C}\right) + Z_{gp}//Z_{in}(H_{dip})$$
(13)

Note that to accurately simulate the large array response we have to choose the absorbers such that  $Z_{in}(H_{dip})$  is equal to  $Z_{fs}$  over the entire frequency range.

# C. Examples

(9)

As an example, consider a unit cell of an  $\infty$ -by- $\infty$  capacitively connected dipole array designed for the 200-1500 MHz range. The dipole elements are placed 9 cm apart and are placed 8.75 cm over a perfect ground plane. From the length of the dipole we make the rough estimate that the reactance of these dipoles is purely inductive and equal to 20 nH. The tip capacitance is set to 1 pF. Using the equivalent circuit of Fig. 2, we show the active scan impedance of a unit cell in an infinite array of these dimensions at broadside in Fig. 3 (blue). Now assume we want to construct the  $\infty$ -by-1 parallel plate waveguide simulator of this array, but due to practical reasons are limited by a total absorber length of  $50\ \mathrm{cm}.$  Following the same assumption as Kraus [9], we take the permittivity of the absorber to be  $\varepsilon_{abs} = 2 - j, \sigma = 0$ . Now, using the formulas as derived in Section II we can calculate the wave impedance as seen by the infinite array element for two different situations. The first one being a pyramidal absorber of 25 cm and the



Fig. 4: Real (solid lines) and imaginary (dashed lines) parts of the wave impedance ( $Z_{in}$  in Fig. 2) as seen by the dipole element in a PPWG simulator of the dimensions given in Section II as function of frequency. Array radiating into a 25 cm pyramidal absorber backed by cavity completely filled with the absorber material of 25 cm and terminated by a perfect electric conductor for boresight scan (blue) and scanned in the H-plane to 45° (green). Also for the array radiating into a 12.5 cm pyramidal absorber backed by cavity completely filled with the absorber material of 37.5 cm and terminated by a perfect electric conductor for boresight scan (green) and scanned in the H-plane to 45° (organe). H-plane to 45° (orange)

second one a pyramidal absorber of 12.5 cm, both of which are backed by a completely filled cavity of absorber up until 50 cm of total length and both terminated by a perfect electric conductor. Fig. 4 shows the wave impedance for both setups when scanned to boresight ( $\theta=\bar{0^o})$  and to  $\theta=45^o$  in the H-plane. As expected, the largest reflections occur at the lowest frequencies since the absorbers are electrically smaller.  $Z_{in}$  approaches  $Z_{fs}$  (eq. 1) for both the boresight scan and  $\theta = 45^{o}$  at the higher frequencies. Furthermore, we can see that a shorter pyramidal cone does not create more reflections from the transition of free space to absorber but does give smaller overall reflections simply by the fact that a larger part of the cavity is filled by the absorber material. Finally in Fig. 3 and Fig. 5 the active scan impedance as calculated by the formulas derived in this paper is shown for broadside and  $\theta = 45^o$  scan, respectively. The increase in the ripple in the active scan impedance upon scanning away from broadside suggest that an even larger or more lossy absorber will be necessary to simulate this array.

# III. CONCLUSION AND FUTURE WORK

The derivation as shown in this paper suggest that the PPWG simulator could be a valid design verification step for very large dense arrays, if the cavity to free space boundary is terminated correctly. In future work we will expand upon the theoretical concept of the parallel-plate waveguide simulator first introduced in this paper. A critical next step will be to move from the 1-by- $\infty$  concept to a practical 1-by-N PPWG



Fig. 5: Real (solid lines) and imaginary (dashed lines) of the active scan impedance of the CCDA PPWG simulator calculated using the equivalent circuit as function of frequency. Array radiating into free space (blue), array radiating into a 25 cm pyramidal absorber backed by cavity completely filled with the absorber material of 25 cm and backed by a perfect conductor (red), array radiating into a 12.5 cm pyramidal absorber backed by cavity completely filled with the absorber material of 37.5 cm and the absorber material of a 25.5 cm and backed by a perfect conductor (green). All for a 45° scan direction in the  $\hat{y}\hat{z}$ -plane.

simulator, i.e. to limit the number of elements and the size of the parallel-plates in the  $\hat{y}$ -direction as indicated in Fig. 2 and investigate how this will effect the response of the array. As a final verification of the PPWG concept a practical prototype will be designed.

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	Conception and	Acquisition of	Data	Analysis and	Interpretation	Final Approval	Total %
	Design	Data and	Conditioning	Statistical	and Discussion		Contribution
		Method	and	Method			
			Manipulation				
Rene Baelemans	75%	100%	100%	100%	100%	0%	79.2%
Acknowledgeme	nt						
acknowledge th	at these represer	nt my contribution	n to the above res	earch output.			
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Adrian Sutinjo	25%	0%	0%	0%	0%	40%	10.8%
Acknowledgeme	nt						
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Bart Smolders	0%	0%	0%	0%	0%	20%	3.33%
Acknowledgeme	nt						
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Signed:							
David Davidson	0%	0%	0%	0%	0%	20%	3.33%
Acknowledgeme	nt						
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Signed:							
Jlf Johannsen	0%	0%	0%	0%	0%	10%	1.67%
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Signed:							
Randall Wayth	0%	0%	0%	0%	0%	10%	1.67%
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	Conception and	Acquisition of	Data	Analysis and	Interpretation	Final Approval	Total %
	Design	Data and	Conditioning	Statistical	and Discussion		Contribution
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Rono Baolomans	70%	100%	100%	100%	100%	0%	78 33%
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David Prinsloo	20%	0%	0%	0%	0%	40%	10%
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Adrian Sutinjo	0%	0%	0%	0%	0%	20%	3.33%
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David Davidson	0%	0%	0%	0%	0%	10%	1.67%
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Randall Wayth	0%	0%	0%	0%	0%	10%	1.67%
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Signed:							

Table 3: Statement of attribution for "Design and Verification of a Capacitively-Connected Dipole Array With a Parallel-Plate Waveguide"

	Conception and Design	Acquisition of Data and	Data Conditioning	Analysis and Statistical	Interpretation and Discussion	Final Approval	Total % Contribution
		Method	and Manipulation	Method			
Rene Baelemans	60%	100%	100%	100%	100%	0%	76.67%
Acknowledgeme	nt		-				
I acknowledge th	nat these represer	nt my contribution	to the above res	earch output.			
Signed:							
Adrian Sutinjo	20%	0%	0%	0%	0%	30%	8.33%
Acknowledgeme	nt						
I acknowledge th	nat these represer	nt my contributior	n to the above res	earch output.			
Signed:							
David Prinsloo	20%	0%	0%	0%	0%	30%	8.33%
Acknowledgeme	nt						
I acknowledge th	nat these represer	nt my contributior	n to the above res	earch output.			
Signed:							
Bart Smolders	0%	0%	0%	0%	0%	20%	3.33%
Acknowledgeme	nt						
I acknowledge th	nat these represer	nt my contributior	n to the above res	earch output.			
Signed:							
David Davidson	0%	0%	0%	0%	0%	20%	3.33%
Acknowledgeme	nt						
I acknowledge th	nat these represer	nt my contributior	n to the above res	earch output.			
Signed:							