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Research Article

Optimization for Nonlinear Uncertain Switched Stochastic Systems with Initial State Difference in Batch Culture Process

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Based on the deterministic description of batch culture expressed in form of switched ordinary differential equations, we introduce a switched stochastic counterpart system with initial state difference together with uncertain switching instants and system parameters to model the process of glycerol biodissimilation to 1,3-propanediol (1,3-PD) induced by *Klebsiella pneumoniae* (*K. pneumoniae*). Important properties of the stochastic system are discussed. Our aim is to obtain the unified switched instants and system parameters under the condition of different initial states. To do this, we will formulate a system identification problem in which these uncertain switched instants and system parameters are regarded as decision variables to be chosen such that the relative error between experimental data and computational results is minimized. Such problem governed by the stochastic system is subject to continuous state inequality constraints and box constraints. By performing a time-scaling transformation as well as introducing the constraint transcription and local smoothing approximation techniques, we convert such problem into a sequence of approximation subproblems. Considering both the difficulty of finding analytical solutions and the complex nature of these subproblems, we develop a parallelized differential evolution (DE) algorithm to solve these approximation subproblems. From an extensive simulation, we show that the obtained optimal switched instants and system parameters are satisfactory with initial state difference.

1. Introduction

1,3-Propanediol (1,3-PD) is an important chemical product with numerous applications in cosmetics, adhesives, lubricants, and medicines. In particular, 1,3-PD has recently been used as a monomer to synthesize a new type of polyester called polyurethanes [1]. There are three common methods of the bioconversion of glycerol by *K. pneumoniae* to 1,3-PD: batch culture [2], continuous culture [3–5], and fed-batch culture [6]. The reason for the necessity of studying batch culture [7] is that 1,3-PD yield, which is defined as the ratio between the formation of 1,3-PD and the consumption of glycerol, is high in batch culture. The high concentration of glycerol leads to the high formation of 1,3-PD and the low formation of by-product in batch culture of glycerol [8]. In particular, note that the high target product yield in batch culture is only applicable to the bioconversion of glycerol. Not all the batch culture is of the high target product yield. Therefore, batch culture, in which the bacteria and substrate will be added to the bioreactor only at the beginning process, is now attracting significant interest in many research areas. Relevant literature includes [9], where identification and robustness analysis of nonlinear multistage enzyme-catalytic dynamical system are researched; the study [10], where sensitivity analysis and identification of kinetic parameters are investigated; the study [11], where strong stability of a nonlinear multistage dynamic system is considered; the study [12], where robust identification of enzymatic nonlinear dynamical systems is studied; the study [13], where modelling and parameter identification for a nonlinear time-delay system are researched; the study [14], where distributionally robust parameter identification of a time-delay dynamical system with stochastic measurements is carried out (many wireless network systems and hybrid dynamical systems [15, 16] have time delays and switchings); the study [17], where dynamic optimization for switched time-delay systems with state-dependent switching conditions is studied; the study [18], where pathway identification using parallel optimization for a nonlinear hybrid system is considered; the study [19], where optimality condition and optimal control for a two-stage nonlinear dynamical system are investigated; the study [20], where bi-objective dynamic optimization of a nonlinear time-delay system is carried out; the study [21], where robust biobjective optimal control is studied. To the best of the authors' knowledge and by surveying mentioned literatures, stochastic influences are not taken into consideration in batch culture.

Biotechnical treatment of microorganisms is commonly described by deterministic systems in form of nonlinear ordinary differential equations [22] to avoid expensive experiments. This description includes an idealization of the technical system component and a qualitative characterization of the biological part. In fact, microbial fermentation cannot commonly avoid stochastic influences reflected on the uncertainty of certain parameters [23]. Since this random phenomenon reveals different patterns and new features, such randomnness will degrade the role of deterministic systems in the research [24]. Therefore, research interest on stochastic dynamic systems has been growing in recent years [25]. Biological phenomena have the dynamical behaviours that are intrinsically erratic, and they are concisely described by a stochastic model, rather than by a deterministic one. In [26, 27], the randomness is introduced by the parameter perturbation and it becomes a standard technique in stochastic population modelling. However, parameter uncertainty is still ignored in their model.

Parameter uncertainty is a key issue in practice because it is difficult (if not impossible) to determine the exact values of many parameters in the dynamic equations describing microbial conversion [28–30]. In this paper, a switched stochastic counterpart with uncertain switched instants and system parameters will be taken to replace the commonly used deterministic description of batch culture with ordinary differential equations to describe the process of glycerol biodissimilation to 1,3-PD by *K. pneumoniae*. In order to estimate the uncertain switched instants and system parameters, we present a system identification problem governed by the stochastic system and subject to continuous state inequality constraints and box constraints to minimize the relative error between experimental data and computational results. In order to handle continuous state inequality constraints, such problem is converted into a sequence of approximation subproblems and one solves them by using time-scaling transformation, constraint transcription, and local smoothing approximation techniques. Since it is a very complicated task to solve these subproblems, we develop a parallelized differential evolution (DE) algorithm to solve these approximation subproblems. Our contribution of this paper includes (1) to propose a new stochastic switched models for batch fermentation to describe the process's randomness characteristics, (2) to establish a optimal identification procedure to identify the parameters, and (3) to design a parallel DE algorithm to save the computational time efficiently. It is observed that the obtained optimal switched instants and system parameters are satisfactory with initial state difference via numerical simulations.

The rest of this paper is organized as follows. In Section 2, a nonlinear switched stochastic dynamical system is formulated to describe the process of glycerol biodissimilation to 1,3-PD by *K. pneumoniae* and some significant properties are discussed. In Section 3, a system identification problem is proposed. In Section 4, time-scaling transformation is performed and approximate subproblems are presented. In Section 5, an optimization algorithm is constructed to solve these subproblems. In Section 6, the numerical results are clarified. Then, conclusion remarks are presented in Section 7.

2. Nonlinear Switched Stochastic System and Its Properties

2.1. Deterministic System. To the best of our knowledge, in the actual batch culture process, there are two different switched instants t_{f_1} and t_{f_2} in different chemical reactions [31]. Accordingly, three different periods have been involved in the typical cell growth of the batch culture ($[0, t_f] \subset \mathbb{R}_+ := [0, +\infty)$) as follows:

(i) The development period (or the first period, denoted by $D_1 := [0, t_{f_1}] \subset \mathbb{R}_+$)

(ii) The growth period (or the second period, denoted by $D_2 := [t_{f_1}, t_{f_2}] \subset \mathbb{R}_+$)

(iii) The stabilization period (or the third period, denoted by $D_3 := [t_{f_2}, t_{f_3} = t_f] \subset \mathbb{R}_+$)

Nomenclature

(i) A^{T} denotes the transposition of the vector or matrix A.

(ii) $I_n := \{1, 2, \ldots, n\}$.

(iii) $u_t = (t_{f_1}, t_{f_2}) \in \mathcal{D} := [a_1, b_1] \times [a_2, b_2] \subset \mathbb{R}_+^2$ denotes switched instants.

(iv) $0 = t_{f_0} < a_1 < b_1 < a_2 < b_2 < t_{f_3} = t_f$.

(v) Based on the sensitivity analysis [10], we take $u_p := ((u_p^1)^{\mathrm{T}}, (u_p^2)^{\mathrm{T}}, (u_p^3)^{\mathrm{T}})^{\mathrm{T}} \in \mathbb{R}^{15}$ as the system parameter vector to be optimized.

(vi) $I_N := \{1, 2, \ldots, N\}$ denotes the set of experiment times in batch culture, where N is the total experiment times.

(vii) $x^{0,l} \in \mathbb{R}^5$ denotes the initial state of the first period of the lth experiment.

(viii) $x^{i-1,l}(t_{f_{i-1}})$, $i \in \{2,3\}$, denote both the end period of the $(i-1)$th stage and the initial state of the ith period for the lth experiment.

(ix) $x^{i,l}(t) = (x_1^{i,l}(t), \dots, x_5^{i,l}(t))^{\mathrm{T}} \in \mathbb{R}^5$ denotes the state trajectory vector whose components are, respectively, the concentrations of biomass, extracellular glycerol, extracellular 1,3-PD, acetate, ethanol of the ith stage of the lth experiment at time $t \in [0, t_f]$.

