School of Surveying and Land Information

An Evaluation of FFT Geoid Determination Techniques and their Application to Height Determination Using GPS in Australia

Kefei Zhang

This thesis is presented as part of the requirements for the award of the Degree of Doctor of Philosophy of the Curtin University of Technology

December, 1997
ABSTRACT

A new, high resolution, high precision and accuracy gravimetric geoid of Australia has been produced using updated data, theory and computational methodologies. The fast Fourier transform technique is applied to the computation of the geoid and terrain effects. The long, medium and short wavelength components of the geoid are determined from the OSU91A global geopotential model, 2'×2' (residual gravity anomalies in a 3° cap and 1'×1' digital terrain model (DTM), respectively.

Satellite altimeter gravity data have been combined with marine gravity data to improve the coverage of the gravity data, and thus the quality of the geoid. The best gridding procedure for gravity data has been studied and applied to the gravity data gridding. It is found that the gravity field of Australia behaves quite differently. None of the free-air, Bouguer or topographic-isostatic gravity anomalies are consistently the smoothest. The Bouguer anomaly is often rougher than the free-air anomaly and thus should be not used for gravity field gridding. It is also revealed that in some regions the topography often contains longer wavelength features than the gravity anomalies.

It is demonstrated that the inclusion of terrain effects is crucial for the determination of an accurate gravimetric geoid. Both the direct and indirect terrain effects need to be taken into account in the precise geoid determination of Australia. The existing AUSGEOID93 could be in error up to 0.7m in terms of the terrain effect only. In addition, a series of formulas have been developed to evaluate the precision of the terrain effects. These formulas allow the effectiveness of the terrain correction and precision requirement for a given DTM to be studied. It is recommended that the newly released 9’×9’ DTM could be more effectively used if it is based on 15’×15’ grid.

It is estimated from comparisons with Global Positioning System (GPS) and Australian Height Datum Data that the absolute accuracy of the new geoid is better than 33cm and the relative precision of the new geoid is better than 10–20cm. This new geoid can support Australian GPS heighting to third-order specifications.
ACKNOWLEDGEMENTS

The author would like to express his profound gratitude to his supervisor, Dr. Will Featherstone, for his continuous support, supervision, constructive comments and discussions throughout the course of his study. Dr. Mike Stewart, my associate supervisor, is highly thanked for his comments, valuable discussions, co-operation and support. The author is indebted to Prof. Graham Lodwick, Head of School of Spatial Sciences, for his continuous support and guidance during this period.

Special thanks are extended to many of my friends and colleagues at the Wuhan Technical University of Surveying and Mapping, but particularly to Professors JN Liu and BZ Tao for their continuous support, guidance and great encouragement during the five years the author worked with them.

The author would like to thank many other individuals and organisations for providing data and software, particularly for Australian Surveying and Land Information Group and Australian Geological Survey Organisation, Professors MG Sideris, R Forsberg, DT Sandwell, AHW Kearsley and JR Gilliland; Mr YC Li, Mr K Alexander, Mr J Steed, Mr M Higgins and Mr M McKay.

The author acknowledges gratefully to Curtin University of Technology for awarding him prestigious scholarships, namely, Overseas Postgraduate Research Scholarship (OPRS) and Curtin University Postgraduate Research Scholarship (CUPRS). The author also greatly acknowledges the indirect financial support from Australian Research Council grant (No:A49331318) endorsed to his supervisor, Dr. Featherstone.

All the secretaries, computer managers and staff at the School of Spatial Sciences, Curtin University of Technology have helped me a lot in various ways. They are sincerely appreciated. The author would like to express his gratitude to all staff in the School, for their friendly help, support and co-operation, particularly to Dr M Tsakari, Dr XL Ding and Dr J Kirby.

Last, but not least, my deepest thanks go to my wife, Suqin Wu, and our daughter, Xi Zhang, for their understanding and continuous support during my three years hard time. Thanks to them for sharing bitter fighting tears and triumphal cheers of a foreign student.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ABSTRACT</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>i</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>viii</td>
</tr>
<tr>
<td>Chapter</td>
<td>xi</td>
</tr>
</tbody>
</table>

## 1. INTRODUCTION

1.1 Background .................................................. 1

1.1.1 Definition and Applications of the Geoid .......... 2

1.1.2 Spectral Geoid Determination ....................... 3

1.1.3 Other Considerations ................................... 4

1.1.4 Global Geoid Models .................................. 5

1.1.5 Above Ground Techniques ............................ 6

1.2 History of the Australian Geoid ....................... 7

1.2.1 Australian Geoids Before 1980 ..................... 7

1.2.2 The Australian Geoid 1991 (AUSGEOID91) .......... 8

1.2.3 The Australian Geoid 1993 (AUSGEOID93) .......... 9

1.3 Expected Problems with AUSGEOID93 .......... 10

1.4 Aims and Objectives of this Research ............. 10

1.5 Outline of the Research ................................ 11

## 2. FUNDAMENTALS OF GEOID DETERMINATION AND GPS HEIGHTING ............................. 16

2.1 Theory of Gravimetric Geoid Determination ........... 17

2.1.1 Introduction ........................................ 17

2.1.2 Geodetic Boundary Value Problems ................ 19

2.1.3 Stokes's Method ................................... 20

2.1.4 Molodensky's Problem .............................. 22

2.1.5 The Relationship Between Stoke's and Molodensky's Problems ............... 24

2.2 Height Determination .................................... 25

2.2.1 The Global Positioning System and Height Determination ................. 25

2.2.2 Geodetic Spirit Levelling and Height Systems .......... 30

2.2.3 The Australian Height Datum (AHD) ................ 32

2.2.4 The Geoid from GPS and Spirit Levelling ............ 33

2.3 Other Geometrical Determinations of the Geoid ......... 34

2.3.1 The Astro-geodetic Method ......................... 35

2.3.2 Satellite Altimetric Method ......................... 35

2.4 Harmonic Expression of the Geopotential ............. 38
Table of Contents (continued)

Chapter                                      Page

2.5 Other Methods to Determine Geoid Undulation ........................................41
2.6 The Terrain Reduction .................................................................41
    2.6.1 Terrain Reduction Methods ......................................................42
    2.6.2 Concluding Statements ..........................................................46
2.7 The General Remove-restore Technique .................................................47
2.8 Summary .........................................................................................49

3. DATA PREPARATION, TEST DATA, THEIR PRE-PROCESSING AND EVALUATION ..................................................................................50
    3.1 Introduction ..................................................................................51
    3.2 Australian Geodetic Datum and World Geodetic System 1984 ..........52
    3.3 Spot Heights and Digital Terrain Model .........................................54
    3.4 Gravity Data, Refinement and Reduction .........................................55
        3.4.1 Gravity Reduction and Gravity Anomaly ....................................57
        3.4.2 Effects of Varying Topographic Density and Actual Gravity
            Gradient ....................................................................................59
        3.4.3 Time Variations of the Gravity Field and Geoid ......................61
    3.5 Altimetric Gravity Anomalies, Evaluation and Combination with Local Gravity Anomalies .................................................................62
        3.5.1 Evaluation of Satellite Altimetry Derived Gravity Anomalies ......63
        3.5.2 Combination with Marine Gravity Anomalies ............................66
    3.6 Geometrical Geoid Undulation from GPS and Spirit Levelling ..........67
        3.6.1 Australian GPS Fiducial Network and National Network ..........67
        3.6.2 Other GPS Networks Used for Comparison ...............................69
        3.6.3 Is GPS/Levelling Really the "Yardstick" for a Gravimetric Geoid?..70
    3.7 Determination of the Best Fitting Global Geopotential Model ..........75
        3.7.1 Introduction ...........................................................................75
        3.7.2 Geopotential Models Available ...............................................77
        3.7.3 Statistical Comparisons and Numerical Analysis ....................78
        3.7.4 Discussion ............................................................................83
    3.8 Summary .......................................................................................85

4. THE FOURIER TRANSFORM IN GEOID DETERMINATION ..............................87
    4.1 Introduction ..................................................................................88
    4.2 Fundamentals of Fourier Transform ...............................................89
        4.2.1 The Two-dimensional Fourier Transform and Convolution ........89
        4.2.2 The Discrete Fourier Transform .............................................90
    4.3 Stokes's Integral by FFT and its Various Kernel Approximations .......91
        4.3.1 Two-dimensional Planar FFT with Discrete and Analytical Spectra...92
        4.3.2 Two-dimensional Spherical Approximate FFT Technique ............92
Table of Contents (continued)

Chapter .......................... Page
4.3.3 One-dimensional Spherical Exact FFT Approach ........................................... 94
4.3.4 Integral Radius (Cap Size) of the Stokes integral by FFT .......................... 94
4.4 Some FFT-related Problems ............................................................................ 95
4.4.1 Effect of Sampling Interval and Spectral Leakage ........................................ 95
4.4.2 Cyclic Convolution and Effect of Padding ................................................... 95
4.4.3 Phase Shifting .............................................................................................. 96
4.5 Numerical Studies on Various Kernel Approximations ............................... 96
4.5.1 Test Data ..................................................................................................... 96
4.5.2 Overall Effects ............................................................................................. 97
4.5.3 Boundary Effects ......................................................................................... 101
4.5.4 Effect of 100 Percent Zero Padding ............................................................ 105
4.5.5 Consistency of the Various Kernel Approximations ................................... 106
4.5.6 Summary .................................................................................................... 109
4.6 Terrain Correction by FFT ................................................................................ 109
4.7 Indirect Effect of Helmert Second Condensation ............................................. 110
4.8 Comments and Discussion ............................................................................... 111
4.9 Summary ......................................................................................................... 112

5. FEATURES OF THE AUSTRALIAN GRAVITY FIELD ........................................ 113
5.1 Introduction ..................................................................................................... 114
5.2 Selection of Test Areas and Profiles ............................................................... 115
5.3 Analysis of Various Gravity Anomaly Surfaces .............................................. 118
5.3.1 Statistical Comparisons .............................................................................. 119
5.3.2 Power Spectral Analysis .............................................................................. 120
5.3.3 Hurst Fractal Dimension Analysis .............................................................. 124
5.3.4 Discussion .................................................................................................. 127
5.4 Profile Analysis ................................................................................................ 127
5.4.1 Visual Comparisons .................................................................................... 128
5.4.2 Statistical Comparisons .............................................................................. 131
5.4.3 Power Spectral Analyses ............................................................................ 133
5.4.4 Hurst Fractal Dimension Analyses ............................................................. 135
5.4.5 Discussion .................................................................................................. 137
5.5 Reasons on the Special Features of the Australian Gravity Field ............... 137
5.6 Discussion ....................................................................................................... 138
5.7 Summary ........................................................................................................ 140

6. GRIDDING THE AUSTRALIAN GRAVITY FIELD ........................................... 141
6.1 Introduction ..................................................................................................... 142
6.2 Continuous Curvature Splines in Tension ....................................................... 144
6.2.1 Minimum Curvature Splines .................................................................... 144
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2.2 Minimum Curvature Splines in Tension</td>
<td>145</td>
</tr>
<tr>
<td>6.3 Bi-cubic Spline Method</td>
<td>147</td>
</tr>
<tr>
<td>6.4 Least Squares Polynomial Fitting</td>
<td>147</td>
</tr>
<tr>
<td>6.5 Moving Weighted Average Method</td>
<td>149</td>
</tr>
<tr>
<td>6.6 Kriging Method</td>
<td>150</td>
</tr>
<tr>
<td>6.7 A Comparison of the Four Gridding Procedures</td>
<td>153</td>
</tr>
<tr>
<td>6.8 Choice of the Optimal Gridding Procedure for the Australian</td>
<td></td>
</tr>
<tr>
<td>Gravity Field</td>
<td></td>
</tr>
<tr>
<td>6.8.1 A Practical Remove-restore Technique for Gravity Gridding</td>
<td>155</td>
</tr>
<tr>
<td>6.8.2 The Test Areas and Strategies</td>
<td>157</td>
</tr>
<tr>
<td>6.8.3 Numerical Comparisons and Analysis</td>
<td>163</td>
</tr>
<tr>
<td>6.9 Discussion</td>
<td>168</td>
</tr>
<tr>
<td>6.10 Summary</td>
<td>169</td>
</tr>
<tr>
<td>7. EVALUATION OF THE TERRAIN CORRECTION AND ITS INDIRECT EFFECT</td>
<td></td>
</tr>
<tr>
<td>7.1 Introduction</td>
<td>171</td>
</tr>
<tr>
<td>7.2 Evaluation of the Total Terrain Effects</td>
<td>172</td>
</tr>
<tr>
<td>7.2.1 Relevant Formulae for Practical Computation</td>
<td>173</td>
</tr>
<tr>
<td>7.2.2 Practical Computation and Statistical Analysis</td>
<td>175</td>
</tr>
<tr>
<td>7.2.3 Conclusions and Recommendations</td>
<td>178</td>
</tr>
<tr>
<td>7.3 Evaluation of the Accuracy of the Terrain Correction</td>
<td></td>
</tr>
<tr>
<td>7.3.1 Derivation of the Formulae</td>
<td></td>
</tr>
<tr>
<td>7.3.2 Numerical Estimations of the Terrain Correction Error</td>
<td></td>
</tr>
<tr>
<td>7.3.3 Effective Grid Size and Accuracy Evaluation of the Terrain</td>
<td></td>
</tr>
<tr>
<td>Corrections</td>
<td></td>
</tr>
<tr>
<td>7.3.4 Discussion</td>
<td></td>
</tr>
<tr>
<td>7.4 Summary</td>
<td></td>
</tr>
<tr>
<td>8. PRACTICAL GRAVIMETRIC GEOID DETERMINATION</td>
<td>199</td>
</tr>
<tr>
<td>8.1 Introduction</td>
<td></td>
</tr>
<tr>
<td>8.2 Computation of a New Australian Geoid</td>
<td>200</td>
</tr>
<tr>
<td>8.2.1 Data Preparation of a New Geoid</td>
<td>200</td>
</tr>
<tr>
<td>8.2.2 Selection of the Best Grid Size for Gravity Anomalies</td>
<td>202</td>
</tr>
<tr>
<td>8.2.3 Selection of Optimal Reference Field</td>
<td>203</td>
</tr>
<tr>
<td>8.2.4 Optimal Gridding Procedure</td>
<td>203</td>
</tr>
<tr>
<td>8.2.5 FFT Related Considerations</td>
<td>204</td>
</tr>
<tr>
<td>8.2.6 Differences from AUSGEOID93</td>
<td>204</td>
</tr>
<tr>
<td>8.3 Low Frequency Differences between Actual Gravity Field and OSU91A</td>
<td></td>
</tr>
<tr>
<td>8.4 Optimal Cap Size in the FFT</td>
<td>207</td>
</tr>
<tr>
<td></td>
<td>212</td>
</tr>
</tbody>
</table>
Table of Contents (continued)

**Chapter** | **Page**
---|---
8.5 Comments on the Effects of the Satellite Altimetry-derived Gravity Data | 215
8.6 Optimal Grid Size for the Geoid Undulation Computation | 216
8.7 Evaluation of the Best FFT Geoid | 218
8.7.1 Terrain Effects on AUSGEOID93 | 218
8.7.2 Removal of the Bias and Tilt | 221
8.7.3 Absolute Accuracy of the Best FFT Geoid | 222
8.7.4 Relative Precision of the Best FFT Geoid | 222
8.8 Discussion | 224
8.9 Summary | 226

9. CONCLUSIONS AND RECOMMENDATIONS | 227
9.1 Objectives of the Research | 228
9.2 Summary of Major Findings | 228
9.2.1 Best Fitting Reference Field | 228
9.2.2 Gravity Data Preparation and Combination with Satellite Altimetric Data | 229
9.2.3 FFT Related Considerations | 229
9.2.4 Special Features of the Australian Gravity Field | 230
9.2.5 Techniques Analysing the Variation of the Gravity Field | 230
9.2.6 Optimal Gridding Procedure for Australian Gravity Field | 230
9.2.7 Evaluation of the Terrain Effects | 231
9.2.8 Theoretical Estimation of the Precision of the Terrain Corrections | 232
9.2.9 Optimal Capsize for Stokes's Integral | 232
9.2.10 Contribution of the Satellite Altimetric Gravity Data | 232
9.2.11 Selection of Grid Size for Gravity Field Gridding | 232
9.2.12 Accuracy Evaluation of the New Geoid | 233
9.3 Conclusions | 233
9.4 Recommendations | 236

REFERENCES | 239

APPENDIX I Data and Results of the National and Local GPS and Levelling Stations | 257
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 The geoid and reference ellipsoid</td>
<td>19</td>
</tr>
<tr>
<td>2.2 The geometry of the Stokes BVP and the Molodensky BVP</td>
<td>24</td>
</tr>
<tr>
<td>2.3 Geoid determination from GPS/levelling</td>
<td>34</td>
</tr>
<tr>
<td>2.4 Geoid determination by Astro-geodetic method</td>
<td>35</td>
</tr>
<tr>
<td>2.5 Geoid undulation determination by satellite altimetry</td>
<td>36</td>
</tr>
<tr>
<td>2.6 Scheme of various terrain reduction methods</td>
<td>43</td>
</tr>
<tr>
<td>2.7 Principle of the Remove-restore technique</td>
<td>47</td>
</tr>
<tr>
<td>2.8 Contributions of different data to regional geoid determination</td>
<td>48</td>
</tr>
<tr>
<td>3.1 Distribution of the marine and land gravity observations in Australia</td>
<td>56</td>
</tr>
<tr>
<td>3.2 Block mean differences between satellite altimetric gravity and marine gravity on a 10'x10' grid</td>
<td>66</td>
</tr>
<tr>
<td>3.3 Station distribution of 59 AFN/ANN sites with optical levelling results on the AHD and the inferred geometric geoid</td>
<td>68</td>
</tr>
<tr>
<td>3.4 Distribution of the GPS/levelling stations in (a) Victoria, (b) Western Australia, (c) Australian Capital Territory</td>
<td>72</td>
</tr>
<tr>
<td>3.5 The effects of the deflection of the vertical on GPS heighting at a station</td>
<td>75</td>
</tr>
<tr>
<td>3.6 Geoid differences between OSU91A and the 59 levelled AFN/ANN/AHD stations</td>
<td>84</td>
</tr>
<tr>
<td>4.1 Geoid differences between OSU91A and that from 2-D planar FFT with discrete kernel for band 180–360</td>
<td>99</td>
</tr>
<tr>
<td>4.2 Geoid difference between OSU91A and that from 2-D planar FFT with analytical kernel for band 180–360</td>
<td>99</td>
</tr>
<tr>
<td>4.3 Geoid difference between OSU91A and that from 2-D spherical FFT with approximate kernel for band 180–360</td>
<td>100</td>
</tr>
<tr>
<td>4.4 Geoid differences between OSU91A and that from the 1-D spherical FFT with exact kernel for band 180–360</td>
<td>100</td>
</tr>
<tr>
<td>4.5 Features of the residual gravity in band 50-360</td>
<td>103</td>
</tr>
<tr>
<td>4.6 Features of the residual gravity in band 90-360</td>
<td>103</td>
</tr>
<tr>
<td>4.7 Features of the residual gravity in band 180-360</td>
<td>104</td>
</tr>
<tr>
<td>4.8 Four equal sub-areas and their common boundaries</td>
<td>106</td>
</tr>
<tr>
<td>4.9 Misfits of various kernels along profiles φ=-26.0° when (a) the whole data area is used and (b) a 3.0° capsize is used</td>
<td>107</td>
</tr>
<tr>
<td>4.10 Misfits of various kernels along profiles λ= 135° when (a) the whole data area is used and (b) a 3.0° capsize is used</td>
<td>108</td>
</tr>
</tbody>
</table>
List of Figures (continued)

**Figure** | **Page**
--- | ---
5.1 Colour image of the free-air gravity anomaly field of continental Australia | 116
5.2 Colour image of the Bouguer gravity field of continental Australia | 117
5.3 Geographical location of the test areas and profiles | 118
5.4 Power spectral distribution of various gravity anomalies in area A1 | 121
5.5 Power spectral distribution of various gravity anomalies in area A2 | 122
5.6 Power spectral distribution of various gravity anomalies in area A3 | 123
5.7 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P1 | 128
5.8 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P2 | 129
5.9 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P3 | 130
5.10 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies at 25°S, P1 | 133
5.11 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies at 26°S, P2 | 134
5.12 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies at 36°S, P3 | 135
6.1 Distribution of the gravity data in area A1 | 158
6.2 Distribution of the gravity data in area A2 | 158
6.3 Distribution of the gravity data in area A3 | 159
6.4a Contour map of free-air gravity anomalies in area A1 | 161
6.4b Contour map of Bouguer gravity anomalies in area A1 | 161
6.4c Contour map of topographic-isostatic gravity anomalies in area A1 | 162
6.4d Contour map of OSU91A residual free-air gravity anomalies in area A1 | 162
7.1 Colour image of the terrain corrections in Australia | 179
7.2 The indirect effect of the terrain corrections over continental Australia | 180
7.3 Total terrain effects on the geoid in Australia | 181
7.4 Profiles of terrain height, indirect effect, terrain correction and geoidal correction along latitude profiles (1) \( \varphi = -24.5^\circ \), (2) \( \varphi = -34^\circ \) | 182
7.5 Profiles of terrain height, indirect effect, terrain correction and geoidal correction along longitude profiles (1) \( \lambda = 126^\circ \), (2) \( \lambda = 150^\circ \) | 183
7.6 Topography and the terrain corrections at an arbitrary point P | 185
7.7 The domain of the terrain correction integral is divided into three parts \( E_0, E_1 \) and \( E_2 \) | 187
List of Figures (continued)

**Figure**  

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.8</td>
<td>Various error contribution coefficients for $\bar{C}=0.5$ mgal, 1 mgal and 3 mgal respectively</td>
<td>194</td>
</tr>
<tr>
<td>8.1</td>
<td>A flow chart for the optimal implementation of the new Australian geoid</td>
<td>201</td>
</tr>
<tr>
<td>8.2</td>
<td>Colour image of the new gravimetric geoid of Australia</td>
<td>205</td>
</tr>
<tr>
<td>8.3</td>
<td>Differences between the new gravimetric geoid and AUSGEOID93</td>
<td>206</td>
</tr>
<tr>
<td>8.4</td>
<td>Scheme of a low pass filter to separate OSU91A effects in low frequency</td>
<td>209</td>
</tr>
<tr>
<td>8.5</td>
<td>Gravity anomaly differences between actual gravity field and OSU91A in long wavelength (&lt;360)</td>
<td>210</td>
</tr>
<tr>
<td>8.6</td>
<td>Geoid undulation differences between actual gravity field and OSU91A in long wavelength (&lt;360)</td>
<td>211</td>
</tr>
<tr>
<td>8.7</td>
<td>STD differences between FFT geoid and GPS/levelling results using different cap sizes both with (+SA) and without the inclusion of satellite altimetry data</td>
<td>214</td>
</tr>
<tr>
<td>8.8</td>
<td>Estimates of relative differences versus baseline lengths between the new geoid and GPS/AHD in (a) WA, (b) Victoria, (C) ACT</td>
<td>223</td>
</tr>
</tbody>
</table>
LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Statistical comparison of gravity anomaly differences between satellite altimetry and marine observations for different block sizes</td>
<td>65</td>
</tr>
<tr>
<td>3.2 Accuracy estimation of the GPS ellipsoidal height differences over different GPS networks</td>
<td>71</td>
</tr>
<tr>
<td>3.3 The separation between the AHD and geoid derived from levelling and oceanographic data</td>
<td>73</td>
</tr>
<tr>
<td>3.4 List of global geopotential models currently available for the computation of the Australian geoid</td>
<td>77</td>
</tr>
<tr>
<td>3.5 Statistics of the &quot;ground truth&quot; data used in this study</td>
<td>78</td>
</tr>
<tr>
<td>3.6 Statistical comparisons of geoid heights between geopotential models and geometrical geoid heights at the 59 AFN/ANN/AHD stations</td>
<td>79</td>
</tr>
<tr>
<td>3.7 Statistical comparisons of geoid heights between geopotential models and geometrical geoid heights at the 18 Victorian GPS/AHD stations</td>
<td>80</td>
</tr>
<tr>
<td>3.8 Statistical comparisons of geoid heights between geopotential models and geometrical geoid heights at the 21 Western Australian GPS/AHD stations</td>
<td>80</td>
</tr>
<tr>
<td>3.9 Statistical comparisons of geoid heights between geopotential models and geometrical geoid heights at the 86 Australian Capital Territory GPS/AHD stations</td>
<td>80</td>
</tr>
<tr>
<td>3.10 Statistical comparisons between high degree geopotential models and free-air gravity anomalies on the Australian continent</td>
<td>82</td>
</tr>
<tr>
<td>3.11 Statistical comparisons between high degree geopotential models and marine gravity anomalies</td>
<td>82</td>
</tr>
<tr>
<td>3.12 Statistical comparisons between high degree geopotential models and satellite-altimetry-derived gravity anomalies</td>
<td>83</td>
</tr>
<tr>
<td>4.1 Statistics of the &quot;ground truth&quot; data in three bands</td>
<td>97</td>
</tr>
<tr>
<td>4.2 Differences between the GGM and FFT geoids using different approximations of Stokes's kernel (band 50-360)</td>
<td>98</td>
</tr>
<tr>
<td>4.3 Differences between the GGM and FFT geoids using different approximations of Stokes's kernel (band 90-360)</td>
<td>98</td>
</tr>
<tr>
<td>4.4 Differences between the GGM and FFT geoids using different approximations of Stokes's kernel (band 180-360)</td>
<td>98</td>
</tr>
<tr>
<td>4.5 Differences of residual FFT geoids using various approximations of Stokes's kernel with 100% zero padding and a 5° boundary excluded in band 1</td>
<td>101</td>
</tr>
<tr>
<td>4.6 Differences of residual FFT geoids using various approximations of Stokes's kernel with 100% zero padding and a 5° boundary excluded in band 2</td>
<td>102</td>
</tr>
<tr>
<td>4.7 Differences of residual FFT geoids using various approximations of Stokes's kernel with 100% zero padding and a 5° boundary excluded in band 3</td>
<td>102</td>
</tr>
</tbody>
</table>
List of Tables (continued)

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.8 Differences of residual FFT geoids using different approximations of Stokes's kernel without zero padding (band 1; 50-360)</td>
<td>105</td>
</tr>
<tr>
<td>4.9 Differences of residual FFT geoids using different approximations of Stokes's kernel without zero padding (band 2; 90-360)</td>
<td>105</td>
</tr>
<tr>
<td>4.10 Differences of residual FFT geoids using different approximations of Stokes's kernel without zero padding (band 3; 180-360)</td>
<td>105</td>
</tr>
<tr>
<td>5.1 Statistics of the free-air and Bouguer anomalies over the whole continent</td>
<td>115</td>
</tr>
<tr>
<td>5.2 Location of the three test areas and the three profiles</td>
<td>118</td>
</tr>
<tr>
<td>5.3 The statistics of various gravity anomaly types in the Hamersley Ranges, Western Australia</td>
<td>119</td>
</tr>
<tr>
<td>5.4 The statistics of various gravity anomaly types in Central Australia</td>
<td>119</td>
</tr>
<tr>
<td>5.5 The statistics of various gravity anomaly types in the Snowy Mountains, Victoria</td>
<td>119</td>
</tr>
<tr>
<td>5.6 Hurst fractals of various gravity anomalies in area A1</td>
<td>125</td>
</tr>
<tr>
<td>5.7 Hurst fractals of various gravity anomalies in area A2</td>
<td>126</td>
</tr>
<tr>
<td>5.8 Hurst fractals of various gravity anomalies in area A3</td>
<td>126</td>
</tr>
<tr>
<td>5.9 Profile statistics in test area A1</td>
<td>131</td>
</tr>
<tr>
<td>5.10 Profile statistics in test area A2</td>
<td>131</td>
</tr>
<tr>
<td>5.11 Profile statistics in test area A3</td>
<td>132</td>
</tr>
<tr>
<td>5.12 Hurst fractals of various gravity anomalies along P1</td>
<td>136</td>
</tr>
<tr>
<td>5.13 Hurst fractals of various gravity anomalies along P2</td>
<td>136</td>
</tr>
<tr>
<td>5.14 Hurst fractals of various gravity anomalies along P3</td>
<td>136</td>
</tr>
<tr>
<td>6.1 A descriptive comparison of the four gridding methods</td>
<td>154</td>
</tr>
<tr>
<td>6.2 Number of data points used for evaluation in the three areas</td>
<td>157</td>
</tr>
<tr>
<td>6.3 The statistics of the various gravity anomaly data used in the three areas</td>
<td>160</td>
</tr>
<tr>
<td>6.4 Reference surface comparisons of smoothness versus roughness</td>
<td>163</td>
</tr>
<tr>
<td>6.5a Accuracy comparisons of various gridding procedures in area A1 based on free-air anomalies</td>
<td>163</td>
</tr>
<tr>
<td>6.5b Accuracy comparisons of various gridding procedures in area A1 based on Bouguer anomalies</td>
<td>164</td>
</tr>
<tr>
<td>6.5c Accuracy comparisons of various gridding procedures in area A1 based on topographic-isostatic gravity anomalies</td>
<td>164</td>
</tr>
<tr>
<td>6.5d Accuracy comparisons of various gridding procedures in area A1 based on residual free-air gravity anomalies relative to OSU91A</td>
<td>164</td>
</tr>
</tbody>
</table>
List of Tables (continued)

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.6a Accuracy comparisons of various gridding procedures in area A2 based on free-air anomalies</td>
<td>165</td>
</tr>
<tr>
<td>6.6b Accuracy comparisons of various gridding procedures in area A2 based on Bouguer anomalies</td>
<td>165</td>
</tr>
<tr>
<td>6.6c Accuracy comparisons of various gridding procedures in area A2 based on topographic-isostatic gravity anomalies</td>
<td>165</td>
</tr>
<tr>
<td>6.6d Accuracy comparisons of various gridding procedures in area A2 based on residual free-air gravity anomalies relative to OSU91A</td>
<td>165</td>
</tr>
<tr>
<td>6.7a Accuracy comparisons of various gridding procedures in area A3 based on free-air anomalies</td>
<td>166</td>
</tr>
<tr>
<td>6.7b Accuracy comparisons of various gridding procedures in area A3 based on Bouguer anomalies</td>
<td>166</td>
</tr>
<tr>
<td>6.7c Accuracy comparisons of various gridding procedures in area A3 based on topographic-isostatic gravity anomalies</td>
<td>166</td>
</tr>
<tr>
<td>6.7d Accuracy comparisons of various gridding procedures in area A3 based on residual free-air gravity anomalies relative to OSU91A</td>
<td>166</td>
</tr>
<tr>
<td>6.8 A summary of the standard deviation results for Areas 1, 2 and 3</td>
<td>167</td>
</tr>
<tr>
<td>6.9 Computer time used for the gridding using different methods</td>
<td>168</td>
</tr>
<tr>
<td>7.1 Statistics of the terrain corrections, direct and indirect geoidal effects, total terrain effects and the differences between geoid and height anomaly for the whole Australia</td>
<td>176</td>
</tr>
<tr>
<td>7.2 Statistics of $C, S[C], \delta N_{\text{bias}}, N_T$ and $\Delta$ for the Western part of Australia</td>
<td>176</td>
</tr>
<tr>
<td>7.3 Statistics of $C, S[C], \delta N_{\text{bias}}, N_T$ and $\Delta$ for the Eastern part of Australia</td>
<td>176</td>
</tr>
<tr>
<td>7.4 Statistics of $C, S[C], \delta N_{\text{bias}}, N_T$ and $\Delta$ for Tasmania</td>
<td>176</td>
</tr>
<tr>
<td>7.5 Examples of the maximum error coefficients based on different effective domains of integration</td>
<td>193</td>
</tr>
<tr>
<td>7.6 RMS error estimation of the terrain correction caused by various errors</td>
<td>193</td>
</tr>
<tr>
<td>7.7 Accuracy estimation of the terrain correction with varying accuracy of computation point and accuracy of DTM</td>
<td>196</td>
</tr>
<tr>
<td>8.1 Geoid undulation differences between the new FFT geoid and AFN/ANN/AHD at 35 coastal stations both with and without the satellite altimetry data</td>
<td>216</td>
</tr>
<tr>
<td>8.2 Differences between the gravimetric and geometric geoid using different grid sizes of the gravity anomalies</td>
<td>217</td>
</tr>
<tr>
<td>8.3 Self-consistency among the geoids produced from different grid sizes</td>
<td>218</td>
</tr>
<tr>
<td>8.4 Statistical results of geoid differences at 59 AFN/ANN stations</td>
<td>219</td>
</tr>
</tbody>
</table>
List of Tables (continued)

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.5 Statistical results of geoid differences at 86 Australian Capital Territory GPS/levelling stations</td>
<td>219</td>
</tr>
<tr>
<td>8.6 Statistical results of geoid differences at 18 Victorian GPS/levelling stations</td>
<td>220</td>
</tr>
<tr>
<td>8.7 Statistical results of geoid differences at 21 Western Australian GPS/levelling stations</td>
<td>220</td>
</tr>
<tr>
<td>8.8 Standard deviation of the discrepancies between the optimal gravimetric geoid and GPS/levelling geoid before and after four parameter fitting</td>
<td>221</td>
</tr>
<tr>
<td>8.9 Precision estimation of the best FFT geoid from different GPS/AHD networks</td>
<td>224</td>
</tr>
<tr>
<td>I-1 Various geoids and terrain effects in 59 AFN/ANN GPS/levelling stations</td>
<td>257</td>
</tr>
<tr>
<td>I-2 Various geoids and terrain effects in 21 WA GPS/levelling stations</td>
<td>258</td>
</tr>
<tr>
<td>I-3 Various geoids and terrain effects in 86 ACT GPS/levelling stations</td>
<td>259</td>
</tr>
<tr>
<td>I-4 Various geoids and terrain effects in 18 Victoria GPS/levelling stations</td>
<td>261</td>
</tr>
</tbody>
</table>
AN EVALUATION OF FFT GEOID DETERMINATION TECHNIQUES AND THEIR APPLICATION TO HEIGHT DETERMINATION USING GPS IN AUSTRALIA

CHAPTER ONE

INTRODUCTION
Chapter 1

INTRODUCTION

1.1 Background

1.1.1 Definition and Applications of the Geoid

The geoid, a level surface of the gravity field which is most close to idealised mean sea-level, is used to express the dimensions of the Earth in geodesy. The geoid also plays an important role in many other geosciences. It is a fundamental reference surface for surveying and mapping from which the elevation of a point is measured. In oceanography, the geoid can be used to determine sea surface topography and the moving features of sea currents (e.g. Englis, 1985; Nerem and Kobinsky, 1994). The offset of the dynamic ocean surface from the geoid is the signal which bears important information about the ocean circulation patterns. In geophysics, the geoid is inverted to study the internal properties and dynamics of the Earth such as crustal density anomalies and Moho depth, and estimate mantle convection (Vanícek and Christou, 1994; Arnalviet and Boavida, 1993; Chase, 1985; Christou et al., 1989).

Triggered by the development of these scientific branches, the precision of the geoid has been greatly improved over the world in recent decades. Examples on a continental scale include the USA (Milbert, 1993, 1995), Canada (Vanícek and Kleusberg, 1987; Vanícek et al., 1995; Sideris and She, 1995), China (Jiang and Guan, 1986; Shi, 1992; Chen, 1996) and Europe (Denker and Torge, 1993; Denker et al., 1994; 1995). Nevertheless, the precision of the modern geoid in some areas leaves much to be desired. It still does not offer sufficient accuracy for all applications. For example, nowadays, the Global Positioning System (GPS) has found broad applications as it can quickly, easily and accurately determine the ellipsoidal height of a point. However, if the geoid undulation is not known precisely, the elevation of the point in the local system can not be derived accurately (see Figure 2.3). Consequently, the combination of a precise geoid with GPS measurements can provide a cost-effective alternative to geodetic levelling. An improved model of the geoid will allow the surveyor to use GPS to its full capacity. However, as stated above, geodetic levelling by GPS is not the only application of the geoid. A broad
range of geodetic, precise engineering, geophysical, and oceanographical applications exist, rendering the need for precise geoid determination methods more pressing than ever (Sideris, 1993).

1.1.2 Spectral Geoid Determination
The precise determination of local geoid undulations by gravimetric means has been a dominant research topic of geodesy in the last ten years. Significant developments have taken place in both the theoretical aspects and the practical improvement of the geoid's precision. The fast Fourier transformation (FFT) method was introduced to compute the geoid and terrain corrections based on a flat Earth model (Forsberg, 1984; 1985). The FFT method is, in practice, always used for a residual field with a harmonic reference field subtracted (e.g. Schwarz et al., 1990). Until Strang van Hees (1990) presented the idea of the spherical FFT, the FFT was always applied in a flat Earth surface approximation, which needs many assumptions (Forsberg and Sideris, 1993). Strang van Hees (ibid.) developed a new approach where the integral is evaluated on the sphere, not on a plane, with only a few approximations. The accuracy of the method was further improved through a "multi-band" solution (Forsberg and Sideris, 1993), allowing virtually rigorous FFT solutions on the sphere in terms of avoiding the planar approximation. Haagmans et al. (1993) further developed this method so that no approximations are necessary at all, much computer memory is saved, and exactly the same result is obtained as would be from straightforward numerical integration.

This one-dimensional FFT method is now in wide use for geoid computation (e.g. Denker et al., 1994; Sideris and She, 1995). However, the periodicity and windowing errors, inherent to all FFT frequency domain convolution approaches, remain in the method, although recent studies show that proper use of zero-padding can relieve this to a certain extent (Sideris and Li, 1993; Tziavos, 1993a; 1996). In addition to the high speed of the FFT method, a significant advantage is that "the effective cap size" (data coverage taken into account) can be dramatically increased compared to conventional methods.
Applications of the FFT method to geoid determination have been illustrated by Denker and Wenzel (1987), who reported cm-geoid results in an area of Northern Germany; by Milbert (1992; 1993; 1995) who computed a 3'x3' FFT geoid covering the entire USA. Another very powerful method to treat a great amount of gravity field data is the Fast Hartley transformation technique, which is more efficient than FFT in both computation speed and memory required. This method has been tested by several authors (Li and Sideris, 1992; Tziavos, 1993b; Li et al., 1995).

1.1.3 Other Considerations
The problem of singular integration, which results from the fact that Stokes's and the terrain effect kernels are infinite at the computation point, have been studied by several authors. The singularity of the Stokes's integral is removed completely through a co-ordinate transformation, as studied by Mather (1973) and Kearsley (1984; 1985), which leads to the so-called 'ring integration' technique (Kearsley, 1986a). The singular integration in both the frequency and space domains can be removed or relieved through a co-ordinate transformation of the integral (Bian, 1992; Bian and Dong, 1991; Klose and Ilk, 1993; Zhang et al., 1995) or through the addition of a constant in the denominator (Sideris, 1985; Schwarz et al., 1990). These developments have improved the accuracy of the numerical integration of the gravity field close to the computation point.

In the modern geodesy community, Stokes's or Molodensky's methods have been modified in form and divided in two parts for the practical computation of the geoid. One is the long and medium wavelength part, which is contributed mostly by satellite information. Another is the short wavelength part, which is contributed by local gravity observations and terrain information. Many authors, such as Sjöberg (1984, 1991; 1994a), Wenzel (1982), Vanícek and Sjöberg (1991), Featherstone (1992), Meissl (1971) and Wong and Gore (1969), have made contributions to these issues to reduce truncation errors or based on different error budget considerations.

Another promising development for geoid computation is the least squares collocation technique (Krarup, 1969; Moritz, 1989) where a variance-covariance matrix is used to combine heterogeneous data. Due to its limitation on computer memory and speed,
and the assumption of the gravity field to be a \textit{stationary process}, this method is not widely used. However, recent development of the fast collocation technique has made this method more applicable (Bottoni and Barzaghi, 1993). Practically equivalent results to the FFT and FHT have been obtained in Europe by Barzaghi \textit{et al.} (1993).

As for practical implementations of the geoid, classical Stokes-like integration has been considered by Kearsley (1986b, 1988a, 1988b), Stewart (1990), Featherstone (1992), Zhang (1990a) and Bian and Zhang (1993), among others. The use of splines for the local representation of the gravity field has been treated in great detail by van Gysen (1994), Bian (1992) and Bian and Zhang (1991b). The integrated geodesy approach (Hein, 1985) has been used with success to model the geoid in areas of very rugged topography by Milbert and Dewhurst (1992).

1.1.4 Global Geoid Models

New developments and advances in gravity field determination from artificial satellites have taken place in the past few decades. The gravitational potential determined from satellite orbit analyses is now of a sufficient quality to give a homogeneous picture of long wavelength geoid components. Satellite-only models in the 1980s have evolved to current models based on a combination of satellite tracking, satellite altimeter, and surface gravity data. The long wavelength global geoid has been updated rapidly, such as: the OSU series from OSU81 (Rapp, 1981a), OSU86E/F (Rapp and Cruz, 1986), OSU89A/B (Rapp and Pavlis, 1990) to OSU91A (Rapp \textit{et al.}, 1991); the GFZ series from GFZ93A/B (Gruber and Anzenhofer, 1993) to GFZ95A (Gruber \textit{et al.}, 1995), and EGM96 (Rapp and Nerem, 1994; Pavlis \textit{et al.}, 1996; Pavlis, 1996). Thus, these combined models (satellite-only models supplemented by terrestrial data) represent the gravity field more and more precisely at medium wavelengths.

The investigation of spherical harmonic tailoring was introduced by Wenzel (1985) and numerically tested by Weber and Zomorrodian (1988), Basic \textit{et al.}, (1989); Li (1993) and Kearsley and Forsberg (1990). The idea of this method is to improve the precision of existing geopotential models when more regional or local gravity data become available. The coefficients of the original model are corrected through an
integration process over the given region. To obtain a good regional application, some tailored geopotential models were developed in Europe (Basic et al., 1989), Australia (Kearsley and Forsberg, 1990) and Canada (Li, 1993). The amount of success usually depends on the density and accuracy of local gravity anomalies available.

Regional gravimetric geoid determinations now use global models, terrestrial gravity observations and the information given by digital terrain models as a standard data input. Today, theoretical models, numerical procedures and data coverage appear to allow us to reach regional "cm" to "dm" level geoid accuracy, especially if combination strategies with satellite altimetry and GPS/levelling control points are applied (Torge, 1990; Vanícek and Christou, 1994; Vermeer, 1996; Forsberg, 1996).

The impact of the GPS on geodetic control surveys has been immense. Most importantly for geoid determination, more and more discrete precise relative geoid undulations can be obtained through the combination of GPS and spirit levelling. The GPS/levelling geoid can be regarded as a different approach to obtain high precise datum control and transformation of the local geoid, and reference of the integrity of various regional geoids (Rapp, 1994a; 1994b; Rapp and Balasubramania, 1992).

1.1.5 Above Ground Techniques

Of importance for most gravity field studies is the success of satellite altimetry. The GEOS-3, SEASAT, and GEOSAT missions have produced increasingly accurate marine geoids with an unprecedented global coverage. The TOPEX/Poseidon (TOPEX/Poseidon Joint Verification Team, 1992; Arabelos and Tziavos, 1994; Nerem et al., 1993) and ERS-1 missions have given further improvements in resolution, accuracy and coverage (Schwintzer et al., 1995). Arabelos and Tziavos (ibid.) recently reported that the standard deviations of the differences between observed and predicted gravity anomalies and geoid undulations were ±5.8 mgal and ±0.09 m respectively in the Mediterranean sea, where ERS-1 and TOPEX/Poseidon data have been combined. Similar (or even higher accuracy) results were reported by Marks et al. (1993), Sandwell et al. (1995), Knudsen et al. (1992), Knudsen and Andersen (1997), Sandwell and Smith (1997) and Marks (1996). Although not all the problems with satellite altimetry have been solved in many parts of the world as yet,
the altimetrically determined marine geoid is known more accurately than the land (or marine) geoid derived from observed gravity anomalies. The increase in altimetric missions makes it possible to present a more detailed gravity field, especially in ocean areas. This will be of benefit to the local gravity field approximation both in offshore and land areas.

Even more promising for the near future is a number of satellite gradiometry developments (Wells, 1984). The gradiometers are sensitive to combinations of the components of the gravity gradient matrix, or the Eötvös-tensor (Sjöberg, 1994a; Arabelos and Tscherning, 1990; Bernard and Touboul, 1989). The gradiometers are expected to measure spectral components of the gravity field at satellite altitude to considerably higher degrees than can be achieved by any other satellite methods (Colombo, 1989; Schrama, 1991). A six-month mission at a satellite altitude of 200 km is expected to yield a global gravity field determination of better than 5 mgal with a spatial resolution of 100 km. Such a type of instrumentation, named GRADIO (Balmino et al., 1984) was planned to be included in ESA’s ARISTOTELES mission (Benz et al., 1989; Visser et al., 1994). Recently, it was proposed that GRADIO should be combined with an onboard GPS frequency standard. While the gradiometer mainly contributes the medium and short wavelength gravity field, the GPS system is expected to significantly contribute to the long wavelength gravity field recovery (Schrama, 1991; Colombo, 1989) because of the improvement in satellite orbital tracking. However, as yet, no dedicated satellite gradiometer mission has been deployed.

Recent results of airborne gravity surveys in Greenland, Antarctica and Switzerland show airborne graviometry at a wavelength of 20 kilometres and with a root mean square accuracy of 3-5 mgal is operational today (Brozena and Peters, 1994; Forsberg and Kenyon, 1994; Schwarz and Wei, 1995). These data can be used to improve the geoid in inaccessible areas, such as mountainous or coastal areas. Tests on the accuracy of the geoid from airborne gravity are given by Schwarz and Li (1996).

1.2 History of the Australian Geoid

1.2.1 Australian Geoids Before 1980

In the late 1960s, the first Australian geoid was determined by Fischer and Slutsky (1967) from astrogeodetic deflections of the vertical at about 600 stations over
Australia. This geoid was relative to the Australian National Spheroid, and used primarily to enable reductions of mean sea level distance to this spheroid. This geoid has a resolution of approximately 55km and the estimated accuracy of the geoid height was about six metres (Kearsley and Govind, 1991).

The first gravimetric geoid for Australia was computed by Mather (1969) using free-air gravity anomalies in Stokes's integral. The data set of anomalies used to represent the outer zones was obtained previously using a combined solution from satellite data and terrestrial gravimetry. The standard deviation of the geoid between the Mather geoid and that of Fischer and Slutsky was estimated to be ±3 metres (Mather, 1969).

In the early 1970s, a map of the geoid in Australia and surrounding oceanic areas was produced on a scale of 1:5,000,000. This map employed Bouguer anomalies for the land area, the precision being estimated at about ±4 metres compared with that of Mather's geoid (Grushinsky and Sazhina, 1971).

Later, Fryer (1971; 1972) produced a geoid based on the Australian Geodetic Datum 1966 by the method of astro-geodetic levelling, but used about 1150 astro-geodetic deflection values with a half degree grid of deflection values interpolated using gravimetrically-determined geoid height values.

In the early 1980s, a gravimetric geoid (JSA82) for South-Asia and the Pacific was given by Allman (1982) and Allman and Veenstra (1984). This geoid was computed based on the World Geodetic System 1972 (WGS72) ellipsoid. It is a combination solution of the GEM10C 180 degree and order geopotential model and 602 Doppler/levelling stations (235 in Australia). The regional biases of this Doppler/levelling corrected geoid are obvious and range between -10 and +10 metres (Allman, 1982).

1.2.2 The Australian Geoid 1991 (AUSGEOID91)

Gilliland (1982; 1983; 1989) and Kearsley (1984; 1985; 1986a; 1986b; 1988b) independently computed the Australian geoid based on geopotential coefficients and gravity observations. The methods used by Kearsley were subsequently used to
produce AUSGEOID91, which was the first continent-wide gravimetric geoid routinely used by surveying authorities in Australia. These geoid undulation values were in terms of WGS84 and were computed on a 10'×10' grid using (Steed and Holtznagel, 1994):

- the OSU89A global geopotential coefficients (Rapp and Pavlis, 1990),
- the 1980 Australian gravity data base from the Australian Geological Survey Organisation (AGSO).

The precision of AUSGEOID91 varies between 2 and 5 ppm of baseline length in most areas and discrepancies and biases of 5-10 ppm are encountered in areas of rugged topography (Featherstone et al., 1997). The accuracy of AUSGEOID91 is approximately 1-2 metres root mean square error (Kearsley and Govind, 1991).

1.2.3 The Australian Geoid 1993 (AUSGEOID93)
A new continent-wide gravimetric geoid over Australia, called AUSGEOID93, was produced by the Australian Surveying and Land Information Group (AUSLIG) in 1993 using the ring integration software developed by Kearsley (1988b). The improvements over AUSGEOID91 to produce AUSGEOID93 are (Steed and Holtznagel, 1994):

- inclusion of the more recent OSU91A geopotential model (Rapp et al., 1991),
- an extended geoid grid,
- refined computation processes.

However, it still uses the 1980 AGSO gravity data.

The absolute accuracy of AUSGEOID93, which is obtained through a comparison with 41 Australian National Network and Fiducial Network sites (Morgan and Manning, 1992), is about 0.4–0.5m (Pearse et al., 1994). Its relative accuracy, compared with that from GPS height differences, is estimated at between ±2–5 ppm of baseline length (or equivalent to third-order levelling) in many instances (Kearsley and Govind, 1991; Steed and Holtznagel, 1994; Featherstone and Alexander, 1996).
1.3 Expected Problems with AUSGEOID93

AUSGEOID93 has been proven superior to all previous geoids in Western Australia by Featherstone and Alexander (1996). A number of advances were made with AUSGEOID93, but the precision is not as high as it could be. The main problems and deficiencies of the AUSGEOID93 are expected to be due to:

- AUSGEOID93 uses the 1980 release of gravity data. Nearly 250,000 new terrestrial gravity data included in the 1992 release were not used;
- AUSGEOID93 is based on OSU91A (Rapp et al., 1991) which was the most up-to-date geopotential model available at that time. Since then, a number of new geopotential models, such as GFZ93A/B (Gruber and Anzenhofer, 1993), GFZ95A (Gruber et al., 1995) and EGM96 (Pavlis, 1996), have been published;
- AUSGEOID93 does not explicitly include digital terrain information (DTM). No Australian-wide DTM was available for the computation of AUSGEOID93. A DTM is believed to be crucial for a gravimetric geoid determination. About six million spot heights are available from AUSLIG and AGSO which correspond to a mean resolution of ~1.1 km;
- Satellite altimetric information was not used offshore Australia;
- Currently, for high accuracy scientific and engineering applications, the precision of AUSGEOID93 is not sufficient, especially in mountainous regions. In high and rugged terrain areas, systematic discrepancies of 10 ppm can often be encountered (Featherstone et al., 1997); and
- Some theoretical and practical problems such as terrain effects, the local spectral features of the gravity field and geoid, as well as appropriate error evaluation, have not been properly solved in Australia.

1.4 Aims and Objectives of this Research

This research will endeavour to develop an optimal method to produce the most accurate gravimetric geoid which is up-to-date for Australia, both in theory and practice. The new geoid may provide an alternative and cost-effective method to geodetic levelling and for many scientific applications, such as geodetic positioning, geodynamics and geophysical exploration. The aim of this research is to enable the
production of a geoid with relative precision of 10cm-20cm, with absolute precision higher than existing geoid and resolution of a few kilometres. With this precision and accuracy, the new geoid is expected to support third order GPS heighting. This is to be verified to some extent using GPS and spirit levelling data.

The aims and objectives of this research are:

- To use the 1992 release of the Australian gravity database, which comprises 637,287 points in computation of a gravimetric geoid;
- To choose the optimal geopotential model to be adopted to give the best reference gravity field;
- To use refined gravity reduction procedures;
- To evaluate the terrain effects to gravimetric geoid determination in Australian context. A detailed DTM will be used for the computation of terrain correction and its associated direct and indirect effects in order to study its effects on the geoid in Australia;
- To study the features of Australian topography and gravity field using alternative techniques, such as power spectral and fractal dimension analyses;
- To select the optimal gridding procedure to be applied to grid gravity anomalies before geoid computation;
- To evaluate and combine satellite altimetry-derived gravity anomalies with the Australian gravity data base to improve the quantity and coverage of the gravity field offshore Australia;
- To apply the FFT algorithm to both geoid and terrain correction computations to effectively and efficiently manipulate the large data sets in Australia.
- To evaluate the precision and accuracy of the new geoid using nation-wide and local GPS/levelling networks.
- To investigate some relevant theoretical and practical problems to give the state-of-the-art Australian geoid.

1.5 Outline of the Research

This research is documented in nine chapters. This, the first chapter, presents an introduction to the whole research project, its background and objectives. Chapters Two to Nine will be briefly described hereafter.
Chapter Two presents the theoretical basis of the geoid and reviews the methodologies for the determination of a gravimetric geoid. These include theory of the conventional Stokes and Molodensky methods, their relationships, existing problems and deficiencies and a discussion to remedy them. Chapter Two also gives the theory and methodology for terrain reductions and the principles of GPS heighting. A generalised remove-restore technique is presented for the inclusion of global geopotential model and terrain data.

Chapter Three describes all the data available for this study and discusses their validation, refinement, evaluation, pre-processing and combination. These data include: the geometric geoid from precise GPS and spirit levelling results; gravity measurements made on land and in marine areas; satellite altimetry-derived gravity anomalies around the Australian coastline; spot heights; and, global geopotential models. The GPS related errors for height difference determination are analysed and an estimation of the precision of the geometric geoid is given.

In addition, Chapter Three discusses some special problems that affect an accurate geoid determination. These include the unification of the Australian Height Datum (AHD); time variations of the gravity field and definition of the geoid in four-dimensional space; the effects of varying topographic density and the differences between normal and actual gravity gradients; and the effect of deflection of the vertical on GPS/levelling.

To accurately determine the gravimetric geoid of Australia, the data availability, their validation, refinement and preparation are crucial, since the quality of the data will directly affect the quality of subsequent geoid determination. Any potential physical and geometrical systematic and gross errors are removed at this stage. Also, the satellite altimetry-derived gravity anomalies are compared with marine gravity data. The possibility and the procedures to combine satellite altimetric information with local gravity data are studied to give a homogeneous data source for subsequent geoid determination. The selection and verification of an optimal geopotential model is also studied to give the best local reference field through comparisons with the geometric geoid, and land, marine and satellite altimetry-derived gravity anomalies.
Chapter Four briefly presents the principles of the Fourier transform and its practical applications to gravity field convolution integrals. The formulae for the computation of geoid and terrain reductions using the FFT technique are summarised. The practical implementation of various approximations of Stokes's kernel using FFT techniques are discussed. The four kinds of approximate kernels are: the simple planar kernel approximation, with both analytical and discrete spectra; the two-dimensional planar approximate kernel; and, the one-dimensional spherical exact kernel. Some problems related to the application of FFT to gravity field convolution integrals, such as selection of data coverage, the effect of zero padding, edge effects, and the consistency of the method itself, are discussed and studied numerically. Numerical simulations on geoid determination by FFT are presented and suggestions regarding applications of the FFT to the Australian geoid are proposed. An optimal procedure for the determination of the Australian gravity field by FFT technique is suggested.

Chapter Five investigates the behaviour of the Australian gravity field. Three test areas are selected which represent different gravity field and topographic features. Apart from statistical comparisons and profile analyses, Fourier power spectra (Champeney, 1973, Brigham, 1988) and Hurst fractal dimension analyses (Russ, 1994) are applied. Fourier spectral analysis and fractal geometry provide a new paradigm for understanding many geophysical phenomena, such as detailed structures and features of the Earth's gravity field and the complex nature of the dynamic Earth.

Chapter Six discusses the gridding theory and methodology to find an optimal procedure for the Australian gravity data. Many numerical procedures in the Earth sciences require data on a regularly spaced grid, especially in the computation of geoid undulations and gravimetric terrain effects by the FFT. In contrast, however, most data are acquired at individual observation points or along traverses, both on land and in marine areas. It is, therefore, necessary to construct estimates of the value of a function on a grid (or its mean value), given observations of the value of the function at arbitrary locations in a plane or in three-dimensional space. The methods used in this study are the moving weighted average, Kriging, minimum curvature
splines and polynomial gridding methods. Three test areas are chosen for this study which represent typical gravity field structure. The free-air, Bouguer and topographic-isostatic gravity anomalies both with and without the OSU91A 360 degree and order field removed are chosen as "platforms" for this study. Firstly, the principles of these methods are briefly described. Then, the ability of each gridding method to recover gravity anomaly at known points in three areas are compared to determine the most suitable gravity anomaly type and method. This testing procedure is a critical issue for subsequent gravity field refinement because any gridding error will directly propagate into the gravimetric geoid determination.

Chapter Seven investigates the gravimetric terrain correction, its effects on geoid and height anomalies, and quantifies the effect of terrain in the computation of the Australian gravimetric geoid. Special studies are given to the topographic effects and its associated indirect effects due to the second Helmert condensation method. The fast Fourier transformation method is applied to both the terrain correction and its indirect effect for computational efficiency over a large area such as Australia. This study is particularly important because all the previous geoids, such as AUSGEOID91 and AUSGEOID93, do not include terrain information in this way. Apart from the numerical tests and practical implementation of the terrain correction, its theoretical accuracy is also evaluated. A series of formulae are derived for the accuracy evaluation in terms of the three-dimensional positional errors of the terrain height and point of interest and the assumption of uniform density.

Chapter Eight summaries optimal procedures for an accurate geoid determination of Australia. The capacity of the OSU91A model to recover the regional gravity anomaly and geoid undulation in long wavelength (>110km) is studied. The optimal capsice for Stokes's integral is also investigated. Moreover, the improvement of the geoid both with and without the satellite altimetry-derived gravity anomalies is assessed. The best grid size of the gravity anomalies used in the geoid computation is also investigated. The accuracy and precision of the new geoid is evaluated using both nation-wide and local GPS/levelling networks, and its improvement relative to the previous generation AUSGEOID93 is also given.
Finally, the research is summarised and conclusions from the results are drawn in Chapter Nine. A number of recommendations are also made for further research of the next generation gravimetric geoid determination of Australia and for other continental areas.
AN EVALUATION OF FFT GEOID DETERMINATION
TECHNIQUES AND THEIR APPLICATION TO HEIGHT
DETERMINATION USING GPS IN AUSTRALIA

CHAPTER TWO

FUNDAMENTALS OF GEOID DETERMINATION AND GPS HEIGHTING
Chapter 2

FUNDAMENTALS OF GEOID DETERMINATION AND GPS HEIGHTING

2.1 Theory of Gravimetric Geoid Determination

2.1.1 Introduction

Following the definition of the geoid by Gauß (1828) and Listing (1873), orthometric heights are referred to a specified equipotential surface, which intersects everywhere with the direction of gravity at right angles and parts of which coincide with the surface of the oceans. The geoid is a gravity equipotential surface to which the elevation of a point can be conveniently referred. The computation of the geoid is based on the solution of the field equation of gravitation which describes gravitation in-the-small (Graaafrend, 1994), and in which the rotating frame of reference is time-independent to a first order approximation.

\[ \nabla \cdot \nabla [V(x, y, z)] = \nabla^2 V(x, y, z) = -4\pi G \rho(x, y, z), \]

(2.1)

where \( \nabla \cdot \) and \( \nabla \) are the divergence and gradient operators, respectively;

\( V \) is the gravitational potential of the Earth;

\( G = 6.67259 \times 10^{-11} \text{m}^3 \text{s}^{-2} \text{kg}^{-1} \) is the gravitational constant (Cohen and Taylor, 1996);

and

\( \rho(x, y, z) \) is the density function of the Earth's mass.

