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Multi-objective optimal design of carbon and glass reinforced hybrid composites based on design rules

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ABSTRACT

An optimisation study on the carbon and glass fibre reinforced hybrid composites in three-point bending is presented in this paper. For both unidirectional and multi-directional hybrid composites, the flexural properties were obtained by Finite Element Analysis (FEA)-based simulation. Several design rules were derived, and optimisation was done by applying these rules, with minimising the cost and weight being the objectives and the flexural strength being the constraint. The results from the design rule-based optimisation were compared with those from the non-dominated sorting GA-II (NSGA-II).

Introduction

Fibre reinforced hybrid composites is made by reinforcing a matrix with two or more types of fibres. For layered composite materials, it is shown from previous research [1–4] that the flexural strength can be improved by hybridising carbon and glass fibres. The strain-to-failure is improved by including higher strain-to-failure glass fibre plies [5]. The existence of hybrid effect can be potentially useful for achieving a balanced cost and weight optimal composite material.

The properties of composites can be obtained analytically by Classical Lamination Theory (CLT) or numerically by Finite Element Analysis (FEA). In our previous research [6–9], the flexural properties of composites were obtained by the analytical approach based on simple CLT. In addition to CLT, the flexural properties of composites can also be obtained by the numerical approach based on FEA [10]. A recent study has shown that CLT underestimates the flexural strength [11].

A composite laminate is usually made of several fibre reinforced laminas or plies. In the design process, the fibre type, fibre orientation and fibre volume fraction of each ply need to be carefully selected to meet the design requirements. Because of the number of design variables, optimisation of composites is not a trivial task. Extensive research has been done on the optimisation of composites. Some reviews are available in the literature [12–14]. Two important design objectives are weight and cost minimisation. These two requirements are usually conflicting and thus trade-off needs to be made. An optimisation problem to minimise the cost and weight of composites is called a multi-objective optimisation problem.

Evolutionary algorithms, e.g., genetic algorithm (GA), are often used for the multi-objective optimisation of composites. A modified version of the NSGA, known as NSGA-II, is one of the most popular MOEAs due to its simplicity and efficiency [15]. NSGA-II has been used in our previous research to minimise the cost and weight of unidirectional [6,7] and multidirectional [8,9] hybrid composites. In these studies, the flexural properties of composites were obtained by the analytical approach based on simple CLT. It is shown that the positive hybrid effects help to reduce the cost and weight of the composite.

Since CLT underestimates the flexural strength, for better accuracy, FEA-based simulation was adopted in this study to find the flexural properties of hybrid composites. When NSGA-II and FEA are coupled, because for each run in NSGA-II, the FEA model needs to be updated, the multi-objective optimisation is preventively time-consuming and infeasible for practical use [11]. Thus, a set of design rules were developed based on the theoretical and numerical analyses in this study. Using these design rules, potential stacking configurations to achieve optimal design were derived. The relationship between the flexural strength and fibre volume fractions were obtained by FEA and regression analysis. Given the required minimum flexural strength, optimisation was done for the hybrid composite under flexural loading with minimum cost and weight as the objective functions. Both unidirectional and multi-directional hybrid composites were studied.

Methodology

Material properties

In this study, the fibre types being included in optimisation are high

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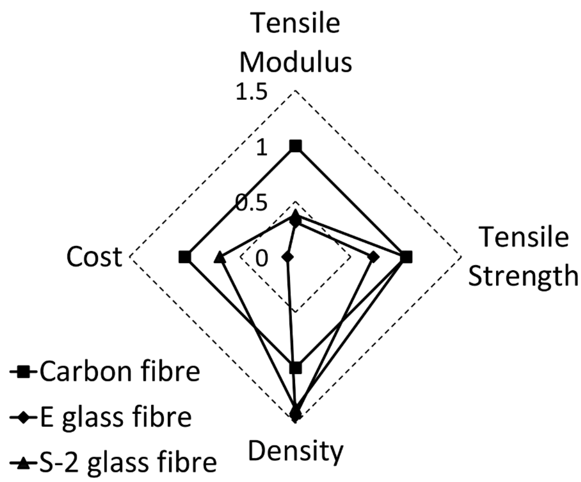

Fig. 1. Material properties of carbon and glass fibres.

strength carbon fibre, S-2 glass and E glass fibres. An epoxy was chosen to be the matrix. Epoxy resins are widely used in composites because of their high strength (tensile, compressive and flexural), good chemical resistance, fatigue resistance, corrosion resistance and electrical resistance [16]. Typical values of the properties of the fibres and epoxy resin can be found in Ref. [1]. The tensile modulus, strength, density and cost of the carbon and glass fibres are shown in Fig. 1. The properties are normalised with reference to those of the carbon fibre. It is seen from Fig. 1 that the carbon fibre has higher tensile modulus than the glass fibres. The carbon fibre has higher cost but lower density than the E glass fibre. The S-2 glass fibre has similar density but much higher cost compared to the E glass fibre. The E glass fibre has lower tensile strength compared to the carbon and S-2 glass fibres.

For each ply, based on the constituent properties and its fibre volume fraction, the ply properties, including the longitudinal modulus E_{11}, the transverse moduli E_{22} and E_{33}, and the shear moduli G_{12}, G_{13} and G_{23}, are derived by Chou [17]. The strength components of composites were derived, and stress-based failure criteria were employed. The failure criterion is defined to be the ratio of the maximum stress and strength.

The weight of a composite material can be characterised by its density. The density of the hybrid composite reinforced by carbon and glass fibres, ρ_c, can be derived based on RoM as follows:

$$\rho_c = \rho_{fc}V_{fc} + \rho_{fg}V_{fg} + \rho_m V_m \quad (1)$$

where ρ_{fc}, ρ_{fg} and ρ_m are the densities of carbon fibre, glass fibre and the matrix, respectively, and V_{fc}, V_{fg} and V_m are the volume fractions of carbon fibre, glass fibre and the matrix, respectively.

The material cost of the hybrid composite, C_c, is given by

$$C_c = C_{fc}V_{fc} + C_{fg}V_{fg} + C_m V_m \quad (2)$$

where C_{fc}, C_{fg} and C_m are the costs of carbon fibre, glass fibre and the matrix, respectively.

FEA-based model

The hybrid composite in this study consists of eight plies and the thickness of each ply is 0.25 mm. The total thickness is 2 mm, the width is 10 mm, and the length is 100 mm. The flexural properties of hybrid composites were evaluated via FEA-based three-point bending simulation in accordance with ASTM D7264 [18]. The composite specimen is supported by two rollers at a span of L and loaded at its mid-span, as shown in Fig. 2. The span-to-thickness ratio was chosen to be 32. The composite is loaded in such way that plies 1 and 8 are in tension and compression, respectively.

The hybrid composite in three-point bending was simulated by FEA using Ansys Workbench. The hybrid composite is modelled as a shell structure and the supporting and loading rollers are modelled as cylindrical solids. Fixed support is applied to the supporting rollers and a pre-

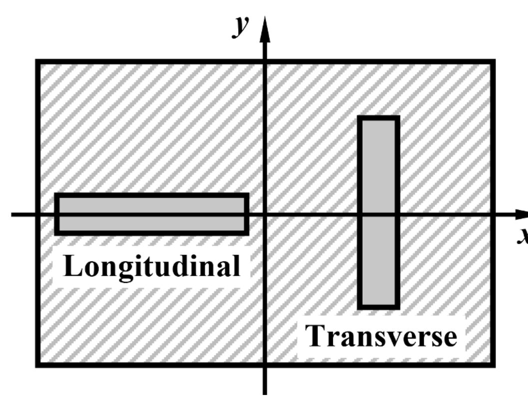

Fig. 3. Loading conditions for unidirectional and multi-directional hybrid composites.

Table 1
Potential layups for unidirectional hybrid composite.