Based on [32], the nonlinear three-period dynamical system governing the batch culture can be described as follows:

$$\dot{x}^{i,l}(t) = h^{i,l}\left(x^{i,l}, u_p^i\right), \quad t \in D_i$$
$$x^{i,l}\left(t_{f_{i-1}}\right) = \begin{cases} x^{0,l}, & i = 1, \\ x^{i-1,l}\left(t_{f_{i-1}}\right), & i \in \{2,3\}. \end{cases} \tag{1}$$

In system (1),

$$h^{i,l}\left(x^{i,l}, u_p^i\right) = \left(\mu^{i,l} x_1^{i,l}, -\left(u_p^i(2) + \frac{\mu^{i,l}}{0.0165}\right) \cdot x_1^{i,l}, \left(u_p^i(3) + 41.2584\mu^{i,l}\right) x_1^{i,l}, \left(u_p^i(4) + 4.541\mu^{i,l}\right) x_1^{i,l}, + \left(u_p^i(5)\, 3.046\mu^{i,l}\right) x_1^{i,l}\right)^{\mathrm{T}}, \tag{2}$$

where the specific cellular growth rate $\mu^{i,l}$ can be expressed as follows:

$$\mu^{i,l} = u_p^i(1) \frac{x_2^{i,l}}{x_2^{i,l} + 0.28} \prod_{j=2}^{5} \left(1 - \frac{x_j^{i,l}}{x_j^*}\right). \tag{3}$$

Let $u_p^0 := [0.67, 2.2, -2.69, -0.97, 5.26]^{\mathrm{T}}$ [32]. The admissible range U_{ad} of system parameters u_p^1, u_p^2 and u_p^3 is defined as

$$U_{ad} := \left[\underline{u}_p, \overline{u}_p\right] = \prod_{j=1}^{5} \left[\underline{u}_p(j), \overline{u}_p(j)\right] = \prod_{j=1}^{5} \left[u_p^0(j) - \frac{\left|u_p^0(j)\right|}{2}, u_p^0(j) + \frac{\left|u_p^0(j)\right|}{2}\right] \subset \mathbb{R}^5. \tag{4}$$

Under anaerobic conditions at 37°C and pH=7.0, the concentrations of biomass, glycerol, and products are restricted in a certain range according to the practical production. $x_* := [0.0001, 0.1, 0, 0, 0]^{\mathrm{T}}$ and $x^* := [15, 2039, 939.5, 1026, 360.9]^{\mathrm{T}}$ [32], where $x_{j*} \geq 0$ and $x_j^* \geq 0$, $j \in I_5$ are the lower and upper bound of $x_j^i(t)$, $i \in I_3$, respectively. So the admissible range S_0 of $x_j^i(t)$, $i \in I_3$, is

$$x^i(t) \in S_0 := [x_*, x^*] = \prod_{j=1}^{5} \left[x_{j*}, x_j^*\right] \subset \mathbb{R}_+^5. \tag{5}$$

2.2. Stochastic System

Nomenclature

(i) $(U_{ad}, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathbb{P})$ denotes a complete probability space, where $\mathscr{F}$ is a σ-algebra of subsets of U_{ad} with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathscr{F}_0$ contains all null subsets $\mathscr{N}$ of U_{ad}).

(ii) $\mathbf{E}(\cdot)$ denotes the expectation operator with respect to some probability measure $\mathbb{P}$.

(iii) For $i \in I_3$,

$$\mathbf{X}^{i,l}(t) := \left(X_1^{i,l}(t), \dots, X_5^{i,l}(t)\right)^{\mathrm{T}} \in S_0, \quad t \in \left[t_{f_{i-1}}, t_{f_i}\right], \tag{6}$$

which denotes a stochastic process whose components $\mathbf{X}_j^{i,l}(t)$, $j \in I_5$, denote the scalar stochastic process on biomass, glycerol, 1,3-PD, acetic acid, and ethanol of the $i_{th} \in I_3$ stage in the $l_{th} \in I_N$ experiment, respectively.

The stochastic counterpart of system (1) can be rewritten in the matrix form

$$\dot{\mathbf{X}}^{i,l}(t) = h^{i,l}\left(\mathbf{X}^{i,l}(t), u_p^i\right) = \mathbf{A}^{i,l}\mathbf{X}(t)^{i,l}, \quad t \in D_i,$$
$$\mathbf{X}^{i,l}\left(t_{f_{i-1}}\right) = \begin{cases} \mathbf{X}_0^l, & i = 1, \\ \mathbf{X}^{i-1,l}\left(t_{f_{i-1}}\right), & i \in \{2,3\}, \end{cases} \tag{7}$$

where $\mathbf{A}^{i,l} := (a_{\iota\kappa}^{i,l})_{5\times 5}$. It is clear to see that

$$\begin{aligned} a_{11}^{i,l} &= \mu^{i,l}, \\ a_{21}^{i,l} &= -u_p^i(2) - \frac{\mu^{i,l}}{0.0165}, \\ a_{31}^{i,l} &= u_p^i(3) + 41.2584\mu^{i,l}, \\ a_{41}^{i,l} &= u_p^i(4) + 4.541\mu^{i,l}, \\ a_{51}^{i,l} &= u_p^i(5) + 3.046\mu^{i,l}, \\ a_{\iota\kappa}^{i,l} &= 0, \quad \iota, \kappa \in I_5, \ \kappa \neq 1. \end{aligned} \tag{8}$$

Each system parameter $a_{\iota\kappa}^{i,l}$ is stochastically perturbed as follows:

$$a_{\iota\kappa}^{i,l} \longleftarrow a_{\iota\kappa}^{i,l} + \sigma_{\iota\kappa}^i \mathrm{d}W^{i,\kappa}(t), \tag{9}$$

where $\mathbf{W}^i(t) := (W^{i,1}(t), \dots, W^{i,5}(t))^{\mathrm{T}} \in \mathscr{F}_t$, $t \in [t_{f_{i-1}}, t_{f_i}]$, $i \in I_3$; its components are Gaussian white noise of the $i_{th} \in I_3$ stage and the given diffusion matrix $\sigma^i = (\sigma_{\iota\kappa}^i)_{5\times 5}$ satisfies the following conditions:

$$\begin{aligned} \sigma_{\iota\iota}^i &> 0, \quad \text{if } 1 \leq \iota \leq 5, \\ \sigma_{\iota\kappa}^i &\geq 0, \quad \text{if } \iota \neq \kappa. \end{aligned} \tag{10}$$

The process of batch culture with perturbations can be formulated as the following nonlinear stochastic dynamical system:

$$\mathrm{d}\mathbf{X}^{i,l}(t) = h^{i,l}\left(\mathbf{X}^{i,l}(t), u_p^i\right)\mathrm{d}t + \sigma^i\left(\mathbf{X}^{i,l}(t)\right)\mathrm{d}\mathbf{W}^i(t), \quad t \in D_i,\ i \in I_3,$$
$$\mathbf{X}^{i,l}\left(t_{f_{i-1}}\right) = \begin{cases}\mathbf{X}_0^l, & i = 1,\\ \mathbf{X}^{i-1,l}\left(t_{f_{i-1}}\right), & i \in \{2,3\},\end{cases} \tag{11}$$

where

(i) $\sigma^i(\mathbf{X}^{i,l}(t)) = \sigma^i\mathbf{X}^{i,l}(t)$,

(ii) $h^l(\mathbf{X}^l(t), u_p) := [h^{1,l}(\mathbf{X}^{1,l}(t), u_p^1), \ldots, h^{3,l}(\mathbf{X}^{3,l}(t), u_p^3)]^{\mathrm{T}}$,

(iii) $\mathbf{E}(\mathbf{X}_0^l) = x_0^l$.

2.3. Properties of Solutions to Stochastic System. In this section, we will discuss some properties of the solutions to the stochastic system. According to the definition of the functions $h^{i,l}(\mathbf{X}^{i,l}(t), u_p^i)$ and $\sigma^i(\mathbf{X}^{i,l}(t))$, $i \in I_3$, $l \in I_N$, defined by Sections 2.1 and 2.2, we can easily carry out the proof of Property 1.

Property 1. The vector-valued functions $h^{i,l}(\mathbf{X}^{i,l}(t), u_p^i)$ and $\sigma^i(\mathbf{X}^{i,l}(t))$, $i \in I_3$, $l \in I_N$, are measurable for $t \in D_i$ and $\mathbf{X}^{i,l} \in S_0$.

Similarly to [26], we have

Property 2. For the given vector-valued functions $h^{i,l}(\mathbf{X}^{i,l}(t), u_p^i)$ and $\sigma^i(\mathbf{X}^{i,l}(t))$, $i \in I_3$, $l \in I_N$, there exist positive constants $\mathscr{K}$ and $\mathscr{K}'$ such that, for $t \in D_i$, the following conditions hold:

(i) uniform Lipschitz condition

$$\left\|h^{i,l}\left(\widehat{\mathbf{X}}^{i,l}, u_p^i\right) - h^{i,l}\left(\check{\mathbf{X}}^{i,l}, u_p^i\right)\right\| + \left\|\sigma^i\left(\widehat{\mathbf{X}}^{i,l}\right) - \sigma^i\left(\check{\mathbf{X}}^{i,l}\right)\right\| \le \mathscr{K}\left\|\widehat{\mathbf{X}}^{i,l} - \check{\mathbf{X}}^{i,l}\right\|, \quad \forall \widehat{\mathbf{X}}^{i,l}, \check{\mathbf{X}}^{i,l} \in S_0; \tag{12}$$

(ii) growth continuous

$$\left\|h^{i,l}\left(\widehat{\mathbf{X}}^{i,l}, u_p^i\right)\right\| + \left\|\sigma^i\left(\widehat{\mathbf{X}}^{i,l}\right)\right\| \le \mathscr{K}'\left(1 + \left\|\widehat{\mathbf{X}}^{i,l}\right\|\right), \quad \forall \widehat{\mathbf{X}}^{i,l} \in S_0, \tag{13}$$

where $\|\cdot\|$ is the Euclidean vector norm.

