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Structural damage identification using improved Jaya algorithm based on sparse regularization and Bayesian inference

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Abstract: Structural damage identification can be considered as an optimization problem, by defining an appropriate objective function relevant to structural parameters to be identified with optimization techniques. This paper proposes a new heuristic algorithm, named improved Jaya (I-Jaya) algorithm, for structural damage identification with the modified objective function based on sparse regularization and Bayesian inference. To improve the global optimization capacity and robustness of the original Jaya algorithm, a clustering strategy is employed to replace solutions with low-quality objective values and a new updated equation is used for the best-so-far solution. The objective function that is sensitive and robust for effective and reliable damage identification is developed through sparse regularization and Bayesian inference and used for optimization analysis with the proposed I-Jaya algorithm. Benchmark tests are conducted to verify the improvement in the developed algorithm. Numerical studies on a truss structure and experimental validations on an experimental reinforced concrete bridge model are performed to verify the developed approach. A limited quantity of modal data, which is distinctively less than the number of unknown system parameters, are used for structural damage identification. Significant measurement noise effect and modelling errors are considered. Damage identification results demonstrate that the proposed method based on the I-Jaya algorithm and the modified objective function based on sparse

regularization and Bayesian inference can provide accurate and reliable damage identification, indicating the proposed method is a promising approach for structural damage detection using data with significant uncertainties and limited measurement information.

Keywords: Improved Jaya algorithm, Bayesian interference, Sparse regularization, Structural damage identification, Uncertainty, Measurement noise.

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1. Introduction

Conducting damage identification and quantification of structures based on measured vibration data is one of the most significant research topics in the area of structural health monitoring (SHM), because it is relevant to assessing the service performance and evaluating the integrity of structures [1]. When structures have damages, alternations are observed in the dynamic vibration characteristics. Therefore, numerous methods have been developed for structural damage identification based on the changes in structural vibration characteristics [2-3].

Basically, these methods can be categorized into two types, relying on the fact that structural damage identification is performed in the frequency domain or the time domain. Frequency domain based methods are developed to identify damages by using structural modal information, such as natural frequencies, mode shapes, damping ratios and other frequency domain data. Pandey and Biswas [4] used the flexibility matrices for damage identification. Shi and Law [5] developed the modal strain energy ratio to locate the structural damages. Yan et al. [6-7] used Principle Component Analysis (PCA) to analyze structural modal data for distinguishing the changes in vibration characteristics due to environmental variations or structural damage. Numerical and experimental studies illustrated that the proposed method can be effective for the linear and nonlinear structures. Furthermore, the spectral approach was also widely applied to address structural damage quantification, especially for nonlinear systems [8].

On the other hand, structural damage identification methods in the time domain have been developed rapidly in the recent years. Lu and Wang [9] proposed an enhanced sensitivity method to perform damage identification, in which a trust-region restriction was introduced to improve the performance of the traditional sensitivity approach. Hu et al. [10] developed a method using the homotopy continuation algorithm to identify the cracks in beam structures, in which acceleration responses were used to formulate the objective function. Li et al. [11] developed a damage identification and optimal sensor placement method for structures under traffic-induced vibrations, based on response reconstruction in the time domain. Recently, the time domain methods have been also developed to conduct the identification of nonlinear structures. Yang et al. [12] developed an adaptive Extended Kalman Filter (EKF) approach to identify the damage in both the linear and nonlinear structures. Xie and Feng [13] applied the Iterated Unscented Kalman Filter (IUKF) for highly nonlinear structures. Experimental results demonstrated that IUKF can be used to provide better state estimation and parameter identification results than Unscented Kalman Filter (UKF). For damage identification in initially nonlinear systems, Shiki et al. [14] used a discrete Volterra model to separate the linear and nonlinear components of the dynamic responses of a system. Afterwards, hypothesis tests were introduced to detect variations in the statistical properties of the damage features. Villani et al [15] adopted the stochastic Volterra series to conduct damage identification for uncertain nonlinear systems, in which the uncertainties were simulated by the variation posed in the linear stiffness and damping coefficient.

However, most of the above mentioned methods require a good guess of the initial system parameters and an accurate estimation of the gradients. Furthermore, difficulties arise when utilizing these methods for the identification of large scale structures when only few measurement data is available. Regularization in the solution would be essential to ensure that the identification results are physically meaningful. Considering that structural damage identification could be viewed as an optimization problem [16], computational intelligence techniques are developed to perform the optimization in structural damage identification, such as the genetic algorithms (GAs), the particle swarm optimizer (PSO), the artificial bee colony algorithms (ABCs), the differential evolution (DE) algorithms, the artificial neural network (ANN), the support vector machine (SVM) and other machine learning methods. These intelligence methods generally make predictions via data instead of the specific formulas. Therefore, they could not only avoid the mentioned shortcomings (requiring good initial values and gradient information), but also enable to perform identification of large-scale and complex structures [17]. Wang [18] developed using the hybrid GA with the Gaussian-Newton method to identify the parameters of both linear and nonlinear structural systems. Guo and Li [19] developed a two-stage damage identification method based on the evidence fusion along with the improved PSO. Later, Chen and Yu [20] employed the PSO integrated with Nelder-Mead method to tackle the damage identification problem. Sun et al. [21] constructed a modified ABC algorithm to perform the identification of structural parameters, in which a nonlinear factor used for improving the convergence performance was introduced to achieve the balance between the global and local searches. Ding et al. [22-23] adopted ABC to identify structural damages and cracks by using the objective function based on natural frequencies and the modal assurance criteria (MAC). Tang et al. [24] proposed DE to identify structural parameters with and without considering noise contamination in the measurement data. Padil et al. [25] demonstrated that ANN is a good choice to solve the damage identification problem considering uncertainties. Bornn et al. [26] developed an approach using the autoregressive SVM to detect the damage in initially nonlinear systems. Santos et al. [27] presented four kernel-based algorithms for damage identification under varying operational and environmental conditions. From the above studies, it can be found that these computational intelligence approaches are promising tools for structural identification, however, challenges still exist, such as

(a) In some studies, the target structures used for investigation have a small number of elements. The uncertainty effect on the final identification results is rarely investigated; and

(b) The performance and robustness of algorithms for the scenarios when only a limited number of measurement data are available and at the same time the data contain significant noise, need to be improved.

Recently a new computational intelligence method, namely Jaya algorithm [28-29], has been developed. Compared with the above-mentioned computational intelligence methods [17-25], the distinct feature of the Jaya algorithm is that there are no special controlling parameters in the algorithm. In contrast for many other methods, GA needs a proper setting of crossover probability, mutation rate and selection operator, and ABC needs proper quantities of onlooker bees, scout bees and parameter 'limit'. Furthermore, compared with other gradient-based algorithms [4-5], the Jaya algorithm has the following superiorities: (i) It is free from sensitivity analysis and initial guess of the parameters; and (ii) It does not require gradient information. When performing the damage identification of structures with a large number of elements, the gradient information may be difficult to obtain or the calculation is time consuming due to the significant computational demand with a large-scale system, which restricts the potential applications of these gradient-based methods. Therefore, it is interesting to develop and extend the Jaya algorithm for structural damage identification. Furthermore, to address the two challenges as mentioned above, modifications are introduced into the standard Jaya algorithm to enhance its global optimization ability and a better objective function that is more robust to identify structural parameters with limited measurement information is proposed. These are the two main contributions of this study.

For addressing the first challenge, a relatively complex truss structure is used in the numerical study. The uncertainty effects considered in this study include the uncertainties in the computational model parameters and the modeling errors. The former one results from a mathematical modeling process of the designed structure, which has parameters that could be subjected to certain level of statistical variations. The latter one denotes that the modelling process could introduce modelling errors, which is widely known as the modeling uncertainties [30-31]. To overcome the influences of these uncertain effects, following the study in Ref. [32], variations in each element stiffness parameters are introduced to simulate the uncertainties. The variation is modelled as a random Gaussian distribution vector with a mean value of 0 and a specific level of standard deviation.

For addressing the second challenge as mentioned, when developing and applying optimization methods for damage identification, studies on developing more reasonable objective functions that are more robust and stable in optimization analysis for damage identification are conducted. To achieve this purpose, one way is to introduce the regularization technique to reform the objective function. Recently, the sparse regularization techniques with the enforcement of the sparsity constraint on the damage locations have been widely investigated and promising results are obtained, since damages are often observed at a few locations while the majority of elements remain intact [33-34]. Furthermore, the traditional objective functions [22-23] are usually ill-posed, and introducing the sparse regularization constraint on the damage identification is beneficial to overcome the ill-posedness in the inverse problems [35]. Another possible way to tackle this challenge is to employ the probabilistic analysis, i.e. based on the Bayesian inference. It considers the complete information relevant to the measured data for statistical inference with an appropriate likelihood function [36]. Bayesian-based methods have been developed for damage identification. For example, Beck et al. [37] presented a Bayesian statistical framework for structural identification and adopted this theory to perform continuous online identification. Later, Bayesian spectral density approaches, Bayesian fast Fourier transform (FFT) methods [38-39], Bayesian-based Monte Carlo method [40] have been further developed for structural damage identification. The results in previous studies [20, 36] demonstrated that the Bayesian inference can be used to enhance the robustness of damage identification. Therefore, to improve the identification with a limited number of measurements of a significant noise effect, a new objective function is proposed by using incomplete modal data and the penalty items considering the sparse regularization and Bayesian inference.

This paper proposes an improved Jaya algorithm (I-Jaya) to conduct damage identification of structures by using vibration measurement data. To enhance the capacity of the developed methodology for the identification of large-scale structures, two modifications are developed based on the standard Jaya algorithm to enhance its global search ability. 1% variation [32] is introduced into the elemental stiffness parameters to simulate the uncertainties in the structure. To improve the identification with a limited number of measurements of a significant noise effect, a new objective function is proposed by using incomplete modal data and the penalty items considering the sparse regularization and Bayesian inference. Classical mathematical benchmarks are utilized to validate the accuracy and improvement of the proposed approach. Numerical investigations on a 121-bar truss structure are performed to demonstrate the accuracy of the developed algorithm with the use of the modified objective function. Experimental validations on a reinforced concrete bridge are conducted to demonstrate the performance of the proposed method.

2. Theoretical background

2.1 Damage identification of structures

Changes in structural system parameters, i.e. stiffness, mass and damping, would introduce the alterations in structural vibration properties. Hence SDI could be conducted based on this fact by using vibration measurement data. Vibration characteristics, such as frequencies and mode shapes of a structure without considering the damping, could be obtained by solving the eigenvalue problem

$$(\mathbf{K}-\omega_i^2\mathbf{M})\cdot\mathbf{\Phi_i}=0 \tag{1}$$

where $\mathbf{K}$ and $\mathbf{M}$ represent the system stiffness and mass matrices, respectively; ω_i and $\mathbf{\Phi_i}$ denote the ith natural frequency and the corresponding mode shape, respectively.

In this study, structural damage is assumed to be only related to the stiffness reduction, since the mass alteration of a structure could be easily inspected [41]. In this case, structural damage would be characterized via a scalar stiffness reduction variable for each element $\alpha_h(h=1,2,...,Nel)$ with the value between 0 and 1 as follows

$$\mathbf{K_d}=\sum_{h=1}^{Nel}(1-\alpha_h)\cdot\mathbf{k_{eh}} \tag{2}$$

where $\mathbf{k_{eh}}$ represents the h th elemental stiffness matrix under the undamaged state; Nel denotes the number of total elements of a structure; $\mathbf{K_d}$ represents the structural stiffness matrix under the damaged state; α_h denotes the elemental stiffness reduction parameter to be identified. It shall be noted that $\alpha_h=1$ implies that this element is totally damaged, and $\alpha_h=0$ means that the element is intact.