Number of glass/epoxy plies	Layup
0	$[0]_{8C}$
1	$[0_G/0_{7C}]$
2	$[0_{2G}/0_{6C}]$
	$[0_{4C}/0_{2G}/0_{2C}]$
3	$[0_{2G}/0_{2C}/0_G/0_{3C}]$
	$[0_G/0_{3C}/0_{2G}/0_{2C}]$
	$[0_{3C}/0_{3G}/0_{2C}]$
4	$[0_{2G}/0_{2C}]_2$
5	$[0_{2G}/0_{2C}/0_{3G}/0_C]$
6	$[0_{2G}/0_C/0_{4G}/0_C]$
	$[0_{3G}/0_C]_2$
7	$[0_{7G}/0_C]$
8	$[0]_{8G}$

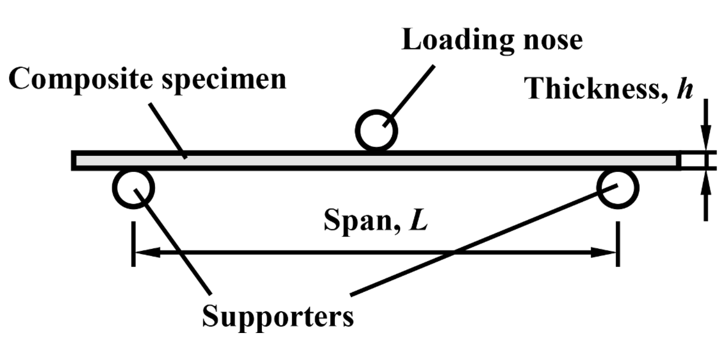

Fig. 2. A composite specimen in three-point bending.

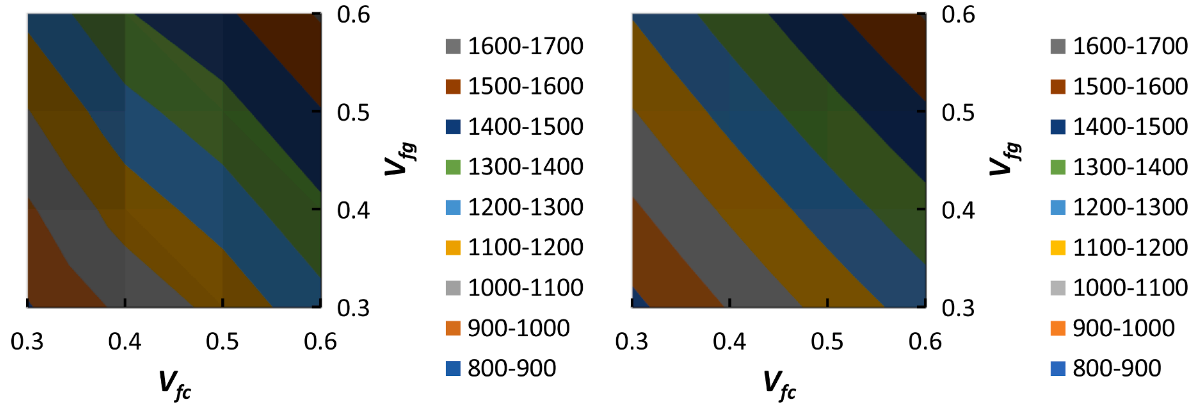

Fig. 4. Response surface of flexural strength for layup $[0_{2G}/0_{2C}/0_G/0_{3C}]$. Left: FEA; right: regression model.

Table 2
Potential layups for multi-directional hybrid composite.

Number of glass/epoxy plies	Layup
0	$[90/0]_{3C}$
	$[90/0_2/90_2/0]_C$
1	$[(90/0/90)_C/0_G/(90/0)_C]$
	$[(90/0)_C/0_G/(90_2/0)_C]$
	$[(90/0_2)_C/90_G/(90/0)_C]$
2	$[(90/0)_G/(90/0)_{2C}]$
	$[(90/0)_C/(90/0)_G/(90/0)_C]$
	$[(90/0)_C/(0/90)_G/(90/0)_C]$
3	$[(90/0)_G/90_C/0_G/(90/0)_C]$
4	$[(90/0)_{2G}/(90/0)_C]$

scribed displacement of 7 mm is applied to the loading roller. Frictional contact is defined between the rollers and the composite specimen. Previous research by the present author [19] shows the contact stress has negligible effect in this case. The dominant failure mode for all similar hybrid composite specimens under three-point bending load is in-plane failure for large span-to-thickness ratios.

Linear static analysis is conducted to simulate the first ply failure. Upon completion of simulation, the reaction force due to the prescribed displacement and the maximum failure criterion are obtained. The failure load is calculated using the reaction force and, which is the ratio of the maximum stress to the failure stress. When the maximum failure criterion is less than 1, no failure occurs, and when the maximum failure criterion is greater than or equal to 1, failure occurs somewhere in the specimen. The flexural strength is obtained by

$$S_F = \frac{3PL}{2bh^2 f} \quad (3)$$

where P is the reaction force, L is the span, b is the width, h is the thickness, and f is the maximum failure criterion. This developed modelling approach was validated against the experimental data [10].

The loading conditions are schematically shown in Fig. 3. For the unidirectional hybrid composite, only one loading direction needs to be considered, i.e., longitudinal. The composite specimen is bent about the y axis. For the multi-directional hybrid composite, two loading conditions are considered. In addition to longitudinal, transverse loading is also considered, i.e., the composite specimen is bent about the x axis.

Design rule-based optimisation

It is shown from previous research that comparable flexural strengths can be achieved when the carbon/epoxy plies are replaced by glass/epoxy plies, i.e., via hybridisation. When a carbon/epoxy ply is replaced by a glass/epoxy ply, the cost will be reduced but the density will increase.

For the unidirectional carbon and glass fibre reinforced hybrid composites, three design rules are derived from previous studies [10, 19].

Rule 1: Glass/epoxy plies should be placed on the compressive side and carbon/epoxy plies should be placed on the tensile side.

Rule 2: Glass/epoxy plies should be placed on the tensile side close to the neutral plane.

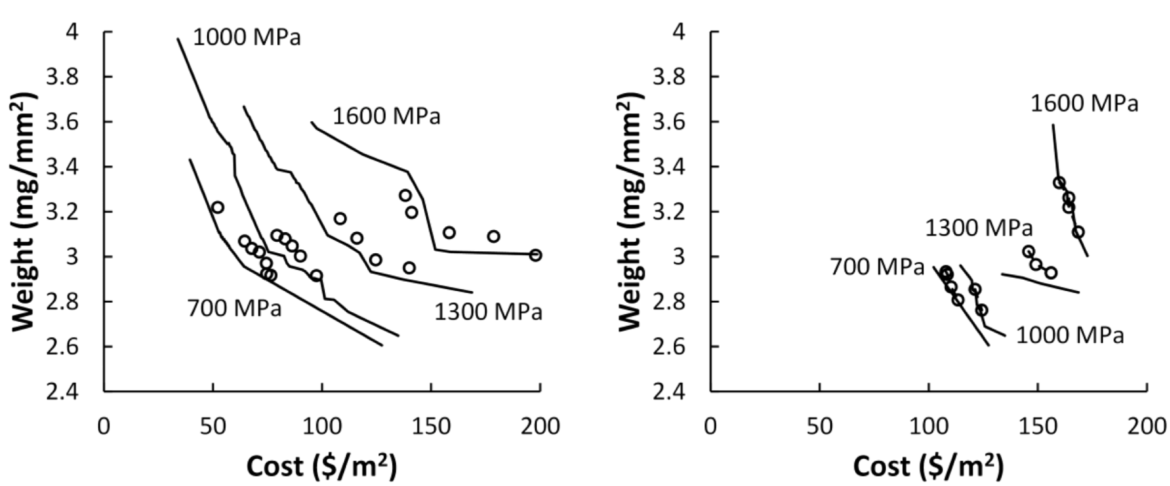

Fig. 5. Pareto fronts of unidirectional hybrid composites reinforced by carbon and E glass fibres (left) and carbon and S-2 glass fibres (right) from design rule-based optimisation (lines) and NSGA-II optimisation (symbols).