The source of gravitation is the mass density field \( \rho(x, y, z) \) which is multiplied by the coupling constant \( G \). The Newton volume integral is a special solution of the field equation of gravity (Graaafrend, 1994). According to Newton's law, the Earth's gravity potential \( W \) at a point on or outside Earth's surface is expressed by the formula:

\[ W = V + \Phi = G \iiint_{V} \frac{\rho dv}{r} + \frac{(1+\omega)^2}{2}, \]

(2.2)

where \( \Phi \) is centrifugal potential of the Earth;

\( dv \) is an element of unit volume inside the Earth of total volume \( V \);

\( r \) is the distance from the mass element to a specified point;

\( \omega \) is the angular velocity of the Earth; and
l is the perpendicular distance from the axis of rotation.

The gravitational potential \( V \) is a single valued, finite, continuous and differentiable function everywhere outside the Earth, where it satisfies Laplace's harmonic condition:

\[
\nabla^2 V = 0.
\]

(2.3)

The surfaces of \( W = \text{constant} \) are called equipotential surfaces of gravity or level surfaces. A surface of a liquid which is at rest in a rotating co-ordinate system is, by its very nature, such an equipotential surface. This leads to the concept of the geoid. The gravity vector \( \vec{g} \) is normal to these equipotential surfaces everywhere and is related to the gravity potential by:

\[
\vec{g} = \nabla W.
\]

(2.4)

The specific equipotential surface \( W(x,y,z) = \text{constant} \), coincident with the mean sea level is, according to Listing (1873), defined as the geoid.

An ellipsoid of rotation is frequently accepted as a second-order approximation of the figure of the Earth (geoid). The geoid undulation is defined as the height of the geoid above (positive) or below (negative) the defined reference ellipsoid and is measured along the ellipsoidal normal. This ellipsoid is, generally, called the reference or mean Earth ellipsoid. The size and shape adopted for this ellipsoid define a geodetic reference system, of which many have been defined in the past. In this study, the World Geodetic System 1984 (WGS84) (Defense Mapping Agency, 1987) which is based on the Geodetic Reference System 1980 (GRS80) (Moritz 1980), has been used. The two systems are essentially the same except the slight difference in \( J_2 \) (dynamical form factor) and rounding errors (Schwartz, 1989). All computational formulae and defining constants for the GRS80 reference ellipsoid and normal gravity can be found in Moritz (1980) and those of WGS84 in Defense Mapping Agency (1987).

The difference between the actual gravity potential \( W \) and the potential of the reference ellipsoid \( U \) is known as the anomalous, disturbing or deviation potential \( T \).

\[
T_{r_0} = W_{r_0} - U_{r_0}.
\]

(2.5)
where \( P_0 \) is referred to a point on the geoid (Figure 2.1). The distance \( P_0Q_0 \) in Figure 2.1 is the geoid-ellipsoid separation and is also referred to as the geoid height or geoid undulation \( N \). The lines \( n \) and \( n' \) are normal lines of the reference ellipsoid and the geoid respectively, \( \theta \) is the deflection of the vertical, \( \gamma \) is the normal gravity vector, and \( \gamma_{r_0} \) is the Earth’s gravity vector.

![Figure 2-1 The geoid and reference ellipsoid (after Heiskanen and Moritz, 1967)](image)

2.1.2 Geodetic Boundary Value Problems

The disturbing potential is a particularly useful concept as it is directly related to the geoid undulation by Bruns’s formula (Heiskanen and Moritz, 1967, p.85):

\[
N = \frac{T_{r_0}}{\gamma}.
\]

(2.6)

The relationship between gravity anomaly, the disturbing potential and its derivative has the form of a differential equation (Heiskanen and Moritz, 1967):

\[
\Delta g = g_{r_0} - \gamma_{a_0} = -\frac{\partial T_{r_0}}{\partial n} + \frac{I}{\gamma_{a_0}} \frac{\partial \gamma}{\partial n} T_{r_0}.
\]

(2.7)

This expression is known as the fundamental equation of physical geodesy. It directly relates the measured quantity \( \Delta g \) to the unknown disturbing potential \( T \) and hence the geoid undulation \( N \) via Bruns’s formula.

Equation (2.7) can be expressed as the following spherical approximation:

\[
-\Delta g = \frac{\partial T_{r_0}}{\partial \gamma_{a_0}} + \frac{2T_{r_0}}{R},
\]

(2.8)

where \( R \) is the mean radius of the spherical approximation of the geoid. Since \( \Delta g \) is assumed to be known at every point on the geoid, a linear combination of \( T \) and
\( \frac{\partial T}{\partial n} \) is given on the geoid. Therefore, the determination of \( T \) forms the third boundary value problem of physical geodesy (Heiskanen and Moritz, 1967). Given gravity anomalies on the boundary surface (\( \Gamma \)), the geodetic boundary value problem (BVP) can be expressed as:

\[
\begin{align*}
\nabla^2 \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} &= 0 & \text{outside } \Gamma \\
T_{r_0} = W_{r_0} - U_{r_0} = \gamma a_0 N & \quad \text{on } \Gamma \\
\Delta g = g_{r_0} - \gamma a_0 & \quad \text{on } \Gamma \\
\Delta g_{r_0} + \left( \frac{\partial T_{r_0}}{\partial \gamma a_0} + \frac{2T_{r_0}}{R} \right) &= 0 & \text{on } \Gamma \\
limit_{r \to \infty} T = 0 & \quad r \to \infty .
\end{align*}
\]

Therefore, the geodetic boundary value problem allows the determination of the geoid and the external gravity field of the Earth from the value of the gravity scalar and the gravity potential given on it (Torge, 1991).

2.1.3 Stokes's Method

Solving the Laplace harmonic equation and corresponding boundary conditions in equation (2.9) yields the following Stokes formula (Stokes, 1849; Heiskanen and Moritz, 1967):

\[
N = \frac{T}{\gamma} = \frac{R}{4\pi \gamma} \int_0^\pi S(\psi) \Delta g d\sigma ,
\]

(2.10)

where \( \psi \) is the spherical distance between the running and computation points;

\( R \) is the mean radius of the spherical Earth (geoid);

\( d\sigma \) is the surface integration element;

\( T \) is the disturbing potential;

\( \Delta g \) is the gravity anomaly on the geoid; and

\( S \) is the Stokes's kernel function, which is given by

\[
S(\psi) = \frac{1}{\sin(\psi/2)} - 6\sin \frac{\psi}{2} + I - 5\cos \psi - 3\cos \psi \ln(\sin \frac{\psi}{2} + \sin^2 \frac{\psi}{2} ),
\]

(2.11)

or in the polynomial form:

\[
S(\psi) = \sum_{n=2}^{\infty} \frac{(2n+1)}{(n-1)} P_n(\cos \psi) ,
\]
and where $P_n(\cos \psi)$ are Legendre's polynomials (zonal harmonics). Stokes's formula is based on a spherical approximation of the Earth, causing relative errors of the order $3 \times 10^{-3}$ (Heiskanen and Moritz, 1967) or absolute errors within one metre (Olliver, 1980). Molodensky et al. (1962), Bjerhammar (1962), Mather (1973), Rapp (1981b) and Cruz (1986), for example, have all derived formulae that take the flattening of the reference ellipsoid into account.

In addition to the assumption that the boundary surface of the geoid is a sphere, there exist some implicit conditions in equation (2.10). First, Stokes's formula represents a global integration where gravity observations are required worldwide. Such a scenario is presently not possible. Therefore, the integral is usually performed over a spherical cap of limited radius $\psi_0$, and the outer zones are accounted for by replacing the local field with a global model.

Secondly, the expression for Stokes's function in terms of Legendre polynomials contains no zero- or first-degree terms, thus suppressing these harmonics in the calculated disturbing potential. The zero-degree term represents the global mean difference of the mass and potential between the Earth and reference ellipsoid. The first-degree terms are only zero if the centre of the reference ellipsoid coincides with the mass centre of the Earth (geocentre). Therefore, it is important that the reference ellipsoid is geocentric.

Thirdly, no mass outside the geoid is assumed in order for the disturbing potential to be harmonic. This poses the biggest theoretical and practical obstacle in Stokes's approach to geoid determination. The presence of topographical masses violates the basic assumption behind this solution, which is the harmonicity of the disturbing potential outside the geoid. Therefore, the topographic masses outside the geoid should be removed by suitable gravity reductions (downward continuation). This will be discussed further in section §2.6.

In principle, the reduction steps presuppose that the density distribution of the topography and the gravity gradient from the geoid to the topographic surface are known. The topographic mass-shift necessitates correction for the so-called indirect
effect. Theoretically, after this correction, any reduction method should provide the unique geoid undulation. In practice, however, only approximate estimates of mass densities are used and the free-air reduction of normal gravity is frequently applied, with each method leading to various degrees of erroneous results. Essentially, each method gives a compensated-geoid or co-geoid.

The first serious attempt to remedy this flaw in the application of Stokes's method can be attributed to Helmert (1884), which was subsequently named Helmert's condensation method. Helmert's condensation method is regarded as the most straightforward method (Vanicek and Martinec, 1994) and is used in this thesis. Several terrain reduction methods will be described in §2.6.

2.1.4 Molodensky's Problem
As stated, the main problem with Stokes's integral is that it requires gravity data be reduced to the geoid. This reduction of gravity data to the geoid necessarily requires knowledge of the mass distribution of the topography above the geoid. Practically, this problem can be circumvented by assigning a reasonable constant density to the topographic masses. Theoretically, however, the problem remains. As Molodensky et al. (1962) have shown, the geodetic boundary value problem for the Earth's surface may be solved without this hypothesis.

Molodensky abandoned the concept of the geoid. Instead, he introduced a new surface, called the quasigeoid, in which the geoid undulation is replaced by the height anomaly. Subsequently, the quasigeoid can be determined from surface gravity data, in contrast to geoid determination, which requires inaccurate gravity reduction to sealevel and a knowledge of topographic density (Sjöberg, 1995).

This is a different approach, with which the shape of the Earth can be determined theoretically, given the gravity potential and gravity vector at all points on the Earth's surface. Molodensky applied Stokes's problem to the ground surface by introducing a supplementary surface, called the telluroid Σ (Figure 2.2). The geoid undulation (N) and gravity anomaly (Δg) corresponding to Stokes's problem now become the height
anomaly ($\zeta$) and gravity disturbance ($\delta g$) corresponding to Molodensky's problem at
the Earth's physical surface.

Molodensky's problem requires the solution of a non-linear equation. After
linearisation, the corresponding boundary conditions can be expressed as follows,
where the telluroid ($\Sigma$) becomes the boundary surface.

$$
\begin{align*}
\nabla^2 T &= \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 & \text{outside } \Sigma \\
T_P &= W_p - U_p = \zeta \gamma_0 & \text{on } \Sigma \\
\delta g_P &= \varepsilon_p - \gamma_0 & \text{on } \Sigma \\
\delta g + \left( \frac{\partial T}{\partial r} + \frac{2T}{R} \right) &= 0 & \text{on } \Sigma \\
\lim_{r \to \infty} T_P &= 0 \\
\end{align*}
$$

(2.12)

The point $P$ refers to the Earth's physical surface.

**Molodensky series solution** There are a number of solutions for the Molodensky
BVP, such as those of Brovar (1964) and Pellinen (1972). They are all in a similar
form to the Molodensky series solution. The height anomaly can typically be
expressed as a series (Moritz, 1989, Brovar, 1964; Li et al., 1995) of the form:

$$
\zeta = \zeta_0 + \zeta_1 + \zeta_2 + \cdots,
$$

(2.13)

where

$$
\zeta_n = \frac{R}{4\pi \gamma_0} \int \Delta g_n S(\psi) d\sigma, \quad \forall \ n = 0, 1
$$

(2.14)

$$
\zeta_n = \frac{R}{4\pi \gamma_0} \int \left[ \frac{\Delta g_{n-1} S(\psi) d\sigma - \frac{R^2}{4\pi \gamma_0} \int \frac{(H - H_F)^2}{l_0^2} \Delta g_{n-1} d\sigma} \right] \Delta g_n d\sigma, \quad \forall \ n \geq 2
$$

(2.15)

and $\Delta g_0$ is the gravity anomaly on the telluroid. The other $\Delta g_n$ terms take the
following form:

$$
\Delta g_1 = \frac{R^2}{4\pi} \int \frac{H - H_F}{l_0} \Delta g_0 d\sigma,
$$

(2.16)

$$
\Delta g_2 = \frac{R^2}{4\pi} \int \frac{H - H_F}{l_0^2} \Delta g_1 d\sigma + \Delta g_0 l_0 \beta,
$$

(2.17)
\[ l_s^b = 2R \sin \frac{\psi}{2}, \]  \hspace{1cm} (2.18)

and where \( \beta \) is the total terrain inclination angle. \( H \) and \( H' \) are the heights of the running point and the point of interest respectively.

2.1.5 The Relationship Between Stokes’s Problem and Molodensky’s Problem

If the values of the gravity anomaly and the disturbing potential are given on the geoid, the BVP becomes Stokes’s problem; if they are given on the terrain surface, the BVP becomes Molodensky’s problem. The gravity anomaly and geoid undulation in Stokes’s problem respectively correspond to the gravity disturbance and height anomaly (referred to telluroid) in Molodensky’s problem. The relations among geoid undulation \((N)\), orthometric height \((H)\), ellipsoidal height \((h)\), height anomaly \((\zeta)\), normal height \((H')\) are shown in Figure 2.2.

[Diagram showing the geometry of the Stokes BVP and the Molodensky BVP.]

Figure 2.2 The geometry of the Stokes BVP and the Molodensky BVP (The relationships among geoid height \((N)\), orthometric height \((H)\), height anomaly \((\zeta)\) and normal height \((H')\))
The geoid undulation and height anomaly are related by (Heiskanen and Moritz, 1967):

\[ \zeta - N = \frac{\bar{g} - \bar{y}}{\bar{y}} \frac{\Delta g_B}{H} \]  

(2.19)

where \((\zeta - N)\) is the difference between the height anomaly and the geoid undulation; \(\bar{g}\) and \(\bar{y}\) are mean gravity and mean normal gravity along the plumblines; \(\Delta g_B\) is the Bouguer gravity anomaly; and \(H\) is the height.

The difference between the geoid and the height anomaly has recently been investigated further by Sjöberg (1995) and Li et al. (1995) and is computed for Australia in Chapter Seven using equation (2.19).

2.2 Height Determination

2.2.1 The Global Positioning System and Height Determination

The Navstar Global Positioning System (GPS) is a passive, 24 hour, all-weather, satellite-based radio navigation and positioning system, which is designed to provide precise three-dimensional position and velocity, as well as time information, on a continuous and world-wide basis. This satellite system is deployed and operated by the United States Department of Defense for their real-time navigation and positioning, but is also available for civilian use. GPS achieved its full operational capability on 31 January 1994 when 24 satellites were successfully operating simultaneously.

With the advent of the GPS, a new age has dawned for surveying and navigation. GPS now makes it possible for ground-based receivers to determine relative positions with a degree of precision previously unachievable by traditional surveying methods. For practical applications in surveying and geodesy, an accuracy of better than ±1ppm (or one millimetre for one kilometre of baseline length) can be achieved. For a full description of the components of the Global Positioning System and standard GPS surveying techniques, the reader is referred to Hofmann-Wellenhof et al. (1994) and Leick (1995).
Two aspects of GPS positioning are particularly important for its application to orthometric height determination. First, relative GPS provides only ellipsoidal height differences in the WGS84 system (Hofmann-Wellenhof et al., 1994). Therefore, to compute the orthometric height, GPS must be used in conjunction with a precise geoid that has been computed in an identical reference system (§2.2.4). Second, primarily due to the geometry of the satellite constellation, the weakest aspect of GPS positioning is the height component (Leick, 1995). However, a number of additional factors can significantly affect the accuracy of GPS heighting. These are:

(a) Atmospheric effects on signal propagation

GPS satellite signals propagate from satellite to receiver through the atmosphere, which can cause a delay and refraction of the signals, resulting in erroneous satellite-receiver range measurements. The ionosphere and troposphere are the layers within the Earth's atmosphere that most adversely affect GPS measurements.

i) The ionosphere

The atmospheric layer from about 60km to 1,000km is called the ionosphere and it is composed of free electrons and ions. These charged particles affect the propagation of electromagnetic waves (Budden, 1985). Ionospheric refraction is frequency dependent (dispersive) with the largest effects during the day and in periods of high solar activity (Klobuchar, 1991). Most ionospheric effects are removed using dual-frequency observations to form the so-called ionospheric free observable (e.g. Hofmann-Wellenhof et al., 1994, p.108). However, higher order effects cannot be removed (Brunner and Gu, 1991). Assuming dual frequency observations, the typical absolute range error caused by second order ionospheric effects has been estimated to be as much as 2cm (Bassiri and Hajj, 1993), although, theoretically, relative positioning techniques remove the majority of these effects over baselines of less than 500km-1000km in length.
ii) The troposphere
The tropospheric delay is due to the presence of gaseous molecules and water vapour in the lower atmosphere (<10km altitude). As it is non-dispersive, it can not be eliminated using dual-frequency observations. Instead, it is corrected using empirical atmospheric models, sometimes constrained by local meteorological conditions. Standard tropospheric models, such as the Saastamoinen (1973) model can adequately remove the component of the troposphere attributed to gaseous molecules (dry component). However, tropospheric delay errors caused by the presence of water vapour (wet component) are more variable and hence more difficult to model. Wet delay errors are considered to be the main limitation on the attainable accuracy of GPS heighting (Brunner and Welsch, 1993). An unmodelled 1cm differential tropospheric delay will lead to approximate 3cm error in the height difference over a baseline (Dodson, 1995, p.67). Advanced GPS processing software tends to solve for unmodelled wet tropospheric delay parameters as part of the estimation process (Shardlow, 1994). Such techniques can estimate the absolute range error caused by the troposphere to two centimetres (Dodson et al., 1996).

Relative positioning techniques remove the majority of tropospheric effects over short baselines (<50km in length). For longer baselines, tropospheric heighting error is strongly dependent on baseline length and modelling strategy used. For example, errors in height of 12mm to 24mm have been reported for a 124km baseline (Dodson et al., 1996).

In general, ionospheric and tropospheric effects can be substantially reduced if the observations are made over relatively short baselines (<20–30km) because of the similarity of the local meteorological conditions. These effects are thus negligible for local short baseline networks. However, heighting errors on larger networks remain strongly dependent on the GPS processing techniques used to model the atmosphere.
(b) Earth body tides, ocean tide and atmospheric loading

The Earth experiences an elastic deformation due to the tidal forces. These are called Earth body tides. The ocean tides and atmospheric loading cause additional deformation by the loading of the water's and atmospheric masses on the Earth. These effects lead to time-dependent variation in station heights.

The variation of the station co-ordinates caused by solid Earth tides can be efficiently implemented using Wahr's theory and its precision is estimated to be better than one centimetre (McCarthy, 1992).

The dynamical effect of the ocean tides can be also efficiently modelled as a periodic variation. Its precision can be estimated to be better than a few millimetres (ibid.). However, if ocean tide loading effects are left unmodelled in GPS data processing, height errors of amplitudes as large as eight centimetres have been reported over baselines of around 300km (Beamson, 1995). Ocean tide loading effects are confined to coastal sites in regions where large tidal ranges exist. For example, the ocean tide range at Broome in Western Australia, is greater than 10m, giving a maximum tidal loading range of 1–2 cm (Dawson, 1996).

A varying atmospheric pressure distribution induces crustal deformation. This effect is analogous to ocean-tide loading. The influence of such surface displacement varies with location and status of the atmosphere (MacMillan and Gipson, 1994). Tests in Europe indicated that this effect can reach 2–5cm in some cases (Sun et al., 1995). Rabel and Schuh (1986) estimated that the seasonal variation of this effects may be many millimetres. Large-scale atmospheric loading part can be fairly precisely modelled (<1mm) (ibid.) but is rarely applied as the effect will only be significant over baselines of several thousand kilometres in length.

As with atmospheric errors, the majority of tidal errors are cancelled out for local short baseline networks. For large networks, modelling is essential to achieve higher precision.
(c) Orbital error of the GPS satellite

Orbital error of the GPS satellite is another factor affecting the precision of the GPS ellipsoidal height. The propagation of orbital error into GPS baseline length can be approximately determined from the rule of thumb given by Wells et al. (1987) as

\[ \frac{\sigma_p}{\rho} = \frac{\sigma_L}{L} \]  

(2.20)

where \( \rho = 20,200 \text{ km} \) is the altitude of GPS satellite;
\( \sigma_p \) is the orbital error of the GPS satellite;
\( L \) is the baseline length; and,
\( \sigma_L \) is the baseline error associated with \( \sigma_p \).

Assuming \( \sigma_p = \pm 0.5-2 \) metres for the precise ephemeris (Zumberge et al., 1995; IGS Central Bureau, 1996) and \( \sigma_p = \pm 10-20 \) metres for the broadcast ephemeris (Hofmann-Wellenhof et al., 1994) respectively, the approximate errors in baseline length are 0.025–0.1ppm and 0.5–1ppm respectively. The corresponding height errors can be 3–5 times worse.

For long baselines, it is preferable to use the precise ephemeris from International GPS Service for Geodynamics through post-processing (IGS Central Bureau, 1996). For high precision networks, such as the Australian Fiducial Network (AFN) and Australian National Network (ANN), GPS orbital corrections are estimated as part of the estimation process using the so-called ‘fiducial technique’ yielding accuracy as low as 0.01ppm (Morgan et al., 1996). Over short baselines, with relative positioning techniques, the orbital errors tend to be negligible.

(d) Phase centre and set-up effects of the antenna

Antenna height errors propagate directly into GPS ellipsoidal height solutions. Generally, set-up errors can be minimised by ensuring careful installation of GPS equipment, strong network configuration, and repeat antenna height measurements in the field. Antenna height measurement can usually be achieved to 1-2mm.

The antenna phase centre is the point in the antenna at which the GPS signals physically appear to arrive. This point varies between different types of antenna and, within a single antenna, varies based on frequency of the received signals, and the
elevation and azimuth of the satellite being observed (e.g. Wu et al., 1993). The magnitude of phase centre variations can be in the order of several centimetres (Schupler et al., 1995). The effect is minimised by ensuring identical antennae, oriented in the same direction, are used over short baselines (<20-50km). For longer baselines the effect must be modelled as antenna phase centre errors do not always cancel in relative positioning, even if the same antennae are used. Processing of long (>200km), mixed antenna baselines can give absolute height errors of greater than 5cm if phase centre models are not applied (Stewart, 1996). Phase centre models for many commonly used antennae have recently been released by the International GPS Service for Geodynamics (IGS Central Bureau, 1996). However, no phase centre models were applied in the processing of the GPS data used in this thesis.

(c) Multipath effects

Multipath is caused by nearby objects which reflect the GPS signals before they reach the antenna. For carrier phase observations, this effect can reach up to 10 centimetres (Hofmann-Wellenhof et al., 1994, p.127). This error is site-dependent and does not cancel with relative positioning. Generally, the receiver and antenna are designed to minimise multipath and for high precision GPS surveys, points are chosen to avoid potential multipath sources. Therefore, multipath effects are usually assumed to be negligible, if appropriate precautions have been taken.

In summary, the precision of GPS ellipsoidal height differences depends on a number of factors, particularly the separation of the stations. Table 3.2 ($\S$3.6) estimates the contribution of the different GPS error sources to GPS ellipsoidal heights, for each GPS network used in this study.

2.2.2 Geodetic Spirit Levelling and Height Systems

On land, spirit levelling has been used to determine the heights relative to a local datum. The measured height difference, or the sum of the spirit levelling increments, depends on the path taken and thus is not in general zero for a circuit:

$$\sum dn = misclosure \neq 0$$

(2.21)

To define a physically rigorous heighting system, gravity measurements are made to define geopotential differences that are path independent.
The physical significance of an orthometric height is that it is directly dependent on the local gravity field (it is a combination of geometric and physical features of the Earth and measured along the plumbline, see Figure 2.2). In practice, this is evaluated from spirit levelled heights conducted along a traverse by measuring the line of sight height difference between two graded staves together with observations of gravity along the levelling route. The levelling instrument is located between the staves and aligned with its gravity vector and only gives geometric heights. This must be converted to orthometric height using observed gravity and the orthometric correction (Torge, 1991).

The orthometric height \( H \) is the distance reckoned along the curved plumb-line from the geoid to the surface point (Heiskanen and Moritz, 1967; Figure 2.2). It is defined mathematically by:

\[
H = \frac{C}{\bar{g}} ,
\]

\[
C = W_o - W_P = -\int_{h_0}^{h} dW = \int_{h_0}^{h} g \, dh ,
\]

\[
\bar{g} = \frac{1}{H} \int_0^H g \, dH ,
\]

where \( C \) is the geopotential number;

\( \delta n \) is the spirit levelled increment; and

\( W_o \) and \( W_P \) are the geopotential at the geoid and point \( P \) respectively.

For the computation of the mean gravity along the plumb line (\( \bar{g} \)), the actual values of gravity are required between geoid and the Earth’s surface. Since a direct measurement of gravity inside the Earth is, generally, not possible, a hypothesis regarding the mass distribution must be formed. The Poincaré-Prey reduction is often assumed (Torge, 1991).

When using the Molodensky approach, the normal height \( H' \) is defined as

\[
H' = \frac{C}{\bar{\gamma}}
\]

\[
\bar{\gamma} = \frac{1}{H'} \int_0^{H'} \gamma dH' \text{ or } \bar{\gamma} = \gamma_0 \left\{ 1 + \frac{H'}{a} \left[ l + f + m - 2 f \sin^2 \phi \right] + \frac{(H')^2}{a} \right\}
\]

with
\[ f = (a - b)/a, \quad \text{and} \quad m = \omega^2 a / \gamma, \]

where \( \gamma \) is the normal gravity at the ellipsoid with latitude \( \phi \);

\( f \) and \( m \) are the geometric and dynamic flattening of the Earth respectively.

Hence, \( H' \) may be computed rigorously. Of most importance, the normal height is
determined without any hypothesis regarding the Earth's internal density distribution.
The relationship of the two height systems is analogous to equation (2.19):
\[ H - H' = \zeta - N \] (2.27)

Heights are relative quantities, often referred to a datum station which is usually
defined by mean sea-level using tide gauge. This point then gives the elevation datum
through a levelling network of accurately surveyed bench marks. The region can then
be more extensively covered by spirit levelling or trigonometrical heighting to give the
height of points relative to the datum station(s).

2.2.3 The Australian Height Datum (AHD)
The Australian Height Datum (AHD) was defined in 1971 by the simultaneous
adjustment of 97,230 kilometres of two-way levelling which comprises the adjusted
heights of 497 junction points forming 261 independent closed traverses (Roelse \textit{et al.}, 1971). Mean sea level between 1966-1968 was assigned the value of zero on the
Australian Height Datum at 30 tide gauges which are distributed at approximate
1,000 km intervals around the mainland coast (Granger, 1972). Due to the presence
of varying sea surface topography which separates each tide gauge from the geoid,
the AHD is not necessarily a level surface of the Earth's gravity field (Mather \textit{et al.},
1976).

The Australian Height Datum in Tasmania was established in 1979, adjusted in 1983,
and is connected to mean sea level at two tide gauge stations. The levelling network
consists of 72 sections between 57 junction points (National Mapping Council, 1986).
The height datums in Tasmania and mainland of Australia are independent for
geodetic levelling purposes. An offset exists between them, and from the studies of
Rapp (1994a) and Coleman \textit{et al.} (1979) is \(-40\mathrm{cm}\) and \(-10\mathrm{cm}\) respectively.
A number of studies have been dedicated to the definition and precision of the AHD (e.g. Roelse et al., 1971; Granger, 1972; Mitchell, 1975; Mather et al., 1976; Coleman et al., 1979; Morgan, 1992; Kearsley et al., 1993; Featherstone, 1995). The AHD is neither an equipotential surface nor strictly a true orthometric height system. The AHD is a normal orthometric height system in which the measured gravity was not used in the computation of the orthometric corrections (Holloway, 1988). The AHD, MSL and geoid agree to approximately one metre level around the Australian coastline (Featherstone, 1995). Therefore, geoid height from GPS in conjunction with spirit levelling (AHD) is not the geoid but another non-parallel non-equipotential surface (Mather et al., 1976; Featherstone, 1995). Fortunately, however, the AHD appears to provide elevations to a standard of third-order levelling over smaller networks (Kearsley et al., 1988; Morgan, 1992). A more detailed discussion is given in Chapter Three (§3.6.3).

2.2.4 The Geoid from GPS and Spirit Levelling

Relative GPS provides ellipsoidal height differences \((h_a - h_b)\) in the WGS84 system. Spirit levelling and gravity measurements provide orthometric height differences above the geoid \((H_a - H_b)\). As seen from Figure 2.3, the geoid undulation difference with respect to the WGS84 ellipsoid, can thus be determined. Conversely, the elevation of a point can be determined from GPS measurements if its geoid undulation is provided (Engelis et al., 1984; Engelis et al., 1985; Erker and Sünkel, 1989). GPS and levelling observations can be used as reference and control for transformations between local and global co-ordinate systems. They provide an important exterior checking and intercomparison criteria for gravimetric geoid determination, and will be used as such in this study.

If the deflection of the verticals (DOV) and the curvature of the plumb line are neglected, then the geoid undulation \(N\), ellipsoidal height \(h\) and orthometric height \(H\) can be related by (see Figure 2.3):

\[
\begin{align*}
H &= h - N & \text{For GPS heighting} \\
N &= h - H & \text{For GPS geoid determination}
\end{align*}
\]  

(2.28)

or in the relative mode
\[ \Delta N = N_B - N_A = (h_B - H_B) - (h_A - H_A) = \Delta h - \Delta H. \]  \hspace{1cm} (2.29)

Equation (2.29) implies that relative GPS in conjunction with spirit levelling can provide precise geoid height differences. This relationship allows the relative precision of gravimetric geoid solutions to be tested, and has been done so by Milbert (1992; 1993; 1995) and Stewart and Hipkin (1989), among many others. However, this does not provide any constraint on the zero-degree term or scale of the gravimetric solution. Ideally, one site in the network should have its height above the ellipsoid and orthometric height known accurately, to give the absolute geoid height (e.g. GPS surveys at tide gauges). If point A has been connected to a tide gauge, then the geoid undulation in point B can be determined by means of GPS. This, in turn, will allow absolute gravimetric geoid control through the network using GPS and address the problem of zero-degree scaling (Dodson et al., 1994; Dodson, 1995). However, this approach requires that the sea surface topography has been modelled accurately at the tide gauge stations.

2.3 Other Geometrical Determinations of the Geoid

Several other techniques exist to produce geoid heights by geometrical means. In practice, these methods generally only give sparse, point estimates at local or regional scales. These are difficult to interpolate and do not contribute a high resolution geoid. Therefore, these point estimates are more commonly used for comparison and control of gravimetric geoid solutions (Featherstone, 1992). However, they are now also used as supplemental information for geoid determination nowadays since more data are available (e.g. Jiang and Duquenne, 1996).
2.3.1 The Astro-geodetic Method

As shown in Figure 2.4, the astro-geodetic method can be used to determine the geoid gradient using vertical deflections (Bomford, 1980; Heiskanen and Moritz, 1967). The deflection of the vertical ($\theta$) is determined in practice by comparing the astrogeodetic and geodetic co-ordinates of the same point. This gives a direct measure of the gradient of the geoid with respect to the ellipsoid at that point. Suppose that two arbitrary points, $A$ and $B$, are separated by a distance $D$, then Helmert's formula for astrogeodetic levelling reads:

$$N_B - N_A = \int_A^B \theta dD = D \star \frac{\theta_A + \theta_B}{2}.$$  \hspace{1cm} (2.30)

![Figure 2.4 Geoid determination by Astro-geodetic method](image)

The precision of the geoid is critically affected by the average separation of the astro-geodetic observations (Fryer, 1970; 1971; 1972). Also, this method generally gives a relative geoid rather than an absolute one because the deflections of the vertical are referred to a local ellipsoid. The propagation of errors becomes significant when the astrogeodetic geoid undulation is required in the areas far away from the geodetic origin. Due to its high expense, low accuracy, time consumption and production of a geoid on a local ellipsoid, this method is rarely used nowadays.

2.3.2 Satellite Altimetric Method

Since the ocean covers 78 percent of the Earth's surface, it is important to include information related to ocean regions. Geodetic observations made on the moving sea surface are difficult and generally less accurate than on land. However, satellite altimetry can measure sea surface topography to a high resolution and precision (Nerem et al., 1995; Zhang and Blais, 1995; Zlotnicki, 1994). This procedure can be
used to estimate the Australian geoid in offshore areas using the principle shown in Figure 2.5.

![Diagram](image)

Figure 2.5 Geoid undulation determination by satellite altimetry

If the absolute position of an altimetric satellite is known with respect to the Earth's geocentre and its altitude above the sea surface measured, then the shape of the geoid with respect to the reference ellipsoid can be derived (cf. equation 2.31). This is the basis of satellite altimeter geoid determination.

\[
N = h_E - h_{ISS} - H
\]  

(2.31)

where 
- \( h_E \) is the satellite height above reference ellipsoid;
- \( h_{ISS} \) is the satellite height above instantaneous sea surface; and
- \( H \) is the height of the instantaneous sea surface.

Several satellite altimetric missions have been launched since the 1970s. The first was on board SKYLAB in 1974, followed by GEOS-3 (1975), SEASAT (1978), GEOSAT (1975), ERS-1 (1990) and TOPEX/Poseidon (1993). The logistics of each mission are obviously different but the principles of geoid determination are common: A satellite-borne altimeter orbits at an altitude of approximately 800 kilometres and precesses in such a way that the altimeter's antenna always faces the Earth, and ground tracks are repeated for data averaging purposes. An electromagnetic signal is emitted from the satellite antenna in pulses of a few hundred Hertz (the radar band).
These are reflected from roughly an one-square kilometre foot-print on the sea surface and the two-way travel time recorded. Each foot-print is separated by approximately seven kilometres because of the speed of the satellite and altimeter pulse frequency, whereas, each track is separated by a larger distance limiting the resolution, which necessitates interpolation.

A radar altimeter measures the two-way travel time of a pulse emitted by an antenna onboard the satellite, scattered back by the ocean surface, and received by the same antenna. It also measures the time history of the backscattered energy (Zlotnicki, 1994). Three main sources of error affect the measurement and have to be accounted for before the geoid height is computed. These are satellite orbit errors, atmospheric refraction (or altimetric distance measurements) and dynamic sea surface topography resulting from tides, waves, currents, atmospheric pressure and water salinity.

Since the first missions, ground-based satellite tracking has improved substantially, and when combined with geopotential model in the dynamic orbit calculation, atmospheric refraction, solar activity and radiation pressure modelling, can today yield the orbit to 10cm RMS precision (e.g. TOPEX/Poseidon). The pulsed signal has to pass through the ionosphere and troposphere twice before being recorded and is therefore affected by refraction. The ionospheric delay is at its maximum during the daytime and especially during periods of high solar activity, causing errors of up to 15cm in the altitude, but can now be modelled fairly accurately (Zlotnicki, 1994). The tropospheric delay is more problematic. It is possible to evaluate for the dry atmosphere (molecule mass uncontaminated by wet vapour) but a realistic estimate of wet atmospheric delay (i.e. due to wet vapour) is very difficult to model. Recently, water vapour radiometer measurements have been made simultaneously from the satellite and used to compute this wet delay; the error may reach a few centimetres (Dodson et al., 1996). These difficulties are common to GPS (§2.2.1).

The actual reflection point of the signal in the foot-print is assumed to be directly below the altimeter but this is susceptible to error over the distances involved. Also, the roughness and reflectivity of the surface is directly dependent upon the local wind speed. This is accounted for by recording the returning signal when reaches a
particular threshold power and studying signal structure to refine the altitude during post-processing. Timing errors can obviously affect the distance measurement but are monitored and can be removed.

The instantaneously determined sea surface shape still includes an element of sea surface topography from oceanic circulation, permanent and time dependent oceanographic effects. The ocean tide, annual cycle of sea level and ocean circulation can be modelled or corrected to less than 10 cm (Zlotnicki, 1994).

The accuracy of altimetric geoid determination has been studied extensively (e.g. Wagner, 1989; Zlotnicki, 1994), and the errors are discussed and evaluated in a number of papers (Shum et al., 1995; Nerem, 1994; Tapley et al., 1994; Ma et al., 1994 and Visser et al., 1993). The precision of the altimetric geoid determination has increased by two orders of magnitude since the first missions, with the ERS-1 and TOPEX/Poseidon satellites producing results at the sub-decimetre level (Nerem et al., 1995)

2.4 Harmonic Expression of the Geopotential

Spherical harmonic expansions of the geopotential are frequently used for modelling the Earth’s gravity field at the global scale. The maximum degree and order of recently available models is 360, corresponding to a resolution (half wavelength) of about 55 kilometres at the equator. The spectral representation of the Earth’s gravity field is used because most of the Earth’s-gravity-field-related quantities can not be expressed in a simple analytical form. Global geopotential models (GGM) are important when they are combined with the short wavelength gravity field e.g. for regional geoid determinations by the remove-restore method.

A gravimetric geoid can be computed from terrestrial gravity measurements using Stokes’s integral formula (§2.1). However, this alone requires gravity data coverage over the entire Earth, which is impractical and virtually impossible at present. In the 1960s and 70s, satellite geodesy provided geodesists with global geopotential models, which are spherical harmonic representations of the long and medium wavelength
component of the Earth's gravity field. Molodensky et al. (1962) proposed the combination of Stokes's integral with a geopotential model. This approach avoids the requirement of global gravity data coverage and reduces the time required for geoid computation. Rapp and Rummel (1975), and others, provided some of the gravimetric geoid computation methods using this combined approach. Therefore, as a higher than second-degree reference surface, a geopotential model plays an important role in the gravimetric determination of the geoid.

The past decade has seen the development of a series of GGMs of increasing degree and order. A number of geopotential harmonic coefficient sets, which have been derived by different authors or research organisations and different data sources, are available, such as the OSU series (Rapp, 1981a; Rapp and Cruz, 1986; Rapp and Pavlis, 1990; Rapp et al., 1991), the GFZ series (Gruber and Anzenhofer, 1993; Gruber et al., 1995; Gruber et al., 1996), the GRIM series (Reigber et al., 1985; Reigber et al., 1992) and the GEM/JGM series (Nerem, 1994; Tapley et al., 1996) and EGM96 (Pavlis et al., 1996; Pavlis, 1996).

If the reference ellipsoid is geocentric, its mass and potential are equal to the mass and potential of the Earth, and its semi-minor axis is coincident with Earth's rotation axis, the disturbing potential of the Earth's gravity field can be expressed as the following harmonic expansion form, in which the zero- and first-order degrees are absent (Heiskanen and Moritz, 1967):

$$T = \frac{GM}{r} \sum_{n=2}^{n_{\text{max}}} \sum_{m=0}^{n} \left( \delta C_{nm} \cos n\lambda + \delta S_{nm} \sin n\lambda \right) \bar{P}_{nm}(\cos \theta),$$

(2.32)

where $T$ is the disturbing potential;

$GM$ is the geocentric gravitational constant;

$r$, $\theta$ and $\lambda$ are the spherical polar co-ordinates of the computation point;

$a$ is the semi-major axis of the reference ellipsoid;

$\bar{P}_{nm}(\cos \theta)$ are the fully normalised associated Legendre functions; and

$\delta C_{nm}$ and $\delta S_{nm}$ are the fully normalised spherical harmonic coefficients of $T$.

The corresponding geoid undulation ($N_{GGM}$) at any point is given by inserting equation (2.32) in Bruns's formula (equation 2.6) to give
\[ N_{GGM} = \frac{GM^{\text{max}}}{\gamma r} \sum_{n=2} a_n \sum_{m=0}^{n} (\delta \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm} (\cos \theta). \] (2.33)

The gravity anomaly ($\Delta g_{GGM}$) can be computed by

\[ \Delta g_{GGM} = \frac{GM^{\text{max}}}{r^2} \sum_{n=2} a_n \sum_{m=0}^{n} (\delta \bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm} (\cos \theta). \] (2.34)

Therefore, a geoid height or gravity anomaly can be computed from a geopotential model for any point using equations (2.33) or (2.34) respectively.

For the determination of mean geoid heights and mean gravity anomalies over a block with a capsise of angular radius $\psi_0$, the Pellinen (1972) smoothing operator should be applied in equations (2.33) and (2.34) (Sjöberg, 1980):

\[ \beta_n = \frac{2n - 1}{n + 1} \cos \psi_0 - \frac{n - 2}{n + 1} \beta_{n-2}; \quad \forall n \geq 2, \] (2.35)

with

\[ \beta_0 = 1, \quad \beta_1 = (1 + \cos \psi_0)/2. \] (2.36)

When using a global geopotential model for regional gravimetric geoid determination, the long and medium wavelength component of the geoid must not be added to the solution twice (Rapp and Rummel, 1975; Kearsley, 1988b). This can be avoided using a routine approach, commonly called the remove-restore technique (§2.7), where the gravity anomalies implied by the geopotential model (equation 2.36) are subtracted from the terrestrial gravity anomalies ($\Delta g$) to yield residual gravity anomalies:

\[ \Delta g' = \Delta g - \Delta g_{GGM}. \] (2.37)

These residual gravity anomalies are utilised in a Stokesian geoid computation to provide residual geoid undulations based on the same spherical harmonic expansion of the same geopotential model. The corresponding long and medium wavelength geoid component from the geopotential model (equation 2.33) is subsequently added or restored to these residual geoid undulations.

In addition to reducing the gravity data and computational requirements, this remove-restore approach reduces the influence of some of the approximations and
assumptions inherent to Stokes’s formula. For instance, the spherical approximation
associated with Stokes’s formula results in an error of 0.3% of the computed geoid
height when referred to an ellipsoid (Heiskanen and Moritz, 1967, p.94). In Australia,
for example, the residual geoid undulations are less than 5m in comparison to the
maximum total geoid undulation of 70m. Therefore, this spherical approximation
error is reduced from 21cm to 1.5cm.

The relation given in Heiskanen and Moritz (1967)

\[ \Delta g_n = \frac{\gamma(n-1)}{R} N_n \]  \quad (2.38)

illustrates that the power of the gravity field is distributed inversely between the geoid
and gravity anomalies for each wavelength. The majority of the power of the geoid’s
signal is contained in the low degrees (long wavelength component), whereas the
majority of the gravity anomaly’s power is held in the medium and short wavelengths
(Schwarz, 1984). Therefore, gravimetric geoid determination is essentially a
redistribution of power within the gravity field spectrum.

2.5 Other Methods to Determine the Geoid Undulation

In addition to the methods mentioned above, the geoid can be determined through
many other approaches, such as collocation (Moritz, 1989; Marti, 1994; de Min,
1995), fast collocation (Bottoni and Barzaghi, 1993; Barzaghi et al., 1993), relativity
(Bjerhammar, 1985; Shen et al., 1993; Shen, 1996), and isostasy (Haxby and
Turcotte, 1978). These methods, whilst acknowledged, will not be discussed further.

2.6 The Terrain Reduction

The Stokes solution to the classical geodetic BVP assumes that there is no mass
external to the geoid and the gravity anomalies are given on the boundary surface of
the geoid. To implement the Stokes integral, one has to take into account the mass
outside the geoid and lower the gravity observations to the geoid through downward
continuation (Heiskanen and Moritz, 1967). Even for the modern Molodensky BVP
(§2.1), practical computation is implemented in a series form of integrals where the
terrain height and gravity anomaly are involved in the integrals (Molodensky et al.,
1962; Schwarz et al., 1990; Sideris, 1994b). As indicated by Moritz (1968), the integrand of linear term in Molodensky's solution is approximately equal to the classical gravimetric terrain correction.

The terrain height observations are an indispensable form of information for gravity field refinement and recovery. They dominate the short wavelength part of the gravity field and thus can be used to model detailed gravity field structure in unobserved positions. However, some geophysical information is also needed to identify heterogeneous masses above and below the geoid for better modelling, such as the state of isostatic equilibrium and density of the topography.

There are a number of advantages to removing the terrain effect. It can compensate for the practical deficiency of the Stokes theory and is also very helpful to the modern Molodensky theory. It is used to derive the values of the gravity observations on the geoid which are required by the Stokes formula. Another important application is to find a smooth reference surface for which improved data prediction can be implemented. Generally, the gravitational effect of the topography and bathymetry are the dominant sources contributing to the high-frequency variation of the local gravity field. By computationally removing the effect of the masses, a more smooth residual field will result. The residual gravity field should be more suitable for gravity field modelling and geoid determination.

2.6.1 Terrain Reduction Methods
There are a number of terrain reduction methods in geodesy. They differ with regard to the way the effect of topography is treated. The commonly used methods are shown in Figure 2.6.
Bouguer Reduction and Terrain Correction. The object of the Bouguer reduction of gravity is the complete removal of the topographic masses. The refined Bouguer reduction implies that all topographic masses are removed to infinity. It can be divided into two parts: terrain correction and Bouguer plate effect. Its formulas can be expressed as follows:

\[
\Delta g_B = G \rho \int_0^H \int_0^r \frac{z - H_p}{r^3} \, dx \, dy \, dz \\
= 2\pi G \rho - G \rho \int_{H_p}^H \frac{z - H_p}{r^3} \, dx \, dy \, dz \\
= 2\pi G \rho H_p - C
\]

(2.39)

where \(H_p\) is the height of the point of interest. \(C\) is the terrain correction, which is always positive and typically one order of magnitude smaller than the simple Bouguer term (Forsberg, 1985). It can be expressed as (Schwarz et al., 1990; Heiskanen and Moritz, 1967):

\[
C(P) = G \int_{H_p}^H \frac{(z - H_p) \, p \, dE \, dz}{r^3}.
\]

(2.40)
with

\[ r^2 = (x_P - x)^2 + (y_P - y)^2 + (H_P - z)^2, \]  

(2.41)

where \( C(P) \) is the terrain correction at point \( P \);

\( (x_P, y_P, H_P) \) and \( (x, y, z) \) are the co-ordinates of the reference point \( P \) and a running point \( Q \) in a local Cartesian system respectively;

\( r \) is the distance between point \( P \) and the running point; and

\( E \) is the surface of the Earth.

The linear, planar expression of the terrain correction is (Schwarz et al., 1990; Moritz, 1968):

\[ C(P) = \frac{G_P}{2} \int \frac{(H - H_P)^2}{r^2} \, dx \, dy \]  

(2.42)

where

\[ r^2 = (x_P - x)^2 + (y_P - y)^2 \]

Generally, after the removal of the irregular topographic effect through equation (2.42), the residual gravity field should be smooth.

**Topographic-Isostatic Compensation** The Bouguer reduction implicitly assumes that the topographic masses are simply superimposed on an essentially homogeneous crust. However, some kind of mass deficiency under the mountains exists as discussed by Torge (1989). This means that the topographic masses are compensated in some way. Two different theories for such a compensation were developed by Pratt and Airy. The Pratt model proposes that below a certain depth, the compensation depth, there is no lateral variation in density. The total mass of the columns is the same, and the difference in height is compensated by a difference in density between the columns. According to Airy, the mountains are floating on fluid substrata of higher density, so that the higher the mountain, the deeper it sinks. The Airy model proposes that the density of the crust is approximately the same and that the mass variation between blocks of varying elevation is compensated by how far the columns extend into the denser mantle.

The object of isostatic reduction of gravity is the regularization of the Earth's crust according to some model of isostasy. The topographic masses are not completely removed as they are in the Bouguer reduction, but are shifted into the interior of the
geoid in order to make up the mass deficiencies that exist under the continents. Theoretically, after topographic-isostatic reduction, the residual gravity anomaly should be, generally, even smoother than the Bouguer reduction. However, this method involves an assumption that the region is in isostatic equilibrium, which is not necessarily true.

**Helmert Condensation Reduction** Following Helmert (1884), the terrain masses can be shifted onto the geoid (removal of masses outside the geoid and restore them onto the geoid surface). Helmert argued that the effect of the condensation of the topographic masses can minimise the geoid distortion and indirect effect. Therefore, Helmert’s second condensation method is the most suitable for a geoid computation (Heiskanen and Moritz, 1967).

In Helmert’s scheme, the topography is condensed to form a surface layer on the geoid. The topographic masses are shifted along the local vertical and the value of the total mass remains unchanged. This will cause an attraction change to the free-air anomalies which is equal to the terrain correction when it is assumed that the free-air anomalies are linearly dependent on the elevation of the topography and a planar approximation is used (Moritz, 1968, Martinec, 1991).

The change in potential due to mass shifting results in an indirect effect on geoid undulation ($\delta N_{ind}$) and a secondary indirect effect on the gravity anomaly ($0.3086\delta N_{ind}$) (Martinec and Vanfcek, 1994a; 1994b). However, the secondary indirect effect is usually negligible (Wichiencharoen, 1982).

The indirect effect can be expressed as:

$$\delta N_{ind} = \frac{V_A - V_C}{\gamma}, \quad (2.43)$$

where $V_A$ and $V_c$ are gravitational potentials of the actual topography and the condensed topography on the geoid. Equation (2.43) can be further expressed as the following approximate form (Mainville et al., 1994; Wichiencharoen, 1982):

$$\delta N_{ind} \approx \frac{-\pi GpH_p^2}{\gamma} - \frac{Gp}{6\gamma} \int \int \frac{H^3 - H_p^3}{r^3} dx dy, \quad (2.44)$$

and Helmert’s gravity reduction is approximated by
\[
\Delta g \mathcal{H} = -\frac{G \rho}{2} \iint \frac{(H - H_p)^2}{r^3} \, dx \, dy = -C
\] (2.45)

Equation (2.44) is not valid when \( H - H_p \) is greater than \( r \) (Heck, 1992; Mainville et al., 1994), which can cause difficulties in steep terrain.

**Residual Terrain Modelling** The residual terrain model (RTM) method assumes that the irregularity of the Earth’s gravity field is caused by the undulation of the topography relative to a reference field which is a smoothed average height surface (Forsberg and Tscherning, 1981; Forsberg, 1984). The advantages are that the reduced anomalies are smooth and the indirect effect is generally very small with the computed geoid surface is less distorted (Forsberg, 1984).

2.6.2 Concluding Statements

The actual gravity field is so complicated that none of the above reductions are perfect nor exact. Therefore, it should be emphasised that for geodetic purposes the reduction methods used must be mathematically precise and self-consistent, and the same model must be used throughout the remove-restore process. Usually the topographic effect can be computed reasonably accurately if reliable and dense elevation data are available and it is assumed that the crustal density is constant.

However, if an imperfection in the removal procedure causes long wavelength components to be present in the residual gravity anomalies, the integral transformation (equation 2.10) can have unpredictable results, although some errors can be cancelled out after the restore procedure. Such long wavelength components may introduce errors in the local geoid which have a larger amplitude than the original errors implied by the remove procedure. It is, therefore, recommended that the best fitting geopotential model should possess a reasonably high degree and order. The parameters and models used in the terrain reduction should effectively represent detailed local gravity field information. The DTM used for gravity field reduction should be refined, and any potential gross error and aliasing effects should be minimised in the terrain reduction.
2.7 The General Remove-restore Technique

The terrain reduction comprises two basic steps, removal of the masses outside the geoid (visible topography) and a small correction due to the shift of the mass (compensation or restore). Similarly, the inclusion of a global geopotential model removes the low-frequency gravity anomalies prior to Stokes integration, then restores the geoid contribution from the same global geopotential model. This is well-known in the geodetic sciences as the remove-restore technique. The principle of the general remove-restore technique is shown in Figure 2.7.

![Diagram of remove-restore technique]

**Figure 2.7 Principle of the Remove-restore Technique**

In Figure 2.7, $L_i^{\text{obs}}(T)$ are observations in random distributed points;

$L_i(T_{\text{mod}})$ and $L_j(T_{\text{mod}})$ are the *remove and restore* values of a functional of $T$ to account for the irregularity of the Earth's gravity field in location $i$ and $j$ respectively;

$L_i^{\text{obs}}(T^c)$ are the residual observations which can be well modelled;

$L_j^{\text{pred}}(T^c)$ are predicted residual values for the unobserved locations; and

$L_j^{\text{pred}}(T)$ are the predicted values in location $j$.

The above remove-restore technique has a broad range of applications. When using this procedure, some frequency bands of the gravity field quantities (especially the long and very short wavelength parts) are removed from the data. The reduced gravity field has a smaller magnitude, and it is generally much smoother. The advantages of this technique are that only a limited data collection area is required for
the evaluation and that a statistical treatment of the residual field is facilitated (Torge, 1991), such as collocation (Moritz, 1989). The Stokes integral, when applying the above remove-restore procedures, can be expressed as

$$N = \frac{R}{4\pi c^2} \int (\Delta g_{FA} - \Delta g_{GGM} - \Delta g_{DTM}) d\sigma + N_{GGM} + N_{DTM} \quad (2.46)$$

or alternatively

$$N = N_{GGM} + N_{\Delta z} + N_{DTM} \quad (2.47)$$

$$\Delta g = \Delta g_{FA} - \Delta g_{GGM} - \Delta g_{DTM} \quad (2.48)$$

where $\Delta g_{DTM}$ is the gravity anomaly correction of the terrain reduction; and

$N_{DTM}$ is the indirect effect on the geoid undulation due to mass shifting.

Using a geopotential model with local gravity data and digital terrain elevation data in geoid computation (e.g. Mainville et al., 1992; Forsberg and Kearsley, 1990; Sideris et al., 1992) has become a commonly used methodology (e.g. Forsberg, 1993; 1994; Schwarz and Sideris, 1993; Zhao, 1989).

The geopotential model information is generally of long and medium wavelength in nature. The contribution of local gravity data is generally of medium and short wavelength nature. The local topographic effect is relatively small, typically of centimetre to decimetre level (Forsberg and Tscherning, 1981) and is short wavelength in nature. These contributions are shown in Figure 2.8. These three different kinds of information have different error characteristic effects on the geoid computation. Another advantage of applying the terrain reduction is to remove the
aliasing effect of the geoid computation. That is, after the addition of the terrain reduction, the short wavelength part of the gravity anomalies due to the irregular terrain effect is not present in the convolution integral (Sideris, 1994a).

2.8 Summary

This Chapter presents the theoretical basics and methodologies for the determination of a gravimetric geoid, reduction of terrain effect and GPS heighting. These include the theory of the conventional Stokes and Molodensky methods, their relationships, existing problems and deficiencies and a discussion to remedy them. In addition, the error sources, precision and accuracy of the GPS ellipsoidal height determination are described.

Theoretically, the determination of the geoid from gravity anomalies is a global integration. In practice, the integration is rarely global, but is carried out over a limited local region. Also, the integrand is no longer the gravity anomaly, but a residual gravity anomaly. Subtracting the model gravity anomalies (remove) is designed to achieve following two objectives: first, that long wavelength gravity components are removed from the locally available data. Therefore, it is assumed implicitly that this allows the range of integration to be restricted to the local region. Secondly, the short wavelength components of the gravity field are removed in order to produce an integrand which is smoother and so more readily integrable.

Essentially, geoid undulation can be divided into three spectral parts: long, medium and short wavelength components. Only if all the three components are precisely determined, is the final geoid precise.
AN EVALUATION OF FFT GEOID DETERMINATION
TECHNIQUES AND THEIR APPLICATION TO HEIGHT
DETERMINATION USING GPS IN AUSTRALIA

CHAPTER THREE

DATA PREPARATION, TEST DATA, THEIR PRE-PROCESSING AND EVALUATION
Chapter 3

DATA PREPARATION, TEST DATA, THEIR PRE-PROCESSING AND EVALUATION

3.1 Introduction

Geoid computation can be considered as a filter with a multiple input and single output (Sideris, 1994a; 1996a). Therefore, to accurately determine the gravimetric geoid of Australia, the data availability, their validation, refinement and preparation are crucial, since the quality of the data will directly affect the quality of subsequent geoid determination. Any potential physical and geometrical systematic and gross errors should be removed. This chapter describes all the data available for this study and discusses their validation, refinement, pre-processing and optimal combination.

The original data include:

- gravity observations both in land (526,091 points) and marine (111,396 points) areas from the Australian Geological Survey Organisation (AGSO),
- gravity anomalies derived from satellite altimetric observations (Sandwell et al., 1995),
- GPS/levelling observations (59 points) at the Australian National Network (ANN) and Fiducial Network (AFN) stations from the Australian Surveying and Land Information Group (AUSLIG),
- three other local GPS/levelling networks in Australian Capital Territory (86 points), Victoria (18 points) and Western Australia (21 points),
- spot heights from AUSLIG (5,089,211),
- high degree and order global geopotential models from the USA, Germany, Canada and the International Geoid Service (IGeS), Italy (>10 sets).

Altimetric observations have been assessed and combined with local gravity data and terrain height data formatted and transformed to World Geodetic System 1984 (WGS84) where necessary. Any erroneous records have been corrected or removed. This procedure gives a homogeneous data source for subsequent geoid determination. The selection and verification of an optimal geopotential model has also been studied to give the best local reference field.
3.2 Australian Geodetic Datum and World Geodetic System 1984

Current national mapping in Australia uses the Australian Map Grid 1966 (AMG66) and 1984 (AMG84), which are projections of co-ordinates from the Australian Geodetic Datum 1966 (AGD66) and 1984 (AGD84), respectively. The AMG66 and AMG84 are based on the Universal Transverse Mercator (UTM) projection and use geometrical constants of the Australian National Spheroid (ANS). The ANS, for all practical purposes, has the same dimensions as the GRS67 reference ellipsoid, which was adopted by the International Association of Geodesy (IAG) at the 1967 General Assembly at Lucerne (IAG, 1971). The origin of Australian geodetic network was fixed at the Johnston Origin (National Mapping Council, 1986). The minor axis of the spheroid was defined as being parallel to the Earth’s mean axis of rotation at the start of 1962. The meridian plane of zero for the ellipsoid was defined as being parallel to the Bureau Internationale de l’Heure (BIH) mean meridian plane near Greenwich (ibid.).

Both AGD66 and AGD84 are the gazetted Australian Geodetic Datums. The co-ordinates differ only because the latter has been readjusted using additional data and improved adjustment techniques. The difference between these two co-ordinate sets varies from about 2 metres in south east Australia to about 5 metres in the north west, but the variation is not uniform (National Mapping Council, 1986; Featherstone, 1994). Because of the heterogeneous nature of AGD66, there are no national transformation parameters available to automatically transform between AGD66 and AGD84 (ibid.). It is, however, possible to compute local parameters based on a comparison of sites in the area which have both AGD66 and AGD84 positions.

A reference system that best fits the whole Earth is an Earth-centred (or geocentric) system, which is updated from time-to-time based on the latest information. The reference system adopted in 1980, known as the Geodetic Reference System 1980 (GRS80) (Moritz, 1980), was used by the United States Defence Mapping Agency as the basis for the World Geodetic System 1984 (WGS84), which is currently used for the GPS satellite navigation system (Defense Mapping Agency, 1987). The parameters of the WGS84 ellipsoid are identical to those of the GRS80 ellipsoid with one minor exception. The coefficient used for the second degree zonal harmonic is
that of the WGS84 Earth Gravitational Model rather than the \( J_2 \) used in GRS80 (Moritz, 1980). The WGS84 ellipsoid parameters are: semi-major axis \( a=6378137\,\text{m} \), inverse flattening \( 1/f=298.257223563 \) (Defense Mapping Agency, 1987).

The Intergovernmental Committee on Surveying and Mapping (ICSM) resolved to adopt an Earth-centred co-ordinate system to be implemented in Australia after the year 2000, replacing the regional AGD which was introduced in 1966 (ICSM, 1994). The geocentric datum of Australia (GDA) is realised through the estimated positions of the Australian Fiducial Network (AFN) and the Australian National Network (ANN) which is tied to the International Terrestrial Reference Frame 1992 (ITRF92) (Manning and Harvey, 1994). The GRS80 ellipsoid has been adopted to provide the reference surface for geodetic curvilinear co-ordinates.