The traditional objective function, denoted as f_{obj1}, is defined based on the alterations of natural frequencies and Modal Assurance Criterion (MAC), which can be given as [20, 22-23]

$$\boldsymbol{\alpha}^*=\arg\min_{\boldsymbol{\alpha}} f_{obj1}(\boldsymbol{\alpha})=\arg\min_{\boldsymbol{\alpha}}(\sum_{i=1}^{NF}\Delta\omega_i^2+\sum_{i=1}^{NM}(1-MAC_i)) \tag{3}$$

with

$$\Delta\omega_i = \frac{|\omega_i^c - \omega_i^m|}{\omega_i^m} \tag{4}$$

$$MAC_i = \frac{(\mathbf{\Phi_i^{cT}} \cdot \mathbf{\Phi_i^m})^2}{\|\mathbf{\Phi_i^c}\|^2 \|\mathbf{\Phi_i^m}\|^2} \tag{5}$$

where ω_i^c and $\mathbf{\Phi_i^c}$ represent the i th calculated natural frequency and mode shape from the finite element model analysis, respectively; ω_i^m and $\mathbf{\Phi_i^m}$ are the corresponding measured natural frequency and mode shape, respectively. NF and NM represent the order numbers of natural frequencies and mode shapes, respectively. The calculated modal data are acquired by using the stiffness parameters $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_{Nel}]$ with the finite element analysis. Generally speaking, SDI is treated as an ill-posed problem with the searching parameters that may have multiple local optimal points [22]. The optimization techniques can be used for identifying the optimal set of parameters that could minimize the objective function. When the input data is limited or even less than the number of unknown parameters to be identified and the data is contaminated with the significant measurement noise, the damage identification becomes much more difficult. To overcome these challenges, it is emerging to investigate and develop robust and powerful algorithms with proper objective functions, which may improve the identification of the complex structures.

2.2 Proposed objective function

2.2.1 The objective function based on sparse regularization

In real situations, structural damages happen usually at a few locations [42]. Therefore the damage vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_{Nel}]$

at least close to zero, except the damaged elements with non-zero entries. When the number of measured data is less than the total number of unknown parameters in the inverse identification, Equation (3) is underdetermined and ill-posed. Therefore, the l_1 regularization technique [33] can be utilized to help solve the underdetermined inverse problem. The objective function based on sparse regularization, denoted as *obj2*, can be defined as

$$\boldsymbol{\alpha}^* = \arg\min_{\boldsymbol{\alpha}} f_{obj2}(\boldsymbol{\alpha}) = \arg\min_{\boldsymbol{\alpha}}(\sum_{i=1}^{NF} \Delta\omega_i^2 + \sum_{i=1}^{NM}(1 - MAC_i) + \lambda \|\boldsymbol{\alpha}\|_1) \tag{6}$$

where $\lambda > 0$ is the regularization parameter and $\|\boldsymbol{\alpha}\|_1$ denotes the l_1 norm of the solution, namely, $\|\boldsymbol{\alpha}\|_1 = \sum_{h=1}^{Nel} |\alpha_h|$. It should be noted that a small λ would pose a higher penalty on the residual term, resulting in an over-fitting solution. Conversely, for a large λ value, it would loss data fidelity. Therefore, the discrepancy principle (DP) rule [33] is employed here to select the optimal regularization parameter λ.

2.2.2 The objective function based on Bayesian inference

Vibration measurement data are usually polluted with the environmental noise, which could be considered as a zero-mean Gaussian white-noise in numerical simulations. The noisy response can be described as [20]

$$\mathbf{X}_{noise} = \mathbf{X}(1 + \varepsilon\mathbf{R}) \tag{7}$$

where $\mathbf{X}_{noise}$ and $\mathbf{X}$ are the noisy and original response vectors, respectively; ε denotes the noise level ranging from 0 to 100%, while $\mathbf{R}$ is a random vector with the standard normal distribution $N(0,1)$.

To improve the capacity of the developed algorithm against the noise effect in the optimization process, the Bayesian inference is introduced to modify the objective function. The

$\boldsymbol{\theta}$) can be obtained via Bayes' theorem [20]

$$p(\boldsymbol{\theta} \mid \mathbf{D}) = c \cdot p(\mathbf{D} \mid \boldsymbol{\theta}) p(\boldsymbol{\theta}) \tag{8}$$

where $p(\boldsymbol{\theta} \mid \mathbf{D})$ represents the PDF of model parameters $\boldsymbol{\theta}$ given the modal data $\mathbf{D}$, $p(\mathbf{D} \mid \boldsymbol{\theta})$ represents the likelihood function given the model parameter $\boldsymbol{\theta}$, and $p(\boldsymbol{\theta})$ denotes the prior PDF of model parameters $\boldsymbol{\theta}$ based on observations and/or modelling assumptions. Specifically, it ought to be noted that modal data $\mathbf{D}$ means the real measured data, such as frequencies and mode shapes. In the Bayesian theory, these measured data are served to obtain the posterior probability density function of the model parameters $\boldsymbol{\theta}$. In this study for structural damage identification, the model parameters denote the element stiffness parameters vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, ..., \alpha_{Nel}]$. Furthermore, the prior distribution of model parameters $\boldsymbol{\theta}$ is assumed as a uniform distribution, which means that their PDFs are a series of constants [20]. c is a constant which enables the integral of $p(\boldsymbol{\theta} \mid \mathbf{D})$ to be 1. Supposing that the Bayesian inference is applied on natural frequencies and taking $\mathbf{D} = [\mathbf{D}_1, \mathbf{D}_2, ..., \mathbf{D}_{N_s}]$ as observed modal data with N_s samples, and $\mathbf{D}_s = [\omega_{1,s}, \omega_{2,s}, ..., \omega_{i,s}]$ denoting the natural frequencies in the s th observation or measurement. The likelihood functions of the modal data are assumed to be independent, and the principle of the maximum entropy is employed as a basis to assume Gaussian distributions for these modal data [20]. Based on this assumption, the PDF of any frequency parameter ($\omega_{i,s}$) can be obtained as

$$p(\omega_{i,s} \mid \boldsymbol{\theta}) = c_1 \exp[-\frac{(\omega_{i,s} - \omega_i^c)^2}{2\sigma_i^2}] \tag{9}$$

where $\omega_{i,s}$ represents the ith frequency in the sth measurement, ω_i^c denotes the ith calculated frequency, and σ_i^2 represents the variance of the ith frequency and can be calculated as

$$\sigma_i^2 = \frac{1}{N_s - 1}\sum_{i=1}^{NF}(\omega_{i,s} - \varpi_i)^2 \tag{10}$$

where ϖ_i represents the mean value of the ith natural frequency.

Since it is assumed the testing obtained modal data are independent, the likelihood in Eq. (8) can be calculated as

$$p(\mathbf{D}\,|\,\boldsymbol{\theta}) = \prod_{s=1}^{N_s} p(\mathbf{D}_s\,|\,\boldsymbol{\theta}) = \prod_{s=1}^{N_s}(\prod_{i=1}^{NF} p(\omega_{i,s}\,|\,\boldsymbol{\theta})) \tag{11}$$

When the prior distribution of natural frequencies is considered as the uniform distribution, substituting Eq. (11) to Eq. (8) can have the final form of $p(\boldsymbol{\theta}\,|\,\mathbf{D})$. It can be calculated as

$$p(\boldsymbol{\theta}\,|\,\mathbf{D}) = c\exp[-\sum_{s=1}^{N_s}\sum_{i=1}^{NF}[\frac{(\omega_{i,s} - \omega_i^c)^2}{2\sigma_i^2}]] \tag{12}$$

Eq. (12) represents the frequency-based Bayesian conditional probability function. The goal of Bayes analysis is to maximize the likelihood probability function $p(\boldsymbol{\theta}\,|\,\mathbf{D})$ based on the test data, which can be converted to minimize the exponent part in Eq. (12). Combining with the item related with the mode shapes in the objective function as described in Eq. (3), the third objective function, denoted as *obj3*, is given as

$$\boldsymbol{\alpha}^* = \arg\min_{\boldsymbol{\alpha}} f_{obj3}(\boldsymbol{\alpha}) = \arg\min(\sum_{s=1}^{N_s}\sum_{i=1}^{NF}[\frac{(\omega_{i,s} - \omega_i^c)^2}{2\sigma_i^2}] + \sum_{i=1}^{NM}(1 - MAC_i)) \tag{13}$$

Comparing with the first objective function defined in Eq. (3), the item relevant to minimizing the difference in natural frequencies is modified based on Bayesian inference. It should be noted that when involving the Bayesian inference in the objective function, significant computational time may be required to obtain the variances. Since natural frequencies are scalars, their covariance values are relatively straightforward to be obtained. However, for the mode shapes, obtaining the covariance matrices will be relatively complex and time-consuming, considering the modal shapes are vectors. To simplify the calculation and increase the efficiency, only the frequencies are considered in the objective function in Eq. (13).

2.2.3 The objective function based on Bayesian inference and sparse regularization

By considering the Bayesian inference and sparse regularization simultaneously, a hybrid objective function, defined as *obj4* expressed below is proposed in this study for SDI,

$$\boldsymbol{\alpha}^* = \arg\min_{\boldsymbol{\alpha}} f_{obj4}(\boldsymbol{\alpha}) = \arg\min_{\boldsymbol{\alpha}}\left(\sum_{s=1}^{N_s}\sum_{i=1}^{NF}\left[\frac{(\omega_{i,s}-\omega_i^c)^2}{2\sigma_i^2}\right] + \sum_{i=1}^{NM}(1-MAC_i) + \lambda\|\boldsymbol{\alpha}\|_1\right) \tag{14}$$

The Bayesian inference is included to improve the robustness [20], and the regularization term is applied to solve the underdetermined inverse problems [36]. The effectiveness and improvement of these objective functions will be compared in this study.

3. Optimization algorithm

3.1 Jaya algorithm

The proposed I-Jaya algorithm in this study is developed and improved based on the standard Jaya algorithm. The standard Jaya algorithm is briefly reviewed here [28-29] for the completeness of this paper. The Jaya algorithm is a new type of heuristic algorithms, inspired by the concept that the feasible solution acquired for a given problem ought to move towards the best solution and avoid the worst solution [28-29]. Specifically, for every feasible solution, the way of generating its offspring is to move closer to the success (i.e. approaching the best solution) and avoid the failure (i.e. escaping from the worst solution). When generating the offspring, the objective function values are compared to decide whether the new solution (offspring) or the previous solution would be selected for the next iteration. Gradually, the algorithm endeavors to become victories by approaching the best solution and therefore it is named after Jaya (a Sanskrit word meaning victory). Compared with other computational intelligence methods, such as ABC, BMO and PSO etc., the distinct feature of the Jaya algorithm is that there are no special controlling parameters in the algorithm. The procedures of operating the Jaya algorithm includes three steps, namely, the initialization, the local search strategy and the greedy selection mechanism, which are briefly described in the following.