By the proof in Property 2, Theorem 5.2 in [33], and Theorem 5.4 in [34], we can get the following interesting properties.

Property 3 (existence and uniqueness). For $\forall l \in I_N$, given the vector-valued functions $h^{i,l}(\mathbf{X}^{i,l}(t), u_p^i)$ and $\sigma^i(\mathbf{X}^{i,l}(t))$, $i \in I_3$, system (11) has a unique solution, denoted by $\mathbf{X}^l(t; \mathbf{X}_0^l, u_t, u_p) := (\mathbf{X}_1^l(t; \mathbf{X}_0^l, u_t, u_p), \ldots, \mathbf{X}_5^l(t; \mathbf{X}_0^l, u_t, u_p))^{\mathrm{T}}$, satisfying the initial condition $\mathbf{X}_0^l, l \in I_N$ on $[0, t_f]$. Furthermore, $\mathbf{X}^l(t; \mathbf{X}_0^l, u_t, u_p)$ is continuous with respect to $(u_t, u_p) \in \mathscr{D} \times U_{ad}^3$ and satisfies the following integral equation:

$$\mathbf{X}^l\left(t; \mathbf{X}_0^l, u_t, u_p\right) = \begin{cases}\mathbf{X}^{1,l}\left(t; \mathbf{X}_0^l, u_p^1\right) = \mathbf{X}_0^l + \int_0^t h^{1,l}\left(\mathbf{X}^{1,l}(s), u_p^1\right)ds + \int_0^t \sigma^1\left(\mathbf{X}^{1,l}(s)\right)d\mathbf{W}^1(s), & t \in D_1,\\ \mathbf{X}^{2,l}\left(t; \mathbf{X}^{1,l}\left(t_{f_1}\right), u_p^2\right) = \mathbf{X}^{1,l}\left(t_{f_1}\right) + \int_{t_{f_1}}^t h^{2,l}\left(\mathbf{X}^{2,l}(s), u_p^2\right)ds + \int_{t_{f_1}}^t \sigma^2\left(\mathbf{X}^{2,l}(s)\right)d\mathbf{W}^2(s), & t \in D_2,\\ \mathbf{X}^{3,l}\left(t; \mathbf{X}^{2,l}\left(t_{f_2}\right), u_p^3\right) = \mathbf{X}^{2,l}\left(t_{f_2}\right) + \int_{t_{f_2}}^t h^{3,l}\left(\mathbf{X}^{3,l}(s), u_p^3\right)ds + \int_{t_{f_2}}^t \sigma^3\left(\mathbf{X}^{3,l}(s)\right)d\mathbf{W}^3(s), & t \in D_3.\end{cases} \tag{14}$$

Property 4 (Markov property and boundedness). For $\forall l \in I_N$, the solution $\mathbf{X}^l(t; \mathbf{X}_0^l, u_t, u_p)$ is a Markov process on the interval $[0, t_f]$ whose initial probability distribution at $t = 0$ is the distribution of $\mathbf{X}_0^l$ and $\mathbf{X}^l(t; \mathbf{X}_0^l, u_t, u_p)$ has continuous paths. Moreover

$$\left(\sup_{0 \le t \le t_f} \mathbf{E}\left\|\mathbf{X}^l\left(t; \mathbf{X}_0^l, u_t, u_p\right)\right\|\right)^2 \le B\left(1 + \mathbf{E}\left\|\mathbf{X}_0^l\right\|^2\right), \tag{15}$$

where the constant B depends only on $\mathscr{K}, \sigma^1, \sigma^2, \sigma^3$, and t_f.

Property 5 (stochastic continuity). Almost all realizations of $\mathbf{X}^l(t; \mathbf{X}_0^l, u_t, u_p)$ are continuous on $[0, t_f]$.

3. System Identification Problem

The system identification problem governed by a stochastic system is generally to adjust the values of switched instants and system parameters so that the discrepancy between predicted and observed system output is as small as possible. The purpose of this section is to establish a sysem identification problem governed by system (11) in batch culture.

In the process of batch culture, we have measured N experimental data. Let $y^l(t)$ be the vector function fitted by experiment data. It denotes the concentration of every extracellular ingredient at time point $t \in [0, t_f]$. The system identification problem is to choose an optimal vector

$(u_t, u_p) \in \mathscr{D} \times U_{ad}^3$ such that the expectation of distinction between stochastic process, denoted by $\mathbf{X}^l(t; \mathbf{X}_0^l, u_t, u_p)$, and $y^l(t)$, $l \in I_N$, is minimized. Hence, the cost function can be defined by

$$J(u_t, u_p) := \mathbf{E}\left[\frac{\sum_{l=1}^{N}\int_0^{t_f}\left(\left\|\mathbf{X}^l(t; \mathbf{X}_0^l, u_t, u_p) - y^l(t)\right\| / \left\|y^l(t)\right\|\right)\mathrm{d}t}{N \times t_f}\right]. \tag{16}$$

Now, for $i \in I_3$, $l \in I_N$, we are in a position to propose a system identification (**SI**) problem as follows:

$$\begin{aligned} \min \quad & J(u_t, u_p) \\ \text{s.t.} \quad & \mathbf{X}^{i,l}\left(t; \mathbf{X}^{i,l}(t_{f_{i-1}}), u_p^i\right) \in S_0, \quad \forall t \in [t_{f_{i-1}}, t_{f_i}], \\ & (u_t, u_p) \in \mathscr{D} \times U_{ad}^3. \end{aligned} \tag{17}$$

The following theorem is to show the identifiability of the **SI** problem.

Theorem 6. *The **SI** problem admits an optimal solution.*

Proof. According to Property 3, we see that $\mathbf{X}^l(t; \mathbf{X}_0^l, u_t, u_p)$ is continuous with respect to $(u_t, u_p) \in \mathscr{D} \times U_{ad}^3$, so the cost function $J(u_t, u_p)$ is continuous on $(u_t, u_p) \in \mathscr{D} \times U_{ad}^3$. Moreover, $\mathscr{D} \times U_{ad}^3$ is a closed bounded set, which indicates that the optimal solution of the **SI** problem exists. Then, the proof is complete. □

4. Time-Scaling Transformation and Approximate Subproblems

The **SI** problem presents two major challenges for the existing numerical solution approaches:

(i) In the **SI** problem, the switched instants t_{f_1} and t_{f_2} are decision variables and need to be optimized. It is cumbersome to integrate the state and costate systems numerically when the switched instants t_{f_1} and t_{f_2} are decision variables.

(ii) The continuous state constraints $\mathbf{X}^{i,l}(t; \mathbf{X}^{i,l}(t_{f_{i-1}}), u_p^i) \in S_0$, $\forall t \in [t_{f_{i-1}}, t_{f_i}]$, $i \in I_3$, $l \in I_N$ are not easy to be satisfied in practice.

For these reasons, these conventional numerical optimization algorithms struggle to handle these challenges. In the section, we use the time-scaling transformation to deal with the variable time points together with the constraint transformation and the local smoothing approximate technique to handle the continuous state constraints.

4.1. Time-Scaling Transformation. Firstly, to handle the first challenge, one way of circumventing the difficulties caused by variable switched instants is to apply *time-scaling transformation* [35]—originally called the control *parameterization enhancing transform*. The time-scaling transformation works by mapping the variable switched instants to fixed points in a new time horizon, thus yielding a new optimization problem with the fixed switched instants. To apply the time-scaling transformation, we first introduce a new **time variables** θ^i as the duration of the i_{th} subinterval in $(t_{f_{i-1}}, t_{f_i}]$. That is,

$$\theta^i := t_{f_i} - t_{f_{i-1}}, \quad i \in I_3. \tag{18}$$

Then, the time-scaling transformation will be implemented by introducing a new time variable $s \in [0, 3]$ and relating s to t through the equation

$$t = \overline{\mu}(s) = \sum_{j=0}^{i-1}\theta^j + \theta^i(s - (i-1)), \quad s \in [i-1, i], \; i \in I_3, \tag{19}$$

where μ is the so-called *time-scaling function* and $\theta^0 = 0$.