WGS84 is an Earth-centred reference system, whereas the ANS, used for the AGD co-ordinates, is offset from the Earth's centre of mass. This results in AGD and WGS84 positions differing by approximately 200 meters over continental Australia (Featherstone, 1996). National transformation parameters are available to transform between AGD84 and WGS84. The seven parameter transformation is a rigorous option for converting 3-D Cartesian co-ordinates from one system to another (ibid.). It includes three origin shifts \((X_0, \, Y_0, \, Z_0)\), three rotations \((r_x, \, r_y, \, r_z)\) and one scaling factor \((K)\). The transformation can be expressed as:

\[
\begin{bmatrix}
X \\
Y \\
Z_{\text{GDA}}
\end{bmatrix} = \begin{bmatrix}
X_0 \\
Y_0 \\
Z_0
\end{bmatrix} + (I + K) \times \begin{bmatrix}
1 & r_z & -r_y \\
r_z & 1 & r_x \\
r_y & -r_x & 1
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z_{\text{AGD}}
\end{bmatrix}, \tag{3.1}
\]

where \([X,Y,Z]^T\) is the co-ordinate vector and the subscripts "\(\text{GDA}\)" and "\(\text{AGD}\)" refer to the co-ordinate systems of GDA and AGD. A new set of the seven parameters is currently being derived using the AFN and ANN networks (Higgins, 1987; 1994) for use in the above equation. However, interim AGD84 to WGS84 parameters, determined by Higgins (1987) as: \(X_0 = -116.00\,\text{m}, \, Y_0 = -50.47\,\text{m}, \, Z_0 = +141.69\,\text{m}, \, r_x = -0.23^\circ, \, r_y = -0.39^\circ, \, r_z = -0.344^\circ\) and \(K=0.0983\) parts per million, have been used in this study.
3.3 Spot Heights and Digital Terrain Model (DTM)

The Australian landscape is strikingly distinctive, with a variety of landforms ranging from extensive plains and plateaus to great expanses of desert, with low tablelands in the arid centre and the west, which together comprise two thirds of the continent. Australia is the lowest, flattest and smallest of all continents. The highest mountain in Australia is 2228 m (Mount Kosciusko) and lowest elevation on land is -15 m (Lake Eyre).

The original Australian spot height database, supplied by the Australian Surveying and Land Information Group (AUSLIG), is based on the AMG66 co-ordinate system and Australian and New Zealand standard. The database includes zone number, easting and northing, offsets of easting, northing and height, horizontal and vertical scale factors and other related information. There are 5,089,211 records in the database. Assuming AGD84 and AGD66 to be the same (§3.2), the AMG84 co-ordinates are transferred to AGD84 geographical co-ordinates and then to the WGS84 system by using Higgins's seven parameters (§3.2).

An additional 637,487 spot heights have been provided by AGSO for the station heights of the gravity observations. These were verified by plotting and comparing the locations of these two data sets. Therefore, there are a total of 5,726,698 records of spot heights to form a digital terrain model (DTM) of Australia. On average, the density of the spot height is approximately one point per 1.3 km² which corresponds to a resolution of 1.1 km. In order to minimise the aliasing effect (§4.3) in the terrain reduction (§2.6), a grid size of 1'×1', which is about 1.8 km resolution, was chosen to produce the digital terrain model (also see §7.3.4). The minimum curvature spline in tension method (Smith and Wessel, 1990; Wessel and Smith, 1991; 1995; §6.2) was used to produce the 1'×1' DTM.

A 9" by 9" (~250m) DTM has recently been released for the Australian continent (Carrol and Morse, 1996). However, this was not available for the present study. Instead, the 1'×1' DTM has been constructed essentially from the same data and is used to estimate the magnitude of terrain effect in the Australian geoid.
3.4 Gravity Data, Refinement and Reduction

The gravity data set used in this research comprises the AGSO 1992 gravity database and has been validated by Featherstone et al. (1997). The spatial resolution of these data is approximately one observation per seven kilometres in Tasmania and South Australia and one observation per 11 kilometres elsewhere (Mather et al., 1976; Gilliland, 1987). However, the resolution is uneven and increases dramatically where detailed gravity surveys have been conducted in areas of geophysical or geological interest (see Figure 3.1). The gravity database contains 11 fields. These fields are survey number, AGD latitude, AGD longitude, gravimeter height, raw gravity, free air anomaly, Bouguer anomaly, ground height, terrain correction, marine flag and original file name indicating the source of each record. All gravity data and their anomalies are in micrometres per second squared (\(\mu m/s^2\)). The approaches used to detect erroneous data were based on the following strategies (Featherstone et al., 1997):

1. Gravity surveys known to be in error from an earlier validation by Gilliland (1987) and confirmed by AGSO;
2. Records without raw gravity observation supplied;
3. The height of a station greater than the spot height from AUSLIG data base and records with an unexplainable difference between the ground and gravimeter heights;
4. Self-contradictory data records, such as marine observations which appeared to be on land or have a positive elevation, and vice versa;
5. Records with non-numeric characters where numeric characters were expected;
6. Duplicated records or records with extremely large or small gravity values;
7. The free-air and Bouguer anomalies given in the database are also checked against the values recomputed from observations of gravity and its coordinates.
Figure 3.1 Distribution of the marine and land gravity observations in Australia (526,648 and 107,755 continental and marine measurements respectively; supplied by AGSO, Mercator projection)
In addition, the likely error records can be found through comparing the checked station with all its adjacent stations. Unrealistic values which show large discrepancies or gradient change were then excluded. After these numerical tests have been applied, any other gross errors remaining in the data could be detected through a visual approach. There are, in total, 638,492 records in the original data set, while 905 records have been detected as containing errors and thus removed after the above validation procedures had been applied. The gravity anomalies were transformed from the AGD to WGS84 system using Higgins's seven parameters (Featherstone, 1993; §3.2). Of the 637,487 records, there are 526,091 in continental and 111,396 in marine areas, respectively. The distribution of the gravity data is shown in Figure 3.1.

3.4.1 Gravity Reduction and Gravity Anomaly

The gravity anomaly (Δg) is defined by:

\[ Δg = g_F - γ_0 \]  \hspace{1cm} (3.2)

where \( g_F \) is gravity on the geoid; and
\( γ_0 \) is normal gravity on the ellipsoid.

However, observations have been made on the topography and these must be reduced to the geoid. The point free-air and Bouguer anomalies have been determined from the gravity observations through following procedures:

a) *Normal gravity formula and free air reduction* The free-air gravity anomalies can be determined by removing the normal gravity (\( γ_0 \)) which is produced by GRS80 ellipsoid. Normal gravity can be determined through Somigliana's closed formula as follows (Moritz, 1980):

\[ γ_0 = \frac{a γ_e \cos^2 φ + b γ_0 \sin^2 φ}{\sqrt{a^2 \cos^2 φ + b^2 \sin^2 φ}} \]  \hspace{1cm} (3.3)

where \( a \) and \( b \) are the semi-major and semi-minor radii of GRS80;
\( γ_e \) and \( γ_0 \) are normal gravity at the equator and the poles of GRS 80, respectively; and
\( φ \) is the GRS80 geodetic latitude of the observation.
If surface gravity anomalies are used in equation (3.2), these are still 'in the air' and thus need to be reduced to the geoid according to the conventional BVP equation (2.7). The free air correction lowers the observation down to the geoid to give the gravity anomaly (Heiskanen and Moritz, 1967):

$$
\Delta g_{FA} = g - \gamma + \delta g^F = g - \gamma + \frac{\partial g}{\partial H} H - \frac{1}{2} \frac{\partial^2 g}{\partial H^2} H^2,
$$

(3.4)

where $g$ is the gravity measurement on the ground;

$$
\frac{\partial g}{\partial H}
$$

is the gradient of actual gravity;

$H$ is the height of gravimeter; and

$\delta g^F$ is the second order free air correction.

In practice, $\frac{\partial g}{\partial H}$ is approximated by the gradient of normal gravity. This approximation causes an error in some cases that will be discussed in §3.4.2. Equation (3.4) can be further expressed in a more practical, second-order form as follows:

$$
\delta g^F = 0.3086 \times H + 0.72 \times 10^{-6} H^2
$$

(3.5)

If the gravimeter was located in water, then the water attraction above the gravimeter should be removed by the following formula:

$$
\delta g^W = 2\pi G \rho_w H
$$

(3.6)

where $\rho_w$ is the uniform density of the sea water, assumed the 1,030 kgm$^{-3}$.

b) The atmospheric correction The gravity observations are taken on the Earth's surface and thus enclosed in the atmosphere, whereas the boundary conditions (§2.1.2) require that there is no mass external to the geoid. To remove the effect of the attraction of atmospheric masses, an atmospheric correction ($\delta g^A$) should be added to the gravity observation (Moritz, 1980; Sjöberg, 1993). This small correction is a function of height and is tabulated by Moritz (1980). The polynomial expression of the atmospheric correction is (Featherstone, 1992, p. 107)

$$
\delta g^A = 0.871 - 1.03 \times 10^{-4} H + 5.31 \times 10^{-9} H^2 - 2.16 \times 10^{-13} H^3 + \cdots
$$

(3.7)

where $H$ is the orthometric height in metres and $\delta g^A$ is in milligals. The atmospherically corrected free-air anomaly is finally expressed as follows:

$$
\Delta g_{FA} = \Delta g + \delta g^F + (-\delta g^\ast) + \delta g^A.
$$

(3.8)
c) **Refined Bouguer correction** The refined Bouguer anomaly ($\Delta g_{BG}$) can be computed through the following formula:

$$
\Delta g_{BG} = \begin{cases} 
\Delta g_{FA} - 2\pi G \rho \times H_{grad} + C & H_{grad} \geq 0 \\
\Delta g_{FA} - 2\pi G (\rho - \rho_w) \times H_{grad} & H_{grad} < 0 \quad \text{(at sea)}
\end{cases}
$$

(3.9)

where, $C$ is the terrain correction; $H_{grad}$ is the ground height of the gravity observation; and $\rho$ and $\rho_w$ are the average densities of the rock and sea water respectively.

The terrain corrections for the marine observations are zero. For a detailed discussion on the determination of the terrain corrections, see §2.6.

**d) Estimation of accuracy of the gravity observations** According to Mather *et al.* (1976, p.62 and p.81) and Dooley and Barlow (1976), the estimated precision of the observed gravity with respect to the Australian gravity network is ±0.5 mgal and the precision of the station elevations is better than ±10 m. This propagates to approximately ±3.1 mgal in the free-air anomalies. A concurrent estimate by Barlow (1977) and Anfiloff *et al.* (1976) is ±0.2–0.3 mgal for the observations and ±4–6 m for the elevations. This propagates to ±1.2-1.8 mgal in the free-air anomalies (Featherstone *et al.*, 1997).

The accuracy is slightly worse for marine gravity measurements, as these were inevitably made on an accelerating platform. Mather *et al.* (1976) estimate that the accuracy is ±2-6 mgal for the AGSO marine gravity measurements. Anfiloff *et al.* (1976) estimate that the RMS is 2–4 mgal for the surveys over the north-west shelf of Australia and about six mgal for the surveys of the continental margins. However, Wessel and Watts (1988) studied cross-over errors in the Lamont Doherty Geological Observatory's gravity data-base, which contains almost all Australian marine observations held by AGSO, and shows that the marine gravity measurements offshore Australia could be in error by over 20 mgal prior to a cross-over adjustment.

### 3.4.2 Effects of Varying Topographic Density and Actual Gravity Gradient

The density of the topographic masses is required for the computation of terrain corrections, Bouguer and topographic-isostatic anomalies (§2.6), and for the
downward continuation of the gravity anomalies (e.g. free-air gravity anomalies). Erroneous topographical density leads to fictitious anomalies and corresponding misinterpretations (Torge, 1989). A mean crustal density of 2,670 kgm$^{-3}$ is conventionally used for topographic masses and was used throughout this study.

For local studies, however, the local rock density should be used in preference. Because of abundant mineral deposits of Australia, the density of topographic masses may vary much (say, 1,300kgm$^{-3}$ to 3,500kgm$^{-3}$, which is partly proved in Chapter 5). The density is determined from geological surveys, such as drilling and seismic results. To achieve a more accurate terrain reduction, a digital density model is required. However, density effects on terrain corrections are relatively small (see §7.3). The density effects on the computation of Bouguer and topographic-isostatic gravity anomalies may be large. However, most of these effects are compensated after the restore process. Dooley (1979) studied a geophysical profile across Australia at 29°S. It is reported that the density curve along profile 29°S is fairly smooth (Dooley, 1979, Figure 2). However, this only represents one profile and does not necessarily reflect the density variation of the whole of Australia. Since the digital density model is not available for this research, the density effects to terrain reductions are recommended for future studies (§9.4).

In the computation of a free-air gravity anomaly, since the actual gravity gradient is not available for most stations, the normal gravity gradient is used for the computation. The height correction formula in equation (3.4) uses normal gravity gradient. The differences between the normal and actual gravity gradients are:

$$
\Delta G = \frac{\partial r}{\partial h} - \frac{\partial g}{\partial h} \approx 3.086 \pm 0.72 \times 10^{-6} H^2 \frac{\partial g}{\partial h}
$$

(3.10)

However, this approximation can cause some errors in some places. According to Wellman et al. (1984; 1985), the vertical gradient of the gravity at six absolute gravity sites varies from 0.274mgalm$^{-1}$ at Hobart to 0.348mgalm$^{-1}$ at Perth. The approximation errors of the actual gravity gradient may cause large error in the free-air reduction and thus gravimetric geoid determination. However, if the gravity anomalies are taken as gravity disturbance on the telluroid as per the Molodensky's theory requires, the normal gravity gradient should be used for upward continuation. The approximation is thus circumvented.
3.4.3 Time Variations of the Gravity Field and Geoid

The Earth's gravity field is changing consistently due to a number of time-dependent factors, such as winds, currents, pressure, salinity, geostrophic effect in the ocean, river discharge, tectonic movement, eruption of volcanoes, temperature change (green-house effect), melting of the icecaps. For these reasons, the gravity measurements made on the Earth's surface are subject to temporal effects and thus need to be reduced to the same epoch. The Australian gravity observations are made over a period of more than 30 years. Therefore, these gravity measurements are referred to different epochs. So theoretically, these observations should be reduced to a common epoch.

According to Listing (1873), the conventional geoid is theoretically defined as an equipotential surface which is most close to mean sea level (MSL). In practice, because of the effects of the sea surface topography, the geoid defined by sea level measurements is not an equipotential surface (Mather, 1975). The MSLs in different oceans depart from an equipotential maximum differences more than one metre and MSL itself is not a constant in one place. The mean sea-level surface is consistently changing (0.5cm/year from Frasetto, 1970). This affects subsequent gravity data reduction to geoid which is in fact in an inconsistent system. The geoid itself is not a fixed surface, so the geoid can be defined as a function of time and space position. That is, the geoid determination thus belongs to a four-dimensional geodetic problem.

Given an arbitrary point \( P \) near or on the Earth, the geopotential at point \( P \) can be expressed as

\[
W(t) = W(t_0) + \delta W(\delta t),
\]

\[
t = t_0 + \delta t,
\]

where \( W(t) \) and \( W(t_0) \) is the geopotential at a initial time \( t_0 \) and time \( t \) respectively;

\( \delta t \) is the time interval.

For an arbitrary constant \( C \), a point set

\[
W = \{ P | W(t_0 + \delta W(\delta t) = C, P \in R^3 \}
\]

(3.13)
constructs a closed and an equipotential surface. For the geoid with geopotential of
\( W_0 \), it can be expressed as
\[
W = W^{(c)} = W_0
\]  (3.14)
The geoid at epoch \( t \) is thus defined.

Since the changing features of the Australian gravity field are not known and the
changes of the gravity field are relatively very small, all the observations in this thesis
are taken as constant and for a single epoch. However, this assumption will introduce
an unknown error in the final gravimetric geoid solution.

3.5 Altimetric Gravity Anomalies, Evaluation and Combination with Local
Gravity Anomalies

As seen from Figure 3.1, the sparse coverage of marine gravity measurements
offshore Australia can also restrict the geoid determination near the coast. This is
because Stokes's formula requires a homogeneous coverage of gravity data around
each computation point. The coast of Australia is densely populated and highly
developed and thus demands the most reliable geoid solution (Featherstone et al.,
1997). Therefore, recent satellite altimetry results may be a supplementary data
source to improve data coverage and quality, and thus improve the geoid near the
coast. A wide data coverage around the coast is also very useful to remove the
boundary effects in the geoid determination by the fast Fourier transform (§4.3).

Many developments and advances in gravity field determination using satellite
techniques have taken place in the past few decades. The gravity potential determined
from satellite orbit analyses is now of a sufficient quality to give a good homogeneous
picture of the long wavelength geoid components (Balmino and Perosanz, 1994). Even
more important for most geophysical interpretations and gravity field studies is
the success of satellite altimetry. The GEOS-3, SEASAT, and GEOSAT missions
have produced an increasingly accurate marine geoid with an unprecedented global
coverage. The TOPEX/Poseidon (Nerem et al., 1995) and ERS-1 missions have
given further improvement in accuracy and coverage (Smith and Sandwell, 1995).
Arabelos and Tziavos (1994) reported that the standard deviations of the differences
between the observed and predicted gravity anomaly and geoid were 3.8 mgal and
0.09 m respectively in the Mediterranean sea where ERS-1 and TOPEX/Poseidon data have been combined. Although not all the problems with satellite altimetry have been solved, in many parts of the world the altimetrically determined marine geoid is known more accurately than the land geoid derived from observed gravity anomalies. The improving data coverage and quality from recent altimetric missions make it possible to present a more detailed gravity field, especially in ocean areas. This will be of great help to local gravity field approximation in land areas and particularly for offshore areas. Also, the release of the formerly classified GEOSAT data south of 30°S (Nerem et al., 1995) will confer significant benefits to Australia.

Sandwell et al. (1995) and Sandwell and Smith (1997) have calculated global gravity anomalies through a combination of satellite altimeter profiles from multiple missions, which includes data from GEOSAT, ERS-1, SEASAT and TOPEX/Poseidon (FTP site: baltica.ucsd.edu). The accuracy and resolution of the derived field is approaching the best fields derived from declassified GEOSAT Geodetic Mission data south of 30°S (Sandwell et al., 1995). The combination of various altimetry surveys used by Sandwell and Smith (1997) provides the view of all ocean basins at a 10 km resolution. In order to avoid a multi-satellite crossover adjustment, sea surface topography profiles were first differentiated, then combined to produce grids of east and north vertical deflection. These were, in turn, converted to gravity anomalies using a planar fast Fourier transform algorithm (Sandwell, 1992). This approach has been shown to be superior to the use of an inverse Stokes’s formula by Olgiati et al. (1995).

3.5.1 Evaluation of Satellite Altimetric Gravity Anomalies
To study the possibility of combining satellite altimetric gravity anomalies with local gravity anomalies, the accuracy of the satellite altimetric gravity anomalies must be evaluated. The accuracy of satellite altimetric gravity anomalies is, in the most accurate cases, 3–6 mgal (e.g. Arabelos and Tziavos, 1994; Nerem et al., 1995; Arabelos and Vemeer, 1996; Sandwell and Smith, 1997). However, the accuracy of the Australian marine gravity observations is 2-6 mgal (§3.4; Gilliland, 1987) and their resolution is about 15–30km per observation on average (see Figure 3.1).
Another consideration is the actual resolution of the satellite altimetric observations around Australia, which is approximately ~20km (Sandwell and Smith, 1997) although the nominal grid size is 2'. Marks (1996) recently compared seven well-navigated shipboard marine gravity profiles with Sandwell's satellite altimetric gravity anomalies grid (Marks et al., 1993), and found the resolution recoverable is 23-30km for the GEOSAT/ERS-1 grid and 26-30km for the GEOSAT-only grid. Sandwell and Smith (1997) expressed that: "The resolution estimates show that the shortest wavelength recoverable in the gravity field from satellite altimetry is about 20 km". For this reason, a block mean comparison method is chosen instead of point to point comparison.

The comparison is made between the satellite altimetric gravity anomalies around Australia and marine gravity anomalies for different grid sizes, namely 6'x6', 10'x10' and 30'x30'. The mean block values for each grid are determined by using following strategy:

- A moving weighted average;
- The root square of distance is used for weight assignment to avoid too much weight being assigned to a near point;
- At least three observations in each quadrant are used to improve the estimate of the average value;
- The radius of the search window is equal to double the grid size (e.g. 35km for 10'x10' grid).

In addition, the satellite altimetric results are expected to be unreliable near coastal areas because of loss of altimeter lock. Therefore, the comparison is performed about one fifth of a degree (~20 km) away from the coastline. One way to exclude the data in the areas of loss of altimeter lock is to combine the marine gravity information with a detailed bathymetric model. However, this method is very time-consuming, taking about 20 days on a Sun Sparcstation 10 workstation. Instead, the DTM/bathymetric depth model was used as a searching machine. Stations with height greater than -100.0m were taken to be too near to the coast. This method was 30 to 100 times
faster than the former one. Table 3.1 lists the results of the comparison for the three different grids.

If the precision of the marine gravity anomalies ($\sigma_{\text{mar}}$) is assumed, a precision of the satellite altimetry-derived gravity anomalies ($\sigma_{\text{sa}}$) can be estimated using the STD of differences between these two types of gravity anomalies ($\sigma_{\text{sa-mar}}$) as follows:

$$\sigma_{\text{sa}} = \sqrt{\sigma_{\text{sa-mar}}^2 - \sigma_{\text{mar}}^2}$$  \hspace{1cm} (3.15)

Considering the factors that affect the accuracy of the marine gravity measurements discussed in §3.4, the accuracy of the 6'|x|6', 10'|x|10' and 30'|x|30' mean marine gravity anomalies are assumed to be 4 mgal, 3 mgal and 1 mgal respectively (note that the block mean accuracies are better than point accuracies). Using equation (3.15), the corresponding STD errors of the satellite altimetry derived mean gravity anomalies are 5.89 mgal for 6'|x|6', 6.34 mgal for 10'|x|10' and 4.20 mgal for 30'|x|30' respectively. The precision estimates are listed in the second last column of Table 3.1. The last column in Table 3.1 is the number of blocks found in the computation process under the above four searching strategies. These precision estimates agree with, and are thus partly proved by, Sandwell and Smith (1997) in the northern mid-Atlantic Ridge (5.8mgal) and the South Atlantic (6.7mgal).

<table>
<thead>
<tr>
<th>Grid size</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD ($\sigma_{\text{sa-mar}}$)</th>
<th>$\sigma_{\text{mar}}$</th>
<th>$\sigma_{\text{sa}}$ (estimated)</th>
<th>No of blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>6'</td>
<td>x</td>
<td>6'</td>
<td>31.92</td>
<td>-79.24</td>
<td>-2.56</td>
<td>7.63</td>
<td>7.12</td>
<td>4.00</td>
</tr>
<tr>
<td>10'</td>
<td>x</td>
<td>10'</td>
<td>25.34</td>
<td>-65.36</td>
<td>-2.55</td>
<td>7.46</td>
<td>7.01</td>
<td>3.00</td>
</tr>
<tr>
<td>30'</td>
<td>x</td>
<td>30'</td>
<td>12.79</td>
<td>-15.13</td>
<td>-0.60</td>
<td>6.37</td>
<td>4.32</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Since the resolutions of the satellite altimetric and the marine data are about 20–30km and 15–30km respectively, the 10'|x|10' gridded data are used to combine with the marine gravity. Another advantage using 10'|x|10' (instead of 6'|x|6') gridded data is to reduce the number of the less accurate satellite altimetric gravity data in the computation of the gravimetric geoid. The misfit difference for the 10'|x|10' grid is shown in Figure 3.2.
Figure 3.2  Block mean differences between satellite altimetric gravity and marine gravity on a $10' \times 10'$ grid (units in mgal, contour interval=2 mgal)

3.5.2 Combination with Marine Gravity Anomalies

From Table 3.1, it is estimated that the satellite altimetry derived mean gravity anomalies have an accuracy of approximately 4--7 mgal for grid sizes $6' \sim 30'$, while block mean marine gravity anomalies have an accuracy of about 1--4 mgal. From Figure 3.2, it is concluded that the differences are not homogenous. The biggest differences occur in the Papua New Guinea Strait ($150^\circ$E, $11^\circ$S), which are most likely caused by unmodelled sea surface topography (Hamon and Greig, 1972) and insufficient time for the altimeter to acquire a good signal. A systematic difference is found in the east ($155^\circ$E, $25^\circ$S) and west ($114^\circ$E, $30^\circ$S). This is probably caused by unmodelled sea surface topography (ibid.).
The strategies used to combine the satellite altimetric gravity anomalies with marine gravity anomalies are:

- in Papua New Guinea Strait area, satellite altimetric data were not used;
- in the areas where both gravity anomalies exist, the weight for satellite altimetric anomalies was determined by their standard deviations and the number of observations;
- in the areas where marine gravity anomalies do not exist, satellite altimetric gravity anomalies were used;

This significantly improves the coverage and quantity of the present Australian marine gravity database.

3.6 Geometrical Geoid Undulation from GPS and Spirit Levelling

3.6.1 Australian GPS Fiducial Network and National Network

The Australian Fiducial Network (AFN) consists of eight permanent, continuously operating, Rogue GPS receivers on the Australian mainland and Tasmania (Manning and Harvey, 1992). The network was initially observed during the International GPS Service for Geodynamics (IGS) Epoch '92 campaign, July to August, 1992. The AFN, in conjunction with six additional sites beyond the Australian mainland, forms the Australian Regional GPS Network (ARGN). The AFN station co-ordinates are based on the ITRF92 at epoch 1994.0, and are estimated to have a precision of a few centimetres (2-4 parts in $10^8$) (Morgan et al., 1994).

The Australian National Network (ANN) consists of 78 GPS campaign points spaced at approximately 500 km intervals across Australia. This network was observed between 1992 and 1994; the first GPS observation campaign being conducted during the IGS Epoch '92 campaign and followed by a further nine days of observations during August-September, 1993. It is estimated that the horizontal and vertical precision of the co-ordinates is less than three centimetres and five centimetres respectively at the 95 percent confidence level (Morgan et al., 1996). Some stations, primarily those that participated in both the 1992 and 1993 epochs, approach the precision of the ARGN sites (ibid.). However, systematic heighting errors of a larger
magnitude may be present at ANN sites, due to the modelling procedures applied during the processing of ANN GPS data. For example, ocean tide loading models and antenna phase centres models (§2.2.1) were not applied in ANN data processing (ibid.). An estimate of the possible magnitude of heighting errors in the ANN is given in Table 3.2.

The geoid undulations derived from the AFN/ANN sites are also susceptible to errors in the AHD (§2.2.3). The geometrical geoid undulations from GPS and AHD heights at the 59 stations that have optical levelling results are used in this study. The relevant information for these ANN and AFN sites is listed in Appendix I. Figure 3.3 shows the distribution of the optically levelled ANN and AFN stations and a geometrical geoid determined via equation (2.28).

Figure 3.3 Station distribution of 59 AFN/ANN sites with optical levelling results on the AHD and the inferred geometric geoid (unit in metres, Mercator projection from WGS84)
3.6.2 Other GPS Networks Used for Comparison

Three local GPS/levelling networks are used for the evaluation of the gravimetric geoid undulation accuracy. The three networks are located in Western Australia (21 stations), Victoria (18 stations) and Australian Capital Territory (86 stations). These three local GPS networks represent a range of topography: smooth in Western Australia, rugged (by Australian standards) in the Australian Capital Territory and medium in Victoria. The distributions of stations in the three local networks are shown in Figure 3.4.

The 21 spirit-levelled Western Australian GPS stations are located in the Merlinleigh Basin area. These stations were occupied with static, single-frequency, relative GPS for a minimum duration of four hours over baseline lengths averaging around 40 km with shortest line of seven kilometres and longest line of 70km (Featherstone et al., 1996). Their AHD heights are provided by existing benchmarks. The precision achieved for the GPS-derived ellipsoidal height was better than 2mm/km relative to AGD84.

The 18 spirit-levelled Victorian GPS stations cover an area of about 300km by 800km with average baseline length of 120km (shortest baseline length is about 30km, longest baseline length is about 250km). These stations were occupied at least four hours using dual-frequency receivers (Ross, 1996). The AHD values are derived from third-order spirit levelling and are connected to the States’ levelling network (ibid.). Formal standard deviations (1σ) on the network adjustment of the GPS vectors are 2–6cm in the horizontal component and 3–8cm in the height component (Ross, 1996).

The Australian Capital Territory GPS network originally consisted of 109 stations and covers an area of 100km by 80km which was supplied by the Australian Capital Territory Government (Higgins et al., 1996). All stations were at least third-order optically levelled. This network consists of 116 baselines ranging from 0.02 km to 16 km with average baseline length of 3km (Higgins, 1996) and is a combination of several separate GPS surveys. These observations have not been adjusted as datum
inconsistencies of several metres were found between surveys (Higgins, 1996). In an attempt to remove the system inconsistencies, only 86 out of 109 stations were used. However, this data set must still be considered unreliable, because it is difficult to describe the exact magnitude of any remaining discrepancies.

Table 3.2 summarises the various GPS height error sources, as described in §2.2, for each GPS network described above. The magnitudes of each error source represent estimates only, based on the information available from the processing of each GPS data set. It should be noted that the total ellipsoidal height errors quoted represent a worst case scenario for each specific network. These values are to be used as a term of reference only for the gravimetric geoid and GPS/levelled geoid comparisons given in Chapter Eight.

An estimation of the spirit levelling results (height differences) in the AHD is also listed in Table 3.2. Third-order geodetic levelling tolerance (12\sqrt{K} mm) and average baseline length are used in this estimation. Note that the levelling accuracy of AHD is about 8\sqrt{K} mm from the national levelling network adjustment (Roelse et al., 1971). Finally, the accuracy estimation on GPS/AHD derived geoid is given using the general law of error propagation of variances.

3.6.3 Is GPS/Levelling Really the "Yardstick" for a Gravimetric Geoid?
As discussed in §2.2 and equation (2.28), GPS/levelling can provide a discrete, precise geoid undulation. This undulation, which has been used as a control on the gravimetric geoid, therefore, should have a higher accuracy. However, the determination of equation (2.28) has the following two limitations. Firstly, the AHD should coincide with real geoid; any systematic errors will propagate into subsequent geoid results directly. Secondly, the ignorance of the deflection of the vertical can cause an error in the geometric geoid determination.
<table>
<thead>
<tr>
<th></th>
<th>AFN/ANN</th>
<th>WA</th>
<th>Victoria</th>
<th>ACT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>mean baseline length</strong></td>
<td>500km</td>
<td>40km</td>
<td>120km</td>
<td>3km</td>
</tr>
<tr>
<td><strong>Ionosphere</strong></td>
<td>&lt; 2cm</td>
<td>0–3cm</td>
<td>negligible</td>
<td>negligible</td>
</tr>
<tr>
<td></td>
<td>(single frequency)</td>
<td>(dual frequency)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Troposphere</strong></td>
<td>0–3cm (single daily tropospheric parameter estimated)</td>
<td>0–3cm (standard dry delay model applied)</td>
<td>0–5cm (standard dry delay model applied)</td>
<td>negligible</td>
</tr>
<tr>
<td><strong>Earth body tides</strong></td>
<td>negligible (model applied)</td>
<td>negligible</td>
<td>&lt; 5mm</td>
<td>negligible</td>
</tr>
<tr>
<td><strong>Ocean tide loading</strong></td>
<td>0–5cm</td>
<td>negligible</td>
<td>negligible</td>
<td>negligible</td>
</tr>
<tr>
<td><strong>Atmospheric loading</strong></td>
<td>&lt; 2cm</td>
<td>negligible</td>
<td>negligible</td>
<td>negligible</td>
</tr>
<tr>
<td><strong>Orbital error</strong></td>
<td>&lt; 0.01ppm, (5mm) (orbital parameters estimated in fiducial adjustment)</td>
<td>broadcast ephemeris 1ppm 4cm</td>
<td>broadcast ephemeris 1ppm 4cm</td>
<td>broadcast ephemeris 1ppm 4cm</td>
</tr>
<tr>
<td><strong>Antenna (setup, phase centre)</strong></td>
<td>0–5cm (phase centre models not applied)</td>
<td>negligible (same type of antennae used on all baselines)</td>
<td>&lt; 1cm (same type of antenna used on all baselines)</td>
<td>negligible (same type of antenna used on all baselines)</td>
</tr>
<tr>
<td><strong>Total ellipsoid height error</strong></td>
<td>&lt; 1cm</td>
<td>&lt; 1cm</td>
<td>&lt; 1cm</td>
<td>&lt; 1cm</td>
</tr>
<tr>
<td><strong>relative AHD height error (12-km tolerance)</strong></td>
<td>8cm</td>
<td>6cm</td>
<td>13cm</td>
<td>1cm</td>
</tr>
<tr>
<td><strong>GPS/AHD geoid height error</strong></td>
<td>27cm</td>
<td>8cm</td>
<td>13cm</td>
<td>2cm</td>
</tr>
<tr>
<td><strong>Comments</strong></td>
<td>published ANN ellipsoidal height error estimated to be 5cm at 95% confidence</td>
<td>geodetic control network for a gravimetric survey</td>
<td>coordinated onto ANN</td>
<td>systematic errors present from combination of unadjusted individual GPS surveys</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Higgins et al. (1996)</td>
</tr>
</tbody>
</table>
Figure 3.4 Distribution of the GPS/levelling stations in (a) Victoria, (b) Western Australia, (c) Australian Capital Territory

(*) is GPS/levelling station, WGS84 system, Linear projection
a) The Australian Height Datum

The minimally constrained adjustment of geodetic levelling generates mean sea level (MSL) heights that differ to the order of one metre at tide gauge stations (Roelse, et al., 1971) and has been controversial (Morgan, 1992). One problem is that the constrained adjustment introduces the distortion directly into the Australian geodetic levelling network. Another problem is that tide gauge measurements of MSL are not a true position of the geoid. Theoretically, the period for the MSL determination should be at least 18.6 years due to the revolution and rotation periods of the Sun and the Moon. Three years’ tide gauge observation is not sufficient enough (this uncertainty may be up to ±10 cm according to Coleman et al. (1979)). A larger problem is the existence of the sea surface topography (SST), which is up to 1-2 metres (Mitchell, 1972; 1975; Coleman et al., 1979), which is not removed. For example, the extrapolated SST value from Laskowski (1983) at the fundamental tide station in Jervis Bay is 0.3±0.2 m.

By definition, the gravity anomaly refers to the geoid which is an unique equipotential surface of the actual gravity field, whereas the AHD has a separation with the mean-sea-level and the geoid. Featherstone (1995) estimated the differences between AHD, MSL and the geoid. Their differences are given approximately in Table 3.3.

<table>
<thead>
<tr>
<th>Datum differences</th>
<th>Separation (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max</td>
</tr>
<tr>
<td>AHD-MSL(levelling)</td>
<td>0.0</td>
</tr>
<tr>
<td>MSL-geoid(oceanography)</td>
<td>0.7</td>
</tr>
<tr>
<td>AHD-geoid(from above)</td>
<td>1.4</td>
</tr>
</tbody>
</table>

However, the determination of MSL is a very complicated task. The MSL is a constantly changing surface, as is the SST (Merry and Vaníček, 1983). The factors that cause variation in MSL include salinity and temperature differences, atmospheric pressure changes, winds, crustal movements, eustatic changes, melting of ice and river discharge (Laskowski, 1983). Also, according to Frasetto (1970), the global mean sea surface appears to be rising at a rate of 0.5cm/year. This, therefore, affects
subsequent gravity data reduction to the geoid which is in fact in an inconsistent system. Naturally, this leads to the concept of the time-varying geoid (§3.4.4).

In summary, the features of the AHD lead to unfavourable GPS/AHD geoid height results. The AHD is neither a geopotential surface nor strictly a true orthometric height (Holloway, 1988; Mitchell, 1990; Featherstone, 1995). The AHD, MSL and geoid agree to approximately one metre level around the Australian coastline. Therefore, a geoid height from GPS in conjunction with spirit levelling (AHD) is not the geoid but another non-parallel non-equipotential surface (Mather et al., 1976; Featherstone, 1995). Fortunately, however, the AHD appears to provide elevations to a standard of third-order levelling homogeneously. These errors will not affect the results obtained using the geometric geoid modelling technique to determine height differences over a short distance (80km or less, for example). Their effects are expected to be linear and are thus absorbed by the interpolation process as pointed out by Gilliland (1986).

b) Effects of the Deflection of the Vertical (DOV) to GPS/Levelling Derived Geoid

Since the geoid undulation is defined as a point P of geoid is projected onto the point Q of the ellipsoid by means of the ellipsoidal normal (Heiskanen and Moritz, 1967), a more accurate expression for geoid undulation from the ellipsoidal height (h) and the orthometric height (H) is (see Figure 3.5):

\[
N = h - (H + \delta H) = h - (H + H\left(\frac{1}{\cos \vartheta} - 1\right)), \quad (3.16)
\]

\[
\delta H = H\left(\frac{1}{\cos \vartheta} - 1\right) = \frac{1}{2} H \cdot \vartheta^2, \quad (3.17)
\]

\[
\vartheta = \xi \cos \alpha + \eta \sin \alpha, \quad (3.18)
\]

where \((\xi, \eta)\) are the components of the DOV in prime vertical and meridian directions respectively, and

\(\vartheta\) is the component of the DOV in a particular direction with geodetic azimuth \(\alpha\).
The effect of deflection of the verticals at both points A and B in Figure 2.4 can thus be expressed as

$$\delta \Delta N_{AB} = \delta H_B - \delta H_A = \frac{1}{2} (H_B \delta \delta^1_B - H_A \delta \delta^1_A).$$  \hfill (3.19)

Inserting the maximum Australian terrain height $H=2228\text{m}$ and deflection of vertical $\delta=60''$ into equation (3.17), the maximum error caused by this approximation is less than $0.5 \text{ mm}$. When used in the relative case, as is done in GPS surveying, this error cancels even further (equation 3.19). Therefore, the approximation is accurate when considering the errors that affect geodetic spirit levelling heights and GPS heights (Table 3.2).

### 3.7 Determination of the Best Fitting Global Geopotential Model (GGM)

#### 3.7.1 Introduction

Precise regional geoid determinations are usually carried out using a global geopotential model (GGM), together with a set of point or mean terrestrial gravity anomalies and topographic information (§2.7). In order to determine the Australian gravimetric geoid to support GPS height determination, it is necessary to first choose the best fitting geopotential model for the Australian region. If a global geopotential model is the best fit to the gravity field of Australia, it is reasonable to expect that its associated gravity anomalies and geoid heights are a best fit to observed gravity data and geometrically derived geoid undulations in the same area. Therefore, a series of GGMs are compared with Australian gravity and GPS/AHD geometric geoid undulation data to determine the best fitting GGM.
Similar comparisons of the fit of earlier GGMs to the Australian gravity field have been studied by Kearsley and Holloway (1989) and Kearsley and Govind (1991), which demonstrate that OSU89A was superior at this time. Since then, the high-degree OSU91A (Rapp et al., 1991), GFZ93A/B (Gruber and Anzenhofer, 1993), GFZ95A (Gruber et al., 1995) and EGM96 (Pavlis, 1996; Pavlis et al., 1996; Rapp and Nerem, 1994) geopotential models have been published. OSU91A was used in the AUSGEOID93 regional gravimetric geoid solution (Steed and Holtznagel, 1994), but it was not shown whether OSU91A was superior to its predecessors. Eight high-degree GGMs listed in Table 3.4 are compared with geometrical geoid heights derived from relative GPS and levelling observations, with observed free-air gravity anomalies on land and at sea, and with free-air gravity anomalies derived from satellite altimetric measurements.

The statistics used for these comparisons comprise maximum, minimum, mean, standard deviation (STD) and the root mean square (RMS) of the differences between the geopotential models and "ground truth" data. The most informative of these statistics, in this case, is the standard deviation because the mean and RMS differences are distorted by the inclusion of the zero-order term (§2.1.3). Therefore, the best fitting GGM will be the one having the lowest standard deviation. However, the mean differences of geoid undulation are of importance for the computation of an absolute geoid or in other words, the unification and intercomparison of the Australian geoid with the international geoid.

The geopotential model that provides the closest statistical fit to these "ground truth" data will be considered to be the most suitable model to adopt for the determination of the Australian gravimetric geoid. Such a verification is important for the gravity field gridding and geoid determination when combining a geopotential model with Stokes's formula because a best fitting model can reduce the impact of the assumptions and approximations inherent to Stokes's formula (§4.5). Small or preferably zero mean residual gravity anomalies are required for Stokes integral by FFT to reduce spectral leakage (§4.5).

This investigation is a logical continuation of the work of Kearsley and Holloway (1989) and Kearsley and Govind (1991). Similar tests have also been reported in
Taiwan by Kao and Bethel (1992) and in Spain by Gil et al. (1994). A paper on these findings was published by Zhang and Featherstone (1995).

3.7.2 Geopotential Models Available

The past two decades have seen the development of a sequence of global geopotential models of increasing spherical harmonic degree and order, and hence resolution. The most recent models are complete to degree and order 360 and can provide long and medium wavelength information of the Earth’s gravity field to a resolution (half wavelength) of approximately 55 km at the equator.

Several global geopotential coefficient models are available for this research from the International Geoid Service (IGeS) in Milano, Italy (Brovelli and Migliaccio, 1994), and Sideris (1996b), Gruber and Anzenhofer (1993) and Gruber et al. (1995). Some of the models used in this study are listed in Table 3.4. Other models held in the School of Surveying and Land Information are GRIM series (Reigber et al., 1985; Reigber et al., 1992; Schwintzer et al., 1990), WGS84-EGM (Kumar, 1984), GEM series (Marsh et al., 1990; Lerch et al., 1994), JGM series (Nerem et al., 1994; Nerem, 1994; Tapley et al., 1996), IFE88E2 (Basic et al., 1989), GPM2 (Wenzel, 1985). The degree of spherical harmonic expansion \( M \) gives the spatial resolution of each model, where half a degree is equivalent to a harmonic expansion of 360.

Table 3.4 List of global geopotential models currently available for the computation of the Australian geoid \( (M \) is the degree of the expansion)

<table>
<thead>
<tr>
<th>model</th>
<th>( M )</th>
<th>reference(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSU81</td>
<td>180</td>
<td>Rapp (1981a)</td>
</tr>
<tr>
<td>OSU86E</td>
<td>360</td>
<td>Rapp and Cruz (1986)</td>
</tr>
<tr>
<td>OSU89B</td>
<td>360</td>
<td>Rapp and Pavlis (1990)</td>
</tr>
<tr>
<td>OSU91A</td>
<td>360</td>
<td>Rapp et al. (1991)</td>
</tr>
<tr>
<td>GFZ93A/B</td>
<td>360</td>
<td>Gruber and Anzenhofer (1993)</td>
</tr>
<tr>
<td>GFZ95A</td>
<td>360</td>
<td>Gruber et al. (1995)</td>
</tr>
<tr>
<td>EGM96</td>
<td>360</td>
<td>Rapp and Nerem (1994); Pavlis et al. (1996)</td>
</tr>
</tbody>
</table>

The EGM96 geopotential model was released at the time of writing this thesis. EGM96 is a collaborative project between Goddard Space Flight Centre (GSFC) and Defense Mapping Agency (DMA) based on DMA’s best 30’×30’ mean gravity
anomalies and GSFC’s extension collection of satellite tracking data (Nerem et al., 1995). Details for the production of this model are given by Pavlis (1996). Tests indicate that the EGM96 is the best fit GGM in many places of the world (Sideris, 1996b). Therefore, EGM96 is also included to see if the new GGM is superior to its predecessors in the Australian area. Note that some parameters used for EGM96 (i.e. \( GM=3986004.415 \times 10^8 \text{m}^2\text{s}^{-2} \) and \( a=6378136.3 \text{m} \)) are different for other GGMs (Pavlis et al., 1996).

3.7.3 Statistical Comparisons and Numerical Analysis

As stated above, the best fit GGM is verified by using the following statistical comparisons:

1. The differences between geoid undulations computed from various geopotential models and geometrically-derived geoid heights at points of the 59 AFN/ANN with optical levelling data, and three other regional GPS/levelling networks on the AHD in Western Australia, Australian Capital Territory and Victoria (§3.2).

2. The differences between gravity anomalies computed from various geopotential models and gravity anomalies derived from the recently verified release (Featherstone et al., 1997) of the AGSO gravity data base. This includes both land and marine gravity observations.

3. The differences between gravity anomalies computed from various geopotential models and gravity anomalies derived from satellite altimetry offshore Australia (Sandwell and Smith, 1997).

The best fit GGM is expected to be best fit to not only the geoid undulations but also to the gravity anomalies in Australia. The statistics of these ‘ground truth’ data are summarised in Table 3.5.

<table>
<thead>
<tr>
<th>Data</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
<th>points</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFN/ANN (m)</td>
<td>71.30</td>
<td>-32.24</td>
<td>12.64</td>
<td>27.06</td>
<td>23.93</td>
<td>59</td>
</tr>
<tr>
<td>land ( \Delta g ) (mgal)</td>
<td>100.59</td>
<td>-91.99</td>
<td>3.23</td>
<td>22.73</td>
<td>22.50</td>
<td>526,091</td>
</tr>
<tr>
<td>marine ( \Delta g ) (mgal)</td>
<td>154.96</td>
<td>-212.56</td>
<td>-3.62</td>
<td>31.66</td>
<td>31.16</td>
<td>111,396</td>
</tr>
<tr>
<td>sat. alt. ( \Delta g ) (mgal)</td>
<td>157.57</td>
<td>-194.57</td>
<td>-3.02</td>
<td>32.38</td>
<td>31.99</td>
<td>1,785,740</td>
</tr>
</tbody>
</table>

(a) Geoid height comparisons at GPS and levelling stations

The GGM geoid height can be easily computed from equation (2.33) using the algorithms of either Rapp (1982) or Rizos (1979). In this case, the routines of Rapp
were utilised as these produce geoid heights at discrete points, whereas Rizos's recursive routines, while more efficient and faster, compute geoid heights on a regular grid. This necessitates that interpolation be used to obtain geoid heights at other than grid points. The statistical differences between each GGM and the "ground truth" geometrical geoid heights are summarised in Tables 3.6 through 3.9.

Since the accuracy of the AFN/ANN/AHD geoid is about 28 cm (§3.6.1), the comparisons in Table 3.6 do not give a firm indication that any global geopotential models GFZ93A/B, GFZ95A, OSU91A and EGM96 perform well in terms of STD fit (within a range of 10cm). However, the OSU91A, GFZ95A and EGM96 geopotential models were computed using different data sources and data processing procedures and have not only lower standard deviations but also smaller mean differences and RMS errors. Therefore, any one of these three geopotential models is recommended to be the best fit GGM to the geometrical geoid in Australia.

Table 3.6 Statistical comparisons of geoid heights between geopotential models and geometrical geoid heights at the 59 AFN/ANN/AHD stations (units in metres)

<table>
<thead>
<tr>
<th>Model</th>
<th>Max (m)</th>
<th>Min (m)</th>
<th>Mean (m)</th>
<th>RMS (m)</th>
<th>STD (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSU81</td>
<td>1.552</td>
<td>-2.131</td>
<td>-0.391</td>
<td>0.94</td>
<td>0.85</td>
</tr>
<tr>
<td>OSU86E</td>
<td>0.984</td>
<td>-2.582</td>
<td>-0.356</td>
<td>0.75</td>
<td>0.66</td>
</tr>
<tr>
<td>OSU89B</td>
<td>1.713</td>
<td>-0.677</td>
<td>0.199</td>
<td>0.52</td>
<td>0.48</td>
</tr>
<tr>
<td>OSU91A</td>
<td>1.189</td>
<td>-0.990</td>
<td>0.080</td>
<td>0.49</td>
<td>0.45</td>
</tr>
<tr>
<td>GFZ93A</td>
<td>1.306</td>
<td>-0.399</td>
<td>0.535</td>
<td>0.67</td>
<td>0.40</td>
</tr>
<tr>
<td>GFZ93B</td>
<td>1.236</td>
<td>-0.643</td>
<td>0.323</td>
<td>0.52</td>
<td>0.41</td>
</tr>
<tr>
<td>GFZ95A</td>
<td>0.955</td>
<td>-0.562</td>
<td>0.111</td>
<td>0.36</td>
<td>0.35</td>
</tr>
<tr>
<td>EGM96</td>
<td>0.851</td>
<td>-0.912</td>
<td>-0.014</td>
<td>0.41</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Considering the accuracy of the three local GPS/AHD networks (§3.6.2), the comparisons in Tables 3.7 through 3.9 indicate that all the global geopotential models GFZ93A/B, GFZ95A, OSU91A and EGM96 perform equally well in terms of STDs. Since these networks are local in nature, their absolute accuracy depends on the accuracy of the local reference stations. Therefore, the mean differences in Tables 3.7 to 3.9 should not be used for comparisons. The STDs from the models OSU91A, GFZ95A, and EGM96 are all approximately equal. Again, any one of these three geopotential models is recommended to be the best fit to the geoid in Australia and it
is hard to discern which one is the best without further comparison with gravity anomalies.

Table 3.7 Statistical comparisons of geoid heights between geopotential models and geometrical geoid heights at the 18 Victorian GPS/AHD stations

<table>
<thead>
<tr>
<th>Model</th>
<th>Max (m)</th>
<th>Min (m)</th>
<th>Mean (m)</th>
<th>RMS (m)</th>
<th>STD (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSU81</td>
<td>1.268</td>
<td>-0.954</td>
<td>-0.068</td>
<td>0.55</td>
<td>0.54</td>
</tr>
<tr>
<td>OSU86E</td>
<td>0.173</td>
<td>-1.245</td>
<td>-0.450</td>
<td>0.56</td>
<td>0.33</td>
</tr>
<tr>
<td>OSU89B</td>
<td>1.055</td>
<td>-0.719</td>
<td>0.234</td>
<td>0.51</td>
<td>0.45</td>
</tr>
<tr>
<td>OSU91A</td>
<td>0.288</td>
<td>-0.682</td>
<td>-0.178</td>
<td>0.30</td>
<td>0.24</td>
</tr>
<tr>
<td>GFZ93A</td>
<td>1.313</td>
<td>-0.153</td>
<td>0.622</td>
<td>0.73</td>
<td>0.39</td>
</tr>
<tr>
<td>GFZ93B</td>
<td>0.955</td>
<td>-0.415</td>
<td>0.339</td>
<td>0.48</td>
<td>0.34</td>
</tr>
<tr>
<td>GFZ95A</td>
<td>0.824</td>
<td>-0.575</td>
<td>0.190</td>
<td>0.42</td>
<td>0.37</td>
</tr>
<tr>
<td>EGM96</td>
<td>0.332</td>
<td>-0.648</td>
<td>-0.039</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 3.8 Statistical comparisons of geoid heights between geopotential models and geometrical geoid heights at the 21 Western Australian GPS/AHD stations

<table>
<thead>
<tr>
<th>Model</th>
<th>Max (m)</th>
<th>Min (m)</th>
<th>Mean (m)</th>
<th>RMS (m)</th>
<th>STD (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSU81</td>
<td>-0.651</td>
<td>-2.241</td>
<td>-1.280</td>
<td>1.38</td>
<td>0.52</td>
</tr>
<tr>
<td>OSU86E</td>
<td>-0.871</td>
<td>-1.688</td>
<td>-1.247</td>
<td>1.27</td>
<td>0.23</td>
</tr>
<tr>
<td>OSU89B</td>
<td>-0.050</td>
<td>0.737</td>
<td>0.242</td>
<td>0.33</td>
<td>0.22</td>
</tr>
<tr>
<td>OSU91A</td>
<td>0.610</td>
<td>0.026</td>
<td>0.261</td>
<td>0.30</td>
<td>0.15</td>
</tr>
<tr>
<td>GFZ93A</td>
<td>0.416</td>
<td>-0.052</td>
<td>0.120</td>
<td>0.16</td>
<td>0.11</td>
</tr>
<tr>
<td>GFZ93B</td>
<td>0.179</td>
<td>-0.290</td>
<td>-0.116</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>GFZ95A</td>
<td>0.089</td>
<td>-0.356</td>
<td>-0.198</td>
<td>0.23</td>
<td>0.11</td>
</tr>
<tr>
<td>EGM96</td>
<td>0.144</td>
<td>-0.383</td>
<td>-0.122</td>
<td>0.18</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 3.9 Statistical comparisons of geoid heights between geopotential models and geometrical geoid heights at the 86 Australian Capital Territory GPS/AHD stations

<table>
<thead>
<tr>
<th>Model</th>
<th>Max (m)</th>
<th>Min (m)</th>
<th>Mean (m)</th>
<th>RMS (m)</th>
<th>STD (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSU81</td>
<td>-0.219</td>
<td>-1.699</td>
<td>-1.208</td>
<td>1.26</td>
<td>0.36</td>
</tr>
<tr>
<td>OSU86E</td>
<td>-1.385</td>
<td>-2.082</td>
<td>-1.693</td>
<td>1.70</td>
<td>0.18</td>
</tr>
<tr>
<td>OSU89B</td>
<td>0.306</td>
<td>-0.259</td>
<td>0.072</td>
<td>0.17</td>
<td>0.15</td>
</tr>
<tr>
<td>OSU91A</td>
<td>-0.671</td>
<td>0.1246</td>
<td>0.912</td>
<td>0.93</td>
<td>0.16</td>
</tr>
<tr>
<td>GFZ93A</td>
<td>0.463</td>
<td>-0.052</td>
<td>0.242</td>
<td>0.29</td>
<td>0.15</td>
</tr>
<tr>
<td>GFZ93B</td>
<td>0.133</td>
<td>-0.446</td>
<td>-0.102</td>
<td>0.19</td>
<td>0.16</td>
</tr>
<tr>
<td>GFZ95A</td>
<td>0.073</td>
<td>-0.514</td>
<td>-0.162</td>
<td>0.23</td>
<td>0.16</td>
</tr>
<tr>
<td>EGM96</td>
<td>-0.733</td>
<td>-1.398</td>
<td>-1.022</td>
<td>1.040</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Comparing Tables 3.6 through 3.9 with Table 3.2, it can be seen that the difference precisions of the GPS/AHD geometrical geoid for different GPS networks are correlated with the different precision of the GPS/AHD results, assuming that the accuracy of the GGMs over Australia is homogeneous. The most precise GPS/AHD
results in Western Australia give the best fit with the GGMs, whilst the least precise GPS/AHD results in AFN/ANN stations give the biggest STD difference. It is also shown that the Australian Capital Territory GPS/AHD results have a similar precision with Western Australia GPS/AHD results inspite of the combination of unadjusted individual surveys (§3.6.2). However, this estimation is very rough due to the precision and resolution to be achievable for current GGMs.

(b) Comparisons with AGSO's free-air gravity anomalies

Again, an optimal fitting GGM is expected to be best fit with both the geometric geoid and the gravity anomalies in the same area. The GGM gravity anomalies are computed using equation (2.34) and the smallest STD of the residuals imply the best fitting gravity anomalies and hence GGM. The gravity anomalies derived from the geopotential models listed in Table 3.4 are compared against continental and marine free-air gravity anomalies around Australia.

The computation of the GGM gravity anomaly at each point in the AGSO gravity data set using Rapp's (1982) routines takes approximately 100 hours on a Sun Sparcstation 10 model 30. Since the GGMs are computed based on global half-degree mean gravity anomalies, half degree block mean values are chosen as the basic computation element for the comparisons. This approach allowed the fast algorithms of Rizos (1979) to be used and reduced the computation time to approximately one hour per GGM. The results for these comparisons are summarised for land gravity data in Table 3.10, and for marine gravity data in Table 3.11.

In Table 3.10, OSU91A provides the second lowest standard deviation with AGSO's land gravity data. Using only the standard deviations, because of the effect of the zero-order term, the OSU89B geopotential model provides the best fit to the land gravity data (Table 3.10). However, the model is only marginally better than OSU91A and should not be used further, as OSU89B contains geophysically predicted gravity data which makes some assumptions of the isostatic equilibrium of the Earth. OSU91A provides lower STD than EGM96.
Table 3.10 Statistical comparisons between high degree geopotential models and free-air gravity anomalies on the Australian continent (units in mgal)

<table>
<thead>
<tr>
<th>Model</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSU81</td>
<td>71.724</td>
<td>-60.552</td>
<td>-2.208</td>
<td>13.592</td>
<td>13.411</td>
</tr>
<tr>
<td>OSU86F</td>
<td>50.996</td>
<td>-39.049</td>
<td>-1.064</td>
<td>7.239</td>
<td>7.161</td>
</tr>
<tr>
<td>OSU86E</td>
<td>51.103</td>
<td>-39.197</td>
<td>-1.084</td>
<td>7.248</td>
<td>7.167</td>
</tr>
<tr>
<td>OSU89B</td>
<td>60.937</td>
<td>-33.987</td>
<td>-0.550</td>
<td>6.127</td>
<td>6.151</td>
</tr>
<tr>
<td>OSU91A</td>
<td>57.663</td>
<td>-33.663</td>
<td>-0.776</td>
<td>6.319</td>
<td>6.272</td>
</tr>
<tr>
<td>GFZ93A</td>
<td>67.713</td>
<td>-33.934</td>
<td>-0.487</td>
<td>6.673</td>
<td>6.655</td>
</tr>
<tr>
<td>GFZ93B</td>
<td>64.291</td>
<td>-33.805</td>
<td>-0.179</td>
<td>6.622</td>
<td>6.620</td>
</tr>
<tr>
<td>GFZ95A</td>
<td>63.555</td>
<td>-36.871</td>
<td>-0.549</td>
<td>6.508</td>
<td>6.485</td>
</tr>
<tr>
<td>EGM96</td>
<td>51.800</td>
<td>-48.356</td>
<td>-1.024</td>
<td>7.021</td>
<td>6.945</td>
</tr>
</tbody>
</table>

In Table 3.11, OSU91A, EGM96 and OSU89B provide similar standard deviations with AGSO's marine gravity data. Because the geoid undulation on land is of most importance and the land gravity measurements are more accurate than the marine gravity, therefore, this comparison should be made in conjunction with Table 3.10. Therefore, OSU91A is recommended to be the best fit GGM.

Table 3.11 Statistical comparisons between high degree geopotential models and marine gravity anomalies (units in mgal)

<table>
<thead>
<tr>
<th>Model</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSU81</td>
<td>104.698</td>
<td>-102.919</td>
<td>-0.741</td>
<td>17.201</td>
<td>17.185</td>
</tr>
<tr>
<td>OSU86F</td>
<td>78.603</td>
<td>-78.878</td>
<td>-1.787</td>
<td>11.558</td>
<td>11.419</td>
</tr>
<tr>
<td>OSU86E</td>
<td>78.451</td>
<td>-78.759</td>
<td>-1.775</td>
<td>11.553</td>
<td>11.416</td>
</tr>
<tr>
<td>OSU89B</td>
<td>63.299</td>
<td>-63.977</td>
<td>-1.680</td>
<td>10.688</td>
<td>10.555</td>
</tr>
<tr>
<td>OSU91A</td>
<td>66.238</td>
<td>-63.892</td>
<td>-1.513</td>
<td>10.546</td>
<td>10.437</td>
</tr>
<tr>
<td>GFZ93B</td>
<td>92.015</td>
<td>-101.747</td>
<td>-1.821</td>
<td>11.685</td>
<td>11.542</td>
</tr>
<tr>
<td>GFZ95A</td>
<td>104.639</td>
<td>-137.108</td>
<td>-1.502</td>
<td>12.833</td>
<td>12.745</td>
</tr>
<tr>
<td>EGM96</td>
<td>52.296</td>
<td>-58.841</td>
<td>-0.630</td>
<td>9.529</td>
<td>9.508</td>
</tr>
</tbody>
</table>

(c) Comparisons with satellite-altimetry-derived gravity anomalies
The satellite-altimetry-derived gravity anomalies around Australia from Sandwell and Smith (1997) are used in this comparison. Half-degree mean gravity anomalies are used for comparisons to reduce computer time and the statistical results are listed in Table 3.12.