Initialization

An initial colony is generated randomly in the search space. This colony contains CS individuals. Each individual in the colony is marked with $\boldsymbol{\theta}_j$. Every individual ($\boldsymbol{\theta}_j$) contains n variables ($\boldsymbol{\theta}_j = [\theta_1, \theta_2, ..., \theta_q, ..., \theta_n]$), which can be created as,

$$\theta_{j,q} = \theta_{j,q}^l + rand(0,1) \cdot (\theta_{j,q}^u - \theta_{j,q}^l) \tag{15}$$

where $\theta_{j,q}$ denotes the qth variable of $\boldsymbol{\theta}_j$; $\theta_{j,q}^u$ and $\theta_{j,q}^l$ are the upper bound and the lower bound of the variable $\theta_{j,q}$. $rand(0,1)$ represents a random number in the range within 0 to 1.

Local search strategy

After creating the initial colony, the local search for these individuals will be carried out. As mentioned before, the core of the Jaya is to pursue success but avoid failure, therefore, the best solution and the worst one in each iteration would be used to formulate the local search strategy for every individual. It is assumed that $\theta_{j,q,G}$ qth dimension of the jth

individual at the Gth generation. The offspring $\theta'_{j,q,G}$ created by this value can be calculated as

$$\theta'_{j,q,G} = \theta_{j,q,G} + r_{1,q,G} \cdot (\theta_{best,q,G} - |\theta_{j,q,G}|) - r_{2,q,G} \cdot (\theta_{worst,q,G} - |\theta_{j,q,G}|) \tag{16}$$

where $r_{1,q,G}$ and $r_{2,q,G}$ are two random numbers located in the $[0,1]$. $\theta_{best,q,G}$ and $\theta_{worst,q,G}$ are the values of the qth variable for the best individual and the worst one, respectively. The second item in Eq. (16) denotes that the trend of the process towards the best solution while the third item represents the tendency of the solution to avoid the worst solution. Afterwards the judgement of boundary condition will be conducted by using

$$\theta'_{j,i,G} = \begin{cases} \theta^{l}_{j,i}, & if\ \theta'_{j,i,G} < \theta^{l}_{j,i} \\ \theta^{u}_{j,i}, & if\ \theta'_{j,i,G} > \theta^{u}_{j,i} \\ \theta'_{j,i,G}, & otherwise \end{cases} \tag{17}$$

Greedy selection mechanism

Extending the above-mentioned local search strategy to all dimensions, it will acquire the new individual $\boldsymbol{\theta}'_{j,G}$. The greedy selection mechanism [41] is applied to determine whether the new individual or the previous one will be selected for the next iteration. Namely, the objective function values of the $\boldsymbol{\theta}_{j,G}$ and $\boldsymbol{\theta}'_{j,G}$ will be compared. The individual with a smaller function value will be kept to the next generation.

$$\boldsymbol{\theta}_{j,G+1} = \begin{cases} \theta'_{j,G}, f(\theta'_{j,G}) \le f(\theta_{j,G}) \\ \theta_{j,G}, otherwise \end{cases} \tag{18}$$

where f denotes the objective function that requires to be minimized. The algorithm will be continually conducted until the termination condition is satisfied, i.e., the maximum objective function evaluation number is reached.

3.2 Improved Jaya algorithm

In the standard Jaya algorithm, it can be found that every mutation as shown in Eq. (16) is relevant to the best-so-far solution and the worst one. Therefore, the whole colony will centralize into the best-so-far solution with iterations, and the colony information may not be fully used. In this case, if the best-so-far solution is trapped into the local minimal, the whole iteration of the algorithm would cease. Besides, from observing the updated strategy as shown in Eq. (16), it is clear that for the best-so-far solution, the second item trying to reach the best solution would make the optimization lose efficiency. Aiming at overcoming these drawbacks, two modifications are developed to enhance the algorithm's performance.

K-means clustering

The K-means clustering is a simple yet powerful tool that organizes a data set (pattern) into a number of groups or clusters. Within every group or cluster, these data or pattern are similar to each other. In other words, clustering technique is a useful tool to discover the inherent pattern in any given dataset [43]. Besides, the clustering centers can be viewed as the representations of these clusters, since their formulations are based on the combinations of other individuals in these clusters. Therefore, to make full use of the colony information, it seems a smart choice to integrate the K-means clustering technique into the standard Jaya algorithm, since the information of the whole colony can be represented through these so-called 'clustering centers'. Furthermore, during early iterations, conducting the K-means clustering is straightforward and this works as a crossover operators that would effectively utilize the colony information, which is beneficial to improve the algorithm's convergence performance [43-44]. The specific procedure of operating clustering mechanism is described as follows

Step 1: $K = 0.1 \cdot CS$ initial clustering centers $\mathbf{C}_1, \mathbf{C}_2, ..., \mathbf{C}_k$ are produced randomly from the CS individuals $[\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, ..., \boldsymbol{\theta}_{cs}]$.

Step 2: The remaining individuals are distributed to these clustering centers according to their distances to these centers. Specifically, $\boldsymbol{\theta}_j$ is assumed to represent a remaining individual in the colony. If and only if it satisfies the distance condition $\| \boldsymbol{\theta}_j - \mathbf{C}_m \| \leq \| \boldsymbol{\theta}_j - \mathbf{C}_p \|$ ($\mathbf{C}_p$ denotes any other clustering centers), the individual $\boldsymbol{\theta}_j$ will belong to the cluster with the clustering center $\mathbf{C}_m$. Based on this rule, other individuals can find their clusters through the comparison with the distances generated from every clustering center. The distance between any two individuals (i.e., $\boldsymbol{\theta}_j$ and $\boldsymbol{\theta}_c$) is determined by the Manhattan distance, given as follows

$$d(\boldsymbol{\theta}_j, \boldsymbol{\theta}_c) = \| \boldsymbol{\theta}_j - \boldsymbol{\theta}_c \| = \sum_{q=1}^{n} abs(\theta_{j,q} - \theta_{c,q}) \tag{19}$$

Step 3: After assigning other individuals to these clustering centers, the new clustering centers $\mathbf{C}_1^{'}, \mathbf{C}_2^{'}, ..., \mathbf{C}_k^{'}$ are calculated by using the following equation

$$\mathbf{C}_m^{'} = \frac{1}{u_m} \sum_{\boldsymbol{\theta}_j \in \mathbf{c}_m} \boldsymbol{\theta}_j, j = 1,2,...,CS \tag{20}$$

where u_m is the number of individuals belonging to the clustering center C_m.

Step 4: Finally, another K parents individuals from the colony will be selected and then combined with the newly-calculated clustering centers as a new set, marked with τ. The individuals' objective function values will be calculated in the set τ, and these values are sorted from the smallest to the largest. The first K individuals would be put in the colony.

The clustering operation is demonstrated herein. The pseudo-code of operating the K-means clustering is shown in Figure 1.

Algorithm 1 Conducting K-means clustering before individuals' updating

1. Randomly select $K = rnd\,\mathrm{int}[0.1 \cdot CS]$ individuals from the colony
2. Calculate the distances between remaining individuals and clustering centers by Eq. (19)
3. Assign remaining individuals to clustering centers based on the nearest distance
4. Calculate the new clustering centers by Eq. (20)
5. Take away K parents individuals from the colony. Sort them together with the newly-calculated clustering centers in the τ
6. Calculate individuals' objective values in the τ and sort them from the smallest to the largest
7. The first K individuals are put in the colony

Figure 1. The pseudo-code of operating the K-means clustering.

A new updating equation for the best solution

In the Jaya algorithm, the best solution in every iteration plays a crucial role in the whole optimization process, because it guides and draws other individuals to its own region. To prevent best solutions from trapping in the local minimal to some extents, a new updating equation that focuses on the global search is introduced here [45]

$$\theta'_{best,q,G} = \theta_{best,q,G} + \varphi_{best,q,G}(\theta_{j,q,G} - \theta_{best,q,G}) \tag{21}$$

where $\theta_{best,q,G}$ denotes the value of the qth dimension of the best solution at the Gth generation and $\theta'_{best,q,G}$ represents its offspring value. $\varphi_{best,q,G}$ is a random number locating in the [0,1]. $\theta_{best,q,G}$ means the value of the qth dimension of an arbitrary individual in the colony. From Eq. (21), a new candidate is generated by removing the old solution towards a randomly chosen one in the colony. Such randomness can enable this search strategy's exploration ability.

The above two modifications for the standard Jaya algorithm are presented. These improvements are easy to operate and do not bring much complexity to the standard Jaya algorithm. The flowchart of the proposed I-Jaya algorithm is shown in Figure 2.

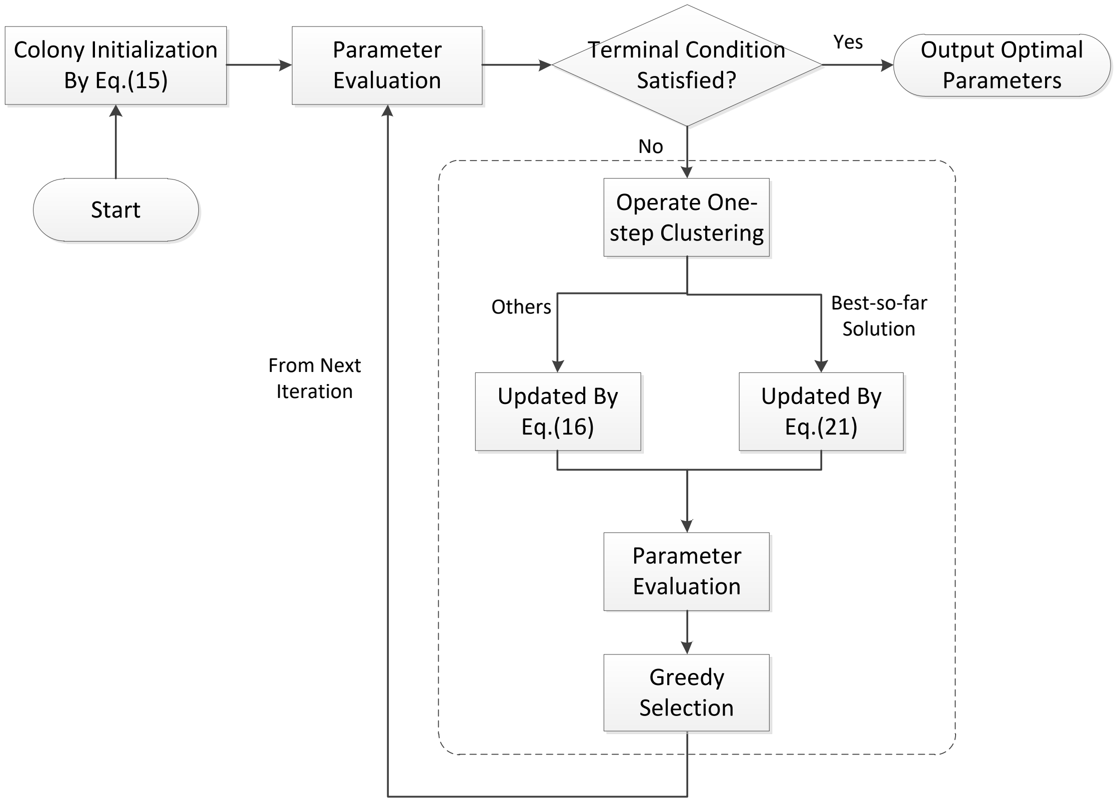

Figure 2. The flowchart of the proposed I-Jaya algorithm.