Note that the time-scaling function is nondecreasing, continuous, and piecewise-linear. Moreover,

$$\frac{\mathrm{d}\overline{\mu}(s)}{\mathrm{d}s} = \theta^i, \quad s \in (i-1, i), \; i \in I_3. \tag{20}$$

The new decision parameters θ^i, $i \in I_3$, will satisfy

$$\sum_{i=1}^{3}\theta^i = t_f, \quad 0 \le \theta^i \le t_f, \; i \in I_3. \tag{21}$$

Let $\theta := [\theta^1, \theta^2, \theta^3]^{\mathrm{T}}$. Any $\theta \in \mathbb{R}^3$ satisfying (21) is called an *admissible duration vector*. Now, since $i - (i-1) = 1$, for $i \in I_3$,

$$\overline{\mu}(\tilde{s}_i) = \sum_{j=1}^{i}\theta^j = t_{f_i}. \tag{22}$$

Therefore, the time-scaling function maps the fixed integer $s = i$ to the switched instant $t = t_{f_i}$.

Let $\widetilde{\mathbf{X}}^{i,l}(s) := \mathbf{X}^{i,l}(\overline{\mu}(s))$ and $\widetilde{\mathbf{W}}^i(s) := \mathbf{W}^i(\overline{\mu}(s))$. If $s \in [i-1, i]$, then $\overline{\mu}(s) \in [t_{f_{i-1}}, t_{f_i}]$. Substituting (19) and (20) into (11) gives

$$\begin{aligned} \mathrm{d}\widetilde{\mathbf{X}}^{i,l}(s) &:= \tilde{h}^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}(s), u_p^i\right)\mathrm{d}s + \tilde{\sigma}^i\left(\widetilde{\mathbf{X}}^{i,l}(s)\right)\mathrm{d}\widetilde{\mathbf{W}}^i(s) \\ &= \mathrm{d}\mathbf{X}^{i,l}(\overline{\mu}(s)) \\ &= \theta^i h^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}, u_p^i\right)\mathrm{d}s \\ &\quad + \theta^i\sigma^i\left(\widetilde{\mathbf{X}}^{i,l}(s)\right)\mathrm{d}\widetilde{\mathbf{W}}^i(s), \quad s \in [i-1, i], \\ \widetilde{\mathbf{X}}^{i,l}(\tilde{s}_{i-1}) &= \begin{cases} \widetilde{\mathbf{X}}_0^l, & i = 1, \\ \widetilde{\mathbf{X}}^{i-1,l}(i-1), & i \in \{2, 3\}. \end{cases} \end{aligned} \tag{23}$$

Let

$$\widetilde{\mathbf{X}}^l(s; \mathbf{X}_0^l, \theta, u_p) = \begin{cases} \widetilde{\mathbf{X}}^{i,l}\left(s; \mathbf{X}_0^l, \theta^i, u_p^i\right), & i = 1, \\ \widetilde{\mathbf{X}}^{i,l}\left(s; \widetilde{\mathbf{X}}^{i-1,l}(\tilde{s}_{i-1}), \theta^i, u_p^i\right), & i \in \{2, 3\}. \end{cases} \tag{24}$$

It denotes the solution of system (23) corresponding to the admissible pair (θ, u_p). Based on (16) and (21) it becomes

$$J\left(\theta, u_p\right) := \mathbf{E}\left[\frac{\sum_{l=1}^{N}\int_0^3\left(\left\|\widetilde{\mathbf{X}}^l\left(s;\mathbf{X}_0^l,\theta,u_p\right)-y^l\left(\overline{\mu}\left(s\right)\right)\right\|/\left\|y^l\left(\overline{\mu}\left(s\right)\right)\right\|\right)\mathrm{d}s}{3N}\right], \tag{25}$$

$$\Phi\left(\theta\right) := \sum_{i=1}^{3}\theta^i - t_f = 0, \quad 0 \le \theta^i \le t_f,\ i \in I_3, \tag{26}$$

where $\Phi : [0, t_f] \longrightarrow \mathbb{R}$, is a given function. Then, for $i \in I_3$, $l \in I_N$, the **SI** problem becomes the transformation system identification (**TSI**) problem as follows:

$$\begin{aligned}
\min.\quad & J\left(\theta, u_p\right)\\
s.t.\quad & \widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right) \in S_0,\\
& \qquad \forall s \in \left[i-1, i\right],\\
& \left(\theta^i, u_p^i\right) \in \left[0, t_f\right] \times U_{ad},\\
& \Phi\left(\theta\right) = 0.
\end{aligned} \tag{27}$$

Note that the time-scaling transformation has replaced the variable **switched instants** t_{f_1} and t_{f_2} in the original approximate problem with conventional decision parameter vector θ^i, $i \in I_3$, in the equivalent problem. Since $i = \{0, 1, 2, 3\}$, in the equivalent problem, are fixed, this problem can be solved readily by any standard gradient-based optimization method or others.

The **TSI** problem is equivalent to the original problem. In fact, if (θ^{i*}, u_p^{i*}), $i \in I_3$, is a solution of the transformed problem, then the optimal switched instants for the **SI** problem are

$$t_{f_i}^* = \sum_{j=1}^{i}\theta^{j*}. \tag{28}$$

4.2. Approximate Problem. In view of the definition of semi-infinite programming problem [17, 36] and the continuous state inequality constraint in the **TSI** problem, we know that the **TSI** problem is a semi-infinite programming problem. An efficient algorithm for solving optimization problem of this type, which involve the so-called *constraint transcription technique*, is discussed in [36]. As a matter of fact, the essential difficulty lies in the judgement of the constraint $\widetilde{\mathbf{X}}^{i,l}(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i) = [\widetilde{X}_1^{i,l}(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i),\ldots,\widetilde{X}_5^{i,l}(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i)]^{\mathrm{T}} \in S_0$, $\forall s \in [i-1,i]$, $i \in I_3$, $l \in I_N$. To overcome these difficulties, let

$$\begin{aligned}
& g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right)\\
&\quad := \widetilde{X}_j^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right) - x_j^*, \quad j \in I_5,\\
& g_{5+j}^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right)\\
&\quad := x_{j*} - \widetilde{X}_j^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right), \quad j \in I_5.
\end{aligned} \tag{29}$$

For $i \in I_3$, $l \in I_N$, the constraint

$$g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right) \le 0, \qquad \forall s \in \left[i-1, i\right], \tag{30}$$

is equivalently transcribed into

$$G\left(\theta^i, u_p^i, i, l\right) := \sum_{j=1}^{10}\int_{i-1}^{\mathrm{i}}\max\left\{0, g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right)\right\}\mathrm{d}s = 0. \tag{31}$$

However, $G(\theta^i, u_p^i, i, l)$ is nonsmooth in (θ^i, u_p^i). Consequently, the standard optimization routines would have difficulties with this type of equality constraints. Next, we replace (31) with

$$\widetilde{G}_\varepsilon\left(\theta^i, u_p^i, i, l\right) := \sum_{j=1}^{10}\int_{i-1}^{i}\varphi_\varepsilon\left(g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right), i, l\right)\mathrm{d}s = 0, \tag{32}$$

where $\varepsilon > 0$ and

$$\varphi_\varepsilon\left(g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right), i, l\right) := \begin{cases} 0, & \text{if } g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right) < -\varepsilon,\\ \dfrac{\left[g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right)+\varepsilon\right]^2}{4\varepsilon}, & \text{if } -\varepsilon \le g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right) \le \varepsilon,\\ g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right), & \text{if } g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}\left(i-1\right),\theta^i,u_p^i\right)\right) > \varepsilon. \end{cases} \tag{33}$$

Note that the equality constraints specified in (32) do not satisfy the constraint qualification. Thus it is not advisable to compute it as such numerically. For this reason, we replace (32) with

$$\widetilde{G}_{\varepsilon,\gamma}\left(\theta^i,u_p^i,i,l\right) := \sum_{j=1}^{10}\int_{i-1}^{i}\varphi_\varepsilon\left(g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right)\right),i,l\right)\mathrm{d}s-\gamma\le 0, \quad (34)$$

where $\gamma > 0$.

For constructing the optimization algorithm, when θ^i and u_p^i are not feasible, we can move θ^i and u_p^i towards the feasible region in the direction of $-\partial\widetilde{G}_{\varepsilon,\gamma}(\theta^i,u_p^i,i,l)/\partial\theta^i$ and $-\partial\widetilde{G}_{\varepsilon,\gamma}(\theta^i,u_p^i,i,l)/\partial u_p^i$, respectively. In this paper, we develop a scheme for computing the gradients of constraint $\widetilde{G}_{\varepsilon,\gamma}(\theta^i,u_p^i,i,l)$ with respect to the parameters θ^i and u_p^i, respectively.