Table 3.12 illustrates that, again, OSU91A is the superior GGM in this region. Note also that the standard deviations are marginally greater than those for the marine
gravity anomalies (Table 3.11). This partly supports the evaluation described in §3.5, whereby the satellite altimetric gravity anomalies have slightly less accuracy than the AGSO marine gravity observations.

Table 3.12 Statistical comparisons between high degree geopotential models and satellite-altimetry-derived gravity anomalies (units in mgal)

<table>
<thead>
<tr>
<th>Model</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSU81</td>
<td>103.992</td>
<td>-116.859</td>
<td>-0.553</td>
<td>16.658</td>
<td>16.649</td>
</tr>
<tr>
<td>OSU86F</td>
<td>67.763</td>
<td>-133.774</td>
<td>-1.007</td>
<td>10.149</td>
<td>10.099</td>
</tr>
<tr>
<td>OSU86E</td>
<td>67.844</td>
<td>-133.638</td>
<td>-1.005</td>
<td>10.143</td>
<td>10.093</td>
</tr>
<tr>
<td>OSU89B</td>
<td>82.736</td>
<td>-130.717</td>
<td>-0.796</td>
<td>10.039</td>
<td>10.007</td>
</tr>
<tr>
<td>OSU91A</td>
<td>83.584</td>
<td>-130.494</td>
<td>-0.609</td>
<td>10.007</td>
<td>9.989</td>
</tr>
<tr>
<td>GFZ93B</td>
<td>87.021</td>
<td>-133.595</td>
<td>-1.192</td>
<td>12.592</td>
<td>12.535</td>
</tr>
<tr>
<td>GFZ95A</td>
<td>101.968</td>
<td>-134.706</td>
<td>-1.055</td>
<td>13.474</td>
<td>13.348</td>
</tr>
<tr>
<td>EGM96</td>
<td>138.577</td>
<td>-112.281</td>
<td>0.022</td>
<td>10.170</td>
<td>10.170</td>
</tr>
</tbody>
</table>

3.7.4 Discussion

Based on the previous statistical comparisons, it can be concluded that the recent degree 360 OSU and GFZ series and EGM96 geopotential models all perform very well in recovering the Australian geoid and gravity anomalies. However, the OSU91A model is consistently the best among these. It can recover geoid heights with a standard deviation of approximately 0.45m and free-air gravity anomalies with a standard deviation of approximately 6 mgal on land, and 10 mgal at sea. Therefore, the OSU91A model is recommended as the most appropriate GGM for Australian regional geoid determination.

Figure 3.6 shows a minimum tension spline surface (Smith and Wessel, 1990; Wessel and Smith, 1991) fitted to the differences between OSU91A and the 59 AFN/ANN/AHD controls. There are evident long wavelength (>500km) differences between these two geoid estimates over Australia. These could be attributed to errors in the geopotential model, errors in the GPS, or the systematic offset between AHD and true geoid (§3.6.3a), or a combination of these factors. At present, it is difficult to isolate the exact source or sources of this error. However, some of the differences are due to spikes or differences at a single station (e.g. station [117°E, 21°S]), which suggests the presence of a gross levelling error (or: AHD error). The long and
medium wavelength (>110km) errors of the OSU91A will be further investigated using gravity data in §8.2.3.

Figure 3.6 Geoid differences between OSU91A and the 59 levelled AFN/ANN/AHD stations (unit=m, Linear projection)

A final observation is on the relative ability for the GGMs to recover gravity and geoid information according to the spectral relation given in equation (2.38). Comparing the geoid undulations in Table 3.5 with those in Table 3.6, it can be concluded that approximately 98 percent of the Australian geoid signal is supplied by the recent, high-degree GGMs. This value reduces slightly to 96 percent for the degree 180 expansion. Conversely, the degree-360 and degree-180 expansions of the gravity anomalies can only remove approximately 67 percent and 50 percent of the terrestrial gravity anomalies, respectively (compare Table 3.5 and Tables 3.6 and 3.7). As expected, this demonstrates that the majority of the geoid's power is contained in the low degree harmonics, whereas the power in the gravity anomalies is held in the intermediate and
high degree harmonics. These observations agree with the gravity field power spectrum given by Schwarz (1984) and equation (2.38).

As shown in Tables 3.6 through 3.12, the EGM96 performs almost equally well with OSU91A, although OSU91A is marginally better than EGM96 for gravity anomaly recovery (except for marine gravity). This is because no significant number of new gravity measurements over Australia were used in EGM96. Therefore, OSU91A is recommended for the best fitting GGM of Australia. However, the best way to determine a locally optimal GGM is, theoretically, to tailor the current GGM using regional refined, denser gravity data (Kearsley and Forsberg, 1990).

3.8 Summary
This chapter describes all the data used in this study. These data include gravity observations on land and in marine areas, satellite-derived gravity anomalies, GPS and spirit levelling data from AFN/ANN stations and three local networks, spot heights and global geopotential models. These data have been validated and evaluated using the procedures described in this chapter.

The precision and accuracy of the geometric geoid, particularly the GPS ellipsoidal height differences and AHD heights, is analysed. It is estimated that the accuracies of the land and marine gravity measurements are 1–2mgal and 2–6mgal respectively. Studies indicate that the accuracy of the satellite altimetry-derived gravity anomalies is approximately 4–7mgal for 10′×10′ block mean values around Australia. This provides the possibility of combining the satellite altimetric gravity anomalies with marine gravity anomalies to improve the coverage and quantity of the marine gravity observations.

A number of GPS/levelling networks are chosen for the evaluation of the best fitting GGM and subsequent gravimetric geoid. It is estimated that the accuracies of these geometric geoids are 28 cm for the whole Australia (AFN/ANN), and 10 cm, 19 cm and 2 cm (plus systematic errors) for Western Australian, Victorian and Australian Capital Territory GPS/AHD networks respectively. An 1′x1′ DTM was produced
using splines in tension from the spot height data base. Furthermore, the best fitting GGM is studied through a series of comparisons with the "ground truth" geoid undulation and gravity data. OSU91A is found to be the best fit geopotential model of Australia.

In addition, some special problems that affect an accurate geoid determination are discussed. These include the unification of the AHD; time variations of the gravity field and definition of the geoid in four-dimensional space; the effects of varying topographic density and the differences between normal and actual gravity gradients; and the effect of deflection of the vertical on GPS/levelling.
AN EVALUATION OF FFT GEOID DETERMINATION TECHNIQUES AND THEIR APPLICATION TO HEIGHT DETERMINATION USING GPS IN AUSTRALIA

CHAPTER FOUR

THE FOURIER TRANSFORM IN GEOID DETERMINATION
Chapter 4

THE FOURIER TRANSFORM IN GEOID DETERMINATION

4.1 Introduction

Generally, large numbers of data are involved in the evaluation of the Earth's gravity field, particularly in geoid determination and terrain reduction. For the interpretation of geophysical data, it is often convenient if the quantity to be analysed is decomposed as a sum of components, a process known as spectral analysis. The Fast Fourier Transform (FFT) technique (Brigham, 1988) and Fast Hartley Transform (FHT) technique (Hartley, 1924; Bracewell, 1984; 1986a) have proven to be very powerful tools for the efficient evaluation of gravity field convolution integrals (e.g. Schwarz et al., 1990; Li, 1993; Tziavos, 1993b; Li and Sideris, 1994b). These techniques can also handle heterogeneous and noisy data (Sideris, 1995), and thus present a very attractive alternative to the classical, time consuming numerical methods.

The theory of Fourier and Hartley transforms has been known for some time. Runge (1905) first presented the FFT concept and Cooley and Tukey (1965) presented an algorithm suitable for implementation on a computer. Hartley (1924) proposed the use of a new kind of transform that, nowadays, when implemented on a computer, is faster and requires less memory than the FFT (Bracewell et al., 1986). Whilst the FFT technique has become a viable, and sometimes indispensable, option in the field of geophysical data interpretation and computation, the FHT is becoming more and more popular in gravity field computation, mainly due to its relative efficiency (Li and Sideris, 1992; Tziavos, 1993b; Sideris and Forsberg, 1991).

This chapter briefly presents the principles of the Fourier transform and its practical application to gravity field convolution integrals. The practical implementation of various approximations of Stokes's kernel are discussed in terms of the FFT. Some problems related to the application of FFT to gravity field convolution integrals, such as the effect of zero padding, edge effects and consistency of the method itself, are also discussed. Moreover, numerical simulations of geoid determination by FFT are also presented (cf. Tziavos, 1996), and recommendations regarding applications of the
FFT to the Australian geoid are made. An optimal procedure for the determination of the Australian gravity field by the FFT technique is suggested. The program used for this computation is a revised version of FFTGEOID, which was originally coded by Y.C. Li and M.G. Sideris of the University of Calgary, Canada.

4.2 Fundamentals of Fourier Transform

4.2.1 The Two-dimensional Fourier Transform and Convolution

The two-dimensional (2-D) Fourier transform or spectrum of an absolute integrable function \( h(x,y) \) is defined to be the integral (Bracewell, 1986b; Champeney, 1973):

\[
\mathbf{F}(h(x, y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) e^{-2\pi i (ux + vy)} \, dx \, dy = H(u, v),
\]

where \( \mathbf{F} \) is the two-dimensional (2-D) Fourier operator;
\( H(u,v) \) is the spectrum of the function \( h(x,y) \);
\( u \) and \( v \) are the wave numbers (spatial circular frequencies) corresponding to the spatial co-ordinates \( x \) and \( y \) respectively; and,
\( i = \sqrt{-1} \) is the imaginary term.

The function \( h(x,y) \) in the space domain can be obtained from its spectrum \( H(u,v) \) by means of the following inverse 2-D Fourier transform:

\[
\mathbf{F}^{-1}(H(u, v)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(u, v) e^{-2\pi i (ux + vy)} \, du \, dv = h(x, y),
\]

where \( \mathbf{F}^{-1} \) is the inverse 2-D Fourier operator. The functions \( h(x,y) \) and \( H(u,v) \) form a Fourier transform pair denoted by:

\[
h(x, y) \xrightarrow{\mathbf{F}} H(u, v) \quad \text{or} \quad H(u, v) \xrightarrow{\mathbf{F}^{-1}} h(x, y).
\]

The convolution (*) of two functions \( h(x,y) \) and \( g(x,y) \) is defined as:

\[
f(x, y) = h(x, y) * g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x', y')g(x'-x, y'-y)dx' \, dy' .
\]

Convolution has the following important properties:

\[
h(x, y) \cdot g(x, y) \xrightarrow{\mathbf{F}} H(u, v) * G(u, v),
\]

\[
f(x, y) = h(x, y) * g(x, y) \xrightarrow{\mathbf{F}} F(u, v) = H(u, v) \cdot G(u, v).
\]

The relationship in equation (4.4) implies that standard gravity field formulae such as Stokes’s integral (equation 2.10), the terrain correction (equation 2.40) and indirect
effect (equation 2.44) can be treated as convolution integrals, and thus evaluated efficiently in the frequency domain. Equation (4.6) indicates that a convolution of two functions in the space domain is equivalent to filtering of one function by the other (multiplication of spectra) in the frequency domain.

4.2.2 The Discrete Fourier Transform (DFT)
The Fourier transform defined by equation (4.1) refers to a continuous data set of infinite length. In reality, a regional gravity or terrain data set is both discrete and finite. Therefore, in practice, the Fourier transform is applied within a limited (finite) window and gridded data. This transform is known as the discrete Fourier transform (DFT).

Suppose that regular gridded data are given in the area \([-X/2, X/2], [-Y/2, Y/2]\) with sampling intervals \(\Delta x\) and \(\Delta y\). Corresponding to the continuous Fourier transform pair (4.1) and (4.2), the DFT pair can be expressed as:

\[
H_D(u_m, v_n) = \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} h(x_k, y_l) e^{-2\pi i (u_m x_k + v_n y_l)} \Delta x \Delta y
\]

(4.7)

\[
h_D(x_k, y_l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} H(u_m, v_n) e^{-2\pi i (u_m x_k + v_n y_l)} \Delta u \Delta v
\]

(4.8)

with

\[
x_k = k\Delta x, \quad (k = 0, 1, \cdots, M - 1)
\]

\[
y_l = l\Delta y, \quad (l = 0, 1, \cdots, N - 1)
\]

\[
u_n = n\Delta v, \quad (n = 0, 1, \cdots, N - 1),
\]

(4.9)

where the subscript '\(D\)' indicates the discrete Fourier transform. The record lengths may be then expressed by:

\[
X = M\Delta x, \quad \text{and} \quad Y = N\Delta y,
\]

(4.10)

where \(M\) and \(N\) are the number of points along the \(x\) and \(y\) directions respectively. Assuming the data to be periodically extended in the 2-D plane, the spectrum also becomes discrete with the following frequency spacings:

\[
\Delta u = \frac{1}{X} = \frac{1}{M\Delta x}, \quad \Delta v = \frac{1}{Y} = \frac{1}{N\Delta y}
\]

(4.11)
According to the Nyquist principle or Shannon’s sampling theorem (Brigham, 1988; Bracewell, 1986b), the spectral estimation of equation (4.7) will only allow the frequencies up to:

\[
\nu_M = \pm \frac{M}{2} \Delta u = \pm \frac{l}{2 \Delta x}, \quad \nu_N = \pm \frac{N}{2} \Delta v = \pm \frac{l}{2 \Delta y}
\]  

(4.12)

to be resolved. This means that the highest frequencies that can be resolved from a data set sampled at \( \Delta x \) and \( \Delta y \) are Nyquist frequencies \( \nu_M \) and \( \nu_N \), or, equally, wavelengths of \( 2\Delta x, 2\Delta y \) in the space domain. This can be regarded as an important concern for subsequent gravity field gridding where grid size should be equal to or less than the average spacing of the gravity observations to preserve the original observed information.

4.3 Stokes’s Integral by FFT and its Various Kernel Approximations

As indicated by Moritz (1968) and others (Sideris, 1994a; 1994b; Schwarz et al., 1990), the planar approximation of Stokes’s integral (2.10) reads

\[
N(x_p, y_p) = \frac{1}{2 \pi \gamma} \left[ \int_{E} \frac{\Delta g dx dy}{l(x, y)} \right] = \frac{1}{2 \pi \gamma} \int \Delta g(x, y) \cdot l(x, y) \, dx \, dy,
\]  

(4.13)

where \( l(x, y) = \sqrt{(x-x_p)^2 + (y-y_p)^2} \), is the planar distance between computation point \((x_p, y_p)\) and the running point \((x, y)\), and \(E\) is a tangent plane through the point of interest \((P)\). This planar approximation formula is valid only in the vicinity of the computation point. To avoid long wavelength errors, the area of local data should not extend to more than several hundreds of kilometres in both directions (Sideris, 1994a). If so, the data should at least be long wavelength filtered by removing the effect of a global model, which is routinely done as part of the remove-restore technique.

In order to apply FFT techniques to the Stokes integral (2.10) in section §2.1.3, the kernel function (2.11) needs some approximations. Equation (4.13) is the planar approximation form of the equation (2.10). In recent years, the study on the kernel approximation has been investigated extensively. This approach has evolved from the simple planar FFT (Schwarz et al., 1990), to the 2-D spherical FFT (Strang van Hees, 1990), to the multi-band FFT (Forsberg and Sideris, 1993), and the precise 1-D FFT.
(Haagmans et al., 1993). The implementation of the various kernel approximations will be discussed next.

4.3.1 Two-dimensional Planar FFT with Discrete and Analytical Spectra

If mean gravity anomalies are given on a $M \times N$ gridded lattice with spacings $\Delta x$ and $\Delta y$ in the $x$ and $y$ directions respectively, the discrete 2-D planar FFT (2DPFFT) form for equation (4.13) is (Tziavos, 1996; Sideris, 1994a):

$$N(i, j) = \frac{1}{2\pi Y} F^{-1} \{ \Delta G \cdot L_N \}, \quad (4.14)$$

where

$$\Delta G(m, n) = F \{ \Delta g(k, l) \} = \Delta x\Delta y \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} \Delta g(k, l) e^{-j2\pi mnk/M + nln/N}, \quad (4.15)$$

$$L_N(m, n) = F \{ l_N(k, l) \} = \Delta x\Delta y \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} l_N(k, l) e^{-j2\pi mnk/M + nln/N}, \quad (4.16)$$

and $(i, j)$, $(m, n)$ and $(k, l)$ express the position of a grid node. Equation (4.14) can also be written as the following analytical kernel spectrum form (Schwarz et al., 1990; Sideris and Li, 1993):

$$N(x, y) = \frac{1}{2\pi Y} F^{-1} \{ F(\Delta g(x, y)) \cdot F(l(x, y)) \}, \quad (4.17)$$

with

$$F(l(x, y)) = L_N(u, v) = 1/(u^2 + v^2)^{1/2}, \quad (4.18)$$

where $u$ and $v$ are the frequencies corresponding to $x$ and $y$ respectively. $L_N$ is called the analytically defined spectrum (Sideris and Li, 1993; Li, 1993). This method is called the 2-D planar FFT method with analytical spectrum (2DPFFTAS). Equation (4.17) can be conveniently implemented by only one direct and one inverse Fourier transform.

4.3.2 Two-dimensional Spherical Approximate FFT Technique

Strang van Hees (1990) proposed to evaluate Stokes's integral on the sphere by means of the spherical 2-D FFT, to reduce the planar approximation errors (2DSAFFT). The discrete Stokes integral can be further expressed as (ibid.)

$$N(\varphi, \lambda) = \frac{K \Delta \varphi \Delta \lambda}{4\pi Y} \sum_{\lambda = \lambda_1}^{\lambda_2} \sum_{\varphi = \varphi_1}^{\varphi_2} S(\psi_{\rho q}) \Delta g(\varphi, \lambda, \varphi) \cos \varphi, \quad (4.19)$$

with

$$S(\psi_{\rho q}) = \frac{1}{s} - 4 - 6s + 10s^2 - (3 - 6s^2) \ln(s + s^2), \quad (4.20)$$
\[ s = \sin \left( \frac{\psi_{pq}}{2} \right), \quad (4.20a) \]

and where \( M \) and \( N \) are the number of parallels and meridians respectively, and \( \varphi \in [\varphi_1, \varphi_M], \lambda \in [\lambda_1, \lambda_N] \) are the latitude and longitude respectively. Equation (4.19) can thus be expressed as:

\[
N(\varphi, \lambda) = \frac{R \Delta \varphi \Delta \lambda}{4 \pi Y} \left\{ S(\psi) \ast \Delta \cos \varphi_m \right\} = \frac{R \Delta \varphi \Delta \lambda}{4 \pi Y} \mathbf{F}^{-1} \left\{ \mathbf{F}(S) \mathbf{F}[\Delta \cos \varphi_m] \right\}, \quad (4.21)
\]

where

\[ \varphi_m = \frac{\varphi_1 + \varphi_M}{2}, \quad \text{and} \quad \cos \varphi_p = \cos \varphi_q = \cos \varphi_m. \quad (4.22) \]

As expressed by Strang van Hees (1990), the term \( \sin^2 \left( \frac{\psi_{pq}}{2} \right) \) reads

\[ \sin^2 \left( \frac{\psi_{pq}}{2} \right) = \sin^2 \left( \frac{\Delta \varphi_{pq}}{2} \right) + \sin^2 \left( \frac{\Delta \lambda_{pq}}{2} \right) \cos^2 \varphi_m, \quad (4.23) \]

or in a more accurate form expressed by Forsberg and Sideris (1993) as

\[ \sin^2 \left( \frac{\psi_{pq}}{2} \right) = \sin^2 \left( \frac{\Delta \varphi_{pq}}{2} \right) + \sin^2 \left( \frac{\Delta \lambda_{pq}}{2} \right) \left( \cos^2 \varphi_m - \sin^2 \frac{\Delta \varphi_{pq}}{2} \right). \quad (4.24) \]

Equation (4.24) is a function of \( \Delta \varphi_{pq} \) and \( \Delta \lambda_{pq} \). Therefore, Stokes' function (4.20) becomes a rotationally symmetric function. The approximation of equation (4.22) makes it possible to compute geoid undulations over a large area on the sphere at all grid points simultaneously by the 2-D FFT.

The approximation equations (4.23) and (4.24) still lead to an error into the geoid computation. The main drawback of both the planar and the spherical 2-D FFT methods is that, due to these approximations in the kernel function, only approximate solutions can be achieved (Farell, 1991; Featherstone, 1992; Forsberg and Sideris, 1993; Sideris, 1995). In order to reduce these approximation errors, Forsberg and Sideris (1993) recommend the multi-band FFT approach to reduce part of these errors. According to Forsberg and Sideris (1993), the region under consideration is divided into a set of equidistant latitude zones \( \varphi_l (l = 1, 2, \ldots) \) and the Stokes kernel is evaluated in different bands to reduce the approximation error. The geoid computation in each band is then merged to produce the geoid over the whole area.
4.3.3 One-dimensional Spherical Exact FFT Approach (1DSEFFT)

Haagmans et al. (1993) propose an exact 1-D FFT approach to evaluate the convolution integral on the sphere without any approximation. This method allows us to use an exact kernel function everywhere in the integration area. The geoid undulation by 1-D approach can be expressed as follows:

\[ N(\varphi_i, \lambda_k) = \frac{R \Delta \varphi \Delta \lambda}{4 \pi Y} \sum_{j=0}^{N-1} \left[ \frac{1}{2} \int_{F} \left[ \Delta g(\varphi_j, \lambda_k) \cos \varphi_j \right] F_j \left[ S(\varphi_j, \varphi_j, \varphi_k) \right] \right], \]  

(4.25)

where, \( \varphi_i = \varphi_1, \varphi_2, \ldots, \varphi_N \), \( S(\varphi_i, \varphi_j, \varphi_k) \) is given by (4.20). For the results on a fixed parallel of latitude \( \varphi_2 \) and using data along a parallel latitude \( \varphi_j \), \( \varphi \) is only a function of \( \lambda_k - \lambda_i \) and \( \Delta g \) is a function of \( \lambda_j \) (Sideris, 1994a; Haagmans et al., 1993).

The main advantage of the 1-D FFT is that it gives the same results as those obtained through direct numerical integration, provided that 100 percent zero padding is applied to eliminate edge effects (Li, 1993). In addition, considerable computer memory can be saved compared to that of the 2-D spherical FFT since only one 1-D complex array is required. Moreover, this approach is far more computationally efficient than the classical direct numerical integration although it is more time consuming than the 2-D FFT integral methods mentioned above, because of the parallel by parallel summation in the computation. Numerical results using the 1-D FFT method can be found in She (1993), Sideris and She (1995) for computation over Canada, in Tziavos (1996) for computation over Europe, and in Featherstone et al. (1996) for computation over a smaller test area in Western Australia.

4.3.4 Integral Radius (Cap Size) of the Stokes integral by FFT

The spectral development of the FFT technique requires the convolution integration covering the whole residual gravity anomaly grid, whereas, direct numerical integration of the Stokes formula uses the residual gravity anomalies over a certain capsize. There is no theoretical objection to use the residual gravity anomalies over a cap in the FFT. However, in order to study different gravity anomaly contributions over different cap sizes, a modification can be made to use only the residual gravity anomalies within a specified cap in the convolution process. Here Stokes's kernel is calculated to a certain capsize beyond which its values are set to zero prior to Fourier transform.
4.4 Some FFT-related Problems

The critical problems related to FFT geoid computation are the effects of the sampling interval, spectral leakage and padding of the original data. Another important consideration is the boundary effect of the various FFT methods mentioned in §4.3.

4.4.1 Effect of Sampling Interval and Spectral Leakage

Obviously, the discrete Fourier or Hartley transform process will introduce errors due to the discretisation and the truncation of the infinite power series. These errors are known as aliasing and leakage. Aliasing is caused by undersampling and can only be minimised by using densely sampled data. Leakage is so called because some energy leaks from the main lobe to its side lobes of power spectrum (Bracewell, 1986b; Brigham, 1988). It is caused by the finite extent of the data length which does not permit longer wavelengths to be accurately represented.

4.4.2 Cyclic Convolution and Effect of Padding

The Fourier method assumes that infinite periodicity of the input data. The truncation of the periodic samples introduces spectral leakage into the computation which causes errors. This effect, according to Bracewell (1986b) and Sideris (1987), is called cyclic convolution. To partially compensate for this error, a padding technique can be applied. Padding applies a number of zeros to the boundaries around the values of the input data matrix, while the kernel values are computed accordingly in the extended areas instead of zero values. Studies indicate that 100 percent zero padding can improve the results (Forsberg and Solheim, 1989; Harrison and Dickinson, 1989). This holds for both planar and spherical Stokes's integrals and terrain correction computation by FFT (Li, 1993; Sideris and Li, 1993). The results obtained in this way are equal to the results from a direct numerical integral. However, without the exact model values, it is difficult to quantify the improvement. Furthermore, the rapid decrease of almost all of the integral kernel functions, such as Stokes's integral and the terrain correction integral, can relieve the problem of cyclic convolution further (Sideris, 1987; Zhao, 1989; Sideris and Li, 1993; Haagmans et al., 1993). This is because the effect of the data in the distant zones is less.
4.4.3 Phase Shifting

Another problem said to be unique to the FFT method by Sideris (1987) and Brigham (1988), is phase shifting. The phase shifting is caused by the origin inconsistency of the co-ordinate system for the data and assumed origin of the subroutine of the FFT algorithm. It can be easily corrected by a co-ordinate translation of the gravity anomalies.

4.5 Numerical Studies on Various Kernel Approximations

To study the effectiveness of the various kernel approximation methods (§4.3) in terms of accuracy, a simulated gravity field has been produced over the whole Australia ($\varphi$: [-47.0°, -5.0°], $\lambda$: [110.0°, 160.0°]). This ‘ground truth’ gravity field is produced through the OSU91A geopotential model which is the best fit geopotential model for Australia (see Chapter 3 and Zhang and Featherstone, 1995).

The gravity anomalies ($\Delta g_{GGM}$) and geoid undulations ($N_{GGM}$) are computed using OSU91A GGM and equations (2.33) and (2.34) in a 5′×5′ grid. These 5′×5′ gravity anomalies (or residual gravity anomalies in a given frequency band) are then input to the FFT geoid computation formulas with different kernel approximations (equations 4.14, 4.17, 4.19 and 4.25) to produce geoid undulation ($N'$). The effectiveness of the various kernel approximation methods can be tested using the differences between the two geoids ($N_{GGM}$ and $N'$). Similar work was conducted by Tziavos (1996) for Europe. This is an application and further investigation to the Australian case.

4.5.1 Test Data

As is the case in the remove-restore technique, a reference field (global geopotential model) is always used to reduce the long wavelength effect of the gravity field. Therefore, the residual gravity field models used are OSU91A geopotential model from degrees 50 to 360, 90 to 360 and 180 to 360; namely band 1 (50–360); band 2 (90–360) and band 3 (180–360). The long wavelength components in the three bands (say degrees less than 50, 90 or 180 respectively) are removed accordingly to reduce the spectral leakage errors and thus a smaller integration area can be used in the Stokes integral.
According to Rapp's rule of thumb (Rapp, 1981a) \( \theta^2 = 180^\circ / n_{max} \), the theoretical side lengths (\( \theta^\circ \)) of the integral cap are 3.6\(^\circ\), 2\(^\circ\) and 1\(^\circ\) corresponding to the three bands. Theoretically, the highest resolution of the OSU91A is, according to Rapp's rule, 0.5\(^\circ\). Also, to reduce aliasing effects in the FFT, a denser grid size of 5' was used, which is one sixth of the OSU91A resolution. This produces 505x601=303505 grid lattice values. Another reason for choosing a 5' grid size is that the resolution of Australian gravity observations is around 5' (see Chapter 3). Therefore, the simulated study results in this section are more relevant to the full practical implementation for the Australian geoid.

4.5.2 Overall Effects

Table 4.1 lists the statistical results of the residual geoid undulations and gravity anomalies from the OSU91A geopotential model corresponding to the above three bands. The magnitude of the geoid undulations in Table 4.1 reduces more quickly than the gravity anomalies with increasing frequency bands from 50 to 180, as was predicted by equation (2.38). In addition, the roughness of the gravity field reduces explicitly from band 1 to band 3. In the case where the 90 degree and order reference gravity field is removed, the results are almost unbiased.

<table>
<thead>
<tr>
<th>OSU91A GGM (5'x5')</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>geoid undulation N (metres)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50-360</td>
<td>9.56</td>
<td>-15.35</td>
<td>-0.030</td>
<td>1.67</td>
<td>1.67</td>
</tr>
<tr>
<td>90-360</td>
<td>6.86</td>
<td>-8.50</td>
<td>0.002</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>180-360</td>
<td>2.47</td>
<td>-2.93</td>
<td>0.001</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>gravity anomaly ( \Delta g ) (mgals)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50-360</td>
<td>193.38</td>
<td>-232.59</td>
<td>-0.210</td>
<td>24.45</td>
<td>24.45</td>
</tr>
<tr>
<td>90-360</td>
<td>160.88</td>
<td>-164.41</td>
<td>0.07</td>
<td>19.95</td>
<td>19.95</td>
</tr>
<tr>
<td>180-360</td>
<td>91.00</td>
<td>-107.27</td>
<td>0.05</td>
<td>11.83</td>
<td>11.83</td>
</tr>
</tbody>
</table>

The small bias of -0.21 mgal for band 50-360 in Table 4.1, is most probably due to a long wavelength effect. Its influence is critical to the results of the residual geoid because it could contribute to leakage in the FFT computation. The significance of this effect can be seen in Table 4.2 where statistical differences are very large, and this is more serious around the boundaries (§4.5.3).
Tables 4.2, 4.3 and 4.4 list the statistical comparisons between the geoid undulations from different kernels in the FFT method and the ‘ground truth’ residual geoid undulations from OSU91A model for the three bands tested. Figures 4.1, 4.2, 4.3 and 4.4 show a representative sample of the error features of the various kernel approximations. Only band 180-360 is shown because other bands give similar results, and are thus omitted.

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>discrete</td>
<td>6.22</td>
<td>-2.42</td>
<td>0.34</td>
<td>0.46</td>
<td>0.31</td>
</tr>
<tr>
<td>analytical</td>
<td>5.74</td>
<td>-2.16</td>
<td>0.33</td>
<td>0.45</td>
<td>0.30</td>
</tr>
<tr>
<td>Spherical</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-D</td>
<td>6.38</td>
<td>-2.44</td>
<td>0.28</td>
<td>0.46</td>
<td>0.37</td>
</tr>
<tr>
<td>2-D</td>
<td>6.35</td>
<td>2.25</td>
<td>-0.30</td>
<td>0.51</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Table 4.3 Differences between the GGM and FFT geoids using different approximations of Stokes’s kernel (band 90-360, with 100% zero padding, unit=m)

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>discrete</td>
<td>1.96</td>
<td>-2.85</td>
<td>-0.12</td>
<td>0.18</td>
<td>0.14</td>
</tr>
<tr>
<td>analytical</td>
<td>1.71</td>
<td>-2.84</td>
<td>-0.11</td>
<td>0.19</td>
<td>0.15</td>
</tr>
<tr>
<td>Spherical</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-D</td>
<td>2.03</td>
<td>-3.17</td>
<td>-0.10</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>2-D</td>
<td>2.04</td>
<td>-3.29</td>
<td>-0.10</td>
<td>0.20</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 4.4 Differences between the GGM and FFT geoids using different approximations of Stokes’s kernel (band 180-360, with 100% zero padding, unit=m)

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>planar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>discrete</td>
<td>0.69</td>
<td>-1.62</td>
<td>-0.08</td>
<td>0.10</td>
<td>0.06</td>
</tr>
<tr>
<td>analytical</td>
<td>0.65</td>
<td>-1.16</td>
<td>-0.08</td>
<td>0.11</td>
<td>0.08</td>
</tr>
<tr>
<td>spherical</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-D</td>
<td>0.74</td>
<td>-1.59</td>
<td>-0.07</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td>2-D</td>
<td>0.74</td>
<td>-1.55</td>
<td>-0.08</td>
<td>0.11</td>
<td>0.08</td>
</tr>
</tbody>
</table>

From Tables 4.2, 4.3 and 4.4, the STD of the geoid differences from the 1-D FFT with the exact spherical kernel are very similar with those from the planar kernels or 2-D FFT with spherical kernel within each specified band. The 1-D FFT with exact kernel and 2-D FFT with planar kernel methods perform equally well. The 2-D FFT with approximate spherical kernel performs slightly less accurately when compared with those from the 1-D FFT and 2-D FFT with exact kernel methods. The 2-D spherical FFT with approximate kernel gives the worst results among the four methods presented.
Figure 4.1 Geoid differences between OSU91A and that from 2-D planar FFT with discrete kernel for band 180–360 (Mercator projection, contour interval=10 cm)

Figure 4.2 Geoid difference between OSU91A and that from 2-D planar FFT with analytical kernel for band 180–360 (Mercator projection, contour interval=10cm)
Figure 4.3 Geoid difference between OSU91A and that from 2-D spherical FFT with approximate kernel for band 180–360 (Mercator projection, contour interval=10 cm)

Figure 4.4 Geoid differences between OSU91A and that from the 1-D spherical FFT with exact kernel for band 180–360 (Mercator projection, contour interval=10 cm)
As discussed in Chapter 2, only the residual gravity anomalies will be used in the Stokes integral and frequencies less than degree and order 360 are expected to be removed by OSU91A model. Therefore, the residual gravity anomalies will contain predominantly high frequency parts of the local gravity field where leakage can be minimised. These figures are also meaningful for the investigation of the edge effects in the FFT techniques in terms of different kernels.

The factor that affects the accuracy of the results is the roughness of the gravity field. When using the smallest residual gravity field in the band 180-360, the maximum, minimum, mean, root mean square and standard deviation are all reduced greatly compared with those from the bands 50-360 and 90-360, as is expected. In terms of the statistical results in Tables 4.2, 4.3 and 4.4, the 1-D FFT method is not explicitly better than other approximate FFT methods, although it is theoretically more rigorous. From Figures 4.1, 4.2, 4.3 and 4.4, it is found that all the results from various kernel approximations are almost identical to each other in the central part of the computation area, except for the 2-D planar FFT with analytical kernel spectrum where some errors occur in the centre of the area. The largest differences consistently occur at the data set boundaries.

4.5.3 Boundary Effects
The largest problem that affects the previous accuracy evaluation of the FFT approximation is the boundary effect. In order to show this more clearly, statistics of the computation results in a sub-area, with a 5° boundary excluded (e.g. \( \varphi: [-42^\circ, -10^\circ], \lambda: [115^\circ, 155^\circ] \)), are listed in Tables 4.5, 4.6 and 4.7.

<table>
<thead>
<tr>
<th>Kernel approximations</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar</td>
<td>Discrete</td>
<td>1.16</td>
<td>-0.22</td>
<td>0.30</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>Analytical</td>
<td>1.08</td>
<td>0.33</td>
<td>-0.29</td>
<td>0.32</td>
</tr>
<tr>
<td>Spherical</td>
<td>1-D</td>
<td>1.12</td>
<td>-0.14</td>
<td>0.23</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>2-D</td>
<td>1.17</td>
<td>-0.30</td>
<td>0.25</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Table 4.6 Differences of residual FFT geoids using various approximations of Stokes's kernel with 100% zero padding and a 5° boundary excluded in band 2 (90-360)

<table>
<thead>
<tr>
<th>Kernel approximations</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discrete</td>
<td>0.11</td>
<td>-0.50</td>
<td>-0.10</td>
<td>0.11</td>
<td>0.05</td>
</tr>
<tr>
<td>Analytical</td>
<td>0.40</td>
<td>-0.59</td>
<td>-0.10</td>
<td>0.13</td>
<td>0.08</td>
</tr>
<tr>
<td>Spherical</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-D</td>
<td>0.01</td>
<td>-0.34</td>
<td>-0.09</td>
<td>0.10</td>
<td>0.05</td>
</tr>
<tr>
<td>2-D</td>
<td>0.21</td>
<td>-0.43</td>
<td>-0.09</td>
<td>0.11</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Table 4.7 Differences of residual FFT geoids using various approximations of Stokes's kernel with 100% zero padding and a 5° boundary excluded in band 3 (180-360)

<table>
<thead>
<tr>
<th>Kernel approximations</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discrete</td>
<td>0.03</td>
<td>-0.20</td>
<td>-0.07</td>
<td>0.08</td>
<td>0.03</td>
</tr>
<tr>
<td>Analytical</td>
<td>0.22</td>
<td>-0.32</td>
<td>-0.07</td>
<td>0.09</td>
<td>0.05</td>
</tr>
<tr>
<td>Spherical</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-D</td>
<td>0.01</td>
<td>-0.18</td>
<td>-0.06</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>2-D</td>
<td>0.07</td>
<td>-0.21</td>
<td>-0.07</td>
<td>0.08</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Comparing Tables 4.5, 4.6 and 4.7 with Tables 4.2, 4.3 and 4.4 respectively, the accuracy of the FFT results in the sub-area is much higher when the 5° boundaries are taken into account. The standard deviations from the three bands can be improved by over 50 percent. The maximum and minimum are improved more apparently. Moreover, the accuracy of the results from various kernels are almost the same in the sub-area. Their differences are a few centimetres. The errors in the boundary areas are heterogeneous (see Figures 4.1–4.4). To explain this feature, the “ground truth” residual gravity anomalies are shown in Figures 4.5–4.7 for each of the three bands.
Figure 4.5 Features of the residual gravity in band 50-360 (unit= mgal; Linear projection)

Figure 4.6 Features of the residual gravity in band 90-360 (unit= mgal; Linear projection)
From Figures 4.5, 4.6 and 4.7, errors in the boundaries are correlated with the roughness of the residual gravity anomalies near the boundaries. Due to the sharp truncation of the residual gravity field in the north-east and north-west areas, the biggest errors occurred in these two parts. This is due to the Gibbs phenomenon. It can also be seen that the boundary effect extends by up to 15 degrees in a rough gravity area such as in the northern part of the test region (Figures 4.2 to 4.4), while in the smooth gravity anomaly area, there is almost no boundary effect (see Figure 4.4). The 2-D planar FFT with analytically defined kernel spectrum is not as good as the other three methods since its results are erroneous both in the central area and around the boundaries. This boundary effect can be reduced by the use of a tailored GGM, for example. Therefore, it is suggested that residual and smoothed gravity anomalies are always to be used in the FFT methods. Also, the analytically-defined kernel spectrum should not be used in the geoid computation.
These edge effects are bigger for the exact 1-D FFT than for other kinds of kernel approximations. It seems that the results from the planar kernel with a discrete spectrum are the least affected. However, the differences are very small and their accuracy is very high (up to a few centimetres). One possible reason for this is that the errors in the kernel linearisation of the Stokes’s formula smooth the truncation effect around the boundaries, and thus the truncation errors affect the smoothed kernel less than for a more precise kernel.

4.5.4 Effect of 100 Percent Zero Padding

In order to study how the zero padding affects the accuracy of the various kernel approximations in the FFT methods, the residual geoids are computed without zero padding and based on the above three bands. The results are listed in Tables 4.8, 4.9 and 4.10 respectively.

| Table 4.8 Differences of residual FFT geoids using different approximations of Stokes’s kernel without zero padding (band 1; 50-360) |
|---|---|---|---|---|---|---|
| Kernel approximations | Max | Min | Mean | RM | STD |
| Planar | Discrete | 6.22 | -2.90 | 0.08 | 0.41 | 0.40 |
| | Analytical | 5.76 | -2.11 | -0.39 | 0.49 | 0.30 |
| Spherical | 1-D | 6.53 | -3.17 | 0.10 | 0.50 | 0.49 |
| | 2-D | 6.50 | -3.07 | 0.10 | 0.53 | 0.52 |

| Table 4.9 Differences of residual FFT geoids using different approximations of Stokes’s kernel without zero padding (band 2; 90-360) |
|---|---|---|---|---|---|---|
| Kernel approximations | Max | Min | Mean | RMS | STD |
| Planar | Discrete | 2.53 | -2.68 | -0.03 | 0.21 | 0.21 |
| | Analytical | 1.69 | -2.85 | -0.13 | 0.20 | 0.15 |
| Spherical | 1-D | 2.65 | -2.98 | -0.02 | 0.25 | 0.25 |
| | 2-D | 2.79 | -3.07 | -0.02 | 0.27 | 0.27 |

| Table 4.10 Differences of residual FFT geoids using different approximations of Stokes’s kernel without zero padding (band 3; 180-360) |
|---|---|---|---|---|---|---|
| Method | Max | Min | Mean | RM | STD |
| planar | discrete | 0.96 | -1.50 | -0.01 | 0.09 | 0.09 |
| | analytical | 0.65 | -1.17 | -0.11 | 0.13 | 0.08 |
| spherical | 1-D | 1.04 | -1.43 | -0.07 | 0.10 | 0.10 |
| | 2-D | 1.08 | -1.49 | -0.08 | 0.10 | 0.10 |
Comparing Tables 4.8, 4.9 and 4.10 with Tables 4.2, 4.3 and 4.4 respectively, it is shown that 100 percent zero padding improves the accuracy of the geoid undulation for all the above three bands and all the four different kernel approximations. This suggests that the 100 percent zero padding should always be applied to the geoid undulation for the four kinds of kernel approximations. The performance improvement through 100 percent zero padding is more than 20 percent in terms of their statistics for all the three bands. Theoretically, this is due to the removal of the effect of circular convolution described by Sideris and Li (1993).

4.5.5 Consistency of the Various Kernel Approximations

In order to further investigate the boundary effect and the consistency of various kernel approximations themselves, the integral area is divided into four sub-blocks and the four sub-areas are overlapped by six degrees along their common boundaries. The four sub-areas are shown in Figure 4.8. The geoid was computed in each area and Figures 4.9 and 4.10 show the misfit along common boundaries in the longitude and latitude directions respectively. The integration is conducted over both the whole area with 100 percent zero padding and for a three degree capsize. If the FFT method is working properly, there should be no misfit.

![Figure 4.8](image)

Figure 4.8 Four equal sub-areas (NW, NE, SE and SW) and their common boundaries ($\phi=-26.0^\circ$, $\lambda=135.0^\circ$)
Figure 4.9 Misfits of various kernels along profiles φ=-26.0° when (a) the whole data area is used and (b) a 3.0° capsize is used (band 180-360, units in metres, 100% zero padding)
Figure 4.10 Misfits of various kernels along profiles $\lambda = 135^\circ$ when (a) the whole data area is used and (b) a $3.0^\circ$ capsize is used (band 180-360, units in metres, 100% zero padding)
From Figures 4.9 and 4.10, it is concluded that all the four kernel approximations have a good consistency in terms of the misfit along boundaries. The amplitude (maximum minus minimum) of the misfit waveforms are slightly improved when a 3° capsize is used (except when the analytical kernel is used), particularly for the 1-D FFT. However, the differences between the values from different neighbouring blocks are very similar and agree to within few centimetres. In other words, the errors cause the misfit are not from the different FFT techniques but from the structure of the gravity anomaly data and its truncation at the boundaries. This suggests that the only way to reduce the boundary effect is to extend the data coverage in the rough area. As such, the combination of gravity observation with satellite altimetry is expected to be more helpful.

4.5.6 Summary

From the above Tables and Figures, it is concluded that the roughness of the gravity anomalies is more critical to get good results than the adoption of different kernel approximations. Residual gravity anomalies that have a small STD are highly recommended. In order to avoid introducing leakage effects, zero-mean residual gravity anomalies should also be used over the computation area. The planar and spherical FFTs (both 1-D and 2-D) give almost equally good results in Australia. However, the 1-D exact spherical FFT should be used for its theoretical completeness, smaller memory requirement and practical stability. Its main drawback is that it is relatively time-consuming. However, as long as a batch model used, it is easily implemented. One hundred percent zero padding is highly recommended since it can improve the results by about 20 percent.

4.6 Terrain Correction by FFT

The planar, linear approximation of the gravimetric terrain correction (2.45) can be expressed as (Schwarz et al. 1990; Forsberg, 1984; 1985; Zhao, 1989; Sideris, 1985)

\[ C(x_p, y_p) = \frac{1}{2} G \rho \int \frac{(h - h_p)^2}{l^2(x, y)} \, dx \, dy \]  \hspace{1cm} (4.26)

This can be further expressed as the following convolution form:

\[ C(x, y) = \frac{1}{2} G \rho \left[ h^2 * l + h_p^2 * l_0 - 2h_p (h * l) \right] \]  \hspace{1cm} (4.27)
\[ l = \Delta x \Delta y \left[ x^2(m, n) + y^2(m, n) \right]^{1/2}, \]  

(4.28)

where \( h = h(x, y), h_p = h(x_p, y_p) \), and \( l_0 \) is a small constant to remove the singularity of function \( l \) (Schwarz et al., 1990). Other methods to relieve or remove the singularity of the terrain corrections are given by Zhang et al. (1995), Bian and Dong (1991) and Klose and Ilk (1993). In the spectral form, this is:

\[
C(x, y) = \frac{1}{2} G \rho \{ F^{-1} \left[ F(h^2(x', y')) \cdot F(l(x', y')) \right] + h_p^2(x', y') \cdot l_0 - 2h_p(x', y') \cdot F^{-1} \left[ F(h(x', y')) \cdot F(l(x', y')) \right] \}.
\]

(4.29)

Recently, Peng et al. (1995) expressed equation (4.29) as a 3-D FFT form which takes into account spatial variations of the topographical density function \( \rho(x, y, z) \) (Sideris, 1994a; Peng et al., 1995). However, this is not used in this study because the topographical density data are not available.

### 4.7 Indirect Effect of Helmert Second Condensation

Given an \( M \times N \) digital terrain height array on the plane, the linear approximation (equation 2.44) of the indirect effect in the Helmert second condensation method (Sideris and She, 1995; Sideris, 1990) can be expressed as following discrete integral form:

\[
\delta N_{\text{ind}} = \frac{-\pi G \rho H_p^2}{\gamma} + \frac{G \rho \Delta x \Delta y H_p^2}{6 \gamma} \sum_{x=x_1}^{x_m} \sum_{y=y_1}^{y_m} \frac{1}{l^3} - \frac{G \rho \Delta x \Delta y}{6 \gamma} \sum_{x=x_1}^{x_m} \sum_{y=y_1}^{y_m} \frac{H^3}{l^3}.
\]

(4.34)

Equation (4.34) can be evaluated using the 2-D discrete FFT, yielding the indirect effects for all grid points in one run. The 2-D discrete FFT form of equation (4.34) is as follows (Sideris and She, 1995):

\[
\delta N_{\text{ind}} = \frac{-\pi G \rho H_p^2}{\gamma} + \frac{G \rho \Delta x \Delta y}{6 \gamma} H_p^2 F^{-1} \left\{ F\left[ \frac{1}{l^3}\right] H^3 \right\} - \frac{G \rho \Delta x \Delta y}{6 \gamma} F^{-1} \left\{ F\left[ \frac{1}{l^3}\right] F[H^3] \right\}.
\]

(4.35)

This 2-D discrete integral can also be expressed as following 1-D form (ibid.)

\[
\delta N_{\text{ind}} = \frac{-\pi G \rho H_p^2}{\gamma} + \frac{G \rho \Delta x \Delta y}{6 \gamma} H_p^2 F^{-1} \left\{ \sum_{x=x_1}^{x_m} \left[ F\left[ \frac{1}{l^3}\right] F[H^3] \right]\right\} - \frac{G \rho \Delta x \Delta y}{6 \gamma} F^{-1} \left\{ \sum_{x=x_1}^{x_m} \left[ F\left[ \frac{1}{l^3}\right] F[H^3] \right]\right\}.
\]

(4.36)
Equation (4.36) is the 1-D FFT formula for the indirect effect evaluation which computes the indirect effect column by column. Similar formulae can be formed to compute the indirect effect row by row. These are quantified in Chapter Seven.

4.8 Comments and Discussion

Many gravity field convolution integrals can be computed extremely efficiently and quickly using an algorithm known as the Fast Fourier Transform (FFT). A full explanation of the principles behind the FFT can be found in Bracewell (1986b). When computing a DFT for \( N \) points, computation time is proportional to \( N^2 \), whereas, computing time is proportional to \( N \ln N \) using the FFT representing a saving in CPU time (Yfantis and Borgman, 1981). The central theme in the original FFT is that \( N \) is factorisable into products of two. Typically, however, the transform is quickest if the number of points in a data set can be factorised into the high powers of a small amount of prime numbers.

Another important property of the FFT is that it requires input data to be on a regular grid and the results are output on a regular grid. The data must therefore be interpolated onto a grid, the spacing of which has been carefully chosen to avoid aliasing effects. A successful interpolation requires detailed, homogeneous data coverage over the interest area.

There are a number of considerations required to get optimal results through the FFT technique. First and most importantly, the input data used should be residual gravity anomalies reduced by a reference field. The removal of the bias in the residual gravity anomalies is important to avoid spectral leakage. One hundred percent zero padding is highly recommended for further improvement of the geoid to remove cyclic convolution errors and edge effects. All the four kernel approximation methods discussed in this chapter can produce similar results and are consistent if 100 percent zero padding applied and residual gravity anomalies are used. The spherical procedures do not clearly outperform the corresponding planar techniques at low or mid latitude areas. However, the 1-D spherical exact FFT is recommended for precise local geoid determination because it does not use kernel approximation. The boundary effects are location dependent. The study shows that the removal of 90 degree and order reference field seems enough for centimetre geoid determination. After excluding a 5° boundary around the computation area, the standard deviation errors
can reach a few centimetres (see Table 4.6). The rougher the residual gravity anomalies, the higher and broader of the boundary effects. In the north part of the Australian test area, it could be up to 15 degrees, with almost no effect in the south.

4.9 Summary
This chapter presents the theoretical basics of Fourier transform and its applications to gravimetric geoid determination. Practical formulas using Stokes's integral by FFT and its various kernel approximations are discussed. In addition, the practical formulas for the computation of the terrain corrections and the indirect effect of Helmert's second condensation are given. Some FFT related problems, such as sampling interval, spectral leakage and padding of the gravity data are discussed and analysed.

Simulated residual gravity and geoid undulation data are produced using the OSU91A geopotential model in the three bands (50–360, 90–360 and 180–360) to test the effectiveness of the Stokes's integral by FFT. Using these "ground truth" data, the overall effects, boundary effects, 100% zero padding of the gravity data, and consistency of the various kernel approximations along common boundaries for the geoid computation have been tested.

It is concluded that gravity field integrals can be computed extremely efficiently and quickly using the FFT. Stokes's integral by FFT can give centimetre precision geoid undulation. However, some special considerations must be applied if high precision geoid is expected. First and most importantly, the input data must be long wavelength filtered using a GGM. The removal of the bias in the residual gravity anomalies is important to reduce spectral leakage. Secondly, one hundred percent zero padding should be used for further improvement of the geoid to remove cyclic convolution errors and edge effects. Moreover, the 1-D spherical FFT is recommended for precise local geoid computation. Furthermore, the boundary effects are location dependent. Larger data coverage and better residual gravity structure (e.g. long and medium wavelengths are effectively removed prior to FFT) are required to produce a precise geoid. The rougher the residual gravity anomalies, the higher and broader the boundary effects.
AN EVALUATION OF FFT GEOID DETERMINATION TECHNIQUES AND THEIR APPLICATION TO HEIGHT DETERMINATION USING GPS IN AUSTRALIA

CHAPTER FIVE

FEATURES OF THE AUSTRALIAN GRAVITY FIELD
Chapter 5

FEATURES OF THE AUSTRALIAN GRAVITY FIELD

5.1 Introduction

The conventional approach to gravity anomaly gridding is to smooth the gravity field using reductions (remove), interpolate these onto a grid, then restore the desired gravity anomalies (§2.6 and §6.7.1). Gravity field gridding is, generally, performed on a reference surface where mathematical and physical relations of the residual observations are simple and thus can be precisely modelled. Since the interpolation errors will directly propagate into the subsequent gravimetric geoid (Chapter Eight), the estimation of the gridded values is very important. Generally, the smoother the reference surface, the more precise the gridding (§6.8).

As one of the world's oldest continents, Australia has experienced a complicated geological history and has a strikingly distinctive landscape. As such, the gravity field of Australia behaves differently to that in other countries. This section of the research studies the special features of the Australian gravity field in terms of the relative roughness of various gravity anomaly types. The gravity anomaly types tested comprise free-air, refined Bouguer and topographic-isostatic gravity anomalies. The topographic-isostatic gravity anomalies were computed based on the Airy model \(D=32\text{km}, \Delta \rho=-400\text{kgm}^{-3}\) and the 'TC' program written by Forsberg (1984).

In the past few decades, there have been many developments and new applications of mathematical techniques for describing the complicated algebraic functions and for analysing empirical continuous data derived from many different types of signals. The most important development of these techniques includes the fast Fourier transform method (§4.2; Cooley and Tukey, 1965), fractal geometry (§5.2; Russ, 1994) and wavelets (Koornwinder, 1993). In addition to power spectral analysis, the techniques of simple statistical comparisons and visualisation are also applied in this thesis. Since the fractal geometry technique is becoming important in the study of various geoscientific phenomena (e.g. Burrough, 1981; Barnsley, 1988; Barton and Pointe, 1995), the Hurst fractal dimension analysis is also briefly studied in this chapter.
5.2 Selection of Test Areas and Profiles

Simple statistical comparisons and visualisation are direct ways for the comparison of data sets. To intuitively present special features of the Australian gravity field, free-air and refined Bouguer gravity anomalies are plotted in Figures 5.1 and 5.2 respectively. Table 5.1 lists the statistical information for these free-air and Bouguer gravity anomalies over the whole of the Australian continent.

<table>
<thead>
<tr>
<th>Anomaly</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free-air</td>
<td>172.8</td>
<td>-121.1</td>
<td>0.4</td>
<td>25.3</td>
<td>23.3</td>
</tr>
<tr>
<td>Bouguer</td>
<td>82.6</td>
<td>-164.0</td>
<td>-24.7</td>
<td>41.3</td>
<td>33.0</td>
</tr>
</tbody>
</table>

After the removal of the gravitational effect of the topographical masses above the geoid, theoretically, the refined Bouguer anomaly surface should be smoother than the free-air anomaly surface (Heiskanen and Moritz, 1967). However, as can be seen from Table 5.1, the standard deviation of the Bouguer anomalies is much higher than that of free-air anomalies over the whole continent. This indicates that the Bouguer anomalies are rougher overall than the free-air anomalies in Australia. This also implies that the application of a constant topographical density model (ρ=2,670kgm⁻³) can not effectively remove the irregularities of the gravity field of the topography in Australia, and thus it is concluded that a complicated geological structure exists.

The Australian free-air anomaly (Figure 5.1) shows a preponderance of negative features in the southwest and more positive features in the north and east flanks. Figure 5.2 shows a largely negative Bouguer anomaly field as a prominent feature of the Australian gravity (Symonds and Willcox, 1976). Comparing Figure 5.1 with Figure 5.2, the reason that Bouguer anomalies have a higher standard deviation than free-air anomalies is that in some subregions, such as area A2 shown on Figure 5.3, the Bouguer anomaly contains higher frequency information than the free-air anomaly. To further investigate this phenomenon, three test areas (A1, A2 and A3) and three profiles (P1, P2 and P3) have been selected which represent typical features of the gravity field. As shown on Figure 5.3 and Table 5.2, these three test areas are located in the west of Australia (the Hamersley Ranges, Western Australia), Central Australia and the south-east of Australia (Snowy Mountains, Victoria).
Figure 5.1 Colour image of the free-air gravity anomaly field of continental Australia (Linear projection)
Figure 5.2 Colour image of the Bouguer gravity field of continental Australia (Mercator projection)
Figure 5.3 Geographical location of the test areas and profiles (Linear projection)

Table 5.2 Location of the three test areas (A1, A2, A3) and the three profiles (P1, P2, P3)

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>P1</th>
<th>A2</th>
<th>P2</th>
<th>A3</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>latitude</td>
<td>24°–26°S</td>
<td>25°S</td>
<td>22°–29°S</td>
<td>26°S</td>
<td>34°–38°S</td>
<td>36°S</td>
</tr>
<tr>
<td>longitude</td>
<td>116°–120°E</td>
<td>126°–132°E</td>
<td>145°–149°E</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that the sizes of the three selected test areas are different. This is simply to show the properties of the gravity anomalies. Since the gravity anomaly types are compared in each area, it does not matter what size the area is.

5.3 Analysis of Various Gravity Anomaly Surfaces

The relative roughness of the free-air, Bouguer and topographic-isostatic gravity anomalies in the three test areas were studied using simple statistical comparison, power spectral analysis and simple Hurst fractal dimension analysis. The rougher a surface is, the higher its STD, power spectra in medium and short wavelengths and Hurst fractal are.
Gridding is first required for the computation of 2-D Fourier spectra by the FFT and the 2-D Hurst fractal dimension. All the three types of gravity anomaly (free-air, Bouguer and topographic-isostatic) were gridded using the minimum curvature spline method (§6.2) and using the same gridding parameters. A two-minute grid size is used which yields grids of 120×60, 240×180 and 120×120 for areas A1, A3 and A3, respectively. The standard deviations of the various gravity anomalies after gridding (STD$_{gd}$) are listed in the second last column of the Tables 5.3, 5.4 and 5.5.

5.3.1 Statistical Comparisons

Statistical comparison is a simple, intuitive method and thus is often used in gravity field studies. The statistical information of the three types of gravity anomalies is listed in Tables 5.3 to 5.5 for areas A1, A2 and A3, respectively.

Table 5.3 The statistics of various gravity anomaly types in the Hamersley Ranges, Western Australia (area A1, λ∈[116°E, 120°E], φ∈[24°S, 26°S])

<table>
<thead>
<tr>
<th>(mgal)</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
<th>STD$_{gd}$</th>
<th>Smoothness</th>
</tr>
</thead>
<tbody>
<tr>
<td>free-air</td>
<td>52.6</td>
<td>-73.7</td>
<td>-7.9</td>
<td>25.7</td>
<td>24.5</td>
<td>23.1</td>
<td>Roughest</td>
</tr>
<tr>
<td>Bouguer</td>
<td>121.5</td>
<td>-100.0</td>
<td>-47.8</td>
<td>64.6</td>
<td>43.4</td>
<td>41.0</td>
<td>Smoothest</td>
</tr>
<tr>
<td>topo-isos</td>
<td>42.2</td>
<td>-75.5</td>
<td>-13.7</td>
<td>27.3</td>
<td>23.6</td>
<td>22.1</td>
<td>Roughest</td>
</tr>
</tbody>
</table>

Table 5.4 The statistics of various gravity anomaly types in Central Australia (area A2, λ∈[126°E, 132°E], φ∈[22°S, 29°S])

<table>
<thead>
<tr>
<th>(mgal)</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
<th>STD$_{gd}$</th>
<th>Smoothness</th>
</tr>
</thead>
<tbody>
<tr>
<td>free-air</td>
<td>146.3</td>
<td>-113.1</td>
<td>-35.2</td>
<td>48.2</td>
<td>33.0</td>
<td>36.7</td>
<td>Smoothest</td>
</tr>
<tr>
<td>Bouguer</td>
<td>163.9</td>
<td>-100.0</td>
<td>12.3</td>
<td>99.0</td>
<td>98.2</td>
<td>76.2</td>
<td>Roughest</td>
</tr>
<tr>
<td>topo-isos</td>
<td>128.4</td>
<td>-115.5</td>
<td>-39.5</td>
<td>52.1</td>
<td>33.9</td>
<td>36.8</td>
<td>Roughest</td>
</tr>
</tbody>
</table>

Table 5.5 The statistics of various gravity anomaly types in the Snowy Mountains, Victoria (area A3, λ∈[145°E, 149°E], φ∈[34°S, 38°S])

<table>
<thead>
<tr>
<th>(mgal)</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
<th>STD$_{gd}$</th>
<th>Smoothness</th>
</tr>
</thead>
<tbody>
<tr>
<td>free-air</td>
<td>173.1</td>
<td>-44.4</td>
<td>5.8</td>
<td>21.1</td>
<td>20.3</td>
<td>32.3</td>
<td>Roughest</td>
</tr>
<tr>
<td>Bouguer</td>
<td>43.62</td>
<td>-81.7</td>
<td>-21.1</td>
<td>27.4</td>
<td>17.5</td>
<td>18.2</td>
<td>Smoothest</td>
</tr>
<tr>
<td>topo-isos</td>
<td>77.0</td>
<td>-42.9</td>
<td>6.1</td>
<td>14.0</td>
<td>12.6</td>
<td>12.5</td>
<td>Smoothest</td>
</tr>
</tbody>
</table>
From Tables 5.3 through 5.5 it is concluded that the relative roughness of the three gravity anomalies is different in each of the three regions (A1, A2 and A3). By comparing the STD, where the smaller STD implies a smoother field, not one of the gravity anomalies is consistently the smoothest. This implies that some special structures and features of the gravity field exist in these areas. Note also that the roughness of the free-air and topographic-isostatic gravity anomalies in both areas A1 and A2 is very similar in terms of their standard deviations (Tables 5.3 and 5.4).