4. Numerical Studies

4.1 Benchmark tests

To investigate the accuracy of using the developed I-Jaya algorithm for tackling optimization problems against the standard Jaya algorithm, classical mathematical benchmark functions with 100 unknown variables [22] are tested here. These functions can be categorized into four types, that is, an uni-modal separable function (Sphere), three multi-modal and non-separable functions (Griewank, Schaffer and Ackley), an uni-modal and non-separable function (Rosenbrock) and a

$CS = 100$ and the termination condition is set as when the total number of function evaluations reaches 10^5. Each case is independently repeated 30 times and the means of objective function values are recorded.

Figure 3 shows the convergence progresses of the mentioned six benchmark functions. It can be clearly observed that the proposed I-Jaya algorithm has a more competitive convergence speed, and a much better accuracy in the solution than the standard Jaya algorithm as shown in Table 1. Because of its excellent performance in dealing with optimization problems, the I-Jaya algorithm will be used to tackle the following SDI problem.

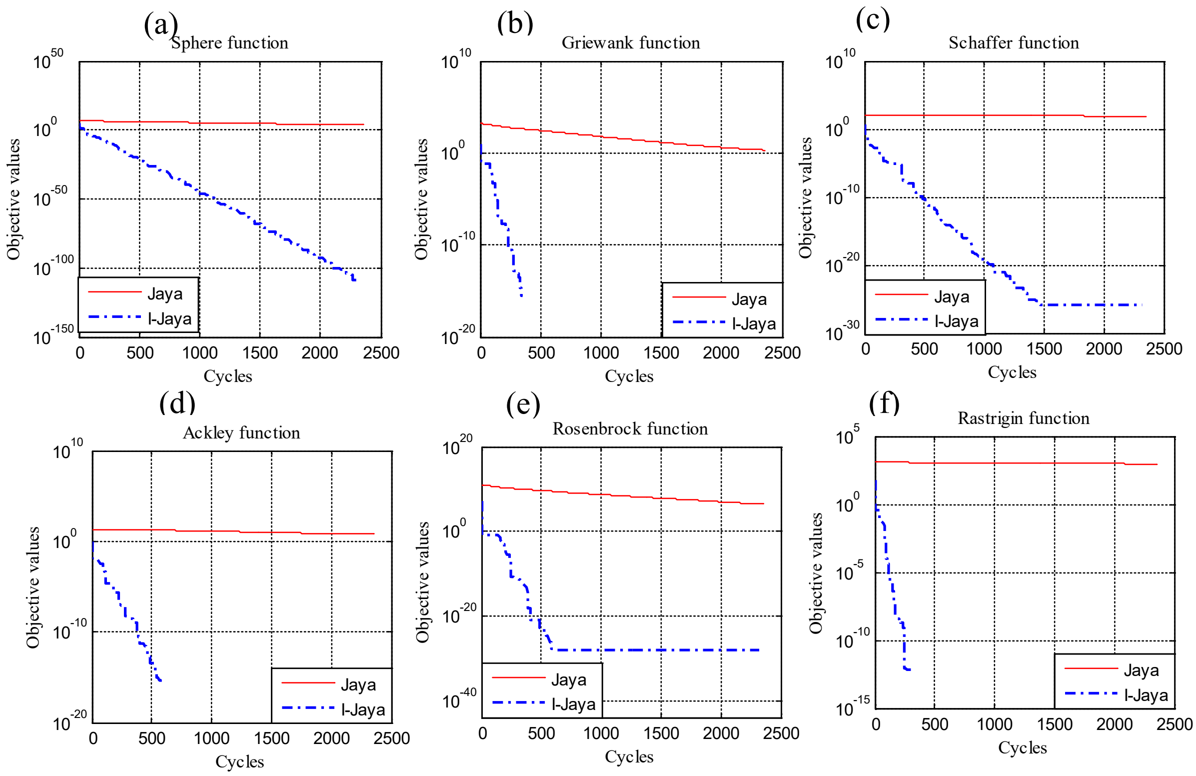

Figure 3. The convergence speed for the classical benchmarks based on Jaya and the proposed I-Jaya algorithm: (a) Sphere; (b) Griewank; (c) Schaffer; (d) Ackley; (e):Rosenbrock; (f) Rastrigin.

Table 1 Statistic results obtained by the Jaya and I-Jaya algorithms for the six classical benchmarks.

Algorithms		Function name					
		Sphere	Griewank	Schaffer	Ackley	Rosenbrock	Rastrigin
Jaya	Mean	3.91E+03	2.11E+00	8.22E+01	5.97E+00	2.62E+06	9.45E+02
	std.	6.67E+02	3.36E-01	4.33E+00	4.96E-01	7.78E+05	6.67E+01
I-Jaya	Mean	1.07E-109	0.00E+00	1.18E-26	0.00E+00	9.86E-29	0.00E+00
	std.	2.88E-110	0.00E+00	6.12E-27	0.00E+00	8.16E-30	0.00E+00

4.2 Numerical simulations

The superiority of the proposed I-Jaya algorithm has been demonstrated in the above benchmark verifications. In this section, a 121-bar truss structure is employed as a numerical example to demonstrate the improvement by using the above-mentioned modified objective functions based on Bayesian inference and sparse regularization to identify the structural damage with a limited quantity of available measurement information. The truss model is shown in Figure 4. Young's modulus, density and Poisson ratio are respectively defined as $E = 70\text{GPa}$, $\rho = 2700\,\text{kg/m}^3$ and $\mu = 0.33$. The boundary conditions of the truss are simulated by three springs with a large stiffness, i.e. $K_{1,1} = 2\times10^{10}\ \text{N/m}$; $K_{1,2} = 2\times10^{10}\ \text{N/m}$; $K_{49,2} = 2\times10^{10}\ \text{N/m}$. The first six natural frequencies and the relevant incomplete mode shapes are used for identification. It should be noted that in the numerical studies, the number of available modal data is less than the number of unknowns. Therefore the damage identification of this structure is an underdetermined inverse problem. Random measurement noises are included in the natural frequencies and mode shapes, respectively, by using Eq. (7). 1% variation with Gaussian distributions is introduced into all the elemental stiffness parameters for simulating the uncertainties. In terms of the parameters setting for I-Jaya, the colony size is $CS = 100$ and the maximum objective function evaluation number is set as 49000. For each damage case, 30 runs are independently conducted to acquire statistical results.

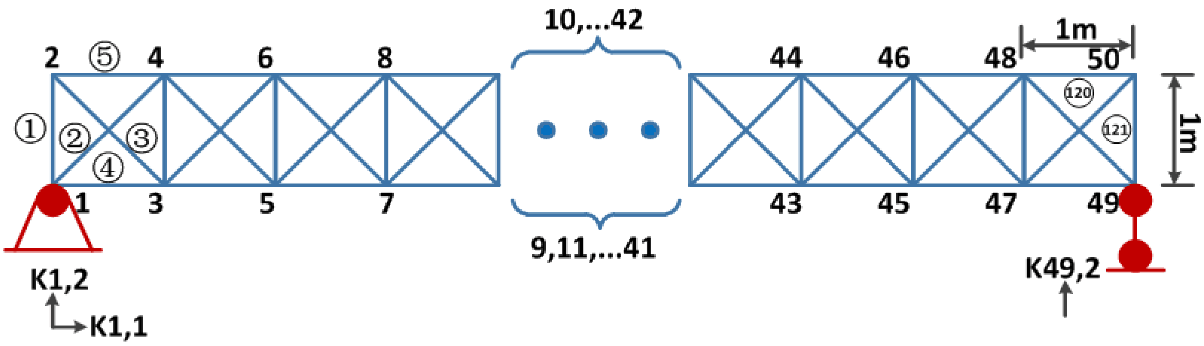

Figure 4. The model of the truss structure.

4.2.1 Performance comparison on different objective functions

The first damage case, denoted as Case 1, is assumed that there is a 15% stiffness reduction in the 10th element, which means $\alpha_{10} = 0.15$.

parameter λ for the *obj2* and *obj4* are set as $2 \cdot 10^{-4}$ and $2 \cdot 10^{-6}$, respectively. The proposed algorithm is used for identification, with different objective functions. Figure 5 shows the iteration processes of the identified damage index α_{10} by using different objective functions. It is observed that using *obj4* converges faster and provides more accurate damage identification results. Figure 6 shows the final damage identification results in all the elements of the truss model by using four different objective functions. The mean values and the variation range with mean values plus and minus standard deviations are shown in Figure 6. The identified damage extents in the 10th element by using four objective functions are also given in Table 2. It can be observed from Figure 6 and Table 2 that the identification results by using *obj4* is the most accurate.

The second damage case, denoted as "Case 2", is assumed with 15% stiffness reductions in the 10th and the 45th element, namely, $\alpha_{10} = \alpha_{45} = 0.15$. The input modal data are the same as those in Case 1. The regularization parameters for the *obj2* and *obj4* are set as $5 \cdot 10^{-4}$ and 10^{-6}, respectively. Figure 7 shows the identification results of Case 2, and Table 2 lists the identified damage extents in the damaged elements. It is clearly observed that when using the *obj1*, a number of significant false identifications are generated. With the sparse regularization term, the false identification by using the *obj2* are greatly reduced. By including Bayesian inference in *obj3*, the identification can be improved as compared with using *obj1*. However, there are still a number of observed false identifications with considerable standard deviations. The identification accuracy by using *obj4* is significantly improved. The identification results from these two damage cases demonstrate the superiority of using both the sparse regularization and Bayesian learning. Using only sparse regularization in *obj2* or Bayesian inference in *obj3* can certainly improve the accuracy and performance in damage identification. However, when sparse regularization and Bayesian inference are used simultaneously in *obj4*, a much more accurate identification is achieved. The identified damage extents are close to the true values with minor standard deviations, and almost no false identification is observed. These identification results verify that the new objective function based on Bayesian inference and sparse regularization can greatly enhance the accuracy and robustness of utilizing the I-Jaya algorithm for SDI.

Table 2. Damage identification results in the numerical studies.

Damage case	Damage location	True value	*Obj1* Mean value	*Obj1* std.	*Obj2* Mean value	*Obj2* std.	*Obj3* Mean Value	*Obj3* std.	*Obj4* Mean value	*Obj4* std.
Case 1	α_{10}	0.15	0.1098	0.0598	0.1362	0.0154	0.1556	0.0014	0.1511	0.0006
Case 2	α_{10}	0.15	0.1351	0.0164	0.1388	0.0155	0.1440	0.0108	0.1486	0.0009
	α_{45}	0.15	0.1202	0.0475	0.1381	0.0132	0.1347	0.0399	0.1507	0.0007

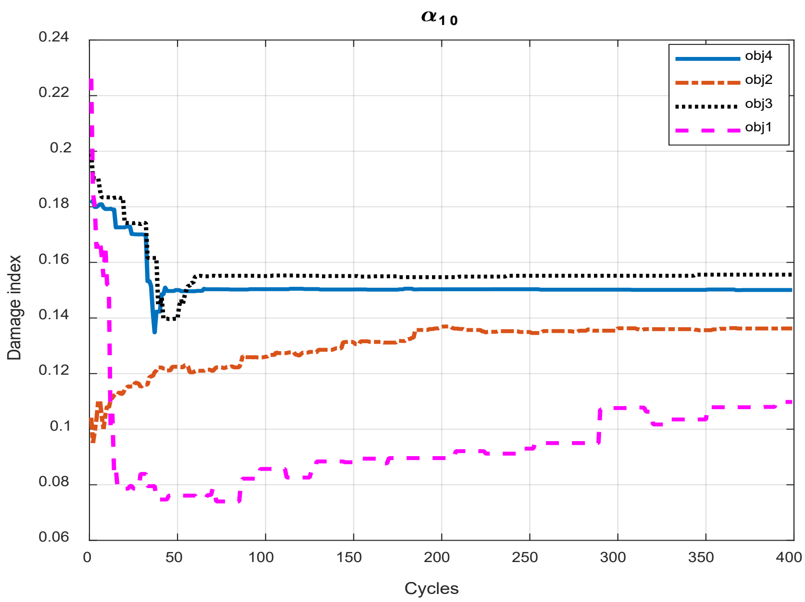

Figure 5. The convergence processes of the identified damage index α_{10} with different objective functions.