Theorem 7. *Given $l \in I_N$ and $i \in I_3$, for each $\varepsilon > 0, \gamma > 0$, the gradients of the constraint functionals $\widetilde{G}_{\varepsilon,\gamma}(\theta^i,u_p^i,i,l)$ with respect to u_p^i and θ^i are*

$$\frac{\partial\widetilde{G}_{\varepsilon,\gamma}\left(\theta^i,u_p^i,i,l\right)}{\partial u_p^i(k)} := \int_{i-1}^{i}\frac{\partial H\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right),u_p^i,\lambda^{i,l}(s),i,l\right)}{\partial u_p^i(k)}ds, \quad k\in I_5, \quad (35)$$

and

$$\frac{\partial\widetilde{G}_{\varepsilon,\gamma}\left(\theta^i,u_p^i,i,l\right)}{\partial\theta^i} := \int_{i-1}^{i}\frac{\partial H\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right),u_p^i,\lambda^{i,l}(s),i,l\right)}{\partial\theta^i}ds, \quad (36)$$

where

$$\begin{aligned}H&\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right),u_p^i,\lambda^{i,l}(s),i,l\right)\\&=\sum_{j=1}^{10}\varphi_\varepsilon\left(g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right)\right),i,l\right)\\&+\left[\lambda^{i,l}(s)\right]^{\mathrm{T}}\left\{\widetilde{h}^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right),u_p^i\right)\right.\\&\left.+\widetilde{\sigma}^i\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right)\right)\dot{\widetilde{\mathbf{W}}}^i(s)\right\},\end{aligned} \quad (37)$$

and

$$\lambda^{i,l}(s)=\left[\lambda_1^{i,l}(s),\dots,\lambda_5^{i,l}(s)\right]^{\mathrm{T}}, \quad (38)$$

is the solution of the costate system (39)

$$\dot{\lambda}^{i,l}(s) = -\frac{\partial H\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right),u_p^i,\lambda^{i,l}(s),i,l\right)}{\partial\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right)}, \quad (39)$$

with the continuous boundary

$$\lambda^{i,l}(i) = [0,0,0,0,0]^{\mathrm{T}}, \quad i\in I_3. \quad (40)$$

Proof. Let $u_p^i \in U_{ad}$ be an arbitrary but fixed vector and $\forall\delta_k \in \mathbb{R},\ k\in I_5$. Define

$$u_p^{i,\varpi}(k) := \left[u_p^i(1),\dots,u_p^i(k)+\varpi\delta_k,\dots,u_p^i(5)\right], \quad (41)$$

where $\varpi > 0$ is an arbitrarily small real number such that $u_{k*} < u_p^i(k)+\varpi\delta_j < u_k^*$, $k\in I_5$. Therefore, $\widetilde{G}_{\varepsilon,\gamma}(\theta^i,u_p^{i,\varpi},i,l)$ can be expressed as

$$\begin{aligned}\widetilde{G}_{\varepsilon,\gamma}&\left(\theta^i,u_p^{i,\varpi}(k),i,l\right) := \gamma\\&+\sum_{j=1}^{10}\int_{i-1}^{i}\varphi_\varepsilon\left(g_j^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^{i,\varpi}\right)\right),i,l\right)\mathrm{d}s+\int_{i-1}^{i}\left[\lambda^{i,l}(s)\right]^{\mathrm{T}}\\&\times\left\{h^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^{i,\varpi}\right),u_p^{i,\varpi}\right)\right.\\&+\widetilde{\sigma}^i\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^{i,\varpi}\right)\right)\dot{\widetilde{\mathbf{W}}}^i(s)\\&\left.-\dot{\widetilde{\mathbf{X}}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^{i,\varpi}\right)\right\}\mathrm{d}s,\end{aligned} \quad (42)$$

where $\lambda^{i,l}$ is yet arbitrary. Hence, it follows that

$$\begin{aligned}\Delta\widetilde{G}_{\varepsilon,\gamma}&\left(\theta^i,u_p^i,i,l\right) := \left.\frac{d\widetilde{G}_{\varepsilon,\gamma}\left(\theta^i,u_p^{i,\varpi}(k),i,l\right)}{d\varpi}\right|_{\varpi=0}\\&=\frac{\partial\widetilde{G}_{\varepsilon,\gamma}\left(\theta^i,u_p^i,i,l\right)}{\partial u_p^i(k)}\delta_k\\&=\int_{i-1}^{i}\left\{\frac{\partial H\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right),u_p^i,\lambda^{i,l}(s),i,l\right)}{\partial\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right)}\right.\\&\cdot\Delta\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right)\\&+\frac{\partial H\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right),u_p^i,\lambda^{i,l}(s),i,l\right)}{\partial u_p^i(k)}\delta_k\\&\left.-\left[\lambda^{i,l}(s)\right]^{\mathrm{T}}\Delta\dot{\widetilde{\mathbf{X}}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right)\right\}\mathrm{d}s,\end{aligned} \quad (43)$$

where $H(\widetilde{\mathbf{X}}^{i,l}(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i),u_p^i,\lambda^{i,l}(s),i,l)$ is defined as in (37). Integrating (43) by parts and combining (37), (38), (39), (40), and (42), we have

$$\frac{\partial\widetilde{G}_{\varepsilon,\gamma}\left(\theta^i,u_p^i,i,l\right)}{\partial u_p^i(k)}\delta_k := \int_{i-1}^{i}\frac{\partial H\left(\widetilde{\mathbf{X}}^{i,l}\left(s;\widetilde{\mathbf{X}}^{i-1,l}(i-1),\theta^i,u_p^i\right),u_p^i,\lambda^{i,l}(s),i,l\right)}{\partial u_p^i(k)}\delta_k\mathrm{d}s. \quad (44)$$

Since δ_k is arbitrary, conclusion (35) of the theorem follows.

Similarly to the proof of the above, we can prove

$$\frac{\partial \widetilde{G}_{\varepsilon,\gamma}\left(\theta^i, u_p^i, i, l\right)}{\partial \theta^i} := \int_{i-1}^{i} \frac{\partial H\left(\widetilde{\mathbf{X}}^{i,l}\left(s; \widetilde{\mathbf{X}}^{i-1,l}(i-1), \theta^i, u_p^i\right), u_p^i, \lambda^{i,l}(s), i, l\right)}{\partial \theta^i} \mathrm{d}s. \quad (45)$$

The proof is complete. □

Then, the **TSI** problem can be approximated by the following problem:

$$\begin{aligned} \mathbf{TSI}_{\varepsilon,\gamma}: \min.\quad & J\left(\theta, u_p\right) \\ s.t.\quad & \widetilde{G}_{\varepsilon,\gamma}\left(\theta^i, u_p^i, i, l\right) \le 0, \quad i \in I_3,\ l \in I_N, \\ & \left(\theta^i, u_p^i\right) \in \left[t_0, t_f\right] \times U_{ad}, \quad i \in I_3, \\ & \Phi(\theta) - \gamma \le 0. \end{aligned} \quad (46)$$

As is shown in [36], we can prove the following theorem.

Theorem 8. *There exists a $\gamma(\varepsilon) > 0$ such that for all γ, $\gamma(\varepsilon) > \gamma > 0$, any feasible decision variables $\theta^*_{\varepsilon,\gamma}$ and $u_{p^*_{\varepsilon,\gamma}}$ to the $\mathbf{TSI}_{\varepsilon,\gamma}$ problem are also feasible decision variables to the **TSI** problem.*

Remark 9. Theorem 8 ensures that the corresponding $\gamma(\varepsilon)$ for each ε in this sequence is finite.

5. Parallel Differential Evolution Algorithm

Since it is usually not possible to derive an analytical solution to system (23), numerical approaches are indispensable, especially for biological fermentation. To solve the **TSI** problem numerically, various optimization routes, such as gradient-based algorithms [37], can be used. Nonetheless, all those techniques are only designed to find local optima. Motivated by the mechanisms of natural selection, differential evolution (DE) was first proposed by Storn and Price [38] in 1997. DE is a recent optimization technique and an exceptionally simple and easy method used for the evolution strategy. It is a significantly fast and robust numerical optimization method and it is more likely to find the true global optimum.

Furthermore, one of main hurdles with solving the **TSI** problem numerically is that there exists a huge number of numerical computations of differential equations. This makes solving the **TSI** problem intolerable by a serial IDE algorithm. To improve computational efficiency, it is natural to construct a parallel optimization algorithm. In 2004, Tasoulis et al. [39] proposed a parallel DE algorithm using a ring-network topology, which can improve both the speed and the performance of the method. This point of view, which has been observed for a wide variety of experiments [40–43], is being gradually accepted by experts in the field of parallel DE.

However, what we need to solve is an optimization problem with both box constraints and continuous state constraints, to which DE cannot be applied directly [41]. To handle such constraints, we introduce the gradients of the constraint functions into our algorithm (see Theorem 7). Definitions of variables are shown in Table 1.