To summarise, simple statistical comparisons demonstrate that the Bouguer and isostatic gravity anomalies are not necessary smoother than free-air gravity anomalies in Australia as is predicted by theory. The roughness of the three kinds of gravity anomaly behaves quite differently in the three test areas. Compared to the conventionally accepted gravity field characteristics, the gravity field of Australia has been found to be unusual.

5.3.2 Power Spectral Analysis

To analyse the spectral characteristics of the gravity anomaly surfaces, and thus the gravity field, the two-dimensional Fourier power spectral analysis method (Brigham, 1988; Champeney, 1973) has been implemented using the program Spectrum. This program was coded based on subroutines FFT3D and FFTCC from the International Mathematical and Statistical Libraries. The power spectra (amplitude) of the various gravity anomalies versus wavelength are plotted in Figures 5.4, 5.5 and 5.6 for each of the test areas (log-to-log scales). The full spectrum (as given by the data window and gravity observations) is not shown; only the region where the relative differences are clear is shown for clarity.
Figure 5.4 Spectral distribution of various gravity anomalies in area A1 (log to log scales)

As seen from Figure 5.4, for area A1, the Bouguer anomaly contains consistently greater power over the wavelengths considered (i.e. the whole window from 4 km to ~430 km) and thus is the roughest. The free-air anomaly contains almost the same power as the topographic-isostatic anomaly when the wavelengths are greater than 30km. However, the free-air anomaly contains slightly more power than the topographic-isostatic anomaly at wavelengths less than 30km (Figure 5.4), particularly at wavelengths less than 10km. Therefore, it can be concluded that the Bouguer anomaly is the roughest and the topographic-isostatic anomaly is the smoothest in this area. These conclusions are consistent with those made from the simple statistical comparisons (§5.3.1).
From Figure 5.5, for area A2, the Bouguer anomaly contains almost consistently greater power over the wavelengths considered (i.e. the whole window from 4 km to ~600 km) and thus is the roughest. The topographic-isostatic anomaly contains slightly greater power than the free-air anomaly between wavelengths 4km and ~14km. However, these two anomalies have almost the same power spectral distribution when wavelengths are greater than 30km. Therefore, it is concluded that the Bouguer anomaly is the roughest and the free-air anomaly is the smoothest in this area, particularly at short wavelengths (i.e. <14km). However, the free-air gravity anomaly contains slightly greater power than topographic-isostatic gravity anomaly between wavelengths ~14km and ~26km (Figure 5.5). It is estimated that the roughness of the topographic-isostatic and free-air gravity anomalies is very similar as implied from their STDs in Table 5.4 (36.7mgal vs. 36.8mgal). These conclusions are also confirmed from simple statistical analysis (§5.3.1).
From Figure 5.6, the free-air anomaly contains almost consistently greater power over the wavelengths considered (i.e. the whole window from 4 km to ~350 km) and thus is the roughest. Overall, the Bouguer anomaly contains more power than the topographic-isostatic anomaly, particularly in wavelengths less than 15km and greater than 40km. Therefore, it is concluded that the free-air anomaly is the roughest and the topographic-isostatic gravity anomaly is the smoothest in this area. This observation agrees with theory. However, there exist some overlaps among these three curves (e.g. between wavelengths 20km and 40km) which indicates that these power spectral distributions have a relatively wide undulation in some wavelengths. It is estimated that the relative roughness of the Bouguer and the topographic-isostatic anomaly is contributed to the spectral differences not only in short wavelengths but also in other wave bands.
To summarise, Fourier power spectral analyses demonstrate that the Bouguer anomalies contain significant power in both areas A1 and A2 over the whole wavelengths, which implies that the Bouguer anomalies are the roughest in these areas. It is shown that the free-air anomaly in area A3 contains significant power over the whole wavelengths, which implies that the free-air anomaly is the roughest in this area.

From the above analyses and the analyses in §5.3.1, it is concluded that power spectral analysis gives results consistent with the statistical comparisons. In addition, the Fourier spectral analysis can give more detailed information for specific frequencies in the gravity field. Of particular importance is the short wavelength information obtained from the spectral distribution analyses which presents a clearer picture of the gravity anomaly spectral structures. This analyses agree completely with the conclusions drawn from §5.3.1.

5.3.3 Hurst Fractal Dimension Analysis

The fractal characterisation technique was initially applied to time-based phenomena, but can also be used for elevation-like profile analysis (Russ, 1994; Mandelbrot and Benoit, 1982; Barnsley, 1988). The best known applications of fractals are to surfaces and profiles (Barton and Pointe, 1995). The fractal dimension of a surface or a line profile can be used to quantify its smoothness through its fractal dimension (ibid.). The larger the fractal dimension, the rougher the surface or profile. There are many different approaches to estimate fractal dimensions (Russ, 1994; Barnsley, 1988). The underlying characteristic of a fractal set is the self-similarity of the scales in the sense that there are large and small scales which maintain some relation between them (Farge et al., 1993, p.355). Therefore, the fractal dimension can be a useful tool to measure the roughness of a profile or a surface in terms of classifying the texture of various gravity anomaly surfaces.

Hurst, or rescaled range analysis of fractals, was initially performed on time-based historical data (Hurst et al., 1965). The basic idea to construct the Hurst fractal is to use a log-log plot of the maximum differences in a given window versus the size of the window (range). The size of this window is progressively increased while the maximum difference is determined accordingly. The slope fitted to the log-log plot of the maximum differences and corresponding window size using linear regression is the slope value ($\hat{S}$). The slope value ($\hat{S}$) thus defined is used to determined the Hurst dimension ($D$).
Assuming the gravity anomalies ($\Delta g$) in a limited area to follow self-similarity, the log maximum differences of the gravity anomalies ($\log \Delta g$) and log differences ($\log L$) for a given window size ($L$) is related by:

$$S_i = \frac{\log_{10}(\Delta g_i)}{\log_{10}(L_i)}.$$  \hfill (5.1)

The Hurst fractal dimension for a surface is

$$D = 3 - \hat{S},$$  \hfill (5.2)

and for a profile is

$$D = 2 - \hat{S}.$$  \hfill (5.3)

Equation (5.1) can be alternatively expressed as:

$$\Delta g_i = L_i^{\hat{S}}.$$  \hfill (5.4)

where $S_i$ is the slope of the log-log graph corresponding to the $i$th window;

$\hat{S}$ is the regression slope of the log-log plot with an RMS regression error $\sigma$;

$D$ is the Hurst fractal dimension for a surface (or a profile); and

$\Delta g_i$ is the maximum differences of the gravity anomalies within the $i$th window size ($L_i$).

Equations (5.1) through (5.3) indicate that the log-log graph yields data points that can be represented by a straight-line whose slope ($\hat{S}$) is related with the Hurst dimension ($D$). The Hurst dimension determines the relative roughness of the surface as a whole. The larger the value of $D$, the rougher of the surface.

Using the technique presented above, the Hurst dimension and the slope of log-log graph of various gravity anomalies for the areas A1, A2 and A3 are listed in Table 5.6, 5.7 and 5.8 respectively. The software used for this computation is the Fractal program coded by Russ (1994).

Table 5.6 Hurst fractals of various gravity anomalies in area A1

<table>
<thead>
<tr>
<th>Areas</th>
<th>Slope($\hat{S}$)</th>
<th>Regression error ($\sigma$)</th>
<th>Hurst dimension ($D$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bouguer</td>
<td>0.257</td>
<td>$\pm 0.046$</td>
<td>2.743</td>
</tr>
<tr>
<td>Free-air</td>
<td>0.651</td>
<td>$\pm 0.185$</td>
<td>2.349</td>
</tr>
<tr>
<td>Topo-isost</td>
<td>0.647</td>
<td>$\pm 0.158$</td>
<td>2.353</td>
</tr>
</tbody>
</table>
From Table 5.6 for area A1, it is concluded that Bouguer anomalies are the roughest. However, it is difficult to tell the difference between the fractal dimensions of free-air and topographic-isostatic gravity anomalies due to the regression errors ($\sigma$). Therefore, their relative roughness is not clearly distinguishable using their Hurst fractal dimensions.

<table>
<thead>
<tr>
<th>Areas</th>
<th>Slope($S$)</th>
<th>Regression error ($\sigma$)</th>
<th>Hurst dimension ($D$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bouguer</td>
<td>0.168</td>
<td>$\pm0.028$</td>
<td>2.832</td>
</tr>
<tr>
<td>Free-air</td>
<td>0.541</td>
<td>$\pm0.056$</td>
<td>2.459</td>
</tr>
<tr>
<td>Topo-isost</td>
<td>0.570</td>
<td>$\pm0.064$</td>
<td>2.430</td>
</tr>
</tbody>
</table>

From Table 5.7, it is concluded that Bouguer anomaly is the roughest in area A2. Again, it is hard to tell which anomaly is the smoothest from the fractals of the free-air and topographic-isostatic gravity anomalies due to the regression errors. To confirm which surface is smoother, a supplementary technique needs to be implemented, such as statistical comparison (§5.3.1) or power spectral analysis (§5.3.2).

<table>
<thead>
<tr>
<th>Areas</th>
<th>Slope($S$)</th>
<th>Regression error ($\sigma$)</th>
<th>Hurst dimension ($D$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bouguer</td>
<td>0.572</td>
<td>$\pm0.153$</td>
<td>2.428</td>
</tr>
<tr>
<td>Free-air</td>
<td>0.493</td>
<td>$\pm0.046$</td>
<td>2.501</td>
</tr>
<tr>
<td>Topo-isost</td>
<td>0.410</td>
<td>$\pm0.037$</td>
<td>2.590</td>
</tr>
</tbody>
</table>

From Table 5.8, again, due to the regression errors in the three regression slopes, it is hard to tell which surface is the roughest or the smoothest. If the regression errors are ignored, the Hurst dimensions may lead to the wrong conclusion (e.g. the Bouguer anomaly is the smoothest and topographic-isostatic is the roughest). However, from the statistical comparisons and Fourier power spectral analysis (Table 5.5 and Figure 5.6), the topographic-isostatic anomaly is the smoothest anomaly and the free-air anomaly is the roughest. Therefore, the Hurst fractal method is not applicable to this area.

From the above analyses, it is concluded that the simple 2-D Hurst fractal method is a less useful tool for quantifying the relative roughness of the gravity anomalies. When
the STDs of the gravity anomalies are close (e.g. area A3), this method may lead to wrong conclusions. For instance, the relative roughness of the free-air and topographic-isostatic anomalies in areas A1 and A2 can not be correctly distinguished if comparisons are made merely based on their Hurst fractal dimensions. Therefore, the 2-D Hurst method is not recommended for quantifying the relative roughness of the gravity field, particularly when the two surfaces have a similar roughness.

5.3.4 Discussion

Of the three methods used to compare the roughness of the free-air, Bouguer and topographic-isostatic gravity anomalies for an area, the direct statistical comparisons and Fourier power spectral analysis methods are the most informative. Statistical analysis is simple and intuitive method for measuring the roughness of a gravity anomaly surface. The power spectral analysis can give the exactly same roughness information as the statistical comparisons. In addition, the power spectral method can also give the relative power contribution in different wave bands.

The Hurst fractal dimension method can only approximately indicate the relative roughness of surfaces. Due to the regression errors in the estimation of the Hurst fractal dimension of the various anomaly surfaces, when the roughness of the two surfaces is very similar (Tables 5.6 and 5.7), the simple 2-D Hurst analysis method is not very informative. Therefore, the simple 2-D Hurst method is not suitable for the roughness analysis of the gravity anomalies, particularly for those surfaces that have similar standard deviations.

5.4 Profile Analysis

In addition to the above two-dimensional analyses of the surfaces, one-dimensional analyses are also conducted on the chosen profiles (§5.2). Each profile was constructed using a 0.1° band which explains some of the jumps in Figure 5.7. This band is the same as that used by Anfiloff (1982). The approaches used for profile analyses are visualisation, statistical comparison, power spectral analysis and the 1-D Hurst fractal dimension.
5.4.1 Visual Comparisons

The profiles of the free-air, Bouguer and topographic-isostatic gravity anomalies and terrain height are plotted for profiles P1, P2 and P3 in Figures 5.7, 5.8 and 5.9 respectively.

![Graph](image)

Figure 5.7 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P1 [25°S, 116°E-120°E]

In Figure 5.7, the profiles of free-air, and terrain height are very similar in shape and they thus have a strong correlation in area A1. The free-air anomaly profile is rougher than other two kinds of anomaly profiles in this area. The Bouguer anomaly profile is the smoothest.
Figure 5.8 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P2 [26°S, 126°E-132°E]

For the profile P2 (Figure 5.8), it is revealed that Bouguer anomalies are relatively rough and some steep gradients exist at the intervals [126.0°E-128.0°E]. The free-air and topographic-isostatic gravity anomalies have a slightly negative correlation with terrain heights. This implies that a complicated geological structure exists in this region and \( \rho=2.670 \text{kgm}^{-3} \) is not representative of the true topographic density. In the intervals [128.0°E-130.0°E] topographic-isostatic anomaly has a slight correlation with terrain height. The topographic-isostatic gravity anomaly profile is the smoothest profile in this area, whereas the free-air gravity anomaly profile is slightly rougher than the topographic-isostatic profile.

The interesting fact is that the terrain profile, in contrast to the gravity anomaly profiles, is relatively smooth which means that the topography is of a longer wavelength in nature than the gravity. Therefore, it is concluded that terrain heights do not have strong correlation with free-air gravity anomalies along this profile at high
(or even medium) frequencies, suggesting that there exists a large density anomaly below this region. From this point of view, the topographical density information is very important for the terrain reduction in the application of the remove-restore techniques (§2.6), and free-air gravity anomaly reduction, where the gradient of the gravity cannot be represented by the normal gravity gradient, can cause errors in the free-air reduction. This will be discussed further in §8.3. In addition, the free-air anomalies are correlated with terrain height, but not as strongly as in areas A1 and A2 (see Figures 5.8 and 5.9).

This finding can be confirmed by Anfiloff (1982), who used eighteen elevation and gravity profiles across Australia to study crust and tectonic processes using sparse gravity observations (~260,000). Anfiloff (ibid.) concluded that the gradient of the Bouguer anomalies along profile 133°E is very steep. Unfortunately, he was not able to give more analysis on several other profiles where similar features can be observed (i.e. profiles 22°S, 24°S, 26°S, 29°S, 133°E and 144°E).

Figure 5.9 Free-air, Bouguer and isostatic-topographic gravity anomalies and terrain along profile P3 (36°S, 145°E–149°E)
As seen from Figure 5.9, the free-air anomalies are highly correlated with the terrain heights in the interval [146.5°E-149°E]. However, all the three types of gravity anomalies are not correlated with the terrain heights in the interval [145°E-146.5°E]. This is most probably due to the relatively flat terrain and complicated geological structure along this interval. If a usual terrain reduction is applied to the gravity observations in this region using terrain heights only, the residual gravity anomaly will become rougher rather than smoother. Terrain and free-air anomalies are very rough in this area. The Bouguer anomaly is smoother than free-air anomaly. The topographic-isostatic anomaly profile is the smoothest.

This analysis will be further supported from spectral analysis of these profiles (§5.4.3). This study shows that the gravity anomalies, not only in the three areas, but also along the three profiles in the three areas, have special features. None of the gravity anomaly types are consistently smoothest in the three test profiles.

5.4.2 Statistical Comparisons

Statistical information for various gravity anomalies and terrain elevation along the three profiles P1, P2 and P3 are listed in Tables 5.9, 5.10 and 5.11 respectively.

<table>
<thead>
<tr>
<th>Profiles</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>STD</th>
<th>Smoothness</th>
</tr>
</thead>
<tbody>
<tr>
<td>free-air (mgal)</td>
<td>-13.5</td>
<td>47.8</td>
<td>8.8</td>
<td>15.8</td>
<td>roughest</td>
</tr>
<tr>
<td>Bouguer (mgal)</td>
<td>-55.6</td>
<td>-19.0</td>
<td>-40.2</td>
<td>9.8</td>
<td>smoothest</td>
</tr>
<tr>
<td>topo-isost (mgal)</td>
<td>-13.6</td>
<td>37.9</td>
<td>4.8</td>
<td>12.9</td>
<td></td>
</tr>
<tr>
<td>terrain (m)</td>
<td>313.7</td>
<td>622.0</td>
<td>446.8</td>
<td>94.1</td>
<td></td>
</tr>
</tbody>
</table>

From Table 5.9, it is concluded that the topographic-isostatic gravity anomaly is not necessarily the smoothest along this profile (P1). The Bouguer and free-air anomalies along profile P1 are the smoothest and roughest respectively.

<table>
<thead>
<tr>
<th>Profiles</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>STD</th>
<th>Smoothness</th>
</tr>
</thead>
<tbody>
<tr>
<td>free-air (mgal)</td>
<td>-65.2</td>
<td>72.8</td>
<td>-31.4</td>
<td>15.4</td>
<td>roughest</td>
</tr>
<tr>
<td>Bouguer (mgal)</td>
<td>-98.9</td>
<td>120.6</td>
<td>-48.4</td>
<td>71.5</td>
<td>smoothest</td>
</tr>
<tr>
<td>topo-isost (mgal)</td>
<td>-67.4</td>
<td>70.0</td>
<td>-37.0</td>
<td>15.2</td>
<td></td>
</tr>
<tr>
<td>terrain (m)</td>
<td>405.3</td>
<td>649.3</td>
<td>524.5</td>
<td>50.4</td>
<td></td>
</tr>
</tbody>
</table>
From Table 5.10, it is concluded that the topographic-isostatic and Bouguer gravity anomalies are the smoothest and the roughest along profile P2, respectively. In addition, the free-air anomalies have a similar roughness with topographic-isostatic anomalies along profile P2. The Bouguer anomaly is much rougher than topographic-isostatic and free-air gravity anomalies as is indicated by simple visualisation. It is concluded that the Bouguer anomalies are rougher than free-air anomalies which indicates that simple removal of the topographic masses can not improve the smoothness of the gravity field. However, after the topographic masses are restored using the Airy model, the topographic-isostatic gravity anomalies become the smoothest although they are marginally smoother than the free-air anomalies.

<table>
<thead>
<tr>
<th>Profiles</th>
<th>Min</th>
<th>Max</th>
<th>Mean</th>
<th>STD</th>
<th>Smoothness</th>
</tr>
</thead>
<tbody>
<tr>
<td>free-air (mgal)</td>
<td>-43.5</td>
<td>117.5</td>
<td>2.7</td>
<td>23.5</td>
<td>roughest</td>
</tr>
<tr>
<td>Bouguer (mgal)</td>
<td>-74.6</td>
<td>-1.9</td>
<td>-20.7</td>
<td>18.6</td>
<td></td>
</tr>
<tr>
<td>topo-isost (mgal)</td>
<td>-16.7</td>
<td>36.9</td>
<td>1.8</td>
<td>10.2</td>
<td>smoothest</td>
</tr>
<tr>
<td>terrain (m)</td>
<td>94.5</td>
<td>1632.0</td>
<td>209.6</td>
<td>298.1</td>
<td></td>
</tr>
</tbody>
</table>

From Table 5.11, it is concluded that the topographic-isostatic and free-air gravity anomalies are the smoothest and the roughest along profile P3, respectively. The inclusion of the terrain information can improve the smoothness of the gravity anomaly surfaces which implies that the gravity anomalies along profile P3 are correlated with topography.

To summarise, simple statistical comparisons for these three profiles indicate that the topographic-isostatic and Bouguer gravity anomalies are not necessarily smoother than the free-air anomalies. This means the free-air gravity anomalies are not always positively correlated with topography. None of the three anomalies are consistently the smoothest or roughest.
5.4.3 Power Spectral Analyses

The power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies on profiles, P1, P2 and P3 are shown on Figures 5.11, 5.12 and 5.13, respectively. This method is very useful to analyse the detailed spectral structure of the gravity field in terms of power distribution versus wavelength. This analysis used the ‘XMGR’ software package held in the School of Surveying and Land Information, Curtin University of Technology. XMGR is public domain software taken from the Internet and runs on a Sun workstation.

![Power spectra graph]

Figure 5.10 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies at 25°S (116°E–120°E), P1

In Figure 5.10, all the three gravity profiles (i.e. Bouguer, free-air and topographic-isostatic gravity anomalies) contain very similar power spectra in the short wavelength components (5km–20km). However, the free-air profile is the roughest and Bouguer anomaly profile is slightly smoother than that of topographic-isostatic gravity anomaly profile.
In Figure 5.11, it is clearly seen that Bouguer anomaly is much rougher than those of free-air and topographic-isostatic anomaly profiles in area A2 for the short and medium frequency parts (10km–300km). The roughness of the free-air profile is very similar to that of the topographic-isostatic anomaly. It is hard to tell which is smoother directly from Figure 5.11. The topographic-isostatic anomaly, however, is slightly smoother than free-air anomaly can be observed if Figure 5.11 is rescaled. This conclusion complies with the previous statistical analysis (§5.2.2a).
Figure 5.12 Power spectra of free-air, Bouguer and isostatic-topographic gravity anomalies at 36°S (145°E–149°E), P3

From profile P3 (Figure 5.12), it is concluded that the free-air anomaly is very rough in the short wavelength range (10km–50km). The Bouguer and topographic-isostatic profiles are relatively smooth in this wavelength band. However, the topographic-isostatic anomaly profile is slightly smoother than that of Bouguer anomaly.

5.4.4 Hurst Fractal Dimension Analyses
The Hurst technique is also applied to study the fractal dimension of the profiles (§5.3.3). The 1-D Hurst fractal dimensions (2-\hat{S}) of various gravity anomaly types along the three profiles are calculated using the Fractal software (Russ, 1994) and their results are listed in Tables 5.12, 5.13 and 5.14 respectively.
Table 5.12 Hurst fractals of various gravity anomalies along P1

<table>
<thead>
<tr>
<th>Areas</th>
<th>Slope(S)</th>
<th>Regression error (σ)</th>
<th>Hurst dimension (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bouguer</td>
<td>0.0080</td>
<td>±0.0020</td>
<td>1.9920</td>
</tr>
<tr>
<td>Free-air</td>
<td>0.0093</td>
<td>±0.0026</td>
<td>1.9907</td>
</tr>
<tr>
<td>Topo-isost</td>
<td>0.0077</td>
<td>±0.0022</td>
<td>1.9923</td>
</tr>
<tr>
<td>Elevation</td>
<td>0.0223</td>
<td>±0.0028</td>
<td>1.9773</td>
</tr>
</tbody>
</table>

As seen from Table 5.12, due to regression errors and the similar magnitude of the Hurst fractals, it is hard to tell which gravity anomaly profile is the smoothest or roughest. Therefore, the 1-D Hurst method is not applicable to this profile.

Table 5.13 Hurst fractals of various gravity anomalies along P2

<table>
<thead>
<tr>
<th>Areas</th>
<th>Slope(S)</th>
<th>Regression error (σ)</th>
<th>Hurst dimension (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bouguer</td>
<td>0.0001</td>
<td>±0.0000</td>
<td>1.9999</td>
</tr>
<tr>
<td>Free-air</td>
<td>0.0015</td>
<td>±0.0001</td>
<td>1.9985</td>
</tr>
<tr>
<td>Topo-isost</td>
<td>0.0015</td>
<td>±0.0001</td>
<td>1.9985</td>
</tr>
<tr>
<td>Elevation</td>
<td>0.0483</td>
<td>±0.0071</td>
<td>1.9527</td>
</tr>
</tbody>
</table>

As seen from Table 5.13, the topographic-isostatic anomaly is the smoothest and the Bouguer anomaly is the roughest. This conclusion is supported from Fourier power spectral analysis and simple statistical comparisons. However, the relative roughness between the Bouguer and the free-air gravity anomalies is not as clear as the information from statistical analysis and spectral analysis. Therefore, only if the regression errors are small, the Hurst method can be used to analyse the relative roughness of the profiles.

Table 5.14 Hurst fractals of various gravity anomalies along P3

<table>
<thead>
<tr>
<th>Areas</th>
<th>Slope(S)</th>
<th>Regression error (σ)</th>
<th>Hurst dimension (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bouguer</td>
<td>0.0384</td>
<td>±0.0047</td>
<td>1.9616</td>
</tr>
<tr>
<td>Free-air</td>
<td>0.0483</td>
<td>±0.0071</td>
<td>1.9517</td>
</tr>
<tr>
<td>Topo-isost</td>
<td>0.0483</td>
<td>±0.0071</td>
<td>1.9517</td>
</tr>
<tr>
<td>Elevation</td>
<td>0.0835</td>
<td>±0.0075</td>
<td>1.9165</td>
</tr>
</tbody>
</table>

From Table 5.14, it is concluded that the Hurst fractal is not very informative. The Hurst fractals of the three gravity anomalies are too close to be distinguishable due to the regression errors.
From the above comparison of the Hurst fractals in the three profiles, it is concluded that the relative degree of roughness among various profiles can only be distinguished approximately from Hurst dimension values. In contrast to the simple statistical comparisons, visual analysis and power spectral analysis, the Hurst fractal leads to conclusions that differ due to the presence of regression errors. Therefore, the simple Hurst fractal dimension is less reliable.

5.4.5 Discussion

Of the three methods used for quantifying the smoothness of various profiles, statistical comparisons and Fourier power spectral analyses are the most informative. The statistical comparison method is simple, intuitive and easily implemented. The power spectral method also performs very well to scrutinise the detailed spectral distribution of various gravity anomaly profiles. The 1-D Hurst fractal analysis method can also give relative roughness information approximately. However, when the smoothness of the profiles is close, due to the regression errors of the slope, the 1-D Hurst method does not perform very well. Therefore, the simple statistical comparison and Fourier power spectral analysis methods are recommended for the analysis of the gravity anomaly profiles.

5.5 Reasons for the Special Features of the Australian Gravity Field

As is demonstrated from §5.2 to §5.4, the Australian gravity field behaves in a complicated manner. Overall, the Bouguer anomaly is more variable than free-air anomaly. None of the free-air, Bouguer and topographic-isostatic gravity anomalies are consistently the smoothest or the roughest in any region.

Generally, gravity is correlated with topography. After removal of the terrain effects, residual gravity anomalies should be smooth. The smoothness of the detailed gravity surface/profile implies that the density variations are either deep-seated, or near-surface but gradual, while the roughness of the detailed gravity surface and/or profile implies that the density variations steeply in the topography either horizontally or vertically, or both. Since the Australian continent is an old continent, it is weathered and has experienced a complicated geological history. The (upper) crust is dominated by old compacted dense rocks and next to young less dense rocks, large density
changes exist dominantly, particularly in central Australia. Large density changes correspond to steep gravity changes. Therefore, the gravity does not necessarily always have a positive correlation with local topography. There exist large density changes in the smooth topography. It is concluded that the gravity is not correlated with topography but with geology and tectonics in some regions.

It is likely that the highly variable gravity field in Australia is due to the complicated geological structure of the continent in conjunction with the relatively smooth topography. This also implies that in some parts of Australia there exist large density anomalies below the continent. To depict detailed structure of the Australian gravity field, a detailed topographic density model together with a digital terrain model are highly desirable.

5.6 Discussion

Due to the long geological history of the Australian continent, being abundant with mineral resources and having large density contrasts, this study shows that the Earth’s gravity field of Australia presents a very special nature. It is concluded that:

The statistics of free-air (Figure 5.1) and Bouguer (Figure 5.2) gravity anomalies over continental Australia indicate that the Bouguer anomalies are more variable, which implies they would introduce larger interpolation errors than free-air anomalies. The power spectra (Figures 5.4, 5.5 and 5.11) show that the Bouguer anomalies, in comparison to the free-air anomalies, contain significant power at both short (tens of km) and medium (hundreds of km) wavelengths in some regions (e.g. A1 and A2).

It is evident that none of the three gravity anomaly types tested in this study are necessarily the smoothest over the Australian continent. This finding has some important applications for the Earth’s gravity field related studies. One important application of the findings is for gravity field gridding (Chapter Six). Another important application is to aid further understanding or describing the detailed structures of the Earth’s gravity field. Therefore, it is recommended that the
smoothest gravity anomaly type should be used for gravity anomaly gridding. This observation may also be pertinent to other parts of the world.

In some regions of Australia, the Bouguer anomalies are not necessarily the optimal gravity anomalies to be used for interpolation, nor are the topographic-isostatic anomalies. The smoothest gravity anomaly type in each subregion in conjunction with an optimal geopotential model should be used for gravity anomaly gridding. This is studied further in Chapter six.

Because of the complexity of the Australian gravity field, a constant topographic density model can not effectively remove irregularities in the gravity field. Therefore, a topographic mass density model or three-dimensional digital density model is highly recommended for future refinement of the Australian gravity field and for gravity reduction.

The statistical comparison method, in combination with the profile and power spectral analysis methods can effectively reveal the detailed gravity field structure and thus provides a more thorough understanding of the gravity field. The power spectral technique is a powerful tool for gravity field analysis in terms of the power distribution along different wave bands.

Fractal geometry is becoming increasingly important in various aspects of the Earth sciences and can, generally, achieve satisfactory results. However, due to the regression error in the computation of the fractal dimension, the simple Hurst fractal method is not applicable when the roughness of the surface or profiles is similar. Further research is recommended for other fractal dimensional analysis approaches to test their effectiveness.

The statistical analysis method is a simple, intuitive and informative method. It can effectively reveal the variability (roughness versus smoothness) of the gravity anomalies. It is, therefore, recommended for quantifying the smoothness of the gravity field and will be used in subsequent chapters.
5.7 Summary

This chapter has investigated the features of the Australian gravity field for both surfaces and profiles. The methods used are simple statistical comparison, power spectrum analyses and the simple Hurst fractal dimension. Three test areas (Hamersley Ranges, Central Australia and Snowy Mountains), which represent extreme features of the gravity field, were chosen to test this. It is demonstrated that the simple statistical analysis and Fourier power spectral methods are the most informative tools for measuring the smoothness of the gravity field.

It is revealed that the topography often contains longer wavelength features than the gravity anomalies in some parts of Australia. It is also demonstrated that not one of the free-air, Bouguer or topographic-isostatic gravity anomalies is consistently the smoothest type in Australia, which has implications for gridding (§6) and subsequent gravimetric geoid determination. The reasons that cause this special feature are: a) the complicated geology generates a very variable gravity field, b) in some regions, the topography is of longer wavelength than the gravity anomalies, c) the constant density of the topographic masses $\rho=2,670\text{kgm}^{-3}$ is not representative.

As one of the world's oldest continents, Australia has experienced a complicated geological history but has an extensively weathered topography. Therefore, the gravity field is variable and not always correlated with the terrain. Bouguer and terrain corrections will not necessarily smooth the gravity field. It is demonstrated that the Bouguer anomaly is more variable than the free-air anomaly in Australia and thus should not necessarily be used for gravity field gridding.
AN EVALUATION OF FFT GEOID DETERMINATION
TECHNIQUES AND THEIR APPLICATION TO HEIGHT
DETERMINATION USING GPS IN AUSTRALIA

CHAPTER SIX

GRIDDING THE AUSTRALIAN GRAVITY FIELD
Chapter 6

GRIDDING THE AUSTRALIAN GRAVITY FIELD

6.1 Introduction

Many numerical procedures in the Earth sciences require data on a regularly spaced grid, especially the computation of geoid undulations and terrain effects by the fast Fourier transform. In contrast, however, most data are acquired at individual observation points or along traverses both on land and in marine areas. It is, therefore, necessary to construct estimates of the value of a function on a grid (or its mean value), given observations of the value of the function at arbitrary locations in a plane or in three-dimensional space. This operation or data treatment process is called gridding (e.g. Smith and Wessel, 1990; Tscherning and Forsberg, 1992).

Gridding can be considered to be a prediction (interpolation or extrapolation in some cases) process. Gridding using only observations at discrete points is an ill-posed problem because it admits an infinite number of solutions. That is, any function passing through the observed data is a solution to the interpolation problem. In order to reduce this ambiguity, a priori information about the behaviour of the sample function is, generally, introduced. The a priori information usually considered is the continuity, the smoothness and some analytical and spatial features of the sampled function. However, the most important consideration is its capability to accurately recover the value at the unsampled point. In other words, the prediction method used is considered accurate and reliable if it maintains the broad and fine features of the original data without introducing undue distortion.

Geodesists use observations of the acceleration due to gravity and position to predict unobserved gravity field-related quantities, notably the geoid. In order to minimise the numerical effort or to apply mathematical and physical relationships, some idealised features of the gravity field are assumed. For example, the gravitational field of the Earth is treated as being:

- smooth (in the sense of having a limited spectral bandwidth),
• a stationary process,
• a sum of a finite number of base functions (e.g. spherical harmonics), and
• locally related (completely inferable using the data in a region or a small cap).

Certainly, none of the above are strictly true and the gravity field is assumed to be simple in some sense. However, if one wants to incorporate geophysical information concerning the Earth such as density information, it seems to be preferable not to claim simplicity for the gravity field, but for its sources (Lehmann, 1993).

There are a few aspects of concern in evaluating a gridding algorithm. The relative importance of each depends on the intended application. The first concern is the global properties of the solution. An a priori known functional form is generally assumed or constructed for the predicted function. The second concern is honouring data constraints; that is to decide whether the method gives exact or approximate values when it passes through observed data locations. The third concern is the method of interpolation or extrapolation in poorly constrained regions. It is in this last area that gridding algorithms differ most and where global properties strongly affect the solution (Smith and Wessel, 1990). In general, all gridding algorithms share the following assumptions:

(a) The function to be gridded is single-valued at any point. This assumption ensures that the function constructs a mathematically simple surface and the predicted value can be uniquely expressed;
(b) The function is everywhere defined, smooth and continuous within the region to be gridded;
(c) The function is positively auto-correlated over some length scale at least as large as the typical spacing between observation points (Davis, 1986; 1987). This means that the quantities, both observed and predicted, are related mathematically or physically, or both.

In addition, any method, in order to be suitable for the treatment of a large volume of data, should be conveniently adaptable to computer use. The desirable interpolating function should be well behaved and assume values not unrealistic when compared to values at known locations (Crain and Bhattacharyya, 1967). This criterion will be used in this chapter.
Several problems are involved in the process of defining a single interpolation function for the entire volume of data over a large area. Firstly, the surface described by such a function will normally be of such a high order as to require considerable computer storage. Secondly, there will be serious problems involved in the inversion of large matrices, which at times may not be well behaved when the data are densely sampled (e.g. least squares collocation). These problems can be avoided by considering only the data points lying within a small area surrounding a particular grid point (a locally related property). Interpolation within the small area could then be done rapidly and sometimes accurately using low-order surfaces. In practice, however, the shape of the area should be large enough so as to include data points on all sides of the grid point, and in order to ensure that the process will truly be interpolation rather than extrapolation.

In this section of the research, the gridding methods of continuous curvature splines in tension, least squares polynomial fitting, bi-cubic splines, moving weighted averages and Kriging have been investigated. Firstly, the principles of these methods are briefly described. Then, the ability of each gridding method to recover gravity at known points in three areas are compared to determine the most suitable gridding approach for subsequent geoid determination.

6.2 Continuous Curvature Splines in Tension

6.2.1 Minimum Curvature Splines

The minimum curvature splines method (Briggs, 1974) interpolates the random observations to be gridded with a surface having continuous second derivatives and minimal total squared curvature (Smith and Wessel, 1990). This method is widely used in the Earth sciences for data analysis and display (e.g. Briggs, 1974; Smith and Wessel, 1990). According to Smith and Wessel (1990), this is an integral method whose advantage is that it assures a solution with the desired properties. In two dimensions, the minimum curvature interpolant is the natural bi-cubic spline. Solving the differential equation is equivalent to a third-order spline, in which the optimum properties of the spline fit are revealed.

Suppose in a two-dimensional function, \( \Delta g = f(x,y) \), the total squared curvature \( C \) is:
\[ C(\Delta g) = \iint \left( \frac{\partial^2 \Delta g}{\partial x^2} + \frac{\partial^2 \Delta g}{\partial y^2} \right)^2 \, dx \, dy \]  

(6.1)

Briggs (1974) proved that if the function \( \Delta g \) makes \( C \) an extremum, then it obeys

\[
\frac{\partial^4 \Delta g}{\partial x^4} + 2 \frac{\partial^4 \Delta g}{\partial x^2 \partial y^2} + \frac{\partial^4 \Delta g}{\partial y^4} = \begin{cases} f_n & x = x_n, y = y_n \\ 0 & \text{otherwise} \end{cases}
\]

(6.2)

If the function \( \Delta g \) obeys (6.2) it minimises \( C \). Here, \( f_n \) are described as the forces acting at \((x_n, y_n), n=1,2,\cdots,N\) if \( \Delta g=f(x, y) \) is the displacement of a thin elastic spline, as defined in the mechanics literature (Boor, 1978). Minimising equation (6.1) leads to following differential equation (Smith and Wessel, 1990):

\[
\nabla^2(\nabla^2 \Delta g) = \sum_i f_i \delta(x-x_i, y-y_i),
\]

(6.3)

where \( \nabla^2 \) is the 2-D gradient operator (Laplacian);

\((x_0, y_0, \Delta g_0)\) are constraining data, and the boundary conditions along an edge are:

\[
\frac{\partial^2 \Delta g}{\partial n^2} = 0 \quad \text{and} \quad \frac{\partial}{\partial n} (\nabla^2 \Delta g) = 0,
\]

(6.4)  (6.5)

where \( \partial/\partial n \) indicates a derivative normal to an edge, with

\[
\frac{\partial^2 \Delta g}{\partial x \partial y} = 0
\]

(6.6)
at the corners. These three conditions in equations (6.4), (6.5) and (6.6) are called free-edge conditions, and under these conditions equation (6.2) has a unique solution with continuous second derivatives.

6.2.2 Minimum Curvature Splines in Tension

Smith and Wessel (1990) further expressed equation (6.3) approximately in the general form:

\[
D \nabla^2(\nabla^2 \Delta g) = \left[ T_{xx} \frac{\partial^2 \Delta g}{\partial x^2} + 2 T_{xy} \frac{\partial^2 \Delta g}{\partial x \partial y} + T_{yy} \frac{\partial^2 \Delta g}{\partial y^2} \right] = q
\]

(6.7)

where from the point of view of elastic force analogy, \( D \) is the constant flexural rigidity of a thin elastic plate;

\( q \) is the vertical normal stress; and

\( T_{xx}, T_{xy} \) and \( T_{yy} \) are the constant horizontal forces per unit vertical length.
Supposing that $T_{xx} = T_{yy} = T$ and $T_{xy} = 0$, equation (6.7) becomes following general spline with tension form *ibid.:

$$\left(1 - T\right) \nabla^2 (\nabla^2 \Delta g) - T \nabla^2 \Delta g = \sum_I f \delta(x - x_i, y - y_i)$$

(6.8)

where $T \in [0, 1]$ is tension parameter.

If $T = 0$, equation (6.8) results in the minimum curvature spline surface with free edges (i.e. a natural bi-cubic spline). Imposing tension at the boundary and the interior means that a compromise is made between the misfit to the data and the curvature of the solution. This method can suppress local maxima and minima between observed data points that may be not desirable, which have been caused by the oscillation of the natural spline (an extraneous inflection point). Therefore, minimum curvature spline with tension provides the flexibility to handle potential field data that arise in different applications of the Earth sciences.

This method was coded as the *Surface* command line by Wessel and Smith (1991; 1995) in the Generic Mapping Tools (GMT) and has been widely used in many geosciences (e.g. Smith and Wessel, 1990; Mendonca and Silva, 1995; Zhang and Featherstone, 1995). The GMT is a public domain software package, based on the UNIX platform, and is composed of more than fifty programs. The GMT can be used to manipulate columns of tabular data, time-series, and gridded data sets and display these data from simple x-y plot to maps and colour, perspective illustrations, for example. Most of the figures in this thesis have been produced using GMT.

This tensioned spline method has been studied for the gridding of gravity anomalies and heights. Theoretically, the minimum curvature spline method is a local, exact and deterministic interpolator. The advantages of this spline method are that the interpolator is totally constrained, with rapid convergence, and involving a relatively light computing load.
6.3 Bi-cubic Spline Method

If the values of a function $\Delta g(x, y)$ are given at the nodes $i=0,1,\ldots,m$ and $j=0,1,\ldots,n$ of a grid, the piecewise polynomial function defined within each rectangular cell $[x_i, x_{i+1}, y_j, y_{j+1}]$ of the grid is of the following form:

$$
\Delta g_{i,j}(x, y) = \sum_{m=0}^{3} \sum_{n=0}^{3} a_{i,j}^{m,n} (x - x_{i-1})^m (y - y_{j-1})^n .
$$

To determine the coefficients of equation (6.9), supplementary conditions must be applied to the function $\Delta g(x, y)$. The bi-cubic spline function method assumes that the values, slopes and curvatures of the function $\Delta g(x, y)$ are continuous at all the given nodes (Davis and David, 1980; Bhattacharyya, 1969; Boor, 1978), that is:

$$
p_{i,j} = \Delta g_x(x_i, y_j) = \left. \frac{\partial (\Delta g)}{\partial x} \right|_{i,j},
$$

$$
q_{i,j} = \Delta g_y(x_i, y_j) = \left. \frac{\partial (\Delta g)}{\partial y} \right|_{i,j},
$$

$$
s_{i,j} = \Delta g_{xy}(x_i, y_j) = \left. \frac{\partial^2 (\Delta g)}{\partial x \partial y} \right|_{i,j},
$$

where the variables with respect to which $\Delta g(x, y)$ has been differentiated are indicated in the subscripts of $\Delta g(x, y)$. Further details of this method are given by Pelto et al. (1968) and Inoue (1986).

The bi-cubic spline method is a simple and fast method for interpolation. However, it requires the input data to be in the form of a regular grid. Therefore, this method is best suited to interpolation from a grid, e.g. to get point geoid undulation from a grid of geoid undulations. This method is used in Chapter Eight to determine point geoid undulation from a grid.

6.4 Least Squares Polynomial Fitting (LSPF)

The mathematical expression for a continuous surface, $\Delta g(x, y)$, which passes through $N$ points $Z_k=\Delta g(x, y)$ ($k=1,2,\ldots,N$) is necessarily complex. It must contain $N$ coefficients which are determined from the data and perhaps others necessitated by boundary (or other) conditions (Crain, 1970). The function of the simplest
mathematical form is the "interpolating polynomial" (ibid.), that is, the polynomial of lowest order \( M \) which passes through the \( N \) points. It reads
\[
\Delta g(x, y) = \sum_{i,j=0}^{M} a_{ij} x^{i} y^{j} \quad (i + j \leq M) ,
\]
(6.11)
where \( i=0,1,2,\ldots,k_{1} \) and \( j=0,1,2,\ldots,k_{2} \);
\( k_{1} \), \( k_{2} \) are the highest degrees of the polynomial development for \( x \) and \( y \) respectively;
\( M = (k_{1}+1)(k_{2}+1) \) is the total number of terms in the equation (6.11)
(Kassim, 1980, p.11); and
\( a_{ij} \) are the coefficients to be determined.

The coefficients \( a_{ij} \) are determined by solving the following set of equations:
\[
\Delta g(x_{i}, y_{j}) = Z_{k} \quad (k = 1,2,\ldots,N) .
\]
(6.12)
Equation (6.11) is a 2-D polynomial approximation. It has some inherent difficulties. Since the polynomial is entirely unconstrained except at the data points, the values attained between the points may be highly unreasonable. For high order polynomials, the solution may oscillate, thus giving inaccurate interpolation. Besides, the higher order terms frequently cause instability to the solution. To avoid inverting such a large matrix, a piecewise 2-D least squares polynomial interpolation is introduced in this study. The following polynomials are used as a local approximation:
\[
\Delta g(x, y, h) = \sum_{i,j=0}^{M_{1}} a_{ij} x^{i} y^{j} + \sum_{k=1}^{M_{2}} b_{k} H^{k} ,
\]
(6.13)
where \( M_{1} \) and \( M_{2} \) are the numbers of terms for \((x,y)\) and \( H \) respectively;
\( H \) is the point height; and
\( b_{k} \) are the coefficients to be determined.

The importance of this method is that:

1. It can take into account local height information, if any, to remove the correlation with heights;
2. It is flexible, involves a lower work load, and is generally stable; and
3. Statistical estimation of the interpolation reliability can be easily achieved.
If \( M_1 + M_2 < N \), its solution is an overdetermined problem. In order to achieve a unique solution, the least squares criterion is applied as follows: Applying \( N \) known point values to (6.13), gives \( N \) observational equations which have \( (M_1 + M_2) \) unknowns. In matrix form, this reads:

\[
\bar{V} = \bar{X} \hat{A} - \bar{\Delta g} \quad (6.14)
\]

\[
\hat{A} = \left( \bar{X}^T P \bar{X} \right)^{-1} \bar{X}^T P \bar{\Delta g} , \quad (6.15)
\]

where \( \bar{X} \) is a coefficient matrix with dimension \( N \times (M_1 + M_2) \);
\( \hat{A} \) is the estimate for the unknown parameter matrix with dimension \( (M_1 + M_2) \times 1 \); and
\( \bar{\Delta g} \) is the constraining gravity anomaly vector with dimension \( N \times 1 \).

The corresponding root mean square estimation of accuracy can be thus expressed as:

\[
\sigma = \sqrt{\frac{\bar{V}^T P\bar{V}}{N - (M_1 + M_2)}} . \quad (6.16)
\]

The least squares polynomial method has a number of limitations and advantages. It requires the surface to be differential, continuous and smooth. It generally has large edge effects and outliers in less constrained areas, and is thus not suitable for gravity field prediction. However, this interpolator is mathematically simple and more suitable for trend analysis.

### 6.5 Moving Weighted Average Method

Any moving average is an expression of the following general model (Crain and Bhattacharyya, 1967; Burrough, 1986):

\[
\bar{\Delta g}_X = \frac{\sum_{k=1}^N W_k \bar{\Delta g}_k}{\sum_{k=1}^N W_k} \quad (6.17)
\]

That is, an estimated grid value (\( \bar{\Delta g}_X \)) is based on the weighted sum of \( N \) adjacent observations \( \Delta g_k \). The nature of the weighting function (\( W_k \)) varies from one moving average scheme to another. Most moving-average techniques consider distance from the estimated point to the observed points in some manner. The schemes differ in how they assign weights to the data. The most commonly used weighting methods are: inverse distance \( (r^{-i}) \) and square of the inverse distance \( (r^{-2}) \). In some cases, other
kinds of weighting methods are used, such as reciprocal exponential functions \( e^{-ar} \) (Delfiner and Dtilhomme, 1975). The resulting surface depends on the function or parameters of the function used, and the size of the domain or window from which the sample data points are drawn (Burrough, 1986).

The advantages of the moving weighted average method are
- it is arithmetically convenient and simple;
- it is expected that remote points should have less influence on the value of the grid point than should the nearby points, thus having a local signature (e.g. locally related);
- it appears to yield rapid convergence; and
- the resulting surface has a good mathematical feature, such as continuous and differentiable.

One of the disadvantages of any weighting method lies in the fact that the value to be predicted is bounded by the minimum and maximum of the sample used for the prediction. Another disadvantage is that the values estimated by moving averages are susceptible to the configuration of the data such as clustering and systematic planar trend or drift in the data points. The latter can be reduced by removing a geopotential model as a trend during the gridding process. Furthermore, the weight used is not optimal and the window domain is chosen somewhat arbitrarily.

### 6.6 The Kriging Method

Kriging attempts not only to estimate the values of a spatially distributed variable, but also to assess the probable error associated with these estimates (Krige, 1978; Matheron, 1963). It is optimal in the sense that it provides estimates of values at unrecorded places without bias \( E(\hat{\mathbf{x}}_0) = 0 \) and with minimum variance (equation 6.21) (Cressie, 1988; Burgess and Webster, 1980). It has been recognised as an important tool in solving a wide variety of problems in different stages of mining for which it was originally designed (Krige, 1978). In addition to mapping, Kriging can be applied as a filter (cf. Wiener's filter) to separate either the low frequency (trend) component or, conversely, the high frequency (nugget effect) component of the spatial variability.
The statistical theory from which Kriging techniques are derived is called the theory of regionalised variables (Deutsch and Journel, 1992). A variable is considered to be a regionalised variable if it varies from one place to another with apparent continuity, but cannot be represented by an ordinary, workable function. Instead, it can be better described by a stochastic surface. Topographic and structural surfaces, and gravity anomalies are all examples of regionalised variables. Therefore, the Kriging method describes the spatial properties of the natural phenomena.

The value of a variable $\Delta g$ at location $X=[x,y]^T$ is given by

$$\Delta g(X) = m(X) + \varepsilon'(X) + \varepsilon''(X)$$

where $m(X)$ is a deterministic function describing the basic structural part of $\Delta g$ at $X$;

$\varepsilon'(X)$ denotes the stochastic, locally varying and spatially related residuals from $m(X)$; and

$\varepsilon''(X)$ is an observation noise which is a spatially independent residual with zero mean and variance $\sigma^2$.

In fact, the Kriging method is a special kind of moving weighted average method where the weights are determined from statistical point of view. The Kriging estimator can be expressed as:

$$\hat{\Delta g}_X = \sum_{k=1}^{N} \lambda_k \Delta g_k$$

with

$$\sum_{k=1}^{N} \lambda_k = 1,$$

where $\hat{\Delta g}_X$ is the estimated value at location $X$. The weight $\lambda_k$ is determined in such a way that the Kriging estimator is an optimal estimator in the sense of being unbiased and having minimum estimation variance (Olea, 1974). That is:

$$\sigma^2_{de} = E[(\hat{\Delta g}_X - \Delta g_X)^2] = \text{minimum}$$

Kriging is used to build models of uncertainty that depend on the data available, in addition to the data configuration. However, stochastic simulation allows an alternative and equally probable realisations of the spatial distribution of the attributes under study. These alternative stochastic simulations provide a measure of uncertainty about the unsampled values taken altogether in space rather than one by one.
Universal Kriging This is a technique for combining the least-squares trend surface with the optimum moving average solution. A model drift form \( m(X) \) in equation (6.18) gives:

\[
\hat{m}(x) = \sum_{i=1}^{p} \beta_i f_i(X)
\]

(6.22)

where \( \beta \) are unknown coefficients;

\( f_i(X) \) are a function of integer powers of \( X \) and known at every point \( x_i \);

\( p \) is the number of terms used to describe the drift; and,

\( \varepsilon'(x) = 0 \) in equation (6.18).

In practice, the drift is often represented either by a linear expression or a quadratic expression (Burrough, 1986). The least squares solution requires that \( \beta \) are so chosen that:

\[
(\overline{\Delta g} - \overline{\hat{\beta} F})^T \overline{W}^{-1}(\overline{\Delta g} - \overline{\hat{\beta} F}) = \text{minimum },
\]

(6.23)

where

\[
\overline{\Delta g} = [\Delta g_1(x_1), \ldots, \Delta g_N(x_N)]^T
\]

(6.24)

and

\[
\overline{F} = f_{ij} = f_j(x_i), \quad (i = 1, 2, \ldots, N; \quad j = 1, 2, \ldots, p)
\]

(6.25)

In equation (6.23), \( \hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_p)^T \) is the estimator of \( \hat{\beta} = (\overline{\beta}_1, \overline{\beta}_2, \ldots, \overline{\beta}_p)^T \) and \( \overline{W} \) is the autocorrelation matrix of the residual. Solving equation (6.21) gives:

\[
\hat{\beta} = (\overline{F}^T \overline{W}^{-1} \overline{F})^{-1} \overline{F}^T \overline{W}^{-1} \overline{\Delta g}
\]

(6.26)

The residuals \( (\overline{\Delta g} - \overline{\hat{\beta}} F) \) from equation (6.23) will then be subject to the moving-average technique of equation (6.19) to give the final solution of the universal Kriging method. Further details can be found in Giltrap (1983), Journel and Huijbregts (1978) and Deutsch and Journel (1992).

Conceptually, Kriging is almost the same as least squares collocation in geodesy which was developed by Krarup (1969) and refined by Moritz (1989). A theoretical discussion on their similarities can be found in Dermanis (1984). The Kriging covariance function is more general than that in collocation, which is expressed as a functional of the disturbing potential only.

Theoretically the Kriging method is an optimal estimator, because it not only takes location and size of the data but also the distribution of the samples which is very
important for anisotropic data, such as the gravity field. Kriging has a number of advantages over most other interpolation methods:

- Kriging can smooth, or regress, estimates based on the proportion of total sample variance accounted for by random "noise" \( \epsilon^2(X) \). The noisier the data set, the less individual samples represent their immediate vicinity, and the more they are smoothed.
- Kriging can take into account the clustering effect. The weight assigned to a sample is lowered to the degree that its information is duplicated by nearby, highly correlated samples. This helps mitigate the impact of oversampling.
- Anisotropy can be taken into account in the Kriging method. When samples are more highly correlated in a particular direction, Kriging weights will be greater for sample in that direction.

The main disadvantage of this method is that it is very time consuming to compute its weight and sometimes higher memory requirement for solving the inverse of a large matrix, as with least squares collocation.

6.7 A Comparison of the Four Gridding Procedures

There are several aspects of concern in evaluating a gridding algorithm, such as whether the algorithm itself is exact or approximate, and its performance in poorly constrained regions. The most important consideration is its accuracy or reliability. That is, the predicted value should represent the actual case well. Another consideration is the computational effort. Less computer load is expected for computational efficiency.

In addition, gravity field prediction is necessarily a complicated task. An accurate prediction procedure generally involves other geophysical or geological information. To derive such a grid directly means that not only a horizontal interpolation, but also a form of downward or upward continuation is performed.

A description of the characteristics, advantages and disadvantages of the various gridding methods used for this study are summarised in Table 6.1.
Table 6.1  A descriptive comparison of the four gridding methods

<table>
<thead>
<tr>
<th>methods</th>
<th>characteristics</th>
<th>assumptions and limitations</th>
<th>advantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>moving weighted average</td>
<td>exact, local and deterministic estimator;</td>
<td>continuous; differential;</td>
<td>arithmetically simple;</td>
</tr>
<tr>
<td>(§6.5)</td>
<td>moderate or heavy computing load;</td>
<td>configuration of data points and size of window;</td>
<td>high local features;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>susceptible to clustering and trend effects;</td>
<td>constrained by maxima/minima;</td>
</tr>
<tr>
<td>minimum curvature spline</td>
<td>exact, local and deterministic estimator;</td>
<td>continuous second derivatives;</td>
<td>rapid convergence;</td>
</tr>
<tr>
<td>(§6.2)</td>
<td>homogeneity;</td>
<td>no adjustable parameters;</td>
<td>flexibility;</td>
</tr>
<tr>
<td></td>
<td>light computing load;</td>
<td>undesirable oscillation in poorly constrained areas</td>
<td>integral method;</td>
</tr>
<tr>
<td>Kriging (§6.6)</td>
<td>exact, local and stochastic estimator;</td>
<td>intrinsic hypothesis;</td>
<td>constrained by maxima/minima;</td>
</tr>
<tr>
<td></td>
<td>very heavy computing load;</td>
<td>continuous;</td>
<td>optimal interpolator;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>non-stationary of data;</td>
<td>best for detailed estimates and errors</td>
</tr>
<tr>
<td></td>
<td></td>
<td>large computing cost;</td>
<td>required;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>a priori information needed;</td>
<td></td>
</tr>
<tr>
<td>least square polynomial</td>
<td>approximate, global and stochastic estimator;</td>
<td>differential; continuous;</td>
<td>simple;</td>
</tr>
<tr>
<td>(§6.4)</td>
<td>homogeneity;</td>
<td>large edge effects;</td>
<td>fast;</td>
</tr>
<tr>
<td></td>
<td>light or moderate computing load;</td>
<td>unconstrained;</td>
<td>best for trend analysis;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>outliers;</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>errors spatially dependent;</td>
<td></td>
</tr>
</tbody>
</table>

6.8 Choice of the Optimal Gridding Procedure for the Australian Gravity Field

To predict the most accurate values in a regular grid from randomly distributed observations, an optimal procedure needs to be chosen. This includes two aspects. One is to choose the best gridding method based on its accuracy and efficiency. The other aspect is to determine an optimal gridding "platform" which has well-behaved and/or smooth features so that interpolation errors can be minimised.

Three test areas have been chosen which present typical gravity field structure of Australia (§5, Figure 5.1). The moving weighted average, Kriging, minimum curvature spline and polynomial gridding methods are compared in this study. Because of the special features of the Australian gravity field (§5.5), free-air, Bouguer and topographic-isostatic gravity anomaly types are tested. In addition, the residual gravity anomalies relative to the degree 360 expansion of the OSU91A model (i.e. the residual free-air, Bouguer and topographic-isostatic gravity anomalies) are also tested because they are used as part of the remove-restore technique. Theoretically, the Bouguer and topographic gravity anomalies should not be further filtered by subtracting a GGM (i.e. OSU91A). However, due to the complicated features of the Australian gravity field as found in Chapter Five, there still exist some medium wavelength spectral components (e.g. n<360) in the residual Bouguer and
topographic-isostatic gravity anomalies implied by a GGM. This scenario can further be solidified in §6.82 (also see Table 6.3). Moreover, the removal of the GGM is simply tested to determine whether it can improve the interpolation as a numerical procedure rather than a physical process. These six reference gravity anomaly types with which gridding is performed are called "platforms" of the gridding hereafter.

The evaluation strategy used is to compare known values with those produced through various gridding procedures are based on different platforms in the three areas. These platforms correspond to different kinds of residual gravity anomalies which are obtained through a remove-restore procedure. The statistical comparisons used are:

- different gridding methods used for the same residual gravity anomalies in the same area;
- same gridding method used for different residual gravity anomalies in different areas;
- same gridding method used for different residual gravity anomalies in the same area;
- different gridding methods used for the same residual gravity anomalies in different areas.