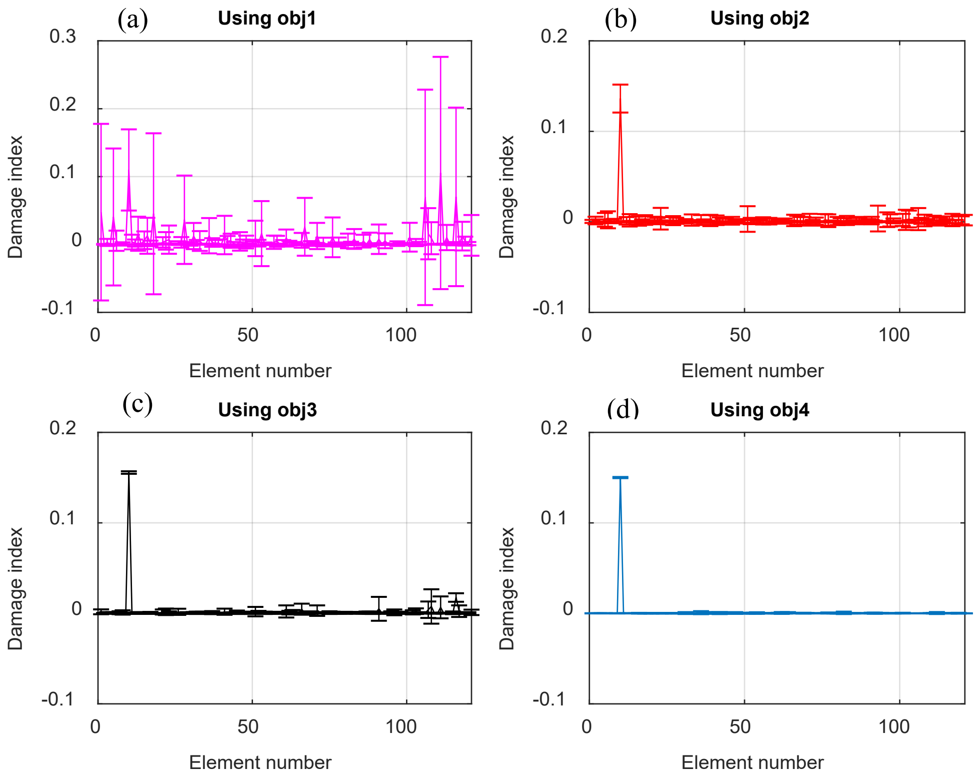

Figure 6. Damage identification results of Case 1 in the numerical study:
(a) using obj1; (b) using obj2; (c) using obj3; (d) using obj4.

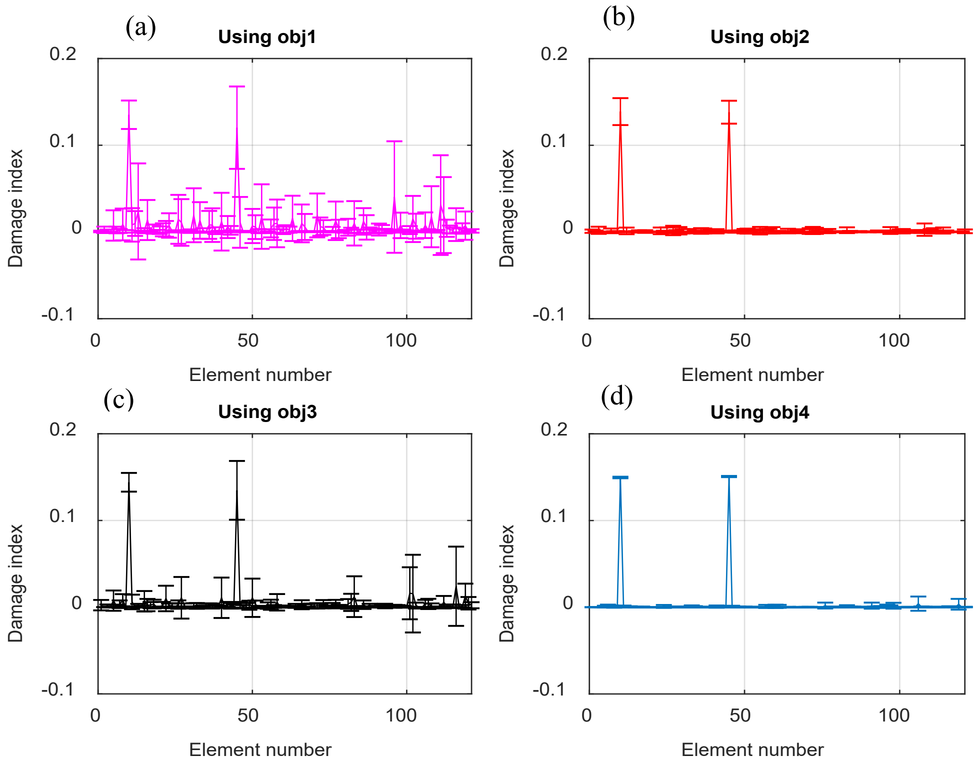

Figure 7. Damage identification results of Case 2 in the numerical study:
(a) using obj1; (b) using obj2; (c) using obj3; (d) using obj4.

4.2.2 Damage identification with limited modal data

The superiority of the proposed objective function *obj4* is demonstrated in the above examples. In this section, only the *obj4* is used to investigate the influence on damage identification by using limited modal data. The third damage case, denoted as Case 3, is assumed with 8% stiffness reductions in the 10^{th}, 45^{th}, and 100^{th} element respectively, which means $\alpha_{10} = \alpha_{45} = \alpha_{100} = 0.08$. The first six natural frequencies and different numbers of mode shape values for these six modes are used for identification. Significant measurement noises are added in natural frequencies and mode shapes with the noise levels of 3% and 5%, respectively. Four scenarios are considered and listed in Table 3. It is noted that the number of available modal data used for identification in each scenario is always less than that of unknown system parameters to be identified. For Scenario 4, a much less number of modal data, that is 36, are used to identify 121 unknown elemental stiffness parameters in this study. The selected regularization parameters based on DP rule [33] as mentioned above are also listed in Table 3. Figure 8 shows the iteration processes of the damage index values on the damaged elements for these four scenarios with different numbers of used modal data. It can be found that after around 200 iterations, all the damage index values converge to the neighborhood of preset values. Figure 9 shows the damage identification results of these four scenarios, and Table 4 lists the identified damage extents in the damaged elements. Accurate identification results are obtained for all the scenarios, indicating that the introduced damages can be well identified by using the proposed algorithm with the sparse regularization and Bayesian inference, even with a small number of available modal data.

Table 3. Used modal data in different scenarios and regularization parameters for Case 3.

Scenario	Number of the used modal data	Quantity of used modal data	λ
1	Six frequencies, and the corresponding mode shapes in the vertical direction at the 2nd,7th,12th,...,47th nodes	66	$2\cdot 10^{-5}$
2	Six frequencies, and the corresponding mode shapes in the vertical direction at the 2nd,10th,18th,...,50th nodes	48	10^{-5}
3	Six frequencies, and the corresponding mode shapes in the vertical direction at the 2nd,11th,20th,...,47th nodes	42	10^{-5}
4	Six frequencies, and the corresponding mode shapes in the vertical direction at the 2nd,12th,23th,34th,45th nodes	36	$1.2\cdot 10^{-5}$

Table 4. Identified damage extents for Case 3 in the numerical studies.

Damage location	True value	Scenario 1		Scenario 2		Scenario 3		Scenario 4	
		Mean value	std.	Mean value	std.	Mean value	std.	Mean value	std.
α_{10}	0.08	0.0791	0.0008	0.0795	0.0014	0.0793	0.0009	0.0782	0.0021
α_{45}	0.08	0.0800	0.0012	0.0795	0.0013	0.0796	0.0016	0.0786	0.0015
α_{100}	0.08	0.0796	0.0013	0.0788	0.0006	0.0784	0.0010	0.0787	0.0013

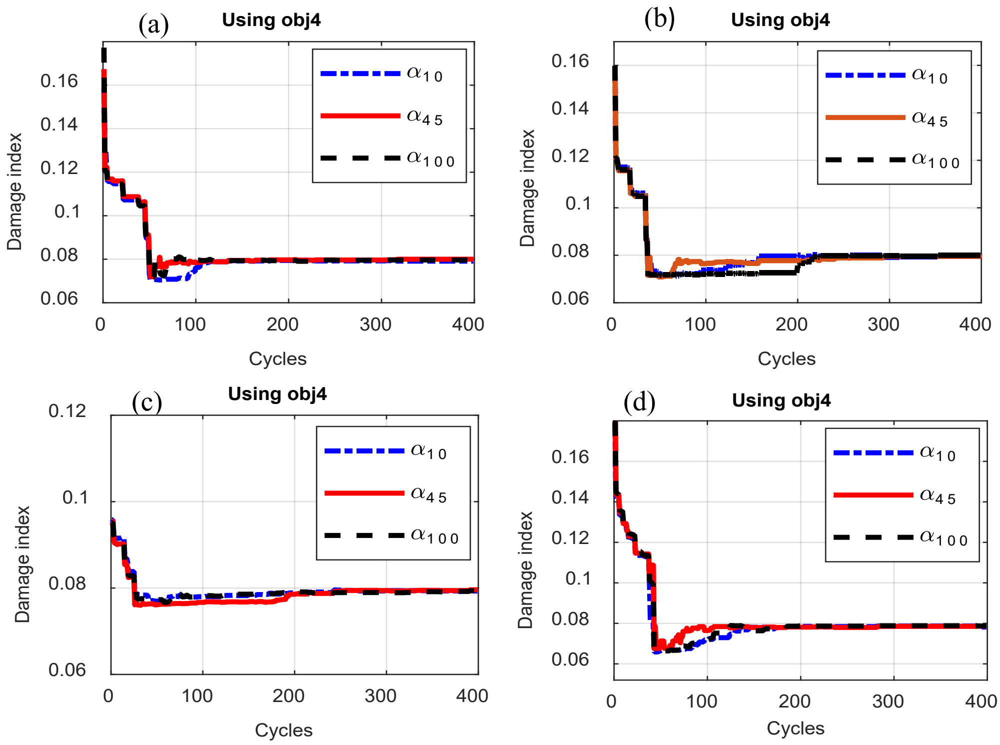

Figure 8. The iteration processes of the damage indices in damaged elements in Case 3: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3; (d) Scenario 4.