In addition, we solve the proposed stochastic identification problem using the following Stochastic Euler-Maruyama (**EM**) scheme. Let $\Delta s^i = (\widetilde{s}_i - \widetilde{s}_{i-1})/Q$ for some positive integer Q, $\tau^i_\iota = \widetilde{s}_{i-1} + \iota \Delta s^i$, $\iota \in I_Q$. Our numerical approximation to $\widetilde{\mathbf{X}}^{i,l}_j(\tau^i_\iota)$ will be denoted $\widetilde{\mathbf{X}}^{i,l}_{j,\iota}$. The **EM** method takes the form

$$\widetilde{\mathbf{X}}^{i,l}_{j,\iota} = \widetilde{\mathbf{X}}^{i,l}_{j,\iota-1} + \widetilde{h}^{i,l}\left(\widetilde{\mathbf{X}}^{i,l}_{j,\iota-1}, u_p^i\right) + \sum_{p=1}^{5} \widetilde{\sigma}^i_{j,p}\left(\widetilde{\mathbf{X}}^{i,l}_{j,\iota-1}\right) \mathrm{d}\mathbf{W}^{i,p}_\iota, \quad (47)$$

$$i \in I_3,\ j \in I_5,\ l \in I_N,\ \iota \in I_Q,$$

where $\mathrm{d}\mathbf{W}^{i,p}_\iota = \mathbf{W}^{i,p}(\tau^i_\iota) - \mathbf{W}^{i,p}(\tau^i_{\iota-1})$ is an independent random variable of the form $\sqrt{\Delta s^i}\mathbf{N}(0,1)$ and $\mathbf{N}(0,1)$ denotes a normally distributed random variable with zero mean and unit variance.

With this in mind, given ε, γ, combining Theorem 8, we propose a parallel modified differential evolution (MDE) algorithm for solving the $\mathbf{TSI}_{\varepsilon,\gamma}$ problem. The main steps of the parallel MDE are as follows. For convenience, the optimization vector is denoted by

$$k := \left(\theta^1, \theta^2, \theta^3, u_p\right)^{\mathrm{T}} := \left(\theta^1, \theta^2, \theta^3, \left(u_p^1\right)^{\mathrm{T}}, \left(u_p^2\right)^{\mathrm{T}}, \left(u_p^3\right)^{\mathrm{T}}\right)^{\mathrm{T}}. \quad (48)$$

Remark 10. The crossover factor is regulated by the following adaptive strategy:

$$CR(\ell) = \begin{cases} r, & \text{if } r_c \le R_c \\ CR(\ell-1), & \text{otherwise,} \end{cases} \quad (49)$$

and the mutation factor $F(\ell)$ is given by

$$F(\ell) = \begin{cases} F_{\min} + r_\ell F_{\max}, & \text{if } r_m \le R_m, \\ F(\ell-1), & \text{otherwise.} \end{cases} \quad (50)$$

Remark 11. In Algorithm 3, ε is a parameter controlling the accuracy of the smoothing approximation; γ is a parameter controlling the feasibility of constraint (5); $\overline{\varepsilon}$ and $\overline{\gamma}$ are two predefined positive parameters ensuring the termination of Algorithm 3; parameters α and β must be chosen less than 1.

6. Numerical Results

The below parameters are derived empirically after numerous experiments: $t_f := 7.75$; system (23) is solved by using Euler method with a step size of (1/72000)(h). In Algorithms 1–3, we select some parameters listed in Table 2 based on [36, 44]. The **TSI** problem is solved by using Algorithms 1–3.

The corresponding optimal system parameters $u_p^{1*}, u_p^{2*}, u_p^{3*}$ and time variables $\theta^{1*}, \theta^{2*}, \theta^{3*}$ are shown

Table 1: Definitions of variables in Section 5.

Variable	Representation	Variable	Representation
$CR(0)$	the maximal iteration taken from [0,1]	$F(0)$	the initial mutation factor taken from [0,1]
$\widetilde{N}$	the total number of genes in the population	M_T	the maximal iteration
$r, r_c, r_\ell, r_{cr}, r_m$	random numbers	R_c, R_m	given constants
$\mathbf{h}(k_{\ell,j})$	the search direction	$\rho(k_{\ell,j})$	the step-size selected by Armijo line search
F_{max}, F_{min}	the maximal and minimal mutation factors	k_*	$[0, 0, 0, \underline{u}_p]$ based on (5)
k^*	$[t_f, t_f, t_f, \overline{u}_p]$		

Table 2: The parameter value of Algorithms 1–3.

Algorithms 1-2	$N := 4$, $\widetilde{N} := 40$; $CR(0) := 0.5$, $F(0) := 0.4$, $F_{max} := 0.9$, $F_{min} := 0.1$, $R_c := 0.3$, $R_m := 0.3$, $M_T := 600$; $Q = 1000$; $\sigma_{i,i} = 0.02$, $\sigma_{i,p} = 0.0004$, $i \neq p$, $i \in I_5$, $p \in I_5$; $x_0^1 = \{0.102, 418.2609, 0, 0, 0, 0, 0, 0\}$, $x_0^2 = \{0.2025, 441.337, 0, 0, 0, 0, 0, 0\}$, $x_0^3 = \{0.173, 402.9348, 0, 0, 0, 0, 0, 0\}$, $x_0^4 = \{0.2245, 509.8913, 0, 0, 0, 0, 0, 0\}$
Algorithm 3	$\varepsilon := 0.1$, $\gamma := 0.01$, $\alpha := 0.1$, $\beta := 0.01$, $\overline{\varepsilon} := 1.0 \times 10^{-7}$, $\overline{\gamma} := 1.0 \times 10^{-7}$

Table 3: The value of u_p^{i*}, θ^{i*}, $i \in I_3$, $t_{f_1}^*$, and $t_{f_2}^*$.

i	u_p^{i*}
1	[0.620069, 2.03605, -1.86244, -0.67078, 7.50586]
2	[0.388043, 1.13123, -3.42823, -0.670187, 7.50937]
3	[0.352169, 1.65974, -3.28238, -0.669891, 7.51112]
	$\theta^{1*} = 2.6$, $\theta^{2*} = 2.4$, $\theta^{3*} = 2.75$; By (18) and (21), $t_{f_1}^* = 2.6$, $t_{f_2}^* = 5$.

Table 4: The relative errors under the optimal switched times and system parameters.

	in the paper			in [12]		
l	$e_1^l(\%)$	$e_2^l(\%)$	$e_3^l(\%)$	$e_1^l(\%)$	$e_2^l(\%)$	$e_3^l(\%)$
$l = 1$	4.20	4.21	16.93	41.87	10.21	19.11
$l = 2$	5.73	6.99	4.89	38.14	13.31	27.87
$l = 3$	4.09	4.40	5.73	43.25	12.02	15.94
$l = 4$	4.80	3.24	4.43	23.47	16.26	24.16

in Table 3. For comparison, the relative errors between the computational values and the experimental data in this work and the ones in [12] are listed in Table 4, in which the relative errors are defined as

$$e_\kappa^l := \mathbf{E}\left\{\frac{\sum_{j=1}^{3Q}\left|\widetilde{\mathbf{X}}_\kappa^l\left(s_j;\mathbf{X}_0^l,\theta,u_p\right)-y_\kappa^l\left(\mu\left(s_j\right)\right)\right|}{\sum_{j=1}^{3Q}\left|y_\kappa^l\left(s_j\right)\right|}\right\}, \quad s_j = j\times\frac{1}{Q},\ \ j=\{1,2,\ldots,3Q\},\ \kappa=1,2,3. \tag{51}$$

From Table 4, we can see that the relative errors are cut down greatly in comparison with the ones in [12]. The errors between experimental data and expectation of sample trajectory with different initial glycerol and biomass concentrations are displayed in Figures 1–4, where the horizontal axes represent time; the left vertical axes represent the concentrations of biomass, the center vertical axes represent the concentrations of glycerol, while the right vertical axes apply for 1,3-PD; the scattergrams denote experimental data, the blue solid lines represent sample trajectory, and the red solid lines display expectation of sample trajectory. The curves in Figures 1–4 are also confirmed that the nonlinear system with optimal kinetic parameters $u_p^{1*}, u_p^{2*}, u_p^{3*}$ and time variables $\theta^{1*}, \theta^{2*}, \theta^{3*}$ can describe the batch culture process reasonably.