Table 3
Selected candidate points from design rule optimisation for carbon and E glass fibre reinforced hybrid composite with minimum flexural strength 700 MPa.

	V_{fc}	V_{fg}	Layup (ply 8 – ply 1)	Flexural strength (MPa)	Cost ($/m^2)	Weight (mg/mm^2)	Hybrid ratio
1	–	0.42	$[0]_{8G}$	703.03	39.46	3.4316	1.0000
2	0.3	0.335	$[0_{7G}/0_{C}]$	703.31	52.75	3.1068	0.8866
3	0.35	0.32	$[0_{7G}/0_{C}]$	703.54	54.71	3.0765	0.8649
4	0.4	0.31	$[0_{7G}/0_{C}]$	707.03	56.55	3.0593	0.8444
5	0.43	0.3	$[0_{7G}/0_{C}]$	701.70	57.75	3.0386	0.8300
6	0.3	0.3	$[0_{3G}/0_{C}]_2$	712.99	64.22	2.9570	0.7500
7	0.3	0.3	$[0_{2G}/0_{C}/0_{4G}/0_{C}]$	804.08	64.22	2.9570	0.7500
8	0.3	0.3	$[0_{2G}/0_{2C}/0_{3G}/0_{C}]$	842.85	74.75	2.8985	0.6250
9	0.3	0.3	$[0_{2G}/0_{2C}]_2$	891.74	85.28	2.8400	0.5000
10	0.3	0.3	$[0_{G}/0_{3C}/0_{2G}/0_{2C}]$	911.50	95.81	2.7815	0.3750
11	0.3	0.3	$[0_{2G}/0_{2C}/0_{G}/0_{3C}]$	891.70	95.81	2.7815	0.3750
12	0.3	0.3	$[0_{2G}/0_{6C}]$	896.31	106.34	2.7230	0.2500
13	0.3	0.3	$[0_{G}/0_{7C}]$	903.03	116.87	2.6645	0.1250
14	0.3	–	$[0]_{8C}$	948.14	127.40	2.6060	0.0000

Rule 3: For sandwich type hybrid composites, carbon/epoxy plies should be placed on the outside and glass/epoxy plies should be placed inside.

It is also noted that Rules 1 and 2 can be combined and Rules 2 and 3 can be combined. In this study, the unidirectional hybrid composite specimen comprises eight plies of 0.25 mm thickness. According to these design rules, potential layups are shown in Table 1. For all layups, from left to right corresponds to from compression (ply 8) to tension (ply 1).

For each potential layup, the fibre volume fractions need to be altered so that the resulting flexural strength is just above the required value. For this purpose, the relationship between the flexural strength and fibre volume fractions is needed. For each given layup, the fibre volume fractions of both carbon/epoxy and glass/epoxy plies were varied between 0.3 and 0.6. A response surface for the flexural strength was then be constructed. As an example, the response surface for layup $[0_{2G}/0_{2C}/0_{G}/0_{3C}]$ is shown in Fig. 4. In Fig. 4, the contour lines show the flexural strengths in MPa. It is seen the flexural strength in general increases with both fibre volume fractions.

The regression model for the flexural strength is given by

$$S_F = c_0 + c_{c1}V_{fc} + c_{c2}V_{fc}^2 + c_{g1}V_{fg} + c_{g2}V_{fg}^2 + c_{cg}V_{fc}V_{fg} \tag{4}$$

where c_0, c_{c1}, c_{c2}, c_{g1}, c_{g2}, and c_{cg} are constants to be determined by Least Squares Estimation (LSE). When determining these constants, constraints are applied so that the flexural strengths from regression formula are less than or equal to those from FEA. The constants for the regression formulas are given in the Appendix.

Given the required flexural strength, the fibre volume fractions of carbon/epoxy and glass/epoxy plies can be obtained by solving the equation $c_{g2}V_{fg}^2 + (c_{g1} + c_{cg}V_{fc})V_{fg} - (S_F - c_0 - c_{c1}V_{fc} - c_{c2}V_{fc}^2) = 0$. If V_{fc} is given, V_{fg} is given by

Table 4
Optimal carbon and E glass fibre reinforced hybrid composites for minimum flexural strength 1000 MPa from design rules.

	V_{fc}	V_{fg}	Layup (ply 8 – ply 1)	Flexural strength (MPa)	Cost ($/m^2)	Weight (mg/mm^2)	Hybrid ratio
1		0.6	$[0]_{8G}$	1004.29	33.92	3.9680	1.0000
2	0.3	0.545	$[0_{7G}/0_{C}]$	1006.44	47.09	3.6543	0.9271
3	0.35	0.525	$[0_{7G}/0_{C}]$	1005.25	49.19	3.6111	0.9130
4	0.41	0.505	$[0_{7G}/0_{C}]$	1004.92	51.60	3.5696	0.8961
5	0.46	0.49	$[0_{7G}/0_{C}]$	1003.56	53.57	3.5393	0.8817
6	0.5	0.48	$[0_{7G}/0_{C}]$	1003.60	55.09	3.5204	0.8705
7	0.54	0.47	$[0_{7G}/0_{C}]$	1001.99	56.61	3.5014	0.8590
8	0.59	0.46	$[0_{7G}/0_{C}]$	1002.06	58.44	3.4842	0.8451
9	0.3	0.525	$[0_{3G}/0_{C}]_2$	1003.18	59.02	3.4599	0.8400
10	0.31	0.52	$[0_{3G}/0_{C}]_2$	1004.73	59.76	3.4523	0.8342
11	0.3	0.48	$[0_{2G}/0_{C}/0_{4G}/0_{C}]$	1002.98	60.06	3.3593	0.8276
12	0.35	0.425	$[0_{2G}/0_{C}/0_{4G}/0_{C}]$	1001.08	64.46	3.2541	0.7846
13	0.4	0.375	$[0_{2G}/0_{C}/0_{4G}/0_{C}]$	1007.78	68.74	3.1601	0.7377
14	0.45	0.33	$[0_{2G}/0_{C}/0_{4G}/0_{C}]$	1021.41	72.90	3.0773	0.6875
15	0.48	0.3	$[0_{2G}/0_{C}/0_{4G}/0_{C}]$	1004.97	75.47	3.0209	0.6522
16	0.39	0.33	$[0_{2G}/0_{2C}/0_{3G}/0_{C}]$	1004.17	82.61	3.0023	0.5851
17	0.4	0.31	$[0_{2G}/0_{2C}/0_{3G}/0_{C}]$	1001.32	83.93	2.9704	0.5636
18	0.41	0.3	$[0_{2G}/0_{2C}/0_{3G}/0_{C}]$	1007.72	85.06	2.9571	0.5495
19	0.35	0.345	$[0_{2G}/0_{2C}]_2$	1015.29	90.84	2.9426	0.4964
20	0.36	0.335	$[0_{2G}/0_{2C}]_2$	1022.92	92.24	2.9348	0.4820
21	0.37	0.32	$[0_{2G}/0_{2C}]_2$	1026.34	93.72	2.9195	0.4638
22	0.3	0.415	$[0_{2G}/0_{2C}/0_{G}/0_{3C}]$	1000.65	94.48	2.9100	0.4536
23	0.38	0.305	$[0_{2G}/0_{2C}]_2$	1030.18	95.20	2.9043	0.4453
24	0.39	0.3	$[0_{2G}/0_{2C}]_2$	1035.85	96.53	2.9039	0.4348
25	0.32	0.39	$[0_{2G}/0_{2C}/0_{G}/0_{3C}]$	1008.69	97.90	2.8998	0.4224
26	0.33	0.38	$[0_{2G}/0_{2C}/0_{G}/0_{3C}]$	1015.43	99.57	2.8975	0.4086
27	0.335	0.3	$[0_{3C}/0_{3G}/0_{2C}]$	1011.58	101.28	2.8126	0.3495
28	0.3	0.415	$[0_{2G}/0_{6C}]$	1002.74	105.45	2.8087	0.3156
29	0.33	0.3	$[0_{4C}/0_{2G}/0_{2C}]$	1013.61	111.97	2.7550	0.2326
30	0.3	0.49	$[0_{G}/0_{7C}]$	1002.39	116.14	2.7353	0.1892
31	0.33		$[0]_{8C}$	1011.73	134.90	2.6486	0.0000

Table 5
Optimal carbon and E glass fibre reinforced hybrid composites for minimum flexural strength 1300 MPa from design rules.