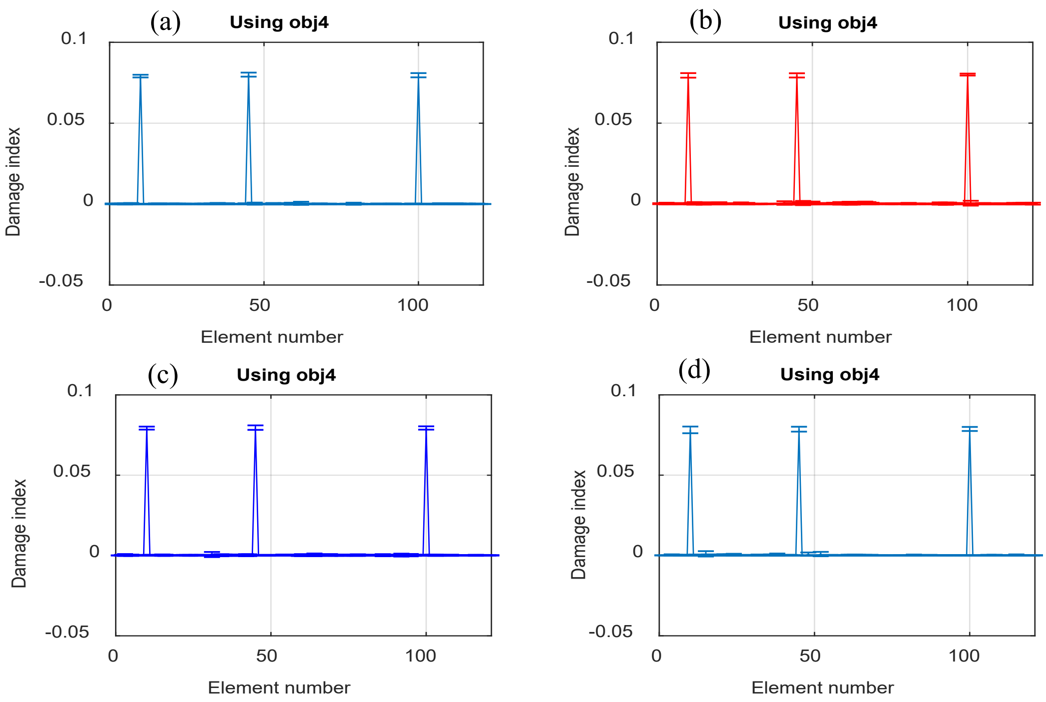

Figure 9. Damage identification results in Case 3 based on different inputs: (a) Scenario 1; (b) Scenario 2; (c) Scenario 3; (d) Scenario 4.

4.3 Comparison with other optimization techniques

In this section, other optimization techniques including GA, Nelder-Mead algorithm [46], and Gaussian bare-bones artificial bee colony (GBABC) algorithm [47] are employed to make comparisons. The fourth damage case, denoted as Case 4, is assumed with 5% stiffness reductions in the 10th, 45th and 100th elements, that is, $\alpha_{10} = \alpha_{45} = \alpha_{100} = 0.05$. This case is defined to simulate minor damage in structures. The used modal data are the same as those defined in Scenario 4 in Table 3, and the objective function *obj4* is used for identification with the regularization parameter defined as $1.8 \cdot 10^{-5}$. Significant measurement noises are added in natural frequencies and mode shapes with the noise levels of 3% and 8%, respectively. Regarding the parameters setting, for GA, the colony size is set as 100. The mutation rate and crossover rate are defined as 0.1 and 0.8, respectively. For the Nelder-Mead algorithm, the initial values are set as 0.1 for all the damage indices, which are quite close to the assumed values. For GBABC, the colony size, parameter '*limit*' and the search tendency 'ST' are set as 100, 6050 and 0.3 respectively, which are the same as those in a previous study [47]. The maximum objective function evaluation number is set as 49000 for all these optimization methods.

Figure 10 shows the final damage identification results by using different optimization methods. It is clearly observed that even with the *obj4*, the state-of-the-art heuristic algorithm (GBABC) and the classical heuristic algorithm (GA) and the traditional optimization algorithm (Nelder-Mead) cannot provide accurate and reliable damage identification results. In contrast, the identified damage results from the proposed I-Jaya algorithm are accurate and reliable. The damage identification results on these three damaged elements are: $\alpha_{10} = 0.0469$

$\alpha_{45} = 0.0478$ with a standard deviation of 0.0011 and $\alpha_{100} = 0.0477$ with a standard deviation of 0.0025, respectively. The identified damage severities are very close to the assumed values. Besides, the false identifications from the proposed approach are minor. The results in this case demonstrate the superiority of the proposed approach to conduct the minor damage identification in structures with the measurement data of significant noise effect, compared with the latest optimization methods. This lays the foundation for the following experimental verification.

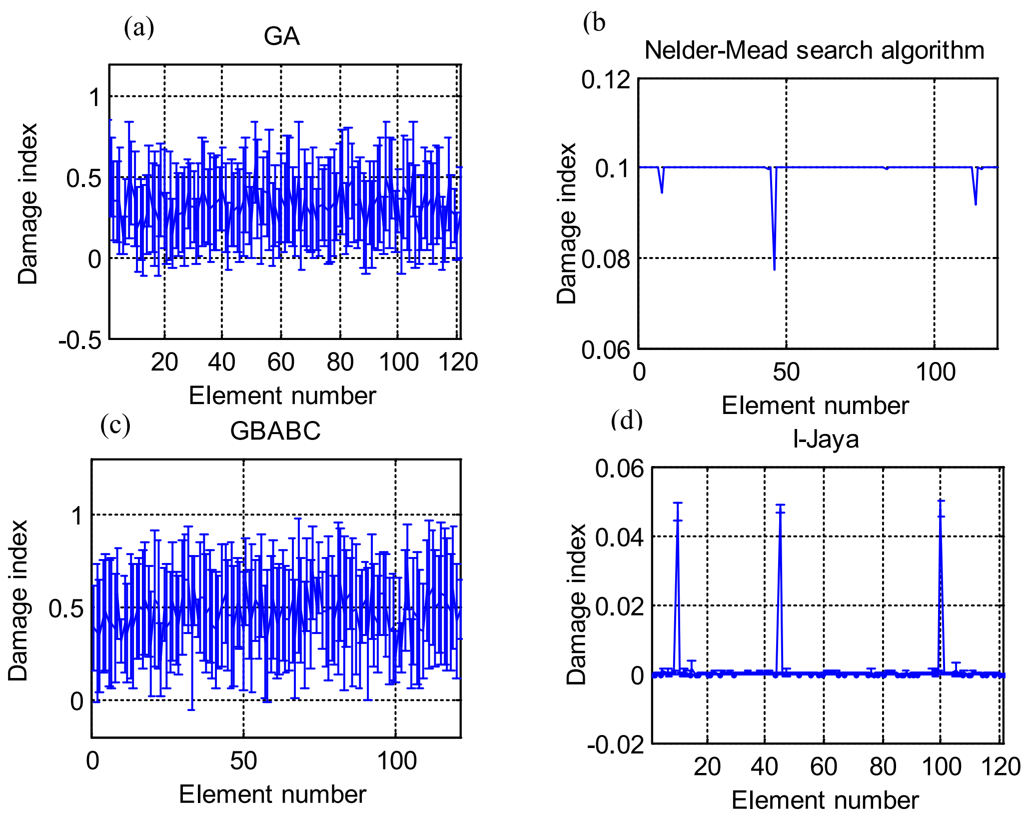

Figure 10. Damage identification results of Case 4 with different optimization methods. (a) GA; (b) Nelder-Mead algorithm; (c) GBABC; (d) I-Jaya.

4.4. Comparison with the standard Jaya algorithm and other methods

In this section, the standard Jaya algorithm and other two methods reported in the literature [48, 49] are employed to identify a new damage case. The results are compared with that obtained by the proposed approach to demonstrate the proposed method. The fifth damage case, denoted as Case 5, is assumed having a 50% stiffness reduction in the 10th element, that is, $\alpha_{10} = 0.5.$, representing a single large damage in the structure.

The standard Jaya and the developed I-Jaya algorithm associated with the *obj4* are used to identify the introduced damage in Case 5. The parameters setting for the Jaya and I-Jaya are the same as those in Cases 1 to 4. The regularization parameter is set as $2 \cdot 10^{-5}$. The used modal data are the same as those in Case 4. Significant measurement noises are assumed in natural frequencies and mode shapes with the noise levels of 3% and 8%, respectively. Figure 11(a) shows the evolutionary process of the mean values of the *obj4* with the two methods. It can be found that the values acquired by the I-Jaya are significantly smaller than those obtained by the Jaya, which indicates the I-Jaya is able to achieve more satisfactory identification results. Figure 12 shows the evolutionary process of the identification damage index α_{10} from the I-Jaya algorithm. After around 320 cycles, the algorithm converges to the neighbourhood of the assumed true damage value. Figure 13 shows the final identification results in all the elements by using different methods. The Jaya algorithm is not able to identify the damages accurately, but the proposed I-Jaya algorithm is capable of identifying the single large damage effectively, with the mean value of 0.5004 and the standard deviation of 0.0002. The results demonstrate the improvement of the proposed modifications on the standard Jaya algorithm.

To further demonstrate the superiority of the proposed method, two methods for structural damage identification, named after the Modified Differential Evolutionary algorithm (MDE) [48]

$CS=100$ and their maximum evaluation times are 49000. For MDE, the threshold value is 0.1; the mutation rate is set as 0.4; the mutation constant is a random number locating in $[0.4,0.9]$. These special parameters setting are the same as those in Ref. [48]. For DPSO, it touches the disturbance mechanism after 100 cycles, and two parameters relevant to the disturbance mechanism are $\varepsilon=10^{-3}$ and $\Delta=10^{-4}$, respectively. Figure 11(b) shows the evolutionary process of the objective values from these two methods. It can be found that their objective function values basically maintain at an order of 10^{-2} after around 100 cycles, which indicates both these two methods would not acquire good identification results for Case 5. The final results obtained from MDE and DPSO are shown in Figure 13. Similar to the standard Jaya algorithm, MDE and DPSO cannot provide accurate damage identification results.

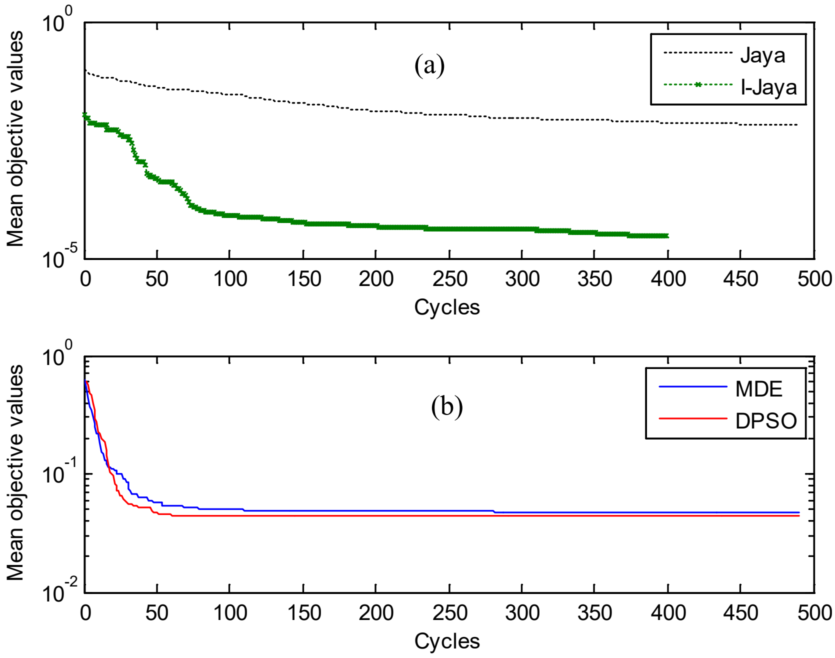

Figure 11. The evolutionary process of the objective values; (a) Jaya and I-Jaya; (b) MDE and DPSO.