7. Conclusion

In this paper, we introduce a switched stochastic counterpart with uncertain switched instants and system parameters to replace the commonly used deterministic description of glycerol biodissimilation to 1,3-PD by *K. pneumoniae* in form of ordinary differential equations. Some important properties of the stochastic system are discussed. Our aim is to identify

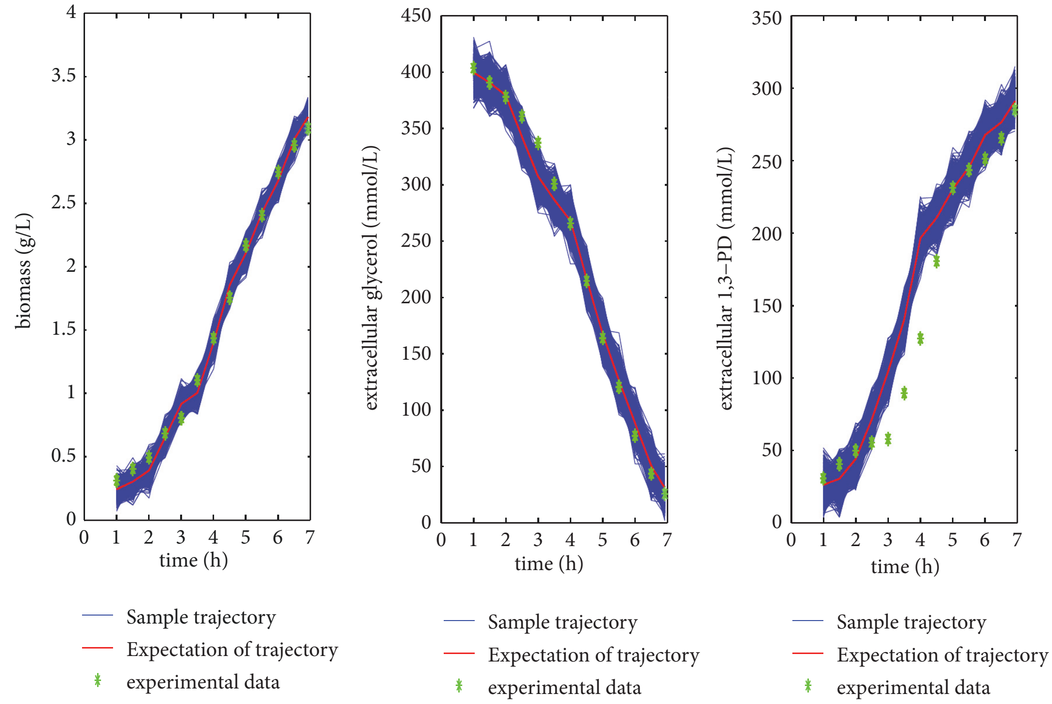

FIGURE 1: The results of initial glycerol concentration of 418.2609 $mmol/L$ and biomass concentration of 0.102 g/L with 1000 sample trajectories.

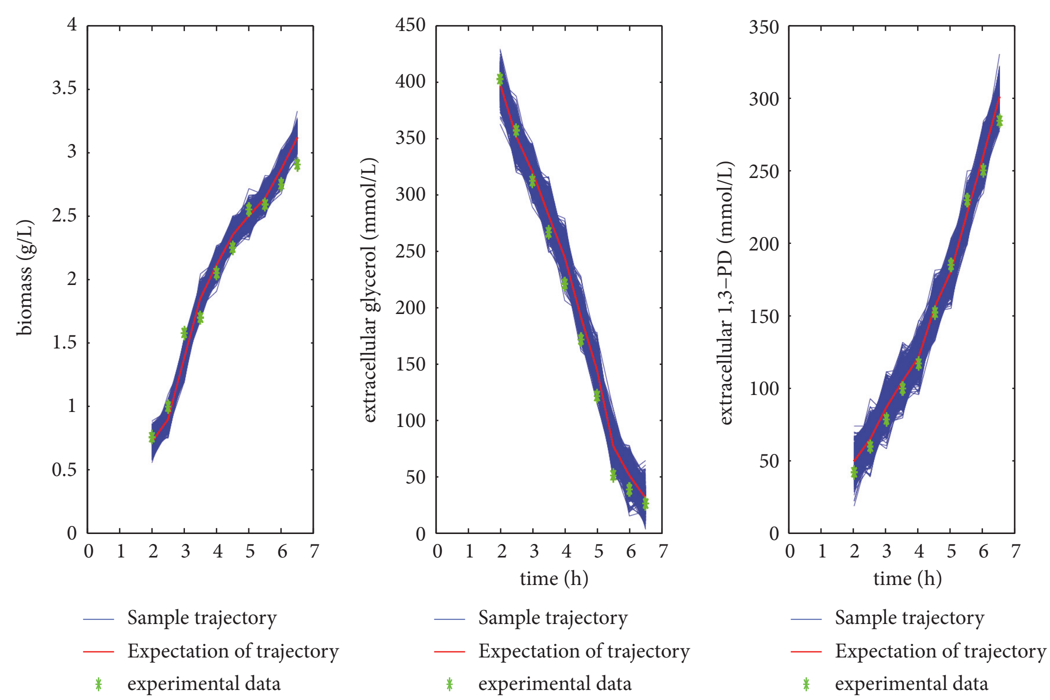

FIGURE 2: The results of initial glycerol concentration of 441.337 $mmol/L$ and biomass concentration of 0.2025g/L with 1000 sample trajectories.

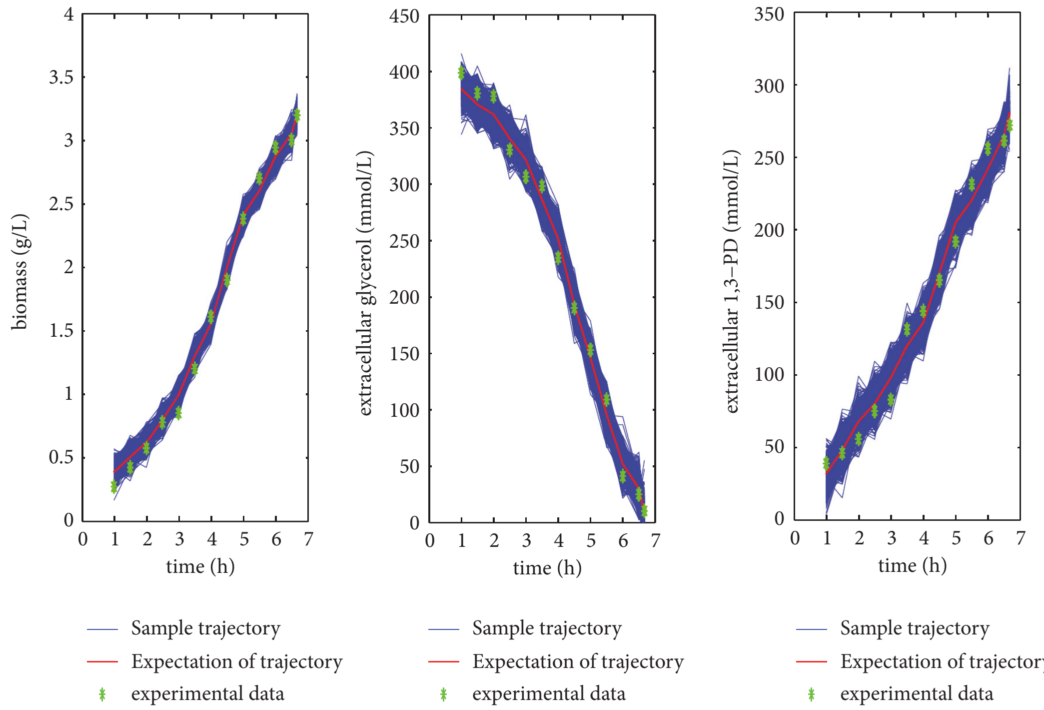

Figure 3: The results of initial glycerol concentration of 402.9348 $mmol/L$ and biomass concentration of $0.173 g/L$ with 1000 sample trajectories.

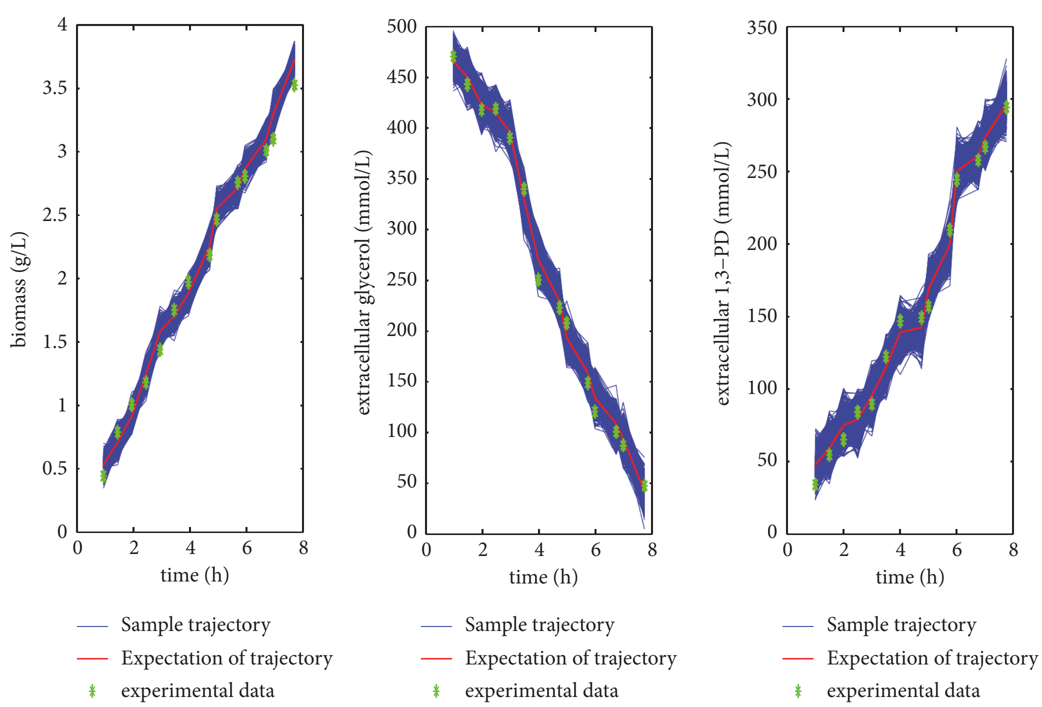

Figure 4: The results of initial glycerol concentration of 509.8913 $mmol/L$ and biomass concentration of $0.2245 g/L$ with 1000 sample trajectories.