	V_{fc}	V_{fg}	Layup (ply 8 – ply 1)	Flexural strength (MPa)	Cost ($/m^2)	Weight (mg/mm^2)	Hybrid ratio
1	0.41	0.6	$[0_{2G}/0_C/0_{4G}/0_C]$	1314.78	64.17	3.6666	0.8145
2	0.45	0.565	$[0_{2G}/0_C/0_{4G}/0_C]$	1313.94	67.47	3.6025	0.7902
3	0.5	0.52	$[0_{2G}/0_C/0_{4G}/0_C]$	1313.32	71.64	3.5197	0.7573
4	0.55	0.48	$[0_{2G}/0_C/0_{4G}/0_C]$	1307.78	75.69	3.4481	0.7236
5	0.6	0.445	$[0_{2G}/0_C/0_{4G}/0_C]$	1316.24	79.62	3.3876	0.6899
6	0.46	0.51	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1304.29	85.71	3.3748	0.6489
7	0.47	0.495	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1300.16	86.93	3.3522	0.6371
8	0.48	0.485	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1302.64	88.06	3.3389	0.6274
9	0.36	0.6	$[0_{2G}/0_{2C}]_2$	1328.02	88.16	3.3296	0.6250
10	0.49	0.475	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1305.32	89.19	3.3256	0.6177
11	0.37	0.59	$[0_{2G}/0_{2C}]_2$	1331.32	89.56	3.3218	0.6146
12	0.5	0.46	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1302.28	90.42	3.3030	0.6053
13	0.55	0.395	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1304.84	96.36	3.2086	0.5448
14	0.6	0.32	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1309.27	102.49	3.0955	0.4706
15	0.52	0.335	$[0_{2C}/0_{2G}]_s$	1300.07	112.24	3.0484	0.3918
16	0.37	0.595	$[0_{2G}/0_{6C}]$	1342.82	117.19	3.0173	0.3490
17	0.47	0.3	$[0_{3C}/0_{3G}/0_{2C}]$	1309.62	122.37	2.9324	0.2769
18	0.465	0.3	$[0_{4C}/0_{2G}/0_{2C}]$	1314.69	137.28	2.8987	0.1770
19	0.465		$[0]_{8C}$	1310.53	168.65	2.8403	0.0000

$$V_{fg} = \frac{-\left(c_{g1} + c_{cg}V_{fc}\right) + \sqrt{\left(c_{g1} + c_{cg}V_{fc}\right)^2 + 4c_{g2}\left(S_F - c_0 - c_{c1}V_{fc} - c_{c2}V_{fc}^2\right)}}{2c_{g2}} \tag{5}$$

For the full carbon fibre composite, the regression model for the flexural strength is given by

$$S_F = c_0 + c_{c1}V_{fc} + c_{c2}V_{fc}^2 \tag{6}$$

Solving the equation, the fibre volume fraction for any given flexural strength is given by

$$V_{fc} = \frac{-c_{c1} + \sqrt{c_{c1}^2 + 4c_{c2}(S_F - c_0)}}{2c_{c2}} \tag{7}$$

The flexural strength of the full glass fibre composite is given by a similar regression formula.

Table 6
Optimal carbon and E glass fibre reinforced hybrid composites for minimum flexural strength 1600 MPa from design rules.

	V_{fc}	V_{fg}	Layup (ply 8 – ply 1)	Flexural strength (MPa)	Cost ($/m^2)	Weight (mg/mm^2)	Hybrid ratio
1	0.58	0.595	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1604.91	95.32	3.5970	0.6310
2	0.59	0.585	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1605.95	96.45	3.5837	0.6230
3	0.6	0.575	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1605.89	97.58	3.5704	0.6150
4	0.6	0.57	$[0_{2G}/0_{2C}]_2$	1607.54	118.62	3.4553	0.4872
5	0.6	0.595	$[0_{2G}/0_{2C}/0_G/0_{3C}]$	1605.81	139.28	3.3774	0.3730
6	0.59	0.6	$[0_{3C}/0_{3G}/0_{2C}]$	1608.88	146.14	3.2554	0.3789
7	0.59	0.3	$[0_{3C}/0_{3G}/0_{2C}]$	1612.95	151.91	3.0319	0.2338
8	0.58	0.3	$[0_{4C}/0_{2G}/0_{2C}]$	1606.12	158.84	3.0212	0.1471
9	0.585		$[0]_{8C}$	1613.23	198.65	3.0107	0.0000

Table 7
Optimal carbon and S-2 glass fibre reinforced hybrid composites for minimum flexural strength 700 MPa from design rules.

	V_{fc}	V_{fg}	Layup (ply 8 – ply 1)	Flexural strength (MPa)	Cost ($/m^2)	Weight (mg/mm^2)	Hybrid ratio
1	0.3	0.3	$[0_{7G}/0_C]$	717.31	102.25	2.9525	0.8750
2	0.3	0.3	$[0_{2G}/0_C/0_{4G}/0_C]$	839.08	105.85	2.9030	0.7500
3	0.3	0.3	$[0_{3G}/0_C]_2$	761.56	105.85	2.9030	0.7500
4	0.3	0.3	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	870.70	109.44	2.8535	0.6250
5	0.3	0.3	$[0_{2G}/0_{2C}]_2$	915.82	113.03	2.8040	0.5000
6	0.3	0.3	$[0_{2G}/0_{2C}/0_G/0_{3C}]$	916.30	116.62	2.7545	0.3750
7	0.3	0.3	$[0_G/0_{3C}/0_{2G}/0_{2C}]$	944.93	116.62	2.7545	0.3750
8	0.3	0.3	$[0_{3C}/0_{3G}/0_{2C}]$	938.29	116.62	2.7545	0.3750
9	0.3	0.3	$[0_{2G}/0_{6C}]$	919.00	120.22	2.7050	0.2500
10	0.3	0.3	$[0_{4C}/0_{2G}/0_{2C}]$	949.25	120.22	2.7050	0.2500
11	0.3	0.3	$[0_G/0_{7C}]$	936.62	123.81	2.6555	0.1250
12	0.3		$[0]_{8C}$	948.14	127.40	2.6060	0.0000

In this study, four minimum flexural strengths, 700, 1000, 1300, and 1600 MPa, were chosen. Given the required minimum flexural strength, the potential layups in Table 1 were investigated by altering the fibre volume fractions. The cost and weight were then recorded. After all layups were investigated, the weight was plotted versus the cost and the lower bound was found to form the Pareto front. It should be noted that for a given layup, more than one fibre volume fraction combination can meet the required flexural strength.

For comparison, optimisation was also conducted by NSGA-II using the built-in optimiser in Ansys Workbench. The optimisation also started from a full carbon fibre composite. The number of initial samples was 83

Table 8
Optimal carbon and S-2 glass fibre reinforced hybrid composites for minimum flexural strength 1000 MPa from design rules.