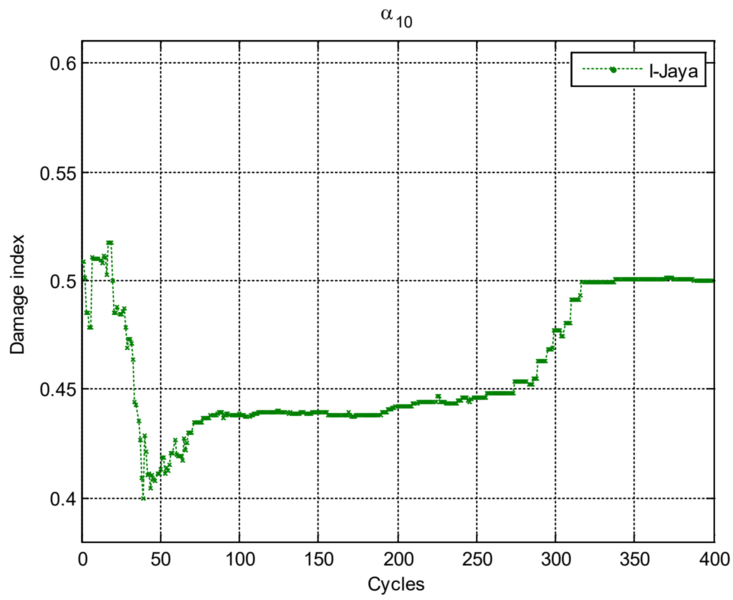

Figure 12. The evolutionary process of the identified damage index in the damaged element based on I-Jaya

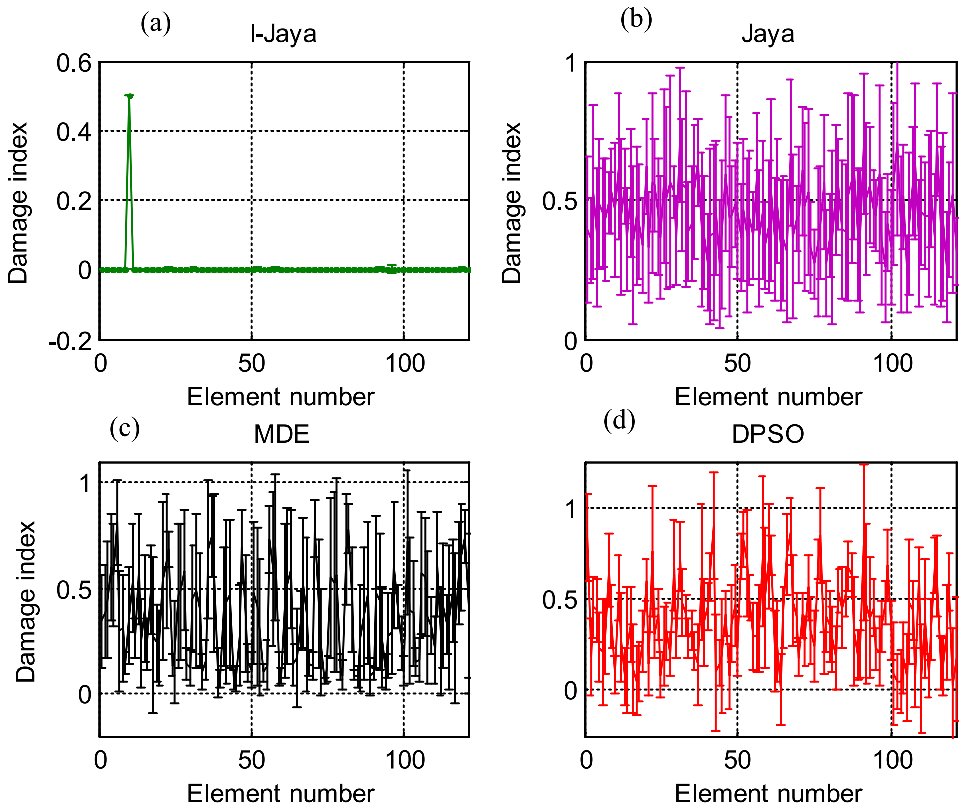

Figure 13. Identification results for Case 5:

(a) I-Jaya; (b) Jaya; (c) MDE; (d) DPSO.

5. Experimental verification

Laboratory studies on a reinforced bridge model are performed to verify the performance of using the developed approach for structural damage detection with experimental testing data.

5.1 Experimental setup and initial model updating

Figure 14 shows the testing structure, which is a simply supported T-type prestressed concrete bridge model, used for validating the effectiveness of using the proposed algorithm and the objective function including sparse regularization and Bayesian inference for damage identification. Figure 15 shows the dimensions of the laboratory model and the placed accelerometer locations for

2.6×10^4 Mpa and 2.7×10^3 $\mathrm{kg/m^3}$. More details of the bridge model can be referred to [50]. Seven accelerometers are located on the top of the bridge model for acquiring the accelerations in the vertical direction, during the hammer impact tests for modal identification.

An initial model updating based on modal information from the intact structure is conducted to generate a baseline for the subsequent damage identification. An initial finite element model of the bridge is built by using flat shell elements, as shown in Figure 16. The finite element model includes 90 elements and 114 nodes with 6 Degrees-of-Freedom (DOFs) at each node. The boundary constraints are simulated by the linear springs. The initial model updating is conducted to adjust the stiffness parameters of the built finite element model by minimizing the difference between the first three natural frequencies acquired from the finite element model analysis and measured from the test. In the initial model updating, the Young's modulus of slab and web of the bridge as well as the support stiffness, that is three parameters in total, are chosen as the parameters to be updated. The proposed I-Jaya algorithm is used to update the initial finite element model. In terms of the parameters setting, CS is set as 30 and the maximum objective function evaluation number is set as 5000. It runs 30 times and the mean values are selected as the final updated parameters. It can be observed from Figure 17 that are there are discrepancies between the analytical frequencies and the measured one. After updating, the calculated natural frequencies from the updated model match very well with the measured ones. The baseline model is used for the following damage identification.

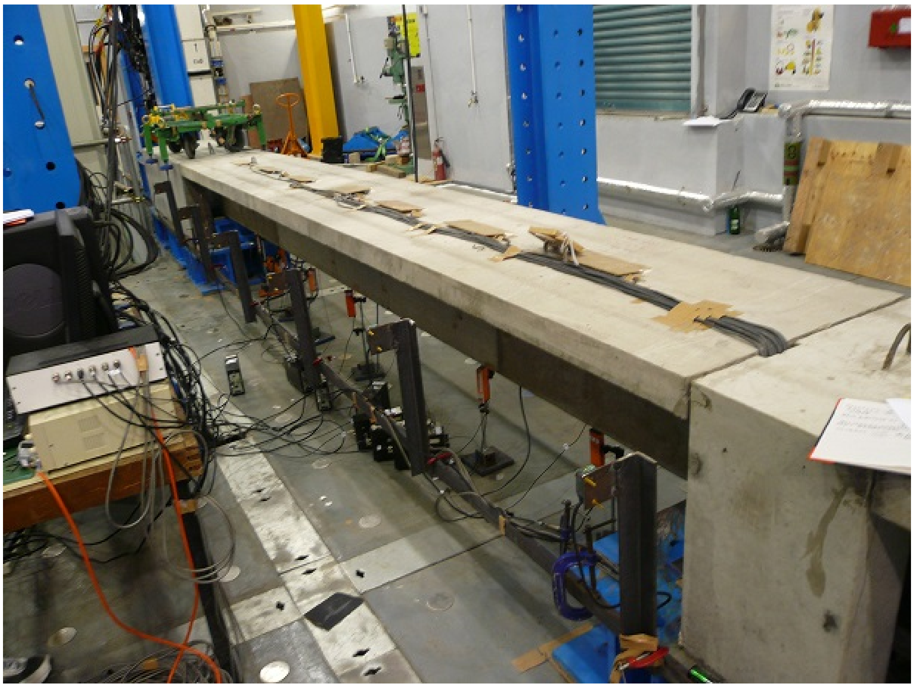

Figure 14. The experimental testing model.

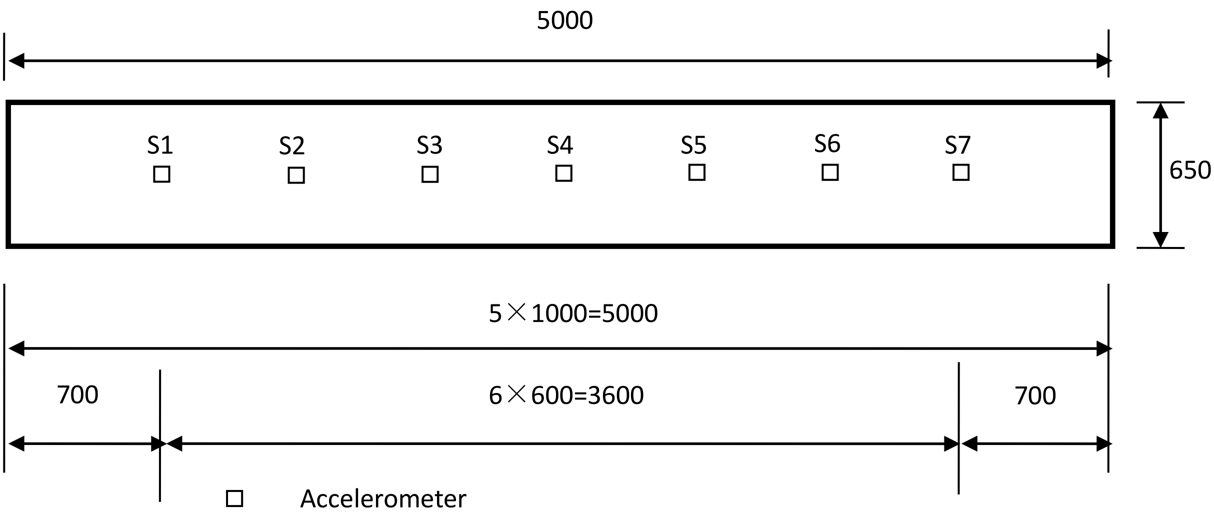

Figure 15. Dimensions (unit: mm) of the experimental concrete bridge model and the placed sensor locations

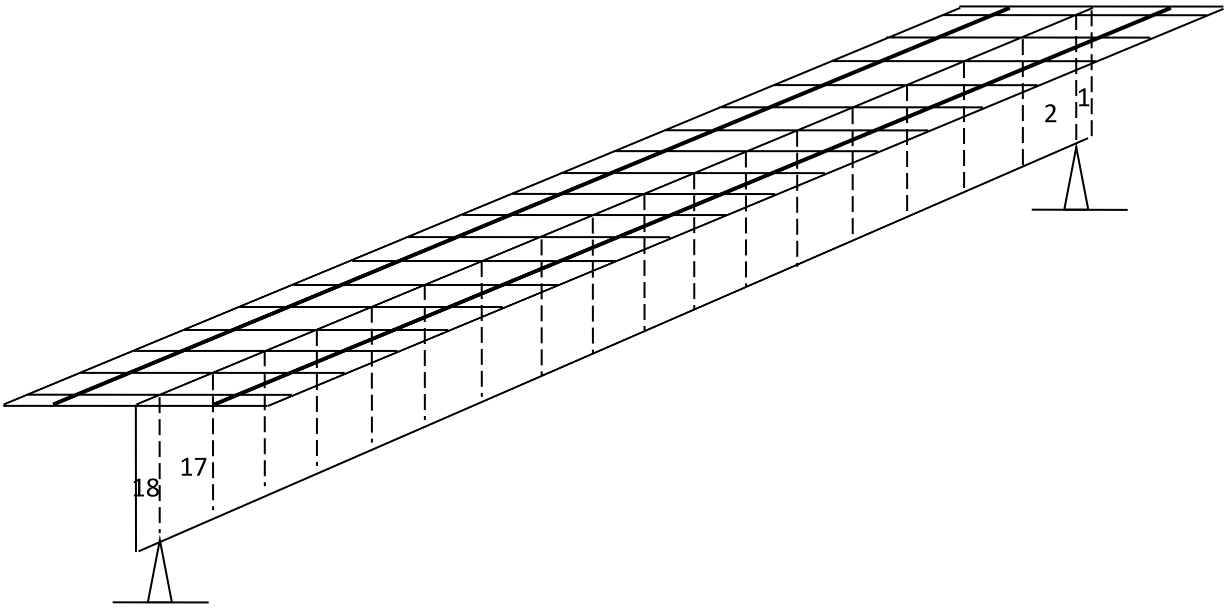

Figure 16. The finite element model of the experimental bridge model.