Require:
The experimental data such as $x_0^l, y^l(\overline{\mu}(s)), s \in [0,3], l \in I_N$;
Ensure:
The value of k_{best} and J_{best};
Randomly generate $\widetilde{N}$ particle positions, denoted by $k_{0,1}, \ldots, k_{0,\widetilde{N}}$. Based on (34), compute the constraint functions, denoted by $\widetilde{G}_{\varepsilon,\gamma}(k_{0,\jmath}, l) := (\widetilde{G}_{\varepsilon,\gamma}(k_{0,\jmath}, 1, l), \widetilde{G}_{\varepsilon,\gamma}(k_{0,\jmath}, 2, l), \widetilde{G}_{\varepsilon,\gamma}(k_{0,\jmath}, 3, l))^{\mathrm{T}}, \jmath \in I_{\widetilde{N}}, l \in I_N$;
for each $\jmath \in I_{\widetilde{N}}$ **do**
  if $\widetilde{G}_{\varepsilon,\gamma}(k_{0,\jmath}, l) > 0, \ l \in I_N$, **then**
    compute $\mathbf{h}(k_{0,\jmath}) := \left.\dfrac{\partial \widetilde{G}_{\varepsilon,\gamma}(k,l)}{\partial k}\right|_{k=k_{0,\jmath}}$, and move $k_{0,\jmath}$ according to $k_{0,\jmath} := k_{0,\jmath} - \rho(k_{0,\jmath})\mathbf{h}(k_{0,\jmath})$, until $\widetilde{G}_{\varepsilon,\gamma}(k_{0,\jmath}, l) \le 0$, with $\mathbf{h}(k_{0,\jmath})$ the search direction and $\rho(k_{0,\jmath})$ the step-size selected by Armijo line search;
  end if
end for
Solve system (23) by (47). Based on (25), compute the cost function, denoted by $J(k_{0,\jmath}), \jmath \in I_{\widetilde{N}}$ and let $J_{best}^0 := J(k_g)$, where $k_g := \arg\min_{\jmath \in I_{\widetilde{N}}}\{J(k_{0,\jmath})\}$;
Let $k_{best} := k_g$ and broadcast (**MPI_Broad**), J_{best}^0, k_{best} and $k_{0,\jmath}, \jmath \in I_{\widetilde{N}}$, to all Processors;
Let $\ell = 1$;
for $\ell = 1; \ell < M_T; \ell + +$ **do**
  if $(\ell \ge M_\varrho$ and $J_{best}^{\ell - M_\varrho} - J_{best}^{\ell} < \varrho_1)$ **then**
    return k_{best} and J_{best};
  end if
  Let $\jmath := 1$;
  for $\jmath = 1; \jmath < \widetilde{N}; \jmath + +$ **do**
    if $\jmath > \widetilde{N}$ **then**
      Let $J_{best}^{\ell} := J(k_g), J_{best} := J(k_g), k_{best} := k_g$;
      Finish the $\jmath_{th}$ loop and start the $(\jmath + 1)_{th}$ loop;
    end if
    Receive (**MPI_Recv**) the updated information of Processor $\jmath$ during iteration ℓ, including $k_{\ell,\jmath}$ and $J(k_{\ell,\jmath})$;
    if $J(k_{\ell,\jmath}) < J(k_g)$ **then**
      Let $k_g := k_{\ell,\jmath}$ and Broadcast (**MPI_Broad**) $k_{\ell,\jmath}$ and the updated k_g to all slave processors;
    else
      Broadcast (**MPI_Broad**) $k_{\ell,\jmath}$ to all slave processors;
    end if
  end for
end for

Algorithm 1: Master processor (Processor 0).

the uncertain switched instants and system parameters under condition of different initial state. For this, taking the relative error between experimental data and computational results as the cost function, we propose a system identification problem governed by the stochastic system and subject to some constraints. In consideration of both the difficulty of finding analytical solutions and the complexity of solution procedure, a parallelized differential evolution algorithm is developed to solve the identification problem. An illustrative numerical example shows the appropriateness of the optimal switched instants and system parameters with initial state difference.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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1: Let $\ell = 1$;
2: **for** $\ell = 1$; $\ell < M_T$; $\ell + +$ **do**
3:   **if** $\ell > M_T$ **then**
4:     **return** Algorithm 1;
5:   **end if**
6:   Randomly choose p_1 and p_2 ($j \neq p_1 \neq p_2$) from $\{1, 2, \ldots, \widetilde{N}\}$ and generate the trial vector

$$k_{\ell,j} = \begin{cases} k_g + F(\ell)[k_{\ell-1,p_1} - k_{\ell-1,p_2}], & \text{if } r_{cr} \leq CR(\ell) \\ k_{\ell-1,j}, & \text{otherwise}; \end{cases}$$

7:   **if** $k_{\ell,j}$ violates boundary constraints **then**
8:     It is reflected back from the bound by

$$k_{\ell,j} = \begin{cases} 2k_* - k_{\ell,j}, & \text{if } k_{\ell,j} \in [-\infty, k_*), \\ 2k^* - k_{\ell,j}, & \text{if } k_{\ell,j} \in [k^*, +\infty); \end{cases}$$

9:   **end if**
10:   Test the value of the constraint functions $\widetilde{G}_{\varepsilon,\gamma}(k_{\ell,j}, l) \coloneqq (\widetilde{G}_{\varepsilon,\gamma}(k_{\ell,j}, 1, l), \widetilde{G}_{\varepsilon,\gamma}(k_{\ell,j}, 2, l), \widetilde{G}_{\varepsilon,\gamma}(k_{\ell,j}, 3, l))^{\mathrm{T}}$, $j \in I_{\widetilde{N}}$, $l \in I_N$. For each $j \in I_{\widetilde{N}}$, if $\widetilde{G}_{\varepsilon,\gamma}(k_{\ell,j}, l) > 0$, $l \in I_N$, then, based on Theorem 7, compute $\mathbf{h}(k_{\ell,j}) \coloneqq \left.\dfrac{\partial \widetilde{G}_{\varepsilon,\gamma}(k,l)}{\partial k}\right|_{k=k_{\ell,j}}$, and move $k_{\ell,j}$ according to $k_{\ell,j} \coloneqq k_{\ell,j} - \rho(k_{\ell,j})\mathbf{h}(k_{\ell,j})$, until $\widetilde{G}_{\varepsilon,\gamma}(k_{\ell,j}, l) \leq 0$;
11:   Solve system (23) by (47) and compute the cost function $J(k_{\ell,j})$, $j \in I_{\widetilde{N}}$ based on (25);
12:   Send (**MPI_Send**) $k_{\ell,j}$ and $J(k_{\ell,j})$, $j \in I_{\widetilde{N}}$ to Master processor;
13: **end for**

Algorithm 2: Slave processors (Processor j, $j = 1, 2, \ldots, \widetilde{N}$).

1: Choose initial values of $\varepsilon > 0, \gamma > 0$;
2: Solve the $\mathbf{TSI}_{\varepsilon,\gamma}$ problem using Algorithm 1 to give $k^*_{\varepsilon,\gamma} \coloneqq (\theta^{1*}, \theta^{2*}, \theta^{3*}, (u_p^{1*})^{\mathrm{T}}, (u_p^{2*})^{\mathrm{T}}, (u_p^{3*})^{\mathrm{T}})^{\mathrm{T}}$;
3: Check feasibility of $g_j^{i,l}(\widetilde{\mathbf{X}}^{i,l}(s; \widetilde{\mathbf{X}}^{i-1,l}(\widetilde{s}_{i-1}), \theta^{i*}, u_p^{i*})) \leq 0$, $j \in I_{10}$, $l \in I_N$ for $\forall s \in [\widetilde{s}_{i-1}, \widetilde{s}_i]$, $i \in I_3$;
4: **if** $k^*_{\varepsilon,\gamma}$ is feasible **then**
5:   Let $\varepsilon \coloneqq \beta\varepsilon$;
6:   **if** $\varepsilon > \overline{\varepsilon}$ **then**
7:     Go to 2;
8:   **else**
9:     **return** $k^*_{\varepsilon,\gamma}$;
10:   **end if**
11: **else**
12:   Let $\gamma \coloneqq \alpha\gamma$;
13:   **if** $\gamma > \overline{\gamma}$ **then**
14:     Go to 2;
15:   **else**
16:     **return** $k^*_{\varepsilon,\gamma}$;
17:   **end if**
18: **end if**

Algorithm 3: Combining Algorithms 1 and 2 with Theorem 8, the **RIP** problem can be solved by the algorithm.

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