	V_{fc}	V_{fg}	Layup (ply 8 – ply 1)	Flexural strength (MPa)	Cost ($/m^2)	Weight (mg/mm^2)	Hybrid ratio
1	0.43	0.305	$[0_{2G}/0_C/0_{4G}/0_C]$	1003.13	114.55	2.9594	0.6803
2	0.4	0.3	$[0_{2G}/0_{2C}/0_{3G}/0_C]$	1014.34	118.81	2.9068	0.5556
3	0.32	0.355	$[0_{2G}/0_{2C}]_2$	1003.49	119.77	2.8936	0.5259
4	0.33	0.34	$[0_{2G}/0_{2C}]_2$	1004.02	119.86	2.8801	0.5075
5	0.34	0.325	$[0_{2G}/0_{2C}]_2$	1004.95	119.96	2.8667	0.4887
6	0.35	0.31	$[0_{2G}/0_{2C}]_2$	1006.32	120.05	2.8532	0.4697
7	0.3	0.385	$[0_{2G}/0_{2C}/0_G/0_{3C}]$	1005.01	121.54	2.8418	0.4350
8	0.335	0.3	$[0_{3C}/0_{3G}/0_{2C}]$	1011.69	122.09	2.7856	0.3495
9	0.3	0.385	$[0_{2G}/0_{6C}]$	1005.99	123.49	2.7632	0.2996
10	0.3	0.4	$[0_G/0_{7C}]$	1000.19	125.74	2.6898	0.1600
11	0.33		$[0]_{8C}$	1011.73	134.90	2.6486	0.0000

Table 9
Optimal carbon and S-2 glass fibre reinforced hybrid composites for minimum flexural strength 1300 MPa from design rules.

	V_{fc}	V_{fg}	Layup (ply 8 – ply 1)	Flexural strength (MPa)	Cost ($/m^2)	Weight (mg/mm^2)	Hybrid ratio
1	0.31	0.6	$[0_{2G}/0_{6C}]$	1313.96	133.66	2.9212	0.3922
2	0.47	0.3	$[0_{3C}/0_{3G}/0_{2C}]$	1309.39	143.19	2.9054	0.2769
3	0.465	0.3	$[0_{4C}/0_{2G}/0_{2C}]$	1314.20	151.15	2.8807	0.1770
4	0.465		$[0]_{8C}$	1310.53	168.65	2.8403	0.0000

and the number of samples each iteration was 83. Up to 10 candidate points were found from optimisation.

Secondly, the multi-directional hybrid composites were considered. The composite specimen comprised six plies of 0.25 mm thickness. Both fibre volume fractions were varied between 0.3 and 0.7. For the purpose of achieving flexural strengths in two directions, half of the plies are in 0° and the other half are in 90° The potential layups are shown in Table 2. For all layups, from left to right corresponds to from compression (ply 6) to tension (ply 1). When the number of glass plies is greater than 5, the required flexural strengths cannot be achieved.

For a given layup, the fibre volume fractions of both carbon/epoxy and glass/epoxy plies were varied between 0.3 and 0.7, and optimisation was done by constructing a response surface in a similar approach for the unidirectional hybrid composite. Five minimum flexural strengths, 500, 600, 700, 800, and 900 MPa, were chosen.

For comparison, optimisation was also conducted by NSGA-II using the built-in optimiser in Ansys Workbench. The optimisation also started from a full carbon fibre composite. The number of initial samples was 116 and the number of samples each iteration was 116. Up to 10 candidate points were found from optimisation.

Table 10
Optimal carbon and S-2 glass fibre reinforced hybrid composites for minimum flexural strength 1600 MPa from design rules.

	V_{fc}	V_{fg}	Layup (ply 8 – ply 1)	Flexural strength (MPa)	Cost ($/m^2)	Weight (mg/mm^2)	Hybrid ratio
1	0.6	0.58	$[0_{2G}/0_C/0_{4G}/0_C]$	1605.16	156.98	3.5849	0.7436
2	0.49	0.595	$[0_{2G}/0_{2C}]_2$	1608.97	159.52	3.3431	0.5484
3	0.5	0.585	$[0_{2G}/0_{2C}]_2$	1608.06	160.00	3.3365	0.5392
4	0.51	0.575	$[0_{2G}/0_{2C}]_2$	1607.35	160.48	3.3299	0.5300
5	0.52	0.565	$[0_{2G}/0_{2C}]_2$	1606.83	160.96	3.3233	0.5207
6	0.53	0.555	$[0_{2G}/0_{2C}]_2$	1606.51	161.44	3.3167	0.5115
7	0.54	0.545	$[0_{2G}/0_{2C}]_2$	1606.40	161.92	3.3101	0.5023
8	0.55	0.535	$[0_{2G}/0_{2C}]_2$	1606.50	162.40	3.3035	0.4931
9	0.56	0.525	$[0_{2G}/0_{2C}]_2$	1606.82	162.88	3.2969	0.4839
10	0.57	0.515	$[0_{2G}/0_{2C}]_2$	1607.36	163.36	3.2903	0.4747
11	0.49	0.6	$[0_{2G}/0_{2C}/0_G/0_{3C}]$	1620.26	163.66	3.2314	0.4235
12	0.5	0.59	$[0_{2G}/0_{2C}/0_G/0_{3C}]$	1618.96	164.64	3.2300	0.4145
13	0.51	0.58	$[0_{2G}/0_{2C}/0_G/0_{3C}]$	1617.84	165.63	3.2286	0.4056
14	0.49	0.6	$[0_{2G}/0_{6C}]$	1623.17	167.41	3.1129	0.2899
15	0.58	0.3	$[0_{4C}/0_{2G}/0_{2C}]$	1605.58	172.72	3.0032	0.1471

Results and discussion

Unidirectional hybrid composites

The Pareto fronts of the unidirectional carbon and E glass fibre reinforced hybrid composite from the design rule-based optimisation and NSGA-II optimisation are shown in Fig. 5. For all required minimum flexural strengths, the design rule-based optimisation gives a wider range of Pareto front than the NSGA-II optimisation. The Pareto front from the NSGA-II optimisation has slightly higher cost and weight compared to that from the design rule-based optimisation.

It is shown that the carbon and E glass fibre reinforced hybrid composite offers lower material cost but higher weight. The largest differences in cost and weight are seen when the required flexural strength is 700 MPa. When the required flexural strength is 1000 or 1300 MPa, E glass fibre offers some optimal designs with low cost, and some of these designs have comparable weight as S-2 glass fibre reinforced hybrid composites. The cost benefit diminishes when the required flexural strength is 1600 MPa, with E glass and S-2 glass fibre reinforced hybrid composites having similar costs and weights.

The selected candidates from optimisation are shown in Tables 3–10. It is seen more candidates have been found for the carbon and E glass fibre reinforced hybrid composite compared to the carbon and S-2 glass fibre reinforced one. For the carbon and E glass fibre reinforced hybrid composite, the full glass layup $[0]_{8G}$ can be one of the candidates when the minimum flexural strength is 700 MPa or 1000 MPa. Layups $[0_{7G}/0_C]$, $[0_{2G}/0_{6C}]$, and $[0_G/0_{7C}]$ are found by using Rule 1. Layups $[0_{3G}/0_C]_2$, $[0_{2G}/0_C/0_{4G}/0_C]$, $[0_{2G}/0_{2C}/0_{3G}/0_C]$, $[0_{2G}/0_{2C}]_2$, and $[0_{2G}/0_{2C}/0_G/0_{3C}]$ are found by applying Rules 1 and 2. Layups $[0_{3C}/0_{3G}/0_{2C}]$ and