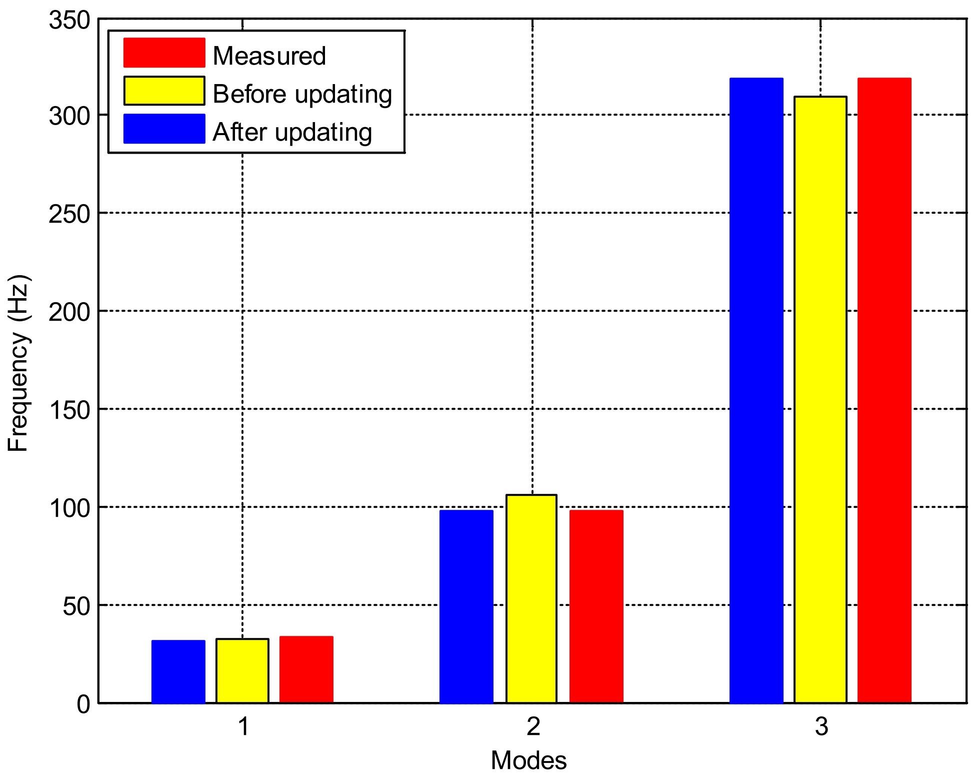

Figure 17. The measured and updated frequencies in the experimental studies

5.2 Damage identification for the testing model

The cracks are introduced by applying the two-point loads at the center of the bridge model. Figure 18 shows the cracks observed in the web elements, under the static loads of 180KN. This is considered as the damaged state, and the stiffness reductions in web elements are to be identified. Specifically, 24 major cracks are observed in the web elements, which are from the 4^{th} element to the 15^{th} element. Table 5 lists the information about the observed typical cracks in the testing model, which are mainly distributed from the 8^{th} element to the 13^{th} element. Modal tests are conducted to obtain the natural frequencies and mode shapes at the sensor locations under the damaged state, as shown in Figure 15. These modal data are used as the input for the damage identification. The damages mainly occur in the web elements of the bridge model, and thus only the web elements are included for the identification. Elements 1 and 18 are however not included as they are outside the supports.

The proposed I-Jaya algorithm and the *obj4* are used to conduct the damage identification. The colony size is $CS = 50$ and the maximum objective function evaluation number is set as 12250. Multiple measurements are required for the *obj4*, when introducing Bayesian inference. Therefore, similar as Ref. [20], natural frequencies are assumed to vary within $\pm 3\%$ of their real values. Limited number of modal data, which is less than the total number of the web elements of the structure, are used to identify the damage. The input data include the first natural frequency and the corresponding mode shape values at the placed sensor locations. Therefore totally 8 measured modal data are used to conduct damage identification in web elements. The regularization parameter is set as $\lambda = 5 \cdot 10^{-6}$, based on DP rule. The damage identification will be independently run 30 times to obtain the statistic results.

Figure 19 shows the final damage identification result. Since there is no analytical formula to relate a number of observed cracks in a reinforced concrete bridge model with the flexural stiffness of elements, it is difficult if not impossible to obtain the accurate damage extents according to the observed cracks, which are shown in Figure 18 and Table 5. Therefore the identified damage pattern in Figure 19 is compared with the observed crack pattern to validate the effectiveness of the proposed approach for damage identification. The identification results demonstrate that the main damage pattern, with the main damage distributed from the 8th element to the 13th element [50], can be identified. Considering that only the first frequency and mode shape are used for identification and the main damage pattern can be identified, the results indicate that the proposed approach has the capacity to identify the damages in the experimental model with limited measurement data.

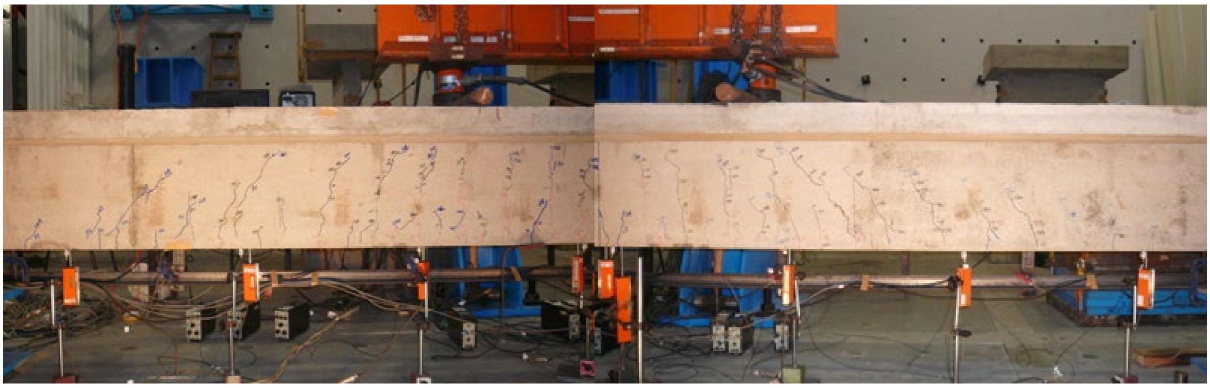

Figure 18. observed cracks in the web elements of the concrete bridge model

Table 5. Reported typical cracks in the tested structure [50]

Distance from left support of beam (mm)	1800	1950	2125	2260	2340	2540	2680
Web element number	8	8	9	9	9	10	10
Crack height(mm)	280	248	273	286	243	244	261
Distance from left support of beam (mm)	2800	2900	3030	3130	3220	3330	3510
Web element number	11	11	12	12	12	13	13
Crack height(mm)	251	120	260	118	274	220	220

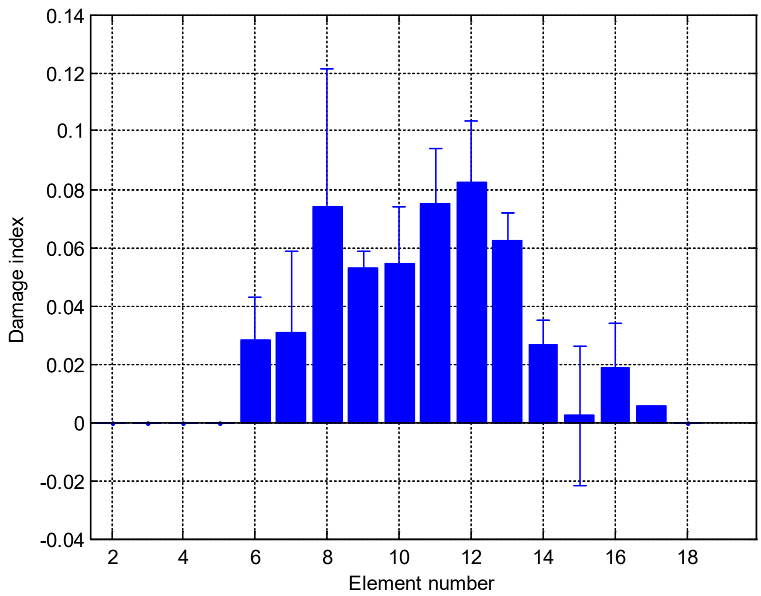

Figure 19. Damage identification results of the testing bridge model in the experimental study with limited input data.

6. Conclusions

This paper proposes a new approach for damage identification of structures based on I-Jaya algorithm. Since the objective function is important in the damage identification, the sparse regularization and Bayesian inference are added in the traditional objective function based on modal data to perform the damage identification and improve the robustness. Numerical studies on a truss structure are conducted to investigate the accuracy and efficiency of the proposed approach and demonstrate the improvement by using different objective functions for damage identification. Experimental studies on a reinforced concrete bridge model are carried out to verify the performance and effectiveness of the proposed approach for damage identification with real experimental testing data. Some conclusions can be drawn as follows

- The proposed I-Jaya algorithm is more efficient to deal with the classical benchmark test functions. The results show an improved performance and global optimization ability compared with the original Jaya algorithm;
- Results in the numerical truss example demonstrate that the modified objective function based on the sparse regularization and Bayesian inference yields more reliable and accurate identification results, compared with the traditional objective functions or traditional objective functions with either sparse regularization or Bayesian inference only;
- Compared with other widely used optimization techniques, such as the Nelder-Mead algorithm, GA and GBABC algorithm, results from the proposed approach demonstrate the superiority in identifying the minor damages under the significant noise effect;
- In the experimental verifications, the proposed approach can be used to perform the initial model updating accurately. For the following damage identification, the developed I-Jaya algorithm and the modified objective function based on sparse regularization and Bayesian

inference can well identify the damage pattern of a reinforced concrete bridge model with few modal data.

- The damage identification results in the numerical and experimental studies well demonstrate that the proposed approach can effectively and accurately identify the damages in the structures, even when the uncertainty effect is significant.

Acknowledgments

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