6.8.1 A Practical Remove-restore Technique for Gravity Gridding

As discussed in §2.7, a commonly used gridding technique involves the following three steps: i) removal of the irregular part of the gravity field on the point reference surface (remove), ii) perform prediction or gridding on the smoothed reference surface ("platform"), iii) restoration of the actual gravity field to get the predictions on the point reference surface (restore). This remove-restore technique used for the gridding of the gravity anomalies can be depicted as the following three steps:

1. The residual gravity anomalies as computed using

\[
\Delta g_{\text{ref}}^{\text{obs}} = \begin{cases} 
\Delta g_{\text{FA}} - \delta g_{\text{obs}} & \text{without GGM removed}, \\
\Delta g_{\text{FA}} - \delta g_{\text{obs}} - \Delta g_{\text{GGM}} & \text{with GGM removed}, 
\end{cases}
\]  

(6.27)

where the subscript 'obs' indicates the reduction is conducted on the observed points rather than the prediction points;

\( \Delta g_{\text{ref}}^{\text{obs}} \) are the residual gravity anomalies on the reference surface;

\( \Delta g_{\text{FA}} \) are free-air gravity anomalies defined by equation (3.8);

\( \Delta g_{\text{GGM}} \) is the gravity anomaly computed via equation (2.34) using the OSU91A geopotential coefficients to degree 360; and
\( \delta_{g_{\text{obs}}} \) is the gravity reduction or corrections used to smooth the gravity anomalies.

In this study, the free-air, Bouguer and topographic-isostatic gravity anomalies are used for the computation of the residual gravity anomalies. Therefore, \( \delta_{g_{\text{obs}}} \) can be further expressed as follows:

\[
\delta_{g_{\text{obs}}} = \begin{cases} 
0 & \text{free-air anomaly,} \\
\delta_{g_{bg}} = 2\pi G \rho H - C & \text{Bouguer anomaly,} \\
\delta_{g_{TI}} & \text{topographic-isostatic anomaly,}
\end{cases}
\] (6.28)

where \( \delta_{g_{TI}} \) is the topographic-isostatic correction, computed by means of a 1’×1’ DTM information which includes removal of direct terrain effect and its isostatic compensation;

\( \delta_{g_{bg}} \) is the Bouguer anomaly correction which includes two parts. One is the removal of a Bouguer plate effect (2\( \pi \)G\( \rho \)H) and the other is the terrain correction (C) (equation 2.40).

2. Interpolate these onto a gridded mesh by the means of an optimal gridding method

\[
\Delta g_{\text{res}}^{\text{pred}} = f(\Delta g_{\text{obs}}^{\text{ref}})
\] (6.29)

where the predicted values \( \Delta g_{\text{res}}^{\text{pred}} \) on a regular grid are expressed as a function \( f \) of the residual gravity anomalies at the observed locations \( \Delta g_{\text{obs}}^{\text{ref}} \). This can be achieved by one of the gridding procedures, described in §6.2 through §6.7.

3. Reconstruct free-air anomalies on this regular grid using the DTM and geopotential model where applicable,

\[
\Delta g_{\text{free}}^{\text{pred}} = \begin{cases} 
\Delta g_{\text{res}}^{\text{pred}} + \delta_{g_{\text{res}}} & \text{without GGM removed,} \\
\Delta g_{\text{res}}^{\text{pred}} + \delta_{g_{\text{res}}} + \Delta g_{\text{GGM}}^{\text{pred}} & \text{with GGM removed,}
\end{cases}
\] (6.30)

where \( \delta_{g_{\text{res}}} \) are the residual gravity anomalies corresponding to the terrain reductions in equation (6.28) at the desired grid points;

\( \Delta g_{\text{GGM}}^{\text{pred}} \) are gravity anomalies from OSU91A in the desired gridded points.

This, and other similar remove-restore methods, assume that the residual gravity anomalies are smooth and, as such, are expected to reduce interpolation errors.
6.8.2 The Test Areas and Strategies

In order to study the best procedure and anomaly type for gridding the Australian gravity field, the three test areas chosen in Chapter Five (Figure 5.1) which correspond to different gravity field structures (§5.3–5.5) and different data distributions (§3.4), are used. As illustrated in Chapter Five, the gravity field in these three areas shows extremes in the behaviour of the Australian gravity field. The simple statistical comparison (mainly STD) is used throughout this chapter because it is simple and informative as demonstrated in Chapter Five.

A number of observed gravity points are selected as reference points (known values) which are compared against to determine an optimal gridding procedure. The program *Pickup* is used to select these reference points in such a way that the locations of these points coincide with the centre of a 1' by 1' grid lattice. These reference points are then excluded from the gridding process. If a gridding method is the best one, it is reasonable to expect that the predicted values are a best fit to these reference point values from a statistical point of view. The data information in these three areas is listed in Table 6.2 and the location and distribution of the data in the three test areas are shown in Figures 6.1, 6.2 and 6.3, respectively.

<table>
<thead>
<tr>
<th>No of points</th>
<th>total</th>
<th>gridding</th>
<th>known</th>
<th>average data density</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2 [129°E–132°E, 23.5°S–26.5°S]</td>
<td>16773</td>
<td>16507</td>
<td>266</td>
<td>1 point/3km</td>
</tr>
<tr>
<td>A3 [146°E–148°E, 35.0°S–37.0°S]</td>
<td>16426</td>
<td>16360</td>
<td>66</td>
<td>1 point/2km</td>
</tr>
</tbody>
</table>

As shown in Figures 6.1 to 6.3, the distribution of the gravity observations is heterogeneous and anisotropic. The gravity observations are clustered in some places (see §3.4 and Figures 6.1–6.3). For instance, in test area A1 the gravity distribution is fairly even except for some observations made along tracks. In test areas A2 and A3, the distribution of the data is apparently not uniform. The gravity observations are clustered in some areas and sparse in other areas, such as in the lower left corner of A3. This may cause some difficulty for the gravity field gridding, as discussed in §6.7.
Figure 6.1 Distribution of the gravity data in area A1
(* is reference point, + is data point, Linear projection)

Figure 6.2 Distribution of the gravity data in area A2
(* is reference point, + is data point, Linear projection)
The statistics of the six gravity anomaly types in the three areas are listed in Table 6.3, and the structures (contour maps) of the free-air (FA), Bouguer (BA), isostatic-topographic (TT) and residual free-air gravity anomalies relative to OSU91A in A1 are shown in Figures 6.4a, 6.4b, 6.4c and 6.4d respectively. The reference points (known data) in A1 are also shown on Figures 6.4a through 6.4d. Since the residual gravity anomaly contour maps for areas A2 and A3 show similar features, they are not presented here.

Reviewing Table 6.3, it is evident that the Bouguer and topographic-isostatic gravity anomalies are not necessarily smoother than free-air anomalies because of the special features of the Australian gravity field (§5.6). This implies that the medium and high frequency components of the gravity field are not effectively removed through the terrain reduction process. This is most probably due to the assumption of a uniform topographic density of $\rho=2,670\text{kgm}^{-3}$. However, the subtraction of OSU91A from the free-air gravity anomalies can greatly improve their smoothness (Table 6.3).
Therefore, residual Bouguer and topographic-isostatic gravity anomalies were also computed. The statistics of the corresponding residual Bouguer (BA-GGM) and topographic-isostatic gravity anomalies (TI-GGM) are also listed in Table 6.3.

Table 6.3 The statistics of the various gravity anomaly data used in the three areas (unit in milligals)

<table>
<thead>
<tr>
<th>Areas</th>
<th>Maximum-minimum</th>
<th>Mean</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1</td>
<td>A2</td>
<td>A3</td>
</tr>
<tr>
<td>FA</td>
<td>126.3</td>
<td>238.7</td>
<td>217.2</td>
</tr>
<tr>
<td>BA</td>
<td>221.5</td>
<td>263.9</td>
<td>107.1</td>
</tr>
<tr>
<td>TI</td>
<td>117.7</td>
<td>224.6</td>
<td>105.5</td>
</tr>
<tr>
<td>FA-GGM</td>
<td>62.3</td>
<td>132.0</td>
<td>219.8</td>
</tr>
<tr>
<td>BA-GGM</td>
<td>265.4</td>
<td>352.5</td>
<td>180.1</td>
</tr>
<tr>
<td>TI-GGM</td>
<td>59.1</td>
<td>142.7</td>
<td>139.9</td>
</tr>
</tbody>
</table>

Notes: 1. FA=free-air anomaly, 2. BA=Bouguer anomaly
3. TI= topographic-isostatic anomaly, 4. GGM=OSU91A detrended

In Table 6.3, the residual gravity anomalies (TI-GGM and BA-GGM) do not outperform the residual free-air anomalies (FA-GGM) in terms of standard deviation except in A1 where the STD of TI-GGM is slightly smaller than that of FA-GGM. The STDs of the residual Bouguer and TI anomalies are rougher than either the anomalies themselves or residual free-air anomalies (except for the STD of the TI-GGM in A1). On the other hand, the computation of these two residual gravity anomalies requires additional terrain and geophysical information (e.g. detailed DTM and isostatic status of the crust) and involves much computer effort to remove the terrain effects.

From Figures 6.4a to 6.4d, the smoothness of different reference surfaces varies as demonstrated by their statistical properties. The residual free-air gravity anomaly surface (FA-GGM) is the smoothest and the Bouguer anomaly surface (BA-GGM) is the roughest in test area A2. However, this changes in the test area A3 where topographic-isostatic gravity anomaly (TI) is the smoothest and BA-GGM is the roughest. A comparison of smoothest versus roughest for various reference surfaces (gridding platforms) in the three areas is summarised in Table 6.4. Again, none of the four kinds of residual anomalies are consistently smooth or rough.
Figure 6.4a Contour map of free-air gravity anomalies in area A1
(Linear projection, contour interval=10mgal)

Figure 6.4b Contour map of Bouguer gravity anomalies in area A1
(Linear projection, contour interval=10mgal)
Figure 6.4c  Contour map of topographic-isostatic gravity anomalies in area A1
(Linear projection, contour interval= 10mgal)

Figure 6.4d  Contour map of OSU91A residual free-air gravity anomalies in area A1
(Linear projection, contour interval= 10mgal)
Table 6.4 Reference surface comparisons of smoothness versus roughness (STD)

<table>
<thead>
<tr>
<th></th>
<th>Smoothest</th>
<th>Smoother</th>
<th>Smooth</th>
<th>Rough</th>
<th>Rougher</th>
<th>Roughest</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>TI-GGM (9.2)</td>
<td>FA-GGM (9.4)</td>
<td>TI (23.6)</td>
<td>FA (25.4)</td>
<td>BA (43.3)</td>
<td>BA-GGM (47.4)</td>
</tr>
<tr>
<td>A2</td>
<td>FA-GGM (12.2)</td>
<td>TI-GGM (12.6)</td>
<td>FA (40.1)</td>
<td>TI (42.2)</td>
<td>BA (96.2)</td>
<td>BA-GGM (129.1)</td>
</tr>
<tr>
<td>A3</td>
<td>TI (12.2)</td>
<td>TI-GGM (14.4)</td>
<td>FA-GGM (15.7)</td>
<td>BA (17.6)</td>
<td>FA (20.1)</td>
<td>BA-GGM (31.1)</td>
</tr>
</tbody>
</table>

From Table 6.4, TI-GGM, FA-GGM and TI are the smoothest anomalies in areas A1, A2 and A3, respectively. However, the TI-GGM and FA-GGM are consistently smooth in the three areas although they are not necessarily always the smoothest. Considering the facts that the TI-GGM involves very heavy computing load, and a detailed digital terrain model and geophysical information for the isostatic equilibrium status are required prior to the computation, this method is not recommended for extensive use. In contrast, however, the FA-GGM involves relatively less computer load and can be easily implemented. Moreover, no terrain and geophysical information is required for its computation. Therefore, it is recommended for subsequent use.

6.8.3 Numerical Comparisons and Analysis

Tables 6.5 (a–d), 6.6 (a–d) and 6.7 (a–d) list the statistical comparisons between reference points and gridded values at same points for the various gridding procedures based on different reference surfaces in areas A1, A2 and A3 respectively. The simple statistical comparison method is used because it is simple and informative as demonstrated in §5.7. A summary of the standard deviation (STD) agreement (with known values) of various gridding procedures in the three areas is given in Table 6.8.

Table 6.5a Accuracy comparisons of various gridding procedures in area A1 based on free-air anomalies (unit in milligals)

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance (1/r^2)</td>
<td>9.98</td>
<td>-10.52</td>
<td>-0.40</td>
<td>5.77</td>
<td>5.70</td>
</tr>
<tr>
<td>Inverse distance (1/r^2)</td>
<td>11.25</td>
<td>-10.21</td>
<td>0.15</td>
<td>5.18</td>
<td>5.09</td>
</tr>
<tr>
<td>Kriging</td>
<td>8.91</td>
<td>-9.74</td>
<td>-0.84</td>
<td>3.88</td>
<td>3.86</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>9.08</td>
<td>-11.27</td>
<td>-0.99</td>
<td>5.55</td>
<td>5.52</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>25.96</td>
<td>-27.84</td>
<td>-0.16</td>
<td>12.79</td>
<td>12.56</td>
</tr>
</tbody>
</table>
Table 6.5b Accuracy comparisons of various gridding procedures in area A1 based on Bouguer anomalies (unit in milligals)

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance (1/r^3)</td>
<td>77.24</td>
<td>-69.39</td>
<td>-5.88</td>
<td>22.46</td>
<td>22.33</td>
</tr>
<tr>
<td>Inverse distance (1/r^2)</td>
<td>87.03</td>
<td>-60.12</td>
<td>-7.70</td>
<td>27.90</td>
<td>27.30</td>
</tr>
<tr>
<td>Kriging</td>
<td>22.73</td>
<td>-66.55</td>
<td>-3.92</td>
<td>17.81</td>
<td>17.70</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>36.98</td>
<td>-57.96</td>
<td>-2.22</td>
<td>17.35</td>
<td>17.18</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>161.72</td>
<td>-55.42</td>
<td>-12.98</td>
<td>51.39</td>
<td>50.64</td>
</tr>
</tbody>
</table>

Table 6.5c Accuracy comparisons of various gridding procedures in area A1 based on topographic-isostatic gravity anomalies (unit in milligals)

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance (1/r^3)</td>
<td>6.04</td>
<td>-10.28</td>
<td>-0.98</td>
<td>5.19</td>
<td>5.15</td>
</tr>
<tr>
<td>Inverse distance (1/r^2)</td>
<td>6.32</td>
<td>-10.36</td>
<td>-0.87</td>
<td>5.45</td>
<td>5.45</td>
</tr>
<tr>
<td>Kriging</td>
<td>5.09</td>
<td>-7.90</td>
<td>-0.79</td>
<td>2.96</td>
<td>2.91</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>5.94</td>
<td>-8.42</td>
<td>-0.82</td>
<td>3.36</td>
<td>3.32</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>19.46</td>
<td>-22.56</td>
<td>-1.24</td>
<td>9.81</td>
<td>9.91</td>
</tr>
</tbody>
</table>

Table 6.5d Accuracy comparisons of various gridding procedures in area A1 based on residual free-air gravity anomalies relative to OSU91A (unit in milligals)

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance (1/r^3)</td>
<td>10.47</td>
<td>-10.42</td>
<td>0.12</td>
<td>5.29</td>
<td>5.29</td>
</tr>
<tr>
<td>Inverse distance (1/r^2)</td>
<td>10.44</td>
<td>-9.85</td>
<td>0.66</td>
<td>5.64</td>
<td>5.59</td>
</tr>
<tr>
<td>Kriging</td>
<td>8.74</td>
<td>-9.12</td>
<td>-0.88</td>
<td>3.81</td>
<td>3.71</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>8.27</td>
<td>-9.47</td>
<td>-1.05</td>
<td>5.23</td>
<td>5.10</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>15.24</td>
<td>-12.88</td>
<td>2.14</td>
<td>7.81</td>
<td>7.51</td>
</tr>
</tbody>
</table>

From Tables 6.5 (a–d), it is concluded that quite different results are obtained using different gridding methods in the same area. When the Bouguer anomaly type is used as the gridding platform, the worst results are obtained. The STDs in Table 6.5b are much larger than their corresponding values in Tables 6.5a, 6.5c and 6.5d because the Bouguer surface is the roughest surface in area A1. The gridding precisions for “platforms” FA, TI and FA-GGM are close for the same gridding algorithm (in the same area) though their roughness varies from STD=9.38 mgal (GGM-FA) to 25.4 mgal (FA). Therefore, it is concluded that the Bouguer anomaly type should not be used for gridding in this area. In addition, the effect of using different gridding methods is not as critical as that of using different platforms.
Table 6.6a Accuracy comparisons of various gridding procedures in area A2 based on free-air anomalies (unit in milligals)

<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance (1/r^2)</td>
<td>28.22</td>
<td>-12.32</td>
<td>0.28</td>
<td>3.67</td>
<td>3.66</td>
</tr>
<tr>
<td>Inverse distance (1/r^3)</td>
<td>36.78</td>
<td>-15.56</td>
<td>0.36</td>
<td>5.66</td>
<td>5.65</td>
</tr>
<tr>
<td>Kriging</td>
<td>17.96</td>
<td>-11.07</td>
<td>0.23</td>
<td>2.63</td>
<td>2.62</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>13.74</td>
<td>-13.77</td>
<td>0.22</td>
<td>2.77</td>
<td>2.76</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>125.5</td>
<td>-27.25</td>
<td>-1.38</td>
<td>25.60</td>
<td>25.57</td>
</tr>
</tbody>
</table>

Table 6.6b Accuracy comparisons of various gridding procedures in area A2 based on Bouguer anomalies (unit in milligals)

<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance (1/r^2)</td>
<td>160.06</td>
<td>-92.20</td>
<td>1.79</td>
<td>20.23</td>
<td>20.15</td>
</tr>
<tr>
<td>Inverse distance (1/r^3)</td>
<td>129.06</td>
<td>-107.89</td>
<td>1.45</td>
<td>21.39</td>
<td>21.34</td>
</tr>
<tr>
<td>Kriging</td>
<td>139.17</td>
<td>-139.90</td>
<td>1.28</td>
<td>18.79</td>
<td>18.75</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>152.41</td>
<td>-145.32</td>
<td>1.48</td>
<td>18.86</td>
<td>18.80</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>78.57</td>
<td>-269.16</td>
<td>-3.51</td>
<td>78.86</td>
<td>78.78</td>
</tr>
</tbody>
</table>

Table 6.6c Accuracy comparisons of various gridding procedures in area A2 based on topographic-isostatic gravity anomalies (unit in milligals)

<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance (1/r^2)</td>
<td>29.02</td>
<td>-13.18</td>
<td>0.30</td>
<td>3.66</td>
<td>3.65</td>
</tr>
<tr>
<td>Inverse distance (1/r^3)</td>
<td>37.83</td>
<td>-13.69</td>
<td>0.42</td>
<td>5.68</td>
<td>5.66</td>
</tr>
<tr>
<td>Kriging</td>
<td>18.34</td>
<td>-12.05</td>
<td>0.26</td>
<td>2.61</td>
<td>2.60</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>13.67</td>
<td>-15.88</td>
<td>0.25</td>
<td>2.80</td>
<td>2.79</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>129.57</td>
<td>-25.81</td>
<td>-0.26</td>
<td>26.59</td>
<td>26.59</td>
</tr>
</tbody>
</table>

Table 6.6d Accuracy comparisons of various gridding procedures in area A2 based on residual free-air gravity anomalies relative to OSU91A (unit in milligals)

<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance (1/r^2)</td>
<td>27.44</td>
<td>-12.86</td>
<td>0.19</td>
<td>3.53</td>
<td>3.53</td>
</tr>
<tr>
<td>Inverse distance (1/r^3)</td>
<td>35.40</td>
<td>-13.62</td>
<td>0.26</td>
<td>5.47</td>
<td>5.46</td>
</tr>
<tr>
<td>Kriging</td>
<td>17.69</td>
<td>-11.07</td>
<td>0.11</td>
<td>2.61</td>
<td>2.60</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>13.58</td>
<td>-15.15</td>
<td>0.07</td>
<td>2.66</td>
<td>2.66</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>79.73</td>
<td>-36.10</td>
<td>-0.78</td>
<td>11.79</td>
<td>11.76</td>
</tr>
</tbody>
</table>

Similarly, from Tables 6.6 (a–d), it is concluded again that quite different results are obtained using different gridding methods in the same area. The Bouguer anomaly platform gives the worst gridding results. The STDs in Table 6.6b are much larger than their corresponding values in Tables 6.6a, 6.6c and 6.6d because the Bouguer surface is the roughest surface in area A2. The gridding precisions for "platforms"
FA, TI and FA-GGM are close for a certain procedure though their STDs vary from 12.22 mgal (FA-GGM) to 42.2 mgal (TI). Therefore, it is concluded that the Bouguer anomaly type should not be used for gridding in this area. The FA-GGM platform is the best platform. This is expected since the platform FA-GGM is the smoothest in this area.

Table 6.7a Accuracy comparisons of various gridding procedures in area A3 based on free-air anomalies (unit in milligals)

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance ($1/r^3$)</td>
<td>20.33</td>
<td>-9.42</td>
<td>1.74</td>
<td>6.40</td>
<td>6.16</td>
</tr>
<tr>
<td>Inverse distance ($1/r^2$)</td>
<td>25.06</td>
<td>-11.48</td>
<td>1.65</td>
<td>7.79</td>
<td>7.61</td>
</tr>
<tr>
<td>Kriging</td>
<td>21.58</td>
<td>-7.75</td>
<td>1.58</td>
<td>5.61</td>
<td>5.39</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>20.08</td>
<td>-9.02</td>
<td>1.96</td>
<td>5.22</td>
<td>5.84</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>45.28</td>
<td>-52.96</td>
<td>-10.32</td>
<td>23.16</td>
<td>20.74</td>
</tr>
</tbody>
</table>

Table 6.7b Accuracy comparisons of various gridding procedures in area A3 based on Bouguer anomalies (unit in milligals)

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance ($1/r^3$)</td>
<td>5.32</td>
<td>-11.08</td>
<td>-0.29</td>
<td>3.10</td>
<td>3.09</td>
</tr>
<tr>
<td>Inverse distance ($1/r^2$)</td>
<td>6.07</td>
<td>-11.29</td>
<td>-0.25</td>
<td>3.40</td>
<td>3.39</td>
</tr>
<tr>
<td>Kriging</td>
<td>5.15</td>
<td>-6.98</td>
<td>-0.06</td>
<td>2.15</td>
<td>2.15</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>5.38</td>
<td>-6.00</td>
<td>0.25</td>
<td>2.13</td>
<td>2.11</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>35.74</td>
<td>-19.33</td>
<td>1.95</td>
<td>12.90</td>
<td>12.75</td>
</tr>
</tbody>
</table>

Table 6.7c Accuracy comparisons of various gridding procedures in area A3 based on topographic-isostatic gravity anomalies (unit in milligals)

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance ($1/r^3$)</td>
<td>12.08</td>
<td>-9.80</td>
<td>0.22</td>
<td>3.54</td>
<td>3.53</td>
</tr>
<tr>
<td>Inverse distance ($1/r^2$)</td>
<td>8.83</td>
<td>-11.84</td>
<td>0.21</td>
<td>3.58</td>
<td>3.57</td>
</tr>
<tr>
<td>Kriging</td>
<td>10.39</td>
<td>-5.35</td>
<td>0.66</td>
<td>2.70</td>
<td>2.62</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>10.93</td>
<td>-5.72</td>
<td>1.03</td>
<td>2.99</td>
<td>2.81</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>25.81</td>
<td>-23.39</td>
<td>-2.29</td>
<td>11.96</td>
<td>11.74</td>
</tr>
</tbody>
</table>

Table 6.7d Accuracy comparisons of various gridding procedures in area A3 based on residual free-air gravity anomalies relative to OSU91A (unit in milligals)

<table>
<thead>
<tr>
<th>Method</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance ($1/r^3$)</td>
<td>21.79</td>
<td>-10.53</td>
<td>1.92</td>
<td>6.91</td>
<td>6.64</td>
</tr>
<tr>
<td>Inverse distance ($1/r^2$)</td>
<td>28.52</td>
<td>-13.00</td>
<td>1.91</td>
<td>8.37</td>
<td>8.15</td>
</tr>
<tr>
<td>Kriging</td>
<td>21.37</td>
<td>-7.65</td>
<td>1.52</td>
<td>5.59</td>
<td>5.38</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>20.24</td>
<td>-8.94</td>
<td>1.86</td>
<td>5.21</td>
<td>5.87</td>
</tr>
<tr>
<td>Polynomial (3rd order)</td>
<td>51.77</td>
<td>-33.19</td>
<td>3.97</td>
<td>17.96</td>
<td>17.51</td>
</tr>
</tbody>
</table>
As shown in Tables 6.7 (a–d), the best results can be obtained if the TI or BA platform is used. The gridding method used is as important as the selection of the gridding platform. The gridding precisions using different methods and different platforms in this area do not change as much as in areas A1 and A2. This is due to the relative roughness of the four gravity anomalies is close which implies the roughness of the platform affects gridding precision most. In addition, both the FA and FA-GGM platforms give similar poor agreement. Among the procedures tested here, the Kriging and minimum curvature spline give similar good results and Kriging is slightly better than minimum curvature spline method.

Table 6.8 gives a summary of the STD results for the various gravity anomaly types in the three test areas using different gridding methods. It is demonstrated that both the smoothness of the reference surface and the gridding method used are important for high precision gridding. However, the selection of gridding platform is more critical than the gridding method itself. The smoother the reference surface, the smaller the STD error of the gridding. When the same platform is used, the Kriging and Minimum Curvature spline methods give similar good results. The highest precisions achieved are 2.9 mgal, 2.6 mgal and 2.1 mgal for areas A1, A2 and A3 respectively. Therefore, it is concluded that the gridding precision in the three test areas is better than 3 mgal if the smoothest platform together with the best gridding method (minimum tension spline) is used.

<table>
<thead>
<tr>
<th>Area No</th>
<th>Free Air</th>
<th>Bouguer</th>
<th>Topo.-Isost.</th>
<th>FA-GGM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1 A2 A3</td>
<td>A1 A2 A3</td>
<td>A1 A2 A3</td>
<td>A1 A2 A3</td>
</tr>
<tr>
<td>Inv. dist. (1/r²)</td>
<td>5.7 3.7 6.2</td>
<td>22.3 20.2 3.1</td>
<td>5.2 3.7 3.5</td>
<td>5.3 3.5 6.6</td>
</tr>
<tr>
<td>Inv. dist. (1/r³)</td>
<td>5.1 5.7 7.6</td>
<td>27.3 21.3 3.4</td>
<td>5.5 5.7 3.6</td>
<td>5.6 5.5 8.2</td>
</tr>
<tr>
<td>Kriging</td>
<td>3.9 2.6 5.4</td>
<td>17.7 18.8 2.2</td>
<td>2.9 2.6 2.6</td>
<td>3.7 2.6 5.4</td>
</tr>
<tr>
<td>Mini. curv.</td>
<td>5.5 2.8 5.8</td>
<td>17.2 18.8 2.1</td>
<td>3.3 2.8 2.8</td>
<td>5.1 2.7 5.9</td>
</tr>
<tr>
<td>Polynomial</td>
<td>12.6 25.6 20.7</td>
<td>50.6 78.8 12.6</td>
<td>9.9 26.6 11.7</td>
<td>7.5 11.8 17.5</td>
</tr>
<tr>
<td>Data</td>
<td>25.4 40.1 20.1</td>
<td>43.3 96.2 17.6</td>
<td>23.6 42.2 12.2</td>
<td>9.4 12.2 15.7</td>
</tr>
</tbody>
</table>

In addition, to compare computational efficiency, the approximate computer time used for the computation using these methods for area A1 is listed in Table 6.9. It is clear that the Kriging method is very time consuming and the minimum curvature
spline method is very computationally efficient, whilst giving similar good results. Therefore, the minimum tension spline method has been chosen to grid the gravity data in this study.

Table 6.9 Computer time used for the gridding using different methods (1°×1° grid)

<table>
<thead>
<tr>
<th>method</th>
<th>approximate CPU time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse distance cubed (1/r^3)</td>
<td>1200</td>
</tr>
<tr>
<td>Inverse distance squared (1/r^2)</td>
<td>900</td>
</tr>
<tr>
<td>Kriging</td>
<td>10800</td>
</tr>
<tr>
<td>Minimum Curvature Spline</td>
<td>210</td>
</tr>
<tr>
<td>3rd order Polynomial</td>
<td>300</td>
</tr>
</tbody>
</table>

6.9 Discussion

As shown in Table 6.8, the numerical tests indicate that different gridding methods lead to quite different precisions in the same area using the same kind of platform, such as the Kriging and polynomial methods in the three areas. Using different platforms in the same area (i.e. the same data distribution), the gridding also leads to a quite different agreement with known values, such as the free-air and Bouguer anomalies in area A2. Generally, the smoother the platform (low STD), the better the agreement with known values. A rough platform (high STD) may lead to large gridding error, such as the Bouguer anomaly in areas A1 and A2. It is also shown that using the same gridding method for different platforms in the same area, the precision of the results is quite different. In addition, different gridding methods lead to quite different results in different areas although the same kind of platform is used, such as the Bouguer anomaly in areas A2 and A3.

In the three areas, not one of the four platforms is consistently the smoothest or the roughest, as found in Chapter Five. The roughness of a surface as determined by statistics determines the accuracy of estimation. When gridding based on different reference surfaces in the same area, the same method leads to quite different results.

Of all the methods used in this section of the research, the polynomial method leads to the worst agreement with known values and thus should not be used for gridding. From the comparisons between different powers of inverse distance in all the three
areas and all four reference surfaces, the gridding accuracy of the moving weighted average is higher when third order inverse distance is used except for free-air anomalies in area A1 (Table 6.5a). This implies that the gravity anomalies possess more power in the short wavelength features (Table 6.8).

Overall, the Kriging and minimum curvature spline methods perform almost equally well. They are the best procedures for all the four reference surfaces (FA, BA, TI and FA-GGM) and in all the three areas (A1, A2 and A3). Although the Kriging method is slightly better than the minimum curvature spline in terms of accuracy, the latter is over fifty times faster. The gridding precision is estimated better than 3 mgal in the three areas if the smoothest platforms are used.

Rigorously, the smoothest surface should be used in each area to achieve the best results. However, the determination of TI and TI-GGM requires a detailed DTM and is computationally very heavy. Moreover, it involves some geophysical information (isostatic model and parameters) which contains some uncertainties. Therefore, TI and TI-GGM reference surface are not recommended for extensive use. In contrast, however, the residual free-air reference surface (with the removal of OSU91A) is fairly smooth in all three areas (Table 6.4), though it is not necessarily always the smoothest. FA-GGM is recommended for the most suitable gridding platform. For these reasons, the FA-GGM platform together with the minimum curvature spline method will be used in the determination of the Australian geoid.

6.10 Summary

This chapter discusses the theory and methodology used to find an optimal gridding procedure for Australian gravity data. The construction of a gridded data set from randomly located observations requires interpolation. This procedure is a critical issue for subsequent gravity field refinement because any error committed at this stage will directly propagate into subsequent geoid determination. The purpose to select an optimal gridding procedure is to use the discrete observational information as much as possible to predict accurate grid values. The methods used for this study are the moving weighted average, Kriging, minimum curvature splines and polynomial gridding methods. Three test areas were chosen for this study which represent typical
gravity field structure. The free-air anomaly, Bouguer and topographic-isostatic
gravity anomalies both with and without the removal of the 360 degree OSU91A are
tested to find the best gridding "platforms".

As the gravity data distribution in Australia is heterogeneous and clustered, it is
critical to find an optimal gridding procedure. Both the gridding method and the
gridding platform are important for an accurate gravity field gridding. However, the
effects of using the inappropriate gravity anomaly type for gridding can be substantial.

It is also concluded that the removal of a low frequency gravity field together with
gravity reductions is also important for gridding. The most appropriate platform
gridding Australian gravity field is the residual free-air gravity anomaly with the
removal of OSU91A.

Among the gridding methods used here, Kriging and minimum curvature splines
methods produced similar results. However, the minimum curvature splines method is
computationally more efficient. Therefore, the minimum curvature spline method is
recommended for the gridding of Australian gravity. When this optimal gridding
procedure is used, the gridding precision of the gravity anomalies can be greater than
3 mgal.
AN EVALUATION OF FFT GEOID DETERMINATION
TECHNIQUES AND THEIR APPLICATION TO HEIGHT
DETERMINATION USING GPS IN AUSTRALIA

CHAPTER SEVEN

EVALUATION OF THE TERRAIN CORRECTION AND ITS INDIRECT EFFECT
CHAPTER 7

EVALUATION OF THE TERRAIN CORRECTION AND ITS INDIRECT EFFECT

7.1 Introduction

Detailed terrain information and gravity anomalies are essential for the accurate gravimetric determination of the geoid, since they generally reveal the high and medium frequency components of the geoid (e.g. Wang and Rapp, 1990; Stewart, 1990; Bian and Zhang, 1991a). However, the terrain height information is important to regional gravity field refinement in the following cases:

1. The height information is used to smooth the gravity field through the remove-restore technique, and thus very helpful for gravity field prediction. Generally, after the removal of the terrain effects, the residual gravity anomaly surface becomes very smooth so that the remove-restore technique can be applied to predict other gravity field quantities and to perform gridding.

2. It is used for the determination of free-air, Bouguer, topographic-isostatic and residual terrain model gravity anomalies (Forsberg, 1984; Forsberg and Tscherning, 1981; §2.6.2).

3. Taking into account terrain effects in the determination of the geoid is extremely important both practically and theoretically (§2.6). Shifting all topographic masses onto the geoid can compensate for some of the deficiencies in the application of Stokes's method. In practice, this is achieved using a digital terrain model to reduce the topographic masses in order to preserve harmonicity, then computing the corresponding indirect effect of this reduction.

4. The terrain information can also provide short-wavelength geoid undulations that are not always sampled by terrestrial gravity observations alone.

5. When combined with detailed topographic density information, terrain height information may be an important alternative to obtaining detailed gravity field information as a by-product of geophysical exploration and prospecting.

The terrain correction plays an important role in accurate gravimetric geoid computations because of its short wavelength nature. Past studies have shown that
taking into account terrain correction in accurate geoid determination is essential, especially in (high) mountainous areas (Wang and Rapp, 1990; Sideris, 1985; Forsberg, 1984; Zhang, 1990b; Bian and Zhang, 1991a). The terrain correction is a systematically positive correction term for the geoid undulation. The corresponding mean undulation correction in some cases is significant (Wang and Rapp, 1990; Zhang, 1990b; Zhang and Bian, 1993). In China, for example, the maximum difference between the geoid and the height anomaly is more than three metres (Zhang, 1990a; 1990b; Bian and Zhang, 1991a).

However, the current Australian geoid AUSGEOID93 does not include detailed terrain information in terms of terrain corrections and indirect effects (§2.6). This chapter investigates the regional gravity field feature of Australia in terms of the terrain correction, its effects on geoid and height anomalies, and validates the effect of terrain in the computation of a new gravimetric geoid (§8.2). The fast Fourier transformation method (§4) is applied to both the terrain correction and its indirect effect for computational efficiency over Australia. In addition, the theoretical accuracy of the terrain corrections is also evaluated. A series of formulae are derived for the accuracy evaluation.

7.2 Evaluation of the Total Terrain Effects

7.2.1 Relevant Formulae for Practical Computation

Following equations (2.10) and (2.14), the practical formulae for geoid computation can be expressed in the following form:

$$N = S[\Delta g - \Delta g_{zgm} + C] + \Delta N_{ggm} + \delta N_{ind},$$

(7.1)

where $C$ is the terrain correction determined by equation (2.46) in §2.1.6; $\delta N_{ind}$ is the indirect effect of the terrain corrections on the geoid and expressed by equation (2.46) in §2.1.6; The subscript 'ggm' expresses quantities derived from the global geopotential model (equations 2.33 and 2.34); and $S$ is the Stokes integral operator expressed by

$$S[\Delta g] = \frac{R}{4\pi} \int_{\sigma} S(\psi) \Delta g d\sigma.$$  

(7.2)

By neglecting the contributions of the global geopotential model and the terrestrial gravity data in equation (7.1) for convenience, the contribution of only terrain
information to conventional gravimetric geoid determination is expressed more simply. This is known as the total terrain effect on the geoid

$$N_T = S[C] + \delta N_{\text{ind}}.$$  \hspace{1cm} (7.3)

The main problem with the classical Stokes integral is that it requires the gravity data be reduced (or downward continued) to the geoid. Such a reduction requires a knowledge of the density distribution in the topography above the geoid. This problem can be circumvented in practice by assigning a reasonable constant density to the topographic masses, thus simplifying the formulae and making them more suitable for evaluation using spectral methods. Theoretically, however, the problem of unknown topographic density remains (Martinec, 1991; Martinec and Vaníček, 1994a; 1994b).

As was discussed in §2.1.4, Molodensky introduced a new surface, called the quasigeoid, in which the concept of the geoid height is replaced by the height anomaly (Figure 2.2). Subsequently, the quasigeoid can be determined more directly from surface gravity data, in contrast to geoid determination, and without knowledge of topographic density variations. There have been a number of suggested solutions to the Molodensky BVP, such as those of Brovar (1964) and Pellinen (1972). These are in a similar form to Molodensky's series solution, where the height anomaly can typically be expressed as (Moritz, 1989; Brovar, ibid; Li et al., 1995):

$$\zeta = \zeta_0 + \zeta_1 + \zeta_2 + \cdots$$  \hspace{1cm} (7.4)

where the terms $\zeta_0, \zeta_1, \zeta_2, \cdots$, can be determined using the formulae (2.14) through (2.18) in §2.1.4.

The difference between the height anomaly and the geoid ($\Delta$) can, thus, be expressed approximately as:

$$\Delta = \zeta - N$$  \hspace{1cm} (7.5)

where $\zeta$ is calculated from equations (7.4) and (2.14) through (2.18) and $N$ is calculated via equation (7.1). Equation (7.5) is an approximate formula for $N$ computed via equation (7.1) is an adjusted geoid (i.e. co-geoid). Alternatively, their separation ($\Delta$) can be estimated from the following formula (Heiskanen and Moritz, 1967; also §2.1.5).
\[
\Delta = \frac{g - \bar{\gamma}}{\bar{\gamma}} H \approx -\frac{\Delta g_B}{\bar{\gamma}} H
\]  
(7.6)

where \( g \) and \( \bar{\gamma} \) are mean actual and normal gravity along their respective normals, and in the approximation, \( \Delta g_B \) and \( H \) are the Bouguer anomaly and the height respectively.

The separation between the geoid and the quasigeoid has been investigated further by Sjöberg (1995) and expressed as a series form of terrain height. In this study, however, the approximation in equation (7.6) is used so as to give an indication of the magnitude of their separation in the Australian context.

7.2.2 Practical Computation and Statistical Analysis

The gravimetric terrain corrections \( (C) \), the contribution to the geoid from these terrain corrections \( (S[C]) \), and their indirect effect on the geoid \( (N_{ind}) \) have each been computed over the entire Australian continent using a 1'×1' digital terrain model (DTM) (§3.3). The one-dimensional fast Fourier transformation (1D-FFT) method (Haagmans et al., 1993; Sideris and She, 1995) was used to evaluate the convolution integral (7.1) without the need for any further approximations. One-hundred percent zero padding has been applied to the edges of the data areas so as to eliminate the effects of circular convolution (§4.4.2; Sideris and Li, 1993; Tziavos, 1996). The terrain correction convolution integral is evaluated efficiently by FFT (§4.6, Schwarz et al., 1990; Sideris, 1990). Figures 7.1 through 7.3 show colour images of the terrain corrections, indirect effect of the terrain corrections and the total terrain effect on gravimetric geoid determination respectively.

The statistics of each of these contributions for the whole of Australia are listed in Table 7.1, together with the terrain heights. However, since the Australian terrain is heterogeneous, with eastern Australia and Tasmania being rough and central and Western Australia being relatively smooth, the statistical features in each region are also given separately in Tables 7.2 through 7.4. The difference between geoid and height anomaly \( (\zeta-N) \) by equation (7.6) is also listed in Tables 7.1~7.4.
Table 7.1 Statistics of the terrain corrections ($C$), direct ($S[C]$), and indirect geoidal effects ($\delta N_{ind}$), total terrain effects ($N_T$) and the differences between geoid and height anomaly ($\Delta$) for the whole Australia (based on a 1′x1′ DTM)

<table>
<thead>
<tr>
<th>Items</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ (mgal)</td>
<td>25.478</td>
<td>0.000</td>
<td>0.100</td>
<td>0.462</td>
<td>0.451</td>
</tr>
<tr>
<td>$S[C]$ (m)</td>
<td>0.793</td>
<td>0.013</td>
<td>0.078</td>
<td>0.125</td>
<td>0.097</td>
</tr>
<tr>
<td>$\delta N_{ind}$ (m)</td>
<td>0.000</td>
<td>-0.215</td>
<td>-0.004</td>
<td>0.009</td>
<td>0.008</td>
</tr>
<tr>
<td>$N_T$ (m)</td>
<td>0.688</td>
<td>0.000</td>
<td>0.140</td>
<td>0.177</td>
<td>0.108</td>
</tr>
<tr>
<td>$\Delta = \zeta - N$ (m)</td>
<td>0.151</td>
<td>-0.048</td>
<td>0.010</td>
<td>0.020</td>
<td>0.018</td>
</tr>
<tr>
<td>$H$ (m)</td>
<td>2126.2</td>
<td>0.000</td>
<td>175.3</td>
<td>266.5</td>
<td>202.7</td>
</tr>
</tbody>
</table>

Table 7.2 Statistics of $C$, $S[C]$, $\delta N_{ind}$, $N_T$ and $\Delta$ for the Western part of Australia ($\lambda < 145^\circ$)

<table>
<thead>
<tr>
<th>Items</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ (mgal)</td>
<td>21.385</td>
<td>0.000</td>
<td>0.044</td>
<td>0.153</td>
<td>0.146</td>
</tr>
<tr>
<td>$S[C]$ (m)</td>
<td>0.341</td>
<td>0.013</td>
<td>0.052</td>
<td>0.066</td>
<td>0.042</td>
</tr>
<tr>
<td>$\delta N_{ind}$ (m)</td>
<td>0.000</td>
<td>-0.082</td>
<td>-0.004</td>
<td>0.007</td>
<td>0.006</td>
</tr>
<tr>
<td>$N_T$ (m)</td>
<td>0.477</td>
<td>0.000</td>
<td>0.122</td>
<td>0.145</td>
<td>0.079</td>
</tr>
<tr>
<td>$\Delta = \zeta - N$ (m)</td>
<td>0.151</td>
<td>-0.048</td>
<td>0.011</td>
<td>0.022</td>
<td>0.019</td>
</tr>
<tr>
<td>$H$ (m)</td>
<td>1259.2</td>
<td>0.000</td>
<td>172.2</td>
<td>254.4</td>
<td>187.5</td>
</tr>
</tbody>
</table>

Table 7.3 Statistics of $C$, $S[C]$, $\delta N_{ind}$, $N_T$ and $\Delta$ for the Eastern part of Australia ($\lambda \geq 145^\circ$)

<table>
<thead>
<tr>
<th>Items</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ (mgal)</td>
<td>25.478</td>
<td>0.000</td>
<td>0.311</td>
<td>0.950</td>
<td>0.898</td>
</tr>
<tr>
<td>$S[C]$ (m)</td>
<td>0.793</td>
<td>0.037</td>
<td>0.205</td>
<td>0.248</td>
<td>0.139</td>
</tr>
<tr>
<td>$\delta N_{ind}$ (m)</td>
<td>0.000</td>
<td>-0.215</td>
<td>-0.006</td>
<td>0.014</td>
<td>0.013</td>
</tr>
<tr>
<td>$N_T$ (m)</td>
<td>0.688</td>
<td>0.003</td>
<td>0.253</td>
<td>0.283</td>
<td>0.129</td>
</tr>
<tr>
<td>$\Delta = \zeta - N$ (m)</td>
<td>0.147</td>
<td>-0.015</td>
<td>0.006</td>
<td>0.011</td>
<td>0.002</td>
</tr>
<tr>
<td>$H$ (m)</td>
<td>2126.2</td>
<td>0.000</td>
<td>192.4</td>
<td>317.4</td>
<td>253.9</td>
</tr>
</tbody>
</table>

Table 7.4 Statistics of $C$, $S[C]$, $\delta N_{ind}$, $N_T$ and $\Delta$ for Tasmania ($144.6^\circ < \lambda < 148.4^\circ$, $-43.9^\circ < \phi < -40.6^\circ$)

<table>
<thead>
<tr>
<th>Items</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$ (mgal)</td>
<td>19.233</td>
<td>0.000</td>
<td>1.170</td>
<td>2.047</td>
<td>1.678</td>
</tr>
<tr>
<td>$S[C]$ (m)</td>
<td>0.365</td>
<td>0.105</td>
<td>0.220</td>
<td>0.229</td>
<td>0.065</td>
</tr>
<tr>
<td>$\delta N_{ind}$ (m)</td>
<td>0.000</td>
<td>-0.108</td>
<td>-0.005</td>
<td>0.013</td>
<td>0.012</td>
</tr>
<tr>
<td>$N_T$ (m)</td>
<td>0.293</td>
<td>0.110</td>
<td>0.206</td>
<td>0.208</td>
<td>0.039</td>
</tr>
<tr>
<td>$\Delta = \zeta - N$ (m)</td>
<td>0.070</td>
<td>-0.012</td>
<td>0.003</td>
<td>0.007</td>
<td>0.00</td>
</tr>
<tr>
<td>$H$ (m)</td>
<td>1522.0</td>
<td>0.000</td>
<td>229.8</td>
<td>372.9</td>
<td>294.5</td>
</tr>
</tbody>
</table>

The statistics summarised in Tables 7.1 through 7.4 indicate that the total terrain effect on the Australian geoid (equation 7.3) reaches its maximum value of 0.688m in eastern Australia. This is to be expected because the topographic heights are both greater and more variable in this region, which is dominated by the Great Dividing
Range (Figure 7.1). These statistics indicate that the current Australian geoid-AUSGEOID93 (Steed and Holtznagel, 1994) could have an error of up to 0.688 m (0.14m RMS) due to the terrain correction effect.

For most parts of Western Australia, the total terrain effect is relatively small (Table 7.2). The mean terrain corrections and indirect effects in the west are typically one-third to one-fourth of those in the east respectively. This is because the terrain in the western part of Australia is relatively lower and smoother, consisting mainly of plateau landforms and gently undulating hills.

Another observation is that the total terrain effect in Tasmania is significant. The mean terrain correction to the gravity anomaly in Tasmania is three times larger than that of eastern Australia, which is indicative of the more rugged terrain in Tasmania. However, the related effects (e.g. $S[C]$, $\delta N_{\text{ind}}$, $N_{\text{r}}$ and $\Lambda$) in Tasmania and eastern Australia have a similar magnitude.

Colour images of the terrain corrections, its direct effect on the geoid and its indirect effect are shown on Figures 7.1 to 7.3 to demonstrate their distribution. As can be seen from Figure 7.1, the terrain corrections are highly correlated with topography and some coast areas where a large terrain gradient exists. The direct and indirect effects have similar features (Figures 7.2 and 7.3). To further explain these observations, four profiles ($\varphi=24.5^\circ$S, $\varphi=34.0^\circ$S, $\lambda=126^\circ$ and $\lambda=150^\circ$) have been randomly selected to show the respective features of the computed quantities. These are depicted in Figures 7.4 and 7.5. From these figures, it is seen that the indirect effect has a strong negative correlation with terrain where it mirrors the terrain height along the profile very well. This manifests because the first term ($\pi G\rho H_2 y / \gamma$) in equation (2.44), which is a quadratic function of $H$, is dominant in most of Australia.

The gravimetric terrain correction profile is also correlated with the terrain height, albeit to a lesser extent. However, there do exist some peaks, which reflect very short-wavelength topographic features. There are two possible factors to contribute to this phenomenon. One results from possible DTM errors where peaks and troughs may exist. The other results from the form of equation (2.45) where the numerator can become considerably larger than the denominator in the presence of steep
topographic gradients (cf. the results in Tasmania) in the vicinity of the point of interest. This has implications for the grid size of the DTM used to compute the terrain corrections, where it becomes unstable (Martinec and Vaníček, 1994b; Martinec, 1991).

7.2.3 Conclusions and Recommendations
The distinctive Australian landscape is the major factor contributing to the behaviour of the gravity field and hence the geoid in this part of the world. From this study and its analyses, it can be concluded that:

1. The direct effect of the terrain on the geoid must be taken into account during Australian geoid computations, especially in eastern Australia and Tasmania, where its effect can exceed several tens of centimetres (maximum of 0.793m).
2. The indirect effect of the terrain reduction on the geoid should be also taken into account, although it is generally small. The shape of the indirect effect mirrors the terrain due to the dominance of the first term in equation (2.47).
3. Conceptually the differences between geoid and quasigeoid should be strictly distinguished. However, their maximum difference in Australia is relatively small (<0.151m), but still significant if Australian Height Datum elevations are to be determined from GPS to a high level of accuracy.
4. The FFT has proven to be an effective method for large area computations of the geoid, terrain correction and its primary and indirect effects on the geoid.

In addition, a detailed 9"×9" DTM is released by Australian Surveying and Land Information Group (AUSLIG) at the time of writing this thesis. For further refined computation of the terrain effects, this newly released detailed DTM should be used in the future.
Figure 7.1 Colour image of the terrain corrections ($C$) in Australia (Mercator projection, colour scale in mgal)
Figure 7.2 The indirect effect ($\delta N_{ind}$) of the terrain corrections over continental Australia (Mercator projection, colour scale in metres)
Figure 7.4 Profiles of terrain height, indirect effect, terrain correction and geoidal correction along latitude profiles (1) $\varphi=-24.5^\circ$, (2) $\varphi=-34^\circ$.
Figure 7.5 Profiles of terrain height, indirect effect, terrain correction and geoidal correction along longitude profiles (1) $\lambda=126^\circ$, (2) $\lambda=150^\circ$
7.3 Evaluation of the Accuracy of the Terrain Correction

As stated in §7.1, the terrain correction plays an important role in accurate gravimetric geoid computation because of its short wavelength features. In addition, the terrain correction plays an important role in the refinement, smoothing and prediction of the Earth's gravity field. However, the accuracy of the terrain correction is affected by errors in a digital terrain model (DTM) and the three-dimensional position of the point of interest, as well as in the uniform density assumption. The question is: how critical are these errors to the computation of the terrain correction?

The computation of the terrain correction is performed through a complicated integration in which several variables are used. These are the 3-D position of both the point of interest and the running point, and the topographic density (see equation 7.7). The DTM heights, generally predicted by applying gridding methods, are indirect measurements, which can be erroneous. Also, the uniform density of the topography (\(\rho = 2670\text{kgm}^{-3}\)) is assumed in almost all geodetic computations. Another consideration is that the position of the gravity observation may be in error as well. Therefore, these erroneous data and assumptions will obviously contaminate the accuracy of the terrain correction.

This section derives a series of formulae to evaluate the accuracy of the terrain correction by considering the errors from these different data sources. This is useful for the error estimation of the terrain correction, if the accuracy of the input data is known. The results can be also applied to the determination of the sampling interval of heights in order to effectively recover terrain information for subsequent geoid computation.

7.3.1 Derivation of the Formulae

Given an arbitrary point \(P\) on the terrain surface (Figure 7.6), the terrain correction is given by (Schwarz et al., 1990; Moritz, 1968; and equation 2.40):

\[
C(P) = G \iint_E \left[ \frac{y - H}{r^3} \rho \, dz \right] dE,
\]

with

\[
r^2 = (x - x_p)^2 + (y - y_p)^2 + (H - z_p)^2,
\]
where \( C(P) \) is terrain correction at point \( P \);

\[(x_P, y_P, H_P) \text{ and } (x, y, z) \text{ are three-dimensional co-ordinates of the point of interest } P \text{ and the roving point } Q \text{ in a local Cartesian system respectively;}
\]
\[\rho = \rho(x, y, z) \text{ is density of topographic masses; and}
\]
\( E \) is Earth's surface.

![Diagram](image)

Figure 7.6 Topography (solid line) and the terrain corrections (dashed line) at an arbitrary point \( P \). A height error \( \Delta H_P \) at point \( P \) causes an error in the terrain correction equal to a thin Bouguer plate \( 2\pi G \rho \Delta H_P \) (solid thin line).

In a linear, planar approximation, \( C(P) \) becomes (Moritz, 1968; Schwarz et al., 1990):

\[
C(P) = \frac{1}{2} G \oint \iint \left( \frac{(H_Q - H_P)'}{r_0^2} \right) dx dy,
\]
where

\[
r_0^2 = (x_P - x)^2 + (y_P - y)^2,
\]

where \( r_0 \) is planar distance between roving point \( Q \) and the point of interest \( P \). This represents the so-called 'linear and planar approximation' to the terrain correction (Moritz, 1968). As pointed out by Schwarz et al. (1990), the accuracy of the linear approximation is usually sufficient (sub-mgal) in a root mean square (RMS) sense although large errors are possible in the vicinity of the point of interest (Forsberg, 1984; Sideris, 1985). However, the approximation error will be considered as an 'algorithm error', instead of an observational error in the measurements.

Reviewing equations (7.7) and (7.9), equation (7.9) can be expressed as a function model of the following variables:

\[
C(P) = f(X_P, H_P, H_Q, \rho).
\]

where
\[
\begin{bmatrix}
H_Q = H_Q(x, y) \\
X_P = [x_p, y_p]
\end{bmatrix}
\]  
(7.12)

This implies that the value of \( C(P) \) is contaminated by errors in quantities \( x_p, y_p, H_P, H_Q \) and \( \rho \). Taking the total differential of equation (7.11) over \( x_p, y_p, H_P, H_Q \) and \( \rho \) gives:

\[
\Delta C(P) = \frac{\partial C(P)}{\partial H_Q} \Delta H_Q + \frac{\partial C(P)}{\partial H_P} \Delta H_P + \frac{\partial C(P)}{\partial X_P} \Delta X_P + \frac{\partial C(P)}{\partial \rho} \Delta \rho = \mathbf{F} \overline{\Delta}
\]  
(7.13)

\[
\Delta X_P = [\Delta x_p, \Delta y_p]^T
\]  
(7.14)

\[
\mathbf{F} = [F_i] = \begin{bmatrix}
\frac{\partial C(P)}{\partial H_Q} & \frac{\partial C(P)}{\partial H_P} & \frac{\partial C(P)}{\partial X_P} & \frac{\partial C(P)}{\partial \rho}
\end{bmatrix}
\]  
(7.15)

\[
\overline{\Delta} = [\Delta H_Q, \Delta H_P, \Delta X_P, \Delta \rho]^T
\]

where \( \Delta H_Q, \Delta H_P, \Delta X_P \) and \( \Delta \rho \) are the errors in \( H_Q, H_P, X_P \) and \( \rho \) respectively.

In this study, the errors of \( x_p, y_p, H_P, H_Q \) and \( \rho \) are assumed to be uncorrelated. Applying the law of error propagation to equation (7.13), the root mean square error of the terrain corrections becomes:

\[
\sigma = \sqrt{\sum_i \sigma_i^2} = \sqrt{F_{H_Q}^2 M_{H_Q}^2 + F_{H_P}^2 M_{H_P}^2 + F_{X_P}^2 M_{X_P}^2 + F_{\rho}^2 M_{\rho}^2} = \sqrt{\sum_i F_i^2 M_i^2}
\]  
(7.16)

with

\[
\overline{M} = [M_i] = [M_{H_Q}, M_{H_P}, M_{X_P}, M_{\rho}],
\]  
(7.17)

where \( F_i \) and \( M_i \) (\( i = H_Q, H_P, X_P, \rho \)) are the corresponding partial derivatives and RMS errors respectively;

\( \sigma \) is the RMS error of the terrain correction; and

\( \sigma_i \) are the RMS effects corresponding to the RMS error contributions \( M_i \).

Since equations (7.7) and (7.9) are singular when \( r \to 0 \) or \( r_o \to 0 \), it is assumed that the computation is performed outside a very small cap \( E_0 \) (the innermost zone), whose radius is \( R_0 \) (see Figure 7.7). A number of authors have contributed to the treatment of the singularity problem in the innermost zone of the integration, such as Heiskanen and Moritz (1967), Sideris (1990), Bian and Dong (1991), Schwarz et al (1990), Klose and Ilk (1993) and Zhang et al. (1995). Furthermore, since the kernel function of the terrain correction decreases rapidly with the increase of integral radius \( \propto l/r^3 \), to apply the terrain correction integration over a spherical cap with radius 80–100km is, generally, sufficient (Zhang, 1990b). Therefore, the domain of the integration \( E \) is
limited to $E_t (r > R_0)$. In practical computations, the error of this approximation is expected to be negligible even in a very rough terrain area (Zhang, 1990b). The area $E_t$ will be called the effective domain of the terrain correction integral (see Figure 7.7).

![Figure 7.7](image)

Figure 7.7 The domain of the terrain correction integral is divided into three parts $E_0 (r < R_0)$, $E_t (R_t > r > R_0)$ and $E_2 (r > R_t)$, where $E_t$ is the effective domain of integration.

**Remark 7.1** The resolution of most currently available DTMs is, generally, less than 500m (or 20°). Therefore, if the computation is performed outside $R_0 = 500m$, this value is directly applicable. However, the assumption of a large $R_0$ is equivalent to assuming the DTM height in area $E_0$ is error free.

**Remark 7.2** For the terrain correction in areas of very rough topography, this approximation can cause some errors in the computation. However, since these terrain corrections are small quantity themselves, this approximation is presumed to be negligible.

**Remark 7.3** As can be seen in §7.3.2a, this method has some theoretical limitations on the selection of the radius $R_0$. However, in practice it can be conceived that the DTM is error free both in areas $E_0$ and $E_2$, or their effects are negligible to the total terrain correction computation. If a large $R_0$ is used, the terrain effects in the innermost area should be taken into account separately.

Hereafter, the error contribution from the errors in the DTM heights ($\Delta H_0$), height of the reference point ($\Delta H_r$), planar co-ordinates ($\Delta X_r$) and the uniform topographic density assumption ($\Delta \rho$) will be studied.
(a) Error contribution of erroneous DTM height $\Delta H_Q$

Terrain reductions are usually computed from DTM heights, which themselves are usually observed photogrammetrically or digitised from a topographic map and thus are subject to errors. Also, the computation is done by means of approximated convolution integral formulas (Forsberg, 1984; Heiskanen and Moritz, 1967). The question now is how to estimate the accuracy of the derived terrain correction from the given accuracy of the DTM heights.

Suppose that $\Delta H_Q$ is the error of the DTM height at the roving point $Q$. According to equation (7.9), the effect of this error on the terrain correction is:

$$\Delta C = K \int_{E_1}^{E_2} \left( \frac{2(H - H_P)\Delta H_Q}{r_0^3} \right) dx dy ,$$

(7.18)

where, $E_1 < r_0 < r_0 < E_1$, and $K = G \rho / 2$ is a constant. Therefore,

$$\Delta C^2 = \left( \frac{K \int_{E_1}^{E_2} \left( \frac{2(H_Q - H_P)\Delta H_Q}{r_0^3} \right) dx dy}{\int_{E_1}^{E_2} \left( \frac{2(H_Q - H_P)\Delta H_Q}{r_0^3} \right) dx dy} \right)^2 \int_{E_1}^{E_2} \left( \frac{2(H_Q - H_P)\Delta H_Q}{r_0^3} \right) dx dy \int_{E_1}^{E_2} \left( \frac{2(H_Q - H_P)\Delta H_Q}{r_0^3} \right) dx dy' .$$

(7.19)

The average of equation (7.19) gives the standard variance of the errors. Equation (7.19) can be simplified by applying the following two basic well-known theorems of integral calculus (Heiskanen and Moritz, 1967, p.272):

1. The symbols that denote variables of integration in a definite integral are irrelevant.
2. Products of definite integrals may be written as one multiple integral.