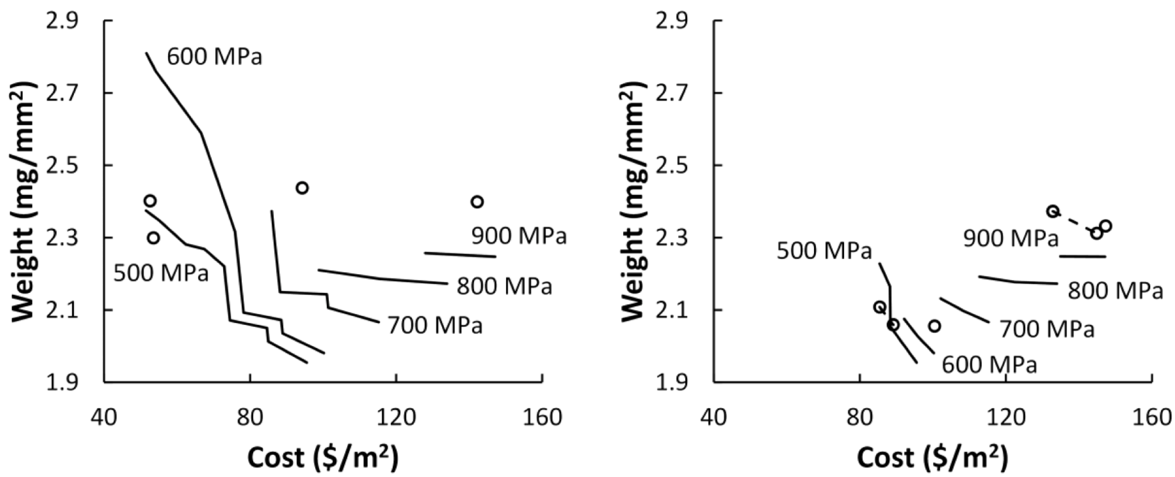

Fig. 6. Pareto fronts of multi-directional hybrid composites reinforced by carbon and E glass fibres (left) and carbon and S-2 glass fibres (right) from design rule-based optimisation (lines) and NSGA-II optimisation (symbols).

Table 11
Summary of layups of candidates for carbon and E glass fibre reinforced hybrid composite from design rule-based optimisation.

Layup	Flexural strength 500 MPa	600 MPa	700 MPa	800 MPa	900 MPa
$[(90/0)_{2G}/(90/0)_C]$	X	X			
$[(90/0)_G/90_C/0_G/(90/0)_C]$	X	X			
$[(90/0)_C/(90/0)_G/(90/0)_C]$	X	X	X		
$[(90/0)_G/(90/0)_{2C}]$	X				
$[(90/0)_C/(0/90)_G/(90/0)_C]$				X	
$[(90/0/90)_C/0_G/(90/0)_C]$	X	X	X		
$[(90/0_2)_C/90_G/(90/0)_C]$				X	
$[(90/0)_C/0_G/(90_2/0)_C]$					X
$[90/0]_{3C}$	X	X	X		
$[90/0_2/90_2/0]_C$				X	X

Table 12
Summary of layups of candidates for carbon and S-2 glass fibre reinforced hybrid composite from design rule-based optimisation.

Layup	Flexural strength 500 MPa	600 MPa	700 MPa	800 MPa	900 MPa
$[(90/0)_{2G}/(90/0)_C]$	X				
$[(90/0)_G/90_C/0_G/(90/0)_C]$	X				
$[(90/0)_C/(90/0)_G/(90/0)_C]$	X	X	X		
$[(90/0)_C/(0/90)_G/(90/0)_C]$	X			X	
$[(90/0/90)_C/0_G/(90/0)_C]$	X	X	X		
$[(90/0_2)_C/90_G/(90/0)_C]$				X	
$[(90/0)_C/0_G/(90_2/0)_C]$					X
$[90/0]_{3C}$	X	X	X		
$[90/0_2/90_2/0]_C$				X	X

$[0_{4C}/0_{2G}/0_{2C}]$ are found by applying Rules 2 and 3. In addition, the full carbon layup $[0]_{8C}$ is also one of the candidates.

The full carbon ($[0]_{8C}$) and sandwich ($[0_{3C}/0_{3G}/0_{2C}]$ and $[0_{4C}/0_{2G}/0_{2C}]$) layups are optimal layups for high required flexural strengths. Layups $[0_{2G}/0_{2C}/0_{3G}/0_C]$ and $[0_{2G}/0_{2C}]$ are optimal layups for all required flexural strengths.

For the carbon and S-2 glass fibre reinforced hybrid composite, it is interesting to note that the carbon layup $[0]_{8C}$ is not a candidate when the minimum required flexural strength is 1600 MPa. $[0_{2G}/0_{6C}]$ is the optimal layup for all required flexural strengths.

Multi-directional hybrid composites

The Pareto fronts of the multi-directional carbon and E glass fibre reinforced hybrid composite from the design rule-based optimisation and NSGA-II optimisation are shown in Fig. 6. For all required minimum flexural strengths, the design rule-based optimisation gives a wider range of Pareto front than the NSGA-II optimisation. It is seen for lower required flexural strengths, e.g., 500 and 600 MPa, the Pareto front from the NSGA-II optimisation has similar cost and weight compared to that from the design rule-based optimisation. However, for higher required flexural strengths, the Pareto front from the NSGA-II optimisation has much higher cost and weight compared to that from the design rule-based optimisation. It is also shown that very few candidates could be found from the NSGA-II optimisation.

When the two types of glass fibres are compared, it is shown that the carbon and E glass fibre reinforced hybrid composite offers a wider range of candidates. The most significant advantage of E glass fibre is the cost. It is seen some low-cost candidates have been found for the carbon and E glass fibre reinforced hybrid composite.

The layups from the design rule-based optimisation are summarised in Tables 11 and 12. When the required flexural strength is between 500 and 700 MPa, the optimal layups can be obtained based on $[90/0]_3$. For the carbon and E glass fibre reinforced hybrid composite, when the required flexural strength is 500 or 600 MPa, glass/epoxy plies can be placed on the compressive side or at the inner of the composite. This also applies to the carbon and S-2 glass fibre reinforced hybrid composite when the required flexural strength is 500 MPa. When the flexural strength is 700 MPa, glass/epoxy plies can only be placed at the inner of the composite. When the required flexural strength is greater than 700 MPa, the optimal layups can be obtained based on $[90/0_2/90_2/0]$.

Table 13
Optimal multi-directional carbon and E glass fibre reinforced hybrid composites from design rules.

	V_{fc}	V_{fg}	Layup (ply 6 – ply 1)	Flexural strength (MPa)	Transverse flexural strength (MPa)	Cost ($/m^2)	Weight (mg/mm^2)	Hybrid ratio
500 MPa								
1	0.3	0.425	[(90/0)$_{2G}$/(90/0)$_C$]	639.74	500.98	51.51	2.3748	0.7391
2	0.35	0.395	[(90/0)$_{2G}$/(90/0)$_C$]	616.88	500.35	55.09	2.3478	0.6930
3	0.3	0.435	[(90/0)$_G$/90$_C$/0$_G$/(90/0)$_C$]	640.97	501.68	62.40	2.2809	0.5918
4	0.35	0.4	[(90/0)$_G$/90$_C$/0$_G$/(90/0)$_C$]	615.31	500.46	67.49	2.2684	0.5333
5	0.3	0.5	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	605.53	568.73	72.95	2.2205	0.6250
6	0.3	0.44	[(90/0)$_G$/(90/0)$_{2C}$]	671.47	501.65	73.41	2.1758	0.4231
7	0.3	0.4	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	602.09	567.93	73.72	2.1460	0.4000
8	0.3	0.3	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	600.07	567.06	74.49	2.0715	0.3333
9	0.3	0.4	[(90/0/90)$_C$/0$_G$/(90/0)$_C$]	598.27	575.05	84.64	2.0503	0.2105
10	0.3	0.3	[(90/0/90)$_C$/0$_G$/(90/0)$_C$]	598.41	575.19	85.02	2.0130	0.1667
11	0.3	–	[90/0]$_{3C}$	598.75	575.34	95.55	1.9545	0.0000
600 MPa								
1	0.37	0.7	[(90/0)$_{2G}$/(90/0)$_C$]	662.20	604.85	51.65	2.8094	0.7910
2	0.4	0.66	[(90/0)$_{2G}$/(90/0)$_C$]	714.37	600.92	54.14	2.7604	0.7674
3	0.375	0.675	[(90/0)$_G$/90$_C$/0$_G$/(90/0)$_C$]	670.47	600.73	66.66	2.5890	0.6429
4	0.33	0.6	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	646.33	605.36	75.93	2.3163	0.4762
5	0.33	0.5	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	641.83	604.68	76.70	2.2418	0.4310
6	0.33	0.4	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	638.55	603.96	77.47	2.1673	0.3774
7	0.33	0.3	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	636.10	603.18	78.24	2.0928	0.3125
8	0.325	0.4	[(90/0/90)$_C$/0$_G$/(90/0)$_C$]	628.62	605.68	88.54	2.0724	0.1975
9	0.325	0.3	[(90/0/90)$_C$/0$_G$/(90/0)$_C$]	628.74	605.83	88.93	2.0352	0.1558
10	0.325	–	[90/0]$_{3C}$	629.22	605.95	100.24	1.9811	0.0000
700 MPa								
1	0.41	0.6	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	745.86	703.48	85.93	2.3731	0.4225
2	0.41	0.5	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	741.41	702.99	86.70	2.2986	0.3788
3	0.41	0.4	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	738.17	703.10	87.47	2.2241	0.3279
4	0.41	0.3	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	735.75	701.80	88.24	2.1496	0.2679
5	0.405	0.4	[(90/0/90)$_C$/0$_G$/(90/0)$_C$]	729.03	705.77	101.04	2.1434	0.1649
6	0.405	0.3	[(90/0/90)$_C$/0$_G$/(90/0)$_C$]	729.13	705.93	101.43	2.1062	0.1290
7	0.405	–	[90/0]$_{3C}$	730.05	705.97	115.24	2.0663	0.0000
800 MPa								
1	0.495	0.3	[(90/0)$_C$/(0/90)$_G$/(90/0)$_C$]	887.89	802.70	98.865	2.2100	0.2326
2	0.495	0.3	[(90/0$_2$)$_C$/90$_G$/(90/0)$_C$]	1034.78	802.72	115.49	2.1861	0.1081
3	0.505	–	[90/0$_2$/90$_2$/0]$_C$	1052.56	801.59	133.99	2.1728	0.0000
900 MPa								
1	0.575	0.3	[(90/0)$_C$/0$_G$/(90$_2$/0)$_C$]	1008.99	902.05	127.99	2.2571	0.0945
2	0.575	–	[90/0$_2$/90$_2$/0]$_C$	1184.81	902.68	147.11	2.2474	0.0000

Table 14
Optimal multi-directional carbon and S-2 glass fibre reinforced hybrid composites from design rules.

	V_{fc}	V_{fg}	Layup (ply 6 – ply 1)	Flexural strength (MPa)	Transverse flexural strength (MPa)	Cost ($/m^2)	Weight (mg/mm^2)	Hybrid ratio
500 MPa								
1	0.3	0.355	[(90/0)$_{2G}$/(90/0)$_C$]	594.88	501.87	85.42	2.2279	0.7030
2	0.3	0.36	[(90/0)$_G$/90$_C$/0$_G$/(90/0)$_C$]	594.49	500.74	88.24	2.1647	0.5455
3	0.3	0.3	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	600.10	567.90	88.37	2.0535	0.3333
4	0.3	0.3	[(90/0)$_C$/(0/90)$_G$/(90/0)$_C$]	648.26	555.94	88.37	2.0535	0.3333
5	0.3	0.3	[(90/0/90)$_C$/0$_G$/(90/0)$_C$]	598.36	575.20	91.96	2.0040	0.1667
6	0.3	–	[90/0]$_{3C}$	598.75	575.34	95.55	1.9545	0.0000
600 MPa								
1	0.33	0.3	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	636.14	603.98	92.12	2.0748	0.3125
2	0.325	0.3	[(90/0/90)$_C$/0$_G$/(90/0)$_C$]	628.70	605.83	95.86	2.0262	0.1558
3	0.325	–	[90/0]$_{3C}$	629.22	605.95	100.24	1.9811	0.0000
700 MPa								
1	0.41	0.3	[(90/0)$_C$/(90/0)$_G$/(90/0)$_C$]	735.82	702.53	102.12	2.1316	0.2679
2	0.405	0.3	[(90/0/90)$_C$/0$_G$/(90/0)$_C$]	729.11	705.93	108.36	2.0972	0.1290
3	0.405	–	[90/0]$_{3C}$	730.05	705.97	115.24	2.0663	0.0000
800 MPa								
1	0.495	0.3	[(90/0)$_C$/(0/90)$_G$/(90/0)$_C$]	896.38	804.50	112.74	2.1920	0.2326
2	0.495	0.3	[(90/0$_2$)$_C$/90$_G$/(90/0)$_C$]	1034.77	804.68	122.43	2.1771	0.1081
3	0.505	–	[90/0$_2$/90$_2$/0]$_C$	1052.56	801.59	133.99	2.1728	0.0000
900 MPa								
1	0.575	0.3	[(90/0)$_C$/0$_G$/(90$_2$/0)$_C$]	1017.55	902.07	134.93	2.2481	0.0945
2	0.575	–	[90/0$_2$/90$_2$/0]$_C$	1184.81	902.68	147.11	2.2474	0.0000

Glass/epoxy plies can only be placed at the inner of the composite. The candidates from optimisation are shown in Tables 13 and 14.

Conclusions

In this paper, an optimisation study on hybrid composites under flexural loading is presented in this paper. One high strength carbon fibre and two types of glass fibres, i.e., S-2 glass and E glass, were chosen to reinforce an epoxy matrix. Several design rules were derived and with which optimisation with minimum cost and weight as the objective functions was done for both unidirectional and multi-directional hybrid composites. It is shown the design-rule based optimisation gives better candidates with respect to the cost and weight when compared to the NSGA-II optimisation. The advantage of the design-rule-based optimisation is it is much faster compared to the NSGA-II optimisation.

The presented research is potentially useful for the design of lightweight structures. For complex composite components, FEA can be applied to simulate the performance under given loadings. The design rules can be applied and optimisation can then be carried out. The time-consuming process of NSGA-II can be avoided.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Tables A1–A4.

Table A1
Regression coefficients for carbon and E glass fibre reinforced unidirectional hybrid composite.

Layup	c_0	c_{c1}	c_{c2}	c_{g1}	c_{g2}	c_{cg}
$[0]_{8C}$	460.47	1262.89	1208.85			
$[0_G/0_{7C}]$	440.90	618.35	1314.27	601.28	−15.63	−229.35
$[0_{2G}/0_{6C}]$	136.57	1541.27	−310.58	945.75	86.94	203.39
$[0_{4C}/0_{2G}/0_{2C}]$	465.87	1224.04	1269.26			
$[0_{2G}/0_{2C}/0_G/0_{3C}]$	151.22	1456.36	−349.18	944.88	62.98	334.95
$[0_G/0_{3C}/0_{2G}/0_{2C}]$	445.47	634.07	1321.96	595.12	−21.71	−219.46
$[0_{3C}/0_{3G}/0_{2C}]$	467.31	1188.87	1266.02			
$[0_{2G}/0_{2C}]_2$	107.97	1745.42	−623.36	830.05	133.86	460.43
$[0_{2G}/0_{2C}/0_{3G}/0_C]$	248.33	1494.79	−56.38	150.57	1065.77	111.83
$[0_{2G}/0_C/0_{4G}/0_C]$	88.57	1524.76	−639.36	786.81	320.91	441.33
$[0_{3G}/0_C]_2$	232.27	810.72	−156.56	489.80	855.43	307.41
$[0_{7G}/0_C]$	229.05	415.65	−421.58	708.75	563.61	840.08
$[0]_{8G}$	196.74			867.83	795.18	

Table A2
Regression coefficients for carbon and S-2 glass fibre reinforced unidirectional hybrid composite.