Given these theorems, the error variance $E(\Delta C^2)$ caused by the errors $\Delta H_Q$ can be expressed as:

$$E(\Delta C^2) = 4 K^2 \int_{E_1}^{E_2} \int_{E_1}^{E_2} E(\Delta H_Q, \Delta H_Q') \frac{(H_Q - H_P)(H_Q' - H_P)}{r_0^3} dx dy dx' dy'$$

(7.20)

where $E(\Delta H_Q, \Delta H_Q') = f(x, y, x', y')$ is the error covariance function of the DTM heights in the sense of collocation; and
\( E \) is the mathematical expectation operator.

If it is assumed that the error of the DTM heights is stationary (Davis, 1986), the error covariance function \( E(\Delta H_Q, \Delta H'_Q) \) is a function of the distance \( r \) between points \( P \) and \( Q \) and thus can be expressed as:

\[
E(\Delta H_Q, \Delta H'_Q) = f(r).
\]  
(7.21)

This theoretical error-covariance function \( f(r) \) can be approximated by a typical model, such as linear, spherical, exponential or Guassian models, which depends on several parameters (Giltrap, 1983). For simplicity, it is assumed that:

\[
\begin{cases}
  f(r) = M_{\Delta H}^2 \left(1 - \frac{r}{R'_1}\right) & r \leq R'_1 \\
  f(r) = 0 & r > R'_1
\end{cases}
\]  
(7.22)

where \( M_{\Delta H}^2 = E(\Delta H_Q, \Delta H_Q) \) is the RMS error square of the local DTM heights; and \( R'_1 \) is the correlation length.

Thus,

\[
\iint_{E_1} E(\Delta H_Q, \Delta H'_Q) dx'dy' = M_{\Delta H}^2 S_{E_1},
\]  
(7.23)

where

\[
S_{E_1} = 2\pi \left(\frac{R'_1^2}{6} - \frac{R'_0^2}{2} + \frac{R^2}{3R'_1}\right).
\]  
(7.24)

Inserting equations (7.23) and (7.24) into (7.20), a simple result is obtained, provided that the error function is subject to the following two implicit assumptions:

1. Only errors of DTM heights at neighbouring points are noticeably correlated; beyond a certain distance \( (R'_1) \) there is no correlation;

2. The accuracy of DTM height is the same for every point in the effective domain of the integration (i.e. area \( E_1 \)).

Equation (7.20) then yields:

\[
E(\Delta C^2) \leq 4K^2 M_{\Delta H}^2 S_{E_1} \int_{E_1} \frac{(H_Q - H_F)^2}{r_0^6} dx dy \leq
\]
\[
4 K^2 M_{\text{H}_0}^2 S_{E_i} \int_{E_i} \frac{(H_0 - H_F)^2}{R_0^3} \, dx \, dy \leq \frac{4 K M_{\text{H}_0}^3 S_{E_i} \overline{C}}{R_0^3}, \tag{7.25}
\]

and

\[
\sigma_{\text{H}_Q} = \sqrt{\frac{4 K S_{E_i} \overline{C}}{R_0^3}} M_{\text{H}_Q}. \tag{7.26}
\]

From equation (7.26), it is evident that the effect of erroneous DTM heights on the terrain correction is a function of the value of the terrain correction itself and the accuracy of the DTM heights.

\textbf{Remark 7.4} Since equation (7.25) is an average process and the average behaviour of error contributions from various error sources are of most interest, \( \overline{C} \) in equation (7.26) should be explained as an average value of the terrain corrections over the area of interest.

(b) Error contribution of erroneous reference point height (\( \Delta H_F \))

Consider a point \( P \) at the Earth's surface (Figure 7.6), the total gravitational effect of the topography (\( g_e \)) can be expressed as:

\[
\begin{align*}
g_e &= 2 \pi G \rho H_F - C(P) \\
g_e &= 2 \pi G \rho (H_F + \Delta H_F) - C(P').
\end{align*} \tag{7.27}
\]

where \( \Delta H_F \) is an individual error in the height of the reference point \( P \). Hence, the contribution of \( \Delta H_F \) is:

\[
C(P') - C(P) = 2 \pi G \rho \Delta H_F. \tag{7.28}
\]

It is obvious from equation (7.28) that the contribution of \( \Delta H_F \) is systematic for point \( P \).

For all the points of interest in an area, their RMS effect is:

\[
\sigma_{H_F} = 2 \pi G \rho M_\rho = 0.1119 M_\rho \tag{7.29}
\]

where \( M_\rho \) is the RMS error of the assumed constant topographic density.
(c) Error contribution of the horizontal position \( X_P \) of the gravity observation

Suppose that the horizontal position of the computation point \( P \) has an error \( \Delta x_P = [\Delta x_P, \Delta y_P] \). From equations (7.9), (7.11) and (7.12)

\[
\Delta C_{x_P} = C(X_P + \Delta X_P, *) - C(X_P, *),
\]

\[X_P = [x_P, y_P]^T, \quad X = [x, y]^T,\]

where "*" expresses other variables used in equation (7.13) for convenience. Applying Taylor's series expansion to equation (7.30) gives:

\[
\Delta C_{x_P} = \left( G \rho \int_{E_1} \left( \frac{(H_0 - H_P)^2}{r_0^3} \frac{3(X - X_P)}{r_0^2} \right) r_0^2 \right) \Delta X_P,
\]

with

\[
\left| \frac{3(X - X_P)}{r_0^2} \right| \leq \frac{3}{R_0}.
\]

Supposing that \( M_x = M_y \) gives

\[
\sigma_{x_P} = \frac{3C}{R_0} M_{x_P} = \frac{3C}{R_0} M_{y_P}.
\]

For the terrain correction at the computation point, the error contribution is systematic. As stated in Remark 7.4, \( C \) in equation (7.34) should also be taken as an average value over the area of interest.

(d) Error contribution of the uniform density assumption (\( \Delta \rho \))

From equation (7.7) or (7.9),

\[
\Delta C_{\rho} = C(*, \rho + \Delta \rho) - C(*, \rho).
\]

Again, "*" expresses other variables used in the equation (7.13). By applying Taylor's series expansion to the above equation gives

\[
\Delta C_{\rho} = K \int_{E_1} \left( \frac{(H_0 - H_P)^2}{r_0^3} \Delta \rho \right) \frac{\Delta \rho}{\rho} dx dy,
\]

Applying the similar procedure to equations (7.19) and (7.20) gives:

\[
\sigma_{\Delta \rho} = \frac{C(P)}{\rho} M_{\Delta \rho}.
\]

Here, the error \( \Delta \rho \) should be taken as the RMS deviation of average crustal density from its adopted uniform value (usually \( \rho = 2670 \text{kgm}^{-3} \)) over the integration area. Otherwise, the term \( \Delta \rho/\rho \) in equation (7.36) can not be moved out of the integration.
For the same reason as in Remark 7.4, $\overline{C}$ is again an average value of the terrain correction.

### 7.3.2 Numerical Estimations of the Terrain Correction Error

Following the above derivations and inserting the constants $\rho = 2670 \text{kgm}^{-3}$ and $G = 6.67259 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$ (Cohen and Taylor, 1996) into the formulae (7.26), (7.29), (7.34) and (7.37), the error contribution coefficients $F_i$ can be summarised as follows:

$$F_{H_0} = \frac{4KSE_1C}{R_0^3},$$  \hspace{1cm} (7.38)

$$F_{H_p} = 2\pi G\rho,$$  \hspace{1cm} (7.39)

$$F_{x_p} = \frac{3C}{R_0},$$  \hspace{1cm} (7.40)

$$F_{\rho} = \frac{C}{\rho}.$$  \hspace{1cm} (7.41)

Recall that formulae (7.38) through (7.41) are based on a linear, planar approximation and can be used to estimate the maximum error contamination of the terrain correction (7.7). The error sources include the 3-D position of the point of interest, the DTM heights and uniform topographical density. It should be reiterated that the terrain correction value $\overline{C}$ in equations (7.38) to (7.41) should be regarded as a local mean value since we are interested in an average, not an individual, effect. It should be also noted that the formulae (7.38) and (7.40) are their maximum possible RMS error estimates, following the adoption of inequality signs.

The different effective domains ($E_l$) and the error correlation length of the DTM heights ($R_i$) are used to investigate their effect on the total terrain correction. Numerical estimates of the error contribution coefficients for a selection of parameters $R_0$ and $R_i$ are listed in Table 7.5. The individual error contributions of the DTM heights, topographical density and 3-D position at the point of interest for typical terrain correction values are plotted in Figure 7.8. Table 7.6 lists the RMS errors corresponding to these error sources.
Table 7.5 Examples of the maximum error coefficients based on different effective domains of integration (for correlation lengths 20 km and 40 km)

<table>
<thead>
<tr>
<th></th>
<th>( R_i = 20) km</th>
<th>( R_i = 40) km</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_0 = 2000) m</td>
<td>( R_0 = 1000) m</td>
</tr>
<tr>
<td>( F_{HG} )</td>
<td>0.0042 ( \sqrt{C} )</td>
<td>0.1217 ( \sqrt{C} )</td>
</tr>
<tr>
<td>( F_{HP} )</td>
<td>0.1119</td>
<td>0.1119</td>
</tr>
<tr>
<td>( F_{XP} )</td>
<td>0.0015 ( \sqrt{C} )</td>
<td>0.0030 ( \sqrt{C} )</td>
</tr>
<tr>
<td>( F_o )</td>
<td>3.7 \times 10^{-4} ( \sqrt{C} )</td>
<td>3.7 \times 10^{-4} ( \sqrt{C} )</td>
</tr>
</tbody>
</table>

Table 7.6 RMS error estimation of the terrain correction caused by various errors \((R_i = 20\) km, \( R_0 = 1000\) m, \( M_{sp} = 200\) kg m\(^{-3}\), unit=mgal)

<table>
<thead>
<tr>
<th>( \sqrt{E(\Delta C^2)} )</th>
<th>( M_{HG} = 0.5) m, ( M_{HP} = 0.1) m</th>
<th>( M_{HG} = 1) m, ( M_{HP} = 0.5) m</th>
<th>( M_{HG} = 2) m, ( M_{HP} = 1) m</th>
<th>( M_{HG} = 5) m, ( M_{HP} = 2) m</th>
<th>( M_{HG} = 10) m, ( M_{HP} = 5) m</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta C ) (mgal)</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>%</td>
</tr>
<tr>
<td>0.1</td>
<td>0.03</td>
<td>0.07</td>
<td>0.14</td>
<td>0.29</td>
<td>0.67</td>
</tr>
<tr>
<td>0.3</td>
<td>0.04</td>
<td>0.09</td>
<td>0.17</td>
<td>0.40</td>
<td>0.87</td>
</tr>
<tr>
<td>0.5</td>
<td>0.06</td>
<td>0.11</td>
<td>0.22</td>
<td>0.49</td>
<td>0.98</td>
</tr>
<tr>
<td>1.0</td>
<td>0.06</td>
<td>0.15</td>
<td>0.28</td>
<td>0.65</td>
<td>1.34</td>
</tr>
<tr>
<td>2.0</td>
<td>0.17</td>
<td>0.23</td>
<td>0.39</td>
<td>0.90</td>
<td>1.8</td>
</tr>
<tr>
<td>5.0</td>
<td>0.39</td>
<td>0.47</td>
<td>0.67</td>
<td>1.43</td>
<td>2.8</td>
</tr>
</tbody>
</table>

From Figure 7.8 and Table 7.6, the error in the DTM heights and the station height are critical, while the effects of the planar positional error of the station and constant density assumption are not. The slopes of the error (e.g. error contribution coefficients \( F_o \)) of the DTM heights and the station height are much steeper than the planar position errors of the reference point and topographical density error. Generally, the error contribution coefficient of the planar position of the reference point is less than one twentieth of the DTM heights and thus negligible. Since the variation in the crustal density is limited (e.g. from \(-1500\) kg m\(^{-3}\) to minerals \(-3500\) kg m\(^{-3}\), Kearey and Brooks, 1984), the effect of uniform density assumption is not critical (e.g. Martinec et al., 1995).

As shown in Table 7.6, the flatter the topography, the higher the accuracy requirement of the DTM heights and station height. For instance, in Western Australia, the mean value of the terrain corrections is 0.044 mgal (Table 7.2), the accuracy requirement for the DTM heights and station height is less than 2 metres. However, the
accuracy of the spot heights is -7.5m (Carrol and Morse, 1996). Otherwise, one has to face the risk of useless terrain correction computation results (even worse, it can mislead the role of terrain effects in geoid computation!). Therefore, it is suggested that for a very flat area, such as Western Australia where the average terrain correction is less than 0.05 mgal (Table 7.2), it is better to simply set terrain correction as zero instead of to compute it using the less accurate DTM heights. This is an important consideration when using the new 9''x9'' DTM in a new Australian geoid.

Figure 7.8 Various error contribution coefficients for $C = 0.5$ mgal, 1mgal and 3mgal respectively (Figures a, b and c), $\sigma_{\text{hp}}$ (solid line), $\sigma_{\text{hQ}}$ (dotted line), $\sigma_{\text{sp}}$ (dashed line) and $\sigma_{\text{dp}}$ ($K_1 = 20$km, $R_0 = 1000$m)
Martinec *et al.* (1995) recently investigated the effect of lateral density variations of the
topographical masses on geoid determination using the Stokes-Helmert scheme. As an
extreme case in which large lateral density contrast (the boundary of rock and water
with density difference of 1670 kg/m$^3$) was introduced. The computation results for the
Lake Superior area of Canada, suggested that the corrections to geoid are fairly small.
This agrees with the findings here. In fact, this numerical conclusion can be implied
from equation (7.41).

7.3.3 Effective Grid Size and Accuracy Evaluation of the Terrain Corrections
As stated earlier, the accuracy of the terrain correction depends on the accuracy of the
DTM heights, three-dimensional position of the point of interest and the variation of the
topographical density due to geological structure. Generally, the accuracy of the DTM
depends on the grid size required and spot heights. If a coarse grid is computed from a
denser grid, the accuracy of the coarse grid will be higher than the original denser grid
from the law of error propagation. For instance, the newly released 9"×9" DTM in
Australia has an RMS accuracy of approximately ±7.5m (Carrol and Morse, 1996). If
the denser 9"×9" DTM is averaged to a coarser 1′×1′ DTM, the RMS accuracy of the
1′×1′ DTM is approximately ±1.2m. Therefore, the study results here can act as a
reference on the determination of an effective grid size for the computation of terrain
correction in a new Australian geoid and a guidance for decision making before
acquiring the DTM.

To study the maximum RMS error in the computation of the terrain correction, the
1′×1′ DTM produced in §3.3 is used. Table 7.7 lists a combined RMS estimation of the
terrain correction using a selection of accuracy estimation of the DTM height and
station height. Since the terrain corrections in the western part of Australia are
essentially zero (mean=0.044mgal), the accuracy is not discussed for Western Australia.
Considering the mean terrain corrections in eastern part of Australia and Tasmania are
0.34 mgal and 1.1 mgal respectively, the mean terrain correction 0.5 mgal will be used
in this discussion.
Table 7.7  Accuracy estimation of the terrain correction with varying accuracy of computation point ($M_{hp}$) and accuracy of DTM ($M_{HQ}$) for $\overline{C} = 0.5$ mgal, $R = 20$ km and $R_0 = 1000$ m (unit in milligals)

<table>
<thead>
<tr>
<th>$M_{hp}$</th>
<th>0.1m</th>
<th>1.0m</th>
<th>3.0m</th>
<th>4.0m</th>
<th>5.0m</th>
<th>7.0m</th>
<th>10.0m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 m</td>
<td>0.01</td>
<td>0.09</td>
<td>0.26</td>
<td>0.34</td>
<td>0.43</td>
<td>0.60</td>
<td>0.86</td>
</tr>
<tr>
<td>1.0 m</td>
<td>0.11</td>
<td>0.14</td>
<td>0.28</td>
<td>0.36</td>
<td>0.44</td>
<td>0.61</td>
<td>0.87</td>
</tr>
<tr>
<td>3.0 m</td>
<td>0.34</td>
<td>0.35</td>
<td>0.42</td>
<td>0.48</td>
<td>0.54</td>
<td>0.69</td>
<td>0.92</td>
</tr>
<tr>
<td>4.0m</td>
<td>0.45</td>
<td>0.46</td>
<td>0.51</td>
<td>0.56</td>
<td>0.62</td>
<td>0.75</td>
<td>0.97</td>
</tr>
<tr>
<td>5.0 m</td>
<td>0.56</td>
<td>0.57</td>
<td>0.62</td>
<td>0.66</td>
<td>0.70</td>
<td>0.82</td>
<td>1.02</td>
</tr>
<tr>
<td>7.0 m</td>
<td>0.78</td>
<td>0.78</td>
<td>0.82</td>
<td>0.86</td>
<td>0.89</td>
<td>0.99</td>
<td>1.16</td>
</tr>
<tr>
<td>10.0 m</td>
<td>1.12</td>
<td>1.12</td>
<td>1.15</td>
<td>1.17</td>
<td>1.20</td>
<td>1.27</td>
<td>1.41</td>
</tr>
</tbody>
</table>

From Table 7.7, in order to achieve a precision of the terrain correction higher than the mean terrain correction over the area (e.g. 0.5 mgal), the precision requirement for DTM and station heights should be greater than 3.0m. For the terrain correction computation on a regular grid through FFT method, the precision of DTM heights is equal to that of station heights. Therefore, for the terrain correction computation of Australia (mainland), the grid size of $9''\times9''$ DTM heights should be not directly used. Instead, a grid size of $15''\times15''$ is recommended. If the precision of the spot heights is assumed to be $\pm 6$m (the precision of the gravity station height is $\pm 4$–6m from Barlow, 1977), the precision of the $1'\times1'$ DTM is approximately 3.0m. Therefore, maximum terrain correction RMS error is less than 0.42 mgal. This justifies the use of the $1'\times1'$ grid size selected.

7.3.4 Discussion

A series of formula have been derived for the accuracy evaluation of the terrain correction, based on a planar, linear approximation formula. This study shows that for the terrain correction computation in a mountainous area ($\overline{C} > 1$ mgal), the DTM errors ($\Delta H_Q$) are more critical to the terrain correction computation than the height accuracy of the point of interest ($\Delta H_p$). In a topographically flat area ($\overline{C} < 0.5$ mgal), the height accuracy of the point of interest ($\Delta H_p$), becomes more critical to the terrain correction computation than the DTM errors ($\Delta H_Q$). However, the errors in the planar position of the point of interest and uniform topographic density assumption...
(Δρ), compared to other sources of errors, are not so critical to the computation of terrain corrections.

The effect of the planar position errors (ΔXp) is less critical than that of the station height and DTM. For the terrain correction computation in a flat terrain areas, the effects from DTM and station height of interest are similar. The effect of assuming a uniform topographical density, although the slope of its error coefficient is large, is not significant even in special cases as shown in Martinec et al. (1995). Therefore, the effects of the average density assumption are, generally, negligible. Moreover, the effect of the errors in the height at the point of interest is systematic in statistical sense. Therefore, the removal of the offset between DTM terrain model and reference point is also important.

The studies suggest that for the terrain correction computation of Australia, the most important consideration is the precision of the DTM heights and the elevation of gravity stations. However, the elevation precision of the gravity observation stations is more important than the precision of DTM heights in mainland Australia. In Tasmania, the precision of the DTM heights is more important that the elevation precision of the gravity observation stations since its mean terrain correction is larger than 1 mgal. The effects of uniform topographical density and planar position error of the gravity station are, generally, negligible.

7.4 Summary

This chapter numerically investigated terrain reductions and their effects on Australian gravimetric geoid determination. The regional gravity field features are also studied in terms of terrain corrections, its effects on geoid and height anomaly. The fast Fourier transformation (FFT) method is used to compute the gravimetric terrain correction and its primary and indirect effects on the geoid, because of its computational efficiency over large areas such as Australia. The addition of terrain corrections is expected to improve upon the current Australian geoid (AUSGEOID93).
It was demonstrated that the information from a detailed digital terrain model (1'×1' DTM) is crucial for a high precision, high resolution geoid determination in terms of terrain corrections. The maximum terrain effect is 0.688 m, and root mean square effect 0.14 m in Australia. Over the rest of Australia, the terrain effects make a small but systematic contribution to the gravimetric geoid. The maximum value of the terrain correction itself is up to 25 mgal (referred to a 1'×1' DTM). This also suggests that the difference between geoid and height anomaly must be considered in precise geoid determination of Australia.

A series of formulae have been also derived to evaluate the accuracy of the terrain correction computation. The terrain correction is generally, contaminated by various errors such as the errors in the DTM heights, 3-D position of the point of interest and uniform topographical density assumption. If the errors in the computation are larger than terrain correction themselves, the addition of the terrain correction is detrimental. Therefore, these formulae are important for the accuracy estimation of the terrain correction. In addition, these formulae can also be applied for the adoption of an effective grid size of the DTM.

Considering the fact that the accuracy of the spot heights, the grid size for an effective computation of the terrain corrections should be equal to or greater than 1'×1' (1.8km). The newly released 9"×9" DTM heights is not recommended for the direct computation of the terrain corrections. Based on the results here, the effective grid size for the 9"×9" detailed DTM should be equal to or greater than 15"×15". Otherwise, its computational errors may spoil the accuracy of the terrain correction itself.
AN EVALUATION OF FFT GEOID DETERMINATION TECHNIQUES AND THEIR APPLICATION TO HEIGHT DETERMINATION USING GPS IN AUSTRALIA

CHAPTER EIGHT

PRACTICAL GRAVIMETRIC GEOID DETERMINATION
Chapter 8

PRACTICAL GRAVIMETRIC GEOID DETERMINATION

8.1 Introduction
In the previous six chapters, a number of theoretical and practical problems have been studied concerning the Australian geoid. This chapter will combine the previous results to produce an accurate gravimetric geoid determination procedure for Australia. An accurate gravimetric geoid will also be produced. The accuracy (both relative and absolute) of the geoid is evaluated against Australian Fiducial and National GPS networks (AFN/ANN) and three other local GPS/levelling networks in Australian Capital Territory, Victoria and Western Australia.

In addition, a number of remaining problems related to the gravimetric geoid determination are also studied. The long wavelength (>110km) features of the OSU91A reference field are studied using spectral analysis and a low pass filter. The optimal cap size for the Stokes integral and its effects on the geoid determination are studied. The contribution of the satellite altimetry-derived gravity anomalies to the determination of the new gravimetric geoid is investigated. In addition, a number of grid sizes are chosen to test the most appropriate grid size of the gravity anomaly for the geoid computation.

It is expected that these recommendations will assist government authorities (e.g. the Australian Surveying and Land Information Group) in implementing a new generation of Australian geoid.

8.2 Computation of a New Australian Geoid
Figure 8.1 shows a flow chart for the implementation of a new Australian gravimetric geoid (i.e. the best FFT geoid). This procedure is mainly based on the following research results:
Figure 8.1 A flow chart for the optimal implementation of the new Australian geoid (the best FFT geoid)
8.2.1 Data Preparation of a New Geoid

Heterogeneous data, including gravity measurements both on land and at sea, spot heights, GPS/levelling data and global geopotential models were evaluated, validated, refined and integrated using the theory and methodologies in Chapter Three.

The updated Australian gravity data base (1992) was provided by Australian Geological Survey Organisation (AGSO) and the spot heights by Australian Surveying and Land Information Group (AUSLIG). The raw data were checked for errors and transformed from the Australian Geodetic Datum to the World Geodetic System 1984 (WGS84). The gravity values were reduced from point height to reference surface (the geoid) using a second order free-air reduction and atmospheric correction (§3.4). It is estimated that the accuracies of gravity on land and at sea (marine gravity) are 1–2 mgal and 2–6 mgal, respectively.

A large coverage of accurate gravity anomalies is required to reduce edge effects in the FFT (§4.5.3). From this point of view, it is important to properly incorporate satellite altimetric information with ground-based gravity data. Satellite altimetry-derived gravity anomalies, where available, were evaluated and then incorporated with marine gravity anomalies to improve the offshore coverage and quantity of the gravity field of Australia (§3.5.2). It has been estimated that the satellite altimetric gravity anomalies around Australia have an actual resolution of 10–20 km and a precision of 4–7 mgal (§3.5). Therefore, 10'×10' grid mean satellite altimetric gravity anomalies were used for the combination with marine gravity data.

As the newly-released 9''×9'' DTM was not available for this research, heights from both spot height and gravity data bases were combined and gridded to 1'×1' DTM for gravity reduction using the minimum curvature spline method (§3.3). The selection of the DTM grid size is based on the resolution (~1.1 km) of the spot heights and the error estimation formulae of the terrain corrections developed in §7.3.

8.2.2 Selection of the Best Grid Size for Gravity Anomalies

According to Shannon's sampling theorem (Brigham, 1988; Bracewell, 1986b), a grid size of 8'×8' should be used for gravity anomalies with average density of 7–8 km per
observation. Since the grid size of terrain correction is 1’×1’, theoretically, the resolution of the terrain corrected gravity anomalies should be 2’×2’ (again using Shannon’s sampling theorem). Because of the heterogeneous density of the gravity observations as demonstrated in §3.4 and §6.7, a denser grid size is required to maintain the direct gravity observational information in denser observed areas. Another important consideration is the practical implementation of the Stokes integral where the kernel function (S(ψ)) is singular and reduces rapidly in the vicinity of the computation point. Therefore, a denser grid size is required for the geoid computation via Stokes’s integral to reduce the discretisation error. A 2’×2’ gridded gravity anomaly, which covers an area of 50°×42° (λ:[110°E, 160°E], φ:[5°S, 42°S]), was used for geoid computation where by a compromise is made between the resolution of the 1’×1’ DTM and the land gravity observations (5’×5’). However, grid sizes of 1’, 2’, 5’, 8’, 10’ and 20’ will be tested in this section of the research to find the most appropriate grid size.

8.2.3 Selection of Optimal Reference Field

To optimally determine long wavelength components of the geoid, the best fitting global geopotential model (GGM) was studied. The selection and verification of an optimal GGM were carried out to give the best local reference field through comparisons with GPS/levelling-derived geoid and gravity anomalies derived from satellite altimetry and continental and marine gravity observations (§3.7). The OSU91A geopotential model (Rapp et al., 1991) to degree 360 was proved to be the best fit to gravity anomalies and good fit to GPS/levelling geoid undulation for Australia (§3.7), and thus has been adopted throughout this research. In addition, a best fit GGM plays an important role in the applications of the generalised remove-restore technique, reducing leakage and boundary effects in the FFT and gravity field gridding. It also reduces the spherical approximation error in Stokes’s formula to less than 2cm. This model will be studied further to scrutinise its errors in the long wavelength components (§8.3).

8.2.4 The Optimal Gridding Procedure

The optimal gridding procedure (§6.8) was applied to reconstruct gravity anomalies in a regular grid (i.e. 2’×2’). The findings of special features of the Australian gravity field
were also incorporated to support the selection of the smoothest residual gravity anomalies. The Bouguer anomaly is much rougher than the free-air anomaly in some areas (§5.1). Rigorously, the TI-GGM, FA-GGM and TI anomaly types should be used in areas A1, A2 and A3, respectively. However, the computer load for the determination of TI and TI-GGM is very heavy. Reviewing Table 6.4, the STDs of the FA-GGM anomaly in the three areas are either the smallest (A2) or very close to the smallest (A1 and A3). To avoid a heavy computing load whilst maintaining a reasonable accuracy, the residual free-air gravity anomalies relative to the OSU91A reference gravity field were used as the gridding platform since it is generally a smooth gravity anomaly type.

8.2.5 FFT Related Considerations

The FFT technique was applied for both Stokes and terrain related formulae because of its computational efficiency over a large area such as Australia. The 1-D exact spherical FFT method (§4.3; Haagmans et al., 1993), was implemented and 100% zero padding was applied to reduce cyclic convolution and windowing errors in the FFT (§4.4).

The terrain correction was applied to gravity anomalies to take into account the terrain effect (§7.2). The indirect effect due to the terrain correction was also applied (§7.2) as a correction in the final geoid (equation 2.47). In addition, a cap size of 3.0° was used for the production of the new FFT geoid. This cap size will be proved to be most appropriate for Australia in next section (§8.4).

8.2.6 Differences from AUSGEOID93

A colour image of the new gravimetric geoid is shown in Figure 8.2. The differences between the new geoid and the existing AUSGEOID93 (Steed and Holtznagel, 1993) over continental Australia are shown in Figure 8.3. These differences were produced by averaging the new geoid onto the same 10′ grid as AUSGEOID93, then subtracting AUSGEOID93.
Figure 8.2 Colour image of the new gravimetric geoid of Australia (Linear projection, unit in meters)
Several features of the new geoid can be observed in Figure 8.3. Assuming that the gravity data sets (1980 and 1992 AGSO releases) to produce the AUSGEOID93 and the new geoid here are predominantly the same. The capsize used for AUSGEOID93 is approximately 0.5°. In Figure 8.3, the steep gradient of the gravity field near the Darling Scarp and central Australia is clearly shown in the new geoid. This implies that there is more high frequency information in the new geoid. This improvement over AUSGEOID93 is mainly contributed to the high resolution gravity data, terrain height information and probably satellite altimetry data.

Larger differences (>0.5m) between the new geoid and AUSGEOID93 are present in northern Australia centred at (20°S, 138°E). This is correlated with low-frequency differences between OSU91A and the actual gravity field (§8.3, Figures 8.4 and 8.5). Some of these large differences may also come from the contribution of the satellite altimetry data in the nearby ocean areas (cf. Figure 3.1). Another reason is that a larger cap size was used in the new geoid (i.e. 3.0° versus 0.5°) which allows more low frequency parts of the gravity field into the new solution. These features are also reflected in Figure 8.6 (§8.3) which implies that the local gravity data can account for some unmodelled errors in the OSU91A.

One interesting finding is that the relatively small differences occur in high terrain correction areas (i.e. the Great Dividing Range), which most probably caused by the coarser resolution of the 10’ grid that filters out some very high frequency parts. In Tasmania, it is shown that more detailed features of the new geoid are very clear (red versus blue). These detailed features are mainly contributed to the local terrain information and probably the information from satellite altimetry data.

8.3 Low Frequency Differences between Actual Gravity Field and OSU91A

The errors in the OSU91A coefficients have two contributions, direct and indirect (cf. equation 2.46). The direct contribution comes from the long wavelength computation of the geoid undulation via equation (2.33). External examination using AFN/ANN/AHD (Figure 3.6) suggests that there exist long wavelength errors in some regions (e.g. northern Australia and near point [139°E, 35S°], §3.7.4).
The indirect contribution of the erroneous GGM is reflected in the computation of residual gravity anomalies (cf. equation 2.48). The residual gravity anomalies may introduce spectral leakage in the FFT computation if low frequencies are present. This may also cause the geoid computation results to vary significantly with different cap sizes (§8.4). Kearsley (1988a) demonstrated that the noisy residual gravity anomalies have large negative impact on geoid determination which correlate with both baseline length and cap size.

To study the long-wavelength differences between the actual gravity field and OSU91A, the free-air gravity anomalies were first detrended to 360 degree, then a low pass filter was used to extract the residual gravity anomalies for wavelengths greater than 110km. Any differences represent gravity features unmodelled by OSU91A. The practical procedure to separate the residual gravity anomalies for wavelengths greater than 110km is demonstrated below.

The free-air gravity anomalies ($\Delta g$) are expressed in spectral form as

$$\Delta g = \sum_{n=2}^{\infty} \Delta g_n$$

(8.1)

The residual gravity anomalies ($\Delta g'$) via equation (2.37) and using OSU91A model can be expressed as

$$\Delta g' = \Delta g - \Delta g_{GGM}$$

$$= \sum_{n=2}^{\infty} \Delta g_n - \sum_{n=2}^{360} \Delta g_{GGM}$$

(8.2)

where $\Delta g_{GGM}^n$ is the $n$th spectral component ($2 \leq n \leq 360$) of the gravity anomaly calculated from OSU91A using equation (2.34); and

$\Delta g_n$ is the $n$th spectral component of the terrestrial gravity anomaly.

As the geopotential model does not perfectly model the low-frequency gravity anomalies, low frequency differences ($\delta g$) remain in the residual gravity anomalies. Therefore,

$$\Delta g' = \sum_{n=361}^{\infty} \Delta g_n + \sum_{n=2}^{360} \delta g_{GGM}^n$$

(8.3)
where $\delta g_n^{GGM}$ is the difference between actual gravity anomalies and OSU91A gravity anomalies in low frequency from degrees 2 to 360.

As shown in Figure 8.4, to quantify the effect of these low-frequency differences on the gravity anomalies, low-pass filtering was used via the 2-D FFT and program 'Spectra'.

![Diagram](image)

Figure 8.4 Scheme of a low pass filter to separate OSU91A effects in low frequency

The low-frequency residual gravity anomalies (i.e. long-wavelength gravity anomaly differences between the actual gravity field and OSU91A) are plotted in Figure 8.5. From Figure 8.5, it is obvious that some long wavelength differences (110km–2000km) exist, particularly around Darwin (131°E, 13°S), the Gulf of Carpentaria (137°E, 15°S) and the Arafura Sea areas (135°E, 11°S). This is most probably due to lack of or inaccurate gravity data around the Indonesian Islands, Coral Sea ([150°–158°E, 10°–20°S]) and the Arafura Sea in the determination of OSU91A. Another significant finding is an approximate 110km–300km wavelength error up to magnitude of 6 mgal near Adelaide (139°E, 35°S) which agrees with the finding in Figure 3.6 from the direct geometric comparison with AFN/ANN/AHD. As these are independent checks on the model, this strongly suggests that there is an error in OSU91A in this region.

An approximate 110km–500km wavelength error of magnitude up to 4 mgal exists along the Darling Fault in Western Australia (116°E, 32°S), which implies that OSU91A can not reflect the steep gravity change and complicated geological structure along the Darling Scarp. Furthermore, some regional errors of wavelength
up to 2000km exist in eastern Australia (e.g. Tasman Basin [151°–160°E, 25°–32°S], Cairns and Coral Sea [150°–158°E, 10°–20°S]). These features agree with the geometrical analysis (Figure 3.6 in §3.7.4), which again suggest these errors exist in OSU91A.

![Gravity anomaly differences between actual gravity field and OSU91A in long wavelength (<360) (contour interval = 2 mgal)](image)

Figure 8.5 Gravity anomaly differences between actual gravity field and OSU91A in long wavelength (<360) (contour interval = 2 mgal)

In order to determine the contribution of these low frequency differences (δg) to the geoid (δN), the 1D FFT was applied to the low-pass filtered residual anomalies. A contour map of the geoid effect of these low-degree differences is shown in Figure 8.6.
Similarly, it is clearly shown in Figure 8.6 that there exist long wavelength differences between the geoids from OSU91A and the actual gravity anomalies, particularly in the Northern Territory, the Arafura Sea and Coral Sea areas. Overall, the magnitude of the geoid undulation difference is less than 0.4m. Note that the differences in Darling Scarp, Adelaide and Tasman Basin are -0.2m, 0.2m and 0.2–0.4m respectively. Comparing Figure 8.6 with Figure 3.6, it is found that the major features of these two figures are similar which strongly suggests that OSU91A has long wavelength descriptive errors in these regions. Fortunately, however, the medium wavelength (110–600km) errors in the OSU91A are improved by gravity data through Stokes’s integral over a capsize of 3.0° (cf. §2.7 from equations 2.46 to 2.48).

To summarise, in order to produce a high accuracy gravimetric geoid, the quality of the global geopotential model seems critical. Ideally, a best fit GGM should completely express the long wavelength components of the local gravity field in terms of geoid undulation and gravity anomalies. Unfortunately, no one of the current
available GGMs possess these two features simultaneously (§3-7 and §3-8). The EGM96 has a good performance in recovering geoid undulation, while OSU91A has a good performance in recovering gravity anomalies in terms of the STD agreement with the half degree gravity anomalies and GPS/levelling derived geometric geoid at 59 AFN/ANN/AHD sites. In fact, these features could not be determined well by the global features of the GGM. Only if high accuracy and resolution local gravity anomalies were used, could the geopotential model achieve this goal. Therefore, a higher precision, tailored GGM using the refined gravity anomalies of Australia, is highly recommended for this purpose.

8.4 Optimal Cap Size in the FFT

Theoretically, if a high degree and order geopotential model, such as OSU91A, is used to provide the long wavelength contribution of the gravity field to the geoid undulation, it should be sufficient to integrate residual gravity anomalies over a cap size which corresponds to the resolution of the GGM. Since 0.5° equal-angular block mean gravity anomalies were used in the production of OSU91A (Rapp et al., 1991), the cap size of the Stokes integration of the residual gravity anomalies required should be equal to 0.5°, according to Rapp’s (1977) rule. Note that the radius of the cap size is even less than this value (e.g. ~0.3° for Australia) considering the difference between ‘equal-angular’ and ‘circle of radius’ blocks (Kearsley, 1984; 1986b).

Past studies in Australia show that different authors tend to claim different optimal cap sizes for the Stokes’s integral, such as Kearsley (1984; 1986a; 1988a), Jaksa et al. (1991), Gilliland (1994a; 1994b). Kearsley (1988a) concluded that, for the highest precision, a cap size of 1° should be used together with a 180 degree and order GGM, which coincides with the cap size from Rapp’s rule. However, Kearsley’s tests in Western Australia and South Australia indicated that the best agreement occurs at 0.2°. Holloway (1988) carried out a similar test in South Australia, where the best agreement with GPS/levelling was obtained for a cap size of 0.3°. Jaksa et al. (1991), using a similar method and OSU89B/OSU86E in Australia over long distances in central Australia, showed that the best results occur at cap size=1.5°. Gilliland (1994b) demonstrated that a 0.5° cap size achieves the best agreement with
GPS/levelling results from a test in Melbourne area using OSU91A to degree and order 360, which also coincides with the capsize from Rapp's rule.

In fact, this becomes complicated by several factors as pointed out by Kearsley (1988a), particularly for the errors in GGM and in the residual gravity anomalies. It has been demonstrated that there exist large differences between the actual gravity data and GGM (6mgal for gravity anomaly, 0.4m for geoid undulation) in the long wavelength (110km-2000km) components of the gravity field (§8.3). Due to the complicated features of the Australian gravity field, the long wavelength components (e.g. 110km-500km) of the gravity field are not necessarily well reflected in a global Earth model. Therefore, the residual gravity anomalies still contain long wavelength components of the gravity field, and the residual gravity anomalies thus vary significantly in different areas as is found in Chapters Five and Six. These errors are probably the key reasons for inconsistent optimal cap size in different areas.

If the residual gravity anomalies contain gravity field information of degrees less than 360 as demonstrated in Figures 8.5 and 8.6, the residual gravity anomalies are biased (and possibly contaminated) by these residual low-frequency components. The spectral leakage effects may become significant in the application of FFT due to the presence of a bias. As seen from Figure 8.6, these differences are large in some regions (~40cm). Assuming a one-milligal constant bias in the gravity anomalies over a capsize of 3.0°, Stokes's integral will give an error of 38cm (Featherstone, 1992, p.149). Therefore, these long wavelength effects are critical to both absolute accuracy and relative precision.

In many FFT geoid determination, the whole data area is used in the convolution integration (e.g. Sideris and Schwarz, 1987). However, in Australia, when the whole data area (110°E≤λ≤160°E, 5°S≤φ≤47°S) is used, the STD of the differences from the AFN/ANN/AHD geometric geoid is of the order of 2-3m (cf. Figure 8.7, outside the window). Therefore, it appears that a limited cap size should be used in the FFT (§4.3.4).

To test for the optimal cap size, the geoid undulation is evaluated for different cap sizes ranging from 0° to 5.0° using the 1-D FFT based on 2°×2° gridded gravity anomalies. These geoid undulations for different cap sizes are then compared with the
GPS/levelling-derived geometric geoid (§3.6) and their STD differences are depicted in Figure 8.7.

![Graph showing STD differences between FFT geoid and GPS/levelling results using different cap sizes both with (+SA) and without the inclusion of satellite altimetry data.

As shown in Figure 8.7, the best agreements occur at different cap sizes for different areas. The cap sizes for the best results achieved are 3.0° for Australia as a whole, 0.2° for Australian Capital Territory, 0.1° for Western Australia and 0.3° for Victoria, respectively. The STD curves in Figure 8.7 generally have peaks and troughs in different locations for different areas (the ‘W’ pattern reported by Kearsley (1988a)). In addition, all the optimal cap sizes for different areas are less than (or equal to) 3.0°.
Moreover, the selection of a cap size is not very crucial for the AFN/ANN sites if a cap size less than 3.0° is used because the curves, on the whole (0°~3.0°), do not change significantly (0~8cm). These conclusions are similar to that of Kearsley (1988a), Holloway (1988) and Gilliland (1994a; 1994b). From the four curves, cap sizes 0.1°~0.5°, generally give the best agreements for the three local GPS/levelling networks. However, for a large cap size, the results may vary significantly. The STD differences increase with the increasing cap size when the cap size is larger than 3.0°. This is mainly due to the errors in gravity anomalies and GGM coefficients propagating into the solution. Therefore, a capsize smaller than 3.0° is recommended for the gravimetric geoid computation of Australia.

To summarise, the optimal capsize changes with locations which is most probably caused by the erroneous features in the long wavelength components of the OSU91A model. The complicated gravity field structure worsens this further (more unmodelled features). Considering the fact that AFN/ANN covers an area of ~7,500,000km² and the three local GPS networks altogether cover an area of 215,000 km², the optimal capsize for Australian-wide geoid should be referred to AFN/ANN. Therefore, when the Australian continent is considered as a whole, the optimal capsize is 3.0°. However, such a choice of optional capsize is dependent on the quality of the GPS/AHD data. Therefore, the conclusion made are restricted to the data available at the time of this study. Moreover, the optimal capsize is different in different regions. The optimal capsizes are 0.1°, 0.3° and 0.2° for Western Australia, Victoria and Australian Capital Territory, respectively.

8.5 Comments on the Effects of the Satellite Altimetry-derived Gravity Data

As demonstrated in section §3.5, the precision of the satellite altimetry-derived gravity anomalies is 4-7 mgal for 10'×10' block mean values which is slightly less than that of marine gravity anomalies (1~3mgal for 10'×10' block mean values, §3.5.1). However, the satellite altimetry data are very useful to improve the quantity and coverage of the terrestrial gravity data particularly in the coastal areas. This section of the research will investigate the effects of the satellite altimetry-derived gravity anomalies to the geoid computation in terms of quality of the geoid.

Using the procedure demonstrated in §3.5.2, the satellite altimetry data were combined with marine gravity data. Figure 8.7 indicates that the inclusion of the satellite altimetry data can improve the accuracy of the geoid particularly for the
AFN/ANN and Victorian sites. The STD at AFN/ANN sites is slightly improved from 0.41m (without satellite altimetry data) to 0.37 m (Figure 8.7).

However, the inclusion of the satellite altimetry data does not improve the gravimetric geoid in Western Australia and Australian Capital Territory. In particular, when cap size is larger than 1.0° the agreement in Western Australia becomes poor (1–10cm).

To further demonstrate the effectiveness of the satellite altimeter data in the geoid undulation computation, 35 of the AFN/ANN stations along the Australian coastline (§3.6.1) were picked out as reference stations. If the inclusion of the satellite altimetric data does contribute to the precise geoid computation significantly, the precision at these stations should be apparently improved. The statistical results of the new geoid with and without the inclusion of the satellite altimetry data at the 35 coastal stations are listed in Table 8.1.

Table 8.1 Geoid undulation differences between the new FFT geoid and AFN/ANN/AHD at 35 coastal stations both with and without the satellite altimetry data (units in metres)

<table>
<thead>
<tr>
<th>Geoid differences</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>New geoid(without SA data )-GPS/AHD</td>
<td>0.85</td>
<td>-1.37</td>
<td>-0.15</td>
<td>0.50</td>
<td>0.48</td>
</tr>
<tr>
<td>New geoid(with SA data )-GPS/AHD</td>
<td>0.70</td>
<td>-1.22</td>
<td>-0.06</td>
<td>0.41</td>
<td>0.40</td>
</tr>
</tbody>
</table>

From Table 8.1, it is concluded that the inclusion of the satellite altimeter gravity data can improve the gravimetric geoid. The improvement is significant at an 85% confidence level using the F-test. All the statistical discrepancies between the new geoid and GPS/AHD geometric geoid (Max, Min, Mean, RMS and STD) are improved after the addition of the satellite altimetric data. The STD and range (maximum minus minimum) improvements are 8 cm and 30 cm respectively.

Therefore, it is concluded that the inclusion of satellite altimetry data improves not only the quantity and coverage of the gravity field, but also quality of the gravity field of Australia (i.e. the precision of the geoid).

8.6 Optimal Grid Size for the Geoid Undulation Computation

In order to test the most appropriate grid size of the residual gravity anomalies for Australian geoid computation, a number of grid sizes are chosen. These are 1’×1’,
2\times2', 5\times5', 8\times8', 10\times10' and 20\times20'. The discrepancies between the geoid undulations produced using each grid size gravity anomalies and the geometric geoid at 59 the AFN/ANN/AHD stations are listed in Table 8.2.

Table 8.2 Differences between the gravimetric and geometric geoid using different grid sizes of the gravity anomalies (units in metres)

<table>
<thead>
<tr>
<th>grid size</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\times1'</td>
<td>0.754</td>
<td>-1.241</td>
<td>0.014</td>
<td>0.378</td>
<td>0.377</td>
</tr>
<tr>
<td>2\times2'</td>
<td>0.728</td>
<td>-1.227</td>
<td>0.013</td>
<td>0.375</td>
<td>0.374</td>
</tr>
<tr>
<td>5\times5'</td>
<td>0.920</td>
<td>-1.261</td>
<td>0.030</td>
<td>0.383</td>
<td>0.381</td>
</tr>
<tr>
<td>8\times8'</td>
<td>0.642</td>
<td>-1.414</td>
<td>0.011</td>
<td>0.389</td>
<td>0.388</td>
</tr>
<tr>
<td>10\times10'</td>
<td>1.237</td>
<td>1.259</td>
<td>0.069</td>
<td>0.457</td>
<td>0.453</td>
</tr>
<tr>
<td>20\times20'</td>
<td>1.418</td>
<td>-1.567</td>
<td>0.078</td>
<td>0.518</td>
<td>0.510</td>
</tr>
</tbody>
</table>

From Table 8.2, the discrepancies between the GPS/AHD and the gravimetric geoid using different grid sizes are different and the second smallest grid size (e.g. 2\times2') gives the best STD fit. When the grid size is larger than the Nyquist frequency (e.g. 10' and 20' grids), the discrepancies become larger and larger (e.g. 8cm and 13cm for 10' and 20' grids respectively). However, when the grid size is less than the Nyquist frequency (\leq8'), the discrepancies are relatively stable (i.e. \sim1cm for STD) and a denser grid size generally gives slightly better results. It is estimated that these differences are mainly caused by the discretisation errors of the Stokes’s kernel. Therefore, a denser grid is recommended for geoid computation to reduce discretisation errors of Stokes’s kernel although aliasing effects may exist for a grid size less than the Nyquist frequency. It is also shown that the grid size is not very critical when it is less than the Nyquist frequency.

Moreover, self-consistency of the solutions based on different grid size gravity anomalies is also tested using geoid heights at coincident grids nodes (a 5° boundary is also excluded at each side to reduce edge effects). The geoid discrepancies at grid nodes between the different solutions (1\times1', 5\times5', 10\times10' and 20\times20') and 2\times2' are listed in Table 8.3.
Table 8.3  Self-consistency among the geoids produced from different grid sizes (a 5° boundary is excluded in each side, units in metres)

<table>
<thead>
<tr>
<th>grid size</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔN (1°-2°)</td>
<td>0.227</td>
<td>-0.206</td>
<td>0.002</td>
<td>0.016</td>
<td>0.016</td>
</tr>
<tr>
<td>ΔN (5°-2°)</td>
<td>0.569</td>
<td>-0.440</td>
<td>0.007</td>
<td>0.047</td>
<td>0.047</td>
</tr>
<tr>
<td>ΔN (8°-2°)</td>
<td>0.816</td>
<td>-0.560</td>
<td>-0.017</td>
<td>0.068</td>
<td>0.066</td>
</tr>
<tr>
<td>ΔN (10°-2°)</td>
<td>1.256</td>
<td>-0.792</td>
<td>0.006</td>
<td>0.158</td>
<td>0.158</td>
</tr>
<tr>
<td>ΔN (20°-2°)</td>
<td>1.470</td>
<td>-1.093</td>
<td>0.021</td>
<td>0.188</td>
<td>0.187</td>
</tr>
</tbody>
</table>

Similarly, when the grid size is less than 8′×8′, the discrepancies at common grid points are smaller than the discrepancies with large grid sizes (i.e. 10′×10′ and 20′×20′). Therefore, the results become worse (16cm to 19cm for STD) when the grid size is greater than the Nyquist frequency. When the grid size is close to or less than Nyquist frequency (i.e. 1′×1′, 5′×5′ and 8′×8′), the discrepancies are very similar (i.e. 1.6cm, 5cm and 7cm for STD). The second densest grid size (2′×2′) is again recommended for the gravimetric geoid computation. However, the results from the densest grid size (1′×1′) are very close to those from 2′×2′ grid size.

8.7 Evaluation of the Best FFT Geoid

There are two ways to determine a new generation geoid. One is to compute the geoid undulation using the techniques depicted in §8.1 (the best FFT geoid). Another is to apply direct and indirect terrain correction effects to the existing AUSGEOID93 (i.e. a terrain corrected AUSGEOID93). As AUSGEOID93 is the geoid currently used in Australia, it is informative to use this to show what improvements have been made by this research.

8.7.1 Terrain Effects on AUSGEOID93

To evaluate the impact of the terrain effects on the Australian geoid, the existing AUSGEOID93 gravimetric geoid (Steed and Holtznagel, 1994) was augmented by the total terrain effect (direct plus indirect effects). This geoid solution is then compared with GPS and spirit levelling data at the 59 AFN/ANN/AHD stations. The statistics of their differences are listed in Table 8.4.
Table 8.4  Statistical results of geoid differences at 59 AFN/ANN stations  
(units in metres)

<table>
<thead>
<tr>
<th>Geoid</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUSGEOID93</td>
<td>0.67</td>
<td>-1.16</td>
<td>-0.22</td>
<td>0.46</td>
<td>0.45</td>
</tr>
<tr>
<td>Terrain corrected AUSGEOID93</td>
<td>0.80</td>
<td>-0.79</td>
<td>0.08</td>
<td>0.42</td>
<td>0.41</td>
</tr>
<tr>
<td>The best FFT geoid</td>
<td>0.72</td>
<td>-1.22</td>
<td>0.01</td>
<td>0.37</td>
<td>0.37</td>
</tr>
</tbody>
</table>

From Table 8.4, the addition of the total terrain effect to the existing AUSGEOID93 free-air co-geoid effectively removes the systematic differences between the gravimetric geoid and the geometric (GPS-levelling) geoid. After adding the total terrain effect to the existing AUSGEOID93, the mean departure of the terrain corrected AUSGEOID93 from GPS and levelling networks reduces from -0.22m to 0.08m. The maximum total terrain effect is 0.526m at the 59 AFN/ANN stations (see Appendix I). However, the standard deviation and root mean square of the differences are not improved significantly (4cm). This could be due to the systematic departures of the Australian Height Datum from the geoid caused by levelling errors, errors in relative GPS, or the resolution of the DTM used, or a combination of them.

Over the rest of Australia, the terrain effects make a small but systematic contribution to the gravimetric geoid, depending on local topography (see Figures 7.1 through 7.5). The maximum total terrain effects are 0.522 m, 0.587 m and 0.085 m for Australian Capital Territory, Victoria and Western Australia, respectively (see Appendix I). Statistical comparisons among AUSGEOID93 both with and without the inclusion of the terrain effects and the best FFT geoid for the three local GPS/levelling networks are listed in Tables 8.5 through 8.7.

Table 8.5  Statistical results of geoid differences at 86 Australian Capital Territory  
GPS/levelling stations (units in metres)

<table>
<thead>
<tr>
<th>Geoid</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUSGEOID93</td>
<td>0.14</td>
<td>-0.28</td>
<td>0.01</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Terrain corrected AUSGEOID93</td>
<td>0.66</td>
<td>0.24</td>
<td>0.50</td>
<td>0.51</td>
<td>0.05</td>
</tr>
<tr>
<td>The best FFT geoid</td>
<td>0.90</td>
<td>0.50</td>
<td>0.77</td>
<td>0.74</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Table 8.6 Statistical results of geoid differences at 18 Victorian GPS/levelling stations (units in metres)

<table>
<thead>
<tr>
<th>Geoid</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUSGEOID93</td>
<td>0.26</td>
<td>-0.78</td>
<td>-0.23</td>
<td>0.42</td>
<td>0.32</td>
</tr>
<tr>
<td>Terrain corrected AUSGEIOD93</td>
<td>0.56</td>
<td>-0.37</td>
<td>0.13</td>
<td>0.28</td>
<td>0.25</td>
</tr>
<tr>
<td>The best FFT geoid</td>
<td>0.47</td>
<td>-0.38</td>
<td>-0.01</td>
<td>0.23</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Table 8.7 Statistical results of geoid differences at 21 Western Australian GPS/levelling stations (units in metres)

<table>
<thead>
<tr>
<th>Geoid</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUSGEIOD93</td>
<td>0.20</td>
<td>-0.20</td>
<td>0.02</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Terrain corrected AUSGEIOD93</td>
<td>0.28</td>
<td>-0.12</td>
<td>0.09</td>
<td>0.15</td>
<td>0.12</td>
</tr>
<tr>
<td>The best FFT geoid</td>
<td>0.19</td>
<td>-0.21</td>
<td>0.05</td>
<td>0.13</td>
<td>0.11</td>
</tr>
</tbody>
</table>

From Tables 8.5 through 8.7, the STD differences between the best FFT geoid and the GPS/levelling-derived geometric geoid vary between networks. Recall the precision estimation of the GPS/AHD in Table 3.2 (§3.6), due to the systematic errors present from the combination of unadjusted individual surveys in the Australian Capital Territory 86 GPS/levelling stations, the STD in Table 8.5 is not improved apparently. However, all the three geoids show very good STD fits with the geometric geoid. The STD differences in Tables 8.6 and 8.7 are most probably caused by the different precisions of the Victorian and Western Australian GPS/levelling networks. A good fit (11cm) between the geoids in Western Australia is obtained where GPS/AHD results have a higher precision (10cm), whereas a relatively poor fit (23cm) in Victoria corresponds to a precision of 19cm GPS/AHD. Therefore, the precision differences are mainly caused by the different GPS/AHD precisions. This is an important consideration when verifying a gravimetric geoid in this way.

Of the three networks, it is concluded that the simple inclusion of the total terrain effects can improve the existing AUSGEIOD93 but not as much as new solution using a capsize of 3.0°. The new gravimetric geoid is, again, superior to the AUSGEIOD93 both with and without addition of the total terrain effects in terms of STD differences, except in ACT where systematic errors are present in the GPS data.
8.7.2 Removal of the Bias and Tilt

A number of factors contribute to the differences between the geometric geoid and gravimetric geoid. These are the errors in the GPS ellipsoidal height, errors in the spirit levelling and AHD, long wavelength errors in OSU91A model, and errors in the residual gravity anomalies.

This standard deviation for all the GPS/levelling networks could be improved by fitting a bias and tilt to the discrepancies. This removal of the trend principally models the systematic errors in the AHD heights, but it might also correct, to a certain extent, long wavelength errors in the GGM geoid model and GPS ellipsoidal heights (Li, 1993). Theoretically, the height datum for spirit levelling should coincide with the gravimetric geoid. In practice, however, this requirement is difficult to meet and thus causes systematic errors between the gravimetric geoid and GPS/levelling-derived geometric geoid. Moreover, as stated earlier ($§$8.2.2), the GGM model may introduce direct and indirect effects to the gravimetric geoid, which will cause a systematic bias and/or tilt in the gravimetric geoid. These two systematic errors can be partially modelled by fitting the discrepancies using a four parameter model (Heiskanen and Moritz, 1967):

\[ N_{\text{fixed}} = N + a\varphi + b\lambda + c\varphi\lambda + d \]

(8.4)

where, \( \varphi \) and \( \lambda \) are latitude and longitude respectively. The four parameters \( a, b, c \) and \( d \) are regression coefficients to be determined.

<table>
<thead>
<tr>
<th></th>
<th>AFN/ANN</th>
<th>ACT</th>
<th>Victoria</th>
<th>WA</th>
</tr>
</thead>
<tbody>
<tr>
<td>std</td>
<td>0.37</td>
<td>0.07</td>
<td>0.23</td>
<td>0.11</td>
</tr>
<tr>
<td>%</td>
<td>0.07</td>
<td>0.23</td>
<td>0.11</td>
<td>0.04</td>
</tr>
<tr>
<td>std (after fitting)</td>
<td>0.33</td>
<td>0.06</td>
<td>0.18</td>
<td>0.04</td>
</tr>
<tr>
<td>%</td>
<td>10%</td>
<td>11%</td>
<td>17%</td>
<td>64%</td>
</tr>
</tbody>
</table>

Table 8.8 shows the results of the comparisons between geoid and GPS/levelling results before and after four parameter fitting at various GPS networks. The standard deviation is improved from 37 cm to 33 cm for the AFN/ANN/AHD, from 23 cm to 18 cm for Victoria, from 11 cm to 4 cm for Western Australia and from 7 cm to 6 cm.
for Australian Capital Territory. However, the removal of bias and tilt does not contribute a significant improvement (10%) for the AFN/ANN/AHD mainly because the errors in the AHD height are heterogeneous and the network spans to the whole continent. The percentage improvements for the two local networks (Victoria and Western Australia) are more significant, up to 64% for Western Australia. This is expected because the systematic errors (or bias and tilt) in a small area are more easily corrected than in a large area. Also note that the percentage improvement for Australian Capital Territory is not very obvious. This is due to the GPS/AHD systematic errors between different GPS surveys (§3.6.2) which can not be modelled by simple four parameter fitting.

8.7.3 Absolute Accuracy of the Best FFT Geoid
The absolute accuracy of the best FFT geoid can be evaluated using the AFN/ANN/AHD derived geometric geoid. Comparing Table 8.8 with Table 8.4, the absolute accuracy of the best FFT geoid has been improved by about 15 cm over AUSGEOID93. The absolute accuracy of the best FFT geoid is STD=0.33 m. However, their mean differences are improved by about 20 cm. In addition, the actual absolute accuracy of the new geoid might be better than 0.3 m since the GPS/levelling results themselves are contaminated by the errors in the spirit levelling and AHD (STD=0.28 m), as described in §8.3 and estimated in Table 3.2 (§3.6.2).