Layup	c_0	c_{c1}	c_{c2}	c_{g1}	c_{g2}	c_{cg}
$[0]_{8C}$	460.47	1262.89	1208.85			
$[0_G/0_{7C}]$	453.40	548.37	1393.33	733.70	−21.54	−276.22
$[0_{2G}/0_{6C}]$	182.34	1826.53	−475.38	395.86	839.59	346.39
$[0_{4C}/0_{2G}/0_{2C}]$	466.37	1220.97	1270.83			
$[0_{2G}/0_{2C}/0_G/0_{3C}]$	161.61	1798.53	−556.95	501.22	696.04	477.96
$[0_G/0_{3C}/0_{2G}/0_{2C}]$	458.35	566.13	1396.67	722.82	−31.55	−255.24
$[0_{3C}/0_{3G}/0_{2C}]$	464.66	1204.22	1248.45			
$[0_{2G}/0_{2C}]_2$	304.43	1566.28	−252.26	24.93	1177.85	563.67
$[0_{2G}/0_{2C}/0_{3G}/0_C]$	289.03	1457.74	−68.27	61.84	1377.77	88.25
$[0_{2G}/0_C/0_{4G}/0_C]$	284.60	1241.99	−139.93	197.81	1240.88	260.51
$[0_{3G}/0_C]_2$	269.65	802.38	−174.38	456.43	1105.98	335.28
$[0_{7G}/0_C]$	273.16	444.90	−421.68	649.44	820.10	883.18
$[0]_{8G}$	234.70			896.18	1053.45	

Table A3
Regression coefficients for carbon and E glass fibre reinforced bi-directional hybrid composite.

Layup	c_0	c_{c1}	c_{c2}	c_{g1}	c_{g2}	c_{cg}
$[90/0]_{3C}$	−25.12	2615.55	−2046.74			
$[90/0_2/90_2/0]_C$	−115.73	2538.77	−1462.55			
$[(90/0/90)_C/0_G/(90/0)_C]$	−10.11	2744.39	−2393.17	−208.97	−102.15	547.21
$[(90/0)_C/0_G/(90_2/0)_C]$	−120.82	2552.07	−1462.71	11.09	13.07	−39.74
$[(90/0_2)_C/90_G/(90/0)_C]$	−103.06	2873.01	−2247.41	−143.53	7.40	294.96
$[(90/0)_G/(90/0)_{2C}]$	176.69	683.02	−706.94	485.45	−153.56	−18.39
$[(90/0)_C/(90/0)_G/(90/0)_C]$	−18.47	2662.77	−2273.87	−137.78	−134.57	497.74
$[(90/0)_C/(0/90)_G/(90/0)_C]$	−104.78	2883.86	−2248.92	−147.37	30.84	266.92
$[(90/0)_G/90_C/0_G/(90/0)_C]$	177.14	761.07	−1087.38	390.62	−480.72	709.58
$[(90/0)_{2G}/(90/0)_C]$	−5.77	801.73	−1121.62	992.83	−946.52	643.48

Table A4
Regression coefficients for carbon and S-2 glass fibre reinforced bi-directional hybrid composite.

Layup	c_0	c_{c1}	c_{c2}	c_{g1}	c_{g2}	c_{cg}
$[90/0]_{3C}$	−25.12	2615.55	−2046.74			
$[90/0_2/90_2/0]_C$	−115.73	2538.77	−1462.55			
$[(90/0/90)_C/0_G/(90/0)_C]$	−0.59	2497.01	−2149.55	−0.35	−363.45	588.47
$[(90/0)_C/0_G/(90_2/0)_C]$	−120.94	2552.67	−1462.87	11.27	13.99	−41.44
$[(90/0_2)_C/90_G/(90/0)_C]$	−145.48	2897.84	−2234.81	−32.26	−90.80	283.09
$[(90/0)_G/(90/0)_{2C}]$	179.17	717.74	−767.44	545.52	−203.77	20.44
$[(90/0)_C/(90/0)_G/(90/0)_C]$	−21.84	2592.87	−2209.31	−37.56	−291.60	535.80
$[(90/0)_C/(0/90)_G/(90/0)_C]$	−98.36	2855.22	−2217.05	−171.57	35.77	312.66
$[(90/0)_G/90_C/0_G/(90/0)_C]$	−210.38	1027.03	−829.67	1677.80	−759.08	−623.25
$[(90/0)_{2G}/(90/0)_C]$	51.53	838.67	−1344.69	869.78	−1092.76	1095.87

References

[1] C. Dong, H.A. Ranaweera-Jayawardena, I.J. Davies, Flexural properties of hybrid composites reinforced by S-2 glass and T700S carbon fibres, Compos. Part B Eng. 43 (2) (2012) 573–581.
[2] C. Dong, J. Duong, I.J. Davies, Flexural properties of S-2 glass and TR30S carbon fiber-reinforced epoxy hybrid composites, Polym. Compos. 33 (5) (2012) 773–781.
[3] C. Dong, I.J. Davies, Flexural properties of glass and carbon fiber reinforced epoxy hybrid composites, Proc. Inst. Mech. Eng. Part L J. Mater. Des. Appl. 227 (4) (2013) 308–317. Part L: Journal of Materials Design and Applications.
[4] C. Dong, Sudarisman, I.J. Davies, Flexural properties of E glass and TR50S carbon fiber reinforced epoxy hybrid composites, J. Mater. Eng. Perform. 22 (1) (2013) 41–49.
[5] P.W. Manders, M.G. Bader, The strength of hybrid glass/carbon fibre composites, J. Mater. Sci. 16 (8) (1981) 2233–2245.
[6] M. Kalantari, C. Dong, I.J. Davies, Multi-objective robust optimisation of unidirectional carbon/glass fibre reinforced hybrid composites under flexural loading, Compos. Struct. 138 (2016) 264–275.
[7] M. Kalantari, C. Dong, I.J. Davies, Multi-objective analysis for optimal and robust design of unidirectional glass/carbon fibre reinforced hybrid epoxy composites under flexural loading, Compos. Part B Eng. 84 (2016) 130–139.
[8] M. Kalantari, C. Dong, I.J. Davies, Effect of matrix voids, fibre misalignment and thickness variation on multi-objective robust optimization of carbon/glass fibre-reinforced hybrid composites under flexural loading, Compos. Part B Eng. 123 (2017) 136–147.
[9] M. Kalantari, C. Dong, I.J. Davies, Multi-objective robust optimization of multi-directional carbon/glass fibre-reinforced hybrid composites with manufacture related uncertainties under flexural loading, Compos. Struct. 182 (2017) 132–142.
[10] C. Dong, Flexural properties of symmetric carbon and glass fibre reinforced hybrid composite laminates, Compos. Part C Open Access 3 (2020), 100047.
[11] C. Dong, Multi-objective optimal design of carbon and glass reinforced hybrid composites under flexural loading, J. Appl. Comput. Mech. (2022).
[12] H. Ghiasi, D. Pasini, L. Lessard, Optimum stacking sequence design of composite materials part I: constant stiffness design, Compos. Struct. 90 (1) (2009) 1–11.
[13] H. Ghiasi, et al., Optimum stacking sequence design of composite materials part II: variable stiffness design, Compos. Struct. 93 (1) (2010) 1–13.
[14] S. Nikbakt, S. Kamarian, M. Shakeri, A review on optimization of composite structures Part I: laminated composites, Compos. Struct. 195 (2018) 158–185.
[15] K. Deb, et al., A fast and elitist multiobjective genetic algorithm: NSGA-II, IEEE Trans. Evol. Comput. 6 (2) (2002) 182–197.
[16] B. Bilyeu, W. Brostow, K.P. Menard, Epoxy thermosets and their applications I: chemical structures and applications, J. Mater. Educ. 21 (1999) 281–286, 5/6.
[17] T.W. Chou, Microstructural Design of Fiber Composites, Cambridge University Press, Cambridge, U.K., 1992.
[18] Standard test method for flexural properties of polymer matrix composite materials, in ASTM D7264/D7264M-15. 2015, ASTM International, West Conshohocken, PA, USA, 2015.
[19] M. Kalantari, C. Dong, I.J. Davies, Numerical investigation of the hybridisation mechanism in fibre reinforced hybrid composites subjected to flexural load, Compos. Part B Eng. 102 (2016) 100–111.