8.7.4 Relative Precision of the Best FFT Geoid
The relative precision of the best FFT geoid can be evaluated using the three local GPS/AHD networks. To access the relative accuracy of the new geoid, the relative differences between the new geoid and the GPS/AHD geometric geoid for baselines in the three local area of the GPS networks are plotted against baseline length (Figure 8.8).
Figure 8.8 Estimates of relative differences versus baseline lengths between the new geoid and GPS/AHD in (a) WA, (b) Victoria, (C) ACT
From Figure 8.8, it is shown that the differences are not obviously correlated with baseline length which implies that precision of the new geoid is homogeneous. From the precision estimation of the geometric geoids ($\sigma_{\text{GPS/AHD}}$) in the three local GPS/levelling networks and the STD differences between the best FFT geoid and GPS/levelling geoid ($\sigma_{\text{df}}$), the precision of the best FFT geoid ($\sigma_{\text{New}}$) can be estimated using the simple error propagation law (equation 8.5). The estimated precisions of the geoid are listed in Table 8.9.

$$\sigma_{\text{New}} = \sqrt{\sigma_{\text{df}}^2 - \sigma_{\text{GPS/AHD}}^2}$$  

(8.5)

<table>
<thead>
<tr>
<th></th>
<th>AFN/ANN</th>
<th>WA</th>
<th>Vic</th>
<th>ACT</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>New geoid-GPS/AHD ($\sigma_{\text{df}}$)</td>
<td>0.37</td>
<td>0.11</td>
<td>0.23</td>
<td>0.07</td>
<td>Tab. 8.8</td>
</tr>
<tr>
<td>(after a 4-parameter fitting)</td>
<td>(0.33)</td>
<td>(0.04)</td>
<td>(0.18)</td>
<td>(0.06)</td>
<td></td>
</tr>
<tr>
<td>GPS ellipsoidal height ($\sigma_{\text{GPS}}$)</td>
<td>0.08</td>
<td>0.06</td>
<td>0.13</td>
<td>0.01?</td>
<td>Tab. 3.2</td>
</tr>
<tr>
<td>relative AHD ($\sigma_{\text{AHD}}$)</td>
<td>0.27</td>
<td>0.08</td>
<td>0.13</td>
<td>0.02</td>
<td>Tab. 3.2</td>
</tr>
<tr>
<td>GPS/AHD ($\sigma_{\text{GPS/AHD}}$)</td>
<td>0.28</td>
<td>0.10</td>
<td>0.19</td>
<td>0.02?</td>
<td>Tab. 3.2</td>
</tr>
<tr>
<td>New geoid ($\sigma_{\text{New}}$)</td>
<td>0.24</td>
<td>0.04</td>
<td>0.13</td>
<td>0.06</td>
<td>eqn (8.5)</td>
</tr>
<tr>
<td>(after fitting)</td>
<td>(0.17)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.06)</td>
<td></td>
</tr>
</tbody>
</table>

From Table 8.9, if the effect of the errors in the GPS ellipsoidal height differences on both the spirit levelling and geometric geoid are assumed the same, it can be concluded that the relative precision of the new geoid is, at least, comparable with third order levelling results. Note that in Table 3.2 the magnitudes of each error source represent approximate estimates only, based on the information available from the processing of each GPS data set. It should be noted that the total ellipsoidal height errors quoted represent a worst case scenario for each specific network. Therefore, the new geoid may be of higher accuracy than these figures suggest.

8.8 Discussion

A new gravimetric geoid of Australia has been produced using the procedures depicted in earlier chapters and the results in this chapter. Through a comparison with the GPS/levelling-derived geometric geoid in the whole of Australia (AFN/ANN),
Australian Capital Territory, Victoria and Western Australia, it is shown that the new geoid is superior to the existing gravimetric geoid, AUSGEOID93. The absolute accuracy of the new geoid has been improved (~8 cm) over AUSGEOID93 and reaches RMS = 0.33 m after four parameter fitting. The analyses also demonstrate that the relative precision of the new geoid achieves 10 cm–30 cm. After a four parameter fit, the STD differences between the new geoid and independent geometric geoid in three local GPS/levelling networks reach 4 cm (WA), 6 cm (Victoria) and 18 cm (ACT) respectively. This precision is comparable with the precision of third order levelling results.

Since the long wavelength components of the gravimetric geoid are directly determined using the best fit GGM model, the precision and accuracy of the best fit GGM is very important. It has been demonstrated that there exist large differences between the OSU91A model and Australian data which can be up to 0.4 m and 6 mgal for geoid undulation and gravity anomaly respectively in Australia. It is thus recommended that a higher precision and accuracy, higher degree geopotential model is required to produce a very precise gravimetric geoid.

It is shown that the geoid precision is critically affected by the cap size of the Stokes integration. It is also demonstrated that overall the optimal capsize for Australia is 3.0°. However, since the low-frequency errors in the OSU91A model and the complicated features of the Australian gravity field, the optimal capsize grid size varies with location. The optimal capsize for three local GPS networks is 0.1°–0.3°.

Since the optimal cap size is different for different regions, more GPS/levelling networks, optimally even spanning whole Australia are recommended to determine optimal cap sizes for different regions. These regional gravimetric geoids are then recommended to be combined with the GPS levelling derived geoids in various regions to give final geoid as proposed by Jiang and Duquenne (1996) to support GPS heighting in a more direct way.

The tests presented here indicate that a denser grid size of the gravity anomalies (but not less than the resolution implied by the detailed DTM) is preferable for an accurate
determination of the gravimetric geoid. However, the selection of grid size is not very
critical when the cap size is less than the Nyquist frequency. A grid size greater than
the Nyquist frequency of the gravity data should not be used for accurate geoid
determination.

This study also shows that the inclusion of the satellite altimetric gravity data can not
only improve the quantity and coverage of the Australian gravity field but also
improve the precision of the geoid (quality). It would appear to degrade the geoid
locally in some areas (e.g. WA), however. This is particularly important for Australia
since it is surrounded by oceans. However, an appropriate evaluation and assessment
of the satellite altimetry data prior to be combined with marine gravity data is
important.

8.9 Summary

This chapter combines the results of earlier chapters to produce an accurate
gravimetric geoid of Australia (the best FFT geoid). The optimal cap size for the
Stokes integral is also studied in order to obtain the best combination of the
information from both OSU91A and local gravity and terrain data via FFT. In
addition, the low-frequency errors of OSU91A, the contribution of the satellite
altimetric gravity data, optimal grid size of the gravity anomalies have also been
studied.

It is demonstrated that the new geoid is superior to existing AUSGEOID93. Its
absolute accuracy is better than 30cm and relative precision is better than 10–20 cm.
It is demonstrated that the new gravimetric geoid is, at worst, comparable with third-
order spirit levelling.
AN EVALUATION OF FFT GEOID DETERMINATION TECHNIQUES AND THEIR APPLICATION TO HEIGHT DETERMINATION USING GPS IN AUSTRALIA

CHAPTER NINE

CONCLUSIONS AND RECOMMENDATIONS
Chapter 9

CONCLUSIONS AND RECOMMENDATIONS

9.1 Objectives of the Research

The main objectives of this research were to develop an optimal method, using state-of-the-art theory, procedures and methodologies, to produce the most accurate and precise gravimetric geoid model of Australia (§1.4). This research has also been orientated to study some related problems such as the theory, data reduction and the algorithmic implementation of gravimetric geoid determination.

The results of this research provide an alternative and cost-effective method to geodetic levelling for Australian GPS height determination and for a wide range of applications, such as surveying and mapping, three-dimensional geodetic positioning, geodynamics, geophysics and ocean sciences. In addition, this research can provide the international geodetic community with guidance on accurate geoid determination over large areas.

9.2 Summary of Major Findings

9.2.1 Best Fitting Reference Field

The OSU91A model was proved to be the best fitting global geopotential model (GGM) for Australia (§3.7). The long wavelength bands of the geoid were optimally determined through the adoption of the best fit GGM (§8.2). The best fit GGM plays a crucial role in the generalised remove-restore technique of the gravity field determination (§2.7). It is also important to reduce spectral leakage for the practical implementation of the Stokes integral via the fast Fourier transform (§4.4.1). In addition, the best fitting GGM is vital for the gravity field gridding for which a smooth, unbiased residual gravity field is desired (§6.8).

Ideally, a best fit GGM should perfectly express the long wavelength components of the local gravity field in terms of geoid undulation and gravity anomalies without computation errors. Although OSU91A is the best fit of all GGM models tested, it has low-frequency differences with the Australian gravity field which contribute up to
6mgal to the gravity anomalies and 40 cm to the geoid with a wavelength ranging from 110km to 1500km respectively (§8.3).

9.2.2 Gravity Data Preparation and Combination with Satellite Altimetric Data
The refinement of the gravity anomalies from raw gravity observations is delicate work and involves a number of measures (§3.4). The raw gravity data both in continental and marine areas were validated, screened, co-ordinate transformed and pre-processed. It was estimated that the accuracies of the land and marine gravity anomalies are 1–2 mgal and 2–6 mgal respectively. Since a large coverage of accurate gravity anomalies is required to reduce edge effects in the FFT (§4.5), it is important to properly incorporate satellite altimetric information with ground-based gravity data (§8.5).

Satellite altimetry-derived gravity anomalies, where available, were evaluated and then incorporated with marine gravity anomalies (§3.5) to improve the offshore coverage and quantity of the gravity field of Australia. It was shown that the satellite altimetric gravity anomalies around Australia have an actual resolution of 10–20km and a precision of 4-7mgal (§3.5). Therefore, a 10'×10' grid of satellite altimetric gravity anomalies was used for the combination with marine gravity data (§3.5.2).

9.2.3 FFT Related Considerations
The gravity field integrals can be computed extremely efficiently and quickly using FFT, particularly over a large area. Stokes's integral by FFT can give centimetre precision geoid undulation if some special considerations are applied (§4.5). First and most importantly, the input data must be long wavelength filtered using the best fit geopotential model. The removal of the bias in the residual gravity anomalies is important to reduce spectral leakage. Second, one hundred percent zero padding should be used for further improvement of the geoid to remove cyclic convolution errors and edge effects (§4.5.4). Moreover, the 1-D spherical FFT is recommended for precise local geoid computation (§4.5.6). Furthermore, the boundary effects are location dependent. The rougher the residual gravity anomalies, the higher and broader the boundary effects. In the northern part of the test area, boundary effects could be up to 15 degrees where OSU91A does not fit the local gravity. There is almost no effect in the southern part of the test area where OSU91A provides a good
fit to local gravity. Larger data coverage and better residual gravity structure (e.g. long and medium wavelengths are effectively removed prior to FFT) are required to produce a precise geoid (§4.5.6).

9.2.4 Special Features of the Australian Gravity Field

Because of the long geological history and weathered topography, the Australian gravity field behaves quite differently to the gravity field in other continental areas of the world. The topography often contains longer wavelength features than the gravity anomalies in some parts of Australia (§5.4.1). None of the free-air, Bouguer or topographic-isostatic gravity anomalies are consistently the smoothest type in Australia (§5.3). This observation implies that there exist large density anomalies in some regions below the Australian continent (§5.5) and that the continent does not satisfy simple model of isostatic compensation.

9.2.5 Techniques Analysing the Variation of the Gravity Field

A number of techniques were used to analyse the variations of the gravity field (§5.3–5.4). The statistical comparison, in combination with direct visualisation and power spectral analysis methods, can effectively reveal the detailed gravity field structure and thus provide an understanding of the gravity field (§5.3.1). The power spectral technique is a powerful tool for gravity field analysis in terms of the power distribution along different wave bands. Due to the regression error in the computation of the fractal dimension, it was found that the simple Hurst fractal method is not informative when the roughness of the surface or profiles is similar.

The statistical analysis method is a simple, intuitive and informative method. It can effectively reveal the variability of the gravity anomalies (§5.6). It is, therefore, recommended for quantifying the smoothness of the gravity field before gridding.

9.2.6 Optimal Gridding Procedure for Australian Gravity Field

Determination of an optimal gridding procedure is very important to give the most accurate estimation of the gravity anomalies on a regular grid. This procedure is a critical issue for subsequent gravity field refinement because any error will directly propagate into the geoid determination. The gridded gravity anomalies should express local gravity field as precisely as possible. The same three test areas were chosen for
this study as in Chapter Five. The optimal gridding procedure included two aspects: optimal gridding method and the selection of the smoothest gridding "platform".

It was demonstrated that none of the six tested gravity anomaly types are consistently the smoothest (§6.8.3) in the three test areas. The best gridding procedure was found to be the minimum curvature spline method together with residual free-air gravity relative to 360 degree OSU91A (§6.8–§6.9). The effect of using the inappropriate gravity anomaly type is significant. The gridding precision of the gravity anomalies was found to be better than 3 mgal when the optimal gridding procedure was applied (§6.9).

9.2.7 Evaluation of the Terrain Effects
To optimally determine the short-wavelength component of the geoid, detailed terrain information is highly desirable. Apart from revealing short-wavelength information about the geoid, terrain information also plays several other roles in geoid determination (§7.1). The computation of terrain effects during the determination of an accurate geoid is extremely important, both practically and theoretically. The terrain corrections were computed based on a detailed digital model (1'x1') which corresponds to a resolution of 1.8 km. The maximum terrain effect is 0.69 m, and root mean square effect 0.14 m in Australia (§7.2.2). The maximum value of the terrain correction itself is up to 25 mgal. Therefore, the existing AUSGEOID93 can have a maximum error of 0.69m in terms of terrain effects only.

Special attention was given to the gravimetric terrain correction and its associated indirect effect due to the second Helmert condensation method. The FFT technique was applied to both the terrain correction and its indirect effect for computational efficiency in a large area such as Australia. Both the direct and indirect effects of the terrain corrections must be taken into account in the geoid computation, particularly in eastern Australia and Tasmania. The maximum direct and indirect effects are 0.79m and -0.22m respectively (§7.2).
9.2.8 Theoretical Estimation of the Precision of the Terrain Corrections

Apart from the numerical tests and practical implementation of the terrain correction, its theoretical accuracy was also evaluated. The theoretical contamination of the digital terrain model errors, three-dimensional station error of the gravity observation, and uniform topographical density assumption to the terrain correction were studied. A series of error estimation formulae were developed (§7.3). This research also has applications in the clarification of the terrain correction effectiveness to geoid determination due to these contamination errors (Table 7.7 and Figure 7.8). Its applications to the Australian context were given (§7.3.4). In addition, the results were applied for the selection of optimal grid size of a DTM to use terrain information most efficiently and effectively (§7.3.4).

9.2.9 Optimal Capsize for Stokes’s Integral

The errors in the long wavelength components of OSU91A and regional gravity anomalies lead to the optimal cap size being location dependent. This results in the precision of the gravimetric geoid being capsize dependent. The optimal cap size for the Stokes integral was studied through comparisons with nation-wide and regional GPS/levelling network stations. The precision of the GPS/AHD, however, can also affect the determination of an optimal capsize. The optimal capsize for continental Australia is 3.0° (§8.4).

9.2.10 Contribution of the Satellite Altimetric Gravity Data

The effectiveness of using current satellite altimetric data in the geoid computation was studied. Gravimetric geoids both with and without the inclusion of the satellite altimetric gravity data were computed and compared with 35 AFN/ANN/AHD controls at coastal stations (§8.5). An improvement of 8cm in the STD was obtained and the improvement is significant at the 85% confidence level. This gives an external confirmation that the satellite altimetric data have been appropriately evaluated and combined with local marine gravity data.

9.2.11 Selection of Grid Size for Gravity Field Gridding

Theoretically, the grid size used for FFT geoid computation can be determined from sampling theory (i.e. the Nyquist frequency). Since the average density of the gravity observations is 7–9km, an 8'-9' grid size should be used. However, due to heterogeneous and anisotropic features of the gravity observation spacing, grid size
such determined causes information to be lost for the denser observed areas, such as South Australia (§3.4). Moreover, both the geoid computation and terrain correction formulas are singular in the immediate vicinity of the computation point and their values change rapidly. A denser grid size can greatly reduce the discretisation error in the integral (§8.6). On the other hand, the grid size should not be too denser to be implemented on the computer. A denser grid size than the Nyquist frequency should be used (§8.6).

9.2.12 Accuracy Evaluation of the New Geoid

The accuracy and precision of the new geoid were evaluated using both nation-wide and local GPS networks and its improvement over the previous generation AUSGEOID93 was also studied.

The STD agreement of the discrepancies between the new geoid and local GPS/levelling networks reaches a few centimetres precision (4cm for Western Australia, 6cm for Australian Capital Territory and 18cm for Victoria) after a four parameter fit (§8.7). The new geoid has approximately 8cm improvement over the current existing geoid AUSGEOID93. Moreover, the nominal resolution of the new geoid is 7km (2’grid) which is much higher than that of AUSGEOID93 (10’) (§8.2).

The absolute accuracy of the gravimetric geoid is higher than 33cm, relative precision is better than 10cm over baselines 4km-40km, 20cm over average baseline 150km (§8.7). However, the actual absolute accuracy may be even better because GPS/levelling results in AFN/ANN station are contaminated by the unfavourable features of the AHD and the errors in the GPS ellipsoidal heights. This new geoid appears able to support GPS heighting at third or even second order tolerance. It is also demonstrated that the unfavourable features of the AHD appear to be not critical for local applications (§8.7.2). Any bias and tilt in the datum can be modelled further by applying simple four-parameter fitting.

9.3 Conclusions

From the theoretical studies and numerical tests in this thesis, the following major conclusions can be drawn:
1. The gravimetric geoid computed in this thesis has a absolute accuracy at worst 33cm (45cm for AUSGEOID93), and a relative precision of several centimetres (4cm in WA, 6cm in ACT and 18cm in Victoria) and nominal resolution of 7km.

2. The information from detailed digital terrain model (1'×1' DTM) is crucial for a high precision, high resolution geoid determination in terms of terrain corrections. Both the direct and indirect effects of the terrain need to be taken into account.

3. The gravity field of Australia is very complicated and presents an exceptional case in relation to the widely accepted view of gravity field structure. The topography often contains longer wavelength features than the gravity anomalies in some parts of Australia. Not one of the free-air, Bouguer or topographic-isostatic gravity anomalies is consistently the smoothest type in Australia. This observation also implies that there exists large density anomalies in some regions below the Australian continent. In addition, this finding of the special feature of the gravity field is also important to geophysicists.

4. Theoretical studies show that for flat terrain areas, such as Western Australia, the error of DTM heights is not as critical as the error in the height of gravity stations. However, the error of DTM heights is critical in topographically rough areas.

5. To recover accurate gravity anomalies on a regular grid, the optimal gridding method and most suitable anomaly type should be applied. The removal of a low frequency gravity field together with gravity reductions is also important for gridding. The smoothest platform is of first importance and always selected prior to gridding, while the gridding method itself is of secondary importance. The effect of using the inappropriate gravity anomalies for gridding is very high.

6. Among the gridding methods used here, minimum curvature spline method has been proved to be an optimal method and thus recommended for the gridding of gravity anomalies.

7. The statistical comparison and power spectral technique are simple and informative methods for the analysis of the variations of the gravity field. Due to the regression errors, the simple Hurst fractal dimension method can only approximately give the relative variation information over different gravity anomaly types and thus is not recommended for further use.
8. Simulation studies show that the boundary effects in the FFT geoid computation are location dependent. The rougher the residual gravity anomalies, the higher and broader the boundary effects.

9. Almost all gravity field related integrals can be computed extremely efficiently and effectively using the FFT if it is properly applied. However, the real gravity field of Australia is complicated and the removal of geopotential model to a high degree is highly recommended. In addition, a large data coverage should be used to remove boundary effect in the FFT, particularly in the north part of Australia.

10. There are a number of considerations to get optimal results through FFT technique. First and most importantly, the input data used should be residual gravity anomalies reduced by an optimal reference field. The removal of the bias in the residual gravity anomalies is also important to avoid spectral leakage. One-hundred percent zero padding is recommended for further improvement of the geoid to remove cyclic convolution errors and edge effects. This is particularly useful for Australian context since the residual gravity anomalies can be well approximated by zero values in the surrounding oceans.

11. All the four kernel approximation methods discussed in this research can produce a similar good result and have a good consistency if 100 percent zero padding is applied and residual gravity anomalies are used. The 1-D spherical exact procedure does not clearly outperform the corresponding planar techniques at low or middle latitude area (e.g. over Australia). However, the 1-D spherical exact FFT is recommended for precise local geoid determination for its theoretical rigour.

12. Studies show that there exist some long wavelength differences between the OSU91A model and Australian data. The effect of these errors to the gravity anomaly and geoid undulation can be up to 6mgal and 0.4m over a range of 110km to 1500km over continental Australia.

13. Due to the errors in OSU91A and GPS/AHD geoid and special features of the Australian gravity field, the optimal capsize for Stokes's integral varies with location. When considering Australia as a whole, the optimal grid size is 3.0°. However, for some local areas such as Western Australia, Victoria, Australian Capital Territory and South Australia, a small capsize (0.1°–0.3°) can give relatively good results.
14. The gravity anomalies from satellite altimetry around Australia were estimated that have a precision of 4-7 mgal (for 10′x10′ grid) which is slightly worse than marine gravity observations. Therefore, the satellite altimetry derived gravity anomalies should be assigned less weight than marine gravity anomalies and mainly used to fill in the areas where marine gravity observations are not presented. Satellite altimetric data can improve not only the quantity and coverage of the gravity field, but also quality of the geoid, particularly at the coast of Australia.

15. A denser grid size (but not less than the resolution of the detailed DTM used) is recommended for the gravity field gridding. The grid size larger than the Nyquist frequency should not be used for a precise geoid determination.

16. The best FFT geoid is able to support GPS heighting at third order level. It is also demonstrated that the unfavourable features of the AHD appears to be not critical for local application. Its bias and tilt can be relieved or even removed further by applying simple four-parameter fitting.

9.4 Recommendations

From the theoretical developments and numerical tests in this thesis, the following recommendations are proposed for future research:

1. A 3-D digital density model is desirable for the gravity data reduction and thus provides the possibility to study the detailed structure of the gravity field. This is particularly useful for Australia since there exists a very complicated gravity field.

2. Further studies on the roughness versus smoothness of the various gravity anomaly types are recommended in small scale (e.g. 1°x1° block) to scrutinise more detailed gravity field information. Ideally, the smoothest gravity anomaly type should be always used as gridding platform in each small area. However, this is a delicate work and more geophysical information is required.

3. The GPS and orthometric height data must be of higher precision if adequate control on the gravimetric geoid is to be achieved. Due to the warping and distortion inherent to the AHD, the AHD heights need further refinement to give a good absolute control of the gravimetric geoid. The systematic consistency including the removal of sea surface topography to correct the distortion of the
unconstrained adjustment height datum need further study. A re-adjustment of
the junction points of the Australian first-order levelling network is also
recommended to be another important measure to use tide gauges, altimetric
geoid and sea surface topography as necessary constraints. This is expected to
improve the quality of AHD.

4. A regional tailored geopotential model, Australian geopotential model (AGM) is
suggested to be studied for next generation gravimetric geoid determination. The
AGM can be determined by applying a tailoring technique to the optimally
In addition, due to the low-frequency differences between the GGM and actual
gravity field, a modified kernel method in the Stokes integral (i.e. a removal of
low degree parts) is recommended to reduce this long wavelength error in future
studies.

5. Satellite altimetry data collection, analysis, assessment and implementation of
marine geoid together with the sea surface topography information and marine
and continental gravity observations are important for further refinement of the
Australian geoid. Satellite altimetry-derived gravity anomalies need properly
assessed in terms of error features and accuracy so as to be optimally combined
with local gravity data. Satellite altimetry and airborne gravity are an alternative
to get more detailed gravity field information and thus important for further
improvements of the gravity field in terms of coverage, quality and quantity. This
is of importance for FFT computation and thus is expected to further improve the
accuracy and precision of the geoid along the coastline of Australia. This is
particularly useful for Australia, since the coastline is densely populated relative
to the interior.

6. The FFT method should be used for its efficiency and effectiveness. However,
proper treatments are required, such as proper padding, minimisation of leakage
and boundary effects, optimal capsize and removal of bias.

7. To study more detailed structures and behaviours of the Australian gravity field,
further work is suggested that more rigorous 2-D or 3-D fractal-dimensional
method should be tested to scrutinise these special features not only in one
direction, but also in all directions.
8. The newly released 9"×9" DTM should be used in the future to recover more detailed gravity field information (accuracy and resolution). However, 15"×15" DTM is recommended for a more efficient and accurate computation of the terrain correction.

9. More high precision regional/local GPS/levelling networks are required for the determination of the optimal cap sizes for different areas. These regional gravimetric geoids can be further combined with GPS/levelling derived geoid using the integrated least square adjustment procedure to give more precise geoid of Australia for various engineering applications.
REFERENCES


Dawson, J. (1996) *Personal communication*, Australian Surveying and Land Information Group, Belconnen, Bruce, ACT.


Intergovernmental Committee on Surveying and Mapping (1994) A New Era for Australia, Information Circular, Inter-governmental Committee on Surveying and Mapping, Belconnen, 6 pp.


International Association of Geodesy Symposium No. 117, Tokyo, Japan, September, pp.429-436.


TOPEX/Poseidon Joint Verification Team (1992) TOPEX/Poseidon Joint Verification Plan. NASA, Jet Propulsion Laboratory publication, 92-9, 102 pp.


Appendix I

Summary of the results at GPS/levelling stations, GPS/levelling derived geometric geoid (\(N_{\text{GPS}}\)), geoid from AUSGEOID93 (\(N_{\text{OZ93}}\)), terrain effect (\(N_T\)) and new geoid (\(N_{\text{New}}\))

Table I-1 Various geoids and terrain effects in 59 AFN/ANN GPS/levelling stations

<table>
<thead>
<tr>
<th>Latitude</th>
<th>Longitude</th>
<th>Leveling (Class/Order)</th>
<th>(N_{\text{GPS}})</th>
<th>(N_{\text{OZ93}})</th>
<th>(N_T)</th>
<th>(N_{\text{New}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>-42.8047</td>
<td>147.4387</td>
<td>C/3</td>
<td>-3.630</td>
<td>-3.694</td>
<td>0.210</td>
<td>-3.306</td>
</tr>
<tr>
<td>-42.8040</td>
<td>147.4400</td>
<td>C/3</td>
<td>-3.576</td>
<td>-3.680</td>
<td>0.210</td>
<td>-3.303</td>
</tr>
<tr>
<td>-42.4953</td>
<td>147.9200</td>
<td>C/3</td>
<td>-3.249</td>
<td>-3.279</td>
<td>0.200</td>
<td>-2.524</td>
</tr>
<tr>
<td>-41.0506</td>
<td>145.9087</td>
<td>C/3</td>
<td>-1.501</td>
<td>-1.419</td>
<td>0.235</td>
<td>-1.258</td>
</tr>
<tr>
<td>-40.8641</td>
<td>144.7094</td>
<td>D/4</td>
<td>-2.670</td>
<td>-2.571</td>
<td>0.198</td>
<td>-2.499</td>
</tr>
<tr>
<td>-40.7535</td>
<td>147.9705</td>
<td>C/3</td>
<td>0.223</td>
<td>0.295</td>
<td>0.224</td>
<td>0.561</td>
</tr>
<tr>
<td>-38.4856</td>
<td>143.9119</td>
<td>A/3</td>
<td>1.513</td>
<td>1.661</td>
<td>0.255</td>
<td>1.816</td>
</tr>
<tr>
<td>-38.3148</td>
<td>141.5259</td>
<td>A/3</td>
<td>-3.481</td>
<td>-3.286</td>
<td>0.199</td>
<td>-3.346</td>
</tr>
<tr>
<td>-37.7574</td>
<td>144.6829</td>
<td>C/3</td>
<td>4.760</td>
<td>4.531</td>
<td>0.330</td>
<td>5.101</td>
</tr>
<tr>
<td>-37.0743</td>
<td>149.9079</td>
<td>C/3</td>
<td>12.126</td>
<td>11.843</td>
<td>0.421</td>
<td>11.791</td>
</tr>
<tr>
<td>-35.7975</td>
<td>145.5538</td>
<td>C/3</td>
<td>9.531</td>
<td>9.376</td>
<td>0.357</td>
<td>9.563</td>
</tr>
<tr>
<td>-35.6364</td>
<td>148.9394</td>
<td>C/3</td>
<td>19.446</td>
<td>18.593</td>
<td>0.488</td>
<td>19.009</td>
</tr>
<tr>
<td>-35.3992</td>
<td>148.9800</td>
<td>A/3</td>
<td>19.118</td>
<td>18.470</td>
<td>0.526</td>
<td>19.276</td>
</tr>
<tr>
<td>-35.1233</td>
<td>138.5797</td>
<td>C/3</td>
<td>-0.252</td>
<td>-0.865</td>
<td>0.207</td>
<td>-0.481</td>
</tr>
<tr>
<td>-35.0783</td>
<td>117.6214</td>
<td>C/3</td>
<td>-31.818</td>
<td>-31.499</td>
<td>0.035</td>
<td>-31.499</td>
</tr>
<tr>
<td>-34.4660</td>
<td>150.8516</td>
<td>A/2</td>
<td>22.292</td>
<td>21.133</td>
<td>0.449</td>
<td>21.072</td>
</tr>
<tr>
<td>-33.9294</td>
<td>141.0027</td>
<td>D/4</td>
<td>4.749</td>
<td>4.933</td>
<td>0.237</td>
<td>4.769</td>
</tr>
<tr>
<td>-33.8742</td>
<td>121.8946</td>
<td>C/3</td>
<td>-29.367</td>
<td>-29.167</td>
<td>0.067</td>
<td>-28.995</td>
</tr>
<tr>
<td>-33.4297</td>
<td>149.5671</td>
<td>A/1</td>
<td>25.146</td>
<td>25.287</td>
<td>0.445</td>
<td>25.788</td>
</tr>
<tr>
<td>-32.3599</td>
<td>145.9910</td>
<td>C/3</td>
<td>19.001</td>
<td>19.122</td>
<td>0.300</td>
<td>18.989</td>
</tr>
<tr>
<td>-32.0453</td>
<td>133.7166</td>
<td>C/3</td>
<td>-9.832</td>
<td>-9.608</td>
<td>0.164</td>
<td>-9.283</td>
</tr>
<tr>
<td>-31.8992</td>
<td>141.4499</td>
<td>C/3</td>
<td>13.630</td>
<td>14.004</td>
<td>0.245</td>
<td>13.773</td>
</tr>
<tr>
<td>-31.8935</td>
<td>138.4245</td>
<td>C/3</td>
<td>7.030</td>
<td>6.959</td>
<td>0.258</td>
<td>6.880</td>
</tr>
<tr>
<td>-31.8435</td>
<td>152.7534</td>
<td>C/3</td>
<td>29.401</td>
<td>29.504</td>
<td>0.450</td>
<td>29.324</td>
</tr>
<tr>
<td>-31.8402</td>
<td>115.9755</td>
<td>C/3</td>
<td>-31.416</td>
<td>-31.632</td>
<td>0.059</td>
<td>-31.375</td>
</tr>
<tr>
<td>-31.8020</td>
<td>115.8852</td>
<td>C/3</td>
<td>-32.235</td>
<td>-32.440</td>
<td>0.056</td>
<td>-31.995</td>
</tr>
<tr>
<td>-31.0553</td>
<td>121.4491</td>
<td>C/3</td>
<td>-22.378</td>
<td>-22.536</td>
<td>0.075</td>
<td>-22.537</td>
</tr>
<tr>
<td>-30.7703</td>
<td>128.9554</td>
<td>C/3</td>
<td>-17.806</td>
<td>-17.132</td>
<td>0.129</td>
<td>-17.104</td>
</tr>
<tr>
<td>-29.0466</td>
<td>115.3470</td>
<td>C/3</td>
<td>-25.207</td>
<td>-24.979</td>
<td>0.059</td>
<td>-25.126</td>
</tr>
<tr>
<td>-28.9991</td>
<td>145.6688</td>
<td>C/3</td>
<td>26.555</td>
<td>26.899</td>
<td>0.280</td>
<td>26.762</td>
</tr>
<tr>
<td>-28.8555</td>
<td>151.2107</td>
<td>C/3</td>
<td>36.468</td>
<td>36.325</td>
<td>0.425</td>
<td>36.580</td>
</tr>
<tr>
<td>-28.5385</td>
<td>131.7396</td>
<td>C/3</td>
<td>-5.401</td>
<td>-4.852</td>
<td>0.170</td>
<td>-5.159</td>
</tr>
<tr>
<td>-28.5249</td>
<td>153.5382</td>
<td>C/3</td>
<td>38.795</td>
<td>38.917</td>
<td>0.410</td>
<td>38.793</td>
</tr>
<tr>
<td>-27.4774</td>
<td>153.0270</td>
<td>A/4</td>
<td>41.675</td>
<td>41.669</td>
<td>0.370</td>
<td>41.687</td>
</tr>
<tr>
<td>-26.5902</td>
<td>148.3884</td>
<td>D/4</td>
<td>37.790</td>
<td>37.632</td>
<td>0.304</td>
<td>37.663</td>
</tr>
<tr>
<td>-26.5344</td>
<td>142.3847</td>
<td>C/3</td>
<td>24.503</td>
<td>24.813</td>
<td>0.242</td>
<td>24.769</td>
</tr>
<tr>
<td>-25.9470</td>
<td>133.2097</td>
<td>C/3</td>
<td>5.582</td>
<td>5.862</td>
<td>0.192</td>
<td>6.218</td>
</tr>
<tr>
<td>-25.7071</td>
<td>122.9096</td>
<td>D/4</td>
<td>-8.687</td>
<td>-8.589</td>
<td>0.111</td>
<td>-8.822</td>
</tr>
<tr>
<td>-25.2142</td>
<td>118.0070</td>
<td>C/3</td>
<td>-15.454</td>
<td>-15.805</td>
<td>0.096</td>
<td>-15.315</td>
</tr>
<tr>
<td>-25.1140</td>
<td>113.7316</td>
<td>C/3</td>
<td>-20.660</td>
<td>-21.191</td>
<td>0.056</td>
<td>-21.179</td>
</tr>
<tr>
<td>-24.9947</td>
<td>128.3126</td>
<td>C/3</td>
<td>1.557</td>
<td>1.552</td>
<td>0.147</td>
<td>1.732</td>
</tr>
</tbody>
</table>
Table I-1 (continued)

<table>
<thead>
<tr>
<th>latitude</th>
<th>longitude</th>
<th>levelling (Class/order)</th>
<th>( N_{\text{GPS}} )</th>
<th>( N_{\text{OZa}} )</th>
<th>( N_t )</th>
<th>( N_{\text{New}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-24.8853</td>
<td>152.3199</td>
<td>/4</td>
<td>47.652</td>
<td>47.365</td>
<td>0.288</td>
<td>47.869</td>
</tr>
<tr>
<td>-23.6814</td>
<td>139.6174</td>
<td>/3</td>
<td>32.409</td>
<td>32.312</td>
<td>0.223</td>
<td>32.943</td>
</tr>
<tr>
<td>-23.6701</td>
<td>133.8855</td>
<td>C/3</td>
<td>15.715</td>
<td>15.852</td>
<td>0.221</td>
<td>15.917</td>
</tr>
<tr>
<td>-23.5722</td>
<td>145.2960</td>
<td>D/4</td>
<td>41.855</td>
<td>41.548</td>
<td>0.269</td>
<td>41.653</td>
</tr>
<tr>
<td>-23.1837</td>
<td>150.6567</td>
<td>D/4</td>
<td>51.418</td>
<td>50.452</td>
<td>0.297</td>
<td>51.152</td>
</tr>
<tr>
<td>-23.1729</td>
<td>150.6796</td>
<td>A/3</td>
<td>51.412</td>
<td>50.441</td>
<td>0.296</td>
<td>51.138</td>
</tr>
<tr>
<td>-21.1167</td>
<td>149.2075</td>
<td>/3</td>
<td>55.729</td>
<td>55.054</td>
<td>0.320</td>
<td>54.996</td>
</tr>
<tr>
<td>-20.9814</td>
<td>117.0972</td>
<td>C/3</td>
<td>-6.656</td>
<td>-7.485</td>
<td>0.112</td>
<td>-7.515</td>
</tr>
<tr>
<td>-20.3431</td>
<td>139.2062</td>
<td>C/3</td>
<td>43.556</td>
<td>42.834</td>
<td>0.216</td>
<td>43.511</td>
</tr>
<tr>
<td>-19.6173</td>
<td>134.1885</td>
<td>C/3</td>
<td>33.343</td>
<td>33.044</td>
<td>0.178</td>
<td>33.188</td>
</tr>
<tr>
<td>-19.3473</td>
<td>146.7752</td>
<td>C/3</td>
<td>58.368</td>
<td>57.447</td>
<td>0.393</td>
<td>57.958</td>
</tr>
<tr>
<td>-17.8894</td>
<td>122.2672</td>
<td>C/3</td>
<td>16.391</td>
<td>16.253</td>
<td>0.118</td>
<td>16.327</td>
</tr>
<tr>
<td>-15.6057</td>
<td>128.2764</td>
<td>C/3</td>
<td>36.099</td>
<td>35.296</td>
<td>0.218</td>
<td>35.877</td>
</tr>
<tr>
<td>-14.8407</td>
<td>135.0618</td>
<td>C/3</td>
<td>50.276</td>
<td>49.336</td>
<td>0.152</td>
<td>49.922</td>
</tr>
<tr>
<td>-12.8437</td>
<td>131.1327</td>
<td>C/3</td>
<td>51.268</td>
<td>50.895</td>
<td>0.118</td>
<td>50.885</td>
</tr>
<tr>
<td>-12.4668</td>
<td>130.8410</td>
<td>C/3</td>
<td>51.815</td>
<td>51.899</td>
<td>0.105</td>
<td>51.531</td>
</tr>
<tr>
<td>-12.4233</td>
<td>130.8796</td>
<td>C/3</td>
<td>52.108</td>
<td>51.906</td>
<td>0.104</td>
<td>51.764</td>
</tr>
<tr>
<td>-10.5841</td>
<td>142.2110</td>
<td>/3</td>
<td>71.301</td>
<td>71.868</td>
<td>0.075</td>
<td>71.038</td>
</tr>
</tbody>
</table>

Table I-2 Various geoids and terrain effects in 21 WA GPS/leveling stations

<table>
<thead>
<tr>
<th>latitude</th>
<th>longitude</th>
<th>( N_{\text{GPS}} )</th>
<th>( N_{\text{OZa}} )</th>
<th>( N_t )</th>
<th>( N_{\text{New}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-24.0173</td>
<td>114.0418</td>
<td>-18.479</td>
<td>-18.426</td>
<td>0.065</td>
<td>-18.329</td>
</tr>
<tr>
<td>-23.1493</td>
<td>114.5066</td>
<td>-14.430</td>
<td>-14.625</td>
<td>0.076</td>
<td>-14.646</td>
</tr>
<tr>
<td>-23.5556</td>
<td>114.2360</td>
<td>-16.649</td>
<td>-16.643</td>
<td>0.070</td>
<td>-16.629</td>
</tr>
<tr>
<td>-24.5481</td>
<td>115.2846</td>
<td>-16.561</td>
<td>-16.593</td>
<td>0.085</td>
<td>-16.499</td>
</tr>
<tr>
<td>-24.5832</td>
<td>115.2903</td>
<td>-16.620</td>
<td>-16.655</td>
<td>0.085</td>
<td>-16.556</td>
</tr>
<tr>
<td>-23.5791</td>
<td>114.2080</td>
<td>-16.763</td>
<td>-16.810</td>
<td>0.069</td>
<td>-16.780</td>
</tr>
<tr>
<td>-23.5678</td>
<td>114.2236</td>
<td>-16.725</td>
<td>-16.722</td>
<td>0.070</td>
<td>-16.701</td>
</tr>
<tr>
<td>-23.4333</td>
<td>114.2800</td>
<td>-16.139</td>
<td>-16.161</td>
<td>0.071</td>
<td>-16.152</td>
</tr>
<tr>
<td>-25.0568</td>
<td>115.3291</td>
<td>-17.286</td>
<td>-17.276</td>
<td>0.080</td>
<td>-17.124</td>
</tr>
<tr>
<td>-24.9566</td>
<td>114.5981</td>
<td>-18.984</td>
<td>-18.836</td>
<td>0.070</td>
<td>-18.795</td>
</tr>
<tr>
<td>-24.9780</td>
<td>114.6515</td>
<td>-18.816</td>
<td>-18.696</td>
<td>0.071</td>
<td>-18.669</td>
</tr>
<tr>
<td>-25.0060</td>
<td>114.7010</td>
<td>-18.693</td>
<td>-18.592</td>
<td>0.071</td>
<td>-18.566</td>
</tr>
<tr>
<td>-24.9776</td>
<td>114.8246</td>
<td>-18.200</td>
<td>-18.082</td>
<td>0.074</td>
<td>-18.103</td>
</tr>
<tr>
<td>-24.9955</td>
<td>114.8497</td>
<td>-18.164</td>
<td>-18.048</td>
<td>0.074</td>
<td>-18.055</td>
</tr>
<tr>
<td>-25.0769</td>
<td>115.0197</td>
<td>-17.848</td>
<td>-17.775</td>
<td>0.076</td>
<td>-17.713</td>
</tr>
<tr>
<td>-25.0574</td>
<td>115.2331</td>
<td>-17.532</td>
<td>-17.457</td>
<td>0.079</td>
<td>-17.330</td>
</tr>
<tr>
<td>-25.1368</td>
<td>115.4560</td>
<td>-17.074</td>
<td>-16.978</td>
<td>0.081</td>
<td>-16.906</td>
</tr>
<tr>
<td>latitude</td>
<td>longitude</td>
<td>$N_{GPS}$</td>
<td>$N_{GZS}$</td>
<td>$N_{N}$</td>
<td>$N_{New}$</td>
</tr>
<tr>
<td>----------</td>
<td>-----------</td>
<td>-----------</td>
<td>-----------</td>
<td>---------</td>
<td>-----------</td>
</tr>
<tr>
<td>-35.1105</td>
<td>149.2402</td>
<td>19.796</td>
<td>19.886</td>
<td>0.465</td>
<td>20.509</td>
</tr>
<tr>
<td>-35.1108</td>
<td>149.2463</td>
<td>19.788</td>
<td>19.899</td>
<td>0.467</td>
<td>20.526</td>
</tr>
<tr>
<td>-35.1109</td>
<td>149.2468</td>
<td>19.813</td>
<td>19.900</td>
<td>0.467</td>
<td>20.527</td>
</tr>
<tr>
<td>-35.1138</td>
<td>149.2342</td>
<td>19.780</td>
<td>19.865</td>
<td>0.465</td>
<td>20.486</td>
</tr>
<tr>
<td>-35.1190</td>
<td>149.2272</td>
<td>19.773</td>
<td>19.837</td>
<td>0.466</td>
<td>20.456</td>
</tr>
<tr>
<td>-35.1192</td>
<td>149.2272</td>
<td>19.805</td>
<td>19.836</td>
<td>0.466</td>
<td>20.454</td>
</tr>
<tr>
<td>-35.1232</td>
<td>149.0459</td>
<td>19.214</td>
<td>19.254</td>
<td>0.478</td>
<td>19.982</td>
</tr>
<tr>
<td>-35.1289</td>
<td>149.0476</td>
<td>19.219</td>
<td>19.241</td>
<td>0.478</td>
<td>19.974</td>
</tr>
<tr>
<td>-35.1478</td>
<td>149.0547</td>
<td>19.212</td>
<td>19.218</td>
<td>0.480</td>
<td>19.967</td>
</tr>
<tr>
<td>-35.1539</td>
<td>149.0587</td>
<td>19.228</td>
<td>19.216</td>
<td>0.478</td>
<td>19.967</td>
</tr>
<tr>
<td>-35.1628</td>
<td>149.0645</td>
<td>19.213</td>
<td>19.213</td>
<td>0.478</td>
<td>19.969</td>
</tr>
<tr>
<td>-35.1652</td>
<td>149.0666</td>
<td>19.223</td>
<td>19.215</td>
<td>0.477</td>
<td>19.971</td>
</tr>
<tr>
<td>-35.1736</td>
<td>149.2552</td>
<td>19.701</td>
<td>19.745</td>
<td>0.471</td>
<td>20.372</td>
</tr>
<tr>
<td>-35.1757</td>
<td>149.0728</td>
<td>19.215</td>
<td>19.215</td>
<td>0.480</td>
<td>19.974</td>
</tr>
<tr>
<td>-35.1774</td>
<td>149.0724</td>
<td>19.235</td>
<td>19.208</td>
<td>0.480</td>
<td>19.970</td>
</tr>
<tr>
<td>-35.1806</td>
<td>149.1325</td>
<td>19.423</td>
<td>19.423</td>
<td>0.477</td>
<td>20.106</td>
</tr>
<tr>
<td>-35.1815</td>
<td>149.2362</td>
<td>19.641</td>
<td>19.681</td>
<td>0.472</td>
<td>20.316</td>
</tr>
<tr>
<td>-35.1836</td>
<td>149.2582</td>
<td>19.685</td>
<td>19.726</td>
<td>0.470</td>
<td>20.351</td>
</tr>
<tr>
<td>-35.1867</td>
<td>149.2581</td>
<td>19.659</td>
<td>19.718</td>
<td>0.470</td>
<td>20.343</td>
</tr>
<tr>
<td>-35.1919</td>
<td>149.2607</td>
<td>19.677</td>
<td>19.710</td>
<td>0.472</td>
<td>20.336</td>
</tr>
<tr>
<td>-35.2087</td>
<td>149.0204</td>
<td>18.981</td>
<td>18.933</td>
<td>0.490</td>
<td>19.787</td>
</tr>
<tr>
<td>-35.2150</td>
<td>149.1810</td>
<td>19.453</td>
<td>19.453</td>
<td>0.478</td>
<td>20.156</td>
</tr>
<tr>
<td>-35.2168</td>
<td>149.1816</td>
<td>19.430</td>
<td>19.448</td>
<td>0.478</td>
<td>20.154</td>
</tr>
<tr>
<td>-35.2171</td>
<td>149.1824</td>
<td>19.438</td>
<td>19.448</td>
<td>0.477</td>
<td>20.154</td>
</tr>
<tr>
<td>-35.2219</td>
<td>149.1747</td>
<td>19.399</td>
<td>19.412</td>
<td>0.480</td>
<td>20.136</td>
</tr>
<tr>
<td>-35.2243</td>
<td>149.1679</td>
<td>19.351</td>
<td>19.388</td>
<td>0.481</td>
<td>20.121</td>
</tr>
<tr>
<td>-35.2264</td>
<td>149.1611</td>
<td>19.337</td>
<td>19.364</td>
<td>0.483</td>
<td>20.106</td>
</tr>
<tr>
<td>-35.2270</td>
<td>149.1566</td>
<td>19.349</td>
<td>19.349</td>
<td>0.484</td>
<td>20.098</td>
</tr>
<tr>
<td>-35.2322</td>
<td>149.1549</td>
<td>19.296</td>
<td>19.323</td>
<td>0.485</td>
<td>20.087</td>
</tr>
<tr>
<td>-35.2325</td>
<td>149.1553</td>
<td>19.296</td>
<td>19.324</td>
<td>0.485</td>
<td>20.089</td>
</tr>
<tr>
<td>-35.2332</td>
<td>149.1563</td>
<td>19.303</td>
<td>19.324</td>
<td>0.485</td>
<td>20.089</td>
</tr>
<tr>
<td>-35.2341</td>
<td>149.1577</td>
<td>19.309</td>
<td>19.325</td>
<td>0.485</td>
<td>20.090</td>
</tr>
<tr>
<td>-35.2351</td>
<td>149.1591</td>
<td>19.295</td>
<td>19.325</td>
<td>0.484</td>
<td>20.091</td>
</tr>
<tr>
<td>-35.2360</td>
<td>149.1605</td>
<td>19.305</td>
<td>19.326</td>
<td>0.484</td>
<td>20.092</td>
</tr>
<tr>
<td>-35.2370</td>
<td>149.1619</td>
<td>19.313</td>
<td>19.326</td>
<td>0.484</td>
<td>20.094</td>
</tr>
<tr>
<td>-35.2380</td>
<td>149.1633</td>
<td>19.331</td>
<td>19.327</td>
<td>0.484</td>
<td>20.095</td>
</tr>
<tr>
<td>-35.2380</td>
<td>149.1813</td>
<td>19.349</td>
<td>19.368</td>
<td>0.459</td>
<td>20.103</td>
</tr>
<tr>
<td>-35.2697</td>
<td>149.1588</td>
<td>19.252</td>
<td>19.190</td>
<td>0.468</td>
<td>20.020</td>
</tr>
<tr>
<td>-35.2865</td>
<td>148.9222</td>
<td>18.469</td>
<td>18.603</td>
<td>0.515</td>
<td>19.367</td>
</tr>
<tr>
<td>-35.3073</td>
<td>149.2594</td>
<td>19.402</td>
<td>19.402</td>
<td>0.476</td>
<td>20.108</td>
</tr>
<tr>
<td>-35.3075</td>
<td>148.8101</td>
<td>18.529</td>
<td>18.529</td>
<td>0.442</td>
<td>19.069</td>
</tr>
<tr>
<td>-35.3121</td>
<td>149.2466</td>
<td>19.385</td>
<td>19.340</td>
<td>0.490</td>
<td>20.097</td>
</tr>
<tr>
<td>-35.3169</td>
<td>149.2436</td>
<td>19.348</td>
<td>19.315</td>
<td>0.491</td>
<td>20.084</td>
</tr>
<tr>
<td>-35.3196</td>
<td>149.2434</td>
<td>19.308</td>
<td>19.307</td>
<td>0.492</td>
<td>20.079</td>
</tr>
<tr>
<td>-35.3232</td>
<td>149.2403</td>
<td>19.284</td>
<td>19.286</td>
<td>0.492</td>
<td>20.068</td>
</tr>
<tr>
<td>-35.3272</td>
<td>149.2379</td>
<td>19.301</td>
<td>19.266</td>
<td>0.492</td>
<td>20.058</td>
</tr>
<tr>
<td>-35.3306</td>
<td>149.2377</td>
<td>19.284</td>
<td>19.256</td>
<td>0.494</td>
<td>20.052</td>
</tr>
</tbody>
</table>
Table I-3 (continued)

<table>
<thead>
<tr>
<th>Latitude</th>
<th>Longitude</th>
<th>NGFS</th>
<th>NO200</th>
<th>NT</th>
<th>Nnew</th>
</tr>
</thead>
<tbody>
<tr>
<td>-35.3308</td>
<td>148.8266</td>
<td>18.621</td>
<td>18.534</td>
<td>0.510</td>
<td>19.167</td>
</tr>
<tr>
<td>-35.3309</td>
<td>148.8309</td>
<td>18.573</td>
<td>18.538</td>
<td>0.513</td>
<td>19.175</td>
</tr>
<tr>
<td>-35.3385</td>
<td>149.2346</td>
<td>19.234</td>
<td>19.234</td>
<td>0.495</td>
<td>20.033</td>
</tr>
<tr>
<td>-35.3466</td>
<td>149.2987</td>
<td>19.472</td>
<td>19.472</td>
<td>0.474</td>
<td>20.065</td>
</tr>
<tr>
<td>-35.3736</td>
<td>148.8040</td>
<td>18.593</td>
<td>18.551</td>
<td>0.476</td>
<td>19.144</td>
</tr>
<tr>
<td>-35.3854</td>
<td>148.8050</td>
<td>18.592</td>
<td>18.546</td>
<td>0.470</td>
<td>19.146</td>
</tr>
<tr>
<td>-35.4328</td>
<td>148.9480</td>
<td>18.538</td>
<td>18.538</td>
<td>0.539</td>
<td>19.315</td>
</tr>
<tr>
<td>-35.4361</td>
<td>148.9464</td>
<td>18.544</td>
<td>18.536</td>
<td>0.541</td>
<td>19.312</td>
</tr>
<tr>
<td>-35.4467</td>
<td>148.9646</td>
<td>18.541</td>
<td>18.538</td>
<td>0.534</td>
<td>19.297</td>
</tr>
<tr>
<td>-35.4496</td>
<td>148.9766</td>
<td>18.540</td>
<td>18.542</td>
<td>0.535</td>
<td>19.304</td>
</tr>
<tr>
<td>-35.4499</td>
<td>148.9884</td>
<td>18.547</td>
<td>18.547</td>
<td>0.532</td>
<td>19.311</td>
</tr>
<tr>
<td>-35.4515</td>
<td>148.9830</td>
<td>18.532</td>
<td>18.544</td>
<td>0.533</td>
<td>19.305</td>
</tr>
<tr>
<td>-35.4516</td>
<td>148.9677</td>
<td>18.549</td>
<td>18.556</td>
<td>0.535</td>
<td>19.292</td>
</tr>
<tr>
<td>-35.4521</td>
<td>148.9886</td>
<td>18.536</td>
<td>18.546</td>
<td>0.533</td>
<td>19.308</td>
</tr>
<tr>
<td>-35.4571</td>
<td>148.9732</td>
<td>18.533</td>
<td>18.536</td>
<td>0.536</td>
<td>19.290</td>
</tr>
<tr>
<td>-35.4622</td>
<td>148.9689</td>
<td>18.516</td>
<td>18.531</td>
<td>0.536</td>
<td>19.277</td>
</tr>
<tr>
<td>-35.5423</td>
<td>149.1357</td>
<td>18.523</td>
<td>18.658</td>
<td>0.522</td>
<td>19.393</td>
</tr>
<tr>
<td>-35.5439</td>
<td>149.1342</td>
<td>18.653</td>
<td>18.653</td>
<td>0.523</td>
<td>19.390</td>
</tr>
<tr>
<td>-35.5552</td>
<td>149.1301</td>
<td>18.506</td>
<td>18.621</td>
<td>0.527</td>
<td>19.375</td>
</tr>
<tr>
<td>-35.5608</td>
<td>149.1216</td>
<td>18.487</td>
<td>18.608</td>
<td>0.526</td>
<td>19.369</td>
</tr>
<tr>
<td>-35.5719</td>
<td>149.1342</td>
<td>18.502</td>
<td>18.583</td>
<td>0.526</td>
<td>19.358</td>
</tr>
<tr>
<td>-35.5803</td>
<td>149.1359</td>
<td>18.510</td>
<td>18.563</td>
<td>0.528</td>
<td>19.351</td>
</tr>
<tr>
<td>-35.5944</td>
<td>149.1381</td>
<td>18.476</td>
<td>18.531</td>
<td>0.519</td>
<td>19.327</td>
</tr>
<tr>
<td>-35.8176</td>
<td>148.8163</td>
<td>18.566</td>
<td>18.562</td>
<td>0.522</td>
<td>19.331</td>
</tr>
<tr>
<td>-35.8184</td>
<td>148.8258</td>
<td>18.568</td>
<td>18.560</td>
<td>0.521</td>
<td>19.323</td>
</tr>
<tr>
<td>-35.8212</td>
<td>148.8407</td>
<td>18.574</td>
<td>18.566</td>
<td>0.520</td>
<td>19.314</td>
</tr>
<tr>
<td>-35.8224</td>
<td>148.8487</td>
<td>18.567</td>
<td>18.561</td>
<td>0.520</td>
<td>19.310</td>
</tr>
<tr>
<td>-35.8224</td>
<td>148.8334</td>
<td>18.563</td>
<td>18.567</td>
<td>0.522</td>
<td>19.317</td>
</tr>
<tr>
<td>-35.8255</td>
<td>148.8574</td>
<td>18.574</td>
<td>18.553</td>
<td>0.519</td>
<td>19.303</td>
</tr>
<tr>
<td>-35.8280</td>
<td>148.8634</td>
<td>18.559</td>
<td>18.547</td>
<td>0.520</td>
<td>19.300</td>
</tr>
<tr>
<td>-35.8318</td>
<td>148.8711</td>
<td>18.562</td>
<td>18.539</td>
<td>0.519</td>
<td>19.292</td>
</tr>
<tr>
<td>-35.8357</td>
<td>148.8770</td>
<td>18.790</td>
<td>18.512</td>
<td>0.518</td>
<td>19.288</td>
</tr>
<tr>
<td>-35.8649</td>
<td>148.9930</td>
<td>18.460</td>
<td>18.405</td>
<td>0.503</td>
<td>19.122</td>
</tr>
<tr>
<td>-35.8852</td>
<td>148.9856</td>
<td>18.422</td>
<td>18.377</td>
<td>0.499</td>
<td>19.112</td>
</tr>
<tr>
<td>-35.8879</td>
<td>148.9858</td>
<td>18.418</td>
<td>18.373</td>
<td>0.498</td>
<td>19.109</td>
</tr>
<tr>
<td>-35.8914</td>
<td>148.9815</td>
<td>18.394</td>
<td>18.371</td>
<td>0.502</td>
<td>19.117</td>
</tr>
<tr>
<td>-35.8960</td>
<td>148.9767</td>
<td>18.381</td>
<td>18.367</td>
<td>0.501</td>
<td>19.121</td>
</tr>
<tr>
<td>-35.8969</td>
<td>148.9666</td>
<td>18.370</td>
<td>18.370</td>
<td>0.477</td>
<td>19.114</td>
</tr>
<tr>
<td>-35.9017</td>
<td>148.9722</td>
<td>18.353</td>
<td>18.362</td>
<td>0.499</td>
<td>19.120</td>
</tr>
<tr>
<td>Latitude</td>
<td>Longitude</td>
<td>$N_{gps}$</td>
<td>$N_{deg}$</td>
<td>$N_{eq}$</td>
<td>$N_{sec}$</td>
</tr>
<tr>
<td>----------</td>
<td>------------</td>
<td>-----------</td>
<td>-----------</td>
<td>----------</td>
<td>-----------</td>
</tr>
<tr>
<td>-38.3536</td>
<td>144.9507</td>
<td>3.168</td>
<td>3.069</td>
<td>0.299</td>
<td>3.020</td>
</tr>
<tr>
<td>-36.1178</td>
<td>140.9782</td>
<td>1.307</td>
<td>1.339</td>
<td>0.217</td>
<td>1.477</td>
</tr>
<tr>
<td>-38.1513</td>
<td>144.6121</td>
<td>2.888</td>
<td>2.651</td>
<td>0.301</td>
<td>3.120</td>
</tr>
<tr>
<td>-37.0103</td>
<td>147.6647</td>
<td>13.873</td>
<td>13.346</td>
<td>0.587</td>
<td>13.739</td>
</tr>
<tr>
<td>-37.6468</td>
<td>148.9789</td>
<td>10.188</td>
<td>9.404</td>
<td>0.415</td>
<td>10.210</td>
</tr>
<tr>
<td>-38.6624</td>
<td>143.4516</td>
<td>0.500</td>
<td>0.649</td>
<td>0.234</td>
<td>0.378</td>
</tr>
<tr>
<td>-37.8403</td>
<td>140.7560</td>
<td>-4.042</td>
<td>-3.805</td>
<td>0.189</td>
<td>-3.569</td>
</tr>
<tr>
<td>-35.9713</td>
<td>143.6198</td>
<td>6.144</td>
<td>5.998</td>
<td>0.282</td>
<td>5.912</td>
</tr>
<tr>
<td>-38.2318</td>
<td>146.9400</td>
<td>5.361</td>
<td>5.194</td>
<td>0.387</td>
<td>5.347</td>
</tr>
<tr>
<td>-36.8790</td>
<td>144.7075</td>
<td>6.713</td>
<td>6.274</td>
<td>0.332</td>
<td>6.889</td>
</tr>
<tr>
<td>-36.6125</td>
<td>142.3582</td>
<td>3.430</td>
<td>3.292</td>
<td>0.248</td>
<td>3.453</td>
</tr>
<tr>
<td>-35.2534</td>
<td>142.8952</td>
<td>5.721</td>
<td>5.751</td>
<td>0.259</td>
<td>5.950</td>
</tr>
<tr>
<td>-38.8500</td>
<td>145.9666</td>
<td>3.174</td>
<td>3.435</td>
<td>0.298</td>
<td>3.327</td>
</tr>
<tr>
<td>-36.3628</td>
<td>145.6962</td>
<td>9.148</td>
<td>8.749</td>
<td>0.388</td>
<td>9.180</td>
</tr>
<tr>
<td>-37.5749</td>
<td>146.1905</td>
<td>9.912</td>
<td>9.630</td>
<td>0.422</td>
<td>9.533</td>
</tr>
<tr>
<td>-36.8543</td>
<td>146.0624</td>
<td>9.783</td>
<td>9.083</td>
<td>0.439</td>
<td>9.477</td>
</tr>
<tr>
<td>-36.0836</td>
<td>147.0966</td>
<td>12.718</td>
<td>11.973</td>
<td>0.490</td>
<td>12.377</td>
</tr>
<tr>
<td>-37.5504</td>
<td>143.0848</td>
<td>3.589</td>
<td>3.353</td>
<td>0.259</td>
<td>3.359</td>
</tr>
</tbody>
</table>